1.	A printed circuit board has 8 different locations in which a component can be placed. If we have 4 <u>different</u> resistors and 2 <u>identical</u> capacitors to place on the board, how many different designs are possible? Assume any component location can accept either a resistor or capacitor.
2.	Let A and B be two events. Suppose the probability that neither A or B occurs is 2/3. What is the probability that one or both occur?
3.	Suppose you are taking a multiple-choice test with c choices for each question. In answering a question on this test, the probability that you know the answer is p. If you don't know the answer, you choose one at random. What is the probability that you knew the answer to a question, given that you answered it correctly?
4.	Suppose that $P(A) = 0.4, P(B) = 0.3$ and $P((A \cup B)^C) = 0.42$. Are A and B independent?
5.	NO PROBLEM 5

6.	Suppose that $X \sim \text{Bin}(n, 0.5)$. Find the probability mass function of $Y = 2X$.
7.	Suppose that the cdf of X is given by: $$F(a) = \begin{cases} 0 & \text{for } a < 0 \\ \frac{1}{5} & \text{for } 0 \le a < 2 \\ \frac{2}{5} & \text{for } 2 \le a < 4 \\ 1 & \text{for } a \ge 4. \end{cases}$$ Determine the pmf of X.

8.	The probability of rain on any given day in June in Cambridge is 0.8. Assuming that the weather on each day is independent of the weather on other days, find the probability that it rains on at least 25 days in June.
9.	**(10 points)** Suppose that the random variable X is equal to the number of hits obtained by a certain baseball player in his next 3 at bats. If $P(X = 1) = 0.3$, $P(X = 2) = 0.2$ and $P(X = 0) = 3P(X = 3)$. Find $E(X)$.
10.	**(10 points)** If the distribution function of X is given by $$F(b) = \begin{cases} 0 & b < 0, \\ 1/3 & 0 \leq b < 1, \\ 2/3 & 1 \leq b < 2, \\ 1 & b \geq 2. \end{cases}$$ Calculate the probability mass function of X.

SOLUTIONS

1. 4 different resistors

2 identical capacitors

Total no. of designs $= P^8_4 \cdot C^4_2 = C^8_2 P^6_4$

$$= \frac{8!}{(8-4)!} \cdot \frac{4!}{2!(4-2)!} = \frac{8!}{2!2!} = \frac{8!}{4} = 10{,}080$$

2. Given

$P(A \cup B)' = 2/3 \leftarrow$ complement of $P(A \cup B)$

$\therefore P(A \cup B) = 1 - P(A \cup B)' = 1 - 2/3 = \underline{1/3}$

$P(A \cup B) \rightarrow$ probability of one or both occurring.

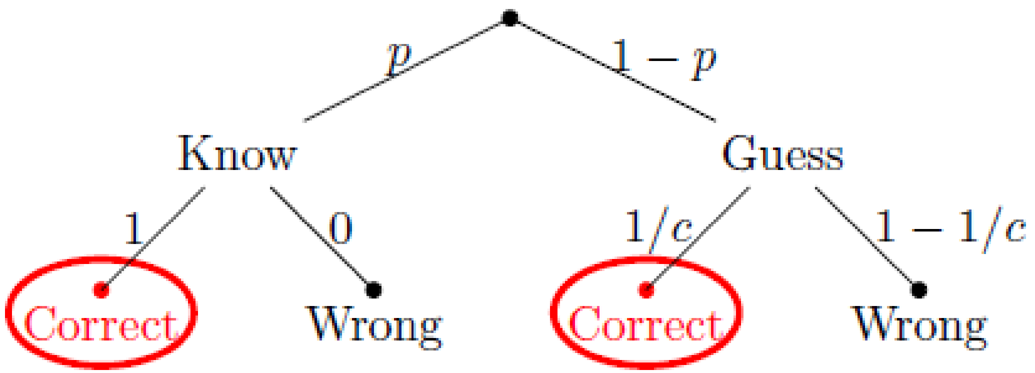
$\boxed{\text{OR CONSIDER ANOTHER WAY}}$

▷ complement of <u>neither</u> A or B occurring

IS ~~B~~ either A or B or both occur

SOLUTION – Review Problem 3

The following tree shows the setting



Let C be the event that you answer the question correctly. Let K be the event that you actually know the answer. The left circled node shows $P(K \cap C) = p$. Both circled nodes together show $P(C) = p + (1-p)/c$. So,

$$P(K|C) = \frac{P(K \cap C)}{P(C)} = \frac{p}{p + (1-p)/c}$$

$P(K|C) \rightarrow$ Probability that correct answer was known given that correct answer was selected

Or we could use the algebraic form of Bayes theorem and the law of total probability: Let G stand for the event that you're guessing. Then we have,
$P(C|K) = 1$, $P(K) = p$, $P(C) = P(C|K)P(K) + P(C|G)P(G) = p + (1-p)/c$. So,

$$P(K|C) = \frac{P(C|K)P(K)}{P(C)} = \frac{p}{p + (1-p)/c}$$

SOLUTION – Review Problem 4

We have $P(A \cup B) = 1 - 0.42 = 0.58$ and we know because of the inclusion-exclusion principle that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Thus,

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.4 + 0.3 - 0.58 = 0.12 = (0.4)(0.3) = P(A)P(B)$$

So A and B are independent.

