Chapter 11

PROBLEM 11.1

For an ideal radiator (hohlraum) with a 10-cm-diameter opening, located in black surroundings at 16°C, (a) calculate the net radiant heat transfer rate for hohlraum temperatures of 100°C and 560°C, (b) the wavelength at which the emission is a maximum, (c) the monochromatic emission at λ_{max}, and (d) the wavelengths at which the monochromatic emission is 1 per cent of the maximum value.

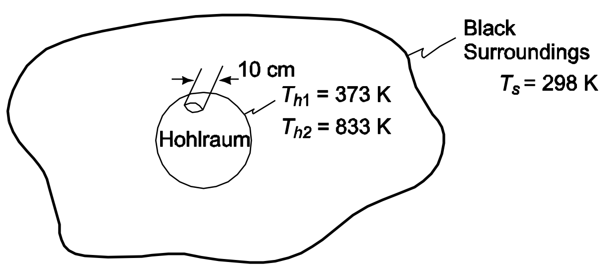
GIVEN

- An ideal radiator (hohlraum) in black surroundings
- Radiator opening diameter (D) = 10 cm = 0.1 m
- Surrounding temperature (T_s) = 16°C = 289 K
- Hohlraum temperatures
  - T_{h1} = 100°C = 373 K
  - T_{h2} = 560°C = 833 K

FIND

(a) The net radiant heat transfer rate (q_r)
(b) The wavelength at which the emission is maximum (λ_{max})
(c) The monochromatic emission at λ_{max} ($E_{\lambda max}$)
(d) The wavelengths at which the monochromatic emission is 1% $E\lambda_{max}$

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

All parts of the problem will first be solved for T_h = 373 K

(a) The net radiative transfer between any two black surfaces is given by Equation (11.47)

$$q_{12} = A_1 F_{12} (E_{b1} - E_{b2})$$

where $A_1 = (\pi/4)D^2$

F_{12} = 1, since surface 1 is surrounded by surface 2.

From Equation (11.3) $E_{b7} = \sigma T_h^4$ and $E_{b2} = \sigma T_s^4$

$$q_{12} = \frac{\pi}{4} D^2 \sigma (T_h^4 - T_s^4) = \frac{\pi}{4} (0.1 \text{ m})^2 \quad 5.67\times10^{-8} \text{ W/(m}^2 \text{ K}^4) \quad [(373 \text{ K})^4 - (289 \text{ K})^4] = 5.51 \text{ W}$$

(b) The wavelength at which the maximum emission occurs for a black body is given by Equation (11.2)

$$\lambda_{max} T_h = 2.898 \times 10^{-3} \text{ m K}$$

$$\lambda_{max} = \frac{2.898 \times 10^{-3}\,\mathrm{m\,K}}{T_h} = \frac{2.898 \times 10^{-3}\,\mathrm{m\,K}}{373\,\mathrm{K}} = 7.77 \times 10^{-6}\ \mathrm{m} = 7.77\ \mu\mathrm{m}$$

(c) The monochromatic emission is given by Equation (11.1)

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}$$

where $C_1 = 3.7415 \times 10^{-16}$ W m^2

$C_2 = 1.4388 \times 10^{-2}$ m K

$$E_{b\lambda max} = \frac{3.7415 \times 10^{-16}\,\mathrm{Wm^2}}{(7.77 \times 10^{-6}\,\mathrm{m})^5\left[\exp\left(\frac{1.4388 \times 10^{-2}\,\mathrm{m\,K}}{(7.77 \times 10^{-6}\,\mathrm{m})(373\,\mathrm{K})}\right) - 1\right]} = 9.29 \times 10^7\ \mathrm{W/m^3}$$

(d) 1% $E_{b\lambda max} = E_{b\lambda} = 9.29 \times 10^5$ W/m^3

$$\frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)} = 9.29 \times 10^5\ \mathrm{W/m^3}$$

$$\frac{3.7415 \times 10^{-16}\,\mathrm{Wm^2}}{\lambda^5\left[\exp\left(\frac{1.4388 \times 10^{-2}\,\mathrm{m\,K}}{(\lambda)(373\,\mathrm{K})}\right) - 1\right]} - 9.29 \times 10^5\ \mathrm{W/m^3} = 0$$

(There will be one solution below λ_{max} and one above λ_{max})

By trial and error $\lambda_1 = 2.55\ \mu$m

$\lambda_2 = 51.4\ \mu$m

Repeating the above proceduur for $T_h = 833$ K (a) 211 W (c) 5.15×10^9 W/m^2

(b) 3.48 μm (d) $\lambda_1 = 1.14\ \mu$m

$\lambda_2 = 23.05\ \mu$m

PROBLEM 11.2

A tungsten filament is heated to 2700 K. At what wavelength is the maximum amount of radiation emitted? What fraction of the total energy is in the visible range (0.4 to 0.75 μm)? Assume that the filament radiates as a gray body.

GIVEN

- Heated tungsten filament
- Filament temperature (T_f) = 2700 K

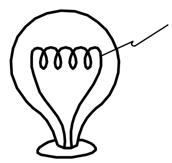
FIND

(a) Wavelength at which the maximum radiation is emitted (λ_{max})
(b) Percentage of radiation in the visible ranger (0.4 to 0.75 μm)

ASSUMPTIONS

- The filament radiates as a gray body

SKETCH



SOLUTION

(a) Since the tungsten is a gray body, the maximum emission occurs at the same wavelength as a black body. From Equation (11.2)

$$\lambda_{max} = \frac{2.898 \times 10^{-3}\,\text{m K}}{T} = \frac{2.898 \times 10^{-3}\,\text{m K}}{2700\,\text{K}} = 1.07 \times 10^{-6} = 1.07\ \mu\text{m}$$

(b) Gray body radiation is proportional to the black body radiation at all wavelengths. Therefore, the percentage of the gray body radiation in the visible spectrum is the same as the percentage of black body radiation in the visible spectrum.

For the limits of the visible range $\lambda_1 T = (0.4 \times 10^{-6}\ \text{m})\ (2700\ \text{K}) = 1.08 \times 10^{-3}\ \text{m K}$

$$\lambda_2 T = (0.75 \times 10^{-6}\ \text{m})\ (2700\ \text{K}) = 2.025 \times 10^{-3}\ \text{m K}$$

From Table 11.1

$$\frac{E_b(0 \rightarrow \lambda_1 T)}{\sigma T^4} = 0.0011$$

$$\frac{E_b(0 \rightarrow \lambda_2 T)}{\sigma T^4} = 0.071$$

The percent within the visible spectrum is

$$\frac{E_b(\lambda_1 T \rightarrow \lambda_2 T)}{\sigma T^4} = 0.071 - 0.0011 = 0.07 = 7.0\%$$

PROBLEM 11.3

Determine the total average hemispherical emissivity and the emissive power of a surface that has a spectral hemispherical emissivity of 0.8 at wavelengths less than 1.5 μm. 0.6 at wavelengths from 1.5 to 2.5 μm, and 0.4 at wavelengths longer than 2.5 μm. The surface temperature is 1111 K.

GIVEN

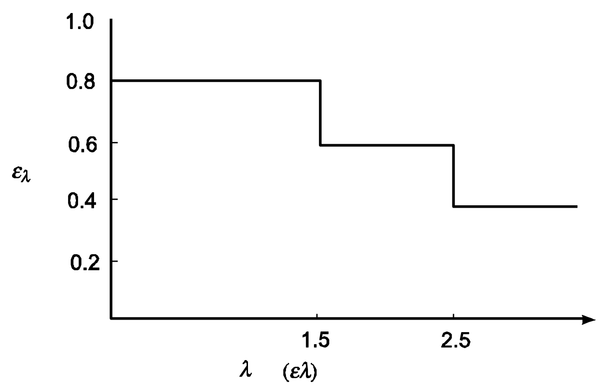
- A surface at temperature (T) = 1111 K
- Spectral hemispherical emittance (ε_λ) = 0.8 for λ < 1.5 μm (ε_1)
  = 0.6 for 1.5 μm < λ < 2.5 μm (ε_2)
  = 0.4 for λ > 2.5 μm (ε_3)

FIND

(a) The total average hemispherical emittance, $\varepsilon(T)$
(b) The emissive power, $E(T)$

SKETCH

The hemispherical emittance is shown graphically below



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

(a) The total average hemispherical emittance is given by Equation (11.31)

$$\varepsilon(\mathrm{T}) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{b\lambda}(\lambda,T)\, d\lambda}{\int_0^\infty E_{b\lambda}(\lambda,T)\, d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{b\lambda}(\lambda,T)\, d\lambda}{\sigma T^4}$$

For λ_1 = 1.5 μm $\quad \lambda_1 T = (1.5 \times 10^{-6}\text{ m})(1111\text{ K}) = 1.67 \times 10^{-3}$ m K

For λ_2 = 2.5 μm $\quad \lambda_2 T = (2.5 \times 10^{-6}\text{ m})(1111\text{ K}) = 2.78 \times 10^{-3}$ m K

Interpolating from Table 11.1

$$\frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} = 0.0266$$

$$\frac{E_b(0 \to \lambda_2 T)}{\sigma T^4} = 0.2234$$

$$\therefore \quad \frac{E_b(\lambda_1 T \to \lambda_2 T)}{\sigma T^4} = 0.2234 - 0.0266 = 0.1968$$

$$\therefore \quad \frac{E_b(\lambda_2 T \to \infty)}{\sigma T^4} = 1 - \frac{E_b(0 \to \lambda_2 T)}{\sigma T^4} = 1 - 0.2234 = 0.7766$$

Therefore

$$\varepsilon\,(T) = \varepsilon_1 \frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} + \varepsilon_2 \frac{E_b(\lambda_1 T \to \lambda_2 T)}{\sigma T^4} + \varepsilon_3 \frac{E_b(\lambda_2 T \to \infty)}{\sigma T^4}$$

$$\varepsilon\,(T) = 0.8\ (0.0266) + 0.6\ (0.1968) + 0.4\ (0.7766) = 0.45$$

(b) From Equation (11.31)

$$\varepsilon\,(T) = \frac{E\,(T)}{E_b(T)} = \frac{E\,(T)}{\sigma T^4}$$

$$E\,(T) = \varepsilon\,(T)\,\sigma\,T^4 = 0.45 \quad 5.67\times10^{-8}\ \mathrm{W/(m^2K^4)} \quad (1111\ \mathrm{K})^4 = 3.89 \times 10^4\ \ \mathrm{W/m^2}$$

PROBLEM 11.4

Shown that (a) $E_{b\lambda}/T^5 = f(\lambda T)$ only. For λT = 5000 μm K, calculate $E_{b\lambda}/T^5$.

GIVEN

- λT = 5000 μm K = 0.005 m K

FIND

Shown that

(a) $E_{b\lambda}/T^6 = f(\lambda T)$

(b) Calculate $E_{b\lambda}/T^5$ for the λT given

SOLUTION

(a) Starting with Equation (11.1)

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}$$

Where: $C_1 = 3.7415 \times 10^{-16}$ W m^2

$C_2 = 1.4388 \times 10^{-2}$ m K

$$\frac{E_{b\lambda}}{T^5} = \frac{C_1}{(\lambda T)^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)} = f(\lambda T)$$

(b) At λT = 0.005 m K

$$\frac{E_{b\lambda}}{T^5} = \frac{3.7415 \times 10^{-16}\,\mathrm{Wm^2}}{(0.005\,\mathrm{m\,K})^5\left[\exp\left(\frac{1.4388 \times 10^{-2}\,\mathrm{m\,K}}{(0.005\,\mathrm{m\,K})}\right) - 1\right]} = 7.14 \times 10^{-6}\ \mathrm{W/(m^3\,K^5)}$$

PROBLEM 11.5

Compute the average emittance of anodized aluminum at 100°C and 650°C from the spectral curve in Figure 9.16. Assume ε_λ = 0.8 for $\lambda > 9\ \mu$m

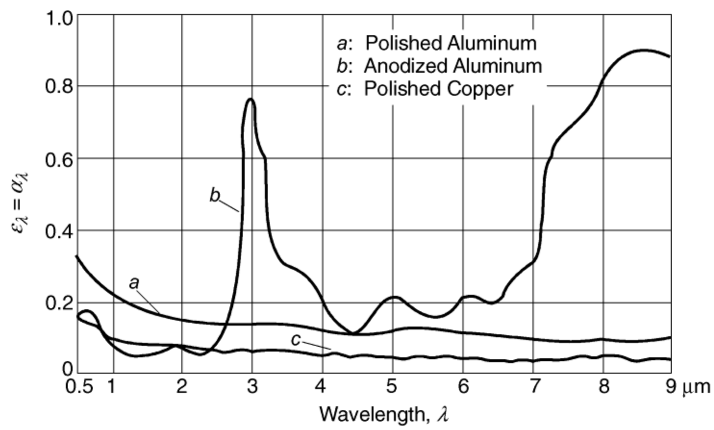
GIVEN

The spectral curve of Figure 11.17 for anodized aluminum

FIND

The average emittance (ε) at (a) 100°C = 373 K, and, (b) 650°C = 923 K

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 10^{-8} W/(m^2 K^4)

SOLUTION

The average emittance is given by Equation (11.31)

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda)E_{b\lambda}(\lambda,T)\,d\lambda}{\int_0^\infty E_{b\lambda}(\lambda,T)\,d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda)E_{b\lambda}(\lambda,T)\,d\lambda}{\sigma T^4} = \frac{\int_0^{\lambda_1} \varepsilon_\lambda(\lambda)E_{b\lambda}(\lambda,T)\,d\lambda}{\sigma T^4} + \varepsilon_\lambda(\lambda)\frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4}$$

The second term can be calculated from the following expression

$$\frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4} = 1 - \frac{E_b(0 \to \lambda_1 T)}{\sigma T^4}$$

At T = 923 K $\qquad \lambda_1 T = (9 \times 10^{-6}\text{ m})\,(923\text{ K}) = 8.31 \times 10^{-3}$ m K

From Table 11.1

$$\frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} = 0.8677$$

$$\therefore\ \varepsilon_\lambda(\lambda)\,\frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4} = 0.8\,(1 - 0.8677) = 0.1058$$

The first term for the average emittance can be approximated by divided Figure 11.17 into 12 segments.

$$\frac{\int_0^{\lambda_1} \varepsilon_\lambda(\lambda) E_{b\lambda}(\lambda,T)\, d\lambda}{\int_0^{\infty} E_{b\lambda}(\lambda,T)\, d\lambda} \approx \sum_{n=0}^{12} \varepsilon_{\lambda n} E_{b\lambda n} \Delta\lambda_n$$

where $\varepsilon_{\lambda n}$ = The average ε for the n^{th} segment

$$E_{b\lambda n} = \frac{C_1}{\lambda_n^5\left(e^{\frac{C_2}{\lambda at}} - 1\right)}$$

$\lambda_n = \lambda$ at the center of segment n

$C_1 = 3.7415 \times 10^{-16}$ W/m^2

$C_2 = 1.4388 \times 10^{-16}$ m K

$\Delta\lambda_n$ = The width of segment n

The values of these quantities are tabulated below for T = 923 K (λ = wavelength at the end of the segment).

n	λ(μm)	λn(μm)	$\varepsilon_{\lambda n}$	$\varepsilon_{\lambda n} E_{\lambda} n\, \Delta\lambda_n$ (W/m^2)
	0.5			
1	1	0.75	0.12	0.2362
2	2	1.5	0.06	90.673
3	2.5	2.25	0.07	222.73
4	3	2.75	0.41	1689.9
5	3.3	3.15	0.51	1318.6
6	4	3.65	0.27	1546.6
7	5	4.5	0.17	1113.8
8	6	5.5	0.18	835.44
9	7	6.5	0.22	709.18
10	8	7.5	0.58	1307.8
11	8.5	8.25	0.86	749.58
12	9	8.75	0.88	649.88
			Sum	10234.4 W/m^2

Therefore, for T = 923 K

$$\varepsilon = \frac{10234\,\text{W/m}^2}{\sigma T^4} + 0.1058 = \frac{10234\,\text{W/m}^2}{5.67 \times 10^{-8}\,\text{W/(m}^2\,\text{K}^4)\ (923\,\text{K})^4} + 0.1058 = 0.35$$

Repeating this procedure for T = 100°C = 373 K yields, ε = 0.677

PROBLEM 11.6

A large body of nonluminous gas at a temperature of 1100°C has emission bands between 2.5 and 3.5 μm and between 5 and 8 μm. At 1100°C, the effective emittance in the first band is 0.8 and in the second 0.6. Determine the emissive power of this gas in W/m^2.

GIVEN

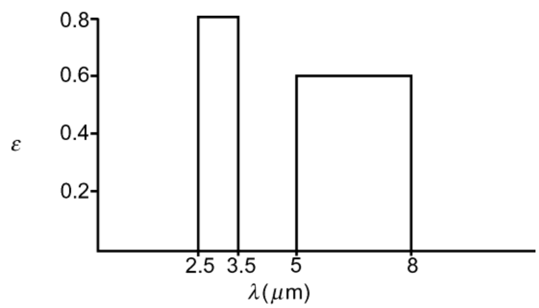
- A large body of nonluminous gas
- Gas temperature (T) = 1100°C = 1373 K
- Emission bands: 2.5 μm < λ_1 < 3.5 μm 5 μm < λ_2 < 8 μm
- Effective emittances: ε_1 = 0.8; ε_2 = 0.6

FIND

- The emissive power (E) in W/m^2

SKETCH

The effective emittance can be represented graphically as shown below



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The total emittance is given by Equation (11.31)

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{b\lambda}(\lambda,T)\,d\lambda}{\int_0^\infty E_{b\lambda}(\lambda,T)\,d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda E_{b\lambda}\,d\lambda}{\sigma T^4} = \varepsilon_1 \frac{E_b(\lambda_{11}T \to \lambda_{12}T)}{\sigma T^4} * + \varepsilon_2 \frac{E_b(\lambda_{21}T \to \lambda_{22}T)}{\sigma T^4}$$

where $\lambda_{11}\,T$ = (2.5 × 10^{-6} m) (1373 K) = 3.4 × 10^{-3} m K

$\lambda_{12}\,T$ = (3.5 × 10^{-6} m) (1373 K) = 4.8 × 10^{-3} m K

$\lambda_{21}\,T$ = (5 × 10^{-6} m) (1373 K) = 6.9 × 10^{-3} m K

$\lambda_{22}\,T$ = (8 × 10^{-6} m) (1373 K) = 11.0 × 10^{-3} m K

From Table 11.1

$$\frac{E_b(0 \to \lambda_{11}T)}{\sigma T^4} = 0.3618$$

$$\frac{E_b(0 \to \lambda_{12}T)}{\sigma T^4} = 0.6076$$

$$\frac{E_b(0 \to \lambda_{21}T)}{\sigma T^4} = 0.8022$$

$$\frac{E_b(0 \to \lambda_{22}T)}{\sigma T^4} = 0.9320$$

$$\frac{E_b(\lambda_{11}T \to \lambda_{12}T)}{\sigma T^4} = 0.6076 - 0.3618 = 0.2458$$

$$\frac{E_b(\lambda_{21}T \to \lambda_{22}T)}{\sigma T^4} = 0.9320 - 0.8022 = 0.1298$$

$$\varepsilon = 0.8(0.2458) + 0.6(0.1298) = 0.2745$$

$$\therefore E = \varepsilon E_b = \varepsilon \sigma T^4 = 0.2745 \;\; 5.67 \times 10^{-8}\ \mathrm{W/(m^2\,K^4)} \;\; (1373)^4 = 5.53 \times 10^4\ \mathrm{W/m^2}$$

PROBLEM 11.7

A flat plate is in a solar orbit 150,000,000 km from the sun. It is always oriented normal to the rays of the sun and both sides of the plate have a finish which has a spectral absorptance of 0.95 at wavelengths shorter than 3 μm and 0.06 at wavelengths longer than 3 μm. Assuming that the sun is a 5550 K blackbody source with a diameter of 1,400,000 km, determine the equilibrium temperature of the plate.

GIVEN

- A flat plate in solar orbit oriented normal to the rays of the sun
- Distance from the sun (R) = 150,000,000 km = 1.5×10^{11} m
- Spectral absorplatace (ε_λ) of both sides $\varepsilon_{\lambda 1} = 0.95$ for $\lambda_1 < 3\ \mu$m
  $\varepsilon_{\lambda 2} = 0.06$ for $\lambda_1 > 3\ \mu$m

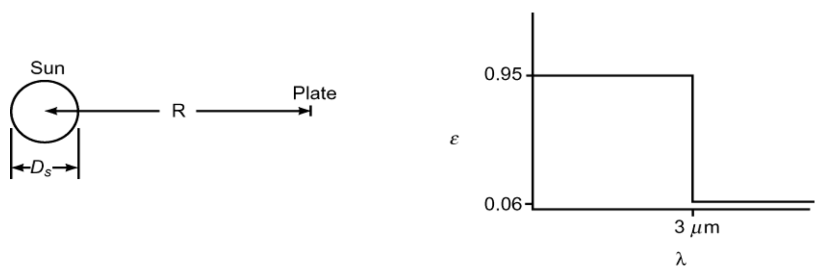
FIND

- The equilibrium temperature of the plate (T_p)

ASSUMPTIONS

- The sun is a black body at T_s = 5550 K
- Sun diameter (D_s) = 1,400,000 km = 1.4×10^9 m

SKETCH



SOLUTION

From Equation (11.31)

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{b\lambda}\, d\lambda \int_0}{\int_0^\infty E_{b\lambda}\, d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{b\lambda}(\lambda, T)\, d\lambda}{\sigma T^4}$$

$$\varepsilon = \varepsilon_{\lambda 1} \frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} + \varepsilon_{\lambda 2} \frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4}$$

For the sun $\lambda_1 T = (3 \times 10^{-6}\text{ m})(5550\text{ K}) = 16.7 \times 10^{-3}$ m K

From Table 11.1

$$\frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} = 0.9764$$

$$\frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4} = 1 - 0.9764 = 0.0236$$

The absorptance of the plate is given by Equation (11.33)

$$\alpha_p = \frac{\int_0^\infty \sigma_\lambda G_\lambda\, d\lambda}{\int_0^\infty G_b\, d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda E_{b\lambda}\, d\lambda}{\sigma T^4} \quad (\text{Since } \alpha_\lambda = \varepsilon_\lambda)$$

$$\alpha_p = \varepsilon_{\lambda 1} \frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} + \varepsilon_{\lambda 2} \frac{E_b(\lambda_1 T \to \infty)}{\sigma T^4} = 0.95\,(0.9764) + (0.06)\,(0.0236) = 0.9290$$

Let q_p = the flux incident on the plate
Energy leaving the sun surface = energy crossing sphere of radius R

$$\sigma T_s^4 A_S = q_p A_R$$
$$\sigma T_s^4 \pi D_s^2 = q_p \, 4 \pi R^2$$
$$q_p - \sigma T_s^4 \frac{D_s^2}{4R^2}$$

Performing the energy balance on the plate
Energy absorbed from sun = energy emitted form both sides of the plate

$$\alpha q_p = 2 \sigma T_P^4 \varepsilon_P$$
$$\alpha \sigma T_s^4 \frac{D_s^2}{4R^2} = 2 \sigma T_P^4 \left(\varepsilon_{\lambda 1} \frac{E_b(0 \to \lambda_1 T_p)}{\sigma T_P^4} + \varepsilon_{\lambda 2} \frac{E_b(\lambda_1 T_P \to \infty)}{\sigma T_P^4} \right)$$
$$T_P = \left[\frac{\sigma T_s^4 D_s^2}{8R\left(\varepsilon_{\lambda 1} \frac{E_b(0 \to \lambda_1 T_p)}{\sigma T_P^4} + \varepsilon_{\lambda 2} \frac{E_b(\lambda_1 T_P \to \infty)}{\sigma T_P^4} \right)} \right]^{\frac{1}{4}}$$
$$\frac{\alpha T_S^4}{8} = \frac{D_s^2}{R^2} \quad \frac{0.9290 (5550\,\text{K})^4}{8} \left(\frac{1.4 \times 10^9}{1.5 \times 10^{11}} \right)^2 = 9.60 \times 10^9 \text{ K}^4$$
$$\therefore T_P = \left[\frac{9.60 \times 10^9 \,\text{K}^4}{(0.95)\left(\frac{E_b(0 \to \lambda_1 T_p)}{\sigma T_P^4} \right) + (0.06) \frac{E_b(\lambda_1 T_P \to \infty)}{\sigma T_P^4} } \right]^{\frac{1}{4}}$$

This can be solved iteratively. For a first guess, let T_P = 500 K.

$$\lambda_1 T_p = (3 \times 10^{-6} \text{ m}) (500 \text{ K}) = 1.5 \times 10^{-3} \text{ m K}$$
$$\frac{E_b(0 \to \lambda_1 T_P)}{\sigma T_P^4} = 0.01376$$
$$\frac{E_b(\lambda_1 T_P \to \infty)}{\sigma T_P^4} = 1 - 0.01376 = 0.98624$$
$$T_p = \left[\frac{9.60 \times 10^9 K^4}{0.95(0.01376) + 0.06(0.98624)} \right]^{\frac{1}{4}} = 604 \text{ K}$$

Repeating this procedure

T_p (K)	λT_p (m K)	$\frac{E_b(0 \to \lambda_1 T_P)}{\sigma T_P^4}$	Tp (K)
604	0.00181	0.0407	562
562	0.00169	0.0286	579
579	0.00173	0.0325	573
573	0.00172	0.0315	575

$$T_p = 575 \text{ K} = 302°\text{C}$$

COMMENTS

Note that the total absorptance (α) = 0.929 but the total emittance (ε) = 0.088.

PROBLEM 11.8

By substituting Equation 11.1 for $E_{b\lambda}$ (*T*) in Equation 11.4 and performing the integration over the entire spectrum, derive a relationship between σ and the constants C_1 and C_2 in Equation 11.1.

GIVEN

- Equation 11.1

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}$$

where $C_1 = 3.7415 \times 10^{-16}$ W m^2 $\quad$ $C_2 = 1.4388 \times 10^{-2}$ m K

- Equation 11.4

$$\int_0^\infty E_{b\lambda}\, d\lambda = \sigma T^4 = E_b$$

FIND

- A relationship between σ and C_1 and C_2

SOLUTION

$$\int_0^\infty E_{b\lambda}\, d\lambda = \int_0^\infty \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)}\, d\lambda = \sigma T^4$$

This can be solved using the transformation of variables.

$$\text{Let } \zeta = \frac{C_2}{\lambda T} \quad \text{and} \quad d\lambda = \left(-\frac{T\lambda^2}{C_2}\right) d\left(\frac{C_2}{\lambda T}\right) = \left(\frac{T\lambda}{C_2}\right)^2 - \frac{C_2}{\lambda}\ d\left(\frac{C_2}{\lambda T}\right)$$

$$\sigma T^4 = \int_0^\infty \frac{C_1}{\lambda^5\left(\frac{e_2}{\lambda T} - 1\right)}\, d\lambda = \int_0^\infty \left(\frac{C_2}{\lambda T}\right)^5 \left(\frac{T}{C_2}\right)^5 \frac{C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)} \left(\frac{T\lambda}{C_2}\right)^2 - \frac{C_2}{\lambda}\ \left(\frac{C_2}{\lambda T}\right)$$

$$\sigma T^4 = -\frac{C_1}{C_2^4}\ T^4 \int_0^\infty \frac{\zeta^3}{e^\zeta - 1}\, d\zeta$$

From a table of integrals

$$\int_0^\infty \frac{\zeta^3}{e^\zeta - 1}\, d\zeta = -\frac{\pi^4}{15}$$

$$\therefore\ \sigma T^4 = \frac{C_1 \pi^4}{15 C_2^4}\ T^4$$

$$\sigma = \frac{C_1 \pi^4}{15 C_2^4} = \frac{(3.7415 \times 10^{-16}\,\text{Wm}^2)\pi^4}{15(1.4388 \times 10^{-2}\,\text{mK})^4} = 5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)$$

PROBLEM 11.9

Determine the ratio of the total hemispherical emissivity to the normal emissivity for a nondiffuse surface if the intensity of emission varies as the cosine of the angle measured from the normal.

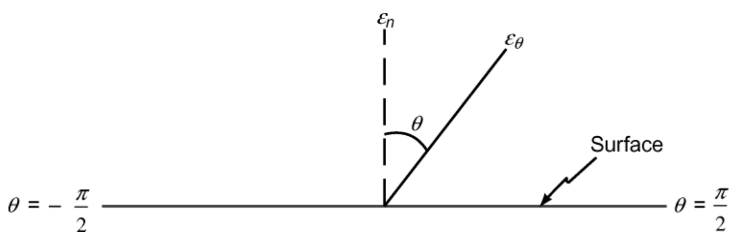
GIVEN

- A nondiffuse surface
- Intensity of emission varies as the cosine of the angle measured from normal

FIND

- The ratio of the total hemispherical emissivity (ε) to the normal emissivity (ε_n)

SKETCH



SOLUTION

From Equation (11.35), the intensity of emission varies with $\cos\theta$, then the emissivity must be proportional to $\cos\theta$

$$\varepsilon_\theta = \varepsilon_n \cos\theta$$

The average hemispherical value is

$$\varepsilon = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \varepsilon_n \cos\theta\, d\theta = \frac{\varepsilon_n}{\pi}\sin\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{2}{\pi}\,\varepsilon_n \sin\frac{\pi}{2} = \frac{2}{\pi}\,\varepsilon_n$$

The ratio $\varepsilon/\varepsilon_n$ is given by

$$\frac{\varepsilon}{\varepsilon_n} = \frac{2}{\pi} = 0.637$$

PROBLEM 11.10

The glass cover for a flat plate solar collector has a spectral transmissivity that may be approximated as shown in the figure. Determine the total effective transmissivity of the glass cover to solar radiation assuming the sun emits that of a blackbody at 5800 K.

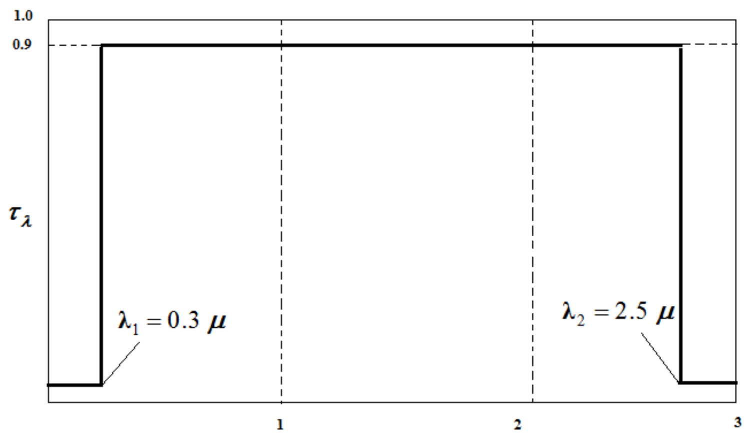
GIVEN

- Glass cover for flat plate solar collector with transmissivity as shown.
- Solar radiation at 5800 K assuming it as blackbody.

FIND

- Total effective transmissivity of glass cover to solar radiation.

SKETCH



SOLUTION

From Equation (11.10) we have,
Total transmissivity

$$\tau = \int_{0.3}^{2.5} \tau_1 E_{b\lambda}(T)d\lambda / E_b = \tau_1 \left(\int_0^{0.3} E_{b\lambda}(T)d\lambda / \left(\sigma T^4\right) - \int_0^{2.5} E_{b\lambda}(T)d\lambda / \left(\sigma T^4\right) \right)$$

$$= \tau_1 \left(E(0-0.3*5600)/\left(\sigma T^4\right) - E(0-0.3*5600)/\left(\sigma T^4\right) \right)$$

For $\lambda_1 T = 0.3*10^{-6}*5600 = 1.68*10^{-3}$ mK

$E(0-0.3*5600)/\left(\sigma T^4\right) = 0.0276$

For $\lambda_2 T = 2.5*10^{-6}*5600 = 14*10^{-3}$ mK

$E(0-2.5*5600)/\left(\sigma T^4\right) = 0.96297$

Thus,

Total transmissivity is $\tau = \tau_1 \left(E(0-0.3*5600)/\left(\sigma T^4\right) - E(0-0.3*5600)/\left(\sigma T^4\right) \right)$

=0.9*(0.96297-0.0276)
= 0.842

PROBLEM 11.11

Estimate the temperature of the Earth if there were no atmosphere to trap solar radiation. The diameter of the Earth is approximately 1.27 $*10^7$ m, and the distance between the sun and the Earth is approximately 1.53$*10^{11}$ m +/- 1.7%. For your calculations, assume that the sun is a point source and that the Earth moves in a circular motion around it. Furthermore, assume that the sun radiates as an equivalent blackbody at a temperature of 5760 K.

PROBLEM 11.12

Repeat Problem 11.11 for the temperature of Mars. In this case, the student should carry out a simple library search to estimate the diameter of Mars and its approximate distance from the sun.

PROBLEM 11.13

The diameter of the sun is $1.39*10^9$ m. Estimate the percentage of the total radiation emitted by the sun, which approximates a blackbody at 5760 K, that is actually intercepted by the Earth. Of the total radiation falling on the Earth, about 70% falls on the ocean. Estimate the amount of radiation from the sun that falls on land, and then estimate the ratio of energy currently used worldwide and the amount of terrestrial solar energy that is available. Discuss why all of the energy cannot be harnessed.

GIVEN

- Diameter of sun (D_s)= $1.39*10^9$ m
- Sun is approximate black body at temperature (T_s) =5760 K
- 70% of radiation falls on the ocean.
- Diameter of the earth (D_e)= $1.27*10^7$ m
- Distance between sun and earth (R_{se})=$1.53*10^{11}$ m

FIND

- Amount of radiation from sun that falls on land
- Ratio of energy currently used worldwide and the amount of terrestrial solar energy that is available.
- Discuss why all energy cannot be harnessed.

ASSUMPTIONS

- The sun's radiation is assumed as ideal black body radiation.

SOLUTION

The total radiation from sun is given by

$$q = \sigma A T^4 = 5.67\times10^{-8}\times\pi*\left(1.39\times10^9\right)^2\left(5760\right)^4 \text{ W}$$

$$q = 3.79*10^{26} \text{ W}$$

The amount of radiation reaching the earth is obtained by using energy balance equation

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\pi D_s^2 \sigma T_s^4 = 4\pi\left[R_{S,e} - D_e/2\right]^2 G_s$$

$$\pi\left(1.39\times10^9\right)^2 5.67\times10^{-8}\times\left(5760\right)^4 = 4\pi\left[1.53\times10^{11} - 6.35\times10^6\right]^2 G_s$$

$$G_s = 1288 \text{ W/m}^2$$

Total radiation intercepted by earth is the circle side facing the sun. Thus

$$q_e = G_s*\frac{\pi D_e^2}{4} = 1288*\frac{\pi\left(1.27\times10^7\right)^2}{4} = 1.63\times10^{17} \text{ W}$$

Thus the percentage of solar radiation intercepted by earth is

$$\frac{3.26\times10^{17}}{3.79\times10^{26}}*100\% = 8.6\times10^{-8} \text{ \%}$$

The amount of energy that falls on land

$q_l = 0.3 \times q_e = 4.89 \times 10^{16}$ W =48900 TW (TeraWatt)

Total energy currently used worldwide(T_w) 12.3 TW

The ratio of energy totally used and solar energy that is available

$= \frac{12.3}{48900} \times 100\%$ = 0.025 %

The reasons why all the energy cannot be harnessed is

- Solar energy spreads all over the surface of earth and it is not possible to harness energy from surfaces everywhere.
- The efficiency of energy that can be converted to solar energy is low.
- Land, gases and atmosphere absorb the solar energy that is incident on earth and cannot be recovered.
- Part of energy incident on earth is reflected which cannot be harnessed.

Problem 11.14

As a result of the atmosphere surrounding the Earth trapping some of the incoming solar radiation, the average temperature of the Earth is approximately 15°C. Estimate the amount of radiation that is trapped by the atmosphere, including CO2 and methane, which provides the shield to maintain the temperature at a level that sustains living organisms. Comment on the current concern about global warming as a result of an increasing percentage of CO_2 and methane in the atmosphere surrounding the Earth.

