University of Cincinnati
College of Engineering and Applied Science
Department of Mechanical & Materials Engineering

Course Code : **MECH-5072C**
Course Title : **EXPERIMENTAL METHODS/ME**

Experiment Number : #3
Experiment Title : **Thermal Radiation**

Date(s) Performed : **Feb 17, 2023**
Date Submitted : **Feb 24, 2023**

Group Code : **F-01**
Group Members : **YI, Hongrui: Objective 1, 2 hours**
: **LIU, Xinran: Objective 3, 2 hours**
YANG, Rui: Objective 2, 2 hours
YOU, Xuzhen: Objective 4, 2 hours

Instructor : **Dr. Sarath Kannan**

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1. Objectives

At the end of this experiment, the students are expected to master:

(1) Inverse Square Law of Light: Prove illuminance of a surface is inversely proportional to the square of the distance from the light source.

(2) Lambert's Cosine Law: Demonstrate Lambert's Cosine Law: the energy radiated by the surface in any direction is equal to the normal radiation times the cosine of the angle between the direction of the radiation and normal to the surface.

(3) Demonstrate the Law of Absorption; light (energy) passing through a filter (non-opaque) reduces its intensity in proportion to the thickness and absorption coefficient of the material.

(4) Fourth Power Law: Show that the intensity of radiation varies as the fourth power of the source temperature (heat source).

2. Theoretical Background

Inverse Square Law of Light:

$$E_r = \varphi X^{-2} \tag{1}$$

Where,

Er : energy received (Lux)

Φ : source energy (Lumens)

X : distance to receiver (Meters)

Lambert's Cosine Law:

$$\mathrm{E}\theta\mathrm{=E0cos}(\theta) \tag{2}$$

Eθ is illuminance at angle θ to normal.

E is illuminance in normal direction.

Intensity of light through a filter plate varies with the absorption coefficient and the thickness of the plate as:

$$I = \ I_o e^{-\alpha x} \tag{3}$$

Where, I_o is the source intensity without plate/obstruction;

I is the intensity through plate/obstruction;

X is the thickness of plate/obstruction.

So the absorption coefficient α can be calculated by:

$$\alpha = \frac{-\ln\left(\frac{I}{I_o}\right)}{x} \tag{4}$$

Fourth power law: Show that the intensity of radiation varies as the fourth power of the source temperature (heat source):

$$q = \sigma\varepsilon(T_s^4 - T_a^4) \tag{5}$$

Where,

$q = Heat\ flux\ in\ W/m^2$

$\sigma = Boltzmann\ constant, 5.67 \times \frac{10^{-8}W}{m^2K^4}\varepsilon$

$\varepsilon = emissivity, fraction\ of\ power\ emitted\ compared\ to\ an\ ideal\ black\ body$

$T_s = surface\ temperature\ (K)$

$T_a = ambient\ temperature\ (K)$

For an ideal black body, since $\varepsilon = 1$:

$$q_b = \sigma(T_s^4 - T_a^4) \tag{6}$$

Also,

$$\varepsilon = \frac{q}{q_b} = \frac{q}{\sigma(T_s^4 - T_a^4)} \tag{7}$$

Where, $q = fR$

f is the correction factor $f = \frac{(0.063^2 + L^2)}{0.063^2}$

R is the radiometer reading

L is the distance from emitter to radiometer

3. Sample Calculation

3.1 Inverse Square Law of Light

$$E_r = \varphi X^{-2}$$

Where,

E_r : energy received (Lux)

Φ : source energy (Lumens)

X : distance to receiver (Meters)

Taking 'Distance 125mm Light Meter 450 Lux' and 'Distance 150mm Light Meter 380 Lux' as an example:

