Problem 7.1

[Difficulty: 2]

7.1 The slope of the free surface of a steady wave in one-dimensional flow in a shallow liquid layer is described by the equation

$$\frac{\partial h}{\partial x} = -\frac{u}{g}\frac{\partial u}{\partial x}$$

Use a length scale, L, and a velocity scale, V_0, to nondimensionalize this equation. Obtain the dimensionless groups that characterize this flow.

Given: Equation describing the slope of a steady wave in a shallow liquid layer

Find: Nondimensionalization for the equation using length scale L and velocity scale V_o. Obtain the dimensionless groups that characterize the flow.

Solution: To nondimensionalize the equation all lengths are divided by the reference length and all velocities are divided by the reference velocity. Denoting the nondimensional quantities by an asterisk:

$$h^* = \frac{h}{L} \qquad x^* = \frac{x}{L} \qquad u^* = \frac{u}{V_0}$$

Substituting into the governing equation: $$\frac{\partial(h^* L)}{\partial(x^* L)} = -\frac{u^* V_0}{g}\frac{\partial(u^* V_0)}{\partial(x^* L)} \qquad \frac{\partial h^*}{\partial x^*} = -\frac{V_0^2}{gL}u^*\frac{\partial u^*}{\partial x^*}$$

The dimensionless group is $\frac{V_0^{\,2}}{g \cdot L}$ which is the square of the Froude number.

Problem 7.2 [Difficulty: 2]

7.2 One-dimensional unsteady flow in a thin liquid layer is described by the equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial h}{\partial x}$$

Use a length scale, L, and a velocity scale, V_0, to nondimensionalize this equation. Obtain the dimensionless groups that characterize this flow.

Given: Equation describing one-dimensional unsteady flow in a thin liquid layer

Find: Nondimensionalization for the equation using length scale L and velocity scale Vo. Obtain the dimensionless groups that characterize the flow.

Solution: To nondimensionalize the equation all lengths are divided by the reference length and all velocities are divided by the reference velocity. Denoting the nondimensional quantities by an asterisk:

$$x^* = \frac{x}{L} \qquad h^* = \frac{h}{L} \qquad u^* = \frac{u}{V_0} \qquad t^* = \frac{t}{L/V_0}$$

Substituting into the governing equation: $\frac{\partial(u^*V_0)}{\partial(t^*L/V_0)} + u^*V_0\frac{\partial(u^*V_0)}{\partial(x^*L)} = -g\frac{\partial(h^*L)}{\partial(x^*L)}$ Simplifying this expression:

$$\frac{V_0^2}{L}\frac{\partial u^*}{\partial t^*} + \frac{V_0^2}{L}u^*\frac{\partial u^*}{\partial x^*} = -g\frac{\partial h^*}{\partial x^*}$$ Thus: $$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} = -\frac{gL}{V_0^2}\frac{\partial h^*}{\partial x^*}$$

The dimensionless group is $\frac{g\cdot L}{V_0^{\ 2}}$ which is the reciprocal of the square of the Froude number.

7.3 In atmospheric studies the motion of the earth's atmosphere can sometimes be modeled with the equation

$$\frac{D\vec{V}}{Dt} + 2\vec{\Omega} \times \vec{V} = -\frac{1}{\rho}\nabla p$$

where $\vec{V}$ is the large-scale velocity of the atmosphere across the Earth's surface, ∇p is the climatic pressure gradient, and $\vec{\Omega}$ is the Earth's angular velocity. What is the meaning of the term $\vec{\Omega} \times \vec{V}$? Use the pressure difference, Δp, and typical length scale, L (which could, for example, be the magnitude of, and distance between, an atmospheric high and low, respectively), to nondimensionalize this equation. Obtain the dimensionless groups that characterize this flow.

Given: Equations for modeling atmospheric motion

Find: Non-dimensionalized equation; Dimensionless groups

Solution:

Recall that the total acceleration is

$$\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}$$

Nondimensionalizing the velocity vector, pressure, angular velocity, spatial measure, and time, (using a typical velocity magnitude V and angular velocity magnitude Ω):

$$\vec{V}^* = \frac{\vec{V}}{V} \qquad p^* = \frac{p}{\Delta p} \qquad \vec{\Omega}^* = \frac{\vec{\Omega}}{\Omega} \qquad x^* = \frac{x}{L} \qquad t^* = t\frac{V}{L}$$

Hence

$$\vec{V} = V\vec{V}^* \qquad p = \Delta p\, p^* \qquad \vec{\Omega} = \Omega\vec{\Omega}^* \qquad x = L\, x^* \qquad t = \frac{L}{V}t^*$$

Substituting into the governing equation

$$V\frac{V}{L}\frac{\partial \vec{V}^*}{\partial t^*} + V\frac{V}{L}\vec{V}^* \cdot \nabla^* \vec{V}^* + 2\Omega V\,\vec{\Omega}^* \times \vec{V}^* = -\frac{1}{\rho}\frac{\Delta p}{L}\nabla p^*$$

The final dimensionless equation is

$$\frac{\partial \vec{V}^*}{\partial t^*} + \vec{V}^* \cdot \nabla^* \vec{V}^* + 2\left(\frac{\Omega L}{V}\right)\vec{\Omega}^* \times \vec{V} = -\frac{\Delta p}{\rho V^2}\nabla p^*$$

The dimensionless groups are

$$\frac{\Delta p}{\rho V^2} \qquad \frac{\Omega L}{V}$$

The second term on the left of the governing equation is the Coriolis force due to a rotating coordinate system. This is a very significant term in atmospheric studies, leading to such phenomena as geostrophic flow.

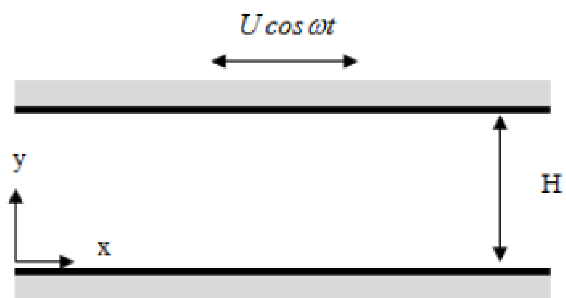
Problem 7.4

(Difficulty 2)

7.4 Fluid fills the space between two parallel plates. The differential equation that describes the instantaneous fluid velocity for unsteady flow with the fluid moving parallel to the walls is

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2}$$

The lower plate is stationary and the upper plate oscillates in the x-direction with a frequency ω and an amplitude in the plate velocity of U. Use the characteristic dimensions to normalize the differential equation and obtain the dimensionless groups that characterize the flow.



P 7.4

Find: Use the characteristic dimensions to normalize the equation and obtain the dimensionless groups.

Solution: This is unidirectional flow with oscillating boundary with characteristic dimensions U and H. We will normalize the variables using these characteristic dimensions as:

$$u^* = \frac{u}{U}$$

$$y^* = \frac{y}{H}$$

$$t^* = \frac{t}{\frac{H}{U}}$$

Substituting these dimensions into the differential equation we have:

$$\rho \frac{\partial (Uu^*)}{\partial \left(\frac{H}{U} t^*\right)} = \mu \frac{\partial^2 (Uu^*)}{\partial (Hy^*)^2}$$

$$\frac{\partial u^*}{\partial t^*} = \frac{\mu}{\rho U H}\frac{\partial^2 u^*}{\partial y^{*2}}$$

On the boundary we have:

At $y = 0, u = 0$ so we have:

$$u^* = 0 \text{ at } y^* = 0$$

At $y = H, u = U \cos \omega t$ so we have:

$$u^* = \cos \omega t \text{ at } y^* = 1$$

We define the Reynolds number as:

$$Re = \frac{\rho U H}{\mu}$$

The differential equation becomes:

$$\frac{\partial u^*}{\partial t^*} = \frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}}$$

The fluid flow is governed by the following two dimensionless group:

$$Re = \frac{\rho U H}{\mu}$$

$$\Pi = \cos \omega t$$

It can be written as:

$$\frac{u}{U} = f\left(\frac{\rho U H}{\mu}, \cos \omega t\right)$$

Problem 7.5

[Difficulty: 4]

7.5 By using order of magnitude analysis, the continuity and Navier–Stokes equations can be simplified to the Prandtl boundary-layer equations. For steady, incompressible, and two-dimensional flow, neglecting gravity, the result is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

Use L and V_0 as characteristic length and velocity, respectively. Nondimensionalize these equations and identify the similarity parameters that result.

Given: The Prandtl boundary-layer equations for steady, incompressible, two-dimensional flow neglecting gravity

Find: Nondimensionalization for the equation using length scale L and velocity scale V_0. Obtain the dimensionless groups that characterize the flow.

Solution: To nondimensionalize the equation all lengths are divided by the reference length and all velocities are divided by the reference velocity. Denoting the nondimensional quantities by an asterisk:

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L} \qquad u^* = \frac{u}{V_0} \qquad v^* = \frac{v}{V_0}$$

Substituting into the continuity equation: $\dfrac{\partial(u^* V_0)}{\partial(x^* L)} + \dfrac{\partial(v^* V_0)}{\partial(y^* L)} = 0$ Simplifying this expression: $\dfrac{V_0}{L}\dfrac{\partial u^*}{\partial x^*} + \dfrac{V_0}{L}\dfrac{\partial v^*}{\partial y^*} = 0$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

We expand out the second derivative in the momentum equation by writing it as the derivative of the derivative. Upon substitution:

$$u^* V_0 \frac{\partial(u^* V_0)}{\partial(x^* L)} + v^* V_0 \frac{\partial(u^* V_0)}{\partial(y^* L)} = -\frac{1}{\rho}\frac{\partial p}{\partial(x^* L)} + \nu\frac{\partial}{\partial(y^* L)}\frac{\partial(u^* V_0)}{\partial(y^* L)}$$

Simplifying this expression yields:

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho V_0^2}\frac{\partial p}{\partial x^*} + \frac{\nu}{V_0 L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Now every term in this equation has been non-dimensionalized except the pressure gradient. We define a dimensionless pressure as:

$$p^* = \frac{p}{\rho V_0^2}$$

Substituting this into the momentum equation: $u^*\dfrac{\partial u^*}{\partial x^*} + v^*\dfrac{\partial u^*}{\partial y^*} = -\dfrac{1}{\rho V_0^2}\dfrac{\partial(p^*\rho V_0^2)}{\partial x^*} + \dfrac{\nu}{V_0 L}\dfrac{\partial^2 u^*}{\partial y^{*2}}$

Simplifying this expression yields:

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{\nu}{V_0 L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

The dimensionless group is $\dfrac{\nu}{V_0 \cdot L}$ which is the reciprocal of the Reynolds number.

Problem 7.6

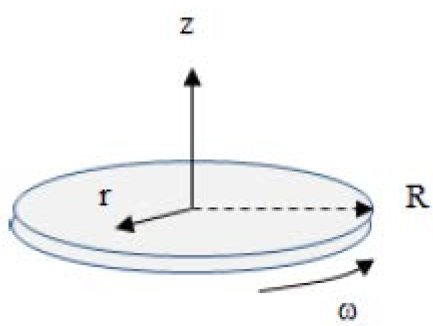
(Difficulty 2)

7.6 Consider a disk of radius R rotating in an incompressible fluid at a speed ω. The equations that describe the boundary layer on the disk are:

$$\frac{1}{r}\left(\frac{\partial (r v_r)}{\partial r}\right) + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(v_r \frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = \mu \frac{\partial^2 v_r}{\partial z^2}$$

Use the characteristic dimensions to normalize the differential equation and obtain the dimensionless groups that characterize the flow.



P 7.6

Find: Use the characteristic dimensions to normalize the equation and obtain the dimensionless groups.

Solution: This is unidirectional flow with oscillating boundary. We will normalize the variable with the following characteristic dimensions. Because there is no characteristic velocity in the problem, we will use the product ωR as the characteristic velocity:

$$v_r^* = \frac{v_r}{\omega R}$$

$$v_\theta^* = \frac{v_\theta}{\omega R}$$

$$v_z^* = \frac{v_z}{\omega R}$$

$$r^* = \frac{r}{R}$$

$$z^* = \frac{z}{R}$$

Substitute these equations in to the differential equation we have:

For the continuity equation:

$$\frac{1}{R}\frac{1}{r^*}\left(\frac{\partial(Rr^*\omega Rv_r^*)}{\partial Rr^*}\right)+\frac{\partial\omega Rv_z^*}{\partial Rz^*}=0$$

Thus

$$\omega\frac{1}{r^*}\left(\frac{\partial(r^*v_r^*)}{\partial r^*}\right)+\omega\frac{\partial v_z^*}{\partial z^*}=0$$

Or

$$\frac{1}{r^*}\left(\frac{\partial(r^*v_r^*)}{\partial r^*}\right)+\frac{\partial v_z^*}{\partial z^*}=0$$

For the r-momentum equation:

$$\rho\left(\omega Rv_r^*\frac{\partial\omega Rv_r^*}{\partial Rr^*}-\frac{(\omega Rv_\theta^*)^2}{Rr^*}+\omega Rv_z^*\frac{\partial\omega Rv_r^*}{\partial Rz^*}\right)=\mu\frac{\partial^2\omega Rv_r^*}{\partial R^2z^{*2}}$$

$$\rho\left(\omega^2Rv_r^*\frac{\partial v_r^*}{\partial r^*}-\omega^2R\frac{(v_\theta^*)^2}{r^*}+\omega^2Rv_z^*\frac{\partial v_r^*}{\partial z^*}\right)=\frac{\mu\omega}{R}\frac{\partial^2v_r^*}{\partial z^{*2}}$$

$$\rho\omega^2R\left(v_r^*\frac{\partial v_r^*}{\partial r^*}-\frac{(v_\theta^*)^2}{r^*}+v_z^*\frac{\partial v_r^*}{\partial z^*}\right)=\frac{\mu\omega}{R}\frac{\partial^2v_r^*}{\partial z^{*2}}$$

$$\left(v_r^*\frac{\partial v_r^*}{\partial r^*}-\frac{(v_\theta^*)^2}{r^*}+v_z^*\frac{\partial v_r^*}{\partial z^*}\right)=\frac{\mu}{\rho\omega R^2}\frac{\partial^2v_r^*}{\partial z^{*2}}$$

Defining the Reynolds number as:

$$Re=\frac{\rho\omega R^2}{\mu}$$

The differential equation becomes:

$$\left(v_r^*\frac{\partial v_r^*}{\partial r^*}-\frac{(v_\theta^*)^2}{r^*}+v_z^*\frac{\partial v_r^*}{\partial z^*}\right)=\frac{1}{Re}\frac{\partial^2v_r^*}{\partial z^{*2}}$$

The Reynolds number characterizes the fluid flow as:

$$\frac{\vec{V}}{\omega R}=f\left(\frac{\rho\omega R^2}{\mu}\right)$$

Problem 7.7

[Difficulty: 2]

7.7 An unsteady, two-dimensional, compressible, inviscid flow can be described by the equation

$$\frac{\partial^2\psi}{\partial t^2}+\frac{\partial}{\partial t}(u^2+v^2)+(u^2-c^2)\frac{\partial^2\psi}{\partial x^2}+(v^2-c^2)\frac{\partial^2\psi}{\partial y^2}+2uv\frac{\partial^2\psi}{\partial x\partial y}=0$$

where ψ is the stream function, u and v are the x and y components of velocity, respectively, c is the local speed of sound, and t is the time. Using L as a characteristic length and c_0 (the speed of sound at the stagnation point) to nondimensionalize this equation, obtain the dimensionless groups that characterize the equation.

Given: Equation for unsteady, 2D compressible, inviscid flow

Find: Dimensionless groups

Solution:

Denoting nondimensional quantities by an asterisk

$$x^*=\frac{x}{L}\qquad y^*=\frac{y}{L}\qquad u^*=\frac{u}{c_0}\qquad v^*=\frac{v}{c_0}\qquad c^*=\frac{c}{c_0}\qquad t^*=\frac{t\,c_0}{L}\qquad \psi^*=\frac{\psi}{L\,c_0}$$

Note that the stream function indicates volume flow rate/unit depth!

Hence

$$x=L\,x^*\qquad y=L\,y^*\qquad u=c_0\,u^*\qquad v=c_0\,v^*\qquad c=c_0\,c^*\qquad t=\frac{L\,t^*}{c_0}\qquad \psi=L\,c_0\,\psi^*$$

Substituting into the governing equation

$$\left(\frac{c_0^3}{L}\right)\frac{\partial^2\psi^*}{\partial t^{*2}}+\left(\frac{c_0^3}{L}\right)\frac{\partial\left(u^{*2}+v^{*2}\right)}{\partial t}+\left(\frac{c_0^3}{L}\right)\left(u^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial x^{*2}}+\left(\frac{c_0^3}{L}\right)\left(v^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial y^{*2}}+\left(\frac{c_0^3}{L}\right)2u^*v^*\frac{\partial^2\psi^*}{\partial x^*\partial y^*}=0$$

The final dimensionless equation is

$$\frac{\partial^2\psi^*}{\partial t^{*2}}+\frac{\partial\left(u^{*2}+v^{*2}\right)}{\partial t}+\left(u^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial x^{*2}}+\left(v^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial y^{*2}}+2u^*v^*\frac{\partial^2\psi^*}{\partial x^*\partial y^*}=0$$

No dimensionless group is needed for this equation!

Problem 7.8

[Difficulty: 2]

7.8 Experiments show that the pressure drop for flow through an orifice plate of diameter d mounted in a length of pipe of diameter D may be expressed as $\Delta p = p_1 - p_2 = f(\rho, \mu, \overline{V}, d, D)$. You are asked to organize some experimental data. Obtain the resulting dimensionless parameters.

Given: Functional relationship between pressure drop through orifice plate and physical parameters

Find: Appropriate dimensionless parameters

Solution: We will use the Buckingham pi-theorem.

1 $\quad \Delta p \quad \rho \quad \mu \quad V \quad D \quad d \qquad$ n = 6 parameters

2 Select primary dimensions M, L, t:

3 $\quad \Delta p \quad \rho \quad \mu \quad V \quad D \quad d$

$$\frac{M}{Lt^2} \quad \frac{M}{L^3} \quad \frac{M}{Lt} \quad \frac{L}{t} \quad L \quad L$$

r = 3 dimensions

4 $\quad \rho \quad V \quad D \qquad$ m = r = 3 repeating parameters

5 We have n - m = 3 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = \Delta p \cdot \rho^a \cdot V^b \cdot D^c \quad \text{Thus:} \quad \left(\frac{M}{L \cdot t^2}\right) \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3 \cdot a + b + c = 0$

t: $-2 - b = 0$

The solution to this system is:

$a = -1 \quad b = -2 \quad c = 0$

$$\Pi_1 = \frac{\Delta p}{\rho \cdot V^2}$$

Check using F, L, t primary dimensions: $\frac{F}{L^2} \cdot \frac{L^4}{F \cdot t^2} \cdot \frac{t^2}{L^2} = 1$ Checks out.

$$\Pi_2 = \mu \cdot \rho^a \cdot V^b \cdot D^c \quad \text{Thus:} \quad \left(\frac{M}{L \cdot t}\right) \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3 \cdot a + b + c = 0$

t: $-1 - b = 0$

The solution to this system is:

$a = -1 \quad b = -1 \quad c = -1$

$$\Pi_2 = \frac{\mu}{\rho \cdot V \cdot D}$$

(This is the Reynolds number, so it checks out)

$\Pi_3 = d \cdot \rho^a \cdot V^b \cdot D^c$ Thus: $L \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $a = 0$

L: $1 + c = 0$

t: $b = 0$

The solution to this system is:

$a = 0 \quad b = 0 \quad c = -1$

$\Pi_3 = \frac{d}{D}$

(This checks out)

Problem 7.9

[Difficulty: 2]

7.9 At very low speeds, the drag on an object is independent of fluid density. Thus the drag force, F, on a small sphere is a function only of speed, V, fluid viscosity, μ, and sphere diameter, D. Use dimensional analysis to determine how the drag force F depends on the speed V.

Given: At low speeds, drag F on a sphere is only dependent upon speed V, viscosity μ, and diameter D

Find: Appropriate dimensionless parameters

Solution: We will use the Buckingham pi-theorem.

1 $\quad F \quad V \quad \mu \quad D \qquad$ n = 4 parameters

2 Select primary dimensions M, L, t:

3 $\quad F \quad V \quad \mu \quad D \qquad$ n = 4 parameters

$$\frac{ML}{t^2} \quad \frac{L}{t} \quad \frac{M}{Lt} \quad L \qquad \text{r = 3 dimensions}$$

4 $\quad V \quad \mu \quad D \qquad$ m = r = 3 repeating parameters

5 We have n - m = 1 dimensionless group. Setting up a dimensional equation:

$$\Pi_1 = F\cdot V^a\cdot\mu^b\cdot D^c \qquad \text{Thus:} \quad \left(\frac{M\cdot L}{t^2}\right)\cdot\left(\frac{L}{t}\right)^a\cdot\left(\frac{M}{L\cdot t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents:

M: $\quad 1 + b = 0$

L: $\quad 1 + a - b + c = 0$

t: $\quad -2 - a - b = 0$

The solution to this system is:

$a = -1 \quad b = -1 \quad c = -1$

$$\Pi_1 = \frac{F}{\mu\cdot V\cdot D}$$

Check using F, L, t primary dimensions: $\quad F\cdot\frac{t}{L}\cdot\frac{L^2}{F\cdot t}\cdot\frac{1}{L} = 1 \quad$ Checks out.

Since the procedure produces only one dimensionless group, it must be a constant. Therefore: $\quad \frac{F}{\mu\cdot V\cdot D} = \text{constant}$

Problem 7.10

[Difficulty: 2]

7.10 We saw in Chapter 3 that the buoyant force, F_B, on a body submerged in a fluid is directly proportional to the specific weight of the fluid, γ. Demonstrate this using dimensional analysis, by starting with the buoyant force as a function of the volume of the body and the specific weight of the fluid.

Given: Functional relationship between buoyant force of a fluid and physical parameters

Find: Buoyant force is proportional to the specific weight as demonstrated in Chapter 3.

Solution: We will use the Buckingham pi-theorem.

1 $F_B \quad V \quad \gamma$ $\qquad$ n = 3 parameters

2 Select primary dimensions F, L, t:

3 $F_B \quad V \quad \gamma$

$F \quad L^3 \quad \dfrac{F}{L^3}$ $\qquad$ r = 2 dimensions

4 $V \quad \gamma$ $\qquad$ m = r = 2 repeating parameters

5 We have n - m = 1 dimensionless group. Setting up dimensional equations:

$$\Pi_1 = F_B \cdot V^a \cdot \gamma^b \qquad \text{Thus:} \qquad F \cdot \left(L^3\right)^a \cdot \left(\frac{F}{L^3}\right)^b = F^0 \cdot L^0$$

Summing exponents:

F: $1 + b = 0$

L: $3 \cdot a - 3 \cdot b = 0$

The solution to this system is: $a = -1 \quad b = -1$

$$\Pi_1 = \frac{F_B}{V \cdot \gamma}$$

Check using M, L, t dimensions: $\dfrac{M \cdot L}{t^2} \cdot \dfrac{1}{L^3} \cdot \dfrac{t^2 \cdot L^2}{M} = 1$

The functional relationship is: $\Pi_1 = C \quad \dfrac{F_B}{V \cdot \gamma} = C$ $\quad$ Solving for the buoyant force: $F_B = C \cdot V \cdot \gamma$ $\quad$ Buoyant force is proportional to γ (Q.E.D.)

Problem 7.11

(Difficulty 1)

7.11 Assume that the velocity acquired by a body falling from rest (without resistance) depends on weight of body, acceleration due to gravity, and distance of fall. Prove by dimensional analysis that $V = C\sqrt{g_n h}$ and is thus independent of the weight of the body.

Find: Prove V is independent of the weight of the body.

Solution:

(1) There are four dimensional parameters: V, W, g_n, h so $n = 4$

(2) Select primary dimensions $M, L\ and\ t$.

(3) We have the following relations:

	V	C	W	g_n	h
M	0	0	1	0	0
L	1	0	1	1	1
t	-1	0	-2	-2	0

All three primary dimensions are represented so r = 3. The number of repeating variables will then be m = r = 3.

The number of groups will be n – r = 4 – 3 = 1. All of the dimensional parameters will be combined into one group.

The group is then assumed to be of the form

$$\Pi = V\, W^a g_n^b h^c$$

So we get:

$$\Pi = \frac{L}{t}\left(\frac{ML}{t^2}\right)^a \left(\frac{L}{t^2}\right)^b (L)^c = M^0 L^0 t^0$$

We evaluate the coefficients by setting the coefficients of each dimension on the left hand side equal to the coefficient for the dimension on the right hand side (which is 0).

$$a = 0$$
$$1 + a + b + c = 0$$
$$-1 - 2a - 2b = 0$$

Solving for the values of b and c

$$b = -\frac{1}{2}$$
$$c = -\frac{1}{2}$$

Thus

$$\Pi = V g_n^{-0.5} h^{-0.5} = \frac{V}{\sqrt{g_n h}}$$

Because there is only one group, it is a constant and we then have

$$V = C\sqrt{g_n h}$$

The coefficient a for the only variable that contains mass (the weight W) is zero, so it is clear that V is independent of the weight of the body.

Problem 7.12

(Difficulty 2)

7.12 Derive by dimensional analysis an expression for the local velocity in established pipe flow through a smooth pipe if this velocity depends only on mean velocity, pipe diameter, distance from pipe wall, and density and viscosity of the fluid.

Find: The appropriate dimensionless groups.

Solution:

(1) There are 6 dimensional parameters : V, V_m, d, y, ρ, μ so $n = 6$

(2) Select primary dimensions $M, L\ and\ t$.

(3) We have the following relation:

	V	V_m	d	y	ρ	μ
M	0	0	0	0	1	1
L	1	1	1	1	-3	-1
t	-1	-1	0	0	0	-1

We have all three dimensions so r = m = 3. We need 3 repeating parameters that include all of the dimensions and we pick the following three repeating parameters:

$$V_m \quad d \quad \rho$$

The number of dimensionless groups will be n – m = 6 – 3 = 3 dimensionless groups: For the first group we have combine the repeating variables with the dimensional parameter V

$$\Pi_1 = V_m{}^a d^b \rho^c V = \left(\frac{L}{t}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c \left(\frac{L}{t}\right) = M^0 L^0 t^0$$

Equating the exponents for each of the dimensions

$$c = 0$$
$$a + b - 3c + 1 = 0$$
$$-a - 1 = 0$$

Solving for the exponents

$$a = -1$$
$$b = 0$$

So the first group is

$$\Pi_1 = \frac{V}{V_m}$$

For the second group we combine with the dimensional parameter y:

$$\Pi_2 = {V_m}^d d^e \rho^f y = \left(\frac{L}{t}\right)^d (L)^e \left(\frac{M}{L^3}\right)^f (L) = M^0 L^0 t^0$$
$$f = 0$$
$$d + e - 3f + 1 = 0$$

Solving for the exponents

$$-d = 0$$
$$e = -1$$

The second group is

$$\Pi_2 = \frac{y}{d}$$

Similarly, for the third group, combining with the dimensional parameter μ

$$\Pi_3 = {V_m}^g d^h \rho^i \mu = \left(\frac{L}{t}\right)^g (L)^h \left(\frac{M}{L^3}\right)^i \left(\frac{M}{L\ t}\right) = M^0 L^0 t^0$$

The exponents are then related as

$$i + 1 = 0$$
$$g + h - 3i - 1 = 0$$
$$-g - 1 = 0$$

Solving for the exponents

$$i = -1$$
$$g = -1$$
$$h = -1$$

The third group is then

$$\Pi_3 = \frac{\mu}{V_m d \rho}$$

The functional relation among the groups is then

$$\frac{V}{V_m} = f\left(\frac{y}{d}, \frac{\mu}{V_m d \rho}\right)$$

Problem 7.13

(Difficulty 2)

7.13 The speed of shallow water waves in the ocean ($e.g.$ $seismic\ sea\ waves\ or\ tsunamis$) depends only on the still water depth and the acceleration due to gravity. Derive an expression for wave speed.

Find: The appropriate dimensionless groups.

Solution:

(1) There are three dimensional parameters V, d, g so $n = 3$

(2) Select primary dimensions $M, L\ and\ t$.

