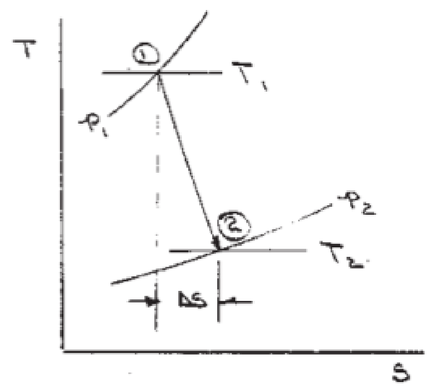
Problem 12.1

(Difficulty: 2)

12.1 Air is expanded in a steady flow process through a turbine. Initial conditions are 1300 ℃ and 2.0 MPa (abs). Final conditions are 500 ℃ and atmospheric pressure. Show this process on a Ts diagram. Evaluate the changes in a internal energy, enthalpy, and specific entropy for this process.

Assumptions: The air behaves as an ideal gas.

Solution: Use the ideal gas relations. The process diagram is



The internal energy and enthalpy changes are

$$\Delta u = u_2 - u_1 = C_v(T_2 - T_1) = 717.4\ \frac{J}{kg \cdot K} \times (500 - 1300)\ K = -574\ \frac{kJ}{kg}$$

$$\Delta h = h_2 - h_1 = C_p(T_2 - T_1) = 1004\ \frac{J}{kg \cdot K} \times (-800\ K) = -803\ \frac{kJ}{kg}$$

To calcualte the entropy change, we use the Tds equation:

$$Tds = dh - vdp = C_p dT - RT\frac{dp}{p}$$

$$ds = C_p\frac{dT}{T} - R\frac{dp}{p}$$

$$s_2 - s_1 = C_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} = 1004\ \frac{J}{kg \cdot K} \times \ln\frac{500 + 273}{1300 + 273} - 286.9\ \frac{J}{kg \cdot K} \times \ln\frac{0.101}{2.0}$$

$$s_2 - s_1 = (-713.3 + 856.6)\frac{J}{kg \cdot K} = 143\ \frac{J}{kg \cdot K}$$

Problem 12.2

[Difficulty: 2]

12.2 Five kilograms of air is cooled in a closed tank from 250 to 50°C. The initial pressure is 3 MPa. Compute the changes in entropy, internal energy, and enthalpy. Show the process state points on a *Ts* diagram.

Given: Cooling of air in a tank

Find: Change in entropy, internal energy, and enthalpy

Solution:

Basic equation:

$$p = \rho \cdot R \cdot T \qquad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$$

$$\Delta u = c_v \cdot \Delta T \qquad \Delta h = c_p \cdot \Delta T$$

Assumptions: 1) Ideal gas 2) Constant specific heats

Given or available data

$$M = 5 \cdot kg \qquad T_1 = (250 + 273) \cdot K \qquad T_2 = (50 + 273) \cdot K \qquad p_1 = 3 \cdot MPa$$

$$c_p = 1004 \cdot \frac{J}{kg \cdot K} \qquad c_v = 717.4 \cdot \frac{J}{kg \cdot K} \qquad k = \frac{c_p}{c_v} \qquad k = 1.4 \qquad R = c_p - c_v \qquad R = 287 \cdot \frac{J}{kg \cdot K}$$

For a constant volume process the ideal gas equation gives $\frac{p_2}{p_1} = \frac{T_2}{T_1}$ $\qquad p_2 = \frac{T_2}{T_1} \cdot p_1 \qquad p_2 = 1.85 \cdot MPa$

Then

$$\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right) \qquad \Delta s = -346 \cdot \frac{J}{kg \cdot K}$$

$$\Delta u = c_v \cdot \left(T_2 - T_1\right) \qquad \Delta u = -143 \cdot \frac{kJ}{kg}$$

$$\Delta h = c_p \cdot \left(T_2 - T_1\right) \qquad \Delta h = -201 \cdot \frac{kJ}{kg}$$

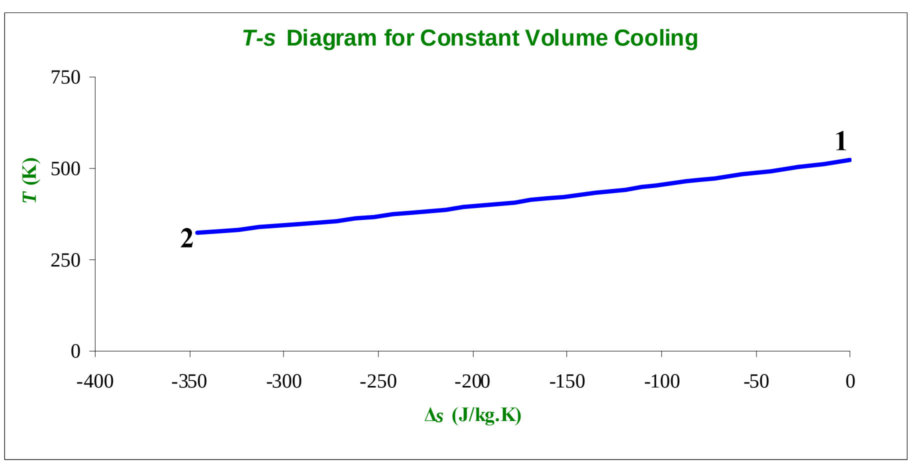
Total amounts are

$$\Delta S = M \cdot \Delta s \qquad \Delta S = -1729 \cdot \frac{J}{K}$$

$$\Delta U = M \cdot \Delta u \qquad \Delta U = -717 \cdot kJ$$

$$\Delta H = M \cdot \Delta h \qquad \Delta H = -1004 \cdot kJ$$

Here is a plot of the T-s diagram:



Problem 12.3

[Difficulty: 3]

12.3 Air is contained in a piston-cylinder device. The temperature of the air is 100°C. Using the fact that for a reversible process the heat transfer $q = \int T ds$, compare the amount of heat (J/kg) required to raise the temperature of the air to 1200°C at (a) constant pressure and (b) constant volume. Verify your results using the first law of thermodynamics. Plot the processes on a *Ts* diagram.

Given: Air in a piston-cylinder

Find: Heat to raise temperature to 1200ºC at a) constant pressure and b) constant volume

Solution:

The data provided, or available in the Appendices, is:

$T_1 = (100 + 273)\cdot K$ $\quad T_2 = (1200 + 273)\cdot K$ $\quad R = 287\cdot\frac{J}{kg\cdot K}$ $\quad c_p = 1004\cdot\frac{J}{kg\cdot K}$ $\quad c_v = c_p - R$ $\quad c_v = 717\cdot\frac{J}{kg\cdot K}$

a) For a constant pressure process we start with $\quad T\cdot ds = dh - v\cdot dp$

Hence, for p = const. $\quad ds = \frac{dh}{T} = c_p\cdot\frac{dT}{T}$

But $\quad \delta q = T\cdot ds$

Hence $\quad \delta q = c_p\cdot dT \quad q = \int c_p\, dT \quad q = c_p\cdot(T_2 - T_1) \quad q = 1104\cdot\frac{kJ}{kg}$

b) For a constant volume process we start $\quad T\cdot ds = du + p\cdot dv$

Hence, for v = const. $\quad ds = \frac{du}{T} = c_v\cdot\frac{dT}{T}$

But $\quad \delta q = T\cdot ds$

Hence $\quad \delta q = c_v\cdot dT \quad q = \int c_v\, dT \quad q = c_v\cdot(T_2 - T_1) \quad q = 789\cdot\frac{kJ}{kg}$

Heating to a higher temperature at constant pressure requires more heat than at constant volume: some of the heat is used to do work in expanding the gas; hence for constant pressure less of the heat is available for raising the temperature.

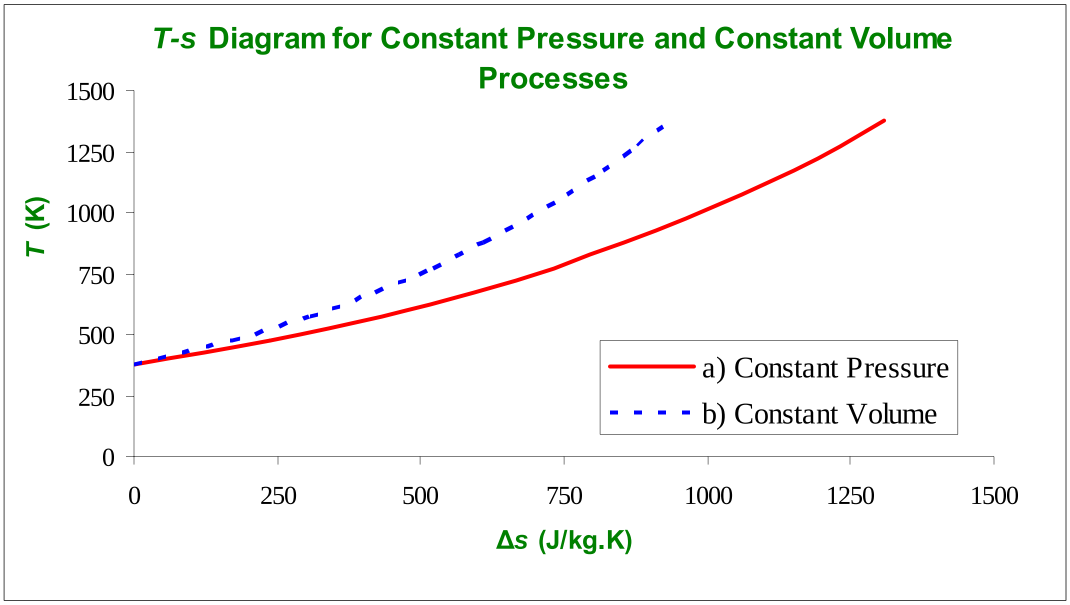
From the first law: Constant pressure: $q = \Delta u + w$ Constant volume: $q = \Delta u$

The two processes can be plotted using Eqs. 11.11b and 11.11a, simplified for the case of constant pressure and constant volume.

a) For constant pressure $\quad s_2 - s_1 = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R\cdot\ln\left(\frac{p_2}{p_1}\right)$ so $\Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right)$

b) For constant volume $\quad s_2 - s_1 = c_v\cdot\ln\left(\frac{T_2}{T_1}\right) + R\cdot\ln\left(\frac{v_2}{v_1}\right)$ so $\Delta s = c_v\cdot\ln\left(\frac{T_2}{T_1}\right)$

The processes are plotted in *Excel* and shown on the next page

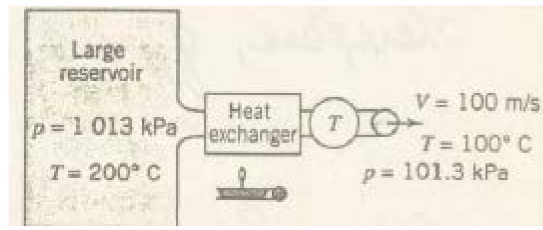


T-s Diagram for Constant Pressure and Constant Volume Processes
T (K)
1500
1250
1000
750
500
250
0
0
250
500
750
1000
1250
1500
Δs (J/kg.K)
a) Constant Pressure
b) Constant Volume

Problem 12.4

(Difficulty 1)

12.4 Calculate the power delivered by the turbine per unit mass of airflow when the transfer in the heat exchanger is zero. Then, how does the power depend on the heat transfer through the exchanger if all other conditions remain the same? Assume air is a perfect gas.



Assumption: Air is an ideal gas. The flow is steady

Find: The energy delivered to the turbine and its dependency on heat transfer.

Solution: Use the energy equation to find the power (eq. 4.56):

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

The flow is steady, there is no shear work, there is no change in elevation, the entering velocity V_1 is zero, and the flow work and internal energy can be combined into the enthalpy. For the situation of zero heat transfer through the heat exchanger we have on a per unit mass basis:

$$-w_s = -w_T = (h_2 - h_1) + \frac{1}{2}V_2^2$$

For ideal gas the enthalpy is related to the specific heat and temperature change:

$$w_T = c_p(T_1 - T_2) - \frac{1}{2}V_2^2$$

Where for air

$$c_p = 1003\ \frac{J}{kg\cdot K}$$

Thus the turbine power is then

$$w_T = 1003\ \frac{J}{kg\cdot K}\times(100\ K) - \frac{1}{2}\left(\left(100\ \frac{m}{s}\right)^2\right) = 95.3\ \frac{kJ}{kg}$$

If the heat transfer through the heat exchanger is Q_H, we have:

$$w_T = c_p(T_1 - T_2) - \frac{1}{2}V_2^2 + Q_H$$

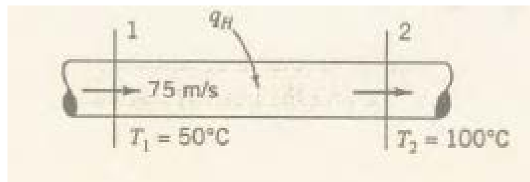
$$w_T = 95.3\ \frac{kJ}{kg} + Q_H$$

If the heat is added to the fluid flow, the energy delivered to the turbine will increase in direct proportion.

Problem 12.5

(Difficulty 1)

12.5 If hydrogen flows as a perfect gas without friction between stations 1 and 2 while $q_H = 7.5 \times 10^5\ \frac{J}{kg}$, find V_2.



Assumption: Hydrogen is an ideal gas. The flow is steady.

Find: The velocity V_2.

Solution: Use the energy equation (eq. 4.56) to find the velocity:

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

The flow is steady, there are no work terms, there is no elevation change, and the internal energy and flow work can be combined into the enthalpy. The energy equation on a per unit mass flow becomes:

$$q_H = (h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2)$$

For ideal gas the enthalpy difference can be related to the specific heat and temperature difference, and we have:

$$q_H = c_p(T_2 - T_1) + \frac{1}{2}(V_2^2 - V_1^2)$$

Where for hydrogen

$$c_p = 14446\ \frac{J}{kg\cdot K}$$

Thus the velocity V_2 is

$$V_2 = \sqrt{2q_H - 2c_p(T_2 - T_1) + V_1^2}$$

$$V_2 = \sqrt{2\times 7.5\times 10^5\ \frac{J}{kg} - 2\times 14446\ \frac{J}{kg\cdot K}\times(50\ K) + \left(75\ \frac{m}{s}\right)^2} = 247\ \frac{m}{s}$$

Problem 12.6

[Difficulty: 3]

12.6 A 1-m^3 tank contains air at 0.1 MPa (abs) and 20°C. The tank is pressurized to 2 MPa. Assuming that the tank is filled adiabatically and reversibly, calculate the final temperature of the air in the tank. Now assuming that the tank is filled isothermally, how much heat is lost by the air in the tank during filling? Which process (adiabatic or isothermal) results in a greater mass of air in the tank?

Given: Air is compressed from standard conditions to fill a tank

Find:
(a) Final temperature of air if tank is filled adiabatically and reversibly
(b) Heat lost if tank is filled isothermally
(c) Which process results in a greater mass of air in the tank

Solution:

The data provided, or available in the Appendices, is:

$$c_p = 1004 \cdot \frac{J}{kg \cdot K} \qquad R = 287 \cdot \frac{J}{kg \cdot K} \qquad c_v = c_p - R \qquad c_v = 717 \cdot \frac{J}{kg \cdot K} \qquad k = \frac{c_p}{c_v} \qquad k = 1.4$$

$$V = 1 \cdot m^3 \qquad p_1 = 0.1 \cdot MPa \qquad T_1 = (20 + 273) \cdot K \qquad p_2 = 2 \cdot MPa$$

Adiabatic, reversible process is isentropic: $T_{2s} = T_1 \cdot \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$ $\qquad T_{2s} = 689.9\,K$

For the isothermal process, we look at the first law: $\Delta u = q - w = c_v \cdot \Delta T$ but $\Delta T = 0$ so: $\Delta u = 0$ and $q = w$

The work is equal to: $w = \int p\, dv = \int \frac{R \cdot T_1}{v} dv = R \cdot T_1 \cdot \int_{v_1}^{v_2} \frac{1}{v} dv = R \cdot T_1 \cdot \ln\left(\frac{v_2}{v_1}\right)$

From Boyle's law: $p_1 \cdot v_1 = p_2 \cdot v_2 \qquad \frac{v_2}{v_1} = \frac{p_1}{p_2}$ substituting this into the above equation: $w = R \cdot T_1 \cdot \ln\left(\frac{p_1}{p_2}\right)$

$w = -252 \cdot \frac{kJ}{kg}$ $\qquad$ Therefore the heat transfer is $q = w = -252 \cdot \frac{kJ}{kg}$ (The negative sign indicates heat loss)

The mass of the air can be calculated from the ideal gas equation of state: $p \cdot V = M \cdot R \cdot T \qquad M = \frac{p_2 \cdot V}{R \cdot T_1} = 23.8\,kg$

So the actual heat loss is equal to: $Q = M \cdot q$ $\qquad Q = -5.99 \times 10^3 \cdot kJ$

The mass in the tank after compression isothermally is: $M_t = 23.8\,kg$

For the isentropic compression: $M = \frac{p_2 \cdot V}{R \cdot T_{2s}} = 10.1\,kg$ $\qquad$ Therefore the isothermal compression results in more mass in the tank.

Problem 12.7

(Difficulty: 2)

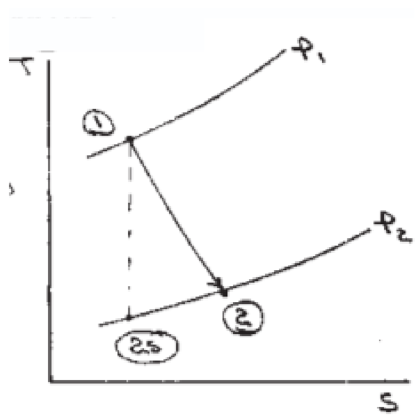
12.7 Air enters a turbine in a steady flow at $0.5\ kg/s$ with negligible velocity. Inlet conditions are $1300\ ℃$ and $2.0\ MPa\ (abs)$. The air is expanded through the turbine to atmospheric pressure. If the actual temperature and velocity at the turbine exit are $500\ ℃$ and $200\ m/s$, determine the power produced by the turbine. Label state points on a Ts diagram for this process.

Assumptions: The air behaves as an ideal gas

Solution: For an isentropic expansion through the turbine,

$$T_{2\ s=c} = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{(k-1)}{k}}$$

The process diagram is



The isentropic outlet temperatures is

$$T_{2\ s=c} = (1300 + 273)K \times \left(\frac{0.101\ MPa}{2.0\ MPa}\right)^{\frac{1.4-1}{1.4}}$$

$$T_{2\ s=c} = 670\ K\ (397\ ℃)$$

The energy equation for the flow between the turbine inlet and outlet is:

$$\dot{W} + \dot{Q} = \dot{m}\left[\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right)\right]$$

Assume $\dot{Q} = 0$, for an ideal gas with constant specific heats,

$$h_2 - h_1 = C_p(T_2 - T_1)$$

$$\dot{W} = \dot{m}\left[C_p(T_2 - T_1) - \frac{V_2^2}{2}\right] = 0.5\ \frac{kg}{s} \times \left[1004\ \frac{N \cdot m}{kg \cdot K} \times (773 - 1573)\ K - \frac{1}{2} \times (200\ \frac{m}{s})^2 \times \frac{N \cdot s^2}{kg \cdot m}\right]$$

$$\dot{W} = -392 \times 10^3\ \frac{N \cdot m}{s}$$

(Negative sign indicates work out).

$$\dot{W}_{out} = 392\ kW$$

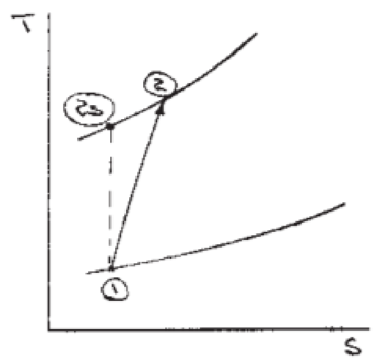
Problem 12.8

(Difficulty: 3)

12.8 Natural gas, with the thermodynamic properties of methane, flows in an underground pipeline of $0.6\ m$ diameter. The gage pressure at the inlet to a compressor station is $0.5\ MPa$; outlet pressure is $8.0\ MPa\ (gage)$. The gas temperature and speed at inlet are $13\ °\mathrm{C}$ and $32\ m/s$, respectively. The compressor efficiency is $\eta = 0.85$. Calculate the mass flow rate of natural gas through the pipeline. Label state points on a Ts diagram for compressor inlet and outlet. Evaluate the gas temperature and speed at the compressor outlet and the power required to drive the compressor.

Assumptions: Methane behaves as an ideal gas.

Solution: Use the ideal gas relations, the continuity equation, and the energy equation. The process diagram is



The mass flow rate is given by:

$$\dot{m} = \rho V A$$

where

$$\rho = \frac{p}{RT}$$

$$\dot{m} = \frac{p_1}{RT_1} V_1 \pi \frac{D^2}{4} = (500 + 101) \times 10^3\ \frac{N}{m^2} \times \frac{kg \cdot K}{518.3\ N \cdot m} \times \frac{1}{286\ K} \times 32\ \frac{m}{s} \times \frac{\pi}{4} \times (0.6\ m)^2$$

$$\dot{m} = 36.7\ \ kg/s$$

For an isentropic compression:

$$T_{2\,s=c} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = 286\ K \times \left(\frac{8.101\ MPa}{0.601\ MPa}\right)^{\frac{1.31-1}{1.31}} = 529\ K$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$T_2 - T_1 = \frac{T_{2s} - T_1}{\eta_c}$$

$$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c} = 286\ K + \frac{(529 - 286)\ K}{0.85}$$

$$T_2 = 572\ K$$

From continuity,

$$\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Assume $A_1 = A_2$, then

$$V_2 = \frac{\rho_1}{\rho_2} V_1 = \frac{p_1}{p_2}\frac{T_2}{T_1} V_1 = \frac{0.601}{8.101} \times \frac{572}{286} \times 32\ \frac{m}{s} = 4.75\ m/s$$

Writing the first law of thermodynamics between compressor inlet and outlet:

$$\dot{W} + \dot{Q} = \dot{m}\left[\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right)\right]$$

Assume $\dot{Q} = 0$,

$$\dot{W} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2)\right] = \dot{m}\left[C_p(T_2 - T_1) + \frac{1}{2}(V_2^2 - V_1^2)\right]$$

$$\dot{W} = 36.7\ \frac{kg}{s}\left[2190\ \frac{N\cdot m}{kg\cdot K} \times (572 - 286)\ K + \frac{1}{2}\left\{(4.75\ \frac{m}{s})^2 - \left(32\ \frac{m}{s}\right)^2 \times \frac{N\cdot s^2}{kg\cdot m}\right\}\right]$$

$$\dot{W} = 36.7[626 \times 10^3 - 501]\ \frac{N\cdot m}{s}$$

$$\dot{W} = 23\ MW$$

Problem 12.9

(Difficulty 1)

12.9 Carbon dioxide flows at a speed of 10 $\frac{m}{s}$ in a pipe and then through a nozzle where the velocity is 50 $\frac{m}{s}$. What is the change in gas temperature between pipe and nozzle? Assume this is an adiabatic flow of a perfect gas.

Assumptions: Carbon dioxide is an ideal gas. The flow is steady and adiabatic.

Find: The change in temperature.

Solution: Use the energy equation (eq. 4.56) to find the temperature change:

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

There is no heat or work transfers, the flow is steady, and there is no change in elevation. The internal energy and flow work are combined into enthalpy and we have:

$$0 = (h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2)$$

For ideal gas we have the enthalpy change related to the temperature change through the specific heat:

$$0 = c_p(T_2 - T_1) + \frac{1}{2}(V_2^2 - V_1^2)$$

Where for carbon dioxide

$$c_p = 858\ \frac{J}{kg\cdot K}$$

Thus

$$(T_2 - T_1) = \frac{(V_1^2 - V_2^2)}{2\ c_p} = \frac{\left(\left(10\ \frac{m}{s}\right)^2 - \left(50\ \frac{m}{s}\right)^2\right)}{2\times 858\ \frac{J}{kg\cdot K}} \times 1\frac{N\ s^2}{kg\ m}\times 1\frac{J}{N\ m}$$

$$\Delta T == -1.39\ K = -1.39℃$$

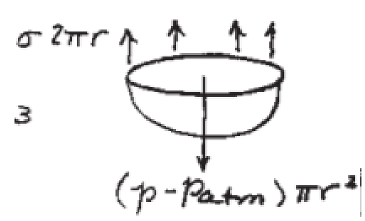
Problem 12.10

(Difficulty: 4)

12.10 In an isothermal process, 0.1 cubic feet of standard air per minute (SCFM) is pumped into a balloon. Tension in the rubber skin of the balloon is given by $\sigma = kA$, where $k = 200\ lbf/ft^3$, and A is the surface area of the balloon in ft^2. Compute the time required to increase the balloon radius from 5 to 7 in.

Assumptions: (1) Standard air, $\rho = 0.0765\ lbm/ft^3$ (2) Ideal gas

Solution: Assume the air behaves as an ideal gas. Use a force balance and the continuity equation. The physical picture is



The mass flow rate is $\dot{m} = \rho_{std} Q = constant$, so the time to increase the mass into the balloon is

$$\Delta t = \frac{\Delta m}{\dot{m}}$$

$$p = \rho R T$$

Then

$$\dot{m} = \rho Q = 0.0765\ \frac{lbm}{ft^3} \times 0.10\ \frac{ft^3}{min} \times \frac{min}{60\ s} = 1.28 \times 10^{-4}\ lbm/s$$

From a force balance on the balloon:

$$(p - p_{atm})\pi r^2 = \sigma 2\pi r = k(4\pi r^2)2\pi r = 8\pi^2 k r^3$$

or

$$p = p_{atm} + 8\pi k r$$

For $r = 5\ in$,

$$p = 14.7 + 8\pi \times 200\ \frac{lbf}{ft^3} \times 5\ in \times \frac{ft^3}{1728\ in^3} = 29.2\ psia$$

$$\rho = \frac{p}{RT} = 29.2\ \frac{lbf}{in^2} \times \frac{lbm \cdot {}^\circ R}{53.3\ ft \cdot lbf} \times \frac{1}{519\ {}^\circ R} \times 144\ \frac{in^2}{ft^2} = 0.152\ \ lbm/ft^3$$

$$\forall = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} \times (5\ in)^3 \times \frac{ft^3}{1728\ in^3} = 0.303\ ft^3$$

$$m = \rho\forall = 0.152\ \frac{lbm}{ft^3} \times 0.303\ ft^3 = 0.0461\ lbm$$

For $r = 7\ in$,

$$p = 14.7 + 8\pi \times 200\ \frac{lbf}{ft^3} \times 7\ in \times \frac{ft^3}{1728\ in^3} = 35.1\ psia$$

Tabulating,

$r\ (in)$	$p\ (psia)$	$\rho\ (lbm/ft^3)$	$\forall\ (ft^3)$	$m\ (lbm)$
5	29.2	0.152	0.303	0.0461
7	35.1	0.183	0.831	0.152

Then

$$\Delta m = m_7 - m_5 = 0.152 - 0.0461\ lbm = 0.106\ lbm$$

and

$$\Delta t = 0.106\ lbm \times \frac{s}{1.28 \times 10^{-4}\ lbm} = 828\ s\ (\approx 14\ min)$$

Problem 12.11

[Difficulty: 3]

12.11 Calculate the speed of sound at 20°C for (a) hydrogen, (b) helium, (c) methane, (d) nitrogen, and (e) carbon dioxide.

Given: Five different gases at specified temperature

Find: Sound speeds for each gas at that temperature

Solution: Basic equation: $c = \sqrt{k \cdot R \cdot T}$

The data provided, or available in the Appendices, is: $T = (20 + 273) \cdot K$

$k_{H2} = 1.41 \quad R_{H2} = 4124 \cdot \frac{J}{kg \cdot K} \qquad k_{He} = 1.66 \quad R_{He} = 2077 \cdot \frac{J}{kg \cdot K}$

$k_{CH4} = 1.31 \quad R_{CH4} = 518.3 \cdot \frac{J}{kg \cdot K} \qquad k_{N2} = 1.40 \quad R_{N2} = 296.8 \cdot \frac{J}{kg \cdot K}$

$k_{CO2} = 1.29 \quad R_{CO2} = 188.9 \cdot \frac{J}{kg \cdot K}$

$c_{H2} = \sqrt{k_{H2} \cdot R_{H2} \cdot T} \qquad c_{H2} = 1305 \frac{m}{s}$

$c_{He} = \sqrt{k_{He} \cdot R_{He} \cdot T} \qquad c_{He} = 1005 \frac{m}{s}$

$c_{CH4} = \sqrt{k_{CH4} \cdot R_{CH4} \cdot T} \qquad c_{CH4} = 446 \frac{m}{s}$

$c_{N2} = \sqrt{k_{N2} \cdot R_{N2} \cdot T} \qquad c_{N2} = 349 \frac{m}{s}$

$c_{CO2} = \sqrt{k_{CO2} \cdot R_{CO2} \cdot T} \qquad c_{CO2} = 267 \frac{m}{s}$

Problem 12.12

[Difficulty: 1]

12.12 An airplane flies at 550 km/hr at 1500 m altitude on a standard day. The plane climbs to 15,000 m and flies at 1200 km/h. Calculate the Mach number of flight in both cases.

