Chapter 5

PROBLEM 5.1

Evaluate the Reynolds number for flow over a tube from the following data

$$\boldsymbol{D} = 6 \text{ cm}$$
$$\boldsymbol{U_\infty} = 1.0 \text{ m/s}$$
$$\boldsymbol{\rho} = 300 \text{ kg/m}^3$$
$$\boldsymbol{\mu} = 0.04 \text{ N s/m}^2$$

GIVEN

D = 6 cm
U_∞ = 1.0 m/s
ρ = 300 kg/m^3
μ = 0.04 N s/m^2

FIND

The Reynolds Number (Re)

SOLUTION

The Reynolds number, from Table 5.3, is

$$Re = \frac{U_\infty L}{\nu} = \frac{U_\infty L \rho}{\mu}$$

The Reynolds number based on the tube diameter is

$$Re = \frac{U_\infty D \rho}{\mu} = \frac{1\,\text{m/s}\ (6\,\text{cm})\ 1\,\text{m/(100 cm)}\ \ 300\,\text{kg/m}^3}{0.04\,\text{(Ns)/m}^2\ \ \text{kg m/(s}^2\text{N)}} = 450$$

PROBLEM 5.2

Evaluate the Prandtl number from the data below

$$\boldsymbol{c_p = 2.1\ \text{kJ/(kg K)}}$$
$$\boldsymbol{k = 3.4\ \text{W/(m K)}}$$
$$\boldsymbol{\mu = 0.45\ \text{kg/(m s)}}$$

GIVEN

c_p= 2.1 kJ/(kg K)
k= 3.4 W/(m K)
μ= 0.45 kg/(m s)

FIND

The Prandtl number (Pr)

SOLUTION

The Prandtl number from Equation (5.8) is

$$Pr = \frac{c_p \mu}{k}$$

$$Pr = \frac{(2100\, J\ /\ (\mathrm{kgK}))(0.45\ /\ (\mathrm{kg}\ /\ (\mathrm{m\,s})))}{3.4 W\ /\ (\mathrm{m}\ K)} = 278$$

PROBLEM 5.3

Evaluate the Nusselt number for flow over a sphere for the following conditions

$$\boldsymbol{D} = \mathbf{0.15\ m}$$
$$\boldsymbol{k} = \mathbf{0.2\ W/(m\ K)}$$
$$\boldsymbol{h_c} = \mathbf{102\ W/(m^2\ K)}$$

GIVEN

D= 0.15 m

k= 0.2 W/(m K)

h_c= 102 W/(m^2 K)

FIND

The Nusselt number (Nu)

SOLUTION

The Nusselt number is given by Equation (5.18)

$$Nu = \frac{h_c L}{k}$$

Based on the diameter of the sphere, the Nusselt number is

$$Nu_D = \frac{h_c D}{k} = \frac{\left(102 W/(m^2\ K)\right)(0.15 m)}{[0.2\,\mathrm{W/(m\,K)}]} = 76.5$$

PROBLEM 5.4

Evaluate the Stanton number for flow over a tube from the data below

$$\mathbf{D = 10\ cm}$$
$$\mathbf{U_\infty = 4\ m/s}$$
$$\boldsymbol{\rho} \mathbf{= 13{,}000\ kg/m^3}$$
$$\boldsymbol{\mu} \mathbf{= 1 \times 10^{-3}\ N\ s/m^2}$$
$$\mathbf{c_p = 140\ J/(kg\ K)}$$
$$\mathbf{h_c = 1000\ W/(m^2\ K)}$$

GIVEN

D = 10 cm
U_∞ = 4 m/s
ρ = 13,000 kg/m^3
$\mu = 1 \times 10^{-3}$ N s/m^2
c_p = 140 J/(kg K)
$\bar{h}_c$ = 1000 W/(m^2 K)

FIND

The Stanton number (*St*) for flow over a tube.

SOLUTION

The Stanton number is given in Table 5.3 as

$$St = \frac{\bar{h}_c}{\rho U_\infty c_p}$$

The Stanton number based on the average heat transfer coefficient is

$$St = \frac{\bar{h}_c}{\rho U_\infty c_p} = \frac{[1000\,\mathrm{W/(m^2K)}]}{13{,}000\,\mathrm{kg/m^3}\;\; 4\,\mathrm{m/s}\;\; 140\,\mathrm{J/(kgK)}\;\; \mathrm{Ws/J}} = 1.37 \times 10^{-4}$$

PROBLEM 5.5

Evaluate the dimensionless groups h_cD/k, $U_\infty D\rho/\mu$, $c_p\ \mu/k$ for water, *n*-Butyl alcohol, mercury, hydrogen, air, and saturated steam at a temperature of 100°C. Let D = 1 m, U_∞ = 1 m/sec, and h_c = 1 W/(m² K).

GIVEN

D = 1 m
U_∞ = 1 m/s
h_c = 1 W/(m² K)

FIND

The dimensionless groups

- $h_c\ D/k$ (Nusselt number)
- $U_\infty D\ \rho/\mu$ (Reynolds number)
- $c_p\ \mu/k$ (Prandtl number)

PROPERTIES AND CONSTANTS

From Appendix 2, at 100°C

Substance	Table Number	Density, ρ (kg/m³)	Specific Heat, c_p (J/kg K)	Thermal Conductivity W/(m K)	Absolute Viscosity $\mu \times 10^6$ (N s/m²)
Water	13	958.4	4211	0.682	277.5
n-Butyl Alcohol	18	753	3241	0.163	540
Mercury	25	13,385	137.3	10.51	1242
Hydrogen	31	0.0661	14,463	0.217	10.37
Air	27	0.916	1022	0.0307	21.673
Saturated Steam	34	0.5977	2034	0.0249	12.10

SOLUTION

For water at 100°C

$$Nu = \frac{h_c D}{k} = \frac{[1\,\text{W}/(\text{m}^2\,\text{K})]\ 1\,\text{m}}{[0.682\,\text{W}/(\text{m\,K})]} = 1.47$$

$$Re_D = \frac{U_\infty D\rho}{\mu} = \frac{1\,\text{m/s}\quad 1\,\text{m}\quad 958.4\,\text{kg/m}^3}{277.5\times10^{-6}\ \text{Ns/m}^2\quad \text{kg\,m}/(\text{s}^2\,\text{N})} = 34 \times 10^6$$

$$Pr = \frac{c_p\mu}{k} = \frac{4211\,\text{J/(kg K)}\quad 277.5\times10^{-6}\,(\text{Ns})/\text{m}^2}{0.682\,\text{W}/(\text{m\,K})\quad \text{Ns}^2/(\text{kg\,m})} = 1.71$$

The dimensionless groups for the other substances can be calculated in a similar manner

Substance	Nu	Re_D	Pr
Water	1.47	3.4×10^6	1.71
n-Butyl Alcohol	6.13	$14. \times 10^6$	10.73
Mercury	0.10	1.1×10^7	0.016
Hydrogen	4.61	6.3×10^3	0.694
Air	32.6	4.2×10^4	0.721
Saturated Steam	40.2	4.9×10^4	0.988

PROBLEM 5.6

A fluid flows at 5 m/s over a wide, flat plate 15-cm-long. For each of the following list, calculate the Reynolds number at the downstream end of the plate. Indicate weather the flow at that point is laminar, transition, or turbulent. Assume all fluids are at 40°C.

(a) Air

(b) CO_2

(c) Water

(d) Engine Oil

GIVEN

A fluid flows over a flat plate
Fluid velocity (U_∞) = 5 m/s
Length of plate (L) = 15 cm = 0.15 m
Fluid temperature = 40°C

FIND

The Reynolds number at the downstream end of the plate (Re_L) for

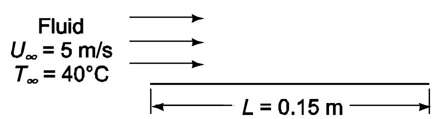
(a) Air
(b) CO_2
(c) Water
(d) Engine Oil

Indicate if the flow is laminar, transitional, or turbulent

ASSUMPTIONS

Steady state

SKETCH



PROPERTIES AND CONSTANTS

At 40°C, the kinematic viscosities of the given fluids are as follows
From Appendix 2, Table 28 for Air (ν_a) = 17.6×10^{-6} m²/s
From Appendix 2, Table 29 for CO_2 (ν_c) = 9.07×10^{-6} m²/s
From Appendix 2, Table 13 for Water (ν_w) = 0.658×10^{-6} m²/s
From Appendix 2, Table 17 for Engine Oil (ν_o) = 240×10^{-6} m²/s

SOLUTION

The Reynolds number, from Table 5.3, is

$$\frac{Re = U_\infty L}{\nu}$$

The transition from laminar to turbulent flow over a plate occurs at a Reynolds number of about 5×10^5.
For air

$$Re_L = \frac{5\,\text{m/s} \;\; 0.15\,\text{m}}{17.6 \times 10^{-6}\,\text{m}^2/\text{s}} = 4.3 \times 10^4 \text{ (Laminar)}$$

For CO_2

$$Re_L = \frac{5\,\text{m/s} \quad 0.15\,\text{m}}{9.07\times10^{-6}\,\text{m}^2/\text{s}} = 8.3 \times 10^4 \text{ (Laminar)}$$

For water

$$Re_L = \frac{5\,\text{m/s} \quad 0.15\,\text{m}}{0.658\times10^{-6}\,\text{m}^2/\text{s}} = 1.1 \times 10^6 \text{ (Turbulent)}$$

For engine oil

$$Re_L = \frac{5\,\text{m/s} \quad 0.15\,\text{m}}{240\times10^{-6}\,\text{m}^2/\text{s}} = 3.1 \times 10^3 \text{ (Laminar)}$$

PROBLEM 5.7

The average Reynolds number for air passing in turbulent flow over a 2-m-long flat plate is 2.4×10^6. Under these conditions, the average Nusselt number was found to be equal to 4150. Determine the average heat transfer coefficient for an oil having thermal properties similar to those in Appendix 2, Table 18 at 30°C at the same Reynolds number and flowing over the same plate.

GIVEN

Turbulent flow of air over a flat plate
Average Reynolds number (Re_L) = 2.4×10^6
Plate length (L) = 2 m
Average Nusselt number (Nu) = 4150

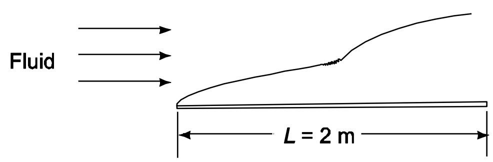
FIND

Average heat transfer coefficient $\bar{h}_c$ for oil flowing at the same Re over the same plate

ASSUMPTIONS

Steady state
Fully developed turbulent flow
Transition from laminar to turbulent flow occurs at $Re_x = 5 \times 10^5$

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 18, for oil at 30°C

Thermal Conductivity (k) = 0.11 W/(m K)

Kinematic Viscosity (v) = 15.4×10^{-6} m²/s

Thermal Diffusivity (α) = 707×10^{-10} m²/s

SOLUTION

The Prandtl number for the oil is

$$Pr = \frac{v}{\alpha} = \frac{15.4\times 10^{-6}\ \text{m}^2/\text{s}}{707\times 10^{-10}\ \text{m}^2/\text{s}} = 218$$

The empirical correlation from Table 5.5 can be used to find the Nusselt number for the oil.

$$Nu_L = 0.036\, Pr^{0.33}\, [Re_L^{0.8} - 23{,}200] \text{ for } Re_L > 5 \times 10^5 \text{ and } Pr > 0.5$$

$$Nu_L = 0.036\,(218)^{0.33}\, [(2.4 \times 10^6)^{0.8} - 23{,}200] = 22{,}100$$

$$\bar{h}_c = \frac{Nu_L\, k}{D} = \frac{22{,}100\ \ 0.11\,\text{W/(m K)}}{2.0\,\text{m}} = 1216\ \text{W/(m}^2\,\text{K)}$$

PROBLEM 5.8

The dimensionless ratio $U_\infty / \sqrt{Lg}$, called Froude number, is a measure of similarity between an ocean-going ship and a scale model of the ship to be tested in a laboratory water channel. A 150 m long cargo ship is designed to run at 36 km/h, and a 1.5 m geometrically similar model is towed in a water channel to study wave resistance. What should be the towing speed in m s^{-1}?

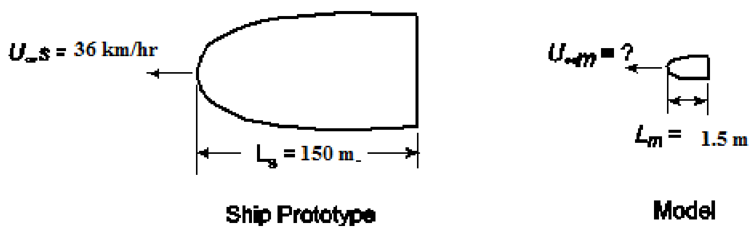
GIVEN

A ship model and its prototype
Froude number = $U_\infty / \sqrt{Lg}$ is a measure of similarity
Ship length (L_s) = 150 m
Ship speed ($U_\infty s$) = 20 knots
Model length (L_m) = 1.5 m

FIND

Model towing speed ($U_{\infty m}$)

SKETCH



SOLUTION

For similar wave shape, the Froude number should be the same for the model and the prototype

$$\frac{U_{\infty m}}{\sqrt{L_m g}} = \frac{U_{\infty s}}{\sqrt{L_s g}} \Rightarrow U_{\infty m} = U_{\infty s} \sqrt{\frac{L_m}{L_s}}$$

$$U_{\infty m} = 36 \text{ km/hr} \sqrt{\frac{1.5m}{150m}} = 3.6 \text{ km/hr} = 1 \text{ m/s}$$

PROBLEM 5.9

When a sphere falls freely through a homogeneous fluid, it reaches a terminal velocity at which the weight of the sphere is balanced by the buoyant force and the frictional resistance of the fluid. Make a dimensional analysis of this problem and indicate how experimental data for this problem could be correlated. Neglect compressibility effects and the influence of surface roughness.

GIVEN

A sphere falling freely through a homogeneous fluid
Terminal velocity occurs when weight is balanced by buoyant force and friction resistance of the fluid

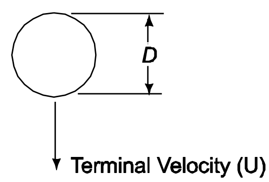
FIND

Make a dimensional analysis and indicate how data may be correlated

ASSUMPTIONS

Compressibility effects are negligible
Influence of surface roughness is negligible

SKETCH



SOLUTION

The variables which must be correlated and their dimensions are shown below

	Variable	Symbol	Dimensions
1.	Acceleration of Gravity	g	$[L/t^2]$
2.	Density Difference	$\rho_s - \rho_f$	$[M/L^3]$
3.	Fluid Density	ρ_f	$[M/L^3]$
4.	Terminal Velocity	U	$[L/t]$
5	Sphere Diameter	D	$[L]$
6.	Fluid Viscosity	μ	$[M/L\,t]$

The density difference was chosen for variable 2 because we anticipate that this difference, rather than the sphere density, will be an important parameter. Clearly, if $\rho_s = \rho_f$, then $U = 0$. The Buckingham π Theorem (Section 5.7.2 and 5.7.3) can be used to correlate the variables. There are 6 variables and 3 primary dimensions. Therefore, 3 dimensionless groups will be found.

$$\pi = g^a\,(\rho_s - \rho_f)^b\,\rho_f^c\,U^d\,D^e\,\mu^f$$

Substituting the primary dimensions into the equation

$$[\pi] = \left[\frac{L}{t^2}\right]^a \left[\frac{M}{L^3}\right]^{b+c} \left[\frac{L}{t}\right]^d [L]^e \left[\frac{M}{Lt}\right]^f = 0$$

Equating the sum of the exponents of each primary dimension to zero:

For M: $b + c + f = 0$ [1]
For L: $a - 3b - 3c + d + e - f = 0$ [2]
For t: $2a + d + f = 0$ [3]

There are 6 unknowns and only 3 equations, therefore, the value of the 3 exponents can be chosen for each π

For π_1, Let $a = 0$, $b = 0$ and $c = 1$

From equation [1] $f = -1$

From equation [3] $d = 1$

From equation [2] $e = 1$

$$\therefore \;\; \pi_1 = U\,D\,\rho_f\,\mu^{-1} = \frac{UD\rho_f}{\mu} = Re_D$$

For π_2, Let $a = 1$, $b = 1$ and $f = 0$

From equation [1] $c = -1$

From equation [3] $d = -2$

From equation [2] $e = 1$

$$\therefore \;\; \pi_2 = g\,(\rho_s - \rho_f)\,U^{-2}\,D\,\rho_f^{-1} = \frac{g \;\; \rho_s - \rho_f \;\; D}{\rho_f U^2} = \frac{g \;\; \rho_s - \rho_f \;\; \frac{x}{6}D^3}{\frac{4}{3}\left(\frac{1}{2}\rho_f U^2\right)\frac{x}{4}D^4}$$

$$\pi_2 = \frac{\text{Weight of sphere in the fluid}}{\frac{4}{3}\ \text{Dynamic pressure} \times \text{cross sectional area of sphere}}$$

$$\pi_2 = \frac{3}{4}\frac{\text{Drag force on sphere / Cross sectional area}}{\text{Dynamic pressure}} = \frac{3}{4}\,C_D \text{ (Drag Coefficient)}$$

For π_3, Let $a = 0$, $b = 1$, and $f = 0$

From equation [1] $c = -1$

From equation [3] $d = 0$

From equation [2] $e = 0$

$$\therefore \;\; \pi_3 = (\rho_s - \rho_f)\,\rho_f^{-1} = \frac{\rho_s - \rho_f}{\rho_f}$$

But this dimensionless group already appears in π_2. (This redundancy could have been avoided had we chosen the weight of the sphere in the liquid in place of the two variables $(\rho_s - \rho_f)$ and g.) Therefore, the experimental data for this problem could be correlated by

$$C_D = f\,(Re_D)$$

PROBLEM 5.10

Experiments have been performed on the temperature distribution in a homogeneous long cylinder (0.1 m diameter, thermal conductivity of 0.2 W/(m K) with uniform internal heat generation. By dimensional analysis, determine the relation between the steady-state temperature at the center of the cylinder T_c the diameter, the thermal conductivity, and the rate of heat generation. Take the temperature at the surface as you datum. What is the equation for the center temperature if the difference between center and surface temperature is 30°C when the heat generation is 3000 W/m^3?

GIVEN

A homogeneous long cylinder with uniform internal heat generation
Diameter (D) = 0.1 m
Thermal conductivity (k) = 0.2 W/(m K)
Difference between surface and center temperature ($T_c - T_s$) = 30°C
Heat generation rate $\dot{q}$ = 3000 W/m^3

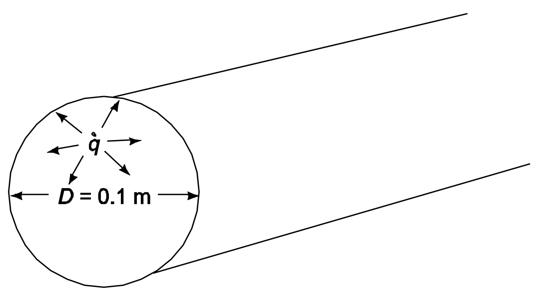
FIND

(a) Relation between center temperature (T_c), diameter (D), thermal conductivity (k), and rate of heat generation $\dot{q}$
(b) Equation for the center temperature for the given data

ASSUMPTIONS

Steady state
One dimensional conduction in the radial direction

SKETCH



SOLUTION

(a) The temperature difference is a function of the variable given

$$T_c - T_s = f(D, k, \dot{q})$$

having the following primary dimensions

$$T_c - T_s \rightarrow [T]$$
$$D \rightarrow [L]$$
$$k \rightarrow \left[\frac{ML}{t^3\,T}\right]$$
$$\dot{q} \rightarrow \left[\frac{M}{Lt^3}\right]$$

Let the unknown function be represented by

$$T_c - T_s = A\,D^a\,k^b\,\dot{q}^{\,c}$$

Where A is a dimensionless constant

$$\therefore \quad [T] = [L]^a \left[\frac{ML}{t^3T}\right]^b \left[\frac{M}{Lt^3}\right]^c$$

Summing the exponents of each primary dimension

For T: $1 = -b \quad \rightarrow \quad b = -1$

For M: $0 = b + c \quad \rightarrow \quad c = -b = 1$

For L: $0 = a + b - c \quad \rightarrow \quad a = c - b = 2$

For t: $0 = -3b + 3c$

$$\therefore \quad T_c - T_s = A\, D^2\, k^{-1}\, \dot{q} = A\, \frac{D^2 \dot{q}}{k}$$

The given data can now be used to evaluate the unknown constant

$$A = \frac{k\left(T_c - T_s\right)}{D^2 \dot{q}} = \frac{\left(0.2\,\mathrm{W/(m\,K)}\right)\left(30°\mathrm{C}\right)}{\left(0.1\,\mathrm{m}\right)^2\left(3000\,\mathrm{W/m^2}\right)} = 0.2$$

The equation for the center temperature is

$$T_c = T_s + 0.2\,\frac{D^2 \dot{q}}{k}$$

PROBLEM 5.11

The convection equations relating the Nusselt, Reynolds, and Prandtl numbers can be rearranged to show that for gases, the heat-transfer coefficient h_c depends on the absolute temperature T and the group $\sqrt{U_\infty/x}$. This formulation is of the form $h_{c,x} = CT^n \sqrt{U_\infty/x}$, where n and C are constants. Indicate clearly how such a relationship could be obtained for the laminar flow case from $Nu_x = 0.332\ Re_x^{0.5}\ Pr^{0.333}$ for the condition $0.5 < Pr < 5.0$. State restrictions on method if necessary.

GIVEN

For laminar flow: $Nu_x = 0.332\ Re_x^{\ 0.5}\ Pr^{0.333}$ for $0.5 < Pr < 5.0$

FIND

Rearrange the given equation to the form $h_{c,x} = C\,T^{\,n} \sqrt{U_\infty/x}$

(State restrictions)

ASSUMPTIONS

Gas behaves as an ideal gas

SOLUTION

From Table 5.3

$$Nu_x = \frac{h_c x}{k} \qquad Re_x = \frac{U_\infty x}{\nu} \qquad Pr = \frac{c_p \mu}{k}$$

$$\frac{h_c x}{k} = 0.332 \left(\frac{U_\infty \times \rho}{\mu}\right)^{\frac{1}{2}} \left(\frac{c_p \mu}{k}\right)^{\frac{1}{3}}$$

By the ideal gas law

$$\rho = P/(RT)$$

Where p = Pressure

R = Gas constant

T = Absolute temperature

$$\therefore \quad h_c = 0.332\ c_p^{\frac{1}{3}}\ \mu^{-\frac{1}{6}}\ k^{\frac{2}{3}} \left(\frac{p}{R}\right)^{\frac{1}{2}} T^{-\frac{1}{2}} \sqrt{\frac{U_\infty}{x}}$$

$$\therefore \quad C = 0.332\ c_p^{\frac{1}{3}}\ \mu^{-\frac{1}{6}}\ k^{\frac{2}{3}} \left(\frac{p}{R}\right)^{\frac{1}{2}}$$

C is constant if the following restrictions apply

Constant pressure

Variation of thermal properties with temperature is negligible

$$\mathbf{h_c = 0.332}\, c_p^{\frac{1}{3}}\ \mu^{-\frac{1}{6}}\ k^{\frac{2}{3}} \left(\frac{p}{R}\right)^{\frac{1}{2}} T^{-\frac{1}{2}} \sqrt{\frac{U_\infty}{x}}$$

PROBLEM 5.12

Replot the data points of Figure 5.9(b) on log-log paper and find an equation approximating the best correlation line. Compare your results with Figure 5.10. Then, suppose that steam at 1 atm and 100°C is flowing across a 5-cm-*OD* pipe at a velocity of 1 m/s. Using the data in Figure 5.10, estimate the Nusselt number, the heat transfer coefficient, and the rate of heat transfer per meter length of pipe if the pipe is at 200°C and compare with predictions from your correlation equation.

GIVEN

Figure 5.9(b) in text
Steam flowing across a pipe
Steam pressure = 1 atm
Steam temperature (T_s) = 100°C
Pipe outside diameter (D) = 5 cm = 0.05 m
Steam velocity (U_∞) = 1 m/s
Pipe temperature (T_p) = 200°C

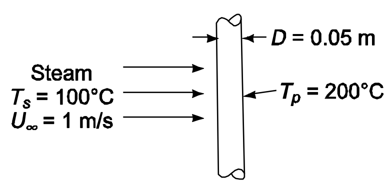
FIND

(a) Replot Figure 5.9(b) on log-log paper and find an equation approximating the best correlation line
(b) Find the Nusselt number (Nu), the heat transfer coefficient (hc), and the rate of heat transfer per unit length (q/L) using Figure 5.10
(c) Compare results with your correlated equation

ASSUMPTIONS

Steady state
Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 35, for steam at 1 atm and 100°C

Thermal conductivity (k) = 0.0249 W/(m K)

Kinematic viscosity (ν) = 20.2×10^{-6} m²/s

Thermal diffusivity (α) = 0.204×10^{-4} m²/s

SOLUTION

(a) The data taken from Figure 5.9(b) is shown below and plotted on a log-log scale

Re	*Nu*	Log *Re*	Log *Nu*
240	9	2.38	0.95
500	12	2.70	1.08
1,000	18	3.00	1.26
1,800	19	3.26	1.28
2,000	20	3.30	1.30
4,100	30	3.61	1.48
7,000	39	3.85	1.59
13,500	62	4.13	1.79
20,000	88	4.30	1.94
28,000	110	4.45	2.04
42,000	135	4.62	2.13
50,000	150	4.70	2.18

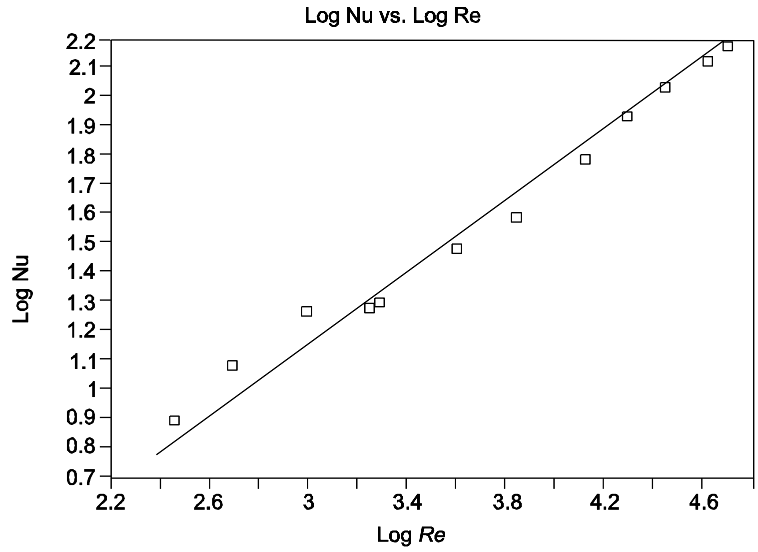


Figure 4.9(b) Variation of Nusselts Number with Reynolds Number on a log-log Scale

Fitting this data with a linear least squares regression yields:

$$\log Nu = 0.615 \log Re_D - 0.687$$

or

$$Nu = 0.21\, Re_D^{0.615}$$

(b) For the given data,

$$Re_D = \frac{U_\infty D}{\nu} = \frac{1.0\,\text{m/s} \quad 0.05\,\text{m}}{20.2\times10^{-6}\,\text{m}^2/\text{s}} = 2475$$

$$Pr = \frac{\nu}{\alpha} = \frac{20.2\times10^{-6}\,\text{m}^2/\text{s}}{0.204\times10^{-6}\,\text{m}^2/\text{s}} = 0.990$$

Although Figure 5.10 applies to Reynolds numbers between 3 and 100, we will apply its results to the larger Reynolds number for this case for the purpose of comparison

$$\frac{Nu_D}{Pr^{0.3}} = 0.82\, Re_D^{0.4}$$

$$Nu_D = 0.82\, Pr^{0.3}\, Re_D^{0.4} = 0.82\,(0.99)_{0.3}\,(2475)^{0.4} = 18.6$$

From Table 5.3

$$Nu_D = \frac{h_c D}{k}$$

$$\therefore \quad h_c = \frac{Nu_D k}{D} = \frac{18.6 \;\; 0.0249\,\text{W/(m K)}}{0.05\,\text{m}} = 9.27\ \text{W/(m}^2\ \text{K)}$$

The rate of convective heat transfer is given by Equation (1.10)

$$q = h_c A\, \Delta T = h_c\, \pi D L\, (T_p - T_s)$$

$$\therefore \quad \frac{q}{L} = h_c\, \pi D\, (T_p - T_s) = \;\; 9.27\,\text{W/(m}^2\,\text{K)} \quad \pi\,(0.05\ \text{m})\,(200°\text{C} - 100°\text{C}) = 145.6\ \text{W/m}$$

(c) The correlation from part (a) yields

$$Nu = 0.21\,(2475)^{0.615} = 25.7$$

$$h_c = \frac{Nu\,k}{D} = \frac{25.7\;\;0.0249\,\mathrm{W/(m\,K)}}{0.05\,\mathrm{m}} = 12.8\ \mathrm{W/(m^2\,K)}$$

$$\frac{q}{L} = h_c\,\pi D\,(T_p - T_s) = \;\;12.8\,\mathrm{W/(m^2K)}\;\;\;\pi\,(0.05\ \mathrm{m})\,(200°\mathrm{C} - 100°\mathrm{C}) = 201.0\ \mathrm{W/m}$$

The results obtained from Figure 5.10 are 28% lower than these results.

COMMENTS

The use of Figure 5.10 for a Reynolds number larger than 100 is inappropriate and in this case leads to a significant underestimation of the heat transfer coefficient. On the other hand, the correlation equation we developed from Figure 5.9(b) is strictly valid for air only. Since the Prandtl number for steam is different than that of air, we introduce an (unknown) error in using the data of Figure 5.9(b) to predict heat transfer to steam.

PROBLEM 5.13

The torque due to the frictional resistance of the oil film between a rotating shaft and its bearing is found to be dependent on the force *F* normal to the shaft, the speed of rotation *N* of the shaft, the dynamic viscosity μ of the oil, and the shaft diameter *D*. Establish a correlation among the variables by using dimensional analysis.

GIVEN

The oil film between a rotating shaft and its bearing

The torque (T) due to frictional resistance is a function of normal force (F), speed of rotation (N), dynamic viscosity (μ), and shaft diameter (D)

FIND

A correlation among the variables

ASSUMPTIONS

Steady state

SOLUTION

The Buckingham π Theorem (Sections 5.7.2 and 5.7.3) can be used to find the correlation. The primary dimensions of the variables are listed below

	Variable	Symbol	Dimensions
1.	Normal Force	F	$[M\,L/t^2]$
2.	Speed of Rotation	N	$[1/t]$
3.	Dynamic Viscosity	μ	$[M/L\,t]$
4.	Shaft Diameter	D	$[L]$
5.	Torque	T	$[M\,L^2/t^2]$

There are 5 variables and 3 primary dimensions. Therefore, two dimensionless groups are needed to correlate the variables

$$\pi = T^a\, F^b\, N^c\, \mu^d\, D^e$$

In terms of the primary dimensions

$$[\pi] = \left[ML^2/t^2\right]^a \left[ML/t^2\right]^b [1/t]^c\,[M/(Lt)]^d\ [L]^e = 0$$

Equation the sum of the exponents of each primary dimension to zero

For μ: $a + b + d = 0$ [1]

For L: $2a + b - d + e = 0$ [2]

For t: $2a + 2b + c + d = 0$ [3]

By inspection of equation [1] and [3]: $c = d$

There are five unknowns but only 3 equations. Therefore, the value of two of the exponents can be chosen for each dimensionless group.

For π_1: Let $a = 1$ and $b = 0$

From equation [1] $d = -1 = c$

From equation [2] $e = -3$

$$\therefore\ \pi_1 = T\,N^{-1}\,\mu^{-1}\,D^{-3} = T\big/\left(N\mu D^3\right)$$

For π_2: Let $a = 0$ and $b = 1$

From equation [1] $d = -1 = c$

From equation [2] $e = -2$

$$\therefore\ \pi_2 = F\,N^{-1}\,\mu^{-1}\,D^{-2} = F\big/\left(N\mu D^2\right)$$

From Equation (5.24)

$$\pi_1 = f(\pi_2) \qquad \therefore\ T\big/\left(N\mu D^3\right) = f\left(F\big/\left(N\mu D^2\right)\right)$$

PROBLEM 5.14

A series of tests in which water was heated while flowing through a 1 m-long electrically heated tube of 1.3 cm *ID* yielded the experimental pressure-drop data shown next.

Mass Flow Rate $\dot{m}$ (kg/s)	Fluid Bulk Temp T_b (°C)	Tube Surface Temp T_s (°C)	Pressure Drop with Heat Transfer Δp_{ht} (kPa)
1.37	32	52	67
0.98	45	94	33
0.82	36	104	23
1.39	37	120	58
0.97	42	140	31

Isothermal pressure-drop data for the same tube are given in terms of the dimensionless friction factor $f = (\Delta p/\rho\,\bar{u}^2)\,(2D/L)g_c$ and Reynolds number based on the pipe diameter, $Re_D = \bar{u}\,D/\nu = 4\dot{m}/\pi D\mu$. The symbol $\bar{u}$ denotes the average pipe velocity.

Re_D	1.71×10^5	1.05×10^5	1.9×10^5	2.41×10^5
f	0.0189	0.0205	0.0185	0.0178

By comparing the isothermal with the non-isothermal friction coefficients at similar bulk Reynolds numbers, derive a dimensionless equation for the non-isothermal friction coefficients in the form

$$f = \text{constant} \times Re_D^n\,(\mu_s/\mu_b)^m$$

where μ_s **= viscosity at surface temperature**

μ_b **= viscosity at bulk temperature**

n **and** m **= empirical constants.**

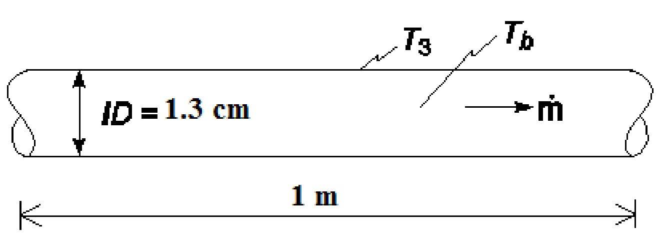
GIVEN

Water flowing through a tube
Isothermal and nonisothermal pressure drop data given above
The dimensionless friction factor (f) = $(\Delta p/\rho\bar{u}^2)\,(2D/L)g_c$
Reynolds number (Re_D) = $4\,\dot{m}/\pi D\,\mu$
Inside tube diameter (D) = 1.3 cm = 0.013 m
Tube length (L) = 1 m

FIND

Dimensionless equation of the form: $f = \text{constant} \times Re_d^n\,(\mu_s/\mu_b)^m$

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water

Temperature (°C)	Abs. Viscosity, $\mu \times 10^6$ (kg/m s)
32	763
45	605.1
36	707.4
37	695
42	631.6
52	541
94	300
104	270
120	235
140	201

SOLUTION

The exponent n will be determined from the isothermal data by the least squares fit for log Re_D vs. log f

$x = \log Re_D$	$y = \log f$
5.23	–1.724
5.02	–1.688
5.28	–1.733
5.38	–1.750

The least squares straight line fit for the data is

$$\log f = -0.823 - 0.172 \log Re_D$$

or

$$f = 0.149\, Re_D^{-0.1715}$$

The data and straight line fit are shown below

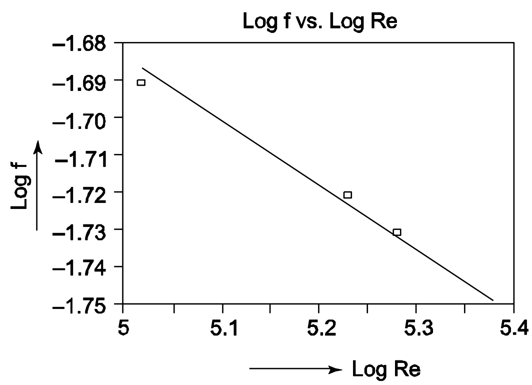


Figure Problem 4.74 (a): Plot of log f with Respect to log Re

Evaluating Re_D (based on the bulk temperature), 0.149 $Re^{-0.1715}$, f (based on the bulk temperature), and μ_s/μ_f for the non-isothermal case

$Re_D \times 10^{-5}$	$0.149\, Re_D^{-0.1715}$	f	μ_s/μ_b
1.73	0.0189	0.0187	0.709
1.55	0.0192	0.0182	0.500
1.11	0.0203	0.0175	0.381
1.91	0.0185	0.0160	0.340
1.45	0.0194	0.0173	0.315

$$\text{Let} \quad y = \log\left(\frac{f}{0.149\, Re_D^{-0.1712}}\right) \text{ and } x = \log\left(\frac{\mu_s}{\mu_b}\right)$$

y	x
–0.0046	–0.150
–0.0232	–0.302
–0.0666	–0.419
–0.0654	–0.469
–0.0520	–0.502

The linear least square fit for this data is

$$y = 0.1649 \times + 0.01976$$

Therefore: $f = 0.156\, Re_D^{-0.1715}\, (\mu_s/\mu_b)^{0.1649}$

The data for x and y and the straight line fit are shown below

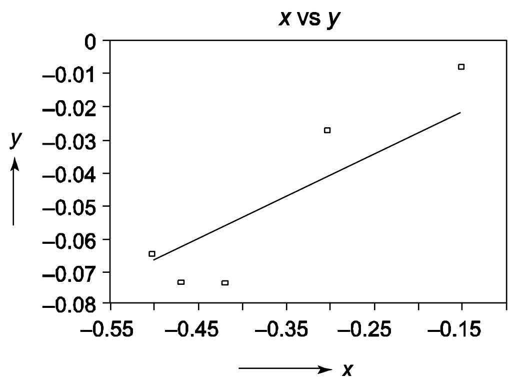


Figure Problem 4.14 (b): Plot of Variables x and y

Comparing the correlation to the experimental data

f, experimental	f, correlation	% difference
0.0187	0.0186	–0.31
0.0182	0.0179	–1.6
0.0175	0.0181	3.4
0.0160	0.0162	1.2
0.0173	0.0168	–2.9

PROBLEM 5.15

The experimental data shown tabulated were obtained by passing *n*-butyl alcohol at a bulk temperature of 15°C over a heated flat plate (0.3-m-long, 0.9-m-wide, surface temperature of 60°C). Correlate the experimental data using appropriate dimensionless numbers and compare the line which best fits the data with Equation 5.38.

