《机械工程中的数值分析技术》

作业

学　　生：易弘睿

学　　号：20186103

专业班级：机械一班

作业编号：2021070607

重庆大学-辛辛那提大学联合学院

二〇二一年七月

Catalog

Lec17 Interpolation

1.1 Question 17.2

17.2 Use Newton's interpolating polynomial to determine y at $x = 3.5$ to the best possible accuracy. Compute the finite divided differences as in Fig. 17.5, and order your points to attain optimal accuracy and convergence. That is, the points should be centered around and as close as possible to the unknown.

x	0	1	2.5	3	4.5	5	6
y	2	5.4375	7.3516	7.5625	8.4453	9.1875	12

The Matlab code is below:

```
%% 7.2
clc;clear all;
x=[1 2.5 3 4.5 5 6];
y=[5.4375 7.3516 7.5625 8.4453 9.1875 12];
xx=3.5;
yint=Newtint(x,y,xx);
function yint = Newtint(x,y,xx)
    n = length(x);
    if length(y)~=n
        error('x and y must be same length');
    end
    b = zeros(n,n);
    b(:,1) = y(:);
    for j = 2:n
        for i = 1:n-j+1
            b(i,j) = (b(i+1,j-1)-b(i,j-1))/(x(i+j-1)-x(i));
        end
    end
    xt = 1;
    yint = b(1,1);
    for j = 1:n-1
        xt = xt*(xx-x(j));
        yint = yint+b(1,j+1)*xt;
```

```
    end
Data=[x(1:5)' b(1:5,:)];
Data1=[x(6)' b(6,:)];
fprintf('------------------------------Result------------
----------------------\n')
fprintf('%s\t %s\t\t %s\t\t %s\t\t %s\t\t %s\t\t %s\t\t\n
','x','y',...
    'first','second','third','fourth','fifth');
fprintf('%0.1f\t %0.3f
\t %0.2e\t %0.2e\t %0.2e\t %0.2e\t %0.2e\n',Data');
fprintf('%0.1f\t %0.1f
\t %0.1e\t %0.1e\t %0.1e\t %0.1e\t %0.1e\n',Data1');
fprintf('-----------------------------------------------------
----------------------\n')
fprintf('The value of y at x=3.5 as obtained from the
code above is %f',yint);
end
```

The output is below:

```
-----------------------------Result--------------------------------
   x    y      first      second       third      fourth       fifth

   1.0  5.438     1.28e+00   -4.27e-01  1.46e-01   -7.62e-06  1.27e-06
   2.5  7.352     4.22e-01   8.34e-02   1.46e-01   -1.27e-06  0.00e+00
   3.0  7.563     5.89e-01   4.48e-01   1.46e-01   0.00e+00   0.00e+00
   4.5  8.445     1.48e+00   8.85e-01   0.00e+00   0.00e+00   0.00e+00
   5.0  9.188     2.81e+00   0.00e+00   0.00e+00   0.00e+00   0.00e+00
   6.0  12.0   0.0e+00    0.0e+00    0.0e+00    0.0e+00    0.0e+00
-------------------------------------------------------------------
The value of y at x=3.5 as obtained from the code above is 7.742167>>
```

1.2 Question 17.9

17.9 Employ inverse interpolation to determine the value of x that corresponds to $f(x) = 0.93$ for the following tabulated data:

x	0	1	2	3	4	5
***f*(*x*)**	0	0.5	0.8	0.9	0.941176	0.961538

Note that the values in the table were generated with the function $f(x) = x^2/(1 + x^2)$.

(a) Determine the correct value analytically.

(b) Use quadratic interpolation and the quadratic formula to determine the value numerically.

(c) Use cubic interpolation and bisection to determine the value numerically.

(a) The Matlab code is below:

```
clc;clear all;
syms x;
x=solve(x^2/(1+x^2)==0.93,x);
x=double(x);
fprintf('The correct value analytically achieved is %f',
x(2));
```

The output is below:

```
The correct value analytically achieved is 3.644957>>
```

(b) The Matlab code is below:

```
clc;clear all;
x=[3,4,5];
y=[0.9 0.941176 0.961538];
syms xb;
fx=0.93;
x=solve(y(1)+(y(2)-y(1))/(x(2)-x(1))*(xb-x(1))+((y(3)-
y(2))/(x(3)-x(2))-(y(2)-y(1))/(x(2)-x(1)))/(x(3)-
x(1))*(xb-x(1))*(xb-x(2))==fx,xb);
x=double(x);
fprintf('The value achieved by quadratic equation is %f',
x(1));
```

The output is below:

```
The value achieved by quadratic equation is 3.672954>>
```

(c) The Matlab code is below:

```
clc;clear all;
x=[3,4,5];
y=[0.9 0.941176 0.961538];
p=polyfit(x,y,3);
func=@(x) 0.0063*x^3-0.0864*x^2+0.4118*x-
0.6585;maxit=30;es= 0.0001;
xl=3 ; xu=4;%lower and upper limits
iter = 0; xr = xl;
while iter<=maxit
    xrold = xr ;
xr = (xl + xu)/2;
if xr ~= 0
ea = abs((xr - xrold)/xr)* 100;
end
test = func(xl)*func(xr);
if test < 0
xu = xr;
elseif test > 0
    xl = xr;
else
    ea = 0;
end
if ea <= es || iter >= maxit
    break;
end
iter = iter + 1;
end
root = xr ;
fprintf('The value achieved by bisection is %f', root);
```

The output is below:

```
The value achieved by bisection is 3.654364>>
```

1.3 Question 17.11

17.11 The following data for the density of nitrogen gas versus temperature come from a table that was measured with high precision. Use first- through fifth-order polynomials to estimate the density at a temperature of 330 K. What is your best estimate? Employ this best estimate and inverse interpolation to determine the corresponding temperature.

***T*, K**	200	250	300	350	400	450
Density, kg/m^3	1.708	1.367	1.139	0.967	0.854	0.759

(a) The Matlab code is below:

```
clc;clear all;
T=[300 350 400 250 450 200];
D=[1.139 0.967 0.854 1.367 0.759 1.708 ];
xx=330;
Newtint(T,D,xx);
function yint = Newtint(x,y,xx)
    n = length(x);
    if length(y)~=n
        error('x and y must be same length');
    end
    b = zeros(n,n);
    b(:,1) = y(:);
    for j = 2:n
        for i = 1:n-j+1
            b(i,j) = (b(i+1,j-1)-b(i,j-1))/(x(i+j-1)-x(i));
        end
    end
    xt = 1;
    yint = b(1,1);
    for j = 1:n-1
        xt = xt*(xx-x(j));
        yint = yint+b(1,j+1)*xt;
    end
Data=[x(1:5)' b(1:5,:)];
```

```
Data1=[x(6)' b(6,:)];
fprintf('--------------------------------Result------------
-----------------------\n')
fprintf('%s\t %s\t\t %s\t\t %s\t\t %s\t\t %s\t\t %s\t\t\n
','x','y',...
   'first','second','third','fourth','fifth');
fprintf('%0.1f\t %0.3f
\t %0.2e\t %0.2e\t %0.2e\t %0.2e\t %0.2e\n',Data');
fprintf('%0.1f\t %0.1f
\t %0.1e\t %0.1e\t %0.1e\t %0.1e\t %0.1e\n',Data1');
fprintf('-------------------------------------------------
-----------------------\n')
fprintf('The value of D at T=330K as obtained from the
code above is %f',yint);
end
```

The output is below:

```
-------------------------------Result-----------------------------------
   x     y       first      second         third      fourth         fifth

   300.0    1.139      -3.44e-03  1.18e-05   4.00e-09   -2.93e-10   -
2.77e-12
   350.0    0.967      -2.26e-03  1.16e-05   -4.00e-08  -1.60e-11
0.00e+00
   400.0    0.854      -3.42e-03  7.60e-06   -3.76e-08  0.00e+00
0.00e+00
   250.0    1.367      -3.04e-03  1.51e-05   0.00e+00   0.00e+00
0.00e+00
   450.0    0.759      -3.80e-03  0.00e+00   0.00e+00   0.00e+00
0.00e+00
   200.0    1.7     0.0e+00     0.0e+00     0.0e+00     0.0e+00     0.0e+00
------------------------------------------------------------------------
The value of D at T=330K as obtained from the code above is 1.029021>>

The values of the density obtained from first to fifth orders
polynomials are 1.0358 1.0287 1.0289 1.0279 1.0290
The best estimate of the density is 1.0289 of third order, because the
values starts oscillating between 1.029 and 1.028 with an order higher
than three.
```

(b)The Matlab code is below:

```
clc;clear all;
```

```
T=[300 350 400 250];
D=[1.139 0.967 0.854 1.367];
TT=(T-mean(T))/std(T);
p=polyfit(TT,D,3)
```

The output is below:

```
p =

    0.0011    0.0479   -0.2222    1.0458

which achieves the coefficients.
```

(c) The Matlab code is below:

```
clear all; clc; close;
T=[300,350,400 , 250];
D=[1.139 ,0.967 ,0.854,1.367];
TT=(T-mean(T))/std(T);
func=@(x) 0.0011*x^3+0.0479*x^2-
0.2222*x+0.0169;maxit=30 ;
es= 0.0001 ;
ea=100;
xl=TT(1) ;
xu=TT(2);
iter = 0;
xr = xl;
while iter<=maxit
   xrold = xr ;
xr = (xl + xu)/2;
if xr ~= 0
ea = abs((xr - xrold)/xr)* 100;
end
test = func(xl)*func(xr);
if test < 0
xu = xr;
elseif test > 0
   xl = xr;
else
   ea = 0;
end
if ea <=es || iter >= maxit
   break;
end
iter = iter + 1;
```

```
end
root = xr ;
root=root*std(T)+mean(T);
fprintf('The the corresponding temperature is %f
K',root);
```

The output is below:

```
The the corresponding temperature is 329.992899 K>>
```