CHAPTER 19

19.1 A table of integrals can be consulted to determine

$$\int \tanh\ dx = \frac{1}{a}\ln\ \cosh ax$$

Therefore,

$$\int_0^t \sqrt{\frac{gm}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}}t\right)\ dt = \sqrt{\frac{gm}{c_d}}\sqrt{\frac{m}{gc_d}}\left[\ln\ \cosh\left(\sqrt{\frac{gc_d}{m}}t\right)\right]_0^t$$

$$\sqrt{\frac{gm^2}{gc_d^2}}\left[\ln\ \cosh\left(\sqrt{\frac{gc_d}{m}}t\right) - \ln\ \cosh(0)\right]$$

Since cosh(0) = 1 and ln(1) = 0, this reduces to

$$\frac{m}{c_d}\ln\ \cosh\left(\sqrt{\frac{gc_d}{m}}t\right)$$

19.2 **(a)** The analytical solution can be evaluated as

$$\int_0^4 (1-e^{-x})\ dx = \left[x+e^{-x}\right]_0^4 = 4+e^{-4}-0-e^{-0} = 3.018316$$

(b) single application of the trapezoidal rule

$$(4-0)\frac{0+0.981684}{2} = 1.963369 \qquad (\varepsilon_t = 34.95\%)$$

(c) composite trapezoidal rule

<u>n = 2:</u>

$$(4-0)\frac{0+2(0.864665)+0.981684}{4} = 2.711014 \qquad (\varepsilon_t = 10.18\%)$$

<u>n = 4:</u>

$$(4-0)\frac{0+2(0.632121+0.864665+0.950213)+0.981684}{8} = 2.93784 \qquad (\varepsilon_t = 2.67\%)$$

(d) single application of Simpson's 1/3 rule

$$(4-0)\frac{0+4(0.864665)+0.981684}{6} = 2.960229 \qquad (\varepsilon_t = 1.92\%)$$

(e) composite Simpson's 1/3 rule (n = 4)

$$(4-0)\frac{0+4(0.632121+0.950213)+2(0.864665)+0.981684}{12} = 3.013449 \qquad (\varepsilon_t = 0.16\%)$$

(f) Simpson's 3/8 rule.

$$(4-0)\frac{0+3(0.736403+0.930517)+0.981684}{8} = 2.991221 \qquad (\varepsilon_t = 0.9\%)$$

(g) Simpson's rules (n = 5):

$$I = (1.6-0)\frac{0+4(0.550671)+0.798103}{6}$$
$$+(4-1.6)\frac{0.798103+3(0.909282+0.959238)+0.981684}{8}$$
$$= 0.80021+2.215604 = 3.015814 \qquad \varepsilon_t = 0.08\%$$

19.3 (a) Analytical solution:

$$\int_0^{\pi/2} (8+4\cos x)\, dx = \left[8x+4\sin x\right]_0^{\pi/2} = 8(\pi/2)+4\sin(\pi/2)-0 = 16.56637$$

(b) Trapezoidal rule (n = 1):

$$I = (1.570796-0)\frac{12+8}{2} = 15.70796 \qquad \varepsilon_t = \left|\frac{16.56637-15.70796}{16.56637}\right| \times 100\% = 5.182\%$$

(c) Trapezoidal rule (n = 2):

$$I = (1.570796-0)\frac{12+2(10.82843)+8}{4} = 16.35861 \qquad \varepsilon_t = 1.254\%$$

Trapezoidal rule (n = 4):

$$I = (1.570796-0)\frac{12+2(11.69552+10.82843+9.530734)+8}{8} = 16.51483 \qquad \varepsilon_t = 0.311\%$$

(d) Simpson's 1/3 rule:

$$I = (1.570796-0)\frac{12+4(10.82843)+8}{6} = 16.57549 \qquad \varepsilon_t = 0.055\%$$

(e) Simpson's rule (n = 4):

$$I = (1.570796-0)\frac{12+4(11.69552+9.530734)+2(10.82843)+8}{12} = 16.56691 \qquad \varepsilon_t = 0.0032\%$$

(f) Simpson's 3/8 rule:

$$I = (1.570796-0)\frac{12+3(11.4641+10)+8}{8} = 16.57039 \qquad \varepsilon_t = 0.024\%$$