GIVEN

The average temperature of the earth is 15^0C.

FIND

- Amount of radiation trapped by atmosphere
- Comment on concern about global warming as a result of increasing percentage of CO_2.

SOLUTION

Solar radiation flux enters the atmosphere at about 235 W/m^2. Out of this about 168 W/m^2 reaches the Earth's surface which is warmed to an average 15°C, while about 67 W/m^2 is absorbed by the atmosphere. Steady state is achieved by emission from atmospheric radiation into space at a rate of about 195 W/m^2 plus radiation directly from the Earth's surface at about 40 W/m^2.

If the greenhouse gases like CO_2 and methane increase in concentration at atmosphere, higher solar radiation flux will be trapped by the gases as a result the steady state atmospheric radiation increases and the temperature of earth increases.

In recent times, due to human activities there is increase in concentration of greenhouse gases like CO_2 and methane. This has led to increase in temperature of earth. Polar ice caps are melting gradually which has led to increase in sea level. This phenomenon has threatened lives on earth. If the trend keeps on increasing the earth will be inhabitable to living beings in near future. Thus there is requirement to curb global warming.

PROBLEM 11.15

A hypothetical PV solar cell in space can utilize solar radiation between 0.8 and 1.1 μm in wavelength. Estimate the maximum theoretical efficiency for this solar cell facing the sun using the ideal blackbody curve of the sun as the source. Supposing that all the radiation outside the spectral range utilized by the solar cell for generating electricity is dissipated into heat, estimate the rate at which a module of solar cells of 1.0 m² area would have to be cooled to maintain the temperature of the module below 90°F.

GIVEN

- PV solar cell utilizing radiation between 0.8 and 1.1 μm.
- Surface area of solar cells (A)= 1.0 m^2

FIND

- Maximum theoretical efficiency of solar cell facing the sun using ideal blackbody curve of the sun.
- Rate at which module of solar cells would have to be cooled to maintain temperature of module below 90^0F or 32^0C.

ASSUMPTIONS

- The sun's radiation can be assumed as ideal black body curve.
- The variation in irradiation is linear for the interval of 0.8 to 1.1 μm.

SOLUTION

From Figure 11.25 for Solar spectral irradiance at sea level for direct beam.

The average spectral irradiation for range of 0.8 to 1.1 μm is 0.6 W/(m^2 nm).

Thus, total radiation that can be utilized by PV solar cell

G= 0.6 W/(m^2 nm) *(1100-800) nm= 180 W/m^2

Total solar radiation incident S= 1368 W/m^2

Thus maximum theoretical solar efficiency (η_{th})= $\frac{180}{1368}$ *100% = 13%

The radiation which is converted to heat is Q_{heat}= 1368-180=1188 W/m^2

Total power dissipated as heat from the Solar cell is P= 1188 W/m^2*1.0 m^2 = 1188 W

PROBLEM 11.16

Repeat Problem 11.15 for a PV module in Arizona on a sunny day at noon in an environment at 100°F. State your assumptions.

PROBLEM 11.17

Estimate the rate at which heat needs to be supplied to an astronaut repairing the Hubble telescope in space. Assume that the emissivity of the spacesuit is 0.5. Describe your model with a simple sketch and clearly state your assumptions.

GIVEN

- An astronaut repairing the Hubble telescope in space.
- Emissivity of spacesuit (ε) = 0.5

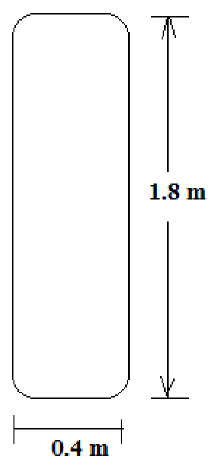
FIND

- Rate at which heat needs to be supplied to the astronaut.

ASSUMPTIONS

- The spacesuit is idealized as a cylindrical object of height 1.8 m and diameter of 0.4 m.
- Temperature of 37^0C is maintained inside the suit.

SKETCH



SOLUTION

We have,

Area of the suit (A)= $2\pi r\left(r+h\right)=2\pi * 0.4\left(0.4+1.8\right)$ = 5.53 m^2

With body temperature of T= 37^0C (310 K) and surrounding temperature of T_s= 0 K,

Heat radiated per unit area is given by

$$q'' = \varepsilon\sigma(T^4 - T_s^4) = 0.5 * 5.67 * 10^{-8} * 310^4 = 261 \text{ W/m}^2$$

Thus total heat radiated from body is

Q_{total}= 261 W/m^2*5.53 m^2 = 1447 W

PROBLEM 11.18

Derive an expression for the geometric shape factor F_{1-2} for a rectangular surface A_1, 1 by 20 m placed parallel to and centered 5 m above a 20-m-square surface A_2.

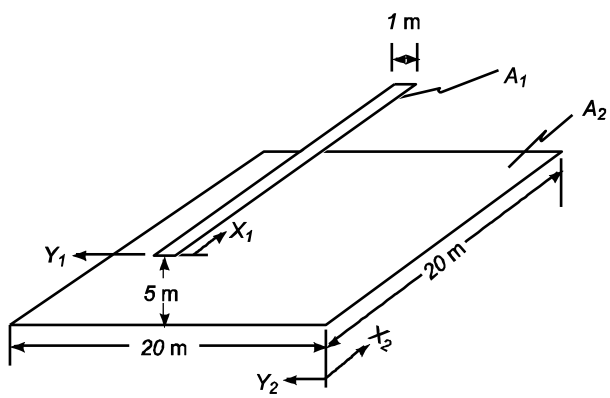
GIVEN

- Rectangular surface A_1 and square surface A_2
- A_1 is parallel to and centered 5 m above A_2
- Dimensions of A_1 = 1 m × 20 m
- A_2 is 20 m square

FIND

- The shape factor F_{12}

SKETCH



SOLUTION

From Equation (11.53)

$$A_1 F_{12} = \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2}\, dA_2\, dA_1$$

Given a differential element dA_1, at $x,y,z = (x_1, y_1, -5\text{ m})$

Given a differential element dA_2, at $x,y,z = (x_2, y_2\ 0)$

The distance between elements (r) is: $r^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (5\text{ m} - 0)^2$

Since the surfaces are parallel: $\cos\theta_1 = \cos\theta_2 = (5\text{ m})/r$

The double integral can be expanded into the following quadruple integral:

$$A_1 F_{12} = \int_{x_1=0}^{20\text{ m}}\int_{y_1=0}^{1\text{m}}\int_{x_2=0}^{20\text{ m}}\int_{y_2=0}^{20\text{ m}} \frac{\left(\frac{5\text{m}}{r}\right)\left(\frac{5\text{m}}{r}\right)}{\pi r^2}\, dy_2\, dx_2\, dy_1\, dx_1$$

$$A_1 F_{12} = \frac{25}{\pi}\int_{x_1=0}^{20\text{ m}}\int_{y_1=0}^{1\text{m}}\int_{x_2=0}^{20\text{ m}}\int_{y_2=0}^{20\text{ m}} \frac{dy_2\ dx_2\ dy_1\ dx_1}{[(x_1 - x_2)^2 + (y_1 - y_2)^2 + 25\text{m}^2]^2}$$

This can be simplified somewhat by trigonometric substitutions, however, it is fairly simple to solve numerically as it is. Let all the dx terms equal Δx, where $20/\Delta x$ and $1/\Delta x$ are both integers. The integral can then be approximated by

$$\frac{25\,\Delta x^4}{\pi}\sum_{ix_1=1}^{20/\Delta x}\sum_{iy_1=1}^{1/\Delta x}\sum_{ix_2=1}^{20/\Delta x}\sum_{iy_2=1}^{20/\Delta x} [(x_1 - x_2)^2 + (y_1 - y_2)^2 + 25\text{ m}^2]^{-2}$$

where $x_1 = (ix_1)(\Delta x) - \Delta x/2 \quad x_2 = (ix_2)(\Delta x) - \Delta x/2$

$y_1 = (iy_1)(\Delta x) - \Delta x/2 \quad y_2 = (iy_2)(\Delta x) - \Delta x/2$

This is implemented in the Pascal program shown below

```
var     dx,dx2,sum,r4,.real;
        ix1,ix2,iy1,iy2,nx1,nx2,ny1,ny2.integer;
        x1,x2,y1,y2:real;
begin
        dx = 1.00;
        dx2 = dx/2;
        nx1 = trunc(20/dx);
        nx2 = trunc(20/dx);
        ny1 = trunc(1/dx);
        ny2 = trunc(20/dx);
        sum = 0.00;
for ix1 = 1 to nx1 do
        begin
          x1 = ix1*dx-dx2;
          write1n(x 1:8.3);
          for iy1 = 1 to ny 1 do
          begin
            y1 = iy1*dx-dx2;
            for ix2 = 1 to nx2 do
            begin
              x2 = ix2*dx-dx2;
              for iy2 = 1 to ny2 do
              begin
                y2 = iy2*dx-dx2;
                r4 = (x1-x2)*(x1-x2) + (y1 - y2)* (y1 - y2) + 25;
                r4 = 1/r4/r4;
                sum = sum + r4;
            end;
          end;
        end;
     end;
     sum = sum* 25/3.14159265/20*dx*dx*dx*dx;
writen(*F12 = Sum:8.4);
end.
```

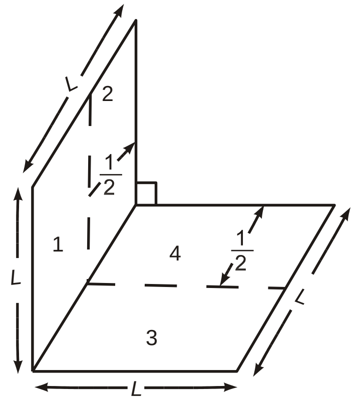
Running this program yields the following result

$$F_{12} = 0.427$$

Comment: If the geometry is approximated as two-dimensional so that the view factor can be calculated using the crossed string method, we get $F_{12} = 0.97$, a significant error.

PROBLEM 11.19

Determine the shape factor F_{1-4} for the geometrical configuration shown below.



GIVEN

- Geometrical configurations shown above

FIND

- The shape factor F_{1-4}

SOLUTION

Let $A_5 = A_1 + A_2$ and $A_6 = A_3 + A_4$

Applying Equation (11.55)

$$A_{12}\, F_{12-34} = A_{12}\, F_{12-3} + A_{12}\, F_{12-4} = A_3\, F_{3-12} + A_4\, F_{4-12}$$

$$A_3\, F_{3-12} = A_3\, F_{3-1} + A_3\, F_{3-2}$$

$$A_4\, F_{4-12} = A_4\, F_{4-1} + A_4\, F_{4-2}$$

Combining these equations $A_{12}\, F_{12-34} = A_3\, F_{3-1} + A_3\, F_{3-2} + A_4\, F_{4-1} + A_4\, F_{4-2}$

By symmetry $F_{1-4} = F_{2-3} = F_{4-1} = F_{3-2}$ and $F_{3-1} = F_{1-3} = F_{2-4} = F_{4-2}$

Also $A_1 = A_2 = A_3 = A_4$ and $A_{12} = 2A_1$

Therefore $A_{12}\, F_{12-34} = 2A_1\,(F_{1-3} + F_{1-4})$

Solving for F_{1-4} $F_{1-4} = F_{12-34} - F_{1-3}$

The shape factors F_{1-3} and F_{12-34} can be determined from Figure 11.31.

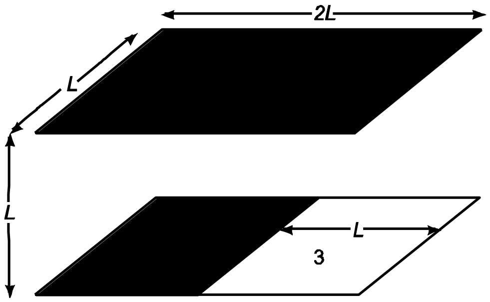
For F_{1-3} $Y = Z = \dfrac{L}{\left(\dfrac{L}{2}\right)} = 2 \quad \rightarrow \quad F_{1-3} = 0.15$

For F_{12-34} $Y = Z = \dfrac{L}{L} = 1 \quad \rightarrow \quad F_{12-34} = 0.20$

$$F_{1-4} = 0.20 - 0.15 = 0.05$$

PROBLEM 11.20

Determine this shape factor F_{1-2} for the geometrical configuration shown



GIVEN

- The geometrical configuration shown above

FIND

- The shape factor F_{1-2}

SOLUTION

From Equation (11.55) $F_{1-23} = F_{1-2} + F_{1-3}$

By symmetry $F_{1-2} = F_{1-3}$

Therefore, $F_{1-23} = 2\,F_{1-2} \rightarrow \; F_{1-2} = 0.5\,F_{1-23}$

The shape factor (F_{1-23} is given in Figure 11.32

$$\frac{y}{D} = \frac{2L}{L} = 2 \text{ and } \frac{x}{D} = \frac{L}{L} = 1$$

From Figure 11.32 $F_{1-23} = 0.30$

$$F_{1-2} = 0.5(0.30) = 0.15$$

PROBLEM 11.21

> **Determine the view factors F_{12} and F_{21} for the following geometries (as shown in the figure): (a) a sphere of diameter, D, inside a cubic box of length L=D, (b) one side of a diagonal partition in a long square duct of side L, and (c) the end and side of a circular tube with a diameter equal to its length.**

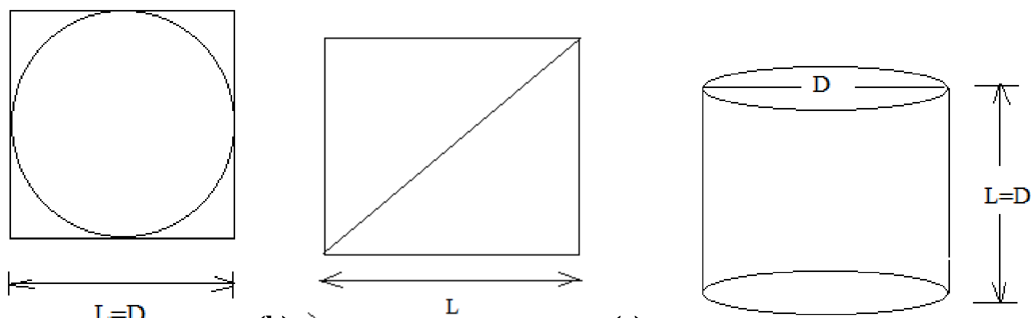
GIVEN

- Geometrical configurations shown above

FIND

- The view factors F_{12} and F_{21}

SKETCH



(a) (b) (c)

SOLUTION

For figure (a)

F_{12}=1, as body (A_1) sphere of diameter D cannot see itself.
Now,

$$A_1F_{12}= A_2F_{21}$$

$$\frac{\pi D^2}{4} * F_{12} = D^2 * F_{21}$$

$$F_{21} = \frac{\pi}{4} = 0.785$$

For figure (b)

F_{12}=1, as body (A_1) diagonal of base L cannot see itself.
Now,

$$A_1F_{12}= A_2F_{21}$$

$$\frac{L^2}{2} * F_{12} = L^2 * F_{21}$$

$$F_{21} = \frac{1}{2} = 0.0.5$$

For figure (c)

F_{12}=1, as body (A_1) circle of base L cannot see itself.
Now,

$$A_1F_{12}= A_2F_{21}$$

$$\frac{\pi L^2}{2} * F_{12} = \pi L^2 * F_{21}$$

$$F_{21} = \frac{1}{2} \mathrm{F}_{12} = 0.5$$

PROBLEM 11.22

A circular ice skating rink, 20 m in diameter, is enclosed by a large hemispherical dome of diameter 30 m. Assuming that the ice is 2 cm thick, estimate the time it takes for the ice to melt if the refrigeration system of the rink fails. Make this calculation first with both the dome and the ice behaving as black bodies at respective temperatures of 20°C and 0°C. Then repeat with the water-ice having an emissivity of 0.3 and losing heat by natural convection to the air in the dome with a convection heat transfer coefficient of 15 W/(m^2 K).

GIVEN

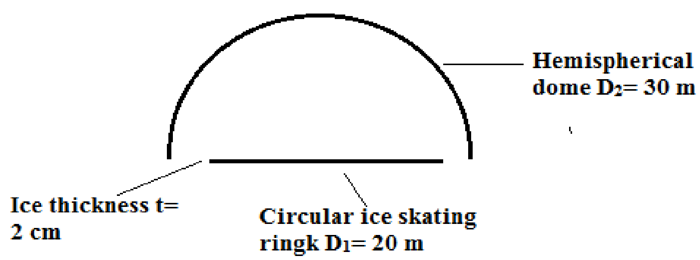
- Circular skating rink enclosed by hemispherical dome
- Diameter of the rink (D_1) = 20 m
- Diameter of hemispherical dome (D_2)= 30 m
- Temperature of dome and surrounding air (T_2)= 20^{0}C
- Temperature of ice (T_1) = 0^{0}C

FIND

Time it takes for ice to melt if the refrigeration system of rink fails considering
(a) both dome and ice behaving as black bodies.
(b) ice having emissivity of 0.3 and loosing heat by natural convection to air in dome with

$$\bar{h}_c = 15 \text{ W/(m}^2 \text{ K)}$$

SKETCH



PROPERTIES AND CONSTANTS

Latent heat of fusion for ice (L_f)= 334 KJ/kg
Stefan-Boltzman's constant (σ)=5.67*10^{-8} W/(m^2 K^4)

SOLUTION

(a) F_{12}= 1 for circular ice rink.

$$A_1 = \frac{\pi D^2}{4} = \frac{\pi * 20^2}{4} = 314.2 \text{ m}^2$$

Net heat transfer from circular rink to dome is given by

$$q_{12} = A_1 F_1 \sigma \left(T_1^4 - T_2^4\right) = 314.2 * 5.67 * 10^{-8} * \left(273^4 - 293^4\right) \text{ W} = -32343 \text{ W} = -32.343 \text{ kW}$$

Thus, heat loss if refrigeration system fails is 32.343 kW.

Mass of ice in circular rink is

m = A_1*t*ρ = 314.2*0.02*917 kg/m^3 = 6257 kg

Thus total energy required to melt the ice is

Q= m*L_f = 334 kJ/kg * 6257 kg = 2090041 kJ

Thus total time required to melt the ice is

$$t = Q / q_{12} = 64621 \text{ seconds} = 18 \text{ hrs.}$$

Thus the time required to completely melt all ice if refrigeration system fails is 18 hrs.

(b) Now, with emissivity of 0.3

Net heat transfer form circular rink to dome is given by.

$$q_{12} = A_1 F_1 \varepsilon\sigma\left(T_1^4 - T_2^4\right) = 314.2*0.3*5.67*10^{-8}*\left(273^4 - 293^4\right) \text{ W} = -9703 \text{ W} = -9.703 \text{ kW}$$

Heat loss due to convection is given by $q_c = \bar{h}_c A_1 (T_1 - T_2) = 15*314.2*(0-20)$ W =- 94260 W

= -94.26 kW

Thus total heat loss rate to when the refrigeration system fails q= q_{12}+q_c = -103.96 kW

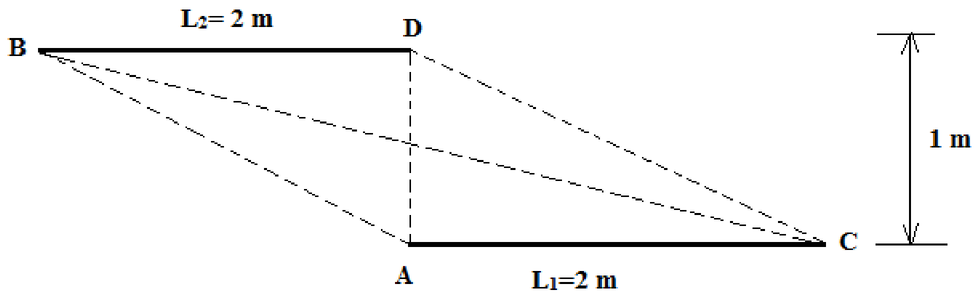
Thus total time required to melt the ice is

$$t = Q / q_{12} = 20{,}104 \text{ seconds} = 5.6 \text{ hrs.}$$

Thus the time required to completely melt all ice if refrigeration system fails is 5.6 hrs.

PROBLEM 11.23

Two parallel plates of infinite extent perpendicular to the page are situated relative to each other as shown in the figure. The upper plate is at 300°C and the lower plate is at 100°C. Determine the net rate of radiation heat transfer from the upper and lower plate assuming the environment is at a temperature below 100°C. Assume that both the lower surface of the upper plate and the upper surface of the lower plate are black.



GIVEN

- Two parallel plates of infinite extent perpendicular to the page.
- Upper plate temperature (T_2) = 300^0C
- Lower plate temperature (T_1)= 100^0C
- Length of the plates $L_1 = L_2 = 2$ m
- Distance between the plates (R) = 1 m

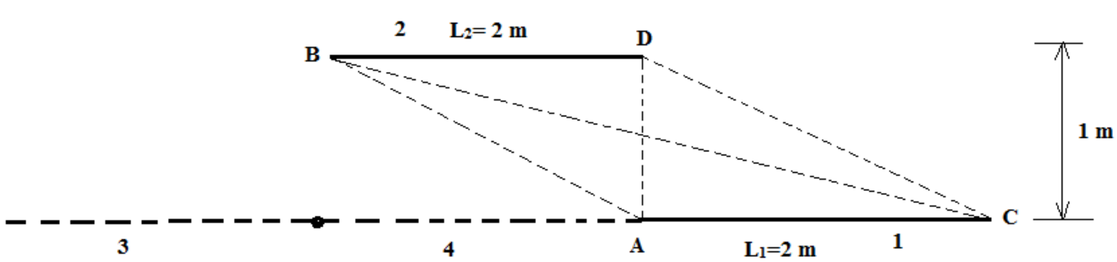
FIND

(a) Net rate of radiation heat transfer from upper and lower plate.

ASSUMPTIONS

- Lower surface of upper plate and upper surface of lower plate are black.

SKETCH



PROPERTIES AND CONSTANTS

The Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

Two imaginary plates 3 and 4 are extended along the plate 1 so that the center of both upper and lower plates align.
By symmetry $F_{21} = F_{23}$

Thus, $F_{21} = 0.5*(F_{2(1,4,3)} - F_{24})$

Now, for plates of infinite extent perpendicular to the plane and of given dimensions

y=6 m D=1 m

y/D= 6

width of plates (x)= ∞,

x/D=∞

From Figure (11.32) Shape factor for directly opposed plane

$F_{2(1,3,4)} = 0.88$

For plates 2 and 4, the shape factor F_{24} is calculated as

y=2 m D=1 m

y/D= 2

width of plates (x)= ∞,

x/D=∞

From Figure (11.32) Shape factor for directly opposed plane

$F_{24} = 0.65$

Thus, the shape factor F_{21} is calculated as

$F_{21} = 0.5*(0.88 - 0.65) = 0.115$

Thus, net radiative heat transfer between upper and lower plates is

$Q/A_2 = F_{21}\sigma(T_2^4 - T_1^4) = 0.115*5.67*10^{-8}*(573^4 - 373^4)$ W/m^2= 576.7 W/m^2

PROBLEM 11.24

Using basic shape-factor definitions, estimate the equilibrium temperature of the planet Mars which has a diameter of 6600 km and revolves around the sun at a distance of 225 × 10^6 km. The diameter of the sun is 1,384,000 km. Assume that both the planet Mars and the sun act as blackbodies with the sun having an equivalent blackbody temperature of 5600 K. Then repeat your calculations assuming that the albedo of Mars (the fraction of the incoming radiation returned to space) is 0.15.

GIVEN

- The planet Mars revolving around the sun
- Diameter of Mars (D_m) = 6600 km
- Distance from the sun (R_{ms}) = 225 × 10^6 km
- Diameter of the sun (D_s) =1,384,000 km

FIND

(a) The equilibrium temperature of Mars (T_m)
(b) T_m assuming that the albedo of Mars = 0.15

ASSUMPTIONS

- Both Mars and the sun act as blackbodies
- The sun has an equivalent blackbody temperature (T_s) of 5600 K

SKETCH

PROPERTIES AND CONSTANTS

The Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The energy from the sun that is incident on Mars can be calculated by integrating Equation (11.50)

$$q_{1-2} = E_{b1} \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2}\, dA_2\, dA_1$$

where θ_1 and θ_2 are the angles shown in Figure 11.28. For this case, since $D_m << R_{ms}$ and $D_s << R_{ms}$, the following approximations can be used cos θ_1 and cos θ_2 = 1, $r = R_{ms}$.

$$dA_1 = \frac{\pi}{4} D_s^2 \quad dA_2 = \frac{\pi}{4} Dm^2$$

Also from Equation (11.3) $E_{b1} = \sigma T_s^4$

$$\therefore\ q_{1-2} = \sigma T_s^4 \left(\frac{D_s D_m}{R_{ms}}\right)^2 \frac{\pi}{16}$$

$$q_{1-2} = \left(5.67\times10^{-8} W/(\mathrm{m}^2\ \mathrm{K}^4)\right)\ (5600\ \mathrm{K})^4 \left[\frac{(1.384*10^9\ \mathrm{m})(6.6*10^6\ \mathrm{m})}{225\times10^9\,\mathrm{m}}\right]^2 \frac{\pi}{16}$$

$$q_{1-2} = 1.805 \times 10^{16}\ \mathrm{W}$$

Case (a)

If Mars behaves as a blackbody, it will absorb all the sun's energy incident on it. For equilibrium, the energy radiated by Mars must equal the incident solar energy

$$q_m = A_2 E_{b2} = \pi D_m^2 \sigma T_m^4 = q_{1-2}$$

Solving for the temperature of Mars

$$T_m = \left(\frac{q_{1-2}}{\pi D_m^2 \sigma}\right)^{0.25} = \left[\frac{\left(1.805\times 10^{16} W\right)}{\pi (6.6*10^6)^2 \left(5.67\times 10^{-8} W/(\mathrm{m}^2 K^4)\right)}\right]^{0.25} = 220 \text{ K}$$

Case (b)

For an albedo of 0.15, Mars absorbs only 85% of the incident solar radiation, therefore

$$T_m = \left(\frac{0.85 q_{1-2}}{\pi D_m^2 \sigma}\right)^{0.25} = 220 \text{ K } (0.85)^{0.25} = 211 \text{ K}$$

PROBLEM 11.25

Two coaxial parallel plate discs are situated relative to each other as shown in the sketch. The diameter of the upper plate is 20 cm, and that of the lower plate is 40 cm. The lower plate is maintained at a uniform temperature of 500 K and the upper temperature is maintained at 456 K. To maintain these relative temperatures, a heater has to be installed in the upper plate. Assuming an environmental temperature of the surroundings at 300 K, determine the power necessary of the heater in the upper plate to maintain the two constant plate temperatures under steady state conditions.

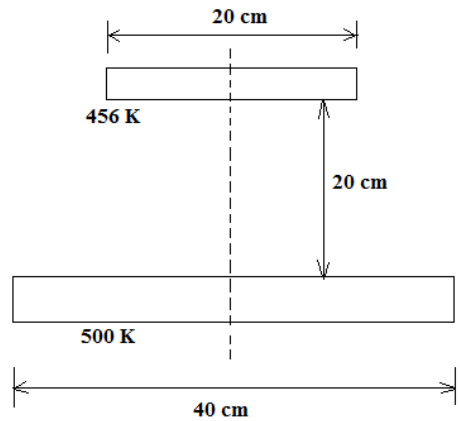
GIVEN

- Two coaxial parallel plate discs at distance of L = 20 cm=0.2 m
- Upper plate diameter (D_1) = 20 cm = 0.2 m
- Lower plate diameter (D_2) = 40 cm = 0.4 m
- Upper plate temperature (T_1) = 456 K
- Lower plate temperature (T_2) = 500 K
- Surrounding temperature (T_{surr}) = 300 K

FIND

(a) Power necessary of the heater in upper plate to maintain the constant plate temperatures under steady state conditions.

SKETCH



PROPERTIES AND CONSTANTS

The Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

Net heat radiation leaving the upper plate is given by

$$q = A_1 F_{12} \sigma \left(T_1^4 - T_2^4\right) + A_1 F_{13} \sigma \left(T_1^4 - T_{surr}^4\right)$$

From Table 11.4, #7 for Two parallel discs of unequal radius a and b separated apart by distance L

$$F_{12} = \frac{1}{2a^2}\left[L^2 + a^2 + b^2 - \sqrt{\left(L^2 + a^2 + b^2\right)^2 - 4a^2 b^2}\right]$$

where a=0.1 m, b= 0.2 m, L= 0.2 m

$$F_{12} = \frac{1}{2*0.1^2}\left[0.2^2 + 0.1^2 + 0.2^2 - \sqrt{\left(0.2^2 + 0.1^2 + 0.2^2\right)^2 - 4*0.1^2*0.2^2}\right]$$

$$F_{12} = \frac{1}{2*0.1^2}\left[0.09 - \sqrt{\left(0.09\right)^2 - 0.0016}\right]$$

$$F_{12} = \frac{1}{2*0.1^2}\left[0.09 - 0.0806\right]$$

$$F_{12} = 0.47$$

Now we have

$$F_{12} + F_{13} = 1$$

$$\Rightarrow \quad F_{13} = 0.53$$

Thus, heat flow rate from top plate is

$$q = A_1 F_{12}\sigma\left(T_1^4 - T_2^4\right) + A_1 F_{13}\sigma\left(T_1^4 - T_{surr}^4\right)$$

$$q = \pi*0.1^2*0.47*5.67*10^{-8}*\left(456^4 - 500^4\right) + \pi*0.1^2*0.53*5.67*10^{-8}*\left(456^4 - 300^4\right)\text{ W}$$

$$q = \pi*0.1^2*0.47*5.67*10^{-8}*\left(456^4 - 500^4\right) + \pi*0.1^2*0.53*5.67*10^{-8}*\left(456^4 - 300^4\right)\text{ W}$$

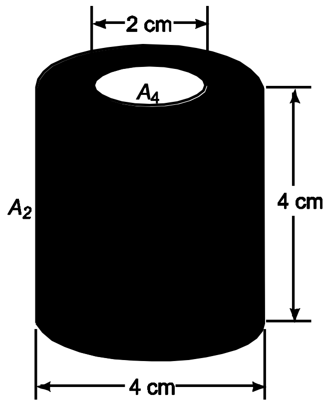
$$q = -16.12 + 33.17\text{ W}$$

$$q = -16.12 + 33.17\text{ W} = 17.05\text{ W}$$

Thus, power necessary of the heater in upper plate to maintain the constant plate temperatures under steady state conditions is 17.05 W

PROBLEM 11.26

A 4-cm-diameter cylindrical enclosure with black surfaces, as shown in the accompanying sketch, has a 2-cm hole in the top cover. Assuming the walls of the enclosure are at the same temperature, determine the percentage of the total radiation emitted from the walls that escapes through the hole in the cover.



GIVEN

- Cylindrical enclosure of black surfaces shown above
- Cylinder diameter (D) = 4 cm = 0.04 m
- Diameter of hole in top (D_h) = 2 cm = 0.02 m

FIND

- The percentage of the total radiation emitted from the wall which will escape through the hole in the cover (F_{e4}).

ASSUMPTIONS

- The walls of the enclosure are at the same temperature

SOLUTION

The total area of the interior of the enclosure (A_e) is

$$A_e = A_1 + A_2 + A_3 = \frac{\pi}{4} D^2 + \pi D L + \frac{\pi}{4} (D^2 - D_h^2)$$

$$A_e = \pi \left[\frac{0.04\,\mathrm{m}}{4}^{2} + (0.04\,\mathrm{m})\,(0.04\mathrm{m}) + \frac{1}{4}[(0.04\,\mathrm{m})^2 - (0.02\,\mathrm{m})^2] \right] = 0.0023\ \pi\,\mathrm{m}^2$$

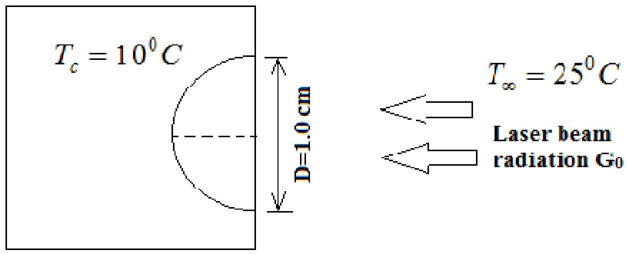
The shape factor between A_4 and the enclosure is unity F_{4e} = 1. From Equation (9.46)

$$A_e\, F_{e4} = A_4\, F_{4e} = A_4$$

$$\therefore\ F_{e4} = \frac{A_4}{A_e} = \frac{\pi D_h^2}{4 A_e} = \frac{\pi (0.02\,\mathrm{m})^2}{4(0.0023\pi\,\mathrm{m}^2)} = 0.044$$

PROBLEM 11.27

A laser beam sends a uniform radiation beam towards the hemispherical opening in a solid housing with dimensions as shown. The surface of the cavity is black and essentially insulated from the surrounding housing material. The hemispherical cavity is a thin-walled metal with a black surface. The temperature of the housing as well as that of the surroundings is 25°C. Determine the radiant flux of the laser beam, G_0 [W/m^2], if a thermocouple located on the hemispherical surface indicates a temperature of 10°C.



GIVEN

- Laser beam sending uniform radiation beam towards the hemispherical opening.
- Temperature of housing and surrounding (T_∞)= 25^0C.
- Temperature difference on thermocouple in hemispherical surface (ΔT_c) = 10^0C.

FIND

- Radiant flux of the laser beam.

ASSUMPTIONS

- Cavity surface is black and perfectly insulated from its mounting material.
- Negligible convection heat transfer from cavity surface.
- Surroundings are large, isothermal.

SOLUTION

The total area of the opening of the hemisphere (A_0) is $\dfrac{\pi D^2}{4}$.

