TEST OF ORDINARY DIFFERENTIAL EQUATIONS

1. Find the solution of the following differential equations.$(8' \times 5)$

(1) $(2x \sin y + 3x^2 y)dx + (x^3 + x^2 \cos y + y^2)dy = 0.$

(2) $\frac{dy}{dx} = \frac{x+y}{x-y}.$

(3) $\frac{dy}{dx} + \frac{1}{x}y = x^3. \quad (x \neq 0).$

(4) $(3x^2 + y)dx + (2x^2 y - x)dy = 0.$

(5) $(x^2 + 1)(y^2 - 1)dx + xy\,dy = 0.$

2.$(8')$ Find the general solution of $y'' + ay = 0.$

3.$(10')$ Solve the differential equation $y''' + 3y'' + 3y' + y = e^{-x}(x - 5).$

4.$(10')$ Solve the differential equation $y'' - 5y' + 6y = xe^{2x}.$

5.$(10')$ Solve the differential equation $y'' + y' - 2y = 3e^x - \frac{1}{2}\sin x.$

6.$(10')$ To find a fourth order linear homogeneous differential equation with constant coefficients with $y_1 = e^x$, $y_2 = 2xe^x$, $y_3 = \sin 3x$, $y_4 = 7\cos 3x$ as particular solutions and find its general solution.

7. $(6' \times 2)$

(1)Solve Bernoulli's equation when $n = 0$; when $n = 1$. (Bernoulli's equation : $y' + p(x)y = q(x)y^n.$)

(2) Show that if $n \neq 0, 1$, then the substitution $z = y^{1-n}$ reduces Bernoulli's equation to a linear equation.