# *Spring 2021*
# *Probability and Statistics, CQU*
# *Midterm Review*
# *Uncertainty*

The equation for calculating the velocity of a stream of air flow using a measuring device called a Pitot Tube is given by:

$$v = \sqrt{\frac{2\Delta p}{\rho}}$$

Where $v$ is the air velocity (m/sec), $\Delta p$ is the difference between the static and total air stream pressures (N/m$^2$) and $\rho$ is the ambient density of air (kg/m$^3$). The density is related to the ambient temperature ($T_a$) and ambient pressure ($P_a$) by the Perfect Gas Law

$$\rho = \frac{P_a}{RT_a}$$

Where $R$ is the gas constant for air ($= 287.058\ J/kg - K$). You are given the following nominal values and uncertainties with units:

$$\Delta p = 2000 \pm 25\ N/m^2$$
$$P_a = 101{,}350 \pm 2100\ N/m^2$$
$$T_a = 293 \pm 0.1\ K$$

Compute the <u>dimensionless</u> uncertainty for the measured air velocity

Which parameter has the largest impact on the (dimensionless) air velocity uncertainty?

**Suggestion:** First, compute the dimensionless uncertainties for $\Delta p$, $P_a$ and $T_a$. Second, apply RSS method to obtain dimensionless uncertainty for measured air velocity

***SOLUTION***

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#9 $v=\sqrt{\frac{2\Delta p}{\rho}}$  $v=v(\Delta p,\rho)$

$\rho=\frac{p_a}{RT_a}$  $R=287\ \frac{J}{kg\cdot K}$  $\rho=\rho(p_a,T_a)$

① $u_v=\left[\left(\underbrace{\frac{\Delta p}{v}\frac{\partial v}{\partial \Delta p}u_{\Delta p}}_{I}\right)^2+\left(\underbrace{\frac{\rho}{v}\frac{\partial v}{\partial \rho}u_{\rho}}_{II}\right)^2\right]^{1/2}$ (2 pts)

(2a) I: $\frac{\Delta p}{v}\frac{\partial v}{\partial \Delta p}u_{\Delta p}=\frac{\Delta p}{\sqrt{\frac{2\Delta p}{\rho}}}\left(\frac{1}{2}\sqrt{\frac{\rho}{2\Delta p}}\cdot\frac{2}{\rho}\right)u_{\Delta p}=\sqrt{\frac{\rho\Delta p}{2}}\frac{1}{\sqrt{2\Delta p\rho}}u_{\Delta p}$

$=\frac{u_{\Delta p}}{2}=\frac{1}{2}\frac{\delta\Delta p}{\Delta p}=\frac{1}{2}\frac{25}{2000}=0.00625$ (2pts)

(2b) II: $\frac{\rho}{v}\frac{\partial v}{\partial \rho}u_\rho=\frac{\rho}{\sqrt{\frac{2\Delta p}{\rho}}}\left(\frac{1}{2}\sqrt{\frac{\rho}{2\Delta p}}\cdot\frac{-2\Delta p}{\rho^2}\right)u_\rho$

$=\cancel{\rho}\sqrt{\frac{\rho}{2\Delta p}}\left(-\sqrt{\frac{\rho}{2\Delta p}}\cdot\frac{\Delta p}{\rho^{\cancel{2}}}\right)u_\rho=-\cancel{\Delta p}\frac{\cancel{\rho}\,u_\rho}{2\cancel{\Delta p}}=-\frac{u_\rho}{2}$

Compute $u_\rho$

③ $u_\rho=\left[\left(\underbrace{\frac{p_a}{\rho}\frac{\partial\rho}{\partial p_a}u_{p_a}}_{III}\right)^2+\left(\underbrace{\frac{T_a}{\rho}\frac{\partial\rho}{\partial T_a}u_{T_a}}_{IV}\right)^2\right]^{1/2}$ (2pts)

(4a) III: $\frac{p_a}{\rho}\frac{\partial\rho}{\partial p_a}u_{p_a}=\frac{\cancel{p_a}}{\frac{\cancel{p_a}}{RT_a}}\frac{1}{RT_a}u_{p_a}=u_{p_a}=\frac{2100}{101,350}=0.021$

(4b) IV: $\frac{T_a}{\rho}\frac{\partial\rho}{\partial T_a}u_{T_a}=\frac{\cancel{T_a}}{\frac{\cancel{p_a}}{R\cancel{T_a}}}\cdot\left(\frac{-\cancel{p_a}}{\cancel{R}T_a^2}\right)u_{T_a}=-u_{T_a}=\frac{0.1}{293}=3.41\times10^{-4}$

⑤ $u_\rho=\left[(0.021)^2+(3.41\times10^{-4})^2\right]^{\frac{1}{2}}=0.021$ (2pts)

use ⑤ → (2b) →

⑥ term II: $\frac{\rho}{v}\frac{\partial v}{\partial\rho}u_\rho=-u_\rho/2=\frac{-0.021}{2}=-0.011$ (2pts)

use (2a), ⑥ → ① →

⑦ $u_v=\left[(0.00625)^2+(-0.011)^2\right]^{1/2}=0.013\approx1.3\%$ (2.5pt.

Dimensionless uncertainty for density
has largest impact on dimensionless uncertainty
for velocity