重庆大学-辛辛那提大学联合学院

学生实验报告

CQU-UC Joint Co-op Institute (JCI)

Student Experiment Report

实验课程名称 Experiment Course Name Circuit and Sensor Experiment

开课实验室（学院）Laboratory (School) College of Mechanical Engineering

学院 School Joint Co-op Institut 年级 2018 专业 ME01 班 Student Group 02

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学年 Academic Year 2021 学期 Semester Summer

成绩 Grade		
教师签名 Signature of Instructor		

批改说明 Marking instructions:

批改说明 Marking instructions:

指导老师请用红色水笔批改，在扣分处标明所扣分数并给出相应理由，在封面的平时成绩处注明成绩。

Supervisors should mark the report with a **red ink pen**. Please write down **the points deducted** for each section when errors arise and specify the corresponding reasons. Please write down **the total grade** in the table on the cover page.

重庆大学-辛辛那提大学联合学院 2016 年制订

University of Cincinnati
College of Engineering & Applied Science
School of Dynamic Systems

Course Code : **20-MECH-3070C**
Course Title : **Circuits and Sensing Lab**

Experiment Number : **2**
Experiment Title : **Bridge Circuits and Strain Gages**

Date(s) Performed : **June 16, 2021**
Date Submitted : **June 23, 2021**

Group Code : **02**
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Instructor : **Wang, Xian**

Experiment Results	=	_______/40
Analysis	=	_______/30
Answers to Questions	=	_______/20
Writing	=	_______/10

TOTAL SCORE **=** **_______/100**

ABSTRACT

In part I and II, the experiment analyzed the bridge circuit, measured the resistance, and applied the bridge circuit. First, it can be inferred that the relationship between the output voltage and the resistance change in the bridge circuit is significant. Next, the output voltage is measured by changing the resistance in R3. At the same time, two theoretical results are obtained by means of differential equation and matrix. From the point of error, the differential equation has smaller error and is more reliable.

In part III, by comparing the experimental data to the theoretical prediction of the strains, it is seen that the stress and strain increase correspondingly as the applied weight increases proportionally and the stress and strain of half bridge is twice that of quarter bridge. Also, the stress and strain of full bridge is four times that of quarter bridge.

In part IV, we will measure the pressure of soft drink cans before and after they are opened. This pressure is determined by measuring the axial and circumferential strain of the tank side wall. By modeling the tank as a long thin-walled cylinder, the pressure is related to the lateral wall strain. From this part, we can see the relationship between the pressure and the stress of the long thin-walled cylinder, and the stress-strain relationship of the biaxial stress.

Table of Contents

1. Objectives

At the end of this experiment, the students are expected to:

- Learn how to build the bridge circuits and know strain gage applications.
- Understand the general principles behind a bridge circuit and how to stabilize an unbalanced bridge.
- Learn how to use a strain gage with a bridge circuit through a trainer (Digital Strain Indicator) and a couple of applications (Cantilever and "soda can").

2. Theoretical Background

For part 1:

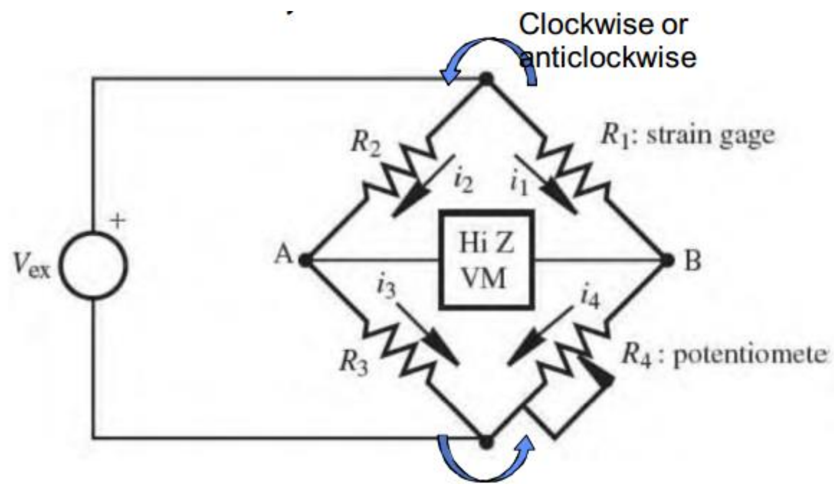


Figure 1 Wheatstone Bridge

For the static balanced mode, when the voltage between nodes A and B is 0.

$$i_1R_1 = i_2R_2$$

$$i_3R_3 = i_4R_4$$

Because the high-input impedance voltmeter between A and B is assumed to draw no current,

$$i_1 = i_4 = \frac{V_{ex}}{R_1 + R_4}$$

$$i_2 = i_3 = \frac{V_{ex}}{R_2 + R_3}$$

If we know R_2 and R_3 accurately and we note the value for R_4 on the precision potentiometer scale, we can accurately calculate the unknown resistance R_1 as

$$\frac{R_1}{R_4} = \frac{R_2}{R_3}$$

$$R_1 = \frac{R_2R_4}{R_3}$$

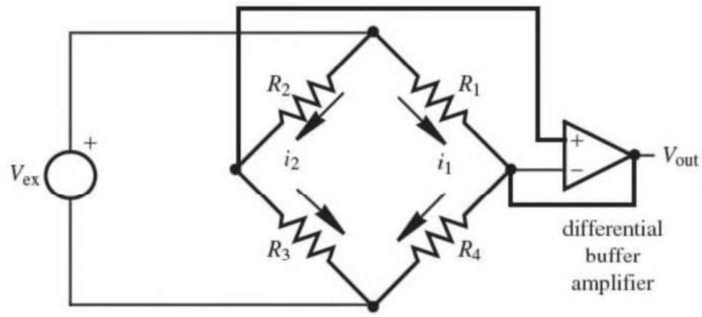


Figure 2 dynamic deflection operation

In dynamic deflection operation, R_1 representing a strain gage. Then changes in R_1 can be determined from changes in the output voltage.

$$V_{out} = i_1 R_1 - i_2 R_2 = i_3 R_3 - i_4 R_4$$
$$V_{ex} = i_1 (R_1 + R_{4)} = i_2 (R_2 - R_3)$$

Then:

$$V_{out} = [\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}] V_{ex}$$

When R_1 changes, as the strain gage is loaded, R_1 is replaced by its new resistance $R_1 + \Delta R_1$.

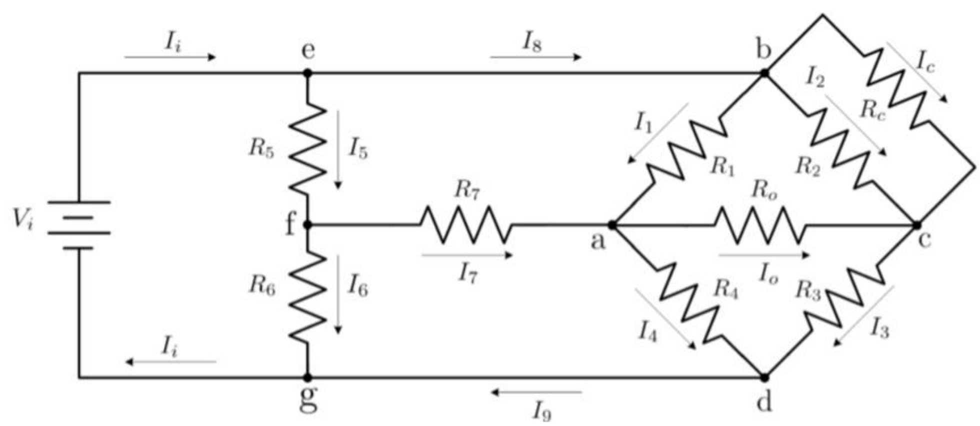
$$V_{out} = V_{ex} [\frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_4} - \frac{R_2}{R_2 + R_3}]$$

Rearranging it:

$$\frac{\Delta R_1}{R_1} = \frac{\frac{R_4}{R_1} (\frac{V_{out}}{V_{ex}} + \frac{R_2}{R_2 + R_3})}{1 - \frac{V_{out}}{V_{ex}} - \frac{R_2}{R_2 + R_3}} - 1$$

By measuring the change in the output voltage ΔV_{out}, students can determine the gage resistance change ΔR_1

Finally, Kirchhoff's law is used to balance the bridge:



According to the nodal method, the equations of circuit are built up as follows.

Bridge resistors: R_1, R_2, R_3, R_4

Potentiometer: R_5+R_6 (R_{b1})

Resistor at pot wiper: R_7 (R_{b2})

DMM input impedance: R_o (typically 1MΩ)

Calibrating resistor: R_c (suggested value 1GΩ)

$$R_{2||c} = R_2||R_c = \frac{R_2 R_c}{R_2 + R_c}$$

$$(R_5 + R_6)I_1 - (R_5)I_2 - (R_6)I_3 = V_i$$

$$(R_1 + R_5 + R_7)I_2 - (R_5)I_1 - (R_7)I_3 - (R_1)I_4 = 0$$

$$(R_4 + R_6 + R_7)I_3 - (R_6)I_1 - (R_7)I_2 - (R_4)I_5 = 0$$

$$\left(R_1 + R_{2||c} + R_o\right)I_4 - (R_1)I_2 - (R_o)I_5 = 0$$

$$(R_3 + R_4 + R_o)I_5 - (R_4)I_3 - (R_o)I_4 = 0$$

$$\vec{x}^T = [I_1\ I_2\ I_3\ I_4\ I_5]$$

$$\vec{b}^T = [V_i\ 0\ 0\ 0\ 0]$$

$$A = \begin{bmatrix} R_5 + R_6 & -R_5 & -R_6 & 0 & 0 \\ -R_5 & R_1 + R_5 + R_7 & -R_7 & -R_1 & 0 \\ -R_6 & -R_7 & R_4 + R_6 + R_7 & 0 & -R_4 \\ 0 & -R_1 & 0 & R_1 + R_{2||c} + R_o & -R_o \\ 0 & 0 & -R_4 & -R_o & R_3 + R_4 + R_o \end{bmatrix}$$

$$\vec{x} = A^{-1}\vec{b}$$

$$V_o = (I_4 - I_5)R_o$$

For part 2:

In this part, students will use a potentiometer-balanced bridge to estimate the resistance of the load resistor used in the familiarization exercises.

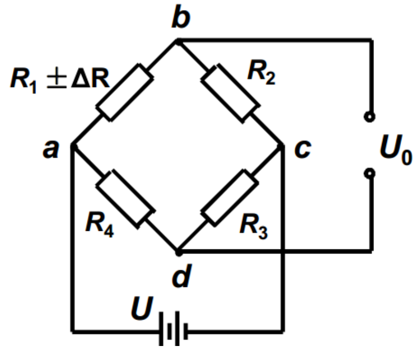


Figure 3 Sensitivity of Bridge

If the bridge is balanced ($\frac{R_2}{R_1} = \frac{R_3}{R_4}$), then:

$$U_o = \frac{R_1R_3 - R_2R_4}{(R_1 + R_2)(R_3 + R_4)}\text{U}$$

Assume R_1 is a strain gage and deformed, $R_1 + \Delta R_1$ replace R_1:

$$U_o = \frac{(R_1 + \Delta R_1)R_3 - R_2R_4}{(R_1 + \Delta R_1 + R_2)(R_3 + R_4)} \approx \frac{\left(\frac{\Delta R_1}{R_1}\right)\left(\frac{R_3}{R_4}\right)}{\left(1 + (\frac{R_2}{R_1})\right)\left(1 + (\frac{R_3}{R_4})\right)}\text{U}$$

If $\frac{R_2}{R_1} = \frac{R_3}{R_4}$ =n, then:

$$U_o \approx \frac{\left(\frac{\Delta R_1}{R_1}\right)\left(\frac{R_3}{R_4}\right)}{\left(1 + (\frac{R_2}{R_1})\right)\left(1 + (\frac{R_3}{R_4})\right)}\text{U}$$