Given: Airplane cruising at two different elevations

Find: Mach numbers

Solution:

Basic equation: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$

Available data $R = 286.9 \frac{J}{kg \cdot K}$ $\quad k = 1.4$

At $z = 1500 \cdot m$ $\quad T = 278.4 \cdot K$ from Table A.3

Hence $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 334 \frac{m}{s}$ $\quad c = 1204 \cdot \frac{km}{hr}$ and we have $V = 550 \cdot \frac{km}{hr}$

The Mach number is $M = \frac{V}{c}$ $\quad M = 0.457$

Repeating at $z = 15000 \cdot m$ $\quad T = 216.7 \cdot K$

Hence $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 295 \frac{m}{s}$ $\quad c = 1062 \cdot \frac{km}{hr}$ and we have $V = 1200 \cdot \frac{km}{hr}$

The Mach number is $M = \frac{V}{c}$ $\quad M = 1.13$

Problem 12.13

(Difficulty: 1)

12.13 Actual performance characteristics of the Lockheed SR-71 "Blackbrid" reconnaissance aircraft never were released. However, it was thought to cruise at $M = 3.3$ at 85,000 ft altitude. Evaluate the speed of sound and flight speed for these conditions. Compare to the muzzle speed of a 30-06 rifle bullet (700 m/s).

Assumptions: Air behaves as an ideal gas

Solution: Use the ideal gas relation for the speed of sound

$$c = \sqrt{kRT}$$

At altitude,

$$z = 85{,}000\ ft \times 0.3048\ \frac{m}{ft} = 25.9\ km$$

From Table A.3,

$$T = 222\ K$$

$$c = \sqrt{kRT} = \left[1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 222\ K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{\frac{1}{2}} = 299\ \ m/s$$

$$V = Mc = 3.3 \times 299\ \ m/s = 987\ \ m/s$$

$$\frac{V}{V_{bullet}} = \frac{987}{700} = 1.41$$

Problem 12.14

(Difficulty 1)

12.14 For a speed of sound in steel of 4300 $\frac{m}{s}$, determine the bulk modulus of elasticity. Compare the modulus of elasticity of steel to that of water. Determine the speed of sound in steel, water, and air at atmospheric conditions. Comment on differences.

Find: the bulk modulus of elasticity

Assumption: Steel is homogeneous

Solution: Use the relation between speed of sound, bulk modulus, and density

$$c = \sqrt{\frac{E_v}{\rho}}$$

Using the specific gravity of steel from Table A.1, we have the bulk modulus as:

$$E_{vs} = c^2\rho = c^2 SG_{steel}\rho_{water} = \left(4300\ \frac{m}{s}\right)^2 \times 7.83 \times 1000\ \frac{kg}{m^3} = 144.8\ \frac{GN}{m^2}$$

The bulk modulus of water from Table A.2 is:

$$E_{vw} = 2.24\ \frac{GN}{m^2}$$

So the modulus of elasticity of water is smaller than that of steel.

For the sound speed of steel:

$$c_s = 4300\ \frac{m}{s}$$

The sound speed of water is then

$$c_w = \sqrt{\frac{E_{vw}}{\rho}} = \sqrt{\frac{2.24\ \frac{GN}{m^2}}{1000\ \frac{kg}{m^3}}} = 1497\ \frac{m}{s}$$

For the sound speed of air at atmospheric conditions:

$$c_a = \sqrt{kRT} = \sqrt{1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 288\ K \times \frac{kg \cdot m}{N \cdot s^2}} = 340\ \frac{m}{s}$$

So we have the following relation:

$$c_a < c_w < c_s$$

It means the sound speed is increasing from gas, liquid to solid.

Problem 12.15

(Difficulty 1)

12.15 Determine and plot the Mach number of an automobile as a function of speed from 25 mph to 100 mph for winter ($T = 0$ °F) and summer ($T = 100$ °F)

Find: Mach numbers

Assumption: Air behaves as an ideal gas

Solution: Use the relation for speed of sound in an ideal gas

$$c = \sqrt{kRT}$$

For the air we have:

$$k = 1.4 \quad and \quad R = 1715\ \frac{ft \cdot lbf}{slug \cdot °R}$$

For the winter conditions we have:

$$c = \sqrt{kRT} = \sqrt{1.4 \times 1715\ \frac{ft \cdot lbf}{slug \cdot °R} \times 459.67\ °R} = \sqrt{1.4 \times 1715\ \frac{ft \cdot lbf}{\frac{lbf \cdot s^2}{ft} \cdot °R} \times 459.67\ °R}$$

$$c = 1051\ \frac{ft}{s}$$

The Mach number is defined as:

$$M = \frac{V}{c}$$

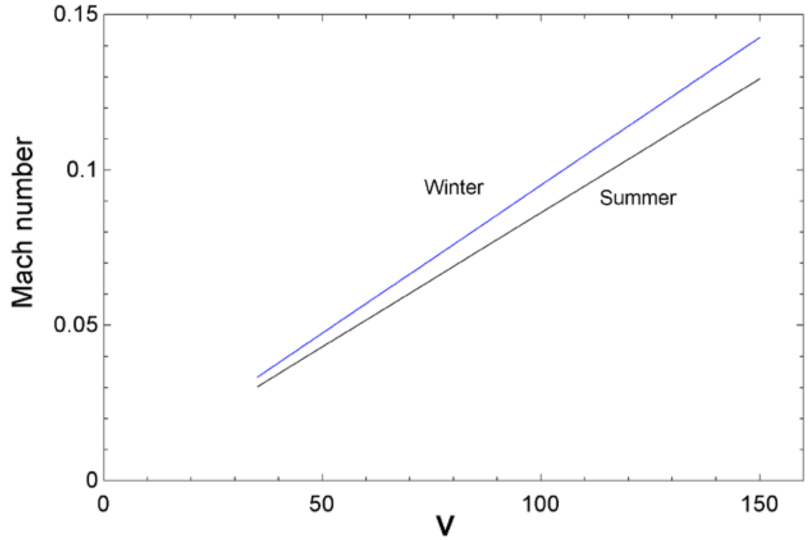
We have the following range:

$$V_{min} = 25\ mph = 36.7\ \frac{ft}{s} \ \ to \ \ V_{max} = 100\ mph = 146.7\ \frac{ft}{s}$$

For the summer we have:

$$c = \sqrt{kRT} = \sqrt{1.4 \times 1715\ \frac{ft \cdot lbf}{slug \cdot °R} \times (100 + 459.67)\ °R} = 1159\ \frac{ft}{s}$$

The plot is shown in the figure:



For the same velocity, the Mach number is higher in winter than summer because the temperature is lower and thus the speed of sound is lower.

Problem 12.16

[Difficulty: 2]

12.16 Investigate the effect of altitude on Mach number by plotting the Mach number of a 500 mph airplane as it flies at altitudes ranging from sea level to 10 km.

Given: Airplane cruising at 550 mph

Find: Mach number versus altitude

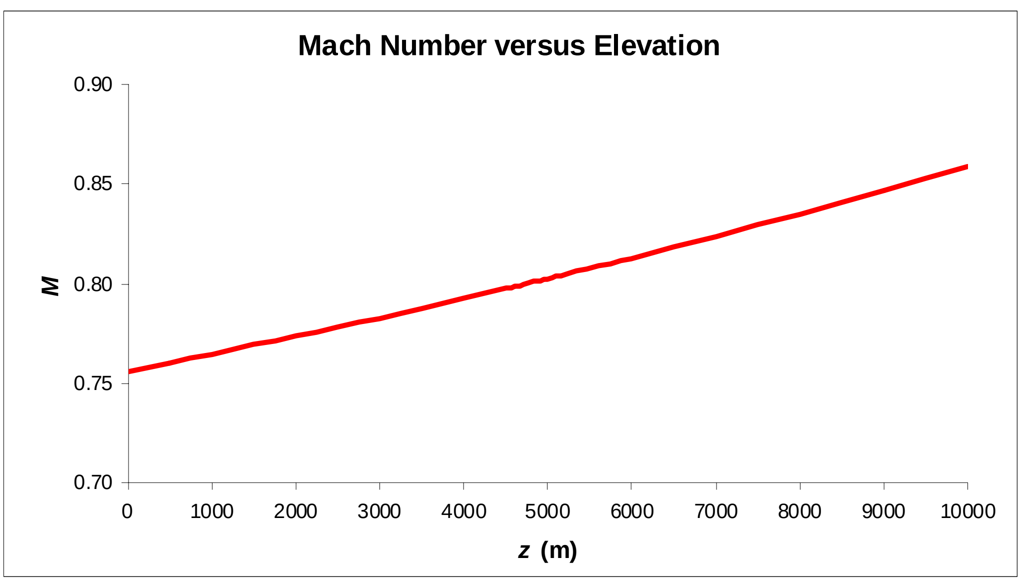
Solution:

Basic equation: $c = \sqrt{k \cdot R \cdot T}$ $\qquad M = \frac{V}{c}$ $\qquad$ Here are the results, generated using *Excel*:

V = 500 mph
R = 286.90 J/kg-K (Table A.6)
k = 1.40

Data on temperature versus height obtained from Table A.3

z (m)	*T* (K)	*c* (m/s)	*c* (mph)	*M*
0	288.2	340	661	0.756
500	284.9	338	658	0.760
1000	281.7	336	654	0.765
1500	278.4	334	650	0.769
2000	275.2	332	646	0.774
2500	271.9	330	642	0.778
3000	268.7	329	639	0.783
3500	265.4	326	635	0.788
4000	262.2	325	631	0.793
4500	258.9	322	627	0.798
5000	255.7	320	623	0.803
6000	249.2	316	615	0.813
7000	242.7	312	607	0.824
8000	236.2	308	599	0.835
9000	229.7	304	590	0.847
10000	223.3	299	582	0.859



Problem 12.17

[Difficulty: 2]

12.17 The grandstand at the Kennedy Space Center is located 3.5 mi away from the Space Shuttle Launch Pad. On a day when the air temperature is 80°F, how long does it take the sound from a blastoff to reach the spectators? If the launch was early on a winter morning, the temperature may be as low as 50°F. How long would the sound take to reach the spectators under those conditions?

Given: Shuttle launch

Find: How long after seeing it do you hear it?

Solution:

Basic equation: $c = \sqrt{k \cdot R \cdot T}$

Assumption: Speed of light is essentially infinite (compared to speed of sound)

The given or available data is $T = (80 + 460) \cdot R$ $\quad L = 3.5 \cdot mi$ $\quad k = 1.4$ $\quad R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$

Hence $c = \sqrt{k \cdot R_{air} \cdot T}$ $\quad c = 1139 \cdot \frac{ft}{s}$

Then the time is $\Delta t = \frac{L}{c}$ $\quad \Delta t = 16.23\,s$

In the winter: $T = (50 + 460) \cdot R$

Hence $c = \sqrt{k \cdot R_{air} \cdot T}$ $\quad c = 1107 \cdot \frac{ft}{s}$

Then the time is $\Delta t = \frac{L}{c}$ $\quad \Delta t = 16.7\,s$

Problem 12.18 [Difficulty: 2]

12.18 Use data for specific volume to calculate and plot the speed of sound in saturated liquid water over the temperature range from 0 to 200°C.

Given: Data on water specific volume

Find: Speed of sound over temperature range

Solution:

Basic equation: $c = \sqrt{\frac{\partial}{\partial \rho} p}$ at isentropic conditions

As an approximation for a liquid $c = \sqrt{\frac{\Delta p}{\Delta \rho}}$ using available data.

We use compressed liquid data at adjacent pressures of 5 MPa and 10 MPa, and estimate the change in density between these pressures from the corresponding specific volume changes

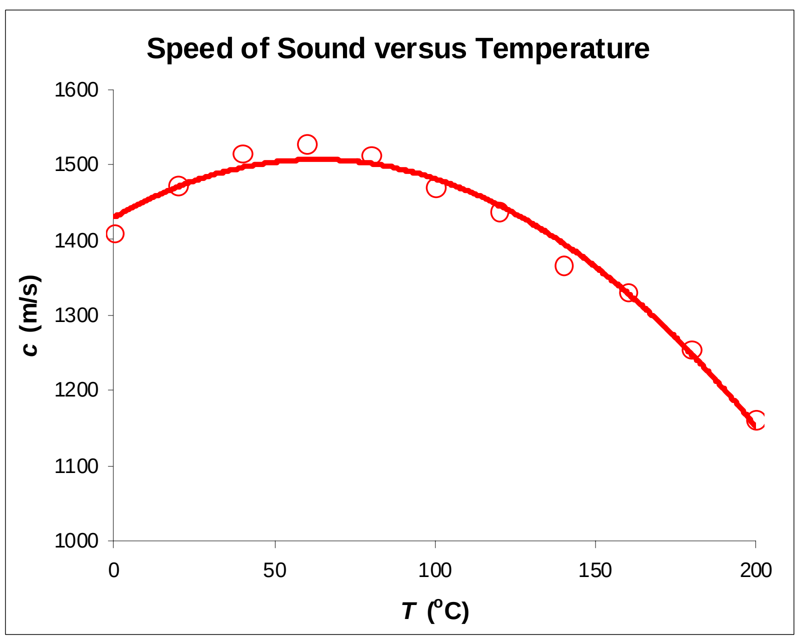
$$\Delta p = p_2 - p_1 \qquad \Delta \rho = \frac{1}{v_2} - \frac{1}{v_1} \qquad \text{and} \qquad c = \sqrt{\frac{\Delta p}{\Delta \rho}} \quad \text{at each temperature}$$

Here are the results, calculated using *Excel*:

p_2 = 10 MPa
p_1 = 5 MPa
ηp = 5 MPa

Data on specific volume versus temperature can be obtained fro any good thermodynamics text (try the Web!)

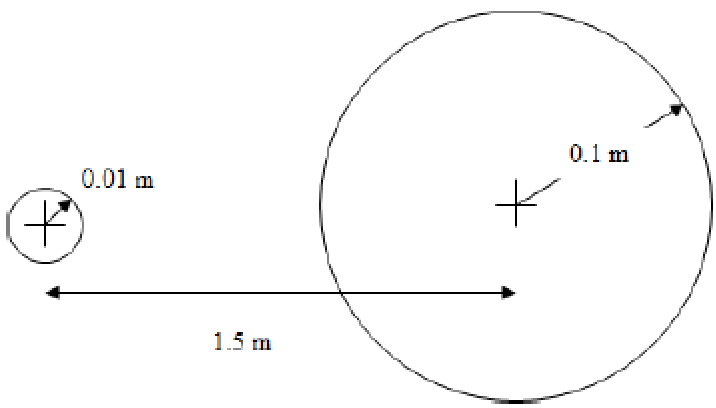
	p_1	p_2		
T (°C)	v (m³/kg)	v (m³/kg)	$\Delta\rho$ (kg/m³)	c (m/s)
0	0.0009977	0.0009952	2.52	1409
20	0.0009996	0.0009973	2.31	1472
40	0.0010057	0.0010035	2.18	1514
60	0.0010149	0.0010127	2.14	1528
80	0.0010267	0.0010244	2.19	1512
100	0.0010410	0.0010385	2.31	1470
120	0.0010576	0.0010549	2.42	1437
140	0.0010769	0.0010738	2.68	1366
160	0.0010988	0.0010954	2.82	1330
180	0.0011240	0.0011200	3.18	1254
200	0.0011531	0.0011482	3.70	1162



Problem 12.19

(Difficulty 1)

12.19 An object traveling in atmospheric air emits two pressure waves at different times. At an instant in time, the waves appear as in the figure. Determine the velocity and Mach number of the object and its current location.



Find: The velocity and Mach number

Assumption: The air is uniform in temperature and is an ideal gas

Solution: Use the relations for the speed of sound and Mach number

We have the equation for the speed of sound in an ideal gas as:

$$c = \sqrt{kRT}$$

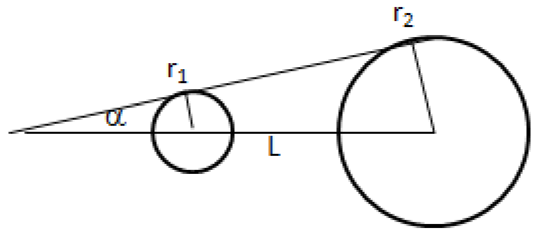
For the air we have:

$$k = 1.4 \quad and \quad R = 287 \frac{J}{kg \cdot K}$$

The speed of sound is then:

$$c = \sqrt{kRT} = \sqrt{1.4 \times 287 \frac{J}{kg \cdot K} \times 288\ K} = 340\ \frac{m}{s}$$

The geometry of the sound waves are



Where $r_1 = 0.1\ m,\ r_2 = 0.01\ m, and\ d = 1.5\ m$

The geometric relation is:

$$\frac{L-d}{L} = \frac{r_2}{r_1} = \frac{1}{10}$$

Or the distance between the two locations is

$$L = \frac{10d}{9} = \frac{10 \times 1.5\ m}{9} = 1.667\ m$$

The current location is $1.667\ m$ from the center of the circle 1.

$$t = \frac{r_1}{c} = \frac{0.1\ m}{340\ \frac{m}{s}} = 0.000294\ s$$

The velocity of the object can be calculated as:

$$V = \frac{L}{t} = \frac{1.667\ m}{0.000294\ s} = 5670\ \frac{m}{s}$$

The Mach number is calculated as:

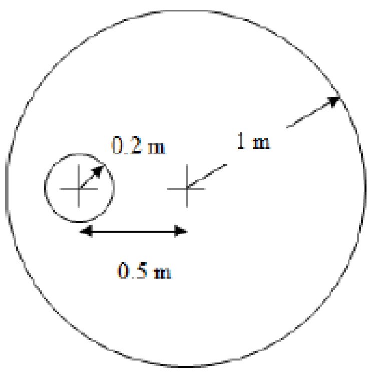
$$M = \frac{V}{c} = \frac{5670\ \frac{m}{s}}{340\ \frac{m}{s}} = 16.67$$

This is supersonic flow.

Problem 12.20

(Difficulty 1)

12.20 An object traveling in atmospheric air emits two pressure waves at different times. At an instant in time, the waves appear as in the figure. Determine the velocity and Mach number of the object and its current location.



Find: The velocity and Mach number

Assumption: The air is uniform in temperature and is an ideal gas

Solution: Use the relations for the speed of sound and Mach number

We have the equation for the speed of sound in an ideal gas as:

$$c = \sqrt{kRT}$$

For the air we have:

$$k = 1.4 \quad and \quad R = 287 \frac{J}{kg \cdot K}$$

$$r_1 = 1\,m$$

$$r_2 = 0.2\,m$$

$$d = 0.5\,m$$

We have the equation for ideal gas as:

$$c = \sqrt{kRT} = \sqrt{1.4 \times 287 \frac{J}{kg \cdot K} \times 288\,K} = 340\ \frac{m}{s}$$

The time after emitting the pressure wave 1 for circle 1 is:

$$t_1 = \frac{r_1}{c} = \frac{1\ m}{340\ \frac{m}{s}} = 0.00294\ s$$

The time after emitting the pressure wave 2 for circle 2 is:

$$t_2 = \frac{r_2}{c} = \frac{0.2\ m}{340\ \frac{m}{s}} = 0.000588\ s$$

So the time between these two pressure waves is:

$$\Delta t = t_1 - t_2 = \ 0.002352\ s$$

The velocity of the object is then:

$$V = \frac{d}{\Delta t} = \frac{0.5\ m}{0.002352\ s} = 213\ \frac{m}{s}$$

The Mach number is calculated as:

$$M = \frac{V}{c} = \frac{213\ \frac{m}{s}}{340\ \frac{m}{s}} = 0.626$$

This is subsonic flow.

The current location of the object (distance from the center of circle 1) is computed by:

$$L = Vt_1 = 213\ \frac{m}{s} \times 0.00294\ s = 0.626\ m$$

Problem 12.21

(Difficulty 1)

12.21 While at the seashore, you observe an airplane that is flying at 10,000 ft. You hear the airplane 8 $seconds$ after it passes directly overhead. Estimate the airplane speed and Mach number. If the airplane had been flying at 30,000 ft, how many seconds would have passed before you heard it?

Find: The velocity and Mach number

Assumption: The air is uniform in temperature and is an ideal gas

Solution: Use the relations for the speed of sound and Mach number

We have the equation for the speed of sound in an ideal gas as:

$$c = \sqrt{kRT}$$

For the air we have:

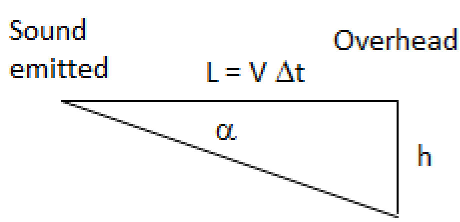
$$k = 1.4 \quad and \quad R = 1715\ \frac{ft \cdot lbf}{slug \cdot {}^\circ R}$$

The speed of sound is then

$$c = \sqrt{kRT} = \sqrt{1.4 \times 1715\ \frac{ft \cdot lbf}{slug \cdot {}^\circ R} \times (459.67 + 59)\ {}^\circ R} = \sqrt{1.4 \times 1715\ \frac{ft \cdot lbf}{\frac{lbf \cdot s^2}{ft} \cdot {}^\circ R} \times 518.67\ {}^\circ R}$$

$$c = 1115\ \frac{ft}{s}$$

We have the following geometric relationships between the time the sound was emitted and when the airplane was overhead:



We have the following geometric relation:

$$\tan\alpha = \frac{h}{x} = \frac{h}{V \cdot \Delta t}$$

We also have the relation between the speed of the airplane, the speed of sound, and the angle of the sound cone:

$$\sin\alpha = \frac{c}{V} = \frac{1}{M}$$

Using trigonometric relations

$$\cos\alpha = \frac{\sin\alpha}{\tan\alpha} = \frac{\frac{c}{V}}{\frac{h}{V\cdot\Delta t}} = \frac{c\cdot\Delta t}{h} = \frac{1115\ \frac{ft}{s}\times 8\ s}{10{,}000\ ft} = 0.892$$

The cone angle is

$$\alpha = 26.9\,°$$

And the Mach number is

$$M = \frac{1}{\sin\alpha} = 2.21$$

The velocity is then

$$V = cM = 1115\ \frac{ft}{s}\times 2.21 = 2460\ \frac{ft}{s}$$

If $h = 30000\ ft$,

$$\alpha = 26.9\,°$$

$$\tan\alpha = \frac{h}{x} = \frac{h}{V\cdot\Delta t}$$

$$\Delta t = \frac{h}{V\cdot\tan\alpha} = \frac{30000\ ft}{2460\ \frac{ft}{s}\times 0.507} = 24.\,s$$

We could also see this different time in terms of geometry. If the height is three times and the angle of the cone is the same, the time must be three times greater too.

Problem 12.22

[Difficulty: 3]

12.22 The temperature varies linearly from sea level to approximately 11 km altitude in the standard atmosphere. Evaluate the *lapse rate*—the rate of decrease of temperature with altitude—in the standard atmosphere. Derive an expression for the rate of change of sonic speed with altitude in an ideal gas under standard atmospheric conditions. Evaluate and plot from sea level to 10 km altitude.

Given: Data on atmospheric temperature variation with altitude

Find: Lapse rate; plot rate of change of sonic speed with altitude

Solution:

The given or available data is: $R_{air} = 286.9 \cdot \frac{J}{kg \cdot K}$ $\quad k = 1.4 \quad T_0 = 288.2 \cdot K \quad T_{10k} = 223.3 \cdot K \quad z = 10000 \cdot m$

For a linear temperature variation $\quad T = T_0 + m \cdot z$

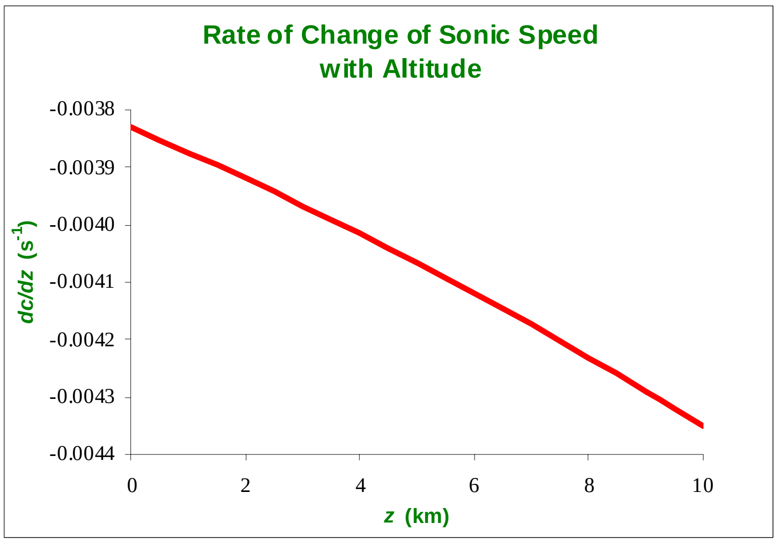
$$\frac{dT}{dz} = m = \frac{T - T_0}{z}$$ which can be evaluated at z = 10 km

$$m = \frac{T_{10k} - T_0}{z} = -6.49 \times 10^{-3} \frac{K}{m}$$

For an ideal gas $\quad c = \sqrt{k \cdot R \cdot T} = \sqrt{k \cdot R \cdot (T_0 + m \cdot z)}$

Hence $\quad \frac{dc}{dz} = \frac{m \cdot k \cdot R}{2 \cdot c}$ $\quad$ Here are the results, calculated using *Excel*:

z (km)	*T* (K)	*dc/dz* (s^{-1})
0	288.2	-0.00383
1	281.7	-0.00387
2	275.2	-0.00392
3	268.7	-0.00397
4	262.2	-0.00402
5	255.8	-0.00407
6	249.3	-0.00412
7	242.8	-0.00417
8	236.3	-0.00423
9	229.8	-0.00429
10	223.3	-0.00435



Problem 12.23

[Difficulty: 3]

12.23 A projectile is fired into a gas (ratio of specific heats $k = 1.625$) in which the pressure is 450 kPa (abs) and the density is 4.5 kg/m^3. It is observed experimentally that a Mach cone emanates from the projectile with 25° total angle. What is the speed of the projectile with respect to the gas?

Given: Projectile fired into a gas, Mach cone formed

Find: Speed of projectile

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \alpha = \text{asin}\left(\frac{1}{M}\right)$ $\quad p = \rho \cdot R \cdot T$

Given or available data $\quad p = 450 \cdot \text{kPa}$ $\quad \rho = 4.5 \cdot \frac{\text{kg}}{\text{m}^3}$ $\quad k = 1.625$ $\quad \alpha = \frac{25}{2} \cdot \text{deg} = 12.5 \cdot \text{deg}$

Combining ideal gas equation of state and the sonic speed: $\quad c = \sqrt{k \cdot \frac{p}{\rho}}$ $\quad c = 403.1 \frac{\text{m}}{\text{s}}$

From the Mach cone angle: $\quad M = \frac{1}{\sin(\alpha)}$ $\quad M = 4.62$ $\quad$ Therefore the speed is: $\quad V = M \cdot c$ $\quad V = 1862 \frac{\text{m}}{\text{s}}$

Problem 12.24

(Difficulty: 1)

12.24 A photograph of a bullet shows a Mach angle of 32°. Determine the speed of the bullet for standard air.

Assumptions: (1) air behaves as an ideal gas (2) constant specific heats

Solution: The ideal gas relations for the speed of sound and for the Mach number and angle are

$$c = \sqrt{kRT}$$

$$\sin\alpha = \frac{1}{M}$$

$$M = \frac{1}{\sin\alpha}$$

$$M = \frac{V}{c}$$

$$V = cM = \frac{c}{\sin\alpha}$$

Since

$$c = \sqrt{kRT}$$

Then

$$V = \frac{1}{\sin\alpha}(kRT)^{\frac{1}{2}} = \frac{1}{\sin 32°}\left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 288\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}}$$

$$V = 642\ m/s\ \ (2110\ ft/s)$$

Problem 12.25

[Difficulty: 2]

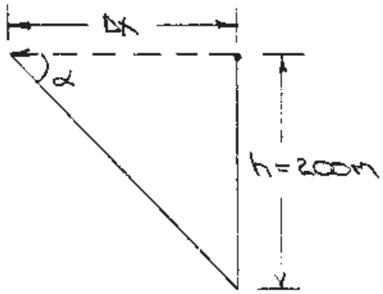
12.25 An F-4 aircraft makes a high-speed pass over an airfield on a day when $T = 35°C$. The aircraft flies at $M = 1.4$ and 200 m altitude. Calculate the speed of the aircraft. How long after it passes directly over point A on the ground does its Mach cone pass over point A?

Solution: Assume T = constant over 200m elevation.

$T = 35\,C = 308\,K$

$$c = \sqrt{kRT} = \left[1.4 \times 287\,\frac{N\cdot m}{kg\cdot K} \times 308\,K \times \frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 352\,m/s$$

$$V = Mc = 1.4 \times 352\,m/s = 493\,m/s \qquad \longleftarrow V$$



From the instant the aircraft is directly overhead until the Mach cone reaches the ground, the plane travels a distance Δx at speed V = 493 m/s.

$$\sin\alpha = \frac{1}{M} = \frac{1}{1.4} = 0.7143$$

$$\alpha = 45.6°$$

$$\frac{h}{\Delta x} = \tan\alpha \qquad \therefore\ \Delta x = \frac{h}{\tan\alpha} = \frac{200\,m}{\tan 45.6°} = 196\,m$$

Since the plane moves at constant speed V

$$\Delta x = V\,\Delta t \qquad \text{and} \qquad \Delta t = \frac{\Delta x}{V} = \frac{196\,m}{493\,m/s}$$

$$\Delta t = 0.398\,s \qquad \longleftarrow \Delta t$$

Problem 12.26

(Difficulty: 2)

12.26 An aircraft passes overhead at 3 km altitude. The aircraft flies at $M = 1.5$; assume air temperature is constant at 20 °C. Find the air speed of the aircraft. A headwind blows at 30 m/s. How long after the aircraft passes directly overhead does its sound reach a point on the ground?

