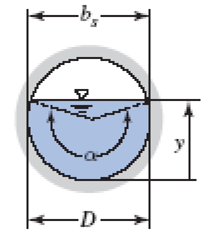
Problem 11.1 [Difficulty: 2]

11.1 Verify the equation given in Table 11.1 for the hydraulic radius of a circular channel. Evaluate and plot the ratio R/D, for liquid depths between 0 and D.



Given: Circular channel

Find: Derive expression for hydraulic radius; Plot R/D versus D for a range of depths

Solution:

The area is (from simple geometry - a segment of a circle plus two triangular sections)

$$A = \frac{D^2}{8}\cdot\alpha + 2\cdot\frac{1}{2}\cdot\frac{D}{2}\cdot\sin\left(\pi - \frac{\alpha}{2}\right)\cdot\frac{D}{2}\cdot\cos\left(\pi - \frac{\alpha}{2}\right) = \frac{D^2}{8}\cdot\alpha + \frac{D^2}{4}\cdot\sin\left(\pi - \frac{\alpha}{2}\right)\cdot\cos\left(\pi - \frac{\alpha}{2}\right)$$

$$A = \frac{D^2}{8}\cdot\alpha + \frac{D^2}{8}\cdot\sin(2\cdot\pi - \alpha) = \frac{D^2}{8}\cdot\alpha - \frac{D^2}{8}\cdot\sin(\alpha) = \frac{D^2}{8}\cdot(\alpha - \sin(\alpha))$$

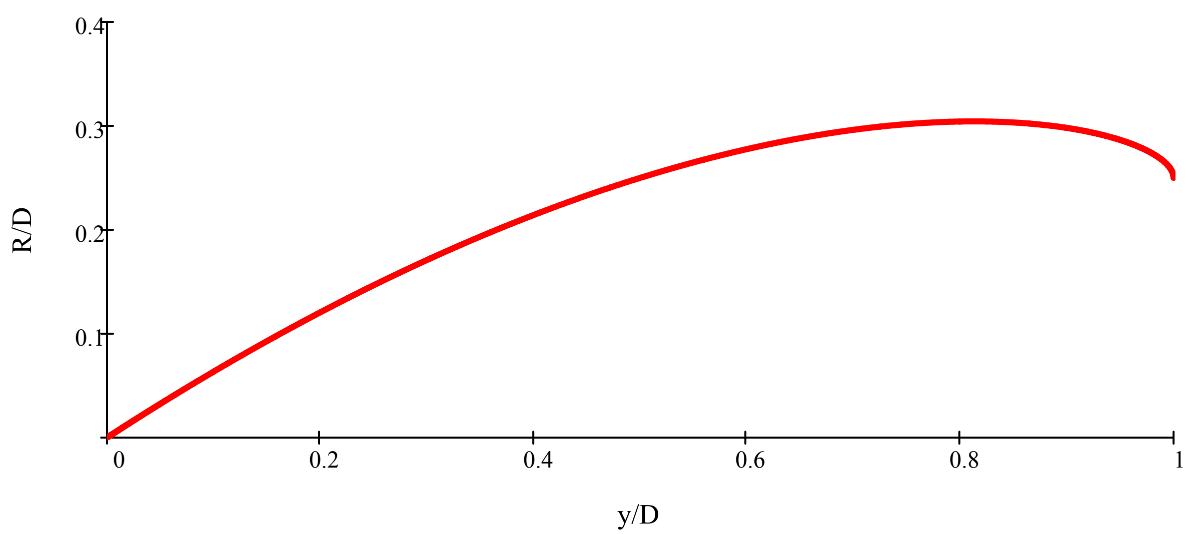
The wetted perimeter is (from simple geometry) $P = \frac{D}{2}\cdot\alpha$

Hence the hydraulic radius is $R = \frac{A}{P} = \frac{\frac{D^2}{8}\cdot(\alpha - \sin(\alpha))}{\frac{D}{2}\cdot\alpha} = \frac{1}{4}\cdot\left(1 - \frac{\sin(\alpha)}{\alpha}\right)\cdot D$

We are to plot $\frac{R}{D} = \frac{1}{4}\cdot\left(1 - \frac{\sin(\alpha)}{\alpha}\right)$

We will need y as a function of α: $y = \frac{D}{2} + \frac{D}{2}\cdot\cos\left(\pi - \frac{\alpha}{2}\right) = \frac{D}{2}\cdot\left(1 - \cos\left(\frac{\alpha}{2}\right)\right)$ or $\frac{y}{D} = \frac{1}{2}\cdot\left(1 - \cos\left(\frac{\alpha}{2}\right)\right)$

The graph can be plotted in *Excel*.



Problem 11.2

[Difficulty: 1]

11.2 A pebble is dropped into a stream of water that flows in a rectangular channel at 2 m depth. In one second, a ripple caused by the stone is carried 7 m downstream. What is the speed of the flowing water?

Given: Pebble dropped into flowing stream

Find: Estimate of water speed

Solution:

Basic equation $c = \sqrt{g \cdot y}$ and relative speeds will be $V_{wave} = V_{stream} + c$

Available data $y = 2 \cdot m$ and $V_{wave} = \frac{7 \cdot m}{1 \cdot s}$ $V_{wave} = 7 \frac{m}{s}$

We assume a shallow water wave (long wave compared to water depth)

$c = \sqrt{g \cdot y}$ so $c = 4.43 \frac{m}{s}$

Hence $V_{stream} = V_{wave} - c$ $V_{stream} = 2.57 \frac{m}{s}$

Problem 11.3

[Difficulty: 3]

11.3 Solution of the complete differential equations for wave motion without surface tension shows that wave speed is given by

$$c = \sqrt{\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi y}{\lambda}\right)}$$

where λ is the wave wavelength and y is the liquid depth. Show that when $\lambda/y \ll 1$, wave speed becomes proportional to $\sqrt{\lambda}$. In the limit as $\lambda/y \to \infty$, $c = \sqrt{gy}$. Determine the value of λ/y for which $c > 0.99\sqrt{gy}$.

Given: Speed of surface waves with no surface tension

Find: Speed when λ/y approaches zero or infinity; Value of λ/y for which speed is 99% of this latter value

Solution:

Basic equation $\quad c = \sqrt{\dfrac{g\cdot\lambda}{2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi\cdot y}{\lambda}\right)}} \quad$ (1)

For $\lambda/y \ll 1$ $\quad \tanh\left(\dfrac{2\cdot\pi\cdot y}{\lambda}\right)$ approaches 1 $\quad \tanh(\infty) \to 1 \quad$ so $\quad c = \sqrt{\dfrac{g\cdot\lambda}{2\cdot\pi}}$

Hence c is proportional to $\sqrt{\lambda}$ $\quad$ so as λ/y approaches ∞ $\quad c = \sqrt{g\cdot y}$

We wish to find λ/y when $\quad c = 0.99\cdot\sqrt{g\cdot y}$

Combining this with Eq 1 $\quad 0.99\cdot\sqrt{g\cdot y} = \sqrt{\dfrac{g\cdot\lambda}{2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi\cdot y}{\lambda}\right)}} \quad$ or $\quad 0.99^2\cdot g\cdot y = \dfrac{g\cdot\lambda}{2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi\cdot y}{\lambda}\right)}$

Hence $\quad 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi\cdot y}{\lambda}\right) = \dfrac{\lambda}{y} \quad$ Letting λ/y = x $\quad$ we find $\quad 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi}{x}\right) = x$

This is a nonlinear equation in x that can be solved by iteration or using *Excel*'s *Goal Seek* or *Solver*

$x = 1 \quad x = 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi}{x}\right) \quad x = 6.16 \quad x = 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi}{x}\right) \quad x = 4.74$

$x = 4.74 \quad x = 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi}{x}\right) \quad x = 5.35 \quad x = 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi}{x}\right) \quad x = 5.09$

$x = 5.09 \quad x = 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi}{x}\right) \quad x = 5.2 \quad x = 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi}{x}\right) \quad x = 5.15$

$x = 5.15 \quad x = 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi}{x}\right) \quad x = 5.17 \quad x = 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi}{x}\right) \quad x = 5.16$

$x = 5.16 \quad x = 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi}{x}\right) \quad x = 5.17 \quad x = 0.99^2\cdot 2\cdot\pi\cdot\tanh\left(\dfrac{2\cdot\pi}{x}\right) \quad x = 5.16$

Hence $\quad \dfrac{\lambda}{y} = 5.16$

Problem 11.4

(Difficulty 1)

11.4 A water flow rate of 250 cfs flows at a depth of 5 $feet$ in a rectangular channel that is 9 $feet$ wide. Determine whether the flow is sub-or supercritical. For this flow rate, determine the depth for critical flow.

Assumption The flow is steady and incompressible

Solution: Use the relations for flow in a channel to determine the characteristics.

The Froude number is defined as:

$$F_r = \frac{V}{\sqrt{g\, y_h}}$$

The velocity in the rectangular channel, which has a width b, is:

$$V = \frac{Q}{A} = \frac{Q}{b\, y_h} = \frac{250\ \frac{ft^3}{s}}{9\ ft \times 5ft} = 5.55\ \frac{ft}{s}$$

The Froude number is

$$F_r = \frac{5.55 ft/s}{\sqrt{32.2 \frac{ft}{s^2} \times 5ft}} = 0.43$$

The Froude number is less than unity and the flow is subcritical.

For the critical depth at this flow rate we have the Froude number equal to unity. Using the continuity expression to relate the velocity to the volume flow rate, width, and depth, we have for the critical depth :

$$y_c = \left(\frac{Q^2}{b^2 g}\right)^{\frac{1}{3}} = \left(\frac{\left(250\ \frac{ft^3}{s}\right)^2}{(9ft)^2 \times 32.2\ \frac{ft}{s^2}}\right)^{\frac{1}{3}} = 2.88\ ft$$

Problem 11.5

(Difficulty 1)

11.5 Determine and plot the relation between water velocity and depth over the range of $V = 0.1\ \frac{m}{s}$ to $10\ \frac{m}{s}$ for Froude numbers of 0.5 $(subcritical)$, 1.0 $(critical)$, and 2 $(supercritical)$. Explain how the flow can be subcritical, critical or supercritical for (a) the same velocity and (b) the same depth.

Assumption The flow is steady and incompressible

Solution: Use the relations for flow in a channel to determine the characteristics.

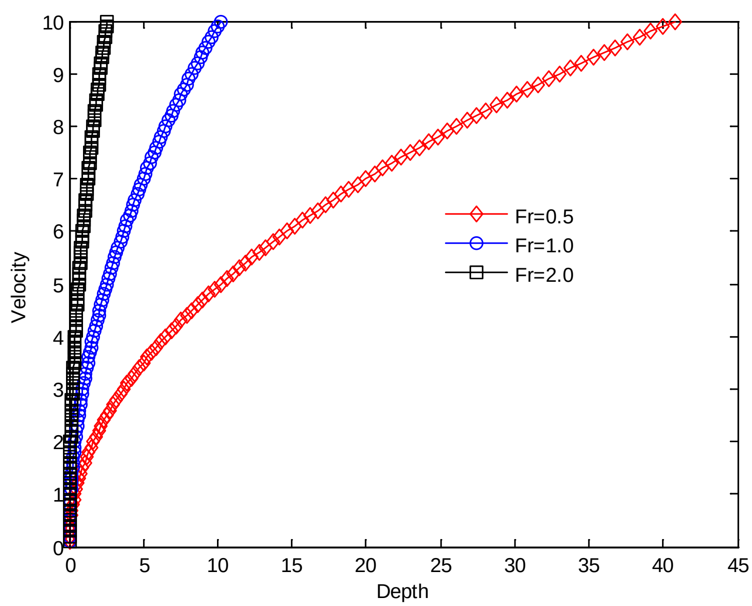
The Froude number is defined as:

$$F_r = \frac{V}{\sqrt{g\ y_h}}$$

The depth is then related to the velocity and Froude number as

$$y_h = \frac{V^2}{Fr^2 g}$$

For the different Froude numbers, we have the plots between the velocity and depth as:



(a) For the same velocity, if we decrease the depth the flow can change from subcritical to critical then supercritical.

(b) For the same depth, if we increase the velocity, the flow can change from subcritical to critical then supercritical.

Problem 11.6

[Difficulty: 1]

11.6 Capillary waves (ripples) are small amplitude and wavelength waves, commonly seen, for example, when an insect or small particle hits the water surface. They are waves generated due to the interaction of the inertia force of the fluid ρ and the fluid surface tension σ. The wavelength is

$$\lambda = 2\pi\sqrt{\frac{\sigma}{\rho g}}$$

Find the speed of capillary waves in water and mercury.

Given: Expression for capillary wave length

Find: Length of water and mercury waves

Solution:

Basic equation $\lambda = 2\cdot\pi\cdot\sqrt{\frac{\sigma}{\rho\cdot g}}$

Available data

Table A.2 (20°C) $SG_{Hg} = 13.55$ $SG_w = 0.998$ $\rho = 1000\cdot\frac{kg}{m^3}$

Table A.4 (20°C) $\sigma_{Hg} = 484\times10^{-3}\cdot\frac{N}{m}$ $\sigma_w = 72.8\times10^{-3}\cdot\frac{N}{m}$

Hence $\lambda_{Hg} = 2\cdot\pi\cdot\sqrt{\frac{\sigma_{Hg}}{SG_{Hg}\cdot\rho\cdot g}}$ $\lambda_{Hg} = 12\,mm$ $\lambda_{Hg} = 0.472\,in$

$\lambda_w = 2\cdot\pi\cdot\sqrt{\frac{\sigma_w}{SG_w\cdot\rho\cdot g}}$ $\lambda_w = 17.1\,mm$ $\lambda_w = 0.675\,in$

Problem 11.7 [Difficulty: 2]

11.7 The Froude number characterizes flow with a free surface. Plot on a log-log scale the speed versus depth for $0.1 \text{ m/s} < V < 3 \text{ m/s}$ and $0.001 < y < 1$ m; plot the line $Fr = 1$, and indicate regions that correspond to tranquil and rapid flow.