Intensity:

$$\emptyset_1 = E_{r_1} \cdot X_1^2 = 700 \cdot 0.100^2 = 7.00lm$$

$$\emptyset_2 = E_{r_2} \cdot X_2^2 = 370 \cdot 0.150^2 = 8.325lm$$

For the plot, log(E_r) versus log(X) is shown below:

$$\log(E_r) = -2\log(X) + \log(\emptyset)$$

Convert this equation into linear relationship, then it is

$$y = kx + \mathrm{c}$$

$$\ln(E_{r_1}) = \ln(700) = 6.55$$

$$\ln(X_1) = \ln(0.100) = -2.30$$

$$\ln(E_{r_2}) = \ln(370) = 5.91$$

$$\ln(X_2) = \ln(0.150) = -1.90$$

Curve fitting:

$$k = \frac{\ln(E_{r_1}) - \ln(E_{r_2})}{\ln(X_1) - \ln(X_2)}$$

3.2 Lambert's Cosine Law

Eθ=E0cos(θ)

Eθ is illuminance at angle θ to normal.

E is illuminance in normal direction.

3.3 Law of Absorption

Intensity of light through a filter plate varies with the absorption coefficient and the thickness of the plate as:

$$I = I_o e^{-\alpha x}$$

Where, I_o is the source intensity without plate/obstruction;

I is the intensity through plate/obstruction;

X is the thickness of plate/obstruction.

So the absorption coefficient α can be calculated by:

$$\alpha = \frac{-\ln\left(\frac{I}{I_o}\right)}{x}$$

Table 1. Variable Optical Density

Filter	No Filter	Pale	Medium	Dark
Light Meter (Lux)	220	210	140	40

Table 2. Variable Filter Thickness

Thickness (mm)	3	6	9
Light Meter (Lux)	140	80	50

Take 3mm thickness Pale plate as an example:

$$\alpha = \frac{-\ln\left(\frac{210}{220}\right)}{0.003} = 15.5067$$

3.4 Fourth Power Law

Fourth power law: Show that the intensity of radiation varies as the fourth power of the source temperature (heat source):

$$q = \sigma\varepsilon(T_s^4 - T_a^4)$$

Where, $q = Heat\ flux\ in\ W/m^2$

$\sigma = Boltzmann\ constant, 5.67 \times 10^{-8} W/m^2K^4$

$\varepsilon = emissivity, fraction\ of\ power\ emitted\ compared\ to\ an\ ideal\ black\ body$

$T_s = surface\ temperature\ (K)$

$T_a = ambient\ temperature\ (K)$

For an ideal black body, since $\varepsilon = 1$:

$$q_b = \sigma(T_s^4 - T_a^4)$$

Also,

$$\varepsilon = \frac{q}{q_b} = \frac{q}{\sigma(T_s^4 - T_a^4)}$$

Where, $q = fR$

f is the correction factor $f = \frac{(0.063^2 + L^2)}{0.063^2}$

R is the radiometer reading

L is the distance from emitter to radiometer

The experiment data shown as follow:

Table 3. Emissivity of Black Plate

Type of Plate	Plate Temp, T_s (°C)	44	46	48	50	52
Black	Radiometer, $R(W/m^2)$	15.1	16.5	18.3	19.8	21.2

Table 4. Emissivity of Silver Plate

Type of Plate	Plate Temp, T_s (°C)	44	46	48	50	52
Silver	Radiometer, $R(W/m^2)$	13.5	14.8	16.3	17.7	19.3

Table 5. Emissivity of Polished Plate

Type of Plate	Plate Temp, T_s (°C)	36	38	40	42	44
Polished	Radiometer, $R(W/m^2)$	2.6	3.0	3.6	3.7	3.9

For correction factor:

$$f = \frac{(0.063^2 + 0.15^2)}{0.063^2} = \frac{2941}{441}$$

For the black body flux, take the black plate at 44°C as an example:

$$q_b = 5.67 \times \frac{10^{-8}W}{m^2K^4}\left[(273.15 + 44)^4K^4 - (273.15 + 19)^4K^4\right] = 160.5894956\ W/m^2$$

For the Corrected Radiometer value, take the black plate at 44°C as an example:

$$q = \frac{2941}{441} \times \frac{15.1W}{m^2} = 100.700907\ W/m^2$$

For the Emissivity, take the black plate as an example:

$$\varepsilon = \frac{\bar{q}}{q_b} = \frac{121.2412\ W/m^2}{190.3622286\ W/m^2} = 0.636897$$