Assumptions: The air temperature is constant with altitude and the headwind is steady.

Solution: Use the ideal gas relations

$$T = constant = 20°\text{C} = 293\ K$$

$$c = (kRT)^{\frac{1}{2}} = \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 293\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 343\ m/s$$

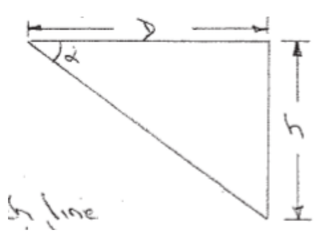
$$V = Mc = 1.5 \times 343\ m/s = 515\ m/s$$

The airspeed is the velocity of the plane relative to the air. The ground speed is then:

$$\bar{V}_p = \bar{V}_{air} + \bar{V}_{pla}$$

$$\bar{V}_p = 485\ m/s$$

The physical picture is



From the instant the aircraft is directly overhead until the Mach cone reaches the ground, the plane travels a distance, D, at speed $\bar{V}_p = 485\ m/s$.

The value of the time, t is then $t = D/\bar{V}_p$. Since the air temperature is constant, the Mach line is straight and $D = h/\tan\alpha$, where $\alpha = \sin^{-1}(1/M)$.

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°$$

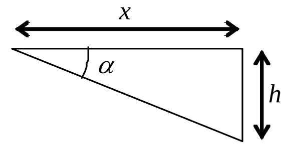
Then the time is

$$t = \frac{D}{V_p} = \frac{h}{\tan \alpha} \frac{1}{V_p} = \frac{3000\ m}{\tan 41.8°} \times \frac{s}{485\ m} = 6.92\ s$$

Problem 12.27

[Difficulty: 3]

12.27 A supersonic aircraft flies at 3 km altitude at a speed of 1000 m/s on a standard day. How long after passing directly above a ground observer is the sound of the aircraft heard by the ground observer?



Given: Supersonic aircraft flying overhead

Find: Time at which airplane heard

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \alpha = \text{asin}\left(\frac{1}{M}\right)$

Given or available data $V = 1000 \cdot \frac{m}{s}$ $\quad h = 3 \cdot km$ $\quad k = 1.4$ $\quad R = 286.9 \frac{J}{kg \cdot K}$

The time it takes to fly from directly overhead to where you hear it is $\Delta t = \frac{x}{V}$

If the temperature is constant then $x = \frac{h}{\tan(\alpha)}$

The temperature is not constant so the Mach line will not be straight. We can find a range of Δt by considering the temperature range

At $h = 3 \cdot km$ we find from Table A.3 that $T = 268.7 \cdot K$

Using this temperature $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 329 \frac{m}{s}$ and $M = \frac{V}{c}$ $\quad M = 3.04$

Hence $\alpha = \text{asin}\left(\frac{1}{M}\right)$ $\quad \alpha = 19.2 \cdot deg$ $\quad x = \frac{h}{\tan(\alpha)}$ $\quad x = 8625 m$ $\quad \Delta t = \frac{x}{V}$ $\quad \Delta t = 8.62 s$

At sea level we find from Table A.3 that $T = 288.2 \cdot K$

Using this temperature $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 340 \frac{m}{s}$ and $M = \frac{V}{c}$ $\quad M = 2.94$

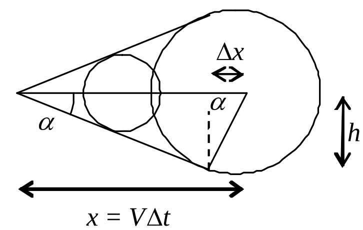
Hence $\alpha = \text{asin}\left(\frac{1}{M}\right)$ $\quad \alpha = 19.9 \cdot deg$ $\quad x = \frac{h}{\tan(\alpha)}$ $\quad x = 8291 m$ $\quad \Delta t = \frac{x}{V}$ $\quad \Delta t = 8.29 s$

Thus we conclude that the time is somwhere between 8.62 and 8.29 s. Taking an average $\Delta t = 8.55 \cdot s$

Problem 12.28

[Difficulty: 3]

12.28 For the conditions of Problem 12.27 find the location at which the sound wave that first reaches the ground observer was emitted.



Given: Supersonic aircraft flying overhead

Find: Location at which first sound wave was emitted

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \alpha = \text{asin}\left(\frac{1}{M}\right)$

Given or available data $V = 1000 \cdot \frac{m}{s}$ $\quad h = 3 \cdot km$ $\quad k = 1.4$ $\quad R = 286.9 \frac{J}{kg \cdot K}$

We need to find Δx as shown in the figure $\quad \Delta x = h \cdot \tan(\alpha)$

The temperature is not constant so the Mach line will not be straight (α is not constant). We can find a range of α and Δx by considering the temperature range

At $h = 3 \cdot km$ we find from Table A.3 that $\quad T = 268.7 \cdot K$

Using this temperature $\quad c = \sqrt{k \cdot R \cdot T}$ $\quad c = 329 \frac{m}{s}$ $\quad$ and $\quad M = \frac{V}{c}$ $\quad M = 3.04$

Hence $\quad \alpha = \text{asin}\left(\frac{1}{M}\right)$ $\quad \alpha = 19.2 \cdot deg$ $\quad \Delta x = h \cdot \tan(\alpha)$ $\quad \Delta x = 1043\, m$

At sea level we find from Table A.3 that $\quad T = 288.2 \cdot K$

Using this temperature $\quad c = \sqrt{k \cdot R \cdot T}$ $\quad c = 340 \frac{m}{s}$ $\quad$ and $\quad M = \frac{V}{c}$ $\quad M = 2.94$

Hence $\quad \alpha = \text{asin}\left(\frac{1}{M}\right)$ $\quad \alpha = 19.9 \cdot deg$ $\quad \Delta x = h \cdot \tan(\alpha)$ $\quad \Delta x = 1085\, m$

Thus we conclude that the distance is somwhere between 1043 and 1085 m. Taking an average $\quad \Delta x = 1064 \cdot m$

Problem 12.29

(Difficulty: 4)

12.29 The Concorde supersonic transport cruised at $M = 2.2$ at 17 km altitude on a standard day. How long after the aircraft passed directly above a ground observer was the sound of the aircraft heard?

Assumptions: The air temperature variation is that of the standard atmosphere.

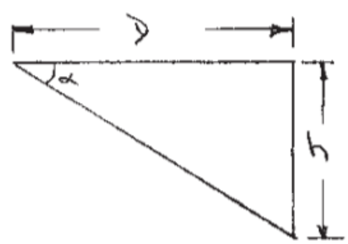
Solution: Use the ideal relations

At 17 km, $T = 216.7\ K$,

$$c = (kRT)^{\frac{1}{2}} = \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 216.7\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 295\ \frac{m}{s}$$

$$V = Mc = 2.2 \times 295\ \frac{m}{s} = 649\ \frac{m}{s}$$

The geometry of the situation is



If the speed of sound were constant all the way to ground, the Mach line would remain straight. The Mach angle , would be constant with:

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{2.2}\right) = 27°$$

Then, from the diagram,

$$D = \frac{h}{\tan\alpha}$$

and

$$t = \frac{D}{V} = \frac{h}{V\tan\alpha} = \frac{17000\ m}{\tan 27°} \times \frac{s}{649\ m} = 51.4\ s$$

However, the speed of sound varies over the altitude because the temperature varies with altitude.

At sea level, $T = 288.2\ K$,

$$c = (kRT)^{\frac{1}{2}} = \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 288.2\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 340\ \frac{m}{s}$$

The corresponding value of Mach number for $V = 649\ \frac{m}{s}$ is:

$$M = \frac{V}{c} = \frac{649\ \frac{m}{s}}{340\ \frac{m}{s}} = 1.91$$

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{1.91}\right) = 31.6°$$

Thus, if the speed of sound were constant (at the sea level) value over the entire altitude, then

$$t = \frac{D}{V} = \frac{h}{V\tan\alpha} = \frac{17000\ m}{\tan 31.6°} \times \frac{s}{649\ m} = 42.6\ s$$

We can obtain a better approximation by considering the variation of temperature with altitude, from Table A.3,

$$11\ km < y < 20\ km$$

$$T = 216.7\ K$$

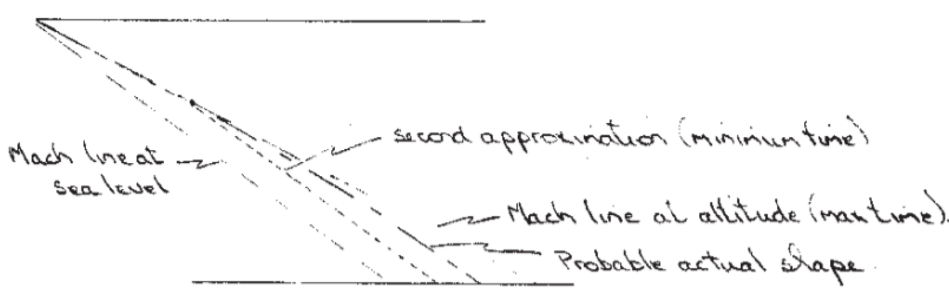
$$0 < y < 11\ km$$

T varies linearly with y,

$$T = T_0 - by$$

$$T_0 = 288.2\ K$$

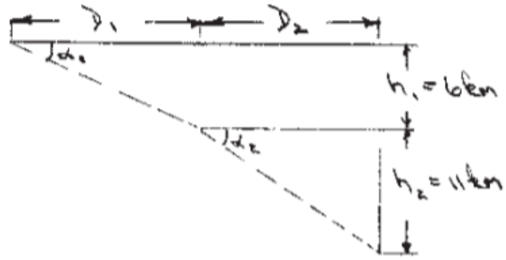
$$b = 6.50\ \frac{K}{km}$$



Since T is constant for $y > y_0 = 11\ km$, the second approximation which assumes the Mach line at sea level for

$$0 < y < 11\ km$$

gives the minimum time.



$$D_1 = \frac{h_1}{\tan\alpha_1} = \frac{6\ km}{\tan 27°} = 11.77\ km$$

$$D_2 = \frac{h_2}{\tan\alpha_2} = \frac{11\ km}{\tan 31.6°} = 17.88\ km$$

$$D = D_1 + D_2$$

$$t = \frac{D}{V} = \frac{29.65\ km}{649\ \frac{m}{s}} = 45.7\ s$$

Consequently,

$$45.7\ s \leq t \leq 51.4\ s$$

Since the two values are reasonably close, it is appropriate to take the average value and say that $t = 48.5\ s$

Problem 12.30 [Difficulty: 2]

12.30 Plot the percentage discrepancy between the density at the stagnation point and the density at a location where the Mach number is M, of a compressible flow, for Mach numbers ranging from 0.05 to 0.95. Find the Mach numbers at which the discrepancy is 1 percent, 5 percent, and 10 percent.

Given: Mach number range from 0.05 to 0.95

Find: Plot of percentage density change; Mach number for 1%, 5% and 10% density change

Solution:

The given or available data is: $k = 1.4$

Basic equation:

$$\frac{\rho_0}{\rho} = \left[1 + \frac{(k-1)}{2}\cdot M^2\right]^{\frac{1}{k-1}} \quad (12.20c) \qquad \text{Hence} \quad \frac{\Delta\rho}{\rho_0} = \frac{\rho_0 - \rho}{\rho_0} = 1 - \frac{\rho}{\rho_0} \quad \text{so} \quad \frac{\Delta\rho}{\rho_0} = 1 - \left[1 + \frac{(k-1)}{2}\cdot M^2\right]^{\frac{1}{1-k}}$$

Here are the results, generated using *Excel*:

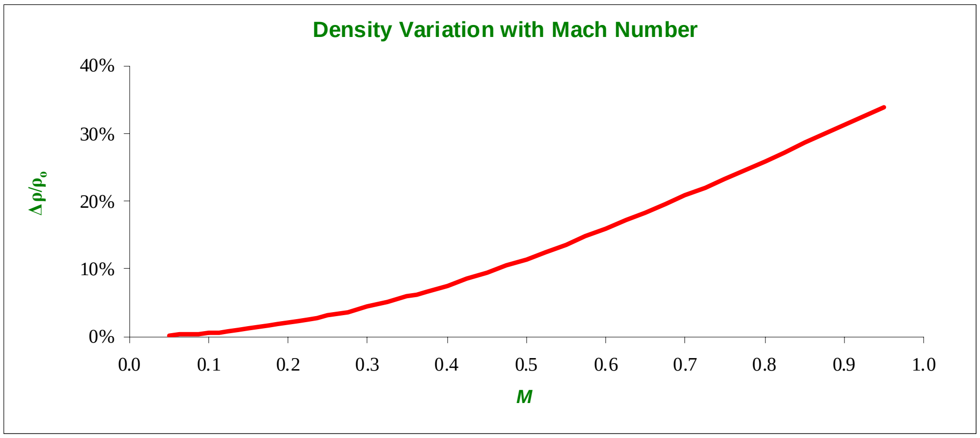
M	$\Delta\rho/\rho_o$
0.05	0.1%
0.10	0.5%
0.15	1.1%
0.20	2.0%
0.25	3.1%
0.30	4.4%
0.35	5.9%
0.40	7.6%
0.45	9.4%
0.50	11%
0.55	14%
0.60	16%
0.65	18%
0.70	21%
0.75	23%
0.80	26%
0.85	29%
0.90	31%
0.95	34%

To find M for specific density changes use *Goal Seek* repeatedly

M	$\Delta\rho/\rho_o$
0.142	1%
0.322	5%
0.464	10%

Note: Based on ρ (not ρ_o) the results are:

0.142	0.314	0.441



Problem 12.31

[Difficulty: 2]

12.31 Compute the air density in the undisturbed air and at the stagnation point of an aircraft flying at 250 m/s in air at 28 kPa and 250°C. What is the percentage increase in density? Can we approximate this as an incompressible flow?

Given: Pressure data on aircraft in flight

Find: Change in air density; whether flow can be considered incompressible

Solution:

The data provided, or available in the Appendices, is:

$k = 1.4 \qquad p_0 = 48 \cdot kPa \qquad p = 27.6 \cdot kPa \qquad T = (-55 + 273) \cdot K$

Governing equation (assuming isentropic flow):

$$\frac{p}{\rho^k} = \text{constant} \qquad (12.12c)$$

Hence

$$\frac{\rho}{\rho_0} = \left(\frac{p}{p_0}\right)^{\frac{1}{k}}$$

so

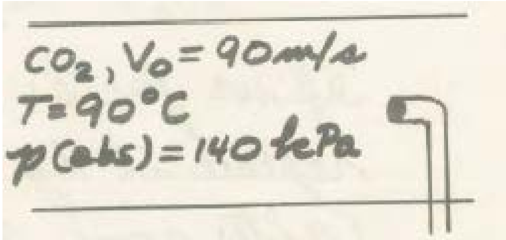
$$\frac{\Delta\rho}{\rho} = \frac{\rho_0 - \rho}{\rho} = \frac{\rho_0}{\rho} - 1 = \left(\frac{p_0}{p}\right)^{\frac{1}{k}} - 1 \qquad \frac{\Delta\rho}{\rho} = 48.5 \cdot \%$$

NOT an incompressible flow!

Problem 12.32

(Difficulty 2)

12.32 Carbon dioxide flows in a duct at a velocity of $90\ \frac{m}{s}$, absolute pressure $140\ kPa$, and the temperature 90 °C. Calculate pressure and temperature on the nose of a small object placed in this flow.



Find: The pressure and temperature on the nose.

Assumptions: Carbon dioxide behaves as an ideal gas. The flow is steady. The flow decelerates isentropically to the stagnation conditions.

Solution: Use the energy equation (4.56)

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

For isentropic flow, we have the relation for the pressure and temperature

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{k}{k-1}}$$

The flow is steady and there is no heat transfer or work and the elevations are the same. The velocity at the nose is zero. The energy equation becomes, where 0 denotes the stagnation condition:

$$0 = (h_0 - h) - \frac{1}{2}V^2$$

For ideal gas the enthalpy is related to the specific heat and we have:

$$0 = c_p(T_0 - T) - \frac{1}{2}V^2$$

Where for carbon dioxide

$$c_p = 858.2\frac{J}{kg\cdot K}$$

Thus the stagnation temperature is

$$T_0 = T + \frac{V^2}{2\,c_p} = 90\;C + \frac{\left(90\;\frac{m}{s}\right)^2}{2 \times 858.2\,\frac{J}{kg \cdot K}} = 90\;C + 4.72\;℃ = \;94.7\;℃ = 367.9\;K$$

For the stagnation pressure we have from the isentropic relation

$$p = p_0 \left(\frac{T}{T_0}\right)^{\frac{k}{k-1}}$$

Where for carbon dioxide

$$k = 1.28$$

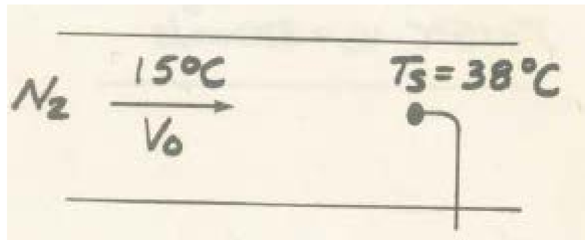
The stagnation pressure is then

$$p = 140\;kPa \times \left(\frac{363.2}{367.9}\right)^{\frac{1.28}{1.28-1}} = 148.5\;kPa$$

Problem 12.33

(Difficulty 1)

12.33 If nitrogen at 15 °C is flowing and the stagnation temperature on the nose of a small object in the flow is measured as 38 °C, what is the velocity in the pipe?



Find: The velocity in the pipe.

Assumptions: Nitrogen behaves as an ideal gas. The flow is steady. The flow decelerates isentropically to the stagnation conditions.

Solution: Use the energy equation (4.56)

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

The flow is steady and there is no heat transfer or work and the elevations are the same. The velocity at the nose is zero. The energy equation becomes, where 0 denotes the stagnation condition:

$$0 = (h_0 - h) - \frac{1}{2}V^2$$

For ideal gas the enthalpy is related to the specific heat and we have:

$$0 = c_p(T_0 - T) - \frac{1}{2}V^2$$

The velocity is then

$$V_0 = \sqrt{2c_p(T_0 - T)}$$

For nitrogen, the specific heat is

$$c_p = 1038\,\frac{J}{kg\cdot K}$$

The velocity is then

$$V_0 = \sqrt{2 \times 1038 \frac{J}{kg \cdot K} \times (39C - 15C) \times \left(\frac{kg\ m}{N\ s^2}\right) \times \left(\frac{N\ m}{J}\right)} = 219\ \frac{m}{s}$$

Problem 12.34

[Difficulty: 2]

12.34 An aircraft cruises at $M=0.65$ at 10 km altitude on a standard day. The aircraft speed is deduced from measurement of the difference between the stagnation and static pressures. What is the value of this difference? Compute the air speed from this actual difference assuming (a) compressibility and (b) incompressibility. Is the discrepancy in airspeed computations significant in this case?

Given: Mach number of aircraft

Find: Pressure difference; air speed based on a) compressible b) incompressible assumptions

Solution:

The data provided, or available in the Appendices, is:

$R = 287\cdot\frac{J}{kg\cdot K}$ $\quad c_p = 1004\cdot\frac{J}{kg\cdot K}$ $\quad k = 1.4$ $\quad M = 0.65$

From Table A.3, at 10 km altitude $\quad T = 223.3\cdot K$ $\quad p = 0.2615\cdot 101\cdot kPa$ $\quad p = 26.4\cdot kPa$

The governing equation for pressure change is: $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ (12.20a)

Hence $\quad p_0 = p\cdot\left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\quad p_0 = 35.1\cdot kPa$

The pressure difference is $\quad p_0 - p = 8.67\cdot kPa$

a) Assuming compressibility $\quad c = \sqrt{k\cdot R\cdot T}$ $\quad c = 300\frac{m}{s}$ $\quad V = M\cdot c$ $\quad V = 195\frac{m}{s}$

b) Assuming incompressibility

Here the Bernoulli equation applies in the form $\quad \frac{p}{\rho} + \frac{V^2}{2} = \frac{p_0}{\rho}$ $\quad$ so $\quad V = \sqrt{\frac{2\cdot(p_0 - p)}{\rho}}$

For the density $\quad \rho = \frac{p}{R\cdot T}$ $\quad \rho = 0.412\frac{kg}{m^3}$ $\quad V = \sqrt{\frac{2\cdot(p_0 - p)}{\rho}}$

Hence $\quad V = 205\frac{m}{s}$

In this case the error at $M = 0.65$ in computing the speed of the aircraft using Bernoulli equation is $\quad \frac{205 - 195}{195} = 5.13\cdot\%$

Problem 12.35

[Difficulty: 2]

12.35 Modern high-speed aircraft use "air data computers" to compute air speed from measurement of the difference between the stagnation and static pressures. Plot, as a function of actual Mach number M, for $M = 0.1$ to $M = 0.9$, the percentage error in computing the Mach number assuming incompressibility (i.e., using the Bernoulli equation), from this pressure difference. Plot the percentage error in speed, as a function of speed, of an aircraft cruising at 12 km altitude, for a range of speeds corresponding to the actual Mach number ranging from $M = 0.1$ to $M = 0.9$.

Given: Flight altitude of high-speed aircraft

Find: Mach number and aircraft speed errors assuming incompressible flow; plot

Solution:

The governing equation for pressure change is:
$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} \qquad (12.20a)$$

Hence
$$\Delta p = p_0 - p = p\cdot\left(\frac{p_0}{p} - 1\right) \qquad \Delta p = p\cdot\left[\left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} - 1\right] \qquad (1)$$

For each Mach number the actual pressure change can be computed from Eq. 1

Assuming incompressibility, the Bernoulli equation applies in the form
$$\frac{p}{\rho} + \frac{V^2}{2} = \frac{p_0}{\rho} \quad \text{so} \quad V = \sqrt{\frac{2\cdot(p_0 - p)}{\rho}} = \sqrt{\frac{2\cdot\Delta p}{\rho}}$$

and the Mach number based on this is
$$M_{incomp} = \frac{V}{c} = \frac{\sqrt{\frac{2\cdot\Delta p}{\rho}}}{\sqrt{k\cdot R\cdot T}} = \sqrt{\frac{2\cdot\Delta p}{k\cdot\rho\cdot R\cdot T}}$$

Using Eq. 1
$$M_{incomp} = \sqrt{\frac{2}{k}\cdot\left[\left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} - 1\right]}$$

The error in using Bernoulli to estimate the Mach number is
$$\frac{\Delta M}{M} = \frac{M_{incomp} - M}{M}$$

For errors in speed:

Actual speed:
$$V = M\cdot c \qquad V = M\cdot\sqrt{k\cdot R\cdot T}$$

Speed assuming incompressible flow:
$$V_{inc} = M_{incomp}\cdot\sqrt{k\cdot R\cdot T}$$

The error in using Bernoulli to estimate the speed from the pressure difference is
$$\frac{\Delta V}{V} = \frac{V_{incomp} - V}{V}$$

The computations and plots are shown below, generated using *Excel*:

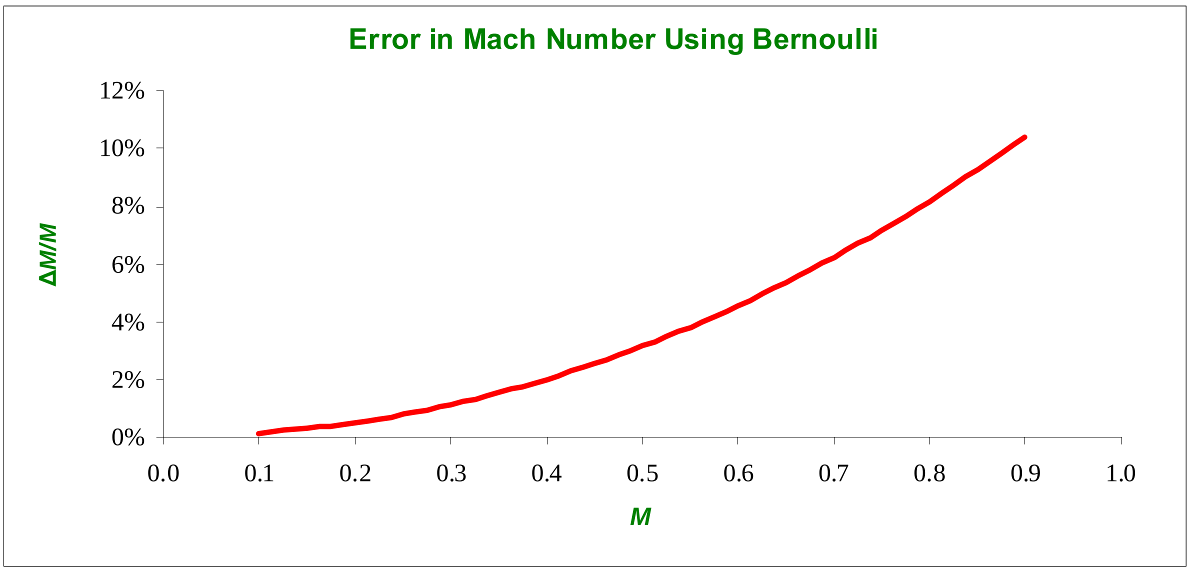
The given or available data is:

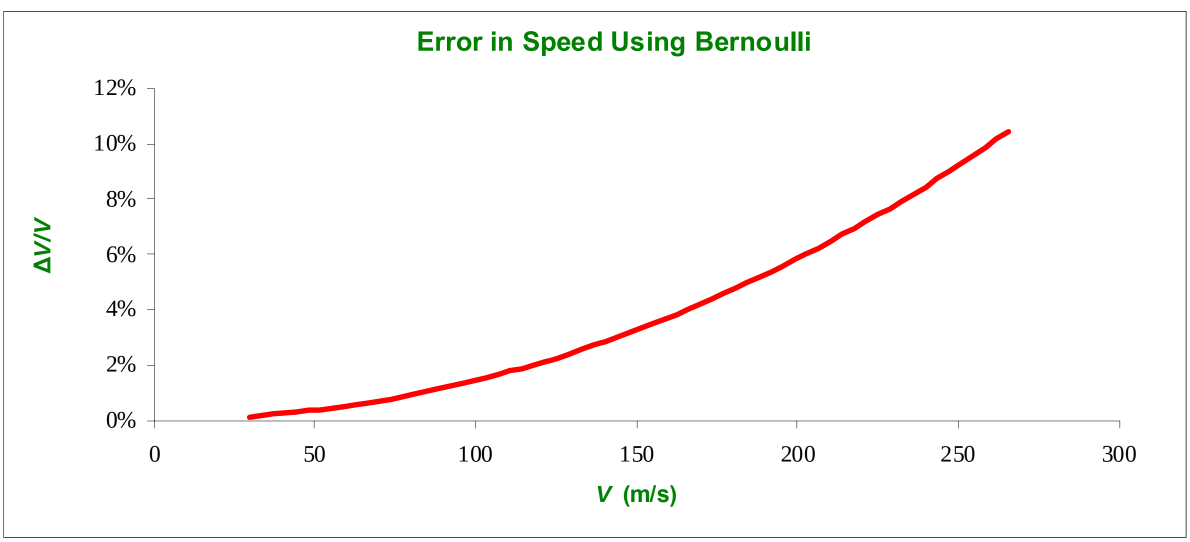
R = 286.9 J/kg.K
k = 1.4
T = 216.7 K (At 12 km, Table A.3)

Computed results:

c = 295 m/s

M	M_{incomp}	$\Delta M/M$	V (m/s)	V_{incomp} (m/s)	$\Delta V/V$
0.1	0.100	0.13%	29.5	29.5	0.13%
0.2	0.201	0.50%	59.0	59.3	0.50%
0.3	0.303	1.1%	88.5	89.5	1.1%
0.4	0.408	2.0%	118	120	2.0%
0.5	0.516	3.2%	148	152	3.2%
0.6	0.627	4.6%	177	185	4.6%
0.7	0.744	6.2%	207	219	6.2%
0.8	0.865	8.2%	236	255	8.2%
0.9	0.994	10.4%	266	293	10.4%





Problem 12.36 [Difficulty: 2]

12.36 A supersonic wind tunnel test section is designed to have $M = 2.5$ at 15°C and 35 kPa (abs). The fluid is air. Determine the required inlet stagnation conditions, T_0 and p_0. Calculate the required mass flow rate for a test section area of 0.175 m^2.