(g) Simpson's rules (n = 5):

$$I = (0.628319-0)\frac{12+4(11.80423+11.23607}{6}$$
$$+(1.570796-0.628319)\frac{11.23607+3(10.35114+9.236068)+8}{8}$$
$$= 7.377818+9.188887 = 16.5667 \qquad \varepsilon_t = 0.002\%$$

19.4 (a) The analytical solution can be evaluated as

$$\int_{-2}^{4} (1-x-4x^3+2x^5)\ dx = \left[x-\frac{x^2}{2}-x^4+\frac{x^6}{3} \right]_{-2}^{4}$$

$$= 4-\frac{4^2}{2}-4^4+\frac{4^6}{3}-(-2)+\frac{(-2)^2}{2}+(-2)^4-\frac{(-2)^6}{3}=1104$$

(b) single application of the trapezoidal rule

$$(4-(-2))\frac{-29+1789}{2}=5280 \qquad (\varepsilon_t=378.3\%)$$

(c) composite trapezoidal rule

<u>*n* = 2:</u> $(4-(-2))\dfrac{-29+2(-2)+1789}{4}=2634 \qquad (\varepsilon_t=138.6\%)$

<u>*n* = 4:</u> $(4-(-2))\dfrac{-29+2(1.9375+(-2)+131.3125)+1789}{8}=1516.875 \qquad (\varepsilon_t=37.4\%)$

(d) single application of Simpson's 1/3 rule

$$(4-(-2))\frac{-29+4(-2)+1789}{6}=1752 \qquad (\varepsilon_t=58.7\%)$$

(e) composite Simpson's 1/3 rule (*n* = 4)

$$(4-(-2))\frac{-29+4(1.9375+131.3125)+2(-2)+1789}{12}=1144.5 \qquad (\varepsilon_t=3.6685\%)$$

(f) Simpson's 3/8 rule.

$$(4-(-2))\frac{-29+3(1+31)+1789}{8}=1392 \qquad (\varepsilon_t=26.09\%)$$

(g) Boole's rule.

$$(4-(-2))\frac{7(-29)+32(1.9375)+12(-2)+32(131.3125)+7(1789)}{90}=1104 \qquad (\varepsilon_t=0\%)$$

19.5 Note that the following numerical results (a and c) are based on the tabulated values. Slightly different results would be obtained if the function is used to generate values with more significant digits.

(a) The analytical solution can be evaluated as

$$\int_{0}^{1.2} e^{-x}\ dx = \left[-e^{-x}\right]_0^{1.2} = -e^{-1.2}-(-e^0)=0.698805788$$

(b) Trapezoidal rule

$$(0.1-0)\frac{1+0.9048}{2}+(0.3-0.1)\frac{0.9048+0.7408}{2}+(0.5-0.3)\frac{0.7408+0.6065}{2}$$

$$+(0.7-0.5)\frac{0.6065+0.4966}{2}+(0.95-0.7)\frac{0.4966+0.3867}{2}+(1.2-0.957)\frac{0.3867+0.3012}{2}$$

$$=0.09524+0.16456+0.13473+0.11031+0.110413+0.085988=0.70124 \quad (\varepsilon_t=0.35\%)$$

(c) Trapezoidal and Simpson's Rules

$$(0.1-0)\frac{1+0.9048}{2}+(0.7-0.1)\frac{0.9048+3(0.7408+0.6065)+0.4966}{8}$$
$$+(1.2-0.7)\frac{0.4966+4(0.3867)+0.3012}{6} = 0.09524+0.408248+0.195383=0.698871 \quad (\varepsilon_t = 0.0093\%)$$

19.6 **(a)** The integral can be evaluated analytically as,

$$\int_{-2}^{2}\left[\frac{x^3}{3}-3y^2x+y^3\frac{x^2}{2}\right]_0^4 dy$$
$$\int_{-2}^{2}\frac{(4)^3}{3}-3y^2(4)+y^3\frac{(4)^2}{2}\,dy$$
$$\int_{-2}^{2}21.33333-12y^2+8y^3\,dy$$
$$\left[21.33333y-4y^3+2y^4\right]_{-2}^{2}$$
$$21.33333(2)-4(2)^3+2(2)^4-21.33333(-2)+4(-2)^3-2(-2)^4=21.33333$$

(b) The composite trapezoidal rule with n = 2 can be used the evaluate the inner integral at the three equispaced values of y,

y = –2: $\quad (4-0)\dfrac{-12+2(-24)-28}{4}=-88$

y = 0: $\quad (4-0)\dfrac{0+2(4)+16}{4}=24$

y = 2: $\quad (4-0)\dfrac{-12+2(8)+36}{4}=40$

These results can then be integrated in y to yield

$$(2-(-2))\frac{-88+2(24)+40}{4}=0$$

which represents a percent relative error of

$$\varepsilon_t=\left|\frac{21.33333-0}{21.33333}\right|\times 100\%=100\%$$

which is not very good.

