University of Cincinnati
College of Engineering and Applied Science
Department of Mechanical & Materials Engineering

Course Code : **20-MECH-4071C**
Course Title : **Measurements & Instrumentation Lab**

should be bold

Experiment Number : #15
Experiment Title : **Dynamic Testing of Mechanical Systems Transient Measurements**

Date(s) Performed : **October 10th&17th,2022**
Date Submitted : **October 24th, 2022**

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Deductions by TA:

Initials	Content	Form/Writ
LB	8	1(lang)
SJ	1	1(form)
DP	4	0

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TOTAL SCORE = 185 /200

ABSTRACT

The experiment is divided into three parts, which are Structural Beam Test, Dynamic analysis of a Cantilever Beam and IEPE Circuit.

In the section of Structural Beam Test, conduct a vibration test on a simple structure (free-free beam) to determine its damped natural frequencies and mode shapes. Impact test approach is used to estimate the FRF and the collected FRF data is used to estimate the first five natural frequencies and mode shapes of the beam being tested. By plotting the collected data, we found that it is easier and more convenient to use the real FRF graphs than the phase FRF graphs. For the torsional mode, it is symmetrical about the centerline of the free-free beam. For the bending mode, the mode shape of points 1-12 and the mode shape of points 13-24 are similar.

do not use personal pronouns

In the section of Dynamic analysis of a Cantilever Beam, use two different sensors (an accelerometer and a strain gage) to demonstrate various methods in which time data can be taken for dynamic systems. Use the curve fitting tool of MATLAB to process the collected data and to estimate undamped natural frequency, damped natural frequency and damping ratio. Then, those data are used to determine the material property $E_\rho = 2.685 \times 10^7\ m^2/s^2$. Combined with the material property and the physical diagram shown by the experiment, a kind of steel (with the specific modulus $E_\rho = 25 \times 10^6\ m^2/s^2$) is considered to use to make the cantilever beam. Finally, compute the weight of the unknown mass (hanger) $m_{hanger} = 0.4687\ lb$ and the effective mass of the beam $m_{beam} = 0.1384\ lb$.

In the section of IEPE Circuit, construct a complex circuit with components that might not be familiar and a circuit diagram similar to what is typically seen in industry manuals.

-1.5; procedue discussion must be more specific

Table of Contents

-1 incorrect

1. Objectives

In the part of structural beam test, it is expected to conduct a vibration test on a simple structure (free-free beam) to determine its damped natural frequencies and mode shapes, utilize a single reference, impact test approach to estimate the frequency response functions, and use the collected FRF data to estimate the first five natural frequencies and mode shapes of the beam being tested. In the part of dynamic analysis of a cantilever beam, combined with mechanical vibration knowledge learned before, it is expected to use two different sensors (an accelerometer and a strain gage) to demonstrate various methods in which time data can be taken for dynamic systems, use the curve fitting tool of MATLAB to process the collected data and to estimate undamped natural frequency, damped natural frequency and damping ratio, and determine the material property (E/ρ) of the cantilever beam and the mass of a hanger attached to the end of the cantilever beam. In the part of IEPE circuit, it is expected to construct a complex circuit with components that might not be familiar and a circuit diagram similar to what is typically seen in industry manuals.

2. Theoretical Background

1.1. Vibrations of a Free-Free Beam

The bending vibrations of a beam are described by the following equation:

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = 0$$

E: Young's Modulus

I: Second moment of area of the cross section

ρ: Density

A: Cross section area of the beam

L: Length of the beam

The solution of the equation above can be written as a standing wave:

$$y(x,t) = w(x)u(t)$$

Separate the spatial and temporal component. This leads to the following characteristic equation that relates the circular frequency ω to the wavenumber k :

$$w^2 = \frac{EI}{\rho A} k^4$$

The spatial part can be written as:

$$w(x) = C_1 \sin(kx) + C_2 \cos(kx) + C_3 \sinh(kx) + C_4 \cosh(kx)$$

For a Free-Free Beam the boundary conditions are (vanishing of force and moment):

$$\begin{cases} w''(0) = 0 \\ w'''(0) = 0 \\ w''(L) = 0 \\ w'''(L) = 0 \end{cases} \rightarrow \begin{cases} -C_2 + C_4 = 0 \\ -C_1 + C_3 = 0 \\ -C_1 \sin(kL) - C_2 \cos(kL) + C_3 \sinh(kL) + C_4 \cosh(kL) = 0 \\ -C_1 \cos(kL) - C_2 \sin(kL) + C_3 \cosh(kL) + C_4 \sinh(kL) = 0 \end{cases}$$

Using the first two equations the 3rd and 4th can be arranged in matrix form:

$$\begin{bmatrix} \sinh(kL) - \sin(kL) & \cosh(kL) - \cos(kL) \\ \cosh(kL) - \cos(kL) & \sin(kL) + \sinh(kL) \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For a non trivial solution the determinant of the matrix has to vanish to get:

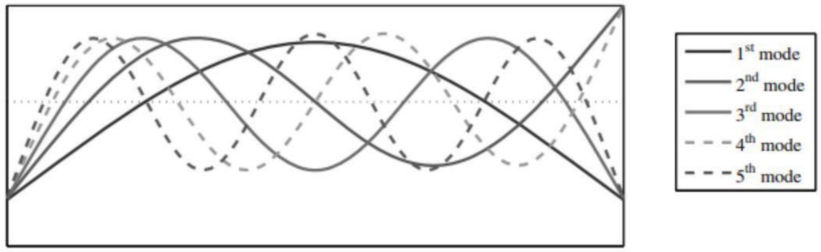
$$\cosh(kL) \cos(kL) = 1$$

The transcendental equation above has infinite solutions. It can be solved numerically. The first five values are reported here:

Mode order n	$k_n L$
0	0
1	4.7300
2	7.8532
3	10.9956
4	14.1371
5	17.2787

The mode shapes are given by the following:

$$w_n(x) = [\sinh(k_n x) + \sin(k_n x)] + \frac{\sin(k_n L) - \sinh(k_n L)}{\cosh(k_n L) - \cos(k_n L)} [\cosh(k_n x) + cos(k_n x)]$$



For the torsional mode, the first torsional mode can be computed as follows:

The shear modulus:

$$G = \frac{E}{2(1+v)}$$

The torsional constant:

$$\gamma = bh^3 \left[\frac{1}{3} - 0.21 \frac{h}{b} \left(1 - \frac{h^4}{12b^4}\right)\right]$$

The polar moment:

$$J_P = \frac{bh}{12}(b^2 + h^2)$$

The first torsional mode:

$$f_{T,1} = \frac{C_T}{2L} = \frac{\sqrt{\frac{G\gamma}{\rho J_P}}}{2L}$$

1.2. Dynamic analysis of a Cantilever Beam

Damped natural frequency:

$$\omega_d = \frac{2\pi}{T}$$

Undamped natural frequency:

$$\omega_n = \frac{w_d}{\sqrt{1-\zeta^2}} = \sqrt{\frac{k}{m}}$$

Damping ratio:

$$\zeta = \text{Estimation by MATLAB curve fitting tool}$$

Undamped natural frequency relative to mode shape:

$$\omega = \frac{\alpha}{L^2}\sqrt{\frac{EI}{A\rho}}\,(\mathrm{rad/s})$$

Values of α :

End conditions	Frequency equation	1st mode	2nd mode	3rd mode	4th mode	5th mode
Clamped–free	$\cos\lambda l \cosh\lambda l = -1$	3.52	22.4	61.7	21.0	199.9
Pinned–pinned	$\sin\lambda l = 0$	9.87	39.5	88.9	157.9	246.8
Clamped–pinned	$\tan\lambda l = \tanh\lambda l$	15.4	50.0	104.0	178.3	272.0
Clamped–clamped or Free-free	$\cos\lambda l \cosh\lambda l = 1$	22.4	61.7	121.0	199.9	298.6

3. Experimentation

3.1 Structural Beam Test

3.1.1 Procedure Objectives

During this experiment, the students are expected to: First, conducting a vibration test on a simple structure (free-free beam) to determine its damped natural frequencies and mode shapes. Second, utilize a single reference, impact test approach to estimate the frequency response functions. Third, use the collected FRF data to estimate the first five natural frequencies and mode shapes of the beam being tested.

3.1.2 Experimental Results

All original data are drawn below:

-5; missing dimensions of the beam and spacing of hitting points

.1; missing units of real, imag, magnitude, phase

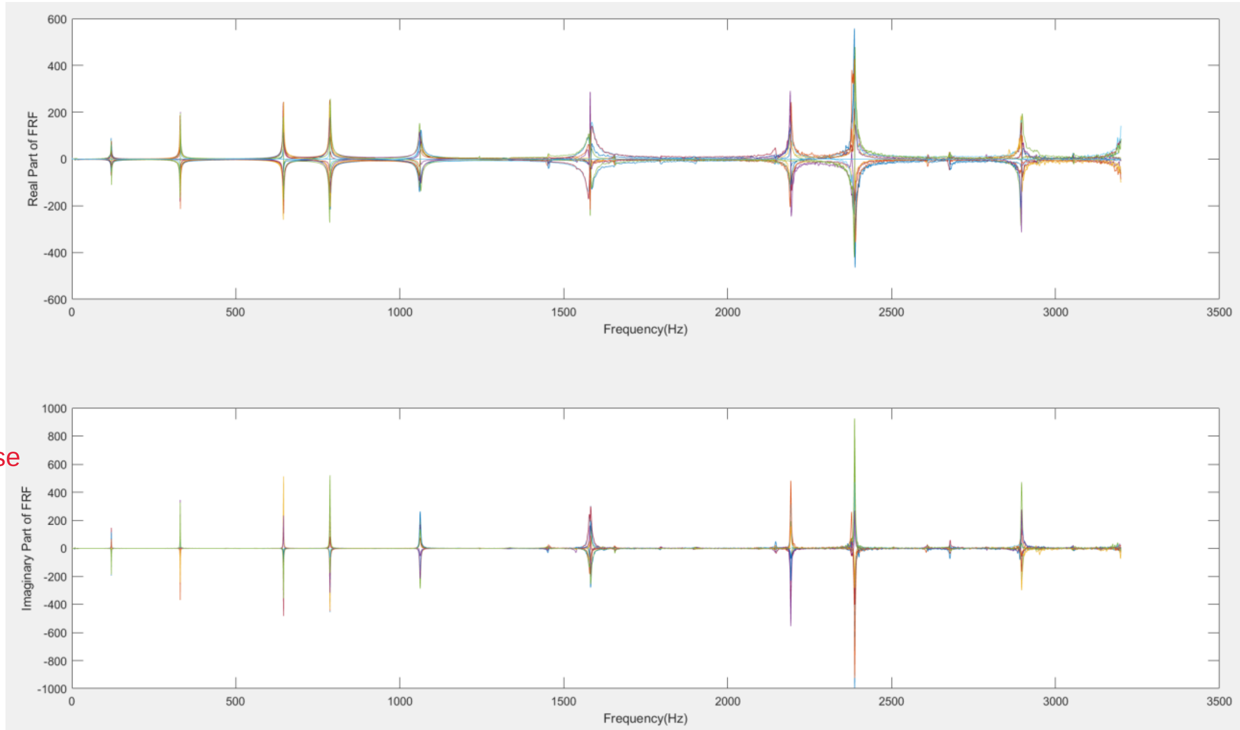


Figure 1 Real and imaginary part of points 1-12

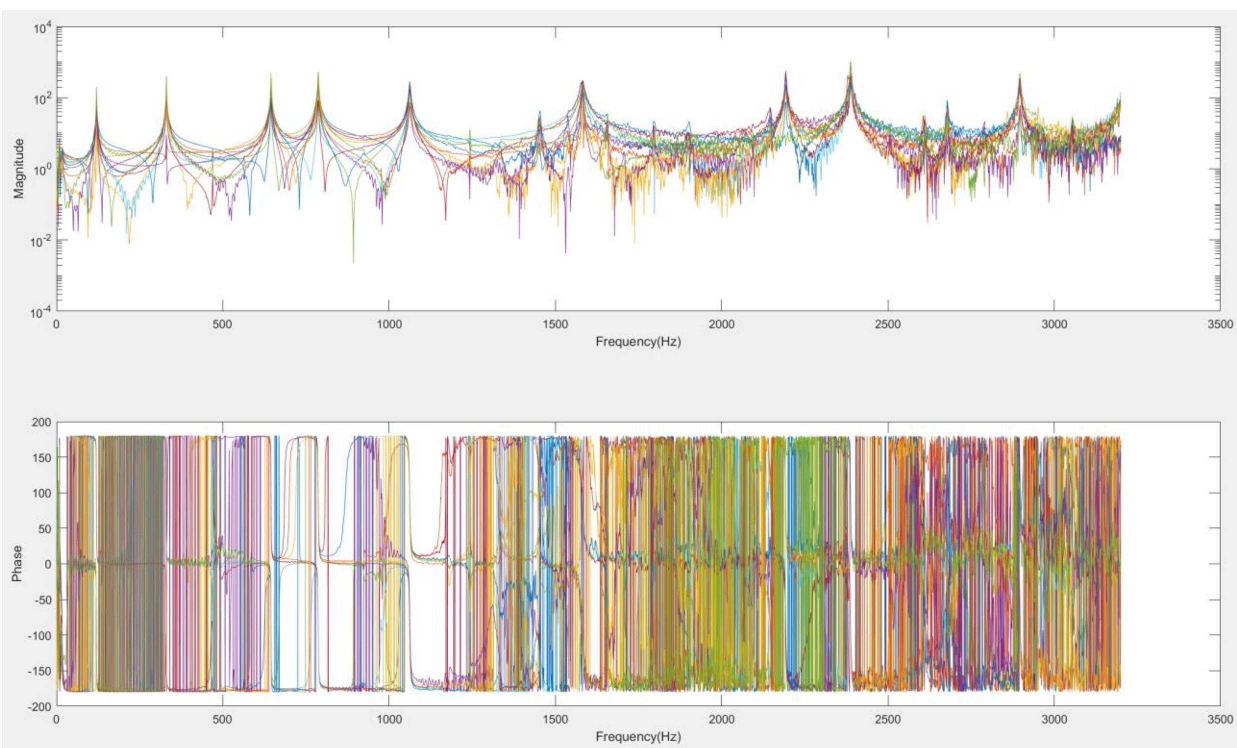


Figure 2 Magnitude and phase of points 1-12

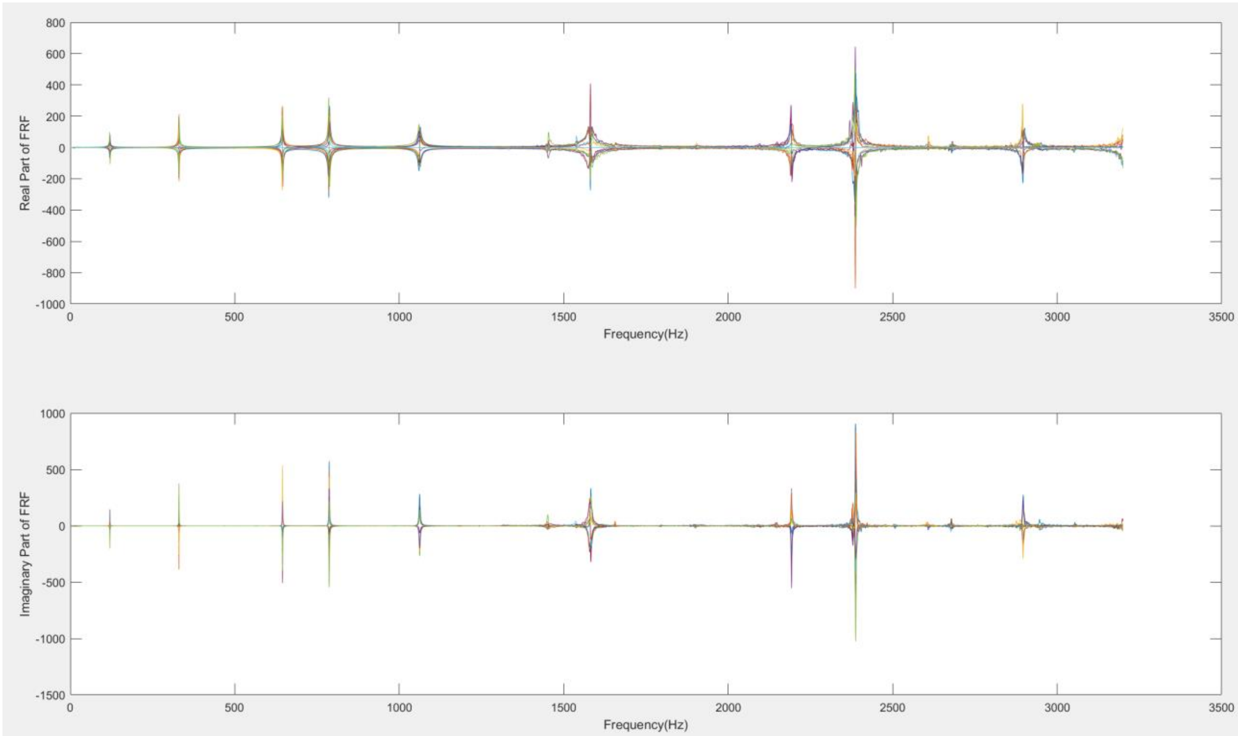


Figure 3 Real and imaginary part of points 13-24

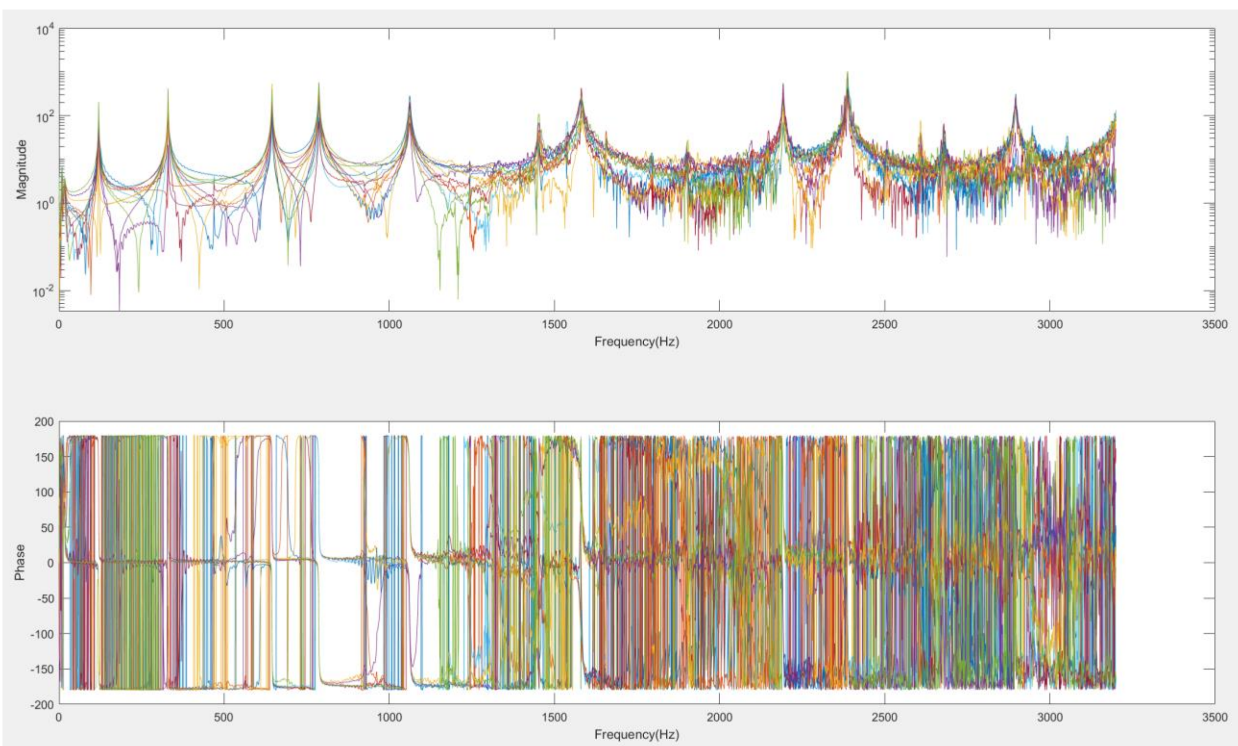


Figure 4 Magnitude and phase of points 13-24

3.1.3 Analysis Questions

A. Using the dimensions and material properties of your beam, estimate the first four bending natural frequencies and the first torsional natural frequency for a free- free beam.

Table 1. Properties and dimensions of the free-free beam

Length (L)	36 in
Width (W)	3 in
Height (H)	0.75 in
Young's Modulus (E)	10442717.56lb/in^2
Density (ρ)	0.0975lbm/in^3
Poisson Ratio (v)	0.34

Sample calculation:

For the 1 st bending natural frequency:

$$I = \frac{WH^3}{12} = \frac{(3\text{in})(0.75\text{in})^3}{12} = 0.1055\text{in}^4$$
$$A = WH = (3\text{in})(0.75\text{in}) = 2.25\text{in}^2$$
$$k_1 = \frac{k_1 L}{L} = \frac{4.7300}{36\text{in}} = 0.1314\text{in}^{-1}$$
$$w_1 = \sqrt{\frac{EI}{\rho A}k^4} = \sqrt{\frac{\left(10442717.56\frac{lb}{in^2}\right)(0.1055\text{in}^4)}{\left(0.0975\frac{lbm}{in^3} * \frac{1}{lbm} * \frac{lb * s^2}{32.174 * 12\text{in}}\right)(2.25\text{in}^2)}(0.1314\text{in}^{-1})^4}$$
$$w_1 = 760.04\frac{\text{rad}}{s} = 120.96\text{ Hz}$$

Similarly, other three bending natural frequencies w_2, w_3 and w_4 can be computed, as shown

Table 2. The first four bending natural frequencies for the free-free beam

Mode order	Bending natural frequencies
1	120.96 Hz
2	333.45 Hz

3	653.69 Hz
4	1080.57 Hz

For the 1st tortional natural frequency:

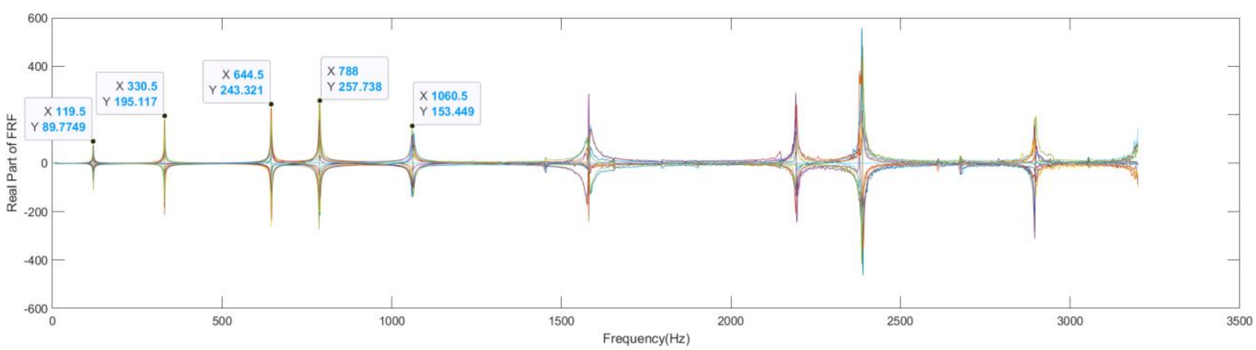
$$G = \frac{E}{2(1+v)} = \frac{10442717.56\,\frac{lb}{in^2}}{2(1+0.34)} = 3896536.4030\,\frac{lb}{in^2}$$

$$\gamma = WH^3\left[\frac{1}{3} - 0.21\frac{H}{W}\left(1-\frac{H^4}{12W^4}\right)\right] = (3)(0.75)^3\left[\frac{1}{3} - 0.21\frac{0.75}{3}\left(1-\frac{(0.75)^4}{12(3)^4}\right)\right] = 0.3555\text{in}^4$$

$$J_P = \frac{WH}{12}(W^2+H^2) = \frac{(3\text{in})(0.75\text{in})}{12}[(3\text{in})^2+(0.75\text{in})^2] = 1.7930\text{in}^4$$

$$f_{T,1} = \frac{C_T}{2L} = \frac{\sqrt{\frac{G\gamma}{\rho J_P}}}{2L} = \frac{\sqrt{\frac{(3896536.4030\,\frac{lb}{in^2})(0.3555\,in^4)}{\left(0.0975\frac{lbm}{in^3} * \frac{1}{lbm} * \frac{lb * s^2}{32.174 * 12\,in}\right)(1.7930\,in^4)}}}{2(36\,in)} = 768.16\,Hz$$

B. Using the Real FRF graphs from your results, estimate the first five undamped natural frequencies.



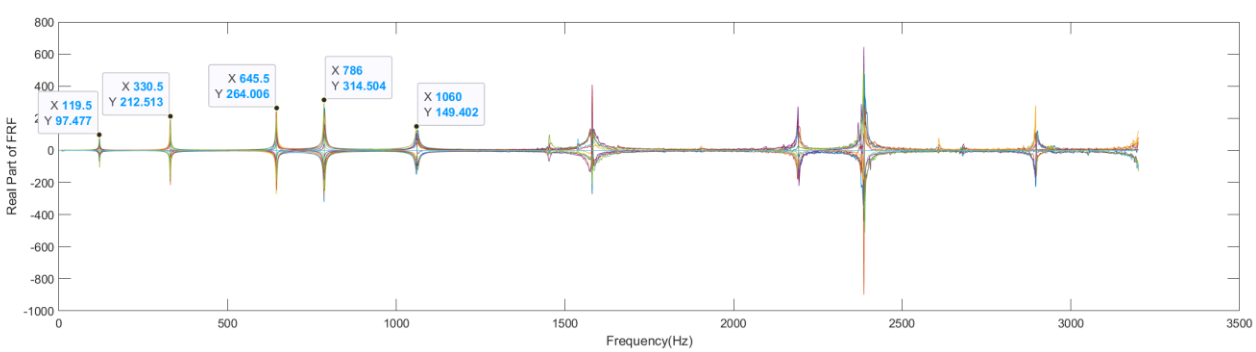


Figure 5. Estimation for the first five undamped natural frequencies

According to **Figure 5. Estimation for the first five undamped natural frequencies**, the first five undamped natural frequencies can be estimated as:

Table 3. Estimation for the first five undamped natural frequencies

Mode order	Type	Undamped natural frequencies
1	Bending	120 Hz
2	Bending	330 Hz
3	Bending	645 Hz
4	Torsional	787 Hz
5	Bending	1062 Hz

C. How do your experimental estimates (question B) compare to the theoretical values (question A)? If there is a significant difference between the analytical and experimental values, what do you think are the causes of the discrepancies?