Given: Wind tunnel at M = 2.5

Find: Stagnation conditions; mass flow rate

Solution:

Basic equations:

$$c = \sqrt{k \cdot R \cdot T} \qquad M = \frac{V}{c} \qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}} \qquad \frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$$

Given or available data

$$M = 2.5 \qquad T = (15 + 273) \cdot K \qquad p = 35 \cdot kPa \qquad A = 0.175 \cdot m^2$$

$$k = 1.4 \qquad R = 286.9 \cdot \frac{J}{kg \cdot K}$$

Then

$$T_0 = T \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right) \qquad T_0 = 648\,K \qquad T_0 = 375 \cdot °C$$

Also

$$p_0 = p \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}} \qquad p_0 = 598 \cdot kPa$$

The mass flow rate is given by $m_{rate} = \rho \cdot A \cdot V$

We need

$$c = \sqrt{k \cdot R \cdot T} \qquad c = 340\,\frac{m}{s} \qquad V = M \cdot c \qquad V = 850\,\frac{m}{s}$$

and also

$$\rho = \frac{p}{R \cdot T} \qquad \rho = 0.424\,\frac{kg}{m^3}$$

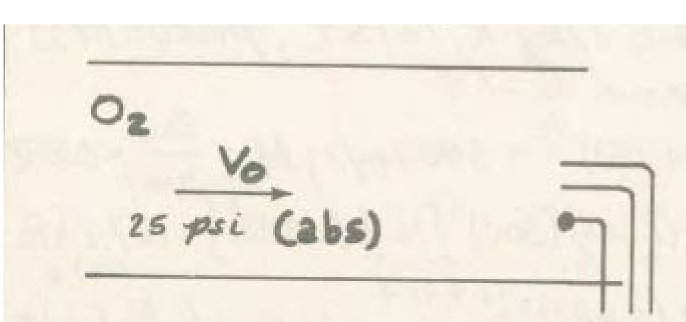
Then

$$m_{rate} = \rho \cdot A \cdot V \qquad m_{rate} = 63.0\,\frac{kg}{s}$$

Problem 12.37

(Difficulty 2)

12.37 Oxygen flows in a passage at a pressure of 25 $psia$. The pressure and temperature on the nose of a small object in the flow are 28 $psia$ and 150 °F, repectively. What is the velocity in the passage?



Find: The velocity V_0 in the passage.

Assumptions: Oxygen behaves as an ideal gas. The flow is steady. The flow decelerates isentropically to the stagnation conditions.

Solution: Use the energy equation (4.56)

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

For isentropic flow, we have the relation for the pressure and temperature

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{k}{k-1}}$$

The flow is steady and there is no heat transfer or work and the elevations are the same. The velocity at the nose is zero. The energy equation becomes, where 0 denotes the stagnation condition:

$$0 = (h_0 - h) - \frac{1}{2}V^2$$

For ideal gas the enthalpy is related to the specific heat and we have:

$$0 = c_p(T_0 - T) - \frac{1}{2}V^2$$

Or the velocity is

$$V = \sqrt{2c_p(T_0 - T)}$$

To find the temperature, we use the isentropic relation

$$T = T_0 \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}$$

Where for oxygen k = 1.4. The static temperature is then

$$T = (150 + 459.7)R \left(\frac{25\ psia}{28\ psia}\right)^{\frac{1.4-1}{1.4}} = 590.3\ R = 130.6\ F$$

The specific heat of oxygen is c_p = 0.2172 Btu/lbm-R

The velocity is then

$$V = \sqrt{2 \times 0.2172\ \frac{Btu}{lbm\ R}(150 - 130.6)\ F \times \left(\frac{lbm\ ft}{lbf\ s^2}\right) \times \left(\frac{778\ ft\ lbf}{Btu}\right)} = 459.6\ \ \frac{ft}{s}$$

Problem 12.38

(Difficulty 2)

12.38 What is the pressure on the nose of a bullet moving through standard sea level air at 300 $\frac{m}{s}$ according to (a) the flow is incompressible and (b) the flow is compressible. Compare results.

Find: The pressure on the nose of a bullet

Assumptions: The speed of the bullet is steady. The air decelerates isentropically to the stagnation conditions.

Solution: Use the Bernoulli and energy equation and isentropic relations

a) For the incompressible assumption, we have the Bernoulli equation between the static and stagnation states as:

$$\frac{p_s}{\rho}+\frac{V_s^2}{2}+gz_s=\frac{p_0}{\rho}+\frac{V_0^2}{2}+gz_0$$

There is no elevation change and the velocity on the nose is zero. We assume the elevation is constant so the Bernoulli equation becomes

$$\frac{p_s}{\rho}+\frac{V_s^2}{2}=\frac{p_0}{\rho}$$

For air we have:

$$\rho=1.225\ \frac{kg}{m^3},\quad k=1.4,\quad c_p=1004\ \frac{J}{kg\ C}$$

We have the absolute pressure on the nose as:

$$p_0=p_s+\frac{\rho V_s^2}{2}=101.3\ kPa+1.225\ \frac{kg}{m^3}\times\frac{\left(300\ \frac{m}{s}\right)^2}{2}=156.4\ kPa$$

b) For the compressible flow case, we have the energy equation

$$\dot{Q}-\dot{W}_s-\dot{W}_{shear}-\dot{W}_{other}=\frac{\partial}{\partial t}\int_{CV}e\rho d\forall+\int_{CS}\left(u+pv+\frac{V^2}{2}+gz\right)\rho\vec{V}\cdot d\vec{A}$$

The flow is steady and there is no heat transfer or work and the elevations are the same. The velocity at the nose is zero. The energy equation becomes:

$$0=(h_0-h_s)-\frac{1}{2}V_s^2$$

For ideal gas the enthalpy is related to the specific heat and we have:

$$0 = c_p(T_0 - T_s) - \frac{1}{2}V_s^2$$

The stagnation temperature is then

$$T_0 = T + \frac{V^2}{2\,c_p} = (20 + 273.2)K + \frac{\left(3000\ \frac{m}{s}\right)^2}{2 \times 1004\,\frac{J}{kg \cdot K}} = 338\ K$$

For isentropic flow, we have the relation for the pressure and temperature

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{k}{k-1}}$$

The stagnation pressure is then

$$p_0 = p\left(\frac{T_0}{T}\right)^{\frac{k}{k-1}} = \ 101.3\ kPa \times \left(\frac{338\ K}{293.2\ K}\right)^{\frac{1.4}{1.4-1}} = 166.7\ \ kPa$$

Problem 12.39

(Difficulty: 2)

12.39 Air flows steadily through a length ((1) denotes inlet and (2) denotes exit) of insulated constant-area duct. Properties change along the duct as a result of friction.

(a) Beginning with the control volume form of the first law of thermodynamics, show that the equation can be reduced to

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = constant$$

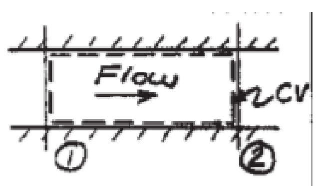
(b) Denoting the constant by h_0 (the stagnation enthalpy), show that for adiabatic flow of an ideal gas with friction:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

(c) For this flow dose $T_{01} = T_{02}$? $p_{01} = p_{02}$? Explain these results.

Assumptions: (1) $\dot{Q} = 0\ (adiabatic)$ (2) $\dot{W}_s = 0$ (3) $\dot{W}_{shear} = 0$ (4) Steady flow (5) Uniform flow at each section (6) Neglect Δz (7) Ideal gas; $h_0 - h = C_p(T_0 - T)$, $C_p = \frac{kR}{k-1}$, $c^2 = kRT$

Solution: Apply the energy equation to the CV shown:



$$\dot{Q} + \dot{W}_s + \dot{W}_{shear} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS} (e + pv)\rho \bar{V} \cdot d\bar{A}$$

Then

$$0 = \left(u_1 + p_1 v_1 + \frac{V_1^2}{2}\right)\{-|\rho_1 V_1 A|\} + \left(u_2 + p_2 v_2 + \frac{V_2^2}{2}\right)\{|\rho_2 V_2 A|\}$$

But $h = u + pv$, and $|\rho_1 V_1 A| = |\rho_2 V_2 A| = |\rho V A| = \dot{m}$, so

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = h + \frac{V^2}{2} = h_0 = constant$$

Thus

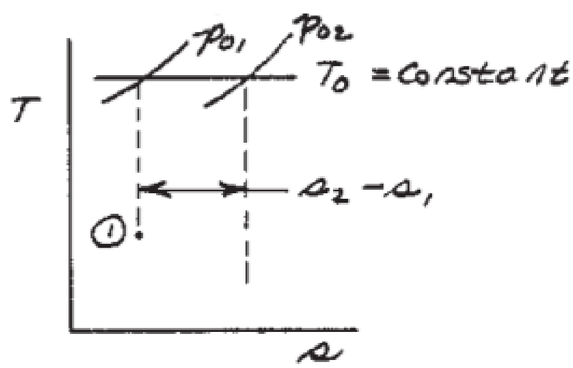
$$C_p T_0 = C_p T + \frac{V^2}{2}$$

$$\frac{T_0}{T} = 1 + \frac{V^2}{2C_p T} = 1 + \frac{(k-1)V^2}{2kRT} = 1 + \frac{k-1}{2}\frac{V^2}{C^2} = 1 + \frac{k-1}{2}M^2$$

From the energy equation,

$$T_{01} = T_{02} = T_0 = constant$$

The process diagram is:



Since flow is frictional,

$$\Omega_2 > \Omega_1$$

Therefore

$$p_{02} < p_{01}$$

Problem 12.40

[Difficulty: 2]

12.40 Air flows in an insulated duct. At point (1) the conditions are $M_1 = 0.1$, $T_1 = 20°\text{C}$, and $p_1 = 1.0$ MPa (abs). Downstream, at point (2), because of friction the conditions are $M_2 = 0.7$, $T_2 = -5.62°\text{C}$, and $p_2 = 136.5$ kPa (abs). (Four significant figures are given to minimize roundoff errors.) Compare the stagnation temperatures at points (1) and (2), and explain the result. Compute the stagnation pressures at points (1) and (2). Can you explain how it can be that the velocity *increases* for this frictional flow? Should this process be isentropic or not? Justify your answer by computing the change in entropy between points (1) and (2). Plot static and stagnation state points on a *Ts* diagram.

Given: Data on air flow in a duct

Find: Stagnation pressures and temperatures; explain velocity increase; isentropic or not?

Solution:

The data provided, or available in the Appendices, is:

$$R = 287 \cdot \frac{J}{kg \cdot K} \qquad c_p = 1004 \cdot \frac{J}{kg \cdot K} \qquad k = 1.4$$

$$M_1 = 0.1 \quad T_1 = (20 + 273) \cdot K \quad p_1 = 1000 \cdot kPa \quad M_2 = 0.7 \quad T_2 = (-5.62 + 273) \cdot K \quad p_2 = 136.5 \cdot kPa$$

For stagnation temperatures:

$$T_{01} = T_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^{\ 2}\right) \qquad T_{01} = 293.6\,K \qquad T_{01} = 20.6 \cdot C$$

$$T_{02} = T_2 \cdot \left(1 + \frac{k-1}{2} \cdot M_2^{\ 2}\right) \qquad T_{02} = 293.6\,K \qquad T_{02} = 20.6 \cdot C$$

(Because the stagnation temperature is constant, the process is adiabatic)

For stagnation pressures:

$$p_{01} = p_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^{\ 2}\right)^{\frac{k}{k-1}} \qquad p_{01} = 1.01 \cdot MPa$$

$$p_{02} = p_2 \cdot \left(1 + \frac{k-1}{2} \cdot M_2^{\ 2}\right)^{\frac{k}{k-1}} \qquad p_{02} = 189 \cdot kPa$$

The entropy change is:

$$\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right) \qquad \Delta s = 480 \cdot \frac{J}{kg \cdot K}$$

Note that

$$V_1 = M_1 \cdot \sqrt{k \cdot R \cdot T_1} \qquad V_1 = 34.3 \frac{m}{s} \qquad V_2 = M_2 \cdot \sqrt{k \cdot R \cdot T_2} \qquad V_2 = 229 \frac{m}{s}$$

Although there is friction, suggesting the flow should decelerate, because the static pressure drops so much, the net effect is flow acceleration!

The entropy increases because the process is adiabatic but irreversible (friction).

From the second law of thermodynamics $ds \geq \frac{\delta q}{T}$: becomes $ds > 0$

Problem 12.41

(Difficulty: 3)

12.41 Consider steady, adiabatic flow of air through a long straight pipe with $A = 0.05\ m^2$. At the inlet (section (1)) the air is at 200 $kPa\ (abs)$, 60 ℃, and 146 m/s. Downstream at section (2), the air is at 95.6 $kPa\ (abs)$ and 280 m/s. Determine $p_{01}, p_{02}, T_{01}, T_{02}$, and the entropy change for the flow. Show state and stagnation points on a T-s diagram.

Assumptions: (1) $\dot{Q} = 0\ (adiabatic)$ (2) $\dot{W}_s = 0$ (3) $\dot{W}_{shear} = 0$ (4) Steady flow (5) Uniform flow at each section (6) $\Delta z = 0$ (7) Ideal gas, $k = 1.4$ (8) $A_1 = A_2 = A = constant$

Solution: Use the continuity and energy equations and the isentropic flow relations

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho\, \bar{V} \cdot d\bar{A}$$

$$\dot{Q} + \dot{W}_s + \dot{W}_{shear} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(e + \frac{p}{\rho}\right)\rho\bar{V} \cdot d\bar{A}$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}$$

The upstream Mach number is found using

$$M_1 = \frac{V_1}{c_1}$$

$$c_1 = (kRT_1)^{\frac{1}{2}} = \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 333\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 366\ m/s$$

$$M_1 = \frac{V_1}{c_1} = \frac{146}{366} = 0.399$$

The upstream stagnation temperature and pressure are

$$T_{01} = T_1\left[1 + \frac{k-1}{2}M_1^2\right] = 333\ K \times [1 + 0.2 \times (0.399)^2] = 344\ K$$

$$p_{01} = p_1 \left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}} = 200\ kPa \times [1 + 0.2 \times (0.399)^2]^{3.5} = 223\ kPa$$

From the energy equation:

$$0 = \left(h_1 + \frac{V_1^2}{2}\right)\dot{m} + \left(h_2 + \frac{V_2^2}{2}\right)\dot{m}$$

Where

$$\dot{m} = \rho A V$$

From continuity,

$$0 = -\rho_1 V_1 A + \rho_2 V_2 A$$

Or, since the duct flow area is constant

$$\rho_1 V_1 = \rho_2 V_2$$

Since the flow is adiabatic

$$h_1 + \frac{V_1^2}{2} = h_{01} = h_2 + \frac{V_2^2}{2} = h_{02}$$

For an ideal gas with constant specific heats,

$$T_{02} = T_{01} = 344\ K$$

From continuity,

$$\rho_2 = \rho_1 \frac{V_1}{V_2} = \frac{p_1}{RT_1}\frac{V_1}{V_2} = 200 \times 10^3\ \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{333\ K} \times \frac{146}{280} = 1.09\ \frac{kg}{m^3}$$

Then the downstream temperature is

$$T_2 = \frac{p_2}{\rho_2 R} = 95.6 \times 10^3\ \frac{N}{m^2} \times \frac{m^3}{1.09\ kg} \times \frac{kg \cdot K}{287\ N \cdot m} = 306\ K\ (33\ ^\circ\mathrm{C})$$

The downstream Mach number is found as

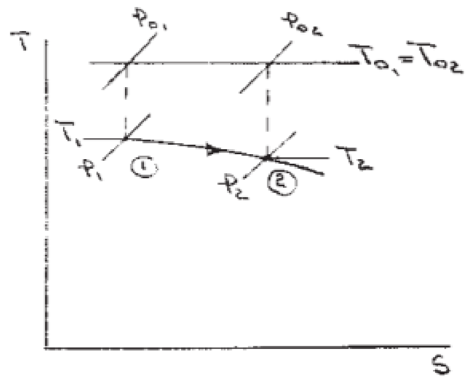
$$c_2 = (kRT_2)^{\frac{1}{2}} = \left(1.4 \times \frac{kg \cdot K}{287\ N \cdot m} \times 306\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 351\ m/s$$

$$M_2 = \frac{V_2}{c_2} = \frac{280}{351} = 0.798$$

The downstream stagnation pressure is

$$p_{02} = p_2 \left[1 + \frac{k-1}{2} M^2\right]^{\frac{k}{k-1}} = 95.6\ kPa[1 + 0.2(0.798)^2]^{3.5}$$

$$p_{02} = 145\ kPa$$



The entropy change is found using the T-ds equations

$$Tds = dh - vdp = c_p dT - \frac{RT}{p} dp$$

$$ds = c_p \frac{dT}{T} - R\frac{dp}{p} = ds_0 = -R\frac{dp_0}{p_0}$$

$$s_{02} - s_{01} = s_2 - s_1 = -R \ln \frac{p_{02}}{p_{01}}$$

$$s_2 - s_1 = -287\ \frac{J}{kg \cdot K} \times \ln \frac{145}{223} = 0.124\ \frac{kJ}{kg \cdot K}$$

Problem 12.42

(Difficulty: 2)

12.42 Air passes through a normal shock in a supersonic wind tunnel. Upstream conditions are $M_1 = 1.8$, $T_1 = 270\ K$, and $p_1 = 10\ kPa\ (abs)$. Downstream conditions are $M_2 = 0.6165$, $T_2 = 413.6\ K$, and $p_2 = 36.13\ kPa\ (abs)$. (Four significant figures are given to minimize roundoff errors.) Evaluate local isentropic stagnation conditions (a) upstream from, and (b) downstream from, the normal shock. Calculate the change in specific entropy across the shock. Plot static and stagnation state points on a Ts diagram.

Assumptions: The flow through the shock is one-dimensional

Solution: Use the isentropic flow and normal shock relations

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{\frac{k}{k-1}}$$

The upstream stagnation temperature and pressure are

$$T_{01} = T_1 \left[1 + \frac{k-1}{2} M_1^2\right] = 270\ K \times [1 + 0.2 \times (1.8)^2] = 445\ K$$

$$p_{01} = p_1 \left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}} = 10.0\ kPa \times [1 + 0.2 \times (1.8)^2]^{3.5} = 57.5\ kPa\ (abs)$$

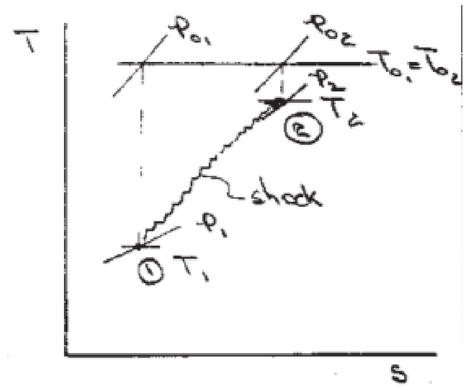
The downstream stagnation temperature is

$$T_{02} = T_2 \left[1 + \frac{k-1}{2} M_2^2\right] = 413.6\ K \times [1 + 0.2 \times (0.6165)^2] = 445\ K$$

Since the flow through the shock is adiabatic, the stagnation temperatures are equal. The downstream stagnation pressure is

$$p_{02} = p_2 \left[1 + \frac{k-1}{2} M_2^2\right]^{\frac{k}{k-1}} = 36.13\ kPa \times [1 + 0.2 \times (0.6165)^2]^{3.5} = 46.7\ kPa\ (abs)$$

The process is



The entropy change is determined from the T-ds equations:

$$Tds = dh - vdp = c_p dT - RT\frac{dp}{p}$$

$$ds = c_p\frac{dT}{T} - R\frac{dp}{p}$$

$$s_2 - s_1 = s_{02} - s_{01} = -R\ln\frac{p_{02}}{p_{01}} = -287\ \frac{J}{kg\cdot K}\times\ln\frac{46.69}{57.46}$$

The specific entropy change is

$$s_2 - s_1 = 59.6\ \frac{J}{kg\cdot K}$$

Problem 12.43

(Difficulty: 3)

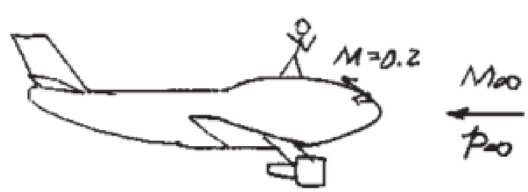
12.43 A Boeing 747 cruise at $M = 0.87$ at an altitude of $13\ km$ on a standard day. A window in the cockpit is located where the external flow Mach number is 0.2 relative to the plane surface. The cabin is pressurized to an equivalent altitude of $2500\ m$ in a standard atmosphere. Estimate the pressure, difference across the window. Be sure to specify the direction of the net pressure force.

Assumptions: (1) Ideal gas (2) Isentropic flow

Solution: Use the isentropic flow relations

$$p_0 = p\left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$$

Consider observer on aircraft : air is decelerated isentropically from $M_\infty = 0.87$ to $M = 0.2$.



From Table A.3:

$Altitude\ (km)$	p/p_0	$p\ (kPa)$
2.5	0.7372	74.7
13.0	0.1636	16.6

For isentropic stagnation:

$$p_0 = p_\infty\left(1 + \frac{k-1}{2}M_\infty^2\right)^{\frac{k}{k-1}} = 16.6\ kPa\left(1 + \frac{1.4-1}{2}(0.87)^2\right)^{3.5} = 27.2\ kPa\ (abs)$$

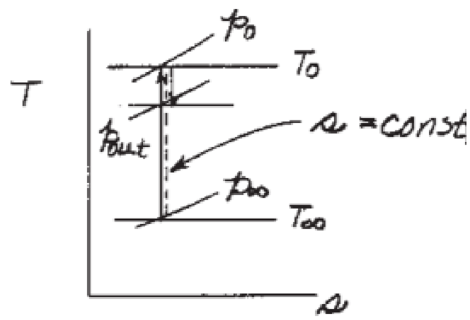
For isentropic stagnation to $M = 0.2$:

$$p_{out} = \frac{p_0}{\left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}} = \frac{27.2\ kPa}{(1 + 0.2(0.2)^2)^{3.5}} = 26.5\ kPa\ (abs)$$

The pressure difference across window is:

$$\Delta p = p_{in} - p_{out} = (74.7 - 26.5)\ kPa = 48.2\ kPa$$

{Inside pressure is higher; window force is toward outside.} The corresponding Ts diagram is:



Problem 12.44 [Difficulty: 2]

12.44 Space debris impact is a real concern for spacecraft. If a piece of space debris were to create a hole of 0.001 in.^{2} area in the hull of the International Space Station (ISS), at what rate would air leak from the ISS? Assume that the atmosphere in the International Space Station (ISS) is air at a pressure of 14.7 psia and a temperature of 65°F.

Given: Air leak in ISS

Find: Mass flow rate

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\quad V_{crit} = \sqrt{\frac{2 \cdot k}{k+1} \cdot R \cdot T_0}$ $\quad \frac{\rho_0}{\rho_{crit}} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}$

The interior conditions are the stagnation conditions for the flow

Given or available data $T_0 = (65 + 460) \cdot R$ $\quad p_0 = 14.7 \cdot psi$ $\quad R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$ $\quad k = 1.4$ $\quad A = 0.001 \cdot in^2$

The density of air inside the ISS would be: $\rho_0 = \frac{p_0}{R_{air} \cdot T_0}$ $\quad \rho_0 = 2.35 \times 10^{-3} \cdot \frac{slug}{ft^3}$

Then $\rho_{crit} = \frac{\rho_0}{\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}}$ $\quad \rho_{crit} = 1.49 \times 10^{-3} \cdot \frac{slug}{ft^3}$ $\quad V_{crit} = \sqrt{\frac{2 \cdot k}{k+1} \cdot R_{air} \cdot T_0}$ $\quad V_{crit} = 1025 \cdot \frac{ft}{s}$

The mass flow rate is $m_{rate} = \rho_{crit} \cdot V_{crit} \cdot A$ $\quad m_{rate} = 1.061 \times 10^{-5} \cdot \frac{slug}{s}$ $\quad m_{rate} = 3.41 \times 10^{-4} \cdot \frac{lbm}{s}$

Problem 12.45

(Difficulty: 1)

12.45 A CO_2 cartridge is used to propel a toy rocket. Gas in the cartridge is pressurized to $45\ MPa\ (gage)$ and is at 25 °C. Calculate the critical conditions (temperature, pressure, and flow speed) that correspond to these stagnation conditions.

Solution: Use the isentropic flow relations

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$$

For CO_2, $k = 1.29$. At critical conditions, $M = 1$. The critical temperature is given by

$$\frac{T_0^*}{T^*} = 1 + \frac{k-1}{2} = 1.145$$

$$T^* = \frac{T_0^*}{1.145} = \frac{298\ K}{1.145} = 260\ K$$

And the critical pressure is

$$\frac{p_0^*}{p^*} = \left[1 + \frac{k-1}{2}\right]^{\frac{k}{k-1}} = [1.145]^{4.448} = 1.826$$

$$p^* = \frac{p_0^*}{1.826} = \frac{45.101\ MPa}{1.826} = 24.7\ MPa\ (abs)$$

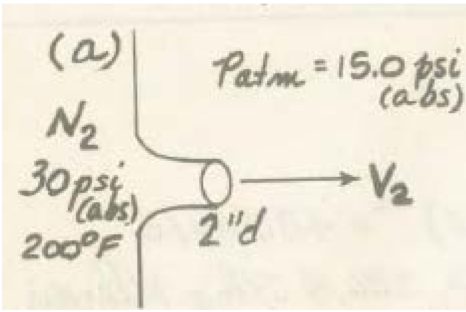
The velocity is then

$$V^* = c^* = (kRT^*)^{\frac{1}{2}} = \left(1.29 \times 189\ \frac{N \cdot m}{kg \cdot K} \times 260\ K\right)^{\frac{1}{2}} = 252\ m/s$$

Problem 12.46

(Difficulty 2)

12.46 Nitrogen flows from a large tank, through a convergent nozzle of 2 in tip diameter, into the atmosphere. The temperature in that tank is 200°F. Calculate the pressure, velocity, temperature, and sonic velocity in the jet. And calculate the flow rate when the tank pressure is (a) 30psi and (b) 25 psi. Barometric pressure is 15.0 psi. What is the lowest tank pressure that will produce sonic velocity in the jet? What is this velocity and what is the flow rate?



Solution:

For nitrogen we have:

$$k = 1.4$$

$$c_p = 6210 \frac{ft \cdot lbf}{slug \cdot °R}$$

$$R = 1773 \frac{ft \cdot lbf}{slug \cdot °R}$$

$$\left(\frac{p_2}{p_1}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = 0.528$$

(a)

$$p_2 = 0.528 \times 30\ psi = 15.84\ psi > p_{atm}$$

Therefore, the flow is sonic at the exit and we have:

$$V_2 = c_2$$

$$p_2 = 15.85\ psi$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$$

$$T_2 = (200 + 459.6)°R \times \left(\frac{15.85\ psi}{30\ psi}\right)^{\frac{0.4}{1.4}} = 550\ °R$$

$$V_2 = c_2 = \sqrt{kRT_2} = \sqrt{1.4 \times 1773\ \frac{ft \cdot lbf}{slug \cdot °R} \times 550\ °R} = 1168\ \frac{ft}{s}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{15.85\ \times 144\ \frac{lbf}{ft^2}}{1773\ \frac{ft \cdot lbf}{slug \cdot °R} \times 550\ °R} = 0.00234\ \frac{slug}{ft^3}$$

The flow rate is:

$$\dot{m} = \rho_2 V_2 A_2 = 0.06\ \frac{slug}{s}$$

(b)

$$p_2 = 0.528 \times 25\ psi = 13.2\ psi < p_{atm}$$

Therefore we have:

$$p_2 = 15.0\ psi$$

$$T_2 = (200 + 459.6)°R \times \left(\frac{15psi}{25\ psi}\right)^{\frac{0.4}{1.4}} = 570\ °R$$

$$c_p T_2 + \frac{V_2^2}{2} = c_p T_s$$

$$V_2 = \sqrt{2c_p(T_s - T_2)} = \sqrt{2 \times 6210\ \frac{ft \cdot lbf}{slug \cdot °R} \times (659.6 - 570)°R} = 1058\ \frac{ft}{s}$$

$$c_2 = \sqrt{kRT_2} = \sqrt{1.4 \times 1773\ \frac{ft \cdot lbf}{slug \cdot °R} \times 570\ °R} = 1189\ \frac{ft}{s}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{15 \times 144\ \frac{lbf}{ft^2}}{1773\ \frac{ft \cdot lbf}{slug \cdot °R} \times 570\ °R} = 0.00214\ \frac{slug}{ft^3}$$

$$\dot{m} = \rho_2 V_2 A_2 = 0.049\ \frac{slug}{s}$$

The lowest tank pressure to yield sonic velocity must be:

$$\left(\frac{15\ psi}{p_1}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = 0.528$$

$$p_1 = 28.41\ psi$$

$$T_2 = (200 + 459.6)°R \times \left(\frac{15psi}{28.41\ psi}\right)^{\frac{0.4}{1.4}} = 550\ °R$$

$$c_2 = \sqrt{kRT_2} = \sqrt{1.4 \times 1773\ \frac{ft \cdot lbf}{slug \cdot °R} \times 550\ °R} = 1168\ \frac{ft}{s}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{15 \times 144\ \frac{lbf}{ft^2}}{1773\ \frac{ft \cdot lbf}{slug \cdot °R} \times 550\ °R} = 0.0022\ \frac{slug}{ft^3}$$

$$\dot{m} = \rho_2 V_2 A_2 = 0.056\ \frac{slug}{s}$$

Problem 12.47

(Difficulty 3)

12.47 Air flows from the atmosphere into an evacuated tank through a convergent nozzle of 38 mm tip diameter. If atmospheric pressure and temperature are 101.3 kPa and 15 ℃, respectively, what vacuum must be maintained in the tank to produce sonic velocity in the jet? What is the flow rate? What is the flow rate when the vacuum is 254 mm of mercury?

Find: The pressure in the tank and the flow rate.