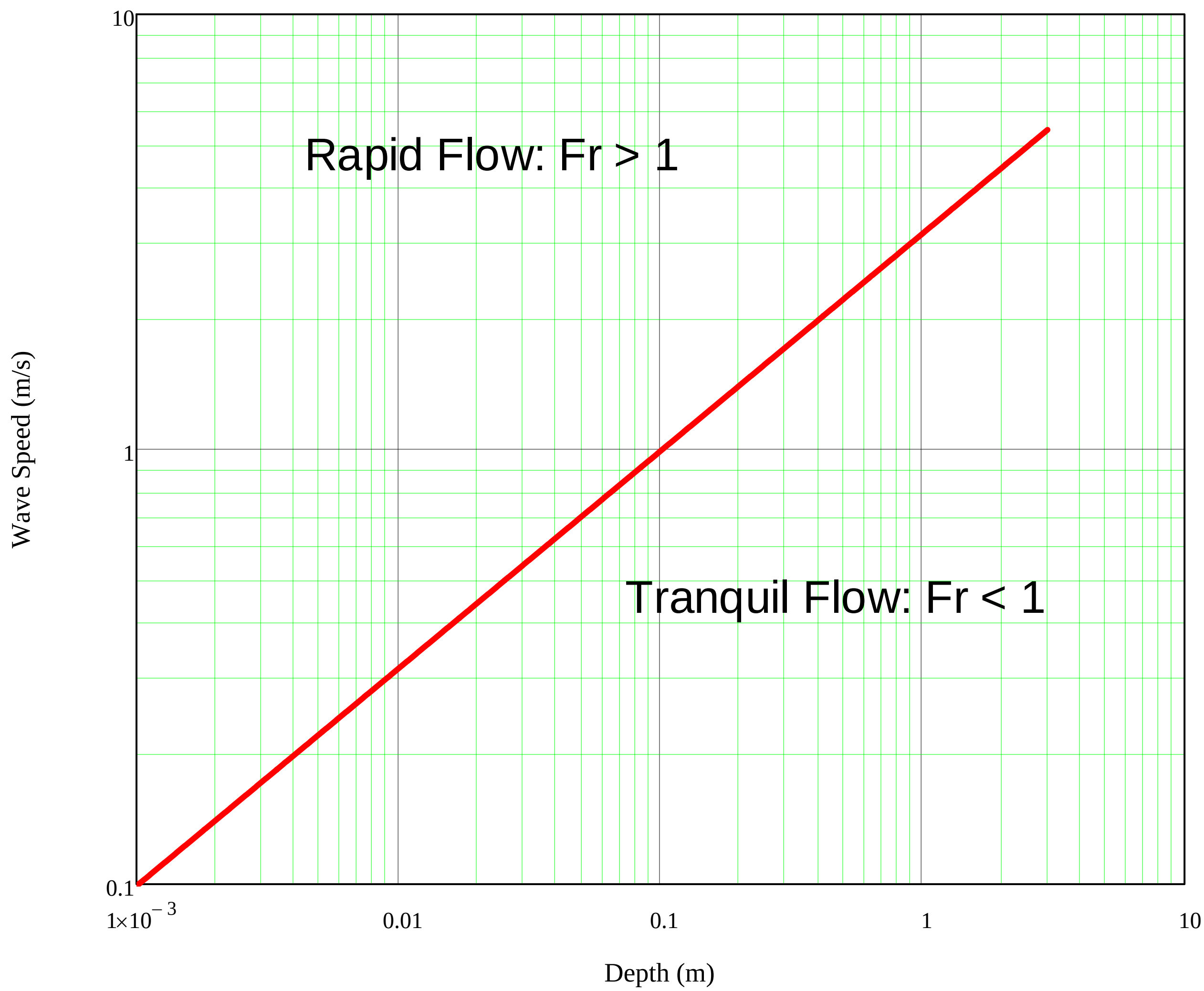
Given: Shallow water waves

Find: Speed versus depth

Solution:

Basic equation $c(y) = \sqrt{g \cdot y}$

We assume a shallow water wave (long wave compared to water depth)



Problem 11.8

(Difficulty 2)

11.8 Consider waves on the surface of a tank of water that travel at 5 $\frac{ft}{s}$. How fast would the waves travel if the tank were on the moon, on Jupiter, or on an orbiting space station? Explain your results.

Assumption: The flow is steady and incompressible.

Assume that the Froude number and the depth of the flow is the same at each location as on the earth.

Solution: Use the relations for flow in a channel to determine the characteristics.

The Froude number is defined as:

$$F_r = \frac{V}{\sqrt{g\, y_h}}$$

The Froude number will be the same for the tank on the earth and on the moon, but the velocity and gravity will be different:

$$F_r = \frac{V_e}{\sqrt{g_e\, y_h}} = \frac{V_m}{\sqrt{g_m\, y_h}}$$

On earth the value of gravity is $g_e = 32.2\ \frac{ft}{s^2}$ and on the moon it is $g_m = 5.37\ \frac{ft}{s^2}$.. The depth is the same and so the velocities are related as

$$\frac{V_e}{\sqrt{g_e}} = \frac{V_m}{\sqrt{g_m}}$$

Or

$$V_m = V_e\sqrt{\frac{g_m}{g_e}} = 5\frac{ft}{s} \times \sqrt{\frac{5.37\ \frac{ft}{s^2}}{32.2\ \frac{ft}{s^2}}} = 2.04\ \frac{ft}{s}$$

On the Jupiter we have for gravity $g_J = 80\ \frac{ft}{s^2}$. Therefor the velocities are

$$\frac{V_e}{\sqrt{g_e}} = \frac{V_J}{\sqrt{g_J}}$$

$$V_J = V_e\sqrt{\frac{g_J}{g_e}} = 5\frac{ft}{s} \times \sqrt{\frac{80\ \frac{ft}{s^2}}{32.2\ \frac{ft}{s^2}}} = 7.88\ \frac{ft}{s}$$

On the orbiting space station gravity is essentially zero. The velocity is then

$$V_o = V_e\sqrt{\frac{g_o}{g_e}} = 5\,\frac{ft}{s} \times \sqrt{\frac{0\ \frac{ft}{s^2}}{32.2\ \frac{ft}{s^2}}} = 0\ \frac{ft}{s}$$

From our results, we can see when we have higher gravitational attraction, the velocity is larger for the same Froude number.

11.9 A submerged body traveling horizontally beneath a liquid surface at a Froude number (based on body length) about 0.5 produces a strong surface wave pattern if submerged less than half its length. (The wave pattern of a surface ship also is pronounced at this Froude number.) On a log-log plot of speed versus body (or ship) length for 1 m/s < V < 30 m/s and 1 m < x < 300 m, plot the line $Fr = 0.5$.

Given: Motion of sumerged body

Find: Speed versus ship length

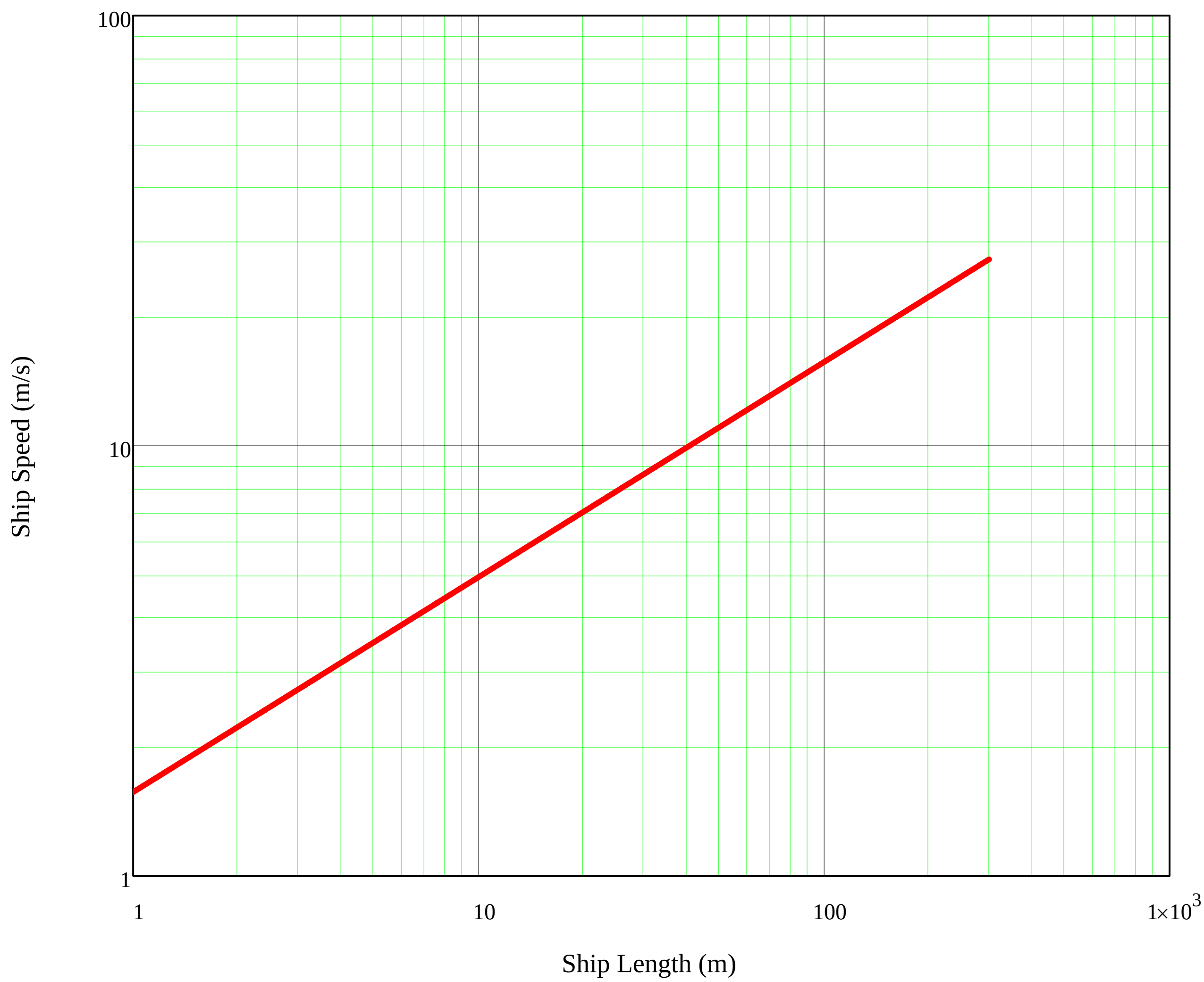
Solution:

Basic equation $c = \sqrt{g \cdot y}$

We assume a shallow water wave (long wave compared to water depth)

In this case we want the Froude number to be 0.5, with $Fr = 0.5 = \frac{V}{c}$ and $c = \sqrt{g \cdot x}$ where x is the ship length

Hence $V = 0.5 \cdot c = 0.5 \cdot \sqrt{g \cdot x}$



Problem 11.10 [Difficulty: 1]

11.10 Water flows in a rectangular channel at a depth of 750 mm. If the flow speed is (a) 1 m/s and (b) 4 m/s, compute the corresponding Froude numbers.

Given: Flow in a rectangular channel

Find: Froude numbers

Solution:

Basic equation $$Fr = \frac{V}{\sqrt{g \cdot y}}$$

Available data $y = 750 \cdot mm$ $\quad V_1 = 1 \cdot \frac{m}{s}$ $\quad V_2 = 4 \cdot \frac{m}{s}$

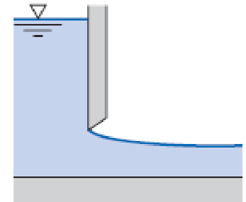
Hence $$Fr_1 = \frac{V_1}{\sqrt{g \cdot y}} \qquad Fr_1 = 0.369$$ Subcritical flow

$$Fr_2 = \frac{V_2}{\sqrt{g \cdot y}} \qquad Fr_2 = 1.47$$ Supercritical flow

Problem 11.11

[Difficulty: 2]

11.11 A partially open sluice gate in a 5-m-wide rectangular channel carries water at 10 m^3/s. The upstream depth is 2.5 m. Find the downstream depth and Froude number.



Given: Data on sluice gate

Find: Downstream depth; Froude number

Solution:

Basic equation: $$\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2 + h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow.

Noting that $p_1 = p_2 = p_{atm}$, (1 = upstream, 2 = downstream) the Bernoulli equation becomes

$$\frac{V_1^2}{2 \cdot g} + y_1 = \frac{V_2^2}{2 \cdot g} + y_2$$

The given data is $b = 5 \cdot m$ $\quad y_1 = 2.5 \cdot m$ $\quad Q = 10 \cdot \frac{m^3}{s}$

For mass flow $Q = V \cdot A$ so $V_1 = \frac{Q}{b \cdot y_1}$ and $V_2 = \frac{Q}{b \cdot y_2}$

Using these in the Bernoulli equation $$\frac{\left(\frac{Q}{b \cdot y_1}\right)^2}{2 \cdot g} + y_1 = \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2 \qquad (1)$$

The only unknown on the right is y_2. The left side evaluates to $\frac{\left(\frac{Q}{b \cdot y_1}\right)^2}{2 \cdot g} + y_1 = 2.53\,m$

To find y_2 we need to solve the non-linear equation. We must do this numerically; we may use the Newton method or similar, or *Excel*'s *Solver* or *Goal Seek*. Here we interate manually, starting with an arbitrary value less than y_1.

For $y_2 = 0.25 \cdot m$ $\quad \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2 = 3.51\,m$ $\quad$ For $y_2 = 0.3 \cdot m$ $\quad \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2 = 2.57\,m$

For $y_2 = 0.305 \cdot m$ $\quad \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2 = 2.50\,m$ $\quad$ For $y_2 = 0.302 \cdot m$ $\quad \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2 = 2.54\,m$

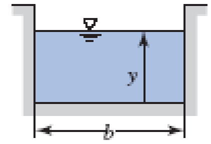
Hence $y_2 = 0.302\,m$ is the closest to three figs.

Then $V_2 = \frac{Q}{b \cdot y_2}$ $\quad V_2 = 6.62\,\frac{m}{s}$ $\quad Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $\quad Fr_2 = 3.85$

Problem 11.12

[Difficulty: 1]

11.12 Find the critical depth for flow at 3 m^3/s in a rectangular channel of width 2.5 m.



Given: Rectangular channel flow

Find: Critical depth

Solution:

Basic equations: $$y_c = \left(\frac{Q^2}{g \cdot b^2}\right)^{\frac{1}{3}}$$

Given data: $b = 2.5 \cdot m$ $\quad Q = 3 \cdot \frac{m^3}{s}$

Hence $$y_c = \left(\frac{Q^2}{g \cdot b^2}\right)^{\frac{1}{3}} \qquad y_c = 0.528\,m$$

Problem 11.13

(Difficulty 3)

11.13 Flow occurs in a rectangular channel of $6\ m$ width and has a specific energy of $3\ m$. Plot accurately the relation between depth and specific energy. Determine from the curve (a) the critical depth (b) the maximum flow rate (c) the flow rate at a depth of $2.4\ m$, and (d) the depths at which a flow rate of $28.3\ \frac{m^3}{s}$ may exist, and the flow condition at these depths.

Assumption: The flow is steady and incompressible

Solution: Use the energy relations for flow in a channel to determine the characteristics.

The specific energy is defined as:

$$E = y + \frac{Q^2}{2gA^2}$$

The area is the product of the width b and depth y. Thus the flow is related to the specific energy as

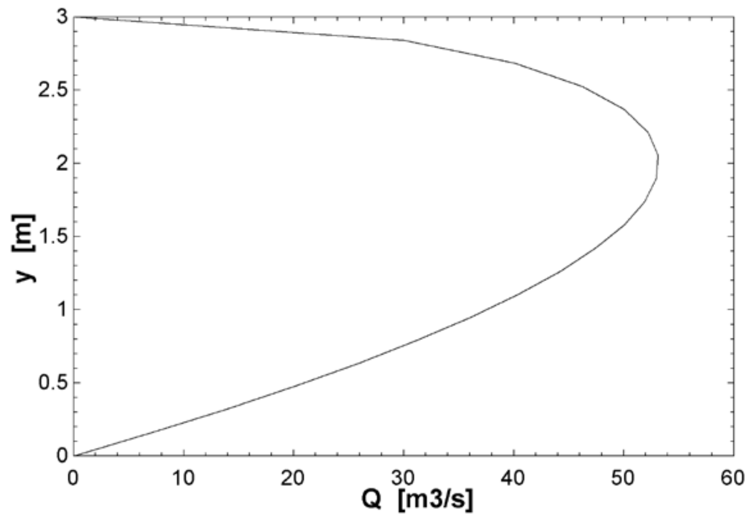
$$Q = \sqrt{2gb^2(y^2E - y^3)}$$

With values

$$Q = \sqrt{2 \times 9.81\ \frac{m}{s^2} \times (6m)^2 \times (y^2 \times 3m - y^3)}$$

$$Q = 6\sqrt{(3y^2 - y^3)}\ \frac{m^3}{s}$$

The relation between depth and specific energy is:



(a) The critical depth occurs when we have:

$$\frac{dQ}{dy} = 0$$

From the curve, the critical depth is $y_c = 2.0\ m$

(b) The maximum flow rate from the curve is $Q_{max} = 53\ \frac{m^2}{s}$

(c) From the curve, the flow rate at $y = 2.4\ m$ is $\mathrm{Q} = 50\ \frac{m^2}{s}$

(d) For a flow rate of 28.3 $\frac{m^3}{s}$ we have depths of 0.7 m (supercritical flow) and 2.9 m (subcritical flow)

Problem 11.14

(Difficulty 2)

11.14 What is the maximum flow rate which may occur in a rectangular channel $2.4\,m$ wide for a specific energy of $1.5\,m$?

Assumption: The flow is steady and incompressible

Solution: Use the energy relations for flow in a channel to determine the characteristics.

The specific energy is defined as:

$$E = y + \frac{Q^2}{2gA^2}$$

The area is the product of the width b and depth y. Thus the flow is related to the specific energy as

$$Q = \sqrt{2gb^2(y^2E - y^3)}$$

With values

$$Q = \sqrt{2 \times 9.81\,\frac{m}{s^2} \times (2.4\,m)^2 \times (y^2 \times 1.5m - y^3)}$$

$$Q = 10.6\sqrt{(1.5\,y^2 - y^3)}\ \frac{m^3}{s}$$

For the maximum flow rate we have that the change in flow rate with respect to depth is zero, or:

$$\frac{dQ}{dy} = 10.6 \times \frac{1}{2} \times \frac{2 \times 1.5y - 3y^2}{\sqrt{(1.5y^2 - y^3)}} = 0$$

The depth is the critical depth y_c. O

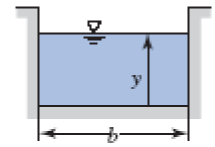
$$3y_c - 3y_c^2 = 0$$
$$y_c = 1.0\ m$$

So the flow rate is then:

$$Q_{max} = 10.6 \times \sqrt{(1.5 \times (1.0\,)^2 - (1.0\,)^3)}\ \frac{m^2}{s} = 7.51\ \frac{m^2}{s}$$

Problem 11.15 [Difficulty: 2]

11.15 A rectangular channel carries a discharge of 10 ft^3/s per foot of width. Determine the minimum specific energy possible for this flow. Compute the corresponding flow depth and speed.



Given: Data on rectangular channel

Find: Minimum specific energy; Flow depth; Speed

Solution:

Basic equation: $E = y + \frac{V^2}{2 \cdot g}$

In Section 11-2 we prove that the minimum specific energy is when we have critical flow; here we rederive the minimum energy point

For a rectangular channel $Q = V \cdot b \cdot y$ or $V = \frac{Q}{b \cdot y}$ with $\frac{Q}{b} = 10 \cdot \frac{\frac{ft^3}{s}}{ft} = \text{constant}$

Hence, using this in the basic equation $E = y + \left(\frac{Q}{b \cdot y}\right)^2 \cdot \frac{1}{2 \cdot g} = y + \left(\frac{Q^2}{2 \cdot b^2 \cdot g}\right) \cdot \frac{1}{y^2}$

E is a minimum when $\frac{dE}{dy} = 1 - \left(\frac{Q^2}{b^2 \cdot g}\right) \cdot \frac{1}{y^3} = 0$ or $y = \left(\frac{Q^2}{b^2 \cdot g}\right)^{\frac{1}{3}}$ $y = 1.46 \cdot ft$

The speed is then given by $V = \frac{Q}{b \cdot y}$ $V = 6.85 \cdot \frac{ft}{s}$

Note that from Eq. 11.22 we also have $V_c = \left(\frac{g \cdot Q}{b}\right)^{\frac{1}{3}}$ $V_c = 6.85 \cdot \frac{ft}{s}$ which agrees with the above

The minimum energy is then $E_{min} = y + \frac{V^2}{2 \cdot g}$ $E_{min} = 2.19 \cdot ft$

Problem 11.16

(Difficulty 1)

11.16 Flow in the channel of Problem 11.15 has a specific energy of 4.5 ft. Compute the alternate depths of this specific energy.

Given: The specific energy E.

Find: The alternate depths.

Solution: Use the specific energy equation for open channel flow:

$$E = y + \frac{V^2}{2g}$$

For a rectangular channel:

$$Q = Vby$$

$$V = \frac{Q}{by}$$

$$\frac{Q}{b} = 10\ \frac{ft^2}{s} = constant$$

Hence we have:

$$E = y + \frac{1}{2g}\left(\frac{Q}{by}\right)^2 = y + \frac{Q^2}{2b^2 g}\frac{1}{y^2} = 4.5\ ft$$

$$g = 32.2\ \frac{ft}{s^2}$$

So we have:

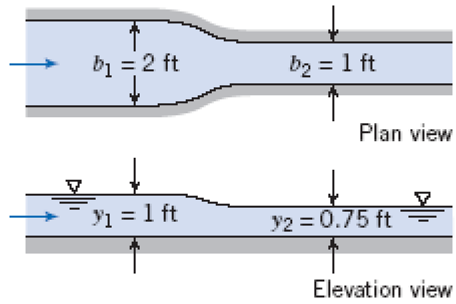
$$y + \frac{1.553\ ft^3}{y^2} - 4.5\ ft = 0$$

Solving this nonlinear implicit equation by matlab we have the alternate depths as:

$y = 0.634\ ft$ or $y = 4.42\ ft$

Problem 11.17

11.17 Consider the Venturi flume shown. The bed is horizontal, and flow may be considered frictionless. The upstream depth is 1 ft, and the downstream depth is 0.75 ft. The upstream breadth is 2 ft, and the breadth of the throat is 1 ft. Estimate the flow rate through the flume.



Given: Data on venturi flume

Find: Flow rate

Solution:

Basic equation: $$\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2$$ The Bernoulli equation applies because we have steady, incompressible, frictionless flow

At each section $Q = V \cdot A = V \cdot b \cdot y$ or $V = \frac{Q}{b \cdot y}$

The given data is $b_1 = 2 \cdot ft$ $y_1 = 1 \cdot ft$ $b_2 = 1 \cdot ft$ $y_2 = 0.75 \cdot ft$

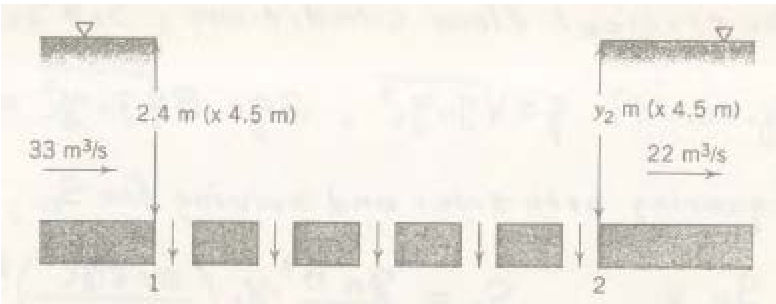
Hence the Bernoulli equation becomes (with $p_1 = p_2 = p_{atm}$) $$\frac{\left(\frac{Q}{b_1 \cdot y_1}\right)^2}{2 \cdot g} + y_1 = \frac{\left(\frac{Q}{b_2 \cdot y_2}\right)^2}{2 \cdot g} + y_2$$

Solving for Q $$Q = \sqrt{\frac{2 \cdot g \cdot (y_1 - y_2)}{\left(\frac{1}{b_2 \cdot y_2}\right)^2 - \left(\frac{1}{b_1 \cdot y_1}\right)^2}}$$ $Q = 3.24 \cdot \frac{ft^3}{s}$

Problem 11.18

(Difficulty 2)

11.18 Eleven cubic meters per second are diverted through ports in the bottom of the channel between sections 1 and 2. Neglecting head losses and assuming a horizontal channel, what depth of water is to be expected at section 2? What channel width at section 2 would be required to produce a depth of 2.5 m ?



Assumption: The flow is steady and incompressible

Solution: Use the energy relations for flow in a channel to determine the characteristics.

The specific energy is defined as:

$$E = y + \frac{Q^2}{2gA^2}$$

For the frictionless flow the specific energy E will be a constant. At section 1 we have:

$$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q_1^2}{2gA_1^2}$$

$$E_1 = 2.4\ m + \frac{\left(33\ \frac{m^3}{s}\right)^2}{2 \times 9.81\ \frac{m}{s^2} \times (2.4\ m \times 4.5\ m)^2} = 2.88\ m$$

At section 2 we have:

$$E_2 = E_1 = 2.88\ m$$

$$E_2 = y_2 + \frac{Q_2^2}{2gA_2^2} = y_2 + \frac{22^2}{2 \times 9.81 \times 4.5 \times 4.5 \times y_2^2} = y_2 + \frac{1.22}{y_2^2} = 2.88\ m$$

Solving this equation for y_2 we have:

$$y_2 = 2.71\ m$$

If the water depth at section 2 is 2.5 m, we have, where b2 is the width:

$$E_2 = y_2 + \frac{Q_2^2}{2gA_2^2} = y_2 + \frac{22^2}{2 \times 9.81 \times 2.5 \times 2.5 \times b_2^2} = 2.5 + \frac{3.95}{b_2^2} = 2.88\ m$$

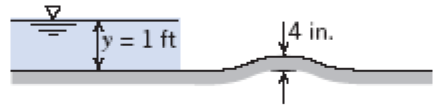
Thus

$$b_2 = 3.22\ m$$

Problem 11.19

[Difficulty: 3]

11.19 A rectangular channel 10 ft wide carries 100 cfs on a horizontal bed at 1.0 ft depth. A smooth bump across the channel rises 4 in. above the channel bottom. Find the elevation of the liquid free surface above the bump.



Given: Data on rectangular channel and a bump

Find: Elevation of free surface above the bump

Solution:

Basic equation: $$\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2 + h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow. Note that at location 2 (the bump), the potential is $y_2 + h$, where h is the bump height

Recalling the specific energy $E = \frac{V^2}{2 \cdot g} + y$ and noting that $p_1 = p_2 = p_{atm}$, the Bernoulli equation becomes $E_1 = E_2 + h$

At each section $Q = V \cdot A = V \cdot b \cdot y$ or $V = \frac{Q}{b \cdot y}$

The given data is $b = 10 \cdot ft$ $y_1 = 1 \cdot ft$ $h = 4 \cdot in$ $Q = 100 \cdot \frac{ft^3}{s}$

Hence we find $V_1 = \frac{Q}{b \cdot y_1}$ $V_1 = 10 \cdot \frac{ft}{s}$

and $E_1 = \frac{V_1^2}{2 \cdot g} + y_1$ $E_1 = 2.554 \cdot ft$

Hence $$E_1 = E_2 + h = \frac{V_2^2}{2 \cdot g} + y_2 + h = \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 + h \quad \text{or} \quad \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = E_1 - h$$

This is a nonlinear implicit equation for y_2 and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We select y_2 so the left side of the equation equals $E_1 - h = 2.22 \cdot ft$

For $y_2 = 1 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 2.55 \cdot ft$ For $y_2 = 1.5 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 2.19 \cdot ft$

For $y_2 = 1.4 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 2.19 \cdot ft$ For $y_2 = 1.3 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 2.22 \cdot ft$

Hence $y_2 = 1.30 \cdot ft$

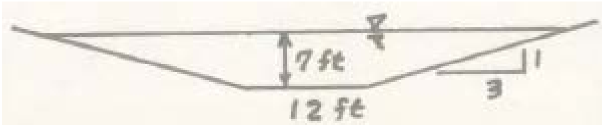
Note that $V_2 = \frac{Q}{b \cdot y_2}$ $V_2 = 7.69 \cdot \frac{ft}{s}$

so we have $Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}}$ $Fr_1 = 1.76$ and $Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $Fr_2 = 1.19$

Problem 11.20

(Difficulty 2)

11.20 At what depths may 800 cfs flow in a trapezoidal channel of base width 12 ft and side slopes of 1(vert.) on 3 (horiz.) if the specific energy is 7 ft ?



