Chapter 8

PROBLEM 8.1

Show that the coefficient of thermal expansion for an ideal gas is 1/*T*, where *T* is the absolute temperature.

GIVEN

- Ideal gas
- Absolute temperature = T

FIND

- Show that the thermal expansion coefficient (β) = $1/T$

SOLUTION

The volumetric thermal expansion coefficient is defined as

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

For an ideal gas

$$pV = mRT \Rightarrow \frac{m}{V} = \rho = \frac{p}{RT}$$

where

p = pressure
V = volume
m = mass
R = gas constant

For a constant pressure

$$\frac{\partial \rho}{\partial T} = -\frac{p}{RT^2}$$

$$\therefore\ \beta = -\frac{1}{\left(\frac{p}{RT}\right)}\left(-\frac{p}{RT^2}\right) = \frac{1}{T}$$

PROBLEM 8.2

> **From its definition and property values in Appendix 2, Table 13, calculate the coefficient of thermal expansion, β, for saturated water at 403 K. Then compare your results with the value in the table.**

GIVEN

- Saturated water at 400 K

FIND

- The thermal expansion coefficient, β, from its definition

SOLUTION

The coefficient of thermal expansion is defined as the change of density with temperature at constant pressure

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

It can be approximated as

$$\beta \cong -\frac{1}{\left(\dfrac{\rho_1 + \rho_2}{2}\right)}\left(\frac{\rho_1 - \rho_2}{T_1 - T_2}\right)$$

For comparison with Appendix 2, Table 13, let T_1 = 393 K, T_2 = 413 K: From the table, ρ_1 = 943.5 kg/m^3, ρ_2 = 926.3 kg/m^3.

$$\beta \cong -\frac{1}{\left(\dfrac{943.5+926.3}{2}\right)\text{kg/m}^3}\left[\frac{(943.5+926.3)\,\text{kg/m}^3}{(393\,\text{K}-413\,\text{K})}\right] = 9.19 \times 10^{-4}\ \text{K}^{-1}$$

The table lists the thermal expansion coefficient at the average of these temperatures (403 K) to be 9.1 $\times$ 10^{-4} 1/K— a difference of about 1%.

PROBLEM 8.3

Using the standard steam tables, calculate the coefficient of thermal expansion, β, from its definition for steam at 450°C and pressures of 10 kPa and 1 MPa. Then compare your results with the value obtained by assuming that steam is a perfect gas and explain the difference.

GIVEN

- Steam
- Temperature = 450°C = 723 K

FIND

The coefficient of thermal expansion at 10 kPa and 10 MPa from

(a) Standard Steam tables
(b) Perfect Gas Law

PROPERTIES AND CONSTANTS

From steam tables

Temperature (K)	Pressure (MPa)	Density (kg/m^3)
673	1	3.262
773	1	2.824
673	0.01	0.03219
773	0.01	0.02803

SOLUTION

(a) The thermal expansion coefficient is given by

$$\beta = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \equiv -\frac{1}{\left(\frac{\rho_1+\rho_2}{2}\right)}\left(\frac{\rho_1-\rho_2}{T_1-T_2}\right)$$

At 1 MPa.: $$\beta \cong -\frac{1}{\left(\frac{3.262+2.824}{2}\right)\text{ kg/m}^3}\left[\frac{(3.262-2.824)\text{ kg/m}^3}{(673\text{K}-773\text{K})}\right] = 1.439\times 10^{-3}\text{ K}^{-1}$$

At 10 kPa: $$\beta \cong -\frac{1}{\left(\frac{0.03219+0.02803}{2}\right)\text{ kg/m}^3}\left[\frac{(0.03219-0.02803)\text{ kg/m}^3}{(673\text{K}-773\text{K})}\right] = 1.381\times 10^{-3}\text{ K}^{-1}$$

(b) For an ideal gas $\rho_1 T_1 = \rho_2 T_2 \Rightarrow \rho_1 = \rho_2 \frac{T_2}{T_1}$

$$\therefore \quad \beta \cong -\frac{1}{\frac{\rho_2}{2}\left(\frac{T_2}{T_1}+1\right)}\,\frac{\rho_2\left(\frac{T_2}{T_1}-1\right)}{T_1-T_2} = \frac{1}{\left(\frac{T_2+T_1}{2}\right)} = \frac{1}{T_{ave}}$$

At T_{ave} = 723 K, $\beta \cong \frac{1}{723\text{K}} = 1.383\times 10^{-3}\,\frac{1}{\text{K}}$ (independent of pressure)

COMMENTS

The result of part (b) correlates better with ρ calculated in part (a) at 10 kPathan at 1 MPa because steam more closely resembles an ideal gas at 10 kPa than at 1 MPa.

PROBLEM 8.4

A long cylinder of 0.1-m-diameter has a surface temperature of 400 K. If it is immersed in a fluid at 350 K, natural convection will occur as a result of the temperature difference. Calculate the Grashof and Rayleigh numbers that will determine the Nusselt number if the fluid is

(a) Nitrogen (b) Air (c) Water

(d) Oil (e) Mercury

GIVEN

- A long cylinder immersed in fluid
- Diameter (D) = 0.1 m
- Surface temperature (T_s) = 400 K
- Fluid temperature (T_∞) = 350 K

FIND

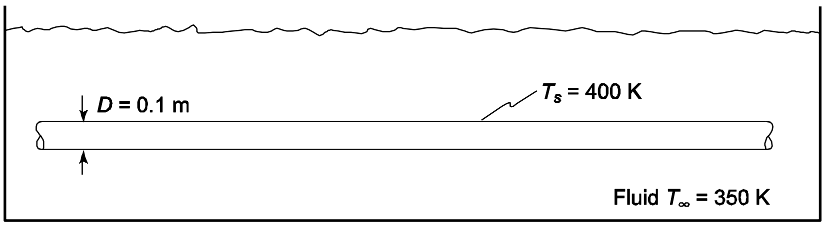
The Grashof number (Gr) and the Rayleigh number (Ra) if the fluid is

(a) Nitrogen (b) Air (c) Water
(d) Oil (e) Mercury

ASSUMPTIONS

- The cylinder is in a horizontal position (the characteristic length is the diameter of the cylinder)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 33, 28, 13, 17 and *26*, at the mean temperature of 375 K

Fluid	Nitrogen	Air	Water	Oil	Mercury
Table number	32	27	13	16	25
Thermal expansion coefficient, β (1/K)	0.00271	0.00268	0.00075	—	—
Kinematic viscosity, $\times 10^{-6}$ m^2/s	23.21	23.67	0.294	20.3	0.0928
Prandtl number (Pr)	0.697	0.71	1.75	2.76	0.0162

Fluid	Oil		Mercury	
Temperature (K)	353	393	323	423
Density, ρ (kg/m^3)	852.0	829.0	13,506	13.264

The thermal expansion coefficients for oil and mercury can be estimated from

$$\beta \cong -\frac{2}{(\rho_1 + \rho_2)}\left(\frac{\rho_1 - \rho_2}{T_1 - T_2}\right)$$

For oil at 373 K $\beta \approx 0.00068$ 1/K
For mercury at 373 K $\beta \approx 0.00018$ 1/K

SOLUTION

The Grashof number based on the cylinder diameter is

$$Gr_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^3}$$

For nitrogen

$$Gr_D = \frac{(9.8\,\text{m/s}^2)\ 0.00271\,\text{K}^{-1}\ (400\,\text{K} - 350\,\text{K})(0.1\text{m})^3}{23.21 \times 10^{-6}\,\text{m}^2/\text{s}^{\ 2}} = 2.46 \times 10^6$$

The Rayleigh number is defined as

$$Ra_D = Gr_D\, Pr = 2.46 \times 10^6\ (0.697) = 1.72 \times 10^6 \text{ (for Nitrogen)}$$

Calculating the Gr_D and Ra_D in a similar manner for the other fluids

Fluid	Gr_D	Ra_D
Nitrogen	2.46×10^6	1.72×10^6
Air	2.34×10^6	1.66×10^6
Water	4.25×10^9	7.44×10^9
Oil	8.08×10^5	2.23×10^6
Mercury	1.02×10^{10}	1.66×10^8

PROBLEM 8.5

Use Fig. 8.3 to determine the Nusselt number and heat transfer coefficient for the conditions given in Problem 8.4.

GIVEN

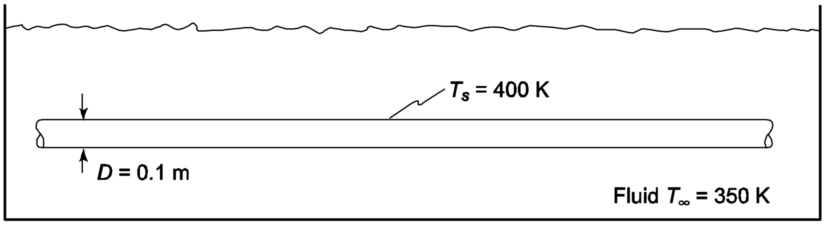
- A long cylinder immersed in a fluid
- Diameter (D) = 0.1 m
- Surface temperature (T_s) = 400 K
- Fluid temperature (T_∞) = 350 K

Fluid	Gr_D	Ra_D
Nitrogen	2.46×10^6	1.72×10^6
Air	2.34×10^6	1.66×10^6
Water	4.25×10^9	7.44×10^9
Oil	8.08×10^5	2.23×10^6
Mercury	1.02×10^{10}	1.66×10^8

FIND

- The Nusselt number (Nu) and heat transfer coefficient (h_c) for each fluid from Figure 8.3

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 33, 28, 13, 17 and 26, at the mean temperature of 375 K

Fluid	Thermal Conductivity, k (W/(m K))
Nitrogen	0.0316
Air	0.0307
Water	0.682
Oil	0.137
Mercury	10.51

SOLUTION

From Problem 8.4, the Rayleigh number for nitrogen is 1.72×10^6. The abscissa of Figure 8.3 is the base 10 log of the Rayleigh number

$$\text{Log}\,(1.72 \times 10^6) = 6.24$$

From Figure 8.3: $\log \overline{Nu}_D \approx 1.25 \rightarrow \overline{Nu}_D = 17.78$

$$\therefore\ \bar{h}_c = \frac{k}{D}\overline{Nu}_D = \frac{0.03156\ \text{W/(m K)}}{0.1\text{m}}\ 17.78 = 5.61\ \text{W/(m}^2\text{K)}$$

Following a similar procedure for the other fluids yields the following results

Fluid	log (Ra_D)	log ($\overline{Nu}_D$)	($\overline{Nu}_D$)	$\bar{h}_c$ (W/(m^2 K))
Nitrogen	6.24	1.25	17.78	5.61
Air	6.22	1.25	17.78	5.46
Water (extrapolating)	9.87	2.2	158	1081
Oil	6.34	1.28	19.1	26.1
Mercury	8.22	1.75	56.2	5910

PROBLEM 8.6

The following equation has been proposed for the heat transfer coefficient in natural convection from long vertical cylinders to air at atmospheric pressure:

$$\bar{h}_c = \frac{536.5\,(T_s - T_\infty)^{0.33}}{T_f}$$

Where T_f = the film temperature = 1/2 ($T_s + T_\infty$) and T_f is in the range 0 to 200°C

The corresponding equation in dimensionless form is

$$\frac{(\bar{h}_c\ L)}{k} = C(GrPr)^m$$

Compare the two equations, to determine those values of *C*, *m* and *n* in the second equation that will give the same results as the first equation.

GIVEN

- Empirical equations shown above

FIND

- Values of C, m and n

ASSUMPTIONS

- Air behaves as an ideal gas

SOLUTION

$$\frac{\bar{h}_c\ L}{k} = C(Gr_L\ Pr)^m$$

But
$$Gr_L = \frac{g\,\beta\,(T_s - T_\infty)\,L^3}{v^2} \quad \text{and} \quad Pr = \frac{c_p\ \mu}{k}$$

$$\therefore\ \frac{\bar{h}_c\ L}{k} = C\left[\frac{g\ \beta\,(T_s - T_\infty)\,L^3\,c_p\ \mu}{v^2 k}\right]^m$$

But
$$v = \frac{\mu}{\rho} \text{ and for an ideal gas: } \beta \cong \frac{1}{T} \text{ and } \rho = \frac{P}{RT}$$

$$\therefore\ \bar{h}_c = C\ \frac{k}{L}\left[\frac{g\,P^2 (T_s - T_\infty)\,L^3\,c_p}{T^3\,\mu\,R^2\,k}\right]^m$$

Equating this to the empirical equation

$$\bar{h}_c = (Ck^{\,1-m}\,g^{\,m}\,p^{\,2m}\,\mu^{-m}\,R^{-2m}\,c_p^{\,m})\,(T_s - T_\infty)^m\,L^{3m-1}\,T^{-3m} = 536.5\,(T_s - T_\infty)^{0.33}\,T^{-1}$$

The exponents of the variables must be the same, so

For $(T_s - T_\infty)$: $m = 0.33$
For L: $3\,m - 1 = 0$
For T: $-3\,m = -1$

The value of the constant C is determined by

$$C\ k^{\frac{2}{3}} g^{\frac{1}{3}} p^{\frac{2}{3}} \mu^{-\frac{1}{3}} R^{-\frac{2}{3}} c_p^{\frac{1}{3}} = 536.5$$

From Appendix 2, Table 28 for dry air at 100°C and one atmosphere

k = 0.0307 W/(m K)

μ = 21.673 × 10^{-6} Ns/m^2

c_p = 1022 J/(kg K)

The gas constant for air (R) = 287 J/(kg K)

The absolute pressure of one atmosphere (P) = 101,000 N/m^2.

$$\therefore\ C \quad 0.0307\,\text{W/(m K)}^{\frac{2}{3}} \quad 9.8\,\text{m/s}^{2\ \frac{1}{3}} \quad 101{,}000\ \text{N/m}^{2\ \frac{2}{3}}$$

$$21.673\times 10^{-6}\ \text{(Ns)/m}^{2\ -\frac{1}{3}} \quad 287\ \text{J/(kg K)}^{-\frac{2}{3}} \quad 1022\ \text{J/(kg K)}^{\frac{1}{3}} = 536.5$$

$$C = 0.142$$

The non-dimensional empirical equation is

$$Nu_L = 0.142\ (Gr_L Pr)^{\frac{1}{2}}$$

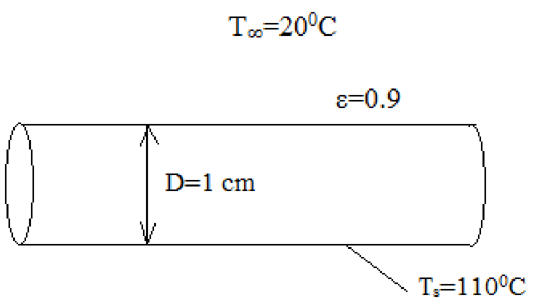
COMMENTS

The units of the constant in the empirical equation must be W/(m^2 K$^{1/3}$).

The non-dimensional empirical equation closely resembles that given by Equation (8.13) for turbulent natural convection from vertical cylinders.

PROBLEM 8.7

Consider the electrical space heater of Example 8.1, where the electric resistance heating coil is enclosed in a straight horizontal polished metallic cylinder that has an outer diameter of 1 cm and an emissivity of 0.9. If its surface temperature is at 110°C and the room air temperature is at 20°C, calculate the total heat loss per unit length of the cylinder due to both convection and radiation. For the purpose of this calculation, assume that the reflector is so designed that it "collects" all of the radiant heat from the back half of the horizontal heater and that it reflects 95% of the incident radiation.



GIVEN

- An electric resistance heating coil enclosed in horizontal metallic cylinder.
- Emissivity (ε) = 0.9
- Diameter of cylinder (D) = 1 cm= 0.01 m
- Surface temperature (T_s) = 110°C= 383 K
- Ambient air temperature (T_∞) = 20°C= 293 K
- Reflects 95% of incident radiation.

FIND

Total heat loss per unit length of cylinder due to convection and radiation.

ASSUMPTIONS

- Air is still
- Surface temperature is uniform and constant

PROPERTIES AND CONSTANTS

From Appendix 2, Tables 28, for dry air at the mean temperature of 65°C

Thermal expansion coefficient (β) = 0.00296 1/K

Thermal conductivity (k) = 0.02825 W/(m K)

Kinematic viscosity (ν) = 19.92 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Grashof number is

$$Gr_L = \frac{g\,\beta\,(T_s - T_\infty)\,D^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)(0.00296/\text{K})(110°\text{C} - 20°\text{C})(0.01\text{m})^3}{\left(19.92\times10^{-6}\text{m}^2\ /\ \text{s}\right)^2} = 6579.3$$

Pr=0.71

Ra=Gr_L*Pr=4671.2

Log_{10}(Ra)=3.67

From Figure 8.3 we have

$\mathrm{Log}_{10}(\mathrm{Nu}) = 0.64$

$\Rightarrow \mathrm{Nu} = 4.36$

$$\frac{\bar{h}_c D}{k} = 4.36$$

$$\bar{h}_c = 4.36\frac{k}{D} = 4.36 * \frac{0.02825}{0.01}$$

$$\bar{h}_c = 12.33 \text{ (W/m}^2\text{ K)}$$

Thus total heat loss per unit length by both convection and radiation is given by

$$\frac{Q}{L} = \pi D \bar{h}_c (T_s - T_\infty) + \pi D \varepsilon \left(T_s^4 - T_\infty^4\right) \text{ W/m}$$

$$\frac{Q}{L} = \pi * 0.01 * 12.33 * (383 - 293) + \pi * 0.01 * 5.67 * 10^{-8} * 0.9 * \left(383^4 - 293^4\right) \text{ W/m}$$

$$\frac{Q}{L} = 34.86 + 22.68 \text{ W/m} = 57.5 \text{ W/m}$$

PROBLEM 8.8

The heat loss from uninsulated hot water pipes installed in homes can be considerable and hence increases the overall energy cost of the home. Consider a 5-m-long section of such a pipe that has an outer diameter of 1.7 cm and a surface temperature of 75°C. If the room air temperature is 20°C, calculate the natural convection heat loss from the pipe section. If the radiation heat losses are to be included as well, determine its contribution as a percentage of the total pipe heat transfer rate; assume that the outer pipe surface has an emissivity of 0.7.

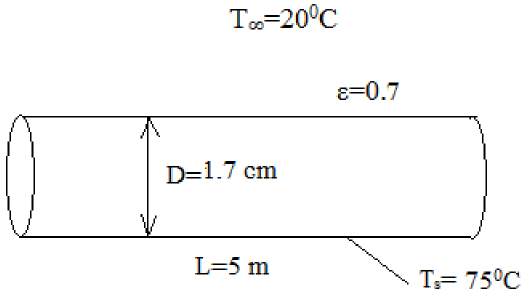
GIVEN

- Uninsulated hot water pipe dissipating heat to surrounding
- Emissivity (ε) = 0.7
- Diameter of cylinder (D) = 1.7 cm= 0.017 m
- Surface temperature (T_s) = 75°C= 348 K
- Ambient air temperature (T_∞) = 20°C= 293 K
- Length of the cylinder (L)= 5 m

FIND

(a) Natural convection heat loss from pipe section.
(b) Contribution of radiation as percentage of total pipe heat transfer rate.

SKETCH



ASSUMPTIONS

- Air is still
- Surface temperature is uniform and constant

PROPERTIES AND CONSTANTS

From Appendix 2, Tables 28, for dry air at the mean temperature of 47.5°C

Thermal expansion coefficient (β) = 0.00312 1/K

Thermal conductivity (k) = 0.02695 W/(m K)

Kinematic viscosity (ν) = 18.27 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

(a) The Grashof number is

$$Gr_L = \frac{g\,\beta\,(T_s - T_\infty)\,D^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00312/\text{K})(75°\text{C} - 20°\text{C})(0.017\text{m})^3}{\left(18.27\times 10^{-6}\text{m}^2/\text{s}\right)^2} = 21380$$

Pr=0.71

$Ra = Gr_L * Pr = 15180$
$Log_{10}(Ra) = 4.18$

From Figure 8.3 we have

$Log_{10}(Nu) = 0.76$
$\Rightarrow Nu = 5.75$

$$\frac{\bar{h}_c D}{k} = 5.75$$

$$\bar{h}_c = 4.36\frac{k}{D} = 5.75 * \frac{0.02695}{0.017} \text{ W/(m}^2\text{ K)}$$

$$\bar{h}_c = 9.12 \text{ W/(m}^2\text{ K)}$$

Thus, heat loss by convection is given by

$$Q_{conv} = \pi D L \bar{h}_c (T_s - T_\infty) \text{ W}$$

$$Q_{conv} = \pi * 0.017 * 5 * 9.12 * (348 - 293) \text{ W}$$

$$Q_{conv} = 134 \text{ W}$$

(b) Heat loss by radiation is given by

$$Q_{rad} = \pi D L \varepsilon \left(T_s^4 - T_\infty^4\right) \text{ W}$$

$$Q_{rad} = \pi * 0.017 * 5 * 0.7 \left(348^4 - 293^4\right) \text{ W}$$

$$Q_{rad} = 77.32 \text{ W}$$

Thus percentage of total heat loss by radiation is

% loss in radiation= $\frac{77.32}{134 + 77.32}$ = 36.6%

PROBLEM 8.9

Overhead electric transmission lines get heated (due to the electric resistivity of the cable material) and lose heat to the ambient air; this transmission energy loss due to heat transfer is referred to as the Joule effect in the conduction material. Consider a 3-cm diameter horizontal cable that dissipates heat at the rate of 50 W per unit length, and that the heat transfer due to surrounding air is only by natural convection. Determine the surface temperature of the cable when (a) in the summer time the air temperature is 35°C and (b) in the winter time the air temperature is 25°C.

GIVEN

- Overhead heated electric transmission lines loosing heat due to convection
- Heat dissipation rate= 50 W/m
- Diameter of transmission lines (D) = 3 cm= 0.03 m
- Ambient air temperature during summer (T_∞)

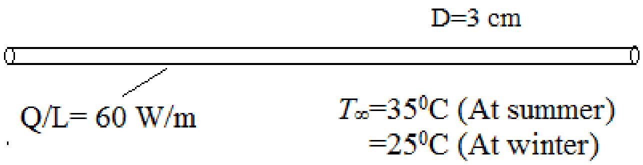
At summer = 35°C

At winter= 25^0C

FIND

(Surface temperature of cable in (a) Summer (b) Winter

SKETCH



ASSUMPTIONS

- Air is still
- Surface temperature is uniform and constant
- Air properties are constant along the range of temperature.

PROPERTIES AND CONSTANTS

From Appendix 2, Tables 28, for dry air at the assumed mean temperature of 50°C

Thermal expansion coefficient (β) = 0.003095 1/K

Thermal conductivity (k) = 0.0272 W/(m K)

Kinematic viscosity (ν) = 18.5×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

(a) The Grashof number is

$$Gr_L = \frac{g\,\beta\,(T_s - T_\infty)\,D^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.003095/\text{K})(T_s - T_\infty)(0.03\text{m})^3}{\left(18.5\times10^{-6}\text{m}^2/\text{s}\right)^2} = 2392.8(T_s\text{-}T_\infty)$$

Pr=0.71

Nusselt number for thin horizontal wire is given by Equation (8.20) we have

$$Nu_D = 0.53(Gr_D\,\text{Pr})^{1/4}$$

$$Nu_D = 0.53(2392.8 * 0.71(T_s - T_\infty))^{1/4}$$

$$Nu_D = 3.4026(T_s - T_\infty)^{1/4}$$

$$\frac{\bar{h}_c D}{k} = 3.4026(T_s - T_\infty)^{1/4}$$

$$\bar{h}_c = 3.4026(T_s - T_\infty)^{1/4} \frac{k}{D} \text{ W/(m}^2\text{ K)}$$

$$\bar{h}_c = 3.085(T_s - T_\infty)^{1/4} \text{ W/(m}^2\text{ K)}$$

Thus, heat loss per unit length by convection is given by

$$Q_{conv} / L = \pi D \bar{h}_c (T_s - T_\infty) = 60 \text{ W/m}$$

$$\pi * 0.03 * 3.085(T_s - T_\infty)^{1/4} * (T_s - T_\infty) = 60 \text{ W/m}$$

$$(T_s - T_\infty)^{5/4} = 206.35$$

(a) For summer T_∞ =35^{0}C

Thus the wire surface temperature is

$$(T_s - 35)^{5/4} = 206.35$$

$$(T_s - 35) = 71.7$$

$$T_s = 106.7^0 C$$

(b) For winter T_∞ =25^{0}C

Thus the wire surface temperature is

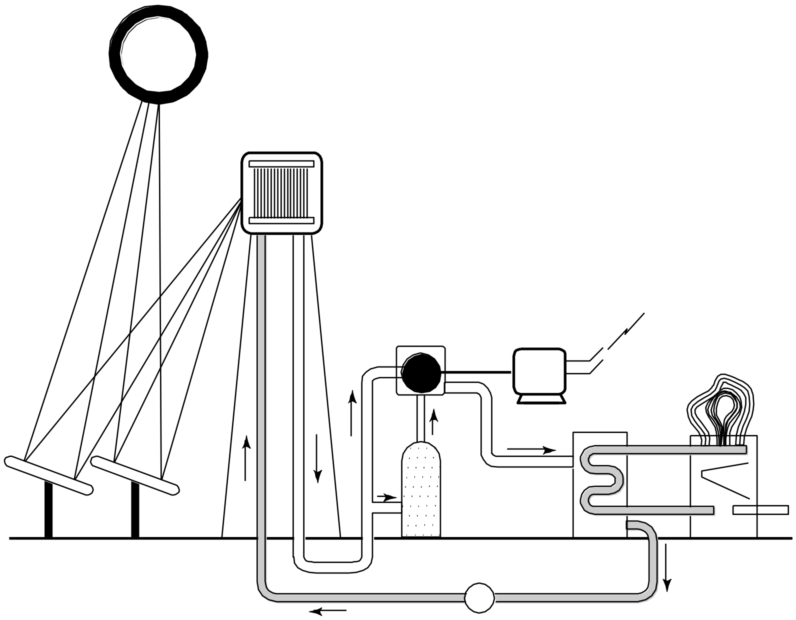
$$(T_s - 25)^{5/4} = 206.35$$

$$(T_s - 25) = 71.7$$

$$T_s = 96.7^0 C$$

PROBLEM 8.10

Solar One, located near Barstow, CA, was the first large-scale (10 MW electric) solar-thermal electric-power-generating plant in the United States. A schematic diagram of the plant is shown below. The receiver may be treated as a cylinder 7 m in diameter and 13.5-m-tall. At the design operating conditions, the average outer surface temperature of the receiver is about 675°C and ambient air temperature is about 40°C. Estimate the rate of heat loss, in MW, from the receiver—via natural convection only—for the temperatures given. What are other mechanisms by which heat may be lost from the receiver?



GIVEN

- A vertical cylinder in air
- Height of cylinder (L) = 13.5 m
- Diameter (D) = 7 m
- Surface temperature (T_s) = 675°C
- Ambient air temperature (T_∞) = 40°C

FIND

(a) The rate of convective heat loss (q_c) in MW
(b) What other mechanisms for heat loss exist?

ASSUMPTIONS

- Air is still
- Surface temperature is uniform and constant

PROPERTIES AND CONSTANTS

From Appendix 2, Tables 28, for dry air at the mean temperature of 357.5°C

Thermal expansion coefficient (β) = 0.00160 1/K

Thermal conductivity (k) = 0.0461 W/(m K)

Kinematic viscosity (ν) = 58.1×10^{-6} m^2/s

Prandtl number (Pr) = 0.72

SOLUTION

The Grashof number is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.0016\,1/\text{K}\ (675°\text{C} - 40°\text{C})(13.5\,\text{m})^3}{58.1\times 10^{-6}\,\text{m}^2/\text{s}^{\ 2}} = 7.26 \times 10^{12}$$

Therefore, the flow is turbulent.

Equation (8.13) gives the Nusselt number for a turbulent boundary layer

$$Nu_L = 0.13\ (Gr_L\,Pr)^{\frac{1}{3}} = 0.13\ [7.26\times 10^{12}(0.72)]^{\frac{1}{3}} = 2256$$

$$\therefore\ \bar{h}_c = \frac{k}{L}\ Nu_L = \frac{0.0461\,\text{W/(m K)}}{13.5\,\text{m}}\ 2256 = 7.70\ \text{W/(m}^2\ \text{K)}$$

(a) The rate of convective heat transfer is

$$q_c = \bar{h}_c\,\pi\,D\,L\,(T_s - T_\infty) = 7.7\ \text{W/(m}^2\ \text{K)}\ \ \pi\,(7\ \text{m})\,(13.5\ \text{m})\,(675°\text{C} - 40°\text{C})$$

$$10^{-6}\,(\text{MW})/\text{W} = 1.45\ \text{MW}$$

(b) Other mechanisms for heat transfer from the surface are

1. Radiation to the surroundings.
2. Conduction to the interior of the cylinder where the heat can be removed by a working fluid.
3. Conduction to the support structure.
4. Forced convection to the ambient air when breezes occur.

PROBLEM 8.11

Compare the rate of heat loss from a human body with the typical energy intake from consumption of food (4.325 MJ/day). Model the body as a vertical cylinder 30 cm in diameter and 1.8-m-high in still air. Assume the skin temperature is 2°C below normal body temperature. Neglect radiation, transpiration cooling (sweating), and the effects of clothing.

GIVEN

- Human body idealized as a vertical cylinder in still air
- Diameter (D) = 30 cm = 0.3 m
- Height (L) = 1.8 m
- Skin temperature (T_s) = 2°C below normal body temperature (37°C) = 35°C

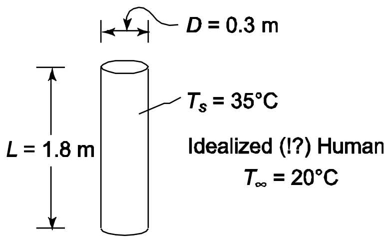
FIND

- Heat loss (q) and compare to consumption of food 4.325 MJ/day

ASSUMPTIONS

- Steady state
- Radiation, transpiration cooling, and clothing effects are negligible
- Ambient air temperature (T_∞) = 20°C
- Heat loss from the top of the cylinder is small compared to that from the sides

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 27.5°C

Thermal expansion coefficient (β) = 0.00333 1/°C

Thermal conductivity (k) = 0.0257 W/(m K)

Kinematic viscosity (ν) = 16.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Grashof number based on the height is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)\,L^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ 0.00333\ 1/\text{K}\ (35°\text{C} - 20°\text{C})(1.8\,\text{m})^3}{\left(16.4 \times 10^{-6}\ \text{m}^2/\text{s}\right)^2} = 1.06 \times 10^{10} > 10^9$$

Therefore, the flow is turbulent.

For turbulent boundary layer, the average heat transfer coefficient is given by Equation (8.13)

$$h_c = 0.13\ \frac{k}{L}\ (Gr_L\,Pr)^{\frac{1}{3}} = 0.13\,\frac{0.0256\ \text{W/(m K)}}{1.8\,\text{m}}[1.06 \times 10^{10}(0.71)]^{\frac{1}{3}} = 3.62\ \text{W/(m}^2\ \text{K)}$$

The rate of convective heat loss from the sides of the cylinder is

$$q_c = h_c\, \pi D\, L\, (T_s - T_\infty) = 3.62\ \mathrm{W/(m^2\ K)}\ \pi\ (0.3\ \mathrm{m})\ (1.8\ \mathrm{m})\ (35°\mathrm{C} - 20°\mathrm{C}) = 92.2\ \mathrm{W}$$

$$\text{Food consumption} = 4.325\ \mathrm{MJ/day} * 10^6\ \mathrm{J/MJ} * \left(\frac{1\ \mathrm{day}}{24\ \mathrm{h}}\right)\left(\frac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right)\ \mathrm{Ws/J} = 50.1\mathrm{W}$$

COMMENTS

The heat loss calculated for the idealized human is about 46% greater than the average food consumption. This point out the importance of clothing.

PROBLEM 8.12

An electric room heater has been designed in the shape of a vertical cylinder 2-m-tall and 30 cm in diameter. For safety, the heater surface cannot exceed 35°C. If the room air is 20°C, find the power rating of the heater in watts.

GIVEN

- An electric heater in the shape of a vertical cylinder
- Heater height (H) = 2 m
- Heater diameter (D) = 30 cm = 0.3 m
- Room air temperature (T_∞) = 20°C = 293 K

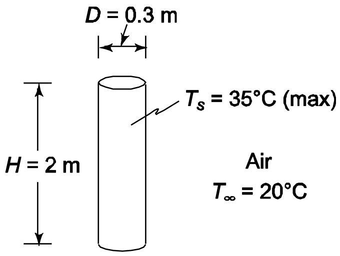
FIND

- The power rating of the heater (q) in watts

ASSUMPTIONS

- Radiation is negligible
- Heat transfer from the top and bottom of the tank is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 27.5°C

Thermal expansion coefficient (β) = 0.00332 1/K
Thermal conductivity (k) = 0.0256 W/(m K)
Kinematic viscosity (ν) = 16.4×10^{-6} m²/s
Prandtl number (Pr) = 0.71

SOLUTION

When the heater surface temperature is 35°C, the Grashof number for the heater sides is

$$Gr_H = \frac{g\,\beta(T_s - T_\infty)H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ 0.00332\ 1/\text{K}\ (35°\text{C} - 20°\text{C})\ (2\ \text{m})^2}{16.4\times 10^{-6}\ \text{m}^2/\text{s}^{\ 2}} = 1.45 \times 10^{10} > 10^9$$

Therefore, the flow is turbulent.

