学院________ 专业、班________ 年级________ 学号________ 姓名________ 考试教室________

公平竞争、诚实守信、严肃考纪、拒绝作弊

密 封 线

重庆大学《工程概率与统计》课程试卷

◉ A卷 ○ B卷

2020 — 2021 学年 第一学期

开课学院：UC 学院 课程号：ENED40801 考试日期：2020.11.30

考试方式： ○ 开卷 ◉ 闭卷 ○ 其他 考试时间： 100 分钟

题号	一	二	三	四	五	六	七	八	九	十	总 分
得分											

考试提示/Instructions

1. 严禁随身携带通讯工具等电子设备参加考试。
2. **考试作弊，留校察看，毕业当年不授学位；请人代考、替他人考试、两次及以上作弊等，属严重作弊，开除学籍。**
3. Do any 8 out of 10 problems listed. If you do more than 8 problems, the top 8 scores will count towards your exam grade.
4. List problems that you complete on this page below (put a circle around the problems that you complete).
5. **Use the work sheets provided after the questions**
   - **Must show ALL work to receive partial credit for your answers**
   - **Put a BOX around all your answers**
6. Copy all answers from Work Sheets to **ANSWER TABLE**
7. Write both your Chinese (Pinyin) Name and English Name on each sheet of paper that you use to complete test
8. **This exam is completely closed book and notes. All equations and reference information that you will need is provided below**

Completed Problems List
（圈出所选 8 题，若全写，圈出“Completed all 10 problems”）

1 2 3 4 5 6 7 8 9 10

Completed all 10 problems

All Problems worth a maximum of 12.5 points.

1. Prove that $\sum_{k=0}^{n} C_k^n = 2^n$

命题人：Steve Ettorre
组题人：Steve Ettorre
审题人：Thomas Huston
命题时间：2020.11.16
教务处制

2. You take a random sample from a population and form a 95% confidence interval for the population mean, *m*. What quantity is guaranteed to fall within the confidence interval that you construct? *(Circle and write down the letter of your selection. Include brief computations or short writeup (1-2 sentences) on work sheet below to justify your selection. NOTE – the correct selection without any justification will receive zero score)*

a) σ

b) μ

c) $\bar{x}$

d) $z_{\alpha=0.025}$

3. Let $x_1, x_2, \dots, x_{100}$ be independent random variables that all follow the same probability distribution (that is, same pdf, mean, variance). The population mean, μ = $E\{x_i\}$ = 12.5, and variance σ^2 = $Var\{x_i\}$ = 100. Find the (approximate) probability that $P(x_1 + x_2 + \cdots + x_{100} > 1229)$ using the central limit theorem

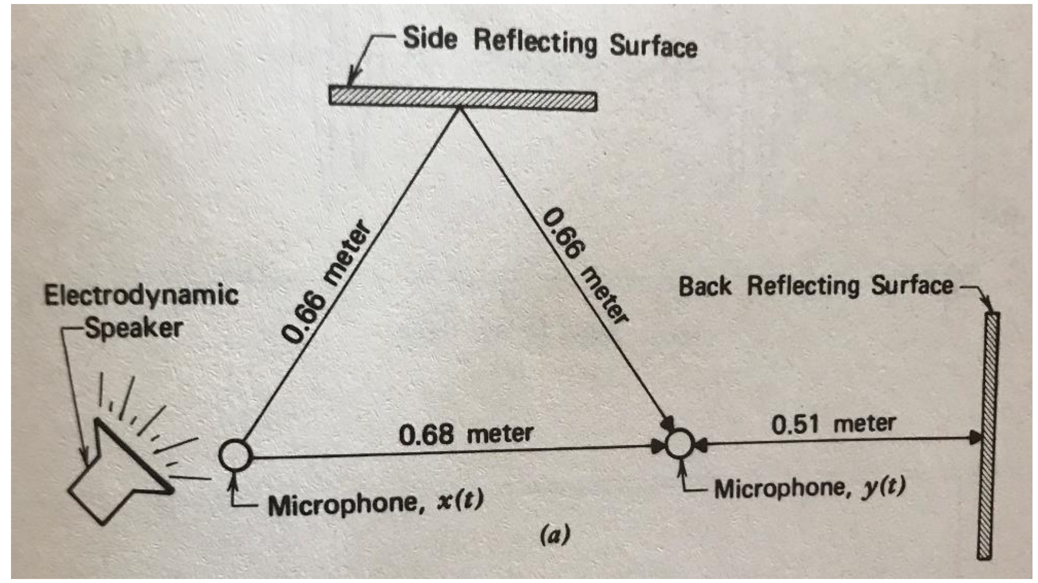
4. Consider the experimental arrangement shown in the figure below. Assume that the speaker is driven by random noise and that the sound waves coming from the speaker behave as spherical waves that travel with constant speed, $c = 340\ m/sec$ (speed of sound in air)

a) Consider the cross-correlation, $R_{xy}(\tau)$. At what values of the lag index, τ will microphone measurements x and y be highly correlated?

b) Consider the cross-correlation, $R_{yx}(\tau)$. Again, at what values of the lag index, τ will microphone measurements x and y be highly correlated?

c) Comment on the lag index values that you obtain from parts (a) and (b). Do they make physical sense? If you only had the xy microphone data and the cross-correlation results and did not know the experimental arrangement, could you (approximately) determine the location of the speaker?