And:

$$U_o = \left(\frac{\Delta R_1}{R_1}\right)\frac{n}{(n+1)^2}\text{U}$$

So sensitivity of bridge is

$$\text{S} = \frac{U_o}{\Delta R_1/R_1} = \frac{n}{(n+1)^2}\text{U}$$

The relationship between strain and sensitivity of strain gage is:

$$\frac{\Delta R_1}{R_1} = F\varepsilon$$

Where F is the Gage Factor. Then:

$$U_o = SK\varepsilon$$

1. Full Bridge

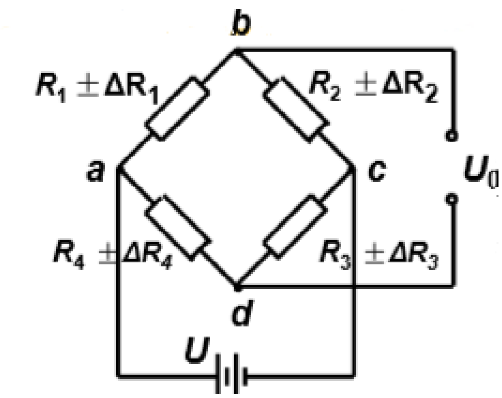


Figure 4 Full Bridge

Each strain gage has change ΔR, then:

$$U_o = \frac{R_1R_3 - R_2R_4}{(R_1 + R_2)(R_3 + R_4)}\mathrm{U}$$

If R_1=R_2=R_3=R_4=R and the bridge is balanced, then:

$$U_o = \frac{U}{4} \times \frac{\Delta R_1 - \Delta R_2 + \Delta R_3 - \Delta R_4}{R}$$

And the sensitivity of strain gage is same(F):

$$U_o = \frac{(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4)UK}{4}$$

If ΔR_1= ΔR_2= ΔR_3= ΔR_4= ΔR, and $R_1 \rightarrow R_1$+ΔR, $R_2 \rightarrow R_2$ +ΔR, $R_3 \rightarrow R_3$+ΔR, $R_4 \rightarrow R_4$ +ΔR

$$U_o = \frac{U\Delta R}{R}, \mathrm{S} = \mathrm{U}$$

2. Half Bridge

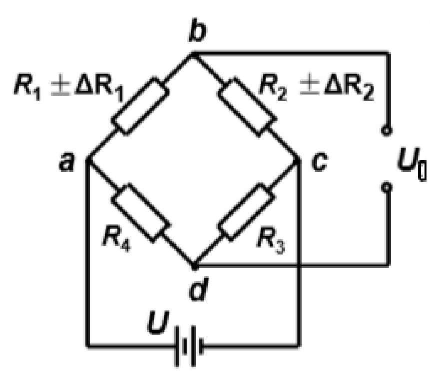


Figure 5 Half Bridge

Each strain gage has change ΔR. (As the same of the full bridge)

If $\Delta R_1 = \Delta R_2 = \Delta R$, $\Delta R_3 = \Delta R_4 = 0$, and $R_1 \rightarrow R_1 + \Delta R$, $R_2 \rightarrow R_2 + \Delta R$

$$U_o = \frac{U\Delta R}{2R}, \mathrm{S} = \frac{\mathrm{U}}{2}$$

3. Quarter Bridge

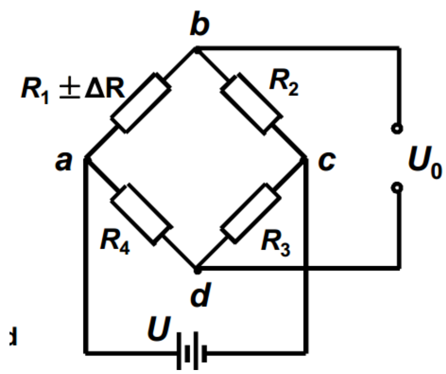


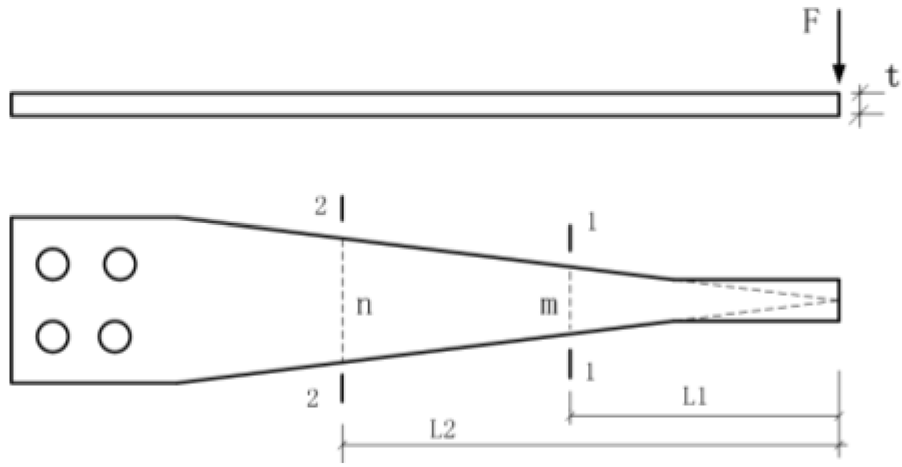
Figure 6 Quarter Bridge

Each strain gage has change ΔR. (As the same of the full bridge)

If $\Delta R_1 = \Delta R$, $\Delta R_2 = \Delta R_3 = \Delta R_4 = 0$, and $R_1 \rightarrow R_1 + \Delta R$

$$U_o = \frac{U\Delta R}{4R}, \mathrm{S} = \frac{\mathrm{U}}{4}$$

For part 3:



The maximum stress of section 1-1:

$$\sigma_1 = \frac{M_1}{w_1} = \frac{F \cdot L_1}{w_1}$$

Section modulus of 1-1:

$$w_1 = \frac{I_1}{\frac{t}{2}} = \frac{\frac{m \cdot t^3}{12}}{\frac{t}{2}} = \frac{m \cdot t^2}{6}$$

The maximum stress of section 2-2:

$$\sigma_2 = \frac{M_2}{w_2} = \frac{F \cdot L_2}{w_2}$$

Section modulus of 2-2:

$$w_2 = \frac{I_2}{\frac{t}{2}} = \frac{\frac{m \cdot t^3}{12}}{\frac{t}{2}} = \frac{m \cdot t^2}{6}$$

$$\sigma_1 = \frac{6F \cdot L_1}{m \cdot t^2}$$

$$\sigma_2 = \frac{6F \cdot L_2}{n \cdot t^2}$$

$$\sigma_1 = \sigma_2$$

$$\frac{L_1}{L_2} = \frac{m}{n}$$

For part 4:

For a long, thin-walled cylinder, the stress is obtained through Hooke's law.

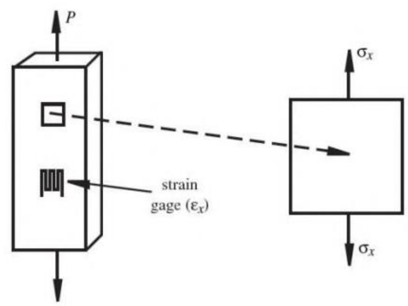
$$\sigma_x = E\varepsilon_x$$

Here, σ_x is given by:

$$\sigma_x = \frac{P}{A}$$

Then

$$P = AE\varepsilon_x$$



For biaxial load, the relation of stress and strain is followed:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E}$$

So

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

For a thin-walled pressure vessel

$$\sigma_x = \frac{pr}{t}$$

$$\sigma_y = \frac{pr}{2t}$$

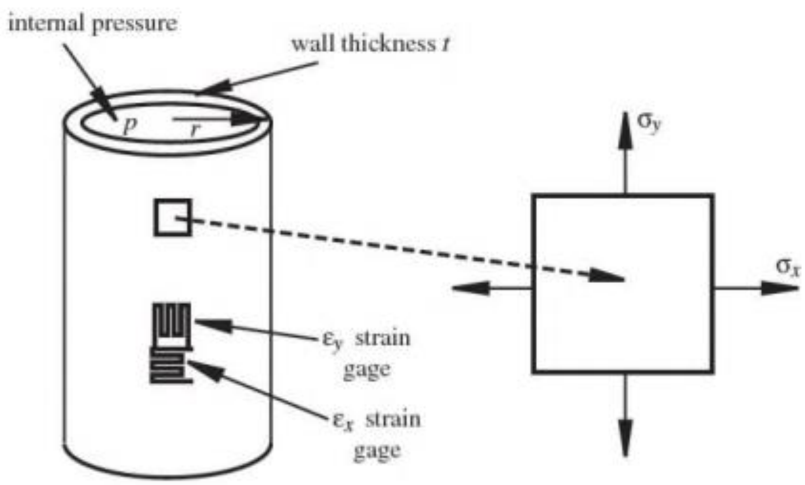
Then we can get the pressure through strain:

$$p = \frac{t\sigma_x}{r} = \frac{tE}{r(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y)$$

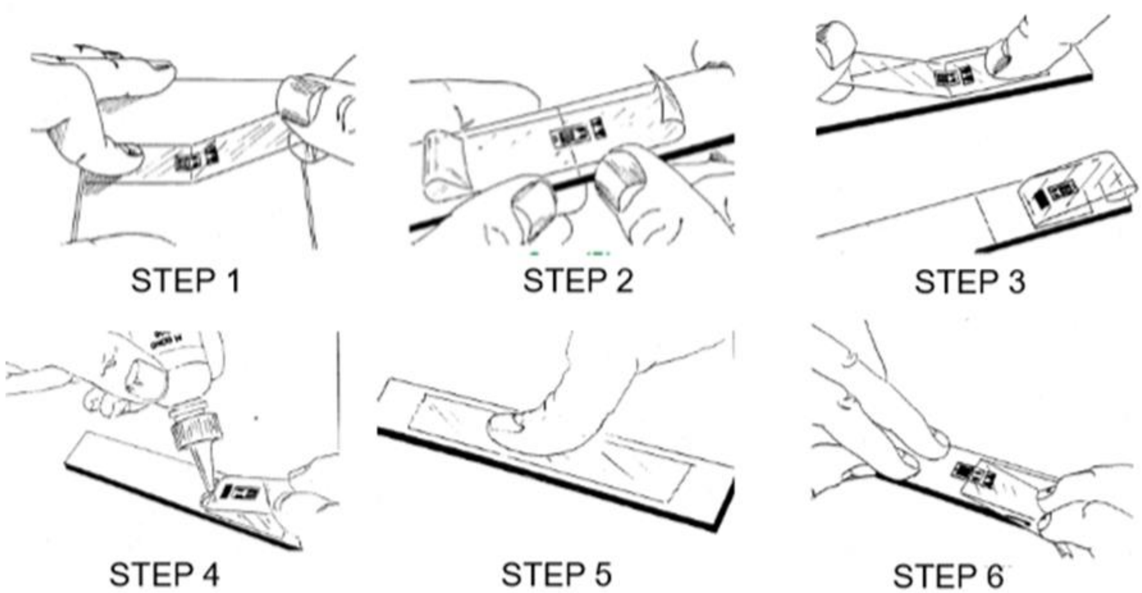
Or

$$p = \frac{2t\sigma_y}{r} = \frac{2tE}{r(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x)$$

Either expression would yield the correct pressure value



Procedures for mounting the strain gages are shown as follows:



3. Experimentation

3.1 Bridge Circuit Analysis & Measurements

3.1.1 Summary of Procedure

a) This step involves analysis and measurements. Use four 100Ω resistors, measure their exact values and record them in Table 1. Construct the bridge circuit of Figure 7 in the breadboard. Note that the output voltage V_O is given by:

$$V_0 = V_1[\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4}]$$

The bridge is balanced when $V_O = 0$.

- Set the power source to provide a voltage of 2 V_{DC} and measure the actual value and record it in Table 1. Apply this voltage to the circuit and measure the output voltage V_O using the DMM.
- Compute for the output voltage V_O with the nominal, as well as, the actual values of the bridge resistors.