The average heat transfer coefficient is given by Equation (8.13)

$$h_c = 0.13\ \frac{k}{L}(Gr_L\,Pr)^{\frac{1}{3}} = 0.13\ \frac{0.0256\ \text{W/(m K)}}{2\,\text{m}}\ [1.45\times 10^{10}(0.71)]^{\frac{1}{3}} = 3.62\ \text{W/(m}^2\ \text{K)}$$

The power rating of the heater must equal the rate of heat transfer from the heater

$$q = h_c\,\pi D\,L\,(T_s - T_\infty) = 3.62\ \text{W/(m}^2\ \text{K)}\quad \pi\,(0.3\ \text{m})\ (2\ \text{m})\ (35°\text{C} - 20°\text{C}) = 102\ \text{W}$$

COMMENTS

This heater would probably not suffice for most applications. Either the surface temperature needs to be raised, of the size of the heater needs to be increased.

PROBLEM 8.13

A long, heated, cylindrical steel rod is removed from a heat treatment furnace and has to be cooled to complete the process. If the surface temperature of the rod is at 150°C and the cooling fluid temperature is maintained at 25°C, what is the minimum diameter of the rod to produce turbulent natural convection heat transfer if it is held in a horizontal position in (a) air and (b) water? Also, determine the initial heat transfer rate by natural convection from a 10-cm diameter rod in each of the two fluids.

GIVEN

- Long heated cylindrical steel rod, cooled by natural convection.
- Surface temperature of rod (T_s=150^{0}C
- Fluid temperature (T_∞) = 25^{0}C

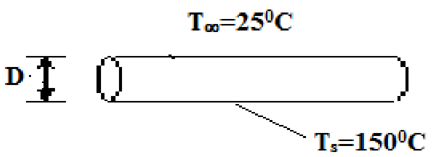
FIND

(a) Minimum diameter of rod to produce turbulent natural convection in (i) Air (ii) water
(b) Initial heat transfer rate by natural convection from 10 cm diameter rod in each fluids.

ASSUMPTIONS

- Radiation is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 87.5°C

Thermal expansion coefficient (β) = 0.00277 1/K
Thermal conductivity (k) = 0.0298 W/(m K)
Kinematic viscosity (ν) = 22.55 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71

From Appendix 2, Table 13, for water at the mean temperature of 87.5°C

Thermal expansion coefficient (β) = 0.000678 1/K
Thermal conductivity (k) = 0.6765 W/(m K)
Kinematic viscosity (ν) = 0.33 × 10^{-6} m^2/s
Prandtl number (Pr) = 67.8

SOLUTION

(a) The Grashof number when the fluid is air at 25^{0}C is

$$Gr_D = \frac{g\,\beta\,(T_s - T_\infty)D^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)(0.00277\ 1/\text{K})(150°\text{C} - 25°\text{C})\ D^3}{\left(22.55\times 10^{-6}\text{m}^2/\text{s}\right)^2} = 6.673*10^9*\text{D}^3$$

Pr=0.71

For Turbulent flow in Natural convection the criterion from figure (8.5) is

$$\text{Ra}_\text{D} = \text{Gr}_\text{D} * \text{Pr} = 10^9$$

$$=> 6.673*10^9*0.71*\text{D}^3 = 10^9$$

$$D^3 = 0.2111\ \mathrm{m}^3$$

D=0.595 m

Thus the required diameter to produce turbulent natural convection in air is 59.5 cm.

The Grashof number when the fluid is water at 25^{0}C is

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{(9.8\ \mathrm{m/s^2})(0.000678\ 1/\mathrm{K})(150°\mathrm{C} - 25°\mathrm{C})\ D^3}{\left(0.33\times 10^{-6}\mathrm{m}^2/\mathrm{s}\right)^2} = 7.626*10^{12}*D^3$$

Pr=67.8

For Turbulent flow in Natural convection the criterion from figure (8.5) is

$$Ra_D = Gr_D * Pr = 10^9$$

$$=> 7.626*10^{12}*67.8*D^3 = 10^9$$

$$=> D^3 = 1.934*10^{-6}\mathrm{m}^3$$

=>D=0.0125 m

Thus the required diameter to produce turbulent natural convection in water is 1.25 cm

(b) For a 10 cm diameter rod in Air

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{(9.8\ \mathrm{m/s^2})(0.00277\ 1/\mathrm{K})(150°\mathrm{C} - 25°\mathrm{C})\ D^3}{\left(22.55\times 10^{-6}\mathrm{m}^2/\mathrm{s}\right)^2} = 6.673*10^9*0.1^3$$

$$Gr_D = 6.673*10^6$$

For a horizontal cylinder in natural convection, the Nusselt number is given by equation (8.20) as

$$\overline{Nu} = 0.53(Gr_D\ Pr)^{1/4}$$

$$\overline{Nu} = 0.53(6.673*10^6*0.71)^{1/4} = 24.72$$

$$\bar{h}_c = Nu\frac{k}{D} = 7.36\ \mathrm{W/(m^2\ K)}.$$

Thus heat transfer rate is

$$q'' = \bar{h}_c(T_s - T_\infty) = 921\ \mathrm{W/m^2}.$$

For a 10 cm diameter rod in water,

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{(9.8\ \mathrm{m/s^2})(0.000678\ 1/\mathrm{K})(150°\mathrm{C} - 25°\mathrm{C})\ D^3}{\left(0.33\times 10^{-6}\mathrm{m}^2/\mathrm{s}\right)^2} = 7.626*10^{12}*0.1^3$$

$$Gr_D = 7.626*10^9$$

For a horizontal cylinder in natural convection, since turbulent Nu equation for natural convection in horizontal cylinders is not available,

The Nusselt number is given by equation (8.24) with Sinθ= 0.

$$\overline{Nu} = 0.47 Gr_D^{1/4}\ Pr^{1/3}$$

$$\overline{Nu} = 0.53(7.676*10^9*67.8)^{1/4} = 266$$

$$\bar{h}_c = Nu\frac{k}{D} = 1803\ \mathrm{W/(m^2\ K)}.$$

Thus heat transfer rate is

$$q'' = \bar{h}_c(T_s - T_\infty) = 225.5\ \mathrm{kW/m^2}.$$

PROBLEM 8.14

Consider a design for a nuclear reactor using natural-convection heating of liquid bismuth as shown. The reactor is to be constructed of parallel vertical plates 1.8 m tall and 1.2 m wide, in which heat is generated uniformly. Estimate the maximum possible heat dissipation rate from each plate if the average surface temperature of the plate is not to exceed 870°C and the lowest allowable bismuth temperature is 315°C.

GIVEN

- Vertical plates with uniform heat generation in bismuth
- Plate height (L) = 1.8 m
- Plate width (w) = 1.2 m
- Maximum average surface temperature (T_s) = 870°C
- Minimum bismuth temperature (T_∞) = 315°C

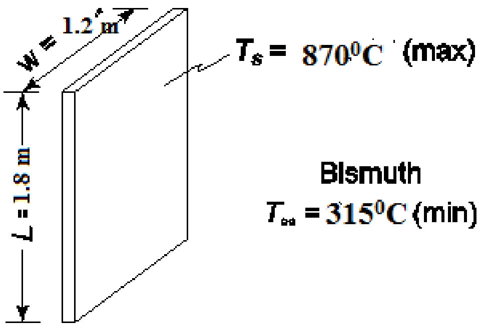
FIND

- Maximum possible heat dissipation rate (q) from each plate

ASSUMPTIONS

- Steady state
- Free convection only
- Edge effects are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 25, for bismuth at the mean temperature of (592.5°C)

Thermal conductivity (k) = 15.58 W/(m K)

Kinematic viscosity (ν) = 0.991×10^{-7} m^2/s

Prandtl number (Pr) = 0.010

Also Density at 538°C (ρ_{538}) = 9739 kg/m^3

Density at 649°C (ρ_{649}) = 9611 kg/m^3

To find the thermal expansion coefficient (β)

$$\beta = \left(\frac{2}{\rho_{538} + \rho_{649}}\right)\left(\frac{\rho_{538} - \rho_{649}}{111°C}\right) = 1.19 \times 10^{-4}\ \frac{1}{K}$$

SOLUTION

The Grashof number based on the vertical length of the plate is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)\,L^3}{\nu^2} = \frac{\left(9.8\,m/\text{s}^2\right)\left(1.19\times10^{-4}\,1/^o\text{C}\right)(870°C - 315°C)\,(1.8\,m)^3}{\left(0.991\times10^{-7}\,m^2\,/\,\text{s}\right)^2} = 3.84 \times 10^{14}$$

The average Nusselt number for a vertical plate in liquid metal for $Gr > 10^9$ is given by Equation (8.13)

$$\overline{Nu}_L = \frac{\bar{h}_c L}{k} = 0.13\;(\underline{Gr}_L Pr)^{\frac{1}{3}}$$

Solving for the heat transfer coefficient

$$\bar{h}_c = 0.13\,\frac{k}{L}(Gr_L Pr)^{\frac{1}{3}} = 0.13\,\frac{\left(15.58\,W\,/\,(mK)\right)}{1.8m}\,[3.84\times10^{14}(0.010)]^{\frac{1}{3}} = 17620\,W/(\text{m}^2\text{K})$$

The maximum rate of heat transfer from both sides of the plate is given by Equation (1.10)

$$q = \bar{h}_c\;A\,\Delta T = \left(17620\,W/(\text{m}^2\;\text{K})\right)\;[2\,(1.8\text{ m})\,(1.2\text{ m})]\,(870°\text{C} - 315°\text{C}) = 4.2 \times 10^7\text{ W}$$

PROBLEM 8.15

A mercury bath at 60°C is to be heated by immersing cylindrical electric heating rods, each 20 cm tall and 2 cm in diameter. Calculate the maximum electric power rating of a typical rod if its maximum surface temperature is 140°C.

GIVEN

- Cylindrical heating rods in a mercury bath
- Mercury temperature (T_∞) = 60°C
- Rod diameter (D) = 2 cm = 0.02 m
- Rod height (L) = 20 cm = 0.2 m
- Maximum surface temperature (T_s) = 140°C

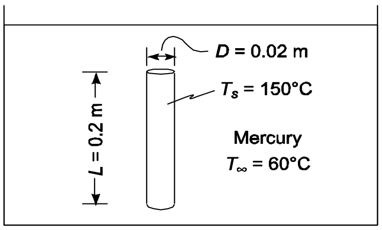
FIND

- The maximum electric power rating $(\dot{q}_e)$ of a rod

ASSUMPTIONS

- Steady state
- The rods are in a vertical position

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 26, for mercury at the mean temperature of 100°C

Thermal conductivity (k) = 10.51 W/(m K)

Kinematic viscosity (ν) = 0.093 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.0162

Also Density at 50°C (ρ_{50}) = 13,506 kg/m^3

Density at 15°C (ρ_{150}) = 13,264 kg/m^3

To find the thermal expansion coefficient (β), $\beta = \left(\dfrac{2}{\rho_{50} + \rho_{150}}\right)\left(\dfrac{\rho_{50} - \rho_{150}}{100°\text{C}}\right)$ = 1.81 × 10^{-4} 1/K

SOLUTION

The Grashof number at the top of the cylinder is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ 1.81\times 10^{-4}\ \text{1/K}\ (140°\text{C} - 60°\text{C})(0.2\,\text{m})^3}{\left(0.093\times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 1.31 \times 10^{11} > 10^9$$

Therefore, the boundary layer is turbulent and the average heat transfer coefficient is given by Equation (8.13)

$$\bar{h}_c = 0.13\ \frac{k}{L}\,(Gr_L\,Pr)^{\frac{1}{3}} = 0.13\ \frac{10.5\ \text{W/(m K)}}{0.2\ \text{m}}\,[1.31\times 10^{11}(0.0162)]^{\frac{1}{3}} = 8776\ \text{W/(m}^2\ \text{K)}$$

The maximum electric power rating of a rod is equal to the maximum rate of heat transfer from a rod

$$\dot{q}_c = q_c = \bar{h}_c\ \pi D\,L\,(T_s - T_\infty) = 8776\ \text{W/(m}^2\ \text{K)}\ \pi(0.02\ \text{m})(0.2\ \text{m})\ (140°\text{C} - 60°\text{C}) = 8823\ \text{W}$$

PROBLEM 8.16

An electric heating blanket is subjected to an acceptance test. It is to dissipate 400 W on the high setting when hanging in air at 20°C. (a) If the blanket is 1.3-m-wide: what is the length required if its average temperature at the high setting is to be 40°C, (b) If the average temperature at the low setting is to be 30°C, what rate of dissipation is possible?

GIVEN

- An electric blanket hanging in air
- Heat dissipation rate (q_h) = 400 W
- Air temperature (T_∞) = 20°C
- Blanket width (w) = 1.3 m
- Average temperatures
  - High (T_{sh}) = 40°C
  - Low (T_{s1}) = 30°C

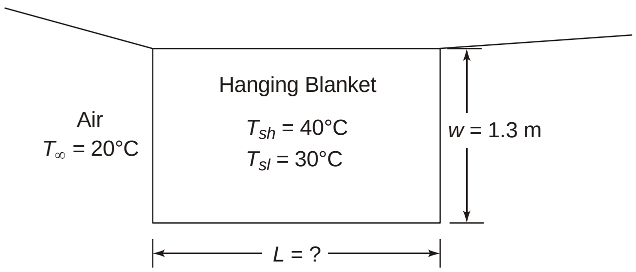
FIND

(a) The length of the blanket (L)
(b) Heat dissipation rate on the low setting (q_1)

ASSUMPTIONS

- Air is still
- Moisture in the air has a negligible effect
- Blanket is hung vertically with its 1.3 m sides vertical

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28 for dry air at the mean temperatures for the two settings

Mean Temperature (°C)	30°C	25°C
Thermal expansion coefficient, β (1/K)	0.00330	0.00336
Thermal conductivity, k (W/(m K))	0.0258	0.0255
Kinematic viscosity, $\nu \times 10^{-6}$ (m^2/s)	16.7	16.2
Prandtl number, Pr	0.71	0.71

SOLUTION

(a) On the high setting, the Grashof number for the blanket is

$$Gr_w = \frac{g\,\beta(T_s - T_\infty)\,w^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.0033\,1/\text{K}\ (40°\text{C} - 20°\text{C})(1.3\,\text{m})^3}{\left(16.7 \times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 5.09 \times 10^9$$

Therefore, the boundary layer is turbulent and the natural convection heat transfer coefficient is given by Equation (8.13)

$$\bar{h}_{ch} = 0.13\ \frac{k}{w}(Gr_w Pr)^{\frac{1}{3}} = 0.13\ \frac{0.0258\ \text{W/(m K)}}{1.3\,\text{m}}[5.09 \times 10^9 (0.71)]^{\frac{1}{3}} = 3.96\ \text{W/(m}^2\ \text{K)}$$

The rate of heat transfer from both sides of the blanket is

$$q_h = \bar{h}_{ch}\ A\ (T_{sh} - T_\infty) = \bar{h}_{ch}\ (2Lw)\ (T_{sh} - T_\infty)$$

Solving for the length of the blanket

$$L = \frac{q_h}{2\,\bar{h}_{ch}\ w(T_{sh} - T_\infty)} = \frac{400\,\text{W}}{2\ 3.96\ \text{W/(m}^2\ \text{K)}\ (1.3\,\text{m})(40°\text{C} - 20°\text{C})} = 1.94\ \text{m}$$

(b) The Grashof number for the blanket on the low setting is

$$Gr_w = \frac{g\,\beta(T_s - T_\infty)\,w^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.00336\,1/\text{K}\ (30°\text{C} - 20°\text{C})(1.3\,\text{m})^3}{16.2 \times 10^{-6}\ \text{m}^2/\text{s}\ ^2} = 2.76 \times 10^9$$

This is still turbulent, therefore

$$\bar{h}_{cl} = 0.13\ \frac{0.0255\ \text{W/(m K)}}{1.3\,\text{m}}[2.76 \times 10^9 (0.71)]^{\frac{1}{3}} = 3.19\ \text{W/(m}^2\text{K)}$$

The heat dissipation rate possible is equal to the rate of convection from both sides

$$q_1 = \bar{h}_{cl}\ 2Lw\ (T_{sl} - T_\infty) = 3.19\ \text{W/(m}^2\ \text{K)}\ (2)\ (1.94\ \text{m})\ (1.3\ \text{m})\ (30°\text{C} - 20°\text{C}) = 161\ \text{W}$$

PROBLEM 8.17

An aluminum sheet, 0.4-m-tall, 1-m-long, and 0.002-m-thick is to be cooled from an initial temperature of 150°C to 50°C by immersing it suddenly in water at 20°C. The sheet is suspended from two wires at the upper corners as shown in the sketch.

(a) Determine the initial and the final rate of heat transfer from the plate.

(b) Estimate the time required.

(Hint: Note that in laminar natural convection, $h \approx \Delta T^{0.25}$)

GIVEN

- A vertical aluminum sheet in water
- Plate dimensions: height (H) = 0.4 m, length (L) = 1 m, thickness (s) = 0.002 m
- Initial plate temperature (T_{si}) = 150°C
- Water temperature (T_∞) = 20°C
- Final plate temperature (T_{sf}) = 50°C

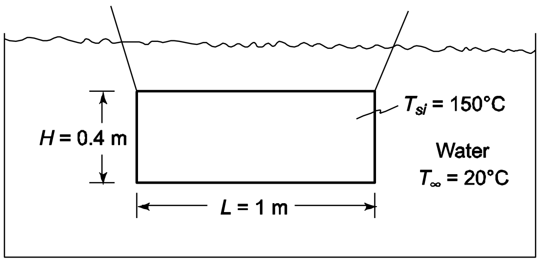
FIND

(a) The initial and final heat transfer rates
(b) The time required

ASSUMPTIONS

- Constant and uniform water temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, for aluminum at the mean temperature of 100°C

Thermal conductivity (k_{a1}) = 238 W/(m K)

Density (ρ) = 2702 kg/m^3

Specific heat (c) = 896 J/(kg J)

From Appendix 2, Table 13, for water at the mean temperatures:

Mean Temperature (°C)	85°C	35°C
Thermal expansion coefficients, β (1/k)	0.00066	0.00034
Thermal conductivity, k (W/(m K))	0.675	0.624
Kinematic viscosity, $\nu \times 10^6$ (m^2/s)	0.337	0.725
Prandtl number, Pr	2.04	4.8

SOLUTION

(a) The Grashof number based on the height of the plate is
Initial

$$Gr_H = \frac{g\,\beta(T_s - T_\infty)\,H^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)\ 0.00066\text{ 1/K }\ (150°\text{C} - 20°\text{C})(0.4\text{ m})^3}{\left(0.337\times 10^{-6}\text{ m}^2/\text{s}\right)^2} = 4.74\times 10^{11}\text{ (Turbulent)}$$

Final

$$Gr_H = \frac{(9.8\ \text{m/s}^2)\ 0.00034\ 1/\text{K}\ (50°\text{C} - 20°\text{C})(0.4\ \text{m})^3}{\left(0.725 \times 10^{-6}\ \text{m}^2/\text{s}\right)^2} = 1.22 \times 10^{10}$$

(Turbulent)

The average heat transfer coefficient from a vertical plate with a turbulent boundary layer is given by Equation (8.13)

$$\bar{h}_c = 0.13\ \frac{K}{H}(Gr_L Pr)^{\frac{1}{3}}$$

Initial

$$\bar{h}_{ci} = 0.13\ \frac{0.675\ \text{W/(m K)}}{0.4\ \text{m}}[4.74 \times 10^{11}(2.04)]^{\frac{1}{3}} = 2169\ \text{W/(m}^2\text{K)}$$

Final

$$\bar{h}_{cf} = 0.13\ \frac{0.624\ \text{W/(m K)}}{0.4\ \text{m}}[1.22 \times 10^{10}(4.8)]^{\frac{1}{3}} = 787\ \text{W/(m}^2\text{K)}$$

The rate of convective heat transfer from the plate is given by Equation (1.10)

$$q_c = \bar{h}_c\ A\ \Delta\ T$$

Initial

$$q_{ci} = 2169\ \text{W/(m}^2\text{K)}\ \ [2\ (0.4\ \text{m})\ (1\ \text{m})]\ (150°\text{C} - 20°\text{C}) = 2.26 \times 10^5\ \text{W}$$

Final

$$q_{cf} = 787\ \text{W/(m}^2\text{K)}\ \ [2\ (0.4\ \text{m})\ (1\ \text{m})]\ (50°\text{C} - 20°\text{C}) = 1.89 \times 10^4\ \text{W}$$

(b) The initial Biot number for the aluminum sheet is

$$Bi = \frac{\bar{h}_{ci}\ S}{2K_{al}} = \frac{2169\ \text{W/(m}^2\text{K)}\ (0.002\ \text{m})}{2\ \ 238\text{W/(m K)}} = 0.009 << 0.1$$

Therefore, the internal thermal resistance of the aluminum sheet is negligible during the entire cool down and the temperature-time history of the sheet is given by Equation (3.3)

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(\frac{-\bar{h}_c\ A_s}{c\,\rho V}t\right)$$

Solving for the time

$$t = \frac{c\,\rho V}{\bar{h}_c\ A_s}\ \ln\left(\frac{T_o - T_\infty}{T - T_\infty}\right) \cong \frac{c\,\rho s}{2\bar{h}_c}\ \ln\left(\frac{T_o - T_\infty}{T - T_\infty}\right)$$

Using the average of the initial and final heat transfer coefficients

$$t = \frac{896\ \text{J/(kg K)}\ \ 2702\ \text{kg/m}^2\ \ (0.002\ \text{m})}{2\ \ 1478\ \text{W/(m}^2\text{K)}\ \ \text{J/(W s)}}\ \ln\left(\frac{150°\text{C} - 20°\text{C}}{50°\text{C} - 20°\text{C}}\right) = 2.4\ \text{s}$$

PROBLEM 8.18

A 0.1-cm-thick square, flat copper plate, 2.5 m × 2.5 m is to be cooled in a vertical position. The initial temperature of the plate is 90°C with the ambient fluid at 30°C. The fluid medium is either atmospheric air or water.

(a) Calculate the Grashof number
(b) Determine the initial heat transfer coefficient
(c) Calculate the initial rate of heat transfer by convection
(d) Estimate the initial rate of temperature change for the plate

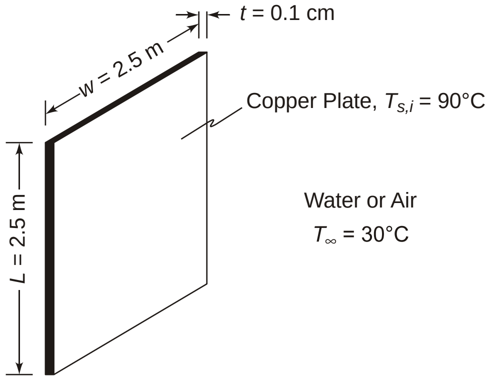
GIVEN

- A vertical flat copper plate in either air or water
- Plate thickness (t) = 0.1 cm = 0.001 m
- Plate dimensions ($L \times w$) = 2.5 m × 2.5 m
- Initial plate temperature ($T_{s,i}$) = 90°C
- Ambient fluid temperature (T_∞) = 30°C

FIND

(a) The Grashof number (Gr)
(b) The rate of heat transfer by convection (q_c)
(c) The initial rate of temperature change $(dT/dt)_{t=0}$

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, for copper

Density (ρ_c) = 8933 kg/m^3

Specific heat (c_c) = 383 J/(kg K)

From Appendix 2, Tables 13 and 28

	Water at 60°C	Air at 60°C
Thermal conductivity, k (W/(m K))	0.657	0.0279
Thermal expansion coefficient, β (1/K)	0.00052	0.003
Kinematic Viscosity $\nu \times 10^6$ m^2/s	0.480	19.4
Prandtl number, Pr	3.02	0.71

SOLUTION

(a) The Grashof number is defined as

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)\,L^3}{\nu^2}$$

$$\text{For water: } Gr_L = \frac{(9.8\,\text{m/s}^2)\ 0.00052\,1/\text{K}\ (90\,°\text{C} - 30\,°\text{C})(2.5\,\text{m})^3}{4.480\times 10^{-6}\,\text{m}^2/\text{s}^{\ 2}} = 2.07 \times 10^{13}$$

$$\text{For air: } Gr_L = \frac{(9.8\,\text{m/s}^2)\ 0.003\,1/\text{K}\ (90\,°\text{C} - 30\,°\text{C})(2.5\,\text{m})^3}{19.4\times 10^{-6}\,\text{m}^2/\text{s}^{\ 2}} = 7.32 \times 10^{10}$$

(b) The average heat transfer coefficient is given by Equation (8.13) (turbulent)

$$\bar{h}_c = 0.13\ \frac{k}{L}(Gr_L Pr)^{\frac{1}{3}}$$

$$\text{For water: } \bar{h}_c = 0.13\ \frac{0.657\ \text{W/(m K)}}{2.5\,\text{m}}[2.07\times 10^{13}(3.02)]^{\frac{1}{3}} = 1356\ \text{W/(m}^2\text{K)}$$

$$\text{For air: } \bar{h}_c = 0.13\ \frac{0.0279\ \text{W/(m K)}}{2.5\,\text{m}}[7.32\times 10^{10}(0.71)]^{\frac{1}{3}} = 5.4\ \text{W/(m}^2\text{K)}$$

The rate of convective heat transfer is given by Equation (1.10)

$$q_c = \bar{h}_c\ A\ \Delta T$$

$$\text{For water: } q_c = 1356\ \text{W/(m}^2\text{K)}\quad [2\ (2.5\ \text{m})^2]\ (90°\text{C} - 30°\text{C}) = 1.017 \times 10^6\ \text{W}$$

$$\text{For air: } q_c = 5.4\ \text{W/(m}^2\text{K)}\quad [2\ (2.5\ \text{m})^2]\ (90°\text{C} - 30°\text{C}) = 4050\ \text{W}$$

(c) Since the sheet is very thin and the thermal conductivity of copper is very high, it is safe to assume that the Biot number is less than 0.1 for both cases. The initial rate of temperature change is given by

$$\left(\frac{dT}{dt}\right)_{t=0} = \frac{q_c}{m\,c} = \frac{q_c}{\rho V\,c} = \frac{q_c}{\rho c\,LW\,t}$$

$$\text{For water: } \left(\frac{dT}{dt}\right)_{t=0} = \frac{1.017\times 10^6\,\text{W}\ \ \text{J/(W s)}}{(8933\,\text{kg/m}^3)\ \ 383\,\text{J/(kg K)}\ \ (2.5\,\text{m})(2.5\,\text{m})(0.001\,\text{m})} = 47.6\ \text{K/s}$$

$$\text{For air: } \left(\frac{dT}{dt}\right)_{t=0} = \frac{4050\ \text{W}\ \ \text{J/(W s)}}{(8933\,\text{kg/m}^3)\ \ 383\,\text{J/(kg K)}\ \ (2.5\,\text{m})(2.5\,\text{m})(0.001\,\text{m})} = 0.19\ \text{K/s}$$

COMMENTS

The initial cooling rate in water is about 250 times that in air.

PROBLEM 8.19

A laboratory apparatus is used to maintain a horizontal slab of ice at -2.2°C so that specimens can be prepared on the surface of the ice and kept close to 0°C. If the ice is 10 cm by 3.8 cm and the laboratory is kept at 16°C, find the cooling rate in watts that the apparatus must provide to the ice.

GIVEN

- A slab of ice in a laboratory
- Ice temperature (T_i) = -2.2°C
- Ice dimensions: 10 cm by 3.8 cm
- Ambient temperature (T_∞) = 16°C

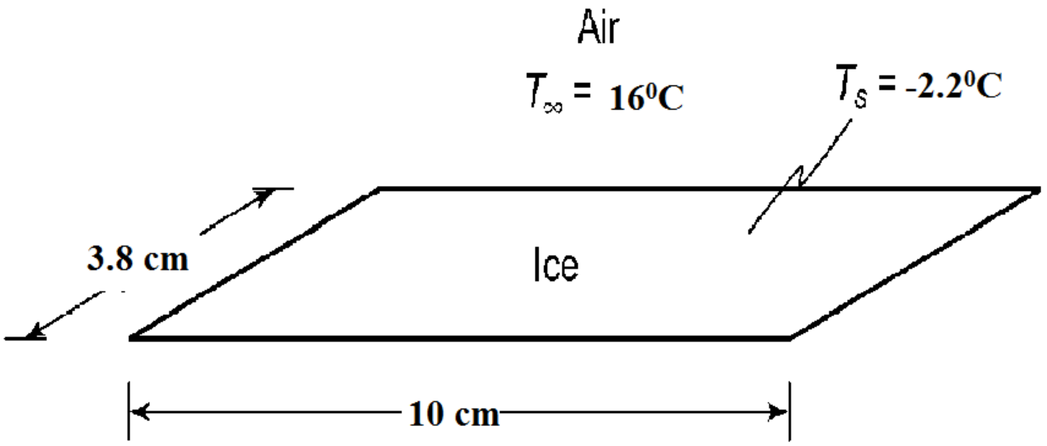
FIND

- The cooling rate (q) in watts

ASSUMPTIONS

- Air in the laboratory is still
- Effects of sublimation are negligible
- Effects of moisture in the air are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 6.9°C

Thermal expansion coefficient (β) = 0.0036 1/K
Thermal conductivity (k) = 0.0242 W/(m K)
Kinematic viscosity (ν) = 14.51*10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

The characteristic length for the ice is

$$L = \frac{A}{P} = \frac{(10\text{ cm})(3.8\,cm)}{2(10\,cm + 3.8\,cm)} = 1.37\text{ cm} = 0.0137\text{ m}$$

The Grashof and Rayleigh numbers based on this length are

$$Gr_L = \frac{g\,\beta(T_\infty - T_i)L^3}{\nu^2} = \frac{(9.8\ m/s^2)(0.003\,6/K)(16\,°C + 2.2\,°C)(0.0137\,m)^3}{\left(1.451\times10^{-5}\,m^2/\ s\right)^2} = 7841.8$$

$$Ra_L = Gr_L\,Pr = 5946\ (0.71) = 5568 \cong 5.6\times10^3$$

This is a case with very low Rayleigh number, which is below 10^5 by two orders of magnitude ($Ra_L = 5.6\times10^3$), and hence poses a problem for estimating the heat transfer coefficient. For $Ra_L \geq 10^5$, the natural convection heat transfer coefficient can be determined from the Nu_L value predicted by Eq. (8.17) [also see Table 8.2]. However, that is not the case here and such situations are indeed encountered in engineering practice. Thus an engineering solution can be obtained by extrapolating and extending the applicability of Eq. (8.17) so as to use it to predict the Nusselt number even for the low Rayleigh number case, and hence an approximation can be obtained as follows:

$$Nu_L = 0.27\ Ra_L^{\frac{1}{4}} = 0.27\ (5568)^{\frac{1}{4}} = 2.33$$

$$\bar{h}_c = Nu_L\ \frac{k}{L} = 2.33\ \frac{\left(0.0242\ W/(m\,K)\right)}{0.0137m} = 4.11\ \ W/(m^2K)$$

The cooling load is

$$q = \bar{h}_c\ A\ (T_\infty - T_i) = \left(4.11\,W/(m^2\ K)\right)(0.1\ m)\,(0.038\,m)\ (16°C - (-2.2°C) = 0.28\ Watt$$

$$q = 0.28\ W$$

PROBLEM 8.20

An electronic circuit board is the shape of a flat plate 0.3 m × 0.3 m in plan-form and dissipates 15 W. It is placed in operation on an insulated surface in a horizontal position or at an angle of 45 degrees to horizontal, in both cases; it is in still air at 25°C. If the circuit fails above 60°C, determine if the two proposed installations are safe.

GIVEN

- A flat plate with insulated back, horizontal or at an angle of 45 degrees in still air
- Plate size ($s \times s$) = 0.3 m × 0.3 m
- Heat generation rate (q_G) = 15 W
- Air temperature (T_∞) = 25°C
- Maximum plate temperature (T_s) = 60°C

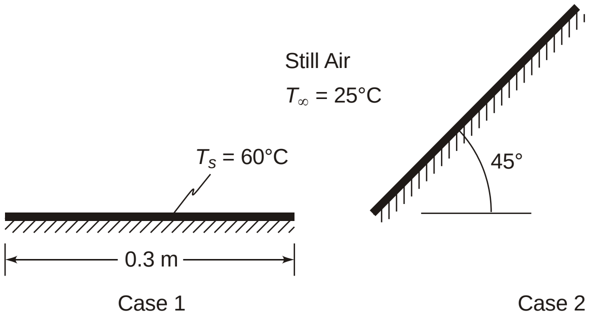
FIND

- If the two plate positions are safe

ASSUMPTIONS

- Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 43°C

Thermal expansion coefficient (β) = 0.00316 1/K

Thermal conductivity (k) = 0.0267 W/(m K)

Kinematic viscosity (ν) = 17.9×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

For the horizontal case, the characteristic length

$$L = \frac{A}{P} = \frac{s^2}{4s} = \frac{s}{4} = \frac{0.3\text{m}}{4} = 0.075 \text{ m}$$

The Grashof and Rayleigh numbers for the flat case at the maximum operating temperature are

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ \ 0.00316\ 1/\text{K}\ \ (60°\text{C} - 25°\text{C})(0.075\,\text{m})^3}{17.9\times10^{-6}\text{m}^2/\text{s}\ \ ^2} = 1.83 \times 10^6$$

$$Ra_L = Gr_L\, Pr = 1.83 \times 10^6\ (0.71) = 1.30 \times 10^6$$

For the inclined case, the characteristic length is the length of the inclined side (0.3 m), the Grashof number for the inclined case is

$$Gr_L = \frac{(9.8\ \text{m/s}^2)\ 0.00316\ 1/\text{K}\ (60°\text{C} - 25°\text{C})(0.3\text{m})^3}{\left(17.9 \times 10^{-6}\ \text{m}^2/\text{s}\right)^2} = 1.17 \times 10^8$$

Case 1: The average Nusselt number is given by Equation (8.15)

$$\overline{Nu} = 0.54\ Ra_L^{\frac{1}{4}} = 0.54\ (1.30 \times 10^6)^{\frac{1}{4}} = 18.23$$

$$\bar{h}_c = \overline{Nu}\ \frac{k}{L} = 18.23\ \frac{0.0267\ \text{W/(m K)}}{0.075\,\text{m}} = 6.49\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer from the plate at T_s = 60°C is

$$q = \bar{h}_c\ A\ (T_s - T_\infty) = 6.49\ \text{W/(m}^2\text{K)}\ (0.3\ \text{m})^2\ (60°\text{C} - 25°\text{C}) = 26.3\ \text{W}$$

Since this is larger than the heat generation rate, the actual surface temperature will be less than 60°C. Case 1 configuration is safe.

