Chapter 6

PROBLEM 6.1

Determine the heat transfer coefficient at the stagnation point and the average value of the heat transfer coefficient for a single 5-cm-*OD*, 60-cm-long tube in cross-flow. The temperature of the tube surface is 260°C, the velocity of the fluid flowing perpendicular to the tube axis is 6 m/s, and temperature of the fluid is 38°C. The following fluids are to be considered (a) air, (b) hydrogen, and (c) water.

GIVEN

- A single tube in cross-flow
- Tube outside diameter (D) = 5 cm = 0.05 m
- Tube length (L) = 60 cm = 0.6 m
- Tube surface temperature (T_s) = 260°C
- Fluid velocity (V) = 6 m/s
- Fluid temperature (T_b) = 38°C

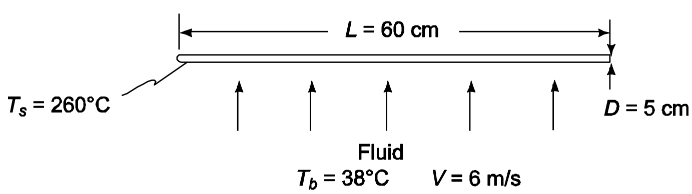
FIND

1. The heat transfer coefficient at the stagnation point (h_{co})
2. The average heat transfer coefficient $\bar{h}_c$ for the following fluids
   (a) air, (b) hydrogen, and (c) water.

ASSUMPTIONS

- Steady state
- Turbulence level of the free stream approaching the tube is low

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the bulk temperature of 38°C

Thermal conductivity (k) = 0.0264 W/(m K)

Kinematic viscosity (ν) = 17.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

and the Prandtl number at the surface temperature

(Pr_s) = 0.71.

From Appendix 2, Table 32, for hydrogen

Thermal conductivity (k) = 0.187 W/(m K)

Kinematc viscosity (ν) = 116.6 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.704

Prandtl number at the surface temperature (Pr_s) = 0.671

From Appendix 2, Table 13, for water

Thermal conductivity (k) = 0.629 W/(m K)
Kinematic viscosity (ν) = 0.685 × 10^{-6} m²/s
Prandtl number (Pr) = 4.5
Prandtl number at the surface temperature (Pr_s) = 0.86

SOLUTION

For air as the fluid
The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{(6\,\text{m/s})(0.05\,\text{m})}{17.4\times10^{-6}\,\text{m}^2/\text{s}} = 17{,}241$$

The heat transfer coefficient at the stagnation point can be calculated by applying Equation (6.2) at $\theta = 0$

$$h_\infty = 114\frac{k}{D}Re_D^{0.5}Pr^{0.4} = 1.14\,\frac{0.0264\,\text{W/(m K)}}{0.05\,\text{m}}\,(17{,}241)^{0.5}(0.71)^{0.4} = 68.9\ \text{W/(m}^2\ \text{K)}$$

The average Nusselt number is given by Equation (6.3)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C Re_D{}^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

For Re_D = 17,241
C = 0.26 m = 0.6
For Pr = 0.71
n = 0.37

$$\overline{Nu}_D = 0.26(17{,}241)^{0.6}(0.71)^{0.37}\left(\frac{0.71}{0.71}\right)^{0.25} = 79.8$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 79.8\,\frac{0.0264\,\text{W/(m K)}}{0.05\,\text{m}} = 42.1\ \text{W/(m}^2\text{K)}$$

Using the properties listed above and applying the methodology above to the other fluids yields the following results

Fluid	Re	h_{co} (W/(m²K))	$\bar{h}_c$ (W/(m²K))
Air	17,241	68.9	42.1
Hydrogen	2572	187.9	96.1
Water	438,000	17,322	20,900

COMMENTS

Since the Reynolds number for water is much higher than the air or hydrogen transition from a laminar to a turbulent boundary layer occurs sooner and the flow over most of the cylinder surface is turbulent. Hence the average heat transfer coefficient over the surface is higher than the heat transfer coefficient at the stagnation point.

PROBLEM 6.2

A mercury-in-glass thermometer at 40°C (*OD* = 1 cm) is inserted through duct wall into a 3 m/s air stream at 66°C. This can be modelled as cylinder in cross-flow, as shown in figure. Estimate the heat transfer coefficient between the air and the thermometer.

GIVEN

- Thermometer in an air stream
- Thermometer temperature (T_s) = 40°C
- Thermometer outside diameter (D) = 1 cm = 0.01 m
- Air velocity (V) = 3 m/s
- Air temperature (T_b) = 66°C

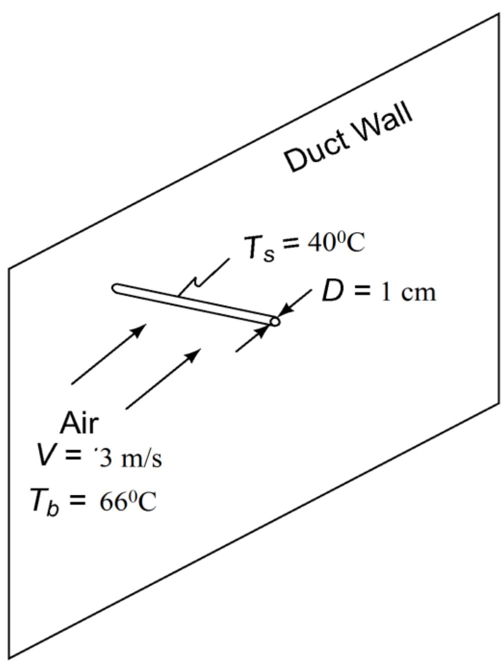
FIND

- The heat transfer coefficient $\bar{h}_c$

ASSUMPTIONS

- Steady state
- Turbulence in the free stream approaching the thermometer is low
- Effect of the duct walls in negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the bulk temperature of 66°C

Thermal conductivity (k) = 0.0282 W/(m K)

Kinematic viscosity (ν) = 20*10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At the thermometer surface temperature of 40°C, the Prandtl number (Pr_s) = 0.71

SOLUTION

The Reynolds number for this case is

$$Re_D = \frac{V\,D}{\nu} = \frac{(3m/s)(0.01m)}{\left(20*10^{-6}\,m^2/\mathrm{s}\right)} = 1500$$

The Nusselt number is given by Equation (6.3)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\,Re_D^{\,m}\,Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where $C = 0.26$

$m = 0.6$

$n = 0.37$

$$\overline{Nu}_D = 0.26(1500)^{0.6}\,(0.71)^{0.37}\left(\frac{0.71}{0.71}\right)^{0.25} = 18.43$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 18.43*\frac{(0.0282\,W/(m\,K))}{0.01\text{ m}} = 52\;W/(m^2\,K)$$

PROBLEM 6.3

> **Steam at 1 atm and 100°C is flowing across a 5-cm-*OD* tube at a velocity of 6 m/s. Estimate the Nusselt number, the heat transfer coefficient, and the rate of heat transfer per meter length of pipe if the pipe is at 200°C.**

GIVEN

- Steam flowing across a tube
- Steam pressure = 1 atm
- Steam bulk temperature (T_b) = 100°C
- Tube outside diameter (D) = 5 cm = 0.05 m
- Steam velocity (V) = 6 m/s
- Pipe surface temperature (T_s) 200°C

FIND

(a) The Nusselt number $\overline{Nu}_D$

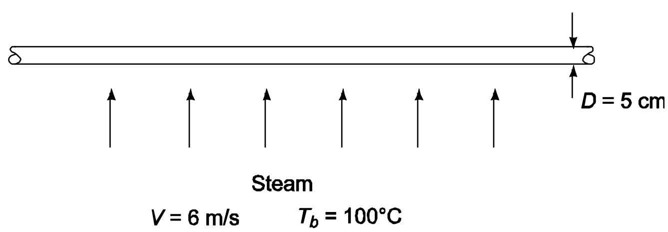
(b) The heat transfer coefficient $\bar{h}_c$

(c) The rate of heat transfer per unit length (q/L)

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 35, for steam at 100°C and 1 atm

Thermal conductivity (k) = 0.0249 W/(m K)

Kinematic viscosity (ν) = 20.2 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.987

At the tube surface temperature of 200°C, the Prandtl number of the steam (Pr_s) = 1.00

SOLUTION

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{6\,\text{m/s} \quad 0.05\,\text{m}}{20.2\times10^{-6}\,\text{m}^2/\text{s}} = 1.49 \times 10^4$$

(a) The Nusselt number for this geometry is given by Equation (6.3)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\,Re_D^{\,m}\,Pr^n\left(\frac{Pr}{Pr_s}\right)^{0.25}$$

For $Re = 1.49 \times 10^4$

$C = 0.26$ $m = 0.6$ $n = 0.37$

$$\overline{Nu}_D = 0.26(1.49 \times 10^4)^{0.6}\,(0.987)^{0.37}\left(\frac{0.987}{1.00}\right)^{0.25} = 82.2$$

(b)

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 82.2\,\frac{0.0249\,\mathrm{W/(m\,K)}}{0.05\,\mathrm{m}} = 40.9\ \mathrm{W/(m^2K)}$$

(c) The rate of heat transfer by convection from the tube is

$$q = \bar{h}_c\ A_t\,(T_s - T_b) = \bar{h}_c\,\pi DL\,(T_s - T_b)$$

$$\frac{q}{L} = 40.9\mathrm{W/(m^2K)}\ \ \pi\,(0.05\ \mathrm{m})\,(200°\mathrm{C} - 100°\mathrm{C}) = 642\ \mathrm{W/m}$$

PROBLEM 6.4

An electrical transmission line of 1.2 cm diameter carries a current of 200 Amps and has a resistance of 3 × 10^{-4} ohm per meter of length. If the air around this line is at 16°C, determine the surface temperature on a windy day, assuming a wind blows across the line at 33 km/h.

GIVEN

- An electrical transmission line on a windy day
- Line outside diameter (D) = 12 cm = 0.012 m
- Current (I) = 200b Amps
- Resistance per unit length (R_e/L) = 3 × 10^{-4} ohm/m
- Air temperature (T_b) = 16°C
- Air velocity (V) = 33 km/h = 9.17 m/s

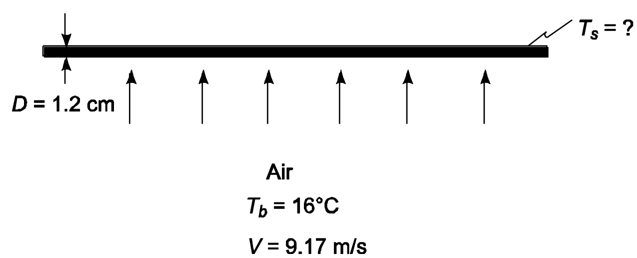
FIND

- The line surface temperature (T_s)

ASSUMPTIONS

- Steady state conditions
- Air flow approaching line has low free-stream turbulence

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 16°C

Thermal conductivity (k) = 0.0248 W/(m K)

Kinematic viscosity (ν) = 15.3 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{(9.17\,\text{m/s})\;0.012\,\text{m}}{15.3\times10^{-6}\,\text{m}^2/\text{s}} = 7192$$

The Nusselt number is given by Equation (6.3). The variation of the Prandtl number with temperature is small enough to be neglected for air

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\,Re_D^{\,m}\,Pr^n$$

For Re_D = 7192

C = 0.26 m = 0.6 n = 0.37

$$\overline{Nu}_D = 0.26(7192)^{0.6}\,(0.71)^{0.37} = 47.2$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 47.2 \frac{0.0248\,\text{W/(m K)}}{0.012\,\text{m}} = 97.5\ \text{W/(m}^2\ \text{K)}$$

The rate of heat transfer by convection must equal the energy dissipation

$$\bar{h}_c\, \pi D\, L\, (T_s - T_b) = I^2\, Re$$

Solving for the tube surface temperature

$$T_s = \frac{I^2 \left(\dfrac{R_e}{L}\right)}{\bar{h}_c \pi D} + T_b = \frac{200\,\text{A}^{\;2}\;\; 3\times10^{-4}\,\text{Ohm/m}\;\;\; \text{W/(A}^2\text{Ohm)}}{97.5\,\text{W/(m}^2\text{K)}\;\; \pi(0.012\,\text{m})} + 16°\text{C} = 19.3°\text{C}$$

COMMENTS

It is assumed that the thermal conductivity is high and thus the surface temperature is approximately uniform.

PROBLEM 6.5

Derive an equation in the form $\bar{h}_c = f(T, D, U_\infty)$ for flow of air over a long horizontal cylinder for the temperature range 0°C to 100°C, using Equation (6.3) as a basis.

GIVEN

- Flow over a long horizontal cylinder
- Air temperature range is 0°C < T < 100°C

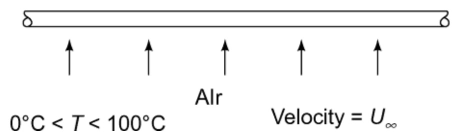
FIND

- An equation in the form $\bar{h}_c = f(T, D, U_\infty)$ based on Equation (6.3)

ASSUMPTIONS

- Steady state
- Prandtl number variation is negligible

SKETCH



SOLUTION

From Appendix 2, Table 28, for dry air the Prandtl number is constant (Pr = 0.71) for the given temperature range. From Equation (6.3), neglecting the variation of Prandtl number term

$$\bar{h}_c = C\frac{k}{D}\, Re_D^m\, Pr^n$$

where n = 0.37 for air and C and m are given in Table 6.1.

To obtain the desired functional relationship, the kinematic viscosity (ν) and thermal conductivity (k) must be expressed as a function of temperature.

From Appendix 2, Table 28

T(°C)	$\nu \times 10^6$ (m²/s)	k W/(m K)
0	13.9	0.0237
20	15.7	0.0251
40	17.6	0.0265
60	19.4	0.0279
80	21.5	0.0293
100	23.6	0.0307

Plotting these data, we see that the relationship is nearly linear in both cases. Therefore, a linear least squares regression line will be fit to the data

$$\nu = 1.38 \times 10^{-5} + 9.67 \times 10^{-8}\, T \qquad (\nu \text{ in m}^2/\text{s},\ T \text{ in °C})$$

$$k = 0.0237 + 7.0 \times 10^{-5}\, T \qquad (k \text{ in W/(m K)},\ T \text{ in °C})$$

Therefore

$$\bar{h}_c = C\,\frac{0.0237 + 7\times10^{-5}T}{D}\left(\frac{U_\infty D}{1.38\times10^{-5} + 9.67\times10^{-8}T}\right)^m (0.71)^{0.37}$$

$$\bar{h}_c = 0.881\ \mathrm{C}\ U_\infty^{\,m}\ D^{m-1}\ \frac{0.0237 + 7\times 10^{-5}\,T}{(1.38\times 10^{-5} + 9.67\times 10^{-8}\,T)^m}$$

where T is in °C

$\bar{h}_c$ is in W/(m^2 K)

and C and M are given in Table 6.1 as a function of Reynolds number

PROBLEM 6.6

Repeat Problem 6.5 for water in the temperature range 10°C to 40°C.

GIVEN

- Water flow over a long horizontal cylinder
- Water temperature range is 10°C < T < 40°C

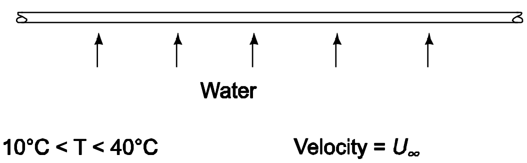
FIND

- An equation in the form $\bar{h}_c = f(T, D, U_\infty)$ based on Equation (6.3)

ASSUMPTIONS

- Steady state
- Temperature difference between water and the cylinder is small enough that the Prandtl number variation is negligible
- The density of water can be considered constant

SKETCH



SOLUTION

Equation (6.3) neglecting the Prandtl number variation

$$\bar{h}_c = C\frac{k}{D} Re^m Pr^n = \frac{k}{D}\left(\frac{U_\infty D\rho}{\mu}\right)^m \left(\frac{c\mu}{k}\right)^n$$

Where C and m are given in Table 6.1 and n = 0.37. Since Pr < 10 for the given temperature range. From Appendix 2, Table 13, for water

T (°C)	k W/(m K)	$\mu \times 10^6$ (Ns/m^2)
10	0.577	1296
15	0.585	1136
20	0.597	993
25	0.606	880.6
30	0.615	792.4
35	0.624	719.8
40	0.633	658.0

Over the given temperature range, the density of water varies only 0.8%. Therefore, the density will be considered constant at its average value: ρ = 996 kg/m^3. Likewise, the variation in specific heat is only 0.5% and its average value is c = 4185 J/(kg K).

Applying a linear least squares regression to k vs. T yields

$$k = 0.588 + 1.89 \times 10^{-3}\, T. \ (T \text{ in °C, } k \text{ in W/(m K)}).$$

Applying a linear least squares regression on log (μ) vs. log (T) yields

$$\log(\mu) = -2.375 - 0.493 \log(T)$$

$$\mu = 0.0042\, T^{-0.493}$$

Substituting these into the expression for h_c

$$\bar{h}_c = C(0.558 + 1.89 \times 10^{-3}T)^{(1-0.37)} D^{m-1} U_\infty^m \; 996\,\text{kg/m}^3 \;{}^m$$

$$(0.0042T^{-0.493})^{(0.37-m)} \; 4185\,\text{J/(kg K)} \;{}^{0.37}$$

$$\bar{h}_c = 21.88(996)^m \, C \, U_\infty^m \, D^{(m-1)}(0.558 + 1.89 \times 10^{-3}\, T)^{0.63}(0.0042\, T^{-0.493})^{(0.37-m)}$$

where $\bar{h}_c$ is in W/(m K) $\quad$ T is in °C

U_∞ is in m/s $\quad$ D is in m

and C and m are given in Table 6.1 as function of Re

PROBLEM 6.7

The Alaska Pipeline carries 2 million barrels of crude oil per day from Prudhoe Bay to Valdez covering a distance of 800 miles. The pipe diameter is 48 in. and it is insulated with 4 in. of fiberglass covered with steel sheeting. Approximately half of the pipeline length is above ground, running nominally in the north-south direction. The insulation maintains the outer surface of the steel sheeting at approximately 10°C. If the ambient temperature averages 0°C and prevailing winds are 2 m/s from the northeast, estimate the total rate of heat loss from the above-ground portion of the pipeline.

GIVEN

- Fiberglass insulated pipe with air flow at 45° to its axis
- Insulation is covered with sheet steel
- Length of pipe above ground (L) = (800 miles)/2 = 400 miles
- Pipe diameter (D_p) = 48 in.
- Insulation thickness (t) = 4 in.
- Sheet steel temperature (T_s) = 10°C
- Average ambient air temperature (T_∞) = 0°C
- Average air velocity (U_∞) = 2 m/s

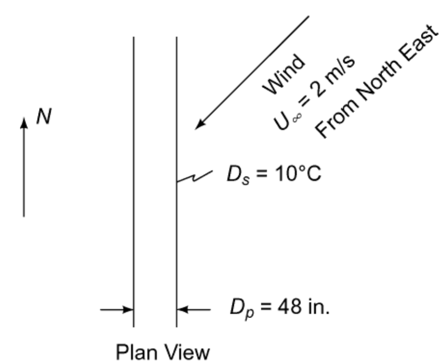
FIND

- The total rate of heat loss from the above ground portion of the pipe (q)

ASSUMPTIONS

- Thermal resistance of the sheet steel as well as contact resistance can be neglected

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 0°C

Thermal conductivity (k) = 0.0237 W/(m K)

Kinematic viscosity (ν) = 13.9×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The outside diameter of the insulated pipe is

$$D = D_p + 2t = [48 \text{ in.} + 2(4 \text{ in.})]\ 0.0254 \text{ m/in.} = 1.422 \text{ m}$$

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(6\,\text{m/s})\ 1.422\,\text{m}}{(13.9\times10^{-6}\,\text{m}^2/\text{s})} = 2.05 \times 10^5$$

Since the air flow is not perpendicular to the pipe axis, Groehn's correlation, Equation (6.4), must be used

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = 0.206\, Pr^{0.36}\, Re_N^{0.63}$$

where $Re_N = Re_D \sin\theta = 2.05 \times 10^5 \sin(45°) = 1.45 \times 10^5$

$$\overline{Nu}_D = 0.206\,(0.71)^{0.36}\,(1.45 \times 10^5)^{0.63} = 325$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 325\,\frac{0.0237\,\text{W/(m K)}}{1.422\,\text{m}} = 5.42\,\text{W/(m}^2\text{K)}$$

The total rate of heat transfer is give by

$$q = \bar{h}_c\, A_t\,(T_s - T_\infty)$$

where A_t = the total transfer area = $\pi D L$

$= \pi\,(1.422\text{ m})\,(400\text{ mi})\,(5280\text{ ft/mi})\,(0.3048\text{ m/ft}) = 2.88 \times 10^6\text{ m}^2$

$q = (5.42\text{ W/(m}^2\text{K)})\,(2.88 \times 10^6\text{ m}^2)\,(10°\text{C} - 0°\text{C}) = 1.56 \times 10^8\text{ W} = 155\text{ MW}$

COMMENTS

The calculation has assumed that there is no significant interaction between the ground and the pipe.

PROBLEM 6.8

An engineer is designing a heating system that consists of multiple tubes placed in a duct carrying the air supply for a building. She decides to perform preliminary tests with a single copper tube, 2-cm-*O.D.*, carrying condensing steam at 100°C. The air velocity in the duct is 5 m/s and its temperature is 20°C. The tube is placed normal to the flow, but it may be advantageous to place the tube at an angle to the air flow and thus increase the heat transfer surface area. It the duct width is 1 m, predict the outcome of the planned tests and estimate how the angle θ will affect the rate of heat transfer. Are there limits?

GIVEN

- A copper tube carrying condensed steam in an air duct
- Tube outside diameter (D) = 2 cm = 0.02 m
- Steam temperature (T_s) = 100°C
- Air velocity (U_∞) = 5 m/s
- Air temperature (T_∞) = 20°C
- Duct width (w) = 1 m

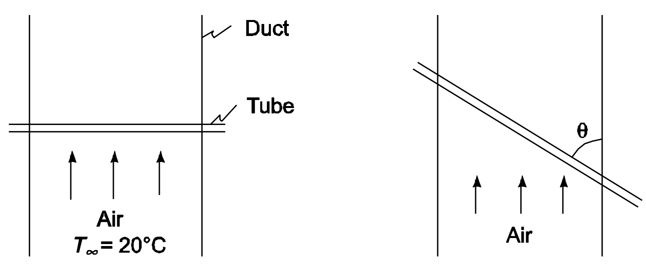
FIND

- Is it more advantageous to have the tubes normal to the air flow or at some angle to the air flow?

ASSUMPTIONS

- Steady state
- Air velocity in the duct is uniform
- Thermal resistance due to steam condensing is negligible
- Thermal resistance of the tube wall is negligible

SKETCH



Normal to flow   At an angle to flow

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 20°C

Thermal conductivity (k) = 0.0251 W/(m K)

Kinematic viscosity (ν) = 15.7 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number based on the tube diameter is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{5\,\text{m/s} \;\; 0.02\,\text{m}}{15.7\times10^{-6}\,\text{m}^2/\text{s}} = 6369$$

For the perpendicular position, the tube length (L) = w = 1 m and the Nusselt number can be calculated using Equation (6.4) with θ = 90°

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = 0.206\, Pr^{0.36}\, Re_N^{0.63}$$

where $Re_N = Re_D \sin(\theta) = Re_D$ for $\theta = 90°$

$$\overline{Nu}_D = 0.206\,(0.71)^{0.36}\,(6369)^{0.63} = 45.4$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 45.4 \frac{0.0251\,\text{W/(m K)}}{0.02\,\text{m}} = 57.0\ \text{W/(m}^2\ \text{K)}$$

The rate of heat transfer is

$$q = \bar{h}_c\, \pi D L\, (T_s - T_\infty) = 57.0\,\text{W/(m}^2\text{K)}\ \ \pi\,(0.02\ \text{m})(1\ \text{m})(100°\text{C} - 20°\text{C}) = 287\ \text{W}$$

For the angled position, the tube length (L) = $w/\sin\theta$. Applying Equation (6.4)

$$\overline{Nu}_D = 0.206\,(0.71)^{0.36}\,(6369 \sin\theta)^{0.63} = 45.38\,(\sin\theta)^{0.63}$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 45.38\,(\sin\theta)^{0.63}\ \frac{0.0251\,\text{W/(m K)}}{0.02\,\text{m}} = 56.95\,(\sin\theta)^{0.63}\ \ \text{W/(m}^2\text{K)}$$

The rate of heat transfer is

$$q = \bar{h}_c\, \pi D L\, (T_s - T_\infty) = \bar{h}_c\, \pi D \frac{\text{w}}{\sin\theta}\,(T\text{s} - T_\infty)$$

$$q = 56.95\ \sin\theta^{\ 0.63}\,\text{W/(m}^2\text{K)}\ \ \pi\,(0.02\ \text{m})\ \frac{1\,\text{m}}{\sin\theta}\,(100°\text{C} - 20°\text{C}) = 286.3\,(\sin\theta)^{-0.37}\ \text{W}$$

The engineer will find that the rate of heat transfer will increase because the heat transfer coefficient decreases with $(\sin\theta)^{0.63}$ but the area increases with $1/\sin\theta$. Therefore, the rate of heat transfer increases with $1/(\sin\theta)^{0.37}$.

PROBLEM 6.9

In a metal manufacturing plant and its heat treatment process, a long, hexagonal copper extrusion (or rod) comes out of heat-treatment furnace at 400°C and is then quenched by immersing it in a 50°C air stream flowing perpendicular to its axis at 10 m/s as depicted in the figure. The surface of the copper has an emissivity of 0.9 due to oxidation in this process. The rod is 3 cm across opposing flat sides, and it has a cross-sectional area of 7.79 cm^2, and a perimeter of 10.4 cm. Determine the time required for the center of the copper to cool to 100°C.

GIVEN

- A long hexagonal copper extrusion in an air stream flowing perpendicular to its axis
- Initial temperature (T_o) = 400°C
- Air temperature (T_∞) = 50°C
- Air velocity (V_∞) = 10 m/s
- Surface emissivity (ε) = 0.9
- Distance across the flats (D) = 3 cm = 0.03 m
- Cross sectional area of the extrusion (A_c) = 7.79 cm^2 = 7.79 × 10^{-4} m^2
- Perimeter of the extrusion (P) = 10.4 cm = 0.104 m

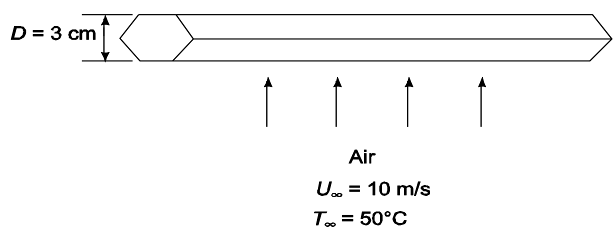
FIND

- The time (t) required for the center of the copper to cool to 100°C

ASSUMPTIONS

- Variations of the copper properties with temperature are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average of the initial and final film temperature of 150°C

Thermal conductivity (k_a) = 0.0339 W/(m K)
Kinematic viscosity (ν) = 29.6 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71

For Appendix 2, Table 12, for copper

Thermal Conductivity (k) = 386 W/(m K) at 250°C
Density (ρ) = 8933 kg/m^3 at 20°C
Specific heat (c) = 383 J/(kg K) at 20°C

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{10\,\text{m/s} \quad 0.02\,\text{m}}{29.6\times10^{-6}\,\text{m}^2/\text{s}} = 10{,}135$$

The Nusselt number for non-circular cross sections in gases by Equation (6.6)

$$\overline{Nu}_D = B\, Re_D^{\,n}$$

where D, B, and n are given by Table 6.2 B = 0.138, n = 0.638

$$\overline{Nu}_D = 0.138\,(10{,}135)^{0.638} = 49.6$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 49.6\,\frac{0.0339\ \text{W/(m K)}}{0.03\,\text{m}} = 56.0\ \text{W/(m}^2\ \text{K)}$$

The characteristic length for determining the Biot number of the rod is defined in Section 2.6.1 as

$$L_c = \frac{\text{volume}}{\text{surface area}} = \frac{LA_c}{LP} = \frac{A_c}{P} = \frac{7.79\times10^{-4}\,\text{m}^2}{0.104\,\text{m}} = 0.0075\ \text{m}$$

The Biot Number, from Table 4.3, is

$$Bi = \frac{\bar{h}_c L_c}{k_c} = \frac{56\,\text{W/(m}^2\text{K)}\ \ 0.0075\,\text{m}}{386\,\text{W/(m K)}} = 0.0011 << 0.1$$

Therefore, the internal thermal resistance of the extrusion may be neglected and lumped parameters may be applied. An energy balance on the extrusion, including radiation, yields the following

$$q = PL[\,\bar{h}_c\,(T - T_\infty) + \varepsilon\,\sigma\,(T^4 - T_\infty^{\,4})] = -\,\rho\,A_c\,L_c\,(dT/dt)$$

This equation must be solved numerically

$$\frac{dT}{dt} = -\frac{P}{\rho A_c c}\,[\,\bar{h}_c\,(T - T_\infty) + \varepsilon\,\sigma\,(T^4 - T_\infty^{\,4})] = -\frac{P}{\rho L_c c}\,[\,\bar{h}_c\,(T - T_\infty) + \varepsilon\,\sigma\,(T^4 - T_\infty^{\,4})]$$

$$\frac{dT}{dt} = \frac{-1}{8933\,\text{kg/m}^3\ \ 0.0075\,\text{m}\ \ 383\,\text{(Ws)/(kg K)}}$$

$$\left[\ 56.0\,\text{W/(m}^2\ \text{K)}\ \ (T - T_\infty) + 0.9\ \ 5.67\times10^{-8}\ \text{W/(m}^2\,\text{K}^4)\ \ [T^4 - T_\infty^{\ 4}]\right]$$

$$\frac{dT}{dt} = -\,0.0022(T - T_\infty) - 1.9887\times10^{-12}\,(T^4 - T_\infty^{\,4})\ \text{K / s}$$

This can be solved numerically using a finite difference method

$$\Delta T = T(t + \Delta T) - T(t) = -\Delta t\{0.0022[T(t) - T_\infty] + 1.9887\times10^{-12}[T(t)^4 - T_\infty^{\,4}]\}$$

$T_\infty = 323$ K, Let $\Delta t = 30$ seconds

t (s)	T (K)		t (s)	T (K)
0	673		390	431
30	638		420	422
60	608		450	415
90	582		480	407
120	559		510	401
150	538		540	395
180	519		570	389
210	503		600	384
240	488		630	380
270	474		660	375
300	462	$\Delta t = 14$ s	674	373.3
330	451			
360	440			
		$t = 674$ s = 11.2 minutes		

PROBLEM 6.10

Repeat Problem 6.9 if the extrusion cross-section is elliptical with the major axis normal to the air flow and same mass per unit length. The major axis of the elliptical cross-section is 5.46 cm and its perimeter is 12.8 cm.