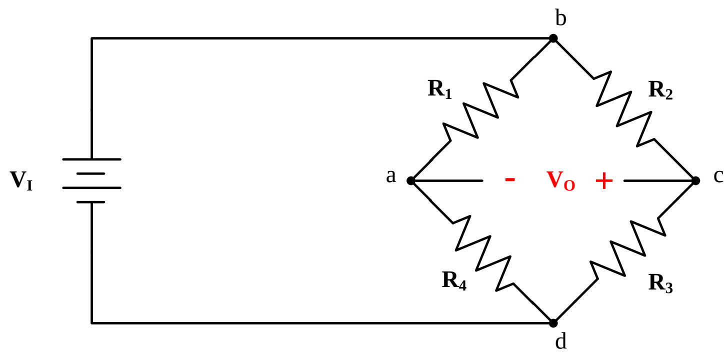


Figure 7 A bridge circuit

b) This step involves analysis. Suppose that a quarter bridge circuit is set up with R_3 as the active arm where the value of the resistor R_3 changes according to the equation: R_3=R_3^0+ΔR_3, where ΔR_3 represents a small change in the value of R_3. Derive an equation for the output voltage V_O in terms of ΔR_3 for the case where the bridge is initially balanced and then there is a change in resistance of the active arm. The equation could be obtained as:

$$V_0 = [\frac{\Delta R_3 R_2^0}{(R_2^0 + R_3^0) + (R_2^0 + R_3^0 + \Delta R_3)}]V_1$$

This equation indicates that the bridge circuit can be used to detect resistance change ΔR_3 by measuring the output voltage induced between junctions a) and c) of Figure 7. A similar circuit can be used with a strain gage with changes in resistance (i.e. represented by ΔR_3) with changing strain. The last equation is used when the circuit is initially balanced using exact resistors. Often the circuit cannot be exactly balanced. The next section introduces a more complicated balancing circuit that is adjustable to balance the bridge before any measurements are made.

c) This step involves analysis and measurements. If the condition R_3^0/R_2^0= R_4^0/R_1^0 is not exactly satisfied, the bridge circuit is not considered in balance. Two extra resistors Rb1 and R_{b2} can be added as shown in Figure 8 to maintain the balance of the circuit (R_{b1} = 10kΩ potentiometer and R_{b2} can vary depending on the required sensitivity of V_O to the potentiometer but 1kΩ will be used for this experiment.) The potentiometer R_{b1} can be adjusted to balance the circuit making V_O equal to zero.

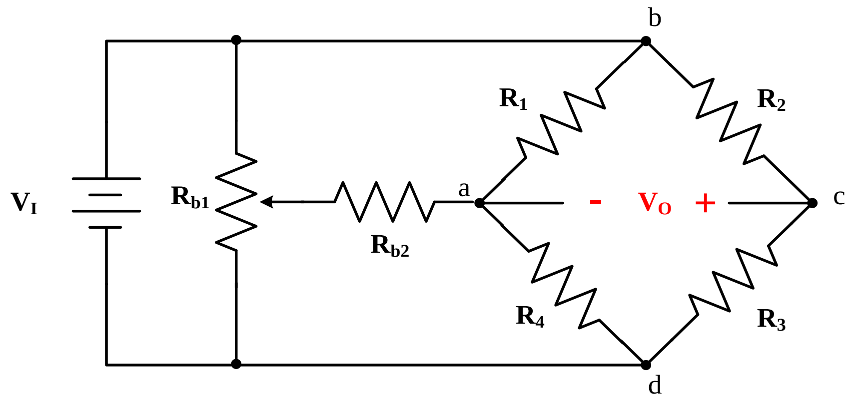


Figure 8 Modified bridge circuit

Setup the modified bridge circuit as shown in Figure 8 and first balance it by adjusting R_{b1}. Temporarily turn off the power source to perform several measurements. The potentiometer may have to be removed from the circuit for the measurements to avoid loading it with resistance from the board and the circuit. Record the resistance measurements in Table 2, then reconnect the potentiometer into the circuit. Measure V_O for several values of the fractional resistance of $\Delta R_3/R_3^0$ in the range of -0.1 to 0.1 (i.e., measure V_O caused by changing R_3). To do this, use the 33Ω, 47Ω, 68Ω and 150Ω resistors (R_3^i) that were provided to you. Replace the original bridge resistor R_3^0 with one of these resistors and record the exact value of resistance and the output voltage each time. Use Table 3 to log the measurements.

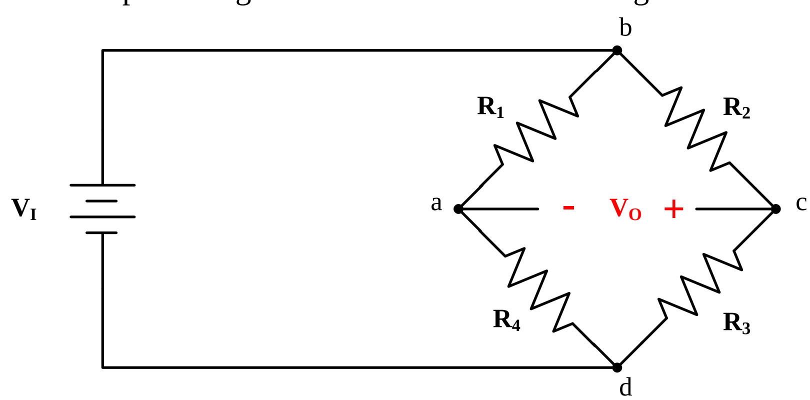


Figure 8-1 A bridge circuit

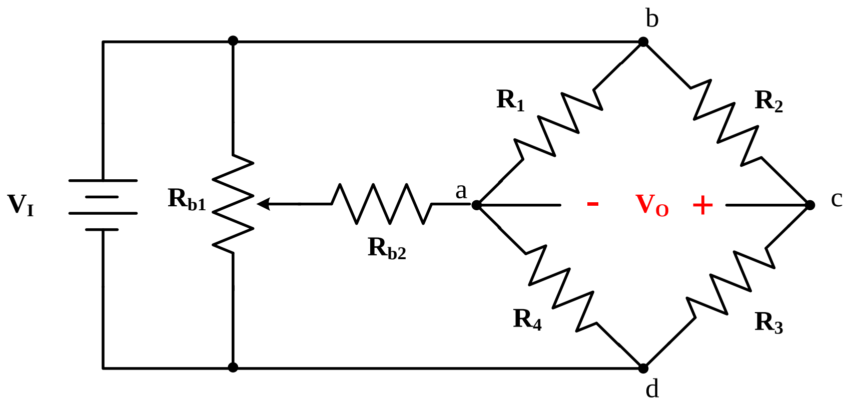


Figure 8-2 Modified bridge circuit

<u>3.1.2 Results</u>

a) Derivation:

$$V_0 = [\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4}]V_1$$

For the real condition, in order to balance the bridge, V_0=0. Thus:

$$\frac{R_3^0}{R_2^0 + R_3^0} - \frac{R_4^0}{R_1^0 + R_4^0} = 0$$

$$(R_2^0 + R_3^0)R_4^0 = (R_1^0 + R_4^0)R_3^0$$

$$R_4^0 R_2^0 = R_1 R_3^0$$

$$\frac{R_3^0}{R_2^0} = \frac{R_4^0}{R_1^0}$$

Table 1. Actual resistance of bridge resistors

Bridge Resistors	Resistance (Ω)
R_1^0	98.5
R_2^0	98.7
R_3^0	98.4
R_4^0	98.5

V_{IN} (MEAS) = 2.05V
V_O (MEAS) = 1.5mV
$V_{O(COMP)-NOM} = 0\text{mV}$

$$V_{O(COMP)-ACT} = \left[\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4}\right] V_1 = \left[\frac{98.4\Omega}{98.7\Omega + 98.4\Omega} - \frac{98.5\Omega}{98.5\Omega + 98.5\Omega}\right] 2.05\text{V} = 1.56\text{mV}$$

$$\varepsilon = \left|\frac{V_{O(COMP)-ACT} - V_{O(MEAS)}}{V_{O(COMP)-ACT}}\right| = \left|\frac{1.56\text{mV} - 1.5\text{mV}}{1.56\text{mV}}\right| = 3.8462\%$$

Comment: The error is 3.8462%, less than 5%, which means the result is good.

b) Derivation:

$$V_0 = [\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4}]V_1$$

For the balanced condition:

$$V_0 = [\frac{R_3^0}{R_2^0 + R_3^0} - \frac{R_4^0}{R_1^0 + R_4^0}]V_1$$

$$\frac{R_3^0}{R_2^0 + R_3^0} = \frac{R_4^0}{R_1^0 + R_4^0}$$

When the R3 changes:

$$V_0 = [\frac{R_3^0 + \Delta R_3}{R_2^0 + R_3^0 + \Delta R_3} - \frac{R_4^0}{R_1^0 + R_4^0}]V_1 = [\frac{R_3^0 + \Delta R_3}{R_2^0 + R_3^0 + \Delta R_3} - \frac{R_3^0}{R_2^0 + R_3^0}]V_1$$

$$= [\frac{(R_3^0 + \Delta R_3)(R_2^0 + R_3^0) - (R_2^0 + R_3^0 + \Delta R_3)R_3^0}{(R_2^0 + R_3^0 + \Delta R_3)(R_2^0 + R_3^0)}]V_1$$

$$V_0 = [\frac{R_3^0 \Delta R_3}{(R_2^0 + R_3^0 + \Delta R_3)(R_2^0 + R_3^0)}]V_1$$

c)

Table 2. Potentiometer (R_{b1}) & R_{b2} Setting after Balancing

	Resistance (Ω) – Circuit	Resistance (Ω) –Matlab
R_5 (R_{b1} across terminals 1 and 2)	5557	5565
R_6 (R_{b1} across terminals 2 and 3)	4460	4475
Total POT Swing, R_{b1} (1-3)	10040Ω	
R_{b2} or R_7	983Ω	

The error of R_5

$$\varepsilon = \left|\frac{R_{circuit} - R_{matlab}}{R_{matlab}}\right| = \left|\frac{5.557 - 5.565}{5.565}\right| = 0.1438\%$$

The error of R_6

$$\varepsilon = \left|\frac{R_{circuit} - R_{matlab}}{R_{matlab}}\right| = \left|\frac{4.46 - 4.475}{4.475}\right| = 0.3352\%$$

The graph between the output voltage and the potentiometer settings

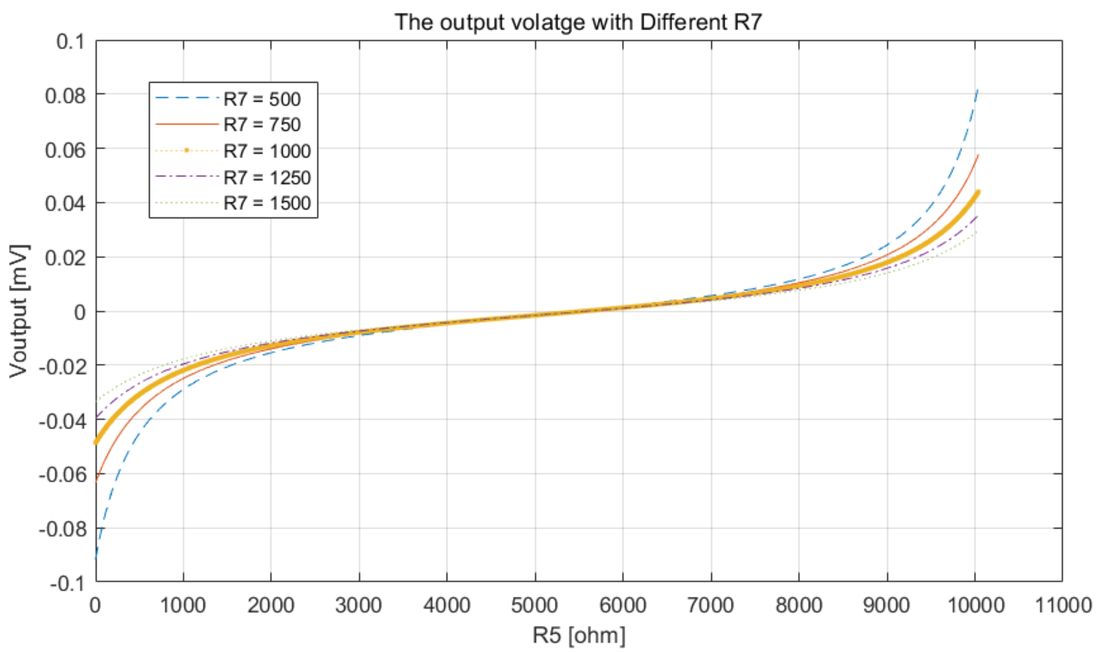
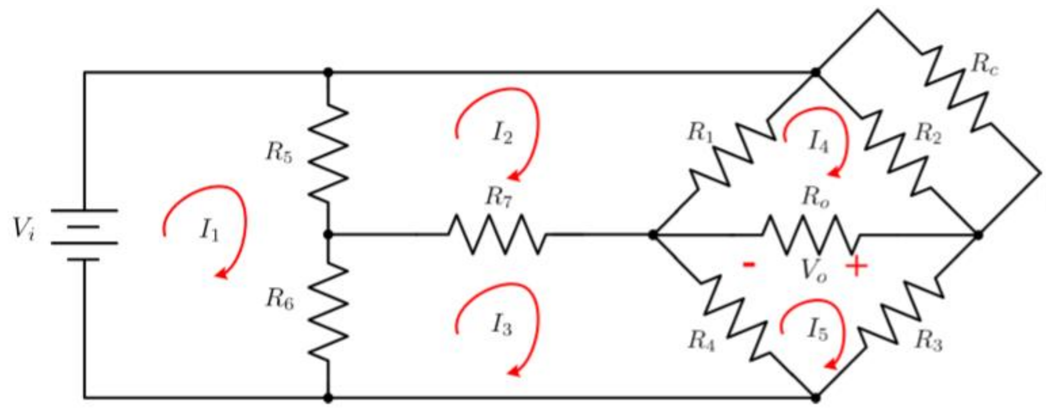