Case 2

$$Gr_L\, Pr \cos\theta = 1.17 \times 10^8\ (0.71) \cos(45°) = 5.87 \times 10^7$$

Therefore, the average heat transfer coefficient is given by Equation (8.14)

$$\bar{h}_c = 0.56\ \frac{k}{L}\,(Gr_L Pr \cos\theta)^{\frac{1}{4}} = 0.56\ \frac{0.0267\ \text{W/(m K)}}{0.3\,\text{m}}(5.87 \times 10^7)^{\frac{1}{4}} = 4.36\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer when T_s = 60°C is

$$q = 4.36\ \text{W/(m}^2\text{K)}\ (0.3\ \text{m})^2\ (60°\text{C} - 25°\text{C}) = 17.7\ \text{W}$$

Since this is also greater than the heat generation rate, the actual temperature will be less than 60°C. Therefore, Case 2 is also safe.

PROBLEM 8.21

Cooled air is flowing through a long sheet metal air conditioning duct, 0.2 m high and 0.3 m wide. If the duct temperature is 10°C and passes through a crawl space under a house at 30°C, estimate

(a) The heat transfer rate to the cooled air per meter length of duct.

(b) The additional air conditioning load if the duct is 20-m-long.

(c) Discuss qualitatively the energy conservation if the duct were insulated with glass wool.

GIVEN

- An air conditioning duct in a crawl space
- Duct height (H) = 0.2 m
- Duct width (w) = 0.3 m
- Duct temperature (T_s) = 10°C
- Ambient temperature (T_∞) = 30°C

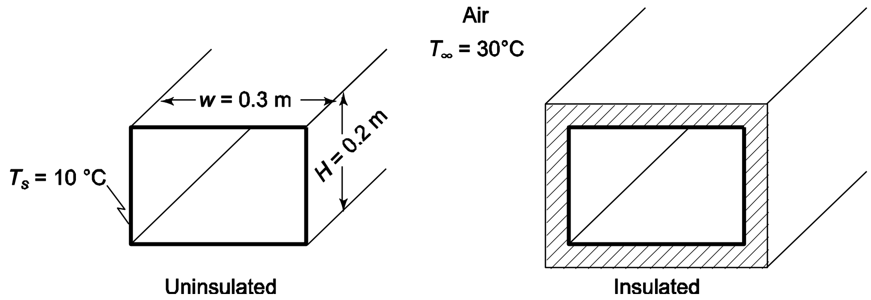
FIND

(a) The heat transfer rate per meter length (q_c/L) to the cooled air in the duct
(b) The additional air conditioning load (q_{20}) if the duct length (L) = 20 m
(c) Discuss qualitatively the energy conservation if the duct were insulated with glass wool

ASSUMPTIONS

- Ambient air is still
- Duct temperature is constant and uniform
- Radiation is negligible
- Edge effects are negligible
- No condensation on the duct surface

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 20°C

Thermal expansion coefficient (β) = 0.00341 1/K

Thermal conductivity (k) = 0.0251 W/(m K)

Kinematic viscosity (ν) = 15.7×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

(a) The duct can be thought of as two vertical and two horizontal cooled flat plates.
For the sides

$$Gr_H = \frac{g\,\beta(T_s - T_\infty)H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ 0.00341\ 1/\text{K}\ (30\,°\text{C} - 10\,°\text{C})(0.2\,\text{m})^3}{\left(15.7\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 2.17 \times 10^7 < 10^9$$

So the flow is laminar.

For the top and bottom, the characteristic length (L_c) is given by

$L_c = A/P = Lw/(2L + 2w)$. Since $L >> w$: $L_c \approx w/2 = 0.15$ m

$$Gr_{Lc} = \frac{g\,\beta(T_s - T_\infty)L_c^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ 0.00341\ 1/\text{K}\ (30\,°\text{C} - 10\,°\text{C})(0.15\,\text{m})^3}{\left(15.7\times10^{-6}\ \text{m}^2/\text{s}\right)^2} = 9.15 \times 10^6 < 10^7$$

The heat transfer coefficient for the vertical sides of the duct is given by Equation (8.12a)

$$\bar{h}_{cs} = 0.68\ Pr^{\frac{1}{2}} \frac{Gr_H^{\frac{1}{4}}}{(0.952 + Pr)^{\frac{1}{4}}} \frac{k}{H} = 0.68\ (0.71)^{\frac{1}{4}} \frac{(2.17\times10^7)^{\frac{1}{4}}}{(0.952 + 0.71)^{\frac{1}{4}}} \frac{0.0251\,\text{W/(m K)}}{0.2\,\text{m}} = 4.32\ \text{W/(m}^2\text{K)}$$

The top is a cooled surface facing upward. The heat transfer coefficient from the top is the same as that for a heated surface facing downward and given by Equation (8.17)

$$\bar{h}_{ct} = 0.27\ \frac{k}{L_c}(Gr_L Pr)^{\frac{1}{4}} = 0.27\ \frac{0.0251\,\text{W/(m K)}}{0.15\,\text{m}}[9.15\times10^6\ (0.71)]^{\frac{1}{4}} = 2.28\ \text{W/(m}^2\text{K)}$$

The heat transfer coefficient for the bottom, a cooled surface facing downward, is given by Equation (8.15) since $Ra_L < 10^7$

$$\bar{h}_{cb} = 0.54\ \frac{k}{w}(Gr_{L_c} Pr)^{\frac{1}{4}} = 0.54\ \frac{0.0251\,\text{W/(m K)}}{0.15\,\text{m}}[9.15\times10^6\ (0.71)]^{\frac{1}{4}} = 4.56\ \text{W/(m}^2\text{K)}$$

The total convective heat transfer to the duct is

$$q_c = [2\bar{h}_{cs}\,H\,L + (\bar{h}_{ct} + \bar{h}_{cb})\,w\,L]\ (T_\infty - T_s)$$

$$\frac{q_c}{L} = 2\ 4.32\ \text{W/(m}^2\text{K)}\ (0.2\,\text{m}) + 2.28\ \text{W/(m}^2\text{K)} + 4.56\ \text{W/(m}^2\text{K)}\ 0.3\ \text{m}\ (30°\text{C} - 10°\text{C}) = 75.6\ \text{W/m}$$

(b) For a 20-m-long duct

$$q_c = \left(\frac{q_c}{L}\right) L = 75.6\ \text{W/m (20 m)} = 1512\ \text{W}$$

(c) The addition of insulation to the outer surface of the duct will have several effects

1. It will increase the outer surface temperature of the duct and decrease the duct wall temperature.
2. The higher surface temperature will lower the natural convection heat transfer coefficient because the temperature difference between the duct and the ambient air will be reduced.
3. The lower convective heat transfer coefficient and the additional conductive thermal resistance of the insulation will lead to a decrease in the rate of heat transfer to the air in the duct. This will reduce the load on the air conditioning system assuming that the crawl space is not to be intentionally cooled.

PROBLEM 8.22

Solar radiation at 600 W/m^2 is absorbed by a black roof inclined at 30°C as shown. If the underside of the roof is well insulated, estimate the maximum roof temperature in T_{inf}= 20°C air.

GIVEN

- Inclined roof, well insulated on the underside
- Incline angle (θ) = 30 degrees
- Air temperature = 20°C
- Solar radiation absorbed (q_s) = 600 W/m^2

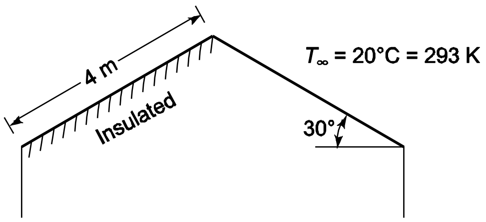
FIND

- The maximum roof temperature

ASSUMPTIONS

- The roof behaves as a black body (ε = 1.0)
- The sky behaves as a black body at 0 K

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5,

The Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4).

SOLUTION

The maximum roof temperature will occur when the air is quiescent. Since the air properties must be evaluated at the mean of the ambient and surface temperatures, as iterative procedure must be used.

Iteration #1

Let T_s = 60°C = 333 K

From Appendix 2, Table 28, for dry air at the mean temperature of 40°C

Thermal expansion coefficient (β) = 0.00319 1/K

Thermal conductivity (k) = 0.0265 W/(m K)

Kinematic viscosity (ν) = 17.6×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The Grashof number is

$$Gr_L = \frac{g\,\beta (T_s - T_\infty) L^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ 0.00319\ 1/\text{K}\ \ (333\text{K} - 293\text{K})(4\,\text{m})^3}{\left(17.6 \times 10^{-6}\ \text{m}^2/\text{s}\right)^2} = 2.58 \times 10^{11}$$

$$Gr_L\, Pr \cos\theta = 2.58 \times 10^{11}\ (0.71) \cos(30°) = 1.59 \times 10^{11}$$

The average convective heat transfer coefficient for this geometry is given by Equation (8.14). Although $Gr_L\ Pr\ \cos\ \theta$ is slightly larger than 10^{11}, Equation (8.14) will be extrapolated by this problem

$$\bar{h}_c = 0.56\ \frac{k}{L}\,(Gr_L\,Pr\cos\theta)^{\frac{1}{4}} = 0.56\ \frac{0.0265\ \text{W/(m K)}}{4\,\text{m}}\,(1.59\times 10^{11})^{\frac{1}{4}} = 2.34\ \text{W/(m}^2\ \text{K)}$$

For steady state, the solar gain must equal the convective and radiative losses

$$\frac{q_s}{A} = \bar{h}_c\ (T_s - T_\infty) + \sigma T_s^4$$

$$600\ \text{W/m}^2 = 2.34\ \text{W/(m}^2\text{K)}\ \ (T_s - 293\ \text{K}) + 5.67\times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)\ (T_s^4)$$

Checking the units then eliminating them for clarity

$$5.67\times 10^{-8}\ T_s^4 + 2.34\ T_s - 1286 = 0$$

By trial and error: T_s = 314 K.

Repeating this procedure for another iteration

T_s = 314 K	$Gr_L\ Pr\ \cos\ \theta = 9.58\times 10^{10}$
T_{mean} = 304 K = 30°C	
β = 0.0033 1/K	h_c = 2.01 W/(m² K)
k = 0.0258 W/(m² K)	T_s = 315 K
$\nu = 16.7\times 10^{-6}$ m²/s	
Pr= 0.71	

The maximum roof temperature: (T_s) = 315 K = 42°C

COMMENTS

The procedure converges quickly because of the 1/4 power in the Nusselt number correlation.

PROBLEM 8.23

A 1-m-square copper plate is placed horizontally on 2-m-high legs. The plate has been coated with a material that provides a solar absorptivity of 0.9 and an infrared emissivity of 0.25. If the air temperature is 30°C, determine the equilibrium temperature on an average clear day in which the solar radiation incident on a horizontal surface is 850 W/m^2.

GIVEN

- A horizontal copper plate in air
- Plate dimensions ($s \times s$) = 1 m × 1 m
- Solar absorptance (α_s) = 0.9
- Infrared emittance (ε) = 0.25
- Air temperature (T_∞) = 30°C = 303 K
- Incident solar radiation (q_s/A) = 850 W/m^2

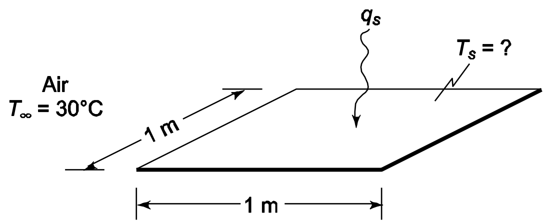
FIND

- Equilibrium Temperature (T_s)

ASSUMPTIONS

- The sky behaves as a black body at 0 K
- The effect of the legs is negligible
- Air is quiescent
- Radiative heat transfer from the bottom of the plate is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5,

The Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4).

SOLUTION

Since the air properties must be evaluated at the mean of the surface and ambient temperatures, an iterative process must be used

1. Guess at the surface temperature.
2. Evaluate the air properties and calculate the Grashof number.
3. Use the appropriate correlation to find the average convective heat transfer coefficients on the top and bottom of the plate.
4. Calculate a new surface temperature.

This process must be repeated until the temperature converges within an acceptable tolerance.

Iteration #1

1. Let T_s = 90°C = 363 K
2. From Appendix 2, Table 28, for dry air at he mean temperature of (60°C)
   Thermal expansion coefficient (β) = 0.00300 1/K
   Thermal conductivity (k) = 0.0279 W/(m K)
   Kinematic viscosity (ν) = 19.4 × 10^{-6} m^2/s
   Prandtl number (Pr) = 0.71

The characteristic length of the plate (L) = A/P = (1 m^2)/(4 m) = 0.25 m
The Grashof and Rayleigh numbers based on the characteristic length are

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)\,L^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\,(0.0031/\text{K})\,(90°\text{C} - 30°\text{C})\,(0.25\,\text{m})^3}{19.4 \times 10^{-6}\,\text{m}^2/\text{s}\ ^2} = 7.32 \times 10^7$$

$$Ra_L = Gr_L\,Pr = 7.32 \times 10^7\,(0.71) = 5.20 \times 10^7$$

3. For the top of the plate, Equation (8.16) gives the average Nusselt number

$$\overline{Nu} = 0.15\ Ra^{\frac{1}{3}} = 0.15\ (5.20 \times 10^7)^{\frac{1}{3}} = 56.00$$

$$\bar{h}_{ct} = \overline{Nu}\,\frac{k}{L} = 56.00\ \frac{0.0279\ \text{W/(m K)}}{0.25\,\text{m}} = 6.25\ \text{W/(m}^2\ \text{K)}$$

For the bottom of the plate, Equation (8.17) gives the average Nusselt number

$$\overline{Nu} = 0.27\ (Ra_L)^{\frac{1}{4}} = 0.27\ (5.20 \times 10^7)^{\frac{1}{4}} = 22.9$$

$$\bar{h}_{cb} = \overline{Nu}\,\frac{k}{L} = 22.9\ \frac{0.0279\ \text{W/(m K)}}{0.25\,\text{m}} = 2.56\ \text{W/(m}^2\ \text{K)}$$

4. For equilibrium, the rate of solar gain must equal the total rate of convective heat transfer from top and bottom and radiative heat transfer from the top surface.

$$\alpha\ \frac{q_s}{A} = (\bar{h}_{ct} + \bar{h}_{cb})\ (T_s - T_\infty) + \frac{1}{2}\ \varepsilon\,\sigma\,(T_s^4 - T_{\text{sky}}^4)$$

$$0.9\ \ 850\ \text{W/m}^2 = [(6.25 + 2.56)\ \text{W/(m}^2\ \text{K)}]\ (T_s - 303\ \text{K}) + 0.25\ \ 5.67 \times 10^{-8}\,\text{W/(m}^2\ \text{K}^4)\ \ (T_s^4 - 0)$$

Checking the units then eliminating them for clarity

$$1.43 \times 10^{-8}\ T_s^4 + 8.81\ T_s - 3434 = 0$$

By trial and error: T_s = 362 K = 89°C.

Since this is very close to the initial guess, another iteration is not necessary. The equilibrium temperature is about 89°C.

COMMENTS

The coating on the plate is called a selective surface and is often used in solar applications for decreasing reradiation losses from absorbers.

PROBLEM 8.24

A 2.5 × 2.5 m steel sheet 1.5-mm-thick is removed from an annealing oven at a uniform temperature of 425°C and placed in a large room at 20°C in a horizontal position. (a) Calculate the rate of heat transfer from the steel sheet immediately after its removal from the furnace, considering both radiation and convection. (b) Determine the time required for the steel sheet to cool to a temperature of 60°C. Hint: This will require numerical integration.

GIVEN

- Horizontal steel sheet in air
- Sheet dimensions = 2.5 m × 2.5 m × 0.0015 m
- Sheet initial temperature (T_{si}) = 425°C = 698 K
- Air temperature (T_∞) = 20°C = 293 K

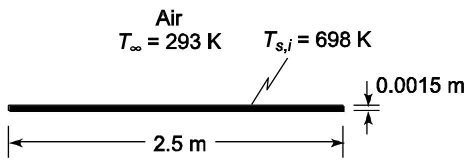
FIND

(a) The initial rate of heat transfer (q)
(b) The time required for the sheet to cool to 60°C (333 K)

ASSUMPTIONS

- The room behaves as a black body at T_∞
- The steel sheet behaves as a black body (ε = 1.0)
- Heat transfer takes place from both top and bottom of the sheet
- The steel is 1% carbon steel
- Heat transfer from the edges of the plate is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5,

The Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

From Appendix 2, Table 10, for 1% carbon steel

Specific heat (c_s) = 473 J/(kg K)

Density (ρ_s) = 7801 kg/m^3

From Appendix 2, Table 28, for dry air at the mean temperature of 496 K (223°C)

Thermal expansion coefficient (β) = 0.00203 1/K

Thermal conductivity (k) = 0.0384 W/(m K)

Kinematic viscosity (ν) = 38.7 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

(a) The characteristic length for this geometry is

$$L = \frac{A}{P} = \frac{S^2}{4S} = \frac{S}{4} = \frac{2.5\,\text{m}}{4} = 0.625\text{ m}$$

The Grashof and Rayleigh numbers are

$$Gr_L = \frac{g\,\beta\,(T_s - T_\infty)\,L^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.00203\,1/\text{K}\ \ (698\,\text{K} - 293\,\text{K})\,(0.625\,\text{m})^3}{38.7 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.31 \times 10^9$$

$$Ra_L = Gr_L\,Pr = 1.31 \times 10^9\,(0.71) = 9.33 \times 10^8$$

The average Nusselt number on the bottom of the plate is given by Equation (8.17)

$$\overline{Nu} = 0.27\ (Ra_L)^{\frac{1}{4}} = 0.27\ (9.33 \times 10^8)^{\frac{1}{4}} = 47.2$$

$$\bar{h}_{ct} = \overline{Nu}\,\frac{k}{L} = 47.2\ \frac{0.0384\ \text{W/(m K)}}{0.625\,\text{m}} = 2.90\ \ \text{W/(m}^2\,\text{K)}$$

The average Nusselt number on the top of the plate is given by Equation (8.16)

$$\overline{Nu} = 0.15\ Ra^{\frac{1}{3}} = 0.15\ (9.33 \times 10^8)^{\frac{1}{3}} = 146.6$$

$$\bar{h}_c = \overline{Nu}\,\frac{k}{L} = 146.6\ \frac{0.0384\ \text{W/(m K)}}{0.625\,\text{m}} = 9.01\ \ \text{W/(m}^2\,\text{K)}$$

The total rate of heat transfer is the sum of the convective and radiative components

$$q_{\text{total}} = (\bar{h}_{ct} + \bar{h}_{cb})\ A\,(T_s - T_\infty) + 2\,A\ \sigma\,(T_s^4 - T_\infty^4)$$

$$q_{\text{total}} = (9.01 + 2.90)\ \text{W/(m}^2\text{K)}\ \ (2.5\ \text{m})^2\,(698\ \text{K} - 293\ \text{K}) + 2\,(2.5\ \text{m})^2$$

$$5.67 \times 10^{-8}\,\text{W/(m}^2\,\text{k}^4)\quad [(698\ \text{K})^4 - (293\ \text{K})^4]$$

$$q_{\text{total}} = 1.94 \times 10^5\ \text{W}$$

(b) As the plate cools, the rate of heat transfer will decrease. The cooling time will be estimated by calculating a new sheet temperature and heat transfer every time period.

The Biot number for the sheet is

$$Bi = \frac{\bar{h}\,s}{2\,k} = \frac{9.01\,\text{W/(m}^2\text{K)}\ \ (0.0015\,\text{m})}{2\ \ 0.0384\ \text{W/(m K)}} = 0.176$$

This is slightly above 0.1. For a first order approximation, we can neglect the thermal resistance in the plate. For the first 20 second interval

$$\text{Total energy loss, } q_{\text{total}}\ (20\text{s}) = \text{m c } \Delta\text{T} = V\,\rho\,c\ (T_{s,i} - T_{s,\,20})$$

Solving for temperature after 20 sec

$$T_{s,20} = T_{s,i} - \frac{q_{\text{total}}\ (20\,\text{s})}{V\,\rho c} = 698\ \text{K} - \frac{1.94 \times 10^5\,\text{W}\ \ \text{J/(W s)}\ \ (20\,\text{s})}{0.0015\,\text{m}\,(2.5\,\text{m})^2\ \ 7801\,\text{kg/m}^3\ \ \ 473\,\text{J/(kg K)}} = 586\ \text{K}$$

This temperature is then used to calculate new transfer coefficients and heat transfer rates as shown above. This procedure is followed until the temperature of the plate is 333 K.

Time (s)	20	40	80	120
T_s (K)	586	528	451	411

T_{mean} (K)	440	411	372	352
β (1/K)	0.00230	0.00244	0.00268	0.00284
k (W/m K)	0.0349	0.0332	0.0307	0.0292
$\nu \times 10^6$ (m²/s)	31.5	28.3	23.6	21.4
Pr	0.71	0.71	0.71	0.71
$Ra_L \times 10^{-9}$	1.15	1.22	1.29	1.24
h_{cb} (W/(m² K))	2.78	2.67	2.51	2.37
h_{ct} (W/(m² K))	8.78	8.50	8.02	7.52
$q_{total} \times 10^{-4}$ (W)	9.99	6.65	3.46	2.24

Time (s)	200	300
T_s (K)	359	330
T_{mean}	326	
β (1/K)	0.00306	
k (W/m K)	0.0267	
$\nu \times 10^6$ (m²/s)	17.9	
Pr	0.71	
$Ra_L \times 10^{-9}$	1.07	
h_{cb} ((W/(m² K))	2.09	
h_{ct} ((W/(m² K))	6.56	
$q_{total} \times 10^{-4}$ (W)	1.01	

Interpolating between 200 and 300 seconds

The time required to reach 333 K is approximately 290 seconds = 4.8 min.

PROBLEM 8.25

A thin electronic circuit board, 0.1 m by 0.1 m in size, is to be cooled in air at 25°C. The board is placed in a vertical position and the back side is well insulated. If the heat dissipation is uniform at 200 W/m², determine the average temperature of the surface of the board cover.

GIVEN

- Vertical circuit board in air
- Back is well insulated
- Board dimensions ($L \times H$) = 0.1 m × 0.1 m
- Air temperature (T_∞) = 25°C
- Uniform heat dissipation rate ($\dot{q}_G/A$) = 200 W/m²

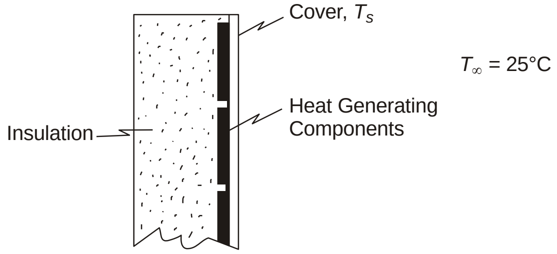
FIND

- The average temperature of the surface of the board (T_s)

ASSUMPTIONS

- Ambient air is still
- The board has reached steady state
- Radiation is negligible

SKETCH



SOLUTION

Since the fluid properties must be evaluated at the average of the surface and ambient temperatures, an iterative procedure is required to calculate the average surface temperature of the cover. For the first iteration, let T_s = 55°C

From Appendix 2, Table 28, for dry air at the mean temperature of 40°C

Thermal expansion coefficient (β) = 0.00319 1/K

Thermal conductivity (k) = 0.0265 W/(m K)

Kinematic viscosity (ν) = 17.6 × 10⁻⁶ m²/s

Prandtl number (Pr) = 0.71

The Grashof number is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\ \mathrm{m/s^2})\ 0.00319\ 1/\mathrm{K}\ (55°\mathrm{C} - 25°\mathrm{C})\,(0.1\mathrm{m})^3}{17.6 \times 10^{-6}\,\mathrm{m^2/s}} = 3.03 \times 10^6 < 10^9$$

For a laminar boundary layer, Equation (8.12a) gives the average heat transfer coefficient

$$\bar{h}_c = 0.68\ Pr^{\frac{1}{2}}\ \frac{Gr_H^{\frac{1}{4}}}{(0.952 + Pr)^{\frac{1}{4}}}\frac{k}{H} = 0.68\ (0.71)^{\frac{1}{2}}\ \frac{(3.03\times 10^6)^{\frac{1}{4}}}{(0.952 + 0.71)^{\frac{1}{4}}}\ \frac{0.0265\ \mathrm{W/(m\,K)}}{0.1\mathrm{m}} = 6.07\ \mathrm{W/(m^2\,K)}$$

The rate of heat generation must equal the rate of convection heat transfer for steady state

$$\frac{\dot{q}_G}{A} = \frac{q_c}{A} = \bar{h}_c \ (T_s - T_\infty)$$

Solving for the surface temperature

$$T_s = T_\infty + \frac{\dot{q}_G}{A} = 25°\text{C} + \frac{200\ (\text{W/m}^2)}{6.07\ \text{W/(m}^2\text{K)}} = 57.9°\text{C}$$

For the second try, let T_s = (55°C + 57.9°C/2 = 56.5°C, i.e. halfway between the first guess and the result of the first interation. This gives T_{mean} = 40.7°C so the property values will change very little. For the second iteration.

T_s = 56.5°C
Mean Temp. = 40.7°C
β = 0.00319 1/K
k = 0.0265 W/(m K)
$\nu = 17.6 \times 10^{-6}$ (m^2/s)
Pr = 0.71
$Gr_L = 3.17 \times 10^6$
$\bar{h}_c$ = 6.15 W/(m^2 K)
T_s = 57.5°C

Therefore, the average surface temperature is about 58°C.

PROBLEM 8.26

A plot of coffee has been allowed to cool to 17°C. If the electrical coffee maker is turned back on, the hot plate on which the pot rests is brought up to 70°C immediately and held at that temperature by a thermostat. Consider the pot to be a vertical cylinder 130 mm in diameter and the depth of coffee in the pot to be 100 mm. Neglect heat losses from the sides and top of the pot. How long will it take before the coffee is drinkable (50°C)? How much did it cost to heat the coffee if electricity costs \$0.05 per kilowatt-hour?

GIVEN

- Coffee pot, idealized as a vertical cylinder, on a hot plate
- Initial temperature of the pot and coffee ($T_{s,i}$) = 17°C
- Hot plate temperature (T_{hp}) = 70°C (constant)
- Pot diameter (D) = 130 mm = 0.13 m
- Depth of coffee (δ) = 100 mm = 0.1 m

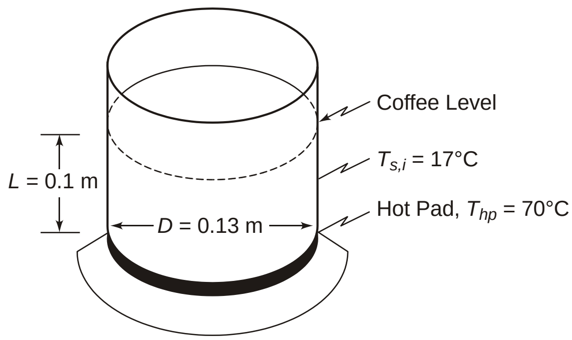
FIND

(a) Time for the coffee to reach 50°C
(b) Cost to heat the coffee if electricity costs \$0.05/kWh

ASSUMPTIONS

- Heat losses from the sides and the top are negligible
- All energy from the hot plate goes into the coffee
- Internal resistance of the coffee is negligible
- Thermal resistance of the bottom of the pot is negligible
- Coffee has the thermal properties of water
- Variation of the thermal properties of the coffee with temperature can be neglected

SKETCH



PROPERTIES AND CONSTANTS

The relevant thermal properties will be evaluated using the average coffee temperature of (17°C + 50°C)/2 = 33.5°C.

From Appendix 2, Table 13, for water

At 33.5°C Density (ρ) = 994.6 kg/m^3
Specific Heat (c) = 4175 J/(kg K)

At the mean temperature of (33.5°C + 70°C)/2 = 51.8°C

Thermal expansion coefficient (β_c) = 0.00047 1/K

Thermal conductivity (k_c) = 0.648 W/(m K)

Kinematic viscosity (ν_c) = 0.549 × 10^{-6} m^2/s

Prandtl number (Pr_c) = 3.5

SOLUTION

(a) The heat transfer coefficient from between the hot plate and the coffee can be evaluated by treating the coffee volume as a horizontal water layer heated from below. The Rayleigh number is

$$Ra_\delta = \frac{g\,\beta(T_{hp} - T_c)\,\delta^3 Pr_c}{\nu_c^2} = \frac{(9.8\,\text{m/s}^2)\;0.00047\,1/\text{K}\;(70°\text{C} - 33.5°\text{C})\,(0.1\text{m})^3(3.5)}{\left(0.549\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 1.95 \times 10^9$$

The Nusselt number for this geometry is given by Equation (8.30b)

$$Nu_\delta = 1 + 1.44\left[1 - \frac{1708}{Ra_\delta}\right]\left[\left(\frac{Re_\delta}{5830}\right)^{\frac{1}{3}} - 1\right] + 2.0\left[\frac{Re_\delta^{\frac{1}{3}}}{140}\right]^{\left[1-\ln\left(Ra_\delta^{\frac{1}{3}}/140\right)\right]}$$

where the notation [ ] indicates that if the quantity inside the bracket is negative, the quantity is to be taken as zero.

$$Nu_\delta = 1 + 1.44\left[1 - \frac{1708}{1.95\times10^9}\right] + \left[\left(\frac{1.95\times10^9}{5830}\right)^{\frac{1}{3}} - 1\right] + 2.0\left[\frac{(1.95\times10^9)^{\frac{1}{3}}}{140}\right]^{\left[1-\ln\left((1.95\times10^9)^{\frac{1}{3}}/140\right)\right]}$$

$$Nu_\delta = 71.0$$

$$h_c = Nu_\delta\,\frac{k_c}{\delta} = 71.0\;\frac{0.648\ \text{W/(m K)}}{0.1\text{m}} = 460\ \ \text{W/(m}^2\,\text{K)}$$

The time required for heating can be calculated from Equation (3.3), solving for the time

$$t = -\frac{c\rho V}{h_c A}\ \ln\left(\frac{T - T_{hp}}{T_o - T_{hp}}\right) = -\frac{c\rho\frac{\pi}{4}D^2\delta}{h_c\frac{\pi}{4}D^2}\ \ln\left(\frac{T - T_{hp}}{T_o - T_{hp}}\right) = -\frac{c\rho\delta}{h_c}\ \ln\left(\frac{T - T_{hp}}{T_o - T_{hp}}\right)$$

$$t_f = \frac{4175\ \text{J/(kg K)}\quad 994.6\ \text{kg/m}^3\ \ (0.1\text{m})}{460\ \text{W/(m}^2\text{K)}\quad \text{J/(W s)}}\ \ln\left[\frac{50°\text{C} - 70°\text{C}}{17°\text{C} - 70°\text{C}}\right] = 880\ \text{s} = 14.7\ \text{min}$$

(b) The total heat transfer from the hot plate during this time is

$$E = \int_0^{t_1} q_t\,dt = \int_0^{t_1} \bar{h}_c\,A_{\text{bottom}}\,[T_{hp} - T(t)]\,dt = h_c\,\frac{\pi}{4}\,D^2 \int_0^{t_1} [T_{hp} - T(t)]\,dt$$

From Equation (3.3)

$$T_{hp} - T(t) = (T_{hp} - T_{si})\,\exp\left(-\frac{\bar{h}_c\,t}{c\rho\delta}\right)$$

Therefore

$$\int_0^{t_1} [T_{hp} - T(t)]\,dt = (T_{hp} - T_{si})\int_0^{t_1} \exp\left(-\frac{\bar{h}_c\,t}{c\rho\delta}\right)dt = (T_{hp} - T_{si})\left(-\frac{c\rho\delta}{h_c}\right)\left[\exp\left(-\frac{\bar{h}_c\,t_f}{c\rho\delta}\right) - 1\right]$$

$$\therefore\ E = -\frac{\pi}{4}\,D^2\,c\,\rho\,\delta\,(T_{hp} - T_{si})\left[\exp\left(-\frac{\bar{h}_c\,t_f}{c\rho\delta}\right) - 1\right]$$

$$E = -\frac{\pi}{4}(0.13\ \text{m})^2 \quad 4175\ \text{J/(kg K)} \quad 994.6\ \text{kg/m}^3 \quad (0.1\ \text{m})$$

$$(70°\text{C} - 17°\text{C}) \left[\exp\left(-\frac{460\ \text{W/(m}^2\text{K)} \ \ (880\,\text{s})}{4175\ \text{J/(kg K)} \quad 994.6\ \text{kg/m}^3 \ \ (0.1\text{m})} \right) - 1 \right]$$

$$E = 181{,}916\ \text{J} \left(\frac{\text{h}}{3600\,\text{s}} \right) \ \ \text{(Ws)/J} \ \ \left(\frac{\text{kW}}{1000\,\text{W}} \right) = 0.051\ \text{kWh}$$

$$\text{Cost} = E \left(\frac{\$0.05}{\text{kWh}} \right) = (0.051\ \text{kWh}) \left(\frac{\$0.05}{\text{kWh}} \right) = \$0.003$$

COMMENTS

The power consumption of the hot plat is about 12.5 watts.
The cost estimate neglects all losses from the hot plate to the ambient air.

