Chapter 10

PROBLEM 10.1

In a heat exchanger, as shown in accompanying figure, air flows over brass tubes of 1.8-cm-*ID* and 2.1-cm-*OD* that contain steam. The convection heat-transfer coefficients on the air and steam sides of the tubes are 70 W/(m^2 K) and 210 W/(m^2 K), respectively. Calculate the overall heat transfer coefficient for the heat exchanger (a) based on the inner tube area, (b) based on the outer tube area.

GIVEN

- Air flow over brass tubes containing steam
- Tube diameters
  - Inside (D_i) = 1.8 cm = 0.018 m
  - Outside (D_o) = 2.1 cm = 0.021 m
- Convective heat transfer coefficients
  - Air side $\bar{h}_c$ = 70 W/(m^2 K)
  - Steam side $\bar{h}_i$ = 210 W/(m^2 K)

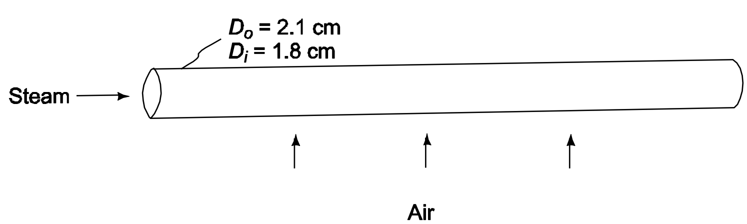
FIND

- The overall heat transfer coefficient for the heat exchanger based on (a) the inner tube area (U_i) and (b) the outer tube area (U_o)

ASSUMPTIONS

- The heat transfer coefficients are uniform over the transfer surfaces

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, the thermal conductivity of brass at 20°C (k_b) = 111 W/(m K)

SOLUTION

(a) The overall heat transfer coefficient based on the inner area is given by Equation (10.4)

$$U_i = \frac{1}{\left(\dfrac{1}{\bar{h}_i}\right) + \left[\dfrac{A_i \ln \frac{r_o}{r_i}}{2\pi k L}\right] + \left(\dfrac{A_i}{A_o \bar{h}_o}\right)}$$

where A_i = inside area = $\pi D_i L$

A_o = outside area = $\pi D_o L$

$$\therefore \qquad U_i = \frac{1}{\left(\frac{1}{\bar{h}_i}\right) + \left[\frac{D_i \ln \frac{D_o}{D_i}}{2k}\right] + \left(\frac{D_i}{D_o \bar{h}_o}\right)}$$

$$U_i = \frac{1}{\frac{1}{210\ \mathrm{W/(m^2\,K)}} + \left[\frac{(0.018\,\mathrm{m})\ln\left[\frac{0.021\,\mathrm{m}}{0.018\,\mathrm{m}}\right]}{2\ 111\,\mathrm{W/(m\,K)}}\right] + \left(\frac{0.018\,\mathrm{m}}{0.021\,\mathrm{m}\ \ 70\ \mathrm{W/(m^2\,K)}}\right)} = 58.8\ \mathrm{W/(m^2\ K)}$$

(b) The overall heat transfer coefficient based on the outer area is given by Equation (10.3)

$$U_o = \frac{1}{\left(\frac{A_o}{A_i \bar{h}_i}\right) + \left[\frac{A_o \ln \frac{r_o}{r_i}}{2\pi k L}\right] + \left(\frac{1}{\bar{h}_o}\right)} = \frac{1}{\left(\frac{D_o}{D_i \bar{h}_i}\right) + \left[\frac{D_o \ln \frac{D_o}{D_i}}{2k}\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\frac{0.021\,\mathrm{m}}{(0.018\,\mathrm{m})\ \ 210\ \mathrm{W/(m^2\,K)}} + \left[\frac{(0.021\,\mathrm{m})\ln\left[\frac{0.021\,\mathrm{m}}{0.018\,\mathrm{m}}\right]}{2\pi\ \ 111\ \mathrm{W/(m\,K)}}\right] + \frac{1}{70\ \mathrm{W/(m^2\,K)}}} = 50.4\ \mathrm{W/(m^2\ K)}$$

PROBLEM 10.2

Repeat Problem 10.1 but assume that a fouling factor of 0.00018 (m² K)/W has developed on the inside of the tube during operation.

GIVEN

- Air flow over brass tube containing steam
- Tube diameters   Inside (D_i) = 1.8 cm = 0.018 m
  Outside (D_o) = 2.1 cm = 0.021 m
- Convective heat transfer coefficients   Air side $\bar{h}_o$ = 70 W/(m² K)
  Steam side $\bar{h}_i$ = 210 W/(m² K)
- Fouling factor on the inside of the tube (R_d) = 0.0018 (m² K)/W

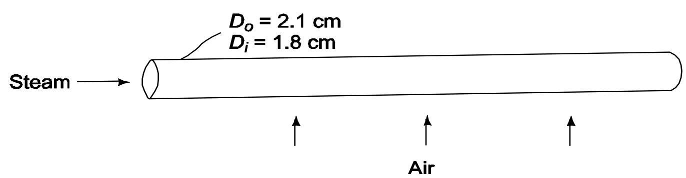
FIND

- The overall heat transfer coefficient for the heat exchanger based on (a) the inner tube area (U_i) and (b) the outer tube area (U_o)

ASSUMPTIONS

- The heat transfer coefficients are uniform over the transfer surfaces

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 10, the thermal conductivity of brass (k_b) = 111 W/(m K)

SOLUTION

From the solution to Problem 10.1 U_i = 58.76 W/(m² K)
U_o = 50.36 W/(m² K)

without fouling. The overall heat transfer coefficient with fouling (U_d) can be calculated by rearranging Equation (10.5b)

$$U_d = 1\Big/\left(R_{di} + \frac{1}{U_i}\right)$$

(a) Based on the inner tube area

$$U_{di} = 1\Big/\left(R_{di} + \frac{1}{U_i}\right) = 1\Big/\left(\left(0.0018\,(\text{m}^2\ \text{K})/\text{W}\right) + \frac{1}{\left(58.8\ \text{W}/(\text{m}^2\ \text{K})\right)}\right) = 53.1\ \text{W}/(\text{m}^2\ \text{K})$$

(b) To base the overall heat transfer coefficient on the outer tube area, the fouling factor must also be based on the outer tube area

$$R_{do} = (A_o/A_i)\,R_{di} = D_o/D_i\,R_{di}$$

$$U_{do} = \frac{1}{R_{do} + \dfrac{1}{U_o}} = \frac{1}{\dfrac{D_o}{D_i}R_{do} + \dfrac{1}{U_o}} = \frac{1}{\dfrac{2.1\text{m}}{1.8\text{m}}\ 0.0018\,(\text{m}^2\ \text{K})/\text{W} + \dfrac{1}{50.4\ \text{W}/(\text{m}^2\ \text{K})}} = 45.5\ \text{W}/(\text{m}^2\ \text{K})$$

PROBLEM 10.3

A light oil flows through a copper tube of 2.6-cm-*ID* and 3.2-cm-*OD*. Air flows perpendicular over the exterior of the tube as shown in the following sketch. The convection heat transfer coefficient for the oil is 120 W/(m² K) and for the air is 35 W/(m² K). Calculate the overall heat transfer coefficient based on the outside area of the tube (a) considering the thermal resistance of the tube, (b) neglecting the resistance of the tube.

GIVEN

- Air flow over a copper tube with oil flow within the tube
- Tube diameters
  - Inside (D_i) = 2.6 cm = 0.026 m
  - Outside (D_o) = 3.2 cm = 0.032 m
- Convective heat transfer coefficients
  - Oil $\bar{h}_i$ = 120 W/(m² K)
  - Air $\bar{h}_o$ = 35 W/(m² K)

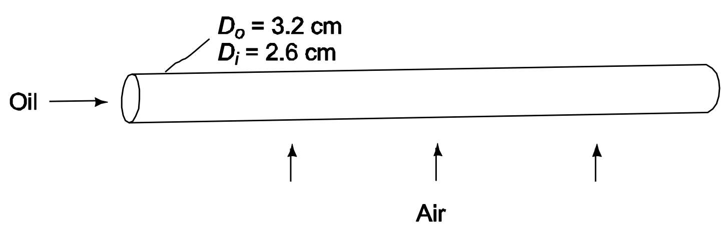
FIND

- The overall heat transfer coefficient based on the outside tube area (U_o), (a) considering the thermal resistance of the tube, and (b) neglecting the tube resistance

ASSUMPTIONS

- Uniform heat transfer coefficients
- Variation of thermal properties is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, the thermal conductivity of copper (k) = 392 W/(m K) (at 127°C)

SOLUTION

(a) The overall heat transfer coefficient based on the outer tube area is given by Equation (10.3)

$$U_o = \frac{1}{\left(\frac{A_o}{A_i \bar{h}_i}\right) + \left[\frac{A_o \ln \frac{r_o}{r_i}}{2\pi k L}\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

where A_i = inside area = $\pi D_i L$

A_o = outside area = $\pi D_o L$

$$\therefore \quad U_o = \frac{1}{\left(\frac{D_o}{D_i \bar{h}_i}\right) + \left[\frac{D_o \ln \frac{D_o}{D_i}}{2k}\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\dfrac{0.032\,\text{m}}{(0.026\,\text{m})\ 120\ \text{W/(m}^2\ \text{K)}} + \left[\dfrac{(0.032\,\text{m})\ln\left[\frac{0.032\,\text{m}}{0.026\,\text{m}}\right]}{2\ \ 392\ \text{W/(m K)}}\right] + \dfrac{1}{35\ \text{W/(m}^2\ \text{K)}}}$$

$$U_o = \frac{1}{(0.01026 + 0.0000085 + 0.02857)\ (\text{m}^2\ \text{K})/\text{W}} = 25.8\ \text{W/(m}^2\ \text{K)}$$

(b) The thermal resistance of the tube wall can be neglected by eliminating the bracketed term in the denominators of the expressions above

$$U_o = \frac{1}{(0.01026 + 0.02857)\ (\text{m}^2\ \text{K})/\text{W}} = 25.8\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

- Neglecting the thermal resistance of the tube wall has a negligible effect on the overall heat transfer coefficient.
- The thermal resistance on the air side is 74% of the overall thermal resistance.

PROBLEM 10.4

Repeat Problem 10.3, but assume that fouling factors of 0.0009 (m^2 K)/W and 0.0004 (m^2 K)/W have developed on the inside and on the outside, respectively.

GIVEN

- Air flow over a copper tube with oil flow within the tube
- Tube diameters: Inside (D_i) = 2.6 cm = 0.026 m
  Outside (D_o) = 3.2 cm = 0.032 m
- Convective heat transfer coefficients: Oil $\bar{h}_i$ = 120 W/(m^2 K)
  Air $\bar{h}_o$ = 35 W/(m^2 K)
- Fouling factors: Inside (R_{di}) = 0.0009 (m^2 K)/W
  Outside (R_{do}) = 0.0004 (m^2 K)/W

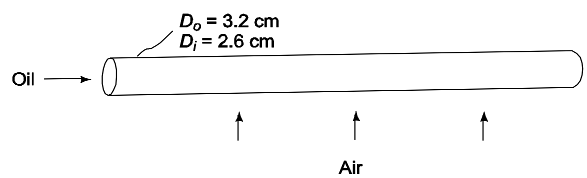
FIND

- The overall heat transfer coefficient based on the outside tube area (U_o), (a) considering the thermal resistance of the tube, and (b) neglecting the tube resistance

ASSUMPTIONS

- Uniform heat transfer coefficients
- Variation of thermal properties is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, the thermal conductivity of copper (k) = 392 W/(m K) (at 127°C)

SOLUTION

From the solution to Problem 10.3 with or without tube wall resistance

$$U_o = 25.75 \text{ (m}^2\text{ K)/W}$$

(a) The overall heat transfer with fouling can be calculated by rearranging Equation (10.5b)

$U_d = 1/(R_d + 1/U)$ where R_d = the total fouling factor

Based on the outside tube area

$$R_d = R_{do} + \frac{A_o}{A_i} R_{di} = R_{do} + \frac{D_o}{D_i} R_{di}$$

$$R_d = 0.0004\,(\text{m}^2\text{K})/\text{W} + \left(\frac{3.2\,\text{m}}{2.6\,\text{m}}\right) 0.0009\,(\text{m}^2\text{K})/\text{W} = 0.001508\,(\text{m}^2\text{ K})/\text{W}$$

$$U_d = 1\Big/\left(\left(0.001508\,(\text{m}^2\text{ K})/\text{W}\right) + \frac{1}{\left(25.8\text{ W}/(\text{m}^2\text{ K})\right)}\right) = 24.8\text{ W}/(\text{m}^2\text{ K})$$

(b) The tube wall resistance is negligible as shown in the solution to Problem 10.3.

COMMENTS

The given fouling factors lead to a 4% decrease in the overall heat transfer coefficient based on the outer tube wall area.

PROBLEM 10.5

Water flowing in a long aluminum tube is to be heated by air flowing perpendicular to the exterior of the tube. The *ID* of the tube is 1.85 cm and its *OD* is 2.3 cm. The mass flow rate of the water through the tube is 0.65 kg/s and the temperature of the water in the tube averages 30°C. The free stream velocity and ambient temperature of the air are 10 m/s and 120°C, respectively. Estimate the overall heat transfer coefficient for the heat exchanger using appropriate correlations from previous chapters. State all your assumptions.

GIVEN

- Air flowing perpendicular to the exterior of an aluminum tube with water flowing within the tube
- Tube diameters
  - Inside (D_i) = 1.85 cm = 0.0185 m
  - Outside (D_o) = 2.3 cm = 0.023 m
- Mass flow rate of water (m_w) = 0.65 kg/s
- Average temperature of the water (T_w) = 30°C
- Air free stream velocity (V_a) = 10 m/s
- Air temperature (T_a) = 120°C

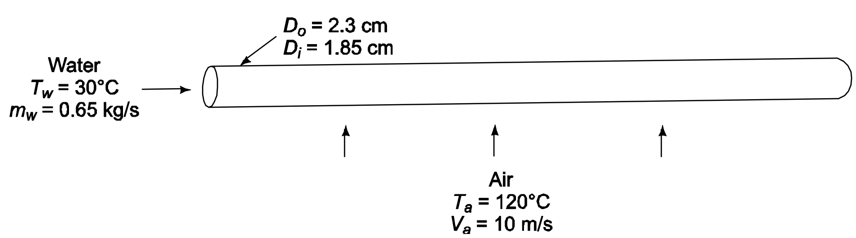
FIND

- The overall heat transfer coefficient (U)

ASSUMPTIONS

- Steady state
- The variation of the Prandtl number of air with temperature is negligible
- The aluminum is pure

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, thermal conductivity of aluminum (k_{al}) = 238 W/(m K) at 75°C
From Appendix 2, Table 13, for water at 30°C

Density (ρw) = 996 kg/m^3
Thermal conductivity (k_w) = 0.615 W/(m K)
Absolute viscosity (μ_w) = 792 × 10^{-6} (Ns)/m^2
Prandtl number (Pr_w) = 5.4

From Appendix 2, Table 28, for dry air at 120°C

Thermal conductivity (k_a) = 0.0320 W/(m K)
Kinematic viscosity (ν) = 26.0 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number on the water side is

$$Re_D = \frac{V_w D_i}{\nu} = \frac{4\dot{m}_w}{\pi D_i \mu} = \frac{4\ \ 0.65\,\text{kg/s}}{\pi(0.0185\,\text{m})\ \ 792\times10^{-6}\ \text{(Ns)/m}^2\ \ \ \text{(kg m)/(Ns}^2)} = 56{,}500 \text{ (Turbulent)}$$

The Nusselt number on the water side is given by Equation (7.61)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(56{,}500)^{0.8}\,(5.4)^{0.4} = 286$$

$$\bar{h}_i = \overline{Nu}_D\,\frac{k_w}{D_i} = 286\,\frac{0.615\,\text{W/(m K)}}{0.0185\,\text{m}} = 9508\,\text{W/(m}^2\,\text{K)}$$

The Reynolds number on the air side is

$$Re_D = \frac{V_a D_o}{\nu_a} = \frac{(10\,\text{m/s})(0.023\,\text{m})}{26\times10^{-6}\,\text{m}^2\text{/s}} = 8846$$

Applying Equation (6.3) and Table 6.1 but neglecting the Prandtl number variation

$$\overline{Nu}_D = 0.26\, Re_D^{0.6}\, Pr^{0.36} = 0.26\,(8846)^{0.6}\,(0.71)^{0.36} = 53.64$$

$$\bar{h}_o = \overline{Nu}_D\,\frac{k_a}{D_o} = 53.64\,\frac{0.0320\,\text{W/(m K)}}{0.023\,\text{m}} = 74.63\,\text{W/(m}^2\,\text{K)}$$

The overall heat transfer coefficient based on the outer tube diameter is given by Equation (10.3)

$$U_o = \frac{1}{\left(\dfrac{A_o}{A_i \bar{h}_i}\right) + \left[\dfrac{A_o \ln\frac{r_o}{r_i}}{2\pi k L}\right] + \left(\dfrac{1}{\bar{h}_o}\right)}$$

where A_i = inside area = $\pi D_i L$

A_o = outside area = $\pi D_o L$

$$\therefore \qquad U_o = \frac{1}{\left(\dfrac{D_o}{D_i \bar{h}_i}\right) + \left[\dfrac{D_o \ln\frac{D_o}{D_i}}{2k}\right] + \left(\dfrac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\dfrac{0.023\,\text{m}}{(0.0185\,\text{m})\ \ 9508\,\text{W/(m}^2\,\text{K)}} + \left[\dfrac{(0.023\,\text{m})\ln\frac{0.023\,\text{m}}{0.0185\,\text{m}}}{2\ \ 238\,\text{W/(m K)}}\right] + \dfrac{1}{74.6\,\text{W/(m}^2\,\text{K)}}} = 73.8\,\text{W/(m}^2\,\text{K)}$$

COMMENTS

The air side thermal resistance accounts for 99% of the total resistance. The water side convective resistance and the conductive resistance of the tube are of the same order of magnitude.

PROBLEM 10.6

Hot water is used to heat air in a double pipe heat exchanger as shown in the following sketch. If the heat transfer coefficients on the water side and on the air side are 550 W/(m^2 K) and 55 W/(m^2 K), respectively, calculate the overall heat transfer coefficient based on the outer diameter. The heat exchanger pipe is 5 cm, schedule 40 steel (*k* = 54 W/m K), with water inside.

GIVEN

- A double pipe heat exchanger with water in inner tube and air in the annulus
- Heat transfer coefficients
  - Water side $\bar{h}_i$ = 550 W/(m^2 K)
  - Air side $\bar{h}_o$ = 55 W/(m^2 K)
- Inner pipe: 5 cm, schedule 40 steel
- Pipe thermal conductivity (k) = 54 W/(m K)

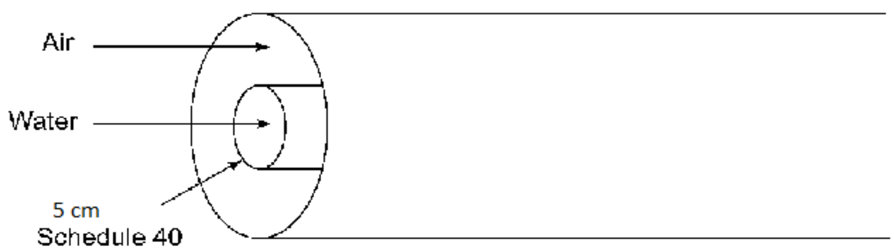
FIND

- The overall heat transfer coefficient based on the outer diameter (U_o).

ASSUMPTIONS

- Uniform heat transfer coefficients

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 41, for 5 cm, schedule 40 pipe

- Outside diameter (D_o) = 2.375 in = 6.033 cm=0.0603 m
- Wall thickness (t) = 0.154 in=0.39 cm=0.0039 m

SOLUTION

Inside diameter (D_i) = $D_o - 2t$ = 6.033 cm – 2(0.39 cm) = 5.253 cm=0.0525 m

The overall heat transfer coefficient based on the outer tube diameter is given by Equation (10.3)

$$U_o = \frac{1}{\left(\frac{A_o}{A_i\bar{h}_i}\right) + \left[\frac{A_o \ln \frac{r_o}{r_i}}{2\pi k L}\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

where A_i = inside area = $\pi D_i L$

A_o = outside area = $\pi D_o L$

$$\therefore \qquad U_o = \frac{1}{\left(\frac{D_o}{D_i \bar{h}_i}\right) + \left[\frac{D_o \ln \frac{D_o}{D_i}}{2k}\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\frac{0.0603m}{(0.0525m)\left(550\,W/(m^2\,\mathrm{K})\right)} + \left[\frac{(0.0603m)\ln\left(\frac{0.0603m}{0.0525m}\right)}{2\left(54\,W/(m^2\ \mathrm{K})\right)}\right] + \frac{1}{55W/(m^2\,\mathrm{K})}}$$

$$U_o = \frac{1}{2.088*10^{-3} + 7.73*10^{-5} + 0.01818}$$

$$U_o = 49.15\ \mathrm{W/(m^2\ K)}$$

PROBLEM 10.7

Repeat Problem 10.6, but assume that a fouling factor of 0.173 (m^2 K) /kW based on the tube outside diameter has developed over time.

GIVEN

- A double pipe heat exchanger with water in inner tube and air in the annulus
- Heat transfer coefficients
  - Water side $\bar{h}_i$ = 550 W/(m^2 K)
  - Air side $\bar{h}_o$ = 55 W/(m^2 K)
- Inner pipe: 2 in., schedule 40, made of steel
- Pipe thermal conductivity (k) = 54 W/(m K)
- Fouling factor (R_d) = 0.173 (m^2 K)/kW= $1.73*10^{-4}$ (m^2 K)/kW, based on the tube outside area

FIND

- The overall heat transfer coefficient based on the outer diameter (U_o)

ASSUMPTIONS

- Uniform heat transfer coefficients
- The given fouling factor is based on the outer tube diameter (D_o)

SOLUTION

From the solution to Problem 10.6: U_o = 49.15W/(m^2 K) without the fouling factor. The overall heat transfer coefficient with the fouling factor (U_d) can be found by rearranging Equation (10.5b)

$$U_d = \frac{1}{R_d + \frac{1}{U}} = \frac{1}{\left(0.000173\,(m^2\ \text{K/W})\right) + \frac{1}{\left(49.15\,W/(\text{m}^2\text{K})\right)}} = 48.73\ \text{W/(m}^2\ \text{K)}$$

$$U_d = 48.73\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

The inclusion of the fouling factor reduces the overall heat transfer coefficient by 1%.

PROBLEM 10.8

> **The heat transfer coefficient on the inside of a copper tube (1.9-cm-*ID* and 2.3-cm-*OD*) is 500 W/(m² K) and 120 W/(m² K) on the outside, but a deposit with a fouling factor of 0.009 (m² K)/W (based on the tube outside diameter) has built up over time. Estimate the percent increase in the overall heat transfer coefficient if the deposit were removed.**

GIVEN

- Heat transfer through a copper tube
- Heat transfer coefficients
  - Inside $\bar{h}_i$ = 500 W/(m² K)
  - Outside $\bar{h}_o$ = 120 W/(m² K)
- Tube diameters
  - Inside (D_i) = 1.9 cm = 0.019 m
  - Outside (D_o) = 2.3 cm = 0.023 m
- Fouling factor (R_d) = 0.009 (m² K)/W

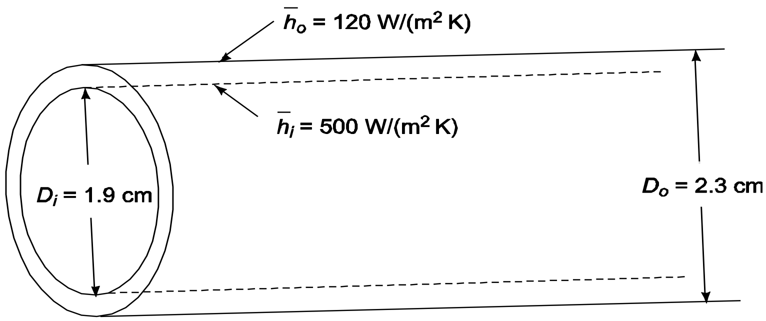
FIND

- Per cent increase in the overall heat transfer coefficient if the deposit were removed

ASSUMPTIONS

- Constant thermal conductivity properties
- The copper is pure

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, the thermal conductivity of copper (k) = 392 W/(m K) at 127°C

SOLUTION

The overall heat transfer coefficient without fouling based on the outside tube area is given by Equation (10.3)

$$U_o = \frac{1}{\left(\frac{A_o}{A_i \bar{h}_i}\right) + \left[\frac{A_o \ln \frac{r_o}{r_i}}{2\pi k L}\right] + \left(\frac{1}{\bar{h}_o}\right)} = \frac{1}{\left(\frac{D_o}{D_i \bar{h}_i}\right) + \left[\frac{D_o \ln \frac{D_o}{D_i}}{2k}\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\frac{0.023\,\text{m}}{(0.019\,\text{m})\ 500\,\text{W/(m}^2\,\text{K)}} + \left[\frac{(0.023\,\text{m})\ln \frac{0.023\,\text{m}}{0.019\,\text{m}}}{2\ 392\,\text{W/(m K)}}\right] + \frac{1}{120\,\text{W/(m}^2\,\text{K)}}} = 92.9\,\text{W/(m}^2\,\text{K)}$$

The overall heat transfer coefficient with fouling is given by Equation (10.5b)

$$U_d = \frac{1}{R_d + \frac{1}{U_o}} = \frac{1}{0.009\,(\mathrm{m^2K})/\mathrm{W} + \frac{1}{92.9\ \mathrm{W/(m^2K)}}} = 50.6\ \mathrm{W/(m^2K)}$$

The percent increase is

$$\frac{U_o - U_d}{U_d} \times 100 = \frac{92.9 - 50.6}{50.6} \times 100 = 84\%$$

PROBLEM 10.9

In a shell-and-tube heat exchanger with $\bar{h}_i = \bar{h}_o$ = 5600 W/(m² K) and negligible wall resistance, by what per cent would the overall heat transfer coefficient (based on the outside area) change if the number of tubes was doubled? The tubes have an outside diameter of 2.5 cm and a tube wall thickness of 2 mm. Assume that the flow rates of the fluids are constant, the effect of temperature on fluid properties is negligible, and the total cross sectional area of the tubes is small compared to the flow area of the shell.

GIVEN

- Shell and tube heat exchanger
- Heat transfer coefficients: $\bar{h}_i = \bar{h}_o$ = 5600 W/(m² K)
- Negligible wall resistance
- Tube outside diameter (D_o) = 2.5 cm = 0.025 m
- Tube wall thickness (t) = 2 mm = 0.002 m

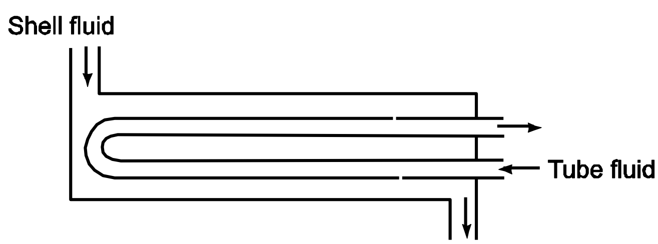
FIND

- The per cent change in the overall heat transfer coefficient if the number of tubes is doubled

ASSUMPTIONS

- Flow rates of the fluids are constant
- The effect of temperature of fluid properties is negligible
- The fluid flow is turbulent in the tubes (this is consistent with the high heat transfer coefficients)

SKETCH



SOLUTION

The inside diameter (D_i) = $D_o - 2t$ = 2.5 cm – 2(0.2cm) = 2.1 cm = 0.021 m

The original overall heat transfer coefficient is given by Equation (10.3). Neglecting wall resistance

$$\frac{1}{U_o} = \frac{A_o}{A_i \bar{h}_i} + \frac{1}{\bar{h}_o} = \frac{D_o}{D_i \bar{h}_i} + \frac{1}{\bar{h}_o} = \left(\frac{2.5}{2.1}\right)\frac{1}{5600\ \mathrm{W/(m^2K)}} + \frac{1}{5600\ \mathrm{W/(m^2K)}}$$

$$U_o = 2557\ \mathrm{W/(m^2\ K)}$$

Since the total cross sectional area of the tubes is small compared tot the flow area of the shell, doubling the number of tubes will have little effect on the shell-side fluid velocity. Therefore, the shell side heat transfer coefficient will not change. Doubling the number of tubes with the same mass flow rate cuts the fluid velocity in the tubes in half. For turbulent flow in tube (from Section 7.5)

$$\bar{h}_c \propto Re_D^{0.8} \quad \Rightarrow \bar{h}_c \propto V^{0.8}$$

$$\frac{\bar{h}_{c,\mathrm{new}}}{\bar{h}_{c,\mathrm{orig}}} = \left(\frac{V_{\mathrm{new}}}{V_{\mathrm{orig}}}\right)^{0.8} = (0.5)^{0.8} = 0.574$$

$$\bar{h}_{c,\mathrm{new}} = 0.574\ \ 5600\ \mathrm{W/(m^2K)} = 3214\ \mathrm{W/(m^2K)}$$

$$\therefore \ \frac{1}{U_{o,\text{new}}} = \left(\frac{2.5}{2.1}\right)\frac{1}{3214\ \text{W/(m}^2\text{K)}} + \frac{1}{5600\ \text{W/(m}^2\text{K)}} = 1822\ \text{W/(m}^2\text{K)}$$

$$\%\ \text{Change} = \frac{U_o - U_{o,\text{new}}}{U_o} \times 100 = \frac{2557 - 1822}{2557} \times 100 = 29\%\ \text{Decrease}$$

PROBLEM 10.10

Water is heated by hot air in a heat exchanger. The flow rate of the water is 12 kg/s and that of the air is 2 kg/s. The water enters at 40°C and the air enters at 460°C. The overall heat transfer coefficient of the heat exchanger is 275 W/(m^2 K) based on a surface area of 14 m^2. Determine the effectiveness of the heat exchanger if it is (a) parallel-flow type or (b) a cross-flow type (both fluids unmixed). Then calculate the heat transfer rate for the two types of heat exchangers described and the outlet temperatures of the hot and cold fluids for the conditions given.

GIVEN

- Water heated by air in a heat exchanger
- Water flow rate $\dot{m}_w$ = 12 kg/s
- Air flow rate $\dot{m}_a$ = 2 kg/s
- Inlet temperatures
  - Water ($T_{w,\text{in}}$) = 40°C
  - Air ($T_{a,\text{in}}$) = 460°C
- Overall heat transfer coefficient (U) = 275 W/(m^2 K)
- Transfer area (A) = 14 m^2

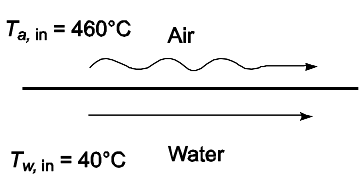
FIND

(a) The effectiveness (e)
(b) The heat transfer rate (q)
(c) The outlet temperature ($T_{w,\text{out}}$, $T_{a,\text{out}}$) for: 1. parallel-flow 2. cross-flow

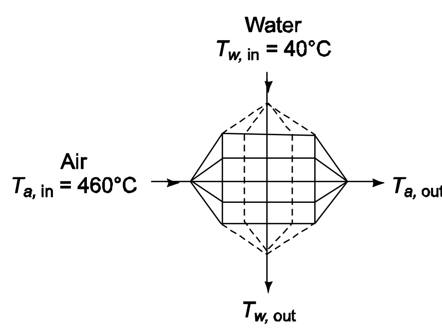
ASSUMPTIONS

- Steady state

SKETCH



1. Parallel Flow



2. Cross-flow

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, the specific heat of dry air (c_{pa}) = 1059 J/(kg K) at 400°C
From Appendix 2, Table 13, the specific heat of water (c_{pw}) = 4178 J/(kg K) at 50°C

SOLUTION

The heat capacity rates are

For air $\quad C_a = \dot{m}_a\, c_{pa} =$ (2 kg/s) 1059 J/(kg K) = 2118 W/K

For water $\quad C_w = \dot{m}_w\, c_{pw} =$ (12 kg/s) 4178 J/(kg K) = 50,136 W/K

(a) The effectiveness for parallel-flow geometry is given by Equation 10.26

$$\mathscr{E} = \frac{1 - \exp\left[-\left(1 + \dfrac{C_{\min}}{C_{\max}}\right)\dfrac{UA}{C_{\min}}\right]}{1 + \dfrac{C_{\min}}{C_{\max}}}$$

$$\mathscr{E} = \frac{1 - \exp\left[-\left(1 + \dfrac{2118}{50{,}136}\right)\dfrac{275\ \mathrm{W/(m^2\,K)}\ \ 14\,\mathrm{m^2}}{2118\ \mathrm{W/K}}\right]}{1 + \dfrac{2118}{50{,}136}} = 0.815$$

For cross-flow, the effectiveness can be taken from Figure 10.21

$$\frac{C_{\min}}{C_{\max}} = \frac{2188}{50{,}136} = 0.042$$

$$NTU = \frac{UA}{C_{\min}} = \frac{(275\ \mathrm{W/(m^2K)}\ \ 14\,\mathrm{m^2}}{2118\ \mathrm{W/(m^2K)}} = 1.82$$

From Figure 10.21: $e \approx 0.83$

(b) The rate of heat transfer is

$$q = \mathsf{E}\, C_{\min}\,(T_{h,\mathrm{in}} - T_{c,\mathrm{in}})$$

For parallel-flow

$$q = 0.815\ \ 2118\ \mathrm{W/K}\ \ (460°\mathrm{C} - 40°\mathrm{C}) = 7.25 \times 10^5\ \mathrm{W} = 725\ \mathrm{kW}$$

For cross-flow

$$q = 0.83\ \ 2118\ \mathrm{W/K}\ \ (460°\mathrm{C} - 40°\mathrm{C}) = 7.38 \times 10^5\ \mathrm{W} = 738\ \mathrm{kW}$$

(c) The outlet temperatures can be calculated from

$$q = \dot{m}\, c_p\, \Delta T = C\,(T_{\mathrm{out}} - T_{\mathrm{in}}) \Rightarrow T_{\mathrm{out}} = T_{\mathrm{in}} + q/C$$

Parallel-flow

Water $$T_{w,\mathrm{out}} = 40°\mathrm{C} + \frac{7.25 \times 10^5\ \mathrm{W}}{50{,}136\ \mathrm{W/K}} = 54°\mathrm{C}$$

Air $$T_{a,\mathrm{out}} = 460°\mathrm{C} - \frac{7.25 \times 10^5\ \mathrm{W}}{2118\ \mathrm{W/K}} = 118°\mathrm{C}$$

Cross-flow

Water $$T_{w,\mathrm{out}} = 40°\mathrm{C} + \frac{7.38 \times 10^5\ \mathrm{W}}{50{,}136\ \mathrm{W/K}} = 55°\mathrm{C}$$

Air $$T_{a,\mathrm{out}} = 460°\mathrm{C} - \frac{7.38 \times 10^5\ \mathrm{W}}{2118\ \mathrm{W/K}} = 112°\mathrm{C}$$

COMMENTS

The cross-flow arrangement improves the heat transfer rate by 1.8%.