(3) We have the following relation:

	V	d	g
M	0	0	0
L	1	1	1
t	-1	0	-2

Mass does not appear in any of the parameters so we need only 2 repeating variables

$$r = 2$$

We pick up the following two repeating parameters that include all of the dimensions:

$$d \quad g$$

The number of dimensionless groups is n – m = n – r = 3 – 2 = 1
This group is then

$$\Pi_1 = d^a g^b V = L^a \left(\frac{L}{t^2}\right)^b \left(\frac{L}{t}\right) = M^0 L^0 t^0$$

Equating the coefficients of the dimensions. There is no exponent for the mass

$$a + b + 1 = 0$$
$$-2b - 1 = 0$$

Solving for the values

$$b = -\frac{1}{2}$$
$$a = -\frac{1}{2}$$

The group is

$$\Pi_1 = \frac{V}{\sqrt{dg}}$$

There is only one group and so it is a constant. The velocity is then

$$V = C\sqrt{d\,g}$$

Problem 7.14 [Difficulty: 2]

7.14 The speed, V, of a free-surface wave in shallow liquid is a function of depth, D, density, ρ, gravity, g, and surface tension, σ. Use dimensional analysis to find the functional dependence of V on the other variables. Express V in the simplest form possible.

Given: That speed of shallow waves depends on depth, density, gravity and surface tension

Find: Dimensionless groups; Simplest form of V

Solution:

Apply the Buckingham Π procedure

① $V \quad D \quad \rho \quad g \quad \sigma$ $\qquad n = 5$ parameters

② Select primary dimensions M, L, t

③ $$\begin{Bmatrix} V & D & \rho & g & \sigma \\ \frac{L}{t} & L & \frac{M}{L^3} & \frac{L}{t^2} & \frac{M}{t^2} \end{Bmatrix}$$ $r = 3$ primary dimensions

④ $g \quad \rho \quad D$ $\qquad m = r = 3$ repeat parameters

⑤ Then $n - m = 2$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = g^a \rho^b D^c V = \left(\frac{L}{t^2}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \frac{L}{t} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{ll|l} M: & b = 0 & b = 0 \\ L: & a - 3b + c + 1 = 0 & c = -\frac{1}{2} \\ t: & -2a - 1 = 0 & a = -\frac{1}{2} \end{array}$$

Hence $\Pi_1 = \dfrac{V}{\sqrt{gD}}$

$$\Pi_2 = g^a \rho^b D^c \sigma = \left(\frac{L}{t^2}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \frac{M}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{ll|l} M: & b + 1 = 0 & b = -1 \\ L: & a - 3b + c = 0 & c = -2 \\ t: & -2a - 2 = 0 & a = -1 \end{array}$$

Hence $\Pi_2 = \dfrac{\sigma}{g\rho D^2}$

⑥ Check using F, L, t as primary dimensions

$$\Pi_1 = \frac{\frac{L}{t}}{\left(\frac{L}{t^2}L\right)^{\frac{1}{2}}} = [1] \qquad \Pi_2 = \frac{\frac{F}{L}}{\frac{L}{t^2}\frac{Ft^2}{L^4}L^2} = [1]$$

The relation between drag force speed V is

$$\Pi_1 = f(\Pi_2) \qquad \frac{V}{\sqrt{gD}} = f\left(\frac{\sigma}{g\rho D^2}\right) \qquad V = \sqrt{gD}\, f\left(\frac{\sigma}{g\rho D^2}\right)$$

Problem 7.15 [Difficulty: 2]

7.15 The boundary-layer thickness, δ, on a smooth flat plate in an incompressible flow without pressure gradients depends on the freestream speed, U, the fluid density, ρ, the fluid viscosity, μ, and the distance from the leading edge of the plate, x. Express these variables in dimensionless form.

Given: Functional relationship between boundary layer thickness and physical parameters

Find: Appropriate dimensionless parameters

Solution: We will use the Buckingham pi-theorem.

1 $\delta \quad x \quad \rho \quad \mu \quad U$ n = 5 parameters

2 Select primary dimensions M, L, t:

3 $\delta \quad x \quad \rho \quad \mu \quad U$

$L \quad L \quad \dfrac{M}{L^3} \quad \dfrac{M}{Lt} \quad \dfrac{L}{t}$ r = 3 dimensions

4 $\rho \quad x \quad U$ m = r = 3 repeating parameters

5 We have n - m = 2 dimensionless groups. Setting up dimensional equations:

$\Pi_1 = \delta \cdot \rho^a \cdot x^b \cdot U^c$ Thus: $L \cdot \left(\dfrac{M}{L^3}\right)^a \cdot L^b \cdot \left(\dfrac{L}{t}\right)^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $0 + a = 0$

L: $1 - 3 \cdot a + b + c = 0$

t: $0 - c = 0$

The solution to this system is:

$a = 0 \quad b = -1 \quad c = 0$

$\Pi_1 = \dfrac{\delta}{x}$

Check using F, L, t dimensions: $(L) \cdot \left(\dfrac{1}{L}\right) = 1$

$\Pi_2 = \mu \cdot \rho^a \cdot x^b \cdot U^c$ Thus: $\left(\dfrac{M}{L \cdot t}\right) \cdot \left(\dfrac{M}{L^3}\right)^a \cdot L^b \cdot \left(\dfrac{L}{t}\right)^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3 \cdot a + b + c = 0$

t: $-1 - c = 0$

The solution to this system is:

$a = -1 \quad b = -1 \quad c = -1$

$\Pi_2 = \dfrac{\mu}{\rho \cdot x \cdot U}$

Check using F, L, t dimensions: $\left(\dfrac{F \cdot t}{L^2}\right) \cdot \left(\dfrac{L^4}{F \cdot t^2}\right) \cdot \left(\dfrac{1}{L}\right) \cdot \left(\dfrac{t^2}{L}\right) = 1$

The functional relationship is:

$\Pi_1 = f(\Pi_2)$

Problem 7.16

[Difficulty: 2]

7.16 The speed, V, of a free-surface gravity wave in deep water is a function of wavelength, λ, depth, D, density, ρ, and acceleration of gravity, g. Use dimensional analysis to find the functional dependence of V on the other variables. Express V in the simplest form possible.

Given: Functional relationship between the speed of a free-surface gravity wave in deep water and physical parameters

Find: The dependence of the speed on the other variables

Solution: We will use the Buckingham pi-theorem.

1 $V \quad \lambda \quad D \quad \rho \quad g$ $\qquad n = 5$ parameters

2 Select primary dimensions M, L, t:

3

V	λ	D	ρ	g	
$\frac{L}{t}$	L	L	$\frac{M}{L^3}$	$\frac{L}{t^2}$	$r = 3$ dimensions

4 $D \quad \rho \quad g$ $\qquad m = r = 3$ repeating parameters

5 We have n - m = 2 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = V \cdot D^a \cdot \rho^b \cdot g^c \qquad \text{Thus:} \qquad \frac{L}{t} \cdot L^a \cdot \left(\frac{M}{L^3}\right)^b \cdot \left(\frac{L}{t^2}\right)^c = M^0 \cdot L^0 \cdot t^0$$

Summing exponents:

M: $b = 0$

L: $1 + a - 3 \cdot b + c = 0$

t: $-1 - 2 \cdot c = 0$

The solution to this system is: $a = -\frac{1}{2} \quad b = 0 \quad c = -\frac{1}{2}$

$$\Pi_1 = \frac{V}{\sqrt{g \cdot D}}$$

Check using F, L, t dimensions: $\left(\frac{L}{t}\right) \cdot \left(\frac{t}{L}\right) = 1$

$$\Pi_2 = \lambda \cdot D^a \cdot \rho^b \cdot g^c \qquad \text{Thus:} \qquad L \cdot L^a \cdot \left(\frac{M}{L^3}\right)^b \cdot \left(\frac{L}{t^2}\right)^c = M^0 \cdot L^0 \cdot t^0$$

Summing exponents:

M: $b = 0$

L: $1 + a - 3 \cdot b + c = 0$

t: $-2 \cdot c = 0$

The solution to this system is: $a = -1 \quad b = 0 \quad c = 0$

$$\Pi_2 = \frac{\lambda}{D}$$

Check using F, L, t dimensions: $L \cdot \frac{1}{L} = 1$

The functional relationship is: $\Pi_1 = f(\Pi_2)$ $\qquad \frac{V}{\sqrt{g \cdot D}} = f\left(\frac{\lambda}{D}\right)$ $\qquad$ Therefore the velocity is: $\qquad V = \sqrt{g \cdot D} \cdot f\left(\frac{\lambda}{D}\right)$

Problem 7.17

(Difficulty 2)

7.17 Derive an expression for the velocity of very small ripples on the surface of a liquid if this velocity depends only on ripple length and density and surface tension of the liquid.

Find: The appropriate dimensionless groups.

Solution:

(1) The dimensional parameters are V, L, ρ, σ so $n = 4$

(2) Select primary dimensions $M, L\ and\ t$.

(3) We have the following relation:

	V	L	ρ	σ
M	0	0	1	1
L	1	1	-3	0
t	-1	0	0	-2

All three dimensions appear so $r = 3$.

We need 3 repeating parameters that include all of the dimensions. We pick up the following three repeating parameters:

$$L \quad \rho \quad \sigma$$

The number of groups is n – m = n – r = 4 – 3 = 1
Thus this group is

$$\Pi_1 = L^a \rho^b \sigma^c V = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{t^2}\right)^c \left(\frac{L}{t}\right) = M^0 L^0 t^0$$

Equating the coefficients of the dimensions

$$b + c = 0$$
$$a - 3b + 1 = 0$$
$$-2c - 1 = 0$$

Solving for the coefficients

$$c = -\frac{1}{2}$$
$$b = \frac{1}{2}$$
$$a = \frac{1}{2}$$

The group is then

$$\Pi_1 = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}$$

Since there is only one group, it is a constant and then the velocity is given by

$$V = C\sqrt{\frac{\sigma}{\rho L}}$$

Problem 7.18

(Difficulty 3)

7.18 Derive an expression for the axial thrust exerted by a propeller if the thrust depends only on forward speed, angular speed, size, and viscosity and density of the fluid. How would the expression change if gravity were a relevant variable in the case of a ship propeller?

Find: The appropriate dimensionless groups.

Solution:

(1) There are 6 dimensional parameters $F, V, \omega, d, \mu, \rho$, g so $n = 7$

(2) Select primary dimensions $M, L\ and\ t$.

(3) We have the following relation:

	F	V	ω	d	μ	ρ	g
M	1	0	0	0	1	1	0
L	1	1	0	1	−1	−3	1
t	−2	−1	−1	0	−1	0	−2

All dimensions are present so the number of repeating variables is $r = 3$. We pick the following three repeating parameters that include all of the dimensions:

$$V \quad d \quad \rho$$

We will have n – m = n – r = 7 – 3 = 4 nondimensional groups if we include gravity. If we do not include gravity, we will have only three groups. For the first group we will combine the force F with the repeating variables

$$\Pi_1 = V^a d^b \rho^c F = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{ML}{t^2} = M^0 L^0 t^0$$

Equating the exponents of the dimensions

$$c + 1 = 0$$
$$a + b - 3c + 1 = 0$$

Solving for the values of the exponents

$$c = -1$$
$$-a - 2 = 0$$
$$a = -2$$
$$b = -2$$

The first group is then

$$\Pi_1 = \frac{F}{\rho V^2 d^2}$$

We combine the viscosity with the repeating variables for the second group:

$$\Pi_2 = V^d d^e \rho^f \mu = \left(\frac{L}{t}\right)^d L^e \left(\frac{M}{L^3}\right)^f \left(\frac{M}{Lt}\right) = M^0 L^0 t^0$$

Solving for the exponents

$$f + 1 = 0$$
$$f = -1$$
$$d + e - 3f - 1 = 0$$
$$-d - 1 = 0$$
$$d = -1$$
$$e = -1$$

$$\Pi_2 = \frac{\mu}{\rho V d}$$

Similarly for the third group that combines ω

$$\Pi_3 = V^g d^h \rho^i \omega = \left(\frac{L}{t}\right)^g L^h \left(\frac{M}{L^3}\right)^i \frac{1}{t} = M^0 L^0 t^0$$

Solving for the exponents

$$i = 0$$
$$g + h - 3i = 0$$
$$-g - 1 = 0$$
$$g = -1$$
$$h = 1$$
$$\Pi_3 = \frac{\omega d}{V}$$

For the three groups that do not include gravity, we have the relation

$$\frac{F}{\rho V^2 d^2} = f\left(\frac{\omega d}{V}, \frac{\mu}{\rho V d}\right)$$

For the fourth group with the addition of gravity g, we have:

$$\Pi_3 = V^j d^k \rho^l g = \left(\frac{L}{t}\right)^j L^k \left(\frac{M}{L^3}\right)^l \left(\frac{L}{t^2}\right) = M^0 L^0 t^0$$

The exponents are then

$$l = 0$$
$$j + k - 3l + 1 = 0$$
$$-j - 2 = 0$$
$$j = -2$$
$$k = 1$$
$$\Pi_4 = \frac{gd}{V^2}$$

So if g is also the variable, the expression for thrust becomes:

$$\frac{F}{\rho V^2 d^2} = f\left(\frac{\omega d}{V}, \frac{\mu}{\rho V d}, \frac{gd}{V^2}\right)$$

Problem 7.19

(Difficulty 2)

7.19 Derive an expression for drag force on a smooth submerged object moving through incompressible fluid if this force depends only on speed and size of object and viscosity and density of the fluid.

Find: The appropriate dimensionless groups.

Solution:

(1) We have the dimensional parameters D, V, L, μ, ρ so $n = 5$ dimensional parameters

(2) Select primary dimensions $M, L\ and\ t$.

(3) We have the following relation:

	D	V	L	μ	ρ
M	1	0	0	1	1
L	1	1	1	-1	-3
t	-2	-1	0	-1	0

All three dimensions are present and r = 3 so we need 3 repeating parameters that include all of the dimensions. We pick up the following three repeating parameters:

$$V \quad L \quad \rho$$

We will have n – m = n – r = 5 – 3 = 2 dimensionless groups. For the first group we will combine the repeating variables with the parameter D:

$$\Pi_1 = V^a L^b \rho^c D = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c L = M^0 L^0 t^0$$

Equating the coefficients for each dimension and solving for the values

$$c + 1 = 0$$
$$c = -1$$
$$a + b - 3c + 1 = 0$$
$$-a - 2 = 0$$
$$a = -2$$
$$b = -2$$

The first group is

$$\Pi_1 = \frac{D}{\rho V^2 L^2}$$

For the second group we combine the repeating variables with the viscosity:

$$\Pi_2 = V^d L^e \rho^f \mu = \left(\frac{L}{t}\right)^d L^e \left(\frac{M}{L^3}\right)^f \frac{M}{Lt} = M^0 L^0 t^0$$

The exponents are then

$$\begin{gathered} f + 1 = 0 \\ f = -1 \\ d + e - 3f - 1 = 0 \\ -d - 1 = 0 \\ d = -1 \\ e = -1 \end{gathered}$$

The second group is

$$\Pi_2 = \frac{\mu}{\rho V L}$$

The relation between the groups is

$$\frac{D}{\rho V^2 L^2} = f\left(\frac{\mu}{\rho V L}\right)$$

Problem 7.20

[Difficulty: 2]

7.20 The energy released during an explosion, E, is a function of the time after detonation t, the blast radius R at time t, and the ambient air pressure p, and density ρ. Determine, by dimensional analysis, the general form of the expression for E in terms of the other variables.

Given: Functional relationship between the energy released by an explosion and other physical parameters

Find: Expression for E in terms of the other variables

Solution: We will use the Buckingham pi-theorem.

1 $E \quad t \quad R \quad p \quad \rho$ — n = 5 parameters

2 Select primary dimensions M, L, t:

3 $E \quad t \quad R \quad p \quad \rho$

$$\frac{M \cdot L^2}{t^2} \quad t \quad L \quad \frac{M}{L \cdot t^2} \quad \frac{M}{L^3}$$

r = 3 dimensions

4 $\rho \quad t \quad R$ — m = r = 3 repeating parameters

5 We have n - m = 2 dimensionless groups. Setting up dimensional equations:

$\Pi_1 = E \cdot \rho^a \cdot t^b \cdot R^c$ Thus: $\frac{M \cdot L^2}{t^2} \cdot \left(\frac{M}{L^3}\right)^a \cdot t^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $1 + a = 0$

L: $2 - 3 \cdot a + c = 0$

t: $-2 + b = 0$

The solution to this system is:

$a = -1 \quad b = 2 \quad c = -5$

$$\Pi_1 = \frac{E \cdot t^2}{\rho \cdot R^5}$$

Check using F, L, t dimensions: $F \cdot L \cdot \frac{L^4}{F \cdot t^2} \cdot t^2 \cdot \frac{1}{L^5} = 1$

$\Pi_2 = p \cdot \rho^a \cdot t^b \cdot R^c$ Thus: $\frac{M}{L \cdot t^2} \cdot \left(\frac{M}{L^3}\right)^a \cdot t^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3 \cdot a + c = 0$

t: $-2 + b = 0$

The solution to this system is:

$a = -1 \quad b = 2 \quad c = -2$

$$\Pi_2 = \frac{p \cdot t^2}{\rho \cdot R^2}$$

Check using F, L, t dimensions: $\frac{F}{L^2} \cdot \frac{L^4}{F \cdot t^2} \cdot t^2 \cdot \frac{1}{L^2} = 1$

The functional relationship is: $\Pi_1 = f(\Pi_2)$

$$\frac{E \cdot t^2}{\rho \cdot R^5} = f\left(\frac{p \cdot t^2}{\rho \cdot R^2}\right) \qquad E = \frac{\rho \cdot R^5}{t^2} \cdot f\left(\frac{p \cdot t^2}{\rho \cdot R^2}\right)$$

Problem 7.21

[Difficulty: 2]

7.21 Measurements of the liquid height upstream from an obstruction placed in an open-channel flow can be used to determine volume flow rate. (Such obstructions, designed and calibrated to measure rate of open-channel flow, are called *weirs*.) Assume the volume flow rate, Q, over a weir is a function of upstream height, h, gravity, g, and channel width, b. Use dimensional analysis to find the functional dependence of Q on the other variables.

Given: Functional relationship between the flow rate over a weir and physical parameters

Find: An expression for Q based on the other variables

Solution: We will use the Buckingham pi-theorem.

1 $Q \quad h \quad g \quad b$ — $n = 5$ parameters

2 Select primary dimensions L, t:

3 $Q \quad h \quad g \quad b$

$\frac{L^3}{t} \quad L \quad \frac{L}{t^2} \quad L$ — $r = 2$ dimensions

4 $h \quad g$ — $m = r = 2$ repeating parameters

5 We have n - m = 2 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = Q \cdot h^a \cdot g^b \qquad \text{Thus:} \qquad \frac{L^3}{t} \cdot L^a \cdot \left(\frac{L}{t^2}\right)^b = L^0 \cdot t^0$$

Summing exponents:

L: $3 + a + b = 0$

t: $-1 - 2 \cdot b = 0$

The solution to this system is: $a = -\frac{5}{2} \qquad b = -\frac{1}{2}$

$$\Pi_1 = \frac{Q}{h^2 \cdot \sqrt{g \cdot h}}$$

Check: $\left(\frac{L^3}{t}\right) \cdot \left(\frac{1}{L}\right)^2 \cdot \left(\frac{t}{L}\right) = 1$

$$\Pi_2 = b \cdot h^a \cdot g^b \qquad \text{Thus:} \qquad L \cdot L^a \cdot \left(\frac{L}{t^2}\right)^b = L^0 \cdot t^0$$

Summing exponents:

L: $1 + a + b = 0$

t: $-2 \cdot b = 0$

The solution to this system is: $a = -1 \qquad b = 0$

$$\Pi_2 = \frac{b}{h}$$

Check: $L \cdot \frac{1}{L} = 1$

The functional relationship is: $\Pi_1 = f(\Pi_2) \qquad \frac{Q}{h^2 \cdot \sqrt{g \cdot D}} = f\left(\frac{b}{h}\right)$ Therefore the flow rate is: $Q = h^2 \cdot \sqrt{g \cdot h} \cdot f\left(\frac{b}{h}\right)$

Problem 7.22

[Difficulty: 2]

7.22 The load-carrying capacity, W, of a journal bearing is known to depend on its diameter, D, length, l, and clearance, c, in addition to its angular speed, ω, and lubricant viscosity, μ. Determine the dimensionless parameters that characterize this problem.

Given: Functional relationship between the load bearing capacity of a journal bearing and other physical parameters

Find: Dimensionless parameters that characterize the problem.

Solution: We will use the Buckingham pi-theorem.

1 $W \quad D \quad l \quad c \quad \omega \quad \mu$ $\qquad$ $n = 6$ parameters

2 Select primary dimensions F, L, t:

3 $W \quad D \quad l \quad c \quad \omega \quad \mu$

$F \quad L \quad L \quad L \quad \dfrac{1}{t} \quad \dfrac{F\cdot t}{L^2}$ $\qquad$ $r = 3$ dimensions

4 $D \quad \omega \quad \mu$ $\qquad$ $m = r = 3$ repeating parameters

5 We have n - m = 3 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = W\cdot D^a\cdot \omega^b\cdot \mu^c \qquad \text{Thus:} \qquad F\cdot L^a\cdot\left(\frac{1}{t}\right)^b\cdot\left(\frac{F\cdot t}{L^2}\right)^c = F^0\cdot L^0\cdot t^0$$

Summing exponents:

F: $1 + c = 0$

L: $a - 2\cdot c = 0$

t: $-b + c = 0$

The solution to this system is:

$a = -2 \qquad b = -1 \qquad c = -1$

$$\Pi_1 = \frac{W}{D^2\cdot\omega\cdot\mu}$$

Check using M, L, t dimensions: $\dfrac{M\cdot L}{t^2}\cdot\dfrac{1}{L^2}\cdot t\cdot\dfrac{L\cdot t}{M} = 1$

By inspection, we can see that: $\Pi_2 = \dfrac{l}{D} \qquad \Pi_3 = \dfrac{c}{D}$

The functional relationship is: $\Pi_1 = f(\Pi_2, \Pi_3)$

$$\frac{W}{D^2\cdot\omega\cdot\mu} = f\left(\frac{l}{D}, \frac{c}{D}\right)$$

Problem 7.23

(Difficulty 2)

7.23 Derive an expression for the drag force on a smooth object moving through compressible fluid if this force depends only on speed and size of object, and viscosity, density and modulus of elasticity of the fluid.

Find: The appropriate dimensionless groups.

Solution:

(1) We have six dimensional parameters D, V, L, μ, ρ, E so $n = 6$ dimensional parameters

(2) Select primary dimensions M, L and t.

(3) We have the following relation:

	D	V	L	μ	ρ	E
M	1	0	0	1	1	1
L	1	1	1	-1	-3	-1
t	-2	-1	0	-1	0	-2

All three dimensions are present so $r = 3$. We need 3 repeating parameters that include all of the dimensions and we will pick the following three repeating parameters:

$$V \quad L \quad \rho$$

We will have n – m = n – r = 6 – 3 = 3 dimensionless variables. The first group will combine the dimension D

$$\Pi_1 = V^a L^b \rho^c D = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c L = M^0 L^0 t^0$$

Equating the exponents of the dimensions and solving for their values

$$c + 1 = 0$$
$$c = -1$$
$$a + b - 3c + 1 = 0$$
$$-a - 2 = 0$$
$$a = -2$$
$$b = -2$$

The first group is then

$$\Pi_1 = \frac{D}{\rho V^2 L^2}$$

For the second group we will combine the viscosity:

$$\Pi_2 = V^d L^e \rho^f \mu = \left(\frac{L}{t}\right)^d L^e \left(\frac{M}{L^3}\right)^f \frac{M}{Lt} = M^0 L^0 t^0$$

The exponents are related as

$$\begin{gathered} f + 1 = 0 \\ f = -1 \\ d + e - 3f - 1 = 0 \\ -d - 1 = 0 \\ d = -1 \\ e = -1 \end{gathered}$$

The second group is then

$$\Pi_2 = \frac{\mu}{\rho V L}$$

The third group combines the modulus of elasticity

$$\Pi_3 = V^g L^h \rho^i E = \left(\frac{L}{t}\right)^g L^h \left(\frac{M}{L^3}\right)^i \frac{M}{Lt^2} = M^0 L^0 t^0$$

Solving for the exponents

$$\begin{gathered} i + 1 = 0 \\ i = -1 \\ g + h - 3i - 1 = 0 \\ -g - 2 = 0 \\ g = -2 \\ h = 0 \end{gathered}$$

The third group is

$$\Pi_3 = \frac{E}{\rho V^2}$$

The relation among the groups is then

$$\frac{D}{\rho V^2 L^2} = f\left(\frac{\mu}{\rho V L}, \frac{E}{\rho V^2}\right)$$

Problem 7.24

(Difficulty 2)

7.24 A circular disk of diameter d and of negligible thickness is rotated at a constant angular speed, ω, in a cylindrical casing filled with a liquid of viscosity μ and density ρ. The casing has an internal diameter D, and there is a clearance y between the surfaces of disk and casing. Derive an expression for the torque required to maintain this speed if it depends only on the foregoing variables.

Find: The appropriate dimensionless groups.

Solution:

(1) There are seven dimensional parameters $T, d, \omega, \mu, \rho, D, y$ so $n = 7$ dimensional parameters

(2) Select primary dimensions $M, L \text{ and } t$.

(3) We have the following relation:

	T	d	ω	μ	ρ	D	y
M	1	0	0	1	1	0	0
L	2	1	0	−1	−3	1	1
t	−2	0	−1	−1	0	0	0

All three dimensions are present so r = 3. We need 3 repeating parameters that include all of the dimensions and we will pick the following three repeating parameters:

$$\omega \quad d \quad \rho$$

We will have n – m = n – r = 7 – 3 = 4 dimensionless groups. For the first group we will combine the repeating variables with the torque T

$$\Pi_1 = \omega^a d^b \rho^c T = \left(\frac{1}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{ML^2}{t^2} = M^0 L^0 t^0$$

Equating the exponents of the dimensions and solving for the exponents

$$c + 1 = 0$$
$$c = -1$$
$$b - 3c + 2 = 0$$
$$b = -5$$
$$-a - 2 = 0$$
$$a = -2$$
$$\Pi_1 = \frac{T}{\rho \omega^2 d^5}$$

For the second group, we will combine the repeating variable with the viscosity

$$\Pi_2 = \omega^d d^e \rho^f \mu = \left(\frac{1}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{M}{Lt} = M^0 L^0 t^0$$

Equating the exponents of the dimensions

$$\begin{gathered} f + 1 = 0 \\ f = -1 \\ e - 3f - 1 = 0 \\ e = -2 \\ -d - 1 = 0 \\ d = -1 \end{gathered}$$

The second group is

$$\Pi_2 = \frac{\mu}{\rho \omega d^2}$$

For the third group we combine the internal diameter

$$\Pi_3 = \omega^g d^h \rho^i D = \left(\frac{1}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c L = M^0 L^0 t^0$$

Solving for the exponents

$$\begin{gathered} g = 0 \\ i = 0 \\ h = 1 \end{gathered}$$

The third group is

$$\Pi_3 = \frac{D}{d}$$

The last group combines the clearance y:

$$\Pi_4 = \omega^j d^k \rho^l y = \left(\frac{1}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c L = M^0 L^0 t^0$$

The exponents are

$$\begin{gathered} j = 0 \\ l = 0 \\ k = 1 \\ \Pi_4 = \frac{y}{d} \end{gathered}$$

The relation among the groups is

$$\frac{T}{\rho\omega^2 d^5} = f\left(\frac{\mu}{\rho\omega d^2}, \frac{D}{d}, \frac{y}{d}\right)$$

Problem 7.25

(Difficulty 2)

7.25 Two cylinders are concentric, the outer one fixed and the inner one movable. A viscous incompressible fluid fills the gap between them. Derive an expression for the torque required to maintain constant-speed rotation of the inner cylinder if this torque depends only on the diameters and lengths of the cylinders, the viscosity and density of the fluid, and the angular speed of the inner cylinder.

Find: The appropriate dimensionless groups.

Solution:

(1) There are eight dimensionless parameters T,d_1, d_2, L_1, L_2, μ, ρ ω so $n = 8$

(2) Select primary dimensions $M, L\ and\ t$.