Assumptions: The flow in the nozzle is steady and isentropic. Air can be treated as an ideal gas

Solution: Use the energy equation and the isentropic relations. The energy equation is:

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

The relation between the pressures and the Mach number for isentropic flow is given by

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}$$

At the throat of the convergent nozzle, the Mach number is unity when the velocity is sonic. The pressure p is then, where the specific heat ratio of the air is 1.4

$$p = \frac{p_0}{\left[1 + \frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}} = \frac{101.3\ kPa}{\left[1 + \frac{1.4-1}{2}1^2\right]^{\frac{1.4}{1.4-1}}} = 53.5\ kPa$$

The vacuum can be calculated as:

$$Vac = (101.3\ kPa - 53.5\ kPa) \times \frac{760\ mm}{101.3\ kPa} = 359\ mm$$

So the vacuum is 359 mm Hg vac.

To calculate the temperature we need to determine the temperature, density, and velocity at the exit. The temperature is determined from the isentropic relation between temperature and pressure, written as:

$$T = T_0\left[\frac{p}{p}\right]^{\frac{k-1}{k}} = 288.3\ K \times \left[\frac{53.5\ kPa}{101.3\ kPa}\right]^{\frac{1.4-1}{1.4}} = 240.2\ K$$

From the ideal gas law we compute the density as

$$\rho = \frac{p}{RT} = \frac{53.5\ kPa}{286.8\ \frac{J}{kg \cdot K} \times 240.1\ K} = 0.777\ \frac{kg}{m^3}$$

We evaluate the velocity from the speed of sound and Mach number. The speed of sound is:

$$c = \sqrt{kRT} = \sqrt{1.4 \times 286.8\ \frac{J}{kg \cdot K} \times 240.1\ K} = 310.4\ \frac{m}{s}$$

The velocity equals the sonic velocity

$$V = c = 310.4\ \frac{m}{s}$$

The mass flow rate can be calculated as:

$$\dot{m} = \rho V A = 0.777\ \frac{kg}{m^3} \times 310.4\ \frac{m}{s} \times \frac{\pi}{4} \times (0.038\ m)^2 = 0.274\ \frac{kg}{s}$$

If the vacuum is 254 mm of mercury, the pressure is:

$$p = 101.3\ kPa - 101.3\ kPa \times \frac{254\ mm}{760\ mm} = 67.4\ kPa$$

The exit temperature is then

$$T = T_0 \left[\frac{p}{p}\right]^{\frac{k-1}{k}} = 288.3\ K \times \left[\frac{67.4\ kPa}{101.3\ kPa}\right]^{\frac{1.4-1}{1.4}} = 256.5\ K$$

The energy equation is:

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V} \cdot d\vec{A}$$

There are no heat or work transfers and the elevation change is zero. The entering velocity is zero and so the energy equation becomes, using the enthalpy as the sum of the internal energy and flow work:

$$0 = (h - h_0) + \frac{1}{2}V^2$$

As the air is an ideal gas we have:

$$0 = c_p(T - T_0) + \frac{1}{2}V^2$$

With the value of cp of 1004 J/kg-K, we have for the velocity

$$V = \sqrt{2c_p(T_0 - T)} = \sqrt{2 \times 1003 \ \frac{J}{kg \cdot K} \times (288.3\ K - 256.5\ K)} = 252.2\ \frac{m}{s}$$

Then density is again calculated using the ideal gas law:

$$\rho = \frac{p}{RT} = \frac{67.4\ kPa}{286.8\ \frac{J}{kg \cdot K} \times 256.5\ K} = 0.916\ \frac{kg}{m^3}$$

The mass flow rate in this case is:

$$\dot{m} = \rho V A = 0.916\ \frac{kg}{m^3} \times 252.2\ \frac{m}{s} \times \frac{\pi}{4} \times (0.038\ m)^2 = 0.262\ \frac{kg}{s}$$

Problem 12.48

(Difficulty 2)

12.48 Oxygen discharges from a tank through a convergent nozzle. The temperature and velocity in the jet are $-20°\text{C}$ and $270\ \frac{m}{s}$, respectively. What is the temperature in the tank? What is the temperature on the nose of a small object in the jet?

Find: The temperature in the tank and the stagnation temperature of the jet.

Assumptions: Oxygen behaves as an ideal gas. The flow is steady. The flow decelerates isentropically to the stagnation conditions.

Solution: Use the energy equation (4.56)

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

For the flow from the tank through the nozzle, the flow is steady and there is no heat or work transfer and the elevations are the same. The velocity in the tank is zero. The energy equation becomes, where 0 denotes the stagnation condition in the tank:

$$0 = (h_0 - h) - \frac{1}{2}V^2$$

For ideal gas the enthalpy is related to the specific heat and we have:

$$0 = c_p(T_0 - T) - \frac{1}{2}V^2$$

For oxygen

$$C_p = 909.2\ \frac{J}{kg\cdot K}$$

Thus

$$T_0 = T + \frac{V^2}{2c_p} = -20\ C + \frac{\left(270\ \frac{m}{s}\right)^2}{2\times 909.2\ \frac{J}{kg\cdot K}} = 20.1\ °\text{C}$$

The temperature in the tank is 20.1.

The flow from the tank through the nozzle and then to stagnation on the nose is adiabatic. On the nose the velocity is zero, so the temperature on the nose is the same as in the tank, and also 20.1 °C.

Problem 12.49

[Difficulty: 1]

12.49 The hot gas stream at the turbine inlet of a JT9-D jet engine is at 1500°C, 140 kPa (abs), and $M = 0.32$. Calculate the critical conditions (temperature, pressure, and flow speed) that correspond to these conditions. Assume the fluid properties of pure air.

Given: Data on hot gas stream

Find: Critical conditions

Solution:

The data provided, or available in the Appendices, is:

$R = 287 \cdot \frac{J}{kg \cdot K}$ $\quad k = 1.4$ $\quad T_0 = (1500 + 273) \cdot K$ $\quad T_0 = 1773 K$ $\quad p_0 = 140 \cdot kPa$

For critical conditions $\quad \frac{T_0}{T_{crit}} = \frac{k+1}{2}$ $\quad T_{crit} = \frac{T_0}{\frac{k+1}{2}}$ $\quad T_{crit} = 1478 K$

$\frac{p_0}{p_{crit}} = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$ $\quad p_{crit} = \frac{p_0}{\left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}}$ $\quad p_{crit} = 74.0 \cdot kPa$ $\quad$ absolute

$V_{crit} = \sqrt{k \cdot R \cdot T_{crit}}$ $\quad V_{crit} = 770 \frac{m}{s}$

Problem 12.50

(Difficulty 2)

12.50 Carbon dioxide discharges from a tank through a convergent nozzle into the atmosphere. If the tank temperature and gage pressure are 38 ℃ and 140 kPa, respectively, what jet temperature, pressure and velocity can be expected? Barometric pressure is 101.3 kPa.

Find: The pressure, temperature, and velocity in the jet

Assumptions: Carbon dioxide behaves as an ideal gas. The flow is steady and isentropic.

Solution: Use the isentropic flow relations.

We first need to check to see if the nozzle is choked. The critical pressure ratio for choked flow is given by

$$\frac{p^*}{p_0} = \left(\frac{k+1}{2k}\right)^{\frac{-k}{k-1}}$$

For the carbon dioxide we have $k = 1.28$. The critical pressure for the exhaust is

$$p^* = p_0 \left(\frac{k+1}{2}\right)^{\frac{-k}{k-1}} = (140 + 101.3)\ kPa \times \left(\frac{1.28+1}{2}\right)^{\frac{-1.28}{1.28-1}} = 2401.3\ kPa \times 0.549 = 132.6\ kPa$$

The discharge pressure for choked flow through the nozzle is higher than atmospheric pressure, so the flow is sonic. The pressure in the jet is then 132.6 kPa

Since the flow is isentropic, we have the relation between temperature and pressure:

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}$$

Or

$$T = T_0 \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}} = (38 + 273.2)K \times (0.549)^{\frac{1.28-1}{1.28}} = 273.0\ K = -0.2\ C$$

Since the discharge velocity is sonic, we have for the speed of sound:

$$c = \sqrt{kRT} = \sqrt{1.28 \times 187.8\ \frac{J}{kg \cdot K} \times 273\ K \times \left(1\ \frac{kg\ m}{N\ s^2}\right) \times \left(1\ \frac{N\ m}{J}\right)} = 256\ \frac{m}{s}$$

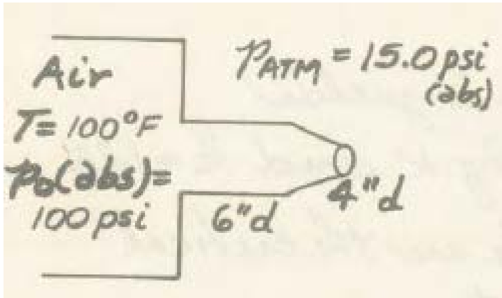
As the jet exhausts at sonic speed

$$V = c = 256\ \frac{m}{s}$$

Problem 12.51

(Difficulty 3)

12.51 Air $(at\ 100°\text{F}\ and\ 100\ psia)$ in a large tank flows into a $6\ in$ pipe, whence it discharges to the atmosphere $(15\ psi)$ through a convergent nozzle of $4\ in$ tip diameter, calculate pressure, temperature , and velocity in the pipe.



Find: The pressure, temperature, and velocity in the jet

Assumptions: Air behaves as an ideal gas. The flow is steady and isentropic.

Solution: Use the isentropic flow relations.

We first need to check to see if the nozzle is choked. The critical pressure ratio for choked flow is given by

$$\frac{p^*}{p_0} = \left(\frac{k+1}{2k}\right)^{\frac{-k}{k-1}}$$

For the air we have $k = 1.4$. The critical pressure for the exhaust is

$$p^* = p_0\left(\frac{k+1}{2}\right)^{\frac{-k}{k-1}} = 100\ psia \times \left(\frac{1.4+1}{2}\right)^{\frac{-1.4}{1.4-1}} = 100\ psia \times 0.528 = 52.8\ psia$$

The discharge pressure for choked flow through the nozzle is higher than atmospheric pressure, so the flow is sonic. The pressure in the jet is then 52.8 psia

Since the flow is choked and isentropic we can use the equations that relate the flow at any location in the channel to the exit flow. The relation between the area and Mach number at any location and the throat area is given by eq 12.30d:

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{\frac{k+1}{2(k-1)}}$$

The ratio of the area in the 6 in pipe to the throat area is

$$\frac{A}{A^*} = \frac{\frac{\pi}{4} \times \left(\frac{6}{12}\ ft\right)^2}{\frac{\pi}{4} \times \left(\frac{4}{12}\ ft\right)^2} = 2.25$$

Using either an equation solver, iteration, or Figure D.1, the Mach number in the 6 in pipe is

$$M = 0.268$$

The temperature in the pipe is given by equation 12.30b or Figure D.1

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

The temperature can be calculated or found using Figure D.1 to be

$$T = 551.7R = 92.0\ F$$

The speed of sound at this temperature is

$$c = \sqrt{kRT} = \sqrt{1.4 \times 1715\ \frac{ft \cdot lbf}{slug \cdot {}^\circ R} \times 551.7\ R \times \left(1\ \frac{slug\ ft}{lbf\ s^2}\right)} = 1151\ \frac{ft}{s}$$

The velocity is then from the definition of Mach number

$$V = Mc = 0.268 \times 1151\ \frac{ft}{s} = 309\ \frac{ft}{s}$$

The pressure is found using equation 12.30 a or in Figure D.1

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{\frac{k}{k-1}}$$

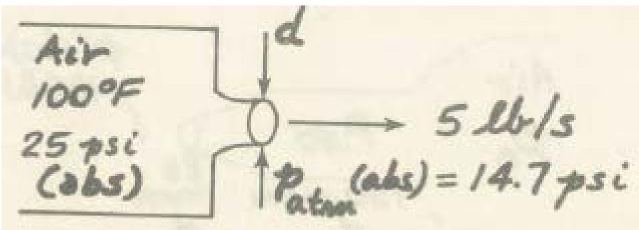
The pressure is found to be

$$p = 95.1\ psia$$

Problem 12.52

(Difficulty 3)

12.52 Calculate the required diameter of a convergent nozzle to discharge 5.0 $\frac{lbf}{s}$ of air from a large tank (in which the temperature is 100 °F) to the atmosphere $(14.7\ psia)$ if the pressure in the tank is:(a) $25.0\ psia$. (b) $30.0\ psia$.



Find: The diameter of a converging nozzle

Assumptions: Air behaves as an ideal gas. The flow is steady and isentropic.

Solution: Use the isentropic flow relations.

We first need to check to see if the nozzle is choked. The critical pressure ratio for choked flow is given by

$$\frac{p^*}{p_0} = \left(\frac{k+1}{2k}\right)^{\frac{-k}{k-1}}$$

a) The critical pressure for the exhaust if the tank pressure is 25 psia

$$p^* = p_0\left(\frac{k+1}{2}\right)^{\frac{-k}{k-1}} = 25\ psia \times \left(\frac{1.4+1}{2}\right)^{\frac{-1.4}{1.4-1}} = 25\ psia \times 0.528 = 13.2\ psia$$

The exhaust pressure is atmospheric and higher than this pressure so flow is not choked and is subsonic throughout. We compute the temperature of the flow at this pressure from the isentropic relation

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}$$

Where k = 1.4. The temperature is

$$T = 480.9\ R = 21.2\ F$$

The relation between temperature and the Mach number is given by

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

Using either an equation solver, iteration, or Figure D.1, the exit Mach number is

$$M = 0.905$$

The speed of sound at this temperature is

$$c = \sqrt{kRT} = \sqrt{1.4 \times 1715\ \frac{ft \cdot lbf}{slug \cdot °R} \times 480.9\ R \times \left(1\ \frac{slug\ ft}{lbf\ s^2}\right)} = 1075\ \frac{ft}{s}$$

The exit velocity is from the definition of Mach number

$$V = Mc = 0.905 \times 1075\ \frac{ft}{s} = 973\ \frac{ft}{s}$$

The density at the exit is determined from the ideal gas relations

$$\rho \quad = \frac{p}{RT} = \frac{14.7 \times 144\ \dfrac{lbf}{ft^2}}{1715\ \dfrac{ft \cdot lbf}{slug \cdot °R} \times 480.9\ R} = 0.00257\ \frac{slug}{ft^3} = 0.0826\ \frac{lbm}{ft^3}$$

The nozzle area is related to the flow rate as

$$\dot{m} = \rho AV = \rho \frac{\pi}{4} D^2 V$$

The diameter needed for 5 lbm/s is then

$$d = 0.282\ ft$$

b) The critical pressure for the exhaust if the tank pressure is 30 psia

$$p^* = p_0 \left(\frac{k+1}{2}\right)^{\frac{-k}{k-1}} = 30\ psia \times 0.528 = 15.8\ psia$$

The discharge pressure for choked flow through the nozzle is higher than atmospheric pressure, so the flow is sonic. We can use the expression for the flow though a choked nozzle(12.32a)

$$\dot{m} = A_t p_0 \sqrt{\frac{k}{R\,T_0}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$

Which for air reduces to (eq 12.32b)

$$\dot{m} = 0.04 \frac{A_t p_0}{\sqrt{T_0}}$$

The area of the nozzle for 5 lbm/s is then

$$A_t \doteq\ m \frac{\sqrt{T_0}}{0.04 p_0} = 0.0274\ ft^2$$

The diameter is then

$$d = 0.187\ ft$$

Problem 12.53 [Difficulty: 2]

12.53 Steam flows steadily and isentropically through a nozzle. At an upstream section where the speed is negligible, the temperature and pressure are 450°C and 6 MPa (abs). At a section where the nozzle diameter is 2 cm, the steam pressure is 2 MPa (abs). Determine the speed and Mach number at this section and the mass flow rate of steam. Sketch the passage shape.

Given: Steam flow through a nozzle

Find: Speed and Mach number; Mass flow rate; Sketch the shape

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\qquad h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

Assumptions: 1) Steady flow 2) Isentropic 3) Uniform flow 4) Superheated steam can be treated as ideal gas

Given or available data $\quad T_0 = (450 + 273) \cdot K \quad p_0 = 6 \cdot MPa \quad p = 2 \cdot MPa$

$D = 2 \cdot cm \quad k = 1.30 \quad R = 461.4 \cdot \frac{J}{kg \cdot K}$

From the steam tables (try finding interactive ones on the Web!), at stagnation conditions

$s_0 = 6720 \cdot \frac{J}{kg \cdot K} \qquad h_0 = 3.302 \times 10^6 \cdot \frac{J}{kg}$

Hence at the nozzle section $\quad s = s_0 = 6720 \cdot \frac{J}{kg \cdot K}$ and $\quad p = 2 \cdot MPa$

From these values we find from the steam tables that $\quad T = 289\,°C \quad h = 2.997 \times 10^6 \cdot \frac{J}{kg} \quad v = 0.1225 \cdot \frac{m^3}{kg}$

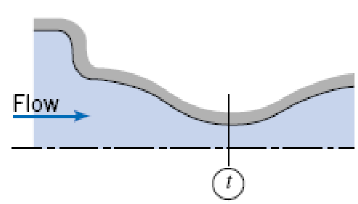
Hence the first law becomes $\quad V = \sqrt{2 \cdot (h_0 - h)} \qquad V = 781 \frac{m}{s}$

The mass flow rate is given by $\quad m_{rate} = \rho \cdot A \cdot V = \frac{A \cdot V}{v} \qquad A = \frac{\pi \cdot D^2}{4} \qquad A = 3.14 \times 10^{-4} m^2$

Hence $\quad m_{rate} = \frac{A \cdot V}{v} \qquad m_{rate} = 2.00 \frac{kg}{s}$

For the Mach number we need $\quad c = \sqrt{k \cdot R \cdot T} \qquad c = 581 \frac{m}{s} \qquad M = \frac{V}{c} \qquad M = 1.35$

The flow is supersonic starting from rest, so must be converging-diverging



Problem 12.54

[Difficulty: 2]

12.54 Nitrogen flows through a diverging section of duct with $A_1 = 0.15\ \text{m}^2$ and $A_2 = 0.45\ \text{m}^2$. If $M_1 = 0.7$ and $p_1 = 450\ \text{kPa}$, find M_2 and p_2.

Given: Data on flow in a passage

Find: Pressure and Mach number at downstream location

Solution:

The given or available data is:

$R =$	296.8	J/kg-K
$k =$	1.4	
$p_1 =$	450	kPa
$M_1 =$	0.7	
$A_1 =$	0.15	m^2
$A_2 =$	0.45	m^2

Equations and Computations:

From M_1 and p_1, and Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)}$$

$p_{01} =$ 624 kPa

From M_1, and Eq. 13.7d
(using built-in function *IsenA* (M, k))

$$\frac{A}{A*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)}$$

$A^*_1 =$ 0.1371 m^2

For isentropic flow ($p_{01} = p_{02},\ A^*_2 = A^*_1$)

$p_{02} =$	624	kPa
$A^*_2 =$	0.1371	m^2
$A_2/A^*_2 =$	3.2831	

From A_2/A^*_2, and Eq. 13.7d
(using built-in function *IsenMsubfromA* (M, k))
Since there is no throat, the flow stays subsonic

$M_2 =$ 0.1797

From M_2 and p_{02}, and Eq. 13.7a
(using built-in function *Isenp* (M, k))

$p_2 =$ 610 kPa

Problem 12.55

[Difficulty: 2]

12.55 At a section in a passage, the pressure is 30 psia, the temperature is 100°F, and the speed is 1750 ft/s. At a section downstream the Mach number is 2.5. Determine the pressure at this downstream location for isentropic flow of air. Sketch the passage shape.

Given: Data on flow in a passage

Find: Pressure at downstream location

Solution:

The given or available data is:

$R =$	53.33	ft·lbf/lbm·°R
$k =$	1.4	
$T_1 =$	560	°R
$p_1 =$	30	psi
$V_1 =$	1750	ft/s
$M_2 =$	2.5	

Equations and Computations:

From T_1 and Eq. 12.18

$$c = \sqrt{kRT}$$

$c_1 =$ 1160 ft/s

Then

$M_1 =$ 1.51

From M_1 and p_1, and Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)}$$

$p_{01} =$ 111 psi

For isentropic flow ($p_{01} = p_{02}$)

$p_{02} =$ 111 psi

From M_2 and p_{02}, and Eq. 13.7a
(using built-in function *Isenp* (M, k))

$p_2 =$ 6.52 psi

Problem 12.56

(Difficulty 2)

12.56 In a given duct flow $M = 2.0$, the velocity undergoes a 20% decrease. What percent change in area was needed to accomplish this? What should be the answer if $M = 0.5$?

Find: Change in area

Assumptions: Air behaves as an ideal gas. The flow is steady and isentropic.

Solution: Use the isentropic flow relations for the relation between area and velocity changes (eq 12.29):

$$\frac{dV}{V} = -\frac{dA}{A}\frac{1}{(M^2 - 1)}$$

Or, for the change in area

$$\frac{dA}{A} = -\frac{dV}{V}(M^2 - 1)$$

For $M = 2.0$ and a 20 % decrease in velocity

$$\frac{dA}{A} = -\,0.2 \times (2^2 - 1) = -0.6 = -60\%$$

So the area needed to decrease by 60%.

For $M = 0.5$,

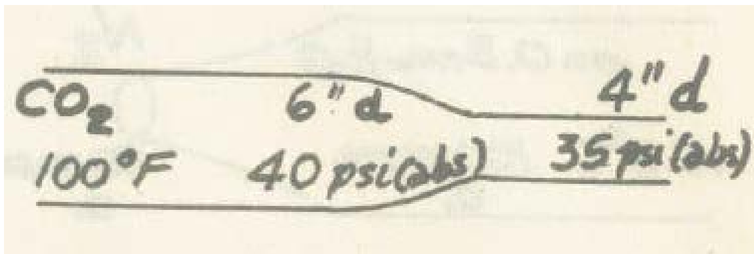
$$\frac{dA}{A} = -0.2 \times (0.5^2 - 1) = 0.15 = 15\%$$

So the area needed to increase by 15%.

Problem 12.57

(Difficulty 2)

12.57 Carbon dioxide flows through a 4 in. construction in a 6 in. pipe. The pressures in the pipe and constriction are 40 psia and 35 psia, respectively. The temperature in the pipe is 100 F. Calculated (a) the flow rate, (b) the temperature in the constriction, (c) the velocities in the pipe and the constriction, and (d) the Mach numbers in the pipe and the constriction.



Given: Data is shown in the figure.

Assumptions: Flow is steady and isentropic. Carbon dioxide behaves as an ideal gas

Solution: Use the ideal gas law, the isentropic relations for an ideal gas, and the relation for isentropic flow through a pipe.

For carbon dioxide $R = 1123\ \frac{lbf \cdot ft}{slug \cdot °R}$ and $k = 1.28$

(a) The density in the pipe is calculated using the ideal gas law as:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{40\ psi}{1123\ \frac{lbf \cdot ft}{slug \cdot °R} \times 100\ °\mathrm{F}} = \frac{5760\ \frac{lbf}{ft^2}}{1123\ \frac{lbf \cdot ft}{slug \cdot °R} \times (459.6 + 100\ )\ °R} = 0.00916\ \frac{slug}{ft^3}$$

For the mass flow rate we have the relation for isentropic flow between two locations:

$$\dot{m} = \frac{A_2}{\sqrt{1 - \left(\frac{p_2}{p_1}\right)^{\frac{2}{k}} \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2k}{k-1} p_1 \rho_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{k}} - \left(\frac{p_2}{p_1}\right)^{\frac{k+1}{k}}\right]}$$

where

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times \left(\frac{4}{12}\ ft\right)^2 = 0.0873\ ft^2$$

So we have the mass flow rate as:

$$\dot{m} = \frac{0.0873\ ft^2}{\sqrt{1-\left(\frac{35}{40}\right)^{\frac{2}{1.28}}\left(\frac{16}{36}\right)^2}} \times \sqrt{\frac{2\times 1.28}{1.28-1} \times 5760\ \frac{\frac{slug\cdot ft}{s^2}}{ft^2} \times 0.00916\ \frac{slug}{ft^3} \times \left[\left(\frac{35}{40}\right)^{\frac{2}{1.28}} - \left(\frac{35}{40}\right)^{\frac{1.28+1}{1.28}}\right]}$$

$$\dot{m} = 0.32\ \frac{slug}{s}$$

(b) For isentropic flow, we have the following relation between temperature and pressure:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = 559.6°R \times \left(\frac{35}{40}\right)^{\frac{1.28-1}{1.28}} = 543.5\ °R = 83.9\ °\text{F}$$

(c) For the velocity in the pipe we use the continuity equation:

$$V_1 = \frac{\dot{m}}{\rho_1 A_1} = \frac{0.32\ \frac{slug}{s}}{0.00916\ \frac{slug}{ft^3} \times \frac{\pi}{4} \times \left(\frac{6}{12}\ ft\right)^2} = 177.8\ \frac{ft}{s}$$

At location 2

$$\rho_2 = \frac{p_2}{RT_2} = \frac{35\ psi}{1123\ \frac{lbf\cdot ft}{slug\cdot °R} \times 83.9\ °\text{F}} = \frac{5040\ \frac{lbf}{ft^2}}{1123\ \frac{lbf\cdot ft}{slug\cdot °R} \times (459.6 + 83.9)\ °R} = 0.00826\ \frac{slug}{ft^3}$$

$$V_2 = \frac{\dot{m}}{\rho_2 A_2} = \frac{0.32\ \frac{slug}{s}}{0.00826\ \frac{slug}{ft^3} \times 0.0873\ ft^2} = 444\ \frac{ft}{s}$$

(d) The sonic velocity in constriction is found from the expression::

$$c_2 = \sqrt{kRT_2} = \sqrt{1.28 \times 1123\ \frac{lbf\cdot ft}{slug\cdot °R} \times 543.5\ °R} = 884\ \frac{ft}{s}$$

The Mach number is:

$$M_2 = \frac{V_2}{c_2} = \frac{444\ \frac{ft}{s}}{884\ \frac{ft}{s}} = 0.502$$

The sonic velocity in the pipe is:

$$c_1 = \sqrt{kRT_1} = \sqrt{1.28 \times 1123\ \frac{lbf\cdot ft}{slug\cdot °R} \times 559.6°R} = 897\ \frac{ft}{s}$$

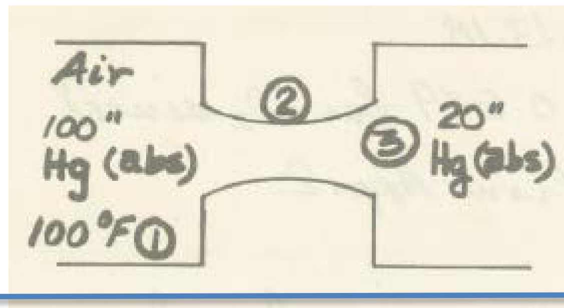
The Mach number is:

$$M_1 = \frac{V_1}{c_1} = \frac{177.8\ \frac{ft}{s}}{897\ \frac{ft}{s}} = 0.198$$

Problem 12.58

(Difficulty 3)

12.58 Five pounds of air per second discharge from a tank through a convergent-divergent nozzle into another tank where a vacuum of 10 in of mercury is maintained. If the pressure and temperature in the upstream tank are 100 in of mercury absolute and 100 °F, respectively, what nozzle-exit diameter must be provided for full expansion? What throat diameter is required? Calculate pressure, temperature, velocity and sonic velocity in throat and nozzle exits. Barometric pressure is 30 in of mercury.



Find: Flow properties and areas

Assumptions: Air can be treated as an ideal gas. The flow is steady and isentropic.

Solution: Use the relations for isentropic flow

We first need to check to see if the nozzle is choked. The critical pressure ratio for choked flow is given by

$$\frac{p^*}{p_0} = \left(\frac{k+1}{2k}\right)^{\frac{-k}{k-1}} = 0.528$$

Corresponding to a throat pressure of 52 in. hg. The exit pressure is lower than this value so the flow through the nozzle will be choked. We can compute the throat area using the isentropic flow relation for choked flow W which for air reduces to (eq 12.32b)

$$\dot{m} = 0.04\frac{A_t p_0}{\sqrt{T_0}}$$

The pressure of 100 in. hg absolute is converted to 49.1 psia. The nozzle area A* is then calculated as

$$A^* = \frac{\dot{m}\sqrt{T_0}}{0.04 p_0} = \frac{5\frac{lbm}{s} \times \sqrt{(100 + 459.7)R}}{0.04 \times 49.1 psia} = 0.0314\ ft^2$$

The corresponding diameter is D = 0.200 ft = 2.4 in.

The temperature of the air at the nozzle is calculated using the isentropic flow relation (eq 12.30b) or Figure D.1

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

At the throat the Mach number is unity. The temperature is then 466.4 R.

The velocity is the speed of sound at this temperature since the Mach number is unity

$$c = \sqrt{kRT} = \sqrt{1.4 \times 1715 \ \frac{ft \cdot lbf}{slug \cdot {}^\circ R} \times 466.4\ R \times \left(1 \ \frac{slug\ ft}{lbf\ s^2}\right)} = 1058 \ \frac{ft}{s}$$

For the exit of the nozzle, we will first compute the Mach number using the relation between pressure and Mach number. The Mach number is found using equation 12.30 a or Figure D.1, where the stagnation pressure is 49.1 psia and the pressure is 20 in. hg, or 9.82 psia

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{\frac{k}{k-1}}$$

The Mach number is found to be 1.71.

The area is found using the relation between the Mach number and critical, or throat, given by eq 12.30d, or Figure D.1:

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2} M^2}{\frac{k+1}{2}}\right]^{\frac{k+1}{2(k-1)}}$$

The exit area of the nozzle is found to be 0.0423 ft2, corresponding to a diameter of 0.232 ft or 2.78 in.