Table 3. Output Voltage with Changing R_3

Trial	Test R_3 (Ω)	Original R_3^0 (Ω)	ΔR_3 (Ω) (Test-Original)	Output Voltage, V_O (mV)				
				Circuit	Equation	Matlab	$\varepsilon_{Circuit,Equation}$	$\varepsilon_{Circuit,Matlab}$
1	33.4	98.4	-65	-504	-492.80	-492.42	2.22%	2.30%
2	47.0	98.4	-51.4	-360	-353.32	-353.02	1.86%	1.94%
3	66.8	98.4	-31.6	-194	-191.23	-191.04	1.43%	1.53%
4	146.5	98.4	48.1	201	196.46	196.27	2.26%	2.35%



Bridge Resistors: R_1, R_2, R_3, R_4

Potentiometer: $R_5 + R_6$ (R_{b1} or R_{pot} – the nominal value is 10 kΩ)

Resistor at Pot Wiper: R_7 (R_{b2} – the nominal value is 1 kΩ)

DMM Input Impedance: R_0 (typically 1 MΩ)

Calibrating Resistor: R_c (suggested value is 1 GΩ)

Simplifying the parallel branch in the bridge network, let

$$R_{2||c} = R_2||R_c = \frac{R_2R_c}{R_2+R_c}$$

The simplified branch reduces the number of meshes to five (5), hence there are five (5) mesh currents:

$$\vec{x}^T = [I_1 \quad I_2 \quad I_3 \quad I_4 \quad I_5]$$

Setting up the mesh current equations:

$$(R_5 + R_6)I_1 - (R_5)I_2 - (R_6)I_3 = V_i$$

$$(R_1 + R_5 + R_7)I_2 - (R_5)I_1 - (R_7)I_3 - (R_1)I_4 = 0$$

$$(R_4 + R_6 + R_7)I_3 - (R_6)I_1 - (R_7)I_2 - (R_4)I_5 = 0$$

$$\left(R_1 + R_{2||c} + R_0\right)I_4 - (R_1)I_2 - (R_0)I_5 = 0$$

$$(R_3 + R_4 + R_0)I_5 - (R_4)I_3 - (R_0)I_4 = 0$$

Using system of linear equations approach, let

$$\vec{b}^T = [V_i \quad 0 \quad 0 \quad 0 \quad 0]$$

Thus,

$$A\vec{x} = \vec{b}$$

The mesh currents can be solved using

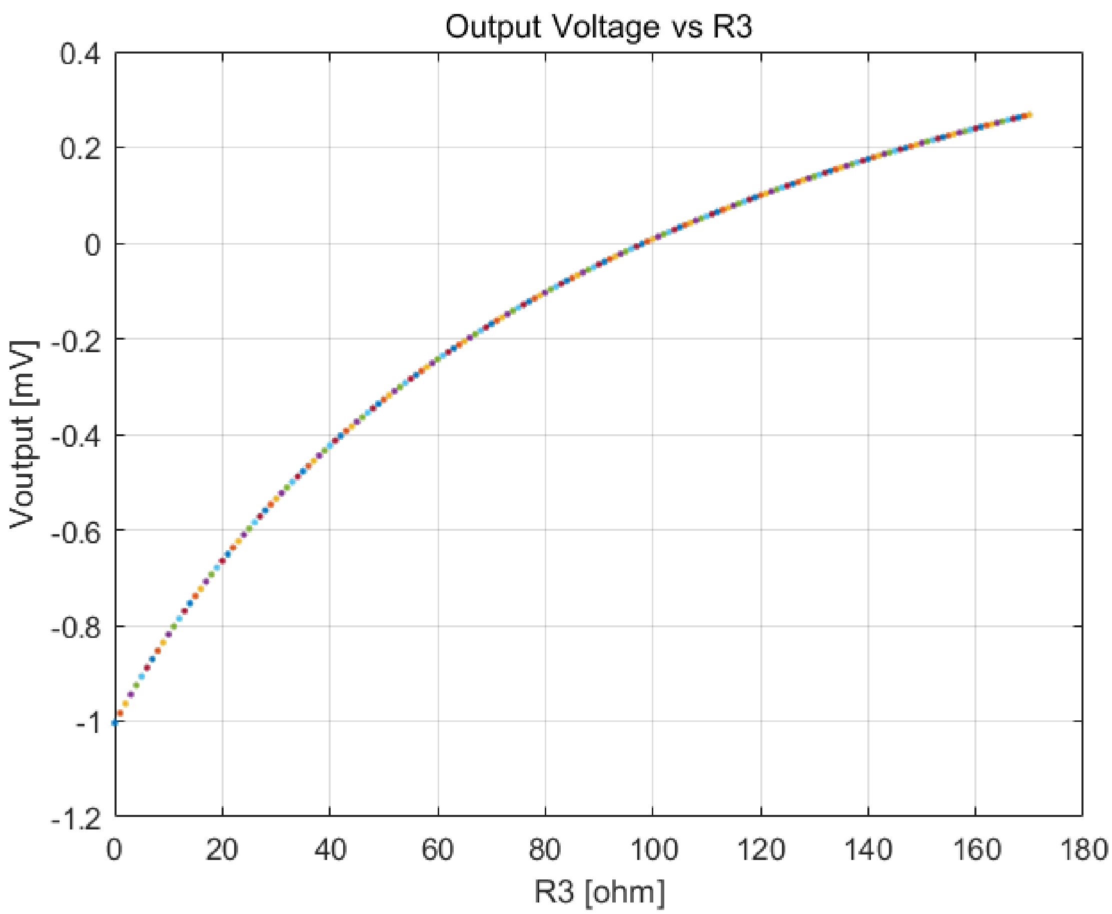
$$\vec{x} = A^{-1}\vec{b}$$

$$\mathbb{A} = \begin{bmatrix} R_5 + R_6 & -R_5 & -R_6 & 0 & 0 \\ -R_5 & R_1 + R_5 + R_7 & -R_7 & -R_1 & 0 \\ -R_6 & -R_7 & R_4 + R_6 + R_7 & 0 & -R_4 \\ 0 & -R_1 & 0 & R_1 + R_{2\|c} + R_o & -R_o \\ 0 & 0 & -R_4 & -R_o & R_3 + R_4 + R_o \end{bmatrix}$$

The output voltage V_O can then be computed as

$$V_0 = (I_4 - I_5)R_0$$

The graph of the output voltage versus R_3:



3.1.3 Analysis & Discussion

In part a, if the resistances of the four bridge resistors are equal when the input voltage is 2000mV, the output voltage is calculated as 0mV. However, there is some error in the actual resistance measured. Therefore, the calculated actual output voltage may also have some errors. The error could have come from two sources. First, because different resistances cause potential changes, the difference in resistance results in non-zero output voltage. Second, the difference between the measured and calculated V_0 may be due to the extra resistance of the circuit, and the other neglected resistances are valid in the real world.

In part b, from the derivation, the relationship between output voltage and changed resistance is obvious.

In part c, the circuit has a matrix function (derived from the node voltage law).

The calculated and measured results of the changes of resistors R_5 and R_6 at both ends of the balance circuit have little error, which proves that the experiment is correct. When the value of R_3 changes, the output voltage will also change accordingly, and the results obtained by derivation and matrix calculation are different. The error between the circuit results and the equation results is approximately 2%, and the error between the circuit results and the matrix results fluctuates between the maximum value of 2.35% and the minimum value of 1.53%. Two theoretical results are obtained through the derivation of equations and the formation of matrices. The error indicates that the error of the derivation equation is smaller than that of MATLAB data, which is more reliable.

3.2 Bridge Circuit Application – Measurement of Load Resistor

3.2.1 Summary of Procedure

a) In this part, we will use a potentiometer-balanced bridge to estimate the resistance of the load resistor used in the familiarization exercises. Setup the bridge circuit of figure using four (4) 100Ω resistors, measure and record their exact values in Table 4.

b) Set the power supply to provide a voltage of 2 V_{DC} and measure the actual value and record it in Table 6. Apply this voltage to the circuit and measure the output voltage V_O using the DMM. Because the bridge circuit is not in balance, add the two extra resistors R_{b1} and R_{b2} shown in Figure 2 to balance of the circuit (R_{b1} = 10kΩ potentiometer and R_{b2} would vary depending on the required sensitivity of V_O to the potentiometer but 1kΩ will be used for this experiment.) Adjust the potentiometer R_{b1} to balance the circuit by making V_O equals to zero.

c) Temporarily turn off the power source to perform several measurements. The potentiometer may have to be removed from the circuit for the measurements to avoid loading it with resistance from the board and the circuit. Record the resistance measurements in Table 5, then reconnect the potentiometer into the circuit.

3.2.2 Results

Table 4. Actual resistance of bridge resistors

Bridge Resistors	Resistance (Ω)
R_1^0	98.5
R_2^0	98.7
R_3^0	98.4
R_4^0	98.5

Table 5. Potentiometer (R_{b1}) & R_{b2} Setting after Balancing

	Resistance (Ω)
R_5 (R_{b1} across terminals 1 and 2)	5557
R_6 (R_{b1} across terminals 2 and 3)	4460
Total POT Swing, R_{b1} (1 and 3)	10040
R_7 (R_{b2})	983

Table 6. Output Voltage with Changing R_3

Test case	Excitation Voltage, V_{ex} (mV)	Output Voltage, V_O (mV)	ΔR_2 (Ω) (from eq'n)	Stamped Value (Ω)
test leads shorted	/	/	/	
precision resistor (1 ohm)	2000	5	0.987	1
load resistor (0.6 ohm)	2000	2.7	0.534	0.5

For the precision resistor:

$$\frac{\Delta R_2}{R_2} = \frac{\frac{R_3}{R_2}\left(\frac{\Delta V_0}{V_1} + \frac{R_2}{R_2 + R_3}\right)}{\left(1 - \frac{\Delta V_0}{V_1} - \frac{R_2}{R_2 + R_3}\right)} - 1 = \frac{\frac{98.4}{98.7}\left(\frac{5}{2000} + \frac{98.7}{98.7 + 98.4}\right)}{\left(1 - \frac{5}{2000} - \frac{98.7}{98.7 + 98.4}\right)} - 1 = 0.0100$$

$$\Delta R_2 = 0.9870\Omega$$

$$\varepsilon = \left|\frac{stamped\ value - \Delta R_2}{stamped\ value}\right| = 1.3\%$$

For the load resistor:

$$\frac{\Delta R_2}{R_2} = \frac{\frac{R_3}{R_2}\left(\frac{\Delta V_0}{V_1} + \frac{R_2}{R_2 + R_3}\right)}{\left(1 - \frac{\Delta V_0}{V_1} - \frac{R_2}{R_2 + R_3}\right)} - 1 = \frac{\frac{98.4}{98.7}\left(\frac{4.7}{2000} + \frac{98.7}{98.7 + 98.4}\right)}{\left(1 - \frac{4.7}{2000} - \frac{98.7}{98.7 + 98.4}\right)} - 1 = 0.00541$$

$$\Delta R_2 = 0.5340\Omega$$

$$\varepsilon = \left|\frac{stamped\ value - \Delta R_2}{stamped\ value}\right| = 6.8\%$$

3.2.3 Analysis & Discussion

Through measurement and calculation, it is found that the errors are relatively small. In calculating the Precision Resistor experiments, the experimenter can get 1.3% of the error. In calculating the Load Resistor experiments, the experimenter can get 6.8% error.