GIVEN

- A long elliptical copper extrusion in an air stream
- Initial temperature (T_o) = 400°C
- Air Temperature (T_∞) = 50°C
- Air velocity (V_∞) = 10 m/s
- Surface emissivity (ε) = 0.9
- Elliptical cross-section with major axis normal to the air flow
- Length of the major axis of the ellipse (D) = 5.46 cm = 0.0546 m
- Perimeter of ellipse (P) = 12.8 cm = 0.128 m
- Same mass per unit length as Problem 6.9

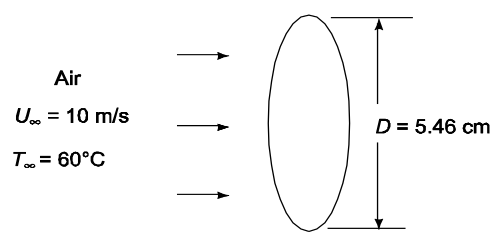
FIND

- The time (t) required for the center of the copper to cool to 100°C

ASSUMPTIONS

- Air flow is perpendicular to the axis of the extrusion
- Variation of the copper properties with temperature is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average of the initial and final film temperature of 150°C

Thermal conductivity (k_a) = 0.0339 W/(m K)
Kinematic viscosity (ν) = 29.6 × 10^{-69} m^2/s
Prandtl number (Pr) = 0.71

From Appendix 2, Table 12, for copper

Thermal conductivity (k) = 386 W/(m K) at 250°C
Density (ρ) = 8933 kg/m^3 at 20°C
Specific heat (c) = 383 J/(kg K) at 20°C

SOLUTION

Since the density of the extrusion in this problem is the same as the previous problem, the same mass per unit length implies the same cross-section area

$$A_{c,\text{ellipse}} = A_{c,\text{hexagon}} = 7.79 \text{ cm}^2 = 7.79 \times 10^{-4} \text{ m}^2$$

Following the same procedure as the solution to Problem 6.9

The Reynolds number is

$$Re_D = U_\infty D/\nu = (10\,\text{m/s})(0.0546\,\text{m})\big/\left(29.6\times10^{-6}\,\text{m}^2/\text{s}\right) = 18{,}446$$

The Nusselt number for non-circular cross sections in gases is given by Equation (6.6)

$$\overline{Nu}_D = B\, Re_D^{\,n}$$

where D, B, and n are given by Table 6.2 $B = 0.085$, $n = 0.804$

(Although the Reynolds number for this case is slightly out of range of Equation (6.6), it will be applied to estimate the Nusselt number)

$$\overline{Nu}_D = 0.085\,(18{,}446)^{0.804} = 229$$

$$\bar{h}_c = \overline{Nu}_D\,(k/D) = 229\,((0.0339\ \text{W/(m K)})/0.0546\,\text{m}) = 142\ \text{W/(m}^2\ \text{K)}$$

The characteristic length for determining the Biot number of the rod is defined in Section 2.6.1 as

$$L_c = \frac{\text{volume}}{\text{surface area}} = \frac{LA_c}{LP} = \frac{A_c}{P} = \frac{7.79\times10^{-4}\,\text{m}^2}{0.128\,\text{m}} = 0.0061\ \text{m}$$

The Biot number, from Table 4.3, is

$$Bi = \frac{\bar{h}_c L_c}{k_c} = \frac{142\,\text{W/(m}^2\text{K)}\ \ 0.0061\,\text{m}}{386\,\text{W/(m K)}} = 0.0022 \ll 0.1$$

Therefore, the internal thermal resistance of the extrusion may be neglected and lumped parameters may be applied. An energy balance on the extrusion, including radiation, yields the following

$$\frac{dT}{dt} = -\frac{1}{\rho L_c c}\,[\,\bar{h}_c\,(T - T_\infty) + \varepsilon\,\sigma\,(T^4 - T_\infty^4)]$$

$$\frac{dT}{dt} = \frac{-1}{8933\,\text{kg/m}^3\ \ 0.0061\,\text{m}\ \ 383\,(\text{W s})/(\text{kg K})}$$

$$\left[\,142\,\text{W/(m}^2\ \text{K)}\ (T - T_\infty) + 0.9\ \ 5.67\times10^{-8}\,\text{W/(m}^2\ \text{K}^4)\ [T^4 - T_\infty{}^4]\right]$$

$$\frac{dT}{dt} = -\,0.0068\,(T - T_\infty) - 2.445\times10^{-12}\,(T^4 - T_\infty^4)\ \text{K/s}$$

This can be solved numerically using a finite difference method

$$\Delta T = T(t+\Delta T) - T(t) = -\Delta t\{0.0068[T(t) - T_\infty] + 2.445\times10^{-12}\,[T(t)^4 - T_\infty^4]\}$$

$$T_\infty = 323\ \text{K},\quad \text{Let } \Delta t = 30 \text{ seconds}$$

	t (s)	T (K)
	0	673
	30	587
	60	525
	90	479
	120	444
	150	418
	180	397
Let Δt = 26 seconds	206	383
Let Δt = 19 seconds	225	375
	$t \approx 225$ s = 3.75 minutes	

COMMENTS

The elliptical extrusion cools more quickly due to both higher convection heat transfer coefficient and more surface area.

PROBLEM 6.11

The human body is typically modelled as a vertical cylinder that is 1.8 m high and is 30 cm in diameter, as shown in the figure. Calculate the average rate of heat loss from this body, which is maintained at 37°C, on a windy day when the airstream has a 5 m/s velocity and is at 35°C. To ascertain "wind chill" effects, compare this result with the heat loss that would occur in "stagnant" conditions, or when it is not windy and the heat transfer is only by natural convection (consider an average heat transfer coefficient of 3.6 W/(m^2 K) for free convection). What is the wind chill effect if the wind got stronger (10 m/s) and colder (25°C)? Even though the natural convection heat transfer coefficient also changes somewhat (as discussed later in Chapter 8), for this calculation consider it to remain the same. Moreover, compare the heat loss in both cases with the typical energy intake, or metabolic heat production from consumption of food, of about 1033 kcal/day and comment upon your results.

GIVEN

- Human body modeled as a cylinder in an air stream
- Body surface temperature (T_s) = 37°C
- Air velocity (V_∞) = 5 m/s
- Air temperature (T_∞) = 35°C
- Cylinder diameter (D) = 30 cm = 0.3 m
- Cylinder height (H) = 1.8 m

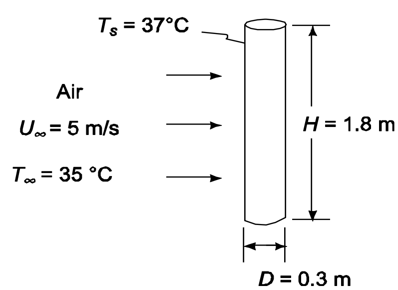
FIND

(a) The heat loss from the idealized human body
(b) Heat loss if the wind speed is 10 m/s and its temperature is 25^{0}C.
(c) Compare with the free convection results of Problem 5.8 and with the typical food consumption rate of 1033 kcal/day

ASSUMPTIONS

- Air velocity is perpendicular to the axis of the cylinder
- Air flow approaching cylinder is laminar
- Heat transfer from the ends can be neglected

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 35°C

Thermal conductivity (k) = 0.0262 W/(m K)

Kinematic viscosity (ν) = 17.1 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At the surface temperature of 37°C Pr_s = 0.71

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{5\,\text{m/s} \quad 0.3\,\text{m}}{17.1\times10^{-6}\,\text{m}^2/\text{s}} = 87{,}719 \qquad \& \qquad \frac{L}{D} = \frac{1.8\,\text{m}}{0.3\,\text{m}} = 6$$

(a) Since $L/D > 4$, its effect on the Nusselt number is negligible and Equation (6.3) may be applied

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\, Re_D^m\, Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where $n = 0.37$ and, from Table 6.1: $C = 0.26$ m = 0.6

$$\overline{Nu}_D = 0.26\,(87{,}719)^{0.6}\,(0.71)^{0.37}\,(1) = 212$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 212\,\frac{0.0262\,\text{W/(m K)}}{0.3\text{m}} = 18.6\,\text{W/(m}^2\text{K)}$$

The rate of heat transfer is

$$q = \bar{h}_c\, \pi D L\,(T_s - T_\infty) = 18.6\,\text{W/(m}^2\text{K)}\;\pi\,(0.3\,M)(1.8\text{ m})(37°\text{C} - 35°\text{C}) = 63.1\text{ W}$$

(b) For U_∞=10 m/s and T_∞= 25^{0}C

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(10\,\text{m/s})(0.3\text{m})}{(17.1\times10^{-6}\,\text{m}^2/\text{s})} = 174{,}238$$

Applying equation (6.3) we have

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\, Re_D^m\, Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where $n = 0.37$ and, from Table 6.1: $C = 0.26$ m = 0.6

$$\overline{Nu}_D = 0.26\,(174238)^{0.6}\,(0.71)^{0.37}\,(1) = 319$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 319 * \frac{0.0262\,\text{W/(m K)}}{0.3\text{m}} = 27.9\,\text{W/(m}^2\text{K)}$$

The rate of heat transfer is

$$q = \bar{h}_c\, \pi D L\,(T_s - T_\infty) = \left(27.9\,\text{W/(m}^2\text{K)}\right) \pi\,(0.3\,M)(1.8\text{ m})(37°\text{C} - 25°\text{C}) = 568.2\text{ W}$$

(c) From Problem 6.8 for natural convection

$$q_{\text{natural}} = 92.2\text{ W}$$

This result is 46% higher than that calculated above. Note that the ambient air temperature in Problem 8.8 is 20°C. The natural convection heat transfer coefficient for that problem was 3.6 W/m^2 K which is only 19% of the value calculated above for forced convection.

The rate of food consumption is

$$\text{Food consumption} = 1033\,\text{kcal/day}\;\; 1000\,\text{cal/kcal}\;\; 4.1868\,\text{J/cal}\left(\frac{1\text{day}}{24\,\text{hr}}\right)\left(\frac{1\,\text{hr}}{3600\,\text{s}}\right)\;(\text{Ws})/\text{J} = 50.1\text{W}$$

This heat transfer rate is 21% lower than that calculated in part (a).

PROBLEM 6.12

A nuclear reactor fuel rod is a circular cylinder 6 cm in diameter. The rod is to be tested by cooling it with a flow of sodium at 205°C and a velocity of 5 cm/s perpendicular to its axis. If the rod surface is not to exceed 300°C, estimate the maximum allowable power dissipation in the rod.

GIVEN

- Cylinder in a cross flow of liquid sodium
- Cylinder diameter (D) = 6 cm = 0.06 m
- Sodium temperature (T_∞) = 205°C
- Sodium velocity (U_∞) = 5 cm/s = 0.05 m/s
- Maximum rod surface temperature (T_s) = 300°C

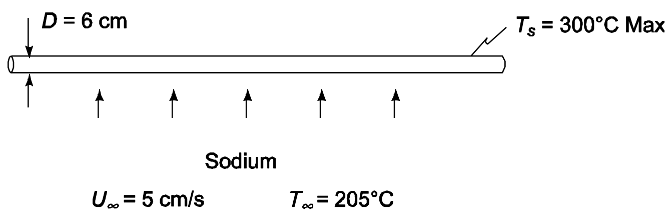
FIND

- The maximum allowable power dissipation $\dot{q}_G$

ASSUMPTIONS

- Steady state
- Turbulence in the sodium flow approaching the rod is low
- Heat generation per unit volume in the rod is uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for sodium at 205°C

Thermal conductivity (k) = 80.3 W/(m K)

Kinematic viscosity (ν) = 4.6×10^{-7} m²/s

Prandtl number (Pr) = 0.0072

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{0.05\,\text{m/s} \;\; 0.06\,\text{m}}{4.6\times10^{-7}\,\text{m}^2/\text{s}} = 6522$$

$Re_D\, Pr$ = 6522 (0.0072) = 47.0

Therefore, Equation (6.7) may be applied

$$\overline{Nu}_D = 1.125\,(Re_D\,Pr)^{0.413} = 1.125\,(47.0)^{0.413} = 5.52$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 5.52\,\frac{(80.3\,\text{W/(m K)})}{0.06\,\text{m}} = 7381\,\text{W/(m}^2\text{K)}$$

The rate of heat transfer at the maximum surface temperature is

$$q = \bar{h}_c A_t (T_s - T_\infty) = \bar{h}_c \pi D L (T_s - T_\infty)$$

$$\frac{q}{L} = 7381 \text{W/(m}^2\text{K)} \; \pi (0.06 \text{ m})(1 \text{ m})(300°\text{C} - 205°\text{C}) = 1.32 \times 10^5 \text{ W/m}$$

The maximum rate of heat generation per unit volume of the rod is

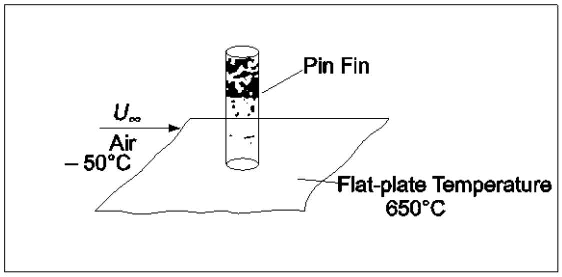
$$\dot{q}_G = \frac{q}{\text{volume}} = \frac{q}{\frac{\pi}{4} D^2 L} = \frac{4}{\pi D^2}\left(\frac{q}{L}\right) = \frac{4}{\pi \; 0.06 \text{ m}^{\;2}} \; 1.32 \times 10^5 \text{ W/m} = 4.67 \times 10^7 \text{ W/m}^3$$

COMMENTS

If the rate of heat generation exceeds the value calculated, the surface temperature will rise to dissipate the energy. Also, non-uniform heat generation can lead to hot spots as will variations in the local value of the heat transfer coefficient around the circumference (see Equation (6.2)).

PROBLEM 6.13

A stainless steel pin fin 5-cm-long, 6-mm-*OD*, extends from a flat plate into a 175 m/s air stream as shown in the sketch. Estimate (a) the average heat transfer coefficient between air and the fin. (b) the temperature at the end of the fin. (c) the rate of heat flow from the fin.



GIVEN

- A stainless steel pin fin in an air stream
- Pin length (L) = 5 cm = 0.05 m
- Pin diameter (D) = 6 mm = 0.006 m
- Air velocity (U_∞) = 175 m/s

FIND

(a) The average heat transfer coefficient ($\bar{h}_c$)
(b) The temperature of the end of the fin (T_L)
(c) The rate of heat flow from the fin (q_f)

ASSUMPTIONS

- Steady state
- Air approaching the fin has negligible turbulence
- Radiative heat transfer is negligible
- Steel is type 304
- Steel properties are uniform

PROPERTIES AND CONSTANTS

Extrapolating from Appendix 2, Table 28, for dry air at –50°C

Thermal conductivity (k) = 0.0202 W/(m K)

Kinematic viscosity (ν) = 9.3×10^{-6} m²/s

Prandtl number (Pr) = 0.71

From Appendix 2, Table 10, for Type 304 stainless steel k_s = 14.4 W/(m K) at 20°C
(Note that figure 1.6 shows very little increase in k for stainless steel in the range of 300°C to 700°C.)

SOLUTION

(a) The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{175\,\text{m/s}\;\;0.006\,\text{m}}{9.3\times10^{-6}\,\text{m}^2/\text{s}} = 1.13 \times 10^5 \qquad \& \qquad \frac{L}{D} = \frac{0.05\,\text{m}}{0.006\,\text{m}} = 8.33 > 4$$

Therefore, Equation (6.3) and Table 6.1 may be used. (Note that Pr/Pr_s = 1)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\, Re_D^m\, Pr^n \qquad c = 0.26 \qquad m = 0.6 \qquad n = 0.37$$

$$\overline{Nu}_D = 0.26\,(1.13 \times 10^5)^{0.6}\,(0.71)^{0.37} = 247$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 247 \frac{0.0202\,\text{W/(m K)}}{0.006\,\text{m}} = 829 \ \text{W/(m}^2\text{K)}$$

(b) From Table 2.1, for a fin of uniform cross-section with convection at the tip, the temperature distribution is

$$\frac{T - T_\infty}{T_s - T_\infty} = \frac{\cosh[m(L-x)] + \left(\frac{\bar{h}_c}{\text{m k}}\right)\sinh[m(L-x)]}{\cosh(m\,L) + \left(\frac{\bar{h}_c}{\text{m k}}\right)\sinh[m\,L]}$$

Where $m = \sqrt{\frac{\bar{h}_c P}{k_s A_c}} = \sqrt{\frac{\bar{h}_c \pi D}{k_s \frac{\pi}{4} D^2}} = \sqrt{\frac{4\bar{h}_c}{k_s D}} = \sqrt{\frac{4\ 829\,\text{W/(m}^2\ \text{K)}}{14.4\,\text{W/(m K)}\ \ 0.006\,\text{m}}} = 196.3\ \text{1/m}$

$$m\,L = 196.3\ \text{1/m}\ (0.05\ \text{m}) = 9.81$$

$$\frac{\bar{h}_c}{\text{m K}} = \frac{829\,\text{W/(m}^2\text{K)}}{196.3(\text{1/m})\ \ 14.4\,\text{W/(m K)}} = 0.2943$$

At $x = L$ $\quad \frac{T - T_\infty}{T_s - T_\infty} = \frac{\cosh(0) + 0.2943\sinh(0)}{\cosh(9.81) + 0.2943\sinh(9.81)} = 0.000085$

$\therefore \quad T = 0.000085\ (T_s - T_\infty) + T_\infty = 0.000085\ (650°\text{C} - 50°\text{C}) - 50°\text{C} = -49°\text{C}$

The tip temperature is practically the same as the ambient temperature.

(c) The rate of heat transfer, from Table 2.1 is

$$q_f = M \frac{\sinh(m\,L) + \left(\frac{\bar{h}_c}{\text{m k}}\right)\cosh(m\,L)}{\cosh(m\,L) + \left(\frac{\bar{h}_c}{\text{m k}}\right)\sinh(m\,L)}$$

where $\quad M = \sqrt{\bar{h}_c P k_s A_a}\ (T_s - T_\infty) = \sqrt{\bar{h}_c \frac{\pi^2}{4} D^3 k_s}\ \ (T_s - T_\infty)$

$$M = \sqrt{829\,\text{W/(m}^2\text{K)}\ \ \frac{\pi^2}{4}\ 0.006\,\text{m}\ \ ^3 14.4\,\text{W/(m K)}}\ \ (650°\text{C} + 50°\text{C}) = 55.94$$

$$q_f = 55.94\ \text{W}\ \frac{\sinh(9.81) + 0.2943\cosh(9.81)}{\cosh(9.81) + 0.2943\sinh(9.81)} = 55.9\ \text{W}$$

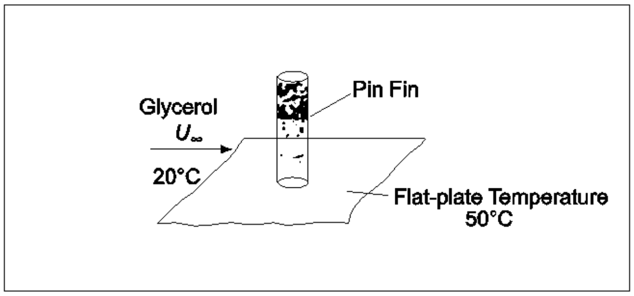
COMMENTS

These results should be considered an estimate due to uncertainty in the air properties.

Also, due to the presence of the surface from which the fin protrudes, the flow is not uniform as assumed by Equation (6.3), therefore, the heat transfer coefficient may vary.

PROBLEM 6.14

Repeat Problem 6.13 with glycerol at 20°C flowing over the fin at 2 m/s. The plate temperature is 50°C.



GIVEN

- A stainless steel pin fin in an air stream
- Pin length (L) = 5 cm = 0.05 m
- Pin diameter (D) = 6 mm = 0.006 m
- Glycerol velocity (U_∞) = 2 m/s
- Glycerol temperature (T_∞) = 20°C
- Plate temperature (T_p) = 50°C

FIND

(a) The average heat transfer coefficient ($\bar{h}_c$)
(b) The temperature of the end of the fin (T_L)
(c) The rate of heat flow from the fin (q_f)

ASSUMPTIONS

- Steady state
- Turbulence in the glycerol approaching the fin is low
- Radiative heat transfer is negligible
- Steel is type 304
- Steel properties are uniform
- Variation of the thermal properties of glycerol and steel with temperature is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 22, for glycerol at 20°C

Thermal conductivity (k) = 0.285 W/(m K)

Kinematic viscosity (ν) = 1175 × 10^{-6} m^2/s

Prandtl number (Pr) = 12,609

From Appendix 2, Table 10, for type 304 stainless steel

$$k_s = 14.4 \text{ W/(m K) at } 20°\text{C}$$

SOLUTION

(a) The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{2\,\text{m/s}\quad 0.006\,\text{m}}{1175\times10^{-6}\text{m}^2/\text{s}} = 10.21$$

Therefore, Equation (6.3) and Table 6.1 may be used. (Note that Pr/Pr_s = 1)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\, Re_D^m\, Pr^n \qquad c = 0.75 \qquad m = 0.4 \qquad n = 0.36$$

$$\overline{Nu}_D = 0.75(10.21)^{0.4}\,(12{,}609)^{0.36} = 56.88$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 56.88\,\frac{0.285\,\mathrm{W/(m\,K)}}{0.006\,\mathrm{m}} = 2701\,\mathrm{W/(m^2K)}$$

(b)

$$m = \sqrt{\frac{4\bar{h}_c}{k_s D}} = \sqrt{\frac{4\ \ 2701\,\mathrm{W/(m^2K)}}{14.4\,\mathrm{W/(m\,K)}\ \ 0.006\,\mathrm{m}}} = 354\ 1/\mathrm{m}$$

$$m\,L = 354\ 1/\mathrm{m}\ \ (0.05\ \mathrm{m}) = 17.7$$

$$\frac{\bar{h}_c}{m\,K} = \frac{2701\,\mathrm{W/(m^2K)}}{354\,1/\mathrm{m}\ \ 14.4\,\mathrm{W/(m\,K)}} = 0.53$$

At $x = L$

$$\frac{T - T_\infty}{T_s - T_\infty} = \frac{\cosh(0) + 0.53\sinh(0)}{\cosh(17.7) + 0.53\sinh(17.7)} = 2.69 \times 10^{-8}$$

Therefore, the tip temperature is practically the same as the ambient glycerol temperature.

(c) The rate of heat transfer, from Table 2.1 is

$$M = \sqrt{2701\,\mathrm{W/(m^2K)}\ \frac{\pi^2}{4}\ \ 0.006\,\mathrm{m}^{\ 3}\ \ 14.4\,\mathrm{W/(m\,K)}}\ \ (50°\mathrm{C} - 20°\mathrm{C}) = 4.32$$

$$q_f = 4.32\ \mathrm{W}\ \frac{\sinh(17.7) + 0.53\cosh(17.7)}{\cosh(17.7) + 0.556\sinh(18.54)} = 4.32\ \mathrm{W}$$

PROBLEM 6.15

> **Water at 180°C enters a bare, 15-m-long, 2.5-cm wrought iron pipe at 3 m/s. If air at 10°C flows perpendicular to the pipe at 12 m/s, determine the outlet temperature of the water. (Note that the temperature difference between the air and the water varies along the pipe.)**

GIVEN

- Wrought-iron pipe with water flow inside and perpendicular air flow outside
- Water entrance temperature ($T_{W,\text{in}}$) = 180°C
- Water velocity (V_W) = 3 m/s
- Pipe length (L) = 15 m
- Pipe diameter (D) = 2.5 cm = 0.025 m
- Air temperature (T_a) = 10°C
- Air velocity (V_a) = 12 m/s

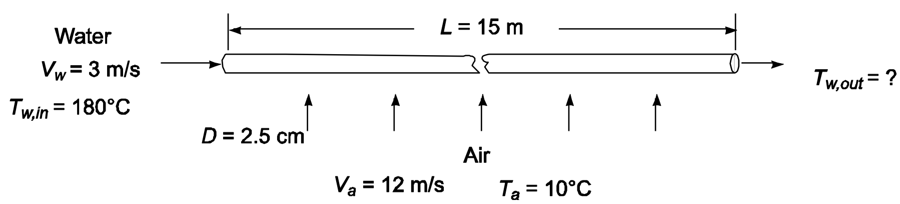
FIND

- Outlet temperature of the water ($T_{W,\text{out}}$)

ASSUMPTIONS

- Steady state
- Air flow approaching pipe is negligible
- Thermal resistance of the pipe is negligible
- The pipe thickness can be neglected

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 10°C

Thermal conductivity (k_a) = 0.0244 W/(m K)

Kinematic viscosity (ν_a) = 17.8 × 10^{-6} m^2/s

Prandtl number (Pr_a) = 0.71

From Appendix 2, Table 13, for water at the entrance temperature of 180°C

Thermal conductivity (k_w) = 0.673 W/(m K)

Kinematic viscosity (ν_w) = 0.173 × 10^{-6} m^2/s

Prandtl number (Pr_w) = 1.01

Density (ρ_w) = 886.6 kg/m^2

Specific Heat (c) = 4396 J/(kg K)

SOLUTION

Air Side:

The Reynolds number on the air side is

$$(Re_D)_{\text{air}} = \frac{V_a D}{\nu_a} = \frac{12\,\text{m/s}\quad 0.025\,\text{m}}{17.8\times10^{-6}\,\text{m}^2/\text{s}} = 16{,}853$$

The Nusselt number is given by Equation 6.3 and Table 6.1

$$\overline{Nu}_{D\ \text{air}} = \frac{\bar{h}_{c\ \text{air}}\, D}{k_a} = C\, Re_D^m\, Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where $C = 0.26$, $m = 0.6$, and $n = 0.37$.

Note that the Prandtl number of air does not change appreciably between the air and water temperatures. Therefore, $Pr/Pr_s = 1$.

$$\overline{Nu}_{D\ \text{air}} = 0.26\ (16{,}853)^{0.6}\ (0.71)^{0.36} = 78.7$$

$$\bar{h}_{c\ \text{air}} = \overline{Nu}_{D\ \text{air}} \frac{k_a}{D} = 78.7\ \frac{0.0244\,\text{W/(m K)}}{0.025\,\text{m}} = 76.8\ \text{W/(m}^2\text{K)}$$

Water Side:

The Reynolds number based on the inlet properties is

$$Re_D = \frac{V_w D}{\nu_w} = \frac{3\,\text{m/s}\quad 0.025\,\text{m}}{0.173\times 10^{-6}\ \text{m}^2/\text{s}} = 4.33 \times 10^5 \text{ (Turbulent)}$$

Applying Equation (6.60)

$$\overline{Nu}_{D\ \text{water}} = \frac{\bar{h}_{c\ \text{water}}\, D}{k_w} = 0.023\, Re_D^{0.8}\, Pr_n \qquad n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_{D\ \text{water}} = 0.023\ (4.33 \times 10^5)^{0.8}\ (1.01)^{0.3} = 746$$

$$\bar{h}_{c\ \text{water}} = \overline{Nu}_{D\ \text{water}} \frac{k_w}{D} = 746\ \frac{0.673\,\text{W/(m K)}}{0.025\,\text{m}} = 20{,}078\ \text{W/(m}^2\text{K)}$$

The overall heat transfer coefficient is

$$\frac{1}{U} = \frac{1}{\bar{h}_{c\ \text{air}}} + \frac{1}{\bar{h}_{c\ \text{water}}} = \frac{1}{76.8\,\text{W/(m}^2\text{K)}} + \frac{1}{20{,}078\,\text{W/(m}^2\text{K)}} = 0.0131\ (\text{m}^2\text{K})/\text{W}$$

$$U = 76.6\,\text{W/(m}^2\text{K)}$$

Let’s assume that the water temperature changes little from the pipe inlet to outlet. Since the air temperature is constant and uniform, the heat transfer from the water is then analogous to the uniform surface temperature analysis of Section 6.2.2 and Equation (6.36) may be applied

$$\frac{T_{w,\text{out}} - T_a}{T_{w,\text{in}} - T_a} = \exp\left(-\frac{UPL}{\dot{m}\,c}\right) = \exp\left(-\frac{U\pi DL}{\frac{\pi}{4} V_w D^2 \rho_w c}\right) = \exp\left(-\frac{4UL}{V_w D \rho_w c}\right)$$

Solving for the water outlet temperature

$$T_{w,\text{out}} = T_a + (T_{w,\text{in}} - T_a)\exp\left(-\frac{4UL}{V_w D \rho_w c}\right)$$

$$T_{w,\text{out}} = 10°\text{C} + (180°\text{C} - 10°\text{C})\exp\left[-\frac{4\ \ 76.6\,\text{W/(m}^2\text{K)}\ \ 15\,\text{m}}{3\,\text{m/s}\ \ 0.025\,\text{m}\ \ 886.6\,\text{kg/m}^3\ \ 4396\,\text{J/(kg K)}\ \ (\text{Ws})/\text{J}}\right]$$

$$T_{w,\text{out}} = 177°\text{C}$$

Therefore, the assumption that the water changes little from pipe inlet to outlet is valid.

