Chapter 9

PROBLEM 9.1

Water at atmospheric pressure is boiling in a pot with a flat copper bottom on an electric range which maintains the surface temperature at 115°C. Calculate the boiling heat transfer coefficient.

GIVEN

- Water at atmospheric pressure boiling in a copper bottom pot
- Surface temperature of the pot bottom (T_s) = 115°C

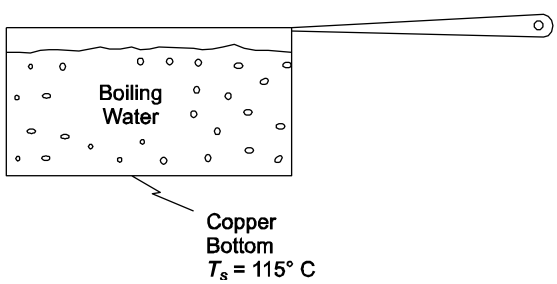
FIND

- The boiling heat transfer coefficient (h_b)

ASSUMPTIONS

- Temperature of the pan bottom is uniform
- The copper is polished

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 1 atm (T_{sat} = 100°C)

Density (ρ_l) = 958.4 kg/m^3

Specific heat (c_l) = 4211 J/(kg K)

Absolute viscosity (μ_l) = 277.5 × 10^{-6} (Ns)/m^2

Prandtl number (Pr_l) = 1.75

Heat of vaporization (h_{fg}) = 2257 kJ/kg = 2.257 × 10^6 J/kg

From Appendix 2, Table 35, the density of steam at 100° (ρ_v) = 0.5977 kg/m^3

From Table 9.2, Surface tension at 100°C (σ) = 0.0589 N/m

From Table 9.1, The coefficient, C_{sf}, for water on emery polished copper = 0.0128

SOLUTION

Assuming the boiling is nucleate boiling, the heat flux, q'', is given by Equation (9.2)

$$\frac{c_l\,\Delta T_x}{h_{fg}\,Pr_l^{\,n}} = C_{sf}\left[\frac{q''}{\mu_l\,h_{fg}}\sqrt{\frac{g_c\,\sigma}{g\,(\rho_l - \rho_v)}}\right]^{0.33}$$

where $\Delta T_x = T_s - T_{sat}$ = 115°C – 100°C = 15°C

g_c = 1.0 (in the SI system)

n = 1.0 for water

g = 9.8 m/s^2

Rearranging

$$q'' = \left(\frac{c_l\, \Delta T_x}{h_{fg} Pr_l^{\,n}\, C_{sf}}\right)^3 \frac{\mu_l\, h_{fg}}{\sqrt{\dfrac{g_c\, \sigma}{g\,(\rho_l - \rho_v)}}}$$

$$q'' = \left(\frac{4211\,\text{J/(kg K)}\ \ (15°\text{C})}{2.257\times 10^6\,\text{J/kg}\ \ (1.75)\,(0.0128)}\right)^3$$

$$\frac{277.5\times 10^{-6}\,(\text{N s})/\text{m}^2\ \ (\text{kg m})/(\text{s}^2\text{N})\ \ 2.257\times 10^6\,\text{J/kg}\ \ (\text{W s})/\text{J}}{\sqrt{\dfrac{0.0589\,\text{N/m}\ \ (\text{kg m})/(\text{s}^2\text{N})}{9.8\,\text{m/s}^2\ \ (958.4-0.5977)\,\text{kg/m}^3}}}$$

$$= 4.87\times 10^5\ \text{W/m}^2$$

The critical heat flux for nucleate boiling is given by Equation (9.4)

$$q''_c = \frac{\pi}{24}\ \rho_v^{0.5}\, h_{fg}\ [\sigma\, g\,(\rho_l - \rho_v)\, g_c]^{\frac{1}{4}}$$

$$q''_c = \frac{\pi}{24}\ \ 0.5977\,\text{kg/m}^3\ ^{0.5}\ \ 2.257\times 10^6\,\text{J/kg}\ \ (\text{Ws})/\text{J}$$

$$0.0589\,\text{J/kg}\ \ (\text{kg m})/(\text{s}^2\text{N})\ \ (9.8\,\text{m/s}^2)\,(958.4-0.5977)\text{kg/m}^3\ ^{\frac{1}{4}}$$

$$q''_c = 1.11\times 10^6\ \text{W/m}^2$$

Since $q'' < q''_c$, the nucleate boiling assumption is valid.

By definition

$$h_b = \frac{q''}{\Delta T_x} = \frac{4.87\times 10^5\,\text{W/m}^2}{15\,\text{K}} = 3.25\times 10^4\ \text{W/(m}^2\text{K)}$$

PROBLEM 9.2

Predict the nucleate-boiling heat transfer coefficient for water boiling at atmospheric pressure on the outside surface of a 1.5-cm-*OD* vertical copper tube 1.5-m-long. Assume the tube-surface temperature is constant at 10 K above the saturation temperature.

GIVEN

- Water boiling at atmospheric pressure on the outside surface of vertical copper tube
- Tube outside diameter (D) = 1.5 cm = 0.015 m
- Tube length (L) = 1.5 m
- Tube surface temperature above saturation temperature (ΔT_x) = 10 K

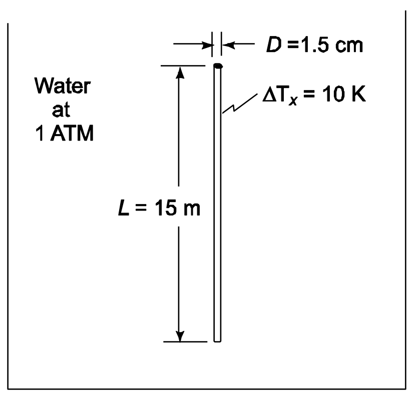
FIND

- The nucleate-boiling heat transfer coefficient (h_b)

ASSUMPTIONS

- The water is at saturation temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 1 atm (T_{sat} = 100°C)

Density (ρ_l) = 958.4 kg/m^3

Specific heat (c_l) = 2411 J/(kg K)

Absolute viscosity (μ_l) = 277.5 × 10^{-6} (Ns)/m^2

Prandtl number (Pr_l) = 1.75

Heat of vaporization (h_{fg}) = 2257 kJ/kg = 2.257 × 10^6 J/kg

From Appendix 2, Table 35, the density of steam at 100°C (ρ_v) = 0.5977 kg/m^3

From Table 9.2, Surface tension at 100°C (σ) = 0.0589 N/m

From Table 9.1, The coefficient, C_{sf}, for water on copper = 0.0130

SOLUTION

As stated near the end of Section 9.2.2, 'the geometric shape of the heating surface has no appreciable effect on the nucleate boiling mechanism'.

Assuming the boiling is nucleate boiling, the heat flux q'', is given by Equation (9.2)

$$\frac{c_l\,\Delta T_x}{h_{fg}\,Pr_l^{\,n}} = C_{sf}\left[\frac{q''}{\mu_l\,h_{fg}}\sqrt{\frac{g_c\,\sigma}{g\,(\rho_l-\rho_v)}}\right]^{0.33}$$

where $\Delta T_x = T_s - T_{sat} = 10$ K

$g_c = 1.0$ (in the SI system)

$n = 1.0$ for water

$g = 9.8$ m/s^2

Rearranging

$$q'' = \left(\frac{c_l\,\Delta T_x}{h_{fg}\,Pr_l^{\,n}\,C_{sf}}\right)^3 \frac{\mu_l\,h_{fg}}{\sqrt{\dfrac{g_c\,\sigma}{g\,(\rho_l-\rho_v)}}}$$

$$q'' = \left(\frac{4211\,\text{J/(kg K)}\;(10\,°\text{C})}{2.257\times10^6\,\text{J/kg}\;(1.75)\,(0.0130)}\right)^3$$

$$\times\frac{277.5\times10^{-6}\,(\text{N s})/\text{m}^2\quad(\text{kg m})/(\text{s}^2\text{N})\quad 2.257\times10^6\,\text{J/kg}\quad(\text{W s})/\text{J}}{\sqrt{\dfrac{0.0589\,\text{N/m}\;(\text{kg m})/(\text{s}^2\text{N})}{9.8\,\text{m/s}^2\;(958.4-0.5977)\,\text{kg/m}^3}}}$$

$$= 1.37\times10^5\ \text{W/m}^2$$

The critical heat flux for nucleate boiling is given by Equation (9.4)

$$q''_c = \left(\frac{\pi}{24}\right)\rho_v^{0.5}\,h_{fg}\,[\sigma\,g\,(\rho_l-\rho_v)\,g_c]^{\frac{1}{4}}$$

$$q''_c = \left(\frac{\pi}{24}\right)\;0.5977\,\text{kg/m}^3\;{}^{0.5}\;\;2.257\times10^6\,\text{J/kg}\quad(\text{Ws})/\text{J}$$

$$0.0589\,\text{J/kg}\quad(\text{kg m})/(\text{s}^2\text{N})\quad 9.8\,\text{m/s}^2\quad(958.4-0.5977)\,\text{kg/m}^3\;\;\frac{1}{4}$$

$$q''_c = 1.11\times10^6\ \text{W/m}^2$$

Since $q'' < q''_c$, the nucleate boiling assumption is valid.

By definition

$$h_b = \frac{q''}{\Delta T_x} = \frac{1.37\times10^5\,\text{W/m}^2}{10\,\text{K}} = 1.37\times10^4\ \text{W/(m}^2\text{K)}$$

PROBLEM 9.3

Estimate the maximum heat flux obtainable with nucleate pool boiling on a clean surface for (a) water at 100 kPa on brass, (b) water at 1 MPa on brass.

GIVEN

- Nucleate pool boiling on a clean surface

FIND

- The maximum heat flux obtainable for (a) water at 1 atm on brass; and (b) water at 10 atm on brass

ASSUMPTIONS

- Water is at saturation temperature

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water from Table 9.2 for surface tension

Pressure (MPa)	0.1	1
Saturation Temperature (°C)	100	180.4
Liquid density, ρ_l (kg/m^3)	958.4	886.1
Heat of Vaporization, h_{fg} (J/kg)	2.257×10^6	2.013×10^6
Vapor density, $\rho_v = 1/v_g$ (kg/m^3)	0.5977	5.22
Surface tension, σ (N/m)	0.0589	0.0422

SOLUTION

The maximum heat flux for nucleate boiling is given by Equation 9.4

$$q''_c = \left(\frac{\pi}{24}\right) \rho_v^{0.5}\, h_{fg}\, [\sigma\, g\, (\rho_l - \rho_v) g_c]^{\frac{1}{4}}$$

Case (a)

$$q''_c = \left(\frac{\pi}{24}\right) \left(0.5977\,\mathrm{kg/m^3}\right)^{0.5} \left(2.257\times 10^6\,\mathrm{J/kg}\right) \left(\mathrm{(Ws)/J}\right)$$

$$\left[\left(0.0589\,\mathrm{J/kg}\right)\left(\mathrm{(kg\,m)/(s^2N)}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(958.4 - 0.5977\right)\mathrm{kg/m^3}\right]^{\frac{1}{4}}$$

$$q''_c = 1.11 \times 10^6\ \mathrm{W/m^2}$$

Case (b)

$$q''_c = \left(\frac{\pi}{24}\right) \left(5.22\,\mathrm{kg/m^3}\right)^{0.5} \left(2.013\times 10^6\,\mathrm{J/kg}\right) \left(\mathrm{(Ws)/J}\right)$$

$$\left[\left(0.0422\,\mathrm{J/kg}\right)\left(\mathrm{(kg\,m)/(s^2N)}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(886.1 - 5.22\right)\mathrm{kg/m^3}\right]^{\frac{1}{4}}$$

$$q''_c = 2.63 \times 10^6\ \mathrm{W/m^2}$$

PROBLEM 9.4

Determine the excess temperature at one-half of the maximum heat flux for the fluid-surface combinations in Problem 9.3.

GIVEN

- Nucleate pool boiling on a clean surface

FIND

The excess temperature (ΔT_x) at one half of the maximum heat flux for (a) water at 10 kPa on brass (b) water at 1 MPa on brass

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water Table 9.2 for surface tension

Pressure (MPa)	0.1	1
Saturation Temperature (°C)	100	180.4
Liquid density, ρ_l (kg/m^3)	958.4	886.1
Heat of Vaporization, h_{fg} (J/kg)	2.257×10^6	2.013×10^6
Vapor density, $\rho_v = 1/V_g$ (kg/m^3)	0.5977	5.22
Surface tension, σ (N/m)	0.0589	0.0422
Specific heat, c_l (J/kg K)	4211	4398
Absolute viscosity, μ_l (Ns/m^2)	277.5×10^{-6}	151.7×10^{-6}
Prandtl number, Pr	1.75	1.01

From Table 9.1, the coefficient, C_{sf}, for water on brass = 0.0060

SOLUTION

Solving Equation (9.2) for the excess temperature

$$\Delta T_x = \frac{C_{sf}\, h_{fg}\, Pr_l^{\,n}}{c_l} \left[\frac{q''}{\mu_l\, h_{fg}} \sqrt{\frac{g_c\, \sigma}{g\,(\rho_l - \rho_v)}} \right]^{0.33}$$

where $n = 10$ for water.

(a) From the solution to Problem 9.3, $q''_{max} = 1.11 \times 10^6$ W/m^2.
For $q'' = q''_{max}/2 = 5.55 \times 10^5$ W/m^2, and water at 1 atm on brass

$$\Delta T_x = \frac{(0.0060)\ 2.257\times10^6 \text{ J/kg}\ (1.75)}{4211 \text{J/(kg K)}}$$

$$\left[\frac{5.55\times10^5 \text{ W/m}^2\ \ \text{J/(W s)}}{277.5\times10^{-6}\,(\text{N s})/\text{m}^2\ \ 2.257\times10^6 \text{ J/(kg)}\ \ (\text{kg m})/(\text{s}^2\text{N})} \sqrt{\frac{0.0589 \text{ N/m}\ \ (\text{kg m})/(\text{s}^2\text{N})}{9.8 \text{m/s}^2\ \ (958.4 - 0.5977)\text{kg/m}^3}} \right]^{0.33}$$

$$\Delta T_x = 7.3 \text{ K}$$

(b) From the solution to Problem 9.3, $q''_{max} = 2.63 \times 10^6$ W/m^2.
For $q'' = q''_{max}/2 = 1.315 \times 10^6$ W/m^2; and water at 10 atm on brass

$$\Delta T_x = \frac{(0.0060)\ 2.013\times10^6 \text{ J/kg}\ (1.01)}{4398 \text{ J/(kg K)}}$$

$$\left[\frac{1.315\times10^6 \text{ W/m}^2\ \ \text{J/(W s)}}{151.7\times10^{-6}\,(\text{N s})/\text{m}^2\ \ 2.013\times10^6 \text{ J/kg}\ \ (\text{kg m})/(\text{s}^2\text{N})} \sqrt{\frac{0.0422 \text{ N/m}\ \ (\text{kg m})/(\text{s}^2\text{N})}{9.8 \text{m/s}^2\ \ (886.1 - 5.22)\text{kg/m}^3}} \right]^{0.33}$$

$$\Delta T_x = 5.8 \text{ K}$$

PROBLEM 9.5

In thermal management or cooling of microchip modules for high-powered computer systems, very high heat fluxes have to be accommodated in the design of the cooling method. For many such cases, immersion cooling via pool boiling in a dielectric fluid is often employed [94]. In a pool boiling experiment to evaluate the cooling behavior from a simulated chip module, water is boiled at atmospheric pressure over a submerged emery-polished copper-plate heater. If the surface excess temperature is measured to be 14.5 K, what is the heat flux dissipated by boiling from the copper plate?

GIVEN

- Water at atmospheric pressure boiling on emery-polished copper-plate heater
- Surface excess temperature (ΔT_x)= 14.5 K

FIND

(a) Heat flux dissipated by boiling from copper plate.

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the properties of water at 100°C are

Density (ρ_l) = 958.4 kg/m^3
Specific heat (c_l) = 4211 J/(kg K)
Absolute viscosity (μ_l) = 277.5 × 10^{-6} kg/ms
Prandtl number (Pr_l) = 1.75
Heat of vaporization (h_{fg}) = 2.257 × 10^6 J/kg
Vapor density (ρ_v) = 0.598 kg/m^3

Table 9.2 gives

Surface tension (σ) = 58.9 × 10^{-3} N/m

SOLUTION

The coefficient for the emery polished copper-plate surface (C_{sf})= 0.0128
Solving Equation (9.2) for q'' we have

$$C_{sf} = \frac{c_l\,\Delta T_x}{h_{fg}\,Pr_l^{\,n}}\left[\frac{q''}{\mu_l\,h_{fg}}\sqrt{\frac{g_c\,\sigma}{g\,(\rho_l-\rho_v)}}\right]^{-0.33}$$

For water, n = 1 in this equation.
So

$$0.0128 = \frac{4211\,\text{J/(kg K)}\ (14.5\,\text{K})}{2.257\times10^6\ \text{J/kg}\ (1.75)}$$

$$\left[\frac{q''}{\left(277.5\times10^{-6}\ \text{kg/(ms)}\right)\left(2.257\times10^6\ \text{J/kg}\right)}\sqrt{\frac{(0.0589\,\text{N/m})}{\left(9.81\text{m/s}^2\right)(958.4-0.598)\,\text{kg/m}^3}}\right]^{-0.33}$$

$$\left[\frac{q''}{\left(277.5\times10^{-6}\ \text{kg/(ms)}\right)\left(2.257\times10^6\ \text{J/kg}\right)}*2.504*10^{-3}\right]=1.76*10^{-3}$$

q''=434 W/m^2

PROBLEM 9.6

Compare the critical heat flux for a flat horizontal surface with that for a submerged horizontal wire of 3-mm-diameter in water at saturation temperature and pressure.

GIVEN

- Flat, horizontal surface and a submerged, horizontal wire

FIND

(a) Critical heat flux for both geometries

ASSUMPTIONS

- The water is at atmospheric pressure

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13

Liquid density (ρ_l) = 958.4 kg/m^3

Vapor density (ρ_v) = 0.598 kg/m^3

and Table 9.2 gives

Surface tension (σ) = 0.0589 N/m

SOLUTION

From Table 9.3, entry #5, the ratio of critical heat fluxes for two geometries is

$$\frac{q''_{\text{max,wire}}}{q''_{\text{max,Z}}} = 0.94\left(\frac{R}{L_b}\right)^{-\frac{1}{4}}$$

The bubble length scale, L_b is calculated from

$$L_b = \sqrt{\frac{\sigma}{g\,(\rho_l - \rho_v)}} = \sqrt{\frac{0.0589\,\text{N/m}\quad(\text{kg m})/(\text{s}^2\text{N})}{9.81\,\text{m/s}^2\;\;(958.4 - 0.598)\;\;\text{kg/m}^3}} = 0.0025 \text{ m} = 2.5 \text{ mm}$$

Since $\dfrac{R}{L_b} = \dfrac{1.5}{2.5} = 0.6$, we have

$$\frac{q''_{\text{max,wire}}}{q''_{\text{max,Z}}} = 0.94\;(0.6)^{\frac{1}{4}} = 1.068$$

The wire has about 7% higher critical heat flux than the horizontal plate.

PROBLEM 9.7

For saturated pool boiling of water on a horizontal plate, calculate the peak heat flux at pressures of 10, 20, 40, 60, and 80 % of the critical pressure p_c. Plot your results as q''_{max}/p_c versus p/p_c. The surface tension of water may be taken as σ = 0.0743 (1 – 0.0026 T), where σ is in newtons per meter and *T* in degree Centigrade. The critical pressure of water is 22.09 MPa.

GIVEN

- Saturated pool boiling of water on a horizontal plate
- Surface tension of water (σ) = 0.0743 (1 – 0.0026 *T*) (Where σ is in N/m and *T* is in °C)
- The critical pressure (p_c) = 22.09 MPa = 2.209×10^4 kPa

FIND

- The peak heat flux, q''_{max} for pressures of 10, 20, 40, 60, and 80 % of the critical pressure, p_c

ASSUMPTIONS

- Steady state
- The plate is clean

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water

Percent of, p_c	10	20	40	60	80
Pressure, p (kPa)	2209	4418	8836[a]	13,254[a]	17,672[a]
Saturation Temperature T_{sat} (°C)	217	256	302	332	355
Liquid density, ρ_l (kg/m^3)	844.6	789.7	709.7	634.0	550.9
Vapor density, ρ_v (kg/m^3)	11.1	22.3	47.9	80.7	128.5
Heat of vaporization, $h_{fg} \times 10^{-6}$ (J/kg)	1.8635	1.6768	1.3844	1.1093	0.803

(a) data from steam tables

SOLUTION

The peak heat flux is given by Equation 9.4

$$q''_c = \left(\frac{\pi}{24}\right) \rho_v^{0.5} h_{fg} \, [\sigma \, g \, (\rho_l - \rho_v) \, g_c]^{\frac{1}{4}}$$

For $p = 0.1\, p_c$:

$$q''_c = \left(\frac{\pi}{24}\right) \; 11.1\,\text{kg/m}^3 \;^{0.5} \quad 1.8635 \times 10^6 \text{ J/kg} \quad (\text{Ws})/\text{J}$$

$$0.0743[1 - 0.0026(217)]\text{N/m} \quad (\text{kg m})/(\text{s}^2\text{N}) \quad 9.8\,\text{m/s}^2 \quad (844.6 - 11.1\text{kg/m}^3 \;^{\frac{1}{4}}$$

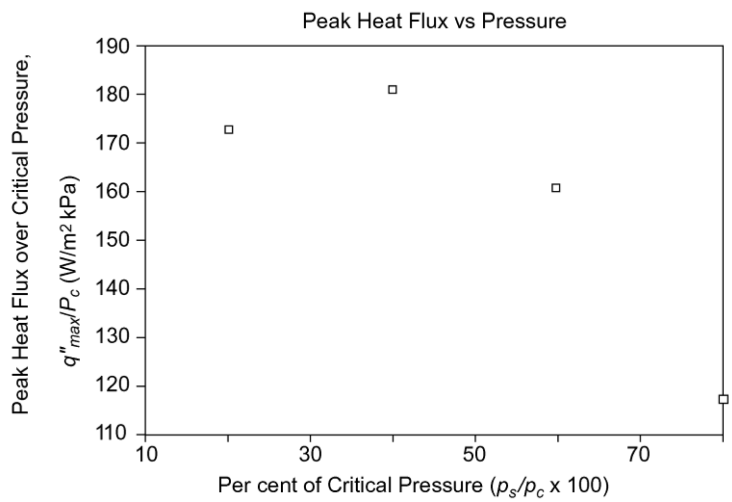
$$q''_c = 3.28 \times 10^6 \text{ W/m}^2$$

$$\frac{q''_{\max}}{P_c} = \frac{3.28 \times 10^6 \text{ W/m}^2}{2.209 \times 10^4 \text{kPa}} = 148 \frac{\text{W/m}^2}{\text{kPa}}$$

Repeating this procedure for the rest of the cases

Per cent of p_c	10	20	40	60	80
$q''_{\max} \times 10^{-6}$ (W/m^2)	3.28	3.83	4.00	3.56	2.63
$q''_{\max}/p_c$ (W/m^2 kPa)	148	173	181	161	119

These results are plotted below



COMMENTS

Note the similarity of the graph to Figure 9.9.

PROBLEM 9.8

For establishing an experimental station that carries out mineral prospecting on the Moon and also houses periodic human visits for space-based explorations, a cooling (or thermal management) system is to be designed for the high-powered electronic systems that are needed to operate the controls. One of the most effective methods for large heat flux dissipation is to use phase change, or boiling, as the primary convection mechanism. For designing one such pressurized system, calculate the maximum heat flux attainable in nucleate boiling with saturated water at 200 kPa pressure. Note that the gravitational field on the Moon surface is 1.622 m/s^2 , which is approximately a little less than one-sixth that on Earth.