The temperature of the flow is found from (eq 12.30b) or Figure D.1

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

The temperature is 353.4 R. The speed of sound at this temperature is found from

$$c = \sqrt{kRT} = 931 \ \frac{ft}{s}$$

For a Mach number of 1.71 at the exit, the velocity is

$$V = M\ c = 1.71 \times 931 \ \frac{ft}{s} = 1574 \ \frac{ft}{s}$$

Problem 12.59

[Difficulty: 3]

12.59 Air flows isentropically through a converging-diverging nozzle from a large tank containing air at 250°C. At two locations where the area is 1 cm^2, the static pressures are 200 kPa and 50 kPa. Find the mass flow rate, the throat area, and the Mach numbers at the two locations.

Given: Data on flow in a nozzle

Find: Mass flow rate; Throat area; Mach numbers

Solution:

The given or available data is:

R =	286.9	J/kg·K			
k =	1.4				
T_0 =	523	K			
p_1 =	200	kPa	p_2 =	50	kPa
A =	1	cm^2			

Equations and Computations:

We don't know the two Mach numbers. We do know for each that Eq. 13.7a applies:

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)}$$

Hence we can write two equations, but have three unknowns (M_1, M_2, and p_0)!

We also know that states 1 and 2 have the same area. Hence we can write Eq. 13.7d twice:

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)}$$

We now have four equations for four unknowns (A^*, M_1, M_2, and p_0)!

We make guesses (using Solver) for M_1 and M_2, and make the errors in computed A^* and p_0 zero.

For:	M_1 =	0.512		M_2 =	1.68			**Errors**
from Eq. 13.7a:	p_0 =	239	kPa	p_0 =	239	kPa		0.00%
and from Eq. 13.7d:	A^* =	0.759	cm^2	A^* =	0.759	cm^2		0.00%
Note that the throat area is the critical area							**Sum**	0.00%

The stagnation density is then obtained from the ideal gas equation

ρ_0 = 1.59 kg/m^3

The density at critical state is obtained from Eq. 13.7a (or 12.22c)

ρ^* = 1.01 kg/m^3

The velocity at critical state can be obtained from Eq. 12.23)

$$V^* = c^* = \sqrt{\frac{2k}{k+1}RT_0}$$

V^* = 418 m/s

The mass flow rate is $\rho^* V^* A^*$

m_{rate} = 0.0321 kg/s

Problem 12.60

(Difficulty: 2)

12.60 Air, at an absolute pressure of $60.0\ kPa$ and $27\ ℃$, enters a passage at $486\ m/s$, where $A = 0.02\ m^2$. At section (2) downstream, $p = 78.8\ kPa\ (abs)$. Assuming isentropic flow, calculate the Mach number at section (2). Sketch the flow passage.

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas

Solution: Use the isentropic flow relations

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$$

$$c = \sqrt{kRT}$$

For isentropic flow,

$$p_{01} = p_{02} = p_0 = constant$$

The upstream Mach number is

$$M_1 = \frac{V_1}{c_1}$$

$$c_1 = (kRT_1)^{\frac{1}{2}} = \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 300\ K\right)^{\frac{1}{2}} = 347\ \ m/s$$

$$M_1 = \frac{V_1}{c_1} = \frac{486}{347} = 1.40$$

The stagnation pressure is

$$p_{01} = p_1\left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}} = 60.0\ kPa[1 + 0.2(1.4)^2]^{3.5} = 191\ kPa$$

The downstream pressure is given by

$$\frac{p_{02}}{p_2} = \left[1 + \frac{k-1}{2}M_2^2\right]^{\frac{k}{k-1}}$$

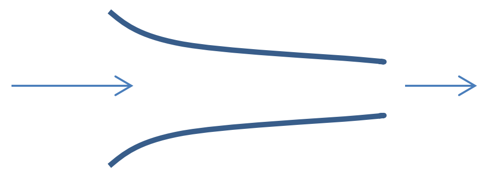
where

$$p_{02} = p_{01}$$

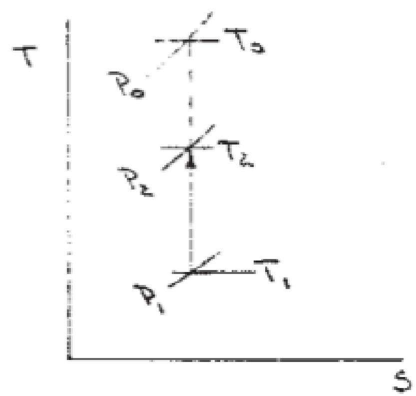
The downstream Mach number is

$$M_2 = \left\{\frac{2}{k-1}\left(\frac{p_0}{p_2}\right)^{\frac{k-1}{k}} - 1\right\}^{\frac{1}{2}} = \left\{\frac{2}{0.4}\left(\frac{191}{78.8}\right)^{0.286} - 1\right\}^{\frac{1}{2}} = 1.20$$

Since $M_2 < M_1$ and both M_1 *and* $M_2 > 1.0$, then passage from (1) to (2) is a supersonic diffuser.



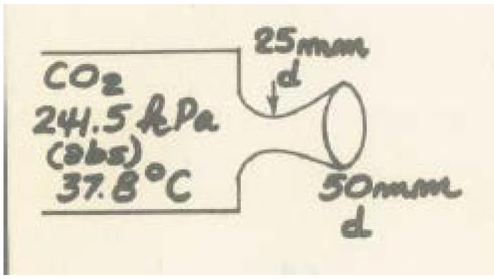
The process is



Problem 12.61

(Difficulty 2)

12.61 Carbon dioxide flows from a tank through a convergent-divergent nozzle of 25 mm throat and 50 mm exit diameter. The absolute pressure and temperature in the tank are 241.5 kPa and 37.8 ℃, respectively. Calculate the mass flow rate when the absolute exit pressure is (a) 172.5 kPa and (b) 221 kPa.



Find: The mass flow rate through the nozzle

Assumptions: Carbon dioxide behaves as an ideal gas. The flow is steady and isentropic.

Solution: Use the isentropic flow relations.

We will first assume that the exit pressure is low enough so that the flow through the nozzle is choked. The critical pressure ratio for choked flow is given by

$$\frac{p^*}{p_0} = \left(\frac{k+1}{2k}\right)^{\frac{-k}{k-1}}$$

For the carbon dioxide we have $k = 1.28$. The critical pressure for the exhaust is

$$p^* = p_0\left(\frac{k+1}{2}\right)^{\frac{-k}{k-1}} = (140 + 101.3)\ kPa \times \left(\frac{1.28+1}{2}\right)^{\frac{-1.28}{1.28-1}} = 2401.3\ kPa \times 0.549 = 132.6\ kPa$$

This pressure is less than the exit pressure so that flow is choked. The flow may not be isentropic in the diverging section as there may be shock waves present, but this does not affect the flow up to the converging section.

$$R = 187.8\ \frac{J}{kg \cdot K}$$

We have the following equation as:

$$\left(\frac{p_2}{p_1}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = 0.549$$

At the throat we have:

$$V_{25} = C_{25}$$

For choked flow we have the relation for flow rate (eq 12.32a):

$$\dot{m} = \frac{A_2 p_1}{\sqrt{T_1}} \sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

$$\dot{m} = \frac{\frac{\pi}{4} \times (0.025\ m)^2 \times 241.5\ kPa}{\sqrt{(37.8 + 273.2)\ K}} \sqrt{\frac{1.28}{187.8\ \frac{J}{kg \cdot K}}\left(\frac{2}{2.28}\right)^{\frac{2.28}{0.28}}} = 0.325\ \frac{kg}{s}$$

This is the flow rate for both exit pressures.

Problem 12.62

(Difficulty 2)

12.62 A convergent-divergent nozzle of 50 mm tip diameter discharges to the atmosphere (103.2 kPa) from a tank in which air is maintained at an absolute pressure and temperature of 690 kPa and 37.8"C, respectively. What is the maximum mass flow rate that can occur through this nozzle? What throat diameter must be provided to produce this mass flow rate?

Find: The maximum flow rate and throat diameter.

Assumptions: Air behaves as an ideal gas. The flow is steady and isentropic.

Solution: Use the isentropic flow relations.

For the maximum flow rate, the flow will be fully expanded so the exit pressure equals atmospheric. Using the relation between pressure and Mach number (eq 12.30a), we can find the exit Mach number from either an equation solver or Figure D.1, where the stagnation pressure is 690 kPa and the exit pressure is 103.2 kPa

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{\frac{k}{k-1}}$$

The Mach number is found to be 1.90.

Using the relation between the area and Mach number at any location and the throat area (eq 12.30d) or Figure D.1, wc can find the relation between the exit area and the throat area:

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2} M^2}{\frac{k+1}{2}}\right]^{\frac{k+1}{2(k-1)}}$$

The ratio A/A* is 1.55. The exit area of the 50 mm diameter nozzle is 0.001963 m2. The nozzle area id then 0.001264 m2, corresponding to a diameter:

$$D_{th} = 0.0412\ m = 41.2\ mm$$

The flow rate is computed for isentropic flow, which for air reduces to (eq 12.32b)

$$\dot{m} = 0.04 \frac{A_t p_0}{\sqrt{T_0}}$$

The flow rate

$$\dot{m} = 1.98\ \frac{kg}{s}$$

Problem 12.63 [Difficulty: 3]

12.63 Air flows adiabatically through a duct. At the entrance, the static temperature and pressure are 310 K and 200 kPa, respectively. At the exit, the static and stagnation temperatures are 294 K and 316 K, respectively, and the static pressure is 125 kPa. Find (a) the Mach numbers of the flow at the entrance and exit and (b) the area ratio A_2/A_1.

Given: Data on flow in a passage

Find: Mach numbers at entrance and exit; area ratio of duct

Solution:

The given or available data is:

$R =$	286.9	J/kg-K
$k =$	1.4	
$T_1 =$	310	K
$p_1 =$	200	kPa
$T_2 =$	294	K
$T_{02} =$	316	K
$p_2 =$	125	kPa

Equations and Computations:

Since the flow is adiabatic, the stagnation temperature is constant:

$T_{01} =$ 316 K

Solving for the Mach numbers at 1 and 2 using Eq. 13.7b
(using built-in function *IsenMfromT*(*Tratio*, *k*))

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

Then

$M_1 =$	0.311	
$M_2 =$	0.612	

Using the ideal gas equation of state, we can calculate the densities of the gas:

$\rho_1 =$	2.249	kg/m^3
$\rho_2 =$	1.482	kg/m^3

From static temperatures and Eq. 12.18 $c = \sqrt{kRT}$

$c_1 =$	352.9	m/s
$c_2 =$	343.6	m/s
$V_1 =$	109.8	m/s
$V_2 =$	210.2	m/s

Since flow is steady, the mass flow rate must be equal at 1 and 2.
So the area ratio may be calculated from the densities and velocities:

$A_2/A_1 =$ 0.792

Note that we can not assume isentropic flow in this problem. While the flow is adiabatic, it is not reversible. There is a drop in stagnation pressure from state 1 to 2 which would invalidate the assumption of isentropic flow.

Problem 12.64

[Difficulty: 2]

12.64 Air flows isentropically through a converging nozzle into a receiver where the pressure is 250 kPa (abs). If the pressure is 350 kPa (abs) and the speed is 150 m/s at the nozzle location where the Mach number is 0.5, determine the pressure, speed, and Mach number at the nozzle throat.

Given: Isentropic air flow in converging nozzle

Find: Pressure, speed and Mach number at throat

Solution:

Basic equations:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2 \qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$$

Given or available data

$$p_1 = 350\cdot kPa \qquad V_1 = 150\cdot\frac{m}{s} \qquad M_1 = 0.5 \qquad p_b = 250\cdot kPa$$

$$k = 1.4 \qquad R = 286.9\cdot\frac{J}{kg\cdot K}$$

The flow will be choked if $p_b/p_0 < 0.528$

$$p_0 = p_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)^{\frac{k}{k-1}} \qquad p_0 = 415\cdot kPa \qquad \frac{p_b}{p_0} = 0.602 \qquad \text{(Not choked)}$$

Hence

$$\frac{p_0}{p_t} = \left(1 + \frac{k-1}{2}\cdot M_t^2\right)^{\frac{k}{k-1}} \quad \text{where} \quad p_t = p_b \qquad p_t = 250\cdot kPa$$

so

$$M_t = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_t}\right)^{\frac{k-1}{k}} - 1\right]} \qquad M_t = 0.883$$

Also

$$V_1 = M_1\cdot c_1 = M_1\cdot\sqrt{k\cdot R\cdot T_1} \quad \text{or} \quad T_1 = \frac{1}{k\cdot R}\cdot\left(\frac{V_1}{M_1}\right)^2 \qquad T_1 = 224\,K \qquad T_1 = -49.1\cdot°C$$

Then

$$T_0 = T_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right) \qquad T_0 = 235\,K \qquad T_0 = -37.9\cdot°C$$

Hence

$$T_t = \frac{T_0}{1 + \frac{k-1}{2}\cdot M_t^2} \qquad T_t = 204\,K \qquad T_t = -69.6\cdot°C$$

Then

$$c_t = \sqrt{k\cdot R\cdot T_t} \qquad c_t = 286\,\frac{m}{s}$$

Finally

$$V_t = M_t\cdot c_t \qquad V_t = 252\,\frac{m}{s}$$

Problem 12.65

[Difficulty: 2]

12.65 Air flows isentropically through a converging nozzle into a receiver in which the absolute pressure is 35 psia. The air enters the nozzle with negligible speed at a pressure of 60 psia and a temperature of 200°F. Determine the mass flow rate through the nozzle for a throat diameter of 4 in.

Given: Air flow in a converging nozzle

Find: Mass flow rate

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\quad p = \rho \cdot R \cdot T$ $\quad \frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

Given or available data $p_b = 35 \cdot psi$ $\quad p_0 = 60 \cdot psi$ $\quad T_0 = (200 + 460) \cdot R$ $\quad D_t = 4 \cdot in$

$k = 1.4$ $\quad R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$ $\quad A_t = \frac{\pi}{4} \cdot D_t^2$ $\quad A_t = 0.0873 \cdot ft^2$

Since $\frac{p_b}{p_0} = 0.583$ is greater than 0.528, the nozzle is not choked and $p_t = p_b$

Hence $M_t = \sqrt{\frac{2}{k-1} \cdot \left[\left(\frac{p_0}{p_t}\right)^{\frac{k-1}{k}} - 1\right]}$ $\quad M_t = 0.912$

and $T_t = \frac{T_0}{1 + \frac{k-1}{2} \cdot M_t^2}$ $\quad T_t = 566 \cdot R$ $\quad T_t = 106 \cdot °F$

$c_t = \sqrt{k \cdot R_{air} \cdot T_t}$ $\quad V_t = c_t$ $\quad V_t = 1166 \cdot \frac{ft}{s}$

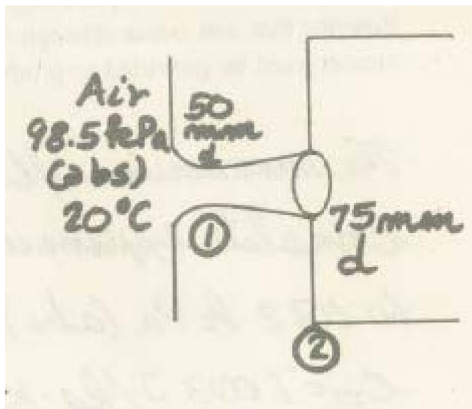
$\rho_t = \frac{p_t}{R_{air} \cdot T_t}$ $\quad \rho_t = 5.19 \times 10^{-3} \cdot \frac{slug}{ft^3}$

$m_{rate} = \rho_t \cdot A_t \cdot V_t$ $\quad m_{rate} = 0.528 \cdot \frac{slug}{s}$ $\quad m_{rate} = 17.0 \cdot \frac{lbm}{s}$

Problem 12.66

(Difficulty 2)

12.66 Atmospheric air (at 98.5 kPa and 20 ℃) is drawn into a vacuum tank through a convergent-divergent nozzle of 50 mm throat diameter and 75 mm exit diameter. Calculate the largest mass flow rate that can be drawn through this nozzle under these conditions.



Find: The maximum flow rate.

Assumptions: Air behaves as an ideal gas. The flow is steady and isentropic.

Solution: Use the isentropic flow relations.

For the maximum flow rate, the flow will be choked. We can compute the flow rate for isentropic flow, which for air reduces to (eq 12.32b)

$$\dot{m} = 0.04\frac{A_t p_0}{\sqrt{T_0}}$$

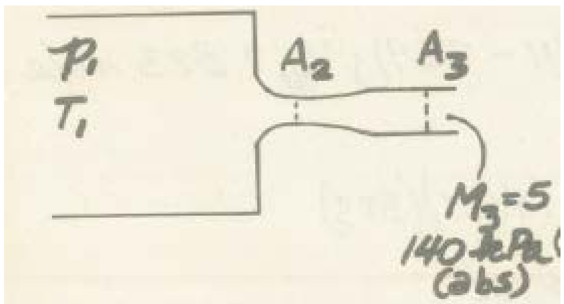
For air we have:

$$\dot{m} = 0.04 \times \frac{\frac{\pi}{4} \times (0.05\ m)^2 \times 98.5 \times 10^3\ Pa}{\sqrt{(273.2 + 20\ K)}} = 0.451\ \frac{kg}{s}$$

Problem 12.67

(Difficulty 2)

12.67 The exit section of a convergent-divergent nozzle is to be used for the test section of a supersonic wind tunnel. If the absolute pressure in the test section is to be 140 kPa, what pressure is required in the reservoir to produce a Mach number of 5 in the test section? For the air temperature to be -20℃ in the test section, what temperature is required in the reservoir? What ratio of throat area to test section area is required to meet these conditions?



Find: The pressure and temperature in the reservoir and the throat diameter

Assumptions: Air behaves as an ideal gas. The flow is steady and isentropic.

Solution: Use the isentropic flow relations.

The reservoir pressure is found using the relation between pressure and Mach number. The stagnation, or reservoir pressure, using equation 12.30 a or Figure D.1, where the exit pressure is 140 kPa and the Mach number is 5

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{\frac{k}{k-1}}$$

The reservoir pressure must be

$$p_0 = 74{,}070\ kPa$$

The temperature of the air in the reservoir is calculated using the isentropic flow relation (eq 12.30b) or Figure D.1

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

The temperature in the reservoir must be

$$T_0 = 1519\ K = 1246\ C$$

The area ratio is found using the relation between the Mach number and critical, or throat, given by eq 12.30d, or Figure D.1:

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{\frac{k+1}{2(k-1)}}$$

The area ratio is found to be

$$\frac{A}{A^*} = 0.040$$

Problem 12.68

(Difficulty: 2)

12.68 Air flowing isentropically through a converging nozzle discharges to the atmosphere. At the section where the absolute pressure is 250 kPa, the temperature is 20 °C and the air speed is 200 m/s. Determine the nozzle throat pressure.

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas

Solution: Use the isentropic flow relations

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$$

$$c = \sqrt{kRT}$$

The nozzle will be choked, i.e. $M_t = 1.0$, if the pressure ratio $\frac{p_b}{p_o} \leq 0.528$. The exit Mach number is

$$M_1 = \frac{V_1}{c_1}$$

$$c_1 = \sqrt{kRT_1} = \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 293\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 343\ m/s$$

$$M_1 = \frac{V_1}{c_1} = \frac{200}{343} = 0.583$$

The stagnation pressure is then

$$p_0 = p_1\left(1 + \frac{k-1}{2}M_1^2\right)^{\frac{k}{k-1}} = 250\ kPa[1 + 0.2(0.583)^2]^{3.5} = 315\ \ kPa$$

The ratio of back pressure to stagnation pressure is

$$\frac{p_b}{p_o} = \frac{101}{315} = 0.321 < 0.528$$

The flow is choked and

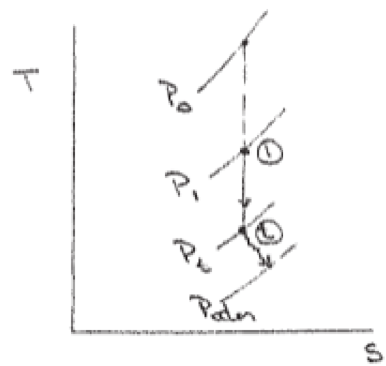
$$M_t = 1.0$$

For $M_t = 1.0$,

$$\frac{p_t}{p_o} = 0.528$$

$$p_t = 0.528 p_o = 0.528 \times 315\ kPa = 166\ kPa$$

The states are



Problem 12.69

(Difficulty: 2)

12.69 Air flows from a large tank ($p = 650\ kPa\ (abs), T = 550°\text{C}$) through a converging nozzle, with a throat area of $600\ mm^2$, and discharges to the atmosphere. Determine the mass rate of flow for isentropic flow through the nozzle.

Assumptions: (1) steady flow (2) isentropic flow in nozzle (3) uniform flow at a section (4) ideal gas

Solution: Use the continuity equation and the isentropic flow relations

$$\dot{m} = \rho V A$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$

$$c = \sqrt{kRT}$$

Since

$$\frac{p_b}{p_0} = \frac{101}{650} = 0.155 < 0.528$$

The nozzle is chocked and $M_t = 1.0$. We need to determine ρ_t and V_t

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$T_t = \frac{T_0}{1 + \frac{k-1}{2} M_t^2} = \frac{823\ K}{1 + 0.2(1.0)^2} = 686\ K$$

$$V_t = M_t c_t = M_t (kRT_t)^{\frac{1}{2}} = 1.0 \times \left(1.4 \times 287\ \frac{N\ \cdot m}{kg \cdot K} \times 686\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 525\ \frac{m}{s}$$

The throat pressure is related to the stagnation pressure as

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$

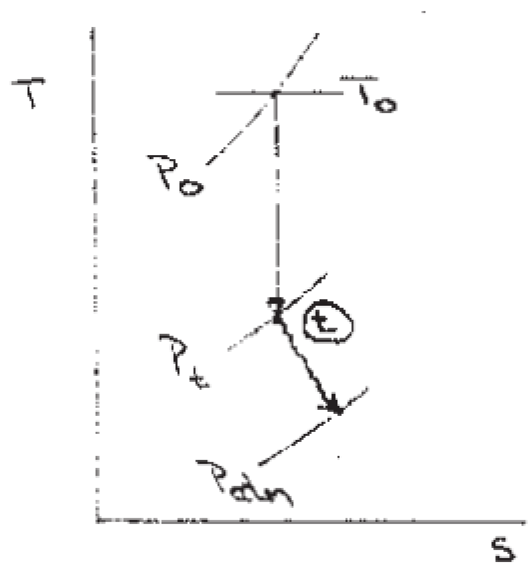
$$p_t = \frac{p_0}{\left(1 + \frac{k-1}{2} M_t^2\right)^{\frac{k}{k-1}}} = \frac{650\ kPa}{[1 + 0.2(1.0)^2]^{3.5}} = 343\ kPa$$

$$\rho_t = \frac{p_t}{RT_t} = 343 \times 10^3\ \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{686\ K} = 1.74\ \frac{kg}{m^3}$$

Finally, the mass flow rate is

$$\dot{m} = \rho_t V_t A_t = 1.74\ \frac{kg}{m^3} \times 525\ \frac{m}{s} \times 6 \times 10^{-4}\ m^2 = 0.548\ \frac{kg}{s}$$

The states are :



Problem 12.70

(Difficulty: 2)

12.70 A converging nozzle is connected to a large tank that contains compressed air at $15\ ℃$. The nozzle exit area is $0.001\ m^2$. The exhaust is discharged to the atmosphere. To obtain satisfactory shadow photograph of the flow pattern leaving the nozzle exit, the pressure in the exit plane must be greater than $325\ kPa\ (gage)$. What pressure is required in the tank? What mass flow rate of air must be supplied if the system is to run continuously? Show static and stagnation state points on a Ts diagram.

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas behavior

Solution: : Use the continuity equation and the isentropic flow relations

$$\dot{m} = \rho A V$$

$$p = \rho R T$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

Since $p_e > p_b$, nozzle is choked and $M_e = 1.0$. The stagnation pressure and temperature are

$$p_0 = p_e \left(1 + \frac{k-1}{2} M_e^2\right)^{\frac{k}{k-1}} = 426\ kPa \times [1 + 0.2]^{3.5} = 806\ kPa$$

$$\frac{T_0}{T_e} = 1 + \frac{k-1}{2} M_e^2$$

The exit pressure is

$$T_e = \frac{T_0}{1.2} = \frac{288\ K}{1.2} = 240\ K$$

The velocity is

$$V_e = c_e = (kRT_e)^{\frac{1}{2}} = \left[1.4 \times 287\ \frac{J}{kg \cdot K} \times 240\ K \times \frac{N \cdot m}{J} \times \frac{kg \cdot m}{N \cdot s^2}\right]^{\frac{1}{2}} = 311\ \frac{m}{s}$$

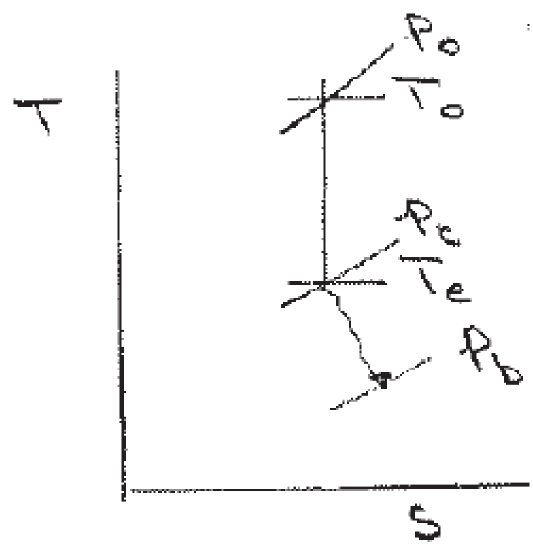
$$\rho_e = \frac{p_e}{RT_e} = \frac{426 \times 10^3 \, \frac{N}{m^2}}{287 \, \frac{J}{kg \cdot K} \times 240 \, K} \times \frac{J}{N \cdot m} = 6.18 \, \frac{kg}{m^3}$$

Then the mass flow rate is

$$\dot{m} = \rho_e V_e A_e = 6.18 \, \frac{kg}{m^3} \times 311 \, \frac{m}{s} \times 0.001 \, m^2 = 1.92 \, \frac{kg}{s}$$

For steady flow, $\dot{m} = 1.92 \, \frac{kg}{s}$ must be supplied to the tank.

The process is



The corresponding volume flow rate of standard air is:

$$Q = \frac{\dot{m}}{\rho_{stard}} = \frac{1.92 \, \frac{kg}{s}}{6.18 \, \frac{kg}{m^3}} = 0.31 \, \frac{m^3}{s}$$

Problem 12.71

[Difficulty: 2]

12.71 Air at 0°C is contained in a large tank on the space shuttle. A converging section with exit area 1×10^{-3} m^2 is attached to the tank, through which the air exits to space at a rate of 2 kg/s. What are the pressure in the tank, and the pressure, temperature, and speed at the exit?

Given: Temperature in and mass flow rate from a tank

Find: Tank pressure; pressure, temperature and speed at exit

Solution:

The given or available data is:

	R =	286.9	J/kg.K
	k =	1.4	
	T_0 =	273	K
	A_t =	0.001	m^2
	m_{rate} =	2	kg/s

Equations and Computations:

Because $p_b = 0$ $\quad p_e = p^*$

Hence the flow is choked!

Hence $\quad T_e = T^*$

From T_0, and Eq. 12.22b

$$\frac{T_0}{T^*} = \frac{k+1}{2} \qquad (12.22b)$$

T^* = 228 K

T_e = 228 K

-45.5 °C

Also	$M_e =$	1	
Hence	$V_e =$	$V^* =$	c_e

From T_e and Eq. 12.18

$$c = \sqrt{kRT} \qquad (12.18)$$

$c_e =$ 302 m/s

Then $V_e =$ 302 m/s

To find the exit pressure we use the ideal gas equation after first finding the exit density.
The mass flow rate is $m_{rate} = \rho_e A_e V_e$

Hence $\rho_e =$ 6.62 kg/m^3

From the ideal gas equation $p_e = \rho_e R T_e$

$p_e =$ 432 kPa

From $p_e = p^*$ and Eq. 12.22a

$$\frac{p_0}{p^*} = \left[\frac{k+1}{2}\right]^{k/(k-1)} \qquad (12.22a)$$

$p_0 =$ 817 kPa

We can check our results:
From p_0, T_0, A_t, and Eq. 13.9a

$$\dot{m}_{\text{choked}} = A_e p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \qquad (13.9a)$$

Then $m_{choked} =$ 2.00 kg/s

$m_{choked} =$ m_{rate} Correct!

Problem 12.72 [Difficulty: 2]

12.72 A large tank initially is evacuated to −10 kPa (gage). (Ambient conditions are 101 kPa at 20°C.) At $t = 0$, an orifice of 5 mm diameter is opened in the tank wall; the vena contracta area is 65 percent of the geometric area. Calculate the mass flow rate at which air initially enters the tank. Show the process on a Ts diagram. Make a schematic plot of mass flow rate as a function of time. Explain why the plot is nonlinear.