Sample calculation of relative error:

$$\text{Error}_{1} = \left|\frac{w_{1,\text{ estimated}} - w_{1,\text{ theoretical}}}{w_{1,\text{ theoretical}}}\right| \times 100\% = \left|\frac{120 - 120.96}{120.96}\right| \times 100\% = 0.79\%$$

Similarly, other four relative errors can be computed, as shown below:

Table 4. Relative errors for estimated values and theoretical values

Mode order	Type	Theoretical	Estimated	Relative error
1	Bending	120.96 Hz	120 Hz	0.79%
2	Bending	333.45 Hz	330 Hz	1.03%
3	Bending	653.69 Hz	645 Hz	1.33%
4	Torsional	768.16 Hz	787 Hz	2.45%
5	Bending	1080.57 Hz	1062 Hz	1.72%

Comparing experimental estimates and the theoretical values, it is not a significant difference between the analytical and experimental values. The maximum relative error is 0.79%, which is small. It shows that the experimental data is accurate, and the estimation method is effective.

There are errors in the data collected by the instrument, which may lead to some errors in the calculated theoretical value. The experimental equipment used is not a perfect free-free beam. In fact, it may have little damping. This will also cause errors in the results.

D. Although the phase FRF graphs could be used to estimate the undamped natural frequencies, why is it better to use the real FRF graphs?

According to Figure 6 and Figure 8, the phase FRF graphs are too messy to be used to estimate the undamped natural frequencies. The phase change is too complex. Useful data is difficult to estimate directly from the graph. Instead, the real FRF graphs are more direct and clearer. The points where the peak is located can be easily found and used for estimation.

E. Using the Imaginary FRF graphs from your results, determine and plot the mode shape for each undamped frequency (scale each vector so that its values are within the range from −1 to 1). Note that the shapes for the bending modes should be similar to those shown in the reference material.

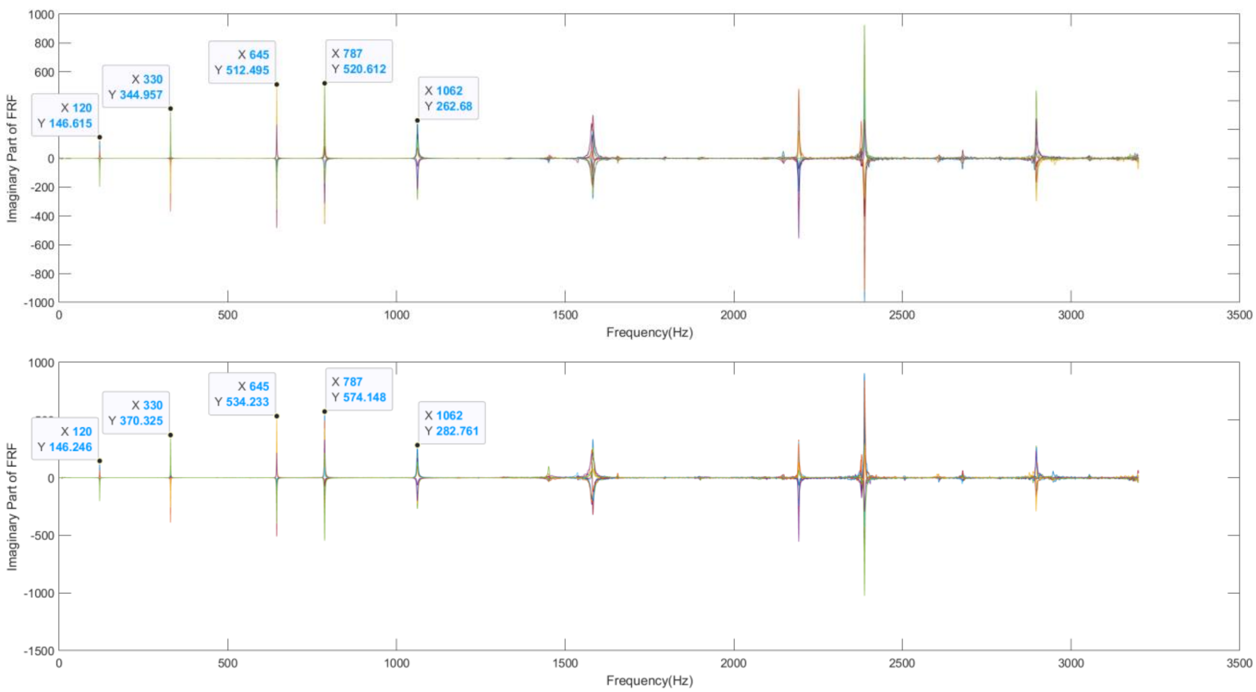


Figure 6. Imaginary Values of FRF Plots

Table 5. Data points for N-th row of H

Mode order	Type	N^{th} row of H
1	Bending	241
2	Bending	661
3	Bending	1291

-1 format

4	Torsional	1575
5	Bending	2125

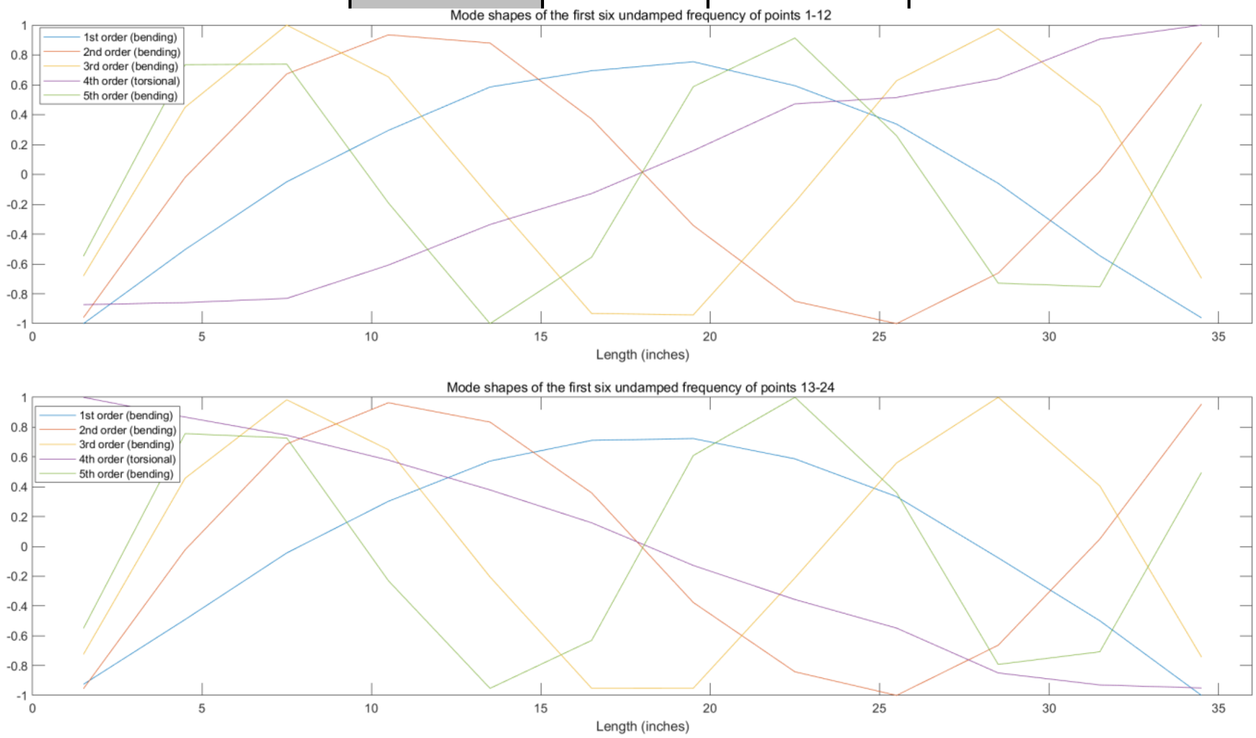
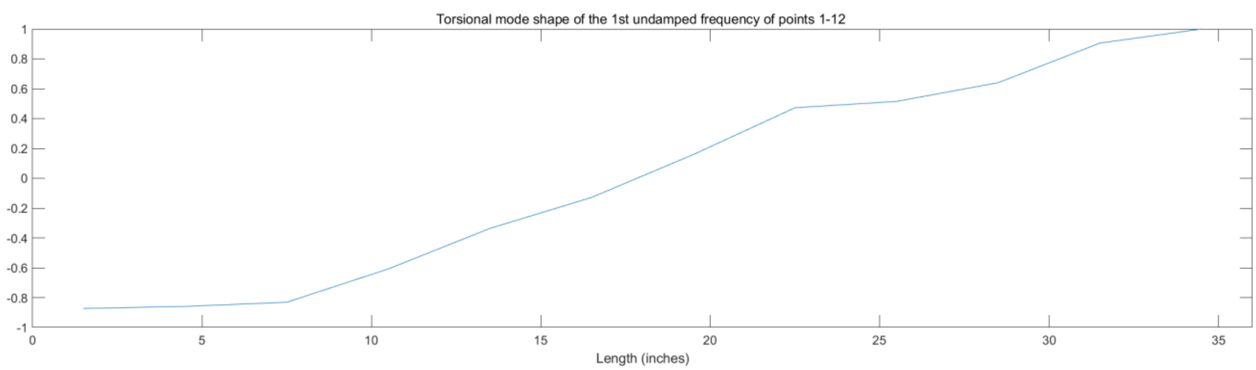


Figure 7. Mode shapes of the first five undamped frequency

F. What does the shape of the torsional mode look like?



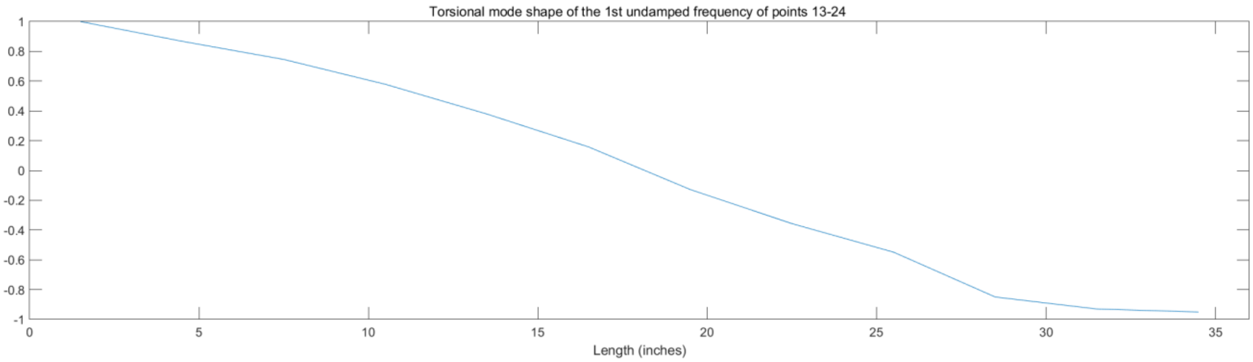


Figure 8. The first torsional mode

The shape of the torsional mode is symmetrical about the centerline of the free-free beam. It increases from one end of the beam and decreases from the other end. Its shape is a curve, and the middle part is similar to a straight line.

G. If theoretical analysis is not available for the object being tested, how would the type of mode (bending, torsion, etc) be determined from each of the plots obtained in Question E?

Through the plots of Question E, it can be found that there are obvious differences in their shape features. For the torsional mode, it is symmetrical about the centerline of the free-free beam. The mode shape of points 1-12 and the mode shape of points 13-24 should be symmetrical. For the bending mode, the mode shape of points $1-12$ and the mode shape of points $13-24$ should be similar.

H. If impact testing was only conducted along the centerline of the beam, could the torsional mode have been detected? Why or why not?

If impact testing was only conducted along the centerline of the beam, the torsional mode could not have been detected. Centerline is the neutral axis. The torsional mode would always be zero, so it cannot be detected.