Errors that may occur during the experiment are as follows:

1. Unstable circuit connection. This will lead to inaccurate experimental measurement data, the error in the calculation process will become larger.

2. Error of measuring instrument itself. The multimeter and other instruments have certain resistance and error. This can cause our readings to be skewed, affecting subsequent calculations.

3.3 Strain Measurement and Analysis on a Cantilever Beam

3.3.1 Summary of Procedure

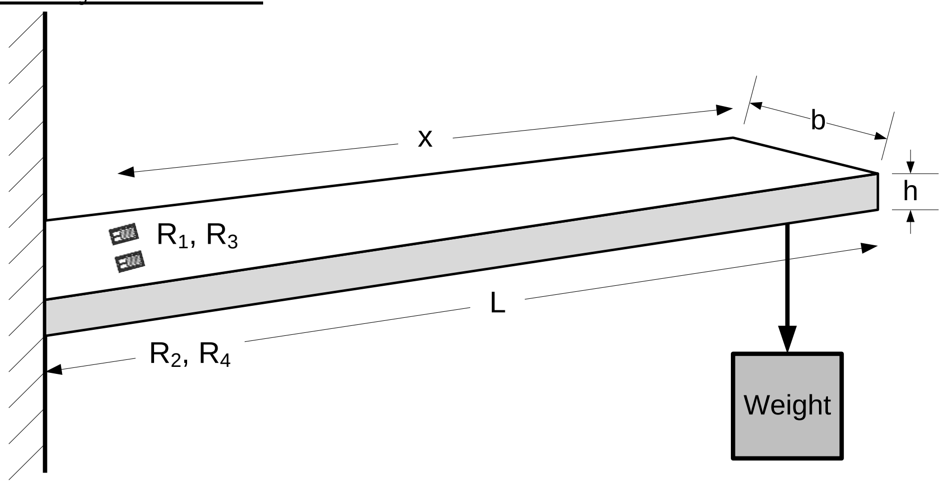
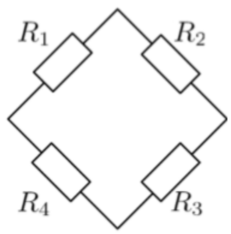
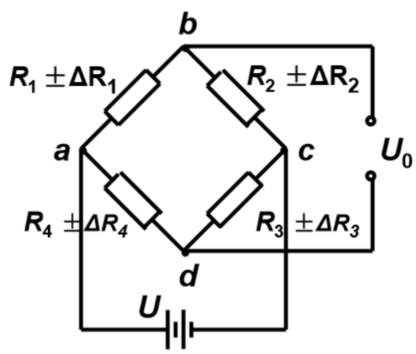


Figure 9 Cantilever Diagram

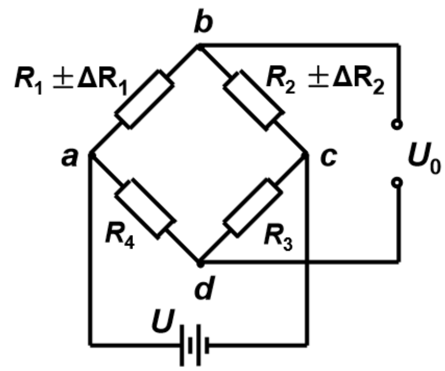
Two strain gages are mounted on the top of a cantilever beam and two strain gages are mounted on the bottom of the beam shown in Figure 9. The gages are near the root of the beam and a weight load is applied to the free end of the cantilever beam. Compute the strain in the beam based on beam theory. Measure the strain corresponding to the different weights at the end of the beam and record the data in Table 8. Compare the experimental data to the theoretical prediction of the strains.



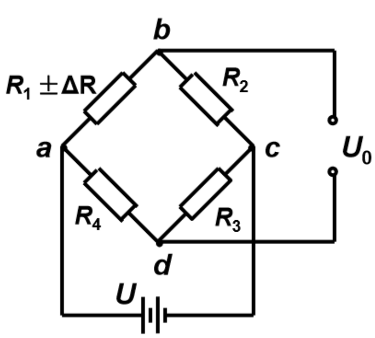
The Bridge Circuit



Full Bridge



Half Bridge



Quarter Bridge

3.3.2 Results

Table 7. Cantilever Dimensions

Parameter	Measurement (mm)
b	33
h	6
x	283
L	415

Young's Modulus for Steel (E, GPa)	200

Table 8-1. Data on Strain Gage Application: Cantilever Beam

Weight (g)	Moment (N·m)	Calculated Values		Measured Values					
				Full Bridge		Half Bridge		Quarter Bridge	
		Stress (MPa)	Strain (με)	Stress (MPa)	Strain (με)	Stress (MPa)	Strain (με)	Stress (MPa)	Strain (με)
1000	2.77623	14	70	14.513	290.26	14.795	147.95	14.728	73.64
2000	5.55246	28	140	30.1785	603.57	34.572	345.72	29.828	149.14
3000	8.32869	42	210	42.4345	848.69	48.021	480.21	43.312	216.56
4000	11.10492	56	280	55.6925	1113.85	63.872	638.72	57.796	288.98

For **Moment** in Table 8-1, taking **Weight=1000g** as sample:

$$\textbf{Moment} = \mathrm{L} \times \mathrm{F} = \mathrm{x} \times m \times \mathrm{g} = 415mm \times 1000\mathrm{g} \times 9.81\frac{m}{s^2} = 2.77623\ N \cdot m$$

For **Calculated Stress** in Table 8-1, taking **Weight=1000g** as sample:

$$\textbf{Stress} = \frac{Moment}{w} = \frac{Moment}{\frac{bh^2}{6}} = \frac{2.77623\ N \cdot m}{\frac{33mm \times (6mm)^2}{6}} = 14\ MPa$$

For **Calculated Strain** in Table 8-1, taking **Weight=1000g** as sample:

$$\textbf{Strain} = \frac{Stress}{E} = \frac{14\ MPa}{200GPa} = 70\ \mu\varepsilon$$

For **Strain of Full Bridge** in Table 8-1, taking **Weight=1000g** as sample:

$$\textbf{Stress} = \textbf{Strain} \times E = 290.26\mu\varepsilon \times 200\mathrm{GPa}/4 = 14.513\ MPa$$

For **Strain of Half Bridge** in Table 8-1, taking **Weight=1000g** as sample:

$$\textbf{Stress} = \textbf{Strain} \times E = 147.95\mu\varepsilon \times 200\text{GPa}/2 = 14.795\ MPa$$

For **Strain of Quarter Bridge** in Table 8-1, taking **Weight=1000g** as sample:

$$\textbf{Stress} = \textbf{Strain} \times E = 73.64\mu\varepsilon \times 200\text{GPa} = 14.728\ MPa$$

$$\%\ \text{ERROR} = \frac{Predicted - Measured}{Predicted} \times 100\%$$

Table 8-2. Errors on Strain Gage Application: Cantilever Beam

Calculated Values		**Measured Values of Quarter Bridge**		**% ERROR**	
Stress (MPa)	**Strain** (με)	**Stress** (MPa)	**Strain** (με)	**of Stress**	**of Strain**
14	70	14.728	73.64	5.2%	5.2%
28	140	29.828	149.14	6.53%	6.53%
42	210	43.312	216.56	3.12%	3.12%
56	280	57.796	288.98	3.2%	3.2%

For **% ERROR of Stress** in Table 10-2, taking **Weight=2000g** as sample:

$$\%\ \text{ERROR} = \frac{Predicted - Measured}{Predicted} \times 100 = \frac{29.828MPa - 28MPa}{28MPa} \times 100 = 6.53\%$$

For **% ERROR of Strain** in Table 10-2, taking **Weight=2000g** as sample:

$$\%\ \text{ERROR} = \frac{Predicted - Measured}{Predicted} \times 100 = \frac{140.00\mu\varepsilon - 149.14\mu\varepsilon}{140.00\mu\varepsilon} \times 100 = 6.53\%$$

3.3.3 Analysis & Discussion

From the table,

For Weight=1000g:

Calculated Stress = 14 MPa ≈ 14.728 MPa = Measured Stress of Quarter Bridge

Calculated Stress = 14 MPa ≈ 17.795 MPa $=\frac{1}{2}\times$ Measured Stress of Half Bridge

Calculated Stress = 14 MPa ≈ 14.513 MPa $=\frac{1}{4}\times$ Measured Stress of Half Bridge

Calculated Strain = 70 με ≈ 73.64 με = Measured Strain of Quarter Bridge

Calculated Strain = 70 με ≈ 73.975 με $=\frac{1}{2}\times$ Measured Strain of Half Bridge

Calculated Strain = 70 με ≈ 72.565 με $=\frac{1}{4}\times$ Measured Strain of Half Bridge

When weight=2000g, 3000g, 4000g, it follows the same rule.

For Full Bridge:

Measured Stress of Weight=2000g: 28 MPa ≈30.1785 MPa$=\frac{1}{4}\times$ Measured Stress of Full Bridge of Weight=2000g

Measured Stress of Weight=3000g: 42 MPa ≈42.4345 MPa$=\frac{1}{4}\times$ Measured Stress of Full Bridge of Weight=3000g

Measured Stress of Weight=4000g: 56 MPa ≈55.6925 MPa$=\frac{1}{4}\times$ Measured Stress of Full Bridge of Weight=4000g

When for the Stress and strain of Half Bridge, Quarter Bridge, and Calculated Values, it follows the same rule.

Since the weight is determined, quarter bridge uses a strain gauge, and the stress calculated from the strain is the actual stress applied by the weight. Half bridge uses two strain gauges, and the resulting stress is twice the stress actually applied by the weight. Full bridge uses four strain gauges, and the resulting force is four times the actual stress applied by the weight. At the same time, as the applied weight increases proportionally, the stress and strain increase correspondingly.

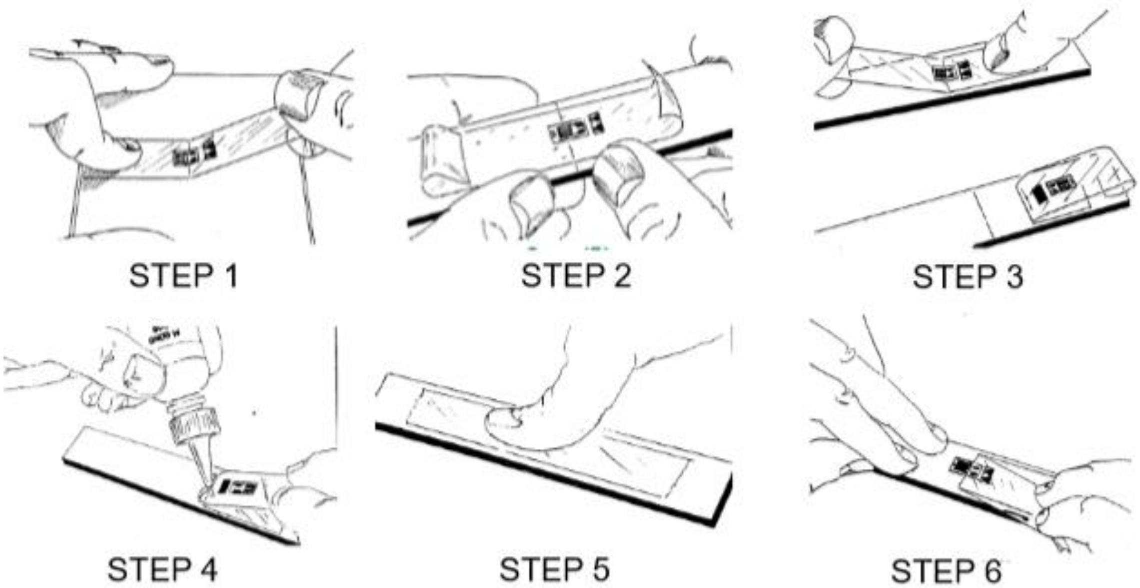
At the same time, there is an error between the measured value and the calculated value. The error comes from: 1. The reading error when measuring dimensions of the beam; 2. The accuracy defect of the measuring tool itself; 3. The system error of the instrument itself; 4. The connection of the measuring line is unstable; 5. In the long-term measurement state, the temperature of the wire rises, which will affect the resistance value of the measurement resistance; 6. The disturbance of the measurement environment.