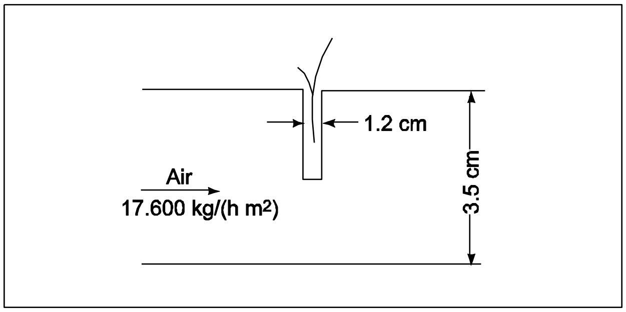
COMMENTS

The average water temperature is 178.5°C. This is not different enough from the inlet temperature to justify another iteration using the water properties at the average water temperature.

Note that the convective thermal resistance of the air is 99.6% of the total thermal resistance.

PROBLEM 6.16

The temperature of air flowing through a 25-cm-diameter duct whose inner walls are at 320°C is to be measured with a thermocouple soldered in a cylindrical steel wall of 1.2-cm-*OD* with an oxidized exterior, as shown in the accompanying sketch. The air flows normal to the cylinder at a mass velocity of 17,600 kg/(h m²). If the temperature indicated by the thermocouple is 200°C, estimate the actual temperature of the air.



GIVEN

- Cylindrical thermocouple wall in an air duct
- Duct diameter (D_d) = 25 cm = 0.25 m
- Duct wall temperature (T_{ds}) = 320°C =m 593 K
- Wall outside diameter (D_w) = 1.2 cm = 0.012 m
- Exterior of wall is oxidized
- Air mass velocity $\dot{m}/A$ = 17,600 kg/(h m²)
- Thermocouple indicated temperature (T_{tc}) = 200°C = 473 K

FIND

- Air temperature (T_∞)

ASSUMPTIONS

- Steady state
- Thermal resistance between the thermocouple and the wall exterior surface is negligible
- Inside of duct behaves as a black body enclosure
- Conduction to the thermocouple wall from the duct wall can be neglected

PROPERTIES AND CONSTANTS

From Appendix 2, Table 7, the emissivity of oxidized steel (ε) = 0.94.

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m² K⁴)

SOLUTION

An iterative solution must be used since the rate of heat transfer will depend on the air properties which are a function of the unknown air temperature. Heat is transferred by radiation from the duct wall to the thermocouple wall and from the thermocouple wall to the air. Therefore, the air temperature will be lower then the thermocouple reading. The rate of heat transfer from the wall to the thermocouple must equal that from the thermocouple to the air

$$\bar{h}_c A\,(T_{tc} - T_a) = \sigma\,\varepsilon\,A\,(T_{ds}^4 - T_{tc}^4)$$

Solving for the air temperature

$$T_a = T_{tc} - \frac{\sigma\,\varepsilon}{\bar{h}_c}\,(T_{ds}^4 - T_{tc}^4)$$

For the first iteration, let T_a = 150°C. From Appendix 2, Table 28, for air at 150°C

Density (ρ) = 0.820 kg/m^3

Thermal conductivity (k) = 0.0339 W/(m K)

Kinematic viscosity (ν) = 29.6 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At the wall temperature of 200°C Pr_s = 0.71

The air velocity (U_∞) is

$$U_\infty = \frac{m}{A\rho} = \frac{17600\,\text{kg/(h m}^2\text{)}}{0.820\,\text{kg/m}^3 \quad 3600\,\text{s/h}} = 5.96\ \text{m/s}$$

The Reynolds number based on the well diameter is

$$Re_D = \frac{U_\infty D_w}{\nu} = \frac{5.96\,\text{m/s} \quad 0.012\,\text{m}}{29.6\times10^{-6}\,\text{m}^2\text{/s}} = 2417$$

The Nusselt number is given by Equation (6.3) and Table 6.1

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\,Re_D^m\,Pr^n\left(\frac{Pr}{Pr_s}\right)^{0.25} \quad \text{where } C = 0.026 \quad m = 0.6 \quad n = 0.37$$

$$\overline{Nu}_D = 0.26\,(2417)^{0.6}\,(0.71)^{0.36}\,(1) = 24.54$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 24.54\,\frac{0.0339\,\text{W/(m K)}}{0.012\,\text{m}} = 69.3\ \text{W/(m}^2\text{K)}$$

The air temperature is

$$T_a = 200°\text{C} - \frac{5.67\times10^{-8}\ \text{W/(m}^2\text{K}^4\text{)} \quad 0.94}{69.3\,\text{W/(m}^2\text{K)}}\,[(593\text{ K})^4 - (473\text{ K})^4] = 143°\text{C}$$

The original guess for T_a is close to the above value. Another iteration using air properties at 143°C would not significantly improve the result.

PROBLEM 6.17

Develop an expression for the ratio of the rate of heat transfer to water at 40°C from a thin flat strip of width $\pi D/2$ and length L at zero angle of attack and a tube of the same length and diameter D in cross-flow with its axis normal to the water flow in the Reynolds number range between 50 and 1000. Assume both surfaces are at 90°C.

GIVEN

- Water flowing over a thin flat strip at zero angle of attack or a tube in crossflow
- Water temperature (T_∞) = 40°C
- Tube diameter = D
- Strip width = $\pi D/2$
- Tube and strip length = L
- Reynolds number: $50 < Re < 1000$

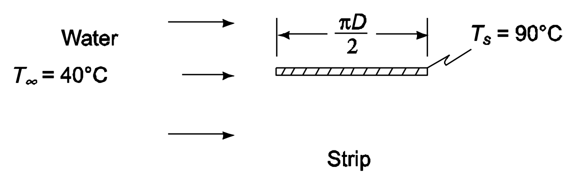
FIND

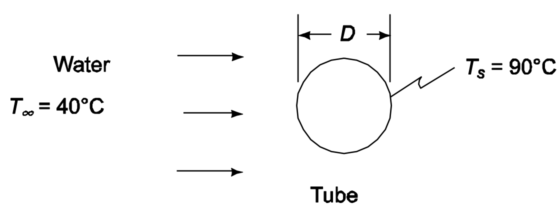
- The ratio of the heat transfer from the strip and that from the cylinder. (q_s/q_t)

ASSUMPTIONS

- Steady state for both cases
- The tube and strip temperatures (T_s) are 90°C

SKETCH





PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 40°C: Prandtl number (Pr) = 4.3
At the surface temperature of 90°C: Pr_s = 1.94

SOLUTION

Note that the heat transfer area (πD) is the same in both cases.

Thin Strip:

The flow over the thin strip is laminar for the Reynolds number given. The Nusselt number is given by Equation (4.38)

$$\overline{Nu}_L = 0.664\ Re_L^{\frac{1}{2}}\ Pr^{\frac{1}{3}}$$

Tube:

The Nusselt number for the tube is given by Equation (6.3) and Table 6.1

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\left(\frac{U_\infty D}{\nu}\right)^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

Re_D	C	m
1 – 40	0.75	0.4
40 – 1 × 10^3	0.51	0.5
1 × 10^3 – 2 × 10^5	0.26	0.6
2 × 10^5 – 1 × 10^6	0.076	0.7

Since the transfer areas and temperature differences are the same, the ratio of the rates of heat transfer is equal to the ratio of the heat transfer coefficients. The heat transfer rate from the strip will be 64% of that from the tube with the same Reynolds number.

PROBLEM 6.18

Repeat Problem 6.17 for air flowing over the same two surfaces in the Reynolds number range between 40,000 and 200,000. Neglect radiation.

GIVEN

- Air flowing over a thin flat strip at zero angle of attack or a tube in crossflow
- Air temperature (T_∞) = 40°C
- Tube diameter = D
- Strip width = $\pi D/2$
- Tube and strip length = L
- Reynolds number : 40,000 < Re < 200,000

FIND

- The ratio of the heat transfer from the strip and that from the cylinder. (q_s/q_t)

ASSUMPTIONS

- Radiative heat transfer is negligible
- Steady state for both cases
- The tube and strip temperatures (T_s) are 90°C

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 40°C

Kinematic viscosity (ν) = 17.6×10^{-6} m²/s

Prandtl number (Pr) = 0.71

at 90°C Pr_s = 0.71

SOLUTION

This solution follows the same procedure as the solution to Problem 6.17

Applying Equation (5.38)

$$\overline{Nu}_L = 0.664\ Re_L^{\frac{1}{2}}\ Pr^{\frac{1}{3}}$$

For the tube, from Equation (6.3) and Table 6.1

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\left(\frac{U_\infty D}{\nu}\right)^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

Re_D	C	m
1 – 40	0.75	0.4
40 – 1×10^3	0.51	0.5
1×10^3 – 2×10^5	0.26	0.6
2×10^5 – 1×10^6	0.076	0.7

but $Pr = Pr_S$. The heat transfer rate from the strip will be 165% of that from the tube with the same Reynolds number.

PROBLEM 6.19

The instruction manual for a hot-wire anemometer states that 'roughly speaking, the current varies as the one-fourth power of the average velocity at a fixed wire resistance'. Check this statement, using the heat transfer characteristics of thin wire in air and water.

GIVEN

- A thin current carrying wire in an air or water stream

FIND

- Show that the current (I) varies as the one-fourth power of the fluid velocity (V_∞) at a fixed resistance (Re_I)

ASSUMPTIONS

- Radiative heat transfer is negligible

SOLUTION

This solution follows the same procedure as the solution to Problem 6.17

Holding the wire resistance constant has the effect of holding the wire temperature (T_s) and therefore, the fluid properties constant. T_s, T_b, A_t, and Re_I are constant.

According to Equation (6.3)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\left(\frac{U_\infty D}{\nu}\right)^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

for 40 < Re < 1000 m = 0.5 $\rightarrow$ I a U_∞1/4

Since the wire diameter is typically a few microns, we expect the Reynolds number to be very low. Therefore, from Table 6.1 m = 0.4 to 0.5 and m/2 = 0.2 to 0.25

Re_D	C	m
1 – 40	0.75	0.4
40 – 1 × 10^3	0.51	0.5
1 × 10^3 – 2 × 10^5	0.26	0.6
2 × 10^5 – 1 × 10^6	0.076	0.7

PROBLEM 6.20

A hot-wire anemometer is used to determine the boundary layer velocity profile in the air flow over a scale model of an automobile. The hot-wire is held in a traversing mechanism that moves the wire in a direction normal to the surface of the model. The hot-wire is operated at constant temperature. The boundary layer thickness is to be defined as the distance from the model surface at which the velocity is 90% of the free stream velocity. If the probe current is I_0, when the hot-wire is held in the free stream velocity, U_∞, what current indicates the edge of the boundary layer? Neglect radiation heat transfer from the hot-wire and conduction from the ends of the wire.

GIVEN

- Thin, electrically-heated constant-temperature wire in air flow near an automobile model
- Boundary layer thickness ° point when velocity (U_y) = 90% free stream velocity V_o
- Probe current at $U_\infty = I_o$

FIND

- Probe current at edge of the boundary layer (I_b)

ASSUMPTIONS

- Radiation is negligible
- Conduction from the ends of the hot-wire is negligible
- Reynolds number is small

SOLUTION

Restating the desire result: What is the current of $V = 0.9\ V_\infty$ in terms of the current I_o at V_o? Since the diameter of the wire will be very small, the Reynolds number will be small. For $1 < Re < 40$ the Nusselt number, use Equation (6.3) and Table 6.1

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\left(\frac{U_\infty D}{\nu}\right)^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

Re_D	C	m
1 – 40	0.75	0.4
40 – 1 × 10^3	0.51	0.5
1 × 10^3 – 2 × 10^5	0.26	0.6
2 × 10^5 – 1 × 10^6	0.076	0.7

U_y is the air velocity at a distance y from the model surface.

The rate of electrical energy dissipation must equal the rate of convective heat transfer. The electrical resistance of the wire (Re_I) is a function of the wire temperature only and is therefore constant in this case.

For $U_y = 0.9\ U_\infty$

$$I = I_o\ (0.9)0.2$$

$$I = 0.979\ I_o$$

The current will be 0.979 I_o at the edge of the boundary layer.

PROBLEM 6.21

A platinum hot-wire anemometer operated in the constant-temperature mode has been used to measure the velocity of a helium stream. The wire diameter is 20 μm, its length is 5 mm, and it is operated at 90°C. The electronic circuit used to maintain the wire temperature has a maximum power output of 5 watts and is unable to accurately control the wire temperature if the voltage applied to the wire is less than 0.5 volt. Compare the operation of the wire in the helium stream at 20°C and 10 m/s with operation in air and water at the same temperature and velocity. The electrical resistance of the platinum at 90°C is 21.6 μΩ - cm.

GIVEN

- A constant temperature platinum hot-wire in a stream of helium
- Wire diameter = 20 μm = 20 * 10^{-6} m
- Wire length (L) = 5 mm = 0.005 m
- Wire temperature (T_w) = 90°C
- Maximum electric power to wire (P_{max}) = 5 W
- Minimum voltage (V_{min}) = 0.5 V
- Helium temperature (T) = 20°C
- Helium velocity (U_{∞}) = 10 m/s
- Resistivity (r_e) = 21.6 μΩ - cm.= 21.6 *10^{-8} Ω - m

FIND

- Compare the operation of the wire in helium to that in air and water

ASSUMPTIONS

- Radiation is negligible

PROPERTIES AND CONSTANTS

From Appendix 2

Fluid	Helium	Air	Water
Table number	31	28	13
Thermal conductivity at 20°C, k (W/(mK))	0.1471	0.0251	0.597
Kinematic viscosity at 20°C, × 10^6 (m^2/s)	122.2	15.7	1.006
Prandtl number at 20°C, Pr	0.70	0.71	7.0
Prandtl number at 90°C, Pr_s	0.71	0.71	1.94

SOLUTION

The Nusselt number is given by Equation (6.3) and Table 6.1

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = C\left(\frac{U_\infty D}{\nu}\right)^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

Re_D	C	m
1 – 40	0.75	0.4
40 – 1 × 10^3	0.51	0.5
1 × 10^3 – 2 × 10^5	0.26	0.6
2 × 10^5 – 1 × 10^6	0.076	0.7

Helium: C = 0.75 m = 0.4 n = 0.37

Air: C = 0.75 m = 0.4 n = 0.37

Water: C = 0.51 m = 0.5 n = 0.37

The rate of convective heat transfer must equal the electrical power dissipated. Therefore, the power capabilities of the unit are sufficient for these conditions in helium and air but not in water. The voltage for the case with air is too low for the device. Therefore, the device will perform adequately only for the helium flow under these conditions.

PROBLEM 6.22

A hot-wire anemometer consists of a 5-mm-long, 5-µm-diameter platinum wire. The probe is operated at constant current of 0.03 amp. The electrical resistivity of platinum is 17 µΩ-cm at 20°C and increases by 0.385% per °C.

(a)If the voltage across the wire is 1.75 volts, determine the velocity of the air flowing across it and the wire temperature if the free-stream air temperature is 20°C.

(b)What are the wire temperature and voltage if the air velocity is 10 m/s?

Neglect radiation and conduction heat transfer from the wire.

GIVEN

- A hot wire in air
- Wire diameter (D) = 5 m = 5×10^{-6} m
- Wire length (L) 5 mm = 0.005 m
- Current (I) = 0.03 A (constant)
- Electrical resistivity (r_{el}) = 17 W cm = 17×10^{-8} W m at 20°C and increases 0.385% per °C.
- Air temperature (T_∞) = 20°C

FIND

(a) The air velocity (U_∞) and the wire temperature (T_w) if the voltage across the wire (V_{el}) =1.75V
(b) The wire temperature (T_w) and voltage (V_{el}) if the air velocity (U_∞) = 10 m/s

ASSUMPTIONS

- Radiative heat transfer is negligible
- Variation of Prandtl number with temperature id negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 20°C

Thermal conductivity (k) = 0.0251 W/(m K)

Kinematic viscosity (ν) = 15.7×10^{-6} m²/s

Prandtl number (Pr) = 0.71

At 90°C: Pr = 0.71

SOLUTION

The electrical resistivity of the wire as a function of temperature is

$$\rho_{el} = \rho_{el,20}\,[1 + 0.00385\,(T_w - 20°\text{C})] \qquad (T_w \text{ in °C})$$

The electrical resistance of the wire is

$$R_{el} = \frac{\rho_{el} L}{A_c} = \frac{4\rho_e L}{\pi D^2} = \frac{4\ 0.005\,\text{m}}{\pi\ 5\times10^{-6}\,\text{m}^{\ 2}}\,[17 \times 10^{-8}\ \Omega\text{m}]\,[1 + 0.00385\,(T_w - 20°\text{C})]$$

$$R_{el} = 43.29\ \Omega\,[1 + 0.00385\,(T_w - 20°\text{C})]$$

(a) The voltage across the wire is given by

$$V_{el} = IR_{el} = I(43.29\ \Omega)\,[1 + 0.00385\,(T_w - 20°\text{C})]$$

Solving for the wire temperature

$$T_w = \frac{1}{0.00385}\left[\frac{V_{el}}{I(43.29\Omega)} - 1\right] + 20°\text{C}$$

$$= \frac{1}{0.00385}\left[\frac{1.75\,\text{volt}}{0.03\,\text{A}(43.29\Omega)} - 1\right] + 20°\text{C} = 110°\text{C}$$

The rate of convective heat transfer for the wire must equal the electrical power dissipated.

$$\bar{h}_c\, \pi D\, L\, (T_w - T_\infty) = V_{el} I$$

$$\bar{h}_c = \frac{V_{el}\, I}{\pi DL(T_w - T_\infty)} = \frac{1.75\,\text{volt}\,(0.03\,\text{A})\ \ \text{W/(volt A)}}{\pi\,(5\times10^{-6}\,\text{m})\,(0.005\,\text{m})\,(110°\text{C} - 20°\text{C})} = 7427\ \text{W/(m}^2\text{K)}$$

Assuming that $1 < Re < 40$, and neglecting variation of Prandtl number, Equation (6.3) and Table 6.1 give the heat transfer coefficient as

$$\bar{h}_c = 0.75\,\frac{k}{D}\left(\frac{U_\infty D}{\nu}\right)^{0.4} Pr^{0.37}$$

Solving for the air velocity

$$U_\infty = \left[\frac{D\,\bar{h}_c}{0.75\,k}\,Pr^{-0.37}\left(\frac{\nu}{D}\right)^{0.4}\right]^{2.5}$$

$$U_\infty = \left[\frac{(5\times10^{-6}\,\text{m})(7427\,\text{W/(m}^2\text{K)})}{0.75(0.0251\,\text{W/(m K)})}(0.71)^{-0.37}\left(\frac{15.7\times10^{-6}\ \text{m}^2/\text{s}}{5\times10^{-6}\,\text{m}}\right)^{0.4}\right]^{2.5} = 23.6\,\text{m/s}$$

Note that $Re_D = \dfrac{U_\infty D}{\nu} = \dfrac{23.6\,\text{m/s}\ \ 5\times10^{-6}\,\text{m}}{15.7\times10^{-6}\,\text{m}^2/\text{s}} = 7.5$ which is in the assumed range.

(b) The Reynolds number at $U_\infty = 10$ m/s is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{10\,\text{m/s}\ \ 5\times10^{-6}\,\text{m}}{15.7\times10^{-6}\,\text{m}^2/\text{s}} = 3.18$$

$$\overline{Nu}_D = 0.75\,(3.18)^{0.4}\,(0.71)^{0.37} = 1.05$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 1.05\,\frac{0.0251\,\text{W/(m K)}}{5\times10^{-6}\,\text{m}} = 5271\,\text{W/(m}^2\text{K)}$$

Balancing the rate of heat transfer and electrical power dissipation

$$\bar{h}_c\,\pi D\, L\,(T_w - T_\infty) = I^2 R_{el} = I^2\,(43.29\,\Omega)\,[1 + 0.00385\,(T_w - 20°\text{C})]$$

$$5271\,\text{W/(m}^2\text{K)}\ \ \pi\,(5\times10^{-6}\,\text{m})\,(0.005\text{m})\,(T_w - 20°\text{C}) = (0.03\ \text{A})^2\,(43.29\,\Omega)\,[1 + 0.00385\,(T_w - 20°\text{C})]$$

$$0.000414\,\text{W/K}\ \ (T_w - 20°\text{C}) = (0.0390\ \text{W})\,[1 + 0.00385\,(T_w - 20°\text{C})]$$

By trial and error $T_w = 168°\text{C}$

$$V_{el} = IR_{el} = (0.03\text{A})\,(43.29\,\Omega)\,[1 + 0.00385(168°\text{C} - 20°\text{C})] = 2.04\ \text{Volts}$$

COMMENTS

Some heat transfer correlations require that air properties be evaluated at the film temperature. This type of correlation would make the calculation of velocity from a given voltage much more difficult since the film temperature changes with the wire temperature. In this case, operation in the constant temperature mode is much simpler because the film temperature is fixed.

PROBLEM 6.23

A 2.5 cm sphere is maintained at 50°C in either an air stream or a water stream, both at 20°C and 2 m/s velocity. Compare the rate of heat transfer and the drag on the sphere for both fluids.

GIVEN

- A sphere in an air stream or a water stream
- Sphere diameter (D) = 2.5 cm = 0.025 m
- Sphere temperature (T_s) = 50°C
- Fluid temperature (T_f) = 20°C
- Fluid velocity (U_∞) = 2 m/s

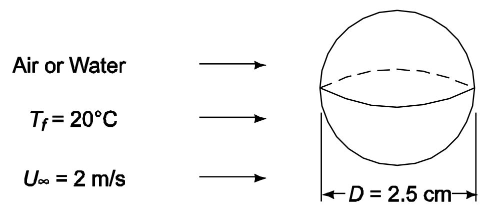
FIND

- The rate of heat transfer (q) and the drag force

ASSUMPTIONS

- Radiation is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2

Fluid	Air	Water
Table Number	28	13
Density at 20°C, ρ (kg/m^3)	1.164	998.2
Thermal conductivity at 20°C, k (W/(m K))	0.0251	0.597
Kinematic Viscosity at 20°C, $\nu \times 10^6$ (m^2/s)	15.7	1.006
Prandtl number at 20°C, Pr	0.71	7.0
Absolute viscosity at 20°C, $\mu_\infty \times 10^6$ (Ns)/m^2	18.240	993
Absolute viscosity at 50°C, $\mu_\infty \times 10^6$ (Ns)/m^2	19.515	555.1

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu}$$

For air

$$Re_D = \frac{2\,\text{m/s} \quad 0.025\,\text{m}}{15.7\times10^{-6}\,\text{m}^2/\text{s}} = 3185$$

For water

$$Re_D = \frac{2\,\text{m/s} \quad 0.025\,\text{m}}{1.006\times10^{-6}\,\text{m}^2/\text{s}} = 49{,}702$$

Equation (6.11) can be applied to both cases

$$Nu_D = 2 + (0.4\, Re_D^{0.5} + 0.06\, Re_D^{\,0.67})\, Pr^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{0.25}$$

For air

$$Nu_D = 2 + (4.0\,(3185)^{0.5} + 0.06\,(3185)^{0.67})\,(0.71)^{0.4} \left(\frac{18.240}{19.515}\right)^{0.25} = 32.8$$

$$h_c = Nu_D \frac{k}{D} = 32.8 \frac{0.0251\,\text{W/(m K)}}{0.025\,\text{m}} = 32.9\ \text{W/(m}^2\text{K)}$$

For water

$$Nu_D = 2 + (0.4\,(49{,}702)^{0.5} + 0.06\,(49{,}702)^{0.7})\,(7.0)^{0.4} \left(\frac{993}{555.1}\right)^{0.25} = 438$$

$$h_c = Nu_D \frac{k}{D} = 438 \frac{0.597\,\text{W/(m K)}}{0.025\,\text{m}} = 10{,}469\ \text{W/(m}^2\text{K)}$$

The rate of heat transfer is $q = h_c\, A\, \Delta T = h_c\, \pi D^2\, (T_s - T_\infty)$

For air

$$q = 32.9\,\text{W/(m}^2\text{K)}\ \pi\,(0.025\text{ m})^2\,(50°\text{C} - 20°\text{C}) = 1.9\text{ W}$$

For water

$$q = 10{,}469\,\text{W/(m}^2\text{K)}\ \pi\,(0.025\text{ m})^2\,(50°\text{C} - 20°\text{C}) = 617\text{ W}$$

The total drag coefficient can be read from Figure 6.6 and is defined in Section 6.2 as

$$C_D = \frac{\text{Drag force}}{\left(\frac{\rho U_\infty^2}{2}\right)\left(\frac{\pi D^2}{4}\right)} \Rightarrow \text{Drag force} = \frac{1}{8} C_D\, \rho\, U_\infty^2\, \pi D^2$$

For air, From Figure 6.6, $C_D = 0.4$

$$\text{Drag force} = \frac{1}{8}(0.4)\ \ 1.164\,\text{kg/m}^3\ \ \ 2\,\text{m/s}^{\ 2}\ \pi\,(0.025\text{ m})^2\ \ (\text{Ns}^2)/(\text{kg m}) = 0.00046\text{ N}$$

For water, From Figure 6.6, $C_D = 0.5$

$$\text{Drag force} = \frac{1}{8}\ (0.5)\ \ 998.2\,\text{kg/m}^3\ \ \ 2\,\text{m/s}^{\ 2}\ \pi\,(0.025\text{m})^2\ \ (\text{Ns}^2)/(\text{kg m}) = 0.49\text{ N}$$

COMMENTS

Note that the heat transfer increases by a factor of 324 in water while the drag force increases by a factor of 1065.

PROBLEM 6.24

Determine the effect of forced convection on heat transfer from an incandescent lamp. What would be the glass temperature be for air velocities of 0.5, 1, 2, and 4 m/s?

GIVEN

- A spherical glass light bulb in air
- Bulb power consumption (P) = 100 W
- 10% of energy is in the form of visible light
- Diameter (D) = 10 cm = 0.1 m
- Bulb emissivity (ε) = 0.85
- Ambient air temperature (T_∞) = 20°C = 293 K

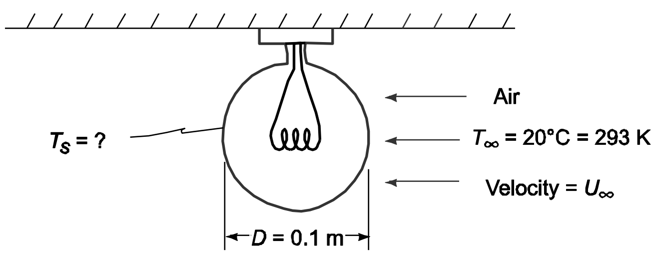
FIND

- The glass temperature (T_s) for air velocities (U_∞) of 0.5, 1, 2, and 4 m/s

ASSUMPTIONS

- The bulb has reached steady state
- The surrounding behave as a black body at T_∞

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4).
From Appendix 2, Table 28, for dry air at 20°C

Thermal conductivity (k) = 0.025 W/(m K)

Kinematic viscosity (ν) = 15.7 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu}$$

For U_∞ = 0.5 m/s $\quad Re_d = \dfrac{0.5\,\text{m/s} \quad 0.1\,\text{m}}{15.7\times10^{-6}\,\text{m}^2/\text{s}}$ = 3185

The Nusselt number is given by Equation (6.9)

$$Nu_D = 0.37\, Re_D^{0.6}$$

$$h_c = Nu_D \frac{k}{D} = 0.37 \frac{k}{D} Re_D^{0.6}$$

For $U_\infty = 0.5$ m/s

$$h_c = 0.37 \frac{0.0251\,\text{W/(m K)}}{0.1\,\text{m}} (3185)^{0.6} = 11.74 \text{ W/(m}^2\text{K)}$$

The rate of convective and radiative heat loss must equal the rate of heat generation

$$q_c + q_r = \pi D^2 [h_c (T_s - T_\infty) + \varepsilon \sigma (T_s^4 - T_\infty^4))] = 0.9 (100 \text{ W}) = 90 \text{ W}$$

For $U_\infty = 0.5$ m/s

$$q_c + q_r = \pi (0.1)^2 \left[11.74\,\text{W/(m}^2\text{K)} \; T_s - 293\text{K} + 0.85 \; 5.67 \times 10^{-8}\,\text{W/(m}^2\text{K}^4) \left[T_s^4 - 293\text{K}^{\;4} \right] \right] = 90 \text{ W}$$

Checking the units, then eliminating them for clarity

$$1.514 \times 10^{-9} T_s^4 + 0.3688\, T_s - 209.2 = 0$$

By trial and error: T_s = 429 K = 156°C

Following the same procedure for the other air velocities yields the following results

Velocity, U_∞ (m/s)	Heat transfer coefficient, h_c (W/m^2 K))	Glass Temperature, T_s (°C)
0.5	11.74	141
1.0	17.79	130
2.0	26.96	104
4.0	40.87	81

PROBLEM 6.25

An experiment was conducted in which the heat transfer from a sphere in sodium was measured. The sphere, 0.5 in. in diameter was pulled through a large sodium bath at a given velocity while an electrical heater inside the sphere maintains the temperature at a set point. The following table gives the results of the experiment

Run #	**1**	**2**	**3**	**4**	**5**
Velocity (m/s)	**3.44**	**3.14**	**1.56**	**3.44**	**2.16**
Sphere Surface Temp (°C)	**478**	**434**	**381**	**350**	**357**
Sodium Bath Temp (°C)	**300**	**300**	**300**	**200**	**200**
Heater Temp (°C)	**486**	**439**	**385**	**357**	**371**
Heat Flux × 10^{-6} (W/m^2)	**14.6**	**8.94**	**3.81**	**11.7**	**8.15**

Determine how well the above data are predicated by the appropriate correlation given in the text. Express your results in terms of the per cent difference between the experimentally determined Nusselt number and that from the equation.

