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HW3

Exercise 1.4

1.(a) $(A+B)^2=(A+B)(A+B)=A^2+AB+BA+B^2$

$(a+b)^2=a^2+2ab+b^2$

$\because AB\neq BA$

$\therefore$ not work

(b) $(A+B)(A-B)=A^2-AB+BA-B^2$

$(a+b)(a-b)=a^2-b^2$

$\because AB\neq BA$

$\therefore$ not work

3. $A=\begin{pmatrix}0&1\\0&1\end{pmatrix}$

$B=\begin{pmatrix}1&1\\0&0\end{pmatrix}$

$AB=\begin{pmatrix}0&0\\0&0\end{pmatrix}$

8. $\frac{1}{2}c_1-\frac{1}{2}c_2-\frac{1}{2}c_3-\frac{1}{2}c_4$

$$=\begin{pmatrix}\frac{1}{4}\\-\frac{1}{4}\\-\frac{1}{4}\\-\frac{1}{4}\end{pmatrix}-\begin{pmatrix}-\frac{1}{4}\\\frac{1}{4}\\-\frac{1}{4}\\-\frac{1}{4}\end{pmatrix}-\begin{pmatrix}-\frac{1}{4}\\-\frac{1}{4}\\\frac{1}{4}\\-\frac{1}{4}\end{pmatrix}-\begin{pmatrix}-\frac{1}{4}\\-\frac{1}{4}\\-\frac{1}{4}\\\frac{1}{4}\end{pmatrix}$$

$$=\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$A^2=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$A^3=A^2\cdot A=A$

$$A^{2n}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$$A^{2n+1}=A=\begin{pmatrix}\frac{1}{2}&-\frac{1}{2}&-\frac{1}{2}&-\frac{1}{2}\\-\frac{1}{2}&\frac{1}{2}&-\frac{1}{2}&-\frac{1}{2}\\-\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}&-\frac{1}{2}\\-\frac{1}{2}&-\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\end{pmatrix}$$

10. (a) $C^T=(A+B)^T=A^T+B^T=A+B=C$

Symmetric

(b) $D^T=(A^2)^T=A^TA^T=AA=A^2=D$

Symmetric

(c) $E^T=(AB)^T=B^TA^T=BA\neq AB=E$

Nonsymmetric

(d) $F^T=(ABA)^T=(BA)^TA^T=A^TB^TA^T=ABA=F$

Symmetric

(e) $G^T=(AB+BA)^T=(AB)^T+(BA)^T$

$=B^TA^T+A^TB^T=AB+BA=G$

Symmetric

(f) $H^T=(AB-BA)^T=(AB)^T-(BA)^T$

$=B^TA^T-A^TB^T\neq AB-BA=H$

Nonsymmetric

17. Suppose A is nonsingular

$AA^{-1} = A^{-1}A = I$

$\because Ax = Ay$

$\therefore A^{-1}Ax = A^{-1}Ay$

$\therefore x = y$

which is a contradiction with $x \neq y$

Hence, A is singular

20 $\because (I-A)(I+A+A^2+\cdots+A^k)$

$= I(I+A+\cdots+A^k) - A(I+A+\cdots+A^k)$

$= (I+A+\cdots+A^k) - (A+A^2+\cdots+A^{k+1})$

$= I - A^{k+1} = I$

$\therefore I-A$ is nonsingular and $(I-A)^{-1} = I+\cdots+A^k$

Exercise 1.5

3.

(a) $A \xrightarrow[r_2 \to r_2]{r_1 \to -2r_1} B$

$I \xrightarrow{r_1 \to -2r_1} \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix} = E$

(b) $A \xrightarrow{r_2 \leftrightarrow r_3} B$

$I \xleftrightarrow{r_2 \leftrightarrow r_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = E$

(c) $A \xrightarrow{r_3 + 2r_2} B$

$I \xrightarrow{r_3 + 2r_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = E$

7. (a) $A \xrightarrow{r_2 - 3r_1} \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$

$\xrightarrow{r_1 - r_2} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$\xrightarrow{\frac{1}{2}r_1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}$

(b) $A = (A^{-1})^{-1} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

8(c) $A \xrightarrow{r_2 + 2r_1} \begin{pmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ -6 & -3 & 4 \end{pmatrix}$

$\xrightarrow{r_3 - 3r_1} \begin{pmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & -6 & -2 \end{pmatrix}$

$\xrightarrow{r_3 + 2r_2} \begin{pmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix} = U$

$L = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -2 & 1 \end{pmatrix}$

$LU = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -2 & 1 \end{pmatrix} \begin{pmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix}$

$= \begin{pmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & 3 & 4 \end{pmatrix} = A$

10.

(e) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{r_1 - r_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

$\xrightarrow{r_2 - r_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

(f) $\begin{pmatrix} 2 & 0 & 5 \\ 0 & 3 & 0 \\ 1 & 0 & 3 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 3 & 0 \\ 2 & 0 & 5 \end{pmatrix}$

$\xrightarrow{\frac{1}{3} r_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 2 & 0 & 5 \end{pmatrix} \xrightarrow{r_3 - 2r_1} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$\xrightarrow{-r_3} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{r_1 - 3r_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 3 & 0 & -5 \\ 0 & 1/3 & 0 \\ -1 & 0 & 2 \end{pmatrix}$

13. Yes.

No. Fo example:

$$\underbrace{\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{elementary}} = \underbrace{\begin{pmatrix} 1 & 0 & 3 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{not elementary}}$$

31. $a_1 + a_2 = a_3 + 2a_4$

$\Rightarrow a_1 + a_2 - a_3 - 2a_4 = 0$

$\because a_1 + a_2 - a_3 - 2a_4 = A(1,1,-1,-2)^T = 0$

$Ax = 0$ has a nontrivial solution

$\therefore A$ is singular