4. Result

4.1 Inverse Square Law of Light

Table 1 Inverse Square Law of Light

Distance (mm)	100	150	200	225	275	300	330	360	400
Light meter(LUX)	700	370	220	180	130	110	90	80	80

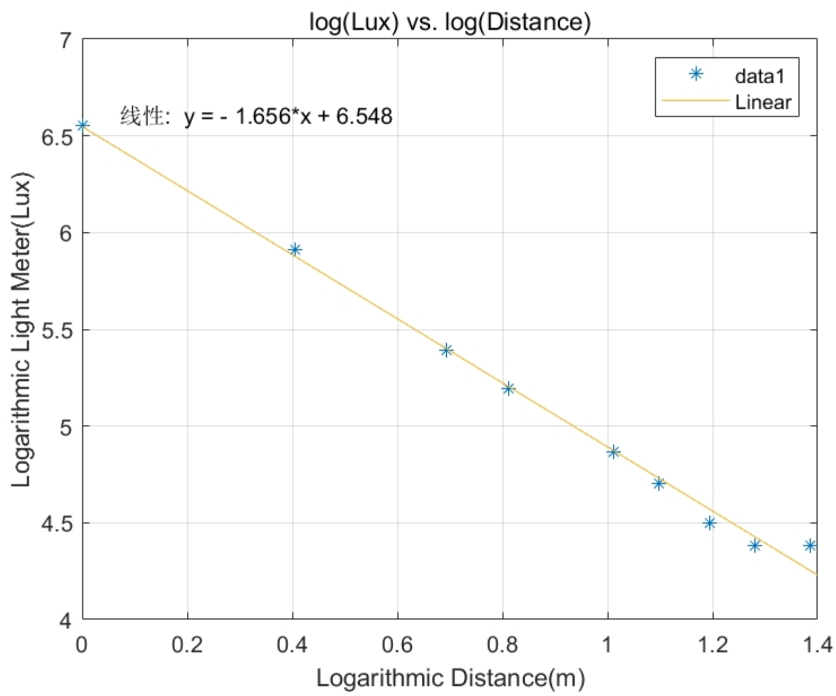

Figure 1.Plot of log(LUX) vs. log(Distance)

4.2 Lambert's Cosine Law

Table 2. Data of Lambert's Cosine Law

Angle	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60
Measured Light meter(LUX)	360	480	500	600	680	700	700	690	670	650	600	500	400
Theoretical Light meter(LUX)	350	450	536	606	658	689	700	689	657	606	536	450	350

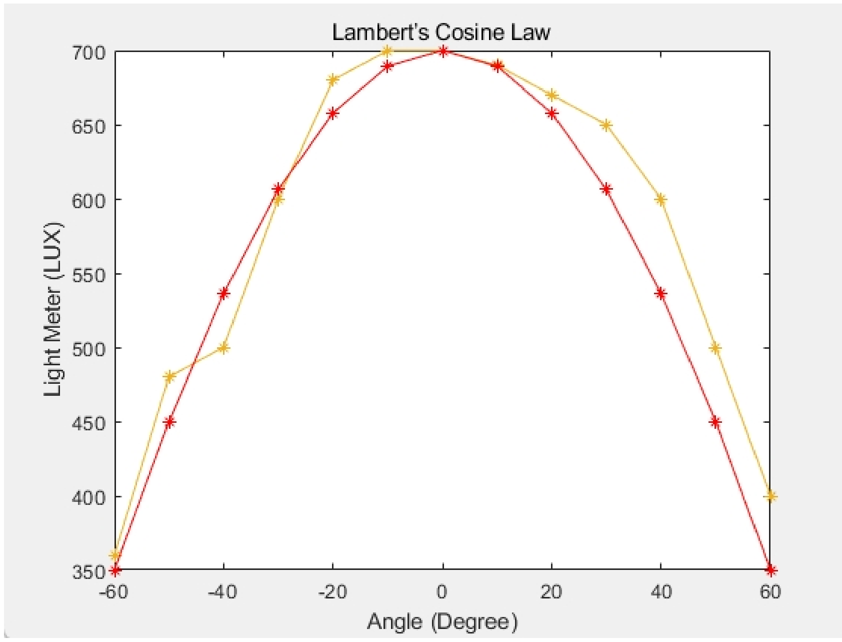

Figure 2. Experimental Light Meter vs. Theoretical Light Meter

4.3 Law of Absorption

Table 6. Absorption coefficient of different filter

Filter	No Filter	Pale	Medium	Dark
Light Meter (Lux)	220	210	140	40
Absorption coefficient (m^(-1))	\	15.5067	150.6617	568.2494