(c) Single applications of Simpson’s 1/3 rule can be used the evaluate the inner integral at the three equispaced values of y,

y = –2: $\quad (4-0)\dfrac{-12+4(-24)-28}{6}=-90.66667$

y = 0: $\quad (4-0)\dfrac{0+4(4)+16}{6}=21.33333$

y = 2: $\quad (4-0)\dfrac{-12+4(8)+36}{6}=37.33333$

These results can then be integrated in y to yield

$$(2-(-2))\frac{-90.66667+4(21.33333)+37.33333}{6}=21.33333$$

which represents a percent relative error of

$$\varepsilon_t=\left|\frac{21.33333-21.33333}{21.33333}\right|\times 100\%=0\%$$

which is perfect

19.7 (a) The integral can be evaluated analytically as,

$$\int_{-4}^{4}\int_{0}^{6}\left[\frac{x^4}{4}-2yzx\right]_{-1}^{3}dy\ dz=\int_{-4}^{4}\int_{0}^{6}\ 20-8yz\ dy\ dz$$

$$\int_{-4}^{4}\int_{0}^{6}\ 20-8yz\ dy\ dz=\int_{-4}^{4}\left[20y-4zy^2\right]_0^6\ dz=\int_{-4}^{4}\ 120-144z\ dz$$

$$\int_{-4}^{4}\ 120-144z\ dz=\left[120z-72z^2\right]_{-4}^{4}=120(4)-72(4)^2-120(-4)+72(-4)^2=960$$

(b) Single applications of Simpson's 1/3 rule can be used the evaluate the inner integral at the three equispaced values of y for each value of z,

<u>z = –4:</u>

y = 0: $(3-(-1))\frac{-1+4(1)+27}{6}=20$

y = 3: $(3-(-1))\frac{23+4(25)+51}{6}=116$

y = 6: $(3-(-1))\frac{47+4(49)+75}{6}=212$

These results can then be integrated in y to yield

$$(6-0)\frac{20+4(116)+212}{6}=696$$

<u>z = 0:</u>

y = 0: $(3-(-1))\frac{-1+4(1)+27}{6}=20$

y = 3: $(3-(-1))\frac{-1+4(1)+27}{6}=20$

y = 6: $(3-(-1))\frac{-1+4(1)+27}{6}=20$

These results can then be integrated in y to yield

$$(6-0)\frac{20+4(20)+20}{6}=120$$

<u>z = 4:</u>

y = 0: $$(3-(-1))\frac{-1+4(1)+27}{6}=20$$

y = 3: $$(3-(-1))\frac{-25+4(-23)+3}{6}=-76$$

y = 6: $$(3-(-1))\frac{-49+4(-47)-21}{6}=-172$$

These results can then be integrated in y to yield

$$(6-0)\frac{20+4(-76)-172}{6}=-456$$

The results of the integrations in y can then be integrated in z to yield a perfect result:

$$(6-0)\frac{696+4(120)-456}{6}=960$$

19.8 (a) The trapezoidal rule can be used to determine the distance

$$I=(2-1)\frac{5+6}{2}+(3.25-2)\frac{6+5.5}{2}+\bullet\bullet\bullet=60.125$$

and the average,

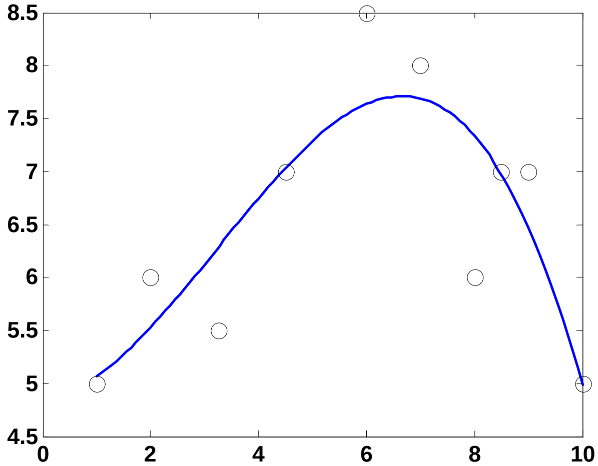
$$\bar{v}=\frac{60.125}{10-1}=6.680556$$

(b) The polynomial can be fit as,

```
>> format long
>> t = [1 2 3.25 4.5 6 7 8 8.5 9 10];
>> v = [5 6 5.5 7 8.5 8 6 7 7 5];
>> p = polyfit(t,v,3)
p =
  -0.01800093017330   0.17531524519646   0.06028850825860   4.85065783294403
```

The cubic can be plotted along with the data,

```
>> tt = linspace(1,10);
>> vv = polyval(p,tt);
>> plot(tt,vv,t,v,'o')
```



This equation can be integrated to yield

$$x = \int_1^{10} -0.018t^3 + 0.1753t^2 + 0.06029t + 4.8066 \; dt$$

$$= \left[-0.0045t^4 + 0.05844t^3 + 0.030144t^2 + 4.8066t \right]_1^{10} = 60.02235$$

19.9

z	$w(z)$	$\rho g w(z)(75 - z)$	$\rho g z w(z)(75 - z)$
75	200	0.0000E+00	0.0000E+00
62.5	190	2.3299E+07	1.4562E+09
50	175	4.2919E+07	2.1459E+09
37.5	160	5.8860E+07	2.2073E+09
25	135	6.6218E+07	1.6554E+09
12.5	130	7.9706E+07	9.9633E+08
0	122	8.9762E+07	0.0000E+00

$$f_t = 75 \frac{8.9762 + 4(7.9706 + 5.8860 + 2.3299) + 2(6.6218 + 4.2919) + 0}{3(6)} \times 10^7 = 3.9812 \times 10^9$$

$$\int_0^D \rho g z w(z)(D - z) \; dz = 75 \frac{0 + 4(0.99633 + 2.2073 + 1.4562) + 2(1.6554 + 2.1459) + 0}{3(6)} \times 10^9 = 1.0934 \times 10^{11}$$

$$d = \frac{1.0934 \times 10^{11}}{3.9812 \times 10^9} = 27.464$$

19.10 (a) Trapezoidal rule:

$$f = 30 \frac{0 + 2(71.653 + 68.456 + 55.182 + 42.176 + 31.479) + 23.200}{2(6)} = 1402.728$$

$$f = \frac{30 \dfrac{0 + 2(358.266 + 684.556 + 827.729 + 843.511 + 786.982) + 696.010}{2(6)}}{1402.728} = \frac{19,245.24}{1402.728} = 13.720 \text{ m}$$

(b) Simpson’s 1/3 rule:

$$f = 30\frac{0+4(71.653+55.182+31.479)+2(68.456+42.176)+23.200}{3(6)} = 1462.867$$

$$f = \frac{30\dfrac{0+4(358.266+827.729+786.982)+2(684.556+843.511)+696.010}{3(6)}}{1462.867} = \frac{19406.75}{1462.867} = 13.266\text{ m}$$

19.11 The values needed to perform the evaluation can be tabulated:

Height *l*, m	Force, *F*(*l*), N/m	*l*×*F*(*l*)
0	0	0
30	340	10,200
60	1,200	72,000
90	1,550	139,500
120	2,700	324,000
150	3,100	465,000
180	3,200	576,000
210	3,500	735,000
240	3,750	900,000

Because there are an even number of equally-spaced segments, we can evaluate the integrals with the multi-segment Simpson's 1/3 rule.

$$F = (240-0)\frac{0+4(340+1550+3100+3500)+2(1200+2700+3200)+3750}{24} = 519{,}100$$

$$I = (240-0)\frac{0+4(10200+139500+465000+735000)+2(72000+324000+576000)+900000}{24}$$

$$= 82{,}428{,}000$$

The line of action can therefore be computed as

$$d = \frac{82{,}428{,}000}{519{,}100} = 158.7902$$

19.12 (a) Analytical solution:

$$M = \int_0^{11} 5+0.25x^2\ dx = \left[5x+0.083333x^3\right]_0^{11} = 165.9167$$

(b) Composite trapezoidal rule:

$$I = (1-0)\frac{5+5.25}{2} + (2-1)\frac{5.25+6}{2} + \bullet\bullet\bullet + (11-10)\frac{30+35.25}{2} = 166.375$$