Velocity (m/s)	**0.089**	**0.305**	**0.488**	**1.14**
Average heat transfer coefficient W/(m^2°C)	**121**	**218**	**282**	**425**

GIVEN

n-butyl alcohol flowing over a heated flat plate
Bulk temperature (T_b) = 15°C
Plate surface temperature (T_p) = 60°C
Plate length (L) = 0.3 m
Plate width (w) = 0.9 m
The experimental data given above

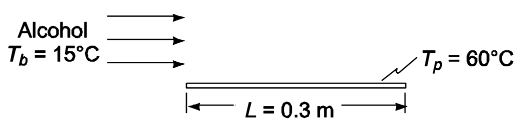
FIND

(a) Correlate the data by appropriate dimensionless numbers
(b) Compare line which best fits the data with Equation 5.38

ASSUMPTIONS

Steady state
Alcohol flows parallel to the length of the plate
Plate temperature is uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 19: For *n*-butyl alcohol at the average of the bulk and surface temperatures (known as the film temperature): 37.5°C.

Absolute viscosity (μ) = 1.92×10^{-3} N s/m^2

Thermal conductivity (k) = 0.166 W/(m K)

Density (ρ) = 796 kg/m^3

Prandtl number (Pr) = 29.4

SOLUTION

(a) The relevant variables and their primary dimensions are listed below

Variable	Symbol	Dimensions
Heat transfer coefficient	$\bar{h}_c$	$[M/t^3\,T]$
Velocity	U_∞	$[L/t]$
Length of plate	L	$[L]$
Absolute viscosity	μ	$[M/Lt]$
Thermal conductivity	k	$[ML/t^3\,T]$
Density	ρ	$[M/L^3]$

Note: Specific heat should be included in this list, but we suspect that it will show up as a Prandtl number which is constant for the series of tests performed. Therefore, we can easily extract its contribution. There are 6 variables are 4 primary dimensions, therefore, they can be correlated with two dimensionless groups. These dimensionless groups can be determined by the Buckingham π Theorem (Sections 5.7.2 and 5.7.3).

$$\pi = \bar{h}_c^{\,a}\, U_\infty^{\,b}\, L^c\, \mu^d\, k^e\, \rho^f$$

Equating the primary dimensions

$$0 = \left[\frac{M}{t^3 T}\right]^a \left[\frac{L}{t}\right]^b [L]^c \left[\frac{M}{Lt}\right]^d \left[\frac{ML}{t^3 T}\right]^e \left[\frac{M}{L^3}\right]^f$$

Equating the sums of the exponents of each primary dimension

For T:	$0 = -a - e$	[1]
For M:	$0 = a + d + e + f$	[2]
For t:	$0 = -3a - b - d - 3e$	[3]
For L:	$0 = b + c - d + e - 3f$	[4]

There are four equations and six unknowns. Therefore, the values of two of the exponents may be chosen for each dimensionless group.

For π_1, Let $f = 1$ $e = 0$

From equation [1]: $a = 0$

From equation [2]: $d = -1$

From equation [3]: $b = 1$

From equation [4]: $c = 1$

$$\therefore \quad \pi^1 = U_\infty L\, \mu^{-1} \rho = \frac{\rho u_\infty L}{\mu} = Re_L$$

For π_2, Let $a = 1$ $d = 0$

From equation [1]: $e = -1$

From equation [2]: $f = 0$

From equation [3]: $b = 0$

From equation [4]: $c = 1$

$$\therefore \quad \pi^2 = \bar{h}_c L\, k^{-1} = \frac{\bar{h}_c L}{k} = Nu$$

The range of Prandtl number is insufficient to get a functional relationship, therefore the data can be correlated by the Nusselt number and the Reynolds number:

$$Nu = f(Re)$$

Calculating Re_L and Nu for each data point

U_∞(m/s)	$\bar{h}_c$ W/(m^2 K)	$Re_L \times 10^{-4}$	$\overline{Nu}$
0.089	121	1.11	218.7
0.305	218	3.79	394.0
0.488	282	6.07	509.6
1.14	425	14.2	768.1

On a log-log plot, these points fall roughly on a straight line

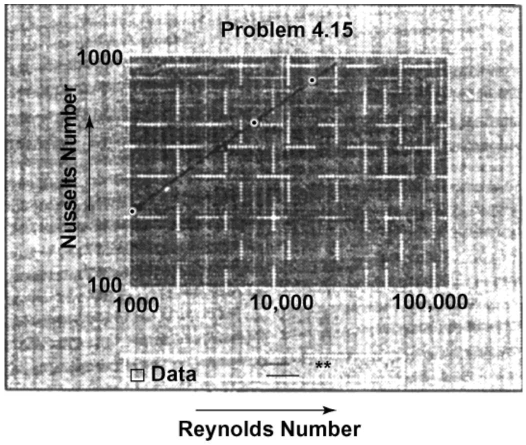


Figure Problem 4.15: Plot of *Nu* vs *Re* on log-log scale

The linear regression gives the following line

$$\log \overline{Nu} = 0.494 \log Re_L + 0.339$$

or

$$\overline{Nu} = 2.185\, Re_L^{\,0.494}$$

(b) For this problem, $Pr = 29.4$. Including this in the correlation

$$\overline{Nu} = 0.708\, Re_L^{0.494}\, Pr^{0.33}$$

Equation 5.38 for laminar flow over a flat plate is

$$\overline{Nu} = 0.664\, Re_L^{0.5}\, Pr^{0.33}$$

which is about 7% less than our experimental data.

PROBLEM 5.16

The test data tabulated on the next page were reduced from measurements made to determine the heat-transfer coefficient inside tubes at Reynolds numbers only slightly above transition and at relatively high Prandtl numbers (as associated with oils). Tests were made in a double-tube exchanger with a counterflow of water to provide the cooling. The pipe used to carry the oils was 1.5 cm-*OD*, 18 BWG, 3 m-long. Correlate the data in terms of appropriate dimensionless parameters.

Test No.	Fluid	h_c	ρu	c_p	k_f	μ_b*10^3	μ_f*10^3
11	10C oil	490	1450	1.971	0.1349	5.63	8.01
19	10C oil	725	2030	1.976	0.1349	5.49	7.85
21	10C oil	1500	3320	2.034	0.1342	3.96	5.75
23	10C oil	810	1445	2.072	0.1337	3.06	4.09
24	10C oil	940	3985	1.896	0.1358	9.82	11.22
25	10C oil	770	1400	2.076	0.1337	3.0	4.81
36	1488 pyranol	800	2425	1.088	0.1273	4.97	6.95
39	1488 pyranol	760	3840	1.088	0.1280	9.5	12.00
45	1488 pyranol	1025	2680	1.088	0.1271	4.25	5.3
48	1488 pyranol	715	5180	1.088	0.1285	16.42	22.00
49	1488 pyranol	600	4370	1.088	0.1285	16.32	18.7

where

h_c = mean surface heat-transfer coefficient based on the mean temperature difference, W/(m^2 K)

ρu = mass velocity, kg/(m^2 s)

c_p = specific heat, kJ/(kg K)

k_f = thermal conductivity, W/(m K) (based on film temperature)

μ_b = viscosity, based on average bulk (mixed mean) temperature, kg/(m s)

μ_f = viscosity, based on average film temperature, kg/(m s)

Hint: Start by correlating *Nu* and Re_D irrespective of the Prandtl numbers, since the influence of the Prandtl number on the Nusselt number is expected to be relatively small. By plotting *Nu* vs. *Re* on log-log paper, one can guess the nature of the correlation equation, $Nu = f_1\,(Re)$. A plot of $Nu/f_1\,(Re)$ vs. *Pr* then reveals the dependence upon *Pr*. For the final equation, the influence of the viscosity variation also is considered.

GIVEN

Oil in a counterflow heat exchanger
Pipe specifications: 1.5 cm-*OD*, 18 BWG
Pipe length (L) = 3 m
The experimental data above

FIND

Correlate the data in terms of appropriate dimensionless parameters

ASSUMPTIONS

The data represents the steady state for each case

PROPERTIES AND CONSTANTS

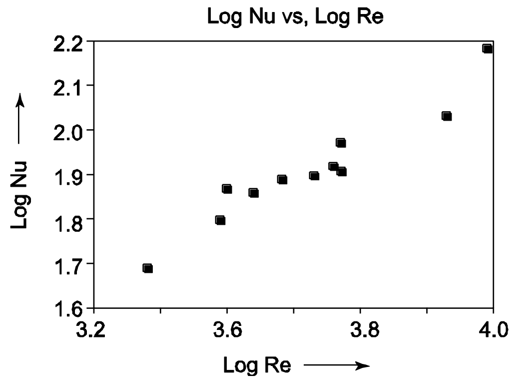
From Appendix 2, Table 42: for 1.5 cm O*D*, 18 BWG tubing, the inside diameter D = 1.338 cm

SOLUTION

The appropriate dimensionless parameters are the average Nusselt number ($Nu = h_c\ D/k_f$). The Reynolds number ($Re_D = \rho u D/\mu_f$) and the Prandtl number ($Pr = C_p\ \mu_f/k$). The values of the dimensionless parameters for each test are listed below.

Test no.	Nu	$Re_D \times 10^{-3}$	Pr	log Nu	log Re
11	49.0	2.41	117.9	1.69	3.38
19	72.3	3.46	115.7	1.86	3.54
21	149.9	7.72	87.7	2.18	3.89
23	81.7	4.73	63.7	1.91	3.67
24	93.3	4.75	157.7	1.97	3.68
25	77.4	3.89	75.1	1.89	3.59
36	84.0	4.66	59.7	1.92	3.67
39	79.4	4.27	102.6	1.90	3.63
45	108.4	6.76	45.6	2.03	3.83
48	74.7	3.16	187.2	1.87	3.50
49	62.5	3.13	159.9	1.80	3.49

Plotting log Nu vs. log Re_D reveals a roughly linear relationship.



Fitting a least squares regression line to the data

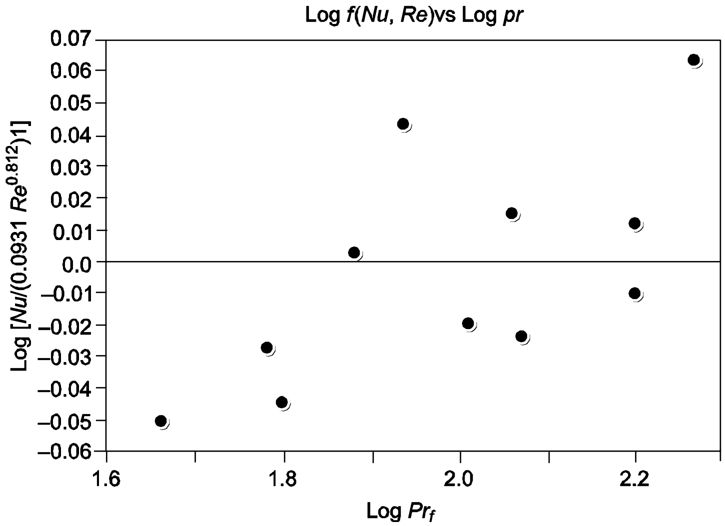
$$\log Nu = -1.0314 + 0.812 \log Re$$

or

$$Nu = 0.0931\, Re^{0.812}$$

The variation of Nu with Pr_f can be determined by plotting log $[Nu/(0.0931\, Re^{0.812})]$ vs. log Pr_f.

log $[Nu/(0.0931\, Re^{0.812})]$	log Pr_f
–0.0252	2.07
0.0165	2.06
0.0501	1.94
–0.0405	1.80
0.0156	2.20
0.0047	1.88
–0.0240	1.78
–0.0171	2.01
–0.0438	1.66
0.0624	2.27
–0.0108	2.20



Although there is considerable scatter in this plot, it does follow a trend of increasing log Pr_f with increasing log $[Nu/(0.0931\ Re^{0.812})]$ and will be fit with a straight least squares regression line. A least squares fit yields

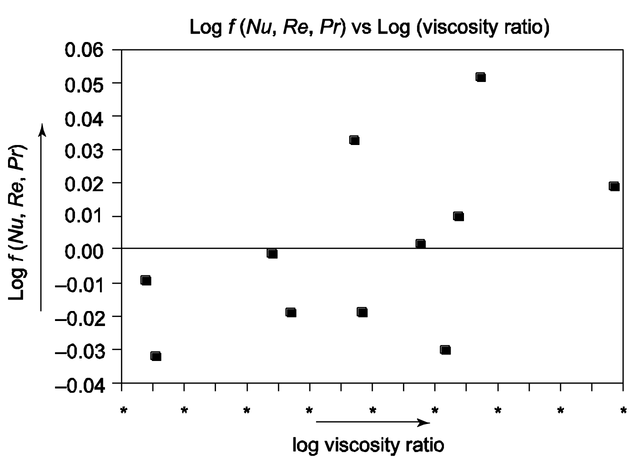
$$\log [Nu/(0.0931\ Re^{0.812})] = -0.2152 + 0.1076 \log Pr_f$$

or

$$Nu = 0.0567\ Re^{0.812}\ Pr_f^{0.108}$$

Plotting $\log [Nu/0.0567\ Re^{0.812}\ Pr_f^{0.108}]$ vs. $\log (\mu_f/\mu_b)$

$\log [Nu/0.0567\ Re^{0.812}\ Pr_f^{0.108}]$	$\log (\mu_f/\mu_b)$
–0.0335	0.153
0.0900	0.157
0.0557	0.164
–0.0200	0.127
–0.0064	0.058
0.0175	0.207
0.00041	0.145
–0.0190	0.104
–0.0076	0.098
0.0323	0.124
0.0334	0.061



Fitting these points with a straight least squares regression line

$$\log\left[\frac{Nu}{0.0567\,Re^{0.812}\,Pr_f^{0.108}}\right] = -\,0.0385 + 0.2993\,\log\left(\frac{\mu_f}{\mu_b}\right)$$

or

$$Nu = 0.0519\,Re^{0.812}\,Pr_f^{0.108}\left(\frac{\mu_f}{\mu_b}\right)^{0.2993}$$

Test No.	Experimental Nu	$0.0432\,Re^{0.828}\,Pr_f^{0.118}\left(\frac{\mu_f}{\mu_b}\right)^{0.3128}$
11	49.0	53.8
19	72.2	72.1
21	149.8	134.8
23	81.7	85.3
24	93.2	90.0
25	77.4	78.3
36	83.9	84.8
39	79.4	81.4
45	108.3	107.8
48	74.7	69.0
49	62.5	64.4

PROBLEM 5.17

A turbine blade with a characteristic length of 1 m is cooled in an atmospheric pressure wind tunnel by air at 40°C and a velocity of 100 m/s. For a surface temperature of 500 K, the cooling rate is found to be 10,000 watts. Use these results to estimate the cooling rate from another turbine blade of similar shape, but with a characteristic length of 0.5 m operating with a surface temperature of 600 K in air at 40°C and a velocity of 200 m/s.

GIVEN

A turbine blade in a wind tunnel
Length of blade (L_1) = 1 m
Air temperature (T_{ai}) = 40°C = 313 K
Air velocity ($U_{\infty 1}$) = 100 m/s
Air pressure = 1 atm
Blade surface temperature (T_s) = 500 K
Cooling rate (q) = 10,000 W

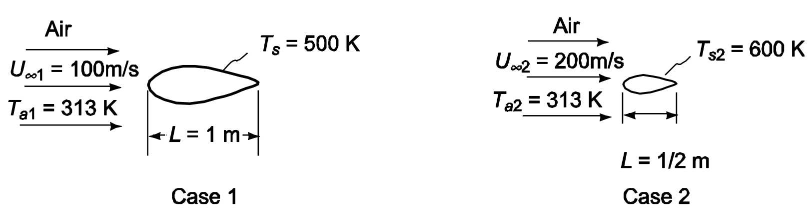
FIND

Cooling rate from a similar blade with a characteristic length (L_2) of 0.5 m and a surface temperature (T_{s2}) of 600 K and a velocity ($U_{\infty 2}$) of 200 m/s.

ASSUMPTIONS

Steady state for both cases
Uniform blade surface temperature
Air temperature is constant and the same in both cases

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the film temperatures

	Case 1	Case 2
	T = 406.5 K	T = 456.5 K
Kinematic viscosity, $\nu \times 10^6$ (m²/s)	27.6	33.2
Thermal conductivity, k W/(m K)	0.0328	0.0360

SOLUTION

Important variables

	Dimensions
Cooling rate, q	$[M\,L^2/t^3]$
Length, L	$[L]$
Air–blade Temperatures ($T_s - T_b$)	$[T]$
Air Velocity, U_∞	$[L/t]$
Kinematic Viscosity, ν	$[L^2/t]$
Thermal Conductivity, k	$[M\,L/t^3\,T]$

The (6 – 4 = 2) dimensionless groups can be determined by the Buckingham p theory

$$\pi = q^a\, L^b\, (T_s - T_b)^c\, U_\infty^d\, \nu^e\, k^f$$

Equating the primary dimensions

$$0 = \left[\frac{ML^2}{t^3}\right]^a [L]^b\,[T]^c \left[\frac{L}{t}\right]^d \left[\frac{L^2}{t}\right]^e \left[\frac{ML}{t^3 T}\right]^f$$

For M: $a + f = 0$

For T: $c - f = 0$

For t: $3a + d + e + 3f = 0$

For L: $2a + b + d + 2e + f = 0$

π_1: Let $a = 0$ and $d = 1 \rightarrow f = 0$; $c = 0$; $e = -1$; $b = 1$

$$\pi_1 = \frac{U_\infty L}{v} = Re_L$$

π_2: Let $a = 1$ and $d = 0 \rightarrow f = -1$; $c = -1$; $e = 0$; $b = -1$

$$\pi_2 = \frac{q}{L\ \ T_s - T_a\ \ k}$$

$$\therefore\ \frac{q}{L\ \ T_s - T_a\ \ k} = f(Re_L)$$

Assume that the function has the form

$$\frac{q}{L\ \ T_s - T_a\ \ k} = Re_L{}^m$$

The data of the larger blade can be used to evaluate m

$$Re_L = \frac{U_\infty L}{v} = \frac{100\,\text{m/s}\ \ 1.0\,\text{m}}{27.6 \times 10^{-6}\,\text{m}^2/\text{s}} = 3.62 \times 10^6$$

$$m = \frac{\log\left[\dfrac{q}{L\ \ T_s - T_a\ \ k}\right]}{\log Re_L} = \frac{\log\left[\dfrac{10{,}000\,\text{W}}{1\text{m}\ \ 500\,\text{K} - 313\,\text{K}\ \ \ 0.0328\,\text{W/(m K)}}\right]}{\log\ \ 3.62 \times 10^6} = 0.490$$

$$\therefore\quad q = L\,k\,(T_s - T_a)\,Re_L{}^{0.49}$$

Applying this to the smaller blade

$$Re_L = \frac{U_\infty L}{v} = \frac{200\,\text{m/s}\ \ 0.5\,\text{m}}{33.2 \times 10^{-6}\,\text{m}^2/\text{s}} = 3.0 \times 10^6$$

$$q = 0.5\ \text{m}\ \ \ 0.0360\,\text{W/(m K)}\ \ \ (600\ \text{K} - 313\ \text{K})\ (3.0 \times 10^6)^{0.49} = 7723\ \text{W}$$

PROBLEM 5.18

The drag on an airplane wing in flight is known to be a function of density of air (ρ), viscosity of air (μ), free-stream velocity (U_∞), characteristic dimension of the wing(S) *and the* shear stress on the surface of the wing (τ_s).

Show that the dimensionless drag

$$\frac{\tau_s}{\rho U_\infty^{\ 2}}$$

can be expressed as a function of the Reynolds number

$$\frac{\rho U_\infty S}{\mu}.$$

GIVEN

An airplane wing in flight

Drag on wing (D) = $f(\rho, \mu, U_\infty, S, \tau_s)$

FIND

$$\text{Show that } \frac{\tau_s}{\rho U_\infty^{\ 2}} = f\left(\frac{\rho U_\infty S}{\mu}\right)$$

SOLUTION

The relevant variables and their dimensions are shown below

Variable	Symbol	Dimensions
Density	ρ	$[M/L^3]$
Viscosity	μ	$[M/Lt]$
Velocity	U_∞	$[L/t]$
Characteristic Dimensions	S	$[L]$
Shear Stress	τ_s	$[M/Lt^2]$

There are 5 variables and 3 primary dimensions. Therefore, the variables can be correlated with 2 dimensionless groups.

Using the Buckingham π theory (Sections 5.7.2 and 5.7.3)

$$\pi = \rho^a \mu^b U_\infty^c S^d \tau_s^e$$

In terms of the primary dimensions

$$0 = \left[\frac{M}{L^3}\right]^a \left[\frac{M}{Lt}\right]^b \left[\frac{L}{t}\right]^c [L]^d \left[\frac{M}{Lt^2}\right]^e$$

Equating the sum of the exponents of each primary dimension to zero

For M: $0 = a + b + e$ [1]

For t: $0 = -b - c - 2e$ [2]

For L: $0 = -3a - b + c + d - e$ [3]

Since there are 5 unknowns and only 3 equations, the value two exponents may be chosen for each dimensionless group

For π_1: Let $e = 1$ and $a = -1$

From equation [1]: $b = 0$

From equation [2]: $c = -2$

From equation [3]: $d = 0$

$$\pi_1 = \rho^{-1} U_\infty^{-2} \tau_s = \frac{\tau_s}{\rho U_\infty^{\ 2}}$$

For π_2: Let $a = 1$ and $b = -1$

From equation [1]: $e = 0$

From equation [2]: $c = 1$

From equation [3]: $d = 1$

$$\pi_2 = \rho\,\mu^{-1}\,U_\infty\,S = \frac{\rho U_\infty S}{\mu}$$

As shown in Equation (5.24)

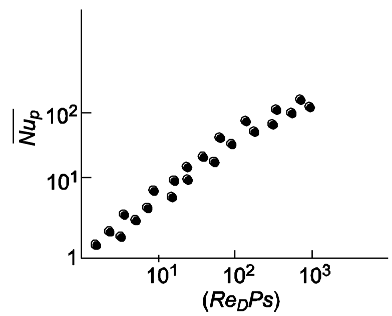
$$\pi_1 = f\,(\pi_2)$$

or

$$\frac{\tau_s}{\rho U_\infty{}^2} = f\left(\frac{\rho U_\infty S}{\mu}\right)$$

PROBLEM 5.19

Suppose that the graph below shows measured values of h_c for air in forced convection over a cylinder of diameter D plotted on a logarithmic graph of Nu_D as a function of $Re_D Pr$.



Write an appropriate dimensionless correlation for the average Nusselt number for these data and state any limitations to your equation.

GIVEN

Forced convection of air over a cylinder
Experimental data given above

FIND

An appropriate dimensionless correlation for the average Nusselt number

SOLUTION

The data lies along an approximately straight line on the log-log graph. Therefore, a straight line fit will be used. Choosing two points on the graph

$$[Nu_D = 1, (Re_D)(Pr) = 1] \text{ and } [Nu_D = 100, (Re_D)(Pr) = 1000]$$

A straight line on the log-log plot is represented by

$$\log(Nu_D) = a \log(Re_D\, Pr) + b$$

Substituting the two points into the equation and solving for a and b

$$\log(1) = a \log(1) + b \rightarrow b = 0$$

$$\log(100) = a \log(1000) + b \rightarrow a = 0.667$$

Therefore

$$\log(Nu_D) = 0.667 \log(Re_D\, Pr)$$

or

$$Nu_D = (Re_D\, Pr)^{0.667}$$

This is based on data in the range $1 < Re_d\, Pr < 10^3$ and is therefore valid only in this range.

PROBLEM 5.20

When viscous dissipation is appreciable, the conservation of energy [Eq. 5.6] must take into account the rate at which mechanical energy is irreversibly converted to thermal energy due to viscous effects in the fluid. This gives rise to an additional term, ϕ, on the right-hand side, the viscous dissipation where

$$\frac{\phi}{\mu} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\ \frac{\partial u}{\partial x}^{\ 2} + \left(\frac{\partial v}{\partial y}\right)^2\right] - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2$$

Apply the resulting equation to laminar flow between two infinite parallel plates, with the upper plate moving at a velocity *U*. Assuming constant physical properties [ρ, c_p, k, μ], obtain expressions for the velocity and temperature distributions. Compare the solutions with the dissipation term included with the results when dissipation is neglected. Find the plate velocity required to produce a 1 K temperature rise in nominally 40°C air relative to the case where dissipation is neglected.

GIVEN

Laminar flow between two infinite parallel plates
Upper plate moves at a velocity U_∞
The viscous dissipation term given above must be used in the conservation of energy equation

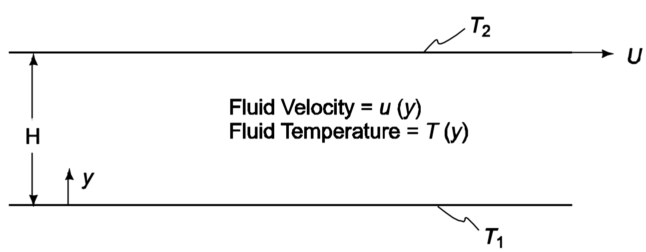
FIND

(a) Expression for velocity and temperature distributions
(b) Compare these to solutions without the dissipation term
(c) Plate velocity that gives a 1 K rise in 40°C air relative to the case without the dissipation term

ASSUMPTIONS

Steady state
Constant physical properties
The plates are at constant temperatures, T_1, T_2

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 40°C

Thermal conductivity (k) = 0.0265 W/(m K)

Absolute viscosity (μ) = 19.1×10^{-6} N s/m^2

SOLUTION

(a) Including the viscous dissipation term in Equation (5.6)

$$\rho\, c_p \left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$+ \mu \left\{\left(\frac{\partial u}{dy} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\ \frac{\partial u}{\partial x}^{\ 2} + \left(\frac{\partial v}{\partial y}\right)^2\right] - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2\right\}$$

Eliminating the terms which are zero for this case

$$0 = k\,\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2$$

(Note that since the left side of Equation (5.6) drops out completely, d^2T/dy^2 is multiplied by k and not by α-see Section 5.4.) For this case, the conservation of momentum Equation (5.5) reduces to:

$$0 = \frac{d^2u}{dy^2}$$

The boundary conditions for these equations are

1. $T = T_1$, $u = 0$ at $y = 0$
2. $T = T_2$, $u = U$ at $y = H$

Integrating the momentum equation twice yields

$$\frac{du}{dy} = c_1 \qquad u(y) = c_1\,y + c_2$$

Applying the first boundary condition: $c_2 = 0$
Applying the second boundary condition: $c_1 = U/H$
Therefore, the velocity distribution between the plates is

$$u(y) = U\,\frac{y}{H} \quad \Rightarrow \quad \frac{du}{dy} = \frac{U}{H}$$

Substituting this into the energy equation yields

$$0 = k\,\frac{d^2T}{dy^2} + \mu\left(\frac{U}{H}\right)^2 \quad \text{or} \quad \frac{d^2T}{dy^2} = -\frac{\mu}{k}\left(\frac{U}{H}\right)^2$$

Integrating twice

$$\frac{dT}{dy} = -\frac{\mu U^2}{k\,H^2}\,y + c_1 \qquad T(y) = -\frac{\mu U^2}{2k\,H^2}\,y_2 + c_1\,y + c_2$$

Applying the first boundary condition: $c_2 = T_1$
Applying the second boundary condition: $c_1 = (T_2 - T_1)/H + (U)/(2\,k\,H)$
Therefore, the temperature distribution is

$$T(y) = -\frac{\mu U^2}{2k\,H^2}\,y^2 + \left[\frac{T_2 - T_1}{H} + \frac{\mu U^2}{2k\,H}\right]y + T_1$$

$$T(y) = T_1 + (T_2 - T_1)\,\frac{y}{H} + \frac{\mu U^2}{2k}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

(b) When dissipation is neglected, the momentum equation, and therefore, the velocity distribution, remain unchanged. Without viscous dissipation, the energy equation is

$$0 = \frac{d^2T}{dy^2}$$

Integrating twice

$$\frac{dT}{dy} = c_1 \qquad T(y) = c_1\,y + c_2$$

From the first boundary condition: $c_2 = T_1$
From the second boundary condition: $c_1 = (T_2 - T_1)/H$

$$\therefore\ T(y) = T_1 + \frac{T_2 - T_1}{H}\, y$$

Including the viscous dissipation term leads to an increase in temperature of

$$\frac{\mu U^2}{2k}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

at each distance y from the lower plate.

(c) This temperature increase is a maximum at

$$\frac{d}{dy}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right] = 0 \quad \Rightarrow \quad y = \frac{H}{2}$$

At this point, the temperature increase is

$$\Delta T = \frac{\mu U^2}{8k} \quad \text{So} \quad U = \sqrt{\frac{8k\,\Delta T}{\mu}}$$

For $\Delta T = 1$ K

$$U = \sqrt{\frac{8\ \ 0.0265\,\mathrm{W/(m\,K)}\ \ (1\mathrm{K})}{19.1\times 10^{-6}\,\mathrm{Ns/m^2}\ \ \mathrm{Ws/Nm}}} = 105\ \mathrm{m/s}$$

PROBLEM 5.21

A journal bearing is idealized as a stationary flat plate and a moving flat plate that moves parallel to it. The space between the two plates is filled by an incompressible fluid. Consider such a bearing with the stationary and moving plates at 10°C and 20°C respectively, the distance between them is 3 mm, the speed of the moving plate is 5 m/s, and there is engine oil between the plates.

(a) Calculate the heat flux to the upper and lower plates

(b) Determine the maximum temperature of the oil.

GIVEN

Journal bearing: Two flat plates, one stationary, one moving with oil between them
Stationary plate temperature (T_s) = 10°C
Moving plate temperature (T_m) = 20°C
Distance between plate (H) = 3 mm = 0.003 m
Speed of the moving plate (U_p) = 5 m/s

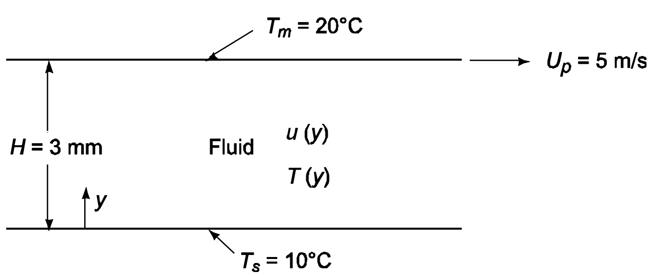
FIND

(a) Heat flux (q/A) for the plates
(b) The maximum temperature of the oil

ASSUMPTIONS

Steady state
Constant physical properties
Negligible edge effects
Oil is incompressible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 17, for engine oil at 15°C

Thermal conductivity (k) = 0.145 W/(m K)
Absolute viscosity (μ) = 1.561 (Ns)/m^2
Prandtl number (Pr) = 196

SOLUTION

(a) The temperature distribution for this geometry was derived in Problem 5.20

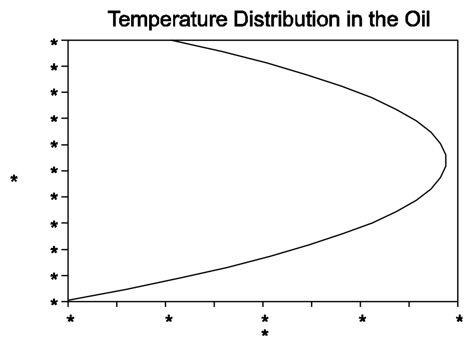
$$T(y) = T_s + (T_m - T_s)\left(\frac{y}{H}\right) + \frac{\mu U^2}{2k}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

For this case

$$T(y) = 10°\text{C} + (10°\text{C})\left(\frac{y}{H}\right) + \frac{1.561(\text{Ns})/\text{m}^2\,(5\,\text{m/s})^2}{2\,(0.145\,\text{W/(m K)})\,(\text{Nm})/(\text{Ws})}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

$$T(y) = 10°\text{C} + (10°\text{C}) \left(\frac{y}{H}\right) + (134.6\text{ K}) \left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

This is plotted below



The first derivative of the temperature is

$$\frac{dT}{dy} = \frac{T_m - T_s}{H} + \frac{\mu U^2}{2k}\left[\frac{1}{H} - 2\frac{y}{H^2}\right]$$

The heat flux at the top plate ($y = H$) is

$$\frac{q}{A} = -k\left.\frac{dT}{dy}\right|_{y=H} = -\frac{k(T_m - T_s)}{H} - \frac{1}{2}\mu U_p^2\left(\frac{1}{H} - \frac{2}{H}\right)$$

$$\frac{q}{A} = \frac{1}{H}\left[\frac{1}{2}\mu U_p^2 - k(T_m - T_s)\right]$$

$$\frac{q}{A} = \frac{1}{0.003\text{m}}\left[\frac{1}{2}\left(1.561(\text{Ns})/\text{m}^2\right)\left(5\text{m/s}\right)^2\left((\text{Ws})/(\text{Nm})\right) - 0.145\text{W}/(\text{m K})(20°\text{C} - 10°\text{C})\right]$$

$$\frac{q}{A} = 6020\text{ W/m}^2 \text{ (into the plate)}$$

The heat flux at the bottom plate ($y = 0$) is

$$\frac{q}{A} = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\left[\frac{T_m - T_s}{H} + \frac{\mu U_p^2}{2kH}\right] = -\frac{1}{H}\left[\frac{1}{2}\mu U_p^2 + k(T_m - T_s)\right]$$

$$\frac{q}{A} = \frac{1}{0.003\text{m}}\left[\frac{1}{2}\left(1.56(\text{Ns})/\text{m}^2\right)\left(5\text{m/s}\right)^2\left((\text{Ws})/(\text{Nm})\right) + 0.145\text{W}/(\text{m K})(20°\text{C} - 10°\text{C})\right]$$

$$\frac{q}{A} = -6980\text{ W/m}^2 \text{ (out of the plate)}$$

(b) The maximum temperature occurs where the first derivative is zero

$$0 = \frac{T_m - T_s}{H} + \frac{\mu U_p^2}{2k}\left[\frac{1}{H} - \frac{2y}{H^2}\right] \Rightarrow \frac{y}{H} = \frac{1}{2} + \frac{k(T_m - T_s)}{\mu U_p^2}$$

$$\frac{y}{H} = \frac{1}{2} + \frac{0.145\text{W}/(\text{m K})(20°\text{C} - 10°\text{C})}{\left(1.561(\text{Ns})/\text{m}^3\right)\left(5\text{m/s}\right)^2\left((\text{Ws})/(\text{Nm})\right)} = 0.537$$

$$y_{max} = 0.537(3 \text{ mm}) = 1.61 \text{ mm}$$

Checking the second derivative

$$\frac{d^2T}{dy^2} = -\frac{\mu U_p^2}{k H}$$

This is negative throughout the region, therefore, $T(y)$ is concave down throughout the region and the temperature at y = 1.6 mm is indeed the maximum temperature. These calculations are verified by the graph of $T(y)$. Inserting y_{max} into the expression for $T(y)$ yields: $T_{max} = T(0.00161 \text{ mm}) = 48.8°\text{C}$.

COMMENTS

The difference in heat fluxes at the plate $\Delta q''$ = 6980 W/m^2 – 6020 W/m^2 = 920 W/m^2 must equal the heat dissipated within the oil.

PROBLEM 5.22

A journal bearing has a clearance of 0.5 mm. The journal has a diameter of 100 mm and rotates at 3600 rpm within the bearing. It is lubricated by an oil having a density of 800 kg/m³, a viscosity of 0.01 kg/ms, and a thermal conductivity of 0.14 W/(m K). If the bearing surface is at 60°C, determine the temperature distribution in the oil film assuming that the journal surface is insulated.

GIVEN

A journal bearing
Diameter (D) = 100 mm = 0.1 m
Clearance (H) = 0.5 mm = 0.0005 m
Rotational speed (ω) = 3600 rpm
Oil properties

- Density (ρ) = 800 kg/m³
- Viscosity (μ) = 0.01 kg/ms
- Thermal conductivity (k) = 0.14 W/(m K)

Temperature of bearing surface (T_b) = 60°C

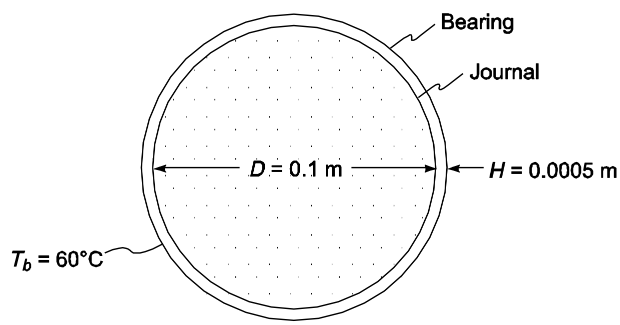
FIND

(a) Temperature distribution in the oil film

ASSUMPTIONS

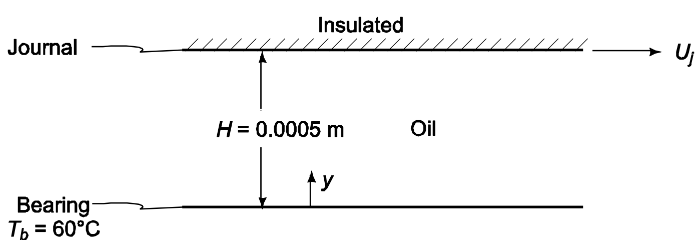
Steady state
Uniform and constant bearing surface temperature
Constant fluid properties
The journal surface is insulated (negligible heat transfer)

SKETCH



SOLUTION

Since the clearance is small compared to the bearing diameter, the bearing may be idealized as parallel flat plates with oil between them, one stationary and one moving



(a) As shown in Problem 5.20, the velocity distribution for this geometry is linear

$$u(y) = U_j \frac{y}{H} \quad \text{where: } U_j = \frac{D}{2}\,\omega = \frac{1}{2}\,(0.1\text{ m})$$

$$3600\,\frac{\text{rotations}}{\text{s}}\quad\frac{2\times\text{rad}}{\text{rotation}}\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 18.85\ \text{m/s}$$

For this geometry, the energy Equation (5.6) with viscous dissipation reduces to

$$k\,\frac{d^2t}{dy^2} = -\mu\left(\frac{du}{dy}\right)^2 = -\mu\left(\frac{U_j}{H}\right)^2$$

With boundary conditions: at $y = 0$ $T = T_b$

at $y = H$ $dT/dy = 0$ (insulated)

Integrating

$$k\,\frac{dt}{dy} = -\mu\left(\frac{U_j}{H}\right)^2 y + c_1$$

Applying the second boundary condition

$$c_1 = \frac{\mu U_j^2}{H} \Rightarrow k\,\frac{dt}{dy} = \mu\,\frac{U_j^2}{H}\,1 - \frac{y}{H}$$

Integrating

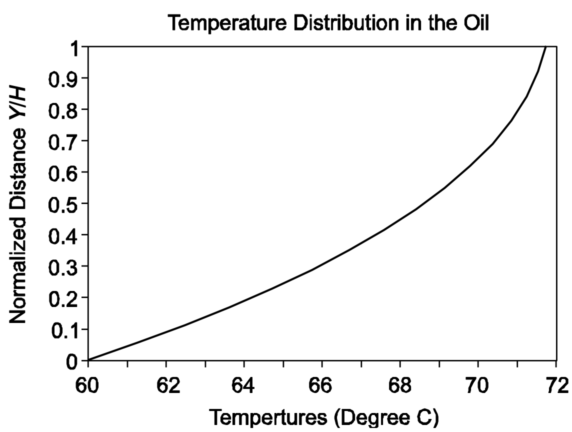
$$kT = \mu\,\frac{U_j^2}{H}\left(y - \frac{y^2}{2H}\right) + c_2 = \mu\,U_j^2\left[\frac{y}{H} - \frac{1}{2}\,\frac{y}{H}^{\,2}\right] + c_2$$

Applying the first boundary condition $c_2 = kT_b$

$$T = T_b + \frac{\mu U_j^2}{k}\left[\frac{y}{H} - \frac{1}{2}\,\frac{y}{H}^{\,2}\right]$$

$$T = 60°\text{C} + \frac{0.01\,\text{kg/sm}\ \ 18.85\,\text{m/s}^{\,2}}{0.14\,\text{W/(m K)}\ \ (\text{kg m}^2)/(\text{Ws}^3)}\left[\frac{y}{H} - \frac{1}{2}\,\frac{y}{H}^{\,2}\right] = 60°\text{C} + (25.38\ \text{K})\left[\frac{y}{H} - \frac{1}{2}\,\frac{y}{H}^{\,2}\right]$$

This is shown graphically below



PROBLEM 5.23

A journal bearing has a clearance of 0.5 mm. The journal has a diameter of 100 mm and rotates at 3600 rpm within the bearing. The journal is lubricated by an oil having a density of 800 kg/m^3, a viscosity of 0.01 kg/(ms), and a thermal conductivity of 0.14 W/(m K). Both the journal and the bearing temperatures are maintained at 60°C. Calculate the rate of heat transfer from the bearing and the power required for rotation per unit length.