PROBLEM 10.11

Exhaust gases from a power plant are used to preheat air in a cross-flow heat exchanger. The exhaust gases enter the heat exchanger at 450°C and leave at 200°C. The air enters the heat exchanger at 70°C, leaves at 250°C, and has a mass flow rate of 10 kg/s. Assume the properties of the exhaust gases can be approximated by those of air. The overall heat transfer coefficient of the heat exchanger is 154 W/(m^2 K). Calculate the heat exchanger surface area required if (a) the air is unmixed and the exhaust gases are mixed and (b) both fluids are unmixed.

GIVEN

- Cross-flow heat exchanger – exhaust gas to air
- Exhaust temperatures
  - $T_{e,\text{in}}$ = 450°C
  - $T_{e,\text{out}}$ = 200°C
- Air temperatures
  - $T_{a,\text{in}}$ = 70°C
  - $T_{a,\text{out}}$ = 250°C
- Air flow rate $\dot{m}_a$ = 10 kg/s
- Overall heat transfer coefficient (U) = 154 W/(m^2 K)

FIND

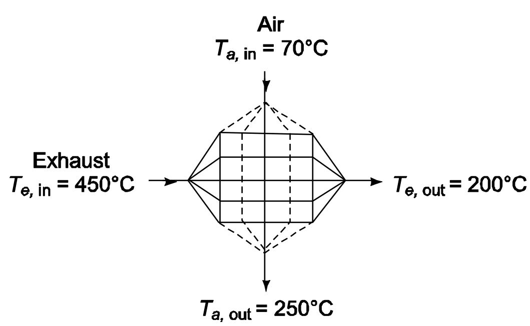
The heat exchanger surface area (A) if

(a) Air is unmixed, exhaust is mixed
(b) Both are unmixed

ASSUMPTIONS

- The properties of the exhaust gases can be approximated by those of air
- The air is in the tube

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, the specific heat of air at the mean temperature of 160°C (c_{pa}) = 1030 J/(kg K)

SOLUTION

For counterflow, from Figure 10.13 $\Delta T_a = T_{e,\text{in}} - T_{a,\text{out}}$ = 450°C – 250°C = 200°C

$\Delta T_b = T_{e,\text{out}} - T_{a,\text{in}}$ = 200°C – 70°C = 130°C

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{200°\mathrm{C} - 130°\mathrm{C}}{\ln\left(\frac{200}{130}\right)} = 162°\mathrm{C}$$

(a) For cross-flow with the exhaust mixed, the *LMTD* must be modified according to Figure 10.16

$$P = \frac{T_{a,\mathrm{out}} - T_{a,\mathrm{in}}}{T_{e,\mathrm{in}} - T_{a,\mathrm{in}}} = \frac{250 - 70}{450 - 70} = 0.47$$

$$Z = \frac{T_{e,\mathrm{in}} - T_{e,\mathrm{out}}}{T_{a,\mathrm{out}} - T_{a,\mathrm{in}}} = \frac{450 - 200}{250 - 70} = 1.4$$

From Figure 10.16, $F = 0.76$

$$\therefore \qquad \Delta T_{\mathrm{mean}} = F(LMTD) = 0.76\ (162°\mathrm{C}) = 123°\mathrm{C}$$

The rate of heat transfer is

$$q = UA\,\Delta T_{\mathrm{mean}} = \dot{m}_a\, c_{pa}\, (T_{a,\mathrm{out}} - T_{a,\mathrm{in}})$$

Solving for the heat exchanger surface area

$$A = \frac{\dot{m}_a c_{pa}(T_{a,\mathrm{out}} - T_{a,\mathrm{in}})}{U \Delta T_{\mathrm{mean}}} = \frac{10\ \mathrm{kg/s}\quad 1030\ \mathrm{J/(kg\,K)}\quad 250°\mathrm{C} - 70°\mathrm{C}}{154\ \mathrm{W/(m^2K)}\ \ (123°\mathrm{C})\ \ \mathrm{J/(Ws)}} = 98\ \mathrm{m}^2$$

(b) For both fluids unmixed, the *LMTD* must be corrected using Figure 10.17: $F = 0.86$

$$\therefore \qquad \Delta T_{\mathrm{mean}} = 0.86\ (162°\mathrm{C}) = 139°\mathrm{C}$$

$$A = \frac{10\ \mathrm{kg/s}\quad 1030\ \mathrm{J/(kg\,K)}\quad 250°\mathrm{C} - 70°\mathrm{C}}{154\ \mathrm{W/(m^2K)}\ \ (139°\mathrm{C})\ \ \mathrm{J/(Ws)}} = 86\ \mathrm{m}^2$$

COMMENTS

The required transfer area is 15% smaller when both fluids are unmixed in this case.

PROBLEM 10.12

A shell-and-tube heat exchanger having one shell pass and four tube passes is shown schematically in the following sketch. The fluid in the tubes enters at 200°C and leaves at 100°C. The temperature of the fluid is 20°C entering the shell and is 90°C leaving the shell. The overall heat transfer coefficient based on the surface area of 12 m^2 is 300 W/(m^2 K). Calculate the heat transfer rate between the fluids.

GIVEN

- A shell-and-tube heat exchanger with one shell pass and four tube passes
- Temperature of fluid in tube
  - $T_{t,\text{in}}$ = 200°C
  - $T_{t,\text{out}}$ = 100°C
- Temperature of fluid in shell
  - $T_{s,\text{in}}$ = 20°C
  - $T_{s,\text{out}}$ = 90°C
- Overall heat transfer coefficient (U) = 300 W/(m^2 K)
- Surface area (A) = 12 m^2

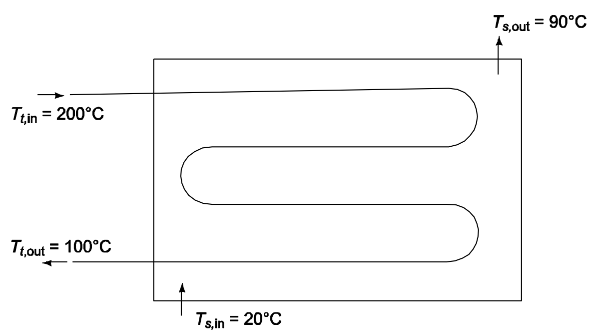
FIND

- The heat transfer rate (q)

ASSUMPTIONS

- Steady state
- Exchanger geometry is countnerflow as shown in Figure 10.14

SKETCH



SOLUTION

A log-mean temperature difference for this counterflow arrangement is

$$LMTD = \Delta T = \frac{\Delta T_a - \Delta T_b}{\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)}$$

From Figure 10.13 $\quad \Delta T_a = T_{t,\text{in}} - T_{s,\text{out}} = 200°\text{C} - 90°\text{C} = 110°\text{C}$

$$\Delta T_b = T_{t,\text{out}} - T_{s,\text{in}} = 100°\text{C} - 20°\text{C} = 80°\text{C}$$

$$\Delta T = \frac{110°\text{C} - 80°\text{C}}{\ln\left(\dfrac{110}{80}\right)} = 94.2°\text{C}$$

This value must be modified to account for the shell-and-the geometry using Figure 10.14

$$P = \frac{T_{t,\text{out}} - T_{t,\text{in}}}{T_{s,\text{in}} - T_{t,\text{in}}} = \frac{100 - 200}{20 - 200} = 0.556$$

$$Z = \frac{T_{s,\text{in}} - T_{s,\text{out}}}{T_{t,\text{out}} - T_{t,\text{in}}} = \frac{20 - 90}{100 - 200} = 0.7$$

From Figure 10.14, $F = 0.85$

$$\therefore \qquad \Delta T_{\text{mean}} = F(\Delta T) = 0.85\ (94.2°\text{C}) = 80.1°\text{C}$$

The heat transfer rate is given by

$$q = U\,A\,\Delta T_{\text{mean}} = \ 300\ \text{W/(m}^2\ \text{K)}\ \ (12\ \text{m}^2)\ (80.1°\text{C}) = 2.88 \times 10^5\ \text{W}$$

COMMENTS

Since ΔT_a is less than 50% greater than ΔT_b, the mean temperature difference may be used in place of the *LMTD* without introducing significant error. Use of the mean temperature difference in this case leads to a heat transfer rate of 2.907×10^5 W (1% higher).

PROBLEM 10.13

> **Oil $c_p = 2.1\,\mathrm{kJ/(kg\,K)}$ is used to heat water in a shell and tube heat exchanger with a single shell and two tube passes. The overall heat transfer coefficient is 525 W/(m^2 K). The mass flow rates are 7 kg/s for the oil and 10 kg/s for the water. The oil and water enter the heat exchanger at 240°C and 20°C, respectively. The heat exchanger is to be designed so that the water leaves the heat exchanger with a minimum temperature of 80°C. Calculate the heat transfer surface area required to achieve this temperature.**

GIVEN

- Oil heats water in a heat exchanger with one shell pass and two tube passes
- Oil specific heat (c_{po}) = 2.1 kJ/(kg K) = 2100 J/(kg K)
- Overall heat transfer coefficient (U) = 525 W/(m^2 K)
- Oil mass flow rate $\dot{m}_o$ = 7 kg/s
- Water mass flow rate $\dot{m}_w$ = 10 kg/s
- Inlet temperatures: Oil ($T_{o,\mathrm{in}}$) = 240° Water ($T_{w,\mathrm{in}}$) = 20°C
- Minimum water outlet temperature ($T_{w,\mathrm{out}}$) = 80°C

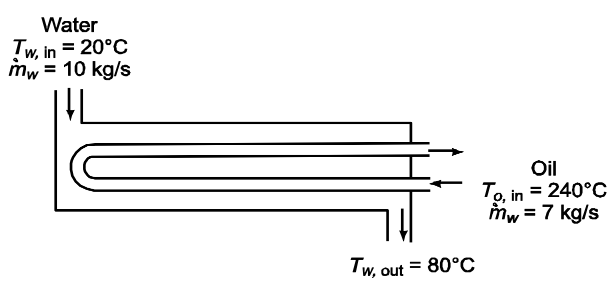
FIND

- The heat transfer area (A) required

ASSUMPTIONS

- Oil is in the tubes

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the average temperature of 50°C (c_{pw}) = 4178 J/(kg K)

SOLUTION

The heat capacity rates are

$$C_o = \dot{m}_o\, c_{po} = (7\ \mathrm{kg/s})\ 2100\,\mathrm{J/(kg\,K)} = 14{,}700\ \mathrm{W/K}$$

$$C_w = \dot{m}_w\, c_{pw} = (10\ \mathrm{kg/s})\ 4178\,\mathrm{J/(kg\,K)} = 41{,}780\ \mathrm{W/K}$$

$$\frac{C_{\min}}{C_{\max}} = \frac{14{,}700}{41{,}780} = 0.35$$

The effectiveness required to achieve $T_{w,\mathrm{out}}$ = 80°C is

$$\mathrm{E} = \frac{C_w\ T_{w,\mathrm{out}} - T_{w,\mathrm{in}}}{C_{\min}\ T_{o,\mathrm{in}} - T_{w,\mathrm{in}}} = \frac{41{,}780\,\mathrm{J/(kg\,K)}\ \ 80^\circ\mathrm{C} - 20^\circ\mathrm{C}}{14{,}700\,\mathrm{J/(kg\,K)}\ \ 240^\circ\mathrm{C} - 20^\circ\mathrm{C}} = 0.775$$

The number of transfer units, NTU, from Figure 10.20: $NTU = (U\,A)/C_{\min} \approx 2.5$

$$\therefore \qquad A = NTU\frac{C_{\min}}{U} = 2.5\,\frac{14{,}700\ \text{W/K}}{525\ \text{W/(m}^2\ \text{K)}} = 70\ \text{m}^2$$

PROBLEM 10.14

A shell-and-tube heat exchanger with two tube passes and a single shell pass is used to heat water by condensing steam in the shell. The flow rate of the water is 15 kg/s and it is heated from 60 to 80°C. The steam condenses at 140°C and the overall heat transfer coefficient of the heat exchanger is 820 W/(m² K). If there are 45 tubes with an *OD* of 2.75 cm, calculate the length of tubes required.

GIVEN

- Shell-and-tube heat exchanger with two tube passes and one shell pass
- Water in tubes, condensing steam in shell
- Water flow rate $\dot{m}_w$ = 15 kg/s
- Water temperatures
  - $T_{w,\text{in}}$ = 60°C
  - $T_{w,\text{out}}$ = 80°C
- Steam temperature (T_s) = 140°C
- Overall heat transfer coefficient (U) = 820 W/(m^2 K)
- Number of tubes (N) = 45
- Tube outside diameter (D) = 2.75 cm = 0.0275 m

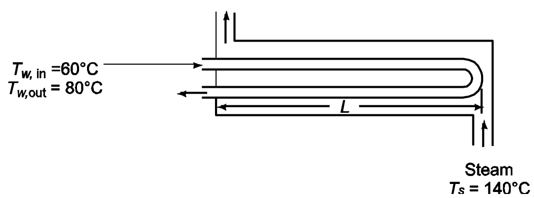
FIND

- The length of tubes (L)

ASSUMPTIONS

- Counterflow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the mean temperature of 70°C (c_{pw}) = 4188 J/(kg K)

SOLUTION

For counterflow, from Figure 10.13 $\Delta T_a = T_s - T_{w,\text{out}}$ = 140°C – 80°C = 60°C

$\Delta T_b = T_s - T_{w,\text{in}}$ = 140°C – 60°C = 80°C

Since ΔT_a is less than 50% greater than ΔT_b, the mean temperature may be used

$$\Delta T_{\text{mean}} = \frac{1}{2}(80°\text{C} + 60°\text{C}) = 70°\text{C}$$

The rate of heat transfer is given by

$$q = UA\,\Delta T_{\text{mean}} = \dot{m}_w\, c_{pw}\,(T_{w,\text{out}} - T_{w,\text{in}})$$

Solving for the transfer area

$$A = \frac{\dot{m}_w c_{pw} \;\; T_{w,\text{out}} - T_{w,\text{in}}}{U \Delta T_{\text{mean}}} = \frac{15\,\text{kg/s} \;\; 4188\,\text{J/(kg K)} \;\; 80°\text{C} - 60°\text{C}}{820\,\text{W/(m}^2\text{K)} \;\; (70°\text{C}) \;\; \text{J/(Ws)}} = 21.9\ \text{m}^3$$

The outside area of the tubes is

$$A = (2\ \text{passes})\, N\, \pi D\, L \;\; \Rightarrow \;\; L = \frac{A}{2N\pi D} = \frac{21.9\,\text{m}^2}{2(45)\pi(0.0275\,\text{m})} = 2.8\ \text{m}$$

If each tube is bent in half to create the two tube passes, than the total tube length is 2 L = 5.6 m

PROBLEM 10.15

Benzene flowing at 12.5 kg/s is to be cooled continuously from 80°C to 54°C by 10 kg/s of water available at 15.5°C. Using Table 10.6, estimate the surface area required for (a) cross-flow with six tube passes and one shell pass with neither of the fluids mixed and (b) a counterflow exchanger with one shell pass and eight tube passes with the colder fluid inside tubes.

GIVEN

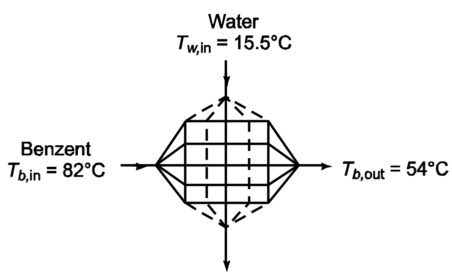
- Benzene coded by water in a heat exchanger
- Benzene flow rate $\dot{m}_b$ = 12.5 kg/s
- Water flow rate $\dot{m}_w$ = 10 mg/s
- Benzene temperatures
  - ($T_{b,\text{in}}$) = 82°C
  - ($T_{b,\text{out}}$) = 54°C
- Water inlet temperature ($T_{w,\text{in}}$) = 15.5°C

FIND

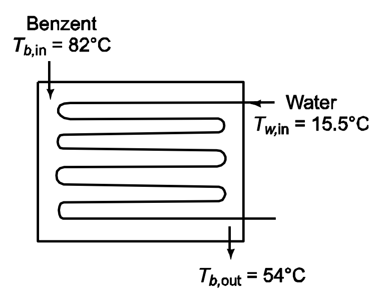
The surface area required for

(a) Cross-flow, 6 tube passes, 1 shell pass, both unmixed
(b) Counterflow, 8 tube passes, 1 shell pass, water in tubes

SKETCH



Case (a)



Case (b)

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at 20°C (c_{pw}) = 4182 J/(kg K)
From Appendix 2, Table 21, the specific heat of Benzene at 68°C (c_{pb}) = 1926 J/(kg K)

SOLUTION

The heat capacity rates are

$$C_b = \dot{m}_b\, c_{pb} = (12.5 \text{ kg/s})\ 1926 \text{ J/(kg K)} = 24{,}075 \text{ W/K}$$

$$C_w = \dot{m}_w\, c_{pw} = (10 \text{ kg/s})\ 4182 \text{ J/(kg K)} = 41{,}820 \text{ W/K}$$

$$\frac{C_{\min}}{C_{\max}} = \frac{24{,}075}{41{,}820} = 0.576$$

The rate of heat transfer is

$$q = \mathsf{E}\, C_{\min}\,(T_{b,\text{in}} - T_{w,\text{in}}) = \dot{m}_b\, c_{pb}\, \Delta T_b = C_{\min}\,(T_{b,\text{in}} - T_{b,\text{out}})$$

Solving for the effectiveness of the heat exchanger

$$\mathsf{E} = \frac{T_{b,\text{in}} - T_{b,\text{out}}}{T_{w,\text{in}} - T_{w,\text{in}}} = \frac{82°\text{C} - 54°\text{C}}{82°\text{C} - 15.5°\text{C}} = 0.42$$

(a) For unmixed cross-flow, the number of transfer units is given by Figure 10.21: $NTU \approx 0.7$.
By definition

$$NTU = \frac{UA}{C_{\min}} \Rightarrow A = NTU \frac{C_{\min}}{U}$$

The overall heat transfer coefficient (U), from Table 10.5 is in the range of 280-850 W/(m^2 K) between water and organic solvents. Therefore, use U = 565 + 50% W/(m^2 K).

$$\therefore \qquad A = 0.7 \frac{(24{,}075 \text{ W/K})}{(565+50\%) \text{ W/(m}^2\text{K)}} = 30 \text{ m}^2 + 50\%$$

(b) NTU for counterflow from Figure 10.20 is $NTU \approx 0.75$

$$\therefore \qquad A = 0.75 \frac{(24{,}075 \text{ W/K})}{(565+50\%) \text{ W/(m}^2\text{K)}} = 32 \text{ m}^2 + 50\%$$

PROBLEM 10.16

> **Water entering a shell-and-tube heat exchanger at 35°C is to be heated to 75°C by an oil. The oil enters at 110°C and leaves at 75°C. The heat exchanger is arranged for counterflow with the water making one shell pass and the oil two tube passes. If the water flow rate is 68 kg per minute and the overall heat transfer coefficient is estimated from Table 10.1 to be 320 W/(m² K), calculate the required heat exchanger area.**

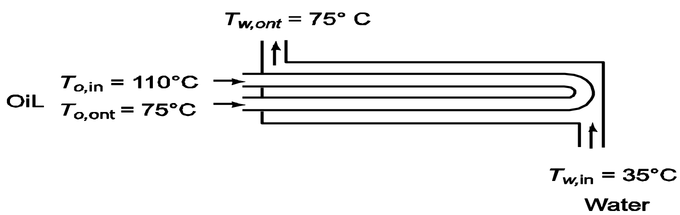
GIVEN

- One shell pass, two tube passes counterflow heat exchanger with water in shell, oil in tube
- Water temperatures $T_{w,in}$ = 35°C
  $T_{w,out}$ = 75°C
- Oil temperatures $T_{o,in}$ = 110°C
  $T_{w,out}$ = 75°C
- Water flow rate $\dot{m}_n$ = 68 kg/min = 1.133 kg/s
- Overall heat transfer coefficient (U) = 320 W/(m² K)

FIND

- The required area (A)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the average temperature of 55°C (c_{pw}) = 4180 J/(kg K)

SOLUTION

For counterflow from Figure 10.13 $\Delta T_a = T_{t,\text{in}} - T_{s,\text{out}}$ = 110°C – 75°C = 35°C

$\Delta T_b = T_{t,\text{out}} - T_{s,\text{in}}$ = 75°C – 35°C = 40°C

$$LMTD = (\Delta T_a - \Delta T_b) \Big/ \ln\left(\frac{\Delta T_a}{\Delta T_b}\right) = (35°\text{C} - 40°\text{C}) \Big/ \ln\left(\frac{35}{40}\right) = 37.4°\text{C}$$

The *LMTD* must be modified according to Figure 10.14

$$P = \frac{T_{o,\text{out}} - T_{o,\text{in}}}{T_{w,\text{in}} - T_{o,\text{in}}} = \frac{75 - 110}{35 - 110} = 0.47$$

$$Z = \frac{T_{w,\text{in}} - T_{w,\text{out}}}{T_{o,\text{out}} - T_{o,\text{in}}} = \frac{35 - 75}{75 - 110} = 1.14$$

From Figure 10.14: $F \approx 0.80$ $\Delta T_{\text{mean}} = F(LMTD)$ = 0.80 (37.4°C) = 29.9°C

The rate of heat transfer is $q = U\,A\,\Delta T_{\text{mean}} = \dot{m}_w\, c_{pw}\,(T_{w,\text{out}} - T_{w,\text{in}})$

Solving for the transfer area

$$A = \frac{\dot{m}_w c_{pw}\ \ T_{w,\text{out}} - T_{w,\text{in}}}{U\,\Delta T_{\text{mean}}} = \frac{1.133\,\text{kg/s}\ \ 4180\,\text{J/(Kg K)}\ \ 75°\text{C} - 35°\text{C}}{320\,\text{W/(m}^2\text{K)}\ \ (29.9°\text{C})\ \ \text{J/(W s)}} = 19.8\ \text{m}^2$$

PROBLEM 10.17

Starting with a heat balance, show that the effectiveness for a counterflow arrangement is

$$\mathrm{E} = \frac{\mathbf{1-exp}\left[-\left(\dfrac{\mathbf{1-}\boldsymbol{C}_{\mathbf{min}}}{\boldsymbol{C}_{\mathbf{max}}}\right)\boldsymbol{NTU}\right]}{\mathbf{1-}\left(\dfrac{\boldsymbol{C}_{\mathbf{min}}}{\boldsymbol{C}_{\mathbf{max}}}\right)\mathbf{exp}\left[-\left(\dfrac{\mathbf{1-}\boldsymbol{C}_{\mathbf{min}}}{\boldsymbol{C}_{\mathbf{max}}}\right)\boldsymbol{NTU}\right]}$$

GIVEN

- Counterflow heat exchanger

FIND

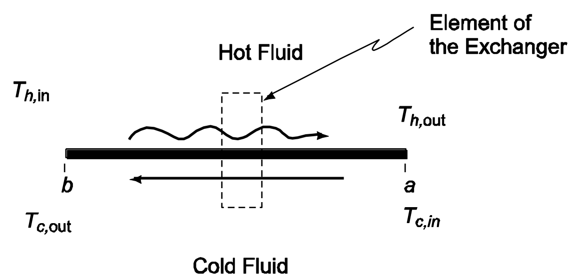
- Show that the effectiveness is

$$\mathrm{E} = \frac{1-\exp\left[-\left(\dfrac{1-C_{\min}}{C_{\max}}\right)NTU\right]}{1-\left(\dfrac{C_{\min}}{C_{\max}}\right)\exp\left[-\left(\dfrac{1-C_{\min}}{C_{\max}}\right)NTU\right]}$$

ASSUMPTIONS

- Heat loss to surroundings in negligible

SKETCH



SOLUTION

A heat balance on an element of the heat exchanger yields

$$dq = -C_h\, dT_h = C_c\, dT_c = U(T_h - T_c)\, dA$$

Rearranging

$$-\frac{C_h}{C_c}\,\frac{dT_h}{T_h - T_c} = -\frac{dT_c}{T_h - T_c} = \frac{U}{C_c}\, dA$$

Note that if $\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2} = \dfrac{C_1}{C_2}$

then $\dfrac{C_1}{C_2} = \dfrac{A_1 + B_1}{A_2 + B_2}$

Therefore

$$\frac{U}{C_c}\, dA = \frac{-\ dT_h - dT_c}{\dfrac{C_c}{C_h}\ T_h - T_c\ -\ T_h - T_c} = \frac{d\ \ T_h - T_c}{\left(1 - \dfrac{C_c}{C_h}\right)\ T_h - T_c}$$

$$\frac{1}{T_h - T_c}\, d(T_h - T_c) = \frac{U}{C_c}\left(1 - \frac{C_c}{C_h}\right) dA$$

Integrating from *a* to *b*

$$\ln\left[\frac{T_{h,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{out}}}\right] = \frac{UA}{C_c}\left(1 - \frac{C_c}{C_h}\right)$$

But, from Equation (10.22)

$$\mathcal{E} = \frac{C_h}{C_{\min}}\,\frac{T_{h,\text{out}} - T_{h,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \frac{C_c}{C_{\min}}\,\frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}}$$

From which

$$T_{h,\text{out}} = T_{h,\text{in}} - \mathcal{E}(T_{h,\text{in}} - T_{c,\text{in}})$$
$$T_{c,\text{out}} = T_{c,\text{in}} + \mathcal{E}(T_{h,\text{in}} - T_{c,\text{in}})$$

Therefore

$$T_{h,\text{out}} - T_{c,\text{in}} = T_{h,\text{in}} - \mathcal{E}(T_{h,\text{in}} - T_{c,\text{in}})\left(\frac{C_{\min}}{C_h}\right) - T_{c,\text{in}} = \left(1 - \mathcal{E}\frac{C_{\min}}{C_h}\right)(T_{h,\text{in}} - T_{c,\text{in}})$$

$$T_{h,\text{in}} - T_{c,\text{out}} = T_{h,\text{in}} - T_{c,\text{in}} - \mathcal{E}(T_{h,\text{in}} - T_{c,\text{in}})\,\frac{C_{\min}}{C_c} = \left(1 - \mathcal{E}\frac{C_{\min}}{C_c}\right)(T_{h,\text{in}} - T_{c,\text{in}})$$

Substituting these into the energy balance

$$\ln\left[\frac{1 - \mathcal{E}\dfrac{C_{\min}}{C_h}}{1 - \mathcal{E}\dfrac{C_{\min}}{C_c}}\right] = \frac{UA}{C_c}\left(1 - \frac{C_c}{C_h}\right) \Rightarrow \frac{1 - \mathcal{E}\dfrac{C_{\min}}{C_h}}{1 - \mathcal{E}\dfrac{C_{\min}}{C_c}} = \exp\left[\frac{UA}{C_c}\left(1 - \frac{C_c}{C_h}\right)\right]$$

Solving for effectiveness

$$1 - \mathcal{E}\frac{C_{\min}}{C_h}\ \exp\left[\left(1 - \frac{C_c}{C_h}\right)\frac{UA}{C_c}\right] = 1 - \mathcal{E}\frac{C_{\min}}{C_c}$$

$$\mathcal{E}\frac{C_{\min}}{C_c} - \mathcal{E}\frac{C_{\min}}{C_h}\ \exp\left[-\left(1 - \frac{C_c}{C_h}\right)\frac{UA}{C_c}\right] = \exp\left[-\left(1 - \frac{C_c}{C_h}\right)\frac{UA}{C_c}\right]$$

$$\mathcal{E} = \frac{1 - \exp\left[-\left(1 - \dfrac{C_c}{C_h}\right)\dfrac{UA}{C_c}\right]}{\dfrac{C_{\min}}{C_c} - \dfrac{C_{\min}}{C_h}\exp\left[-\left(1 - \dfrac{C_c}{C_h}\right)\dfrac{UA}{C_c}\right]}$$

Define $NTU = UA/C_{\min}$

Case (a) If $C_c = C_{\min} \rightarrow C_h = C_{\max}$

Then:

$$\mathcal{E} = \frac{1 - \exp\left[-\left(\dfrac{1 - C_{\min}}{C_{\max}}\right)NTU\right]}{1 - \left(\dfrac{C_{\min}}{C_{\max}}\right)\exp\left[-\left(\dfrac{1 - C_{\min}}{C_{\max}}\right)NTU\right]}$$

Case (b) If $C_c = C_{\max} \rightarrow C_h = C_{\min}$

$$\mathrm{E} = \frac{1-\exp\left[-\left(1-\frac{C_{\max}}{C_{\min}}\right)\frac{UA}{C_{\max}}\right]}{\frac{C_{\max}}{C_{\min}}-\exp\left[-\left(1-\frac{C_{\max}}{C_{\min}}\right)\frac{UA}{C_{\max}}\right]}$$

Multiplying the numerator and denominator by

$$\exp\left[\left(1-\frac{C_{\max}}{C_{\min}}\right)\frac{UA}{C_{\max}}\right] = \exp\left[\left(1-\frac{C_{\max}}{C_{\min}}\right)\left(\frac{C_{\max}}{C_{\min}}\right)NTU\right] = \exp\left[\left(1-\frac{C_{\max}}{C_{\min}}\right)NTU\right]$$

yields the following result

$$\mathrm{E} = \frac{\exp\left[-\left(\frac{1-C_{\max}}{C_{\min}}\right)NTU\right]-1}{\frac{C_{\min}}{C_{\max}}\exp\left[-\left(\frac{1-C_{\max}}{C_{\min}}\right)NTU\right]-1}$$

$$\mathrm{E} = \frac{1-\exp\left[-\left(\frac{1-C_{\min}}{C_{\max}}\right)NTU\right]}{1-\left(\frac{C_{\min}}{C_{\max}}\right)\exp\left[-\left(\frac{1-C_{\min}}{C_{\max}}\right)NTU\right]}$$

PROBLEM 10.18

In a shell of a shell and tube heat exchanger with two shell passes and eight tube passes, 12.6 kg/s of water is heated in the shell from 80^{0}C to 150°C. Hot exhaust gases having roughly the same physical properties as air enter the tubes at 340°C and leave at 180°C. The total surface, based on the outer tube surface, is 930 m^2. Determine (a) the log-mean temperature difference if the heat exchanger were simple counterflow type, (b) the correction factor *F* for the actual arrangement, (c) the effectiveness of the heat exchanger, (d) the average overall heat transfer coefficient.

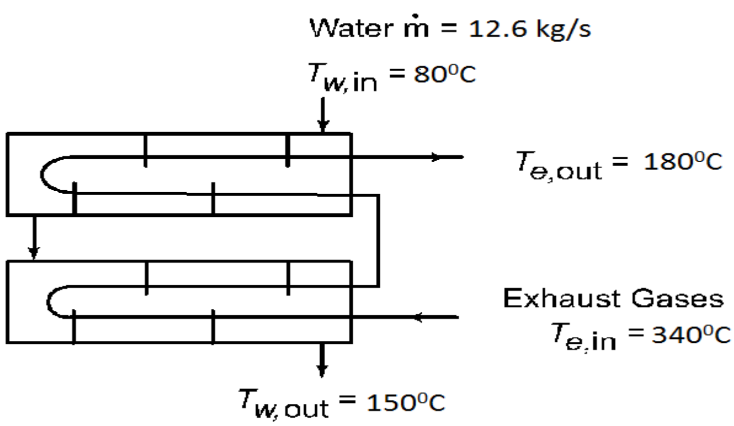
GIVEN

- A tubular heat exchanger with 2 shell passes and 8 tube passes
- Water in the shell, exhaust gases in the tubes
- Exhaust gases have roughly the same physical properties as air
- Water flow rate $\dot{m}_w$ = 12.6 kg/s
- Water temperatures
  - $T_{w,\text{in}}$ = 80°C
  - $T_{w,\text{out}}$ = 150°C
- Exhaust temperatures
  - $T_{e,\text{in}}$ = 340°C
  - $T_{e,\text{out}}$ = 180°C
- Surface are (A) = 930 m^2

FIND

(a) *LMTD* if the exchanger is a simple counterflow type
(b) Correction *F* for the actual arrangement
(c) Effectiveness (e)
(d) The overall heat transfer coefficient (U)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the mean temperature of 130°C (c_{pw}) = 4244.5 W/(kg K).

SOLUTION

(a) From Figure 10.13 for simple counterflow $\Delta T_a = T_{e,\text{in}} - T_{w,\text{out}}$ = 340°C – 150°C = 190°C

$\Delta T_b = T_{e,\text{out}} - T_{w,\text{in}}$ = 180°C – 80°C = 100°C

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{190°C - 100°C}{\ln\left(\frac{190}{100}\right)} = 140°C$$

(b) The *LMTD* must be modified as shown in Figure 10.15

$$P = \frac{T_{e,\text{out}} - T_{e,\text{in}}}{T_{w,\text{in}} - T_{e,\text{in}}} = \frac{180-340}{80-340} = 0.62$$

$$Z = \frac{T_{w,\text{in}} - T_{w,\text{out}}}{T_{e,\text{out}} - T_{e,\text{in}}} = \frac{80-150}{180-340} = 0.27$$

From Figure 10.15, $F = 0.97$

(c) To find the effectiveness, the heat capacity rate of the exhaust must first be determined. An energy balance yields

$$C_e\,(T_{e,\text{in}} - T_{e,\text{out}}) = C_w\,(T_{w,\text{out}} - T_{w,\text{in}})$$

where $C_w = \dot{m}_w\, c_{pw} = 12.6\ \text{kg}/s\,(4244.5\,J/(\text{kg}\ K)) = 53480.7\,W/K$

$$C_h = C_e = C_w\,\frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_{e,\text{in}} - T_{e,\text{out}}} = 53480.7\ \text{W/K}\left(\frac{150-80}{340-180}\right) = 23397.8\,W/K = C_{\min}$$

The effectiveness is given by Equation (10.22a)

$$\text{E} = \frac{C_h\ \ T_{h,\text{in}} - T_{h,\text{out}}}{C_{\min}\ \ T_{h,\text{in}} - T_{c,\text{in}}} = \frac{T_{e,\text{in}} - T_{e,\text{out}}}{T_{e,\text{in}} - T_{w,\text{out}}} = \frac{340-180}{340-150} = 0.84 = 84\%$$

(d) The rate of heat transfer is

$$q = U\,A\,\Delta T_{\text{mean}} = U\,A\,F\,(LMTD) = C_w\,(T_{w,\text{out}} - T_{w,\text{in}})$$

Solving for the overall heat transfer coefficien

$$U = \frac{C_w\ \ T_{w,\text{out}} - T_{w,\text{in}}}{AF\ \ LMTD} = \frac{(53480\,W/\text{K})(150°C - 80°C)}{900\,m^2\,(0.97)(140°C)} = 30.6\ \text{W/(m}^2\ \text{K)}$$

PROBLEM 10.19

In gas turbine recuperators, the exhaust gases are used to heat the incoming air and C_{min}/C_{max} is therefore approximately equal to unity. Show that for this case $e = NTU/(1 + NTU)$ for counterflow and $e = 1/2\ (1 - e^{-2NTU})$ for parallel flow.