No Problem 5

SOLUTION – Review Problem 6

For $y = 0, 2, 4, \ldots, 2n$,

$X \to \text{Binomial}(n, 0.5)$

$p \equiv$ probability of success $= 1/2$

$\therefore P(X = x_0) = C^n_{x_0} \left(\tfrac{1}{2}\right)^{x_0} \left(\tfrac{1}{2}\right)^{n-x_0}$

$= C^n_{x_0} (1/2)^n$

let $Y = 2X$, $X = Y/2$

$P(Y = y_0) = P(X = y_0/2)$

$= \boxed{C^n_{y_0/2} (1/2)^n}$

$$P(Y = y) = P(X = \frac{y}{2}) = \binom{n}{y/2} \left(\frac{1}{2}\right)^n .$$

SOLUTION – Review Problem 7

The jumps in the distribution function are at 0, 2, 4. The value of $p(a)$ at a jump is the height of the jump:

a	0	2	4
$p(a)$	1/5	1/5	3/5

SOLUTION – Review Problem 8

success → rains

$p(\text{success}) = 0.8$

— June has 30 days ($n = 30$)

$P(\text{rain on } x_0 \text{ days in June})$

$$= C^{\cancel{n}\,30}_{x_0}(0.8)^{x_0}(0.2)^{30-x_0}$$

$\therefore P(\text{rain on at least 25 days in June})$

$$= C^{30}_{25}(0.8)^{25}(0.2)^{5} + C^{30}_{26}(0.8)^{26}(0.2)^{4}$$

$$+ \ldots + C^{30}_{29}(0.8)^{29}(0.2)^{1} + C^{30}_{30}(0.8)^{30}(0.2)$$

$$= 0.172 + 0.133 + 0.0785 + 0.0337 + 0.00928 + 0.00124$$

$$\boxed{= 0.428}$$

SOLUTION – Review Problem 9

I. **(10 points)** Suppose that the random variable X is equal to the number of hits obtained by a certain baseball player in his next 3 at bats. If $P(X = 1) = 0.3$, $P(X = 2) = 0.2$ and $P(X = 0) = 3P(X = 3)$. Find $E(X)$.

Solution: Since the probabilities sum to 1, we have that

$$\begin{aligned} 1 &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) \\ &= P(X = 1) + P(X = 2) + 4P(X = 3) \\ &= 0.5 + 4P(X = 3) \end{aligned}$$

Hence,

$$P(X = 3) = 0.5/4 = 1/8 = 0.125. \text{ and } \; P(X = 0) = 3P(X = 3) = 3/8 = 0.375.$$

Now,

$$\begin{aligned} E(X) &= 0 \times P(X = 0) + 1 \times P(X = 1) + 2 \times P(X = 2) + 3 \times P(X = 3) \\ &= 1(0.3) + 2(0.2) + 3(0.125) = 1.075. \end{aligned}$$

SOLUTION – Review Problem 10

II. **(10 points)** If the distribution function of X is given by

$$F(b) = \begin{cases} 0 & b < 0, \\ 1/3 & 0 \leq b < 1, \\ 2/3 & 1 \leq b < 2, \\ 1 & b \geq 2. \end{cases}$$

Calculate the probability mass function of X.

Solution: Let us denote by p_X the probability mass function of X, then for every $i = 0, 1, 2$

$$p_X(i) = P(X = i) = F(i) - \lim_{b \to i-} F(b).$$

Hence,

$$\begin{aligned} p_X(0) &= F(0) - \lim_{b \to 0-} F(b) \\ &= 1/3 - 0 = 1/3. \end{aligned}$$

$$\begin{aligned} p_X(1) &= F(1) - \lim_{b \to 1-} F(b) \\ &= 2/3 - 1/3 = 1/3. \end{aligned}$$

$$\begin{aligned} p_X(2) &= F(2) - \lim_{b \to 2-} F(b) \\ &= 1 - 2/3 = 1/3. \end{aligned}$$

Hence the mass function is given by $p_X(0) = p_X(1) = p_X(2) = 1/3$.