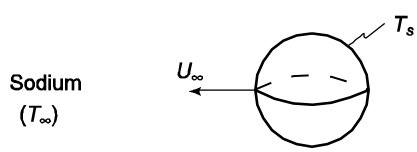
GIVEN

- A sphere is pulled through a sodium bath at a given velocity
- Sphere diameter = 0.5 in. = 0.0127 m
- Sphere temperature is kept constant
- Experimental data given above

FIND

- The standard deviation between the data and the appropriate correlation

SKETCH



SOLUTION

The Correlation of the heat transfer rate will be illustrated with Run #1. The film temperature (T_f) = (T_s + T_∞)/2 = (478°C + 300°C)/2 = 389°C.

From Appendix 2, Table 27, for sodium at 389°C

Thermal conductivity (k) = 71.6 W/(m K)

Kinematic viscosity (ν) = 3.08 × 10^{-7} m^2/s

Prandtl number (Pr) = 0.0050

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{3.44\,\text{m/s} \quad 0.0127\,\text{m}}{2.75\times10^{-7}\,\text{m}^2/\text{s}} = 1.42 \times 10^5$$

The Nusselt number for spheres in liquid metals for

$3.6 \times 10^4 < Re_D < 2 \times 10^5$ is given by Equation (6.14)

$$\overline{Nu}_D = 2 + 0.386\,(RePr)^{\frac{1}{2}} = 2 + 0.386\,\left[1.42\times10^5(0.005)\right]^{\frac{1}{2}} = 12.28$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 12.28\,\frac{71.6\,\text{W/(m K)}}{0.0127\,\text{m}} = 69{,}230\ \text{W/(m}^2\text{K)}$$

The heat flux from the sphere is

$$\frac{q}{A} = \bar{h}_c (T_s - T_\infty) = 69,230\,\text{W/(m}^2\,\text{K)} \quad (478°\text{C} - 300°\text{C}) = 1.23 \times 10^7\,\text{W/m}^2$$

Similarly for the other test runs

Run #	1	2	3	4	5
Film Temp. (°C)	389	367	341	275	279
k (W/(m K))	71.6	72.6	73.8	77.0	76.8
$\nu \times 10^7$ (m^2/s)	3.08	3.19	3.42	3.99	3.96
Pr	0.0050	0.0052	0.0055	0.0063	0.0063
$Re_D \times 10^{-5}$	1.42	1.25	0.579	1.09	0.693
$\bar{h}_c$ W/(m^2 K)	69,230	67,690	51,649	73,463	60,868
$q/A \times 10^{-6}$ (W/m^2)	12.3	9.07	4.18	11.0	9.56
exper. $q/A \times 10^{-6}$ (W/m^2)	14.6	8.94	3.18	11.7	8.15
Percent difference (%)	–15.8	+1.5	+9.7	–5.9	–17.3

PROBLEM 6.26

A copper sphere initially at a uniform temperature of 132°C is suddenly released at the bottom of a large bath of bismuth at 500°C. The sphere diameter is 1 cm and it rises through the bath at 1 m/s. How far will the sphere rise before its center temperature is 300°C? What is its surface temperature at that point? (The sphere has a thin nickel plating to protect the copper from the bismuth.)

GIVEN

- A copper sphere with a thin nickel plating rising through a bath of bismuth
- Initial copper temperature (T_o) = 132°C (uniform)
- Bismuth temperature (T_∞) = 500°C
- Ascent velocity (U_∞) = 1 m/s
- Sphere diameter (D) = 1 cm = 0.01 m

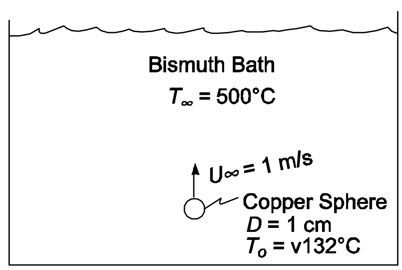
FIND

(a) Distance sphere will rise before its center temperature, $T(o,t)$ = 300°C
(b) The sphere surface temperature at that time, $T(r_o,t)$

ASSUMPTIONS

- Thermal resistance of the nickel plating is negligible
- Thermal properties of the copper can be considered uniform and constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 25, for Bismuth at the initial film temperature of 316°C

Thermal conductivity (k_b) = 16.44 W/(m K)

Kinematic viscosity (ν) = 1.57 × 10^{-7} m^2/s

Prandtl number (Pr) = 0.014

From Appendix 2, Table 12, for copper

Thermal conductivity (k_c) = 388 W/(m K) at its mean temperature of 216°C

Specific heat (c) = 383 J/(kg K) at 20°C

Density (ρ) = 8933 kg/m^3 at 20°C

Thermal diffusivity (α) = 116.6 × 10^{-6} m^2/s

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{1\,\text{m/s} \quad 0.01\,\text{m}}{1.57\times10^{-7}\ \text{m}^2/\text{s}} = 6.37 \times 10^4$$

Applying Equation (6.14)

$$\overline{Nu}_D = 2 + 0.386\ (RePr)^{\frac{1}{2}} = 2 + 0.386\left[6.37\times10^4(0.014)\right]^{\frac{1}{2}} = 13.52$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 13.53\ \frac{16.44\,\text{W/(m K)}}{0.01\,\text{m}} = 2.22\times10^4\ \text{W/(m}^2\ \text{K)}$$

(a) The Biot number for the sphere is

$$Bi = \frac{\bar{h}_c r}{k_s} = \frac{2.22\times10^4\ \text{W/(m}^2\text{K)}\quad 0.005\,\text{m}}{388\,\text{W/(m K)}} = 0.287 > 0.1$$

Therefore, internal thermal resistance is significant and the chart solutions of Figure 2.44 must be used.

$$\frac{T(0,t)-T_\infty}{T_o-T_\infty} = \frac{300°\text{C}-500°\text{C}}{132°\text{C}-500°\text{C}} = 0.543$$

$$\text{and}\qquad \frac{1}{Bi} = \frac{1}{0.287} = 3.48$$

From Figure 2.44

$$Fo = \frac{\alpha t}{r_o^{\,2}} = 0.75$$

$$\therefore\qquad t = 0.75\ \frac{r_o^{\,2}}{\alpha} = 0.75\ \frac{0.005\,\text{m}^{\ 2}}{116.6\times10^{-6}\ \text{m}^2/\text{s}} = 0.16\ \text{s}$$

The distance (x) the sphere will rise during this time is

$$x = U_\infty t = 1\ \text{m/s}\ (0.16\text{s}) = 0.16\ \text{m} = 16\ \text{cm}$$

(b) The surface temperature can be determined from Figure 2.44

$$\frac{1}{Bi} = 3.48 \text{ and } \frac{r}{r_o} = 1 \qquad \Rightarrow \frac{T(r_o,t)-T_\infty}{T(0,t)-T_\infty} = 0.84$$

$$T(r_o,t) = 0.84\ (300°\text{C} - 500°\text{C}) + 500°\text{C} = 332°\text{C}$$

PROBLEM 6.27

A spherical water droplet of 1.5-mm-diameter is freely falling in atmospheric air. Calculate the average convection heat transfer coefficient when the droplet has reached its terminal velocity. Assume that the water is at 50°C and the air is at 20°C. Neglect mass transfer and radiation.

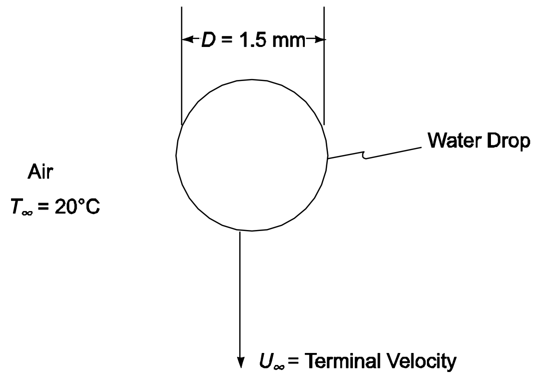
GIVEN

- A spherical water droplet freely falling in atmospheric air
- Drop diameter (D) = 1.5 mm = 0.0015 m
- Water drop temperature (T_d) = 50°C
- Air temperature (T_∞) = 20°C

FIND

- The average heat transfer coefficient at terminal velocity ($\bar{h}_c$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 20°C

Thermal conductivity (k) = 0.0251 W/(m K)

Kinematic viscosity (ν) = 15.7 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

Density (ρ_a) = 1.164 kg/m^3

From Appendix 2, Table 13, for water at 50°C

Density (ρ_w) = 988.1 kg/m^3

SOLUTION

The weight of the water droplet is

$$W = (\text{mass})\ g_g = (\text{Volume})\ \rho_w\ g_g = \frac{\pi}{6} D^3\ \rho_w\ g_g = \frac{\pi}{6} (0.0015\text{m})^3\ \ 988.1\,\text{kg/m}^3\ \ \ 9.81\,\text{m/s}^2$$

$$W = 1.713 \times 10^{-5}\ \ \text{kg m}\ \ /\text{s}^2 = 1.713 \times 10^{5}\ \text{N}$$

Terminal velocity occurs when the droplet's weight is balance by the viscous drag force which is given in Section 6.2 and Figure 6.7.

$$W = C_D \left(\frac{\rho_a U_\infty{}^2}{2} \right) \left(\frac{\pi D^2}{4} \right)$$

Solving for the velocity

$$U_\infty = \left(\frac{8\,\mathrm{W}}{C_D \pi D^2 \rho_a}\right)^{\frac{1}{2}} = \left[\frac{8\ 1.13\times10^{-5}\,(\mathrm{kg\,m})/\mathrm{s}^2}{C_D \pi\ 0.0015\,\mathrm{m}\ ^2\ 1.164\,\mathrm{kg/m^3}}\right]^{\frac{1}{2}}$$

$$U_\infty = \ 4.081\,\mathrm{m/s}\ C_D^{-\frac{1}{2}}$$

But C_D is a function of U_∞ through the Reynolds number and Figure 6.6 by trial and error

U_∞ (m/s)	Re_D	C_D	4.081 $C_D^{-1/2}$ (m/s)
10	955	0.44	6.15
6.0	588	0.55	5.50
5.5	525	0.59	5.3
5.3	506	0.60	5.3

Terminal velocity ≈ 5.3 m/s

The Nusselt number is given by Equation (6.12) as

$$\overline{Nu}_D = 2 + \left(\frac{Re_D}{4} + 3\times10^{-4}\,Re_D^{1.6}\right)^{\frac{1}{2}}$$

$$\overline{Nu}_D = 2 + \left(\frac{506}{4} + 3\times10^{-4}(506)^{1.6}\right)^{\frac{1}{2}} = 13.53$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 13.53\ \frac{0.0251\,\mathrm{W/(m\,K)}}{0.0015\,\mathrm{m}} = 226\ \mathrm{W/(m^2K)}$$

COMMENT

In this solution, the effect of evaporation has been neglected.

PROBLEM 6.28

In a lead-shot tower, spherical 0.95-cm-diameter *BB* shots are formed by drops of molten lead which solidify as they descend in cooler air. At the terminal velocity, i.e., when the drag equals the gravitational force, estimate the total heat transfer coefficient if the lead surface is at 171°C, the surface of the lead has an emissivity of 0.63, and the air temperature is 16°C. Assume C_D = 0.75 for the first trial calculation.

GIVEN

- Spherical lead-shot falling through the air at terminal velocity
- Shot diameter (D) = 0.95 cm = 0.0095 m
- Lead surface temperature (T_s) = 171°C = 494 K
- Lead surface emissivity (ε) = 0.63
- Air temperature (T_∞) = 16°C 289 K
- Assume C_D = 0.75 for the first trial calculation

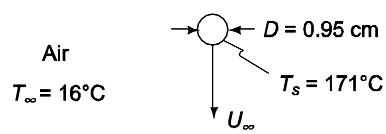
FIND

- The total average heat transfer coefficient (h_{total}).

ASSUMPTIONS

- The surroundings act as a black body enclosure at T_∞

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 16°C

Thermal conductivity (k) = 0.0248 W/(m K)

Kinematic viscosity (ν) = 15.3×10^{-6} m²/s

Prandtl number (Pr) = 0.71

Density (ρ) = 1.182 kg/m³

From Appendix 2, Table 12, the density of lead (ρ_L) = 11,340 kg/m³

From Appendix 1, Table 15, the Stephan-Boltzmann constant(σ) = 5.67×10^{-8} W/(m² K⁴)

SOLUTION

The weight of the lead shot is

$$W = (\text{mass})g_g = (\text{Volume})\ \rho\, g_g = \frac{\pi}{6} D^3\ \rho\, g_g = \frac{\pi}{6}\ (0.0095\text{m})^3\ \ 11{,}340\,\text{kg/m}^3\ \ \ 9.81\,\text{m/s}^2$$

$$W = 0.0499\ (\text{kg m})/\text{s}^2 = 0.0499\ \text{N}$$

Terminal velocity occurs when the weight is balanced by the drag force which is given Section 6.2

$$W = \text{Drag Force} = C_D \left(\frac{\rho_a U_\infty^{\ 2}}{2}\right)\left(\frac{\pi D^2}{4}\right)$$

Solving for the terminal velocity

$$U_\infty = \left(\frac{8W}{C_D \pi D^2 \rho_a}\right)^{\frac{1}{2}} = \left[\frac{8\ 0.0499\,(\text{kg m})/\text{s}^2}{C_D \pi\ 0.0095\,\text{m}^{\ 2}\ 1.182\,\text{kg/m}}\right]^{\frac{1}{2}}$$

$$U_\infty = \ 34.51\,\text{m/s}\ \ C_D^{-\frac{1}{2}}$$

Using the recommended drag coefficient for the first iteration

$$U_\infty = 34.51\ \text{m / s}\,(0.75)^{-\frac{1}{2}} = 39.9\ \text{m/s}$$

$$Re_D = \frac{U_\infty D}{\nu} = \frac{39.9\,\text{m/s}\ \ 0.0095\,\text{m}}{15.3 \times 10^{-6}\ \text{m}^2/\text{s}} = 24{,}745$$

From Figure 6.6, for Re_D = 24,745, C_D = 0.47

Repeating this procedure for further iterations

Iteration #	2	3
C_D	0.47	0.48
U_∞ (m/s)	50.34	49.81
Re_D	31,259	30,931
C_D (from Figure 6.7)	0.48	0.48

The Nusselt number is given by Equation (6.12)

$$\overline{Nu}_D = 2 + \left(\frac{Re_D}{4} + 3 \times 10^{-4}\, Re_D^{\ 1.6}\right)^{\frac{1}{2}}$$

$$\overline{Nu}_D = 2 + \left(\frac{30{,}391}{4} + 3 \times 10^{-4} (30{,}391)^{1.6}\right)^{\frac{1}{2}} = 113$$

$$\bar{h}_c = \overline{Nu}_D\, \frac{k}{D} = 113\, \frac{0.0248\,\text{W/(m K)}}{0.0095\,\text{m}} = 295\ \text{W/(m}^2\text{K)}$$

The total rate of heat transfer can be used to calculate the total heat transfer coefficient as follows

$$q_{\text{total}} = \bar{h}_{\text{total}}\ A\ (T_s - T_\infty) = q_c + q_r = \bar{h}_c\, A\ (T_s - T_\infty) + \varepsilon\, \sigma A\ (T_s^4 - T_\infty^4)$$

$$\bar{h}_{\text{total}} = \bar{h}_c + \frac{\varepsilon \sigma \left(T_s^{\ 4} - T_\infty^{\ 4}\right)}{\left(T_s - T_\infty\right)}$$

$$\bar{h}_{\text{total}} = \ 295\,\text{W/(m}^2\text{K)}\ + \frac{0.63\ \ 5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4)\ \ [(494\,\text{K})^4 - (289\,\text{K})^4\,]}{494\,\text{K} - 289\,\text{K}}$$

$$\bar{h}_{\text{total}} = \ (295 + 9.1)\text{W/(m}^2\text{K)}\ \ = 304\ \text{W/(m}^2\text{K)}$$

COMMENTS

97% of the heat transfer is due to convection.

PROBLEM 6.29

> **A copper sphere 2.5 cm in diameter is suspended by a fine wire in the center of an experimental hollow cylindrical furnace whose inside wall is maintained uniformly at 430°C. Dry air at a temperature of 90°C and a pressure of 1.2 atm is blown steadily through the furnace at a velocity of 14 m/s. The interior surface of the furnace wall is black. The copper is slightly oxidized, and its emissivity is 0.4. Assuming that the air is completely transparent to radiation, calculate for the steady state (a) the convection heat transfer coefficient between the copper sphere and the air, and (b) the temperature of the sphere.**

GIVEN

- A copper sphere suspended in a furnace with air flowing over it
- Sphere diameter (D) = 2.5 cm = 0.025 m
- Sphere emissivity (ε) = 0.4
- Furnace wall temperature (T_w) = 430°C = 703 K
- Air temperature (T_a) = 90°C = 363 K
- Air pressure (p_a) = 1.2 atm
- Air velocity (U_∞) = 14 m/s

FIND

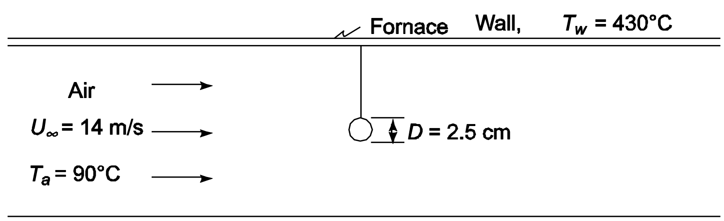
(a) The convective heat transfer coefficient $\left(\bar{h}_c\right)$

(b) The temperature of the sphere (T_s)

ASSUMPTIONS

- Steady state
- The air behaves as an ideal gas
- The furnace can be treated as a black body enclosure
- Only the density of the air varies with temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 90°C and 1 atm pressure

Density (ρ) = 0.942 kg/m^3

Thermal conductivity (k) = 0.0300 W/(m K)

Absolute viscosity (μ) = 21.232 × 10^{-6} (Ns)/m^2

Prandtl number (pr) = 0.71

From Appendix 1, Table 15, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The density of the air at 1.2 atm can be calculated from Boyle's law

$$\frac{P_1}{P_{1.2}} = \frac{\rho_1}{\rho_{1.2}} \Rightarrow \rho_{1.2} = \rho_1 \frac{P_{1.2}}{P_1} = 0.942 \text{ kg/m}^3 \, \frac{1.2\,\text{atm}}{1\,\text{atm}} = 1.130\,\text{kg/m}^3$$

The kinematic viscosity is

$$\nu = \frac{\mu}{\rho} = \frac{21.232\times10^{-6}\,(\mathrm{N\,s})/\mathrm{m}^2 \quad (\mathrm{kg\,m})/(\mathrm{Ns}^2)}{1.130\ \mathrm{kg/m}^3} = 18.8 \times 10^{-6}\ \mathrm{m}^2/\mathrm{s}$$

(a) The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{14\ \mathrm{m/s} \quad 0.025\ \mathrm{m}}{18.8\times10^{-6}\ \mathrm{m}^2/\mathrm{s}} = 18{,}617$$

The convective Nusselt number can be estimated using Equation (6.12)

$$\overline{Nu}_D = 2 + \left(\frac{Re_D}{4} + 3\times10^{-4}\,Re_D^{\,1.6}\right)^{\frac{1}{2}}$$

$$\overline{Nu}_D = 2 + \left(\frac{18{,}617}{4} + 3\times10^{-4}(18{,}617)^{1.6}\right)^{\frac{1}{2}} = 83.8$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 83.8\,\frac{0.03\ \mathrm{W/(m\,K)}}{0.025\ \mathrm{m}} = 100.6\ \mathrm{W/(m^2K)}$$

(b) In steady state, the sphere temperature will be between T_a and T_w and the convective loss to the air must equal the radiative gain from the furnace walls

$$\bar{h}_c A\,(T_s - T_\infty) = \varepsilon\,\sigma\,(T_\infty^{\,4} - T_s^{\,4})$$

$$100.6\ \mathrm{W/(m^2K)} \quad (T_s - 363\ \mathrm{K}) = 0.4 \quad 5.67\times10^{-8}\ \mathrm{W/(m^2K^4)} \quad [(703\ \mathrm{K})^4 - T_s^{\,4}]$$

Checking the units, then eliminating them for clarity

$$2.28\times10^{-8}\,T_s^{\,4} + 100.6\,T_s - 42{,}087 = 0$$

By trial and error: T_s = 412 K = 139°C

PROBLEM 6.30

A method for measuring the convective heat transfer from spheres has been proposed. A 20 mm diameter copper sphere with an embedded electrical heater is to be suspended in a wind tunnel. A thermocouple inside the sphere measures the sphere surface temperature. The sphere is supported in the tunnel by a type 304 stainless steel tube 5 mm OD, a 3 mm ID and 20-cm length. The steel tube is attached to the wind tunnel wall in such a way that no heat is transferred through the wall. For this experiment, examine the magnitude of the correction that must be applied to the sphere heater power to account for conduction along the support tube. The air temperature is 20°C and the desired range of Reynolds numbers is 10^3 to 10^5.

GIVEN

- A heater copper sphere supported by a steel tube in a wind tunnel
- Sphere diameter (D_s) = 20 mm = 0.02 m
- Tube diameters
  - Outside (D_{to}) = 5 mm = 0.005 m
  - Inside (D_{ti}) = 3 mm = 0.003 m
- Tube length (L) = 20 cm = 0.2 m
- There is no heat transfer between the tube and the wall
- Air temperature (T_∞) = 20°C
- Reynolds number range: $10^3 < Re_{Ds} < 10^5$

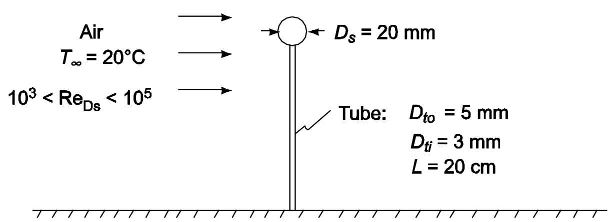
FIND

- The correction to the heater power to account for conduction along the support tube

ASSUMPTIONS

- Steady state
- Contact resistance between the sphere and the tube is negligible
- The effect of the boundary layer near the wind tunnel wall on the heat transfer from the tube is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 20°C

Thermal conductivity (k) = 0.0251 W/(m K)

Kinematic viscosity (ν) = 15.7 × 10^{-6} m²/s

Prandtl number (Pr) = 0.71

From Appendix 2, Table 10, for type 304 stainless steel: k_s = 14.4 W/(m K)

SOLUTION

Equation (6.9) can be used to estimate the heat transfer coefficient on the sphere

$$\bar{h}_{cs} = 0.37 \frac{k}{D_s} Re_{D_s}^{0.6}$$

At $Re_{D_s} = 10^3$ $\quad \bar{h}_{cs} = 0.37 \dfrac{0.0251\,\text{W/(m K)}}{0.02\,\text{m}} (10^3)^{0.6} = 29.3\,\text{W/(m}^2\text{K)}$

At $Re_{D_s} = 10^5$ $\qquad \bar{h}_{cs} = 0.37 \dfrac{0.0251\,\text{W/(m K)}}{0.02\,\text{m}} (10^5)^{0.6} = 464\,\text{W/(m}^2\text{K)}$

The rate of heat transfer from the sphere, neglecting the influence of the tube, would be

$$q_s = \bar{h}_{cs}\, \pi D_s^2\, (T_s - T_\infty) = 0.37 \frac{k}{D} Re_{D_s}^{0.6}\, \pi D_s^2\, (T_s - T_\infty)$$

The heat transfer coefficient on the tube can be calculated using Equation 6.3 and Table 6.1 (Note that $Pr/Pr_s \approx 1$ for air in the anticipated temperature range)

$$\bar{h}_{ct} = 0.26 \frac{k}{D_{to}} Re_{D_{to}}^{0.6}\, Pr^{0.37} = 0.26 \frac{k}{D_{to}} Re_{D_s}^{0.6} \left(\frac{D_{to}}{D_s}\right)^{0.6} Pr^{0.37}$$

At $R_{D_s} = 10^3$ $\bar{h}_{ct} = 0.26 \dfrac{0.0251\,\text{W/(m K)}}{0.005\,\text{m}} (10^3)^{0.6} \left(\dfrac{0.005\,\text{m}}{0.02\,\text{m}}\right)^{0.6} (0.71)^{0.37} = 31.6\,\text{W/(m}^2\text{K)}$

At $R_{D_s} = 10^5$ $\bar{h}_{ct} = 501\,\text{W/(m}^2\text{K)}$

The tube can be modeled as a fin of uniform cross-section with an adiabatic tip protruding from the sphere. The cross-sectional area of the tube (A_f) and perimeter of the air (P) are

$$A_f = \frac{\pi}{4}(D_{to}^2 - D_{ti}^2) = \frac{\pi}{4}\,[(0.005\text{ m})^2 - (0.003)^2] = 1.256 \times 10^{-5}\text{ m}^2$$

$$P = \pi D_{to} = \pi\,(0.005\text{ m}) = 0.0157\text{ m}$$

The rate of heat transfer from the tube for a given sphere temperature is given Table 2.1 as

$$q_t = M \tanh(mL)$$

where $\quad m = \sqrt{\bar{h}_{ct} P / (k_s A_f)} \quad$ & $\quad M = \sqrt{\bar{h}_{ct} P k_s A_f}\,(T_s - T_\infty)$

The fraction correction to the power data due to the tube is

$$\frac{q_t}{q_s} = \frac{\sqrt{\bar{h}_{ct} P k_s A_f}\,\tanh\left(L\sqrt{\dfrac{\bar{h}_{ct} P}{k_s A_f}}\right)}{\bar{h}_{cs}\pi D_s^2}$$

At $Re_{DS} = 10^3$

$$\sqrt{\bar{h}_{ct} P k_s A_f} = \sqrt{31.6\,\text{W/(m}^2\text{K)}\ \ 0.0157\,\text{m}\ \ 14.4\,\text{W/(m K)}\ \ 1.256\times10^{-5}\,\text{m}^2} = 0.0095\,\text{W/K}$$

$$\frac{q_t}{q_s} = \frac{0.0095\,\text{W/K}\ \tanh\left(0.2\,\text{m}\sqrt{\dfrac{31.6\,\text{W/(m}^2\text{K)}\ \ 0.0157\,\text{m}}{14.4\,\text{W/(m K)}\ \ 1.256\times10^{-5}\,\text{m}^2}}\right)}{29.3\,\text{W/(m K)}\ \ \pi\ \ 0.02\,\text{m}^{\ 2}} = 0.258 = 25.8\%\text{ correction}$$

At $Re_{D_s} = 10^5$

$$\frac{q_t}{q_a} = 0.065 = 6.5\%\text{ correction}$$

PROBLEM 6.31

Estimate (a) the heat transfer coefficient for a spherical fuel droplet injected into a diesel engine at 80°C and 90 m/s. The oil droplet is 0.025 mm in diameter, the cylinder pressure is 4800 kPa, and the gas temperature is 944 K. (b) Estimate the time required to heat the droplet to its self-ignition temperature of 600°C.

GIVEN

- An oil droplet injected into a diesel engine
- Initial droplet temperature (T_o) = 80°C
- Injection velocity (U_d) = 90 m/s
- Droplet diameter (D) = 0.025 mm = 2.5×10^{-5} m
- Cylinder pressure = 4800 kPa
- Gas temperature (T_∞) = 944 K = 671°C

FIND

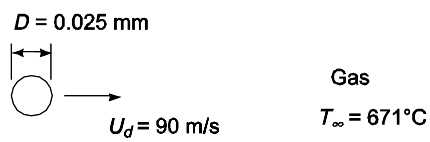
(a) The heat transfer coefficient ($\bar{h}_c$)

(b) The time (t) required for the drop to reach 300°C

ASSUMPTIONS

- Radiative heat transfer is negligible
- The gas has the properties of air and behaves as an ideal gas
- Only the density of the gas is affected by pressure
- Variation of the thermal conductivity of the oil with temperature is negligible
- Fuel has the same properties as unused engine oil

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 671°C and 1 atm (101 kPa) pressure

Density (ρ) = 0.382 kg/m^3
Thermal conductivity (k) = 0.0616 W/(m K)
Absolute viscosity (μ) = 40.121×10^{-6} Ns/m^2
Prandtl number (Pr) = 0.73

Extrapolating from Appendix 2, Table 17, for unused engine oil at the average temperature of 300°C

Density (ρ_o) = 726.3 kg/m^3
Thermal conductivity (k_o) = 0.126 W/(m K)
Specific heat (c) =3094 J/(kg K)

SOLUTION

The density of the air at 4800 kPa can be calculated from Boyle's law

$$p_1 V_1 = p_2 V_2 \Rightarrow \frac{P_1}{P_2} = \frac{\rho_1}{\rho_2} \Rightarrow \rho_2 = \rho_1 \frac{P_2}{P_1} = 0.382\,\text{kg/m}^3 \quad \frac{4800\,\text{kPa}}{101\,\text{kPa}} = 18.15\,\text{kg/m}^3$$

The Reynolds number is

$$Re_D = \frac{U_d D \rho}{\mu} = \frac{90\,\text{m/s} \quad 2.5\times10^{-5}\,\text{m} \quad 18.15\,\text{kg/m}^3}{40.121\times10^{-6}\,(\text{Ns})/\text{m}^2} = 1018$$

(a) The Nusselt number can be calculated from Equation (6.12)

$$\overline{Nu}_D = 2 + \left(\frac{Re_D}{4} + 3\times10^{-4}\, Re_D^{\,1.6}\right)^{\frac{1}{2}}$$

$$\overline{Nu}_D = 2 + \left(\frac{1018}{4} + 3\times10^{-4}(1018)^{1.6}\right)^{\frac{1}{2}} = 18.55$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 18.55\,\frac{0.0616\,\text{W/(m K)}}{2.5\times10^{-5}\,\text{m}} = 4.57\times10^{4}\ \text{W/(m}^2\text{K)}$$

(b) The Biot number for the droplet is

$$Bi = \frac{\bar{h}_c D}{2k_s} = \frac{\left(4.57\times10^{4}\ \text{W/(m}^2\text{K)}\right)\left(2.5\times10^{-5}\ \text{m}\right)}{2\left(0.126\,\text{W/(m K)}\right)} = 4.54 >> 0.1$$

Therefore, the internal resistance of the oil drop cannot be neglected and the chart solution of Figure 2.40 must be used. Assuming the heat transfer coefficient is constant, the ratio of the rate of heat transfer at time t and initially is

$$\frac{Q(t)}{Q_i} = \frac{T(r_o, t) - T_\infty}{T_o - T_\infty} = \frac{600°\text{C} - 671°\text{C}}{80°\text{C} - 671°\text{C}} = 0.12$$

From Figure 2.40, for $Bi = 4.54$

$$(Bi)^2\, Fo = \frac{\bar{h}_c^{\,2}\,\alpha\, t}{k_o^{\,2}} = \frac{\bar{h}_c^{\,2}\, t}{k_o \rho c} = 7.9$$

Solving for the time

$$t = \frac{7.9\left(0.126\,\text{W/(m K)}\right)\left(726.3\,\text{kg/m}^3\right)\left(3094\,\text{J/(kg K)}\right)\left((\text{W s})/\text{J}\right)}{\left(4.57\times10^{4}\ \text{W/(m}^2\text{K)}\right)^2}$$

$$t = 10.7\times10^{-4}\ \text{s} = 1.07\ \mu\text{s}$$

PROBLEM 6.32

Heat transfer from an electronic circuit board is to be determined by placing a model for the board in a wind tunnel. The model is a 15-cm-square plate with embedded electrical heaters. The wind from the tunnel air is delivered at 20°C. Determine the average temperature of the model as a function of power dissipation for an air velocity of 2.5 and 10 m/s. The model is pitched 30° and yawed 10° with respect to the flow direction. The surface of the model acts as a blackbody.