PROBLEM 8.27

A laboratory experiment has been performed to determine the natural-convection heat transfer correlation for a horizontal cylinder of elliptical cross section in air. The cylinder is 1-m-long, has a hydraulic diameter of 1 cm, a surface area of 0.0314 m^2, and is heated internally by electrical resistance heating. Recorded data include power dissipation, cylinder surface temperature, and ambient air temperature. The power dissipation has been corrected for radiation effects:

$T_s - T_\infty$ (°C)	q (W)
15.2	**4.60**
40.7	**15.76**
75.8	**34.29**
92.1	**43.74**
127.4	**65.62**

Assume that all air properties may be evaluated at 27°C and determine the constants in the correlation equation: $Nu = C\,(Gr\,Pr)^m$

GIVEN

- A horizontal, elliptical cylinder in air
- Hydraulic diameter (D_h) = 1 cm = 0.01 m
- Length (L) = 1 m
- Cylinder surface area (A_s) = 0.0314 m^2
- Experimental data for ($T_s - T_\infty$) and q shown above

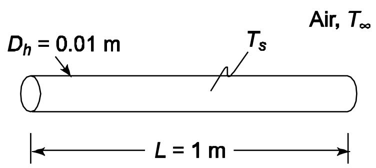
FIND

- The constants in the correlation equation $Nu = C(Gr\,Pr)^m$

ASSUMPTIONS

- All air properties may be evaluated at 27°C

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 27°C

Thermal expansion coefficient (β) = 0.00333 1/K

Thermal conductivity (k) = 0.0256 W/(m K)

Kinematic viscosity (ν) = 16.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Nusselt and Grashof number for the points are given by the following equation

$$\overline{Nu}_D = \frac{h_c D_h}{k} = \frac{D_h}{k}\left[\frac{q}{A_s\,(T_s - T_\infty)}\right]$$

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)\,D_h^3}{\nu^2}$$

Tabulating these and their logarithms for the experimental data

$\overline{Nu}_D$	$(Gr_D\,Pr) \times 10^{-3}$	$\log \overline{Nu}_D$	$\log (Gr_D\,Pr)$
3.76	1.31	0.575	3.11
4.81	3.51	0.682	3.55
5.62	6.53	0.750	3.81
5.91	7.93	0.772	3.90
6.40	10.98	0.806	4.04

Performing a least squares fit on the data yields

$$\log \overline{Nu}_D = 0.250 \log (Gr_D\,Pr) - 0.204$$

or

$$\overline{Nu}_D = 0.63\,(Gr_D\,Pr)^{0.25}$$

PROBLEM 8.28

A long, 2-cm-diameter horizontal copper pipe carries dry saturated steam at 120 kPa absolute pressure. The pipe is contained within an environmental testing chamber in which the ambient air pressure can be adjusted from 50 kPa to 200 kPa absolute, while the ambient air temperature is held constant at 20°C. What is the effect of this pressure change on the rate of condensate flow per meter length of pipe? Assume that the pressure change does not affect the absolute viscosity, thermal conductivity, or specific heat of the air.

GIVEN

- A long horizontal copper pipe carrying saturated steam within an environmental testing chamber
- Outside diameter (D) = 2 cm = 0.02 m
- Steam pressure = 120 kPa
- Ambient pressure range (P) = 50 to 200 kPa
- Ambient air temperature (T_∞) = 20°C

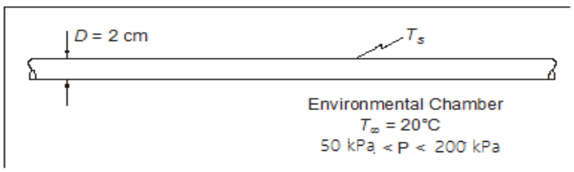
FIND

- Effect of ambient pressure change on rate of condensate flow per meter length of pipe

ASSUMPTIONS

- Pressure change has no effect on absolute viscosity, thermal conductivity, or specific heat of the air
- Air is still
- Chamber temperature is held constant while pressure is changed
- Convective thermal resistance on the inside of the pipe is negligible
- Thermal resistance of the copper pipe is negligible
- The air behaves as an ideal gas

SKETCH



PROPERTIES AND CONSTANTS

From standard steam tables: For saturated steam 0.12 MPa the heat of vaporization (h_{fg}) = 2238 kJ/kg, and the temperature (T_s) = 105°C.

From Appendix 2, Table 28, for dry air at the mean temperature of 62.5°C and one atmosphere

Thermal expansion coefficient (β) = 0.00298 1/K

Thermal conductivity (k) = 0.0281 W/(m K)

Absolute viscosity (μ) = 20.02×10^{-6} (N s)/m^2

Prandtl number (Pr) = 0.71

Density (ρ) = 1.018 kg/m^3

For an ideal gas

$$\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2} \Rightarrow \rho_2 = \frac{P_2}{P_1}\,\rho_1$$

$$\text{At } P = 50 \text{ kPa: } \rho = \left(\frac{0.5}{1}\right) 1.018 \text{ kg/m}^3 = 0.509 \text{ kg/m}^3 \Rightarrow \nu = \frac{\mu}{\rho} = 3.93 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{At } P = 2.0 \text{ atm: } \rho = \left(\frac{2.0}{1}\right) 1.018 \text{ kg/m}^3 = 2.036 \text{ kg/m}^3 \Rightarrow \nu = \frac{\mu}{\rho} = 9.83 \times 10^{-6} \text{ m}^2/\text{s}$$

SOLUTION

The Grashof number based on the pipe diameter is

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)\,D^3}{\nu^2}$$

$$\text{At 0.5 atm: } Gr_D = \frac{(9.8 \text{ m/s}^2)\,(0.00298 \text{ 1/K})\,(105°\text{C} - 20°\text{C})\,(0.02 \text{ m})^3}{(3.93 \times 10^{-5} \text{ m}^2/\text{s})^2} = 1.29 \times 10^4$$

$$\text{At 200 kPa: } Gr_D = \frac{(9.8 \text{ m/s}^2)\,(0.00298 \text{ 1/K})\,(105°\text{C} - 20°\text{C})\,(0.02 \text{ m})^3}{(9.83 \times 10^{-6} \text{ m}^2/\text{s})^2} = 2.05 \times 10^5$$

The Nusselt number for a horizontal cylinder is given by Equation (8.20). (All requirements are satisfied at both pressures.)

$$\overline{Nu}_D = 0.53\,(Gr_D\,Pr)^{\frac{1}{4}}$$

$$\text{At 0.5 atm: } \overline{Nu}_D = 0.53\,[1.29 \times 10^4\,(0.71)]^{\frac{1}{4}} = 5.18$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 5.18 \left(\frac{0.0281 \text{ W/(m K)}}{0.2 \text{ m}}\right) = 7.28 \text{ W/(m}^2\text{K)}$$

$$\text{At 2.0 atm: } \overline{Nu}_D = 0.53\,[2.05 \times 10^5\,(0.71)]^{\frac{1}{4}} = 10.35$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{L} = 10.35 \left(\frac{0.0281 \text{ W/(m K)}}{0.02 \text{ m}}\right) = 14.54 \text{ W/(m}^2\text{K)}$$

The rate of heat transfer per meter length of pipe is

$$\frac{q_c}{L} = \bar{h}_c\,\pi D\,(T_s - T_\infty)$$

$$\text{At 50 kPa: } \frac{q_c}{L} = (7.28 \text{ W/(m}^2\text{K)})\,(\pi)\,(0.02 \text{ m})\,(105°\text{C} - 20°\text{C}) = 38.9 \text{ W/m}$$

$$\text{At 200 kPa: } \frac{q_c}{L} = (14.54 \text{ W/(m}^2\text{K)})\,(\pi)\,(0.02 \text{ m})\,(105°\text{C} - 20°\text{C}) = 77.7 \text{ W/m}$$

It is clear that raising the ambient pressure from 50 kPa to 200 kPa will double the flow of condensation $(\dot{m}_c)$ per meter of pipe

$$\text{At 50 kPa: } \frac{\dot{m}_c}{L} = \frac{\frac{q_c}{L}}{h_{fg}} = \frac{38.9 \text{ W/m}}{(2238 \text{ kJ/kg})\,(1000 \text{ J/kJ})\,((\text{W s})/\text{J})} = 1.74 \times 10^{-5} \text{ kg/s} = 1.04 \text{ g/min}$$

$$\text{At 200 kPa: } \frac{\dot{m}_c}{L} = \frac{77.7 \text{ W/m}}{(2238 \text{ kJ/kg})\,(1000 \text{ J/(kJ)})\,(\text{Ws/J})} = 3.47 \times 10^{-5} \text{ kg/s} = 2.08 \text{ g/min}$$

PROBLEM 8.29

Compare the rate of condensate flow from the pipe in Problem 8.28 (air pressure = 200 kPa) with that for a 3.89-cm-*OD* pipe and 200 kPa air pressure. What is the rate of condensate flow if the 2 cm pipe is submerged in a 20°C constant-temperature water bath?

GIVEN

- A long horizontal copper pipe carrying saturated steam within an environmental testing chamber or a water bath
- Steam pressure = 120 kPa
- Ambient pressure (P) = 2 atm
- Ambient air or water temperature (T_∞) = 20°C

FIND

Rate of condensate flow for

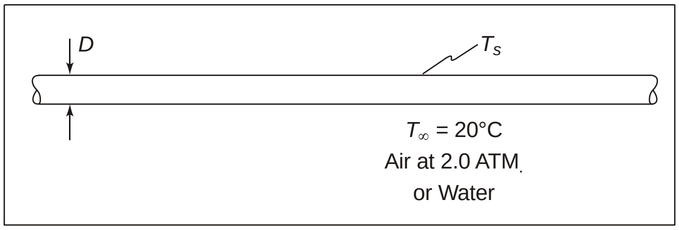
(a) Diameter (D) = 3.89 cm = 0.0389 m Fluid is air at 2.0 atm

(b) Diameter (D) = 2 cm = 0.02 m Fluid is water at T_∞ = 20°C

ASSUMPTIONS

- Pressure change has no effect on absolute viscosity, thermal conductivity, or specific heat of the air
- Air is still
- Convective thermal resistance on the inside of the pipe is negligible
- Thermal resistance of the copper pipe is negligible
- The air behaves as an ideal gas

SKETCH



PROPERTIES AND CONSTANTS

From standard steam tables: For saturated steam at 0.12 MPa, the heat of vaporization (h_{fg}) = 2238 kJ/kg, and the temperature (T_s) = 105°C.

From Appendix 2, Table 28, for dry air at the mean temperature of 62.5°C and one atmosphere

Thermal expansion coefficient (β) = 0.00298 1/K

Thermal conductivity (k) = 0.0281 W/(m K)

Prandtl number (Pr) = 0.71

From Problem 8.28: at P = 2.0 Atm, Kinematic viscosity (ν) = 9.83×10^{-6} Ns/m^2

From Appendix 2, Table 13, for water at the mean temperature of 62.5°C and one atmosphere

β = 0.00053 1/K $\quad$ k = 0.659 W/(m K)

$\nu = 0.461 \times 10^{-6}$ m^2/s $\quad$ Pr = 2.89

SOLUTION

The Grashof number is

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)\,D^3}{\nu^2}$$

Case (a): $$Gr_D = \frac{(9.8\ \mathrm{m/s^2})\ 0.00298\ 1/\mathrm{K}\ \ (105°\mathrm{C} - 20°\mathrm{C})\,(0.0389\,\mathrm{m})^3}{\left(9.83\times 10^{-6}\,\mathrm{m^2/s}\right)^2} = 1.51\times 10^6$$

Case (b): $$Gr_D = \frac{(9.8\ \mathrm{m/s^2})\ 0.00053\ 1/\mathrm{K}\ \ (105°\mathrm{C} - 20°\mathrm{C})\,(0.02\,\mathrm{m})^3}{\left(0.461\times 10^{-6}\,\mathrm{m^2/s}\right)^2} = 1.66\times 10^7$$

Both cases fall within the range of requirements for the use of Equation (8.20)

$$\overline{Nu}_D = 0.53\ (Gr_D\,Pr)^{\frac{1}{4}}$$

Case (a) $$\overline{Nu}_D = 0.53\ [1.51\times 10^6\ (0.71)]^{\frac{1}{4}} = 17.1$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 17.1\ \frac{0.0281\ \mathrm{W/(m\,K)}}{0.0389\,\mathrm{m}} = 12.3\ \ \mathrm{W/(m^2K)}$$

Case (b) $$\overline{Nu}_D = 0.53\ [1.66\times 10^7\ (2.89)]^{\frac{1}{4}} = 44.1$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 44.1\ \frac{0.659\ \mathrm{W/(m\,K)}}{0.02\ \mathrm{m}} = 1453\ \ \mathrm{W/(m^2K)}$$

The condensate flow rate per meter of pipe is given by

$$\frac{\dot{m}_c}{L} = \frac{\frac{q_c}{L}}{h_{gf}} = \frac{\bar{h}_c\,\pi D\,(T_s - T_\infty)}{h_{fg}}$$

Case (a): $$\frac{\dot{m}_c}{L} = \frac{12.3\ \mathrm{W/(m^2K)}\ \ \pi\,(0.0389\,\mathrm{m})\,(105°\mathrm{C} - 20°\mathrm{C})}{2238\ \mathrm{kJ/kg}\ \ 1000\ \mathrm{J/kJ}\ \ \ \mathrm{W\,s/J}} = 5.7\times 10^{-5}\ \mathrm{kg/s} = 3.42\ \mathrm{g/min}$$

Case (b): $$\frac{\dot{m}_c}{L} = \frac{1453\ \mathrm{W\,(m^2K)}\ \ \pi\,(0.02\,\mathrm{m})\,(105°\mathrm{C} - 20°\mathrm{C})}{2238\ \mathrm{kJ/kg}\ \ 1000\ \mathrm{J/(kJ)}\ \ \ \mathrm{W\,s/J}} = 3.47\times 10^{-3}\ \mathrm{kg/s} = 208\ \mathrm{g/min}$$

COMMENTS

The rate of condensate flow from Problem 8.28 with a 2-cm-diameter pipe in air at 200 kPa. is 2.1 g/min. A change in the fluid from air to water leads to a much larger increase in the rate of condensate flow (100 times) than an increase in the pipe diameter to 3.89 cm (1.6 times).

PROBLEM 8.30

A thermocouple (0.8-mm-*OD*) is located horizontally in a large enclosure whose walls are at 37°C. The enclosure is filled with a transparent quiescent gas that has the same properties as air. The electromotive force (emf) of the thermocouple indicates a temperature of 230°C. Estimate the true gas temperature if the emissivity of the thermocouple is 0.8.

GIVEN

- Horizontal thermocouple in a large enclosure
- Thermocouple outside diameter (D) = 0.8 mm = 0.0008 m
- Enclosure wall temperature (T_e) = 37°C = 310 K
- Gas is enclosure is quiescent and has the same properties as air
- Thermocouple reading (T_{tc}) = 230°C = 503 K
- Thermocouple emissivity (ε) = 0.8

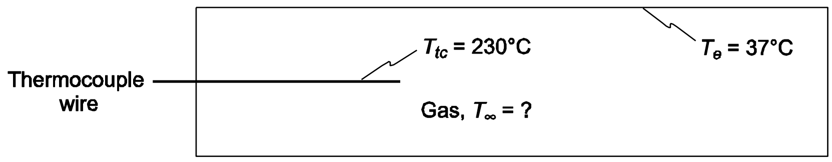
FIND

- True gas temperature (T_∞)

ASSUMPTIONS

- Enclosure behaves as a black body
- Conduction along the thermocouple out of the enclosure is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5 The Stephan-Boltzmann Constant (σ) = 5.67×10^{-8} W/(m^2 K^4).

SOLUTION

An iterative procedure is required. For the first iteration, let T_∞ = 300°C = 573 K.
From Appendix 2, Table 28, for dry air at the mean temperature of 265°C

Thermal expansion coefficient (β) = 0.00188 1/K

Thermal conductivity (k) = 0.0408 W/(m K)

Kinematic viscosity (ν) = 44.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The Grashof number based on the thermocouple diameter is

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.00188\,1/\text{K}\ (300°\text{C} - 230°\text{C})(0.0008\,\text{m})^3}{\left(44.4\times10^{-6}\text{m}^2/\text{s}\right)^2} = 0.335$$

The Rayleigh number is

$$Ra_D = Gr_D\,(Pr) = (0.335)\,(0.71) = 0.238$$

$\log Ra_D = -0.624$

From Figure 8.3 $\log Nu \approx -0.05 \rightarrow Nu = 0.89$

$$h_c = Nu \frac{k}{D} = 0.89 \frac{0.0408 \text{ W/(m K)}}{0.0008 \text{ m}} = 45 \text{ W/(m}^2\text{K)}$$

For steady state, the rate of convection to the thermocouple must equal the rate of radiation from the thermocouple

$$h_c A (T_\infty - T_{tc}) = \varepsilon \sigma A (T_{tc}^4 - T_e^4)$$

Solving for the gas temperature

$$T_\infty = T_{tc} + \frac{\varepsilon\sigma}{h_c} (T_{tc}^4 - T_e^4)$$

$$T_\infty = 503 \text{ K} + \frac{0.8 \; 5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4)}{45 \text{ W/(m}^2\text{K)}} [(503 \text{ K})^4 - (310 \text{ K})^4] = 559 \text{ K}$$

Using this as the beginning of another iteration

$T_{\text{mean}} = 259°\text{C}$

$\beta = 0.00190$ 1/K

$k = 0.0405$ W/(m K)

$\nu = 43.5 \times 10^{-6}$ m^2/s

$Pr = 0.71$

$Ra_D = 0.20$

$h_c = 43$ W/(m^2 K)

$T_\infty = 561$ K

Therefore, the true gas temperature is about 560 K = 287°C.

PROBLEM 8.31

A sphere 20 cm in diameter containing liquid air (–140°C) is covered with 5-cm-thick glass wool (50 kg/m^3 density) with an emissivity of 0.8. Estimate the rate of heat transfer to the liquid air from the surrounding air at 20°C by convection and radiation. How would you reduce the heat transfer?

GIVEN

- A sphere containing liquid air covered with glass wool
- Sphere diameter (D_s) = 20 cm = 0.2 m
- Liquid air temperature (T_a) = – 140°C = 133 K
- Surrounding air temperature (T_∞) = 20°C = 293 K
- Insulation thickness (s) = 5 cm = 0.05 m
- Insulation emissivity (ε) = 0.8

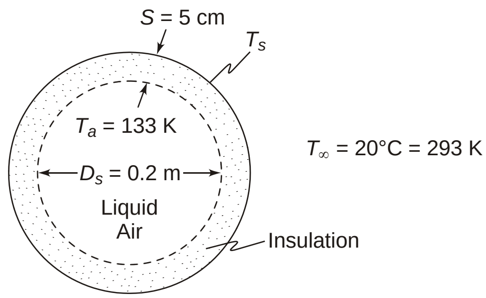
FIND

- Rate of heat transfer from liquid air to surrounding air (q)
- How can this be reduced?

ASSUMPTIONS

- Steady state conditions
- The surroundings behave as a black body enclosure at T_∞
- Surrounding air is still
- Thermal resistance of the convection inside the sphere and of the container wall are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4).
From Appendix 2, Table 11, the thermal conductivity of glass wool (k_i) = 0.037 W/(m K).

SOLUTION

The natural convection heat transfer coefficient on the exterior of the insulation depends on the exterior temperature of the insulation (T_s), an iterative procedure is therefore required. For the first iteration, let T_s = – 20°C (253 K)

From Appendix 2, Table 28, for dry air at the mean temperature of 0°C

Thermal expansion coefficient (β) = 0.00366 1/K

Thermal conductivity (k) = 0.0237 W/(m K)

Kinematic viscosity (ν) = 13.9×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The characteristic length for the sphere is

$$L^+ = \frac{A}{\left(\frac{4A_{\text{Horz}}}{\pi}\right)^{\frac{1}{2}}} = \frac{\pi D_i^2}{D_i} = \pi D_i = \pi\ (D_s + 2s) = \pi\ [0.2\text{ m} + 2(0.05\text{ m})] = 0.942\text{ m}$$

The Grahsof and Rayleigh numbers based on this length are

$$Gr_{L+} = \frac{g\,\beta(T_s - T_\infty)\,(L^+)^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)\ 0.00366\ 1/\text{K}\ \ (20°\text{C} - 20°\text{C})(0.942\text{ m})^3}{13.9\times10^{-6}\text{m}^2/\text{s}^{\ 2}} = 6.21\times10^9$$

$$Ra_{L+} = Gr_{L+}\,Pr = 6.21\times10^9\,(0.71) = 4.41\times10^9$$

Although the empirical relation of Equation (8.25) extends only to $Ra^+ = 1.5 \times 10^9$, it will be extrapolated here to estimate the Nusselt number

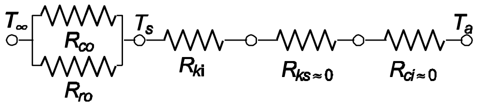
$$Nu^+ = 5.75 + 0.75\left[\frac{Ra^+}{F\ (Pr)}\right]^{0.252}$$

where $$F(Pr) = \left[1+\left(\frac{0.49}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}} = \left[1+\left(\frac{0.49}{0.71}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}} = 2.88$$

$$\therefore\ Nu^+ = 5.75 + 0.75\left[\frac{4.41\times10^9}{2.88}\right]^{0.252} = 160.5$$

$$\bar{h}_c = Nu^+\ \frac{k}{L^+} = 160.5\ \frac{0.0237\text{ W/(m K)}}{0.942\text{ m}} = 4.04\ \text{W/(m}^2\text{K)}$$

The thermal circuit for the sphere is shown below



where R_{ci} = interior convective resistance (negligible)

R_{ks} = conductive resistance of the container (negligible)

R_{ki} = conductive resistance of the insulation

R_{co} = exterior convective resistance

R_{ro} = exterior radiative resistance

From Equation (2.51)

$$R_{ki} = \frac{r_o - r_i}{4\pi k_i\, r_o\, r_i} \quad \text{where } r_o = \frac{D_s}{2} + s = 0.1\text{ m} + 0.05\text{ m} = 0.15\text{ m and } r_i = \frac{D_s}{2} = 0.1\text{ m}$$

$$\therefore\ R_{ki} = \frac{0.15\text{m} - 0.1\text{m}}{4\pi\ \ 0.037\text{ W/(m K)}\ \ (0.15\text{m})\,(0.1\text{m})} = 7.17\ \text{K/W}$$

From Equation (1.14)

$$R_{co} = \frac{1}{\bar{h}_c\,A} = \frac{1}{\bar{h}_c\,4\pi r_o^2} = \frac{1}{4.04\text{ W/(m}^2\text{K)}\ \ 4\pi(0.15\text{m})^2} = 0.875\ \text{K/W}$$

The exterior radiative resistance is

$$R_{ro} = \frac{T_\infty - T_s}{4\pi r_o^2\ \varepsilon\sigma\,(T_\infty^4 - T_s^4)} = \frac{293\text{K} - 253\text{K}}{4\pi(0.15\text{m})^2\ (0.8)\ 5.67\times10^{-8}\text{ W/(m}^2\text{K}^4)\ \ [(293\text{K})^4 - (253\text{K})^4]}$$

$$R_{ro} = 0.953 \text{ K/W}$$

The net resistance for the thermal network is $R_t = R_{ki} + R_o$ where

$$R_o = \frac{R_{co}\ R_{ro}}{R_{co} + R_{ro}} = \frac{(0.875\text{K/W})\,(0.953\text{K/W})}{0.875\text{K/W} + 0.953\text{K/W}} = 0.46 \text{ K/W}$$

$$R_t = 7.17 \text{ K/W} = 0.46 \text{ K/W} = 7.63 \text{ K/W}$$

The rate of heat transfer is given by

$$q = \frac{T_\infty - T_a}{R_t} = \frac{293\text{K} - 133\text{K}}{7.63\text{K/W}} = 20.97 \text{ W}$$

The accuracy of the insulation surface temperature guess can be checked from

$$q = \frac{T_\infty - T_{so}}{R_o} = \frac{293\text{K} - 253\text{K}}{0.46\text{K/W}} = 86.9 \text{ W} > 20.97 \text{ W}$$

Therefore, we need to reduce T_{so}. However, notice that nearly 94% of the total thermal resistance is due to the insulation. This means that adjusting T_{so} has little effect on the total rate of heat transfer. It also means that the heat gain by the liquid air can be most easily reduced by increasing the thickness of insulation, selecting an insulation with lower thermal conductivity, or both.

PROBLEM 8.32

Only 10% of the energy dissipated by the tungsten filament of an incandescent lamp is in the form of useful visible light. Consider a 100 W lamp with a 10 cm spherical glass bulb. Assuming an emissivity of 0.85 for the glass and ambient air temperature of 20°C, what is the temperature of the glass bulb?

GIVEN

- A spherical glass light bulb in air
- Bulb power consumption (P) = 100 W
- 10% of energy is in the form of visible light
- Diameter (D) = 10 cm = 0.1 m
- Bulb emissivity (ε) = 0.85
- Ambient temperature (T_∞) = 20°C = 293 K

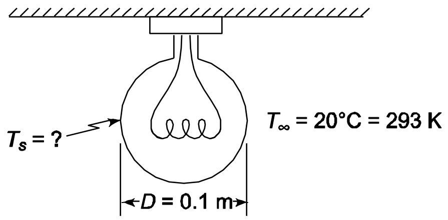
FIND

- The temperature of the glass bulb (T_s)

ASSUMPTIONS

- Ambient air is till
- The bulb has reached steady state
- The surrounding behave as a black body at T_∞

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5 the Stephan-Boltzmann constant (σ) = 5.7×10^{-8} W/(m^2 K^4).

SOLUTION

The rate of heat transfer by convection and radiation from the bulb must equal the rate of heat generation.

$$q_c + q_r = \pi D^2 \left[\bar{h}_c \ (T_s - T_\infty) + \varepsilon \, \sigma (T_s^4 - T_\infty^4) \right] = 0.9 \ (100 \text{ W}) = 90 \text{ W}$$

Since the fluid properties depend on the surface temperature, an iterative procedure must be used. For the first iteration, let T_s = 100°C = 373 K.

From Appendix 2, Table 28, for dry air at the mean temperature of 60°C

Thermal expansion coefficient (β) = 0.00300 1/K

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The characteristic length for a 3-D body is given by

$$L^+ = \frac{A}{\left(\frac{4A_{\text{Horz}}}{\pi}\right)^{\frac{1}{2}}} = \frac{\pi D^2}{\left(\frac{4\left(\frac{\pi}{4}D^2\right)}{\pi}\right)^{\frac{1}{2}}} = \pi D = \pi\ (0.1\text{ m}) = 0.314\text{ m}$$

The Grashof and Rayleigh numbers are

$$Gr_{L+} = \frac{g\,\beta(T_s - T_\infty)\,(L^+)^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)\ 0.0031/\text{K}\ (100°\text{C} - 20°\text{C})(0.314\text{m})^3}{19.4\times10^{-6}\text{m}^2/\text{s}^{\ 2}} = 1.94\times10^8$$

$$Ra_{L+} = Gr_{L+} + Pr = 1.94\times10^8\ (0.71) = 1.38\times10^8$$

Equation (8.25) correlates data for 3-D bodies including spheres for $200 < Ra_{L+} < 1.5\times10^9$

$$Nu^+ = 5.75 + 0.75\left[\frac{Ra^+}{F\ (Pr)}\right]^{0.252}$$

$$\text{where}\quad F(Pr) = \left[1+\left(\frac{0.49}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}} = \left[1+\left(\frac{0.49}{0.71}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}} = 2.88$$

$$\therefore\ Nu^+ = 5.75 + 0.75\left[\frac{1.38\times10^8}{2.88}\right] = 70.4$$

$$\bar{h}_c = Nu^+\ \frac{k}{L^+} = 70.4\ \frac{0.0279\text{ W/(m K)}}{0.314\text{ m}} = 6.26\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer by convection and radiation must equal the heat generation rate

$$q_c + q_r = \pi(0.1\text{ m})^2\left[6.26\text{ W/(m}^2\text{K)}(T_s - 293\text{K}) + 0.85\ 5.67\times10^{-8}\text{W/(m}^2\text{K}^4)\ (T_s^4 - (293\text{K})^4)\right] = 90\text{ W}$$

Checking the units then eliminating them for clarity

$$0.197\ T_s + 1.514\times10^{-9}\ T_s^4 - 158.8 = 0$$

By trial and error: T_s = 460 K = 187°C.

The results of further iterations are tabulated below

Iteration #	2	3
T_s (K)	459	457
T_{mean} (K)	376	375
β (1/K)	0.00266	0.00270
k (W/(m^2 K))	0.0309	0.0308
$\nu\times10^6$ (m^2/s)	24.0	23.8
Pr	0.71	0.71
$Ra^+\times10^{-8}$	1.86	1.68
h_c (W/(m^2 K))	7.43	7.23
T_s (°C)	181	182

The bulb temperature, therefore, is approximately 182°C.

COMMENTS

Note that radiative transfer accounts for about 66% of the total heat transfer from the bulb.

PROBLEM 8.33

A 2-cm-diameter bare aluminum electric power transmission line with an emissivity of 0.07 carries 500 A at 400 kV. The wire has an electrical resistivity of 1.72 micro-ohms cm^2/cm at 20°C and is suspended horizontally between two towers separated by 1 km. Determine the surface temperature of the transmission line if the air temperature is 20°C. What fraction of the dissipated power is due to radiation heat transfer?

GIVEN

- An aluminum electric power transmission line suspended horizontally
- Emissivity (ε) in air = 0.3
- Line diameter (D) = 2 cm = 0.02 m
- Current (I) = 500 amp
- Voltage (V) = 400 kV
- Electrical resistivity (ρ_e) = 1.72 Ω cm^2/cm at 20°C
- Space between towers (L) = 1 km
- Air temperature (T_∞) = 20°C = 293 K

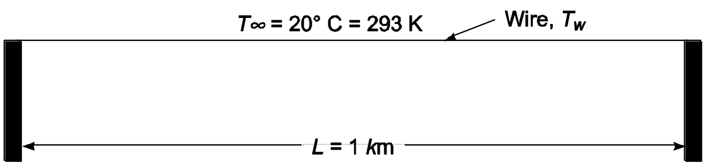
FIND

(a) Surface temperature of wire (T_w)
(b) Fraction of dissipated power due to radiation

ASSUMPTIONS

- Steady state
- The wire radiates to the surroundings which behave as a black body enclosure at T_∞

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/m^2 K^4.

SOLUTION

The power dissipation is given by Ohm's Law

$$P = I^2 R_c = I^2 \frac{\rho}{A_c} = \frac{4I^2\rho}{\pi D^2} = \frac{4\,(500\,\text{Amps})^2\ 1.72\times10^{-6}\,\text{ohm}\,\text{cm}^2/\text{cm}}{\pi\,(2\,\text{cm})^2}$$

$$= 0.1368\ \text{W/cm} = 13.68\ \text{W/m}$$

This must equal the rate of heat transfer by convection and radiation per meter

$$P = \pi D\,[h_c\,(T_w - T_\infty) + \varepsilon_w\,\sigma\,(T_w^4 - T_\infty^4)]$$

Since h_c varies with T_w, an iterative procedure must be used. For the first iteration, let T_w = 60°C.

From Appendix 2, Table 28, for dry air at the mean temperature of 40°C

Thermal expansion coefficient (β) = 0.00319 1/K

Thermal conductivity (k) = 0.0265 W/(m K)

Kinematic viscosity (ν) = 17.6×10^{-6} m²/s

Prandtl number (Pr) = 0.71

The Grashof number based on the wire diameter is

$$Gr_D = \frac{g\,\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\;0.00319\,1/\text{K}\;\;(60\,°\text{C} - 20\,°\text{C})\,(0.02\,\text{m})^3}{17.6\times10^{-6}(\text{m}^2/\text{s})^{\;2}} = 3.23\times10^4$$

The Nusselt number for this geometry and Grashof number is given by Equation (8.20)

$$Nu_D = 0.53\;(Gr_D Pr)^{\frac{1}{4}} = 0.53\;[3.23\times10^4(0.71)]^{\frac{1}{4}} = 6.52$$

$$h_c = Nu_D\;\frac{k}{D} = 6.52\;\frac{0.0265\,\text{W/(m K)}}{0.02\,\text{m}} = 8.64\;\;\text{W/(m}^2\,\text{K)}$$

$$\therefore\; P = 13.68\;\text{W/m} = \pi\,(0.02\;\text{m})$$

$$\left[8.64\;\text{W/(m}^2\;\text{K)}\,(T_w - 293\text{K}) + 0.07\;\;5.67\times10^{-8}\;\;\text{W/(m}^2\text{K}^4)\;\;[T_w{}^4 - (293\text{K})^4]\right]$$

Checking the units then eliminating them for clarity

$$3.97\times10^{-9}\;T_w{}^4 + 8.64\;T_w - 2778 = 0$$

By trial and error T_w = 317 K = 44°C

Performing further iterations

Iteration #	2	3
T_w (°C)	44	46.6
Mean Temp. (°C)	32	33.33
β (1/K)	0.00328	0.00326
k W/(m K)	0.0259	0.0260
$\nu \times 10^6$ (m²/s)	16.8	17.0
Pr	0.71	0.71
$Gr_D \times 10^{-4}$	2.19	2.35
Nu_D	5.92	6.02
h_c (W/(m² K))	7.66	7.83
T_w (°C)	47	46

The equilibrium surface temperature is 46°C.

(b) The rate of heat transfer by convection is

$$\frac{q_c}{L} = h_c\,\pi D\,(T_w - T_\infty) - 7.83\;\text{W/(m}^2\text{K)}\;\;\pi\;\;(0.02\;\text{m})\;(46°\text{C} - 20°\text{C}) = 12.79\;\text{W/m}$$

The rate of heat transfer by radiation is

$$\frac{q_r}{L} = \varepsilon\,\sigma\,\pi D(T_s{}^4 - T_\infty{}^4) = 0.07\;\;\;5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\;\;\pi\;\;(0.02\;\text{m})$$

$$[(319\;\text{K})^4 - (293\;\text{K})^4] = 0.74\;\text{W}$$

As a check on the results

$$\frac{q_c}{L} + \frac{q_r}{L} = 12.79\;\text{W/m} + 0.74\;\text{W/m} = 13.53\;\text{W/m} \cong P$$

The fraction of the power dissipation by radiation is

$$\frac{\frac{q_r}{L}}{P} = \frac{0.74}{13.53} = 0.055 = 5.5\%$$

PROBLEM 8.34

An 20 cm-diameter horizontal steam pipe carries 1.66 kg/min of dry saturated steam at 120°C. If ambient air temperature is 20°C, determine the rate of condensate flow at the end of 3 m of pipe. Use an emissivity of 0.85 for the pipe surface. If heat losses are to be kept below 1 per cent of the rate of energy transport by the steam, what thickness of fiberglass insulation is required? The rate of energy transport by the steam is the heat of condensation of the steam flow. The heat of vaporization of the steam is 2210 kJ/kg.

GIVEN

- A horizontal steam pipe in air
- Pipe outside diameter (D) = 20 cm= 0.2 m
- Mass flow rate of steam (ms) = 1.66 kg/min=0.0277 kg/s
- Steam temperature (T_s) = 120°C =393 K
- Ambient air temperature (T_∞) = 20°C= 293 K
- Emissivity of pipe surface (ε) = 0.85
- Heat of vaporization (h_{fg}) = 2210 kJ/kg

FIND

(a) Rate of condensate flow (mc) at the end of 3 m of pipe.
(b) Thickness of fiberglass insulation (S) to keep loss below 1% of the energy transport by steam

ASSUMPTIONS

- Steady state
- Air is still
- Thermal resistance of the convection in the pipe and of the pipe wall are negligible
- The surroundings behave as an enclosure at T_∞
- Insulation is foil covered, its emissivity ≈ 0.0

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Appendix 2, Table 11, Thermal conductivity of fiberglass (ki) = 0.035 W/(m K)
From Appendix 2, Table 28, for dry air at the mean temperature of 70°C

Thermal expansion coefficient (β) = 0.00291 1/K

Thermal conductivity (k) = 0.0288 W/(m K)

Kinematic viscosity (ν) = 17.1×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

(a) The Grashof number for the uninsulated pipe is

$$Gr_D = 5.43 \times 10^7$$

The rate of heat transfer from 3 m of the pipe(considering 1% loss in energy transport) is

$$q = 611.4 \text{ W}$$

This must equal the convection from the surface of the insulation (radiation is negligible) and the conduction through the insulation.

$$q = h_c\, \pi (D + 2s)\, L(T_{si} - T_\infty) = \frac{T_{si} - T_\infty}{R_k} = 611.4 \text{ W}$$

where $$R_k = \frac{\ln\left(\frac{D+2s}{D}\right)}{2\pi\, Lk_i}$$

Rearranging to eliminate the insulation thickness

$$s = \frac{q}{2h_c\,\pi\, L(T_{si} - T_\infty)} - \frac{D}{2} \qquad q = \frac{2\pi\, Lk_i\,(T_{si} - T_\infty)}{\ln\left(\frac{h_c\,\pi DL(T_{si} - T_\infty)}{q}\right)}$$

Since h_c depends on the insulation surface temperature T_{si}, an iterative procedure must be used.
For the first iteration, let T_{si} =50°C.
From Appendix 2, Table 28, for dry air at the mean temperature of 35°C

Thermal expansion coefficient (β) = 0.00323 1/K
Thermal conductivity (k) = 0.0261 W/(m K)
Kinematic viscosity (ν) = $17.1 * 10^{-6}$ m^2/s
Prandtl number (Pr) = 0.71

Assuming the insulation is thin compared to the pipe radius

$$Gr_D = \frac{g\,\beta\,(T_s - T_\infty)D^3}{\nu^2} = \frac{(9.81 m/\text{s}^2)(0.00323\ 1/\text{K})(50°\text{C} - 20°\text{C})(0.2)^3}{\left(17.1 * 10^6\ m^2/s\right)^2}$$

$$Gr_D = 2.6 \times 10^7$$

$$h_c = 0.53\ \frac{(0.0261\ \text{W}/(m\,K))}{0.2}\left[2.6 \times 10^7\,(0.71)\right]^{\frac{1}{4}} = 4.534\ \text{W}/(\text{m}^2\text{K})$$

$$611.4\ \text{W} = \frac{2\pi\,(3m)\left(0.035\ \text{W}/(\text{m}\ K)\right)(T_{si} - 20°\text{C})}{\ln\left[\frac{\left(4.534\,W/(\text{m}^2\text{K})\right)\pi\,(0.2)\,(3)\,(T_{si} - 20°\text{C})}{611.4\,W}\right]}$$

Checking the units then eliminating them for clarity

$$\frac{0.66T_{si} - 13.2}{\ln(0.01397T_{si} - 0.279)} - 611.4 = 0$$

By trial and error: T_{si} = 97.8°C
Performing further iterations

Iteration #	2	3	4
Mean Temp. (°C)	15.3	10.4	11.6
β (1/K)	0.00306	0.00306	0.00306
k (W/(m K))	0.0278	0.0271	0.0273
ν (m^2/s)	$19.4*10^{-6}$	$18.6*10^{-6}$	$18.8*10^{-6}$
Pr	0.71	0.71	0.71
$Gr_D \times 10^{-7}$	5.16	4.32	4.57
h_c (W/(m^2 K))	5.62	5.25	5.36
T_{si} (°C)	80.5	85.1	83.7

The surface temperature of the insulation is about 84°F

$$\therefore\ s = \frac{611.4\,W}{2\left(5.36\,W/(\text{m}^2\ K)\right)\pi\,(3m)\,(84°\text{C} - 20°\text{C})} - \frac{0.2}{2} = 0.0054\ \text{m} = 5.4\ \text{mm}$$

Thus about a 5.5 mm of insulation is required.

PROBLEM 8.35

A long steel rod (2 cm in diameter, 2-m-long) has been heat-treated and quenched to a temperature of 100°C in an oil bath. To cool the rod further, it is necessary to remove it from the bath and expose it to room air. Will the faster cool-down result from cooling the cylinder in the vertical or horizontal position? How long will the two methods require to allow the rod to cool to 40°C in 20°C air?

GIVEN

- A long steel rod in air
- Diameter (D) = 2 cm = 0.02 m
- Length (L) = 2 m
- Initial temperature ($T_{s,i}$) = 100°C
- Air temperature (T_∞) = 20°C

FIND

(a) Is it faster to cool the rod vertically or horizontally?
(b) Time for rod to cool to 40°C in each position

ASSUMPTIONS

- Steel is 1% carbon

PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, for 1% carbon steel

Thermal conductivity (ks) = 43 W/(m K)
Specific heat (c) = 473 J/(kg K)
Density (ρ) = 7801 kg/m^3

From Appendix 2, Table 28, for dry air at the initial mean temperature of 60°C

Thermal expansion coefficient (β) = 0.003 1/K
Thermal conductivity (k) = 0.0279 W/(m K)
Kinematic viscosity (ν) = 19.4 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

As the temperature of the rod decreases, the heat transfer coefficient will also decrease. Therefore, a rough numerical integration will be used to estimate the cooling time.

Note that the air properties must be evaluated at each step.

Time (min)	5	10	15	20	30	40	42	60	76
Vertical h_c (W/(m^2 K))	5.96	5.80	5.65	5.50	5.34	5.06	4.78	4.29	
DT (°C)	7.8	6.8	6.0	5.3	9.4	7.3	11.6	5.7	
New T_s (°C)	92.2	85.4	79.4	74.0	64.6	57.3	45.7	40.0	
Horizontal h_c (W/(m^2 K))	10.15	9.73	9.37	9.02	8.67	8.02	7.54		
DT (°C)	13.2	10.6	8.6	7.0	11.5	7.6	1.4		
New T_s (°C)	86.8	76.2	67.6	60.6	49.1	41.5	40.1		

Cooling time: about 76 minutes in the vertical position, about 42 minutes in the horizontal position

An alternate method of solution uses the average heat transfer coefficients and the time-temperature history given by Equation (3.3). Evaluating the heat transfer coefficients when the rod has reached 40°C

Vertical: h_{cv}, final = 4.22 W/(m^2 K) → hcv, ave = 5.09 W/(m^2 K)

Horizontal: h_{ch}, final = 7.77 W/(m^2 K) $\rightarrow$ h_{ch}, ave = 8.96 W/(m^2 K)

The time required for the rod to cool to the temperature T_f is calculated by rearranging Equation (2.89)

Similarly for the horizontal position: t = 2854 s = 48 min.

This more approximate technique yields cooling times about 10-14% greater than the numerical technique shown above.

PROBLEM 8.36

In petroleum processing plants, it is often necessary to pump highly viscous liquids such as asphalt through pipes. To keep pumping costs within reason, the pipelines are electrically heated to reduce the viscosity of the asphalt. Consider a 15-cm-*OD* uninsulated pipe and an ambient temperature of 20°C. How much power per meter of pipe length is necessary to maintain the pipe at 50°C? If the pipe is insulated with 5 cm of fiberglass insulation, what is the power requirement?

GIVEN

- An electrically heated pipe
- Diameter (D) = 15 cm = 0.15 m
- Pipe surface temperature (T_{sp}) = 50°C

FIND

(a) Power per meter (q_e/L) required with no insulation
(b) Power per meter required with 5 cm (0.05 m) of fiberglass insulation

ASSUMPTIONS

- The pipe is horizontal and in quiescent air
- Radiative heat transfer is negligible
- No heat is transferred to the fluid in the pipe

PROPERTIES AND CONSTANTS

From Appendix 2, Table *28*, for dry air at the mean temperature of 35°C

Thermal expansion coefficient (β) = 0.00325 1/K
Thermal conductivity (k) = 0.0262 W/(m K)
Kinematic viscosity (ν) = 17.1×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

From Appendix 2, Table 11, the thermal conductivity of fiberglass (k_{fg}) = 0.035 W/(m K)

SOLUTION

The correlation for the average heat transfer coefficient for this geometry is given by Equation (8.20). (Note that the criteria of 103 < Gr_D < 10^9 and Pr > 0.5 is satisfied.)

The thermal properties needed to evaluate h_c must be calculated at the mean of T_{si} and T_∞. Therefore, an iterative process is required.

For iteration #1, let T_{si} = 35°C

From Appendix 2, Table 28, for dry air at the mean temperature of 27.5°C

Thermal expansion coefficient (β) = 0.00333 1/K
Thermal conductivity (k) = 0.0256 W/(m K)
Kinematic viscosity (ν) = 16.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

Equation (8.20) gives the heat transfer coefficient

Performing further iterations using the same Procedure

Iteration #	2	3
T_{si} (°C)	23.9	25.2
Mean Temp. (°C)	22.0	22.6
β (1/K)	0.00339	0.00338
k (W/(m K))	0.0252	0.0253
$\nu \times 10^6$ (m^2/s)	15.9	15.9
Pr	0.71	0.71
$Gr_D \times 10^{-6}$	8.01	10.6
h_c (W/(m^2 K))	2.61	2.81
$L\ R_c$ (m K)/W	0.488	0.452
T_{si} (°C)	25.2	24.9

COMMENTS

The insulation has reduced the rate of heat loss by 84%.

PROBLEM 8.37

Estimate the rate of convective heat transfer across a 1-m-tall double-pane window assembly in which the outside pane is at 0°C and the inside pane is at 20°C. The panes are spaced 2.5 cm apart. What is the thermal resistance (‘*R*’ value) of the window if the rate of radiative heat flux is 84 W/m^2?

GIVEN

- Double-pane window assembly
- Height (H) = 1 m
- Spacing (δ) = 2.5 cm = 0.025 m
- Pane temperatures
  - Inside (T_i) = 20°C
  - Outside (T_o) = 0°C
- Radiative heat flux (q_r/A) = 84 W/m^2

FIND

(a) The rate of convective heat transfer (q_c/A)
(b) The thermal resistance (R)

ASSUMPTIONS

- Steady state
- Conduction through the window frame is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 10°C

Thermal expansion coefficient (β) = 0.00354 1/K
Thermal conductivity (k) = 0.0244 W/(m K)
Kinematic viscosity (ν) = 14.8 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

(a) The aspect ratio for the window is

$$\frac{H}{\delta} = \frac{1\,\text{m}}{0.025\,\text{m}} = 40$$

The Grashof and Rayleigh numbers based on the spacing are

$$Ra_\delta = Gr_\delta Pr = 4.95 \times 10^4\ (0.71) = 3.51 \times 10^4$$

$$Gr_\delta = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.00354\,1/\text{K}\ (20°\text{C} - 0°\text{C})(0.025\,\text{m})^3}{14.8 \times 10^{-6}\ \text{m}^2/\text{s}\ ^2} = 4.95 \times 10^4$$

$$h_c = Nu_\delta \frac{k}{\delta} = 1.89\ \frac{0.0244\ \text{W/(m K)}}{0.025\,\text{m}} = 1.85\ \text{W/(m}^2\text{K)}$$

$$Nu_\delta = 0.42\ Ra_\delta^{0.25}\ Pr^{0.012} \left(\frac{H}{\delta}\right)^{-0.3} = 0.42\ (3.51 \times 10^4)^{0.25}\ (0.71)^{0.012}\ (40)^{-0.3} = 1.89$$

The Nusselt number for an enclosed space with $H/\delta = 40$ and $10^9 < Ra_\delta\, 10^7$ is given by Equation (8.29a)
The rate of heat transfer by convection is given by

$$\frac{q_c}{A} = h_c\ \ T_i - T_o = 1.85\,\text{W/(m}^2\text{K)}\ \ (20°\text{C} - 0°\text{C}\ \ = 37.0\ \text{W}$$

(b) The 'R' value must satisfy the following equation

$$\frac{q_{\text{total}}}{A} = \frac{T_i - T_o}{R} = \frac{q_c}{A} + \frac{q_r}{A} - R = \frac{T_i - T_o}{\frac{q_c}{A} + \frac{q_r}{A}} = \frac{20°C - 0°C}{(37+84)\ \text{W/m}^2} = 0.165\ \text{m}^2\text{K/W}$$

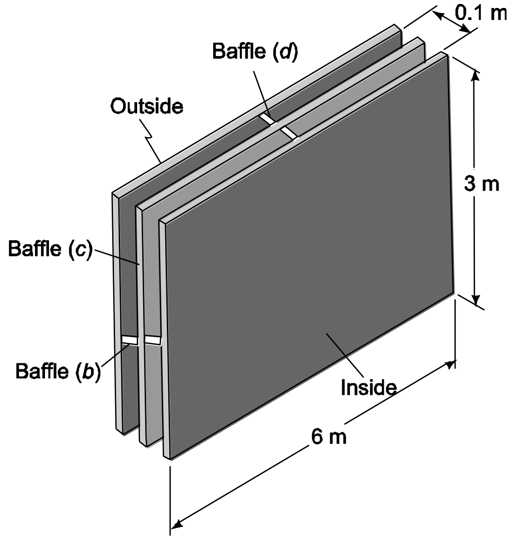
The 'R' value is usually expressed in English units

$$0.165\,\text{m}^2\text{K/W}\left(\frac{0.5275\ \text{h°F/Btu}}{\text{K/W}}\right)\ 10.764\,\text{ft}^2/\text{m}^2 = 0.94\ \text{h ft}^2\text{°F/Btu}$$

The R value is approximately 1.

PROBLEM 8.38

An architect is asked to determine the heat loss through a wall of a building constructed as shown in the sketch. The space between wall is 10 cm and contains air. If the inner surface is at 20°C and the outer surface is at – 8°C, (a) estimate the heat loss by natural convection. Then determine the effect of placing a baffle (b) horizontally at the mid-height of the vertical section (*B*), (c) vertically at the center of the horizontal section (*C*), and (d) vertically half-way between the two surfaces (*D*).



GIVEN

- Air filled wall construction as shown above
- Inner wall temperature (T_i) = 20°C
- Outer wall temperature (T_o) = – 8°C
- Wall spacing (δ) = 10 cm = 0.1 m
- Wall height (L) = 3 m
- Wall width (w) = 6 m

FIND

The rate of heat loss by natural convection (q_c) for the wall

(a) without baffles
(b) with a horizontal baffle at a mid-height of the wall-baffle *B*
(c) with a vertical baffle at the center of the horizontal section-baffle *C*
(d) with a vertical baffle midway between the walls-baffle *D*

ASSUMPTIONS

- Wall temperatures are constant and uniform
- Steady state conditions
- Baffle thickness is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 6°C

Thermal expansion coefficient (β) = 0.00359 1/K

Thermal conductivity (k) = 0.0241 W/(m K)

Kinematic viscosity (ν) = 14.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

(a) The Grashof and Rayleigh numbers based on the space between the walls (d) are

$$Ra_\delta = Gr_\delta\, Pr = 4.75 \times 10^6\ (0.71) = 3.37 \times 10^6$$

$$Gr_\delta = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2} = \frac{(9.8\ \mathrm{m/s^2})\ \ 0.00359\ 1/\mathrm{K}\ \ (20°\mathrm{C} + 8°\mathrm{C})(0.1\mathrm{m})^3}{14.4 \times 10^{-6}\mathrm{m^2/s}\ \ ^2} = 4.75 \times 10^6$$

The aspect ratio (L/δ) = (3 m)/(0.1 m) = 30

The correlation for this geometry is given by Equation (8.29a)

$$Nu_\delta = 0.42\ Ra_\delta^{0.25}\ Pr^{0.012}\left(\frac{L}{\delta}\right)^{-0.3} = 0.42(3.37 \times 10^6)^{0.25}\ (0.71)^{0.012}\ (30)^{-0.3} = 6.46$$

$$h_c = Nu_\delta\ \frac{k}{\delta} = 6.46\ \ \frac{0.0241\ \mathrm{W/(m\ K)}}{0.1\mathrm{m}} = 1.56\ \ \mathrm{W/(m^2K)}$$

The rate of heat loss is

$$q = h_c\, A\ (T_i - T_o) = \ \ 1.56\ \mathrm{W/(m^2K)}\ \ \ (3\ \mathrm{m})\ (6\ \mathrm{m})\ (20°\mathrm{C} + 8°\mathrm{C}) = 786\ \mathrm{W}$$

$$h_c = Nu\delta\ \frac{k}{\delta} = 7.95\ \ \frac{0.0241\ \mathrm{W/(m\ K)}}{0.1\mathrm{m}} = 1.92\ \ \mathrm{W/(m^2K)}$$

(b) With baffles at mid-height, the Rayleigh number is unchanged, but L = 1.5 m, L/δ = (1.5 m)/(0.1 m) = 15

$$Nu_\delta = 0.42\ (3.37 \times 10^6)^{0.25}\ (0.71)^{0.012}\ (15)^{-0.3} = 7.95$$

$$q = \ \ 1.92\ \mathrm{W/(m^2K)}\ \ \ (3\ \mathrm{m})\ (6\ \mathrm{m})\ (20°\mathrm{C} + 8°\mathrm{C}) = 966\ \mathrm{W}$$

These baffles actually increase the rate of heat transfer by 23%.

(d) The temperature of the vertical baffles is assumed to be approximately equal to the average of the wall temperatures (6°C). From Appendix 2, Table 28, for dry air at the mean temperatures for the two enclosed spaces:

Mean Temperature (°C)	– 1°C (estimated) 13°C	
β (1/K)	0.00365	0.00350
k (W/(m K))	0.0236	0.0246
N × 10^6 (m^2/s)	13.5	15.1
Pr	0.71	0.71

The Rayleigh numbers for the two sections are

$$Ra_\delta = Gr_\delta\, Pr = \frac{g\beta(T_s - T_\infty)\left(\dfrac{\delta}{2}\right)^3\ Pr}{\nu^2}$$

For the inside section

$$Ra_\delta = \frac{(9.8\,\text{m/s}^2)\ 0.0035\ 1/\text{K}\ \ (20°\text{C} - 6°\text{C})(0.05\,\text{m})^3(0.71)}{15.1\times 10^{-6}\,\text{m}^2/\text{s}^{\ 2}} = 1.87 \times 10^5$$

For the outside section

$$Ra_\delta = \frac{(9.8\,\text{m/s}^2)\ 0.00365\ 1/\text{K}\ \ (6°\text{C} + 8°\text{C})(0.05\,\text{m})^3(0.71)}{13.5\times 10^{-6}\,\text{m}^2/\text{s}^{\ 2}} = 2.44 \times 10^5$$

$$R_{ci} = \frac{1}{h_c A} = \frac{1}{1.25\ \text{W/(m}^2\text{K)}\ \ (3\,\text{m})(6\,\text{m})} = 0.0443\ \text{K/W}$$

The aspect ratio is $L/\delta = 3/0.05 = 60$

$$Nu_\delta = 0.42\ (1.87 \times 10^5)^{0.25}\ (0.71)^{0.012}\ (60)^{-0.3} = 2.55$$

$$h_c = 2.55\ \frac{0.0246\ \text{W/(m K)}}{0.05\,\text{m}} = 1.25\ \ \text{W/(m}^2\text{K)}$$

Although this is beyond the range of the correlation, Equation (8.29a) will be used to estimate the Nusselt numbers

$$R_{ci} = \frac{1}{h_c A} = \frac{1}{1.28\ \text{W/(m}^2\text{K)}\ \ (3\,\text{m})(6\,\text{m})} = 0.0432\ \text{K/W}$$

Inside section

$$h_c = 2.72\ \frac{0.0236\ \text{W/(m K)}}{0.05\,\text{m}} = 1.28\ \ \text{W/(m}^2\text{K)}$$

Outside section

$$Nu_\delta = 0.42\ (2.44 \times 10^5)^{0.25}\ (0.71)^{0.012}\ (60)^{-0.3} = 2.72$$

These two thermal resistances are in series: therefore, the total resistance is their sum and the rate of heat transfer through the wall is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_i - T_o}{R_{ci} + R_{co}} = \frac{20°\text{C} + 8°\text{C}}{0.0443\ \text{K/W} + 0.0432\ \text{K/W}} = 319.8\ \text{W}$$

This represents a 59% decrease in rate of heat transfer from the unbaffled case.

(d) Since the width of the enclosed space does not enter into the calculation of the heat transfer coefficient, this baffle will have no effect on the rate of heat transfer.

PROBLEM 8.39

A flat plate solar collector of 3 m × 5 m area has an absorber plate that is to operate at a temperature of 70°C. To reduce heat losses, a glass cover is placed 0.05 m from the absorber. Its operating temperature is estimated to be 35°C. Determine the rate of heat loss from the absorber if the 3 m edge is tilted at angles of inclination from the horizontal of 0°, 30°, and 60°.

GIVEN

- A flat plate solar collector
- Area = 3 m × 5 m
- Absorber temperature (T_a) = 70°C
- Glass cover temperature (T_c) = 30°C
- Distance between absorber and cover (δ) = 0.05 m

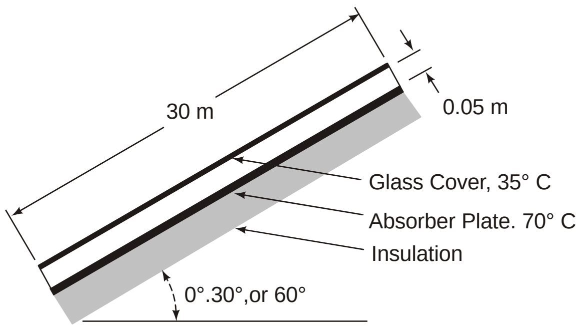
FIND

Heat loss by natural convection from the absorber of angles (θ) of (a) 0°, (b) 30°, and (c) 60° from the horizontal

ASSUMPTIONS

- The space is air filled

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 52.5°C

Thermal expansion coefficient (β) = 0.00307 1/K
Thermal conductivity (k) = 0.0274 W/(m K)
Kinematic viscosity (ν) = 18.7 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

The Grashof and Rayleigh numbers for this geometry are

$$Gr_\delta = \frac{g\,\beta(T_s - T_\infty)\delta^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.00307\ 1/\text{K}\ (70°\text{C} - 35°\text{C})(0.05\,\text{m})^3}{(18.7 \times 10^{-6}\text{m}^2/\text{s})^2} = 3.76 \times 10^5$$

$$Ra_\delta = Gr_\delta\, Pr = 3.76 \times 10^5\,(0.71) = 2.67 \times 10^5$$

The heat transfer coefficient is given by Equation (8.31), where the quantities enclosed by [ ] are to be set to zero if they are negative: At θ = 0°.

Since the aspect ratio (L/δ) = 3/0.05 = 60, the critical angle is 70°

$$h_c = 2.75 \text{ W/(m}^2\text{K)}$$

The rate of natural convective heat transfer is given by

$$q_c = h_c A(T_a - T_c) = 2.75 \text{ W/(m}^2\text{K)} \ (3 \text{ m}) (5 \text{ m}) (70°\text{C} - 35°\text{C}) = 1444 \text{ W/m}$$

Performing a similar calculation for the other angles yields the following results

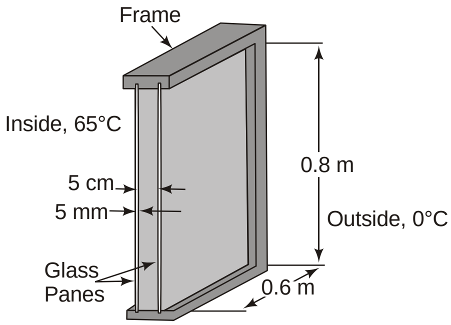
Angle, θ (degrees)	h_c (W/(m^2 K))	q_c (W)
0	2.75	1444
30	2.65	1390
60	2.32	1221

COMMENTS

Heat transfer by radiation will also be significant in this case.

PROBLEM 8.40

Determine the rate of heat loss through a double paned window, as shown in the sketch, if the inside room temperature is 65°C and the average outside air is 0°C during December. Neglect the effect of the window frame.



If the house is electrically heated at a cost of \$0.06/(kW hr), estimate the savings achieved with a double-pane compared to a single-pane window during December.

GIVEN

- A double-paned window as shown
- Inside room temperature ($T_{\infty i}$) = 65°C
- Outside air temperature ($T_{\infty o}$) = 0°C
- Cost of heating = \$0.06/(kW hr)

FIND

(a) The rate of heat loss through a double paned window
(b) The saving of double paning over single paning

ASSUMPTIONS

- Saving can be based on steady state analysis
- Inside and outside air is still
- Radiative heat transfer is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 11, thermal conductivity of window glass (k_g) = 0.81 W/(m K)

SOLUTION

A single glazed window will be analyzed first

$$Gr_H = \frac{g\beta(T_s - T_\infty)\,H^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.00311\ 1/\text{K}\ (65°\text{C} - 32.5°\text{C})(0.8\,\text{m})^3}{\left(18.4\times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 1.50\times 10^9$$

Since the temperature of the glass is unknown, an iterative procedure is required. For the first iteration, let T_g = 32.5°C.

From Appendix 2, Table 28, for dry air

Mean Temperature (°C)	16.3	48.8
Thermal expansion coefficient, β (1/K)	0.00346	0.00311
Thermal conductivity, k (W/(m K))	0.0248	0.0271
Kinematic viscosity, $\nu \times 10^6$ (m²/s)	15.4	18.4
Prandtl Number, Pr	0.71	0.71

The Grashof number based on the window height is
Inside

$$Gr_H = \frac{g\,\beta\,(T_s - T_\infty)\,H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ \ 0.00311\ 1/\text{K}\ \ (65°\text{C} - 32.5°\text{C})\,(0.8\,\text{m})^3}{18.4 \times 10^{-6}\,\text{m}^2/\text{s}\ \ ^2} = 1.50 \times 10^9$$

Outside

$$Gr_H = \frac{g\,\beta\,(T_s - T_\infty)\,H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ \ 0.00346\ 1/\text{K}\ \ (32.5°\text{C} - 0°\text{C})\,(0.8\,\text{m})^3}{15.4 \times 10^{-6}\ \text{m}^2/\text{s}\ \ ^2} = 2.38 \times 10^9$$

Since $Gr_H > 10^9$, the Nusselt numbers are given by Equation (8.13)

$$Nu_L = 0.13 \quad Gr_L Pr^{\frac{1}{3}}$$

Inside

$$Nu_L = 0.13\left[2.38 \times 10^9 (0.71)\right]^{\frac{1}{3}} = 154.8$$

$$h_c = Nu_L\ \frac{k}{L} = 154.8\ \frac{0.0248\ \text{W/(m K)}}{0.8\,\text{m}} = 4.80\ \text{W/(m}^2\text{K)}$$

Outside

$$Nu_L = 0.13\left[2.38 \times 10^9 (0.71)\right]^{\frac{1}{3}} = 154.8$$

$$h_c = Nu_L\ \frac{k}{L} = 154.8\ \frac{0.0248\ \text{W/(m K)}}{0.8\,\text{m}} = 4.80\ \text{W/(m}^2\text{K)}$$

The rate of convection inside and outside must be the same

$$h_{ci}\,A\,(T_{\infty i} - T_g) = h_{co}\,A\,(T_g - T_{\infty o})$$

Solving for the glass temperature

$$T_g = \frac{h_{ci}\,T_{\infty i} + h_{co}\,T_{\infty o}}{h_{ci} + h_{co}} = \frac{4.5\ \text{W/(m}^2\ \text{K)}\ \ (65°\text{C}) +\ 4.8\ \text{W/(m}^2\ \text{K)}\ \ (0°\text{C})}{(4.5 + 4.8)\ \text{W/(m}^2\text{K)}} = 31.5°\text{C}$$

This is close enough to the initial guess that the heat transfer coefficients will not be re-calculated. The thermal circuit for the single glazed window is shown below
where

$$R_{co} = \frac{1}{h_{co}A} = \frac{1}{4.8\ \text{W/(m}^2\text{K)}\ \ (0.8\,\text{m})(0.6\,\text{m})} = 0.434\ \text{K/W}$$

$$R_k = \frac{t_g}{k_g A} = \frac{0.005\,\text{m}}{0.81\ \text{W/(m K)}\ \ (0.8\,\text{m})(0.6\,\text{m})} = 0.0129\ \text{K/W}$$

$$R_{ci} = \frac{1}{h_{ci}A} = \frac{1}{4.5\ \text{W/(m}^2\text{K)}\ \ (0.8\,\text{m})(0.6\,\text{m})} = 0.463\ \text{K/W}$$

The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_{\infty i} - T_{\infty o}}{R_{co} + R_k + R_{ci}} = \frac{65°\text{C} - 0°\text{C}}{(0.434 + 0.0129 + 0.463)\,\frac{\text{K}}{\text{W}}} = 71.4\ \text{W}$$

For the double glazed case, the temperature of the inside and outside panes must be estimated for the first iteration. Let T_{go} = 16°C and T_{gi} = 49°C (by symmetry).

From Appendix 2, Table 28, for dry air

	Outside	Enclosure	Inside
Mean Temperature (°C)	8	32.5	57
β (1/K)	0.00356	0.00327	0.00303
k (W/(m K))	0.0243	0.0260	0.0277
$\nu \times 10^6$ (m²/s)	14.6	16.9	19.1
Pr	0.71	0.71	.071

The Grashof numbers are

Inside

$$Gr_H = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ 0.00303\ 1/\text{K}\ (65°\text{C} - 49°\text{C})(0.8\,\text{m})^3}{\left(19.1 \times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 6.67 \times 10^8$$

Outside

$$Gr_H = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ 0.00356\ 1/\text{K}\ (16°\text{C} - 0°\text{C})(0.8\,\text{m})^3}{\left(14.6 \times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 1.34 \times 10^9$$

Enclosed space

$$Gr_\delta = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ 0.00327\ 1/\text{K}\ (49°\text{C} - 16°\text{C})(0.5\,\text{m})^3}{\left(16.9 \times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 4.63 \times 10^5$$

$$Ra_\delta = Gr_\delta Pr = 4.63 \times 10^5\ (0.71) = 3.29 \times 10^5$$

Using the correlation in Equation (8.13)

Inside

$$Nu_L = 0.13\ (Gr_L Pr)^{\frac{1}{3}} = 0.13\left[6.67 \times 10^8 (0.71)\right]^{\frac{1}{3}} = 101$$

$$h_{ci} = Nu_L\ \frac{k}{L} = 101\ \frac{0.0277\ \text{W/(m K)}}{0.8\ \text{m}} = 3.51\ \text{W/(m}^2\text{K)}$$

Outside

$$Nu_L = 0.13\left[1.34 \times 10^9 (0.71)\right]^{\frac{1}{3}} = 128$$

$$h_{co} = Nu_L\ \frac{k}{L} = 128\ \frac{0.0243\ \text{W/(m K)}}{0.8\ \text{m}} = 3.88\ \text{W/(m}^2\text{K)}$$

Enclosed space: H/δ = 0.8 m/0.05 m = 16. Although Pr < 1, the correlation in Equation (8.29a) will be applied to estimate the Nusselt number for the enclosed space

$$Nu_\delta = 0.42\ Ra_\delta^{0.25}\ Pr^{0.012}\left(\frac{H}{\delta}\right)^{-0.3} = 0.42\ (3.29 \times 10^5)^{0.25}\ (0.71)^{0.012}\ (16)^{-0.3} = 4.36$$

$$H_{c\delta} = Nu_\delta\ \frac{k}{L} = 4.36\ \frac{0.0260\ \text{W/(m K)}}{0.05\ \text{m}} = 2.27\ \text{W/(m}^2\text{K)}$$

The thermal circuit for the double glazed window is shown below

The thermal resistance of the glass (Rk_g) is the same as the single glazed case. The remaining thermal resistances are

$$R_{ci} = \frac{1}{h_{ci} A} = \frac{1}{3.51\,\mathrm{W/(m^2\,K)}\;(0.8\,\mathrm{m})\;(0.6\,\mathrm{m})} = 0.594\ \mathrm{K/W}$$

$$R_{c\delta} = \frac{1}{h_{c\delta} A} = \frac{1}{2.27\,\mathrm{W/(m^2 K)}\;(0.8\,\mathrm{m})\;(0.6\,\mathrm{m})} = 0.918\ \mathrm{K/W}$$

$$R_{co} = \frac{1}{h_{co} A} = \frac{1}{3.88\,\mathrm{W/(m^2 K)}\;(0.8\,\mathrm{m})\;(0.6\,\mathrm{m})} = 0.537\ \mathrm{K/W}$$

The rate of heat transfer is

$$q = \frac{\Delta T}{R_{\mathrm{total}}} = \frac{T_{\infty i} - T_{\infty o}}{R_{ci} + R_c\delta + R_{co} + 2R_{kg}} = \frac{65°\mathrm{C} - 0°\mathrm{C}}{[0.594 + 0.918 + 0.537 + 2\,(0.0129)]\,\mathrm{K/W}} = 31.3\ \mathrm{W}$$

The inner and outer glass temperatures can be checked by the convection equations for those surfaces
Inside

$$q = h_{ci}\, A\, (T_i - T_{gi})$$

$$T_{gi} = T_i - \frac{q}{h_{ci} A} = 65°\mathrm{C} - \frac{31.3\ \mathrm{W}}{3.51\,\mathrm{W/(m^2 K)}\;(0.8\,\mathrm{m})(0.6\,\mathrm{m})} = 46.4°\mathrm{C}$$

Outside

$$T_{go} = T_o + \frac{q}{h_{co} A} = 0°\mathrm{C} + \frac{31.3\ \mathrm{W}}{3.88\,\mathrm{W/(m^2 K)}\;(0.8\,\mathrm{m})\;(0.6\,\mathrm{m})} = 16.8°\mathrm{C}$$

These are close enough to the initial guesses that another iteration is not warranted.

The savings of the double glazed over the single glazed window are

$$\text{Savings} = (q_{\mathrm{double}} - q_{\mathrm{single}})\ (\text{Cost of heating})$$

$$\text{Savings} = (71.4\ \mathrm{W} - 31.3\ \mathrm{W})\left(\frac{\$0.06}{\mathrm{kWh}}\right)\left(\frac{\mathrm{kW}}{1000\,\mathrm{W}}\right)\ 24\,\mathrm{h/day} = \frac{\$0.06}{\mathrm{day}}$$

This one small double glazed window saves 6 cents per day.

COMMENTS

The two surfaces of each pane of glass will actually be at a slightly different temperature. This can be neglected because the convective resistance are an order of magnitude greater than the conductive resistance of the glass.

PROBLEM 8.41

> **Calculate the rate of heat transfer between a pair of concentric horizontal cylinders 20 mm and 126 mm in diameter. The inner cylinder is maintained at 37°C and the outer cylinder is maintained at 17°C.**

GIVEN

- Concentric cylinders
- Smaller diameter (D_i) = 20 mm = 0.02 m
- Larger diameter (D_o) = 126 mm = 0.126 m
- Inner cylinder temperature (T_i) = 37°C
- Outer cylinder temperature (T_o) = 17°C

FIND

The rate of heat transfer (q)

ASSUMPTIONS

- Steady state
- The space between the cylinders is filled with air
- Radiative heat transfer is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 27°C

Thermal expansion coefficient (β) = 0.00333 1/K
Thermal conductivity (k) = 0.0256 W/(m K)
Kinematic viscosity (ν) = 16.4×10^{-6} m²/s
Prandtl number (Pr) = 0.71

SOLUTION

$$b = \frac{D_o - D_i}{2} = \frac{0.126\,\text{m} - 0.02\,\text{m}}{2} = 0.053 \text{ m}$$

The Grashof number based on the space between the cylinders is

$$Gr_b = \frac{g\,\beta\,(T_s - T_\infty)\,b^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\;0.00333\text{ 1/K}\;(37°\text{C} - 17°\text{C})(0.053\,\text{m})^3}{\left(16.4 \times 10^{-6}\ \text{m}^2/\text{s}\right)^2} = 3.61 \times 10^5$$

The Rayleigh number is

$$Ra_b = Gr_b\,Pr = 3.61 \times 10^5\,(0.71) = 2.57 \times 10^5$$

To use the correlation given in Equation (8.33), the following criteria must be satisfied

$$10 \le \left[\frac{\ln\left(\dfrac{D_o}{D_i}\right)}{b^{\frac{3}{4}}\left(\dfrac{1}{D_i^{\frac{3}{5}}} + \dfrac{1}{D_o^{\frac{3}{5}}}\right)^{\frac{5}{4}}}\right]^4 Ra_b < 10^7$$

$$\left[\frac{\ln\left(\frac{0.126}{0.02}\right)}{(0.053)^{\frac{3}{4}}\left(\frac{1}{(0.02)^{\frac{3}{5}}}+\frac{1}{(0.126)^{\frac{3}{5}}}\right)^{\frac{5}{4}}}\right]^4 (2.57 \times 10^5) = (0.6196)^4\,(2.57 \times 10^5) = 3.79 \times 10^4$$

The effective thermal conductivity of the air in the gap between the cylinders is given by Equation (8.33)

$$k_{eff} = 0.386\, k \left[\frac{\ln\left(\frac{D_o}{D_i}\right)}{b^{\frac{3}{4}}\left(\frac{1}{D_i^{\frac{3}{5}}}+\frac{1}{D_o^{\frac{3}{5}}}\right)^{\frac{5}{4}}}\right]^4 \left(\frac{Pr}{0.861+Pr}\right)^{\frac{1}{4}} Ra_b^{\frac{1}{4}}$$

$$k_{eff} = 0.386 \quad 0.0386\ \text{W/(m K)} \quad [0.6196] \left(\frac{0.71}{0.861+0.71}\right)^{\frac{1}{4}} \quad 2.57\times 10^5 \ {}^{\frac{1}{4}} = 0.113\ \text{W/(m K)}$$

The rate of heat transfer is given by Equation (2.38)

$$q_k = \frac{T_o - T_i}{R_{th}}$$

where R_{th} is given by substituting k_{eff} for k in Equation (2.39)

$$R_{th} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\,\Pi\,\text{L}\,\text{k}_{\text{eff}}} = \frac{\ln\left(\frac{0.126}{0.02}\right)}{2\,\Pi\,\text{L}\ \ 0.113\ \text{W/(m}^2\ \text{K)}} = 2.59\ \frac{1}{L}\ \text{mK/W}$$

$$\therefore\ q_k = \frac{37°\text{C} - 17°\text{C}}{2.59\,\frac{1}{L}\ \text{(mK/W)}} \Rightarrow \frac{q_k}{L} = 7.72\ \text{W/m}$$

PROBLEM 8.42

Two long concentric horizontal aluminum tubes of 0.2 m and 0.25 m diameter are maintained at 300 K and 400 K respectively. The space between the tubes is filled with nitrogen. If the surfaces of the tubes are polished to prevent radiation, estimate the rate of heat transfer for gas pressure of (a) 1 MPa and (b) 10 kPa in the annulus.

GIVEN

- Two concentric horizontal aluminum tubes with nitrogen between them
- Diameters
  - D_i = 0.2 m
  - D_o = 0.25 m
- Temperatures
  - T_i = 300 K
  - T_o = 400 K
- Surface of tubes is polished

FIND

The rate of heat transfer for

(a) Pressure (p_a) = 1 MPa
(b) Pressure (p_b) = 10 kPa

ASSUMPTIONS

- Steady state conditions
- Radiative heat transfer is negligible
- Only the density of the nitrogen is affected by the pressure

PROPERTIES AND CONSTANTS

From Appendix 2, Table 33, for Nitrogen at one atmosphere and the mean temperature of 350 K

Thermal expansion coefficient (β) = 0.00292 1/K
Thermal conductivity (k) = 0.02978 W/(m K)
Absolute viscosity (μ) = 19.91 × 10^{-6} (Ns)/m^2
Density (ρ) = 0.9980 Kg/m^3
Prandtl number (Pr) = 0.702

Correcting the density for Pressure

(a) At p_a = 1 MPa: $\dfrac{\rho_a}{\rho} = \dfrac{p_a}{1}$ atm = 10 $\quad \rho_a = 9.98\ \text{kg/m}^3 - \nu_a = \dfrac{\mu}{\rho_a} = 1.99 \times 10^{-6}\ \text{m}^2/\text{s}$

(b) At p_b = 10 kPa: $\dfrac{\rho_b}{\rho} = \dfrac{p_b}{1}$ atm = 0.1 $\quad \rho_a = 0.0998\ \text{kg/m}^3 - \nu_b = \dfrac{\mu}{\rho_b} = 199 \times 10^{-6}\ \text{m}^2/\text{s}$

SOLUTION

The gap between the cylinders (b) = ($D_o - D_i$)/2 = 0.025 m

The Grashof and Rayleigh numbers based on the gap between the cylinders (b) are

Case (a)

$$Gr_b = \frac{g\,\beta\,(T_s - T_\infty)\,b^3}{\nu^2} = \frac{(9.8\ \text{m/s}^2)\ 0.00292\ 1/\text{K}\ (400\,\text{K} - 300\,\text{K})(0.025\,\text{m})^3}{(1.99 \times 10^{-6}\,\text{m}^2/\text{s})^2} = 1.13 \times 10^7$$

$$Ra_b = Gr_b\,Pr = 1.13 \times 10^7\ (0.702) = 7.93 \times 10^6$$

Case (b) $Gr_b = 1.13 \times 10^3$ $Ra_b = 793$

To use the correlation of Equation (8.33), the following criteria must be met

For case (b)

$$10 \le \left[\frac{\ln\left(\frac{D_o}{D_i}\right)}{b^{\frac{3}{4}}\left(\frac{1}{D_i^{\frac{3}{5}}} + \frac{1}{D_o^{\frac{3}{5}}}\right)^{\frac{5}{4}}} \right]^4 Ra_b < 10^7$$

$$\left[\frac{\ln\left(\frac{0.25}{0.2}\right)}{(0.025)^{\frac{3}{4}}\left(\frac{1}{(0.2)^{\frac{3}{5}}} + \frac{1}{(0.25)^{\frac{3}{5}}}\right)^{\frac{5}{4}}} \right]^4 (793) = (0.4839)^4\ (793) = 59.8$$

For case (a): $(0.4838)^4\ (7.93 \times 10^6) = 4.35 \times 10^5$

Therefore, the condition is met for both cases.

The effective thermal conductivity of the gap is given by Equation (8.33)

$$k_{\text{eff}} = 0.386\ k \left[\frac{\ln\left(\frac{D_o}{D_i}\right)}{b^{\frac{3}{4}}\left(\frac{1}{D_i^{\frac{3}{5}}} + \frac{1}{D_o^{\frac{3}{5}}}\right)^{\frac{5}{4}}} \right] \left(\frac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}} Ra_b^{\frac{1}{4}}$$

Case(a): $k_{\text{eff}} = 0.386\ \ 0.02978\ \text{W/(mK)}\ \ [0.4839]\ \left(\frac{0.702}{0.861+0.702}\right)^{\frac{1}{4}} (793\times 10^6)^{\frac{1}{4}} = 0.242\ \ \text{W/(mK)}$

Case(b): $k_{\text{eff}} = 0.386\ \ 0.02978\ \text{W/(mK)}\ \ [0.4839]\ \left(\frac{0.702}{0.861+0.702}\right)^{\frac{1}{4}} (793)^{\frac{1}{4}} = 0.0242\ \ \text{W/(mK)}$

The rate of heat transfer is given by Equations (2.38) and (2.39)

$$q = \frac{\Delta T}{R_{th}} = \frac{2\pi L k_{\text{eff}} (T_o - T_i)}{\ln\left(\frac{r_o}{r_i}\right)}$$

$$\frac{q}{L} = \frac{2\pi (400\,\text{K} - 300\,\text{K})}{\ln\left(\frac{0.25}{0.2}\right)}\ k_{\text{eff}} = (2815.7\text{K})\ k_{\text{eff}}$$

Case (a) q/L = 681 W/m   Case (b): q/L = 68.1 W/m

PROBLEM 8.43

A solar collector design consists of several parallel tubes each enclosed concentrically in an outer tube that is transparent to solar radiation. The tubes are thin walled with the inner and outer cylinder diameters of of 0.10 and 0.15 m respectively. The annular space between the tubes is filled with air at atmospheric pressure. Under operating condition the inner and outer tube surface temperatures are 70°C and 30°C respectively.

(a) What is the convective heat loss per meter of tube length?
(b) If the emissivity of the outer surface of the inner tube is 0.2 and the outer cylinder behaves as though it were a black body, estimate the radiation loss.
(c) Discuss design options for reducing the total heat loss.

GIVEN

- Thin walled concentric tubes with air atmospheric pressure between them
- Inner tube diameter (D_i) = 0.1 m
- Outer tube diameter (D_o) = 0.15 m
- Inner tube temperature (T_i) = 70°C = 343 K
- Outer tube temperature (T_o) = 30°C = 303 K
- Outer surface emissivity of inner tube (ε) = 0.2

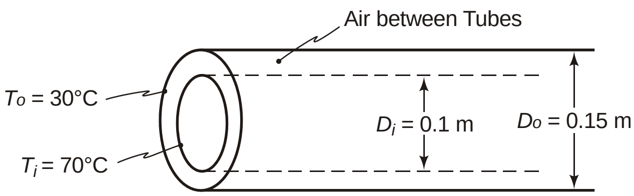
FIND

(a) The convective loss pe meter of tube (q_c/L)
(b) The radiative loss (q_r/L)
(c) Discuss design options for reducing the total heat loss

ASSUMPTIONS

- Steady state
- Tubes are horizontal

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, The Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Appendix 2, Table 28, for dry air at the mean temperature of 50°C

Thermal expansion coefficient (β) = 0.00310 1/K
Thermal conductivity (k) = 0.0272 W/(m K)
Kinematic viscosity (ν) = 18.5×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

(a) The characteristic length for the Problem is the air gap

$$b = \frac{D_o - D_i}{2} = \frac{0.15\,\text{m} - 0.1\,\text{m}}{2} = 0.025 \text{ m}$$

The Rayleigh number based on the characteristic length is

$$Ra_b = Gr_b\, Pr = \frac{g\,\beta (T_s - T_\infty)\, b^3 Pr}{\nu^2}$$

$$= \frac{(9.8\ \text{m/s}^2)\ 0.0031\ 1/\text{K}\ (70°\text{C} - 30°\text{C})(0.025\,\text{m})^3 (0.71)}{18.5 \times 10^{-6}\,\text{m}^2/\text{s}\ ^2} = 3.94 \times 10^4$$

The correlation for this geometry is given in Equation (8.33). Its use is restricted to the following condition

$$10 \le \left[\ln\left(\frac{D_0}{D_i}\right) \Big/ \left(b^4 \left(\frac{1}{D_i^{3/5}} + \frac{1}{D_0^{3/5}} \right) \right) \right]^4 Ra_b < 10^7$$

$$\left[\ln\left(\frac{0.15}{0.1}\right) \Big/ \left(0.025^4 \left(\frac{1}{0.1^{3/5}} + \frac{1}{0.15^{3/5}} \right) \right) \right]^4 (3.94 \times 10^4) = (0.556)^4\, (3.94 \times 10^4) = 3.77 \times 10^3$$

Therefore, the condition is met.

The effective thermal conductivity of the gap is

$$k_{\text{eff}} = 0.386\, k \left[\ln\left(\frac{D_0}{D_i}\right) \Big/ \left(b^4 \left(\frac{1}{D_i^{3/5}} + \frac{1}{D_0^{3/5}} \right) \right) \right] \left(\frac{Pr}{0.861 + Pr} \right)^{\frac{1}{4}} Ra_b^{\frac{1}{4}}$$

$$k_{\text{eff}} = 0.386\ \ 0.0272\ \text{W/(m K)}\ \ [0.556] \left(\frac{0.71}{0.861 + 0.71} \right)^{\frac{1}{4}} (3.94 \times 10^4)^{\frac{1}{4}} = 0.0674\ \ \text{W/(m K)}$$

The convective heat transfer per unit length across the gap is given by Equation (2.38) and (2.39)

$$\frac{q_c}{L} = \frac{(T_i - T_o)\, 2\pi\, k_{\text{eff}}}{\ln\left(\frac{D_o}{D_i}\right)} = \frac{(70°\text{C} - 30°\text{C})\, 2\pi \left(0.0674\ \text{W/(m K)}\right)}{\ln\left(\frac{0.15}{0.1}\right)} = 41.8\ \text{W/m}$$

(b) Since the inner tube is completely surrounded by the outer tube, the radiative heat transfer is given by Equation (1.17)

$$\frac{q_r}{L} = \pi D_i\, \varepsilon\, \sigma (T_i^4 - T_o^4) = \pi (0.1\ \text{m})\ (0.2)\ \ 5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)\ \ [(343\ \text{K})^4 - (303\ \text{K})^4] = 19.3\ \text{W}$$

The total rate of heat transfer is the sum of the convective and radiative components

$$\frac{q_{\text{total}}}{L} = \frac{q_c}{L} + \frac{q_r}{L} = 41.8\ \text{W/m} + 19.3\ \text{W/m} = 61.1\ \text{W/m}$$

(c) Evacuating the space between the tubes would eliminate the convective heat transfer and thereby reduce the total rate of heat transfer by 67%. The heat loss could be further decreased by decreasing the emissivity of both cylinders.

PROBLEM 8.44

Liquid oxygen at -183°C is stored in a thin walled spherical container with an outside diameter of 2 m. This container is surrounded by another sphere of 2.5 m inside diameter to reduce heat loss. The inner spherical surface has an emissivity of 0.05 and the outer sphere is black. Under normal operation the space between the spheres is evacuated but an accident resulted in a leak in the outer sphere and the space is filled with air at 100 kPa. If the outer sphere is at 25°C, compare the heat losses before and after the accident.

GIVEN

- A sphere filled with liquid oxygen surrounded by a larger sphere
- Sphere diameters
  - D_i = 2 m
  - D_o = 2.5 m
- Emissivity of inner sphere (ε) = 0.05
- Outer sphere temperature (T_o) = 25°C = 298 K
- Liquid oxygen temperature (T_i) = –183° = 90 K
- Outer sphere is black

FIND

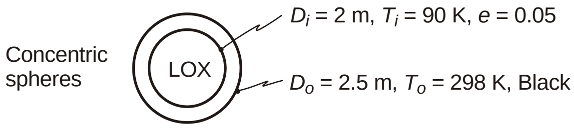
The rate of heat loss with

(a) A vacuum between the spheres
(b) Air at 100 kPa between the spheres

ASSUMPTIONS

- Steady state
- The internal convective resistance and the resistance of the inner sphere wall are negligible

SKETCH



PROPERTIES AND CONSTANTS

The thermal expansion coefficient (β) ≈ $1/T$ = 1/(194 K) = 0.0052 1/K

Extrapolating from Appendix 2, Table 27, for dry air at the mean temperature of –79°C from values at 0°C and 20°C

Thermal conductivity (k) = 0.018 W/(m K)
Kinematic viscosity (ν) = 6.8×10^{-6} m²/s
Prandtl number (Pr) = 0.71

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m² K⁴)

SOLUTION

(a) With the space evacuated, there will only be radiative heat transfer as given by Equation (1.17)

$$q_r = A_i\, \varepsilon_i\, \sigma (T_o^4 - T_i^4) = \pi D_i^2\, \varepsilon_1\, \sigma (T_o^4 - T_i^4)$$

$$q_r = \pi (2\text{m})^2\, (0.05)\;\; 5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)\;\; [(298\text{ K})^4 - (90\text{K})^4] = 278.6\text{ W}$$

The characteristic length for the problem is: $b = (D_o - D_i)/2$ = (2.5 m – 2.0 m)/2 = 0.25 m

The Rayleigh number is

$$Ra_b = Gr_b\,Pr = \frac{g\beta(T_o - T_i)\,b^3 Pr}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.0052\ 1/\text{K}\ \ (298\,\text{K} - 90\,\text{K})(0.25\,\text{m})^3(0.71)}{6.8\times10^{-6}\,\text{m}^2/\text{s}\ \ ^2} = 2.54\times10^9$$

(b) The following criteria must be satisfied to use Equation (8.34) for the convective heat transfer

$$10 \le \left[\frac{b}{(D_o - D_i)^4\left(D_i^{-\frac{7}{5}} + D_o^{-\frac{7}{5}}\right)^5}\right] Ra_b < 10^7$$

$$\left[\frac{0.25\,\text{m}}{(2\,\text{m})(2.5\,\text{m})\ ^4\left((2.5\,\text{m})^{-\frac{7}{5}} + (2.5\,\text{m})^{-\frac{7}{5}}\right)^5}\right] 2.54\times10^9 = (0.0032879)\ (2.54\times10^9) = 8.36\times10^6$$

Therefore, the condition is met.

The effective thermal conductivity of the air space is

$$k_{\text{eff}} = 0.74\,k\left[\frac{b^{\frac{1}{4}}}{D_oD_i\left(D_i^{-\frac{7}{5}} + D_o^{-\frac{7}{5}}\right)^{\frac{5}{4}}}\right] Ra_b^{\frac{1}{4}}\left(\frac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}}$$

$$k_{\text{eff}} = 0.74\ \ 0.018\,\text{W/(mK)}\left[\frac{(0.25\,\text{m})^{\frac{1}{4}}}{(2\,\text{m})(2.5\,\text{m})\left((2\,\text{m})^{-\frac{7}{5}} + (2.5)^{-\frac{7}{5}}\right)^{\frac{5}{4}}}\right](2.54\times10^9)^{\frac{1}{4}}\left(\frac{0.71}{0.861 + 0.71}\right)^{\frac{1}{4}}$$

$$k_{\text{eff}} = 0.059\ \ \text{W/(m\,K)}$$

The total rate of heat transfer will be the sum of the convective and radiative heat transfer

$$q = q_c + q_r = \frac{T_o - T_i}{R_{\text{eff}}} + q_r$$

Where R_{eff} is given by Equation (2.51)

$$R_{\text{eff}} = \frac{r_o - r_i}{4\pi k_{\text{eff}}\ r_o r_i} = \frac{D_o - D_i}{2\pi k_{\text{eff}}\ D_oD_i} = \frac{0.5\,\text{m}}{2\pi\ \ 0.059\ \text{W/(m\,K)}\ \ (2\,\text{m})(2.5\,\text{m})} = 0.270\ \text{K/W}$$

$$q = \frac{298\text{K} - 90\text{K}}{0.270\ \text{K/W}} + 278.6\ \text{W} = 771.1\ \text{W} + 278.6\text{W} = 1050\ \text{W}$$

The leak causes the rate of heat loss to increase 3.8 times

COMMENTS

The rate of convective heat transfer is about 73% of the total rate of heat transfer.

PROBLEM 8.45

The surfaces of two concentric spheres having radii of 75 and 100 mm are maintained at 325 K and 275 K, respectively.

(a) If the space between the spheres is filled with nitrogen at 500 kPa, estimate the convection heat transfer rate.

(b) If both sphere surfaces are black, estimate the total rate of heat transfer between them.

(c) Suggest ways to reduce the heat transfer.

GIVEN

- Concentric spheres with nitrogen between them
- Nitrogen pressure (p) = 500 kPa
- Sphere radii
  - Inner sphere (r_i) = 75 mm = 0.075 m
  - Outer sphere (r_o) = 100 mm = 0.1 m
- Sphere temperatures
  - Inner sphere (T_i) = 325 K
  - Outer sphere (T_o) = 275 K
- Both spheres are black

FIND

(a) Convective heat transfer (q_c)
(b) Total heat transfer (q_{total})
(c) Suggest ways to reduce the heat transfer

ASSUMPTIONS

- Sphere temperatures are constant and uniform
- Only the density of the nitrogen is affected significantly by pressure
- The nitrogen behaves as an ideal gas

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann consant (σ) = 5.67×10^{-8} W/(m^2 K^4)

From Appendix 2, Table 33, for Nitrogen at atmospheric pressure and the mean temperature of 300 K

Thermal expansion coefficient (β) = 0.00333 1/K
Thermal conductivity (k) = 0.02620 W/(m K)
Absolute viscosity (μ) = 17.84×10^{-6} N s/m^2
Density (ρ) = 1.142 kg/m^3
Prandtl number (Pr) = 0.713

The density of the nitrogen at 500 kPa can be calculated from the ideal gas law

$$\rho_2 = \frac{p_2}{p_1}\,\rho_1 = \frac{5\,\text{atm}}{1\,\text{atm}}\;1.1421\,\text{kg/m}^3 \;= 5.7105\ \text{kg/m}^3$$

$$\therefore \quad \text{The kinematic viscosity } (\nu) = \frac{\mu}{\rho} = \frac{17.84\times10^{-6}\,(\text{Ns})/\text{m}^2}{5.7105\,\text{kg/m}^3} = 3.124\ 10^{-6}\ \text{m}^2/\text{s}$$

SOLUTION

(a) The effective thermal conductivity of the nitrogen is given by Equation (8.34)

$$k_{\text{eff}} = 0.74\, k \left[b^{\frac{1}{4}} \Big/ \left(D_o D_i \left(D_i^{-\frac{7}{5}} + D_o^{-\frac{7}{5}} \right)^{\frac{5}{4}} \right) \right] Ra_b^{\frac{1}{4}} \left(\frac{Pr}{0.861 + Pr} \right)^{\frac{1}{4}}$$

where $b = r_o - r_i = 25 \text{ mm} = 0.025 \text{ m}$

$$Ra_b = Gr_b\, Pr = \frac{g\beta(T_i - T_o)\, b^3 Pr}{\nu^2} = \frac{(9.8 \text{ m/s}^2)\ 0.00333 \text{ 1/K}\ (325\text{K} - 275\text{K})(0.025\text{m})^3(0.713)}{3.124 \times 10^{-6} \text{m}^2/\text{s}^{\ 2}}$$

$$Ra_b = 1.86 \times 10^6$$

The following condition must be met to use the above correlation

$$10 \le \left[b \Big/ \left((D_o D_i)^4 \left(D_i^{-\frac{7}{5}} + D_o^{-\frac{7}{5}} \right)^5 \right) \right] Ra_b < 10^7$$

$$\left[0.025\text{m} \Big/ \left([(0.2\text{m})(0.15\text{m})]^4 \left((0.15\text{m})^{-\frac{7}{5}} + (0.2\text{m})^{-\frac{7}{5}} \right)^5 \right) \right] 1.86 \times 10^6 = 7.59$$

$\times\ 10^3$

Therefore, the condition is met.

$$k_{\text{eff}} = 0.74\ \ 0.0262 \text{ W/(m K)} \left[(0.025\text{m})^{\frac{1}{4}} \Big/ \left((0.2\text{m})(0.15\text{m}) \left((0.15\text{m})^{-\frac{7}{5}} + (0.2\text{m})^{-\frac{7}{5}} \right)^{\frac{5}{4}} \right) \right]$$

$$(1.86 \times 10^6)^{\frac{1}{4}} \left(\frac{0.713}{0.861 + 0.713} \right)^{\frac{1}{4}}$$

$$k_{\text{eff}} = 0.148 \text{ W/(m K)}$$

The thermal resistance of the nitrogen is given by Equation (2.51)

$$R_{\text{eff}} = \frac{r_o - r_i}{4\pi k_{\text{eff}}\, r_o r_i} = \frac{0.1\text{m} - 0.075\text{m}}{4\pi\ 0.148 \text{ W/(m K)}\ (0.1\text{m})(0.075\text{m})} = 1.792 \text{ K/W}$$

The rate of convective heat transfer is given by

$$q_c = \frac{\Delta T}{R_{\text{eff}}} = \frac{325\text{K} - 275\text{K}}{1.792 \text{ K/W}} = 27.9 \text{ W}$$

(b) The radiative heat transfer from a black body to a black body enclosure is given by Equation (1.16)

$$q_r = A_1\, \sigma(T_1^4 - T_2^4) = 4\, \pi r_i\, \sigma(T_1^4 - T_2^4)$$

$$q_r = 4\, \pi (0.075 \text{ m})^2\ 5.67 \times 10^{-8} \text{W/(m}^2\text{K}^4)\ \ ((325\text{K})^4 - (275\text{K})^4) = 21.8 \text{ W}$$

The total rate of heat transfer is the sum of the radiative and convective heat transfer

$$q_{\text{total}} = q_r + q_c = 21.8 \text{ W} + 27.9 \text{ W} = 49.7 \text{ W}$$

(c) The rate of heat transfer could be reduced in several ways, including

- Coating the spheres to reduce their emissivity, thereby decreasing the rate of radiative heat transfer.

- Partially or totally evacuating the space between the spheres to decrease the rate of convective heat transfer

PROBLEM 8.46

The refrigeration system for an indoor ice rink is to be sized by an HVAC contractor. The refrigeration system has a COP (coefficient of performance) of 0.5. The ice surface is estimated to be –2°C and the ambient air is 24°C. Determine the size of the refrigeration system in kW required for a 110 m diameter circular ice surface.

GIVEN

- Round ice rink
- Diameter (D) = 110 m
- Ice surface temperature (T_s) = – 2°C
- Air temperature (T_∞) = 24°C
- COP of refrigeration system = 0.5

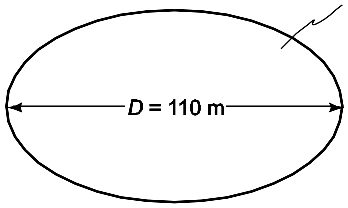
FIND

- Size of the refrigeration system required

ASSUMPTIONS

- Air is quiescent
- The effects of sublimation are negligible
- Radiation heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 11°C

Thermal expansion coefficient (β) = 0.00352 1/K
Thermal conductivity (k) = 0.0245 W/(m K)
Kinematic viscosity (ν) = 14.9×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

The characteristic length (L) for the ice rink is

$$L = \frac{A}{P} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4} = \frac{110\,\text{m}}{4} = 27.5\text{ m}$$

The grashof and Rayleigh numbers are

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.00352\ (1/\text{K})\ \ (24°\text{C} + 2°\text{C})(27.5\,\text{m})^3}{\left(14.9\times 10^{-6}\text{m}^2/\text{s}\right)^2} = 8.40 \times 10^{13}$$

$$Ra_L = Gr_L\,Pr = 8.4 \times 10^{13}\,(0.71) = 5.97 \times 10^{13}$$

Although this is beyond the range of available horizontal plate correlations, the correlation will be extended to estimate the Nusselt number for the ice rink. The correlation for a cooled surface facing downward is Equation (8.16)

$$\overline{Nu}_L = 0.15\ Ra_L^{\frac{1}{3}} = 0.15\ (5.97 \times 10^{13})^{\frac{1}{3}} = 5862$$

$$\bar{h}_c = \overline{Nu}_L\ \frac{k}{L} = 5862\ \frac{0.0245\ \text{W/(m K)}}{27.5\,\text{m}} = 5.22\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer to the rink is

$$q_c = \bar{h}_c\ A\ (T_\infty - T_s) = \bar{h}_c\ \frac{\pi}{4}\ D^2\ (T_s - T_\infty)$$

$$q_c = \quad 5.22\ \text{W/(m}^2\text{K)}\ \ \frac{\pi}{4}\ (110\ \text{m})^2\ (24°\text{C} + 2°\text{C}) = 1.29 \times 10^6\ \text{W} = 1290\ \text{kW}$$

The size of the refrigeration unit (q_{ref}) is

$$q_{\text{ref}} = \frac{q_c}{\text{COP}} = \frac{1290\,\text{kW}}{0.5} = 2580\ \text{kW}$$

PROBLEM 8.47

Consider the problem described in Example 8.5. Show that the transient heating of the water in the pan, assuming the water to be well-mixed and thermally homogenous at any instant in time, can be expressed by the following:

$$\rho V c_p \left(\frac{T_{water,i} - T_{water,t}}{t_i - t} \right)_{water} = \bar{h}_c A \left(100 - T_{water,t} \right)$$

where *V* is the volume of water in the pan and *A* is the area of the bottom surface of the pan. Solve this equation (numerically or otherwise) to determine the time required to heat the water to (a) 50°C and (b) 80°C. Also, determine the total heat transfer to the water in the pan in each case.

GIVEN

- A covered pan kept in stove-top burner
- Depth of pan (L)= 8 cm
- Pan bottom surface temperature (T_s) = 100°C
- Initial water temperature at top (T_∞) = 20°C
- Diameter of the pan (D) = 15 cm

FIND

- Show the expression for heat flow through convection to pan
- Time required and total heat transfer to water while heating the water to 50^0C and 80^0C

ASSUMPTIONS

- Radiation heat transfer is negligible

SKETCH

Refer to Example 8.5

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 60°C

Thermal expansion coefficient (β) = 5.18×10^{-4} 1/K
Thermal conductivity (k) = 0.657 W/(m K)
Kinematic viscosity (ν) = 0.478×10^{-6} m^2/s
Prandtl number (Pr) = 3.02
Density(ρ)= 982.8 kg/m^3
Specific heat capacity (c_p)=4182.8 J/(kg K)

SOLUTION

At any instant of time rate of heat transferred through natural convection from bottom surface to top= Rate of heat gained by water for rise in temperature

If $T_{t,\ water}$ is temperature of water at given instant t and $T_{i,water}$ is water temperature after small time interval t_i, then

Rate of heat gained by water during that instant = $m c_p \left(\frac{T_{water,i} - T_{water,t}}{t_i - t} \right)$

Where m is the mass of water and c_p is the specific heat capacity of water.

If $\bar{h}_c$ is heat transfer coefficient for natural convection at the particular instant then

Heat transferred by natural convection from bottom to top surface= $\bar{h}_c A(100 - T_{water,t})$

Thus $\rho V c_p \left(\dfrac{T_{water,i} - T_{water,t}}{t_i - t} \right)_{water} = \bar{h}_c A\left(100 - T_{water,t}\right)$ where $m = \rho V$

The grashof and Rayleigh numbers are

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8 \text{ m/s}^2)\left(5.18*10^{-4} \text{ (1/K)}\right)(100 - T_{water,t})(0.08)^3}{\left(0.478 \times 10^{-6} \text{m}^2 / \text{s}\right)^2}$$

$$= 1.1375 \times 10^7 \ (100 - T_{water,t})$$

$$Ra_L = Gr_L\,Pr = 1.1375 \times 10^7 \ (100 - T_{water,t}) *3.02$$

$$= 3.435 \times 10^7 (100 - T_{water,t})$$

The Nusselt number is then calculated as

$$N\bar{u} = 1 + 1.44\left[1 - \frac{1708}{Ra}\right] + \left[\left(\frac{Ra}{5830}\right)^{1/3} - 1\right] + 2.0\left[\frac{Ra^{1/3}}{140}\right]^{[1 - \ln(Ra^{1/3}/140)]}$$

$$\bar{h}_c = \frac{Nu * k}{L} = 8.213\left(1 + 1.44\left[1 - \frac{1708}{Ra}\right] + \left[\left(\frac{Ra}{5830}\right)^{1/3} - 1\right] + 2.0\left[\frac{Ra^{1/3}}{140}\right]^{[1 - \ln(Ra^{1/3}/140)]}\right)$$

Substituting the values obtained from Example 8.5 and solving we get

$$\rho V c_p \left(\frac{T_{water,i} - T_{water,t}}{t_i - t} \right)_{water} = \bar{h}_c A\left(100 - T_{water,t}\right)$$

$$982.8 * 0.08 * 4182.8 * \left(\frac{T_{water,i} - T_{water,t}}{t_i - t} \right)_{water} = \bar{h}_c \left(100 - T_{water,t}\right)$$

$$40042\left(\frac{T_{water,i} - T_{water,t}}{t_i - t} \right)_{water} = \left(100 - T_{water,t}\right)\left(1 + 1.44\left[1 - \frac{1708}{Ra}\right] + \left[\left(\frac{Ra}{5830}\right)^{1/3} - 1\right] + 2.0\left[\frac{Ra^{1/3}}{140}\right]^{[1 - \ln(Ra^{1/3}/140)]}\right)$$

$$T_{water,i} = T_{water,t} + \frac{\Delta t}{40042}\left(100 - T_{water,t}\right)\left(1 + 1.44\left[1 - \frac{1708}{Ra}\right] + \left[\left(\frac{Ra}{5830}\right)^{1/3} - 1\right] + 2.0\left[\frac{Ra^{1/3}}{140}\right]^{[1 - \ln(Ra^{1/3}/140)]}\right)$$

```
% MATLAB code for determining time required for heating water at pan to
% desired temperature.
Td=50; % Desired temperature
Tw=20; % initial water temperature
```

```
delt=0.5; % time difference
t=0; % initial time
while Tw<Td
Ra=3.435*10^7*(100-Tw); % Rayleigh's number
Tw=Tw+delt/40042*(100-Tw)*(1+1.44*(1-1708/Ra)+((Ra/5830)^(1/3)-
1)+2*(Ra^(1/3)/140)^(1-log(Ra^(1/3)/140)));
% New temperature calculated after delt instant
t=t+delt; % total time required
end
```

The above equation is solved iteratively in MATLAB (codes attached) to get the following results.

(i) Time required to heat the water to 50^0C is 256.5 seconds.

(ii) Time required to heat the water to 80^0C is 881.5 seconds.

(b) Total heat transfer to water in each case:

For temperature rise of 50^0C (using properties at $T_f=35^0$C)

$$Q = \rho V c_p (T - T_i) \text{ Joules}$$

$$Q = \rho * \pi * \frac{D^2}{4} * L * c_p (T - T_i) \text{ Joules}$$

$$Q = 994.1 * \pi * \frac{(0.15)^2}{4} * 0.08 * 4175(50-20) \text{ J}$$

$$Q = 994.1 * \pi * \frac{(0.15)^2}{4} * 0.08 * 4175(50-20) \text{ J}$$

$$Q = 176023 \text{ Joule}$$

$$Q = 176.023 \text{ kJ}$$

For temperature rise of 80^0C (using properties at $T_f=50^0$C)

$$Q = \rho V c_p (T - T_i) \text{ Joules}$$

$$Q = \rho * \pi * \frac{D^2}{4} * L * c_p (T - T_i) \text{ Joules}$$

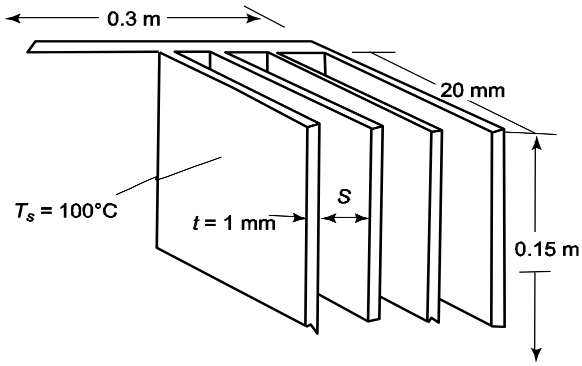
$$Q = 988.1 * \pi * \frac{(0.15)^2}{4} * 0.08 * 4178 * (80-20) \text{ J}$$

$$Q = 350212 \text{ Joule}$$

$$Q = 350.21 \text{ kJ}$$

PROBLEM 8.48

An electronic device is to be cooled in air at 20°C by an array of equally spaced vertical rectangular fins as shown in the sketch below. The fins are made of aluminum and their average temperature, T_s, is 100°C.



Estimate **(a) The optimum spacing, *s***
(b) The number of fins
(c) The rate of heat transfer from one fin
(d) The total rate of heat dissipation
(e) Is the assumption of a uniform fin temperature justified?

GIVEN

- Electronic device with vertical aluminum fins in air
- Air temperature (T_∞) = 20°C
- Average fin temperature (T_s) = 100°C

FIND

(a) The optimum spacing (s)
(b) The number of fins
(c) The rate of heat transfer from one fin
(d) The total rate of heat dissipation
(e) Is the assumption of a uniform fin temperature justified?

ASSUMPTIONS

- Steady state
- Uniform fin temperature
- The air is still
- Heat transfer from the top and bottom of the fins is negligible
- The heat transfer coefficient on the wall area between the fins is approximately the same as on the fins

PROPERTIES AND CONSTANTS

From Appendix 2, Table 12
For aluminum: Thermal conductivity (k_{al}) = 239 W/(m K) at 100°C
From Appendix 2, Table 28, for dry air at the mean temperature of 60°C

Thermal expansion coefficient (β) = 0.00300 1/K

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Grashof number for the fins, based on vertical height of the fin (L) is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\text{ m/s}^2)\ 0.003\text{ 1/K}\ \ (100°\text{C} - 20°\text{C})\,(0.15\text{ m})^3}{19.4\times10^{-6}\text{ m}^2/\text{s}\ ^2} = 2.11 \times 10^7$$

Therefore, the Rayleigh number is

$$Ra_L = Gr_L\, Pr = 2.11 \times 10^7\,(0.71) = 1.50 \times 10^7$$

(a) The optimum fin spacing (s) is given by Equation (8.56a)

$$s = \frac{2.7}{P^{0.25}}$$

$$\text{where}\quad P = \frac{Ra_L}{L^4} = \frac{1.50\times10^7}{(0.15\text{ m})^4} = 2.96 \times 10^{10}\ \text{1/m}^4$$

therefore, $s = 0.0065$ m = 6.5 mm

(b) Let n = the number of fins on the device, then

$$n\,t + (n-1)\,s = 0.3\text{ m}$$

$$n = \frac{0.3\text{ m} + s}{s+t} = \frac{0.3\text{ m} + 0.0065\text{ m}}{0.0065\text{ m} + 0.001\text{ m}} = 40.9$$

40 fins will fit on the device with optimum spacing.

(c) The average heat transfer coefficient over a fin is given in Table 8.1

$$h_c = \frac{k}{s}\left[\frac{576}{P^2\,s^8} + \frac{2.873}{P^{\frac{1}{2}}\,s^2}\right]^{-\frac{1}{2}}$$

$$h_c = \frac{0.0279\text{ W/(m K)}}{0.0065\text{ m}}\left[\frac{576}{2.96\times10^{10}\text{ 1/m}^4\ ^2\,(0.0065\text{ m})^3} + \frac{2.87}{2.96\times10^{10}\text{ 1/m}^4\ ^{\frac{1}{2}}\,(0.0065\text{ m})}\right]^{-\frac{1}{2}}$$

$$h_c = 5.78\text{ W/(m}^2\text{K)}$$

The rate of heat transfer from a single fin is

$$q_f = h_c\,A_f\,(T_s - T_\infty) = 5.78\ \text{W/(m}^2\text{K)}\ [0.15\text{ m }(0.041\text{ m})]\,(100°\text{C} - 20°\text{C}) = 2.84\text{ W}$$

(d) The total rate of heat dissipation is the sum of the heat transfer from the fins and the heat transfer from the wall area between the fins

$$q_{\text{total}} = \nu\,q_f + (\nu - 1)\,h_c\,A_w\,(T_s - T_\infty)$$

$$q_{\text{total}} = 40\,(2.84\text{ W}) + 39\ \ 5.78\text{ W/(m}^2\text{K)}\ \ (0.15\text{ m})(0.0065\text{ m})\,(100°\text{C} - 20°\text{C})$$

$$q_{\text{total}} = 113.6\text{ W} + 17.6\text{ W} = 131.2\text{ W}$$

(e) From Table 2.1, if the heat transfer from the tips of the fins is neglected, the temperature distribution along each fin is

$$\frac{T(x)-T_\infty}{T(0)-T_\infty} - \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

The temperature change along the fin is

$$\frac{T(0)-T(L)}{T(0)-T_\infty} = 1 - \frac{1}{\cosh(mL)}$$

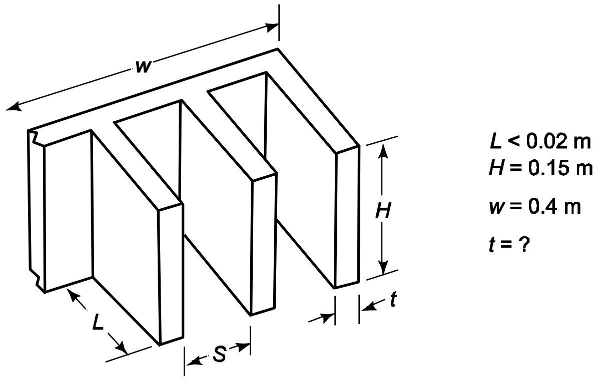
$$\text{where } m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{5.78\ \text{W/(m}^2\text{K)}[2(0.15\,\text{m} + 0.001\,\text{m})]}{0.001\,\text{m}(0.15\,\text{m})\ \ 239\ \text{W/(m K)}}} = 6.98\ \text{m}^{-1} L = 0.02\ \text{m}$$

$$\frac{T(0)-T(L)}{T(0)-T_\infty} = 1 - \frac{1}{\cosh[(6.98\,\text{m}^{-1})(0.02\,\text{m})]} = 0.966$$

Therefore, the assumption of an isothermal fin is justified.

PROBLEM 8.49

An electronic device is to be cooled by natural convection in atmospheric air at 20°C. The device generates 50 W internally and only one of its external surfaces is suitable for attaching fins. The surface available for attaching cooling fins is 0.15-m-tall and 0.4-m-wide. The maximum length of a fin perpendicular to the surface is limited to 0.02 m and the temperature at the base of the fin is not to exceed 70°C in one design and 100°C in another.



Design an array of fins spaced at a distance s from each other so that the boundary layers do not interfere with each other appreciably and maximum rate of heat dissipation is approached. For the evaluation of this spacing, assume that the fins are at a uniform temperature. Then select a thickness *t* that will provide good fin efficiency and ascertain which base temperature is feasible.

(For complete thermal analyses see ASME J. Heat Transfer, 1977, p. 369, *J*. Heat Transfer, 1979, p. 569, and *J*. Heat Transfer, 1984, p. 116.)

GIVEN

- An electronic device with vertical aluminum fins in air
- Air temperature (T_∞) = 20°C
- Heat generation ($\dot{q}_G$) = 50 W
- Height of surface (H) = 0.15 m
- Width of surface (w) = 0.4 m
- Maximum fin length (L_f) = 0.02 m
- Maximum base temperatures: T_{b1} = 70°C
  T_{b2} = 100°C
- Fin spacing = s
- Fin thickness = t

FIND

(a) Fin spacing such that the boundary layers do not interfere
(b) Select a fin thickness that gives a good fin efficiency and ascertain which base temperature is feasible

ASSUMPTIONS

- The fins are at a uniform temperature equal to the base temperature

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperatures for each of the base temperatures

Mean Temperature (°C)	45°C	60°C
Thermal expansion coefficient, β (1/K)	0.00314	0.003
Thermal conductivity, k (W/(m K))	0.0269	0.0279
Kinematic viscosity, $\nu \times 10^{-6}$ (m^2/s)	18.1	19.4
Prandtl number, Pr	0.71	0.71

From Appendix 2, Table 12, the thermal conductivity of aluminum in the range of 70 to 100°C (k_a) = 240 (W/(m K))

SOLUTION

(a) The boundary layer thickness on a vertical flat plate is given by Equation (8.11b)

$$\delta(x) = 4.3 \times \left[\frac{Pr + 0.56}{Pr^2 + Gr_x}\right]^{\frac{1}{4}}$$

The fin spacing (s) must be twice the boundary thickness at the top of the fin ($x = H$) to avoid boundary layer interference

$$s = 2\,\delta(H) = 8.6\,H\left[\frac{Pr + 0.56}{Pr^2 + Gr_H}\right]^{\frac{1}{4}}$$

where $Gr_H = \dfrac{g\,\beta(T_s - T_\infty)H^3}{\nu^2}$

For T_b = 70°C: $Gr_H = \dfrac{(9.8\text{ m/s}^2)\ 0.00314\ (1/\text{K})\ (70°\text{C} - 20°\text{C})(0.15\text{ m})^3}{18.1 \times 10^{-6}\text{m}^2/\text{s}^{\ 2}} = 1.59 \times 10^7$

$$s = 8.6\ (0.15\text{ m})\left[\frac{0.71 + 0.56}{0.71^2(1.59 \times 10^7)}\right]^{\frac{1}{4}} = 0.026\text{m} = 2.6\text{ cm}$$

For T_b = 100°C: $Gr_H = \dfrac{(9.8\text{ m/s}^2)\ 0.003\ (1/\text{K})\ (100°\text{C} - 20°\text{C})(0.15\text{ m})^3}{19.4 \times 10^{-6}\text{m}^2/\text{s}^{\ 2}} = 2.11 \times 10^7$

$$s = 8.6\ (0.15\text{ m})\left[\frac{0.71 + 0.56}{0.71^2(2.11 \times 10^7)}\right]^{\frac{1}{4}} = 0.024\text{m} = 2.4\text{ cm}$$

Let the fin spacing (s) = 2.5 cm.

The average Nusselt number of the fins is given by Equation (8.12b)

$$\overline{Nu}_H = 0.68\ Pr^{\frac{1}{2}}\ \frac{Gr_H^{\frac{1}{4}}}{(0.952 + Pr)^{\frac{1}{4}}}$$

For T_b = 70°C: $\overline{Nu}_H = 0.68\ (0.71)^{\frac{1}{2}}\ \dfrac{(1.59 \times 10^7)^{\frac{1}{4}}}{(0.952 + 0.71)^{\frac{1}{4}}} = 31.87$

$$\bar{h}_c = \overline{Nu}_H \frac{k}{H} = 31.87 \frac{0.0269 \ \text{W/(m K)}}{0.15\,\text{m}} = 5.71 \ \text{W/(m}^2\text{K)}$$

$$\text{For } T_b = 100°\text{C:} \ \overline{Nu}_H = 0.68 \ (0.71)^{\frac{1}{2}} \frac{(2.11 \times 10^7)^{\frac{1}{4}}}{(0.952 + 0.71)^{\frac{1}{4}}} = 34.20$$

$$\bar{h}_c = \overline{Nu}_H \frac{k}{H} = 34.2 \frac{0.0269 \ \text{W/(m K)}}{0.15\,\text{m}} = 6.36 \ \text{W/(m}^2\text{K)}$$

(b) The fin efficiency is approximated by Equation (2.72).
For a 'good' fin efficiency, let $\eta_f = 0.99$

$$0.99 = \frac{\tanh\sqrt{W}}{\sqrt{W}} \Rightarrow \quad W = 0.0289$$

$$\text{where } W = \frac{2\bar{h}_c L_c^2}{k_a t} \qquad L_c = L_f + \frac{t}{2}$$

$$\text{For } T_b = 70°\text{C: } W = \frac{2 \ 5.71\,\text{W/(m}^2\text{K)} \ \ 0.02\,\text{m} + t/2^{\ 2}}{240 \ \text{W/(m K)} \ t} = 0.0289 \Rightarrow t = 0.0007 \text{ m} = 0.7 \text{ mm}$$

$$\text{For } T_b = 100°\text{C: } W = \frac{2 \ 6.36\,\text{W/(m}^2\text{K)} \ \ 0.02\,\text{m} + t/2^{\ 2}}{240 \ \text{W/(m K)} \ t} = 0.0289 \Rightarrow t = 0.0008 \text{ m} = 0.8 \text{ mm}$$

Let t = 0.75 mm for either case.
The number of fins on the device (N) is given by

$$N\,t + (N-1)\,s = w \quad \Rightarrow \quad N = \frac{w+s}{t+s} = \frac{0.4\,\text{m} + 0.025\,\text{m}}{0.00075\,\text{m} + 0.025\,\text{m}} = 16.5$$

There will be 17 fins. The surface area of the fins and wall area between them is

$$A = H\,[N(2L_f + t) + (N-1)s] = 0.15\text{m}[17(0.04\text{m} + 0.00075\text{m}) + 16(0.025\text{m})] = 0.164\text{m}^2$$

The rate of heat transfer is

$$q = hc\,A\,(T_b - T_\infty)$$

$$\text{For } T_b = 70°\text{C} \qquad q = 5.71 \ \text{W/(m}^2\,\text{K)} \ (0.164\text{m}^2)\,(70°\text{C} - 20°\text{C}) = 46.8 \text{ W} < q_G$$

$$\text{For } T_b = 100°\text{C} \qquad q = 6.36 \ \text{W/(m}^2\,\text{K)}\,(0.164\text{m}^2)\,(100°\text{C} - 20°\text{C}) = 83.4 \text{ W} > q_G$$

The 100°C base temperature is feasible; the 70°C base temperature is not.

COMMENTS

The optimum spacing from Equation (8.56a) is 0.0017 m for t = 0.00075 m indicating the surface area gained outweighs the reduction in heat transfer due to the interference of the boundary layers. The rate of heat transfer with this spacing and a base temperature of 70°C is 47.4 W. Still not quite adequate.

PROBLEM 8.50

The electronic controls of a medical imaging device are housed in a compartment such that it dissipates heat to the ambient from a flat vertical plate surface, 15 cm wide and 20 cm high, to which fins are attached so as to enhance the heat transfer and improve the cooling of the electric systems. If 10 fins made of aluminum with a rectangular cross section are attached that extend over the full vertical height of the plate surface, where each fin is 0.15 cm thick with equal spacing of 1.5 cm along the width of the plate, determine the optimum fin height and the rate of heat transfer to the surrounding air. The room air temperature is at 20°C and the base temperature of the plate surface to which the fins are attached should not exceed 70°C.

GIVEN

- An electronic device with vertical aluminum fins in air
- Air temperature (T_∞) = 20°C
- Vertical Height of fin (H) = 0.2 m
- Width of surface (w) = 0.15 m
- Fin thickness(t) = 0.0015 m
- Maximum base temperatures: T_b= 70°C
- Fin spacing (s)= 1.5 cm=0.015 m

FIND

(a) Optimum fin height perpendicular to surface.
(b) Rate of heat transfer to surrounding air.

ASSUMPTIONS

- The fins are at a uniform temperature equal to the base temperature

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperatures for each of the base temperatures

Mean Temperature (°C)	45°
Thermal expansion coefficient, β (1/K)	0.00314
Thermal conductivity, k (W/(m K))	0.0269
Kinematic viscosity, $\nu \times 10^{-6}$ (m^2/s)	18.1
Prandtl number, Pr	0.71

From Appendix 2, Table 12, the thermal conductivity of aluminum in the range of around 70°C (k_a) = 240 (W/(m K))

SOLUTION

(a) The boundary layer thickness on a vertical flat plate is given by Equation (8.11b)

$$\delta(x) = 4.3 \times \left[\frac{Pr + 0.56}{Pr^2 + Gr_x}\right]^{\frac{1}{4}}$$

Let, L be the optimum fin height.

The fin spacing (s) must be twice the boundary thickness at the top of the fin ($x = H$) to avoid boundary layer interference

$$s = 2\,\delta(H) = 8.6\,H\;\left[\frac{Pr + 0.56}{Pr^2 + Gr_H}\right]^{\frac{1}{4}}$$

where $Gr_H = \dfrac{g\,\beta(T_s - T_\infty)H^3}{\nu^2}$

For $T_b =$ 70°C: $Gr_H = \dfrac{(9.8\ \text{m/s}^2)(0.00314\ (1/\text{K}))(70°\text{C} - 20°\text{C})(0.2)^3}{\left(18.1\times10^{-6}\text{m}^2/\text{s}\right)^2} = 3.757 \times 10^7$

The average Nusselt number of the fins is given by Equation (8.12b)

$$\overline{Nu}_H = 0.68\ Pr^{\frac{1}{2}}\ \frac{Gr_H^{\frac{1}{4}}}{(0.952 + Pr)^{\frac{1}{4}}}$$

$$\text{For } T_b = 70°\text{C:}\ \overline{Nu}_H = 0.68\ (0.71)^{\frac{1}{2}}\ \frac{(3.757\times10^7)^{\frac{1}{4}}}{(0.952 + 0.71)^{\frac{1}{4}}} = 39.51$$

$$\bar{h}_c = \overline{Nu}_H\ \frac{k}{H} = 23.63\frac{\left(0.0269\ \ \text{W/(m K)}\right)}{0.0343\,\text{m}} = 18.53\ \ \text{W/(m}^2\text{K)}$$

The rate of heat transfer is

$$q = hc\ A\ (T_b - T_\infty)$$

$A= H\ [N(2L_f + t)] = 0.2$ m[10(2L + 0.0015 m)] = 4L+0.003 m^2

For $T_b =$ 70°C $\quad q = 18.53\ \text{W/(m}^2\ \text{K)}\ (2\text{L}+0.03)\ (70°\text{C} - 20°\text{C})$

$= 926.5(2\text{L}+0.003)$

From fin analysis in chapter 2

Fin perimeter (P)=2t+2H=0.403

Fin cross section area (A)=H*t=0.2*0.0015= 3*10^{-4}

$$Q = 10\sqrt{\bar{h}_c PkA} * \eta_f\left(T_s - T_\infty\right) = \quad 10\sqrt{18.53*0.403*240*3*10^{-4}} * (70-20)\eta_f = 366.5\eta_f$$

Now measuring efficiency for each value of L from Figure (2.21) and substituting the values of L and corresponding η_f in above two equation.

L= 10 cm = 65 %

L (cm)	$L_c^{3/2}(\bar{h}/kA_m)^{1/2}$	η_f	q(W)	Q(W)
10 cm	0.717	70%	188	238
12 cm	0.861	63%	225.1	230.8
12.5 cm	0.896	62.5%	234	229

Thus the required fin legth is about 12.3 cm.

(b) The rate of heat transfer to the surrounding from total fins is given by Q=$366.5\eta_f$ =230 W

PROBLEM 8.51

Estimate the rate of heat transfer from one side of a 2-m-diameter disk with a surface temperature of 50°C rotating at 600 rev/min in 20°C air.

GIVEN

- A disk rotating in air
- Diameter (D) = 2 m
- Rotational speed (ω) = 600 rev/min
- Air temperature (T_∞) = 20°C
- Surface temperature (T_s) = 50°C

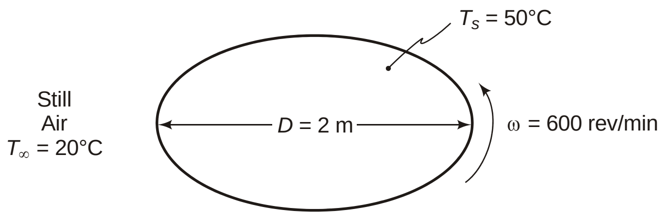
FIND

- The rate of heat transfer from one side (q)

ASSUMPTIONS

- The heat transfer has reached steady state
- The disk is horizontal
- Air is still

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 35°C

Thermal expansion coefficient (β) = 0.00325 1/K
Thermal conductivity (k) = 0.0262 W/(m K)
Kinematic viscosity (ν) = 17.1×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

The rotational Reynolds number for the disk is

$$Re_\omega = \frac{\omega D^2}{\nu} = \frac{(600\ \text{rev/min})\ 2\pi\ \text{rad/rev}\ \ 2\,\text{m}^{\ 2}}{17.1\times 10^{-6}\,\text{m}^2/\text{s}\ \ 60\ \text{s/min}} = 1.47 \times 10^7 > 10^6 \text{ (turbulent)}$$

The critical Reynolds number is given in Section 8.4

$$Re_\omega = 1 \times 10^6 = \frac{4 r_c^2\,\omega}{\nu} \quad \Rightarrow \quad r_c = \sqrt{\frac{(1\times 10^6)\,\nu}{4\omega}}$$

$$= \sqrt{\frac{1\times 10^6\ \ 17.1\times 10^{-6}\,\text{m}^2/\text{s}\ \ \ 60\ \text{s/min}}{4\ \ 600\ \text{rev/min}\ \ \ 2\pi\ \text{rad/rev}}} = 0.26 \text{ m}$$

The average heat transfer coefficient is given by Equation (8.47)

$$\bar{h}_c = \frac{k}{r_o}\left\{0.36\left(\frac{\omega r_o^2}{\nu}\right)^{\frac{1}{2}}\left(\frac{r_c}{r_o}\right)^2 + 0.015\left(\frac{\omega r_o^2}{\nu}\right)^{0.8}\left(1-\left(\frac{r_c}{r_o}\right)^{2.6}\right)\right\}$$

Since $\omega D^2/\nu = 1.47 \times 10^7$, $\omega\, r_o^2/\nu = 3.67 \times 10^6$ and $r_c/r_o = 0.26$. So

$$\bar{h}_c = \frac{0.0262\ \text{W/(m K)}}{1\text{m}}\left[(0.36)(3.67\times 10^6)^{\frac{1}{2}}(0.26)^2 + (0.015)(3.67\times 10^6)^{0.8}(1-(026)^{2.6})\right]$$

$$= 69.3\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer is

$$q = h_c\, A\, (T_s - T_\infty) = h_c\, \pi\, r_o^2\, (T_s - T_\infty) = 69.3\ \text{W/(m}^2\text{K)}\quad \pi\, (1\ \text{m})^2\, (50°\text{C} - 20°\text{C}) = 6535\ \text{W}$$

PROBLEM 8.52

A sphere 0.1-m-diameter is rotating at 20 rpm in a large container of CO_2 at atmospheric pressure. If the sphere is at 60°C and the CO_2 at 20°C, estimate the rate of heat transfer.

GIVEN

- A rotating sphere in carbon dioxide at atmospheric pressure
- Diameter (D) = 0.1 m
- Speed of rotation (ω) = 20 rev/min
- Sphere temperature (T_s) = 60°C
- CO_2 temperature (T_∞) = 20°C

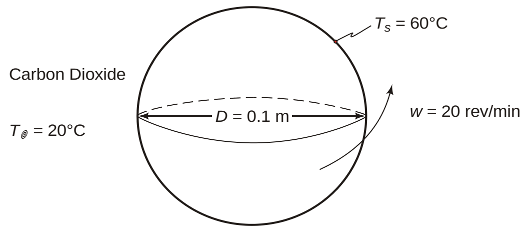
FIND

- The rate of heat transfer

ASSUMPTIONS

- Steady state conditions
- The carbon dioxide is still
- Radiation is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for CO_2 at the mean temperature of 40°C

Thermal expansion coefficient (β) = 0.00319 1/K

Thermal conductivity (k) = 0.0176 W/(m K)

Kinematic viscosity (ν) = 9.0×10^{-6} m^2/s

Prandtl number (Pr) = 0.77

SOLUTION

Converting the rotational speed to radians per second

$$\omega = (20\,\text{rev/min})(2\pi\,\text{rad/rev})/60\,\text{s/min} = 2.09\ \text{s}^{-1}$$

The rotational Reynolds number for the sphere is

$$\omega = (20\,\text{rev/min})(2\pi\,\text{rad/rev})/60\,\text{s/min} = 2.09\ \text{s}^{-1}$$

All requirements are met for the correlation presented in Equation (8.50)

$$\overline{Nu}_D = 0.43\, Re_\omega^{0.5}\, Pr^{\,0.4} = 0.43\,(2322)^{0.5}\,(0.77)^{0.4} = 18.67$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 18.67\,\frac{0.0176\ \text{W/(m K)}}{0.1\text{m}} = 3.29\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer by natural convection is given by

$$q_c = \bar{h}_c\,A\,(T_s - T_\infty) = \bar{h}_c\,\pi D^2\,(T_s - T_\infty) = 3.29\ \text{W/(m}^2\ \text{K)}\quad \pi\,(0.1\ \text{m})^2\,(60°\text{C} - 20°\text{C}) = 4.13\ \text{W}$$

PROBLEM 8.53

A mild steel (1% carbon), 2-cm-*OD* shaft, rotating in 20°C air at 20,000 rev/min, is attached to two bearings 0.7 m apart. If the temperature at the bearings is 90°C, determine the temperature distribution along the shaft. Hint: Show that for the high rotational speeds Equation (8.44) approaches: $\overline{Nu}_D = 0.086\,(\pi D^2\omega/\nu)^{0.7}$

GIVEN

- A mild steel shaft rotating in air between two bearings
- Shaft diameter (D) = 2 cm = 0.02 m
- Rotational speed (ω) = 20,000 rev/min
- Air temperature (T_∞) = 20°C
- Length of shaft (L) = 0.7 m
- Bearing temperatures (T_b) = 90°C

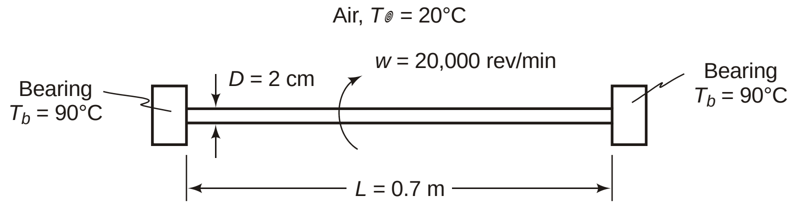
FIND

- The temperature distribution along the shaft

ASSUMPTIONS

- The rod has reached steady state
- Radiation is negligible
- The shaft is horizontal

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, thermal conductivity of 1% carbon steel (k_s) = 43 W/(m K)

SOLUTION

The Nusselt number for this geometry is given by Equation (8.44)

$$\overline{Nu}_D = 0.11\,(0.5\,Re_\omega^2 + Gr_D\,Pr)^{0.35}$$

Where $$Re_\omega = \frac{\pi\,\omega\,D^2}{\nu}$$

Evaluating the air properties at the mean of the air and bearing temperatures (55°C); from Appendix 2, Table 28

Thermal expansion coefficient (β) = 0.00305 1/K
Thermal conductivity (k) = 0.0276 W/(m K)
Kinematic viscosity (ν) = 19.0×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

The rotational Reynolds number is

$$Re_\omega = \frac{\pi\,(2000\text{ rev/min})\,(2\pi\text{ rad/rev})\,(0.02\text{ m})^2}{(19.0\times10^{-6}\text{ m}^2/\text{s})\,(60\text{ s/min})} = 1.39\times10^5$$

$$Gr_D\,Pr = \frac{g\,\beta(T_b - T_\infty)\,D^3\,Pr}{\nu^2} = \frac{(9.8\text{ m/s}^2)\,(0.00305\text{ 1/K})\,(90°\text{C} - 20°\text{C})(0.02\text{ m})^3(0.71)}{(19.0\times10^{-6}\text{ m}^2/\text{s})^2} = 3.29\times10^4$$

For this problem, $0.5\, Re_\omega^2 >> Gr_D\, Pr$ because of the high rotational speed, therefore. $Gr_D\, Pr$ can be neglected and the Nusselt number is given by

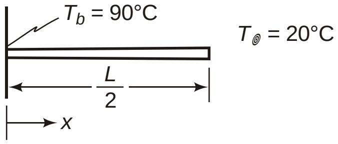
$$\overline{Nu}_D = 0.11\,(0.5\, Re_\omega^2)^{0.35} = 0.0863\ 1 Re_\omega^{0.7}$$

Based on the average of the air and bearing temperatures

$$\overline{Nu}_D = 0.0863\,(1.39 \times 10^5)^{0.7} = 344$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 344\ \frac{0.0276\ \text{W/(m K)}}{0.02\,\text{m}} = 474\ \text{W/(m}^2\text{K)}$$

By symmetry, the axial conduction at the center of the shaft must be zero and the shaft can be treated as two pin fins with adiabatic tips as shown below



The temperature distribution for this configuration is given in Table 2.1

$$\frac{T - T_\infty}{T_b - T_s} = \frac{\cosh[m(L_f - x)]}{\cosh(mL_f)} \qquad \left(L_f = \frac{\text{L}}{2}\right)$$

$$\text{where m} = \sqrt{\frac{h_c P}{k_s A_c}} = \sqrt{\frac{h_c\, \pi D}{k_s \pi / 4\, D^2}} = \sqrt{\frac{4 h_c}{D k_s}} = \sqrt{\frac{4\ \ 474\ \text{W/(m}^2\text{K)}}{0.02\,\text{m}\ \ 43\ \text{W/(m K)}}} = 47.0\ 1/\text{m}$$

$$T = T_\infty + (T_b - T_s)\left[\frac{\cosh[m(L_f - x)]}{\cosh(mL_f)}\right] = 20°\text{C} + (90°\text{C} - 20°\text{C})\left[\frac{\cosh[47.0\ 1/\text{m}(0.35\,\text{m} - x)]}{\cosh[(47.0\ 1/\text{m})(0.35\,\text{m})]}\right]$$

$$T = 20°\text{C} + (1.0 \times 10^{-5}\ °\text{C} \cosh(16.5 - 47.0\, x)$$

The average rod temperature is given by $\quad T_{\text{ave}} = \dfrac{1}{\frac{L}{2}} \displaystyle\int_0^{\frac{L}{2}} T\, dx$

Let $A = 1.0 \times 10^{-5}$ °C and $y = 16.5 - 47.0\, x$ then: $dy = -\,47.0\, dx$

when $\times = \dfrac{L}{2}$, $y = 0$

when $\times = 0$, $y = 16.5$

$$T_{\text{ave}} = \frac{-\,2}{47.0\,L} \int_{16.5}^{0} [20 + \text{A} \cosh(y)]\, dy = \frac{-\,2}{47.0\,L}\ [20\, y + \text{A} \sinh(y)]^{0}_{16.5}$$

$$T_{\text{ave}} = \frac{-\,2}{47.0\,L}\ [-20(16.5) - (1.0 \times 10^{-5}) \sinh(16.5)] = 24.5°\text{C}$$

Using the mean of the air temperature and the average shaft temperature to evaluate the air properties and re-evaluating the temperature profile

$T_{\text{mean}} = 22.2$°C

$k = 0.0253$ W/(m K)

$\nu = 15.9 \times 10^{-6}$ m^2/s

$Re_\omega = 1.6 \times 10^5$

$h_c = 491$ W/(m^2 K)

$m = 47.8$ 1/m

$$T = 20°\text{C} + (7.59 \times 10^{-6}\ °\text{C}) \cosh(16.7 - 47.8\, x)$$

$T_{\text{ave}} = 24.0$°C

where x = distance in meters from a bearing up to L/2.

PROBLEM 8.54

Reconsider the problem of cooling of a rotating rod (shaft) by natural convection described in Example 8.6. Determine the shaft rotation at which the heat transfer contribution due to the rotation convection is about the same as that due to natural convection, and calculate the total heat transfer rate at this condition. Also, graph the variation of *q*, as well as its two components (*qr* and *qnc*) with shaft rpm ranging from 3 rpm to 15 rpm, and comment upon the results.

GIVEN

- Cooling of rotational shaft by natural convection
- Rod diameter (D) = 20 cm = 0.2 m
- Air temperature (T_∞) = 20°C
- Temperature of rotating rod (T_b) = 1000°C

FIND

- Shaft rotation at which heat transfer contribution due to rotational convection is same as that due to natural convection.
- Total heat transfer rate at this condition.
- Graph of variation of q and qr and qnc with shaft rpm.

ASSUMPTIONS

- Radiation is negligible
- The shaft is horizontal

PROPERTIES AND CONSTANTS

Thermal expansion coefficient (β) = 0.003 1/K

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Nusselt number for this geometry is given by Equation (8.44)

$$\overline{Nu}_D = 0.11\,(0.5\,Re_\omega^2 + Gr_D\,Pr)^{0.35}$$

Where $$Re_\omega = \frac{\pi\omega D^2}{\nu}$$

The heat transfer contribution due to rotational convection becomes equal to heat transfer due to natural convection when two RHS terms in equation (8.44) become equal. That is

$$0.5\,\mathrm{Re}_\omega^2 = Gr_D\,\mathrm{Pr}$$

$$0.5\left(\frac{\pi\,\omega\,D^2}{\nu}\right)^2 = \frac{g\beta\Delta T D^3}{\nu^2}\,\mathrm{Pr}$$

$$\omega^2 = \frac{g\beta\Delta T}{0.5\pi^2 D}\,\mathrm{Pr}$$

$$\omega^2 = \frac{9.8*0.003*80}{0.5\pi^2*0.2}*0.71$$

$$\omega^2 = 1.69 \quad => \quad \omega = 1.3 \text{ rad/s} = 12.42 \text{ rpm}$$

Thus at 12.42 rpm heat transfer contribution due to rotational convection becomes equal to heat transfer due to natural convection.

Heat transfer rate at this condition:

$$\overline{Nu}_D = 0.11\ (Gr_D\, Pr + Gr_D\, Pr)^{0.35}$$

$$\overline{Nu}_D = 0.11 \left(\frac{2g\beta\Delta T D^3}{\nu^2} \text{Pr} \right)^{0.35}$$

$$\overline{Nu}_D = 0.11 \left(\frac{2*9.8*0.003*80*0.2^3}{(19.4*10^{-6})^2} 0.71 \right)^{0.35}$$

$$\overline{Nu}_D = 0.11 *559.6 \quad = 61.56$$

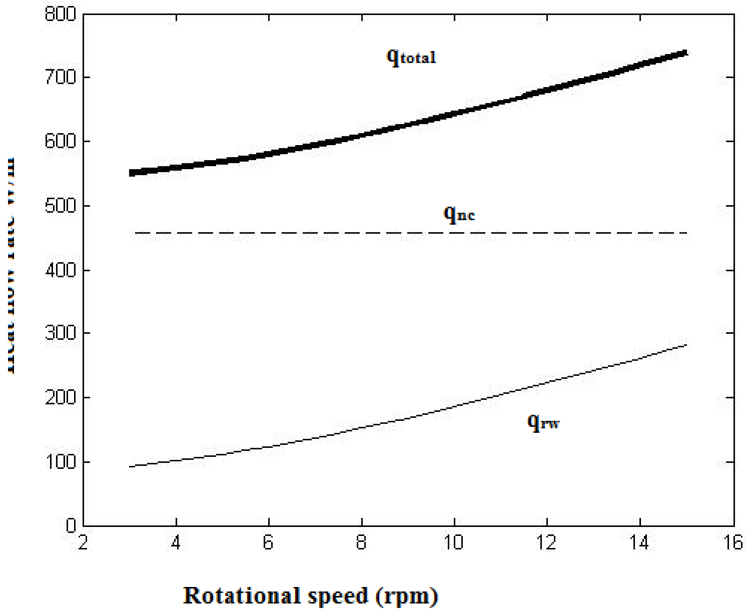
$$\bar{h}_c = \overline{Nu}_D \ \frac{k}{D} = 61.6 \ \frac{0.0279 \text{ W/(m K)}}{0.2 \text{ m}} = 8.59 \ \text{W/(m}^2\text{K)}$$

Thus the rate of heat transfer per unit length is

$$q/L = \bar{h}_c * \pi D * (T_s - T_b)$$

$$q/L = 8.59 * \pi * 0.2 * (100 - 20) \ \text{W/m}$$

$$q/L = 431 \ \text{W/m}$$



COMMENTS

The effect of heat transfer due to natural convection and rotation individually do not sum up to the combined effect due to both. Thus q_{nc} and q_{rw} do not become equal at 12.41 rpm as stated above. The combined effect due to these two is less than the individual effects.

PROBLEM 8.55

Consider a vertical 20-cm-tall flat plate at 120°C suspended in a fluid at 100°C. If the fluid is being forced past the plate from above, estimate the fluid velocity for which natural convection becomes negligible (less than 10%) in: (a) mercury (b) air (c) water.

GIVEN

- A vertical flat plate suspended in a fluid
- Plate temperature (T_s) = 120°C
- Fluid temperature (T_∞) = 100°C
- Fluid is being forced past the plate from above
- Plate height (H) = 20 cm = 0.2 m

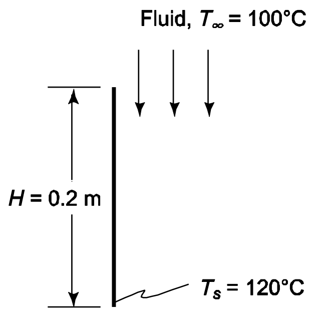
FIND

- The fluid velocity (U_∞) for which natural convection has a less than 10% effect in
  (a) mercury (b) air (c) water

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 26, 28 and 13

Fluid (at 110°C)	Mercury	Air	Water
Thermal expansion coefficient, β (1/K)	0.000182	0.00262	0.00080
Kinematic viscosity, $\nu \times 10^6$ m²/s	0.0913	24.8	0.269
Prandtl number, Pr	0.016	0.71	1.59

SOLUTION

From Equation (8.55), for laminar forced convection over a flat plate, the effect of buoyancy will be less than 10% if

$$Gr_H < 0.150\, Re_H^2 \Rightarrow \frac{g\,\beta (T_s - T_\infty) H^3}{\nu^2} < 0.150 \left(\frac{U_\infty H}{\nu}\right)^2$$

Solving for the fluid velocity

$$U_\infty > [6.67\, g\,\beta (T_s - T_\infty) H]^{\frac{1}{2}}$$

$$U_\infty > \left[6.67 \left(9.8\ \text{m/s}^2\right) \beta\ (1/\text{K})\ (120°\text{C} - 100°\text{C})(0.2\ \text{m})\right]^{1/2} = 16.17\ \beta^{1/2}\ \text{m/s}$$

(a) For mercury: $U_\infty < 16.17\,(0.000182)^{1/2} = 0.22$ m/s

(b) For air: $U_\infty < 16.17\ (0.00262)^{1/2} = 0.83$ m/s

(c) For water: $U_\infty < 16.17\ (0.0008)^{1/2} = 0.46$ m/s

The Reynolds numbers for these fluid velocities are

(a) For mercury: $Re_H = \dfrac{(0.22\text{ m/s})(0.2\text{ m})}{0.0913\times10^{-6}\text{m}^2/\text{s}} = 4.82 \times 10^5$

(b) For air: $Re_H = \dfrac{(0.83\text{ m/s})(0.2\text{ m})}{24.8\times10^{-6}\text{m}^2/\text{s}} = 6.69 \times 10^3$

(c) For water: $Re_H = \dfrac{(0.46\text{ m/s})(0.2\text{ m})}{0.269\times10^{-6}\text{m}^2/\text{s}} = 3.42 \times 10^5$

These Reynolds numbers are all within the laminar regime (mercury is approaching the transition to turbulence). Therefore, the use of Equation (8.55) was valid.

PROBLEM 8.56

Suppose a thin vertical flat plate 60-cm-high and 40-cm-wide, is immersed in a fluid flowing parallel to is surface. If the plate is at 40°C and the fluid at 10°C, estimate the Reynolds number at which buoyancy effects are essentially negligible for heat transfer from the plate if the fluid is: (a) mercury, (b) air, and (c) water. Then calculate the corresponding fluid velocity for the three fluids.

GIVEN

- A thin flat plate immersed in a fluid flowing parallel to its surfaces
- Plate height (H) = 60 cm = 0.6 m
- Plate width (w) = 40 cm = 0.4 m
- Plate temperature (T_s) = 40°C
- Fluid temperature (T_∞) = 10°C

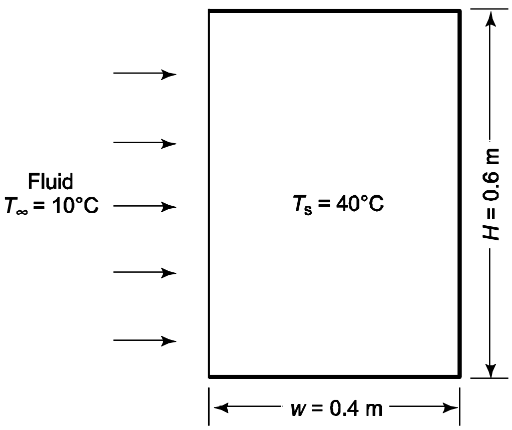
FIND

- The Reynolds number and corresponding fluid velocity (U_∞) for buoyancy effects to be negligible, if the fluid is: (a) mercury, (b) air, (c) water

ASSUMPTIONS

- Steady state conditions

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Tables 26, 28 and 13 at the mean temperature of 25°C

Fluid	Mercury	Air	Water
Thermal expansion coefficient, β (1/K)	—	0.00336	0.000255
Kinematic viscosity, $\nu \times 10^6$ m²/s	0.112	16.2	0.884
Density, ρ (kg/m³):	13,628 (0°C) 13,506 (50°C)		

The thermal expansion coefficient of mercury can be estimated from

$$\beta \cong \frac{2}{\rho_0 + \rho_{50}} = \left(\frac{\rho_0 - \rho_{50}}{273\text{K} - 323\text{K}}\right) = \frac{2}{(13,658 + 13,506)\ \text{kg/m}^3}\left(\frac{(13,658 - 13,506)\ \text{kg/m}^3}{273\text{K} - 323\text{K}}\right)$$

$$= 0.00018\ 1/\text{K}$$

SOLUTION

The Grashof number based on height is

$$Gr_H = \frac{g\,\beta(T_s - T_\infty)\,H^3}{\nu^2}$$

For mercury

$$Gr_s = \frac{(9.8\,\text{m/s}^2)\;0.00018\;1/\text{K}\;(40°\text{C} - 10°\text{C})(0.6\,\text{m})^3}{\left(0.112\times10^{-6}\,\text{m}^2/\text{s}\right)^2} = 9.11\times10^{11}$$

For this geometry, the ratio that must be satisfied for the natural convection to have an essentially negligible effect is given at the end of Section 8.5 as

$$\frac{Gr_H}{Re_w^2} < 0.7 \Rightarrow Re_w = \frac{U_\infty\,w}{\nu} > 1.20\;Gr_H^{\frac{1}{2}}$$

For mercury

$$Re_w > 1.20(9.11\times10^{11})^2 = 1.15\times10^6$$

$$\therefore\; U_\infty = Re_w\,\frac{\nu}{w} = 1.15\times10^6\,\frac{0.112\times10^{-6}\,\text{m}^2/\text{s}}{0.4\,\text{m}} = 0.321\ \text{m/s}$$

Applying a similar analysis to the other fluids yields the following results

Fluid	Mercury	Air	Water
Gr_H	9.11×10^{11}	8.13×10^{8}	2.07×10^{10}
Re_w	1.15×10^{6}	3.42×10^{4}	1.72×10^{5}
U_∞ (m/s)	0.32	5.54	0.382

PROBLEM 8.57

A vertical isothermal plate 30-cm-high is suspended in an atmosphere air stream flowing at 2 m/s in a vertical direction. If the air is at 16°C, estimate the plate temperature for which the natural-convection effect on the heat transfer coefficient will be less than 10%.

GIVEN

- A vertical isothermal plate is an atmospheric air stream
- Plate height (L) = 30 cm = 0.3m
- Air velocity (U_∞) = 2 m/s (vertically)
- Air temperature (T_∞) = 16°C

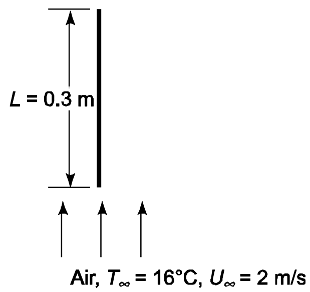
FIND

- The plate temperature (T_s) for which natural convection effect on the heat transfer coefficient will be less than 10%.

ASSUMPTIONS

- Steady State

SKETCH



SOLUTION

The average of the air and plate surfaces must be used to evaluate the fluid properties. Since the surface temperature is not known, the problem will first be solved by guessing the plate surface temperature. This temperature will be used to evaluate fluid properties. The resulting plate temperature will be used to update the fluid properties. For the first iteration, let T_s = 30°C. Therefore, the fluid properties will be evaluated at (30°C + 16°C)/2 = 23°C. From Appendix 2, Table 28

Thermal expansion coefficient (β) = 0.00338 1/K

Kinematic viscosity (ν) = 16.0×10^{-6} m^2/s

The Reynolds number for the top of the plate is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(2\ \text{m/s})(0.3\text{m})}{16.0 \times 10^{-6}\text{m}^2/\text{s}} = 3.75 \times 10^4 < 5 \times 10^5 \text{ (laminar)}$$

By Equation (8.55), the natural convection effect will be less then 10% when

$$Gr_L < 0.150\, Re_L^2 \;\Rightarrow\; \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} < 0.150 \left(\frac{U_\infty L}{\nu}\right)^2$$

Solving for the surface temperature

$$T_s < T_\infty + \frac{0.150\,U_\infty^2}{L\,g\,\beta} = 16°\text{C} + \frac{0.15\ \ 2\ \text{m/s}^{\ 2}}{0.3\text{m}\ \ (9.8\ \text{m/s}^2)\ \ 0.00338\ 1/\text{K}} = 76.4°\text{C}$$

Re-evaluating the thermal equation coefficient at the mean temperature of 46.2°C

β = 0.00313 1/K

$$T_s = 16°\text{C} + \frac{0.15\ \ 2\ \text{m/s}\ ^{2}}{0.3\,\text{m}\ \ 9.8\,\text{m/s}^2\ \ \ 0.00313\ 1/\text{K}} = 81.2°\text{C}$$

Performing one more iteration: At T_{avg} = 48.6°C, β = 0.00311 1/K

$$T_s = 16°\text{C} + \frac{0.15\ \ 2\ \text{m/s}\ ^{2}}{0.3\,\text{m}\ \ 9.8\,\text{m/s}^2\ \ \ 0.00311\ 1/\text{K}} = 81.6°\text{C}$$

For all surface temperatures

$$T_s \leq 81.6°\text{C}$$

natural convection heat transfer will contribute less than 10% to the total heat transfer.

PROBLEM 8.58

A horizontal disk 1 m in diameter rotates in air at 25°C. If the disk is at 100°C, estimate the number of revolutions per minute at which natural convection for a stationary disk becomes less than 10% of the heat transfer for a rotating disk.

GIVEN

- A rotating horizontal disk in air
- Diameter (D) = 1 m
- Air temperature (T_∞) = 25°C
- Disk temperature (T_s) = 100°C

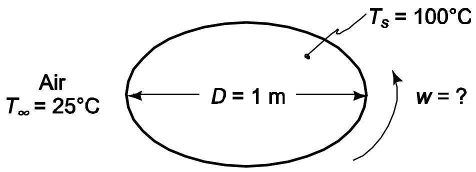
FIND

- The rotational speed (ω) at which natural convection becomes less than 10% of the thermal effects of rotation

ASSUMPTIONS

- Steady state conditions

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 62.5°C

Thermal expansion coefficient (β) = 0.00297 1/K

Thermal conductivity (k) = 0.0281 W/(m K)

Kinematic viscosity (ν) = 19.7×10^{-6} m²/s

Prandtl number (Pr) = 0.71

SOLUTION

The characteristic length for free convection from the stationary disk is

$$L_c = \frac{A}{P} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4} = 0.25 \text{ m}$$

The Rayleigh number is

$$Ra_{Lc} = Gr_{Lc}\,Pr = \frac{g\,\beta(T_s - T_\infty)\,L_c^3\,Pr}{\nu^2}$$

$$= \frac{(9.8\,\text{m/s}^2)\;0.00297\,\text{1/K}\;(100\,°\text{C} - 25\,°\text{C})(0.25\,\text{m})^3(0.71)}{19.7\times 10^{-6}\,\text{m}^2/\text{s}^{\;\;2}} = 6.24 \times 10$$

The Nusselt number for a static disk is given by Equation (8.16)

$$Nu_{Lc} = 0.15\;Ra_{Lc}^{\frac{1}{3}} = 0.15\;(6.24\times 10^7)^{\frac{1}{3}} = 59.50$$

$$h_{\text{stat}} = Nu_{Lc}\ \frac{k}{L_c} = 59.50\ \frac{0.0281\ \text{W/(m K)}}{0.25\,\text{m}} = 6.69\ \text{W/(m}^2\text{K)}$$

Assuming the rotational speed is high enough to product turbulent flow, the Nusselt number is given by Equation (8.47)

$$\bar{h}_c = \frac{k}{r_o}\left\{0.36\left(\frac{\omega\, r_o^2}{\nu}\right)^{\frac{1}{2}}\left(\frac{r_c}{r_o}\right)^2 + 0.015\left(\frac{\omega\, r_o^2}{\nu}\right)^{0.8}\left(1-\left(\frac{r_c}{r_o}\right)^{2.6}\right)\right\}$$

where $\quad \dfrac{4 r_c^2 \omega}{\nu} = 10^6$

Since

$$h_{\text{rot}} = 10 \times 6.69 = 66.9\ \text{W/(m}^2\text{K)}$$

we have

$$\frac{h_{\text{rot}}\, r_o}{k} = 1190$$

This can be written as

$$0.36\left(\frac{Re}{4}\right)^{\frac{1}{2}}\frac{Re_c}{Re} + 0.015\left(\frac{Re}{4}\right)^{0.8}\left(1-\left(\frac{Re_c}{Re}\right)^{1.3}\right) = 1190$$

By trial and error: $Re = 5.64 \times 10^6$ (which is turbulent) and $\omega = 111$ rad/s = 1060 rpm.
Note that $r_c = 0.211$ m.

PROBLEM 8.59

A gas-fired industrial furnace is used to generate steam. The furnace is a 3 m cubic structure and the interior surfaces are completely covered with boiler tubes transporting pressurized wet steam at 150°C. It is desired to keep the furnace losses to 1% of the total heat input of 1 MW. The outside of the furnace can be insulated with a blanket-type mineral wool insulation [*k* = 0.13 W/(m °C)], which is protected by a polished metal sheet outer shell. Assume the floor of the furnace is insulated. What is the temperature of the metal shell sides? What thickness of insulation is required?

GIVEN

- An insulated cubic furnace with steam filled tubes on the inner walls
- Steam temperature (T_{st}) = 150°C
- Length of a side of the furnace (L) = 3 m
- Thermal conductivity of mineral wool insulation (k_i) = 0.13 W/(m°C)
- Insulation is protected by metal sheet outer shell
- Furnace losses (q_c) = 1% of total heat input
- Total heat input (q_{in}) = 1 MW = 106 W

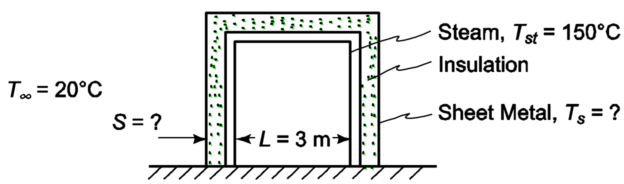
FIND

(a) Temperature of the metal sheel sides (T_s)
(b) The thickness of insulation (s) required

ASSUMPTIONS

- Steady state operation
- Thermal resistance of the convection within the steam pipes, the steam pipe walls, the furnace walls, and the metal shell negligible compared to that of the insulation
- Air outside the furnace is still
- The floor is well insulated —heat loss is negligible
- Temperature of the metal shell is uniform
- Ambient temperature (T_∞) = 20°C (293 K)
- Edge effects are negligible
- The emissivity of the polished metal shell (ε) = 0.05 (see Table 9.2)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann Constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

(a) Since the Grashof number on the outside of the metal shell will depend on the temperature of the metal shell, an iterative procedure is required. For the first iteration, let T_s = 100°C (373 K).

From Appendix 2, Table 28, for dry air at the mean temperature of 60°C

Thermal expansion coefficient (β) = 0.00300 1/K

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The Grashof number for the four sides of the furnace, assuming the insulation thickness is small compared to 3 m, is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.003\ 1/\text{K}\ (100°\text{C} - 20°\text{C})(3\,\text{m})^3}{19.4 \times 10^{-6}\text{m}^2/\text{s}^{\ 2}} = 1.69 \times 10^{11}$$

The heat transfer from the furnace will be calculated by treating the sides as vertical flat plates and the top as a horizontal flat plate facing upward. From Equation (8.13), the heat transfer coefficient for the sides is

$$\bar{h}_{cs} = 0.13\ \frac{k}{L}\ (Gr_L\,Pr)^{\frac{1}{3}} = 0.13\ \frac{0.0279\ \text{W/(m K)}}{3\text{m}}\ [1.69 \times 10^{11}(0.71)]^{\frac{1}{3}} = 5.96\ \text{W/(m}^2\text{K)}$$

The characteristic dimension for the top of the furnace (L_c) is

$$L_c = \frac{A}{P} = \frac{L^2}{4L} = \frac{L}{4} = 0.75\ \text{m}$$

The Grashof and Rayleigh numbers based on this dimension are

$$Gr_{Lc} = \frac{(9.8\,\text{m/s}^2)\ 0.00188\ (1/\text{K})\ (100°\text{C} - 20°\text{C})(0.75\text{m})^3}{19.4 \times 10^{-6}\text{m}^2/\text{s}^{\ 2}} = 2.64 \times 10^9\ \text{(turbulent)}$$

$$Ra_{Lc} = Gr_{Lc}\,Pr = 2.64 \times 10^9\ (0.71) = 1.87 \times 10^9$$

The average Nusselt number is given by Equation (8.16)

$$\overline{Nu}_{Lc} = 0.15\ Ra_{Lc}^{\frac{1}{3}} = 0.15\ (1.87 \times 10^9)^{\frac{1}{3}} = 184.9$$

$$\bar{h}_{ct} = \overline{Nu}_{L_c}\ \frac{k}{L_c} = 184.9\ \frac{0.0279\ \text{W/(m K)}}{0.75\text{m}} = 6.88\ \text{W/(m}^2\text{K)}$$

The rate of convection and radiation must be 1% of the total heat input

$$q_c = q_r = (\bar{h}_{cs}\ A_s + \bar{h}_{ct}\ A_t)\ (T_s - T_\infty) + \varepsilon\,\sigma A\ (T_s^4 - T_\infty^4) = 0.01\ q_{\text{in}}$$

where A_s = the area of the sides = $4(3\text{ m})^2 = 36\text{ m}^2$
A_t = the area of the top = $(3\text{ m})^2 = 9\text{ m}^2$
$A = A_s + A_t = 45\text{ m}^2$

$$5.96\,\text{W/(m}^2\text{K)}\ (36\,\text{m}^2) + 6.88\,\text{W/(m}^2\text{K)}\ (9\,\text{m}^2)\ h\ (T_s - 293\text{ K}) + 0.05$$
$$5.67 \times 10^{-8}\,\text{W/(m}^2\text{K}^4)\ \ (45\text{ m}^2)\ (T_s^4 - (293\text{ K})^4)$$
$$= 0.01\ (10^6\text{ W})$$

By trial and error: T_s = 327 K = 54°C

Following the same procedure for other iterations

Iteration #	2	3	4
T_s (°C)	51	64	60
Mean Temp. (°C)	35.5	42	40
β (1/K)	0.00324	0.00317	0.00319
k (W/(m K))	0.0262	0.0266	0.0265
$\nu \times 10^6$ (m²/s)	17.2	17.8	17.6
Pr	0.71	0.71	0.71
h_{cs} (W/(m² K))	4.54	5.02	4.89

h_{ct} (W/(m^2 K))	5.23	5.79	5.65
T_s (°C)	64	60	61

Therefore, the surface temperature (T_s) ≈ 61°C

(b) The rate of conductive heat transfer through the insulation must also be 1% of the input heat

$$q_k = \frac{Ak_i}{S}(T_{st} - T_s) = 0.01\, q_{\text{in}}$$

Solving for the insulation thickness

$$s = \frac{Ak_i}{0.01 q_{\text{in}}}(T_{st} - T_s) = \frac{45\,\text{m}^2\ \ 0.13\ \text{W/(m K)}}{0.01(10^6\,\text{W})}(150°\text{C} - 61°\text{C}) = 0.052\text{m} = 5.2\text{ cm}$$

COMMENTS

The insulation thickness is small compared to the length of a side of the furnace, therefore, neglecting the edge effects or effect on the exterior surface area should not introduce appreciable error.

PROBLEM 8.60

A 0.15-m-square circuit board is to be cooled in a vertical position, as shown in the sketch. The board is insulated on one side while on the other, 100 closely spaced square chips are mounted, each of which dissipated 0.06 W of heat. The board is exposed to air at 25°C and the maximum allowable chip temperature is 60°C. Investigate the following cooling options

(a) Natural convection

(b) Air cooling with upward flow at a velocity of 0.5 m/s

(c) Air cooling with downward flow at the velocity of 0.5 m/s

GIVEN

- Square vertical circuit board insulated on one side, chips on the other side
- Length of each side (L) = 0.15 m
- Heat dissipation per chip (q) = 0.06 W
- Number of chips (N) = 100
- Ambient air temperature (T_∞) = 25°C
- Maximum allowable chip temperature (T_s) = 60°C

FIND

Investigate the following cooling options

(a) Natural convection

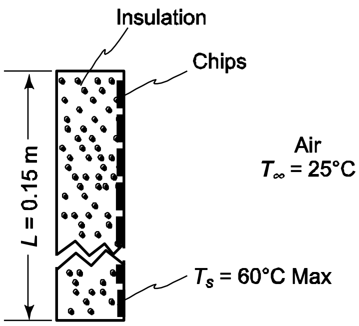
(b) Forced air cooling with an upward air velocity (U_∞) = 0.5 m/s

(c) Forced air cooling with a downward air velocity (U_∞) = 0.5 m/s

ASSUMPTIONS

- Steady state
- Uniform surface temperature
- Radiative heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the mean temperature of 42.5°C

Thermal expansion coefficient (β) = 0.00317 1/K

Thermal conductivity (k) = 0.0267 W/(m K)

Kinematic viscosity (ν) = 17.8×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The rate of heat generation per unit area is

$$\frac{q_g}{A} = \frac{N\,q}{L^2} = \frac{100(0.06\,\text{W})}{(0.15\,\text{m})^2} = 266.7\ \text{W/m}^2$$

(a) The Grashof number is

$$Gr_L = \frac{g\,\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ 0.00317\ (1/\text{K})\ (60°\text{C} - 25°\text{C})(0.15\,\text{m})^3}{\left(17.8\times10^{-6}\text{m}^2/\text{s}\right)^2} = 1.16\times10^7$$

The Nusselt number for natural convection is given by Equation (8.12b)

$$(Nu_L)_{\text{free}} = 0.68\ Pr^{\frac{1}{2}}\ \frac{Gr_L^{\frac{1}{4}}}{(0.952 + Pr)^{\frac{1}{4}}} = 0.68\ (0.71)^{\frac{1}{2}}\ \frac{(1.16\times10^7)^{\frac{1}{4}}}{(0.952 + 0.71)^{\frac{1}{4}}} = 29.44$$

$$(h_c)_{\text{free}} = Nu_L\ \frac{k}{L} = 29.44\ \frac{0.0267\ \text{W/(m K)}}{0.15\,\text{m}} = 5.24\,\text{W/(m}^2\text{K)}$$

The rate of convective heat transfer must equal the rate of heat generation if

$$\frac{q_c}{A} = h_c\,(T_s - T_\infty) = 5.24\,\text{W/(m}^2\text{K)}\ \ (60°\text{C} - 25°\text{C}) = 183.4\ \text{W/m}^2 < \frac{q_g}{A}$$

Since this is lower than the heat generation rate, the actual surface temperature will be higher than the maximum of 60°C.

Therefore, natural convection alone will not keep the chips cool enough.

(b) The Reynolds number for $U_\infty = 0.5$ m/s is

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(0.5\,\text{m/s})(0.15\,\text{m})}{17.8\times10^{-6}\text{m}^2/\text{s}} = 4.21\times10^3 < 5\times10^5\ \text{(laminar)}$$

From Equation (8.45) the relative importance of natural and forced convection is indicated by the following ratio

$$\frac{Gr_L}{Re_L^{\,2}} = \frac{1.16\times10^7}{(4.21\times10^3)^2} = 0.65$$

Since $(Gr_L/Re_L^2) \approx 1$, natural and forced convection are of the same order of magnitude. The average Nusselt number can be estimated from Equation (8.57)

$$Nu = \left[(Nu_{\text{forced}})^3 + (Nu_{\text{free}})^3\right]^{\frac{1}{3}}$$

In this case, the natural convective flow is in the same direction as the forced convection flow; therefore, the plus sign is appropriate.

The forced convection Nusselt number is given by Equation (4.38)

$$(Nu_L)_{\text{forced}} = 0.664\ Re_L^{\frac{1}{2}}\ Pr^{\frac{1}{3}} = 0.664\ (4.21\times10^3)^{\frac{1}{2}}\ (0.71)^{\frac{1}{3}} = 38.44$$

$$\therefore\ N_u = [(38.44)^3 + (29.44)^3]^{\frac{1}{3}} = 43.50$$

$$h_c = Nu_L\ \frac{k}{L} = 43.50\ \frac{0.0267\ \text{W/(m K)}}{0.15\,\text{m}} = 7.74\ \text{W/(m}^2\text{K)}$$

$$\frac{q_c}{A} = h_c\,(T_s - T_\infty) = 7.74\,\text{W/(m}^2\text{K)}\ \ (60°\text{C} - 25°\text{C}) = 271.0\,\text{W/m}^2 > \frac{q_g}{A}$$

Therefore, this configuration is adequate to keep the chip surface temperature below 60°C.

(c) In this configuration, the free convective flow opposes the forced convection

$$N_u = [(Nu_{\text{forced}})^3 - (Nu_{\text{free}})^3]^{\frac{1}{3}} = [(38.44)^3 - (29.44)^3]^{\frac{1}{3}} = 31.51$$

$$h_c = Nu_L \frac{k}{L} = 31.51 \frac{0.0267 \text{ W/(m K)}}{0.15 \text{ m}} = 5.61 \text{ W/(m}^2\text{K)}$$

$$\frac{q_c}{A} = h_c (T_s - T_\infty) = 5.16 \text{ W/(m}^2\text{K)} \quad (60°\text{C} - 25°\text{C}) = 196.3 \text{ W/m}^2 < \frac{q_g}{A}$$

Therefore, this configuration will not keep the chips cool enough.