(3) We have the following relation:

	T	$d_1\ d_2$	$L_1\ L_2$	μ	ρ	ω
M	1	0	0	1	1	0
L	2	1	1	-1	-3	0
t	-2	0	0	-1	0	-1

All of the primary dimensions appear and we have r= 3. We need 3 repeating parameters that include all of the dimensions and we pick the following three repeating parameters:

$$\omega \quad d_1 \quad \rho$$

There will be n – m = n – r = 8 – 3 = 5 dimensionless groups. For the first group we will combine the torque T:

$$\Pi_1 = \omega^a d_1^b \rho^c T = \left(\frac{1}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{ML^2}{t^2} = M^0 L^0 t^0$$

Equating the exponents of the dimensions and solving for the value of the exponents

$$c + 1 = 0$$
$$c = -1$$
$$b - 3c + 2 = 0$$
$$b = -5$$
$$-a - 2 = 0$$
$$a = -2$$

The first group is then

$$\Pi_1 = \frac{T}{\rho\omega^2 d_1^5}$$

For the second group, we will combine the repeating variables with the viscosity:

$$\Pi_2 = \omega^d d_1^e \rho^f \mu = \left(\frac{1}{t}\right)^d L^e \left(\frac{M}{L^3}\right)^f \frac{M}{Lt} = M^0 L^0 t^0$$

Equating the exponents of the dimensions

$$\begin{gathered} f + 1 = 0 \\ f = -1 \\ e - 3f - 1 = 0 \\ e = -2 \\ -d - 1 = 0 \\ d = -1 \end{gathered}$$

The second group is

$$\Pi_2 = \frac{\mu}{\rho\omega d_1^2}$$

The third group combines the diameter d_2:

$$\Pi_3 = \omega^g d_1^h \rho^i d_2 = \left(\frac{1}{t}\right)^g L^h \left(\frac{M}{L^3}\right)^i L = M^0 L^0 t^0$$

Equating the exponents and solving for their value

$$\begin{gathered} g = 0 \\ i = 0 \\ h = -1 \end{gathered}$$

So the third group is

$$\Pi_3 = \frac{d_2}{d_1}$$

The fourth group is similar to the third with the parameter L_1:

$$\Pi_4 = \omega^j d_1^k \rho^l L_1 = \left(\frac{1}{t}\right)^j L^k \left(\frac{M}{L^3}\right)^l L = M^0 L^0 t^0$$

The solution is the same as for the third group

$$\begin{gathered} j = 0 \\ l = 0 \\ k = -1 \end{gathered}$$

And the fourth group is

$$\Pi_4 = \frac{L_1}{d_1}$$

The fifth group is similar to the previous two groups, and could be obtained by inspection

$$\Pi_5 = \omega^m d_1^n \rho^o L_2 = \left(\frac{1}{t}\right)^m L^n \left(\frac{M}{L^3}\right)^o L = M^0 L^0 t^0$$

Solving for the exponents

$$m = 0$$
$$o = 0$$
$$n = -1$$

The fifth group is

$$\Pi_5 = \frac{L_2}{d_1}$$

The functional relation between the groups is

$$\frac{T}{\rho\omega^2 d_1^5} = f\left(\frac{\mu}{\rho\omega d_1^2}, \frac{d_2}{d_1}, \frac{L_1}{d_1}, \frac{L_2}{d_1}\right)$$

Problem 7.26 **(In Excel)**

[Difficulty: 2]

7.26 The time, t, for oil to drain out of a viscosity calibration container depends on the fluid viscosity, μ, and density, ρ, the orifice diameter, d, and gravity, g. Use dimensional analysis to find the functional dependence of t on the other variables. Express t in the simplest possible form.

Given: That drain time depends on fluid viscosity and density, orifice diameter, and gravity

Find: Functional dependence of t on other variables

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is:	$n = 5$
The number of primary dimensions is:	$r = 3$
The number of repeat parameters is:	$m = r = 3$
The number of Π groups is:	$n - m = 2$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents a, b, and c for each.

REPEATING PARAMETERS: Choose ρ, g, d

	M	L	t
ρ	1	-3	
g		1	-2
d		1	

Π GROUPS:

	M	L	t
t	0	0	1

Π_1: $a = $ **0**, $b = $ **0.5**, $c = $ **-0.5**

	M	L	t
μ	1	-1	-1

Π_2: $a = $ **-1**, $b = $ **-0.5**, $c = $ **-1.5**

The following Π groups from Example 7.1 are not used:

M	L	t
0	0	0

Π_3: $a = $ **0**, $b = $ **0**, $c = $ **0**

M	L	t
0	0	0

Π_4: $a = $ **0**, $b = $ **0**, $c = $ **0**

Hence $\quad \Pi_1 = t\sqrt{\frac{g}{d}} \quad$ and $\quad \Pi_2 = \frac{\mu}{\rho g^{\frac{1}{2}} d^{\frac{3}{2}}} \rightarrow \frac{\mu^2}{\rho^2 g d^3} \quad$ with $\quad \Pi_1 = f(\Pi_2)$

The final result is $\quad t = \sqrt{\frac{d}{g}}\, f\left(\frac{\mu^2}{\rho^2 g d^3}\right)$

Problem 7.27 [Difficulty: 2]

7.27 You are asked to find a set of dimensionless parameters to organize data from a laboratory experiment, in which a tank is drained through an orifice from initial liquid level h_0. The time, τ, to drain the tank depends on tank diameter, D, orifice diameter, d, acceleration of gravity, g, liquid density, ρ, and liquid viscosity, μ. How many dimensionless parameters will result? How many repeating variables must be selected to determine the dimensionless parameters? Obtain the Π parameter that contains the viscosity.

Given: Functional relationship between the time needed to drain a tank through an orifice plate and other physical parameters

Find: (a) the number of dimensionless parameters
(b) the number of repeating variables
(c) the Π term which contains the viscosity

Solution: We will use the Buckingham pi-theorem.

1 $\tau \quad h_0 \quad D \quad d \quad g \quad \rho \quad \mu$ $\qquad$ $n = 7$ parameters

2 Select primary dimensions M, L, t:

3 $\tau \quad h_0 \quad D \quad d \quad g \quad \rho \quad \mu$

$T \quad L \quad L \quad L \quad \frac{L}{t^2} \quad \frac{M}{L^3} \quad \frac{M}{L \cdot t}$ $\qquad$ $r = 3$ dimensions

4 $\rho \quad d \quad g$ $\qquad$ $m = r = 3$ repeating parameters $\qquad$ We have $n - m = 4$ dimensionless groups.

5 Setting up dimensional equation including the viscosity:

$\Pi_1 = \mu \cdot \rho^a \cdot d^b \cdot g^c$ $\qquad$ Thus: $\frac{M}{L \cdot t} \cdot \left(\frac{M}{L^3}\right)^a \cdot L^b \cdot \left(\frac{L}{t^2}\right)^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3 \cdot a + b + c = 0$

t: $-1 - 2 \cdot c = 0$

The solution to this system is:

$a = -1 \qquad b = -\frac{3}{2} \qquad c = -\frac{1}{2}$

$$\Pi_1 = \frac{\mu}{\rho \cdot d^{\frac{3}{2}} \cdot g^{\frac{1}{2}}}$$

Check using F, L, t dimensions: $\frac{F \cdot t}{L^2} \cdot \frac{L^4}{F \cdot t^2} \cdot \frac{1}{L^{\frac{3}{2}}} \cdot \frac{t}{L^{\frac{1}{2}}} = 1$

Problem 7.28

[Difficulty: 3]

7.28 A continuous belt moving vertically through a bath of viscous liquid drags a layer of liquid, of thickness h, along with it. The volume flow rate of liquid, Q, is assumed to depend on μ, ρ, g, h, and V, where V is the belt speed. Apply dimensional analysis to predict the form of dependence of Q on the other variables.

Given: Functional relationship between the flow rate of viscous liquid dragged out of a bath and other physical parameters

Find: Expression for Q in terms of the other variables

Solution: We will use the Buckingham pi-theorem.

1 $Q \quad \mu \quad \rho \quad g \quad h \quad V$ — $n = 6$ parameters

2 Select primary dimensions M, L, t:

3 $Q \quad \mu \quad \rho \quad g \quad h \quad V$

$$\frac{L^3}{t} \quad \frac{M}{L\cdot t} \quad \frac{M}{L^3} \quad \frac{L}{t^2} \quad L \quad \frac{L}{t}$$ — $r = 3$ dimensions

4 $\rho \quad V \quad h$ — $m = r = 3$ repeating parameters

5 We have n - m = 3 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = Q\cdot\rho^a\cdot V^b\cdot h^c \qquad \text{Thus:} \qquad \frac{L^3}{t}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents:

M: $a = 0$

L: $3 - 3\cdot a + b + c = 0$

t: $-1 - b = 0$

The solution to this system is:

$a = 0 \qquad b = -1 \qquad c = -2$

$$\Pi_1 = \frac{Q}{V\cdot h^2}$$

Check using F, L, t dimensions: $\frac{L^3}{t}\cdot\frac{t}{L}\cdot\frac{1}{L^2} = 1$

$$\Pi_2 = \mu\cdot\rho^a\cdot V^b\cdot h^c \qquad \text{Thus:} \qquad \frac{M}{L\cdot t}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3\cdot a + b + c = 0$

t: $-1 - b = 0$

The solution to this system is:

$a = -1 \qquad b = -1 \qquad c = -1$

$$\Pi_2 = \frac{\mu}{\rho\cdot V\cdot h}$$

Check using F, L, t dimensions: $\frac{F\cdot t}{L^2}\cdot\frac{L^4}{F\cdot t^2}\cdot\frac{t}{L}\cdot\frac{1}{L} = 1$

$\Pi_3 = g \cdot \rho^a \cdot V^b \cdot h^c$ Thus: $\frac{L}{t^2} \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $a = 0$

L: $1 - 3 \cdot a + b + c = 0$

t: $-2 - b = 0$

The solution to this system is:

$a = 0 \quad b = -2 \quad c = 1$

$$\Pi_3 = \frac{g \cdot h}{V^2}$$

Check using F, L, t dimensions: $\frac{L}{t^2} \cdot L \cdot \frac{t^2}{L^2} = 1$

The functional relationship is: $\Pi_1 = f(\Pi_2, \Pi_3)$

$$\frac{Q}{V \cdot h^2} = f\left(\frac{\rho \cdot V \cdot h}{\mu}, \frac{V^2}{g \cdot h}\right) \qquad Q = V \cdot h^2 \cdot f\left(\frac{\rho \cdot V \cdot h}{\mu}, \frac{V^2}{g \cdot h}\right)$$

Problem 7.29

(Difficulty 2)

7.29 Derive an expression for the frictional torque exerted on the journal of a bearing if this torque depends only on the diameters of journal and bearing, their axial lengths (these are the same), viscosity of the lubricant, angular speed of the journal, and the transverse load (force) on the bearing.

Find: The appropriate dimensionless groups.

Solution:

(1) There are seven dimensional parameters $T, d_1, d_2, H, \mu, \omega, F$ so $n = 7$s

(2) Select primary dimensions M, L and t.

(3) We have the following relation:

	T	d_1	d_2	H	μ	ω	F
M	1	0	0	0	1	0	1
L	2	1	1	1	-1	0	1
t	-2	0	0	0	-1	-1	-2

All of the dimensions are present so $r = 3$. We will have 3 repeating parameters and we pick the following three repeating parameters that include all of the dimensions:

$$d_1 \quad \omega \quad F$$

There will be n – m = n – r = 7 – 3 = 4 dimensionless groups. For the first group we will combine the repeating variables with the torque:

$$\Pi_1 = d_1^a \omega^b F^c T = L^a \left(\frac{1}{t}\right)^b \left(\frac{ML}{t^2}\right)^c \frac{ML^2}{t^2} = M^0 L^0 t^0$$

We equate the exponents of the dimensions and solve for their values

$$c + 1 = 0$$
$$c = -1$$
$$a + c + 2 = 0$$
$$a = -1$$
$$-b - 2c - 2 = 0$$
$$b = 0$$

The first group is then

$$\Pi_1 = \frac{T}{Fd_1}$$

For the second group we will combine the repeating variables with the diameter d_2:

$$\Pi_2 = d_1^d \omega^e F^f d_2 = L^d \left(\frac{1}{t}\right)^e \left(\frac{ML}{t^2}\right)^f L = M^0 L^0 t^0 f = 0$$

Equating the exponents of the dimensions and solving for their values

$$d + f + 1 = 0$$
$$d = -1$$
$$-e - 2f = 0$$
$$e = 0$$

The second group is

$$\Pi_2 = \frac{d_2}{d_1}$$

Similarly, for the third group with the length H

$$\Pi_3 = d_1^g \omega^h F^i H = L^g \left(\frac{1}{t}\right)^h \left(\frac{ML}{t^2}\right)^i L = M^0 L^0 t^0$$
$$g = -1$$
$$h = 0$$
$$i = 0$$

The third group is similar to the second group and could have been determined by inspection

$$\Pi_3 = \frac{L}{d_1}$$

For the fourth group we combine the viscosity with the repeating variables:

$$\Pi_4 = d_1^j \omega^k F^l \mu = L^a \left(\frac{1}{t}\right)^b \left(\frac{ML}{t^2}\right)^c \frac{M}{tL} = M^0 L^0 t^0$$

Equating the exponents of the dimensions and solving for them

$$l + 1 = 0$$
$$l = -1$$
$$j + l - 1 = 0$$
$$j = 2$$
$$-k - 2l - 1 = 0$$
$$k = 1$$

The fourth group is

$$\Pi_4 = \frac{\mu \omega d_1^2}{F}$$

The relation among the groups is

$$\frac{T}{Fd_1} = f\left(\frac{d_2}{d_1}, \frac{L}{d_1}, \frac{\mu\omega d_1^2}{F}\right)$$

Problem 7.30

(Difficulty 3)

7.30 Tests on the established flow of six different liquids in smooth pipes of various sizes yield the following data: Make a dimensional analysis of this problem and a plot of the resulting dimensionless numbers as ordinate and abscissa. What conclusion may be drawn from the plot?

Diameter mm	Velocity m/s	Viscosity mPa s	Density Kg/m^3	Wall shear stress Pa
300	2.26	862	1247	51.2
250	2.47	431	1031	33.5
150	1.22	84.3	907	5.41
100	1.39	44.0	938	9.67
50	0.20	1.5	861	0.162
25	0.36	1.0	1000	0.517

Find: Make a dimensional analysis and draw a conclusion from the plot.

Solution: Use dimensional analysis to determine the relation. First, determine the dimensionless parameters of the problem

(1) There five variables in the table, so there are 5 dimensional parameters: $\tau, V, d, \rho, \mu.$ $n = 5$

(2) Select primary dimensions $M, L \; and \; t.$

(3) We have the following relations:

	τ	V	d	ρ	μ
M	1	0	0	1	1
L	-1	1	1	-3	-1
t	-2	-1	0	0	-1

All of the dimensions are present and so $r = 3$. We need 3 repeating parameters that include all of the dimensions and we pick the following three repeating parameters:

$$V \quad d \quad \rho$$

There will be n – m = n – r = 5 – 3 = 2 dimensionless groups. The first group will combine the repeating variables with the wall shear stress:

$$\Pi_1 = V^a d^b \rho^c \tau = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{M}{L\,t^2} = M^0 L^0 t^0$$

Equating the exponents of the dimensions and solving for the values

$$c+1=0$$
$$c=-1$$
$$a+b-3c-1=0$$
$$-a-2=0$$
$$a=-2$$
$$b=0$$

The first group is

$$\Pi_1 = \frac{\tau}{\rho V^2}$$

For the second group we combine the repeating variables with the viscosity:

$$\Pi_2 = V^d d^e \rho^f \mu = \left(\frac{L}{t}\right)^d L^e \left(\frac{M}{L^3}\right)^f \frac{M}{Lt} = M^0 L^0 t^0$$

Equating the exponents of the dimensions and solving for them

$$f+1=0$$
$$f=-1$$
$$d+e-3f-1=0$$
$$-d-1=0$$
$$d=-1$$
$$e=-1$$

The second group is

$$\Pi_2 = \frac{\mu}{\rho V d}$$

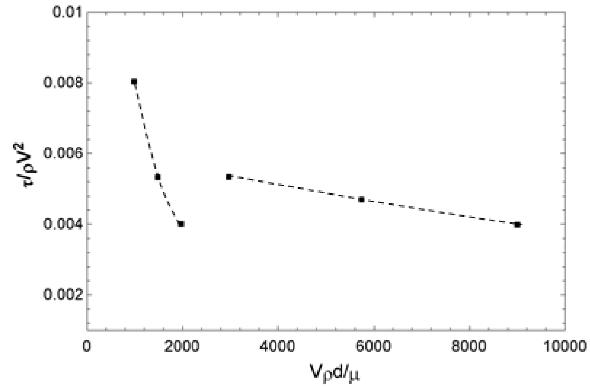
The relation between the two groups is:

$$\frac{\tau}{\rho V^2} = f\left(\frac{\mu}{\rho V d}\right)$$

The data in the table are combined into the two dimensionless groups in the table below. The second group is recognized as the inverse of the Reynolds number, so the Reynolds number is calculated:

$\frac{\tau}{\rho V^2}$	$\frac{\rho V d}{\mu}$
0.00804	981
0.00533	1477
0.00401	1969
0.00534	2963
0.00470	5740
0.00399	9000

The data are plotted in the following figure



The plot demonstrates a universal relationship for all pipes and all fluid between $\frac{\tau}{\rho V^2}$ and $\frac{\mu}{\rho V d}$ for established flow in smooth cylindrical pipes. The discontinuity at $\frac{\rho V d}{\mu} = 2100$ delineates laminar and turbulent zones, and the different trends of the curves suggest a large difference between the physical laws governing the two regimes.

Note too that you could not draw such a conclusion from the data alone. It needs to be put in nondimensional form for us to draw conclusions.

Problem 7.31

[Difficulty: 2]

7.31 The power, $\mathscr{P}$, required to drive a fan is believed to depend on fluid density, ρ, volume flow rate, Q, impeller diameter, D, and angular velocity, ω. Use dimensional analysis to determine the dependence of $\mathscr{P}$ on the other variables.

Given: Functional relationship between the power required to drive a fan and other physical parameters

Find: Expression for P in terms of the other variables

Solution: We will use the Buckingham pi-theorem.

1 $\quad P \quad \rho \quad Q \quad D \quad \omega$ $\qquad$ n = 5 parameters

2 Select primary dimensions M, L, t:

3 $\quad P \quad \rho \quad Q \quad D \quad \omega$

$$\frac{M\cdot L^2}{t^3} \qquad \frac{M}{L^3} \qquad \frac{L^3}{t} \qquad L \qquad \frac{1}{t}$$

r = 3 dimensions

4 $\quad \rho \quad D \quad \omega$ $\qquad$ m = r = 3 repeating parameters

5 We have n - m = 2 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = P\cdot\rho^a\cdot D^b\cdot\omega^c \qquad \text{Thus:} \qquad \frac{M\cdot L^2}{t^3}\cdot\left(\frac{M}{L^3}\right)^a\cdot L^b\cdot\left(\frac{1}{t}\right)^c = M^0\cdot L^0\cdot t^0$$

Summing exponents:

M: $1 + a = 0$

L: $2 - 3\cdot a + b = 0$

t: $-3 - c = 0$

The solution to this system is:

$a = -1 \quad b = -5 \quad c = -3$

$$\Pi_1 = \frac{P}{\rho\cdot D^5\cdot\omega^3}$$

Check using F, L, t dimensions: $\dfrac{F\cdot L}{t}\cdot\dfrac{L^4}{F\cdot t^2}\cdot\dfrac{1}{L^5}\cdot t^3 = 1$

$$\Pi_2 = Q\cdot\rho^a\cdot D^b\cdot\omega^c \qquad \text{Thus:} \qquad \frac{L^3}{t}\cdot\left(\frac{M}{L^3}\right)^a\cdot L^b\cdot\left(\frac{1}{t}\right)^c = M^0\cdot L^0\cdot t^0$$

Summing exponents:

M: $a = 0$

L: $3 - 3\cdot a + b = 0$

t: $-1 - c = 0$

The solution to this system is:

$a = 0 \quad b = -3 \quad c = -1$

$$\Pi_2 = \frac{Q}{D^3\cdot\omega}$$

Check using F, L, t dimensions: $\dfrac{1}{t}\cdot L\cdot\dfrac{t}{L} = 1$

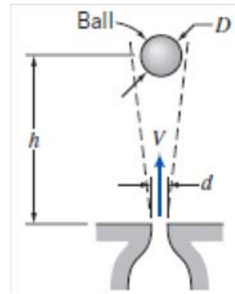
The functional relationship is: $\Pi_1 = f(\Pi_2)$

$$\frac{P}{\rho\cdot D^5\cdot\omega^3} = f\left(\frac{Q}{D^3\cdot\omega}\right) \qquad P = \rho\cdot D^5\cdot\omega^3\cdot f\left(\frac{Q}{D^3\cdot\omega}\right)$$

Problem 7.32

[Difficulty: 3]

7.32 The sketch shows an air jet discharging vertically. Experiments show that a ball placed in the jet is suspended in a stable position. The equilibrium height of the ball in the jet is found to depend on D, d, V, ρ, μ, and W, where W is the weight of the ball. Dimensional analysis is suggested to correlate experimental data. Find the Π parameters that characterize this phenomenon.



Given: Functional relationship between the height of a ball suported by a vertical air jet and other physical parameters

Find: The Π terms that characterize this phenomenon

Solution: We will use the Buckingham pi-theorem.

1 $h \quad D \quad d \quad V \quad \rho \quad \mu \quad W$ $\qquad$ n = 7 parameters

2 Select primary dimensions M, L, t:

3
h	D	d	V	ρ	μ	W
L	L	L	$\frac{L}{t}$	$\frac{M}{L^3}$	$\frac{M}{L \cdot t}$	$\frac{M \cdot L}{t^2}$

r = 3 dimensions

4 $\rho \quad V \quad d$ $\qquad$ m = r = 3 repeating parameters

5 We have n - m = 4 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = h \cdot \rho^a \cdot V^b \cdot d^c \qquad \text{Thus:} \qquad L \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$$

Summing exponents:

M: $a = 0$

L: $1 - 3 \cdot a + b + c = 0$

t: $-b = 0$

The solution to this system is:

$a = 0 \qquad b = 0 \qquad c = -1$

$$\Pi_1 = \frac{h}{d}$$

Check using F, L, t dimensions: $L \cdot \frac{1}{L} = 1$

$$\Pi_2 = D \cdot \rho^a \cdot V^b \cdot d^c \qquad \text{Thus:} \qquad L \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$$

Summing exponents:

M: $a = 0$

L: $1 - 3 \cdot a + b + c = 0$

t: $-b = 0$

The solution to this system is:

$a = 0 \qquad b = 0 \qquad c = -1$

$$\Pi_2 = \frac{D}{d}$$

Check using F, L, t dimensions: $L \cdot \frac{1}{L} = 1$

$\Pi_3 = \mu \cdot \rho^a \cdot V^b \cdot d^c$ Thus: $\frac{M}{L \cdot t} \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3 \cdot a + b + c = 0$

t: $-1 - b = 0$

The solution to this system is:

$a = -1 \quad b = -1 \quad c = -1$

$$\Pi_3 = \frac{\mu}{\rho \cdot V \cdot d}$$

Check using F, L, t dimensions: $\frac{F \cdot t}{L^2} \cdot \frac{L^4}{F \cdot t^2} \cdot \frac{t}{L} \cdot \frac{1}{L} = 1$

$\Pi_4 = W \cdot \rho^a \cdot V^b \cdot d^c$ Thus: $\frac{M \cdot L}{t^2} \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $1 + a = 0$

L: $1 - 3 \cdot a + b + c = 0$

t: $-2 - b = 0$

The solution to this system is:

$a = -1 \quad b = -2 \quad c = -2$

$$\Pi_4 = \frac{W}{\rho \cdot V^2 \cdot d^2}$$

Check using F, L, t dimensions: $F \cdot \frac{L^4}{F \cdot t^2} \cdot \frac{t^2}{L^2} \cdot \frac{1}{L^2} = 1$

Problem 7.33 **(In Excel)** [Difficulty: 3]

7.33 The diameter, d, of bubbles produced by a bubble-making toy depends on the soapy water viscosity, μ, density, ρ, and surface tension, σ, the ring diameter, D, and the pressure differential, Δp, generating the bubbles. Use dimensional analysis to find the Π parameters that characterize this phenomenon.

Given: Bubble size depends on viscosity, density, surface tension, geometry and pressure

Find: Π groups

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is:	$n = 6$
The number of primary dimensions is:	$r = 3$
The number of repeat parameters is:	$m = r = 3$
The number of Π groups is:	$n - m = 3$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents a, b, and c for each.

REPEATING PARAMETERS: Choose ρ, Δp, D

	M	L	t
ρ	1	-3	
Δp	1	-1	-2
D		1	

Π GROUPS:

	M	L	t
d	0	1	0

Π_1: $a =$ **0**, $b =$ **0**, $c =$ **-1**

	M	L	t
μ	1	-1	-1

Π_2: $a =$ **-0.5**, $b =$ **-0.5**, $c =$ **-1**

	M	L	t
σ	1	0	-2

Π_3: $a =$ **0**, $b =$ **-1**, $c =$ **-1**

	M	L	t
	0	0	0

Π_4: $a =$ **0**, $b =$ **0**, $c =$ **0**

Hence $$\Pi_1 = \frac{d}{D} \qquad \Pi_2 = \frac{\mu}{\rho^{\frac{1}{2}}\Delta p^{\frac{1}{2}} D} \rightarrow \frac{\mu^2}{\rho \Delta p D^2} \qquad \Pi_3 = \frac{\sigma}{D\Delta p}$$

Note that the Π_1 group can be obtained by inspection

Problem 7.34

[Difficulty: 3]

7.34 Choked-flow nozzles are often used to meter the flow of gases through piping systems. The mass flow rate of gas is thought to depend on nozzle area A, pressure p, and temperature T upstream of the meter, and the gas constant R. Determine how many independent Π parameters can be formed for this problem. State the functional relationship for the mass flow rate in terms of the dimensionless parameters.

Given: Functional relationship between the mass flow rate of gas through a choked-flow nozzle and other physical parameters

Find: (a) How many independent Π terms that characterize this phenomenon
(b) Find the Π terms
(c) State the functional relationship for the mass flow rate in terms of the Π terms

Solution: We will use the Buckingham pi-theorem.

1 m A p T R (Mathcad can't render dots!) n = 5 parameters

2 Select primary dimensions M, L, t:

3

m	A	p	T	R
$\frac{M}{t}$	L^2	$\frac{M}{L \cdot t^2}$	T	$\frac{L^2}{t^2 \cdot T}$

r = 4 dimensions

4 p A T R m = r = 4 repeating parameters We have n - m = 1 dimensionless group.

5 Setting up dimensional equations:

$$\Pi_1 = m \cdot p^a \cdot A^b \cdot T^c \cdot R^d \qquad \text{Thus:} \quad \frac{M}{t} \cdot \left(\frac{M}{L \cdot t^2}\right)^a \cdot \left(L^2\right)^b \cdot T^c \cdot \left(\frac{L^2}{t^2 \cdot T}\right)^d = M^0 \cdot L^0 \cdot t^0 \cdot T^0$$

Summing exponents:

M: $1 + a = 0$

L: $-a + 2 \cdot b + 2 \cdot d = 0$

t: $-1 - 2 \cdot a - 2 \cdot d = 0$

T: $c - d = 0$

The solution to this system is:

$a = -1 \quad b = -1 \quad c = \frac{1}{2} \quad d = \frac{1}{2}$

$$\Pi_1 = \frac{m}{p \cdot A} \cdot \sqrt{R \cdot T}$$

Check using F, L, t dimensions: $\frac{F \cdot t}{L} \cdot \frac{L^2}{F} \cdot \frac{1}{L^2} \cdot \frac{L}{\frac{1}{t \cdot T^{\frac{1}{2}}}} \cdot T^{\frac{1}{2}} = 1$

The functional relationship is: $\Pi_1 = C \qquad \frac{m}{p \cdot A} \cdot \sqrt{R \cdot T} = C$ So the mass flow rate is: $m = C \cdot \frac{p \cdot A}{\sqrt{R \cdot T}}$

Problem 7.35

[Difficulty: 3]

7.35 A large tank of liquid under pressure is drained through a smoothly contoured nozzle of area A. The mass flow rate is thought to depend on nozzle area, A, liquid density, ρ, difference in height between the liquid surface and nozzle, h, tank gage pressure, Δp, and gravitational acceleration, g. Determine how many independent Π parameters can be formed for this problem. Find the dimensionless parameters. State the functional relationship for the mass flow rate in terms of the dimensionless parameters.

Given: Functional relationship between the mass flow rate of liquid from a pressurized tank through a contoured nozzle and other physical parameters

Find: (a) How many independent Π terms that characterize this phenomenon
(b) Find the Π terms
(c) State the functional relationship for the mass flow rate in terms of the Π terms

Solution: We will use the Buckingham pi-theorem.

1 $\quad m \quad A \quad \rho \quad h \quad \Delta p \quad g$ $\qquad$ $n = 6$ parameters

2 $\quad$ Select primary dimensions M, L, t:

3 $\quad m \quad A \quad \rho \quad h \quad \Delta p \quad g$

$$\frac{M}{t} \quad L^2 \quad \frac{M}{L^3} \quad L \quad \frac{M}{L \cdot t^2} \quad \frac{L}{t^2}$$

$r = 3$ dimensions

4 $\quad \rho \quad A \quad g$ $\qquad$ $m = r = 3$ repeating parameters $\qquad$ We have $n - m = 3$ dimensionless groups.