GIVEN

A journal bearing
Diameter (D) = 100 mm = 0.1 m
Clearance (H) = 0.5 mm = 0.0005 m
Rotational speed (ω) = 3600 rpm
Oil properties

- Density (ρ) = 800 kg/m^3
- Viscosity (μ) = 0.01 kg/(ms)
- Thermal conductivity (k) = 0.14 W/(m K)

Temperature of both surface (T_b) = 60°C

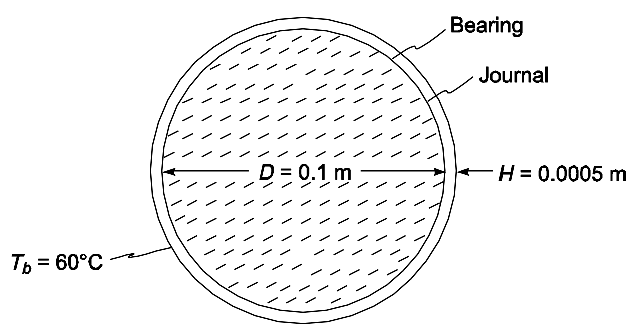
FIND

(a) The rate of heat transfer from the bearing
(b) The power required for rotation per unit length

ASSUMPTIONS

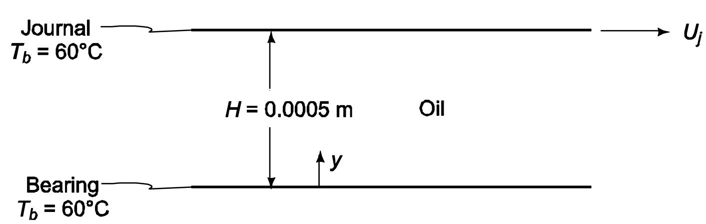
Steady state
Uniform and constant surface temperatures
Constant fluid properties

SKETCH



SOLUTION

Since the clearance is small compared to the bearing diameter, the bearing may be idealized as parallel flat plates with oil between them, one stationary and one moving



The temperature distribution for this geometry was derived in Problem 5.20

$$T(y) = T_b + (T_j - T_b)\ \frac{y}{H} + \frac{m U_j^2}{2k}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

From Problem 5.22: U_j = 18.85 m/s

$$T(y) = 60°\text{C} + \frac{0.1\,\text{kg/(ms)}\ (18.85\,\text{m/s})^2}{2\ (0.14\,\text{W/(m K)})\ \ (\text{kg m}^2)/(\text{Ws})^2}\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

$$T(y) = 60°\text{C} + (126.9\ \text{K})\left[\frac{y}{H} - \left(\frac{y}{H}\right)^2\right]$$

The rate of heat transfer to the bearing is given by Fourier's Law

$$q = -kA\ \frac{dt}{dy}\Big|_{y=0} = -k\,[\pi(D + 2H)\,L]\left[\frac{\mu U_j^2}{k H}\ \left(1 - \frac{y}{H}\right)\right]_{y=0}$$

$$\frac{q}{L} = -\pi\ \mu\, U_j^2\ \left(\frac{D}{H} + 2\right)$$

$$\frac{q}{L} = -\pi\ (0.01\,\text{kg/ms})\ (18.85\,\text{m/s})^2\left(\frac{0.1\,\text{m} + 0.001\,\text{m}}{0.0005\,\text{m}}\right)\ (\text{W s}^2)/(\text{kg m}^2) = -\ 2235\ \text{W/m}$$

(into the bearing)

The power required to turn the journal is the product of the drag force on the journal and the journal speed

$$P = F\,U_j = \tau_w\,A\,U_j = \pi\,D\,L\,U_j\ \tau_w$$

The shear stress (τ) is given by Equation (5.2)

$$\frac{P}{L} = \pi\,D\,U_j\left(\mu\frac{du}{dy}\right)_{y=H} = \pi\,D\,U_j\,\mu\ \frac{U_j}{H} = \frac{1}{H}\ \pi\,D\,\mu\,U_j^2$$

$$\frac{P}{L} = \frac{1}{0.0005\,\text{m}}\ \pi(0.1\ \text{m})\ \ (0.01\,\text{kg/(ms)})\ \ (18.85\,\text{m/s})^2\ \ (\text{Ws}^3)/(\text{kg m}^2) = 2233\ \text{W/m}$$

COMMENTS

Note that for conservation of energy, the power required (P/L) should be equal to the heat loss (q/L). The slight difference (0.09%) in the results is due to the difference in surface area of journal and the bearing which was not incorporated into the analysis.

PROBLEM 5.24

Engine oil at 100°C flows over and parallel to a flat surface at a velocity of 3 m/s. Calculate the thickness of the hydrodynamic boundary layer at a distance 0.3 m from the leading edge of the surface.

GIVEN

Engine oil flows over a flat surface
Engine oil temperature (T_b) = 100°C
Engine oil velocity (U_∞) = 3 m/s

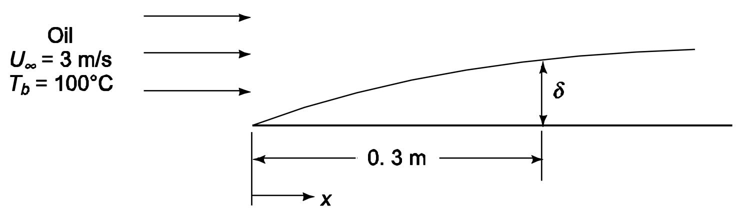
FIND

The hydrodynamic boundary layer thickness (δ) at a distance 0.3 m from the leading edge

ASSUMPTIONS

Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 17, for engine oil at 100°C

Kinematic viscosity (ν) = 20.3×10^{-6} m²/s

SOLUTION

The local Reynolds 0.3 m from the leading edge based on the bulk fluid temperature is

$$Re_x = \frac{U_\infty x}{\nu} = \frac{3.0\,\text{m/s}\ 0.3\,\text{m}}{20.3\times10^{-6}\ \text{m}^2/\text{s}} = 4.43 \times 10^4$$

Since $Re_x < 5 \times 10^5$, the boundary layer is laminar. The boundary layer thickness for laminar flow over a flat plate is given by Equation (5.28)

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5\ 0.3\,\text{m}}{\sqrt{4.43\times10^4}} = 7.1 \times 10^{-3}\ \text{m} = 7.1\ \text{mm}$$

PROBLEM 5.25

Assuming a linear velocity distribution and a linear temperature distribution in the boundary layer over a flat plate, derive a relation between the thermal and hydrodynamic boundary-layer thicknesses and the Prandtl number.

GIVEN

Boundary layer over a flat plate

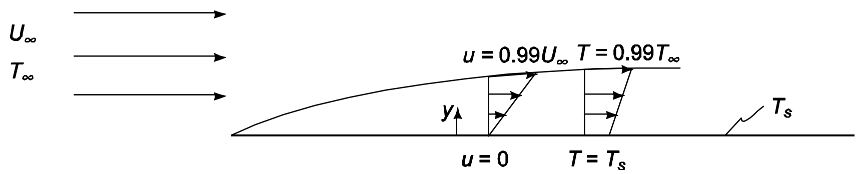
FIND

A relation between the thermal and hydrodynamic boundary-layer thicknesses and the Prandtl number

ASSUMPTIONS

Linear velocity and temperature distributions in the boundary layers

SKETCH



SOLUTION

Let Absolute viscosity of the fluid = μ

Plate surface temperature = T_s

Bulk fluid temperature = T_∞

Bulk fluid viscosity = U_∞

Density of the fluid = ρ

Thermal diffusivity of the fluid = α

The linear velocity profile will be used to solve the integral momentum equation first. The integral energy equation will then be solved and combined with the momentum solution.

Linear velocity profile: $u = u_o + ay$

Subject to $u = 0$ at $y = 0 \rightarrow u_o = 0$

$u = 0.99\, U_\infty \approx U_\infty$ at $y = \delta \rightarrow a = U_\infty/\delta$

therefore $u = (U_\infty/\delta)y$

Substituting this into the integral momentum equation for a laminar boundary layer (Equation 5.42)

$$\frac{d}{dx}\int_0^b \rho\left(\frac{U_\infty}{\delta}\, y\right)\left[U_\infty - \frac{U_\infty}{\delta}\, y\right] dy = \tau_w = \mu\left.\frac{du}{dy}\right|_{y=0}$$

(The wall shear stress (τ_w) is defined by Equation (4.2))

In this case, du/dy = constant = U_∞/δ

Integrating

$$\frac{d}{dx}\left[\frac{U_\infty\rho}{\delta}\left(\frac{U_\infty}{2}\, y^2 - \frac{U_\infty}{3\delta}\, y^3\right)\right]_0^b = \mu\,\frac{U_\infty}{\delta}$$

$$\frac{d}{dx}\left[\frac{U_\infty{}^2\rho\delta}{6}\right] = \mu\,\frac{U_\infty}{\delta}$$

$$\delta\, d\delta = \frac{6\mu}{\rho U_\infty}\, dx$$

Integrating

$$\frac{1}{2}\,\delta^2 = \frac{6\mu}{\rho U_\infty}\,x + c$$

At $x = 0$, $\delta = 0 \rightarrow c = 0$

$$\delta = \left(\frac{12\mu x}{\rho U_\infty}\right)^{\frac{1}{2}} = 3.46 \times \left(\frac{\mu}{\rho U_\infty x}\right)^{\frac{1}{2}} = 3.46 \times\ Re_x^{-\frac{1}{2}}$$

Linear temperature profile: $T = T_o + by$

Subject to $\quad T = T_s$ at $y = 0 \rightarrow T_o = T_s$

$T = 0.99\ T_\infty \approx T_\infty$ at $y = \delta_t \rightarrow b = \dfrac{T_\infty - T_s}{\delta_t}$

$$\therefore \quad T = T_s + \frac{T_\infty - T_s}{\delta_t}\,y$$

Substituting this and the expression for U into the integral energy equation of the laminar boundary layer for low speed flow (Equation 5.44)

$$\frac{d}{dx}\int_0^{\delta_t}\left[T_\infty - \left(T_s + \frac{T_\infty - T_s}{\delta_t}\,y\right)\right]\left(\frac{U_\infty}{\delta}\,y\right)dy - a\left[\frac{d}{dy}\left(T_s + \frac{T_\infty - T_s}{\delta_t}\,y\right)\right]_{y=0} = 0$$

$$\frac{d}{dx}\int_0^{\delta_t}\frac{U_\infty}{\delta}\ T_\infty - T_s\ \left[y - \frac{1}{\delta_t}\,y^2\right]dy - \alpha\ \frac{T_\infty - T_s}{\delta_t} = 0$$

Integrating

$$\frac{d}{dx}\left[\frac{U_\infty}{\delta}\ T_\infty - T_s\ \left(\frac{1}{2}y^2 - \frac{1}{3\delta_t}\,y^3\right)\right]_0^{\delta_t} = \alpha\ \frac{T_\infty - T_s}{\delta_t}$$

$$\frac{d}{dx}\left[U_\infty\,\frac{\delta_t^{\,2}}{6\delta}\right] = \frac{\alpha}{\delta_t}$$

Let $\quad \zeta = \dfrac{\delta_t}{\delta}$

Then $\quad \dfrac{d}{dx}\left[U_\infty\,\dfrac{\zeta^2\delta}{6}\right] = \dfrac{\alpha}{\delta\zeta}$

or $\ \delta\dfrac{d\delta}{dx} = \dfrac{6\alpha}{U_\infty\zeta^3}$ (ζ is independent of x)

Substituting Equation [1] into this expression

$$\frac{6\mu}{\rho U_\infty} = \frac{6\alpha}{U_\infty\zeta^3} \Rightarrow \zeta^3 = \left(\frac{\delta_t}{\delta}\right)^3 = \frac{\alpha\rho}{\mu} = \frac{1}{Pr}$$

$$\frac{\delta}{\delta_t} = Pr^{0.33}$$

PROBLEM 5.26

The average friction coefficient for flow over a 0.6-m-long plate is 0.01. What is the value of the drag force in N per m width of the plate for the following fluids: (a) air at 15°C, (b) steam at 100°C and atmospheric pressure, (c) water at 40°C, (d) mercury at 100°C, and (e) *n*-Butyl alcohol at 100°C?

GIVEN

Flow over a plate
Friction coefficient (C_f) = 0.01
Length of plate (L) = 0.6 m

FIND

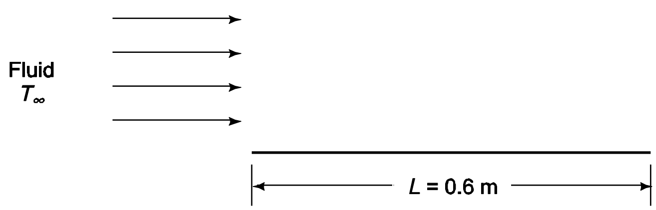
The value of the drag force (D) in N per meter width of the plate for

(a) Air at 15°C
(b) Steam at 100°C and atmospheric pressure
(c) Water at 40°C
(d) Mercury at 100°C
(e) *N*-Butyl alcohol at 100°C

ASSUMPTIONS

Steady state
Fully developed turbulent flow

SKETCH



PROPERTIES AND CONSTANTS

The following information is from Appendix 2

Substance	Table Number	Temperature (°C)	Kinematic Viscosity $\nu \times 10^6$ m²/s	Density, ρ kg/m³
(a) Air	27	1 5	15.3	1.19
(b) Steam	34	100	20.2	0.5977
(c) Water	13	40	0.658	992.2
(d) Mercury	25	100	0.0928	13.385
(e) *n*-Butyl Alcohol	18	100	0.69	751

SOLUTION

Assuming the boundary layer is laminar, the average friction is given by Equation (5.31)

$$\bar{C}_f = 1.33 \left(\frac{U_\infty L}{\nu}\right)^{-\frac{1}{2}} \Rightarrow U_\infty = \left(\frac{1.33}{\bar{C}_f}\right)^2 \frac{\nu}{L}$$

Therefore, the Reynolds number at the end of the plate is

$$Re_1 = U_\infty \frac{L}{\nu} = \left[\left(\frac{1.33}{\bar{C}_f}\right)^2 \frac{\nu}{L}\right]\frac{L}{\nu} = \left(\frac{1.33}{0.01}\right)^2 = 1.77 \times 10^4 < 5 \times 10^5$$

Therefore, the assumption that the boundary layer is laminar is valid.

The drag force on the plate is

$$D = \bar{\tau}_w\ A$$

The wall shear stress (τ_w) is related to the friction coefficient by Equation (5.13)

$$\bar{\tau}_w = \bar{C}_f\ \frac{1}{2}\rho U_\infty^2$$

$$\therefore\ D = \bar{C}_f\ \frac{1}{2}\rho\left[\left(\frac{1.33}{\bar{C}_f}\right)^2 \frac{\nu}{L}\right]^2 A_s = \frac{1.565}{\bar{C}_f^{\,3}}\rho\ \frac{\nu}{L}^{\ 2}\ L\ w$$

$$\frac{D}{w} = \frac{1.565}{\bar{C}_f^3}\frac{\rho}{L}\nu^2$$

$$\text{(a) Air: } \frac{D}{w} = \frac{1.565}{(0.01)^3}\left(\frac{1.190\,\text{kg/m}^3}{0.6\,\text{m}}\right)\ 15.3\times10^{-6}\,\text{m}^2/\text{s}^{\ 2} = 7.3\times10^{-4}\ \text{N/m}$$

$$\text{(b) Steam: } \frac{D}{w} = \frac{1.565}{(0.01)^3}\left(\frac{0.5977\,\text{kg/m}^3}{0.6\,\text{m}}\right)\ 20.2\times10^{-6}\,\text{m}^2/\text{s}^{\ 2} = 6.4\times10^{-4}\ \text{N/m}$$

$$\text{(c) Water: } \frac{D}{w} = \text{s}\,\frac{1.565}{(0.01)^3}\left(\frac{992.2\,\text{kg/m}^3}{0.6}\right)\ 0.658\times10^{-6}\,\text{m}^2/\text{s}^{\ 2} = 1.1\times10^{-3}\ \text{N/m}$$

$$\text{(d) Mercury: } \frac{D}{w} = \frac{1.565}{(0.01)^3}\left(\frac{13{,}385\,\text{kg/m}^3}{0.6\,\text{m}}\right)\ 0.0928\times10^{-6}\,\text{m}^2/\text{s}^{\ 2} = 3.0\times10^{-4}\ \text{N/m}$$

$$\text{(e) Alcohol: } \frac{D}{w} = \frac{1.565}{(0.01)^3}\left(\frac{751\,\text{kg/m}^3}{0.6\,\text{m}}\right)\ 0.69\times10^{-6}\,\text{m}^2/\text{s}^{\ 2} = 9.3\times10^{-4}\ \text{N/m}$$

PROBLEM 5.27

Experimental measurements of the temperature distribution during the flow of air at atmospheric pressure over the wing of an airplane indicate that the temperature distribution near the surface can be approximated by a linear equation

$$\boldsymbol{(T - T_s) = a\, y\, (T_\infty - T_s)}$$

where $\boldsymbol{a}$ **= a constant = 2 m^{-1}**

$\boldsymbol{T_s}$ **= surface temperature, *K***

$\boldsymbol{T_\infty}$ **= free stream temperature, *K***

$\boldsymbol{y}$ **= perpendicular distance from surface (mm)**

(a) Estimate the convective heat transfer coefficient if T_s = 50°C and T_∞ = – 50°C.

(b) Calculate the heat flux in W/m^2

GIVEN

Air flow over an airplane wing
Temperature distribution is given by the expression above
Surface temperature (T_s) = 50°C
Ambient temperature (T_∞) = – 50°C

FIND

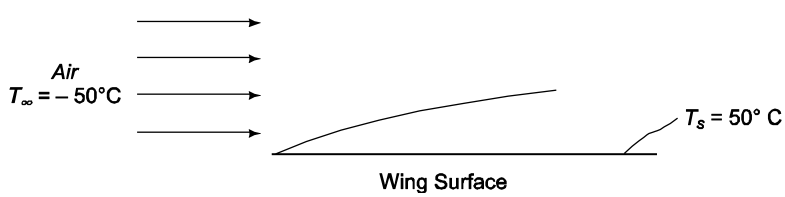
(a) The convective heat transfer coefficient $\bar{h}_c$

(b) The heat flux (q/A) in W/m^2

ASSUMPTIONS

Steady state conditions
Uniform surface temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for air at 0°C, the thermal conductivity (k) = 0.0237 W/(m K)

SOLUTION

(a) The heat transfer coefficient is given by Equation (5.1)

$$\bar{h}_c = \frac{-k_f}{T_s - T_\infty} \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

where k_f is the thermal conductivity of the fluid. Evaluate at the average of the bulk fluid temperature and the surface temperature. (This average is called the film temperature).
For this problem

$$\frac{T_s + T_\infty}{2} = \frac{50°\text{C} - 50°\text{C}}{2} = 0°\text{C}; \qquad k_f = 0.0237 \text{ W/(m K)}$$

For the temperature distribution, we find

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = a\,(T_\infty - T_s)$$

$$\therefore \qquad \bar{h}_c = \frac{-k_f}{T_s - T_\infty}\, a\,(T_\infty - T_s) = a\,k_f = 2(1/\text{m})\ \ 0.0237\,\text{W/(m K)}\ \ = 0.0474\ \text{W/(m}^2\ \text{K)}$$

(b) The rate of heat transfer is given by

$$q = \bar{h}_c\, A\,(T_s - T_\infty)$$

$$\frac{q}{A} = \bar{h}_c\,(T_s - T_\infty) = \ \ 0.0474\,\text{W/(m}^2\,\text{K)}\ \ (50°\text{C} + 50°\text{C}) = 4.74\ \text{W/m}^2$$

PROBLEM 5.28

For flow over a slightly curved isothermal surface, the temperature distribution inside the boundary layer δ_t may be approximated by the polynomial

$T(y) = a + by + cy^2 + dy^3$ ($y < \delta_t$) where y is the distance normal to the surface.

(a) **By applying appropriate boundary conditions, evaluate the constants a, b, c, and d.**

(b) **Then obtain a dimensionless relation for the temperature distribution in the boundary layer.**

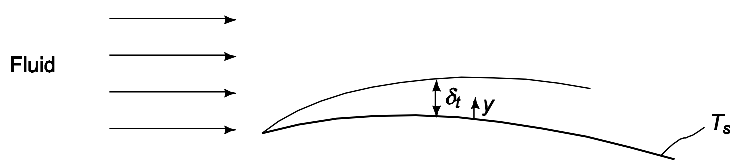
GIVEN

Flow over a slightly curved isothermal surface
Polynomial temperature distribution: $T(y) = a + by + cy^2 + dy^3$

FIND

(a) The values for a, b, c, and d
(b) A dimensionless relation for the temperature distribution in the boundary layer

SKETCH



SOLUTION

Let: Bulk fluid temperature = T_∞
Temperature of the surface = T_s

(a) The boundary conditions (BC) are

1. $T = T_s$ at $y = 0$
2. $T = 0.99\ T_\infty \approx T_\infty$ at $y = \delta_t$
3. $\frac{dT}{dy} = 0$ at $y = \delta_t$ (zero heat flux)
4. $\frac{d^2T}{dy^2} = 0$ at $y = 0$ (see Section 4.9.1)

From $B.C.$ 1: $a = T_s$
From $B.C.$ 2: $T_\infty = a + b\delta_t + c\delta_t^2 + d\delta_t^3$
From $B.C.$ 3: $0 = b + 2c\delta_t + 3d\delta_t^2$
From $B.C.$ 4: $0 = c$

Solving this set of 4 equations with 4 unknowns yields

$$a = T_s$$

$$b = \frac{3\ T_\infty - T_s}{2\delta_t}$$

$$c = 0$$

$$d = \frac{T_s - T_\infty}{2\delta_t^3}$$

(b) Substituting the constants into the temperature distribution

$$T = T_s + \frac{3}{2}\frac{T_\infty - T_s}{\delta_t}\,y + \frac{T_s - T_\infty}{2\delta_t^3}\,y^3$$

$$\frac{T - T_s}{T_\infty - T_s} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

Let θ = dimensionless temperature = $\dfrac{T - T_s}{T_\infty - T_s}$

ζ = dimensionless distance = $\dfrac{y}{\delta_t}$

Then

$$\theta = \frac{3}{2}\,\zeta - \frac{1}{2}\,\zeta^3$$

PROBLEM 5.29

Air at 20°C flows at 1 m/s between two parallel flat plates spaced 5 cm apart. Estimate the distance from the entrance to the point at which the hydrodynamic boundary layers meet.

GIVEN

Air flows between two parallel flat plates
Air speed (U_∞) = 1 m/s
Distance between the plates (D) = 5 cm = 0.05 m
Air temperature = 20°C

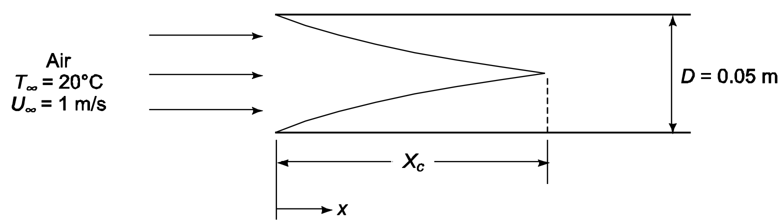
FIND

The distance from the entrance (X^1) where the boundary layers meet

ASSUMPTIONS

Steady state
Laminar flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 20°C
Kinematic viscosity (ν) = 15.7 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

If the boundary layer is laminar, the hydrodynamic boundary layer thickness is given by Equation (5.28)

$$\delta = 5x\big/\sqrt{Re_x} = 5\,(\nu x/U_\infty)^{1/2}$$

The boundary layers will meet when $\delta = D/2$

$$\frac{D}{2} = 5\,(\nu x_c/U_\infty)^{1/2}$$

Solving for distance x_c, $\quad x_c = \dfrac{D^2 U_\infty}{100\,\nu} = \dfrac{(0.05\,\mathrm{m})^2\,(1\,\mathrm{m/s})}{100\,(15.7\times10^{-6}\,\mathrm{m^2/s})} = 1.59$ m

The Reynolds number at x_c = 1.59 m is

$$Re_{x_c} = \frac{U_\infty x_c}{\nu} = \frac{(1\,\mathrm{m/s})\,(1.59\,\mathrm{m})}{15.7\times10^{-6}\,\mathrm{m^2/s}} = 1.0\times10^5 < 5\times10^5$$

COMMENTS

Since $Re < 5 \times 10^5$, the assumption of a laminar boundary layer is valid. If the Reynolds number were in the turbulent regime, the problem would have to be reworked.

PROBLEM 5.30

Air 1000°C flows at an inlet velocity of 2 m/s between two parallel flat plates spaced 1 cm apart. Estimate the distance from the entrance to the point where the boundary layers meet.

GIVEN

Air flows between two parallel flat plates
Air velocity (U_∞) = 2 m/s
Plate spacing (S) = 1 cm = 0.01 m

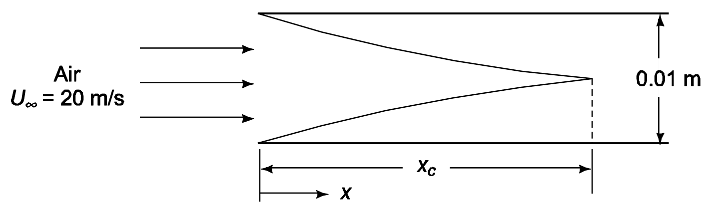
FIND

The distance from the entrance (x_c) where the boundary layers meet

ASSUMPTIONS

Steady flow
Air is dry and at a temperature of 1000°C

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 1000°C
The kinematic viscosity (ν) = 181×10^{-6} m^2/s

SOLUTION

The boundary layers meet when

$$\delta_{x_c} = \frac{1}{2} S = 0.005 \text{ m}$$

Assuming the flow is laminar, the boundary layer thickness is given by Equation (5.28)

$$\delta_x = \frac{5x}{\sqrt{Re_x}} = 5x \left(\frac{\nu}{U_\infty x}\right)^{0.5} \Rightarrow x = \frac{\delta_x{}^2 U_\infty}{25\nu}$$

$$x_c = \frac{0.005\,\text{m}^{\;2}\;\; 2\,\text{m/s}}{25\;\; 181\times10^{-6}\,\text{m}^2/\text{s}} = 0.011 \text{ m}$$

Checking the laminar flow assumption
$Re_{xc} < 5 \times 10^5$, therefore, the flow is laminar. The boundary layers meet at x = 0.011 m.

PROBLEM 5.31

For liquid metals with Prandtl numbers much less the unity, the hydrodynamic boundary layer is much thinner than the thermal boundary layer. As a result, one may assume that the velocity in the boundary layer is uniform [$u = U_\infty$ and $v = 0$]. Starting with Equation (5.7b), show that the energy equation and its boundary condition are analogous to those for a semi-infinite slab with a sudden change in surface temperature (see Equation (3.22)). Then show that the local Nusselt number is given by

$$Nu_x = 0.565\ (Re_x\ Pr)^{0.5}$$

Compare this relation with the equation for liquid metals in Table 5.5.

GIVEN

Liquid metal flowing over a flat plate

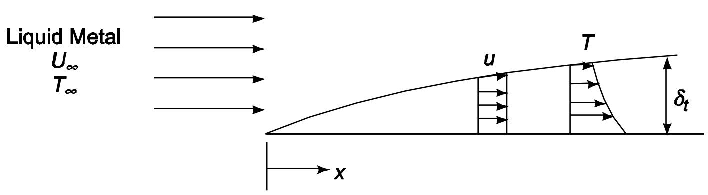
FIND

(a) Show that the energy Equation (5.7b) and its boundary conditions are analogous to those for a semi-infinite slab with sudden change in surface temperature (Equation (3.22))
(b) Show that $Nu_x = 0.564\ (Re_x\ Pr)^{0.5}$
(c) Compare this relation with the equation in Table 5.5 for liquid metals.

ASSUMPTIONS

Steady state
Uniform velocity in the boundary layer: $u = U_\infty,\ v = 0$
Mercury is at room temperature (20°C)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 26, for mercury at 20°C

Kinematic viscosity (ν) = 0.114×10^{-6} m²/s

Prandtl number (Pr) = 0.0249

SOLUTION

(a) Substituting $u = U_\infty$ and $v = 0$ into the energy Equation (5.7b)

$$U_\infty \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2} \qquad [1]$$

The three dimensional conduction equation is given by Equation (2.6)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_G}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

For a semi-infinite slab with no internal heat generation $q_G = 0$ and the temperature varies only with x and t

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad [2]$$

This is analogous to the energy equation [1] with the following substitutions: $y \to x$ and $x/U_{\infty} \to t$. The boundary conditions for the energy equation and the conduction equation for a semi-infinite slab subjected to a step change in surface temperature are

Energy Equation	Conduction Equation
$T(o, x) = T_w$	$T(o, t) = T_s$
$T(y, o) = T_{\infty}$	$T(x, o) = T_i$

(b) Both the equations and the boundary conditions are analogous, therefore, the solution for a semi-infinite solid (Equation (3.22)) can be used as a solution to the liquid metal flow problem with the appropriate substitution of variables

$$q_i(x) = \frac{k\ T_w - T_{\infty}}{\sqrt{\dfrac{\pi \alpha x}{U_{\infty}}}} = k(T_w - T_{\infty})\ \frac{1}{\sqrt{\pi}}\left(\frac{U_{\infty}}{\alpha x}\right)^{\frac{1}{2}}$$

$$Nu_x = \frac{hx}{k} = \frac{\dot{q}\ x\ x}{k\ T_w - T_{\infty}} = \frac{1}{\sqrt{\pi}}\left(\frac{U_{\infty} x}{\alpha}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{\pi}}\left(\frac{U_{\infty} x}{\nu}\frac{\nu}{\alpha}\right)^{\frac{1}{2}}$$

$$Nu_x = 0.564\ (Re_x Pr)^{\frac{1}{2}}$$

(c) The above equation agrees with the equation given in Table 5.5 for liquid metals.

PROBLEM 5.32

A heat exchanger for heating liquid mercury is under development. The exchanger is visualized as a 15 cm long and 0.3 m wide flat plate. The plate is maintained at 70°C, and the mercury flows parallel to the short side at 15°C with a velocity of 0.3 m/s. (a) Find the local friction coefficient at the midpoint of the plate and the total drag force on the plate. (b) Determine the temperature of the mercury at a point 10 cm from the leading edge and 1.25 mm. from the surface of the plate. (c)
Calculate the Nusselt number at the end of the plate.

GIVEN

Mercury flowing over a flat plate
Temperature of mercury (T_∞)) = 15°C
Velocity (U_∞)) = 0.3 m/s
Plate length (L) = 15 cm= 0.15 m
Plate width = 0.3 m
Plate surface temperature (T_s) = 70°C

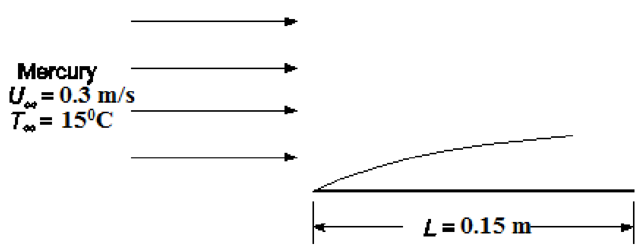
FIND

(a) Local friction coefficient (C_{fx}) at the middle point of the plate (x = 7.5 cm) and the total drag force (D) on the plate
(b) Temperature of the mercury 10 cm from the leading edge and 1.25 mm. from the surface of the plate
(c) The Nusselt number (Nu_L) at the end of the plate

ASSUMPTIONS

Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 26, for mercury at the average of T_∞ and T_s (42.5°C)

Thermal conductivity (k) = 9.24 W/(m K)

Kinematic viscosity (ν) = 1.06 × 10^{-7} m^2/s

Prandtl number (Pr) = 0.0216

Density (ρ) = 13522 kg/m^3

SOLUTION

The Reynolds number at the end of the plate is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(0.3m/s)(0.15m)}{1.06\times10^{-7}\,m^2/\text{s}} = 4.25 \times 10^5 < 5 \times 10^5$$

Therefore, the boundary layer is laminar over the entire plate.

(a) The local friction coefficient for a laminar boundary layer is given by Equation (5.30)

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$$

At $\quad x = 0.075$ m: $Re_x = \frac{U_\infty x}{\nu} = \frac{(0.3m/s)(0.075m)}{1.02\times10^{-7}\ m^2/s} = 2.20 \times 10^5$

$$\therefore \quad C_{fx} = \frac{0.664}{\sqrt{2.18\times10^5}} = 1.41 \times 10^{-3}$$

The total drag force on the plate is the product of the average shear stress and the area. The shear stress is related to the friction coefficient by Equation (5.30)

$$D = \tau_s A = \frac{1}{2}\rho U_\infty^2\ \overline{C_f}\ A = \rho U_\infty^2 C_{fL} A = \rho U_\infty^2 \frac{0.664}{\sqrt{Re_L}} A$$

$$D = 13522\ \text{kg/m}^3 (0.3m/s)^2 \frac{0.664}{\sqrt{4.25\times10^5}} (0.15\text{ m})(0.3\text{ m}) = 0.0558\text{ N}$$

(b) The laminar thermal boundary layer thickness is given by Equations (5.32) and (5.28)

$$\delta_{th} = \frac{\delta}{Pr^{\frac{1}{3}}} = \frac{5x}{Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}}$$

At $\quad x = 0.1$ m: $Re_x = \frac{(0.3m/s)(0.1m)}{1.02\times10^{-7}\ m^2/s} = 2.94\times 10^5$

$$\delta_{th} = \frac{5(0.1)}{(2.94\times10^5)^{\frac{1}{2}}(0.0216)^{\frac{1}{3}}} = 3.31*10^{-3}\text{ m} = 3.31\text{ mm}$$

Therefore, a point y=1.25 mm from the plate surface is inside of the thermal boundary layer and the temperature of the mercury is

$$\frac{T-T_\infty}{T_s-T_\infty} = \frac{3}{2}\frac{y}{\partial_t} - \frac{1}{2}\left(\frac{y}{\partial_t}\right)^3 \quad => \quad \frac{T-15^0C}{70^0C-15^0C} = \frac{3}{2}\left(\frac{1.25}{3.31}\right) - \frac{1}{2}\left(\frac{1.25}{3.31}\right)^3$$

$$\frac{T-15^0C}{70^0C-15^0C} = 0.54 \quad => \quad T-15^0C-=0.54*55^0C$$

$$T = 44.7^0C$$

(c) The local Nusselt number for a laminar boundary layer is given by Equation (5.37)

$$Nu_{x=L} = 0.332\ Pr^{\frac{1}{3}} Re_L^{\frac{1}{2}} = 0.332\ (0.0216)^{\frac{1}{3}} (4.37\times10^5)^{\frac{1}{2}} = 61.1$$

PROBLEM 5.33

The integral method also can be applied to turbulent flow conditions if experimental data for the wall shear stress are available. In one of the earliest attempts to analyze turbulent flow over a flat plate, Ludwig Prandtl proposed in 1921 the following relations for the dimensionless velocity and temperature distributions

$$\frac{u}{U_\infty} = \frac{y}{\delta}^{\frac{1}{7}}$$

$$\frac{(T - T_\infty)}{(T_s - T_\infty)} = 1 - \left(\frac{y}{\delta_t}\right)^{\frac{1}{7}} \qquad (T_s > T > T_\infty)$$

From experimental data, an empirical relation relating the shear stress at the wall with boundary layer thickness is

$$\tau_s = \frac{0.023\,\rho U_\infty^2}{Re_\delta^{0.25}} \qquad \text{where } Re_\delta = \frac{U_\infty\,\delta}{\nu}$$

Following the approach outlined in Section 5.9.1 for laminar conditions, substitute the above relations in the boundary layer momentum and energy integral equations and derive equations for:

(a) The boundary layer thickness

(b) The local friction coefficient, and

(c) The local Nusselt number.

Assume $\delta = \delta_t$ and discuss the limitations of your results.