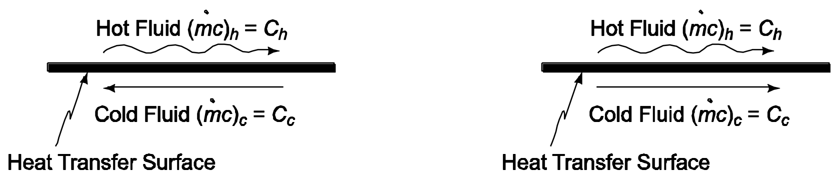
GIVEN

- Gas turbine recuperator
- $C_{min}/C_{max} \approx 1$

FIND

Show that (a) $e = NTU/(1 + NTU)$ for counterflow
(b) $e = 1/2\ (1 - e^{-2NTU})$ for parallel flow

SKETCH



SOLUTION

(a) From the solution of Problem 10.17: for counterflow

$$\mathsf{E} = \frac{1 - \exp[-\ 1 - C^*\ \ NTU]}{1 - C^* \exp[-\ 1 - C^*\ \ NTU]}$$

where $C^* = C_{min}/C_{max}$ For $C^* = 1$, e is undefined

Applying L'Hopital's rule

$$\mathsf{E}_{C^* \to 1} = \lim_{C^* \to 1} \frac{f(C^*)}{g(C^*)} = \lim_{C^* \to 1} \frac{f'(C^*)}{g''(C^*)}$$

$$\mathsf{E}_{C^* \to 1} = \lim_{C^* \to 1} \frac{-NTU \exp[-(1 - C^*)NTU]}{-C^*NTU \exp[-(1 - C^*)NTU] - \exp[-(1 - C^*)NTU]}$$

$$\mathsf{E} = \frac{NTU}{1 + NTU}$$

(b) For parallel flow, the effectiveness is given by Equation (10.26)

$$\mathsf{E} = \frac{1 - \exp[-(1 + C_{min}/C_{max})\, NTU]}{1 + \left(\dfrac{C_{min}}{C_{max}}\right)}$$

For $C_{min}/C_{max} = 1$

$$\mathsf{E} = \frac{1 - \exp\ -2NTU}{2} = \frac{1}{2}(1 - e^{-2NTU})$$

PROBLEM 10.20

In a single-pass counterflow heat exchanger, 4536 kg/h of water enter at 15°C and cool 9071 kg/h of an oil having a specific heat of 2093 J/(kg K) from 93 to 65°C. If the overall heat transfer coefficient is 284 W/(m² °C), determine the surface area required.

GIVEN

- Oil and water in a single-pass counterflow heat exchanger
- Water flow rate $\dot{m}_w$ = 4536 kg/h = 1.26 kg/s
- Oil flow rate $\dot{m}_o$ = 9071 kg/h = 2.52 kg/s
- Inlet temperatures: Water ($T_{w,\text{in}}$) = 15°C
  Oil ($T_{o,\text{in}}$) = 93°C
- Oil outlet temperature ($T_{o,\text{out}}$) = 65°C
- Oil specific heat (c_{po}) = 2093 J/(kg °C)
- Overall heat transfer coefficient (U) = 284 W/(m² °C)

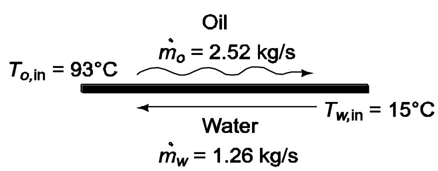
FIND

- The surface area (A) required

ASSUMPTIONS

- Steady state
- Constant thermal properties

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at 20°C (c_{pw}) = 4182 J/(kg K)

SOLUTION

The outlet temperature of the water can be determined from an energy balance

$$\dot{m}_w c_w (T_{w,\text{out}} - T_{w,\text{in}}) = \dot{m}_o c_o (T_{o,\text{in}} - T_{o,\text{out}})$$

$$T_{w,\text{out}} = T_{w,\text{in}} + \frac{\dot{m}_o c_{po}}{\dot{m}_w c_{pw}} (T_{o,\text{in}} - T_{o,\text{out}})$$

$$T_{w,\text{out}} = 15°\text{C} + \frac{(2.52 \text{ kg/s})(2093 \text{ J/(kg K)})}{(1.26 \text{ kg/s})(4182 \text{ J/(kg K)})} (93°\text{C} - 65°\text{C}) = 43°\text{C}$$

From Figure 10.13 $\Delta T_a = T_{o,\text{in}} - T_{w,\text{out}} = 93°\text{C} - 43°\text{C} = 50°\text{C}$

$\Delta T_b = T_{o,\text{out}} - T_{w,\text{in}} = 65°\text{C} - 15°\text{C} = 50°\text{C}$

Therefore, $\Delta T_{\text{mean}} = 50°\text{C}$

The rate of heat transfer is

$$q = U A \Delta T_{\text{mean}} = \dot{m}_o c_{po} (T_{o,\text{in}} - T_{o,\text{out}})$$

$$\therefore \quad A = \frac{\dot{m}_o c_{po} \;\; T_{o,\text{in}} - T_{o,\text{out}}}{U \Delta T_{\text{mean}}} = \frac{(2.52 \text{ kg/s})(2093 \text{ J/(kg k)})(93°\text{C} - 65°\text{C})}{(284 \text{ W/(m}^2\text{K)})(50°\text{C})(\text{J/(W s)})} = 10.4 \text{ m}^2$$

PROBLEM 10.21

A steam-heated single-pass tubular preheater is designed to raise 5.6 kg/s of air from 20^0C to 75°C, using saturated steam at 2.6 MPa(abs). It is proposed to double the flow rate of air and, in order to be able to use the same heat exchanger and achieve the desired temperature rise, it is proposed to increase the steam pressure. Calculate the steam pressure necessary for the new conditions and comment on the design characteristics of the new arrangement.

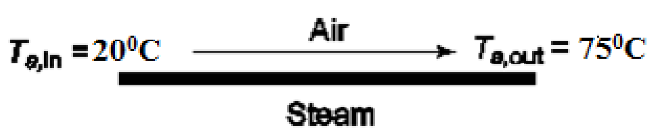
GIVEN

- Steam-heated single-pass tubular preheated heating air
- Air flow rate $(\dot{m}_a)$ = 5.6 kg/s
- Air temperature
  - $T_{a,\text{in}}$ = 20°C
  - $T_{a,\text{out}}$ = 75°C
- Saturated steam pressure (p_s) = 2.6 MPa = 2.6×10^6 N/m^2

FIND

- The steam pressure necessary for a double $\dot{m}_a$ with the same temperature rise

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the saturation temperature (T_s) of steam at 2.6×10^6 N/m^2 = 225°C

SOLUTION

The heat capacity rate of the steam is virtually infinite, therefore, $C_{\min}/C_{\max}$ = 0. The effectiveness must be determined using Equation (10.22b)

$$\mathsf{E} = \frac{C_c}{C_{\min}} \frac{T_{a,\text{out}} - T_{a,\text{in}}}{T_s - T_{a,\text{in}}} \qquad \text{where } C_c = C_a = C_{\min}$$

$$\therefore \mathsf{E} = \frac{75-20}{225-20} = 0.268$$

Examination of $e = e\,(NTU)$ when $C_{\min}/C_{\max} = 0$ in Figures 10.18 and 10.19 or in Equation (10.26) and the solution of Problem 10.19 reveals that $e = e\,(NTU)$ is the same of both counterflow and parallel flow when $C_{\min}/C_{\max} = 0$. For $e = 0.268$, Figure 10.18 gives $NTU = (U\,A)/C_{\min} \approx 0.25$.

Doubling the flow rate of air doubles its heat capacity rate ($C_{\min}$). Therefore, the NTU is halved. For the new flow rate: $NTU = 0.125$.

From Figure 10.18 for $NTU = 0.125$, $C_{\min}/C_{\max} = 0 \rightarrow e = 0.15$

The rate of heat transfer, from Equation (10.23) is

$$q = e\,C_{\min}\,(T'_s - T_{s,\text{in}}) = \dot{m}_a\,c_{pa}\,(T_{a,\text{out}} - T_{a,\text{in}}) = C_{\min}\,(T_{a,\text{out}} - T_{a,\text{in}})$$

Solving for the steam temperature required.

$$T'_s = T_{a,\text{in}} + \frac{T_{a,\text{out}} - T_{a,\text{in}}}{\mathsf{E}} = 20°\text{C} + \frac{75^0 C - 20^0 C}{0.15} = 387°\text{C}$$

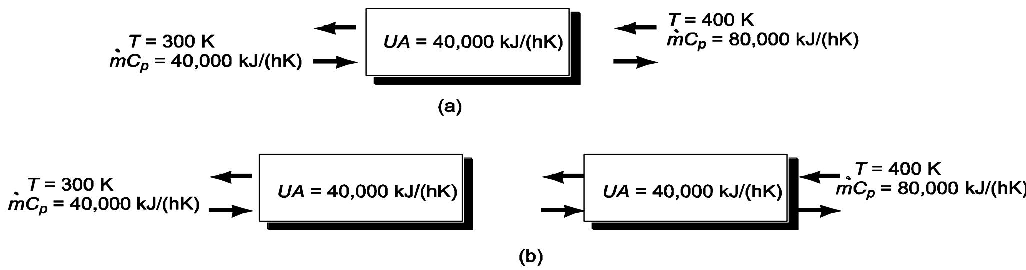
The steam temperature required for the doubled air flow rate is 387°C. From Appendix 2, Table 13, for T_s = 387°C, the saturation pressure cannot be calculated as it is much larger than saturation temperature range of upto 300^0C.

COMMENTS

Pressure increases rapidly with steam temperature, therefore, the solution is only practical if the equipment is designed to operate safely at high pressure.

PROBLEM 10.22

For safety reasons, a heat exchanger performs as shown in (a) of accompanying figure. An engineer suggests that it would be wise to double the heat transfer area so as to double the heat transfer rate. The suggestion is made to add a second, identical exchanger as shown in (b). Evaluate this suggestion, that is, show whether or not the heat transfer rate would double.



GIVEN

Case 1

Heat exchanger as shown above.

- Overall heat transfer coefficient times the transfer area (UA) = 40,000 kJ/(hr K)
- Heat capacity rates
  - C_h = 80,000 kJ/(h K)
  - C_c = 40,000 kJ/(h K)
- Entering temperatures
  - T_b,in = 400 K
  - T_c,in = 300 K

Case 2

Two of the same heat exchanges as shown above in Figure B.

FIND

- Does the heat transfer rate double?

ASSUMPTIONS

- Heat exchangers are simple counterflow geometry

SOLUTIONS

Case 1 $\dfrac{C_{\min}}{C_{\max}} = \left(\dfrac{40{,}000}{80{,}000}\right) = 0.5$

The number of transfer units is: $NTU = (UA)/C_{\min}$ = (40,000)/(40,000) = 1.0

From Figure 10.19: $e_1 = 0.54$

Case 2

For this case, the transfer area is doubled, therefore

$$UA_{\text{total}} = 2(UA) = 80{,}000 \text{ kJ/(h K)}$$

$$NTU = (80{,}000)/(40{,}000) = 2.0$$

From Figure 10.19: $e_2 = 0.78$

Applying Equation 10.23

$$\frac{q_2}{q_1} = \frac{\mathrm{E}_2\, C_{\min}\left(T_{h,\text{in}} - T_{c,\text{in}}\right)}{\mathrm{E}_1\, C_{\min}\left(T_{h,\text{in}} - T_{c,\text{in}}\right)} = \frac{\mathrm{E}_2}{\mathrm{E}_1} = \frac{0.78}{0.54} = 1.44$$

The rate of heat transfer does not double. It is increased by only 44%.

PROBLEM 10.23

In a single-pass counterflow heat exchanger, 1.25 kg/s of water enter at 15°C and cools 2.5 kg/s of an oil having a specific heat of 2093 J/(kg K) from 95°C to 65°C. If the overall heat transfer coefficient is 280 W/(m² K), determine the surface area required.

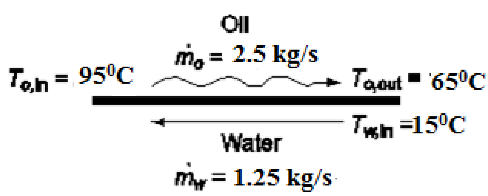
GIVEN

- Water cooling oil in a single-pass counterflow heat exchanger
- Water flow rate $(\dot{m}_w)$ = 1.25 kg/s
- Oil flow rate $(\dot{m}_o)$ = 2.5 kg/s
- Oil specific heat (c_{po}) = 2093 J/(kg K)
- Water inlet temperature ($T_{w,\text{ in}}$) = 15°C
- Oil temperature
  - $T_{o,\text{ in}}$ = 95°C
  - $T_{o,\text{ out}}$ = 65°C
- Overall heat transfer coefficient (U) = 280 W/(m² K)

FIND

- The surface area (A) required

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water (c_{pw}) = 4187 J/(kg K)

SOLUTION

The heat capacity rates are

$$\text{where } C_w = \dot{m}_w\, c_{pw} = 1.25 \text{ kg/s } (4187\, J/(\text{kg K})) = 5233 \text{ W/K}$$

$$\text{where } C_o = \dot{m}_o\, c_{po} = 2.5 \text{ kg/s } (2093\, J/(\text{kg K})) = 5233 \text{ W/K}$$

Therefore, $C_{\min}/C_{\max} = 1.0$

The outlet temperature of the water can be determined from an energy balance

$$q = C_o\,(T_{o,\text{in}} - T_{o,\text{out}}) = C_w\,(T_{w,\text{out}} - T_{w,\text{in}})$$

$$T_{w,\text{out}} = T_{w,\text{in}} + (T_{o,\text{in}} - T_{o,\text{out}}) = 15°C + 95°C – 65°C = 45°C$$

From Figure 10.13, for counterflow

$$\Delta T_a = T_{o,\text{in}} - T_{w,\text{out}} = 95°C – 45°C = 50°C$$

$$\Delta T_b = T_{o,\text{out}} - T_{w,\text{in}} = 65°C – 15°C = 50°C$$

(Note that $\Delta T_a = \Delta T_b$ because $C_w = C_o$)

Therefore, ΔT_{means} = 50°C

The rate of heat transfer is $\quad q = U\,A\,\Delta T_{\text{mean}} = C_o\,(T_{o,\text{in}} - T_{o,\text{out}})$

Solving for the transfer area

$$A = \frac{C_o(T_{o,\text{in}} - T_{o,\text{out}})}{U\,\Delta T_{\text{mean}}} = \frac{(5233\,W/\text{K})(95°C-65°C)}{(280\,W/(\text{m}^2\text{ K}))(50°C)} = 11.2 \text{ m}^2$$

PROBLEM 10.24

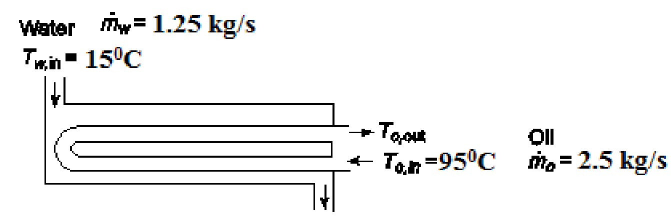
Determine the outlet temperature of oil in Problem 10.23 for the same initial temperatures of the fluids if the flow arrangement is one shell pass and two tube passes, but with the same total area and average overall heat transfer coefficient as the unit in Problem 10.23.

- Water cooling oil in a tube and shell heat exchanger
- Water flow rate $(\dot{m}_w)$ = 1.25 kg/s
- Oil flow rate $(\dot{m}_o)$ = 2.5 kg/s
- Oil specific heat (c_{po}) = 2093 J/(kg K)
- Water inlet temperature: ($T_{w,\text{in}}$) = 15°C
- Oil inlet temperature: $T_{o,\text{in}}$ = 95°C
- Overall heat transfer coefficient (U) = 280 W/(m^2 K)
- One shell and two tube passes
- Same surface area as Problem 10.23: A = 11.2 m^2

FIND

- The oil outlet temperature ($T_{o,\text{out}}$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water (c_{pw}) = 4187 W/(kg K)

SOLUTION

From the solution to Problem 10.23 $\quad C_w = C_o$ = 5233 W/K

$$\frac{C_{\min}}{C_{\max}} = 1.0$$

The number of transfer units is

$$NTU = \frac{UA}{C_{\min}} = \frac{\left(280\, W/(\text{m}^2\ \text{K})\right)\left(11.2\, m^2\right)}{(5233\, W/\text{K})} = 0.60$$

From Figure 10.20: $e \approx 0.35$

From Equation (10.22a) (for $C_h/C_{\min}$ = 1.0)

$$\mathsf{E} = \frac{T_{o,\text{in}} - T_{o,\text{out}}}{T_{o,\text{in}} - T_{w,\text{in}}}$$

$$T_{o,\text{ out}} = T_{o,\text{ in}} - \mathscr{E}\,(T_{o,\text{ in}} - T_{w,\text{ on}}) = 95°\text{C} - 0.35\,(95°\text{C} - 15°\text{C}) = 67°\text{C}$$

COMMENTS

The outlet oil temperature is approximately the same as the previous problem because the effectiveness is not improved significantly by an additional pass for small values of *NTU*.

PROBLEM 10.25

Carbon dioxide at 427°C is to be used to heat 12.6 kg/s of pressurized water from 37°C to 148°C while the gas temperature drops 204°C. For an overall heat transfer coefficient of 57 W/(m^2 K), compute the required area of the exchanger in square feet for (a) parallel flow, (b) counterflow, (c) a 2-4 reversed current exchanger, and (d) crossflow, gas mixed.

GIVEN

- CO_2 heating water in a heat exchanger
- CO_2 temperatures
  - $T_{g,\text{in}}$ = 427°C
  - $T_{g,\text{out}}$ = 427 – 204 = 223°C
- Water temperatures
  - $T_{w,\text{in}}$ = 37°C
  - $T_{w,\text{out}}$ = 148°C
- Water flow rate $(\dot{m}_w)$ = 12.6 kg/s
- Overall heat transfer coefficient (U) = 57 W/(m^2 K)

FIND

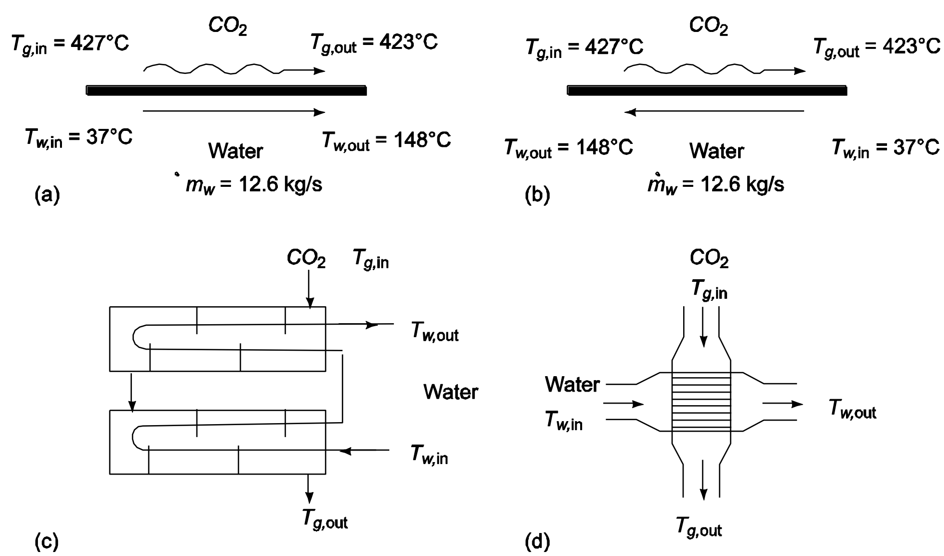
The area (A) required for:

(a) parallel-flow (c) A 2- 4 reversed current exchanger
(b) counterflow (d) Crossflow, gas mixed

ASSUMPTIONS

- In configuration (c), the gas is in the shell side

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the mean temperature of 92°C (c_{pw}) = 4205 J/(kg K)

SOLUTION

(a) For parallel flow, from Figure 10.12 $\Delta T_a = T_{g,\text{ in}} - T_{w,\text{in}}$ = 427°C – 37°C = 390°C
$\Delta T_b = T_{g,\text{out}} - T_{w,\text{out}}$ = 223°C – 148°C = 75°C

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\dfrac{\Delta T_a}{\Delta T_b}} = \frac{390°\text{C} - 75°\text{C}}{\ln\left(\dfrac{390}{75}\right)} = 191°\text{C}$$

The rate of heat transfer is

$$q = U\,A\,(LMTD) = \dot{m}_w\;c_{pw}\,(T_{w,\text{out}} - T_{w,\text{in}})\text{s}$$

Solving for the transfer area

$$A = \frac{\dot{m}_w C_{pw}(Tw,\ \text{out} - T_{w,\text{in}}}{U(LMTD)} = \frac{12.6\ \text{kg/s}\quad 4205\ \text{J/(kg K)}\quad 148°\text{C} - 37°\text{C}}{57\ \text{W/(m}^2\text{K)}\ \ (191°\text{C})\ \ \text{J/(Ws)}}$$

$$= 541\ \text{m}^2 = 5827\ \text{ft}^2$$

(b) For counterflow, from Figure 10.13 $\Delta T_a = T_{g,\text{in}} - T_{w,\text{out}} = 427°\text{C} - 148°\text{C} = 279°\text{C}$
$\Delta T_b = T_{g,\text{out}} - T_{w,\text{in}} = 223°\text{C} - 37°\text{C} = 186°\text{C}$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\dfrac{\Delta T_a}{\Delta T_b}} = \frac{279°\text{C} - 186°\text{C}}{\ln\left(\dfrac{279}{186}\right)} = 229°\text{C}$$

Similarly

$$A = \frac{12.6\ \text{kg/s}\quad 4205\ \text{J/(kg K)}\quad 148°\text{C} - 37°\text{C}}{57\ \text{W/(m}^2\text{K)}\ \ (229°\text{C})\ \ \text{J/(Ws)}} = 450\ \text{m}^2 = 4850\ \text{ft}^2$$

(c) The counterflow *LMTD* must be corrected using Figure 10.15 for this configuration

$$P = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_{g,\text{in}} - T_{w,\text{in}}} = \frac{148 - 37}{427 - 37} = 0.28$$

$$= \frac{T_{g,\text{in}} - T_{g,\text{out}}}{T_{w,\text{out}} - T_{w,\text{in}}} = \frac{427 - 223}{148 - 37} = 1.84$$

From Figure 10.15 $F \approx 0.97$

$$\therefore\ \Delta T_{\text{mean}} = F(\Delta T) = 0.97\ (229°\text{C}) = 222°\text{C}$$

$$A = \frac{12.6\ \text{kg/s}\quad 4205\ \text{J/(kg K)}\ \ (148°\text{C} - 37°\text{C})}{57\ \text{W/(m}^2\text{K)}\ \ (222°\text{C})\ \ \text{J/(Ws)}} = 465\ \text{m}^2 = 5003\ \text{ft}^2$$

(d) The counterflow *LMTD* must be modified using Figure 10.16 for crossflow, gas mixed.
For $P = 0.28$, $Z = 1.84 \rightarrow F = 0.93$

$$\therefore\ \Delta T_{\text{mean}} = F(\Delta T) = 0.93\ (229°\text{C}) = 213°\text{C}$$

$$A = \frac{12.6\ \text{kg/s}\quad 4205\ \text{J/(kg K)}\quad 148°\text{C} - 37°\text{C}}{57\ \text{W/(m}^2\text{K)}\ \ (213°\text{C})\ \ \text{J/(Ws)}} = 484\ \text{m}^2 = 5210\ \text{ft}^2$$

COMMENTS

A simple counterflow heat exchanger requires the least transfer area for this case.

PROBLEM 10.26

> **An economizer is to be purchased for a power plant. The unit is to be large enough to heat 7.5 kg/s of pressurized water from 71 to 182°C. There are 26 kg/s of flue gases (c_p = 1000 J/(kg K)) available at 426°C. Estimate (a) the outlet temperature of the flue gases and (b) the heat transfer area required for a counterflow arrangement if the overall heat transfer coefficient is 57 W/(m² K).**

GIVEN

Counterflow heat exchanger - flue gases heating water
Water flow rate $(\dot{m}_w)$ = 7.5 kg/s
Water temperatures

- $T_{w,\text{in}}$ = 71°C
- $T_{w,\text{out}}$ = 182°C

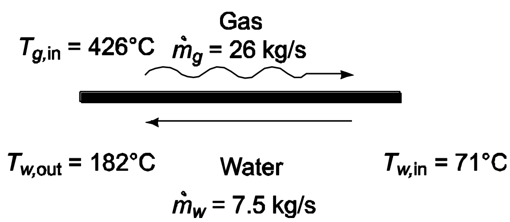
Gas flow rate ($\dot{m}_g$) = 26 kg/s

Gas specific heat (c_{pg}) = 1000 J/(kg K)
Gas inlet temperature ($T_{g,\text{in}}$) = 426°C
Overall heat transfer coefficient (U) = 57 W/(m² K)

FIND

(a) Outlet gas temperature $T_{g,\text{out}}$
(b) Heat transfer area (A) required

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the mean temperature of 126.5°C (c_{pw}) = 4240 J/(kg K)

SOLUTION

The heat capacity rates are

$$C_w = \dot{m}_w \, c_{pw} = 7.5 \text{ kg/s} \;\; 4240 \text{ J/(kg K)} = 31{,}801 \text{ W/K}$$

$$C_g = \dot{m}_g \, c_{pg} = 26 \text{ kg/s} \;\; 1000 \text{ J/(kg K)} = 26{,}000 \text{ W/K}$$

(a) A heat balance yields

$$C_g\,(T_{g,\text{in}} - T_{g,\text{out}}) = C_w\,(T_{w,\text{out}} - T_{w,\text{in}})$$

Solving for the outlet gas temperature

$$T_{g,\text{out}} = T_{g,\text{in}} - \frac{C_w}{C_g}\, T_{w,\text{out}} - T_{w,\text{in}}) = 426°\text{C} - \frac{31{,}801}{26{,}000}\;(182°\text{C} - 71°\text{C}) = 290°\text{C}$$

(b) From Figure 10.13 for counterflow $\quad \Delta T_a = T_{g,\text{in}} - T_{w,\text{out}} = 426°\text{C} - 182°\text{C} = 244°\text{C}$
$\Delta T_b = T_{g,\text{out}} - T_{w,\text{in}} = 290°\text{C} - 71°\text{C} = 219°\text{C}$

$$\Delta T_{\text{mean}} = LMTD = \frac{\Delta T_a - \Delta T_b}{\frac{\Delta T_a}{\Delta T_b}} = \frac{244°\text{C} - 219°\text{C}}{\ln\left(\frac{244}{219}\right)} = 231°\text{C}$$

The rate of heat transfer is

$$q = U\,A\,\Delta T_{\text{mean}} = C_w(T_{w,\text{out}} - T_{w,\text{in}})$$

$$A = \frac{C_w(T_{w,\text{out}} - T_{w,\text{in}})}{U\Delta T_{\text{mean}}} = \frac{31{,}801\ \text{W/K} \quad 182°\text{C} - 71°\text{C}}{57\ \text{W/(m}^2\text{K)} \quad 231°\text{C}} = 268\ \text{m}^2$$

PROBLEM 10.27

Water flowing through a pipe is heated by steam condensing on the outside of the pipe. (a) Assuming a uniform overall heat transfer coefficient along the pipe, derive an expression for the water temperature as a function of distance from the entrance. (b) For an overall heat transfer coefficient of 570 W/(m² K), based on the inside diameter of 5 cm, a steam temperature of 104°C, and water-flow rate of 0.063 kg/s, calculate the length required to raise the water temperature from 15.5 to 65.5°C.

GIVEN

- Water flowing through a pipe steam condensing on the outside

FIND

(a) An expression for the water temperature as a function of distance from the entrance, $T_w(x)$

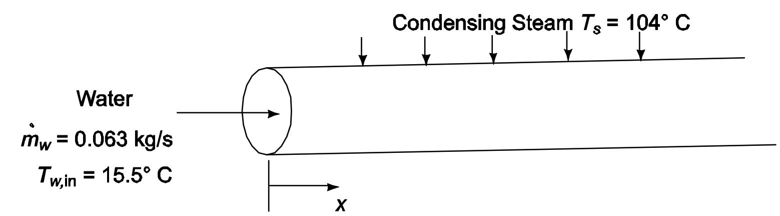
(b) For Overall heat transfer coefficient (U) = 570 W/(m² K)
Inside diameter (D) = 5 cm = 0.05 m
Steam temperature (T_s) = 104°C
Water flow rate ($\dot{m}_w$) = 0.063 kg/s
Water temperatures: $T_{w,\text{in}}$ = 15.5°C $T_{w,\text{out}}$ = 65.5°C

Find the length (L) required

ASSUMPTIONS

- A uniform overall heat transfer coefficient

SKETCH

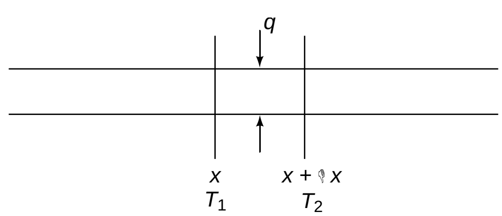


PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the mean temperature of 40°C (c_{pw}) = 4175 J/(kg K)

SOLUTION

(a) Consider an element of the exchanger as shown below



An energy balance on the element yields

$$\dot{m}_w\, c_{pw}\, \Delta T = U\, A\, [T_s - T(x)]$$

where $$\Delta T = T_2 - T_1 = \left(T_1 + \frac{dT}{dx}\Delta x\right) - T_1 = \frac{dT}{dx}\ \Delta x$$

and $\quad A = \pi D \,\Delta x$

As $\Delta x \to 0$, $T_1 \to T$

$$\therefore \qquad \dot{m}_w \; c_{pw} \; \frac{dT}{dx} = U \,\pi D \,(T_s - T)$$

$$\left(\frac{1}{T - T_s}\right) dT = -\,\frac{U \,\pi D}{\dot{m}_w \, c_{pw}} \, dx$$

Integrating from 0 to X

$$\ln\left(\frac{T_w(x) - T_s}{T_{w,\text{in}} - T_s}\right) = -\,\frac{U \,\pi D}{\dot{m}_w \, c_{pw}} \, x$$

$$T_w(x) = T_s + (T_{w,\text{in}} - T_s) \exp\left(-\frac{U \pi D x}{\dot{m}_w c_{pw}}\right)$$

(b) Solving for the distance X

$$x = -\,\frac{\dot{m}_w C_{pw}}{U \,\pi D} \; \ln\left(\frac{T_w(x) - T_s}{T_{w,\text{in}} - T_s}\right)$$

$$L = -\,\frac{0.063\,\text{kg/s} \quad 4175\,\text{J/(kg k)}}{570\,\text{W/(m}^2\text{K)} \quad \text{J/(Ws)} \;\; \pi(0.05\,\text{m})} \; \ln\left(\frac{65.5°\text{C} - 104°\text{C}}{15.5°\text{C} - 104°\text{C}}\right) = 2.45\text{ m}$$

PROBLEM 10.28

Water at a rate of 0.32 L/s and a temperature of 27°C enters a No. 18 BWG 1.6 cm condenser tube made of nickel chromium steel $(\mathbf{k = 26\ W/(m\,K)})$. The tube is 3 m long and its outside is heated by steam condensing at 50°C. Under these conditions, the average heat transfer coefficient on the water side is 10 kW/(m^2 K), and the heat transfer coefficient on the steam side may be taken as 11.3 kW/(m^2 K). On the interior of the tube, however, there is a scale having a thermal conductance equivalent to 5.6 kW/(m^2 K). (a) Calculate the overall heat transfer coefficient *U* per square meter of exterior surface area. (b) Calculate the exit temperature of the water.

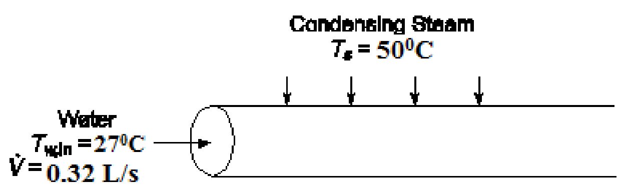
GIVEN

- A nickel chromium steel condenser tube with water inside and condensing steam outside
- Water flow rate $\dot{v}$ = 0.32 L/s= 3.2*10^{-4} m^3/s
- Water inlet temperature ($T_{w,\text{in}}$) = 27°C
- Tube: No. 18 BWG 1.6 cm
- Steel thermal conductivity (k_{st}) = 26 W/(m K)
- Tube length (L) = 3 m
- Steam temperature (T_s) = 50°C
- Average water- side transfer coefficient $(\bar{h}_i)$ = 10 kW/(m^2 K)
- Average steam transfer coefficient $(\bar{h}_o)$ = 11.3 kW/(m^2 K)
- Interior scaling conductance ($1/R_i$) = 5.6 kW/(m^2 K)

FIND

(a) The overall heat transfer coefficient (U) based on exterior surface area
(b) The outlet water temperature ($T_{w,\text{in}}$)

SKETCH



PROPERTIES AND CONSTANT

From Appendix 2, Table 42, for No. 18 BWG 1.6 cm tube

Inside Diameter (D_i) = 1.34 cm

Outside Diameter (D_o) = 1.6 cm

From Appendix 2, Table 13, the density of water at 30°C (ρ) = 995.7 kg/m^3; the specific heat (c_{pw}) = 4176 W/(kg K)

SOLUTION

(a) The overall heat transfer coefficient can be calculated from Equation (10.6)

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + R_o + R_k + \frac{R_i A_o}{A_i} + \frac{A_o}{A_i \bar{h}_i}$$

where R_o = exterior scaling resistance = 0

$A_o = \pi D_o L$

$A_i = \pi D_i L$

$$R_k = \frac{A_o \ln \frac{D_o}{D_i}}{2\pi L k} = \frac{D_o \ln \frac{D_o}{D_i}}{2k}$$

$$\frac{1}{U_d} = \frac{1}{\left(11300\, W/(\mathrm{m^2K})\right)} + \frac{0.016 m\left(\frac{0.016}{0.0134}\right)}{2\left(26\, W/(\mathrm{m^2K})\right)} + \left(\frac{0.016}{0.0134}\right)$$

$$\frac{1}{\left(5600\, W/(\mathrm{m^2K})\right)} + \left(\frac{0.016}{0.0134}\right)\left(\frac{1}{10000 W/(\mathrm{m^2K})}\right)$$

$$U_d = 1268.2 \text{ W/(m}^2\text{ K)}$$

(b) The number of transfer units is

$$NTU = \frac{UAc}{C_{\min}} = \frac{U\pi D_o L}{\dot{m}_w c_{pw}} = \frac{U\pi D_o L}{\dot{v}\rho c_{pw}}$$

$$NTU = \frac{\left(1268.2\, W/(\mathrm{m^2K})\right)\pi(0.016\, m)(3\, m)}{\left(3.2 * 10^{-4}\ \mathrm{m^3/s}\right)\left(995.7\, kg/\mathrm{m^3}\right)\left(4176\, J/(\mathrm{kg\ K})\right)} = 0.144$$

Since the heat capacity rate of the steam is essentially infinite, $C_{\min}/C_{\max} = 0$.
From Figure 10.18 or 10.19 (both are the same for $C_{\min}/C_{\max} = 0$): $e \approx 0.15$
From Equation (10.13b)

$$\mathsf{E} = \frac{C_c}{C_{\min}}\frac{T_{w,\mathrm{out}} - T_{w,\mathrm{in}}}{T_s - T_{w,\mathrm{in}}} = \frac{T_{w,\mathrm{out}} - T_{w,\mathrm{in}}}{T_s - T_{w,\mathrm{in}}}$$

$$T_{w,\mathrm{out}} = T_{w,\mathrm{in}} + \mathsf{E}\,(T_s - T_{w,\mathrm{in}}) = 27°\mathrm{C} + 0.15\,(50°\mathrm{C} - 27°\mathrm{C}) = 30.5°\mathrm{C}$$

PROBLEM 10.29

It is proposed to preheat the water for a boiler with flue gases from the boiler stack. The flue gases are available at the rate of 0.25 kg/s at 150°C, with a specific heat of 1000 J/(kg K). The water entering the exchanger at 15°C at the rate of 0.05 kg/s is to be heated at 90°C. The heat exchanger is to be of the reversed current type with one shell pass and 4 tube passes. The water flows inside the tubes which are made of copper (2.5 cm-*ID*, 3.0 cm-*OD*). The heat transfer coefficient at the gas side is 115 W/(m^2 K), while the heat transfer coefficient on the water side is 1150 W/(m^2 K). A scale on the water side offers an additional thermal resistance of 0.002 (m^2 K)/W. (a) Determine the overall heat transfer coefficient based on the outer tube diameter. (b) Determine the appropriate mean temperature difference for the heat exchanger. (c) Estimate the required tube length. (d) What would be the outlet temperature and the effectiveness if the water flow rate is doubled, giving a heat transfer coefficient of 1820 W/(m^2 K)?