Assumption The flow is steady and incompressible

Solution: Use the energy relations for flow in a channel to determine the characteristics.

The specific energy is defined as:

$$E = y + \frac{Q^2}{2gA^2}$$

The area as a function of depth is given by

$$A = 12y + \frac{y \cdot 3y}{2} \times 2$$

The specific energy is then:

$$E = y + \frac{Q^2}{2g\left(12y + \frac{y \cdot 3y}{2} \times 2\right)^2} = y + \frac{Q^2}{2g(12y + 3y^2)^2}$$

Thus for a specific energy of 7 ft

$$y + \frac{800^2}{2 \times 32.2 \times (12y + 3y^2)^2} = 7$$

Solving this equation we have two possibilities:

$$y = 2.42\ ft \text{ (supercritical flow)}$$

$$y = 6.80\ ft \text{ (subcritical flow)}$$

Problem 11.21 [Difficulty: 3]

11.21 At a section of a 10-ft-wide rectangular channel, the depth is 0.3 ft for a discharge of 20 ft³/s. A smooth bump 0.1 ft high is placed on the floor of the channel. Determine the local change in flow depth caused by the bump.

Given: Data on rectangular channel and a bump

Find: Local change in flow depth caused by the bump

Solution:

Basic equation: $$\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2 + h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow. Note that at location 2 (the bump), the potential is $y_2 + h$, where h is the bump height

Recalling the specific energy $E = \frac{V^2}{2 \cdot g} + y$ and noting that $p_1 = p_2 = p_{atm}$, the Bernoulli equation becomes $E_1 = E_2 + h$

At each section $Q = V \cdot A = V \cdot b \cdot y$ or $V = \frac{Q}{b \cdot y}$

The given data is $b = 10 \cdot ft$ $y_1 = 0.3 \cdot ft$ $h = 0.1 \cdot ft$ $Q = 20 \cdot \frac{ft^3}{s}$

Hence we find $V_1 = \frac{Q}{b \cdot y_1}$ $V_1 = 6.67 \cdot \frac{ft}{s}$

and $E_1 = \frac{V_1^2}{2 \cdot g} + y_1$ $E_1 = 0.991 \cdot ft$

Hence $$E_1 = E_2 + h = \frac{V_2^2}{2 \cdot g} + y_2 + h = \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 + h$$ or $$\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = E_1 - h$$

This is a nonlinear implicit equation for y_2 and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We select y_2 so the left side of the equation equals $E_1 - h = 0.891 \cdot ft$

For $y_2 = 0.3 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.991 \cdot ft$ For $y_2 = 0.35 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.857 \cdot ft$

For $y_2 = 0.33 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.901 \cdot ft$ For $y_2 = 0.334 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.891 \cdot ft$

Hence $y_2 = 0.334 \cdot ft$ and $\frac{y_2 - y_1}{y_1} = 11.3 \cdot \%$

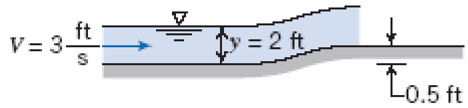
Note that $V_2 = \frac{Q}{b \cdot y_2}$ $V_2 = 5.99 \cdot \frac{ft}{s}$

so we have $Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}}$ $Fr_1 = 2.15$ and $Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $Fr_2 = 1.83$

Problem 11.22

[Difficulty: 3]

11.22 Water, at 3 ft/s and 2 ft depth, approaches a smooth rise in a wide channel. Estimate the stream depth after the 0.5 ft rise.



Given: Data on wide channel

Find: Stream depth after rise

Solution:

Basic equation:
$$\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2 + h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow. Note that at location 2 (the bump), the potential is $y_2 + h$, where h is the bump height

Recalling the specific energy $E = \frac{V^2}{2 \cdot g} + y$ and noting that $p_1 = p_2 = p_{atm}$, the Bernoulli equation becomes $E_1 = E_2 + h$

At each section $Q = V \cdot A = V_1 \cdot b \cdot y_1 = V_2 \cdot b \cdot y_2$ $\qquad V_2 = V_1 \cdot \frac{y_1}{y_2}$

The given data is $y_1 = 2 \cdot ft$ $\qquad V_1 = 3 \cdot \frac{ft}{s}$ $\qquad h = 0.5 \cdot ft$

Hence $E_1 = \frac{V_1^2}{2 \cdot g} + y_1$ $\qquad E_1 = 2.14 \cdot ft$

Then $E_1 = E_2 + h = \frac{V_2^2}{2 \cdot g} + y_2 + h = \frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 + h$ or $\frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 = E_1 - h$

This is a nonlinear implicit equation for y_2 and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We select y_2 so the left side of the equation equals $E_1 - h = 1.64 \cdot ft$

For $y_2 = 2 \cdot ft$ $\qquad \frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 = 2.14 \cdot ft$ $\qquad$ For $y_2 = 1.5 \cdot ft$ $\qquad \frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 = 1.75 \cdot ft$

For $y_2 = 1.3 \cdot ft$ $\qquad \frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 = 1.63 \cdot ft$ $\qquad$ For $y_2 = 1.31 \cdot ft$ $\qquad \frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 = 1.64 \cdot ft$

Hence $y_2 = 1.31 \cdot ft$

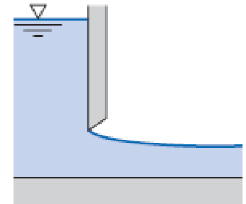
Note that $V_2 = V_1 \cdot \frac{y_1}{y_2}$ $\qquad V_2 = 4.58 \cdot \frac{ft}{s}$

so we have $Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}}$ $\qquad Fr_1 = 0.37$ $\qquad$ and $\qquad Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $\qquad Fr_2 = 0.71$

Problem 11.23

[Difficulty: 2]

11.23 A horizontal rectangular channel 3 ft wide contains a sluice gate. Upstream of the gate the depth is 6 ft; the depth downstream is 0.9 ft. Estimate the volume flow rate in the channel.



Given: Data on sluice gate

Find: Flow rate

Solution:

Basic equation: $$\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow.

Noting that $p_1 = p_2 = p_{atm}$, (1 = upstream, 2 = downstream) the Bernoulli equation becomes

$$\frac{V_1^2}{2 \cdot g} + y_1 = \frac{V_2^2}{2 \cdot g} + y_2$$

The given data is $b = 3 \cdot ft$ $\quad y_1 = 6 \cdot ft$ $\quad y_2 = 0.9 \cdot ft$

Also $Q = V \cdot A$ so $V_1 = \frac{Q}{b \cdot y_1}$ and $V_2 = \frac{Q}{b \cdot y_2}$

Using these in the Bernoulli equation $$\frac{\left(\frac{Q}{b \cdot y_1}\right)^2}{2 \cdot g} + y_1 = \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2$$

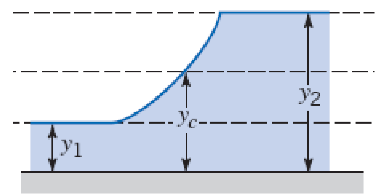
Solving for Q $$Q = \sqrt{\frac{2 \cdot g \cdot b^2 \cdot y_1^2 \cdot y_2^2}{y_1 + y_2}} \qquad Q = 49.5 \cdot \frac{ft^3}{s}$$

Note that $$V_1 = \frac{Q}{b \cdot y_1} \qquad V_1 = 2.75 \cdot \frac{ft}{s} \qquad Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}} \qquad Fr_1 = 0.198$$

$$V_2 = \frac{Q}{b \cdot y_2} \qquad V_2 = 18.3 \cdot \frac{ft}{s} \qquad Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}} \qquad Fr_2 = 3.41$$

Problem 11.24 [Difficulty: 2]

11.24 A hydraulic jump occurs in a rectangular channel 4.0 m wide. The water depth before the jump is 0.4 m and 1.7 m after the jump. Compute the flow rate in the channel, the critical depth, and the head loss in the jump.



Given: Data on rectangular channel and hydraulic jump

Find: Flow rate; Critical depth; Head loss

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\qquad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$ $\qquad y_c = \left(\frac{Q^2}{g\cdot b^2}\right)^{\frac{1}{3}}$

The given data is $b = 4\cdot m$ $\qquad y_1 = 0.4\cdot m$ $\qquad y_2 = 1.7\cdot m$

We can solve for Fr_1 from the basic equation $\qquad \sqrt{1 + 8\cdot Fr_1^{\,2}} = 1 + 2\cdot\frac{y_2}{y_1}$

$$Fr_1 = \sqrt{\frac{\left(1 + 2\cdot\frac{y_2}{y_1}\right)^2 - 1}{8}} \qquad Fr_1 = 3.34 \qquad \text{and} \qquad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$$

Hence $\qquad V_1 = Fr_1\cdot\sqrt{g\cdot y_1} \qquad V_1 = 6.62\,\frac{m}{s}$

Then $\qquad Q = V_1\cdot b\cdot y_1 \qquad Q = 10.6\cdot\frac{m^3}{s}$

The critical depth is $\qquad y_c = \left(\frac{Q^2}{g\cdot b^2}\right)^{\frac{1}{3}} \qquad y_c = 0.894\,m$

Also $\qquad V_2 = \frac{Q}{b\cdot y_2} \qquad V_2 = 1.56\,\frac{m}{s} \qquad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}} \qquad Fr_2 = 0.381$

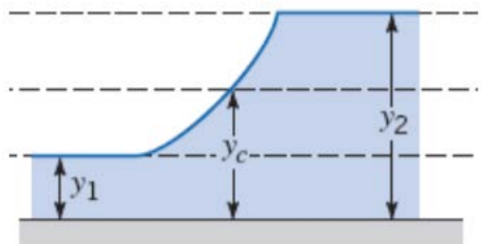
The energy loss is $\qquad H_l = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right) \qquad H_l = 0.808\,m$

Note that we could used $\qquad H_l = \frac{(y_2 - y_1)^3}{4\cdot y_1\cdot y_2} \qquad H_l = 0.808\,m$

Problem 11.25

(Difficulty 1)

11.25 A hydraulic jump occurs in a wide horizontal channel. The discharge is 2 m^3/s per meter of width. The upstream depth is 500 mm. Determine the depth of the jump.



Given: Data on wide channel and hydraulic jump

Find: Jump depth

Solution: Use the basic equation for the depths before and after a hydraulic jump:

$$\frac{y_2}{y_1} = \frac{1}{2} \cdot \left(-1 + \sqrt{1 + 8 \cdot Fr_1^2}\right)$$

The given data is:

$$\frac{Q}{b} = 2\ \frac{\frac{m^3}{s}}{m}$$

$$y_1 = 500\ mm$$

Also

$$Q = V \cdot A = V \cdot b \cdot y$$

Hence

$$V_1 = \frac{Q}{b \cdot y_1} = \frac{2\ \frac{m^2}{s}}{0.5\ m} = 4\ \frac{m}{s}$$

$$Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}} = \frac{4\ \frac{m}{s}}{\sqrt{9.81\ \frac{m}{s^2} \times 0.5\ m}} = 1.806$$

Then we have:

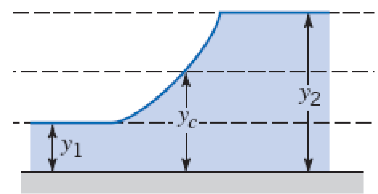
$$y_2 = \frac{y_1}{2} \cdot \left(-1 + \sqrt{1 + 8 \cdot Fr_1^2}\right)$$

$$y_2 = 1.05\ m$$

Problem 11.26

[Difficulty: 2]

11.26 A hydraulic jump occurs in a rectangular channel. The flow rate is 200 ft^3/s, and the depth before the jump is 1.2 ft. Determine the depth behind the jump and the head loss, if the channel is 10 ft wide.



Given: Data on wide channel and hydraulic jump

Find: Jump depth; Head loss

Solution:

Basic equations:
$$\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right) \qquad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$$

The given data is
$$Q = 200\cdot\frac{ft^3}{s} \qquad b = 10\cdot ft \qquad y_1 = 1.2\cdot ft$$

Also
$$Q = V\cdot A = V\cdot b\cdot y$$

Hence
$$V_1 = \frac{Q}{b\cdot y_1} \qquad V_1 = 16.7\cdot\frac{ft}{s} \qquad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}} \qquad Fr_1 = 2.68$$

Then
$$y_2 = \frac{y_1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right) \qquad y_2 = 3.99\cdot ft$$

$$V_2 = \frac{Q}{b\cdot y_2} \qquad V_2 = 5.01\cdot\frac{ft}{s} \qquad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}} \qquad Fr_2 = 0.442$$

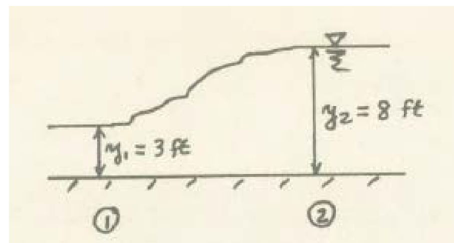
The energy loss is
$$H_l = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right) \qquad H_l = 1.14\cdot ft$$

Note that we could use
$$H_l = \frac{(y_2 - y_1)^3}{4\cdot y_1\cdot y_2} \qquad H_l = 1.14\cdot ft$$

Problem 11.27

(Difficulty 2)

11.27 The depths of water upstream and downstream from a hydraulic jump on the horizontal "apron" downstream from a spillway structure are observed to be approximately 3 ft and 8 ft. If the structure is 200 ft long (perpendicular to the direction of the flow), about how much horsepower is being dissipated in this jump?



Assumption: The flow is steady and incompressible

Solution: Use the energy relations for a hydraulic jump to determine the power dissipated.

