MECH3011 Test #1, Spring 2021 Name:_

1. Multiple choice questions: [5 points each]

C i) Identical drum A and B are being tested for strength by continuously filling w/ water to the same height.

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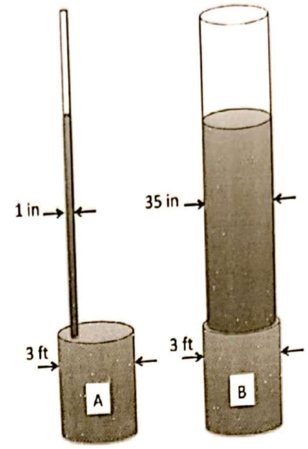
a) Drum A should burst first
b) Drum B should burst first
c) Drum A and B should burst at same time
d) If additional info is needed, what is it



A ii) Consider the figure of two dams below, what is the correct statement

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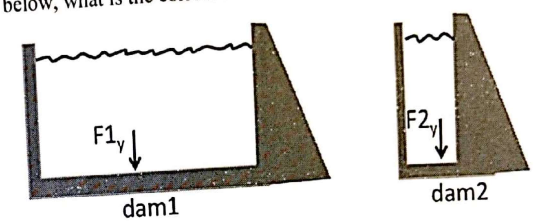
a) $F1_y > F2_y$
b) $F1_y < F2_y$
c) $F1_y = F2_y$
d) Cannot be determined



B iii) $u = 4xy^3$; $v = -4x^3y$.
The set of equations above represents a:

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a) two-dimensional incompressible flow
b) two-dimensional compressible flow
c) uncertain

C iv) A 0.3 m by 0.5 m rectangular air duct carries a flow of 0.45 m^3/s at a density of 2 k/m^3. The velocity in the duct is

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a) 1.5 m/s
b) 0.9 m/s
c) 3 m/s
d) Infinite m/s

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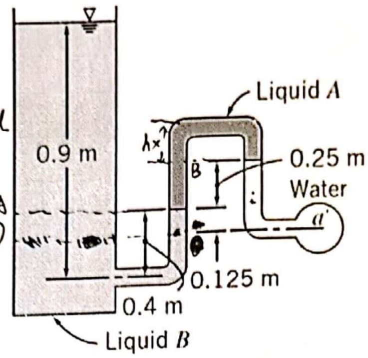
2. Determine the gage pressure in kPa at point a, if liquid A has SG = 1.20 and liquid B has SG = 0.75. The liquid surrounding point a is water, and the tank on the left is open to the atmosphere. ρ_{water} = 1,000 kg/m^3. [25pts]

assumption: Steady state.

Incompressible fluid

$g = 9.8 \, m/s^2$



At A:

$P_A = \rho_B g h_A$

~~$P_B = \rho_A g(\Delta x + 0.25) + \rho_B g \cdot 0.125$~~

for point B: $P_B + \rho_A g \cdot 0.25 = P_A$

$P_B = P_A - \rho_A g \cdot 0.25$

At a:

~~$P_a = \rho_A g h_x + \rho_{water} g$~~

$P_a = P_B + \rho_{water} g (0.25 + 0.125)$

$\Rightarrow P_a = \rho_B g h_A - \rho_A g \cdot 0.25 + \rho_{water} g \cdot (0.375)$, $\rho_A = 1.2 \rho_{water}$

$\rho_B = 0.75 \rho_{water}$

$h_A = (0.9 - 0.4) m = 0.5 m$

$\Rightarrow P_a = (0.75 \cdot 0.5m - 1.2 \cdot 0.25m + 0.375m) \rho_{water} g$

$= 4410$ Pa

$= 4.41$ kPa

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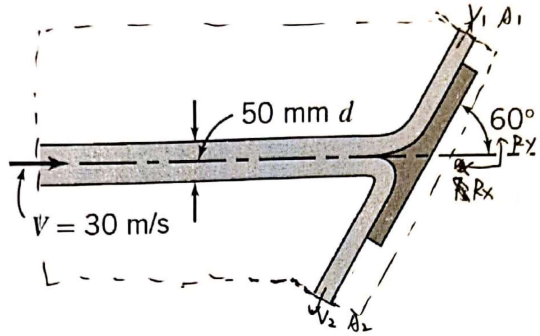
3. This water jet of 50 mm diameter moving at 30 m/s is divided in half by a "splitter" on the stationary flat plate. Calculate the magnitude and direction of the force on the plate. Assume that flow is in a horizontal plane. $\rho_{water} = 999$ kg/m^3. [25pts]

assumption:

Steady state,

incompressible fluid.