Table 7. Absorption coefficient of different thickness

Thickness (mm)	3	6	9
Light Meter (Lux)	140	80	50
Absorption coefficient (m^(-1))	150.6617	168.6002	164.6227

4.4 Fourth Power Law

Table 8. Emissivity of Black Plate

Plate Temp, T_s (℃)	**Radiometer, $R(W/m^2)$**	**Black Body flux, $q_b(W/m^2)$**	**Corrected Radiometer value, $q(W/m^2)$**	**Emissivity,ε**
44	15.1	160.5894956	100.7009	

46	16.5	175.1969022	110.0374	
48	18.3	190.0815239	122.0415	
50	19.8	205.246846	132.0449	
52	21.2	220.6963754	141.3814	
Average		190.3622286	121.2412	0.636897

Table 9. Emissivity of Silver Plate

Plate Temp, T_s (℃)	**Radiometer, $R(W/m^2)$**	**Black Body flux, $q_b(W/m^2)$**	**Corrected Radiometer value, $q(W/m^2)$**	**Emissivity, ε**
44	13.5	160.5894956	90.03061	
46	14.8	175.1969022	98.70023	
48	16.3	190.0815239	108.7036	
50	17.7	205.246846	118.0401	
52	19.3	220.6963754	128.7104	
Average		190.3622286	108.837	0.571736

Table 10. Emissivity of Polished Plate

Plate Temp, T_s (℃)	**Radiometer, $R(W/m^2)$**	**Black Body flux, $q_b(W/m^2)$**	**Corrected Radiometer value, $q(W/m^2)$**	**Emissivity, ε**
36	2.6	104.8630761	17.33923	
38	3	118.3960399	20.0068	
40	3.6	132.1924952	24.00816	
42	3.7	146.2558405	24.67506	
44	3.9	160.5894956	26.00884	
Average		132.4593895	22.40762	0.169166

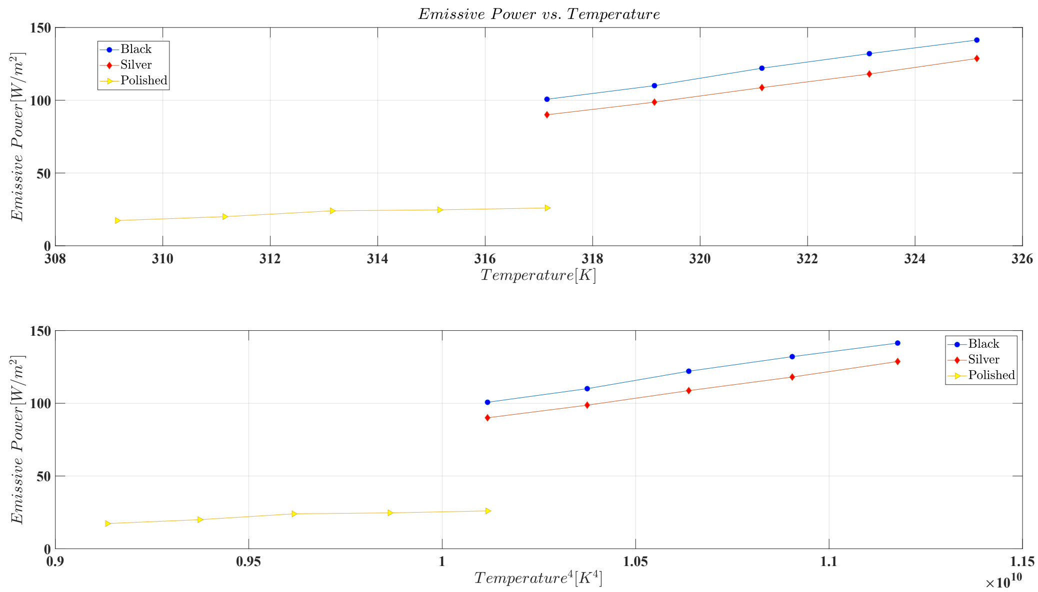

Figure 3. Emissive Power vs. Temperature

5. Discussion and Conclusions

5.1. Inverse Square Law of Light

(1) Comment on how slope of logarithmic Lux vs. Distance plot compares to the ideal value of -2.