GIVEN

- A Model square electronic circuit board pitched 30° and yawed 10° in a wind tunnel
- Length of a side (L) = 15 cm = 0.15 m
- Air temperature (T_∞) = 20°C = 293 K
- Air velocity (U_∞) = 2.5 m/s or 10 m/s

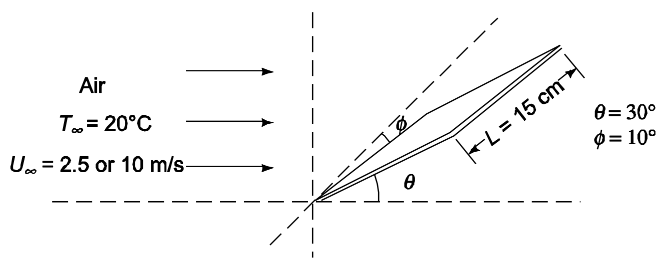
FIND

- The average surface temperature of the model (T_s) as a function of power dissipation ($\dot{Q}$)

ASSUMPTIONS

- Steady state
- The surface of the model acts as a black body (ε = 1.0)
- The wind tunnel acts as a black body enclosure at the air temperature.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 20°C

Kinematic viscosity (ν) = 15.7×10^{-6} m²/s

Prandtll number (Pr) = 0.71

Density (ρ) = 1.164 kg/m³

Specific heat (c) = 1012 J/(kg K)

From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m² K⁴)

SOLUTION

The Reynolds number is

For U_∞ = 2.5 m/s

$$Re_L = \frac{U_\infty L}{\nu} = \frac{2.5\,\text{m/s} \quad 0.15\,\text{m}}{15.7\times10^{-6}\,\text{m}^2/\text{s}} = 2.39 \times 10^4$$

For U_∞ = 10 m/s

$$Re_L = \frac{(10\,\text{m/s})(0.15\,\text{m})}{\left(15.7\times10^{-6}\,\text{m}^2/\text{s}\right)} = 9.55 \times 10^4$$

The pitch and yaw angles as well as the Reynolds numbers fall within the range of Equation (6.18)

$$\left(\frac{\bar{h}_c}{c\rho U_\infty}\right) Pr^{\frac{2}{3}} = 0.930\ Re_L^{-\frac{1}{2}}$$

For U_{inf} = 2.5 m/s

$$\bar{h}_c = 0.930\ c\ \rho\ U_\infty\ Pr^{-\frac{2}{3}} Re_L^{-\frac{1}{2}} = 0.930\ \ 1012\,\text{J/(kg K)}\ \ \ 1.164\,\text{kg/m}^3$$

$$2.5\,\text{m/s}\ \ \left(2.39\times10^4\right)^{-\frac{1}{2}} (0.71)^{-\frac{2}{3}}\ \ \text{(Ws)/J}$$

$$\bar{h}_c = 22.3\ \ \text{W/(m}^2\text{K)}$$

For U_{inf} = 10 m/s

$$\bar{h}_c = 44.5\ \ \text{W/(m}^2\text{K)}$$

The rate of power dissipation is the sum of the convective and radiative losses

$$\dot{Q} = \bar{h}_c\ 2L^2\,(T_s - T_\infty) + \sigma 2L^2\,(T_s^4 - T_\infty^4)$$

At U_{inf} = 2.5 m/s

$$\dot{Q} = \ \ 22.3\,\text{W/(m}^2\text{K)}\ \ 2(0.15\text{m})^2\,(T_s - 293\text{ K}) + \ \ 5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\ \ 2(0.15\text{m})^2\,[T_s^4 - (293\text{ K})^4]$$

Checking the units, then eliminating for clarity

$$2.565\times10^{-9}\,T_s^4 + 1.0035\,T_s - 312.8 = \dot{Q}$$

($\dot{Q}$ in watts, T_s in K)

Similarly, for U_∞ = 10 m/s

$$2.565\times10^{-9}\,T_s^4 + 2.0025\,T_s - 605.5 = \dot{Q}$$

($\dot{Q}$ in watts, T_s in K)

PROBLEM 6.33

An electronic circuit contains a power resistor that dissipates 1.5 W. The designer wants to modify the circuitry in such a way that it will be necessary for the resistor to dissipate 2.5 W. The resistor is in the shape of a disk 1 cm in diameter and 0.6-mm-thick. Its surface is aligned with a cooling air flow at 30°C and 10 m/s velocity, i.e., the vertical axis of the disc is in cross-flow with the cooling air. The resistor lifetime becomes unacceptable if its surface temperature exceeds 90°C. Is it necessary to replace the resistor for the new circuit?

GIVEN

- A heat generating resistor disk with its surface aligned with a cooling airflow
- Heat generation rate ($\dot{Q}_G$) = 2.5 W
- Disk diameter (D) = 1 cm = 0.01 m
- Disk thickness (t) = 0.6 mm = 0.0006 m
- Air temperature (T_∞) = 30°C
- Air velocity (U_∞) = 10 m/s
- Maximum surface temperature (T_s) = 90°C

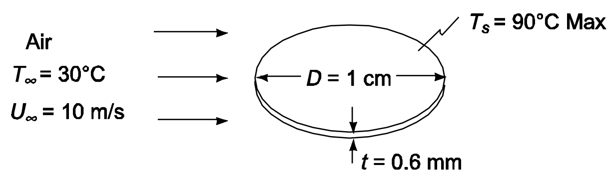
FIND

- Is it necessary to replace the resistor?

ASSUMPTIONS

- Steady state
- Radiation is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the maximum film temperature of 60°C

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number based on the diameter is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{10\,\text{m/s}\ \ 0.01\,\text{m}}{19.4\times10^{-6}\,\text{m}^2/\text{s}} = 5155$$

$$\frac{t}{D} = \frac{0.0006\,\text{m}}{0.01\,\text{m}} = 0.06$$

The Nusselt number for the geometry is given by Equation (6.19)

$$\overline{Nu}_D = 0.591\ Pr^{\frac{1}{3}}\,Re_D^{0.564} = 0.591\ (0.71)^{\frac{1}{3}}\ (5155)^{0.564} = 65.42$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 65.42\,\frac{0.0279\,\text{W/(m K)}}{0.01\,\text{m}} = 182.5\,\text{W/(m}^2\text{K)}$$

The rate of heat transfer at the maximum surface temperature of 90°C is

$$q = \bar{h}_c A\,(T_s - T_\infty) = \bar{h}_c \left[2\left(\frac{\pi}{4} D^2\right) + \pi D\, t \right] (T_s - T_\infty)$$

$$q = 182.5\,\mathrm{W/(m^2K)} \left[\frac{\pi}{2}(0.01)^2 + \pi\ (0.01\ \mathrm{m})\ (0.0006\ \mathrm{m}) \right] (90°\mathrm{C} + 30°\mathrm{C}) = 1.93\ \mathrm{W} < \dot{Q}$$

Therefore, the surface temperature must be greater than 90°C to dissipate the required 2.5 Watts. The resistor must be replaced.

PROBLEM 6.34

Suppose the resistor in Problem 6.33 is rotated so that its axis is aligned with the flow. What is the maximum permissible power dissipation?

GIVEN

- A heat generating resistor disk with its axis aligned with a cooling airflow
- Heat generation rate ($\dot{Q}_G$) = 2.5 W
- Disk diameter (D) = 1 cm = 0.01 m
- Disk thickness (t) = 0.6 mm = 0.0006 m
- Air temperature (T_∞) = 30°C
- Air velocity (U_∞) = 10 m/s
- Maximum surface temperature (T_s) = 90°C

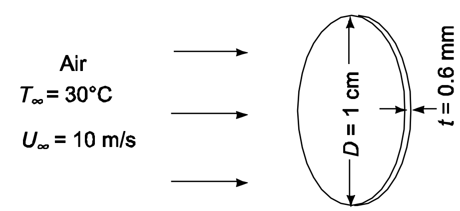
FIND

- The maximum permissible power dissipation ($\dot{Q}_G$)

ASSUMPTIONS

- Heat transfer from the edges is negligible
- The heat flux is equal from both faces

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the maximum film temperature of 60°C

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

From Appendix 2, Table 28, for dry air at the free stream temperature of 30°C

Thermal conductivity (k) = 0.0258 W/(m K)

Kinematic viscosity (ν) = 16.7×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number based on the film temperature is unchanged from Problem 6.33 Re_{Df} = 5155

The Reynolds number based on the free stream conditions is Re_{Dfs} = 5988

The Nusselt number for the upstream face of the disk is given by Equation (6.17) using the free stream properties

$$\overline{Nu}_D = 1.05\ Re_{Dfs}^{\frac{1}{2}}\ Pr^{0.36} = 1.05\,(5988)^{\frac{1}{2}}\,(0.71)^{0.37} = 71.58$$

$$\bar{h}_{cu} = \overline{Nu}_D \frac{k}{D} = 71.58 \frac{0.0258\,\mathrm{W/(mK)}}{0.01\,\mathrm{m}} = 184.7\,\mathrm{W/(m^2K)}$$

The Nusselt number for the downstream face can be estimated from Equation (6.15) (using the properties at film temperature) because the flow behind the disk will be separated and the separated region behind a normal flat plate will be similar

$$\overline{Nu}_D = 0.20\ Re_{Df}^{\frac{2}{3}} = 0.20\ (5155)^{\frac{2}{3}} = 59.69$$

$$\bar{h}_{cd} = \overline{Nu}_D \frac{k}{D} = 59.69\ \frac{0.0279\,\mathrm{W/(m\,K)}}{0.01\,\mathrm{m}} = 166.5\ \mathrm{W/(m^2K)}$$

The maximum power dissipation for the whole chip is

$$\dot{Q}_{\max} = \bar{h}_{cd} + \bar{h}_{cu} \quad \frac{\pi}{4} D^2\,(T_s - T_\infty)$$

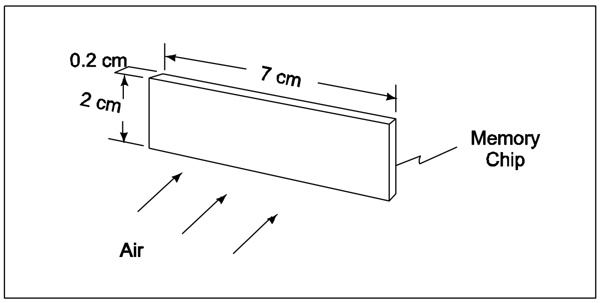
$$\dot{Q}_{\max} = 166.5\,\mathrm{W/(m^2K)} + 184.7\,\mathrm{W/(m^2K)} \quad \frac{\pi}{4}(0.01\ \mathrm{m})^2\ (90°\mathrm{C} - 30°\mathrm{C}) = 1.65\ \mathrm{W}$$

COMMENTS

The circuit designer cannot solve the problem by reorienting the resistor. The upstream face of the disk has about the same heat transfer coefficient as the surface of the disk aligned with the flow. However, the downstream face has significantly lower heat transfer coefficient because it is in a separated flow regime.

PROBLEM 6.35

To decrease the size of personal computer mother boards, designers have turned to a more compact method of mounting memory chips on the board. The single in-line memory modules, as they are called, essentially mount the chips on their edges so that their thin dimension is horizontal, as shown in the sketch below. For safe operation (so that the VLSI microchips do not "burn out"), their maximum temperature always has to be <=75°C under all operating conditions. Assuming that the memory board, housing the chip modules, is also at the same temperature, determine the maximum power that is dissipated from the chips if the board is cooled by air at 20°C. The air flows perpendicular to the board face, as shown in the figure, with a velocity of 10 m/s. Consider the heat loss from both the front face and back face of the memory-modules board. Is it beneficial (dissipate more heat) if the board was aligned parallel to the air flow? To model the board, in both cases, consider an equivalent "square" of the same rectangular face area and determine the appropriate length scale.



GIVEN

- Computer memory chip in an air stream as shown above
- Chip temperature (T_s) = 90°C
- Air temperature (T_∞) = 20°C
- Air velocity (U_∞) = 10 m/s

FIND

- The maximum power dissipation ($\dot{Q}_G$)

ASSUMPTIONS

- Radiative heat transfer is negligible
- Use of Equation (6.18) for a non-square surface will not introduce significant error
- Heat transfer from all four edges of the chip is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the free stream temperature of 60°C

Density (ρ) = 1.025 kg/m^3

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4×10^{-6} m^2/s

Prandtl number (Pr) = 0.71

Specific heat (c) = 1017 J/(kg K)

From Appendix 2, Table 28, for dry air at the film temperature of 75°C

Thermal conductivity (kf) = 0.0290 W/(m K)

Kinematic viscosity (ν_f) = 21.0×10^{-6} m^2/s

Prandtl number (Pr_f) = 0.71

SOLUTION

Front of chip:

The Reynolds number for the front of the chip will be based on a characteristic length that is equal to the length of the side of a square with the same area and on the properties evaluated at the free stream temperature.

$$L_{\text{eq}} = \sqrt{(7\,\text{cm})(2\,\text{cm})} = 3.74 \text{ cm} = 0.0374 \text{ m}$$

$$Re_{L_{\text{eq}}} = \frac{U_\infty L_{\text{eq}}}{\nu} = \frac{10\,\text{m/s} \quad 0.0374\,\text{m}}{19.4\times 10^{-6}\,\text{m}^2/\text{s}} = 19{,}278$$

Applying Equation (6.18) as the only correlation available to estimate the heat transfer coefficient on the front of the chip

$$\bar{h}_{cf} = 0.930\, c\,\rho\, U_\infty Pr^{-\frac{2}{3}}\, Re_L^{-\frac{1}{2}}$$

$$= 0.930 \;\; 1017\,\text{J/(kg K)} \;\; 1.025\,\text{kg/m}^3 \;\; 10\,\text{m/s} \;\; (19{,}278)^{-\frac{1}{2}} (0.71)^{-\frac{2}{3}} \;\; \text{(Ws)/J}$$

$$\bar{h}_{cf} = 87.7 \;\text{W/(m}^2\text{K)}$$

Back of the chip:

The Reynolds number based on the properties evaluated at the film temperature is

$$Re_{L_{eq}} = \frac{U_\infty L_{eq}}{\nu} = \frac{10\,\text{m/s} \quad 0.0374\,\text{m}}{21.0\times 10^{-6}\,\text{m}^2/\text{s}} = 17{,}809$$

Applying Equation (6.15) (using the properties evaluated film temperature) to estimate the Nusselt number on the back of the chip

$$\overline{Nu}_D = 0.20\, Re_D^{\frac{2}{3}} = 0.20 \;(17{,}809)^{\frac{2}{3}} = 136.3$$

$$\bar{h}_{cb} = \overline{Nu}_{L_{eq}} \frac{k}{L_{eq}} = 143.8\,\frac{0.0290\,\text{W/(m K)}}{0.0374\,\text{m}} = 105.7\,\text{W/(m}^2\text{K)}$$

The maximum power dissipation is equal to the total rate of heat transfer

$$\dot{Q}_G = q_c = \left(\bar{h}_{cf} + \bar{h}_{cb}\right) A_{\text{frontal}}\,(T_s - T_\infty)$$

$$\dot{Q}_G = (87.7 + 105.7)\,\text{W/(m}^2\text{ K)} \;(0.07\text{ m})\,(0.02\text{ m})\,(90°\text{C} - 60°\text{C})$$

$$\dot{Q}_G = 8.1 \text{ W}$$

COMMENTS

The rate of heat transfer would be greater if the long axis of the chip were aligned with the air velocity.

PROBLEM 6.36

A long, half-round cylinder is placed in an air stream with its flat face down-stream. An electrical resistance heater inside the cylinder maintains the cylinder surface temperature at 50°C. The cylinder diameter is 5 cm, the air velocity is 31.8 m/s, and the air temperature is 20°C. Determine the power input of the heater per unit length of cylinder. Neglect radiation heat transfer.

GIVEN

- A long, half-round cylinder in an air stream with its flat surface downstream
- Cylinder surface temperature (T_s) = 50°C
- Cylinder diameter (D) = 5 cm = 0.05 m
- Air velocity (U_∞) = 31.8 m/s
- Air temperature (T_∞) = 20°C

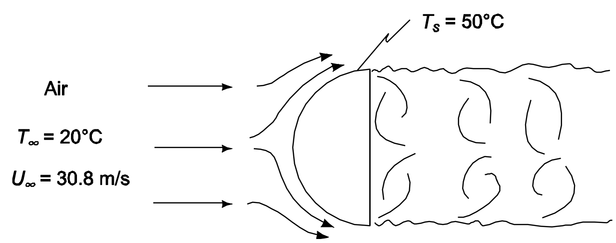
FIND

- Power input to the heater ($\dot{Q}_G / L$)

ASSUMPTIONS

- Steady state
- Radiative heat transfer is negligible
- Flow separates at the cylinder edges as shown below

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the film temperature of 35°C

Thermal conductivity (k) = 0.0262 W/(m K)

Kinematic viscosity (ν) = 17.1 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At the surface temperature of 50°C: Pr_s = 0.71

SOLUTION

The Reynolds number based on the film temperature is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(31.8\,\text{m/s})(0.05\,\text{m})}{17.1\times10^{-6}\,\text{m}^2/\text{s}} = 9.30 \times 10^4$$

This is within the range of the correlation of Equation (6.16), for the downstream surface

$$\overline{Nu}_D = 0.16\, Re_D^{\frac{2}{3}} = 0.16\,(9.30\times10^4)^{\frac{2}{3}} = 328.4$$

$$\bar{h}_{cd} = \overline{Nu}_D \frac{k}{D} = 328.4\,\frac{0.0262\,\text{W/(m K)}}{0.05\,\text{m}} = 172.1\,\text{W/(m}^2\text{K)}$$

For the half-round upstream face, the average Nusselt number can be estimated by numerically integrating the curve of Figure 6.7 for Re_D = 101,300

$$\overline{Nu}_D = \frac{1}{90°}\int_0^{90°} Nu_{D\theta}\, d\theta = \frac{1}{90°} \text{ (Area under the } Nu_{D\theta} \text{ Vs } \theta \text{ curve from } \theta = 0 \text{ to } 90°)$$

Performing the integration by the trapezoidal rule

$$\overline{Nu}_D = \frac{1}{90°}\,[20°\,(320) + 20°\,(310) + 20°\,(280) + 20°(190) + 10°\,(150)] = 260$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 260\;\frac{0.0262\,\text{W/(m K)}}{0.05\,\text{m}} = 136.2\;\text{W/(m}^2\text{K)}$$

The power input to the heater must equal the rate of convective heat loss

$$Q_G = (\bar{h}_{cd}\;A_d + \bar{h}_{cu}\;A_u)\,(T_s - T_\infty) = \left[\bar{h}_{cd}\,(DL) + \bar{h}_{cu}\left(\frac{\pi}{2}DL\right)\right](T_s - T_\infty)$$

$$\frac{Q_G}{L} = \left(\bar{h}_{cd} + \frac{\pi}{2}\bar{h}_{cu}\right) D(T_s - T_\infty) = \left[\left(172.1 + \frac{\pi}{2}(136.2)\right)\text{W/(m}^2\text{K)}\right]\;(0.05\text{ m})$$

$$(50°\text{C} - 20°\text{C}) = 579\;\text{W/m}$$

COMMENT

The use of Equation 6.3 and Table 6.1 rather than Figure 6.7 to estimate the upstream heat transfer coefficient leads to a transfer coefficient 11% lower and a final result 6% lower than those presented above.

PROBLEM 6.37

To reduce the carbon footprint of building energy consumption, photovoltaic (PV) cell panels are increasingly being used to provide a solar, self-sustaining source of electricity in both homes and large buildings. However, the semiconductor material that converts the solar light energy to electricity does not function efficiently at elevated temperatures, and the performance of a hot PV cell degrades. Externally mounted panels often rely for their cooling on atmospheric air. One such PV panel that has a square face (1 m * 1 m), as shown in the figure, is mounted on the side face of a building with air flowing over across it in parallel. If the maximum allowable temperature of the PV panel always has to be <= 70°C, calculate the rate of convection heat loss from the face of the panel when air at 25°C flows over it with a velocity of 1.0 m/s. What is the heat loss if the wind speed decreased to 0.25 m/s? Also, do the results change significantly if the airstream is not parallel to the panel but has a pitch and yaw angle, relative to the panel alignment, or strikes it perpendicularly? Note that the heat loss in many ways, gives the PV panel designers an estimate of the typical range of electricity generation and the PV cell operating efficiency.

GIVEN

- Air flows over PV panel surface of 1m*1m in parallel
- Air temperature (T_a) = 25°C
- Air velocity (U_s) = 1 m/s
- Maximum allowable temperature of PV panel (T_w) = 70°C

FIND

- Rate of Convection heat loss when the velocity is 1 m/s.
- Rate of heat loss when air velocity is reduced to 0.25 m/s
- Change in results if airstream is not parallel but strikes at an angle perpendicularly?

ASSUMPTIONS

- Steady state

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 50°C

Thermal conductivity (k) = 0.0272 W/(m K)

Kinematic viscosity (ν) = 18.5 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At T_w: Pr_s = 0.71

SOLUTION

Reynold's Number for parallel air flow with flow velocity of 1 m/s is

$$Re_D = \frac{UL}{\nu} = \frac{1*1}{18.5*10^{-6}} = 54054$$

Thus Nusselt Number for flow over flat plate in laminar region Re<5*10^5 fro Pr<1 is given by

$$\overline{Nu_D} = 1.124\,\mathrm{Re}_d^{1/2}\,\mathrm{Pr}^{1/2}$$

$$\overline{Nu_D} = 220.2$$

$$\frac{\overline{h_c}L}{k} = 220.2$$

$$\overline{h_c} = 220.2*0.0272 = 6\ \mathrm{W/(m^2\ K)}$$

Thus the rate of heat loss through the panel is

$$\dot{q} = \bar{h}_c A(T_w - T_a) \qquad => \qquad \dot{q} = 6*1*1*(70-25)W$$

$$\dot{q} = 270W$$

Reynold's Number for parallel air flow with flow velocity of 1 m/s is

$$\mathrm{Re_D} = \frac{UL}{\nu} = \frac{0.25*1}{18.5*10^{-6}} = 13513$$

Thus Nusselt Number for flow over flat plate in laminar region Re<5*10^5 fro Pr<1 is given by

$$\overline{Nu_D} = 1.124\,\mathrm{Re}_d^{1/2}\,\mathrm{Pr}^{1/2} \qquad => \qquad \overline{Nu_D} = 1.124*(13513)^{1/2}(0.71)^{1/2}$$

$$\frac{\bar{h}_c L}{k} = 110.1$$

$$\bar{h}_c = 110.1*0.0272 = 3 \text{ W/(m}^2\text{ K)}$$

Thus the rate of heat loss through the panel is

$$\dot{q} = \bar{h}_c A(T_w - T_a) \qquad => \qquad \dot{q} = 3*1*1*(70-25)W$$

$$\dot{q} = 135W$$

If the air strikes perpendicularly to the panel.

$$L_{\text{eq}} = \sqrt{(1m)(1\text{m})} = 1 \text{ m}$$

For air velocity of 1 m/s

$$Re_{L_{\text{eq}}} = \frac{U_\infty L_{\text{eq}}}{\nu} = \frac{(1\text{m/s})(1m)}{(18.5\times10^{-6}\text{ m}^2/\text{s})} = 54054$$

Applying Equation (6.18) as the only correlation available to estimate the heat transfer coefficient on the front of the chip

$$\bar{h}_{cf} = 0.930\, c\, \rho\, U_\infty Pr^{-\frac{2}{3}}\, Re_L^{-\frac{1}{2}}$$

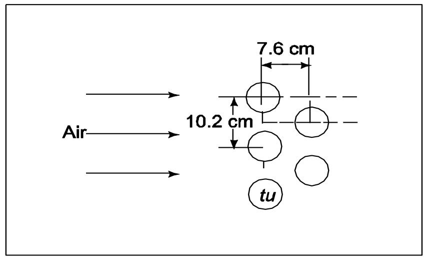
$$= 0.930\,(1017\,\text{J/(kg K)})(1.025\,\text{kg/m}^3)(1\text{m/s})\ (54,054)^{-\frac{1}{2}}(0.71)^{-\frac{2}{3}}\ \text{(Ws)/J}$$

$$\bar{h}_{cf} = 5.24\ \text{W/(m}^2\text{K)}$$

Thus if the air stream strikes perpendicularly instead of parallel to the plate heat transfer slightly decreases because of decrease in heat transfer coefficient.

PROBLEM 6.38

A multi-tube heat exchanger is used in a process plant to pre-heat air before it enters a combustion chamber, using low pressure steam that flows inside the tubes and condenses. The tube bundle is configured with 6-cm outer diameter tubes in a staggered arrangement as shown in the accompanying figure. The condensing steam in the tubes maintains their outer surface temperature at 117°C, and air at 60° flows across the tube bank with a free stream velocity of 1.0 m/s. Determine the average heat transfer coefficient for air.



GIVEN

- Air flow through the tube bank shown
- Air temperature (T_a) = 60°C
- Air velocity (U_s) = 1 m/s
- Tube outside diameter (D) = 6 cm = 0.06 m
- Tube wall temperature (T_w) = 117°C

FIND

- The average heat transfer coefficient ($\bar{h}_c$)

ASSUMPTIONS

- Steady state

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 60°C

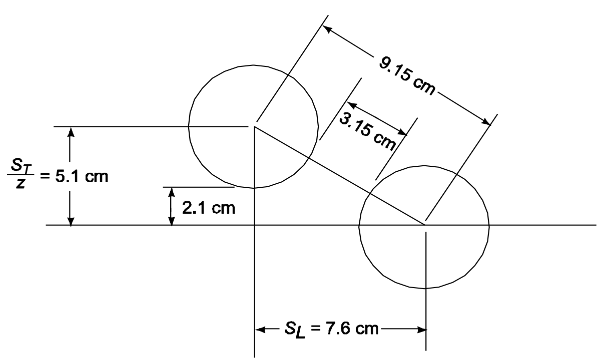
Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At T_w: Pr_s = 0.71

SOLUTION



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = U_s \frac{5.1}{2.1} = 2.43 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_{\max} D}{\nu} = \frac{2.43\,\text{m/s} \quad 0.06\,\text{m}}{19.4 \times 10^{-6}\,\text{m}^2/\text{s}} = 7515 \qquad \text{(Transition region)}$$

$$\frac{S_T}{S_L} = \frac{10.2\,\text{cm}}{7.6\,\text{cm}} = 1.34 < 2$$

Therefore, Equation (6.27) is applicable

$$\overline{Nu}_D = 0.35\left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{0.60}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.35(1.34)^{0.2}\,(7515)^{0.60}\,(0.71)^{0.36}\,(1) = 69.4$$

However, this Nusselt is applicable only to tube banks of ten or more rows. Since there are only two rows in this case, the average Nusselt number can be estimated by multiplying the above result by 0.75 as shown in Table 6.3.

$$\overline{Nu}_D = 0.75\,(69.4) = 52.05$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 52.05 \frac{0.0279\,\text{W/(m K)}}{0.06\,\text{m}} = 24.2 \text{ W/(m}^2\text{K)}$$

The pressure drop is given by by $\Delta p = f * \dfrac{\rho U_{\max}^2}{2 g_c} * N$

Where f is calculated from Figure (6.24)

For S_T/S_L=1.34 and Re_D=7515 from the figure we get x=1

For S_T/D= 7.5/6=1.22 and Re_D=7515 from figure (6.24)

f/x=0.91

Thus f =0.91

Pressure drop $\Delta p = 0.91 * \dfrac{1.025 * 5^2}{2*1} * 2$ = 23.3 Pa.

PROBLEM 6.39

Reconsider the heat exchanger of Problem 6.38 with an inline tube arrangement, where the centerlines of the tubes are spaced 7.5 cm apart, both in the longitudinal and transverse directions. Compare the results with those for the staggered tube arrangement and comment upon the difference. Also, which of the two arrangements is expected to have a larger pressure drop?

