考试教室________ 姓名________ 学号________ 年级________ 专业、班________ 学院________

公平竞争、诚实守信、严肃考纪、拒绝作弊

密 封 线

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重庆大学《Kinematics and Kinetics》课程试

◉ A卷 ○ B卷

2016 —2017 学年 第 二 学期

开 课 学 院：机 械 工 程 学 院 课 程 号： ME30821 考试日期：

考试方式：○ 开卷 ◉ 闭卷 ○ 其他 考试时间：120 分钟

题号	一	二	三	四	五	六	七	八	九	十	总 分
得分											

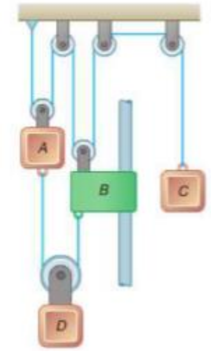
考试提示

1.严禁随身携带通讯工具等电子设备参加考试；

2.考试作弊，留校察看，毕业当年不授学位；请人代考、替他人考试、两次及以上作弊等，属严重作弊，开除学籍。

一、（15 分）

The system shown starts from rest, and each component moves with a constant acceleration. If the relative acceleration of block C with respect to collar B is 60 mm/s² upward and the relative acceleration of block D with respect to block A is 110 mm/s² downward, determine (a) the velocity of block C after 3s, (b) the change in position of block D after 5 s.



Positive direction Downward

$$2x_A + 2x_B + x_C = const$$
$$x_D - x_A + x_D - x_B = const$$

$$a_C - a_B = 60$$
$$a_D - a_A = -110$$

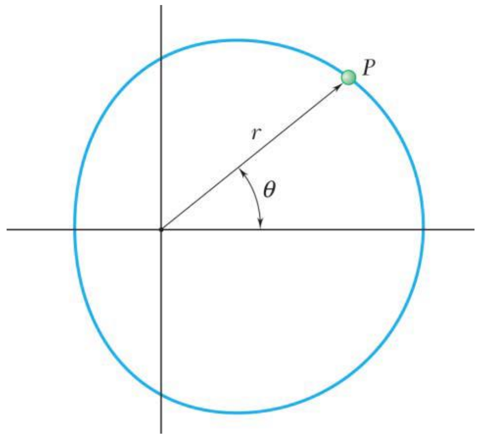
$$a_B = -100mm/s^2 \quad a_A = 120mm/s^2 \quad a_C = -40mm/s^2 \quad a_D = 10mm/s^2$$

$$v_C = v_0 + a_C t = -120mm/s$$

$$\Delta x_D = \frac{1}{2} a_D t^2 = 125mm$$

二、（20 分）

The path of a particle P is a limaçon. The motion of the particle is defined by the relations $r = b(2+\cos \pi t)$ and $\theta = \pi t$, where t and θ are expressed in seconds and radians, respectively. Determine (a) the velocity and the acceleration of the particle when $t = 2s$, (b) the value of θ for which the magnitude of the velocity is maximum.



SOLUTION

Differentiate the expressions for r and θ with respect to time.

$$r = b(2+\cos\pi t),$$
$$\dot{r} = -\pi b\sin\pi t,$$
$$\ddot{r} = -\pi^2 b\cos\pi t$$
$$\theta = \pi t,$$
$$\dot{\theta} = \pi,$$
$$\ddot{\theta} = 0$$

(*a*) At $t = 2$ s, $\sin\pi t = 0, \quad \cos\pi t = 1$

$$r = 3b, \quad \dot{r} = 0, \quad \ddot{r} = -\pi^2 b, \quad \theta = 2\pi \text{ rad}, \quad \dot{\theta} = \pi \text{ rad/s}$$

$v_r = \dot{r} = 0, \quad v_\theta = r\dot{\theta} = 3\pi b,$ $\qquad \mathbf{v} = 3\pi b\mathbf{e}_\theta$ ◀

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\pi^2 b - (3b)\pi^2 = -4\pi^2 b$$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0,$ $\qquad \mathbf{a} = -4\pi^2 b\mathbf{e}_r$ ◀

(*b*) Values of θ for which v is maximum.