3.1.4 General Discussion

Combined with the data recorded in the experiment and the data and diagram analysis in the Analysis Questions part below, the experiment was successful. The error between the undamped natural frequency estimated by the experimental data and the theoretical value is small. The mode shape drawn from the experimental data is similar to the theoretical plot. The error may come from the operation error and the measurement error of the experimental instrument.

When estimating underdamped natural frequencies, it is easier and more convenient to use the real FRF graphs than the phase FRF graphs. The phase FRF graphs are messy. The phase change is complex. Useful data is difficult to estimate directly from the graph. Instead, the real FRF graphs are more direct and clearer. The points where the peak is located can be easily found and used for estimation.

For the torsional mode, it is symmetrical about the centerline of the free-free beam. The mode shape of points 1-12 and the mode shape of points 13-24 should be symmetrical. If impact testing is only conducted along the centerline of the beam, the torsional mode can not have been detected because centerline is the neutral axis. For the bending mode, the mode shape of points $1-12$ and the mode shape of points $13-24$ should be similar.

3.2 Dynamic Analysis of a Cantilever Beam

3.2.1 Procedure Objectives

-1; 3-5 sentences

During this experiment, combined with mechanical vibration knowledge learned before, it is expected to use two different sensors (an accelerometer and a strain gage) to demonstrate various methods in which time data can be taken for dynamic systems, use the curve fitting tool of MATLAB to process the collected data and to estimate undamped natural frequency, damped natural frequency and damping ratio, and determine the material property (E/ρ) of the cantilever beam and the mass of a hanger attached to the end of the cantilever beam.

3.2.2 Experimental Results

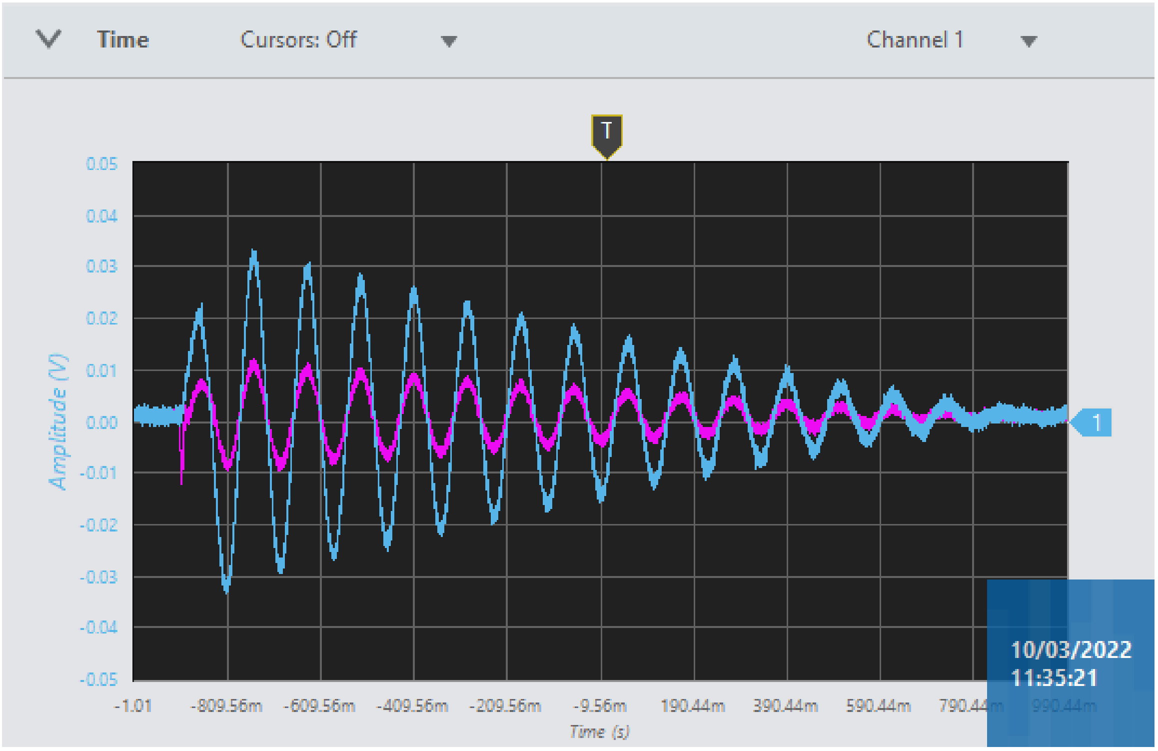
Beam dimensions:

Table 6. Beam Dimensions

-1; incorrect beam dimensions

Length (L)	12.785 inches
Width (W)	0.75 inches
Height (H)	0.25 inches

Raw data:



Trigger settings	Values
Mode	Auto
Type	Immediate
Acquisition delay	Disabled
Position	-9.558ms

Horizontal & acquisition settings	Values
Time per division	200ms
Acquisition	Decimate
Sampling	16 kS/s

Channel settings	Channel 1 values	Channel 2 values
Volts per division	10mV	10mV
Coupling	AC	AC
Probe attenuation	1x	1x
Vertical offset	0V	0V
Vertical position	0V	0V

Figure 9. Oscilloscope Screenshot for Case A (Beam, Hanger, and Weight)

Trigger settings	Values
Mode	Auto
Type	Immediate
Acquisition delay	Disabled
Position	-9.558ms

Horizontal & acquisition settings	Values
Time per division	100ms
Acquisition	Decimate
Sampling	32 kS/s

Channel settings	Channel 1 values	Channel 2 values
Volts per division	10mV	10mV
Coupling	AC	AC
Probe attenuation	1x	1x
Vertical offset	0V	0V
Vertical position	0V	0V

Figure 10. Oscilloscope Screenshot for Case B (Beam, Hanger)

Trigger settings	Values
Mode	Auto
Type	Immediate
Acquisition delay	Disabled
Position	-9.558ms

Horizontal & acquisition settings	Values
Time per division	50ms
Acquisition	Decimate
Sampling	64 kS/s

Channel settings	Channel 1 values	Channel 2 values
Volts per division	10mV	10mV
Coupling	AC	AC
Probe attenuation	1x	1x
Vertical offset	0V	0V
Vertical position	0V	0V

Figure 11. Oscilloscope Screenshot for Case C (Beam only)

3.2.3 Analysis Questions

A. In this experiment, similar free response data is obtained from two different sensors. Why are the locations chosen for these sensors different?

The locations of two different sensors are determined according to their respective measurement characteristics. The displacement and acceleration of the free end change more and the strain of the free end changes less. The acceleration and displacement of the fixed end are smaller and the strain of the fixed end changes more. Therefore, when strain indicator box is considered to be installed at the fixed end, the measurement is better. When accelerometer is considered to be installed at the free end, the measurement is better.

B. Using Matlab's curve fit tool to fit the strain gage data to an exponentially damped sine wave, estimate the following parameters for each case:

- **Damped natural frequency**
- **Damping ratio**
- **Undamped natural frequency**

(1) Case A: Beam and Hanger with 4lb weight

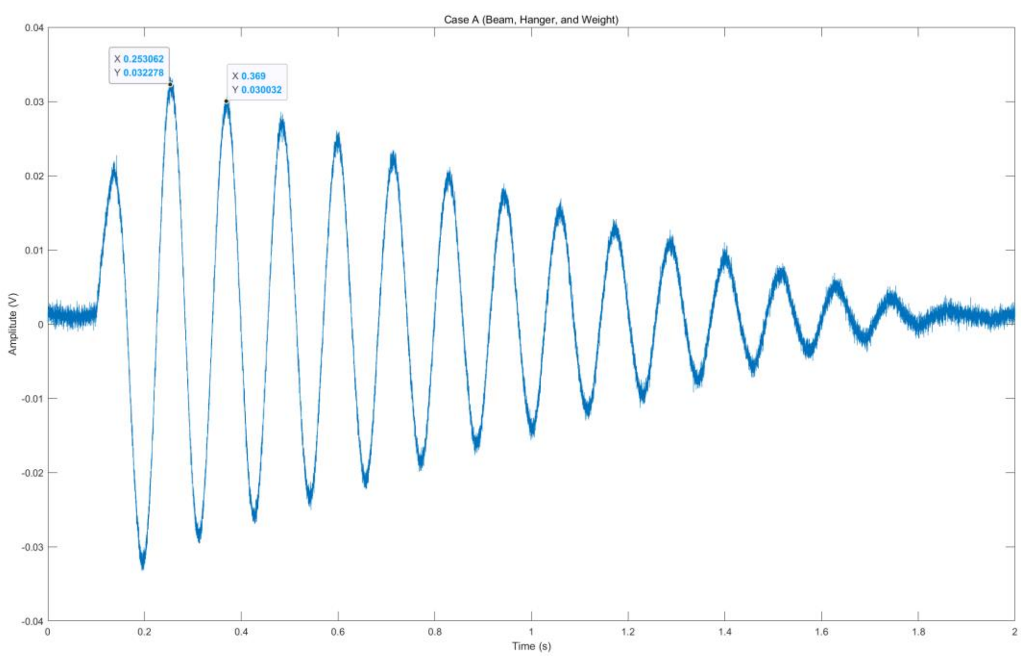


Figure 12. Strain Indicator Box Response for Case A

According to the matlab present, $\omega d = \frac{1}{(369-253.062)ms} = 8.625Hz$.

By fitting the Strain indicator box response following:

$y = a * exp(-b * 8.625 * 2 * pi * x/(sqrt(1 - b^2))) * sin(2 * pi * x * 8.625 + c) + d$

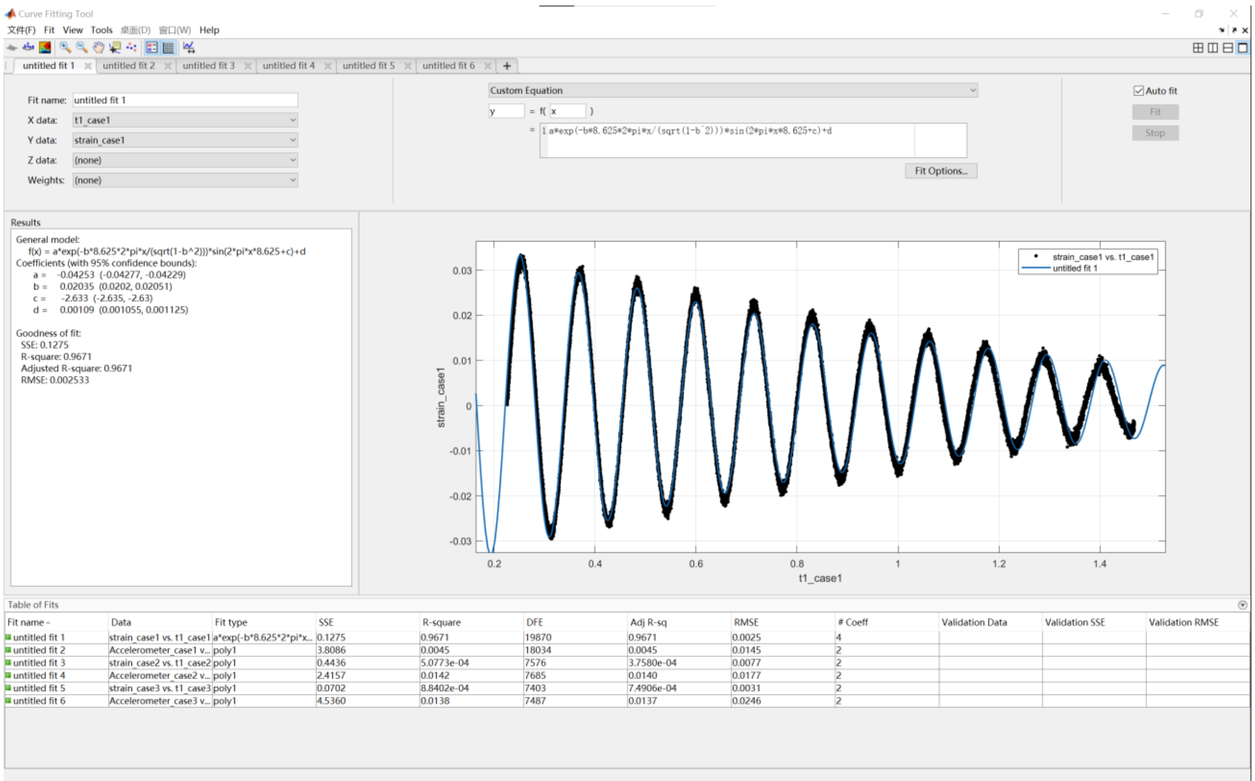


Figure 13. Curve Fitting for Strain Indicator Box for Case A

Then, $\xi = b = 0.02035$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{8.625Hz}{\sqrt{1-0.02035^2}} = 8.627Hz$$

