易弘睿

20186103

1a

a. $\ddot{y} - 4\dot{y} + 3y = 9t^2 + 4, \quad y(0) = 6,\ \dot{y}(0) = 8$

$\ddot{y} - 4\dot{y} + 3y = 0$

$r^2 - 4r + 3 = 0$

$(r-3)(r-1) = 0$

$r_1 = 3 \quad r_2 = 1$

$Y_c = c_1 e^{3t} + c_2 e^{t}$

Assume $Y_p = At^2 + Bt + C$

$\dot{Y}_p = 2At + B$

$\ddot{Y}_p = 2A$

$2A - 4(2At+B) + 3(At^2+Bt+C) = 9t^2+4$

$2A - 8At - 4B + 3At^2 + 3Bt + 3C = 9t^2+4$

$3At^2 + (3B-8A)t + (2A - 4B + 3C) = 9t^2 + 4$

$A = 3,\ B = 8,\ C = 10$

$Y = c_1 e^{3t} + c_2 e^{t} + 3t^2 + 8t + 10$

$\begin{cases} y(0) = c_1 + c_2 + 10 = 6 \\ \dot{y}(0) = 3c_1 + c_2 + 8 = 8 \end{cases} \Rightarrow \begin{cases} c_1 = 2 \\ c_2 = -6 \end{cases}$

$\Rightarrow Y = 2e^{3t} - 6e^{t} + 3t$

d. $\ddot{y} + 6\dot{y} + 9y = 27e^{-6t}, \quad y(0) = -2,\ \dot{y}(0) = 0$, plot the solution, $0 < t < 3$ sec.

$\ddot{y} + 6\dot{y} + 9y = 0$

$r^2 + 6r + 9 = 0$

$(r+3)^2 = 0$

$r_1 = r_2 = -3$

$Y_c = c_1 e^{-3t} + c_2 t e^{-3t}$

Assume $Y_p = A e^{-6t}$

$Y_p' = -6A e^{-6t}$

$Y_p'' = 36A e^{-6t}$

$36A e^{-6t} + 6\cdot(-6)A e^{-6t} + 9A e^{-6t} = 27 e^{-6t}$

$A = 3$

$Y = 3e^{-6t} + c_1 e^{-3t} + c_2 t e^{-3t}$

$\begin{cases} y(0) = 3 + c_1 + c_2 = -2 \\ \dot{y}(0) = -18e^{-6t} - 3c_1 e^{-3t} + c_1 e^{-3t} + c_2 t(-3e^{-3t}) \end{cases}$

$= -18 - 3c_1 + c_1 = 0$

$\begin{cases} c_1 = -9 \\ c_2 = 4 \end{cases}$

$\Rightarrow Y = 3e^{-6t} - 9c_1 e^{-3t} + 4te^{-3t}$

f. $\dot{y} + a\,y = b\cos\omega_1 t + c\sin\omega_1 t, \quad y(0) = y_0$

$$\dot{y} + ay = 0$$

$$r + a = 0$$

$$r = -a$$

$$Y_c = C_1 e^{-at}$$

$$Y_p = A\cos w_1 t + B\sin w_1 t$$

$$Y_p' = -Aw_1\sin w_1 t + Bw_1\cos w_1 t$$

$$-Aw_1\sin w_1 t + Bw_1\cos w_1 t + a(A\cos w_1 t + B\sin w_1 t) = b\cos w_1 t + c\sin w_1 t$$

$$\begin{cases} Bw_1 + aA = b \\ -Aw_1 + aB = c \end{cases} \Rightarrow \begin{cases} A = \dfrac{ab - cw_1}{w_1^2 + a^2} \\ B = \dfrac{ac + bw_1}{w_1^2 + a^2} \end{cases}$$

$$Y = C_1 e^{-at} + \frac{ab - cw_1}{w_1^2 + a^2}\cos w_1 t + \frac{bw_1 + ac}{w_1^2 + a^2}\sin w_1 t$$

