Chapter 3

PROBLEM 3.1

Consider a flat plate or a plane wall with a thickness L and long cylinder of radius r_0. Both of these are made of materials such that they can be treated as lumped capacitances (Bi<0.1). Show that in each case, the characteristic length ℓ_c, defined as ℓ_c=(V/A_s), can be approximated as (L/2) and (r_0/2) respectively.

GIVEN

- Plane wall with a thickness L
- Long cylinder of radius r_0
- Both cylinder and plane wall can be treated as lumped capacitances (Bi<0.1)

SHOW

- Characteristic length can be approximated as (L/2) and (r_0/2) respectively.

ASSUMPTIONS

- Both plane wall and cylinder can be treated as lumped capacitances.

SOLUTION

We have,

$\ell_c = (V/A_s)$ =For a plane wall thickness L, width B and height H

V= LBH and A_s=2BH

$\ell_c = (V/A_s)$ = LBH/2BH= L/2

For a long cylinder with height h and radius r_0

$V=\pi r_0^2 h$ and $A_s = 2\pi r_0 h$

$\ell_c = (V/A_s) = \pi r_0^2 h / 2\pi r_0 h = r_0/2$

PROBLEM 3.2

> **High-strength stainless steel is required for use in building structures and equipment (e.g. , cranes). It is produced by heat treating quench-hardened steel in a process called tempering that reduces brittleness and imparts toughness. In a production facility, alloy steel plates (k=50 W/(m K), c=460 J/(kg K) and ρ=7865 kg/m^3) of thickness 3 cm have to be tempered in a convective oven by heating them to 550^{0}C. If the plates are initially at 40^{0}C and the air inside the heat treating oven is at 700^{0}C with a convective heat transfer coefficient of 45 W/(m^2 K), determine how long the plate has to remain in the oven.**

GIVEN

- Alloy steel plate of thickness L=3 cm=0.03 m
- Tempering in oven to 550^{0}C
- Initial temperature of the plate (T_o) = 40°C
- Heat treating oven temperature (T_∞) = 700°C
- Convective heat transfer coefficient ($\bar{h}_c$) = 45 W/(m^2 K)

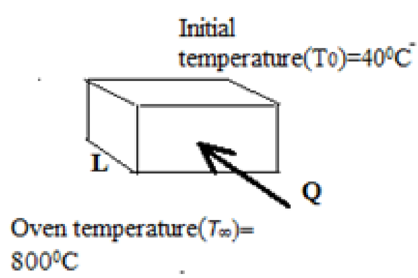
FIND

- The time required for the plate to reach 550^{0}C.

ASSUMPTIONS

- Constant thermal conductivity
- End effects are negligible

SKETCH



PROPERTIES AND CONSTANTS

As per the information provided:
Thermal conductivity (k) = 50 W/(m K)
Specific heat (c) = 460 J/(kg K)
Density (ρ) = 7865 kg/m^3

SOLUTION

For a flat plate we have L_c=L/2= 0.015 m

Biot number(Bi)= $\bar{h}_c$ L_c/k=45*0.015/50=0.0135 which is << 0.1

Thus the flat plate can be considered as lumped capacitance.
Therefore, the internal resistance of the rod is negligible.
The temperature-time history of the rod, from Equation (3.3) is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(\frac{\bar{h}_c A_s}{c\rho V} t\right)$$

$$\frac{\bar{h}_c A_s}{c\rho V} = \frac{\bar{h}_c * 2WH}{c\rho * LWH} = \frac{2\bar{h}_c}{c\rho L} = \frac{2*45}{460*7865*0.03} = 0.000829 \text{ s}^{-1}$$

$$\frac{T_\infty - T}{T_\infty - T_o} = \exp\left[-\left(0.000829\frac{1}{S}\right)t\right]$$

Solving for the time

$$t = -(1206 \text{ s}) \ln\left(\frac{T_\infty - T}{T_\infty - T_o}\right)$$

The time required to reach 550°C is

$$t = -(1206 \text{ s}) \ln\left(\frac{700 - 550}{700 - 40}\right) = 121\text{s}$$

$$t = -(1206 \text{ s}) \ln\left(\frac{150}{660}\right) = 1206*1.48 \text{ s} = 1787 \text{ seconds.}$$

PROBLEM 3.3

> **Quenching is a rapid cooling process by which many metallic alloys are hardened. Brass (a copper alloy: 70 Cu, 30 Zn) plates that are 4 mm thick are quenched in a water bath at a heat treating plant. If a brass plate is initially at 450^{0}C determine the quench time needed for the plate to attain 90^{0}C. The water bath temperature is 80^{0}C and has an average convective heat transfer coefficient of 45 W/(m^2 K), determine how long the plate has to remain in the oven.**

GIVEN

Brass alloy plate of thickness L=4 mm=0.004 m

Initial temperature of the plate (T_o) = 450°C

Water bath temperature (T_∞) = 80°C

Convective heat transfer coefficient ($\bar{h}_c$) = 45 W/(m^2 K)

FIND

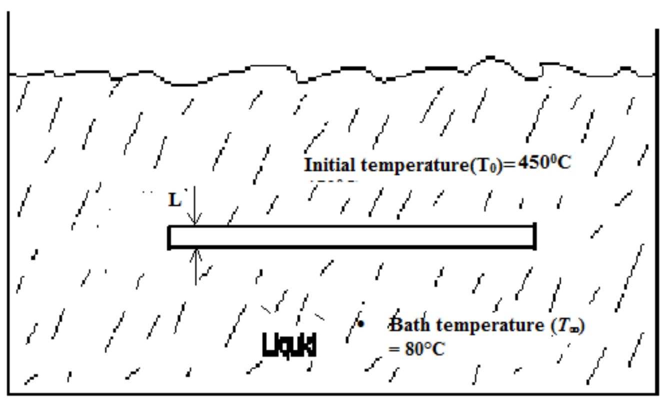
The quench time required for the plate to reach 90^{0}C.

ASSUMPTIONS

Constant thermal conductivity

End effects are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, for Brass:

Thermal conductivity (k) = 111 W/(m K)

Specific heat (c) = 385 J/(kg K)

Density (ρ) = 8522 kg/m^3

SOLUTION

For a flat plate we have L_c=L/2= 0.002 m

Biot number(Bi)= $\bar{h}_c$ L_c/k=45*0.002/111=0.008 which is << 0.1

Thus the flat plate can be considered as lumped capacitance.

Therefore, the internal resistance of the plate is negligible.

The temperature-time history of the plate, from Equation (3.3) is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(\frac{\bar{h}_c A_s}{c\rho V} t\right)$$

$$\frac{\bar{h}_c A_s}{c\rho V} = \frac{\bar{h}_c * 2WH}{c\rho * LWH} = \frac{2\bar{h}_c}{c\rho L} = \frac{2*45}{385*8522*0.004} = 0.006857 \text{ s}^{-1}$$

$$\frac{T_\infty - T}{T_\infty - T_o} = \exp\left[-\left(0.006857 \frac{1}{S}\right) t\right]$$

Solving for the time

$$t = -(145.8 \text{ s}) \ln\left(\frac{T_\infty - T}{T_\infty - T_o}\right)$$

The quench time required to reach 90°C for the plate is

$$t = -(145.8 \text{ s}) \ln\left(\frac{80 - 90}{80 - 450}\right)$$

$$t = -(145.8 \text{ s}) \ln\left(\frac{150}{660}\right) = 145.8*3.61 \text{ s} = 526 \text{ seconds.}$$

PROBLEM 3.4

A thermocouple is made up of a spherical bead bimetallic junction at the end of two very thin wires of different materials. For example, in a type-T thermocouple, a copper wire and a constantan wire, with wire diameters ranging from 0.5 mm to 0.025 mm, are fused together to make a spherical bead that is also of a similar order-of-magnitude diameter. For such a thermocouple, determine the appropriate diameter of the spherical bead junction so as to ensure a response time of less than 2 s ($\tau_t \le 2$ s) in water, which is at 40°C and has a convective heat transfer coefficient of $\bar{h} = 80$ W/m^2 K. Consider the bead junction to have properties of Cu, and to be initially at 150°C. Also, for the calculated bead diameter, what would be the response time in air at 40°C and $\bar{h} = 10$ W/m^2 K?

GIVEN

- Copper and constanton wire with diameter range from 0.5 mm to 0.025 mm fused together
- Response time $\tau_t \le 2$ s
- Water bath temperature (T_∞) = 40°C
- Convective heat transfer coefficient ($\bar{h}_c$) = 80 W/(m^2 K)
- Initial temperature of bead junction (T_0)=150^0

FIND

- Appropriate diameter of spherical bead junction to ensure response time of less than 2 seconds.
- Response time in air at 40°C and $\bar{h} = 10$ W/m^2 K for calculated bead diameter.

ASSUMPTIONS

- Constant thermal conductivity
- End effects are negligible

PROPERTIES AND CONSTANTS

From Table 12, Appendix 2, we get for Copper

k_s =391 W/m K

c = 383 J/kg K

ρ=8930 kg/m^3

SOLUTION

The surface area and volume of the junction beads are

$A_s=\pi D^2$

$V=\frac{1}{6}\pi D^3$

The Biot number in water is $\quad Bi= \frac{\bar{h}D}{6k}$

The surface area and volume of the junction beads are

$A_s=\pi D^2$

$V=\frac{1}{6}\pi D^3$

The Biot number in water is $\quad Bi= \frac{\bar{h}D}{6k}$

For 0.5 mm diameter D=5*10^{-4} m

$Bi=\frac{80*5*10^{-4}}{6*391}$ =1.7*10^{-5} << 0.1

For 0.025 mm diameter D=2.5*10^{-5} m

$Bi=\frac{80*2.5*10^{-5}}{6*391}$ =8.5*10^{-6} << 0.1

As Bi is <<< 0.1, internal resistance can be neglected and system can be considered as lumped capacitance.

From equation (3.3)

$$\frac{T-T_\infty}{T_o-T_\infty}=e^{-\frac{h A_s}{c\rho V}t} = e^{-\frac{h\,4\pi r_o^2}{c\rho\frac{4}{3}\pi r_o^3}t} = e^{-\frac{3h}{c\rho\, r_o}t}$$

$$Bi.Fo=\frac{hA_s}{c\rho V}t=\frac{80*6}{383*8930*D}=\frac{1.40*10^{-4}}{D}$$

$$\frac{Bi.Fo}{t}=\frac{hA_s}{c\rho V}=\frac{80*6}{383*8930*D}$$

Also from equation 3.10

$$\tau_t=\frac{t}{Bi.Fo} \text{ <= 2 s}$$

Considering 2 s as time constant we get

$$\frac{1}{2}=\frac{1.40*10^{-4}}{D}$$

D= 2.8*10^{-4} m= 0.28 mm

For air at 40^{0}C and h=10 W/(m^2 K) for given diameter D= 2.8*10^{-4} m we have

Bi.Fo= $\frac{hA_s}{c\rho V}t$

$\tau_t = \frac{t}{Bi.Fo} = \frac{c\rho V}{hA_s}$

$= \frac{383*8930*2.8*10^{-4}}{6*10}$ s

=16 seconds

Bi.Fo= $\frac{hA_s}{c\rho V}t = \frac{80*6}{383*8930*D} = \frac{1.40*10^{-4}}{D}$

$\frac{\text{Bi.Fo}}{\text{t}} = \frac{hA_s}{c\rho V} = \frac{80*6}{383*8930*D}$

Also from equation 3.10

$\tau_t = \frac{t}{Bi.Fo}$ <= 2 s

Considering 2 s as time constant we get

$\frac{1}{2} = \frac{1.40*10^{-4}}{D}$

D= $2.8*10^{-4}$ m= 0.28 mm

PROBLEM 3.5

In a ball bearing production facility, steel balls that are each of 15 mm in diameter are annealed by first heating them to 870^{0}C and then slowly cooling in air to 125^{0}C. If the cooling air stream temperature is 60^{0}C, and it has a convective heat transfer coefficient of 35 W/(m^2 K), determine the time required for the cooling.

GIVEN

- Steel balls of diameter D=15 mm=0.015 mm
- Initial temperature of balls before cooling (T_o) =870°C
- Cooling air stream temperature (T_∞) = 60°C
- Convective heat transfer coefficient ($\bar{h}_c$) = 35 W/(m^2 K)

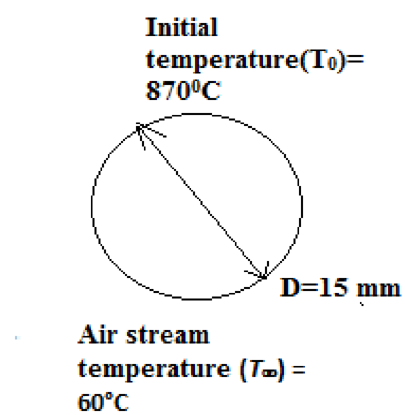
FIND

- The time required for cooling ball bearing to 125^{0}C.

ASSUMPTIONS

- Constant thermal conductivity
- End effects are negligible

SKETCH



PROPERTIES AND CONSTANTS

For steel ball:
Thermal conductivity (k) = 50 W/(m K)
Specific heat (c) = 460 J/(kg K)
Density (ρ) = 7865 kg/m^3

SOLUTION

For a spherical ball bearing we have L_c=D/6= 0.0025 m
Biot number (Bi)= $\bar{h}_c$ L_c/k=35*0.0025/50=0.00175 which is << 0.1
Thus, the ball bearing can be considered as lumped capacitance.
Therefore, the internal resistance of the ball bearing is negligible.
The temperature-time history of the ball bearing, from Equation (3.3) is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(\frac{\bar{h}_c A_s}{c\rho V} t\right)$$

$$\frac{\bar{h}_c A_s}{c\rho V} = \frac{\bar{h}_c * \pi D^2}{c\rho * \pi D^3 / 6} = \frac{6\bar{h}_c}{c\rho D} = \frac{6*35}{460*7865*0.015} = 0.003869 \text{ s}^{-1}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left[-\left(0.003869 \frac{1}{S}\right) t\right]$$

Solving for the time

$$t = -(258.4 \text{ s}) \ln \frac{T - T_\infty}{T_o - T_\infty}$$

The time required to reach 550°C is

$$t = -(258.4 \text{ s}) \ln \left(\frac{125 - 60}{870 - 60}\right)$$

$$t = -(258.4 \text{ s}) \ln \left(\frac{65}{810}\right) = 258.4*2.52 \text{ s} = 651.8 \text{ seconds.}$$

PROBLEM 3.6

A 0.6-cm-diameter mild steel rod at 38°C is suddenly immersed in a liquid at 93°C with $\bar{h}_c$ = 110 W/(m^2 K). Determine the time required for the rod to warm to 88°C.

GIVEN

- A mild steel rod is suddenly immersed in a liquid
- Rod diameter (D) = 0.6 cm = 0.006 m
- Initial temperature of the rod (T_o) = 38°C
- Liquid temperature (T_∞) = 93°C
- Heat transfer coefficient ($\bar{h}_c$) = 113.5 W/(m^2 K)

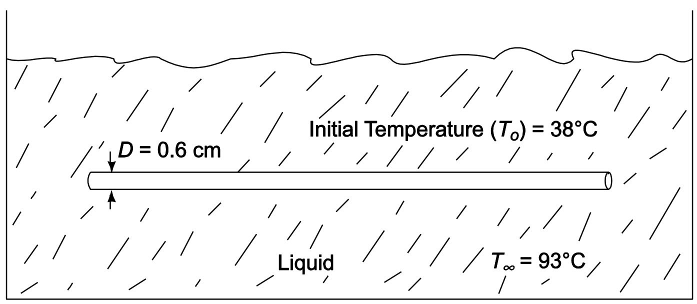
FIND

- The time required for the rod to warm to 88°C

ASSUMPTIONS

- The rod is 1% carbon steel
- Constant thermal conductivity
- End effects are negligible
- The rod is very long compared to its diameter
- There is radial conduction only in the rod

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10: For 1% carbon steel at 20°C:

Thermal conductivity (k) = 43 W/(m K)
Specific heat (c) = 473 J/(kg K)
Density (ρ) = 7801 kg/m^3
Thermal diffusivity (α) = 1.172 × 10^{-5} m^2/s. [$\alpha = k/\rho c$].

SOLUTION

The Biot number is calculated first to check if the internal resistance is negligible

$$B_{\mathrm{i}} = \frac{\bar{h}_c D}{4k} = \frac{110\,\mathrm{W/(m^2K)}\ (0.006\,\mathrm{m})}{4\ \ 43\,\mathrm{W/(m^2\ K)}} = 0.0038 << 0.1$$

Therefore, the internal resistance of the rod is negligible.

The temperature-time history of the rod, from Equation (3.3) is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(\frac{\bar{h}_c A_s}{c\rho V} t\right)$$

$$\frac{\bar{h}_c A_s}{c\rho V} = \frac{\bar{h}_c \pi DL}{c\rho \frac{\pi}{4} D^2 L} = \frac{4\bar{h}_c}{c\rho D} = \frac{4\ 100\text{W/(m}^2\text{K)}\ \text{(J/Ws)}}{(473\text{W/kg K})\ 7801\,\text{Kg/m}^3\ \ (0.006\,\text{m})} = 0.020\ 1/S$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left[-\ 0.020\frac{1}{S}\ t\right]$$

Solving for the time

$$t = -(50.3\text{ s})\ \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

The time required to reach 88°C is

$$t = -(50.3\text{ s})\ \ln\left(\frac{88 - 93}{38 - 93}\right) = 121\text{s}$$

COMMENTS

The analysis has assumed that the heat capacity of the liquid is much larger than that of the rod and thus the liquid temperature remains constant.

PROBLEM 3.7

In a metal wire manufacturing facility, continuously drawn copper wire with a 2.5 mm diameter is annealed by heating it from its initial temperature of 45^{0}C to 400^{o}C in a soaking oven. The oven inside air temperature is 700^{0}C with an average heat transfer coefficient of $\overline{h_c}$ =45 W/(m^2 K). Estimate the soaking annealing time required. Also if the oven is 3.5 m long and the drawn wire is continuously fed through the oven (entering at one end and pulled out at the other longitudinal end), what should be the drawing speed or velocity?

GIVEN

- Copper wire with diameter (D)= 2.5 mm=0.0025 m
- Initial temperature of the wire(T_o) = 45°C
- Oven air temperature(T_∞) = 700°C
- Heat transfer coefficient ($\bar{h}_c$) = 45 W/(m^2 K)
- Length of the oven(L)=3.5 m

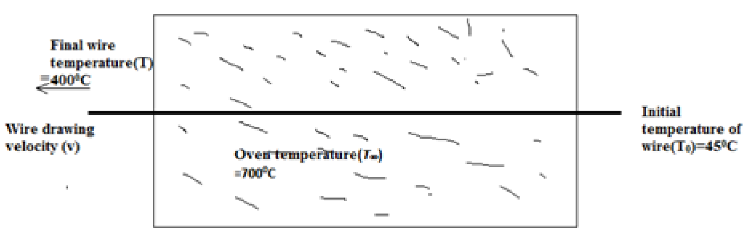
FIND

- The soaking annealing time required for the wire to 400°C
- Drawing speed through the oven(v)

ASSUMPTIONS

- Constant thermal conductivity
- End effects are negligible
- The wire is very long compared to its diameter
- There is radial conduction only in the wire

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For copper Thermal conductivity (k) = 396 W/(m K)
Density (ρ) = 8933 kg/m^3
Specific heat (c) = 383 J/kg

SOLUTION

(a) The Biot number is calculated first to check if the internal resistance is negligible

$$B_i = \frac{\bar{h}_c D}{4k} = \frac{\left(45\text{W/(m}^2\text{K)}\right)(0.0025\,\text{m})}{4\left(396\,\text{W/(m}^2\text{ K)}\right)} = 0.000071 << 0.1$$

Therefore, the internal resistance of the rod is negligible.

The temperature-time history of the rod, from Equation (3.3) is

$$\frac{T-T_\infty}{T_o-T_\infty} = \exp\left(\frac{\bar{h}_c A_s}{c\rho V}t\right)$$

$$\frac{\bar{h}_c A_s}{c\rho V} = \frac{\bar{h}_c \pi DL}{c\rho\frac{\pi}{4}D^2L} = \frac{4\bar{h}_c}{c\rho D} = \frac{4\left(45\,\mathrm{W/(m^2K)}\right)(\mathrm{J/Ws})}{(383\,\mathrm{W/kg\ K})\left(8933\,\mathrm{Kg/m^3}\right)(0.0025\,\mathrm{m})} = 0.02104\ 1/S$$

$$\frac{T-T_\infty}{T_o-T_\infty} = \exp\left[-\left(0.02104\frac{1}{S}\right)t\right]$$

Solving for the time

$$t = -(47.53\ \mathrm{s})\ \ln\left(\frac{T-T_\infty}{T_o-T_\infty}\right)$$

The time required to reach 88°C is

$$t = -(47.53\ \mathrm{s})\ \ln\left(\frac{400-700}{45-700}\right)$$

$$t = -(47.53\ \mathrm{s})\ \ln\left(\frac{300}{655}\right) = 47.53*0.78\ \mathrm{s} = 37\ \text{seconds}$$

(b) Drawing speed through oven

Length of the oven(L)=3.5 m

Time required for soaking annealing to 400^0C(t)=37 seconds

Thus drawing speed(v)=L/t= 3.5 m/37 s=0.095 m/s =9.5 cm/s

PROBLEM 3.8

Green coffee beans, after harvesting, are dried and roasted in a fluidized-bed roaster. This type of roaster has hot air typically at 250^{0}C, and the air blows through a screen or perforated plate above which the beans float in an agitated suspension (fluidization) in the hot air as they get heated. As the beans heat up, they get dried and roasted to the required condition. They are typically flat ellipsoids that have an average volume of 44 mm^3 and an average surface area of 66 mm^2. For a flavorful and aromatic medium roast, the beans have to be heated to 200^{0}C. If the beans are initially at 22^{0}C at the time they enter the roaster and the convective heat transfer coefficient of the heating air is 20 W/(m^2 K), determine the required roasting time. Coffee bean thermophysical properties are k=0.184 W/(m K), c=2.564 kJ/(kg K) and ρ=552 kg/m^3.