5 $\quad$ Setting up dimensional equations:

$$\Pi_1 = m \cdot \rho^a \cdot A^b \cdot g^c \qquad \text{Thus:} \qquad \frac{M}{t} \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(L^2\right)^b \cdot \left(\frac{L}{t^2}\right)^c = M^0 \cdot L^0 \cdot t^0$$

Summing exponents:

M: $\quad 1 + a = 0$

L: $\quad -3 \cdot a + 2 \cdot b + c = 0$

t: $\quad -1 - 2 \cdot c = 0$

The solution to this system is:

$$a = -1 \qquad b = -\frac{5}{4} \qquad c = -\frac{1}{2}$$

$$\Pi_1 = \frac{m}{\rho \cdot A^{\frac{5}{4}} \cdot g^{\frac{1}{2}}}$$

Check using F, L, t dimensions: $\quad \frac{F \cdot t}{L} \cdot \frac{L^4}{F \cdot t^2} \cdot \frac{1}{L^{\frac{5}{2}}} \cdot \frac{t}{L^{\frac{1}{2}}} = 1$

$$\Pi_2 = h \cdot \rho^a \cdot A^b \cdot g^c \qquad \text{Thus:} \qquad L \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(L^2\right)^b \cdot \left(\frac{L}{t^2}\right)^c = M^0 \cdot L^0 \cdot t^0$$

Summing exponents:

M: $\quad a = 0$

L: $\quad 1 - 3 \cdot a + 2 \cdot b + c = 0$

t: $\quad -2 \cdot c = 0$

The solution to this system is:

$$a = 0 \qquad b = -\frac{1}{2} \qquad c = 0$$

$$\Pi_2 = \frac{h}{\sqrt{A}}$$

Check using F, L, t dimensions: $\quad L \cdot \frac{1}{L} = 1$

$\Pi_3 = \Delta p \cdot \rho^a \cdot A^b \cdot g^c$ Thus: $\frac{M}{L \cdot t^2} \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(L^2\right)^b \cdot \left(\frac{L}{t^2}\right)^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3 \cdot a + 2 \cdot b + c = 0$

t: $-2 - 2 \cdot c = 0$

The solution to this system is:

$a = -1 \qquad b = -\frac{1}{2} \qquad c = -1$

$$\Pi_3 = \frac{\Delta p}{\rho \cdot g \cdot \sqrt{A}}$$

Check using F, L, t dimensions: $\frac{F}{L^2} \cdot \frac{L^4}{F \cdot t^2} \cdot \frac{t^2}{L} \cdot \frac{1}{L} = 1$

The functional relationship is: $\Pi_1 = f\left(\Pi_2, \Pi_3\right)$ $\frac{m}{\rho \cdot A^{\frac{5}{4}} \cdot g^{\frac{1}{2}}} = f\left(\frac{h}{\sqrt{A}}, \frac{\Delta p}{\rho \cdot g \cdot \sqrt{A}}\right)$

So the mass flow rate is:

$$m = \rho \cdot A^{\frac{5}{4}} \cdot g^{\frac{1}{2}} \cdot f\left(\frac{h}{\sqrt{A}}, \frac{\Delta p}{\rho \cdot g \cdot \sqrt{A}}\right)$$

Problem 7.36

[Difficulty: 3]

7.36 Spin plays an important role in the flight trajectory of golf, Ping-Pong, and tennis balls. Therefore, it is important to know the rate at which spin decreases for a ball in flight. The aerodynamic torque, T, acting on a ball in flight, is thought to depend on flight speed, V, air density, ρ, air viscosity, μ, ball diameter, D, spin rate (angular speed), ω, and diameter of the dimples on the ball, d. Determine the dimensionless parameters that result.

Given: Functional relationship between the aerodynamic torque on a spinning ball and other physical parameters

Find: The Π terms that characterize this phenomenon

Solution: We will use the Buckingham pi-theorem.

1 $T \quad V \quad \rho \quad \mu \quad D \quad \omega \quad d$ $\qquad$ n = 7 parameters

2 Select primary dimensions M, L, t:

3 $T \quad V \quad \rho \quad \mu \quad D \quad \omega \quad d$

$$\frac{M \cdot L^2}{t^2} \quad \frac{L}{t} \quad \frac{M}{L^3} \quad \frac{M}{L \cdot t} \quad L \quad \frac{1}{t} \quad L$$ r = 3 dimensions

4 $\rho \quad V \quad D$ $\qquad$ m = r = 3 repeating parameters

5 We have n - m = 4 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = T \cdot \rho^a \cdot V^b \cdot D^c \qquad \text{Thus:} \quad \frac{M \cdot L^2}{t^2} \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$$

Summing exponents:

M: $1 + a = 0$

L: $2 - 3 \cdot a + b + c = 0$

t: $-2 - b = 0$

The solution to this system is:

$a = -1 \qquad b = -2 \qquad c = -3$

$$\Pi_1 = \frac{T}{\rho \cdot V^2 \cdot D^3}$$

Check using F, L, t dimensions: $F \cdot L \cdot \frac{L^4}{F \cdot t^2} \cdot \frac{t^2}{L^2} \cdot \frac{1}{L^3} = 1$

$$\Pi_2 = \mu \cdot \rho^a \cdot V^b \cdot D^c \qquad \text{Thus:} \quad \frac{M}{L \cdot t} \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3 \cdot a + b + c = 0$

t: $-1 - b = 0$

The solution to this system is:

$a = -1 \qquad b = -1 \qquad c = -1$

$$\Pi_2 = \frac{\mu}{\rho \cdot V \cdot D}$$

Check using F, L, t dimensions: $\frac{F \cdot t}{L^2} \cdot \frac{L^4}{F \cdot t^2} \cdot \frac{t}{L} \cdot \frac{1}{L} = 1$

$\Pi_3 = \omega \cdot \rho^a \cdot V^b \cdot D^c$ Thus: $\frac{1}{t} \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $a = 0$

L: $-3 \cdot a + b + c = 0$

t: $-1 - b = 0$

The solution to this system is:

$a = 0 \qquad b = -1 \qquad c = 1$

$$\Pi_3 = \frac{\omega \cdot D}{V}$$

Check using F, L, t dimensions: $\frac{1}{t} \cdot L \cdot \frac{t}{L} = 1$

$\Pi_4 = d \cdot \rho^a \cdot V^b \cdot D^c$ Thus: $L \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $a = 0$

L: $1 - 3 \cdot a + b + c = 0$

t: $-b = 0$

The solution to this system is:

$a = 0 \qquad b = 0 \qquad c = -1$

$$\Pi_4 = \frac{d}{D}$$

Check using F, L, t dimensions: $\frac{1}{L} \cdot L = 1$

The functional relationship is: $\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4)$

$$\frac{T}{\rho \cdot V^2 \cdot D^3} = f\left(\frac{\mu}{\rho \cdot V \cdot D}, \frac{\omega \cdot D}{V}, \frac{d}{D}\right)$$

Problem 7.37

[Difficulty: 3]

7.37 The power loss, $\mathcal{P}$, in a journal bearing depends on length, l, diameter, D, and clearance, c, of the bearing, in addition to its angular speed, ω. The lubricant viscosity and mean pressure are also important. Obtain the dimensionless parameters that characterize this problem. Determine the functional form of the dependence of $\mathcal{P}$ on these parameters.

Given: Functional relationship between the power loss in a journal bearing and other physical parameters

Find: The Π terms that characterize this phenomenon and the function form of the dependence of P on these parameters

Solution: We will use the Buckingham pi-theorem.

1 $\quad P \quad l \quad D \quad c \quad \omega \quad \mu \quad p$ $\qquad$ n = 7 parameters

2 $\quad$ Select primary dimensions F, L, t:

3 $\quad P \quad l \quad D \quad c \quad \omega \quad \mu \quad p$

$$\frac{F\cdot L}{t} \quad L \quad L \quad L \quad \frac{1}{t} \quad \frac{F\cdot t}{L^2} \quad \frac{F}{L^2}$$

r = 3 dimensions

4 $\quad D \quad \omega \quad p$ $\qquad$ m = r = 3 repeating parameters

5 $\quad$ We have n - m = 4 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = P\cdot D^a\cdot\omega^b\cdot p^c \qquad \text{Thus:} \qquad \frac{F\cdot L}{t}\cdot L^a\cdot\left(\frac{1}{t}\right)^b\cdot\left(\frac{F}{L^2}\right)^c = F^0\cdot L^0\cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

F: $1 + c = 0$ $\qquad$ $a = -3 \quad b = -1 \quad c = -1$ $\qquad$ $\Pi_1 = \dfrac{P}{D^3\cdot\omega\cdot p}$

L: $1 + a - 2\cdot c = 0$

t: $-1 - b = 0$

$$\Pi_2 = l\cdot D^a\cdot\omega^b\cdot p^c \qquad \text{Thus:} \qquad L\cdot L^a\cdot\left(\frac{1}{t}\right)^b\cdot\left(\frac{F}{L^2}\right)^c = F^0\cdot L^0\cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

F: $c = 0$ $\qquad$ $a = -1 \quad b = 0 \quad c = 0$ $\qquad$ $\Pi_2 = \dfrac{l}{D}$

L: $1 + a - 2\cdot c = 0$

t: $-b = 0$

$$\Pi_3 = c\cdot D^a\cdot\omega^b\cdot p^c \qquad \text{Thus:} \qquad L\cdot L^a\cdot\left(\frac{1}{t}\right)^b\cdot\left(\frac{F}{L^2}\right)^c = F^0\cdot L^0\cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

F: $c = 0$ $\qquad$ $a = -1 \quad b = 0 \quad c = 0$ $\qquad$ $\Pi_3 = \dfrac{c}{D}$

L: $1 + a - 2\cdot c = 0$

t: $-b = 0$

$\Pi_4 = \mu \cdot D^a \cdot \omega^b \cdot p^c$ Thus: $\frac{F \cdot t}{L^2} \cdot L^a \cdot \left(\frac{1}{t}\right)^b \cdot \left(\frac{F}{L^2}\right)^c = F^0 \cdot L^0 \cdot t^0$

Summing exponents:

F: $1 + c = 0$

L: $-2 + a - 2 \cdot c = 0$

t: $1 - b = 0$

The solution to this system is:

$a = 0 \quad b = 1 \quad c = -1$

$$\Pi_4 = \frac{\mu \cdot \omega}{p}$$

6 Check using M, L, t dimensions: $\frac{M \cdot L^2}{t^3} \cdot \frac{1}{L^3} \cdot t \cdot \frac{L \cdot t^2}{M} = 1 \quad L \cdot \frac{1}{L} = 1 \quad L \cdot \frac{1}{L} = 1 \quad \frac{M}{L \cdot t} \cdot \frac{1}{t} \cdot \frac{L \cdot t^2}{M} = 1$

The functional relationship is: $\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4) \quad \frac{P}{\omega \cdot p \cdot D^3} = f\left(\frac{l}{D}, \frac{c}{D}, \frac{\mu \cdot \omega}{p}\right) \qquad P = \omega \cdot p \cdot D^3 \cdot f\left(\frac{l}{D}, \frac{c}{D}, \frac{\mu \cdot \omega}{p}\right)$

Problem 7.38

[Difficulty: 3]

7.38 The thrust of a marine propeller is to be measured during "open-water" tests at a variety of angular speeds and forward speeds ("speeds of advance"). The thrust, F_T, is thought to depend on water density, ρ, propeller diameter, D, speed of advance, V, acceleration of gravity, g, angular speed, ω, pressure in the liquid, p, and liquid viscosity, μ. Develop a set of dimensionless parameters to characterize the performance of the propeller. (One of the resulting parameters, gD/V^2, is known as the *Froude speed of advance*.)

Given: Functional relationship between the thrust of a marine propeller and other physical parameters

Find: The Π terms that characterize this phenomenon

Solution: We will use the Buckingham pi-theorem.

1 $\quad F_T \quad \rho \quad D \quad V \quad g \quad \omega \quad p \quad \mu$ $\qquad$ n = 8 parameters

2 $\quad$ Select primary dimensions F, L, t:

3 $\quad F_T \quad \rho \quad D \quad V \quad g \quad \omega \quad p \quad \mu$

$$\frac{M\cdot L}{t^2} \quad \frac{M}{L^3} \quad L \quad \frac{L}{t} \quad \frac{L}{t^2} \quad \frac{1}{t} \quad \frac{M}{L\cdot t^2} \quad \frac{M}{L\cdot t}$$

r = 3 dimensions

4 $\quad \rho \quad V \quad D$ $\qquad$ m = r = 3 repeating parameters

5 $\quad$ We have n - m = 5 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = F_T\cdot\rho^a\cdot V^b\cdot D^c \qquad \text{Thus:} \quad \frac{M\cdot L}{t^2}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

M: $\quad 1 + a = 0$ $\qquad a = -1 \quad b = -2 \quad c = -2$

L: $\quad 1 - 3\cdot a + b + c = 0$

t: $\quad -2 - b = 0$

$$\Pi_1 = \frac{F_T}{\rho\cdot V^2\cdot D^2}$$

$$\Pi_2 = g\cdot\rho^a\cdot V^b\cdot D^c \qquad \text{Thus:} \quad \frac{L}{t^2}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

M: $\quad a = 0$ $\qquad a = 0 \quad b = -2 \quad c = 1$

L: $\quad 1 - 3\cdot a + b + c = 0$

t: $\quad -2 - b = 0$

$$\Pi_2 = \frac{g\cdot D}{V^2}$$

$$\Pi_3 = \omega\cdot\rho^a\cdot V^b\cdot D^c \qquad \text{Thus:} \quad \frac{1}{t}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

M: $\quad a = 0$ $\qquad a = 0 \quad b = -1 \quad c = 1$

L: $\quad -3\cdot a + b + c = 0$

t: $\quad -1 - b = 0$

$$\Pi_3 = \frac{\omega\cdot D}{V}$$

$$\Pi_4 = p\cdot\rho^a\cdot V^b\cdot D^c \qquad \text{Thus:} \qquad \frac{M}{L\cdot t^2}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3\cdot a + b + c = 0$

t: $-2 - b = 0$

The solution to this system is:

$a = -1 \qquad b = -2 \qquad c = 0$

$$\Pi_4 = \frac{p}{\rho\cdot V^2}$$

$$\Pi_5 = \mu\cdot\rho^a\cdot V^b\cdot D^c \qquad \text{Thus:} \qquad \frac{M}{L\cdot t}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3\cdot a + b + c = 0$

t: $-1 - b = 0$

The solution to this system is:

$a = -1 \qquad b = -1 \qquad c = -1$

$$\Pi_5 = \frac{\mu}{\rho\cdot V\cdot D}$$

6 Check using F, L, t dimensions: $F\cdot\frac{L^4}{F\cdot t^2}\cdot\frac{t^2}{L^2}\cdot\frac{1}{L^2} = 1 \qquad \frac{L}{t^2}\cdot L\cdot\frac{t^2}{L^2} = 1 \qquad \frac{1}{t}\cdot L\cdot\frac{t}{L} = 1 \qquad \frac{F}{L^2}\cdot\frac{L^4}{F\cdot t^2}\cdot\frac{t^2}{L^2} = 1 \qquad \frac{F\cdot t}{L^2}\cdot\frac{L^4}{F\cdot t^2}\cdot\frac{t}{L}\cdot\frac{1}{L} = 1$

7.39 The rate dT/dt at which the temperature T at the center of a rice kernel falls during a food technology process is critical—too high a value leads to cracking of the kernel, and too low a value makes the process slow and costly. The rate depends on the rice specific heat, c, thermal conductivity, k, and size, L, as well as the cooling air specific heat, c_p, density, ρ, viscosity, μ, and speed, V. How many basic dimensions are included in these variables? Determine the Π parameters for this problem.

Given: That the cooling rate depends on rice properties and air properties

Find: The Π groups

Solution:

Apply the Buckingham Π procedure

① dT/dt c k L c_p ρ μ V $n = 8$ parameters

② Select primary dimensions M, L, t and T (temperature)

③ $r = 4$ primary dimensions

$$\begin{array}{cccccccc} dT/dt & c & k & L & c_p & \rho & \mu & V \\ \dfrac{T}{t} & \dfrac{L^2}{t^2T} & \dfrac{ML}{t^2T} & L & \dfrac{L^2}{t^2T} & \dfrac{M}{L^3} & \dfrac{M}{Lt} & \dfrac{L}{t} \end{array}$$

④ V ρ L c_p $m = r = 4$ repeat parameters

Then $n - m = 4$ dimensionless groups will result. By inspection, one Π group is c/c_p. Setting up a dimensional equation,

$$\Pi_1 = V^a \rho^b L^c c_p^d \frac{dT}{dt} = \left(\frac{L}{t}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \left(\frac{L^2}{t^2T}\right)^d \frac{T}{t} = T^0 M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{rl|l} T: & -d + 1 = 0 & d = 1 \\ M: & b = 0 & b = 0 \\ L: & a - 3b + c + 2d = 0 & a + c = -2 \rightarrow c = 1 \\ t: & -a - 2d - 1 = 0 & a = -3 \end{array}$$

Hence $\Pi_1 = \dfrac{dT}{dt}\dfrac{Lc_p}{V^3}$

By a similar process, we find $\Pi_2 = \dfrac{k}{\rho L^2 c_p}$ and $\Pi_3 = \dfrac{\mu}{\rho L V}$

Hence

$$\frac{dT}{dt}\frac{Lc_p}{V^3} = f\left(\frac{c}{c_p}, \frac{k}{\rho L^2 c_p}, \frac{\mu}{\rho L V}\right)$$

Problem 7.40

[Difficulty: 4]

7.40 When a valve is closed suddenly in a pipe with flowing water, a water hammer pressure wave is set up. The very high pressures generated by such waves can damage the pipe. The maximum pressure, p_{max}, generated by water hammer is a function of liquid density, ρ, initial flow speed, U_0, and liquid bulk modulus, E_v. How many dimensionless groups are needed to characterize water hammer? Determine the functional relationship among the variables in terms of the necessary Π groups.

Given: Functional relationship between the maximum pressure experienced in a water hammer wave and other physical parameters

Find: (a) The number of Π terms that characterize this phenomenon
(b) The functional relationship between the Π terms

Solution: We will use the Buckingham pi-theorem.

1 $\quad p_{max} \quad \rho \quad U_0 \quad E_V$ $\qquad$ n = 4 parameters

2 Select primary dimensions M, L, t:

3 $\quad p_{max} \quad \rho \quad U_0 \quad E_V$

$$\frac{M}{L\cdot t^2} \quad \frac{M}{L^3} \quad \frac{L}{t} \quad \frac{M}{L\cdot t^2}$$

r = 3 dimensions

4 $\quad \rho \quad U_0$ $\qquad$ m = 2 repeating parameters because p_{max} and E_V have the same dimensions.

We have n - m = 2 dimensionless groups.

5 Setting up dimensional equations:

$$\Pi_1 = p_{max}\cdot\rho^a\cdot U_0^b \qquad \text{Thus:} \qquad \frac{M}{L\cdot t^2}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t}\right)^b = M^0\cdot L^0\cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

M: $\quad 1 + a = 0$ $\qquad a = -1 \quad b = -2$ $\qquad \Pi_1 = \dfrac{p_{max}}{\rho\cdot U_0^2}$

L: $\quad -1 - 3\cdot a + b = 0$

t: $\quad -2 - b = 0$

$$\Pi_2 = E_V\cdot\rho^a\cdot U_0^b \qquad \text{Thus:} \qquad \frac{M}{L\cdot t^2}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t}\right)^b = M^0\cdot L^0\cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

M: $\quad 1 + a = 0$ $\qquad a = -1 \quad b = -2$ $\qquad \Pi_2 = \dfrac{E_V}{\rho\cdot U_0^2}$

L: $\quad -1 - 3\cdot a + b = 0$

t: $\quad -2 - b = 0$

6 Check using F, L, t dimensions: $\quad \dfrac{F}{L^2}\cdot\dfrac{L^4}{F\cdot t^2}\cdot\dfrac{t^2}{L^2} = 1 \quad \dfrac{F}{L^2}\cdot\dfrac{L^4}{F\cdot t^2}\cdot\dfrac{t^2}{L^2} = 1$

The functional relationship is: $\quad \Pi_1 = f(\Pi_2)$ $\quad$ Thus:

$$\frac{p_{max}}{\rho\cdot U_0^2} = f\left(\frac{E_V}{\rho\cdot U_0^2}\right)$$

Problem 7.41 [Difficulty: 3]

7.41 An airship is to operate at 20 m/s in air at standard conditions. A model is constructed to $\frac{1}{20}$ scale and tested in a wind tunnel at the same air temperature to determine drag. What criterion should be considered to obtain dynamic similarity? If the model is tested at 75 m/s, what pressure should be used in the wind tunnel? If the model drag force is 250 N, what will be the drag of the prototype?

Given: Airship is to operate at 20 m/s in air at standard conditions. A 1/20 scale model is to be tested in a wind tunnel at the same temperature to determine drag.

Find:
(a) Criterion needed to obtain dynamic similarity
(b) Air pressure required if air speed in wind tunnel is 75 m/s
(c) Prototype drag if the drag on the model is 250 N

Solution: Dimensional analysis predicts: $\frac{F}{\rho \cdot V^2 \cdot L^2} = f\left(\frac{\rho \cdot V \cdot L}{\mu}\right)$ Therefore, for dynamic similarity, it would follow that:

$$\frac{\rho_m \cdot V_m \cdot L_m}{\mu_m} = \frac{\rho_p \cdot V_p \cdot L_p}{\mu_p}$$

Since the tests are performed at the same temperature, the viscosities are the same. Solving for the ratio of densities:

$$\frac{\rho_m}{\rho_p} = \frac{V_p}{V_m} \cdot \frac{L_p}{L_m} \cdot \frac{\mu_m}{\mu_p} = \frac{20}{75} \times 20 \times 1 = 5.333$$ Now from the ideal gas equation of state: $\rho = \frac{p}{R \cdot T}$ Thus:

$$p_m = p_p \cdot \frac{\rho_m}{\rho_p} \cdot \frac{T_p}{T_m} \qquad p_m = 101 \cdot kPa \times 5.333 \times 1 \qquad p_m = 5.39 \times 10^5 \, Pa$$

From the force ratios: $\frac{F_p}{\rho_p \cdot V_p^2 \cdot L_p^2} = \frac{F_m}{\rho_m \cdot V_m^2 \cdot L_m^2}$ Thus: $F_p = F_m \cdot \frac{\rho_p}{\rho_m} \cdot \left(\frac{V_p}{V_m}\right)^2 \cdot \left(\frac{L_p}{L_m}\right)^2$

Substituting known values: $F_p = 250 \cdot N \times \frac{1}{5.333} \times \left(\frac{20}{75}\right)^2 \times (20)^2$ $\qquad F_p = 1.333 \cdot kN$

Problem 7.42

(Difficulty 2)

7.42 An airplane wing of 3 m chord length moves through still air at 15 °C and 101.3 kPa at a speed of 320 $\frac{km}{h}$. A 1: 20 scale model of this wing is placed in a wind tunnel, and dynamic similarity between a model and prototype is desired. (a) What velocity is necessary in a tunnel where the air has the same pressure and temperature as that in flight? (b) What velocity is necessary in a variable-density wind tunnel where absolute pressure is 1400 kPa and temperature is 15 °C? (c) At what speed must the model move through water (15 °C) for dynamic similarity?

Find: The wind velocity for dynamic similarity between a model and prototype

Solution:

(a) For the dynamic similarity we have the Reynolds number as:

$$\left(\frac{\rho VL}{\mu}\right)_m = \left(\frac{\rho VL}{\mu}\right)_p$$

As the air has same temperature and pressure, so the air properties are the same. For dynamic similarity we then have

$$(VL)_m = (VL)_p$$

The length scale is

$$\frac{L_p}{L_m} = 20$$

The prototype velocity is

$$V_p = 320\ \frac{km}{h} = 88.9\ \frac{m}{s}$$

Thus the model velocity must be

$$V_m = 1778\ \frac{m}{s}$$

This is a supersonic velocity and in a different regime from that for the prototype

(b) In this case we have the same viscosity for the model and prototype:

$$\mu_m = \mu_p$$

Using the ideal gas law we have:

$$\left(\frac{\rho}{p}\right)_m = \left(\frac{\rho}{p}\right)_p$$

With the specified pressures, the density ratio is

$$\rho_m = \frac{p_m}{p_p}\rho_p = \frac{1400}{101.3}\rho_p$$

Thus, for dynamic similarity

$$\left(\frac{\rho VL}{\mu}\right)_m = \left(\frac{\rho VL}{\mu}\right)_p$$

Or

$$(\rho VL)_m = (\rho VL)_p$$

The model velocity would then be

$$V_m = V_p \frac{\rho_p L_p}{\rho_m L_m} = 88.9\ \frac{m}{s} \times \frac{101.3}{1400} \times 20 = 128.7\ \frac{m}{s}$$

This is a reasonable velocity, but the wind tunnel pressure is about 15 atmospheres and very high.

(c) The properties for the model of water at 15 ℃ are:

$$\rho_m = 998\ \frac{kg}{m^3}$$
$$\mu_m = 1.139 \times 10^{-3}\ Pa \cdot s$$

We have for the prototype:

$$\rho_p = 1.225\ \frac{kg}{m^3}$$
$$\mu_p = 1.789 \times 10^{-5}\ Pa \cdot s$$

Using the similarity for Reynolds number:

$$V_m = V_p \frac{\rho_p L_p \mu_m}{\rho_m L_m \mu_p} = 88.9\ \frac{m}{s} \times \frac{1.225\ \frac{kg}{m^3}}{998\ \frac{kg}{m^3}} \times 20 \times \frac{1.139 \times 10^{-3}\ Pa \cdot s}{1.789 \times 10^{-5}\ Pa \cdot s}$$
$$V_m = 138.8\ \frac{m}{s}$$

This is a very high velocity for water flow.

Problem 7.43

(Difficulty 2)

7.43 A flat plate 1.5 m long and 0.3 m wide is towed at 3 $\frac{m}{s}$ in a towing basin containing water at 20 ℃, and the drag force is observed to be 14 N. Calculate the dimensions of similar plate which will yield dynamically similar conditions in an airstream (101.4 kPa and 15 ℃) having a velocity of 18 $\frac{m}{s}$. What drag force may be expected on this plate?

Find: The dimensions of a model and the drag force D_p.

Solution:

For the dynamic similarity we have equal Reynolds numbers for the model and protype:

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

For the prototype in water:

$$\rho_p = 998\ \frac{kg}{m^3}$$

$$\mu_p = 1.002 \times 10^{-3}\ Pa \cdot s$$

$$L_p = 1.5\ m$$

$$V_p = 3\ \frac{m}{s}$$

For the model in air the properties are

$$\rho_m = 1.225\ \frac{kg}{m^3}$$

$$\mu_m = 1.789 \times 10^{-5}\ Pa \cdot s$$

$$V_m = 18\ \frac{m}{s}$$

The length of the model for dynamic similarity is then

$$L_m = \frac{\mu_m}{\rho_m V_m} \frac{\rho_p V_p L_p}{\mu_p} = L_p \frac{\mu_m}{\mu_p} \frac{\rho_p}{\rho_m} \frac{V_p}{V_m}$$

$$L_m = 1.5\ m \times \frac{1.789 \times 10^{-5} Pa\ s}{1.002 \times 10^{-3}\ Pa \cdot s} \times \frac{998\ \frac{kg}{m^3}}{1.225\ \frac{kg}{m^3}} \times \frac{3\ \frac{m}{s}}{18\ \frac{m}{s}}$$

$$L_p = 3.64\ m$$

Under this dynamical similar condition, we have:

$$\frac{D}{\rho V^2 A^2} = f\left(\frac{\rho V L}{\mu}\right)$$

From the geometric similarity,

$$\frac{L_m}{b_m} = \frac{L_p}{b_p}$$

$$b_p = \frac{b_m}{L_m} L_p = \frac{0.3\ m}{1.5\ m} \times 3.64\ m = 0.73\ m$$

We also have:

$$\left(\frac{D}{\rho V^2 A}\right)_m = \left(\frac{D}{\rho V^2 A}\right)_p$$

The drag force will be:

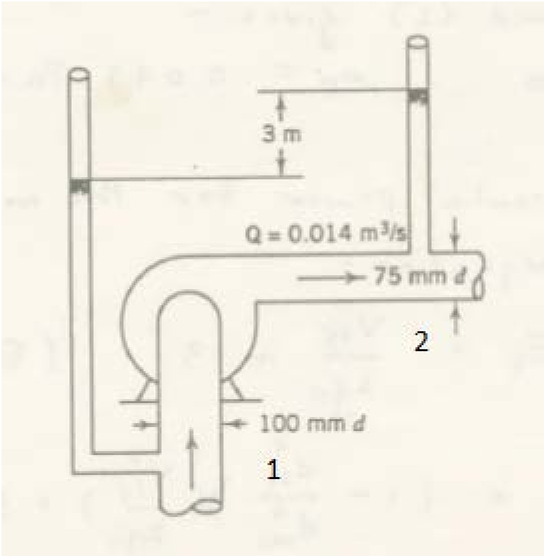
$$D_m = D_p \frac{\rho_m}{\rho_p} \frac{V_m^2}{V_p^2} \frac{A_m}{A_p} = 14N \times \frac{1.225\ \frac{kg}{m^3}}{998\ \frac{kg}{m^3}} \times \frac{18m/s^2}{3m/s^2} \times \frac{3.64m \times 0.73m}{1.5m \times 0.3m}$$

$$D_m = 3.65\ N$$

Problem 7.44

(Difficulty 3)

7.44 This $1:12$ pump model ($using\ water\ at$ $15\ ℃$) simulates a prototype for pumping oil of specific gravity 0.9. The input to the model is $0.522\ kW$. Calculate the viscosity of the oil and the prototype power for complete dynamic similarity between model and prototype.



