1 Error and Uncertainty

(1) System error-> inacccurate

(2) Random error-> imprecision

(3) Blunders-> outliers

The rms error (E_{rms}) is calculated by measured variables and constants through **Equation 1**. The nominal value has the same number of significant digits as nominal value of measured data (lowest value), but the rms error value has the same number of decimal places as nominal value.

$$E_{rms} = \sqrt{\left(\frac{\partial X}{\partial v_1}\Delta v_1\right)^2 + \left(\frac{\partial X}{\partial v_2}\Delta v_2\right)^2 + \cdots + \left(\frac{\partial X}{\partial v_n}\Delta v_n\right)^2} \qquad (1)$$

<u>Attention:</u>

Write function only in terms of measured variables

150±1%, error value: 1.5

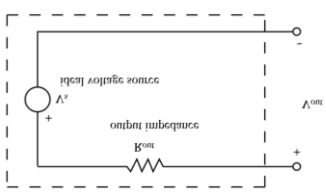
2 Non-ideal Device

(1) Real voltage source-> small output impedance

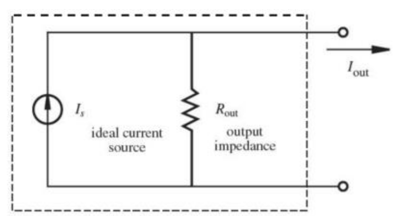
(2) Real current source-> large output impedance

(3) Real voltmeter-> large input impedance   affect lager resistor measurement

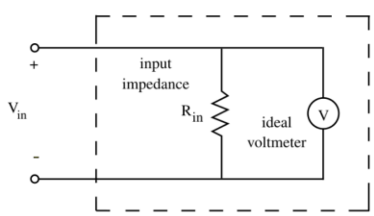
(4) Real ammeter-> small input impedance   affect small resistor measurement



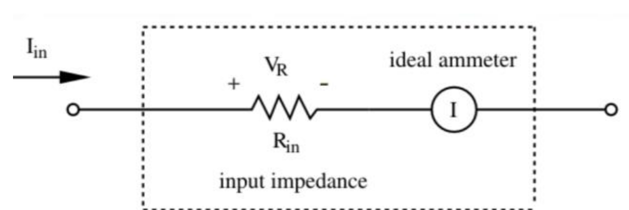
(1)



(2)



(3)



(4)

3 Oscilloscope Usage

(1) AC coupling-> block DC component (low frequency), need t=5τ to achieve stable.

Use: accelerometer, measure vibration (improve resolution), measure small fluctuations

(2) Chassis ground->Channel-后的原件被短路

4.1 First order system for RC circuit (τ, ω_b)

4.1.1 Time domain analysis

A first-order system is described by **Equation 1**. The coefficient ratio on the left side of **Equation 1** is called the time constant and is defined in **Equation 2**.

$$\frac{A_1}{A_0}\frac{dX_{\text{out}}}{dt}+X_{\text{out}}=\frac{B_0}{A_0}X_{\text{in}} \tag{1}$$

$$k=\frac{B_0}{A_0}\,(\text{static sensitivity}) \qquad \tau=\frac{A_1}{A_0}\,(\text{time constant}) \tag{2}$$

Therefore, the first-order system equation can be written as **Equation 3**. **Equation 4** describes the solution for X_{out}, and its plot is shown in **Figure 1**.

$$\tau\frac{dX_{out}}{dt}+X_{out}=KA_{in} \tag{3}$$

$$X_{out}(t)=KA_{in}\left(1-e^{-t/\tau}\right) \tag{4}$$

If the time is equal to τ, then the value of X_{out} can be calculated by **Equation 5**.

$$X_{\text{out}}(\tau)=KA_{\text{in}}(1-e^{-1})=0.632KA_{\text{in}} \tag{5}$$

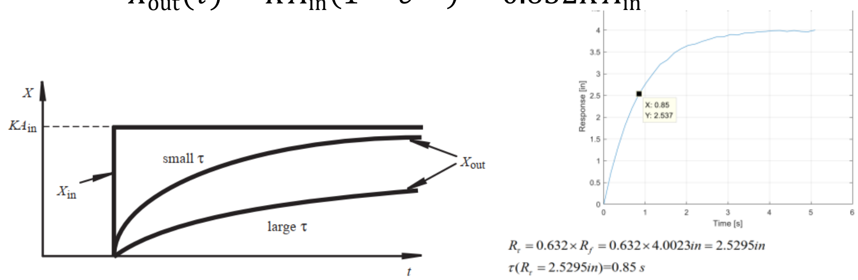


Figure 1. Time response for the first-order system.