(2) Case B: Beam and Hanger

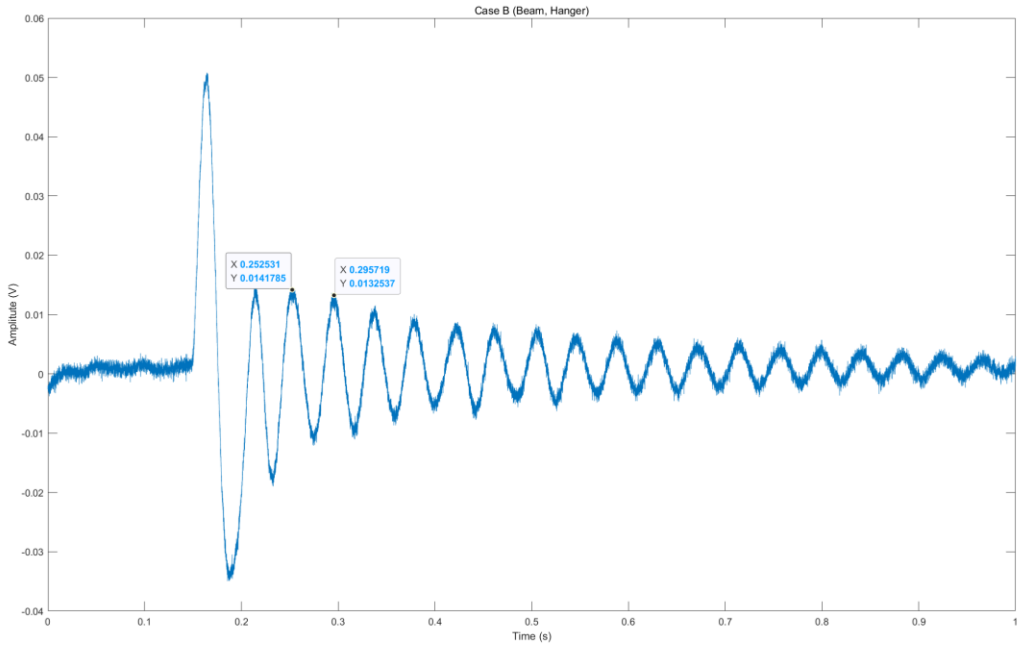


Figure 14. Strain Indicator Box Response for Case B

According to the matlab present, $\omega_d = \frac{1}{(295.719-252.531)ms} = 23.15Hz$.

By fitting the Strain indicator box response following:

$y = a * exp(-b * 23.15 * 2 * pi * x/(sqrt(1 - b^2)) * sin(2 * pi * x * 23.15 + c) + d$

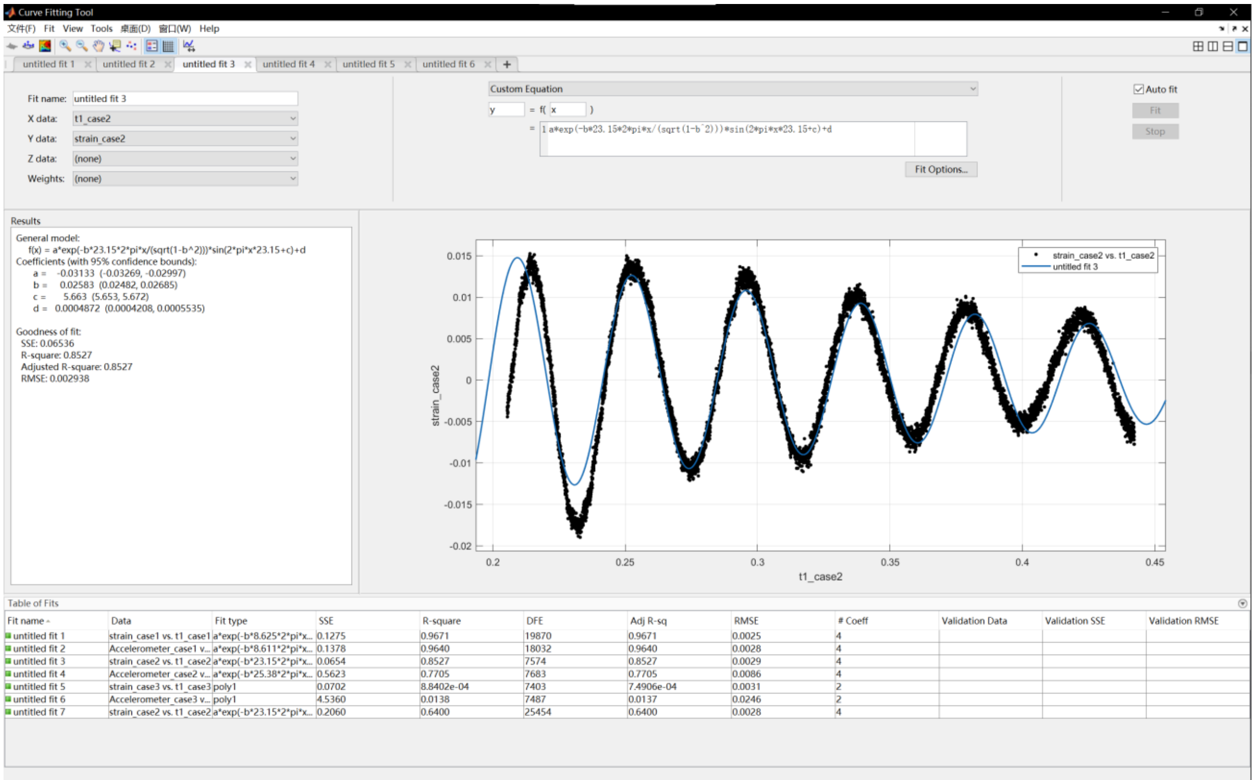


Figure 15. Curve Fitting for Strain Indicator Box for Case B

Then, $\xi = b = 0.02583$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{23.15Hz}{\sqrt{1-0.02583^2}} = 23.16Hz$$

(3) Case C: Beam only

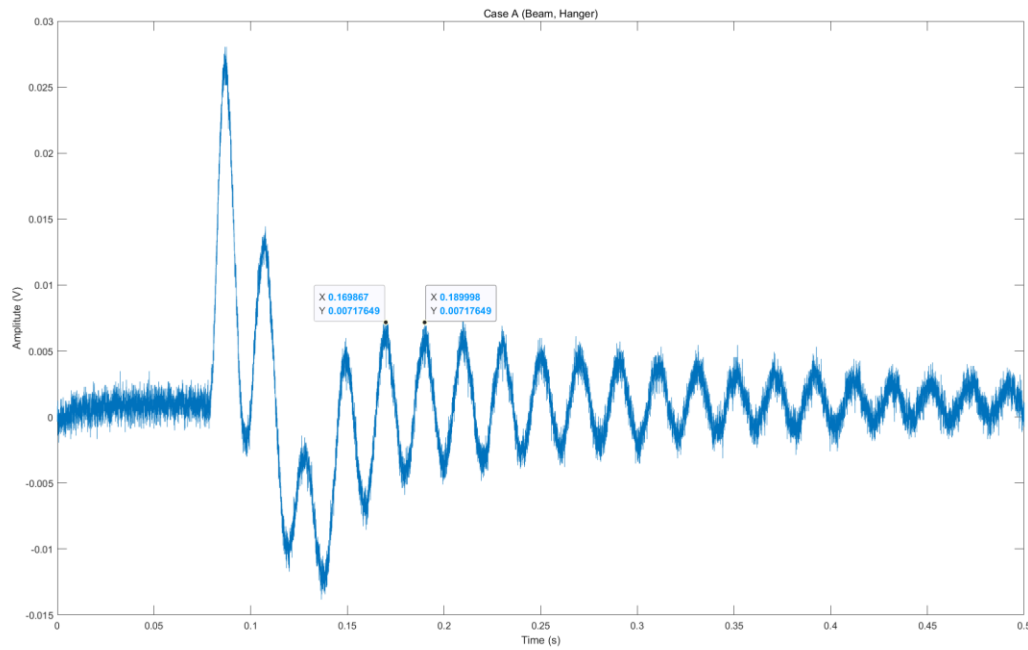
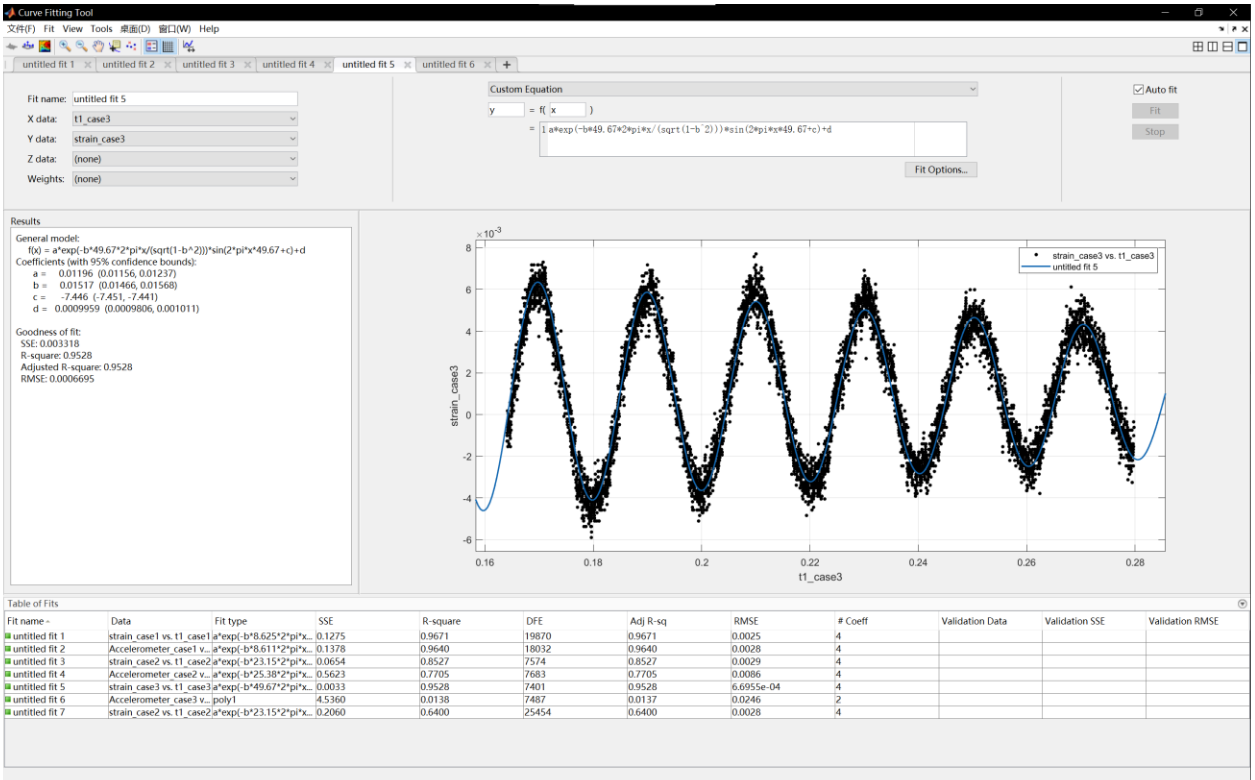


Figure 16. Strain Indicator Box Response for Case C

According to the matlab present, $\omega_d = \frac{1}{(189.998-169.867)ms} = 49.67Hz$.