$$y(0) = C_1 + \frac{ab - cw_1}{w_1^2 - a^2} = y_0$$

$$\Rightarrow C_1 = y_0 - \frac{ab - cw_1}{w_1^2 - a^2}$$

$$Y = \left(y_0 + \frac{cw_1 - ab}{w_1^2 - a^2}\right)e^{-at} + \frac{ab - cw_1}{w_1^2 + a^2}\cos w_1 t + \frac{bw_1 + ac}{w_1^2 + a^2}\sin w_1 t$$

g. $\ddot{y} + \omega_n^{\,2} y = b\cos\omega_1 t, \quad y(0) = y_0, \ \dot{y}(0) = 0$

$$\ddot{y} + w_n^2 y = 0$$

$$r^2 + w_n^2 = 0$$

$$r = \pm w_n i$$

$$Y_c = C_1 e^{w_n i t} + C_2 e^{-w_n i t}$$

Assume $Y_p = A\cos w_1 t$

$$Y_p' = -Aw_1\sin w_1 t$$

$$Y_p'' = -Aw_1^2\cos w_1 t$$

$$-Aw_1^2 + Aw_n^2 = b$$

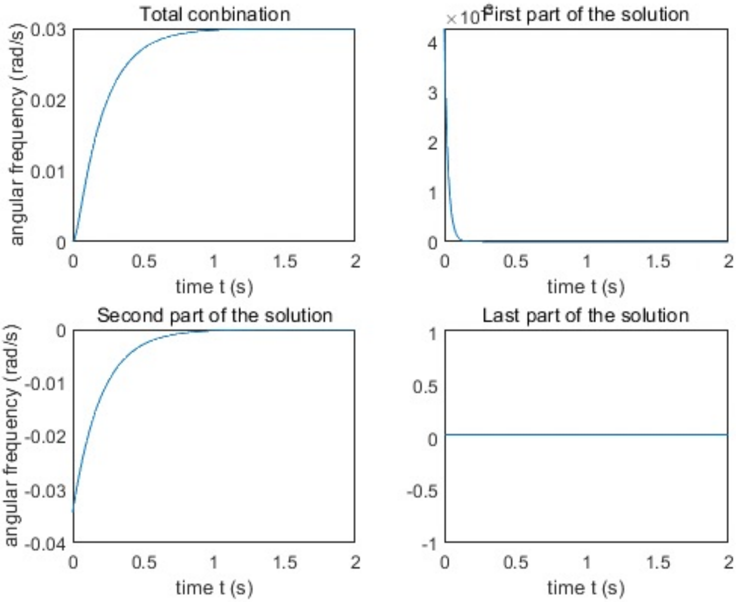
$$A = \frac{b}{w_n^2 - w_1^2}$$

$$Y = \frac{b}{w_n^2 - w_1^2}\cos w_1 t + C_1 e^{w_n i t} + C_2 e^{-w_n i t}$$

$$\begin{cases} y(0) = \dfrac{b}{w_n^2 - w_1^2} + C_1 + C_2 = y_0 \\ \dot{y}(0) = C_1 w_n - C_2 w_n = 0 \end{cases} \Rightarrow \begin{cases} C_1 = \dfrac{y_0}{2} - \dfrac{b}{2(w_n^2 - w_1^2)} \\ C_2 = \dfrac{y_0}{2} - \dfrac{b}{2(w_n^2 - w_1^2)} \end{cases}$$

$$\Rightarrow Y = \frac{b}{w_n^2 - w_1^2}\cos w_1 t + \left(\frac{y_0}{2} - \frac{b}{2(w_n^2 - w_1^2)}\right)\left(e^{w_n i t} + e^{-w_n i t}\right)$$

2a - 3



Total conbination
angular frequency (rad/s)
time t (s)
First part of the solution
time t (s)
Second part of the solution
angular frequency (rad/s)
time t (s)
Last part of the solution
time t (s)