Find: The viscosity of the oil μ_p and the prototype power.

Solution: Use similitude to determine the parameters of the prototype.

Using the continuity relation, the velocity at the outlet is

$$V_m = \frac{Q}{A} = \frac{0.014\ \frac{m^3}{s}}{\frac{1}{4} \times \pi \times (0.075\ m)^2} = 3.2\ \frac{m}{s}$$

We have the following properties for the model:

$$\rho_m = 998\ \frac{kg}{m^3}$$

$$\mu_m = 1.139 \times 10^{-3}\ Pa \cdot s$$

$$\rho_p = 0.9 \times 998\ \frac{kg}{m^3} = 898\ \frac{kg}{m^3}$$

For dynamic similarity, we need to have equal Reynolds number and Froude numbers. The Reynolds number dynamic similarity means that:

$$\left(\frac{Vd\rho}{\mu}\right)_m = \left(\frac{Vd\rho}{\mu}\right)_p$$

The Froude number dynamic similarity means that:

$$\left(\frac{V^2}{dg}\right)_m = \left(\frac{V^2}{dg}\right)_p$$

From the Reynolds number similarity

$$\frac{3.2\ \frac{m}{s} \times 0.075\ m \times 998\ \frac{kg}{m^3}}{1.139 \times 10^{-3}\ Pa \cdot s} = \frac{V_p \times (0.075\ m \times 12) \times \left(998\ \frac{kg}{m^3}\right)}{\mu_p}$$

And from the Froude number similarity

$$\frac{\left(3.2\ \frac{m}{s}\right)^2}{\left(0.075\ m \times 9.81\ \frac{m}{s^2}\right)} = \frac{\left(V_p\right)^2}{\left(0.075\ m \times 12 \times 9.81\ \frac{m}{s^2}\right)}$$

Using the Froude number similarity we can find the prototype velocity

$$V_p = 11.09\ \frac{m}{s}$$

With this value of velocity, we can find the prototype viscosity

$$\mu_p = 0.043\ Pa \cdot s$$

We now apply the First Law of Thermodynamics equation for the model from the pipe at location 1 $(100\ mm\ dia)$ to the pipe at location 2 $(75\ mm)$. The velocity at the outlet is the model velocity used in the Reynolds number. For the ideal flow without losses (pump inefficiency) we have

$$\frac{V_1^2}{2g} + \frac{\dot{W}_{m\,\iota}}{\dot{m}} + z_1 = \frac{V_m^2}{2g} + z_2$$

The velocity at location 1 is

$$V_1 = V_m \frac{d_2^2}{d_1^2} = 3.2\ \frac{m}{s} \times \frac{(0.075\ m)^2}{(0.1\ m)^2} = 1.8\ \frac{m}{s}$$

The difference in elevation is given by the manometers. These measure the pressure, but also reflect the height difference

$$z_m - z_1 = 3\ m$$

The ideal pump power per unit mass flow is then

$$\frac{\dot{W}_{m\,\iota}}{\dot{m}} = \frac{\left(3.2\ \frac{m}{s}\right)^2 - \left(1.8\frac{m}{s}\right)^2}{2 \times 9.81\ \frac{m}{s^2}} + 3\ m = 3.35\ m$$

The total ideal power for the model is then

$$\dot{W}_{m\,\iota} = Q_m \gamma \frac{\dot{W}_{m\,\iota}}{\dot{m}} = 0.014\ \frac{m^3}{s} \times 9810\ \frac{N}{m^3} \times 3.35\ m = 460\ W$$

The pump efficiency is the ratio of the ideal model power to the input:

$$\eta_m = \frac{W_{m\,i}}{W_m} = \frac{460\ W}{522\ W} = 0.881$$

We now compute the ideal power for the prototype. Because of dynamic similarity, the power per unit mass flow will be the same. The prototype is 12 times as large as the model, so:

$$\frac{\dot{W}_{p\,\iota}}{\dot{m}} = \frac{\dot{W}_{m\,\iota}}{\dot{m}} \times 12 = 3.35\ m \times 12 = 40.2\ m$$

The outlet diameter is also 12 time larger, so the volume flow rate for the prototype is

$$Q_p = 11.09\ \frac{m}{s} \times \frac{\pi}{4} \times (0.075\ m \times 12)^2 = 7.05\ \frac{m^3}{s}$$

The ideal prototype power is then

$$W_{p\,i} = 0.9 Q_p \frac{\dot{W}_{p\,i}}{\dot{m}} \gamma = 0.9 \times 7.05\ \frac{m^3}{s} \times 40.2\ m \times 9810\ \frac{N}{m^3} = 2.50\ MW$$

The efficiency is assumed to be the same. The prototype power is then

$$W_p = \frac{W_{p\,i}}{\eta} = \frac{2.50\ MW}{0.881} = 2.84\ MW$$

Problem 7.45

[Difficulty: 5]

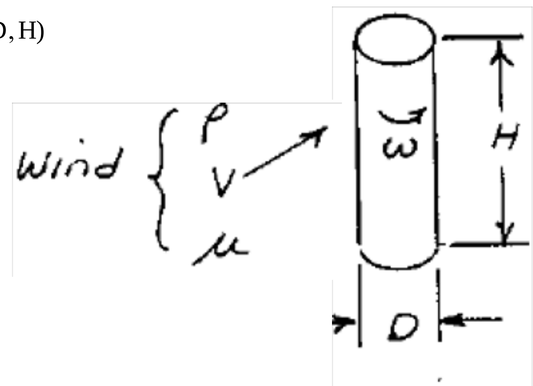
7.45 An ocean-going vessel is to be powered by a rotating circular cylinder. Model tests are planned to estimate the power required to rotate the prototype cylinder. A dimensional analysis is needed to scale the power requirements from model test results to the prototype. List the parameters that should be included in the dimensional analysis. Perform a dimensional analysis to identify the important dimensionless groups.

Given: Vessel to be powered by a rotating circular cylinder. Model tests are planned to determine the required power for the prototype.

Find: (a) List of parameters that should be included in the analysis

(b) Perform dimensional analysis to identify the important dimensionless groups

Solution: From an inspection of the physical problem: $P = f(\rho, \mu, V, \omega, D, H)$



We will now use the Buckingham pi-theorem to find the dimensionless groups.

1 $\quad P \quad \rho \quad \mu \quad V \quad \omega \quad D \quad H \qquad$ n = 7 parameters

2 $\quad$ Select primary dimensions M, L, t:

3 $\quad P \quad \rho \quad \mu \quad V \quad \omega \quad D \quad H$

$\quad \dfrac{M\cdot L^2}{t^3} \quad \dfrac{M}{L^3} \quad \dfrac{M}{L\cdot t} \quad \dfrac{L}{t} \quad \dfrac{1}{t} \quad L \quad L \qquad$ r = 3 dimensions

4 $\quad \rho \quad \omega \quad D \qquad$ m = r = 3 repeating parameters

5 $\quad$ We have n - m = 4 dimensionless groups. Setting up dimensional equations:

$\Pi_1 = P\cdot\rho^a\cdot\omega^b\cdot D^c$ $\qquad$ Thus: $\dfrac{M\cdot L^2}{t^3}\cdot\left(\dfrac{M}{L^3}\right)^a\cdot\left(\dfrac{1}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$

Summing exponents: $\qquad$ The solution to this system is:

M: $1 + a = 0 \qquad a = -1 \quad b = -3 \quad c = -5 \qquad \Pi_1 = \dfrac{P}{\rho\cdot\omega^3\cdot D^5}$

L: $2 - 3\cdot a + c = 0$

t: $-3 - b = 0$

$\Pi_2 = \mu\cdot\rho^a\cdot\omega^b\cdot D^c$ $\qquad$ Thus: $\dfrac{M}{L\cdot t}\cdot\left(\dfrac{M}{L^3}\right)^a\cdot\left(\dfrac{1}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$

Summing exponents: $\qquad$ The solution to this system is:

M: $1 + a = 0 \qquad a = -1 \quad b = -1 \quad c = -2 \qquad \Pi_2 = \dfrac{\mu}{\rho\cdot\omega\cdot D^2}$

L: $-1 - 3\cdot a + c = 0$

t: $-1 - b = 0$

$\Pi_3 = V\cdot\rho^a\cdot\omega^b\cdot D^c$ $\qquad$ Thus: $\dfrac{L}{t}\cdot\left(\dfrac{M}{L^3}\right)^a\cdot\left(\dfrac{1}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$

Summing exponents: $\qquad$ The solution to this system is:

M: $a = 0 \qquad a = 0 \quad b = -1 \quad c = -1 \qquad \Pi_3 = \dfrac{V}{\omega\cdot D}$

L: $1 - 3\cdot a + c = 0$

t: $-1 - b = 0$

$\Pi_4 = H\cdot\rho^a\cdot\omega^b\cdot D^c$ Thus: $L\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{1}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$

Summing exponents:

M: $a = 0$

L: $1 - 3\cdot a + c = 0$

t: $-b = 0$

The solution to this system is:

$a = 0 \quad b = 0 \quad c = -1$

$\Pi_4 = \frac{H}{D}$

6 Check using F, L, t dimensions: $\frac{F\cdot L}{t}\cdot\frac{L^4}{F\cdot t^2}\cdot t^3\cdot\frac{1}{L^5} = 1 \quad \frac{F\cdot t}{L^2}\cdot\frac{L^4}{F\cdot t^2}\cdot t\cdot\frac{1}{L^2} = 1 \quad \frac{L}{t}\cdot t\cdot\frac{1}{L} = 1 \quad L\cdot\frac{1}{L} = 1$

The functional relationship is: $\Pi_1 = f\left(\Pi_2, \Pi_3, \Pi_4\right)$

$$\frac{P}{\rho\cdot\omega^3\cdot D^5} = f\left(\frac{\mu}{\rho\cdot\omega\cdot D^2}, \frac{V}{\omega\cdot D}, \frac{H}{D}\right)$$

Problem 7.46

[Difficulty: 2]

7.46 On a cruise ship, passengers complain about the noise emanating from the ship's propellers (probably due to turbulent flow effects between the propeller and the ship). You have been hired to find out the source of this noise. You will study the flow pattern around the propellers and have decided to use a 1:9-scale water tank. If the ship's propellers rotate at 100 rpm, estimate the model propeller rotation speed if (a) the Froude number or (b) the Reynolds number is the governing dimensionless group. Which is most likely to lead to the best modeling?

Given: Flow around ship's propeller

Find: Model propeller speed using Froude number and Reynolds number

Solution:

Basic equations: $Fr = \dfrac{V}{\sqrt{g \cdot L}}$ $\qquad$ $Re = \dfrac{V \cdot L}{\nu}$

Assumptions: (a) The model and the actual propeller are geometrically similar
(b) The flows about the propellers are kinematically and dynamically similar

Using the Froude number $\quad Fr_m = \dfrac{V_m}{\sqrt{g \cdot L_m}} = Fr_p = \dfrac{V_p}{\sqrt{g \cdot L_p}}$ $\quad$ or $\quad \dfrac{V_m}{V_p} = \sqrt{\dfrac{L_m}{L_p}}$ $\qquad (1)$

But the angular velocity is given by $\quad V = L \cdot \omega$ $\quad$ so $\quad \dfrac{V_m}{V_p} = \dfrac{L_m}{L_p} \cdot \dfrac{\omega_m}{\omega_p}$ $\qquad (2)$

Comparing Eqs. 1 and 2 $\quad \dfrac{L_m}{L_p} \cdot \dfrac{\omega_m}{\omega_p} = \sqrt{\dfrac{L_m}{L_p}}$ $\qquad \dfrac{\omega_m}{\omega_p} = \sqrt{\dfrac{L_p}{L_m}}$

The model rotation speed is then $\quad \omega_m = \omega_p \cdot \sqrt{\dfrac{L_p}{L_m}}$ $\qquad \omega_m = 100 \cdot rpm \times \sqrt{\dfrac{9}{1}}$ $\qquad \omega_m = 300 \cdot rpm$

Using the Reynolds number $\quad Re_m = \dfrac{V_m \cdot L_m}{\nu_m} = Re_p = \dfrac{V_p \cdot L_p}{\nu_p}$ $\quad$ or $\quad \dfrac{V_m}{V_p} = \dfrac{L_p}{L_m} \cdot \dfrac{\nu_m}{\nu_p} = \dfrac{L_p}{L_m}$ $\qquad (3)$

(We have assumed the viscosities of the sea water and model water are comparable)

Comparing Eqs. 2 and 3 $\quad \dfrac{L_m}{L_p} \cdot \dfrac{\omega_m}{\omega_p} = \dfrac{L_p}{L_m}$ $\qquad \dfrac{\omega_m}{\omega_p} = \left(\dfrac{L_p}{L_m}\right)^2$

The model rotation speed is then $\quad \omega_m = \omega_p \cdot \left(\dfrac{L_p}{L_m}\right)^2$ $\qquad \omega_m = 100 \cdot rpm \times \left(\dfrac{9}{1}\right)^2$ $\qquad \omega_m = 8100 \cdot rpm$

Of the two models, the Froude number appears most realistic; at 8100 rpm serious cavitation will occur, which would invalidate the similarity assumptions. Both flows will likely have high Reynolds numbers so that the flow becomes independent of Reynolds number; the Froude number is likely to be a good indicator of static pressure to dynamic pressure for this (although cavitation number would be better).

Problem 7.47

[Difficulty: 3]

7.47 A $\frac{1}{5}$-scale model of a torpedo is tested in a wind tunnel to determine the drag force. The prototype operates in water, has 533 mm diameter, and is 6.7 m long. The desired operating speed of the prototype is 28 m/s. To avoid compressibility effects in the wind tunnel, the maximum speed is limited to 110 m/s. However, the pressure in the wind tunnel can be varied while holding the temperature constant at 20°C. At what minimum pressure should the wind tunnel be operated to achieve a dynamically similar test? At dynamically similar test conditions, the drag force on the model is measured as 618 N. Evaluate the drag force expected on the full-scale torpedo.

Given: A torpedo with D = 533 mm and L = 6.7 m is to travel at 28 m/s in water. A 1/5 scale model of the torpedo is to be tested in a wind tunnel. The maximum speed in the tunnel is fixed at 110 m/s, but the pressure can be varied at a constant temperature of 20 deg C.

Find: (a) Minimum pressure required in the wind tunnel for dynamically similar testing.
(b) The expected drag on the prototype if the model drag is 618 N.

Solution: The problem may be stated as: $F = f(\rho, V, D, \mu)$ From the Buckingham pi theorem, we expect 2 Π terms:

$$\frac{F}{\rho \cdot V^2 \cdot D^2} = g(Re) \quad \text{where} \quad Re = \frac{\rho \cdot V \cdot D}{\mu}$$

Matching Reynolds numbers between the model and prototype flows: $\frac{\rho_m \cdot V_m \cdot D_m}{\mu_m} = \frac{\rho_p \cdot V_p \cdot D_p}{\mu_p}$ Thus: $\rho_m = \rho_p \cdot \frac{V_p}{V_m} \cdot \frac{D_p}{D_m} \cdot \frac{\mu_m}{\mu_p}$

At 20 deg C: $\mu_p = 1.00 \times 10^{-3} \cdot \frac{N \cdot s}{m^2}$ and $\mu_m = 1.81 \times 10^{-5} \cdot \frac{N \cdot s}{m^2}$ So substituting in values yields:

$$\rho_m = 998 \cdot \frac{kg}{m^3} \times \frac{28}{110} \times \frac{5}{1} \times \frac{1.81 \times 10^{-5}}{1.00 \times 10^{-3}} \qquad \rho_m = 23.0 \cdot \frac{kg}{m^3}$$

From the ideal gas equation of state: $p_m = \rho_m \cdot R \cdot T_m$

Substituting in values: $p_m = 23.0 \cdot \frac{kg}{m^3} \times 287 \cdot \frac{N \cdot m}{kg \cdot K} \times 293 \cdot K \times \frac{Pa \cdot m^2}{N}$ $\qquad p_m = 1.934 \cdot MPa$

If the conditions are dynamically similar: $\frac{F_m}{\rho_m \cdot V_m^2 \cdot D_m^2} = \frac{F_p}{\rho_p \cdot V_p^2 \cdot D_p^2}$ Thus: $F_p = F_m \cdot \frac{\rho_p}{\rho_m} \cdot \left(\frac{V_p}{V_m}\right)^2 \cdot \left(\frac{D_p}{D_m}\right)^2$

Substituting in known values: $F_p = 618 \cdot N \times \frac{998}{23.0} \times \left(\frac{28}{110}\right)^2 \times \left(\frac{5}{1}\right)^2$ $\qquad F_p = 43.4 \cdot kN$

Problem 7.48

(Difficulty 3)

7.48 A flow rate of $0.18\ \frac{m^3}{s}$ of water $20\ ℃$ discharges a $0.3\ m$ pipe through a $0.15\ m$ nozzle into the atmosphere. The axial force component exerted by water on the nozzle is $3\ kN$. If frictional effects may be ignored, what corresponding force will be exerted on a $4{:}\ 1$ prototype of nozzle and pipe discharging $1.13\ \frac{m^3}{s}$ of air ($101.4\ kPa\ and\ 15\ ℃$) to the atmosphere? If frictional effects are included, the axial component is $3.56\ kN$. What flow rate of air is then required for dynamic similarity? What is the corresponding force on the nozzle discharging air?

Find: (a) The force F_{air}. (b) The flow rate Q and F_{air} with frictional effects.

Solution:

(a) For dynamic similarity for pressure force of frictionless flow we have the equality of the pressure coefficient:

$$\left(\frac{\Delta p}{\rho V^2}\right)_m = \left(\frac{\Delta p}{\rho V^2}\right)_p$$

Using the continuity equation to replace the velocity with the volume flow rate

$$Q = A\ V$$

So the dynamic similarity becomes

$$\left(\frac{\Delta p A^2}{\rho Q^2}\right)_m = \left(\frac{\Delta p A^2}{\rho Q^2}\right)_p$$

The product of the pressure difference and the area is the force

$$F = \Delta p\ A$$

So for dynamic similarity we have

$$\left(\frac{\mathrm{F}A}{\rho Q^2}\right)_m = \left(\frac{\mathrm{F}A}{\rho Q^2}\right)_p$$

We have the following properties for the force on model. We take the water flow as the model and the air flow as the prototype:

$$\rho_m = 998\ \frac{kg}{m^3}$$

$$A_m = \frac{\pi}{4} \times (0.15\ m)^2 = 0.0177\ m^2$$

$$Q_m = 0.18\ \frac{m^3}{s}$$

$$F_m = 3\ kN$$

And for the prototype

$$\rho_p = 1.225\ \frac{kg}{m^3}$$

$$A_p = \frac{\pi}{4} \times (0.15\ m \times 4)^2 = 0.283\ m^2$$

$$Q_p = 1.13\ \frac{m^3}{s}$$

Thus we have for the force on the prototype

$$\left(\frac{\mathrm{F}A}{\rho Q^2}\right)_m F_p = F_m \frac{A_m}{A_p}\frac{\rho_p}{\rho_m}\frac{Q_p^2}{Q_m^2} = 3kN \times \frac{0.0177\ m^2}{0.283\ m^2} \times \frac{1.225\ \frac{kg}{m^3}}{998\ \frac{kg}{m^3}} \times \frac{\left(1.13\ \frac{m^3}{s}\right)^2}{\left(0.18\ \frac{m^3}{s}\right)^2}$$

$$F_p = F_{air} = 9.08\ N$$

(b) With friction, we need to have the dynamic similarity for Reynolds number and Froude number. For the Reynolds number:

$$\left(\frac{\rho \mathrm{VD}}{\mu}\right)_m = \left(\frac{\rho \mathrm{VD}}{\mu}\right)_p$$

And for the Froude number

$$\left(\frac{\rho \mathrm{QD}}{A\mu}\right)_m = \left(\frac{\rho \mathrm{QD}}{A\mu}\right)_p$$

We have the properties

$$D_m = 0.15\ m$$
$$\mu_m = 1.002 \times 10^{-3}\ Pa \cdot s$$
$$D_p = 0.6\ m$$
$$\mu_p = 1.789 \times 10^{-5}\ Pa \cdot s$$

We have the velocity for the model as

$$V_m = \frac{Q_m}{A_m} = \frac{0.18\ \frac{m^3}{s}}{\frac{\pi}{4} \times (0.15\ m)^2} = 10.9\ \frac{m}{s}$$

The prototype model velocity is then

$$V_p = V_m \frac{\rho_m}{\rho_p} \frac{\mu_p}{\mu_m} \frac{L_m}{L_p} = 10.9\ \frac{m}{s} \times \frac{998\ \frac{kg}{m^3}}{1.225\ \frac{kg}{m^3}} \times \frac{1.789 \times 10^{-5}\ Pa \cdot s}{1.002 \times 10^{-3}\ Pa \cdot s} \times \frac{1}{4}$$

$$V_p = 39.6\ \frac{m}{s}$$

Thus the prototype flow rate is

$$Q_p = V_p A_p = 39.6\ \frac{m}{s} \times 0.283\ m^2 = 10.47\ \frac{m^3}{s}$$

Finally, we have for the force for dynamic similarity:

$$\left(\frac{\mathrm{F}A}{\rho Q^2}\right)_m = \left(\frac{\mathrm{F}A}{\rho Q^2}\right)_p$$

Or

$$F_p = F_m \times \frac{\rho_p}{\rho_m} \times \left(\frac{Q_p}{Q_m}\right)^2 \times \frac{A_m}{A_p}$$

where

$$F_m = 3.56\ kN$$

The force on the prototype is then

$$F_p = 3.56\ kN \times \frac{1.225\ \frac{kg}{m^3}}{998\ \frac{kg}{m^3}} \times \left(\frac{10.47\ \frac{m^3}{s}}{0.18\ \frac{m^3}{s}}\right)^2 \times \frac{0.0177\ m^2}{0.283\ m^2}$$

$$F_p = F_{air} = 925\ N$$

Problem 7.49

(Difficulty 2)

7.49 A force of $9\ N$ is required to tow a $1:50$ ship model at $4.8\ \frac{km}{h}$. Assuming the same water in towing basin and sea, calculate the corresponding speed and force in the prototype if the flow is dominated by: (a) density and gravity (b) density and surface tension (c) density and viscosity

Find: The corresponding speed V_p and F_p.

Solution:

(a) If the flow is dominated by density and gravity, for dynamic similarity we must have equal Froude numbers. It is easier to work with the square of the Froude number:

$$\left(\frac{V^2}{gL}\right)_m = \left(\frac{V^2}{gL}\right)_p$$

Or, since it is the same gravity for the model and prototype

$$V_p = V_m \times \left(\frac{L_p}{L_m}\right)^{1/2} = 4.8\ \frac{km}{h} \times \left(\frac{50}{1}\right)^{1/2}$$

Thus the velocity is

$$V_p = 33.9\ \frac{km}{h}$$

We also have that for dynamic similarity that the drag coefficients are equal

$$\left(\frac{F}{\rho V^2 L^2}\right)_m = \left(\frac{F}{\rho V^2 L^2}\right)_p$$

The density is the same for both the model and prototype and so we have

$$F_p = F_m \times \left(\frac{L_p}{L_m}\right)^2 \times \left(\frac{V_p}{V_m}\right)^2 = 9N \times \left(\frac{50}{1}\right)^2 \times \left(\frac{33.9\ \frac{km}{h}}{4.8\ \frac{km}{h}}\right)^2$$

Or the force is:

$$F_p = 1122\ \ kN$$

(b) If the flow is governed by the density and surface tension we have equal Weber numbers for dynamic similarity:

$$\left(\frac{\rho V^2 L}{\sigma}\right)_m = \left(\frac{\rho V^2 L}{\sigma}\right)_p$$

The surface tension and density are the same for both model and prototype so we have for the velocity

$$V_p = V_m \times \left(\frac{L_m}{L_p}\right)^{1/2} = 4.8\ \frac{km}{h} \times \left(\frac{1}{50}\right)^{1/2}$$

$$V_p = 0.68\ \frac{km}{h}$$

which is much lower than the speed for Froude number similarity

For the force, we again use that the force coefficients are the same for dynamic similarity:

$$\left(\frac{F}{\rho V^2 L^2}\right)_m = \left(\frac{F}{\rho V^2 L^2}\right)_p$$

So we get:

$$F_p = F_m \times \left(\frac{L_p}{L_m}\right)^2 \times \left(\frac{V_p}{V_m}\right)^2 = 9N \times \left(\frac{50}{1}\right)^2 \times \left(\frac{0.68\ \frac{km}{h}}{4.8\ \frac{km}{h}}\right)^2$$

$$F_p = 452\ N$$

(c) If the flow is dominated by the density and viscosity, the Reynolds numbers are the same for dynamic similarity:

$$\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p$$

Or, since the density and viscosity are the same for both model and prototype

$$V_p = V_m \times \left(\frac{L_m}{L_p}\right) = 4.8\ \frac{km}{h} \times \left(\frac{1}{50}\right)$$

$$V_p = 0.096\ \frac{km}{h}$$

Again, because the force coefficients are then equal:

$$\left(\frac{F}{\rho V^2 L^2}\right)_m = \left(\frac{F}{\rho V^2 L^2}\right)_p$$

Or

$$F_p = F_m \times \left(\frac{L_p}{L_m}\right)^2 \times \left(\frac{V_p}{V_m}\right)^2 = 9N \times \left(\frac{50}{1}\right)^2 \times \left(\frac{0.096\ \frac{km}{h}}{4.8\ \frac{km}{h}}\right)^2$$

or

$$F_p = 9\ N$$

We see that the force we predict on the model depends very heavily on what we assume the flow is dominated by. In this case the forces differ by 10^6!

Problem 7.50 [Difficulty: 2]

7.50 An airplane wing, with chord length of 1.5 m and span of 9 m, is designed to move through standard air at a speed of 7.5 m/s. A $\frac{1}{10}$-scale model of this wing is to be tested in a water tunnel. What speed is necessary in the water tunnel to achieve dynamic similarity? What will be the ratio of forces measured in the model flow to those on the prototype wing?

Given: Model of wing

Find: Model test speed for dynamic similarity; ratio of model to prototype forces

Solution:

We would expect $F = F(l, s, V, \rho, \mu)$ where F is the force (lift or drag), l is the chord and s the span

From Buckingham Π
$$\frac{F}{\rho \cdot V^2 \cdot l \cdot s} = f\left(\frac{\rho \cdot V \cdot l}{\mu}, \frac{l}{s}\right)$$

For dynamic similarity
$$\frac{\rho_m \cdot V_m \cdot l_m}{\mu_m} = \frac{\rho_p \cdot V_p \cdot l_p}{\mu_p}$$

Hence
$$V_m = V_p \cdot \frac{\rho_p}{\rho_m} \cdot \frac{l_p}{l_m} \cdot \frac{\mu_m}{\mu_p}$$

From Table A.8 at 20°C $\mu_m = 1.01 \times 10^{-3} \cdot \frac{N \cdot s}{m^2}$ From Table A.10 at 20°C $\mu_p = 1.81 \times 10^{-5} \cdot \frac{N \cdot s}{m^2}$

$$V_m = 7.5 \cdot \frac{m}{s} \times \left(\frac{1.21 \cdot \frac{kg}{m^3}}{998 \cdot \frac{kg}{m^3}}\right) \times \left(\frac{10}{1}\right) \times \left(\frac{1.01 \times 10^{-3} \cdot \frac{N \cdot s}{m^2}}{1.81 \times 10^{-5} \cdot \frac{N \cdot s}{m^2}}\right) \qquad V_m = 5.07 \frac{m}{s}$$

Then
$$\frac{F_m}{\rho_m \cdot V_m^2 \cdot l_m \cdot s_m} = \frac{F_p}{\rho_p \cdot V_p^2 \cdot l_p \cdot s_p} \qquad \frac{F_m}{F_p} = \frac{\rho_m}{\rho_p} \cdot \frac{V_m^2}{V_p^2} \cdot \frac{l_m \cdot s_m}{l_p \cdot s_p} = \frac{998}{1.21} \times \left(\frac{5.07}{7.5}\right)^2 \times \frac{1}{10} \times \frac{1}{10} = 3.77$$

Problem 7.51

[Difficulty: 3]

7.51 A water pump with impeller diameter 24 in. is to be designed to move 15 ft^3/s when running at 750 rpm. Testing is performed on a 1:4 scale model running at 2400 rpm using air (68°F) as the fluid. For similar conditions (neglecting Reynolds number effects), what will be the model flow rate? If the model draws 0.1 hp, what will be the power requirement of the prototype?