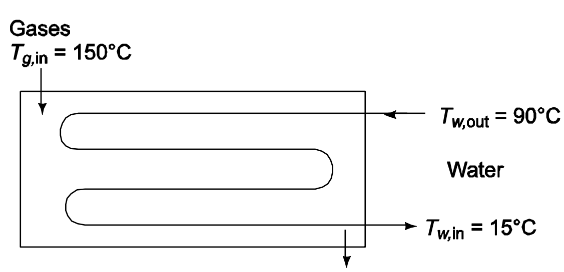
GIVEN

- Reverse current heat exchanger - 1 shell pass , 4 tube passes
- Water in tubes, flue gases in shell
- Copper tubes
  - Inside diameter (D_i) = 2.5 cm = 0.025 m
  - Outside diameter (D_o) = 3.0 cm = 0.03 m
- Specific heat of gases (c_{pg}) = 1000 J/(kg K)
- Gas inlet temperature
  - $T_{g,\text{in}}$ = 150°C
- Water temperatures
  - $T_{w,\text{in}}$ = 15°C
  - $T_{w,\text{out}}$ = 90°C
- Gas flow rate $(\dot{m}_g)$ = 0.25 kg/s
- Water flow rate $(\dot{m}_w)$ = 0.05 kg/s
- Tubes are copper
- Gas side heat transfer coefficient $(\bar{h}_c)$ = 115 W/(m^2 K)
- Water side heat transfer coefficient $(\bar{h}_i)$ = 1150 W/(m^2 K)
- Scaling resistance on the water side (R_i) = 0.002 (m^2 K)/W

FIND

(a) The overall heat transfer coefficient (U_o) based on the outside tube diameter
(b) The appropriate mean temperature difference (ΔT_{mean})
(c) The required tube length (L)
(d) The outlet temperature and effectiveness if the water flow rate were doubled, making $\bar{h}_i$ = 1820 W/(m^2 K)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at the average temperature of 52.5°C (c_{pw}) = 4179 J/kg K

From Appendix 2, Table 12, the thermal conductivity of copper (k) = 392 W/(m K) at 127°C

SOLUTION

(a) The overall heat transfer coefficient is given by Equation (10.6)

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + R_o + R_k + \frac{R_i\, A_o}{A_i} + \frac{A_o}{A_i\, \bar{h}_i}$$

Where: $R_o = 0$

$A_o = \pi\, D_o\, L$

$A_i = \pi\, D_i\, L$

$$R_k = \frac{A_o \ln \frac{D_o}{D_i}}{2\pi k L} = \frac{D_o \ln \frac{D_o}{D_i}}{2k}$$

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} + R_i\,\frac{D_o}{D_i} + \frac{D_o}{D_i\, \bar{h}_i}$$

$$\frac{1}{U_d} = \frac{1}{115\ \mathrm{W/(m^2K)}} + \frac{0.03\,\mathrm{m}\ln\left(\frac{3}{2.5}\right)}{2\ \ 392\ \mathrm{W/(m\ K)}} + 0.002\,(\mathrm{m^2\ K})/\mathrm{W}\ \ \frac{3}{2.5} + \frac{3\,\mathrm{cm}}{1150\ \mathrm{W/(m^2K)}\ \ 2.5\,\mathrm{cm}}$$

$$U_d = 82.3\ \mathrm{W/(m^2\ K)}$$

(b) The outlet temperature of the gases can be determined from an energy balance

$$\dot{m}_s\ c_{pg}\,(T_{g,\mathrm{in}} - T_{g,\mathrm{out}}) = \dot{m}_w\ c_{pw}\,(T_{w,\mathrm{out}} - T_{w,\mathrm{in}})$$

$$T_{g,\mathrm{out}} = T_{g,\mathrm{in}} - \frac{\dot{m}_w c_{pw}}{\dot{m}_g c_{pg}}\ (T_{w,\mathrm{out}} - T_{w,\mathrm{in}})$$

$$T_{g,\mathrm{out}} = 150°\mathrm{C} - \frac{0.05\ \mathrm{kg/s}\ \ 4179\ \mathrm{J/(kg\,K)}}{0.25\ \mathrm{kg/s}\ \ 100\ \mathrm{J/(kg\,K)}}\ (90°\mathrm{C} - 15°\mathrm{C}) = 87°\mathrm{C}$$

From Figure 10.13 for a simple counterflow heat exchanger

$$\Delta T_a = T_{g,\mathrm{in}} - T_{w,\mathrm{out}} = 150°\mathrm{C} - 90°\mathrm{C} = 60°\mathrm{C}$$

$$\Delta T_b = T_{g,\mathrm{out}} - T_{w,\mathrm{in}} = 87°\mathrm{C} - 15°\mathrm{C} = 72°\mathrm{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\frac{\Delta T_a}{\Delta T_b}} = \frac{60°\mathrm{C} - 72°\mathrm{C}}{\ln\left(\frac{60}{72}\right)} = 66°\mathrm{C}$$

This must be corrected using Figure 10.14

$$P = \frac{T_{w,\mathrm{out}} - T_{w,\mathrm{in}}}{T_{g,\mathrm{in}} - T_{w,\mathrm{in}}} = \frac{90-15}{150-15} = 0.56$$

$$Z = \frac{T_{g,\mathrm{in}} - T_{g,\mathrm{out}}}{T_{w,\mathrm{out}} - T_{w,\mathrm{in}}} = \frac{150-87}{90-15} = 0.84$$

From Figure 10.14, $F = 0.78$

$$\therefore\ \Delta T_{\mathrm{mean}} = F(LMTD) = 0.78\ (66°\mathrm{C}) = 51°\mathrm{C}$$

(c) The rate of heat transfer is

$$q = U A_o \Delta T_{\text{means}} = \dot{m}_w \; c_{pw} \,(T_{w,\text{out}} - T_{w,\text{in}})$$

$$L = \frac{\dot{m}_w C_{pw}}{\pi D_o U \,\Delta T_{\text{mean}}}(T_{w,\text{out}} - T_{w,\text{in}}) = \frac{0.05\,\text{kg/s} \quad 4179\,\text{J/(kg K)}}{(0.03\,\text{m}) \;\; 82.3\,\text{W/(m}^2\text{K)} \;\; (51°\text{C})} \;\; (90°\text{C} - 15°\text{C}) = 39.6 \text{ m}$$

Length of each tube pass = $L/4$ = 9.9 m

(d) For a doubled water flow rate, h_i = 1820 W/(m^2 K) similarly to part (a)

$$\frac{1}{U_d} = \frac{1}{115\,\text{W/(m}^2\text{ K)}} + \frac{0.03\,\text{m}\ln\left(\frac{3}{2.5}\right)}{2 \;\; 392\,\text{W/(m}^2\text{K)}} + (0.002\,(\text{m}^2\text{K})/\text{W})\left(\frac{3}{2.5}\right) + \frac{3\,\text{cm}}{1820\,\text{W/(m}^2\text{ K)} \;\; (2.5\,\text{cm})}$$

$$U_d = 85.0 \text{ W/(m}^2\text{ K)}$$

The heat capacity rates are

$$C_w = \dot{m}_w \; c_{pw} = (0.1 \text{ kg/s}) \;\; 4179\,\text{J/(kg K)} \quad = 418 \text{ W/K}$$

$$C_g = \dot{m}_g \; c_{pg} = (0.25 \text{ kg/s}) \;\; 1000\,\text{J/(kg K)} \quad = 250 \text{ W/K}$$

$$\frac{C_{\min}}{C_{\max}} = \frac{250}{418} = 0.60$$

The number of transfer units is

$$NTU = \frac{UA_o}{C_{\min}} = \frac{U\pi D_o L}{C_{\min}} = \frac{85.0\,\text{W/(m}^2\text{K)} \;\; \pi \;\; 0.03\,\text{m} \quad 39.6\,\text{m}}{250\,\text{W/(m}^2\text{K)}} = 1.27$$

From Figure 10.20 $e \approx 0.57$

The outlet temperature can be calculated from Equation (10.22b)

$$\text{E} = \frac{C_w}{C_{\min}} \frac{\left(T_{w,out} - T_{w,in}\right)}{\left(T_{g,in} - T_{w,in}\right)}$$

$$T_{w,\text{out}} = T_{w,\text{in}} + \text{E}\,\frac{C_{\min}}{C_w}\,(T_{g,\text{in}} - T_{w,\text{in}}) = 15°\text{C} + 0.57\left(\frac{250}{418}\right)(150°\text{C} - 15°\text{C}) = 61°\text{C}$$

PROBLEM 10.30

Hot water is to be heated from 10 to 30°C, at the rate of 300 kg/s by atmospheric pressure steam in a single-pass shell-and-tube heat exchanger consisting of 1-in. schedule 40 steel pipe. The surface coefficient on the steam side is estimated to be 11,350 W/(m^2 K). An available pump can deliver the desired quantity of water provided the pressure drop through the pipes does not exceed 15 psi. Calculate the number of tubes in parallel and the length of each tube necessary to operate the heat exchanger with the available pump.

GIVEN

Single-pass shell-and-tube heat exchanger
Water is heated by atmosphere steam
Water temperatures

- $T_{w,\text{in}}$ = 10°C
- $T_{w,\text{out}}$ = 30°C

Water flow rate $(\dot{m}_w)$ = 300 kg/s
Inner tube: 1 in. schedule 40 steel pipe
Maximum water pressure drop (Δ_p) = 15 psi = 103,422 N/m^2
Steam side heat transfer coefficient $(\bar{h}_o)$ = 11,350 W/(m^2 K)

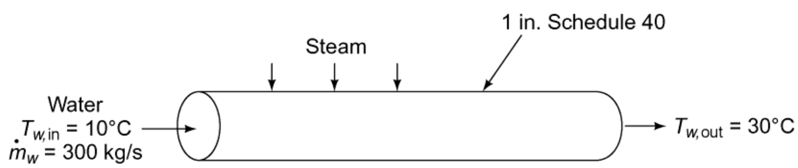
FIND

(a) The number of tubes in parallel (N)
(b) The length of each tube (L)

ASSUMPTIONS

The tube is smooth
The tube is 1% carbon steel
Uniform pipe surface temperature
Fully developed flow in pipe
Water flow is turbulent to insure good heat transfer

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 41, for 1 in. schedule 40 pipe

Inside diameter (D_i) = 1.049 in. = 0.0266 m
Outside diameter (D_o) = 1.315 in. = 0.0334 m

From Appendix 2, Table 13, the saturation temperature of steam at 1 atm = 100°C
From Appendix 2, Table 13, for water at the average temperature of 20°C

Absolute viscosity (μ) = 993 × 10^{-6} Ns/m^2
Density (ρ) = 998.2 kg/m^3
Specific heat (c_{pw}) = 4182 J/(kg K)
Thermal conductivity (k_w) = 0.597 W/(m K)
Prandtl number (Pr) = 7.0

From Appendix 2, Table 10, the thermal conductivity of 1% carbon steel (k_s) = 43 W/(m K).

SOLUTION

The Reynolds number for the water flow through the pipes is

$$Re_D = \frac{VD_i}{\nu} = \frac{4\dot{m}}{N\pi D_i \mu} = \frac{4\ \ 300\,\text{kg/s}}{N\pi\ \ 0.0266\,\text{m}\ \ \ 993\times10^{-6}\,(\text{Ns})/\text{m}^2\ \ \ (\text{kg m})/(\text{Ns}^2)} = (1.45\times10^{7})\ \frac{1}{N}$$

The friction factor for turbulent flow through smooth tubes for $10^5 < Re < 10^6$ is given by Equation (7.57)

$$f = 1.84\, Re^{-0.2} = 1.84\,(1.45\times10^{7}\, N^{-1})^{-0.2} = 0.068\, N^{\,0.2}$$

The pressure drop through the tube is given by Equation (7.13)

$$\Delta p = f\,\frac{L}{D_i}\,\frac{\rho V^2}{2g_c} = 1.84\left(\frac{4\dot{m}}{N\pi D_i\mu}\right)^{-0.2}\frac{L}{D_i}\,\frac{\text{p}}{2g_c}\left(\frac{4\dot{m}}{4\pi \text{D}_\text{i}^2\rho}\right) = 11.16\left(\frac{\dot{m}}{\pi}\right)^{1.8}\frac{\mu^{0.2}}{\rho D^{4.8}}\,LN^{-1.8}$$

$$\frac{L}{N^{1.8}} = 0.0896\,\Delta p\left(\frac{\pi}{\dot{m}}\right)^{1.8}\frac{\rho D^{4.8}}{\mu^{0.2}} = 0.0896\,(103{,}422\ \text{N/m}^2)\ \ (\text{kg m})/(\text{s}^2\text{N})$$

$$\left(\frac{\pi}{300\ \text{kg/s}}\right)^{1.8}\frac{993\,\text{kg/s}\ \ (0.0266\,\text{m})^{48}}{\left[993\times10^{-6}\ \ (\text{Ns})/\text{m}^2\ \ \ (\text{kg m})/(\text{s}^2\text{N})\ \right]}$$

$$\frac{L^{1.8}}{N} = 8.26\times10^{-4}\ \text{m} \rightarrow L = (2.75\times10^{-4})\, N^{1.8}$$

The heat capacity rate of the condensing steam is essentially infinite, therefore, $C_{\min}/C_{\max} = 0$. The effectiveness, from Equation (10.22b) is

$$\mathscr{E} = \frac{C_w}{C_{\min}}\,\frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_s - T_{w,\text{in}}} = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_s - T_{w,\text{in}}} = \frac{30°\text{C} - 10°\text{C}}{1000°\text{C} - 10°\text{C}} = 0.22$$

From Figure 10.18 or 10.19, $NTU \approx 0.25$

$$NTU = \frac{UA}{C_{\min}} = \frac{U_o A_o}{\dot{m}_w c_{pw}} = \frac{U_o N\pi D_o L}{\dot{m}_w c_{pw}}$$

The overall heat transfer coefficient (U_o) is given by Equation (10.3)

$$U_O = \frac{1}{\left(\frac{A_o}{A_i h_i}\right) + \left[A_o \ln\left(\frac{\frac{r_o}{r_i}}{2k_s L}\right)\right] + \left(\frac{1}{\bar{h}_o}\right)} = \frac{1}{\left(\frac{D_o}{D_i h_i}\right) + \left[D_o \ln\left(\frac{\frac{D_o}{D_i}}{2k_s L}\right)\right] + \left(\frac{1}{\bar{h}_o}\right)}$$

For fully developed turbulent flow with constant surface temperature, the Nusselt number in the pipe is given by Equation (7.61)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \text{ where } n = 0.4 \text{ for heating}$$

$$\bar{h}_i = \overline{Nu}_D\,\frac{k_w}{D_i} = 0.023\,\frac{k_w}{D_i}\left(\frac{4\dot{m}}{N\pi D_i}\right)^{0.8} Pr^{0.4}$$

Substituting these and $L = 2.75\times10^{-4}\, N^{1.8}$ m into the expression for NTU

$$NTU = \frac{N\,\pi\,D_o\;2.75\times10^{-4}\,\text{m}\;N^{1.8}}{\dot{m}_w c_{pw}\left[\dfrac{D_o}{D_i\left[\left(\dfrac{0.023\,k_w}{D_i}\right)\;1.45\times10^{7}\,N^{-1}\;{}^{0.8}\,(Pr)^{0.4}\right]}+\left(D_o\ln\dfrac{\frac{D_o}{D_i}}{2\,k_s L}\right)+\left(\dfrac{1}{\bar{h}_o}\right)\right]}$$

$$0.25 = \;300\,\text{kg/s}\;\;4182\,\text{J/(kg K)}$$

$$\frac{\pi(0.0334\,\text{m})\left(2.75\times10^{-4}\,\text{m}\right)N^{1.8}}{(300\,\text{kg/s})(4182\,\text{J/(kg k)})\left[\left(\dfrac{0.03340}{0.0266}\right)\dfrac{1}{(0.023)\dfrac{0.597\,\text{W/(mK)}}{0.0266\,\text{m}}\left(\dfrac{1.45\times10^{7}}{N}\right)^{0.8}7^{0.4}}+\dfrac{0.0334\ln\dfrac{0.0034}{0.0026}}{2\;\;43\,\text{W/(mK)}}+\dfrac{1}{11{,}350\,\text{W/(m}^2\text{K)}}\right]}$$

Canceling all units

$$0.25 = \frac{2.30\times10^{11}\,N^{2.8}}{2.084\times10^{-6}\,N^{0.8}+1.77\times10^{-4}}$$

By trial and error, $N = 220$

Therefore

$$L = 2.75\times10^{-4}\,(220)^{1.8} = 4.5\text{ m}$$

PROBLEM 10.31

Water flowing at a rate of 12.6 kg/s is to be cooled from 90 to 65°C by means of an equal flow rate of cold water entering at 40°C. The water velocity with the such that the overall coefficient of heat transfer *U* is 2300 W/(m² K). Calculate the heat-exchanger surface area (in square meters) needed for each of the following arrangements: (a) parallel flow, (b) counterflow, (c) a multi-pass heat exchanger with the hot water making one pass through a well-baffled shell and the cold water making two passes through the tubes, and (d) a crossflow heat exchanger with both sides unmixed.

GIVEN

- Warm water cooled by cold water in a heat exchanger
- Both flow rates ($\dot{m}_c = \dot{m}_w$) = 12.6 kg/s
- Water temperatures
  - $T_{h,\text{in}}$ = 90°C
  - $T_{h,\text{out}}$ = 65°C
  - $T_{c,\text{in}}$ = 40°C
- Overall heat transfer coefficient (U) = 2300 W/(m² K)

FIND

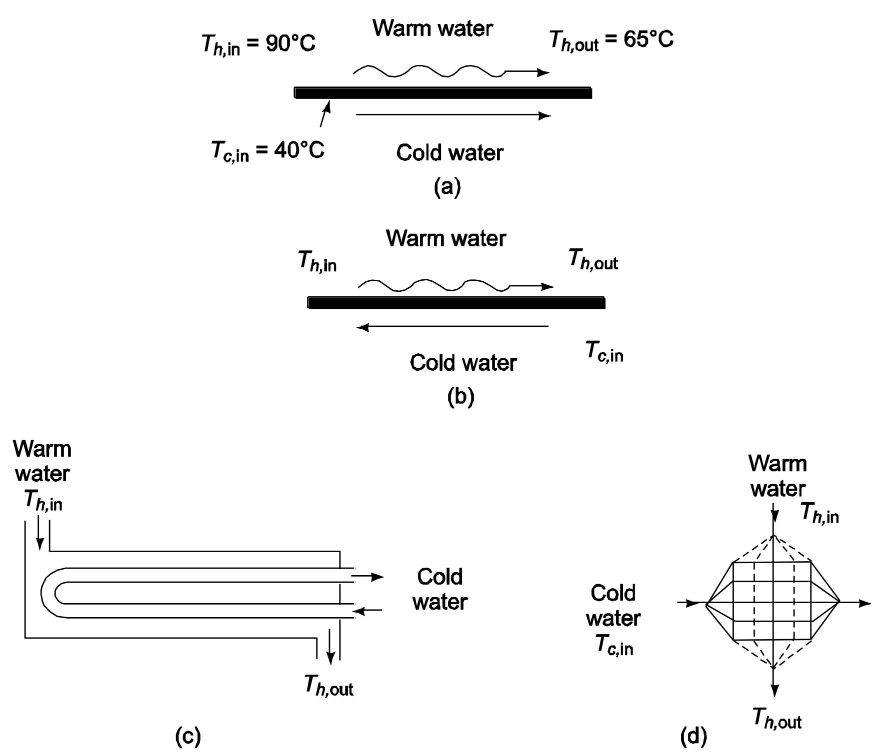
The transfer area (A) for

(a) Parallel flow	(c) Tube-and-shell; 1 hot shell pass, 2 cold tube passes
(b) Counterflow	(d) Crossflow–both unmixed

ASSUMPTIONS

- The specific heat is constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water in the temperature range of interest (c_p) = 4187 J/(kg K)

SOLUTION

Since $\dot{m}_h = \dot{m}_c$ and $c_{ph} = c_{pc} \Rightarrow \dfrac{C_{\min}}{C_{\max}} = 1.0$

Also $\Delta T_h = \Delta T_c \Rightarrow T_{c,\text{out}} = T_{c,\text{in}} + \Delta T_h = 40°\text{C} + 25°\text{C} = 65°\text{C}$

The effectiveness of the heat exchanger, from Equation (10.22a) is

$$\text{E} = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \frac{90°\text{C} - 65°\text{C}}{90°\text{C} - 40°\text{C}} = 0.50$$

(a) Figure 10.17 shows that infinite NTU would be required to reach e = 0.5 for $C_{\min}/C_{\max} = 1$ with a parallel flow configuration. Therefore, parallel flow is not practical. For $C_{\min}/C_{\max} = 1$, Equation (10.26) reduces to

$$\text{E} = \frac{1}{2}\left(1 - e^{-2NTU}\right) \quad \text{For } \mathscr{E} = 0.5\text{: } e^{-NTU} = 0 \; NTU \to \infty$$

However, for practical purposes, $e = 0.5$ at $NTU = 2.5$

$$\therefore A = NTU \frac{C_{\min}}{U} = 2.5 \frac{52{,}756 \text{ W/K}}{2300 \text{ W/(m}^2\text{K)}} = 57.3 \text{ m}^3$$

(b) From Figure (10.19), for $e = 0.5$ and $C_{\min}/C_{\max} = 1.0$: $NTU = 1.1$

$$NTU = \frac{UA}{C_{\min}} \text{ where } C_{\min} = \dot{m} \; 8 \; c_p = (12.6 \text{ kg/s}) \; 4187 \text{ J/(kg K)} = 52.756 \text{ W/K}$$

Solving for the area

$$A = NTU \frac{C_{\min}}{U} = 1.1 \frac{52{,}756 \text{ W/K}}{2300 \text{ W/(m}^2\text{K)}} = 25.2 \text{ m}^2$$

(c) From Figure (10.20), $NTU = 1.3$

$$A = 1.3 \frac{52{,}756 \text{ W/K}}{2300 \text{ W/(m}^2\text{K)}} = 29.8 \text{ m}^2$$

(d) From Figure (10.21), $NTU = 1.2$

$$A = 1.2 \frac{52{,}756 \text{ W/K}}{2300 \text{ W/(m}^2\text{K)}} = 27.5 \text{ m}^2$$

PROBLEM 10.32

Water flowing at a rate of 10 kg/s through 50 tubes double-pass shell and tube heat exchanger heats air that flows through the shell side. The length of the brass tubes is 6.7 m and they have an outside diameter of 2.6 cm and an inside diameter of 2.3 cm. The heat transfer coefficient of the water and air are 470 W/(m² K) and 210 W/(m² K), respectively. The air enters the shell at a temperature of 15°C and a flow rate of 16 kg/s. The temperature of the water as it enters the tubes is 75°C. Calculate (a) the heat exchanger effectiveness, (b) the heat transfer rate to the air, and (c) the outlet temperature of the air and water.

GIVEN

- Shell-and-tube heat exchanger - one shell, two tube passes
- Water in brass tubes, air in shell
- Water flow rate $(\dot{m}_w)$ = 10 kg/s
- Number of double passes (N) = 50
- Tube length (L) = 6.7 m
- Tube diameters
  - D_o = 2.6 cm = 0.026 m
  - D_i = 2.3 cm = 0.023 m
- Heat transfer coefficient
  - Water $(\bar{h}_i)$ = 470 W/(m² K)
  - Air $(\bar{h}_o)$ = 210 W/(m² K)
- Air inlet temperature ($T_{a,\text{in}}$) = 15°C
- Air flow rate $(\dot{m}_a)$ = 16 kg/s
- Water inlet temperature ($T_{w,\text{in}}$) = 75°C

FIND

(a) Effectiveness (e)

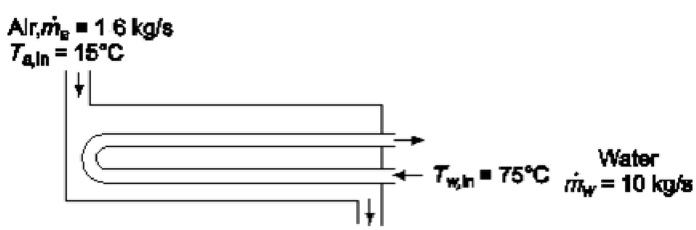
(b) The heat transfer rate (q)

(c) Outlet temperatures ($T_{a,\text{out}}$, $T_{w,\text{out}}$)

ASSUMPTIONS

- Tube length includes both passes

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water at 75°C (c_{pw}) = 4190 J/(kg K)

From Appendix 2, Table 28, the specific heat of air at 15°C (c_{pa}) = 1012 J/(kg K)

From Appendix 2, Table 10, the thermal conductivity of brass (k_b) = 111 W/(m K)

SOLUTION

The heat capacity rates are

$$C_w = \dot{m}_w\ c_{pw} = (10\ \text{kg/s})\ \ 4190\ \text{J/(kg K)}\ \ = 41{,}900\ \text{W/K}$$

$$C_a = \dot{m}_a\ c_{pa} = (16\ \text{kg/s})\ \ 1012\ \text{J/(kg K)}\ \ = 16192\ \text{W/K}$$

$$\frac{C_{\min}}{C_{\max}} = \frac{16192}{41{,}900} = 0.39$$

(a) The overall heat transfer coefficient is given by Equation (10.3)

$$U_o = \frac{1}{\left(\frac{A_o}{A_i \bar{h}_i}\right) + \left(A_o \ln\left(\frac{\frac{r_o}{r_i}}{2\pi k L}\right)\right) + \left(\frac{1}{\bar{h}_o}\right)} = \frac{1}{\left(\frac{D_o}{D_i \bar{h}_i}\right) + \left(D_o \ln\left(\frac{\frac{D_o}{D_i}}{2\pi k L}\right)\right) + \left(\frac{1}{\bar{h}_o}\right)}$$

$$U_o = \frac{1}{\frac{(0.026\,\text{m})}{(0.023\text{m})\ \ 470\ \text{W/(m}^2\text{K)}} + \frac{(0.026\text{m})\ \ \ln(0.026\text{m}/0.023\text{m})}{2\ \ 111\ \text{W/(m K)}} + \frac{1}{210\ \text{W/(m}^2\text{K)}}} = 139\ \text{W/(m}^2\ \text{K)}$$

The transfer area is $A = N(\pi D_o L) = 50[\pi(0.026\ \text{m})(6.7\ \text{m})] = 27.36\ \text{m}^2$
The Number of transfer units is

$$NTU = \frac{UA}{C_{\min}} = \frac{\left(139\ \text{W/(m}^2\text{K)}\right)(27.36\,\text{m}^2)}{\left(16192\ \text{W/(m}^2\text{K)}\right)} = 0.235$$

From Figure 10.20 for $C_{\min}/C_{\max} = 0.4$ and NTU = 0.235, e ≈ 0.20 = 20%
(b) The rate of heat transfer is given by Equation (10.23)

$$q = \mathscr{E}\, C_{\min}\ (T_{h,\text{in}} - T_{c,\text{in}}) = 0.2\ \left(16192\ \text{W / (m}^2\text{K)}\right)\ (75°\text{C} - 15°\text{C}) = 194{,}304\ \text{W}$$

(c) For the water

$$q = C_w\ (T_{w,\text{in}} - T_{w,\text{out}})$$

$$\therefore\quad T_{w,\text{out}} = T_{w,\text{in}} - \frac{q}{C_w} = 75°\text{C} - \frac{194{,}304\ \text{W}}{(41{,}900\ \text{(W/ K)})} = 70.3°\text{C}$$

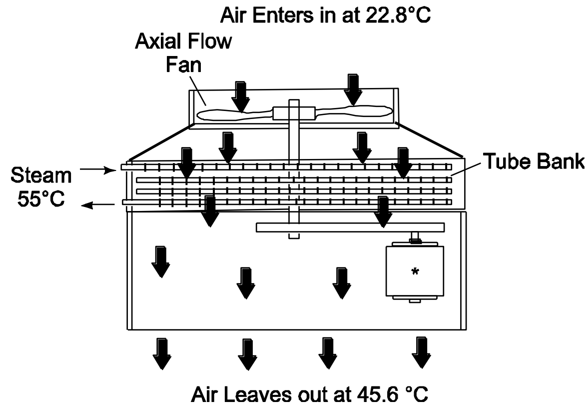
For the air

$$q = C_a\ (T_{a,\text{out}} - T_{a,\text{in}})$$

$$\therefore\quad T_{a,\text{out}} = T_{a,\text{in}} + \frac{q}{C_a} = 15°\text{C} + \frac{194{,}304\text{W}}{(16192\ \text{(W/ K)})} = 27°\text{C}$$

PROBLEM 10.33

An air-cooled low-pressure-steam condenser is shown in following figure.



The tube bank is four rows deep in the direction of air flow and there are total of 80 tubes. The tubes have *ID* = 2.2 cm and *OD* 2.5 cm and are 9-m-long with circular fins on the outside. The tube-plus-fin area is 16 times the bare tube area (i.e., the fin area is 15 times the bare tube area, neglect the tube surface covered by fins). The fin efficiency is 0.75. Air flows past the outside of the tubes. On a particular day, the air enters at 22.8°C and leaves at 45.6°C. The air flow rate is 3.4×10^5 kg/h.

The steam temperature is 55°C and has a condensing coefficient of 10^4 W/(m^2 K). The steam-side fouling coefficient is 10^4 W/(m^{2}K). The tube wall conductance per unit area is 10^5 W/(m^{2}K). The air-side fouling resistance is negligible. The air-side-film heat transfer coefficient is 285 W/(m^{2}K). (Note this value has been corrected for the number of transverse tube rows.)

(a) What is the log-mean temperature difference between the two streams?
(b) What is the rate of heat transfer?
(c) What is the rate of steam condensation?
(d) Estimate the rate of steam condensation if there were no fins.

GIVEN

- The condenser shown above
- Number of tubes (N) = 80 and number of rows (N_r) = 4 (in air flow direction)
- Tube diameters D_i = 2.2 cm = 0.022 m
  D_o = 2.5 cm = 0.025 m
- Tube length (L) = 9 m
- Air temperature $T_{a,\text{in}}$ = 22.8°C
  $T_{a,\text{out}}$ = 45.6°C
- Air flow rate $\dot{m}_a$ = 3.4×10^5 kg/h = 94.4 kg/s
- Steam temperature = 55°C (constant)
- Fin area = 15 (tube area)
- Fin efficiency (η_f) = 0.75
- Steam side Transfer coefficient $\bar{h}_i$ = 10^4 W/(m^2 K)
  Fouling coefficient ($1/R_i$) = 10^4 W/(m^2 K)
- Tube wall conductance per unit area ($1/R_k$) = 10^5 W/(m^2 K)
- Air side: Transfer coefficient $\bar{h}_o$ = 285 W/(m^2 K)
- Fouling resistance on the air side is negligible

FIND

(a) The log-mean temperature difference ($LMTD$)
(b) The rate of heat transfer (q)
(c) The rate of steam condensation $\dot{m}_c$

(d) Estimate the rate of steam condensation if there were no fins $\dot{m}_{c2}$

ASSUMPTIONS

- Air side transfer coefficient is the same with or without fins
- Tube surface covered by the fins is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average temperature of 34.2°C, the specific heat (c_{pa}) = 1013 J/(kg K)

From Appendix 2, Table 13, for steam at a saturation temperature of 55°C, the heat of vaporization (h_{fg}) = 2600 (kJ/kg)

SOLUTION

(a) From Figure 10.10 $\Delta T_a = T_s - T_{a,\text{in}} = 55°\text{C} - 22.8°\text{C} = 32.2°\text{C}$

$\Delta T_b = T_s - T_{a,\text{out}} = 55°\text{C} - 45.6°\text{C} = 9.4°\text{C}$

$$LMTD = (\Delta T_a - \Delta T_b)\Big/\ln\left(\frac{\Delta T_a}{\Delta T_b}\right) = (32.2°\text{C} - 9.4°\text{C})\Big/\ln\left(\frac{32.2}{9.4}\right) = 18.5°\text{C}$$

(b) The overall heat transfer coefficient is given by Equation (10.6) for the base tube area

$$\frac{1}{U_{d,\text{bare}}} = \frac{1}{\bar{h}_o} + R_o + R_k + \frac{R_i A_o}{A_i} + \frac{A_o}{A_i \bar{h}_i} = \frac{1}{\bar{h}_o} + 0 + \frac{1}{\left(\frac{1}{R_k}\right)} + \frac{D_o}{D_i}\left(\frac{1}{\left(\frac{1}{R_i}\right)} + \frac{1}{\bar{h}_i}\right)$$

$$\frac{1}{U_{d,\text{bare}}} = \frac{1}{285\ \text{W/(m}^2\text{K)}} + \frac{1}{10^5\ \text{W/(m}^2\text{K)}} + \left(\frac{2.5}{2.2}\right)\left(\frac{1}{10^4\left(\text{W/(m}^2\text{ K)}\right)} + \frac{1}{10^4\left(\text{W/(m}^2\text{ K)}\right)}\right)$$

$U_{d,\text{bare}} = 267\ \text{W/(m}^2\text{ K)}$

The rate of heat transfer for the bare tubes above is

$$q_b = U A\,(LMTD) = U_d(N\,\pi\, D_o L)(LMTD)$$

$$q_b = 267\ \text{W/(m}^2\text{K)}\ \ 80\ \pi\,(0.025\text{ m})\,(9\text{ m})\,(18.5°\text{C}) = 2.79 \times 10^5\ \text{W}$$

If the entire fin area was at the same temperature as the exterior of the tube wall, the rate of heat transfer from the fin would be

$$Q'_f = U A_{\text{fin}}\,(LMTD) = U_d\,(15\,A_t)\,LMTD = 15\,q_b$$

The actual rate of heat transfer from the fins is

$$q_f = \eta_f\, q'_f = 15\ \eta_f\, q_b$$

The total rate of heat transfer is

$$q = q_b + q_f = q_b\,(1 + 15\eta_f) = 2.79 \times 10^5\,[1 + 15(0.75)] = 3.42 \times 10^6\ \text{W}$$

(c) The rate of condensation is

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{3.42\times 10^6\ \text{W}\,(\text{J/(Ws)})}{(2600\ \text{kJ/kg})(1000\ \text{J/kJ})} = 1.32\ \text{kg/s}$$

(d) If there were no fins, the rate of heat transfer would be that from the bare tube alone. Therefore

$$\dot{m}_{c2} = \frac{q_b}{h_{fg}} = \frac{2.79\times 10^5\text{W}\ \ \text{J/(Ws)}}{2600\ \text{kJ/kg}\ \ \ 1000\ \text{J/kJ}} = 0.11\ \text{kg/s}$$

COMMENTS

The rate of condensate flow without the fins is only 8% of that with fins.