GIVEN

Turbulent flow over a flat plate
Velocity and temperature distributions as given above
Shear stress at the wall as given above

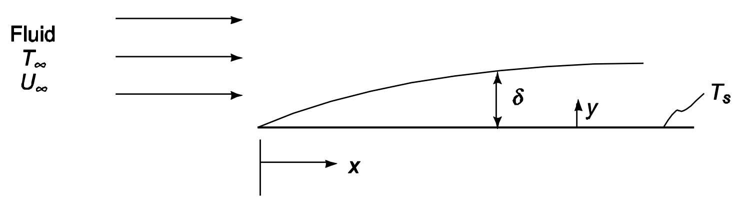
FIND

(a) The boundary layer thickness (δ)
(b) The local friction coefficient (C_f)
(c) The local Nusselt number (Nu_x)

ASSUMPTIONS

Steady state conditions
The hydrodynamic and thermal boundary layer thicknesses are equal

SKETCH



SOLUTION

(a) Substituting the relation for the velocity distribution and shear stress at the wall into the integral momentum equation (Equation (5.42))

$$\frac{d}{dx}\int_0^{\delta_t} \rho U_\infty \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \left[U_\infty - U_\infty\left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right] dy = \tau_w = 0.023\ \frac{\rho U_\infty{}^2}{Re_\delta{}^{\frac{1}{4}}}$$

Integrating

$$\frac{d}{dx}\left[\frac{\rho U_\infty{}^2}{\delta^{\frac{1}{7}}}\left(\frac{7}{8}\delta^{\frac{8}{7}}-\frac{7}{9\delta^{\frac{1}{7}}}\delta^{\frac{9}{7}}\right)\right] = +\,0.023\,\rho\,U_\infty^2\left(\frac{\rho U_\infty\delta}{\mu}\right)^{-\frac{1}{4}}$$

$$\frac{d}{dx}\left(\frac{7}{8}\delta-\frac{7}{9}\delta\right) = 0.023\left(\frac{\rho U_\infty}{\mu}\right)^{-\frac{1}{4}}\delta^{-\frac{1}{4}}$$

$$\delta^{\frac{1}{4}}\,d\delta = 0.023\left(\frac{72}{7}\right)\left(\frac{\rho U_\infty}{\mu}\right)^{-\frac{1}{4}}dx$$

Integrating

$$\frac{4}{5}\delta^{\frac{5}{4}} = 0.023\left(\frac{72}{7}\right)\left(\frac{\rho U_\infty}{\mu}\right)^{-\frac{1}{4}} x + C$$

at $x = 0$, $\delta = o \rightarrow C = 0$

$$\delta = \frac{\left[\frac{5}{4}\;0.023\left(\frac{72}{7}\right)\right]^{\frac{4}{5}} x^{\frac{4}{5}}}{\left(\frac{\rho U_\infty}{\mu}\right)^{\frac{1}{5}}}$$

$$\frac{\delta}{x} = 0.377\;Re_x^{-\frac{1}{5}}$$

(b) Substituting the shear stress at the wall and the expression for δ into Equation (5.51)

$$C_{fx} = \frac{2\tau_s}{\rho U_\infty{}^2} = \frac{2\left[0.023\rho U_\infty{}^2\left(\frac{U_\infty\delta}{\nu}\right)^{-\frac{1}{4}}\right]}{\rho U_\infty{}^2} = 0.046\left(\frac{U_\infty}{\nu}\right)^{-\frac{1}{4}}\left(\frac{0.377\,x}{\left(\frac{U_\infty x}{\nu}\right)^{\frac{1}{5}}}\right)^{-\frac{1}{4}}$$

$$C_{fx} = 0.059\;Re^{-\frac{3}{10}}$$

(c) Substituting the velocity and temperature distributions into the integral energy equation, Equation (5.44): Note that

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{q_c}{-kA} = -\frac{h_c}{k}(T_s - T_\infty) \text{ and } \delta = \delta_t$$

$$\frac{d}{dx}\int_0^{\delta_t} T_s - T_\infty\left[1-\left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right]U_\infty\left(\frac{y}{\delta}\right)^{\frac{1}{7}}dy - \alpha\left(\frac{\partial T}{\partial y}\right)_{y=0} = 0$$

$$(T_s - T_\infty)\,U_\infty\frac{d}{dx}\int_0^{\delta}\left(\left(\frac{y}{\delta}\right)^{\frac{1}{7}}-\left(\frac{y}{\delta}\right)^{\frac{2}{7}}\right)dy = -\,\alpha\,\frac{h_c}{k}(T_s - T_\infty) = \frac{h_c}{\rho c}(T_s - T_\infty)$$

$$U_\infty\frac{d}{dx}\left(\frac{7}{72}\delta\right) = \frac{h_c}{\rho c}$$

From part (a)

$$\delta = 0.377 \left(\frac{U_\infty x}{\nu}\right)^{-\frac{1}{5}} x = 0.377 \left(\frac{U_\infty}{\nu}\right)^{-\frac{1}{5}} x^{\frac{4}{5}}$$

$$\therefore \quad \frac{d\delta}{dx} = \frac{4}{5}\left(\frac{U_\infty}{\nu}\right)^{-\frac{1}{5}} x^{-\frac{1}{5}} = 0.3016 \; Re_x^{-\frac{1}{5}}$$

$$\therefore \quad \frac{h_c}{\rho c} = 0.0293 \, U_\infty \; Re_x^{-\frac{1}{5}}$$

$$\frac{h_c x}{k} = Nu = 0.0293 \; \frac{U_\infty x}{k} \; Re_x^{-\frac{1}{5}} \; \rho \, c$$

$$Nu = 0.0293 \; \frac{U_\infty x \rho}{\mu} \; Re_x^{-\frac{1}{5}} \; Pr$$

$$Nu \approx 0.0293 \; Re_x^{\frac{4}{5}} \; Pr$$

COMMENTS

Note that the assumption that the hydrodynamic and thermal boundary layer thicknesses are equal will only be valid if $Pr \approx 1$.

PROBLEM 5.34

A thin, flat plate is placed in an atmospheric pressure air stream that flows parallel to it at a velocity of 5 m/s. The temperature at the surface of the plate is maintained uniformly at 200°C, and that of the main air stream is 30°C. Calculate the temperature and horizontal velocity at a point 30 cm from the leading edge and 4 mm above the surface of the plate.

GIVEN

A thin plate in an air stream at atmospheric pressure
Air velocity (U_∞) = 5 m/s
Plate surface (T_s) = 200°C (uniform)
Air temperature (T_∞) = 30°C

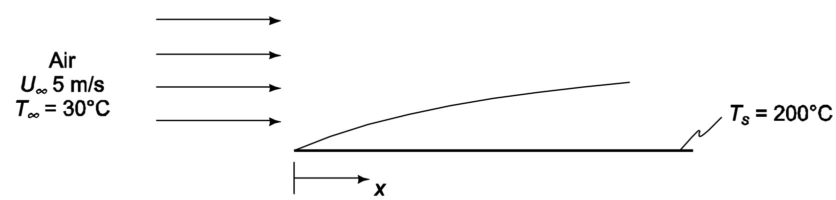
FIND

The air temperature and horizontal velocity at x = 30 cm = 0.3 m and 4 mm = 0.004 m above the plate

ASSUMPTIONS

Steady state
Moisture in the air is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average of T_s and T_∞ (115°C)

Kinematic viscosity (ν) = 25.4×10^{-6} m²/s

Prandtl number (Pr) – 0.71

SOLUTION

The Reynolds number at x = 0.3 m is

$$Re_x = \frac{U_\infty x}{\nu} = \frac{5\,\text{m/s}\quad 0.3\,\text{m}}{25.4\times10^{-6}\,\text{m}^2/\text{s}} = 5.91 \times 10^4 < 5 \times 10^5$$

Therefore, the boundary layer is laminar. The laminar hydrodynamic boundary layer thickness is given by Equation (5.28)

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5\ \ 0.3\,\text{m}}{\sqrt{5.91\times10^4}} = 0.0062\ \text{m}$$

The thermal boundary layer thickness is given in Equation (5.32)

$$\delta_{th} = \frac{\delta}{Pr^{\frac{1}{3}}} = \frac{0.0062\,\text{m}}{(0.71)^{\frac{1}{3}}} = 0.007\ \text{m}$$

Therefore, the point of interest is within both the hydrodynamic and thermal boundary layers. Figures 5.11 and 5.13 can be used to find the velocity and temperature at x = 0.3 m, y = 0.004 m. The abscissa of Figure 5.11 is

$$\frac{y}{x}\sqrt{Re_x} = \frac{0.004\,\text{m}}{0.3\,\text{m}}\sqrt{5.91\times10^4} = 3.24$$

From Figure 5.11

$$\frac{u}{U_\infty} \approx 0.87$$

$$\therefore \quad u = 0.87\, U_\infty = 0.82 \quad 5\,\text{m/s} \quad = 4.4\ \text{m/s}$$

The abscissa for Figure 5.13 is

$$\frac{y}{x}\sqrt{Re_x}\ Pr^{\frac{1}{3}} = 3.24\ (0.71)^{\frac{1}{3}} = 2.89$$

From Figure 5.13

$$\therefore \quad T = T_s + 0.78\,(T_\infty - T_s) = 30°\text{C} + 0.78\,(200°\text{C} - 30°\text{C}) = 163°\text{C}$$

PROBLEM 5.35

An array of 16 silicon chips arranged in 2 rows is insulated at the bottom and cooled by air flowing in forced convection over the top. The array is located either with its long side or its short side facing the cooling air. If each chip is 10 mm × 10 mm in surface area and dissipates the same power, calculate the rate of maximum power dissipation permissible for both possible arrangements if the maximum permissible surface temperature of the chips is 100°C. What would be the effect of a turbulator on the leading edge to trip the boundary layer into turbulent flow? The air temperature is 30°C and its velocity is 25 m/s.

GIVEN

An array of 16 silicon chips cooled by air flowing over the top with bottom insulated.
Dimensions of each chip = 10 mm × 10 mm = 0.01 m × 0.01 m
Array is 2 rows, 8 chips per row
Each chip dissipates the same power ($\dot{q}_G/A$)
Maximum surface temperature (T_s) = 100°C
Air temperature (T_∞) = 30°C
Air velocity (U_∞) = 25 m/s

FIND

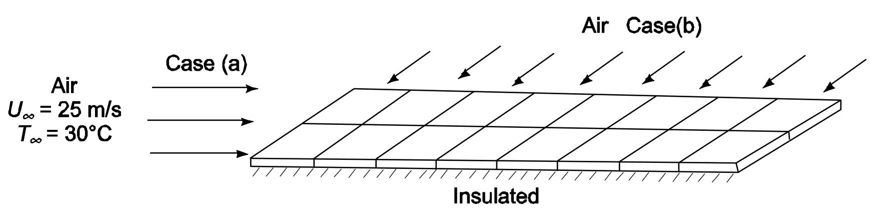
The maximum power dissipation permissible (q_G/A) for

(a) The long side facing the air flow
(b) The short side facing the air flow
(c) The effect of a turbulator on the leading edge

ASSUMPTIONS

Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28 for dry air at the film temperature (65°C)

Kinematic viscosity (ν) = 19.9 × 10^{-6} m^2/s

Thermal conductivity (k) = 0.0283 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

(a) With the long edge facing the air flow, L = (2) (0.01 m) = 0.02 m

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(25\,\text{m/s})(0.02\,\text{m})}{19.9\times10^{-6}\,\text{m}^2/\text{s}} = 2.51 \times 10^4 < 5 \times 10^5 \text{ (Laminar)}$$

The local heat transfer coefficient for laminar flow is given by Equation (5.37)

$$h_{cx} = \frac{k}{x}\, 0.332\, Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

The transfer coefficient decreases with increasing x and it will be minimum at $x = L$. The permissible heat generation rate for a given maximum surface temperature will therefore be limited to the heat flux from the plate at $x = L$

$$\left.\frac{q_G}{A}\right|_{\max} = \left.\frac{q_G}{A}\right|_{x=L} = h_{cL}\,(T_s - T_\infty) = \frac{k}{L}\;0.332\;Re_L^{\frac{1}{2}}\,Pr^{\frac{1}{3}}\;(T_s - T_\infty)$$

$$\left.\frac{q_G}{A}\right|_{\max} = \frac{0.0283\,\mathrm{W/(m\,K)}}{0.02\,\mathrm{m}}\;0.332\;(2.5\times10^4)^{\frac{1}{2}}(0.71)^{\frac{1}{3}}\,(100°\mathrm{C} - 30°\mathrm{C}) = 4650\ \mathrm{W/m^2}$$

$$= 0.465\ \mathrm{W/chip}$$

(b) With the short edge facing the air flow, L = (8) (0.01 m) = 0.08 m

$$Re_L = \frac{U_\infty\,L}{\nu} = \frac{(25\,\mathrm{m/s})\,(0.08\,\mathrm{m})}{19.9\times10^{-6}\,\mathrm{m^2/s}} = 1.01\times10^5\ \text{(still laminar)}$$

As shown in part (a)

$$\left.\frac{q_G}{A}\right|_{\max} = \frac{0.0283\,\mathrm{W/(m\,K)}}{0.08\,\mathrm{m}}\;0.332\;(1.01\times10^5)^{\frac{1}{2}}\;(0.71)^{\frac{1}{3}}\,(100°\mathrm{C} - 30°\mathrm{C}) = 2330\ \mathrm{W/m^2}$$

$$= 0.233\ \mathrm{W/chip}$$

(c) Assuming the boundary layer is turbulent over the entire array, the local heat transfer coefficient is given by Equation (5.81)

$$h_{cx} = 0.0288\;\frac{k}{x}\,Re_x^{0.8}\,Pr^{\frac{1}{3}}$$

Therefore, the maximum heat generation rate is

$$(\dot{q}_G/A)_{\max} = 0.0288\;(k/L)\;Re_L^{0.8}\;Pr^{\frac{1}{3}}\;(T_s - T_\infty)$$

For the long edge facing the air flow

$$(\dot{q}_G/A)_{\max} = 0.0288\;(0.0283\,\mathrm{W/(m\,K)}/0.02\,\mathrm{m})\,(2.5\times10^4)^{0.5}\;(0.71)^{\frac{1}{3}}\,(100°\mathrm{C} - 30°\mathrm{C})$$

$$(\dot{q}_G/A)_{\max} = 0.842\;\frac{\mathrm{W}}{\mathrm{chip}}$$

For the short edge facing the air flow

$$(\dot{q}_G/A)_{\max} = 0.0288\;(0.0283\,\mathrm{W/(m\,K)}/0.08\,\mathrm{m})\;(1.01\times10^5)^{0.8}\;(0.71)^{\frac{1}{3}}\,(100°\mathrm{C} - 30°\mathrm{C})$$

$$(\dot{q}_G/A)_{\max} = 0.641\ \mathrm{W/chip}$$

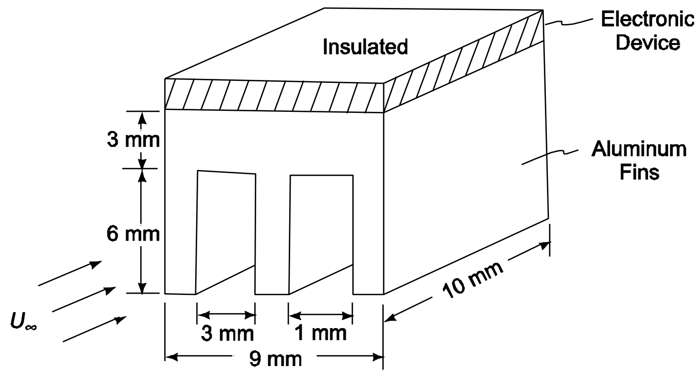
COMMENTS

Orienting the short edge rather than the long edge into the air flow allows about a doubling of the heat generation rate for the laminar case and about a 31% increase in the turbulent case. Note that the heat transfer coefficient decreases less rapidly with x for a turbulent boundary layer than it does for a laminar boundary layer.

The turbulator allows an increase in the heat generation rate of about 80% for the long edge oriented towards the air flow.

PROBLEM 5.36

An electronic device is to be cooled by air flowing over aluminum fins attached to its lower surface as shown



U_∞ = 10 m/s

T_∞ Air = 20°C

The device dissipates 5 W and the thermal contact resistance between the lower surface of the device and the upper surface of the cooling fin assembly is 0.1 cm^2 K/W. If the device is at a uniform temperature and insulated at the top, estimate that temperature under steady state.

GIVEN

Electronic device attached to aluminum fins as shown above
Air velocity (U_∞) = 10 m/s
Air temperature (T_∞) = 20°C
The device temperature is uniform
Top is insulated
Heat dissipation from the device ($\dot{q}_G$) = 5 W
Contact resistance between the device and the fins (R_i) = 0.1 cm^2 K/W

FIND

The steady state temperature of the device (T_{device})

ASSUMPTIONS

Fin material is pure aluminum
Heat transfer is one dimensional through the 3 mm thickness of aluminum
Heat loss through the insulation is negligible
Convection from the fins may be approximated as parallel flow over a flat plate
Heat loss from the edges of the device is negligible
Heat loss from the front and back edges of the fins is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, for aluminum at 127°C: Thermal conductivity (k_a) = 240 W/(m K)
From Appendix 2, Table 28, for dry air at 20°C

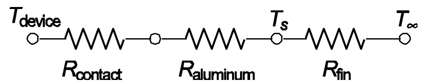
Kinematic viscosity (ν) = 15.7 × 10^{-6} m^2/s

Thermal conductivity (k) = 0.0251 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

The thermal circuit for heat flow from the device to the air is shown below



The Reynolds number at the trailing edge of the device is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(10\,\text{m/s})\,(0.01\,\text{m})}{15.7\times10^{-6}\,\text{m}^2/\text{s}} = 6.37\times10^3 \text{ (Laminar)}$$

The average heat transfer coefficient over the aluminum fins is given by Equation (5.38)

$$h_c = \frac{k}{L}\; 0.664\; Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{0.0251\,\text{W/(m K)}}{0.01\,\text{m}}\; 0.664\,(6.37\times10^3)^{\frac{1}{2}}(0.71)^{\frac{1}{3}} = 118.7\ \text{W/(m}^2\,\text{K)}$$

The rate of heat transfer from a single fin with convection over the tip is given in Table 2.1

$$q = M\,\frac{\sinh(m\,L_f) + \left(\frac{h}{m\,k_a}\right)\cosh(m\,L_f)}{\cosh(m\,L_f) + \left(\frac{h}{m\,k_a}\right)\sinh(m\,L_f)} \qquad (L_f = 0.006\text{ m})$$

where $\quad L_f = 0.006$ m $\quad$ and $\quad m = \sqrt{\dfrac{h_c\,P}{k_a\,A_c}}$

P = fin perimeter = 20 mm = 0.02 m (Neglecting front and back surfaces.)
A_c = fin cross sectional area = (0.01 m) (0.001 m) = 1×10^{-5} m^2

$$\therefore\ m = \sqrt{\frac{118.7\,\text{W/(m}^2\,\text{K)}\,(0.02\,\text{m})}{240\,\text{W/(m K)}\,(1\times10^{-5}\text{m}^2)}} = 31.4\ \text{m}^{-1}$$

$$m\,L_f = 31.4\frac{1}{\text{m}}\ (0.006\text{ m}) = 0.189$$

$$M = (T_s - T_\infty)\,\sqrt{\bar{h}_c P\,k_a A_c}$$

$$M = (T_s - T_\infty)\,\sqrt{118.7\,\text{W/(m}^2\text{K)}(0.02\,\text{m})\ \ 240\,\text{W/(m K)}\ \ (1\times10^{-5}\text{m}^2)} = 0.076\,(T_s - T_\infty)\ \text{W/K}$$

$$\frac{\bar{h}_c}{m\,k_a} = \frac{118.7\,\text{W/(m}^2\,\text{K)}}{31.4\,\text{m}^{-1}\ \ 240\,\text{W/(m K)}} = 0.0158$$

The rate of heat transfer from a single fin is

$$q_f = 0.076\,(T_s - T_\infty)\ \text{W/K}\ \frac{\sinh(0.189) + 0.0158\cosh(0.189)}{\cosh(0.189) + 0.0158\sinh(0.189)} = 0.0153\,(T_s - T_\infty)\ \text{W/K}$$

The total heat transfer is the sum of the heat transfer from the aluminum base not covered by the fins and the heat transfer from three fins. This must equal the heat generation rate

$$q = h_c\,A_b\,(T_s - T_\infty) + 3\ \ 0.0153(R_s - T_\infty)\,\text{W/K} = \dot{q}_G$$

Where A_b = 2(0.003 m) (0.01 m) = 0.6×10^{-4} m^2
Solving for T_s

$$T_s = T_\infty + \frac{\dot{q}_G}{h_c A_b + 0.046\,\text{W/K}} = 20°\text{C} + \frac{5\,\text{W}}{118.7\,\text{W/(m}^2\,\text{K)}(0.6\times 10^{-4}\,\text{m}^2) + 0.046\,\text{W/K}} = 114°\text{C}$$

The device temperature is given by

$$T_{\text{device}} = T_s + \dot{q}_G\ (R_{\text{contact}} + R_{\text{aluminum}})$$

where

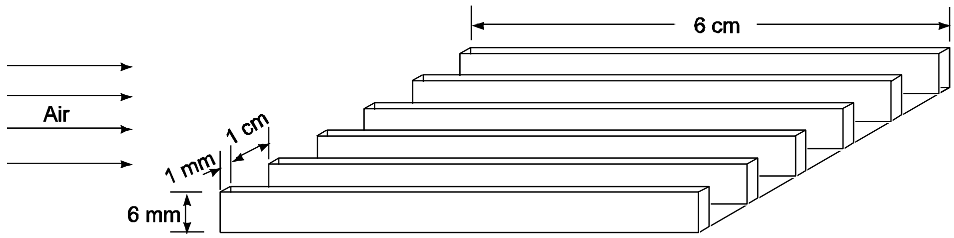
$$\text{R}_{\text{aluminum}} = \frac{t}{A\,k_a} = \frac{0.003\,\text{m}}{(0.01\text{m})\,(0.009\,\text{m})\ \ 240\,\text{W/(m K)}} = 0.1389\ \text{K/W}$$

$$R_{\text{contact}} = \frac{R_i}{A} = \frac{0.1(\text{cm}^2\ \text{K)/W}}{(1\text{cm})(0.9\,\text{cm})} = 0.1111\ \text{K/W}$$

$$\therefore\ T_{\text{device}} = 114°\text{C} + 5\ \text{W}\ (0.1111 + 0.1389)\ \text{K/W} = 115°\text{C}$$

PROBLEM 5.37

The six identical aluminum fins shown lin the sketch are attached to an electronic device for cooling. Cooling air is available at a velocity of 5 m/s from a fan at 20°C. If the average temperature at the base of a fin is not to exceed 100°C, estimate the maximum permissible power dissipation for the device.



GIVEN

Aluminum fins with air flowing over them
Air velocity (U_∞) = 5 m/s
Air temperature (T_∞) = 20°C
Maximum average temperature of the fin base (T_s) = 100°C

FIND

The maximum permissible power dissipation q

ASSUMPTIONS

Steady state

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the film temperature (60°C)

Kinematic viscosity (ν) = 19.4×10^{-6} m²/s

Thermal conductivity (k) = 0.0279 W/(m K)

Prandtl number (Pr) = 0.071

From Appendix 2, Table 12, for aluminum at 100°C

Thermal conductivity (k_{al}) = 239 W/(m K)

SOLUTION

The Reynolds number at the trailing edge of the fins is

$$Re_L = \frac{U_\infty\, L}{\nu} = \frac{(5\,\text{m/s})(0.06\,\text{m})}{19.4 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.55 \times 10^4 < 1 \times 10^5 \text{ (Laminar)}$$

The average transfer coefficient for a laminar boundary layer from Equation (5.38) is

$$h_c = \frac{k}{L}\, 0.664\, Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{0.0279\,\text{W/(m K)}}{0.06\,\text{m}}\, 0.664\,(1.55 \times 10^4)^{\frac{1}{2}} (0.71)^{\frac{1}{3}} = 34.3\ \text{W/(m}^2\ \text{K)}$$

The maximum permissible heat generation is equal to the sum of the heat loss from the fins and heat loss from the wall area between the fins. The heat loss from a single fin is given in Table 2.1

$$q_f = M\, \frac{\sinh(mL_f) + \ h/mk_a\ \cosh(mL_f)}{\cosh(mL_f) + \ h/mk_a\ \sinh(mL_f)}$$

where L_f = 0.006 m and m = $\sqrt{\dfrac{h_c P}{k_a A_c}}$

P = fin perimeter = 2(0.06 m) + 2(0.001 m) = 0.122 m

A_c = fin cross sectional area = (0.001 m) (0.06 m) = 6×10^{-5} m^2

$$\therefore\ m = \sqrt{\frac{34.3\,\text{W/(m}^2\,\text{K)}(0.122\,\text{m})}{239\,\text{W/(m K)}(6\times 10^{-5}\,\text{m}^2)}} = 17.1\ \text{m}^{-1}$$

$$m\,L_f = 17.1\ \text{m}^{-1}\ (0.006\ \text{m}) = 0.10$$

$$M = (T_b - T_\infty)\ \sqrt{h_c P k_a A_c}$$

$$M = (100°\text{C} - 20°\text{C})\sqrt{34.3\,\text{W/(m}^2\text{K)}(0.122\,\text{m})\ \ 239\,\text{W/(m K)}\ \ (6\times 10^{-5}\text{m}^2)} = 19.6\ \text{W}$$

$$\frac{h_c}{m\,k_a} = \frac{34.3\,\text{W/(m}^2\,\text{K)}}{17.1\text{m}^{-1}\ \ 239\,\text{W/(m K)}} = 0.0084$$

$$\therefore\ q_f = 19.6\ \text{W}\ \frac{\sinh(0.1) + 0.0084\cosh(0.1)}{\cosh(0.1) + 0.0084\sinh(0.1)} = 1.94\ \text{W}$$

Summing the heat loss from the six fins and the five wall areas

$$q = 6q_f + 5q_w = 6q_f + 5h_c\,A_w\,(T_s - T_\infty)$$

$$q = 6\ (1.94\ \text{W}) + 5\left(34.3\frac{\text{W}}{\text{m}^2\text{K}}\right)\ (0.01\ \text{m})\ (0.06\ \text{m})\ (100°\text{C} - 20°\text{C})$$

$$= 11.6\ \text{W} + 8.2\ \text{W} = 19.8\ \text{W}$$

COMMENTS

The fins account for about 60% of the total heat transfer. The rate of heat transfer without the fins would be about 9.2 W—less than half of that with the fins. If the entire fin temperature was assumed to be at the base temperature, the calculated heat loss rate would be 21.0 W—about 4% higher than that calculated above. This means that the fin efficiency is very high and L_f could therefore be increased to increase the heat dissipation quite effectively.

PROBLEM 5.38

A 2.5 cm diameter, 15 cm long transit rod (k = 0.97 W/(m K), ρ = 1647 kg/m^3, c = 837 J/(kg K) on the end of a 2.5 cm-diameter wood rod at a uniform temperature of 100°C is suddenly placed into a 16°C, 30 m/s air stream flowing parallel to the axis of the rod. Estimate the average center line temperature of the transit rod 8 min after cooling starts. Assume radial heat conduction, but include radiation losses, based on an emissivity of 0.90, to black surroundings at air temperature.

GIVEN

Transit rod on the end of a wood rod with air flowing parallel to the axis
Transit properties

- Thermal conductivity (k_t) = 0.97 W/(m K)
- Density (ρ_t) = 1647 kg/m^3
- Specific heat (c_t) = 837 J/(kg K)

Rod diameter (D) = 2.5 cm=0.025 m
Transit rod length = 15 cm=0.15 m
Initial temperature (T_o) = 100^{0}C
Air temperature (T_∞) = 16^{0}C
Air speed (U_∞) = 30 m/s
Rod emissivity (ε) = 0.90

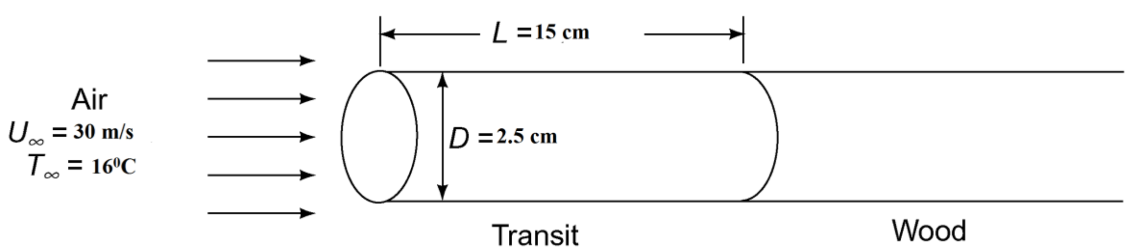
FIND

The average temperature of the center of the rod after 8 min.

ASSUMPTIONS

Radial heat conduction only – neglect end effects
Wood rod acts as an insulator and only provides support for the transit
Surroundings behave as a black body at the ambient temperature
Convective heat transfer can be approximated as a flat plate
Constant thermal properties of the rod and the air

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the initial film temperature (136°F)

Kinematic viscosity (ν) = 19.2*10^{-6} m^2/s

Thermal conductivity (k) = 0.0278 W/(m K)

Prandtl number (Pr) = 0.71

From Appendix 1, Table 5

The Stephen-Boltzmann Constant (σ) = 5.67 × 10^{-8} W/ m^2 K^4)

SOLUTION

The convective heat transfer coefficient between the rod and the air will be determined by treating the rod as a flat plate.

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(30m/\text{s})(0.15m)}{19.2*10^{-6}\, m^2/s} = 2.34 \times 10^5 \text{ (Laminar)}$$

From Equation (5.38) for a laminar boundary layer

$$h_c = \frac{k}{L} 0.664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{(0.0278W/(mK))}{0.15m} 0.664\,(2.34\times10^5)^{\frac{1}{2}}\,(0.71)^{\frac{1}{3}} = 53.1 \text{ W/(m}^2\text{ K)}$$

The overall heat transfer coefficient satisfies the following equation

$$\frac{q_{\text{total}}}{A} = h_t\,(T_o - T_\infty) = \frac{q_c}{A} + \frac{q_r}{A} = h_c\,(T_o - T_\infty) + \varepsilon\,\sigma\,(T_o^4 - T_\infty^4)$$

$$h_t = \frac{h_c\ T_o - T_\infty\ + \varepsilon\sigma\ T_o{}^4 - T_\infty{}^4}{T_o - T_\infty}$$

$$h_t = \frac{\left(53.1W/(\text{m}^2\,K)\right)(373K - 289K) + 0.9\left(5.67\times10^{-8}\text{W/(m}^2\,K^4)\right)\left[(373K)^4 - (289K)^4\right]}{373K - 289K}$$

$$= 60.62 \text{ W/(m}^2\text{ K)}$$

The Biot number for the rod is given in Table 2.3

$$Bi = \frac{h_c r_o}{k_t} = \frac{\left(60.62W/(m^2\,K)\right)[0.0125]}{0.97W/(mK)} = 0.78 > 0.1$$

Therefore, internal resistance is significant and a chart solution must be used: Figure 3.10 applies to a long cylinder. At t = 8 min, the Fourier number is

$$Fo = \frac{\alpha t}{r_o{}^2} = \frac{k_t t}{\rho_t c_t r_o{}^2} = \frac{[0.97W/(mK)](8*60s)}{\left(1647kg/m^3\right)(837J/(kgK))[0.0125m]^2} = 2.16$$

From Figure 3.10 for F_o = 2.16 and $1/Bi$ = 1.27

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = 0.07$$

Solving for the centerline temperature at t = 8 min ($T_{(0,t)}$)

$$T(0,t) = T_\infty + 0.07\,(T_o - T_\infty) = 16°\text{C} + 0.07\,(100°\text{C} - 16°\text{C}) = 21.9°\text{C}$$

COMMENTS

Note that absolute temperatures must be used in radiative equations. The overall heat transfer coefficient is actually decreasing slightly as the rod cools. At 8 min, the rod surface temperature would be about 20.5°C leading to a heat transfer coefficient of 57.2 W/(m^{2}K).

PROBLEM 5.39

In a manufacturing operation, a long strip of sheet metal is transported on a conveyor at a velocity of 2 m/s while a coating on its top surface is to be cured by radiant heating. Suppose that infrared lamps mounted above the conveyor provide a radiant flux of 2500 W/m^2 on the coating. The coating absorbs 50% of the incident radiant flux, has an emissivity of 0.5, and radiates to surrounding at a temperature of 25°C. In addition, the coating also loses heat by convection though a heat transfer coefficient between both the upper and lower surface and the ambient air which are assumed to be at the same temperature as the environment. Estimate the temperature of the coating under steady state conditions, and consider that at any instant in this continuous curing process a strip length of only 3.5 m is exposed to heating.

GIVEN

A long strip of sheet metal on a conveyor
Velocity (U_∞) = 2 m/s
Radiant flux on upper surface (q_{lamps}/A) 2500 W/m^2
Coating absorbs 50 of incident radiant flux, absorptivity (α) = 0.5
Surroundings temperature (T_∞) = 25°C = 298 K
Emissivity of coating (ε) = 0.5

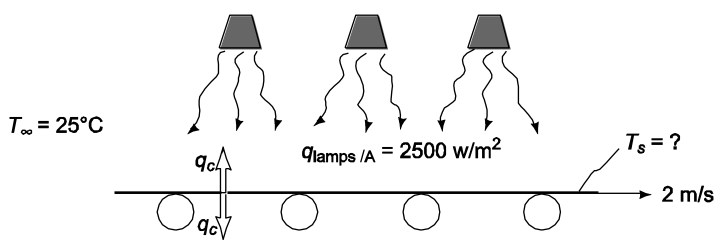
FIND

The temperature of the coating (T_s)

ASSUMPTIONS

Steady state
The thermal resistance of the sheet metal is negligible
The surroundings behave as a blackbody
The ambient air is still
The ambient temperature is constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 25°C (as a first guess)

Kinematic viscosity (ν) = 16.2 × 10^{-6} m^2/s

Thermal conductivity (k) = 0.0255 W/(m K)

Prandtl number (Pr) = 0.71

From Appendix 1, Table 5,

The Stephen-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The length of metal strip (L_c) needed to reach the critical Reynolds number is given by

$$Re_L \frac{U_\infty\ L_c}{\nu} = 5 \times 10^5 \Rightarrow L_c = \frac{5\times 10^5\,\nu}{U_\infty} = \frac{5\times 10^5\ \ 16.2\times 10^{-6}\ \text{m}^2/\text{s}}{2}\ \text{m/s} = 4.1\text{m}$$

Considering the metal strip length exposed to heating as 3.5 m, the flow will be laminar. The estimate of surface temperature will be based on the average convective heat transfer coefficient in the laminar region which is given by Equation (5.38)

$$h_c = \frac{k}{L}\, 0.664\, Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{0.0255\,\mathrm{W/(m\,K)}}{4.1\,\mathrm{m}}\, 0.664\,(5\times10^5)^{\frac{1}{2}}\ (0.71)^{\frac{1}{3}} = 2.61\ \mathrm{W/(m^2\,K)}$$

By the conservation of energy, of steady state

$$\alpha\left(\frac{q_{\text{lamps}}}{A}\right) = \frac{q_r}{A} + 2\ \frac{q_c}{A}$$

$$\text{where } \frac{q_r}{A} = \varepsilon\,\sigma\,(T_s^4 - T_\infty^4)\ \text{[Equation (1.17)]}$$

$$\frac{q_c}{A} = h_c\,(T_s - T_\infty)\ \text{[Equation (1.10)]}$$

$$\therefore\ \alpha\left(\frac{q_{\text{lamps}}}{A}\right) = \varepsilon\,\sigma\ \ (T_s^4 - T_\infty^4) + 2\ h_c\,(T_s - T_\infty)$$

$$0.5\ \ 2500\,\mathrm{W/m^2}\ = 0.5\ \ 5.67\times10^{-8}\,\mathrm{W/(m^2K^4)}\ \ [T_s^4 - (298\ \mathrm{K})^4] + 2\ \ 2.61\,\mathrm{W/(m^2\,K)}\ \ (T_s - 298\ \mathrm{K})$$

Checking the units for consistency, then dropping them for clarity

$$2.835\times10^{-8}\ T_s^4 + 5.22\ T_s - 3029 = 0$$

Solving by trial and error

$$T_s = 417\ \mathrm{K} = 144°\mathrm{C}$$

COMMENTS

Note that absolute temperatures must by used in the radiation equation.

Heat loss from the sheet is nearly equally divided between radiation and convection.

The air properties where taken at the ambient temperature. The estimate could be improved by evaluating the properties at the corrected film temperature, (25°C + 144°C)/2 = 135°C and calculating a new steady state surface temperature.

Because of the small forced convection component in this problem, natural convection from the metal strip may be important. Natural convection will be covered in Chapter 7.

PROBLEM 5.40

In a sheet-metal rolling and stamping factory, a 2.5-mm thick strip of stainless steel is hot rolled, as is schematically depicted in the figure. The strip emerges from the furnace and the rolling mill with a temperature of 900°C with a speed of 10 m/s, and is exposed to atmospheric air at 30°C. Determine the time-dependent rate of change in the temperature of the strip at a distance of 0.75 m from the leading tip of the rolled steel strip. In modeling this problem, neglect all radiation heat transfer but consider *lumped capacitance* for conduction within the strip with convection heat transfer from both the top and bottom surfaces. Also, at what *x*-location from the leading tip is the minimum cooling rate and why? Where is the maximum cooling rate?

GIVEN

Thickness of the strip (t)= 2.5 mm = 0.00025 m
Temperature of the strip (T_0)= 900^0C
Speed of the strip (U_∞)= 10 m/s
Ambient temperature (T_∞)= 30^0C

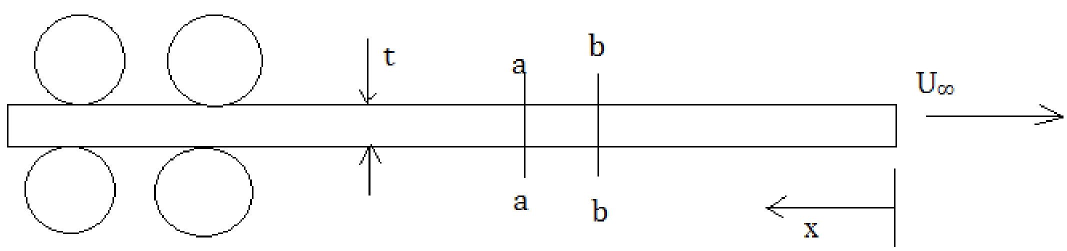
FIND

Time dependent rate of change of temperature of strip at distance 0.75 m from the leading tip of strip.
Location from leading edge at which there is minimum cooling rate.
Location from leading edge at which there is maximum cooling rate.