Given: Model of water pump

Find: Model flow rate for dynamic similarity (ignoring Re); Power of prototype

Solution:

From Buckingham Π $\quad \dfrac{Q}{\omega \cdot D^3} \quad$ and $\quad \dfrac{P}{\rho \cdot \omega^3 \cdot D^5}$

where Q is flow rate, ω is angular speed, d is diameter, and ρ is density (these Π groups will be discussed in Chapter 10)

For dynamic similarity $\quad \dfrac{Q_m}{\omega_m \cdot D_m^3} = \dfrac{Q_p}{\omega_p \cdot D_p^3}$

Hence $\quad Q_m = Q_p \cdot \dfrac{\omega_m}{\omega_p} \cdot \left(\dfrac{D_m}{D_p}\right)^3$

$$Q_m = 15 \cdot \frac{ft^3}{s} \times \left(\frac{2400}{750}\right) \times \left(\frac{1}{4}\right)^3 \qquad Q_m = 0.750 \cdot \frac{ft^3}{s}$$

From Table A.8 at 68°F $\quad \rho_p = 1.94 \cdot \dfrac{slug}{ft^3}$ $\quad$ From Table A.9 at 68°F $\quad \rho_m = 0.00234 \cdot \dfrac{slug}{ft^3}$

Then $\quad \dfrac{P_m}{\rho_m \cdot \omega_m^3 \cdot D_m^5} = \dfrac{P_p}{\rho_p \cdot \omega_p^3 \cdot D_p^5}$

$$P_p = P_m \cdot \frac{\rho_p}{\rho_m} \cdot \left(\frac{\omega_p}{\omega_m}\right)^3 \cdot \left(\frac{D_p}{D_m}\right)^5$$

$$P_p = 0.1 \cdot hp \times \frac{1.94}{0.00234} \times \left(\frac{750}{2400}\right)^3 \times \left(\frac{4}{1}\right)^5 \qquad P_p = 2.59 \times 10^3 \cdot hp$$

Note that if we had used water instead of air as the working fluid for the model pump, it would have drawn 83 hp. Water would have been an acceptable working fluid for the model, and there would have been less discrepancy in the Reynolds number.

Problem 7.52

[Difficulty: 3]

7.52 A model hydrofoil is to be tested at 1:20 scale. The test speed is chosen to duplicate the Froude number corresponding to the 60-knot prototype speed. To model cavitation correctly, the cavitation number also must be duplicated. At what ambient pressure must the test be run? Water in the model test basin can be heated to 130°F, compared to 45°F for the prototype.

Given: A 1:20 model of a hydrofoil is to be tested in water at 130 deg F. The prototype operates at a speed of 60 knots in water at 45 deg F. To model the cavitation, the cavitation number must be duplicated.

Find: Ambient pressure at which the test must be run

Solution: To duplicate the Froude number between the model and the prototype requires: $\frac{V_m}{\sqrt{g \cdot L_m}} = \frac{V_p}{\sqrt{g \cdot L_p}}$ Thus:

$$V_m = V_p \cdot \sqrt{\frac{L_m}{L_p}} \qquad V_m = 60 \cdot \text{knot} \cdot \sqrt{\frac{1}{20}} \qquad V_m = 13.42 \cdot \text{knot}$$

To match the cavitation number between the model and the prototype: $\frac{p_m - p_{vm}}{\frac{1}{2} \cdot \rho_m \cdot V_m^2} = \frac{p_p - p_{vp}}{\frac{1}{2} \cdot \rho_p \cdot V_p^2}$ Therefore:

$$p_m = p_{vm} + \left(p_p - p_{vp}\right) \cdot \frac{\rho_m}{\rho_p} \cdot \left(\frac{V_m}{V_p}\right)^2$$

Assuming that the densities are equal: $p_m = p_{vm} + \left(p_p - p_{vp}\right) \cdot \left(\frac{V_m}{V_p}\right)^2$

From table A.7: at 130 deg F $p_{vm} = 2.23 \cdot \text{psi}$ at 45 deg F $p_{vp} = 0.15 \cdot \text{psi}$ Thus the model pressure is:

$$p_m = 2.23 \cdot \text{psi} + (14.7 \cdot \text{psi} - 0.15 \cdot \text{psi}) \cdot \left(\frac{13.42}{60}\right)^2 \qquad p_m = 2.96 \cdot \text{psi}$$

Problem 7.53

(Difficulty 3)

7.53 A ship 120 m long moves through freshwater at 15 ℃ at 32 $\frac{km}{h}$. A 1: 100 model of this ship is to be tested in a towing basin containing a liquid of specific gravity 0.92. What viscosity must this liquid have for both Reynolds' and Froude's laws to be satisfied? At what velocity must the model be towed? What propulsive force on this ship corresponds to a towing force of 9N in the model?

Find: The liquid viscosity of the model μ_m. The velocity of the model V_m. The force on the ship F_p.

Solution:

For dynamic similarity we must have equal Froude and Reynolds numbers. It is easier to work with the square of the Froude number. For the Froude number

$$\left(\frac{V^2}{gL}\right)_m = \left(\frac{V^2}{gL}\right)_p$$

For the Reynolds number

$$\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p$$

From Froude number similarity, since gravity is the same for both model and prototype

$$V_m = V_p \times \left(\frac{L_m}{L_p}\right)^{1/2} = 32\ \frac{km}{hr} \times \left(\frac{1}{100}\right)^{1/2}$$

$$V_m = 3.2\ \frac{km}{hr}$$

For Reynolds number similarity, we need to account for the difference in density. For dynamic similarity we have for the viscosity

$$\mu_m = \mu_p \times \left(\frac{\rho_m}{\rho_p}\right) \times \left(\frac{V_m}{V_p}\right) \times \left(\frac{L_m}{L_p}\right)$$

with

$$L_p = 120\ m$$

$$\rho_p = 998\ \frac{kg}{m^3}$$

$$\mu_p = 1.139 \times 10^{-3}\ Pa \cdot s$$

$$L_m = 1.2\ m$$

$$\rho_m = 0.92 \times 998\ \frac{kg}{m^3} = 918\ \frac{kg}{m^3}$$

The viscosity of the liquid for the model is

$$\mu_m = 1.139 \times 10^{-3}\ Pa \cdot s \times \left(\frac{918\ \frac{kg}{m^3}}{998\ \frac{kg}{m^3}}\right) \times \left(\frac{3.2\ \frac{km}{hr}}{32\ \frac{km}{hr}}\right) \times \left(\frac{1}{100}\right)$$

The viscosity must be

$$\mu_m = 1.05 \times 10^{-6}\ Pa \cdot s$$

This is a very low viscosity and the fluid must be as “thin” as air.

For the force, we have that the drag coefficients are equal:

$$\left(\frac{F}{\rho V^2 L^2}\right)_m = \left(\frac{F}{\rho V^2 L^2}\right)_p$$

$$F_p = F_m \times \left(\frac{L_p}{L_m}\right)^2 \times \left(\frac{V_p}{V_m}\right)^2 = 9N \times \left(\frac{100}{1}\right)^2 \times \left(\frac{32\ \frac{km}{hr}}{3.2\ \frac{km}{hr}}\right)^2$$

$$F_m = 9\ N$$

Thus

$$F_p = 9 \times 10^6 N$$

Problem 7.54

(Difficulty 3)

7.54 A $1:30$ scale model of a cavitating overflow structure is to be tested in a vacuum tank wherein the pressure is maintained at $2.0\ psia$. The prototype liquid is water at 70 °F. The barometric pressure on the prototype is $14.5\ psia$. If the liquid to be used in the model has vapor pressure of $1.5\ psia$, what values of density, viscosity , and surface tension must it have for complete dynamic similarity between model and prototype?

Find: The parameters for completing the dynamic similarity.

Solution:

For the dynamic similarity we have dynamic similarity with the Froude number, Reynolds number and Euler number. The Froude number squared is easier to work with, and is

$$\left(\frac{V^2}{gL}\right)_m = \left(\frac{V^2}{gL}\right)_p$$

The Reynolds number relation is written in terms of the kinematic viscosity

$$\left(\frac{VL}{\nu}\right)_m = \left(\frac{VL}{\nu}\right)_p$$

The Euler number relation is

$$\left(\frac{\Delta p}{\frac{1}{2}\rho V^2}\right)_m = \left(\frac{\Delta p}{\frac{1}{2}\rho V^2}\right)_p$$

And the Weber number is

$$\left(\frac{\rho V^2 L}{\sigma}\right)_m = \left(\frac{\rho V^2 L}{\sigma}\right)_p$$

The prototype is water at 70 °F. The properties are

$$\text{Vapor pressure} \qquad p_v = 0.36\ psia$$

$$\rho_p = 1.934\ \frac{slug}{ft^3}$$

$$\nu_p = 1.059 \times 10^{-5}\ \frac{ft^2}{s}$$

$$\sigma_p = 0.00498\ \frac{lbf}{ft}$$

From the Euler number, we have the pressure difference for the prototype

$$\Delta p_p = 14.5\, psia - 0.36\, psia = 14.14\, psi$$

And for the model the pressure difference is:

$$\Delta p_m = 2.0\, psia - 1.5\, psia = 0.5\, psi$$

From Euler number similarity we have for the density we have:

$$\rho_m = \rho_p \times \left(\frac{\Delta p_m}{\Delta p_p}\right) \quad \times \left(\frac{V_p}{V_m}\right)^2$$

And from the Froude number similarity we have

$$\frac{V_p}{V_m} = \left(\frac{L_p}{L_m}\right)^{1/2}$$

So the density for the model can be calculated as

$$\rho_m = \rho_p \times \left(\frac{\Delta p_m}{\Delta p_p}\right) \quad \times \left(\frac{L_p}{L_m}\right) = 1.934\ \frac{slug}{ft^3} \times \frac{0.5\, psi}{14.14\, psi} \times \frac{30}{1}$$

$$\rho_m = 2.05\ \frac{slug}{ft^3}$$

From Reynolds number similarity we have for the kinematic viscosity, where we use the relation for the velocities in terms of lengths from the Froude number

$$\nu_m = \nu_p \times \left(\frac{V_m}{V_p}\right) \quad \times \left(\frac{L_m}{L_p}\right) = \left(\frac{L_m}{L_p}\right)^{3/2} = 1.059 \times 10^{-5}\ \frac{ft^2}{s} \times \left(\frac{1}{30}\right)^{3/2}$$

$$\nu_m = 6.44 \times 10^{-8}\ \frac{ft^2}{s}$$

With the density of the model fluid, we have the viscosity

$$\mu_m = \nu\, \rho = 6.44 \times 10^{-8}\ \frac{ft^2}{s} \times 2.05\ \frac{slug}{ft^3} = 1.32 \times 10^{-7}\ \frac{lbf \cdot s}{ft^2}$$

From the Weber number similarity we will get the surface tension:

$$\left(\frac{\rho V^2 L}{\sigma}\right)_m = \left(\frac{\rho V^2 L}{\sigma}\right)_p$$

Thus

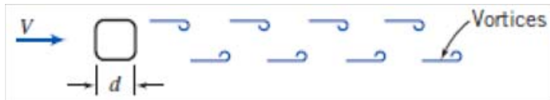
$$\sigma_m = \sigma_p \times \left(\frac{\rho_m}{\rho_p}\right) \times \left(\frac{V_m}{V_p}\right)^2 \times \left(\frac{L_m}{L_p}\right) = \sigma_p \times \left(\frac{\rho_m}{\rho_p}\right) \times \left(\frac{L_m}{L_p}\right)^2$$

$$\sigma_m == 0.00498\ \frac{lbf}{ft} \times \left(\frac{1}{30}\right)^2 = 5.86 \times 10^{-6}\ \frac{lbf}{ft}$$

Problem 7.55

[Difficulty: 3]

7.55 In some speed ranges, vortices are shed from the rear of bluff cylinders placed across a flow. The vortices alternately leave the top and bottom of the cylinder, as shown, causing an alternating force normal to the freestream velocity. The vortex shedding frequency, f, is thought to depend on ρ, d, V, and μ. Use dimensional analysis to develop a functional relationship for f. Vortex shedding occurs in standard air on two cylinders with a diameter ratio of 2. Determine the velocity ratio for dynamic similarity, and the ratio of vortex shedding frequencies.



Given: The frequency of vortex shedding from the rear of a bluff cylinder is a function of ρ, d, V, and μ. Vortex shedding occurs in standard air on two cylinders with a diameter ratio of 2.

Find:
(a) Functional relationship for f using dimensional analysis
(b) Velocity ratio for vortex shedding
(c) Frequency ratio for vortex shedding

Solution: We will use the Buckingham pi-theorem.

1 $f \quad \rho \quad d \quad V \quad \mu$ $\qquad n = 5$ parameters

2 Select primary dimensions F, L, t:

3 $f \quad \rho \quad d \quad V \quad \mu$

$\frac{1}{t} \quad \frac{M}{L^3} \quad L \quad \frac{L}{t} \quad \frac{M}{L\cdot t}$ $\qquad r = 3$ dimensions

4 $\rho \quad V \quad d$ $\qquad m = r = 3$ repeating parameters

5 We have n - m = 2 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = f\cdot\rho^a\cdot V^b\cdot d^c \qquad \text{Thus:} \quad \frac{1}{t}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents:

M: $a = 0$

L: $-3\cdot a + b + c = 0$

t: $-1 - b = 0$

The solution to this system is:

$a = 0 \qquad b = -1 \qquad c = 1$

$$\Pi_1 = \frac{f\cdot d}{V}$$

$$\Pi_2 = \mu\cdot\rho^a\cdot V^b\cdot d^c \qquad \text{Thus:} \quad \frac{M}{L\cdot t}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents:

M: $1 + a = 0$

L: $-1 - 3\cdot a + b + c = 0$

t: $-1 - b = 0$

The solution to this system is:

$a = -1 \qquad b = -1 \qquad c = -1$

$$\Pi_2 = \frac{\mu}{\rho\cdot V\cdot d}$$

6 Check using F, L, t dimensions: $\frac{1}{t}\cdot\frac{t}{L}\cdot L = 1 \qquad \frac{F\cdot t}{L^2}\cdot\frac{L^4}{F\cdot t^2}\cdot\frac{t}{L}\cdot\frac{1}{L} = 1$

The functional relationship is: $\Pi_1 = f(\Pi_2)$ $\frac{f \cdot d}{V} = f\left(\frac{\rho \cdot V \cdot d}{\mu}\right)$

To achieve dynamic similarity between geometrically similar flows, we must duplicate all but one of the dimensionless groups:

$$\frac{\rho_1 \cdot V_1 \cdot d_1}{\mu_1} = \frac{\rho_2 \cdot V_2 \cdot d_2}{\mu_2} \qquad \frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} \cdot \frac{d_2}{d_1} \cdot \frac{\mu_1}{\mu_2} = 1 \times \frac{1}{2} \times 1 \qquad \frac{V_1}{V_2} = \frac{1}{2}$$

Now if $\frac{\rho_1 \cdot V_1 \cdot d_1}{\mu_1} = \frac{\rho_2 \cdot V_2 \cdot d_2}{\mu_2}$ it follows that: $\frac{f_1 \cdot d_1}{V_1} = \frac{f_2 \cdot d_2}{V_2}$ and $\frac{f_1}{f_2} = \frac{d_2}{d_1} \cdot \frac{V_1}{V_2} = \frac{1}{2} \times \frac{1}{2}$ $\frac{f_1}{f_2} = \frac{1}{4}$

Problem 7.56 [Difficulty: 3]

7.56 A $\frac{1}{8}$-scale model of a tractor-trailer rig is tested in a pressurized wind tunnel. The rig width, height, and length are $W = 0.305$ m, $H = 0.476$ m, and $L = 2.48$ m, respectively. At wind speed $V = 75.0$ m/s, the model drag force is $F_D = 128$ N. (Air density in the tunnel is $\rho = 3.23$ kg/m^3.) Calculate the aerodynamic drag coefficient for the model. Compare the Reynolds numbers for the model test and for the prototype vehicle at 55 mph. Calculate the aerodynamic drag force on the prototype vehicle at a road speed of 55 mph into a headwind of 10 mph.

Given: 1/8-scale model of a tractor-trailer rig was tested in a pressurized wind tunnel.

Find:
(a) Aerodynamic drag coefficient for the model
(b) Compare the Reynolds numbers for the model and the prototype vehicle at 55 mph
(c) Calculate aerodynamic drag on the prototype at a speed of 55 mph into a headwind of 10 mph

Solution: We will use definitions of the drag coefficient and Reynolds number.

Governing Equations:

$$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A} \qquad \text{(Drag Coefficient)}$$

$$Re = \frac{\rho\cdot V\cdot L}{\mu} \qquad \text{(Reynolds Number)}$$

Assume that the frontal area for the model is: $A_m = W_m\cdot H_m \quad A_m = 0.305\cdot m \times 0.476\cdot m \qquad A_m = 0.1452\cdot m^2$

The drag coefficient would then be: $C_{Dm} = 2 \times 128\cdot N \times \frac{m^3}{3.23\cdot kg} \times \left(\frac{s}{75.0\cdot m}\right)^2 \times \frac{1}{0.1452\cdot m^2} \times \frac{kg\cdot m}{N\cdot s^2} \qquad C_{Dm} = 0.0970$

From the definition of Re: $\frac{Re_m}{Re_p} = \frac{\rho_m}{\rho_p}\cdot\frac{V_m}{V_p}\cdot\frac{L_m}{L_p}\cdot\frac{\mu_p}{\mu_m}$ Assuming standard conditions and equal viscosities:

$$\frac{Re_m}{Re_p} = \frac{3.23}{1.23} \times \left(75\cdot\frac{m}{s} \times \frac{hr}{55\cdot mi} \times \frac{mi}{5280\cdot ft} \times \frac{ft}{0.3048\cdot m} \times \frac{3600\cdot s}{hr}\right) \times \frac{1}{8} \times 1 = 1 \qquad Re_m = Re_p$$

Since the Reynolds numbers match, assuming geometric and kinetic similarity we can say that the drag coefficients are equal:

$F_{Dp} = \frac{1}{2}\cdot C_D\cdot\rho_p\cdot V_p^{\,2}\cdot A_p$ Susbstituting known values yields:

$$F_{Dp} = \frac{1}{2} \times 0.0970 \times 1.23\cdot\frac{kg}{m^3} \times \left[(55 + 10)\frac{mi}{hr} \times \frac{5280\cdot ft}{mi} \times \frac{0.3048\cdot m}{ft} \times \frac{hr}{3600\cdot s}\right]^2 \times 0.1452\cdot m^2 \times 8^2 \times \frac{N\cdot s^2}{kg\cdot m} \qquad F_{Dp} = 468\,N$$

Problem 7.57

[Difficulty: 2]

7.57 On a cruise ship, passengers complain about the amount of smoke that becomes entrained behind the cylindrical smoke stack. You have been hired to study the flow pattern around the stack, and have decided to use a 1:15 scale model of the 15-ft smoke stack. What range of wind tunnel speeds could you use if the ship speed for which the problem occurs is 12 to 24 knots?

Given: Flow around cruise ship smoke stack

Find: Range of wind tunnel speeds

Solution:

For dynamic similarity $\frac{V_m \cdot D_m}{\nu_m} = \frac{V_p \cdot D_p}{\nu_p}$ or $V_m = \frac{D_p}{D_m} \cdot V_p = \frac{15}{1} \cdot V_p = 15 \cdot V_p$

Since $1 \cdot \text{knot} = 1 \cdot \frac{\text{nmi}}{\text{hr}}$ and $1 \cdot \text{nmi} = 6076.1 \cdot \text{ft}$

Hence for

$$V_p = 12 \cdot \frac{\text{nmi}}{\text{hr}} \times \frac{6076.1 \cdot \text{ft}}{\text{nmi}} \times \frac{\text{hr}}{3600 \cdot \text{s}} \qquad V_p = 20.254 \cdot \frac{\text{ft}}{\text{s}} \qquad V_m = 15 \times 20.254 \cdot \frac{\text{ft}}{\text{s}} \qquad V_m = 304 \cdot \frac{\text{ft}}{\text{s}}$$

$$V_p = 24 \cdot \frac{\text{nmi}}{\text{hr}} \times \frac{6076.1 \cdot \text{ft}}{\text{nmi}} \times \frac{\text{hr}}{3600 \cdot \text{s}} \qquad V_p = 40.507 \cdot \frac{\text{ft}}{\text{s}} \qquad V_m = 15 \times 40.507 \cdot \frac{\text{ft}}{\text{s}} \qquad V_m = 608 \cdot \frac{\text{ft}}{\text{s}}$$

Note that these speeds are very high - compressibility effects may become important, since the Mach number is no longer much less than 1!

Problem 7.58

(Difficulty 3)

7.58 When a sphere of 0.25 mm diameter and specific gravity 5.54 is dropped in water at 25 ℃ it will attain a constant velocity of 0.07 $\frac{m}{s}$. What specific gravity must a 2.5 mm sphere have so that when it is dropped in crude oil (25 ℃) the two flows will be dynamically similar when the terminal velocity is attained ?

Find: The specific gravity for the prototype SG_p.

Solution:

For the dynamic similarity with a falling sphere, we will have dynamic similarity if the Reynolds numbers are equal:

$$\left(\frac{\rho V d}{\mu}\right)_m = \left(\frac{\rho V d}{\mu}\right)_p$$

The model falls in water and the properties are

$$\rho_m = 998\ \frac{kg}{m^3}$$

$$\mu_m = 0.89 \times 10^{-3}\ Pa \cdot s$$

The properties of the sphere are

$$d_m = 0.25\ mm$$

$$SG_m = 5.54$$

And the terminal velocity is

$$V_m = 0.07\ \frac{m}{s}$$

The prototype falls through crude oil and the properties are

$$\rho_p = 858\ \frac{kg}{m^3}$$

$$\mu_p = 6.8 \times 10^{-3}\ Pa \cdot s$$

The prototype sphere diameter is

$$d_p = 2.5\ mm$$

The prototype sphere velocity is then computed using the equality of the Reynolds numbers

$$V_p = V_m \times \left(\frac{\rho_m}{\rho_p}\right) \times \left(\frac{L_m}{L_p}\right) \times \left(\frac{\mu_p}{\mu_m}\right) = V_m = 0.07\ \frac{m}{s} \times \left(\frac{998\ \frac{kg}{m^3}}{858\ \frac{kg}{m^3}}\right) \times \left(\frac{0.25\ mm}{2.5\ mm}\right) \times \left(\frac{6.8 \times 10^{-3}\ Pa \cdot s}{0.89 \times 10^{-3}\ Pa \cdot s}\right)$$

$$V_p = 0.062\ \frac{m}{s}$$

At the terminal velocity condition, the buoyance force and drag force will balance the weight:

$$W - F_B = D$$

We don't know what the drag force is, but we know how the difference between the weight and the buoyancy force for the model is related to the properties of the sphere and the water:

$$(W - F_B)_m \propto (SG_m \rho_{water} - \rho_{water}) g d_m^3$$

And for the prototype, the same realtion is true:

$$(W - F_B)_p = \left(SG_p \rho_{water} - \rho_{oil}\right) g d_p^3$$

For dynamic similarity, the crag coefficients are equal

$$\left(\frac{D}{\rho V^2 L^2}\right)_m = \left(\frac{D}{\rho V^2 L^2}\right)_p$$

Or, with the relation for the drag in terms of the sphere and fluid properties:

$$\frac{(SG_m \rho_{water} - \rho_{water}) g d_m^3}{\rho_m V_m^2 d_m} = \frac{\left(SG_p \rho_{water} - \rho_{oil}\right) g d_p^3}{\rho_p V_p^2 d_p}$$

The specific gravity of the sphere is then:

$$SG_p = \frac{1}{\rho_{water}} \left(\rho_{oil} + (SG_m \rho_{water} - \rho_{water}) \frac{d_m^2}{d_p^2}\ \frac{\rho_p V_p^2}{\rho_m V_m^2}\right)$$

Which yields the value for specific gravity as

$$SG_p = 1.166$$

Problem 7.59

(Difficulty 2)

7.59 The flow about a 150 mm artillery projectile which travels at 600 $\frac{m}{s}$ through still air at 30 ℃ and absolute pressure 101.4 kPa is to be modeled in a high-speed wind tunnel with a 1: 6 model. If the wind tunnel air has a temperature of −18 ℃ and absolute pressure of 68.9 kPa, what velocity is required? If the drag force on the model is 35 N, what is the drag force on the prototype if skin friction may be neglected?

Find: The velocity V_m and the drag force D_p.

For dynamic similarity for high speed flow, the Mach numbers are equal:

$$\frac{V_m}{c_m} = \frac{V_p}{c_p}$$

We calculate the sound speed from the relation for ideal gases.

$$c = \sqrt{kRT}$$

For the model, the speed of sound is

$$T_m = -18\ ℃$$

$$c_m = \sqrt{1.4 \times (286.8) \times (273 - 18)}\ \frac{m}{s} = 320\ \frac{m}{s}$$

And for the prototype

$$T_p = 30\ ℃$$

$$c_p = \sqrt{1.4 \times (286.8) \times (273 + 30)}\ \frac{m}{s} = 349\frac{m}{s}$$

Under Mach number dynamic similarity, the model velocity is

$$V_m = V_p \frac{c_m}{c_p} = 600\ \frac{m}{s} \times \frac{320\ \frac{m}{s}}{349\frac{m}{s}}$$

$$V_m = 550\ \frac{m}{s}$$

For dynamic similarity, the drag coefficients are equal:

$$\left(\frac{D}{\rho V^2 d^2}\right)_m = \left(\frac{D}{\rho V^2 d^2}\right)_p$$

Or, the drag for the model is given by

$$D_p = D_m \times \left(\frac{d_p}{d_m}\right)^2 \times \left(\frac{V_p}{V_m}\right)^2 \times \left(\frac{\rho_p}{\rho_m}\right)$$

The model is a 1:6 scale model so the model diameter is

$$d_m = d_p \frac{1}{6} = 150\ mm \times \frac{1}{6} = 25\ mm$$

The densities of the air for the model an prototype depend on the pressure and temperature according to the ideal gas law

$$\rho = \frac{p}{RT}$$

$$\rho_m = \frac{68900}{286.8 \times (-18 + 273)}\ \frac{kg}{m^3} = 0.942\ \frac{kg}{m^3}$$

$$\rho_p = \frac{101400}{286.8 \times (30 + 273)}\ \frac{kg}{m^3} = 1.167\ \frac{kg}{m^3}$$

Thus the drag force on the prototype is

$$D_p = 35N \times \left(\frac{150\ mm}{25\ mm}\right)^2 \times \left(\frac{600\ \frac{m}{s}}{550\ \frac{m}{s}}\right)^2 \times \left(\frac{1.167\ \frac{kg}{m^3}}{0.942\ \frac{kg}{m^3}}\right)$$

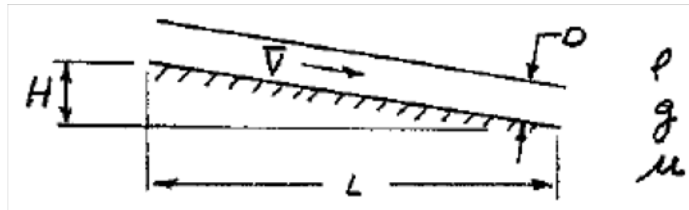
Thus

$$D_p = 1858\ N$$

Problem 7.60

[Difficulty: 4]

7.60 Your favorite professor likes mountain climbing, so there is always a possibility that the professor may fall into a crevasse in some glacier. If that happened today, and the professor was trapped in a slowly moving glacier, you are curious to know whether the professor would reappear at the downstream drop-off of the glacier during this academic year. Assuming ice is a Newtonian fluid with the density of glycerine but a million times as viscous, you decide to build a glycerin model and use dimensional analysis and similarity to estimate when the professor would reappear. Assume the real glacier is 15 m deep and is on a slope that falls 1.5 m in a horizontal distance of 1850 m. Develop the dimensionless parameters and conditions expected to govern dynamic similarity in this problem. If the model professor reappears in the laboratory after 9.6 hours, when should you return to the end of the real glacier to provide help to your favorite professor?



Given: Model the motion of a glacier using glycerine. Assume ice as Newtonian fluid with density of glycerine but one million times as viscous. In laboratory test the professor reappears in 9.6 hours.

Find: (a) Dimensionless parameters to characterize the model test results
(b) Time needed for professor to reappear

Solution: We will use the Buckingham pi-theorem.