3.4 Strain Gage Application ("soft drink can" experiment)

3.4.1 Summary of Procedure

a) Prepare the soda can surface for mounting: 1) use the rough grit sandpaper to remove the paint off a "sufficient" section of the can 2) Use the fine grit sandpaper to smoothen the surface. 3) Wipe off the paint and metal filing/dust using the paper towel. 4) Carefully wipe the prepared surface with the soft cloth that has been moistened with acetone.

b) Mount the strain gages following the follow steps:



c) Install two strain gauges on the soft drink can as shown in the diagram. Note that one strain gage measures axial strain and the other measures circumferential strain. Solder the leads and check the resistance of each pair of leads.

Figure 10 Strain gage locations for "soda can" experiment

d) Connect the strain gauge to a separate l/4 bridge configuration. Determine the effect of "squeezing the soft drink can" on the internal pressure. Shake the can for a few minutes before proceeding to the next operation.

e) When we are sure that each gauge is active and properly balanced, release the pressure in the can without spraying the equipment.

f) Record the strain measurement results during the test. Further, measure and record the thickness, diameter, length, size and the like.

g) Using the strain measurements of part (d), estimate the pressure in the soft drink can before it was opened.

3.4.2 Results

Table 9. Can Dimensions

Parameters	Measurement
Wall Thickness (mm)	0.08
Diameter (mm)	65.97
Height (mm)	99.36

Table 10. Material Properties

Parameters	Value
Young’s Modulus for Al (E, GPa)	69.0
Poisson Ratio (υ)	0.35

Table 11. Strain Gage Measurements

Parameters	Strain (με)		Resistance (ohms)	
	“While squeezing”	“Just after opening”	Nominal	Measured
Axial Strain Gage	3213.28	3044.4.9	350	352
Circumferential Strain Gage	3532.14	2018.53	350	352

Table 12. Class Strain Measurements and Pressure Calculation "Just after opening"

Group Number	ε_A ($\mu\varepsilon$)	ε_C ($\mu\varepsilon$)	Pressure (kPa)
1	133.19	1235.82	244.57
2	60	1556.56	300.86
3	3166.1	340	276.18
4	249.24	1370.01	277.91
5	4030.81	2577.18	380.27
6	866.88	130.06	174.00
Average	1417.7033	1401.8725	322.49
Standard Deviation	1583.6279	834.5602	77.5233

$$\varepsilon_C = 1556.56\mu\varepsilon \quad \varepsilon_A = 60\ \mu\varepsilon$$

$$\sigma = \frac{E}{1-\nu^2}(\varepsilon_C + \nu\varepsilon_A) = \frac{pr}{t}$$

$$p = \frac{\sigma t}{r} = \frac{Et}{r(1-\nu^2)}(\varepsilon_C + \nu\varepsilon_A)$$

$$P = \frac{69GPa \times 0.08mm}{33mm\left(1 - 0.35^2\right)}(1556.56\mu\varepsilon + 0.35 \times 185.7\mu\varepsilon) = 300.72kPa$$

<u>3.4.3 Analysis & Discussion</u>

In this part of experiment, the experimenter calculated both axial and circumferential strains decreased after opening the can. Since this is a biaxial load, they use the stress-strain relationship to represent the biaxial stress. This calculation gives a pressure of 300.72 kPa after opening the can. By observing the data in Table14, the experimenter observed that the value for ε_A they measured is relatively smaller than other groups, which shows that there must be some error in this process. In the welding process, the error of strain gauge position and operation irregularities will lead to data deviation. Reading errors should also be taken into account.

4. Answers/Solutions to Questions

Wheatstone Bridge Experiment

i) Was the resistance easier to measure using the bridge circuit than it was using the Fluke in the familiarization exercise? Why or why not?

Answer: Yes, of course. Since the change of resistance is usually extremely small, the DMM cannot measure its resistance values. But it can be easily measured by using Wheatstone bridge because it can translate the tiny change of resistance into recognized change of voltage.

ii) How well does your estimate compare to the printed precision value stamped on the precision resistor and the load resistor?

Answer: The estimate value is a little lower than the stamped value, whose errors are 23.68% and 29.76%. The reason for the large conjecture error may be the instability of the readings.

iii) How significant was the lead wire resistance compared to the resistance of the precision resistor and the load resistor?

Answer: The lead wire is also important since it is used to balance the voltage. However, on the other side, the lead resistance will also bring errors to the final results. The resistors of the bridge will be affected. Hence, students need to use it appropriately.

Cantilever Beam Experiment

i) Can you use the strain measurements from gage 1 and gage 2 in the cantilever beam experiment (Part 2.4) to calculate Poisson's ratio for each value of the load F?

Answer:

Yes, of course. $\frac{U_0}{U} = \frac{\Delta R}{R}$, $\varepsilon = \frac{\sigma}{E}$

$$\frac{\frac{\Delta R}{R}}{\boldsymbol{\varepsilon}} = 1 + \nu$$

Poisson's ratio: $\nu = \frac{\frac{\Delta R}{R}}{\mathbf{2\varepsilon}} - \frac{1}{2} = \frac{\frac{U_0}{U}}{\mathbf{2\varepsilon}} - \frac{1}{2}$

ii) Calculate the stress and strain on the top and bottom of the beam at the strain gage locations.

Answer:

The stress and strain on the top of the beam at the strain gage locations:

$$y = \frac{h}{2}$$

$$Stress = \sigma = \frac{My}{I} = \frac{M\frac{h}{2}}{\frac{bh^3}{12}} = \frac{6M}{bh^2} = \frac{6mgx}{bh^2} = \frac{6 \times 283mm \times 1000g \times 9.81\frac{m}{s^2}}{33mm \times (6mm)^2} = 14.02MPa$$

$$Strain = \varepsilon = \frac{\sigma}{E} = \frac{14.02\ MPa}{200GPa} = 70.1\ \mu\varepsilon$$

The stress and strain on the bottom of the beam at the strain gage locations:

$$\mathrm{y} = -\frac{h}{2}$$

$$Stress = \sigma = \frac{My}{I} = -\frac{M\frac{h}{2}}{\frac{bh^3}{12}} = -\frac{6M}{bh^2} = -\frac{6mgx}{bh^2} = -\frac{6 \times 283mm \times 1000g \times 9.81\frac{m}{s^2}}{30mm \times (6mm)^2} = -14.02MPa$$

$$Strain = \varepsilon = \frac{\sigma}{E} = \frac{-14.02\ MPa}{200GPa} = -70.1\ \mu\varepsilon$$

iii) Compare these results with strain readings from 1/4, 1/2, and full bridges. Explain the true strain indicated by each bridge. Plot both the theoretical and experimental results on the same graph (stress vs. load).

Answer:

For Weight=1000g:

Calculated Stress = 14 MPa ≈ 14.728 MPa = Measured Stress of Quarter Bridge

Calculated Stress = 14 MPa ≈ 17.795 MPa =$\frac{1}{2}$ × Measured Stress of Half Bridge

Calculated Stress = 14 MPa ≈ 14.513 MPa =$\frac{1}{4}$ × Measured Stress of Half Bridge

Calculated Strain = 70 με ≈ 73.64 με = Measured Strain of Quarter Bridge

Calculated Strain = 70 με ≈ 73.975 με =$\frac{1}{2}$ × Measured Strain of Half Bridge

Calculated Strain = 70 με ≈ 72.565 με =$\frac{1}{4}$ × Measured Strain of Half Bridge

When weight=2000g, 3000g, 4000g, it follows the same rule.

For Full Bridge:

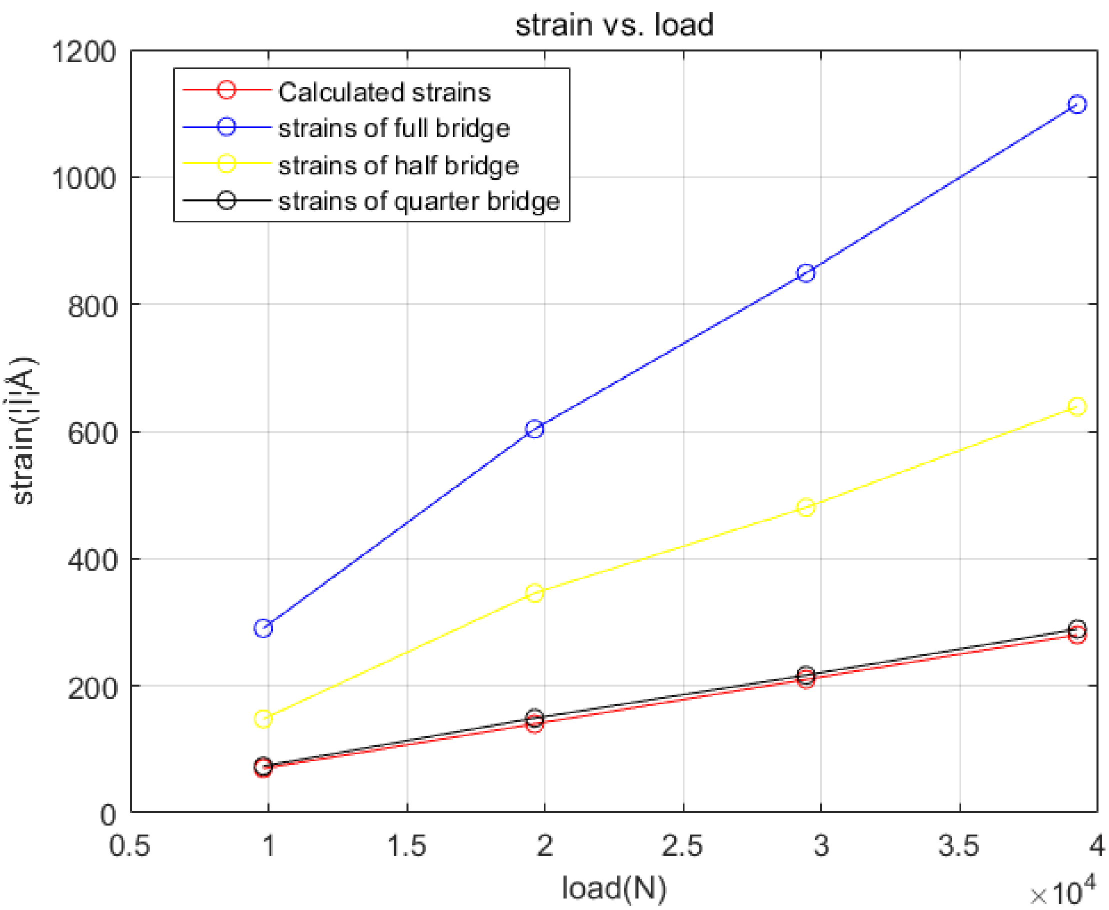
Measured Stress of Weight=2000g: 28 MPa ≈30.1785 MPa=$\frac{1}{4}$ × Measured Stress of Full Bridge of Weight=2000g