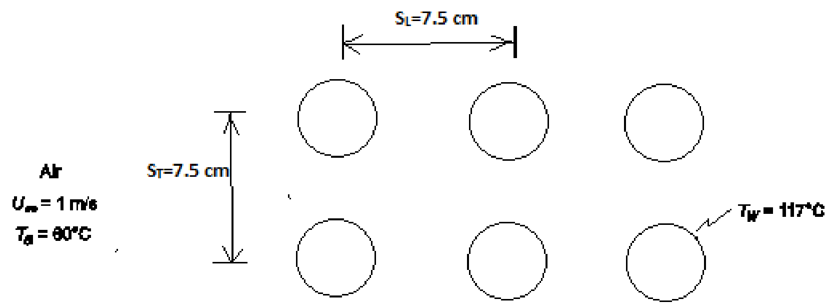
GIVEN

- Air flow through a tube bank
- Tube spacing (S) = 7.5 cm = 0.075 m
- Air temperature (T_a) = 60°C
- Air velocity (U_s) = 1 m/s
- Tube outside diameter (D) = 6 cm = 0.06 m
- Tube wall temperature (T_w) = 117°C

FIND

- The average heat transfer coefficient ($\bar{h}_c$)
- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 60°C

Thermal conductivity (k) = 0.0279 W/(m K)

Kinematic viscosity (ν) = 19.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

At T_w $\quad Pr_s$ = 0.71

SOLUTION

Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity for inline arrangement is

$$U_{\max} = U_s \frac{S_T}{S_T - D} = 5.0 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_{\max} D}{\nu} = \frac{5.0\,\text{m/s} \quad 0.06\,\text{m}}{19.4 \times 10^{-6}\,\text{m}^2/\text{s}} = 15{,}464 \qquad \text{(Transition region)}$$

Applying Equation (6.26)

$$Nu_D = 0.27 Re_D^{0.63}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.27*(15{,}464)^{0.60}\,(0.71)^{0.36}\,(1) = 104$$

Adjusting the Nusselt number for the first two rows

$$Nu_D = 0.75\,(104) = 77.70$$

$$h_c = Nu_D \frac{k}{D} = 77.70\,\frac{0.0279\,\mathrm{W/(m\,K)}}{0.06\,\mathrm{m}} = 36.1\,\mathrm{W/(m^2K)}$$

The pressure drop is given by by $\Delta p = f * \dfrac{\rho U_{\max}^2}{2g_c} * N$

Where f is calculated from Figure (6.23) for inline tubes

For $(S_T/D-1)(S_L/D-1)=1/16= 0.0625$ and $Re_D=15464$ from the figure we get x=6

For $S_L/D= 7.5/6=1.25$ and $Re_D=15464$ from figure (6.23)

f/x=0.52

Thus f =3.12

$$\Delta p = 3.12 * \frac{1.025*5^2}{2*1} * 2 = 79 \text{ Pa.}$$

COMMENT

The change in the geometry from Problem 6.38 lead to a 50% increase in the heat transfer coefficient and 240 % increase in pressure drop. Thus inline tube arrangement is expected to have higher pressure drop.

PROBLEM 6.40

Reconsider the problem described in Example 6.5, where methane gas at 20°C flows with an upstream velocity of 10 m/s over a staggered-arrangement tube bundle, with 5 rows of tubes facing the gas flow, and where their surface temperature is maintained at 50°C. Determine the effect of tube spacing (S_L and S_T) on the average heat transfer coefficient and the pressure drop for the tube bank. Consider different combinations of S_L and S_T with the following dimensions: S_L= 6 cm, 10 cm, and 12 cm; S_T= 8 cm and 10 cm. Comment on your results by suggesting the tube spacing that is preferred for this cross-flow bundle.

GIVEN

- Methane gas flow through a tube bank
- Methane gas temperature (T_a) = 20^0C
- Upstream velocity (U_s) = 10 m/s
- Tube outside diameter (D) = 4 cm = 0.04 m
- Tube surface temperature (T_w) = 50°C

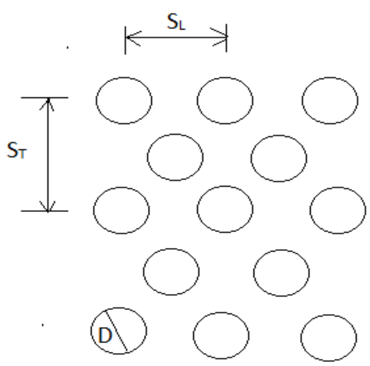
FIND

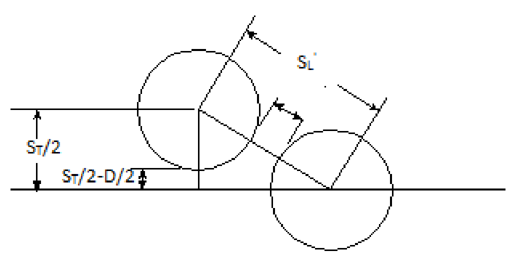
- Average heat transfer coefficient and pressure drop for different combination of tube bank spacing and suggest preferable tube spacing.

ASSUMPTIONS

- Steady state

SKETCH





PROPERTIES AND CONSTANTS

From Appendix 2, Table 36, for methane gas at 20°C

Density(ρ)=0.668 kg/m^3

Thermal conductivity (k) = 0.0332 W/(m K)

Kinematic viscosity (ν) = 16.27×10^{-6} m^2/s

Prandtl number (Pr) = 0.73

At tube temperature of T_w=50^0C

Pr_s = 0.73

Thus Pr/Pr_s=1

Considering S_L=10 cm and S_T= 8 cm

U_{max}=10* $\dfrac{S_T/2}{S_T/2 - D/2}$ =10*4/2= 20 m/s

Re_D= $\dfrac{U_{\max} D}{\nu} = \dfrac{20 * 0.04}{16.27 * 10^{-6}}$ = 49170

Which is a transition regime

S_T/S_L=8/10= 0.8 <2 for Staggered tubes using equation (6.27) we get

$$Nu_D = 0.35\left(\frac{S_T}{S_L}\right)^{0.2} \mathrm{Re}_D^{0.6}\,\mathrm{Pr}^{0.36}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{0.25} \quad => \quad Nu_D = 0.35(0.8)^{0.2} * 49170^{0.6} * 0.73^{0.36}(1)^{0.25}$$

$$Nu_D = 130.63 \quad => \quad \frac{\overline{h_c} d}{k} = 130.63$$

$$\overline{h_c} = 108.4 W/(m^2 K)$$

Now pressure drop is given by $\Delta p = f * \dfrac{\rho U_{\max}^2}{2 g_c} * N$

Where f is calculated from Figure (6.24)

For S_T/S_L=0.8 and Re_D=49170 from the figure we get x=1.1

For S_T/D= 8/4=2 and Re_D=49170

f/x=0.3

f=0.33

Thus pressure loss

$$\Delta p = 0.33 * \frac{0.668 * (20)^2}{2 * 1} * 5 = 440 \text{ Pa}$$

For S_T=10

U_{max}=10* $\dfrac{S_T/2}{S_T/2 - D/2}$ =10* $\dfrac{5}{5-2}$ = 16.67 m/s

Re_D= $\dfrac{U_{\max} D}{\nu} = \dfrac{16.67 * 0.04}{16.27 * 10^{-6}}$ = 40975

Similarly, the problem is solved for each combination of S_T and S_L

S_T	S_L	Nu_D	h_c(W/(m²K)	f	Δp(Pa)
8	6	216	179	0.36	167
8	10	130	108.6	0.30	220.5
8	12	188	156.2	0.33	240.5
10	6	202.6	168.2	0.23	106.7
10	10	183	152	0.25	116
10	12	176.4	146.4	0.26	121

Considering heat transfer rate and pressure drop across the tube bundles use of tube bundles with S_T=10 cm and S_L=6 cm is preferable.

PROBLEM 6.41

Carbon dioxide gas at 1 atmosphere pressure it to be heated from 25°C to 75°C by pumping it through a tube bank at a velocity of 4 m/s. The tubes are heated by steam condensing within them at 200°C. The tubes have 10 mm OD, are in an in–line arrangement, have a longitudinal spacing of 15 mm and a transverse spacing of 17 mm. If 13 tube rows are required, what is the average heat transfer coefficient and what is the pressure drop of the carbon dioxide?

GIVEN

- In-line tube bank: condensing steam inside, CO_2 outside
- CO_2 temperatures
  - In: ($T_{g,\text{in}}$) = 25°C
  - Out: ($T_{g,\text{out}}$) = 75°C
- CO_2 velocity (U_s) = 4 m/s
- Steam temperature (T_s) = 200°C
- Tube outside diameter (D) = 10 mm = 0.01 m
- Longitudinal spacing (S_L) = 15 mm = 0.015 m
- Transverse spacing (S_T) = 17 mm = 0.017 m
- Number of tubes (N) = 13

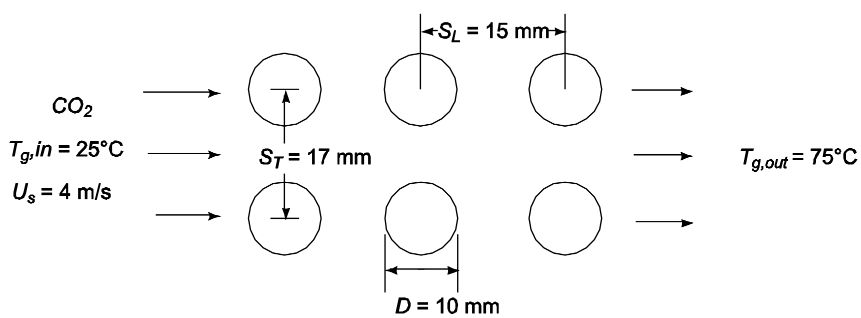
FIND

(a) The average heat transfer coefficient (h_c)
(b) The CO_2 pressure drop (Δp)

ASSUMPTIONS

- Steady state
- The thermal resistances of the condensing steam and the tube walls are negligible (tube wall temperature = T_s)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for carbon dioxide at the average temperature of 50°C

Density (ρ) = 1.6772 kg/m^3

Thermal conductivity (k) = 0.01836 W/(m K)

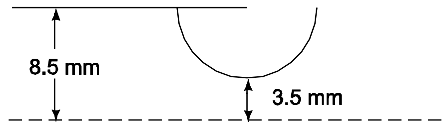
Kinematic viscosity (ν) = 9.64×10^{-6} m^2/s

Prandtl number (Pr) = 0.763

Specific heat (c) = 884 J/(kg K)

At the tube surface temperature of 200°C: Pr_s = 0.712

SOLUTION



The maximum CO_2 velocity is

$$U_{\max} = U_s \frac{8.5\,\text{mm}}{3.5\,\text{mm}} = 9.71\ \text{m/s}$$

The Reynolds number is

$$Re_D = \frac{U_s D}{\nu} = \frac{9.71\,\text{m/s}\quad 0.01\,\text{m}}{9.64\times10^{-6}\,\text{m}^2/\text{s}} = 10{,}077 \qquad\qquad \text{(Transition regime)}$$

$$\frac{S_T}{S_L} = \frac{17\,\text{mm}}{15\,\text{mm}} = 1.133$$

(a) The Nusselt number for this geometry is obtained from Equation (6.26)

$$\overline{Nu}_D = 0.27\, Re_D^{0.63}\, Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.27\ (10{,}077)^{0.63}\ (0.763)^{0.36} \left(\frac{0.763}{0.712}\right)^{0.25} = 82.9$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 82.9\, \frac{0.01863\,\text{W/(m K)}}{0.01\,\text{m}} = 154.5\ \text{W/(m}^2\text{K)}$$

(No correction is needed since $N > 10$)

(b) From Equation (6.34)

$$\Delta p = f\, \frac{\rho U^2_{\max}}{2 g_c} N$$

The pressure drop coefficient (f) is contained in Figure 6.23

$$\left(\frac{S_T}{D} - 1\right)\left(\frac{S_L}{D} - 1\right) = \left(\frac{17\,\text{mm}}{10\,\text{mm}} - 1\right)\left(\frac{15\,\text{mm}}{10\,\text{mm}} - 1\right) = 0.35 \qquad \Rightarrow x = 3$$

For $S_L/D = 1.5$ and $Re_D = 10^4$, from Figure 6.23: $f/x \approx 0.5$

$$f = 0.5\ (3) = 1.5$$

$$\Delta p = 1.5\, \frac{1.6772\,\text{kg/m}^3 \quad 9.7\,\text{m/s}^{\ 2}}{2} (13)\ \ (\text{N s}^2)/(\text{kg m}) \quad (\text{Pa m}^2)/\text{N} = 1.5 \times 10^3\ \text{Pa}$$

PROBLEM 6.42

Compare the rate of heat transfer and the pressure drop for an in-line and a staggered arrangement of a tube bank consisting of 300 tubes, 1.8 m long and 2.5 cm *OD*. The tubes are to be arranged in 15 rows with longitudinal and transverse spacing of 5 cm. The tube surface temperature is 95°C and water at 35°C is flowing at a mass rate of 5400 kg/s over the tubes.

GIVEN

- Water flowing over an in-line and a staggered tube bank
- Number of tubes (N_t) = 300
- Length of tubes (L) = 1.8 m
- Tube outside diameter (D) = 2.5 cm= 0.025 m
- Number of rows (N) = 15
- Normal and parallel spacing = 5 cm=0.05 m
- Tube surface temperature (T_t) = 95°C
- Water inlet temperature ($T_{w,\text{in}}$) = 35°C
- Mass flow rate of water ($\dot{m}$) = 5400 kg/s

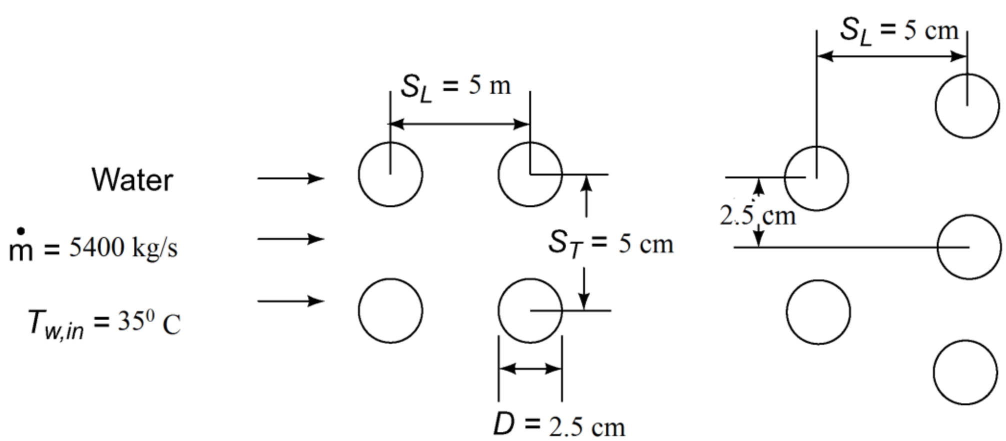
FIND

(a) Compare the rate of heat transfer (q) and
(b) The pressure drop (Δp) for the two configurations

ASSUMPTIONS

- Steady state
- Tube temperature is uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 13, for water at the inlet temperature of 35°C

Density (ρ) = 994.1 kg/m^3

Thermal conductivity (k) = 0.624 W/(m K)

Absolute viscosity (μ) = 719.8$\times$ 10^{-6} N/(m^2 s)

Prandtl number (Pr) = 4.8

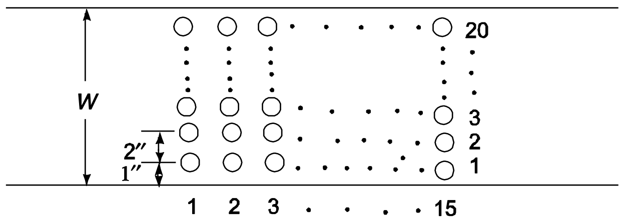
Specific heat (c) = 4175 J/(kg K)

At the tube temperature of 95°C

$Pr_s = 1.88$

SOLUTION

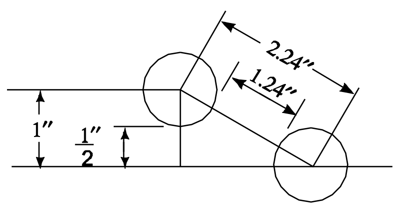
The water velocity can be calculated with the help of the sketch below.



W = 19(5 cm) + 5 cm = 100 cm = 1 m

Therefore, the water velocity is

$$U_s = \frac{\dot{m}}{\rho A} = \frac{(5400\, kg/\text{s})}{\left(994.1 kg/m^3\right)(1m)(1.8m)} = 3.02\,\text{m/s}$$



Therefore, for the staggered configuration, the minimum free area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = U_s\frac{1.0}{0.5} = 6.04\,\text{m/s}$$

This is also the maximum velocity for the in-line case.

The Reynolds number for either case is

$$Re_D = \frac{U_{\max} D\rho}{\mu} = \frac{(6.04\, m/\text{s})(0.025m) * 994.1kg / m^3}{\left(719.8\times10^{-6}\, kg/(m\text{s})\right)} = 2.085 \times 10^5$$

(Turbulent)

(a) The Nusselt number for the in-line case is given by Equation (6.29)

$$\overline{Nu}_D = 0.021\, Re_D^{0.84}\, Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.021(2.085 \times 10^5)^{0.84}\, (4.5)^{0.36} \left(\frac{4.5}{1.88}\right)^{0.25} = 1333$$

$$\bar{h}_{c\ IL} = \overline{Nu}_D\, \frac{k}{D} = 1333\,\frac{(0.624W/(mK))}{0.025m} = 33272\ \text{W/(}m^2\text{K)}$$

The Nusselt number for the staggered case is given by Equation (6.30)

$$\overline{Nu}_D = 0.022\, Re_D^{0.84}\, Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.022\, (2.085 \times 10^5)^{0.84}\, (4.5)^{0.36} \left(\frac{4.5}{1.88}\right)^{0.25} = 1397$$

$$\bar{h}_{c\ ST} = \overline{Nu}_D\, \frac{k}{D} = 1397\,\frac{(0.624W/(mK))}{0.025m} = 34862\,\text{W/(}m^2\text{K)}$$

These heat transfer coefficient differ by only 5%, therefore, the rate of heat transfer for the two tube banks will be nearly equal.

(b) These pressure drop is given by Equation (6.34)

$$\Delta p = f \frac{\rho U_{max}^{2}}{2} N$$

In-line bank:

$$\left(\frac{S_T}{D_o} - 1\right)\left(\frac{S_L}{D_o} - 1\right) = 1$$

From Figure 6.23: $x = 1$, $S_L/D = 2$, $f/x \approx 0.19 \rightarrow f = 0.19$

$$(\Delta p)_{IL} = 0.19 \frac{\left(994.1\, kg/m^3\right)(6.04\, m/\mathrm{s})^2\,(15)}{2} = 9066 \text{ Pa}$$

Staggered tube bank:

From Figure 6.24: x = 1.1. $S_T/D = 2$, $f/x \approx 0.16 \rightarrow f = 0.18$

$$(\Delta p)_{ST} = (\Delta p)_{IL} \left(\frac{0.18}{0.19}\right) = 8599 \text{ Pa}$$

For nearly the same rate of heat transfer, the staggered bank has slightly lower pressure drop.

COMMENTS

These results are only true because the flow is turbulent. Greater difference would occur at lower Reynolds numbers.

PROBLEM 6.43

Consider a heat exchanger consisting of 12.5 mm OD copper tubes in a staggered arrangement with transverse spacing 25 mm, longitudinal spacing 30 mm, and 9 tubes in the longitudinal direction. Condensing steam at 150°C flows inside the tubes. The heat exchanger is used to heat a stream of air flowing at 5 m/s from 20°C to 32°C. What is the average heat transfer coefficient and pressure drop for the tube bank?

GIVEN

- Staggered copper tube bank with condensing steam inside tubes and air flowing over the outside
- Tube outside diameter (D) = 12.5 mm = 0.0125 m
- Transverse spacing (S_T) = 25 mm = 0.025 m
- Longitudinal spacing (S_L) = 30 mm = 0.03 m
- Number of rows of tubes (N) = 9
- Steam temperature (T_s) = 150°C
- Air velocity (U_s) = 5 m/s
- Air temperature: ▪ $T_{a,\text{in}}$ = 20°C
  ▪ $T_{a,\text{out}}$ = 32°C

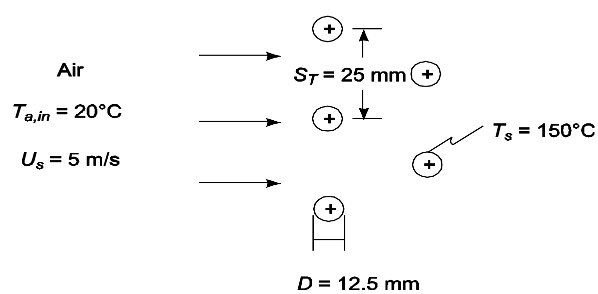
FIND

(a) The average heat transfer coefficient (h_c)
(b) The pressure drop (Δp)

ASSUMPTIONS

- Steady state
- Thermal resistance of the condensing steam and the copper tube is negligible. Therefore, the tube surface temperature = T_s

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average temperature of 26°C

Density (ρ) = 1.157 kg/m^3

Thermal conductivity (k) = 0.0251 W/(m K)

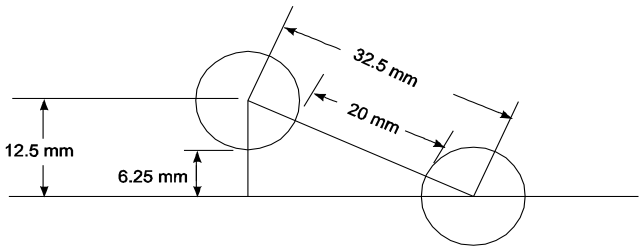
Kinematic viscosity (ν) = 15.9 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

Specific heat (c) = 1012 J/(kg K)

At the tube temperature of 150°C: Pr_s = 0.71

SOLUTION



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = U_s \frac{12.5}{6.25} = 10 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_s D}{\nu} = \frac{10\,\text{m/s} \quad 0.0125\,\text{m}}{15.9\times10^{-6}\ \text{m}^2/\text{s}} = 7862 \qquad \text{(Transition regime)}$$

$$\frac{S_T}{S_L} = \frac{25\,\text{mm}}{30\,\text{mm}} = 0.833$$

(a) Applying Equation (6.26)

$$\overline{Nu}_D = 0.35\left(\frac{S_T}{S_L}\right)0.2\ Re_D^{0.60}\ Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.35\ (0.833)^{0.2}\ (7862)^{0.60}\ (0.71)^{0.36}\ (1) = 64.9$$

However, this Nusselt is applicable only to tube banks of ten or more rows. Since there are only 9 rows in this case, the average Nusselt number can be estimated by multiplying the above result by 0.99 as shown in Table 6.3.

$$\overline{Nu}_D = 0.99\ (64.9) = 64.3$$

$$\bar{h}_c = \overline{Nu}_D\ \frac{k}{D} = 64.3\,\frac{0.0251\,\text{W/(m K)}}{0.0125\,\text{m}} = 129\ \text{W/(m}^2\text{K)}$$

(b) The pressure drop is given by Equation (6.34)

$$\Delta p = f\ \frac{\rho\ U_{\max}^{\ 2}}{2}\ N$$

From Figure 6.24 For $\frac{S_T}{S_L}$ = 0.833, Re = 7862 $\rightarrow x = 1$

For $\frac{S_D}{D} = \frac{25}{12.5} = 2 \rightarrow \frac{f}{x} = 0.4 \rightarrow f = 0.4$

$$\Delta p = \frac{(0.4)\ 1.157\,\text{kg/m}^3 \quad 10\,\text{m/s}}{2}\ (9)\ \ (\text{N s}^2)/(\text{kg m}) \quad (\text{Pa m}^2)/\text{N} \quad = 208 \text{ Pa}$$

PROBLEM 6.44

A blood warmer is often used in certain clinical transfusion procedures. While safely storing and preserving blood requires refrigeration, cold blood when used in rapid transfusion procedures could cause clinically dangerous hypothermia in a patient. A new compact and miniature heat exchanger has been proposed that is used "inline" with the transfusion delivery system. It consists of a multi-tube bundle of 5-mm diameter tubes arranged in an equilateral-triangular array with tube centers 7.5 mm apart. Blood flows inside the tubes and is warmed by a heated airstream that is in external cross flow such that the tube surface is maintained at 40°C. The inlet air flow to the tube array is at 70°C with a free stream velocity of 2 m/s, and there are five lateral or transverse rows in the bundle. Determine the average heat transfer coefficient and pressure drop in the airstream. Also, what is the change in both of these quantities if the tube center-to-center pitch is reduced to 6.0 mm?

GIVEN

- Heated air flowing over an equilateral staggered tube bank
- Air temperatures
  - $T_{air,\text{in}}$ = 70°C
- Number of tube rows (N) = 5
- Tube diameter (D) = 5 mm= 0.005 m
- Center to center pitch(S) = 7.5 mm=0.0075 m
- Free stream velocity (U_a) = 2 m/s
- Tube surface temperature (T_t) = 40°C

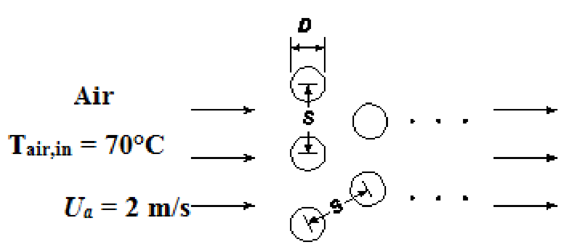
FIND

- The average heat transfer coefficient ($\bar{h}_c$) and pressure drop in airstream.
- Change in average heat transfer coefficient and pressure drop when center to center pitch is reduced to 6 mm.

ASSUMPTIONS

- Steady state
- The correlation of Equation 6.38 can be applied to a pitch-to-diameter ratio of 1.5
- Tube wall temperature is uniform and constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for air at temperature of 70°C

Density (ρ) = 0.9965 kg/m^3

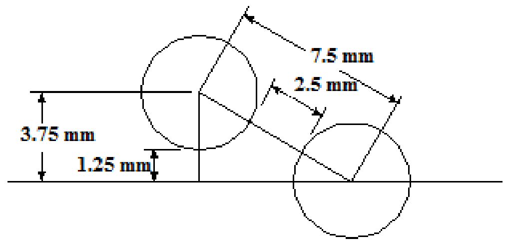
Specific heat (c)= 1018 J/(kg K)

Thermal conductivity (k) = 0.0286 W/(m K)

Kinematic viscosity (ν) = 20.45*10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = U_a \frac{3.75}{1.25} = 6 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_s D}{\nu} = \frac{(6\,\text{m/s})(0.005\,\text{m})}{\left(20.45\times10^{-6}\,\text{m}^2/\text{s}\right)} = 1467$$

- S_T=7.5 mm, $S_L = \sqrt{7.5^2 - 3.75^2}$ = 6.5 mm=0.0065 m
  S_T/S_L=7.5/6.5= 1.15

(a) Applying Equation (6.27)

$$\overline{Nu}_D = 0.35\left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{0.60}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.35\,(1.15)^{0.2}\,(1467)^{0.60}\,(0.71)^{0.36}\,(1) = 25.26$$

However, this Nusselt number is applicable only to tube banks of ten or more rows. Since there are only 5 rows in this case, the average Nusselt number can be estimated by multiplying the above result by 0.92 as shown in Table 6.3.

$$\overline{Nu}_D = 0.92\,(25.26) = 23.25$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 23.25^* \frac{(0.0286\,\text{W/(m K)})}{0.005\,\text{m}} = 133\ \text{W/(m}^2\text{K)}$$

(b) The pressure drop is given by Equation (6.34)

$$\Delta p = f\,\frac{\rho\,U_{\max}^{\ \ 2}}{2}\,N$$

From Figure 6.24 For $\dfrac{S_T}{S_L}$ = 1.15, Re = 1467 $\rightarrow x$ = 1

For $\dfrac{S_D}{D} = \dfrac{7.5}{5}$ = 1.5 $\rightarrow \dfrac{f}{x}$ = 3.9 $\rightarrow f$ = 0.39

$$\Delta p = \frac{(0.39)\left(0.9965\,\text{kg/m}^3\right)(6\,\text{m/s})}{2}(5)\ (\text{N s}^2)/(\text{kg m})\quad (\text{Pa m}^2)/\text{N} = 5.82\ \text{Pa}$$

When center to center pitch is reduced to 6 mm

The minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{max} = U_a \frac{3}{0.5} = 12 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_s D}{\nu} = \frac{(12\,\text{m/s})(0.005\,\text{m})}{\left(20.45\times10^{-6}\ \text{m}^2/\text{s}\right)} = 2934$$

- S_T=7.5 mm, S_L= $\sqrt{6^2-3^2}$ = 5.2 mm=0.0065 m
  S_T/S_L=6/5.2= 1.15

(a) Applying Equation (6.27)

$$\overline{Nu}_D = 0.35\left(\frac{S_T}{S_L}\right)^{0.2} Re_D^{0.60}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{0.25} = 0.35\ (1.15)^{0.2}\ (2934)^{0.60}\ (0.71)^{0.36}\ (1) = 38.3$$

However, this Nusselt is applicable only to tube banks of ten or more rows. Since there are only 5 rows in this case, the average Nusselt number can be estimated by multiplying the above result by 0.92 as shown in Table 6.3.

$$\overline{Nu}_D = 0.92\ (38.3) = 35.2$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 35.2 * \frac{(0.0286\,\text{W/(m K)})}{0.005\,\text{m}} = 201\ \text{W/(m}^2\text{K)}$$

(b) The pressure drop is given by Equation (6.34)

$$\Delta p = f\,\frac{\rho\ U_{max}^{\ \ 2}}{2} N$$

From Figure 6.24 For $\frac{S_T}{S_L}$ = 1.15, Re = 2934 $\rightarrow$ x = 1

For $\frac{S_D}{D} = \frac{6}{5} = 1.2 \rightarrow \frac{f}{x} = 0.8 \rightarrow f = 0.8$

$$\Delta p = \frac{(0.8)\left(0.9965\,\text{kg/m}^3\right)(6\,\text{m/s})}{2}(5)\ \ (\text{N s}^2)/(\text{kg m})\quad (\text{Pa m}^2)/\text{N} = 11.95\ \text{Pa}$$

Thus average heat transfer coefficient increased by 51 % and pressure drop increased by 100 % when the center to center pitch is reduced to 6 mm.