GIVEN

- Green coffee beans with average volume V=44 mm^3 and average surface area A_s=66 mm^2
- Initial temperature of beans(T_o) = 22°C
- Roaster hot air temperature(T_∞) = 250°C
- Heat transfer coefficient ($\bar{h}_c$) = 20 W/(m^2 K)
- Length of the oven(L)=3.5 m

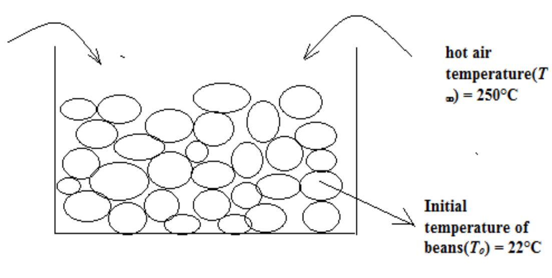
FIND

- The heating time required for beans to reach to 200°C

ASSUMPTIONS

- Constant thermal conductivity
- End effects are negligible

SKETCH



PROPERTIES AND CONSTANTS

Given

For beans	Thermal conductivity (k) = 0.184 W/(m K)
Density (ρ) = 552 kg/m^3
Specific heat (c) = 2.564 kJ/kg=2564 J/kg

SOLUTION

The characteristic length of beans is given by

L_c=V/A_s

L_c=44/66=0.667 mm=0.000667 m

The Biot number is calculated first to check if the internal resistance is negligible

$$B_i = \frac{\bar{h}_c L_c}{4k} = \frac{20*0.00667}{4*0.184} = 0.018125 << 0.1$$

Therefore, the internal resistance of beans is negligible.
The temperature-time history of beans from Equation (3.3) is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(\frac{\bar{h}_c A_s}{c\rho V} t\right)$$

$$\frac{\bar{h}_c A_s}{c\rho V} = \frac{\bar{h}_c}{c\rho L_c} = \frac{20}{552*2564*0.000667} = 0.0212\ 1/S$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(0.02121/S*t\right)$$

Solving for the time

$$t = -(47.2\text{ s})\ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

The time required to reach 88°C is

$$t = -(47.2\text{ s})\ln\left(\frac{200-250}{22-250}\right)$$

$$t = -(47.2\text{ s})\ln\left(\frac{50}{228}\right) = 47.2*1.5\text{ s} = 71\text{ seconds}$$

PROBLEM 3.9

The heat transfer coefficients for the flow of 26.6°C air over a 1.25 cm diameter sphere are measured by observing the temperature-time history of a copper ball of the same dimension. The temperature of the copper ball (c = 376 J/(kg K), ρ = 8928 kg/m³) was measured by two thermocouples, one located in the center, and the other near the surface. The two thermocouples registered, within the accuracy of the recording instruments, the same temperature at any given instant. In one test run, the initial temperature of the ball was 66°C and the temperature decreased by 7°C in 1.15 min. Calculate the heat transfer coefficient for this case.

GIVEN

- A copper ball with air flowing over it
- Ball diameter (D) = 1.25 cm = 0.0125 m
- Air temperature (T_∞) = 26.6°C
- Specific heat of ball (c) = 376 J/(kg K)
- Density of the ball (P) = 8928 kg/m^3
- Thermocouples in the center and the surface registered the same temperature
- Initial temperature of the ball (T_o) = 66°C
- Lapse time = 1.15 min = 69 s
- The temperature decrease ($T_o - T_f$) = 7°C

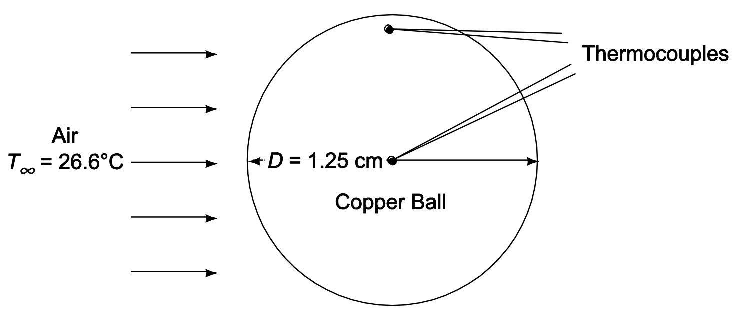
FIND

- The heat transfer coefficient $\bar{h}_c$

ASSUMPTIONS

- The heat transfer coefficient remains constant during the cooling period.

SKETCH



SOLUTION

Since the thermocouples register essentially the same temperature, the internal resistance of the ball is small compared to the external resistance and the ball can be treated with the lumped heat capacity method.

From Equation (3.3) the temperature-time history is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(-\frac{\bar{h}_c A}{c\rho V} t\right) = \exp\left(-\frac{\bar{h}_c (\pi D^2)}{c\rho \dfrac{\pi}{6} - D^3} t\right) = \exp\left(\frac{-6\bar{h}_c}{c\rho D} t\right)$$

Solving for the heat transfer coefficient

$$\bar{h}_c = \frac{c\rho D}{6*t} \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$\bar{h}_c = -\frac{[376\text{J/(kg K)}]\ 8928\,\text{kg/m}^3\ (0.0125\,\text{m})}{6(69\,\text{s})\ \text{J/(Ws)}} \ln\left(\frac{(66°\text{C} - 7°\text{C}) - 26.6°\text{C}}{66°\text{C} - 26.6°\text{C}}\right)$$

$$= 19.8\ \text{W/(m}^2\,\text{K)}$$

COMMENTS

The value is an average over the cooling period.

The procedure described by this problem can be used to evaluate heat transfer coefficients for odd shaped object experimentally.

PROBLEM 3.10

A spherical shell satellite (3-m-*OD*, 1.25-cm-thick stainless steel walls) re-enters the atmosphere from outer space. If its original temperature is 38°C, the effective average temperature of the atmosphere is 1093°C, and the effective heat transfer coefficient is 115 W/(m^2 °C), estimate the temperature of the shell after reentry, assuming the time of reentry is 10 min and the interior of the shell is evacuated.

GIVEN

- A spherical stainless steel satellite reentering the atmosphere
- Outside diameter (D) = 3 m
- Wall thickness (L) = 1.25 cm = 0.0125 m
- Its original temperature (T_o) = 38°C
- The effective temperature of the atmosphere (T_∞) = 1093°C
- The effective heat transfer coefficient $\bar{h}_c$ = 115 W/(m^2 °C)
- The time of reentry (t_r) = 10 min = 600 s
- The interior of the shell is evacuated

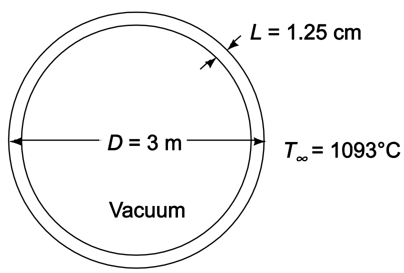
FIND

- The temperature of the shell after reentry (T_f)

ASSUMPTIONS

- Exterior heat transfer is uniform over the shell
- Assume radiation heat transfer is allowed for in the heat transfer coefficient

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, for stainless steel at 20°C

Thermal conductivity (k) = 14.4 W/(m K)
Density (ρ) = 7817 kg/m^3
Specific heat(c) = 461 J/(kg K)

SOLUTION

Since the thickness of the shell is much smaller than the shell radius, the wall can be treated as a plane wall. To estimate the importance of internal thermal resistance, the Biot number is calculated first

$$B_i = \frac{\bar{h}_c L}{k_s} = \frac{[115\,\mathrm{W/(m^2\,°C)}](0.0125\,\mathrm{m})}{14.4\,\mathrm{W/(m\,K)}} = 0.099 < 0.1$$

Therefore, the internal resistance is less than 10% of the external resistance and may be neglected. The temperature-time history of the satellite is given by Equation (3.3):

$$\frac{T - T_\infty}{T_s - T_\infty} = \exp\left(-\frac{\bar{h}_c A_s}{c\rho V} t\right) = \exp\left(-Bi\,Fo\right)$$

$$Bi\,Fo = \left(\frac{\bar{h}_c L}{k_s}\right)\left(\frac{\alpha t}{L^2}\right) = \frac{\bar{h}_c A_s}{c\rho V} t = \frac{\bar{h}_c \pi D^2 t}{c\rho \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{D}{2} - L\right)^3\right]}$$

$$Bi\,Fo = \frac{[115\text{W/(m}^2\text{K)}](3\text{m})^2 \ \text{J/(Ws)} \ t}{[461\text{J/(kg K)}] \ 7817\text{kg/m}^3 \ \frac{4}{3}[(1.5\text{m})^3 - (1.5\text{m} - 0.0125\text{m})^3]}$$

$$= 0.0025t \text{ (t in seconds)}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-0.0025t}$$

$$T = T_\infty + (T_o - T_\infty)e^{-0.0025t}$$

$$T_f = 1093°\text{C} + (38°\text{C} - 1093°\text{C})\, e^{-0.0025(600)} = 868°\text{C}$$

COMMENTS

The analysis has neglected thermodynamic heating during reentry.

PROBLEM 3.11

In the heat treating process in a sheet-metal manufacturing plant, brass plates are annealed by first heating them in an industrial furnace oven the inside of which is maintained at a constant uniform temperature of 550°C. If the surface of the brass plate, which has a thickness of 2.5 cm, is initially at a uniform temperature of 25°C, determine the time required for heating the plate surface to 300°C after the plate has been placed inside the oven. Consider that heat transfer from the oven gases to the plate is by convection and that $\bar{h}_c = 135$ W/m² K.

GIVEN

- Brass plate of thickness L=2.5 cm=0.025 m
- Initial temperature of the plate (T_o) = 25°C
- Heat treating oven temperature (T_∞) = 550°C
- Convective heat transfer coefficient ($\bar{h}_c$) = 135 W/(m^2 K)

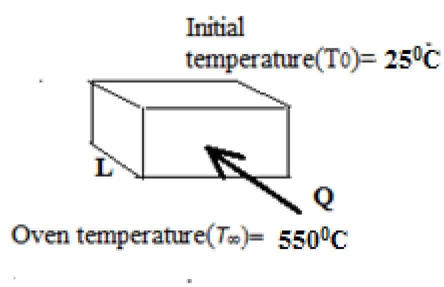
FIND

- The time required for the plate to reach temperature of 300^{0}C.

ASSUMPTIONS

- Constant thermal conductivity
- End effects are negligible

SKETCH



PROPERTIES AND CONSTANTS

As per the information provided:

Thermal conductivity (k) = 50 W/(m K)

Specific heat (c) = 460 J/(kg K)

Density (ρ) = 7865 kg/m^3

SOLUTION

For a flat plate we have L_c=L/2= 0.0125 m

Biot number(Bi)= $\bar{h}_c$ L_c/k=45*0.0125/50=0.0135 which is << 0.1

Thus the flat plate can be considered as lumped capacitance.
Therefore, the internal resistance of the rod is negligible.

The temperature-time history of the rod, from Equation (3.3) is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(\frac{\bar{h}_c A_s}{c\rho V} t\right)$$

$$\frac{\bar{h}_c A_s}{c\rho V} = \frac{\bar{h}_c * 2WH}{c\rho * LWH} = \frac{2\bar{h}_c}{c\rho L} = \frac{2*45}{460*7865*0.03} = 0.000829 \text{ s}^{-1}$$

$$\frac{T_\infty - T}{T_\infty - T_o} = \exp\left[-\left(0.000829\frac{1}{S}\right)t\right]$$

Solving for the time

$$t = -(1206 \text{ s}) \ln\left(\frac{T_\infty - T}{T_\infty - T_o}\right)$$

The time required to reach 550°C is

$$t = -(1206 \text{ s}) \ln\left(\frac{700 - 550}{700 - 40}\right) = 121\text{s}$$

$$t = -(1206 \text{ s}) \ln\left(\frac{150}{660}\right) = 1206*1.48 \text{ s} = 1787 \text{ seconds.}$$

PROBLEM 3.12

A spherical stainless steel vessel at 93°C contains 45 kg of water initially at the same temperature. If the entire system is suddenly immersed in ice water, determine (a) the time required for the water in the vessel to cool to 16°C, and (b) the temperature of the walls of the vessel at that time. Assume that the heat transfer coefficient at the inner surface is 17 W/(m² K), the heat transfer coefficient at the outer surface is 22.7 W/(m² K), and the wall of the vessel is 2.5-cm-thick.

GIVEN

- A spherical stainless steel vessel of water is suddenly immersed in ice water
- Initial temperature of vessel and water (T_i) = 93°C
- Mass of water in the vessel (m) = 45 kg
- The inner heat transfer coefficient $\bar{h}_{ci}$ = 17 W/(m² K)
- The outer heat transfer coefficient $\bar{h}_{co}$ = 22.7 W/(m² K)
- The vessel wall thickness (L) = 2.5 cm = 0.025 m

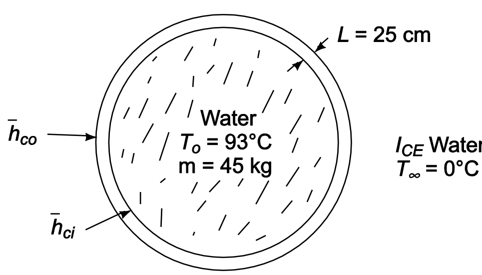
FIND

(a) The time required for the water in the vessel to cool to 16°C
(b) The temperature of the walls of the vessel at that time (T_{sf})

ASSUMPTIONS

- The water in the vessel is well mixed, therefore its temperature is uniform
- The vessel is completely filled with water

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10
For stainless steel: The thermal conductivity (k_s) = 14.4 W/(m K)
Density (ρ) = 7817 kg/m³
Specific heat (c) = 461 J/(kg K)

SOLUTION

If the vessel is completely filled with water

$$V = \frac{m_w}{\rho} = \frac{\pi}{6} D_1^3$$

$$D_i = \left(\frac{6\, m_w}{\pi\, \rho}\right)^{\frac{1}{3}} = \left(\frac{6(45\,\text{kg})}{\pi(1000\,\text{kg/m}^3)}\right)^{\frac{1}{3}} = 0.44\text{ m}$$

$$D_o = D_i + 2\,L = 0.44\text{ m} + 2\,(0.025\text{ m}) = 0.49\text{ m}$$

The internal resistance of the water can be neglected since the water is assumed to be well mixed. The importance of the internal resistance of the vessel wall is indicated by the Biot number of the vessel wall. The characteristic length for the vessel wall is

$$L = \frac{\text{volume}}{\text{Surface area}} = \frac{\frac{\pi}{6}(D_o^3 - D_i^3)}{\pi(D_o^2 + D_i^2)} = \frac{1}{6}\frac{(0.49\,\text{m})^3 - (0.44\,\text{m})^3}{(0.49\,\text{m})^2 + (0.44\,\text{m})^2} = 0.0125\ \text{m}$$

$$\therefore\ Bi = \frac{\bar{h}\,L}{k_s} = \frac{\frac{1}{2}(\bar{h}_{ci} + \bar{h}_\infty)\,L}{k_s} = \frac{\frac{1}{2}(17 + 22.7)\,[\text{W/(m}^2\text{K)}]\,(0.0125\,\text{m})}{14.4\,\text{W/(m K)}} = 0.017 < 0.1$$

Therefore, the vessel and its contents can be treated as a lumped capacitance and the system approximated two lumped capacitances as covered in Section 2.6.1 of the text.

(a) The temperature-time history of the water in the vessel is given by Equation (3.12)

$$\frac{T_w - T_\infty}{T_0 - T_\infty} = \frac{m_2}{m_2 - m_1}e^{m_1 t} - \frac{m_1}{m_2 - m_1}e^{m_2 t}$$

where T_w = temperature of the water, a function of time

$m_1 = 0.5\ \{-(k_1 + k_2 + k_3) + [(k_1 + k_2 + k_3)^2 - 4k_1\,k_3]^{0.5}\}$

$m_2 = 0.5\ \{-(k_1 + k_2 + k_3) - [(k_1 + k_2 + k_3)^2 - 4k_1\,k_3]^{0.5}\}$

$$k_1 = \frac{\bar{h}_{ci} A_i}{\rho_w c_w V_i} = \frac{\bar{h}_{ci}\,\pi D_i^2}{\rho_w c_w \frac{\pi}{6} D_i^3} = \frac{6\,\bar{h}_{ci}}{\rho_w c_w\ D_i} = \frac{6\ \ 17\,\text{W/(m}^2\text{K)}}{1000\text{kg/m}^3\ \ 4187\text{J/(kg K)}\ \ (0.44\,\text{m})} = 5.53 \times 10^{-5}\ 1/\text{s}$$

$$k_2 = \frac{\bar{h}_{ci} A_i}{\rho_s c_s V_s} = \frac{\bar{h}_{ci}\,\pi D_i^2}{\rho_s c_s \frac{\pi}{6}(D_o^3 - D_i^3)} = \frac{17\,\text{W/(m}^2\text{K)}\ \ (0.44\,\text{m})^2}{7817\text{kg/m}^3\ \ 461\text{J/(kg K)}\ \ 1/6\ \ (0.49^3 - 0.044^3)\,\text{m}^3} = 1.69 \times 10^{-4}\ 1/\text{s}$$

$$k_3 = \frac{\bar{h}_{co}\ A_o}{\rho_s c_s V_s} = \frac{\bar{h}_{co}\,\pi D_o^2}{\rho_s c_s \frac{\pi}{6}(D_o^3 - D_i^3)} = \frac{22.7\,\text{W/(m}^2\text{K)}\ \ (0.49\,\text{m})^2}{7817\text{kg/m}^3\ \ 461\,\text{J/(kg/K)}\ \ 1/6\ \ (0.49^3 - 0.044^3)\,\text{m}^3} = 2.79 \times 10^{-4}\ 1/\text{s}$$

$$k_1 + k_2 + k_3 = 5.04 \times 10^{-4}\ \text{s}^{-1}$$

$$4k_1\,k_3 = 6.17 \times 10^{-8}\ \text{s}^{-1}$$

$$m_1 = -3.28 \times 10^{-5}\ \text{s}^{-1}$$

$$m_2 = -4.71 \times 10^{-4}\ \text{s}^{-1}$$

$$m_2 - m_1 = 4.38 \times 10^{-4}\ \text{s}^{-1}$$

The temperature-time history of the water is

$$\frac{T_w - T_\infty}{T_o - T_\infty} = \frac{-4.71 \times 10^{-4}}{-4.38 \times 10^{-4}}e^{-3.28 \times 10^{-5}\frac{1}{s}\ t} - \frac{-3.28 \times 10^{-5}}{-4.38 \times 10^{-4}}e^{-4.71 \times 10^{-4}\frac{1}{s}\ t}$$

For the water to cool to 16°C

$$\frac{16°\text{C} - 0°\text{C}}{93°\text{C} - 0°\text{C}} = 0.1720 = 1.075\,E^{-3.28 \times 10^{-5}\frac{1}{s}\ t} - 0.075\,E^{-4.71 \times 10^{-4}\frac{1}{s}\ t}$$

By trial and error: t = 55,870 s = 15.5 hours

(b) The energy balance for the fluid is given by Equation (3.11a)

$$-(c\,\rho\,V)_w\,\frac{dT_w}{dt} = \bar{h}_i\,A_i\,(T_w - T_s)$$

Differentiating the temperature-time history

$$\frac{dT_w}{dt} = (T_0 - T_\infty)\left[\frac{m_1\,m_2}{m_2 - m_1}e^{m_1 t} - \frac{m_1\,m_2}{m_2 - m_1}e^{m_1 t}\right] = (T_0 - T_\infty)\,\frac{m_1 - m_2}{m_2 - m_1}(e^{m_1 t} - e^{m_2 t})$$

Substituting this into the energy balance for the fluid

$$-\,(c\,\rho\,V)_w(T_0 - T_\infty)\,\frac{m_1\,m_2}{m_2 - m_1}(e^{m_1 t} - e^{m_2 t}) = \bar{h}_{ci}\,A_\mathrm{i}\,(T_w - T_s)$$

$$T_0 = T_w + \frac{(\mathrm{cm})_w}{\bar{h}_{ci}\,A_i}\,(T_0 - T_\infty)\,\frac{m_1\,m_2}{m_2 - m_1}(e^{m_1 t} - e^{m_2 t})$$

$$T_\mathrm{s} = 16°\mathrm{C} + \frac{[4187\,\mathrm{J/(kg\,K)}](45\,\mathrm{kg})}{[17\,\mathrm{W/(m^2K)}]\pi(0.44\,\mathrm{m})^2}\,(93°\mathrm{C} - 0°\mathrm{C})\,\frac{-3.28\times10^{-5}(1/\mathrm{s})\;-4.71\times10^{-4}(1/\mathrm{s})}{-4.38\times10^{-4}(1/\mathrm{s})}$$

$$\times\;e^{-3.28\times10^{-5}(1/\mathrm{s})\,(55870\mathrm{a})} - e^{-4.71\times10^{-4}\,(1/\mathrm{s})\,(55870\mathrm{a})}$$

$$T_s = 6.4\ \mathrm{s}$$

PROBLEM 3.13

A copper wire, 0.8 mm *OD*, 5 cm long, is placed in an air stream whose temperature rises at a rate given by T_{air} = (10+ 14*t*)°C, where t is the time in seconds. If the initial temperature of the wire is 10°C, determine its temperature after 2 s, 10 s and 1 min. The heat transfer coefficient between the air and the wire is 40 W/(m² K).