The energy balance of the system gives

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$A_0 G_0 + A_0 G_{surf} - A_0 E_b\left(T_c\right) = 0$$

All the radiation energy entering and leaving the cavity passes through hypothetical surface. So T_c can be treated as black disk.

$$G_{surf} = E_b(T_{surf}) \text{ and } E_b = \sigma T^4$$

Thus, the energy balance becomes

$$G_0 + \sigma(25+273)^4 K^4 - \sigma(25+10+273)^4 K^4 = 0$$

$$G_0 = 5.67*10^{-8}*(308^4 - 298^4) \text{ W/m}^2$$

$$G_0 = 63.1 \text{ W/m}^2$$

PROBLEM 11.28

Show that the temperature of the re-radiating surface T_r in Figure 11.41 is

$$\boldsymbol{T_R} = \left(\frac{A_1 F_{1R} T_1^4 + A_2 F_{2R} T_2^4}{AF_{1R} + A_2 F_{2R}}\right)^{\frac{1}{4}}$$

GIVEN

- Figure 11.41 shown below

FIND

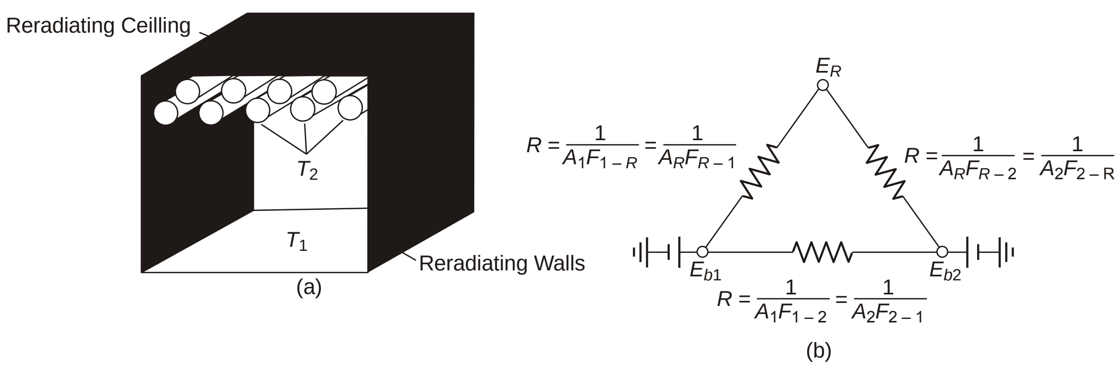
- Show that the temperature of the re-radiating surface T_R, is

$$T_R = \left(\frac{A_1 F_{1R} T_1^4 + A_2 F_{2R} T_2^4}{AF_{1R} + A_2 F_{2R}}\right)^{\frac{1}{4}}$$

ASSUMPTIONS

- Steady state
- The re-radiating surface temperature is uniform

SKETCH



SOLUTION

The following equation can be written from the thermal circuit

$$q_{R-2} = A_2\, F_{2R}\,(E_R - E_{b2}) = \sigma A_2\, F_{2R}\,(T_R^4 - T_2^4)$$

$$q_{r-1} = A_1\, F_{1R}\,(E_R - E_{b1}) = \sigma A_1\, F_{1R}\,(T_R^4 - T_1^4)$$

For steady state, heat flows must sum to zero

$$q_{R-2} + q_{R-1} = 0$$

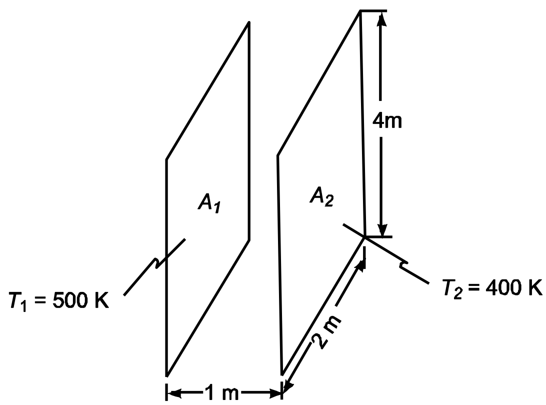
$$\sigma(A_2\, F_{2R}\, T_R^4 - A_2\, F_2^R\, T_2^4 + A_1\, F_{1R}\, T_R^4 - A_1\, F_{1R}\, T_1^4) = 0$$

$$T_R^4\,(A_1\, F_{1R} + A_2\, F_{2R}) = A_1\, F_{1R}\, T_1^4 + A_2\, F_{2R}\, T_2^4$$

$$T_R = \left(\frac{A_1 F_{1R} T_1^4 + A_2 F_{2R} T_2^4}{AF_{1R} + A_2 F_{2R}}\right)^{\frac{1}{4}}$$

PROBLEM 11.29

In the construction of a space platform, two structural members of equal size with surfaces that are considered black are placed relative to each other as shown schematically below. Assuming that the left member attached to the platform is at 500 K, the other is at 400 K and that the surroundings are treated as though black at 0 K, calculate (a) the rate at which the warmer surface must be heated to maintain its temperature, (b) the rate of heat loss from the cooler surface to the surroundings, (c) the net rate of heat loss to the surroundings for both members.



GIVEN

- Two black surfaces as shown above on a space platform

FIND

(a) The rate of heating of the warmer surface (q_1)
(b) Net rate of heat loss to the surroundings (q_s)
(c) The rate of heat loss from the cooler surface to the surroundings (q_{2s})

ASSUMPTIONS

- Steady state
- The surroundings behave as a blackbody enclosure at T_s = 0 K
- The plate also lose heat from their back surface

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The shape factor F can be read off Figure 11.33 line 3

$$\text{Ratio} = (2\text{ m})/(1\text{ m}) = 2 \rightarrow F_{12} \approx 0.51$$

By symmetry, $F_{21} = F_{12}$.
Since neither $A1$ nor $A2$ can see itself $F_{11} = F_{22} = 0$.
The shape factors for any given surface must sum to unity

$$F_{11} + F_{12} + F_{1s} = 1 \quad \rightarrow \quad F_{1s} = 1 - F_{12} = 0.49$$

$$F_{21} + F_{12} + F_{2s} = 1 \quad \rightarrow \quad F_{2s} = 1 - F_{21} = 1 - F_{12} = 0.49$$

(a) The rate of heating of A_1 must equal the net rate of heat transfer from A_1 which is the sum of the net heat transfer rate to A_2 and the heat transfer to the surroundings

$$q_1 = q_{12} + q_{1s} = \sigma A_1 \left[F_{12}\left(T_1^4 - T_2^4\right) + F_{1s}\,T_1^4 + T_1^4\right] = \sigma A_1 \left[\left(F_{12} + F_{1s} + 1\right) T_1^4 - F_{12}\,T_2^4\right]$$

$$q_1 = \sigma A_1 (2T_1^4 - F_{12}\, T_2^4) = \left[5.67 \times 10^{-8}\ \mathrm{W/(m^2K^4)}\right] (2\ \mathrm{m})\ (4\ \mathrm{m})$$

$$[2(500\ \mathrm{K})^4 - 0.51(400\ \mathrm{K})^4] = 5.08 \times 10^4\ \mathrm{W}$$

(b) The rate of heat loss of A_2 to its surroundings is given by Equation (11.47)

$$q_{2s} = A_2\, F_{2s}\, (E_{b2} - E_{bs}) + A_2\, E_{b2} = A_2\, (F_{2s} + 1)\, \sigma T_2^4$$

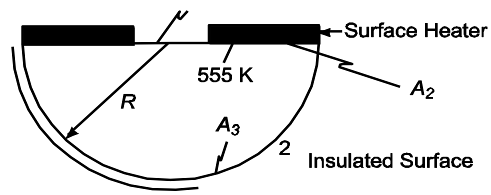
$$q_{2s} = (2\ \mathrm{m})\ (4\ \mathrm{m})\ (1.49) \left[5.67 \times 10^{-8}\ \mathrm{W/(m^2\,K^4)}\right] (400\ \mathrm{K})^4 = 17300\ \mathrm{W}$$

(c) The net rate of heat loss to the surroundings equals the total heat loss from both members. Less q_{12}

$$q_{\text{net}} = 50800 + 17300 - \sigma A_1\, F_{12}\, (T_1^4 - T_2^4) = 59543\ \mathrm{W}$$

PROBLEM 11.30

A radiation source is to be built, as shown in the diagram, for an experimental study of radiation. The base of the hemisphere is to be covered by a circular plate having a centered hole of radius *R*/2. The underside of the plate is to be held at 555 K by heaters embedded in its surface. The heater surface is back. The hemispherical surface is well-insulated on the outside. Assume gray diffuse processes and uniform distribution of radiation. (a) Find the ratio of the radiant intensity at the opening to the intensity of emission at the surface of the heated plate. (b) Find the radiant energy loss through the opening in watts for *R* = 0.3 m. (c) Find the temperature of the hemispherical surface.



GIVEN

- A radiation source as shown above
- Radius of hole = $R/2$
- Temperature of underside of plate (T_2) = 555 K
- Underside of plate is black
- Hemispherical surface is well insulated on the outside

FIND

(a) The ratio of the radiant intensity at the opening to the intensity at the surface of the heated plate (G_1/E_{b2})
(b) The radiant energy loss through the opening (q_1) in watts for R = 0.3 m
(c) The temperature of the hemispherical surface (T_3)

ASSUMPTIONS

- Gray diffuse processes
- Uniform distribution of radiation
- Radiation entering A_1 is negligible, i.e., A_1 as a black body at 0 K
  Heat loss through insulation is negligible

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, The Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The problem consists of radiative exchange between two black surfaces and a gray surface. It can be solved by simplifying Equation (11.69) which applies to gray surfaces

$$A_1 G_1 = J_1 A_1 F_{11} + J_2 A_2 F_{21} + J_3 A_3 F_{31}$$
$$A_2 G_2 = J_1 A_1 F_{12} + J_2 A_2 F_{22} + J_3 A_3 F_{32}$$
$$A_3 G_3 = J_1 A_1 F_{13} + J_2 A_2 F_{23} + J_3 A_3 F_{33}$$

The radiosities, from Equation (11.66) are

$$J_1 = \rho_1 G_1 + \varepsilon_1 E_{b1}$$
$$J_2 = \rho_2 G_2 + \varepsilon_2 E_{b2}$$
$$J_3 = \rho_3 G_3 + e_3 E_{b3}$$

Let the opening the surface 1, the heater surface be surface 2, and the hemisphere be surface 3.
Since A_1 and A_2 cannot see themselves or each other: $F_{11} = F_{22} = F_{12} = F_{21} = 0$
Since A_1 and A_2 are black $\varepsilon_1 = \varepsilon_2 = 1$ and $\rho_1 = \rho_2 = 0$
Neglecting radiation entering A_1 $E_{b1} = 0 \rightarrow$ $J_1 = 0$

In steady state, surface A_3 has no net heat gain or loss $q_3 = 0$. Applying Equation (11.67)

$$0 = A_3 (J_3 - G_3) \rightarrow J_3 = G_3$$

Incorporating these simplifications into the above equations

(1) $A_1 G_1 = G_3 \, 0 A_3 F_{31}$

(2) $A_2 G_2 = G_3 A_3 F_{32}$

(3) $A_3 G_3 = E_{b2} A_2 F_{23} + G_3 A_3 F_{33} \quad \rightarrow \quad A_3 G_3 = (E_{b2} A_2 F_{23})/(1 - F_{33})$

(a) Combining Equations (1) and (3)

$$\frac{G_1}{E_{b2}} = \frac{A_2}{A_1} \frac{F_{23} F_{31}}{1 - F_{33}}$$

The shape factors must sum to unity: $F_{31} + F_{32} + F_{33} = 1$

$$\therefore \frac{G_1}{E_{b2}} = \frac{A_2}{A_1} \frac{F_{23} F_{31}}{F_{31} + F_{32}}$$

From examination of the geometry, it is clear that $F_{13} = 1$ and $F_{23} = 1$

And from Equation 11.46 $\quad A_1 F_{13} = A_3 F_{31} \rightarrow F_{31} = \dfrac{A_1}{A_3}$

$$A_2 F_{23} = A_3 F_{32} \rightarrow F_{32} = \frac{A_1}{A_3}$$

$$\frac{G_1}{E_{b2}} = \frac{A_2}{A_1} \frac{\left(\frac{A_1}{A_3}\right)}{\left(\left(\frac{A_1}{A_3}\right) + \left(\frac{A_2}{A_3}\right)\right)} = \frac{A_2}{A_1 + A_2} = \frac{\pi\left(R^2 - \left(\frac{R}{2}\right)^2\right)}{\pi R^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

(b) The radiation energy loss at the opening is given by the irradiance of surface 1

$$q_1 = G_1 A_1 = \left(\frac{G_1}{E_{b2}}\right) E_{b2} A_1 = \left(\frac{G_1}{E_{b2}}\right) \sigma T_2^4 \frac{\pi}{4} R^2$$

$$q_1 = \left(\frac{3}{4}\right) 5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4) \; (555 \text{ K})^4 \frac{\pi}{4} (0.3\text{m})^2 = 285 \text{ W}$$

(c) From Equation (1)

$$G_3 = \frac{G_1 A_1}{F_{31} A_3} = \left(\frac{G_1}{E_{b2}}\right) E_{b2} \frac{A_1}{A_3} \left(\frac{A_3}{A_1}\right) = \frac{3}{4} \sigma T_2^4$$

Since $J_3 = G_3$, Equation 11.66 yields

$$G_3 = \rho_3 G_3 + \varepsilon_3 E_{b3} \rightarrow G_3 = \frac{\varepsilon_3}{1 - \rho_3} E_{b3}$$

But A_3 is opaque, so $\theta_3 = 0$ and from Equations (11.23) and (11.30)

$$(1 - \rho_3) = \varepsilon_3$$

$$\therefore G_3 = E_{b3} = \sigma T_3^4$$

Combining these two equations

$$\sigma T_3^4 = \frac{3}{4} \sigma T_2^4$$

$$T_3 = \left(\frac{3}{4} T_2^4\right)^{0.25} = \left(\frac{3}{4} (555 \text{K})^4\right)^{0.25} = 516 \text{ K}$$

PROBLEM 11.31

A large slab of steel 0.1-m-thick contains a 0.1-m-diameter circular hole, whose axis is normal to the surface. Considering the sides of the hole to be black, specify the rate of radiative heat loss from the hole. The plate is at 811 K, the surroundings are at 300 K.

GIVEN

- A large slab of steel with a hole whose axis is normal to the surface
- Slab thickness (S) = 0.1 m
- Hole diameter (D) = 0.1 m
- Plate temperature (T_1) = 811 K
- Temperature of surrounding (T_∞) = 300 K

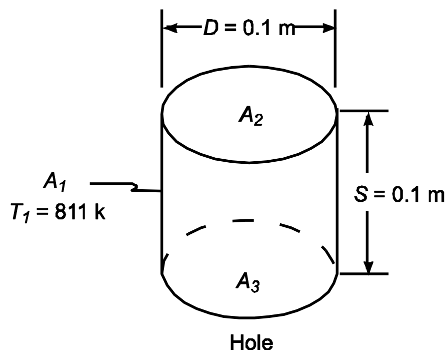
FIND

- The rate of radiative heat loss from the hole (q_r)

ASSUMPTIONS

- The sides of the hole are black

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

From Equation (11.69), for A_2 $\quad A_2\, G_2 = J_1\, A_1\, F_{12} + J_2\, A_2\, F_{22} + J_3\, A_3\, F_{32}$

From Equation (11.66) $\quad J_1 = \rho_1\, G_1 + \varepsilon_1\, E_{b1}$

$$J_2 = \rho_2\, G_2 + \varepsilon_2\, E_{b2}$$

$$J_3 = \rho_3\, G_3 + \varepsilon_3\, E_{b3}$$

Since all surfaces behave as blackbodies $\rho_1 = \rho_2 = \rho_3 = 0$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$

Therefore

$$J_1 = E_{b1} \quad J_2 = E_{b2} \quad \text{and} \quad J_3 = E_{b3} = E_{b2} \text{ (by symmetry)}$$

Substituting these into the above equation yields

$$A_2\, G_2 = E_{b1}\, A_1\, F_{12} + E_{b2}\, (A_2\, F_{22} + A_3\, F_{32})$$

Since A_2 cannot see itself $F_{22} = 0$

$$G_2 = Eb_1\,\frac{A_1}{A_2}\,F_{12} + E_{b2}\,\frac{A_3}{A_2}\,F_{32}$$

By symmetry $F_{12} = F_{13}$

The sum of the shape factors from one surface must be unity

$$F_{11} + F_{12} + F_{13} = 1 \quad \rightarrow \quad F_{11} + 2\,F_{12} = 1$$

$$F_{21} + F_{22} + F_{23} = 1 \quad \rightarrow \quad F_{21} = 1 - F_{23}$$

From Equation (9.46)

$$A_1\,F_{12} = A_2\,F_{21} \rightarrow F_{12} = \frac{A_2}{A_1}\,F_{21} = \frac{A_2}{A_1}\,(1 - F_{23})$$

and $$A_2\,F_{23} = A_3\,F_{32} \rightarrow F_{32} = \frac{A_2}{A_3}\,F_{23}$$

$$G_2 = E_{b1}\,\frac{A_1}{A_2}\left(\frac{A_2}{A_1}(1 - F_{23})\right) + E_{b2}\,\frac{A_3}{A_2}\left(\frac{A_2}{A_3}F_{23}\right) = E_{b1}\,(1 - F_{23}) + E_{b2}\,F_{23}$$

$$= \sigma\,[T_1^4(1 - F_{23}) + T_\infty^4\,F_{23}]$$

The shape factor F_{23} can be determined from Figure (11.33) for D/S = 0.1 m/0.1 m = 1.0, and for disks with direct radiation, curve 1 applies and the shape factor $F_{23} \approx 0.19$.

$$G_2 = \;\; 5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)\,[(811\ \text{K})^4\,(1 - 0.19) + (300\ \text{K})^4\,(0.19)] = 1.996 \times 10^4\ \text{W/m}^2$$

The rate of heat transfer through A_2 is given by Equation (11.67)

$$q_2 = A_2\,(J_2 - G_2) = \frac{\pi}{4}\,D^2\,(E_{b2} - G_2) = \frac{\pi}{4}\,D^2\,(\sigma\,T_2^4 - G_2)$$

$$q_2 = \frac{\pi}{4}\,(0.1\ \text{m})^2 \quad 5.67\times10^{-8}\,\text{W/(m}^2\ \text{K}^4)\;\;(300\,\text{K})^4 - 1.996\times10^4\,\text{W/m}^2 \quad = -151\ \text{W}$$

The negative sign indicates net heat loss through A_2. By symmetry, the total energy leaving the hole is

$$q_{\text{total}} = q_2 + q_3 = 2\,q_2 = 2(151\ \text{W}) = 302\ \text{W}$$

PROBLEM 11.32

> **A 15 cm black disk is placed halfway between two black 3-m-diameter disks that are 7 m apart with all disk surfaces parallel to each other. If the surroundings are 0 K, determine the temperature of the two larger disks required to maintain the smaller disk at 540°C.**

GIVEN

- A black disk (A_1) halfway between two other black disks (A_2 & A_3)
- Diameter of A_1 (D_1) = 15 cm = 0.15 m
- Diameter of A_2 and A_3: ($D_2 = D_3$) = 3 m
- Distance between A_2 and A_3 ($2L$) = 7 m
- Surrounding temperature (T_∞) = 0 K
- Temperature of A_1 (T_1) = 540°C = 813 K

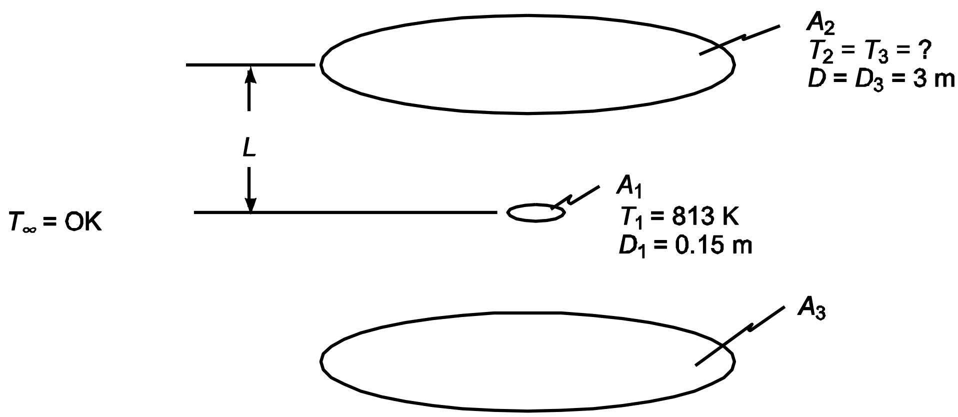
FIND

- The temperature A_2 and A_3 required

ASSUMPTIONS

- A_2 and A_3 are at the same temperature ($T_2 = T_3$)
- Steady state conditions

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The shape factor for the geometry is given in Table 11.4 as

$$F_{12} = \frac{1}{2a^2}\left[L^2 + a^2 + b^2 - \sqrt{(L^2 + a^2 + b^2)^2 - 4a^2 b^2}\right]$$

where $a = D_1/2$ and 0.075 m $\quad b = D_2/2 = 1.5$ m $\quad L = 3.5$ m

By symmetry $F_{13} = F_{12} = 0.155$.

The sum of the shape factors, including the shape factor with the surroundings must be unity

$F_{12} + F_{13} + F_{1\infty} = 1 \quad \rightarrow \quad F_{1\infty} = 1 - 2\,(F_{12}) = 1 - 2\,(0.155) = 0.690$

The net rate of heat transfer from A_1 to A_2 is given by Equation (11.47)

$$q_{1-2} = A_1 F_{12}\,(E_{b1} - E_{b2}) = \sigma A_1 F_{12}\,(T_1^4 - T_2^4)$$

Similarly

$$q_{1-3} = \sigma A_1 F_{13}\,(T_1^4 - T_3^4) = \sigma A_1 F_{12}\,(T_1^4 - T_2^4)$$

$$q_{1-\infty} = \sigma A_7 F_{1\infty} (T_1^4 - T_\infty^4) = \sigma A_1 F_{1\infty} T_1^4$$

For steady state, these rates of heat transfer must sum to zero

$$q_{1-2} + q_{1-3} + q_{1-\infty} = 0$$

$$\sigma A_1 [2 F_{12} (T_1^4 - T_2^4) + F_{1\infty} T_1^4] = 0$$

Solving for T_2

$$5.67*10^{-8}* \frac{\pi(0.15)^2}{4} [2 *0.155* (813^4 - T_2^4) + 0.690* 813^4] = 0$$

$$T_2 = 1089 \text{ K} = T_3$$

The net rate of heat transfer from A_1 to A_2 is given by Equation (11.47)

$$q_{1-2} = A_1 F_{12}(E_{b1} - E_{b2}) = \sigma A_1 F_{12} (T_1^4 - T_2^4)$$

$$= 5.67*10^{-8}*\frac{\pi(0.15)^2}{4}*0.155*(813^4 - 1089^4) \text{ W}$$

$$= -150.5 \text{ W}$$

PROBLEM 11.33

Show that the effective conductance. $A_1\ \overline{F}_{1\text{-}2}$ for two black parallel plates of equal area connected by re-radiating walls at a constant temperature is

$$\boldsymbol{A_1\ \overline{F}_{1\text{-}2} = A_1\left(\frac{1+F_{1-2}}{2}\right)}$$

GIVEN

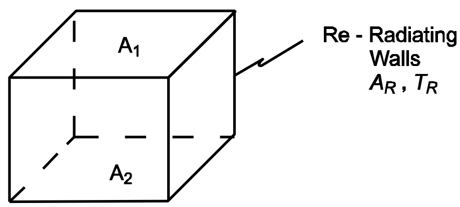
- Two black parallel planes of equal area connected by re-radiating walls at a constant temperature

FIND

- Show that

$$A_1\ \overline{F}_{1-2} = A_1\ \left(\frac{1+F_{1-2}}{2}\right)$$

SKETCH



SOLUTION

From Equation (11.79)

$$A_1\ \overline{F}_{1-2} = A_1\left[F_{1-2} + \frac{1}{\dfrac{1}{F_{1-R}} + \dfrac{A_1}{A_2\,F_{2-R}}}\right]$$

Since A_1 and A_2 cannot see themselves, $F_{1-1} = F_{2-2} = 0$.

The shape factors from a single surface must sum to unity

$F_{1-1} + F_{1-2} + F_{1-R} = 1 \quad \rightarrow \quad F_{1-R} = 1 - F_{1-2}$

$F_{2-1} + F_{2-2} + F_{2-R} = 1 \quad \rightarrow \quad F_{2-R} = 1 - F_{2-1}$

From Equation (11.46) $A_1\ F_{1-2} = A_2\ F_{2-1} \rightarrow\ F_{1-2} = F_{2-1} \rightarrow\ F_{2-R} = F_{1-R} = 1 - F_{1-2}$

Substituting this and $A_1 = A_2$ into the expression for $A_1\ \overline{F}_{12}$

$$A_1\ \overline{F}_{1-2} = A_1\left[F_{1-2} + \frac{1}{\dfrac{1}{1-F_{1-2}} + \dfrac{1}{1-F_{1-2}}}\right] = A_1\left[F_{1-2} + \frac{1}{\left(\dfrac{2}{1-F_{1-2}}\right)}\right] = A_1\left[F_{1-2} + \frac{1-F_{1-2}}{2}\right]$$

$$= A_1\left(\frac{1+F_{1-2}}{2}\right)$$

PROBLEM 11.34

Calculate the net radiant-heat-transfer rate if the two surfaces in Problem 11.18 are black and are connected by a refractory surface with an area of 500 m². A_1 is at 555 K and A_2 is at 278 K. What is the refractory surface temperature?

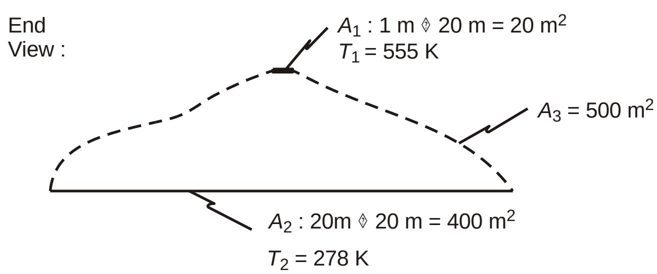
GIVEN

- Rectangular surfaces A_1 and A_2 connected by a refractory surface A_3
- A_1 is parallel to and centered 5 m above A_2
- Dimensions of A_1 = 1 m × 20 m
- A_2 is 20 m^2
- A_3 is 500 m^2
- Temperature of A_1 (T_1) = 555 K
- Temperature of A_2 (T_2) = 278 K
- A_1 and A_2 are black

FIND

(a) The net radiating heat transfer rate (q_1)
(b) The refractory surface temperature (T_3)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

From the solution to Problem 11.18

$$F_{12} = 0.427$$

For the black surfaces A_1 and A_2

$$\rho_1 = \rho_2 = 0 \quad \text{and} \quad \varepsilon_1 = \varepsilon_2 = 1$$

From Equation (11.66)

$$J_1 = E_{b1} \quad \text{and} \quad J_2 = E_{b2}$$

Since $q_3 = 0$, from Equation (11.67)

$$J_3 = G_3$$

From Equation (11.66)

$$G_3 = \frac{\varepsilon_3}{1-\rho_3} E_{b3} = E_{b3}$$

Since neither A_1 nor A_2 can see themselves, $F_{11} = F_{22} = 0$

With these simplifications, Equation (11.69) reduces to

[1] $A_1 G_1 = Eb_2 A_2 F_{21} + E_{b3} A_3 F_{31}$

[2] $A_2 G_2 = E_{b1} A_1 F_{12} + E_{b3} A_3 F_{32}$

[3] $A_3 G_3 = E_{b1} A_1 F_{13} + E_{b2} A_2 F_{23} + E_{b3} A_3 F_{33}$

From Equation [3] $\quad E_{b3} = \dfrac{R_{b1}\, A_1 F_{13} + F_{b2}\, A_2 F_{23}}{A_3\,(1 - F_{33})}$

The shape factors are calculated below

$$A_2\, F_{21} = A_1\, F_{12} \rightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{20\,\text{m}^2}{400\,\text{m}^2}\ (0.427) = 0.0214$$

$$F_{11} + F_{12} + F_{13} = 1 \rightarrow F_{13} = 1 - F_{12} = 1 - 0.427 = 0.573$$

$$A_3\, F_{31} = A_1\, F_{13} \rightarrow F_{31} = \frac{A_1}{A_3} F_{13} = \frac{20\,\text{m}^2}{500\,\text{m}^2}\ (0.573) = 0.0229$$

$$F_{21} + F_{22} + F_{31} = 1 \rightarrow F_{23} = 1 - F_{21} = 1 - 0.0214 = 0.9786$$

$$A_3\, F_{32} = A_2\, F_{23} \rightarrow F_{32} = \frac{A_2}{A_3} F_{23} = \frac{400\,\text{m}^2}{500\,\text{m}^2}\ (0.9786) = 0.7829$$

$$F_{31} + F_{32} + F_{33} = 1 \rightarrow F_{33} = 1 - F_{31} - F_{32} = 1 - 0.0229 - 0.7829 = 0.1942$$

Solving part (b) first

$$E_{b3} = \sigma\, T_3{}^4 = \frac{\sigma T_1^4\, A_1\, F_{13} + \sigma T_2^4\, A_2\, F_{23}}{A_3\,(1 - F_{33})}$$

$$T_3 = \left(\frac{T_1^4\, A_1\, F_{13} + T_2^4\, A_2\, F_{23}}{A_3\,(1 - F_{33})} \right)^{0.25}$$

$$T_3 = \left[\frac{(555\,\text{K})^4 (20\,\text{m}^2)(0.573) + (278\,\text{K})^4 (400\text{m}^2)(0.9786)}{500\,\text{m}^2 (1 - 0.1942)} \right]^{0.25} = 304\ \text{K}$$

(a) The rate of heat loss from $A1$ is given by Equation (11.67)

$$q_1 = A_1\,(J_1 - G_1) = A_1\,(E_{b1} - G_1)$$

From Equation [1]

$$G_1 = \frac{\sigma (T_2^4\, A_2\, F_{21} + T_3^4\, A_3\, F_{31}}{A_1}$$

$$\therefore\ q_1 = \sigma\,(T_1{}^4\, A_1 - T_2{}^4\, A_2\, F_{21} - T_3{}^4\, A_3\, F_{31})$$

$$q_1 = 5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)\ \ [(555\ \text{K})^4\,(20\ \text{m}^2) - (278\ \text{K})^4\,(400\ \text{m}^2)$$

$$(0.0214) - (304\ \text{K})^4\,(500\ \text{m}^2)\ (0.0229)]$$

$$q_1 = 99{,}150\ \text{W (loss)}$$

As a check

$$q_2 = A_2\,(E_{b2} - G_2) = \sigma\,[T_2{}^4 A_2 - T_1{}^4\, A_1\, F_{12} - T_3{}^4\, A_3\, F_{32}] = -\,100{,}040\ \text{W (gain)}$$

PROBLEM 11.35

A black sphere (2.5-cm in diameter) is placed in a large infrared heating oven whose walls are maintained at 370°C. The temperature of the air in the oven is 90°C and the heat-transfer coefficient for convection between the surface of the sphere and the air is 30 W/(m^2 K). Estimate the net rate of heat flow to the sphere when its surface temperature is 35°C.

GIVEN

- A black sphere in a large infrared heating oven
- Sphere diameter (D) = 2.5 cm = 0.025 m
- Oven wall temperature (T_2) = 370°C = 643 K
- Oven air temperature (T_∞) = 90°C = 363 K
- Convective heat transfer coefficient (h_c) = 30 W/(m^2 K)
- Sphere surface temperature (T_1) = 35°C = 308 K

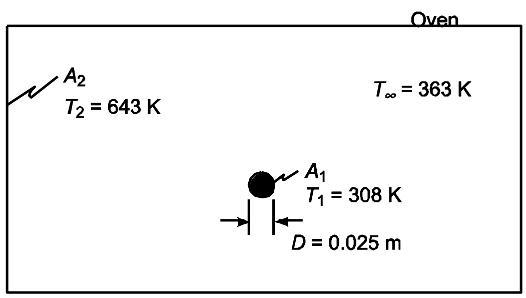
FIND

- The net rate of heat flow to the sphere (q_{total})

ASSUMPTIONS

- The oven walls are diffuse

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/m^{2}K^4

SOLUTION

Since the sphere is black: $\rho_1 = 0$; $\varepsilon_1 = 1$

From Equation (11.66) $\quad J_1 = E_{b1}$ and $J_2 = \rho_2 G_2 + \varepsilon_2 E_{b2}$

From Equation (11.69) $\quad$ [1] $A_1 G_1 = J_1 A_1 F_{11} + J_2 A_2 F_{21}$

[2] $A_2 G_2 = J_1 A_1 F_{12} + J_2 A_2 F_{22}$

Since A_1 cannot see itself, $F_{11} = 0$

Also $\quad F_{12} = 1 \quad \to \quad A_1 F_{12} = A_2 F_{21} \quad \to \quad F_{21} = \dfrac{A_1}{A_2}$

Solving for J_2

$$J_2 = \rho_2 \left(E_{b1} \left(\frac{A_1}{A_2} \right) F_{12} + J_2 F_{22} \right) + \varepsilon_2 E_{b2}$$

$$J_2 = \frac{E_{b1} \left(\dfrac{A_1}{A_2} \right) \rho_2 + \varepsilon_2 E_{b2}}{1 - \rho_2 F_{22}}$$

$$\therefore \; A_1 G_1 = \frac{A_2 F_{21}\left(E_{b1}\left(\frac{A_1}{A_2}\right)\rho_2 + \varepsilon_2 E_{b2}\right)}{1 - \rho_2 F_{22}}$$

From Equation (11.67)

$$q_1 = A_1 (J_1 - G_1) = A_1 E_{b1} - \frac{A_2\left(\frac{A_1}{A_2}\right)\left(E_{b1}\left(\frac{A_1}{A_2}\right)\rho_2 + \varepsilon_2 E_{b2}\right)}{1 - \rho_2 F_{22}}$$

since $\varepsilon_2 = (1 - \rho_2)$, this simplifies to

$$q_1 = \frac{A_1(1 - \rho_2)(E_{b1} - E_{b2})}{1 - \rho_2 F_{22}}$$

Since A_2 is very large compared to A_1: $F_{22} \approx 1.0$ and $q_1 = A_1(E_{b1} - E_{b2}) = \sigma A_1(T_1^4 - T_2^4)$

The total rate of heat transfer is the sum of the convective and radiative heat transfer

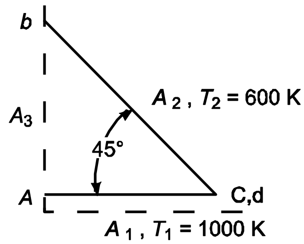
$$q_{total} = q_c + q_1 = A_1 \left[h_c (T_\infty - T_1) + \sigma (T_2^4 - T_1^4)\right]$$

$$q_{total} = p\,(0.025\text{ m})^2 \quad 30\text{ W/(m}^2\text{K)}\;(363\text{K} - 308\text{K}) + \;5.67 \times 10^{-8}\text{ W/(m}^2\text{K}^4)\;[(643\text{K})^4 - (308\text{K})^4]$$

$$q_{total} = 21\text{ W}$$

PROBLEM 11.36

The wedge-shaped cavity shown in the accompanying sketch consists of two long strips joined along one edge. Surface 1 is 1-m-wide, has an emittance of 0.4, and has a temperature of 1000 K. The other wall has a temperature of 600 K. Assuming gray diffuse processes and uniform flux distribution, calculate the rate of energy loss from surface 1 and 2 per meter length.