PROBLEM 10.34

Design (i.e., determine the overall area and a suitable arrangement of shell and tube passes) a tubular-feed water heater capable of heating 2,300 kg/h of water from 21^{0}C to 90°C. The following specification are given (a) saturated steam at 920 kPa absolute pressure is condensing on the outer tube surface, (b) heat transfer coefficient on steam side is 6800 W/(m^2 K), (c) tubes are of copper, 2.5 cm, 2.3-cm-*ID*, 24 m long, and (d) water velocity is 0.8 m/s.

GIVEN

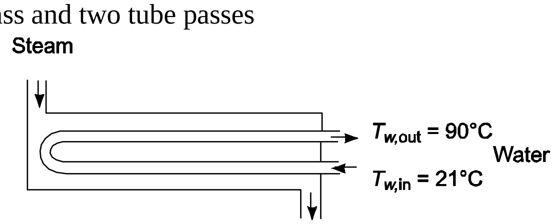
- A tubular-feed water heater, condensing saturated steam on outside
- Water flow rate $\dot{m}_w$ = 2300 kg/h = 0.639 kg/s
- Water temperatures $T_{w,\text{in}}$ = 21°C
  $T_{w,\text{out}}$ = 90°C
- Steam temperature = 920 kPa absolute
- Heat transfer coefficient on steam side $\bar{h}_o$ = 6800 W/(m^2 K)
- Tubes are copper D_i = 2.3 cm = 0.023 m
  D_o = 2.5 cm = 0.025 m
- Water velocity (V) = 0.8 m/s

FIND

- The transfer area (A) and a suitable arrangement of shell and tube passes

SKETCH

Assuming a single shell pass and two tube passes



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, the thermal conductivity of copper at 127°C (k) = 392 W/(m K)
From Appendix 2, Table 13, the saturation temperature of steam at 920 kPa (T_s) = 176°C
From Appendix 2, Table 13, for water at the average temperature of 56°C

Density (ρ) = 984.9 kg/m^3
Specific area (c_{pw}) = 4181 J/(kg K)
Thermal conductivity (k_w) = 0.653 W/(m K)
Kinematic viscosity (ν) = 0.510 × 10^6 m^2/s
Prandtl number (Pr) = 3.23

SOLUTION

The Reynolds number for the water flow is

$$Re_D = \frac{VD_i}{\nu} = \frac{(0.8\,\text{m/s})(0.023\,\text{m})}{(0.510\times10^{-6}\ \text{m}^2/\text{s})} = 3.61 \times 10^4$$

For turbulent flow in a tube, the Nusselt number is given by Equation (7.61)

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\ (6.31 \times 10^6)^{0.8}\ (3.23)^{0.4} = 163$$

$$\bar{h}_i = \overline{Nu}_D \frac{k_w}{D} = 163 \frac{0.653\,\text{W/(m K)}}{0.023\,\text{m}} = 4617\,\text{W/(m}^2\text{K)}$$

The overall heat transfer coefficient is given by Equation (10.3)

$$U_o = 1 \Bigg/ \left[\left(\frac{A_o}{A_i \bar{h}_i} \right) + \left(A_o \ln\left(\frac{\frac{r_o}{r_i}}{2\pi k L} \right) \right) + \left(\frac{1}{\bar{h}_o} \right) \right] = 1 \Bigg/ \left[\left(\frac{D_o}{D_i \bar{h}_i} \right) + \left(D_o \ln\left(\frac{\frac{D_o}{D_i}}{2k} \right) \right) + \left(\frac{1}{\bar{h}_o} \right) \right]$$

$$U_o = 1 \Bigg/ \left[\frac{(0.025\,\text{m})}{(0.023\,\text{m})\left(4617\,\text{W/(m}^2\,\text{K)}\right)} + \frac{(0.025\,\text{m})\left(\ln \frac{0.025}{0.023}\right)}{2\left(392\,\text{W/(m K)}\right)} + \frac{1}{\left(6800\,\text{W/(m}^2\,\text{K)}\right)} \right] = 2596\,\text{W/(m}^2\text{K)}$$

From Figure 10.9 $\quad \Delta T_a = T_s - T_{w,\text{in}} = 176°\text{C} - 21°\text{C} = 155°\text{C}$

$\Delta T_b = T_s - T_{w,\text{out}} = 176°\text{C} - 90°\text{C} = 86°\text{C}$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{155°\text{C} - 86°\text{C}}{\ln \frac{155}{86}} = 117°\text{C}$$

The *LMTD* must be corrected using Figure (10.13)

$$P = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_{s,\text{in}} - T_{w,\text{in}}} = \frac{90 - 21}{176 - 21} = 0.445$$

$$Z = \frac{T_{s,\text{in}} - T_{s,\text{out}}}{T_{w,\text{out}} - T_{w,\text{in}}} = 0$$

From Figure (10.13), $F = 1.0$. Due to the constant temperature of the condensing steam, this arrangement is as effective as a pure counterflow heat exchanger.

The rate of heat transfer is given by

$$q = U_o A_o (LMTD) = \dot{m}_w c_{pw} (T_{w,\text{in}} - T_{w,\text{out}})$$

Solving for the outer tube area required

$$A_o = \frac{\dot{m}_w c_{pw} \; T_{w,\text{in}} - T_{w,\text{out}}}{U_o \; LMTD} = \frac{0.639\,\text{kg/s} \;\; 4181\,\text{J/(kg K)} \;\; 90°\text{C} - 21°\text{C}}{2596\,\text{W/(m}^2\,\text{K)} \;\; (177°\text{C}) \;\; \text{J/(Ws)}} = 0.61\text{ m}^2$$

The number of tubes (N) required for the water to have the given velocity and flow rate is given by

$$\dot{m}_w = V \rho A_{\text{flow}} = V \rho N \frac{\pi}{4} D_i^2$$

$$N = \frac{4\dot{m}_w}{V \rho \pi D_i^2} = \frac{4 \;\; 0.639\,\text{kg/s}}{(0.8\,\text{m/s}) \;\; 984.9\,\text{kg/m}^3 \;\; \pi(0.023\,\text{m})^2} = 1.95 \approx 2$$

If there are two tubes each making two passes, the length of each pass (L_p) is determined from

A_o = (2 tubers) (2 passes) $L_p \, \pi \, D_o$

$$L_p = \frac{A_o}{4\pi D_o} = \frac{0.61\,\text{m}^2}{4\pi(0.025\,\text{m})} = 1.94\text{ m}$$

The tube length is 2 L_p = 3.88 m

The tube length is 2 L_p = 3.88 m

Heat exchanger specifications:

- – Shell and tube design with two tubes
- – One shell pass, two tube passes
- – Length of each tube pass = 1.87 m
- – Length of each tube = 3.88 m

PROBLEM 10.35

Two engineers are having an argument about the efficiency of a tube-side multipass heat exchanger compared to a similar exchanger with a single tube-side pass. Smith claims that for a given number of tubes and rate of heat transfer, more area is required in a two-pass exchanger than in a one-pass, because the effective temperature difference is less. Jones, on the other hand, claims that because the tube-side velocity and hence coefficient is higher, less area is required in a two-pass exchanger.

With the conditions given below, which engineer is correct? Which case would you recommend, or what changes in the exchanger would you recommend?

Exchanger specifications
– 200 tube passes total
– 1 in.-*O.D* copper tubes, 16 B.W.G.
Tube-side fluid
Water entering at 16°C, leaving at 28°C, with a rate of 225,000 kg/h.
Shell-side fluid
Mobiltherm 600, entering at 50°C, leaving at 33°C.
Shell side coefficient = 1700 W/(m² K)

GIVEN

- Tube and shell heat exchanger - water in tubes, Mobiltherm 600 in shell
- Number of tube passes (N_p) = 200
- Tubes are 1 in copper 16 B.W.G.
- Water flow rate $\dot{m}_w$ = 225,000 kg/h = 62.5 kg/s
- Water temperatures
  - $T_{w,\text{in}}$ = 16°C
  - $T_{w,\text{out}}$ = 28°
- Mobiltherm temperatures
  - $T_{m,\text{in}}$ = 50°C
  - $T_{m,\text{out}}$ = 33°C
- Shell side heat transfer coefficient $\bar{h}_o$ = 1700 W/(m² K)

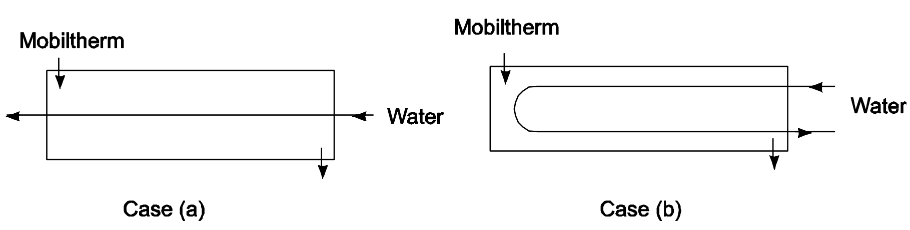
FIND

- Which required less transfer area: (a) single tube pass (b) Two tube passes?

ASSUMPTIONS

- Thermal resistance of copper tube wall is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 42, for 1 in 16 B.W.G. tubes, the diameters are

D_i = 0.870 in. = 0.0221 m

D_o = 1.0 in. = 0.0254 m

From Appendix 2, Table 13, for water at the average temperature of 22°C

Thermal conductivity (k) = 0.601 W/(m K)
Kinematic viscosity (ν) = 0.957 × 10^{-6} m^2/s
Prandtl number (Pr) = 6.6
Density (ρ) = 998 kg/m^3
Specific heat (c_{pw}) = 4180 J/(kg K)

From Appendix 2, Table 23, the specific heat of Mobiltherm 600 at its average temperature of 42°C (c_{pm}) = 1654 J/(kg K)

SOLUTION

For case (a) number of flow passages (N) = Total passes/(passes per tube) = 200/1 = 200.
For case (b) N = 200/2 = 100.
The water velocity (V) is determined by

$$V = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho N \ \frac{\pi}{4} \ D_i^2} = \frac{4\dot{m}}{N \pi D_i^2}$$

Case (a) $\quad V_a = \dfrac{4 \ \ 62.5\,\text{kg/s}}{200 \ \ 998\,\text{kg/m}^3 \ \ \pi(0.0221\,\text{m})^2} = 0.816$ m/s

Case (b) $\quad V_b = 2\,V_a = 2 \ \ 0.816$ m/s $= 1.63$ m/s

The Reynolds number is

$$Re_D = \frac{VD_i}{\nu}$$

Case (a)

$$Re_{Da} = \frac{0.0816\,\text{m/s} \quad 0.0221\,\text{m}}{0.957 \times 10^{-6}\,\text{m}^2/\text{s}} = 18{,}851$$

Case (b)

$$Re_{Db} = 2\,Re_{Da} = 37{,}701$$

The Nusselt number for turbulent flow in a tube is given by Equation (7.61)

$$Nu_D = 0.023\,Re_D^{0.8}\,Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

Case (a)

$$\overline{Nu}_{Da} = 0.023\,(18{,}851)^{0.8}\,(6.6)^{0.4} = 129$$

$$\bar{h}_{ia} = \overline{Nu}_{Da}\,\frac{k}{D_i} = 129\,\frac{0.601\,\text{W/(m K)}}{0.0221\,\text{m}} = 3502\ \text{W/(m}^2\ \text{K)}$$

Case (b)

$$\overline{Nu}_{Db} = 0.023\,(37{,}701 \times 10^6)^{0.8}\,(6.6)^{0.4} = 224$$

$$\bar{h}_{ib} = 224\,\frac{0.601\,\text{W/(m K)}}{0.0221\,\text{m}} = 6108\,\text{W/(m}^2\text{K)}$$

The overall heat transfer coefficient, neglecting the tube wall resistance is

$$\frac{1}{U_o} = \frac{D_o}{D_i \bar{h}_i} + \frac{1}{\bar{h}_o}$$

Case (a)

$$\frac{1}{U_o} = \left(\frac{0.0254}{0.0221}\right)\frac{1}{3502\ \mathrm{W/(m^2K)}} + \frac{1}{1700\ \mathrm{W/(m^2K)}} \Rightarrow U_o = 1091\ \mathrm{W/(m^2K)}$$

Case (b)

$$\frac{1}{U_o} = \left(\frac{0.0254}{0.0221}\right)\frac{1}{6108\ \mathrm{W/(m^2K)}} + \frac{1}{1700\ \mathrm{W/(m^2\ K)}} \Rightarrow U_o = 1288\ \mathrm{W/(m^2K)}$$

The Log-mean temperature difference for counterflow, from Figure 10.10 is

$$\Delta T_a = T_{m,\mathrm{in}} - T_{w,\mathrm{out}} = 50°\mathrm{C} - 28°\mathrm{C} = 22°\mathrm{C}$$

$$\Delta T_b = T_{m,\mathrm{out}} - T_{w,\mathrm{in}} = 33°\mathrm{C} - 16°\mathrm{C} = 17°\mathrm{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{22°\mathrm{C} - 17°\mathrm{C}}{\ln \frac{22}{17}} = 19.4°\mathrm{C}$$

For case (a): $\Delta T_{\mathrm{mean}} = LMTD = 19.4°\mathrm{C}$

For case (b): The *LMTD* must be corrected using Figure 10.14

$$P = \frac{T_{w,\mathrm{out}} - T_{w,\mathrm{in}}}{T_{m,\mathrm{in}} - T_{w,\mathrm{in}}} = \frac{28-16}{50-16} = 0.35$$

$$Z = \frac{T_{m,\mathrm{in}} - T_{m,\mathrm{out}}}{T_{w,\mathrm{out}} - T_{w,\mathrm{in}}} = \frac{50-33}{28-16} = 1.41$$

From Figure 10.14, $F = 0.91$

$$\therefore \qquad \Delta T_{\mathrm{mean}} = F(LMTD) = 0.91\ (19.4°\mathrm{C}) = 17.7°\mathrm{C}$$

The rate of heat transfer is

$$q = U_o A_o \Delta T_{\mathrm{mean}} = \dot{m}_w c_{pw}(T_{w,\mathrm{out}} - T_{w,\mathrm{in}})$$

$$\therefore \qquad A_o = \frac{\dot{m}_w c_{pw}\ \ T_{w,\mathrm{out}} - T_{w,\mathrm{in}}}{U_o \Delta T_{\mathrm{mean}}}$$

Case (a)

$$A_o = \frac{62.5\ \mathrm{kg/s}\ \ \ 4180\ \mathrm{J/(kg\,K)}\ \ \ 28°\mathrm{C} - 16°\mathrm{C}}{1091\ \mathrm{W/(m^2K)}\ \ (19.4°\mathrm{C})\ \ \mathrm{J/(W\,s)}} = 148\ \mathrm{m^2}$$

Case (b)

$$A_o = \frac{62.5\ \mathrm{kg/s}\ \ \ 4180\ \mathrm{J/(kg\,K)}\ \ \ 28°\mathrm{C} - 16°\mathrm{C}}{1288\ \mathrm{W/(m^2K)}\ \ (17.7°\mathrm{C})\ \ \mathrm{J/(W\,s)}} = 138\ \mathrm{m^2}$$

COMMENTS

For these operating conditions, the double-pass heat exchanger requires about 8% less area because although the mean temperature difference for the double pass is 9% less than that for the single pass, the overall heat transfer coefficient is 18% greater.

PROBLEM 10.36

A horizontal shell-and-tube heat exchanger is used to condense organic vapors. The organic vapors condense on the outside of the tubes, while water is used as the cooling medium on the inside of the tubes. The condenser tubes are 1.9-cm-*O.D.*, 1.6-cm-*ID* copper tubes, 2.4 m in length. There are a total of 768 tubes.

The water makes four passes through the exchanger.

Test data obtained when the unit was first placed into service are as follows

Water rate = 3700 l/min
Inlet water temperature = 29°C
Outlet water temperature = 49°C
Organic-vapor condensation temperature = 118°C

After 3 months of operation, another test, made under the same conditions as the first (i.e., same water rate and inlet temperature and same condensation temperature) showed that the exit water temperature was 46°C.

(a) What is the tube-side-fluid (water) velocity?
(b) What is the effectiveness, *e-NTU*, of the exchanger at the time of the first and second test.
(c) Assuming no changes in either the inside transfer coefficient on the condensing coefficient, negligible shell-side fouling, and no fouling at the time of the first test, estimate the tube-side fouling coefficient at the time of the second test.

GIVEN

- A shell-and-tube exchanger, organic vapors condensing in shell, water in copper tubes
- Tube diameters
  - D_o = 1.9 cm = 0.019 m
  - D_i = 1.6 cm = 0.016 m
- Tube length (L) = 2.4 m
- Number of tubes (N) = 768
- Number of tube passes (N_p) = 4
- Water flow rate $\dot{v}_w$ = 3700 1/min = 3.7 m^3/min
- Water temperatures
  - $T_{w,\text{in}}$ = 29°C
  - $T_{w,\text{out}}$ = 49°C
- Organic vapor condensation temperature (T_c) = 118°C
- After 3 months: $T_{w,\text{out}}$ = 46°C

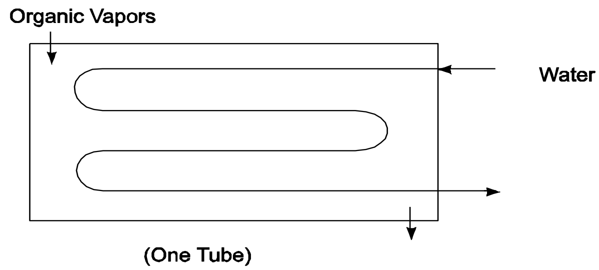
FIND

(a) Water velocity (V_w)
(b) The effectiveness (e) at the time of both tests
(c) Fouling coefficient ($1/R_i$) at the time of the second test

ASSUMPTIONS

- No fouling at the time of the first test
- No change in the inside and outside heat transfer coefficients
- Negligible shell-side fouling
- Length given is the length of one tube - all four passes

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 40°C

Density (ρ) = 992 kg/m^3

Specific heat (c_{pw}) = 4175 J/(kg K)

Thermal conductivity (k) = 0.633 W/(m K)

Kinematic viscosity (ν) = 0.658 × 10^{-6} m^2/s

Prandtl number (Pr) = 4.3

From Appendix 2, Table 12, the thermal conductivity of copper (k_c) = 392 W/(m K) at 127°C.

SOLUTION

(a) The water velocity is

$$V_w = \frac{\dot{v}_w}{A_{\text{flow}}} = \frac{\dot{v}_w}{N\frac{\pi}{4}D_i^2} = \frac{4\ \ 3.7\ \text{m}^3/\text{min}}{(768)\,\pi\,(0.016\,\text{m})^2\,(60\,\text{s/min})} = 0.40\ \text{m/s}$$

(b) The heat capacity rate of the condensing vapor is essentially infinite. The heat capacity rate of the water is

$$C_w = \dot{m}_w c_{pw} = \dot{v}_w \rho\, c_{pw} = \frac{(3.7\ \text{m}^3/\text{min})}{(60\,\text{s}/\text{min})}\ \ 992\ \text{kg/m}^3\ \ \ 4175\ \text{J/(kgK)}\ \ = 255{,}450\ \text{W/K}$$

From Equation (10.13b) with $C_c = C_{\min}$

$$\mathsf{E} = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_c - T_{w,\text{in}}}$$

No scaling $\qquad \mathsf{E} = \dfrac{49-29}{118-29} = 0.22 = 22\%$

With scaling $\qquad \mathsf{E} = \dfrac{46-29}{118-29} = 0.19 = 19\%$

(c) For $C_{\min}/C_{\max} = 0$, the effectiveness of parallel and counterflow exchangers is the same and Equation (10.26) reduces to

$$\mathsf{E} = 1 - e^{-NTU} \Rightarrow NTU = \frac{U_o A_o}{C_{\min}} = -\ln(1-\mathsf{E})$$

Solving for the overall heat transfer coefficient

$$U_o = -\frac{C_{\min}}{A_o}\ln(1-\mathsf{E}) = -\frac{C_{\min}}{N\pi D_o L}\ln(1-\mathsf{E})$$

No scaling

$$U_o = -\frac{(255,450 \text{ W/K})}{(768)\pi \;\; 0.019 \text{m} \quad 2.4 \text{m}} \ln(1 - 0.22) = 577 \text{ W/(m}^2\text{K)}$$

Similarly for scaling U_o = 489 W/(m^2 K)

From Equation 10.5

$$R_D = \frac{1}{U_d} - \frac{1}{U} = \frac{1}{489 \text{ W/(m}^2\text{K)}} - \frac{1}{577 \text{ W/(m}^2\text{K)}} = 0.000312 \text{ W/(m}^2\text{K)}$$

$$\frac{1}{R_D} = 3206 \;\; \text{W/(m}^2\text{K)}$$

PROBLEM 10.37

A shell-and-tube heat exchanger is to be used to cool 25.2 kg/s of water from 38°C to 32°C. The exchanger has one shell-side pass and two tube side passes. The hot water flows through the tubes and the cooling water flows through the shell. The cooling water enters at 24°C and leaves at 32°C. The shell-side (outside) heat transfer coefficient is estimated to be 5678 $\mathbf{W/(m^2\,K)}$. Design specifications require that the pressure drop through the tubes be as close to 13.8 kPa as possible and that the tubes be 18 BWG copper tubing (1.24 mm wall thickness), and each pass is 4.9 m long. Assume that the pressure losses at the inlet and outlet are equal to one and one half of a velocity heat ($\rho V^2/g_c$), respectively. For these specifications, what tube diameter and how many tubes are needed?

GIVEN

A water-to-water shell-and-tube exchanger, hot water in tubes, cooling water in shell
One shell and two tube passes
Hot water flow rate $\dot{m}_h$ = 25.2 kg/s
Water temperatures

- Hot: $T_{h,\text{in}}$ = 38°C $T_{h,\text{out}}$ = 32°C
- Cold: $T_{c,\text{in}}$ = 24°C $T_{c,\text{out}}$ = 32°C

Shell-side transfer coefficient $\bar{h}_o$ = 5678 W/(m^2 K)
Pressure drop (Δp) = 13.8 kPa
Tube wall thickness (t) = 1.24 mm = 0.00124 m
Tube length per pass (L_p) = 4.9 m

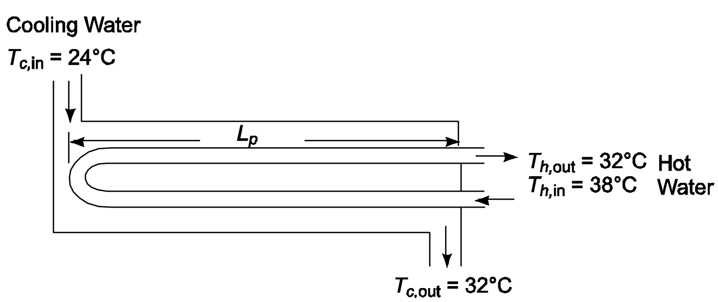
FIND

The tube diameter (D_o) and number of tubes (N)

ASSUMPTIONS

Pressure losses at inlet and outlet (Δp_{ii}) = 1.5 ($\rho\, V^2/g_c$)
Variation of thermal properties with temperature is negligible
Fouling resistance is negligible
Thermal resistance of the tube walls is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 30°C

Density (ρ) = 995.7 kg/m^3

Specific heat (c_p) = 4176 J/(kg K)

Thermal conductivity (k) = 0.615 W/(m K)

Kinematic viscosity (ν) = 0.805 × 10^{-6} m^2/s

Prandtl number (Pr) = 5.4

SOLUTION

From Figure 10.10 $\quad \Delta T_a = T_{h,\text{in}} - T_{c,\text{out}} = 38°\text{C} - 32°\text{C} = 6°\text{C}$

$$\Delta T_b = T_{h,\text{out}} - T_{c,\text{in}} = 32°\text{C} - 24°\text{C} = 8°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)} = \frac{6°\text{C} - 8°\text{C}}{\ln\left(\dfrac{6}{8}\right)} = 7°\text{C}$$

This must be corrected using Figure 10.14

$$P = \frac{T_{h,\text{out}} - T_{h,\text{in}}}{T_{c,\text{in}} - T_{h,\text{in}}} = \frac{32-38}{24-38} = 0.43$$

$$Z = \frac{T_{c,\text{in}} - T_{c,\text{out}}}{T_{h,\text{out}} - T_{h,\text{in}}} = \frac{24-32}{32-38} = 1.33$$

From Figure 10.14, $F = 0.78$

$$\therefore \qquad \Delta T_{\text{mean}} = F(LMTD) = 0.78\ (7°\text{C}) = 5.5°\text{C}$$

An iterative solution is required. For a first guess, let the tubing be 1-in.-*OD*. From Appendix 2, Table 42, for 1-in. BWG 18 tubing: $D_i = 0.902$ in. = 0.0229 m; $D_o = 0.0254$ m. Assume from Table 10.1 that the overall heat transfer coefficient $U_o = 1700$ W/(m^2 K). The rate of heat transfer is

$$q = U_o A_o \Delta T_{\text{mean}} = \dot{m}_h c_{ph} (T_{h,\text{in}} - T_{h,\text{out}})$$

$$U_o [N \pi D_o (2L_p)] \Delta T_{\text{mean}} = \dot{m}_h c_{ph} (T_{h,\text{in}} - T_{h,\text{out}})$$

$$N = \frac{\dot{m}_h c_{ph}}{U_o \pi D_o 2L_p} \frac{T_{h,\text{in}} - T_{h,\text{out}}}{\Delta T_{\text{mean}}} = \frac{25.2\ \text{kg/s} \quad 4176\ \text{J/(kg K)}}{1700\ \text{W/(m}^2\text{K)}\ \ \pi\ \ 0.0254\,\text{m}\ \ 2\ \ 4.9\,\text{m}} \frac{38°\text{C} - 32°\text{C}}{55°\text{C}} = 86 \text{ tubes}$$

The water velocity in the tubes for 86 tubes is

$$V = \frac{\dot{m}}{\rho A_i} = \frac{4\dot{m}}{\rho N \pi D_i^2} = \frac{4\ \ 25.2\ \text{kg/s}}{995.7\ \text{kg/m}^3\ \ (86)\pi\ \ 00229\,\text{m}\ ^2} = 0.715 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{VD_i}{\nu} = \frac{(0.715\ \text{m/s})(0.0229\,\text{m})}{0.805 \times 10^{-6}\ \text{m}^2/\text{s}} = 20{,}326 \text{ (Turbulent)}$$

From Equation (7.61)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\ (20{,}326)^{0.8}\ (5.4)^{0.4} = 248$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D_i} = 248 \frac{0.615\ \text{W/(m K)}}{0.0229\,\text{m}} = 6655\ \text{W/(m}^2\text{K)}$$

The overall heat transfer coefficient, neglecting fouling and tube wall thermal resistance, from Equation (10.3), is

$$\frac{1}{U_o} = \frac{1}{\bar{h}_o} + \frac{D_o}{D_i}\frac{1}{\bar{h}_i} = \frac{1}{5678\ \text{W/(m}^2\ \text{K)}} + \left(\frac{254}{229}\right)\frac{1}{6655\ \text{W/(m}^2\text{K)}}$$

$$U_o = 2917\ \text{W/(m}^2\ \text{K)}$$

The pressure drop through the tube is obtained by adding the inlet and outlet pressure drops to Equation 7.13

$$\Delta p = \left(f \frac{L}{D_i} + 1.5 \right) \frac{\rho V^2}{2 g_c} \qquad \text{(where } L = 2\, L_p = 9.8 \text{ m)}$$

The friction factor *f*, is given by Equation (7.57) for turbulent flow

$$f = \frac{0.184}{Re_D^{\ 0.2}} = \frac{0.184}{(20,326)^{0.2}} = 0.0253$$

$$\Delta p = \left((0.0243) \left(\frac{9.8\,\text{m}}{0.0229\,\text{m}} \right) + 1.5 \right) \frac{995.7\,\text{kg/m}^3 \quad 0.715\,\text{m/s}^{\ 2}}{1} \quad (\text{s}^2\text{N})/(\text{kg m})$$

$$= 6275\,\text{N/m}^2 = 6.3 \text{ kPa}$$

There is about half of the required pressure drop. Therefore, smaller tubes should be used. For a second iteration, let the tubes be 3/4 in. 18 BWG tubes

From Appendix 2, Table 42

$$D_i = 1.66 \text{ cm} = 0.0166 \text{ m} \qquad D_o = 1.19 \text{ cm} = 0.019 \text{ m}$$

Following the same procedure shown above but using U_o = 2000 W/(m^2 K) yields

N = 98 tubes

V = 1.19 m/s

$\bar{h}_i$ = 5452 W/(m^2 K)

U_o = 2582 W/(m^2 K)

Δp = 22.4 kPa

Performing the procedure for the same tubes but using the U_o derived above 2502 W/(m^2 K) yields

N = 76

V = 1.53 m/s

This will give an even higher pressure drop, therefore, use the 1 in. 18 BWG tubes.

PROBLEM 10.38

A shell-and-tube heat exchanger with the characteristics given below is to be used to heat 27,000 kg/h of water before it is sent to a reaction system. Saturated steam at 239 kPa absolute pressure is available as the heating medium and will be condensed without subcooling on the outside of the tubes. From previous experience, the steam-side condensing coefficient can be assumed constant and equal to 11,300 W/(m^2 K).

If the water enters at 16°C, at what temperature will it leave the exchanger? Use reasonable estimates for fouling coefficients.

Exchanger specifications

- **Tubes – 2.5-cm-*OD*, 2.3-cm-*ID*, horizontal copper tubes in six vertical rows**
- **Tube length = 2.4 m**
- **Total number of tubes = 52**
- **Number of tube-side passes = 2**

GIVEN

Shell-and-tube heat exchanger - water in copper tubes, saturated steam is shell

Water flow rate $\dot{m}_w$ = 27,000 kg/h = 7.5 kg/s

Steam pressure = 2.36 atm = 239 kPa

Steam-side coefficient $\bar{h}_o$ = 11,300 W/(m^2 K)

Water entrance temperature: $T_{w,\text{in}}$ = 16°C

Tube diameters

- D_o = 2.5 cm = 0.025 m
- D_i = 2.3 cm = 0.023 m

Tube length (L) = 2.4 m
Number of tubes (N) = 52
Number of tube passes = 2

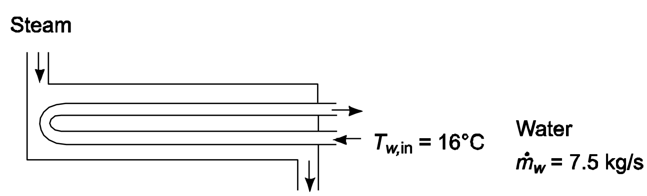
FIND

The water exit temperature ($T_{w,\text{out}}$)

ASSUMPTIONS

Length given is total tube length for both passes

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the temperature of saturated steam at 239 kPa (T_a) = 125°C

From Appendix 2, Table 13, for water at 20°C

Thermal conductivity (k) = 0.597 W/(m K)

Kinematic viscosity (ν) = 1.006 × 10^{-6} m^2/s

Prandtl number (Pr) = 7.0

Density (ρ) = 998.2 kg/m^3

Specific heat (c_p) = 4182 J/(kg K)

From Appendix 2, Table 12, the thermal conductivity of copper (k_c) = 392 W/(m K) at 127°C

SOLUTION

Tube side transfer coefficient:
The water velocity is

$$V = \frac{\dot{m}_w}{\rho A_{\text{flow}}} = \frac{4\dot{m}}{\rho N \pi D_i^2} = \frac{4\ 7.5\ \text{kg/s}}{998.2\ \text{kg/m}^3\ \ (52)\pi(0.023\,\text{m})^2} = 0.348\ \text{m/s}$$

The Reynolds number is $Re_D = \frac{VD_i}{\nu} = \frac{(0.348\ \text{m/s})(0.023\,\text{m})}{1.006\times10^{-6}\ \text{m}^2/\text{s}} = 7956$ (Turbulent)

From Equation (7.61) $\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr^n$ where $n = 0.4$ for heating

$$\overline{Nu}_D = 0.023\ (7956)^{0.8}\ (7.0)^{0.4} = 66.1$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D_i} = 66.1 \frac{0.597\ \text{W/(m K)}}{0.023\,\text{m}} = 1716\ \text{W/(m}^2\text{K)}$$

From Table 10.2: A reasonable fouling factor on the water side (R_i) ≈ 0.0002 (m² K)/W and on the steam side (R_o) ≈ 0.00009 (m² K)/W.
The overall heat transfer coefficient is given by Equation (10.6)

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + R_o + R_k + \frac{R_i A_o}{A_i} + \frac{A_o}{A_i \bar{h}_i} = \frac{1}{\bar{h}_o} + R_o + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k_c} + \frac{D_o}{D_i}\left(R_i + \frac{1}{\bar{h}_i}\right)$$

$$\frac{1}{U_d} = \frac{1}{11{,}300\ \text{W/(m}^2\text{K)}} + 0.00009\ (\text{m}^2\ \text{K})/\text{W} + \frac{(0.025\,\text{m})\ln\left(\frac{25}{23}\right)}{2(392\ \text{W/(m K)})}$$

$$+\left(\frac{25}{23}\right)\left(\left(0.0002\ (\text{m}^2\text{K)/W}\right) + \frac{1}{\left(1716\ \text{W/(m}^2\text{K)}\right)}\right)$$

$$U_d = 969\ \text{W/(m}^2\ \text{K)}$$

The heat capacity rate of the condensing steam is essentially infinite. The heat capacity rate of the water is

$$C_w = \dot{m}_w c_{pw} = (7.5\ \text{kg/s})\ \ 4182\ \text{J/(kg K)}\ \ = 31{,}365\ \text{W/K}$$

The number of transfer units is

$$NTU = \frac{U_o A_o}{C_{\min}} = \frac{U_o N \pi D_o L}{C_w} = \frac{969\ \text{W/(m}^2\ \text{K)}\ \ (52)\pi(0.025\,\text{m})(2.4\,\text{m})}{31{,}365\ \text{W/K}} = 0.30$$

For $C_{\min}/C_{\max} = 0$, $NTU = 0.30$. From Figure 10.20, $e = 0.24$
The outlet temperature can be calculated from Equation (10.22b). Note: $C_c = C_{\min}$.

$$\mathsf{E} = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_s - T_{w,\text{in}}}$$

$$T_{w,\text{out}} = T_{w,\text{in}} + \mathscr{E}\,(T_s - T_{w,\text{in}}) = 16°\text{C} + 0.24\ (125°\text{C} - 16°\text{C}) = 42°\text{C}$$

PROBLEM 10.39

Determine the appropriate size of a shell-and-tube heat exchanger with two tube passes and one shell pass to heat 8.82 kg/s of pure ethanol from 15.6 to 60°C. The heating medium is saturated steam at 152 kPa condensing on the outside of the tubes with a condensing coefficient of 15,000 W/(m²K). Each pass of the exchanger has 50 copper tubes with an *OD* of 1.91 cm and a wall thickness of 0.211 cm. For the sizing, assume the header cross-sectional area per pass is twice the total inside tube cross-sectional area. The ethanol is expected to foul the inside of the tubes with a fouling coefficient of $5678\ \text{W/(m}^2\,\text{K)}$. After the size of the heat exchanger, i.e., the length of the tubes, is known, estimate the frictional pressure drop using the inlet loss coefficient of unity. Then estimate the pumping power required with a pump efficiency of 60% and the pumping cost per year with \$0.10 per kw-hr.