(c) Composite Simpson's rule:

$$I = (2-0)\frac{5+4(5.25)+6}{6} + (4-2)\frac{6+4(7.25)+9}{6} + \bullet\bullet\bullet + (11-8)\frac{21+3(25.25+30)+35.25}{8} = 165.9167$$

19.13 We can set up a table that contains the values comprising the integrand

x, cm	ρ, g/cm^3	A_c, cm^2	$\rho \times A_c$, g/cm
0	4	100	400
400	3.95	103	406.85

600	3.89	106	412.34
800	3.8	110	418
1200	3.6	120	432
1600	3.41	133	453.53
2000	3.3	150	495

We can integrate this data using a combination of the trapezoidal and Simpson's rules,

$$I = (400-0)\frac{400+406.85}{2} + (800-400)\frac{406.85+4(412.34)+418}{6}$$
$$+(2000-800)\frac{418+3(432+453.53)+495}{8} = 861{,}755.8 \text{ g} = 861.7558 \text{ kg}$$

19.14 We can set up a table that contains the values comprising the integrand

***t*, hr**	***t*, d**	**rate (cars/4 min)**	**rate (cars/d)**
7:30	0.312500	18	6480
7:45	0.322917	23	8280
8:00	0.333333	14	5040
8:15	0.343750	24	8640
8:45	0.364583	20	7200
9:15	0.385417	9	3240

We can integrate this data using a combination of Simpson's 3/8 and 1/3 rules. This yields the number of cars that go through the intersection between 7:30 and 9:15 (1.75 hrs),

$$I = (0.34375-0.3125)\frac{6480+3(8280+5040)+8640}{8} + (0.385417-0.34375)\frac{8640+4(7200)+3240}{6}$$
$$= 215.1563 + 282.5 = 497.6563 \text{ cars}$$

The number of cars going through the intersection per minute can be computed as

$$\frac{497.6563 \text{ cars}}{1.75 \text{ hr}}\frac{\text{hr}}{60 \text{ min}} = 4.7396\frac{\text{cars}}{\text{min}}$$

19.15 We can use Simpson's 1/3 rule to integrate across the *y* dimension,

<u>*x* = 0:</u> $I = (4-0)\frac{-2+4(-4)-8}{6} = -17.3333$

<u>*x* = 4:</u> $I = (4-0)\frac{-1+4(-3)-8}{6} = -14$

<u>*x* = 8:</u> $I = (4-0)\frac{4+4(1)-6}{6} = 1.3333$

<u>*x* = 12:</u> $I = (4-0)\frac{10+4(7)+4}{6} = 28$

These values can then be integrated along the *x* dimension with Simpson's 3/8 rule:

$$I = (12-0)\frac{-17.3333+3(-14+1.3333)+28}{8} = -41$$

19.16
```
>> t=[0 10 20 30 35 40 45 50];
>> Q=[4 4.8 5.2 5.0 4.6 4.3 4.3 5.0];
>> c=[10 35 55 52 40 37 32 34];
>> Qc=Q.*c;
>> M=trapz(t,Qc)
M =
  9.5185e+003
```

The problem can also be solved with a combination of Simpson's 3/8 and 1/3 rules:

```
>> M=(30-0)*(Qc(1)+3*(Qc(2)+Qc(3))+Qc(4))/8;
>> M=M+(50-30)*(Qc(4)+4*(Qc(5)+Qc(7))+2*Qc(6)+Qc(8))/12
M =
  9.6235e+003
```

Thus, the answers are 9.5185 g (trapezoidal rule) and 9.6235 g (Simpson's rules).

19.17 A table can be set up to hold the values that are to be integrated:

y, **m**	***H***, **m**	***U***, **m/s**	***UH***, **m^2/s**
0	0.5	0.03	0.015
2	1.3	0.06	0.078
4	1.25	0.05	0.0625
5	1.7	0.12	0.204
6	1	0.11	0.11
9	0.25	0.02	0.005

The cross-sectional area can be evaluated using a combination of Simpson's 1/3 rule and the trapezoidal rule:

$$A_c = (4-0)\frac{0.5+4(1.3)+1.25}{6} + (6-4)\frac{1.25+4(1.8)+1}{6} + (9-6)\frac{1+0.25}{2} = 4.633333 + 3.15 + 1.875 = 9.658333 \text{ m}^2$$

