Hongrui Yi

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公平竞争、诚实守信、严肃考纪、拒绝作弊

密 封 线

# 重庆大学《System Dynamics and Vibrations》课程试卷

◉ A卷 ○ B卷

2020 — 2021 学年 第 二 学期

开课学院：UC 联合 课程号：ME30880 考试日期：2021.8.2

考试方式：○ 开卷 ◉ 闭卷 ○ 其他 考试时间：120 分钟

| 题 号 | 一 | 二 | 三 | 四 | 五 | 六 | 七 | 八 | 九 | 十 | 总 分 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 得 分 | | | | | | | | | | | |

## 考试提示

1.严禁随身携带通讯工具等电子设备参加考试；

2.考试作弊，留校察看，毕业当年不授学位；请人代考、替他人考试、两次及以上作弊等，属严重作弊，开除学籍。

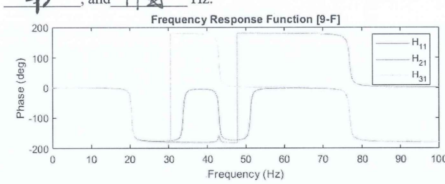
**Instructions**

- IMPORTANT: Write down your name in all pages
- Write down all steps clearly to obtain full credit
- You are allowed to have a calculator and a writing utensil
- Equation sheet: one side of a page
- In problem 3, keep at least 5 significant figures



1. 6 a） Fill-in the blank question: [2pts each]

In the following FRF phase plot of a 3-degree of freedom system, we can identify that the natural frequencies are approximately 20, 45, and 77 Hz.




b） Some True / False questions: [4pts each]

In a multi-degree of freedom problem,

4 T/F To estimate the natural frequency, identify the zero real response location (in the frequency domain)

4 T/F The log decrement technique has more advantages over the quadrature method.

4 T/F To estimate the mode shape, take the value of the peak imaginary part (at the natural frequency) for each input (force) location keeping the output (response) location fixed.

命题人：Pablo Mora 组题人：Pablo Mora 审题人：Thomas Huston 命题时间：2021.07 教务处制

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4　Ⓣ/F　Implying that at resonance the response is 90o behind the forcing function, the imaginary part of the frequency response is maximum and the real part is zero.

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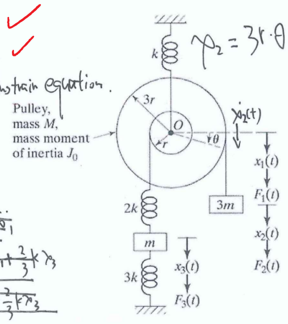
2. For the system in the following figure, develop the equations of motion by the Lagrange's Method. Express the equations of motion in matrix form. Ignore gravity in potential energy. Hint: Express the equations of motion in terms of x1, x2, and x3. [28pts]

$$\begin{cases} T = \frac{1}{2}J_0\dot\theta^2 + \frac{1}{2}M\dot x_1^2 + \frac{1}{2}(3m)\dot x_2^2 + \frac{1}{2}m\dot x_3^2 \\ U = \frac{1}{2}kx_1^2 + \frac{1}{2}(2k)(\cancel{r\theta + x_3})^2 + \frac{1}{2}(3k)x_3^2 \end{cases}$$

$x_2 = 3r\cdot\theta$

$\because \boxed{\theta = \frac{x_2}{3r},\ \dot\theta = \frac{\dot x_2}{3r},\ \ddot\theta = \frac{\ddot x_2}{3r}}$ → Constrain equation.

$$\Rightarrow \begin{cases} T = \left(\frac{J_0}{18r^2} + \frac{3}{2}m\right)\dot x_2^2 + \frac{1}{2}M\dot x_1^2 + \frac{1}{2}m\dot x_3^2 \\ U = \frac{1}{2}kx_1^2 + k\left(\frac{1}{3}x_2 + x_3\right)^2 + \frac{3}{2}kx_3^2 \end{cases}$$




#1 For $x_1$:

$\frac{\partial T}{\partial \dot x_1} = \cancel{\left(\frac{J_0}{9r^2}+3m\right)\dot x_1}\ M\dot x_1 \qquad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot x_1}\right) = \cancel{\left(\frac{J_0}{9r^2}+3m\right)\ddot x_1}\ M\ddot x_1$

$\frac{\partial U}{\partial x_1} = kx_1 \cancel{+ 2k\left(\frac{1}{3}x_1 + x_3\right)\cdot\frac{1}{3} = kx_1 + \frac{2}{9}kx_1 + \frac{2}{3}kx_3}$

$\cancel{= \frac{11}{9}kx_1 + \frac{2}{3}kx_3}$

#2 For $x_2$:

$\cancel{\frac{\partial T}{\partial \dot x_2} = 3m\dot x_2 \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot x_2}\right) = 3m\ddot x_2}$

$\frac{\partial T}{\partial \dot x_2} = \left(\frac{J_0}{9r^2} + 3m\right)\dot x_2 \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot x_2}\right) = \left(\frac{J_0}{9r^2} + 3m\right)\ddot x_2$

$\frac{\partial U}{\partial x_2} = 2k\left(\frac{1}{3}x_2 + x_3\right)\cdot\frac{1}{3} = \frac{2}{9}kx_2 + \frac{2}{3}kx_3$

#3 For $x_3$:

$\frac{\partial T}{\partial \dot x_3} = m\dot x_3 \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot x_3}\right) = m\ddot x_3$

$\frac{\partial U}{\partial x_3} = 2k\left(\frac{1}{3}x_2 + x_3\right) + 3kx_3 = \frac{2}{3}kx_2 + 5kx_3$