GIVEN

- Nucleate boiling with saturated water
- Pressure = 200 kPa
- Gravitational field (g) = 1.622 m/s^2

FIND

- The maximum heat flux (q''_{max})

ASSUMPTIONS

- Steady state conditions
- Nucleate pool boiling

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 200 kPa

Saturation temperature (T_s): 120.5°C

Water density, (ρ_l): 943.5 kg/m^3

Heat of vaporization, (h_{fg}): 2201 kJ/kg

Vapor density, ($\rho_v = 1/v_g$): 1.13 kg/m^3

From Table 9.2, the surface tension (σ) = 0.0547 N/m

SOLUTION

The maximum heat flux is given by Equation (9.4)

$$q''_{max} = \frac{\pi}{24}\ \rho_v^{0.5}\, h_{fg}\ [\sigma\, g\,(\rho_l - \rho_v)\, g_c]^{\frac{1}{4}}$$

$$q''_{max} = \left(\frac{\pi}{24}\right)\ 1.13\,\mathrm{kg/m^3}\ ^{0.5}\ \ 2.201\times 10^6\ \mathrm{J/kg}\quad \mathrm{(Ws)/J}$$

$$\left[(0.0547\ \mathrm{J/kg})\left((\mathrm{kg\,m})/(\mathrm{s^2 N})\right)\left(1.622\ \mathrm{m/s^2}\right)(943.5-1.13)\mathrm{kg/m^3}\right]^{\frac{1}{4}}$$

$$q''_{max} = 9.26\times 10^5\ \mathrm{W/m^2}$$

PROBLEM 9.9

Prepare a graph showing the effect of subcooling between 0 and 50°C on the maximum heat flux calculated in Problem 9.8.

GIVEN

- Nucleate boiling with saturated water
- Pressure = 2 atm
- Gravitational field = 1/10 that of earth
- From Problem 9.8, the maximum heat flux ($q''_{max,sat}$) = 9.26×10^5 W/m^2

FIND

- Prepare a graph showing the effect of sub cooling ($T_{sat} - T_{liquid}$) between 0 and 50°C on q''_{max}

ASSUMPTIONS

- Steady state conditions
- Nucleate pool boiling

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 2 atm (2.0264×10^5 N/m^2)

Saturation temperature (T_s): 120.5°C

Water density, (ρ_l): 943.5 kg/m^3

Heat of vaporization, (h_{fg}): 2201 J/kg

Vapor density, ($\rho_v = 1/v_g$): 1.13 kg/m^3

Thermal conductivity (k_l) = 0.685 W/(m K)

Thermal diffusivity (α_l) = 0.171×10^{-6} m^2/s

From Table 9.2, the surface tension (σ) = 0.0547 N/m

Acceleration due to gravity on earth (g_e) = 9.8 m/s^2

SOLUTION

For subcooling, the effect on the maximum heat flux is given by Equation (9.5)

$$q''_{max} = q''_{max,sat}\left(1+\left[\frac{2k_l(T_{sat}-T_{liquid})}{\sqrt{\pi\alpha_l\,\tau}}\right]\frac{24}{\pi h_{fg}\rho_v}\left[\frac{\rho_v^2}{g_c\,\sigma\,g(\rho_l-\rho_v)}\right]^{\frac{1}{4}}\right)$$

$$\text{where } \tau = \left(\frac{\pi}{3}\right)\sqrt{2\pi}\left[\frac{g_c\sigma}{g(\rho_l-\rho_v)}\right]^{\frac{1}{2}}\left[\frac{\rho_v^2}{g_c\,\sigma\,g(\rho_l-\rho_v)}\right]^{\frac{1}{4}}$$

$$\tau = \left(\frac{\pi}{3}\right)\sqrt{2\pi}\left[\frac{(\text{kg m})/(\text{s}^2\text{N})\;\;0.0547\text{ N/m}}{0.98\text{ m/s}^2\;\;(943.5-1.13)\text{kg/m}^3}\right]^{\frac{1}{2}}$$

$$\left[\frac{1.13\text{ kg/m}^3\;\;^2}{(\text{kg m})/(\text{s}^2\text{N})\;\;\;0.0547\text{ N/m}\;\;\;0.98\text{ m/s}^2\;\;(943.5-1.13)\text{ kg/m}^3}\right]^{\frac{1}{4}} = 0.008055\text{ s}$$

$$q''_{max} = \left(9.26\times10^5\text{ W/m}^2\right)$$

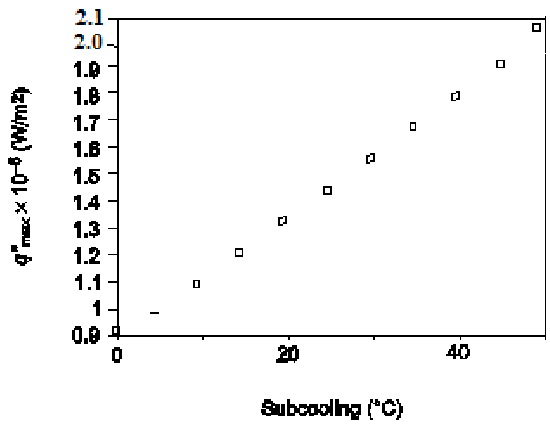
$$\left[1+\left[\frac{2\;\;0.685\text{ W/(m K)}\;\;(T_{sat}-T_{liquid})}{\sqrt{\pi\;\;0.171\times10^{-6}\text{ m}^2\text{/s}\;\;(0.008055\text{s})}}\right]\frac{24}{\pi\;\;2.201\times10^6\text{ J/kg}\;\;\;1.13\text{ kg/m}^3\;\;\;(\text{W s})/\text{J}}\right.$$

$$\left[\frac{1.13\ \text{kg/m}^3\ ^{2}}{(\text{kg m})/(\text{s}^2\text{N})\ \ 0.0547\ \text{N/m}\ \ \ 0.98\ \text{m/s}^2\ \ (943.5-1.13)\text{kg/m}^3}\right]\Bigg]$$

$$q''_{max} = \left(9.26\times10^5\ \text{W/m}^2\right)\ [1 + 0.02551\ (T_{sat} - T_{liquid})]$$

This is tabulated and graphed fro different values of ($T_{sat} - T_{liquid)}$ below

$T_{sat} - T_{liquid}$ (°C)	$q''_{max} \times 10^{-6}$ (W/m^2)
0	0.926
5	1.04
10	1.16
15	1.28
20	1.39
25	1.51
30	1.63
35	1.75
40	1.86
45	1.98
50	2.1



PROBLEM 9.10

A thin-walled horizontal copper tube of 0.5-cm-*OD* is placed in a pool of water at atmospheric pressure and 100°C. Inside the tube, an organic vapor is condensing and the outside surface temperature of the tube is uniform at 232°C. Calculated the average heat transfer coefficient on the outside of the tube.

GIVEN

- A horizontal copper tube in a pool of water at atmospheric pressure
- Tube outside diameter (D) = 0.5 cm = 0.005 m
- Water temperature (T_w) = 100°C = 373 K
- Tube outside surface temperature (T_t) = 232°C = 505 K (uniform)

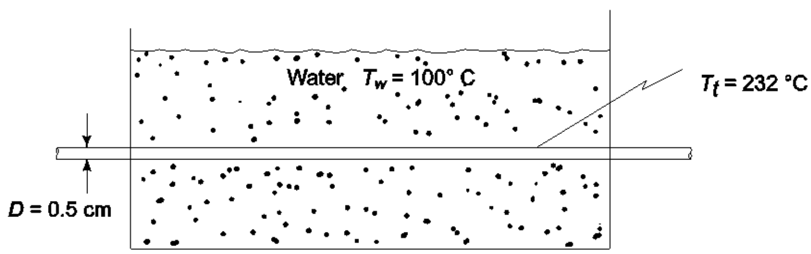
FIND

- The average heat transfer coefficient (h_{total})

ASSUMPTIONS

- Steady state
- The copper tube is polished

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 1 atm, 100°C (saturation temperature)

Density (ρ_l) = 958.4 kg/m^3

Specific heat (c_l) = 4211 J/(kg K)

Absolute viscosity (μ_l) = 277.5 × 10^{-6} (Ns)/m^2

Prandtl number (Pr) = 1.75

Heat of vaporization (h_{fg}) = 2257 kJ/kg = 2.257 × 10^6 J/kg

From Appendix 2, Table 35, for steam at 100°C

Density ρ_v = 0.5977 kg/m^3

Specific heat (c_v) = 2034 J/(kg K)

Thermal conductivity (k_v) = 0.0249 W/(m K)

Absolute viscosity (μ_v) = 12.10 × 10^{-6} (Ns)/m^2

From Table 9.2, the surface tension (σ) = 0.0589 N/m

From Table 9.1, for water on polished copper, the constant C_{sf} = 0.0128

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

From Table 9.2, the emissivity of polished copper (ε) ≈ 0.04.

SOLUTION

The excess temperature $\Delta T_x = T_t = T_{sat} = 232°C - 100°C = 132°C$. This high an excess temperature will probably lead to film boiling. This can be checked by calculating the nucleate boiling heat flux and comparing it to the critical flux. The nucleate boiling heat flux is given by Equation (9.2)

$$\frac{c_l \Delta T_x}{h_{fg} Pr_l^n} = C_{sf}\left[\frac{q''}{\mu_l h_{fg}}\sqrt{\frac{g_c\,\sigma}{g(\rho_l - \rho_v)}}\right]^{0.33}$$

Rearranging $$q'' = \left(\frac{c_l\,\Delta T_x}{h_{fg} Pr_l^n C_{sf}}\right)^{\left(\frac{1}{0.33}\right)} \frac{\mu_l h_{fg}}{\sqrt{\frac{g_c\,\sigma}{g(\rho_l - \rho_v)}}}$$

$$q'' = \left(\frac{4211\text{ J/(kg K)}\ (132°\text{C})}{2.257\times 10^6\text{ J/kg}\ (1.75)(0.0128)}\right)^{\left(\frac{1}{0.33}\right)}$$

$$\frac{277.5\times 10^{-6}\ \text{(N s)/m}^2\ \ \text{(kg m)/(s}^2\text{N)}\ \ 2.257\times 10^6\text{ J/kg}\ \ \text{(W s)/J}}{\sqrt{\frac{0.0589\text{ N/m}\ \ \text{(kg m)/(s}^2\text{N)}}{9.8\text{ m/s}^2\ \ (958.4 - 0.5977)\text{kg/m}^3}}}\ \ 3.57\times 10^8\ \text{W/m}^2$$

The critical heat flux for nucleate boiling is given by Equation (9.4)

$$q''_c = \left(\frac{\pi}{24}\right)\rho_v^{0.5} h_{fg}\left[\sigma\, g(\rho_l - \rho_v)\, g_c\right]^{\frac{1}{4}}$$

$$q''_c = \left(\frac{\pi}{24}\right)\ 0.5977\text{ kg/m}^3\ ^{0.5}\ \ 2.257\times 10^6\text{ J/kg}\ \ \text{(Ws)/J}$$

$$0.0589\text{ J/kg}\ \ \text{(kg m)/(s}^2\text{N)}\ \ 9.8\text{ m/s}^2\ \ (958.4 - 0.5977)\text{kg/m}^3\ ^{\frac{1}{4}}$$

$$q''c = 1.11\times 10^6\ \text{W/m}^2$$

Since $q'' > q''_{max}$, film boiling will exist. The conductive heat transfer coefficient for film boiling on tubes is given by Equation (9.6)

$$\bar{h}_c = 0.62\left(\frac{g(\rho_l - \rho_v)\rho_v k_v^3[h_{fg} + 0.68 c_{pv}\,\Delta t_x]}{D\,\mu_v \Delta t_x}\right)^{\frac{1}{4}}$$

$$\bar{h}_c = 0.62\left[\frac{9.8\text{ m/s}^2\ \ (958.4 - 0.5977)\text{kg/m}^3\ \ 0.5977\text{ kg/m}^3\ \ 0.0249\text{ W/(m K)}\ ^3}{(0.005\text{ m})\ \ 12.10\times 10^{-6}\ \text{(N s)/m}^2\ \ \text{(kg m)/(s}^2\text{N)}\ \ (132°\text{C})}\right.$$

$$\left.\frac{2.257\times 10^6\text{ J/kg} + 0.68\ \ 2034\text{ J/(kg K)}\ \ (132°\text{C})\ \ \text{(W s)/J}}{}\right]^{\frac{1}{4}}$$

$$\bar{h}_c = 250\ \text{W/(m}^2\text{K)}$$

The radiative heat transfer coefficient is given by Equation (9.9)

$$\bar{h}_r = \sigma_r\,\varepsilon\left(\frac{T_t^4 - T_w^4}{T_t - T_w}\right) = 5.67\times 10^{-8}\ \text{W/(m}^2\text{K}^4)$$

$$(0.04)\left(\frac{(505\text{K})^4 - (373\text{K})^4}{(505\text{K}) - (373\text{K})}\right) = 0.78\ \text{W/(m}^2\text{K)}$$

The total heat transfer coefficient is given by Equation (9.8)

$$h_{total} = \bar{h}_c + 0.75\ \bar{h}_r = [250 + (0.75)(0.78)]\ \text{W/(m}^2\text{K)} = 251\ \text{W/(m}^2\text{K)}$$

COMMENTS

Note that the contribution of radiation is very small due to the low emissivity of the polished copper surface.

PROBLEM 9.11

> **In boiling (and condensation) heat transfer, the convection coefficient, h_c, is expected to depend on the difference between surface and saturation temperature $\Delta T = (T_{surface} - T_{saturation})$, the body force arising from the density difference between liquid and vapor, $g(\rho_l - \rho_v)$, the latent heat, h_{fg}, the surface tension, σ, a characteristic length of the system, L, and the thermophysical properties of the liquid or vapor: ρ, c, k, μ. Thus we can write**
>
> **$h = h\{\Delta T, g(\rho_l - \rho_v), h_{fg}, \sigma, L, \rho, c, k, \mu\}$**
>
> **Determine (a) the number of dimensionless groups necessary to correlate experimental data, and (b) appropriate dimensionless groups that should include the Prandtl number, the Jakob number, and the Bond number ($g\Delta\rho L^2/\sigma$).**

GIVEN

- Heat transfer coefficient as a function of several dimensional quantities

FIND

(a) The number of dimensionless groups necessary to correlate experimental data
(b) The appropriate dimensionless groups including *Pr*, *Ja*, and *Bo*

SOLUTION

(a) We have 10 physical quantities and 4 dimensions (Mass, Length, Time, and Temperature) therefore, there must be 10 – 4 = 6 dimensionless groups necessary to correlate experimental data for boiling or condensation.
(b) We are given four of these dimensionless groups

$$\pi_1 = Nu = \frac{hL}{k}$$

$$\pi_2 = Pr = \frac{\mu c}{k}$$

$$\pi_3 = Ja = \frac{c\Delta T}{h_{fg}}$$

$$\pi_4 = Bo = \frac{g\Delta\rho L^2}{\sigma}$$

so there must be two other dimensionless groups.
For either of these dimensionless groups, we can write

$$\pi = \Delta T^a \, (g\Delta\rho)^b \, h_{fg}^{\ c} \sigma^d L^e \rho^f c^g k^h \mu^i$$

or in terms of the dimension of each of the physical quantities we have

$$[\pi] = [T]^a \left[\frac{M}{L^2 t^2}\right]^b \left[\frac{L^2}{t^2}\right]^c \left[\frac{M}{t^2}\right]^d [L]^e \left[\frac{M}{L^3}\right]^f \left[\frac{L^2}{T t^2}\right]^g \left[\frac{ML}{T t^3}\right]^h \left[\frac{M}{L t}\right]^i$$

In order for either of the two new groups to be dimensionless, the following four equations in the powers (a, b, …) must be satisfied

$[T]^0 \quad \Rightarrow \quad a - g - h = 0 \qquad (1)$

$[L]^0 \quad \Rightarrow \quad -2b + 2c + e - 3f + 2g + h - i = 0 \qquad (2)$

$[t]^0 \quad \Rightarrow \quad -2b - 2c - 2d - 2g - 3h - i = 0 \qquad (3)$

$[M]^0 \quad \Rightarrow \quad b + d + f + h + i = 0 \qquad (4)$

Since there are 4 equations in 9 unknowns, we are free to select 5 of these powers. The following table will help determine which powers to select

	π_2	π_3	π_4	π_5	π_6
	Pr	*Ja*	*Bo*		
a	0	1	0		
b	0	0	1		
c	0	–1	0		
d	0	0	–1		
e	0	0	2		
f	0	0	0		
g	1	1	0		
h	–1	0	0		
i	1	0	0		

For π_5, let's set $a = c = d = g = 0$ and $f = 1$ to ensure that our solution vector is not proportional to those for the previous π,*s*. Then equation (1) gives $h = 0$, equation (2) gives $-2b + e - 3 - i = 0$, equation (3) gives $-2b - i = 0$ and equation (4) gives $b + 1 + i = 0$. These can be solved to find $b = 1$, $i = -2$, and $e = 3$. So we have

$$\pi_5 = \frac{g\Delta\rho L^3\rho}{\mu^2}$$

For π_6, let's $a = d = i = 1$, and $c = g = 0$. This will ensure that this vector is not proportional to the others. The four equations can then be solved to give $h = 1$, $b = -3$, $f = 0$, and $e = -6$, giving

$$\pi_6 = \frac{\Delta T\sigma k\mu}{(g\Delta\rho)^3 L^6}$$

PROBLEM 9.12

Environmental concerns have recently motivated the search for replacements for chlorofluorocarbon refrigerants. An experiment has been devised to determine the applicability of such a replacement. A silicon chip is bonded to the bottom of a thin copper plate as shown in the sketch below. The chip is 0.2-cm-thick and has a thermal conductivity of 125 W/(m K). The copper plate is 0.1 cm thick and there is no contact resistance between the chip and the copper plate. This assembly is to be cooled by boiling a saturated liquid refrigerant on the copper surface. The electronic circuit on the bottom of the chip generates heat uniformly at a flux of $q'' = 5 \times 10^4$ W/m². Assume that the sides and the bottom of the chip are insulated. Calculate the steady state temperature at the copper surface and the bottom of the chip, as well as the maximum heat flux in pool boiling, assuming that the boiling coefficient, C_{sf}, is the same as for *n*-pentane on lapped copper. The physical properties of this new coolant are: T_{sat} = 60°C, c_p = 1100 J/(kg K), $h_{fg} = 8.4 \times 10^4$ J/kg, ρ_l = 1620 kg/m³, ρ_v = 13.4 kg/m³, σ = 0.081 N/m, $\mu_l = 4.4 \times 10^{-4}$ kg/(ms) and Pr_l = 9.0.

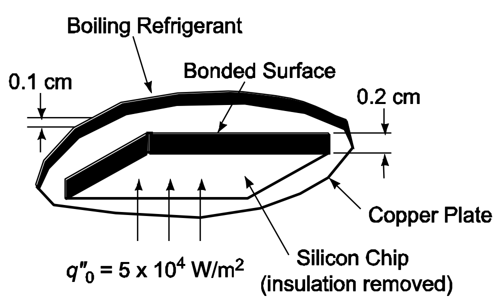
GIVEN

- A new refrigerant boiling on top of a copper plate, cooling a silicon chip
- The refrigerant is a saturated liquid
- Properties of the refrigerant and C_{sf}
- No contact resistance between the copper plate and the chip
- Uniform heat flux produced by the chip is 5×10^4 W/m²

FIND

(a) Steady state temperature at the copper surface
(b) Steady state temperature at the bottom of the chip
(c) Maximum heat flux in pool boiling

SKETCH



PROPERTIES AND CONSTANTS

From Table 9.1, the boiling coefficient, C_{sf}, for *n*-pentane boiling on lapped copper is C_{sf} = 0.0049

SOLUTION

The right side of the sketch above shows the thermal circuit. The heat generated at the bottom of the chip is transferred by conduction up through the chip and through the copper plate and is then transferred to the boiling refrigerant on top of the chip.

Assuming that the heat flux does not exceed the critical heat flux, we can determine the temperature drop as the excess temperature. $\Delta Tx = T_{copper} - T_{sat}$, from Equation (9.2)

$$\frac{c_l\,\Delta T_x}{h_{fg}Pr_l^{\,n}} = C_{fs}\left[\frac{q''}{\mu_l\,h_{fg}}\sqrt{\frac{g_e\,\sigma}{g\,(\rho_l-\rho_v)}}\right]^{0.33}$$

Since the refrigerant is not water, n = 1.7, so the right side of the above equation is

$$(0.0049)\left[\frac{5\times10^4\ \text{W/m}^2}{4.4\times10^{-4}\ \text{kg/(ms)}\quad 8.4\times10^4\ \text{J/kg}}\sqrt{\frac{1\,\text{(kg m)/(N s}^2)\quad 0.081\,\text{N/m}}{9.81\,\text{m/s}^2\ \ (1620-13.4)\,\text{kg/m}^3}}\right]^{0.33} = 0.007093$$

and

$$\Delta T_x = \frac{(0.007093)\ \ 8.4\times10^4\ \text{J/kg}\ \ (9^{1.7})}{1100\ \text{J/(kg K)}} = 22.7\ \text{K}$$

(a) Therefore, $T_{\text{copper}} = T_{\text{sat}} + \Delta T_x$ = 60°C + 22.7 = 82.7°C.
Now, the thermal conductivity of copper at ~90°C is, from Figure 1.7, k_{copper} = 400 W/(m K), so the thermal resistance of the copper is

$$R_{\text{copper}} = \left(\frac{t}{k}\right)_{\text{copper}} = \frac{10^{-3}\text{m}}{400\ \text{W/(m K)}} = 2.5\times10^{-6}\ (\text{m}^2\text{K})/\text{W}$$

The temperature drop across the copper plate is

$$\Delta T_{\text{copper}} = q_o''\,R_{\text{copper}} = \ 5\times10^4\ \text{W/m}^2 \quad 2.5\times10^{-6}\ (\text{m}^2\text{K})/\text{W} \ = 0.125\text{K}$$

The thermal resistance of the chip is

$$R_{\text{chip}} = \left(\frac{t}{k}\right)_{\text{chip}} = \frac{(2\times10^{-3}\text{m})}{125\ \text{W/m}} = 1.6\times10^{-5}\ (\text{m}^2\text{K})/\text{W}$$

So the temperature drop across the chip is

$$\Delta T_{\text{chip}} = q_o''\,R_{\text{chip}} = \ 5\times10^4\ \text{W/m}^2 \quad 1.6\times10^{-5}\ (\text{m}^2\text{K})/\text{W} \ = 0.8\ \text{K}$$

(b) and the temperature of the bottom of the chip is therefore T_{chip} = 82.7+ 0.125+ 0.8 = 83.65°C.
The maximum heat flux can be calculated from Equation (9.4)

$$q''_{\max} = \left(\frac{\pi}{24}\right)\rho_v^{\frac{1}{2}}\ h_{fg}\ [\sigma g g_c\,(\rho_l-\rho_v)]^{\frac{1}{4}}$$

or
(c)

$$q''_{\max} = \left(\frac{\pi}{24}\right)\ 13.4\ \text{kg/m}^3\ ^{\frac{1}{2}}\ \ 8.4\times10^4\ \text{J/kg}$$

$$0.081\,\text{N/m}\quad 9.81\,\text{m/s}^2\quad 1\,\text{(kg m)/(N s}^2)\ \ (1620-13.4)\ \ \text{kg/m}^3\ \ ^{\frac{1}{4}} = 240{,}590\ \text{W/m}^2$$

(Using Lienhard's recommendation, this critical heat flux would be about 11% larger.)
Since ΔT_x is proportional to (heat flux)$^{0.33}$, we can recalculate the chip temperature as follows

$$T'_{\text{chip}} = 60 + 22.7\left(\frac{240{,}590}{50{,}000}\right)^{0.33} + 0.125 + 0.8 = 99.0°\text{C}$$

This temperature could be compared to the maximum permissible chip operating temperature to determine the maximum permissible chip power dissipation.

PROBLEM 9.13

It has recently been proposed by Andraka el al. of Sandia National Laboratories, Albuquerque, in Sodium Reflux Pool-Boiler Solar Receiver On-Sun Test Results (SAND89-2773, June 1992), that the heat flux from a parabolic dish solar concentrator could be delivered effectively to a Stirling engine by a liquid-metal pool boiler. The sketch below shows a cross-section of the pool boiler receiver assembly. Solar flux is absorbed on the concave side of a hemispherical absorber dome, boiling molten sodium metal on the convex side of the dome. The sodium vapor condenses on the engine heater tube as shown near the top of the figure. Condensing sodium transfers its latent heat to the engine working fluid that circulates inside the tube. Calculations indicate that a maximum heat flux of 75 W/cm^2 delivered by the solar concentrator to the absorber dome is to be expected.

After the receiver had been tested for about 50 hours, a small spot on the absorber dome suddenly melted and the receiver failed. Is it possible that the critical flux for the boiling sodium was exceeded? Use the following properties for the sodium: $\tilde{\rho}$ = 0.056 kg/m^3, ρ_l = 779 kg/m^3, h_{fg} = 4.039 × 10^6 J/kg, σ_l = 0.138 N/m, μ_l = 1.8 × 10^{-4} kg/ms.