Conservation of mass: $\frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho\, \vec{V}_{xyz} \cdot d\vec{A} = 0$, $A = \pi \frac{(0.05)^2}{4} = 1.96 \times 10^{-3}\, m^2$

$\rightarrow \quad -V \cdot A + V_1 A_1 + V_2 A_2 = 0$

$V_1 A_1 + V_2 A_2 = 0.059\ m^3/s$

$\rightarrow$ divided in half, so $A_1 V_1 = A_2 V_2 = 0.029\ m^3/s$

~~$V_1 = V_2 = 15 m/s$~~

Momentum equation in x: $F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} \vec{V}_x \rho_w d\forall + \int_{CS} \vec{V}_x \cdot \rho_w \vec{V}_{x} d\vec{A}$

$R_x = -u^2 \rho_w \cdot A + u_1 \rho_w V_1 A_1 - u_2 \rho_w V_2 A_2$

$u_1 = V_1 \cos 60°$, $u_2 = V_2 \cos 60°$ ~~$A_1 V_1 = A_2 V_2$~~

$\rightarrow -R_x = -30^2 m/s \cdot 999 kg/m^3 \cdot 1.96 \times 10^{-3} m^2 + \cancel{30 m/s}\ 15 m/s \cos 60° \cdot 999 kg/m^3 \cdot 0.029 m^3/s$

$- 15 m/s \cos 60° \cdot 999 kg/m^3 \cdot 0.029 m^3/s$

$R_x = \cancel{58.7412}$ N

Momentum equation in y: $R_y = V_1 \rho_w V_1 A_1 \sin 60° - V_2 \rho_w V_2 A_2 \sin 60°$

$= 0$

$\Rightarrow$ Force on the plate is ~~58.7412~~ N in the ~~$-x$~~ direction.

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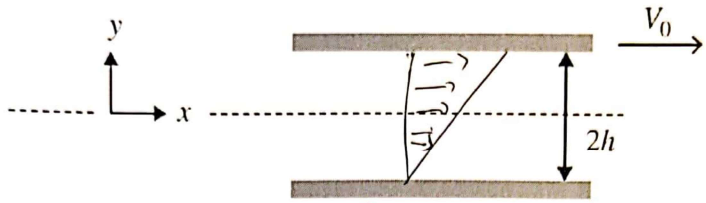
4. Consider a steady, laminar, fully developed incompressible flow between two infinite parallel plates separated by a distance $2h$ as shown. The top plate moves with a velocity V_0. You can neglect the effect of gravity. Derive:



a) an expression for the pressure gradient in the y-direction, [15 points]

$$x:\ \rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$y:\ \rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho g_y-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$z:\ \rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho g_z-\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

for 2-D flow = $w=0$, no z component, neglect the gravity: $g_x=g_y=g_z=0$ 15

for incompressible flow: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$, $\rho=C$, for steady state: $\frac{\partial}{\partial t}=0$

for fully developed flow: $\frac{\partial u}{\partial x}=0$

$\rightarrow \frac{\partial v}{\partial y}=0 \Rightarrow$ ~~$\rho u\frac{\partial v}{\partial x}=-\frac{\partial P}{\partial y}+\mu\frac{\partial^2 v}{\partial x^2}$~~ $\quad \frac{\partial P}{\partial y}=\mu\frac{\partial^2 v}{\partial x^2}-\rho u\frac{\partial v}{\partial x}$

b) the velocity profile. [15 points] the velocity on the bottom plate is 0, so $v=0$

$\rightarrow \frac{\partial P}{\partial y}=0$ ✓

in x: $\frac{\partial P}{\partial x}=\mu\frac{\partial^2 u}{\partial y^2}$ ✓

with $\frac{\partial P}{\partial y}=0$, P will not depend on y. 9

with two infinite parallel plates, $\rightarrow$