By fitting the Strain indicator box response following:

$y = a * exp(-b * 49.67 * 2 * pi * x/(sqrt(1 - b\hat{}2))) * sin(2 * pi * x * 49.67 + c) + \text{d}$



Fit name	Data	Fit type	SSE	R-square	DFE	Adj R-sq	RMSE	# Coeff	Validation Data	Validation SSE	Validation RMSE
untitled fit 1	strain_case1 vs. t1_case1	a*exp(-b*8.625*2*pi*x...	0.1275	0.9671	19870	0.9671	0.0025	4			
untitled fit 2	Accelerometer_case1 v...	a*exp(-b*8.611*2*pi*x...	0.1378	0.9640	18032	0.9640	0.0028	4			
untitled fit 3	strain_case2 vs. t1_case2	a*exp(-b*23.15*2*pi*x...	0.0654	0.8527	7574	0.8527	0.0029	4			
untitled fit 4	Accelerometer_case2 v...	a*exp(-b*25.38*2*pi*x...	0.5623	0.7705	7683	0.7705	0.0086	4			
untitled fit 5	strain_case3 vs. t1_case3	a*exp(-b*49.67*2*pi*x...	0.0033	0.9528	7401	0.9528	6.6955e-04	4			
untitled fit 6	Accelerometer_case3 v...	poly1	4.5360	0.0138	7487	0.0137	0.0246	2			
untitled fit 7	strain_case2 vs. t1_case2	a*exp(-b*23.15*2*pi*x...	0.2060	0.6400	25454	0.6400	0.0028	4			

Figure 17. Curve Fitting for Strain Indicator Box for Case C

Then, $\xi = b = 0.01517$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{49.67Hz}{\sqrt{1-0.01517^2}} = 49.68Hz$$

C. Using Matlab's curve fit tool to fit the accelerometer data to an exponentially damped sine wave, estimate the following parameters for each case:

- **Damped natural frequency**
- **Damping ratio**
- **Undamped natural frequency**

Case A: Beam and Hanger with 4lb weight

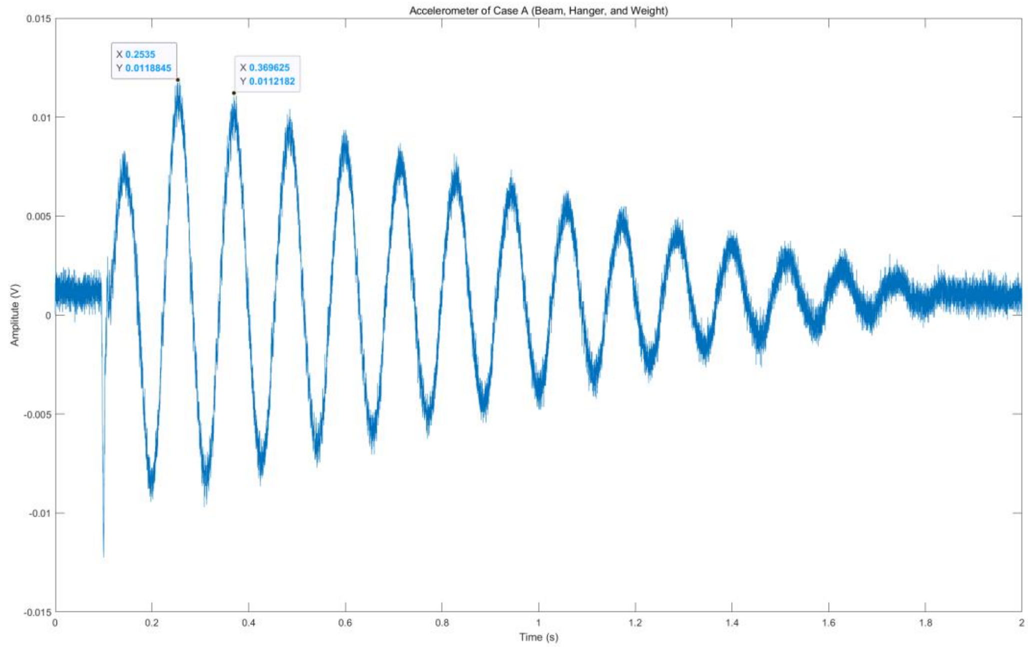


Figure 18. Accelerator Response for Case A

According to the matlab present, $\omega_d = \frac{1}{(369.625-253.5)ms} = 8.611Hz$.

By fitting the Strain indicator box response following:

$y = a * exp(-b * 8.611 * 2 * pi * x/(sqrt(1 - b^2)) * sin(2 * pi * x * 8.611 + c) + d$

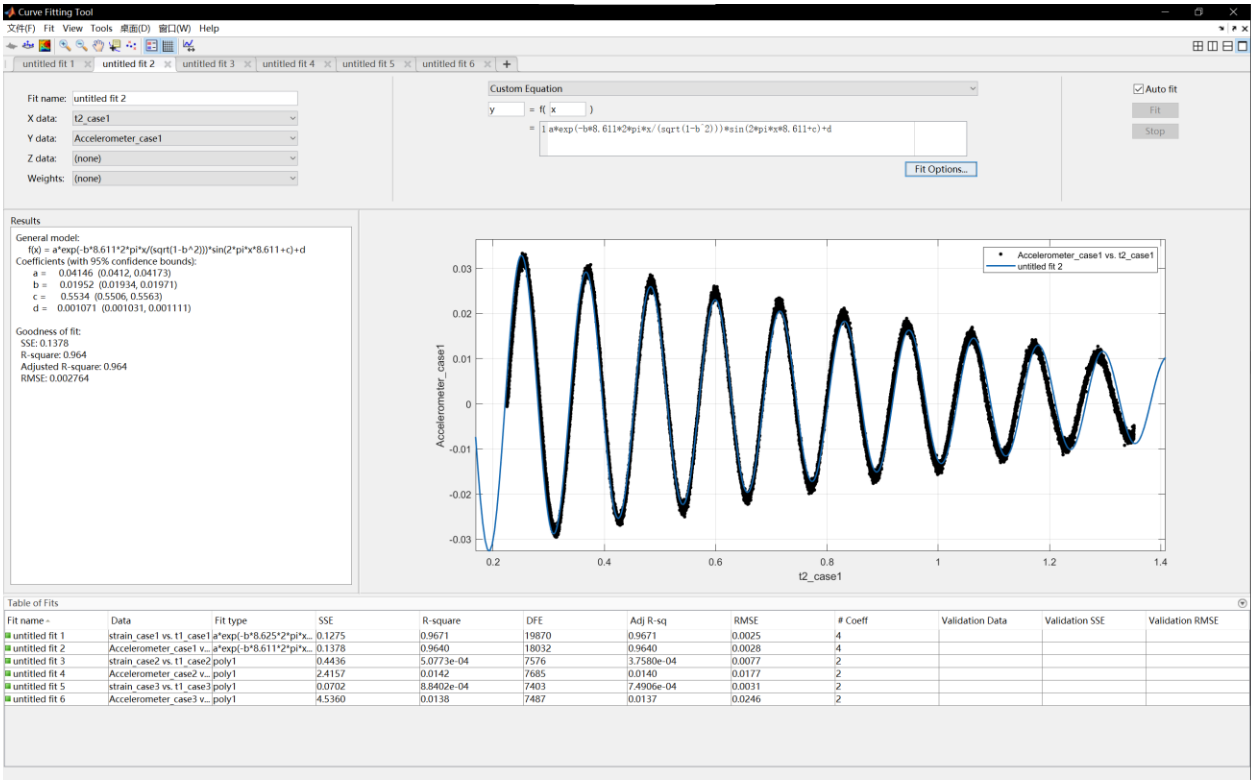


Figure 19. Curve Fitting for Accelerometer for Case A

Then, $\xi = b = 0.02035$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{8.611Hz}{\sqrt{1-0.01952^2}} = 8.613Hz$$

Case B: Beam and Hanger

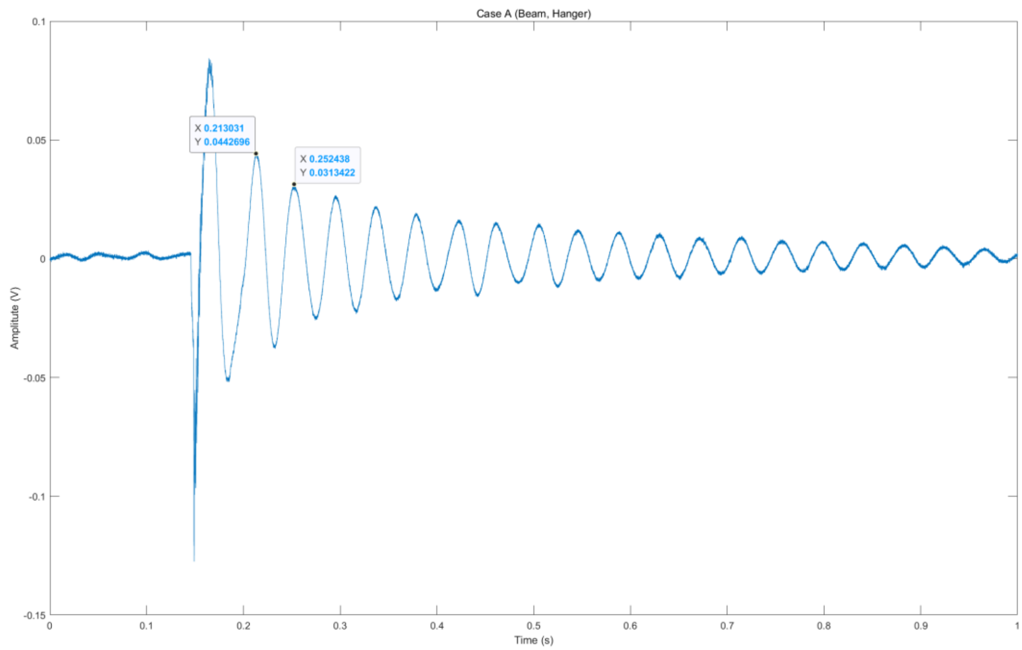


Figure 20. Accelerator Response for Case B

According to the matlab present, $\omega_d = \frac{1}{(252.438-213.031)ms} = 25.38Hz$.

By fitting the Strain indicator box response following:

$y = a * exp(-b * 25.38 * 2 * pi * x/(sqrt(1 - b^2))) * sin(2 * pi * x * 25.38 + c) + \mathrm{d}$

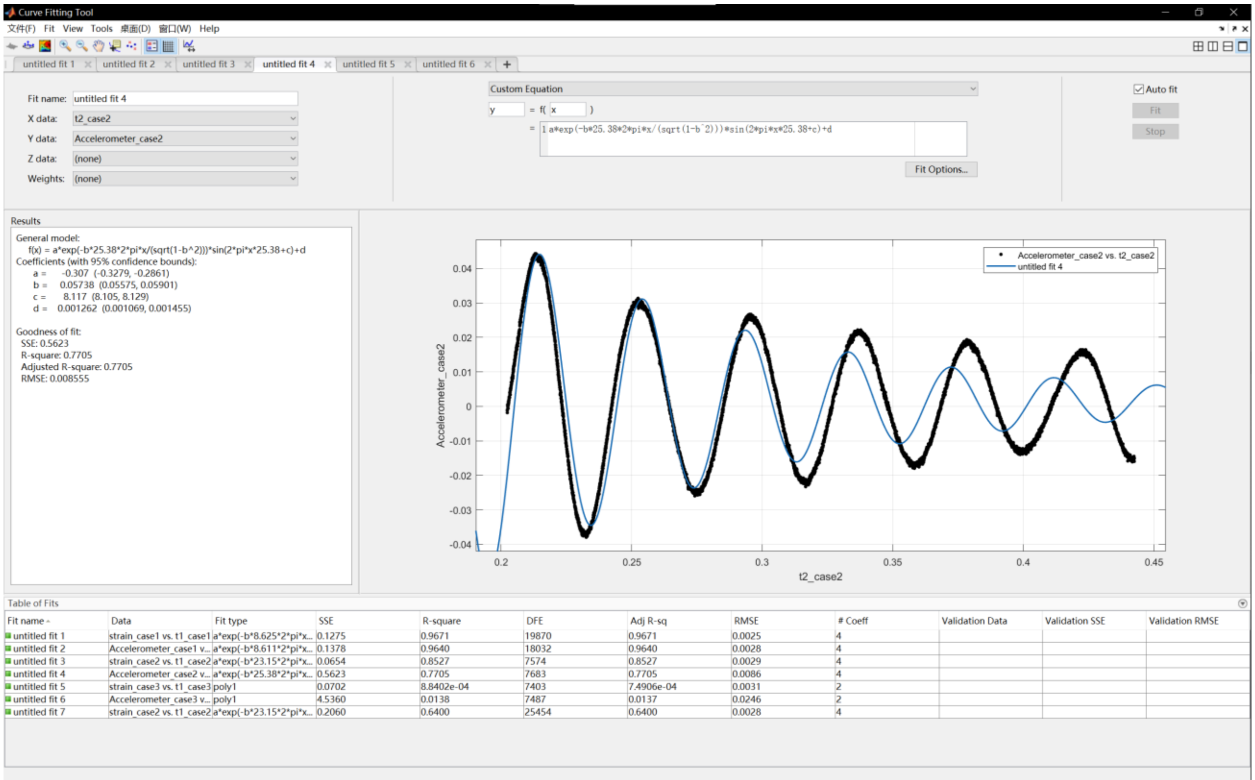


Figure 21. Curve Fitting for Accelerator for Case B

Then, $\xi = b = 0.05738$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{25.38Hz}{\sqrt{1-0.05738^2}} = 25.42Hz$$