GIVEN

- A copper wire is placed in an air stream
- Wire diameter (D) = 0.8 mm=$8*10^{-4}$ m
- Wire length (L) = 5 cm=0.05 m
- Air stream temperature is: $T_{air} = (10 + 14t)$°C
- The initial temperature of the wire (T_o) = 10°C
- The heat transfer coefficient $(\bar{h}_c)$ = 40 W/(m² K)

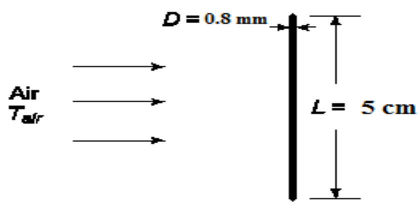
FIND

- The wire temperature after 2 s, 10 s and 1 min

ASSUMPTIONS

- Constant and uniform heat transfer coefficient

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For copper at 127°C

- Thermal conductivity (k) = 383 W/(m K)
- Density (ρ) = 8933 kg/m³
- Specific heat (c) = 383 J/(kg K)

SOLUTION

The Biot number for this problem is

$$Bi = \frac{\bar{h}_c D}{2\,k_c} = \frac{(40\,W/(\text{m}^2\,°\text{K}))(8*10^{-4})}{2\,(383\,W/(\text{m}\,K))} = 4*10^{-5} << 0.1$$

Therefore the internal resistance of the wire can be neglected.

The temperature-time history of the wire can be calculated from the energy balance, Equation (3.1)

$$-\,c\,\rho\,V\,dT = \bar{h}\;A_s\,(T - T_\infty)\,dt$$

but $T_\infty = T_{air} = 10 + 14t$

$$\therefore -c\,\rho\,V\,dT = \bar{h}\,A_s\,(T - 10 + 14t)\,dt$$

Rearranging

$$\frac{dT}{dt} = \frac{\bar{h}\,A_s}{c\rho V}\,(10 + 14t - T)$$

$$\text{let } m = \frac{\bar{h}\,A_s}{c\rho V} = \frac{\bar{h}\,(\pi DL)}{c\rho\,\frac{\pi}{4}D^2 L} = \frac{4\,\bar{h}}{c\rho\,D} = \frac{4\left(40\ \text{W/(m}^2\ \text{K)}\right)}{[383J/(kgK)]\left(8993\ \text{kg/m}^3\right)*(8*10^{-4}m)} = 0.0585\ \text{s}^{-1}$$

$$\therefore \frac{dT}{dt} + m\,T = 2m\,(5 + 7t)$$

This is a linear, first order, non-homogeneous differential equation with a homogeneous solution of $T = c\,e^{-mt}$ and a particular solution $T = c_o + c_1\,t$. Therefore, the general solution has the form:

$$T = c_o + c_1\,t + c_2\,e^{-mt}$$

$$\frac{dT}{dt} = c_1 - c_2\,m\,e^{-mt}$$

$$\frac{dT}{dt} + mT = c_1 - c_2\,m\,e^{-mt} + c_o\,m + c_1\,m\,t + c_2\,m\,e^{-mt} = 2\,m\,(5 + 7\,t)$$

$$c_o\,m + c_1 + c_1\,m\,t = 2\,m\,(5t + 7)$$

$$c_1\,m\,t = 14\,m\,t \Rightarrow c_1 = 14$$

$$c_o\,m + c_1 = c_o\,m + 14 = 10\,\text{m} \Rightarrow c_o = 10 - \frac{14}{m}$$

Substituting these back into the assumed solution yields

$$T = 10 - \frac{14}{m} + 25\,t + c_2\,e^{-mt}$$

Applying the initial condition: $T = 10°\text{C}$ when $t = 0$

$$10 = 10 - \frac{14}{m} + c_2 \Rightarrow c_2 = \frac{14}{m}$$

Therefore, the temperature-time history of the wire is

$$T = 10 + 14t + \frac{14}{m}\,(e^{-mt} - 1)$$

Evaluating the wire temperature at the requested times

$$\text{At } t = 2\ \text{sec: } T = 10 + 28 + \frac{14}{0.0585}\,e^{-0.0585\,(2)} - 1) = 11.5°\text{C}$$

$$\text{At } t = 10\ \text{sec: } T = 10 + 140 - \frac{25}{0.0585}\,(e^{-0.0585\,(10)} - 1) = 44°\text{C}$$

$$\text{At } t = 1\ \text{min} = 60\ \text{sec: } T = 10 + (14)\,(60) - \frac{25}{0.0585}\,(e^{-0.0585\,(60)} - 1) = 618°\text{C}$$

COMMENT

Radiation from the wire will become important well before 60 sec has elapsed.

PROBLEM 3.14

A thin-wall cylindrical vessel (1 m in diameter) is filled to a depth of 1.2 m with water at an initial temperature of 15°C. The water is well stirred by a mechanical agitator. Estimate the time required to heat the water to 50°C if the tank is suddenly immersed into oil at 105°C. The overall heat transfer coefficient between the oil and the water is 284 W/(m^2 K), and the effective heat transfer surface area is 4.2 m^2.

GIVEN

- A thin wall cylindrical vessel filled with water is suddenly immersed into oil
- Diameter of vessel (D) = 1 m
- Depth of water is vessel = 1.2 m
- Initial temperature (T_o) = 15°C
- Final temperature (T_f) = 50°C
- Oil temperature (T_∞) = 105°C
- The overall heat transfer coefficient between the oil and water $(\bar{h})$ = 284 W/(m^2 K)
- The effective heat transfer surface area (A) = 4.2 m^2

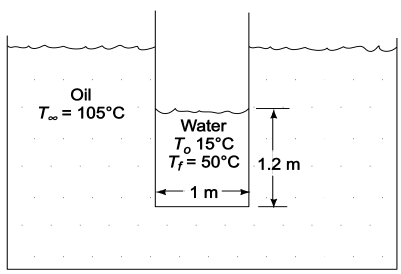
FIND

- The time required to heat the water to 50°C

ASSUMPTIONS

- The thermal capacitance of the cylindrical vessel is negligible
- The temperature of the water is uniform
- The oil temperature remains constant

SKETCH



PROPERTIES AND CONSTANTS

Specific heat of water (c) = 4187 J/(kg K)
Density of water (ρ) = 1000 kg/m^3

SOLUTION

From Equation (3.3), the temperature-time relationship is

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(-\frac{\bar{h} A_s}{c\rho V} t\right)$$

Solving for the time

$$t = \frac{-c\rho V}{\bar{h} A_s} \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t = \frac{-\ 4187\text{J/(kg K)} \ \ 1000\text{kg/m}^3 \ \ [\pi(0.5\,\text{m})^2(1.2\,\text{m})]}{[284\,\text{W/(m}^2\text{K)}](4.2\,\text{m}^2)(\text{J/(W s)})} \ln\left(\frac{50°\text{C} - 105°\text{C}}{15°\text{C} - 105°\text{C}}\right)$$

$$= 1629 \text{ s} = 27 \text{ min}$$

PROBLEM 3.15

A thin-wall jacketed tank, heated by condensing steam at one atmosphere contains 91 kg of agitated water. The heat transfer area of the jacket is 0.9 m^2 and the overall heat transfer coefficient *U* = 227 W/(m^2 K) based on that area. Determine the heating time required for an increase in temperature from 16°C to 60°C.

GIVEN

- A thin wall jacketed tank, heated by condensing steam
- Steam pressure = one atmosphere
- Mass of water in the tank = 91 kg
- The heat transfer area (A) = 0.9 m^2
- The overall heat transfer coefficient (U) = 227 W/(m^2 K) based on that area
- Temperature increases from 16°C to 60°C

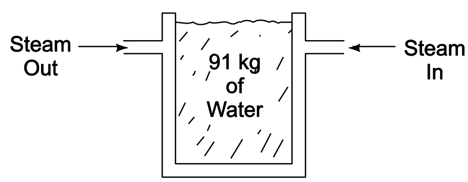
FIND

- Determine the heating time required

ASSUMPTIONS

- Uniform water temperature due to agitation
- Thermal capacitance of the tank wall is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13
The specific heat of water (c) = 4187 J/(kg K)
Temperature of saturated steam at 1 atmosphere (1.01×10^5 ρa) = 100°C

SOLUTION

The temperature-time history for this system is given by Equation (3.3).

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(-\frac{UA_s}{c\rho V}t\right) = \exp\left(-\frac{UA_s}{c\rho\left(\frac{m}{\rho}\right)}t\right) = \exp\left(-\frac{UA_s}{c\,m}t\right)$$

Solving this expression for the time

$$t = -\frac{\text{cm}}{UA_s}\ln\left(\frac{T_f - T_\infty}{T_o - T_\infty}\right) = -\frac{[4187\text{J/(kg K)}](91\text{kg})\ (\text{Ws/J})}{[227\text{W/(m}^2\text{K)}](0.9\,\text{m}^2)}\ \ln\left(\frac{60°\text{C} - 100°\text{C}}{16°\text{C} - 100°\text{C}}\right) = 1384 \text{ s} = 23 \text{ seconds}$$

PROBLEM 3.16

A large 2.54-cm.-thick copper plate is placed between two air streams. The heat transfer coefficient on the one side is 28 W/(m^2 K) and on the other side is 57 W/(m^2 K). If the temperature of both streams is suddenly changed from 38°C to 93°C, determine how long it will take for the copper plate to reach a temperature of 82°C.

GIVEN

- A large copper plate between two air streams whose temperatures suddenly change
- Plate thickness ($2L$) = 2.54 cm = 0.0254 m
- The heat transfer coefficients are
  - $\bar{h}_{c1}$ = 28 W/(m^2 K)
  - $\bar{h}_{c2}$ = 57 W/(m^2 K)
- Air temperature changes from 38°C to 93°C

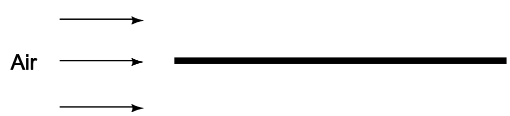
FIND

- How long it will take for the copper plate to reach a temperature of 82°C

ASSUMPTIONS

- The initial temperature of the plate is 38°C
- The plate can be treated as an infinite slab

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12

For copper Thermal conductivity (k) = 396 W/(m K) at 63°C
Density (ρ) = 8933 kg/m^3
Specific heat (c) = 383 J/kg

SOLUTION

The Biot number for this case, using the larger of the heat transfer coefficients is

$$Bi = \frac{\bar{h}_c\, L}{k} = \frac{[57\,\text{W/(m}^2\text{K)}]\ 0.0254/2\ \text{m}}{396\,\text{W/(m K)}} = 0.002 << 0.1$$

Therefore, the internal resistance of the slab can be neglected (the temperature of the slab remains uniform) and the temperature-time history can be calculated from an energy balance

Change in internal energy = heat flow from both sides

$$-c\,\rho\,V\,dT = \bar{h}_{c1}\,A\,(T - T_\infty)\,dt + \bar{h}_{c2}\,A\,(T - T_\infty)\,dt$$

$$-c\,\rho\,V\,dT = (\,\bar{h}_{c1} + \bar{h}_{c2}\,)\,A\,(T - T_\infty)\,dt$$

Rearranging

$$\frac{dT}{T - T_\infty} = \frac{d\,(T - T_\infty)}{T - T_\infty} = -\,\frac{(\bar{h}_{c1} + \bar{h}_{c2})}{c\rho V}\,dt$$

Integrating between a temperature of T_o at time = 0 to a temperature of T at time = t yields

$$\ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right) = \frac{(\bar{h}_{c1} + h_{c2})\,A}{c\rho V}\;t = \frac{(\bar{h}_{c1} + h_{c2})\,A}{c\rho\,(2\,LA)}\;t$$

Solving this for the time

$$t = -\,\frac{2\,Lc\,\rho}{\bar{h}_{c1} + \bar{h}_{c2}}\;\ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t = \frac{0.0254\,\text{m}\;\;383\text{J/(kg K)}\;\;\text{(Ws)/J}\;\;8933\text{kg/m}^3}{(28 + 57)\text{W/(m}^2\text{K)}}\;\ln\left(\frac{82°\text{C} - 93°\text{C}}{30°\text{C}\; - 93°\text{C}}\right)$$

$$t = 1645\text{ s} = 27\text{ min}$$

COMMENTS

Because heat transfer is occurring at both sides of the slab, the characteristic length in the Biot number is approximately half of the slab's thickness. However, since the heat transfer coefficients on the two surfaces are not equal, the center plane is not equivalent to an insulated surface.

PROBLEM 3.17

A 1.4-kg aluminum household iron has a 500 W heating element. The surface area is 0.046 m^2. The ambient temperature is 21°C and the surface heat transfer coefficient is 11 W/(m^2 K). How long after the iron is plugged in will its temperature reach 104°C?

GIVEN

- An aluminum household iron
- Mass of the iron (M) = 1.4 kg
- Power output $(\dot{Q}_G)$ = 500 W
- Surface area (A_s) = 0.046 m^2
- The ambient temperature (T_∞) = 21°C
- The heat transfer coefficient $(\bar{h}_c)$ = 11 W/(m^2 K)

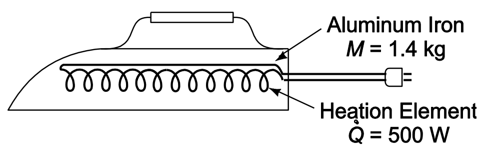
FIND

- How long after the iron is plugged in will its temperature reach 104°C

ASSUMPTIONS

- Constant heat transfer coefficient
- The mass given is for the heated aluminum portion only

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12

For aluminum

Thermal conductivity (k) = 240 W/(m K) at 127°C

Specific heat (c) = 896 J/(kg K)

SOLUTION

To calculate the Biot number for this problem, we must first calculate the characteristic length

$$L = \frac{\text{Volume}}{\text{Surface area}} = \frac{\frac{M}{\rho}}{A_s} = \frac{M}{\rho A_s} = \frac{1.4\text{ kg}}{(2702\text{kg/m}^3)(0.046\text{ m}^2)} = 0.0113\text{ m}$$

The Biot number is

$$Bi = \frac{\bar{h}_c L}{k} = \frac{[11\text{ W/(m}^2\text{ K)}](0.0113\text{ m})}{240\text{W/(m K)}} = 0.0005 < 0.1$$

Therefore, the lumped capacity method may be used. The energy balance for the iron is

Change in internal energy = heat generation – net heat flow from the iron.

$$c\,\rho\,V\,dT = \dot{Q}_G - \bar{h}_c\,A_s\,(T - T_\infty)\,dt$$

$$\text{Let } \Theta = T - T_\infty \text{ and } m = \frac{\bar{h}_c A_s}{c\rho V} = \frac{\bar{h}_c A_s}{c\rho\left(\frac{M}{\rho}\right)} = \frac{\bar{h}_c A_s}{cM}$$

Then the heat balance can be written

$$\frac{d\Theta}{dt} + m\,\Theta = \frac{\dot{Q}_G}{cM}$$

This is a linear, first order, non-homogeneous differential equation. The solution to the homogeneous equation is $\theta_h = c\, e^{-mt}$ and a particular solution is $\theta_p = c$. The general solution is the sum of the homogeneous and particular solutions

$$\Theta = c_1 + c_2\, e^{-mt}$$

Integrating

$$\frac{d\Theta}{dt} = -c_2\, m\, e^{-mt} = -m\,(\Theta - c_1) \text{ (From the previous equation)}$$

Substituting this into the heat balance

$$-m\,(\Theta - c_1) + m\,\Theta = \frac{\dot{Q}_G}{cM} \Rightarrow c_1 = \frac{\dot{Q}_G}{M\,cM}$$

Applying the initial condition, $\theta = 0$ at $t = 0$ yields

$$-c_2\, m = c_1\, m \Rightarrow c_2 = -c_1 = \frac{\dot{Q}_G}{M\,cM}$$

Therefore, the temperature-time history of the iron is given by

$$\Theta = \frac{\dot{Q}_G}{m\,cM}\,(1 - e^{-mt})$$

Solving for t

$$t = -\frac{1}{m}\ln\left(1 - \frac{\Theta m\, cM}{\dot{Q}_G}\right)$$

$$m = \frac{\bar{h}_c A_s}{cM} = \frac{[11\,\text{W/(m}^2\,\text{K)}](0.046\,\text{m}^2)}{[896\,\text{J/(kg K)}](1.4\,\text{kg})\ \text{(Ws)/J}} = 4.034 \times 10^{-4}\ \text{s}^{-1}$$

$$t = -\frac{1}{4.034\times 10^{-4}\,\text{s}^{-1}}\ln\left[1 - \frac{(104°\text{C} - 21°\text{C})\ 4.034\times 10^{-4}\text{s}^{-1}\ \ 896\,1/(\text{kg K})\ \ \ \text{Ws}\ /\text{J}\ \ (1.4\,\text{kg})}{500\,\text{W}}\right]$$

$$t = 217\ \text{s} = 3.6\ \text{min}$$

PROBLEM 3.18

A small aluminum sphere of diameter D, initially at a uniform temperature T_o, is immersed in a liquid whose temperature, T_∞, varies sinusoidally according to

$$T_\infty - T_m = A \sin(\omega t)$$

where: T_m = time-averaged temperature of the liquid

A = amplitude of the temperature fluctuation

ω = frequency of the fluctuations

If the heat transfer coefficient between the fluid in the sphere, $\bar{h}_a$, is constant and the system may be treated as a 'lumped capacity,' derive an expression for the sphere temperature as a function of time.

GIVEN

- A small aluminum sphere is immersed in a liquid whose temperature varies sinusoidally
- Diameter of sphere = D
- Liquid temperature variation: $T_\infty - T_m = A \sin(\omega t)$
- The heat transfer coefficient = $\bar{h}_a$ (constant)
- The system may be treated as a 'lumped capacity'

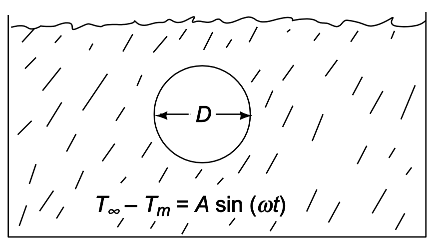
FIND

- An expression for the sphere temperature as a function of time

ASSUMPTIONS

- Constant thermal conductivity

SKETCH



SOLUTION

Let k = thermal conductivity of sphere

ρ = density of sphere

c = specific heat of sphere

An energy balance on the sphere yields

Change in internal energy = heat transfer to liquid

$$\rho c \frac{dT}{dt} = \bar{h}_a A_s (T - T_\infty)$$

$$\frac{dT}{dt} = \frac{h_s A_s}{\rho c V} [T - T_m - A \sin(\omega t)]$$

$$\text{Let } m = \frac{\bar{h}_s A_s}{\rho c V} = \frac{\bar{h}_s \pi d^2}{\rho c \frac{\pi}{6} d^3} = \frac{6 \bar{h}_s}{\rho c D} \text{ and } \Theta = T - T_m$$

$$\frac{d\Theta}{dt} + m\,\Theta = m A_s \sin(\omega t)$$

This is a first order, linear, non-homogeneous differential equation. The general solution is the sum of the homogeneous solution and a particular solution. The homogeneous solution is determined by the characteristic equation, found by substituting $\theta = e^{\lambda t}$ into the homogeneous equation

$$\lambda e^{\lambda t} + m e^{\lambda t} = 0 \qquad (\lambda = -m)$$

The homogeneous solution is $\theta_h = Ce^{-mt.}$

As a particular solution, try $\theta_p = K \cos(\omega t) + M \sin(\omega t)$, substituting θ_p and its derivative into the energy balance

$$-\omega K \sin(\omega t) + M \omega \cos(\omega t) + m K \cos(\omega t) + m M \sin(\omega t) = m A \sin(\omega t)$$

$$(M\omega + mK) \cos(\omega t) - (\omega K - mM) \sin(\omega t) = m A_s \sin(\omega t)$$

$$\therefore M\omega + mK = 0 \Rightarrow M = -\frac{mK}{\omega}$$

$$\text{and } \omega K - m M = -m A_s \quad \Rightarrow \omega K + \frac{mK}{\omega} m = -m A_s$$

$$\therefore K = \frac{M A_s \omega}{\omega^2 + m^2} \text{ and } M = \frac{m^2 A_s}{\omega^2 + m^2}$$

Therefore, the general solution is

$$\Theta = C e^{-mt} + \frac{M A_s}{\omega^2 + m^2} [(-\omega \cos(\omega t) + m \sin(\omega t)]$$

At $t = 0$, $T = T_o$ and $\theta = \theta_o = T_o - T_m$

$$\Theta_o = C - \frac{m A_s \omega}{\omega^2 + m^2} \Rightarrow C = \Theta_o + \frac{m A_s \omega}{\omega^2 + m^2}$$

The dimensionless temperature distribution is

$$\frac{\Theta}{\Theta_o} = \left(1 + \frac{m A_s \omega}{\Theta_o(\omega^2 + m^2)}\right) e^{-mt} + \frac{m A_s}{\omega^2 + m^2} [(m \sin(\omega t) - \omega \cos(\omega t)]$$

PROBLEM 3.19

A wire of perimeter *P* and cross-sectional area A emerges from a die at a temperature *T* (above ambient temperature) and with a velocity *U*. Determine the temperature distribution along the wire in the steady state if the exposed length downstream from the die is quite long. State clearly and try to justify all assumptions.

GIVEN

- A wire emerging from a die at a temperature (T) above ambient
- Wire perimeter = P
- Cross-sectional area = A
- Wire emerges at a temperature T above ambient
- Wire velocity = U

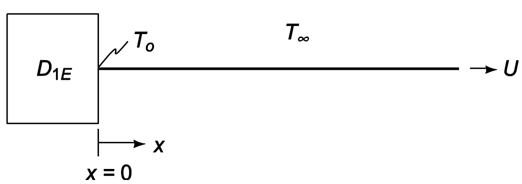
FIND

- The temperature distribution along the wire in the steady state if the exposed length downstream from the die is quite long. State clearly and try to justify all assumptions

ASSUMPTIONS

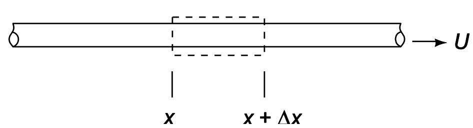
- Ambient temperature is constant at T_∞
- Heat transfer coefficient between the wire and the air is uniform and constant at h_c
- The material properties of the wire are constant
  - Thermal conductivity = k
  - Thermal diffusivity = α
- Axial conduction only
- Wire temperature is uniform at a cross section (negligible internal thermal resistance)

SKETCH



SOLUTION

Consider a control volume around the wire



Performing an energy balance on the control volume

Conduction into volume + Energy carried into the volume by the moving wire = Conduction out of volume + Convection to the environment + Energy carried out of the volume by the moving wire.

$$-k A \left.\frac{dT}{dx}\right|_x + U A \rho c\, T(x) = -k A \left.\frac{dT}{dx}\right|_{x+\Delta x} + \bar{h}_c\, P\, \Delta x (T - T_\infty) + U A \rho c\, T(x + \Delta x)$$

$$\frac{\left.\frac{dT}{dx}\right|_x - \left.\frac{dT}{dx}\right|_{x+\Delta x}}{\Delta x} = \frac{\frac{\rho c}{k} U[T(x+\Delta x) - T(x)]}{\Delta x} + \frac{\bar{h}_c P}{k A} (T - T_\infty)$$

letting $\Delta x \to 0$

$$\frac{d^2 T}{dx^2} = \frac{U}{\alpha}\frac{dT}{dx} + \frac{\bar{h}_c P}{k\ A} (T - T_\infty)$$

$$\text{Let } \theta = T - T_\infty \text{ and } m = \frac{\bar{h}_c P}{k\ A} = \frac{\bar{h}_c\ \pi D}{k \frac{\pi}{4} D^2} = \frac{4\,\bar{h}_c}{k D}$$

Then

$$\frac{d^2\theta}{dx^2} - \frac{U}{\alpha}\frac{d\theta}{dx} - m\ \theta = 0$$

This is a linear, differential equation with constant coefficients. The solution has the following form

$$\theta = c_1\ e^{s_1 x} + c_2\ e^{s_2 x}$$

Substituting this solution and its derivatives into the differential equation:

$$s_1^2\ c_1\ e^{s_1 x} + s_2^2\ c_2\ e^{s_2 x} - \frac{U}{\alpha}(s_1\ c_1\ e^{s_1 x} + s_2\ c_2\ e^{s_2 x}) - m\ (c_1\ e^{s_1 x} + c_2\ e^{s_2 x}) = 0$$

$$s_1^2 - \frac{U}{\alpha}\ s_1 - m = 0 \Rightarrow s_1 = \frac{1}{2}\left(\frac{U}{\alpha}\sqrt{\left(\frac{U}{\alpha}\right)^2 + 4\,\mathrm{m}}\right)$$

$$s_2^2 - \frac{U}{\alpha}\ s_2 - m = 0 \Rightarrow s_2 = \frac{1}{2}\left(\frac{U}{\alpha}\sqrt{\left(\frac{U}{\alpha}\right)^2 + 4\,\mathrm{m}}\right)$$

$$\therefore \text{ Let } s_1 = \frac{1}{2}\left(\frac{U}{\alpha} + \sqrt{\left(\frac{U}{\alpha}\right)^2 + 4\,\mathrm{m}}\right) \text{ and } s_2 = \frac{1}{2}\left(\frac{U}{\alpha} - \sqrt{\left(\frac{U}{\alpha}\right)^2 + 4\,\mathrm{m}}\right)$$

The boundary conditions for the problem are

1. $\theta = \theta_o$ at $x = 0$
2. $\theta \to 0$ at $x \to \infty$

Applying the first boundary condition

$$\theta_o = c_1 + c_2$$

Since, by inspection, s_1 must be positive, for the second boundary condition to be satisfied, the constant c_1 must be zero. Therefore, the temperature distribution in the wire is

$$\theta = \theta_o\ e^{s_2 t}$$

or

$$T = T_\infty + (T_o - T_\infty) \exp\left[\frac{x}{2}\ \frac{U}{\alpha} \sqrt{\frac{U}{\alpha}^2 + 4\,\mathrm{m}}\ \right]$$

PROBLEM 3.20

Dielectric heating, also known as RF or high frequency heating, is a process in which a high-frequency alternating electric field or microwave electromagnetic radiation heats a dielectric material. An important application of this phenomenon is in the heating of food in a microwave oven. Microwave ovens heat food by bombarding it with electromagnetic radiation in the microwave spectral range, causing polarized molecules in the field to rotate and build up thermal energy, thus cooking or heating the food. However, when the food is initially frozen, because the molecules in the solid do not rotate readily, volumetric heat generation is an order of magnitude less than if the material were in the liquid form. Microwave power not absorbed in the food is dissipated to the microwave generator in the form of heat. Estimate the time it takes to heat a 2 lb roast, initially at a temperature of -20°C, after it is placed in a 1-kW total power microwave oven with an interior temperature of 30°C. Assume the meat has a spherical shape and a heat transfer coefficient of 12 W/(m^2 K) from its surface. First estimate the time required to reach a uniform temperature of 0°C when the water in the meat is in the form of ice. Assume that 4% of the oven power is absorbed in the food. After the meat thaws (all ice is melted), the meat increases in temperature. Now estimate the time required for the meat to reach 75°C if 90% of the microwave oven power is absorbed in the meat. Assume that the physical properties of meat are the same as those of liquid water.

GIVEN

- Wight of the roast (m) =2 lb=0.907 kg
- Initial temperature (T_0)=-20^0C
- Microwave oven power(P)= 1000 W
- Microwave interior temperature (T_∞)=30^0C
- Heat transfer coefficient ($\bar{h}_c$)= 12 W/(m^2 K)

FIND

- Time required reaching uniform temperature of 0^0C considering 4% of oven power is absorbed in food.
- Time required to reach 75^0C is 90% of oven power is absorbed in food when ice is melted.

ASSUMPTIONS

- Ambient temperature is constant at T_∞
- Beef has properties of water and ice.
- Heat transfer coefficient is constant at h_c
- The material properties of the wire are constant
    - Thermal conductivity = k
    - Thermal diffusivity = α

SOLUTION

Applying conservation of energy in roast we have

$$h_c A_s(T_\infty - T) + \dot{q} = mc\frac{dT}{dt}$$

The initial condition is $T(0)=T_0$

Defining $\Theta = T - T_\infty - \dfrac{\dot{q}}{h_c A_s}$ the above equation becomes

$$\frac{d\theta}{dt} = -\frac{h_c A_s}{mc}\theta$$

$$\int_{\theta_0}^{\theta_t}\frac{d\theta}{\theta} = -\frac{h_c A_s}{mc}\int_0^t dt$$

$$\ln\left(\frac{\theta(t)}{\theta(0)}\right)=\frac{h_c A_s}{mc}t \quad \Rightarrow \quad \ln\left(\frac{T-T_\infty-\frac{\dot{q}}{h_c A_s}}{T_0-T_\infty-\frac{\dot{q}}{h_c A_s}}\right)=\frac{h_c A_s}{mc}t$$

m = ρV = ρ 4 /3*π r_0^3 => $r_0=\left(\frac{3m}{4\pi\rho}\right)^{1/3}=\left(\frac{3*0.907}{4\pi*920}\right)^{1/3}$=0.062 m

Thus Surface Area (A_s)=$4\pi r_0^2$=4*π*$(0.062)^2$=0.0483 m^2

(a) When the roast is frozen the heat generation rate is $\dot{q}$=0.9P= 900 W

Considering the heating till the temperature reaches 0^0C and substituting the values where in above equation where T=0^0C we get,

$$\ln\left(\frac{0-30-\frac{50}{12*0.0483}}{-20-30-\frac{50}{12*0.0483}}\right)=\left(\frac{12*0.0483}{0.907*2040}\right)t$$

$$\ln\left(\frac{-116.26}{-136.26}\right)=-0.00031t$$

ln(0.858)= -0.00031*t

t=512 seconds= 8.5 minutes.

(b) When the roast is melted the heat generation rate is $\dot{q}$=0.05P= 50 W

Considering the heating till of roast from 0^0C till the temperature reaches 75^0C and substituting the values where in above equation where T=0^0C we get,

$$\ln\left(\frac{75-30-\frac{900}{12*0.0483}}{0-30-\frac{900}{12*0.0483}}\right)=-\left(\frac{12*0.0483}{0.907*4180}\right)t \quad \Rightarrow \ln\left(\frac{1507.8}{1582.8}\right)=-\left(\frac{12*0.0483}{0.907*4180}\right)t$$

ln(0.9526)=0.000152*t
t=319 seconds
=5.32 minutes

PROBLEM 3.21

In the vulcanization of tires, the carcass is placed into a jig and steam at 149°C is admitted suddenly to both sides. If the tire thickness is 2.5 cm, the initial temperature is 21°C, the heat transfer coefficient between the tire and the steam is 150 W/(m^2 K), and the specific heat of the rubber is 1650 J/(kg K), estimate the time required for the center of the rubber to reach 132°C.

GIVEN

- Tire suddenly exposed to steam on both sides
- Steam temperature (T_∞) = 149°C
- Tire thickness (2L) = 2.5 cm = 0.025 m
- Initial tire temperature (T_o) = 21°C
- The heat transfer coefficient (h_c) = 150 W/(m^2 K)
- The specific heat of the rubber (c) = 165 J/(kg K)

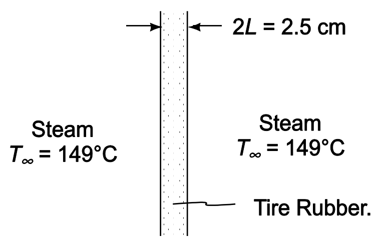
FIND

- The time required for the central layer to reach 132°C

ASSUMPTIONS

- Shape effects are negligible, tire can be treated as an infinite plate

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
For bana rubber

Thermal conductivity (k) = 0.465 W/(m K) at 20°C
Density (ρ) = 1250 g/m^3

SOLUTION

The significance of the internal resistance is determined from the Biot number

$$Bi = \frac{\bar{h}_c L}{k} = \frac{[150\,\text{W/(m}^2\text{K)}]\left(\dfrac{0.025}{2}\,\text{m}\right)}{0.465\,\text{W/(m K)}} = 4.0 >> 0.1$$

Therefore, the internal resistance is significant and the approximate solution will be used.

At center of rubber x=0, from equation (3.43a) we have

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty} = C_1 e^{-\delta_1^2 \tau} \cos(\delta_1 x / L)$$

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=C_1e^{-\delta_1^2\tau}\cos(0)$$

$$\frac{132-149}{21-149}=C_1e^{-\delta_1^2\tau}$$

From Table (3.1) for Infinite Slab

For Bi=4.0, $\delta_1=1.2646$ and $C_1=1.2287$

Thus the above equation becomes

$$0.13=1.2287*e^{-(1.2646)^2\tau}$$

$$e^{-(1.2646)^2\tau}=0.1058$$

$$-(1.2646)^2\tau=\ln(0.1058)$$

$$-(1.6)\tau=-2.246$$

$$\tau=1.40$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{k\,t}{\rho c\,L^2} = 1.4$$

Solving for the time

$$t=\frac{\rho cL^2Fo}{k} = \frac{[1250\,(\text{kg/m}^3)](1650\text{ J / (kg K)})(\,(\text{W s)/J})(\,0.025/2\text{ m})^2\ (1.4)}{0.465\,\text{W/(m K)}}$$

$t =$ 969 s = 16.2 min

PROBLEM 3.22

A 2.5-cm-thick sheet of plastic initially at 21°C is placed between two heated steel plates that are maintained at 138°C. The plastic is to be heated just long enough for its midplane temperature to reach 132°C. If the thermal conductivity of the plastic is 1.1×10^{-3} W/(m K), the thermal diffusivity is 2.7×10^{-6} m²/s, and the thermal resistance at the interface between the plates and the plastic is negligible, calculate: (a) the required heating time, (b the temperature at a plane 0.6 cm from the steel plate at the moment the heating is discontinued, and (c) the time required for the plastic to reach a temperature of 132°C 0.6 cm from the steel plate.

GIVEN

- A sheet of plastic is placed between two heated steel plates
- Sheet thickness ($2L$) = 2.5 cm = 0.025 m
- Initial temperature (T_o) = 21°C
- Temperature of steel plates (T_s) = 138°C
- Heat until mid-plane temperature of sheet (T_c) = 132°C
- The thermal conductivity of the plastic (k) = 1.1×10^{-3} W/(m K)
- The thermal diffusivity (α) = 2.7×10^{-6} m²/s
- The thermal resistance at the interface between the plates and the plastic is negligible

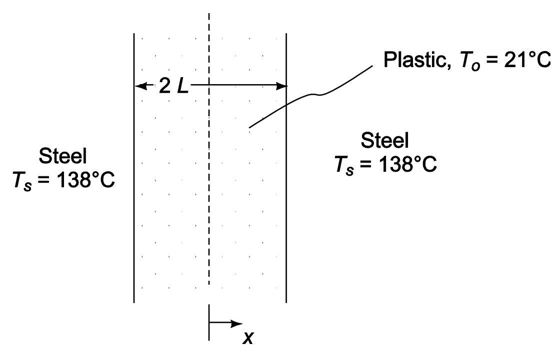
FIND

(a) The required heating time
(b) The temperature at a plane 0.6 cm from the steel plate at the moment the heating is discontinued
(c) The time required for the plastic to reach a temperature of 132°C 0.6 cm from the steel.

ASSUMPTIONS

- The initial temperature of the sheet is uniform
- The temperature of the steel plates is constant
- The thermal conductivity of the sheet is constant

SKETCH



SOLUTION

The approximate solutions apply to convective boundary conditions but it can be applied to this problem by letting $h_c \to \infty$. Therefore, $Bi = \infty$.

(a) To find the time required to heat the midplane from 21°C to 132°C, first calculate

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{132°\text{C} - 138°\text{C}}{21°\text{C} - 138°\text{C}} = 0.0513$$

At center x=0, from equation (3.43a) we have

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty}=C_1 e^{-\delta_1^2\tau}\cos(\delta_1 x/L) \quad \Rightarrow \quad \frac{T(0,t)-T_\infty}{T_0-T_\infty}=C_1 e^{-\delta_1^2\tau}\cos(0)$$

From Table (3.1) for Infinite Slab

For Bi = ∞, $\delta_1 = 1.5708$ and $C_1 = 1.2732$

Thus the above equation becomes

$$0.0513 = 1.2732 * e^{-(1.5708)^2\tau}$$

$$e^{-(1.5708)^2\tau} = 0.04029 \quad \Rightarrow \quad -(1.5708)^2\tau = \ln(0.04029)$$

$$-(2.4674)\tau = -3.21 \quad \Rightarrow \quad \tau = 1.30$$

$$F_0 = \tau = \frac{\alpha t}{r_o^2} = \frac{k t}{\rho c L^2} = 1.3$$

Solving for the time

$$t = \frac{Fo L^2}{\alpha} = \frac{1.3\left(\frac{0.025}{2}\text{ m}\right)^2}{27\times10^{-6}\text{m}^2/\text{s}} = 75 \text{ sec}$$

At center of rubber x=0, from equation (3.43a) we have

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty}=C_1 e^{-\delta_1^2\tau}\cos(\delta_1 x/L)$$

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=C_1 e^{-\delta_1^2\tau}\cos(0)$$

From Table (3.1) for Infinite Slab

For Bi = ∞., $\delta_1 = 1.5708$ and $C_1 = 1.2732$

Thus the above equation becomes

$$0.0513 = 1.2732 * e^{-(1.5708)^2\tau}$$

$$e^{-(1.5708)^2\tau} = 0.04029 \quad \Rightarrow \quad -(1.5708)^2\tau = \ln(0.04029)$$

$$-(2.4674)\tau = -3.21 \quad \Rightarrow \quad \tau = 1.30$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{k t}{\rho c L^2} = 1.3$$

Solving for the time

$$t = \frac{Fo L^2}{\alpha} = \frac{1.3\left(\frac{0.025}{2}\text{ m}\right)^2}{27\times10^{-6}\text{m}^2/\text{s}} = 75 \text{ sec}$$

(b) At 0.6 cm from the steel plate

$$x = L - 0.006 \text{ m} = 0.0125 \text{ m} - 0.006 \text{ m} = 0.0065 \text{ m} \Rightarrow \frac{x}{L} = \frac{0.0065\,\text{m}}{0.0125\,\text{m}} = 0.52$$

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty} = C_1 e^{-\delta_1^2\tau}\cos(\delta_1 x/L)$$

$$\frac{T(0.65cm,t)-T_\infty}{T_0-T_\infty} = 1.2732 * e^{-(1.5708)^2*1.3}\cos(1.5708*0.52)$$

$$\frac{T(0.65cm,t)-T_\infty}{T_0-T_\infty} = 0.0352$$

$$T(0.65cm,t) = 0.0352\left(T_0-T_\infty\right)+T_\infty$$

$$T(0.65cm,t) = 0.0352\left(21-138\right)+138\,^{0}\text{C}$$

$$T(0.65cm,t) = 133.8^0 C$$

(c) When the temperature 0.6 cm from the steel plate is 132°C

From equation (3.43a) we have

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty} = C_1 e^{-\delta_1^2\tau}\cos(\delta_1 x/L)$$

$$x = L - 0.006 \text{ m} = 0.0125 \text{ m} - 0.006 \text{ m} = 0.0065 \text{ m} \Rightarrow \frac{x}{L} = \frac{0.0065\,\text{m}}{0.0125\,\text{m}} = 0.52$$

$$\frac{132-138}{21-138} = 1.2732 * e^{-(1.5708)^2\tau}\cos(1.5708*0.52)$$

$$0.05128 = 0.8715 * e^{-(1.5708)^2\tau}$$

$$-(1.5708)^2\tau = \ln(0.05884) \qquad => -(1.5708)^2\tau = \ln(0.05984)$$

$$\tau = 1.14$$

We have,

$$t = \frac{\tau L^2}{\alpha} = \frac{1.14\left(\frac{0.025}{2}\text{ m}\right)^2}{2.7\times10^{-6}\text{m}^2/\text{s}} = 67 \text{ sec}$$

PROBLEM 3.23

In the inspection of a sample of meat intended for human consumption, it was found that certain undesirable organisms were present. To make the meat safe for consumption, it is ordered that the meat be kept at a temperature of at least 121°C for a period of at least 20 min during the preparation. Assume that a 2.5-cm.-thick slab of this meat is originally at a uniform temperature of 27°C, that it is to be heated from both sides in a constant temperature oven; and that the maximum temperature meat can withstand is 154°C. Assume furthermore that the surface coefficient of heat transfer remains constant and is 10 W/(m^2 K). The following data can be assumed for the sample of meat: specific heat = 4184 J/(kg K); density = 1280 kg/m^3; thermal conductivity = 0.48 W/(m K). Calculate the oven temperature and the minimum total time of heating required to fulfill the safety regulation.

GIVEN

- A slab of meat is heated in constant temperature over
- Meat be kept at a temperature of at least 121°C for a period of at least 20 min during the preparation
- Slab thickness ($2L$) = 2.5 cm = 0.025 m
- Initial uniform temperature (T_o) = 27°C
- The maximum temperature meat can withstand is 154°C
- Specific heat (c) = 4184 J/(kg K)
- Density (ρ) = 1280 kg/m^3
- Thermal conductivity (k) = 0.48 W/(m K)

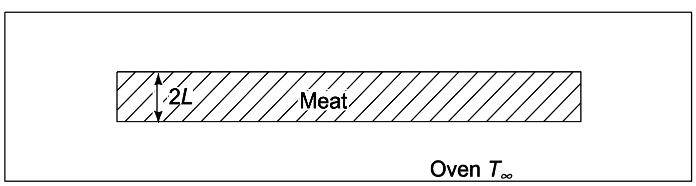
FIND

- The minimum total time of heating required to fulfill the safety regulation

ASSUMPTIONS

- The surface heat transfer coefficient ($\bar{h}_c$)= 10 W/(m K)
- Edge effects are negligible
- One dimensional conduction

SKETCH



SOLUTION

The Biot number for the meat is

$$Bi = \frac{\bar{h}_c L}{k} = \frac{[10\ \text{W/(m}^2\ \text{K)}]\left(\dfrac{0.025}{2}\ \text{m}\right)}{0.48\text{W/(m K)}} = 0.26 > 0.1$$

Therefore, the internal resistance is significant and the transient conduction charts will be used to find a solution.

The highest temperature will occur at the surface of the meat while the lowest will occur at the center of the meat. Therefore, the maximum possible oven temperature (T_∞) can be obtained from approximate solutions.

For Bi=0.26,

From Table 3.1 for infinite slab we have

$\delta_1 = 0.4862$ & $C_1 = 1.03824$

From equation (3.43a) for x= L, we have

$$\frac{T(L,t)-T_\infty}{T_0-T_\infty} = 1.03824 * e^{-\delta_1^2 * 0} \cos(0.4862 * L/L)$$

$$\frac{T(L,t)-T_\infty}{T_0-T_\infty} = 1.03824 * e^{-\delta_1^2 * 0} \cos(0.4862 * L/L)$$

$$\frac{T(L,t)-T_\infty}{T(0,t)-T_\infty} = 0.91 \quad => \quad T_\infty = \frac{(T(L,t)-0.91T(0,t))}{1.0-0.92} = \frac{(154^{\circ}\mathrm{C}-0.91*121^{\circ}\mathrm{C})}{0.09} = 500°\mathrm{C}$$

The actual oven temperature must be less than this so the center temperature can remain above 121°C without the surface temperature exceeding 154°C. The oven temperature and cooking time must be found by iterating the steps below

1. Pick an oven temperature.
2. Use Figure approximate solution to find the Fourier number which determines the time required for the center temperature to reach 121°C.
3. Add 20 min to the time and calculate a new Fourier number.
4. Use the new Fourier number and approximate solution to find the center temperature at the end of the cooking period.
5. Use $(T(r_o, t) - T_\infty)/(T(0, 2t) - T_\infty) = 0.91$ to find the surface temperature at the end of the cooking period.

1. For the first iteration, let the oven temperature (T_∞) = 300°C.

2. $$\frac{T(0,t)-T_\infty}{T_o-T_\infty} = \frac{121^{\circ}\mathrm{C}\text{-}300^{0}C}{27^{\circ}\mathrm{C}\text{-}300^{0}C} = 0.656 = 1.03824 * e^{-(0.4862)^2 * \tau}$$

=> τ = *1.95*

From Figure 3.9 (a) $$Fo = \frac{\alpha t}{r_o^2} = \frac{k\,t}{\rho c L^2} = 1.95$$

Solving for the time for the center to reach 121°C:

$$t = \frac{Fo\,\rho c L^2}{k} = \frac{1.95(4187\,\mathrm{kg/m^3})(1280\,\mathrm{J/(kg\ K)})(0.0125\,\mathrm{m})^2}{0.48\,\mathrm{W/(m^2\ K)}} = 3900 \text{ sec}$$

3. After 20 min (1200s) cooking time: t =5100, F_o = 2.4.

4. From approximate solutions for τ = 2.4, Bi=0.26

$$\frac{T(0,t)-T_\infty}{T_o-T_\infty} = 1.03824 * e^{-(0.4862)^2 * 2.4} = 0.58$$

$$T(0, t) = T_\infty + 0.58\,(T_o - T_\infty) = 300°\mathrm{C} + 0.58\,(27°\mathrm{C} - 300°\mathrm{C}) = 142°\mathrm{C}$$

5. $$\frac{T(L,t)-T_\infty}{T(0,t)-T_\infty} = 1.03824 * e^{-(0.4862)^2 * 2.4} \cos(0.4862) = 0.52$$

$$T(L, t) = T_\infty + 0.9\,(T_0 - T_\infty) = 300°\mathrm{C} + 0.52\,(27°\mathrm{C} - 300°\mathrm{C}) = 158°\mathrm{C}$$

Therefore, an oven temperature of 300°C is too high. The following iterations were performed using the same procedure

Oven Temperature	Fo	Time to Reach 121°C	Fo for 20 min	$\dfrac{T(0,t)-T_\infty}{T_\infty - T_\infty}$	T_o (°C)	T_L (°C)
300°C	1.7	3400 s	2.4	0.55	142	158
200°C	3.2	5578 s	3.9	0.37	136	142
150°C	2.4	4182 s	3.1	0.48	143	156

Therefore, the oven temperature should be set at 250°C and the meat should be heated for a total of 5100 s + 1200 s = 6300 s = 105 mins.

PROBLEM 3.24

A frozen-food company freezes its spinach by first compressing it into large slabs and then exposing the slab of spinach to a low-temperature cooling medium. The large slab of compressed spinach is initially at a uniform temperature of 21°C; it must be reduced to an average temperature over the entire slab of –34°C. The temperature at any part of the slab, however, must never drop below –51°C. The cooling medium which passes across both sides of the slab is at a constant temperature of –90°C. The following data may be used for the spinach: density = 80 kg/m^3; thermal conductivity = 0.87 W/(m K); specific heat = 2100 J/(kg K). Present a detailed analysis outlining a method estimate the maximum thickness of the slab of spinach that can be safely cooled in 60 min.

GIVEN

- Large slabs of spinach are exposed to a low-temperature cooling medium
- Initial uniform temperature (T_o) = 21°C
- Average temperature must be reduced to –34°C
- The temperature at any part must never drop below –51°C
- Cooling medium temperature (T_∞) = –90°C
- Density of spinach (ρ) = 80 kg/m^3
- Thermal conductivity (k) = 0.87 W/(m K)
- Specific heat (c) = 2100 J/(kg K)

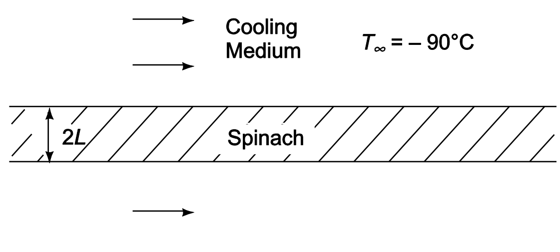
FIND

- Present a detailed analysis outlining a method to estimate the maximum thickness of the slab of spinach that can be safely cooled in 60 min

ASSUMPTIONS

- One dimensional conduction through the slab
- Constant and uniform thermal properties
- The average temperature within the slab is equal to the average of the center and surface temperatures

SKETCH



SOLUTION

The average temperature for a slab with constant thermal conductivity is arithmetic average of center and end temperature. Thus, for a final average temperature in the slab of –34°C, and a final surface temperature of –51°C, the final center temperature must be

$$T(0, t) = 2\,T_{\text{Ave}} - T(L, t) = 2(-34°\text{C}) + 51°\text{C} = -17°\text{C}$$

Figure 3.9(b) can be used to find the Biot number for the spinach slab

$$\frac{T(L,t)-T_\infty}{T(0,t)-T_\infty} = \frac{-51^{\circ}\mathrm{C}-(-90^{\circ}\mathrm{C})}{-17^{\circ}\mathrm{C}-(-90^{\circ}\mathrm{C})} = 0.53$$

From Figure (3.9b) $1/Bi = 0.6$.

Figure (3.9a) can be used to find the Fourier number

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty} = \frac{-17^{\circ}\mathrm{C}-(-90^{\circ}\mathrm{C})}{-21^{\circ}\mathrm{C}-(-90^{\circ}\mathrm{C})} = 0.66$$

From Figure (3.9a) using above information, $F_o = 0.5$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k\,t}{\rho c L^2}$$

Solving for L

$$L = \left(\frac{k\,t}{Fo\,\rho c}\right)^{0.5} = \left(\frac{[0.87\,\mathrm{W/(m\,K)}](\mathrm{J/(W\,s)})\,(60\,\mathrm{min})(60\,\mathrm{s/min})}{0.5\left(80\,\mathrm{kg/m^3}\right)(2100\,\mathrm{J/(kg\,K)})}\right)^{0.5} = 0.19\,\mathrm{m}$$

The thickness of the slab of spinach that can be cooled in 60 minutes is $2L = 0.38$ m = 38 cm.

The heat transfer coefficient needed to achieve this cooling can be calculated from the Biot number

$$Bi = \frac{\overline{h_c}\,L}{k} \Rightarrow \overline{h_c} = Bi\,\frac{k}{L} = \frac{1}{0.6}\,\frac{0.87\,\mathrm{W/(m\,K)}}{0.19\,\mathrm{m}} = 7.6\ \mathrm{W/(m^2K)}$$

ALTERNATIVE METHOD

The approximate solution for slab of infinite length is given by.

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty} = C_1 e^{-\delta_1^2\tau}\cos(\delta_1 x/L)$$

For x=L we have

$$\frac{-51+90}{-17+90} = C_1 e^{-\delta_1^2\tau}\cos(\delta_1 * L/L)$$

$0.53 = C_1 e^{-\delta_1^2\tau}\cos(\delta_1)$ ………………………………………………… (i)

For x=0 at same time we have

$$\frac{-17+90}{-17+90} = C_1 e^{-\delta_1^2\tau}\cos(\delta_1 * 0)$$

$1 = C_1 e^{-\delta_1^2\tau}$ …………………………………………………………………….(ii)

Dividing equn (i) by (ii) we have

$\cos(\delta_1) = 0.53$ => $\delta_1 = 1.007$

From table 3.1

Equivalent to $\delta_1 = 1.007$ Bi= 1.67

For infinite slab with Bi=1.67 we have

$\delta_1 = 1.007$ & $C_1 = 1.1587$

From equation (3.43a) for x= 0, we have

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty} = 1.1587 * e^{-(1.007)^2 * \tau} \cos(0)$$

$$\frac{T(0,t)-T_\infty}{T_\infty - T_\infty} = \frac{-17^\circ\text{C} - (-90^\circ\text{C})}{-21^\circ\text{C} - (-90^\circ\text{C})} = 0.66 = 1.1587 * e^{-(1.007)^2 * \tau}$$

$$\tau = \frac{\ln(0.5696)}{-1.007^2} = 0.55$$

$$\tau = Fo = \frac{\alpha t}{r_o^2} = \frac{k\,t}{\rho c L^2} = 0.55$$

Solving for L

$$L = \left(\frac{k\,t}{Fo\,\rho c}\right)^{0.5} = \left(\frac{[0.87\,\text{W/(m K)}](\text{J/(W s)})(60\text{ min})(60\text{ s/min})}{0.55\left(80\text{ kg/m}^3\right)(2100\text{ J/(kg K)})}\right)^{0.5} = 0.18\text{ m}$$

The thickness of the slab of spinach that can be cooled in 60 minutes is $2L$ = 0.36 m = 36 cm.
The heat transfer coefficient needed to achieve this cooling can be calculated from the Biot number

$$Bi = \frac{\overline{h_c}\,L}{k} \Rightarrow \overline{h_c} = Bi\,\frac{k}{L} = \frac{1}{0.6}\,\frac{0.87\,\text{W/(m K)}}{0.18\text{ m}} = 8.2\ \text{W/(m}^2\text{K)}$$

COMMENTS

The heat transfer coefficients is on the low side of the range for free convection in air (see Table 1.2).

As the value for 1/Bi and F_0 obtained from graph are not accurate due to lack of accurate reading, thus values obtained from approximate solutions are reliable.

Note that if the heat transfer coefficient is greater than 8.2 W/(m^2 K), the surface temperature of the spinach will drop below –51°C before the average temperature is lowered to –34°C.

PROBLEM 3.25

In a curing process for plastic and other polymer-based materials, a plastic sheet, which initially is 25°C, is placed on a hot metal surface that is maintained at 250°C. If the back (or opposite) side of the 5-mm-thick plastic film is considered to be insulated, how long does it take for the back side to reach temperature of 90°C. Assume the following plastic properties: *c* = 1450 J/(kg K), *k*= 0.23 W/(m K), and ρ= 1975 kg/m³.

GIVEN

- Initial temperature of plastic sheet (T_0)= 25^0C
- Sheet thickness (2*L*) = 5 mm = 0.005 m
- Opposite side or back is insulated
- Metal surface temperature (T_∞)= 250^0C
- The thermal conductivity of the plastic (*k*) = 0.23 W/(m K)
- Density(ρ) = 1975 kg/m^3
- Specific heat capacity (c)= 1450 J/(kg K)

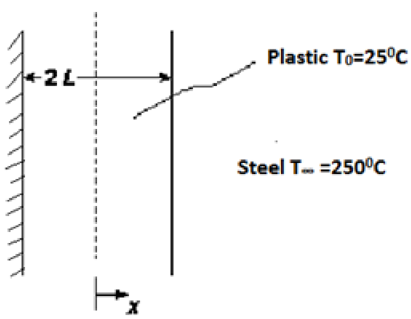
FIND

Time taken for backside to reach temperature of 90^0C

ASSUMPTIONS

- The initial temperature of the sheet is uniform
- The temperature of the steel plates is constant
- The thermal conductivity of the sheet is constant

SKETCH



SOLUTION

For one-dimensional time dependent equation in conduction we have

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

The solution for such infinite plate conduction problem is given by equation (3.43a)

$$\theta = \frac{T(x,t) - T_\infty}{T_0 - T_\infty} = C_1 e^{-\delta_1^2 \tau} \cos(\delta_1 x / L)$$

Where $C_1 = \dfrac{2Sin\delta_1}{\delta_1 + \sin\delta_1 \cos\delta_1}$

And $\delta_1 Tan\delta_1 = Bi = \dfrac{hL}{k}$

we have for insulated plate L_c=0.005 m

Bi=$hL_c/k=\infty$ for conduction without resistance

From Table (3.1)

For Bi=∞ we have $\delta_1 = 1.5708$ and C_1=1.2732

$\theta = \dfrac{T(x,t) - T_\infty}{T_0 - T_\infty} = C_1 e^{-\delta_1^2 \tau} \cos(\delta_1 x / L)$ were x=0 at insulated back. Thus

$\theta = \dfrac{90-250}{25-250} = 1.2732 * e^{-1.5708^2 * \tau}$

$\dfrac{0.711}{1.2732} = e^{-1.5708^2 * \tau}$

-2.467*τ= ln(0.6055)

-2.467*τ= -0.5016

τ =0.2033

We have $\tau = \dfrac{t\alpha}{L^2}$ where α=k/ρc= 0.23/(1975*1450)=$8.03*10^{-8}$

$0.2033 = \dfrac{t * 8.03 * 10^{-8}}{0.005^2}$

t=$0.2033*0.005^2/(8.03*10^{-8})$ seconds

t= 63.29 seconds.

PROBLEM 2.26

A stainless steel cylindrical billet [k = 14.4 W/(m K), $\alpha = 3.9 \times 10^{-6}$ m^2/s] is heated to 593°C preparatory to a forming process. If the minimum temperature permissible for forming is 482°C, how long can the billet be exposed to air at 38°C if the average heat transfer coefficient is 85 W/(m^2 K)? The shape of the billet is shown in the sketch.

GIVEN

- A stainless steel cylindrical billet exposed to air
- Thermal conductivity (k) = 14.4 W/(m K)
- Thermal diffusivity (α) = 3.9×10^{-6} m^2/s
- Initial temperature (T_o) = 593°C
- The minimum temperature permissible for forming is 482°C
- Air temperature (T_∞) = 38°C
- Average heat transfer coefficient $(\bar{h}_c)$ = 85 W/(m^2 K)

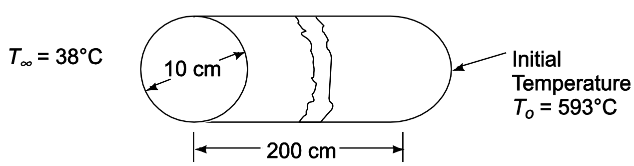
FIND

- How long may the billet be exposed to the air?

ASSUMPTIONS

- End effects are negligible
- Constant heat transfer coefficient
- Conduction in the radial direction only
- Uniform thermal properties

SKETCH



SOLUTION

The Biot number is calculated to determine if internal resistance is significant

$$Bi = \frac{\bar{h}_c\, r_o}{k} = \frac{[85\ \text{W/(m}^2\text{K)}](0.05\ \text{m})}{14.4\ \text{W/(m K)}} = 0.3 > 0.1$$

Therefore, internal resistance is important, and approximate solution is used.

The approximate solution is used for the problem. The approach will be as follows:

1. Apply $(T_{r,t} - T_\infty)/(T_o - T_\infty)$ and the Biot number to find the Fourier number using Equation (3.44a) and Table 3.1.
2. Use the Fourier number to find the time it takes for the surface to cool to the given minimum surface temperature.

For infinite cylinder with Bi=0.3 we have

$$\delta_1 = 0.7465 \ \ \& \ \ C_1 = 1.0712$$

From equation (3.44a) for r= r_0, we have

$$\frac{T(r,t)-T_\infty}{T_0-T_\infty}=C_1 * e^{-(\delta_1)^2 * \tau} J_0(\delta_1 r / r_0)$$

$$\frac{T(r_0,t)-T_\infty}{T_0-T_\infty}=C_1 * e^{-(\delta_1)^2 * \tau} J_0(\delta_1 r_0 / r_0)$$

The surface temperature must not fall below 482°C

$$\frac{T(r_o,t)-T_\infty}{T_o-T_\infty} = \frac{482°C - 38°C}{593°C - 38°C} = 0.80$$

$$0.80 = 1.0712 * e^{-(0.7465)^2 * \tau} J_0(0.7465)$$

$$0.80 = 1.0712 * e^{-0.5572 * \tau} * 0.8649$$

$$\ln(0.8635) = -0.5572 * \tau$$

$$\tau = 0.26$$

Solving for the time

$$t = \frac{\tau r_o^2}{\alpha} = \frac{0.26(0.05\text{ m})^2}{3.9\times10^{-6}(\text{m}^2/\text{s})} = 166\text{ s} = 2.8\text{ min}$$

PROBLEM 3.27

A long copper cylinder 0.6-m in diameter and initially at a uniform temperature of 38°C is placed in a water bath at 93°C. Assuming that the heat transfer coefficient between the copper and the water is 1248 W/(m^2 K), calculate the time required to heat the center of the cylinder to 66°C. As a first approximation, neglect the temperature gradient within the cylinder, then repeat your calculation without this simplifying assumption and compare your results.

GIVEN

- A long copper cylinder is placed in a water bath
- Diameter of cylinder (D) = 0.6 m
- Initial temperature (T_o) = 38°C
- Water bath temperature (T_∞) = 93°C
- The heat transfer coefficient $(\bar{h}_c)$ = 1248 W/(m^2 K)

FIND

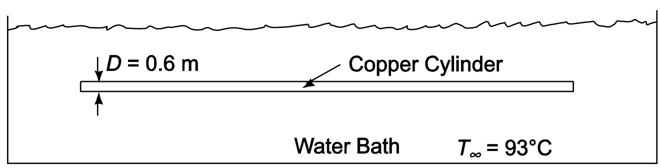
Calculate the time required to heat the center of the cylinder to 66°C assuming

(a) Negligible temperature gradient within the cylinder
(b) Without this simplification, then
(c) Compare your results

ASSUMPTIONS

- Neglect end effects
- Radial conduction only

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For copper

Thermal conductivity (k) = 396 W/(m K) at 63°C
Density (ρ) = 8933 kg/m^3
Specific heat (c) = 383 J/(kg K)
Thermal diffusivity (α) = 1.166 × 10^{-4} m^2/s

SOLUTION

(a) For a negligible temperature gradient within the cylinder, the temperature-time history is given by Equation (3.3)

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{\bar{h}_c A_s}{c\rho V}t} = e^{-\frac{\bar{h}_c \pi D L}{c\rho \frac{\pi}{4} D^2 L}t} = e^{-\frac{4\bar{h}_c}{c\rho D}t}$$

Solving for the time

$$t = -\frac{c\rho D}{4h_c} \ln \left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t = \frac{[383\,\text{J/(kg K)}]\ \text{W s/J}\quad 8933(\text{kg/m}^3)\ (0.6\,\text{m})}{4\ 1248\,\text{W/(m}^2\ \text{K)}} \ln \left(\frac{66°\text{C} - 93°\text{C}}{38°\text{C} - 93°\text{C}}\right)$$

$t = 293$ sec = 4.9 min

(b) The approximate solution method can be used to take the temperature gradient within the cylinder into account.

$$Bi = \frac{\bar{h}_c r_o}{k} = \frac{[1248\,\text{W/(m}^2\text{K)}](0.3\text{m})}{396\,\text{W/(m K)}} = 0.95 \Rightarrow \frac{1}{Bi} = 1.1$$

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \left(\frac{66°\text{C} - 93°\text{C}}{38°\text{C} - 93°\text{C}}\right) = 0.49$$

From equation (3.44a) for infinite cylinder, we have

$$\frac{T(r,t) - T_\infty}{T_0 - T_\infty} = C_1 * e^{-(\delta_1)^2 * \tau} J_0(\delta_1 r / r_0)$$

For Bi=0.95 for Infinite cylinder, From Table (3.1) we have

$\delta_1 = 1.2303$ and $C_1 = 1.19865$

Thus for r_0=0 we have,

$$\frac{T(0,t) - T_\infty}{T_0 - T_\infty} = 1.19865 * e^{-(1.2303)^2 * \tau} J_0(0)$$

$$0.49 = 1.19865 * e^{-(1.2303)^2 * \tau} * 1$$

$$e^{-(1.2303)^2 * \tau} = 0.4088$$

$$-(1.2303)^2 * \tau = \ln(0.4088)$$

$$\tau = 0.59$$

$$Fo = \frac{\alpha t}{r_o^2} = 0.59$$

Solving for the time

$$t = \frac{Fo\, r_o^2}{\alpha} = \frac{0.59(0.3\text{m})^2}{1.166 \times 10^{-4}\,\text{m}^2 / \text{s}} = 455\ \text{s} = 7.6\ \text{min}$$

(c) The lumped capacity method (a) underestimates the required time by 35%.

COMMENTS

Since the Biot number is of the order of magnitude of unity, we could not expect that the lumped capacity assumption is valid.

PROBLEM 3.28

A long wooden rod at 38°C with a 2.5 cm OD is placed into an airstream at 600°C. The heat transfer coefficient between the rod and air is 28.4 W/(m^2 K). If the ignition temperature of the wood is 427°C, ρ = 800 kg/m^3, k = 0.173 W/(m K), and c = 2500 J/(kg K), determine the time between initial exposure and ignition of the wood.

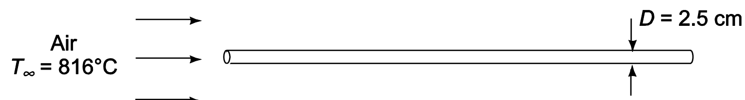
GIVEN

- A long wooden rod is placed into an airstream
- Rod outside diameter (D) = 2.5 cm = 0.025 m
- Initial temperature of the rod (T_o) = 38°C
- Temperature of the airstream (T_∞) = 816°C
- The heat transfer coefficient (h_c) = 28.4 W/(m^2 K)
- The ignition temperature of the wood (T_I) = 427°C
- Density of the rod (ρ) = 800 kg/m^3
- Thermal conductivity (k) = 0.173 W/(m K)
- Specific heat (c) = 2500 J/(kg K)

FIND

- The time between initial exposure and ignition of the wood

SKETCH



SOLUTION

The Biot number for the rod is

$$Bi = \frac{\overline{h_c}\, r_o}{2k} = \frac{[28.4\ \text{W/(m}^2\ \text{K)}]\left(\dfrac{0.025}{2}\ \text{m}\right)}{2*0.173\ \text{W/(m K)}} = 1.025 > 0.1$$

Therefore, the internal thermal resistance of the rod is significant and the approximate solutions will be used.

For approximate solution in infinite cylinder for $Bi = \dfrac{\overline{h_c}\, r_o}{k} = \dfrac{[28.4\ \text{W/(m}^2\ \text{K)}]\left(\dfrac{0.025}{2}\ \text{m}\right)}{0.173\ \text{W/(m K)}} = 2.05 > 0.1$ from Table (3.1) we have

$$\frac{T(r,t)-T_\infty}{T_0-T_\infty} = C_1 * e^{-(\delta_1)^2 * \tau} J_0(\delta_1 r / r_0)$$

For Bi=0.95 for Infinite cylinder, From Table (3.1) we have

$\delta_1 = 1.60896$ and $C_1 = 1.342435$

For $r = r_0$

$$\frac{T(r_0,t)-T_\infty}{T_0-T_\infty} = C_1 * e^{-(\delta_1)^2*\tau} J_0(\delta_1 r_0 / r_0)$$

$$\frac{427-600}{38-600} = 1.3424 * e^{-(1.609)^2*\tau} J_0(1.609)$$

$$0.3078 = 1.3424 * e^{-(1.609)^2*\tau} * 0.4502$$

$$e^{-(1.609)^2*\tau} = 0.5093$$

$$-(1.609)^2 * \tau = \ln(0.5093)$$

$$\tau = 0.26$$

$$Fo = \frac{\alpha t}{r_o^2} = 0.26$$

Solving for the time

$$t = \frac{Fo\, r_o^2}{\alpha} = \frac{Fo \rho c r_o^2}{k} = \frac{0.26\left(800 \text{ kg/m}^3\right)\left(2500 \text{ J/(kg K)}\right)\left(\frac{0.025}{2}\text{ m}\right)^2}{0.173 \text{ W/(m}^2\text{ K)}} = 469 \text{ sec} = 7.82 \text{ min}$$

PROBLEM 3.29

A mild-steel cylindrical billet, 25-cm in diameter, is to be raised to a minimum temperature of 760°C by passing it through a 6-m-long strip type furnace. If the furnace gases are at 1538°C and the overall heat transfer coefficient on the outside of the billet is 68 W/(m^2 K), determine the maximum speed at which a continuous billet entering at 204°C can travel through the furnace.

GIVEN

- A mild-steel cylindrical billet is passed through a furnace
- Diameter of billet = 25 cm = 0.25 m
- Billet is to be raised to a minimum temperature of 760°C
- Length of furnace = 6 m
- Temperature of furnace gases (T_∞) = 1538°C
- The overall heat transfer coefficient $\bar{h}_c$ = 68 W/(m^2 K)
- Initial temperature of billet (T_o) = 204°C

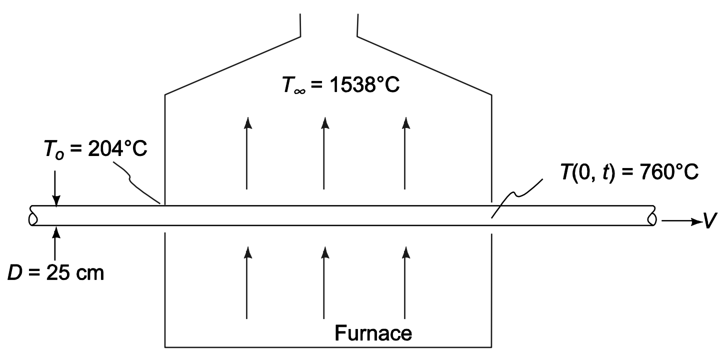
FIND

- The maximum speed at which a continuous billet can travel through the furnace

ASSUMPTIONS

- The heat transfer coefficient is constant
- Billet is 1% carbon steel
- Radial conduction only in the billet, neglect axial conduction

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10

For 1% carbon steel Thermal conductivity (k) = 43 W/(m K)

Thermal diffusivity (α) = 1.172 × 10^{-5} m^2/s

SOLUTION

The Biot number for the billet is

$$Bi = \frac{\bar{h}r_o}{K} = \frac{[68\ \text{W/(m}^2\ \text{K)}]\ 0.125\,\text{m}}{43\ \text{W/(m K)}} = 0.198 > 0.1$$

Therefore, internal resistance is significant and we cannot use the lumped parameter method, approximate solution must be used.

The billet must obtain a centerline temperature of 760°C, therefore

From Equation (3.44a) for r=0 we have

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=C_1*e^{-(\delta_1)^2*\tau}J_0(\delta_1*0/r_0)$$

$$\frac{760-1538}{204-1538}=C_1*e^{-(\delta_1)^2*\tau}J_0(0)$$

For Bi=0.198 for Infinite cylinder, From Table (3.1) we have

$\delta_1=0.6170$ and $C_1=1.048$

From Appendix 2 Table 43 $J_0(0)$=1

$$0.583=1.048*e^{-(0.617)^2*\tau}*1$$

$$-(0.617)^2*\tau=\ln(0.5563)$$

$$\tau=1.54$$

$$Fo = \frac{\alpha t}{r_o^{\,2}} = 1.54$$

Solving for the time

$$t = \frac{For_o^{\,2}}{\alpha} = \frac{1.54\,(0.125\,\mathrm{m})^2}{1.172\times10^{-5}\,\mathrm{m^2\ /\ s}} = 2052\ \mathrm{s}$$

The maximum speed of the billet is

$$V = \frac{\text{Length of furnace}}{\text{time needed}} = \frac{6\,\mathrm{m}}{2052\,\mathrm{s}} = 0.003\ \mathrm{m/s}$$

PROBLEM 3.30

A solid lead cylinder 0.6-m in diameter and 0.6-m long, initially at a uniform temperature of 121°C, is dropped into a 21°C liquid bath in which the heat transfer coefficient $\bar{h}_c$ is 1135 W/(m^2 K). Plot the temperature-time history of the center of this cylinder and compare it with the time histories of a 0.6 m-diameter, infinitely long lead cylinder and a lead slab 0.6 m-thick.

GIVEN

- A solid lead cylinder dropped into a liquid bath
- Cylinder diameter (D) = 0.6 m
- Cylinder (L) = 0.6 m
- Initial uniform temperature (T_o) = 121°C
- Liquid bath temperature (T_∞) = 21°C
- Heat transfer coefficient $\bar{h}_c$ = 1135 W/(m^2 K)

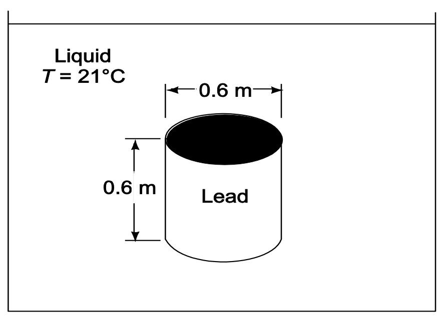
FIND

(a) Plot the temperature-time history of the cylinder center
(b) Compare it with the time history of a 0.6 m diameter, infinitely long lead cylinder
(c) Compare it with the time history of a lead slab 0.6 m thick

ASSUMPTIONS

- Two dimensional conduction within the cylinder
- Constant and uniform properties
- Constant liquid bath temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For lead Thermal conductivity (k) = 34.7 W/(m K) at 63°C
Density (ρ) = 11340 kg/m^3
Specific heat (c) = 129 J/(kg K)
Thermal diffusivity (α) = 24.1 × 10^{-6} m^2/s

SOLUTION

The Biot number based on radius is

$$Bi = \frac{\bar{h}_c r_o}{K} = \frac{[1135\text{W/(m}^2\text{ K)}]\ 0.3\text{m}}{34.7\text{W/(m K)}} = 9.81 > 0.1$$

Therefore, internal resistance is significant.

(a) This two-dimensional system required a product solution. From Table 3.3 the product solution is

$$\frac{\theta_p(x,r)}{\theta_o} = P(x)\,C(r)$$

Where $P(x) = \frac{\theta(x,t)}{\theta_o}$ for an infinite plate

$$C(r) = \frac{\theta(r,t)}{\theta_o} \text{ for a long cylinder}$$

Since the length of the cylinder is the same as its diameter, the Biot number based on length is the same as that based on radius

$$1/Bi = 1/9.81 = 0.102$$

The Fourier number is $Fo = \frac{\alpha t}{L/2 \text{ or } r_o)^2} = \frac{24.1\times10^{-6}\,\text{m}^2/\text{s}\;t}{0.3\text{m}^{\;2}} = 0.000268\,t\,\text{s}^{-1}$

The temperature of the center of the cylinder ($x = 0$, $r = 0$) is determined by calculating the Fourier number for that time, finding $P(0)$ on Figure 3.10, finding $C(0)$ on Figure 3.10, and applying

$$\frac{\theta_p(0,0)}{\theta_o} = \frac{T(0,0)-T_\infty}{T_o - T_\infty} = P(x)\,C(r)$$

$$T(0,0) = T_\infty + P(x)\,C(r)\,(T_o - T_\infty)$$

(b) The center temperature for a long cylinder is

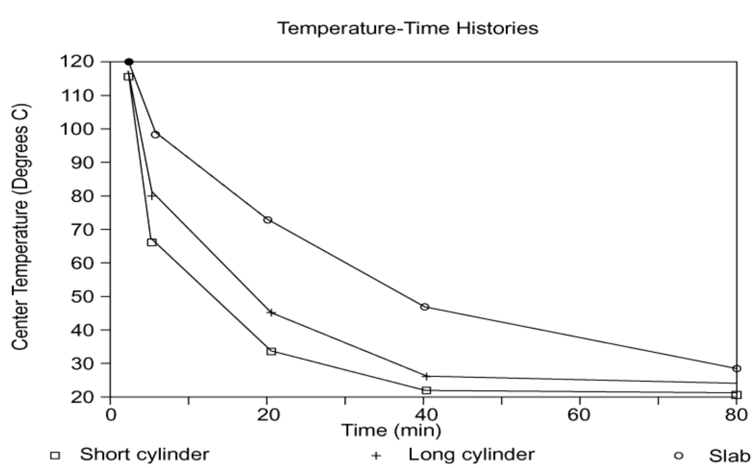
$$T(r = o, t) = T_\infty + C(o)\,(T_o - T_\infty)$$

(c) The center temperature for a slab is

$$T(x = o, t) = T_\infty + P(o)\,(T_o - T_\infty)$$

The temperature-time histories of these three cases are tabulated and plotted below

					$T(0, 0)$ (°C)		
Time(s)	(min)	Fo	$P(0)$	$C(0)$	(a) Short Cylinder	(b) Long Cylinder	(c) Slab
120	2	0.03	0.99	0.95	115	116	120
300	5	0.08	0.78	0.60	68	81	99
1200	20	0.32	0.52	0.24	33	45	73
4800	80	1.28	.075	.033	21	14	29



PROBLEM 3.31

A long 0.6-m-*OD* 347 stainless steel (*k* = 14 W/(m K)) cylindrical billet at 16°C room temperature is placed in an oven where the temperature is 260°C. If the average heat transfer coefficient is 170 W/(m² K), (a) estimate the time required for the center temperature to increase to 232°C by using the appropriate chart and (b) determine the instantaneous surface heat flux when the center temperature is 232°C.

GIVEN

- A long cylindrical billet placed in an oven
- Billet outside diameter = 0.6 m
- Thermal conductivity (k) = 14 W/(m K)
- Initial temperature (T_i) = 16°C
- Oven temperature (T_∞) = 260°C
- The average heat transfer coefficient $\bar{h}_c$ = 170 W/(m² K)
- Center temperature increases to 232°C

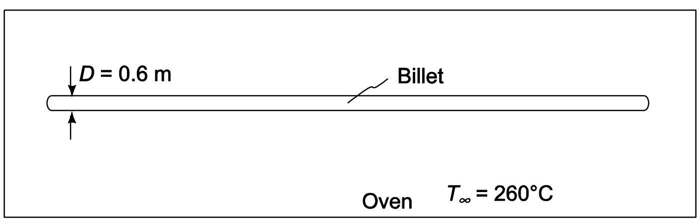
FIND

(a) The time required using the appropriate chart
(b) The instantaneous surface heat fluxes when the center temperature is 232°C

ASSUMPTIONS

- Radial conduction only in billet
- Uniform and constant properties

SKETCH



SOLUTION

(a) The Biot number for the billet is

$$Bi = \frac{\bar{h}_c r_o}{K} = \frac{[170\,\text{W/(m}^2\ \text{K)}]\ 0.3\,\text{m}}{14\,\text{W/(m K)}} = 3.643 > 0.1$$

$$\frac{T(0,t_f) - T_\infty}{T_o - T_\infty} = \frac{232°\text{C} - 260°\text{C}}{16°\text{C} - 260°\text{C}} = 0.115$$

From Equation (3.44a) for r=0 we have

$$\frac{T(0,t) - T_\infty}{T_0 - T_\infty} = C_1 * e^{-(\delta_1)^2 * \tau} J_0(\delta_1 * 0 / r_0)$$

For Bi=3.643 for Infinite cylinder, From Table (3.1) we have

$$\delta_1 = 1.865 \text{ and } C_1 = 1.4517$$

$$0.115 = 1.4517 * e^{-(1.865)^2 * \tau} J_0(0)$$

From Appendix 2 Table 43 $J_0(0) = 1$

$$0.115 = 1.4517 * e^{-(1.865)^2 * \tau}$$

$$-(1.865)^2 * \tau = \ln(0.0792)$$

$$\tau = 0.73$$

$$Fo = \frac{\alpha t}{r_o^2} = 0.73$$

Solving for the time

$$t = \frac{Fo r_o^2}{\alpha} = \frac{0.73(0.3\,\text{m})^2}{0.387 \times 10^{-5}\,\text{m}^2/\text{s}} = 16976 \text{ s} = 283 \text{ min} = 4.7 \text{ hr}$$

(b) The surface temperature is needed to find the surface heat flux. For $r = r_o$, from approximate solution using Equation (3.44a) we have

From Equation (3.44a) for $r=r_0$ we have

$$\frac{T(r_0,t) - T_\infty}{T_0 - T_\infty} = C_1 * e^{-(\delta_1)^2 * \tau} J_0(\delta_1 * r_0 / r_0)$$

$$\frac{T(r_0,t) - T_\infty}{T_0 - T_\infty} = 1.4517 * e^{-(1.865)^2 * 0.73} J_0(1.865)$$

From Appendix 2, Table 43 $J_0(1.865) = 0.30217$

$$\frac{T(r_0,t) - T_\infty}{T_0 - T_\infty} = 1.4517 * e^{-(1.865)^2 * 0.73} * 0.30217$$

$$\frac{T(r_o,t) - T_\infty}{T_0 - T_\infty} = 0.035$$

$$T(r_o, t) = T_\infty + 0.035\,(T(0, t) - T_\infty) = 260°\text{C} + 0.035\,(16°\text{C} - 260°\text{C}) = 251.5°\text{C}$$

The instantaneous surface flux is

$$\frac{q}{A} = \bar{h}\,[T_\infty - T(r_o, t)] = 170\ \text{W/(m}^2\ \text{K)}\ (251°\text{C} - 260°\text{C}) = 1428\ \text{W/m}^2$$

PROBLEM 3.32

Repeat Problem 3.31(a), but assume that the billet is only 1.2-m-long and the average heat transfer coefficient at both ends is 136 W/(m^2 K).

GIVEN

- A cylindrical billet placed in an over
- Billet outside diameter = 0.6 m
- Thermal conductivity (k) = 14 W/(m K)
- Initial temperature (T_o) = 16°C
- Oven temperature (T_∞) = 260°C
- The average heat transfer coefficient $\bar{h}_{cs}$ = 170 W/(m^2 K)
- Increase of the center temperature is 232°C
- Billet length (2L) = 1.2 m
- Heat transfer coefficient at the ends $\bar{h}_{ce}$ = 136 W/(m^2 K)

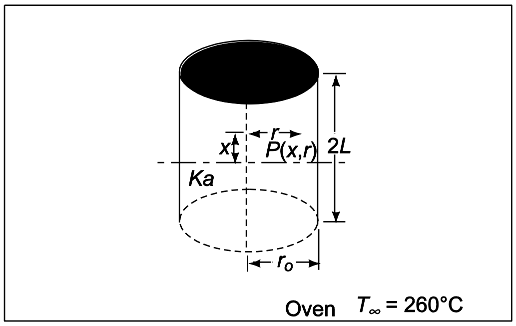
FIND

- The time required using the appropriate charts

ASSUMPTIONS

- Two dimensional conduction within the billet
- Constant and uniform thermal properties
- Constant oven temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10 For Type 304 stainless steel

Thermal diffusivity (α) = 0.387 × 10^{-5} m^2/s

SOLUTION

From Table 2.4, the solution for this geometry is

$$\frac{\theta_p \; x, r}{\theta_o} = P(x)\, C(r)$$

where

$$P(x) = \frac{\theta(x,t)}{\theta_o} \text{ for an infinite plate (Figure 3.9)}$$

$$C(r) = \frac{\theta(r,t)}{\theta_o} \text{ for a long cylinder (Figure 3.10)}$$

$$\frac{\theta_p(0,0)}{\theta_o} = \frac{T(0,0)-T_\infty}{T_o-T_\infty} = \frac{232°\text{C} - 260°\text{C}}{16°\text{C} - 260°\text{C}} = 0.11 = P(0)\,C(0)$$

For the infinite plate solution

$$(Bi)_x = \frac{\bar{h}_{ce}L}{k} = \frac{[136\ \text{W/(m}^2\ \text{K)}]\ (0.6\,\text{m})}{14\ \text{W/(m K)}} = 5.83 \Rightarrow \frac{1}{Bi} = 0.17$$

$$Fo = \frac{\alpha t}{L^2} = \frac{0.387\times10^{-5}\,\text{m}^2/\text{s}}{(0.6\,\text{m})^2}\,t = 1.075 \times 10^{-5}\,t\ \text{s}^{-1}$$

For the long cylinder solution

$$(Bi)_r = \frac{\bar{h}_{cs}r_o}{k} = \frac{[170\ \text{W/(m}^2\ \text{K)}]\ (0.3\,\text{m})}{14\ \text{W/(m K)}} = 3.54 \Rightarrow \frac{1}{Bi} = 0.28$$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{0.387\times10^{-5}\,\text{m}^2/\text{s}}{(0.3\,\text{m})^2}\,t = 4.3 \times 10^{-5}\,t\ \text{s}^{-1}$$

The time required to reach a product solution of 0.115 is found through trial and error.

Time(s)	(min)	Fo_x	$P(0)$	Fo_r	$C(0)$	$P(0)C(0)$
6,000	100	0.065	0.99	0.26	0.37	0.0366
12,000	200	0.13	0.82	0.52	0.17	0.139
15,000	250	0.16	0.54	0.645	0.10	0.054
13,000	217	0.14	0.60	0.56	0.15	0.090
12,500	208	0.134	0.70	0.538	0.208	0.11

The time required is approximately 208 min or 3.4 hours.

COMMENTS

The uncertainty in the solution is high because of the difficulty reading Figure 3.9 at very low Fourier numbers. For higher accuracy, the differential equations that describe the problem would have to be solved.

PROBLEM 3.33

Consider a heat treatment process in which steel rods with a 10 cm diameter at an initial temperature of 600°C are inserted into an oil bath at 25°C. Assuming a convection coefficient of 400 W/(m^2 K) between the oil and the rod, estimate how long it takes for the centerline of the rod to cool to 60°C. If the steel rod, which is 1 m long, is withdrawn from the bath at this point in the process, i.e., when the centerline temperature has reached 60°C, estimate the rate at which heat must be extracted from the oil to maintain its temperature constant during the process of treating 25 rods per hour.

GIVEN

- Heat treatment of steel rod inserted in oil bath
- Rod outside diameter = 0.1 m
- Initial temperature (T_0) = 600°C
- Oil bath temperature (T_∞) = 25°C
- The average heat transfer coefficient $\bar{h}_c$ = 400 W/(m^2 K)

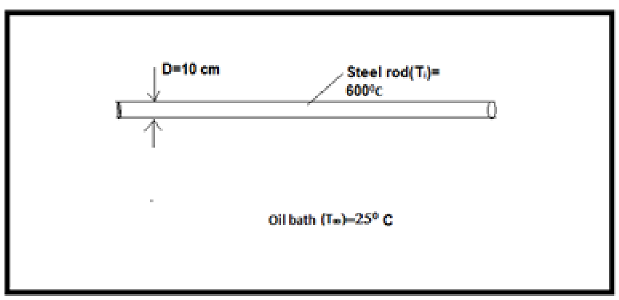
FIND

- Time it takes for centerline of the rod to cool to 60^{0}C.
- Rate at which heat must be extracted from oil to maintain its temperature constant during the process of treating 25 rods per hour.

ASSUMPTIONS

- Radial conduction only in billet
- Uniform and constant properties

SKETCH



PROPERTIES

From Appendix 2, Table 10, for steel

- Thermal conductivity (k) = 43 W/(m K)
- Specific heat of steel (c) = 473 J/(kg K)
- Density of steel (ρ) = 7840 kg/m^3

SOLUTION

(a) The Biot number for the steel rod based on r_0 is

$$B_i = \frac{h_c r_0}{k} = \frac{400 * 0.1}{2 * 43} = 0.466 > 0.1 \text{ thus lumped capacitance cannot be used.}$$

$\frac{T(0,t)-T_\infty}{T_o-T_\infty}$ = $\frac{60-25}{600-25}$ = 0.06086

Using approximate solutions, from Equation (3.44a) for r=0 we have

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=C_1 * e^{-(\delta_1)^2 * \tau} J_0(\delta_1 * 0 / r_0)$$

For Bi=0.466 for Infinite cylinder, From Table (3.1) we have

$\delta_1 = 0.915$ and $C_1 = 1.107$

$$0.06086 = 1.107 * e^{-(0.915)^2 * \tau} J_0(0)$$

From Appendix 2 Table 43 $J_0(0)$=1

$$e^{-(0.915)^2 * \tau} = 0.05498$$
$$-(0.915)^2 * \tau = \ln(0.05498)$$
$$\tau = 3.46$$

Fo = $\frac{\alpha t}{r_o{}^2}$ = 3.46

Solving for the time

$$t = \frac{Fo r_o{}^2}{\alpha} = \frac{2.7*(0.05)^2}{\frac{43}{473*7840}} = 747 \text{ s} = 12.45 \text{ mins}$$

The surface temperature is needed to find the surface heat flux. Thus using approximate solution

From Equation (3.44a) for r=r$_0$ we have

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=C_1 * e^{-(\delta_1)^2 * \tau} J_0(\delta_1 * r_0 / r_0)$$
$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=1.107 * e^{-(0.915)^2 * 3.46} J_0(0.915)$$

From Appendix 2 Table 43 $J_0(0.915)$=0.8012

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=1.107 * e^{-(0.915)^2 * 3.46} * 0.8012$$
$$\frac{T(r_o,t)-T_\infty}{T_0-T_\infty} = 0.04896$$

$T(r_o, t)$ = 25 + 0.04896 ($T_0 - T_\infty$) = 25°C + 0.04896 (600°C – 25°C) = 53.2°C

Equation (3.47) can be used to calculate the heat transferred from one ball during the cooling time:

$$\frac{Q}{Q_0}=1-\frac{2J_1(\delta_1)}{\delta_1}C_1 e^{-\delta_1^2 \tau}$$

$$\frac{Q}{Q_0}=1-\frac{2J_1(0.915)}{0.915}*1.107*e^{-(0.915)^2*3.46}$$

From Appendix 2 Table 43 $J_1(0.915)$ =0.41103

$$\frac{Q}{Q_0}=1-\frac{2*0.41103}{0.915}*1.107*e^{-(0.915)^2*3.46}$$ =1-0.055=0.945

$Q_i = \rho c \pi r_o^2 L(T_o - T_\infty)$= 7840*473*$\pi*0.05^2$ *1(600-25)= 15465 KJ

$Q(t)$ =0.945 Q_i = 0.945 (15465 J) = 14614 k J

The rate of heat that must be extracted to maintain oil temperature constant during treatment of 25 rods per hour is

q =(Balls/hr) (Energy/ball) = $\frac{[25(1/\mathrm{h})](14614\ k\mathrm{J})}{3600(\mathrm{s/h})}$ = 101.5 KW

PROBLEM 3.34

Ball bearings are to be hardened by quenching them in a water bath at a temperature of 37°C. You are asked to devise a continuous process in which the balls roll from a soaking oven at a uniform temperature of 870°C into the water, where they are carried away by a rubber conveyer belt. The rubber conveyor belt, however, is not satisfactory if the surface temperature of the balls leaving the water is above 90°C. If the surface coefficient of heat transfer between the balls and the water may be assumed to be equal to 590 W/(m^2 K), (a) find an approximate relation giving the minimum allowable cooling time in the water as a function of the ball radius for balls up to 1.0-cm in diameter, (b) calculate the cooling time, in seconds, required for a ball having a 2.5-cm-diameter, and (c) calculate the total amount of heat in watts which has to be removed from the water bath in order to maintain a uniform temperature if 100,000 balls of 2.5-cm-diameter are to be quenched per hour.

GIVEN

- Ball bearings quenched in a water bath
- Water bath temperature (T_∞) = 37°C
- Initial temperature of the balls (T_o) = 870°C
- Final surface temperature of the balls (T_f) = 90°C
- Heat transfer coefficie $(\bar{h}_c)$ = 590 W/(m^2 K)

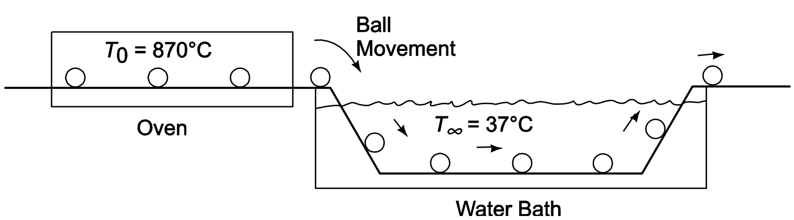
FIND

(a) An approximate relation giving the minimum allowable cooling time in the water as a function of the ball radius for balls upto 1.0 cm in diameter
(b) The cooling time, in seconds, required for a ball having a 2.5-cm-diameter
(c) The total amount of heat in watts which would have to be removed from the water bath in order to maintain its temperature uniform if 100,000 balls of 2.5 cm diameter are to be quenched per hour

ASSUMPTIONS

- The ball bearings are 1% carbon steel

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10
For 1% carbon steel
Thermal conductivity (k) = 43 W/(m K)
Density (ρ) = 7.801 kg/m^3
Specific heat (c) = 473 J/(kg K)
Thermal diffusivity (α) = 1.172 × 10^{-5} m^2/s

SOLUTION

(a) For 1.0-cm-diameter balls

$$Bi = \frac{\bar{h}_c\ r_o}{3k} = \frac{[590\,\text{W/(m}^2\ \text{K)}]\,(0.005\,\text{m})}{3*43\,\text{W/(m K)}} = 0.023 < 0.1$$

Therefore, a lumped capacity method can be used for balls less than 1 cm in diameter. The time temperature history of the ball is given by Equation 3.3

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{h A_s}{c\rho V}t} = e^{-\frac{h\,4\pi r_o^2}{c\rho\frac{4}{3}\pi r_o^3}t} = e^{-\frac{3h}{c\rho\, r_o}t}$$

Solving for the minimum cooling time

$$t = -\frac{c\rho\, r_o}{3\,h}\ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t = -\frac{[473\,\text{J/(kg K)}]\ \text{Ws/J}\quad 7801\,(\text{kg/m}^3)\ r_o}{3\ \ 590\,\text{W/(m}^2\ \text{K)}}\ln\left(\frac{90°\text{C} - 37°\text{C}}{870°\text{C} - 37°\text{C}}\right) = 5743\ r_o\ \text{s/m}$$

(b) For balls having a diameter of 2.5 cm

$$Bi = \frac{h_c r_0}{3k} = \frac{[590\,\text{W/(m}^2\ \text{K)}](0.0125\,\text{m})}{3*43\,\text{W/(m K)}} = 0.0573 < 0.1$$

The internal resistance is not significant.

Therefore, a lumped capacity method can be used for balls less than 1 cm in diameter. The time temperature history of the ball is given by Equation 3.3

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{h A_s}{c\rho V}t} = e^{-\frac{h\,4\pi r_o^2}{c\rho\frac{4}{3}\pi r_o^3}t} = e^{-\frac{3h}{c\rho\, r_o}t}$$

Solving for the minimum cooling time

$$t = -\frac{c\rho\, r_o}{3\,h}\ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t \qquad = -\frac{c\rho\, r_o}{3\,h}\ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$t = -\frac{[473\,\text{J/(kg K)}]\ \text{Ws/J}\quad 7801\,(\text{kg/m}^3)\ r_o}{3\ \ 590\,\text{W/(m}^2\ \text{K)}}\ln\left(\frac{90°\text{C} - 37°\text{C}}{870°\text{C} - 37°\text{C}}\right) = 5743\ r_o\ \text{s/m} = 5743*0.0125\ \text{m s/m}$$

t= 72 seconds.

$$\tau = \frac{\alpha t}{r_0^2} = \tau = \frac{1.172*10^{-5}*72}{0.0125^2} = 5.4$$

(c) We have for approximate solutions:

$$\text{Bi}=\frac{h_c r_0}{k} = \frac{[590\,\text{W/(m}^2\ \text{K)}](0.0125\,\text{m})}{43\,\text{W/(m K)}} = 0.17$$

From Table 3.1 for sphere for Bi=0.17

$\delta_1 = 0.6942$ and $C_1 = 1.05038$

From approximate solution equation (3.48) for sphere

$$\frac{Q}{Q_0} = 1 - \frac{3(\sin\delta_1 - \delta_1\cos\delta_1)}{\delta_1^3} C_1 e^{-\delta_1^2 \tau}$$

$$\frac{Q}{Q_0} = 1 - \frac{3(\sin(0.6942) - (0.6942)\cos(0.6942))}{(0.6942)^3} * 1.05038 * e^{-(0.6942)^2 * 5.4}$$

$Q/Q_0 = 0.93$

From Table 3.3

$$Q_i = \rho\, c\, \frac{4}{3}\, \pi\, r_o^3\, (T_o - T_\infty) = [7801\,(\text{kg/m}^3)](473\ \text{J/kg K})\ \frac{4}{3}\, \pi\, (0.0125\ \text{m})^3\, (870°\text{C} - 37°\text{C}) = 25{,}150\ \text{J}$$

$$\therefore\ Q(t) = 0.93\ Q_i = 0.93\ (25{,}159\ \text{J}) = 23{,}390\ \text{J}$$

The amount of heat needed to quench 100,000 balls per hour is

$$q = \text{(Balls/hr) (Energy/ball)} = \frac{[100{,}000\,(1/\text{h})](23{,}390\,\text{J})}{3600\,(\text{s/h})} = 650{,}000\ \text{W}$$

PROBLEM 3.35

Estimate the time required to heat the center of a 1.5-kg roast in a 163°C oven to 77°C. State your assumptions carefully and compare your results with cooking instructions in a standard cookbook.

GIVEN

- A roast in an oven
- Mass of the roast (m) = 1.5 kg
- Oven temperature (T_∞) = 163°C
- Final temperature of the roast's center (T_f) = 77°C

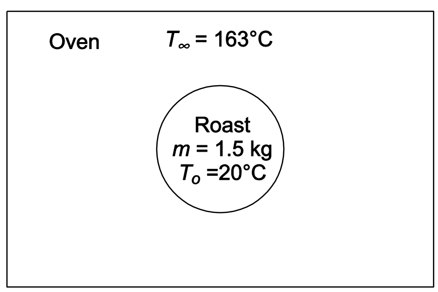
FIND

- The time required to heat the roast

ASSUMPTIONS

- The shape of the roast can be approximated by a sphere
- The roast temperature is initially uniform at (T_o) = 20°C
- The properties of the roast are approximately those of water
  - Thermal conductivity (k) = 0.5 W/(m K)
  - Density (ρ) = 1000 kg/m^3
  - Specific heat = 4000 J/(kg K)
- A uniform heat transfer coefficient of $(\bar{h}_c)$ = 18 W/(m^2 K) exists between the roast and the oven air (midline of the range for free convection given in Table 1.4.)

SKETCH



SOLUTION

With the assumptions listed above, the radius of the spherical roast is given by

$$V = \frac{m}{\rho} = \frac{4}{3}\pi r_o^3 \Rightarrow r_o = \left(\frac{3\,\text{m}}{4\pi\rho}\right)^{\frac{1}{3}} = \left(\frac{3(1.5\ \text{kg})}{4\pi\ 1000\ (\text{kg/m}^3)}\right)^{\frac{1}{3}} = 0.071\ \text{m}$$

Approximate solutions can be used to find the Fourier number. The following parameters (which are listed in Table 3.2) are needed

$$Bi = \frac{\bar{h}_c\ r_o}{k} = \frac{[18\ \text{W/(m}^2\text{K)}](0.071\ \text{m})}{0.5\ \text{W/(m K)}} = 2.56$$

$$\frac{\theta(0,t)}{\theta_o} = \frac{T - T_\infty}{T_o - T_\infty} = \frac{77°\text{C} - 163°\text{C}}{20°\text{C} - 163°\text{C}} = 0.60$$

Using approximate solution

From equation (3.45a) we have

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty}=C_1e^{-\delta_1^2\tau}\frac{\sin(\delta_1 r/r_0)}{\delta_1 r/r_0}$$

For the center r=0, the term becomes indeterminate, so using L Hospital's rule we get.

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty}=C_1e^{-\delta_1^2\tau}\cos(\delta_1 r/r_0)$$

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=C_1e^{-\delta_1^2\tau}\cos(\delta_1 * 0/r_0)$$

For Bi=2.56 From Table 3.1 for sphere we have

$$\delta_1=2.1745 \text{ and } C_1=1.56$$

$$0.60=1.56*e^{-(2.1745)^2\tau}*1$$

$$-(2.1745)^2\tau=\ln(0.3846)$$

$$\tau=0.20=F_0$$

Solving for the time

$$t=\frac{r_o^2\ Fo}{\alpha}=\frac{r_o^2\ Fo\,\rho c}{k}$$

$$t=\frac{(0.071\text{m})^2(0.2)\ 1000\cdot\text{kg/m}^3\ \ 4000\text{J/(kg K)}}{0.5\text{W/m K}\ \ \text{J/Ws}}$$

$$t=8065\text{ s}=134\text{ min}$$

The Better Homes and Gardens Cookbook recommends cooking a Standing Rib Roast with the oven set at 325°F (163°C) for 27-30 minutes per pound to achieve a center temperature of 170°F (77°C) which is considered well done.

This calculation yielded 134 minutes for 1.5 kg (3.3 lbs) or 40 minutes per pound. The discrepancy is probably due to inaccuracies in the assumed properties of the roast.

PROBLEM 3.36

A steel sphere with a diameter of 7.6 cm is to be hardened by first heating it to a uniform temperature of 870°C and then quenching it in a large bath of water at a temperature of 38°C. The following data apply

surface heat transfer coefficient $\bar{h}$ = 590 W/(m^2 K)
thermal conductivity of steel = 43 W/(m K)
specific heat of steel = 628 J/(kg K)
density of steel = 7840 kg/m^3

Calculate: (a) time elapsed in cooling the surface of the sphere to 204°C and (b) time elapsed in cooling the center of the sphere to 204°C.

GIVEN

- A steel sphere is quenched in a large water bath
- Diameter (D) = 7.6 cm = 0.076 m
- Initial uniform temperature (T_o) = 870°C
- Water temperature (T_∞) = 38°C
- Surface heat transfer coefficient (h) = 590 W/(m^2 K)
- Thermal conductivity of steel (k) = 43 W/(m K)
- Specific heat of steel (c) = 628 J/(kg K)
- Density of steel (ρ) = 7840 kg/m^3

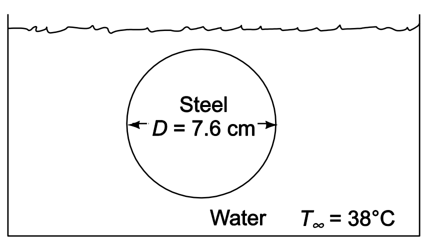
FIND

(a) Time elapsed in cooling the surface of the sphere to 204°C
(b) Time elapsed in cooling the center of the sphere to 204°C

ASSUMPTIONS

- Constant water bath temperature, thermal properties, and transfer coefficient

SKETCH



SOLUTION

The importance of the internal resistance can be determined from the Biot number

$$Bi = \frac{\bar{h}_c\, r_o}{k} = \frac{[590\ \mathrm{W/(m^2\ K)}]\left(\dfrac{0.076}{2}\ \mathrm{m}\right)}{43\ \mathrm{W/(m\ K)}} = 0.52 > 0.1$$

Therefore, the internal resistance is significant and an approximate solution will be used.

Using approximate solution
From equation (3.45a) we have

$$\frac{T(r,t)-T_\infty}{T_0-T_\infty}=C_1 e^{-\delta_1^2\tau}\frac{\sin(\delta_1 r/r_0)}{\delta_1 r/r_0}$$

For Bi=0.52 From Table 3.1 for sphere we have

$$\delta_1 = 1.1853 \text{ and } C_1 = 1.1498$$

For $r=r_0$ we have

$$\frac{204-38}{870-38}=1.1498*e^{-(1.1853)^2\tau}\frac{\sin(1.1853*r_0/r_0)}{1.1853*r_0/r_0}$$

$$\frac{204-38}{870-38}=1.1498*e^{-(1.1853)^2\tau}\frac{\sin(1.1853*r_0/r_0)}{1.1853*r_0/r_0}$$

$$0.2=1.1498*e^{-(1.1853)^2\tau}\frac{\sin(1.1853)}{1.1853}$$

$$e^{-(1.1853)^2\tau}=0.2225$$

$$-(1.1853)^2\tau=\ln(0.2225)$$

$$\tau=1.07$$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k\,t}{\rho c\, r_o^2} = 1.07$$

Solving for the time

$$t = \frac{Fo\,\rho c\, r_o^2}{k} = \frac{1.07*\left(7840\text{kg/m}^3\right)\left(628\text{J/(kg K)}\right)\left(\frac{0.076}{2}\text{ m}\right)^2}{43\text{W/(m}^2\text{K)}} = 176 \text{ s} = 2.95 \text{ min}$$

(For the surface temperature to reach 204°C)

(b) For a center temperature of 204°C

$$\frac{T(0,t)-T_\infty}{T_o-T_\infty} = \frac{204°\text{C}-38°\text{C}}{870°\text{C}-38°\text{C}} = 0.20$$

Using approximate solution, from equation (3.45a) we have

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty}=C_1 e^{-\delta_1^2\tau}\frac{\sin(\delta_1 r/r_0)}{\delta_1 r/r_0}$$

For the center r=0, the term becomes indeterminate, so using L Hospital's rule we get.

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty}=C_1 e^{-\delta_1^2\tau}\cos(\delta_1 r/r_0)$$

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=C_1 e^{-\delta_1^2\tau}\cos(\delta_1 *0/r_0)$$

For Bi=0.52 From Table 3.1 for sphere we have

$$\delta_1 = 1.1853 \text{ and } C_1 = 1.1498$$

$$0.2 = 1.1498 * e^{-(1.1893)^2 \tau} * 1$$

$$e^{-(1.1893)^2 \tau} = 0.1739$$

$$-(1.1893)^2 \tau = \ln(0.1739)$$

$$\tau = 1.24 = F_0$$

Now, the time taken is calculated as

$$t = \frac{1.24\left(7840 \text{kg/m}^3\right)\left(628 \text{J/(kg K)}\right)\left(\frac{0.076}{2}\text{ m}\right)^2}{43 \text{W/(m}^2\text{ K)}} = 205 \text{ s} = 3.42 \text{ min}$$

(For the center temperature to reach 204°C)

PROBLEM 3.37

A monster turnip (assumed spherical) weighing in at 0.45 kg is dropped into a cauldron of water boiling at atmospheric pressure. If the initial temperature of the turnip is 17°C, how long does it take to reach 92°C at the center? Assume that

$$\bar{h}_c = 1700 \text{ W/(m}^2\text{ K)} \qquad c_\rho = 3900 \text{ J/(kg K)}$$

$$k = 0.52 \text{ W/(m K)} \qquad \rho = 1040 \text{ kg/m}^3$$

GIVEN

- A turnip is dropped into boiling water
- Mass of turnip (M) = 0.45 kg
- Water is boiling at atmospheric pressure
- Initial temperature of the turnip (T_o) = 17°C

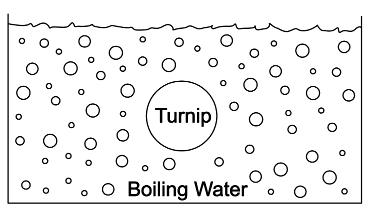
FIND

- Time needed to reach 92°C at the center

ASSUMPTIONS

- Heat transfer coefficient (h_c) = 1700 W/(m^2 K)
- Specific heat ($c\rho$) = 3900 J/(kg K)
- Thermal conductivity (k) = 0.52 W/(m K)
- Density (ρ) = 1040 kg/m^3
- The specific heat of the turnip is constant
- Altitude is sea level, therefore, temperature of boiling water (T_∞) = 100°C
- One dimensional conduction in the radial direction

SKETCH



SOLUTION

The radius of the turnip is given by

$$\text{Volume} = \frac{4}{3}\pi r_o^3 = \frac{M}{\rho} \Rightarrow r_o = \left(\frac{3M}{4\pi\rho}\right)^{\frac{1}{3}} = \left(\frac{3(0.45\,\text{kg})}{4\pi\ 1040\,\text{kg/m}^3}\right)^{\frac{1}{3}} = 0.047\text{ m}$$

The Biot number is

$$Bi = \frac{\bar{h}_c r_o}{k} = \frac{[1700\,\text{W/(m}^2\text{ K)}](0.047\text{ m})}{0.52\,\text{W/(m K)}} = 153 > 0.1$$

Therefore, internal resistance is significant and the approximate solution method will be used.

$$\frac{T(0,t)-T_\infty}{T_o-T_\infty} = \frac{92^\circ\text{C}-100^\circ\text{C}}{17^\circ\text{C}-100^\circ\text{C}} = 0.096$$

Using approximate solution, from equation (3.45a) we have

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty} = C_1 e^{-\delta_1^2\tau}\,\frac{\sin(\delta_1 r/r_0)}{\delta_1 r/r_0}$$

For the center r=0, the term becomes indeterminate, so using L Hospital's rule we get.

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty} = C_1 e^{-\delta_1^2\tau}\cos(\delta_1 r/r_0)$$

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty} = C_1 e^{-\delta_1^2\tau}\cos(\delta_1 * 0/r_0)$$

For Bi=153 which can be considered ∞ From Table 3.1 for sphere we have

$$\delta_1 = 3.1416 \text{ and } C_1 = 2$$

$$0.096 = 2 * e^{-(3.1416)^2\tau}$$

$$-(3.1416)^2\tau = \ln(0.048)$$

$$\tau = 0.3 = F_0$$

Solving for the time

$$t = \frac{Fo\,\rho c r_o^2}{k} = \frac{0.3\left(1040\text{ kg/m}^3\right)(3900\text{ J/(kg K)})(0.047\text{ m})^2}{0.52\text{ W/(m}^2\text{ K)}}$$

$$t = 5168\text{ s} = 86\text{ min} = 1.44\text{ hours}$$

PROBLEM 3.38

An egg, which for the purposes of this problem can be assumed to be a 5-cm-diameter sphere having the thermal properties of water, is initially at a temperature of 4°C. It is immersed in boiling water at 100°C for 15 min. The heat transfer coefficient from the water to the egg may be assumed to be 1700 W/(m² K). What is the temperature of the egg center at the end of the cooking period?

GIVEN

- An egg is immersed in boiling water
- Initial temperature (T_o) = 4°C
- Temperature of boiling water (T_∞) = 100°C
- Time that the egg is in the water (t) = 15 min. = 900 s
- The heat transfer coefficient (h_c) = 1700 W/(m^2 K)

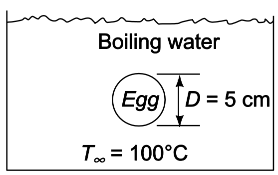
FIND

- The temperature of the egg center at the end of the cooking period

ASSUMPTIONS

- The egg is a sphere of diameter (D) = 5 cm = 0.05 m
- The egg has the thermal properties of water (From Appendix 2, Table 13)
  - Thermal conductivity (k) = 0.682 W/(m K)
  - Density (ρ) = 958.4 kg/m^3
  - Specific Heat (c) = 4211 J/(kg K)

SKETCH



SOLUTION

The Biot number for the egg is

$$Bi = \frac{\overline{h_c}\, r_o}{k} = \frac{[1700\ \text{W/(m}^2\ \text{K)}]\,(0.025\ \text{m})}{0.682\ \text{W/(m K)}} = 62.3 > 0.1$$

Therefore, the internal resistance is significant. Approximate solution can be used to solve the problem. The Fourier number at t = 900 s is

$$Fo = \frac{\alpha t}{r_o^2} = \frac{k\,t}{\rho c r_o^2} = \frac{0.682\ \text{W/(m K)}\ (900s)}{[4211\ \text{J/(kg K)}]((\text{W s})/\text{J})\left(958.4\ \text{kg/m}^3\right)(0.025m)^2} = 0.24$$

Using approximate solution, from equation (3.45a) we have

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty} = C_1 e^{-\delta_1^2 \tau}\,\frac{\sin(\delta_1 r/r_0)}{\delta_1 r/r_0}$$

For the center r=0, the term becomes indeterminate, so using L Hospital's rule we get.

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty}=C_1e^{-\delta_1^2\tau}\cos(\delta_1 r/r_0)$$

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=C_1e^{-\delta_1^2\tau}\cos(\delta_1 * 0/r_0)$$

For Bi=62.3 which can be considered ∞ From Table 3.1 for sphere we have

$$\delta_1=3.08685 \text{ and } C_1=1.997$$

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=1.997 * e^{-(3.08685)^2*0.24} * 1$$

$$\frac{T(0,t)-T_\infty}{T_0-T_\infty}=0.203$$

$$T(0,t)=0.203(4-100)+100$$

$$T(0,t)=80.5^0C$$

PROBLEM 3.39

In the experimental determination of the heat transfer coefficient between a heated steel ball and crushed mineral solids, a series of 1.5% carbon steel balls were heated to a temperature of 700°C and the center temperature-time history of each was measured with a thermocouple as it cooled in a bed of crushed iron ore that was placed in a steel drum rotating horizontally at about 30 rpm. For a 5-cm-diameter ball, the time required for the temperature difference between the ball center and the surrounding ore to decrease from 500°C initially to 250°C was found to be 64, 67, and 72 s, respectively, in three different test runs. Determine the average heat transfer coefficient between the ball and the ore. Compare the results obtained by assuming the thermal conductivity to be infinite with those obtained by taking the internal thermal resistance of the ball into account.

GIVEN

- Heat steel balls are put in crushed iron ore
- Balls are 1.5% carbon steel balls
- Initial temperature of balls (T_o) = 700°C
- Ball diameter = 5 cm = 0.05 m
- Temperature difference between the ball center and the ore
- Center temperature of the balls decreases from 500°C to 250°C
- Time taken was found to be 64, 67, and 72 s, respectively, in three different test runs

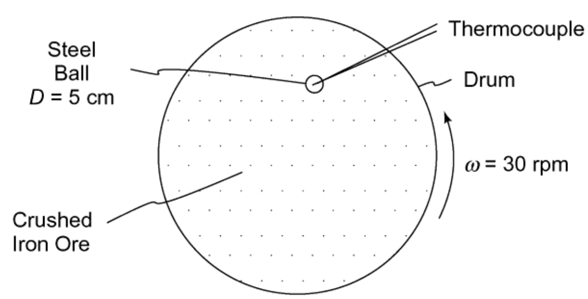
FIND

The average heat transfer coefficient between the ball and the ore.
Compare the results obtained
(a) by assuming the thermal conductivity to be infinite with
(b) those obtained by taking the internal thermal resistance of the ball into account

ASSUMPTIONS

- Temperature of the iron ore is uniform and constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10

For 1.5% carbon steel	Thermal conductivity (k) = 36 W/(m K)
Density (ρ) = 7753 kg/m^3
Specific heat (c) = 486 J/(kg K)
Thermal diffusivity (α) = 0.97×10^{-5} m^2/s

SOLUTION

(a) Assuming the internal resistance of the balls is negligible. The temperature-time history is given by Equation (3.3)

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{h_c A_s}{c\rho V}t} = e^{-\frac{h_c \pi D^2}{c\rho \frac{\pi}{4} D^3}t} = e^{-\frac{6 h_c}{c\rho D}t}$$

Solving for the heat transfer coefficient

$$h_c = \frac{c\rho D}{6t} \ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right)$$

$$h_c = \frac{[486\,\text{J/(kg K)}]\ \text{(Ws)/J}\ \ 7753\,\text{kg/m}^3\ \ (0.05\,\text{m})}{6t} \ln\left(\frac{250^\circ\text{C}}{500^\circ\text{C}}\right) = \frac{21{,}765}{t}\ \text{Ws/(m}^2\ \text{K)}$$

For the three test runs: t = 64 s → h_c = 340 W/(m^2 K)

t = 67 s → h_c = 325 W/(m^2 K)

t = 72 s → h_c = 302 W/(m^2 K)

The average heat transfer coefficient is 322 W/(m^2 K)

(b) The chart method will be used to take the internal thermal resistance into account. Figure 3.11 (a) can be used to determine the Biot number for the balls

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{250^\circ\text{C}}{500^\circ\text{C}} = 0.5$$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{0.97 \times 10^{-5}\text{m}^2/\text{s}(t)}{(0.025\,\text{m})^2}$$

For the three test runs: t = 64 s → Fo = 0.99

t = 67 s → Fo = 1.04

t = 72 s → Fo = 1.12

Figure 3.11 (a) is not detailed enough to distinguish between the first two test runs

For the first two runs: F_o = 1.0 → 1/Bi = 4.0 Bi = 0.25

For the third run: F_o = 1.1 → 1/Bi = 4.2 Bi = 0.238

The average Bi number = [2(0.250) = 0.263]/3 = 0.246 = (h_c r_o)/k

Solving for the transfer coefficient

$$h_c = \frac{Bi\,k}{r_o} = \frac{0.246\ \ 36\text{W/(m K)}}{0.025\,\text{m}} = 354\ \text{W/(m}^2\ \text{K)}$$

Neglecting the internal resistance resulted in a calculated heat transfer coefficient 9% lower than using the chart method.

PROBLEM 3.40

Surface hardening of metallic machine components, such as ball and roller bearings, is carried out in a heat treatment process where the surface temperature is increased to a desirable level without altering the internal temperature substantially. In one such operation, steel ball (spherical) bearings with a diameter of 25 mm, initially at 25°C, are immersed in a very high temperature (1000°C) molten electrolyte-solution bath and quickly removed when the temperature at a depth of 1 mm from the outer surface of the steel ball attains 725°C. If the convective heat transfer coefficient between the steel ball and the bath liquid is assumed to be 4.95 kW/(m^2 K), determine the time required for attaining the surface hardening condition.

GIVEN

- Steel ball bearing diameter (d)= 25 mm=0.025 m
- Initial temperature (T_o) = 25°C
- Temperature of electrolyte solution (T_∞) = 1000°C
- The heat transfer coefficient (h_c) = 4.95 kW/(m^2 K) = 4950 W/(m^2 K)
- Radius of the surface where temperature of 725^0C is attained= 11.5 mm

FIND

- Time required for attaining temperature of 725^0C at depth of 1 mm from outer surface.

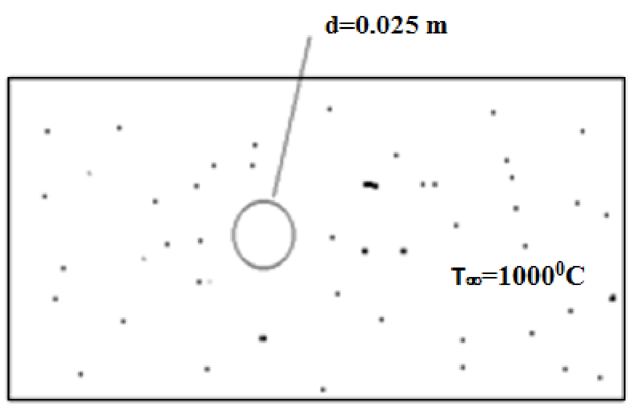
ASSUMPTIONS

constant bath temperature, thermal properties and heat transfer coefficient

PROPERTIES

From Appendix 2 Table 10
thermal conductivity of steel = 43 W/(m K)
specific heat of steel = 473 J/(kg K)
density of steel = 7840 kg/m^3

SKETCH



SOLUTION

The Biot number for the steel ball based on r_0 is

$$Bi = \frac{h_c r_0}{k} = \frac{4950 * 0.0125}{43} = 1.44 >> 0.1$$

Therefore, the internal resistance is significant Approximate solutions can be used to solve the problem.
For Bi=1.44, r/r_0=11.5/12.5= 0.92 we have

$$\frac{T(r,t)-T_\infty}{T(0,t)-T_\infty}=\frac{725-1000}{25-1000} = 0.282$$

Using approximate solution, from equation (3.45a) we have

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty}=C_1 e^{-\delta_1^2\tau}\frac{\sin(\delta_1 r/r_0)}{\delta_1 r/r_0}$$

$$0.282=C_1 e^{-\delta_1^2\tau}\frac{\sin(\delta_1 * 0.92)}{\delta_1 * 0.92}$$

For Bi=1.44 From Table 3.1 for sphere we have

$$\delta_1 = 1.7722 \text{ and } C_1 = 1.364$$

$$0.282 = 1.364 * e^{-(1.7722)^2\tau}\frac{\sin(1.7722*0.92)}{1.7722*0.92}$$

$$0.282 = 1.364 * e^{-(1.7722)^2\tau}\frac{\sin(1.7722*0.92)}{1.7722*0.92}$$

$$e^{-(1.7722)^2\tau} = 0.3371$$

$$-(1.7722)^2\tau = \ln(0.3371)$$

$$\tau = 0.34$$

t= $\dfrac{0.34*7840*473*(0.0125)^2}{43}$ seconds

t= 4.58 seconds.

PROBLEM 3.41

Estimate the depth in moist soil at which the annual temperature variation will be 10% of that at the surface.

GIVEN

- Moist soil

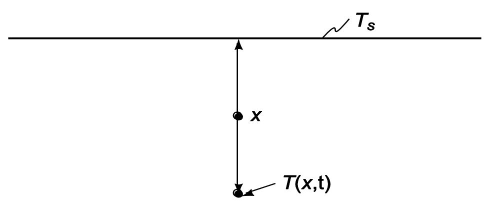
FIND

- The depth in moist soil at which the annual temperature variation will be 10 per cent of that at the surface

ASSUMPTIONS

- Conduction is one dimensional
- The soil has uniform and constant properties
- Annual temperature variation can be treated as a step change in surface temperature with a 6 month response time

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11

For wet soil Thermal conductivity (k) = 2.60 W/(m K) at 20°C

Density (ρ) = 1500 kg/m^3

Thermal diffusivity (α) = 0.0414×10^{-5} m^2/s

SOLUTION

The geometry of this problem is a semi infinite solid as covered in Section 3.4. The transient temperature for a change in surface temperature is given by Equation (3.54)

$$\frac{T_{(x,t)} - T_s}{T_i - T_s} = \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Where T_i is the temperature of the soil until the surface temperature is increased to T_s. For an annual temperature variation of less than 10% of that of the surface

$$T(x, t) - T_i = 0.1\,(T_s - T_i) \text{ at } t = 6 \text{ months}$$

$$T(x, t) = 0.1T_s + 0.9T_i$$

Therefore $T(x, t) - Ts = 0.1T_s + 0.9T_i - T_s = 0.9\,(T_i - T_s)$

$$\frac{T_{(x,t)} - T_s}{T_i - T_s} = 0.9 = \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$\operatorname{erf}\left(\frac{x}{2\sqrt{[0.0414 \times 10^{-5}\,(\text{m}^2\text{s})](0.5 \text{ year})\ 365\,(\text{days/year})\ \ 24\,(\text{h/day})\ \ 3600(\text{s/h})}}\right) = 0.9$$

$$\text{erf}\left(\frac{x}{5.110\text{ m}}\right) = 0.9$$

From Appendix 2, Table 43

$$\text{erf}\,(1.16) = 0.9$$

$$\therefore \quad \frac{x}{5.110\text{ m}} = 1.16$$

$$x = 6\text{ m}$$

PROBLEM 3.42

A large billet of steel initially at 260°C is placed in a radiant furnace where the surface temperature is held at 1200°C. Assuming the billet to be infinite in extent, compute the temperature at point *P* shown in the accompanying sketch after 25 min have elapsed. The average properties of steel are: k = 28 W/(m K), ρ = 7360 kg/m^3, and c = 500 J/(kg K).

GIVEN

- A large billet of steel is placed in a radiant furnace
- Initial temperature (T_o) = 260°C
- Surface temperature of billet in the oven (T_s) = 1200°C
- Lapse time (t) = 25 min = 1500 s
- Thermal conductivity (k) = 28 W/(m K)
- Density (ρ) = 7360 kg/m^3
- Specific heat (c) = 500 J/(kg K)

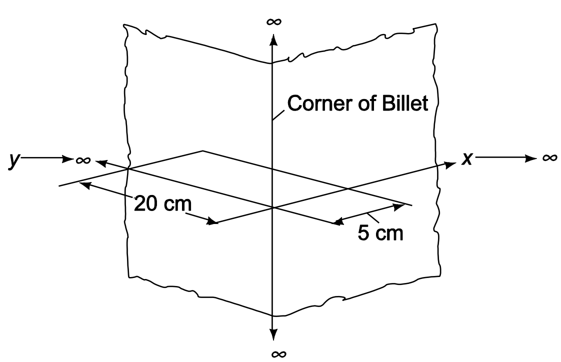
FIND

- The temperature at point P shown in the accompanying sketch

ASSUMPTIONS

- The billet infinite in extent

SKETCH



SOLUTION

From Table 2.4, the solution for a one quarter infinite solid is

$$\frac{\theta_p \quad x, y}{\theta_o} = \frac{T(x, y, t) - T_s}{T_o - T_s} = S(x)\, S(y)$$

Where $S(x)$ and $S(y)$ are solutions for a semi-infinite solid, which are given for a constant surface temperature by Equation (3.54)

$$\frac{T(x,t) - T_\infty}{T_o - T_\infty} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Therefore, the solution to this problem is

$$\frac{T(x,y,t)-T_\infty}{T_o-T_\infty} = \mathrm{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)\ \mathrm{erf}\left(\frac{y}{2\sqrt{\alpha t}}\right)$$

$$T(x, y, t) = T_s + (T_o - T_s)\left[\mathrm{erf}\left(\frac{x}{M}\right)\mathrm{erf}\left(\frac{y}{M}\right)\right]$$

where

$$M = 2\sqrt{\alpha t} = 2\,\frac{k\,t}{\rho c} = 2\sqrt{\frac{[28\,\mathrm{W/(m\,K)}]\ 1500\,\mathrm{s}}{(7360\,\mathrm{kg/m^2})\ 500\,\mathrm{J/(kg\,K)}}} = 0.2137$$

$$\therefore\quad T\,(0.05\text{ m, }0.2\text{ m, }1500\text{ s}) = 1200°\mathrm{C} = (260°\mathrm{C} - 1200°\mathrm{C})\ \mathrm{erf}\left(\frac{0.05\,\mathrm{m}}{0.2137\,\mathrm{m}}\right)\mathrm{erf}\left(\frac{0.2\,\mathrm{m}}{0.2137\,\mathrm{m}}\right)$$

Using Appendix 2, Table 43 for the error function values

$$T\,(0.05\text{ m, }0.2\text{ m, }1500\text{ s}) = 1200°\mathrm{C} - 940°\,(0.259)\,(0.814) = 1002°\mathrm{C}$$

PROBLEM 3.43

It is a well-known physiological phenomenon that some materials feel cooler to the touch than others. This is important in the design of instruments for the use of operators, especially in a space station. Experiments conducted at NASA have shown that different materials, e.g., aluminum and plastic, though in identical size and temperature feel very different to the touch. Consider two plates, one made of aluminum and the other of plastic, each at an initial temperature of 25°C. Determine which of the two will feel hotter to the touch of your finger if it is initially at 32°C. Assume that your finger has a density of 1000 kg/m^3, specific heat of 4180 J/(kg K), and thermal conductivity of 0.625 W/(m K). Also, properties of aluminum are given in Table 12 (Appendix 2), and plastic is considered to have a density of 1990 kg/m^3, specific heat of 1470 J/(kg K), and thermal conductivity of 0.21 W/(m K).

GIVEN

- Initial temperature of plates (T_i)= 25^{0}C
- Initial temperature of finger (T_o) = 32°C
- Density of finger (ρ_f)= 1000 kg/m^3
- Specific heat of finger (c_f)=4180 J/(kg K)
- Thermal conductivity of finger (k_f)= 0.625 W/(m K)
- Density of plastic (ρ_p)=1990 kg/m^3
- Specific heat of plastic (c_p)=1470 J/(kg K)
- Thermal conductivity of plastic (k_p)= 0.21 W/(m K)

FIND

which of the aluminum or plastic feel hotter to touch to your finger.

ASSUMPTIONS

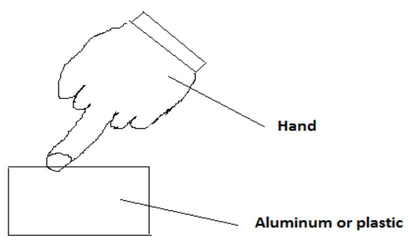
- constant thermal properties
- Negligible contact resistance

The finger and plate behave as semi infinite solid.

PROPERTIES

For aluminum from appendix 2 Table 21
Thermal conductivity (k) = 240 W/(m K)
Specific heat (c) = 896 J/(kg K)
Density of steel = 2702 kg/m^3

SKETCH



SOLUTION

From equation (3.54) for change in surface temperature.

$$q_s''(t) = \frac{k(T_s - T_i)}{\sqrt{\pi k t}}$$

For plastic plate heat flux is given by

$$q_{sp}''(t) = \frac{k_p(T_s - T_{i,p})}{\sqrt{\pi \alpha_p t}}$$

For finger heat flux is given by

$$q_{sh}''(t) = \frac{k_h(T_s - T_{i,h})}{\sqrt{\pi \alpha_h t}}$$

Equilibrium surface temperature is attained when

$$q_{sp}''(t) = q_{sf}''(t)$$

$$\frac{k_p(T_s - T_{i,p})}{\sqrt{\pi \alpha_p t}} = \frac{k_h(T_s - T_{i,f})}{\sqrt{\pi \alpha_f t}}$$

Solving the above equation for T_s substituting $\alpha = k/\rho c$ we get

$$T_s = \frac{(k\rho c)_p^{1/2} T_{i,p} + (k\rho c)_f^{1/2} T_{i,f}}{(k\rho c)_p^{1/2} + (k\rho c)_f^{1/2}}$$

Substituting the properties for plastic and hand given above we get

$$T_s = \frac{(0.21*1990*1470)_p^{1/2} * 25 + (0.625*4180*1000)_h^{1/2} * 32}{(0.21*1990*1470)_p^{1/2} + (0.625*4180*1000)_h^{1/2}}$$

$$T_s = \frac{783.7*25 + 1616*32}{(783.7 + 1616)}$$

$T_s = 29.7^0C$

Similarly for Aluminum plate when touched by hand the surface temperature is

$$T_s = \frac{(k\rho c)_A^{1/2} T_{i,A} + (k\rho c)_f^{1/2} T_{i,f}}{(k\rho c)_A^{1/2} + (k\rho c)_f^{1/2}}$$

$$T_s = \frac{(242*896*1892)_A^{1/2} * 25 + (0.625*4180*1000)_h^{1/2} * 32}{(242*896*1892)_A^{1/2} + (0.625*4180*1000)_h^{1/2}}$$

$$T_s = \frac{6106*25 + 1616*32}{(6106 + 1616)}$$

$T_s = 26.4^0C$

Thus plastic will feel hotter than Aluminum when touched with finger after a while.

COMMENTS

The material with higher $(k\rho c)^{1/2}$ will have higher energy storage capacity and thus changes its temperature gradually in response to outer temperature.