GIVEN

- Sodium pool-boiler solar receiver
- Failure after about 50 hours operation
- Expected peak heat flux was 75 W/cm^2 = 750,000 W/m^2

FIND

(a) Whether the critical heat flux could have been exceeded

ASSUMPTIONS

- To first order, the absorber dome can be treated as flat, horizontal surface

PROPERTIES AND CONSTANTS

As given in the problem statement, the pertinent properties of the sodium vapor and liquid are

Heat of vaporization (h_{fg}) = 4.039 × 10^6 J/kg

Vapor density (ρ_v) = 0.056 kg/m^3

Liquid density (ρ_l) = 779 kg/m^3

Surface tension (σ) = 0.138 N/m

SOLUTION

The critical heat flux can be calculated from Equation (9.4)

$$q''_{\max,Z} = \left(\frac{\pi}{24}\right)\ \rho_v^{\frac{1}{2}}\ h_{fg}\ [\sigma g\,(\rho_l - \rho_v) g_c]^{\frac{1}{4}}$$

For the property values listed above we have

$$q''_{\max,Z} = \left(\frac{\pi}{24}\right)\ \left(0.056\,\text{kg/m}^3\right)^{\frac{1}{2}}\ \left(4.039\times 10^6\ \text{J/kg}\right)$$

$$\left[0.138\,\text{N/m}\ \left(9.81\,\text{m/s}^2\right)\ (779-0.056)\ \text{kg/m}^3\ \ (\text{kg m})/(\text{Ns}^2)\right]^{\frac{1}{4}} = 712{,}970\ \text{W/m}^2$$

If we use Lienhard and Dhir's recommendation that the factor $\pi/24$ be replaced by 0.149, the critical heat flux would be

$$q''_{\max,Z} = 712{,}970 \left(\frac{0.149}{\left(\frac{\pi}{24} \right)} \right) = 811{,}550 \ \text{W/m}^2$$

which exceeds the expected maximum flux. Therefore, exceeding the critical heat flux is a possible factor in the failure of the receiver.

COMMENTS

The correlation for critical heat flux used above has not necessarily been tested with liquid metals so the result should be used with some caution.

PROBLEM 9.14

Calculate the peak heat flux for nucleate pool boiling of water at 300 kPa pressure and 110°C on clean copper.

GIVEN

- Nucleate pool boiling of water on clean copper
- Pressure = 3 atm
- Water temperature (T_w) = 390°C

Find

- The peak heat flux (q''_{max})

ASSUMPTIONS

- Steady state

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 3 atm (3.04×10^5 Pa) pressure

Saturation temperature (T_{sat}) = 133°C

Liquid density (ρ_l) = 932.3 kg/m^3

Vapor density ($\rho_v = 1/v_g$) = 1.55 kg/m^3

Thermal conductivity (k_l) = 0.684 W/(m K)

Heat of vaporization (h_{fg}) = 2164 kJ/kg = 2.164×10^6 J/kg

Thermal diffusivity (α_l) = 0.172×10^{-6} m^2/s

Absolute viscosity (μ_l) = 213.0×10^{-6} (Ns)/m^2

Prandtl number (pr_l) = 1.30

From Table 9.2, the surface tension at 133°C (σ) = 0.0522 N/m

SOLUTION

The maximum heat flux for water at saturation temperature is given by Equation (9.4)

$$q''_{\text{max,sat}} = \left(\frac{\pi}{24}\right)\ \rho_v^{0.5}\, h_{fg}\ [\sigma\, g\,(\rho_l - \rho_v)\, g_c]^{\frac{1}{4}}$$

$$q''_{\text{max,sat}} = \left(\frac{\pi}{24}\right)\ 1.55\ \text{kg/m}^3\ ^{0.5}\ \ 2.164 \times 10^6\ \text{J/kg}\ \ \ (\text{Ws})/\text{J}$$

$$0.0522\ \text{J/kg}\ \ (\text{kg m})/(\text{s}^2\text{N})\ \ 9.8\ \text{m/s}^2\ \ (932.3 - 1.55)\text{kg/m}^3\ ^{\frac{1}{4}}$$

$$q''_{\text{max,sat}} = 1.65 \times 10^6\ \ \text{W/m}^2$$

The maximum heat flux for a subcooled liquid is given by Equation (9.5)

$$q''_{\text{max}} = q''_{\text{max,sat}} \left(1 + \left[\frac{2k_l\,(T_{\text{sat}} - T_{\text{liquid}})}{\sqrt{\pi\,\alpha_l\,\tau}}\right]\frac{24}{\pi h_{fg}\,\rho_v}\left[\frac{\rho_v^2}{g_c\sigma g\,(\rho_l - \rho_v)}\right]^{\frac{1}{4}}\right)$$

$$\text{where } \tau = \left(\frac{\pi}{3}\right)\ \sqrt{2\pi}\ \left[\frac{g_c\sigma}{g\,(\rho_l - \rho_v)}\right]^{\frac{1}{2}}\ \left[\frac{\rho_v^2}{g_c\sigma g\,(\rho_l - \rho_v)}\right]^{\frac{1}{4}}$$

$$t_{(\tan)} = \left(\frac{\pi}{3}\right)\sqrt{2\pi}\left[\frac{(\text{kg m})/(\text{s}^2\text{N})\quad 0.0522\ \text{N/m}}{9.8\ \text{m/s}^2\ \ (932.3-1.55)\text{kg/m}^3}\right]^{\frac{1}{2}}$$

$$\left[\frac{1.55\ \text{kg/m}^3\ ^2}{(\text{kg m})/(\text{s}^2\text{N})\quad 0.0522\ \text{N/m}\quad 9.8\,\text{m/s}^2\ \ (932.3-1.55)\ \text{kg/m}^3}\right]^{\frac{1}{4}} = 0.00167\ \text{s}$$

$$q''_{\max} = \ \ 1.65\times 10^6\ \text{W/m}^2$$

$$\left[1+\left[\frac{2\ \ 0.684\ \text{W/(m K)}\ \ (133°\text{C}-110°\text{C})}{\sqrt{\pi\ \ 0.172\times 10^{-6}\ \text{m}^2/\text{s}\ \ (0.00167\text{s})}}\right]\frac{24}{\pi\ \ 2.164\times 10^6\ \text{J/kg}\quad 1.55\ \text{kg/m}^3\quad (\text{Ws})/\text{J}}\left[\frac{1.55\ \text{kg/m}^3\ ^2}{(\text{kg m})/(\text{s}^2\text{N})\quad 0.0522\ \text{N/m}\quad 9.8\ \text{m/s}^2\ \ (932.3-1.55)\,\text{kg/m}^2}\right]^{\frac{1}{4}}\right]$$

$$q''_{\max} = 2.69\times 10^6\ \text{W/m}^2$$

PROBLEM 9.15

In a metal alloy manufacturing and heat treatment plant and in its immersion quenching process, steel plates are first heated in a furnace and then quenched (or cooled) in a coolant bath to obtain the desired steel properties and surface hardness. In one such process a flat stainless steel plate 0.6-cm-thick, 7.5-cm-wide and 0.3-m-long is immersed horizontally at an initial temperature of 980°C in a large water bath at 100°C and at atmospheric pressure. Determine how long it will take this plate to cool to 540°C.

GIVEN

- A flat stainless steel plate is immersed horizontally in water
- Plate thickness (s) = 0.6 cm = 0.006 m
- Plate width (w) = 7.5 cm = 0.075 m
- Plate length (L) = 0.3 m
- Initial plate temperature (T_{pi}) = 980°C = 1253 K
- Pressure = 1 atm
- Water bath temperature (T_b) = 100°C (saturation) = 373 K

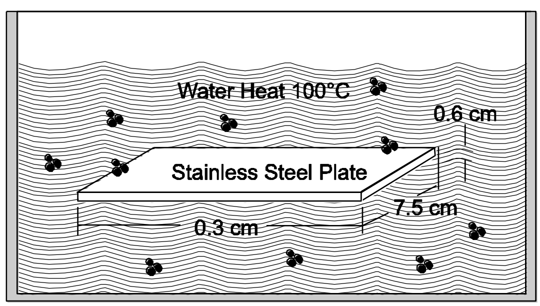
FIND

- Time for plate to cool to T_{pf} = 540°C = 813 K

ASSUMPTIONS

- The heat fluxes from the bottom and top of the plate are equal
- The plate is polished

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the pool temperature of 100°C

Density (ρ) = 958.4 kg/m^3

Thermal conductivity (k) = 0.682 W/(m K)

Absolute viscosity (μ) = 277 × 10^{-6} (Ns)/m^2

Prandtl number (Pr) = 1.75

Specific heat (c) = 4211 J/(kg K)

Heat of vaporization (h_{fg}) = 2.257 × 10^6 J/kg

From Appendix 2, Table 35, for steam at 100°C

k_v = 0.0249 W/(m K) c_v = 2034 J/(kg K) μ_v = 12.10 × 10^{-6} (Ns)/m^2

ρ = 0.5977 kg/m^3

From Table 9.2, The surface tension, σ, for water @ 100°C = 0.0589 N/m

From Table 9.1, The coefficient, C_{sf}, for water on mechanically polished stainless steel = 0.0132

From Table 11.3, the emissivity of polished stainless steel at the average temperature of (980°C + 540°C)/2 = 760°C (ε) = 0.22

From Appendix 1, Table 5, the Stephen-Botzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

From Appendix 2, Table 10, for type 304 stainless steel

(k_{steel}) = 14.4 W/(m K) ρ_{steel} = 7817 kg/m^3 c_{steel} = 461 J/(kg K).

SOLUTION

As the plate cools, the heat flux from the plate will diminish. Therefore, the heat flux will be assumed to be constant over a small time step, then the plate temperature will be updated. This procedure will be repeated until the plate temperature drops to 540°C.

The initial excess temperatures, $\Delta T_x = T_{pi} - T_{\text{sat}}$ = 980°C – 100°C = 880°C.

Assuming nucleate boiling, the heat flux is given by Equation (9.2)

$$\frac{c_l\,\Delta T_x}{h_{fg}\,Pr_l^n} = C_{sf}\left[\frac{q''}{\mu_l\,h_{fg}}\sqrt{\frac{g_c\,\sigma}{g\,(\rho_l-\rho_v)}}\right]^{0.33}$$

Solving for the heat flux

$$q'' = \left(\frac{c_l\,\Delta T_x}{h_{fg}Pr_l^n\,C_{sf}}\right)^3\frac{\mu_l\,h_{fg}}{\sqrt{\dfrac{g_c\,\sigma}{g\,(\rho_l-\rho_v)}}}$$

$$q'' = \left(\frac{4211\,\text{J/(kg K)}\ \ (880\,°\text{C})}{2.257\times10^6\,\text{J/kg}\ \ (1.75)\,(0.0132)}\right)^3\frac{277.5\times10^{-6}\,(\text{N s})/\text{m}^2\ \ (\text{kg m})/(\text{s}^2\text{N})\ \ 2.257\times10^6\ \text{J/kg}\ \ \ (\text{W s})/\text{J}}{\sqrt{\dfrac{0.0589\,\text{N/m}\ \ (\text{kg m})/(\text{s}^2\text{N})}{9.8\,\text{m/s}^2\ \ \ (958.4-0.5977)\text{kg/m}^3}}}$$

$$= 8.96\times10^{10}\ \text{W/m}^2$$

The critical heat flux for nucleate boiling is given be Equation (9.4)

$$q''_c = \frac{\pi}{24}\rho_v^{0.5}\,h_{fg}\,[\sigma\,g\,(\rho_l-\rho_v)\,g_c]^{\frac{1}{4}}$$

$$q''_c = \left(\frac{\pi}{24}\right)\ \ 0.5977\,\text{kg/m}^3\ ^{0.5}\ \ 2.257\times10^6\,\text{J/kg}\ \ \ (\text{Ws})/\text{J}$$

$$0.0589\,\text{J/kg}\ \ \ (\text{kg m})/(\text{s}^2\text{N})\ \ \ 9.8\,\text{m/s}^2\ \ (958.4-0.5977)\,\text{kg/m}^3\ ^{\frac{1}{4}}$$

Since $q'' > q''_c$, The nucleate boiling assumption is invalid and film boiling will occur. Note that at the final plate temperature, $T_{b,f}$ = 540°C, ΔT_x = 440°C, and q'' = 1.12 × 10^{10} > q''_c . Therefore, film boiling will occur during the entire cooling period.

For film boiling on flat horizontal surfaces, the conduction heat transfer coefficient is given by Equation (9.7)

$$\bar{h}_c = \ 0.59 + 0.69\frac{\lambda}{D}\ \left\{\frac{g\,(\rho_l-\rho_v)\rho_v\,k_v^3\,[h_{fg}+0.68\,c_v\Delta T_x]}{\lambda\,\mu_v\,\Delta T_x}\right\}^{\frac{1}{4}}$$

$$\text{where } \lambda = 2\pi\left[\frac{g_c\,\sigma}{g\,(\rho_l - \rho_c)}\right]^{\frac{1}{2}} = 2\pi\left[\frac{(\text{kg m})/(\text{s}^2\text{N})\quad 0.0589\ \text{N/m}}{9.8\ \text{m/s}^2\ \ (958.4 - 0.5977)\ \text{kg/m}^3}\right]^{\frac{1}{2}} = 0.01574\ \text{m}$$

For a flat plate, $D \to \infty$, therefore $\lambda/D \to 0$

$$\bar{h}_c = \left[\frac{9.8\ \text{m/s}^2\ \ (958.4 - 0.5977)\ \text{kg/m}^3\ \ 0.5977\ \text{kg/m}^3\quad 0.0249\ \text{W/(m K)}^{\ 3}}{0.01574\ \text{m}\ \ 12.1\times 10^{-6}\ (\text{N s})/\text{m}^2\ \ \Delta T_x}\right.$$

$$\left.\underline{\left[2.257\times 10^6\ \text{J/kg} + 0.68\ \ 2034\ \text{J/(kg K)}\ \ \Delta T_x\right]}\right]^{\frac{1}{4}}$$

$$\bar{h}_c = 15.32\left[\frac{2.257\times 10^6 + 1383.1\Delta T_x}{\Delta T_x}\right]^{\frac{1}{4}}\ \ \text{W/(m}^2\text{K)}\ \ (\Delta T_x \text{ in °C})$$

Initially

$$\bar{h}_c = 15.32\left[\frac{2.257\times 10^6 + 1383.1(880°\text{C})}{(880°\text{C})}\right]^{\frac{1}{4}} = 121.4\ \ \text{W/(m}^2\text{K)}$$

The radiation heat transfer coefficient is given by Equation (9.9)

$$\bar{h}_r = \sigma\,\varepsilon\left(\frac{T_p^4 - T_{\text{sat}}^4}{T_p - T_{\text{sat}}}\right) = 5.67\times 10^{-8}\ \text{W/(m}^2\text{K}^4)\ \ (0.22)\left(\frac{T_p^4 - (373\text{K})^4}{T_p - (373\text{K})}\right)$$

Initially

$$\bar{h}_r = 1.247\times 10^{-8}\ \text{W/(m}^2\text{K}^4)\ \ \left(\frac{(1253\text{K})^4 - (373\text{K})^4}{(1253\text{K}) - (373\text{K})}\right) = 34.7\ \ \text{W/(m}^2\text{K)}$$

The total heat transfer coefficient is given by Equation (9.8)

$$h_{\text{total}} = \bar{h}_c + 0.75\ \bar{h}_r = [121.4 + (0.75)\,(34.7)]\ \ \text{W/(m}^2\text{K)} = 147.4\ \ \text{W/(m}^2\text{K)}$$

The initial rate of heat transfer is

$$q = \bar{h}_{\text{total}}\ A\,(T_p - T_b) = \bar{h}_{\text{total}}\ \ 2H\,L\,(T_p - T_b)$$

$$q = 147.4\ \text{W/(m}^2\text{K)}\ \ 2\,(0.075\ \text{m})(0.3\ \text{m})\,(980°\text{C} - 100°\text{C}) = 5838\ \text{W}$$

The initial Biot number is

$$Bi = \frac{h_{\text{total}}\,s}{2\,k_{\text{steel}}} = \frac{147.4\ \text{W/(m}^2\text{K)}\ \ (0.006\,\text{m})}{2\ \ 14.4\ \text{W/(m K)}} = 0.03 < 0.1$$

Therefore, the internal resistance of the steel is negligible. The change in the plate temperature is given by

$$\Delta T = \frac{q\,\Delta t}{m_{\text{steel}}\,c_{\text{steel}}} = \frac{q\,\Delta t}{\rho_{\text{steel}}\,(\text{volume})\,c_{\text{steel}}} = \frac{q\,\Delta t}{H\ L\ s\ \rho_{\text{steel}}\ c_{\text{steel}}}$$

For Δt = 5 s

$$\Delta T = \frac{5838\,\text{W}(5\,\text{s})}{(0.075\,\text{m})(0.3\,\text{m})(0.006\,\text{m})\;7817\,\text{kg/m}^3\;\;461\,\text{J/(kg K)}\;\;(\text{W s})/\text{J}} = 60\text{ K}$$

Therefore, after 5 s

$$T_p = T_{pi} - \Delta T = 980°\text{C} - 60°\text{C} = 920°\text{C}$$

Repeating this procedure: for Δt = 5 s

Time (s)	T_p (°C)	h_{total} (W/m^2 K)	q (W)	ΔT (K)
0	980	147.4	5838	60
5	920	145.7	5378	55.3
10	864.7	144.6	4858	49.9
15	814.8	143.8	4626	47.5
20	767.2	143.4	4307	44.3
25	723.0	143.4	4019	41.3
30	681.7	143.5	3757	38.6
35	643.1	143.9	3518	36.2
40	606.9	144.6	3299	33.9
45	573.0	145.4	3096	31.8
50	540			

The plate will cool to 540°C in approximately 50 seconds.

PROBLEM 9.16

Calculate the heat transfer coefficient for film boiling of water on a 1.3 cm horizontal tube if the tube temperature is 550°C and the system is placed under pressure of 50 kPa.

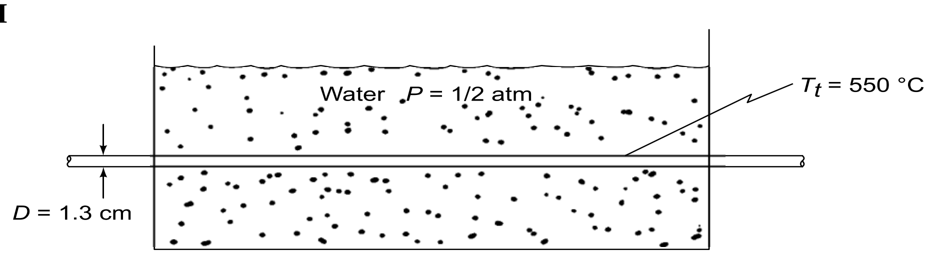
GIVEN
- Film boiling of water on a horizontal tube
- Tube outside diameter (D) = 1.3 cm = 0.013 m
- Tube temperature (T_t) = 550°C
- Pressure = 50 kPa = 50,000 N/m^2

FIND
- The heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state
- Tube temperature is uniform and constant
- The viscosity, specific heat, and thermal conductivity of the vapor can be approximated by those of steam at 1 atm pressure
- Radiation across the vapor film is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 50,000 N/m^2

Saturation temperature (T_{sat}): 81.5°C

Water density, (ρ_l): 970.6 kg/m^3

Heat of vaporization, (h_{fg}): 2304 kJ/kg = 2.304×10^6 J/kg

Vapor density, ($\rho_v = 1/v_g$): 0.3070 kg/m^3

Extrapolating from Appendix 2, Table 35, for steam at 1 atm and 81.5°C

Absolute viscosity (μ_v) = 10.49×10^{-6} (Ns)/m^2

Thermal conductivity (k_v) = 0.0257 W/(m K)

Specific heat (c_v) = 1965 J/(kg K)

SOLUTION

The heat transfer coefficient for film boiling on tubes is given by Equation (9.6)

$$\bar{h}_c = 0.62 \left(\frac{g(\rho_l - \rho_v)\rho_v k_v^3 [h_{fg} + 0.68 c_{pv} \Delta T_x]}{D \mu_v \Delta T_x} \right)^{\frac{1}{4}}$$

$$\bar{h}_c = 0.62 \left[\frac{9.8\,\text{m/s}^2\ (970.6 - 0.3070)\,\text{kg/m}^3\ 0.3070\,\text{kg/m}^3\ \ 0.0257\,\text{W/(m K)}^{\ 3}}{(0.013\,\text{m})\ 10.49 \times 10^{-6}\ (\text{N s})/\text{m}^2\ \ (\text{kg m})/(\text{s}^2\text{N})\ (550\text{°C} - 81.5\text{°C})} \right.$$

$$\left. \frac{2.304 \times 10^6\ \text{J/kg} + 0.68\ 1965\,\text{J/(kg K)}\ (550\text{°C} - 81.5\text{°C})\ \ (\text{W s})/\text{J}}{} \right]^{\frac{1}{4}}$$

$$\bar{h}_c = 135\ \text{W/(m}^2\text{K)}$$

PROBLEM 9.17

A metal-clad electrical heating element of cylindrical shape, as shown in the sketch below, is immersed in water at atmospheric pressure. The element is a 5-cm-OD, and heat generation produces a surface temperature of 300°C. Estimate the heat flux under steady state conditions and the rate of heat generation per unit length.

GIVEN

- Cylindrical, electrical heating element
- Heat generation produces a 300°C surface temperature

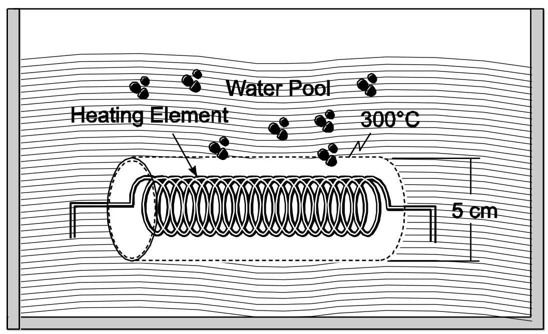
FIND

(a) Heat flux
(b) Rate of heat generation

ASSUMPTIONS

- The system operates at atmospheric pressure
- The heater surface is black

SKETCH



PROPERTIES AND CONSTANTS

Inspection of the boiling curve in Figure (9.1) indicates that at 300°C surface temperature, the heating system must operate in the film boiling regime. Hence, a vapor layer covers the surface of the heater and it is appropriate to evaluate the physical properties at the mean film temperature of (100 + 300)/2 = 200°C.

From Appendix 2, Table 35

Vapor density (ρ_v) = 0.4673 kg/m^3

Specific heat (c_{pv}) = 1982 J/(kg K)

Absolute viscosity (μ_v) = 1.61×10^{-5} kg/(ms)

Thermal conductivity (k_v) = 0.032 W/(m K)

From Appendix 2, Table 13

Heat of vaporization (h_{fg}) = 2.257×10^6 J/kg

Liquid density (ρ_l) = 958.4 kg/m^3

SOLUTION

We find the convection heat transfer coefficient from Equation (9.6)

$$\bar{h}_c = 0.62\left\{\frac{g(\rho_l-\rho_v)\rho_v k_v^3[h_{fg}+0.68c_{pv}\,\Delta T_x]}{D\mu_v\,\Delta T_x}\right\}^{\frac{1}{4}}$$

so

$$\bar{h}_c = 0.62\left\{\frac{9.81\,\mathrm{m/s^2}\ \ (958.4-0.4673)\ \mathrm{kg/m^3}\ \ 0.4673\,\mathrm{kg/m^3}\ \ 0.032\,\mathrm{W/(m\,K)}^{\ 3}}{(0.05\,\mathrm{m})\ 1.61\times10^{-5}\,\mathrm{kg/(ms)}\ \ (200\,\mathrm{K})}\right.$$

$$\left.\frac{2.257\times10^{6}\ \mathrm{J/kg}\ \ +0.68\ \ 1982\,\mathrm{J/(kg\,K)}\ \ (200\,\mathrm{K})}{}\right\}^{\frac{1}{4}}$$

$$= 135\ \ \mathrm{W/(m^2K)}$$

The radiation heat transfer coefficient can be determined from Equation (9.9)

$$\bar{h}_r = \sigma\varepsilon_s\left(\frac{T_s^4-T_{\mathrm{sat}}^4}{T_s-T_{\mathrm{sat}}}\right) = 5.67\times10^{-8}\ \mathrm{W/(K^4m^2)}\ \ (1)\ \left(\frac{(573^4-373^4)(\mathrm{K}^4)}{(573-373)(\mathrm{K})}\right) = 25.1\ \ \mathrm{W/(m^2K)}$$

The total heat transfer coefficient is from Equation (9.8)

$$\bar{h}_{\mathrm{total}} = \bar{h}_c + 0.75\ \bar{h}_r = 135.0\,\mathrm{W/(m^2K)}\ \ +0.75\ \ 25.1\,\mathrm{W/(m^2K)}\ \ = 154\ \ \mathrm{W/(m^2K)}$$

(a) The heat flux is then

$$q'' = \bar{h}_{\mathrm{total}}\ \Delta T = 154\ \mathrm{W/(m^2K)}\ \ (200\ \mathrm{K}) = 30{,}761\ \mathrm{W/m^2}$$

(b) The rate of heat generation per unit length is

$$q_L = \bar{h}_{\mathrm{total}}\ \Delta T\pi D = 154\ \mathrm{W/(m^2K)}\ \ (200\ \mathrm{K})\ (\pi)\ (0.05\ \mathrm{m}) = 4838\ \ \mathrm{W/m}$$

PROBLEM 9.18

Calculate the maximum safe heat flux in the nucleate-boiling regime for water flowing at a velocity of 15 m/s through a 1.2-cm-*ID* tube 0.31-m-long if the water enters at 100 kPa pressure and 100°C saturated liquid.