For a hydraulic jump, we have the relation between the height after the jump and before::

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right)$$

Where the Froude number before the jump is

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}}$$

So we have the following relation, where the velocity upstream of the jump V_1 is unknown:

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\frac{V_1^2}{gy_1}}\right)$$

Or, with the values for depth

$$\frac{8\,ft}{3\,ft} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\frac{V_1^2}{32.2\,\frac{ft}{s^2} \times 3\,ft}}\right)$$

Solving this equation we have:

$$V_1 = 21.7\ \frac{ft}{s}$$

The flow rate is then:

$$q = V_1 b y_1 = 21.7\ \frac{ft}{s} \times 200\ ft \times 3\ ft = 13020\ \frac{ft^3}{s}$$

The velocity at section 2 is:

$$V_2 = \frac{Q}{b y_2} = \frac{13020\ \frac{ft^3}{s}}{200 ft \times 8\ ft} = 8.15\ \frac{ft}{s}$$

The head loss of the jump is calculated as:

$$H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$H_l = \left(3\ ft + \frac{\left(21.7\ \frac{ft}{s}\right)^2}{2 \times 32.2\ \frac{ft}{s^2}}\right) - \left(8\ ft + \frac{\left(8.15\ \frac{ft}{s}\right)^2}{2 \times 32.2\ \frac{ft}{s^2}}\right) = 1.28\ ft$$

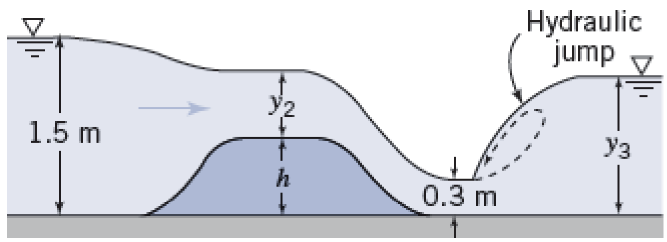
The power loss can be calculated as:

$$P_{loss} = Q H_l l = 13020 \frac{ft^3}{s} \times 62.4\ \frac{lbf}{ft^3} \times 1.28\ ft = 1.04 \times 10^6\ \frac{lbf \cdot ft}{s} = 1890\ \text{hp}$$

Problem 11.28

(Difficulty 3)

11.28 Calculate y_2, h, and y_3 for this two-dimensional flow picture. State any assumptions clearly.



Assumption: The flow is steady and incompressible

There are no losses in the flow between the upstream section and the section just before the hydraulic jump.

We have the specific energy equation in terms of q, the flow per unit width. Location a is upstream location and location b is at location y_2:

$$E_a = y_a + \frac{q^2}{2gy_a^2}$$

$$E_b = y_b + \frac{q^2}{2gy_b^2}$$

Because there are no losses, the specific energy at these two locations is equal

$$E_a = E_b$$

Thus

$$y_a + \frac{q^2}{2gy_a^2} = y_b + \frac{q^2}{2gy_b^2}$$

With values

$$1.5\ m + \frac{q^2}{2 \times 9.81\ \frac{m}{s^2} \times (1.5\ m)^2} = 0.3\ m + \frac{q^2}{2 \times 9.81\ \frac{m}{s^2} \times (0.3\ m)^2}$$

Solving this equation we have:

$$q = 1.486\ \frac{m^2}{s}$$

We assume that the critical depth occurs at the hump, and so we have:

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{\left(1.486\ \frac{m^2}{s}\right)^2}{9.81\ \frac{m}{s^2}}\right)^{\frac{1}{3}} = 0.608\ m$$

$$y_2 = y_c = 0.608\ m$$

The value of the specific energy is then

$$E_a = 1.5\ m + \frac{\left(1.486\ \frac{m^2}{s}\right)^2}{2 \times 9.81\ \frac{m}{s^2} \times (1.5\ m)^2} = 1.55\ m$$

Because this is the location of critical flow, the specific energy is a minimum:

$$E_{min} = y_c + \frac{q^2}{2gy_c^2} = 0.913\ m$$

The specific energy is the sum of the minimum specific energy and the height of the hump

$$E_a = E_{min} + h$$

Thus the height of the hump is

$$h = 0.637\ m$$

For the jump, we have the relation between the depths before and after the jump:

$$\frac{y_3}{y_b} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right)$$

Where the Froude number is the upstream value

$$Fr_b = \frac{V_b}{\sqrt{gy_b}}$$

So we have:

$$\frac{y_3}{y_b} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\frac{V_b^2}{gy_b}}\right)$$

Solving for the velocity

$$V_b = \frac{q}{y_b} = \frac{1.486\ \frac{m^2}{s}}{0.3\ m} = 4.95\ \frac{m}{s}$$

Thus the height is

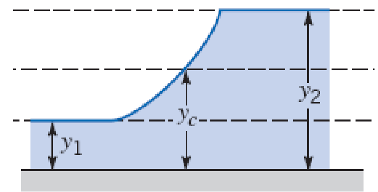
$$y_3 = \frac{1}{2} y_b \left(-1 + \sqrt{1 + 8 \frac{V_b^2}{g y_b}} \right)$$

Or

$$y_3 = \frac{1}{2} \times 0.3\ m \times \left(-1 + \sqrt{1 + 8 \frac{\left(4.95\ \frac{m}{s}\right)^2}{9.81\ \frac{m}{s^2} \times 0.3\ m}} \right) = 1.083\ m$$

Problem 11.29 [Difficulty: 2]

11.29 The hydraulic jump may be used as a crude flow meter. Suppose that in a horizontal rectangular channel 5 ft wide the observed depths before and after a hydraulic jump are 0.66 and 3.0 ft. Find the rate of flow and the head loss.



Given: Data on wide channel and hydraulic jump

Find: Flow rate; Head loss

Solution:

Basic equations: $$\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\ 2}}\right) \qquad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\ 2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\ 2}}{2\cdot g}\right)$$

The given data is $b = 5\cdot ft$ $\quad y_1 = 0.66\cdot ft$ $\quad y_2 = 3.0\cdot ft$

We can solve for Fr_1 from the basic equation $$\sqrt{1 + 8\cdot Fr_1^{\ 2}} = 1 + 2\cdot\frac{y_2}{y_1}$$

$$Fr_1 = \sqrt{\frac{\left(1 + 2\cdot\frac{y_2}{y_1}\right)^2 - 1}{8}} \qquad Fr_1 = 3.55 \qquad \text{and} \qquad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$$

Hence $$V_1 = Fr_1\cdot\sqrt{g\cdot y_1} \qquad V_1 = 16.4\cdot\frac{ft}{s}$$

Then $$Q = V_1\cdot b\cdot y_1 \qquad Q = 54.0\cdot\frac{ft^3}{s}$$

Also $$V_2 = \frac{Q}{b\cdot y_2} \qquad V_2 = 3.60\cdot\frac{ft}{s} \qquad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}} \qquad Fr_2 = 0.366$$

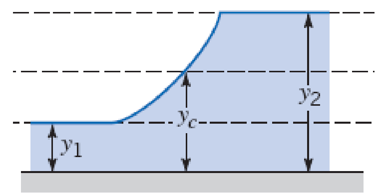
The energy loss is $$H_l = \left(y_1 + \frac{V_1^{\ 2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\ 2}}{2\cdot g}\right) \qquad H_l = 1.62\cdot ft$$

Note that we could use $$H_l = \frac{\left(y_2 - y_1\right)^3}{4\cdot y_1\cdot y_2} \qquad H_l = 1.62\cdot ft$$

Problem 11.30

[Difficulty: 2]

11.30 A hydraulic jump occurs on a horizontal apron downstream from a wide spillway, at a location where depth is 0.9 m and speed is 25 m/s. Estimate the depth and speed downstream from the jump. Compare the specific energy downstream of the jump to that upstream.



Given: Data on wide spillway flow

Find: Depth after hydraulic jump; Specific energy change

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\quad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$

The given data is $y_1 = 0.9\cdot m$ $\quad V_1 = 25\frac{m}{s}$

Then Fr_1 is $Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$ $\quad Fr_1 = 8.42$

Hence $y_2 = \frac{y_1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\quad y_2 = 10.3\,m$

Then $Q = V_1\cdot b\cdot y_1 = V_2\cdot b\cdot y_2$ $\quad V_2 = V_1\cdot\frac{y_1}{y_2}$ $\quad V_2 = 2.19\frac{m}{s}$

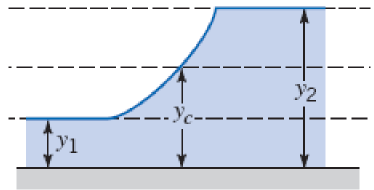
For the specific energies $E_1 = y_1 + \frac{V_1^{\,2}}{2\cdot g}$ $\quad E_1 = 32.8\,m$

$E_2 = y_2 + \frac{V_2^{\,2}}{2\cdot g}$ $\quad E_2 = 10.5\,m$ $\quad \frac{E_2}{E_1} = 0.321$

The energy loss is $H_l = E_1 - E_2$ $\quad H_l = 22.3\,m$

Note that we could use $H_l = \frac{(y_2 - y_1)^3}{4\cdot y_1\cdot y_2}$ $\quad H_l = 22.3\cdot m$

11.31 A hydraulic jump occurs in a rectangular channel. The flow rate is 50 m³/s, and the depth before the jump is 2 m. Determine the depth after the jump and the head loss, if the channel is 1 m wide.



Given: Data on rectangular channel flow

Find: Depth after hydraulic jump; Specific energy change

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^2}\right)$ $H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^2}{2\cdot g}\right) - \left(y_2 + \frac{V_2^2}{2\cdot g}\right)$

The given data is $y_1 = 0.4\cdot m$ $b = 1\cdot m$ $Q = 6.5\frac{m^3}{s}$

Then $Q = V_1\cdot b\cdot y_1 = V_2\cdot b\cdot y_2$ $V_1 = \frac{Q}{b\cdot y_1}$ $V_1 = 16.3\frac{m}{s}$

Then Fr_1 is $Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$ $Fr_1 = 8.20$

Hence $y_2 = \frac{y_1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^2}\right)$ $y_2 = 4.45\,m$

and $V_2 = \frac{Q}{b\cdot y_2}$ $V_2 = 1.46\frac{m}{s}$

For the specific energies $E_1 = y_1 + \frac{V_1^2}{2\cdot g}$ $E_1 = 13.9\,m$

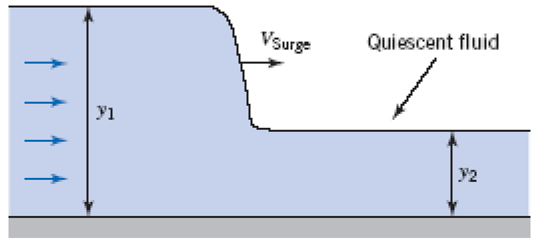
$E_2 = y_2 + \frac{V_2^2}{2\cdot g}$ $E_2 = 4.55\,m$

The energy loss is $H_l = E_1 - E_2$ $H_l = 9.31\,m$

Note that we could use $H_l = \frac{(y_2 - y_1)^3}{4\cdot y_1\cdot y_2}$ $H_l = 9.31\cdot m$

Problem 11.32 [Difficulty: 3]

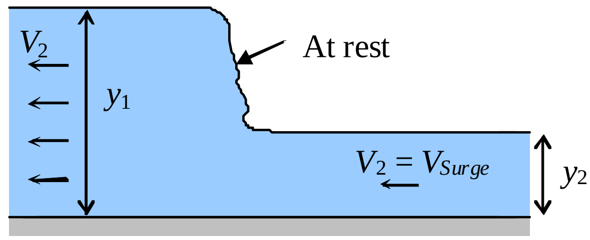
11.32 A positive surge wave, or moving hydraulic jump, can be produced in the laboratory by suddenly opening a sluice gate. Consider a surge of depth y_2 advancing into a quiescent channel of depth y_1. Obtain an expression for surge speed in terms of y_1 and y_2



Given: Surge wave

Find: Surge speed

Solution:



Basic equations: $$\frac{V_1^2 \cdot y_1}{g} + \frac{y_1^2}{2} = \frac{V_2^2 \cdot y_2}{g} + \frac{y_2^2}{2}$$

(This is the basic momentum equation for the flow)

$$V_1 \cdot y_1 = V_2 \cdot y_2 \qquad \text{or} \qquad \frac{V_1}{V_2} = \frac{y_2}{y_1}$$

Then $$y_2^2 - y_1^2 = \frac{2}{g} \cdot \left(V_1^2 \cdot y_1 - V_2^2 \cdot y_2\right) = \frac{2 \cdot V_2^2}{g} \cdot \left[\left(\frac{V_1}{V_2}\right)^2 \cdot y_1 - y_2\right] = \frac{2 \cdot V_2^2}{g} \cdot \left[\left(\frac{y_2}{y_1}\right)^2 \cdot y_1 - y_2\right]$$

$$y_2^2 - y_1^2 = \frac{2 \cdot V_2^2}{g} \cdot \left(\frac{y_2^2}{y_1} - y_2\right) = \frac{2 \cdot V_2^2 \cdot y_2}{g} \cdot \frac{(y_2 - y_1)}{y_1}$$

Dividing by $(y_2 - y_1)$ $$y_2 + y_1 = 2 \cdot \frac{V_2^2}{g} \cdot \frac{y_2}{y_1} \qquad \text{or} \qquad V_2^2 = \frac{g}{2} \cdot y_1 \cdot \frac{(y_2 + y_1)}{y_2}$$

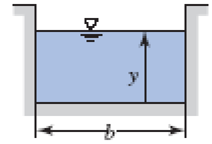
$$V_2 = \sqrt{\frac{g \cdot y_1}{2} \cdot \left(1 + \frac{y_1}{y_2}\right)}$$

But $$V_2 = V_{Surge} \qquad \text{so} \qquad V_{Surge} = \sqrt{\frac{g \cdot y_1}{2} \cdot \left(1 + \frac{y_1}{y_2}\right)}$$

Problem 11.33

[Difficulty: 1]

11.33 A 2-m-wide rectangular channel with a bed slope of 0.0005 has a depth of flow of 1.5 m. Manning's roughness coefficient is 0.015. Determine the steady uniform discharge in the channel.