Case C: Beam only

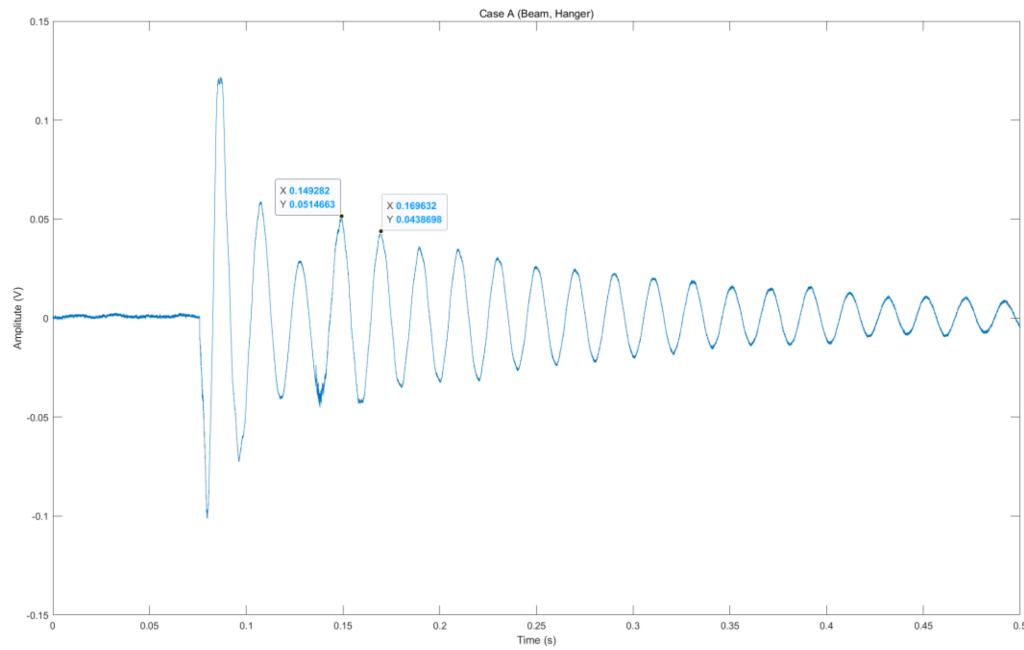


Figure 22. Accelerator Response for Case C

According to the matlab present, $\omega_d = \frac{1}{(169.632-149.282)ms} = 49.14Hz.$

By fitting the Strain indicator box response following:

$y = a * exp(-b * 49.14* 2 * pi * x/(sqrt(1 - b\text{^}2))) * sin(2 * pi * x * 49.14+ c) + \mathrm{d}$

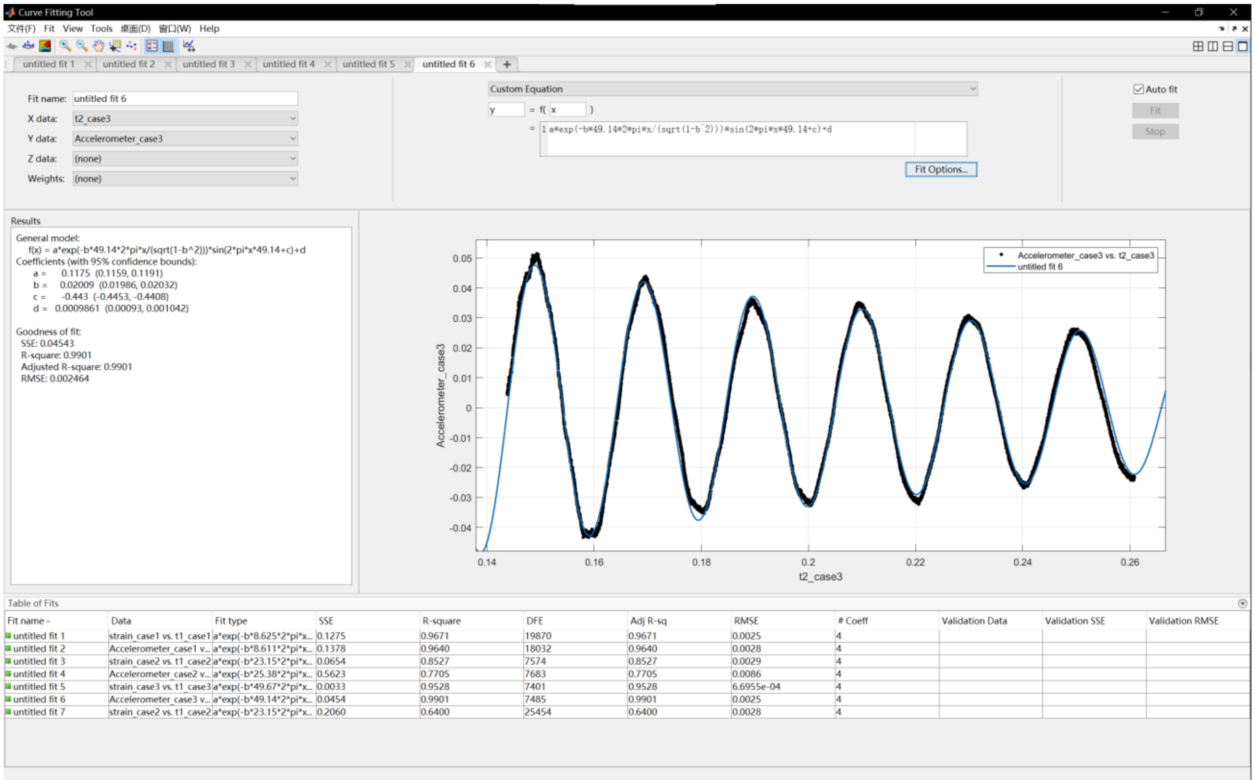


Figure 23. Curve Fitting for Accelerator for Case C

Then, $\xi = b = 0.02009$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{49.14Hz}{\sqrt{1-0.02009^2}} = 49.15Hz$$

-1; missing quantitative comparison

D. Compare the estimated values of natural frequency and damping ratio obtained from each type of sensor. Comment on the similarities and/or differences in values.

The undamped natural frequencies calculated by obtained data from different sensors are very close. In cases A and C, the damping ratios calculated from the data collected by the two sensors are also close. In case B, the calculated damping ratio are quite different. The damping ratio estimated by the data of accelerometer is significantly greater than that estimated by the data of strain indicator box.

From the results, although the damping ratio in case B is quite different, the finally calculated undamped natural frequency is still similar. The reason is that all damping ratios are too small to have a small impact on the calculation results. The error may come from the error of acquisition instrument and curve fitting

For the rest of the analysis questions, only one set of sensor results needs to be used (strain gage or accelerometer).

The results of strain gage are selected to use.

E. Using the measurements of the beam, the results of Case C, and the theoretical beam equation for the first bending mode, determine the specific modulus (E/ρ) for the beam.

End conditions	Frequency equation	1st mode	2nd mode	3rd mode	4th mode	5th mode
Clamped–free	$\cos \lambda l \cosh \lambda l = -1$	3.52	22.4	61.7	21.0	199.9
Pinned–pinned	$\sin \lambda l = 0$	9.87	39.5	88.9	157.9	246.8
Clamped–pinned	$\tan \lambda l = \tanh \lambda l$	15.4	50.0	104.0	178.3	272.0
Clamped–clamped or Free-free	$\cos \lambda l \cosh \lambda l = 1$	22.4	61.7	121.0	199.9	298.6

Figure 24. Value of Alpha

According to **Figure 24. Value of Alpha**,

$$\alpha = 3.52$$

Table 7. Data Processing result record

	Case A		Case B		Case C	
	Strain	Accelerometer	Strain	Accelerometer	Strain	Accelerometer
Damped Natural Frequency (Hz)	8.625	8.611	23.15	25.38	49.76	49.14
Undamped Natural Frequency (Hz)	8.627	8.613	23.16	25.42	49.68	49.15
Damping Ratio	0.02035	0.02035	0.02583	0.05738	0.01517	0.02009

According to **Table 7. Data Processing result record**,

$$\omega = 49.76Hz = 312.65rad/s$$

According to **Table 6. Beam Dimensions**,

$$L = 12.875in = 0.327025m$$

$$W = 0.75in = 0.01905m$$

$$H = 0.25in = 0.0635m$$

$$A = WH = (0.01905m)(0.00635) = 1.2097 \times 10^{-4}m^2$$

$$I = \frac{WH^3}{12} = \frac{(0.01905m)(0.00635)^3}{12} = 4.0648 \times 10^{-10}m^4$$

Original equation:

$$\omega = \frac{\alpha}{L^2}\sqrt{\frac{EI}{A\rho}}$$

Do some conversion to compute the specific modulus (E/ρ):

$$\frac{E}{\rho} = \frac{A}{I}\left(\frac{\omega L^2}{\alpha}\right) = \frac{1.2097 \times 10^{-4}m^2}{4.0648 \times 10^{-10}m^4}\left(\frac{(312.65rad/s)(0.327025m)^2}{3.52}\right)^2 = 2.685 \times 10^7 m^2/s^2$$

F. Based on your calculated value for E/ρ and observations in lab, what material do you believe was used to make the cantilever beam? Comment on any discrepancies between the published specific modulus and your calculated value

Combined with the calculated result ($\frac{E}{\rho} = 2.685 \times 10^7 m^2/s^2$) and the physical diagram (some rust can be seen on the surface of the beam) shown by the experiment, a kind of steel (with the specific modulus $\frac{E}{\rho} = 25 \times 10^6 m^2/s^2$) is considered to use to make the cantilever beam.

In the experiment, the cantilever beam has some oxidization on the surface. In laboratory environment, the steel would get oxidized easily. Hence, there is a discrepancy between the published specific modulus and our calculated value.

G. Using the estimated parameters from the all three cases, determine the weight of the hanger and the effective mass of the beam.

Case A:

$$\omega_A = \sqrt{\frac{k}{m_{beam+} + m_{hanger} + m_{weight}}}$$

Case B:

$$\omega_B = \sqrt{\frac{k}{m_{beam} + m_{hanger}}}$$

Case C:

$$\omega_C = \sqrt{\frac{k}{m_{beam}}}$$

Unknown: k, m_{beam} and m_{hanger}

Known: $\omega_A = 8.625Hz$, $\omega_B = 23.15Hz$, $\omega_C = 49.76Hz$, $\omega_C = 49.76Hz$, $m_{weight} = 4lb$

According to above, the matrices can be showed as:

$$\begin{bmatrix} 8.625^2 & 8.625^2 & -1 \\ 23.76^2 & 23.76^2 & -1 \\ 49.76^2 & 0 & -1 \end{bmatrix} \begin{bmatrix} m_{beam} \\ m_{hanger} \\ k \end{bmatrix} = \begin{bmatrix} -8.625^2 \times 4 \\ 0 \\ 0 \end{bmatrix}$$

Solve for $k = 342.72lbf/in$, $m_{beam} = 0.1384lb$ and $m_{hanger} = 0.4687lb$

3.2.4 General Discussion

Combined with the results of data processing and the data and diagram analysis in the Analysis Questions part below, the experiment was successful. The overall error is small. In case 2, the calculated damping ratio are quite different. The damping ratio estimated by the data of accelerometer is significantly greater than that estimated by the data of strain indicator box. Damping ratios are too small to have a small impact on the calculation results. The error may come from the error of acquisition instrument and curve fitting.

The locations of two different sensors are determined according to their respective measurement characteristics. Strain indicator box is more efficient when located on the fixed end. Accelerometer is more efficient when located on the free end.

3.3 IEPE Circuit

3.2.1 Procedure Objectives

During this experiment, it is expected to construct a complex circuit with components that might not be familiar and a circuit diagram similar to what is typically seen in industry manuals.