GIVEN

- The wedge shaped cavity shown above
- Width of A_1 (W_1) = 1 m
- Emittance of A_1 (ε_1) = 0.4
- Temperature of A_1 (T_1) = 1000 K
- Temperature of A_2 (T_2) = 600 K
- A_2 is black

FIND

- The rate of energy loss from A_1 and A_2 per meter length (q_1/L and q_2/L)

ASSUMPTIONS

- Enclosure temperature (T_e) = 0 K
- Gray diffuse processes
- Uniform flux distribution

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

Width of A_3 (W_3) = 1 m

Width of A_2 (W_2) = $\sqrt{(1\text{m})^2 + (1\text{m})^2} = \sqrt{2}\,\text{m}$

The crossed-string methods can be used to calculate F_{12}. From Equation (11.54)

$$F_{12} = \frac{(ab + cb) - (ab + cd)}{2W_1} = \frac{(1\,\text{m} + \sqrt{2}\,\text{m}) - (1\,\text{m} - 0)}{2(1\,\text{m})} = \frac{\sqrt{2}}{2}$$

From Equation (11.46)

$$A_1 F_{12} = A_2 F_{21} \quad \rightarrow \quad F_{21} = \left(\frac{A_1}{A_2}\right) F_{12}$$

Since neither A_1 nor A_2 can see itself, $F_{11} = F_{22} = 0$

Since A_2 is black: $\rho_2 = 0$ and $\varepsilon_2 = 1$

From Equation (11.66) $J_1 = \rho_1 G_1 + \varepsilon_1 E_{b1} = \rho_1 G_1 + \varepsilon_1 \sigma T_1^4$

$$J_2 = E_{b2} = \sigma T_2^4$$

From Equation (11.69) for A_1

$$A_1\, G_1 = J_1\, A_1\, F_{11} + J_2\, A_2\, F_{21} = E_{b2}\, A_2\, F_{21} = \sigma\, T_2^4\, A_1\, F_{21}$$

Solving for G_1

$$G_1 = \sigma\, T_2^4\, \frac{A_1}{A_2} F_{12} = 5.67\times 10^{-8}\ \mathrm{W/(m^2K^4)}\ (600\mathrm{K}^4)\frac{(1\mathrm{m})L}{(\sqrt{2}\,\mathrm{m})L}\left(\frac{\sqrt{2}}{2}\right) = 3674\,\mathrm{W/m^2}$$

From Equation (11.67)

$$q_1 = A_1\,(J_1 - G_1) = A_1\,[(\rho_1\, G_1 + \varepsilon_1\, \sigma\, T_1^4) - G_1]$$

Since $\rho_1 = 1 - \varepsilon_1$

$$\frac{q_1}{L} = W_1\, \varepsilon_1\,(G_1 + \sigma\, T_1^4) = (1\mathrm{m})\,(0.4)\quad 3674\,\mathrm{W/(m^2\,K)} + 5.67\times 10^{-8}\,\mathrm{W/(m^2K^4)}\ (1000\,\mathrm{K})^4$$

$$\frac{q_1}{L} = 24{,}150\ \mathrm{W/m}\ \ (\text{loss})$$

From Equation (11.69) for A_2

$$A_2\, G_2 = J_1\, A_1\, F_{12} + J_2\, A_2\, F_{22} = (\rho_1\, G_1 + \varepsilon_1\, \sigma\, T_1^4)\, A_1\, F_{12}$$

$$G_2 = (\rho_1\, G_1 + \varepsilon_1\, \sigma\, T_1^4)\left(\frac{A_1}{A_2}\right) F_{12}$$

From Equation (11.67)

$$q_2 = A_2\,(J_2 - G_2) = W_2\, L\left[E_{b2} - (\rho_1\, G_1 + \varepsilon_1\, \sigma T_1^4)\frac{A_1}{A_2} F_{12}\right]$$

Since $\rho_1 = 1 - \varepsilon_1$

$$\frac{q_2}{L} = W_2\left[\sigma T_2^4 - \left(\frac{A_1}{A_2}\right) F_{12}[(1-\varepsilon_1)\, G_1 + \varepsilon_1\, \sigma T_1^4]\right]$$

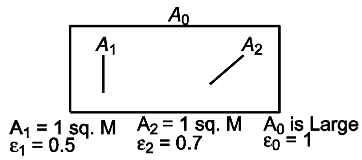
$$\frac{q_1}{L} = (\sqrt{2}\,\mathrm{m})$$

$$\left[\left(5.67\times 10^{-8}\,\mathrm{W/(m^2K^4)}\right)(600\,\mathrm{K})^4 - \left(\frac{1}{\sqrt{2}}\right)\frac{\sqrt{2}}{2}[(1-0.4)\left(3674\,\mathrm{W/(m^3K)}\right) + 0.4\left(5.67\times 10^{-8}\,\mathrm{W/(m^2K^4)}\right)(1000\,\mathrm{K})^4]\right]$$

$$\frac{q_2}{L} = -7204\ \mathrm{W/m}\ (\text{gain})$$

PROBLEM 11.37

Derive an equation for the net rate of radiant heat transfer from surface 1 in the system shown in the accompanying sketch. Assume that each surface is at a uniform temperature and that the geometrical shape factor F_{1-2} is 0.1.



GIVEN

- The system shown above

FIND

- An expression for the net rate of radiant heat transfer from surface 1 (q_1)

ASSUMPTIONS

- Steady state
- A_1 and A_2 are gray, A_0 is black
- Each surface is at a uniform temperature
- The shape factor $F_{12} = 0.1$

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

All of the shape factors for the problem can be expressed in terms of F_{12} using Equation (11.46) and the fact that all shape factors from a given surface must sum to unity.

Also

$$A_1 = A_2 \text{ and } \frac{A_1}{A_0} = \frac{A_2}{A_0} \approx 0$$

$$A_1 F_{12} = A_2 F_{21} \rightarrow F_{21} = F_{12} = 0.1$$

$$F_{10} + F_{11} + F_{12} = 1 \text{ and } F_{11} = 0 \rightarrow F_{10} = 1 - F_{12} = 0.9$$

$$F_{20} + F_{21} + F_{22} = 1 \text{ and } F_{22} = 0 \rightarrow F_{20} = 1 - F_{21} = 0.9$$

$$A_1 F_{10} = A_0 F_{01} \rightarrow F_{01} = \left(\frac{A_1}{A_0}\right) F_{10} \approx 0$$

$$A_2 F_{20} = A_0 F_{02} \rightarrow F_{02} = \left(\frac{A_2}{A_0}\right) F_{20} \approx 0$$

$$F_{00} + F_{01} + F_{02} = 1 \rightarrow F_{00} = 1$$

The net rate of heat transfer from surface 1 is given by Equation (11.67)

$$q_1 = A_1 (J_1 - G_1)$$

Where the radiosity (J_1) and the irradiation (G_1) can be calculated using Equations (11.69) and (11.66)

$$[1]\ A_1 G_1 = J_0 A_0 F_{01} + J_1 A_1 F_{11} + J_2 A_2 F_{21} = J_2 A_2 F_{21}$$

$$[2]\ A_2 G_2 = J_0 A_0 F_{02} + J_1 A_1 F_{12} + J_2 A_2 F_{22} = J_1 A_1 F_{12}$$

$$[3]\ J_1 = \rho_1 G_1 + \varepsilon_1 E_{b1}$$

$$[4]\ J_2 = \rho_2 G_2 + \varepsilon_2 E_{b2}$$

Substituting [4] into [1]

$$A_1 G_1 = (\rho_2 G_2 + \varepsilon_2 E_{b2}) A_2 F_{21}$$

Substituting [2] and [3] into this Equation

$$A_1\, G_1 = [\rho_2(\rho_1\, G_1 + \varepsilon_1\, E_{b1}) \left(\frac{A_1}{A_2}\right) F_{12} + \varepsilon_2\, E_{b2}]\, A_2\, F_{21}$$

$$G_1 = \frac{\rho_2\, \varepsilon_1\, E_{b1}\, A_1\, F_{12}\, F_{21} + \varepsilon_2\, E_{b2}\, A_2\, F_{21}}{A_1 - \rho_2\, \rho_1\, A_1\, F_{21}\, F_{12}}$$

$$q_1 = A_1\,(J_1 - G_1) = A_1[(\rho_1\, G_1 + e_1\, E_{b1}) - G_1] = A_1[\varepsilon_1\, E_{b1} + G_1\,(\rho_1 - 1)]\, A_1(\varepsilon_1\, E_{b1} - G_1\varepsilon_1)$$

$$q_1 = A_1\, e_1\,(E_{b1} - G_1) = A_1\, \varepsilon_1 \left[E_{b1} - \left(\frac{\rho_2\, \varepsilon_1\, E_{b1}\, A_1\, F_{12}\, F_{21} + \varepsilon_2\, E_{b2}\, A_2\, F_{21}}{A_1 - \rho_2\, \rho_1\, A_1\, F_{21}\, F_{12}}\right)\right]$$

where $\rho 1 = 1 - \varepsilon_1 = 0.5\ \rho_2 = 1 - \varepsilon_2 = 0.3$ and $E_{bi} = \sigma\, T_i^4$

$$\therefore\ q_1 = \varepsilon_1\, \sigma \left[A_1\, T_1^4 - \left(\frac{\rho_2\, \varepsilon_1\, A_1\, F_{12}\, F_{21}\, T_1^4 + \varepsilon_2\, A_2\, F_{21}\, T_2^4}{1 - \rho_2\, \rho_1\, F_{21}\, F_{12}}\right)\right]$$

$$q_1 = (0.5)\ \ 5.67 \times 10^{-8}\ \mathrm{W/(m^2K^4)}$$

$$\left[(1\mathrm{m}^2)\, T_1^4 - \left(\frac{(0.3)(0.5)(1\mathrm{m}^2)(0.1)(0.1)\, T_1^4 + (0.7)(1\mathrm{m}^2)(0.1)\, T_2^4}{1 - (0.5)(0.3)(0.1)(0.1)}\right)\right]$$

$$q_1 = \ 2.83 \times 10^{-8}\ \mathrm{W/K^4}\ \ T_1^4 - \ 1.98 \times 10^{-9}\ \mathrm{W/K^4}\ \ T_2^4$$

PROBLEM 11.38

Two 1.5 m-square and parallel flat plates are 30 cm apart. Plate A_1 is maintained at a temperature of 1100 K and A_2 at 500 K. The emissivities of the plates are 0.5 and 0.8, respectively. Considering the surroundings black at 0 K and including multiple inter-reflections, determine (a) the net radiant exchange between the plates and (b) the heat input required by surface A_1 to maintain its temperature. The outer-facing surfaces of the plates are adiabatic.

GIVEN

- Two square parallel flat plates, 30 cm apart
- Temperature of A_1 (T_1) = 1100 K
- Temperature of A_2 (T_2) = 500 K
- Emittances: $\varepsilon_1 = 0.5$ $\varepsilon_2 = 0.8$
- Surroundings are black at (T_3) = 0 K

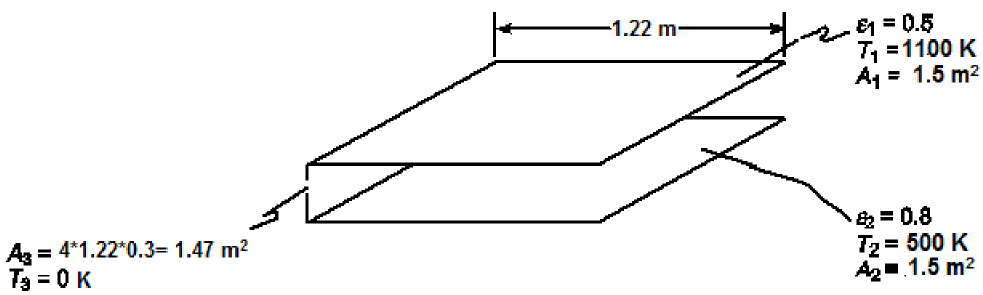
FIND

Including multiple inter-reflections, determine:

(a) The net radiant exchange ($q_{1\text{–}2}$)

(b) The heat input at surface A_1 (q_1) required to maintain its temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The gap between the plates can be considered to be a blackbody square cylindrical surface as shown in the sketch.

$$\rho_3 = 0 \quad \text{and} \quad \varepsilon_3 = 1 \rightarrow \text{ From Equation (11.66) } J_3 = E_{b3} = \sigma T_3^4 = 0$$

Also, since neither A_1 nor A_2 can see itself, $F_{11} = F_{22} = 0$.

From Equation (11.46) $A_1 F_{12} = A_2 F_{21} \rightarrow F_{12} = F_{21}$ (This is apparent from the symmetry of the problem).

The radiosities J_1 and J_2 are needed to calculate the rate of radiant heat transfer and can be determined using Equations (11.69) and (11.66)

From Equation (11.69) [1] $A_1 G_1 = J_1 A_1 F_{11} + J_2 A_2 F_{21} + J_3 A_3 F_{31} = J_2 A_2 F_{21}$

[2] $A_2 G_2 = J_1 A_1 F_{12} + J_2 A_2 F_{22} + J_3 A_3 F_{32} = J_1 A_1 F_{12}$

[3] $A_3\, G_3 = J_1\, A_1\, F_{13} + J_2\, A_2\, F_{23} + J_3\, A_3\, F_{33} = J_1\, A_1\, F_{13} + J_2\, A_2\, F_{23}$

From Equation (11.66) [4] $J_1 = \rho_1\, G_1 + \varepsilon_1\, E_{b1}$

[5] $J_2 = \rho_2\, G_2 + \varepsilon_2\, E_{b2}$

[6] $J_3 = 0$

Substituting Equations [4] and [5] into [1] and [2] $A_1\, G_1 = (\rho_2\, G_2 + \varepsilon_2\, E_{b2})\, A_2\, F_{21}$

$A_2\, G_2 = (\rho_1\, G_1 + \varepsilon_1\, E_{b1})\, A_1\, F_{12}$

Substituting G_1 from the first equation into the second and using $F_{21} = F_{12}$ yields

$$A_2\, G_2 = \left(\rho_1 \frac{A_2}{A_1} F_{12}\, (\rho_2 G_2 + \varepsilon_2 E_{b2}) + \varepsilon_1 E_{b1} \right) A_1\, F_{12}$$

since $A_1 = A_2$ and $E_{bi} = \sigma\, T_i^4$

$$G_2 = \frac{\sigma F_{12}\, (\varepsilon_2\, T_2^4\, F_{12}\, \rho_1 + \varepsilon_1 T_1^4)}{1 - (F_{12})^2 \rho_2\, \rho_1}$$

The shape factor F_{12} can be determined from Figure 11.32: for $x/D = y/D = 5 \rightarrow F_{12} = 0.71$
Also

$$\delta_1 = 1 - \varepsilon_1 = 0.5 \text{ and } \rho_2 = 1 - \varepsilon_2 = 0.2$$

$$\therefore\ G_2 = \left(5.67 \times 10^{-8}\, W/(m^2\, K^4)\right)(0.71)\left[\frac{0.8(500\,K)^4 (0.71)(0.5) + (0.5)(1100\,K)^4}{1 - (0.71)^2 (0.5)(0.2)}\right] = 31787\ W/\text{m}^2$$

From Equation [5] $J_2 = \rho_2\, G_2 + \varepsilon_2\, E_{b2} = \rho_2\, G_2 + \varepsilon_2\, \sigma\, T_2^4$

$$J_2 = 0.2\ \left(31{,}787 W/\text{m}^2\right) + 0.8\ \left(5.67 \times 10^{-8} W/(m^2 K^4)\right)\ (500\text{ K})^4 = 9192\text{ W/m}^2$$

From Equation [1] $G_1 = F_{21}\, J_2 = F_{12}\, J_2 = 0.71\left(9192 W/m^2\right) = 6435\text{ W/m}^2$ and from Equation [4]

$$J_1 = \rho\, G_1 + \varepsilon_1\, \sigma\, T_1^4 = 0.5\left(6435 W/\text{m}^2\right) + 0.5\left(5.67 \times 10^{-8} W/(m^2\, K^4)\right)(1100\text{ K})^4$$

$$= 46254\ W/\text{m}^2$$

(a) The net radiant exchange is given by Equation (11.73)

$$q_{1-2} = (J_1 - J_2)\, A_1\, F_{12} = (46254 - 6435)\text{ W/m}^2 * (1.5\text{ m}^2)\,(0.71) = 42407\text{ W}$$

(b) The required input to surface A_1 is equal to the rate of radiative loss from surface A_1 which is given by Equation (11.67)

$$q_1 = A_1\, (J_1 - G_1) = 1.5\text{ m}^2\, (46{,}254 - 6435)\ W/m^2 = 59{,}724\text{ W}$$

PROBLEM 11.39

In order to conserve energy, the inner surface of double-glazed windows are treated with a low emissivity coating that reduces the emissivity of the uncoated surface from 0.95 to 0.5 for the coated surface. If the temperatures of the two glass panes are 20°C and 0°C, respectively, determine the reduction in heat transfer in a coated double-glazed window as compared with the uncoated one.

GIVEN

- Double glazed windows treated with low emissivity coating that reduce emissivity of uncoated surface from ε_1 =0.95 to ε_2= 0.5.
- Temperature of glass pane 1 (T_1)= 20^0C = 293 K
- Temperature of glass plane 2 (T_2) = 0^0C = 273 K

FIND

Reduction in heat transfer in coated double glazed window as compared to uncoated one.

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

When the double glazed window is not coated heat transfer between the surfaces is

$$q_1 = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_1} - 1} = \frac{5.67*10^{-8}*(293^4 - 273^4)}{\frac{1}{0.95} + \frac{1}{0.95} - 1} = 93.1 \text{ W/m}^2$$

When the double glazed window is coated heat transfer between the surfaces is

$$q_2 = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_2} - 1} = \frac{5.67*10^{-8}*(293^4 - 273^4)}{\frac{1}{0.5} + \frac{1}{0.5} - 1} = 34.3 \text{ W/m}^2$$

Thus the reduction in heat transfer q= q_1-q_2 = 58.8 W/m^2

PROBLEM 11.40

Two concentric spheres 0.2 m and 0.3-m in diameter, with the space between them evacuated, are to be used to store liquid air (133 K). If the surfaces of the spheres have been flashed with aluminum and the liquid air has a talent heat of vaporization of 209 kJ/kg, determine the number of kilograms of liquid air evaporated per hour.

GIVEN

- Two concentric spheres with the space between them evacuated and liquid air in the inner sphere
- Diameters
  - $D_1 = 0.2$ m
  - $D_2 = 0.3$ m
- Liquid air temperature (T_a) = 133 K
- Room temperature (T_∞) = 293 K
- Surfaces of the spheres have been flashed with aluminum
- Heat of vaporization of liquid air (h_{fg}) = 209 kJ/kg = 209,000 J/kg

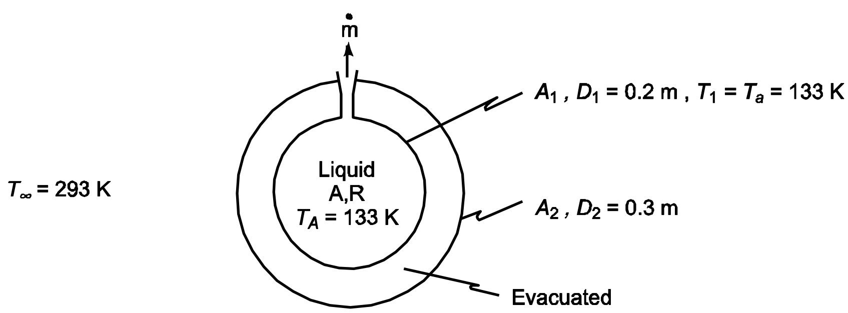
FIND

- The number of kilograms of liquid air evaporated per hour ($\dot{m}$)

ASSUMPTIONS

- Steady state
- Convective thermal resistance between the liquid air and interior sphere is negligible
- Thermal resistance of the sphere walls is negligible
- Natural convection on the exterior is negligible
- The room behaves as a blackbody enclosure
- The thickness of the sphere walls in negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

From Table 11.3, the hemispherical emissivity of the spheres will be approximated by that for oxidized aluminum at 310 K: $\varepsilon = \varepsilon_1\ \varepsilon_2 = 0.11$

SOLUTION

Since A_2 completely surrounds A_1 and A_1 cannot see itself, $F_{12} = 1.0$ and $F_{11} = 0$

From Equation (11.46)

$$A_2\,F_{21} = A_1\,F_{12} \rightarrow\ F_{21} = \frac{A_1}{A_2} = \frac{\pi D_1^2}{\pi D_2^2} = \left(\frac{D_1}{D_2}\right)^2 = \left(\frac{0.2}{0.3}\right)^2 = 0.444$$

The shape factors from a given surface must sum to unity

$$F_{21} + F_{22} = 1 \rightarrow\ \ F_{22} = 1 - F_{21} = 0.556$$

Also

$$\rho = \rho_1 = \rho_2 = 1 - \varepsilon = 0.89$$

The net rate of heat transfer from A_1 to A_2 must equal the rate of heat transfer from the exterior sphere to the surroundings

$$q = q_{12} = \sigma\,\varepsilon\,A_2\,(T_2^4 - T_\infty^4) \qquad [1]$$

The rate of heat transfer between the spheres is given by Equation (11.75)

$$q_{12} = A_1\,F_{12}\,(E_{b1} - E_{b2}) = A_1\,F_{21}\,\sigma\,(T_1^4 - T_2^4) \qquad [2]$$

where f_{12} is given in Equation (11.76) for concentric spheres.

$$F_{12} = \frac{1}{\left(1 - \frac{\varepsilon_1}{\varepsilon_1}\right) + 1 + \left(\frac{A_1}{A_2}\right)\left(1 - \frac{\varepsilon_1}{\varepsilon_2}\right)}$$

$$\frac{A_1}{A_2} = \frac{\pi D_1^2}{\pi D_2^2} = \left(\frac{D_1}{D_2}\right)^2$$

$$F_{12} = \frac{1}{\left(\left(\frac{1-0.11}{0.11}\right) + 1 + \left(\frac{0.2}{0.3}\right)^2\left(\frac{1-0.11}{0.11}\right)\right)} = 0.0788$$

$$A_1\ F_{12}\ \sigma\,(T_1^4 - T_2^4) = \sigma\,\varepsilon\,A_2\,(T_2^4 - T_\infty^4)$$

Solving Equations [1] and [2] for T_2

$$T_2 = \left[\frac{\left(\frac{A_1}{A_2}\right)\left(\frac{F_{12}}{\varepsilon}\right)T_1^4 + T_\infty^4}{1 + \left(\frac{A_1}{A_2}\right)\left(\frac{F_{12}}{\varepsilon}\right)}\right]^{0.25}$$

where $\frac{A_1}{A_2}\ \frac{F_{12}}{\varepsilon} = \left(\frac{0.2}{0.3}\right)^2 \frac{0.0788}{0.11} = 0.3185$

$$T_2 = \left(\frac{0.3185\,(133\text{K})^4 + (293\text{K})^4}{1 + 0.3185}\right)^{0.25} = 274\ \text{K}$$

The rate of evaporation of the liquid air is

$$\dot{m} = \frac{q}{h_{fg}} = \frac{\sigma\,\varepsilon\,A_2(T_\infty^4 - T_2^4)}{h_{fg}} = \frac{5.67\times10^{-8}\ \text{W}/(\text{m}^2\text{K}^4)\ \pi\,(0.11)(0.3\text{m})^2\,[(293\text{K})^4 - (274\text{K})^4]}{209{,}000\,\text{J/kg}\quad(\text{Ws})/\text{J}\quad \text{h}/3600\,\text{s}}$$

$$\dot{m} = 0.053\ \text{kg/h}$$

COMMENT

The rate of evaporation would be reduced to 0.024 kg/h if the two aluminum surfaces in the evacuated space could remain polished so that $\varepsilon_1 = \varepsilon_2 = 0.04$.

PROBLEM 11.41

Determine the steady-state temperatures of two radiation shields placed in the evacuated space between two infinite planes at temperatures of 555 K and 278 K. The emissivity of all surfaces is 0.8.

GIVEN

- Two radiant shields placed in the evacuated space between two infinite planes
- Temperature of the planes
  - $T_1 = 555$ K
  - $T_4 = 278$ K
- Emissivity of all surface (ε) = 0.8

FIND

- The steady state temperatures of the shields (T_2, T_3)

ASSUMPTIONS

- All surfaces are gray and diffuse

SKETCH

SOLUTION

Since the space is evacuated, convection and conduction are negligible. Since the surfaces are simply infinite planes, equivalent conductance $A_1\, f_{12}$ can be used.

The net rate of heat transfer from A_1 to A_2 is given by Equation (11.75)

$$q_{12} = A_1\, F_{12}\,(E_{b1} - E_{b2})$$

For steady state $q_{12} = q_{23} = q_{34}$

Also, because of all the emittances and areas are identical

$$F_{12} = F_{23} = F_{34}$$

Therefore $\quad E_{b1} - E_{b2} = E_{b2} - E_{b3} = E_{b3} - E_{b4}$

$$E_{b2} = 0.5\,(E_{b1} + E_{b3}) \quad \text{and} \quad E_{b3} = 0.5(E_{b2} + E_{b4})$$

Solving for E_{b2}

$$E_{b2} = \frac{1}{2}\left[E_{b1} + \frac{1}{2}(E_{b2} + E_{b4})\right] \to \quad E_{b2} = \frac{4}{3}\left[\frac{1}{2}E_{b1} + \frac{1}{4}E_{b4}\right] = \sigma T_2^4$$

$$T_2 = \left[\frac{1}{3\sigma}(2\sigma T_1^4 + \sigma T_4^4)\right]^{0.25} = \left[\frac{1}{3}(2T_1^4 + T_4^4)\right]^{0.25} = \left[\frac{1}{3}[2(555\,\text{K})^4 + (278\,\text{K})^4]\right]^{0.25} = 505\text{ K}$$

Similarly $\quad T_3 = \left[\frac{1}{3}(2T_4^4 + T_1^4)\right]^{0.25} = \left[\frac{1}{3}[2(278\,\text{K})^4 + (555\,\text{K})^4]\right]^{0.25} = 434\text{ K}$

PROBLEM 11.42

Three thin sheets of polished aluminum are placed parallel to each other so that the distance between them is very small compared to the size of the sheets. If one of the outer sheets is at 280°C, and the other outer sheet is at 60°C, calculate the temperature of the intermediate sheet and the net rate of heat flow by radiation. Convection may be ignored.

GIVEN

- Three thin sheets of polished aluminum parallel to each other
- The distance between the sheets is small compared to the size of the sheets
- Outer sheet temperatures
  - T_1 = 280°C = 553 K
  - T_2 = 60°C = 333 K
- Convection may be ignored

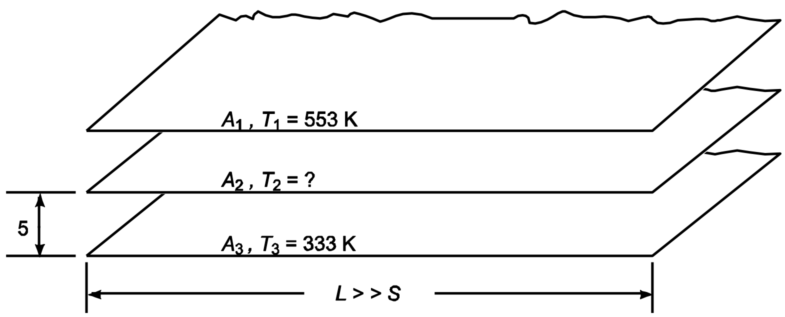
FIND

(a) The temperature of the intermediate sheet
(b) The net rate of heat flow by radiation

ASSUMPTIONS

- Steady state
- All surfaces are gray

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, The Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)
From Table 11.3, the emissivity of polished aluminum at the average temperature of 443 K ($\varepsilon = \varepsilon_1 = \varepsilon_2 = \varepsilon_3$) = 0.05

SOLUTION

The plates may be approximated by infinite parallel planes, therefore, the shape factors are, $F_{12} = F_{21} = F_{23} = F_{32} = 1.0$

(a) For steady state, the net heat flow from surface 2, from Equation (11.75) must be zero

$$q_2 = q_{21} + q_{23} = A_2\, \mathscr{F}_{21}\, (E_{b2} - E_{b1}) + A_2\, \mathscr{F}_{23}\, (E_{b2} - E_{b3}) = 0$$

By symmetry

$$\mathscr{F}_{21} = \mathscr{F}_{23}$$

Therefore $\quad 2\, E_{b2} - E_{b1} - E_{b3} = 0 \;\rightarrow\; 2\, T_7^4 = T_7^4 + T_7^4$

Solving for T_2

$$T_2 = \left[\frac{T_1^4 + T_3^4}{2}\right]^{0.25} = \left[\frac{(553\text{K})^4 + (333\text{K})^4}{2}\right]^{0.25} = 480 \text{ K}$$

(b) From Equation (11.78) for infinitely large parallel plates

$$\mathcal{F}_{12} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{1}{\frac{2}{\varepsilon} - 1} = \frac{1}{\left(\frac{2}{0.05}\right) - 1} = 0.0256$$

The rate of heat transfer is

$$q = q_{12} = A_1 \mathscr{F}_{12} (E_{b1} - E_{b2})$$

$$\frac{q}{A} = \mathcal{F}_{12}\, \sigma (T_1^4 - T_2^4) = 0.0256 \quad 5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4) \quad [(553\ \text{K})^4 - (480\ \text{K})^4] = 59\ \text{W/m}^2$$

PROBLEM 11.43

For each of the following situations, determine the rate of heat transfer between two 1m by 1 m parallel flat plates placed 0.2 m apart and connected by re-radiating walls. Assume that plate 1 is maintained at 1500 K and plate 2 at 500 K. (a) Plate 1 has an emissivity of 0.9 over the entire spectrum and plate 2 has an emissivity of 0.1. (b) Plate 1 has an emissivity of 0.1 between 0 and 2.5 μm and an emissivity of 0.9 at wavelengths longer than 2.5 μm, while plate 2 has an emissivity of 0.1 over the entire spectrum. (c) The emissivity of plate 1 is the same as in part (b), and plate 2 has an emissivity of 0.1 between 0 and 4.0 μm and an emissivity of 0.9 at wavelengths larger than 4.0 μm.

GIVEN

- Two parallel flat plates connected by re-radiating walls
- Plates dimensions: 1 m × 1 m
- Distance between plates (s) = 0.2 m
- Plate temperatures
  - T_1 = 1500 K
  - T_2 = 500 K

FIND

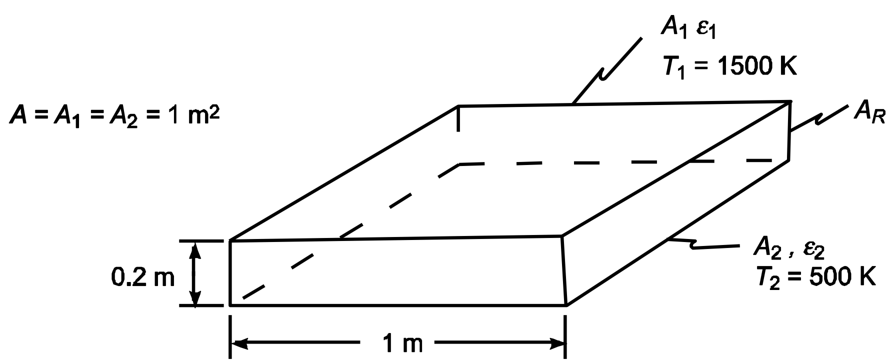
The rate of heat transfer between plates if

(a) The emissivity of plates 1 (ε_7)= 0.9 and the emissivity of plates 2 (ε_2) = 0.1
(b) $\varepsilon_1 = 0.1$ for $0 < \lambda < 2.5\ \mu$m; $\varepsilon_1 = 0.9$ for $\lambda > 2.5\ \mu$m; $\varepsilon_2 = 0.1$
(c) ε_1 is same as (b); $\varepsilon_2 = 0.1$ for $0 < \lambda < 4.0\ \mu$m, and $\varepsilon_2 = 0.9$ for $\lambda > 4.0\ \mu$m

ASSUMPTIONS

- Convective heat transfer is negligible
- The re-radiating surface is gray
- Steady state conditions

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The shape factor F_{12} is given by Figure 11.32 $x/D = y/D$ = (1 m)/(0.2 m) = 5 $\rightarrow$ $F_{12} \approx 0.71$

From Equation (11.46) $A_1 F_{12} = A_2 F_{21} \rightarrow$ $F_{21} = F_{21} = 0.71$

Since neither A_1 nor A_2 can see itself, $A_{11} = A_{22} = 0$

The shape factors from a given surface must sum to zero

$$F_{11} + F_{12} + F_{1R} = 1 \rightarrow F_{1R} = 1 - F_{12} = 0.29$$

$$F_{21} + F_{22} + F_{2R} = 1 \rightarrow F_{2R} = 1 - F_{21} = 0.29$$

The rate of heat transfer is given by Equation (11.80)

$$q_{12} = A_1 F_{12}\, \sigma (T_1^4 - T_2^4)$$

where $A_1 F_{12}$ is given by Equation (11.79)

$$A_1 F_{12} = \frac{1}{\frac{1}{A_1}\left(\frac{1}{\varepsilon_1}-1\right)+\frac{1}{A_2}\left(\frac{1}{\varepsilon_2}-1\right)+\frac{1}{A_1\bar{F}_{12}}}$$

$$\text{where } A_1\ \bar{F}_{12} = A_1\left(F_{12}+\frac{1}{\frac{1}{F_{1R}}+\frac{A_1}{A_2F_{2R}}}\right) = 1\text{ m}^2\left(0.71+\frac{1}{\frac{1}{0.29}+\frac{1}{0.29}}\right) = 0.855\text{ m}^2$$

(a) For $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.9$

$$A_1 F_{12} = \left[\frac{1}{1\text{m}^2}\ \frac{1}{0.1}-1\ +\frac{1}{1\text{m}^2}\ \frac{1}{0.9}-1\ \ \frac{1}{0.855\text{m}^2}\right]^{-1} = 0.0973\text{ m}^2$$

$$q_{12} = (0.0973\text{ m}^2)\ 5.67\times10^{-8}\text{ W/(m}^2\text{K}^4)\ [(1500\text{ K})^4 - (500\text{ K})^4] = 2.76\times10^4\text{ W}$$

(b) For $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.1$

$$A_1 F_{12} = \left[\frac{1}{1\text{m}^2}\left(\frac{1}{0.1}-1\right)+\frac{1}{0.855\text{m}^2}\right]^{-1} = 0.052\text{ m}^2$$

Following the procedure demonstrated in Section 11.7.2

For band 1: $0 < \lambda < 2.5\ \mu$m, $A_1 f_{12} = 0.052$ m^2

$$\lambda T_1 = (2.5\times 10^{-6}\text{ m})\ (1500\text{ K}) = 3.8\times 10^{-3}\text{ m K}$$

From Table 11.2 $\dfrac{E_b(0\rightarrow\lambda T)}{\sigma T^4} = 0.4434$

$$\lambda T_2 = (2.5\times 10^{-6}\text{ m})\ (500\text{ K}) = 1.3\times 10^{-3}\text{ m K}$$

From Table 11.2 $\dfrac{E_b(0\rightarrow\lambda T)}{\sigma T^4} = 0.004963$

$$q_{12}\Big|_0^{2.5\text{m}} = A_1 F_{12}\ (\varepsilon_1 = 0.1,\ \varepsilon_2 = 0.1)\left[\frac{E_b(0\rightarrow\lambda T_1)}{\sigma T_1^4}\sigma T_1^4-\frac{E_b(0\rightarrow\lambda T_2)}{\sigma T_2^4}\sigma T_2^4\right]$$

$$q_{12}\Big|_\infty^{2.5\text{m}} = 0.052\text{ m}^2\ \ 5.67\times10^{-8}\text{ W/(m}^2\text{K}^4)\ \ [0.4434\ (1500\text{ K})^4 - 0.004963\ (500\text{ K})^4] = 6618\text{ W}$$

For band 2: $2.5\ \mu < \lambda$

$$q_{12}\Big|_{2.5\text{m}}^\infty = A_1\ \mathscr{F}_{12}\ (\varepsilon_1 = 0.9,\ \varepsilon_2 = 0.1)\left[\frac{E_b(\lambda T_1\rightarrow\infty)}{\sigma T_1^4}\sigma T_1^4-\frac{E_b(\lambda T_2\rightarrow\infty)}{\sigma T_2^4}\sigma T_2^4\right]$$

$$q_{12}\Big|_t^\infty = 0.0973\text{ m}^2\ \ 5.67\times10^{-8}\text{ W/(m}^2\text{K}^4)\ \ [(1-0.4434)(1500\text{ K})^4 - (1-0.004963)\ (500\text{ K})^4]$$

$$q_{12}\Big|_{2.5\text{m}}^\infty = 15{,}202\text{ W}$$

The total rate of heat transfer is the sum of the rate of heat transfer in the two bands

$$q_{12,\text{total}} = q_{12}\Big|_0^{2.5\text{m}} + q_{12}\Big|_{2.5\text{m}}^{\infty} = 6618 \text{ W} + 15{,}202 \text{ W} = 2.18 \times 10^4 \text{ W}$$

(c) For this case, the spectrum must be broken into three bands

$0 < \lambda < 2.5\ \mu\text{m}$	$\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.1$, $A_1 f_{12} = 0.052 \text{ m}^2$
$2.5\ \mu\text{m} > \lambda, 4\ \mu\text{m}$	$\varepsilon_1 = 0.9$, $\varepsilon_2 = 0.1$, $A_1 f_{12} = 0.0973 \text{ m}^2$
$4\ \mu\text{m} < \text{l}$	ε_1 0.9, $\varepsilon_2 = 0.9$

For $\lambda > 4\ \mu\text{m}$

$$A_1 \mathscr{F}_{12} = \left[\frac{2}{1\text{m}^2}\left(\frac{1}{0.9} - 1\right) + \frac{1}{0.855\text{m}^2}\right]^{-1} = 0.719 \text{ m}^2$$

At $\lambda = 4\ \mu\text{m}$

$$\lambda T_1 = (4 \times 10^{-6} \text{ m})\ (1500 \text{ K}) = 6 \times 10^{-3} \text{ m K}$$

From Table 11.2: $\dfrac{E_b(0 \to \lambda T_1)}{\sigma T_1^4} = 0.7379$

$$\lambda T_2 = (4\times 10^{-6} \text{ m})\ (500 \text{ K}) = 2 \times 10^{-3} \text{ m K}$$

From Table 11.2: $\dfrac{E_b(0 \to \lambda T_2)}{\sigma T_2^4} = 0.0667$

Band 1

Same as part (b) $q_{12}\Big|_0^{2.5\text{m}} = 6618 \text{ W}$

Band 2

$2.5\ \mu\text{m} < \text{l} < 4\ \mu\text{m}$

$$q_{12}\Big|_{2.5}^{4} = A_1 \mathscr{F}_{12}\,(\varepsilon_1 = 0.9, \varepsilon_2 = 0.1)$$

$$\left[\left(\frac{E_b(0 \to 4T_1)}{\sigma T_1^4} - \frac{E_b(0 \to 2.5T_1)}{\sigma T_1^4}\right)\sigma T_1^4 \left(\frac{E_b(0 \to 2.5T_2)}{\sigma T_2^4} - \frac{E_b(0 \to 2.5T_2)}{\sigma T_2^4}\right)\sigma T_2^4\right]$$

$$q_{12}\Big|_0^{2.5\text{m}} = 0.0973 \text{ m}^2\ \ 5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4)\ \ [(0.7379 - 0.4434)\ (1500 \text{ K})^4 - (0.0667 - 0.004963)\ (500 \text{ K})^4]$$

$$q_{12}\Big|_0^{2.5\text{m}} = 27908 \text{ W}$$

Band 3

$\lambda > 4\ \mu\text{m}$

$$q_{12}\Big|_4^{\infty} = A_1 \mathscr{F}_{12}\,(\varepsilon_1 = 0.9, \varepsilon_2 = 0.9)\left[\left(1 - \frac{E_b(0 \to 4T_1)}{\sigma T_1^4}\right)\sigma T_1^4 - \left(1 - \frac{E_b(0 \to 4T_2)}{\sigma T_2^4}\right) s\, T_2^4\right]$$

$$q_{12}\Big|_4^{\infty} = 0.719 \text{ m}^2\ \ 5.67 \times 10^{-8} \text{W/(m}^2\text{K}^4)\ \ [(1 - 0.7379)\ (1500 \text{ K})^4 - (1 - 0.0667\ (500 \text{ K})^4]$$

$$q_{12}\Big|_0^{2.5\text{m}} = 204{,}006 \text{ W}$$

$$q_{12,\text{total}} = (6618 + 27908 + 204{,}006) \text{ W} = 2.39 \times 10^5 \text{ W}$$

PROBLEM 11.44

An inventor claims that a solar collector consisting of two absorber plates connected by a glass surface configured in an equilateral triangle (shown in the sketch) is superior to a collector consisting of two parallel plates (also shown in the sketch). The collector is configured as a long duct through which air is blown, and the absorber plates have a black surface whereas the emissivity of the glass cover is 0.9. Determine the net rate at which radiation is transferred to the cover due to the exchange with the absorber plates if during operation T_1=25°C, T_2= 60°C, and T_3=70°C. For the parallel plate configuration, the temperature of the absorber plate is 70°C.

GIVEN

- Absorber plates and cover plate arranged in parallel plate and equilateral configuration
- Emissivity of glass cover (ε_1)= 0.9
- Surface of absorber plates is black.
- Temperature of absorber plate for parallel configuration (T_2)= 70^0C

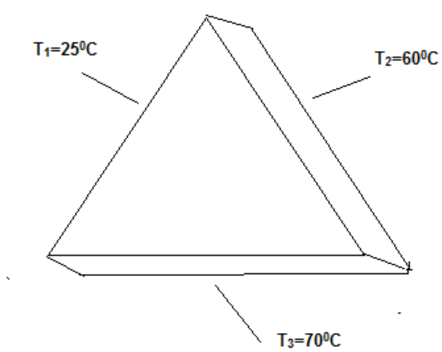
FIND

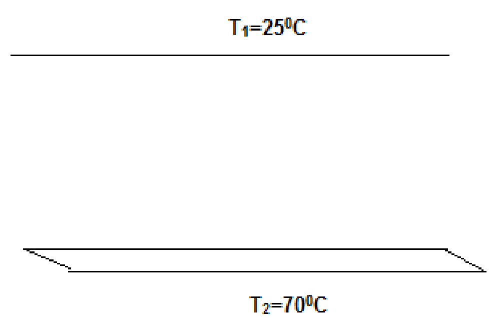
Net rate at which radiation is transferred to the cover.

ASSUMPTIONS

- Convective heat transfer is negligible
- Steady state conditions

SKETCH





PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

From equation 11.72 we have

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/(A_1\varepsilon_1)}$$

Also from equation 11.73

$$q_1 = q_{1-2} + q_{1-3} = (J_1 - J_2) A_1 F_{1-2} + (J_1 - J_3) A_1 F_{1-3}$$

Equating above two equations we have

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/(A_1\varepsilon_1)} = (J_1 - J_2) A_1 F_{1-2} + (J_1 - J_3) A_1 F_{1-3}$$

From symmetry

$$F_{12} = F_{13} = 0.5 \;\&\; J_2 = E_{b2} \text{ and } J_3 = E_{b3}$$

Solving the above equation substituting these values we have

$$E_{b1} - J_1 = \frac{(1-\varepsilon_1)}{(A_1\varepsilon_1)}\left((J_1 - J_2)A_1F_{1-2} + (J_1 - J_3)A_1F_{1-3}\right)$$

$$E_{b1} - J_1 = \frac{(1-0.9)}{(0.9)} * 0.5\left((J_1 - E_{b2}) + (J_1 - E_{b3})\right)$$

$$E_{b1} - J_1 = \frac{(1-0.9)}{(0.9)} * 0.5\left((J_1 - E_{b2}) + (J_1 - E_{b3})\right)$$

$$E_{b1} - J_1 = 0.055\left((J_1 - E_{b2}) + (J_1 - E_{b3})\right)$$

$$1.11J_1 = 0.055E_{b2} + 0.055E_{b3} + E_{b1}$$

$$1.11J_1 = 0.055 * 5.67 * 10^{-8}((333K)^4 + (343K)^4) + 5.67 * 10^{-8} * (298K)^4$$

$$J_1 = 476.4 \text{ W/m}^2$$

Thus net heat transfer from cover plate to absorber plates is.

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/(A_1\varepsilon_1)} = \frac{5.67 * 10^{-8} * (298K)^4 - 476.4}{(1-0.9)/(l * 1 * 0.9)} \text{ W/m}^2$$

$$q_1 = -266l \text{ W}$$

For a parallel plate configuration

From equation 11.72 we have

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/(A_1\varepsilon_1)}$$

Also from equation 11.73

$$q_1 = q_{1-2} = (J_1 - J_2)A_1F_{1-2}$$

Equating above two equations we have

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/(A_1\varepsilon_1)} = (J_1 - J_2)A_1F_{1-2}$$

We have F_{1-2}= 1 and J_2= E_{b2} . Thus

$$E_{b1} - J_1 = \frac{(1-\varepsilon_1)}{(A_1\varepsilon_1)}\left((J_1 - J_2)A_1F_{1-2}\right)$$

$$E_{b1} - J_1 = \frac{(1-0.9)}{(0.9)} * 1\left((J_1 - E_{b2})\right)$$

$$E_{b1} - J_1 = 0.111(J_1 - E_{b2})$$

$$1.111J_1 = 0.111E_{b2} + E_{b1}$$

$$1.11J_1 = 0.111*5.67*10^{-8}(343K)^4 + 5.67*10^{-8}*(298K)^4$$

$$J_1 = 480.6 \text{ W/m}^2$$

Thus net heat transfer from cover plate to absorber plates is.

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/(A_1\varepsilon_1)} = \frac{5.67*10^{-8}*(298K)^4 - 480.6}{(1-0.9)/(l*1*0.9)} \text{ W/m}^2$$

$$q_1 = -301l \text{ W}$$

PROBLEM 11.45

A manned spacecraft capsule has a shape of a cylinder 2.5 m in diameter and 9-m-long. The air inside the capsule is maintained at 20°C and the convection-heat-transfer coefficient on the interior surface is 17 W/(m^2 K). Between the outer skin and the inner surface is a 15 cm layer of glass-wool insulation having a thermal conductivity of 0.017 W/(m K). If the emissivity of the skin is 0.05 and there is no aerodynamic heating or irradiation from astronomical bodies, calculate the total heat transfer rate into space at 0 K.

GIVEN

- A glass-wool insulated cylinder in space filled with air
- Diameter (D) = 2.5 cm
- Length (L) = 9 m
- Air temperature (T_a) = 20°C = 293 K
- Interior convective heat transfer coefficient (h_c) = 17 W/(m^2 K)
- Insulation thickness (t) = 15 cm = 0.15 m
- Thermal conductivity of insulation (k) = 0.017 W/(m K)
- Emissivity of the skin (ε) = 0.05
- No aerodynamic heating or irradiation from astronomical bodies

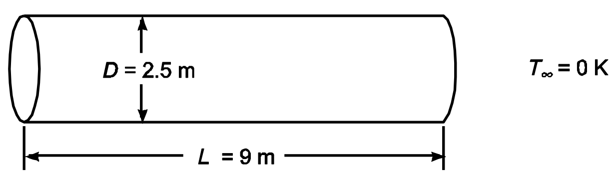
FIND

- The total rate of heat transfer into space at T_∞ = 0 K

ASSUMPTIONS

- Steady state
- Thermal resistance of the capsule walls is negligible compared to that of the insulation

SKETCH



PROPERTIES AND CONSTANTS

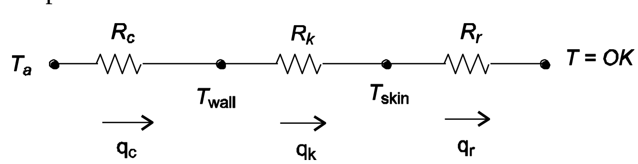
From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

Since $D >> t$ the effect of the cylinder's curvature can be neglected. The total surface area is

$$A = \pi D L + 2\,\frac{\pi}{4} D^2 = \pi D \left(L + \frac{D}{2}\right) = \pi\ (2.5\text{m})(9\text{m} + 1.25\text{m}) = 80.5\ \text{m}^2$$

The thermal circuit for the problem is shown below



where R_c = convective thermal resistance

R_k = conductive thermal resistance of the insulation

R_r = radiative thermal resistance

q_c = convective heat transfer rate to the interior wall = $h_c A\,(T_a - T_{\text{wall}})$

q_k = conductive heat transfer rate to the insulation = $(k/t)A(T_{\text{wall}} - T_{\text{skin}})$

q_r = radiative heat transfer from the skin = $\sigma\,\varepsilon\,T_{\text{skin}}^4$

For steady state, all three rates of heat transfer must be equal

$$h_c(T_a - T_{\text{wall}}) = \frac{k}{t}(T_{\text{wall}} - T_{\text{skin}}) = \sigma\,\varepsilon\,T_{\text{skin}}^4$$

solving for the wall temperature

$$T_{\text{wall}} = \frac{T_a + \dfrac{k}{t\bar{h}_c}T_{\text{skin}}}{1 + \dfrac{k}{t\bar{h}_c}} \qquad \text{Let } B = \frac{k}{t\bar{h}_c} = \frac{0.017\,\text{W/(m K)}}{0.15\,\text{m}\;\;17\,\text{W/(m}^2\text{K)}} = 0.00667$$

$$\therefore \quad \bar{h}_c\left(T_a - \frac{T_a + BT_{\text{skin}}}{1+B}\right) = \sigma\,\varepsilon\,T_{\text{skin}}^4$$

$$\sigma\,\varepsilon\,T_{\text{skin}}^4 - \bar{h}_c\left(T_a - \frac{T_a + BT_{\text{skin}}}{1+B}\right) = 0$$

$$5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\;\;(0.05)T_{\text{skin}}^4 - \;17\,\text{W/(m}^2\text{K)}\;\left(293\,\text{K} - \frac{293\,\text{K} + 0.00667\;T_{\text{skin}}}{1+0.00667}\right) = 0$$

Checking the units, then eliminating them for clarity

$$2.835\times10^{-9}\,T_{\text{skin}}^4 + 0.1126\,T_{\text{skin}} - 33.0 = 0$$

By trial and error

$$T_{\text{skin}} = 227\text{ K}$$

$$q = \sigma\,\varepsilon\,A\,T_{\text{skin}}^4 = \;5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\;(0.05)\,(80.5\text{m}^2)\,(227\text{ K})^4 = 610\text{ W}$$

PROBLEM 11.46

A 1 m × 1 m square solar collector is placed on the roof of a house. The collector receives a solar radiation flux of 800 W/m^2. Assuming that the surroundings act as a blackbody at an effective sky temperature of 30°C, calculate the equilibrium temperature of the collector (a) assuming its surface is black and the conduction and convection are negligible, and (b) assuming that the collector is horizontal and loses heat by natural convection.

GIVEN

- A square solar collector on the roof of a house
- Collector dimensions = 1 m × 1 m
- Solar flux on collector (q_s) = 800 W/m^2

FIND

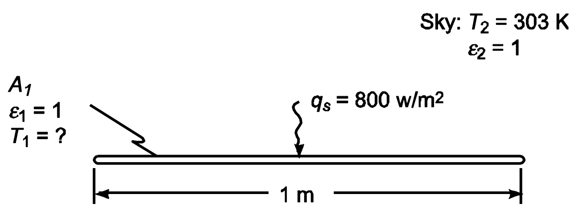
The equilibrium temperature of the collector (T_1) assuming

(a) The collector surface is black (ε_1 = 1) and conduction and convection are negligible
(b) The collector is horizontal and loses heat by natural convection

ASSUMPTIONS

- Steady state conditions
- The surroundings act as a blackbody at an effective sky temperature (T_2) = 30°C = 303 K
- The surrounding air temperature (T_∞) = T_2 = 303 K

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

(a) For equilibrium, the heat loss flux by radiation to the sky must equal the incident solar flux

$$q_s = \sigma\,\varepsilon_1\,(T_1^4 - T_2^4)$$

Solving for the collector temperature

$$T_1 = \left(\frac{q_s}{\sigma\,\varepsilon_1} + T_2^4\right)^{0.25} = \left[\frac{800\,\mathrm{W/m^2}}{5.67\times10^{-8}\,\mathrm{W/(m^2K^4)}\;\;1} + \;303\,\mathrm{K}\;^4\right]^{0.25} = 387\text{ K} = 114°\text{C}$$

(b)The heat loss flux by radiation and convection must equal the incident solar flux

$$q_s = \sigma\,\varepsilon_1\,(T_1^4 - T_2^4) + h_c\,(T_1 - T_\infty)$$

The natural convection heat transfer coefficient depends on the collector temperature, T_1. Therefore, an iterative solution is required. Natural convection will tend to lower the collector temperature calculated in part (a). For the first iteration, let T_1 = 363 K.

From Appendix 2, Table 28, for dry air at the film temperature of (363 K + 303 K) = 333 K

Thermal expansion coefficient (β) = 0.00300 1/K

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The Raleigh number is

$$Ra_L = Gr_L\,Pr = \frac{g\beta\ \Delta T\ L^3 Pr}{\nu^2} = \frac{9.8\,\text{m/s}\ \ 0.0031/\text{K}\ \ 363\,\text{K} - 303\,\text{K}\ \ 1\,\text{m}^{\ 3}\ \ 0.71}{19.4\times 10^{-6}\,\text{m}^2/\text{s}} = 3.33 \times 10^9$$

The Nusselt number for a horizontal plate with upper surface heated in this Raleigh number range is given by Equation (8.16)

$$\overline{Nu}_L = 0.15\ Ra_L^{\frac{1}{3}} = 0.15\ (3.33\times 10^9)^{\frac{1}{3}} = 224$$

$$\bar{h}_c = \overline{Nu}_L\,\frac{k}{L} = 224\ \frac{0.0279\,\text{W/(m K)}}{1\,\text{m}} = 6.25\,\text{W/(m}^2\,\text{K)}$$

Using this in the energy balance

$$800\,\text{W/m}^2 = 5.67\times 10^{-8}\,\text{W/(m}^2\text{K}^4)\ (1)\,[T_1^4 - (303\text{ K})^4] + 6.25\,\text{W/(m}^2\text{K)}\ (T_1 - 303\text{ K})$$

$$5.67 \times 10^{-8}\,T_1^4 + 6.25\,T_1 - 3171.7 = 0$$

By trial and error T_1 = 358 K

The properties of air will not change enough to justify another iteration.

PROBLEM 11.47

A thin layer of water is placed in a pan 1 m in diameter in the desert. The upper surface is exposed to 300 K air and the convection heat transfer coefficient between the upper surface of the water and the air is estimated to be 10 W/(m^2 K). The effective sky temperature depends on atmospheric conditions and is often assumed to be 0 K for a clear night and 200 K for a cloudy night. Calculate the equilibrium temperature of the water on a clear night and a cloudy night.

GIVEN

- A thin layer of water in a circular pan in the desert
- Pan diameter (D) = 1 m
- Air temperature (T_∞) = 300 K
- Convective heat transfer coefficient (h_c) = 10 W/(m^2 K)
- Effective sky temperature (T_2) = 0 K for a clear night, 200 K for a cloudy night

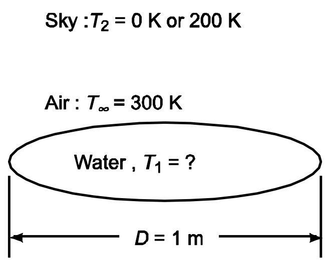
FIND

- The equilibrium temperature of the water (T_1) (a) on a clear night and (b) on a cloudy night

ASSUMPTIONS

- Steady state conditions
- The effect of the sides of the pan is negligible
- Heat transfer to the ground is negligible
- Edge losses are negligible
- Neglect losses due to evaporation

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
From Table 11.3, the emissivity of water (ε) ≈ 0.96

SOLUTION

For equilibrium, the heat gain by convection to the water must equal the heat loss by radiation

$$\bar{h}_c (T_\infty - T_1) = \varepsilon\, \sigma (T_1^4 - T_2^4)$$

(a) For T_2 = 0 K $\qquad \varepsilon\, \sigma\, T_1^4 - \bar{h}_c (T_\infty - T_1) = 0$

$$(0.96)\ \ 5.67\times10^{-8}\,\mathrm{W/(m^2K^4)}\ \ T_1^4 - \ 10\,\mathrm{W/(m^2K)}\ \ (300\ \mathrm{K} - T_1) = 0$$

Checking units, then eliminating for clarity

$$5.443 \times 10^{-8}\, T_1^4 + 10\, T_1 - 3000 = 0$$

By trial and error T_1 = 271 K = –2°C (water will freeze)

(b) For T_2 = 200 K

$$5.443 \times 10^{-8}\, T_1^4 + 10\, T_1 - 3087.1 = 0$$

$$T_1 = 277\ \mathrm{K} = 4°\mathrm{C}$$

PROBLEM 11.48

A Package of electronic equipment is enclosed in a sheet-metal box which has a 0.3 m square base and is 0.15 m high. The equipment uses 1200 W of electrical power and is placed on the floor of a large room. The emissivity of the walls of the box is 0.80 and the room air and the surrounding temperature is 21°C. Assuming that the average temperature of the container wall is uniform, estimate that temperature.

GIVEN

- A sheet metal box of electronics in a large room
- Box dimensions: 0.3 m × 0.3 m × 0.15 m high
- Power dissipation of electronics $\dot{q}_G$ = 1200 W
- Emissivity of the walls of the box (ε) = 0.80
- Room air and surrounding temperature (T_∞) = 21°C = 294 K

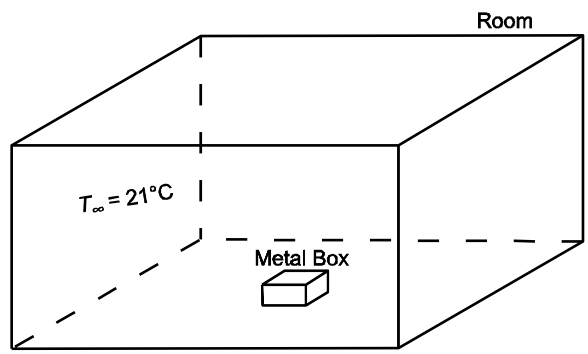
FIND

- The average temperature of the container walls (T_b)

ASSUMPTIONS

- The average temperature of the container walls is uniform
- Steady state
- The room behaves as a blackbody enclosure
- Heat loss from the bottom of the box is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The heat loss by natural convection and radiation from the box must equal the rate of electrical power dissipation.

$$q_{c,T} + q_{c,\text{sides}} + q_r = \dot{q}_G$$

$$(A_{\text{top}}\, h_{c,\text{top}} + A_{\text{sides}}\, h_{c,\text{sides}})\,(T_b - T_\infty) + \sigma\,\varepsilon\,(A_{\text{top}} + A_{\text{sides}})\,(T_b^4 - T_\infty^4)$$

The natural convection heat transfer coefficients are dependent on Tb, therefore, an iterative solution must be used. The initial guess for the box temperature will be based on the box temperature neglecting convection

$$q_r = \dot{q}_G$$

$$\sigma\,\varepsilon\,(A_{\text{top}} + A_{\text{sides}})\,(T_b^4 - T_\infty^4) = \dot{q}_G$$

$$T_b = \left(\frac{\dot{q}_G}{\sigma\,\varepsilon\,(A_{\text{top}} + A_{\text{sides}})} + T_\infty^{\;4} \right)^{0.25}$$

$$T_b = \left(\frac{1200\,\text{W}}{5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\ (0.80)\ (0.09\,\text{m}^2 + 0.18\,\text{m}^2)} + (294\,\text{K})^4 \right)^{0.25} = 570\ \text{K}$$

Natural convection will cause the box temperature to be lower than this value. For a first guess, let T_b = 500 K

From Appendix 2, Table 28, for dry air at the film temperature of 397 K (124°C)

Thermal expansion coefficient (β) = 0.00254 1/K

Thermal conductivity (k) = 0.0322 W/(m K)

Kinematic viscosity (ν) = 26.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

The Grashof number, based on the length of a side of the top of the box is

$$Gr_L = \frac{g\beta\,(T_b - T_\infty)\,L^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)(0.00254\ 1/\text{K})(500\,\text{K} - 294\,\text{K})(0.3\,\text{m})^3}{(26.4\times10^{-6}\ \text{m}^2/\text{s})} = 1.98 \times$$

10^8

The Nusselt number for the top of the box is given by Equation (8.16)

$$\overline{Nu}_L = 0.15\ (Gr_L\ Pr)^{\frac{1}{3}} = 0.15\ [(1.98\times10^8)\ (0.71)]^{\frac{1}{3}} = 78.0$$

$$\bar{h}_{c,\text{top}} = \overline{Nu}_L\ \frac{k}{L} = 78.0\,\frac{0.0322\,\text{W/(m K)}}{0.3\text{m}} = 8.37\,\text{W/(m}^2\text{K)}$$

The Grashof number for the sides of the box is

$$Gr_H = \frac{g\beta\,(T_b - T_\infty)\,H^3}{\nu^2} = \frac{(9.8\,\text{m/s}^2)(0.00254\ 1/\text{K})(500\,\text{K} - 294\,\text{K})(0.15\,\text{m})^3}{(26.4\times10^{-6}\ \text{m}^2/\text{s})^2} = 2.48 \times$$

10^7

The Nusselt number is given by Equation (8.12b)

$$\overline{Nu}_H = 0.68\ Pr^{\frac{1}{2}}\ \frac{Gr_H^{\frac{1}{4}}}{\left(0.952 + Pr^{\frac{1}{4}}\right)} = 0.68\,(0.71)^{\frac{1}{2}}\ \left(\frac{2.48\times10^7}{0.952 + 0.71}\right)^{\frac{1}{4}} = 35.62$$

$$\bar{h}_{c,\text{sides}} = \overline{Nu}_H\ \frac{k}{H} = 35.62\,\frac{0.0322\,\text{W/(m K)}}{0.15\,\text{m}} = 7.65\,\text{W/(m}^2\ \text{K)}$$

Substituting these into the energy balance

$$\left[(0.09\,\text{m}^2)\ (8.37\,\text{W/(m}^2\text{K)}) + (0.18\,\text{m}^2)\ (7.65\,\text{W/(m}^2\text{K)})\right]\ (T_b - 294\ \text{K})$$

$$+\ (5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4))\ \ (0.8)(0.27\ \text{m}^2)\ [T_b^4 - (294\ \text{K})^4] = 1200\ \text{W}$$

Checking the units, then eliminating for clarity

$$1.225 \times 10^{-8}\ T_b^4 + 2.130\ T_b - 1918 = 0$$

By trial and error: T_b = 510 K

Performing another iteration yields the following results

Film temperature = 402 K

k = 0.0325 W/(m K)

β = 0.00251 1/K

$\nu = 27.1 \times 10^{-6}$ m^2/s

Pr = 0.71

$h_{c,\text{top}}$ = 8.41 W/(m^2 K)

$h_{c,\text{sides}}$ = 7.69 W/(m^2 K)

T_b = 510 K = 237°C

COMMENTS

Note that neglecting natural convection leads to an error of 60 K.

PROBLEM 11.49

Liquid nitrogen is stored in a dewar (see the sketch) made of two concentric spheres with the space between them evacuated. The inner sphere has an outside diameter of 1 m and the space between the two spheres is 0.1 m. The surfaces of both spheres are gray with an emissivity of 0.2. If the saturation temperature for nitrogen at atmospheric pressure is 78 K and its latent heat of vaporization is 2 × 10^5 J/kg, estimate its boil-off rate under the following conditions

(a) The outer sphere is at 300 K.

(b) The outer surface of the surrounding sphere is black and loses heat by radiation to surroundings at 300 K. Assume convection is negligible.

(c) Repeat item (b) but include the effect of heat loss by natural convection.

GIVEN

- Liquid nitrogen in two concentric spheres with the space between them evacuated
- Inner sphere diameter (D_i) = 1 m
- Space between spheres (s) = 0.1 m
- Both surfaces are gray with equal emissivites ($\varepsilon_1 = \varepsilon_2$) = 0.2
- Saturation temperature of nitrogen (T_n) = 78 K
- Latent heat of vaporization of nitrogen (h_{fg}) = 2 × 10^5 J/kg

FIND

The boil-off rate $\dot{m}$ under the following conditions

(a) Outer sphere temperature (T_2) = 300 K

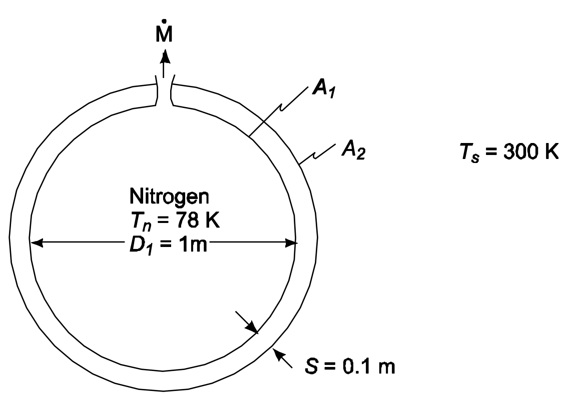
(b) Outer surface of outer sphere is black (ε_o = 1) and loses heat by radiation to surroundings at (T_s) = 300 K, convection is negligible, and

(c) Repeat part (b) but include natural convection

ASSUMPTIONS

- Steady state
- Thermal resistance of the sphere walls is negligible
- Thermal resistance between the nitrogen and the inner sphere is negligible ($T_1 = T_n$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

$D_2 = D_1 + 0.2 = 1.2$ m

(a) The heat transfer is given by Equation (11.75)

$$q_{12} = A_1 \mathscr{F}_{12} (E_{b1} - E_{b2}) = \pi D_i^2 \mathscr{F}_{12} \sigma (T_1^4 - T_2^4)$$

where f_{12} for concentric spheres is given by Equation (11.76)

$$\mathscr{F}_{12} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1}+1+\frac{A_1}{A_2}\frac{1-\varepsilon_2}{\varepsilon_2}} = \frac{1}{\frac{1-0.2}{0.2}+1+\left(\frac{\pi\ 1\text{m}^2}{\pi\ 1.2\text{m}^2}\right)\frac{1-0.2}{0.2}} = 0.129$$

$$q_{12} = \pi (1\text{ m})^2 (0.129)\ \ 5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\ \ [(78\text{ K})^4 - (300\text{ K})^4]$$

$$q_{12} = -185.3\text{ W (heat gained by nitrogen)}$$

The boil-off rate of nitrogen is given by

$$\dot{m} = \frac{q_{12}}{h_{fg}} = \frac{185.3\text{W J/(Ws)}\ \ 3600\text{s/h}}{2\times10^5\text{ J/kg}} = 3.3\text{ kg/h}$$

(b) A heat balance on the outer sphere yields

$$q_{12} = q_{2s}$$

$$A_1 F_{12}\, \sigma (T_1^4 - T_2^4) = A_2\, \sigma (T_2^4 - T_s^4)$$

$$T_2 = \left[\frac{T_s^4 + \frac{A_1}{A_2}F_{12}T_1^4}{1+\frac{A_1}{A_2}F_{12}}\right]^{0.25} = \left[\frac{(300\text{K})^4 + \frac{\pi(1\text{m})^2}{\pi(1.2\text{m})^2}\ 0.129\ \ 78\text{K}^{\ 4}}{1+\frac{\pi(1\text{m})^2}{\pi(1.2\text{m})^2}\ 0.129}\right]^{0.25} = 294\text{ K}$$

$$\therefore\quad q_{12} = \pi(1\text{m})^2(0.129)\ \ 5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\ \ [(78\text{ K})^4 - (294\text{ K})^4] = -170.8\text{W}$$

$$\dot{m} = \frac{q_{12}}{h_{fg}} = \frac{170\text{W J/(Ws)}\ \ 3600\text{s/h}}{2\times10^5\text{J/kg}} = 3.1\text{ kg/h}$$

(c) A heat balance on the sphere yields: $q_{12} = q_{2s} + q_c$

$$A_1 \mathscr{F}_{12}\, \sigma (T_1^4 - T_2^4) = A_2\, \sigma (T_2^4 - T_s^4) + h_c A_2 (T_2 - T_s)$$

The natural convection heat transfer coefficient, h_c, depends on the temperature T_2, therefore, an iterative solution is required. For the first iteration, let T_2 = 296 K.

From Appendix 2, Table 28, for dry air at the film temperature of (296 K + 300 K)/2 = 295 K = 25°C

Thermal expansion coefficient (β) = 0.00336 1/K
Thermal conductivity (k) = 0.0255 W/(m K)
Kinematic viscosity (ν) = 16.2×10^{-6} m²/s
Prandtl number (Pr) = 0.71

The Nusselt number for 3-D bodies is given by Equation (8.25)

$$Nu^+ = 5.75 + 0.7511\left[\frac{Ra^+}{F\ \ Pr}\right]0.252$$

$$\text{where}\quad F(Pr) = \left[1+\left(\frac{0.49}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}} = 2.876$$

$$L^+ = \frac{A}{\left(\frac{4A_{\text{horz}}}{\pi}\right)^{0.5}} = \frac{\pi D_2^2}{\left[\frac{4}{\pi}\left(\frac{\pi}{4}D_2^2\right)\right]^{0.5}} = \pi D = \pi(1.2\text{m}) = 3.77\text{ m}$$

The Rayleigh number is

$$Ra^+ = Gr^+Pr = \frac{g\beta\ \Delta T\ \ (L^+)^3\ Pr}{\nu^2} = \frac{(9.8\,\text{m/s}^2)\ \left(0.00336\,\frac{1}{\text{K}}\right)\ (4\,\text{K})\ (3.77\,\text{m})^3\ (0.71)}{(16.2\times10^{-6}\,\text{m}^2/\text{s})^2} = 1.91\times10^{10}$$

Although this is outside of the Rayleigh number range for the above correlation, the correlation will be used to estimate the Nusselt number for lack of a better method

$$Nu^+ = 5.75 + 0.75\left(\frac{1.91\times10^{10}}{2.876}\right)^{0.252} = 229$$

$$h_c = Nu^+\ \frac{K}{L^+} = 229\ \frac{0.0255\,\text{W/(m K)}}{3.77\,\text{m}} = 1.55\ \text{W/(m}^2\text{K)}$$

Using this value in the heat balance

$$\pi(1\text{m})^2(0.129)\ \ (5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4))\ \ [(78\ \text{K})^4 - T_2^4] = \pi(1.2\text{m})^2$$

$$\left[\ (5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4))\ [T_2^4 - (300\,\text{K})^4] + (1.55\,\text{W/(m}^2\text{K)})\ (T_2 - 300\,\text{K})\right]$$

$$2.795\times10^{-7}\ T_2^4 + 7.01\ T_2 - 4182 = 0$$

By trial and error

$$T_2 = 295\ \text{K}$$

The effect of natural convection is negligible

$$\dot{m} = 3.1\ \text{kg/h}$$

PROBLEM 11.50

An 0.2-m-*OD* oxidized steel pipe at a surface temperature of 756 K passes through a large room in which the air and the walls are at 38°C. If the heat transfer coefficient by convection from the surface of the pipe to the air in the room is 28 W/(m² K), estimate the total heat loss per meter length of pipe.

GIVEN

- An oxidized steel pipe passes through a large room
- Pipe outside diameter (D) = 0.2 m
- Pipe surface temperature (T_s) = 756 K
- Air and wall temperature (T_∞) = 38°C = 311 K
- Convective heat transfer coefficient (h_c) = 28 W/(m² K)

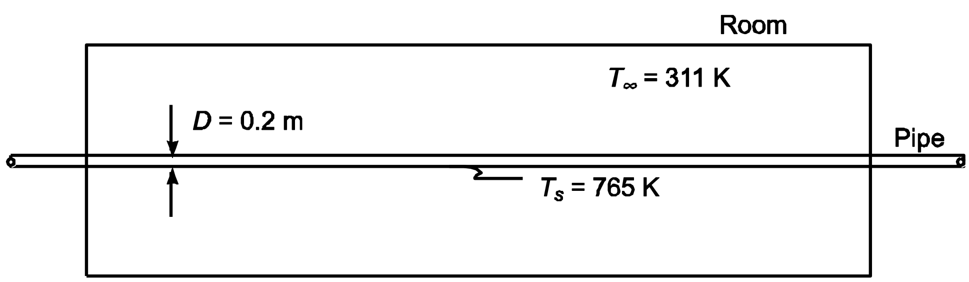
FIND

- The total heat loss per meter of pipe (q/L)

ASSUMPTIONS

- Steady state
- The walls of the room are black (ε_w = 1.0)
- The H_2O and CO_2 in the room air are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m² K⁴)
From Table 9.2, the emissivity of oxidized steel (ε_s) ≈ 0.80

SOLUTION

The total rate of heat transfer is the sum of the convective and radiative rates

$$q = h_c A_t (T_s - T_\infty) + \sigma \varepsilon_s A (T_s^4 - T_\infty^4)$$

where $A = \pi D L$

$$\frac{q}{L} = \pi D [h_c(T_s - T_\infty) + \sigma \varepsilon_s (T_s^4 - T_\infty^4)]$$

$$\frac{q}{L} = \pi (0.2\text{m}) \left[28\,\text{W/(m}^2\text{K)}\ (756\,\text{K} - 311\,\text{K}) + 5.67 \times 10^{-8}\,\text{W/(m}^2\text{K}^4)\ (0.80)\left[(756\,\text{K})^4 - (311\,\text{K})^4\right]\right]$$

$$\frac{q}{L} = 1.68 \times 10^4 \text{ W/m}$$

PROBLEM 11.51

A 6-mm-thick sheet of polished 304 stainless steel is suspended in a comparatively large vacuum-drying oven with black walls. The dimensions of the sheet are 30 cm × 30cm, and its specific heat is 565 J/(kg K). If the walls of the oven are uniformly at 150°C and the metal is to be heated from 10 to 120°C, estimate how long the sheet should be left in the oven if (a) heat transfer by convection may be neglected and (b) the heat transfer coefficient is 3 W/(m² K).

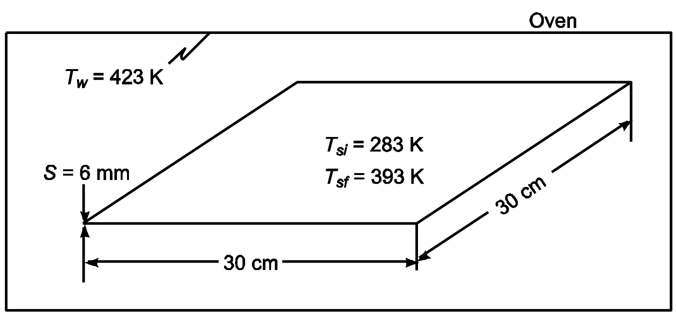
GIVEN

- A sheet of polished stainless steel in a large vacuum drying oven with black walls
- Sheet thickness (s) = 6 mm = 0.006 m
- Sheet dimensions = 30 cm × 30 cm = 0.3 m × 0.3 m
- Specific heat of the sheet (c) = 565 J/(kg K)
- Oven wall temperature (T_w) = 150°C = 423 K
- Sheet temperatures
    - Initial (T_{si}) = 10°C = 283 K
    - Final (T_{sf}) = 120°C = 393 K

FIND

- How long the sheet should be left in the oven if (a) convection may be neglected and, (b) the convective heat transfer coefficient (h_c) = 3 W/(m² K)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m² K⁴)

From Appendix 2, Table 10, the thermal conductivity of type 304 stainless steel (k_s) = 14.4 W/(m K) and its density (ρ_s) = 7817 kg/m³

From Table 11.3, the emissivity of polished stainless steel at the average temperature of 65°C (338 K) (ε_g) = 0.15

SOLUTION

(a) Neglecting convection, the rate of heat transfer is given by

$$q_r = \sigma\, \varepsilon\, A\, (T_w^4 - T_s^4)$$

The radiative heat transfer coefficient is given by Equation (11.89)

$$h_r = \frac{q_r}{A\; T_w - T_s} = \sigma\, \varepsilon \left[\frac{T_w{}^4 - T_s{}^4}{T_w - T_s} \right]$$

For the final conditions

$$h_{rf} = 5.67\times10^{-8}\ \mathrm{W/(m^2K^4)}\ (0.15) \left[\frac{(423)^4 - (393\mathrm{K})^4}{(423\mathrm{K} - 393\mathrm{K})} \right] = 2.31\ \mathrm{W/(m^2K)}$$

For the initial conditions

$$h_{rf} = 1.56\,\text{W/(m}^2\text{K)}$$

The Biot number based on half of the sheet thickness is

$$Bi_{\max} = \frac{\overline{h}_{r,\max}\, s}{2\,K_s} = \frac{2.31\,\text{W/(m}^2\text{K)}\ (0.006\,\text{m})}{2\ 14.4\,\text{W/(m K)}} = 0.0005 << 0.1$$

Therefore, the internal thermal resistance of the steel sheet may be neglected. The temperature change of the sheet over a small time step is given by

$$\Delta T = \frac{q\Delta T}{m\,c} = \frac{\sigma\,\varepsilon\,A\ T_w^4 - T_s^4\ \Delta T}{\rho(\text{volume})\,c} = \frac{2\,\sigma\,\varepsilon\ T_w^4 - T_s^4\ \Delta T}{\rho\,s\,c}$$

$$\Delta T = \frac{2\ 5.67\times 10^{-8}\,\text{W/(m}^2\text{K}^4)\ (0.15)\ \text{J/(Ws)}}{7817\,\text{kg/m}^3\ (0.006\,\text{m})\ 565\,\text{J/(kg K)}}\,[(423\text{ K})^4 - T_s^4]\,\Delta t = (0.0206 - 6.42\times 10^{-13}\,T_s^4)\,\Delta t$$

As the plate heats up, the rate of heat transfer will diminish. Therefore, the following numerical solution will be followed until $T_s = T_{sf}$:

1. Let Δt = 20 min = 1200 s
2. Calculate ΔT using T_{si}
3. Update T_s: $T_s = T_{si} + \Delta T$
4. Use the new T_s to calculate a new ΔT and repeat the procedure

t (min)	ΔT (K)	T_s (K)
0	---	282
20	19.8	301.8
40	18.3	320.1
60	16.6	336.6
80	14.8	351.4
100	12.9	364.3
120	11.1	375.4
140	9.4	384.4
160	7.8	392.5
162	0.6	393.1

The time required = 162 min = 2.7 hours.

(b) The rate of heat transfer by radiation and convection is

$$q = q_c + q_r = h_c\,A\,(T_w - T_s) + \sigma\,\varepsilon\,A\,(T_w^4 - T_s^4)$$

$$\Delta T = \frac{(q_c + q_r)\,\Delta T}{m\,c} = \frac{2\,h_c\,(T_w - T_s) + 2\,\sigma\,\varepsilon\,(T_w^4 - T_s^4)}{\rho\,s\,c}\,\Delta t$$

$$\Delta T = \frac{2\ 3\text{W/(m}^2\text{K)}\ \ 423\text{K} - T_s\ + 2\ 5.67\times 10^{-8}\,\text{W/(m}^2\text{K}^4)\ \ 0.15\left[\ 423\text{K}^{\,4} - T_s^{\,4}\right]}{7817\,\text{kg/m}^3\ \ 0.006\,\text{m}\ \ 565\,\text{J/(kg K)}\ \ (\text{Ws})/\text{J}}\,\Delta t$$

$$\Delta T = (-6.419\times 10^{-13}\,T_s^4 - 0.000226\,T_s + 0.1163)\ \ \text{K/s}\ \ \Delta t$$

Following the procedure of part (a), Let Δt = 10 min initially

t (min)	ΔT (K)	T_s (K)
0	---	282
10	29.1	311.1
20	24.0	335.1
30	19.5	354.6
40	15.6	370.2
50	12.3	382.5
60	9.7	392.1
61	0.88	393.1

Time required = 61 min = 1.02 hour.

PROBLEM 11.52

Calculate the equilibrium temperature of a thermocouple in a large air duct if the air temperature is 1367 K, the duct-wall temperature 533 K, the emissivity of the thermocouple 0.5, and the convective heat transfer coefficient, h_c, is 114 W/(m^2 K).

GIVEN

- A thermocouple in a large air duct
- Air temperature (T_a) = 1367 K
- Duct wall temperature (T_d) = 533 K
- The emissivity of the thermocouple (ε_{tc}) = 0.5
- The convective heat transfer coefficient (h_c) = 114 W/(m^2 K)

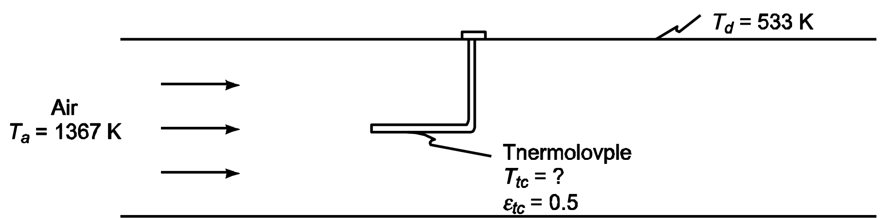
FIND

- The equilibrium temperature of the thermocouple (T_{tc})

ASSUMPTIONS

- Conduction along the thermocouple is negligible
- The walls of the duct are black
- The CO_2 and H_2O in the air are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

In steady state, the heat gain by convection must equal the heat loss by radiation

$$h_c\, A\, (T_a - T_{tc}) = \sigma\, \varepsilon_{tc}\, A\, (T_{tc}^4 - T_d^4)$$

$$114\,\text{W/(m}^2\text{K)} \quad (1367\text{K} - T_{tc}) = \quad 5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4) \quad (0.5)\, [T_{tc}^4 - (533\text{ K})^4]$$

Checking in units, then eliminating them for clarity

$$2.835 \times 10^{-8}\, T_{tc}^4 + 114\, T_{tc} - 158{,}126 = 0$$

By trial and error

T_{tc} = 1066 K.

COMMENTS

Assuming the purpose of the thermocouple is to measure the temperature of the air flowing in the duct, we have an error of 301 K. This so-called thermocouple radiation error can be reduced by increasing the convective heat transfer coefficient via higher air velocity, by reducing the thermocouple emissivity, or by the addition of a radiation shield, see Problem 11.53.

PROBLEM 11.53

Repeat Problem 11.52 with the addition of a radiation shield with emissivity ε_s = 0.1.

GIVEN

- A thermocouple surrounded by a radiation shield in a large air duct
- Air temperature (T_s) = 1367 K
- Duct wall temperature (T_d) = 533 K
- The emissivity of the thermocouple (ε_{tc}) = 0.5
- The convective heat transfer coefficient (h_c) = 114 W/(m^2 K)
- Shield emissivity (ε_s) = 0.1

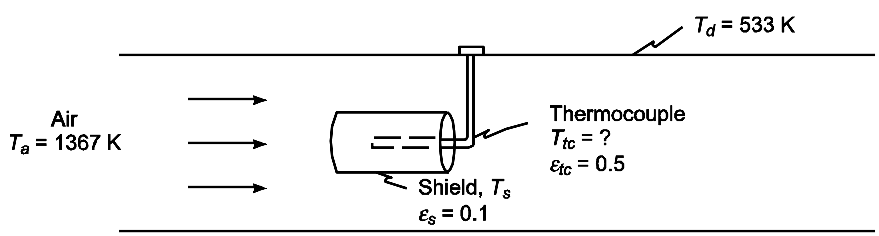
FIND

- The equilibrium temperature of the thermocouple (T_{tc})

ASSUMPTIONS

- Conduction along the thermocouple is negligible
- The walls of the duct are black
- The CO_2 and H_2O in the air are negligible
- The heat transfer coefficient on the inside and outside of the shield is h_c
- The conductive thermal resistance of the shield is negligible
- The view factor between the shield and the thermocouple ≈ 1
- The surface area of the shield is large compared to that of the thermocouple
  Shield and thermocouple are gray
- The thermocouple and shield can be approximated by infinitely long concentric cylinders
- Convective heat transfer between the shield and thermocouples is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/ (m^2 K^4)

SOLUTION

Let A_s = the inside area of the shield ≈ the outside area of the shield.
A heat balance on the radiation shield yields

$$q_s + \overline{h_{cs}}\, 2\, A_s\, (T_a - T_s) = \sigma\, \varepsilon_s\, A_s\, (T_s^4 - T_d^4)$$

Where q_s is the radiative heat transfer to the shield from the thermocouple which is given by Equation (11.74) for long concentric cylinders

$$q_s = \frac{A_{tc}\ E_{btc} - E_{bs}}{\dfrac{1}{\varepsilon_{tc}} + \dfrac{A_{tc}}{A_s}\left(\dfrac{1-\varepsilon_s}{\varepsilon_s}\right)} \qquad \text{but } \frac{A_{tc}}{A_s} << 1$$

$$\therefore \quad q_s = A_{tc}\, \varepsilon_{tc}\, \sigma\, (T_{tc}^4 - T_s^4)$$

Substituting this into the energy balance and dividing by A_s

$$\frac{A_{tc}}{A_s}\,\varepsilon_{tc}\,\sigma\,(T_{tc}^4 - T_s^4) + 2\,\overline{h_{cs}}\,(T_a - T_s) = \sigma\,\varepsilon_s\,(T_s^4 - T_d^4)$$

$$\therefore \qquad 2\,h_{cs}\,(T_a - T_s) \approx \sigma\,\varepsilon_s\,(T_s^4 - T_d^4)$$

This shown that since the thermocouple is small compared to the shield, the effect of the thermocouple wire on the shield temperature can be neUglected

$$2\ 114\,\mathrm{W/(m^2K)}\ \ (1367\mathrm{K} - T_s) = \ 5.67\times10^{-8}\,\mathrm{W/(m^2K^4)}\ \ (0.1)\ [T_s^4 - (533\ \mathrm{K})^4]$$

$$5.67 \times 10^{-7}\ T_s^4 + 228\ T_{tc} - 312{,}134 = 0$$

By trial and error

$$T_s = 1298\ \mathrm{K}.$$

Performing a heat balance on the thermocouple

$$\overline{h_c}\,A_{tc}\,(T_s - T_{tc}) = \sigma\,\varepsilon_{tc}\,A_{tc}\,(T_{tc}^4 - T_s^4)$$

$$114\,\mathrm{W/(m^2K)}\ \ (1367\mathrm{K} - T_{tc}) = \ 5.67\times10^{-8}\ \mathrm{W/(m^2K^4)}\ \ (0.5)\ [T_{tc}^4 - (1298\ \mathrm{K})^4]$$

$$2.835 \times 10^{-8}\ T_{tc}^4 + 114\ T_{tc} - 236{,}311 = 0$$

By trial and error

$$T_{tc} = 1319\ \mathrm{K}.$$

COMMENTS

The thermocouple error has been reduced from 301 K to 48 K by use of the radiation shield.

PROBLEM 11.54

A thermocouple is used to measure the temperature of a flame in a combustion chamber. If the thermocouple temperature is 1033 K and the walls of the chamber are at 700 K, what is the error in the thermocouple reading due to radiation to the walls? Assume all surfaces are black and the convection coefficient is 568 W/(m² K) on the thermocouple.

GIVEN

- A thermocouple in a combustion chamber flame
- Thermocouple temperature (T_{tc}) = 1033 K
- Chamber wall temperature (T_w) = 700 K
- Convection coefficient (h_c) = 568 W/(m² °C)
- All surfaces are black ($\varepsilon = 1.0$)

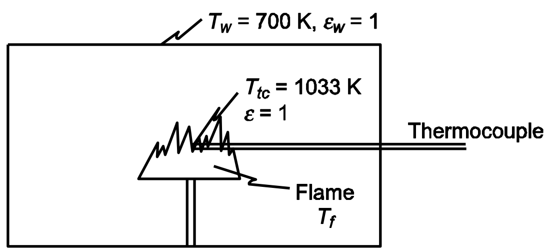
FIND

- The error in the thermocouple reading due to radiation to the walls

ASSUMPTIONS

- Conduction along the thermocouple is negligible
- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m² K⁴)

SOLUTION

In steady state, the rate of heat gain by convection from the flame must equal the heat loss by radiation to the walls:

$$h_c A (T_f - T_{tc}) = \sigma \varepsilon A (T_{tc}^4 - T_w^4) \qquad [\varepsilon = 1]$$

Solving for the flame temperature

$$T_f = T_{tc} + \frac{\sigma}{h_c}(T_{tc}^4 - T_w^4) = 1033\text{ K} + \frac{5.678\times10^{-8}\text{ W/(m}^2\text{K}^4)}{568\text{ W/(m}^2\text{K)}}\left[(1033\text{K})^4 - (700\text{K})^4\right] = 1123\text{ K}$$

$$\text{Error} = T_f - T_{tc} = 1123 - 1033 = 90\text{ K}$$

PROBLEM 11.55

A metal plate is placed in the sunlight. The incident radiant energy G is 780 W/m^2. The air and the surroundings are at 10°C. The heat transfer coefficient by natural convection from the upper surface of the plate is 17 W/(m^2 K). The plate has an average emissivity of 0.9 at solar wavelengths and 0.1 at long wavelengths. Neglecting conduction losses on the lower surface, determine the equilibrium temperature of the plate.

GIVEN

- A metal plate is sunlight
- Incident radiant energy (G) = 780 W/m^2
- Temperature of air and surroundings (T_∞) = 10°C = 283 K
- Natural convection heat transfer coefficient (h_c) = 17 W/(m^2 K)
- Plate emissivity (ε) = 0.9 at solar wavelengths, 0.1 at long wavelengths

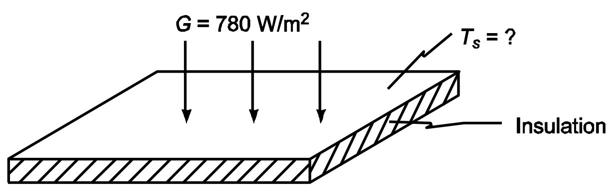
FIND

- The equilibrium temperature of the plate (T_p)

ASSUMPTIONS

- Steady state
- Conduction losses on the lower surface of the plate are negligible
- The surroundings behave as a blackbody enclosure

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The plate will absorb the solar radiation with the absorptivity (α) = ε = 0.9 according to Kirchoff's Law. However, it will radiate to its surroundings at longer infrared wavelengths with ε = 0.1. The heat gain from solar radiation must equal the heat flux loss by radiation and convection at steady state

$$\alpha G = h_c (T_p - T_\infty) + \sigma \varepsilon (T_p^4 - T_\infty^4)$$

$$(0.9)\ \ 780\,\text{W/m}^2 = 17\,\text{W/(m}^2\text{K)}\ (T_p - 283\text{K}) + 5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\ (0.1)\,[T_p^4 - (283\text{ K})^4]$$

$$5.67 \times 10^{-9}\, T_p^4 + 17\, T_p - 5549.37 = 0$$

By trial and error

$$T_p = 323 \text{ K} = 50°\text{C}$$

PROBLEM 11.56

A 60 cm-square section of panel heater is installed in the corner of the ceiling of a room having a 2.7 m-by-3.6 m floor area with an 2.4 m ceiling. If the surface of the heater, made from oxidized iron, is at 147°C and the walls and the air of the room are at 20°C in the steady state, determine (a) the rate of heat transfer to the room by radiation, (b) the rate of heat transfer to the room by convection (h_c = 11 W/(m^2 K), (c) the cost of heating the room per day if the cost of electricity is 7 cents per kWh.

GIVEN

- A 60 cm square panel heater in the corner of the ceiling of a room
- Room dimensions: 2.7 m× 3.6 m × 2.4 m high
- Heater has oxidized iron surface
- Surface temperature (T_s) = 147°C= 420 K
- Room air and walls (T_∞) = 20°C = 293 K
- Convective heat transfer coefficient = 11 W/(m^2 K)

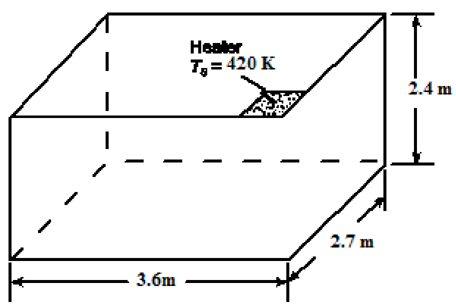
FIND

(a) Rate of radiative heat transfer to the room (q_r)
(b) Rate of convective heat transfer to the room (q_c)
(c) Cost of heating the room at $0.07/kWhr

ASSUMPTIONS

- The walls of the room are black (ε_w = 1)
- Steady state conditions
- Effect of H_2O and CO_2 in the air is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)
From Table 9.2, the emissivity of cast oxidized iron at 420 K (ε_s) = 0.64

SOLUTION

(a) Since the view factor of the heater to the room is unity, the rate of heat transfer by radiation is

$$q_r = \sigma\, \varepsilon_s\, A\, (T_s^4 - T_\infty^4)$$

where A = area of heater = 0.6 m * 0.6 m = 0.36 m^2

$$q_r = \left(5.67\times10^{-8}\, W/(\mathrm{m}^2\ \mathrm{K}^4)\right)(0.64)\,(0.36\ \mathrm{m}^2)\,[(420\ \mathrm{K})^4 - (293\ \mathrm{K})^4] = 310\ \mathrm{W}$$

(b) The rate of heat transfer by convection is

$$q = h_c A (T_s - T_\infty) = \left(11 W/(\mathrm{m}^2\ \mathrm{K})\right) 0.36\ \mathrm{m}^2 * (420\ \mathrm{K} - 293\ \mathrm{K}) = 503\ \mathrm{W}$$

(c) $$\text{Cost} = (q_r + q_c)\ (\text{energy cost})$$

$$\text{Cost} = \left((310+503)\ W\right)\left(0.07\ \$/(\mathrm{kWh})\right)\left(\mathrm{kW}/(1000\,\mathrm{W})\right)\left(24\mathrm{h}/\mathrm{day}\right)$$

$$= \$\ 19.5/\mathrm{day}$$

PROBLEM 11.57

In a manufacturing process, a fluid is transported through a cellar maintained at a temperature of 300 K. The fluid is contained in a pipe having an external diameter of 0.4 m and whose surface has an emissivity of 0.5. To reduce heat losses, the pipe is surrounded by a thin shielding pipe having an *ID* of 0.5 m and an emissivity of 0.3. The space between the two pipes is effectively evacuated to minimize heat losses and the inside pipe is at a temperature of 550 K. (a) Estimate the heat loss from the liquid per meter length, (b) If the fluid inside the pipe is an oil flowing at a velocity of 1 m/s, calculate the length of pipe for a temperature drop of 1 K.

GIVEN

- Fluid in concentric pipes, with the space between the pipes evacuated, running through a cellar space
- Cellar temperature (T_∞) = 300 K
- External diameter of inner pipe (D_1) = 0.4 m
- Emissivity of outer pipe surface (ε_1) = 0.5
- Inside diameter of outer pipe (D_2) = 0.5 m
- Emissivity of inner pipe (ε_2) = 0.3
- Inside pipe temperature (T_1) = 550 K

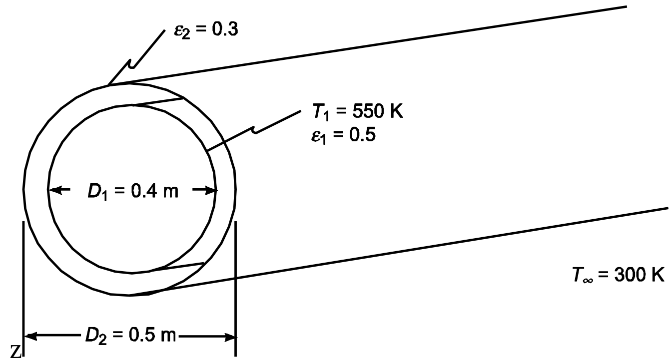
FIND

(a) The heat loss from the liquid per meter length (q/L)
(b) The length of pipe for a temperature drop of 1 K if the fluid is oil flowing at a velocity (V) = 1 m/s

ASSUMPTIONS

- Steady state
- Convection between the pipes is negligible
- The thermal resistance of the pipe walls is negligible
- The thickness of the outer pipe wall is negligible (Inside surface area ≈ Outside surface area)
- Area of the cellar is large compared to the pipe so that cellar behaves as a blackbody enclosure at T_∞
- Oil has the thermal properties of unused engine oil
- The temperature of the inner pipe is constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)
Extrapolating Appendix 2, Table 16, for unused engine oil at 550 K

Density (ρ) = 742 kg/m^3

Specific heat (c) 2998 J/(kg K)

SOLUTION

The rate of heat transfer between the pipes is given by Equation (11.75)

$$q_{12} = A_1 \mathscr{F}_{12} (E_{b1} - E_{b2}) = A_1 \mathscr{F}_{12}\, \sigma (T_1^4 - T_2^4)$$

where F_{12} is given for infinite concentric cylinders by Equation (11.76)

$$\mathscr{F}_{12} = \frac{1}{\dfrac{1-\varepsilon_1}{\varepsilon_1} + 1 + \dfrac{A_1}{A_2}\dfrac{1-\varepsilon_2}{\varepsilon_2}} = \frac{1}{\dfrac{1-0.5}{0.5} + 1 + \left(\dfrac{\dfrac{\pi}{4}\ 0.4\,\mathrm{m}\ L}{\dfrac{\pi}{4}\ 0.5\,\mathrm{m}\ L}\right)\dfrac{1-0.3}{0.3}} = 0.259$$

The rate of heat transfer from the outer pipe to the surroundings is the sum of the rates of convective and radiative heat transfer

$$q_{2\infty} = \overline{h_c}\, A_2 (T_2 - T_\infty) = \sigma\, \varepsilon_2\, A_2 (T_2^4 - T_\infty^4)$$

An energy balance on the outer pipe yields

$$q_{12} = q_{2\infty}$$

$$\mathscr{F}_{12}\, \sigma (T_1^4 - T_2^4) = \overline{h_c}\ \frac{A_2}{A_1} (T_2 - T_\infty) + \sigma\, \varepsilon_2 \frac{A_2}{A_1} (T_2^4 - T_\infty^4)$$

$$0.259\ \ 5.67\times10^{-8}\,\mathrm{W/(m^2K^4)}\ \ [(550\ \mathrm{K})^4 - T_2^4]$$

$$= \overline{h_c}\left(\frac{0.5}{0.4}\right) (T_2 - 300\mathrm{K}) +\ \ 5.67\times10^{-8}\,\mathrm{W/(m^2K^4)}\ \ (0.3)\ \left(\frac{0.5}{0.4}\right) [T_2^4 - (300\ \mathrm{K})^4]$$

Checking the units, then eliminating them for clarity

$$3.595 \times 10^{-8}\, T_2^4 + 1.25\, h_c\, T_2 - 375\, h_c - 375\, h_c - 1516 = 0$$

Since the value of the natural convection heat transfer coefficient, h_c, depends on T_2, an iterative solution must be used. For the first iteration, let T_2 = 400 K.

The Grashof number is

$$Gr_D = \frac{g\beta\ \Delta T\ \ D^3}{\nu_a^{\ 2}} = \frac{9.8\,\mathrm{m/s^2}\ \ (0.00283\,1/\mathrm{K})(100\,\mathrm{K})(0.5\,\mathrm{m})^3}{\left(21.5\times10^{-6}\ \mathrm{m^2/s}\right)^2} = 7.5 \times 10^8$$

$$Gr_D\, Pr = 7.5 \times 10^8\, (0.71) = 5.32 \times 10^8$$

The Nusselt number for this geometry is given by Equation (8.20)

$$\overline{Nu_D} = 0.53\ (Gr_D\ Pr)^{\frac{1}{4}} = 0.53\ (5.32\times10^8)^{\frac{1}{4}} = 80.5$$

$$\overline{h_c} = \overline{Nu_D}\ \frac{k}{D} = 80.5\,\frac{0.0293\,\mathrm{W/(m\,K)}}{0.5\,\mathrm{m}} = 4.72\,\mathrm{W/(m^2K)}$$

Substituting this value into the energy balance yields

$$3.595 \times 10^{-8}\, T_2^4 + 5.896\, T_2 - 3285 = 0$$

By trial and error

$$T_2 = 400\ \mathrm{K}$$

(a) The rate of heat transfer from the liquid is

$$\frac{q}{L} = \pi D_1\, \mathscr{F}_{12}\, \sigma (T_1^4 - T_2^4) = \pi (0.4 \text{ m}) (0.259) \quad 5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4) \quad [(550 \text{ K})^4 - (400 \text{ K})^4]$$

$$\frac{q}{L} = 1216 \text{ W/m}$$

(b) The length of pipe for a temperature drop (ΔT) of 1 K can be determined from the following

$$\frac{q}{L} = \frac{\dot{m}\, c\, \Delta T}{L} = \frac{\rho V A_c\, c \Delta T}{L} = \frac{\rho V \left(\frac{\pi}{4}\right) D_1^2\, c \Delta T}{L}$$

Solving for L

$$L = \frac{\pi \rho V D_1^2\, c \Delta T}{4\left(\frac{q}{L}\right)} = \frac{\pi \quad 742 \text{ kg/m}^3 \quad 1 \text{ m/s} \quad 0.4 \text{ m}^{\ 2} \quad 2998 \text{ J/(kg K)} \quad 1 \text{K}}{4 \quad 1216 \text{ W/m} \quad \text{J/(Ws)}} = 230 \text{ m}$$

PROBLEM 11.58

Forty-five kilograms of carbon dioxide is stored in a high-pressure cylinder that is 25 cm in diameter (*OD*), 1.2 m long and 1.2 cm-thick. The cylinder is fitted with a safety rupture diaphragm designed to fail at 14 MPa (with the specified charge, this pressure will be reached when the temperature increases to 50°C). During a fire, the cylinder is completely exposed to the irradiation from flames at 1097°C (ε = 1.0). For the specified conditions, c = 2.5 kJ/(kg K) for CO_2. Neglecting the convective heat transfer, determine the length of the time the cylinder may be exposed to this irradiation before the diaphragm will fail if the initial temperature is 21°C and (a) the cylinder is bare oxidized steel(ε = 0.79), (b) the cylinder is painted with aluminum paint (ε = 0.30).

GIVEN

- CO_2 in a high pressure cylinder exposed to flames
- Mass of CO_2 (mg) = 45 kg
- Cylinder dimensions
  - Outside Diameter (D) = 25 cm = 0.25 m
  - Length (L) = 1.2 m
  - Thickness (s) = 1.2 cm = 0.012 m
- Rupture diaphragm fails at 14 MPa (T_{gf} = 50°C = 323 K)
- Temperature of flames (T_f) = 1097°C = 1370 K (ε_f = 1.0)
- Specific heat of CO_2 (c_v) = 2.5 kJ/(kg K)
- Initial temperature (T_{gf}) = 21°C = 394 K

FIND

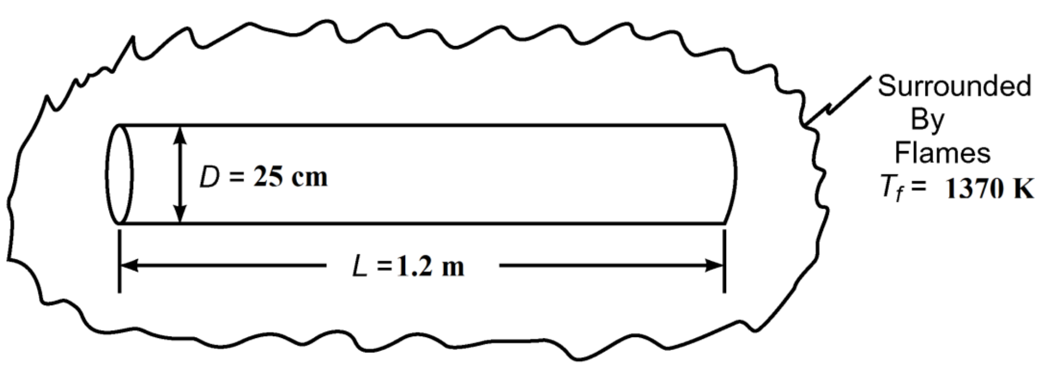
The time for the diaphragm to fail if the cylinder is

(a) bare oxidized steel (ε_s = 0.79) or
(b) painted with aluminum paint (ε_s = 0.30)

ASSUMPTIONS

- Convective heat transfer is negligible
- Cylinder is 1% carbon steel
- Irradiation is constant and uniform over the entire cylinder
- Quasi-steady state
- Thermal resistance between the gas and the cylinder is negligible ($T_s = T_g$)
- Variation of specific heat of gas and cylinder with temperature is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

From Appendix 2, Table 10, for 1% carbon steel at 21°C.

Density (ρ_s) = 7801 kg/m^3

Specific heat (c_s) = 473 J/(kg K)

SOLUTION

The exterior surface area of the cylinder is

$$A_s = \pi D L + 2\ \frac{\pi}{4} D_2 = \frac{\pi}{2} D\,(2\,L + D) = \left(\frac{\pi}{2}\right)(0.25\text{ m})\left(2(1.2\text{ m})+(0.25\text{ m})\right) = 1.041\text{ m}^2$$

The mass of steel in the cylinder is

$$m_s = \rho_s\,(\text{volume}) = \rho_s\,A_s\,s = \left(7801\,kg/m^3\right)(1.041\text{ m}^2)\ (0.012\ \ m) = 97.5\text{ kg}$$

Performing an energy balance on the gas

heat input = rate of increase in enthalpy

$$\sigma\,\varepsilon_s\,A_s\,(T_f^4 - T_g^4) = (m_g\,c_v + m_s\,c_s)\ \frac{dT_g}{d\,t} \qquad [T_g = T_s]$$

Solving for the rate of change of the gas temperature

$$\frac{dT_g}{d\,t} = \frac{\sigma\,\varepsilon_s\ A_s}{m_g\ c_v + m_s\ c_a}\ (T_f^4 - T_g^4)$$

$$\frac{dT_g}{d\,t} = \frac{\left(5.67\times10^{-8}\ W/(m^2K^4)\right)\varepsilon_s\left(1.041m^2\right)}{(45kg)(2500\,J/(kgK))+(97.5kg)(473\,J/(kgK))}\ [(1370\text{ K})^4 - T_g^4]$$

$$\frac{dT_g}{d\,t} = 3.72\times10^{-13}\ \varepsilon_s\,(3.52\times10^{12} - T_g^4)\ \ \text{K}/s$$

Case (a)

Initially $$\frac{dT_g}{d\,t} = 3.72\times10^{-13}\,(0.79)[3.52\times10^{12} - (294\text{ K})^4] = 1.035\text{ K}/s$$

Finally $$\frac{dT_g}{d\,t} = 3.72\times10^{-13}\,(0.79)[3.52\times10^{12} - (323\text{ K})^4] = 6601°\text{F/h} = 1.03\text{ K/s}$$

The rate of change of the gas temperature is essentially constant, therefore, the time required for the gas to reach 120°F is

$$t = \frac{T_{gf} - T_{go}}{\frac{dT}{d\,t}} = \frac{323K - 294K}{\left(\frac{1.035\text{K}}{\text{s}}\right)} = 28\text{ s}$$

Case (b)

$$\frac{dT_g}{d\,t} = 3.72\times10^{-13}\,(0.30)[3.52\times10^{12} - (294\text{ K})^4] = 0.39\text{ K/s}$$

$$t = \frac{323\text{ K} - 294K}{0.39\,K/\text{s}} = 74\text{ s}$$

PROBLEM 11.59

A hydrogen bomb may be approximated by a fireball at a temperature of 7200 K according to a report published in 1950 by the Atomic Energy Commission. (a) Calculate the total rate of radiant-energy emission in watts, assuming that the gas radiates as a blackbody and has a diameter of 1.5 km, (b) If the surrounding atmosphere absorbs radiation below 0.3 μm, determine the per cent of the total radiation emitted by the bomb that is absorbed by the atmosphere, (c) Calculate the rate of irradiation on a 1 m^2 area of the wall of a house 40 km from the center of the blast if the blast occurs at an altitude of 16 km and the wall faces in the direction of the blast, (d) Estimate the total amount of radiation absorbed assuming that the blast lasts approximately 10 sec and that the wall is covered by a coat of red paint, (e) If the wall were made of oak whose flammability limit is estimated to be 650 K and that had a thickness of 1 cm, determine whether or not the wood would catch on fire. Justify your answer by an engineering analysis stating carefully all assumptions.

GIVEN

- A hydrogen bomb fireball
- Fireball temperature (T_1) = 7200 K
- Surrounding atmosphere absorbs radiation below 0.3 μm
- The blast occurs at an altitude (H) of 16 km = 16,000 m

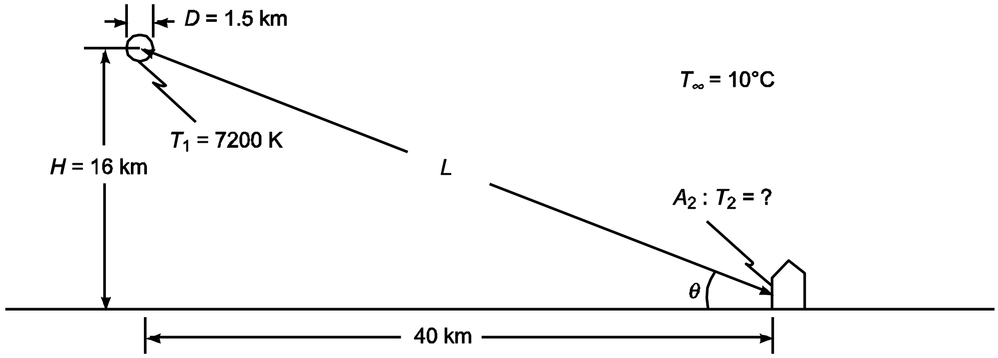
FIND

(a) The total rate of radiant-energy emission in watts (q_r)
(b) The per cent of the total radiation absorbed by te atmosphere
(c) The rate of irradiation on a 1 m^2 area of the wall of a house 40 km (40,000 m) from the center of the blast and facing the blast (G_2)
(d) Total amount of radiation absorbed if the blast lasts 10 seconds and the wall is covered with red paint
(e) If the walls are oak with a flammability limit of 650 K and a thickness (s) of 1 cm, will the wood catch fire?

ASSUMPTIONS

- The gas radiates as a blackbody
- Diameter of the fireball (D) = 1.5 km
- The air and surrounding temperature (T_∞) = 10°C
- The surroundings behave as a blackbody enclosure
- The heat transfer from the oak walls to its surroundings during the 10 seconds of irradiation can be neglected
- The house wall is initially at the surroundings temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

From Table 11.3 the emissivity of red paint at short wavelengths (ε_{2s}) = 0.74
the emissivity of red paint at long wavelengths (ε_{21}) = 0.97

From Appendix 2, Table 11, for oak
Specific heat (c) = 2390 J/(kg K)
Thermal conductivity (k_s) = 0.19 W/(m K)
Density (ρ) ≈ 700 kg/m^3
Thermal diffusivity (α_{th}) ≈ 0.011×10^{-5} m^2/s

SOLUTION

(a) The total rate of radiation emission is the blackbody emissive power, from Equation (11.3), times the area

$$q_1 = E_{b1}\, A = \sigma\, T_1^4\, \pi D^2 = \left(5.67\times10^{-8}\,\mathrm{W/(m^2K^4)}\right)(7200\text{ K})^4\, \pi(1500\text{ m})^2 = 1.08 \times 10^{15}\text{ W}$$

(b) For $\lambda = 0.3\ \mu$m, $T\lambda = (7200\text{ K})(0.3 \times 10^{-6}\text{ m}) = 2.16 \times 10^{-3}$ m K

The fraction of energy absorbed is the fraction, e of the total radiation below 0.3 μm which can be read directly from Table 11.2

$$\%\text{ absorbed by atmosphere} = \frac{E_{b1}(0 \to \lambda T)}{\sigma T^4} \times 100 = (0.09406)(100) = 9.4\%$$

(c) The distance between the house and the fireball center (L) is

$$L = \sqrt{(16\,\text{km})^2 + (40\,\text{km})^2} = 43\text{ km}$$

The energy calculated in part (a) will spread evenly in all directions from the fireball. Therefore, the flux at the distance $L = q_1/A_{sL}$ where A_{sL} is the surface area of a sphere of radius L

$$\frac{q_1}{4\,\pi\, L^2} = \frac{1.08\times10^{15}\,\text{W}}{4\,\pi\,(43{,}000\,\text{m})^2} = 46{,}480\ \text{W/m}^2$$

However, the atmosphere will absorb 9.4% of this energy.

$$\text{Energy flux at wall} = 46{,}480\ \left(46{,}480\,\text{W/m}^2\right)(1 - 0.094) = 42{,}110\ \text{W/m}^2$$

The angle between this flux and the (normal to the) wall surface, θ, is given by

$$\tan\theta = \frac{16\,\text{km}}{40\,\text{km}} \quad\Rightarrow\quad \theta = 21.8°$$

Therefore, the irradiation on the wall is

$$G_2 = (42{,}110\ \text{W/m}^2)\cos\theta = 39{,}100\ \text{W/m}^2$$

(d) By Kirchoff's law, the absorptivity (α_2) = ε_2

$$\text{Energy absorbed} = G_2\, \varepsilon_2\, t = 39{,}100\ \left(39{,}100\,\text{W/m}^2\right)\left(\text{J/(Ws)}\right)(0.74)\,(10\text{s})\left(\text{kJ/(1000J)}\right) = 289\ \text{kJ/m}^2$$

(e) The radiative heat transfer coefficient (hr) is given by

$$G_2 = \overline{h_r}\,(T_f - T_s) \quad\rightarrow\quad \overline{h_r} = \frac{G_2}{T_f - T_s}$$

Since $T_s \ll T_f$, the heat transfer coefficient will not vary much as the T_f changes. To estimate hr, let T_f = 500 K

$$\therefore \quad \overline{h_r} = \frac{39{,}100\ \mathrm{W/m^2}}{7200\,\mathrm{K} - 500\,\mathrm{K}} = 5.84\ \mathrm{W/(m^2K)}$$

The Biot number for the wall is

$$Bi = \frac{\overline{h_r}\, s}{2k} = \frac{5.84\ \mathrm{W/(m^2K)}\ (0.01\,\mathrm{m})}{2\ \ 0.19\,\mathrm{W/(m\,K)}} = 0.154 > 0.1$$

Therefore, the internal thermal resistance of the oak is significant and the chart solution for Figure 3.9 will be used to estimate the surface temperature of the oak after 10 seconds: ($L = s/2 = (0.01\ \mathrm{m})/2 = 0.005\ \mathrm{m}$)

The Fourier number is

$$Fo = \frac{\alpha_{th}\, t}{L^2} = \frac{0.011 \times 10^{-5}\ \mathrm{m^2/s}\ \ (10\,\mathrm{s})}{(0.005\,\mathrm{m})^2} = 0.044$$

$$\frac{1}{Bi} = 6.5$$

From Figure 3.9

$$\frac{T(0,t) - T_t}{T_o - T_t} \approx 1.0$$

(The center of the oak is still at the initial temperature after 10 s).

From Figure 3.9(b) for $x/L = 1.0$

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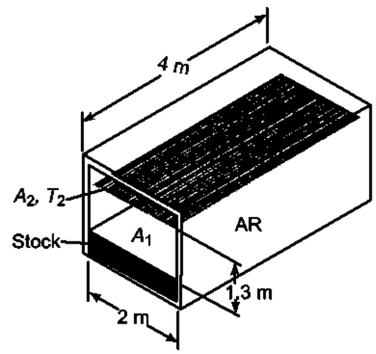
where $T(L,t)$ = the surface temperature of the wall after 10 seconds of exposure to the radiation.

$$\therefore \quad T(L,t) = T_f + 0.92\ (T_o - T_f) = 7200\mathrm{K} + 0.92\ (283\mathrm{K} - 7200\mathrm{K}) = 836\mathrm{K}$$

Therefore, the walls will catch on fire.

PROBLEM 11.60

An electric furnace is to be used for batch heating a certain material with specific heat of 670 J/(kg K) from 20 to 760°C. The material is placed on the furnace floor which is 2m × 4m in area as shown in the accompanying sketch. The side walls of the furnace are made of a refractory material. Parallel to the plane of the roof, but several inches below it, a grid of round resistor rods is installed. The resistors are 13 mm in diameter and are spaced 5 cm center to center. The resistor temperature is to be maintained at 1100°C, under these conditions the emissivity of the resistor surface is 0.6. If the top surface of the stock is assumed to have an emissivity of 0.9, estimate the time required for heating a 6 metric ton batch. External heat losses from the furnace may be neglected, the temperature gradient through the stock can be considered negligibly small, and steady-state conditions can be assumed.



GIVEN

- Batch heating of material in the furnace shown above
- Specific heat of material (c) = 670 Jkg K
- Material temperatures
  - Initial (T_{1i}) = 20°C = 293 K
  - Final (T_{hf}) = 760°C = 1033 K
- Furnace dimensions: 2 m × 4 m × 1.3 m high
- Side walls are refractory material
- Resistor rod diameter (D_r) = 13 mm = 0.013 m
- Resistor center to center distance (s) = 5 cm = 0.05 m
- Resistor temperature (T_2) = 1100°C = 1373 K
- Emissivity of the resistor surface (ε_2) = 0.6
- Emissivity of the material surface (ε_1) = 0.9
- Mass of material (m) = 6 metric tons = 6000 kg

FIND

- The time required (t) for heating the 6 metric ton batch

ASSUMPTIONS

- Quasi-steady state conditions
- External heat losses are negligible
- Temperature gradient through the material is negligible (negligible internal thermal resistance)
- Material is gray
- Convective heat transfer is negligible

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The shape factor F_{21} can be read off Figure 11.34: For s/D = 50/13 = 3.85 and one row: $F_{21} \approx 0.60$. Note that $A_1 = A_2$, therefore, $F_{21} = F_{12}$.

The sum of the shape factors from a given surface must sum to unity

$$F_{11} + F_{12} + F_{1R} = 1 \quad \rightarrow \quad F_{1R} = 1 - F_{12}$$

$$F_{21} + F_{12} + F_{2R} = 1 \quad \rightarrow \quad F_{2R} = 1 - F_{21} = 1 - F_{12}$$

The rate of radiative heat transfer, between two gray surfaces connected by re-radiating surfaces is given by Equation (11.80)

$$q_{12} = A_1 \mathscr{F}_{21}\, \sigma (T_2^4 - T_1^4)$$

where $A_2 f_{21}$ is given by Equation (11.79), note that $A_1 = A_2 = A$

$$A_1 \mathscr{F}_{21} = \frac{1}{\frac{1}{A}\left(\frac{1}{\varepsilon_2}-1\right)+\frac{1}{A}\left(\frac{1}{\varepsilon_1}-1\right)+\frac{1}{A\overline{F_{21}}}}$$

$$\text{where} \quad A\,\overline{F_{21}} = A\left(F_{21}+\frac{1}{\frac{1}{F_{2R}}+\frac{1}{AF_{1R}}}\right) = A\left[F_{12}+\frac{1-F_{12}}{2}\right]$$

$$A\,\mathscr{F}_{21} = \frac{1}{\left(\frac{1}{\varepsilon_1}-1\right)+\left(\frac{1}{\varepsilon_2}-1\right)+\left[F_{12}+\frac{1-F_{12}}{2}\right]} = \frac{(4\text{m})(2\text{m})}{\left(\frac{1}{0.9}-1\right)+\left(\frac{1}{0.6}-1\right)+\left[0.6+\frac{1-0.6}{2}\right]}$$

$$A\,\mathscr{F}_{21} = (8\text{m}^2)\,(0.634) = 5.07\text{ m}^2$$

The temperature changes in the material is given by

$$\Delta T_1 = \frac{q_{21}\Delta t}{mc} = \frac{A\mathscr{F}_{21}\sigma\left(T_2^4 - T_1^4\right)\Delta t}{mc}$$

$$\Delta T_1 = \frac{\left(5.07\,\text{m}^2\right)\left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\left[(373\,\text{K})^4 - T_1^4\right]\Delta t}{(6000\,\text{kg})\left(670\ \text{J/(kg K)}\right)\left(\text{(Ws)/J}\right)}$$

$$\Delta T_1 = \left[\left(0.2541(\text{K/s})\right) - \left(7.15\times10^{-14}\ 1/(\text{K}^3\text{s})\right) T_1^4\right]\Delta t$$

As T_1 increases, the rate of heat transfer will decrease. Therefore, the equation above will be solved for a chosen time increment and the temperature T_1 will then be updated. This procedure will be repeated until T_1 = 760°C = 1033 K.

Let Δt = 5 min = 300 s initially

	Time (min)	ΔT_1 (K)	T_1 (K)
	0	----	293
	5	76.1	369.1
	10	75.8	444.9
	15	75.4	520.3
	20	74.7	595.0
	25	73.5	668.5
	30	72.0	740.5
	35	69.8	810.3
Let Δt = 1 min	40	67.0	877.3
	45	63.5	940.8
	50	59.4	1000.2
	51	11.0	1011.2
	52	10.8	1021.9
	53	10.6	1032.5

The time required ≈ 53 min.

PROBLEM 11.61

A rectangular flat water tank is placed on the roof of a house with its lower portion perfectly insulated. A sheet of glass whose transmission characteristics are tabulated below is placed 1 cm above the water surface. Assuming that the average incident solar radiation is 630 W/m^2, calculate the equilibrium water temperature for a water depth of 12 cm if the heat transfer coefficient at the top of the glass is 8.5 W/(m^2 K) and the surrounding air temperature of 20°C. Disregard intereflections.

τ_λ of glass	**= 0 for wavelength from 0 to0.35 μm**
	= 0.92 for wavelength from 0.35 to 2.7 μm
	= 0 for wavelength larger than 2.7 μm
ρ_λ of glass	**= 0.08 for all wavelengths**

GIVEN

- A glass covered water tank on the roof of a house
- Lower portion of tank is perfectly insulated
- Distance between glass cover and water surface (δ) = 1 cm = 0.01 m
- Average incident solar radiation (I_s) = 630 W/m^2
- Water depth = 12 cm = 0.12 m
- Heat transfer coefficient on the top of the glass (h_{co}) = 8.5 W/(m^2 K)
- Surrounding air temperature (T_∞) = 20°C = 293 K
- Transmissivity of glass (λ_λ) = 0 for 0 < λ < 0.35 μm
  = 0.92 for 0.35 < λ < 2.7 μm
  = 0 for λ > 2.7 m m
- Reflectivity of Glass (ρ_λ) = 0.08

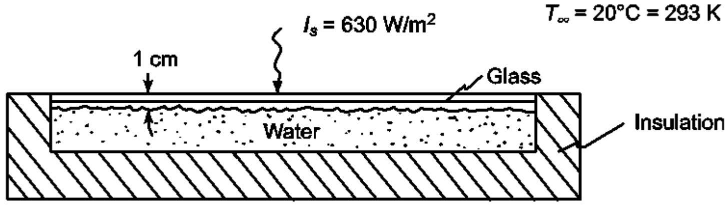
FIND

- The equilibrium temperature of the water (T_w)

ASSUMPTIONS

- The effect of inter-reflections is negligible
- The water temperature is uniform (internal resistance of the water is negligible)
- Steady state conditions
- I_s value given is normal to the glass surface
- The water absorbs all the radiation reaching it
- Water behaves as a blackbody
- The conductive thermal resistance of the glass is negligible
- The sky behaves as a blackbody enclosure at T_{sky} = 0 K
- The sun is blackbody at 6000 K (see Table 9.2)
- The shape factor between the surface and the glass can be taken to be unity
- The air properties are the same as dry air properties
- The glass acts as a black surface for the reradiated energy

SKETCH

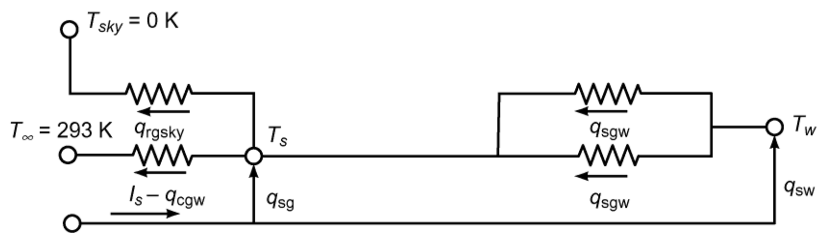


PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The thermal circuit for the problem is shown below



where q_r = radiative heat transfer flux

q_c = convective heat transfer flux

q_s = solar radiation

The radiative heat transfer from the water through the glass to the sky ($q_{rw,\text{sky}}$) ≈ 0 because the majority of the radiation from the water will be at long wavelengths for which the transmissivity of the glass is zero.

An expression can be written for each of the heat fluxes

$$q_{rg\text{sky}} = \sigma\, \varepsilon_g\, (T_g^4 - T_{\text{sky}}^4) = \sigma\, T_g^4$$

$$q_{rwg} = \sigma\, \varepsilon_w\, (T_w^4 - T_g^4) = \sigma\, (T_w^4 - T_g^4)$$

$$q_{eg\infty} = h_{co}\, (T_g - T_\infty)$$

$$q_{cwg} = h_\infty\, (T_w - T_g)$$

$$q_{cg} = (1 - \rho_\lambda)(1 - \tau_\lambda)\, I_s =$$

$$(1-\rho)\left[\, 1-\tau_{(0-0.35)}\ \frac{E_{b(0-0.35)}}{\sigma T^4} I_s + 1-\tau_{(0.35-2.7)}\ \frac{E_{b(0.35-2.7)}}{\sigma T^4} I_s + 1-\tau_{(2.7\to 0)}\ \frac{E_{b(2.7\to 0)}}{\sigma T^4} I_s \right]$$

$$q_{sw} = (1 - \rho_\lambda)\, \tau_\lambda\, I_{s\lambda} = (1 - \rho_\lambda)\, \tau_{(0.35\,\to\,2.7)}\ \frac{E_{b(0.35-2.7)}}{\sigma T^4}\ I_s$$

Only the last two expressions are frequency dependent. From Table 11.2

For $\lambda T = (0.35 \times 10^{-6}\text{ m})(6000\text{ K}) = 2.1 \times 10^{-3}$ m K

$$\frac{E_b(0 \to 0.35T)}{\sigma T^4} = 0.08382$$

For $\lambda T = (2.7 \times 10^{-6}\text{ m})(6000\text{ K}) = 16.2 \times 10^{-3}$ m K

$$\frac{E_b(0 \to 2.7T)}{\sigma T^4} = 0.9746$$

$$\frac{E_b(0.35T \to 2.7T)}{\sigma T^4} = 0.9746 - 0.08382 = 0.8908$$

$$\therefore \qquad q_{sw} = (1 - 0.08)(0.92)(0.8908)\ \ 630\ \text{W/m}^2\ \ = 475\ \text{W/m}^2$$

$$q_{sg} = (1 - 0.08)[(1)(0.08382)(630) + (1 - 0.92)(0.8908)(630) + (1)(1 - 0.9746)(630)] = 105\ \text{W/m}^2$$

The natural convection Nusselt number between the glass and water is given by Equation (8.30)

$$\overline{Nu_\delta} = 1 + 1.44\left[1-\frac{1708}{Ra_\delta}\right]^{\bullet} + \left[\left(\frac{Ra_\delta}{5830}\right)^{\frac{1}{3}}-1\right]^{\bullet} \qquad 1700 < Ra_\delta < 10^8$$

where the notation [ ]· indicates that if the quantity inside the brackets is negative, the quantity is to be taken as zero. The Rayleigh number is given by

$$Ra_\delta = Gr_\delta Pr = \frac{g\beta\ T_w - T_\delta\ \ \delta^3 Pr}{\nu_a{}^2}$$

Since both T_w and T_g are unknown, an iterative solution must be used. For the first iteration, let T_w = 80°C and T_g = 40°C.

From Appendix 2, Table 28, for dry air at the average temperature of $(T_w + T_g)/2$ = 60°C

Thermal expansion coefficient (β) = 0.00300 1/K

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m²/s

Prandtl number (Pr) = 0.71

$$Ra_\delta = \frac{(9.8\,\mathrm{m/s^2})(0.003\,1/K)(40°\mathrm{C})(0.01\mathrm{m})^3(0.71)}{(19.4\times10^{-6}\mathrm{m^2/s})^2} = 2219$$

$$\overline{Nu_\delta} = 1 + 1.44\left[1-\frac{1708}{2219}\right]^{\bullet} + \left[\left(\frac{2219}{5830}\right)^{\frac{1}{3}}-1\right]^{\bullet} = 1 + 0.3316 + 0 = 1.33$$

$$\overline{h_{c\delta}} = \overline{Nu_\delta}\,\frac{k}{\delta} = 1.33\,\frac{0.0279\,\mathrm{W/(m\,K)}}{0.01\mathrm{m}} = 3.72\,\mathrm{W/(m^2K)}$$

An energy balance on the glass plate yields

$$Q_{sg} + q_{cwg} + q_{rwg} = q_{rg\mathrm{sky}} + q_{cg\infty}$$

$$Q_{sg} + \overline{h_{c\delta}}\,(T_w - T_g) + \sigma(T_w{}^4 - T_g{}^4) = \sigma T_g{}^4 + h_{co}\,(T_g - T_\infty)$$

Rearranging

$$\overline{h_\infty} + \overline{h_{c\delta}}\quad T_g = q_{sg} + \overline{h_{co}}\,T_\infty + \overline{h_{c\delta}}\,T_w + \sigma(T_w{}^4 - 2T_g{}^4)$$

$$[1]\quad T_g = K_1 + K_2\,T_w + K_3\,(T_w{}^4 - 2\,T_g{}^4)$$

where

$$K_1 = \frac{q_{sg} + \overline{h_{co}}\,T_\infty}{\overline{h_{co}} + \overline{h_{c\delta}}} = \frac{105\,\mathrm{W/m^2} + 8.5\,\mathrm{W/(m^2K)}\ (293\mathrm{K})}{8.5+3.72\ \ \mathrm{W/(m^2K)}} = 212.4\ \mathrm{K}$$

$$K_2 = \frac{\overline{h_{c\delta}}}{\overline{h_{co}} + \overline{h_{c\delta}}} = \frac{3.72}{8.5+3.72} = 0.3044$$

$$K_3 = \frac{\sigma}{\overline{h_\infty} + \overline{h_{c\delta}}} = \frac{5.67\times10^{-8}\,\mathrm{W/(m^2K^4)}}{8.5+3.72\ \ \mathrm{W/(m^2K)}} = 4.64\times10^{-9}\ 1/\mathrm{K}^3$$

An energy balance on the water yields

$$q_{sw} = q_{rwg} + q_{cwg} = \sigma (T_w^4 - T_g^4) + \bar{h}_{c\delta} \; (T_w - T_g)$$

Rearranging

$$[2] \quad T_w = K_4 - K_5 \, (T_w^4 - T_g^4) + T_g$$

where

$$K_4 = \frac{q_{sw}}{\bar{h}_{c\delta}} = \frac{475 \, \text{W/m}^2}{3.72 \, \text{W/(m}^2 \, \text{K)}} = 127.7 \text{ K}$$

$$K_5 = \frac{\sigma}{\bar{h}_{c\delta}} = \frac{5.67 \times 10^{-8} \, \text{W/(m}^2\text{K}^4)}{3.72 \, \text{W/(m}^2\text{K)}} = 1.524 \times 10^{-8} 1/\text{K}^3$$

These two simultaneous 4th order equations may by solved iteratively as follows:

1. Guess values of T_w and T_g
2. Iterative equation [2] to generate a new value of T_w
3. Using this value of T_w, iterate equation [1] to generate a new rate for T_g.
4. Repeat the procedure until the difference between the values of T_w and T_g for successive iterations is below a chosen tolerance.

This procedure is implemented in the Pascal program shown below

```
var
   Tw1,Tw,Tg1,Tg,Diff_g,Diff_w:real;
Const
   K1 = 212.4;
   K2 = 0.3044;
   K3 = 4.64E-9;
   K4 = 127.7;
   K5 = 1.524E-8;
   gain = 0.4;
Begin
   {Let Tw1 = 353 K and Tg1 = 313 K be the initial guesses}
   Tw:=0.0;
   Tg:=0.0;
   Tw1:=353.0;
   Tg1:=313.0;
   Repeat
      Repeat
         {Iterate equation [2] to calculate a new Tw}
         Tw:=K4 - K5*(Tw1*Tw1* Tw1* Tw1 - Tg1* Tg1* Tg1* Tg1)+ Tg1;
         Diff_w:=Tw-Tw1;
         Tw1:=Tw1+gain*Diff_w;
      Until abs(Diff_w) < 0.1;
      Repeat
         {Iterate equation [1] to calculate a new Tg}
         Tg:=K1 + K2*Tw + K3*(Tw* Tw* Tw* Tw - 2.0*Tg1* Tg1* Tg1 Tg1);
         Diff_g:=Tg-Tg1;
         Tg1:=Tg1+gain*Diff_g;
      Until abs(Diff_g) < 0.1;
      Tw:=K4 - K5*(Tw1* Tw1* Tw1* Tw1* - Tg1* Tg1* Tg1* Tg1*) + Tg1;
      Diff_w:=abs(Tw-Tw1);
      Tw1:=Tw;
   Until Diff_w < 0.1;
   Writeln(' Tw = ',Tw:6.1, K  Tg = ',Tg:6.1,' K');
end.
```

(Note: the gain factor slows down the convergence but is often necessary for non-linear problems)

The output from the first run of this program is

$T_w = 345.4$ K $\quad T_g = 304.2$ K

These values are different enough from the initial guesses that another iteration will be performed:

Mean temperature = 324.8 K

$\beta = 0.00308$ 1/K

$k = 0.0273$ W/(m K)

$\nu = 18.66 \times 10^{-6}$ m^2/s

$Pr = 0.71$

$Ra_\delta = 2536$

$\bar{h}_{c\delta} = 4.01$ W/(m^2 K)

$K_1 = 207.42$ K

$K_2 = 0.3205$

$K_3 = 4.532 \times 10^{-9}$ 1/K^3

$K_4 = 118.45$ K

$K_5 = 1.414 \times 10^{-8}$ 1/K^3

Running the program again, the new constants and the above temperatures as the initial guesses yields:

$T_w = 344.5$ K $\quad T_g = 304.1$ K

The water temperature is approximately 71.5°C.

COMMENTS

Consistent with our assumption, he glass temperature is low enough so that all radiation emitted from the glass will be beyond 2.7 μm so that the glass can be considered black.

PROBLEM 11.62

Mercury is to be evaporated at 317°C in a furnace. The mercury flows through a 25.4 mm BWG No. 18 gauge 304 stainless-steel tube, that is placed in the center of the furnace. The furnace cross section, perpendicular to the tube axis, is a square 20 cm by 20 cm. The furnace is made of brick having an emissivity of 0.85, and its walls maintained uniformly at 977°C. If the convection heat transfer coefficient on the inside of the tube is 2.8 kW/(m^2 K) and the emittance of the outer surface of the tube is 0.60, calculate the rate of heat transfer per meter of tube, neglecting convection within the furnace.

GIVEN

- Mercury flow through a tube in the center of a furnace
- Mercury temperature (T_m) = 317°C = 590 K
- Tube specification: 25.4 mm BWG no 18 gauge stainless steel
- Furnace cross section is 20 cm × 20 cm = 0.2 m × 0.2 m
- Furnace emissivity (ε_2) = 0.85
- Furnace wall temperature (T_2) = 977°C = 1250 K
- Tube interior heat transfer coefficient (h_{ci}) = 2800 W/(m^2 K)
- Tube exterior emissivity (ε_1) = 0.60

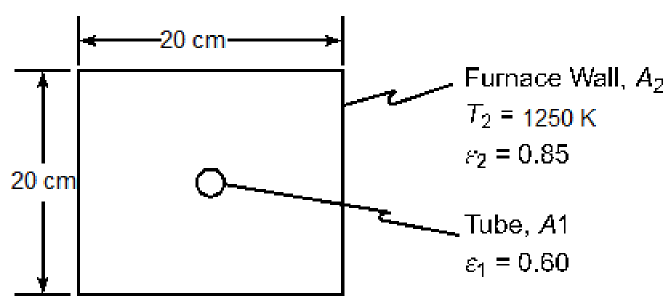
FIND

- The rate of heat transfer per meter of tube

ASSUMPTIONS

- Steady state
- Convection within the furnace is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)
From Appendix 2, Table 42, for 25.4 mm 18 BWG tubes

Inside diameter (D_i) = 22.9 mm= 0.0229 m

Outside diameter (D_o) = 25.4 mm = 0.0254 m

From Appendix 2, Table 10, for type 304 stainless steel,the thermal conductivity (k_s) = 14.4 W/(m K)

SOLUTION

The tube and furnace can be thought of as two infinitely long concentric gray cylinders. The rate of radiative heat transfer is given by Equation (11.75)

$$q_{12} = A_1 \mathcal{F}_{12} (E_{b1} - E_{b2}) = \sigma A_2 \mathcal{F}_{12} (T_1^4 - T_2^4)$$

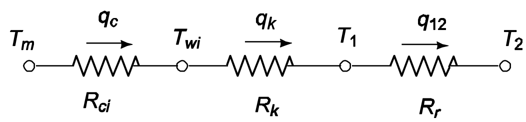
From Equation (11.76)

$$\mathcal{F}_{12} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{A_1}{A_2}\frac{1-\varepsilon_2}{\varepsilon_2}}$$

where $$\frac{A_1}{A_2} = \frac{\pi D_o}{4(0.2m)} = \frac{\pi(0.0254m)}{4(0.2m)} = 0.01$$

$$\mathcal{F}_{12} = \frac{1}{\frac{1-0.6}{0.6} + 1 + 0.1\frac{1-0.85}{0.85}} = 0.595$$

The thermal circuit for this problem is shown below



where

R_{ci} = Convective thermal resistance inside the tube
R_k = Conductive thermal resistance of the tube wall
R_r = Radiative thermal resistance
q_c = Convective heat transfer to the tube wall interior = $h_{ci} A_i (T_m - T_{wi})$
q_k = Conductive heat transfer through the tube wall = $\dfrac{2\pi k_s L}{\ln\left(\frac{D_o}{D_i}\right)} (T_{wi} - T_1)$

q_{1-2} = Radiative heat transfer = $\sigma A_1 f_{12} (T_1^4 - T_2^4)$

For steady state, these three heat transfer rates must be equal

$$h_{ci} A_i (T_m - T_{wi}) = \frac{2\pi k_s L}{\ln \frac{D_o}{D_i}} (T_{wi} - T_1) = \sigma A_1 \mathcal{F}_{12} (T_1^4 - T_2^4)$$

Solving for T_{w1} from the first part of the equation

$$T_{wi} = \frac{\bar{h}_{ci} A_i T_m + \frac{2\pi k_s L}{\ln \frac{D_o}{D_i}} T_1}{\bar{h}_{ci} A_i + \frac{2\pi k_s}{\ln \frac{D_o}{D_i}}}$$

Substituting this into the second part of the equation to solve for T_1 yields

$$\bar{h}_{ci} A_i \left[T_m - \frac{\bar{h}_{ci} A_i T_m + \frac{2\pi k_s L}{\ln \frac{D_o}{D_i}} T_1}{\bar{h}_{ci} A_i + \frac{2\pi k_s}{\ln \frac{D_o}{D_i}}} \right] - \sigma A_1 \mathcal{F}_{12} (T_1^4 - T_2^4) = 0$$

$$\frac{2\pi k_s L}{\ln \frac{D_o}{D_i}} = \frac{2\pi(14.4\,W/(m\text{ K}))L}{\ln\left(\frac{0.0254}{0.0229}\right)} = 873.2\,L\ W/K$$

$$\bar{h}_{ci} A_i = \bar{h}_{ci}\, \pi D_i L = \left(2800\, W/(m^2\mathrm{K})\right) \pi\, (0.0229\ \mathrm{m})\, L = 201.4\, L\ \ W/K$$

$$(201.4L\, (W/K)) \left[590\, K - \frac{(201.4\, L W/K)(590\, K) + (873.2\, L W/\mathrm{K})(T_1)}{(201.4\, L\, W/\mathrm{K}) + (873.2\, W/\mathrm{K})}\right]$$

$$-\left(5.67\times 10^{-8}\ W/(m^2 K^4)\right) \left[\pi (0.0254 m) L\right] (0.595)\ [T_1^4 - (1250\ \mathrm{K})^4] = 0$$

Checking the units, then eliminating them for clarity

$$-2.69\times 10^{-9}\ T_1^4 - 163.71\ T_1 + 118{,}826 = 0$$

By trial and error

$$T_1 = 657\ \mathrm{K}$$

$$\frac{q}{L} = \sigma \frac{A_1}{L} \mathscr{F}_{12}\, (T_1^4 - T_2^4) = \left(5.67\times 10^{-8}\ W/(m^2 K^4)\right) \left[\pi (0.0254\mathrm{m})\right]\ (0.595)\ [(657\ \mathrm{K})^4 - (1250\ \mathrm{K})^4]$$

$$q/L = -\ 10203\ \mathrm{W/m}$$

COMMENTS

Negative sign in the answer indicates heat is transferred to the mercury. Note that $f_{12} \approx \varepsilon_1$, because $A_2 >> A_1$. For $A_1/A_2 = 0$ Equation (11.76) reduces to $f_{12} = \varepsilon_1$.

PROBLEM 11.63

A 2.5-cm-diameter cylindrical refractory crucible for melting lead is to be built for thermocouple calibration. An electrical heater immersed in the metal is shut off at some temperature above the melting point. The fusion-cooling curve is obtained by observing the thermocouple emf as a function of time. Neglecting heat losses through the wall of the crucible, estimate the cooling rate (W) for the molten lead surface (melting point 327.3°C, surface emissivity 0.8) if the crucible depth above the lead surface is (a) 2.5 cm, (b) 17 cm. Assume that the emissivity of the refractory surface is unity and the surroundings are at 21°C, (c) Noting that the crucible would hold about 0.09 kg of lead for which the heat of fusion is 23,260 J/kg, comment on the suitability of the crucible for the purpose intended.