Given: Rectangular channel flow

Find: Discharge

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $b = 2 \cdot m$ and depth $y = 1.5 \cdot m$ we find from Table 11.1

$A = b \cdot y \qquad A = 3.00 \cdot m^2 \qquad R_h = \frac{b \cdot y}{b + 2 \cdot y} \qquad R_h = 0.600 \cdot m$

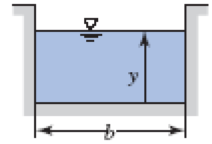
Manning's roughness coefficient is $n = 0.015$ and $S_b = 0.0005$

$Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}} \qquad Q = 3.18 \cdot \frac{m^3}{s}$

Problem 11.34

[Difficulty: 3]

11.34 Determine the uniform flow depth in a rectangular channel 2.5 m wide with a discharge of 3 m^3/s. The slope is 0.0004 and Manning's roughness factor is 0.015.



Given: Data on rectangular channel

Find: Depth of flow

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $b = 2.5 \cdot m$ and flow rate $Q = 3 \cdot \frac{m^3}{s}$ we find from Table 11.1 $A = b \cdot y$ $R = \frac{b \cdot y}{b + 2 \cdot y}$

Manning's roughness coefficient is $n = 0.015$ and $S_b = 0.0004$

Hence the basic equation becomes $Q = \frac{1}{n} \cdot b \cdot y \cdot \left(\frac{b \cdot y}{b + 2 \cdot y}\right)^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$

Solving for y $y \cdot \left(\frac{b \cdot y}{b + 2 \cdot y}\right)^{\frac{2}{3}} = \frac{Q \cdot n}{b \cdot S_b^{\frac{1}{2}}}$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below, to make the left side evaluate to $\frac{Q \cdot n}{b \cdot S_b^{\frac{1}{2}}} = 0.900$.

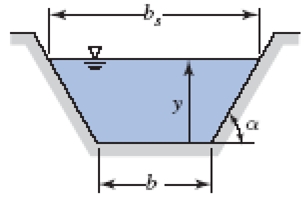
For $y = 1$ (m) $y \cdot \left(\frac{b \cdot y}{b + 2 \cdot y}\right)^{\frac{2}{3}} = 0.676$ For $y = 1.2$ (m) $y \cdot \left(\frac{b \cdot y}{b + 2 \cdot y}\right)^{\frac{2}{3}} = 0.865$

For $y = 1.23$ (m) $y \cdot \left(\frac{b \cdot y}{b + 2 \cdot y}\right)^{\frac{2}{3}} = 0.894$ For $y = 1.24$ (m) $y \cdot \left(\frac{b \cdot y}{b + 2 \cdot y}\right)^{\frac{2}{3}} = 0.904$

The solution to three figures is $y = 1.24$ (m)

Problem 11.35 [Difficulty: 3]

11.35 Determine the uniform flow depth in a trapezoidal channel with a bottom width of 8 ft and side slopes of 1 vertical to 2 horizontal. The discharge is 100 ft^3/s. Manning's roughness factor is 0.015 and the channel bottom slope is 0.0004.



Given: Data on trapzoidal channel

Find: Depth of flow

Solution:

Basic equation: $Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $b = 8 \cdot ft$ $\quad \alpha = \mathrm{atan}\left(\frac{1}{2}\right)$ $\quad \alpha = 26.6\,deg$ $\quad Q = 100 \cdot \frac{ft^3}{s}$ $\quad S_0 = 0.0004$

$n = 0.015$

Hence from Table 11.1 $\quad A = y \cdot (b + y \cdot \cot(\alpha)) = y \cdot (8 + 2 \cdot y)$ $\quad R_h = \frac{y \cdot (b + y \cdot \cot(\alpha))}{b + \frac{2 \cdot y}{\sin(\alpha)}} = \frac{y \cdot (8 + 2 \cdot y)}{8 + 2 \cdot y \cdot \sqrt{5}}$

Hence $\quad Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}} = \frac{1.49}{0.015} \cdot y \cdot (8 + 2 \cdot y) \cdot y \cdot \left[\frac{y \cdot (8 + 2 \cdot y)}{8 + 2 \cdot y \cdot \sqrt{5}}\right]^{\frac{2}{3}} \cdot 0.0004^{\frac{1}{2}} = 100$ (Note that we don't use units!)

Solving for y $\quad \frac{[y \cdot (8 + 2 \cdot y)]^{\frac{5}{3}}}{\left(8 + 2 \cdot y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 50.3$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

For $y = 2$ (ft) $\quad \frac{[y \cdot (8 + 2 \cdot y)]^{\frac{5}{3}}}{\left(8 + 2 \cdot y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 30.27$ $\qquad$ For $y = 3$ (ft) $\quad \frac{[y \cdot (8 + 2 \cdot y)]^{\frac{5}{3}}}{\left(8 + 2 \cdot y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 65.8$

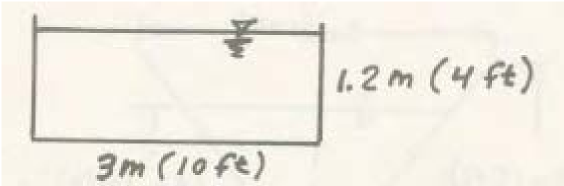
For $y = 2.6$ (ft) $\quad \frac{[y \cdot (8 + 2 \cdot y)]^{\frac{5}{3}}}{\left(8 + 2 \cdot y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 49.81$ $\qquad$ For $y = 2.61$ (ft) $\quad \frac{[y \cdot (8 + 2 \cdot y)]^{\frac{5}{3}}}{\left(8 + 2 \cdot y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 50.18$

The solution to three figures is $\quad y = 2.61$ (ft)

Problem 11.36

(Difficulty 1)

11.36 Water flows uniformly at a depth of 1.2 m in a rectangular canal 3 m wide, laid on a slope of 1 m per 1000 m. What is the mean shear stress on this sides and bottom of the canal?



Assumption: The flow is uniform, steady and incompressible

Solution: Use the relation between shear stress and gravity for a sloped channel to determine the stress.

$$\tau_0 = \gamma R S_0$$

For this problem:

$$S_0 = \frac{1\ m}{1000\ m} = 0.001$$

The hydraulic radius R is

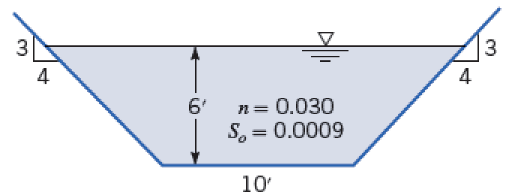
$$R = \frac{A}{P} = \frac{3\ m \times 1.2\ m}{3\ m + 1.2\ m \times 2} = 0.667\ m$$

Thus the shear stress is

$$\tau_0 = \gamma R S_0 = 9810\ \frac{N}{m^3} \times 0.667\ m \times 0.001 = 6.54\ Pa$$

Problem 11.37

11.37 This large uniform open channel flow is to be modeled without geometric distortion in the hydraulic laboratory at a scale of 1 to 9. What flow rate, bottom slope, and Manning n will be required in the model?



Solution:

The Froude number relationship is used to model open channel flows:

$$\left(\frac{V}{\sqrt{gD}}\right)_p = \left(\frac{V}{\sqrt{gD}}\right)_m$$

$$\left(\frac{Q}{\sqrt{gD^5}}\right)_p = \left(\frac{Q}{\sqrt{gD^5}}\right)_m$$

Thus

$$Q_m = Q_p\left(\frac{D_m}{D_p}\right)^{\frac{5}{2}}$$

From equation 11.11 we have:

$$Q = \frac{1.49}{n}AR^{\frac{2}{3}}S_0^{\frac{1}{2}}$$

$$A = 10\,ft \times 6\,ft + 2 \times \left(\frac{1}{2} \times 6\,ft \times 6\,ft \times \frac{4}{3}\right) = 108\,ft^2$$

$$P = 10\,ft + 2 \times 6ft \times \sqrt{1 + \left(\frac{4}{3}\right)^2} = 30.0\,ft$$

$$R = \frac{A}{P} = \frac{108\,ft^2}{30.0\,ft} = 3.6\,ft$$

Thus

$$Q_p = \frac{1.49}{0.030} \times 108\ ft^2 \times (3.6\ ft)^{\frac{2}{3}} \times (0.0009)^{\frac{1}{2}} = 378\ \frac{ft^3}{s}$$

$$Q_m = 378\ \frac{ft^3}{s} \times \left(\frac{1}{9}\right)^{\frac{5}{2}} = 1.56\ \frac{ft^3}{s}$$

For geometric similarity we have:

$$(S_0)_m = (S_0)_p = 0.0009$$

We also have:

$$\frac{Q_p}{Q_m} = \frac{\frac{1.49}{n_p} A_p R_p^{\frac{2}{3}} S_{0p}^{\frac{1}{2}}}{\frac{1.49}{n_m} A_m R_m^{\frac{2}{3}} S_{0m}^{\frac{1}{2}}} = \left(\frac{D_p}{D_m}\right)^{\frac{5}{2}}$$

$$\frac{n_m}{n_p} \left(\frac{D_p}{D_m}\right)^2 \left(\frac{D_p}{D_m}\right)^{\frac{2}{3}} = \left(\frac{D_p}{D_m}\right)^{\frac{5}{2}}$$

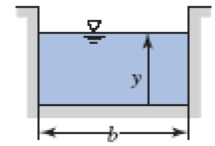
$$\frac{n_m}{n_p} = \left(\frac{D_p}{D_m}\right)^{-\frac{1}{6}} = (9)^{-\frac{1}{6}}$$

So we have:

$$n_m = n_p (9)^{-\frac{1}{6}} = 0.0208$$

Problem 11.38 [Difficulty: 1]

11.38 A rectangular flume built of timber is 3 ft wide. The flume is to handle a flow of 90 ft^3/s at a normal depth of 6 ft. Determine the slope required.



Given: Data on flume

Find: Slope

Solution:

Basic equation: $Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $b = 3 \cdot ft$ and depth $y = 6 \cdot ft$ we find

$A = b \cdot y$ $\quad A = 18 \cdot ft^2$ $\quad R_h = \frac{b \cdot y}{b + 2 \cdot y}$ $\quad R_h = 1.20 \cdot ft$

For wood (not in Table 11.2) a Google search finds n = 0.012 to 0.017; we use $n = 0.0145$ with $Q = 90 \cdot \frac{ft^3}{s}$

$$S_b = \left(\frac{n \cdot Q}{1.49 \cdot A \cdot R_h^{\frac{2}{3}}}\right)^2 \qquad S_b = 1.86 \times 10^{-3}$$

Problem 11.39

(Difficulty 1)

11.39 A channel with square cross section is to carry 20 m^3/s of water at normal depth on a slope of 0.003. Compare the dimensions of the channel required for (a) concrete and (b) masonry.

Given: Data on square channel

Find: Dimensions for concrete and masonry cement

Solution: Use the empirical Manning relation for flow in an open channel

$$Q = \frac{1}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$$

Note that this is an "engineering equation", to be used without units!

For a square channel of width b we find:

$$A = b^2$$

$$R = \frac{b \cdot y}{b + 2 \cdot y} = \frac{b^2}{3b} = \frac{b}{3}$$

$$Q = \frac{1}{n} \cdot b^2 \cdot \left(\frac{b}{3}\right)^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$$

$$b = \left(\frac{nQ(3)^{\frac{2}{3}}}{S_b^{\frac{1}{2}}}\right)^{\frac{3}{8}}$$

The given data is:

$$Q = 20\ \frac{m^3}{s}$$

$$S_b = 0.003$$

For concrete, (assuming large depth), the Manning coefficient is $n = 0.013$

The value of the depth is

$$b = 2.36\ m$$

For masonry, (assuming large depth), the Manning coefficient is $n = 0.03$

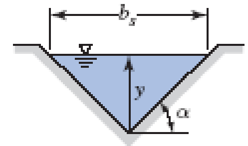
The value of the depth for this material is now

$$b = 3.23\ m$$

Problem 11.40

[Difficulty: 1]

11.40 A triangular channel with side angles of 45° is to carry 10 m^3/s at a slope of 0.001. The channel is concrete. Find the required dimensions.



Given: Data on triangular channel

Find: Required dimensions

Solution:

Basic equation: $$Q = \frac{1}{n}\cdot A\cdot R_h^{\frac{2}{3}}\cdot S_b^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For the triangular channel we have $\alpha = 45\cdot deg$ $\quad S_b = 0.001$ $\quad Q = 10\cdot\frac{m^3}{s}$

For concrete (Table 11.2) $n = 0.013$ (assuming y > 60 cm: verify later)

Hence from Table 11.1 $A = y^2\cdot\cot(\alpha) = y^2$ $\quad R_h = \frac{y\cdot\cos(\alpha)}{2} = \frac{y}{2\cdot\sqrt{2}}$

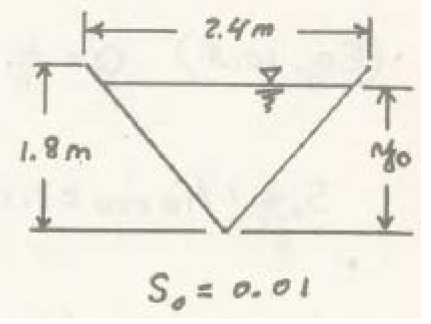
Hence $$Q = \frac{1}{n}\cdot A\cdot R_h^{\frac{2}{3}}\cdot S_b^{\frac{1}{2}} = \frac{1}{n}\cdot y^2\cdot\left(\frac{y}{2\cdot\sqrt{2}}\right)^{\frac{2}{3}}\cdot S_b = \frac{1}{n}\cdot y^{\frac{8}{3}}\cdot\left(\frac{1}{8}\right)^{\frac{1}{3}}\cdot S_b^{\frac{1}{2}} = \frac{1}{2\cdot n}\cdot y^{\frac{8}{3}}\cdot S_b^{\frac{1}{2}}$$

Solving for y $$y = \left(\frac{2\cdot n\cdot Q}{\sqrt{S_b}}\right)^{\frac{3}{8}}$$ $y = 2.20\,m$ (The assumption that y > 60 cm is verified)

Problem 11.41

(Difficulty 2)

11.41 A flume of timber has its cross section an isosceles triangle (apex down) of 2.4 m base and 1.8 m altitude. At what depth will $5\,\frac{m^3}{s}$ flow uniformly in this flume if it is laid on a slope of 0.01?



Assumption: The flow is uniform, steady and incompressible

Solution: Use the Manning equation to find the depth.