ASSUMPTIONS

All the properties are constant.
Heat radiation is negligible.
Negligible longitudinal conduction.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, for stainless steel

Thermal conductivity (*k*) = 14.4 W/(m K)
Density (ρ) = 7817 kg/m^3
Specific heat(*c*) = 461 J/(kg K)
From Table 28, Appendix 2 for Atmospheric Air
Dynamic viscosity for air at film temperature ν = 73*10^{-6} m^2/s
Thermal conductivity (k)= 0.052 W/(m K)
Prandtl Number (Pr)=0.72

SOLUTION

Considering energy balance in section aa-bb of the stainless steel strip we get
Heat lost in conduction from section aa to bb= Heat transferred through convection to surrounding**.**

$$\frac{-dE_{cond}}{dt} = \dot{Q}_{conv}$$

For the surface area A_s of section with strip thickness of t we get

$$-\rho t A_s c_p \frac{dT}{dt} = h_x * 2A_s (T - T_\infty)$$

$$\frac{dT}{dt} = \frac{-2h_x (T - T_\infty)}{c_p \rho t}$$

Considering the strip temperature to be $T=T_0$ for continuous process

$$\frac{dT}{dt} = \frac{-2h_x (T_0 - T_\infty)}{c_p \rho t}$$

$$= \frac{-2h_x (900 - 30)}{461 * 7817 * 0.0025}$$

$= -0.193\ h_x$

At a distance of x=0.75 m from leading edge, the Reynold's number is given by

$Re_x = \frac{U_\infty x}{\nu} = \frac{10 * 0.75}{73 * 10^{-6}} = 1.03*10^5$

Now we have

$Nu = \frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3}$

$h_x = 0.332 \frac{k}{x} Re_x^{1/2}\, Pr^{1/3}$

$= 0.332 * \frac{0.052}{0.75} * \left(1.03 * 10^5\right)^{1/2} * (0.72)^{1/3}$

$=6.62\ W/(m^2\ K)$

Thus at x=0.75 m

$\frac{dT}{dt} = -0.193\ h_x = -0.193*6.62 = -1.27$ K/s

Rate of cooling reduces with distance from leading edge due to reduction in convective heat transfer coefficient which varies as $x^{-1/2}$ and it again increases due to turbulence in turbulent region. Thus cooling rate is minimum at the point of transition.

Transition occurs at $Re_x = 5*10^5$

or $\frac{U_\infty x}{\nu} = 5*10^5$ => $x = 5*10^5 * \frac{\nu}{U_\infty}$

$x = 5*10^5 * 73*10^{-6}/10$

x=3.65 m

Maximum cooling rate occurs just at the leading edge of the strip as convection heat transfer coefficient is maximum at this point.

PROBLEM 5.41

A flat plate heat exchanger operates in a nitrogen atmosphere at a pressure of about 10^4 N/m^2 and 38°C. The flat-plate heat exchanger was originally designed to operate in air at 1 atmosphere and 38°C turbulent flow. Estimate the ratio of heat-transfer coefficients in air to that in nitrogen, assuming forced circulation cooling of the flat plate surface at the same velocity in both cases.

GIVEN

Flat plate heat exchangers in turbulent flow in air and nitrogen
Nitrogen pressure = 10^4 N/m^2
Nitrogen and air temperature (T_∞) = 38°C
Air pressure = 1 atm = 101,300 N/m^2

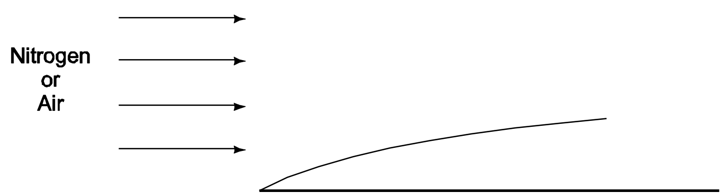
FIND

Ratio of the heat transfer coefficients

ASSUMPTIONS

Forced circulation coding of the plate surfaces at the same velocity in both cases
Steady state
Moisture in the air is negligible
The laminar portion of the boundary layer is negligible
Variation of Nitrogen properties with pressure is negligible
Nitrogen behaves as an ideal gas

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 38°C

Kinematic viscosity (ν_a) = 17.4×10^{-6} m^2/s

Thermal conductivity (k_a) = 0.0264 W/(m K)

Prandtl number (Pr_a) = 0.71

From Appendix 2, Table, for nitrogen at 38°C and 1 atm

Density (ρ_1) = 1.110 kg/m^3

Absolute viscosity (μ_n) = 18.3×10^{-6} kg/m s

Thermal conductivity (k_a) = 0.02699 W/(m K)

Prandtl number (Pr_n) = 0.711

The nitrogen density at $p = 10^4$ Pa can be determined as follows

$$\frac{P_1}{\rho_1} = \frac{P_2}{\rho_2} \Rightarrow \rho_2 = \rho_1 \frac{P_2}{P_1} = 1.110 \text{ kg/m}^3 \left(\frac{10^4}{101{,}300}\right) = 0.1096 \text{ kg/m}^3$$

Therefore, the kinematic viscosity of the nitrogen is

$$\nu = \frac{\mu}{\rho} = \frac{18.3\times10^{-6}\,\text{kg/ms}}{0.1096\,\text{kg/m}^3} = 167 \times 10^{-6} \text{ m}^2\text{/s}$$

SOLUTION

The average heat transfer coefficient for a turbulent boundary layer is given by Equation (5.82)

$$h_c = 0.036\ \frac{k}{L}\ Pr^{\frac{1}{3}}\ Re_L^{0.8} = 0.036\ \frac{k}{L}\ Pr^{\frac{1}{3}}\left(\frac{U_\infty L}{\nu}\right)^{0.8}$$

The ratio of the heat transfer coefficient in air to the heat transfer coefficient in nitrogen is

$$\frac{h_{ca}}{h_{cn}} = \frac{k_a}{k_n}\left(\frac{Pr_a}{Pr_n}\right)^{\frac{1}{3}}\left(\frac{\nu_n}{\nu_a}\right)^{0.8} = \frac{0.0264}{0.027}\left(\frac{0.71}{0.711}\right)^{\frac{1}{3}}\left(\frac{167\times 10^{-6}}{17.4\times 10^{-6}}\right)^{0.8} = 5.67$$

The heat transfer coefficient in air is 6 times greater than that in the low pressure nitrogen.

PROBLEM 5.42

Air at 320 K with a free stream velocity of 10 m/s is used to cool small electronic devices mounted on a printed circuit board as shown in the sketch. Each device is 5 mm × 5 mm square in planform and dissipates 60 mW. A turbulator is located at the leading edge to trip the boundary layer so that it will become turbulent. Assuming that the lower surface of the electronic devices are insulated, estimate the surface temperature at the center of the fifth device on the circuit board.

GIVEN

Air flows over small electronic devices
Air temperature (T_∞) = 320 K
Air velocity (U_∞) = 10 m/s
Dimensions of each device = 5 mm × 5 mm = 0.005 m × 0.005 m
Power dissipation per device ($\dot{q}_G$) = 60 milliwatts = 0.06 W
There is a turbulator at the leading edge

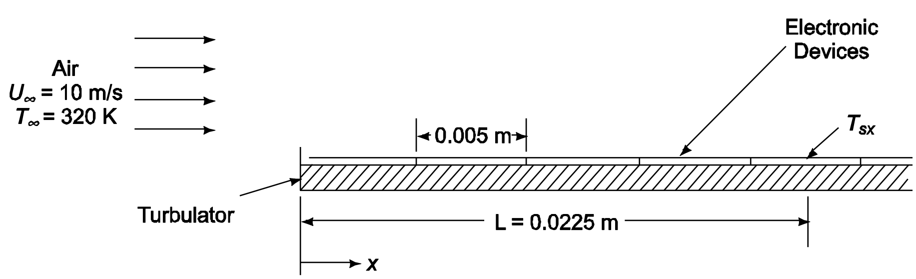
FIND

The surface temperature (T_{sx}) at the center of the fifth device

ASSUMPTIONS

Steady state
Lower surface of the devices is insulated (negligible heat loss)
The devices are placed edge-to-edge on the board
The boundary layer is turbulent from the leading edge on
The bulk fluid temperature is constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average of 320 K

Kinematic viscosity (ν) = 18.2×10^{-6} m²/s

Thermal conductivity (k) = 0.0270 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

The center of the fifth chip is 0.0225 m from the leading edge. The Reynolds number at this point is

$$Re_x = \frac{U_\infty x}{\nu} = \frac{10\,\text{m/s}\ (0.0225\,\text{m})}{18.2\times10^{-6}\ \text{m}^2/\text{s}} = 1.24 \times 10^4$$

Although this would normally be a laminar boundary layer, in this case, it will be turbulent due to the turbulator at the leading edge. For a turbulent boundary layer, the local heat transfer coefficient is given by Equation (5.81)

$$h_{cx} = \frac{k}{x}\, 0.0288\, Re_x^{0.8}\, Pr^{\frac{1}{3}} = \frac{0.0270\,\text{W/(m K)}}{0.0225\,\text{m}}\, 0.0288\, (1.24 \times 10^4)^{0.8}\, (0.71)^{\frac{1}{3}} = 57.9\ \text{W/(m}^2\ \text{K)}$$

For steady state, the rate of convective heat flux at x = 0.0225 m must equal the rate of heat generation per unit surface area

$$\frac{q_{cx}}{A} = h_{cx}\,(T_{sx} - T_\infty) = \frac{q_G}{A}$$

Solving for the surface temperature

$$T_{sx} = T_\infty + \frac{1}{h_{cx}}\frac{q_G}{A} = 320\ \text{K} + \frac{1}{57.9\,\text{W/(m}^2\ \text{K)}}\left(0.06\,\text{W/chip}\,\frac{1\,\text{chip}}{(0.005\,\text{m})\,(0.005\,\text{m})}\right)$$

$$= 361\ \text{K} = 88°\text{C}$$

The film temperature is therefore (320 K + 361 K)/2 = 341 K. Performing another iteration using air properties evaluated at 341 K yields the following results

$$\nu = 20.2 \times 10^{-6}\ \text{m}^2/\text{s}$$

$$k = 0.0285\ \text{W/(m K)}$$

$$Pr = 0.71$$

$$Re_x = 11{,}117$$

$$h_{cx} = 56.1$$

$$T_{sx} = 363\ \text{K} = 90°\text{C}$$

PROBLEM 5.43

A refrigeration truck is traveling at 130 km/h on a desert highway where the air temperature is 50°C. The body of the truck is idealized as a rectangular box, 3-m-wide, 2.1-m-high, and 6-m-long, at a surface temperature 10°C. Assume that (1) the heat transfer from the front and back of the truck may be neglected (2) the stream does not separate from the surface, and that (3) the boundary layer is turbulent over the whole surface. If a one ton refrigerating capacity is required for every 3600 W of heat loss, calculate the required tonnage of the refrigeration unit.

GIVEN

A refrigeration truck traveling on a desert highway
Speed of truck (U_∞) = 130 km/h
Air temperature (T_∞) = 50°C
Truck may be idealized as a box: 3-m-wide, 2.1-m-high, 6-m-long
Truck surface temperature (T_s) = 10°C
One ton of refrigeration unit is needed for every 3600 W of heat loss

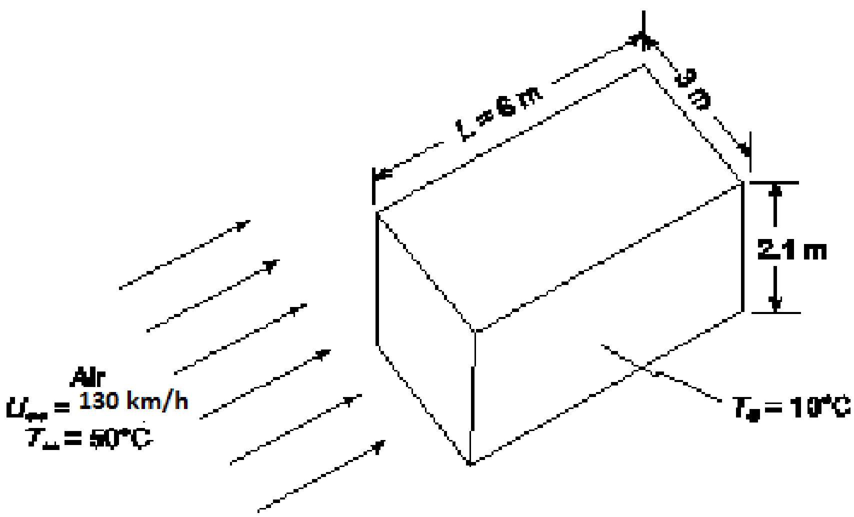
FIND

The required tonnage of the refrigeration unit

ASSUMPTIONS

The heat transfer from the front and back of the truck is negligible
Air stream does not separate from the surface
The boundary layer is turbulent over the whole surface
Moisture of the air is negligible
Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average of T_∞ and T_s (30°C)

Kinematic viscosity (ν) = 16.7×10^{-6} m^2/s

Thermal conductivity (k) = 0.0258 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

The sides of the truck can be visualized as flat plates with air flowing over them. The Reynolds number at the back of the truck is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{130\,(km/h)\,(6\,m)\,(1000\,m/\,km)}{16.7\times10^{-6}\,m^2/s\,(3600\,s/h)} = 1.297 \times 10^7$$

The total area of the sides, top, and bottom of the truck is

$$A = 2\,(6\text{ m})\,(3\text{ m}) + 2\,(6\text{ m})\,(2.1\text{ m}) = 61.2\text{ m}^2$$

The average heat transfer coefficient over the truck with a turbulent boundary layer is given by Equation (5.82)

$$h_c = \frac{k}{L}\ 0.036\ Pr^{\frac{1}{3}}\ Re_L^{0.8}$$

$$h_c = \frac{0.0258\,W/(m\,K)}{6\,m}\ 0.036\ (0.71)^{\frac{1}{3}}\ (1.28 \times 10^7)^{0.8} = 67.0\text{ W/(m}^2\text{ K)}$$

The rate of convective heat transfer to the truck is

$$q = h_c\,A\,(T_s - T_\infty) = 67.0\text{ W/(m}^2\text{ K)}\ (61.2\text{ m}^2)\ (50°\text{C} - 10°\text{C}) = 1.64 \times 10^5\text{ W/m}$$

The tonnage required to cool the truck is

$$\text{Tonnage} = 1.64 \times 10^5\text{ W}\left(\frac{1\,\text{ton}}{3600\text{ W}}\right) = 45.5\text{ tons}$$

COMMENTS

Solar gain may increase the required tonnage on a sunny day depending on the emissivity of the truck surface.

Including the laminar portion of the boundary layer would result in an average heat transfer coefficient of 64 W/(m^2 K) and a 5% decrease in the calculated required tonnage.

PROBLEM 5.44

At the equator, where the sun is approximately overhead at noon, a near optimum orientation for a flat plate solar hot water heater is in the horizontal position. Suppose a 4 m × 4 m solar collector for domestic hot water use is mounted on a horizontal roof as shown in the accompanying sketch. The surface temperature of the glass cover is estimated to be 40°C, and air at 20°C is blowing over the roof at a velocity of 15 mph. Estimate the heat loss by convection from the collector to the air when the collector is mounted

(a) at the leading edge of the roof [L_c = 0] and,

(b) at a distance of 10 m from the leading edge.

GIVEN

A solar collector on a flat, horizontal roof
Collector area ($L \times w$) = 4 m × 4 m
Glass cover surface temperature (T_s) = 40°C
Air temperature (T_∞) = 20°C
Air velocity (U_∞) = 15 mph

FIND

The heat loss by convection (q_c) from the collector when it is mounted

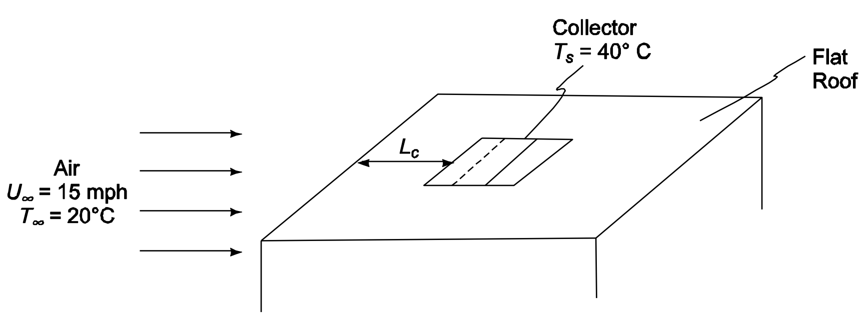
(a) at the leading edge of the roof (L_c = 0)

(b) at a distance of 10 m from the leading edge (L_c = 10 m)

ASSUMPTIONS

Steady state
The collector surface is connected smoothly to the roof surface
Uniform collector surface temperature
Moisture in the air is negligible
Neglect radiation heat transfer losses

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the film temperature (30°C)

Kinematic viscosity (ν) = 16.7×10^{-6} m^2/s

Thermal conductivity (k) = 0.0258 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

(a) For L_c = 0, the Reynolds number at the trailing edge of the collector is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{15\,\text{mi/h}\,(4\,\text{m})\;5280\,\text{ft/mi}\;\;0.3048\,\text{m/ft}}{16.7 \times 10^{-6}\,\text{m}^2\text{/s}\;\;3600\,\text{s/h}} = 1.61 \times 10^6 > 5 \times 10^5$$

(Turbulent)

The average convective heat transfer coefficient over the collector with a mixed boundary layer is given by Equation (5.83)

$$h_c = \frac{k}{L}\;0.036\;Pr^{\frac{1}{3}}\;[Re_L^{0.8} - 23{,}200]$$

$$h_c = \frac{0.0258\,\text{W/(m K)}}{4\,\text{m}}\;0.036\;(0.71)^{\frac{1}{3}}\;[(1.61 \times 10^6)^{0.8} - 23{,}200] = 14.3\ \text{W/m}^2\,\text{K}$$

The rate of convective heat loss from the collector is:

$$q = h_c\,A\,(T_s - T_\infty) = 14.3\;\;14.3\,\text{W/(m}^2\,\text{K)}\;\;(4\ \text{m})\,(4\ \text{m})\,(40°\text{C} - 20°\text{C}) = 4572\ \text{W/m}$$

(b) For L_c = 10 m, the boundary layer will be turbulent over the entire collector surface. The average heat transfer coefficient over the collector can be calculated by integrating the local heat transfer coefficient, Equation (4.81), between L_c and $L_c + L$ and dividing by the length of the collector L

$$h_c = \frac{1}{L}\int_{L_c}^{L+L_c} h_{cx}\,dx = \frac{1}{L}\int_{L_c}^{L+L_c} \frac{k}{x}\;0.0288\,Pr^{\frac{1}{3}}\left(\frac{U_\infty x}{\nu}\right)^{0.8} dx$$

$$= 0.0288\;\frac{k}{L}\,Pr^{\frac{1}{3}}\;\frac{U_\infty}{\nu}^{0.8}\;\int_{L_c}^{L+L_c} x^{-0.2}$$

$$h_c = 0.0288\;\frac{k}{L}\,Pr^{\frac{1}{3}}\left(\frac{U_\infty}{\nu}\right)^{0.8}\;1.25\,[(L + L_c)^{0.8} - L_c^{0.8}]$$

$$h_c = \quad 0.036\;\frac{0.0258\,\text{W/(m K)}}{4\,\text{m}}(0.71)^{\frac{1}{3}}\left[\frac{15\,\text{mi/h}\;\;5280\,\text{ft/mi}\;\;0.3048\,\text{m/ft}}{16.7 \times 10^{-6}\,\text{m}^2\text{/s}\;\;3600\,\text{s/h}}\right]^{0.8}[(14\ \text{m})^{0.8} - (10\ \text{m})^{0.8}]$$

$= 12.3$ W/(m^2 K)

The rate of convective heat loss from the collector is

$$q = h_c\,A\,(T_s - T_\infty) = [12.3\ \text{W/(m}^2\ \text{K)}]\,(4\ \text{m})\,(4\ \text{m})\,(40°\text{C} - 20°\text{C}) = 3928\ \text{W/m}$$

COMMENTS

Note that placing the collector 10 m from the leading edge of the roof lowers the rate of convective loss by about 14% because the local convective coefficient is largest at the leading edge.

PROBLEM 5.45

The air conditioning system in a new Chevrolet Van for use in desert climates is to be sized. The system is to maintain an interior temperature of 20°C when the van travels at 100 km/h through dry air at 30°C at night. If the top of the van is idealized as a flat plate 6-m-long and 2-m-wide, and the sides as flat plates 3-m-tall and 6-m-long, estimate the rate of which heat must be removed from the interior to maintain the specified comfort conditions. Assume the heat transfer coefficient on the inside of the van wall is 10 W/(m^2 K).

GIVEN

A Chevrolet van traveling through dry air
Interior van temperature (T_i) = 20°C
Velocity (U_∞) = 100 km/h
Air temperature (T_∞) = 30°C
Top dimensions = 6-m-long, 2-m-wide
Side dimensions = 6-m-long, 3-m-wide

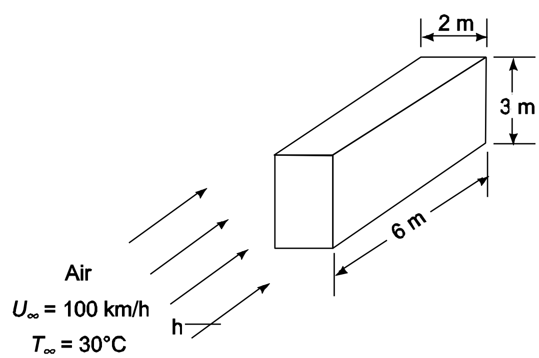
FIND

The rate of which heat must be removed (q)

ASSUMPTIONS

Heat gain from the front, back, and bottom of the van is negligible
Radiative heat transfer is negligible
Van walls are insulated
Thermal resistance of the sheet metal van walls is negligible
The interior heat transfer coefficient $(\bar{h}_c)$ = 10 W/(m^2 K)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at film temperature (25°C)

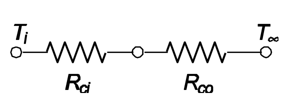
Kinematic viscosity (ν) = 16.2 × 10^{-6} m^2/s

Thermal conductivity (k) = 0.0255 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

The thermal circuit for the van walls and top is shown below



The average heat transfer coefficient on the outside of the van (h_{co}) can be calculated by treating the top and sides as flat plates and length (L) = 6 m

$$Re_L = \frac{U_\infty L}{\nu} = \frac{100\,\text{km/h}\;1000\,\text{m/km}\;(6\,\text{m})}{16.2\times10^{-6}\,\text{m}^2/\text{s}\;3600\,\text{s/h}} = 1.03\times10^7 > 5\times10^5$$

(Turbulent)

For a mixed boundary layer, Equation (5.83) gives the average heat transfer coefficient on the outside of the van

$$h_{co} = 0.036\,\frac{k}{L}Pr^{\frac{1}{3}}\,[Re_L^{0.8} - 23{,}200]$$

$$h_{co} = 0.036\,\frac{0.0255\,\text{W/(mK)}}{6\,\text{m}}(0.71)^{\frac{1}{3}}\,[(1.03\times10^7)^{0.8} - 23{,}200] = 52.5\ \text{W/(m}^2\ \text{K)}$$

The value of the thermal resistances are

Outside

$$R_{co} = \frac{1}{h_o A_o} = \frac{1}{52.5\,\text{W/(m}^2\,\text{K)}\;[2(3\,\text{m})(6\,\text{m}) + 2\,\text{m}(6\,\text{m})]} = 0.00040\ \text{K/W}$$

Inside

$$R_{ci} = \frac{1}{h_i A_i} = \frac{1}{10\,\text{W/(m}^2\,\text{K)}[2(3\,\text{m})(6\,\text{m}) + 2\,\text{m}(6\,\text{m})]} = 0.00208\ \text{K/W}$$

The rate at which heat must be removed is equal to the convective heat gain

$$q = \frac{\Delta T}{R_{co} - R_{ci}} = \frac{30°\text{C} - 20°\text{C}}{(0.0004 + 0.002.8)\,\text{K/W}} = 4032\ \text{W}$$

COMMENTS

Radiative heat transfer may not be negligible depending on the color of the van and the temperature of the night sky.

PROBLEM 5.46

Twenty-five square computer chips, each 10 mm * 10 mm in size and 1 mm thick, are mounted 1 mm apart on an insulating plastic substrate as shown below. The chips are to be cooled by nitrogen flowing along the length of the row at 240°C and atmospheric pressure to prevent their temperature from exceeding 30°C. The design is to provide for a dissipation rate of 30 mW per chip. Estimate the minimum free-stream velocity required to provide safe operating conditions for every chip in the array by considering that the chips disturb and trip the boundary layer to be turbulent.

GIVEN

A row of 25 square computer chips with nitrogen flowing over them
Chip dimensions = 10 mm × 10 mm = 0.01 m × 0.01 m
Chip thickness = 1 mm = 0.001 m
Maximum temperature of the chips (T_s) = 30°C
Nitrogen temperature (T_∞) = – 40°C
Heat dissipation = 30 m W/chip = 0.03 W/chip

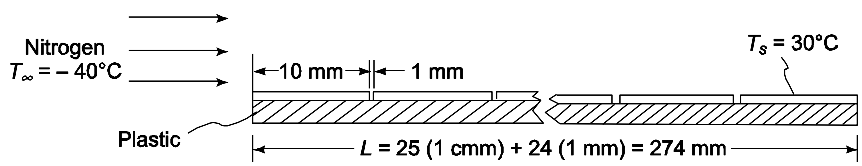
FIND

The minimum free stream velocity (U_∞)

ASSUMPTIONS

Steady state
Heat transfer from the edge of the chips is negligible
The chips trip the boundary layer into turbulence
Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 33, for nitrogen at the film temperature (– 5°C)

Kinematic viscosity (ν) = 22.5×10^{-6} m^2/s
Thermal conductivity (k) = 0.03106 W/(m K)
Prandtl number (Pr) = 0.698

SOLUTION

The heat flux from the chips is

$$\frac{\dot{q}_G}{A} = \frac{0.03\,\text{W/chip}}{(0.01)^2\ \text{m}^2/\text{chip}} = 300\ \text{W/m}^2$$

The local heat transfer coefficient for a turbulent boundary layer is given by Equation (5.81)

$$h_{cx} = 0.0288\ \frac{k}{x} Pr^{\frac{1}{3}} \left(\frac{U_\infty x}{\nu}\right)^{0.8}$$

Since h_{cx} decreases with increasing x, the lowest heat transfer coefficient occurs at the trailing edge of the array. Therefore, the minimum velocity needed o keep the trailing edge of the array at 30°C will be determined by conditions at $x = L$. The convective heat flux at the trailing edge is

$$\frac{q_c}{A} = h_{cL}\,(T_s - T_\infty) = \frac{q_G}{A}$$

$$\frac{q_G}{A} = 0.0288\,\frac{k}{L}\,Pr^{\frac{1}{3}}\left(\frac{U_\infty L}{\nu}\right)^{0.8}(T_s - T_\infty)$$

Solving for free stream velocity

$$U_\infty = 84.29\,\frac{\nu}{L}\left[\frac{q_G}{A}\,Pr^{-\frac{1}{3}}\,\frac{L}{k\,(T_s - T_\infty)}\right]^{1.25}$$

$$U_\infty = 84.29\,\frac{22.5\times10^{-6}\,\mathrm{m^2/s}}{0.274\,\mathrm{m}}$$

$$\left[300\,\mathrm{W/m^2}(0.698)^{-\frac{1}{3}}\,\frac{0.274\,\mathrm{m}}{0.03106\,\mathrm{W/(m\,K)}\;(30°\mathrm{C} + 40°\mathrm{C})}\right]^{1.25} = 0.75\ \mathrm{m/s}$$

The Reynolds number at the trailing edge for this velocity is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(0.75\,\mathrm{m/s})(0.274\,\mathrm{m})}{22.5\times10^{-6}\,\mathrm{m^2/s}} = 9133$$

COMMENTS

Assuming the boundary layer is laminar would lead to higher U_∞, (1.35 m/s).

PROBLEM 5.47

The wing of an airplane has a polished aluminum skin. At a 1500 m altitude, it absorbs 100 W/m² by solar radiation. Assuming that the interior surface of the wing's skin is well insulated and the wing has a chord of 6 m length, i.e., *L* = 6 m, estimate the equilibrium temperature of the wing at a flight speed of 150 m/s at distances of 0.1 m, 1 m, and 5 m from the leading edge. Discuss the effect of a temperature gradient along the chord.

GIVEN

Airplane wing with polished aluminum skin
Altitude = 1500 m
Absorbed solar radiation (q_{sol}/A) = 100 W/m^2
Cord length of wing (L) = 6 m
Flight speed (U_∞) = 150 m/s

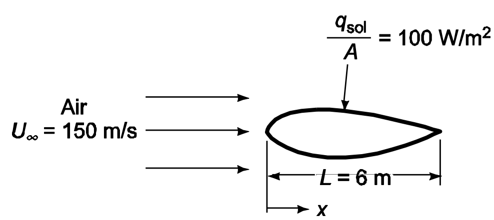
FIND

(a) Equilibrium temperature at x = 0.1 m, 1 m, and 5 m
(b) Discuss the effect of temperature gradient along the wing

ASSUMPTIONS

Steady state
Inside surface of wing is well insulated, so heat loss from the inner surface is negligible
Radiative loss from the wing surface is negligible
Flight speed given is air speed not ground speed
Variation of air properties with pressure is negligible
Neglect aerodynamic heating

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 38 at 1500 m altitude the air temperature (T_∞) = 5°C and the density of the air (ρ) = 1.06 kg/m^3
From Appendix 2, Table 28, for dry air at 1 atm and 5°C

Absolute viscosity (μ) = 17.7 × 10^{-6} N s/m^2

Thermal conductivity (k) = 0.0273 W/(m K)

Prandtl number (Pr) = 0.71

The kinematic viscosity at 1500 m is: $\nu = \mu/\rho = 16.7 \times 10^{-6}$ m^2/s

SOLUTION

(a) The Reynolds numbers at the desired locations are

$$\text{At } x = 0.1\text{ m: } Re_x = \frac{U_\infty x}{\nu} = \frac{150\,\text{m/s}\ (0.1\text{m})}{16.7\times10^{-6}\text{m}^2/\text{s}} = 0.89\times10^6$$

$$\text{At } x = 1\text{ m: } Re_x = \frac{150\,\text{m/s}\ (1\text{m})}{16.7\times10^{-6}\,\text{m}^2/\text{s}} = 8.98\times10^6$$

$$\text{At } x = 5\text{ m: } Re_x = \frac{150\,\text{m/s}\ (5\,\text{m})}{16.7\times10^{-6}\,\text{m}^2/\text{s}} = 4.49\times10^7$$

The boundary layer is turbulent at all these locations. For a turbulent boundary layer, the local heat transfer coefficient is given by Equation (5.81)

$$h_{cx} = \frac{k}{x}\ 0.0288\ Re_x^{\,0.8}\ Pr^{\frac{1}{3}}$$

$$\text{At } x = 0.1\text{ m: } h_{cx} = \frac{0.0237\,\text{W/(m K)}}{0.1\,\text{m}}\ 0.0288\ (0.89\times10^6)^{0.8}\ (0.71)^{\frac{1}{3}} = 350\text{ W/(m}^2\text{ K)}$$

$$\text{At } x = 1\text{ m: } h_{cx} = \frac{0.0237\,\text{W/(m K)}}{1\,\text{m}}\ 0.0288\ (9.98\times10^6)^{0.8}\ (0.71)^{\frac{1}{3}} = 222\text{ W/(m}^2\text{ K)}$$

$$\text{At } x = 5\text{ m: } h_{cx} = \frac{0.0237\,\text{W/(m K)}}{5\,\text{m}}\ 0.0288\ (4.49\times10^7)^{0.8}\ (0.71)^{\frac{1}{3}} = 161\text{ W/(m}^2\text{ K)}$$

The local convective heat loss from the wing must equal the radiative heat gain for equilibrium to exist

$$\frac{q_{cx}}{A} = h_{cx}\,(T_s - T_\infty) = \frac{q_{\text{sol}}}{A}$$

Solving for the wing surface temperature

$$T_s = T_\infty + \frac{1}{h_{cx}}\frac{q_{\text{sol}}}{A}$$

$$\text{At } x = 0.1\text{ m: } T_s = 5°\text{C} + \frac{1}{350\,\text{W/(m}^2\text{ K)}}\ 100\,\text{W/m}^2 = 5.29°\text{C}$$

$$\text{At } x = 1\text{ m: } T_s = 5°\text{C} + \frac{1}{222\,\text{W/(m}^2\text{K)}}\ 100\,\text{W/m}^2 = 5.45°\text{C}$$

$$\text{At } x = 5\text{ m: } T_s = 5°\text{C} + \frac{1}{161\,\text{W/(m}^2\text{K)}}\ 100\,\text{W/m}^2 = 5.62°\text{C}$$

In all three cases, the film temperature is very nearly 5°C, so our choice of 5°C for calculating the air properties is justified.

(b) Conduction along the aluminum skin will effectively smooth out these small temperature differences.

PROBLEM 5.48

It has been proposed to tow icebergs from the polar region to the Middle East in order to supply potable water to arid regions there. A typical iceberg suitable for towing should be relatively broad and flat. Consider an iceberg 0.25-km-thick and 1-km-square. This iceberg is to be towed at 1 km/h over a distance of 6000 km through water whose average temperature during the trip is 8°C. Assuming that the interaction of the iceberg with its surrounding can be approximated by the heat transfer and friction at its bottom surface, calculate the following parameters:

(a) The average rate at which ice will melt at the bottom surface.

(b) The power required to tow the iceberg at the designated speed.

(c) If towing energy costs are approximately 50 cents per kilowatt hour of power and the cost of delivering water at the destination can also be approximated by the same figure, calculate the cost of fresh water.

The latent heat of fusion of the ice is 334 kJ/kg and its density is 900 kg/m^3.

GIVEN

An ice sheet being towed through water

Ice sheet dimensions: 1 km × 1 km × 0.25 km = 1000 m × 1000 m × 250 m

Towing speed (U_∞) = 1 km/h = 1000 m/h

Distance towed = 6000 km = 6×10^6 m

Average water temperature (T_∞) = 8°C

Towing cost = \$.50/kWh

Latent heat of fusion of the ice (h_{sf}) = 334 kJ/kg

Density of he ice (ρ_i) = 900 kg/m^3

FIND

(a) Average melt rate (m) at the bottom surface

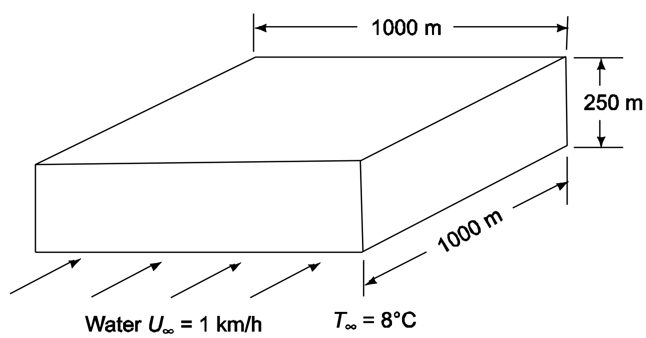
(b) Power (P) required to tow the iceberg

(c) Cost of delivered water (= towing cost)

ASSUMPTIONS

Heat transfer and friction of the sides of the iceberg are negligible

Properties of sea water are the same as fresh water sketch



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the film temperature (4°C)

Kinematic viscosity (ν) = 1.586×10^{-6} m^2/s

Thermal conductivity (k) = 0.566 W/(m K)

Density (ρ_w) = 1000 kg/m^3

Prandtl number (Pr) = 11.9

SOLUTION

(a) The Reynolds number at the trailing edge of the iceberg is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(1000\,\text{m/h})(1000\,\text{m})}{(1.586\times10^{-6}\,\text{m}^2/\text{s})\;3600\,\text{s/h}} = 1.75 \times 10^8 > 5 \times 10^5$$

Therefore, the flow is turbulent. The Reynolds number is large enough that the laminar region of the boundary layer will be neglected. The average heat transfer coefficient over the iceberg bottom is given by Equation (5.82)

$$h_c = 0.036\,\frac{k}{L}Pr^{\frac{1}{3}}\,Re_L^{0.8} = 0.036\,\frac{0.566\,\text{W/(m K)}}{1000\,\text{m}}(11.9)^{\frac{1}{3}}\,(1.75 \times 10^8)^{0.8} = 182.8\ \text{W/(m}^2\ \text{K)}$$

The average rate of convective heat transfer from the bottom of the iceberg is

$$q = h_c A\,(T_s - T_\infty) = 182.8\,\text{W/(m}^2\,\text{K)}\ (1000\ \text{m})\ (1000\ \text{m})\ (8°\text{C} - 0°\text{C}) = 1.46 \times 10^9\ \text{W}$$

This will cause the ice to melt at an average rate (m) given by

$$m = \frac{q}{h_{sf}} = \frac{1.46\times10^9\ \text{W}\ \ \text{J/(Ws)}}{334\,\text{kJ/kg}\ \ 1000\,\text{J/kJ}} = 4370\ \text{kg/s}$$

(b) The power required to tow the iceberg is the product of the drag force on the bottom of the iceberg and the towing speed

$$P = D\,U_\infty = \tau_s\,A\,U_\infty$$

but by definition [Equation (4.13)]

$$C_f = 2\,\tau_s\big/(\rho_w\,U_\infty^2) \quad\Rightarrow\quad \tau_s = 0.5\,\rho_w\,U_\infty^2\,C_f$$

The friction coefficient for turbulent flow, $5 \times 10^5 < Re < 10^7$, is given by Equation (5.78b). Although this relation has not been verified for $Re > 10^7$, it will be extrapolated to $Re = 1.75 \times 10^8$ for this problem

$$C_f = 0.072\,Re_L^{-\frac{1}{5}} \quad\Rightarrow\quad P = 0.036\,Re_L^{-\frac{1}{5}}\,\rho_w\,A\,U_\infty^3$$

$$P = 0.036\,(1.75 \times 10^8)^{-0.2}\ \ 1000\,\text{kg/m}^3\ \ (1000\ \text{m})\ (1000\ \text{m})\left[\frac{1000\,\text{m/h}}{3600\,\text{s/h}}\right]^3$$

$$= 1.73 \times 10^4\,\frac{\text{kg m}}{\text{s}^2}\frac{\text{m}}{\text{s}} = 17.3\ \text{kW}$$

(c) The cost of towing (C) per unit mass of delivered ice is

$$\text{Cost} = \frac{\text{Total cost}}{\text{Mass delivered}} = \frac{\text{(Towing power) (Towing time) (Towing energy cost)}}{\text{Initial mass} - \text{(Rate of melting) (Towing time)}}$$

$$\text{Towing time} = 6000\,\text{km}/(1\,\text{km/h}) = 6000\ \text{h}$$

$$\text{Initial mass} = (\text{Volume}\ (\rho_i) = (1000\ \text{m})(1000\ \text{m})(250\ \text{m})\ \ 900\,\text{kg/m}^3 = 2.25 \times 10^{11}\ \text{kg}$$

$$\therefore\ \text{Cost} = \frac{17.3\,\text{kW}\,(6000\,\text{h})\,(\$.50/\text{kWh})}{2.25\times10^{11}\,\text{kg} - (4370\,\text{kg/s})\,(6000\,\text{h})\ \ 3600\,\text{s/h}} = 4 \times 10^{-7}\ \frac{\$}{\text{kg}}$$

COMMENTS

About 42% of the ice melts during the journey. There are 3.79 kg of water in a gallon, therefore, the transportation costs are $1 for every 6.6 million gallons of water.