GIVEN

- Nucleate boiling of water flowing through a tube
- Water velocity (V) = 15 m/s
- Tube inside diameter (D) = 1.2 cm = 0.012 m
- Tube length (L) = 0.31 m
- Water pressure (p) = 1 atm
- Water temperature (T_w) = 100°C

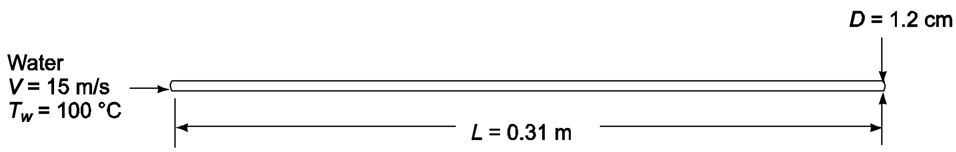
FIND

- The maximum safe heat flux in the nucleate boiling regime (q''_{max})

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 100°C, 1 atm

Density (ρ_l) = 958.4 kg/m^3

Thermal conductivity (k_l) = 0.682 W/(m K)

Absolute viscosity (μ_l) = 277.5 × 10^{-6} (Ns)/m^2

Prandtl number (Pr_l) = 1.75

Specific heat (c_l) = 4211 J/(kg K)

Kinematic viscosity (ν_l) = 0.294 × 10^{-6} m^2/s

Heat of vaporization (h_{fg}) = 2.275 × 10^6 J/kg

Enthalpy of saturated vapor (h_g) = 2.676 × 10^6 J/kg

Enthalpy of saturated liquid (h_b) = 0.419 × 10^6 J/kg

From Appendix 2, Table 35, for steam at 100°C: ρ_v = 0.5977 kg/m^3

From Table 9.2: Surface tension at 100°C (s) = 0.0589 N/m

From Table 9.1: For water on copper, C_{sf} = 0.0130

SOLUTION

Assuming that by ‘safe’ we mean that the critical heat flux is not exceeded, we can use the Griffith correlation, Figure 9.17. The critical pressure is P_c = 218.3 atm, P/P_c = 1/218.3 = 0.0046. From the figure, the ordinate is therefore 6000

$$6000 = \frac{41.5\, q_{max}''}{(h_g - h_b)\rho_v \left[\left(\frac{\rho_l - \rho_v}{\mu_l} \right) g \left(\frac{k_l}{\rho_l\, c_l} \right)^2 \right]^{\frac{1}{3}} F}$$

Since $T_s = T_b$, the parameter F simplifies to

$$F = 1 + 10^{-6} \left(\frac{UD}{v_l} \right) = 1 + 10^{-6} \left(\frac{15\ \text{m/s}\ (0.012\,\text{m})}{0.249 \times 10^{-6}\ \text{m}^2/\text{s}} \right) = 1.612$$

The maximum heat flux is then given by

$$q''_{max} = \frac{6000}{41.5}\ (h_g - h_b)\ \rho_v \left[\left(\frac{\rho_l - \rho_v}{\mu_l} \right) g \left(\frac{k_l}{\rho_l\, c_l} \right)^2 \right]^{\frac{1}{3}} F$$

$$= \left(\frac{6000}{41.5} \right) (2.676 - 0.419)\ \ 10^6\ \text{J/kg}\quad 0.5977\,\text{kg/m}^3$$

$$\left[\frac{(958.4 - 0.5977)\ \text{kg/m}^3}{277.5 \times 10^{-6} (\text{N s})/\text{m}^2}\ 9.8\,\text{m/s}^2 \left(\frac{0.682\ \text{W/(m K)}}{958.4\ \text{kg/m}^3\quad 4211\,\text{J/(kg K)}} \right)^2 \right]^{\frac{1}{3}}\ 1$$

$$q''_{max} = 3.108 \times 10^6\ \text{W/m}^2$$

PROBLEM 9.19

During the 1980s, solar thermal electric technology was commercialized with the installation of 350 MW of electrical power capacity in the California desert. The technology involved heating a heat transfer oil in receiver tubes placed at the focus of line-focus, parabolic trough solar concentrators. The heat transfer oil was then used to generate steam which, in turn, powered a steam turbine/electrical generator. Since the transfer of heat from the oil to the steam creates a temperature drop and a resulting loss in thermal efficiency, alternatives are being considered for future plant. In one alternative, steam would be generated directly inside the receiver tubes. Consider an example in which a heat flux of 50,000 W/m^2 is absorbed on the outside surface of a 12.7 mm I.D., stainless steel 316 tube with a wall thickness of 1.245 mm. Inside the tube, saturated liquid water at 300°C flows at a rate of 100 kg/hr. Determine the tube wall temperature if the steam quality is to be increased to 0.5. Assume the viscosity of steam at operating pressure is $\mu_v = 2.0 \times 10^{-5}$ kg/(ms). Neglect heat losses from the outside of the receiver. See Goswami, Kreith, and Kreider Goswami et al. [95] for a system description.

GIVEN

- 12.7 mm i.d. tube with flowing, boiling water inside
- 50,000 W/m^2 heat flux at tube o.d.
- 100 kg/hr water enters the tube at saturated liquid conditions, 300°C
- Absolute viscosity of the steam is $\mu_v = 2 \times 10^{-5}$ kg/(ms)
- Tube heat losses are negligible

FIND

(a) Tube wall temperature if steam quality is to be 0.5 at tube exit

ASSUMPTIONS

- The method of Chen is applicable

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for T_{sat} = 300°C we have

Heat of vaporization (h_{fg}) = 1.403×10^6 J/kg

Saturation pressure (P_{sat}) = 8.592×10^6 N/m^2

Vapor density (ρ_v) = 46.3 kg/m^3

Liquid density (ρ_l) = 712.5 kg/m^3

Liquid absolute viscosity (μ_l) = 92.2×10^{-6} kg/(ms)

Liquid Prandtl number (Pr_l) = 0.98

Liquid thermal conductivity (k_l) = 0.564 W/(m K)

Liquid specific heat (c_l) = 5694 J/(kg K)

and from Table 9.2

Surface tension (σ) = 0.0143 N/m

SOLUTION

We will follow the method of Chen described in Section 9.3.2. The tube flow area is $A_f = \pi D_i^2/4$ = 0.000127 m^2 and the tube outside radius is r_o = 12.7/2 mm + 1.245 = 7.6 mm. Then

$$G = \frac{\dot{m}}{A_f} = 100\ \text{kg/h} \quad \frac{1}{(0.000127\,\text{m}^2)} \quad \text{h}/3600\,\text{s} \quad = 218.7\ \text{kg/(m}^2\text{s)}$$

The heat flux at the inner tube wall is

$$q''_i = q''_o \frac{r_o}{r_i} = 50{,}000 \frac{7.6}{\frac{12.7}{2}} = 59{,}842 \text{ W/m}^2$$

The convective component of the heat transfer coefficient is

$$h_c = 0.023 \left[\frac{G(1-\times)D}{\mu_l}\right]^{0.8} Pr_l^{0.4} \frac{k_l}{D} F$$

We are interested in conditions at the end of the tube where the quality, x, is 0.5.

$$h_c = 0.023 \left[\frac{218.7 \text{ kg/(sm}^2) (1-0.5)(0.0127 \text{ m})}{92.2\times10^{-6} \text{ kg/(ms)}}\right]^{0.8} 0.98^{0.4} \frac{0.564 \text{ W/(m K)}}{(0.0127 \text{ m})} F = 2229F \text{ W/(m}^2\text{K)}$$

To find F, we must first find X from

$$\frac{1}{X_{tt}} = \left(\frac{x}{1-x}\right)^{0.9} \left(\frac{\rho_l}{\rho_v}\right)^{0.5} \left(\frac{\mu_v}{\mu_l}\right)^{0.1}$$

$$\frac{1}{X_{tt}} = \left(\frac{0.5}{1-0.5}\right)^{0.9} \left(\frac{712.5}{46.3}\right)^{0.5} \left(\frac{2\times10^{-5}}{92.2\times10^{-6}}\right)^{0.1} = 3.37 \text{ or } X_{tt} = 0.297$$

then F is

$$F = 2.35 \left(\frac{1}{X_{tt}} + 0.213\right)^{0.736} = 6.01$$

and the convective heat transfer coefficient is

$$h_c = 2229\, F = 2229 \text{ W/(m}^2\text{K)} \ (6.01) = 13{,}391 \text{ W/(m}^2\text{K)}$$

Now, the boiling heat transfer coefficient is given by Equation (9.12)

$$h_b = 0.00122 \left(\frac{(0.564)^{0.79}(5694)^{0.45}(712.5)^{0.49}(1)^{0.25}}{(0.0143)^{0.5}(92.2\times10^{-6})^{0.29}(1.403\times10^{6})^{0.24}(46.3)^{0.24}}\right) \Delta T_x^{0.24}\, \Delta p_{\text{sat}}^{0.75}\, S$$

In this equation, we check for SI units and ΔP_{sat} is in N/m^2. We have

$$h_b = 1.567\, \Delta T_x^{0.24}\, \Delta p_{\text{sat}}^{0.75}\, S$$

To find S, we need Re_{TP}

$$Re_{TP} = \frac{G(1-x)D}{\mu_l} F^{1.25} \times 10^{-4}$$

so

$$Re_{TP} = \frac{218.7 \text{ kg/(sm}^2) (1-0.5)(0.0127 \text{ m})}{92.2\times10^{-6} \text{ kg/(ms)}} (6.01)^{1.25} \times 10^{-4} = 14.17$$

and we find S from

$$S = (1 + 0.12\, Re_{TP}^{1.14})^{-1} = 0.2886$$

So the expression for the boiling heat transfer coefficient is

$$h_b = (1.567)\,(0.288)\Delta T_x^{0.24}\,\Delta p_{\text{sat}}^{0.75} = 0.4522\Delta T_x^{0.24}\,\Delta p_{\text{sat}}^{0.75}$$

From Table 13, we can approximate the relationship between saturation pressure and temperature

$$\frac{\Delta p_{\text{sat}}}{\Delta T_{\text{sat}}} = \frac{(85.917 - 64.191) \times 10^5 \ \text{N/m}^2}{20\,\text{K}} = 108{,}630 \ (\text{N/m}^2)/\text{K}$$

So

$$\Delta p_{\text{sat}} \approx 108{,}630\ \Delta T_{\text{sat}}$$

when ΔT_{sat} is expressed in K and ΔP_{sat} is expressed in N/m^2. Assuming that the excess temperature, ΔT_x is small, the above expression can be used to find Δp_{sat} by substituting ΔT_x for ΔT_{sat} in the above expression. Given this, we can further simplify the expression for the boiling heat transfer coefficient

$$h_b = 0.4522\Delta T_x^{0.24}\,(108{,}630\ \Delta T_x)^{0.75} = 2706\Delta T_x^{0.99} \approx 2706\Delta T_x$$

According to Chen, the two heat transfer coefficients can be added

$$h = h_c + h_b = 13{,}319 + 2706\Delta T_x$$

Since the heat flux can be written as

$$q'' = h\Delta T_x$$

we have the following relationship between the heat flux and the excess temperature

$$q'' = 13391\ \Delta T_x + 2706\ \Delta T_x^2 = 59{,}842$$

Solving this quadratic equation for the excess temperature we find

$$\Delta T_x = 2.84\ \text{K}$$

So we were justified in assuming that the excess temperature is small.

Finally, we need to calculate the temperature drop across the tube wall. From Equation (2.39)

$$L2\pi r_o q'' = \frac{\Delta T_{\text{wall}}}{\dfrac{\ln\left(\frac{r_0}{r_1}\right)}{2\pi k L}}$$

From Figure 1.7, the thermal conductivity of the 316 stainless steel is $k_{ss} = 17$ W/(mK).

Solving for the wall temperature drop

$$\Delta T_{\text{wall}} = \frac{r_o\, q''_o \ln\left(\dfrac{r_o}{r_i}\right)}{k_{ss}} = \frac{(0.0076\,\text{m})\ \ 50{,}000\ \text{W/m}^2\ \ \ln\ 7.5/6.35}{17\ \text{W/(m K)}} = 3.72\ \text{K}$$

The total temperature drop from the tube wall outer surface to the boiling water is

$$\Delta T_{\text{total}} = 2.8 + 3.7 = 6.5\ \text{K}$$

and the tube wall outer surface temperature is therefore

$$T_{\text{tube,outer}} = 300 + 6.5 = 306.5°\text{C, say } 307°\text{C}$$

PROBLEM 9.20

In space-based systems, such as the International Space Station, for example, highly efficient cooling systems are deployed that involve force convection boiling. In a laboratory experiment to develop one such system, a test liquid, which has the physical properties given, flows inside a vertical tube of 1.905 cm (or 0.75 in) inner diameter with a mass flux (or mass velocity) of 680 kg/(m^2 s). Determine the heat transfer coefficient at a point along the length of the tube at which the quality is 0.3. The wall temperature at this location was measured to be 180^{0}C.

ρ_l =567 kg/m^3	**ρ_v =18.1 kg/m^3**
c_l =2.73 kJ/(kg K)	**c_v =2.36 kJ/(kg K)**
μ_l =156 *10^{-6} kg/(m s)	**μ_v =7.11 *10^{-6} kg/(m s)**
k_l =0.086 W/(m K)	**σ_l =8.2*10^{-3} N/m**
T_{sat}= 163.8°C	**p_{sat} =310.3 kPa**

***p_{sat}* , corresponding to wall temperature**
$(T_S$ = 180°C$)$ = 416.6 kPa

GIVEN

- 1.905 cm vertical tube with flowing, boiling water inside
- Test liquid with properties given above
- Mass flux (G)= 680 kg/(m^2 s)

FIND

(a) Heat transfer coefficient at a point along the length of the tube at which the quality is 0.3

ASSUMPTIONS

- The method of Chen is applicable
- Prandtl number of the fluid is Pr_l=1.0

PROPERTIES AND CONSTANTS

From problem statement above, for T_{sat} = 163.8°C we have

Heat of vaporization (h_{fg}) = 1.403 × 10^6 J/kg
Saturation pressure (P_{sat}) = 3.103 × 10^5 N/m^2
Vapor density (ρ_v) = 18.1 kg/m^3
Liquid density (ρ_l) = 567 kg/m^3
Liquid absolute viscosity (μ_l) = 156× 10^{-6} kg/(ms)
Liquid thermal conductivity (k_l) = 0.086 W/(m K)
Liquid specific heat (c_l) = 2730 J/(kg K)
Surface tension (σ) = 0.0082 N/m

SOLUTION

We will follow the method of Chen described in Section 9.3.2.

$$G = 680 \text{ kg/(m}^2\text{s)}$$

$$\mathrm{Pr}_l = \frac{\mu_l c_p}{k_l} = \frac{156*10^{-6}*2730}{0.086} = 4.95$$

The convective component of the heat transfer coefficient is

$$h_c = 0.023\left[\frac{G(1-\times)D}{\mu_l}\right]^{0.8} Pr_l^{0.4}\ \frac{k_l}{D} F$$

We are interested in conditions at the end of the tube where the quality, x, is 0.5.

$$h_c = 0.023\left[\frac{\left(680\ \text{kg/(sm}^2)\right)(1-0.3)(0.01905\,\text{m})}{\left(156\times10^{-6}\ \text{kg/(ms)}\right)}\right]^{0.8} 4.95^{0.4}\ \frac{(0.086\ \text{W/(m K)})}{(0.01905\,\text{m})}\ F = 1275.5F\ \ \text{W/(m}^2\text{K)}$$

To find F, we must first find X from

$$\frac{1}{X_{tt}} = \left(\frac{x}{1-x}\right)^{0.9}\left(\frac{\rho_l}{\rho_v}\right)^{0.5}\left(\frac{\mu_v}{\mu_l}\right)^{0.1}$$

$$\frac{1}{X_{tt}} = \left(\frac{0.3}{1-0.3}\right)^{0.9}\left(\frac{567}{18.1}\right)^{0.5}\left(\frac{7.11\times10^{-6}}{156\times10^{-6}}\right)^{0.1} = 1.92 \text{ or } X_{tt} = 0.522$$

then F is

$$F = 2.35\left(\frac{1}{X_{tt}} + 0.213\right)^{0.736} = 4.10$$

and the convective heat transfer coefficient is

$$h_c = 672.7\ F = \left(672.7\ \text{W/(m}^2\text{K)}\right)\ (4.10) = 2760\ \ \text{W/(m}^2\text{K)}$$

We have ΔT_x=180-163.8= 16.2^{0}C

Δp_{sat}=416.6-310.3= 106.3 kPa= 1.063*10^5 Pa

Now, the boiling heat transfer coefficient is given by Equation (9.12)

$$h_b = 0.00122\left(\frac{(0.086)^{0.79}(2730)^{0.45}(567)^{0.49}(1)^{0.25}}{(0.0082)^{0.5}(156\times10^{-6})^{0.29}(1.403\times10^{6})^{0.24}(18.1)^{0.24}}\right) 16.2^{0.24}\ (1.063*10^5)^{0.75} *S$$

$$h_b = \left(\frac{113.2}{0.4266}\right) * 14.01*S = 3717.6*S$$

To find S, we need Re_{TP}

$$Re_{TP} = \frac{G(1-x)D}{\mu_l}\ F^{1.25}\times10^{-4}$$

So $$Re_{TP} = \frac{\left(680\ \text{kg/(sm}^2)\right)(1-0.3)(0.01905\,\text{m})}{\left(156\times10^{-6}\ \text{kg/(ms)}\right)}\ (4.1)^{1.25}\times10^{-4} = 5.81$$

and we find S from

$$S = (1 + 0.12\ Re_{TP}^{1.14})^{-1} = 0.528$$

So the expression for the boiling heat transfer coefficient is

h_b= 3717.6*S=3717.6*0.528= 1964 W/ (m^2 K)

Thus, heat transfer coefficient at a point along the length of the tube at which the quality is 0.3

$$h = h_c + h_b = 4724\ \text{W/(m}^2\ \text{K)}$$

PROBLEM 9.21

Calculate the average heat transfer coefficient for film-type condensation of water at pressures of 10 kPa and 101 kPa for (a) a vertical surface 1.5-m-high (b) the outside surface of a 1.5-cm-OD vertical tube 1.5-m-long (c) the outside surface of a 1.6-cm-*OD* horizontal tube 1.5-m-long and (d) a 10-tube vertical bank of 1.6-cm-*OD* horizontal tubes 1.5-m-long. In all cases, assume that the vapor velocity is negligible and that the surface temperatures are constant at 11°C below saturation temperature.

GIVEN

- Film condensation of water

FIND

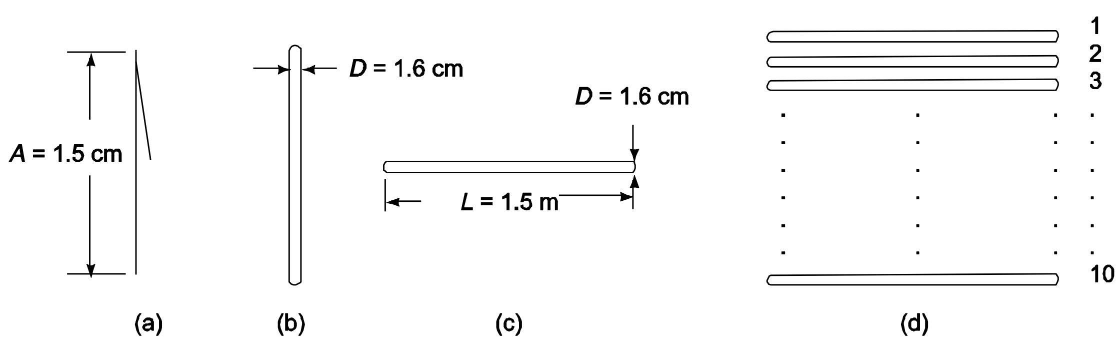
The average heat transfer coefficient at pressure of 10 kPa and 101 kPa for

(a)	A vertical surface of height (H) = 1.5 m	
(b)	The outside surface of a vertical tube	Outside diameter (D) = 1.6 cm = 0.016 m Height (H) = 1.5 m
(c)	The outside surface of a horizontal tube	Outside diameter (D) = 1.6 cm = 0.016m Length (L) = 1.5 m
(d)	A 10 tube vertical bank of horizontal tubes	Outside diameter (D) = 1.6 cm Length (L) = 1.5 m

ASSUMPTIONS

- Steady state
- Vapor velocity is negligible
- Surface temperatures (T_s) are constant at 11°C below saturation temperature
- Film thickness is much smaller than the pipe diameter
- Laminar condensate flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the saturation temperatures for water at

101 kPa (T_{sv1}) = 100°C, therefore $T_s = T_{sv} - 11°C = 89°C$

10 kPa (T_{sv2}) = 45.3°C, therefore, T_s = 34.3°C

The film temperatures, as given in Section 9.4.1 are

$T_{\text{film1}} = T_s + 0.25\,(T_{sv} - T_s) = 89°C + 0.25\,(11°C) = 91.8°C$

T_{film2} = 34.3°C+ 0.25 (11°C) = 37.1°C

From Appendix 2, Table 13, for water at the film temperatures

Film Temperature, °C	91.8	37.1

Density, ρ_l (kg/m^3)	963.8	993.3
Thermal conductivity, k (W/(m K))	0.678	0.628
Absolute viscosity, $\mu_l \times 10^6$ (Ns/m^2)	310.0	693.8
Vapor density, $\rho_v = 1/v_g$ (kg/m^3)	0.4468	0.0427
Heat of vaporization, $h_{fg} \times 10^{-6}$ (J/kg)	2.278	2.413
Specific heat, c_{pl} (J/(kg K))	4204	4175

SOLUTION

The solution will first be worked for p = 101 kPa

(a) The average heat transfer coefficient on a vertical plate is given by Equation (9.21)

$$\bar{h}_c = 0.943 \left[\frac{\rho_l(\rho_l - \rho_v) g\, h'_{fg}\, k^3}{\mu_l L (T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

where $h'_{fg} = h_{fg} + \frac{3}{8}\, c_{pl}\,(T_{sv} - T_s) = 2.278 \times 10^6 \text{ J/(kg)} + \frac{3}{8}\, 4204 \text{ J/(kg K)}\ (11°\text{C})$

$= 2.295 \times 10^6$ J/kg

Rohsenow's analysis showed that h'_{fg} should be replaced by $h_{fg} + 0.68\ c_{pl}\ (T_{sv} - T_s)$ if $c_{pl}\,(T_{sv} - T_s)/h_{fg} < 1$

$$\frac{c_{pl}(T_{sv} - T_s)}{h'_{fg}} = \frac{4204 \text{ J/(kg K)}\ (11°\text{C})}{2.295 \times 10^6 \text{ J/kg}} = 0.0201 < 1$$

Therefore, the Rohsenow results will be used

$h''_{fg} = h_{fg} + 0.68 c_{p1}\,(T_{sv} - T_s) = 2.278 \times 10^6 \text{ J/(kg)} + 0.68\ 4204 \text{ J/(kg K)}\ (11°\text{C}) = 2.309 \times 106\ \text{J/kg}$

$\bar{h}_c = 0.943$

$$\left[\frac{963.8 \text{ kg/m}^3\ (963.8 - 0.4463) \text{ kg/m}^3\ 9.8 \text{ m/s}^2\ \ 2.309 \times 10^6 \text{ J/kg}\ \ (\text{W s})/\text{J}\ \ 0.678 \text{ W/(m K)}^{\ 3}}{310.0 \times 10^{-6}\ (\text{N s})/\text{m}^2\ \ (\text{kg m})/(\text{s}^2\text{N})\ (1.5 \text{ m})(11°\text{C})} \right]^{\frac{1}{4}}$$

$$\bar{h}_c = 5641\ \text{W/(m}^2\text{K)}$$

(b) For vertical tubes large in diameter compared to the film thickness, the heat transfer coefficient is the same as a vertical flat plate. Therefore, h_c = 5651 W/(m^2 K).