There is another way to find the time constant. If the equation is rearranged as **Equation 6**, the value on the right side of **Equation 6** is defined as Z value. Then, the derivative of Z is the minus reciprocal of time constant as shown in **Equation 7**.

$$Z=ln\left(1-\frac{X_{\text{out}}}{KA_{\text{in}}}\right)=-\frac{t}{\tau} \tag{6}$$

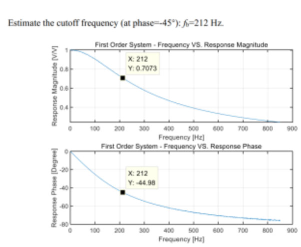
$$\frac{dZ}{dt}=-\frac{1}{\tau} \tag{7}$$

4.1.2 Frequency domain analysis(-90°,-20dB)

From frequency domain, the response is shown in **Equation 8**. The (cut-off frequency *rad/s*) break frequency, located at point where phase angle is -45°, is the inverse of time constant in **Equation 9**.

$$H(\omega)=\frac{X_{out}(\omega)}{X_{in}(\omega)}=\frac{B_o}{j\omega A_1+A_o}=\frac{1}{1+j\omega\tau} \tag{8}$$

$$\tau=\frac{1}{\omega_b} \tag{9}$$



(ωb 在-45°处，时间常数为 ωb 的倒数)

4.2 Second order system for RLC circuit (ς, ω_{peak}, ω_n)

If the system has moderate damping ($0.1< \varsigma <0.7$), percent overshoot method can be used for time analysis, and peak frequency is used for frequency analysis. If the system is small damping ($\varsigma<0.1$), log decrement method can be used for time analysis, and half-Power method can be used for frequency analysis.

4.2.1 Time domain analysis

In second order system, the standard form is shown in **Equation 10**. In RLC circuit, the parameters are determined by **Equation 11**, **Equation 12** and **Equation 13**.

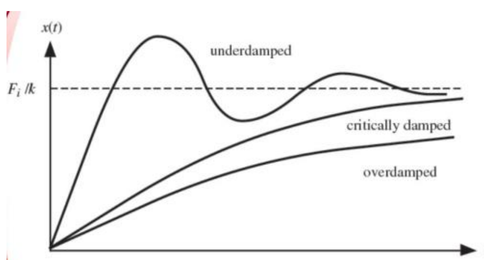
$$\frac{a_2}{a_0}\frac{d^2X_{out}}{dt^2}+\frac{a_1}{a_0}\frac{dX_{out}}{dt}+X_{out}=\frac{b_0}{a_0}X_{in} \tag{10}$$

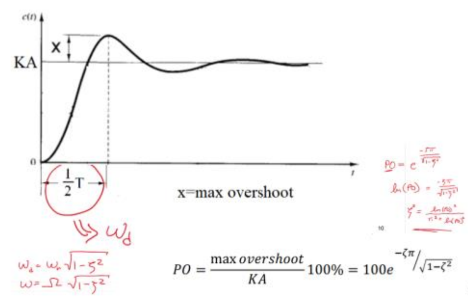
$$X_{\text{out}}(t)=1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\phi\right)$$

$$LC\frac{d^2V_o(t)}{dt^2}+RC\frac{dV_o(t)}{dt}+V_o(t)=V_i(t) \tag{11}$$

$$\frac{1}{{\omega_n}^2}=\frac{a_2}{a_0}=LC\Rightarrow\omega_n=\frac{1}{\sqrt{LC}} \tag{12}$$

$$\frac{2\xi}{\omega_n}=\frac{a_1}{a_0}=RC\Rightarrow\xi=\frac{\omega_n RC}{2}=\frac{R}{2}\sqrt{\frac{C}{L}} \tag{13}$$





(1) The parameters determined by percent overshoot method is shown in by **Equation 14**, **Equation 15** and **Equation 16**.