PROBLEM 5.49

A thin flat plate 15 cm square is tested for drag in a wind tunnel with air at 30 m/s, 100 kpa(abs), and 16°C flowing across and parallel to the top and bottom surfaces. The observed total drag force is 0.06 N. Using the definition of friction coefficient, Equation (5.13), and the Reynolds analogy, calculate the rate of heat transfer from this plate when the surface temperature is maintained at 120°C.

GIVEN

Air flow over the top and bottom of a thin plate
Plate dimensions = 15 cm × 15 cm = 0.15 m × 0.15 m
Air speed (U_∞) = 30 m/s
Air pressure = 100 kpa= 10^5 Pa
Air temperature (T_∞) = 16°C
Total drag force (D) = 0.06 N
Surface temperature (T_s) = 120°C

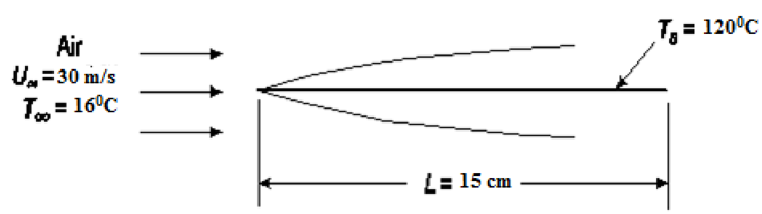
FIND

The rate of heat transfer (q)

ASSUMPTIONS

Steady state
Constant and uniform plate temperature
Radiation is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for air at the average of T_s and T_∞ (68°C)

Kinematic viscosity (ν) = 20.45*10^{-6} m^2/s
Thermal conductivity (k) = 0.0285 W/(m K)
Density (ρ) = 0.9965 kg/m^3
Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number at the end of the plate is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(30\,m/s)(0.15m)}{20.45*10^{-6}\,m^2/s} = 2.2 \times 10^5 \text{ (Laminar)}$$

Equation (5.13)

$$\bar{C}_f = \frac{2\tau_s}{pU_\infty^2}$$

The total drag force on both sides of the plate is

$$D = 2\,A\,\tau_s \quad \Rightarrow \quad \tau_s = \frac{D}{2\,A}$$

where A = the area of one side of the plate.

$$\bar{C}_f = \frac{D}{\rho A U_\infty^2}$$

The Reynolds analogy, corrected for Prandtl numbers other than unity, is given in Equation (5.40)

$$Nu_x = \frac{h_{cx}\,x}{k} = \frac{C_{fx}}{2} Re_x Pr^{\frac{1}{3}}$$

$$\frac{\nu}{U_\infty k\,Pr^{\frac{1}{3}}}\ h_{cx}\,x = \frac{C_{fx}}{2}$$

Averaging this over the length of the plate yields

$$\frac{\nu}{U_\infty k\,Pr^{\frac{1}{3}}}\ \frac{1}{L}\int_0^L h_{cx}\,dx = \frac{1}{2\,L}\int_0^L C_{fx}\,dx$$

$$\frac{\nu}{U_\infty k\,Pr^{\frac{1}{3}}}\ h_c = \frac{1}{2}\,C_f$$

$$\therefore \qquad \bar{h}_c = \frac{k}{L}\left(\frac{C_t}{2}\right)\left(\frac{U_\infty L}{\nu}\right) Pr^{\frac{1}{3}} = \frac{k}{L}\left(\frac{D}{2\rho A U_\infty^2}\right) Re_L Pr^{\frac{1}{3}}$$

$$\bar{h}_c = \frac{(0.0285 W/(m K))}{0.15 m}\ \frac{0.06 N}{2\left(0.9965 kg / m^3\right)(0.15\,m)\,(0.15\,m)(30\,m/\mathrm{s})^2}\ (2.2 \times 10^5)\,(0.71)^{\frac{1}{3}}$$

$$= 55.43\ \mathrm{W/(m^2\ K)}$$

The rate of heat transfer from both sides of the plate is

$$q = 2\ \bar{h}_c\ A\,(T_s - T_\infty) = 2 * 55.43 (W / (m^2 K)\ (0.15\ \mathrm{m})\ (0.15\ \mathrm{m})\ (120°\mathrm{C} - 16^0\mathrm{C}) = 259.4\ \mathrm{W}$$

COMMENTS

If radiation is included, assuming the plate behaves as a blackbody, and is totally enclosed by the wind tunnel which behaves as a blackbody at 60°C, the rate of heat transfer would be

$$q = q_c + q_r = q_c + A\ \sigma\,(T_s^4 - T_\infty^4)$$

$$q = 259.4\ \mathrm{W} + (0.15\ \mathrm{m})\ (0.15\ \mathrm{m})\left(5.67\times10^{-8} W/(m^2\ K^4)\right)[(393\ \mathrm{K})^4 - (289\ \mathrm{K})^4]$$

$$= (259.4 + 21.53)\ \mathrm{W}$$

$$q = 280.93\ \mathrm{W}$$

(9% higher than the results neglecting radiation)

PROBLEM 5.50

A thin flat plate 15-cm-square is suspended from a balance into a uniformly flowing stream of engine oil in such a way that the oil flows parallel to and along both surfaces of the plate. The total drag on the plate is measured and found to be 55.5 N. If the oil flows at the rate of 15 m/s with a temperature of 45°C, calculate the heat-transfer coefficient using the Reynolds analogy.

GIVEN

Engine oil flowing along a thin flat plate
Plate dimensions = 15 cm × 15 cm = 0.15 m × 0.15 m
Engine oil velocity (U_∞) = 15 m/s
Engine oil temperature (T_∞) = 45°C
Total drag force (D) 55.5 N

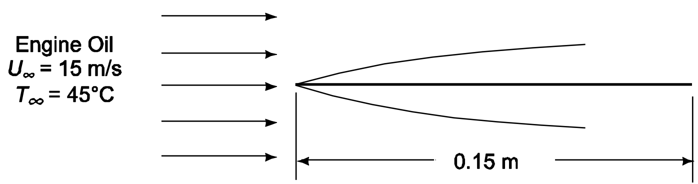
FIND

The heat transfer coefficient (h_c)

ASSUMPTIONS

Steady state
Constant fluid properties
Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 17, for unused engine oil at (45°C):

Kinematic viscosity (ν) = 201×10^{-6} m²/s
Thermal conductivity (k) = 0.143 W/(m K)
Density (ρ) = 873.1 kg/m³
Prandtl number (Pr) = 24.2

SOLUTION

The Reynolds number is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{15\,\text{m/s}\ (0.15\,\text{m})}{201\times 10^{-6}\,\text{m}^2/\text{s}} = 1.12 \times 10^4 \text{ (Laminar)}$$

The friction coefficient is defined in Equation (5.13)

$$\overline{C}_f = \frac{2\tau}{\rho U_\infty^2}$$

The drag force on both sides of the plate is

$$D = 2 A \tau \quad \Rightarrow \quad \tau = \frac{D}{2A}$$

Where A = the area of one side of the plate.

$$\therefore\ \bar{C}_f = \frac{D}{\rho A U_\infty^2}$$

The Reynolds analogy relates the heat transfer coefficient and the friction coefficient in Equation (5.40) (corrected for Prandtl numbers other than unity)

$$Nu_x = \frac{h_{cx} x}{k} = \frac{1}{2}\ C_{fx}\, Re_x\ Pr^{\frac{1}{3}}$$

$$\frac{\nu}{U_\infty k\, Pr^{\frac{1}{3}}}\ h_{cx} = \frac{C_{fx}}{2}$$

Averaging this over the length of the plate yields:

$$\frac{\nu}{U_\infty k\, Pr^{\frac{1}{3}}} \frac{1}{L}\int_0^L\ h_{cx}\, dx = \frac{1}{2L}\int_0^L\ C_{fx}\, dx$$

$$\frac{\nu}{U_\infty k\, Pr^{\frac{1}{3}}}\ \bar{h}_c = \frac{1}{2}\bar{C}_f$$

$$\therefore\quad \bar{h}_c = \frac{k}{L}\left(\frac{\bar{C}_f}{2}\right)\left(\frac{U_\infty L}{\nu}\right) Pr^{\frac{1}{3}} = \frac{k}{L}\left(\frac{D}{2\rho A U_\infty^2}\right)\ Re_L\ Pr^{\frac{1}{3}}$$

$$\bar{h}_c = \frac{0.143\,\text{W/(m K)}\ (55.5\,\text{N})\ \text{kg m/(Ns}^2)}{2\,(0.15\,\text{m})\ 873.1\,\text{kg/m}^3\ (0.15\,\text{m})\,(0.15\,\text{m})\ 15\,\text{m/s}^{\ 2}}\ (1.12 \times 10^4)\,(24.2)^{\frac{1}{3}}$$

$$= 194\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

Since the plate is submerged in the engine oil, the assumption that radiative heat transfer is negligible is valid. If the plate were in a gas, this assumption may not be valid. (See Problem 5.49.)

PROBLEM 5.51

> **For a study on global warming, an electronic instrument has to be designed to map the CO_2 absorption characteristics of the Pacific Ocean. The instrument package resembles a flat plate with a total (upper and lower) surface area of 2 m^2. For safe operation, its surface temperature must not exceed the ocean temperature by more than 2°C. To monitor the temperature of the instrument package, which is towed by a ship moving at 20 m/s, the tension in the towing cable is measured. If the tension is 400 N, calculate the maximum permissible heat generation rate from the instrument package.**

GIVEN

A flat plate towed through water
Total surface area (A) = 2 m^2
Speed through the water (U_∞) = 20 m/s
Towing cable tension (T) = 400 N
Maximum plate surface temperature – ocean temperature (ΔT_{max}) = 2°C

FIND

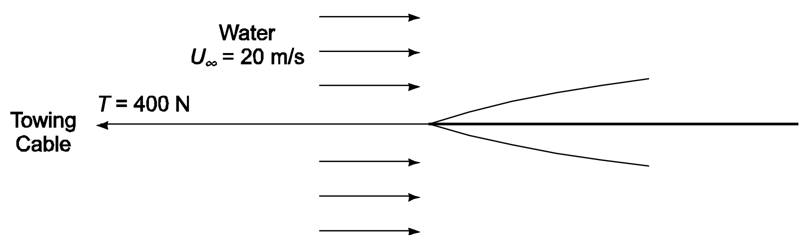
The maximum permissible heat generation rate $(\dot{q}_G)$

ASSUMPTIONS

Steady state
Edge effects are negligible
Effects due to the towing cable are negligible
The speed given is speed relative to the water
The water temperature is about 20°C
The length of the plate in the direction of motion is not known

SKETCH

The plate can be visualized as stationary with the water moving over it



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 20°C

Kinematic viscosity (ν) = 1.006×10^{-6} m^2/s
Thermal conductivity (k) = 0.597 W/(m K)
Density (ρ) = 998.2 kg/m^3
Prandtl number (Pr) = 7.0

SOLUTION

The Drag force on the plate is equal to the tension on the cable

$$T = D = \tau_s A \quad \Rightarrow \quad \tau_s = \frac{T}{A}$$

The friction coefficient is defined in Equation (5.13) as

$$\overline{C_f} = \frac{2\tau_s}{\rho U_\infty^2} = \frac{2T}{\rho A U_\infty^2} = \frac{2\ 400(\text{kg m/s}^2)}{998.2(\text{kg/m}^3)\ (2\text{ m}^2)\ 20\text{ m/s}^{\ 2}} = 1.002 \times 10^{-3}$$

The maximum possible heat generation rate is equal to the rate of heat transfer at the maximum permissible temperature difference

$$q_G = q_c = h_c\, \Delta T_{\max}$$

(a) Assuming the boundary layer is laminar, Equations (5.38) and (5.31) give the average heat transfer coefficient and friction coefficient

$$h_c = \frac{k}{L}\ 0.664 \left(\frac{U_\infty L}{\nu}\right)^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

$$C_f = 1.33 \ \frac{U_\infty L}{\nu}^{-\frac{1}{2}}$$

These equations can be combined to eliminate the length of the plate (L) which is not known

$$h_c = 0.664\ \frac{C_f}{1.33\ \dfrac{U_\infty L}{\nu}^{-\frac{1}{2}}} \frac{k}{L}\left(\frac{U_\infty L}{\nu}\right)^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{1}{2}\ C_f\, k\ \frac{U_\infty}{\nu} Pr^{\frac{1}{3}}$$

(b) Assuming the boundary layer is turbulent and the laminar region can be neglected, the heat transfer coefficient can be taken from Equation (5.82) and the friction coefficient from Equation (5.78b)

$$h_c = \frac{k}{L}\ 0.036\ Pr^{\frac{1}{3}}\left(\frac{U_\infty L}{\nu}\right)^{0.8}$$

$$C_f = 0.072\ \ \frac{U_\infty L}{\nu}^{-0.2}$$

Combining these to eliminate L

$$\bar{h}_c = \frac{1}{2}\overline{C}_f\, k \frac{U_\infty}{\nu} Pr^{\frac{1}{3}}$$

This relationship is valid for the average heat transfer and friction coefficients for both laminar and turbulent boundary layers. Therefore, regardless of Reynolds number

$$\bar{h}_c = \frac{1}{2}\ (1.002 \times 10^{-3})\ \ 0.597\text{ W/(m K)} \left(\frac{20\text{ m/s}}{1.006 \times 10^{-6}\text{ m}^2/\text{s}}\right)(7.0)^{\frac{1}{3}} = 1.14 \times 10^4\text{ W/(m}^2\text{ K)}$$

$$\therefore \qquad \dot{q}_G = 1.14 \times 10^1\text{ W/(m}^2\text{ K) (2 m}^2\text{) (2°C)} = 4.55 \times 10^4\text{ W} = 45.5\text{ kW}$$

PROBLEM 5.52

For flow of gas over a flat surface that has been artificially roughened by sand-blasting, the local heat transfer by convection is correlated by the dimensionless reaction

$$Nu_x = 0.05\ Re_x^{0.9}$$

(a) Derive a relationship for the average heat transfer coefficient in flow over a plate of length *L*.

(b) Assuming the analogy between heat and momentum transfer to be valid, derive a relationship for the local friction coefficient.

(c) If the gas is air at a temperature at 400 K flowing at a velocity of 50 m/s, estimate the heat flux 1 m from the leading edge for a plate surface temperature of 300 K.

GIVEN

Gas flow over a roughened flat surface
Local Nusselt number $Nu_x = 0.05\ Re_x^{0.9}$

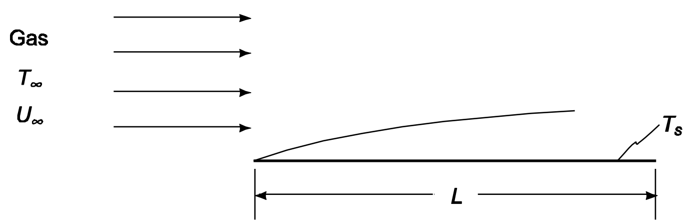
FIND

(a) Average heat transfer coefficient $(\bar{h}_c)$ for a plate of length L
(b) The local friction coefficient (C_{fx})
(c) If gas is air at temperature (T_∞) = 400 K and velocity (U_∞) = 50 m/s, estimate flux (q_x/A) at 1m from the leading edge for a plate surface temperature (T_s) = 300 K.

ASSUMPTIONS

Steady state
She analogy between heat and momentum transfer is valid

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the film temperature of 350 K

Thermal conductivity (k) = 0.0292 W/(m K)

Kinematic viscosity (ν) = 21.4 × 10^{-6} m²/s

SOLUTION

(a) The Nusselt number is defined in Table 5.3.

$$Nu = \frac{h_c D}{k} = 0.05\ Re_x^{0.9} = 0.05 \left(\frac{U_\infty x}{\nu}\right)^{0.9}$$

$$\therefore \quad h_{cx} = 0.05\ k \left(\frac{U_\infty}{\nu}\right)^{0.9} x^{-0.1}$$

The average heat transfer coefficient is obtained by integrating the local heat transfer coefficient between $X = 0$ and $X = L$

$$h_c = \frac{1}{L}\int_0^L 0.05\, k \left(\frac{U_\infty}{\nu}\right)^{0.9} x^{-0.1}\, dx = 0.05\, k \left(\frac{U_\infty}{\nu}\right)^{0.9} \frac{1}{0.9} L^{-0.1}$$

$$h_c = 0.056 \frac{k}{L} Re_L^{0.9}$$

(b) The relationship between the heat transfer and friction coefficients is given in Equation (5.77)

$$\frac{C_{fx}}{2} = \frac{Nu_x}{Re_x Pr^{\frac{1}{3}}} = \frac{0.05\, Re_x^{0.9}}{Re_x Pr^{\frac{1}{3}}}$$

$$C_{fx} = 0.1\, Re_x^{-0.1}\, Pr^{-\frac{1}{3}}$$

(c) From part (a)

$$h_{cx} = 0.05 \quad 0.0291\,\mathrm{W/(m\,K)} \left(\frac{50\,\mathrm{m/s}}{21.2\times10^{-6}\,\mathrm{m^2/s}}\right) (1\ \mathrm{m})^{-0.1} = 791\ \mathrm{W/(m^2\ K)}$$

The heat flux is

$$\frac{q_x}{A} = h_{cx}\,(T_s - T_\infty) = 791 \quad 791\,\mathrm{W/(m^2\ K)}\ (400\ \mathrm{K} - 300\ \mathrm{K}) = 79100\ \mathrm{W/m^2} = 79.1\ \mathrm{kW/m^2}$$

PROBLEM 5.53

Repeat Problem 5.40, but consider the 2.5-mm thick stainless steel strip to emerge from the rolling mill with a speed of 35 m/s. With the temperature of the strip and the surrounding air, respectively, of 900°C and 30°C, determine the time-dependent rate of change of the strip temperature at a distance of 0.75 m and 1.5 m from the leading tip. Comment on your results and the estimated cooling rate compared to that in the previous problem.

GIVEN

Thickness of the strip (t)= 2.5 mm = 0.00025 m
Temperature of the strip (T_0) = 900^0C
Speed of the strip (U_∞) = 35 m/s
Ambient temperature (T_∞) = 30^0C

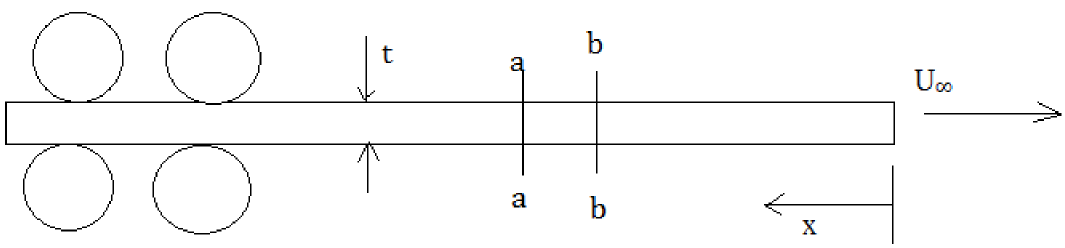
FIND

Time dependent rate of change of temperature of strip at distance 0.75 m from the leading tip of strip.
Location from leading edge at which there is minimum cooling rate.
Location from leading edge at which there is maximum cooling rate.

ASSUMPTIONS

All the properties are constant.
Heat radiation is negligible.
Negligible longitudinal conduction.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, for stainless steel

Thermal conductivity (*k*) = 14.4 W/(m K)
Density (ρ) = 7817 kg/m^3
Specific heat(*c*) = 461 J/(kg K)
From Table 28, Appendix 2 for Atmospheric Air
Dynamic viscosity for air at film temperature ν = 73*10^{-6} m^2/s
Thermal conductivity (k)= 0.052 W/(m K)
Prandtl Number (Pr)=0.72

SOLUTION

Considering energy balance in section aa-bb of the stainless steel strip we get
Heat lost in conduction from section aa to bb= Heat transferred through convection to surrounding.

$$\frac{-dE_{cond}}{dt} = \dot{Q}_{conv}$$

For the surface area A_s of section with strip thickness of t we get

$$-\rho t A_s c_p \frac{dT}{dt} = h_x * 2A_s (T - T_\infty)$$

$$\frac{dT}{dt} = \frac{-2h_x(T-T_\infty)}{c_p\rho t}$$

Considering the strip temperature to be $T=T_0$ for continuous process

$$\frac{dT}{dt} = \frac{-2h_x(T_0-T_\infty)}{c_p\rho t} = \frac{-2h_x(900-30)}{461*7817*0.0025} = -0.193\ h_x$$

At a distance of x=0.75 m from leading edge, the Reynold's number is given by

$$Re_x = \frac{U_\infty x}{\nu} = \frac{35*0.75}{73*10^{-6}} = 3.6*10^5$$

Now we have

$$Nu = \frac{h_x x}{k} = 0.332\,Re_x^{1/2}\,Pr^{1/3} \quad => \quad h_x = 0.332\frac{k}{x}Re_x^{1/2}\,Pr^{1/3}$$

$$h_x = 0.332*\frac{0.052}{0.75}*\left(3.6*10^5\right)^{1/2}*(0.72)^{1/3}$$

=12.37 W/(m^2 K)

Thus at x=0.75 m

$$\frac{dT}{dt} = -0.193\ h_x = -0.193*12.37 = -2.39\ K/s$$

At a distance of x=1.5 m from leading edge, the Reynold's number is given by

$$Re_x = \frac{U_\infty x}{\nu} = \frac{35*1.5}{73*10^{-6}} = 7.2*10^5$$

Now we have

$$Nu = \frac{h_x x}{k} = 0.332\,Re_x^{1/2}\,Pr^{1/3}$$

$$h_x = 0.332\frac{k}{x}Re_x^{1/2}\,Pr^{1/3}$$

$$= 0.332*\frac{0.052}{1.5}*\left(7.2*10^5\right)^{1/2}*(0.72)^{1/3}$$

=8.75 W/(m^2 K)

Thus at x=1.5 m

$$\frac{dT}{dt} = -0.193\ h_x = -0.193*8.75 = -1.7\ K/s$$

The estimated cooling rate is higher than in Problem 5.40 as velocity of strip is higher.

PROBLEM 5.54

Hydrogen at 15°C and at a pressure of 1 atm is flowing along a flat plate at a velocity of 3 m/s. If the plate is 0.3-m-wide and at 71°C, calculate the following quantities at x = 0.3 m and at the distance corresponding to the transition point, i.e., $Re_x = 5 \times 10^5$. (Use properties at 43°C.)

(a) Hydrodynamic boundary layer thickness, in cm.

(b) Thickness of thermal boundary layer, in cm.

(c) Local friction coefficient, dimensionless.

(d) Average friction coefficient, dimensionless.

(e) Drag force, in N.

(f) Local convective-heat-transfer coefficient, in W/(m² °C).

(g) Average convective-heat-transfer coefficient, in W/(m² °C).

(h) Rate of heat transfer, in W.

GIVEN

Hydrogen flowing over a flat plate
Hydrogen temperature (T_∞) = 15°C
Hydrogen pressure = 1 atm
Velocity (U_∞) = 3 m/s
Plate temperature (T_w) = 71°C
Width of plate = 0.3 m

FIND

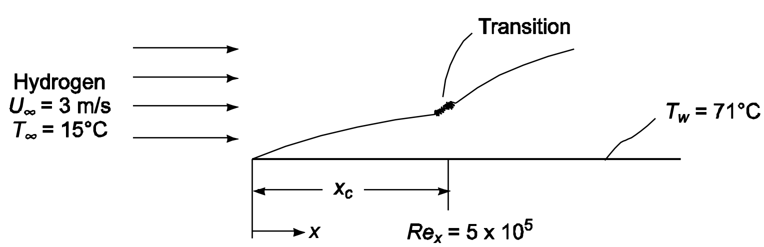
At x = 0.3 m and x_c ($Re_{xc} = 5 \times 10^5$) find

(a) Hydrodynamic boundary layer thickness (δ) in cm
(b) Thickness of thermal boundary layer (δ_t) in cm
(c) Local friction coefficient (C_{fx})
(d) Average friction coefficient (C_f)
(e) Drag force (D) in N
(f) Local convective-heat-transfer coefficient (h_{cx}) in W/(m² °C)
(g) Average convective-heat-transfer coefficient (h_c) in W/(m² °C)
(h) Rate of heat transfer (q) in W

ASSUMPTIONS

Steady state
Constant fluid properties

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 32, for hydrogen at 43°C

Kinematic viscosity (ν) = 119.9×10^{-6} m²/s

Prandtl number (Pr) = 0.703

Density (ρ) = 0.07811 kg/m³

Thermal conductivity (k) = 0.190 W/(m K)

SOLUTION

Transition to turbulence occurs around $Re_x = (U_\infty x_c)/\nu = 5 \times 10^5$

$$\therefore \quad x_c = \frac{510^5\,\nu}{U_\infty} = \frac{5\times 10^5 \;\; 119.9\times 10^{-6}\ \mathrm{m^2/s}}{3}\ \mathrm{m/s} = 20.0\ \mathrm{m}$$

The Reynolds number at $x = 0.3$ m is

$$Re_{0.3} = \frac{U_\infty x}{\nu} = \frac{3\,\mathrm{m/s} \quad 0.3\,\mathrm{m}}{119.9\times 10^{-6}\ \mathrm{m^2/s}} = 7506$$

(a) The hydrodynamic boundary layer thickness is given by Equation (5.28)

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

For $x = 0.3$ m: $\quad \delta = \dfrac{5 \;\; 0.03\,\mathrm{m}}{\sqrt{7506}} = 0.017\ \mathrm{m} = 1.7\ \mathrm{cm}$

For $x = 20$ m: $\quad \delta = \dfrac{5 \;\; 20\,\mathrm{m}}{\sqrt{5\times 10^5}} = 0.14\ \mathrm{m} = 14\ \mathrm{cm}$

(b) The thermal boundary layer thickness, from the empirical relation of Equation (5.32) is

$$\delta_t = \frac{\delta}{Pr^{\;1/3}}$$

For $x = 0.3$ m: $\quad \delta_t = \dfrac{1.7\,\mathrm{cm}}{(0.703)^{1/3}} = 1.91\ \mathrm{cm}$

For $x = 20$ m: $\quad \delta_t = \dfrac{14\,\mathrm{cm}}{(0.703)^{1/3}} = 15.7\ \mathrm{cm}$

(c) The local friction coefficient is given by Equation (5.30)

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$$

For $x = 0.3$ m $\quad C_{fx} = \dfrac{0.664}{\sqrt{7506}} = 0.0077$

For $x = 20$ m $\quad C_{fx} = \dfrac{0.664}{\sqrt{5\times 10^5}} = 0.00094$

(d) The average friction coefficient is given by Equation (5.31)

$$\overline{C_f} = \frac{1}{L}\int_0^L C_{fx}\,dx = \frac{1}{L}\left(2L\frac{0.664}{Re_L}\right) = 2\,C_{fL} \text{ (in the laminar regime)}$$

For the plate between $x = 0$ and $x = 0.3$ m: $\quad \overline{C_f} = 2(0.0077) = 0.0154$

For the plate between $x = 0$ and $x = 20$ m: $\quad \overline{C_f} = 2(0.0094) = 0.00188$

(e) The drag force is the product of the wall shear stress (τ_s) and the wall area (A). The wall shear stress is given in terms of the friction coefficient in Equation (5.30)

$$\tau_s = \frac{1}{2}\rho U_\infty^2 C_{fx} \Rightarrow D = \int_0^L w\tau_s\, dx = \frac{1}{2}\rho U_\infty^2 A \frac{1}{L}\int_0^L C_{fx}\, dx = \frac{1}{2}\rho A U_\infty^2 C_f$$

For the plate area between $x = 0$ and $x = 0.3$ m

$$D = \frac{1}{2}(0.3\text{ m})(0.3\text{ m})\left(0.07811\,\text{kg/m}^3\right)\left(3\,\text{m/s}\right)^2(0.0154) = 0.00049\text{ N}$$

For the plate area between $x = 0$ and $x = 20$ m

$$D = \frac{1}{2}(0.3\text{ m})(20\text{ m})\left(0.07811\,\text{kg/m}^3\right)\left(3\,\text{m/s}\right)^2(0.00188) = 0.0040\text{ N}$$

(f) The local heat transfer coefficient is given in Equation (5.36)

$$h_{cx} = 0.332\,\frac{k}{x}\,Re_x^{\frac{1}{2}}\,Pr^{\frac{1}{3}}$$

For $x = 0.3$ m $\quad h_{cx} = 0.332\left(\frac{0.19\,\text{W/(m K)}}{0.3\,\text{m}}\right)(7506)^{\frac{1}{2}}(0.703)^{\frac{1}{3}} = 16.2\text{ W/(m}^2\text{ K)} = 16.2\text{ W/(m}^2\,^\circ\text{C)}$

For $x = 20$ m $\quad h_{cx} = 0.332\left(\frac{0.19\,\text{W/(m K)}}{20\,\text{m}}\right)(5\times10^5)^{\frac{1}{2}}(0.703)^{\frac{1}{3}} = 1.98\text{ W/(m}^2\text{ K)} = 1.98\text{ W/(m}^2\,^\circ\text{C)}$

(g) The average heat transfer coefficient is twice the local heat transfer coefficient at the end of the plate length as shown in Equation (5.39)

For the plate area from $x = 0$ to $x = 0.3$ m

$$h_c = 2\left(16.2\,\text{W/(m}^2\,^\circ\text{C)}\right) = 32.4\text{ W/(m}^2\,^\circ\text{C)}$$

For the plate area from $x = 0$ to $x = 20$ m

$$h_c = 2\left(1.98\,\text{W/(m}^2\,^\circ\text{C)}\right) = 3.96\text{ W/(m}^2\,^\circ\text{C)}$$

(h) The rate of heat transfer is

$$q = h_c A\,(T_w - T_\infty)$$

For the plate area between $x = 0$ and $x = 0.3$ m

$$q = \left(32.4\,\text{W/(m}^2\,^\circ\text{C)}\right)(0.3\text{ m})(0.3\text{ m})(71^\circ\text{C} - 15^\circ\text{C}) = 163\text{ W}$$

For the plate area between $x = 0$ and $x = 20$ m

$$q = \left(3.96\,\text{W/(m}^2\,^\circ\text{C)}\right)(0.3\text{ m})(20\text{ m})(71^\circ\text{C} - 15^\circ\text{C}) = 1330\text{ W}$$

COMMENTS

Note that the local heat transfer coefficient decreases with distance from the leading edge.

PROBLEM 5.55

Repeat Problem 5.54, parts (d), (e), (g), and (h) for x = 4.0 m and U_∞ = 80 m/s, (a) taking the laminar boundary layer into account and (b) assuming that the turbulent boundary layer starts at the leading edge.

GIVEN

Hydrogen flowing over a flat plate
Hydrogen temperature (T_∞) = 15°C
Hydrogen pressure = 1 atm
Velocity (U_∞) = 80 m/s
Plate temperature (T_w) = 71°C
Width of plate = 0.3 m

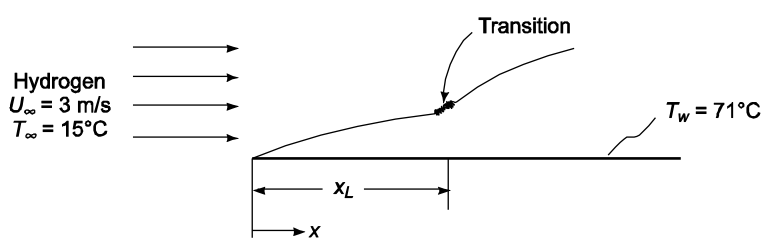
FIND

Calculate the quantities below for x = 4.0 and
(A) Assuming turbulent boundary layer starts at the leading edge
(B) Taking laminar boundary layer into account
- (a) Rate of heat transfer, in W
- (b) Drag force (D) in N
- (c) Average convective heat transfer coefficient $\bar{h}_c$ in W/(m² °C)
- (d) Rate of heat transfer (q) in Watts

ASSUMPTIONS

Steady state
Constant fluid properties

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 32, for hydrogen at 43°C
Kinematic viscosity (ν) = 119.9×10^{-6} m²/s
Prandtl number (Pr) = 0.703
Density (ρ) = 0.07811 kg/m³
Thermal conductivity (k) = 0.190 W/(m K)

SOLUTION

The transition to a turbulent boundary layer occurs at

$$Re_x = \frac{U_\infty x_c}{\nu} = 5 \times 10^5 \Rightarrow x_c = \frac{5\times10^5\,\nu}{U_\infty} = \frac{5\times10^5\ 119.9\times10^{-6}\ \text{m}^2/\text{s}}{80\,\text{m/s}} = 0.75 \text{ m}$$

$$\text{At} \quad x = 4.0 \text{ m}: Re_x = \frac{80\,\text{m/s}\ 4.0\,\text{m}}{119.9\times10^{-6}\,\text{m}^2/\text{s}} = 2.67 \times 10^6$$

which is beyond transition and is in the turbulent regime.

(a) The average friction coefficient between x = 0 and x = L = 4.0 m

(A) Turbulent, Equation (5.78b)

$$\overline{C_f}_T = 0.072\ Re_L^{-\frac{1}{5}} = 0.072\,(2.67\times10^6)^{-\frac{1}{5}} = 3.75 \times 10^{-3}$$

(B) Mixed, Equation (5.80)

$$\overline{C_f}_M = 0.072\left(Re_L^{-\frac{1}{5}} - \frac{0.0464x_c}{L}\right) = 0.072\left((2.67\times10^6)^{-\frac{1}{5}} - \frac{0.0464(0.75\,\text{m})}{4.0\,\text{m}}\right) = 3.13 \times 10^{-3}$$

(b) The drag force on the plate between $x = 0$ and $x = L$ is given by

$$D = \tau_s A = \overline{C_f}\,\frac{\rho U_\infty{}^2}{2} A$$

(A) Turbulent

$$D_T = \frac{1}{2}\,(3.75 \times 10^{-3})\ \left(0.07811\,\text{kg/m}^3\right)\left(80\,\text{m/s}\right)^2 (0.3\text{ m})\,(4.0\text{ m}) = 1.12\text{ N}$$

(B) Mixed

$$D_M = \frac{1}{2}\,(3.13 \times 10^{-3})\ \left(0.07811\,\text{kg/m}^3\right)\left(80\,\text{m/s}\right)^2 (0.3\text{ m})\,(4.0\text{ m}) = 0.94\text{ N}$$

(c) The average heat transfer coefficient between $x = 0$ and $x = L = 4.0$ m

(A) Turbulent, Equation (5.82)

$$(h_c)_T = \frac{k}{L}\,0.036\ Pr^{\frac{1}{3}}\,Re_L^{0.8} = \frac{0.19\,\text{W/(m}^2\,°\text{C)}}{4.0\,\text{m}}\ 0.036\ (0.703)^{\frac{1}{3}}\,(2.67 \times 10^6)^{0.8} = 210.5\ \text{W/(m}^2\ °\text{C)}$$

(B) Mixed, Equation (5.83)

$$(h_c)_M = \frac{k}{L}\,0.036\ Pr^{\frac{1}{3}}\ (Re_L^{0.8} - 23{,}200)$$

$$(h_c)_M = \frac{0.19\,\text{W/(m}^2\,°\text{C)}}{4.0\,\text{m}}\ 0.036\ \left(0.703\right)^{\frac{1}{3}}\ [(2.67 \times 10^6)^{0.8} - 23{,}200] = 175.2\ \text{W/(m}^2\ °\text{C)}$$

(d) The rate of heat transfer

$$q = h_c\,A\,(T_w - T_\infty)$$

(A) Turbulent

$$(q)_T = \left(210.0\,\text{W/(m}^2\,°\text{C)}\right)(0.3\text{ m})\,(4.0\text{ m})\,(71°\text{C} - 15°\text{C}) = 14{,}150\text{ W}$$

(B) Mixed

$$(q)_M = \left(175.2\,\text{W/(m}^2\,°\text{C)}\right)(0.3\text{ m})\,(4.0\text{ m})\,(71°\text{C} - 15°\text{C}) = 11{,}770\text{ W}$$

COMMENTS

Neglecting to take the laminar portion of the boundary layer into account led to a 20% overestimation in the rate of heat transfer from the plate.

PROBLEM 5.56

Determine the rate of heat loss from the wall of a building resulting from a 16 km/h wind blowing parallel to its surface. The wall is 24 m long and 6 m high, its surface temperature is 27°C, and the temperature of the ambient air is 4°C.

GIVEN

The wall of a building with wind blowing parallel to its surface
Wind speed (U_∞) = 16 km/h = 4.44 m/s
Length of wall (L) = 24 m
Height of wall (H) = 6 m
Surface temperature (T_w) = 27°C
Ambient air temperature (T_∞) = 4°C

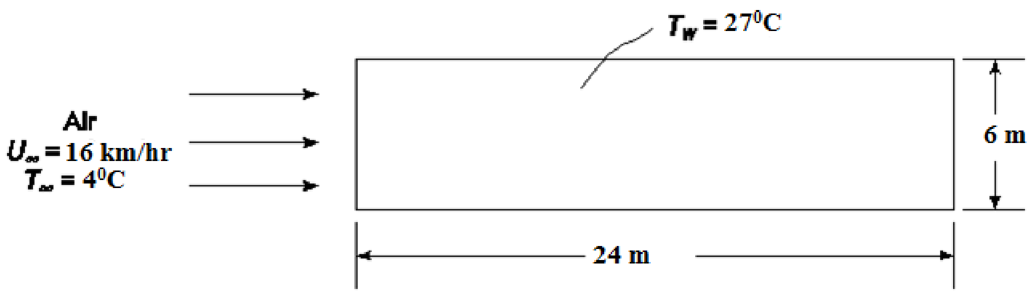
FIND

The rate of heat loss (q).

ASSUMPTIONS

Steady state
There is negligible moisture in the air
The wind blows along the length of the wall and parallel to it
Radiative loss is negligible.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average of the wall and ambient temperatures (16°C)

Kinematic viscosity (ν) = 15.3×10^{-6} m^2/s

Thermal conductivity (k) = 0.0247 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

Transition from laminar to turbulent boundary layer occurs at

$$Re_x = \frac{U_\infty x_c}{\nu} = 5 \times 10^5 \quad \Rightarrow x_c = \frac{5\times10^5\left(15.3*10^{-6}\,m^2/s\right)}{4.44 m/s} = 1.72 \text{ m}$$

Therefore, the boundary layer will be mixed and the average convective heat transfer coefficient is given by Equation (5.83)

$$h_c = \frac{k}{L}\, 0.036\; Pr^{\frac{1}{3}}\, (Re_1{}^{0.8} - 23{,}200)$$

where $$Re_L = \frac{U_\infty L}{\nu} = \frac{4.44(m/s)(24m)}{15.3*10^{-6}\,m^2/s} = 6.97 \times 10^6$$

$$\therefore \quad h_c = 0.036 \frac{[0.0247 W/(\mathrm{m} K)]}{24} (0.71)^{\frac{1}{3}} [(6.97 \times 10^6)^{0.8} - 23{,}200] = 9.09 \text{ W/(m}^2 \text{ K)}$$

The rate of convective heat loss from the wall is

$$q = h_c A (T_w - T_\infty) = [9.09 W/(m^2 K)] (6 \text{ m}) (24 \text{ m}) (27^0\text{C} - 4°\text{C}) = 30110 \text{ W}$$

COMMENTS

Treating the whole boundary layer as turbulent, (Equation (5.82)) would lead to a rate of heat loss 8% higher than the mixed boundary layer solution shown above.

