

Hongrui Yi

a) Dice 1      1      2      3      4      5      6  
                   $\frac{1}{6}$      $\frac{1}{6}$      $\frac{1}{6}$      $\frac{1}{6}$      $\frac{1}{6}$      $\frac{1}{6}$

Dice 1 - Dice 2    -5   -4   -3   -2   -1   0   1   2   3   4   5   6  
 $\frac{1}{36} \times$     1    2    3    4    5    6    5    4    3    2    1

$$E(X) = (0 \times \frac{1}{36} + 1 \times \frac{1}{36} + 2 \times \frac{2}{36} + 3 \times \frac{3}{36} + 4 \times \frac{4}{36} + 5 \times \frac{5}{36} + 6 \times \frac{6}{36}) \times 3$$

$$= \frac{21.5}{36} = \underline{5.833}$$

$$b) E(X^2) = \frac{1}{36} \times 1 + \frac{2}{36} \times 2 + \frac{3}{36} \times 3 + \frac{4}{36} \times 4 + \frac{5}{36} \times 5 + \frac{6}{36} \times 6 + \frac{5}{36} \times 7 + \frac{4}{36} \times 8 + \frac{3}{36} \times 9$$

$$+ \frac{2}{36} \times 10 + \frac{1}{36} \times 12 + \frac{2}{36} \times 15 + \frac{3}{36} \times 16 + \frac{4}{36} \times 18 + \frac{5}{36} \times 20 + \frac{6}{36} \times 24$$

$$+ \frac{5}{36} \times 25 + \frac{4}{36} \times 30 + \frac{3}{36} \times 36 = \frac{441}{36} = \underline{12.25}$$

$$c) \text{Var}(X) = E(X^2) - E^2(X)$$

$$E(X^2) = \frac{1}{36} \times (0^2 \times \frac{1}{36} + 1^2 \times \frac{1}{36} + 2^2 \times \frac{2}{36} + \dots + 5^2 \times \frac{5}{36})$$

$$= \frac{18.5}{36} = 52.5$$

$$\text{Var}(X) = 52.5 - 5.833^2 = 18.473 \quad \sigma(\sqrt{\text{Var}(X)}) = \underline{4.298}$$

$$d) E(Y) = (\frac{1}{36} \times 1^2 + \frac{2}{36} \times 2^2 + \frac{3}{36} \times 3^2 + \dots + \frac{1}{36} \times 36^2) = \frac{8281}{36}$$

$$\text{Var}(Y) = \frac{8281 - 441 \times \frac{441}{36}}{36} = 79.97$$

$$\sigma = \sqrt{\text{Var}(Y)} = \underline{8.943}$$

$$e) \text{Cov} = E(X, Y) - E(X) \cdot E(Y)$$

$$= (2 \times 3 \times \frac{1}{36} + 3 \times 6 \times \frac{2}{36} + 4 \times 9 \times \frac{3}{36} + \dots + 3 \times 30 + \frac{2}{36})$$

$$= \frac{2184}{36} = 60.67$$

$$\text{Cov} = 60.67 - 5.833 \times 12.25 = \underline{-10.79}$$

	1	2	3	4	5	6	7	8	9	10	12	15	16	18	20	24	25	30	36
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$$f) \rho = \frac{\text{Cov}}{\sigma_X \cdot \sigma_Y} = \frac{-10.79}{4.298 \times 8.943} = \underline{-0.28}$$