PROBLEM 6.45

Estimate the heat transfer coefficient for liquid sodium at 540°C flowing over a 10-row staggered-tube bank of 2.5 cm-diameter tubes arranged in an equilateral-triangular array with a 1.5 pitch-to-diameter ratio. The entering velocity is 0.6 m/s, based on the area of the shell, and the tube surface temperature is 200°C. The outlet sodium temperature is 310°C.

GIVEN

- Liquid sodium flowing over an equilateral staggered tube bank
- Sodium temperatures
  - $T_{s,\text{in}}$ = 540°C
  - $T_{s,\text{out}}$ = 310°C
- Number of tube rows (N) = 10
- Tube diameter (D) = 2.5 cm = 0.025 m
- Pitch to diameter ratio (S/D) = 1.5
- Entering velocity (U_s) = 0.6 m/s
- Tube surface temperature (T_t) = 200°C

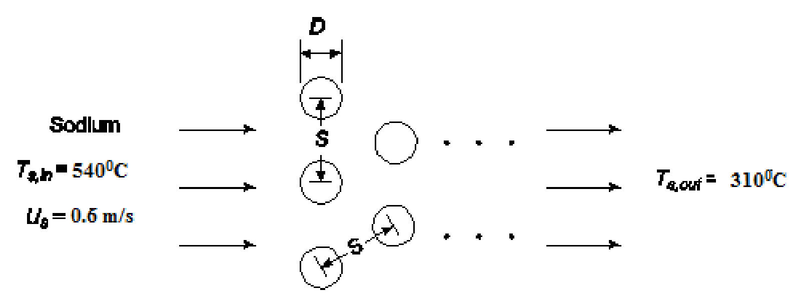
FIND

- Estimate the heat transfer coefficient ($\bar{h}_c$)

ASSUMPTIONS

- Steady state
- The correlation of Equation 6.38 can be applied to a pitch-to-diameter ratio of 1.5
- Tube wall temperature is uniform and constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for sodium at the average temperature of 425°C

Density (ρ) = 847 kg/m^3

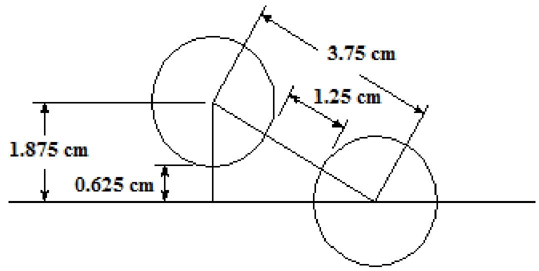
Specific heat (c)=1285 W/(kg K)

Thermal conductivity (k) = 70.1W/(m K)

Kinematic viscosity (ν) = 2.92*10^{-7} m^2/s

Prandtl number (Pr) = 0.0047

SOLUTION



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = U_s \frac{1.875}{0.625} = 1.8\, m/\mathrm{s}$$

The Reynolds number for the tube bank is

$$Re_D = \frac{U_s D}{\nu} = \frac{(1.8\, m/\mathrm{s})(0.025 m)}{\left(2.92 * 10^{-7}\, m^2/\mathrm{h}\right)} = 154{,}109$$

The Reynolds number is out of the range of applicability of Equation (6.35). However, it is the only correlation available for liquid metals in this geometry

$$\overline{Nu}_D = 4.03 + 0.228\, (Re_D\, Pr)^{0.67} = 4.03 + 0.228[154{,}109\ (0.0047)]^{0.67} = 2.8$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 22.8\ \frac{(70.1 W/(mK))}{0.025 m} = 64{,}000\ \mathrm{W/(h\ m^2\ K)}$$

No correction is needed since $N \geq 10$.

PROBLEM 6.46

> **Liquid mercury at a temperature of 315°C flows at a velocity of 10 cm/s over a staggered bank of 5/8-in. 16 BWG stainless steel tubes, arranged in an equilateral triangular array with a pitch-to-diameter ratio of 1.375. If water at 2 atm pressure is being evaporated inside the tubes, estimate the average rate of heat transfer to the water per meter length of the bank, if the bank is 10 rows deep and contains 60 tubes. The boiling heat transfer coefficient is 20,000 W/(m² K).**

GIVEN

- Liquid mercury flow over an equilaterally staggered tube bank
- Inlet mercury temperature ($T_{m,\text{in}}$) = 315°C
- Mercury velocity (U_s) = 10 cm/s = 0.1 m/s
- Tubes are 5/8 in., 26 BWG stainless steel
- Pitch to diameter ratio (S/D) = 1.375
- Water at 2 atm pressure is being evaporated within the tubes
- Number of rows of tubes (N) = 10
- Number of tubes (N_t) = 60
- The boiling heat transfer coefficient ($\bar{h}_b$) = 20,000 W/(m^2 K)

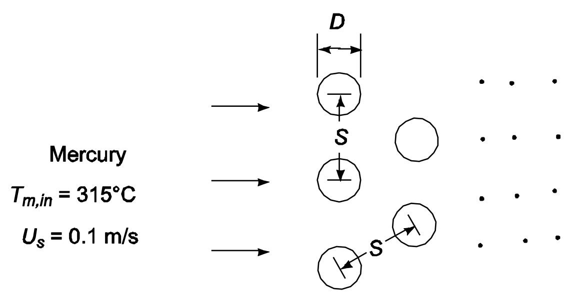
FIND

- The average rate of heat transfer per meter length of the bank (q/L)

ASSUMPTIONS

- Steady state
- Tubes are type 304 stainless steel
- Temperature change of the mercury across the tube bank is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 26, for mercury at the inlet temperature of 315°C

Density (ρ) = 12,847 kg/m^3

Thermal conductivity (k) = 14.02 W/(m K)

Kinematic viscosity (ν) = 0.0673×10^{-6} m^2/s

Prandtl number (Pr) = 0.0083

Specific heat (c) = 134.0 J/(kg K)

From Appendix 2, Table 13, the saturation temperature of water at 2 atm (2.02×10^5 Pa) is T_w = 120°C

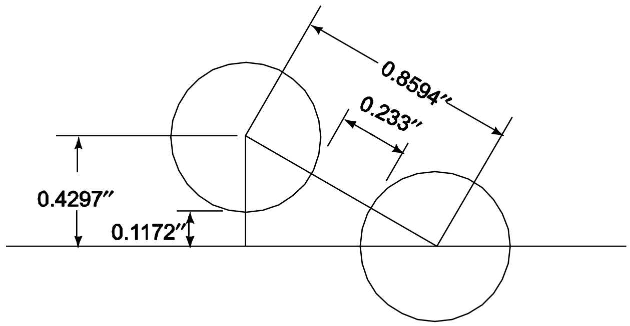
From Appendix 2, Table 42, for 5/8 in. 16 BWG tubes

Outside diameter (D_o) 5/8 in. = 0.0159 m

Inside diameter (D_i) = 0.495 in. = 0.0126 m

From Appendix 2, Table 10, the thermal conductivity of type 304 stainless steel (k_s) = 14.4 W/(m K) at 20°C

SOLUTION



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = U_s \frac{0.4297}{0.1172} = 0.37 \text{ m/s}$$

The Reynolds number is

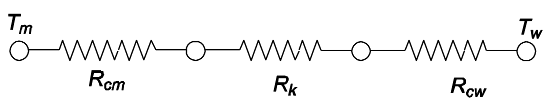
$$Re_D = \frac{U_s D}{\nu} = \frac{0.37\,\text{m/s} \;\; 0.0159\,\text{m}}{0.0673\times10^{-6}\,\text{m}^2/\text{s}} = 87{,}414$$

Applying the correlation of Equation (6.35)

$$\overline{Nu}_D = 4.03 + 0.228\,(Re_D\,Pr)^{0.67} = 4.03 + 0.228\,[87{,}414(0.0083)]^{0.67} = 22.8$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D_o} = 22.8 \frac{14.02\,\text{W/(m K)}}{0.0159\,\text{m}} = 20{,}147 \;\; \text{W/(m}^2\text{K)}$$

The thermal circuit for the problem is shown below



where

R_{cw} = Thermal resistance of the boiling water

$$= \frac{1}{\bar{h}_b A_i} = \frac{1}{\bar{h}_b N_t \pi D_i L} = \frac{1}{20{,}000\,\text{W/(m}^2\text{K)} \;\; 60\pi \;\; 0.0126\,\text{m} \;\; L} = 2.11\times10^{-5}\,(\text{m K})/\text{W} \left(\frac{1}{L}\right)$$

R_k = Conductive resistance of the tube wall

$$R_k = \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_s} = \frac{\ln\left(\frac{0.0159\,\text{m}}{0.0126\,\text{m}}\right)}{2\pi L \;\; 14.4\,\text{W/(m K)}} = 0.00257\,(\text{m K})/L \left(\frac{1}{L}\right)$$

R_{cm} = Convective resistance of the mercury side

$$R_{cm} = \frac{1}{\bar{h}_c A_i} = \frac{1}{\bar{h}_c N_t \pi D_o L} = \frac{1}{20{,}147\,\mathrm{W/(m^2K)}\ \ 60\ \ \pi\ \ 0.0159\,\mathrm{m}\ \ L} = 1.656\times10^{-5}(\mathrm{m\,K})/L\ \left(\frac{1}{L}\right)$$

$$R_{\text{total}} = R_{cw} + R_k + R_{cm} = (2.11\times10^{-5} + 0.00257 + 1.656\times10^{-5})\ (\mathrm{m\,K})/L\ \left(\frac{1}{L}\right)$$

$$= 0.00259(\mathrm{m\,K})/\mathrm{W}\ \left(\frac{1}{L}\right)$$

The rate of heat transfer to the steam is

$$\frac{q}{L} = \frac{\Delta T}{LR_{\text{total}}} = \frac{315°\mathrm{C} - 120°\mathrm{C}}{0.00259\,(\mathrm{m\,K})/\mathrm{W}} = 75{,}300\ \mathrm{W/m}$$

COMMENT

Note that the thermal resistance of the tube wall is 99% of the total resistance.

PROBLEM 6.47

An attractive method of energy conservation in industrial settings is to utilize waste heat from largescale power generating equipment. In one such scheme, the exhaust gases from the diesel engine of an industrial power generator are channeled through a cross-flow, compact, waste-heat recovery boiler to produce process steam. In one such heat exchanger, a finned tube bundle is configured with tubes of 2.5 cm outer diameter that are arranged in a staggered equilateral- triangular array with tube center-to-center pitch of 7.5 cm. Engine exhaust gas (thermal properties of which are taken to be the same as those for air) at 400°C is in cross-flow over the tube bank with a free stream velocity of 2.0 m/s. In order to enhance the gas-side convective heat transfer, the tubes have thin circular fins on their outer surface and the finned surface extension ratio ε=1.15. Due to the steam generation process inside the tubes, their outer surface temperature is 175°C. If there are 10 rows of tubes in the flow direction (N_L = 10), calculate the average heat transfer coefficient and pressure drop for the gas flow stream.

GIVEN

- Heated air flowing over an equilateral staggered tube bank
- Exhaust gas temperature
  - $T_{air,\text{in}}$ = 400°C
- Number of tube rows (N_L) = 10
- Tube diameter (D) = 2.5 cm= 0.025 m
- Center to center pitch(S) = 7.5 cm=0.075 m
- Free stream velocity (U_a) = 2 m/s
- Tube surface temperature (T_t) = 175°C
- Finned surface extension ration (ε)= 1.15

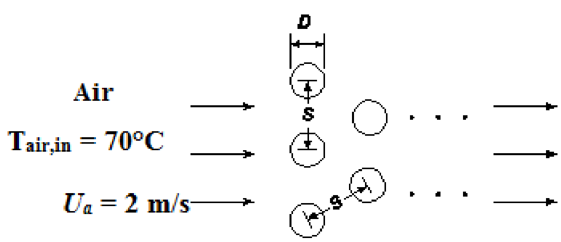
FIND

- The average heat transfer coefficient ($\bar{h}_c$) and pressure drop in gas flow stream.

ASSUMPTIONS

- Steady state
- Tube wall temperature is uniform and constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for air at temperature of 400°C

Density (ρ) = 0.508 kg/m^3

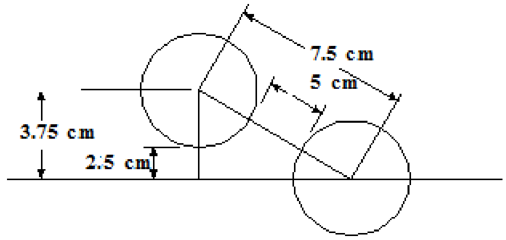
Specific heat (c)= 1059 J/(kg K)

Thermal conductivity (k) = 0.0485 W/(m K)

Kinematic viscosity (ν) = 64.6*10^{-6} m^2/s

Prandtl number (Pr) = 0.72

SOLUTION



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = U_a \frac{3.75}{2.5} = 3 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_s D}{\nu} = \frac{(3\,\text{m/s})(0.025\,\text{m})}{\left(64.6\times10^{-6}\,\text{m}^2/\text{s}\right)} = 1160$$

- S_T=7.5 cm, S_L= $\sqrt{7.5^2 - 3.75^2}$ = 6.5 cm=0.065 m
  S_T/S_L=7.5/6.5= 1.15

Applying Equation (6.40) for finned tube bundle in cross flow Re_D=1160 for staggered tube arrangement

$$\overline{Nu}_D = C_2 \,\text{Re}_D^a \,\text{Pr}^b \left(S_T/S_L\right)^{0.2} \left(p_f/D\right)^{0.18} \left(h_f/D\right)^{-0.14} \left(\text{Pr}/\text{Pr}_w\right)^{0.25}$$

Where for Re_D=1160

C_2=0.192, a=0.65, b=0.36.

Since, the fin parameters are not given and only surface extension ratio is given.

We have,

Surface extension ratio (ε)=Total surface area with fins/ Surface area without fins

$$\varepsilon = \frac{D_0^2 - D_i^2}{2D_i p_f} + 1$$ neglecting the thickness of the fins.

Since all the parameters are not given we should consider this problem as design data problem and select the suitable parameters for fin. For, surface extension ratio of 1.15 taking D_0 as 2.85 we have

h_f =0.175 cm, $p_f = 2.5$ cm

Thus above equation gives

$$\overline{Nu}_D = 0.192 * 1160^{0.65} * 0.72^{0.36} (1.15)^{0.2} (1)^{0.18} (0.07)^{-0.14} (1)^{0.25}$$

$$\overline{Nu}_D = 25$$

Nusselt number is applicable to tube banks of ten or more rows. Since there are 10 rows in this case, correction is not required.

$$\overline{Nu}_D = 25$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 25* \frac{(0.0485\,\text{W/(m K)})}{0.025\,\text{m}} = 48.5\ \text{W/(m}^2\text{K)}$$

(b) The pressure drop is given by Equation (6.34)

$$\Delta p = Eu\ \rho (U_{\max})^2 c_z N_L$$

Where Eu= $0.068\varepsilon^{0.5}\left(\dfrac{S_T-1}{S_L-1}\right)^{-0.4} = 0.068*1.15^{0.5}*\left(\dfrac{7.5-1}{6.5-1}\right)^{-0.4}$ =0.068

From Table (6.4)

C_z=1 for N_L=10

Δp= Eu $\rho(U_{\max})^2 c_z$ N_L=0.068*0.508*3^2*1*10=3.10 Pa

PROBLEM 6.48

Reconsider the heat exchanger of Problem 6.38 with an externally finned-tube bundle used to enhance the air-side heat transfer coefficient in the pre-heater. If thin circular fins are attached on the outside of the tubes such that the surface extension ratio ε=1.12, determine the average air-flow heat transfer coefficient for this finned-tube bank. Compare the results with those for the un-finned or bare tube bank and comment upon the extent of heat transfer enhancement achieved by adding fins.

GIVEN

- Air temperature (T_a) = 60°C
- Air velocity (U_s) = 1 m/s
- Tube outside diameter (D) = 6 cm = 0.06 m
- Tube wall temperature (T_w) = 117°C
- Surface extension ratio (ε)=1.12

FIND

- The average heat transfer coefficient ($\bar{h}_c$) and extent of heat transfer enhancement achieved by adding fins.

ASSUMPTIONS

- Steady state

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 60°C

Thermal conductivity (k) = 0.0279 W/(m K)

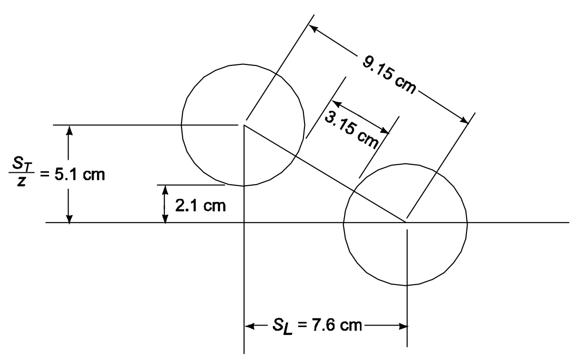
Kinematic viscosity (ν) = 19.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

Density (ρ)=1.025 kg/m^3

At T_w: Pr_s = 0.71

SOLUTION



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{max} = U_s \frac{5.1}{2.1} = 2.43 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_{max} D}{\nu} = \frac{2.43\,\text{m/s} \quad 0.06\,\text{m}}{19.4\times10^{-6}\,\text{m}^2/\text{s}} = 7515 \qquad \text{(Transition region)}$$

$$\frac{S_T}{S_L} = \frac{10.2\,\text{cm}}{7.6\,\text{cm}} = 1.34 < 2$$

Applying Equation (6.40) for finned tube bundle in cross flow Re_D=7515 for staggered tube arrangement

$$\overline{Nu}_D = C_2 \,\text{Re}_D^a \,\text{Pr}^b \left(S_T/S_L\right)^{0.2} \left(p_f/D\right)^{0.18} \left(h_f/D\right)^{-0.14} \left(\text{Pr}/\text{Pr}_w\right)^{0.25}$$

Where for Re_D=7515

C_2=0.192, a=0.65, b=0.36.

Since, the fin parameters are not given and only surface extension ratio is given.
We have,
Surface extension ratio (ε)=Total surface area with fins/ Surface area without fins

$\varepsilon = \dfrac{D_0^2 - D_i^2}{2D_i p_f} + 1$ neglecting the thickness of the fins.

Since all the parameters are not given we should consider this problem as design data problem and select the suitable parameters for fin. For, surface extension ratio of 1.15 taking D_0 as 2.85 we have

h_f =0.42 cm, $p_f = 7.5$ cm

Thus above equation gives

$$\overline{Nu}_D = 0.192*7515^{0.65}*0.71^{0.36}1.34^{0.2}\left(1.25\right)^{0.18}\left(0.07\right)^{-0.14}\left(1\right)^{0.25}$$

$$\overline{Nu}_D = 89.9$$

Nusselt number is applicable to tube banks of ten or more rows. Since there are 10 rows in this case, correction is not required.

$$\overline{Nu}_D = 89.9$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 25*\frac{(0.0279\,\text{W/(m K)})}{0.06\,\text{m}} = 41.8\,\text{W/(m}^2\text{K)}$$

(b) The pressure drop is given by Equation (6.34)

$$\Delta p = Eu\;\rho(U_{\max})^2\, c_z\, N_L$$

Where Eu= $0.068\varepsilon^{0.5}\left(\dfrac{S_T - 1}{S_L - 1}\right)^{-0.4} = 0.068*1.12^{0.5}*\left(\dfrac{10.2-1}{7.6-1}\right)^{-0.4}$ =0.063

From Table (6.4)

C_z=1 for N_L=2

Δp= Eu $\rho(U_{\max})^2\, c_z$ N_L=0.063*1.025*2.43^2*1*2= 0.763 Pa

PROBLEM 6.49

Reconsider the heat exchanger of Problem 6.47, and explore the effect of tube center-to-center spacing in the equilateral array of the finned tubes. Consider the tube spacing of 8.75 cm and 10 cm. What is the extent of change in the average heat transfer coefficient and pressure drop? Likewise, what is the extent of change in the thermal and hydrodynamic performance if a square pitch (inline arrangement is considered) with tube center-to-center spacing of 7.5 cm?

GIVEN

- Heated air flowing over an equilateral staggered tube bank
- Exhaust gas temperature
  - $T_{air,\text{in}}$ = 400°C
- Number of tube rows (N_L) = 10
- Tube diameter (D) = 2.5 cm= 0.025 m
- Center to center pitch(S) = 7.5 cm=0.075 m
- Free stream velocity (U_a) = 2 m/s
- Tube surface temperature (T_t) = 175°C
- Finned surface extension ration (ε)= 1.15

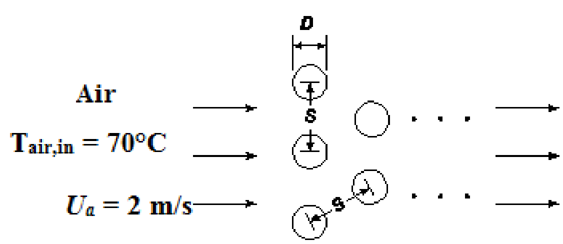
FIND

- The average heat transfer coefficient ($\bar{h}_c$) and pressure drop in gas flow stream.

ASSUMPTIONS

- Steady state
- Tube wall temperature is uniform and constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for air at temperature of 400°C

Density (ρ) = 0.508 kg/m^3

Specific heat (c)= 1059 J/(kg K)

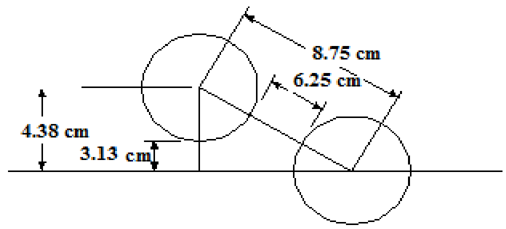
Thermal conductivity (k) = 0.0485 W/(m K)

Kinematic viscosity (ν) = 64.6*10^{-6} m^2/s

Prandtl number (Pr) = 0.72

SOLUTION

For spacing of 8.75 cm



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{max} = U_a \frac{4.38}{3.13} = 2.8 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_s D}{\nu} = \frac{(2.8\,\text{m/s})(0.025\,\text{m})}{\left(64.6\times10^{-6}\,\text{m}^2/\text{s}\right)} = 1083$$

- S_T=8.75 cm, S_L= $\sqrt{8.75^2 - 4.38^2}$ = 7.57 cm

S_T/S_L=8.75/7.57= 1.155

Applying Equation (6.40) for finned tube bundle in cross flow Re_D=1083 for staggered tube arrangement

$$\overline{Nu}_D = C_2\,\text{Re}_D^a\,\text{Pr}^b\left(S_T/S_L\right)^{0.2}\left(p_f/D\right)^{0.18}\left(h_f/D\right)^{-0.14}\left(\text{Pr}/\text{Pr}_w\right)^{0.25}$$

Where for Re_D=1160

C_2=0.192, a=0.65, b=0.36.

Since, the fin parameters are not given and only surface extension ratio is given.

We have,

Surface extension ratio (ε)=Total surface area with fins/ Surface area without fins

$\varepsilon = \dfrac{D_0^2 - D_i^2}{2D_i p_f} + 1$ neglecting the thickness of the fins.

Since all the parameters are not given we should consider this problem as design data problem and select the suitable parameters for fin. For, surface extension ratio of 1.15 taking D_0 as 2.85 we have

h_f =0.175 cm, $p_f = 2.5$ cm

Thus above equation gives

$$\overline{Nu}_D = 0.192 * 1083^{0.65} * 0.72^{0.36}\left(1.155\right)^{0.2}\left(1\right)^{0.18}\left(0.07\right)^{-0.14}\left(1\right)^{0.25}$$

$\overline{Nu}_D$ = 23.9

Nusselt number is applicable to tube banks of ten or more rows. Since there are 10 rows in this case, correction is not required.

$$\overline{Nu}_D = 23.9$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 23.9*\frac{(0.0485\,\text{W/(m K)})}{0.025\,\text{m}} = 46.4\ \text{W/(m}^2\text{K)}$$

(b) The pressure drop is given by Equation (6.36)

$$\Delta p = Eu\ \rho(U_{\max})^2 c_z N_L$$

Where Eu= $0.068\varepsilon^{0.5}\left(\frac{S_T-1}{S_L-1}\right)^{-0.4} = 0.068*1.15^{0.5}*\left(\frac{8.75-1}{7.57-1}\right)^{-0.4}$ =0.0683

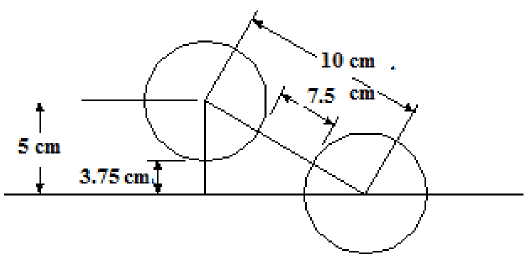
From Table (6.4)

C_z=1 for N_L=10

Δp= Eu $\rho(U_{\max})^2 c_z$ N_L=0.0683*0.508*2.8^2*1*10=2.72 Pa

Thus there is reduction in heat transfer by 4.4% and decrease in pressure drop by 12% with increase in spacing compared to Problem (7.47)

For spacing of 10 cm



Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is

$$U_{\max} = \text{U}_a \frac{5}{3.75} = 2.67 \text{ m/s}$$

The Reynolds number is

$$Re_D = \frac{U_s D}{\nu} = \frac{(2.67\,\text{m/s})(0.025\,\text{m})}{\left(64.6\times10^{-6}\,\text{m}^2/\text{s}\right)} = 1033$$

- S_T=10 cm, S_L= $\sqrt{10^2-5^2}$ = 8.66 cm

S_T/S_L=10/8.66= 1.155

Applying Equation (6.40) for finned tube bundle in cross flow Re_D=1033 for staggered tube arrangement

$$\overline{Nu}_D = C_2\,\text{Re}_D^a\,\text{Pr}^b\left(S_T/S_L\right)^{0.2}\left(p_f/D\right)^{0.18}\left(h_f/D\right)^{-0.14}\left(\text{Pr}/\text{Pr}_w\right)^{0.25}$$

Where for Re_D=1033

C_2=0.192, a=0.65, b=0.36.

Since, the fin parameters are not given and only surface extension ratio is given.

We have,

Surface extension ratio (ε)=Total surface area with fins/ Surface area without fins

$$\varepsilon = \frac{D_0^2 - D_i^2}{2D_i p_f} + 1$$ neglecting the thickness of the fins.

Since all the parameters are not given we should consider this problem as design data problem and select the suitable parameters for fin. For, surface extension ratio of 1.15 taking D_0 as 2.85 we have

h_f =0.175 cm, $p_f = 2.5$ cm

Thus above equation gives

$$\overline{Nu}_D = 0.192*1033^{0.65}*0.72^{0.36}\left(1.155\right)^{0.2}\left(1\right)^{0.18}\left(0.07\right)^{-0.14}\left(1\right)^{0.25}$$

$$\overline{Nu}_D = 23.2$$

Nusselt number is applicable to tube banks of ten or more rows. Since there are 10 rows in this case, correction is not required.

$$\overline{Nu}_D = 23.2$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 23.2*\frac{(0.0485\,\text{W/(m K)})}{0.025\,\text{m}} = 45 \text{ W/(m}^2\text{K)}$$

(b) The pressure drop is given by Equation (6.36)

$$\Delta p = Eu\ \rho(U_{\max})^2 c_z N_L$$

Where Eu= $0.068\varepsilon^{0.5}\left(\frac{S_T - 1}{S_L - 1}\right)^{-0.4} = 0.068*1.15^{0.5}*\left(\frac{8.75-1}{7.57-1}\right)^{-0.4}$ =0.0683

From Table (6.4)

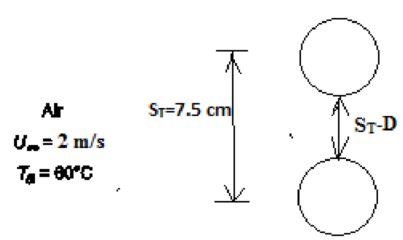
C_z=1 for N_L=10

Δp= Eu $\rho(U_{\max})^2 c_z N_L$=0.0683*0.508*2.67²*1*10=2.47 Pa

Thus there is reduction in heat transfer by 7% and decrease in pressure drop by 20% with increase in spacing compared to Problem (7.47)

When square pitch of 7.5 cm spacing is considered.

Therefore, the minimum free flow area is between adjacent tubes in a row, and the maximum air velocity is



$U_{max} = U_a \frac{7.5}{7.5-2.5} = 3$ m/s

The Reynolds number is

$$Re_D = \frac{U_{max} D}{\nu} = \frac{(3\,\text{m/s})(0.025\,\text{m})}{\left(64.6\times10^{-6}\,\text{m}^2/\text{s}\right)} = 1160$$

- S_T=7.5 cm, S_L=7.5 cm

S_T/S_L= 1

Applying Equation (6.38) for finned tube bundle in cross flow Re_D=1160 for inline tube arrangement

$$\overline{Nu}_D = 0.303\varepsilon^{-0.375}\,\text{Re}_D^{0.625}\,\text{Pr}^{0.36}\left(\text{Pr}/\text{Pr}_w\right)^{0.25}$$

$$\overline{Nu}_D = 0.303*1.15^{-0.375}*1160^{0.625}*0.72^{0.36}\left(1\right)^{0.25}$$

$\overline{Nu}_D$=21

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 21*\frac{(0.0485\,\text{W/(m K)})}{0.025\,\text{m}} = 40.8\ \text{W/(m}^2\text{K)}$$

(b) The pressure drop is given by Equation (6.36)

$$\Delta p = Eu\ \rho(U_{max})^2 c_z N_L$$

Where Eu= $0.068\varepsilon^{0.5}\left(\frac{S_T-1}{S_L-1}\right)^{-0.4} = 0.068*1.15^{0.5}*\left(\frac{7.5-1}{7.5-1}\right)^{-0.4}$ =0.073

From Table (6.4)

C_z=1 for N_L=10

Δp= Eu $\rho(U_{max})^2 c_z N_L$=0.073*0.508*3^2*1*10=3.33 Pa

When inline tube arrangement is considered instead of staggered tubes there is reduction in heat transfer by 16% and increase in pressure drop by 7.4 %.