Based on the theoretical conclusion, the ideal value of the slope should be -2. Comparing with the realistic result where the slope is -1.67, the error is 16.5%.

$$Error = \frac{-2-(-1.656)}{-2} \times 100\% = 17.2\%$$

The error cannot be ignored if the experiment has an acceptable error below 5%.

(2) Comment on sources of error.

Reasons which cause this error can be the following factors:

1. The experiment is not operated in a darkroom. More light entrance the light meter if the lab is conducted in the darkroom and this can impact the result.
2. The light source is not an ideal point light source. In this condition, parallel light exists and this makes light meter receive more energy.
3. Limited experimental data. Figure shows that more data can improve the result precision.

5.2. Lambert's Cosine Law

(1) Comment how theoretical values compare to actual values for Lamberts Cosine Law.

There is little deviation between the actual data and the theoretical data. Prove that Lambert's law of cosines is correct.

(2) Comment on sources of error.

In the experiment, the data is larger when the angle is positive, and the data is smaller when the angle is negative. This may be due to the fact that the negative angle will be partly blocked by the cabinet in the experiment, so the received light will be less and the data will be smaller.

5.3 Law of Absorption

(1) Comment how Absorption coefficient rises from light to dark plates.

According to the result, the absorption coefficient rises from light to dark plates in a rapid way. This is because according to the equation used to calculate the absorption coefficient, when the intensity through plate increases, an exponential growth will be brought to the result.

(2) Comment how Absorption coefficient stays constant for different stacks of the same shade plate

According to the result, the absorption coefficient stays the same for different stacks of the same shade plate. The intensity through plate decreases while the thickness enlarges. That is because the absorption coefficient is a characteristic of the plates, it will not be changing with the thickness of the material.

5.4 Fourth Power Law

(1) Comment how emitted power increases with temperature.

As Figure 3 shown, as the temperature increases the emitted power would generally increase. It also demonstrated that emitted power is linear related to the T^4, which proved the Fourth Power Law.

(2) Comment how recorded emissivity values compare to accepted ones.

From the online source:

https://www.omega.co.uk/literature/transactions/volume1/emissivitya.html

The emissivity of black paint plate is 0.96

$$error_{Black} = \frac{0.94 - 0.636897}{0.94} \times 100\% = 32.245\%$$

The emissivity of silver plate is 0.6

$$error_{silver} = \frac{0.6 - 0.571736}{0.6} \times 100\% = 4.7107\%$$

The emissivity of Roughly Polished Aluminium plate is 0.18

$$error_{Polished} = \frac{0.18 - 0.169166}{0.18} \times 100\% = 6.0189\%$$

(3) Comment on sources of error.

The experiments were not conducted in a fully insulated environment, the emitted power could caused mainly by the heat source. The lights in the laboratory could also be a heat source for the experiment plate, which could have significant impact on the experiment results. Besides, the experimental data is limited. More data in different temperatures should increase the accuracy of the result.