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

From the Table 11.1, we have for the Manning equation:

$$n = 0.013$$

The width b at the water surface is:

$$\frac{b}{2.4\ m} = \frac{y_0}{1.8\ m}$$

$$b = \frac{4}{3} y_0$$

So the area and perimeter can be calculated as:

$$A = \frac{\frac{4}{3} y_0 \cdot y_0}{2} = \frac{2}{3} y_0^2$$

$$P = 2\sqrt{\left(\frac{2}{3} y_0\right)^2 + y_0^2} = 2 y_0 \sqrt{\frac{13}{9}} = 2.40 y_0$$

Thus the hydraulic radius is

$$R = \frac{A}{P} = \frac{\frac{2}{3}y_0^2}{2.40y_0} = 0.278y_0$$

The slope is

$$S_0 = 0.01$$

Then we have from Manning equation

$$5\ \frac{m^3}{s} = \frac{1}{0.013} \times \frac{2}{3} y_0^2 (0.278y_0)^{\frac{2}{3}} \times (0.01)^{\frac{1}{2}}$$

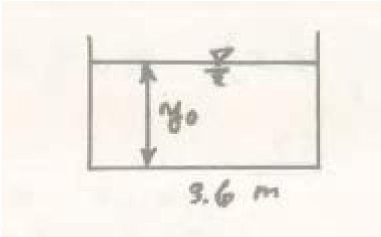
Solving this equation for the depth y_0 we have:

$$y_0 = 1.364\ m$$

Problem 11.42

(Difficulty 2)

11.42 At what depth will 4.25 $\frac{m^3}{s}$ flow uniformly in a rectangular channel 3.6 m wide lined with rubble masonry and laid on a slope of 1 in 4000?



Assumption: The flow is uniform, steady and incompressible

Solution: Use the Manning equation to find the depth.

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

From the Table 11.1, we have for the Manning equation:

$$n = 0.025$$

The flow area and perimeter are calculated as:

$$A = 3.6 y_0 \ m^2$$

$$P = 2y_0 + 3.6 \, m$$

Thus the hydraulic radius is

$$R = \frac{A}{P} = \frac{3.6 y_0}{2y_0 + 3.6}$$

The slope is

$$S_0 = \frac{1}{4000}$$

Then we have from the Manning equation:

$$4.25 \ \frac{m^3}{s} = \frac{1}{0.025} \times 3.6 y_0 \times \left(\frac{3.6 y_0}{2y_0 + 3.6}\right)^{\frac{2}{3}} \times \left(\frac{1}{4000}\right)^{\frac{1}{2}}$$

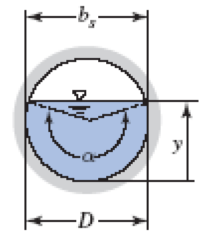
Solving this equation for y_0 we have for the depth:

$$y_0 = 1.95\ m$$

Problem 11.43

[Difficulty: 2]

11.43 A semicircular trough of corrugated steel, with diameter $D = 1$ m, carries water at depth $y = 0.25$ m. The slope is 0.01. Find the discharge.



Given: Data on semicircular trough

Find: Discharge

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the semicircular channel $D = 1 \cdot m$ $y = 0.25 \cdot m$ $S_b = 0.01$

Hence, from geometry $\alpha = 2 \cdot \text{asin}\left(\frac{y - \frac{D}{2}}{\frac{D}{2}}\right) + 180 \cdot deg$ $\alpha = 120 \cdot deg$

For corrugated steel, a Google search leads to $n = 0.022$

Hence from Table 11.1 $A = \frac{1}{8} \cdot (\alpha - \sin(\alpha)) \cdot D^2$ $A = 0.154\, m^2$

$R_h = \frac{1}{4} \cdot \left(1 - \frac{\sin(\alpha)}{\alpha}\right) \cdot D$ $R_h = 0.147\, m$

Then the discharge is $Q = \frac{1}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}} \cdot \frac{m^3}{s}$ $Q = 0.194 \frac{m^3}{s}$

Problem 11.44

(Difficulty 2)

11.44 A rectangular flume built of concrete with 1 ft per 1000 ft slope is 6 ft wide. Water flows at a normal depth of 3 ft. The flume is fitted with a new plastic film liner. Find the new depth of flow if the discharge remains constant.

Given: Data on flume with plastic liner

Find: Depth of flow

Solution: Use the empirical Manning equation for flow in an open channel:

$$Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$$

Note that this is an "engineering equation", to be used without units!

For a rectangular channel of width b and depth y, we find:

$$A = by$$

$$R = \frac{b \cdot y}{b + 2 \cdot y}$$

For a concrete lined channel, the value of the Manning coefficient is $n = 0.013$

The slope of the channel is $S_b = 0.001$

The flow rate is then given by

$$Q = \frac{1.49}{0.013} \cdot by \cdot \left(\frac{b \cdot y}{b + 2 \cdot y}\right)^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$$

$$Q = \frac{1.49}{0.013} \times (3 \times 6) \times \left(\frac{3 \times 6}{6 + 2 \times 3}\right)^{\frac{2}{3}} \times (0.001)^{\frac{1}{2}} = 85.5\ \frac{ft^3}{s}$$

For a new plastic film liner flume, the Manning coefficient is $n = 0.01$.

The flow rate is given by the same relation but with the new Manning coefficient. The flow rate is the same value as previously

$$Q = \frac{1.49}{0.01} \cdot 6y \cdot \left(\frac{6y}{6 + 2 \cdot y}\right)^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$$

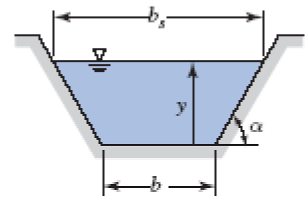
$$6y \cdot \left(\frac{6y}{6 + 2 \cdot y}\right)^{\frac{2}{3}} = \frac{Q}{149 \cdot S_b^{\frac{1}{2}}} = \frac{85.5}{149 \times \sqrt{0.001}} = 18.15$$

Solving the implicit nonlinear equation by matlab we have:

$$y = 2.47\ ft$$

Problem 11.45 [Difficulty: 3]

11.45 Water flows in a trapezoidal channel at a flow rate of 10 m^3/s. The bottom width is 2.4 m, the sides slope at 1:1, and the bed slope is 0.00193. The channel is excavated from bare soil. Find the depth of the flow.



Given: Data on trapzoidal channel

Find: New depth of flow

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $b = 2.4 \cdot m$ $\quad \alpha = 45 \cdot deg$ $\quad Q = 10 \cdot \frac{m^3}{s}$ $\quad S_b = 0.00193$

For bare soil (Table 11.2) $n = 0.020$

Hence from Table 11.1 $A = y \cdot (b + \cot(\alpha) \cdot y) = y \cdot (2.4 + y)$ $\quad R = \dfrac{y \cdot (b + y \cdot \cot(\alpha))}{b + \dfrac{2 \cdot y}{\sin(\alpha)}} = \dfrac{y \cdot (2.4 + y)}{2.4 + 2 \cdot y \cdot \sqrt{2}}$

Hence $$Q = \frac{1}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}} = \frac{1}{0.020} \cdot y \cdot (2.4 + y) \cdot \left[\frac{y \cdot (2.4 + y)}{2.4 + 2 \cdot y \cdot \sqrt{2}}\right]^{\frac{2}{3}} \cdot 0.00193^{\frac{1}{2}} = 10$$ (Note that we don't use units!)

Solving for y $$\frac{[y \cdot (2.4 + y)]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 4.55$$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

For $y = 1.5$ (m) $\quad \dfrac{[y \cdot (2.4 + y)]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 5.37$ $\qquad$ For $y = 1.4$ (m) $\quad \dfrac{[y \cdot (2.4 + y)]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 4.72$

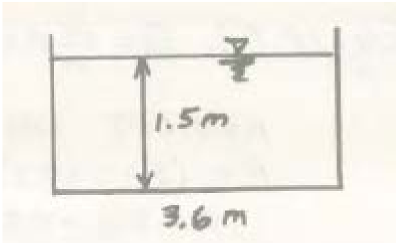
For $y = 1.35$ (m) $\quad \dfrac{[y \cdot (2.4 + y)]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 4.41$ $\qquad$ For $y = 1.37$ (m) $\quad \dfrac{[y \cdot (2.4 + y)]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 4.536$

The solution to three figures is $y = 1.37$ (m)

Problem 11.46

(Difficulty 2)

11.46 What slope is necessary to carry 11 $\frac{m^3}{s}$ uniformly at a depth of 1.5 m in a rectangular channel 3.6 m wide, having $n = 0.017$?



Assumption The flow is uniform, steady and incompressible

Solution: Use the Manning equation to find the slope.

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

The flow area area and perimeter are calculated as:

$$A = 3.6\ m \times 1.5\ m = 5.4\ m^2$$

$$P = 2 \times 1.5\ m + 3.6\ m = 6.6\ m$$

Thus the hydraulic radius is

$$R = \frac{A}{P} = \frac{5.4\ m^2}{6.6\ m} = 0.818\ m$$

The Manning coefficient is n = 0.017. Then using the Manning equation we have:

$$\frac{1}{0.017} \times 5.4\ m^2 \times (0.818\ m)^{\frac{2}{3}} \times S_0^{\frac{1}{2}} = 11\ \frac{m^3}{s}$$

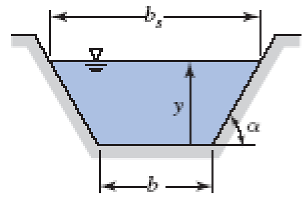
Solving this equation for the slope S_0 we have:

$$S_0 = 0.00157$$

Problem 11.47

[Difficulty: 3]

11.47 The channel of Problem 11.45 has 0.00193 bed slope. Find the normal depth for the given discharge after a new plastic liner ($n = 0.010$) is installed.



Given: Data on trapzoidal channel

Find: New depth of flow

Solution:

Basic equation: $$Q = \frac{1}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $b = 2.4 \cdot m$ $\quad \alpha = 45 \cdot deg$ $\quad Q = 7.1 \cdot \frac{m^3}{s}$ $\quad S_b = 0.00193$

For bare soil (Table 11.2) $n = 0.010$

Hence from Table 11.1 $A = y \cdot (b + y \cdot \cot(\alpha)) = y \cdot (2.4 + y)$ $$R_h = \frac{y \cdot (b + y \cdot \cot(\alpha))}{b + \frac{2 \cdot y}{\sin(\alpha)}} = \frac{y \cdot (2.4 + y)}{2.4 + 2 \cdot y \cdot \sqrt{2}}$$

Hence $$Q = \frac{1}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}} = \frac{1}{0.010} \cdot y \cdot (2.4 + y) \cdot \left[\frac{y \cdot (2.4 + y)}{2.4 + 2 \cdot y \cdot \sqrt{2}}\right]^{\frac{2}{3}} \cdot 0.00193^{\frac{1}{2}} = 7.1$$ (Note that we don't use units!)

Solving for y $$\frac{[y \cdot (2.4 + y)]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 1.62$$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with a shallower depth than that of Problem 11.49.

For $y = 1$ (m) $$\frac{[y \cdot (2.4 + y)]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 2.55$$

For $y = 0.75$ (m) $$\frac{[y \cdot (2.4 + y)]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 1.53$$

For $y = 0.77$ (m) $$\frac{[y \cdot (2.4 + y)]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 1.60$$

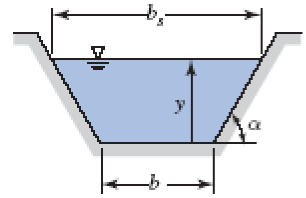
For $y = 0.775$ (m) $$\frac{[y \cdot (2.4 + y)]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 1.62$$

The solution to three figures is $y = 0.775$ (m)

Problem 11.48

[Difficulty: 3]

11.48 For a trapezoidal shaped channel ($n = 0.014$ and slope $S_b = 0.0002$ with a 20-ft bottom width and side slopes of 1 vertical to 1.5 horizontal), determine the normal depth for a discharge of 1000 cfs.



Given: Data on trapezoidal channel

Find: Normal depth

Solution:

Basic equation: $$Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $b = 20 \cdot ft$ $\quad \alpha = atan\left(\frac{1}{1.5}\right)$ $\quad \alpha = 33.7\,deg$ $\quad Q = 1000 \cdot \frac{ft^3}{s}$

$S_0 = 0.0002 \quad n = 0.014$

Hence from Table 11.1 $\quad A = y \cdot (b + y \cdot cot(\alpha)) = y \cdot (20 + 1.5 \cdot y)$ $\quad R_h = \dfrac{y \cdot (b + y \cdot cot(\alpha))}{b + \dfrac{2 \cdot y}{sin(\alpha)}} = \dfrac{y \cdot (20 + 1.5 \cdot y)}{20 + 2 \cdot y \cdot \sqrt{3.25}}$

Hence $$Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}} = \frac{1.49}{0.014} \cdot y \cdot (20 + 1.5 \cdot y) \cdot \left[\frac{y \cdot (20 + 1.5 \cdot y)}{20 + 2 \cdot y \cdot \sqrt{3.25}}\right]^{\frac{2}{3}} \cdot 0.0002^{\frac{1}{2}} = 1000$$ (Note that we don't use units!)

Solving for y $$\frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{\left(20 + 2 \cdot y \cdot \sqrt{3.25}\right)^{\frac{2}{3}}} = 664$$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

For $y = 7.5$ (ft) $$\frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{\left(20 + 2 \cdot y \cdot \sqrt{3.25}\right)^{\frac{2}{3}}} = 684$$ For $y = 7.4$ (ft) $$\frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{\left(20 + 2 \cdot y \cdot \sqrt{3.25}\right)^{\frac{2}{3}}} = 667$$

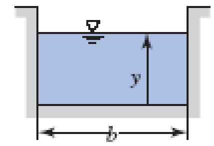
For $y = 7.35$ (ft) $$\frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{\left(20 + 2 \cdot y \cdot \sqrt{3.25}\right)^{\frac{2}{3}}} = 658$$ For $y = 7.38$ (ft) $$\frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{\left(20 + 2 \cdot y \cdot \sqrt{3.25}\right)^{\frac{2}{3}}} = 663$$

The solution to three figures is $y = 7.38$ (ft)

Problem 11.49

[Difficulty: 1]

11.49 Compute the critical depth for the channel in Problem 11.33



Given: Rectangular channel flow

Find: Critical depth

Solution:

Basic equations: $y_c = \left(\frac{Q^2}{g \cdot b^2}\right)^{\frac{1}{3}}$ $\quad Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$

For a rectangular channel of width $b = 2 \cdot m$ and depth $y = 1.5 \cdot m$ we find from Table 11.1

$A = b \cdot y$ $\quad A = 3.00 \cdot m^2$ $\quad R_h = \frac{b \cdot y}{b + 2 \cdot y}$ $\quad R_h = 0.600 \cdot m$

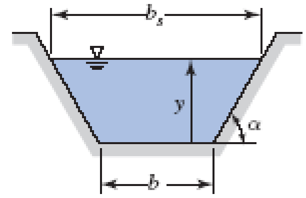
Manning's roughness coefficient is $n = 0.015$ and $S_b = 0.0005$

$Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$ $\quad Q = 3.18 \cdot \frac{m^3}{s}$

Hence $y_c = \left(\frac{Q^2}{g \cdot b^2}\right)^{\frac{1}{3}}$ $\quad y_c = 0.637\,m$

Problem 11.50 [Difficulty: 3]

11.50 A trapezoidal canal lined with brick has side slopes of 2:1 and bottom width of 10 ft. It carries 600 ft^3/s at critical speed. Determine the critical slope (the slope at which the depth is critical).