$$v_r = \dot{r} = -\pi b\sin\pi t$$
$$v_\theta = r\dot{\theta} = -b(2+\cos\pi t)\pi$$
$$v^2 = v_r^2 + v_\theta^2 = \pi^2 b^2\left[\sin^2\pi t + (2+\cos\pi t)^2\right]$$
$$= \pi^2 b^2\left[\sin^2\pi t + 4 + 4\cos\pi t + \cos^2\pi t\right]$$
$$= \pi^2 b^2(5 + 4\cos\pi t)$$

is maximum when $\cos\pi t = 1$ or $\pi t = 0, \ 2\pi, \ 4\pi, \ 6\pi,$ etc

$\theta = \pi t,$ hence $\qquad \theta = 2N\pi, \ N = 0, 1, 2, \ldots$ ◀

$\xrightarrow{+}$: $0 = -444.13\cos 30° + 0.15\alpha_{BD}\cos\beta + 40.376\sin\beta$

$\alpha_{BD} = 2597.0 \text{ rad/s}^2$

$+\downarrow$: $a_D = -444.13\sin 30° - (0.15)(2597.0)\sin\beta + 40.376\cos\beta$

$= -296 \text{ m/s}^2 \qquad \mathbf{a}_P = \mathbf{a}_D \qquad \mathbf{a}_P = 296 \text{ m/s}^2 \uparrow$ ◀

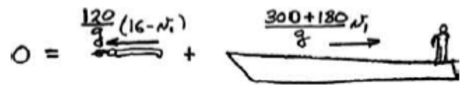
三、（15 分）

A 180 *lb* man and a 120 *lb* woman stand at opposite ends of a 300 *lb* boat, ready to dive, each with a 16 *ft/s* velocity relative to the boat. Determine the velocity of the boat after they have both dived, if (*a*) the woman dives first, (*b*) the man dives first.

SOLUTION

(*a*) Woman dives first:

Conservation of momentum:



$$-\frac{120}{g}(16-v_1)+\frac{300+180}{g}v_1=0$$

$$v_1=\frac{(120)(16)}{600}=3.20\text{ ft/s}\longrightarrow$$

Man dives next. Conservation of momentum:

$$\frac{300+180}{g}v_1=-\frac{300}{g}v_2+\frac{180}{g}(16-v_2)$$

$$v_2=\frac{480v_1-(180)(16)}{480}=2.80\text{ ft/s}\qquad \mathbf{v}_2=2.80\text{ ft/s}\longleftarrow ◀$$

(*b*) Man dives first:

Conservation of momentum:

$$\frac{180}{g}(16-v_1')-\frac{300+120}{g}v_1'=0$$

$$v_1'=\frac{(180)(16)}{600}=4.80\text{ ft/s}\longleftarrow$$

Woman dives next. Conservation of momentum:

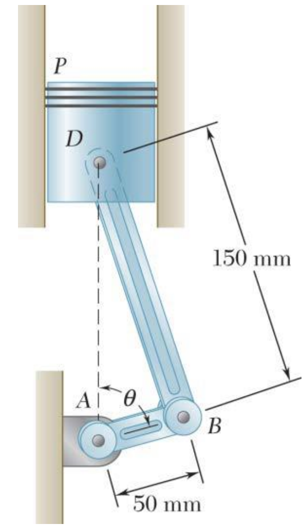
$$-\frac{300+120}{g}v_1'=\frac{300}{g}v_2'+\frac{120}{g}(16-v_2')$$

$$v_2'=\frac{-420v_1'+(120)(16)}{420}=-0.229\text{ ft/s}$$

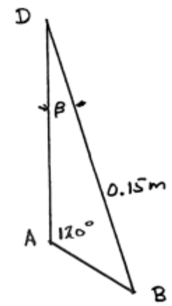
$$\mathbf{v}_2'=0.229\text{ ft/s}\longleftarrow ◀$$

四、（15 分）

Knowing that crank *AB* rotates about Point *A* with a constant angular velocity of 900 rpm clockwise, determine the acceleration of the piston *P* when $\theta=120°$. 15.120



SOLUTION



Law of sines. $\dfrac{\sin\beta}{0.05} = \dfrac{\sin 120^\circ}{0.15}, \quad \beta = 16.779^\circ$

Velocity analysis. $\omega_{AB} = 900 \text{ rpm} = 30\pi \text{ rad/s}$

$$\mathbf{v}_B = 0.05\omega_{AB} = 1.5\pi \text{ m/s} \measuredangle 60^\circ$$

$$\mathbf{v}_D = v_D \downarrow \qquad \omega_{BD} = \omega_{BD}$$

$$\mathbf{v}_{D/B} = 0.15\omega_{BD} \measuredangle \beta$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$[v_D \downarrow] = [1.5\pi \measuredangle 60^\circ] + [0.15\omega_{BD} \measuredangle \beta]$$