APPENDICES

A – MATLAB Code (or Excel, other computational software, etc)

```
%% LAB3 OB2 Lambert s Cosine Law

clear all; clc

angle = [-60 -50 -40 -30 -20 -10 0 10 20 30 40 50 60 ];
lux = [360 480 500 600 680 700 700 690 670 650 600 500 400];
theory = 700*cosd(angle);
figure(1)
plot(angle,lux,'*-')
hold on
plot(angle,theory,'r*-')
xlabel('Angle (Degree)')
ylabel('Light Meter (LUX)')
title('Lambert's Cosine Law')

disp(theory)
%% Lab 3
close all
clear all
clc
set(0,'DefaultAxesFontName', 'Times New Roman')
set(0,'DefaultAxesFontSize', 22)
set(0,'defaultlinelinewidth',0.5)
set(0,'DefaultLineMarkerSize', 8)
set(0,'defaultAxesFontWeight','bold')

%% Code
%% OB 4
T1 = [44 46 48 50 52]+273.15;
T2 = [36 38 40 42 44]+273.15;
q1 = [100.7009 110.0374 122.0415 132.0449 141.3814];
q2 = [90.03061 98.70023 108.7036 118.0401 128.7104];
q3 = [17.33923 20.0068 24.00816 24.67506 26.00884];

figure
subplot(2,1,1)
plot(T1,q1,'o-','MarkerFaceColor','b')
hold on
plot(T1,q2,'d-','MarkerFaceColor','r')
plot(T2,q3,'>-','MarkerFaceColor','y')
xlabel(['$ Temperature{[K]} $'],'interpreter','latex')
ylabel(['$ Emissive\ Power{[W/m^2]} $'],'interpreter','latex')
title(['$ Emissive\ Power\ vs.\ Temperature $'],'interpreter','latex')
legend(['Black'],['Silver'],['Polished'],'interpreter','latex')
grid on
subplot(2,1,2)
plot(T1.^4,q1,'o-','MarkerFaceColor','b')
hold on
plot(T1.^4,q2,'d-','MarkerFaceColor','r')
plot(T2.^4,q3,'>-','MarkerFaceColor','y')
xlabel(['$ Temperature^4{[K^4]} $'],'interpreter','latex')
```

```
ylabel(['$ Emissive\ Power{[W/m^2]} $'],'interpreter','latex')
legend(['Black'],['Silver'],['Polished'],'interpreter','latex')
grid on
```

44	15.1	160.5894956	100.7009	
46	16.5	175.1969022	110.0374	
48	18.3	190.0815239	122.0415	
50	19.8	205.246846	132.0449	
52	21.2	220.6963754	141.3814	
		190.3622286	121.2412	0.636897
44	13.5	160.5894956	90.03061	
46	14.8	175.1969022	98.70023	
48	16.3	190.0815239	108.7036	
50	17.7	205.246846	118.0401	
52	19.3	220.6963754	128.7104	
		190.3622286	108.837	0.571736
36	2.6	104.8630761	17.33923	
38	3	118.3960399	20.0068	
40	3.6	132.1924952	24.00816	
42	3.7	146.2558405	24.67506	
44	3.9	160.5894956	26.00884	
		132.4593895	22.40762	0.169166

B – Scanned Lab Notes

Lab 3: Thermal Radiation

1) Inverse Square Law

Distance(mm)	100	150	200	225	275	300	330	360	400
Light meter (Lux)	700	370	220	180	130	110	90	80	80

2) Lambert's Cosine Law

Angle (deg)	0	10	20	30	40	50	60	-10	-20	-30	-40	-50	-60
Light Meter (Lux)	700	~~700~~ 690	~~700~~ 670	650	600	500	400	700	680	600	500	480	360

3) Law of Absorption

a) Variable Optical Density

Filter	No Filter	Pale	Medium	Dark
Light Meter (Lux)	220	210	140	40

b) Variable Filter Thickness

Thickness (mm)	3	6	9
Light Meter (Lux)	140	80	50

4) Fourth Power Law/Emissivity of a given plate

Ambient Temperature: 19 °C

Distance of Radiometer Senser from heated plate, L: 150 mm

Type of Plate	Plate Temp., Ts (°C)	44	46	48	50	52
Black	Radiometer, R (W/m^2)	15.1	16.5	18.3	19.8	21.2

Type of Plate	Plate Temp., Ts (°C)	44	46	48	50	52
Silver	Radiometer, R (W/m^2)	13.5	14.8	16.3	17.7	19.3

Type of Plate	Plate Temp., Ts (°C)	36	38	40	42	44
Polished	Radiometer, R (W/m^2)	2.6	3.0	3.6	3.7	3.9

Report Group: F-1

Date: 2/17/2023

TA Signature: