

$$\mu_{\text{water}} = 2.34 \times 10^{-4} \frac{\text{lb} \cdot \text{s}}{\text{ft}^2}$$

$$1 \text{ mile} = 5280 \text{ ft}$$

$$1 \frac{\text{slug} \cdot \text{ft}}{\text{s}^2} = 1 \text{ lbf}$$

$$g = 32.2 \frac{\text{ft}}{\text{s}^2}$$

1. European Intercity Express trains operate at speeds of up to 280 km/h. Suppose that a train is 120 m long. Treat the sides and top of the train as one smooth flat plate 9 m wide. When the train moves through still air at sea level, calculate

- a. the possible length of the laminar boundary layer, [5 pts]

$$V = 280 \text{ km/h} = \frac{280000 \text{ m}}{\text{h}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 77.78 \text{ m/s}$$

$$\rho_{\text{air}} = 1.225 \text{ kg/m}^3, \mu_{\text{air}} = 1.789 \times 10^{-5} \text{ Pa} \cdot \text{s}$$

$$\therefore Re = \frac{\rho V L}{\mu} < 5 \times 10^5$$

$$L < \frac{5 \times 10^5 \mu}{\rho V}$$

$$L < \frac{5 \times 10^5 \times 1.789 \times 10^{-5}}{1.225 \times 77.78} \text{ m}$$

$$0.094 \text{ m}$$

- b. the thickness of this laminar layer at its down-stream end, [5 pts]

$$\text{for laminar flow: } \delta_{\text{lam}} = \frac{5x}{\sqrt{Re_x}}, \quad Re_x = \frac{\rho V x}{\mu}, \quad x = 120 \text{ m} = L$$

$$\rightarrow \delta_{\text{lam}} = \frac{5 \times 120}{\sqrt{1.225 \times 77.78 \times 120 / 1.789 \times 10^{-5}}} = 0.094 \text{ m}$$

- c. the thickness of the boundary layer at the rear end of the train, [10 pts]

$$\text{for turbulent flow: } \delta = \frac{0.382x}{Re_x^{1/4}}, \quad x = 120 \text{ m} = L$$

$$Re_x = \frac{\rho V L}{\mu} = \frac{1.225 \times 77.78 \times 120}{1.789 \times 10^{-5}} = 6.391 \times 10^8$$

$$\rightarrow \delta = \frac{0.382 \times 120}{(6.391 \times 10^8)^{1/4}} = 0.095 \text{ m}$$

- d. the viscous drag force on the train in kN. [5 pts]

$$\therefore Re_x = 6.391 \times 10^8, \quad A = 120 \text{ m} \times 9 \text{ m} = 1080 \text{ m}^2$$

$$\therefore C_D = \frac{0.455}{(1 + 0.27 Re_x^{-1/4})^2} = 1.66 \times 10^{-3}$$

$$\rightarrow F_D = \frac{1}{2} \rho_{\text{air}} V^2 A C_D = \frac{1}{2} \times 1.225 \text{ kg/m}^3 \times (77.78 \text{ m/s})^2 \times 1080 \text{ m}^2 \times 1.66 \times 10^{-3}$$

$$= 6639.6 \text{ N}$$

$$= 6.64 \text{ kN}$$

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2. Water flows through a 2-in-diameter tube that suddenly contracts to 1 in diameter. The pressure drop across the contraction is 0.5 psi. Determine the volume flow rate in ft^3/min . [25pts]

assume there is no head loss, incompressible flow, $z_1 = z_2$

$$\frac{p_1}{\rho_{\text{water}}} + \frac{v_1^2}{2} + gz_1 = \frac{p_2}{\rho_{\text{water}}} + \frac{v_2^2}{2} + gz_2$$

$$\frac{d}{dt} \int_{CV} \rho dV + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$$

$$\rightarrow Q_1 = Q_2, A_1 v_1 = A_2 v_2, A_1 = \left(\frac{2}{12} \text{ft}\right)^2 \pi, A_2 = \left(\frac{1}{12} \text{ft}\right)^2 \pi$$

$$\therefore \rho_{\text{water}} = 1.938 \text{ slug/ft}^3, z_1 = z_2$$

$$\rightarrow \frac{A_2}{A_1} = \frac{1}{4}$$

$$\rightarrow \frac{p_1 - p_2}{\rho_{\text{water}}} = \frac{v_2^2 - v_1^2}{2} = \left(1 - \frac{A_2^2}{A_1^2}\right) \frac{v_2^2}{2}$$