Given: Isentropic air flow into a tank

Find: Initial mass flow rate; Ts process; explain nonlinear mass flow rate

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$ $\qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\qquad m_{rate} = \rho \cdot A \cdot V$

Given or available data $\quad p_0 = 101 \cdot kPa \qquad p_b = p_0 - 10 \cdot kPa \qquad p_b = 91 \cdot kPa \qquad T_0 = (20 + 273) \cdot K$

$k = 1.4 \qquad R = 286.9 \cdot \frac{J}{kg \cdot K} \qquad D = 5 \cdot mm$

Then $\quad A = \frac{\pi}{4} \cdot D^2 \qquad A_{vena} = 65 \cdot \% \cdot A \qquad A_{vena} = 12.8 \cdot mm^2$

The flow will be choked if $p_b/p_0 < 0.528$ $\qquad \frac{p_b}{p_0} = 0.901 \qquad$ (Not choked)

Hence $\quad \frac{p_0}{p_{vena}} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad$ wher e $\quad p_{vena} = p_b \qquad p_{vena} = 91 \cdot kPa$

so $\quad M_{vena} = \sqrt{\frac{2}{k-1} \cdot \left[\left(\frac{p_0}{p_{vena}}\right)^{\frac{k-1}{k}} - 1\right]} \qquad M_{vena} = 0.389$

Then $\quad T_{vena} = \frac{T_0}{1 + \frac{k-1}{2} \cdot M_{vena}^2} \qquad T_{vena} = 284\,K \qquad T_{vena} = 11.3 \cdot °C$

Then $\quad c_{vena} = \sqrt{k \cdot R \cdot T_{vena}} \qquad c_{vena} = 338 \frac{m}{s}$

and $\quad V_{vena} = M_{vena} \cdot c_{vena} \qquad V_{vena} = 131 \frac{m}{s}$

Also $\quad \rho_{vena} = \frac{p_{vena}}{R \cdot T_{vena}} \qquad \rho_{vena} = 1.12 \frac{kg}{m^3}$

Finally $\quad m_{rate} = \rho_{vena} \cdot A_{vena} \cdot V_{vena} \qquad m_{rate} = 1.87 \times 10^{-3} \frac{kg}{s}$

The Ts diagram will be a vertical line (T decreases and s = const). After entering the tank there will be turbulent mixing (s increases) and the flow comes to rest (T increases). The mass flow rate versus time will look like the curved part of Fig. 13.6b; it is nonlinear because V AND ρ vary

Problem 12.73

(Difficulty: 3)

12.73 Air flows isentropically through a converging nozzle attached to a large tank, where the absolute pressure is 171 kPa and the temperature is 27 ℃. At the inlet section the Mach number is 0.2. The nozzle discharges to the atmosphere; the discharge area is 0.015 m^2. Determine the magnitude and direction of the force that must be applied to hold the nozzle in place.

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas

Solution: Use the continuity and momentum equations and the isentropic flow relations

$$F_{sx} = p_1 A_1 - p_2 A_2 - p_{atm}(A_1 - A_2) - R_x = \dot{m}(V_2 - V_1)$$

$$\dot{m} = \rho A V = const$$

$$p = \rho R T$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

Evaluate the outlet Mach number to see if the flow is choked.

$$M_2 = \left\{\frac{2}{k-1}\left(\frac{p_0}{p_t}\right)^{\frac{k-1}{k}} - 1\right\}^{\frac{1}{2}} = \left\{\frac{2}{0.4}\left(\frac{171}{101}\right)^{0.286} - 1\right\}^{\frac{1}{2}} = 0.901$$

The outlet Mach number is less than unity and the flow not choked. The properties at the outlet are

$$T_2 = \frac{T_0}{1 + \frac{k-1}{2} M_2^2} = \frac{300\ K}{[1 + 0.2(0.901)^2]} = 258\ K$$

$$V_2 = M_2 c_2 = M_2 (k R T_2)^{\frac{1}{2}} = 0.901 \times \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 258\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 290\ \frac{m}{s}$$

$$\rho_2 = \frac{p_2}{R T_2} = \frac{101 \times 10^3\ \frac{N}{m^2}}{287\ \frac{N \cdot m}{kg \cdot K} \times 258\ K} = 1.36\ \frac{kg}{m^3}$$

The mass flow rate is

$$\dot{m} = \rho_2 V_2 A_2 = 1.36\ \frac{kg}{m^3} \times 290\ \frac{m}{s} \times 0.015\ m^2 = 5.92\ \frac{kg}{s}$$

The inlet properties are then

$$T_1 = \frac{T_0}{1 + \frac{k-1}{2} M_1^2} = \frac{300\ K}{[1 + 0.2(0.2)^2]} = 298\ K$$

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{\frac{1}{2}} = 0.2 \times \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 298\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 69.2\ \frac{m}{s}$$

$$p_1 = \frac{p_0}{\left(1 + \frac{k-1}{2} M_1^2\right)^{\frac{k}{k-1}}} = \frac{171\ kPa}{[1 + 0.2(0.2)^2]^{3.5}} = 166\ kPa$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{166 \times 10^3\ \frac{N}{m^2}}{287\ \frac{N \cdot m}{kg \cdot K} \times 298\ K} = 1.94\ \frac{kg}{m^3}$$

$$A_1 = \frac{\dot{m}}{\rho_1 V_1} = \frac{5.92\ \frac{kg}{s}}{1.94\ \frac{kg}{m^3} \times 69.2\ \frac{m}{s}} = 0.0441\ m^2$$

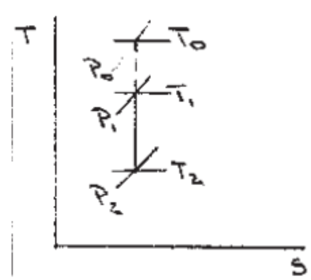
Using the x-momentum equation, the force is

$$R_x = p_1 A_1 - p_2 A_2 - p_{atm}(A_1 - A_2) - \dot{m}(V_2 - V_1) = p_{1g} A_1 - \dot{m}(V_2 - V_1)$$

$$R_x = (166 - 101) \times 10^3\ \frac{N}{m^2} \times 0.0441\ m^2 - 5.92\ \frac{kg}{s} \times \left(290\ \frac{m}{s} - 69.2\ \frac{m}{s}\right)$$

$$R_x = 1560\ N\ (to\ the\ left)$$

The states are



Problem 12.74 [Difficulty: 3]

12.74 Air enters a converging-diverging nozzle at 2 MPa (abs) and 313 K. At the exit of the nozzle, the pressure is 200 kPa (abs). Assume adiabatic, frictionless flow through the nozzle. The throat area is 20 cm^2. What is the area at the nozzle exit? What is the mass flow rate of the air?

Given: Air flow through a converging-diverging nozzle

Find: Nozzle exit area and mass flow rate

Solution:

The given or available data is:

R =	286.9	J/kg-K
k =	1.4	
p_0 =	2	MPa
T_0 =	313	K
p_e =	200	kPa
A_t =	20	cm^2

Equations and Computations:

Using the stagnation to exit static pressure ratio, we can find the exit Mach number:
(using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)}$$

M_e = 2.1572

From M_e, and Eq. 13.7d
(using built-in function *IsenA* (M, k))

$$\frac{A}{A*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)}$$

A_e/A^* = 1.9307

At the throat the flow is sonic, so At = A*. Therefore:

A_e = 38.6 cm^2

To find the mass flow rate at the exit, we will use the choked flow equation:
From p_0, T_0, A_t, and Eq. 13.9a

$$\dot{m}_{\text{choked}} = A_e p_0 \sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \qquad (13.9a)$$

m = 17.646 kg/s

Problem 12.75

(Difficulty: 3)

12.75 A converging nozzle is bolted to the side of a large tank. Air inside the tank is maintained at a constant $50\ psia$ and $100\ °\mathrm{F}$. The inlet area of the nozzle is $10\ in^2$ and the exit area is $1\ in^2$. The nozzle discharges to the atmosphere. For isentropic flow in the nozzle, determine the total force on the bolts, and indicate whether the bolts are in tension or compression.

Assumptions: (1) steady flow (2) isentropic flow in nozzle (3) uniform flow at a section (4) $F_{Bx} = 0$ (5) $V_1 \approx 0$

Solution: Use the continuity and momentum equations and the isentropic flow relations

$$F_{sx} = p_1 A_1 - p_2 A_2 - p_{atm}(A_1 - A_2) - R_x = \dot{m}(V_2 - V_1)$$

$$\dot{m} = \rho A V = const$$

$$p = \rho R T$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

First check to see if the flow is choked,

$$\frac{p_b}{p_0} = \frac{14.7}{50} = 0.294 < 0.528$$

Therefore the nozzle is choked.

$$M_2 = 1.0$$

$$p_2 = 0.528 p_0 = 0.528 \times 50\ psia = 26.4\ psia$$

$$T_2 = \frac{T_0}{1 + \frac{k-1}{2} M_2^2} = \frac{560\ °R}{[1 + 0.2]} = 467\ °R$$

$$\rho_2 = \frac{p_2}{R T_2} = 26.4\ \frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3\ ft \cdot lbf} \times \frac{1}{467\ °R} \times 144\ \frac{in^2}{ft^2} = 0.153\ \frac{lbm}{ft^3}$$

The velocity is

$$V_2 = M_2 c_2 = M_2 (kRT_2)^{\frac{1}{2}} = 1.0 \times \left(1.4 \times 53.3 \ \frac{ft \cdot lbf}{lbm \cdot {}^\circ R} \times 467 \ {}^\circ R \times 32.2 \ \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{\frac{1}{2}}$$

$$V_2 = 1060 \ \frac{ft}{s}$$

The flow rate is

$$\dot{m} = \rho_2 V_2 A_2 = 0.153 \ \frac{lbm}{ft^3} \times 1060 \ \frac{ft}{s} \times 1.0 \ in^2 \times \frac{ft^2}{144 \ in^2} = 1.13 \ \frac{lbm}{s}$$

Using the momentum equation in the x-direction, the force is

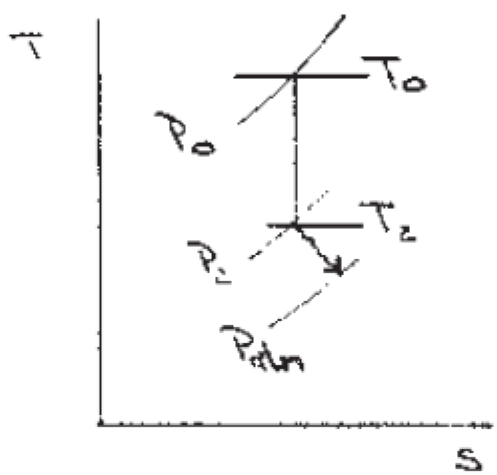
$$R_x = p_1 A_1 - p_2 A_2 - p_{atm}(A_1 - A_2) - \dot{m}(V_2 - V_1) = p_{1g} A_1 - p_{2g} A_2 - \dot{m} V_2$$

$$R_x = (50 - 14.7) \ \frac{lbf}{in^2} \times 10 \ in^2 - (26.4 - 14.7) \ \frac{lbf}{in^2} \times 1 \ in^2 - 1.13 \frac{lbm}{s} \times 1060 \ \frac{ft}{s} \times \frac{slug}{32.2 \ lbm} \times \frac{lbf \cdot s^2}{ft \cdot slug}$$

$$R_x = 304 \ lbf$$

Since R_x acts to the left on CV, bolts are in tension.

The states are:



Problem 12.76

(Difficulty: 4)

12.76 A jet transport aircraft, with pressurized cabin, cruises at 11 km altitude. The cabin temperature and pressure initially are at 25 ℃ and equivalent to 2.5 km altitude. The interior volume of the cabin is 25 m^3. Air escapes through a small hole with effective flow area of $0.002\ m^2$. Calculate the time required for the cabin pressure to decrease by 40 percent. Plot the cabin pressure as function of time.

Assumptions: (1) model flow as isentropic flow through a converging nozzle (2) assume uniform properties within the cabin, isentropic expansion (3) ideal gas behavior

Solution: Use the continuity equation and the isentropic flow relations

$$\frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A} = 0$$

$$\frac{p}{\rho^k} = constant$$

$$p = \rho RT$$

Stagnation conditions within the cabin are:

$$T_i = 298\ K$$

$$p_i = p_{atm}\ at\ 2.5\ km = 74.7\ kPa$$

$$p_f = 0.60 p_i = 44.8\ kPa$$

The external pressure is

$$p_b = p_{atm}\ at\ 11\ km = 22.7\ kPa$$

Then

$$\frac{p_b}{p_i} = 0.304$$

and

$$\frac{p_b}{p_f} = 0.507$$

Flow is choked with the conditions in cabin being stagnation conditions.

From continuity,

$$\frac{\partial}{\partial t}\int_{CV} \rho d\forall = -\int_{CS} \rho \bar{V} \cdot d\bar{A} = -\rho_e V_e A_e$$

$$\forall \frac{d\rho}{dt} = -\rho_e V_e A_e$$

For choked flow, $M_e = 1.0$

$$\frac{\rho}{\rho_e} = \left[1 + \frac{k-1}{2} M_e^2\right]^{\frac{1}{k-1}} = (1.20)^{2.5} = 1.5774$$

$$\rho_e = 0.6339\rho$$

$$\frac{T}{T_e} = 1 + \frac{k-1}{2} M_e^2 = 1.2$$

$$T_e = 0.8333T$$

$$V_e = (kRT_e)^{\frac{1}{2}} = (kR)^{\frac{1}{2}}(0.8333T)^{\frac{1}{2}} = 0.9129(kR)^{\frac{1}{2}}(T)^{\frac{1}{2}}$$

Then

$$\forall \frac{d\rho}{dt} = -\rho_e V_e A_e = -0.63309\rho(0.9129)(kR)^{\frac{1}{2}}(T)^{\frac{1}{2}} A_e$$

$$\forall \frac{d\rho}{dt} = -0.5787(kR)^{\frac{1}{2}} A_e \rho (T)^{\frac{1}{2}}$$

For an isentropic expansion, ρ and T can be related as:

$$\frac{p}{\rho^k} = constant = \frac{\rho RT}{\rho^k}$$

$$\rho^{(1-k)} T = constant$$

Then

$$\rho^{(1-k)} T = \rho_i^{(1-k)} T_i$$

$$T = T_i \left(\frac{\rho_i}{\rho}\right)^{(1-k)}$$

$$T^{\frac{1}{2}} = \frac{T_i^{\frac{1}{2}}}{\rho_i^{\frac{k-1}{2}}} \rho^{\frac{k-1}{2}}$$

Substituting we obtain:

$$\forall \frac{d\rho}{dt} = -0.5787(kR)^{\frac{1}{2}} A_e \rho \frac{T_i^{\frac{1}{2}}}{\rho_i^{\frac{k-1}{2}}} \rho^{\frac{k-1}{2}}$$

$$\frac{d\rho}{dt} = -0.5787 \frac{(kR)^{\frac{1}{2}}}{\forall} A_e \frac{T_i^{\frac{1}{2}}}{\rho_i^{\frac{k-1}{2}}} \rho^{\frac{k+1}{2}} = -c_1 \rho^{\frac{k+1}{2}}$$

$$\frac{d\rho}{\rho^{\frac{k+1}{2}}} = -c_1 dt$$

Where

$$c_1 = 0.5787 \frac{(kR)^{\frac{1}{2}}}{\forall} A_e \frac{T_i^{\frac{1}{2}}}{\rho_i^{\frac{k-1}{2}}}$$

To integrate, we write:

$$-c_1 t = \int \rho^{-\frac{k+1}{2}} d\rho = [\frac{1}{1-\frac{k+1}{2}} \rho^{1-\frac{k+1}{2}}]_{\rho_i}^{\rho_f} = \left[\frac{2}{(1-k)} \rho^{\frac{(1-k)}{2}}\right]_{\rho_i}^{\rho_f}$$

$$-c_1 t = \frac{2}{(k-1)} \left[\rho_i^{\frac{(1-k)}{2}} - \rho_f^{\frac{(1-k)}{2}}\right] = \frac{2}{(k-1)} \rho_i^{\frac{(1-k)}{2}} \left[1 - \left(\frac{\rho_f}{\rho_i}\right)^{\frac{(1-k)}{2}}\right]$$

$$c_1 t = \frac{2}{(k-1)} \rho_i^{\frac{(1-k)}{2}} \left[\left(\frac{\rho_i}{\rho_f}\right)^{\frac{(k-1)}{2}} - 1\right]$$

$$0.5787 \frac{(kR)^{\frac{1}{2}}}{\forall} A_e \frac{T_i^{\frac{1}{2}}}{\rho_i^{\frac{k-1}{2}}} t = \frac{2}{(k-1)} \rho_i^{\frac{(1-k)}{2}} \left[\left(\frac{\rho_i}{\rho_f}\right)^{\frac{(k-1)}{2}} - 1\right]$$

$$0.5787 \frac{(kR)^{\frac{1}{2}}}{\forall} A_e T_i^{\frac{1}{2}} t = \frac{2}{(k-1)} \left[\left(\frac{\rho_i}{\rho_f}\right)^{\frac{(k-1)}{2}} - 1\right]$$

Since

$$\frac{p}{\rho^k} = constant$$

$$\frac{\rho_i}{\rho_f} = \left(\frac{\rho_i}{\rho_f}\right)^{\frac{1}{k}}$$

$$0.5787\,\frac{(kR)^{\frac{1}{2}}}{\forall}A_e T_i^{\frac{1}{2}} t = \frac{2}{(k-1)}\left[\left(\frac{\rho_i}{\rho_f}\right)^{\frac{(k-1)}{2k}} - 1\right] \tag{1}$$

Substituting numerical values,

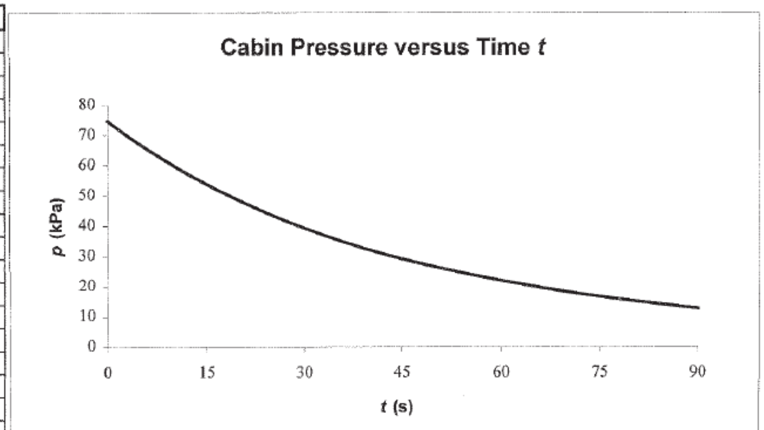
$$0.5787\left[1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 298\ K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{\frac{1}{2}} \times 0.002\ m^2 \times \frac{1}{25\ m^3}\,t = \frac{2}{0.4}\left[\left(\frac{1}{0.6}\right)^{0.1429} - 1\right]$$

$$0.01602t = 0.3786$$

$$t = 23.6\ s$$

Equation 1 is plotted using *Excel*
Note that it's easier to compute *t* from *p* values!

t (s)	*p* (kPa)
0.000	74.7
1.03	73
2.27	71
3.56	69
4.89	67
6.26	65
7.69	63
9.17	61
10.7	59
12.3	57
14.0	55
15.7	53
17.5	51
19.4	49
21.4	47
23.6	**44.8**
25.6	43
27.9	41
30.4	39
33.0	37
35.7	35
38.6	33
41.8	31
45.2	29
48.8	27
52.8	25
57.2	23
62.0	21
67.4	19
73.5	17
80.5	15
90.0	12.7



Problem 12.77

(Difficulty: 2)

12.77 A converging-diverging nozzle, with a throat area of 2 in^2, is connected to a large tank in which air is kept as a pressure of 80 $psia$ and a temperature of 60 °F. If the nozzle is to operate at design conditions (flow is isentropic) and the ambient pressure outside the nozzle is 12.9 psi, calculate the exit area of the nozzle and the mass flow rate.

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas

Solution: Use the isentropic flow relations:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2} M^2}{1 + \frac{k-1}{2}}\right]^{(k+1)/2(k-1)}$$

We have:

$$M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_1}\right)^{k-1/k} - 1\right]\right\}^{\frac{1}{2}} = \left\{\frac{2}{0.4}\left[\left(\frac{80}{12.9}\right)^{0.286} - 1\right]\right\}^{\frac{1}{2}} = 1.85$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$T_1 = \frac{T_0}{1 + \frac{k-1}{2} M_1^2} = \frac{520\ °R}{1 + 0.2(1.85)^2} = 309\ °R$$

The velocity is then found rom the Mach number

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{\frac{1}{2}} = 1.85\left(1.4 \times 53.3\ \frac{ft \cdot lbf}{lbm \cdot °R} \times 309\ °R \times 32.2\ \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{\frac{1}{2}} = 1594\ \frac{ft}{s}$$

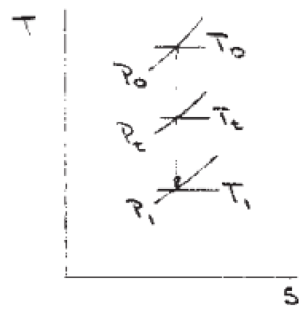
$$\rho_1 = \frac{p_1}{RT_1} = 12.9\ \frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3\ ft \cdot lbf} \times \frac{1}{309\ °R} \times \frac{144\ in^2}{ft^2} = 0.113\ \frac{lbm}{ft^3}$$

Since $M_1 = 1.85$, nozzle must be choked and $M_t = 1.0$. $A_2 = A^*$.

For $M_1 = 1.85$, from Eq. 12.6, $A_1/A^* = 1.496$; $A_1 = 2.99\ in^2$.

$$\dot{m} = \rho_1 V_1 A_1 = 0.113\ \frac{lbm}{ft^3} \times 1594\ \frac{ft}{s} \times 2.99\ in^2 \times \frac{ft^2}{144\ in^2} = 3.74\ \frac{lbm}{s}$$

On a T-s plane



Problem 12.78

(Difficulty: 2)

12.78 Air, at a stagnation pressure of 7.20 MPa (abs) and a stagnation temperature of 1100 K, flows isentropically through a converging-diverging nozzle having a throat area of 0.01 m^2. Determine the speed and the mass flow rate at the downstream section where the Mach number is 4.0.

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas

Solution: Use the continuity equation and the isentropic flow relations:

$$\dot{m} = \rho V A$$

$$p = \rho R T$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$

$$T_1 = \frac{T_0}{1 + \frac{k-1}{2} M_1^2} = \frac{1100\ K}{1 + 0.2(4.0)^2} = 262\ K$$

The velocity is

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{\frac{1}{2}} = 4.0 \times \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 262\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 1300\ \frac{m}{s}$$

Since $M_1 = 4.0$, nozzle must be choked and $M_t = 1.0$. The relation for choked flow is

$$p_t = \frac{p_0}{\left(1 + \frac{k-1}{2} M_t^2\right)^{\frac{k}{k-1}}} = \frac{7.2 \times 10^6\ Pa}{(1 + 0.2(1.0)^2)^{3.5}} = 3.80\ MPa$$

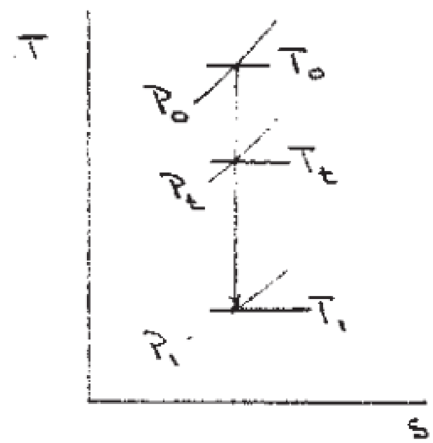
$$T_t = \frac{T_0}{1 + \frac{k-1}{2} M_t^2} = \frac{1100\ K}{1 + 0.2(1.0)^2} = 917\ K$$

$$\rho_t = \frac{p_t}{RT_t} = 3.80 \times 10^6\ \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{917\ K} = 14.4\ \frac{kg}{m^3}$$

$$V_t = M_t c_t = M_t (kRT_2)^{\frac{1}{2}} = 1.0 \times \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 917\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 607\ \frac{m}{s}$$

$$\dot{m} = \rho_t V_t A_t = 14.4\ \frac{kg}{m^3} \times 607\ \frac{m}{s} \times 0.01\ m^2 = 87.4\ \frac{kg}{s}$$

The states are



Problem 12.79

(Difficulty: 3)

12.79 A small rocket motor, fueled with hydrogen and oxygen, is tested on a thrust stand at a simulated altitude of $10\ km$. The motor is operated at chamber stagnation conditions of $1500\ K$ and $8.0\ MPa\ (gage)$. The combustion product is water vapor, which may be treated as an ideal gas. Expansion occurs through a converging-diverging nozzle with design Mach number of 3.5 and exit area of $700\ mm^2$. Evaluate the pressure at the nozzle exit plane. Calculate the mass flow rate of exhaust gas. Determine the force exerted by the rocket motor on the thrust stand.

Assumptions: (1) steady flow (2) isentropic flow (3) ideal gas behavior $k = 1.3$, $R = 461\ J/kg \cdot K$

Solution: Use the continuity equation and the ideal gas relations

$$\dot{m} = \rho V A$$

$$p = \rho R T$$

At $10\ km$ altitude,

$$p_b = 26.5\ kPa$$

Evaluate design pressure at exit,

$$\frac{p_0}{p} = \left[1 + \frac{(k-1)}{2} M^2\right]^{\frac{k}{k-1}}$$

$$p_d = \frac{8.10 \times 10^6\ Pa}{[1 + 0.15(3.5)^2]^{4.333}} = 88.3\ kPa\ (abs)$$

Since $p_b < p_d$, $p_e = p_d = 88.3\ kPa$.

$$\frac{T_0}{T_e} = 1 + \frac{k-1}{2} M_e^2$$

$$T_e = \frac{T_0}{1 + \frac{k-1}{2} M^2} = \frac{1500\ K}{1 + 0.15(3.5)^2} = 529\ K$$

$$\rho_e = \frac{p_e}{R T_e} = 88.3 \times 10^3\ \frac{N}{m^2} \times \frac{kg \cdot K}{461\ N \cdot m} \times \frac{1}{529\ K} = 0.362\ \frac{kg}{m^3}$$

$$V_e = Mc_e = 3.5\left[1.30 \times 461\ \frac{N\cdot m}{kg\cdot K} \times 529\ K \times \frac{kg\cdot m}{N\cdot s^2}\right]^{\frac{1}{2}} = 1970\ \frac{m}{s}$$

$$\dot{m} = \rho_e V_e A_e = 0.362\ \frac{kg}{m^3} \times 1970\ \frac{m}{s} \times 700\ mm^2 \times \frac{m^2}{10^6\ mm^2} = 0.499\ \frac{kg}{s}$$

To determine force on test stand apply x momentum equation to a CV around the rocket motor:

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} V_x \rho d\forall + \int_{CS} V_x \rho \bar{V}\cdot d\bar{A}$$

$$R_x + p_b A_e - p_e A_e = \dot{m} V_e$$

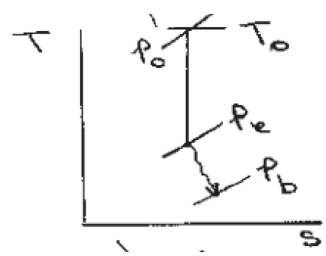
Force on test stand is $K_x = -R_x$

$$K_x = -R_x = -\dot{m} V_e - A_e(p_e - p_b)$$

$$K_x = -0.499\ \frac{kg}{s} \times 1970\ \frac{m}{s} \times \frac{N\cdot s^2}{kg\cdot m} - 700\ mm^2 \times \frac{m^2}{10^6\ mm^2} \times (88.3 - 26.5) \times 10^3\ \frac{N}{m^2}$$

$$K_x = -1026\ N\ (to\ left)$$

The process is



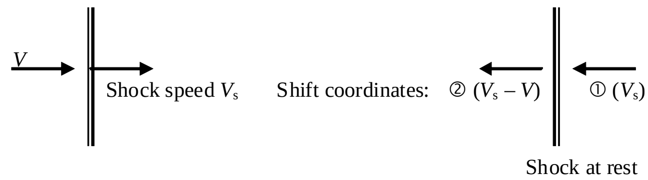
Problem 12.80

[Difficulty: 3]

12.80 Testing of a demolition explosion is to be evaluated. Sensors indicate that the shock wave generated at the instant of explosion is 30 MPa (abs). If the explosion occurs in air at 20°C and 101 kPa, find the speed of the shock wave, and the temperature and speed of the air just after the shock passes. As an approximation assume $k = 1.4$. (Why is this an approximation?)