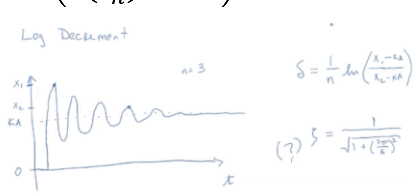
$$\omega_d=\omega_n\times\sqrt{1-\zeta^2} \tag{14}$$

$$PO=\frac{\text{Max overshoot}}{\text{KA}}\times 100\%=100e^{\frac{-\varsigma\pi}{\sqrt{1-\zeta^2}}} \tag{15}$$

$$\zeta=\sqrt{\frac{\ln^2(PO)}{\pi^2+\ln^2(PO)}} \tag{16}$$

(2) The parameters determined by log decrement method is calculated in **Equation 17**, where n is the number of periods. KA is the final value.

$$\zeta=\frac{1}{2\pi n}ln\left(\frac{x(t_0)-KA}{x(t_n)-KA}\right) \tag{17}$$



4.2.2 Frequency domain analysis (-180°,-40dB)

In the RLC circuit, the frequency response function is shown in **Equation 18**, and the gain is calculated by **Equation 19**.

$$\frac{V_o(\omega)}{V_i(\omega)} = \frac{-j\dfrac{1}{\omega C}}{-j\dfrac{1}{\omega C} + R + j\omega L} = \frac{1}{1-\left(\dfrac{\omega}{\omega_n}\right)^2 + 2\zeta\left(\dfrac{\omega}{\omega_n}\right)j} \tag{18}$$

$$Gain = 20\,log_{10}\left(\left|\frac{V_o}{V_i}\right|\right) \tag{19}$$

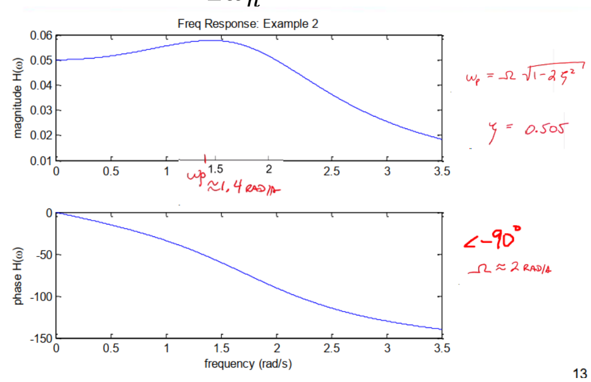
(1) Parameters determined by the peak frequency combined with undamped natural frequency is shown in **Equation 20**, **Equation 21** and **Equation 14**.

$$\omega_{\text{peak}} = \omega_n\sqrt{1-2\zeta^2} \tag{20}$$

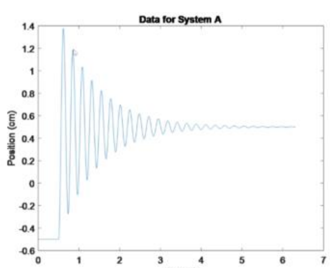
$$\xi = \sqrt{\left[1-\left(\frac{\omega_{peak}}{\omega_n}\right)^2\right] \times 0.5} \tag{21}$$

(2) Parameters determined by the half power method is calculated though **Equation 22** and **Equation 14**. ωH 和 ωL 在纵坐标为 0.707×H(ω)的两点。

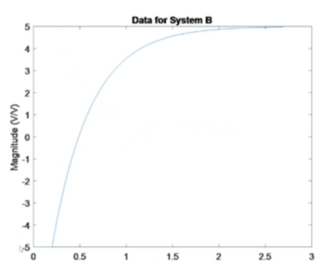
$$\zeta = \frac{\omega_H - \omega_L}{2\omega_n} \tag{22}$$



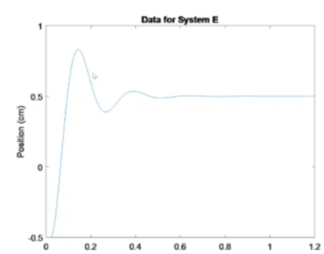
Ωp（magnitude 最大）和 ωn（-90°）都可从图上找出



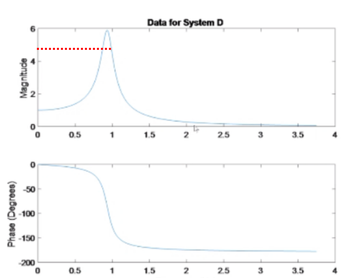
Log decrement (time, 2nd)



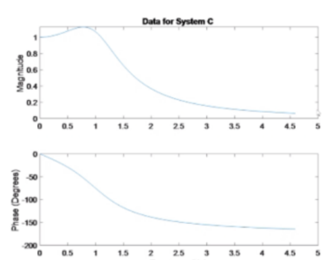
0.632 (time data, 1st)



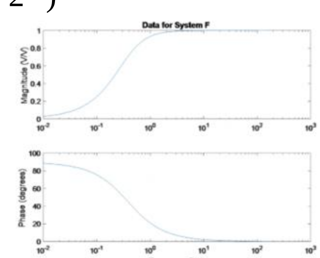
Percentage overshoot (time, 2nd)



Half power (frequency,2nd)

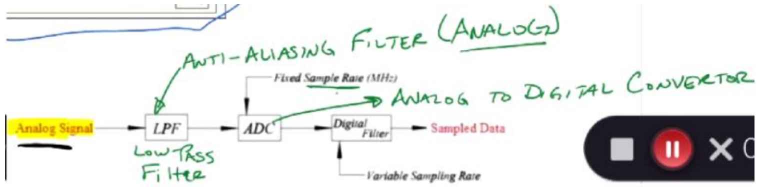


Peak frequency (frequency,2nd)



0.707 (frequency,1st)

5 Analog to Digital Conversion



5.1 Quantization

Quantization is the transformation of a continuous analog input into a set of discrete output states. Amplitude resolution, Q, is directly related to the number of bits, n, used to digitally approximate the analog value in the ADC.

$$N = 2^n$$

$$Q = \frac{V_{max} - V_{min}}{N}$$

5.2 Sampling

Shannon's sampling theorem and the concept of aliasing. Sampling rate, f_s, refers to the number of samples per unit of time and is typically reported in Hz for most measurement systems. Therefore, the time between samples (sampling period) can be found

$$\Delta t = {}^{1}\!/_{f_s}$$

5.3 Shannon's sampling theorem

In order to fully represent a measured signal, Shannon's sampling theorem states that the sample rate must be at least two times the maximum frequency component in the signal. This ensures that there are at least two data points per period of the highest frequency of interest. Nyquist frequency, $fNyq$, is defined as half of the sampling rate and is the maximum frequency that can be properly sampled per Shannon's sampling theorem.

$$f_s = \frac{1}{\Delta t} = 2f_{Nyq}$$

$$f_{Nyq} \geq f_{max}$$

Quiz:

The error that results in violating the conditions of a good FFT is referred to as **Leakage**.

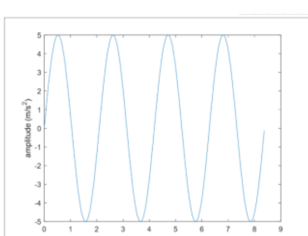
The frequency of aliased data is **lower than the actual frequency**.

The process of evaluating an analog signal at discrete instants in time is referred to as **Sampling.**

The process used to minimize the error in violating the conditions of a good FFT is **Windowing**.

The requirements for an FFT are: (1) the signal must be periodic in the observation time.

(2) all transient data is captured in the observation time



(1)

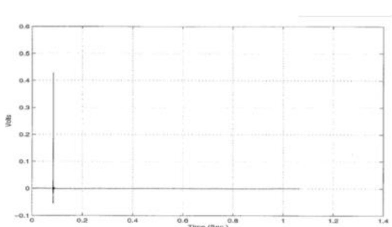


Figure 11. Typical Force Signal (Impact) - Time Domain

(2)