(c) The heat transfer coefficient for horizontal tubes is given by Equation (9.23)

$$\bar{h}_c = 0.725 \left[\frac{\rho_l(\rho_l - \rho_v) g\, h'_{fg}\, k^3}{\mu_l D (T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

$\bar{h}_c = 0.725$

$$\left[\frac{963.8 \text{ kg/m}^3\ (963.8 - 0.4463) \text{kg/m}^3\ 9.8 \text{m/s}^2\ \ 2.309 \times 10^6 \text{J/kg}\ \ (\text{Ws})/\text{J}\ \ 0.678 \text{W/(m K)}^{\ 3}}{310.0 \times 10^{-6}\ (\text{N s})/\text{m}^2\ \ (\text{kg m})/(\text{s}^2\text{N})\ (0.016 \text{ m})(11°\text{C})} \right]^{\frac{1}{4}}$$

$$\bar{h}_c = 13{,}495\ \text{W/(m}^2\text{K)}$$

(d) The heat transfer coefficient on the tube bank is given by Equation (9.24)

$$\bar{h}_c = 0.778\left[1+0.2\frac{c_p(T_{sv}-T_s)}{h_{fg}}(N-1)\right]\left[\frac{\rho_l(\rho_l-\rho_v)g\,h'_{fg}\,k^3}{\mu_l\,N\,D(T_{sv}-T_s)}\right]^{\frac{1}{4}}$$

$$\text{Provided } \frac{(N-1)\,c_p(T_{sv}-T_s)}{h_{fg}} < 2$$

$$\frac{(10-1)\ 4204\ \text{J/(kg K)}\ (11°\text{C})}{2.278\times10^6\ \text{J/kg}} = 0.183 < 2$$

$$\bar{h}_c = 0.728\ [1 + 0.2\ (0.183)]$$

$$\left[\frac{963.8\ \text{kg/m}^3\ (963.8-0.4463)\ \text{kg/m}^3\ 9.8\ \text{m/s}^2\ \ 2.309\times10^6\ \text{J/kg}\ \ (\text{Ws})/\text{J}\ \ 0.678\ \text{W/(m K)}^{\ 3}}{310.0\times10^{-6}\ (\text{N s})/\text{m}^2\ \ (\text{kg m})/(\text{s}^2\text{N})\ 10(0.016\,\text{m})(11°\text{C})}\right]^{\frac{1}{4}}$$

$$\bar{h}_c = 7899\ \ \text{W/(m}^2\text{K)}$$

Repeating this procedure for p = 10 kPa and tabulating all of the heat transfer coefficients in W/(m^2 K)

Pressure (kPa)	101	10
Case (a)	5641	4484
Case (b)	5641	4484
Case (c)	13,495	10,728
Case (d)	7899	6265

PROBLEM 9.22

The inside surface of a 1-m-long vertical 5-cm-*ID* tube is maintained at 120°C. For saturated steam at 350 kPa condensing inside, estimate the average heat transfer coefficient and the condensation rate, assuming the steam velocity is small.

GIVEN

- Steam condensing inside a vertical tube
- Tube length (L) = 1 m
- Tube inside diameter (D) = 5 cm = 0.05 m
- Tube surface temperature (T_s) = 120°C
- Steam pressure (p) = 350 kPa

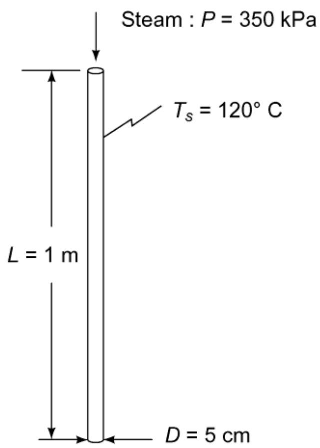
FIND

- The average heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state
- The steam velocity is small
- Film condensation occurs

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 350 kPa

Saturation temperature (T_{sv}) = 138.6°C

Liquid density (ρ_l) = 927.5 kg/m^3

Vapor density ($\rho_v = 1/v_g$) = 1.87 kg/m^3

Thermal conductivity (k_l) = 0.684 W/(m K)

Heat of vaporization (h_{fg}) = 2148 kJ/kg = 2.148×10^6 J/kg

Absolute viscosity (μ_l) = 203.4×10^{-6} (Ns)/m^2

Specific heat (c_{pl}) = 4255 J/(kg K)

SOLUTION

The condensate layer thickness (δ) at the bottom of the tube ($x = L$) can be estimated using Equation (9.17)

$$\delta = \left[\frac{4\mu_l\, k \times (T_{sv} - T_s)}{g\,\rho_l\,(\rho_l - \rho_v) h'_{fg}}\right]^{\frac{1}{4}}$$

where $h'_{fg} = h_{fg} + \frac{3}{8} c_{pl}\,(T_{sv} - T_s)$

$$h'_{fg} = \; 2.148 \times 10^6 \text{ J/kg} \; + \frac{3}{8} \; 4255\,\text{J/(kg K)} \; (138.6°\text{C} - 120°\text{C}) = 2.178 \times 10^6 \text{ J/kg}$$

$$\delta = \left[\frac{4\; 203.4 \times 10^{-6}\,(\text{N s})/\text{m}^2 \;\; (\text{kg m})/(\text{s}^2\text{N}) \;\; 0.684\,\text{W/(m K)} \;\; \text{J/(W s)} \;\; (1\text{m})(138.6°\text{C} - 120°\text{C})}{9.8\,\text{m/s}^2 \;\; 927.5\,\text{kg/m}^3 \;\; (927.5 - 1.87)\,\text{kg/m}^3 \;\; 2.178 \times 10^6\,\text{J/kg}}\right]^{\frac{1}{4}}$$

$= 1.5 \times 10^{-4}$ m

Since the condensate layer is much smaller than the tube diameter, Equation (9.21) can be used to estimate the average heat transfer coefficient.

$$\bar{h}_c = 0.943 \left[\frac{\rho_l(\rho_l - \rho_v) g\, h'_{fg}\, k^3}{\mu_l\, L\,(T_{sv} - T_s)}\right]^{\frac{1}{4}}$$

Rohsenow's analysis showed that h'_{fg} should be replaced by $h'_{fg} + 0.68\; c_{pl}\; (T_{sv} - T_s)$ if $c_{pl}\;(T_{sv} - T_s)/h'_{fg} < 1$

$$\frac{c_{pl}(T_{sv} - T_s)}{h'_{fg}} = \frac{4255\,\text{J/(kg K)} \;\; (138.6°\text{C} - 120°\text{C})}{2.178 \times 10^6 \text{ J/kg}} = 0.0360 < 1$$

$$h'_{fg} = h_{fg} + 0.68\, c_{pl}\,(T_{sv} - T_s) = 2.148 \times 10^6 \text{ J/kg}$$

$$+\, 0.68 \;\; 4255\,\text{J/(kg K)} \;\; (138.6°\text{C} - 120°\text{C}) = 2.202 \times 10^6 \text{ J/kg}$$

$\bar{h}_c = 0.943$

$$\left[\frac{927.5\,\text{kg/m}^3 \;\; (927.5 - 1.87)\,\text{kg/m}^3 \;\; 9.8\,\text{m/s}^2 \;\; 2.202 \times 10^6\,\text{J/kg} \;\; (\text{W s})/\text{J} \;\; 0.684\,\text{W/(m K)}^{\;3}}{203.4 \times 10^{-6}\,(\text{N s})/\text{m}^2 \;\; (\text{kg m})/(\text{s}^2\text{N}) \;\; (1\text{m})(138.6°\text{C} - 120°\text{C})}\right]^{\frac{1}{4}}$$

$$\bar{h}_c = 5933 \;\text{W/(m}^2\text{K)}$$

The above analysis assumes the condensate layer is laminar. This assumption can be checked by checking the Reynolds number at the bottom of the tube. With the aid of Equation (9.14), the Reynolds number can be written as

$$Re_\delta = \frac{4\Gamma_c}{\mu_l} = \frac{4\rho_l^2 g\,\delta^3}{3\mu_l^2}$$

Substituting Equation (9.17) for δ yields

$$Re_\delta = \frac{4\rho_l^2 g}{3\mu_l^2}\left[\frac{4\mu_l\, k_l\, L\,(T_{sv} - T_s)}{g\,\rho_l^2\, h'_{fg}}\right]^{\frac{3}{4}} = 985 < 2000$$

Therefore, the laminar assumption is valid.

PROBLEM 9.23

A horizontal 2.5-cm-*OD* tube is maintained at a temperature of 27°C on its outer surface. Calculate the average heat transfer coefficient if saturated steam at 12 kPa is condensing on this tube.

GIVEN

- Saturated steam condensing on a horizontal tube
- Tube outside diameter (D) = 2.5 cm = 0.025 m
- Tube outer surface temperature (T_s) = 27°C
- Steam pressure (p) = 12 kPa

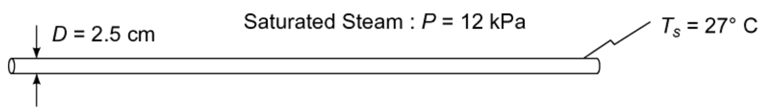
FIND

- The average heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state
- Film condensation occurs

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 12 kPa

Saturation temperature (T_s) = 49.3°C

Liquid density (ρ_l) = 988.4 kg/m^3

Vapor density ($\rho_v = 1/v_g$) = 0.0797 kg/m^3

Thermal conductivity (k_l) = 0.646 W/(m K)

Heat of vaporization (h_{fg}) = 2384 kJ/kg = 2.384 × 10^6 J/kg

Absolute viscosity (μ_l) = 562.1 × 10^{-6} (Ns)/m^2

Specific heat (c_l) = 4178 J/(kg K)

SOLUTION

The heat transfer coefficient for this geometry is given by Equation (9.23)

$$\bar{h}_c = 0.725 \left[\frac{\rho_l(\rho_l - \rho_v)\, g h'_{fg}\, k^3}{\mu_l D (T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

where $h'_{fg} = h_{fg} + 3/8\, c_{pl}\,(T_{sv} - T_s) = 2.384 \times \text{J/kg} + (3/8)\ \ 4178\,\text{J/(kg K)}\ \ (49.3°\text{C} - 27°\text{C})$

$$= 2.419 \times 10^6\ \text{J/kg}$$

$\bar{h}_c$ = 0.725

$$\left[\frac{988.4\,\text{kg/m}^3\ \ (988.4 - 0.0797)\,\text{kg/m}^3\ \ 9.8\,\text{m/s}^2\ \ \ 2.419 \times 10^6\,\text{J/kg}\ \ \ (\text{Ws})/\text{J}\ \ \ 0.646\,\text{W/(m K)}\ ^3}{562.1 \times 10^{-6}\,(\text{N s})/\text{m}^2\ \ \ (\text{kg m})/(\text{s}^2\text{N})\ \ (0.025\,\text{m})(49.3°\text{C} - 27°\text{C})} \right]^{\frac{1}{4}}$$

$$= 8613\ \ \text{W/(m}^2\text{K)}$$

PROBLEM 9.24

Repeat Problem 9.23 for a tier of six horizontal 2.5-cm-*OD* tubes under similar thermal conditions.

GIVEN

- Saturated steam condensing on horizontal tubes
- Tube outside diameter (D) = 2.5 cm = 0.025 m
- Tube outer surface temperature (T_s) = 27°C
- Steam pressure (p) = 12 kPa
- Number of tubes (n) = 6

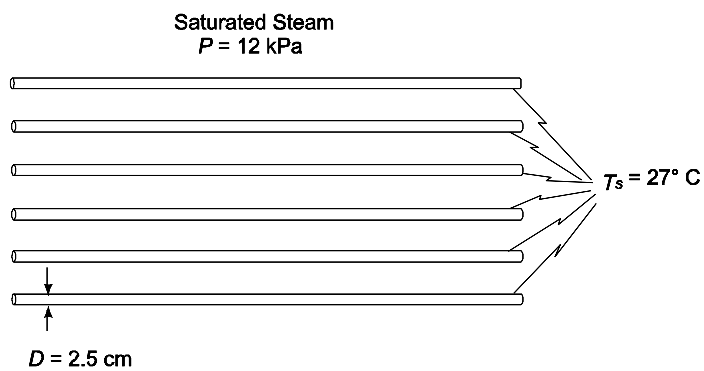
FIND

- The average heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state
- Film condensation occurs

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 12 kPa

Saturation temperature (T_s) = 49.3°C

Liquid density (ρ_l) = 988.4 kg/m^3

Vapor density ($\rho_v = 1/v_g$) = 0.0797 kg/m^3

Thermal conductivity (k_l) = 0.646 W/(m K)

Heat of vaporization (h_{fg}) = 2384 kJ/kg = 2.384 × 10^6 J/kg

Absolute viscosity (μ_l) = 562.1 × 10^{-6} (Ns)/m^2

Specific heat (c_l) = 4178 J/(kg K)

SOLUTION

The average heat transfer coefficient for the tube bank is given by Equation (9.24)

$$\bar{h}_c \;=\; 0.728\left[1+0.2\frac{c_p\,(T_{sv}-T_s)}{h_{fg}}(N-1)\right]\left[\frac{\rho_l\;\;\rho_l-\rho_v\;\;g h'_{fg} k^3}{\mu_l N\,D\;\;T_{sv}-T_s}\right]^{\frac{1}{4}}$$

where $h'_{fg} = h_{fg} + \dfrac{3}{8}\,c_{pl}(T_{sv} - T_s) = \;\; 2.384\times10^6\ \mathrm{J/kg} \;+\; 4178\,\mathrm{J/(kg\,K)}\;\;(49.3°\mathrm{C} - 27°\mathrm{C})$

$$= 2.419 \times 10^6 \text{ J/kg}$$

Provided $\dfrac{(N-1)c_p(T_{sv} - T_s)}{h_{fg}} < 2$

$$\frac{(6-1)\ 4178 \text{ J/(kg K)}\ (49.3°\text{C} - 27°\text{C})}{2.384 \times 10^6 \text{ J/kg}} = 0.1954 < .2$$

$$\bar{h}_c = 0.728\ [1 + 0.2\ (0.1954)]$$

$$\left[\frac{988.4 \text{ kg/m}^3\ (988.4 - 0.0797)\text{kg/m}^3\ 9.8 \text{ m/s}^2\quad 2.419 \times 10^6 \text{ J/kg}\quad (\text{Ws})/\text{J}\quad 0.646 \text{ W/(m K)}^{\ 3}}{562.1 \times 10^{-6} \text{ (N s)/m}^2\quad (\text{kg m})/(\text{s}^2\text{N})\ \ 6(0.025 \text{ m})(49.3°\text{C} - 27°\text{C})}\right]^{\frac{1}{4}}$$

$$\bar{h}_c = 5742 \text{ W/(m}^2\text{K)}$$

PROBLEM 9.25

Saturated steam at 34 kPa condenses on a 1-m-tall vertical plate whose surface temperature is uniform at 60°C. Compare the average heat transfer coefficient and the value of the coefficient 1/3, 2/3, and 1 m from the top. Also, find the maximum plate height for which the condensate film will remain laminar.

GIVEN

- Saturated steam condensing on a vertical plate
- Steam pressure (p) = 34 kPa
- Plate height (L) = 1 m
- Plate surface temperature (T_s) = 60°C (uniform)

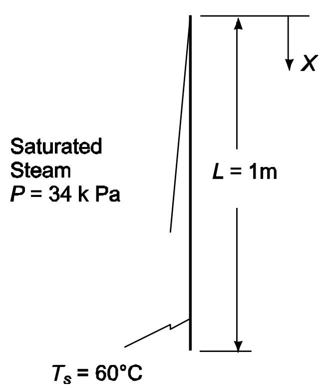
FIND

(a) The average heat transfer coefficient (h_c)
(b) The local heat transfer coefficient (h_α) at x = 1/3 L, 2/3 L, and L
(c) The maximum height for which the condensate film will remain laminar

ASSUMPTIONS

- Steady state
- Film condensation occurs

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated water at 34 kPa

Saturation temperature (T_s) = 71.8°C

Liquid density (ρ_l) = 976.6 kg/m^3

Vapor density ($\rho_v = 1/v_g$) = 0.21 kg/m^3

Thermal conductivity (k_l) = 0.668 W/(m K)

Specific heat (c_l) = 4188 J/(kg K)

Heat of vaporization (h_{fg}) = 2329 kJ/kg = 2.329 × 10^6 J/kg

Absolute viscosity (μ_l) = 3.998 × 10^{-4} (N s)/m^2

SOLUTION

(a) The average heat transfer coefficient is given by Equation (9.21)

$$\bar{h}_c = 0.943 \left[\frac{\rho_l(\rho_l - \rho_v)\, g h'_{fg}\, k^3}{\mu_l\, L\,(T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

where $h'_{fg} = h_{fg} + \frac{3}{8}\, c_{pl}(T_{sv} - T_s)$

$$h'_{fg} = (2.329 \times 10^6 \text{ J / Kg}) + \; 4188\,\text{J/(kg K)}\;\; (71.8°\text{C} - 60°\text{C}) = 2.348 \times 10^6 \;\text{J/(kg K)}$$

For Equation (9.21), Rohsenow recommends h_{fg} be replaced by h_{fg} + 0.68 c_{p1} $(T_{sv} - T_s)$ if $c_{pl}\,(T_{sv} - T_s)/h'_{fg} < 1$

$$\frac{C_{pl}(T_{sv} - T_s)}{h'_{fg}} = \frac{4188\,\text{J/kg K}\;\; (11.8°\text{C})}{2.348 \times 10^6\,\text{J/kg}} = 0.021 < 1$$

$$h'_{fg} = h_{fg} + 0.68\, c_{pl}\,(T_{sv} - T_s) = (2.329 \times 10^6\,\text{J/kg}) + 0.68\;\; 4188\,\text{J/(kg K)}\;\; (11.8°\text{C})$$

$$= 2.363 \times 10^6 \;\text{J/kg}$$

$\bar{h}_c$ = 0.943

$$\left[\frac{976.6\,\text{kg/m}^3\;\; (976.6 - 0.21)\,\text{kg/m}^3\;\; 9.8\,\text{m/s}^2\;\;\; 2.363 \times 10^6\,\text{J/kg}\;\;\; (\text{Ws})/\text{J}\;\;\; 0.668\,\text{W/(m K)}^{\;3}}{3.998 \times 10^{-4}\,(\text{N s})/\text{m}^2\;\;\; (\text{kg m})/(\text{s}^2\text{N})\;\; (1\,\text{m})(11.8°\text{C})}\right]^{\frac{1}{4}}$$

$$\bar{h}_c = 5763 \;\; \text{W/(m}^2\text{K)}$$

(b) The local heat transfer coefficient is given by Equation (9.18)

$$h_x = \left[\frac{\rho_l(\rho_l - \rho_v)\,g h'_{fg}\,k^3}{4\mu_1\,x(T_{sv} - T_s)}\right]^{\frac{1}{4}} = \bar{h}_c\;\left(\frac{L}{4x}\right)^{\frac{1}{4}}\;\left(\frac{1}{0.943}\right)$$

At $x = \frac{1}{3}L$ $\qquad h_x = \;\; 5763\,\text{W/(m}^2\text{K)}\;\; \left(\frac{3}{4}\right)^{\frac{1}{4}}\;\left(\frac{1}{0.943}\right) = 5687 \;\; \text{W/(m}^2\text{K)}$

At $x = \frac{2}{3}L$ $\qquad h_x = \;\; 5763\,\text{W/(m}^2\text{K)}\;\; \left(\frac{3}{8}\right)^{\frac{1}{4}}\;\left(\frac{1}{0.943}\right) = 4782 \;\; \text{W/(m}^2\text{K)}$

At $x = L$ $\qquad h_x = \;\; 5763\,\text{W/(m}^2\text{K)}\;\; \left(\frac{1}{4}\right)^{\frac{1}{4}}\;\left(\frac{1}{0.943}\right) = 4321 \;\; \text{W/(m}^2\text{K)}$

(c) Turbulence occurs when $Re_\delta = \frac{4\Gamma_c}{\mu_l} = 2000$

Combining the definition of the Reynolds number with Equation (9.14)

$$Re_\delta = \frac{4\rho_l^2 g \delta^3}{3\mu_l^2}$$

Solving this for the critical film thickness

$$\delta_c = \left[\frac{3 Re_{\delta c}\mu_l^2}{4\rho_l^2\, g}\right]^{\frac{1}{3}} = \left[\frac{3(2000)\;\; 3.998 \times 10^{-4}\,(\text{N s})/\text{m}^{2\;\;\;2}\;\;\; (\text{kg m})/(\text{s}^2\text{N})^2}{4\;\; 976.6\,\text{kg/ m}^2\;\;\; 9.8\,\text{m/s}^2}\right]^{\frac{1}{3}} = 0.000295 \text{ m}$$

Solving Equation (9.17) for the distance x down a flat plate at which the film thickness is δ:

$$x = \frac{\delta^4 g\,\rho_l(\rho_l - \rho_v)\,h'_{fg}}{4\mu_l\,k(T_{sv} - T_s)}$$

$$x = \frac{(0.000295\,\text{m})\;\; 9.8\,\text{m/s}^2\;\;\; 976.6\,\text{kg/m}^3\;\;\; 976.6 - 0.21\,\text{kg/m}^3\;\;\; 2.363 \times 10^6\,\text{J/kg}}{4\;\; 3.998 \times 10^{-4}\,(\text{N s})/\text{m}^2\;\;\; (\text{kg m})/(\text{s}^2\text{N})\;\;\; 0.668\,\text{W/(m K)}\;\;\; \text{J/(W s)}\;\;\; (11.8°\text{C})} = 13.2 \text{ m}$$

PROBLEM 9.26

At a pressure of 490 kPa, the saturation temperature of sulfur dioxide (SO_2) is 32°C, the density is 1350 kg/m^3, the latent heat of vaporization is 343 kJ/kg, the absolute viscosity is 3.2×10^{-4} (Ns)/m^2, the specific heat is 1445 J/(kg K) and the thermal conductivity is 0.192 W/(m K). If the SO_2 is to be condensed at 490 kPa on a 20 cm flat surface that is inclined at an angle at 45° and, whose temperature is maintained uniformly at 24°C, calculate (a) the thickness of the condensate film 1.3 cm from the bottom, (b) the average heat transfer coefficient and (c) the rate of condensation in kilograms per hour.

GIVEN

- SO_2 condensing on a flat surface inclined 45°
- Pressure (p) = 490 kPa
- SO_2 properties
  - Saturation temperature (T_{sv}) = 32°C
  - Liquid density (ρ_l) = 1350 kg/m^3
  - Heat of vaporization (h_{fg}) = 343 kJ/kg = 343, 000 J/kg
  - Absolute viscosity (μ_l) = 3.2×10^{-4} (Ns)/m^2
  - Specific heat (c_{pl} = 1445 J/(kg K))
  - Thermal conductivity (k) = 0.192 W/(m K)
- Surface temperature (T_s) = 24°C (uniform)
- Length of inclined edge of surface (L) = 20 cm = 0.2 m

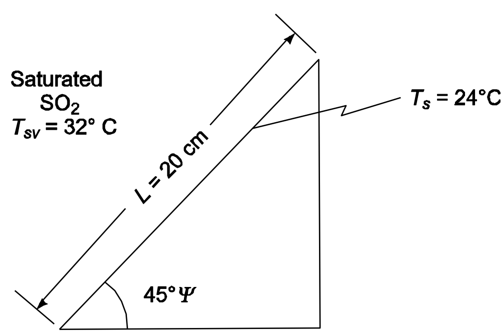
FIND

(a) Condensate film thickness (δ) at 1.3 cm from the bottom ($x = L - 1.3$ cm = 0.187 m)
(b) The average heat transfer coefficient (h_c), and
(c) The rate of condensation (m) in kg/h

ASSUMPTIONS

- Steady state
- Laminar condensate flow
- Vapor density is negligible compared to the liquid density
- Interfacial shear and momentum effects are negligible

SKETCH



SOLUTION

(a) Assuming the condensate element shown in Figure 9.19 is on an inclined plane at an angle ψ with the horizontal, the force balance on the element becomes

$$(\delta - y)\,(\rho_l - \rho_v)\,g\,\sin\psi = \mu_l\,\frac{du}{dy}$$

The constant sin ψ can be carried through the derivation shown in Section 9.4.1 to yield the following version of Equation (9.17)

$$\delta = \left[\frac{4\mu_l\, k\, x(T_{sv} - T_s)}{g \sin\psi\, \rho_l(\rho_l - \rho_v) h'_{fg}} \right]^{\frac{1}{4}}$$

$$\text{where } h'_{fg} = h_{fg} + \frac{3}{8}\, c_{pl}\,(T_{sv} - T_s)$$

$$h'_{fg} = 343{,}000\ \text{J/kg} + \frac{3}{8}\ 1445\ \text{J/(kg K)}\ (32°\text{C} - 24°\text{C}) = 347{,}335\ \text{J/kg}$$

Neglecting the vapor density, the condensate film thickness at x = 0.187 m is

$$\delta = \left[\frac{4\ 3.2\times10^{-4}(\text{N s})/\text{m}^2\ \ (\text{kg m})/(\text{s}^2\text{N})\ \ 0.192\ \text{W/(m K)}\ \ \text{J/(W s)}\ (0.187\ \text{m})(8°\text{C})}{9.8\ \text{m/s}^2\ (\sin 45°)\ 1350\ \text{kg/m}^3{}^{\ 2}\ \ 347{,}335\ \text{J/kg}} \right]^{\frac{1}{4}} = 9.57 \times 10^{-5}\ \text{m}$$

(b) The average heat transfer coefficient is given by Equation (9.22a)

$$\bar{h}_c = 0.943 \left[\frac{\rho_l(\rho_l - \rho_v)\, g h'_{fg}\, k^3 \sin\psi}{\mu_l\, L(T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

$$\bar{h}_c = 0.943 \left[\frac{1350\ \text{kg/m}^3{}^{\ 2}\ \ 9.8\ \text{m/s}^2\ \ 347{,}335\ \text{J/kg}\ \ (\text{W s})/\text{J}\ \ 0.0192\ \text{W/(m K)}^{\ 3}}{3.2\times10^{-4}\ (\text{N s})/\text{m}^2\ \ (\text{kg m})/(\text{s}^2\text{N})(0.2\ \text{m})(8°\text{C})} \right]^{\frac{1}{4}} = 2631\ \text{W/(m}^2\text{K)}$$

(c) An energy balance yields

$$\dot{m}\, h_{fg} = \bar{h}_c\ A\,(T_{sv} - T_s) = h_c\, L\, w\,(T_{sv} - T_s)$$

$$\frac{\dot{m}}{w} = \frac{\bar{h}_c\, L(T_{sv} - T_s)}{h_{fg}} = \frac{2631\ \text{W/(m}^2\text{K)}\ \ \text{J/(W s)}\ \ 3600\ \text{s/h}\ (0.2\ \text{m})(8°\text{C})}{343{,}000\ \text{J/kg}}$$

$$\frac{\dot{m}}{w} = 44.2\ \text{kg/h}\ \text{ per meter width}$$

PROBLEM 9.27

Repeat Problem 9.26 part (b) and (c) but assume that condensation occurs on a 5 cm-*OD* horizontal tube.

GIVEN

- SO_2 condensing on a horizontal tube
- Tube outside diameter (D) = 5 cm = 0.05 m
- Pressure (p) 490 kPa
- SO_2 properties
    - Saturation temperature (T_{sv}) = 32°C
    - Liquid density (ρ_l) = 1350 kg/m^3
    - Heat of vaporization (h_{fg}) = 343 kJ/kg = 343,000 J/kg
    - Absolute viscosity (μ_l) = 3.2×10^{-4} (N s)/m^2
    - Specific heat (c_{pl}) = 1445 J/(kg K)
    - Thermal conductivity (k) = 0.192 W/(m K)
- Surface temperature (T_s) = 24°C (uniform)

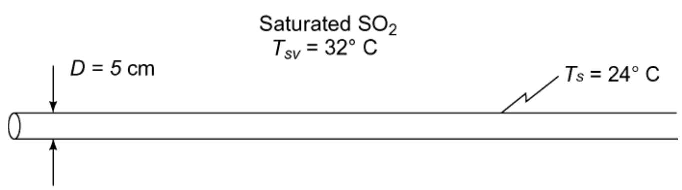
FIND

(a) Condensate film thickness (δ) at 1.3 cm from the bottom ($x = L - 1.3$ cm = 0.187 m)
(b) The average heat transfer coefficient (h_c)
(c) The rate of condensation ($\dot{m}$) in kg/h

ASSUMPTIONS

- Steady state
- Laminar condensate flow
- Vapor density is negligible compared to the liquid density

SKETCH



SOLUTION

(a) (See solution for (a) in Problem 9.26.)
(b) The average heat transfer coefficient is given by Equation (9.23)

$$\bar{h}_c = 0.725 \left[\frac{\rho_l(\rho_l - \rho_v)\, g h'_{fg} k^3}{\mu_l\, D(T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

where $\quad h'_{fg} = h_{fg} + \frac{3}{8} c_{pl}\,(T_{sv} - T_s)$

$$h'_{fg} = \ 343{,}000\ \text{J/kg} \ \ + \ 3/8 \ \ 1445\ \text{J/(kg K)} \ \ (32°\text{C} - 24°\text{C}) = 347{,}000\ \ \text{J/kg}$$

Neglecting the vapor density compared to the liquid density

$$\bar{h}_c = 0.725 \left[\frac{1350\ \text{kg/m}^3 \ \ 9.8\ \text{m/s}^2 \ \ 347{,}335\ \text{J/kg} \ \ (\text{W s})/\text{J} \ \ 0.192\ \text{W/(m K)}\ ^3}{3.2 \times 10^{-4}\ (\text{N s})/\text{m}^2 \ \ (\text{kg m})/(\text{s}^2\text{N}) \ (0.05\text{m})(8°\text{C})} \right]^{\frac{1}{4}} = 3120\ \ \text{W/(m}^2\text{K)}$$

(c) An energy balance yields

$$\dot{m}\, h_{fg} = \bar{h}_c\, A\, (T_{sv} - T_s) = \bar{h}_c\, \pi\, D\, L\, (T_{sv} - T_s)$$

$$\frac{\dot{m}}{L} = \frac{\bar{h}_c \pi D (T_{sv} - T_s)}{h_{fg}} = \frac{3131\,\mathrm{W/(m^2K)} \quad \mathrm{J/(Ws)} \quad 3600\,\mathrm{s/h}\ \ \pi(0.05\,\mathrm{m})(8°\mathrm{C})}{343{,}000\,\mathrm{J/kg}}$$

$$\frac{\dot{m}}{w} = 41.2\ \ \mathrm{kg/h}\ \ \text{per meter length}$$

PROBLEM 9.28

In problem 9.11, it was indicated that the Nusselt number for condensation depends on the Prandtl number and four other dimensionless groups including the Jacob number, the Bond number, and a nameless group resembling the Grashof number, $\rho g\ (\rho_l - \rho_v)L^3/\mu^2$. Give a physical explanation of each of these 3 groups and explain when you expect *Bo* and *Ja* to exert a significant influence and when their respective influence is negligible.

GIVEN

- Three of the dimensionless groups upon which the condensation Nusselt number depends

FIND

(a) Physical explanation for the three groups
(b) When *Ja* and *Bo* are important

SOLUTION

The Jacob number is

$$Ja = \frac{c_p \Delta T}{h_{fg}}$$

and it scales the maximum sensible heat that the liquid can absorb to the latent heat absorbed by the liquid during boiling. For most liquids, *Ja* is small. For large values of the excess temperature, ΔT_x, *Ja* could become significant.

The Bond number is

$$Bo = \frac{g\,\Delta\rho L^2}{\sigma}$$

and it scales the gravitational force to the surface tension force. For water at atmospheric pressure, as an example,

$$Bo \sim \frac{9.81\,\text{m/s}^2 \quad 1000\,\text{kg/m}^3 \quad L^2}{100\,\text{N/m}} = 98\,L^2$$

when L is given in meters. So, if the length scale for a given problem is the order of 0.1 m = 10 cm, the Bond number is order 1 and these forces are comparable. Clearly, for problems involving large length scales, the Bond number will be >> 1.

The nameless dimensionless group is

$$N_{??} = \frac{g\Delta\rho L^3 \rho}{\mu^2}$$

and, like the Grashof number, it scales the buoyant force to the viscous force. For water at atmospheric pressure, we find that $N_{??} >> 1$ if $L \sim 0.1$ m, so for typical cases, the buoyant forces will be much larger than the viscous forces.

PROBLEM 9.29

Saturated methyl chloride at 430 kPa(abs) condenses on a horizontal 10*10 bank of tubes. The 5 cm-*OD* tubes are equally spaced and are 10 cm apart center-to-center on rows and columns. If the surface temperature of the tubes is maintained at 7°C by pumping water through them, calculate the rate of condensation of methyl chloride in kg/(m s).

The properties of saturated methyl chloride at 430 kPa are shown in the list that follows:

Saturation temperature = 16°C

Heat of vaporization = 390 kJ/ kg

Liquid density = 936 kg/m^3

Liquid specific heat = 1.6 kJ/(kg K)

Liquid absolute viscosity = 2 × 10^{-4} kg/(m s)

Liquid thermal conductivity = 0.17 W/(m K)

GIVEN

- Saturated methyl chloride condensing on a ten-by-ten bank of horizontal tubes
- Pressure = 430 kPa
- Tube outside diameter (D) = 5 cm= 0.05 m
- Tube center-to-center spacing (s) = 10 cm= 0.1 m
- Methyl chloride properties given above
- Tube surface temperature (T_s) = 7^{0}C

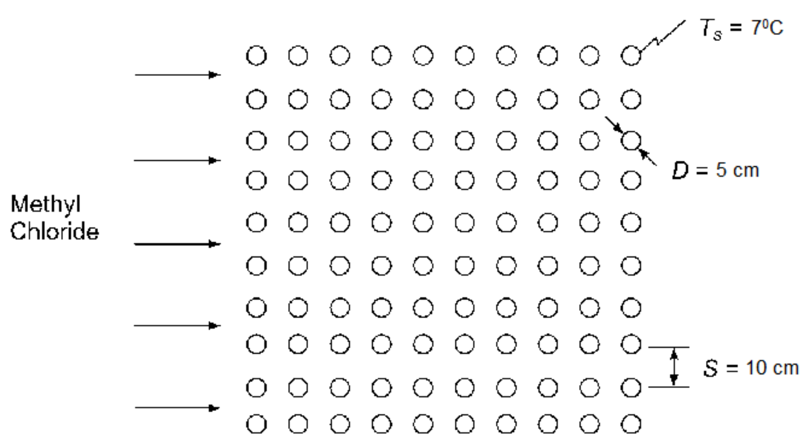
FIND

- The rate of condensation ($\dot{m}$ /L) in kg/(m s)

ASSUMPTIONS

- Steady state
- Laminar flow condensation
- Interfacial shear is negligible
- Tube surface temperature is uniform and constant
- Vapor density is negligible compared to the liquid density

SKETCH



SOLUTION

The average heat transfer coefficient for a vertical row of tubes including liquid subcooling is given by Equation (9.24)

$$\bar{h}_c = 0.728\left[1+0.2\frac{c_p(T_{sv}-T_s)}{h_{fg}}(N-1)\right]\left[\frac{\rho_l(\rho_l-\rho_v)\,g h'_{fg}\,k^3}{\mu_l\,N\,D(T_{sv}-T_s)}\right]^{\frac{1}{4}}$$

Where N = the number of tubes in a vertical row = 10

$$\text{where } h'_{fg} = h_{fg} + \frac{3}{8}\,c_{pl}\,(T_{sv}-T_s)$$

$$h'_{fg} = (390000\ J/kg) + \frac{3}{8}\,(1600\ J/(kg\ \mathrm{K}))\,(16°\mathrm{C}-7°\mathrm{C}) = 395400\ \mathrm{J}/kg$$

Assuming the vapor density is negligible compared to the liquid density

$$\bar{h}_c = 0.728\left[1+0.2\,\frac{(1600\,J/(kgK))(16°\mathrm{C}-7°\mathrm{C})}{(390000\ J/kg)}\,(10-1)\right]$$

$$\left[\frac{(936\,kg/m^3)^2\,(9.81\,m/s^2)(0.17\,W/(\mathrm{m}\,K))^3\,(395400\ J/kg)}{10(0.05\ \mathrm{m})(2\times10^{-4}\ \mathrm{kg}/(m\ \mathrm{s}))(16°\mathrm{C}-7°\mathrm{C})}\right]^{\frac{1}{4}}$$

$$\bar{h}_c = 0.728^*\ 1.066^*2075\ \mathrm{W/(m^2\ K)}$$

$$\bar{h}_c = 1611\ W/(m^2\ \mathrm{K})$$

The rate of heat transfer is

$$q = \bar{h}_c\ A_t\,(T_{sv}-T_s) = \bar{h}_c\ N_{\mathrm{total}}\,\pi D\,T\,(T_{sv}-T_s)$$

The rate of condensate flow is given by

$$\dot{m} = \frac{q}{h_{fg}}$$

$$\frac{\dot{m}}{L} = \frac{\bar{h}_c}{h_{fg}}\ N_{\mathrm{total}}\,\pi D\,(T_{sv}-T_s) = \frac{(1611\,W/(m^2\ \mathrm{K}))}{(390000\ J/kg)}\ (100)\ \pi\ 0.05\ \mathrm{m}\ (16°\mathrm{C}-7°\mathrm{C}) = 0.584\,\mathrm{kg}/(m\,s)$$

PROBLEM 9.30

A vertical rectangular water duct 1-m-high and 0.1-m-deep shown in the sketch is placed in an environment of saturated steam at atmospheric pressure. If the outer surface of the duct is about 50°C, estimate the rate of steam condensation per unit length.

GIVEN

- Water-cooled rectangular duct, 1-m-high, 0.1-m-deep
- Duct surface is 50°C
- Steam environment

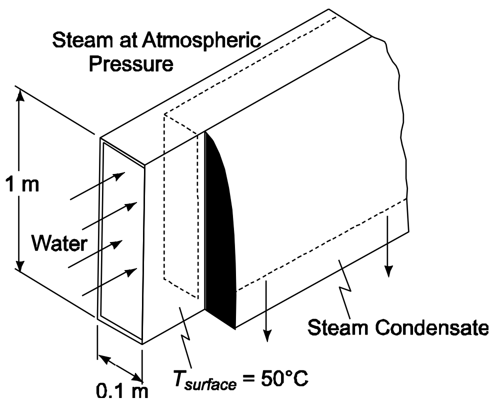
FIND

(a) Rate of steam condensation per unit length of the duct

ASSUMPTIONS

- The steam is at 1 atmosphere pressure
- Condensation from the horizontal duct surfaces can be neglected
- Laminar film condensation on the vertical surfaces (must be checked)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the properties of saturated liquid at the mean temperature of 75°C are

Density (ρ_l) = = 974.9 kg/m^3

Specific heat (c_l) = 4190 J/(kg K)

Thermal conductivity (k_l) = 0.671 W/(m K)

Absolute viscosity (μ_l) = 3.77 × 10^{-4} kg/ms

Heat of vaporization (h_{fg}) = 2.257 × 10^6 J/kg

From Appendix 2, Table 35, the properties of the saturated vapor at the saturation temperature of 100°C are

Density (ρ_v) = 0.597 kg/m^3

SOLUTION

The mean flow of condensate per unit duct length is from Equation (9.14)

$$\Gamma_c = \frac{\rho_l(\rho_l - \rho_v)\delta^3 g}{3\mu_l}$$

and the film thickness can be found from Equation (9.17)

$$\delta = \left[\frac{4\,\mu_l k_l \times (T_{sv} - T_s)}{g\,\rho_l(\rho_l - \rho_v)h'_{fg}}\right]^{\frac{1}{4}}$$

where

$$h'_{fg} = h_{fg} + 0.68\, c_l \Delta T = \left(2.257 \times 10^6 \text{ J/kg}\right) + (0.68)\left(4190 \text{ J/(kg K)}\right)(100 - 50)(\text{K}) = 2.4 \times 10^6 \text{ J/kg}$$

Calculating the condensate film thickness at the bottom edge of the duct, we have

$$\delta = \left[\frac{(4)\left(3.77 \times 10^{-4} \text{kg/(ms)}\right)\left(0.671 \text{W/(mK)}\right)(1\text{m})(50\text{K})}{\left(9.81\text{m/s}^2\right)\left(974.9\text{kg/m}^3\right)(974.9 - 0.597)(\text{kg/m}^3)(2.4 \times 10^6 \text{J/kg})}\right]^{\frac{1}{4}} = 2.18 \times 10^{-4} \text{ m}$$

Now, we can calculate the film flow per unit duct length

$$\Gamma = \frac{(974.9\text{ kg/m}^3)(974.9 - 0.597)(\text{kg/m}^3)(2.18 \times 10^{-4}\text{ m})^3(9.81\text{m/s}^2)}{(3)(3.77 \times 10^{-4}\text{kg/(ms)})} = 0.085 \text{ kg/ms}$$

Doubling this to account for both sides of the duct, we have 0.171 kg/m s for the rate of condensate flow, per unit duct length.

To confirm that the Reynolds number for the condensate film flow is laminar

$$Re = \frac{4\Gamma_c}{\mu_l} = \frac{(4)\left(0.085\text{kg/(ms)}\right)}{3.77 \times 10^{-4}\text{kg/(ms)}} = 902$$

PROBLEM 9.31

The 1-m-long, tube-within-a-tube heat exchanger, as shown in the sketch, is used to condense steam at 2 atmospheres in the annulus. Water flows in the inner tube, entering at 90°C. The inner tube is made of copper with a 1.27 cm OD and 1.0 cm ID. (a) Estimate the water flow rate required to keep its outlet temperature below 100°C. (b) Estimate the pressure drop and the pumping power for the water in the heat exchanger, neglecting inlet and outlet losses.

GIVEN

- Tube-within-a-tube condenser
- Cooling water flowing in the inner tube
- Steam at 2 atm condensing inside the annulus

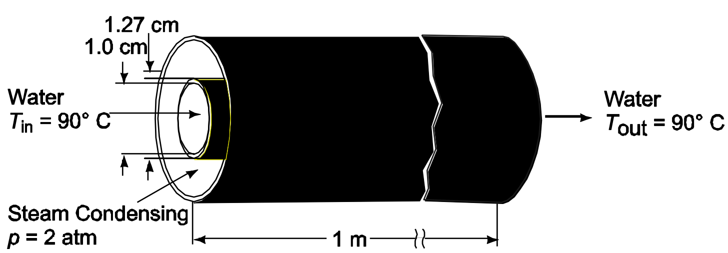
FIND

(a) Coolant water flow rate to maintain coolant outlet temperature below 100°C
(b) Coolant pressure drop and pumping power

ASSUMPTIONS

- Steady conditions
- The heat exchanger is horizontal

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the properties of water at 2 atm are

Saturation temperature, T_{sv} = 121°C

Liquid density, ρ_l = 944 kg/m^3

Liquid specific heat, c_{pl} = 4232 J/(kg K)

Liquid thermal conductivity, k_l = 0.685 W/(mK)

Liquid viscosity, $\mu_l = 2.35 \times 10^{-4}$ kg/(ms)

Heat of vaporization, $h_{fg} = 2.202 \times 10^6$ J/kg

Vapor density, ρ_v = 1.12 kg/m^3

SOLUTION

(a) We can use Equation (9.23) to calculate the average condensing heat transfer coefficient for a horizontal tube

$$\bar{h}_c = 0.725 \left[\frac{\rho_l (\rho_l - \rho_v)\, g h'_{fg}\, k^3}{D \mu_l \,(T_{sv} - T_s)} \right]^{\frac{1}{4}}$$

where

$$h'_{fg} = h_{fg} + 0.68\, c_{pl}\,(T_{sv} - T_s)$$

Assuming that the average coolant temperature is 95°C and neglecting temperature drop across the copper tube, we have T_s = 95°C. Then

$$h'_{fg} = \left(2.202\times10^6\ \text{J/kg}\right) + (0.68)\left(4232\ \text{J/(kg K)}\right)(121-95)(\text{K}) = 2.277\times10^6\ \text{J/kg}$$

The average condensing heat transfer coefficient is then

$$\bar{h}_c = 0.725\left[\frac{\left(944\ \text{kg/m}^3\right)(944-1.12)\ \text{kg/m}^3\left(9.81\ \text{m/s}^2\right)\left(2.277\times10^6\ \text{J/kg}\right)\left(0.685\ \text{W/(m K)}\right)^3}{(0.0127\ \text{m})\left(2.35\times10^{-4}\ \text{kg/(ms)}\right)(121-95)(\text{K})}\right]^{\frac{1}{4}}$$

$$= 12{,}282\ \text{W/(m}^2\text{K)}$$

Performing a heat balance on the cooling water

$$\dot{m}\, c_{pl}(T_{\text{water,out}} - T_{\text{water,in}}) = \bar{h}_c\, \pi DL(T_{sv} - T_s)$$

we can solve for the coolant mass flow

$$\dot{m} = \frac{12{,}282\ \text{W/(m}^2\text{K)}\ (\pi)(0.0127\ \text{m})(1\ \text{m})(121-95)(\text{K})}{4232\ \text{J/(kg K)}\ (100-90)(\text{K})} = 0.30\ \text{kg/s}$$

(b) To determine the pressure drop and pumping power, we need to determine the Reynolds number for the coolant flow

$$Re_w = \frac{4\dot{m}}{\pi\,\mu_l D}$$

At the average bulk coolant temperature of 95°C, Table 13 gives

μ_l = 2.97 × 10^{-4} kg/(ms)

ρ_l = 961 kg/m^3

$$Re_w = \frac{(4)\left(0.30\ \text{kg/s}\right)}{(\pi)\left(2.97\times10^{-4}\ \text{kg/(ms)}\right)(0.01\ \text{m})} = 128{,}610$$

Assuming the tube is smooth, the friction from Figure 7.17 is

$$F = 0.0165$$

and Equation (7.13) gives the pressure drop

$$\Delta p = f\,\frac{L\rho U^2}{D\,2g_c}$$

The mean flow velocity for the coolant is

$$U = \frac{\dot{m}}{\frac{\rho\pi D^2}{4}} = \frac{0.3\ \text{kg/s}}{\left(961\ \text{kg/m}^3\right)\left(\pi(0.01\ \text{m})^2/4\right)} = 3.97\ \text{m/s}$$

The pressure drop is then

$$\Delta p = (0.0165)\left(1\ \text{m}/(0.01\ \text{m})\right)\frac{\left(961\ \text{kg/m}^3\right)\left(3.97\ \text{m/s}\right)^2}{(2)\left(\text{kg m})/(\text{s}^2\text{N})\right)} = 12{,}500\ \text{N/m}^2$$

The pumping power can be determined from Equation (7.19)

$$P_{\text{pumping}} = \Delta p\,\frac{\dot{m}}{\rho} = 12{,}500\ \text{N/m}^2\left(\frac{0.30\ \text{kg/s}}{961\ \text{kg/m}^3}\right) = 3.9\ (\text{N m})/\text{s} = 3.9\ \text{W}$$

PROBLEM 9.32

A one-pass condenser-heat exchanger, shown in the sketch, has 64 tubes arranged in a square array with 8 tubes per line. The tubes are 1.22 m long and are made of copper with an outside diameter of 1.27 cm. They are contained in a shell at atmospheric pressure. Water flows inside the tubes whose outside wall temperature is 98°C. Calculate (a) the rate of steam condensation and (b) the temperature rise of the water if the flow rate per tube is 0.0454 kg/s.