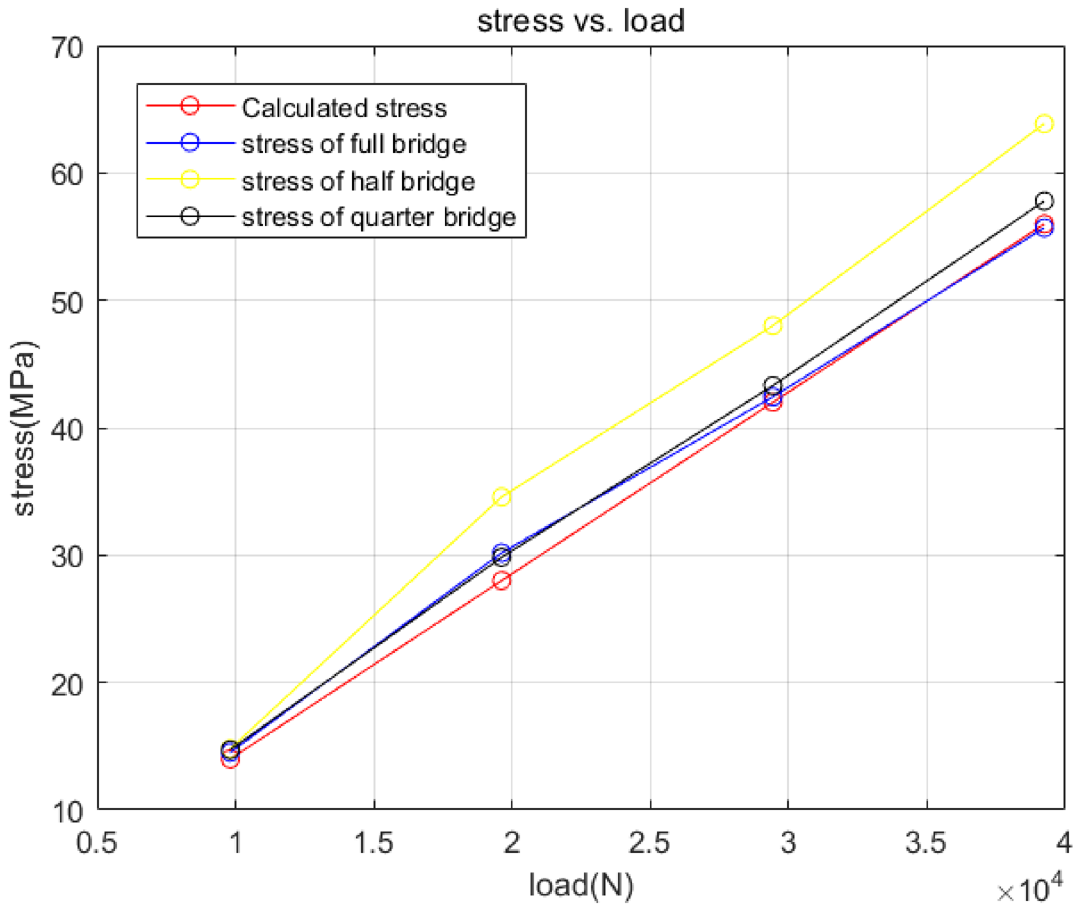
Measured Stress of Weight=3000g: 42 MPa ≈42.4345 MPa=$\frac{1}{4}$ × Measured Stress of Full Bridge of Weight=3000g

Measured Stress of Weight=4000g: 56 MPa ≈55.6925 MPa=$\frac{1}{4}$ × Measured Stress of Full Bridge of Weight=4000g

When for the Stress and strain of Half Bridge, Quarter Bridge, and Calculated Values, it follows the same rule.

By comparing the experimental data to the theoretical prediction of the strains, it is can be seen that the stress and strain increase correspondingly as the applied weight increases proportionally and the stress and strain of half bridge is twice that of quarter bridge. Also, the stress and strain of full bridge is four times that of quarter bridge.





Pressure Experiment

i) Refer to the soft drink can experiment. Use the strain data from each lab group in your section to determine the pressure in each soft drink can before it was opened. Calculate the average pressure Pavg and the standard deviation of the pressure Pstd measurements taken by your lab section.

Answer: Refer to the soft drink can experiment. Use the strain data from each lab group in your section to determine the pressure in each soft drink can before it was opened. Calculate the average pressure $\boldsymbol{P_{avg}}$ and the standard deviation of the pressure $\boldsymbol{P_{std}}$ measurements taken by your lab section.

$$P_{avg} = \frac{P_1 + P_2 + P_3 + P_4 + P_5 + P_6}{6} = 322.49\ kPa$$

$$P_{std} = \sqrt{\frac{1}{6}\sum_{i=1}^{6}(P_i - P_{avg})^2} = 77.5233\ kPa$$

ii) When you squeezed the soft drink can, did the pressure increase significantly? Explain.

Answer: When you squeezed the soft drink can, did the pressure increase significantly? Explain. There is a rise in pressure, but not significantly.

The rise will be reduced when the volume of the tank is tightened and the gases and liquids in the tank will not change. When we squeeze a can of soft drink, one of the strains decreases while the other increases. The change in tension is not obvious, as tightening the tin to some extent requires a great deal of effort. It's not easy to do. Since the experimenters will use these two voltages to calculate the pressure, the pressure will not increase significantly. That is, the volume of the tank will shrink and the gases and liquids in the tank will not change.

iii) Calculate the ratio of axial to circumferential strain in the can. Is this result consistent with the model of a long thin-walled cylinder? Comment on any inconsistencies.

Answer: Calculate the ratio of axial to circumferential strain in the can. Is this result consistent with the model of a long thin-walled cylinder? Comment on any inconsistencies.

$$\text{ratio} = \frac{\varepsilon_a}{\varepsilon_c} = \frac{0.00006}{0.00155656} = 0.038548$$

Compared with theoretical $\nu = 0.35$, the result doesn't consist with the long thin-walled cylinder very well. The error is relatively huge.

The error can be introduced by the fact that the axial and circumferential strain gauges are not tightly fitted. Or the instability of the circuit and the values on screen.

iv) Calculate the ratio of principal stresses in the can. Compare this ratio to that for a long thin walled cylinder. Comment on any inconsistencies

Answer: Calculate the ratio of principal stresses in the can. Compare this ratio to that for a long thin walled cylinder. Comment on any inconsistencies.

$$\sigma_a = \frac{E}{1-\nu^2}(\varepsilon_a + \nu\varepsilon_t) = \frac{69 \times 10^9}{1-0.35^2}(0.00006 + 0.35 \times 0.00155656) = 47.805 \text{ MPa}$$

$$\sigma_c = \frac{E}{1-\nu^2}(\varepsilon_t + \nu\varepsilon_a) = \frac{69 \times 10^9}{1-0.35^2}(0.00155656 + 0.35 \times 0.00006) = 138.910 \text{ MPa}$$

$$\text{stress ratio} = \frac{\text{Axial Stress}}{\text{Circumferential Stress}} = \frac{58.661\text{MPa}}{130.993\text{MPa}} = 0.4478$$

Compared with theoretical $\nu = 0.35$, the result doesn't consist with the long thin-walled cylinder very well, which is bigger than 0.35. The error can also be introduced by the fact that the axial and circumferential strain gauges are not tightly fitted. Or the instability of the circuit and the values on screen.

5. Conclusions

Having performed the experiment, and after a thorough analysis of the data, the following points are therefore concluded:

- The first two parts of the experiment analyzed the bridge circuit, measured the resistance, and applied the bridge circuit. First, it can be inferred that the relationship between the output voltage and the resistance change in the bridge circuit is significant. Next, the output voltage is measured by changing the resistance in R3. At the same time, two theoretical results are obtained by means of differential equation and matrix. From the point of error, the differential equation has smaller error and is more reliable.
- In part 3, the stress and strain increase correspondingly as the applied weight increases proportionally and the stress and strain of half bridge is twice that of quarter bridge. Also, the stress and strain of full bridge is four times that of quarter bridge.
- In part 4, the experimenter calculated both axial and circumferential strains decreased after opening the can. Since this is a biaxial load, they use the stress-strain relationship to represent the biaxial stress. This calculation gives a pressure of 436.64 kPa after opening the can. By observing the data in Table14, the experimenter observed that the value for ε_A they measured is relatively smaller than other groups, which shows that there must be some error in this process. In the welding process, the error of strain gauge position and operation irregularities will lead to data deviation. Reading errors should also be taken into account.

APPENDICES

A – EQUIPMENT LIST

Table 15. Part 1 Equipment List

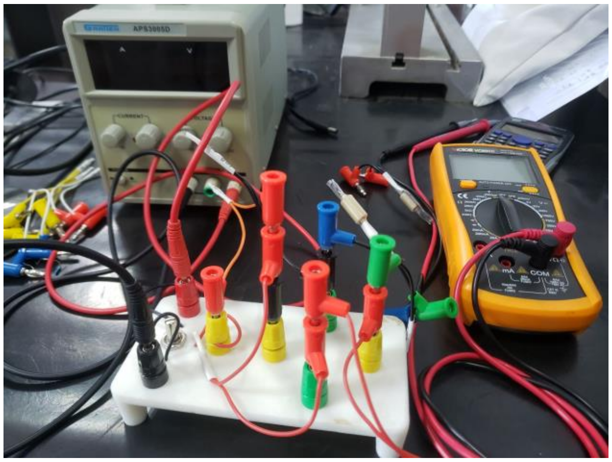
Equipment Description	Model Number	Serial Number
VICTOR Digital Multimeter	VC890D	
GRATTEN Power Source	APS3005D	

Table 16. Part 2 & 3 Equipment List

Equipment Description	Model Number	Serial Number
Digital Strain Indicator	MIK-LCSI	
Switch & Balance Unit		

Table 17. Part 4 Equipment List

Equipment Description	Model Number	Serial Number
VICTOR Digital Multimeter	VC890D	
Digital Strain Indicator	MIK-LCS1	
Switch & Balance Unit		

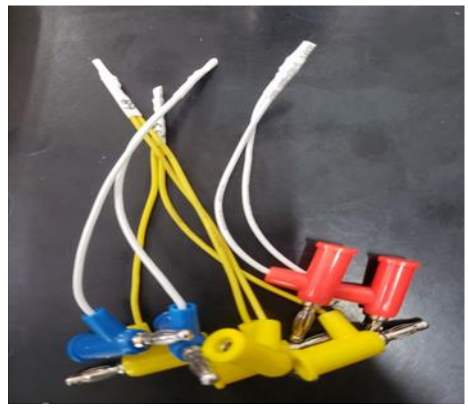


GRATTEN Power Source & Wheatstone Bridge Panel & Resistors

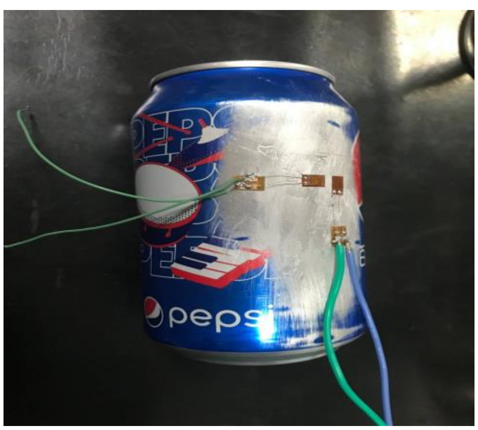
VICTOR Digital Multimeter

Cantilever Beam

Bridge circuit



Resistor



The Soda can

The weight

B – Lab Notes

Group Number		Names		Date	
Expt. Number	2	Expt. Title	Bridge Circuits and Strain Gages		

Part 1: a)

Table 1. Actual Resistance of Bridge Resistors

Bridge Resistors	Resistance (Ω)
R_1^0	98.5
R_2^0	98.7
R_3^0	98.4
R_4^0	98.5

Derivation: $V_O = \left[\frac{R_3}{R_2+R_3} - \frac{R_4}{R_1+R_4}\right]V_I$

$$V_0 = \left[\frac{98.5}{98.5+98.2} - \frac{98.5}{98.5+98.5}\right] \cdot 2.05$$

$$= 1.36\,mV$$

$$\frac{R_3^0}{R_2^0} = \frac{R_4^0}{R_1^0}$$

$V_{IN(MEAS)}$ = 2.05

From the circuit, $V_{O(MEAS)}$ = 1.3mV

Comments: ______

By computation, $V_{O(COMP)\text{-}NOM}$ = 0 and $V_{O(COMP)\text{-}ACT}$ = 1.56mV

Comments: ______

b)

Derivation: $V_O = \left[\frac{R_3}{R_2+R_3} - \frac{R_4}{R_1+R_4}\right]V_I$

$$= \left(\frac{R_3+\Delta R_3}{R_2+R_3+\Delta R_3} - \frac{R_3}{R_1+R_2}\right]V_I = \left[\frac{R_2\Delta R_3}{(R_2+R_3+\Delta R_3)(R_2+R_3)}\right] \cdot V_I$$

$$V_0 = \left[\frac{R_2^0\Delta R_3}{(R_2^0+R_3^0+\Delta R_3)(R_2^0+R_3^0)}\right]V_I$$

c)