Given: Normal shock due to explosion

Find: Shock speed; temperature and speed after shock



Solution:

Basic equations:

$$M_2^2 = \frac{M_1^2 + \frac{2}{k-1}}{\left(\frac{2\cdot k}{k-1}\right)\cdot M_1^2 - 1} \qquad V = M\cdot c = M\cdot\sqrt{k\cdot R\cdot T}$$

$$\frac{p_2}{p_1} = \frac{2\cdot k}{k+1}\cdot M_1^2 - \frac{k-1}{k+1} \qquad \frac{T_2}{T_1} = \frac{\left(1 + \frac{k-1}{2}\cdot M_1^2\right)\cdot\left(k\cdot M_1^2 - \frac{k-1}{2}\right)}{\left(\frac{k+1}{2}\right)^2\cdot M_1^2}$$

Given or available data $\quad k = 1.4 \quad R = 286.9\cdot\frac{J}{kg\cdot K} \quad p_2 = 30\cdot MPa \quad p_1 = 101\cdot kPa \quad T_1 = (20 + 273)\cdot K$

From the pressure ratio $\quad M_1 = \sqrt{\left(\frac{k+1}{2\cdot k}\right)\cdot\left(\frac{p_2}{p_1} + \frac{k-1}{k+1}\right)} \qquad M_1 = 16.0$

Then we have $\quad T_2 = T_1\cdot\frac{\left(1 + \frac{k-1}{2}\cdot M_1^2\right)\cdot\left(k\cdot M_1^2 - \frac{k-1}{2}\right)}{\left(\frac{k+1}{2}\right)^2\cdot M_1^2} \qquad T_2 = 14790\,K \qquad T_2 = 14517\cdot°C$

$$M_2 = \sqrt{\frac{M_1^2 + \frac{2}{k-1}}{\left(\frac{2\cdot k}{k-1}\right)\cdot M_1^2 - 1}} \qquad M_2 = 0.382$$

Then the speed of the shock ($V_s = V_1$) is $\quad V_1 = M_1\cdot\sqrt{k\cdot R\cdot T_1} \qquad V_1 = 5475\,\frac{m}{s} \qquad V_s = V_1 \qquad V_s = 5475\,\frac{m}{s}$

After the shock (V_2) the speed is $\quad V_2 = M_2\cdot\sqrt{k\cdot R\cdot T_2} \qquad V_2 = 930\,\frac{m}{s}$

But we have $\quad V_2 = V_s - V \qquad V = V_s - V_2 \qquad V = 4545\,\frac{m}{s}$

These results are unrealistic because at the very high post-shock temperatures experienced, the specific heat ratio will NOT be constant! The extremely high initial air velocity and temperature will rapidly decrease as the shock wave expands in a spherical manner and thus weakens.

Problem 12.81

(Difficulty: 2)

12.81 A total-pressure probe is placed in a supersonic wind tunnel where $T = 530\ °R$ and $M = 2.0$. A normal shock stands in front of probe. Behind the shock, $M_2 = 0.577$ and $p_2 = 5.76\ psia$. Find (a) the downstream stagnation pressure and stagnation temperature and (b) all fluid properties upstream from the shock. Show static and stagnation state points and the process path on a Ts diagram.

Assumptions: (1) steady flow (2) ideal gas (3) uniform flow at a section

Solution: Use the normal shock and ideal gas relations:

Across a shock

$$p(1 + kM^2) = constant$$

The ideal gas relations are

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}}$$

Across the shock, $p(1 + kM^2) = constant$:

$$p_1 = p_2 \frac{(1 + kM_2^2)}{1 + kM_1^2} = 5.76\ psia \times \frac{[1 + 1.4(0.5774)^2]}{[1 + 1.4(2)^2]} = 1.28\ psia$$

$$\rho_1 = \frac{p_1}{RT_1} = 1.28\ \frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3\ ft \cdot lbf} \times \frac{1}{530\ °R} \times 144\ \frac{in^2}{ft^2} = 0.00653\ \frac{lbm}{ft^3}$$

The velocity is determined from the Mach number

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{\frac{1}{2}} = 2.0 \left[1.4 \times 53.3\ \frac{ft \cdot lbf}{lbm \cdot °R} \times 530\ °R \times 32.2\ \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{\frac{1}{2}} = 2260\ \frac{ft}{s}$$

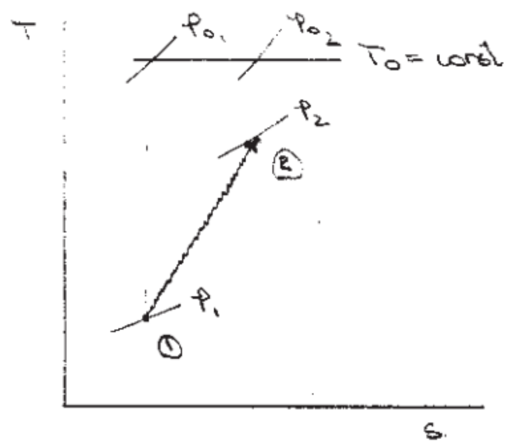
Then

$$T_{o1} = T_1 \left(1 + \frac{k-1}{2} M_1^2\right) = 530\ °R[1 + 0.2(2)^2] = 954\ °R$$

$$p_{01} = p_1\left(1+\frac{k-1}{2}M_1^2\right)^{\frac{k}{k-1}} = 1.28\,psia \times (1.8)^{3.5} = 10.0\,psia$$

$$p_{02} = p_2\left(1+\frac{k-1}{2}M_2^2\right)^{\frac{k}{k-1}} = 5.76\,psia[1+0.2(0.5774)^2]^{3.5} = 7.22\,psia$$

$$T_{o2} = T_{o1} = 954\,°R$$



Use the normal shock functions in Appendix D:

For $M_1 = 2.0$, from App D.4

$$M_2 = 0.577\ (12.34\,b)$$

$$\frac{p_2}{p_1} = 4.50\ (12.36)$$

$$p_1 = 1.28\,psia$$

$$\frac{p_{02}}{p_{01}} = 0.721\ (12.37)$$

For $M_1 = 2.0$, from App D.1

$$\frac{p_1}{p_{01}} = 0.1278$$

$$p_{01} = 10.0\,psia$$

$$p_{02} = 0.721 p_{01} = 7.21\,psia$$

$$\frac{T_1}{T_0} = 0.556\ (4.17b)$$

$$T_{01} = 954\,°R$$

Note: In using the table, it is not necessary to know the downstream Mach number.

Problem 12.82

(Difficulty: 2)

12.82 Air flows steadily through a long, insulated constant-area pipe. At section (1), $M_1 = 2.0$, $T_1 = 140\ \text{°F}$, and $p_1 = 35.9\ psia$. At section (2), downstream from a normal shock, $V_2 = 1080\ \frac{ft}{s}$. Determine the density and Mach number at section (2). Make a qualitative sketch of the pressure distribution along the pipe.

Assumptions: (1) steady flow (2) uniform flow at a section (3) $\dot{Q} = \dot{W}_s = \dot{W}_{shear} = 0$ (4) ideal gas (5) $A_1 = A_2 = A$

Solution: Use the energy equation and the relations for an ideal gas

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

The upstream velocity is

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{\frac{1}{2}} = 2.0 \left(1.4 \times 53.3\ \frac{ft \cdot lbf}{lbm \cdot °R} \times 600\ °R \times 32.2\ \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{\frac{1}{2}} = 2400\ \frac{ft}{s}$$

Using the energy equation, the temperature at 2 is

$$T_2 = T_1 + \frac{1}{2c_p}(V_1^2 - V_2^2)$$

$$T_2 = 600\ °R + \frac{1}{2}[(2.4)^2 - (1.08)^2] \times 10^6\ \frac{ft^2}{s^2} \times \frac{lbm \cdot °R}{0.24\ Btu} \times \frac{Btu}{778\ ft \cdot lbf} \times \frac{slug}{32.2\ lbm} \times \frac{lbf \cdot s^2}{slug \cdot ft}$$

$$T_2 = 982\ °R$$

Determine the Mach number at 2

$$M_2 = \frac{V_2}{c_2}$$

$$c_2 = (kRT_2)^{\frac{1}{2}} = \left(1.4 \times 53.3\ \frac{ft \cdot lbf}{lbm \cdot °R} \times 982\ °R \times 32.2\ \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{\frac{1}{2}} = 1540\ \frac{ft}{s}$$

$$M_2 = \frac{1080}{1540} = 0.701$$

From continuity,

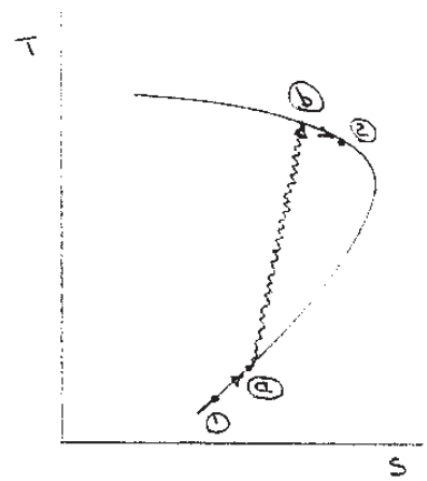
$$\rho_1 V_1 = \rho_2 V_2$$

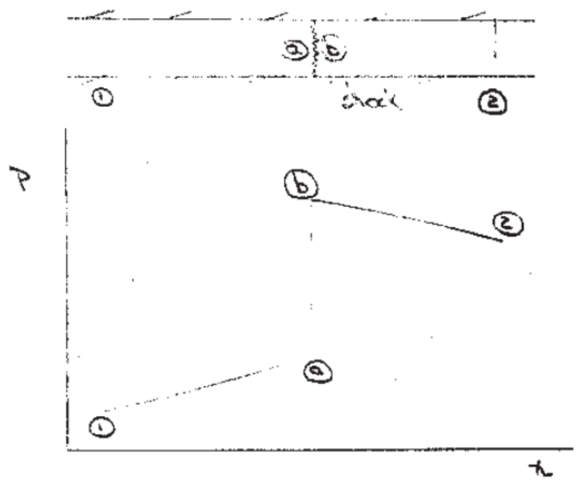
The density is then

$$\rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{V_1}{V_2}\frac{p_1}{RT_1}$$

$$\rho_2 = \frac{2400}{1080} \times 35.9\ \frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3\ ft \cdot lbf} \times \frac{1}{600\ °R} \times 144\ \frac{in^2}{ft^2} = 0.359\ \frac{lbm}{ft^3}$$

The process is

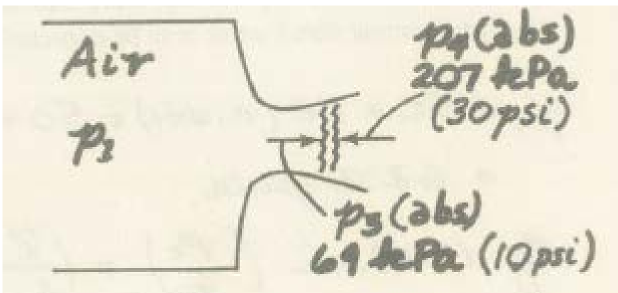




Problem 12.83

(Difficulty 2)

12.83 Air discharges through a convergent-divergent nozzle which is attached to a large reservoir. At a point in the nozzle in a normal shock wave is detected across which the absolute pressure jumps from 69 to 207 kPa. Calculate the pressures in the throat of the nozzle and in the reservoir.



Find: The pressure at the throat and in the reservoir

Assumptions: Air can be treated as an ideal gas. The flow is steady.

Solution: Use the relations for a normal shock wave and for compressible flow

We will use the relation between the upstream and downstream pressures first to find the upstream Mach number. Then, we will assume the flow is isentropic between the reservoir and upstream of the shock wave. We can then use isentropic relations to obtain the throat and reservoir pressures.

The upstream Mach number in terms of pressure is given by equation 13.20d or Figure D.2

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_1^2 - \frac{k-1}{k+1} = \frac{207\ kPa}{69\ kPa}$$

The upstream Mach number is

$$M_1 = 1.65$$

The pressure in the reservoir is the stagnation pressure for the flow from the reservoir to upstream of the shock wave. We can use equation 12.30a or Figure D.1 to find the stagnation pressure:

$$\frac{p_0}{p_1} = \left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}}$$

The stagnation pressure is

$$p_0 = 315\ kPa$$

The pressure in the nozzle throat is the critical pressure, given by:

$$\left(\frac{p_{throat}}{p_0}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = 0.528$$

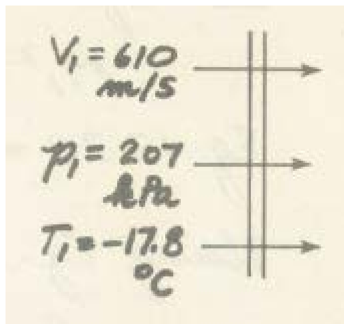
The throat pressure is then

$$p_2 = 166\ kPa$$

Problem 12.84

(Difficulty 2)

12.84 A normal shock wave exists in an air flow. The absolute pressure, velocity and temperature just upstream from the wave are $207\ kPa$, $610\ \frac{m}{s}$, and $-17.8\ ℃$, respectively. Calculate the pressure, velocity, temperature, and sonic velocity just downstream from the shock wave.



Find: The properties downstream of the shock wave

Assumptions: Air can be treated as an ideal gas. The flow is steady.

Solution: Use the relations for a normal shock wave

We need to find first the upstream Mach number. Then from the normal shock wave relations we can find the downstream properties.

The upstream sonic velocity is calculated as:

$$c_1 = \sqrt{kRT_1} = \sqrt{1.4 \times 286.8\ \frac{J}{kg \cdot K} \times (273.2 - 17.8)K} = 320.2\ \frac{m}{s}$$

And the upstream Mach number is

$$M_1 = \frac{V_1}{c_1} = \frac{610\ \frac{m}{s}}{320.2\ \frac{m}{s}} = 1.905$$

The downstream Mach number for a normal shock wave is given in terms of the upstream Mach number by eq 12.43 a or Figure D.2:

$$M_2^2 = \frac{1 + \frac{k-1}{2} M_1^2}{k M_1^2 - \frac{k-1}{2}}$$

This yields

$$M_2 = 0.595$$

With the upstream Mach number and upstream pressure, we can find the down stream pressure and temperature from equations 13.20d and 13.20c or Figure D.2

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_1^2 - \frac{k-1}{k+1}$$

The downstream pressure is

$$p_2 = 842\ kPa$$

The temperature is given by

$$\frac{T_2}{T_1} = \frac{\left(1 + \frac{k-1}{2}M_1^2\right)\left(kM_1^2 - \frac{k-1}{2}\right)}{\left(\frac{k+1}{2}\right)^2 M_1^2} = 1.612$$

The downstream temperature is

$$T_2 = 412\ K = 145\ C$$

The speed of sound at this temperature is

$$c_2 = \sqrt{kRT_2} = 406\ \frac{m}{s}$$

The velocity after the shock wave is then

$$V_2 = c_2 M_2 = 0.595 \times 406\ \frac{m}{s} = 242\ \frac{m}{s}$$

Problem 12.85

[Difficulty: 3]

12.85 Air approaches a normal shock at $V_1 = 900$ m/s, $p_1 = 50$ kPa, and $T_1 = 220$ K. What are the velocity and pressure after the shock? What would the velocity and pressure be if the flow were decelerated isentropically to the same Mach number?

Given: Air approaching a normal shock

Find: Pressure and velocity after the shock; pressure and velocity if flow were decelerated isentropically

Solution:

The given or available data is:

	$R =$	286.9	J/kg-K
	$k =$	1.4	
	$V_1 =$	900	m/s
	$p_1 =$	50	kPa
	$T_1 =$	220	K

Equations and Computations:

The sonic velocity at station 1 is:

$c_1 =$ 297.26 m/s

So the Mach number at 1 is:

$M_1 =$ 3.028

Downstream of the normal shock wave, the Mach number is:

$M_2 =$ 0.4736

The static pressure and temperature ratios are:

$p_2/p_1 =$ 10.528

$T_2/T_1 =$ 2.712

So the exit temperature and pressure are:

$p_2 =$ 526 kPa

$T_2 =$ 596.6 K

At station 2 the sound speed is:

$c_2 =$ 489.51 m/s

Therefore the flow velocity is:

$V_2 =$ 232 m/s

If we decelerate the flow isentropically to

$M_{2s} =$ 0.4736

The isentropic pressure ratios at station 1 and 2s are:

$p_0/p_1 =$ 38.285

$p_0/p_{2s} =$ 1.166

$p_{2s}/p_1 =$ 32.834

So the final pressure is:

$p_{2s} =$ 1642 kPa

The temperature ratios are:

$T_0/T_1 =$ 2.833

$T_0/T_{2s} =$ 1.045

$T_{2s}/T_1 =$ 2.712

So the final temperature is:

$T_{2s} =$ 596.6 K

The sonic velocity at station 2s is:

$c_{2s} =$ 489.51 m/s

Therefore the flow velocity is:

$V_{2s} =$ 232 m/s

Problem 12.86

[Difficulty: 2]

12.86 Air approaches a normal shock at $M_1 = 2.5$, with $T_{0_1} = 1250°$R and $p_1 = 20$ psia. Determine the speed and temperature of the air leaving the shock and the entropy change across the shock.

Given: Normal shock

Find: Speed and temperature after shock; Entropy change

Solution:

The given or available data is:

$R =$	53.33	ft·lbf/lbm·R	0.0685	Btu/lbm·R
$k =$	1.4			
$c_p =$	0.2399	Btu/lbm·R		
$T_{01} =$	1250	^{o}R		
$p_1 =$	20	psi		
$M_1 =$	2.5			

Equations and Computations:

From $p_1 = \rho_1 R T_1$ $\quad \rho_1 =$ 0.0432 slug/ft^3

$V_1 =$ 4334 ft/s

Using built-in function *IsenT* (M,k):

$T_{01}/T_1 =$ 2.25 $\quad T_1 =$ 556 ^{o}R

96 ^{o}F

Using built-in function *NormM2fromM* (M,k):

$M_2 =$ 0.513

Using built-in function *NormTfromM* (M,k):

$T_2/T_1 =$ 2.14 $\quad T_2 =$ 1188 ^{o}R

728 ^{o}F

Using built-in function *NormpfromM* (M,k):

$p_2/p_1 =$ 7.13 $\quad p_2 =$ 143 psi

From $V_2 = M_2\sqrt{kRT_2}$ $\quad V_2 =$ 867 ft/s

From $$\Delta s = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

$\Delta s =$ 0.0476 Btu/lbm·R

37.1 ft·lbf/lbm·R

Problem 12.87

(Difficulty: 2)

12.87 Air undergoes a normal shock. Upstream, $T_1 = 35\ ^\circ\mathrm{C}$, $p_1 = 229\ kPa\ (abs)$, and $V_1 = 704\ \frac{m}{s}$. Determine the temperature and stagnation pressure of the air stream leaving the shock.

Assumptions: (1) steady flow (2) uniform flow at a section (3) $\dot{Q} = \dot{W}_s = \dot{W}_{shear} = 0$ (4) ideal gas (5) $A_1 = A_2 = A$ (6) $F_{Bx} = 0$ (7) no friction force

Solution: Use the ideal gas and normal shock functions

$$M_1 = \frac{V_1}{c_1}$$

$$c_1 = (kRT_1)^{\frac{1}{2}} = \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 308\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{\frac{1}{2}} = 352\ \frac{m}{s}$$

$$M_1 = \frac{704}{352} = 2$$

From App. D.1

$$\frac{p_1}{p_{o1}} = 0.1278$$

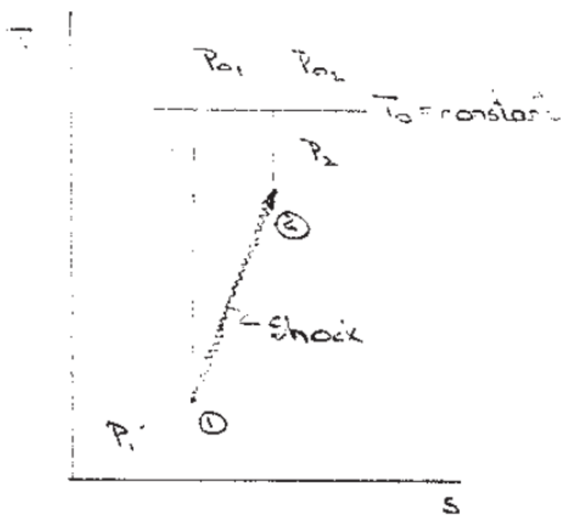
From App. D.4

$$\frac{p_{o2}}{p_{o1}} = 0.7209$$

$$\frac{T_2}{T_1} = 1.687$$

$$T_2 = 1.687 \times 308\ K = 520\ K$$

$$p_{o2} = \frac{p_{o2}}{p_{o1}} \frac{p_{o1}}{p_1} p_1 = 0.7209 \times \frac{1}{0.1278} \times 229\ kPa = 1.29\ MPa\ (abs)$$

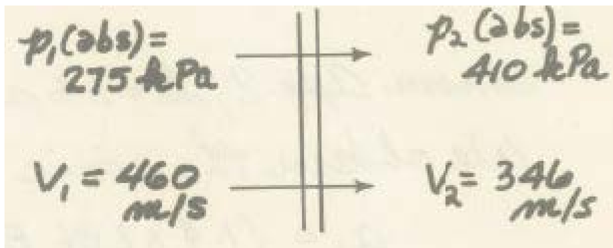


T
P_{01}
P_{02}
T_0 = constant
P_2
2
Shock
1
P_1
s

Problem 12.88

(Difficulty 2)

12.88 If through a normal shock wave (in air), the absolute pressure rises from 275 to 410 kPa and the velocity diminishes from 460 to 346 $\frac{m}{s}$, what temperature are to be expected upstream and downstream from the wave?



Find: The temperatures upstream and downstream of the shock wave

Assumptions: Air can be treated as an ideal gas. The flow is steady.

Solution: Use the relations for a normal shock wave

We will use the relation between the upstream and downstream pressures to find the upstream Mach number, and then the downstream Mach number. With the Mach numbers we can find the temperature.

The upstream Mach number in terms of pressure is given by equation 13.20d or Figure D.2

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_1^2 - \frac{k-1}{k+1} = \frac{410\ kPa}{275\ kPa}$$

The upstream Mach number is

$$M_1 = 1.19$$

The speed of sound is then

$$c_1 = M_1 V_1 = 386\frac{m}{s} \times 1.19 = \sqrt{kRT_1}$$

The upstream temperature is then

$$T_1 = 371\ K$$

The downstream Mach number is related to the upstream Mach number through eq 12.43 a or Figure D.2:

$$M_2^2 = \frac{1 + \frac{k-1}{2}M_1^2}{kM_1^2 - \frac{k-1}{2}}$$

The downstream Mach number is then

$$M_2 = 0.846$$

The speed of sound is then

$$c_2 = M_2 V_2 = 346 \frac{m}{s} \times 0.846 = \sqrt{kRT_1}$$

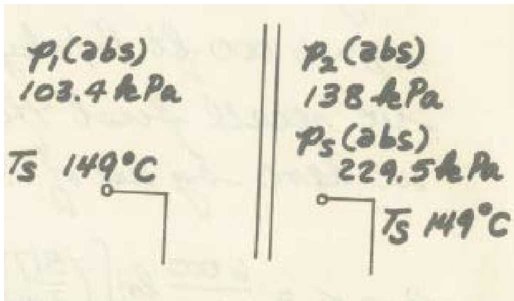
The downstream temperature is then

$$T_1 = 417\ K$$

Problem 12.89

(Difficulty 3)

12.89 The stagnation temperature in an air flow is 149 ℃ upstream and downstream from a normal shock wave. The absolute stagnation pressure downstream from the shock wave is 229.5 kPa. Through the wave the absolute pressure rises from $103.4\ to$ 138 kPa. Determine the velocities upstream and downstream from the wave.



Find: The velocities upstream amd downstream of the shock wave

Assumptions: Air can be treated as an ideal gas. The flow is steady.

Solution: Use the relation between static and stagnation properties and the relation for a normal shock wave

We can find the upstream Mach number from the static pressures using equation 13.20d or Figure D.2

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_1^2 - \frac{k-1}{k+1} = \frac{138\ kPa}{103.4\ kPa}$$

The Mach number is

$$M_1 = 1.134$$

The downstream Mach number for a normal shock wave is given in terms of the upstream Mach number by eq 12.43 a or Figure D.2:

$$M_2^2 = \frac{1 + \frac{k-1}{2}M_1^2}{kM_1^2 - \frac{k-1}{2}}$$

This yields

$$M_2 = 0.886$$

We can now compute the upstream velocity from the definitions of Mach number and stagnation temperature. The upstream static temperature can be related to the velocity and Mach number as

$$M_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{kRT_1}}$$

Or, the static temperature is

$$T_1 = \frac{V_1^2}{kRM_1^2}$$

The definition of the stagnation temperature is:

$$c_p T_0 = c_p T_1 + \frac{V_1^2}{2} = c_p \frac{V_1^2}{kRM_1^2} + \frac{V_1^2}{2}$$

The velocity V_1 is then

$$V_1 = \sqrt{\frac{c_p T_s}{\frac{c_p}{kRM_1^2} + \frac{1}{2}}} = \sqrt{\frac{1003\ \frac{J}{kg \cdot K} \times (273.2 + 149)K}{\frac{1003\ \frac{J}{kg \cdot K}}{1.4 \times 286.8\ \frac{J}{kg \cdot K} \times 1.134^2} + \frac{1}{2}}} = 417\ \frac{m}{s}$$

We can use a similar approach for the downstream velocity. The downstream Mach number is given as

$$M_2 = \frac{V_2}{c_2} = \frac{V_2}{\sqrt{kRT_2}}$$

Or, solving for the temperature

$$T_2 = \frac{V_2^2}{kRM_2^2}$$

Again, the stagnation temperature in terms of the downstream properties is:

$$c_p T_0 = c_p T_2 + \frac{V_2^2}{2} = c_p \frac{V_2^2}{kRM_2^2} + \frac{V_2^2}{2}$$

Or, the velocity V_2 is

$$V_2 = \sqrt{\frac{c_p T_s}{\frac{c_p}{kRM_2^2} + \frac{1}{2}}} = \sqrt{\frac{1003\ \frac{J}{kg \cdot K} \times (273.2 + 149)K}{\frac{1003\ \frac{J}{kg \cdot K}}{1.4 \times 286.8\ \frac{J}{kg \cdot K} \times 0.886^2} + \frac{1}{2}}} = 339\ \frac{m}{s}$$

Problem 12.90

(Difficulty: 2)

12.90 A supersonic aircraft cruises at $M = 2.2$ at $12\ km$ altitude. A pitot tube is used to sense pressure for calculating air speed. A normal shock stands in front of the tube. Evaluate the local isentropic stagnation conditions in front of the shock. Estimate the stagnation pressure sensed by the pitot tube. Show static and stagnation state points and the process path on a Ts diagram.

Assumptions: (1) steady flow (2) uniform flow at a section (3) thin shock (4) ideal gas

Solution: Use the compressible flow functions in Appendix D for the solution.

Use table A.3 to determine properties at state (1). At $12\ km$ altitude,

$$T_1 = 216.7\ K$$

$$p_1 = 19.4\ kPa$$

From App. D.1 for $M_1 = 2.2$,

$$\frac{T_1}{T_{o1}} = 0.5081$$

$$T_{o1} = 426\ K$$

$$\frac{p_1}{p_{o1}} = 0.09352$$

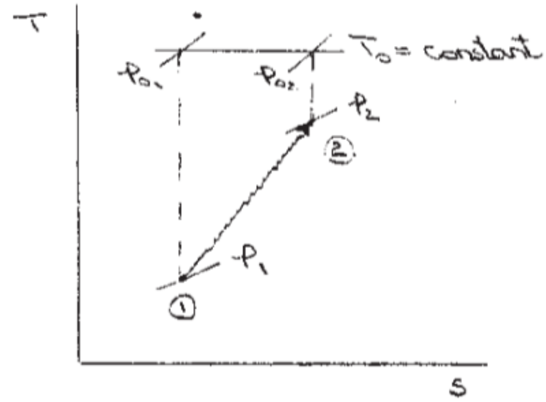
$$p_{o1} = 207\ kPa\ (abs)$$

From APP. D.4, for $M_1 = 2.2$, $M_2 = 0.5471$

$$\frac{p_{o2}}{p_{o1}} = 0.6281$$

$$p_{o2} = 130\ kPa\ (abs)$$

The process is



Problem 12.91

(Difficulty: 2)

12.91 The Concorde supersonic transport flew at $M = 2.2$ at 20 km altitude. Air is decelerated isentropically by the engine inlet system to a local Mach number of 1.3. The air passed through a normal shock and was decelerated further to $M = 0.4$ at the engine compressor section. Assume, as a first approximation, that this subsonic diffusion process was isentropic and use standard atmosphere data for freestream conditions. Determine the temperature, pressure, and stagnation pressure of the air entering the engine compressor.

Assumptions: (1) steady flow (2) ideal gas (3) flow is isentropic across the shock

Solution: Use the compressible flow functions tabulated in Appendix D

At 20 km altitude,

$$T_\infty = 217\ K$$

$$p_\infty = 5.53\ kPa$$

$$M_\infty = 2.2$$

From App D.1,

$$\frac{T_\infty}{T_o} = 0.5081$$

$$\frac{p_\infty}{p_o} = 0.09352$$

$$T_o = 427\ K$$

$$p_o = 59.1\ kPa$$

$M_1 = 1.3$, From App D.1,

$$\frac{T_1}{T_o} = 0.7474$$

$$\frac{p_1}{p_o} = 0.3604$$

$$T_1 = 318\ K$$

$$p_1 = 21.3\ kPa$$

From App D.4,

$$M_2 = 0.786$$

$$\frac{p_{o2}}{p_{o1}} = 0.9794$$

$$\frac{T_2}{T_1} = 1.191$$

$$\frac{p_2}{p_1} = 1.805$$

$$p_{o2} = 57.9\ kPa$$

$$T_2 = 380\ K$$

$$p_2 = 38.4\ kPa$$

$$p_{o3} = p_{o2} = 57.9\ kPa\ (abs)$$

$M_3 = 0.4$, From App D.1,

$$\frac{T_3}{T_o} = 0.9690$$

$$\frac{p_3}{p_{o3}} = 0.8956$$

$$T_3 = 414\ K$$

$$p_3 = 0.8956 p_{o3} = 0.8956 p_{o2} = 51.9\ kPa\ (abs)$$