The flow can be evaluated in a similar fashion:

$$Q = (4-0)\frac{0.015+4(0.078)+0.0625}{6} + (6-4)\frac{0.0625+4(0.234)+0.11}{6} + (9-6)\frac{0.11+0.005}{2}$$

$$= 0.259667 + 0.3695 + 0.1725 = 0.801667 \ \frac{\text{m}^3}{\text{s}}$$

19.18 First, we can estimate the areas by numerically differentiating the volume data. Because the values are equally spaced, we can use the second-order difference formulas from Fig. 23.1 to compute the derivatives at each depth. For example, at the first depth, we can use the forward difference to compute

$$A_s(0) = -\frac{dV}{dz}(0) = -\frac{-1,963,500 + 4(5,105,100) - 3(9,817,500)}{8} = 1,374,450 \text{ m}^2$$

For the interior points, second-order centered differences can be used. For example, at the second point at (z = 4 m),

$$A_s(4) = -\frac{dV}{dz}(4) = -\frac{1,963,500 - 9,817,500}{8} = 981,750 \text{ m}^2$$

The other interior points can be determined in a similar fashion

$$A_s(8) = -\frac{dV}{dz}(8) = -\frac{392,700 - 5,105,100}{8} = 589,050 \text{ m}^2$$
$$A_s(12) = -\frac{dV}{dz}(12) = -\frac{0 - 1,963,500}{8} = 245,437.5 \text{ m}^2$$

For the last point, the second-order backward formula yields

$$A_s(16) = -\frac{dV}{dz}(16) = -\frac{3(0) - 4(392,700) + 1,963,500}{8} = -49,087.5 \text{ m}^2$$

Since this is clearly a physically unrealistic result, we will assume that the bottom area is 0. The results are summarized in the following table along with the other quantities needed to determine the average concentration.

***z*, m**	***V*, m^3**	***c*, g/m^3**	***A$_s$*, m^2**	***c*×*A$_s$***
0	9817500	10.2	1374450.0	14019390
4	5105100	8.5	981750.0	8344875
8	1963500	7.4	589050.0	4358970
12	392700	5.2	245437.5	1276275
16	0	4.1	0	0

The necessary integrals can then be evaluated with the multi-segment Simpson's 1/3 rule,

$$\int_0^z A_s(z)\,dz = (16-0)\frac{1,374,450 + 4(981,750 + 245,437.5) + 2(589,050) + 0}{12} = 9,948,400 \text{ m}^3$$
$$\int_0^z c(z)A_s(z)\,dz = (16-0)\frac{14,019,390 + 4(8,344,875 + 1,276,275) + 2(4,358,970) + 0}{12} = 81,629,240 \text{ g}$$

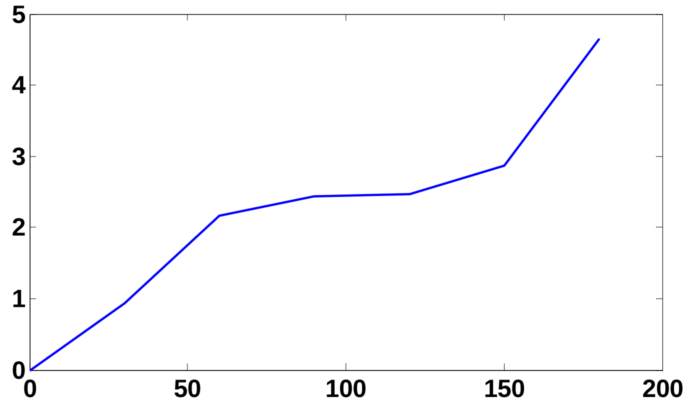
The average concentration can then be computed as

$$\bar{c} = \frac{\int_0^Z c(z)A_s(z)\,dz}{\int_0^Z A_s(z)\,dz} = \frac{81,629,240}{9,948,400} = 8.205263\frac{\text{g}}{\text{m}^3}$$

19.19 The following script can be written to solve this problem:

```
format short g
x=[0 1 2.8 3.9 3.8 3.2 1.3];
th=[0 30 60 90 120 150 180];
F=cos(th*pi/180);
W=cumtrapz(x,F)
plot(th,W)

W =
    0    0.93301     2.1624     2.4374     2.4624     2.8722      4.645
```