$$v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho_{\text{water}}(1 - \frac{A_2^2}{A_1^2})}} = \sqrt{\frac{2 \times 0.5 \text{ lbf/in}^2 \cdot \left(\frac{144 \text{ in}^2}{\text{ft}^2}\right)^2}{1.938 \text{ slug/ft}^3 (1 - \frac{1}{4})}}$$

$$= 8.904 \text{ ft/s}$$

$$\begin{aligned} \rightarrow A_2 v_2 = Q &= \left(\frac{1}{12} \text{ft}\right)^2 \pi \times 8.904 \text{ ft/s} = 15.48 \times 10^{-3} \text{ ft}^3/\text{s} \\ &= 15.48 \times 10^{-3} \text{ ft}^3/\text{s} \times \frac{60 \text{ s}}{1 \text{ min}} \\ &= 0.9288 \text{ ft}^3/\text{min} \end{aligned}$$

3. The unsteady lift force F_L on a cylinder is a function of the diameter D , the cylinder length l , the vortex shedding frequency f , the free stream velocity V_∞ , the free stream density ρ , the free stream speed of sound c_∞ , and the viscosity μ . Use D , V_∞ , and ρ as the repeating parameters.
- What is the dependent parameter? [3pts]
 - What is the total number of variables? [3pts]
 - What is the number of fundamental (primary) dimensions? [3pts]
 - How many dimensionless parameters will result? [3pts]
 - Obtain the π parameter that contains the frequency, f . Note: dimension for f is 1/time. [13pts]

$D \quad l \quad f \quad V_\infty \quad \rho \quad c_\infty \quad \mu \rightarrow n=7$

repeating parameter = D, V_∞, ρ

(a) ~~dependent parameters~~ = ~~l, f, c_∞, μ~~ 0

(b) ~~total number of variables~~ = ~~$n=7$~~ 0

(c) $\therefore D \rightarrow L \quad V_\infty \rightarrow L/t \quad \rho \rightarrow \frac{M}{L^3}$
 \rightarrow primary dimensions: $M, L, t \rightarrow r=3$ 3

(d) $\therefore \begin{matrix} l & f & c_\infty & \mu \\ L & \frac{1}{t} & \frac{L}{t} & \frac{M}{Lt} \end{matrix} \quad \therefore m=r=3$ 3

$\rightarrow n-m=4$ 4 so, four dimensionless parameters will result.

(e) $\pi = \rho^a V_\infty^b D^c f \rightarrow (\frac{M}{L^3})^a (\frac{L}{t})^b (L)^c (\frac{1}{t}) = M^0 L^0 t^0$

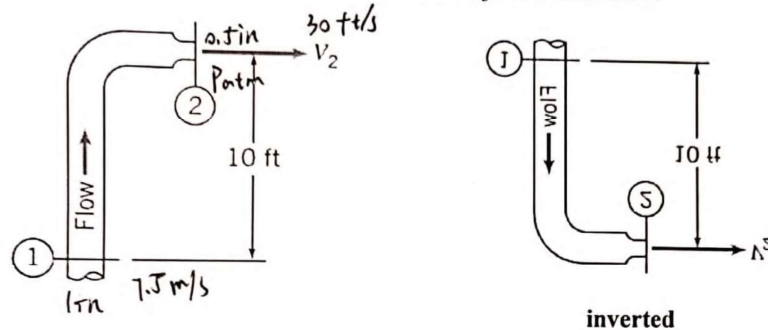
$\rightarrow M: a=0, L: b+c-3a=0, t: -b-1=0$

$\Rightarrow a=0, b=-1, c=1$ so, $\pi = \frac{Df}{V_\infty}$ 13

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4. Fill-in the blank questions: [5pts each]

Water flows steadily up the vertical 1-in-diameter pipe and out the nozzle, which is 0.5 in in diameter, discharging to atmospheric pressure. The stream velocity at the nozzle exit must be 30 ft/s. From here:



- the velocity at section ① is 7.5 ft/s,
- the minimum gage pressure required at section ① is 144.63 psi,
- if the device were inverted, the required minimum gage pressure at section ① would be 193.56 psi.

Your car runs out of gas unexpectedly and you siphon gas from another car. The height difference for the siphon is 1 ft. The hose diameter is 0.5 in. Assume the tank free surface is decreasing very slowly and is exposed to the atmosphere.

- The flow velocity at the syphon exit is 8.025 ft/s.
- The gasoline flow rate is 0.011 ft³/s.