Components $\xrightarrow{+}$: $0 = -1.5\pi\cos 60^\circ - 0.15\omega_{BD}\cos\beta$

$$\omega_{BD} = -\frac{1.5\pi\cos 60^\circ}{0.15\cos\beta} = 16.4065 \text{ rad/s}$$

Acceleration analysis. $\alpha_{AB} = 0$

$$\mathbf{a}_B = 0.05\omega_{AB}^2 = (0.05)(30\pi)^2 = 444.13 \text{ m/s}^2 \measuredangle 30^\circ$$

$$\mathbf{a}_D = a_D \downarrow \qquad \alpha_{BD} = \alpha_{BD}$$

$$\mathbf{a}_{D/B} = [0.15\alpha_{AB} \measuredangle \beta] + [0.15\omega_{BD}^2 \measuredangle \beta]$$

$$= [6\alpha_{BD} \measuredangle \beta] + [40.376 \measuredangle \beta]$$

$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$ Resolve into components.

(Continued)

$\xrightarrow{+}$: $0 = -444.13\cos 30^\circ + 0.15\alpha_{BD}\cos\beta + 40.376\sin\beta$

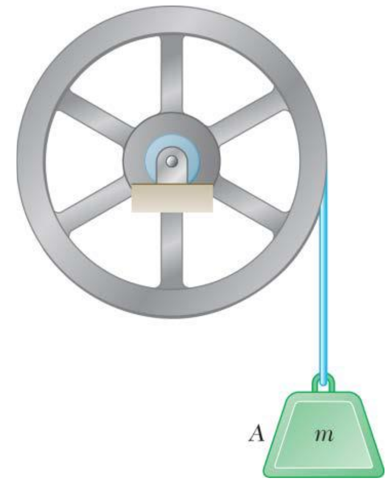
$$\alpha_{BD} = 2597.0 \text{ rad/s}^2$$

$+\downarrow$: $a_D = -444.13\sin 30^\circ - (0.15)(2597.0)\sin\beta + 40.376\cos\beta$

$= -296 \text{ m/s}^2 \qquad \mathbf{a}_P = \mathbf{a}_D \qquad \mathbf{a}_P = 296 \text{ m/s}^2 \uparrow$ ◀

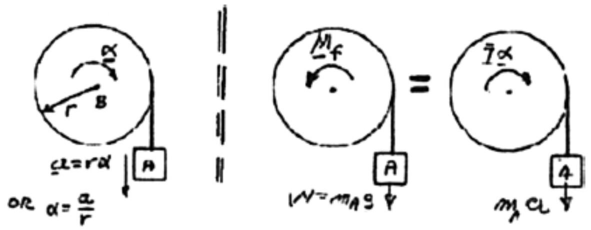
五、（15分）

In order to determine the mass moment of inertia of a flywheel of radius 600 mm, a 12-kg block is attached to a wire that is wrapped around the flywheel. The block is released and is observed to fall 3 m in 4.6 s. To eliminate bearing friction from the computation, a second block of mass 24 kg is used and is observed to fall 3 m in 3.1 s. Assuming that the moment of the couple due to friction remains constant, determine the mass moment of inertia of the flywheel. 16.32



SOLUTION

Kinematics: Kinetics:



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad (m_A y)r - M_f = \bar{I}\alpha + (m_A a)r$$

$$m_A gr - M_f = \bar{I}\frac{a}{r} + m_A ar \qquad (1)$$

Case 1:

$$y = 3\text{ m}$$
$$t = 4.6\text{ s}$$
$$y = \frac{1}{2}at^2$$
$$3\text{ m} = \frac{1}{2}a(4.6\text{ s})^2$$
$$a = 0.2836\text{ m/s}^2$$
$$m_A = 12\text{ kg}$$

Substitute into Eq. (1)

$$(12\text{ kg})(9.81\text{ m/s}^2)(0.6\text{ m}) - M_f = \bar{I}\left(\frac{0.2836\text{ m/s}^2}{0.6\text{ m}}\right) + (12\text{ kg})(0.2836\text{ m/s}^2)(0.6\text{ m})$$

$$70.632 - M_f = \bar{I}(0.4727) + 2.0419 \qquad (2)$$

Case 2:

$$y = 3\text{ m}$$
$$t = 3.1\text{ s}$$
$$y = \frac{1}{2}at^2$$
$$3\text{ m} = \frac{1}{2}a(3.1\text{ s})^2$$
$$a = 0.6243\text{ m/s}^2$$
$$m_A = 24\text{ kg}$$