To here:
19.20 The work can be computed as

$$W = \int_0^{30} (1.6x - 0.045x^2)\cos(-0.00055x^3 + 0.0123x^2 + 0.13x)\, dx$$

For the 4-segment trapezoidal rule, we can compute values of the integrand at equally-spaced values of x with h = 7.5. The results are summarized in the following table,

x	$F(x)$	$\theta(x)$	$F(x)\cos\theta(x)$	Trap Rule
0	0	0	0	
7.5	9.46875	1.434844	1.283339	4.812522
15	13.875	2.86125	-13.333330	-45.187464
22.5	13.21875	2.887031	-12.792760	-97.972837
30	7.5	0.12	7.446065	-20.050109
			Sum $\rightarrow$	-158.39789

The finer-segment versions can be generated in a similar fashion. The results are summarized below:

Segments	*W*
4	-158.398
8	-159.472
16	-157.713

The computation can be also implemented with a tool like MATLAB's `quad` function,

```
>> F=@(x) (1.6*x-0.045*x.^2).*cos(-0.00055*x.^3+0.0123*x.^2+0.13*x);
>> W=quad(F,0,30)
W =
 -157.0871
```

Here is how the solution can be developed with the `trapz` function

```
>> format short g
>> x=[0:0.1:30];
>> Fx=1.6*x-.045*x.^2;
>> th=-0.00055*x.^3+0.0123*x.^2+0.13*x;
>> Fcos=Fx.*cos(th);
>> W=trapz(x,Fcos)
```

```
W =
      -157.09
```

19.21 The mass can be computed as

$$m = \int_0^r \rho(r) A_s(r)\, dr$$

The surface area of a sphere, $A_s(r) = 4\pi r^2$, can be substituted to give

$$m = \int_0^r \rho(r) 4\pi r^2\, dr$$

The average density is equal to the mass per volume, where the volume of a sphere is

$$V = \frac{4}{3}\pi R^3$$

where R = the sphere's radius. For this problem, V = 0.0041888 cm^3. The integral can be evaluated by a combination of trapezoidal and Simpson's rules as outlined in the following table

***r*, mm**	***r*, cm**	**ρ (g/cm^3)**	**A_s (cm^2)**	**$\rho\times A_s$**	**Integrals**	**Method**
0	0	6	0	0		
0.12	0.012	5.81	0.00181	0.010514		
0.24	0.024	5.14	0.007238	0.037204		
0.36	0.036	4.29	0.016286	0.069867	0.000959	← Simpson's 3/8 rule
0.49	0.049	3.39	0.030172	0.102283		
0.62	0.062	2.7	0.048305	0.130424	0.002641	← Simpson's 1/3 rule
0.79	0.079	2.19	0.078427	0.171755	0.002569	← Trapezoidal rule
0.86	0.086	2.1	0.092941	0.195176		
0.93	0.093	2.04	0.108687	0.221721		
1	0.1	2	0.125664	0.251327	0.004394	← Simpson's 3/8 rule
				mass →	0.010562	

Therefore, the mass is 0.010562 g and the average density is $\bar{\rho} = 0.010562 / 0.0041888 = 2.521393$ g/cm^3.

An alternative would be to use the trapezoidal rule. This can be done using MATLAB and the `trapz` function,

```
format short g
r=[0 0.12 0.24 0.36 0.49 0.62 0.79 0.86 0.93 1];
rho=[6 5.81 5.14 4.29 3.39 2.7 2.19 2.1 2.04 2];
r=r*0.1;  %convert radius to cm
integrand=rho*4*pi.*r.^2;
m=trapz(r,integrand)
V=4/3*pi*max(r).^3;
density=m./V
```

When this script is run the results are:

```
mass =
     0.010591
density =
       2.5284
```

19.22 The following script can be used to implement these equations and solve this problem with MATLAB.

```
clear,clc,clf
format short g
rr=[0 1100 1500 2450 3400 3630 4500 5380 6060 6280 6380];
rho=[13 12.4 12 11.2 9.7 5.7 5.2 4.7 3.6 3.4 3];
subplot(2,1,1)
plot(rr,rho,'o-')
r=rr*1e3;  %convert km to m
rho=rho*1e6/1e3;  %convert g/cm3 to kg/m3
integrand=rho*4*pi.*r.^2;
mass=cumtrapz(r,integrand);
EarthMass=max(mass)
EarthVolume=4/3*pi*max(r).^3
EarthDensity=EarthMass/EarthVolume/1e3
subplot(2,1,2)
plot(rr,mass,'o-')
```

The results are

```
EarthMass =
  6.1087e+024
EarthVolume =
  1.0878e+021
EarthDensity =
        5.6156
```