Table 2. Potentiometer (R_{b1}) & R_{b2} Setting after Balancing

	Resistance (Ω) – Circuit	Resistance (Ω) –Matlab
R_5 (R_{b1} across terminals 1 and 2)	5557	5565
R_6 (R_{b1} across terminals 2 and 3)	4460	4475
Total POT Swing, R_{b1} (1-3, red-black)	10040	
R_7 (R_{b2})	983Ω	

Table 3. Output Voltage with Changing R_3

Trial	Test R_3 (Ω)	Original R_3^0 (Ω)	ΔR_3 (Ω) (Test-Original)	Output Voltage, V_0 (mV)		
				Circuit	Equation	Matlab
1	33.4	98.4	-65	-549	-493.80	-492.41
2	47.0	98.4	-51.4	-360	-331.32	-335.02
3	66.8	98.4	-31.6	-194	-191.23	-191.4
4	146.5	98.4	48.1	201	194.6	192.7

TA initials: ______ Date: ______

Lab Notes for Experiment 2 – **Bridge Circuits and Strain Gages** (revised by ______) Page 1

Part 2: a)

Table 4. Actual Resistance of Bridge Resistors

Bridge Resistors	Resistance (Ω)
R_1^0	98.5
R_2^0	98.7
R_3^0	98.4
R_4^0	98.5

b)

Table 5. Potentiometer (R_{b1}) & R_{b2} Setting after Balancing

	Resistance (Ω)
R_5 (R_{b1} across terminals 1 and 2)	5557
R_6 (R_{b1} across terminals 2 and 3)	4460
Total POT Swing, R_{b1} (1 and 3)	10040
R_7 (R_{b2})	983

c)

Table 6. Output Voltage with Change in R_2

Test case	Excitation Voltage, V_{ex} (mV)	Output Voltage, V_O (mV)	ΔR_2 (Ω) (from eq'n)	Stamped Value (Ω)
~~test leads shorted~~				
precision resistor 1.1Ω	2000	5	0.987	1Ω
load resistor 0.6Ω	2000	6.7	0.534	0.5Ω

$$\frac{\Delta R_2}{R_2} = \frac{\frac{R_3}{R_2}\left(\frac{\Delta V_{out}}{V_{ex}} + \frac{R_2}{R_2 + R_3}\right)}{\left(1 - \frac{\Delta V_{out}}{V_{ex}} - \frac{R_2}{R_2 + R_3}\right)} - 1$$

Part 4:

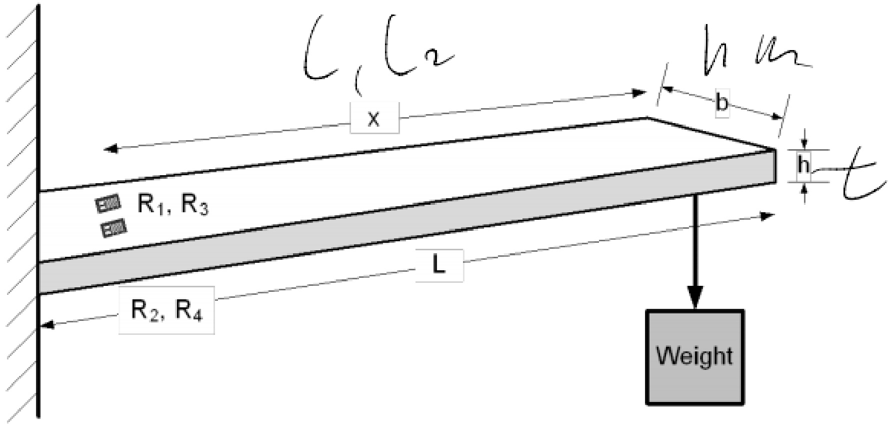


Figure 2. Cantilever Diagram

Table 9. Cantilever Dimensions

Parameter	Measurement (in)
b	33.0 mm
h	6 mm
x	283 mm
L	615 mm

Young's Modulus for Steel (E, in 10^6 psi)	200 GPa

Table 10. Data on Strain Gage Application: Cantilever Beam

Weight (lbs) kg	Moment (lbs*in)	Calculated Values		Measured Values					
				Full Bridge		Half Bridge		Quarter Bridge	
		Stress (psi)	Strain (με)	Stress (psi) Pa	Strain (με)	Stress (psi) Pa	Strain (με)	Stress (psi) Pa	Strain (με)
~~1.406~~ 1	2.776~3	1.4×10^{7}	70	1.4513×10^{7}	280.26	1.4185×10^{7}	141.85	1.4728×10^{7}	73.64
~~3.073~~ 2	5.55246	2.8×10^{7}	140	3.1785×10^{7}	603.57	3.4572×10^{7}	345.72	2.8828×10^{7}	148.14
~~4.739~~ 3	8.32869	4.2×10^{7}	210	4.24145×10^{7}	848.68	4.8021×10^{7}	480.21	4.3312×10^{7}	216.56
~~6.406~~ 4	11.1492	5.6×10^{7}	280	5.5695×10^{7}	1113.85	6.3872×10^{7}	638.72	5.7796×10^{7}	288.98

Origin 红 3575.1 Squeeze 3532.14 打开 2018.53
白 3104.4 3213.28 3044.4

Part 5:

白色纵向
绿色横向.

Table 11. Can Dimensions

Parameters	Measurement
Wall Thickness (mm)	0.08
Diameter (mm)	65.97
Height (mm)	99.36

$\frac{1}{4}\pi(65.97\times10^{-3})^2 \cdot 69\times10^{9} \cdot 60\times10^{-6}$

60

Table 12. Material Properties

Parameters	Value
Young's Modulus for Al (E, GPa)	69.0
Poisson Ratio (υ)	0.35

Table 13. Strain Gage Measurements

	Strain (με)		Resistance (ohms)	
	"While squeezing"	"Just after opening"	Nominal	Measured
Axial Strain Gage	3213.28	3044.4	350	352
Circumferential Strain Gage	3532.14	2018.53	350	352

Table 14. Class Strain Measurements and Pressure Calculation "Just after opening"

Group Number	ε_A (με)	ε_C (με)	Pressure (kPa)
1	133.18	1235.82	244.57
2	60	1556.56	300.72
3	3166.1	349	309.22
4	248.24	1372.01	277.91
5	4035.81	2517.18	380.27
6	866.88	130.06	174.00
AVERAGE	1417.69	1201.605	281.14

Sample Computations:

TA initials: ________ Date: ________

Lab Notes for Experiment 2 – **Bridge Circuits and Strain Gages** (revised by ________) Page 5

C – Matlab Code

```
%% Part1
% Table 1
clear;clc
R1=98.5;R2=98.7;R3=98.4;R4=98.5;
R32=R3/R2;
R41=R4/R1;
v0=((R32/(1+R32))-(R41/(1+R41)))*2.05*1000

% Table 2
close all; clc; clear;
s = 0;
R = 500:250:1500;
Ro = 1e6;
R1 = 98.5;
R2 = 98.7;
R3 = 98.4;
R4 = 98.5;
Rc = 1e9;
R2c = R2*Rc/(R2+Rc);
Vo = zeros(5,10041);
for k = 500:250:1500
    s = s+1;
    R7 = k;
    t = 0;
    for R5 = 0:1:10040
        R6 = 10070 - R5;
        Vi = 2.01;
        A = [R5+R6, -R5, -R6, 0, 0;-R5, R1+R5+R7, -R7, -R1, 0; -R6, -R7, R4+R6+R7, 0, -
R4; 0, -R1, 0, R1+R2c+Ro, -Ro; 0, 0, -R4, -Ro, R3+R4+Ro];
        b=[2.01; 0; 0; 0; 0];
        t = t + 1;
        i = inv(A)*b;
        V0(t)=(i(4)-i(5))*Ro;
    end
    Vo(s,:) = V0;
end
R5 = 0:1:10040;
plot(R5,Vo(1,:),'--',R5,Vo(2,:),'-',R5,Vo(3,:),':.',R5,Vo(4,:),'-.',R5,Vo(5,:),':')
title('The output volatge with Different R7');
xlabel('R5 [ohm]');
ylabel('Voutput [mV]');
```

```
xlim([0 11000]);
grid;
legend('R7 = 500','R7 = 750','R7 = 1000','R7 = 1250','R7 = 1500');

% Table 3
% Code of Table 3 Equation
clear;clc;format long;
% Basic Data
R2=98.7;R3=98.5;R=[33.4 47.0 66.8 146.5];
% Calculate V0
for i=1:4
    V(i)=((R2*(R(i)-R3))/((R2+R3+(R(i)-R3))*(R2+R3)))*2;
end
disp(V);

% Code of Table 3 Matlab
clear;clc;format long;
R1=98.5;R2=98.7;R3=[33.4 47 66.8 146.5];R4=98.5;R5=5557;R6=4460;R7=985;
Rc=1000000000;R0=100000; R2c=(R2*Rc)/(R2+Rc);
for i=1:4; A=[R5+R6 -R5 -R6 0 0;-R5 R1+R5+R7 -R7 -R1 0;-R6 -R7 R4+R6+R7,...
        0 -R4;0 -R1 0 R1+R2c+R0 -R0;0 0 -R4 -R0 R3(i)+R4+R0];
    b=[2;0;0;0;0];
    I=inv(A)*b;
    V0(i)=(I(4)-I(5))*R0;
end
disp(V0);
```

```
%% part2
% shiyan2-1/2
close all; clear; clc;
R = 0:1:170;
for i = 1:length(R)
    t=i;

    Ro = 1e6;
    R1 = 98.5;
    R2 = 98.7;
    R3 = 98.4;
    R4 = 98.5;
    Rc = 1e9;
    R2c = R2*Rc/(R2+Rc);
    R3 = R(i);
    R(1,i) = R3;
```

```
    R5 = 5557;
    R6 = 4460;
    R7 = 983;
    A = [R5+R6, -R5, -R6, 0, 0; -R5, R1+R5+R7, -R7, -R1, 0; -R6, -R7, R4+R6+R7, 0, -
R4; 0, -R1, 0, R1+R2c+Ro, -Ro; 0, 0, -R4, -Ro, R3+R4+Ro];
    b=[2.01;0;0;0;0];
    i=inv(A)*b;
    Vo=(i(4)-i(5))*Ro;
    V(1,t) = Vo;
    plot(R3,Vo,'.'); grid
    title('Output Voltage vs R3');
    xlabel('R3 [ohm]');
    ylabel('Voutput [mV]');
    hold on;
end
```

```
%% part3
load=[1000*9.81 2000*9.81 3000*9.81 4000*9.81];
strainC=[70 140 210 280];
strainf=[290.26 603.57 848.69 1113.85];
strainh=[147.95 345.72 480.21 638.72];
strainq=[73.64 149.14 216.56 288.98];
figure(1)
plot(load,strainC,'r-o')
hold on
plot(load,strainf,'b-o')
hold on
plot(load,strainh,'y-o')
hold on
plot(load,strainq,'k-o')
legend('Calculated strains','strains of full bridge','strains of half bridge','strains of quarter
bridge','Location','Best');
xlabel('load(N)');
ylabel('strain(????)');
title('strain vs. load');
grid on;
load=[1000*9.81 2000*9.81 3000*9.81 4000*9.81];
stressC=[14 28 42 56];
stressf=[14.513 30.1785 42.4345 55.6925];
stressh=[14.795 34.572 48.021 63.872];
stressq=[14.728 29.828 43.312 57.796];
```

```
figure(2)
plot(load,stressC,'r-o')
hold on
plot(load,stressf,'b-o')
hold on
plot(load,stressh,'y-o')
hold on
plot(load,stressq,'k-o')
legend('Calculated stress','stress of full bridge','stress of half bridge','stress of quarter
bridge','Location','Best');
xlabel('load(N)');
ylabel('stress(MPa)');
title('stress vs. load');
grid on;
```

```
%% part4
clear; close all; clc
%a = rand(100,1)
Ea = [133.19 60 3166.1 249.24 4030.81 866.88]
Ec = [1235.82 1556.56 340 1370.01 2577.18 130.06]
P = [244.57 300.86 309.22 277.91 380.27 174.00]
% calculate standard deviation --- sd
sd_Ea = std(Ea)
sd_Ec = std(Ec)
sd_P = std(P)
```