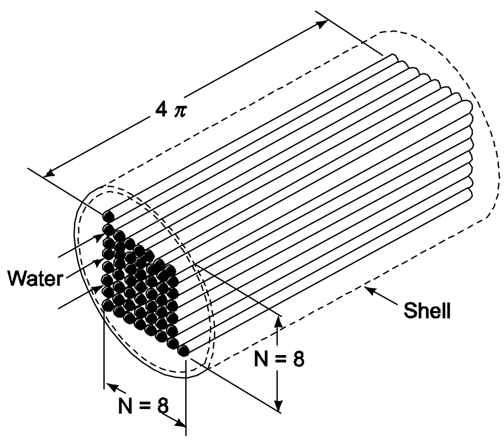
GIVEN

- One-pass condenser heat exchanger with 64 tubes in a square array

FIND

(a) Rate of steam condensation
(b) Water temperature rise

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the properties of saturated water at the film temperature (100 + 98)/2 = 99°C are

Density (ρ_l) = 958 kg/m^3

Absolute viscosity (μ_l) = 2.78 × 10^{-4} kg/(ms)

Thermal conductivity (k_l) = 0.682 W/(mK)

Specific heat (c_l) = 4211 J/(kg K)

Heat of vaporization (h_{fg}) = 2.257 × 10^6 J/kg

SOLUTION

Wall temperature= 98°C

Saturation temperature = 100°C

Tube OD= 0.0127 m

Tube length = 1.22 m

Water flow rate= 0.0454 kg/s

From Equation (9.24), we can obtain the average heat transfer coefficient

$$\bar{h}_c = 0.728[1 + 0.2(N-1)Ja]\left[\frac{g\rho_l(\rho_l - \rho_v)k^3 h'_{fg}}{N D\mu_l(T_{sv} - T_s)}\right]^{\frac{1}{4}}$$

where

$$Ja = \frac{cp(T_{sv} - T_s)}{h_{fg}} = \frac{4211\text{ J/(kg K)}\ (2\text{ K})}{2.257 \times 10^6\text{ J/kg}} = 0.00373$$

and

$$h'_{fg} = h_{fg} + c_{pl}\,(T_{sv} - T_s) = 2.257 \times 10^6\text{ J/ kg} + \ 4211\text{ J/(kg K)}\ (2\text{K}) = 2.265 \times 10^6\text{ J/kg}$$

Neglecting the vapor density compared to the liquid density, the quantity in the right bracket in the equation for the heat transfer coefficient is

$$\left[\frac{(9.81\text{m/s}^2)(958\text{kg/m}^3)^2\ \ 0.68\text{W/(mK)}\ ^3(2.265 \times 10^6\text{ J/kg})}{(8)(0.0127\text{m})\ \ 2.78 \times 10^{-4}\text{kg/(ms)}\ \ (2\text{K})}\right]^{\frac{4}{4}} = 18{,}355\ \text{W/(m}^2\text{K)}$$

so

$$\bar{h}_c = (0.728)[1 + (0.2)(7)(0.00373)][18{,}355] = 13{,}432\ \text{W/m}^2$$

The tube surface area is

$$N\pi DL = (64)(\pi)(0.0127\text{ m})(1.22\text{ m}) = 3.12\text{m}^2$$

The rate of heat transfer is therefore

$$q = \bar{h}_c\ A\,(T_{sv} - T_s) = (13{,}432\text{ W/m}^2\text{K})(3.12\text{m}^2)(2\text{K}) = 83{,}688\text{ W}$$

(a) The flow rate of condensate is then

$$\dot{m}_c = \frac{q}{h'_{fg}} = \frac{83{,}688\text{W}}{2.265 \times 10^6\text{(Ws)/kg}} = 0.03685\ \text{kg/s}$$

The heat transfer per tube is 83,688/64 W = 1308 W and this must equate to the increase in sensible heat in the cooling water, giving for the water temperature rise

(b)

$$\Delta T_w = \frac{1308\text{W}}{0.0454\text{kg/s}\ \ \ 4211\text{J/(kgK)}} = 6.8\text{K}$$

PROBLEM 9.33

Estimate the cross-sectional area required for a 30-cm-long methanol-nickel heat pipe to transport 30 W at atmospheric pressure.

GIVEN

- Methanol-nickel heat pipe
- 30-cm-long
- Atmospheric pressure

FIND

(a) Cross-sectional area required to transport 30 W

ASSUMPTIONS

- The type of wick to be used is a threaded artery wick
- 100°C operation

SOLUTION

Table 9.6 gives

$$q''_{\text{axial}} = 0.45 \text{ W/cm}^2$$

Since the total heat transported by the heat pipe is

$$q = q''_{\text{axial}} A$$

where A is the desired cross-sectional area, we have

$$A = \frac{q}{q''_{\text{axial}}}$$

or

$$A = \frac{(30\,W)}{(0.45\text{ W/cm}^2)} = 66.7 \text{ cm}^2$$

The heat pipe diameter is

$$D_o = \sqrt{\frac{4A}{\pi}} = 9.22 \text{ cm}$$

PROBLEM 9.34

Design a heat pipe cooling system for a spherical satellite that dissipates 5000 W/m^3, has a surface area of 5 m^2, and cannot exceed a temperature of 120°C. All the heat must be dissipated by radiation into space. State all your assumptions.

GIVEN

- Spherical satellite, 5 m^2 surface area
- Dissipates 5000 W/m^3
- Maximum temperature of 102°C
- All heat rejection is by radiation to space

FIND

(a) A design for a heat pipe cooling system

ASSUMPTIONS

- Temperature drop between the satellite interior and the heat pipe evaporator is < 20°C
- Neglect vapor pressure drop in the heat pipe

SOLUTION

Since the satellite has a 5 m^2 surface area, the radius of the spherical satellite is

$$4\pi r_s^2 = 5\text{m}^2 \Rightarrow r_s = 0.631\text{m}^3$$

from which the satellite volume is

$$V_{\text{satellite}} = \frac{4}{3}\pi r_s^3 = 1.05\text{m}^3$$

and the total power dissipated by the satellite, and therefore by the heat pipe cooling system, is

$$q = 5000\ \text{W/m}^3\ (1.05\text{m}^3) = 5260\ \text{W}$$

Since we have assumed that the temperature drop between the satellite interior and the heat pipe evaporator is less than 20°C, we can safely operate the heat pipe at 100°C.

From Figure 9.24, water has the highest figure of merit, M, at the desired temperature of 373 K and it should operate satisfactorily at the desired temperature. From that figure we find

$$M = \frac{\sigma_l\,\rho_l\,h_{fg}}{\mu_l} = 4 \times 10^4\ \text{kW/cm}^2$$

Since we have neglected vapor neglected vapor pressure drop, and since there is no gravitational head, Equation (9.40) simplifies to

$$\frac{2\sigma_l\cos\theta}{r_c} = \frac{\mu_l\,L_{\text{eff}}\,q}{\rho_l\,K_w\,A_w\,h_{fg}}$$

Let's try a 200 mesh nickel wick. Table 38 in Appendix 2 gives the pore size, r_c = 0.004 cm and the wick permeability, $K_w = 0.62 \times 10^{-10}$ m^2. Let us also assume perfect wetting of the wick by the water, giving $\theta = 0$.

Rearranging the previous equation to solve for the geometry of the heat pipe

$$\frac{L_{\text{eff}}}{A_w} = \frac{\sigma_l\,\rho_l h_{fg}}{\mu_l}\frac{2}{r_c}\frac{K_w}{q} = M\,\frac{2}{r_c}\frac{K_w}{q}$$

$$\frac{L_{\text{eff}}}{A_w} = 4 \times 10^4 \times 10^3 \text{ W/cm}^2 \left(\frac{2}{0.004 \text{ cm}}\right) \frac{(0.62 \times 10^{-10} \text{ m}^2)}{5260 \text{ W}} \frac{10^4 \text{ cm}^2}{\text{m}^2} = 2.4 \text{ cm}^{-1}$$

For a reasonably sized heat pipe, let L_{eff} = 30 cm. Then the total wick cross-sectional area required is A_w = 30 cm/ 2.4 cm^{-1} = 12.5 cm^2. Assuming that we need N pipe of diameter D and thickness t, we have

$$12.5 \text{ cm}^2 = \pi DtN$$

If the exterior surface of the heat pipe condenser section is black, the following equation gives the required surface area of the condenser section

$$q = A_{\text{cond}}\, \sigma (T^4_{\text{cond}} - T^4_{\text{space}})$$

This equation assumes that the heat pipes are separated sufficiently that they all have a near-unity shape factor with respect to space.

Using T_{cond} = 100°C = 373 K and assuming T_{space} = 0 K and we can solve for the condenser surface area as follows

$$A_{\text{cond}} = \frac{5260 \text{ W}}{5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4) \ (373 \text{K})^4} = 4.79 \text{ m}^2$$

If the condenser length is L_{cond}, then

$$N\pi DL_{\text{cond}} = 4.79 \text{ m}^2$$

Let L_{cond} = 10 cm giving $N\pi D$ = 47.9 m = 4790 cm. Since we found previously that πDtN = 12.5 cm^2, we can solve for the wick thickness t

$$t = \frac{12.5 \text{cm}^2}{4790 \text{cm}} = 0.0026 \text{ cm}$$

Choose a pipe diameter of 3 cm, then

$$N = \frac{47.9 \text{m}}{\pi (0.03 \text{m})} = 508 \text{ pipes required}$$

Since $L_{\text{eff}} = L + (L_{\text{cond}} + L_{\text{evap}})/2$, letting the evaporator and condenser lengths be the same, 10 cm, we find L = 20 cm and the overall length is 20 + 10 + 10 = 40 cm. We summarize the cooling system below

Pipe diameter = 3 cm

Pipe length = 40 cm

Condenser length = 10 cm

Evaporator length = 10 cm

Adiabatic section length = 20 cm

Wick: 0.0026 cm thickness of 200 mesh nickel powder

Working fluid: water

Pressure: atmospheric

Number of pipes required: 508

PROBLEM 9.35

Compare the axial heat flux achievable by a heat pipe using water as the working fluid with that of a solid silver rod. Assume that both are 20-cm-long, that the temperature difference for the rod from end to end is 100°C and that the heat pipe operates at atmospheric pressure. State your other assumptions.

GIVEN

- Silver rod and a heat pipe, both 20-cm-long
- End-to-end temperature difference of 100°C
- Heat pipe operates at atmospheric pressure

FIND

(a) Axial heat flux for both rods

ASSUMPTIONS

- Neglect heat pipe vapor pressure drop
- Horizontal operation for the heat pipe
- Perfect wetting of the heat pipe fluid, $\theta = 0$

PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, the conductivity for silver at 63°C is k_{silver} = 424 W/(mK)

SOLUTION

The axial heat flux for the silver rod is

$$q'' = \frac{k\Delta T}{L} = \frac{424\ \text{W/(m K)}\ (100\ \text{K})}{0.2\ \text{m}} = 212{,}000\ \text{W/m}^2$$

With the above assumptions, Equation (9.40) simplifies to

$$\frac{2\sigma_l}{r_c} = \frac{\mu_l\, L_{\text{eff}}\, q}{\rho_l\, K_w\, A_w\, h_{fg}}$$

At the operating conditions, water is a satisfactory working fluid. From Figure 9.24, for an operating temperature near 100°C, we have for the figure of merit, M

$$M = \frac{\rho_l\, \sigma_l\, h_{fg}}{\mu_l} = 4 \times 10^4\ \text{kW/cm}^2$$

Rearranging Equation (9.40) to find the heat transport

$$q = M\left(\frac{2K_w A_w}{r_c L_{\text{eff}}}\right)$$

We have to make some assumptions about the wick material and thickness. Let's try 120 mesh nickel, with thickness t = 0.01 cm. Table 38 gives for the wick pore radius r_c = 0.019 cm and for the wick permeability, $K_w = 3.5 \times 10^{-10}$ m^2. We must also assume a length for the condenser and evaporator sections. Since the total length is 20 cm, a reasonable length for the condenser and evaporator is 8 cm. Then L = 20 – 8 – 8 = 4 cm and

$$L_{\text{eff}} = L + (L_{\text{cond}} + L_{\text{evap}})/2 = 12\ \text{cm}$$

Unlike the silver rod, we must select a diameter for the heat pipe, Let's try D = 2 cm giving a total corss-sectional area of $A_{\text{pipe}} = \pi 2^2/4 = 3.14$ cm^2. The wick cross-sectional area is

$$A_w = \pi D t = (\pi)\ (2\text{cm})\ (0.01\ \text{cm}) = 0.0628\ \text{cm}^2$$

We can now calculate the heat transport

$$q = \quad 4\times10^{7}\ \mathrm{W/cm^{2}} \quad \frac{(2)(3.5\times10^{-10}\,\mathrm{m^{2}})(0.0628\,\mathrm{cm^{2}})}{(0.019\,\mathrm{cm})(12\,\mathrm{cm})}\left(\frac{10^{4}\mathrm{cm^{2}}}{\mathrm{m^{2}}}\right) = 77\ \mathrm{W}$$

The heat flux for the heat pipe is therefore

$$q''_{\mathrm{pipe}} = \frac{77\,\mathrm{W}}{3.14\,\mathrm{cm^{2}}} = 245{,}000\ \mathrm{W/m^{2}}$$

This is slightly more than the heat flux for the silver rod so the performance of the two methods of heat transport seems similar. However, the heat pipe provides two advantages: (i) it will undoubtedly cost much less, and (ii) it is isothermal, that is, the temperature difference from one end to the other will be very small. This means that we can afford some temperature drop at either end of the heat pipe to get the heat from the heat source into the evaporator section and out of the condenser section to the heat sink.

PROBLEM 9.36

Show that the dimensionless equation for ice formation at the outside of a tube of radius r_o is

$$t^{\bullet} = \frac{r^{\bullet 2}}{2}\ \ln r^{\bullet} + \left(\frac{1}{2R^{\bullet}} \frac{1}{4}\right)\ (r^{\bullet 2} - 1)$$

where

$$r^{\bullet} = \frac{\varepsilon + r_o}{r_o} \quad R^{\bullet} = \frac{h_o\, r_o}{k} \quad t^{\bullet} = \frac{(T_f - T_{\infty})kt}{p\, L\, r_o^2}$$

Assume that the water is originally at the freezing temperature T_f, that the cooling medium inside the tube surface is just below the freezing temperature at a uniform temperature T_s, and that h_o is the total heat transfer coefficient between the cooling medium and the pipe-ice interface. Also show the thermal circuit.

GIVEN

- Water freezing on the outside of a tube
- Tube radius = r_o
- Cooling medium temperature = T_{∞} (uniform)
- Heat transfer coefficient between the cooling medium and the pipe-ice interface = h_o

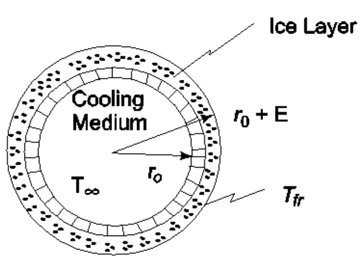
FIND

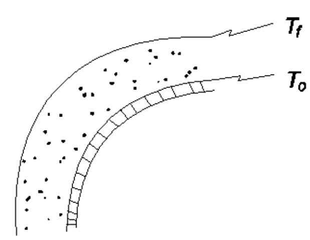
- Draw the thermal circuit and show that the dimensionless equation for ice formation is as shown above

ASSUMPTIONS

- Steady state
- The thermal capacitance of the ice layer is negligible
- T_{∞} is constant
- The water is at the freezing temperature, T_f
- The properties of the ice are uniform

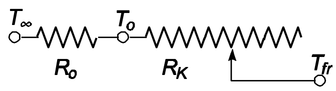
SKETCH





SOLUTION

The thermal circuit is shown below



where

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o 2\pi r_o L}$$

$$R_k = \frac{\ln\left(\frac{r_o+\varepsilon}{r_o}\right)}{2\pi L k}$$

The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{fr} - T_\infty}{R_o + R_k} = 2\,\pi L\, r_o \,\frac{T_{fr} - T_\infty}{\frac{1}{h_o} + \frac{r_o}{k}\ln\left(\frac{r_o+\varepsilon}{r_o}\right)}$$

This is the heat flow rate which removes the latent heat of fusion necessary for freezing the ice as shown by Equation (9.34)

$$q = A\,\rho\, L_f\,\frac{d\varepsilon}{dt} = 2\,\pi\,(r_o + \varepsilon)\,L\,\rho\,L_f\,\frac{d\varepsilon}{dt}$$

where ρ_{Lf} is the latent heat.
Combining these two equations

$$r_o\,\frac{T_{fr} - T_\infty}{\frac{1}{h_o} + \frac{r_o}{k}\ln\left(\frac{r_o+\varepsilon}{r_o}\right)} = (r_o + \varepsilon)\,\rho L_f\,\frac{d\varepsilon}{dt}$$

Rearranging and using $d\varepsilon = r_o\, d\left(\frac{r_o+\varepsilon}{r_o}\right)$

$$\frac{k\,(T_{fr} - T_\infty)}{r_o^2 \rho L_f}\,dt = \left[\frac{k}{r_o h_o} + \ln\left(\frac{r_o+\varepsilon}{r_o}\right)\right]\left(\frac{r_o+\varepsilon}{r_o}\right) d\left(\frac{r_o+\varepsilon}{r_o}\right)$$

$$\text{Let } t^\bullet = \frac{(T_f - T_\infty)kt}{\rho L r_o^2} \quad \rightarrow \quad dt^\bullet = \frac{(T_{fr} - T_\infty)k}{\rho L^2\, r_o^2}\,dt$$

$$r^\bullet = \frac{\varepsilon + r_o}{r_o}$$

$$R^\bullet = \frac{h_o r_o}{k}$$

Expressing the above equation in terms of these dimensionless parameters:

$$dt^\bullet = \left(\frac{1}{R^\bullet} + \ln r^\bullet\right) r^\bullet\, dr^\bullet$$

Integrating

$$\int_0^{t^\bullet} dt^\bullet = \int_1^{r^\bullet}\left(\frac{1}{R^\bullet} + \ln r^\bullet\right) r^\bullet\, dr^\bullet$$

$$t^\bullet = \frac{r^{\bullet 2}}{2R^\bullet}\,\frac{1}{2R^\bullet} + r^{\bullet 2}\left(\frac{\ln r^\bullet}{2} - \frac{1}{4}\right) + \frac{1}{4}$$

$$t^{\bullet} = \frac{r^{\bullet 2}}{2} \ln r^{\bullet} + \left(\frac{1}{2R^{\bullet}} - \frac{1}{4}\right)(r^{\bullet 2} - 1)$$

PROBLEM 9.37

In the manufacture of can ice, cans having inside dimensions of 27.5 x 55 x 125 cm with 2.5 cm inside taper are filled with water and immersed in brine at a temperature of -12°C. For the purpose of a preliminary analysis, the actual ice can be considered as an equivalent cylinder having the same cross-sectional area as the can, and end effects can be neglected. The overall conductance between the brine and the inner surface of the can is 225 W/(m^2 K). Determine the time required to freeze the water and compare with the time necessary if the brine circulation rate is increased to reduce the thermal resistance of the surface to one-tenth of the value specified above. The latent heat of fusion of ice is 334 kJ/kg, its density is 912 kg/m^3, and its thermal conductivity is 2.2 W/(m K).

GIVEN

- Ice formation within a can immersed in a brine solution
- Can dimensions: 27.5 cm × 55 cm × 125 cm (with a 2.5 cm taper)
- Brine temperature (T_∞) = -12°C
- Overall heat transfer coefficient between the brine and the outer surface of the can (h_o) = 225 W/(m^2 K)
- Ice properties
  - Latent heat of fusion (L_f) = 334kJ/kg
  - Density (ρ) = 912 kg/m^3
  - Thermal conductivity (k) = 2.2 W/(m K)

FIND

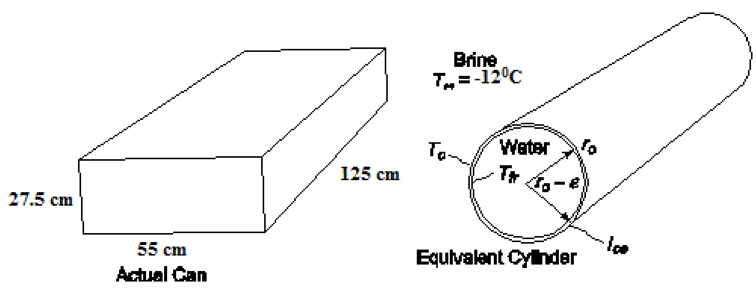
The time required to freeze the water in the can for

(a) the given h_o, and (b) for one-tenth the resistance

ASSUMPTIONS

- Steady state
- The capacitance of the layer can be neglected
- The brine temperature is constant and uniform
- The can can be treated as a cylinder having the same cross-sectional area
- End effects are negligible

SKETCH

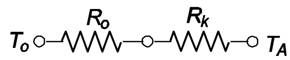


SOLUTION

The effective radius of the equivalent cylinder is

$$r_o = \frac{1}{2}\sqrt{\frac{4A}{\pi}} = \frac{1}{2}\sqrt{\frac{4(27.5\text{ cm})(55\,cm)}{\pi}} = 22\text{ cm} = 0.22\text{ m}$$

The thermal circuit for the problem is shown below



Where $R_o = \dfrac{1}{h_o A_o} = \dfrac{1}{h_o 2\pi r_o L}$ $\qquad R_k = \dfrac{\ln\left(\dfrac{r_o+\varepsilon}{r_o}\right)}{2\pi L k}$

The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{fr} - T_\infty}{R_o + R_k} = 2\,\pi L\, r_o \frac{T_{fr} - T_\infty}{\dfrac{1}{h_o} + \dfrac{r_o}{k}\ln\left(\dfrac{r_o+\varepsilon}{r_o}\right)}$$

This is the heat flow rate which removes the latent heat of fusion necessary for freezing, as shown by Equation (9.56)

$q = A\,\rho\, L_f \dfrac{d\varepsilon}{dt} = -\,2\,\pi\,(r_o + \varepsilon)\,L\,\rho\,L_f \dfrac{d\varepsilon}{dt}$ $\qquad$ where ρL_f is the latent heat.

Combining these equations $\quad \dfrac{k\,(T_{fr} - T_\infty)}{r_o^2\,\rho L_f}\,dt = \quad -\left[\dfrac{k}{r_o h_o} + \ln\left(\dfrac{r_o+\varepsilon}{r_o}\right)\right]\left(\dfrac{r_o+\varepsilon}{r_o}\right) d\left(\dfrac{r_o+\varepsilon}{r_o}\right)$

Let $\quad t^\bullet = \dfrac{(T_f - T_\infty)kt}{\rho L r_o^2} \to dt^\bullet = \dfrac{(T_{fr} - T_\infty)k}{\rho L^2 r_o^2}\,dt \qquad r^\bullet = \dfrac{\varepsilon + r_o}{r_o} \qquad R^\bullet = \dfrac{h_o r_o}{k}$

$$dt^\bullet = -\left(\frac{1}{R^\bullet} + \ln r^\bullet\right) r^\bullet\, dr^\bullet$$

Integrating $\quad \displaystyle\int_0^{t^\bullet} dt^\bullet = \int_1^{r^\bullet}\left(\frac{1}{R^\bullet} + \ln r^\bullet\right) r^\bullet\, dr^\bullet$

$$t^\bullet = \frac{r^{\bullet 2}}{2R^\bullet} - \frac{1}{2R^\bullet} + r^{\bullet 2}\left(\frac{\ln r^\bullet}{2} - \frac{1}{4}\right) + \left(\frac{1}{4}\right)$$

$$t^\bullet = \frac{r^{\bullet 2}}{2}\ln r^\bullet + \left(\frac{1}{2R^\bullet} - \frac{1}{4}\right)(r^{\bullet 2} - 1)$$

All the ice is frozen when $\varepsilon = r_o \to r^* = 0$

At $\quad r^* = 0 \qquad t^* = \dfrac{1}{2R^*} + \dfrac{1}{4}$

$$\frac{k\,(T_{fr} - T_\infty)}{r_o^2\rho L_f}\,t = \frac{1}{2\left(\dfrac{h_o r_o}{k}\right)} + \left(\frac{1}{4}\right)$$

$$t = \frac{r_o^2\rho L_f}{k\,(T_{fr} - T_\infty)}\left(\frac{k}{2h_o r_o} + \frac{1}{4}\right)$$

(a) For h_o = 225 W/(m² K)

$$t = \frac{(0.22\,\text{m})^2\left(912\,kg/m^3\right)(334000\,J/kg)}{(2.2\ \text{W/(m}\,K))(0°\text{C} - (-12°\text{C}))}\left[\frac{2.2\,W/(m\,K)}{2\left(225\,W/(m^2\,K)\right)(0.22\,m)} + \frac{1}{4}\right]$$

$$t = 558448\ (0.0222 + 0.25) = 152020\ \text{sec} = 42.2\ \text{h}$$

(b) If the thermal resistance is one tenth of part (a), that is the same as saying the heat transfer coefficient is increased ten-fold: h_o = 2250 W/(m² °K).

$$t = 558448\,(0.0022 + 0.25) = 140840 \text{ sec} = 39.1 \text{ h}$$

PROBLEM 9.38

Estimate the time required to freeze vegetables in thin, tin cylindrical containers 15 cm in diameter. Air at –12°C is blowing at 4 m/s over the cans, which are stacked to form one long cylinder. The physical properties of the vegetables before and after freezing may be taken as those of water and ice, respectively.

GIVEN

- The freezing of vegetables in thin tin cylindrical cans with air flowing over the cans
- Container diameter (D) = 15 cm = 0.15 m
- Air temperature (T_∞) = –12°C
- Air velocity (U_∞) = 4 m/s
- Vegetables have the same properties as water and ice

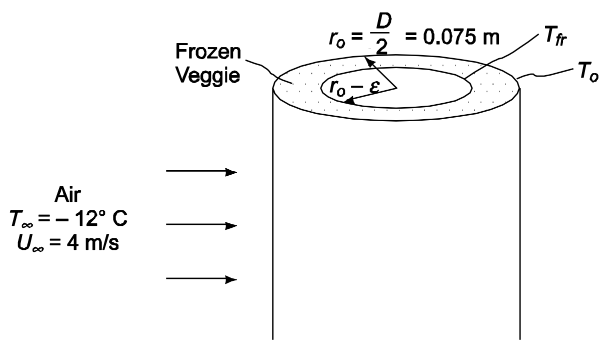
FIND

- The time (t) to freeze the vegetables

ASSUMPTIONS

- Air temperature is constant
- Thermal resistance of the tin can is negligible
- Thickness of the tin can is negligible
- Thermal capacitance of the frozen vegetable layer is negligible

SKETCH



PROPERTIES AND CONSTANTS

The ice property values given in the problem statement of Problem 9.37 are

Latent heat of fusion (L_f) = 333.780 J/kg

Density (ρ) = 918 kg/m^3

Thermal conductivity (k) = 2.22 W/(m K)

Extrapolating for Appendix 2, Table 28, for dry air at –12°C

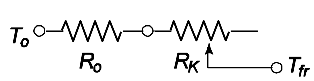
Thermal conductivity (k_a) = 0.0229 W/(m K)

Kinematic viscosity (ν_a) = 12.8 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The thermal circuit for the problem is shown below



where

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o\, 2\pi r_o\, L}$$

$$R_k = \frac{\ln\left(\frac{r_o + \varepsilon}{r_o}\right)}{2\pi L k}$$

The Reynolds number of the air flow is

$$Re_D = \frac{U_\infty D}{\nu_a} = \frac{4\ \text{m/s}\ (0.15\,\text{m})}{12.8\times 10^{-6}\ \text{m}^2/\text{s}} = 4.69 \times 10^4$$

The Nusselt number for the geometry is given by Equation (6.3)

$$\overline{Nu}_D = \frac{\bar{h}_o D}{k_a} = C\, Re_D^m\, Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where, for $Re_D = 4.69 * 10^4$

$C = 0.26$, $m = 0.6$, and $n = 0.37$

Since the surface temperature is between T_∞ and T_{fr}, the Prandtl number evaluated at the surface temperature (Pr_s) will be 0.71 and $Pr/Pr_s = 1.0$

$$\overline{Nu}_D = 0.26\,(4.69 \times 10^4)^{0.6}\,(0.71)^{0.37} = 145.9$$

$$\bar{h}_o = \overline{Nu}_D\ \frac{k_a}{D} = 145.9\ \frac{0.0229\ \text{W/(m K)}}{0.15\,\text{m}} = 22.3\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{fr} - T_\infty}{R_\text{o} + R_k} = 2\,\pi L\, r_o\ \frac{T_{fr} - T_\infty}{\frac{1}{h_o} + \frac{r_o}{k}\ln\left(\frac{r_o + \varepsilon}{r_o}\right)}$$

This is the heat flow rate which removes the latent heat of fusion necessary for freezing, as shown by Equation (9.56)

$$q = A\,\rho\, L_f\ \frac{d\varepsilon}{dt} = -2\ \pi\,(r_o + \varepsilon)\, L\,\rho\, L_f\ \frac{d\varepsilon}{dt}$$

where ρL_f is the latent heat.

Combining these Equations

$$\frac{k\,(T_{fr} - T_\infty)}{r_o^2\,\rho\, L_f}\,dt = -\left[\frac{k}{r_o h_o} + \ln\left(\frac{r_o + \varepsilon}{r_o}\right)\right]\left(\frac{r_o + \varepsilon}{r_o}\right) d\left(\frac{r_o + \varepsilon}{r_o}\right)$$

$$\text{Let}\qquad t^\bullet = \frac{(T_f - T_\infty)kt}{\rho L\, r_o^2} \to dt^\bullet = \frac{(T_{fr} - T_\infty)k}{\rho L^2\, r_o^2}\,dt \qquad r^\bullet = \frac{\varepsilon + r_o}{r_o} \qquad R^\bullet = \frac{h_o\, r_o}{k}$$

$$dt^\bullet = -\left(\frac{1}{R^\bullet} + \ln r^\bullet\right)\ r^\bullet\, dr^\bullet$$

Integrating

$$\int_0^{t^\bullet} dt^\bullet = \int_1^{r^\bullet} \left(\frac{1}{R^\bullet} + \ln r^\bullet\right) r^\bullet \, dr^\bullet$$

$$t^\bullet = \frac{r^{\bullet 2}}{2R^\bullet} - \frac{1}{2R^\bullet} + r^{\bullet 2}\left(\frac{\ln r^\bullet}{2} - \frac{1}{4}\right) + \frac{1}{4}$$

$$t^\bullet = \frac{r^{2\bullet}}{2} \ln r^\bullet + \left(\frac{1}{2R^\bullet} - \frac{1}{4}\right)(r^{\bullet 2} - 1)$$

All the ice is frozen when $\varepsilon = r_o \rightarrow r^* = 0$

At $\quad r^* = 0$

$$t^* = \frac{1}{2R^*} + \left(\frac{1}{4}\right)$$

$$\frac{k\,(T_{fr} - T_\infty)}{r_o^2 \rho L_f}\, t = \frac{1}{2\left(\frac{h_o\, r_o}{k}\right)} + \left(\frac{1}{4}\right)$$

$$t = \frac{r_o^2\, \rho L_f}{k\,(T_{fr} - T_\infty)} \left(\frac{k}{2\, h_o\, r_o} + \frac{1}{4}\right)$$

$$t = 64{,}698 \text{ s } (0.6636 + 0.25) = 59{,}113 \text{ s} = 16.4 \text{ h}$$

PROBLEM 9.39

Estimate the time required for nocturnal radiation to freeze a 3 cm thickness of water with ambient air and initial water temperature at 4°C. Neglect evaporation effects.

GIVEN

- Water exposed to nocturnal radiation and air
- Initial water temperature (T_{wi}) and ambient air temperature (T_∞) = 4°C

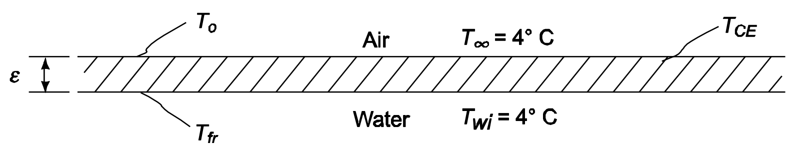
FIND

- The time (t_f) required to freeze an ice layer of thickness (ε_f) = 3 cm = 0.03 m

ASSUMPTIONS

- The thermal capacitance of the ice is negligible
- The energy required to lower the temperature of the water to the freezing point is negligible compared to the latent heat of fusion
- Natural convection from the upper and lower surfaces of the ice layer is negligible
- Effective sky temperature (T_s) = 0 K
- Ice surface behaves as a blackbody (ε_r = 1.0)

SKETCH



PROPERTIES AND CONSTANTS

The ice property values given in the problem statement of Problem 9.37 are:

Latent heat of fusion (L_f) = 333,780 J/kg

Density (ρ) = 918 kg/m^3

Thermal conductivity (k) = 2.22 W/(m K)

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The analysis of Section 9.6 can be applied to this problem by substituting the radiative heat transfer coefficient, h_r, for h_o.

However, h_r is a function of the ice surface temperature, T_o

$$h_r = \frac{\sigma(T_o^4 - R_s^4)}{T_o - T_s} = \sigma T_o^3$$

As ice first begins to form, the surface temperature is the same as the freezing temperature ($T_o - T_{fr}$) and the radiative heat transfer coefficient is

$$h_{ri} = \sigma T_{ft}^3 = \left(5.67 \times 10^{-8}\,\mathrm{W/(m^2K^4)}\right)(273\ \mathrm{K})^3 = 1.154\ \mathrm{W/(m^2K)}$$

The final surface temperature can be calculated by equating the rate of conduction through the ice layer with the rate of radiation from the surface of the ice

$$\frac{k}{\varepsilon} = (T_{fr} - T_o) = \sigma T_o^4$$

$$\frac{2.22\,\mathrm{W/(m\,K)}}{0.03\,\mathrm{m}}\,(273\ \mathrm{K} - T_o) = \left(5.67 \times 10^{-8}\ \mathrm{W/(m^2K^4)}\right) T_o^4$$

By trial and error, T_o = 269 K
Therefore, the final value of h_r is

$$h_{rf} = \sigma T_o^3 = \left(5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4)\right) (269 \text{ K})^3 = 1.104 \text{ W/(m}^2\text{K)}$$

Since the variation of hr is only about 5%. h_r will be considered constant at the average value of 1.13 W/(m^2 K). The time required is given by Equation (9.61)

$$\varepsilon^+ = -1 + \sqrt{1 + 2t^+}$$

where

$$\varepsilon^+ = \frac{h_r \varepsilon}{k} = \frac{\left(1.13 \text{ W/(m}^2\text{K)}\right)(0.03 \text{m})}{2.22 \text{ W/(m K)}} = 0.0153$$

$$t^+ = t\, h_r^2 \frac{T_{fr} - T_s}{\rho L_f k}$$

Solving for t

$$t = \frac{\rho L_f k}{2 \text{h}_r^2 (T_{fr} - T_s)} [(\varepsilon^+ + 1)^2 - 1]$$

$$= \frac{\left(918 \text{ kg/m}^3\right)\left(333{,}780 \text{ J/kg}\right)\left(\text{(W s)/J}\right)\left(2.22 \text{ W/(m K)}\right)}{2\left(1.13 \text{ W/(m}^2\text{K)}\right)^2 (273 \text{K} - 0 \text{ K})} [0.0153 + 1)^2 - 1]$$

$$t = 30{,}084 \text{ s} = 8.4 \text{ h}$$

PROBLEM 9.40

The temperature of a 100 m diameter cooling pond is 7°C on a winter day. If the air temperature suddenly drops to –7°C, calculate the thickness of ice formed after three hours.

GIVEN

- A round cooling pond on a winter day
- Initial temperature (T_I) = 7°C
- Air temperature (T_∞) drops to –7°C
- Pond diameter (D) = 100 m

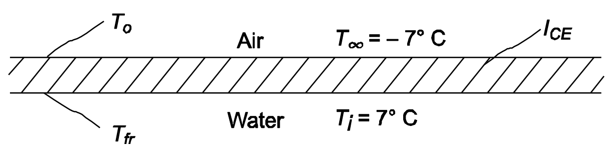
FIND

- The thickness of ice (ε) formed after 3 hours

ASSUMPTIONS

- The thermal capacitance of the ice layer is negligible
- Radiative heat transfer is negligible
- Bulk water temperature remains constant at 7°C
- Air temperature is constant at –7°C
- The air and water are still

SKETCH



PROPERTIES AND CONSTANTS

The ice property values given in the problem statement of Problem 9.37 is

Latent heat of fusion (L_f) = 333,780 J/kg

Density (ρ) = 918 kg/m^3

Thermal conductivity (k) = 2.22 W/(m K)

Extrapolating from Appendix 2, Table 28, for dry air at the estimated film temperature of –3.5°C

Density (ρ_a) = 1.267 kg/m^3

Thermal expansion coefficient (β_a) = 0.00370 1/K

Thermal conductivity (k_a) = 0.0235 W/(m K)

Kinematic viscosity (ν_a) = 13.6 × 10^{-6} m^2/s

Prandtl number (Pr_a) = 0.71

From Appendix 2, Table 13, for water at the estimated film temperature of 3.5°C

Density (ρ_w) = 1000 kg/m^3

Thermal expansion coefficient (β_w) = –0.12 × 10^{-4} 1/K

Thermal conductivity (k_w) = 0.565 W/(m K)

Kinematic viscosity (ν_w) = 1.611 × 10^{-6} m^2/s

Prandtl number (Pr_w) = 12.1

SOLUTION

The convective heat transfer coefficient on the air side (h_o) and the water side (h_ε) must be calculated before the analysis of Section 9.6 can be applied.

Water side

The characteristic length for the pond is

$$L = \frac{A_s}{P} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4} = 25 \text{ m}$$

The Rayleigh number based on this length is

$$Ra_L = Gr_L\, Pr = \frac{g\, \beta_w (T_l - T_{fr}) L^3 Pr}{\nu_w^2}$$

$$Ra_L = \frac{\left(9.8\,\text{m/s}^2\right)\left(-0.12 \times 10^{-4}\,1/\text{K}\right)(7°\text{C} - 0°\text{C})\left(25\,\text{m}\right)^3 (12.1)}{\left(1.611 \times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = -6.12 \times 10^{13}$$

The minus sign indicates that even though we are cooling the water from above, the buoyancy effect is like that for the heating from above case.

Although this is out of its Rayleigh number range, Equation (8.17) will be used, for lack of a more appropriate correlation, to estimate the Nusselt number

$$\overline{Nu}_L = 0.27\, Ra_L^{\frac{1}{4}} = 0.27\,(6.12 \times 10^{13})^{\frac{1}{4}} = 755$$

$$\bar{h}_c = \overline{Nu}_L \frac{k}{L} = 755 \left(\frac{0.565\,\text{W/(m K)}}{25\,\text{m}}\right) = 17.1 \ \text{W/(m}^2\text{K)}$$

Air side

The heat transfer coefficient on the air side (h_o) will depend on the ice surface temperature (T_o) which changes as the ice thickens. The transfer coefficient will be approximated as constant with the surface temperature equal to the freezing temperature. With these simplifications, the Rayleigh number is

$$Ra_L = \frac{\left(9.8\,\text{m/s}^2\right)\left(0.00370\,1/\text{K}\right)(7°\text{C})(25\,\text{m})^3(0.71)}{\left(13.6 \times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 1.52 \times 10^{13}$$

Once again, the correlation of Equation (8.16) will be extended to estimate the Nusselt number

$$\overline{Nu}_L = 0.15\, Ra_L^{\frac{1}{3}} = 0.15\,(1.52 \times 10^{13})^{\frac{1}{3}} = 3718$$

$$\bar{h}_o = \overline{Nu}_L \frac{k}{L} = 3718 \left(\frac{0.0235\,\text{W/(m K)}}{25\,\text{m}}\right) = 3.5 \ \text{W/(m}^2\,\text{K)}$$

Applying Equation (9.63)

$$t^+ = -\frac{1}{(R^+T^+)^2} \ln\left(1 + \frac{R^+T^+\varepsilon^+}{1 + R^+T^+}\right) + \frac{\varepsilon^+}{R^+T^+}$$

where

$$\varepsilon^+ = \frac{\bar{h}_o \varepsilon}{k}$$

$$R^+ = \frac{\bar{h}_c}{\bar{h}_o} = \frac{17.1}{3.5} = 4.89$$

$$T^+ = \frac{T_l - T_{fr}}{T_{fr} - T_\infty} = \frac{7-0}{0+7} = 1.0$$

$$t^+ = t\, h_o{}^2 \frac{T_{fr} - T_\infty}{\rho L_f k} = (3\text{hr})\ 3600\,\text{s/h}\quad 3.5\,\text{W/(m}^2\text{K)}\ ^2$$

$$\frac{(0°\text{C} + 7°\text{C})}{918\,\text{kg/m}^3\quad 333{,}780\,\text{J/kg}\quad (\text{W s})/\text{J}\quad 2.22\,\text{W/(m K)}} = 0.00136$$

$$0.00136 = -\frac{1}{(4.89)^2}\ \ln\left[1 + \frac{4.89\,\varepsilon^+}{1+4.89}\right] + \frac{\varepsilon^+}{4.89}$$

By trial and error

$$\varepsilon^+ = 0.008$$

$$\therefore\ \varepsilon = \varepsilon^+ \frac{k}{\bar{h}_o} = 0.008\ \frac{2.22\,\text{W/(m K)}}{3.5\,\text{W/(m}^2\text{K)}} = 0.0051\text{ m} = 5.1\text{ mm}$$

PROBLEM 9.41

On a rainy Monday afternoon, a wealthy banker calls Sherlock Holmes to arrange a breakfast appointment for the following day to discuss the collection of a loan from farmer Joe. When Holmes arrives at the home of the banker at 9 a.m. Tuesday, he finds the body of the banker in his kitchen. The farmer's house is located on the other side of a lake, approximately 10 km from the banker's home. Since there is no convenient road between the home of the farmer and that of the banker, Holmes phones the police to question the farmer. The police arrive at the farmer's home within the hour and interrogate him about the death of the banker. The farmer claims to have been home all night. The tires on his truck were dry and he explains that his boots were moist and soiled because he had been fishing at the lake early in the morning. The police then phone Holmes to eliminate farmer Joe as a murder suspect because he could not have been at the banker's home since Holmes spoke to him. Holmes then calls the local weather bureau and learns that, although the temperature had been between 2°C and 5°C for weeks, it had dropped to –30°C quite suddenly on Monday night. Remembering that a 3 cm layer of ice can support a man, Holmes takes out his slide rule and heat transfer text, lights his pipe, makes a few calculations, and then phones the police to arrest farmer Joe. Why?

GIVEN

- Murder suspect with flakey alibi

FIND

(a) How Sherlock Holmes was able to disprove the alibi

SOLUTION

Holmes has evidently surmised that with the cold snap, the lake could have frozen to a sufficient thickness of 3 cm. to support the farmer on his way to murder the banker. In addition, the frozen lake would have precluded the farmer from fishing that morning. Basically, we need to determine if a 10 km lake, originally at 5°C or cooler, could have frozen to a thickness of 3 cm overnight after exposure to an air temperature of –30°C.

We can use Equation (9.63) to determine the ice thickness as a function of time. The following parameters are given

Liquid temperature, T_1 = 5°C (this is conservative since the range was 2°C to 5°C)

Freezing temperature, T_{fr} = 0°C

Air temperature, T_∞ = –30°C

The properties of ice from Appendix 2, Table 11 are

Heat of fusion, $L_f = 3.3 \times 10^5$ J/kg

Thermal conductivity, k_{ice} = 2.2 W/(mK)

Density, ρ_{ice} = 913 kg/m^3

In addition to the above parameters, we need to calculate the heat transfer coefficient at the air-ice interface, $\bar{h}_o$ and at the ice-water interface $\bar{h}_\varepsilon$. For the air-ice interface, we assume conservatively that there is no wind. Also note that the ice is warmer than the air and faces up, so we can use Equation (8.15) or (8.16) to determine $\bar{h}_o$.

At the mean temperature of (0 + –30)/2 = –15°C, we have for the air properties from Appendix 2, Table 28 (via extrapolation from data at 0°C and 20°C to –15°C)

$g\beta/\nu^2 = 2.22 \times 10^8$ (K m^3)$^{-1}$

Prandtl number, Pr = 0.71

thermal conductivity, k_{air} = 0.023 W/(mK)

Further, let us assume that the lake is approximately circular, 10 km in diameter. The length scale required in Equation (8.15) or (8.16) is

$$L = \frac{\text{area of lake}}{\text{perimeter of lake}} = \frac{\frac{\pi (10^4\,\text{m})^2}{4}}{\pi 10^4\,\text{m}} = 2500\ \text{m}$$

So, the Rayleigh number is

$$Ra_L = \frac{g\,\beta \Delta T\, L^3 Pr}{\nu^2} = (2.22 \times 10^8\ (\text{m}^3\text{K})^{-1})\ (30\text{K})(2500\text{m})^3(0.71) = 7.38 \times 10^{19}$$

which is well beyond the restriction on Equation (8.16). In lieu of a correlation equation for such a large Rayleigh number, we will use Equation (5.16) and take note of the assumption.

Then the mean Nusselt number is

$$\overline{Nu}_L = 0.15\, Ra_L^{\frac{1}{3}} = 6.28 \times 10^5$$

and the heat transfer coefficient is

$$\bar{h}_o = \frac{k_{\text{air}}}{L}\ \overline{Nu}_L = \frac{0.023\,\text{W/(m K)}\ (6.28 \times 10^5)}{(2500\,\text{m})} = 5.8\ \text{W/(m}^2\text{K)}$$

Equation (8.16) shows that the heat transfer coefficient is independent of L as long as we are in the turbulent regime. That is, $\bar{h}_o$ = 5.8 W/(m^{2}K) regardless of the Rayleigh number in the turbulent regime and we expect that this value of $\bar{h}_o$ would be a reasonable estimate for the 10 km lake surface.

At the bottom surface of the ice, we have a cooled surface facing down, therefore, the same equations apply. For water at the mean temperature of(0 + 5)/2 = 2.5°C, we have from Appendix 2, Table 13

$g\beta/\nu^2 = 0.551 \times 10^9\ (\text{m}^3\text{K})^{-1}$ (from 10°C data)

Prandtl number, Pr = 12.6

thermal conductivity, k_{water} = 0.563 W/(mK)

so

$$Ra_L = \frac{g\,\beta \Delta T\, L^3 Pr}{\nu^2} = (0.551 \times 10^9 (\text{m}^3\text{K})^{-1})\ (5\text{K})(2500\text{m})^3(12.6) = 5.42 \times 10^{20}$$

The Nusselt number is

$$Nu_L = 0.15\ Ra_L^{\frac{1}{3}} = 1.22 \times 10^6$$

and the heat transfer coefficient is

$$\bar{h}_c = \frac{k_{\text{water}}}{L}\ \overline{Nu}_L = \frac{0.563\,\text{W/(m K)}\ (1.22 \times 10^6)}{(2500\,\text{m})} = 275\ \text{W/(m}^2\text{K)}$$

The same comments on $\bar{h}_o$ apply to $\bar{h}_\varepsilon$.

Now, to use Equation (9.62) we need the following parameters

$$R^+ = \frac{\bar{h}_c}{\bar{h}_o} = \frac{275}{2.8} = 47.4$$

$$T^+ = \frac{T_l - T_{fr}}{T_{fr} - T_\infty} = \frac{5 - 0}{0 - (-30)} = 0.166$$

$$R^+T^+ = (47.4)(0.166) = 7.89$$

To calculate how long is needed to freeze the 3 cm layer, we have ε = 0.03 m, so

$$\varepsilon^+ = \frac{\bar{h}_o \varepsilon}{k_{\text{ice}}} = \frac{5.8\ \text{W/(m}^2\text{K)}\ (0.03\,\text{m})}{2.2\ \text{W/(m\,K)}} = 0.079$$

Equation (9.63) then given for the generalized time

$$t^+ = -\frac{1}{7.89^2}\ \ln\left(1+\frac{(7.89)(0.079)}{1+7.89}\right) + \frac{0.079}{7.89} = 0.0089$$

The definition of the generalized time is

$$t^+ = t\bar{h}_o^2\ \frac{T_{fr} - T_\infty}{\rho_{\text{ice}} L_f\ k_{\text{ice}}} = 0.0089$$

Solving for the dimensional time we have

or

$$t = \frac{(0.0089)\ 913\,\text{kg/m}^3\ \ 3.3\times10^5\ \ (\text{W s})/\text{kg}\ \ 2.2\ \text{W/(m\,K)}}{5.8\ \text{W/(m}^2\text{K)}^{\ 2}\ \ 0-(-30)\ \ (\text{K})} = 5861\ \text{s} = 1.63\ \text{h}$$

Therefore, less than 2 hours would be required for the lake to freeze and this is the information that led Sherlock Holmes to ask for the arrest of Farmer Joe.