PROBLEM 5.57

Water at a velocity of 2.5 m/s flows parallel to a 1-m-long horizontal, smooth and thin flat plate. Determine the local thermal and hydrodynamic boundary-layer thicknesses, and the local friction coefficient at the midpoint of the plate. What is the rate of heat transfer from one side of the plate to the water per unit width of the plate if the surface temperature is kept uniformly at 150°C, and the temperature of the main water stream is 15°C?

GIVEN

Water flows over a smooth and thin flat plate
Water velocity (U_∞) = 2.5 m/s
Length of plate (L) = 1 m
Surface temperature (T_s) = 150°C
Water temperature (T_∞) = 15°C

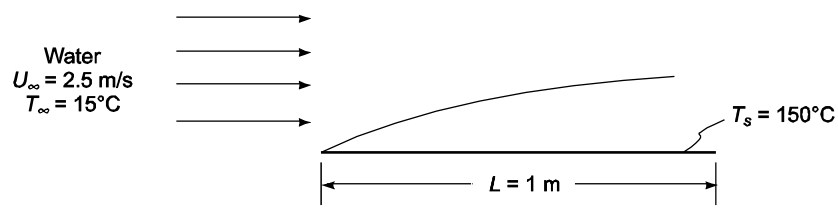
FIND

(a) Local thermal and hydrodynamic boundary layer thicknesses (δ, δ_{th}) and the local friction coefficient (C_{fx}) at the midpoint of the plate (x = 0.5 m)
(b) Heat transfer from the plate per unit width (q/w)

ASSUMPTIONS

Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the average of T_∞ and T_s (83°C)

Kinematic viscosity (ν) = 0.343×10^{-6} m²/s
Thermal conductivity (k) = 0.675 W/(m K)
Prandtl number (Pr) = 2.08

SOLUTION

The Reynolds number at x = 0.5 m is

$$Re_x = \frac{U_\infty x}{\nu} = \frac{2.5\ \text{m/s}\ \ 0.5\,\text{m}}{0.343\times10^{-6}\ \text{m}^2/\text{s}} = 3.46 \times 10^6 > 5 \times 10^5$$

Therefore, the boundary layer is turbulent. The hydrodynamic boundary layer thickness for a turbulent boundary layer is given by Equation (5.79)

$$\delta_x = 0.37 \left(\frac{\nu}{U_\infty}\right)^{\frac{1}{5}} x^{4/5} = 0.37 \left[\frac{0.343\times10^{-6}\ \text{m}^2/\text{s}}{2.5\,\text{m/s}}\right]^{1/5} (0.5\,\text{m})^{\frac{4}{5}} = 0.0092\ \text{m} = 9.1\text{mm}$$

The thermal boundary layer thickness for a turbulent boundary layer is also given by Equation (5.79)

$$\delta_{thx} = \delta_x = 9.1\ \text{mm}$$

The local friction factor for a turbulent boundary layer is given by the empirical Equation (5.78a) (for $5 \times 10^5 < Re < 10^7$)

$$C_{fx} = 0.0576\ Re^{-\frac{1}{5}} = 0.0576\ (3.64 \times 10^6)^{-\frac{1}{5}} = 2.81 \times 10^{-3}$$

(b) The heat transfer coefficient for a mixed boundary layer with transition at $Re = 5 \times 10^5$ is given by Equation (4.83)

$$h_c = \frac{k}{L} 0.036\ Pr^{\frac{1}{3}}\ [Re_L^{0.8} - 23{,}200] \qquad \text{where: } Re_L = \frac{2.5\,\text{m/s}\ 1\,\text{m}}{0.343 \times 10^{-6}\,\text{m}^2/\text{s}} = 7.28 \times 10^6$$

$$h_c = \frac{0.675\,\text{W/(m K)}}{1\,\text{m}}\ 0.036\,(2.08)^{\frac{1}{3}}\ [(7.28 \times 10^6)^{0.8} - 23{,}200] = 8560\ \text{W/(m}^2\ \text{K)}$$

The rate of heat transfer from the plate is

$$q = h_c\, A\, (T_s - T_\infty)$$

$$\therefore \quad \frac{q}{w} = h_c\, L\, (T_s - T_\infty) = 8560\,\text{W/(m}^2\text{K)}\ (1\ \text{m})\ (150°\text{C} - 15°\text{C}) = 1.16 \times 10^6\ \text{W/m}$$

COMMENTS

Treating the entire boundary layer as turbulent would lead to an overestimation of the rate of heat transfer of about 12%.

PROBLEM 5.58

The surface temperature of a thin, flat plate located parallel to an air stream is 90°C. The free stream velocity is 60 m/s, and the temperature of the air is 0°C. The plate is 60-cm-wide and 45-cm-long in the direction of the air stream. Neglecting the end effect of the plate and assuming that the flow in the boundary layer changes abruptly from laminar to turbulent at a transition Reynolds number of $Re_{tr} = 4 \times 10^5$, find

(a) the average heat transfer coefficient in the laminar and turbulent regions

(b) the rate of heat transfer for the entire plate, considering both sides

(c) the average friction coefficient in the laminar and turbulent regions

(d) the total drag force

Also plot the heat transfer coefficient and local friction coefficient as a function of the distance from the leading edge of the plate.

GIVEN

Air flow over a flat plate
Plate surface temperature (T_s) = 90°C
Air velocity (U_∞) = 60 m/s
Air temperature (T_∞) = 0°C
Plate length (L) = 45 cm = 0.45 m
Plate width = 60 cm = 0.6 m

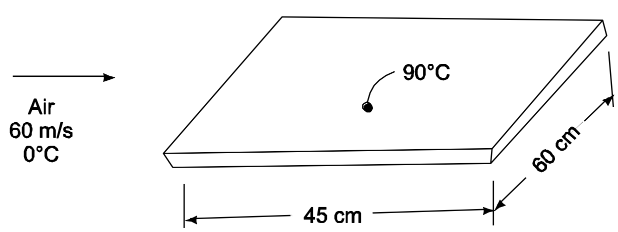
FIND

(a) The average heat transfer coefficient in the laminar (h_{cL}) and turbulent (h_{cT}) regions
(b) The rate of heat transfer for the entire plate, considering both sides (both sides)
(c) The average friction coefficient in the laminar (C_{fL}) and turbulent (C_{fT}) regions
(d) The total drag force (D)
(e) Plot the heat transfer coefficient (h_{cx}) and local friction coefficient (C_{fx}) as a function of the distance from the leading edge (x).

ASSUMPTIONS

Steady state
End effect of the plate is negligible
Boundary layer changes from laminar to turbulent at $Re = 4 \times 10^5$

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average of T_s and T_∞ (45°C)

Kinematic viscosity (ν) = 18.1×10^{-6} m²/s

Thermal conductivity (k) = 0.0269 W/(m K)

Prandtl number (Pr) = 0.71

Density (ρ) = 1.075 kg/m³

SOLUTION

The transition to turbulence occurs at

$$Re_x = \frac{U_\infty x_c}{\nu} = 4 \times 10^5 \Rightarrow x_c = \frac{4\times10^5\,\nu}{U_\infty} = \frac{4\times10^5\ 18.1\times10^{-6}\,\mathrm{m^2/s}}{60} = 0.121\ \mathrm{m}$$

The Reynolds number at the end of the plate is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{60\,\mathrm{m/s}\ \ 0.45\,\mathrm{m}}{18.1\times10^{-6}\,\mathrm{m^2/s}} = 1.49 \times 10^6$$

(a) For the laminar region, h_{cx} is given by Equation (5.56) and the average heat transfer coefficient is

$$h_{cL} = \frac{1}{x_c}\int_0^{x_c} \frac{k}{x} 0.33 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}\,dx = \frac{k}{x_c} 0.66\, Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

$$h_{cL} = \frac{0.0269\,\mathrm{W/(m\,K)}}{0.121\,\mathrm{m}}\ 0.66\ (4\times10^5)^{\frac{1}{2}} (0.71)^{\frac{1}{3}} = 82.8\ \mathrm{W/(m^2\,K)}$$

For the turbulent region, h_{cx} is given by Equation (5.81)

$$h_{cT} = \frac{1}{L-x_c}\int_{x_c}^{L} \frac{k}{x} 0.0288 Re_x^{0.8} Pr^{\frac{1}{3}}\,dx = \frac{k}{L-x_c}\ 0.036\,(Re_L^{0.8} - Re_{x_c}^{0.8})\,Pr^{\frac{1}{3}}$$

$$h_{cT} = \frac{0.0269\,\mathrm{W/(m\,K)}}{0.45\,\mathrm{m}-0.121\,\mathrm{m}}\ 0.036\,[(1.49\times10^6)^{0.8} - (4\times10^5)^{0.8}]\ (0.71)^{\frac{1}{3}} = 148.6\ \mathrm{W/(m^2\,K)}$$

(b) The total heat transfer is the sum of the heat transfer from both regions.

$$q = q_{\mathrm{Lam}} + q_{\mathrm{Turb}} = (h_{cL} A_L + h_{cT} A_T)\,(T_s - T_\infty)$$

$$q = \left[\ 82.8\,\mathrm{W/(m^2K)}\ \ 0.121\,\mathrm{m}\ \ 0.6\,\mathrm{m} + 148.6\,\mathrm{W/(m^2K)}\ \ 0.45\,\mathrm{m}-0.121\,\mathrm{m}\ \ 0.6\,\mathrm{m}\ \right](90°\mathrm{C} - 0°\mathrm{C})$$

$$q = 3131\ \mathrm{W}$$

For both sides $\quad q_{\mathrm{Total}} = 2\,q = 6362\ \mathrm{W}$

(c) The average friction coefficient in the laminar region is given by Equation (5.31)

$$\overline{C_{fL}} = 1.33\,Re_{x_c}^{-\frac{1}{2}} = 1.33\,(4\times10^5)^{-\frac{1}{2}} = 0.00210$$

The local friction coefficient in the turbulent region is given by Equation (5.78a). The average friction coefficient in the turbulent region is

$$\overline{C_{fL}} = \frac{k}{L-x_c}\int_{x_c}^{L} C_{fx}\,dx = \frac{1}{L-x_c}\int_{x_c}^{L} 0.0576\left(\frac{U_\infty x}{\nu}\right)^{-\frac{1}{3}} dx = \frac{0.0576}{L-x_c}\left(\frac{U_\infty}{\nu}\right)^{-\frac{1}{5}} \frac{5}{4}\left(L^{\frac{4}{5}} - x_c^{\frac{4}{5}}\right)$$

$$\overline{C_{fL}} = \frac{0.0576}{0.45\,\mathrm{m}-0.121\,\mathrm{m}}\left(\frac{60\,\mathrm{m/s}}{18.1\times10^{-6}\,\mathrm{m^2/s}}\right)^{-\frac{1}{5}} 1.25\,[(0.45\ \mathrm{m})^{0.8} - (0.121)^{0.8}] = 0.00373$$

(d) The drag force is

$$D = \tau_s A \qquad \text{where: } \tau_s = \overline{C_f}\,\frac{1}{2}\rho\,U_\infty^2 A_s \qquad \text{from Equation (5.13)}$$

For both sides of the plate

$$D = \rho\,U_\infty^2\ \ \overline{C_{fL}}\,A_L + \overline{C_{fT}}\,A_T$$

(e) For the laminar region, $0 < x < 0.121$ m, Equation (5.56) gives the heat transfer coefficient

$$h_{cx} = \frac{k}{x} 0.33 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}} = \frac{0.0269\,\text{W/(m K)}}{x} 0.33 \left(\frac{60\,\text{m/s}}{18.1 \times 10^{-6}} \right)^{\frac{1}{2}} x^{\frac{1}{2}} (0.71)^{\frac{1}{3}} = 14.41\,\text{W/m}^{\frac{3}{2}}\,\text{K} \quad x^{-\frac{1}{2}}$$

For the turbulent region, 0.121 m < x < 0.45 m, from Equation (5.81)

$$h_{cx} = \frac{k}{x} 0.0288 \; Pr^{\frac{1}{3}} Re_x^{0.8} = \; 113.7\,\text{W/(m}^2\,\text{K)} \quad x^{-0.2}$$

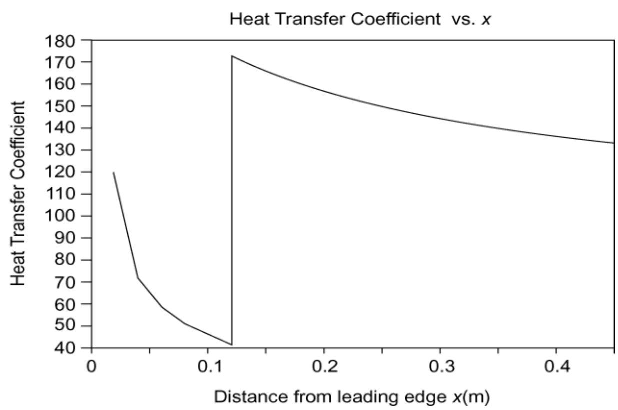
The friction coefficient for the laminar region (Equation (5.30)) is

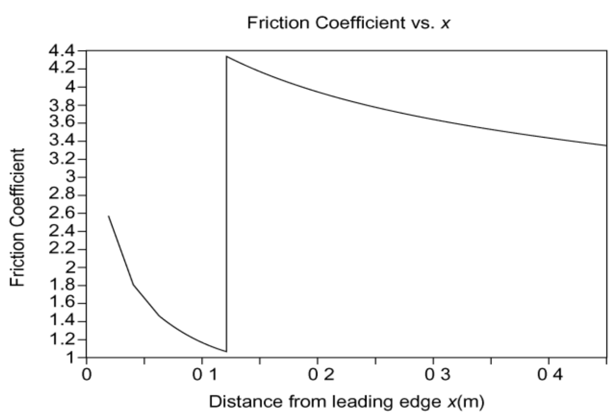
$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} = 3.65 \times 10^{-4} \; x^{-\frac{1}{2}}$$

For the turbulent region (Equation (5.78a))

$$C_{fx} = 0.0576 \; Re^{-\frac{1}{5}} = 2.859 \times 10^{-3} \; x^{-\frac{1}{5}}$$

The variations of the heat transfer coefficient and friction coefficient with distance from the leading edge are plotted below





PROBLEM 5.59

An aluminum cooling fin for a heat exchanger is situated parallel to an atmospheric pressure air stream. The fin is 0.075-m-high, 0.005-m-thick, and 0.45 m in the flow direction. Its base temperature is 88°C, and the air is at 10°C. The velocity of the air is 27 m/s. Determine the total drag force and the total rate of heat transfer from the fin to the air.

GIVEN

Air flow over a heat exchanger fin
Fin length (L) = 0.45 m
Fin height (w) = 0.075
Fin thickness = 0.005 m
Fin base temperature (T_b) = 88°C
Air temperature (T_∞) = 10°C
Air velocity (U_∞) = 27 m/s

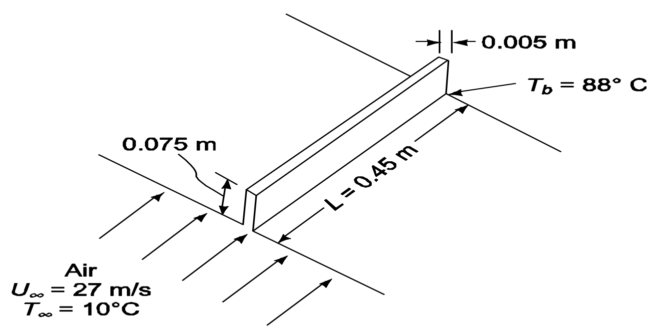
FIND

(a) The total drag force (D) on the fin
(b) The total rate of heat transfer (q) from the fin to the air

ASSUMPTIONS

Steady state
Edge effects are negligible
Both sides of the fin are exposed to the air
Transition to a turbulent boundary layer occurs at $Re_x = 5 \times 10^5$
Fin thickness is negligible
Radiation is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average of T_b and T_∞ (49°C)
Kinematic viscosity (ν) = 18.4×10^{-6} m²/s
Thermal conductivity (k) = 0.0271 W/(m K)
Density (ρ) = 1.061 kg/m³
Prandtl number (Pr) = 0.71
From Appendix 2, Table 12, for aluminum at the average of T_b and T_∞ (49°C)
Thermal conductivity (k) = 238 W/(m K)

SOLUTION

The Reynolds number at the end of the fin is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{27\,\text{m/s}\ (0.45\,\text{m})}{18.4\times10^{-6}\,\text{m}^2/\text{s}} = 6.60 \times 10^5 > 5 \times 10^5$$

The boundary layer is turbulent at the end of the fin. The transition to turbulence occurs at

$$Re_{x_c} = \frac{U_\infty \, x_c}{\nu} = 5 \times 10^5 \Rightarrow x_c = \frac{5\times10^5 \; \nu}{U_\infty} = \frac{5\times10^5 \; 18.4\times10^{-6}\,\mathrm{m^2/s}}{27}\frac{\mathrm{m}}{\mathrm{s}} = 0.341 \text{ m}$$

(a) The average friction factor (C_f) for a mixed boundary layer is given by Equation (5.80)

$$\overline{C}_f = 0.072 \left(\mathrm{Re}_L^{-\frac{1}{5}} \frac{0.0464\,x_c}{L} \right) = 0.072\left[(6.6\times10^5)^{-\frac{1}{5}} - \frac{0.0464\,(0.341\mathrm{m})}{0.45\,\mathrm{m}} \right] = 0.00240$$

The drag force on both sides of the plate (using Equation (5.13) for the shear stress at the wall) is

$$D = 2\;\tau_s\,A = C_f\,\rho\,U_\infty^2\,A = 0.0024\;\; 1.061\,\mathrm{kg/m^3}\;\; 27\,\mathrm{m/s}^{\;2}\,(0.075 \text{ m})\,(0.45 \text{ m}) = 0.063 \text{ N}$$

(b) The average heat transfer coefficient (h_c) for a mixed boundary layer is given in Equation (5.83)

$$h_c = \frac{k}{L}\; 0.036\; Pr^{\frac{1}{3}}\; [Re_L^{0.8} - 23{,}200] = \frac{0.0271\,\mathrm{W/(m\,K)}}{0.45\,\mathrm{m}}\; 0.036\; (0.71)^{\frac{1}{3}}\; [(6.6\times10^5)^{0.8} - 23{,}200\}$$

$$h_c = 42.7 \text{ W/(m}^2\text{ K)}$$

The rate of heat transfer from a fin of uniform cross section and convection from the tip is given in Table 2.1

$$q = M\; \frac{\sinh(m L_f) + \left(\dfrac{h}{m k_a}\right)\cosh(m L_f)}{\cosh(m L_f) + \left(\dfrac{h}{m k_a}\right)\sinh(m L_f)} \qquad \text{where } m \equiv \sqrt{\frac{\overline{h}_c\,P}{k_a\,A_c}}$$

P = perimeter = 2(0.45 m + 0.005 m) = 0.91 m

A_c = cross sectional area = (0.005 m) (0.45 m) = 0.00225 m^2

L_f = 0.075 m

$$\therefore\; m = \sqrt{\frac{42.7\,\mathrm{W/(m^2\,K)}\;\;(0.91\mathrm{m})}{238\,\mathrm{W/(m\,K)}\;\;(0.00225\,\mathrm{m^2})}} = 8.52\;\frac{1}{\mathrm{m}} \qquad => \qquad m\,L_f = 8.52\;\frac{1}{\mathrm{m}}\;\;(0.075 \text{ m}) = 0.639$$

$$M = (T_b - T_\infty)\;\sqrt{\overline{h}_c\,P\,k_a\,A_c} = (88°\mathrm{C} - 10°\mathrm{C})\sqrt{42.7\,\mathrm{W/(m^2K)}\;\;(0.91\mathrm{m})\;\;238\,\mathrm{W/(m\,K)}\;\;(0.0025\,\mathrm{m^2})}$$

$$M = 356 \text{ W}$$

$$\frac{\overline{h}_c}{m\,k_a} = \frac{42.7\,\mathrm{W/(m^2\,K)}}{8.52\dfrac{1}{3}\;\;238\,\mathrm{W/(m\,K)}} = 0.0211$$

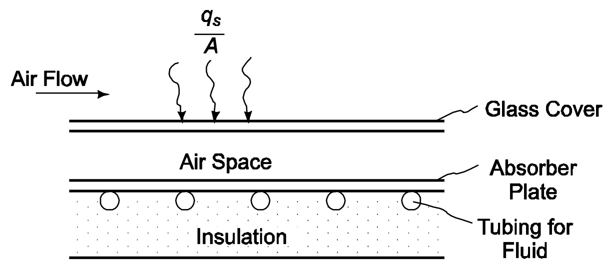
$$\therefore\; q_f = 356 \text{ W}\;\frac{\sinh(0.639) + 0.0211\cosh(0.639)}{\cosh(0.639) + 0.0211\sinh(0.639)} = 206 \text{ W}$$

COMMENTS

If the entire fin was assumed to be at the base temperature, the rate of heat transfer from the fin would be about 225 W, about 9% higher than calculated above. The high conductivity of the fin material makes this installation very thermally efficient, i.e., η_f = 91%.

PROBLEM 5.60

A 4-m^2 flat plate solar collector for domestic hot water heating is shown schematically. Solar radiation at a rate of 750 W/m^2 is incident on the glass cover which transmits 90% of the incident flux. Water flows through the tubes soldered to the backside of the absorber plate, entering with a temperature of 25°C. The Glass cover has a temperature of 27°C in the steady state and radiates heat with an emissivity of 0.92 to the sky at –50°C. In addition, the glass cover losses heat by convection to air at 20°C flowing over its surface at 20 mph.



(a) **Calculate the rate at which heat is collected by the working fluid, i.e., the water in the tubes, per unit area of the absorber plate.**

(b) **Calculate the collector efficiency η_c defined as the ratio of useful energy transferred to the water in the tubes to the solar energy incident on the collector cover plate.**

(c) **Calculate the outlet temperature of the water if its flow rate through the collector is 0.02 kg/s. The specific heat of the water is 4179 J/(kg K).**

GIVEN

A flat plate solar collector with air flowing over it
Collector dimensions = 2 m × 2 m
Incident solar flux = 750 W/m^2
Glass cover transmits 90% of solar flux
Water enters tubes at a temperature (T_{wi} = 25°C)
Glass cover steady state temperature (T_s) = 27°C = 300 K
Emissivity of glass cover (ε) = 0.92
Sky temperature ($T_{\infty r}$) = –50°C = 223 K
Ambient air temperature ($T_{\infty c}$) = 20°C = 293 K
Air speed (U_∞) = 20 mi/h
Water flow rate (m) = 0.02 kg/s
Specific heat of water (c_p) = 4179 J/(kg K)

FIND

(a) Heat flux to the water q_w/A

(b) Collector efficiency (η_c) = $\dfrac{\text{energy to the water}}{\text{incident solar energy}}$

(c) Outlet temperature of the water (T_{wo})

ASSUMPTIONS

Steady State
Radiative heat transfer between the absorber and the glass plate is negliagible
The absorber plate absorbs all the incident solar radiation
Radiative heat transfer from the absorber plate, through the glass to the sky, is negligible
Heat transfer through the back and sides of the collector is negligible
Solar radiation blocked by the collector frame is negligible

Glass cover and absorber temperatures are uniform
The solar energy absorbed by the glass is negligible
The sky behaves as a blackbody

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the film temperature (23.5°C)

Kinematic viscosity (ν) = 16.0×10^{-6} m²/s

Thermal conductivity (k) = 0.0295 W/(m K)

Prandtl number (Pr) = 0.71

From Appendix 1, Table 5

The Stephan-Boltzmann Constant (σ) = 5.67×10^{-8} W/(m² K⁴)

SOLUTION

The glass cover absorbs solar energy and also absorbs energy from the absorber plate. The cover loses heat to ambient by convection to the air and by reradiation. An energy balance on the glass cover will allow us to determine the rate of heat transfer from the absorber plate to the glass cover plate. The absorber plate absorbs solar energy but loses some of this to the glass cover plate as described above. Therefore, an energy balance on the absorber plate will allow us to determine the rate at which energy is absorbed by the absorber plate. Based on the assumptions listed above, this will equal the rate at which energy is delivered to the water.

(a) Energy balance on the glass cover plate

Radiative flux to the sky (q_r/A) + Convective flux to the air (q_c/A) = Net energy gain from the absorber plate (q_{A-G}/A) + Solar energy absorbed (q_s/A)

$$\text{Where } \frac{q_r}{A} = \varepsilon\,\sigma\,(T_s^4 - T_{\infty r}^4) \qquad \text{[Equation (1.17)]}$$

$$\frac{q_c}{A} = \bar{h}_c\,(T_s - T_{\infty c}) \qquad \text{[Equation (1.10)]}$$

$$\frac{q_s}{A} = (0.1)\ \left(750\,\text{W/m}^2\right) = 75\ \text{W/m}^2$$

$$\frac{q_{A-G}}{A} = \varepsilon\,\sigma\,(T_s^4 - T_{\infty r}^4) + h_c\,(T_s - T_{\infty c}) - q_s/A$$

The Reynolds number at the trailing edge of the glass cover plate is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(20\,\text{mi/h})\,(2\,\text{m})\,(5280\,\text{ft/mi})\,(0.3048\,\text{m/ft})}{\left(16.2\times10^{-6}\,\text{m}^2/\text{s}\right)\,(3600\,\text{s/h})} = 1.10 \times 10^6 > 5 \times 10^5$$

Therefore, the boundary layer is mixed and the average convective heat transfer coefficient is given by Equation (5.83)

$$h_\infty = 0.036\ \frac{k}{L}\,Pr^{\frac{1}{3}}\,[Re_L^{0.8} - 23{,}200]$$

$$h_c = 0.036\,\frac{0.0295\,\text{W/(m K)}}{2\,\text{m}}\,(0.71)^{\frac{1}{3}}\,[(1.11 \times 10^6)^{0.8} - 23{,}200] = 21.7\ \text{W/(m}^2\text{ K)}$$

$$\frac{q_{A-G}}{A} = 0.92\ \left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)\ [(300\text{ K})^4 - (223\text{ K})^4] + 21.7\ \text{W/(m}^2\text{ K)}$$

$$(300\text{ K} - 293\text{ K}) - 75\ \text{W/(m}^2\text{ K)}$$

$$\frac{q_{A-G}}{A} = (294 + 152 - 75)\ \text{W/m}^2 = 371\ \text{W/m}^2$$

Energy balance on the absorber plate
heat flux to the water (q_w/A) = solar gain (0.9 q_s/A) heat flux to glass cover (q_{A-G}/A)

$$\frac{q_w}{A} = 0.9\ \ 750\,\text{W/m}^2\ \ - 371\ \text{W/m}^2 = 304\ \text{W/m}^2$$

(b) Collector efficiency

$$\eta_c = \frac{\frac{q_w}{A}}{\frac{q_s}{A}} = \frac{304}{750} = 0.41 = 41\%$$

(c) $$q_w = m\,c_p\,(T_{wo} - T_{wi}) = \left(\frac{q_w}{A}\right)A$$

Solving for the water outlet temperature

$$T_{wo} = T_{wi} + \left(\frac{q_w}{A}\right)\frac{A}{m\,c_p} = 25°\text{C} + 304\ \ 304\,\text{W/m}^2\ \ \ \text{J/Ws}\ \ \frac{2\,\text{m}\ \ 2\,\text{m}}{0.02\,\text{kg/s}\ \ 4179\,\text{J/(kg K)}} = 39.5°\text{C}$$

PROBLEM 5.61

A fluid at temperature T_∞ is flowing at a velocity U_∞ over a flat plate that is at the same temperature as the fluid for a distance x_0 from the leading edge, but at a higher temperature T_s beyond this point. Show by means of the integral boundary-layer equations that ζ, the ratio of the thermal boundary-layer thickness to the hydrodynamic boundary-layer thickness, over the heated portion of the plate is approximately

$$\zeta \approx Pr^{-\frac{1}{3}}\left[1-\left(\frac{x_o}{x}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}$$

if the flow is laminar.

GIVEN

Laminar flow over a flat plate
Fluid temperature = T_∞
Fluid velocity = U_∞
Plate temperature = T_∞ for $x < X_o$
Plate temperature = T_s for $x > X_o$

FIND

Show that

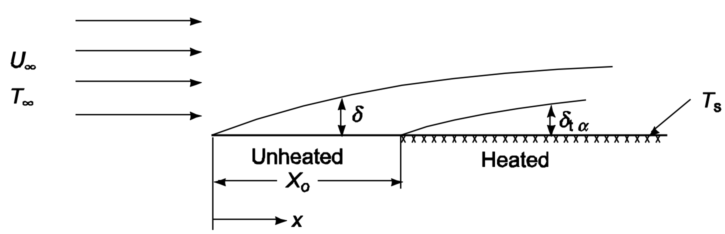
$$\zeta \approx Pr^{-\frac{1}{3}}\left[1-\left(\frac{x_o}{x}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}$$

over the heated portion of the plate

ASSUMPTIONS

Steady state
The temperature distribution is a third-order polynomial: $T - T_s = ay + cy^3$
Property value changes due to the temperature profile do not affect the hydrodynamic boundary layer.

SKETCH



SOLUTION

The velocity and temperature distributions given in Equations (5.46) and (5.53) are valid for this problem

$$\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \qquad \text{for } x > 0$$

$$\frac{T - T_s}{T_\infty - T_s} = \frac{3}{2}\frac{y}{\delta_t} - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 \qquad \text{for } x > x_o$$

The integral energy equation is given by Equation (5.44)

$$\frac{d}{dx}\int_0^{\delta_t} T_\infty - T \;\; u\,dy - \alpha\left(\frac{\partial T}{\partial y}\right)_{y=0} = 0$$

As shown in Section 4.9.1, for the above velocity and temperature distributions

$$\int_0^{\delta_t} T_\infty - T \;\; u\,dy = (T_\infty - T_s)\, U_\infty\, \delta\left(\frac{3}{20}\zeta^2 - \frac{3}{280}\zeta^4\right)$$

where $\zeta = \dfrac{\delta_t}{\delta}$

Also

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = (T_\infty - T_s)\left[\frac{3}{2}\frac{1}{\delta_t} - \frac{3}{2}\frac{1}{\delta_t^3}y^2\right]_{y=0} = \frac{3}{2}\frac{1}{\delta_t}(T_\infty - T_s) = \frac{3}{2}\frac{1}{\zeta\delta}(T_\infty - T_s)$$

Substituting these expressions into the energy equation

$$(T_\infty - T_s)\, U_\infty\, \frac{d}{dx}\left[\delta\left(\frac{3}{20}\zeta^2 - \frac{3}{280}\zeta^4\right)\right] = \frac{3}{2}\frac{\alpha}{\zeta\delta}(T_\infty - T_s)$$

The hydrodynamic boundary layer begins at $X = 0$, but the thermal boundary layer does not begin until $X = X_o$. It will be assumed, therefore, that $\delta_t < \delta \rightarrow \zeta < 1$, therefore, the term $3/280\ \zeta^4$ will be neglected, leaving

$$\frac{3}{20}U_\infty \frac{d}{dx}(\delta\,\zeta^2) = \frac{3}{2}\frac{\alpha}{\zeta\delta}$$

$$\frac{1}{10}U_\infty\,\zeta\,\delta\left(2\delta\zeta\frac{d\zeta}{dx} + \zeta^2\frac{d\delta}{dx}\right) = \alpha$$

$$\frac{1}{10}U_\infty\left(2\delta^2\zeta^2\frac{d\zeta}{dx} + \delta\zeta^3\frac{d\delta}{dx}\right) = \alpha$$

As shown in Equation (5.50)

$$\delta = \sqrt{\frac{280}{13}}\left(\frac{U_\infty x}{\nu}\right)^{-\frac{1}{2}} x$$

$$\therefore \quad \frac{d\delta}{dx} = \frac{1}{2}\sqrt{\frac{280}{13}}\left(\frac{U_\infty x}{\nu}\right)^{-\frac{1}{2}}$$

Substituting these into the energy equation

$$\frac{1}{10}U_\infty\left[\frac{560}{13}\left(\frac{\nu}{U_\infty x}\right)x^2\zeta^2\frac{d\zeta}{dx} + \zeta^3\,\frac{140}{13}\left(\frac{\nu}{U_\infty x}\right)x\right] = \alpha$$

$$\zeta^3 + 4x\,\zeta^2\frac{d\zeta}{dx} = \frac{13\alpha}{14\nu} = \frac{13}{14Pr} \approx \frac{1}{Pr}$$

Let $\lambda = \zeta^3 \qquad \therefore \quad \dfrac{d\lambda}{dx} = 3\zeta^2 \dfrac{d\zeta}{dx}$

$$\lambda + \frac{4}{3} \times \frac{d\lambda}{dx} = \frac{1}{Pr}$$

The solution to this differential equation is the sum of the homogeneous solution and a particular solution. A particular solution is $\lambda = 1/Pr$. The homogeneous solution can be found by assuming $\lambda = x^m$

$$x^m + \frac{4}{3} x\,(m\,x^{m-1}) = 0$$

$$\left(1 + \frac{4}{3} m\right) x^m = 0$$

$$m = -\frac{3}{4}$$

Therefore, the solution to the differential equation is

$$\lambda = \frac{1}{Pr} + Cx^{-\frac{3}{4}}$$

The constant C can be evaluated by the condition that at $x = x_o$, $\delta_t = 0 \rightarrow \zeta = 0 \rightarrow \lambda = 0$

$$0 = \frac{1}{Pr} + Cx^{-\frac{3}{4}} \Rightarrow C = -\frac{1}{Pr} x_o^{\frac{3}{4}}$$

$$\lambda = \frac{1}{Pr}\left[1 - \left(\frac{x_o}{x}\right)^{\frac{3}{4}}\right]$$

$$\zeta = \lambda^{\frac{1}{3}} = Pr^{-\frac{1}{3}}\left[1 - \left(\frac{x_o}{x}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}$$

PROBLEM 5.62

A flat plate solar collector is installed flat-flush on the surface of a roof that is inclined 30° from the vertical normal and facing south in order to maximize its performance. The solar collector is a 2.5 m * 2.0 m rectangle and is placed at a distance of 1.5 m from one edge of the roof, as shown in the figure. If the cover plate of the solar collector is at 25°C when ambient air at 15°C is flowing parallel to it from the direction of the uncovered leading section of the roof and with a velocity of 2 m/s, estimate the rate of heat loss from the solar collector panel.

GIVEN

Air flow over a flat collector plate with unheated starting length
Air temperature (T_∞) = 15°C = 288 K
Cover plate temperature (T_s)= 25^{0}C = 298 K
Air velocity (U_∞) = 2 m/s
Solar collector dimension= 2.5 m * 2 m
Distance of collector from leading edge of roof (ζ) = 1.5 m

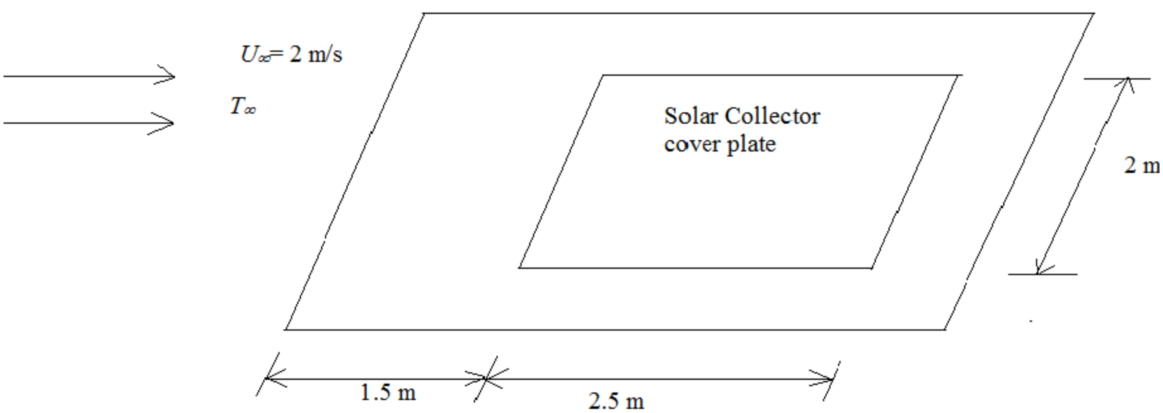
FIND

Rate of heat loss from solar collector panel.

ASSUMPTIONS

Steady state
Transition to turbulence occurs at $Re_x^\bullet$ = 5*10^5
Radiation heat transfer is negligible because of the low emissivity of the plate

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at one atmosphere and at the film temperature 293 K

Kinematic viscosity (ν) = 16.0 × 10^{-6} m^2/s

Thermal conductivity (k) = 0.0295 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

Reynold's number for the roof and solar collector cover plate of total length L= 4 m is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{2*4}{16*10^{-6}} = 5*10^5 <= Re_{x,c}$$

The laminar boundary layer occurs through the roof and cover plate and heat transfer occurs starting from 1.5 m of the leading edge.

From Eq (5.84) for laminar flow we have

$$Nu_x/Nu_{x,\zeta=0} = \left\{1-\left(\frac{\varsigma}{x}\right)^{3/4}\right\}^{-1/3}$$

$$h_x = 0.332\left(\frac{U_\infty}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3}\, x^{-1/2} k \left\{1-\left(\frac{\varsigma}{x}\right)^{3/4}\right\}^{-1/3}$$

$$q = \int_\xi^L q^{"} dA = \int_\xi^L h_x (T_s - T_\infty) dA$$

$$= \int_\xi^L 0.332(T_s - T_\infty)\left(\frac{U_\infty}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3}\, x^{-1/2} k \left\{1-\left(\frac{\varsigma}{x}\right)^{3/4}\right\}^{-1/3} dx$$

$$= 0.332(298-288)\left(\frac{2}{16*10^{-6}}\right)^{1/2} (0.71)^{1/3} * 0.0295 \int_{1.5}^{4} x^{-1/2} \left\{1-\left(\frac{1.5}{x}\right)^{3/4}\right\}^{-1/3} dx$$

$$= 30.9 * \int_{1.5}^{4} x^{-1/2} \left\{1-\left(\frac{1.5}{x}\right)^{3/4}\right\}^{-1/3} dx \ \mathrm{W}$$

Solving the above integral in **Mathmatica ®** we get

q= 30.9* 2.59 W
=80 W
Thus the rate of heat loss from solar collector panel is 80 W.

PROBLEM 5.63

A flat plate test section, which is 0.5 m wide and has an electrically heated surface installed 0.5 m away from the leading edge, as depicted in the schematic given, is to be designed for a boundary-layer measurement experiment. The electrically heated strip, which is 10 cm long and 5 mm deep, is insulated at its bottom surface as well as both wide-end edges. If the surface of this electrical heater strip is to be maintained at a uniform temperature of 120°C when atmospheric pressure air at 20°C flows over it at 20 m/s, estimate the rate of heat generation necessary in the heated strip in order to maintain its surface temperature uniformly.

GIVEN

Air flow over a flat plate with an electrical heater strip
Air temperature (T_∞) = 20°C = 293 K
Heater strip temperature (T_s)= 120^0C = 393 K
Air velocity (U_∞) = 20 m/s
Electrically heated strip dimension =10 m * 5 mm
Distance of collecter from leading edge of roof (ζ) = 0.5 m

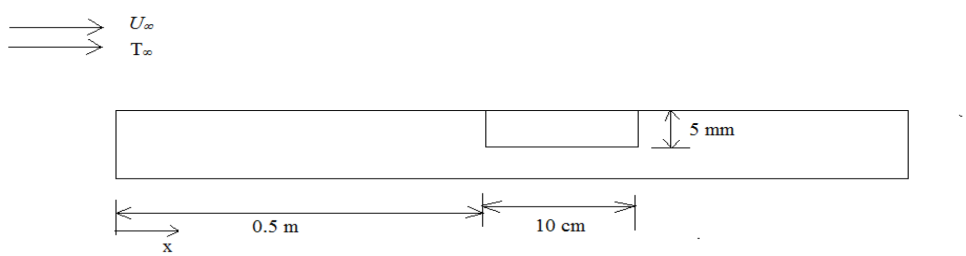
FIND

Rate of heat generation necessary in heated strip in order to maintain its surface temperature uniformly.

ASSUMPTIONS

Steady state
Transition to turbulence occurs at $Re_x^{\bullet}$ = $5*10^5$
Radiation heat transfer is negligible because of the low emissivity of the plate

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at one atmosphere and at the film temperature 343 K

Kinematic viscosity (ν) = 20.5 × 10^{-6} m^2/s

Thermal conductivity (k) = 0.0286 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

Reynold's number for the roof and solar collector cover plate of total length L= 0.6 m is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{20*0.6}{20.5*10^{-6}} = 5.85*10^5 > Re_{x,c}$$

Transition length is where flow changes from laminar to turbulent.

$$L_c = \frac{Re_{L,c}\nu}{U_\infty} = \frac{5*10^5*20.5*10^{-6}}{20} = 0.5125 \text{ m}$$

From Equn (5.84) for laminar flow we have $Nu_x / Nu_{x,\xi=0} = \left\{1-\left(\frac{\varsigma}{x}\right)^{3/4}\right\}^{-1/3}$

From Equation (5.86)
For constant heat flux Nu number for laminar flow is given by
$Nu_x = 0.453 * \mathrm{Re}_x^{0.5}\, \mathrm{Pr}^{1/3}$
and for turbulent flow is given by
$Nu_x = 0.0308 * \mathrm{Re}_x^{0.8}\, \mathrm{Pr}^{1/3}$
Thus h_x

$$h_x \text{ (laminar)} = 0.453\left(\frac{U_\infty}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3}\, x^{-1/2} k \left\{1-\left(\frac{\varsigma}{x}\right)^{3/4}\right\}^{-1/3}$$

$$h_x \text{ (turbulent)}\ 0.0308\left(\frac{U_\infty}{\nu}\right)^{0.8} \mathrm{Pr}^{1/3}\, x^{-0.2} k \left\{1-\left(\frac{\varsigma}{x}\right)^{3/4}\right\}^{-1/3}$$

$$q = \int_\xi^L q'' dA = \int_\xi^L h_x (T_s - T_\infty) dA$$

$$= \int_\xi^{L_c} 0.453(T_s - T_\infty)\left(\frac{U_\infty}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3}\, x^{-1/2} k \left\{1-\left(\frac{\varsigma}{x}\right)^{3/4}\right\}^{-1/3} dx \quad +$$

$$\int_{L_c}^{L} 0.0308(T_s - T_\infty)\left(\frac{U_\infty}{\nu}\right)^{0.8} \mathrm{Pr}^{1/3}\, x^{-0.2} k \left\{1-\left(\frac{\varsigma}{x}\right)^{3/4}\right\}^{-1/3} dx$$

$$= \quad 0.453(393-293)*0.0286*\left(\frac{20}{20.5*10^{-6}}\right)^{1/2} (0.71)^{1/3} \int_{0.5}^{0.5125} x^{-1/2} \left\{1-\left(\frac{\varsigma}{x}\right)^{3/4}\right\}^{-1/3} dx \quad +$$

$$0.0308(393-293)\left(\frac{20}{20.5*10^{-6}}\right)^{0.8} (0.71)^{1/3} * 0.0286 * \int_{0.5125}^{0.6} x^{-0.2} \left\{1-\left(\frac{0.5}{x}\right)^{3/4}\right\}^{-1/3} dx$$

$$= 1141.6 * \int_{0.5}^{0.5125} x^{-1/2} \left\{1-\left(\frac{\varsigma}{x}\right)^{3/4}\right\}^{-1/3} dx + 4861* \int_{0.5125}^{0.6} x^{-0.2} \left\{1-\left(\frac{0.5}{x}\right)^{3/4}\right\}^{-1/3} dx \ \mathrm{W}$$

Solving the above integrals in Mathematica® we get
q= 1141.6*0.1+ 4861*0.245 W
=114.2+ 1191 W
=1305.2 W
Rate of heat generation necessary in heated strip in order to maintain its surface temperature uniformly is 1305 W.

PROBLEM 5.64

A highly polished chromium flat plate is placed in a high-speed wind tunnel to simulate flow over the fuselage of a supersonic aircraft. The air flowing in the wind tunnel is at a temperature of 0°C, a pressure of 3500 N/m^2, and a velocity parallel to the plate of 800 m/s. What temperature is the adiabatic wall temperature in the laminar region and how long is the laminar boundary layer?

GIVEN

High speed air flow over a flat plate
Air temperature (T_∞) = 0°C = 273 K
Air pressure = 3500 N/m^2
Air velocity (U_∞) = 800 m/s

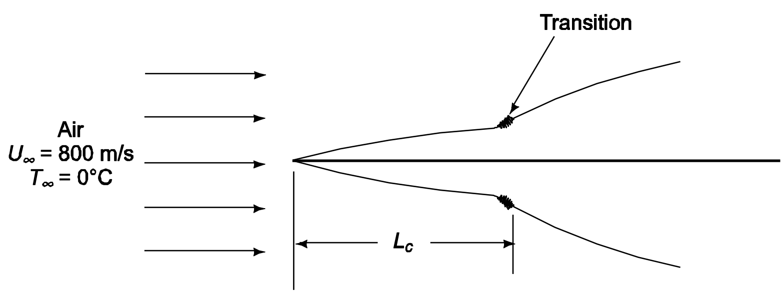
FIND

(a) Adiabatic wall temperature (T_{as})
(b) Length of laminar boundary layer (L_c)

ASSUMPTIONS

Steady state
Transition to turbulence occurs at $Re_x^\bullet = 10^5$
The air behaves as an ideal gas
Radiation heat transfer is negligible because of the low emissivity of the plate

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at one atmosphere and at the bulk temperature
(0°C) Prandtl number (Pr) = 0.71
Specific heat (C_p) = 1011 J/kg K
From Appendix 1, Table 5 g_c = 1.000 kg m/N s^2 (by definition)

SOLUTION

(a) The stagnation temperature is given by Equation (5.91)

$$T_o = T_\infty + \frac{U_\infty^{\;2}}{2g_cC_p} = 273\text{ K} + \frac{(800\,\text{m/s})^2}{2\,(1(\text{kg m})/(\text{Ns}^2))\,(1011\text{J/(kg K)})\,((\text{Nm})/\text{J})} = 273\text{ K} + 317\text{ K} = 590\text{ K}$$

The recovery factor in the laminar region is $Pr^{1/2}$. The adiabatic surface temperature is given by Equation (5.94)

$$\frac{T_{as} - T_\infty}{T_o - T_\infty} = r = Pr^{\frac{1}{2}}$$

$$T_{as} = T_\infty + Pr^{\frac{1}{2}}\,(T_o - T_\infty) = 273\text{ K} + (0.71)^{\frac{1}{2}}\,(590\text{ K} - 273\text{ K}) = 540\text{ K}$$

(b) The reference temperature (T), which must be used in evaluating the Reynolds number, is given by Equation (5.97)

$$T^{\bullet} = T_\infty + 0.5\,(T_s - T_\infty) + 0.22\,(T_{as} - T_\infty)$$

A surface temperature must be assumed to evaluate T. Assuming $T_s = T_\infty = 273$ K

$$T^{\bullet} = 273\text{ K} + 0.5\,(0) + 0.22\,(540\text{ K} - 273\text{ K}) = 332\text{ K}$$

The length of the laminar boundary layer (L_c) is given by

$$Re^{\bullet}_{L_c} = \frac{U_\infty L_c \rho^{\bullet}}{\mu^{\bullet}} = 10^5 \quad \Rightarrow \quad L_c = \frac{10^5\,\mu^{\bullet}}{U_\infty \rho^{\bullet}}$$

The density at the given pressure and reference temperature can be determined from the ideal gas law

$$\rho^{\bullet} = \frac{P}{R_a T^{\bullet}} \qquad \text{where} \qquad R_a = \text{The gas constant for air} = 287\text{ J/(kg K)}$$

$$\rho^{\bullet} = \frac{3500\,\text{N/m}^2}{287\,\text{J/(kg K)}\;\;\text{Nm/J}\;\;332\,\text{K}} = 0.0367\text{ kg/m}^3$$

From Appendix 2, Table 28, for dry air at $T^{\bullet} = 332$ K, the absolute viscosity (μ) = 19.3×10^{-6} Ns/m^2

$$\therefore \quad L_c = \frac{10^5\;\;19.3\times10^{-6}\;\;\text{kg m/(s}^2\text{N)}}{800\,\text{m/s}\;\;0.0367\,\text{kg/m}^3} = 0.066\text{m} = 6.6\text{ cm}$$

COMMENTS

For a more accurate estimation of the length of the laminar region, the average heat transfer coefficient from Equation (4.99) can be used to find the surface temperature. The surface temperature can be used to generate a new reference temperature which is used to find the length L_c. This procedure would be repeated until the value of L_c converges.

PROBLEM 5.65

Air at a static temperature of 21°C and a static pressure of 0.7 kPa(abs) flows at zero angle of attack over a thin electrically heated flat plate at a velocity of 240 m/s. If the plate is 10 cm long in the direction of flow and 60 cm in the direction normal to the flow, determine the rate of electrical heat dissipation necessary to maintain the plate at an average temperature of 55°C.

GIVEN

High speed air flow over a heated flat plate
Air static temperature (T_A) = 21°C
Air static pressure (P) = 0.7 kPa(abs)
Air velocity (U_∞) = 240 m/s
Plate length (L) = 10 cm. = 0.1 m
Plate width (w) = 60 cm = 0.6 m
Average plate temperature (T_s) = 55°C

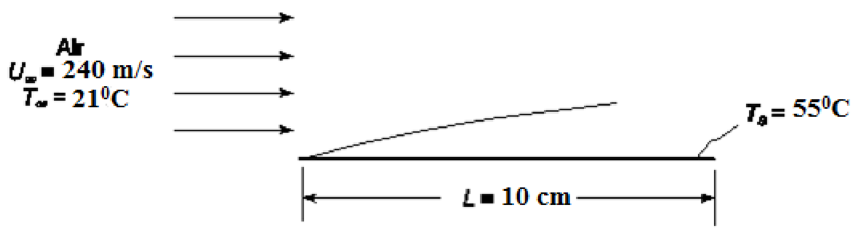
FIND

The rate of electrical heat dissipation $\dot{q}_G$ to maintain the specified plate temperature

ASSUMPTIONS

Steady state
Air behaves as an ideal gas
Air flows on one side of the plate only
Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the free-stream temperature of 21°C

Specific heat (c_p) = 1012 J/(kg K)

Prandtl number (Pr) = 0.71

SOLUTION

The stagnation (T_o) and adiabatic surface (T_{as}) temperatures must be calculated to find the reference temperature (T). From Equation (5.91)

$$T_o = T_\infty + \frac{U_\infty^{\ 2}}{2g_c C_p} = 21°\text{C} + \frac{(240 m/\text{s})^2}{2\left(1 \text{ kgm/(Ns}^2)\right)(1012 J/(kg\,°\text{K}))} = 49.5°\text{C}$$

Assuming the flow is laminar, $r = Pr^{1/2}$ and the adiabatic surface temperature is given by Equation (4.94)

$$T_{as} = T_\infty + Pr^{\frac{1}{2}}(T_o - T_\infty) = 21°\text{C} + (0.71)^{\frac{1}{2}}(49.5°\text{C} - 21^0\text{C}) = 45°\text{C}$$

From Equation (5.97)

$$T^{\bullet} = T_\infty + 0.5\,(T_s - T_\infty) + 0.22\,(T_{as} - T_\infty)$$

$$T^{\bullet} = 21°\text{C} + 0.5\,(55°\text{C} - 21°\text{C}) + 0.22\,(45°\text{C} - 21°\text{C}) = 43.3°\text{C} = 316\text{ K}$$

The density of air at the reference temperature can be calculated from the ideal gas law

$$\rho^{\bullet} = \frac{P}{R_a T^{\bullet}} \quad \text{where} \qquad R_a = \text{The gas constant for air} = 59.1\text{ J/(kg K)}$$

$$\rho^{\bullet} = \frac{\left(0.7*10^3\,N/m^2\right)}{(287\,J/(\text{kg R}))(316\,K)} = 7.7 \times 10^{-3}\text{ kg/m}^3$$

From Appendix 2, Table 28, for dry air at the reference temperature (110°F), the absolute viscosity $\mu^{\bullet}$ = 19.35 × 10^{-6} kg/(m s), the Prandtl number $Pr^{\bullet}$ = 0.71

The Reynolds number at the trailing edge of the plate is

$$Re^{\bullet}_{L_c} = \frac{U_\infty L_c \rho^{\bullet}}{\mu^{\bullet}} = \frac{(240\,m/\text{s})(0.1m)\left(7.7\times10^{-3}\,kg/m^3\right)}{\left(19.35\times10^{-6}\,kg_{\text{m}}/(m\text{s})\right)} = 9550 < 10^5$$

Therefore, the laminar flow assumption is valid.
The average heat transfer coefficient over the plate can be calculated by averaging Equation (4.99)

$$h_c = \int_0^L 0.332\,c_p\,\rho^{\bullet}\,U_\infty\,(Re_x^{\bullet})^{-\frac{1}{2}}\,(Pr^{\bullet})^{-\frac{2}{3}}\,dx = 0.664\,c_p\,\rho^{\bullet}\,U_\infty\,(Re_L^{\bullet})^{-\frac{1}{2}}\,(Pr^{\bullet})^{-\frac{2}{3}}$$

$$h_c = 0.664\,(1012\,J/(kgK))\,\left(7.7\times10^{-3} kg/m^3\right)(240\,m/\text{s})\;\;(9550)^{-\frac{1}{2}}(0.71)^{-\frac{2}{3}}$$

$$= 15.96\text{ W/(m}^2\text{ K)}$$

The electrical heat dissipation required is equal to the convective heat transfer rate

$$q_G = q_c = h_c\,A\,(T_s - T_{as}) = 15.96\text{ W/(m}^2\text{ K)}\,(0.1m)\,(0.6\text{ m})\,(55^0\text{C} - 45°\text{C})$$

$$= 9.6\text{ W}$$

PROBLEM 5.66

Heat rejection from high-speed racing automobiles is a problem because the required heat exchangers generally create additional drag. Integration of heat rejection into the skin of the vehicle has been proposed for a car to be tested at the Bonneville Salt Flats. Preliminary tests are to be performed in a wind tunnel on a flat plate without heat rejection. Atmospheric air in the tunnel is at 10°C and flows at 250 ms^{-1} over the 3 m long thermally non-conducting flat plate. What is the plate temperature 1 m downstream from the leading edge? How much does this temperature differ from that which exists 0.005 m from the leading edge?

GIVEN

High speed air flow over a thermally non-conducting flat plate
Plate length (L) = 3 m
Air temperature (T_∞) = 10°C
Air speed (U_∞) = 250 m/s
Air pressure = 1 atmosphere

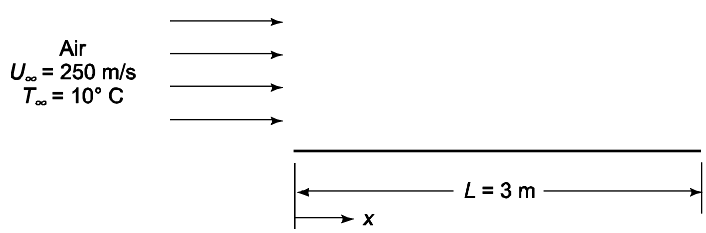
FIND

(a) The plate temperature (T_s) at x = 1 m
(b) Temperature difference between x = 1 m and x = 0.005 m

ASSUMPTIONS

Steady state
Air is on only one side of the plate

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 10°C

Kinematic viscosity (ν) = 14.8 × 10^{-6} m^2/s

Prandlt number (Pr) = 0.71

Specific heat (c_p) = 1011 J/(kg K)

SOLUTION

Since the surface is nonconducting, its surface temperature is equal to the adiabatic surface temperature (T_{as})

$$\text{At} \quad x = 1\text{ m:} \quad Re_x = \frac{U_\infty x}{\nu} = \frac{250\,\text{m/s} \;\; 1\,\text{m}}{14.8\times10^{-6}\,\text{m}^2/\text{s}} = 1.69 \times 10^7 \text{ (Turbulent)}$$

$$\text{At} \quad x = 0.005\text{ m:} \quad Re_x = \frac{U_\infty x}{\nu} = \frac{250\,\text{m/s} \;\; 1\,\text{m}}{14.8\times10^{-6}\,\text{m}^2/\text{s}} = 8.45 \times 10^4 \text{ (Laminar)}$$

The stagnation temperature is given by Equation (5.91)

$$T_o = T_\infty + \frac{U_\infty^{\ 2}}{2g_c c_p} = 10°\text{C} + \frac{(250\,\text{m/s})^2}{2\,(1\,\text{kg m/(N s}^2))\,(1011\,\text{J/(kg K)})\,(\text{N m})/\text{J}} = 41°\text{C}$$

The adiabatic surface temperature is given by Equation (4.93)

$$T_{as} = T_{oc} + r\,(T_o - T_\infty)$$

(a) For the turbulent region, $r = Pr^{1/3}$

$$\therefore \quad \text{At} \quad x = 1\text{ m: } T_{as} = 10°\text{C} + (0.71)^{\frac{1}{3}}\,(41°\text{C} - 10°\text{C}) = 38°\text{C}$$

(b) For the laminar region, $r = pr^{1/2}$

$$\therefore \quad \text{At} \quad x = 0.005\text{m: } T_{as} = 10°\text{C} + (0.71)^{\frac{1}{2}}\,(41°\text{C} - 10°\text{C}) = 36°\text{C}$$

The temperature difference between x =1 m and x = 0.005 m is 2°C.

PROBLEM 5.67

Air at 15°C and 0.01 atmosphere pressure flows at a velocity of 250 m/s over a thin flat strip of metal that is 0.1 m long in the direction of flow,. Determine (a) the surface temperature of the plate at equilibrium and (b) the rate of heat removal required per meter width if the surface temperature is to be maintained at 30°C.

GIVEN

High speed air flow over a thin flat strip of metal
Air temperature (T_∞) = 15°C = 288 K
Air pressure (P) = 0.01 atm = 1013 N/m^2
Metal strip length (L) = 0.1 m
Air velocity (U_∞) = 250 m/s

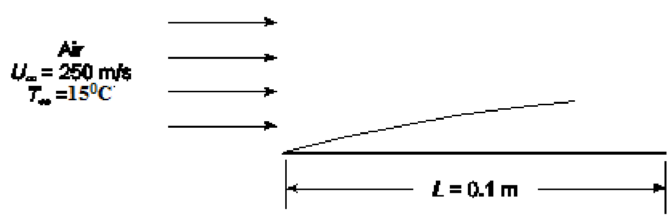
FIND

(a) Equilibrium surface temperature (T_s)
(b) Rate of heat removal per unit width (q/w) for T_s = 30°C

ASSUMPTIONS

Air flows over one side of the strip only
Air behaves as an ideal gas

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 15°C

Absolute viscosity (μ) = 18.0 × 10^{-6} Ns/m^2
Prandtl number (Pr) = 0.71
Specific heat (c_p) = 1012 J/(kg K)

SOLUTION

(a) The density of air at the given pressure and temperature can be calculated from the ideal gas law

$$\rho = \frac{P}{R_a T} \qquad \text{where} \qquad R_a = \text{The gas constant for air} = 287\ \text{J/(kg K)}$$

$$\rho = \frac{1013\,\text{N/m}^2}{287\,\text{J/(kg K)} \quad \text{N m/J} \quad 288\,\text{K}} = 0.0123\ \text{kg/m}^3$$

$$Re_L = \frac{U_\infty L \rho}{\mu} = \frac{250\,\text{m/s} \quad 0.1\,\text{m} \quad 0.0123\,\text{kg/m}^3}{18.0\times10^{-6}\,\text{N s/m}^2 \quad \text{kg m/(N s}^2)} = 1.69 \times 10^4\ \text{(Laminar)}$$

The stagnation temperature is given by Equation (5.91)

$$T_o = T_\infty + \frac{U_\infty^{\ 2}}{2g_c c_p} = 15°\text{C} + \frac{(250\,\text{m/s})^2}{2\left(1\,\text{kg m/(N s}^2)\right)(1012\,\text{J/(kg K)})(\text{Nm/J})} = 46°\text{C}$$

At equilibrium with no heat removal, the surface temperature is equal to the adiabatic surface temperature given by Equation (5.93) with $r = Pr^{1/2}$.

$$T_{as} = T_\infty + Pr^{\frac{1}{2}}(T_o - T_\infty) = 15°\text{C} + (0.71)^{\frac{1}{2}}(46°\text{C} - 15°\text{C}) = 41°\text{C}$$

(b) The reference temperature, Equation (5.97), must be used for the non-adiabatic case

$$T^\bullet = T_\infty + 0.5\,(T_s - T_\infty) + 0.22\,(T_{as} - T_\infty)$$

$$T^\bullet = 15°\text{C} + 0.5\,(30°\text{C} - 15°\text{C}) + 0.22\,(41°\text{C} - 15°\text{C}) = 28°\text{C} = 301\text{ K}$$

From Appendix 2, Table 28, for dry air at 28°C

Absolute Viscosity $\mu^\bullet = 18.6 \times 10^{-6}$ N s/m²

Prandtl number $Pr^\bullet = 0.71$

The density, from the ideal gas law

$$\rho^\bullet = \frac{1013\,\text{N/m}^2}{287\,\text{J/(kg K)}\quad \text{N m/J}\quad 301\,\text{K}} = 0.0117\text{ kg/m}^3$$

$$Re_L^{\ \bullet} = \frac{U_\infty L_c \rho^\bullet}{\mu^\bullet} = \frac{250\,\text{m/s}\quad 0.1\,\text{m}\quad 0.0117\,\text{kg/m}^3}{18.0\times10^{-6}\,\text{N s/m}^2\quad \text{kg m/(N s}^2)} = 1.57 \times 10^4 \text{ (Laminar)}$$

Averaging Equation (5.99) over the length of the plate yields

$$h_c = \frac{1}{L}\int_0^L 0.332\, c_p\, \rho^\bullet U_\infty (Re_x^{\ \bullet})^{-\frac{1}{2}} (Pr^\bullet)^{-\frac{2}{3}}\, dx = 0.664\, c_p\, \rho^\bullet U_\infty (Re_L^{\ \bullet})^{-\frac{1}{2}} (Pr^\bullet)^{-\frac{2}{3}}$$

$$h_c = 0.664\ \ 1013\,\text{J/(kg K)}\quad 0.0117\,\text{kg/m}^3\quad 250\,\text{m/s}\quad \text{N m/J}\quad (1.57\times10^4)^{-\frac{1}{2}}(0.71)^{-\frac{2}{3}} = 19.7\text{ W/(m}^2\text{ K)}$$

The heat removal rate must equal the rate of heat gain from the air to maintain a constant surface temperature

$$\frac{q}{A} = h_c\,(T_s - T_{as}) \Rightarrow q/W = h_c L\,(T_s - T_{as}) = 19.7\text{ W/(m}^2\text{ K)}\,(0.1\text{ m})\,(30°\text{C} - 43°\text{C}) = -25.6\text{ W/m}$$

The negative sign indicates heat gained by the plate.

PROBLEM 5.68

A flat plate is placed in a supersonic wind tunnel with air flowing over it at a Mach number of 2.0, a pressure of 25,000 N/m^2, and an ambient temperature of –15°C. If the plate is 30-cm-long in the direction of flow, calculate the cooling rate per unit area that is required to maintain the plate temperature below 120°C.

GIVEN

High speed air flow over a flat plate
Mach number (M) = 2.0
Air pressure (P) = 25,000 N/m^2
Ambient temperature (T_∞) = –15°C = 258 K
Plate length (L) = 30 cm = 0.3 m

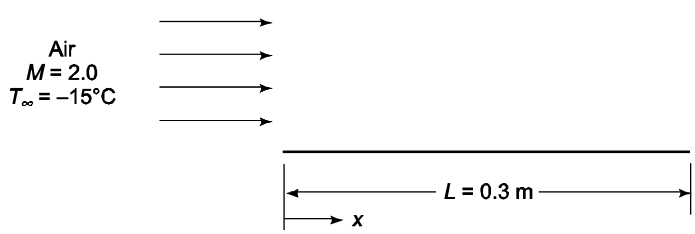
FIND

Cooling rate per unit area (q_c/A) to keep plate temperature (T_s) below 120°C

ASSUMPTIONS

Steady state
Air behaves as an ideal gas
Negligible radiative heat transfer
Air flows over only one side of the plate

SKETCH



PROPERTIES AND CONSTANTS

From Section 5.13, the specific heat ration for air (γ) = 1.4. The gas constant for air (R_a) = 287 J/(kg K).

SOLUTION

The acoustic velocity (a) is given by Equation (5.89)

$$a_\infty = \sqrt{\gamma\, R_a\, T_\infty} = \sqrt{1.4 \;\; 287\,\text{J/(kg K)} \quad \text{N m/J} \quad \text{kg m/(N s}^2) \;\; 258\,\text{K}} = 322 \text{ m/s}$$

$$U_\infty = M\, a_\infty = 2.0 \quad 322\,\text{m/s} \quad = 644 \text{ m/s}$$

The stagnation temperature, from Equation (5.92)

$$T_o = T_\infty \left[1 + \frac{\gamma - 1}{2} M^2\right] = 258 \text{ K} \left[1 + \frac{1.4 - 1}{2}\, 4.0\right] = 464 \text{ K} = 191°\text{C}$$

Assuming the flow is turbulent, $r = Pr^{1/3}$ and the adiabatic surface temperature (T_{as}) is given by Equation (5.93)

$$T_{as} = T_\infty + Pr^{\frac{1}{3}}\,(T_o - T_\infty) = -15°\text{C} + (0.71)^{\frac{1}{3}}\,[191°\text{C} - (-15°\text{C})] = 169°\text{C}$$

The reference temperature ($T^\bullet$) is given by Equation (5.97)

$$T^{\bullet} = T_{\infty} + 0.5\,(T_s - T_{\infty}) + 0.22\,(T_{as} - T_{\infty})$$

$$T^{\bullet} = -15°\text{C} + 0.5\,(120°\text{C} + 15°\text{C}) + 0.22\,(169°\text{C} + 15°\text{C}) = 93°\text{C} = 366\text{ K}$$

From Appendix 2, Table 28, for dry air at the reference temperature (93°C)

Absolute viscosity ($\mu^{\bullet}$) = 21.36×10^{-6} Ns/m^2

Prandtl number ($Pr^{\bullet}$) = 0.71 Specific heat (c_p) = 1021 J/(kg J)

The density can be calculated using the ideal gas law

$$\rho^{\bullet} = \frac{P}{R_a\,T^{\bullet}} \quad \text{where } R_a = \text{The gas constant for air} = 287\text{ J/(kg J)}$$

$$\rho^{\bullet} = \frac{25{,}000\,\text{N/m}^2}{287\,\text{J/(kg J)}\,(\text{N m/J})\,(366\,\text{K})} = 0.238\text{ kg/m}^3$$

$$\text{At } L = 0.3\text{ m } Re_L^{\bullet} = \frac{U_{\infty}\,L\,\rho^{\bullet}}{\mu^{\bullet}} = \frac{(644\,\text{m/s})(0.3\,\text{m})\;0.238\,\text{kg/m}^3}{21.36\times 10^{-6}\,\text{Ns/m}^2\;\;\text{kg m/(Ns}^2)} = 2.15\times 10^6$$

Therefore, the assumption of turbulence is valid and the average heat transfer coefficient, neglecting the laminar portion of the boundary layer, can be calculated by averaging Equation (4.100) from $x = 0$ to $x = L$

$$h_c = \frac{1}{L}\int_0^L 0.0288\,c_p\,\rho^{\bullet}\,U_{\infty}\,(Re_x^{\bullet})^{-0.2}\,(Pr^{\bullet})^{-\frac{2}{3}}\,dx = 0.036\,c_p\,\rho^{\bullet}\,U_{\infty}\,(Re_L^{\bullet})^{-0.2}\,(Pr^{\bullet})^{-\frac{2}{3}}$$

$$h_c = 0.036\;\;1021\,\text{J/(kg K)}\;\;0.238\,\text{kg/m}^3\;\;644\,\text{m/s}\;\;\text{N m/J}\;\;(2.15\times 10^6)^{-0.2}\,(0.71)^{-\frac{2}{3}} = 383\,\text{W/(m}^2\,\text{K)}$$

The rate of cooling must equal the rate of heat loss by convection, given by Equation (4.98)

$$\frac{q_c}{A} = h_c\,(T_s - T_{as}) = 383\text{ W/(m}^2\text{ K)}\,(120°\text{C} - 169°\text{C}) = -18{,}770\text{ W/m}^2$$

The negative sign indicates heat is being transferred to the plate from the air.

COMMENTS

The length of the laminar boundary layer is determined by

$$Re_{L_c}^{\bullet} = \frac{U_{\infty}\,L_c\,\rho^{\bullet}}{\mu^{\bullet}} = 10^5 \quad \Rightarrow \quad L_c = \frac{10^5\,\mu^{\bullet}}{U_{\infty}\,\rho^{\bullet}} = 0.014\text{ m} << 0.3\text{ m}$$

Therefore, neglecting the laminar region does not introduce significant error.

PROBLEM 5.69

A satellite reenters the earth's atmosphere at a velocity of 2700 m/s. Estimate the maximum temperature the heat shield would reach if the shield material is not allowed to ablate and radiation effects are neglected. The temperature of the upper surface of the atmosphere –50°C.

GIVEN

High speed air flow over a satellite
Velocity (U_∞) = 2700 m/s
Air temperature (T_∞) = – 50°C

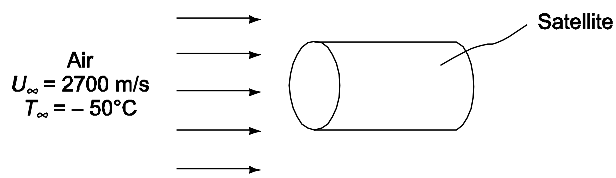
FIND

Maximum heat shield temperature (T_s)

ASSUMPTIONS

Radiative heat transfer is negligible
Shield material does not ablate
Shield can be approximated as a flat plate
Boundary layer is turbulent

SKETCH



PROPERTIES AND CONSTANTS

Extrapolating from Appendix 2, Table 28, for dry air at –50°C

Prandtl number (Pr) = 0.71

Specific heat = 1000 J/(kg K)

SOLUTION

The stagnation temperature is given by Equation (5.91)

$$T_o = T_\infty + \frac{U_\infty^2}{2\, g_c\, c_p} = -50°\text{C} + \frac{\left(2700\,\text{m/s}\right)^2}{2\left(1\,\text{kg m/(N s}^2)\right)\left(1000\,\text{J/(kg K)}\right)\left(\text{N m/J}\right)} = 3595°\text{C}$$

With no ablation or heat removal, the surface temperature of the satellite will be the adiabatic surface temperature given in Equation (5 .93) where $r = Pr^{1/3}$ for turbulent flow

$$T_{as} = T_\infty + Pr^{\frac{1}{3}}\,(T_o - T_\infty) = -50°\text{C} + (0.71)^{\frac{1}{3}}\,(3295°\text{C} + 50°\text{C}) = 3200°\text{C}$$

PROBLEM 5.70

A scale model of an airplane wing section is tested in a wind tunnel at a Mach number of 1.5. The air pressure and temperature in the test section are 20,000 N/m^2 and –30°C, respectively. If the wing section is to be kept at an average temperature of 60°C, determine the rate of cooling required. The wing model may be approximated by a flat plate of 0.3 length in the flow direction.

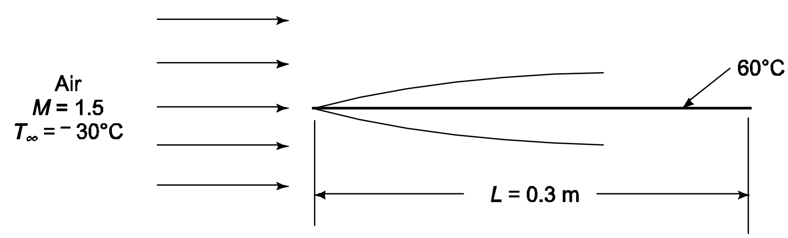
GIVEN

High speed air flow over an airplane wing section
Mach number (M_∞) = 1.5
Air pressure (P) = 20,000 N/m^2
Air temperature = –30°C = 243 K
Average wing surface temperature = 60°C = 333 K
Wing may be approximated as a flat of length (L) = 0.3 m

FIND

The cooling rate (q/A) required

SKETCH



PROPERTIES AND CONSTANTS

Extrapolating from Appendix 2, Table 28, for dry air at the ambient temperature,

The Prandtl number (Pr) = 0.71.

From Section 5.13, the specific heat ratio (γ) = 1.4
The gas constant for air (R_a) = 287 J/(kg K)

SOLUTION

The stagnation temperature is given by Equation (5.92)

$$T_o = T_\infty \left[1+\frac{\gamma-1}{2}M^2\right] = 243\text{ K}\left[1+\frac{1.4-1}{2}(1.5)^2\right] = 352\text{ K} = 179°\text{C}$$

The air speed (U_∞) is calculated from

$$U_\infty = M_\infty a_\infty = M_\infty\sqrt{\gamma R_a T_\infty} = 1.5\sqrt{1.4\;\;287\text{J/(kg K)}\;\;\text{N m/J}\;\;\text{kg m/(N s}^2)\;\;243\text{K}}$$

$$= 469\text{ m/s}$$

The adiabatic surface temperature, from Equation (5.93) is

$$T_{as} = T_\infty + r\,(T_o - T_\infty)$$

Assuming the boundary layer is turbulent, $r = Pr^{1/3}$

$$T_{as} = T_\infty + Pr^{\frac{1}{3}}\,(T_o - T_\infty) = 243\text{ K} + (0.71)^{\frac{1}{3}}\,(352\text{ K} - 243\text{ K}) = 340\text{ K}$$

The reference temperature is given by Equation (5.97)

$$T^{\bullet} = T_\infty + 0.5\,(T_s - T_\infty) + 0.22\,(T_{as} - T_\infty)$$

$$T^{\bullet} = 243\text{ K} + 0.5\,(333\text{ K} - 243\text{ K}) + 0.22\,(340\text{ K} - 243\text{ K}) = 309\text{ K}$$

From Appendix 2, Table 28, for dry air at 309 K

Absolute viscosity ($\mu^{\bullet}$) = 18.9×10^{-6} Ns/m^2

Prandtl number ($Pr^{\bullet}$) = 0.71

Specific heat ($C_p^{\bullet}$) = 1014 J/(kg K)

The density of the air can be calculated from the ideal gas law

$$\rho^{\bullet} = \frac{P}{R_a T^{\bullet}} \quad \text{Where: } R_a = \text{The gas constant for air 287 J/(kg K)}$$

$$\rho^{\bullet} = \frac{20{,}000\,\text{N/m}^2}{287\,\text{J/(kg K)}\,(\text{N m/J})\,(309\,\text{K})} = 0.226\text{ kg/m}^3$$

$$\text{At } L = 0.3\text{ m } Re_L^{\bullet} = \frac{U_\infty\, L\rho^{\bullet}}{\mu^{\bullet}} = \frac{(469\,\text{m/s})(0.3\,\text{m})\;\;0.226\,\text{kg/m}^3}{18.9 \times 10^{-6}\,\text{N s/m}^2\quad \text{kg m/(Ns}^2)} = 1.68 \times 10^6$$

Therefore, the assumption that the boundary layer is turbulent is valid.

The average heat transfer coefficient over the wing can be calculated by averaging Equation (5.100), assuming constant thermal properties:

$$h_c = \frac{1}{L}\int_0^L 0.0288\, c_p\, \rho^{\bullet}\, U_\infty\, (Re_x^{\bullet})^{-0.2} (Pr^{\bullet})^{-\frac{2}{3}}\, dx = 0.036\, c_p\, \rho^{\bullet}\, U_\infty\, (Re_L^{\bullet})^{-0.2} (Pr^{\bullet})^{-\frac{2}{3}}$$

$$h_c = 0.036\;\; 1014\,\text{J/kg}\quad 0.226\,\text{kg/m}^2\quad 469\,\text{m/s}\quad \text{N m/J}\;\; (1.68 \times 10^6)^{-0.2}\,(0.71)^{-\frac{2}{3}}$$

$$= 277\text{ W/(m}^2\text{ K)}$$

The rate of heat transfer from the wing is given by Equation (5.98)

$$\frac{q_c}{A} = h_c\,(T_s - T_{as}) = 277\text{ W/(m}^2\text{ K)}\,(333\text{ K} - 340\text{ K}) = -1940\text{ W/m}^2$$

The negative sign indicates that heat is being transferred from the air to the wing. Therefore, 1940 W/m^2 must be removed to maintain the wing at 60°