Exercise 5.1

1.(a) $\|v\| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{14}$ $\|w\| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{126}$

$v^T w = 2\cdot 6 + 1\cdot 3 + 3\cdot 9 = 42$

$\cos\theta = \frac{42}{\sqrt{14}\cdot\sqrt{126}} = 1$ $\theta = 0$

(b) $\|v\| = \sqrt{2^2 + 3^2} = \sqrt{13}$ $\|w\| = \sqrt{3^2 + 2^2} = \sqrt{13}$

$v^T w = 2\cdot 3 + (-3)\cdot 2 = 0$

$\cos\theta = 0$ $\theta = \frac{\pi}{2}$

(c) $\|v\| = \sqrt{4^2 + 1^2} = \sqrt{17}$ $\|w\| = \sqrt{3^2 + 2^2} = \sqrt{13}$

$v^T w = 4\cdot 3 + 1\cdot 2 = 14$

$\cos\theta = \frac{14}{\sqrt{17}\cdot\sqrt{13}} = \frac{14\sqrt{221}}{221}$ $\theta = \arccos\left(\frac{14\sqrt{221}}{221}\right)$

(d) $\|v\| = \sqrt{2^2 + 1^2 + 3^2 + 1^2} = \sqrt{14}$ $\|w\| = \sqrt{1^2 + 3^2 + 2^2 + 4^2} = \sqrt{21}$

$v^T w = (-1)\cdot 1 + 1\cdot 3 + 3\cdot 2 + 1\cdot 4 = 8$

$\cos\theta = \frac{8}{\sqrt{14}\cdot\sqrt{21}} = \frac{4\sqrt{6}}{147}$ $\theta = \arccos\left(\frac{4\sqrt{6}}{147}\right)$

3.(a) $p = \frac{x^T y}{y^T y} y = \frac{3\cdot 1 + 4\cdot 0}{1\cdot 1 + 0\cdot 0}(1,0)^T = (3,0)^T$

$x - p = (3,4)^T - (3,0)^T = (0,4)^T$

$(x-p)^T p = 0$ $(x-p)\perp p$.

(b) $p = \frac{x^T y}{y^T y} y = \frac{3\cdot 1 + 5\cdot 1}{1\cdot 1 + 1\cdot 1}(1,1)^T = (4,4)^T$

$x - p = (3,5)^T - (4,4)^T = (-1,1)^T$

$(x-p)^T p = 0$ $(x-p)\perp p$.

(c) $p = \frac{x^T y}{y^T y} y = \frac{2\cdot 1 + 4\cdot 1 + 3\cdot 1}{1\cdot 1 + 1\cdot 1 + 1\cdot 1}(1,1,1)^T = (3,3,3)^T$

$x - p = (2,4,3)^T - (3,3,3)^T = (-1,1,0)^T$

$(x-p)^T p = 0$ $(x-p)\perp p$.

(d) $p = \frac{x^T y}{y^T y} y = \frac{2\cdot 1 - 3\cdot 2 - 4\cdot 1}{1\cdot 1 + 2\cdot 2 + 1\cdot 1}(1,2,-1)^T = (-2,-4,2)^T$

$x - p = (2,-3,4)^T - (-2,-4,2)^T = (4,-1,2)^T$

$(x-p)^T p = 0$ $(x-p)\perp p$.

4. $|\cos\theta| \leq 1$ $|x^T y| = \|x\|\cdot\|y\|\cdot|\cos\theta| \leq 6$

6. $y = -\frac{1}{2}x + b$ $(5,2)$

$\Rightarrow 2 = -\frac{5}{2} + b \Rightarrow b = \frac{9}{2}$

$\begin{cases} y = 2x + 1 \\ y = -\frac{1}{2}x + \frac{9}{2} \end{cases} \Rightarrow \begin{cases} x = \frac{7}{5} \\ y = \frac{19}{5} \end{cases} \Rightarrow Q\left(\frac{7}{5}, \frac{19}{5}\right)$

$w = (3,4)^T$ $v = (1,2)^T$

7. $Q^T = \frac{v^T w}{w^T w} w = \frac{1\cdot 3 + 2\cdot 4}{3\cdot 3 + 4\cdot 4}(3,4)^T = \left(\frac{33}{25}, \frac{44}{25}\right)^T$

$\left\| \left(\frac{33}{25}, \frac{44}{25}\right)^T - (1,2)^T \right\| = 0.4$

9. $P_1 = (2,3,1)$ $P_2 = (5,4,3)$ $P_3(3,4,4)$

$x = \vec{P_1 P_2} = (5,4,3)^T - (2,3,1)^T = (3,1,2)^T$ $y = \vec{P_1 P_3} = (3,4,4)^T - (2,3,1)^T = (1,1,3)^T$

$N = x \times y = (1,-7,2)^T$

$(x-2) - 7(y-3) + 2(z-1) = 0$

17.(a) $\cos\theta = \frac{x^T y}{\|x\|\cdot\|y\|} = -\frac{1}{2}$ $\theta = \frac{\pi}{3}$

(b) $\|x-y\| = \|(0,2,-6,3)\| = \sqrt{4+36+9} = 7$