NOTE: as implied in the figure, sound waves that are incident to the side reflecting surface at some angle are reflected with the same angle



5. The continuous random variable x, has a cumulative distribution function (cdf) modeled as

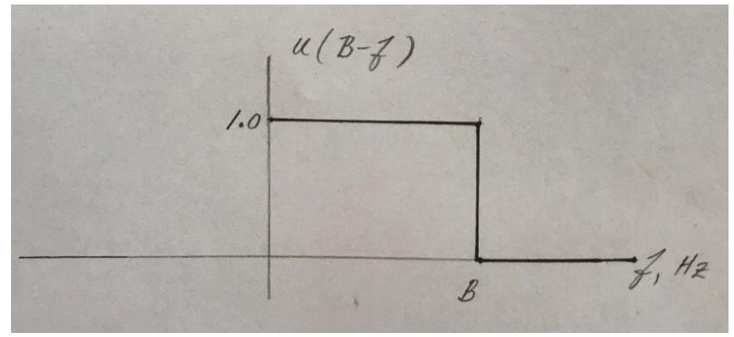
$$F(x) = \begin{cases} 0 & x \le -4 \\ \dfrac{x+4}{9} & -4 \le x \le 5 \\ 1 & x \ge 5 \end{cases}$$

a) Determine the probability density function (pdf), $f(x)$

b) Compute the **characteristic function ($\boldsymbol{\phi(k)}$)** of $f(x)$, defined as $\phi(k) = E\{e^{jkx}\} = \int_{-\infty}^{\infty} e^{jkx} f(x)dx$, where $k \equiv$ wavenumber, rad/m (assumes x has units of meters) and $j = \sqrt{-1}$. Observe that the characteristic function, $\phi(k)$ is the Fourier Transform of $f(x)$. Apply integral definition above to compute $\phi(k).U$se Euler's identity: $e^{j\theta} = cos\theta + jsin\theta$ to express the characteristic function in terms of its Real and Imaginary parts

c) What is the value of $\phi(k = 0)$?

6. Let, $n(t)$ represent a zero-mean, bandlimited, random noise process. Its power spectrum is given by $G_{nn}(f) = Ku(B - f), V^2/Hz$, where, $K \rightarrow$ const and $u(B - f) \rightarrow$ unit step function shown graphically as



Determine the expression for the auto-correlation function,

$R_{nn}(\tau)$ for $n(t)$

7. Show that the following are Fourier Transform pairs ($T \rightarrow$ const, $j = \sqrt{-1}$)

a) $\cos(2\pi fT)\,X(f) \Leftrightarrow 1/2[x(t-T) + x(t+T)]$

b) $\sin(2\pi fT)\,X(f) \Leftrightarrow -j/2[x(t-T) - x(t+T)]$

8. Suppose that against a certain opponent the number of points that the Chongqing University basketball team scores follows a normal distribution with unknown mean, *m* and unknown variance, σ^2. Also suppose that over the course of the last *10* games between the two teams, Chongqing University scored the following point totals: *59, 62, 59, 74, 70, 61, 62, 66, 62, 75*

a) Compute a *95% t*–confidence interval for *m*

b) Now suppose that you learn that $\sigma^2 = 36$. Compute a *95% z*–confidence interval for *m*. How does this compare to the interval in (a)?

9. In the lecture slides, we studied the simple asset price model for modeling stock price fluctuations. A similar model is used for simulating commodity prices (*com-mod-i-ty* – 4 English syllables). In the marketplace, commodities include things such as oil, metals, farm and agricultural products. The model is given by

$$dC = \mu dt + \sigma dX, \qquad C(0) = C_o$$

where, $C(X,t)$ is the commodity price. Similar to the simple asset price model (but not exactly the same), the (constant) coefficients, μ and σ are related to drift and diffusion price movements. $X(t)$ is a Wiener process.

a) Apply the Ito Formula to derive the exact solution for this stochastic ODE

b) Determine expressions for $E\{C\}$ and $Var\{C\}$

10. A random sample of size $n_1 = 16$ is selected from a normal population with a $\mu_1 = 75$ and $\sigma_1 = 8$. A second random sample of size $n_2 = 9$ is taken from another, independent normal population with $\mu_2 = 70$ and standard deviation $\sigma_2 = 12$. Let $\bar{x}_1$ and $\bar{x}_2$ be the two sample mean values. Compute the (approximate) probabilities in parts (a) and (b).

a) The probability that $(\bar{x}_1 - \bar{x}_2) > 4$

b) The probability that $3.5 \leq (\bar{x}_1 - \bar{x}_2) \leq 5.5$