Equation Sheet

Heat Transfer Exam 1

Conduction:

- $q_k = -kA\frac{dT}{dx}$
- Plane wall: $q_k = \frac{\Delta T}{R_k} = K_k \Delta T$
- $R_k = \frac{L}{Ak}$

Convection:

- $q_c = \bar{h}_c A \Delta T$
- $R_c = \frac{1}{\bar{h}_c A}$

Radiation:

- $q_r = A_1 \varepsilon_1 \sigma (T_1^4 - T_2^4)$
- $q_r = A \bar{h}_r \Delta T$
- $\bar{h}_r = \frac{\varepsilon_1 \sigma (T_1^4 - T_2^4)}{T_1 - T_2'}$

$\sigma = 5.67 \times 10^{-8} \left(\frac{W}{m^2 K^4}\right)$

$Resistance\ in\ Parallel$: $R_{total} = \frac{R_A R_B}{R_A + R_B}$

$Overall\ Heat\ Transfer\ Coefficient$: $UA = \frac{1}{R_{total}}$

Conduction Equation: $\nabla * (k \nabla T) + \dot{q}_G = \rho c \frac{\partial T}{\partial t}$

- $Rectangular\ Cordinates$: $\frac{d^2T}{dx^2} = 0$
- $Cylindrical\ Cordinates$: $\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$
- $Spherical\ Cordinates$: $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$

Plane wall:

- $T(x) = \left(\frac{T_1 - T_2}{L}\right)x + T_1$ (w/o $\dot{q}_G$)
- $T(x) = -\frac{\dot{q}_G}{2k}x^2 + \frac{T_2 - T_1}{L}x + \frac{\dot{q}_G L}{2k}x + T_1$ (w/ uniform $\dot{q}_G$)
  - $T(x) = \frac{\dot{q}_G L^2}{2k}\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] + T_1$ (for $T_2 = T_1$)
  - $T_{max} = T_1 + \frac{\dot{q}_G L^2}{8k}$

Cylinder:

- $\frac{T(r)-T_i}{T_o-T_i} = \frac{\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)}$
- $R_{th} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi Lk}$
- $T = T_o + \frac{\dot{q}_G r_o^2}{4k}\left[1 - \left(\frac{r}{r_o}\right)^2\right]$ (solid cyl. w/ uniform $\dot{q}_G$)
  - $\frac{T(r)-T_o}{T_{max}-T_0} = 1 - \left(\frac{r}{r_o}\right)^2$
- $T(r) = T_o + \frac{\dot{q}_G}{4k}(r_0^2 - r^2) + \frac{\ln\left(\frac{r}{r_o}\right)}{\ln\left(\frac{r}{r_i}\right)}\left[\frac{\dot{q}_G}{4k}(r_0^2 - {r_i}^2) + T_o - T_i\right]$ (hallow cyl.)
- $\frac{T(r)- T_\infty}{T_\infty} = \frac{\dot{q}_G r_o}{4\bar{h}_c T_\infty}\left\{2 + \frac{\bar{h}_c r_o}{k}\left[1 - \left(\frac{r}{r_0}\right)^2\right]\right\}$ (solid cyl. immersed in Fluid)
- $r = r_{cr} = \frac{k}{h_\infty}$ (cylinder) $r = r_{cr} = \frac{2k}{h_\infty}$ (sphere)

Sphere:

- $T(r) - T_i = (T_o - T_i)\frac{r_o}{r_o - r_i}\left(1 - \frac{r_i}{r}\right)$
- $R_{th} = \frac{r_o - r_i}{4\pi k r_o r_i}$

Fins:

TABLE 2.2 Equations for temperature distribution and rate of heat transfer for fins of uniform cross section[a]

Case	Tip Condition ($x = L$)	Temperature Distribution, θ/θ_s	Fin Heat Transfer Rate, q_{fin}
1	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	M
2	Adiabatic: $\left.\frac{d\theta}{dx}\right\vert_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
3	Fixed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_s)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\frac{\cosh mL - (\theta_L/\theta_s)}{\sinh mL}$
4	Convection heat transfer: $\bar{h}_c\theta(L) = -k\left.\frac{d\theta}{dx}\right\vert_{x=L}$	$\frac{\cosh m(L-x) + (\bar{h}_c/mk)\sinh m(L-x)}{\cosh mL + (\bar{h}_c/mk)\sinh mL}$	$M\frac{\sinh mL + (\bar{h}_c/mk)\cosh mL}{\cosh mL + (\bar{h}_c/mk)\sinh mL}$

[a] $\theta \equiv T - T_\infty$
$\theta_s \equiv \theta(0) = T_s - T_\infty$
$m^2 \equiv \frac{\bar{h}_c P}{kA}$
$M = \sqrt{\bar{h}_c PAk}\,\theta_s$

Transient Conduction - Negligible Internal Resistance (Lumped Cap.)

- $\frac{T-T_\infty}{T_0-T_\infty} = \exp\left[-\left(\frac{\bar{h}A_s}{c\rho V}\right)t\right] = \exp(-Bi * Fo)$

$$\tau_t = \left(\frac{1}{\bar{h}A_s}\right)(c\rho V) \qquad Bi = \frac{\bar{h}l_c}{k} \qquad Fo = \frac{\alpha t}{l_c^2}$$

Slab: $l_c = \frac{L}{2}$ Cylinder: $l_c = \frac{r_o}{2}$ Sphere: $l_c = \frac{r_o}{3}$

Transient Conduction - Spatial Temperature Distribution (one-term app.)

Situation	Infinite Plate or Slab, Width 2L	Infinitely Long Cylinder, Radius r_0	Sphere, Radius r_0
Biot number	$\frac{\bar{h}_c L}{k}$	$\frac{\bar{h}_c r_0}{k}$	$\frac{\bar{h}_c r_0}{k}$
Fourier number	$\frac{\alpha t}{L^2}$	$\frac{\alpha t}{r_0^2}$	$\frac{\alpha t}{r_0^2}$

Infinite Slab or Plane Wall:

- $\theta = \frac{T_{(x,t)} - T_\infty}{T_o - T_\infty} = C_1 e^{-\delta_1^2\left(\frac{\alpha t}{L^2}\right)} \cos\left(\delta_1 \frac{x}{L}\right)$ $C_1 = \frac{2sin\delta_1}{\delta_1 + sin\delta_1 cos\delta_1}$

Infinite Cylinder:

- $\theta = \frac{T_{(r,t)} - T_\infty}{T_0 - T_\infty} = C_1 e^{-\delta_1^2\left(\frac{\alpha t}{r_0^2}\right)} J_0\left(\delta_1 \frac{r}{r_0}\right)$ $C_1 = \frac{2J_1(\delta_1)}{\delta_1[J_0^2(\delta_1) - J_1^2(\delta_1)]}$

Sphere:

- $\theta = \frac{T_{(r,t)} - T_\infty}{T_o - T_\infty} = C_1 e^{-\delta_1^2\left(\frac{\alpha t}{r_o^2}\right)} \frac{\sin\left(\delta_1 \frac{r}{r_o}\right)}{\delta_1 r/r_o}$ $C_1 = \frac{2(sin\delta_1 - \delta_1 cos\delta_1)}{\delta_1 - sin\delta_1 cos\delta_1}$

- $\frac{Q}{Q_o} = 1 - \frac{sin\delta_1}{\delta_1} C_1 e^{-\delta_1^2 \tau}$ Infinite Slab
- $\frac{Q}{Q_o} = 1 - \frac{2J_1(\delta_1)}{\delta_1} C_1 e^{-\delta_1^2 \tau}$ Infinite Cylinder
- $\frac{Q}{Q_o} = 1 - \frac{3(sin\delta_1 - \delta_1 cos\delta_1)}{\delta_1^3} C_1 e^{-\delta_1^2 \tau}$ Sphere

$$Q_o = c\rho V(T_0 - T_\infty)$$

TABLE 3.1 Eigenvalues and coefficients used in the one-term approximate solutions for one-dimensional transient heat conduction in an infinite slab or plate of thickness $2L$ ($\mathrm{Bi} = \bar{h}L/k$), an infinite cylinder of radius r_o ($\mathrm{Bi} = \bar{h}r_o/k$), and a sphere of radius r_o ($\mathrm{Bi} = \bar{h}r_o/k$).

	Infinite Slab		Infinite Cylinder		Sphere	
Bi	δ_1	C_1	δ_1	C_1	δ_1	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5705	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793

TABLE 43 The zeroth- (J_0) and first-order (J_1) Bessel functions of the first kind.

δ	$J_0(\delta)$	$J_1(\delta)$
0	1	0
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1	0.7652	0.4401

Convection Heat Transfer

$$Re_x = \frac{\rho U_\infty x}{\mu} = \frac{U_\infty x}{v} \qquad Pr = \frac{c_p \mu}{k} = \frac{v}{\alpha}$$

$$Nu_x = \frac{h_c x}{k_f} \qquad St_x = \frac{Nu_x}{Re_x\, Pr}$$

Friction coefficient: $C_{fx} = \frac{\tau_s}{\rho U_\infty^2/2}$ $\qquad \overline{C_f} = \frac{\bar{\tau}}{\rho U_\infty^2/2}$

Flat Plate:

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} \qquad \overline{C_f} = \frac{1}{L}\int_0^L C_{fx}dx = \frac{1.33}{\sqrt{Re_L}} \qquad \frac{\delta}{\delta_{th}} = Pr^{1/3}$$

$$q_c'' = -0.332k\frac{Re_x^{1/2}Pr^{1/3}}{x}(T_\infty - T_s)$$

$$q = 0.664kRe_L^{1/2}Pr^{1/3}b(T_s - T_\infty)$$

$$h_{cx} = 0.332\frac{k}{x}Re_x^{1/2}Pr^{1/3} \qquad \bar{h}_c = 2h_{c\,(x=L)}$$

$$Nu_x = \frac{h_{cx}x}{k}0.332Re_x^{1/2}Pr^{1/3} \qquad \overline{Nu}_L = 0.664Re_L^{1/2}Pr^{1/3}$$

- For Pr < 1: $\quad Nu_x = 0.565\sqrt{Re_x Pr}$

Turbulent Flow over Plane Surfaces $\qquad St_x Pr^{2/3} = \frac{C_{fx}}{2}$

$$C_{fx} = \frac{0.0576}{Re_x^{1/5}} \qquad \overline{C_f} = \frac{1}{L}\int_0^L C_{fx}dx = \frac{0.072}{Re_L^{1/5}} \qquad (\delta/x) = 0.37/Re_x^{0.2}$$

Mixed Laminar–Turbulent Boundary Layer

$$\overline{C_f} = \frac{0.072}{Re_L}\left(Re_L^{4/5} - 23{,}200\right) \qquad Nu_x = \frac{h_{cx}x}{k} = 0.0288Pr^{1/3}\left(\frac{U_\infty x}{v}\right)^{0.8}$$

For fully turbulent B.L.: $\qquad \overline{Nu}_L = \frac{\bar{h}_c L}{k} = 0.036Pr^{1/3}Re_L^{0.8}$

For mixed laminar-turbulent B.L.: $\qquad \overline{Nu}_L = 0.036Pr^{\frac{1}{3}}(Re_L^{\frac{4}{5}} - 23{,}200)$