Given: Data on trapezoidal canal

Find: Critical slope

Solution:

Basic equations: $Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$ and $A = y \cdot b + y \cdot \cot(\alpha)$ $R_h = \frac{y \cdot (b + y \cdot \cot(\alpha))}{b + \frac{2 \cdot y}{\sin(\alpha)}}$

Note that the Q equation is an "engineering" equation, to be used without units!

Available data $b = 10 \cdot ft$ $\alpha = \operatorname{atan}\left(\frac{2}{1}\right)$ $\alpha = 63.4 \, deg$ $Q = 600 \cdot \frac{ft^3}{s}$

For brick, a Google search gives $n = 0.015$

For critical flow $y = y_c$ $V_c = \sqrt{g \cdot y_c}$

so $Q = A \cdot V_c = \left(y_c \cdot b + y_c \cdot \cot(\alpha)\right) \cdot \sqrt{g \cdot y_c}$ $\left(y_c \cdot b + y_c \cdot \cot(\alpha)\right) \cdot \sqrt{g \cdot y_c} = Q$ with $Q = 600 \cdot \frac{ft^3}{s}$

This is a nonlinear implicit equation for y_c and must be solved numerically. We can use one of a number of numerical root finding techniques, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with the given depth

For $y_c = 5$ (ft) $\left(y_c \cdot b + y_c \cdot \cot(\alpha)\right) \cdot \sqrt{g \cdot y_c} = 666$ For $y_c = 4.5$ (ft) $\left(y_c \cdot b + y_c \cdot \cot(\alpha)\right) \cdot \sqrt{g \cdot y_c} = 569$

For $y_c = 4.7$ (ft) $\left(y_c \cdot b + y_c \cdot \cot(\alpha)\right) \cdot \sqrt{g \cdot y_c} = 607$ For $y_c = 4.67$ (ft) $\left(y_c \cdot b + y_c \cdot \cot(\alpha)\right) \cdot \sqrt{g \cdot y_c} = 601$

Hence $y_c = 4.67$ (ft) and $A_{crit} = y_c \cdot b + y_c \cdot \cot(\alpha)$ $A_{crit} = 49.0$ (ft^2)

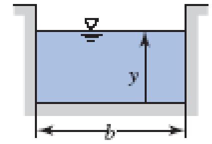
$R_{hcrit} = \frac{y_c \cdot \left(b + y_c \cdot \cot(\alpha)\right)}{b + \frac{2 \cdot y_c}{\sin(\alpha)}}$ $R_{hcrit} = 2.818$ (ft)

Solving the basic equation for S_c $Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$ $S_{bcrit} = \left(\frac{n \cdot Q}{1.49 \cdot A_{crit} \cdot R_{hcrit}^{\frac{2}{3}}}\right)^2$ $S_{bcrit} = 0.00381$

Problem 11.51

[Difficulty: 2]

11.51 An optimum rectangular storm sewer channel made of unfinished concrete is to be designed to carry a maximum flow rate of 100 ft³/s, at which the flow is at critical condition. Determine the channel width and slope.



Given: Data on optimum rectangular channel

Find: Channel width and slope

Solution:

Basic equations: $Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}}$ and from Table 11.3, for optimum geometry $b = 2 \cdot y_n$

Note that the Q equation is an "engineering" equation, to be used without units!

Available data $Q = 100 \cdot \frac{ft^3}{s}$ $n = 0.015$ (Table 11.2)

Hence $A = b \cdot y_n = 2 \cdot y_n^2$ $R_h = \frac{A}{P} = \frac{2 \cdot y_n^2}{y_n + 2 \cdot y_n + y_n} = \frac{y_n}{2}$

We can write the Froude number in terms of Q

$$Fr = \frac{V}{\sqrt{g \cdot y}} = \frac{Q}{A \cdot \sqrt{g \cdot y}} = \frac{Q}{2 \cdot y_n^2 \cdot \sqrt{g} \cdot y_n^{\frac{1}{2}}} \quad \text{or} \quad Fr = \frac{Q}{2 \cdot \sqrt{g} \cdot y_n^{\frac{5}{2}}}$$

Hence for critical flow, Fr = 1 and $y_n = y_c$, so $1 = \frac{Q}{2 \cdot \sqrt{g} \cdot y_c^{\frac{5}{2}}}$ or $Q = 2 \cdot \sqrt{g} \cdot y_c^{\frac{5}{2}}$

Hence $y_c = \left(\frac{Q}{2 \cdot \sqrt{g}}\right)^{\frac{2}{5}}$ $y_c = 2.39$ (ft) and $b = 2 \cdot y_c$ $b = 4.78$ (ft)

Then $Q = \frac{1.49}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_b^{\frac{1}{2}} = \frac{1.49}{n} \cdot 2 \cdot y_c^2 \cdot \left(\frac{y_c}{2}\right)^{\frac{2}{3}} \cdot S_c^{\frac{1}{2}}$ or $Q = \frac{1.49 \cdot 2^{\frac{1}{3}}}{n} \cdot y_c^{\frac{8}{3}} \cdot S_c^{\frac{1}{2}}$

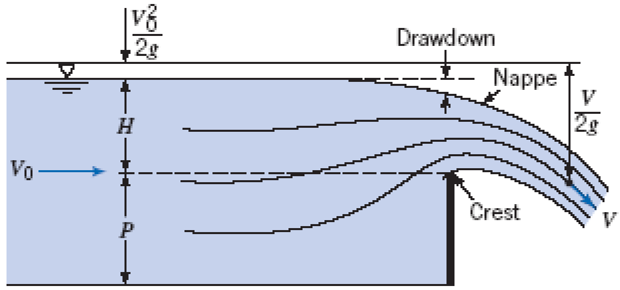
Hence $S_c = \left(\frac{n \cdot Q}{1.49 \cdot 2^{\frac{1}{3}} \cdot y_c^{\frac{8}{3}}}\right)^2$ $S_c = 0.00615$

Using $n = 0.013$ $S_c = \left(\frac{n \cdot Q}{1.49 \cdot 2^{\frac{1}{3}} \cdot y_c^{\frac{8}{3}}}\right)^2$ $S_c = 0.00462$

Problem 11.52

[Difficulty: 1]

11.52 For a sharp-crested suppressed weir ($C_w \approx 3.33$) of length $B = 8.0$ ft, $P = 2.0$ ft, and $H = 1.0$ ft, determine the discharge over the weir. Neglect the velocity of approach head.



Given: Data on rectangular, sharp-crested weir

Find: Discharge

Solution:

Basic equation: $Q = C_w \cdot b \cdot H^{\frac{3}{2}}$ where $C_w = 3.33$ and $b = 8 \cdot ft$ $P = 2 \cdot ft$ $H = 1 \cdot ft$

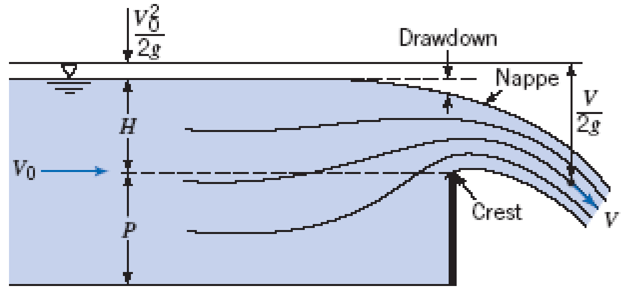
Note that this is an "engineering" equation, to be used without units!

$$Q = C_w \cdot b \cdot H^{\frac{3}{2}} \qquad Q = 26.6 \frac{ft^3}{s}$$

Problem 11.53

[Difficulty: 3]

11.53 A rectangular sharp-crested weir with end contractions is 1.5 m long. How high should the weir crest be placed in a channel to maintain an upstream depth of 2.5 m for 0.5 m^3/s flow rate?



Given: Data on rectangular, sharp-crested weir

Find: Required weir height

Solution:

Basic equations: $Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot b' \cdot H^{\frac{3}{2}}$ where $C_d = 0.62$ and $b' = b - 0.1 \cdot n \cdot H$ with $n = 2$

Given data: $b = 1.5 \cdot m$ $\quad Q = 0.5 \cdot \frac{m^3}{s}$

Hence we find

$$Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot b' \cdot H^{\frac{3}{2}} = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot (b - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}}$$

Rearranging $\quad (b - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = \frac{3 \cdot Q}{2 \cdot \sqrt{2 \cdot g} \cdot C_d}$

This is a nonlinear implicit equation for H and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

The right side evaluates to $\frac{3 \cdot Q}{2 \cdot \sqrt{2 \cdot g} \cdot C_d} = 0.273 \cdot m^{\frac{5}{2}}$

For $H = 1 \cdot m$ $\quad (b - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 1.30 \cdot m^{\frac{5}{2}}$ $\quad$ For $H = 0.5 \cdot m$ $\quad (b - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.495 \cdot m^{\frac{5}{2}}$

For $H = 0.3 \cdot m$ $\quad (b - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.237 \cdot m^{\frac{5}{2}}$ $\quad$ For $H = 0.35 \cdot m$ $\quad (b - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.296 \cdot m^{\frac{5}{2}}$

For $H = 0.34 \cdot m$ $\quad (b - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.284 \cdot m^{\frac{5}{2}}$ $\quad$ For $H = 0.33 \cdot m$ $\quad (b - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.272 \cdot m^{\frac{5}{2}}$

For $H = 0.331 \cdot m$ $\quad (b - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.273 \cdot m^{\frac{5}{2}}$ $\quad H = 0.331\, m$

But from the figure $\quad H + P = 2.5 \cdot m$ $\quad P = 2.5 \cdot m - H$ $\quad P = 2.17\, m$

Problem 11.54

(Difficulty 2)

11.54 What depth of water must exist behind a rectangular sharp-crested weir 1.5 m wide and 1.2 m high, when a flow of 0.28 $\frac{m^3}{s}$ over it? What is the velocity of approach?

Assumption The flow is uniform, steady and incompressible

Solution: Use the flow rate relation for a weir to find the depth of water and the velocity of approach. If the velocity of approach is negligible, the relation is

$$Q = C_w b H^{3/2}$$

Where P is the height of the weir and H is the height of the water surface over the weir. In SI units, the weir coefficient is $C_w = 1.84$.

The height H is then calculated as

$$H = \left(\frac{Q}{C_w b}\right)^{2/3} = \left(\frac{0.28}{1.84 \times 1.5}\right)^{2/3} = 0.218\ m$$

The approach velocity is related to the height of the weir and water surface as

$$Q = b(H + P)V_a$$

or

$$V_a = \frac{Q}{b(H + P)}$$

The approach velocity is then

$$V_a = \frac{Q}{b(H + P)} = \frac{0.28}{1.5 \times (0.218 + 1.2)} = 0.132\ \frac{m}{s}$$

The flow rate when the velocity of approach is not negligible is

$$Q = C_w b \left(H + \frac{V_a^2}{2g}\right)^{3/2}$$

The height H is 0.218 m. The velocity head is

$$\frac{V_a^2}{2g} = \frac{(0.132\ \ m/s)^2}{2 \times 9.8\ {}^{m}/_{s^2}} = 0.00088\ m$$

The velocity head is negligible compared to the height of water and so the approach velocity can be assumed to be negligible.

Problem 11.55

(Difficulty 1)

11.55 A broad-crested weir $0.9\ m$ high has a flat crest and a coefficient of 1.6. If this weir is $6\ m$ wide and the head on it is $0.46\ m$, what will the flow rate be?

Assumption: The flow is uniform, steady and incompressible

Solution: Use the flow rate relation for a weir to find the depth of water and the velocity of approach. If the velocity of approach is negligible, the relation is

$$Q = C_w b H^{3/2}$$

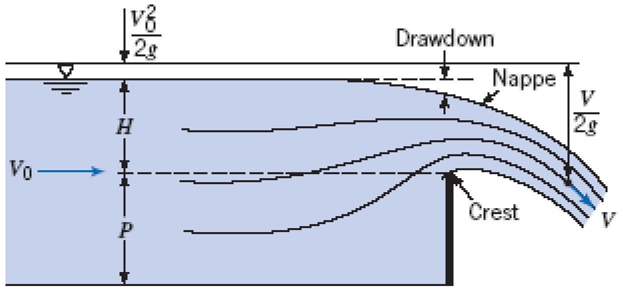
Where H is the height of the water surface over the weir. The weir coefficient is given as $C_w = 1.6$. The flow rate is then

$$Q = C_w b H^{3/2} = 1.6 \times 6 \times 0.46^{3/2} = 3.00\ \frac{m^3}{s}$$

Problem 11.56

[Difficulty: 1]

11.56 The head on a 90° V-notch weir is 1.5 ft. Determine the discharge.



Given: Data on V-notch weir

Find: Discharge

Solution:

Basic equation: $Q = C_w \cdot H^{\frac{5}{2}}$ where $H = 1.5 \cdot ft$ $\quad C_w = 2.50$ for $\theta = 90 \cdot deg$

Note that this is an "engineering" equation in which we ignore units!

$$Q = C_w \cdot H^{\frac{5}{2}} \qquad Q = 6.89 \frac{ft^3}{s}$$