1 $V \quad \rho \quad g \quad \mu \quad D \quad H \quad L$ $\qquad$ n = 7 parameters

2 Select primary dimensions F, L, t:

3 $V \quad \rho \quad g \quad \mu \quad D \quad H \quad L$

$$\frac{L}{t} \quad \frac{M}{L^3} \quad \frac{L}{t^2} \quad \frac{M}{L\cdot t} \quad L \quad L \quad L$$

r = 3 dimensions

4 $\rho \quad g \quad D$ $\qquad$ m = r = 3 repeating parameters

5 We have n - m = 4 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = V\cdot\rho^a\cdot g^b\cdot D^c \qquad \text{Thus:} \quad \frac{L}{t}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t^2}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

M: $a = 0$ $\qquad$ $a = 0 \quad b = -\frac{1}{2} \quad c = -\frac{1}{2}$

L: $1 - 3\cdot a + b + c = 0$

t: $-1 - 2\cdot b = 0$

$$\Pi_1 = \frac{V}{\sqrt{g\cdot D}}$$

$$\Pi_2 = \mu\cdot\rho^a\cdot g^b\cdot D^c \qquad \text{Thus:} \quad \frac{M}{L\cdot t}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{L}{t^2}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

M: $1 + a = 0$ $\qquad$ $a = -1 \quad b = -\frac{1}{2} \quad c = -\frac{3}{2}$

L: $-1 - 3\cdot a + b + c = 0$

t: $-1 - 2\cdot b = 0$

$$\Pi_2 = \frac{\mu}{\rho\sqrt{g\cdot D^3}}$$

(This is a gravity-driven version of Reynolds #)

$\Pi_3 = H \cdot \rho^a \cdot g^b \cdot D^c$ Thus: $L \cdot \left(\frac{M}{L^3}\right)^a \cdot \left(\frac{L}{t^2}\right)^b \cdot L^c = M^0 \cdot L^0 \cdot t^0$

Summing exponents:

M: $a = 0$

L: $-1 - 3 \cdot a + b + c = 0$

t: $-2 \cdot b = 0$

The solution to this system is:

$a = 0 \quad b = 0 \quad c = -1$

$$\Pi_3 = \frac{H}{D}$$

By inspection we can see that $\Pi_4 = \frac{L}{D}$

6 Check using F, L, t dimensions: $\frac{L}{t} \cdot \frac{t}{\frac{1}{L^{\frac{1}{2}}}} \cdot \frac{1}{L^{\frac{1}{2}}} = 1$ $\frac{F \cdot t}{L^2} \cdot \frac{L^4}{F \cdot t^2} \cdot \frac{t}{L^{\frac{1}{2}}} \cdot \frac{1}{L^{\frac{3}{2}}} = 1$ $L \cdot \frac{1}{L} = 1$

The functional relationship would be: $\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4)$ Matching the last two terms insures geometric similarity.

For dynamic similarity: $\frac{\mu_m}{\rho_m \cdot \sqrt{g_m \cdot D_m^3}} = \frac{\mu_p}{\rho_p \cdot \sqrt{g_p \cdot D_p^3}}$ From Tables A.1 and A.2: $SG_{ice} = 0.92$ $SG_{glycerine} = 1.26$

Therefore: $\frac{D_m}{D_p} = \left(\frac{\mu_m}{\mu_p} \cdot \frac{\rho_p}{\rho_m}\right)^{\frac{2}{3}} = \left(\frac{1}{10^6} \times \frac{0.92}{1.26}\right)^{\frac{2}{3}} = 8.11 \times 10^{-5}$ Since we have geometric similarity, the last two terms must match for model and prototype:

So $\frac{L_m}{L_p} = 8.11 \times 10^{-5}$ $L_m = 1850 \cdot m \times 8.11 \times 10^{-5}$ Matching the first Π term: $\frac{V_m}{V_p} = \sqrt{\frac{D_m}{D_p}} = 0.00900$

$L_m = 0.1500\,m$

The time needed to reappear would be: $\tau = \frac{L}{V}$ Thus: $\tau_m = \frac{L_m}{V_m}$ $V_m = \frac{L_m}{\tau_m}$ Solving for the actual time:

$$\tau_p = \frac{L_p}{V_p} = \frac{L_m}{V_m} \cdot \frac{L_p}{L_m} \cdot \frac{V_m}{V_p} = \tau_m \cdot \frac{L_p}{L_m} \cdot \frac{V_m}{V_p} \quad \tau_p = 9.6 \cdot hr \times \frac{1}{8.11 \cdot 10^{-5}} \times 0.00900 \times \frac{day}{24 \cdot hr}$$

$$\tau_p = 44.4 \cdot day$$

Your professor will be back before the end of the semester!

Problem 7.61 [Difficulty: 3]

7.61 A 1:50-scale model of a submarine is to be tested in a towing tank under two conditions: motion at the free surface and motion far below the surface. The tests are performed in freshwater. On the surface, the submarine cruises at 24 knots. At what speed should the model be towed to ensure dynamic similarity? Far below the surface, the sub cruises at 0.35 knot. At what speed should the model be towed to ensure dynamic similarity? What must the drag of the model be multiplied by under each condition to give the drag of the full-scale submarine?

Given: A scale model of a submarine is to be tested in fresh water under two conditions:
1 - on the surface
2 - far below the surface

Find: (a) Speed for the model test on the surface
(b) Speed for the model test submerged
(c) Ratio of full-scale drag to model drag

Solution: On the surface, we need to match Froude numbers: $\dfrac{V_m}{\sqrt{g \cdot L_m}} = \dfrac{V_p}{\sqrt{g \cdot L_p}}$ or: $V_m = V_p \cdot \sqrt{\dfrac{L_m}{L_p}}$

Thus for 1:50 scale: $V_m = 24 \cdot \text{knot} \times \sqrt{\dfrac{1}{50}}$ $\qquad V_m = 3.39 \cdot \text{knot}$ or $V_m = 1.75 \cdot \dfrac{m}{s}$

When submerged, we need to match Reynolds numbers: $\dfrac{\rho_m \cdot V_m \cdot L_m}{\mu_m} = \dfrac{\rho_p \cdot V_p \cdot L_p}{\mu_p}$ or: $V_m = V_p \cdot \dfrac{\rho_p}{\rho_m} \cdot \dfrac{L_p}{L_m} \cdot \dfrac{\mu_m}{\mu_p}$

From Table A.2, $SG_{seawater} = 1.025$ and $\mu_{seawater} = 1.08 \times 10^{-3} \cdot \dfrac{N \cdot s}{m^2}$ at 20ºC. Thus for 1:50 scale:

$V_m = 0.35 \cdot \text{knot} \times \dfrac{1.025}{0.998} \times \dfrac{50}{1} \times \dfrac{1.08 \times 10^{-3}}{1.00 \times 10^{-3}}$ $\qquad V_m = 19.41 \cdot \text{knot}$ or $V_m = 9.99 \cdot \dfrac{m}{s}$

Under dynamically similar conditions, the drag coefficients will match: $\dfrac{F_{Dm}}{\frac{1}{2} \cdot \rho_m \cdot V_m^2 \cdot A_m} = \dfrac{F_{Dp}}{\frac{1}{2} \cdot \rho_p \cdot V_p^2 \cdot A_p}$

Solving for the ratio of forces: $\dfrac{F_{Dp}}{F_{Dm}} = \dfrac{\rho_p}{\rho_m} \cdot \left(\dfrac{V_p}{V_m}\right)^2 \cdot \dfrac{A_p}{A_m} = \dfrac{\rho_p}{\rho_m} \cdot \left(\dfrac{V_p}{V_m} \cdot \dfrac{L_p}{L_m}\right)^2$ Substituting in known values:

For surface travel: $\dfrac{F_{Dp}}{F_{Dm}} = \dfrac{1.025}{0.998} \times \left(\dfrac{24}{3.39} \times \dfrac{50}{1}\right)^2 = 1.29 \times 10^5$ $\qquad \dfrac{F_{Dp}}{F_{Dm}} = 1.29 \times 10^5$ (on surface)

For submerged travel: $\dfrac{F_{Dp}}{F_{Dm}} = \dfrac{1.025}{0.998} \times \left(\dfrac{0.35}{19.41} \times \dfrac{50}{1}\right)^2 = 0.835$ $\qquad \dfrac{F_{Dp}}{F_{Dm}} = 0.835$ (submerged)

Problem 7.62 [Difficulty: 3]

7.62 Consider water flow around a circular cylinder, of diameter D and length l. In addition to geometry, the drag force is known to depend on liquid speed, V, density, ρ, and viscosity, μ. Express drag force, F_D, in dimensionless form as a function of all relevant variables. The static pressure distribution on a circular cylinder, measured in the laboratory, can be expressed in terms of the dimensionless pressure coefficient; the lowest pressure coefficient is $C_p = -2.4$ at the location of the minimum static pressure on the cylinder surface. Estimate the maximum speed at which a cylinder could be towed in water at atmospheric pressure, without causing cavitation, if the onset of cavitation occurs at a cavitation number of 0.5.

Given: The drag force on a circular cylinder immersed in a water flow can be expressed as a function of D, l, V, ρ, and μ. Static pressure distribution can be expressed in terms of the pressure coefficient. At the minimum static pressure, the pressure coefficient is equal to -2.4. Cavitation onset occurs at a cavitation number of 0.5.

Find: (a) Drag force in dimensionless form as a function of all relevant variables
(b) Maximum speed at which a cylinder could be towed in water at atmospheric pressure without cavitation

Solution: The functional relationship for drag force is: $F_D = F_D(D, l, V, \rho, \mu)$ From the Buckingham Π-theorem, we have 6 variables and 3 repeating parameters. Therefore, we will have 3 dimensionless groups. The functional form of these groups is:

$$\frac{F_D}{\rho \cdot V^2 \cdot D^2} = g\left(\frac{l}{D}, \frac{\rho \cdot V \cdot D}{\mu}\right)$$

The pressure coefficient is: $C_P = \dfrac{p - p_{inf}}{\frac{1}{2} \cdot \rho \cdot V^2}$ and the cavitation number is: $Ca = \dfrac{p - p_v}{\frac{1}{2} \cdot \rho \cdot V^2}$

At the minimum pressure point $p_{min} = p_{inf} + \frac{1}{2} \cdot \rho \cdot V_{max}{}^2 \cdot C_{Pmin}$ where $C_{Pmin} = -2.4$

At the onset of cavitation $p_{min} = p_v + \frac{1}{2} \cdot \rho \cdot V_{max}{}^2 \cdot Ca$ where $Ca = 0.5$

Equating these two expressions: $p_{inf} + \frac{1}{2} \cdot \rho \cdot V_{max}{}^2 \cdot C_{Pmin} = p_v + \frac{1}{2} \cdot \rho \cdot V_{max}{}^2 \cdot Ca$ and if we solve for Vmax:

$V_{max} = \sqrt{\dfrac{2(p_{inf} - p_v)}{\rho \cdot (Ca - C_{Pmin})}}$ At room temperature (68 deg F): $p_v = 0.339 \cdot psi$ $\rho = 1.94 \cdot \dfrac{slug}{ft^3}$

Substituting values we get:

$$V_{max} = \sqrt{2 \times (14.7 - 0.339) \cdot \frac{lbf}{in^2} \times \frac{ft^3}{1.94 \cdot slug} \times \frac{1}{[0.5 - (-2.4)]} \times \frac{slug \cdot ft}{lbf \cdot s^2} \times \frac{144 \cdot in^2}{ft^2}} \qquad V_{max} = 27.1 \cdot \frac{ft}{s}$$

Problem 7.63 [Difficulty: 3]

7.63 A $\frac{1}{10}$-scale model of a tractor-trailer rig is tested in a wind tunnel. The model frontal area is $A_m = 0.1\ \text{m}^2$. When tested at $V_m = 75$ m/s in standard air, the measured drag force is $F_D = 350$ N. Evaluate the drag coefficient for the model conditions given. Assuming that the drag coefficient is the same for model and prototype, calculate the drag force on a prototype rig at a highway speed of 90 km/hr. Determine the air speed at which a model should be tested to ensure dynamically similar results if the prototype speed is 90 km/hr. Is this air speed practical? Why or why not?

Given: Model of tractor-trailer truck

Find: Drag coefficient; Drag on prototype; Model speed for dynamic similarity

Solution:

For kinematic similarity we need to ensure the geometries of model and prototype are similar, as is the incoming flow field

The drag coefficient is $$C_D = \frac{F_m}{\frac{1}{2}\cdot\rho_m\cdot V_m^2\cdot A_m}$$

For air (Table A.10) at 20°C $$\rho_m = 1.21\cdot\frac{kg}{m^3} \qquad \mu_p = 1.81\times 10^{-5}\cdot\frac{N\cdot s}{m^2}$$

$$C_D = 2\times 350\cdot N\times\frac{m^3}{1.21\cdot kg}\times\left(\frac{s}{75\cdot m}\right)^2\times\frac{1}{0.1\cdot m^2}\times\frac{N\cdot s^2}{kg\cdot m} \qquad C_D = 1.028$$

This is the drag coefficient for model and prototype

For the rig $$F_p = \frac{1}{2}\cdot\rho_p\cdot V_p^2\cdot A_p\cdot C_D \quad \text{with} \quad \frac{A_p}{A_m} = \left(\frac{L_p}{L_m}\right)^2 = 100 \qquad A_p = 10\cdot m^2$$

$$F_p = \frac{1}{2}\times 1.21\cdot\frac{kg}{m^3}\times\left(90\cdot\frac{km}{hr}\times\frac{1000\cdot m}{1\cdot km}\times\frac{1\cdot hr}{3600\cdot s}\right)^2\times 10\cdot m^2\times 1.028\times\frac{N\cdot s^2}{kg\cdot m} \qquad F_p = 3.89\cdot kN$$

For dynamic similarity $$\frac{\rho_m\cdot V_m\cdot L_m}{\mu_m} = \frac{\rho_p\cdot V_p\cdot L_p}{\mu_p} \qquad V_m = V_p\cdot\frac{\rho_p}{\rho_m}\cdot\frac{L_p}{L_m}\cdot\frac{\mu_m}{\mu_p} = V_p\cdot\frac{L_p}{L_m}$$

$$V_m = 90\cdot\frac{km}{hr}\times\frac{1000\cdot m}{1\cdot km}\times\frac{1\cdot hr}{3600\cdot s}\times\frac{10}{1} \qquad V_m = 250\,\frac{m}{s}$$

For air at standard conditions, the speed of sound is $c = \sqrt{k\cdot R\cdot T}$

$$c = \sqrt{1.40\times 286.9\cdot\frac{N\cdot m}{kg\cdot K}\times(20+273)\cdot K\times\frac{kg\cdot m}{s^2\cdot N}} \qquad c = 343\,\frac{m}{s}$$

Hence we have $$M = \frac{V_m}{c} = \frac{250}{343} = 0.729$$ which indicates compressibility is significant - this model speed is impractical (and unnecessary)

Problem 7.64

[Difficulty: 4]

7.64 The power, $\mathscr{P}$, required to drive a fan is assumed to depend on fluid density ρ, volume flow rate Q, impeller diameter D, and angular speed ω. If a fan with $D_1 = 8$ in. delivers $Q_1 = 15$ ft^3/s of air at $\omega_1 = 2500$ rpm, what size diameter fan could be expected to deliver $Q_2 = 88$ ft^3/s of air at $\omega_2 = 1800$ rpm, provided they were geometrically and dynamically similar?

Given: Power to drive a fan is a function of ρ, Q, D, and ω.

Condition 1: $D_1 = 8\cdot in$ $\quad \omega_1 = 2500\cdot rpm$ $\quad Q_1 = 15\cdot\frac{ft^3}{s}$ $\qquad$ Condition 2: $Q_2 = 88\cdot\frac{ft^3}{s}$ $\quad \omega_2 = 1800\cdot rpm$

Find: Fan diameter for condition 2 to insure dynamic similarity

Solution: We will use the Buckingham pi-theorem.

1 $\quad$ P $\quad$ ρ $\quad$ Q $\quad$ D $\quad$ ω $\qquad$ n = 5 parameters

2 $\quad$ Select primary dimensions M, L, t:

3 $\quad$ P $\quad$ ρ $\quad$ Q $\quad$ D $\quad$ ω

$$\frac{M\cdot L^2}{t^3} \quad \frac{M}{L^3} \quad \frac{L^3}{t} \quad L \quad \frac{1}{t}$$

r = 3 dimensions

4 $\quad$ ρ $\quad$ D $\quad$ ω $\qquad$ m = r = 3 repeating parameters

5 $\quad$ We have n - m = 2 dimensionless groups. Setting up dimensional equations:

$\Pi_1 = P\cdot\rho^a\cdot D^b\cdot\omega^c$ $\qquad$ Thus: $\frac{M\cdot L^2}{t^3}\cdot\left(\frac{M}{L^3}\right)^a\cdot L^b\cdot\left(\frac{1}{t}\right)^c = M^0\cdot L^0\cdot t^0$

Summing exponents: $\qquad$ The solution to this system is:

M: $1 + a = 0$ $\qquad a = -1 \quad b = -5 \quad c = -3$ $\qquad \Pi_1 = \frac{P}{\rho\cdot\omega^3\cdot D^5}$

L: $2 - 3\cdot a + b = 0$

t: $-3 - c = 0$

$\Pi_2 = Q\cdot\rho^a\cdot D^b\cdot\omega^c$ $\qquad$ Thus: $\frac{L^3}{t}\cdot\left(\frac{M}{L^3}\right)^a\cdot L^b\cdot\left(\frac{1}{t}\right)^c = M^0\cdot L^0\cdot t^0$

Summing exponents: $\qquad$ The solution to this system is:

M: $a = 0$ $\qquad a = 0 \quad b = -3 \quad c = -1$ $\qquad \Pi_2 = \frac{Q}{\omega\cdot D^3}$

L: $3 - 3\cdot a + b = 0$

t: $-1 - c = 0$

6 $\quad$ Check using F, L, t dimensions: $\frac{F\cdot L}{t}\cdot\frac{L^4}{F\cdot t^2}\cdot\frac{1}{L^5}\cdot t^3 = 1 \quad \frac{L^3}{t}\cdot t\cdot\frac{1}{L^3} = 1$ $\qquad$ Thus the relationship is: $\frac{P}{\rho\cdot\omega^3\cdot D^5} = f\left(\frac{Q}{\omega\cdot D^3}\right)$

For dynamic similarity we must have geometric and kinematic similarity, and: $\frac{Q_1}{\omega_1\cdot D_1^{\,3}} = \frac{Q_2}{\omega_2\cdot D_2^{\,3}}$ $\quad$ Solving for D_2

$$D_2 = D_1\cdot\left(\frac{Q_2}{Q_1}\cdot\frac{\omega_1}{\omega_2}\right)^{\frac{1}{3}} \qquad D_2 = 8\cdot in \times\left(\frac{88}{15}\times\frac{2500}{1800}\right)^{\frac{1}{3}} \qquad D_2 = 16.10\cdot in$$

Problem 7.65

[Difficulty: 3]

7.65 Over a certain range of air speeds, V, the lift, F_L, produced by a model of a complete aircraft in a wind tunnel depends on the air speed, air density, ρ, and a characteristic length (the wing base chord length, $c = 150$ mm). The following experimental data is obtained for air at standard atmospheric conditions:

V (m/s)	10	15	20	25	30	35	40	45	50
F_L (N)	2.2	4.8	8.7	13.3	19.6	26.5	34.5	43.8	54

Plot the lift versus speed curve. By using *Excel* to perform a trendline analysis on this curve, generate and plot data for the lift produced by the prototype, which has a wing base chord length of 5 m, over a speed range of 75 m/s to 250 m/s.

Given: Data on model of aircraft

Find: Plot of lift vs speed of model; also of prototype

Solution:

V_m (m/s)	10	15	20	25	30	35	40	45	50
F_m (N)	2.2	4.8	8.7	13.3	19.6	26.5	34.5	43.8	54.0

This data can be fit to

$$F_m = \frac{1}{2}\cdot\rho\cdot A_m\cdot C_D\cdot V_m^2 \qquad \text{or} \qquad F_m = k_m\cdot V_m^2$$

From the trendline, we see that

$k_m = 0.0219$ N/(m/s)^2

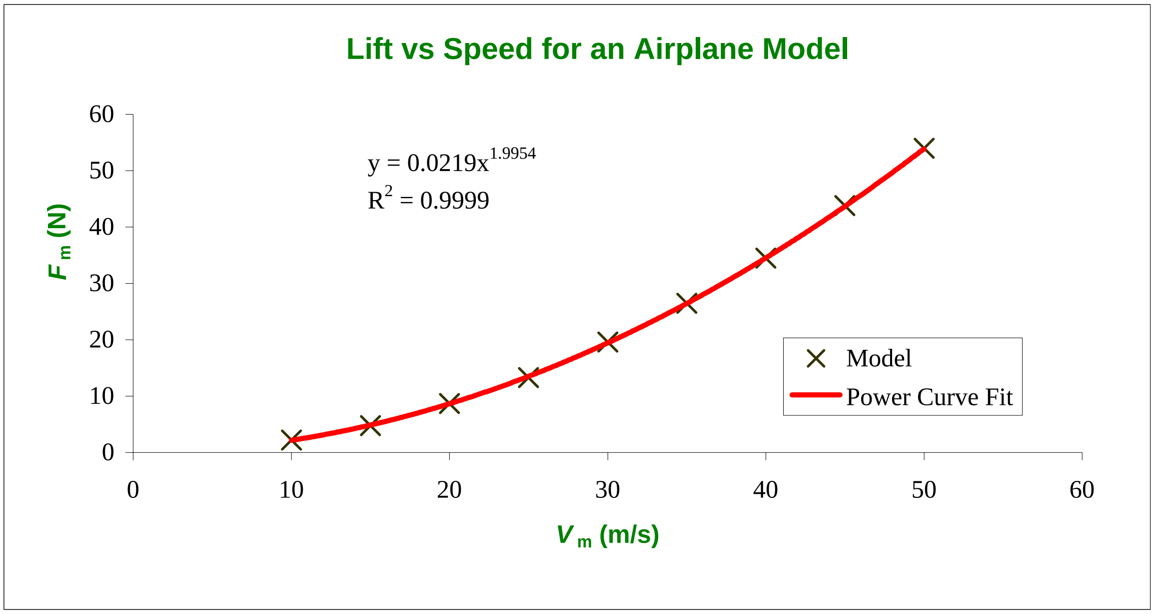
(And note that the power is 1.9954 or 2.00 to three signifcant figures, confirming the relation is quadratic)

Also, $k_p = 1110\ k_m$

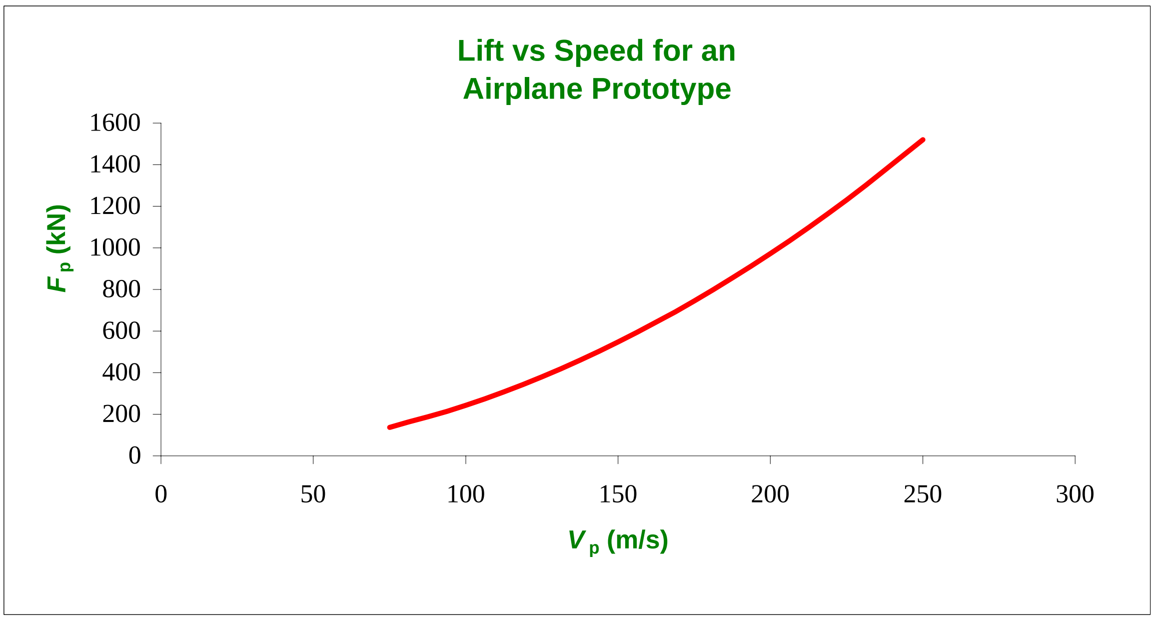
Hence,

$k_p =$ 24.3 N/(m/s)^2 $\qquad F_p = k_p V_m^2$

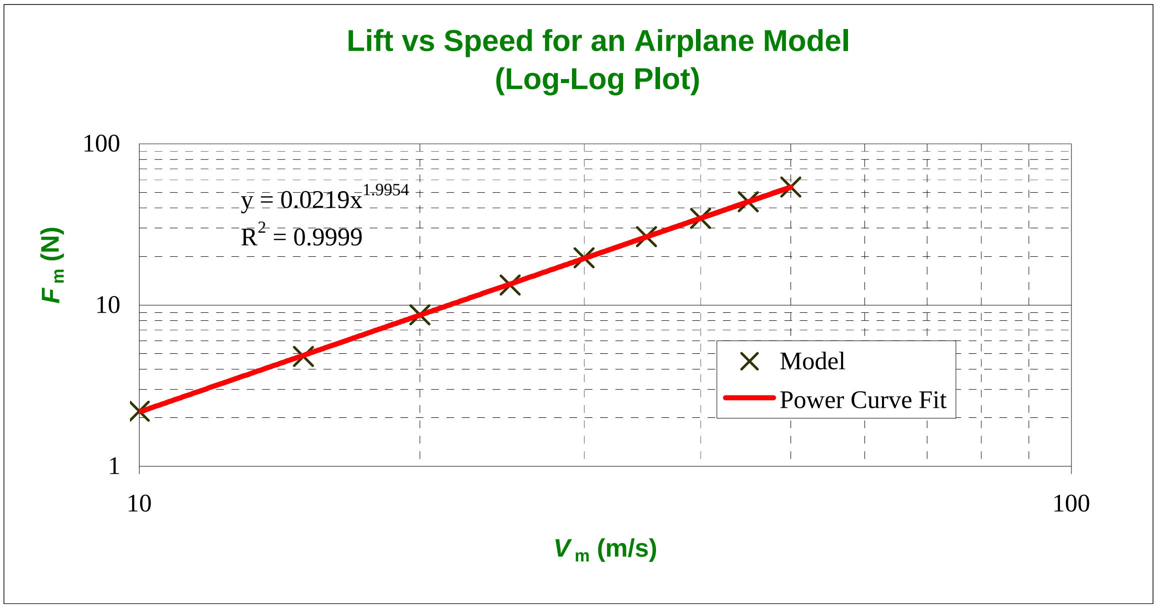
V_p (m/s)	75	100	125	150	175	200	225	250
F_p (kN) (Trendline)	137	243	380	547	744	972	1231	1519

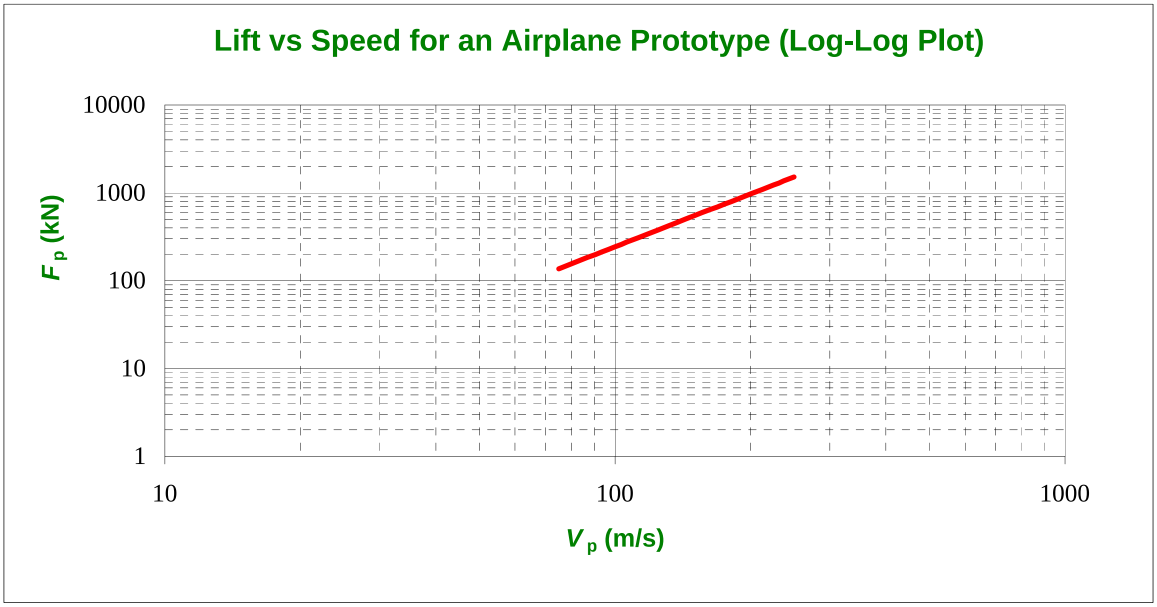


Lift vs Speed for an Airplane Model
y = 0.0219x^1.9954
R^2 = 0.9999
Model
Power Curve Fit
F m (N)
V m (m/s)
0
10
20
30
40
50
60



Lift vs Speed for an Airplane Prototype
F p (kN)
V p (m/s)
0
200
400
600
800
1000
1200
1400
1600
0
50
100
150
200
250
300


Lift vs Speed for an Airplane Model
(Log-Log Plot)
y = 0.0219x^1.9954
R^2 = 0.9999
Model
Power Curve Fit
100
10
1
10
100
F m (N)
V m (m/s)


Lift vs Speed for an Airplane Prototype (Log-Log Plot)
10000
1000
100
10
1
10
100
1000
F p (kN)
V p (m/s)

Problem 7.66 [Difficulty: 2]

7.66 The pressure rise, Δp, of a liquid flowing steadily through a centrifugal pump depends on pump diameter D, angular speed of the rotor ω, volume flow rate Q, and density ρ. The table gives data for the prototype and for a geometrically similar model pump. For conditions corresponding to dynamic similarity between the model and prototype pumps, calculate the missing values in the table.