PROBLEM 6.50

One method of storing solar energy for use during cloudy days, or at night, is to store it in the form of sensible heat in a rock bed. Suppose such a rock bed has been heated to 70°C and it is desired to heat a stream of air by blowing it through the bed. If the air inlet temperature is 10°C and the mass velocity of the air in the bed is 0.5 kg/(s m^2), how long must the bed be in order for the initial outlet air temperature to be 65°C? Assume that the rocks are spherical, 2 cm in diameter, and that the bed void fraction is 0.5. (Hint: The surface area of the rocks per unit volume of the bed is $(6/D_p)(1 - \varepsilon)$.)

GIVEN

- Packed bed of rocks with air blowing through it
- Initial temperature of rocks (T_r) = 70°C
- Inlet air temperature ($T_{a,\text{in}}$) = 10°C
- Mass velocity of air ($\dot{m}$ /A) = 0.5 kg/(s m^2)
- Outlet air temperature ($T_{a,\text{out}}$) = 65°C
- Surface area per unit volume (A_s/V) = $(6/D_p)(1 - \varepsilon)$

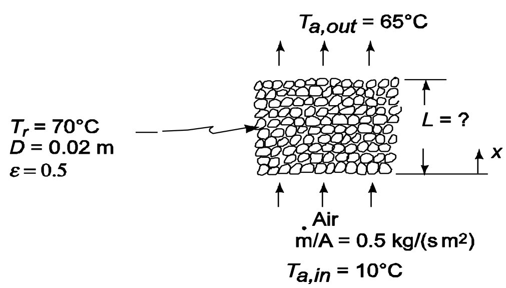
FIND

- Length of bed required (L)

ASSUMPTIONS

- Rocks are spherical with diameter (D) = 2 cm = 0.02 m
- Void fraction of the bed (ε) = 0.5
- Rock temperature remains practically constant

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average temperature of 37.5°C

Density (ρ) = 1.101 kg/m^3

Thermal conductivity (k) = 0.0263 W/(m K)

Kinematic viscosity (ν) = 17.4 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

Specific heat (c_{pa}) = 10147 J/(kg K)

SOLUTION

The Whitaker definition of the Reynolds number is

$$Re_{D_p} = \frac{D_p U_s}{\nu(1-\varepsilon)}$$

$$\text{where} \qquad D_p = \frac{6\,(\text{volume})}{\text{surface area}} = \frac{6\left(\frac{\pi}{6}D^3\right)}{\pi D^2} = D \text{ (for spherical packing)}$$

$$U_s = \frac{m}{A\rho} = \frac{0.5\,\text{kg/(s m}^2)}{1.101\,\text{kg/m}^3} = 0.454 \text{ m/s}$$

$$\therefore \qquad Re_{D_p} = \frac{0.02\,\text{m} \quad 0.45\,\text{m/s}}{17.4\times10^{-6}\,\text{m}^2/\text{s} \quad 1-0.5} = 1044$$

The heat transfer coefficient is given by Equation (6.41)

$$\frac{\bar{h}_c D_p}{k} = \frac{1-\varepsilon}{\varepsilon}\left(0.5\,Re_{D_p}^{\frac{1}{2}} + 0.2\,Re_{D_p}^{\frac{2}{3}}\right) Pr^{\frac{1}{3}}$$

$$\frac{\bar{h}_c D_p}{k} = \frac{1-0.5}{0.5}\left[0.5(1044)^{\frac{1}{2}} + 0.2(1044)^{\frac{2}{3}}\right](0.71)^{\frac{1}{3}} = 32.77$$

$$\bar{h}_c = 32.77\,\frac{0.0263\,\text{W/(m K)}}{0.02\,\text{m}} = 43.1 \text{ W/(m}^2\text{K)}$$

A local energy balance on the air flow through the bed yields

$$\frac{\dot{m}}{A}\,c_{pa}\,[T(x+\Delta x) - T(x)] = \bar{h}_c\,\frac{A_s}{V}\,dx\,[T_r - T(x)]$$

$$\text{where} \qquad \frac{A_s}{V} = \frac{6}{D_p}(1-\varepsilon) = \frac{6}{0.02\,\text{m}}(1-0.5) = 150 \text{ 1/m}$$

$$0.5\,\text{kg/(m}^2\text{s)} \quad 1014\,\text{(Ws)/(kg K)} \quad [T(x+\Delta x) - T(x)] = \quad 43.1\,\text{W/(m}^2\text{K)} \quad 150\,\text{1/m} \;\; dx\,[T_r - T(x)]$$

Checking the units, then eliminating them for clarity

$$T(x+\Delta x) = T(x) + 12.75\,\Delta x\,[T_r - T(x)]$$

Let Δx = 1 cm = 0.01 m

Applying the above equation iteratively until the temperature reaches 65°C yields the following results

	x (m)	$T(x)$ (°C)
	0.00	10.00
	0.01	17.65
	0.02	24.32
	0.03	30.14
	0.04	35.22
	0.05	39.66
	0.06	43.53
	0.07	46.91
	0.08	49.85
Let Δx = 0.02 m	0.10	54.99
	0.12	58.81
	0.14	61.66
	0.16	63.79
	0.18	65.31
	L = 0.18 m = 18 cm	

PROBLEM 6.51

Suppose the rock bed in Problem 6.50 has been completely discharged and the entire bed is at 10°C. Hot air at 90°C and 0.2 m/s is then used to recharge the bed. How long does it take until the first rocks are back up to 70°C and what is the total heat transfer from the air to the bed?

GIVEN

- Packed bed of rocks with air blowing through it
- Inlet air temperature ($T_{a,\text{in}}$) = 90°C
- Mass velocity of air ($\dot{m}/A$) = 0.5 kg/s m^2
- Surface area per unit volume (A_s/V) = $(6/D_p)(1-\varepsilon)$
- Length of bed (from solution to Problem 6.50) (L) = 19 cm = 0.19 m
- Initial temperature of rocks ($T_{r,i}$) = 10°C

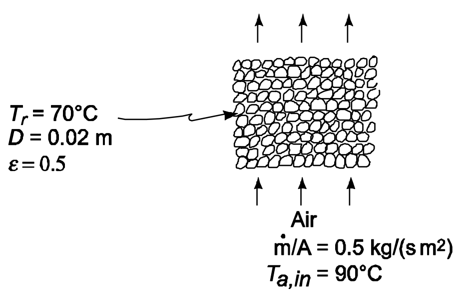
FIND

(a) The time required (t) for $T_{r,\max}$ = 70°C
(b) The rate of heat transfer (q) for that time

ASSUMPTIONS

- Rocks are spherical with diameter (D) = 2 cm = 0.02 m
- Void fraction of the bed (ε) = 0.5
- The rock has the density and thermal conductivity of granite
- The specific heat of the rock is approximately the same as brick or concrete: c = 840 J/(kg K)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 90°C

Density (ρ) = 0.942 kg/m^3

Thermal conductivity (k) = 0.0300 W/(m K)

Kinematic viscosity (ν) = 22.6 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

From Appendix 2, Table 11, for granite

Density (ρ) = 2750 kg/m^3

Thermal conductivity (k_r) = 3.0 W/(m K)

SOLUTION

The air velocity is given by

$$U_s = \frac{\dot{m}}{A\rho} = \frac{0.5\,\text{kg/(m}^2\text{s)}}{0.942\,\text{kg/m}^3} = 0.531\,\text{m/s}$$

$$\therefore \qquad Re_{D_p} = \frac{D_p U_s}{\nu(1-\varepsilon)} = \frac{(0.02\,\text{m})(0.531\,\text{m/s})}{(22.6\times10^{-6}\,\text{m}^2/\text{s})(1-0.5)} = 940$$

Applying Equation 6.41

$$\frac{\bar{h}_c D_p}{k} = \frac{1-\varepsilon}{\varepsilon}\left(0.5 Re_{D_p}^{\frac{1}{2}} + 0.2 Re_{D_p}^{\frac{2}{3}}\right) Pr^{\frac{1}{3}}$$

$$\frac{\bar{h}_c D_p}{k} = \frac{1-0.5}{0.5}\left[0.5(940)^{\frac{1}{2}} + 0.2(940)^{\frac{2}{3}}\right](0.71)^{\frac{1}{3}} = 30.8$$

$$\bar{h}_c = 30.8\,\frac{0.0300\,\text{W/(m K)}}{0.02\,\text{m}} = 46.2\,\text{W/(m}^2\text{K)}$$

The upstream rocks in the bed will heat up most quickly because they are exposed to air at the inlet temperature of 90°C. The Biot number for a rock is

$$Bi = \frac{\bar{h}_c D}{2k_r} = \frac{(46.2\,\text{W/(m}^2\text{K)})(0.02\,\text{m})}{2\,(3.0\,\text{W/(m K)})} = 0.15 > 0.1$$

Therefore, the internal thermal resistance of the rocks cannot be neglected and the chart solution of Figure 2.44 must be used.

$$\frac{T(0,t)-T_\infty}{T_o - T_\infty} = \frac{70°\text{C} - 90°\text{C}}{10°\text{C} - 90°\text{C}} = 0.25$$

$$\frac{1}{Bi} = \frac{1}{0.15} = 6.66$$

From Figure 3.11 (a)

$$Fo = \frac{\alpha\, t}{r_o^{\,2}} = 3$$

Solving for the time

$$t = \frac{Fo\, r_o^{\,2}}{\alpha} = \frac{Fo\, r_o^{\,2}\rho_r c}{k_r} = \frac{3\,(0.1\text{m})^2\,(2750\,\text{kg/m}^3)\,(840\,\text{J/(kg K)})\,(\text{(W s)/J})}{3.0\,\text{W/(m K)}}$$

$$t = 23{,}100 \text{ s} = 6.4 \text{ hours}$$

It will take 6.4 hours for the center of the upstream rocks to reach 70°C.

COMMENTS

Since an average solar radiation is available for more than 6 hours in sunny climates, the rock storage appears to be sized properly

PROBLEM 6.52

An automotive catalytic convertor is a packed bed in which a platinum catalyst is coated on the surface of small alumina spheres. A metal container holds the catalyst pellets and allows engine exhaust gases to flow through the bed of pellets. The catalyst must be heated by the exhaust gases to 300°C before the catalyst helps combust unburned hydrocarbons in the gases. The time required to achieve this temperature is critical because unburned hydrocarbons emitted by the vehicle during a cold start comprise a large fraction of the total emissions from the vehicle during an emission test. A fixed volume of catalyst is required but the shape of the bed can be modified to increase the heat-up rate. Compare the heat-up time for a bed 5-cm-diameter and 20-cm-long with one 10 cm diameter and 5-cm-long. The catalyst pellets are spherical, 5-mm-diameter, have a density of 2 g/cm^3, thermal conductivity of 12 W/(m K) and specific heat of 1100 J/(kg K). The packed-bed void fraction is 0.5. Exhaust gas from the engine is at a temperature of 400°C, a flow rate of 6.4 gm/s, and has the properties of air.

GIVEN

- A packed bed catalytic converter comprised of platinum coated alumina spheres with exhaust gases flowing through them
- Two possible bed geometries
  - Case *a*: Diameter (D_b) = 5 cm = 0.05 m
    Length (L) = 20 cm = 0.2 m
  - Case *b*: Diameter (D_b) = 10 cm = 0.1 m
    Length (L) = 5 cm = 0.05 m
- Sphere density (ρ_p) = 2 g/cm^3 = 2000 kg/m^3
- Sphere diameter (D_p) = 5 mm = 0.005 m
- Sphere thermal conductivity (k_p) = 12 W/(m K)
- Void fraction (ε) = 0.5
- Sphere specific heat (c_p) = 1100 J/(kg K)
- Engine exhaust gas temperature (T_g) = 400°C
- Engine exhaust mass flow rate ($\dot{m}$) = 6.4 g/s = 0.0064 kg/s
- Engine exhaust has the properties of air

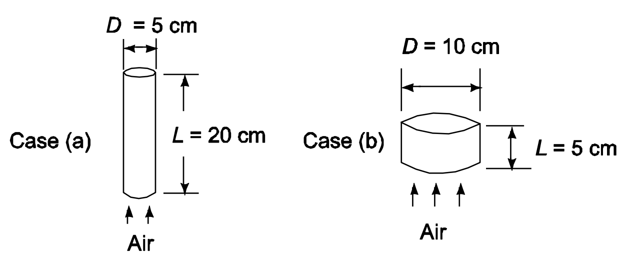
FIND

- The heat-up time (t) for the pellet surface temperature (T_p) to reach 300°C

ASSUMPTIONS

- The initial temperature of the bed (T_o) = 20°C

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 400°C

- Density (ρ) = 0.508 kg/m^3
- Thermal conductivity (k) = 0.0485 W/(m K)
- Absolute viscosity (μ) = 32.754 × 10^{-6} (Ns)/m

- Prandtl number (Pr) = 0.72

SOLUTION

The Reynolds number is

$$Re_{D_p} = \frac{D_p U_s}{\nu(1-\varepsilon)} = \frac{4\,\mathrm{m}\,D_p}{\pi D_b{}^2 \mu(1-\varepsilon)}$$

Case (a) $$Re_{D_p} = \frac{4\ 0.0064\,\mathrm{kg/s}\quad 0.005\,\mathrm{m}}{\pi\ 0.05\,\mathrm{m}\ ^2\ \ 32.754\times10^{-6}(\mathrm{N\,s})/\mathrm{m}\quad (\mathrm{kg\,m})/(\mathrm{N\,s^2})\quad 1-0.05} = 995$$

Case (b) Re_{D_p} = 249

Applying Equation (6.41)

$$\frac{\bar{h}_c D_p}{k} = \frac{1-\varepsilon}{\varepsilon}\left(0.5 Re_{D_p}{}^{\frac{1}{2}} + 0.2 Re_{D_p}{}^{\frac{2}{3}}\right) Pr^{\frac{1}{3}}$$

Case (a) $$\bar{h}_{ca} = \frac{0.0485\,\mathrm{W/(m\,K)}}{0.005\,\mathrm{m}}\left(\frac{1-0.5}{0.5}\right)\left[0.5(995)^{\frac{1}{2}} + 0.2(995)^{\frac{2}{3}}\right](0.71)^{\frac{1}{3}} = 310.4\ \mathrm{W/(m^2\,K)}$$

Case (b) $\bar{h}_{cb}$ = 137.4 W/(m^2 K)

The Bio number for case (a) is

$$Bi = \frac{\bar{h}_c D_p}{2k_p} = \frac{310.4\,\mathrm{W/(m^2K)}\quad 0.005\,\mathrm{m}}{2\ \ 12\,\mathrm{W/(m\,K)}} = 0.065 < 0.1$$

Therefore, the internal resistance of the spheres is negligible in both cases and the lumped parameter analysis of Section 2.6.1 can be used. The upstream portion of the bed will reach 300°C first. Since they will be continuously exposed to 400°C air at a constant heat transfer coefficient, Equation (3.3) may be applied. Solving Equation 3.3 for time

$$t = -\frac{c_p \rho_p V}{\bar{h}_c A_s}\ln\left(\frac{T-T_g}{T_o-T_g}\right) \quad \text{where} \quad \frac{V}{A_s} = \frac{\frac{\pi}{6}D_p{}^3}{\pi D_p{}^2} = \frac{D_p}{6}$$

For case (a)

$$t = -\frac{1100\,\mathrm{J/(kg\,K)}\quad 2000\,\mathrm{kg/m^3}}{310.4\,\mathrm{W/(m^2K)}\quad \mathrm{J/(Ws)}}\left(\frac{0.005\,\mathrm{m}}{6}\right)\ln\left(\frac{300°\mathrm{C}-400°\mathrm{C}}{20°\mathrm{C}-400°\mathrm{C}}\right) = 7.9\ \mathrm{s}$$

For case (b) t = 17.8 s

COMMENTS

The short configuration takes more than twice as long to heat up because of the lower heat transfer coefficient due to the lower gas velocity through the bed.

Once the front row of the bed reaches 300°C, catalytic combustion will occur and quickly heat the rest of the packed bed.

PROBLEM 6.53

In many metal machining operations (cutting or turning) the contact surface at the interface of the tool and metal part is cooled by a liquid coolant that is delivered on the surface by a single round jet. Water at 25°C is used as coolant in one such jet-cooling system, and it flows at the rate of 0.01 kg/s through a circular nozzle with a diameter of 3.0 mm. Determine the average heat transfer coefficient and the rate of cooling from a 5 mm region on the surface of the part being machined. The surface temperature in this area is at 40°C.

GIVEN

- Liquid coolant is delivered on the surface by single round jet.
- Nozzle diameter (d)=3 mm = 0.003 m
- Mass flow rate of water ($\dot{m}$)= 0.01 kg/s
- Temperature of water jet (T_w)= 25^0C
- Surface temperature of the area (T_s)= 40^0C

FIND

Average heat transfer coefficient and rate of cooling from a 5 mm region on surface of part being machined.

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 20°C

Viscosity (μ) = $8.806*10^{-4}$ N s/m^2

Thermal conductivity (k) = 0.606 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

Reynold's number for jet is (Re_d)= $\dfrac{4\dot{m}}{\pi d \mu}$ = = $\dfrac{4*0.01}{\pi*0.003*8.806*10^{-4}}$ = 4820

$r_v = 0.141 d \,\mathrm{Re}_d^{1/3}$ =7.1 mm

$\dfrac{r_c}{d} = 1200\,\mathrm{Re}_d^{-0.422} = 1200*4820^{-0.422}$ =33.5

For constant temperature surface with r<r_v. Nusselt number is given by equation (6.56) as

$$Nu_d = 0.619\,\mathrm{Re}_d^{1/3}\,\mathrm{Pr}^{1/3}\,\hat{r}^{-1/2}$$

Where $\hat{r} = \dfrac{r}{d}\dfrac{1}{\mathrm{Re}_d^{1/3}} = \dfrac{5}{3}\dfrac{1}{4820^{1/3}}$ =0.0986

$Nu_d = 0.619*4820^{1/3}*6.1^{1/3}*0.0986^{-1/2}$ = 60.8

$\dfrac{h_c d}{k}$ =60.8

$$h_c = \frac{60.8 * 0.606}{0.003} = 12290 \text{ W/(m}^2\text{ K)}$$

Rate of cooling $\left(q^{"}\right) = h_c(T_s - T_w) = 12290 * (40 - 25) = 184354$ W/m^2

Thus the average heat transfer coefficient and rate of cooling are 12290 W/(m^2 K) and 184354 W/m^2 respectively.

PROBLEM 6.54

A microprocessor chip (10 mm *10 mm square) is to be cooled by an impinging circular air jet that is directed on the chip surface by a 5-mm diameter nozzle, placed at a distance of 10 mm. The electrical activity in the microcircuits of the chip render a uniform surface heat flux condition. For safe operation of the microprocessor, the chip-surface temperature should not exceed 90°C. If air at 20°C flows through the nozzle with a velocity of 10 m/s, and the maximum heat dissipated by the chip is 1.25 W, is this jet-impingement cooling system able to maintain the entire chip surface at a safe operating temperature?

GIVEN

- Air jet impinges on microprocessor chip
- Nozzle diameter (d)=5 mm = 0.005 m
- Microprocessor chip area= 10 mm*10 mm
- Chip distance from nozzle(z)= 10 mm=0.01 m
- Flow velocity (v)= 10 m/s
- Temperature of fluid jet (T_a)= 20^0C
- Maximum heat dissipated by chip= 1.25 W
- Chip surface temperature should not exceed 90^0C

FIND

Whether the jet impinging cooling system be able to maintain the entire chip surface to safe operating temperature.

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 20°C

Density(ρ)= 1.164 kg/m^3

Viscosity (μ) = 18.24*10^{-6} N s/m^2

Thermal conductivity (k) = 0.0251 W/(m K)

Prandtl number (Pr) = 0.71

SOLUTION

For circular round jet when the jet hits center of 10 mm by 10 mm chip. The farthest distance from the center of the chip is r= $5\sqrt{2}$ mm= 7.1 mm=7.1*10^{-3} mm.

The cooling rate at this point must be greater than heat flux generated by the chip.

Reynold's number for jet is (Re_d)= $\dfrac{\rho v d}{\mu} = \dfrac{1.164*10*0.005}{18.24*10^{-6}}$ = 3190

$$\frac{r}{d} = \frac{7.1}{5} = 1.42$$

$$\frac{r_v}{d} = 0.1773*\mathrm{Re}_d^{1/3} = 0.1773*3190^{1/3} = 2.60$$

Thus, 0.8< $\dfrac{r}{d} < \dfrac{r_v}{d}$ for SRJ with uniform heat flux. Average nusselt number is given by equation (6.50) as

$$\overline{Nu_d} = 0.632\,\mathrm{Re}_d^{1/2}\,\mathrm{Pr}^{1/3}\left(\frac{d}{r}\right)^{1/2} = 0.632*3190^{1/2}*0.71^{1/3}\left(\frac{0.005}{0.0071}\right)^{1/2}$$

$=26.72$

$$\frac{\bar{h}_c d}{k} = 26.72$$

$\bar{h}_c$ = 134 W/(m^2 K)

Total cooling from the area of 10 mm*10 mm is

$$\dot{q} = \bar{h}_c A(T_{\max} - T_a)$$

= 134*10*10*10^{-6}*(90-20) W

=0.94 W

Since the heat generated by chip is more than the maximum cooling effect produced by the jet, the cooling system cannot maintain entire chip at safe operating temperature.

PROBLEM 6.55

Reconsider the problem described in Example 6.7, where water jets from a nozzle that has a diameter of 6 mm and impinges on a disk of 4 cm in diameter that is subjected to a uniform heat flux of 70,000 W/m^2. Instead of water, consider using air and ethylene glycol as the cooling fluids with the same flow rate (0.008 kg/s) and temperature (20°C) in each case, and determine the temperature on the surface of the disk at radial distances of 3mm and 12 mm from the axis of the jet. Compare the results with those for water obtained in Example 6.7 and comment on them.

GIVEN

- Water jet impinges on a disk.
- Nozzle diameter (d)=6 mm = 0.006 m
- Disk diameter = 4 cm =0.04 m
- Flowrate of fluid ($\dot{m}$)= 0.008 kg/s
- Temperature of fluid jet (T_j)= 20^0C

FIND

(a) Temperature on the surface of disk at radial distance of 3 mm and 12 mm from axis of jet.
(b) Compare results with those of water obtained from example 6.7

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 20°C

Viscosity (μ) = 18.24*10^{-6} N s/m^2

Thermal conductivity (k) = 0.0251 W/(m K)

Prandtl number (Pr) = 0.71

From Appendix 2, Table 22, for organic compounds (Ethylene Glycol)

Viscosity (μ) = 199*10^{-4} N s/m^2

Thermal conductivity (k) = 0.258 W/(m K)

Prandtl number (Pr) = 183.7

SOLUTION

For dry air

$$\therefore \qquad Re_D = \frac{4\dot{m}}{\pi d \mu} = \frac{4*0.008}{\pi*0.006*18.24*10^{-6}} = 93073$$

For r=3 mm, r/d=0.003/0.006= 0.5< 0.8

We have,

Radius at which flow becomes turbulent (r_c) is given by

$$\frac{r_c}{d} = 1200\,\mathrm{Re}_d^{-0.422}$$

$$r_c = 1200*93073^{-0.422}*0.006 = 0.057 \text{ m}$$

From equation (6.54)

$$Nu_d = \frac{8\,\mathrm{Re}_d\,\mathrm{Pr}\,f}{49\left(\frac{b}{d}\right)+28\left(\frac{r}{d}\right)^2 f}$$

Where

$$f = \frac{C_f/2}{1.07+12.7(\mathrm{Pr}^{2/3}-1)\sqrt{C_f/2}} \text{ and } C_f = 0.073\,\mathrm{Re}_d^{-1/4}\left(\frac{r}{d}\right)^{1/4}$$

$$\frac{b}{d} = \frac{0.02091}{\mathrm{Re}_d^{1/4}}\left(\frac{r}{d}\right)^{5/4} + C\left(\frac{d}{r}\right) \text{ with C}= 0.1713+\frac{5.147}{\mathrm{Re}_d}\left(\frac{r_c}{d}\right)-\frac{0.02091}{\mathrm{Re}_d^{1/4}}\left(\frac{r_c}{d}\right)^{1/4}$$

We have

$$\text{C}= 0.1713+\frac{5.147}{93073}\left(\frac{0.057}{0.006}\right)-\frac{0.02091}{93073^{1/4}}\left(\frac{0.057}{0.006}\right)^{1/4} =0.1713+0.00053+0.0021=0.17393$$

$$\frac{b}{d} = \frac{0.02091}{93073^{1/4}}\left(\frac{0.003}{0.006}\right)^{5/4} + 0.17393\left(\frac{0.006}{0.003}\right) =0.0005+0.34786=0.358$$

$$C_f = 0.073*93073^{-1/4}\left(\frac{0.003}{0.006}\right)^{1/4} =0.0035$$

$$f = \frac{0.0035/2}{1.07+12.7(0.71^{2/3}-1)\sqrt{0.0035/2}} = \frac{0.00175}{1.07-0.1084} =0.00182$$

$$Nu_d = \frac{962.15}{17.55} = 54.8$$

$$\frac{h_c d}{k} = 54.8$$

$$h_c = 54.8*\frac{0.0251}{0.006} = 229 \text{ W/(m}^2\text{ K)}$$

$$T_s=T_i+\frac{70000}{229} =20+305.6= 325.6^0\text{C}$$

For r=12 mm

$$\text{C}= 0.1713+\frac{5.147}{93073}\left(\frac{0.057}{0.006}\right)-\frac{0.02091}{93073^{1/4}}\left(\frac{0.057}{0.006}\right)^{1/4} =0.1713+0.00053+0.0021=0.17393$$

$$\frac{b}{d} = \frac{0.02091}{93073^{1/4}}\left(\frac{0.012}{0.006}\right)^{5/4} + 0.17393\left(\frac{0.006}{0.012}\right) =0.0028+0.087=0.0905$$

$$C_f = 0.073 * 93073^{-1/4}\left(\frac{0.012}{0.006}\right)^{1/4} = 0.005$$

$$f = \frac{0.005/2}{1.07 + 12.7(0.71^{2/3} - 1)\sqrt{0.005/2}} = \frac{0.0025}{1.07 - 0.13} = 0.00267$$

$$Nu_d = \frac{8\,\mathrm{Re}_d\,\mathrm{Pr}\,f}{49\left(\frac{b}{d}\right) + 28\left(\frac{r}{d}\right)^2 f}$$

$$Nu_d = \frac{8 * 93073 * 0.71 * 0.00267}{49 * 0.0905 + 0.299} = 298.1$$

$$Nu_d == 298.1$$

$$\frac{h_c d}{k} = 298.1$$

$$h_c = 298.1 * \frac{0.0251}{0.006} = 1247 \text{ W/(m}^2\text{ K)}$$

$$T_s = T_i + \frac{70000}{1247} = 20 + 56.13 = 76.13^0\mathrm{C}$$

For Ethanol Glycol

$$\therefore \quad Re_D = \frac{4\dot{m}}{\pi d \mu} = \frac{4 * 0.008}{\pi * 0.006 * 199 * 10^{-4}} = 853$$

For r=3 mm, r/d=0.003/0.006= 0.5< 0.8

From equation (6.47) we have,

$$Nu_d = 0.797\,\mathrm{Re}_d^{1/2}\,\mathrm{Pr}^{1/3} = 0.797 * 853^{1/2} * 183.7^{1/3}$$

=132.3

$$\frac{h_c d}{k} = 132.3$$

$$h_c = 298.1 * \frac{0.258}{0.006} = 12818 \text{ W/(m}^2\text{ K)}$$

$$T_s = T_i + \frac{70000}{12818} = 20 + 5.46 = 25.46^0\mathrm{C}$$

(b) For r=12 mm $r_v = 0.1773 * 853^{1/3} * 0.006$ =0.01 m

$r > r_v$

Since, region IV is not valid for fluids with Pr>4.86,

In region III, Nusselt number is given by equation (6.52) as

$$Nu_d = \frac{0.407\,\mathrm{Re}_d^{1/3}\,\mathrm{Pr}^{1/3}\left(\frac{d}{r}\right)^{2/3}}{\left[0.173\left(\frac{d}{r}\right)^2 + \frac{5.147}{\mathrm{Re}_d}\left(\frac{r}{d}\right)\right]^{2/3}\left[\frac{1}{2}\left(\frac{r}{d}\right)^2 + c\right]^{1/3}}$$ where

$c = -5.051*10^{-5}\mathrm{Re}^{2/3} = -5*10^{-3}$

$$Nu_d = \frac{0.407 * 853^{1/3} * 183.7^{1/3}\left(\frac{0.006}{0.012}\right)^{2/3}}{\left[0.173\left(\frac{0.006}{0.012}\right)^2 + \frac{5.147}{853}\left(\frac{0.012}{0.006}\right)\right]^{2/3}\left[\frac{1}{2}\left(\frac{0.012}{0.006}\right)^2 - 0.005\right]^{1/3}}$$

$$Nu_d = \frac{13.82}{0.14518 * 1.26}$$

$$Nu_d = \frac{1.382}{0.14518 * 1.26} = 75.5$$

$$\frac{h_c d}{k} = 75.5$$

$$h_c = 75.5 * \frac{0.258}{0.006} = 3246.5 \text{ W/(m}^2\text{ K)}$$

$$T_s = T_i + \frac{70000}{3246.5} = 20 + 5.46 = 41.56^0\mathrm{C}$$