GIVEN

- Shell-and-tube heat exchanger, ethanol in copper tubes, steam in shell
- One shell pass and two tube passes
- Ethanol flow rate $\dot{m}_e$ = 8.82 kg/s
- Ethanol temperatures
  - $T_{e,\text{in}}$ = 15.6°C
  - $T_{e,\text{out}}$ = 60°C
- Steam pressure = 152 kPa
- Number of tubes (N) = 50
- Tube outside diameter (D_o) = 1.91 cm = 0.0191 m
- Tube wall thickness (t) = 0.211 cm = 0.00211 m
- Header area per pass = 2 (total inside cross-sectional area)
- Tube side fouling coefficient ($1/R_i$) = 5678 W/(m² K)
- Shell-side transfer coefficient $\bar{h}_o$ = 15,000 W/(m² K)

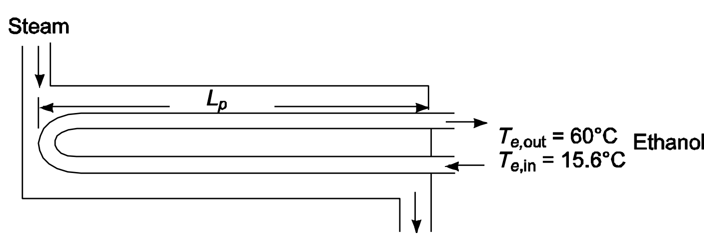
FIND

(a) Size: length of one pass (L_p)
(b) The frictional pressure drop (Δp)
(c) The pumping power required (P_p) with a pump efficiency (η_p) = 60%
(d) Pumping cost per year for energy cost of \$0.10/kw-hr

ASSUMPTIONS

- The variation of thermal properties with temperature is negligible
- Shell side fouling is negligible
- The tubes are smooth
- Entrance pressure drop effects are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the temperature of saturated steam at 152 kPa (T_s) = 110°C

From Appendix 2, Table 22, for ethanol (ethy1 alcohol) at 20°C

Density (ρ) = 790 kg/m^3

Thermal conductivity (k) = 0.182 W/(m K)

Absolute viscosity (μ) = 12.0 × 10^{-4} (Ns)/m^2

Prandtl number (Pr) = 16.29

Specific heat (c_p) = 2470 J/(kg K)

From Appendix 2, Table 12, the thermal conductivity of copper (k_c) = 392 W/(m K) at 127°C

SOLUTION

The inside diameter of the tubes is

$$D_i = D_o - 2t = 1.91 \text{ cm} - 2(0.211 \text{ cm}) = 1.49 \text{ cm} = 0.0149 \text{ m}$$

The Reynolds number for the ethanol flow is

$$Re_D = \frac{VD_i}{\nu} = \frac{4\dot{m}}{N\pi D_i \mu} = \frac{4(8.82 \text{ kg/s})}{(52)\pi\ 0.0149\text{ m}\ \ 12.0\times10^{-4}(\text{N s})/\text{m}^2\ \ (\text{kg m})/(\text{N s}^2)} = 12{,}078 \text{ (Turbulent)}$$

From Equation (7.61)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(12{,}078)^{0.8}\,(16.29)^{0.4} = 129.4$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D_i} = 129.4\,\frac{0.182 \text{ W/(mK)}}{0.0149\text{ m}} = 1581 \text{ W/(m}^2\text{K)}$$

The overall heat transfer coefficient with fouling is given by Equation (10.6)

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + R_o + R_k + \frac{R_i A_o}{A_i} + \frac{A_o}{A_i \bar{h}_i} = \frac{1}{\bar{h}_o} + 0 + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k_c} + \frac{D_o}{D_i}\left(R_i + \frac{1}{\bar{h}_i}\right)$$

$$\frac{1}{U_d} = \frac{1}{15{,}000 \text{ W/(m}^2\text{K)}} + \frac{(0.0191\text{m})\ln\frac{191}{149}}{2\ 392 \text{ W/(m K)}} + \left(\frac{191}{149}\right)\left(\frac{1}{5678 \text{ W/(m}^2\text{K)}} + \frac{1}{1581 \text{ W/(m}^2\text{K)}}\right)$$

$$U_d = 901 \text{ W/(m}^2\text{ K)}$$

The heat capacity rate of the steam is essentially infinite. The heat capacity rate of the ethanol is

$$C_e = \dot{m}_e c_p = (8.82 \text{ kg/s})\ \ 2470 \text{ J/(kg K)} = 21{,}785 \text{ W/K}$$

From Figure 10.10 $\quad \Delta T_a = T_s - T_{e,\text{out}} = 110°\text{C} - 15.6°\text{C} = 94.4°\text{C}$

$\Delta T_b = T_s - T_{e,\text{in}} = 110°\text{C} - 60°\text{C} = 50°\text{C}$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{94.4°\text{C} - 50°\text{C}}{\ln\ \frac{94.4}{50}} = 69.9°\text{C}$$

Because $Z = 0$; $F = 1$ and $\Delta T_{\text{mean}} = LMTD$.

The rate of heat transfer is

$$q = U_o A_o \Delta T_{\text{mean}} = C_e (T_{e,\text{out}} - T_{e,\text{in}})$$

$$U_o (N\pi D_o L)\, \Delta T_{\text{mean}} = C_e (T_{e,\text{out}} - T_{e,\text{in}})$$

Solving for the length

$$L = \frac{C_e}{U_o N \pi D_o}\frac{T_{e,\text{out}} - T_{e,\text{in}}}{\Delta T_{\text{mean}}} = \frac{21,785\ \text{W/K}}{901\ \text{W/(m}^2\text{K)}\ (52)\pi\ \ 0.0191\text{m}}\ \frac{60°\text{C} - 15.6°\text{C}}{69.9°\text{C}} = 4.92\ \text{m}$$

$$L_p = \frac{4.92\ \text{m}}{2} = 2.46\ \text{m}$$

The effectiveness, from Equation (10.22b) is

$$\mathsf{E} = \frac{T_{e,\text{out}} - T_{e,\text{in}}}{T_s - T_{e,\text{in}}} = \frac{60°\text{C} - 15.6°\text{C}}{110°\text{C} - 15.6°\text{C}} = 0.47$$

From Figure 10.20, $NTU \approx 0.7$

$$NTU = \frac{U_o A_o}{C_{\min}} = \frac{U_o N \pi D_o L}{C_{\min}}$$

Solving for the length

$$L = \frac{NTU\ C_{\min}}{U_o N \pi D_o} = \frac{0.7\ \ 21,785\ \text{W/K}}{901\ \text{W/(m}^2\text{K)}\ (52)\pi\ \ 0.0191\text{m}} = 5.42\ \text{m}$$

The length of one pass = L/(# of passes) = (5.42 m)/2 = 2.71 m

This method relies on reading the low end of Figure 10.19 and is probably less accurate than the *LMTD* method.

(b) From Equation (7.13) the pressure drop is

$$\Delta p = f\,\frac{L}{D_i}\frac{\rho V^2}{2g_c} = f\,\frac{L}{D_i}\frac{\rho}{2g_c}\left(\frac{4\dot{m}_e}{N\pi\rho D_i^2}\right)^2$$

Where the friction factor, f, is given for turbulent flow by Equation (7.57)

$$f = 0.184\ Re_D^{-0.2} = 0.184\ (12,078)^{-0.2} = 0.0281$$

$$\Delta p = 0.0281\ \frac{4.92\ \text{m}}{0.0149\ \text{m}}\ \frac{790\ \text{kg/m}^3}{2}\left(\frac{4\ \ 8.82\ \text{kg/s}}{52\pi\ \ 790\ \text{kg/m}^3\ \ (0.0149\ \text{m})^2}\right)^2\ (\text{s}^2\ \text{N})/(\text{kg m}) = 5557\ \text{N/m}^2$$

(c) The pumping power required is

$$P_p = \frac{\dot{v}\Delta p}{\eta_p} = \frac{\dot{m}_e}{\eta_p \rho}\Delta p = \frac{8.82\ \text{kg/s}}{0.6\ \ 790\ \text{kg/m}^3}\ \ 5557\ \text{N/m}^2\ \ (\text{W s})/(\text{N m}) = 103\ \text{W}$$

(d) The cost to run the pump is

$$\text{Cost} = \left(\frac{\$0.10}{\text{kWh}}\right)(103\ \text{W})\left(\frac{1\ \text{kW}}{1000\ \text{W}}\right)\left(\frac{24\ \text{h}}{\text{day}}\right)\left(\frac{365\ \text{days}}{\text{year}}\right) = \$91/\text{year}$$

PROBLEM 10.40

A counterflow regenerator is used in a gas turbine power plant to preheat the air before it enters the combustor. The air leaves the compressor at a temperature of 350°C. Exhaust gas leaves the turbine at 700°C. The mass flow rates of air and gas are 5 kg/s. Take the c_p of air and gas to be equal to 1.05 kJ/(kg K). Determine the required heat transfer area as a function of the regenerator effectiveness, if the overall heat transfer coefficient is 75 W/(m^2 K).

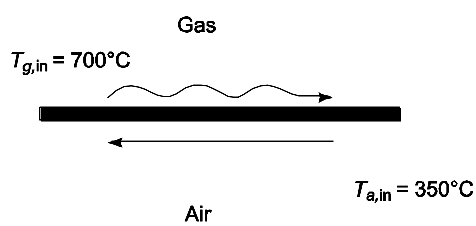
GIVEN

- Counterflow air-to-gas heat exchanger
- Entering temperatures
  - $T_{a,\text{in}}$ = 350°C
  - $T_{g,\text{in}}$ = 700°C
- Mass flow rates: $\dot{m}_a = \dot{m}_g = 5$ kg/s
- Specific heats: $c_{pa} = c_{pg}$ = 1.05 kJ/(kg K) = 1050 J/(kg K)
- Overall heat transfer coefficient (U) = 75 W/(m^2 K)

FIND

- The heat transfer area (A) as a function of the effectiveness (e)

SKETCH



SOLUTION

From Equations (10.23) and (10.17)

$$q = \varepsilon\, C_{\min}\,(T_{g,\text{in}} - T_{a,\text{in}}) = U\,A\,\Delta T \;\Rightarrow\; A = \frac{\varepsilon\, C_{\min}\;\; T_{h,\text{in}} - T_{a,\text{in}}}{U\,\Delta T}$$

Since $m_a\ c_{pa} = m_g\ c_{pg}$, the temperature difference between the gas and air remain constant and $\Delta T = T_{g,\text{in}}\ T_{a,\text{out}}$. The heat capacity rates are equal, therefore, Equation (10.22b) reduces to

$$\varepsilon = \frac{T_{a,\text{out}} - T_{a,\text{in}}}{T_{g,\text{in}} - T_{a,\text{in}}} \Rightarrow T_{a,\text{out}} = T_{a,\text{in}} + \varepsilon\,(T_{g,\text{in}} - T_{a,\text{in}})$$

$$\therefore \qquad \Delta T = T_{g,\text{in}} - T_{a,\text{in}} - \varepsilon\,(T_{g,\text{in}} - T_{a,\text{in}}) = (T_{g,\text{in}} - T_{a,\text{in}})\,(1 - \varepsilon)$$

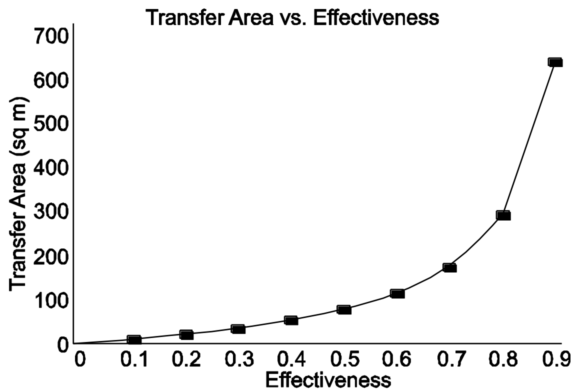
Substituting this into the expression for area

$$A = \frac{\varepsilon\, C_{\min}}{U(1-\varepsilon)} = \frac{\varepsilon\, \dot{m}\, c_p}{U(1-\varepsilon)}$$

$$A = \left(\frac{\varepsilon}{1-\varepsilon}\right) \frac{5\ \text{kg/s}\quad 1050\ \text{J/(kg K)}}{75\ \text{W/(m}^2\ \text{K)}\quad \text{J/(W s)}} = 70\ \text{m}^2 \left(\frac{\varepsilon}{1-\varepsilon}\right)$$

This is tabulated and plotted below

e	A (m^2)
0	0
0.1	7.8
0.2	17.5
0.3	30
0.4	47
0.5	70
0.6	105
0.7	163
0.8	280
0.9	630
1.0	∞



COMMENTS

This problem can also be solved by calculating the number of transfer units for a given area then reading the effectiveness off Figure 10.19.

PROBLEM 10.41

Determine the heat transfer area requirements of Problem 10.40 if (a) 1-2 shell and tube, (b) an unmixed crossflow, and (c) a parallel flow heat exchanger are used, respectively.

GIVEN

- An air-to-gas heat exchanger
- Entering temperatures
  - $T_{a,\text{in}}$ = 350°C
  - $T_{g,\text{in}}$ = 700°C
- Mass flow rates: $m_a = m_g$ = 5 kg/s
- Specific heats: $c_{pa} = c_{pg}$ = 1.05 kJ/(kg K) = 1050 J/(kg K)
- Overall heat transfer coefficient (U) = 75 W/(m^2 K)

FIND

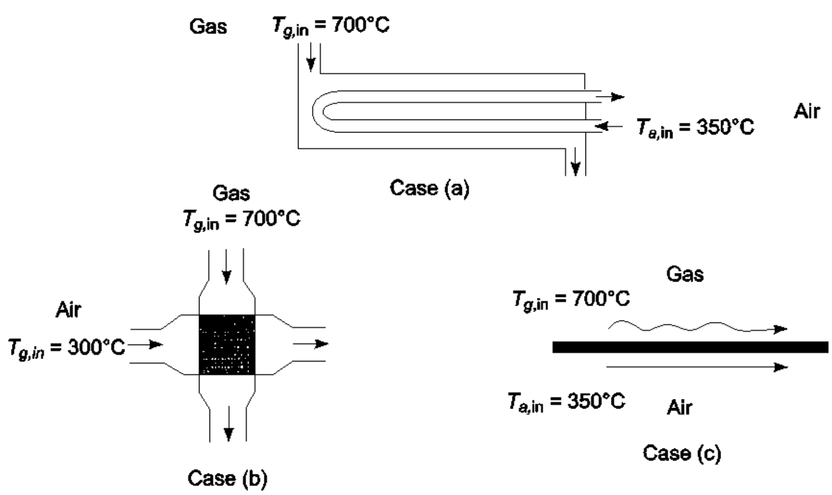
The heat transfer area (A) as a function of the effectiveness (e) for

(a) A 1-2 shell and tube heat exchanger
(b) An unmixed crossflow heat exchanger
(c) A parallel flow heat exchanger

ASSUMPTIONS

- For case (a) the air is in the tubes

SKETCH



SOLUTION

As shown in the solution to Problem 10.40 for counterflow

$$\Delta T = (T_{g,\text{in}} - T_{a,\text{in}})(1 - e)$$

This must be corrected for case (a) and (b) by Figures 10.14 and 10.17 where

$$P = \frac{T_{a,\text{out}} - T_{a,\text{in}}}{T_{g,\text{in}} - T_{a,\text{in}}}$$

From Equation (10.22b)

$$T_{a,\text{out}} = T_{a,\text{in}} + e\,(T_{g,\text{in}} - T_{a,\text{in}})$$

Therefore

$P = e$

Since $C_g = C_a$, $Z = 1$

The solution for parts (a) and (b) are the same as for Problem 10.41 expect that the mean temperature (ΔT) must be multiplied by the factor F with the following results

$$A = \frac{\mathcal{E}\,\dot{m}_w c_p}{UF(1-\mathcal{E})} = \frac{70\,\text{m}^2}{F}\left(\frac{\mathcal{E}}{1-\mathcal{E}}\right)$$

where F is from Figure 10.14 for part (a) and from Figure 10.17 for part (b) where $P = e$ and $Z = 1.0$.

(c) For parallel flow

$$\Delta T = LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)}$$

Where:

$$\Delta T_a = T_{g,\text{in}} - T_{a,\text{in}}$$

$$\Delta T_b = T_{g,\text{out}} - T_{a,\text{out}} = [T_{g,\text{in}} - \mathcal{E}\,(T_{g,\text{in}} - T_{a,\text{in}})] - [T_{a,\text{in}} + \mathcal{E}\,(T_{g,\text{in}} - T_{a,\text{in}})]$$

$$= (T_{g,\text{in}} - T_{a,\text{in}})(1 - 2\,\mathcal{E})$$

$$\Delta T_a - \Delta T_b = (T_{g,\text{in}} - T_{a,\text{in}})(2\,\mathcal{E})$$

$$\frac{\Delta T_a}{\Delta T_b} = \frac{1}{1-2\mathcal{E}}$$

$$\therefore \qquad LMTD = \frac{2\mathcal{E}\left(T_{g,\text{in}} - T_{a,\text{in}}\right)}{\ln\left(\dfrac{1}{1-2\mathcal{E}}\right)}$$

From Equation (10.23) and (10.17)

$$q = \mathcal{E}\, C_{\min}\,(T_{g,\text{in}} - T_{a,\text{in}}) = U\,A\,\frac{2\mathcal{E}\left(T_{g,\text{in}} - T_{a,\text{in}}\right)}{\ln\left(\dfrac{1}{1-2\mathcal{E}}\right)}$$

$$A = \frac{C_{\min}}{2U}\ln\left(\frac{1}{1-2\mathcal{E}}\right) = \frac{5\ \text{kg/s}\quad 1050\ \text{J/(kg K)}}{2\ \ 75\ \text{W/(m}^2\text{K)}}\ \ln\left(\frac{1}{1-2\mathcal{E}}\right) = 35\ \text{m}^2\ \ln\left(\frac{1}{1-2\mathcal{E}}\right)$$

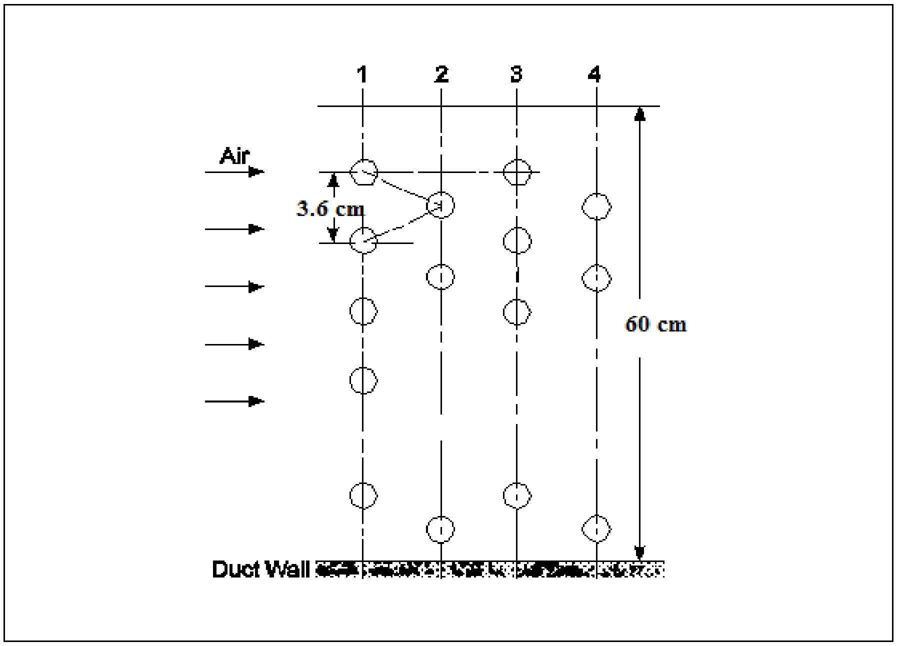
Tabulating these results

e	$F(a)$*	$F(b)$**	$A(a)$ (m^2)	$A(b)$ (m^2)	$A(c)$ (m^2)
0	1.0	1.0	0	0	0
0.1	1.0	1.0	7.7	7.7	7.7
0.2	0.99	0.98	17.9	17.9	17.9
0.3	0.97	0.97	30.4	30.9	32.1
0.4	0.92	0.94	50.7	49.6	56.3
0.5	0.8	0.91	87.5	76.9	∞
0.57	0.5	0.86	186	108	
0.6	NA	0.84		125	
0.7	NA	0.70		233	
0.8	NA	0.5		560	
0.9	NA	NA			
1.0	NA	NA			

* From Figure 10.14
** From Figure 10.17
NA - Data not available from the figures

PROBLEM 10.42

A small space heater is constructed of 1.25 cm, 18-gauge brass tubes that are 60 cm. The tubes are arranged in equilateral, staggered triangles on 3.6 cm centers, four rows of 15 tubes each. A fan blows 0.95 m^3/s of atmospheric pressure air at 21°C uniformly over the tubes (see sketch). Estimate: (a) heat transfer rate (b) exit temperature of the air (c) rate of steam condensation, assuming that saturated steam at 115 kPa(abs) inside the tubes as the heat source. State your assumptions. Work parts *a*, *b*, and *c* of this problem by two methods. First use the *LMTD*, which requires a trial-and-error or graphical solution then use the effectiveness methods.



GIVEN

A small heater made of 4 rows of 15 tubes each as shown above
Tubes: 1.25 cm, 18 gauge brass
Tube length (L) = 60 cm
Distance between tube centers (2 S_T) = 3.6 cm
Air flow rate ($\dot{V}$) = 0.95 m^3/s
Air inlet temperature ($T_{a,\text{in}}$) = 21°C
Saturated steam inside the tubes at pressure (p_s) = 115 kPa (abs)

FIND

Using both the *LMTD* method and *e* method find: (a) The heat transfer rate (q)
(b) Air exit temperature ($T_{a,\text{out}}$)
(c) Rate of steam condensation $\dot{m}_c$

PROPERTIES AND CONSTANTS

From Appendix 2, Table 42, 18 gauge tubes have a wall thickness (t) = 0.049 in.=0.124 cm
From Appendix 2, Table 10, the thermal conductivity of brass (k_b) = 111 W/(m K)
From Appendix 2, Table 13, the temperature of saturated steam at 115 kPa T_s = 103°C and the heat of evaporation (h_{fg}) = 2249 J/kg.
From Appendix 2, Table 28, for air at an estimated mean temperature of 40°C

Specific heat (c_{pa}) = 1014 J/(kg K)

Kinematic viscosity (ν) = 17.6*10^{-6} m^2/s

Prandtl number = 0.71

Thermal conductivity (k_a) = 0.0265 W/(m K)

Density (ρ) = 1.092 kg/m^3

At the steam temperature of 103°C, Pr_s = 0.71.

SOLUTION

The tube diameters are D_o = 1.25 cm=0.0125 m

$D_i = D_o - 2t$ = 1.25 cm – 2(0.124 cm) = 1 cm = 0.01 m

From Table 10.1, the heat transfer coefficient for the condensing steam (h_i) ≈ 5000 – 30,000 W/(m^2K) let h_i = 17,000 W/(m^2 K).

The heat transfer coefficient for the air flow over the tube bank can be calculated as shown in Chapter 7. The Reynolds number for this geometry is

$$Re_D = \frac{V_{\max} D}{\nu} = \frac{\dot{v}_a}{A_{\min}}\frac{D_o}{\nu} = \frac{\dot{v}_a D_o}{[16 \; S_T - D_o \;\; + D_o]L\nu}$$

$$Re_D = \frac{(0.95 \text{ m}^3/s)*(0.0125 \text{ m})}{\left[16(0.036-0.0125)+0.0125\right] \text{ m}*(0.6 \text{ m})*(17.6*10^{-6} \text{ m}^2/s)} = 2894$$

(Transition Regime)

The Nusselt number is given by Equation (6.27)

$$\overline{Nu}_D = 0.35\left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{0.6}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where S_L = Longitudinal spacing = $\sqrt{(2S_T)^2 - (S_T)^2}$ = 3.12 cm=0.0312 m

$$\overline{Nu}_D = 0.35\left(\frac{0.018}{0.0312}\right)^{0.2} (2894)^{0.6}\,(0.71)^{0.36} = 33$$

$$\bar{h}_o = \overline{Nu}_D\,\frac{k_a}{D_o} = 33*\frac{0.0265 \text{ W/(m K)}}{0.0125 \text{ m}} = 70.1 \text{ W/(m}^2\text{ K)}$$

The overall heat transfer coefficient is given by Equation (10.3)

$$\frac{1}{U_o} = \frac{A_o}{A_i\bar{h}_i} + \frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} + \frac{1}{\bar{h}_o} = \frac{D_o}{D_i\bar{h}_i} + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{1}{\bar{h}_o}$$

$$\frac{1}{U_o} = \frac{0.0125 \text{ m}}{0.01 \text{ m}*17{,}000 \text{ W/(m}^2\text{ K)}} + \frac{0.0125 \text{ m}*\ln\left(\frac{0.0125}{0.001}\right)}{2*111 \text{ W/(m K)}}\text{n} + \frac{1}{70.1 \text{ W/(m}^2\text{ K)}}$$

$$\frac{1}{U_o} = (0.0000735 + 0.0000125 + 0.01426) \text{ (m}^2\text{ K)/W}$$

U_o = 66.6 W/(m^2 K)

LMTD method

From Figure 10.10 $\quad \Delta T_a = T_s - T_{a,\text{in}} = 103°\text{C} - 21°\text{C} = 82°\text{C}$

$\Delta T_b = T_s - T_{a,\text{out}}$

To find the *LMTD*, the air outlet temperature must be known, therefore, an iterative solution is required. For the first iteration, let $T_{a,\text{out}} = 38°\text{C}$.

$$\Delta T_b = 103°\text{C} - 38°\text{C} = 65°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)} = \frac{147°\text{F} - 117°\text{F}}{\ln\left(\dfrac{147}{117}\right)} \quad \frac{82^0 C - 65^0 C}{\ln\left(\dfrac{82}{65}\right)} = 73.2^0\text{C}$$

The total transfer area is

$$A = (\text{Number of tubes})\ \pi D_o L = (4)(15)\ \pi\,(0.0125\text{ m})\ (0.6\text{ m}) = 1.412\text{ m}^2$$

The rate of heat transfer is given by Equation (10.17)

$$q = U A\, \Delta T = U A\, (LMTD) = 66.6\text{ W/(m}^2\text{ K)}\ (1.412\text{m}^2)\ (73.2°\text{C}) = 6892\text{ W}$$

The outlet air temperature can be calculated from

$$q = \dot{m}_a\, c_{pa}\, (T_{a,\text{out}} - T_{a,\text{in}}) = \dot{v}_a\, \rho\, c_{pa}\, (T_{a,\text{out}} - T_{a,\text{in}})$$

$$T_{a,\text{out}} = T_{a,\text{in}} + \frac{q}{\dot{v}_a \rho c_{pa}} = 21°\text{C} + \frac{6892\text{ W}}{0.95\text{ m}^3/s * 1.092\text{ kg/m}^3 * 1014\text{ J/(kg K)}} = 27.5°\text{C}$$

Following a similar procedure for a second iteration yields

Mean air temperature = 24.5°F

$\rho = 1.115\text{ kg/m}^3$

$c_{pa} = 1012\text{ J/(kg K)}$

$LMTD = 62°\text{C}$

(a) $q = 7533\text{ W}$

(b) $T_{a,\text{out}} = 28°\text{C}$

The rate of steam condensation is given by

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{7533\text{ W}}{2249 * 10^3\text{ J/kg}} = 3.35*10^{-3}\text{ kg/s}$$

The effectiveness method

The heat rate of the steam is essentially infinite. The heat rate of the air is

$$C_a = \dot{m}_a\, c_{pa} = \dot{v}_a\, \rho\, c_{pa} = 0.95\text{ m}^3/s * 1.092\text{ kg/m}^3 * 1014\text{ J/(kg K)} = 1052\text{ W/K}$$

$$\frac{C_{\text{mixed}}}{C_{\text{unmixed}}} = 0$$

The number of transfer units is

$$NTU = \frac{U A}{C_{\min}} = \frac{66.6\text{ W/(m}^2\text{ K)} * 1.412\text{ m}^3}{1052\text{ W/K}} = 0.089$$

For cross flow, one fluid mixed (air), other fluid unmixed (steam),

from Figure 10.22, $e \approx 6\%$.

(a)

$$q = \mathscr{E}\, C_{\min}\,(T_{h,\text{in}} - T_{c,\text{in}}) = 0.06 * 1052 \text{ W/K} * (103°\text{C} - 21°\text{C}) = 5175.8 \text{ W}$$

Applying Equation (10.22b)

$$\mathscr{E} = \frac{C_a}{C_{\min}} \frac{T_{a,\text{out}} - T_{a,\text{in}}}{T_s - T_{a,\text{in}}}$$

$$T_{a,\text{out}} = T_{a,\text{in}} + \mathscr{E}\,(T_s - T_{a,\text{in}}) = 21°\text{C} + 0.06\,(103°\text{C} - 21°\text{C}) = 26°\text{C}$$

(c) The steam condensation rate is

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{18,346 \text{ Btu/h}}{974 \text{ Btu/1b}} \quad \frac{5175.8 \text{ W}}{2249 * 10^3 \text{ J/kg}} = 2.3*10^{-3} \text{ kg/s}$$

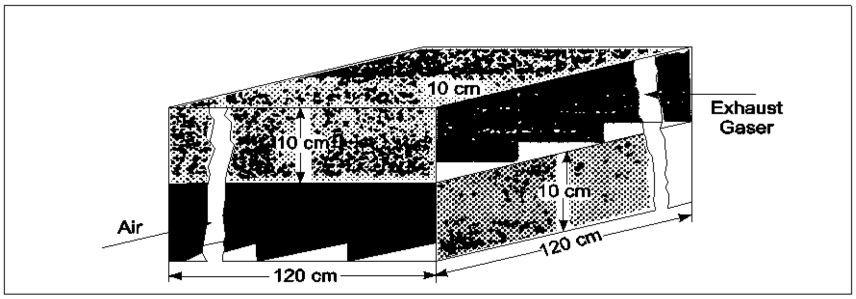
COMMENTS

Although the effectiveness method is more direct for this type of problem, its accuracy is poor due to the low value of *NTU*. The *LMTD* method requires an iterative procedure but gives much better accuracy.

The heat transfer coefficient for the condensing steam will be discussed in more detail in Chapter 10. For this problem, the thermal resistance of the condensing steam is less than 1% of the total thermal resistance, therefore, a rough estimate is adequate.

PROBLEM 10.43

A one-tube pass cross-flow heat exchanger is considered for recovering energy from the exhaust gases of a turbine-driven engine. The heat exchanger is constructed of flat plates, forming an egg-crate pattern as shown in the sketch. The velocities of the entering air (10°C) and exhaust gases (425°C) are both equal to 61 m/s. Assuming that the properties of the exhaust gases are the same as those of the air, estimate the overall heat transfer coefficient *U* for a path length of 1.2 m, neglecting the thermal resistance of the intermediate metal wall. Then determine the outlet temperature of the air, comment on the suitability of the proposed design, and if possible, suggest improvements. State your assumptions.



GIVEN

- The heat exchanger shown above
- Air and exhaust velocities ($V_a = V_e$) = 61 m/s
- Inlet temperatures
    - Air ($T_{a,\text{in}}$) = 10°C
    - Exhaust ($T_{e,\text{in}}$) = 425°C
- Path length (L) = 1.2 m

FIND

(a) The overall heat transfer coefficient (U)
(b) The outlet temperature of the air ($T_{a,\text{out}}$)

ASSUMPTIONS

Steady state
Exhaust gas properties are the same as air
Thermal resistance of the metal walls is negligible
Thermal properties can be evaluated at the average temperature
Heat losses from the exterior walls are negligible
Dividing walls within one side of the exchanger do not participate in the heat transfer process

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for air at the average inlet temperature of about 200°C

Density (ρ) = 0.723 kg/m^3
Thermal conductivity (k) = 0.0370 W/(m K)
Kinematic viscosity (ν) = 35.5 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71
Specific heat (c_p) = 1035 J/(kg K)

SOLUTION

(a) The Reynolds number at the flow is

$$Re_{D_h} = \frac{VD_h}{\nu}$$

where D_h = Hydraulic diameter = $\dfrac{4A}{P} = \dfrac{4(0.1\,\text{m})^2}{4(0.1\,\text{m})}$ = 0.1 m

$$Re_{D_h} = \frac{(61\,\text{m/s})(0.1\,\text{m})}{35.5\times10^{-6}\,\text{m}^2/\text{s}} = 1.72 \times 10^5 \text{ (Turbulent)}$$

The Nusselt number for turbulent flow through ducts is given by Equation (7.61)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \text{ where } n = 0.4 \text{ for heating, 0.3 for cooling}$$

For the air being heated

$$\overline{Nu}_D = 0.023\,(1.72 \times 10^5)^{0.8}\,(0.71)^{0.4} = 309.5$$

$$\bar{h}_a = \overline{Nu}_D \frac{k}{D_h} = 309.5 \frac{0.0370\,\text{W/(mK)}}{0.1\,\text{m}} = 114.5\ \text{W/(m}^2\,\text{K)}$$

For the exhaust being cooled

$$\overline{Nu}_D = 0.023\,(1.72 \times 10^5)^{0.8}\,(0.71)^{0.3} = 320.3$$

$$\bar{h}_e = 118.5\ \text{W/(m}^2\,\text{K)}$$

The overall heat transfer coefficient is

$$\frac{1}{U} = \frac{1}{\bar{h}_e} + \frac{1}{\bar{h}_a} = \frac{1}{114.5\,\text{W/(m}^2\text{K)}} + \frac{1}{118.5\,\text{W/(m}^2\text{K)}}$$

$$U = 58.2\ \text{W/(m}^2\,\text{K)}$$

(b) The heat capacity of both fluids is

$$C = \dot{m}\, c_p = V \rho A_c\, c_p = (60\ \text{m/s})\ \ 0.723\,\text{kg/m}^3\ \ (1.2\ \text{m})\,(0.1\ \text{m})\ \ 118.5\,\text{W/(m}^2\text{K)}\ \ \ (\text{Ws})/\text{J}\ \ = 5477\ \text{W/K}$$

The number of transfer units is

$$NTU = \frac{U A_t}{C_{\min}} = \frac{58.2\ \text{W/(m}^2\text{K)}\ (1.2\,\text{m})(1.2\,\text{m})}{5477\ \text{W/(m}^2\text{K)}} = 0.015$$

From Figure 10.21, $e \approx 1\%$

Rearranging Equation (10.22b) ($C_{\min}/C_{\max} = 1$)

$$T_{a,\text{out}} = T_{a,\text{in}} + \mathscr{E}(T_{e,\text{in}} - T_{a,\text{in}}) = 10°\text{C} + 0.01\,(425°\text{C} - 10°\text{C}) = 14°\text{C}$$

COMMENTS

The accuracy of the air outlet temperature is low because the effectiveness is very low and difficult to read on Figure 10.21. Greater accuracy could be achieved by using the *LMTD* and iterating.