Substitute into Eq. (1):

$$(24\text{ kg})(9.81\text{ m/s}^2)(0.6\text{ m}) - M_f = \bar{I}\left(\frac{0.6243\text{ m/s}^2}{0.6\text{ m}}\right) + (24\text{ kg})(0.6243\text{ m/s}^2)(0.6\text{ m})$$

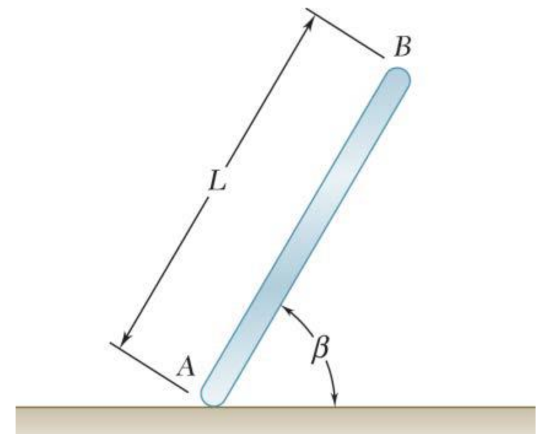
$$141.264 - M_f = \bar{I}(1.0406) + 8.9899 \qquad (3)$$

Subtract Eq. (1) from Eq. (2) to eliminate M_f

$$70.632 = \bar{I}(1.0406 - 0.4727) + 6.948$$
$$63.684 = \bar{I}(0.5679)$$
$$\bar{I} = 112.14\text{ kg}\cdot\text{m}^2 \qquad \bar{I} = 112.1\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

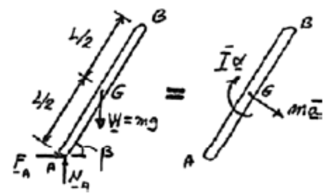
六、（20 分）

The uniform rod AB of weight W is released from rest when $\beta = 70°$. Assuming that the friction force between end A and the surface is large enough to prevent sliding, determine immediately after release (*a*) the angular acceleration of the rod, (b) the normal reaction at A, (c) the friction force at A. 16.157



SOLUTION

We note that rod rotates about A. $\omega = 0$



$$\bar{I} = \frac{1}{12}mL^2$$

$$\bar{a} = \frac{L}{2}\alpha$$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}$: $\quad mg\left(\frac{L}{2}\cos\beta\right) = \bar{I}\alpha + (m\bar{a})\frac{L}{2}$

$$\frac{1}{2}mgL\cos\beta = \frac{1}{12}mL^2\alpha + \left(m\frac{L}{2}\alpha\right)\frac{L}{2}$$

$$= \frac{1}{3}mL^2\alpha$$

$$\alpha = \frac{3}{2}\frac{g\cos\beta}{L} \qquad (1)$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}$: $\quad F_A = m\bar{a}\sin\beta$

$$F_A = m\frac{L}{2}\alpha\sin\beta = m\frac{L}{2}\left(\frac{3}{2}\frac{g\cos\beta}{L}\right)\sin\beta$$

$$F_A = \frac{3}{4}mg\sin\beta\cos\beta \qquad (2)$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$: $\quad N_A - mg = -m\bar{a}\cos\beta = -m\left(\frac{L}{2}\alpha\right)\cos\beta$

$$N_A - mg = -m\frac{L}{2}\left(\frac{3}{2}\frac{g\cos\beta}{L}\right)\cos\beta$$

$$N_A = mg\left(1 - \frac{3}{4}\cos^2\beta\right) \qquad (3)$$

(Continued)

For $\beta = 70°$:

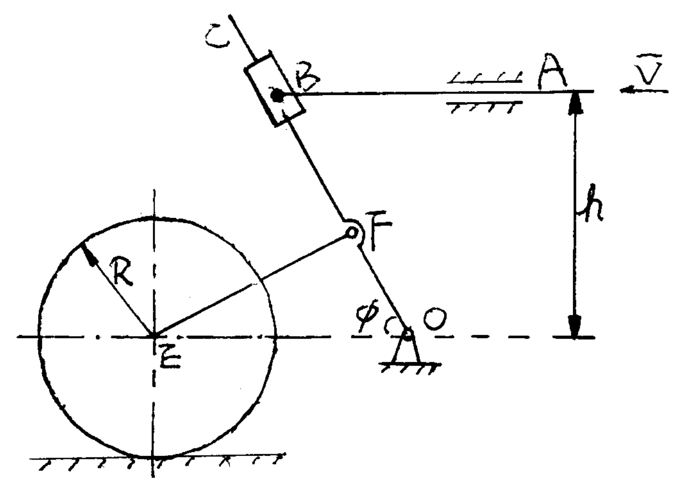
(a) Eq. (1): $\alpha = \frac{3}{2}\frac{g\cos 70°}{L}$ $\qquad \alpha = 0.513\frac{g}{L}$ ◀

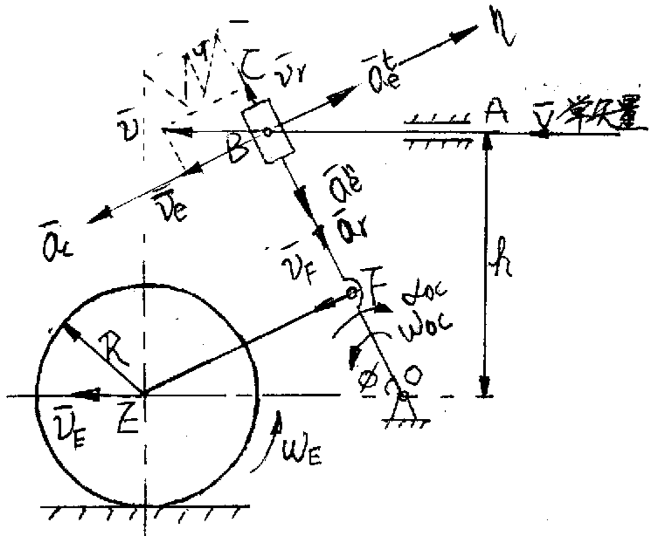
(b) Eq. (3): $N_A = mg\left(1 - \frac{3}{4}\cos^2 70°\right)$ $\qquad \mathbf{N}_A = 0.912mg\uparrow$ ◀

(c) Eq. (2) $F_A = \frac{3}{4}mg\sin 70°\cos 70°$ $\qquad \mathbf{F}_A = 0.241mg \longrightarrow$ ◀

七、Bonus（20 分）

For the structure as shown, given：OF = 4h/9, R = $\sqrt{3}$ h/3 and motion of the roller E is pure rolling without sliding. The rod AB has a constant velocity of $\bar{v}$ pointing to the left. At the position of φ= 60°, EF ⊥ OC. Please calculate (1) the instantaneous angular velocity of the rod OC: ω_{OC} and the roller E: ω_E, and (2) the angular acceleration of the roller E: α_{OC} .





由点的速度合成定理

$$\bar{v}_a = \bar{v}_e + \bar{v}_r$$

大小 v ? ?

方向 √ √ √

由速度平行四边形得

$$v_r = v\cos 60^\circ = \frac{v}{2}$$

$$v_e = v\sin 60^\circ = \frac{\sqrt{3}}{2}v$$

从而得

$$\omega_{OC} = \frac{v_e}{\overline{OB}} = \frac{3v}{4h} \quad \text{rad/s}$$

则

$$v_F = \omega_{OC}\cdot\overline{OF} = \frac{1}{3}v$$

又由速度投影定理

$$v_E\cos 30^\circ = v_F$$

得

$$v_E = {v_F}\big/{\cos 30^\circ} = \frac{2\sqrt{3}}{9}v$$

进而得

$$\omega_E = {v_E}\big/{R} = \frac{2v}{3h} \quad \text{rad/s}$$

$$\omega_{EF} = \frac{v_F}{\overline{FG}} = \frac{v}{4h} \quad \text{rad/s}$$

再进行加速度分析，仍与速度分析一样选取动点与动系，由点的加速度合成定理

$$\bar{a}_a = \bar{a}_e^t + \bar{a}_e^n + \bar{a}_r + \bar{a}_C$$

大小 0 ? $\omega_{OF}^2\cdot\overline{OF}$? $2\omega_{OF}\cdot v_r$

方向 √ √ √ √ √

利用加速度合成图，将上式向 η 轴投影，得

$$0 = a_e^t + 0 + 0 - a_C$$

得

$$a_e^t = a_C = \frac{3v^2}{4h} \quad \text{m/s}^2$$

从而得

$$\alpha_{OC} = \frac{a_e^t}{\overline{OF}} = 0.366 \quad \text{rad/s}^2$$