Hence:
$$\begin{cases} M\ddot x_1 + kx_1 = F_1(t) & ① \\ \left(\frac{J_0}{9r^2} + 3m\right)\ddot x_2 + \frac{2}{9}kx_2 + \frac{2}{3}kx_3 = F_2(t) & ② \\ m\ddot x_3 + \frac{2}{3}kx_2 + 5kx_3 = F_3(t) & ③ \end{cases}$$

Matrix form:

$$\begin{bmatrix} M & 0 & 0 \\ 0 & \frac{J_0}{9r^2}+3m & 0 \\ 0 & 0 & m \end{bmatrix}\begin{Bmatrix} \ddot x_1 \\ \ddot x_2 \\ \ddot x_3 \end{Bmatrix} + \begin{bmatrix} k & 0 & 0 \\ 0 & \frac{2}{9}k & \frac{2}{3}k \\ 0 & \frac{2}{3}k & 5k \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{Bmatrix}$$

3. An industrial machine of mass 453.4 kg is supported on springs with a static deflection of 0.508 cm. If the machine has a rotating unbalance of 0.2303 kg·m. Gravity is 9.81 m/s2. Determine:

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a) the dynamic amplitude at 1200 rpm, [13pts]

$m = 453.4\,kg \quad \Delta x = 0.508\,cm \quad m_0E = 0.2303\,kg\cdot m \quad g = 9.81\,m/s^2$

$mg = k\Delta x \Rightarrow k = \frac{mg}{\Delta x} = \frac{453.4\times 9.81}{0.508\times10^{-2}} = \boxed{875561.811\,N/m}$

$\omega = 1200\,rpm = 1200\times\frac{2\pi}{60}\,rad/s = 40\pi\,rad/s.$

$\boxed{H(\omega) = \frac{1}{j\omega c} = \frac{1}{2j\zeta k} = \frac{X}{m_0E\omega^2} \quad \Rightarrow \left|\frac{1}{j\omega c}\right| = \left|\frac{X}{m_0E\cdot\omega^2}\right| \quad \Rightarrow}$

$|H(\omega)| = \left|\frac{1}{-m\omega^2 + \cancel{j\omega c} + k}\right| = \left|\frac{X}{m_0E\omega^2}\right|$

$\Rightarrow |X| = \left|\frac{m_0E\omega^2}{-m\omega^2 + k}\right| = \left|\frac{0.2303\times(40\pi)^2}{-453.4\times(40\pi)^2 + 875561.811}\right|$

$= \boxed{\cancel{5.0856\times10^{-4}}\ 5.787\times10^{-4}\,m}$

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b) the force transmitted to the floor at this same speed. [12pts]

$$\left|\frac{F_T}{F}\right| = \left|\frac{j\omega c + k}{-m\omega^2 + j\omega c + k}\right| \qquad F_T = \left|\frac{F_T}{F}\right| \cdot m_e E \cdot \omega^2$$

$$= \left|\frac{8755.811}{-453.4\times(40\pi)^2 + 8755.811}\right|$$

$$= \frac{1}{1.2244\times 10^{-3}}\ 0.1393265 = \boxed{13.93265\%}$$

$$F_T = \frac{0.1393265}{1.2244\times 10^{-3}} \times 0.2303 \times (40\pi)^2$$

$$= \cancel{4.4528\,N}$$

$$= \boxed{506.6858\,N}$$

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4.If an arbitrary force f(t) is applied to an undamped oscillator that has initial conditions other than zero (x0, v0), show that the solution must be of the form: [25pts]

$$x(t) = x_0\cos(\Omega t) + \frac{v_0}{\Omega}\sin(\Omega t) + \frac{1}{m\Omega}\int_0^t f(\xi)\sin(\Omega(t-\xi))\,d\xi$$

Hint: you can begin from the free response or the free response to initial conditions, and then express the combined response.

For an arbitrary force $f(t)$, with initial conditions:

$$x(t) = \int_0^t f(\xi)\, h(t-\xi)\, d\xi$$

$$= \int_0^t f(\xi)\cdot \frac{1}{m\omega_r}\sin[\omega_r(t-\xi)]\, d\xi$$

$$= \frac{1}{m\omega_r}\int_0^t f(\xi)\sin[\omega_r(t-\xi)]\, d\xi$$

$\because$ undamped

$\therefore \sigma_r = c = 0,\ \omega_r = \Omega$

Hence, $\boxed{x(t) = \frac{1}{m\Omega}\int_0^t f(\xi)\sin[\Omega(t-\xi)]\, d\xi}$ → specific solution

For the homogeneous eq:

Let $x(t) = X e^{\sigma_r t}$

$x(t) = e^{\sigma_r t}[2X_{Re}\cos(\omega_r t) - 2X_{Im}\sin(\omega_r t)]$

Assume $A = 2X_{Re},\ B = -2X_{Im}$

$x(t) = e^{\sigma_r t}[A\cos(\omega_r t) + B\sin(\omega_r t)]$

$\dot{x}(t) = \sigma_r e^{\sigma_r t}[A\cos(\omega_r t) + B\sin(\omega_r t)] + \omega_r e^{\sigma_r t}[-A\sin(\omega_r t) + B\cos(\omega_r t)]$

I.C.: $\begin{cases} x(0) = A = x_0 \\ \dot{x}(0) = \omega_r \cdot B = v_0 \Rightarrow B = \frac{v_0}{\omega_r} = \frac{v_0}{\Omega} \end{cases}$

$\boxed{x(t) = x_0\cos(\Omega t) + \frac{v_0}{\Omega}\sin(\Omega t)}$ → homogeneous solution

Hence, $x(t) = x_0\cos(\Omega t) + \frac{v_0}{\Omega}\sin(\Omega t) + \frac{1}{m\Omega}\int_0^t f(\xi)\sin(\Omega(t-\xi))\,d\xi$