The small effectiveness is due to the small *NTU*. The *NTU* can be increased by using small ducts to increase the overall heat transfer coefficient or redesign the exchanger to increase the transfer area.

PROBLEM 10.44

A shell-and-tube counterflow heat exchanger is to be designed for heating an oil from 27^{0}C to 82°C. The heat exchanger has two tube passes and one shell pass. The oil is to pass through 3.8 cm schedule 40 pipes at a velocity of 1 m/s and steam is to condense at 102°C on the outside of the pipes. The specific heat of the oil is 1.8 kJ/(kg K) and its mass density is 925 kg/m^3. The steam-side heat transfer coefficient is approximately 10 kW/(m^2 K), and the thermal conductivity of the metal of the tubes is 30 W/ (m K). The results of previous experiments giving the oil-side heat transfer coefficients for the same pipe size at the same oil velocity as those to be used in the exchanger are shown below

ΔT (°C)	**75**	**64**	**53**	**42**	**20**	**-**
T_{oil} (°C)	**27**	**38**	**49**	**60**	**71**	**82**
h_{cf} (W/(m K))	**80**	**85**	**100**	**140**	**250**	**540**

(a) Find the overall heat transfer coefficient *U*, based on the outer surface area at the point where the oil is 38°C, (b) Find the temperature of the inside surface of the pipe when the oil temperature is 38°C. (c) Find the required length of the tube bundle.

GIVEN

A shell-and-tube counterflow heat exchanger - oil in tubes, steam is shell

Oil temperatures

- $T_{o,\text{in}}$ = 27°C
- $T_{o,\text{out}}$ = 82°C

Tubes: 3.8 cm schedule 40 pipes

Oil velocity (V_o) = 1 m/s

Steam temperature (T_s) = 102°C

Oil specific heat (c_{po}) = 1800 J/(kg K)

Oil density (ρ) = 925 kg/m^3

Steam side heat transfer coefficient ($\bar{h}_s$) = 10 kW/(m^2 K)

Thermal conductivity of the tube material (k_t) = 30 W/(m K)

Experimental data above was taken at the same oil velocity

FIND

(a) The overall heat transfer coefficient (U_o) at the point where the oil is 38°C

(b) The inside pipe surface temperature (T_{wi}) when the oil temperature is 38°C

(c) The required length of the tube bundle

PROPERTIES AND CONSTANTS

From Appendix 2, Table 41, for 3.8 cm schedule 40 pipe

D_i = 1.61 in. = 4.1 cm=0.041 m

D_o = 1.9 in. = 4.8 cm= 0.048 m

SOLUTION

(a) The overall heat transfer coefficient based on the outside tube area is given by Equation (10.3)

$$\frac{1}{U_o} = \frac{A_o}{A_i \bar{h}_i} + \frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{1}{\bar{h}_o} = \frac{D_o}{D_i \bar{h}_i} + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{1}{\bar{h}_s}$$

$$\frac{1}{U_o} = \frac{0.048\text{ m}}{(0.041\text{ m})(85\text{ W/(m}^2\text{ K)}} + \frac{(0.048\text{ m})\ln\left(\frac{0.048}{0.041}\right)}{2*30\text{ W/(m}^2\text{ K)}} + \frac{1}{10000\text{ W/(m}^2\text{ K)}}$$

$$\frac{1}{U_o} = (0.01377 + 0.000126 + 0.0001)\ (\text{m}^2\ \text{K})/\text{W}$$

$$U_o = 71.5\ \text{W}/(\text{m}^2\ \text{K})$$

(b) The rate of heat transfer from the oil to the inner pipe surface must equal the rate of heat transfer between the oil and the steam.

$$\bar{h}_{ci}\ A_i\ (T_{wi} - T_o) = U\,A_o\ (T_s - T_o)$$

$$T_{wi} = T_o + \frac{D_o U}{D_i \bar{h}_{ci}}\ (T_s - T_o) = 38°\text{C} + \frac{0.048}{0.041}\left(\frac{71.5}{85}\right)(102°\text{C} - 38°\text{C}) = 101°\text{C}$$

(c) The heat capacity rate of the steam is essentially infinite, therefore, $C_{\min}/C_{\max} = 0$

The effectiveness is given by Equation (10.13b) ($C_c = C_{\min}$).

$$\mathscr{E} = \frac{C_c}{C_{\min}} = \frac{T_{o,out} - T_{o,in}}{T_s - T_{o,in}} = \frac{82 - 27}{102 - 27} = 0.73$$

From Figure 10.20 $NTU = 1.4$

$$NTU = \frac{U_o\,A_o}{C_{\min}} = \frac{U_o\,A_o}{\dot{m}_o C_{po}} = \frac{U_o\,A_o}{V_o\,\rho\,A_c\,c_{po}} = \frac{U_o \pi\,D_o L}{V_o\,\rho \frac{\pi}{4} D_i^2 c_{po}}$$

$$\therefore\ L = \frac{NTU\,V_o \rho c_{po}\,D_i^2}{4U_o D_o} = \frac{1.4(1\ \text{m/s})(925\ \text{kg/m}^3)(1800\ \text{J/(kg K)})(0.041\ \text{m})^2}{4(71.5\ \text{W}/(\text{m}^2\ \text{K}))(0.048\ \text{m})} = 285.4\ \text{m}$$

The length of each pass of a double tube pass would need to be $L/2 = 142.7$ m.

PROBLEM 10.45

A shell-and-tube heat exchanger in an ammonia plant is preheating 1132 m^3 of atmospheric pressure nitrogen per hour from 21^{0}C to 65°C using steam condensing at 138,000 N/m^2. The tube in the heat exchanger have an inside diameter of 2.5 cm. In order to change from ammonia synthesis to methanol synthesis, the same heater is to be used to preheat carbon monoxide from 21^{0}C to 77°C, using steam condensing at 241,000 N/m^2. Calculate the flow rate which can be anticipated from this heat exchanger in kg of carbon monoxide per second.

GIVEN

- Shell-and-tube heat exchanger - nitrogen in tubes, condensing steam in shell
- Nitrogen volumetric flow rate $(\dot{V}_n)$ = 1132 m^3/h = 0.3144 m^3/s
- Nitrogen temperatures
  - $T_{n,\text{in}}$ = 21°C
  - $T_{n,\text{out}}$ = 65°C
- Steam pressure = 138,000 N/m^2
- Tube inside diameter (D_i) = 2.5 cm
- Same heat exchanger is then used with carbon monoxide:
- Carbon monoxide temperatures
  - $T_{c,\text{in}}$ = 21°C
  - $T_{c,\text{out}}$ = 77°C
- New steam pressure = 241 N/m^2

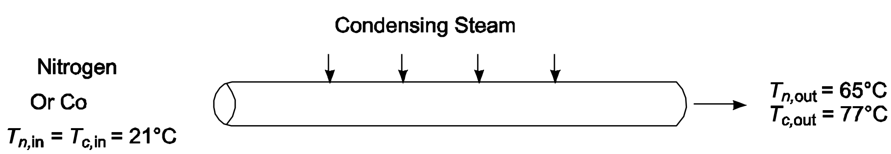
FIND

- The flow rate of carbon dioxide $(\dot{m}_c)$

ASSUMPTIONS

- Two or a multiple of two shell passes
- Thermal resistance of the condensing steam and the tube wall are a small fraction of the total thermal resistance

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the saturation temperature of steam (T_s) is

T_s = 107°C at 138,000 N/m^2

T_s = 125°C at 241,000 N/m^2

32, the saturation temperature of steam (T_s) is

From Appendix 2, Table 33, for nitrogen at the average temperature of 43°C

Density (ρ_n) = 1.096 kg/m^3

Specific heat (c_{pn}) = 1042 J/(kg K)

Thermal conductivity (k_n) = 0.02734 W/(m K)

Absolute viscosity (μ_n) = 18.5 × 10^{-6} (Ns)/m^2

From Appendix 2, Table 30, for the CO at its average temperature of 49°C

Specific heat (c_{pc}) = 1042 J/(kg K)

Thermal conductivity (k_c) = 0.0268 W/(m K)

Absolute viscosity (μ_c) = 18.83×10^{-6} (N s)/m^2

SOLUTION

The data with nitrogen will be used to calculate the overall heat transfer coefficient which will then be modified and applied to the carbon monoxide case.

With nitrogen

The heat capacity rate of the steam is essentially infinite, therefore, $C_{\min}/C_{\max} = 0$.

Applying Equation (10.22b) $C_{\min} = C_c$

$$\mathrm{E} = \frac{T_{n,\text{out}} - T_{n,\text{in}}}{T_s - T_{n,\text{in}}} = \frac{65 - 21}{107 - 21} = 0.51$$

From Figure 10.20, $NTU = U\,A/C_{\min} = 0.75$

The heat capacity rate of the nitrogen is

$$C_n = C_{\min} = \dot{m}_n\, c_{pn} = \dot{v}_n\, \rho_n\, c_{pn} = (0.3144\ \text{m}^3/\text{s})\ \ 1.096\ \text{kg/m}^3\quad 1042\ \text{J/(kg K)}\quad (\text{Ws})/\text{J} = 359.1\ \text{W/K}$$

$$\therefore\ U\,A = NTU\,(C_{\min}) = 0.75\quad 359.1\ \text{W/K} = 269.3\ \text{W/K}$$

With CO

Assuming that the flow of either gas is turbulent, the overall heat transfer coefficient is

$$U_O \propto h_i \propto k\,Re^{0.8} \qquad \text{Since } Pr \approx 0.71 \text{ for either gas}$$

$$\text{so } U_o \propto k\left(\frac{\dot{m}}{\mu}\right)^{0.8}$$

Since the properties of the two gases are very close, $U_c \approx U_n$

The effectiveness of the heat exchanger with the carbon monoxide is

$$\mathrm{E} = \frac{T_{n,\text{out}} - T_{n,\text{in}}}{T_s - T_{n,\text{in}}} = \frac{77°\text{C} - 21°\text{C}}{125°\text{C} - 21°\text{C}} = 0.54 \approx \mathscr{E}_n$$

$$\therefore\ \dot{m}_c \approx \dot{m}_n = \rho\ \dot{v}_n = (1.096\ \text{kg/m}^3)\quad 0.3144\ \text{m}^3/\text{s} = 0.34\ \text{kg/s}$$

PROBLEM 10.46

In an industrial plant a shell-and-tube heat exchanger is heating pressurized dirty water at the rate of 38 kg/s from 60 to 110°C by means of steam condensing at 115°C on the outside of the tubes. The heat exchanger has 500 steel tubes (*ID* = 1.6 cm, *OD* = 2.1 cm) in a tube bundle which is 9-m-long. The water flows through the tubes while the steam condenses in the shell. If it may be assumed that the thermal resistance of the scale on the inside pipe wall is unaltered when the mass rate of flow is increased and that changes in water properties with temperature are negligible, estimate (a) the heat transfer coefficient on the water side and (b) the exit temperature of the dirty water if its mass rate of flow is doubled.

GIVEN

- Shell-and-tube heat exchanger - dirty water in steel tubes, steam condensing in shell
- Water flow rate $(\dot{m}_w)$ = 38 kg/s
- Water temperatures
  - $T_{w,\text{in}}$ = 60°C
  - $T_{w,\text{out}}$ = 110°C
- Steam temperature (T_s) = 115°C
- Number of tubes (N) = 500
- Tube diameters
  - D_i = 1.6 cm = 0.016 m
  - D_o = 2.1 cm = 0.021 m
- Tube bundle length (L) = 9 m

FIND

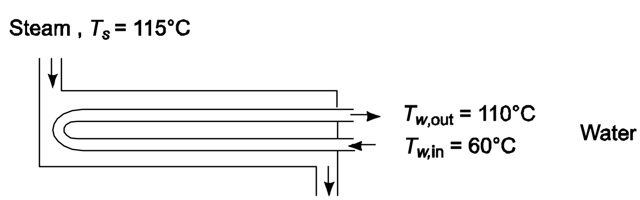
(a) The heat transfer coefficient on the water side $(\bar{h}_i)$

(b) The exit temperature of the dirty water ($T_{w,\text{out}}$) if the mass flow rate $(\dot{m}_w)$ is doubled

ASSUMPTIONS

- The thermal resistance of the scale in the pipe is unaltered when the mass flow rate is increased
- Changes in water properties with temperature are negligible
- Two, or a multiple of two, passes
- The dirty water has the same thermal properties as clean water

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the average temperature of 85°C

Specific heat (c_{pw}) = 4198 J/(kg K)

Thermal conductivity (k) = 0.675 W/(m K)

Absolute viscosity (μ) = 337 × 10^{-6} (N s)/m^2

Prandtl number (Pr) = 2.04

From Appendix 2, Table 10, the thermal conductivity of 1% carbon steel (k_s) = 43 W/(m K) (at 20°C)

SOLUTION

(a) The Reynolds number for flow in the tubes is

$$Re_D = \frac{VD_i}{\nu} = \frac{4\dot{m}}{N\pi D_i \mu} = \frac{4\,(38\text{ kg/s})}{(500)\,\pi\,(0.016\text{ m})\;\;337\times10^{-6}\text{ (Ns)/m}^2\;\;(\text{kg m})/(\text{Ns}^2)} = 17{,}946 \text{ (Turbulent)}$$

Applying Equation (7.61) for turbulent flow in tubes

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \text{ where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(17{,}946)^{0.8}\,(2.04)^{0.4} = 77.4$$

$$\bar{h}_i = \overline{Nu}_D\,\frac{k}{D} = 77.4\;\frac{0.675\text{ W/(m K)}}{0.016\text{ m}} = 3265\ \text{W/(m}^2\text{K)}$$

(b) The scaling resistance can be calculated from the water temperature data
From Figure (10.10)

$$\Delta T_a = T_s - T_{w,\text{in}} = 115°\text{C} - 60°\text{C} = 55°\text{C}$$

$$\Delta T_b = T_s - T_{w,\text{out}} = 115°\text{C} - 110°\text{C} = 5°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)} = \frac{55°\text{C} - 5°\text{C}}{\ln\left(\dfrac{55}{5}\right)} = 21°\text{C}$$

This must be corrected for use in a shell-and-tube heat exchanger according to Figure 10.14. But since $Z = 0$ for condensers, $F = 1$ and $\Delta T_{\text{mean}} = LMTD$, the rate of heat transfer is

$$q = U_o A_o \Delta T_{\text{mean}} = \dot{m}_w\, c_{pw}\,(T_{w,\text{out}} - T_{w,\text{in}})$$

$$\therefore\ U_o = \frac{\dot{m}_w\, c_{pw}}{2N\pi D_o L\,\Delta T_{\text{mean}}}\;(T_{w,\text{out}} - T_{w,\text{in}})$$

$$\therefore\ U_o = \frac{38\text{ kg/s}\quad 4198\text{ J/(kg)}\quad (\text{Ws})/\text{J}}{2(500)\,\pi\,(0.021\text{m})(9\text{m})(21°\text{C})}\,(110°\text{C} - 60°\text{C}) = 640\text{ W/(m}^2\text{ K)}$$

Applying Equation (10.6) ($R_o = 0$)

$$\frac{1}{U_o} = \frac{1}{\bar{h}_o} + R_k + \frac{R_i A_o}{A_i} + \frac{A_o}{A_i \bar{h}_i} = \frac{1}{\bar{h}_o} + R_k + \frac{R_i D_o}{D_i} + \frac{D_o}{D_i \bar{h}_i}$$

Solving for the sum of the scaling, conductive, and outer convective resistances

$$\frac{1}{\bar{h}_o} + R_k + \frac{R_i D_o}{D_i} = \frac{1}{U_o} - \frac{D_o}{D_i \bar{h}_i} = \frac{1}{640\text{ W/(m}^2\text{K)}} - \left(\frac{2.1}{1.6}\right)\;\frac{1}{3265\text{ W/(m}^2\text{K)}} = 0.000116\ (\text{m}^2\text{K})/\text{W}$$

For a double flow rate, the Reynolds number is doubled: $Re_D = 35{,}592$

$$\therefore\ \overline{Nu}_D = 0.023\,(35{,}892)^{0.8}\,(2.04)^{0.4} = 135$$

$$\bar{h}_i = \overline{Nu}_D\,\frac{k}{D} = 135\;\frac{0.675\text{ W/(m K)}}{0.016\text{ m}} = 5685\ \text{W/(m}^2\text{K)}$$

The new overall heat transfer coefficient is

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \text{ where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(17{,}946)^{0.8}\,(2.04)^{0.4} = 77.4$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D} = 77.4 \frac{0.675\ \text{W/(m K)}}{0.016\ \text{m}} = 3265\ (\text{m}^2\ \text{K})/\text{W}$$

From Figure 10.20, $e = 1$

From Equation (10.13b)

$$T_{w,\text{out}} = T_{w,\text{in}} + \mathscr{E}(T_s - T_{w,\text{in}}) = 60°\text{C} + 1\ (115°\text{C} - 60°\text{C}) = 115°\text{C}$$

PROBLEM 10.47

Liquid benzene (specific gravity = 0.86) is to be heated in a counterflow concentric-pipe heat exchanger from 30^{0}C to 90°C. For a tentative design, the velocity of the benzene through the inside pipe (*ID* = 2.7 cm; *OD* = 3.3 cm) can be taken as 8 m/s. Saturated process steam at 1.38 × 10^6 N/m^2 is available for heating. Two methods of using this steam are proposed (a) Pass the process steam directly through the annulus of the exchanger; this would require that the letter be designed for the high pressure. (b) Throttle the steam adiabatically to 138,000 N/m^2 before passing it through the heater. In both cases, the operation would be controlled so that saturated vapor enters and saturated water leaves the heater. As an approximation, assume that for both cases the heat transfer coefficient for condensing steam remains constant at 12,800 W/(m^2 K), that the thermal resistance of the pipe wall is negligible, and that the pressure drop for the steam is negligible. If the inside diameter of the other pipe is 5 cm, calculate the mass rate of flow of steam (kg/s per pipe) and the length of heater required for each arrangement.

GIVEN

A concentric pipe, counterflow heat exchanger–benzene in inner tube; saturated steam in annulus
Specific gravity of benzene (s.g.) = 0.86
Benzene temperatures

- $T_{b,\text{in}}$ = 30°C
- $T_{b,\text{out}}$ = 90°C

Pipe diameters

- D_{ii} = 2.7 cm = 0.027 m
- D_{io} = 3.3 cm = 0.033 m
- D_o = 5 cm = 0.05 m

Benzene velocity (V_b) = 2.5 m/s
Saturated steam pressure = 1.38 × 10^6 N/m^2
Saturated vapor enters condenser and saturated water leaves

FIND

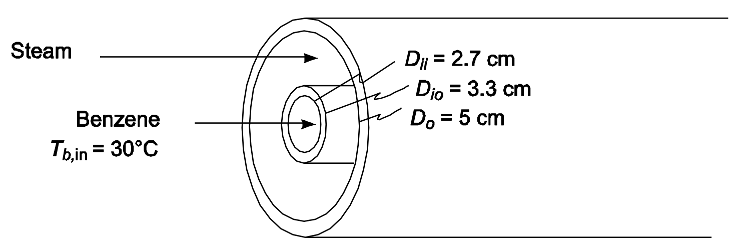
The mass flow rate of steam $(\dot{m}_s)$ and the length of the heater (L) for

(a) Passing steam directly through condenser, and
(b) Steam throttled adiabatically to 138,000 N/m^2 before the heater

ASSUMPTIONS

The heat transfer coefficient on the steam side $(\bar{h}_o)$ = 12,800 W/(m^2 K)
The thermal resistance of the pipe wall is negligible
The pressure drop for the steam is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the saturation temperature and heat of vaporization of steam at

1.38 × 10^6 N/m^2 (T_{sa}) = 194°C h_{fga} = 1963 kJ/kg
1.38 × 10^5 N/m^2 (T_{sb}) = 108°C h_{fgb} = 2236 kJ/kg

From Appendix 2, Table 21, for benzene at the average temperature of 60°C

Specific heat (c_p) = 1908 J/(kg K)

Thermal conductivity (k) = 0.149 W/(m K)

Kinematic viscosity (ν) = 0.485 × 10^{-6} m^2/s

Prandtl number (Pr) = 4.6

Density (ρ) = 859 kg/m^3

SOLUTION

The Reynolds number of the benzene flow is

$$Re_D = \frac{V_b D_{ii}}{\nu} = \frac{(2.5\,\text{m/s})(0.027\,\text{m})}{0.485\times10^{-6}\,\text{m}^2/\text{s}} = 1.39 \times 10^5 \text{ (Turbulent)}$$

The Nusselt number can be calculated using Equation (7.61)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \text{ where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(1.39 \times 10^5)^{0.8}\,(4.6)^{0.4} = 551$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D_i} = 551 \frac{0.149\,\text{W/(m K)}}{0.027\,\text{m}} = 3044 \text{ W/(m}^2\text{ K)}$$

The overall heat transfer coefficient, neglecting wall resistance is

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + \frac{A_o}{A_i \bar{h}_i} = \frac{1}{\bar{h}_o} + \frac{D_o}{D_i \bar{h}_i} = \frac{1}{12{,}800\,\text{W/(m}^2\text{K)}} + \left(\frac{3.3}{2.7}\right)\frac{1}{3041\,\text{W/(m}^2\text{K)}}$$

$$U_o = 2085 \text{ W/(m}^2\text{ K)}$$

(a) T_s = 194°C and $C_{\min}/C_{\max} = 0$
From Equation (10.22b) ($C_c = C_{\min}$)

$$\mathsf{E} = \frac{T_{b,\text{out}} - T_{b,\text{in}}}{T_s - T_{b,\text{in}}} = \frac{90 - 30}{194 - 30} = 0.37$$

From Figure 10.18 or 10.19 NTU = 0.5

$$NTU = \frac{UA}{C_{\min}} = \frac{U_o\,\pi\,D_{io}\,L}{\dot{m}_b\,c_p} = \frac{U_o\,\pi\,D_{io}\,L}{\rho V_b \frac{\pi}{4} D_{ii}^2\,c_p}$$

$$\therefore\quad L = \frac{NTU\,\rho V_b D_{ii}^2\,c_p}{4 U_o D_{io}} = \frac{0.5\;\;859\,\text{kg/m}^3\;\;\;2.5\,\text{m/s}\;\;(0.027\,\text{m})^2\;\;1908\,\text{J/(kg K)}}{4\;\;2085\,\text{W/(m}^2\text{K)}\;\;\;\text{J/(Ws)}\;\;(0.033\,\text{m})} = 5.4 \text{ m}$$

The rate of heat transfer is

$$q = \mathsf{E}\,C_{\min}\,(T_s - T_{b,\text{in}}) = \dot{m}_s\; h_{\text{fga}}$$

$$\dot{m}_s = \frac{\mathsf{E}\,C_{\min}}{h_{fga}}\,(T_s - T_{b,\text{in}}) = \frac{\mathsf{E}\,\rho V_o \frac{\pi}{4} D_{ii}^2\,c_p}{h_{fga}}\,(T_s - T_{b,\text{in}})$$

$$\dot{m}_s = \frac{0.37\;\;859\,\text{kg/m}^3\;\;\;2.5\,\text{m/s}\;\;(0.027)^2\;\;1908\,\text{J/(kg K)}}{1963\,\text{kJ/kg}\;\;\;1000\,\text{J/(kJ)}}\,(194°\text{C} - 30°\text{C}) = 0.073 \text{ kg/s}$$

(b) T_s = 108°C $C_{\min}/C_{\max} = 0$ e = (90 – 30)/(108 – 30) = 0.77

From Figure 10.18 or 10.19 $NTU \approx 1.5$

If U_o remains the same and $C_{\min}$ is the same as case (a), then

$$L = \frac{NTU\,(\text{a})}{NTU\,(\text{b})}\; L\,(\text{a}) = \frac{1.5}{0.5}\,(5.4\text{ m}) = 16.2\text{ m}$$

$$\dot{m}_s = \frac{0.77\;\; 859\,\text{kg/m}^3\;\;\; 2.5\,\text{m/s}\;\; \frac{\pi}{4}(0.027)^2\;\; 1908\,\text{J/(kg K)}}{2236\,\text{kJ/kg}\;\;\; 1000\,\text{J/(kJ)}}\;(108°\text{C} - 30°\text{C}) = 0.063\text{ kg/s}$$

COMMENTS

Throttling the steam reduces the flow rate of steam required by 14% but increases the size of the condenser by 300%. Economic considerations are needed to choose between the two options.

PROBLEM 10.48

Calculate the overall heat transfer coefficient and the rate of heat flow from the hot gasses to the cold air in the cross flow tube-bank of heat exchanger shown in the accompanying illustration for the following operating conditions

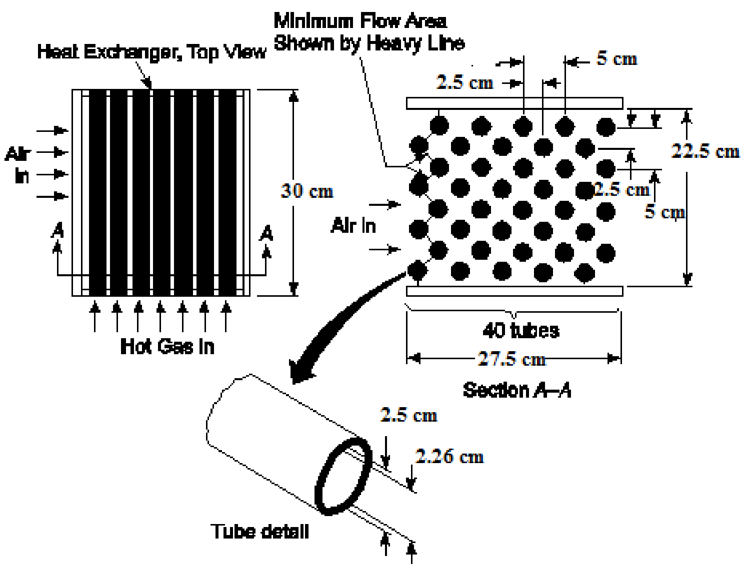
Air flow rate = 0.4 kg/s.

Hot gas flow rate = 0.65 kg/s.

Temperature of hot gasses entering exchanger = 870°C.

Temperature of cold air entering exchanger = 40°C.

Both gases are approximately at atmospheric pressure.



GIVEN

- The crossflow tube bank heat exchanger shown above
- Air flow rate $(\dot{m}_a) = 0.4$ kg/s
- Gas flow rate $(\dot{m}_g) = 0.65$ kg/s
- Entrance temperatures
  - Air ($T_{a,\text{in}}$) = 40°C
  - Gas ($T_{g,\text{in}}$) = 870°C
- Both gases are at 1 atm pressure

FIND

(a) The overall heat transfer coefficient
(b) The rate of heat transfer (q)

ASSUMPTIONS

- The hot gases have the same thermal properties as air
- No scaling
- Thermal resistance of the tube walls can be neglected

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the entering temperatures

Temperature	40°C	870°C

Thermal conductivity, k (W/(m K))	0.0263	0.0704
Specific heat, c_p (J/(kg K))	1014	1123
Density, ρ (kg/m^3)	1.092	0.312
Absolute viscosity, μ (N s/m^2)	19.1×10^{-6}	45.1×10^{-6}
Prandtl number	0.71	0.73

SOLUTION

(a) Heat transfer coefficient inside tubes $\bar{h}_i$

The Reynolds number in the tubes is

$$Re_D = \frac{V_g D_i}{v_g} = \frac{4\dot{m}}{\pi D_i \mu} = \frac{4(0.65\ kg/s)}{\pi(0.0226\ \text{m})(45.1\times10^{-6}\ Ns/\text{m})} = 8.14 \times 10^5 \text{ (Turbulent)}$$

The Nusselt number for turbulent flow in a tube is given by Equation (6.61)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr_n \text{ where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\,(8.14 \times 10^5)^{0.80}\,(0.73)^{0.3} = 1120$$

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D_i} = 1120\, \frac{(0.0704\,W/(\text{m K}))}{0.0226 \text{ m}} = 3489 \text{ W/(m}^2\text{ K)}$$

Heat transfer coefficient outside the tubes $\bar{h}_o$

The velocity of the air based on the minimum flow area is

$$V_{\max} = \frac{\dot{m}}{\rho A_{\min}}$$

From the sketch

$$A_{\min} = [7\,(S'_L - D_o) + 0.025 \text{ m}]L \qquad \text{where } S'_L = \sqrt{(2.5 \text{ cm})^2 + (2.5 \text{ cm})^2}$$

$$= 3.53 \text{ cm} = 0.0353 \text{ m}$$

$$\therefore A_{\min} = [7\,(0.0353 - 0.025) + 0.025]\ 0.3 \text{ m} = 0.02913 \text{ m}^2$$

$$\therefore V_{\max} = \frac{(0.4\ kg/s)}{(1.092\ kg/m^3)(0.02913 m^2)} = 12.57 \text{ m/s}$$

The Reynolds number based on the minimum flow area is

$$Re_D = \frac{V_{\max} D}{v} = \frac{(12.57\ m/s)(0.025\ m)}{(17.6\times10^{-6}\ m^2/\text{s})} = 17855 \text{ (Turbulent)}$$

Applying Equation (6.27)

$$Nu_D = 0.035 \left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{0.6} \Pr^{0.36} \left(\frac{\Pr}{\Pr_s}\right)^{0.25} = 0.035 \left(\frac{1.25}{2.16}\right)^{0.2} (17855)^{0.6}$$

$$(0.71)^{0.36} = 98.6$$

$$h_o = Nu_D \frac{k}{D_o} = 98.6\, \frac{(0.0263 W/(\text{m } K))}{(0.025 \text{ m})} = 103.8\ W/(\text{m}^2 \text{ K})$$

The overall heat transfer coefficient, neglecting the thermal resistance of the tube wall, is

$$\frac{1}{U_d} = \frac{1}{\bar{h}_o} + \frac{A_o}{A_i \bar{h}_i} = \frac{1}{\bar{h}_o} + \frac{D_o}{D_i \bar{h}_i} = \frac{1}{(103.8\, W/(\mathrm{m}^2\, K))} + \left(\frac{1}{0.902}\right)\frac{1}{(3489\, W/(\mathrm{m}^2\, K))}$$

$$U_o = 100.5 \text{ W/(m}^2 \text{ K)}$$

(b) The heat capacity rates are

$$C_a = \dot{m}_a\, c_{pa} = (0.4 \text{ kg/s})(1014\, W/(\text{kg}\, K)) = 405.6 \text{ W/K}$$

$$C_g = \dot{m}_g\, c_{pg} = (0.65 \text{ kg/s})(1123\, J/(kg\, K)) = 730 \text{ W/K}$$

$$\frac{C_{\text{mixed}}}{C_{\text{unmixed}}} = \frac{C_a}{C_g} = \frac{405.6}{730} = 0.55$$

The number of transfer units is

$$NTU = \frac{U_o A_o}{C_{\min}} = \frac{U_o N \pi D_o L}{C_{\min}} = \frac{(100.5\, W/(\mathrm{m}^2 \text{ K}))(40)\pi(0.025\, m)(0.3 \text{ m})}{(405.6\, W/\text{K})} =$$

0.233

From Figure 10.22, e ≈ 0.25

The rate of heat transfer is given by Equation (10.23)

$$q = \varepsilon\, C_{\min}\,(T_{g,\text{in}} - T_{a,\text{in}}) = 0.25\ (405.6\, W/K)\ (870°\text{C} - 40°\text{C}) = 84162 \text{ W}$$

PROBLEM 10.49

> **An oil having a specific heat of 2100 J/(kg K) enters an oil cooler at 82°C at the rate of 2.5 kg/s. The cooler is a counterflow unit with water as the coolant, the transfer area being 28 m^2 and the overall heat transfer coefficient is 570 W/(m^2 K). The water enters the exchanger at 27°C. Determine the water rate required if the oil is to leave the cooler at 38°C.**

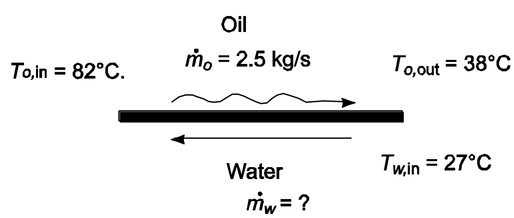
GIVEN

- Counterflow heat exchanger - water cools oil
- Oil specific heat (c_{po}) = 2100 J/(kg K)
- Oil temperatures
  - $T_{o,\text{in}}$ = 82°C
  - $T_{o,\text{out}}$ = 38°C
- Oil flow rate ($\dot{m}_o$) = 2.5 kg/s
- Transfer area (A) = 28 m^2
- The overall heat transfer coefficient (U) = 570 W/(m^2 K)
- Water inlet temperature ($T_{w,\text{in}}$) = 27°C

FIND

- The water flow rate ($\dot{m}_w$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the specific heat of water (c_{pw}) = 4175 J/(kg K) at 40°C

SOLUTION

The heat rate of the oil is

$$C_o = \dot{m}_o \; c_{po} = (2.5 \text{ kg/s}) \quad 2100 \text{ J/(kg K)} \quad = 5250 \text{ W/K}$$

Assuming $C_o = C_{\text{min}}$, the effectiveness, from Equation (10.22a) is

$$\text{E} = \frac{T_{o,\text{in}} - T_{o,\text{out}}}{T_{o,\text{in}} - T_{w,\text{in}}} = \frac{82 - 38}{82 - 27} = 0.80$$

Combining Equations (10.23) and (10.16)

$$q = e\, C_{\text{min}} \,(T_{o,\text{in}} - T_{w,\text{in}}) = U\,A\,(LMTD)$$

Solving for the log mean temperature difference

$$LMTD = \frac{\text{E}\, C_{\text{min}}}{U\,A}\,(T_{o,\text{in}} - T_{w,\text{in}}) = \frac{0.8 \;\; 5250 \text{ W/K}}{570 \text{ W/(m}^2\text{ K)} \;\; (28\,\text{m}^2)} \;(80°\text{C} - 27°\text{C}) = 14.5°\text{C}$$

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)} = \frac{(T_{o,\text{in}} - T_{w,\text{out}}) - (T_{w,\text{out}} - T_{w,\text{in}})}{\ln\left(\dfrac{T_{o,\text{in}} - T_{w,\text{out}}}{T_{o,\text{out}} - T_{w,\text{in}}}\right)}$$

$$14.5°C = \frac{(82°C - T_{w,\text{out}}) - (38°C - 27°C)}{\ln\left(\frac{82 - T_{w,\text{out}}}{38 - 27}\right)} = \frac{71°C - T_{w,\text{out}}}{\ln\left(\frac{82 - T_{w,\text{out}}}{11}\right)}$$

By trial and error $T_{w,\text{ou}}\text{t} = 63°C$

The flow rate of water can be calculated from an energy balance

$$\dot{m}_o \ c_{po} \ (C_{o,\text{in}} - T_{o,\text{out}}) = \dot{m}_w \ c_{pw} \ (T_{w,\text{out}} - T_{w,\text{in}})$$

$$\dot{m}_w = \dot{m}_o \left(\frac{c_{po}}{c_{pw}}\right)\left(\frac{T_{o,\text{in}} - T_{w,\text{out}}}{T_{o,\text{out}} - T_{w,\text{in}}}\right) = (2.5 \text{ kg/s})\left(\frac{2100}{4175}\right)\left(\frac{82 - 38}{63 - 27}\right) = 1.54 \text{ kg/s}$$

The heat capacity rate of the water is

$$C_w = \dot{m}_w \ c_{pw} = 1.54 \text{ kg/s} \quad 4175 \text{ J/(kg K)} \quad = 6417 \text{ W/K}$$

Therefore, the assumption that $C_o = C_{\text{min}}$ is valid.