GIVEN

- A cylindrical refractory crucible filled with molten lead
- Cylinder diameter (D) = 2.5 cm
- Melting point of lead (T_1) = 327.2°C = 600.3 K
- Surface emissivity of lead (ε_1) = 0.8
- Mass of lead in crucible (m) ≈ 0.09
- Heat of fusion of lead (h_{fg}) = 23,260 J/kg

FIND

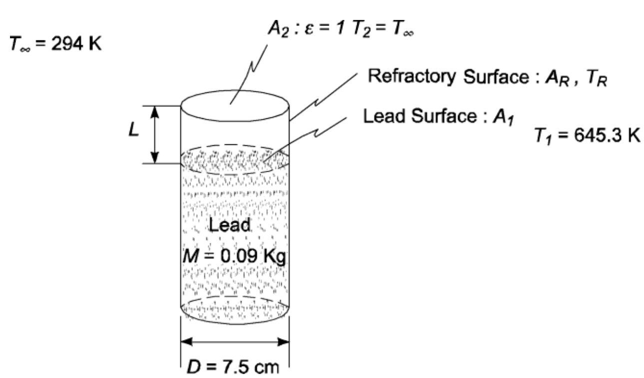
The cooling rate (q) if the crucible depth above the lead surface (L) is

(a) 2.5 cm = 0.025 m
(b) 17 cm = 0.17 m
(c) Comment on the suitability of the crucible for thermocouple calibration

ASSUMPTIONS

- Heat loss through the wall of the crucible is negligible
- The emissivity of the refractory surface (crucible wall above the lead) is unity (ε_2 = 1)
- The surroundings behave as a blackbody enclosure
- The temperature of the refractory surface is uniform at T_R

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

From Appendix 2, Table 28, for air at the film temperature of $(T_1 + T_\infty)/2$ = 447.2 K = 174.2 C

Thermal expansion coefficient (β) = 0.00226 1/K

Thermal conductivity (k) = 0.0354 W/(m K)

Kinematic viscosity (ν) = 32.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The total cooling rate is the sum of natural convection and radiation

$$q = h_c A_1 (T_1 - T_\infty) + q_{12}$$

where q_{12} is the radiative heat transfer between the two surfaces connected by a refractory wall and is given by Equation (9.80)

$$q_{12} = A_1 f_{12}\, \sigma (T_1^4 - T_2^4)$$

where $A_1 f_{12}$ is given by Equation (11.79) (Note that $\varepsilon_2 = 1.0$ and $A_2 = A_1$)

$$A_1 \mathscr{F}_{12} = \frac{1}{\dfrac{1}{A_1}\left(\dfrac{1}{\varepsilon_1}-1\right)+\dfrac{1}{A_2}\left(\dfrac{1}{\varepsilon_2}-1\right)+\dfrac{1}{A_1\overline{F_{12}}}} = \frac{1}{\dfrac{1}{A_1}\left(\dfrac{1}{\varepsilon_1}-1\right)+\dfrac{1}{A_1\overline{F_{12}}}}$$

where $$A_1\overline{F_{12}} = A_1\left(F_{12}+\frac{1}{\dfrac{1}{F_{1R}}+\dfrac{A_1}{A_2F_{2R}}}\right) = A_1\left(F_{12}\,\frac{1}{\dfrac{1}{F_{1R}}+\dfrac{1}{F_{2R}}}\right)$$

The shape factor is given in Table 11.4 #6 by letting $a = b = D/s$ and

$$F_{12} = \frac{2}{D^2}\left(L^2+\frac{D^2}{2}-\sqrt{\left(L^2+\frac{D^2}{2}\right)^2-\frac{D^4}{4}}\right)$$

For Case (a) $$F_{12} = \frac{2}{(2.5)^2}\left((2.5)^2+\frac{(2.5)^2}{2}-\sqrt{\left((2.5)^2+\frac{(2.5)^2}{2}\right)^2-\frac{(2.5)^4}{4}}\right) = 0.17$$

For Case (b) $$F_{12} = \frac{2}{(2.5)^2}\left((17)^2+\frac{(2.5)^2}{2}-\sqrt{\left((17)^2+\frac{(2.5)^2}{2}\right)^2-\frac{(2.5)^4}{4}}\right) = 0.0053$$

By symmetry

$$F_{21} = F_{12}$$

The shape factors from a given surface must sum to unity

$$F_{11} + F_{12} + F_{1R} = 1 \quad\rightarrow\quad F_{1R} = 1 - F_{12}$$

$$F_{21} + F_{12} + F_{2R} = 1 \quad\rightarrow\quad F_{2R} = 1 - F_{21} = 1 - F_{12}$$

$$\therefore A_1\overline{F_{12}} = A_1\left(F_{12}+\frac{1-F_{12}}{2}\right)$$

$$A_1 \mathscr{F}_{12} = \frac{A_1}{\left(\dfrac{1}{\varepsilon_1}-1\right)+\dfrac{1}{\left(F_{12}+\dfrac{1-F_{12}}{2}\right)}}$$

$$q = A_1\left[\bar{h}_c\,(T_1 - T_\infty) + \frac{\sigma}{\left(\frac{1}{\varepsilon_1}-1\right)+\frac{1}{F_{12}+\frac{1-F_{12}}{2}}}\,(T_1^4 - T_2^4)\right]$$

The heat transfer coefficient, h_c, can be calculated from Equation (8.15) or (8.16)

$$Ra_D = Gr_D\,Pr = \frac{g\beta\,(T_1 - T_\infty)\,D^3 Pr}{\nu^2}$$

$$Ra_D = \frac{(9.8\ \text{m/s}^2)\,(0.00226\ 1/\text{K})(600.3\,\text{K} - 294\,\text{K})(0.025\,\text{m})^3(0.71)}{(32.4\times10^{-6}\ \text{m}^2/\text{s})^2} = 7.17\times10^4$$

Although this is slightly below its lower Rayleigh number range, Equation (5.15) will be used to estimate the Nusselt number

$$\overline{Nu}_D = 0.54\ Ra_D^{\frac{1}{4}} = 0.54\,(7.17\times10^4)^{\frac{1}{4}} = 8.84$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 8.84\,\frac{0.0354\ \text{W/(m K)}}{0.025\,\text{m}} = 12.5\ \text{W/(m}^2\text{K)}$$

(a)

$$q = \frac{\pi}{4}(0.025\ \text{m})^2\Bigg[\ (12.5\,\text{W/(m}^2\text{K)})\,(600.3\,\text{K} - 294\,\text{K})$$

$$+\frac{5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)}{\left(\frac{1}{0.8}-1\right)+\frac{1}{0.17+\frac{1-0.17}{2}}}[\,(600.3\,\text{K})^4 - (294\,\text{K})^4]\Bigg]$$

$$q = 3.26\ \text{W}$$

(b) $F_{12} = 0.0053 \;\rightarrow\; q = 3.4$ W

(c) The time required for the lead to solidify at the cooling rate (q) of 3.62 W is given by

$$t = \frac{m\,h_{fg}}{q} = \frac{(0.09\,\text{kg})(23{,}260\,\text{J/kg})}{(3.62\,\text{W})\,(\text{J/(W s)})} = 579\ \text{s} = 9.6\text{min}$$

The technician would have about 9.6 minutes to do the calibration. This should be enough time to accomplish the task.

PROBLEM 11.64

A spherical satellite circling the sun is to be maintained at a temperature of 25°C. The satellite rotates continuously and is covered partly with solar cells having a gray surface with an absorptivity of 0.1. The rest of the sphere is to be covered by a special coating which has an absorptivity of 0.8 for solar radiation and an emissivity of 0.2 for the emitted radiation. Estimate the portion of the surface of the sphere which can be covered by solar cells. The solar irradiation may be assumed to be 1,420 W/m^2 of surface perpendicular to the rays of the sun.

GIVEN

- A spherical satellite partially covered with solar cells is orbiting the sun
- Satellite temperature (T_s) = 25°C = 298 K
- Solar cell absorptivity (α_c) = 0.1
- Absorptivity of the rest of the satellite (α_{2s}) = 0.8
- Emissivity of the rest of the satellite (ε_s) = 0.2

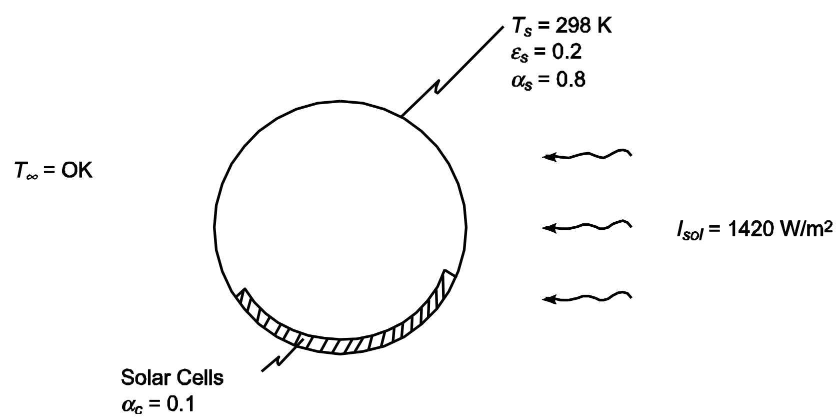
FIND

- The portion of the surface which can be covered by solar cells

ASSUMPTIONS

- Ambient temperature (T_∞) = 0 K
- Quasi steady state
- Solar irradiation (I_{so1}) = 1420 W/m^2 of the surface perpendicular to the rays of the sun
- Satellite and cell surfaces are gray
- Surface temperature of the satellite is uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

Let x be the fraction of the total satellite surface area (A_T) which is covered by solar cells. By definition, the absorptivity is the fraction of the total irradiation absorbed by a body

$$\alpha = \frac{\frac{q_{\text{in}}}{A}}{I_{\text{sol}}}$$

$$\therefore \quad q_{\text{in}} = \alpha\, I_{\text{so1}}\, A = I_{\text{so1}}\,(\alpha_s A_s + \alpha_c A_c) = I_{\text{so1}}\,(\alpha_c \times A_T + \alpha_s\,(1 - x)\, A_T)$$

$$q_{\text{in}} = I_{\text{so1}}\, A_T\, [\alpha_s + (\alpha_c - \alpha_s)\, x]$$

The rate of radiation from he satellite is the emissive power of the satellite from Equation (11.34) multiplied by its area

$$q_{\text{out}} = E\, A = \varepsilon\, \sigma A\, T_s^4 = \sigma\, T_s^4\, (\varepsilon_c \times A_T + \varepsilon_s\, (1 - x)\, A_T) = \sigma A_T\, T_s^4\, [\varepsilon_s + (\varepsilon_c - \varepsilon_s)\, x]$$

For steady state $\quad q_{\text{in}} = q_{\text{out}}$

$$I_{\text{so1}}\, A_T\, [\alpha_s + (\alpha_c - \alpha_s)\, x] = \sigma A_T\, T_s^4\, [\varepsilon_s + (\varepsilon_c - \varepsilon_s)\, x]$$

Solving for the fraction covered by solar cells

$$x = \frac{\left(\frac{I_{\text{sol}}}{\sigma T_s^4}\right) \alpha_s - \varepsilon_s}{\varepsilon_c - \varepsilon_s \; - \left(\frac{I_{\text{sol}}}{\sigma T_s^4}\right) \alpha_c - \alpha_s}$$

$$\frac{I_{\text{sol}}}{\sigma T_s^4} = \frac{1420\,\text{W/m}^2}{5.67 \times 10^{-8}\,\text{W/(m}^2\text{K}^4)\;\; (298\,\text{K})^4} = 3.176$$

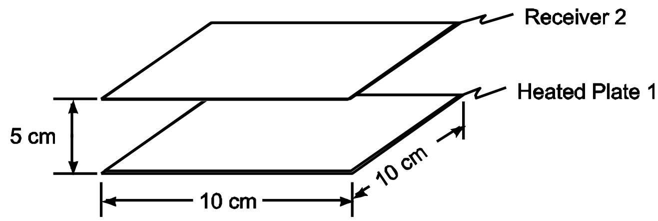
$$\varepsilon_c = 1 - \alpha_c = 1 - 0.1 = 0.9$$

$$x = \frac{3.176\;\; 0.8\;\; - 0.2}{0.9 - 0.2\;\; - 3.176\;\; 0.1 - 0.8} = 0.801$$

80.1% of the satellite can be covered by solar cells.

PROBLEM 11.65

A 10 cm square, electrically heated plate is placed in a horizontal position 5 cm below a second plate of the same size as shown schematically.



The heating surface is gray (emissivity = 0.8) while the receiver has a black surface. The lower plate is heated uniformly over its surface with a power input of 300 W. Assuming that heat losses from the backs of the radiating surface and the receiver are negligible and that the surroundings are at a temperature of 27°C, calculate the following

(a) The temperature of the receiver
(b) The temperature of the heated plate.
(c) The net radiation heat transfer between the two surfaces.
(d) The net radiation loss to the surroundings.
(e) Estimate the effect of natural convection between the two surfaces on the rate of heat transfer.

GIVEN

- A square heated plate below a second plate of equal size as shown above
- Plate size = 10 cm × 10 cm = 0.1 m × 0.1 m
- Distance between plates (L) = 5cm = 0.05 m
- Heated surface (A_1) is gray with an emissivity (ε_1) = 0.8
- Receiver (A_2) is black (ε_2 = 1.0)
- Heater power input $\dot{q}_G$ = 300 W

FIND

(a) The temperature of the receiver (T_2)
(b) The temperature of the heated transfer (T_1)
(c) The net radiation heat transfer (q_{12})
(d) The net radiation loss to the surroundings (q_s)
(e) Estimate the effect of natural convection

ASSUMPTIONS

- Steady state
- Heat losses from the back of each plate are negligible
- Temperature of surroundings (T_3) = 27°C = 300 K
- The surroundings behave as a blackbody enclosure

PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

In steady state, the net rate of heat transfer from the heater must be equal to the power input

$$q_1 = \dot{q}_G = 300 \text{ W}$$

The net rate of heat transfer to the receiver in steady state must be zero: $q_2 = 0$
Also, since the receiver and the surroundings are black

$$J_2 = E_{b2} = \sigma T_2^4 \quad \text{and} \quad J_3 = E_{b3} = \sigma T_3^4$$

The shape factor F_{12} can be read from Figure 11.32

From Figure 11.32 $x/D = y/D = 10/5 = 2 \quad \rightarrow \quad F_{12} = 0.43$

Since $A_1 = A_2$, from Equation (11.46) $F_{21} = F_{12}$ (This is also clear from the symmetry of the problem).

Since neither A_1 nor A_2 can see itself, $F_{11} = F_{22} = 0$

The shape factors from any given surface must sum to unity

$$F_{11} + F_{12} + F_{13} = 1 \quad \rightarrow \quad F_{13} = 1 - F_{12} = 0.57$$

$$F_{21} + F_{12} + F_{23} = 1 \quad \rightarrow \quad F_{23} = 1 - F_{21} = 1 - F_{12} = 0.57$$

From Equation (9.46)

$$F_{31} = \frac{A_1}{A_3} F_{13} = \frac{A_1}{A_3} (1 - F_{12}) = F_{32}$$

The net rate transfer from a gray surface is given be Equation (11.67)

[1] $q_1 = A_1 (J_1 - G_1)$

[2] $q_2 = A_2 (J_2 - G_2) = A_2 (\sigma T_2^4 - G_2) = 0$

From Equation (9.69)

[3] $A_1 G_1 = J_1 A_1 F_{11} + J_2 A_2 F_{21} + J_3 A_3 F_{31} = A_2 F_{12}\, \sigma T_2^4 + A_1(1 - F_{12})\, \sigma T_3^4$

[4] $A_2 G_2 = J_1 A_1 F_{12} + J_2 A_2 F_{22} + J_3 A_3 F_{32} = J_1 A_1 F_{12} + A_1(1 - F_{12})\, \sigma T_3^4$

Combining Equations [2] and [4]

$$\sigma T_2^4 = G_2 = J_1 F_{12} + (1 - F_{12})\, \sigma T_3^4$$

$$[5] \qquad J_1 = \left(\frac{\sigma}{F_{12}}\right) [T_2^4 - (1 - F_{12})\, T_3^4]$$

Substituting Equation [5] and Equation [3] into [1]

$$q_1 = A_1 \left[\frac{\sigma}{F_{12}}\left[T_2^4 - (1 - F_{12})\, T_3^4\right] - \left[F_{12}\, \sigma T_2^4 + (1 - F_{12})\, \sigma T_3^4\right] \right]$$

Solving for T_2

$$T_2 = \left[\frac{(1 - F_{12})\left(\left(\frac{1}{F_{12}}\right) + 1\right) T_3^4 + \left(\frac{q_1}{(\sigma A_1)}\right)}{\frac{1}{F_{12}} - F_{12}} \right]^{0.25}$$

$$T_2 = \left[\frac{(1 - 0.43)\left(\left(\frac{1}{0.43}\right) + 1\right) (300\,\text{K})^4 + \frac{300\,\text{W}}{5.67 \times 10^{-8}\,\text{W/(m}^2\text{K}^4)\ (0.1\text{m})(0.1\text{m})}}{\left(\left(\frac{1}{0.43}\right) - 0.43\right)} \right]^{0.25}$$

$$T_2 = 732\ \text{K} = 459°\text{C}$$

(b) From Equation [3]

$$G_1 = \sigma[F_{12}\, T_2^4 + (1 - F_{12})\, T_3^4] = 5.67 \times 10^{-8}\,\text{W/(m}^2\text{K}^4)\ [0.43\ (732\ \text{K})^4 + (1 - 0.43)(300\ \text{K})^4] = 7261.7\ \text{W/m}^2$$

From Equation [5]

$$J_1 = \frac{5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)}{0.43}\ [(732\ \text{K})^4 - (1 - 0.43)\ (300\ \text{K})^4] = 37{,}249\ \text{W/m}^2$$

Applying Equation (11.66) $J_1 = \rho_1 G_1 + \varepsilon_1 E_{b1} = (1 - \varepsilon_1)\, G_1 + \varepsilon_1\, \sigma T_1^4$

$$T_1 = \left[\frac{1}{\varepsilon_1 \sigma} [J_1 - (1 - \varepsilon_1)\, G_1] \right]^{0.25}$$

$$T_1 = \left[\frac{1}{0.43\ 5.67\times10^{-8}\,\mathrm{W/(m^2K^4)}}\ (37{,}249\,\mathrm{W/m^2}) - (1-0.43)(7261.7\,\mathrm{W/m^2})\ \right]^{0.25}$$

$$T_1 = 1080\ \mathrm{K} = 807°\mathrm{C}$$

(c) The net rate of heat transfer between A_1 and A_2 is given by Equation (11.73)

$$q_{12} = (J_1 - J_2)\, A_1\, F_{12} = (J_1 - \sigma T_2^4)\, A_1\, F_{12}$$

$$q_{12} = \left[37{,}249\ \mathrm{W/m^2} - 5.67\times10^{-8}\ \mathrm{W/(m^2K^4)}\ \ (732\,\mathrm{K})^4\right](0.1\mathrm{m})\ (0.1\mathrm{m})\ (0.43) = 90.2\ \mathrm{W}$$

(d) Applying the conservation of energy of both plates and the surroundings yields

$$q_1 + q_2 + q_3 = 0 \quad \rightarrow \quad q_3 = -q_1 - q_2 = -300\ \mathrm{W} - 0 = -300\ \mathrm{W}$$

The surroundings gain 300 watts from the plates.

(e) An estimate of the natural convection heat transfer will be made by treating the heater and receiver as a horizontal enclosed space, heated from below with the surface temperature calculated above.

From Appendix 2, Table 28, for dry air at the average temperature of (459°C + 807°C)/2 = 633°C

Thermal expansion coefficient (β) = 0.00116 1/K

Thermal conductivity (k) = 0.0599 W/(m K)

Kinematic viscosity (ν) = 108 × 10^{-6} $\mathrm{m^2/s}$

Prandtl number (Pr) = 0.73

The Rayleigh number is

$$Ra_\delta = Gr_\delta\, Pr = \frac{g\beta\ \Delta T\ \delta^3 Pr}{\nu_a^{\,2}} = \frac{9.8\,\mathrm{m/s^2}\ \ 0.00116\,\mathrm{1/K}\ \ 807°\mathrm{C} - 459°\mathrm{C}\ \ (0.05\,\mathrm{m})^3 (0.73)}{108\times10^{-6}\,\mathrm{m^2/s}\ ^2} = 30{,}949$$

Applying Equation (8.30a)

$$\overline{Nu}_\delta = 1 + 1.44\left[1 - \frac{1708}{Ra_\delta}\right]^{\bullet} + \left[\left(\frac{Ra_\delta}{5830}\right)^{\frac{1}{3}} - 1\right]^{\bullet}$$

where $[\ \ ]^{\bullet}$ indicates that the quantity in the brackets should be taken to be zero if it is negative.

$$\overline{Nu}_\delta = 1 + 1.44\left[1 - \frac{1708}{30{,}949}\right]^{\bullet} + \left[\left(\frac{30{,}949}{5830}\right)^{\frac{1}{3}} - 1\right]^{\bullet} = 3.11$$

$$\bar{h}_c = \overline{Nu}_\delta\ \frac{k}{\delta} = 3.11\,\frac{0.0599\,\mathrm{W/(m\,K)}}{0.05\,\mathrm{m}} = 3.72\,\mathrm{W/(m^2K)}$$

The rate of heat transfer by convection is given by

$$q = \bar{h}_c\, A\, (\Delta T) = \ 3.72\,\mathrm{W/(m^2K)}\ \ (0.1\mathrm{m})(0.1\mathrm{m})(807°\mathrm{C} - 459°\mathrm{C}) = 13\ \mathrm{W}$$

COMMENTS

The natural convection heat transfer rate is only about 4% of the total heat transfer rate. Therefore, the estimate of natural convection is probably adequate.

PROBLEM 11.66

A small sphere (2.5 cm-diam) is placed in a heating oven. The oven cavity is a 30 cm cube filled with air at 101 kPa(abs); it contains 3 % water vapor at 810 K, and its walls are at 1370 K. The emissivity of the sphere is equal to 0.44 – 0.00018 *T*, where *T* is the surface temperature in K. When the surface temperature of the sphere is 810 K determine (a) the total irradiation received by the walls of the oven from the sphere, (b) the net heat transfer by radiation between the sphere and the walls of the oven, and (c) the radiant heat transfer coefficient.

GIVEN

- A small sphere in a 30 cm cubic heat oven filled with air
- Sphere diameter (D) = 30 cm – 2.5 cm= 27.5 cm
- Air pressure = 101 kPa(abs) = $1.01*10^5$ Pa
- Air contains 3% water vapor
- Air temperature (T_m) = 810 K
- Oven wall temperature (T_2) = 1370 K
- Sphere emissivity (ε_1) = 0.44 – 0.00018 T (T in K)
- Sphere surface temperature (T_1) = 810 K

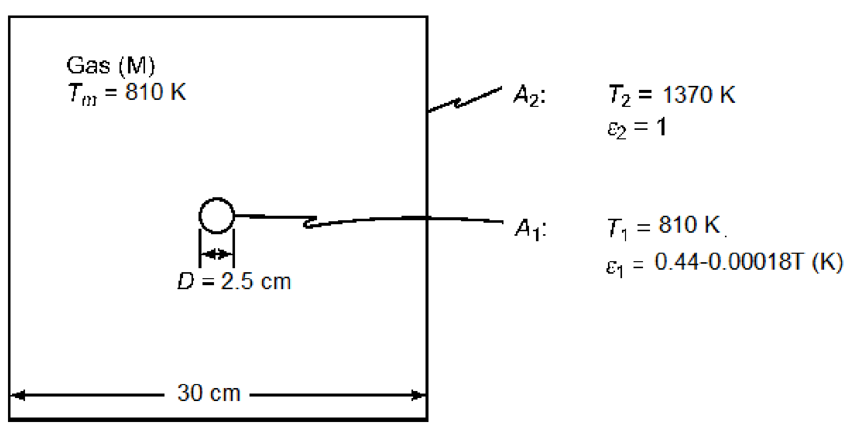
FIND

(a) Total irradiation received by the walls from the sphere (q_2)
(b) The net heat transfer by radiation between the sphere and the walls (q_{12})
(c) The radiant heat transfer coefficient (h_r)

ASSUMPTIONS

- The gas is a gray body
  The oven walls are black (ε_2 = 1)
- The sphere is near the center of the oven

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K)

SOLUTION

(a) At T_1 = 810 ε_1 = 0.44 – 0.00018 (810) = 0.3
The surface area of the sphere is

$$A_1 = \pi D^2 = \pi\,(0.025\text{ m})^2 = 0.00196\text{ m}^2$$

Since the air and the oven completely enclose the sphere, F_{12} = 1.0 and F_{1g} = 1.0

From Section 11.7.3, the portion of the total radiation leaving the sphere that is received by the walls (q_{1R2}) = $J_1 A_1 F_{12} \tau_m$ where τ_m is the transmissivity of the air and the radio sity of the sphere, J_1, is given by Equation 11.72

$$J_1 = E_{b1} - q_1 \frac{1-\varepsilon_1}{A_1 \varepsilon_1}$$

Since the air temperature is the same as the sphere temperature, there will be no net heat transfer between the air and the sphere and the heat transfer from the sphere (q_1) will be the same as the net heat transfer between the sphere and the walls (q_{12}). Simplifying Equation (11.88) with the shape factors above and

$A_1 << A_2$, $\varepsilon_2 = 1$ yields

$$q_{1-2} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 \left(\tau_m + \frac{1}{\varepsilon_m}\right)}}$$

The beam length to calculate τ_m and ε_m can be found in Table 11.5. Since the sphere is near the center of the cube, one half the beam length for a cube will be used

$$L_{eff} = \frac{L_{cube}}{2} = \frac{\left(\frac{2}{3}\right)(\text{edge length})}{2} = (0.3m/3) = 0.1 \text{ m}$$

The partial pressure of the water vapor is

$$P_{H2O} = 3\%\ p_{air} = 0.03(1 \text{ atm}) = 0.03 \text{ atm}$$

The emissivity of the water vapor (ε_m) can be calculated from Figure 11.52 where T_m = 810 K

$$P_{H2O}\, L = (0.03 \text{ atm})\ (0.1 \text{ m}) = 0.0030 \text{ atm m} \Rightarrow \varepsilon_m = 0.012$$

By Kirchoff's radiation law

$$\alpha_m = \varepsilon_m = 0.012$$

Also $\tau_m = 1 - \varepsilon_m = 1 - 0.012 = 0.988$

$$q_{1-2} = \frac{\left(5.67\times10^{-8}\, W/(m^2\, K^4)\right)[(810K)^4 - (1370\,K)^4]}{\frac{1-0.3}{0.3(0.00196\,m^2)} + \frac{1}{0.00196\,m^2\left(0.988 + \frac{1}{0.012}\right)}} = -146.5 \text{ W}$$

(b) The net radiation between the sphere and walls is 146.5 W from the walls to the sphere.

(a) $J_1 = \left(5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\right)\ (810 \text{ K})^4 - (-146.5W)\ \frac{1-0.3}{(0.00196\,m^2)0.3} = 198811\ \left(W/m^2\right)$

$$q_{1R2} = J_1 A_1 F_{12}\ \tau_m = 198811\ \left(\text{W}/(m^2)\right)\ (0.00196 \text{ m}^2)\ (1.0)\ (0.988) = 385 \text{ W}$$

(c) The radiative heat transfer coefficient must satisfy the following equation

$$q_{12} = \bar{h}_r\ A_1\, \Delta_T = \bar{h}_r\ A_1\, (T_2 - T_1)$$

$$\bar{h}_r = \frac{q_{12}}{A_1(R_2 - T_1)} = \frac{146.5W}{(0.00196m^2)(1370K - 810K)} = 133.5\ \text{W/(m}^2\ K)$$

PROBLEM 11.67

A 0.61 m radius hemisphere (811 K surface temperature) is filled with a gas mixture containing 6.67 % CO_2 and water vapor at 0.5 % relative humidity at 533 K and 2-atm pressure. Determine the emissivity and absorptivity of the gas, and the net rate of radiant heat flow to the gas.

GIVEN

- A hemisphere filled with a gas mixture
- Hemisphere radius (r) = 0.61 m
- Hemisphere surface temperature (T_1) = 811 K
- Gas temperature (T_m) = 533 K
- Gas pressure (p_T) = 2 atm
- Gas mixture: 6.67% of CO_2 and water vapor at R.H. = 0.5%

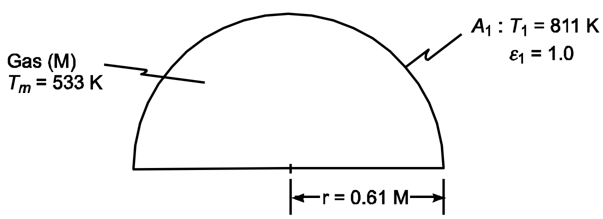
FIND

(a) The emissivity (ε_m) and absorptivity (τ_m) of the gas
(b) The net rate of radiant heat flow to the gas (q_{1-m})

ASSUMPTIONS

- The hemisphere surface is black ($\varepsilon_1 = 1$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

From Appendix 2, Table 13, the saturation pressure of water at 533 K ($P_{sat,H2O}$) = 4.694×10^6 N/m^2

SOLUTION

(a) The path length for the hemisphere is

$$L = 3.4\ \frac{\text{volume}}{\text{area}} = 3.4\ \frac{\frac{1}{2}\left(\frac{4}{3\pi r^3}\right)}{2\pi r^2 + \pi r^2} = 3.4\left(\frac{2}{9}\right) r = 0.461 \text{ m}$$

For a relative humidity of 0.5% , the partial pressure of the water vapor is

$$p_{H2O} = \text{(R.H.)}\ (p_{sat,H2O}) = (0.005)\quad 46.940 \times 10^5 \text{ N/m}^2 \left(\frac{1\text{atm}}{101{,}330\,\text{N/m}^2}\right) = 0.232 \text{ atm}$$

$$p_{H2O}\, L = (0.232 \text{ atm})\ (0.461 \text{ m}) = 0.107 \text{ atm m}$$

From Figure 11.52, for 1 atm pressure $(\varepsilon_{H2O})_{pT=1}$ = 0.18

This must be corrected for the total pressure

$$\frac{p_{H2O} + p_t}{2} = \frac{0.232\,\text{atm} + 2\,\text{atm}}{2} = 1.116 \text{ atm}$$

In Figure 11.54, $C_{H2O} \approx 1.5$

Repeating this procedure for the CO_2

Partial pressure

$$p_{CO2} = p_T\,(6.67\%) = 2 \text{ atm } (0.0667 \text{ m}) = 0.733 \text{ atm}$$

$$p_{CO2}\,L = (0.133 \text{ atm})(0.461 \text{ m}) = 0.061 \text{ atm m}$$

From Figure 11.53, $(\varepsilon_{CO2})_{pT=1} \approx 0.085$

From Figure 11.55, $C_{CO2} \approx 1.25$

The emissivity of the mixture (ε_m) is given by Equation (11.94)

$$\varepsilon_m = C_{H2O}\,(\varepsilon_{H2O})_{pT=1} + C_{CO2}\,(\varepsilon_{CO2})_{pT=1} - \Delta\varepsilon$$

where Δe is determined by interpolating between values from Figure 11.56

$$\frac{p_{H2O}}{p_{CO2} + p_{H2O}} = \frac{0.232}{0.1334 + 0.232} = 0.635$$

$$p_{CO2}\,L = p_{H2O}\,L = (0.061 + 0.107 \text{ atm m} = 0.168 \text{ atm m}$$

At $T_m = 400$ K $\Delta\varepsilon \approx 0.006$ and at $T_m = 811$ K: $\Delta\,\varepsilon \approx 0.007$

Therefore

At $T_m = 811$ K: $\Delta\varepsilon \approx 0.0063$

$$\varepsilon_m = 1.5(0.18) + (1.25)(0.085) - 0.0063 = 0.37$$

$$\tau_m = 1 - \varepsilon_m = 1 - 0.37 = 0.63$$

(b) To evaluate the rate of heat transfer, the emissivity and absorptivity must be evaluated at the surface temperature

$$\text{At } T_s = 811 \text{ K:}\quad p_{H2O}\,L = (0.107 \text{ atm m})\,\frac{T_s}{T_{H2O}} = 0.107 \text{ atm m})\left(\frac{811}{533}\right) = 0.163 \text{ atm m}$$

From Figure (11.52), $\varepsilon'_{H2O} \approx 0.19$

From Equation (11.95)

$$a_{H2O} = C_{H2O}\,\varepsilon'_{H2O}\left(\frac{T_{H2O}}{T_s}\right)^{0.45} = 1.5(0.19)\left(\frac{533}{811}\right)^{0.45} = 0.236$$

At $T_s = 811$ K: $P_{CO2}\,L = (0.061 \text{ atm m})\left(\frac{811}{533}\right) = 0.093$

From Figure (11.53), $\varepsilon'_{CO2} \approx 0.11$

$$\alpha_{CO2} = 1.25(0.11)\left(\frac{533}{811}\right)^{0.65} = 0.105$$

The total absorptivity is the sum of the H_2O and CO_2 absorptivities

$$\alpha_G = 0.236 + 0.105 = 0.341$$

The rate of heat transfer is given by Equation (11.97)

$$q_r = \sigma A_G\,(\varepsilon_G\,T_G^4 - \alpha_G\,T_s^4) = \sigma\,(2\pi r^2 + \pi r^2)\,(\varepsilon_m\,T_m^4 - \alpha_G\,T_7^4)$$

$$q_r = 5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\ [3\pi(0.61 \text{ m})^2][0.37(533 \text{ K})^4 - 0.34(811 \text{ K})^4]$$

$$q_r = -2.33 \times 10^4 \text{ W (from the surface to the gas)}$$

PROBLEM 11.68

> **Two infinitely large black plane surfaces are 0.3 m apart and the space between them is filled by an isothermal gas mixture at 811 K and atmospheric pressure consisting of 25% CO_2, 25% H_2O, and 50% N_2 by volume. If one of the surfaces is maintained at 278 K and the other at 1390 K respectively, calculate**
>
> **(a) the effective emissivity of the gas at its temperature**
>
> **(b) the effective absorptivity of the gas to radiation from the 1390 K surface**
>
> **(c) the effective absorptivity of the gas to radiation from the 278 K surface**
>
> **(d) the net rate of heat transfer to the gas per square meter of surface area**

GIVEN

- Two infinitely large black plane surfaces with an isothermal gas mixture between them
- Distance between surfaces (s) = 0.3 m
- Gas mixture temperature (T_m) = 811 K
- Gas mixture pressure = 1 atm
- Gas contents: 25% CO_2, 25% H_2O, 50 % N_2 by volume
- Surface temperatures
  - T_1 = 278 K
  - T_2 = 1390 K

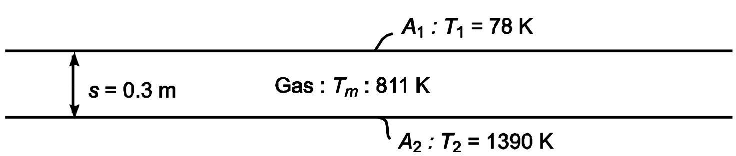
FIND

(a) The effective emissivity of the gas at its temperature (ε_{mix})
(b) The effective absorptivity of the gas to radiation from A_1
(c) The effective absorptivity of the gas to radiation from A_2
(d) The net rate of heat transfer to the gas per square meter of surface are (q_m/A)

ASSUMPTIONS

- Steady state
- Convection is negligible

SKETCH



SOLUTION

(a) The partial pressures of the CO_2 and H_2O are both 0.25 atm. The equivalent mean hemi- spherical beam length, L, is from Table 11.5

$$L = 2s = 0.6\text{m}$$

$$p_{CO2}\,L = p_{H2O}\,L = 0.25 \text{ atm } (0.6\text{m}) = 0.15 \text{ atm m}$$

From Figure 9.46 $(\varepsilon_{H2O})_{pT} \approx 0.18$ From Figure 9.47 $(\varepsilon_{CO2})_{pT} \approx 0.11$

$$p_{CO2}\,L + p_{H2O}\,L = 2(0.15 \text{ atm m}) = 0.30 \text{ atm m and } \frac{p_{H2O}}{p_{H2O}+p_{CO2}} = \frac{0.25}{0.50} = 0.5$$

From Figure 11.56 $\Delta\varepsilon \approx 0.014$

From Equations (11.94), (11.93a), and (11.93b)

$$\varepsilon_{\text{mix}} = C_{H2O}(\varepsilon_{H2O})_{p_T} + C_{CO2}(\varepsilon_{CO2})_{p_T} - \Delta\varepsilon = (1)\,0.18 + (1)\,0.11 - 0.014 = 0.276$$

(b) To find the absorptivity to radiation from A_1, the emittances of the H_2O and CO_2 must first be evaluated at T_1 = 278 K.

Using the procedure shown above with: $p_{H2O}\, L\left(\frac{T_s}{T_{H2O}}\right) = 0.15\left(\frac{278\,\text{K}}{811\,\text{K}}\right) = 0.051$

From Figure 11.52, $\varepsilon'_{H2O} \approx 0.11$ From Figure 11.53, $\varepsilon'_{CO2} \approx 0.085$

Applying Equation (11.95)

$$\alpha_{H2O} = C_{H2O}\, \varepsilon'_{H2O}\left(\frac{T_{H2O}}{T_s}\right)^{0.45} = (1)\,(0.11)\left(\frac{811}{278}\right)^{0.45} = 0.178$$

Applying Equation (11.96)

$$\alpha_{CO2} = C_{CO2}\, \varepsilon'_{CO2}\left(\frac{T_{CO2}}{T_s}\right)^{0.65} = (1)\,(0.085)\left(\frac{811}{278}\right)^{0.65} = 0.171$$

$$\alpha_1 = \alpha_{H2O} + \alpha_{CO2} = 0.178 + 0.171 = 0.349$$

(c) Repeating this procedure for T_2 = 1390 K and $p_{H2O}\, L\, (T_s/T_{H2O}) = 0.257$

From Figure 11.52, $\varepsilon'_{H2O} \approx 0.17$ From Figure 11.53, $\varepsilon'_{CO2} \approx 0.14$

$$\alpha_{H2O} = (0.17)\left(\frac{811}{1390}\right)^{0.45} = 0.133 \quad \text{and} \quad \alpha_{CO2} = (0.14)\left(\frac{811}{1390}\right)^{0.65} = 0.099$$

$$\alpha_2 = 0.133 + 0.099 = 0.232$$

(d) The rate of heat flow from the gas to A_1 is given by Equation (11.97)

$$\frac{q_{r1}}{A}\, \sigma(\varepsilon_{mix}\, T_m^4 - \alpha_1\, T_1^4) = 5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\ [0.276\ (811\ \text{K})^4 - 0.349\ (278\ \text{K})^4] = 6651\ \text{W/m}^2$$

The rate of heat flow from the gas to A_2 is

$$\frac{q_{r2}}{A}\, \sigma(\varepsilon_{mix}\, T_m^4 - \alpha_2\, T_2^4) = 5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4)\ [0.276(811\ \text{K})^4 - 0.232(1390\ \text{K})^4] = 42{,}336\ \text{W/m}^2$$

The net rate of heat transfer from the gas is

$$\frac{q_m}{A} = \frac{q_{r1}}{A} + \frac{q_{r2}}{A} = (6651 - 42{,}336)\ \text{W} = -35684\ \text{W (gain to gas)}$$