3.2.2 Experimental Results

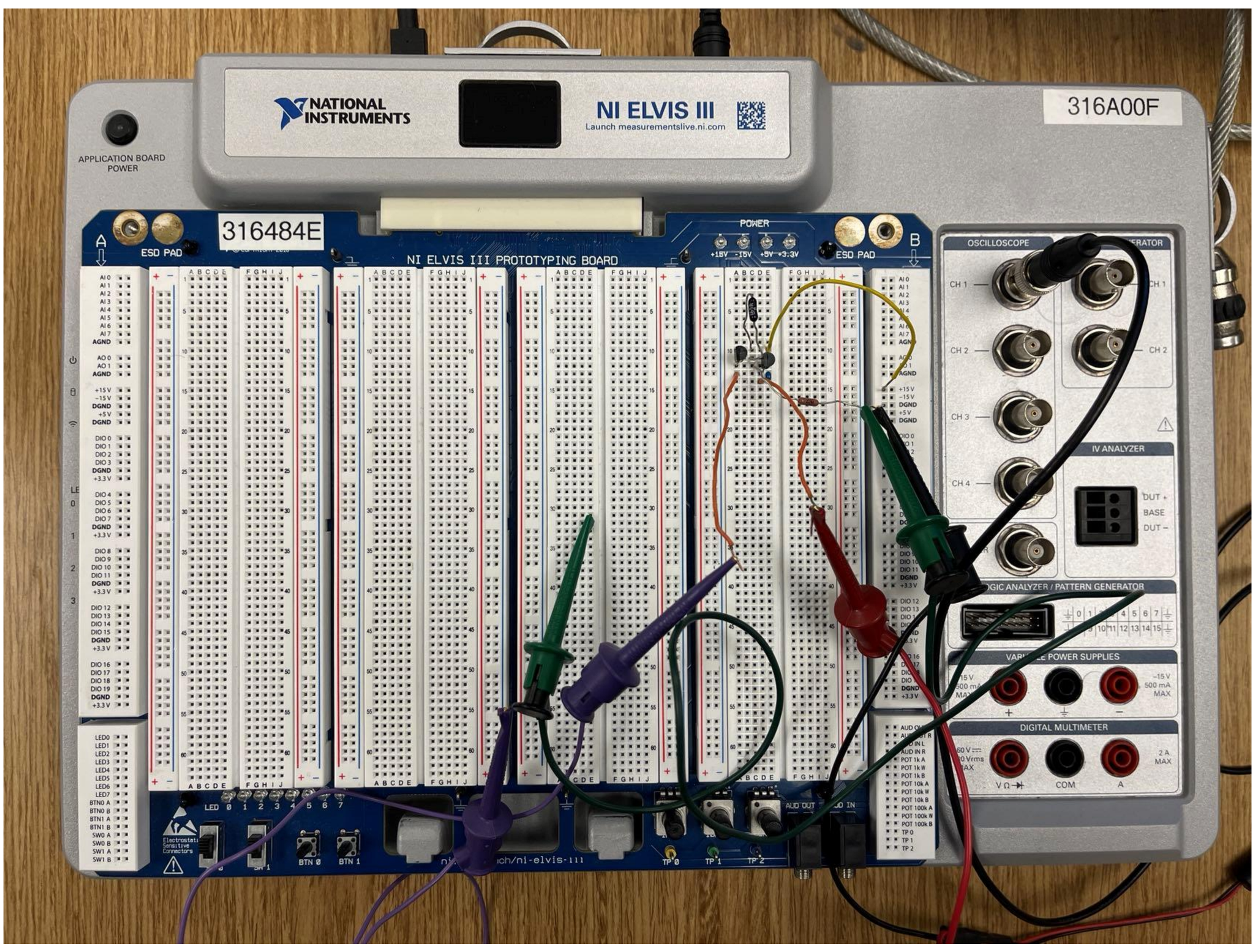


Figure 25. Circuit connection diagram of IEPE constant current supply

4. Conclusions

Having performed the experiment, and after a thorough analysis of the data, the following points are therefore concluded: About section 2, when estimating underdamped natural frequencies, it is easier and more convenient to use the real FRF graphs than the phase FRF graphs. The phase FRF graphs are messy. The phase change is complex. Useful data is difficult to estimate directly from the graph. Instead, the real FRF graphs are more direct and clearer. The points where the peak is located can be easily found and used for estimation. For the torsional mode, it is symmetrical about the centerline of the free-free beam. The mode shape of points 1-12 and the mode shape of points 13-24 should be symmetrical. If impact testing is only conducted along the centerline of the beam, the torsional mode cannot have been detected because centerline is the neutral axis. For the bending mode, the mode shape of points 1-12 and the mode shape of points 13-24 should be similar. About section 3, the locations of two different sensors are determined according to their respective measurement characteristics. Strain indicator box is more efficient when located on the fixed end. Accelerometer is more efficient when located on the free end. Combined with the calculated result (E_ρ = 2.685 × 10^7 m^2/s^2) and the physical diagram (some rust can be seen on the surface of the beam) shown by the experiment, a kind of steel (with the specific modulus E_ρ = 25 × 10^6 m^2/s^2) is considered to use to make the cantilever beam. Using the data from all three cases, compute the weight of the unknown mass (hanger) m_{hanger} = 0.4687 lb and the effective mass of the beam m_{beam} = 0.1384 lb.

APPENDICES

-1; missing equipments

A – EQUIPMENT LIST

Table 8. Equipment List

Equipment Description	Model Number	Serial Number
Digital Multimeter	Fluke 115	
Prototyping Base	NI ELVIS III	316A00F
Protoyping Board for Circuit	NI ELVIS III	316484E
Protoyping Board for Beam Test	NI ELVIS III	3169FF9
Sensor Signal Conditioner	48E09	00032938
Cantilever		2
Strain Indicator	P-3500	0121324
Accelerator	333B30	LW56751

B – MATLAB Code (or Excel, other computational software, etc)

Section 1

```
clear;clc;close all;
% reading in files
% note that "squeeze" removes the third dimension from each variable to create a vector of
values

for ii = 1:24 %number of files (data points) to be loaded
   dummy = sprintf ('2_2_3_%d', ii); %creates string variable of file name
   A = load (dummy); %loads file, which is a structure variable
   H (:,ii) = squeeze (A.Measurements. FRF.H) ; %stores FRF portion of structure
   Coh (:,ii) = squeeze (A.Measurements.COH.gamma2); %stores coherence portion of
structure
end

f = squeeze (A.Measurements.FRF.freq) ; %stores one frequency vector since it is the same
for all files
x = linspace(0,3200,5000);
y = zeros(size(x));

H1 = H(:,1:12);
H2 = H(:,13:24);
magn1 = abs(H1);
phase1 = rad2deg(angle(H1));
magn2 = abs(H2);
phase2 = rad2deg(angle(H2));

figure(1)
subplot(2,1,1)
plot(f,real(H1));
xlabel('Frequency(Hz)')
ylabel('Real Part of FRF')
hold on
plot(x,y)
subplot(2,1,2)
plot(f,imag(H1));
xlabel('Frequency(Hz)')
ylabel('Imaginary Part of FRF')
figure(2)
subplot(2,1,1)
semilogy(f,magn1);
xlabel('Frequency(Hz)')
ylabel('Magnitude')
subplot(2,1,2)
plot(f,phase1)
```

```
xlabel('Frequency(Hz)')
ylabel('Phase')

figure(3)
subplot(2,1,1)
plot(f,real(H2));
xlabel('Frequency(Hz)')
ylabel('Real Part of FRF')
hold on
plot(x,y)
subplot(2,1,2)
plot(f,imag(H2));
xlabel('Frequency(Hz)')
ylabel('Imaginary Part of FRF')
figure(4)
subplot(2,1,1)
semilogy(f,magn2);
xlabel('Frequency(Hz)')
ylabel('Magnitude')
subplot(2,1,2)
plot(f,phase2)
xlabel('Frequency(Hz)')
ylabel('Phase')


figure (6)
subplot(2,1,1)
plot(1.5:3:34.5,imag(H1(241,1:12))/max(abs(imag(H1(241,1:12)))))
hold on
plot(1.5:3:34.5,imag(H1(661,1:12))/max(abs(imag(H1(661,1:12)))))
plot(1.5:3:34.5,imag(H1(1291,1:12))/max(abs(imag(H1(1291,1:12)))))
plot(1.5:3:34.5,imag(H1(1575,1:12))/max(abs(imag(H1(1575,1:12)))))
plot(1.5:3:34.5,imag(H1(2125,1:12))/max(abs(imag(H1(2125,1:12)))))
title('Mode shapes of the first six undamped frequency of points 1-12 ')
legend('1st order (bending)','2nd order (bending)','3rd order (bending)','4th order
(torsional)','5th order (bending)')
xlabel('Length (inches)')
axis([0 36 -1 1])
subplot(2,1,2)
plot(1.5:3:34.5,imag(H2(241,1:12))/max(abs(imag(H2(241,1:12)))))
hold on
plot(1.5:3:34.5,imag(H2(661,1:12))/max(abs(imag(H2(661,1:12)))))
plot(1.5:3:34.5,imag(H2(1291,1:12))/max(abs(imag(H2(1291,1:12)))))
plot(1.5:3:34.5,imag(H2(1575,1:12))/max(abs(imag(H2(1575,1:12)))))
plot(1.5:3:34.5,imag(H2(2125,1:12))/max(abs(imag(H2(2125,1:12)))))
```

```
title('Mode shapes of the first six undamped frequency of points 13-24 ')
legend('1st order (bending)','2nd order (bending)','3rd order (bending)','4th order
(torsional)','5th order (bending)')
xlabel('Length (inches)')
axis([0 36 -1 1])

figure(7)
subplot(2,1,1)
plot(1.5:3:34.5,imag(H1(1575,1:12))/max(abs(imag(H1(1575,1:12)))))
title('Torsional mode shape of the 1st undamped frequency of points 1-12 ')
xlabel('Length (inches)')
axis([0 36 -1 1])
subplot(2,1,2)
plot(1.5:3:34.5,imag(H2(1575,1:12))/max(abs(imag(H2(1575,1:12)))))
title('Torsional mode shape of the 1st undamped frequency of points 13-24 ')
xlabel('Length (inches)')
axis([0 36 -1 1])
```

Section 3.2

```
%% Section 3.2
clear;close all;clc
%% 3_2_A_Strain indicator box
% Load data
t1_case1=xlsread('3_2_2_A_Second.csv','A2:A32002');
strain_case1=xlsread('3_2_2_A_Second.csv','B2:B32002');
% Plot
figure(1)
plot(t1_case1,strain_case1);
hold on
xlim([0 2]);
xlabel('Time (s)');ylabel('Amplitute (V)');
title('Strain Indicator Box of Case A (Beam, Hanger, and Weight)');
% Select data
t1_case1=xlsread('3_2_2_A_Second.csv','A3615:A23488');
strain_case1=xlsread('3_2_2_A_Second.csv','B3615:B23488');
% Fit data
cftool(t1_case1,strain_case1)
%% 3_2_A_Accelerometer
t2_case1=xlsread('3_2_2_A_Second.csv','D2:D32002');
Accelerometer_case1=xlsread('3_2_2_A_Second.csv','E2:E32002');
% Plot
figure(2)
plot(t2_case1,Accelerometer_case1);
hold on
xlim([0 2]);
```

```
xlabel('Time (s)');ylabel('Amplitute (V)');
title('Accelerometer of Case A (Beam, Hanger, and Weight)');
% Select data
t2_case1=xlsread('3_2_2_A_Second.csv','A3600:A21635');
Accelerometer_case1=xlsread('3_2_2_A_Second.csv','B3600:B21635');
% Fit data
cftool(t2_case1,Accelerometer_case1)
%% 3_2_B_Strain indicator box
t1_case2=xlsread('3_2_2_B_Third.csv','A2:A32001');
strain_case2=xlsread('3_2_2_B_Third.csv','B2:B32001');
% Plot
figure(3)
plot(t1_case2,strain_case2);
hold on
xlim([0 1]);
xlabel('Time (s)');ylabel('Amplitute (V)');
title('Case B (Beam, Hanger)');
% Select data
t1_case2=xlsread('3_2_2_B_Third.csv','A6545:A32002');
strain_case2=xlsread('3_2_2_B_Third.csv','B6545:B32002');
% Fit data
cftool(t1_case2,strain_case2)
%% 3_2_B_Accelerometer
t2_case2=xlsread('3_2_2_B_Third.csv','D2:D32001');
Accelerometer_case2=xlsread('3_2_2_B_Third.csv','E2:E32001');
% Plot
figure(4)
plot(t2_case2,Accelerometer_case2);
hold on
xlim([0 1]);
xlabel('Time (s)');ylabel('Amplitute (V)');
title('Case A (Beam, Hanger)');
% Select data
t2_case2=xlsread('3_2_2_B_Third.csv','D6477:D14163');
Accelerometer_case2=xlsread('3_2_2_B_Third.csv','E6477:E14163');
% Fit data
cftool(t2_case2,Accelerometer_case2)
%% 3_2_C_Strain indicator box
t1_case3=xlsread('3_2_2_C_Second.csv','A2:A32002');
strain_case3=xlsread('3_2_2_C_Second.csv','B2:B32002');
% Plot
figure(5)
plot(t1_case3,strain_case3);
hold on
xlim([0 0.5]);
```

```
xlabel('Time (s)');ylabel('Amplitute (V)');
title('Case A (Beam, Hanger)');
% Select data
t1_case3=xlsread('3_2_2_C_Second.csv','A10500:A17904');
strain_case3=xlsread('3_2_2_C_Second.csv','B10500:B17904');
% Fit data
cftool(t1_case3,strain_case3)

%% 3_2_C_Accelerometer
t2_case3=xlsread('3_2_2_C_Second.csv','D2:D32002');
Accelerometer_case3=xlsread('3_2_2_C_Second.csv','E2:E32002');
% Curve fitting
% Plot
figure(6)
plot(t2_case3,Accelerometer_case3);
hold on
xlim([0 0.5]);
xlabel('Time (s)');ylabel('Amplitute (V)');
title('Case A (Beam, Hanger)');
% Select data
t2_case3=xlsread('3_2_2_C_Second.csv','D9200:D16688');
Accelerometer_case3=xlsread('3_2_2_C_Second.csv','E9200:E16688');
% Fit data
cftool(t2_case3,Accelerometer_case3)
```

C – Scanned Lab Notes

Lab 3

Base: 316A00F   Board: 316484E

00032938 (serial)

Board: 3169FF9   PCB: 480E09 (model) model.

Cantilever: 2   Strain Indicator: 0121324 (P-3500)

Size: 12 3/4" × $\frac{1}{4}$" × $\frac{3}{4}$"

week 2

ACCEL. ~~PCB Piezotronics~~: 333B30 (model) LW56751 (SN)

NI ELVIS II: 1568EAD

~~ELVIS III: 316A005~~

Hammer: PCB086C03   LW36729 (SN)

Ruler: office Depot

ICP sensor signal conditionor: 480E09 (modal).

① 00032971 ② 00032974 (SN)

Beam: 36" × 3" × $\frac{3}{4}$"   spacing: ~~7 5/8~~ 3 in

CWF 10/17/22