PROBLEM 10.50

While flowing at the rate of 1.25 kg/s in a simple counterflow heat exchanger, dry air is cooled from 65 to 38°C by means of cold air which enters at 15°C and flows at a rate of 1.6 kg/s. It is planned to lengthen the heat exchanger so that 1.25 kg/s. of air can be cooled from 65 to 26°C with a counterflow current of air at 1.6 kg/s entering at 15°C. Assuming that the specific heat of the air is constant, calculate the ratio of the length of the new heat exchanger to the length of the original.

GIVEN

A simple adiabatic air-to-air counter flow heat exchanger:

Case 1

- Warm air temperatures
  - $T_{h,\text{in}}$ = 65°C
  - $T_{h,\text{out}}$ = 38°C
- Air flow rates
  - $\dot{m}_h$ = 1.25 kg/s
  - $\dot{m}_c$ = 1.6 kg/s
- Cold air inlet temperature ($T_{c,\text{in}}$) = 15°C

After lengthening heat exchanger:

Case 2

- Warm air temperatures
  - $T_{h,\text{in}}$ = 65°C
  - $T_{h,\text{out}}$ = 26°C
- Air flow rates
  - $\dot{m}_h$ = 1.25 kg/s
  - $\dot{m}_c$ = 1.6 kg/s
- Cold air inlet temperature ($T_{c,\text{in}}$) = 15°C

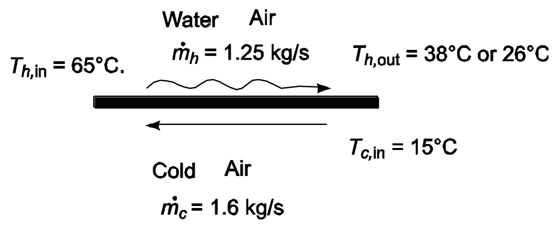
FIND

- The ratio of the length of the new heat exchanger to the length of the original

ASSUMPTIONS

- The specific heat of air is constant
- The overall heat transfer coefficient (U) is the same in both cases

SKETCH



SOLUTION

For both cases

$$\frac{C_{\min}}{C_{\max}} = \frac{C_h}{C_c} = \frac{\dot{m}_h c_p}{\dot{m}_c c_p} = \frac{\dot{m}_h}{\dot{m}_c} = \frac{1.25}{1.6} = 0.78$$

The effectiveness, from Equation (10.22a), is

$$\mathsf{E} = \frac{C_h}{C_{\min}} \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}}$$

Case 1

$$e_1 = \frac{65 - 38}{65 - 15} = 0.54$$

Case 2

$$e_2 = \frac{65 - 26}{65 - 15} = 0.78$$

From Figure 10.19 $\quad NTU_1 = 1.1 \qquad NTU_2 = 2.5$

$$\frac{NTU_1}{NTU_2} = \frac{\dfrac{U_2 A_2}{C_{\min 2}}}{\dfrac{U_1 A_1}{C_{\min 1}}} \qquad \text{But } U_1 = U_2 \quad \text{and} \quad C_{\min 1} = C_{\min 2}$$

$$\therefore \quad \frac{A_2}{A_1} = \frac{NTU_2}{NTU_1} = \frac{2.5}{1.1} = 2.3$$

Since the area is directly proportional to the length.

$$\frac{L_2}{L_1} = 2.3$$

PROBLEM 10.51

Saturated steam at 137 kPa condenses on the outside of a 2.6-m-length of copper tubing heating 5 kg/hr of water flowing in the tube. The water temperatures, measured at 10 equally spaced stations along the tube length are

Station	1	2	3	4	5	6	7	8	9	10	11
Temp °C	18	43	57	67	73	78	82	85	88	90	92

Calculate (a) average overall heat transfer coefficient U_o based on the outside tube area; (b) average water-side heat transfer coefficient *hw* (assume steamside coefficient at h_s = 11,000 W/(m^2 K)), (c) local overall coefficient U_x based on the outside tube area for each of the 10 sections between temperature stations, and (d) local waterside coefficients h_{wx} for each of the 10 sections. Plot all items vs. tube length. Tube dimensions: *ID* = 2 cm, *OD* = 2.5 c. Temperature station 1 is at tube entrance and station 11 is at tube exit.

GIVEN

- Saturated steam condensing on copper tubing with water flowing within
- Steam pressure = 1.35 atm = 136,755 N/m^2
- Tube length (L) = 2.6 m
- Water flow rate $\dot{m}_w$ = 5 kg/h = 0.00139 kg/s
- Water temperatures given above as a function of distance along pipe
- Tube diameters
  - D_i = 2 cm = 0.02 m
  - D_o = 2.5 cm = 0.025 m

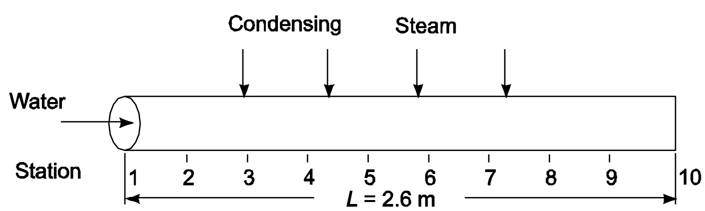
FIND

(a) Average overall heat transfer coefficient based on the outside tube area (U_o)
(b) Average water-side transfer coefficient $\bar{h}_w$
(c) Local overall coefficient (U_x) for each of the 10 sections
(d) Local water-side coefficient h_{wx} for each of the 10 sections
Plot all items vs. tube length

ASSUMPTIONS

- The steam-side heat transfer coefficient $\bar{h}_s$ = 11,000 W/(m^2 K)
- No scaling resistance
- Variation of the specific heat of water is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for saturated steam at 136,755 N/m^2: T_s = 107°C
For water at the average temperature of 55°C, the specific heat (c_{pw}) = 4180 J/(kg K)
From Appendix 2, Table 12, the thermal conductivity of copper (k) = 392 W/(m K) at 127°C

SOLUTION

(a) The heat capacity rate of the water is

$$C_w = \dot{m}_w c_{pw} = (0.00139\,\text{kg/s}\;\; 4180\,\text{J/(kg K)} \;\; = 5.81\ \text{W/K}$$

Since the heat capacity rate of the steam is essentially infinite, $C_{min}/C_{max} = 0$.
The effectiveness of the heat exchanger is given by Equation (10.22b) ($C_c = C_{min}$).

$$\varepsilon = \frac{T_{w,\text{out}} - T_{w,\text{in}}}{T_s - T_{w,\text{in}}} = \frac{92 - 18}{107 - 18} = 0.83$$

For $C_{min}/C_{max} = 0$, the effectiveness for parallel or counterflow are the same and Equation (10.26) reduces to

$$e = 1 - e^{-NTU} \Rightarrow NTU = -\ln(1 - e) = -\ln(1 - 0.83) = 1.77$$

$$NTU = \frac{U_o A_o}{C_{\min}} \Rightarrow U_o = \frac{NTU\, C_{\min}}{A_o} = \frac{NTU\, C_{\min}}{\pi D_o L} = \frac{1.77\;\; 5.81\,\text{W/K}}{\pi\;\; 0.025\,\text{m}\;\;\; 2.6\,\text{m}} = 50.4\ \text{W/(m}^2\text{K)}$$

(b) From Equation (10.3)

$$\frac{1}{U_o} = \frac{A_o}{A_i \bar{h}_i} + \frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{1}{\bar{h}_o} = \frac{D_o}{D_i \bar{h}_w} + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{1}{\bar{h}_s}$$

$$\therefore \qquad \frac{1}{\bar{h}_w} = \frac{D_i}{D_o}\left(\frac{1}{U} - \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k} - \frac{1}{\bar{h}_s}\right)$$

$$\frac{1}{\bar{h}_w} = \left(\frac{1}{(50.4\ \text{W/(m}^2\text{K)})} - \frac{(0.025\,\text{m})\ln\left(\frac{2.5}{2}\right)}{2(392\ \text{W/(m K)})} - \frac{1}{(11{,}000\ \text{W/(m}^2\ \text{K)})}\right)$$

(c) Treating the first section as a separate heat exchanger and following the procedure of part (a)
$C_{max}/C_{min} = 0$, $e = (43 - 18)/(107 - 18) = 0.28$, $NTU = -\ln(1 - 0.28) = 0.33$

$$\therefore \qquad U_x = \frac{NTU\, C_{\min}}{A_o} = \frac{NTU\, C_{\min}}{\pi D_o L} = \frac{0.33\;\; 5.81\,\text{W/K}}{\pi\;\; 0.025\,\text{m}\;\;\; 0.26\,\text{m}} = 93.9\ \text{W/(m}^2\text{K)}$$

This procedure must be repeated for each section. The results are tabulated below section (d).

(d) Following the procedure of section (b), the only value that changes is the overall heat transfer coefficient

$$\frac{1}{h_{w1}} = \frac{2}{2.5}\left(\frac{1}{93.9\ \text{W/(m}^2\text{K)}} - 0.000098\,(\text{m}^2\,\text{K})/\text{W}\right)$$

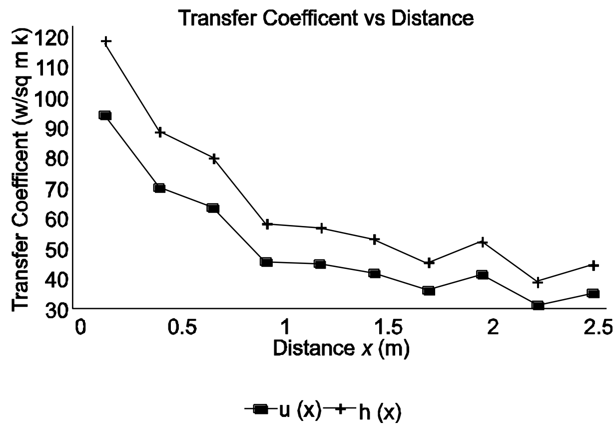
$$h_{w1} = 118.5\ \text{W/(m}^2\ \text{K)}$$

Repeating parts (c) and (d) for each section yields

Section	1	2	3	4	5	6	7	8	9	10
x (m)	0.13	0.39	0.65	0.91	1.17	1.43	1.69	1.95	2.21	2.47
U_x (W/(m^2 K))	93.9	70.2	63.5	46.2	45.3	42.2	36.4	41.7	31.6	35.6
h_{wx} (W/(m^2 K))	118.5	88.4	79.9	58.0	56.9	53.0	45.7	52.3	39.6	44.7

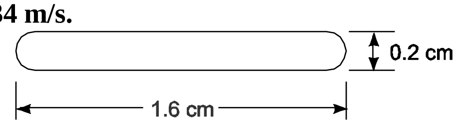
where x = distance from inlet to midpoint of section.

Plotting this data on a single graph



PROBLEM 10.52

Calculate the water side heat transfer coefficient and the coolant pressure drop per unit length of tube for the core of a compact air-to-water intercooler for a 3.7 MW gas turbine plant. The water flows inside of a flattened aluminum tube having the cross-section shown belowThe inside diameter of the tube before it was flattened was 1.23 cm with a wall thickness (*t*) of 0.025 cm. The water enters the tube at 15.6°C and leaves at 26.7°C at a velocity of 1.34 m/s.



GIVEN

Water flow in a flattened tube as shown above
Inlet temperature (T_i) = 15.6°C
Outlet temperature (T_o) = 26.7°C
Water velocity (V) = 1.34 m/s

FIND

(a) The heat transfer coefficient on the inside of the tubes $\bar{h}_c$

(b) The pressure drop per unit length ($\Delta p/L$)

ASSUMPTIONS

Steady state
Fully developed flow

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the average temperature of 21.1°C

Thermal conductivity (k) = 0.599 W/(m K)
Kinematic viscosity (ν) = 0.979 × 10^{-6} m^2/s
Density (ρ) = 998.0 kg/m^3
Prandtl number (Pr) = 6.80

SOLUTION

The hydraulic diameter of the flattened tube is

$$D_h = \frac{4A}{P} = \frac{4[\,1.6\,\text{cm} - 0.2\,\text{cm} \quad 0.2\,\text{cm} + \pi(0.1\,\text{cm})^2\,]}{\pi \ 1.23\,\text{cm}} = 0.322\ \text{cm} = 0.00322\ \text{m}$$

The Reynolds number based on the hydraulic diameter is

$$Re_{D_h} = \frac{VD_h}{\nu} = \frac{(1.34\ \text{m/s})\ 0.00322\,\text{m}}{0.979\times 10^{-6}\ \text{m}^2/\text{s}} = 4407\ \text{(turbulent)}$$

(a) The Nusselt number for turbulent flow is given by Equation (7.61)

$$\overline{Nu} = 0.023\ Re_{D_h}^{\ 0.8}\ Pr^{0.4} = 0.023\ (4407)^{0.8}\ (608)^{0.4} = 40.7$$

$$\bar{h}_c = \overline{Nu}\ \frac{k}{D_h} = 40.7\,\frac{0.599\ \text{W/(m K)}}{0.00322\,\text{m}} = 7580\ \text{W/(m}^2\text{K)}$$

(b) The friction factor for turbulent flow in smooth tubes is given by Equation (7.57)

$$f = 0.184\ Re_{D_h}^{\ -0.2} = 0.184\ (4407)^{-0.2} = 0.0344$$

The pressure drop is given by Equation (7.13)

$$\frac{\Delta P}{L} = \frac{f}{D_h}\frac{\rho V^2}{2g_c} = \frac{0.0344}{0.0322\text{m}}\ \frac{998.0\ \text{kg/m}^3\quad 1.34\ \text{m/s}^{\ 2}}{2\ \ (\text{kg m})/(\text{s}^2\text{N})\quad \text{N}/(\text{m}^2\text{Pa})} = 9560\ \text{Pa/m}$$

PROBLEM 10.53

An air-to-water compact heat exchanger is to be designed to serve as an intercooler for a 3.7 MW gas turbine plant. The exchanger is to meet the following heat transfer and pressure drop performance specifications

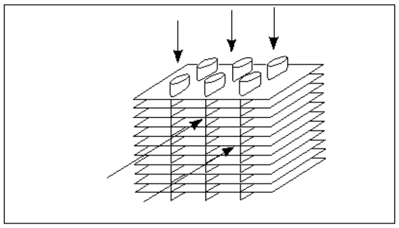
Air-side Operating Conditions

Flow rate	**25.2 kg/s**
Inlet Temperature	**400 K**
Outlet Temperature	**300 K**
Inlet Pressure(p_1)	**2.05×10^5 N/m^2**
Pressure Drop Ratio ($\Delta p/p_1$)	**7.6%**

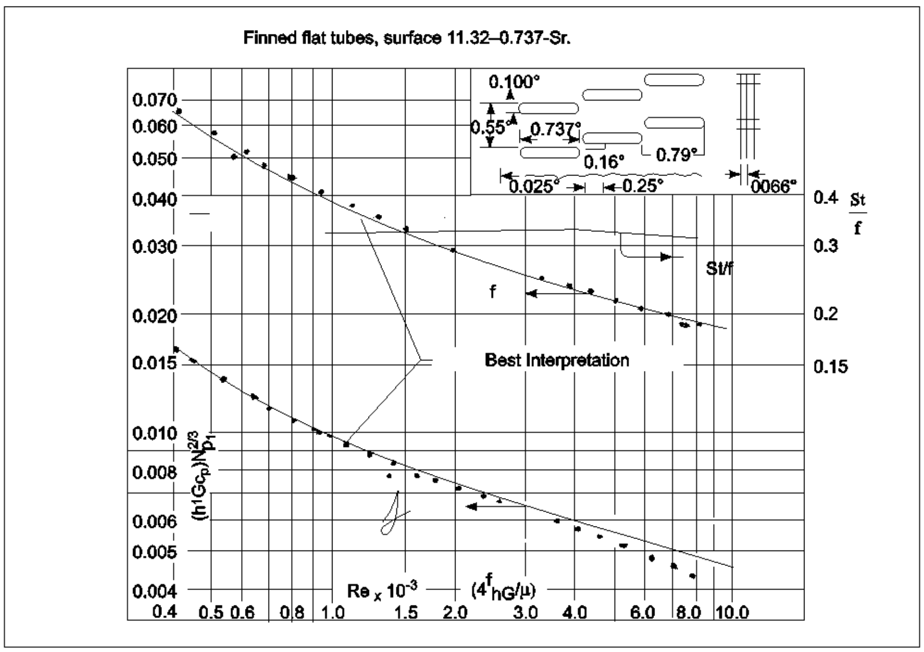
Water-side Operating Conditions

Flow rate	**50.4 kg/s**
Inlet Temperature	**289 K**

The exchanger is to have a cross-flow configuration with both fluids unmixed. The heat exchanger surface proposed for the exchanger consists of flattened tubes with continuous aluminum fins specified as a 11.32– 0.737 – SR surface in Ref. 10. The heat exchanger is shown schematically below.



The measured heat transfer and friction characteristic for this exchanger surface are shown in the graph below



Geometrical details for the proposed surface are

Air side **Flow passage hydraulic radius (r_h) = 0.0878 cm**

Total transfer area/total volume (a_{air}) = 886 m^2/m^3

Free flow area/frontal area (s) = 0.780

Fin area/total area (A_f/A) = 0.845

Fin metal thickness (t) = 0.0001 m

Fin length (1/2 distance between tubes, L_f) = 0.00572 m

Water side **Tubes as given in Problem 10.53**

Water-side transfer area/total volume (a_{H2O}) = 138 m²/m³

The design should specify the core size, the air flow frontal area, and the flow length. The water velocity inside the tubes is 1.34 m/s. See Problem 10.53 for the calculation of the water side heat transfer coefficient.

Note: (i) the free-flow area is defined such that the mass velocity, G, is the air mass flow rate per unit free flow area, (ii) the core pressure drop is given by $\Delta p = fG^2L/2\rho r_h$ where L is the length of the core in the air flow direction, (iii) the fin length, L_f, is defined such that $L_f = 2A/P$ where A is the fin cross-sectional area for heat conduction and P is the effective fin perimeter.

GIVEN

- Air-to-Water Intercooler with the geometry and requirements specified above
- From Problem 10.53: Water side convective heat transfer coefficient ($h_{c,H2O}$) = 7580 W/(m² K)

FIND

(a) The air flow frontal area (A_{air})
(b) The flow length (L)
(c) The core size

ASSUMPTIONS

- Steady state
- Entrance effects are negligible
- Flow acceleration effects are negligible
- Negligible fouling resistance
- Negligible variation in thermal resistance
- The thermal resistance of the tube wall is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for air at the mean temperature of 77°C

Specific heat (c) = 1019 J/(kg K)

Density (ρ) = 0.977 kg/m³

Prandtl number (Pr) = 0.71

Absolute viscosity (μ) = 20.6 × 10⁶ (Ns)/m²

From Appendix 2, Table 13, for water at 20°C, c = 4182 J/(kg K)

From Appendix 2, Table 12, the thermal conductivity of aluminum at 320 K (k_a) = 238 W/(m K)

SOLUTION

The outlet water temperature is given by the conservation of energy

$$\dot{m}_{H2O}\ c_{H2O}\ (T_{in} - T_{out})_{H2O} = \dot{m}_{air}\ c_{air}\ (T_{in} - T_{out})_{air}$$

$$T_{out,H2O} = T_{in,H2O} + \frac{\dot{m}c_{\ air}}{\dot{m}c_{\ H2O}}\ (T_{in} - T_{out})_{air} = 289\text{ K} + \frac{25.2\ \ 1019}{50.4\ \ 4182}(400\text{ K} - 300\text{ K}) = 301\text{ K}$$

The effectiveness required for the specified performance is given by Equation (10.22a)

$$E = \frac{C_h\ (T_{h,\text{in}} - T_{h,\text{out}})}{C_{\min}\ (T_{h,\text{in}} - T_{c,\text{in}})}$$

Since $C_h = C_{\min}$

$$E = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \frac{400\,\text{K} - 300\,\text{K}}{400\,\text{K} - 289\,\text{K}} = 0.90$$

The heat capacity rate ratio is

$$\frac{C_{\min}}{C_{\max}} = \frac{C_{\text{air}}}{C_{\text{H2O}}} = \frac{(\dot{m}c)_{\text{air}}}{(\dot{m}c)_{\text{H2O}}} = \frac{25.2\ (1019)}{50.4\ (4182)} = 0.122$$

From Figure 10.21 for cross-flow heat exchangers with $e = 0.9$ and $C_{\min}/C_{\max} = 0.122$, $NTU_{\max} = 2.75 = U_{\text{air}} A_{\text{air}}/C_{\min}$.

The solution will require iteration. For the first iteration, let $Re_D = 10^4 = 4\ r_h\ G/\mu$

Solving for the mass velocity

$$G = \frac{Re_d\,\mu}{4r_h} = \frac{10^4\ (20.6\times10^{-6}\ \text{kg/ms})}{4\ (0.000878\,\text{m})} = 58.7\ \text{kg}/(\text{m}^2\text{s})$$

From the graphical data at $Re = 10^4$

$$\frac{\bar{h}}{Gc_p} Pr^{\frac{2}{3}} \approx 0.0045 \qquad f \approx 0.018$$

$$\therefore\ \bar{h} = 0.0045\ Gc_p\ Pr^{-\frac{2}{3}} = (0.0045)\ (58.7\ \text{kg}/(\text{m}^2\text{s}))\ (1019\ (\text{Ws})/(\text{kg K}))\ (0.71)^{-\frac{2}{3}} = 337\ \text{W}/(\text{m}^2\text{K})$$

Since $\quad G = \dfrac{\dot{m}_a}{A_{\text{free flow.}}}$

$$A_{\text{free flow}} = \frac{\dot{m}_a}{G} = \frac{25.2\ \text{kg/s}}{58.7\ \text{kg}/(\text{m}^2\text{s})} = 0.429\ \text{m}^2$$

$$A_{\text{frontal}} = \frac{A_{\text{free flow}}}{\sigma} = \frac{0.429\,\text{m}^2}{0.78} = 0.55\ \text{m}^2$$

We must calculate the fin efficiency per Chapter 2

$$m = \sqrt{\frac{h_a P}{kA}} = \sqrt{\frac{2h_a}{k_f L_f}} = \sqrt{\frac{2\ (337\ \text{W}/(\text{m}^2\text{K}))}{(283\,\text{W}/(\text{m K}))\ (0.0001\,\text{m})}} = 154\ \text{m}^{-1}$$

$$m\,L_f = 154\ \text{m}^{-1}\ (0.0572\ \text{m}) = 0.883$$

The fin efficiency from Equation (2.72) is

$$\eta_f = \frac{\tanh\ (mL_f)}{mL_f} = 0.80$$

The total fin efficiency can be calculated from Equation (2.73)

$$\eta_t = 1 - \frac{A_f}{A}\,(1 - \eta_f) = 1 - 0.845\ (1 - 0.80) = 0.83$$

and the overall heat transfer coefficient from Equation (2.74) is

$$U_{\text{air}} = \left(\frac{1}{\eta_t h_a} + \frac{1}{\left(\dfrac{\alpha_{\text{H2O}}}{\alpha_{\text{air}}}\right) h_{\text{H2O}}} \right)^{-1} = \left(\frac{1}{0.83\left(337\ \text{W/(m}^2\text{K)}\right)} + \frac{1}{\left(\dfrac{42.1}{270}\right)\left(7580\ \text{W/(m}^2\text{K)}\right)} \right)^{-1}$$

$$A_{\text{air}} = \frac{NTU\ \dot{m}_a c_{pa}}{U_{air}} = \frac{(2.75)(25.2\ \text{kg/s})\ \ 1019\ \text{Ws/(kg K)}}{227\ \text{W/(m}^2\text{K)}} = 311\ \text{m}^2$$

$$\text{Heat exchanger volume } V = \frac{A_{\text{air}}}{\alpha_{\text{air}}} = \frac{311\,\text{m}^2}{886\ \text{m}^2/\text{m}^3} = 0.35\ \text{m}^3$$

$$\text{Core length } L = \frac{V}{A_{\text{frontal}}} = \frac{0.35\,\text{m}^3}{0.55\,\text{m}^2} = 0.64\ \text{m}$$

The core pressure drop is

$$\Delta p = f\,\frac{G^2}{2\rho}\,\frac{L}{r_h} = \frac{0.018\ \ 58.5\ \text{kg/(m}^2\text{s)}^{\ 2}}{2\ \ 0.977\ \text{kg/m}^3}\ \frac{0.64\,\text{m}}{0.000878\,\text{m}} = 22{,}980\ \ \text{N/m}^2$$

$$\frac{\Delta p}{p_1} = \frac{22{,}980}{205{,}000} = 11\% > 7.6\%\ \text{(Too High)}$$

Repeating this procedure until $\Delta p/p_1 = 7.6\%$ yields the following results

$Re = 8400$	$A_{\text{free}} = 0.513\ \text{m}^2$	$A_{\text{air}} = 342\ \text{m}^2$
$h\ Pr^{\frac{2}{3}}/G\ c_p = 0.0047$	$A_{\text{frontal}} = 0.657\ \text{m}^2$	$V = 0.386\ \text{m}^3$
$f = 0.019$	$\eta_f = 0.821$	$L = 0.587\ \text{m}$
$G = 49.14\ \text{kg/(m}^2\ \text{s)}$	$\eta_{oa} = 0.849$	$\Delta p = 15{,}685\ \text{N/m}^2$
$h = 295\ \text{W/(m}^2\ \text{K)}$	$U_{\text{air}} = 207\ \text{W/(m}^2\ \text{K)}$	$\dfrac{\Delta p}{\Delta p_1} = 0.077$ as required

PROBLEM 10.54

Microchannel compact heat exchangers can be used to cool high heat flux micro-electronic devices. The sketch below shows a schematic view of a typical microchannel heat sink. Micro-fabrication techniques can be used to mass produce aluminum channels and fins with the following dimensions:

$$\mathbf{W_c = W_w = 50\ micrometers}$$
$$\mathbf{b = 200\ micrometers}$$
$$\mathbf{L = 1.0\ cm}$$
$$\mathbf{t = 100\ micrometers}$$

Assuming there are a total of 100 fins and that water at 30°C is used as the cooling medium at a Reynolds number of 2000 estimate

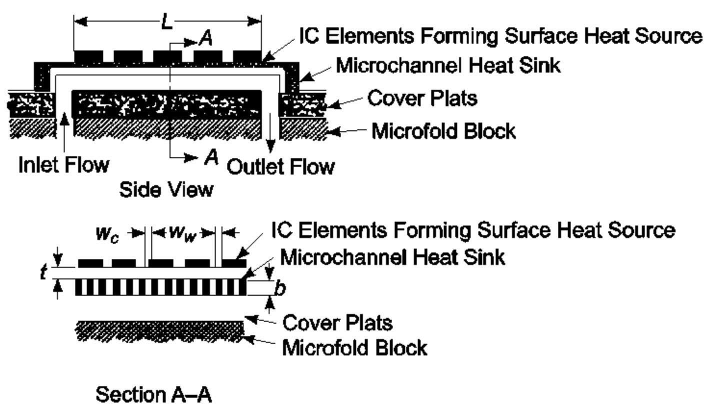
(a) The water flow rate through all the channels

(b) The Nusselt number

(c) The heat transfer coefficient

(d) The effective thermal resistance between the *IC* elements forming the heat source and the cooling water

(e) The rate of heat dissipation allowable if the temperature difference between source and water is not to exceed 100 K



GIVEN

- An aluminum microchannel heat exchanger as shown above
- $w_c = w_w = 50$ micrometers
- $b = 200$ micrometers
- $L = 1.0$ cm
- $t = 100$ micrometers
- Total number of fins (N) = 100
- Cooling water temperature (T_w) = 30°C
- Reynolds number (Re) = 2000

FIND

(a) The water flow rate $\dot{m}$ through all the channels

(b) The Nusselt number $\overline{Nu}$

(c) The heat transfer coefficient $\bar{h}_c$

(d) The effective thermal resistance (R_{eff}) between the IC elements forming the heat source and the cooling water

(e) The rate of heat dissipation (Q_{heatsink}) allowable if the temperature difference between source and water ($T_{\text{IC}} - T_{\text{fluid}}$) is not to exceed 100 K

ASSUMPTIONS

- Steady state
- Uniform and constant heat generation
- The heat generation chip is the same size as the heat exchanger
- A conducting paste has been applied between the heat sink and the IC to eliminate contact resistance
- The cover plate is an insulator

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 30°C

Absolute viscosity (μ) = 792.4×10^{-6} (N s)/m^2

Thermal conductivity (k) = 0.615 W/(m K)

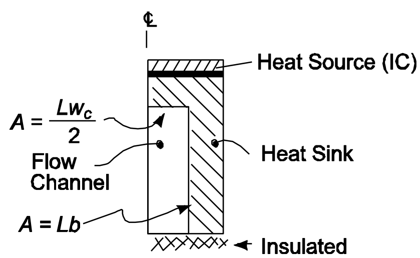
Specific heat (c_p) = 4176 J/(kg K)

From Appendix 2, Table 12

The thermal conductivity of aluminum at 30°C (k_w) = 238 W/(m K)

SOLUTION

A cross-section of the flow channel is shown schematically below



Since we assume there is no contact resistance between the IC and the heat sink, the top of the heat sink is at the IC temperature, T_{IC}. Heat is transferred by conduction through the heat sink directly from the IC to the area on top of the flow channel (area $Lw_c/2$) to the coolant and also along the tall portion of the heat sink to area Lb to the coolant. This latter part of the heat sink acts as a fin because the temperature of this part of the heat sink will decrease as we move down from the IC to the cover plate. As described in Chapter 2, this temperature decrease can be accounted for by the fin efficiency. Given the average heat transfer coefficient, $\bar{h}_c$ in the flow channel and the temperature of the heat sink at the top of the flow channel, T_{TOP}, we can write the rate of heat transfer to the coolant for the half flow channel shown in the sketch above as

$$q = \bar{h}_c L \left(\frac{w_c}{2} + b\eta_f \right) (T_{TOP} - T_{fluid})$$

where η_f is the fin efficiency of the heat sink.

The temperature drop from the IC to the top of the flow channel can be estimated by

$$\frac{k_w L \;\; \frac{w_w + w_w}{2} \;\; T_{IC} - T_{TOP}}{t} = q$$

solving for T_{TOP}

$$T_{TOP} = T_{IC} - \frac{2tq}{k_w \;\; w_w + w_c \;\; L}$$

We can now eliminate T_{TOP} from the equation for q

$$q = \frac{\bar{h}_c L\left(\dfrac{w_c}{2+b\eta_f}\right)(T_{\text{IC}} - T_{\text{fluid}})}{1+\dfrac{\bar{h}_c L\left(\dfrac{w_c}{2+b\eta_f}\right)2t}{k_w (w_w + w_c) L}}$$

so the effective thermal resistance is given by

$$R_{\text{eff}} \frac{T_{\text{IC}} - T_{\text{fluid}}}{q} = \frac{1+\dfrac{\bar{h}_c\left(\dfrac{w_c}{2+b\eta_f}\right)2t}{k_w (w_w + w_c)}}{L\bar{h}_c\left(\dfrac{w_c}{2+b\eta_f}\right)}$$

We find the average heat transfer coefficient as follows. The hydraulic diameter of the channel is

$$D_h = \frac{4A_c}{P} = \frac{4bw_c}{2(b+w_c)} = \frac{4(2\times10^{-4})(5\times10^{-5})}{2(2\times10^{-4}+5\times10^{-5})} = 8 \times 10^{-5}\ \text{m}$$

(a) The total mass flow rate can be calculated from the definition of the Reynolds number

$$Re_{D_h} = \frac{D_h\rho}{\mu} U_\infty = \frac{D_h\rho}{\mu}\left(\frac{\dot{m}}{Nbw_c\rho}\right) = \frac{D_h\dot{m}}{Nbw_c\mu}$$

$$m = \frac{ReNbw_c\mu}{D_h} = \frac{2000\ (100)\ (2\times10^{-4}\ \text{m})\ (5\times10^{-5}\ \text{m})\ (792.4\times10^{-6}\ (\text{Ns})/\text{m}^2)\ (\text{kg m})/(\text{Ns}^2)}{8\times10^{-5}\text{m}} = 0.020\ \text{kg/s}$$

(b) The aspect ratio of the channels is b/w_c = 200/50 = 4. The length-to-hydraulic diameter ratio is L/D_h = (0.01 m)/(8 × 10^{-5} m) = 125, therefore, the flow in the channels should be fully developed and we find the Nusselt number, from Table 7.1. If we assume that the fin efficiency will be high, then it is safe to assume that the flow chanenl is isothermal at any cross section. (In this argument, we are neglecting the fact that the channel is insulated from below). Therefore, we need Nu_{H1} which is 5.33.

(c)
$$h_c = \frac{k}{D_h} Nu = \frac{0.615\ \text{W/(m K)}}{8\times10^{-5}} (5.33) = 40{,}974\ \text{W/(m}^2\text{K)}$$

The fin efficiency can be determined from Equation (2.71)

$$\eta_f = \frac{\tanh\sqrt{\dfrac{\bar{h}_c P L^2{}_f}{kA}}}{\sqrt{\dfrac{\bar{h}_c P L^2{}_f}{kA}}}$$

For this fin we have

$$\bar{h}_c = 40{,}974\ \text{W/(m}^2\text{K)}$$

$$P = 2L$$

$$L_f = 200 \times 10^{-6}\ \text{m}$$

$$A = w_w L$$

First, calculate

$$\sqrt{\frac{\bar{h}_c P L^2{}_f}{kA}} = \sqrt{\frac{40{,}974\ \mathrm{W/(m^2K)}\ \ 2L\ \ {200\times10^{-6}\ \mathrm{m}}^{\ 2}}{238\ \mathrm{W/(m\,K)}\ \ 50\times10^{-6}\ \mathrm{m}\ \ L}} = 0.524$$

and then

$$\eta_f = \frac{\tanh\ 0.524}{0.524} = 0.92$$

Our assumption that the flow channel all is isothermal is fairly good.

(d) We can now quantify the effective thermal resistance. First calculate the quantity

$$\bar{h}_c\left(\frac{w_c}{2+b\eta_f}\right) = 40{,}974\ \left(\mathrm{W/(m^2\,K)}\right)(25\times10^{-6}\ \mathrm{m} + (200\times10^{-6}\ \mathrm{m})\ (0.92)) = 8.56\ \ \mathrm{W/(m\,K)}$$

so

$$R_{\mathrm{eff}} = \frac{1+\dfrac{8.56\ \mathrm{W/(m\,K)}\ \ \ 2\ \ \ 100\times10^{-6}\ \mathrm{m}}{238\ \mathrm{W/(m\,K)}\ \ \ 100\times10^{-6}\ \mathrm{m}}}{8.56\ \mathrm{W/(m\,K)}\ \ (0.01\,\mathrm{m})} = 12.5\ \mathrm{K/W}$$

(e) The heat transfer for the half channel is therefore

$$q = \frac{100\,\mathrm{K}}{12.5\,\mathrm{K/W}} = 7.98\ \mathrm{W}$$

and for the entire heat sink

$$Q_{\mathrm{heatsink}} = 7.98\times2\times100 = 1597\ \mathrm{W}$$