Variable	Prototype	Model
Δp	52.5 kPa	
Q		0.0928 m^3/min
ρ	800 kg/m^3	999 kg/m^3
ω	183 rad/s	367 rad/s
D	150 mm	50 mm

Given: Information relating to geometrically similar model test for a centrifugal pump.

Find: The missing values in the table

Solution: We will use the Buckingham pi-theorem.

1 $\Delta p \quad Q \quad \rho \quad \omega \quad D$ n = 5 parameters

2 Select primary dimensions M, L, t:

3 $\Delta p \quad Q \quad \rho \quad \omega \quad D$

$$\frac{M}{L\cdot t^2} \quad \frac{L^3}{t} \quad \frac{M}{L^3} \quad \frac{1}{t} \quad L$$

r = 3 dimensions

4 $\rho \quad \omega \quad D$ m = r = 3 repeating parameters

5 We have n - m = 2 dimensionless groups. Setting up dimensional equations:

$$\Pi_1 = \Delta p\cdot\rho^a\cdot\omega^b\cdot D^c \qquad \text{Thus:} \quad \frac{M}{L\cdot t^2}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{1}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents: The solution to this system is:

M: $1 + a = 0$ $a = -1 \quad b = -2 \quad c = -2$ $\Pi_1 = \dfrac{\Delta p}{\rho\cdot\omega^2\cdot D^2}$

L: $-1 - 3\cdot a + c = 0$

t: $-2 - b = 0$

$$\Pi_2 = Q\cdot\rho^a\cdot\omega^b\cdot D^c \qquad \text{Thus:} \quad \frac{L^3}{t}\cdot\left(\frac{M}{L^3}\right)^a\cdot\left(\frac{1}{t}\right)^b\cdot L^c = M^0\cdot L^0\cdot t^0$$

Summing exponents: The solution to this system is:

M: $a = 0$ $a = 0 \quad b = -1 \quad c = -3$ $\Pi_2 = \dfrac{Q}{\omega\cdot D^3}$

L: $3 - 3\cdot a + c = 0$

t: $-1 - b = 0$

6 Check using F, L, t dimensions: $\dfrac{F}{L^2}\cdot\dfrac{L^4}{F\cdot t^2}\cdot t^2\cdot\dfrac{1}{L^2} = 1$ $\quad \dfrac{L^3}{t}\cdot t\cdot\dfrac{1}{L^3} = 1$ Thus the relationship is: $\dfrac{\Delta p}{\rho\cdot\omega^2\cdot D^2} = f\left(\dfrac{Q}{\omega\cdot D^3}\right)$

The flows are geometrically similar, and we assume kinematic similarity. Thus, for dynamic similarity:

$$\text{If} \quad \frac{Q_m}{\omega_m \cdot D_m^{\,3}} = \frac{Q_p}{\omega_p \cdot D_p^{\,3}} \quad \text{then} \quad \frac{\Delta p_m}{\rho_m \cdot \omega_m^{\,2} \cdot D_m^{\,2}} = \frac{\Delta p_p}{\rho_p \cdot \omega_p^{\,2} \cdot D_p^{\,2}}$$

From the first relation: $Q_p = Q_m \cdot \frac{\omega_p}{\omega_m} \cdot \left(\frac{D_p}{D_m}\right)^3$ $\quad Q_p = 0.0928 \cdot \frac{m^3}{min} \times \frac{183}{367} \times \left(\frac{150}{50}\right)^3$ $\quad Q_p = 1.249 \cdot \frac{m^3}{min}$

From the second relation: $\Delta p_m = \Delta p_p \cdot \frac{\rho_m}{\rho_p} \cdot \left(\frac{\omega_m}{\omega_p} \cdot \frac{D_m}{D_p}\right)^2$ $\quad \Delta p_m = 52.5 \cdot kPa \times \frac{999}{800} \times \left(\frac{367}{183} \times \frac{50}{150}\right)^2$ $\quad \Delta p_m = 29.3 \cdot kPa$

Problem 7.67

[Difficulty: 3]

7.67 An axial-flow pump is required to deliver 0.75 m³/s of water at a head of 15 J/kg. The diameter of the rotor is 0.25 m, and it is to be driven at 500 rpm. The prototype is to be modeled on a small test apparatus having a 2.25 kW, 1000 rpm power supply. For similar performance between the prototype and the model, calculate the head, volume flow rate, and diameter of the model.

Given: Model of water pump

Find: Model head, flow rate and diameter

Solution:

From Buckingham Π $\quad \dfrac{h}{\omega^2 \cdot D^2} = f\left(\dfrac{Q}{\omega \cdot D^3}, \dfrac{\rho \cdot \omega \cdot D^2}{\mu}\right)$ and $\dfrac{P}{\omega^3 \cdot D^5} = f\left(\dfrac{Q}{\omega \cdot D^3}, \dfrac{\rho \cdot \omega \cdot D^2}{\mu}\right)$

Neglecting viscous effects $\quad \dfrac{Q_m}{\omega_m \cdot D_m^{\,3}} = \dfrac{Q_p}{\omega_p \cdot D_p^{\,3}}$ then $\dfrac{h_m}{\omega_m^{\,2} \cdot D_m^{\,2}} = \dfrac{h_p}{\omega_p^{\,2} \cdot D_p^{\,2}}$ and $\dfrac{P_m}{\omega_m^{\,3} \cdot D_m^{\,5}} = \dfrac{P_p}{\omega_p^{\,3} \cdot D_p^{\,5}}$

Hence if $\quad \dfrac{Q_m}{Q_p} = \dfrac{\omega_m}{\omega_p} \cdot \left(\dfrac{D_m}{D_p}\right)^3 = \dfrac{1000}{500} \cdot \left(\dfrac{D_m}{D_p}\right)^3 = 2 \cdot \left(\dfrac{D_m}{D_p}\right)^3 \qquad (1)$

then $\quad \dfrac{h_m}{h_p} = \dfrac{\omega_m^{\,2}}{\omega_p^{\,2}} \cdot \dfrac{D_m^{\,2}}{D_p^{\,2}} = \left(\dfrac{1000}{500}\right)^2 \cdot \dfrac{D_m^{\,2}}{D_p^{\,2}} = 4 \cdot \dfrac{D_m^{\,2}}{D_p^{\,2}} \qquad (2)$

and $\quad \dfrac{P_m}{P_p} = \dfrac{\omega_m^{\,3}}{\omega_p^{\,3}} \cdot \dfrac{D_m^{\,5}}{D_p^{\,5}} = \left(\dfrac{1000}{500}\right)^3 \cdot \dfrac{D_m^{\,5}}{D_p^{\,5}} = 8 \cdot \dfrac{D_m^{\,5}}{D_p^{\,5}} \qquad (3)$

We can find P_p from $\quad P_p = \rho \cdot Q \cdot h = 1000 \cdot \dfrac{kg}{m^3} \times 0.75 \cdot \dfrac{m^3}{s} \times 15 \cdot \dfrac{J}{kg} = 11.25 \cdot kW$

From Eq 3 $\quad \dfrac{P_m}{P_p} = 8 \cdot \dfrac{D_m^{\,5}}{D_p^{\,5}}$ so $D_m = D_p \cdot \left(\dfrac{1}{8} \cdot \dfrac{P_m}{P_p}\right)^{\frac{1}{5}}$ $\quad D_m = 0.25 \cdot m \times \left(\dfrac{1}{8} \times \dfrac{2.25}{11.25}\right)^{\frac{1}{5}}$ $\quad D_m = 0.120\,m$

From Eq 1 $\quad \dfrac{Q_m}{Q_p} = 2 \cdot \left(\dfrac{D_m}{D_p}\right)^3$ so $Q_m = Q_p \cdot 2 \cdot \left(\dfrac{D_m}{D_p}\right)^3$ $\quad Q_m = 0.75 \cdot \dfrac{m^3}{s} \times 2 \times \left(\dfrac{0.12}{0.25}\right)^3$ $\quad Q_m = 0.166 \dfrac{m^3}{s}$

From Eq 2 $\quad \dfrac{h_m}{h_p} = 4 \cdot \left(\dfrac{D_m}{D_p}\right)^2$ so $h_m = h_p \cdot 4 \cdot \left(\dfrac{D_m}{D_p}\right)^2$ $\quad h_m = 15 \cdot \dfrac{J}{kg} \times 4 \times \left(\dfrac{0.12}{0.25}\right)^2$ $\quad h_m = 13.8 \cdot \dfrac{J}{kg}$

Problem 7.68 [Difficulty: 3]

7.68 A model propeller 1 m in diameter is tested in a wind tunnel. Air approaches the propeller at 50 m/s when it rotates at 1800 rpm. The thrust and torque measured under these conditions are 100 N and 10 N·m, respectively. A prototype 8 times as large as the model is to be built. At a dynamically similar operating point, the approach air speed is to be 130 m/s. Calculate the speed, thrust, and torque of the prototype propeller under these conditions, neglecting the effect of viscosity but including density.

Given: Data on model propeller

Find: Speed, thrust and torque on prototype

Solution: We will use the Buckingham Pi-theorem to find the functional relationships between these variables. Neglecting the effects of viscosity:

1 $F \quad T \quad \rho \quad V \quad D \quad \omega$ $\qquad n = 6$ parameters

2 Select primary dimensions M, L, t:

3 $F \quad T \quad \rho \quad V \quad D \quad \omega$

$$\frac{M\cdot L}{t^2} \quad \frac{M\cdot L^2}{t^2} \quad \frac{M}{L^3} \quad \frac{L}{t} \quad L \quad \frac{1}{t}$$ $r = 3$ dimensions

4 $\rho \quad D \quad \omega$ $\qquad m = r = 3$ repeating parameters

5 We have n - m = 3 dimensionless groups. Setting up dimensional equations:

$\Pi_1 = F\cdot\rho^a\cdot D^b\cdot\omega^c$ Thus: $\frac{M\cdot L}{t^2}\cdot\left(\frac{M}{L^3}\right)^a\cdot L^b\cdot\left(\frac{1}{t}\right)^c = M^0\cdot L^0\cdot t^0$

Summing exponents: The solution to this system is:

M: $1 + a = 0$

L: $1 - 3\cdot a + b = 0$

t: $-2 - c = 0$

$a = -1 \quad b = -4 \quad c = -2$

$$\Pi_1 = \frac{F}{\rho\cdot D^4\cdot\omega^2}$$

$\Pi_2 = T\cdot\rho^a\cdot D^b\cdot\omega^c$ Thus: $\frac{M\cdot L^2}{t^2}\cdot\left(\frac{M}{L^3}\right)^a\cdot L^b\cdot\left(\frac{1}{t}\right)^c = M^0\cdot L^0\cdot t^0$

Summing exponents: The solution to this system is:

M: $1 + a = 0$

L: $2 - 3\cdot a + b = 0$

t: $-2 - c = 0$

$a = -1 \quad b = -5 \quad c = -2$

$$\Pi_2 = \frac{T}{\rho\cdot D^5\cdot\omega^2}$$

$\Pi_3 = V\cdot\rho^a\cdot D^b\cdot\omega^c$ Thus: $\frac{L}{t}\cdot\left(\frac{M}{L^3}\right)^a\cdot L^b\cdot\left(\frac{1}{t}\right)^c = M^0\cdot L^0\cdot t^0$

Summing exponents:

M: $0 + a = 0$

L: $1 - 3 \cdot a + b = 0$

t: $-1 - c = 0$

The solution to this system is:

$a = 0 \qquad b = -1 \qquad c = -1$

$$\Pi_3 = \frac{V}{D \cdot \omega}$$

6 Check using F, L, t dimensions: $F \cdot \frac{L^4}{F \cdot t^2} \cdot \frac{1}{L^4} \cdot t^2 = 1 \qquad F \cdot L \cdot \frac{L^4}{F \cdot t^2} \cdot \frac{1}{L^5} \cdot t^2 = 1 \qquad \frac{L}{t} \cdot \frac{1}{L} \cdot t = 1$

For dynamically similar conditions:

$$\frac{V_m}{D_m \cdot \omega_m} = \frac{V_p}{D_p \cdot \omega_p} \qquad \text{Thus:} \qquad \omega_p = \omega_m \cdot \frac{V_p}{V_m} \cdot \frac{D_m}{D_p} \qquad \omega_p = 1800 \cdot \text{rpm} \times \frac{130}{50} \times \frac{1}{8}$$

$$\omega_p = 585 \cdot \text{rpm}$$

$$\frac{F_m}{\rho_m \cdot D_m^{\,4} \cdot \omega_m^{\,2}} = \frac{F_p}{\rho_p \cdot D_p^{\,4} \cdot \omega_p^{\,2}} \qquad \text{Thus:} \qquad F_p = F_m \cdot \frac{\rho_p}{\rho_m} \cdot \left(\frac{D_p}{D_m}\right)^4 \cdot \left(\frac{\omega_p}{\omega_m}\right)^2 \qquad F_p = 100 \cdot \text{N} \times \frac{1}{1} \times \left(\frac{8}{1}\right)^4 \times \left(\frac{585}{1800}\right)^2$$

$$F_p = 43.3 \cdot \text{kN}$$

$$\frac{T_m}{\rho_m \cdot D_m^{\,5} \cdot \omega_m^{\,2}} = \frac{T_p}{\rho_p \cdot D_p^{\,5} \cdot \omega_p^{\,2}} \qquad \text{Thus:} \qquad T_p = T_m \cdot \frac{\rho_p}{\rho_m} \cdot \left(\frac{D_p}{D_m}\right)^5 \cdot \left(\frac{\omega_p}{\omega_m}\right)^2 \qquad T_p = 10 \cdot \text{N} \cdot \text{m} \times \frac{1}{1} \times \left(\frac{8}{1}\right)^5 \times \left(\frac{585}{1800}\right)^2$$

$$T_p = 34.6 \cdot \text{kN} \cdot \text{m}$$

Problem 7.69

[Difficulty: 3]

7.69 Consider again Problem 7.38. Experience shows that for ship-size propellers, viscous effects on scaling are small. Also, when cavitation is not present, the nondimensional parameter containing pressure can be ignored. Assume that torque, T, and power, $\mathcal{P}$, depend on the same parameters as thrust. For conditions under which effects of μ and p can be neglected, derive scaling "laws" for propellers, similar to the pump "laws" of Section 7.5, that relate thrust, torque, and power to the angular speed and diameter of the propeller.

Given: For a marine propeller (Problem 7.38) the thrust force is: $F_T = F_T(\rho, D, V, g, \omega, p, \mu)$

For ship size propellers viscous and pressure effects can be neglected. Assume that power and torque depend on the same parameters as thrust.

Find: Scaling laws for propellers that relate thrust, power and torque to other variables

Solution: We will use the Buckingham pi-theorem. Based on the simplifications given above:

1 $\quad F_T \quad P \quad T \quad \rho \quad D \quad V \quad g \quad \omega$ $\qquad$ n = 8 parameters

2 $\quad$ Select primary dimensions F, L, t:

3

F_T	P	T	ρ	D	V	g	ω
F	$\frac{F \cdot L}{t}$	$F \cdot L$	$\frac{F \cdot t^2}{L^4}$	L	$\frac{L}{t}$	$\frac{L}{t^2}$	$\frac{1}{t}$

r = 3 dimensions

4 $\quad \rho \quad \omega \quad D$ $\qquad$ m = r = 3 repeating parameters

5 $\quad$ We have n - m = 5 dimensionless groups (3 dependent, 2 independent). Setting up dimensional equations:

$$\Pi_1 = F_T \cdot \rho^a \cdot \omega^b \cdot D^c \qquad \text{Thus:} \qquad F \cdot \left(\frac{F \cdot t^2}{L^4}\right)^a \cdot \left(\frac{1}{t}\right)^b \cdot L^c = F^0 \cdot L^0 \cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

F: $\quad 1 + a = 0$ $\qquad a = -1 \quad b = -2 \quad c = -2$

L: $\quad -4 \cdot a + c = 0$

t: $\quad 2 \cdot a - b = 0$

$$\Pi_1 = \frac{F_T}{\rho \cdot \omega^2 \cdot D^4}$$

$$\Pi_2 = P \cdot \rho^a \cdot \omega^b \cdot D^c \qquad \text{Thus:} \qquad \frac{F \cdot L}{t} \cdot \left(\frac{F \cdot t^2}{L^4}\right)^a \cdot \left(\frac{1}{t}\right)^b \cdot L^c = F^0 \cdot L^0 \cdot t^0$$

Summing exponents: $\qquad$ The solution to this system is:

F: $\quad 1 + a = 0$ $\qquad a = -1 \quad b = -3 \quad c = -5$

L: $\quad 1 - 4 \cdot a + c = 0$

t: $\quad -1 + 2 \cdot a - b = 0$

$$\Pi_2 = \frac{P}{\rho \cdot \omega^3 \cdot D^5}$$

$\Pi_3 = T\cdot\rho^a\cdot\omega^b\cdot D^c$ Thus: $F\cdot L\cdot\left(\frac{F\cdot t^2}{L^4}\right)^a\cdot\left(\frac{1}{t}\right)^b\cdot L^c = F^0\cdot L^0\cdot t^0$

Summing exponents:

F: $1 + a = 0$

L: $1 - 4\cdot a + c = 0$

t: $2\cdot a - b = 0$

The solution to this system is:

$a = -1 \quad b = -2 \quad c = -5$

$$\Pi_3 = \frac{T}{\rho\cdot\omega^2\cdot D^5}$$

$\Pi_4 = V\cdot\rho^a\cdot\omega^b\cdot D^c$ Thus: $\frac{L}{t}\cdot\left(\frac{F\cdot t^2}{L^4}\right)^a\cdot\left(\frac{1}{t}\right)^b\cdot L^c = F^0\cdot L^0\cdot t^0$

Summing exponents:

F: $a = 0$

L: $1 - 4\cdot a + c = 0$

t: $-1 + 2\cdot a - b = 0$

The solution to this system is:

$a = 0 \quad b = -1 \quad c = -1$

$$\Pi_4 = \frac{V}{\omega\cdot D}$$

$\Pi_5 = g\cdot\rho^a\cdot\omega^b\cdot D^c$ Thus: $\frac{L}{t}\cdot\left(\frac{F\cdot t^2}{L^4}\right)^a\cdot\left(\frac{1}{t}\right)^b\cdot L^c = F^0\cdot L^0\cdot t^0$

Summing exponents:

F: $a = 0$

L: $1 - 4\cdot a + c = 0$

t: $-1 + 2\cdot a - b = 0$

The solution to this system is:

$a = 0 \quad b = -1 \quad c = -1$

$$\Pi_5 = \frac{g}{\omega^2\cdot D}$$

6 Check using M, L, t dimensions: $\frac{M\cdot L}{t^2}\cdot\frac{L^3}{M}\cdot t^2\cdot\frac{1}{L^4} = 1 \quad \frac{M\cdot L^2}{t^3}\cdot\frac{L^3}{M}\cdot t^3\cdot\frac{1}{L^5} = 1 \quad \frac{M\cdot L^2}{t^2}\cdot\frac{L^3}{M}\cdot t^2\cdot\frac{1}{L^5} = 1$

$\frac{L}{t}\cdot t\cdot\frac{1}{L} = 1 \quad \frac{L}{t^2}\cdot t^2\cdot\frac{1}{L^2} = 1$

Based on the dependent and independent variables, the "scaling laws" are:

$$\frac{F_T}{\rho\cdot\omega^2\cdot D^4} = f_1\left(\frac{V}{\omega\cdot D},\frac{g}{\omega^2\cdot D}\right)$$

$$\frac{P}{\rho\cdot\omega^3\cdot D^5} = f_2\left(\frac{V}{\omega\cdot D},\frac{g}{\omega^2\cdot D}\right)$$

$$\frac{T}{\rho\cdot\omega^2\cdot D^5} = f_3\left(\frac{V}{\omega\cdot D},\frac{g}{\omega^2\cdot D}\right)$$

Problem 7.70

[Difficulty: 2]

7.70 Closed-circuit wind tunnels can produce higher speeds than open-circuit tunnels with the same power input because energy is recovered in the diffuser downstream from the test section. The *kinetic energy ratio* is a figure of merit defined as the ratio of the kinetic energy flux in the test section to the drive power. Estimate the kinetic energy ratio for the 40 ft × 80 ft wind tunnel at NASA-Ames described on page 267.

Given: Kinetic energy ratio for a wind tunnel is the ratio of the kinetic energy flux in the test section to the drive power

Find: Kinetic energy ratio for the 40 ft x 80 ft tunnel at NASA-Ames

Solution: From the text: $P = 125000 \cdot hp$ $V_{max} = 300 \cdot \frac{nmi}{hr} \times \frac{6080 \cdot ft}{nmi} \times \frac{hr}{3600 \cdot s}$ $V_{max} = 507 \cdot \frac{ft}{s}$

Therefore, the kinetic energy ratio is: $KE_{ratio} = \dfrac{m \cdot \frac{V^2}{2}}{P} = \dfrac{(\rho \cdot V \cdot A) \cdot V^2}{2 \cdot P} = \dfrac{\rho \cdot A \cdot V^3}{2 \cdot P}$ Assuming standard conditions and substituting values:

$$KE_{ratio} = \frac{1}{2} \times 0.00238 \cdot \frac{slug}{ft^3} \times (40 \cdot ft \times 80 \cdot ft) \times \left(507 \cdot \frac{ft}{s}\right)^3 \times \frac{1}{125000 \cdot hp} \times \frac{hp \cdot s}{550 \cdot ft \cdot lbf} \times \frac{lbf \cdot s^2}{slug \cdot ft}$$

$$KE_{ratio} = 7.22$$

Problem 7.71 [Difficulty: 3]

7.71 A 1:16 model of a bus is tested in a wind tunnel in standard air. The model is 152 mm wide, 200 mm high, and 762 mm long. The measured drag force at 26.5 m/s wind speed is 6.09 N. The longitudinal pressure gradient in the wind tunnel test section is $-11.8\ \mathrm{N/m^2/m}$. Estimate the correction that should be made to the measured drag force to correct for horizontal buoyancy caused by the pressure gradient in the test section. Calculate the drag coefficient for the model. Evaluate the aerodynamic drag force on the prototype at 100 km/hr on a calm day.

Given: A 1:16 scale model of a bus (152 mm x 200 mm x 762 mm) is tested in a wind tunnel at 26.5 m/s. Drag force is 6.09 N. The axial pressure gradient is $-11.8\ \mathrm{N/m^2/m}$.

Find:
(a) Horizontal buoyancy correction
(b) Drag coefficient for the model
(c) Aerodynamic drag on the prototype at 100 kph on a calm day.

Solution: The horizontal buoyancy force is the difference in the pressure force between the front and back of the model due to the pressure gradient in the tunnel:

$$F_B = (p_f - p_b)\cdot A = \frac{dp}{dx}\cdot L_m\cdot A_m \quad \text{where:} \quad A_m = 152\cdot\mathrm{mm}\times 200\cdot\mathrm{mm} \quad A_m = 30400\cdot\mathrm{mm}^2$$

Thus: $$F_B = -11.8\cdot\frac{\mathrm{N}}{\mathrm{m}^2\cdot\mathrm{m}}\times 762\cdot\mathrm{mm}\times 30400\cdot\mathrm{mm}^2\times\left(\frac{\mathrm{m}}{1000\cdot\mathrm{mm}}\right)^3 \qquad F_B = -0.273\ \mathrm{N}$$

So the corrected drag force is: $F_{Dc} = 6.09\cdot\mathrm{N} - 0.273\cdot\mathrm{N}$ $\quad F_{Dc} = 5.817\ \mathrm{N}$

The corrected model drag coefficient would then be: $$C_{Dm} = \frac{F_{Dc}}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A_m}$$ Substituting in values:

$$C_{Dm} = 2\times 5.82\cdot\mathrm{N}\times\frac{\mathrm{m}^3}{1.23\cdot\mathrm{kg}}\times\left(\frac{\mathrm{s}}{26.5\cdot\mathrm{m}}\right)^2\times\frac{1}{30400\cdot\mathrm{mm}^2}\times\left(\frac{1000\cdot\mathrm{mm}}{\mathrm{m}}\right)^2\times\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{N}\cdot\mathrm{s}^2} \qquad C_{Dm} = 0.443$$

If we assume that the test was conducted at high enough Reynolds number, then the drag coefficient should be the same at both scales, i.e.: $C_{Dp} = C_{Dm}$

$$F_{Dp} = \frac{1}{2}\cdot\rho\cdot V^2\cdot A_p\cdot C_{Dp} \quad \text{where} \quad A_p = 30400\cdot\mathrm{mm}^2\cdot 16^2\cdot\left(\frac{\mathrm{m}}{1000\cdot\mathrm{mm}}\right)^2 \qquad A_p = 7.782\cdot\mathrm{m}^2$$

$$F_{Dp} = \frac{1}{2}\times 1.23\cdot\frac{\mathrm{kg}}{\mathrm{m}^3}\times\left(100\cdot\frac{\mathrm{km}}{\mathrm{hr}}\times\frac{1000\cdot\mathrm{m}}{\mathrm{km}}\cdot\frac{\mathrm{hr}}{3600\times\mathrm{s}}\right)^2\times 7.782\cdot\mathrm{m}^2\times 0.443\times\frac{\mathrm{N}\cdot\mathrm{s}^2}{\mathrm{kg}\cdot\mathrm{m}} \qquad F_{Dp} = 1.636\cdot\mathrm{kN}$$

(The rolling resistance must also be included to obtain the total tractive effort needed to propel the vehicle.)

Problem 7.72

[Difficulty: 5]

7.72 The propagation speed of small-amplitude surface waves in a region of uniform depth is given by

$$c^2 = \left(\frac{2\pi\sigma}{\lambda\rho} + \frac{g\lambda}{2\pi}\right)\tanh\frac{2\pi h}{\lambda}$$

where h is depth of the undisturbed liquid and λ is wavelength. Explore the variation in wave propagation speed for a free-surface flow of water. Find the operating depth to minimize the speed of capillary waves (waves with small wavelength, also called ripples). First assume wavelength is much smaller than water depth. Then explore the effect of depth. What depth do you recommend for a water table used to visualize compressible flow wave phenomena? What is the effect of reducing surface tension by adding a surfactant?

Discussion: The equation given in Problem 7.2 contains three terms. The first term contains surface tension and gives a speed inversely proportional to wavelength. These terms will be important when small wavelengths are considered.

The second term contains gravity and gives a speed proportional to wavelength. This term will be important when long wavelengths are considered.

The argument of the hyperbolic tangent is proportional to water depth and inversely proportional to wavelength. For small wavelengths this term should approach unity since the hyperbolic tangent of a large number approaches one.

The governing equation is: $c^2 = \left(\frac{\sigma}{\rho}\cdot\frac{2\cdot\pi}{\lambda} + \frac{g\cdot\lambda}{2\cdot\pi}\right)\cdot\tanh\left(\frac{2\cdot\pi\cdot h}{\lambda}\right)$

The relevant physical parameters are: $g = 9.81\cdot\frac{m}{s^2}$ $\rho = 999\cdot\frac{kg}{m^3}$ $\sigma = 0.0728\cdot\frac{N}{m}$

A plot of the wave speed versus wavelength at different depths is shown here: