Problem 10.1 [Difficulty: 2]

10.1 The geometry of a centrifugal water pump is $r_1 = 10$ cm, $r_2 = 20$ cm, $b_1 = b_2 = 4$ cm, $\beta_1 = 30°$, $\beta_2 = 15°$, and it runs at speed 1600 rpm. Estimate the discharge required for axial entry, the power generated in the water in watts, and the head produced.

Given: Geometry of centrifugal pump

Find: Estimate discharge for axial entry; Head

Solution:

Basic equations:
$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$$
$$H = \frac{\dot{W}_m}{\dot{m} g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$

The given or available data is

$\rho = 999 \cdot \frac{kg}{m^3}$ $\quad r_1 = 10 \cdot cm$ $\quad r_2 = 20 \cdot cm$ $\quad b_1 = 4 \cdot cm$ $\quad b_2 = 4 \cdot cm$

$\omega = 1600 \cdot rpm$ $\quad \beta_1 = 30 \cdot deg$ $\quad \beta_2 = 15 \cdot deg$

From continuity $\quad V_n = \frac{Q}{2 \cdot \pi \cdot r \cdot b} = w \cdot \sin(\beta)$ $\quad w = \frac{V_n}{\sin(\beta)}$

From geometry $\quad V_t = U - w \cdot \cos(\beta) = U - \frac{V_n}{\sin(\beta)} \cdot \cos(\beta) = U - \frac{Q}{2 \cdot \pi \cdot r \cdot b} \cdot \cot(\beta)$

For an axial entry $\quad V_{t1} = 0$ so $\quad U_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1) = 0$

Using given data $\quad U_1 = \omega \cdot r_1$ $\quad U_1 = 16.755 \frac{m}{s}$

Hence $\quad Q = 2 \cdot \pi \cdot r_1 \cdot b_1 \cdot U_1 \cdot \tan(\beta_1)$ $\quad Q = 0.2431 \cdot \frac{m^3}{s}$

To find the power we need U_2, V_{t2}, and m_{rate}

The mass flow rate is $\quad m_{rate} = \rho \cdot Q$ $\quad m_{rate} = 242.9 \frac{kg}{s}$

$U_2 = \omega \cdot r_2$ $\quad U_2 = 33.5 \frac{m}{s}$

$V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2)$ $\quad V_{t2} = 15.5 \frac{m}{s}$

Hence $\quad W_m = (U_2 \cdot V_{t2} - U_1 \cdot V_{t1}) \cdot m_{rate}$ $\quad W_m = 1.258 \times 10^5 \cdot \frac{J}{s}$ $\quad W_m = 126 \cdot kW$

The head is $\quad H = \frac{W_m}{m_{rate} \cdot g}$ $\quad H = 52.8 \, m$

Problem 10.2

(Difficulty 1)

10.2 The relevant variables for a turbomachine are $\dot{W}$, D, ω, Q, h, T, and ρ. Find the resulting Π-groups when D, ω, and ρ are the repeating variables Discuss the meaning of each Π obtained.

Find: The appropriate dimensionless groups.

Solution:

(1) There are seven dimensional parameters $\dot{W}$, D, ω, Q, h, T, and ρ, so n = 7

(2) Select primary dimensions M, L and t.

(3) We have the following dimensions for the variables:

	D	ω	ρ	$\dot{W}$	Q	h	T
M	0	0	1	1	0	0	1
L	1	0	-3	2	3	2	2
t	0	-1	0	-3	-1	-2	-2

All of the dimensions are present so $r = 3$. The repeating parameters D, ω, and ρ and they include all of the dimensions. There will be n – m = n – r = 7 – 3 = 4 dimensionless groups. For the first group we will combine the repeating variables with the power:

$$\Pi_1 = D^a \omega^b \rho^c \dot{W} = (L)^a \left(\frac{1}{t}\right)^b \left(\frac{M}{L^3}\right)^c \frac{ML^2}{t^3} = M^0 L^0 t^0$$

We equate the exponents of the dimensions and solve for their values

$$a = -5, \qquad b = -3, \qquad c = -1$$

The first group is then

$$\Pi_1 = \frac{\dot{W}}{\rho\, \omega^3 D^5}$$

This is a dimensionless power group.

For the second group we will combine the repeating variables with the volume flow rate Q:

$$\Pi_2 = D^a \omega^b \rho^c Q = (L)^a \left(\frac{1}{t}\right)^b \left(\frac{M}{L^3}\right)^c \frac{L^3}{t} = M^0 L^0 t^0$$

Equating the exponents of the dimensions and solving for their values

$$a = -3, \qquad b = -1, \qquad c = 0$$

The second group is

$$\Pi_2 = \frac{Q}{\omega\, D^3}$$

This is a dimensionless flow rate through the machine.

For the third group with the energy per unit mass h

$$\Pi_3 = D^a \omega^b \rho^c \mathrm{h} = (L)^a \left(\frac{1}{t}\right)^b \left(\frac{M}{L^3}\right)^c \frac{L^2}{t^2} = \; M^0 L^0 t^0$$

We have

$$a = -2, \qquad b = -2, \qquad c = 0$$

The third group becomes

$$\Pi_3 = \frac{h}{\omega^2\, D^2}$$

Which is a dimensionless group that describes the energy or head of the machine.

For the fourth group we combine the torque T:

$$\Pi_4 = D^a \omega^b \rho^c \mathrm{T} = (L)^a \left(\frac{1}{t}\right)^b \left(\frac{M}{L^3}\right)^c \frac{M\, L^2}{t^2} = \; M^0 L^0 t^0$$

We have

$$a = -5, \qquad b = -2, \qquad c = -1$$

The fourth group becomes

$$\Pi_4 = \frac{T}{D^5 \omega^2\, \rho}$$

The groups are

$$\Pi_1 = \frac{\dot{\mathrm{W}}}{\rho\, \omega^3 D^5}, \;\; \Pi_2 = \frac{Q}{\omega\, D^3}, \qquad \Pi_3 = \frac{h}{\omega^2\, D^2}, \qquad \Pi_4 = \frac{T}{D^4 \omega^2\, \rho}$$

We can put these groups in more familiar form by combining the groups:

$$\Pi = \frac{T}{D^5 \omega^2\, \rho} \cdot \left(\frac{Q}{\omega\, D^3}\right)^{-1} \cdot \left(\frac{h}{\omega^2\, D^2}\right)^{-1} = \frac{\omega T}{\rho Q h}$$

This is the efficiency of a turbomachine (Eq 10.4c)

Similarly

$$\Pi = \left(\frac{\omega^6\, D^3 h}{h^3} \cdot \frac{Q^2}{\omega^2\, D^6}\right)^{1/4} = \frac{\omega Q^{1/2}}{h^{3/4}}$$

This is the specific speed, eq. 7.22a (also in Chapter 10)

Lastly, we can combine the group for power

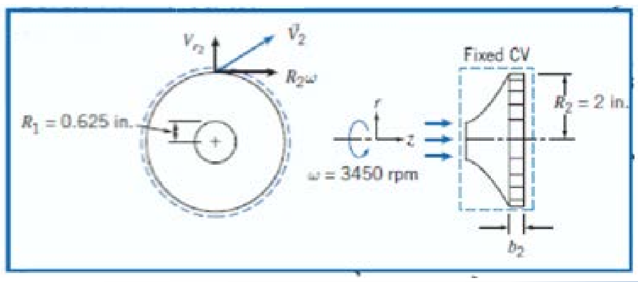
$$\Pi = \frac{\dot{\mathrm{W}}}{\rho\ \omega^3 D^5} \cdot \frac{\omega\ D^3}{Q} = \frac{\dot{\mathrm{W}}}{\rho\ \omega^2 Q D^2}$$

This is the dimensionless power coefficient, eq 10.8.

Problem 10.3

(Difficulty: 2)

10.3 Consider the centrifugal pump impeller dimensions given in Example 10.1. Estimate the ideal head rise and mechanical power input if the outlet blade angle is changed to 60°, 70°, 80°, or 85°.



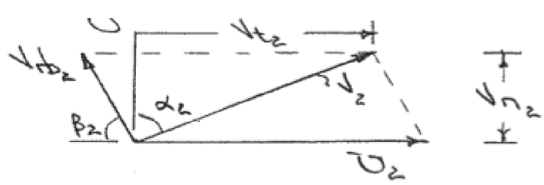
Assumptions: (1) axial flow at the inlet so $V_t = 0$ (2) at blade outlet the flow is uniform and leaves tangent to blade

Solution: Use the fluid machinery relations for power and head

$$\dot{W}_M = (U_2 V_{t2} - U_1 V_{t1})\dot{m}$$

$$H = \frac{1}{g}(U_2 V_{t2} - U_1 V_{t1})$$

The exit velocity diagram is:



From continuity,

$$V_{n2} = \frac{Q}{2\pi r_2 b_2} = V_r b_2 \sin\beta_2$$

$$V_r b_2 = \frac{V_{n2}}{\sin\beta_2}$$

From geometry,

$$V_{t2} = U_2 - V_r b_2 \cos\beta_2 = U_2 - \frac{V_{n2}}{\sin\beta_2}\cos\beta_2 = U_2 - V_{n2}\cot\beta_2$$

Substituting numerical values, for $\beta = 60°$,

$$U_2 = \omega r_2 = 3450\ \frac{rev}{min} \times 2\pi\ \frac{rad}{rev} \times \frac{min}{60\ s} \times 2\ in \times \frac{ft}{12\ in} = 60.2\ \frac{ft}{s}$$

$$V_{n2} = \frac{Q}{2\pi r_2 b_2} = \frac{1}{2\pi} \times 150\ \frac{gal}{min} \times \frac{min}{60\ s} \times \frac{ft^3}{7.48\ gal} \times \frac{1}{2.0\ in} \times \frac{1}{0.383\ in} \times \frac{144\ in^2}{ft^2} = 10.0\ \frac{ft}{s}$$

$$V_{t2} = U_2 - V_{n2} \cot \beta_2 = 60.2\ \frac{ft}{s} - 10.0\ \frac{ft}{s} \times \cot 60° = 54.4\ \frac{ft}{s}$$

$$\dot{m} = \rho Q = 1.94\ \frac{slug}{ft^3} \times 150\ \frac{gal}{min} \times \frac{min}{60\ s} \times \frac{ft^3}{7.48\ gal} = 0.648\ \frac{slug}{s}$$

The head is then

$$H = \frac{1}{g} U_2 V_{t2} = \frac{s^2}{32.2\ ft} \times 60.2\ \frac{ft}{s} \times 54.4\ \frac{ft}{s} = 102\ ft$$

The power is

$$\dot{W}_M = \dot{m} U_2 V_{t2} = \dot{m} g H = 0.648\ \frac{slug}{s} \times 32.2\ \frac{ft}{s^2} \times 102\ ft \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{hp \cdot s}{550\ ft \cdot lbf}$$

$$\dot{W}_M = 3.87\ hp$$

For $\beta = 70°$:

$$V_{t2} = 56.6\ \frac{ft}{s}$$

$$H = 106\ ft$$

$$\dot{W}_M = 4.02\ hp$$

For $\beta = 80°$:

$$V_{t2} = 58.4\ \frac{ft}{s}$$

$$H = 109\ ft$$

$$\dot{W}_M = 4.15\ hp$$

For $\beta = 85°$:

$$V_{t2} = 59.3\ \frac{ft}{s}$$

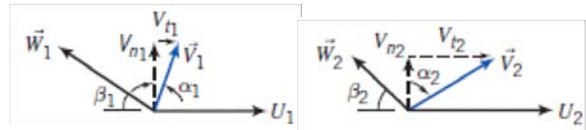
$$H = 111\ ft$$

$$\dot{W}_M = 4.21\ hp$$

Problem 10.4 [Difficulty: 2]

10.4 Dimensions of a centrifugal pump impeller are

Parameter	Inlet, Section (1)	Outlet, Section (2)
Radius, r (in.)	15	45
Blade width, b (in.)	4.75	3.25
Blade angle, β (deg)	40	60



The pump is driven at 575 rpm and the fluid is water. Calculate the theoretical head and mechanical power if the flow rate is 80,000 gpm.

Given: Geometry of centrifugal pump

Find: Theoretical head; Power input for given flow rate

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$

The given or available data is

$\rho = 1.94 \cdot \frac{slug}{ft^3}$ $\quad r_1 = 15 \cdot in$ $\quad r_2 = 45 \cdot in$ $\quad b_1 = 4.75 \cdot in$ $\quad b_2 = 3.25 \cdot in$

$\omega = 575 \cdot rpm$ $\quad \beta_1 = 40 \cdot deg$ $\quad \beta_2 = 60 \cdot deg$ $\quad Q = 80000 \cdot gpm$ $\quad Q = 178 \cdot \frac{ft^3}{s}$

From continuity $\quad V_n = \frac{Q}{2 \cdot \pi \cdot r \cdot b} = V_{rb} \cdot \sin(\beta)$ $\quad V_{rb} = \frac{V_n}{\sin(\beta)}$

From geometry $\quad V_t = U - V_{rb} \cdot \cos(\beta) = U - \frac{V_n}{\sin(\beta)} \cdot \cos(\beta) = U - \frac{Q}{2 \cdot \pi \cdot r \cdot b} \cdot \cot(\beta)$

Using given data $\quad U_1 = \omega \cdot r_1$ $\quad U_1 = 75.3 \cdot \frac{ft}{s}$ $\quad U_2 = \omega \cdot r_2$ $\quad U_2 = 226 \cdot \frac{ft}{s}$

$V_{t1} = U_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1)$ $\quad V_{t1} = 6.94 \cdot \frac{ft}{s}$

$V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2)$ $\quad V_{t2} = 210 \cdot \frac{ft}{s}$

The mass flow rate is $\quad m_{rate} = \rho \cdot Q$ $\quad m_{rate} = 346 \cdot \frac{slug}{s}$

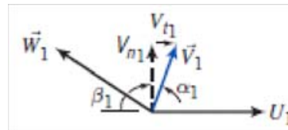
Hence $\quad W_m = (U_2 \cdot V_{t2} - U_1 \cdot V_{t1}) \cdot m_{rate}$ $\quad W_m = 1.62 \times 10^7 \cdot \frac{ft \cdot lbf}{s}$ $\quad W_m = 2.94 \times 10^4 \cdot hp$

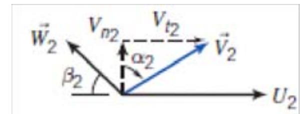
The head is $\quad H = \frac{W_m}{m_{rate} \cdot g}$ $\quad H = 1455 \cdot ft$

Problem 10.5 [Difficulty: 2]

10.5 Dimensions of a centrifugal pump impeller are

Parameter	Inlet, Section (1)	Outlet, Section (2)
Radius, r (in.)	3	9.75
Blade width, b (in.)	1.5	1.125
Blade angle, β (deg)	60	70





The pump is driven at 1250 rpm while pumping water. Calculate the theoretical head and mechanical power input if the flow rate is 1500 gpm.

Given: Geometry of centrifugal pump

Find: Theoretical head; Power input for given flow rate

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$

The given or available data is

$\rho = 1.94\cdot\frac{slug}{ft^3}$ $\quad r_1 = 3\cdot in$ $\quad r_2 = 9.75\cdot in$ $\quad b_1 = 1.5\cdot in$ $\quad b_2 = 1.125\cdot in$

$\omega = 1250\cdot rpm$ $\quad \beta_1 = 60\cdot deg$ $\quad \beta_2 = 70\cdot deg$ $\quad Q = 1500\cdot gpm$ $\quad Q = 3.34\cdot\frac{ft^3}{s}$

From continuity $\quad V_n = \frac{Q}{2\cdot\pi\cdot r\cdot b} = V_{rb}\cdot\sin(\beta)$ $\quad V_{rb} = \frac{V_n}{\sin(\beta)}$

From geometry $\quad V_t = U - V_{rb}\cdot\cos(\beta) = U - \frac{V_n}{\sin(\beta)}\cdot\cos(\beta) = U - \frac{Q}{2\cdot\pi\cdot r\cdot b}\cdot\cot(\beta)$

Using given data $\quad U_1 = \omega\cdot r_1$ $\quad U_1 = 32.7\cdot\frac{ft}{s}$ $\quad U_2 = \omega\cdot r_2$ $\quad U_2 = 106.4\cdot\frac{ft}{s}$

$V_{t1} = U_1 - \frac{Q}{2\cdot\pi\cdot r_1\cdot b_1}\cdot\cot(\beta_1)$ $\quad V_{t1} = 22.9\cdot\frac{ft}{s}$

$V_{t2} = U_2 - \frac{Q}{2\cdot\pi\cdot r_2\cdot b_2}\cdot\cot(\beta_2)$ $\quad V_{t2} = 104\cdot\frac{ft}{s}$

The mass flow rate is $\quad m_{rate} = \rho\cdot Q$ $\quad m_{rate} = 6.48\cdot\frac{slug}{s}$

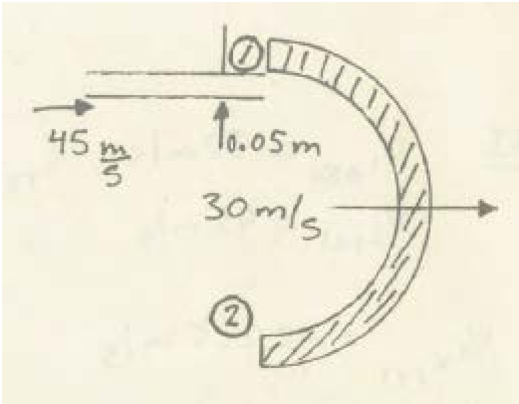
Hence $\quad W_m = (U_2\cdot V_{t2} - U_1\cdot V_{t1})\cdot m_{rate}$ $\quad W_m = 66728\cdot\frac{ft\cdot lbf}{s}$ $\quad W_m = 121\cdot hp$

The head is $\quad H = \frac{W_m}{m_{rate}\cdot g}$ $\quad H = 320\cdot ft$

Problem 10.6

(Difficulty 1)

10.6 The blade is one of a series. Calculate the force exerted by the jet on the blade system.



Find The force on the blade

Assumption: The flow is water and is steady and incompressible. The velocity of the fluid is constant as it turns.

Solution: Use the linear momentum equation to find the force. For the x direction

$$F_x = (V_{2abs} - V_{1abs})\rho Q$$

We use a stationary reference. There is a series of blades so the flow rate through the blades is based on the absolute velocity. The volumetric flow rate is:

$$Q = V_{jet}A = 45\ \frac{m}{s} \times \frac{\pi}{4} \times (0.05\ m)^2 = 0.0884\ \frac{m^3}{s}$$

The absolute velocity entering is the jet velocity.

$$V_{1abs} = 45\ \frac{m}{s}$$

We need to find the absolute velocity leaving. The fluid velocity along the blade surface is the relative velocity

$$V_{rel} = V_{jet} - U = 45\ \frac{m}{s} - 30\ \frac{m}{s} = 15\ \frac{m}{s}$$

At the exit, the relative velocity is in the negative x direction. The absolute velocity leaving is

$$V_{2abs} = V_{rel} + U = -15\ \frac{m}{s} + 30\ \frac{m}{s} = 15\ \frac{m}{s}$$

The fluid is moving in the direction of the blade. The density of the water is

$$\rho = 1000\ \frac{kg}{m^3}$$

The momentum equation is:

$$F_x = (V_{2abs} - V_{1abs})\rho Q = \left(15\ \frac{m}{s} - 45\ \frac{m}{s}\right) \times 1000\ \frac{kg}{m^3} \times 0.0884\ \frac{m^3}{s} = -2650\ N$$

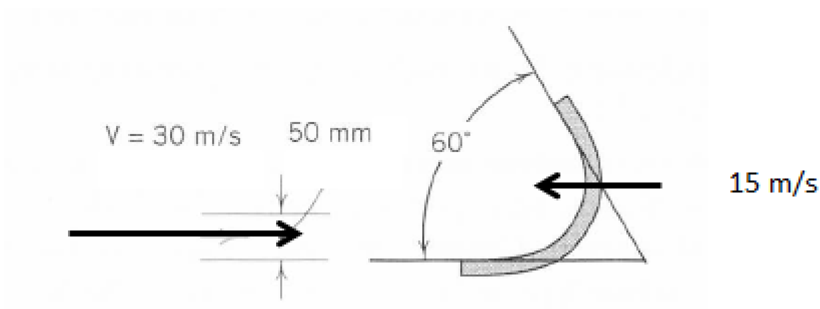
The force on the blade is:

$$F_x' = 2650\ N$$

Problem 10.7

(Difficulty 2)

10.7 This blade is one of a series. What force is required to move the series horizontally against the direction of the jet of water at a velocity 15 $\frac{m}{s}$? What power is required to accomplish this motion?



Find The force on the blade and the power required.

Assumption: The flow is steady and incompressible. The velocity of the fluid is constant as it turns.

Solution: Use the linear momentum equation to find the force. For the x direction

$$F_x = (V_{2abs} - V_{1abs})\rho Q$$

We use a stationary reference. There is a series of blades so the flow rate through the blades is based on the absolute velocity. The volumetric flow rate is:

$$Q = VA = 30\ \frac{m}{s} \times \frac{\pi}{4} \times (0.05\ m)^2 = 0.059\ \frac{m^3}{s}$$

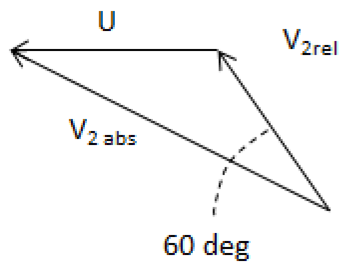
The absolute velocity entering is the jet velocity.

$$V_{1abs} = 30\ \frac{m}{s}$$

We need to find the absolute velocity leaving. The fluid velocity along the blade surface is the relative velocity

$$V_{rel} = V_{jet} - U = 30\ \frac{m}{s} - \left(-15\ \frac{m}{s}\right) = 45\ \frac{m}{s}$$

The vector relation for the leaving velocity is



The relative velocity at the exit is :

$$V_{2rel} = -45\ \frac{m}{s}$$

The component of the exit relative velocity in the x direction is

$$V_{2xrel} = V_{2rel}\ cos(60\ deg) = \ -22.5\ \frac{m}{s}$$

The absolute velocity in the x-direction leaving is

$$V_{2abs} = U + V_{2xrel} = \ -15\ \frac{m}{s} - 22.5\ \frac{m}{s} = \ -37.5\ \frac{m}{s}$$

The density of water is $\rho = 1000\ \frac{kg}{m^3}$

The force on the blade is then:

$$F_x = (V_{2abs} - V_{1abs})\rho Q = \left(-37.5\ \frac{m}{s} - 30\ \frac{m}{s}\right) \times 1000\ \frac{kg}{m^3} \times 0.059\ \frac{m^3}{s} = -3980\ N$$

The power required to accomplish the motion is:

$$p = F_x \cdot U = 3980\ N \times 15\ \frac{m}{s} = 59.7\ kW$$

There is also a force in the y-direction that is found using the momentum equation in the y direction. The absolute velocity of the fluid leaving in the y-direction is

$$V_{2y} = V_{2rel}\ sin(60\ deg) = \ 39\ \frac{m}{s}$$

The y-force is

$$F_y = (V_{2yabs} - 0)\rho Q = \left(39\ \frac{m}{s}\right) \times 1000\ \frac{kg}{m^3} \times 0.059\ \frac{m^3}{s} = 2300\ N$$

The resultant force is calculated as:

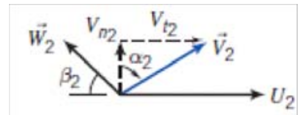
$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-3980\ N)^2 + (2300\ N)^2} = 4600\ N$$

In a centrifugal pump, the force in the y-direction would be opposed by the force of the bearings. The power would be based only on the x-direction force.

Problem 10.8

[Difficulty: 2]

10.8 A centrifugal water pump, with 15 cm diameter impeller and axial inlet flow, is driven at 1750 rpm. The impeller vanes are backward-curved ($\beta_2 = 65°$) and have axial width $b_2 = 2$ cm. For a volume flow rate of 225 m³/hr determine the theoretical head rise and power input to the pump.



Given: Geometry of centrifugal pump

Find: Theoretical head; Power input for given flow rate

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$

The given or available data is

$\rho = 1000\cdot\frac{kg}{m^3}$ $\quad r_2 = 7.5\cdot cm$ $\quad b_2 = 2\cdot cm$ $\quad \beta_2 = 65\cdot deg$

$\omega = 1750\cdot rpm$ $\quad Q = 225\cdot\frac{m^3}{hr}$ $\quad Q = 0.0625\frac{m^3}{s}$

From continuity $\quad V_{n2} = \frac{Q}{2\cdot\pi\cdot r_2\cdot b_2}$ $\quad V_{n2} = 6.63\frac{m}{s}$

From geometry $\quad V_{t2} = U_2 - V_{rb2}\cdot\cos(\beta_2) = U_2 - \frac{V_{n2}}{\sin(\beta_2)}\cdot\cos(\beta_2)$

Using given data $\quad U_2 = \omega\cdot r_2$ $\quad U_2 = 13.7\frac{m}{s}$

Hence $\quad V_{t2} = U_2 - \frac{Q}{2\cdot\pi\cdot r_2\cdot b_2}\cdot\cot(\beta_2)$ $\quad V_{t2} = 10.7\frac{m}{s}$ $\quad V_{t1} = 0$ (axial inlet)

The mass flow rate is $\quad m_{rate} = \rho\cdot Q$ $\quad m_{rate} = 62.5\frac{kg}{s}$

Hence $\quad W_m = U_2\cdot V_{t2}\cdot m_{rate}$ $\quad W_m = 9.15\cdot kW$

The head is $\quad H = \frac{W_m}{m_{rate}\cdot g}$ $\quad H = 14.9\cdot m$

Problem 10.9

(Difficulty: 3)

10.9 Consider the centrifugal pump impeller dimensions given in Example 10.1. Construct the velocity diagram for shockless flow at the impeller inlet, if b=constant. Calculate the effective flow angle with respect to the radial impeller blades for the case of no inlet swirl. Investigate the effects on flow angle of (a) variations in impeller width and (b) inlet swirl velocities.

Assumptions: The flow is stead and the flow enters the impeller at the blade angle.

Solution: Use the velocity vector relations.

The flow rate is

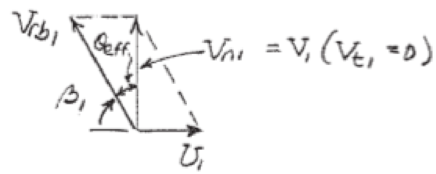
$$Q = 150\ \frac{gal}{min} \times \frac{ft^3}{7.48\ gal} \times \frac{min}{60\ s} = 0.334\ \frac{ft^3}{s}$$

$$r_1 = 0.0521\ ft$$

$$b = 0.0319\ ft$$

$$w = 3450\ \frac{rev}{min} \times 2\pi\ \frac{rad}{rev} \times \frac{min}{60\ s} = 361\ \frac{rad}{s}$$

The velocity vector diagram at the inlet is



From continuity, the normal velocity at the inlet is:

$$V_{n1} = \frac{Q}{2\pi r_1 b_1} = \frac{1}{2\pi} \times 0.334\ \frac{ft^3}{s} \times \frac{1}{0.0521\ ft} \times \frac{1}{0.0319\ ft} = 32.0\ \frac{ft}{s}$$

$$U_1 = \omega r_1 = 361\ \frac{rad}{s} \times 0.0521\ ft = 18.8\ \frac{ft}{s}$$

$$\beta_1 = \tan^{-1}\frac{V_{n1}}{U_1} = \tan^{-1}\left(\frac{32.0}{18.8}\right) = 59.6°$$

Thus for straight radial vanes,

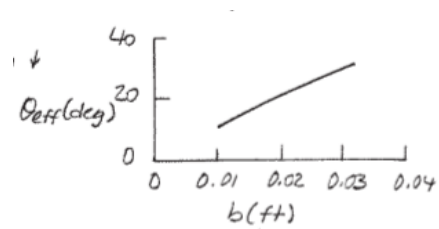
$$\theta_{eff} = \frac{\pi}{2} - \beta_1 = 90° - 59.6° = 30.4°$$

To change θ_{eff}: (a) vary b with no inlet swirl;

$$V_1 = V_{n1} = \frac{Q}{2\pi r_1 b_1}$$

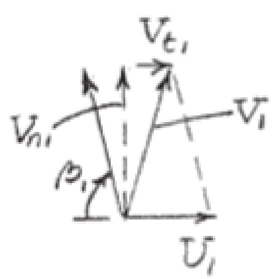
$$\beta_1 = \tan^{-1}\frac{Q}{2\pi r_1 b_1 U_1}$$

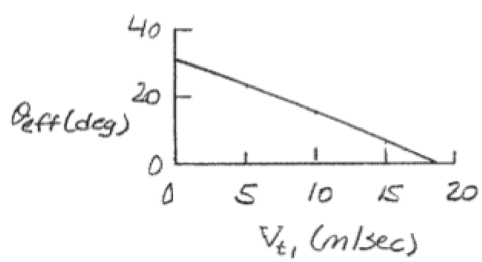
So we have β_1 increases as b_1 decreases.



$$\theta_{eff} = 90° - \beta_1$$

(b) Vary inlet swirl (V_{t1}) with $b = 0.0319\ ft$.





$$\beta_1 = \tan^{-1}\frac{V_{n1}}{U_1 - V_{t1}}$$

So β_1 increases as V_{t1} increases.

$$\theta_{eff} = 90° - \beta_1$$

Problem 10.10

(Difficulty: 3)

10.10 A centrifugal water pump designed to operate at 1300 rpm has dimensions

Parameter	Inlet	Outlet
Radius, r (mm)	100	175
Blade width, b (mm)	10	7.5
Blade angle, β (deg)		40

Draw the inlet velocity diagram for a volume flow rate of 35 L/s. Determine the inlet blade angle for which the entering velocity has no tangential component. Draw the outlet velocity diagram. Determine the outlet absolute flow angle (measured relative to the normal direction). Evaluate the hydraulic power delivered by the pump, if its efficiency is 75 percent. Determine the head developed by the pump.

Assumptions: The flow is steady and one-dimensional through the passages

Solution: Apply continuity and the Euler turbomachine equation.

$$V_n = \frac{Q}{2\pi rb}$$

$$\dot{W}_m = \rho Q(U_2 V_{t2} - U_1 V_{t1})$$

The angular speed is

$$\omega = 1300\ \frac{rev}{min} \times 2\pi\ \frac{rad}{rev} \times \frac{min}{60\ s} = 136\ \frac{rad}{s}$$

$$U_1 = 13.6\ \frac{m}{s}$$

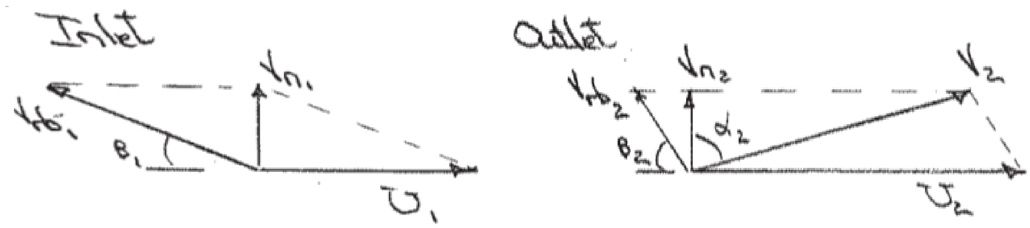
$$U_2 = 23.8\ \frac{m}{s}$$

The normal velocities entering and leaving the impeller are

$$V_{n1} = \frac{Q}{2\pi r_1 b_1} = \frac{1}{2\pi} \times 35\ \frac{L}{s} \times \frac{m^3}{10^3\ L} \times \frac{1}{0.1\ m} \times \frac{1}{0.01\ m} = 5.57\ \frac{m}{s}$$

$$V_{n2} = \frac{r_1 b_1}{r_2 b_2} V_{n1} = \frac{100}{175} \times \frac{10}{7.5} \times 5.57\ \frac{m}{s} = 4.24\ \frac{m}{s}$$

The velocity vector diagrams are



$$\tan\beta_1 = \frac{V_{n1}}{U_1}, \qquad \beta_1 = \tan^{-1}\left(\frac{5.57}{13.6}\right), \qquad \beta_1 = 22.3°$$

From the outlet diagram,

$$V_{t2} = U_2 - V_{n2}\cot\beta_2 = 23.8\ \frac{m}{s} - 4.24\ \frac{m}{s} \times \frac{1}{\tan 40°}$$

$$V_{t2} = 18.8\ \frac{m}{s}$$

$$\alpha_2 = \tan^{-1}\frac{V_{t2}}{V_{n2}} = \tan^{-1}\left(\frac{18.8\ \frac{m}{s}}{4.24\ \frac{m}{s}}\right) = 77.3°$$

The power is then

$$\dot{W}_m = \rho Q(U_2V_{t2} - U_1V_{t1}) = 999\ \frac{kg}{m^3} \times 35\ \frac{L}{s} \times \frac{m^3}{10^3\ L} \times \left[23.8\ \frac{m}{s} \times 18.8\ \frac{m}{s} - 0\right]\frac{N \cdot s^2}{kg \cdot m} \times 0.75$$

$$\dot{W}_m = 11.7\ kW$$

The head is

$$H = \frac{\dot{W}_m}{\rho g Q} = \frac{U_2V_{t2} - U_1V_{t1}}{g} = 23.8\ \frac{m}{s} \times 18.8\ \frac{m}{s} \times \frac{s^2}{9.81\ m} \times 0.75 = 34.2\ m$$

Problem 10.11

(Difficulty 2)

10.11 A series of blades, such as in Example 10.13, moving in the same direction as a water jet of 25 mm diameter and of velocity 46 $\frac{m}{s}$, deflects the jet 75° from its original direction. What relation between the blade velocity and blade angle must exist to satisfy this condition? What is the force on the blade system?

Find The force on the blade

Assumption: The flow is water and is steady and incompressible. The velocity of the fluid is constant as it turns.

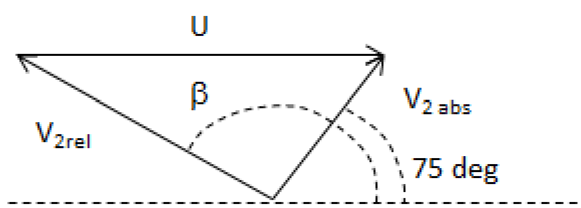
Solution: Use the vector relation among velocities and the linear momentum equation to find the force. For the x direction

$$F_x = (V_{2abs} - V_{1abs})\rho Q$$

We use a stationary reference. There is a series of blades so the flow rate through the blades is based on the absolute velocity. The volumetric flow rate is:

$$Q = V_{jet}A = 46\ \frac{m}{s} \times \frac{\pi}{4} \times (0.025\ m)^2 = 0.023\ \frac{m^3}{s}$$

The velocity vector diagram is



We have for the y-direction components of velocity

$$V_{2abs} \sin 75° = V_{2rel} \sin(180° - \beta)$$

And for the x-direction components

$$V_2 \cos 75° = U - V_{2rel} \cos(180° - \beta)$$

The velocity of the water does not change as it flows along the blade surface. The relative velocity leaving is then also

$$V_{2rel} = 46\ \frac{m}{s} - U$$

The required relation between the blade velocity and blade angle is:

$$\tan 75° = \frac{V_{2rel}\sin(180° - \beta)}{U - V_{2rel}\cos(180° - \beta)} = \frac{(46 - U)\sin(180° - \beta)}{\left(U - \left(46\ \frac{m}{s} - U\right)\cos(180° - \beta)\right)} = 3.73$$

For a given blade velocity U, this relation allows the desired blade angle to be calculated.

The forces on the blades system can be calculated from the x- and y-momentum equations as:

$$F_x = \rho Q(V_1 - u)(1 - \cos\beta)$$

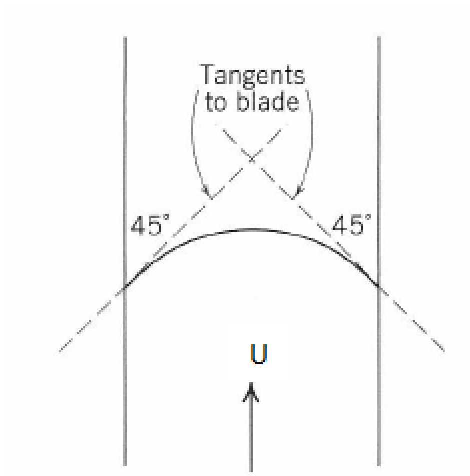
$$F_y = \rho Q V_2 \sin 75°$$

$$F = \sqrt{F_x^2 + F_y^2}$$

Problem 10.12

(Difficulty 3)

10.12 In passing through this blade system, the absolute jet velocity decreases from 41.5 to 22.5 $\frac{m}{s}$. If the flow rate is 57 $\frac{L}{s}$ of water, calculate the power transferred to the blade system and the vertical force component exerted on the blade system.



Find The force on the blade and the power required.

Assumption: The flow is steady and incompressible. The velocity of the fluid is constant as it turns.

Solution: Use the velocity vector relations and the linear momentum equation to find the force. For the x direction (direction of motion):

$$F_x = (V_{2abs} - V_{1abs})\rho Q$$

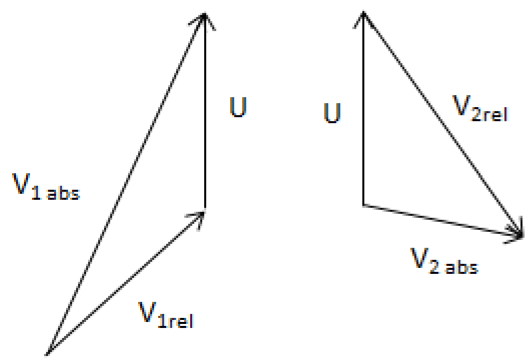
Because we know the absolute velocities, we can compute the power from the energy equation

$$P = \frac{\rho Q}{2}(V_1^2 - V_2^2)$$

Where $V_1 = 41.5\ \frac{m}{s}$, $V_2 = 22.5\ \frac{m}{s}, and\ Q = 57\ \frac{L}{s} = 0.057\ \frac{m^3}{s}$. The power is

$$P = \frac{\rho Q}{2}(V_1^2 - V_2^2) = \frac{998\ \frac{kg}{m^3} \times 0.057\ \frac{m^3}{s}}{2} \times \left(\left(41.5\ \frac{m}{s}\right)^2 - \left(22.5\ \frac{m}{s}\right)^2\right) = 34.6\ kW$$

The velocity diagrams at the inlet and outlet are



The force on the fluid is calculated as:

$$F_x = \rho Q(-V_{1rel}\cos 45° - V_{2rel}\cos 45°)$$

$$V_{1rel} = V_{2rel} = V_{rel}$$

The force on the blade is:

$$F_y = \rho Q(2V_{rel}\cos 45°)$$

And the power is, in terms of the unknown blade velocity U

$$P = F_y U = 2\rho Q U V_{rel}\cos 45°$$

Or

$$UV_{rel} = \frac{P}{2\rho Q\cos 45°} = \frac{34.6\ kW}{2 \times 998\ \frac{kg}{m^3} \times 0.057\ \frac{m^3}{s} \times \frac{1}{\sqrt{2}}} = 430.1\ \frac{m^2}{s^2}$$

From the trigonometric relations for the velocities, we obtain another expression for U and V_{rel}:

$$U^2 + 2UV_{rel}\cos 45° + {V_{rel}}^2 = V_1^2$$

or

$$U^2 + {V_{rel}}^2 = 1114\ \frac{m^2}{s^2}$$

Solving these two simultaneous equations we have two solutions. One is

$$U = 14.25\ \frac{m}{s}\ \ and\ \ V_{rel} = 30.2\ \frac{m}{s}$$

The force is then

$$F_x = 2 \times 998\ \frac{kg}{m^3} \times 0.057\ \frac{m^3}{s} \times 30.2\ \frac{m}{s} \times \frac{1}{\sqrt{2}} = 2430\ N$$

The second set of velocities are

$$U = 30.2\ \frac{m}{s}\ and\ \ V_{rel} = 14.25\frac{m}{s}$$

The force for this velocity solution is

$$F_x = 2 \times 998\ \frac{kg}{m^3} \times 0.057\ \frac{m^3}{s} \times 14.25\frac{m}{s} \times \frac{1}{\sqrt{2}} = 1146\ N$$

Problem 10.13 [Difficulty: 3]

10.13 A centrifugal pump runs at 1750 rpm while pumping water at a rate of 50 L/s. The water enters axially, and leaves tangential to the impeller blades. The impeller exit diameter and width are 300 mm and 10 mm, respectively. If the pump requires 45 kW, and is 75 percent efficient, estimate the exit angle of the impeller blades.

Given: Data on a centrifugal pump

Find: Estimate exit angle of impeller blades

Solution:

The given or available data is $\rho = 999 \cdot \frac{kg}{m^3}$ $\quad Q = 50 \cdot \frac{L}{s}$ $\quad W_{in} = 45 \cdot kW$ $\quad \eta = 75 \cdot \%$

$\omega = 1750 \cdot rpm$ $\quad b_2 = 10 \cdot mm$ $\quad D = 300 \cdot mm$

The governing equation (derived directly from the Euler turbomachine equation) is $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$

For an axial inlet $V_{t1} = 0$ hence $V_{t2} = \frac{W_m}{U_2 \cdot \rho \cdot Q}$

We have $U_2 = \frac{D}{2} \cdot \omega$ $\quad U_2 = 27.5 \frac{m}{s}$ and $W_m = \eta \cdot W_{in}$ $\quad W_m = 33.8 \cdot kW$

Hence $V_{t2} = \frac{W_m}{U_2 \cdot \rho \cdot Q}$ $\quad V_{t2} = 24.6 \frac{m}{s}$

From continuity $V_{n2} = \frac{Q}{\pi \cdot D \cdot b_2}$ $\quad V_{n2} = 5.31 \frac{m}{s}$

With the exit velocities determined, β can be determined from exit geometry

$$\tan(\beta) = \frac{V_{n2}}{U_2 - V_{t2}} \quad \text{or} \quad \beta = \operatorname{atan}\left(\frac{V_{n2}}{U_2 - V_{t2}}\right) \quad \beta = 61.3 \cdot deg$$

Problem 10.14

[Difficulty: 3]

10.14 A centrifugal water pump designed to operate at 1200 rpm has dimensions

Parameter	Inlet	Outlet
Radius, *r* (mm)	90	150
Blade width, *b* (mm)	10	7.5
Blade angle, *β* (deg)	25	45

Determine the flow rate at which the entering velocity has no tangential component. Draw the outlet velocity diagram, and determine the outlet absolute flow angle (measured relative to the normal direction) at this flow rate. Evaluate the hydraulic power delivered by the pump if its efficiency is 70 percent. Determine the head developed by the pump.

Given: Data on a centrifugal pump

Find: Flow rate for zero inlet tangential velocity; outlet flow angle; power; head developed

Solution:

The given or available data is $\rho = 999 \cdot \frac{kg}{m^3}$ $\qquad \omega = 1200 \cdot rpm$ $\qquad \eta = 70 \cdot \%$

$r_1 = 90 \cdot mm$ $\qquad b_1 = 10 \cdot mm$ $\qquad \beta_1 = 25 \cdot deg$ $\qquad r_2 = 150 \cdot mm$ $\qquad b_2 = 7.5 \cdot mm$ $\qquad \beta_2 = 45 \cdot deg$

The governing equations (derived directly from the Euler turbomachine equation) are

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$$

$$H = \frac{\dot{W}_m}{\dot{m} g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$

We also have from geometry $\quad \alpha_2 = \operatorname{atan}\left(\frac{V_{t2}}{V_{n2}}\right) \qquad (1)$

From geometry $\quad V_{t1} = 0 = U_1 - V_{rb1} \cdot \cos(\beta_1) = r_1 \cdot \omega - \frac{V_{n1}}{\sin(\beta_1)} \cdot \cos(\beta_1)$

and from continuity $\quad V_{n1} = \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1}$

Hence $\quad r_1 \cdot \omega - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1 \cdot \tan(\beta_1)} = 0$ $\qquad Q = 2 \cdot \pi \cdot r_1^{\,2} \cdot b_1 \cdot \omega \cdot \tan(\beta_1)$ $\qquad Q = 29.8 \cdot \frac{L}{s}$ $\qquad Q = 0.0298 \frac{m^3}{s}$

The power, head and absolute angle α at the exit are obtained from direct computation using Eqs. 10.2b, 10.2c, and 1 above

$U_1 = r_1 \cdot \omega$ $\qquad U_1 = 11.3 \frac{m}{s}$ $\qquad U_2 = r_2 \cdot \omega$ $\qquad U_2 = 18.8 \frac{m}{s}$ $\qquad V_{t1} = 0 \cdot \frac{m}{s}$

From geometry $\quad V_{t2} = U_2 - V_{rb2} \cdot \cos(\beta_2) = r_2 \cdot \omega - \frac{V_{n2}}{\sin(\beta_2)} \cdot \cos(\beta_2)$

and from continuity $\quad V_{n2} = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2}$ $\qquad V_{n2} = 4.22 \frac{m}{s}$

Hence $$V_{t2} = r_2 \cdot \omega - \frac{V_{n2}}{\tan(\beta_2)} \qquad V_{t2} = 14.6\,\frac{m}{s}$$

Using these results in Eq. 1 $$\alpha_2 = \mathrm{atan}\left(\frac{V_{t2}}{V_{n2}}\right) \qquad \alpha_2 = 73.9\cdot deg$$

Using them in Eq. 10.2b $$W_m = \left(U_2 \cdot V_{t2} - U_1 \cdot V_{t1}\right) \cdot \rho \cdot Q \qquad W_m = 8.22 \cdot kW$$

Using them in Eq. 10.2c $$H = \frac{1}{g} \cdot \left(U_2 \cdot V_{t2} - U_1 \cdot V_{t1}\right) \qquad H = 28.1\,m$$

This is the power and head assuming no inefficiency; with $\eta = 70\%$, we have (from Eq. 10.4c)

$$W_h = \eta \cdot W_m \qquad W_h = 5.75 \cdot kW$$

$$H_p = \eta \cdot H \qquad H_p = 19.7\,m$$

(This last result can also be obtained from Eq. 10.4a $W_h = \rho \cdot Q \cdot g \cdot H_p$)

Problem 10.15

[Difficulty: 1]

10.15 Kerosene is pumped by a centrifugal pump. When the flow rate is 350 gpm, the pump requires 18 hp input, and its efficiency is 82 percent. Calculate the pressure rise produced by the pump. Express this result as (a) feet of water and (b) feet of kerosene.

Given: Data on centrifugal pump

Find: Pressure rise; Express as ft of water and kerosene

Solution:

Basic equations: $\eta = \dfrac{\rho \cdot Q \cdot g \cdot H}{W_m}$

The given or available data is $\rho_w = 1.94 \cdot \dfrac{slug}{ft^3}$ $\quad Q = 350 \cdot gpm \quad Q = 0.780 \cdot \dfrac{ft^3}{s}$

$W_m = 18 \cdot hp \quad \eta = 82 \cdot \%$

Solving for H $\quad H = \dfrac{\eta \cdot W_m}{\rho_w \cdot Q \cdot g} \quad H = 166.8 \cdot ft$

For kerosene, $\quad SG = 0.82 \quad H_k = \dfrac{\eta \cdot W_m}{SG \cdot \rho_w \cdot Q \cdot g} \quad H_k = 203 \cdot ft$

Problem 10.16 [Difficulty: 4]

10.16 In the water pump of Problem 10.8, the pump casing acts as a diffuser, which converts 60 percent of the absolute velocity head at the impeller outlet to static pressure rise. The head loss through the pump suction and discharge channels is 0.75 times the radial component of velocity head leaving the impeller. Estimate the volume flow rate, head rise, power input, and pump efficiency at the maximum efficiency point. Assume the torque to overcome bearing, seal, and spin losses is 10 percent of the ideal torque at $Q = 0.065\ \mathrm{m^3/s}$.

Given: Geometry of centrifugal pump with diffuser casing

Find: Flow rate; Theoretical head; Power; Pump efficiency at maximum efficiency point

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$

The given or available data is

$$\rho = 1000\cdot\frac{kg}{m^3} \qquad r_2 = 7.5\cdot cm \qquad b_2 = 2\cdot cm \qquad \beta_2 = 65\cdot deg$$

$$\omega = 1750\cdot rpm \qquad \omega = 183\cdot\frac{rad}{s}$$

Using given data $\quad U_2 = \omega\cdot r_2 \qquad U_2 = 13.7\frac{m}{s}$

Illustrate the procedure with $\quad Q = 0.065\cdot\frac{m^3}{s}$

From continuity $\quad V_{n2} = \dfrac{Q}{2\cdot\pi\cdot r_2\cdot b_2} \qquad V_{n2} = 6.9\frac{m}{s}$

From geometry $\quad V_{t2} = U_2 - V_{rb2}\cdot\cos(\beta_2) = U_2 - \dfrac{V_{n2}}{\sin(\beta_2)}\cdot\cos(\beta_2)$

Hence $\quad V_{t2} = U_2 - \dfrac{Q}{2\cdot\pi\cdot r_2\cdot b_2}\cdot\cot(\beta_2) \qquad V_{t2} = 10.5\frac{m}{s} \qquad V_{t1} = 0$ (axial inlet)

$$V_2 = \sqrt{V_{n2}^2 + V_{t2}^2} \qquad V_2 = 12.6\frac{m}{s}$$

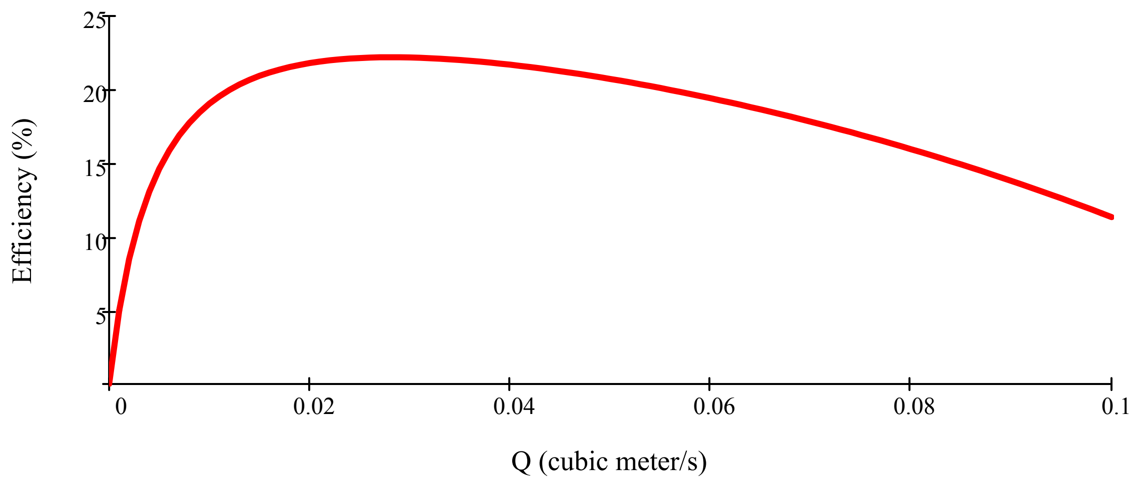
$$H_{ideal} = \frac{U_2\cdot V_{t2}}{g} \qquad H_{ideal} = 14.8\cdot m$$

$$T_{friction} = 10\cdot\%\cdot\frac{W_{mideal}}{\omega} = 10\cdot\%\cdot\frac{\rho\cdot Q\cdot H_{ideal}}{\omega}$$

$$T_{friction} = 10\cdot\%\cdot\frac{Q\cdot\rho\cdot g\cdot H_{ideal}}{\omega} \qquad T_{friction} = 5.13\cdot N\cdot m$$

$$H_{actual} = 60\cdot\%\cdot\frac{V_2^{\ 2}}{2\cdot g} - 0.75\cdot\frac{V_{n2}^{\ 2}}{2\cdot g} \qquad H_{actual} = 3.03\,m$$

$$\eta = \frac{Q\cdot\rho\cdot g\cdot H_{actual}}{Q\cdot\rho\cdot g\cdot H_{ideal} + \omega\cdot T_{friction}} \qquad \eta = 18.7\cdot\%$$



The above graph can be plotted in Excel. In addition, Solver can be used to vary Q to maximize η. The results are

$$Q = 0.0282\cdot\frac{m^3}{s} \qquad \eta = 22.2\cdot\% \qquad H_{ideal} = 17.3\,m \qquad H_{actual} = 4.60\,m$$

$$W_m = Q\cdot\rho\cdot g\cdot H_{ideal} + \omega\cdot T_{friction} \qquad W_m = 5.72\cdot kW$$

Problem 10.17

(Difficulty: 2)

10.17 Use data from Appendix C to choose points from the performance curves for a Peerless horizontal split case Type 16A18B pump at 705 and 880 nominal rpm. Obtain and plot curve-fits of total head versus delivery for this pump, with an 18.0-in-diameter impeller.

Solution: Put the data from Figs. C.9 (705 rpm) and C.10 (880 rpm) in tabular form:

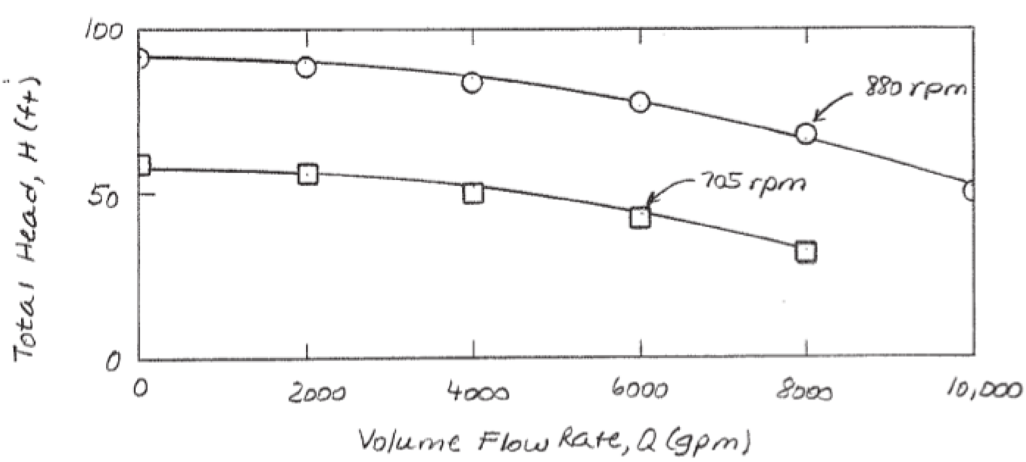
At 705 rpm:

$Q\ (gpm)$	0	2000	4000	6000	8000
$H\ (ft)$	59	56	50	43	32
Curve-fit:	$\hat{H}\ (ft) = 57.8 - 4.09 \times 10^{-7}\ [Q\ (gpm)]^2$;			$r^2 = 0.994$	
$\hat{H}\ (ft)$	57.8	56.2	51.3	43.1	31.6

At 880 rpm:

$Q\ (gpm)$	0	2000	4000	6000	8000	10000
$H\ (ft)$	92	89	84	78	68	50
Curve-fit:	$\hat{H}\ (ft) = 91.5 - 4.01 \times 10^{-7}\ [Q\ (gpm)]^2$;				$r^2 = 0.992$	
$\hat{H}\ (ft)$	91.5	89.9	85.1	77.1	65.9	51.5

Plot the data:



Problem 10.18

[Difficulty: 3]

10.18 Data from tests of a water suction pump operated at 2000 rpm with a 12-in. diameter impeller are

Flow rate, Q (cfm)	36	50	74	88	125
Total head, H (ft)	190	195	176	162	120
Power input, $\mathcal{P}$ (hp)	25	30	35	40	46

Plot the performance curves for this pump; include a curve of efficiency versus volume flow rate. Locate the best efficiency point and specify the pump rating at this point.

Given: Data on suction pump

Find: Plot of performance curves; Best effiiciency point

Solution:

Basic equations: $\eta_p = \dfrac{P_h}{P_m}$ $\quad P_h = \rho\cdot Q\cdot g\cdot H$ (Note: Software cannot render a dot!)

$\rho =$ 1.94 slug/ft^3

Fitting a 2nd order polynomial to each set of data we find

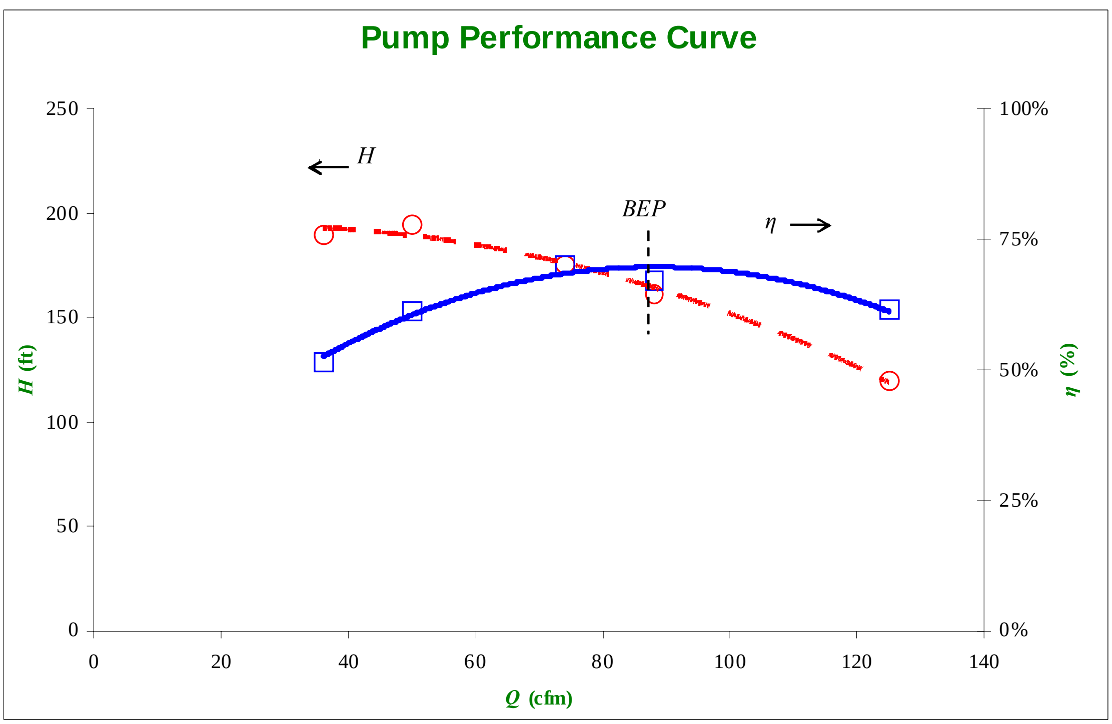
$H = -0.00759Q^2 + 0.390Q + 189.1$

$\eta = -6.31\times10^{-5}Q^2 + 0.01113Q + 0.207$

Q (cfm)	H (ft)	$\mathcal{P}_m$ (hp)	$\mathcal{P}_h$ (hp)	η (%)
36	190	25	12.9	51.7%
50	195	30	18.4	61.5%
74	176	35	24.6	70.4%
88	162	40	27.0	67.4%
125	120	46	28.4	61.7%

Finally, we use Solver to maximize η by varying Q:

Q (cfm)	H (ft)	η (%)
88.2	164.5	69.8%



Problem 10.19

(Difficulty 3)

10.19 A centrifugal pump impeller having $r_1 = 50\ mm, r_2 = 150\ mm,$ and width $b = 37.5\ mm$ is to pump 225 $\frac{L}{s}$ of water and supply $12.2\ J$ of energy to each newton of fluid. The impeller rotates at 1000 $\frac{r}{min}$. What blade angles are required? What power is required to drive this pump? Assume radial flow at the inlet of the impeller.

Find: The angles of the blade for the power input.

Assumption: The flow is steady and incompressible. The velocity of the fluid is constant as it turns.

Solution: Use the velocity vector relations and the angular momentum equation to find the torque and power. The torque is given by:

$$T = \rho Q (V_{t2} r_2 - V_{t1} r_1)$$

And the power is given by

$$\dot{W} = T\omega$$

Because we know the power input, we can work backwards to find the blade angles. The power is given in terms of the energy input to the fluid

$$\dot{W} = \rho\ Q\ \Delta e = \ 9800\ \frac{N}{m^3} \times 0.225 \frac{m^3}{s} \times \frac{12.2\ J}{N} = 26.9\ kW$$

The rotating speed is

$$\omega = \frac{1000\ rpm \times 2\pi}{60} = 104.7\ \frac{rad}{s}$$

The torque is then

$$T = \frac{26.9\ kW}{104.7\ \frac{rad}{s}} = 257\ N \cdot m$$

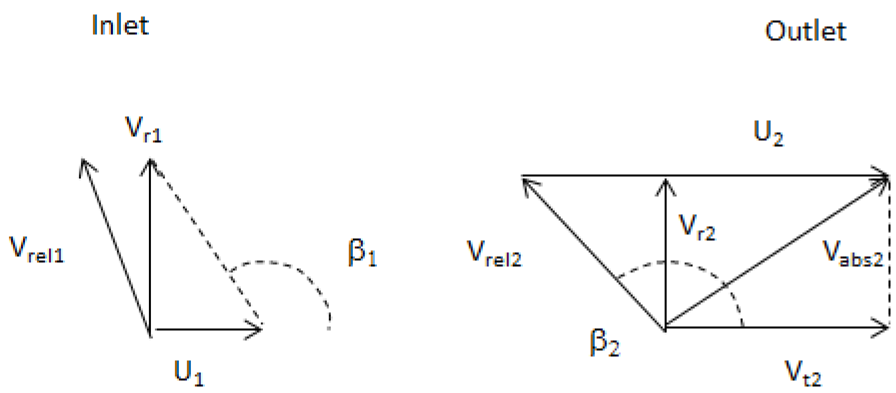
For the inlet flow in the radial direction, there is no tangential velocity. From the relation for the torque, we then have

$$T = \rho Q (V_{t2} r_2 - V_{t1} r_1) = \rho Q V_{t2} r_2$$

The tangential velocity at the exit must then be

$$V_{t2} = \frac{T}{\rho Q r_2} = \frac{257\ N \cdot m}{998\ \frac{kg}{m^3} \times 0.225\frac{m^3}{s} \times 0.15\ m} = 7.61\ \frac{m}{s}$$

The velocity vector relations then are:



We have the following for this problem:

$$Q = 225\ \frac{L}{s} = 0.225\frac{m^3}{s},\quad r_1 = 50\ mm = 0.05\ m, \qquad r_2 = 150\ mm = 0.15\ m,$$
$$b = 37.5\ mm = 0.0375\ m$$

The flow area is calculated as:

$$A_1 = 2\pi r_1 b = 2\pi \times 0.05\ m \times 0.0375\ m = 0.0118\ m^2$$

$$A_2 = 2\pi r_2 b = 2\pi \times 0.15\ m \times 0.0375\ m = 0.0353\ m^2$$

The radial velocities are then:

$$V_{r1} = \frac{Q}{A_1} = \frac{0.225\frac{m^3}{s}}{0.0118\ m^2} = 19.1\ \frac{m}{s}$$

$$V_{r2} = \frac{Q}{A_2} = \frac{0.225\frac{m^3}{s}}{0.0353\ m^2} = 6.37\ \frac{m}{s}$$

Since the impeller rotates at 1000 rpm, the impeller velocity at the inlet and outlet are:

$$U_1 = \omega r_1 = 104.7\ \frac{rad}{s} \times 0.05\ m = 5.24\ \frac{m}{s}$$

$$U = \omega r_2 = 104.7\ \frac{rad}{s} \times 0.15\ m = 15.71\ \frac{m}{s}$$

For the inlet, from the geometry of the velocity diagram

$$\tan(\pi - \beta_1) = \frac{V_{r1}}{U_1} = \frac{19.1\ \frac{m}{s}}{5.24\ \frac{m}{s}} = \ 3.645$$

The inlet angle is then

$$\beta_1 = 105\ °$$

For the outlet, from the geometry of the velocity diagram

$$\tan\left(\beta_2 - \frac{\pi}{2}\right) = \frac{U_2 - V_{t2}}{V_{r2}} = \frac{15.71\ \frac{m}{s} - 7.61\ \frac{m}{s}}{6.37\ \frac{m}{s}} = 1.272$$

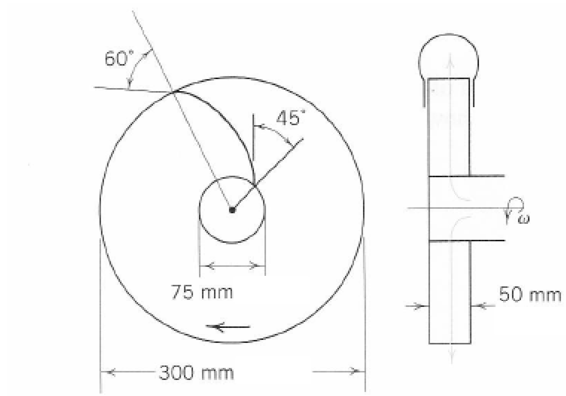
The outlet angle is then

$$\beta_2 = 141.8\ °$$

Problem 10.20

(Difficulty 2)

10.20 A centrifugal pump impeller having dimensions and angles as shown rotates at $500\ \frac{r}{min}$. Assuming a radial direction of velocity at the blade entrance, calculate the flow rate , the pressure difference between inlet and outlet of blades, and the torque and power required to meet these conditions.



Find: The flow rate, pressure difference, torque, and power input.

Assumption: The flow is steady and incompressible. The velocity of the fluid is constant as it turns.

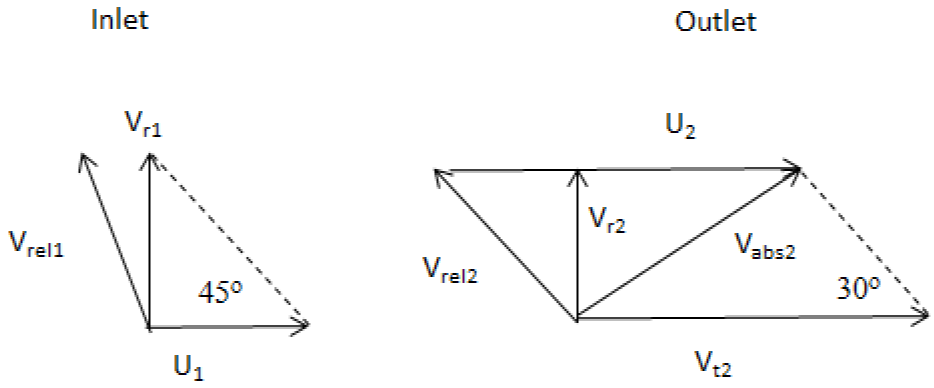
Solution: Use the velocity vector relations and the angular momentum equation to find the torque and power. The torque is given by:

$$T = \rho Q (V_{t2} r_2 - V_{t1} r_1)$$

And the power is given by

$$\dot{W} = T\omega$$

The velocity vector diagrams are



We have the following:

$$r_1 = \frac{0.075\ m}{2} = 0.0375\ m, \qquad r_2 = \frac{0.3\ m}{2} = 0.15\ m, \qquad b = 0.05\ m$$

The blade velocities are calculated as:

$$U_1 = \omega r_1 = \frac{500 \times 2\pi}{60}\frac{rad}{s} \times 0.0375\ m = 1.96\ \frac{m}{s}$$

$$U_2 = \omega r_2 = \frac{500 \times 2\pi}{60}\frac{rad}{s} \times 0.15\ m = 7.85\ \frac{m}{s}$$

The inlet blade angle is 45°, therefore:

$$V_1 = V_{r1} = U_1 = 1.96\ \frac{m}{s}$$

The flow rate is calculated as:

$$Q = V_{r1}A_1 = V_{r1}2\pi r_1 b = 1.96\ \frac{m}{s} \times 2\pi \times 0.0375\ m \times 0.05\ m = 0.023\ \frac{m^3}{s}$$

From the flow rate we have:

$$V_{r2} = \frac{Q}{A_2} = \frac{0.023\ \frac{m^3}{s}}{2\pi \times 0.15\ m \times 0.05\ m} = 0.488\ \frac{m}{s}$$

$$V_{t2} = U_2 - \frac{V_{r2}}{\tan 30°} = 7.0\ \frac{m}{s}$$

$$V_2 = \sqrt{V_{t2}^2 + V_{r2}^2} = \sqrt{\left(7.0\ \frac{m}{s}\right)^2 + \left(0.488\ \frac{m}{s}\right)^2} = 7.02\ \frac{m}{s}$$

$$V_{t1} = 0$$

The torque on the fluid can be calculated as:

$$T = \rho Q(V_{t2}r_2 - V_{t1}r_1) = \rho Q V_{t2} r_2$$

$$T = 998\ \frac{kg}{m^3} \times 0.023\ \frac{m^3}{s} \times 7.0\ \frac{m}{s} \times 0.15\ m = 24.1\ N \cdot m$$

Then we have:

$$\dot{W} = T\omega = 24.1\ N \cdot m \times \frac{500 \times 2\pi}{60}\ \frac{rad}{s} = 1262\ W$$

The head is calculated as:

$$H = \frac{\dot{W}}{\rho g Q} = \frac{1262\ W}{9800\ \frac{N}{m^3} \times 0.023\ \frac{m^3}{s}} = 5.6\ m$$

Applying the Bernoulli equation from section 1 to section 2 we have:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + H = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$P_2 - P_1 = \frac{\rho V_1^2}{2} - \frac{\rho V_2^2}{2} + \gamma H = 32.2\ kPa$$

Problem 10.21

[Difficulty: 3]

10.21 An axial-flow fan operates in sea -level air at 1350 rpm and has a blade tip diameter of 3 ft and a root diameter of 2.5 ft. The inlet angles are $\alpha_1 = 55°$, $\beta_1 = 30°$, and at the exit $\beta_2 = 60°$. Estimate the flow volumetric flow rate, horsepower, and the outlet angle, α_2.

Given: Data on axial flow fan

Find: Volumetric flow rate, horsepower, flow exit angle

Solution:

Basic equations:
$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$

The given or available data is

$$\rho = 0.002377\cdot\frac{\text{slug}}{\text{ft}^3} \quad \omega = 1350\cdot\text{rpm} \quad d_{tip} = 3\cdot\text{ft} \quad d_{root} = 2.5\cdot\text{ft} \quad \alpha_1 = 55\cdot\text{deg} \quad \beta_1 = 30\cdot\text{deg} \quad \beta_2 = 60\cdot\text{deg}$$

The mean radius would be half the mean diameter: $r = \frac{1}{2}\cdot\frac{d_{tip} + d_{root}}{2}$ $\quad r = 1.375\cdot\text{ft}$

Therefore, the blade speed is: $U = r\cdot\omega$ $\quad U = 194.39\cdot\frac{\text{ft}}{\text{s}}$

From velocity triangles we can generate the following two equations: $V_1\cdot\cos(\alpha_1) = w_1\cdot\sin(\beta_1)$ (axial component)

$V_1\cdot\sin(\alpha_1) + w_1\cdot\cos(\beta_1) = U$ (tangential component)

Combining the two equations: $V_1 = \dfrac{U}{\sin(\alpha_1) + \dfrac{\cos(\alpha_1)}{\tan(\beta_1)}}$ $\quad V_1 = 107.241\cdot\frac{\text{ft}}{\text{s}}$ $\quad w_1 = V_1\cdot\dfrac{\cos(\alpha_1)}{\sin(\beta_1)}$ $\quad w_1 = 123.021\cdot\frac{\text{ft}}{\text{s}}$

So the entrance velocity components are: $V_{n1} = V_1\cdot\cos(\alpha_1)$ $\quad V_{n1} = 61.511\cdot\frac{\text{ft}}{\text{s}}$ $\quad V_{t1} = V_1\cdot\sin(\alpha_1)$ $\quad V_{t1} = 87.846\cdot\frac{\text{ft}}{\text{s}}$

The volumetric flow rate would then be: $Q = V_{n1}\cdot\frac{\pi}{4}\cdot\left(d_{tip}^{\ 2} - d_{root}^{\ 2}\right)$ $\quad Q = 132.9\cdot\frac{\text{ft}^3}{\text{s}}$

Since axial velocity does not change: $V_{n2} = V_{n1}$

The exit speed relative to the blade is: $w_2 = \dfrac{V_{n2}}{\sin(\beta_2)}$ $\quad w_2 = 71.026\cdot\frac{\text{ft}}{\text{s}}$ so the tangential component of absolute velocity is:

$V_{t2} = U - w_2\cdot\cos(\beta_2)$ $\quad V_{t2} = 158.873\cdot\frac{\text{ft}}{\text{s}}$ Into the expression for power: $W_m = U\cdot(V_{t2} - V_{t1})\cdot\rho\cdot Q$ $\quad W_m = 7.93\cdot\text{hp}$

The flow exit angle is: $\alpha_2 = \text{atan}\left(\dfrac{V_{t2}}{V_{n2}}\right)$ $\quad \alpha_2 = 68.8\cdot\text{deg}$

Problem 10.22 [Difficulty: 3]

10.22 Data measured during tests of a centrifugal pump driven at 3000 rpm are

Parameter	Inlet, Section ①	Outlet, Section ②
Gage pressure, p (psi)	12.5	
Elevation above datum, z (ft)	6.5	32.5
Average speed of flow, $\overline{V}$ (ft/s)	6.5	15

The flow rate is 65 gpm and the torque applied to the pump shaft is 4.75 lbf·ft. The pump efficiency is 75 percent, and the electric motor efficiency is 85 percent. Find the electric power required, and the gage pressure at section ②.

Given: Data on centrifugal pump

Find: Electric power required; gage pressure at exit

Solution:

Basic equations: $\dot{W}_h = \rho Q g H_p$

$$H_p = \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_{\text{discharge}} - \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_{\text{suction}} \qquad \eta_p = \frac{\dot{W}_h}{\dot{W}_m} = \frac{\rho Q g H_p}{\omega T}$$

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $\quad \omega = 3000 \cdot \text{rpm}$ $\quad \eta_p = 75 \cdot \%$ $\quad \eta_e = 85 \cdot \%$ $\quad Q = 65 \cdot \text{gpm}$ $\quad Q = 0.145 \cdot \frac{\text{ft}^3}{\text{s}}$

$T = 4.75 \cdot \text{lbf} \cdot \text{ft}$ $\quad p_1 = 12.5 \cdot \text{psi}$ $\quad z_1 = 6.5 \cdot \text{ft}$ $\quad V_1 = 6.5 \cdot \frac{\text{ft}}{\text{s}}$ $\quad z_2 = 32.5 \cdot \text{ft}$ $\quad V_2 = 15 \cdot \frac{\text{ft}}{\text{s}}$

$$H_p = \frac{\omega \cdot T \cdot \eta_p}{\rho \cdot Q \cdot g} \qquad H_p = 124 \cdot \text{ft}$$

$$p_2 = p_1 + \frac{\rho}{2} \cdot \left(V_1^{\,2} - V_2^{\,2}\right) + \rho \cdot g \cdot \left(z_1 - z_2\right) + \rho \cdot g \cdot H_p \qquad p_2 = 53.7 \cdot \text{psi}$$

Also $\quad W_h = \rho \cdot g \cdot Q \cdot H_p \qquad W_h = 1119 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}} \qquad W_h = 2.03 \cdot \text{hp}$

The shaft work is then $\quad W_m = \frac{W_h}{\eta_p} \qquad W_m = 1492 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}} \qquad W_m = 2.71 \cdot \text{hp}$

Hence, electrical input is $\quad W_e = \frac{W_m}{\eta_e} \qquad W_e = 1756 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}} \qquad W_e = 2.38 \cdot \text{kW}$

Problem 10.23 [Difficulty: 2]

10.23 A small centrifugal pump, when tested at $N = 2875$ rpm with water, delivered $Q = 0.016\ \mathrm{m^3/s}$ and $H = 40$ m at its best efficiency point ($\eta = 0.70$). Determine the specific speed of the pump at this test condition. Sketch the impeller shape you expect. Compute the required power input to the pump.

Given: Data on small centrifugal pump

Find: Specific speed; Sketch impeller shape; Required power input

Solution:

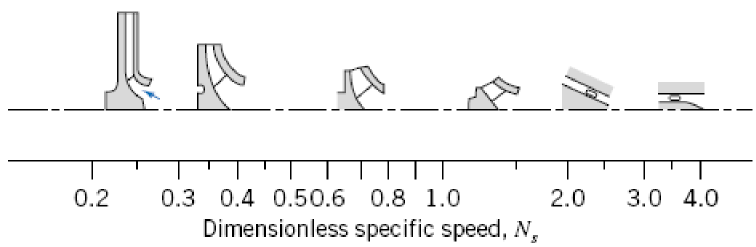
Basic equation: $N_S = \dfrac{\omega Q^{1/2}}{h^{3/4}}$ $\qquad \eta_p = \dfrac{\dot{W}_h}{\dot{W}_m} = \dfrac{\rho Q g H_p}{\omega T}$

The given or available data is

$\rho = 1000 \cdot \dfrac{\mathrm{kg}}{\mathrm{m}^3}$ $\qquad \omega = 2875 \cdot \mathrm{rpm}$ $\qquad \eta_p = 70 \cdot \%$ $\qquad Q = 0.016 \cdot \dfrac{\mathrm{m}^3}{\mathrm{s}}$ $\qquad H = 40 \cdot \mathrm{m}$

Hence $\quad h = g \cdot H \qquad h = 392\,\dfrac{\mathrm{m}^2}{\mathrm{s}^2}$ $\qquad$ (H is energy/weight. h is energy/mass)

Then $\quad N_S = \dfrac{\omega \cdot Q^{\frac{1}{2}}}{h^{\frac{3}{4}}} \qquad N_S = 0.432$



From the figure we see the impeller will be centrifugal

The power input is $\quad W_m = \dfrac{W_h}{\eta_p} \qquad W_m = \dfrac{\rho \cdot Q \cdot g \cdot H}{\eta_p} \qquad W_m = 8.97 \cdot \mathrm{kW}$

Problem 10.24

(Difficulty 3)

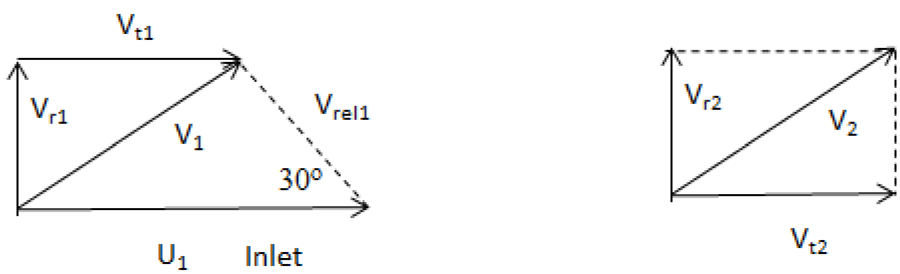
10.24 If the impeller of the problem 10.20 rotates between horizontal planes of infinite extent and the flow rate is 25 $\frac{L}{s}$, what rise of pressure may be expected between one point having $r = 150\ mm$ and another having $r = 225\ mm$?

Find The pressure rise

Assumption: The flow is steady and incompressible. The velocity of the fluid is constant as it turns.

Solution: Use the velocity vector relations and the Bernoulli equation to determine the pressure rise.

The velocity vector diagram is



We recalculate the velocities from Problem 10.20. The new parameters are:

$$r_1 = 0.15\ m, \qquad r_2 = 0.225\ m,\ \ and\ \ Q = 25\ \frac{L}{s} = 0.025\ \frac{m^3}{s}$$

The new radial velocities are

$$V_{r2} = \frac{Q}{2\pi r_2 b} = \frac{0.025\ \frac{m^3}{s}}{2\pi \times 0.225\ m \times 0.05\ m} = 0.354\ \frac{m}{s}$$

$$V_{r1} = \frac{Q}{2\pi r_1 b} = \frac{0.025\ \frac{m^3}{s}}{2\pi \times 0.15\ m \times 0.05\ m} = 0.531\ \frac{m}{s}$$

The rotational speed is

$$\omega = \frac{2\pi \times 500\ rpm}{60} = 52.3\ \frac{rad}{s}$$

The blade velocity is

$$U_1 = \omega r_1 = 7.85\ \frac{m}{s}$$

The entrance conditions for this problem are at $r_1 = 0.15\ m$ so V_1 has a tangential component:

$$V_{t1} = U_1 - V_{r1}\cot 30°$$

$$V_{t1} = 7.85\ \frac{m}{s} - 0.531\ \frac{m}{s} \times \frac{1}{\sqrt{3}} = 6.93\ \frac{m}{s}$$

$$V_1 = \sqrt{V_{t1}^2 + V_{r1}^2} = \sqrt{\left(6.93\ \frac{m}{s}\right)^2 + \left(0.531\ \frac{m}{s}\right)^2} = 6.95\ \frac{m}{s}$$

There is no torque exerted on the fluid between sections 1 and 2. Since

$$P = T\omega = 0$$

Therefore

$$T = 0 = \rho Q(V_{t2}r_2 - V_{t1}r_1)$$

Therefore

$$V_{t2}r_2 = V_{t1}r_1$$

$$V_{t2} = \frac{V_{t1}r_1}{r_2} = \frac{6.93\ \frac{m}{s} \times 0.15\ m}{0.225\ m} = 4.62\ \frac{m}{s}$$

The absolute velocity V_1 is then

$$V_1 = \sqrt{V_{t2}^2 + V_{r2}^2} = \sqrt{\left(4.62\ \frac{m}{s}\right)^2 + \left(0.354\ \frac{m}{s}\right)^2} = 4.63\ \frac{m}{s}$$

Applying the Bernoulli equation from section 1 to section 2 we have:

$$p_1 + \frac{\rho V_1^2}{2} = p_2 + \frac{\rho V_2^2}{2}$$

The pressure rise is

$$p_2 - p_1 = \frac{\rho V_1^2}{2} - \frac{\rho V_2^2}{2} = \frac{998\ \frac{kg}{m^3}}{2} \times \left(\left(6.95\ \frac{m}{s}\right)^2 - \left(4.63\ \frac{m}{s}\right)^2\right) = 13.4\ kPa$$

Problem 10.25

(Difficulty 1)

10.25 At the outlet of a pump impeller of diameter 0.6 m and width 150 mm, the (absolute) velocity is observed to be 30 $\frac{m}{s}$ at an angle of 60° with a radial line. Calculate the torque exerted on the impeller.

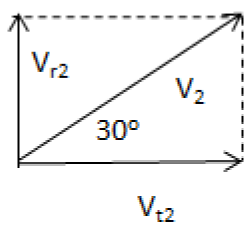
Find The torque

Assumption: The flow is steady and incompressible. The velocity of the entering fluid is int the radial direction.

Solution: Use the velocity vector relations and the angular momentum equation to find the torque.

$$T = \rho Q (V_{t2} r_2 - V_{t1} r_1)$$

The velocity vector diagram at the outlet is



We have the following:

$$r_2 = \frac{0.6\ m}{2} = 0.3\ m, \qquad b = 0.15\ m$$

The radial velocity at the exit is then

$$V_{r2} = 30\ \frac{m}{s} \sin 30° = 15\ \frac{m}{s}$$

And the tangential velocity is then

$$V_{t2} = 30\ \frac{m}{s} \cos 30° = 25.98\ \frac{m}{s}$$

The flow rate is calculated as:

$$Q = V_{r2} 2\pi r b = 15\ \frac{m}{s} \times 2\pi \times 0.3\ m \times 0.15\ m = 4.24\ \frac{m^3}{s}$$

The torque on the fluid for an inlet radial velocity is then:

$$T = \rho Q(V_{t2} r) = 33.1\ kN \cdot m$$

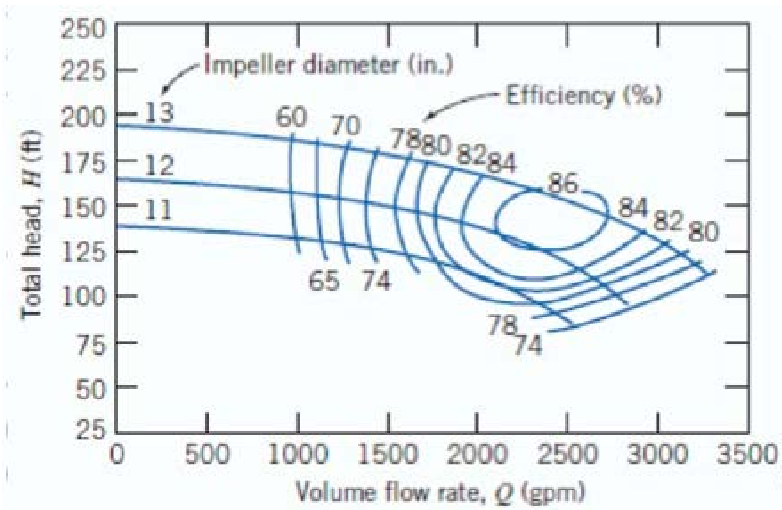
The torque on the impeller is opposite in sign:

$$T_i = -33.1\ kN \cdot m$$

Problem 10.26

(Difficulty: 2)

10.26 Typical performance curves for a centrifugal pump, tested with three different impeller diameters in a single casting, are shown. Specify the flow rate and head produced by the pump and its best efficiency point with a 12-in diameter impeller. Scale these data to predict the performance of this pump when tested with 11 in. and 13 in. impellers. Comment on the accuracy of the scaling procedure.



Solution: From the graph, BEP occurs for the 12 in impeller at $Q \cong 2200\ gpm$ and $H \cong 130\ ft$.

The scaling rules are:

$$Q_2 = Q_1 \left(\frac{D_2}{D_1}\right)^3$$

$$Q_{11} = 2200\ gpm \times \left(\frac{11}{12}\right)^3 = 1690\ gpm$$

$$Q_{13} = 2200\ gpm \times \left(\frac{13}{12}\right)^3 = 2800\ gpm$$

$$H_2 = H_1 \left(\frac{D_2}{D_1}\right)^2$$

$$H_{11} = 130\ ft \times \left(\frac{11}{12}\right)^2 = 109\ ft$$

$$H_{13} = 130\ ft \times \left(\frac{13}{12}\right)^2 = 153\ ft$$

Thus BEP_{11} is at $Q = 1690\ gpm$, $H = 109\ ft$. BEP_{13} is at $Q = 2800\ gpm$, $H = 153\ ft$.

The complete scaling rules tend to move the volume flow rate too far. Accuracy would be improved using $Q_2 = Q_1(D_2/D_1)^2$. Since the impeller width does not change, and $H_2 = H_1(D_2/D_1)^2$. Since $H \cong V^2$ with these modified rules.

$$\left(Q_{11,}\ H_{11}\right) = 1850\ gpm, 109\ ft$$

and

$$\left(Q_{13,}\ H_{13}\right) = 2580\ gpm, 153\ ft$$

These modified scaling points are closer to the measured BEPs.

Problem 10.27

[Difficulty: 3]

10.27 A pump with $D=500$ mm delivers $Q=0.725$ m^3/s of water at $H=10$ m at its best efficiency point. If the specific speed of the pump is 1.74, and the required input power is 90 kW, determine the shutoff head, H_0, and best efficiency, η. What type of pump is this? If the pump is now run at 900 rpm, by scaling the performance curve, estimate the new flow rate, head, shutoff head, and required power.

Given: Data on a pump

Find: Shutoff head; best efficiency; type of pump; flow rate, head, shutoff head and power at 900 rpm

Solution:

The given or available data is

$$\rho = 999\cdot\frac{\text{kg}}{\text{m}^3} \qquad N_s = 1.74 \qquad D = 500\cdot\text{mm} \qquad Q = 0.725\frac{\text{m}^3}{\text{s}} \qquad H = 10\cdot\text{m} \qquad W_m = 90\cdot\text{kW} \qquad \omega' = 900\cdot\text{rpm}$$

The governing equations are

$$W_h = \rho\cdot Q\cdot g\cdot H \qquad N_s = \frac{\omega\cdot Q^{\frac{1}{2}}}{h^{\frac{3}{4}}} \qquad H_0 = C_1 = \frac{U_2^{\,2}}{g}$$

Similarity rules:

$$\frac{Q_1}{\omega_1\cdot D_1^{\,3}} = \frac{Q_2}{\omega_2\cdot D_2^{\,3}} \qquad \frac{h_1}{\omega_1^{\,2}\cdot D_1^{\,2}} = \frac{h_2}{\omega_2^{\,2}\cdot D_2^{\,2}} \qquad \frac{P_1}{\rho_1\cdot\omega_1^{\,3}\cdot D_1^{\,5}} = \frac{P_2}{\rho_2\cdot\omega_2^{\,3}\cdot D_2^{\,5}}$$

$$h = g\cdot H = 98.1\frac{\text{J}}{\text{kg}} \quad \text{Hence} \quad \omega = \frac{N_s\cdot h^{\frac{3}{4}}}{Q^{\frac{1}{2}}} \qquad \omega = 63.7\cdot\frac{\text{rad}}{\text{s}} \qquad W_h = \rho\cdot Q\cdot g\cdot H = 71.0\,\text{kW} \qquad \eta_p = \frac{W_h}{W_m} = 78.9\,\%$$

The shutoff head is given by

$$H_0 = \frac{U_2^{\,2}}{g} \qquad U_2 = \frac{D}{2}\cdot\omega \qquad U_2 = 15.9\frac{\text{m}}{\text{s}} \quad \text{Hence} \quad H_0 = \frac{U_2^{\,2}}{g} \qquad H_0 = 25.8\,\text{m}$$

with $D_1 = D_2$:

$$\frac{Q_1}{\omega_1} = \frac{Q_2}{\omega_2} \quad \text{or} \quad \frac{Q}{\omega} = \frac{Q'}{\omega'} \qquad Q' = Q\cdot\frac{\omega'}{\omega} = 1.073\frac{\text{m}^3}{\text{s}} \qquad \frac{h_1}{\omega_1^{\,2}} = \frac{h_2}{\omega_2^{\,2}} \quad \text{or} \quad \frac{H}{\omega^2} = \frac{H'}{\omega'^2} \qquad H' = H\cdot\left(\frac{\omega'}{\omega}\right)^2 = 21.9\,\text{m}$$

Also

$$\frac{H_0}{\omega^2} = \frac{H'_0}{\omega'^2} \qquad H'_0 = H_0\cdot\left(\frac{\omega'}{\omega}\right)^2 \qquad H'_0 = 56.6\,\text{m}$$

$$\frac{P_1}{\rho\cdot\omega_1^{\,3}} = \frac{P_2}{\rho\cdot\omega_2^{\,3}} \quad \text{or} \quad \frac{W_m}{\omega^3} = \frac{W'_m}{\omega'^3} \qquad W'_m = W_m\cdot\left(\frac{\omega'}{\omega}\right)^3 \qquad W'_m = 292\cdot\text{kW}$$

Problem 10.28

[Difficulty: 3]

10.28 At its best efficiency point ($\eta = 0.87$), a mixed-flow pump, with $D = 16$ in., delivers $Q = 2500$ cfm of water at $H = 140$ ft when operating at $N = 1350$ rpm. Calculate the specific speed of this pump. Estimate the required power input. Determine the curve-fit parameters of the pump performance curve based on the shutoff point and the best efficiency point. Scale the performance curve to estimate the flow, head, efficiency, and power input required to run the same pump at 820 rpm.

Given: Data on a pump at BEP

Find:
(a) Specific Speed
(b) Required power input
(c) Curve fit parameters for the pump performance curve.
(d) Performance of pump at 820 rpm

Solution:

The given or available data is

$\rho = 1.94 \cdot \frac{slug}{ft^3}$ $\quad \eta = 87\%$ $\quad D = 16 \cdot in$ $\quad Q = 2500 \cdot cfm$ $\quad H = 140 \cdot ft$ $\quad \omega = 1350 \cdot rpm$ $\quad \omega' = 820 \cdot rpm$

The governing equations are $\quad N_s = \frac{\omega \cdot \sqrt{Q}}{(g \cdot H)^{0.75}}$ $\quad W_h = \rho \cdot Q \cdot g \cdot H$ $\quad W = \frac{W_h}{\eta}$ $\quad H_0 = \frac{U_2^{\,2}}{g}$

The specific speed is: $\quad N_s = 1.66$

The power is: $\quad W = 761 \cdot hp$

At shutoff $\quad U_2 = \frac{D}{2} \cdot \omega$ $\quad U_2 = 94.248 \cdot \frac{ft}{s}$ $\quad$ Therefore: $\quad H_0 = \frac{U_2^{\,2}}{g}$ $\quad H_0 = 276.1 \cdot ft$

Since $\quad H = H_0 - A \cdot Q^2$ $\quad$ it follows that $\quad A = \frac{H_0 - H}{Q^2}$ $\quad A = 2.18 \times 10^{-5} \cdot \frac{min^2}{ft^5}$

Another way to write this is: $\quad H(ft) = 276.1 - 2.18 \times 10^{-5} \cdot Q(cfm)^2$

$\omega' = 820 \cdot rpm$ $\quad H'_0 = H_0 \cdot \left(\frac{\omega'}{\omega}\right)^2$ $\quad$ and $\quad A' = A$ $\quad$ Thus: $\quad H'_0 = 101.9 \cdot ft$ $\quad A' = 2.18 \times 10^{-5} \cdot \frac{min^2}{ft^5}$

At BEP: $\quad Q' = Q \cdot \left(\frac{\omega'}{\omega}\right)$ $\quad Q' = 1519 \cdot cfm$ $\quad H' = H \cdot \left(\frac{\omega'}{\omega}\right)^2$ $\quad H' = 51.7 \cdot ft$ $\quad \eta' = \eta = 87 \cdot \%$

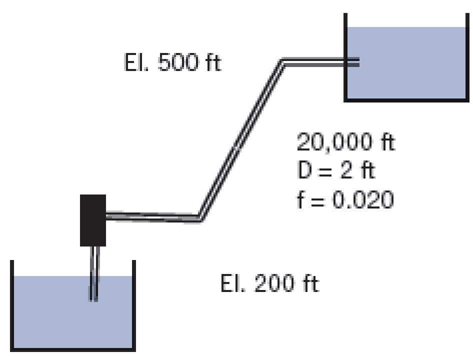
$W_m = W \cdot \left(\frac{\omega'}{\omega}\right)^3$ $\quad W_m = 170.5 \cdot hp$

At

Problem 10.29

(Difficulty 2)

10.29 Using the performance curves in Appendix C, select the smallest diameter Peerless 8AE20G pump operating at 1770 rpm that will deliver a flow of at least 2000 gpm for the pipeline shown. Determine the actual flow rate and the pump electrical power requirement.



Find The smallest pump for this application.

Assumption: The flow is steady and incompressible.

Solution: Use the energy equation to determine the required head and use the pump characteristics to select a pump from Appendix C.

Applying the energy equation to the system between location 1 (the lower reservoir) and 2 (the upper reservoir) we have:

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + H_p = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + H_l$$

Where the head loss is given by

$$h_l = f\frac{L}{D}\frac{V^2}{2g}$$

For a desired flow of 2000 gpm, the velocity in the pipe is

$$V = \frac{Q}{A} = \frac{2000\ gpm \times 0.0223\frac{ft^3}{s - gpm}}{\frac{\pi}{4} \times 2} = 1.42\frac{ft}{s}$$

The head loss is

$$h_l = 0.02 \times \frac{20{,}000ft}{2ft} \times \frac{\left(1.42\frac{ft}{s}\right)^2}{2 \times 32.2\frac{ft}{s^2}} = 6.25ft$$

For locations 1 and 2 we have

$$p_1 = p_{2,}\ V_1 = V_2 = 0,\ z_1 = 100\ ft,\ z_2 = 500\ ft$$

The energy equation becomes

$$100\ ft + H_p = 400\ ft + 6.25ft$$

The desired head is then

$$H_p = 400\ ft - 100ft + 6.25ft = 306ft$$

From Appendix C Fig. C.7 for the Peerless 8AE20G pump operating at 1770 rpm, the 18 in. diameter pump will produce a head of 320 ft at a flow rate of 2000 gpm. This is the appropriate pump to select.

The actual flow rate will be slightly larger than 2000 gpm because the head produced is larger. Selecting a large flow rate, determining the new head loss and required head, and then comparing against the pump performance curve will yield the flow rate that matches the head requirement. Through iteration, the actual flow rate will be about 2500 gpm and the total head requirement is 310 ft.

The electrical pump power is given by

$$\dot{W} = \frac{\rho Q H_p}{\eta}$$

The efficiency at 2500 gpm is 79 %. The power is then

$$\dot{W} = \frac{1.94\frac{slug}{ft^3} \times 2500gpm \times 0.0223\frac{ft^3}{s - gpm} \times 310ft \times 32.2\frac{ft}{s^2}}{0.79} = 138{,}000\ \frac{ft - lbf}{s}$$

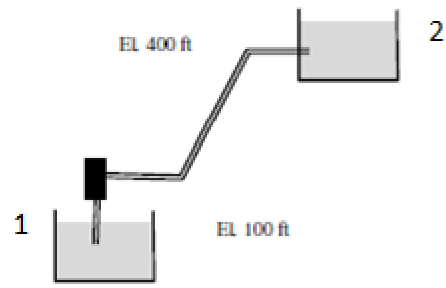
Or

$$\dot{W} = 251\ hp$$

Problem 10.30

(Difficulty 2)

10.30 A pump (Peerless 8AE20G, Appendix C) operates at 1770 rpm. It has a 20-in impeller and supplies the pipeline below while operating at maximum efficiency. Find the pipeline loss coefficient K in the equation $h_L = KQ^2$ for this condition. Neglect local losses. If two of these pumps operate in parallel, what is the flow rate between the two reservoirs? Assume the pipeline *K* value remains unchanged.



Find: The pipeline loss coefficient and the flow rate

Assume: The flow is steady and frictionless

Solution: Use the given pump characteristics and the energy equation. Applying the energy equation to the system between locations 1 and 2 we have:

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + H_p = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + KQ^2$$

Where we represent the loss of the system as KQ^2. For locations 1 and 2 we have

$$p_1 = p_{2,}\ V_1 = V_2 = 0,\ z_1 = 100\ ft,\ z_2 = 400\ ft$$

The energy equation becomes

$$100\ ft + H_p = 400\ ft + KQ^2$$

Using Figure C.7 for a 20-in diameter impeller, at maximum efficiency, the pump produces a head of 350 ft at a flow rate of 3500 gpm. The value of the coefficient K is then determined as

$$K = \frac{100\ ft + 350\ ft - 400\ ft}{(3500\ gpm)^2} = 4.08 \times 10^{-6}\,\frac{ft}{gpm^2}$$

If two pumps operate in parallel, each pump handles only one-half of the flow, but still needs to meet the total head. For each pump then, the energy equation is

$$100\ ft + H_p = 400\ ft + K\left(\frac{Q}{2}\right)^2 = 400\ ft + \frac{K}{4}Q^2$$

We use Figure C.7 and trial and error. We can produce a table of the total head as a function of flow rate. For example, for a flow rate of Q = 4000 gpm the value of the head is

$$H_p = 300\ ft + \frac{4.08 \times 10^{-6} \frac{ft}{gpm^2}}{4}(4000\ gpm)^2 = 316\ ft$$

At this flow rate, the pump can produce 400 ft, so this flow rate is too low. Similarly, for other flow rates, and extrapolating the performance in Figure c.7:

Flow(gpm)	Required Head (ft)	Pump head (ft)
4000	316	400
5000	325	340
6000	336	240

With two pumps in series, the flow will be about 5200 gpm.

Problem 10.31 [Difficulty: 3]

10.31 A pumping system must be specified for a lift station at a wastewater treatment facility. The average flow rate is 110 million liters per day and the required lift is 10 m. Non-clogging impellers must be used; about 65 percent efficiency is expected. For convenient installation, electric motors of 37.5 kW or less are desired. Determine the number of motor/pump units needed and recommend an appropriate operating speed.

Given: Data on pumping system

Find: Number of pumps needed; Operating speed

Solution:

Basic equations: $W_h = \rho \cdot Q \cdot g \cdot H \qquad \eta_p = \dfrac{W_h}{W_m}$

The given or available data is

$\rho = 1000 \cdot \dfrac{kg}{m^3} \qquad Q_{total} = 110 \times 10^6 \cdot \dfrac{L}{day} \qquad Q_{total} = 1.273 \dfrac{m^3}{s} \qquad H = 10 \cdot m \qquad \eta = 65 \cdot \%$

Then for the system $\quad W_h = \rho \cdot Q_{total} \cdot g \cdot H \qquad W_h = 125 \cdot kW$

The required total power is $\quad W_m = \dfrac{W_h}{\eta} \qquad W_m = 192 \cdot kW$

Hence the total number of pumps must be $\dfrac{192}{37.5} = 5.12$, or at least six pumps

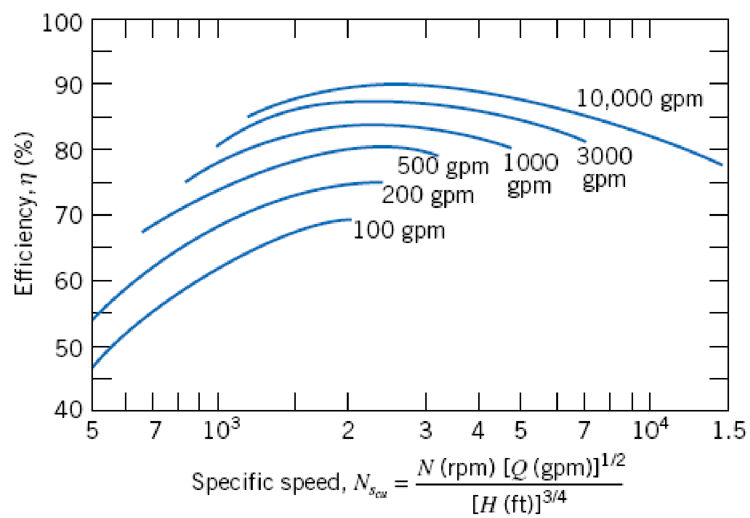
The flow rate per pump will then be $\quad Q = \dfrac{Q_{total}}{6} \qquad Q = 0.212 \dfrac{m^3}{s} \qquad Q = 212 \cdot \dfrac{L}{s}$

From Fig. 10.15 the peak effiiciency is at a specific speed of about

$N_{Scu} = 2000$

We also need $\quad H = 32.8 \cdot ft \qquad Q = 3363 \cdot gpm$

Hence $\quad N = N_{Scu} \cdot \dfrac{H^{\frac{3}{4}}}{Q^{\frac{1}{2}}} \qquad N = 473$



The nearest standard speed to $N = 473$ rpm should be used

Problem 10.32

[Difficulty: 2]

10.32 A centrifugal water pump operates at 1750 rpm; the impeller has backward-curved vanes with $\beta_2 = 60°$ and $b_2 = 1.25$ cm. At a flow rate of 0.025 m^3/s, the radial outlet velocity is $V_{n_2} = 3.5$ m/s. Estimate the head this pump could deliver at 1150 rpm.

Given: Data on centrifugal pump

Find: Head at 1150 rpm

Solution:

Basic equation:

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$

The given or available data is

$$\rho = 1000 \cdot \frac{kg}{m^3} \qquad Q = 0.025 \cdot \frac{m^3}{s} \qquad \beta_2 = 60 \cdot deg \qquad b_2 = 1.25 \cdot cm$$

$$\omega = 1750 \cdot rpm \qquad \omega' = 1150 \cdot rpm \qquad V_{n2} = 3.5 \cdot \frac{m}{s}$$

From continuity $$V_{n2} = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2}$$

Hence $$r_2 = \frac{Q}{2 \cdot \pi \cdot b_2 \cdot V_{n2}} \qquad r_2 = 0.0909\,m \qquad r_2 = 9.09 \cdot cm$$

Then $$V'_{n2} = \frac{\omega'}{\omega} \cdot V_{n2} \qquad V'_{n2} = 2.30\,\frac{m}{s}$$

Also $$U'_2 = \omega' \cdot r_2 \qquad U'_2 = 11.0\,\frac{m}{s}$$

From the outlet geometry $$V'_{t2} = U'_2 - V'_{n2} \cdot \cos(\beta_2) \qquad V'_{t2} = 9.80\,\frac{m}{s}$$

Finally $$H' = \frac{U'_2 \cdot V'_{t2}}{g} \qquad H' = 10.9\,m$$

Problem 10.33 [Difficulty: 3]

10.33 A set of eight 30-kW motor-pump units is used to deliver water through an elevation of 30 m. The efficiency of the pumps is specified to be 65 percent. Estimate the delivery (liters per day) and select an appropriate operating speed.

Given: Data on pumping system

Find: Total delivery; Operating speed

Solution:

Basic equations: $W_h = \rho \cdot Q \cdot g \cdot H \qquad \eta_p = \dfrac{W_h}{W_m}$

The given or available data is

$$\rho = 1000 \cdot \frac{kg}{m^3} \qquad W_m = 30 \cdot kW \qquad H = 30 \cdot m \qquad H = 98.425 \cdot ft \quad \eta = 65 \cdot \%$$

Then for the system $W_{mTotal} = 8 \cdot W_m \qquad W_{mTotal} = 240 \cdot kW$

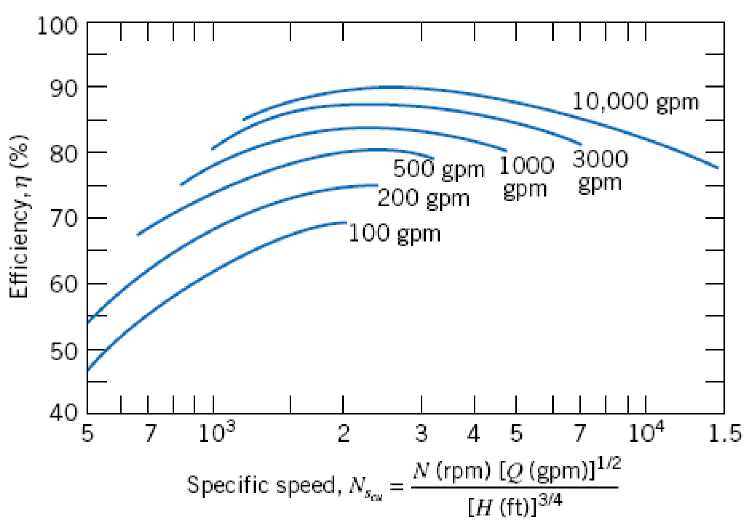
The hydraulic total power is $W_{hTotal} = W_{mTotal} \cdot \eta \qquad W_{hTotal} = 156 \cdot kW$

The total flow rate will then be $Q_{Total} = \dfrac{W_{hTotal}}{\rho \cdot g \cdot H} \qquad Q_{Total} = 0.53 \cdot \dfrac{m^3}{s} \qquad Q_{Total} = 4.58 \times 10^7 \cdot \dfrac{L}{day}$

The flow rate per pump is $Q = \dfrac{Q_{Total}}{8} \qquad Q = 0.066 \cdot \dfrac{m^3}{s} \qquad Q = 1051 \cdot gpm$

From Fig. 10.15 the peak effiiciency is at a specific speed of about

$$N_{Scu} = 2500$$



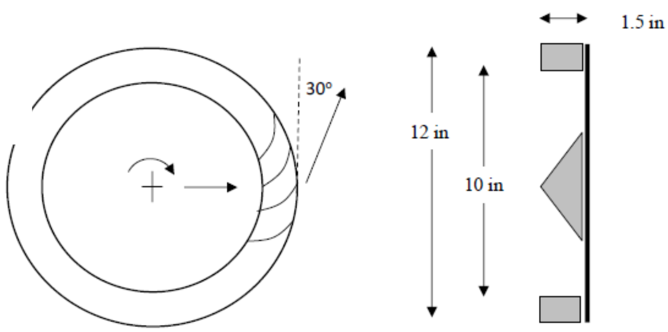
Hence $N = N_{Scu} \cdot \dfrac{H^{\frac{3}{4}}}{Q^{\frac{1}{2}}} \qquad N = 2410$

The nearest standard speed to $N = 2410$ rpm should be used

Problem 10.34

(Difficulty 2)

10.34 A blower has a rotor with 12 in outside diameter and 10 in inside diameter with 1.5 in high rotor blades. The flow rate through the blower is 500 $\frac{ft^3}{min}$ at a rotor speed of 1800 rpm. The air at blade inlet is radial and the discharge angle is 30° from the tangential direction. Determine the power required by the blower motor.



Find The power input.

Assumption: The flow is steady and incompressible.

Solution: Use the angular momentum equation to find the torque and power. The torque is given by:

$$T = \rho Q(V_{t2}r_2 - V_{t1}r_1)$$

And the power is given by

$$\dot{W} = T\omega$$

As the absolute velocity of air at blade inlet is radial we have:

$$T_{shaft} = \dot{m}(r_2 V_{t2})$$

The mass flow rate is calculated as:

$$\dot{m} = \rho Q = 0.00238\ \frac{slug}{ft^3} \times \frac{500}{60}\frac{ft^3}{s} = 0.0198\ \frac{slug}{s}$$

The rotor velocity is:

$$U_2 = \omega r_2 = 1800 \times \frac{2\pi}{60\ s} \times \frac{6}{12}\ ft = 94.3\ \frac{ft}{s}$$

The relative velocity in the radial direction is calculated from the flow rate as:

$$Q = V_{rn2}A = V_{rn2}2\pi r_2 b$$

$$V_{rn2} = \frac{Q}{2\pi r_2 b} = \frac{\frac{500}{60}\frac{ft^3}{s}}{2\pi \times \frac{6}{12}\ ft \times \frac{1.5}{12}\ ft} = 21.2\ \frac{ft}{s}$$

The relative velocity in the tangential direction is:

$$tan30° = \frac{V_{rn2}}{V_{rt2}}$$

$$V_{rt2} = \frac{V_{rn2}}{tan30°} = \frac{21.2\ \frac{ft}{s}}{\frac{1}{\sqrt{3}}} = 36.7\ \frac{ft}{s}$$

So the absolute velocity in the tangential direction is:

$$V_{t2} = U_2 - V_{rt2} = 94.3\ \frac{ft}{s} - 36.7\ \frac{ft}{s} = 57.6\ \frac{ft}{s}$$

The torque is calculated as:

$$T_{shaft} = \dot{m}(r_2 V_{t2}) = 0.0198\ \frac{\frac{lbf \cdot s^2}{ft}}{s} \times \left(\frac{6}{12}\ ft \times 57.6\ \frac{ft}{s}\right) = 0.57\ lbf \cdot ft$$

The power required by blower motor is:

$$\dot{W}_m = \omega T_{shaft} = 1800 \times \frac{2\pi}{60\ s} \times 0.57\ lbf \cdot ft = 107.4\ \frac{lbf \cdot ft}{s} = 0.195\ hp$$

Problem 10.35

(Difficulty 2)

10.35 A centrifugal water pump has an impeller with an outer diameter of 14 in, a blade height of 1 in. It rotates at 1200 rpm. The flow enters parallel to the axis of rotation and leaves at an angle of 35° with an absolute exit velocity of 75 $\frac{ft}{s}$. Determine the water flow rate, the torque, the horsepower required, and the pressure rise.

Find The flow rate, torque, power input, and pressure rise.

Assumption: The flow is steady and incompressible.

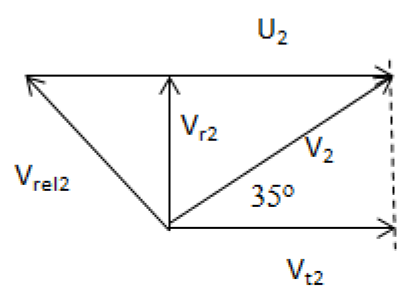
Solution: Use the velocity vector relations and the angular momentum equation to find the torque and power. The torque is given by:

$$T = \rho Q (V_{t2} r_2 - V_{t1} r_1)$$

And the power is given by

$$\dot{W} = T\omega$$

The velocity vector diagram at the exit is



The velocity in the radial direction is given by

$$V_{r2} = V_2 \sin 35° = 43\ \frac{ft}{s}$$

And the velocity in the tangential direction is

$$V_{t2} = V_2 \cos 35° = 61.4 \frac{ft}{s}$$

The water flow rate is calculated as:

$$\dot{m} = \rho Q = \rho 2\pi r_2 b V_{rn2} = 1.94\ \frac{slug}{ft^3} \times 2\pi \times \frac{7}{12}\ ft \times \frac{1}{12}\ ft \times 43\ \frac{ft}{s} = 25.5\ \frac{slug}{s}$$

The torque is given by:

$$T_{shaft} = \dot{m}(r_2 V_{t2} - r_1 V_{t1})$$

As the inlet is in the axial direction so we have:

$$V_{t1} = 0$$

The torque is then

$$T_{shaft} = \dot{m} r_2 V_{t2} = 25.5\ \frac{\frac{lbf \cdot s^2}{ft}}{s} \times \frac{7}{12}\ ft \times 61.4\ \frac{ft}{s} = 913\ lbf \cdot ft$$

The horsepower required is:

$$\dot{W} = \omega T_{shaft} = 1200 \times \frac{2\pi}{60\ s} \times 913\ lbf \cdot ft = 114700\ \frac{lbf \cdot ft}{s} = 208\ hp$$

The head rise is calculated as:

$$H_p = \frac{\dot{W}}{\rho g Q}$$

Using the energy equation

$$H_p = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_2 - \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_1$$

And neglecting the velocity and height change we have:

$$H_p = \left(\frac{p}{\rho g}\right)_2 - \left(\frac{p}{\rho g}\right)_1$$

The pressure rise is calculated by:

$$p_2 - p_1 = \rho g H_p = \rho g \frac{\dot{W}}{\rho g Q} = \frac{\dot{W}}{Q}$$

$$p_2 - p_1 = \frac{114700\ \frac{lbf \cdot ft}{s}}{2\pi \times \frac{7}{12}\ ft \times \frac{1}{12}\ ft \times 43\ \frac{ft}{s}} = 8730\ \ \frac{lbf}{ft^2} = 60.6\ psi$$

Problem 10.36

(Difficulty: 2)

10.36 Appendix C contains area bound curves for pump model selection and performance curves for individual pump models. Use these data to verify the similarity rules for a Peerless Type 4AE12 pump, with impeller diameter $D = 11.0\ in$, operated at 1750 and 3550 nominal rpm.

Solution: From Figs C.4 and C.5 at the best efficiency point (BEP):

$N\ (rpm)$	$Q\ (gpm)$	$H\ (ft)$	$\dot{W}_m\ (hp)$	$\eta\ (\%)$
1750	470	104	17	73^+
3550	970	430	135	74^+

The similarity rules are:

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}$$

$$\frac{H_1}{\omega_1^2 D_1^2} = \frac{H_2}{\omega_2^2 D_2^2}$$

$$\frac{P_1}{\omega_1^3 D_1^5} = \frac{P_2}{\omega_2^3 D_2^5}$$

$$\eta_1 = \eta_2$$

Evaluating, with $D_1 = D_2$,

$$Q_1 = Q_2 \frac{\omega_1}{\omega_2} = 970\ gpm \times \frac{1750\ rpm}{3550\ rpm} = 478\ gpm$$

$$H_1 = H_2 \left(\frac{\omega_1}{\omega_2}\right)^2 = 430\ ft \times \left(\frac{1750\ rpm}{3550\ rpm}\right)^2 = 104\ ft$$

$$P_1 = P_2 \left(\frac{\omega_1}{\omega_2}\right)^3 = 135\ hp \times \left(\frac{1750\ rpm}{3550\ rpm}\right)^3 = 16.2\ hp$$

$$\eta_1 = \eta_2 = 0.74^+$$

Comparing shows excellent agreement.

Problem 10.37

(Difficulty: 3)

10.37 Consider the Peerless Type 16A18B horizontal split case centrifugal pump. Use these performance data to verify the similarity rules for (a) impeller diameter change and (b) operating speeds of 705 and 880 rpm (note the scale change between speeds).

Solution: From Figs. C.9 and C.10 at the best efficiency point (BEP):

$N\ (rpm)$	$D\ (in)$	$Q\ (gpm)$	$H\ (ft)$	$\dot{W}_m\ (hp)$	$\eta\ (\%)$
705	18.0	6250	42	76	86^+
	17.0	5850	37	63	86^+
	16.0	5600	32	54	86
880	18.0	7900	69	155	87^+
	17.0	7400	59	125	87^+
	16.0	7100	50	105	85

The similarity rules are:

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}$$

$$\frac{H_1}{\omega_1^2 D_1^2} = \frac{H_2}{\omega_2^2 D_2^2}$$

$$\frac{P_1}{\omega_1^3 D_1^5} = \frac{P_2}{\omega_2^3 D_2^5}$$

$$\eta_1 = \eta_2$$

Evaluating with $\omega_1 = \omega_2 = 705\ rpm$,

$$Q_{17} = Q_{18}\left(\frac{17}{18}\right)^3 = 5270\ gpm$$

$$Q_{16} = 4390\ gpm$$

$$H_{17} = H_{18}\left(\frac{17}{18}\right)^2 = 37.5\ ft$$

$$H_{16} = 33.2\ ft$$

$$P_{17} = P_{18}\left(\frac{17}{18}\right)^5 = 57.1\ hp$$

$$P_{16} = 42.2\ hp$$

$$\eta = constant$$

At 880 rpm,

$$Q_{17} = 6660\ gpm$$

$$Q_{16} = 5550\ gpm$$

$$H_{17} = 61.5\ ft$$

$$H_{16} = 54.5\ ft$$

$$P_{17} = 116\ hp$$

$$P_{16} = 86\ hp$$

$$\eta = constant$$

Evaluating with $D_1 = D_2 = 18\ in$:

$$Q_{705} = Q_{880}\left(\frac{705}{880}\right) = 6330\ gpm$$

$$H_{705} = H_{880}\left(\frac{705}{880}\right)^2 = 44.3\ ft$$

$$P_{705} = P_{880}\left(\frac{705}{880}\right)^3 = 79.7\ hp$$

$$\eta = constant$$

Comparing results with data shows at constant speed:

(1) flow rate scales poorly (2) head scales well (3) power scales poorly with changes in diameter

Comparing results with data shows at constant diameter:

All quantities scale well with changes in speed.

Flow rate scaling may be improved using the modified procedure discussed in the chapter, in which $Q \sim D^2$ and $P \sim D^4$.

Problem 10.38

(Difficulty: 3)

10.38 Use data from Appendix C to verify the similarity rules for the effect of changing the impeller diameter of a Peerless Type 4AE12 pump operated at 1750 and 3550 nominal rpm.

Solution: From Figs. C.4 and C.5 at the best efficiency point (BEP):

$N\ (rpm)$	$D\ (in)$	$Q\ (gpm)$	$H\ (ft)$	$\dot{W}_m\ (hp)$	$\eta\ (\%)$
1705	10.0	455	87	13	73^+
	11.0	470	104	17	73^+
	12.12	500	123	22	73^+
3550	10.0	930	360	115	74^+
	11.0	970	430	135	74^+
	12.12	1030	500	180	74^+

The similarity rules are:

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}$$

$$\frac{H_1}{\omega_1^2 D_1^2} = \frac{H_2}{\omega_2^2 D_2^2}$$

$$\frac{P_1}{\omega_1^3 D_1^5} = \frac{P_2}{\omega_2^3 D_2^5}$$

$$\eta_1 = \eta_2$$

Evaluating with $\omega_1 = \omega_2 = 1750\ rpm$,

$$Q_{10} = Q_{11}\left(\frac{10}{11}\right)^3 = 353\ gpm$$

$$Q_{12} = Q_{11}\left(\frac{12.12}{11}\right)^3 = 629\ gpm$$

$$H_{10} = 86.0\ ft$$

$$H_{12} = 126.0\ ft$$

$$P_{10} = 10.6\ hp$$

$$P_{12} = 27.6\ hp$$

$$\eta = constant$$

Evaluating, with $\omega_1 = \omega_2 = 3550\ rpm$,

$$Q_{10} = 729\ gpm$$

$$Q_{12} = 1300\ gpm$$

$$H_{10} = 355\ ft$$

$$H_{12} = 522\ ft$$

$$P_{10} = 83.8\ hp$$

$$P_{12} = 219\ hp$$

$$\eta = constant$$

Comparing results with data shows:

(1) flow rate is scaled poorly (2) head is scaled well (3) power is scaled poorly (because flow rate is scaled poorly).

Better results are obtained using the modified scaling rules (see Section 10.3); then

$$Q \sim D^2$$

so

$$Q_{10} = 389\ gpm$$

and

$$P \sim D^4$$

so

$$P_0 = 11.6\ hp$$

at 1750 rpm.

And

$$Q_{10} = 802\ gpm$$

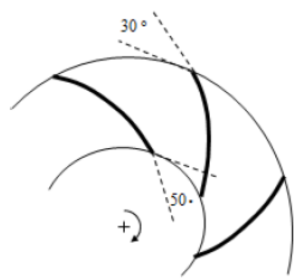
$$P_{10} = 92.2\ hp$$

at 3550 rpm.

Problem 10.39

(Difficulty 3)

10.39 A centrifugal water pump has an impeller with backward curved vanes and an inner diameter of $0.1\ m$, an outer diameter of $0.25\ m$, and a blade height of $4\ cm$. It operates at $1200\ rpm$.Water enters the impeller at the blade angle of $50\ deg$. and leaves at the blade angle of $50\ deg$. The volume flow rate is $0.18\ \frac{m^3}{s}$. Determine the shaft torque and power. Determine the pressure rise when the fluid velocity leaving the pump diffuser is the same as that entering.



Find: Shaft torque an power

Assumptions: The flow is steady and incompressible

Solution: Use the relations for the power and torque of a centrifugal machine and the relations for velocity vectors.

The shaft torque is given by:

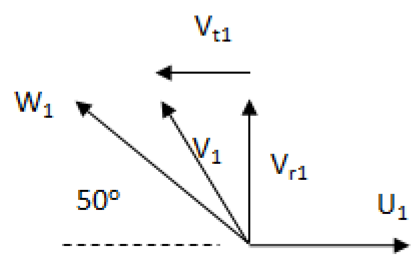
$$T_{shaft} = \dot{m}(r_2 V_{t2} - r_1 V_{t1})$$

The shaft power is given by

$$\dot{W} = \omega T_{shaft}$$

To calculate the torque and power, we need to determine the velocities and the inlet and outlet of the rotor.

At the inlet, the relation among the velocity vectors is:



The volumetric flow rate is:

$$Q = 0.18\ \frac{m^3}{s}$$

The velocity in the radial direction is calculated from the flow rate as:

$$V_{r1} = \frac{Q}{2\pi r_1 b} = \frac{0.18\ \frac{m^3}{s}}{2\pi \times \frac{0.1\ m}{2} \times 0.04\ m} = 14.32\ \frac{m}{s}$$

The relative velocity in the tangential direction is:

$$W_1 = \frac{V_{r1}}{\tan 50°} = \frac{14.32\ \frac{m}{s}}{\tan 50°} = 12.02\ \frac{m}{s}$$

The rotor velocity at the inlet is calculated as:

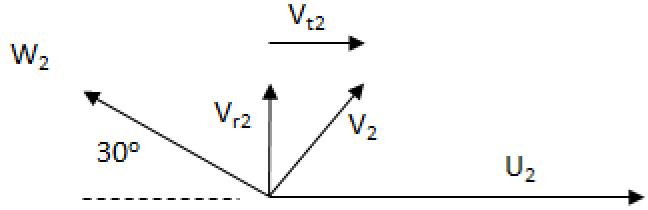
$$U_1 = \omega r_1 = 1200 \times \frac{2\pi}{60\ s} \times 0.05\ m = 6.28\ \frac{m}{s}$$

The absolute velocity in the tangential direction is:

$$V_{t1} = U_1 - V_{r1} = -5.74\ \frac{m}{s}$$

The direction is the same as the relative velocity in the tangential direction.

At the exit we have the relation among the velocities as



The velocity in the radial direction at the exit is:

$$V_{r2} = \frac{Q}{2\pi r_2 b} = \frac{0.18\ \frac{m^3}{s}}{2\pi \times \frac{0.25\ m}{2} \times 0.04\ m} = 5.73\ \frac{m}{s}$$

The relative velocity in the tangential direction is:

$$W = \frac{V_{r2}}{\tan 30°} = \frac{5.73\ \frac{m}{s}}{\tan 30°} = 8.33\ \frac{m}{s}$$

The rotor velocity at the outlet is calculated as:

$$U_2 = \omega r_2 = 1200 \times \frac{2\pi}{60\ s} \times \frac{0.25\ m}{2} = 15.71\ \frac{m}{s}$$

The absolute velocity in the tangential direction is:

$$V_{t2} = U_2 - V_{r2} = 15.71\ \frac{m}{s} - 8.33\ \frac{m}{s} = 7.37\ \frac{m}{s}$$

The direction is the same as the rotor velocity.

The mass flow rate is calculated as:

$$\dot{m} = \rho Q = 1000\ \frac{kg}{m^3} \times 0.18\ \frac{m^3}{s} = 180\ \frac{kg}{s}$$

The shaft torque is calculated as:

$$T_{shaft} = \dot{m}(r_2 V_{t2} - r_1 V_{t1})$$

$$T_{shaft} = 180\ \frac{kg}{s} \times \left(\frac{0.25\ m}{2} \times 7.37\ \frac{m}{s} - \left(-\frac{0.1\ m}{2} \times 5.74\ \frac{m}{s}\right)\right) = 217\ N \cdot m$$

The power is calculated as:

$$\dot{W} = \omega T_{shaft} = 1200 \times \frac{2\pi}{60\ s} \times 217\ N \cdot m = 27.3\ kW$$

If the velocity leaving the pump is the same as entering we have the pressure rise as:

$$p_2 - p_1 = \frac{\dot{W}}{Q} = \frac{27.3\ kW}{0.18\ \frac{m^3}{s}} = \frac{27.3\ \frac{kN \cdot m}{s}}{0.18\ \frac{m^3}{s}} = 152\ kPa$$

Problem 10.40

(Difficulty: 4)

10.40 Catalog data for a centrifugal water pump at design conditions are $Q = 250\ gpm$ and $\Delta p = 18.6\ psi$ at $1750\ rpm$. A laboratory flume requires $200\ gpm$ at $32\ ft$ of head. The only motor available develops $3\ hp$ at $1750\ rpm$. Is this motor suitable for the laboratory flume? How might the pump motor match be improved?

Solution: To obtain efficiency and pump power requirement, find specific speed:

$$H = \frac{\Delta P}{\rho g} = 18.6\ \frac{lbf}{in^2} \times \frac{ft^3}{62.4\ lbf} \times 144\ \frac{in^2}{ft^2} = 42.9\ ft$$

$$Q = 250\ \frac{gal}{min} = 0.557\ cfs$$

$$Ns_{cu} = \frac{NQ^{\frac{1}{2}}}{H^{\frac{3}{4}}} = \frac{1750\ rpm \times (250\ gpm)^{\frac{1}{2}}}{(42.9\ ft)^{\frac{3}{4}}} = 1650$$

From Fig. 10.15, $\eta \approx 0.73$. Thus

$$\dot{W}_m = \frac{\dot{W}_\eta}{\eta} = \frac{\rho Q g H}{\eta} = \frac{1}{0.73} \times 62.4\ \frac{lbf}{ft^3} \times 0.557\ \frac{ft^3}{s} \times 42.9\ ft \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 3.71\ hp$$

The motor is not suitable to drive the pump directly.

The pump at 1750 rpm produces more head and flow than necessary. It may be run at reduced speed, e.g., by using a belt drive.

To produce the flow rate of

$$Q_f = 200\ gpm$$

Solve for the speed

$$\frac{Q_p}{\omega_p D_p^3} = \frac{Q_f}{\omega_f D_f^3}$$

$$\omega_f = \frac{200}{250} \times 1750\ rpm = 1400\ rpm$$

To produce a head of

$$H_f = 32\ ft$$

Solve for the speed

$$\frac{H_p}{\omega_p^2 D_p^2} = \frac{H_f}{\omega_f^2 D_f^2}$$

$$\omega_f = \sqrt{\frac{H_f}{H_p}}\,\omega_p = \sqrt{\frac{32}{42.9}} \times 1750 = 1510\ rpm$$

At 1510 rpm the power requirement will be given by

$$\frac{P_p}{\omega_p^3 D_p^5} = \frac{P_f}{\omega_f^3 D_f^5}$$

So

$$P_f = P_p\left(\frac{\omega_f}{\omega_p}\right)^3 = 3.71\ hp \times \left(\frac{1510}{1750}\right)^3 = 2.38\ hp$$

This is well within the capability of the 3 hp motor. Therefore run pump at 1510 rpm.

Problem 10.41

[Difficulty: 3]

10.41 A 1/3 scale model of a centrifugal water pump, when running at $N_m = 5100$ rpm, produces a flow rate of $Q_m = 1\ \mathrm{m^3/s}$ with a head of $H_m = 5.4$ m. Assuming the model and prototype efficiencies are comparable, estimate the flow rate, head, and power requirement if the design speed is 125 rpm.

Given: Data on a model pump

Find: Prototype flow rate, head, and power at 125 rpm

Solution:

Basic equation: $W_h = \rho \cdot Q \cdot g \cdot H$ and similarity rules

$$\frac{Q_1}{\omega_1 \cdot D_1^3} = \frac{Q_2}{\omega_2 \cdot D_2^3} \qquad \frac{h_1}{\omega_1^2 \cdot D_1^2} = \frac{h2}{\omega_2^2 \cdot D_2^2} \qquad \frac{P_1}{\rho_1 \cdot \omega_1^3 \cdot D_1^5} = \frac{P_2}{\rho_2 \cdot \omega_2^3 \cdot D_2^5}$$

The given or available data is $N_m = 100 \cdot \mathrm{rpm}$ $\quad N_p = 125 \cdot \mathrm{rpm}$ $\quad \rho = 1000 \cdot \frac{\mathrm{kg}}{\mathrm{m}^3}$

$Q_m = 1 \cdot \frac{\mathrm{m}^3}{\mathrm{s}}$ $\quad H_m = 4.5 \cdot \mathrm{m}$

From Eq. 10.8a $\quad W_{hm} = \rho \cdot Q_m \cdot g \cdot H_m \qquad W_{hm} = 44.1 \cdot \mathrm{kW}$

From Eq. 10.19a (with $D_m/D_p = 1/3$)

$$\frac{Q_p}{\omega_p \cdot D_p^3} = \frac{Q_m}{\omega_m \cdot D_m^3} \quad \text{or} \quad Q_p = Q_m \cdot \frac{\omega_p}{\omega_m} \cdot \left(\frac{D_p}{D_m}\right)^3 = 3^3 \cdot Q_m \cdot \frac{\omega_p}{\omega_m}$$

$$Q_p = 27 \cdot Q_m \cdot \frac{N_p}{N_m} \qquad Q_p = 33.8 \frac{\mathrm{m}^3}{\mathrm{s}}$$

From Eq. 10.19b (with $D_m/D_p = 1/3$)

$$\frac{h_p}{\omega_p^2 \cdot D_p^2} = \frac{h_m}{\omega_m^2 \cdot D_m^2} \quad \text{or} \quad \frac{g \cdot H_p}{\omega_p^2 \cdot D_p^2} = \frac{g \cdot H_m}{\omega_m^2 \cdot D_{pm}^2}$$

$$H_p = H_m \cdot \left(\frac{\omega_p}{\omega_m}\right)^2 \cdot \left(\frac{D_p}{D_m}\right)^2 = 3^2 \cdot H_m \cdot \left(\frac{\omega_p}{\omega_m}\right)^2 \qquad H_p = 9 \cdot H_m \cdot \left(\frac{N_p}{N_m}\right)^2 \qquad H_p = 63.3\,\mathrm{m}$$

From Eq. 10.19c (with $D_m/D_p = 1/3$)

$$\frac{P_p}{\rho \cdot \omega_p^3 \cdot D_p^5} = \frac{P_m}{\rho \cdot \omega_m^3 \cdot D_m^5} \quad \text{or} \quad W_{hp} = W_{hm} \cdot \left(\frac{\omega_p}{\omega_m}\right)^3 \cdot \left(\frac{D_p}{D_m}\right)^5 = 3^5 \cdot W_{hm} \cdot \left(\frac{\omega_p}{\omega_m}\right)^3$$

$$W_{hp} = 243 \cdot W_{hm} \cdot \left(\frac{N_p}{N_m}\right)^3 \qquad W_{hp} = 20.9 \cdot \mathrm{MW}$$

Problem 10.42

[Difficulty: 2]

10.42 Sometimes the variation of water viscosity with temperature can be used to achieve dynamic similarity. A model pump delivers 0.10 m^3/s of water at 15°C against a head of 27 m, when operating at 3600 rpm. Determine the water temperature that must be used to obtain dynamically similar operation at 1800 rpm. Estimate the volume flow rate and head produced by the pump at the lower-speed test condition. Comment on the *NPSH* requirements for the two tests.

Given: Data on a model pump

Find: Temperature for dynamically similar operation at 1800 rpm; Flow rate and head; Comment on NPSH

Solution:

Basic equation: $Re_1 = Re_2$ and similarity rules $\frac{Q_1}{\omega_1 \cdot D_1^3} = \frac{Q_2}{\omega_2 \cdot D_2^3}$ $\quad \frac{H_1}{\omega_1^2 \cdot D_1^2} = \frac{H_2}{\omega_2^2 \cdot D_2^2}$

The given or available data is $\omega_1 = 3600 \cdot rpm$ $\quad \omega_2 = 1800 \cdot rpm$ $\quad Q_1 = 0.1 \cdot \frac{m^3}{s}$ $\quad H_1 = 27 \cdot m$

From Table A.8 at 15°C $\quad \nu_1 = 1.14 \times 10^{-6} \cdot \frac{m^2}{s}$

For D = constant $\quad Re_1 = \frac{V_1 \cdot D}{\nu_1} = \frac{\omega_1 \cdot D \cdot D}{\nu_1} = Re_2 = \frac{\omega_2 \cdot D \cdot D}{\nu_2}$ or $\nu_2 = \nu_1 \cdot \frac{\omega_2}{\omega_1}$ $\quad \nu_2 = 5.7 \times 10^{-7} \frac{m^2}{s}$

From Table A.8, at $\nu_2 = 5.7 \times 10^{-7} \frac{m^2}{s}$, we find, by linear interpolation

$$T_2 = 45 + \frac{(50 - 45)}{(5.52 - 6.02)} \cdot (5.70 - 6.02) \qquad T_2 = 48 \text{ degrees C}$$

From similar operation $\frac{Q_1}{\omega_1 \cdot D^3} = \frac{Q_2}{\omega_2 \cdot D^3}$ or $Q_2 = Q_1 \cdot \frac{\omega_2}{\omega_1}$ $\quad Q_2 = 0.0500 \cdot \frac{m^3}{s}$

and also $\frac{H_1}{\omega_1^2 \cdot D^2} = \frac{H_2}{\omega_2^2 \cdot D^2}$ or $H_2 = H_1 \cdot \left(\frac{\omega_2}{\omega_1}\right)^2$ $\quad H_2 = 6.75\,m$

The water at 48°C is closer to boiling. The inlet pressure would have to be changed to avoid cavitation. The increase between runs 1 and 2 would have to be $\Delta p = p_{v2} - p_{v1}$ where p_{v2} and p_{v1} are the vapor pressures at T_2 and T_1. From the steam tables:

$$p_{v1} = 1.71 \cdot kPa \qquad p_{v2} = 11.276 \cdot kPa \qquad \Delta p = p_{v2} - p_{v1} \qquad \Delta p = 9.57 \cdot kPa$$

Problem 10.43 [Difficulty: 4]

10.43 A large deep fryer at a snack-food plant contains hot oil that is circulated through a heat exchanger by pumps. Solid particles and water droplets coming from the food product are observed in the flowing oil. What special factors must be considered in specifying the operating conditions for the pumps?

Discussion: Any solid particles must be able to pass through the pumps without clogging. If the particles are large, this may require larger than normal clearances within the pumps.

If the water droplets flashed to steam, they would form local pockets of water vapor. The pockets of water vapor would disrupt the flow patterns in the pumps in the same way as cavitation in a homogeneous liquid. To prevent this "cavitation" from occurring, static pressure everywhere in the flow circuit must be maintained above the saturation pressure of the water droplets at the temperature of the flowing oil.

The net positive suction head at the pump inlets must be sufficiently high to prevent any problems from occurring within the pumps themselves.

The solid particles may act as nucleation sites, which would foster the development of vapor pockets in the flow. This might increase the net positive suction head required by the pump above that measured in tests using water. The system must be sized to maintain a large net positive suction head at the design flow rate.

Finally, the viscosity of the oil must be considered. If viscosity is high, pump performance will be degraded compared to pumping water. Then a larger pump must be specified to handle the flow requirement of the hot oil circulation system.

Problem 10.44 [Difficulty: 2]

10.44 Data from tests of a pump operated at 1450 rpm, with a 12.3-in. diameter impeller, are

Flow rate, Q (cfm)	20	40	60	80	100	120	140
Net positive suction head required, $NPSR$ (ft)	7.1	8.0	8.9	10.3	11.8	12.3	16.9

Develop and plot a curve-fit equation for $NPSHR$ versus volume flow rate in the form $NPSHR = a + bQ^2$, where a and b are constants. If the $NPSHA = 20$ ft, estimate the maximum allowable flow rate of this pump.

Given: Data on a NPSHR for a pump

Find: Curve fit; Maximum allowable flow rate

Solution: The results were generated in *Excel*:

Q (cfm)	Q^2	$NPSHR$ (ft)	$NPSHR$ (fit)
20	4.00E+02	7.1	7.2
40	1.60E+03	8.0	7.8
60	3.60E+03	8.9	8.8
80	6.40E+03	10.3	10.2
100	1.00E+04	11.8	12.0
120	1.44E+04	14.3	14.2
140	1.96E+04	16.9	16.9

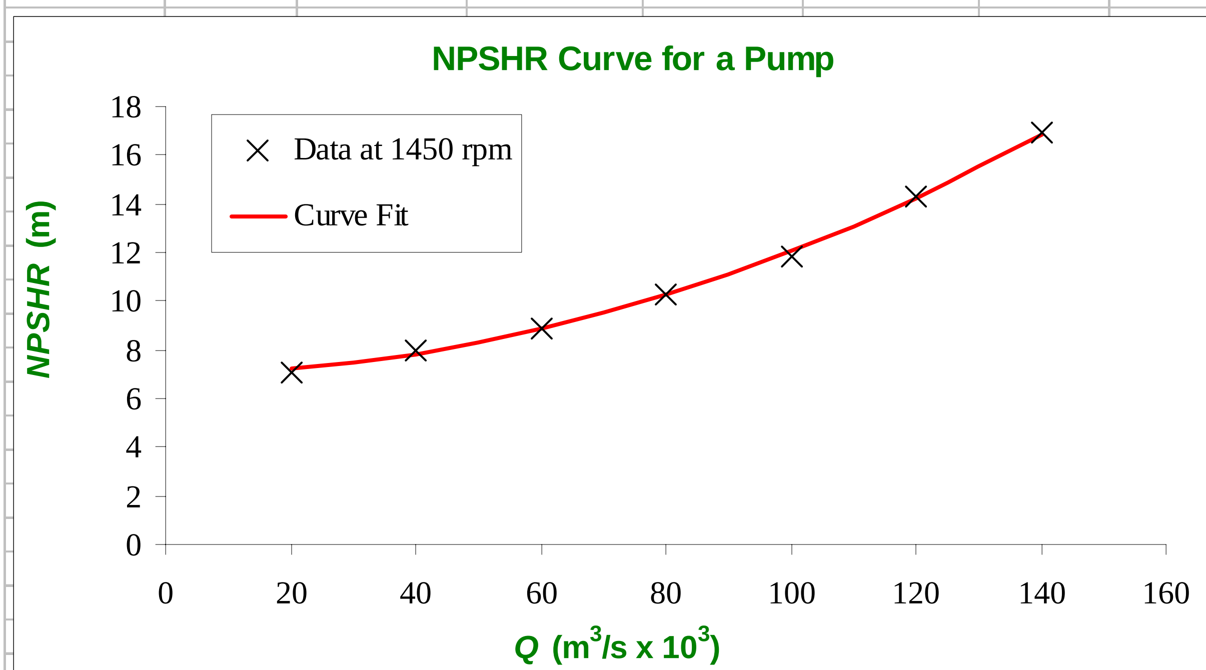
The fit to data is obtained from a least squares fit to $NPSHR = a + bQ^2$

a = 7.04 ft

b = 5.01E-04 ft/(cfm)^2

Q (cfm)	$NPSHR$ (ft)
160.9	20.00

Use *Goal Seek* to find Q!



Problem 10.45

[Difficulty: 3]

10.45 A four-stage boiler feed pump has suction and discharge lines of 10 cm and 7.5 cm inside diameter. At 3500 rpm, the pump is rated at 0.025 m^3/s against a head of 125 m while handling water at 115°C. The inlet pressure gage, located 50 cm below the impeller centerline, reads 150 kPa. The pump is to be factory certified by tests at the same flow rate, head rise, and speed, but using water at 27°C. Calculate the *NPSHA* at the pump inlet in the field installation. Evaluate the suction head that must be used in the factory test to duplicate field suction conditions.

Given: Data on a boiler feed pump

Find: NPSHA at inlet for field temperature water; Suction head to duplicate field conditions

Solution:

Basic equation: $NPSHA = p_t - p_v = p_g + p_{atm} + \frac{1}{2}\cdot\rho\cdot V^2 - p_v$

Given or available data is $D_s = 10\cdot cm$ $\quad D_d = 7.5\cdot cm$ $\quad H = 125\cdot m$ $\quad Q = 0.025\cdot\frac{m^3}{s}$

$p_{inlet} = 150\cdot kPa$ $\quad p_{atm} = 101\cdot kPa$ $\quad z_{inlet} = -50\cdot cm$ $\quad \rho = 1000\cdot\frac{kg}{m^3}$ $\quad \omega = 3500\cdot rpm$

For field conditions $p_g = p_{inlet} + \rho\cdot g\cdot z_{inlet}$ $\quad p_g = 145\cdot kPa$

From continuity $V_s = \frac{4\cdot Q}{\pi\cdot D_s^2}$ $\quad V_s = 3.18\frac{m}{s}$

From steam tables (try Googling!) at 115°C $\quad p_v = 169\cdot kPa$

Hence $NPSHA = p_g + p_{atm} + \frac{1}{2}\cdot\rho\cdot V_s^2 - p_v$ $\quad NPSHA = 82.2\cdot kPa$

Expressed in meters or feet of water $\quad \frac{NPSHA}{\rho\cdot g} = 8.38m$ $\quad \frac{NPSHA}{\rho\cdot g} = 27.5\cdot ft$

In the laboratory we must have the same NPSHA. From Table A.8 (or steam tables - try Googling!) at 27°C $\quad p_v = 3.57\cdot kPa$

Hence $p_g = NPSHA - p_{atm} - \frac{1}{2}\cdot\rho\cdot V_s^2 + p_v$ $\quad p_g = -20.3\cdot kPa$

The absolute pressure is $p_g + p_{atm} = 80.7\cdot kPa$

Problem 10.46

(Difficulty: 2)

10.46 A centrifugal pump, operating at $N = 2265\ rpm$, lifts water between two reservoirs connected by $300\ ft$ of $6\ in$ and $100\ ft$ of $3\ in$ cast-iron pipe in series. The gravity lift is $25\ ft$. Estimate the head requirement, power needed, and hourly cost of electrical energy to pump water at $200\ gpm$ to the higher reservoir. Assume that electricity costs $12\ c/kWhr$ and that the electric motor efficiency is 85 percent.

Assumptions: (1) $p_3 = p_4 = p_{atm}$, $V_3 = V_4 = 0$ (2) neglect minor losses

Solution: Apply the energy equation with head loss to the total system for steady, incompressible flow using (3) and (4) at reservoir surfaces.

$$\frac{p_3}{\rho g} + \alpha_3 \frac{V_3^2}{2g} + z_3 + H_a = \frac{p_4}{\rho g} + \alpha_4 \frac{V_4^2}{2g} + z_4 + \frac{h_{lT}}{g}$$

$$h_{lT} = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2} + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2}$$

Then, as the inlet and outlet pressures and velocities are equal, the head loss is given by

$$H_a = z_4 - z_3 + f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$

$$V_1 = \frac{Q}{A_1} = 200\ \frac{gal}{min} \times \frac{ft^3}{7.48\ gal} \times \frac{min}{60\ s} \times \frac{4}{\pi (0.5\ ft)^2} = 2.27\ \frac{ft}{s}$$

$$V_2 = 9.08\ \frac{ft}{s}$$

For water at 59 °F, $\upsilon = 1.23 \times 10^{-5}\ \frac{ft^2}{s}$,

$$Re_1 = \frac{V_1 D_1}{\upsilon} = 2.27\ \frac{ft}{s} \times \frac{10}{12}\ ft \times \frac{s}{1.23 \times 10^{-5}\ ft^2} = 9.23 \times 10^4$$

$$Re_2 = 1.85 \times 10^5$$

From Table 8.1 for cost iron, $e = 0.00085\ ft$,

$$\frac{e}{D_1} = 0.0017$$

$$\frac{e}{D_2} = 0.0034$$

From Eq. 8.37, solving to find the friction factor:

$$f_1 = 0.0244$$

$$f_2 = 0.0278$$

Substituting into the expression for head loss:

$$H_a = 25\ ft + 0.0244 \times \frac{300 \times 12}{6} \times \left(\frac{2.27}{2}\ \frac{ft}{s}\right)^2 \times \frac{s^2}{32.2\ ft} + 0.0278 \times \frac{100 \times 12}{3} \times \left(\frac{9.08}{2}\ \frac{ft}{s}\right)^2 \times \frac{s^2}{32.2\ ft}$$

$$H_a = 40.4\ ft$$

The specific speed is

$$N_{sac} = \frac{NQ^{\frac{1}{2}}}{H^{\frac{3}{4}}} = \frac{2265 \times (200)^{\frac{1}{2}}}{(40.4\ ft)^{\frac{3}{4}}} = 2000$$

From Fig. 10.15, the efficiency is

$$\eta_p = 0.75$$

Then the power is

$$P_m = \frac{\dot{W}_h}{\eta_p} = \frac{\rho Q g H}{\eta_p}$$

$$P_m = \frac{1}{0.75} \times 200\ \frac{gal}{min} \times \frac{ft^3}{7.48\ gal} \times \frac{min}{60\ s} \times 62.4\ \frac{lbf}{ft^3} \times 40.4\ ft \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 2.72\ hp$$

$$Cost = C\ P_e$$

Since the cost is

$$C = 0.12\ \$\ /kW \cdot hr$$

$$\eta_m = 0.85$$

Then

$$P_e = \frac{P_m}{\eta_m}$$

$$Cost = C\frac{P_m}{\eta_m} = \frac{0.12\ \$}{kW \cdot hr} \times \frac{2.72\ hp}{0.85} \times 0.746\frac{kW}{hp} = 0.28\ \frac{\$}{hr}$$

Problem 10.47 [Difficulty: 3]

10.47 A centrifugal pump is installed in a piping system with $L=300$ m of $D=40$ cm cast-iron pipe. The downstream reservoir surface is 15 m lower than the upstream reservoir. Determine and plot the system head curve. Find the volume flow rate (magnitude and direction) through the system when the pump is not operating. Estimate the friction loss, power requirement, and hourly energy cost to pump water at 1 m³/s through this system.

Given: Pump and reservoir system

Find: System head curve; Flow rate when pump off; Loss, Power required and cost for 1 m³/s flow rate

Solution:

Basic equations: $\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} - h_p \qquad h_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + \Sigma K \cdot \frac{V^2}{2}$ (K for the exit)

where points 1 and 2 are the reservoir free surfaces, and h_p is the pump head

Note also $H = \frac{h}{g}$ Pump efficiency: $\eta_p = \frac{W_h}{W_m}$

Assumptions: 1) $p_1 = p_2 = p_{atm}$ 2) $V_1 = V_2 = 0$ 3) $\alpha_2 = 0$ 4) $z_1 = 0$, $z_2 = -15 \cdot m$ 4) $K = K_{ent} + K_{ent} = 1.5$

From the energy equation $-g \cdot z_2 = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} - h_p + K \cdot \frac{V^2}{2} \qquad h_p = g \cdot z_2 + f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + K \cdot \frac{V^2}{2} \qquad H_p = z_2 + f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + K \cdot \frac{V^2}{2 \cdot g}$

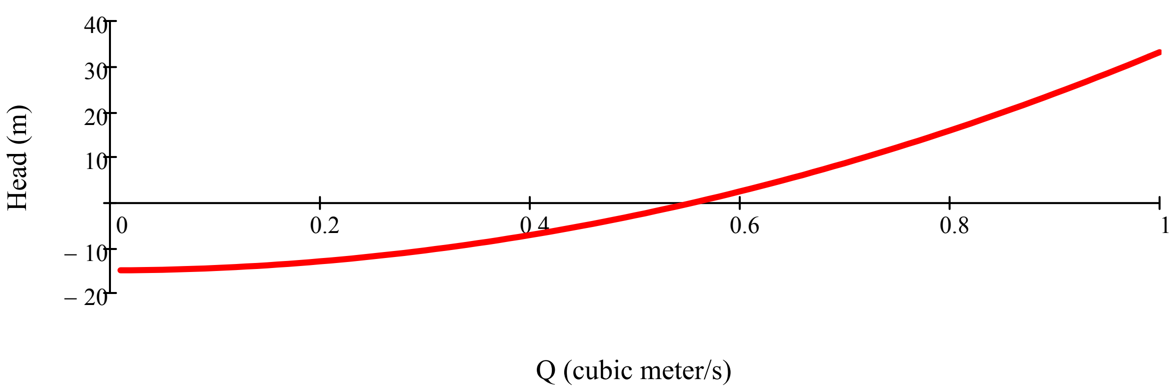
Given or available data $L = 300 \cdot m \qquad D = 40 \cdot cm \qquad e = 0.26 \cdot mm$ (Table 8.1)

$\rho = 1000 \cdot \frac{kg}{m^3} \qquad \nu = 1.01 \times 10^{-6} \cdot \frac{m^2}{s}$ (Table A.8)

The set of equations to solve for each flow rate Q are

$$V = \frac{4 \cdot Q}{\pi \cdot D^2} \qquad Re = \frac{V \cdot D}{\nu} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \qquad H_p = z_2 + f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + K \cdot \frac{V^2}{2 \cdot g}$$

For example, for $Q = 1 \cdot \frac{m^3}{s} \qquad V = 7.96 \cdot \frac{m}{s} \qquad Re = 3.15 \times 10^6 \qquad f = 0.0179 \qquad H_p = 33.1 \cdot m$



The above graph can be plotted in Excel. In Excel, Solver can be used to find Q for $H_p = 0$ $Q = 0.557\,\frac{m^3}{s}$ (Zero power rate)

At $Q = 1\cdot\frac{m^3}{s}$ we saw that $H_p = 33.1\cdot m$

Assuming optimum efficiency at $Q = 1.59\times 10^4\cdot gpm$ from Fig. 10.15 $\eta_p = 92\cdot\%$

Then the hydraulic power is $W_h = \rho\cdot g\cdot H_p\cdot Q$ $W_h = 325\cdot kW$

The pump power is then $W_m = \frac{W_h}{\eta_p}$ $W_m\cdot 2 = 706\cdot kW$

If electricity is 10 cents per kW-hr then the hourly cost is about \$35

If electricity is 15 cents per kW-hr then the hourly cost is about \$53

If electricity is 20 cents per kW-hr then the hourly cost is about \$71

Problem 10.48

(Difficulty: 2)

10.48 Part of the water supply for the South Rim of Grand Canyon National Park is taken from the Colorado River [54]. A flow rate of 600 gpm, taken from the river at elevation 3734 ft, is pumped to a storage tank atop the South Rim at 7022 ft elevation. Part of the pipeline is above ground and part is in a hole directionally drilled at angles up to 70° from the vertical; the total pipe length is approximately 13200 ft. Under steady flow operating conditions, the frictional head loss is 290 ft of water in addition to the static lift. Estimate the diameter of the commercial steel pipe in the system. Compute the pumping power requirement if the pump efficiency is 61 percent.

Assumptions: (1) $p_1 = p_2 = p_{atm}$, $\bar{V}_1 = V_2 = 0$ (2) neglect minor losses

Solution: Apply the energy equation with head loss to the total system for steady, incompressible flow using (1) and (2) at inlet and reservoir surface respectively.

$$\frac{p_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + H_a = \frac{p_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \frac{h_{lT}}{g}$$

$$h_{lT} = f \frac{L}{D} \frac{V^2}{2}$$

The pressure and velocities are equal at both the entrance and exit, and the head loss is then given by:

$$H_a = z_2 - z_1 + \frac{h_{lT}}{g}$$

$$\frac{h_{lT}}{g} = f \frac{L}{D} \frac{V^2}{2g} = 290\ ft$$

Since

$$f = f(Re, e/D)$$

and D is unknown, we must iterate. For commercial steel, $e = 0.00015\ ft$,

The iteration procedure is to assume D, calculate V, Re; determine f (Eq.8.37 a,b); calculate h_{lT}/g and compare to value of 290 ft. A table of the guess values for diameter and the resulting head are given below:

D (in.)	V (ft/s)	Re	f_0	$f^{-0.5}$	f	h_{lT}/g (ft)
12	1.70	1.46E+06	0.0139	8.517	0.0138	8.2
10	2.45	1.75E+06	0.0141	8.433	0.0141	20.8
8	3.83	2.19E+06	0.0146	8.306	0.0145	65.3
6	6.81	2.92E+06	0.0153	8.111	0.0152	289

Thus

$$D = 6.0\ in$$

Alternatively, these relations could be programmed into an equation solver and the solution for D computed directly.

The total pump head is:

$$H_a = 7022 - 3734 + 290 = 3578\ ft$$

The pump power is:

$$P_m = \frac{\dot{W}_h}{\eta_p} = \frac{\rho g Q H}{\eta_p}$$

$$P_m = \frac{1}{0.61} \times 62.4\ \frac{lbf}{ft^3} \times 600\ \frac{gal}{min} \times \frac{ft^3}{7.48\ gal} \times \frac{min}{60\ s} \times 3578\ ft \times \frac{hp \cdot s}{550\ ft \cdot lbf}$$

$$P_m = 890\ hp$$

Problem 10.49

[Difficulty: 3]

10.49 A pump transfers water from one reservoir to another through two cast-iron pipes in series. The first is 3000 ft of 9 in. pipe and the second is 1000 ft of 6 in. pipe. A constant flow rate of 75 gpm is tapped off at the junction between the two pipes. Obtain and plot the system head versus flow rate curve. Find the delivery if the system is supplied by the pump of Example 10.6, operating at 1750 rpm.

Given: Data on pump and pipe system

Find: Delivery through system

Solution:

Governing Equations:

For the pump and system

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{l_T} - \Delta h_{\text{pump}} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = K \frac{\bar{V}^2}{2} \qquad (8.40a)$$

and the pump head (in energy/mass) is given by (from Example 10.6)

$$H_{\text{pump}}(\text{ft}) = 55.9 - 3.44 \times 10^{-5} \cdot Q(\text{gpm})^2$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0,\ V_1 = V_2 = 0,\ z_1 = z_2$) we have

$$0 = h_{lT} - \Delta h_{\text{pump}}$$

$$h_{lT} = g \cdot H_{\text{system}} = \Delta h_{\text{pump}} = g \cdot H_{\text{pump}}$$

or $$H_{lT} = H_{\text{pump}} \qquad (1)$$

where $$H_{lT} = \left(f_1 \cdot \frac{L_1}{D_1} + K_{\text{ent}}\right) \cdot \frac{V_1^{\,2}}{2 \cdot g} + \left(f_2 \cdot \frac{L_2}{D_2} + K_{\text{exit}}\right) \cdot \frac{V_2^{\,2}}{2}$$

Results generated in *Excel* are shown on the next page.

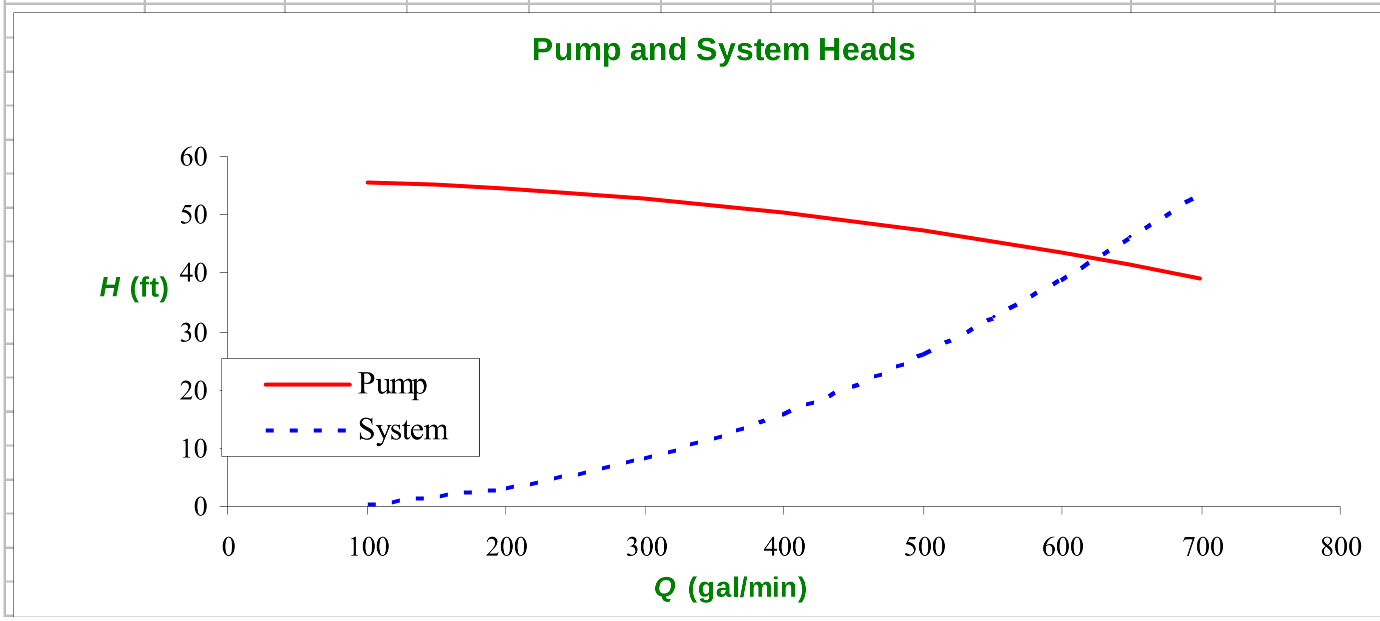
Given or available data:

$L_1 =$	3000	ft	$\nu =$	1.23E-05	ft^2/s (Table A.7)
$D_1 =$	9	in	$K_{ent} =$	0.5	(Fig. 8.14)
$L_2 =$	1000	ft	$K_{exp} =$	1	
$D_2 =$	6	in	$Q_{loss} =$	75	gpm
$e =$	0.00085	ft (Table 8.1)			

The system and pump heads are computed and plotted below.
To find the operating condition, *Goal Seek* is used to vary Q_1
so that the error between the two heads is zero.

Q_1 (gpm)	Q_2 (gpm)	V_1 (ft/s)	V_2 (ft/s)	Re_1	Re_2	f_1	f_2	H_{lT} (ft)	H_{pump} (ft)
100	25	0.504	0.284	30753	11532	0.0262	0.0324	0.498	55.6
200	125	1.01	1.42	61506	57662	0.0238	0.0254	3.13	54.5
300	225	1.51	2.55	92260	103792	0.0228	0.0242	8.27	52.8
400	325	2.02	3.69	123013	149922	0.0222	0.0237	15.9	50.4
500	425	2.52	4.82	153766	196052	0.0219	0.0234	26.0	47.3
600	525	3.03	5.96	184519	242182	0.0216	0.0233	38.6	43.5
700	625	3.53	7.09	215273	288312	0.0215	0.0231	53.6	39.0

Q_1 (gpm)	Q_2 (gpm)	V_1 (ft/s)	V_2 (ft/s)	Re_1	Re_2	f_1	f_2	H_{lT} (ft)	H_{pump} (ft)	Error)
627	552	3.162	6.263	192785	254580	0.0216	0.0232	42.4	42.4	0%



Problem 10.50

[Difficulty: 3]

10.50 Performance data for a pump are

H (ft)	179	176	165	145	119	84	43
Q (gpm)	0	500	1000	1500	2000	2500	3000

Estimate the delivery when the pump is used to move water between two open reservoirs, through 1200 ft of 12 in. commercial steel pipe containing two 90° elbows and an open gate valve, if the elevation increase is 50 ft. Determine the gate valve loss coefficient needed to reduce the volume flow rate by half.

Given: Data on pump and pipe system

Find: Delivery through system, valve position to reduce delivery by half

Solution:

Governing Equations:

For the pump and system

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} - \Delta h_{pump}$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2}$$

$$h_{l_m} = K\frac{\bar{V}^2}{2}$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or $$H_{lT} + \Delta z = H_{pump}$$

where $$H_{lT} = \left[f\cdot\left(\frac{L}{D} + 2\cdot\frac{L_e}{D_{elbow}} + \frac{L_e}{D_{valve}}\right) + K_{ent} + K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

The calculations performed using *Excel* are shown on the next page:

Given or available data **(Note: final results will vary depending on fluid data selected)**:

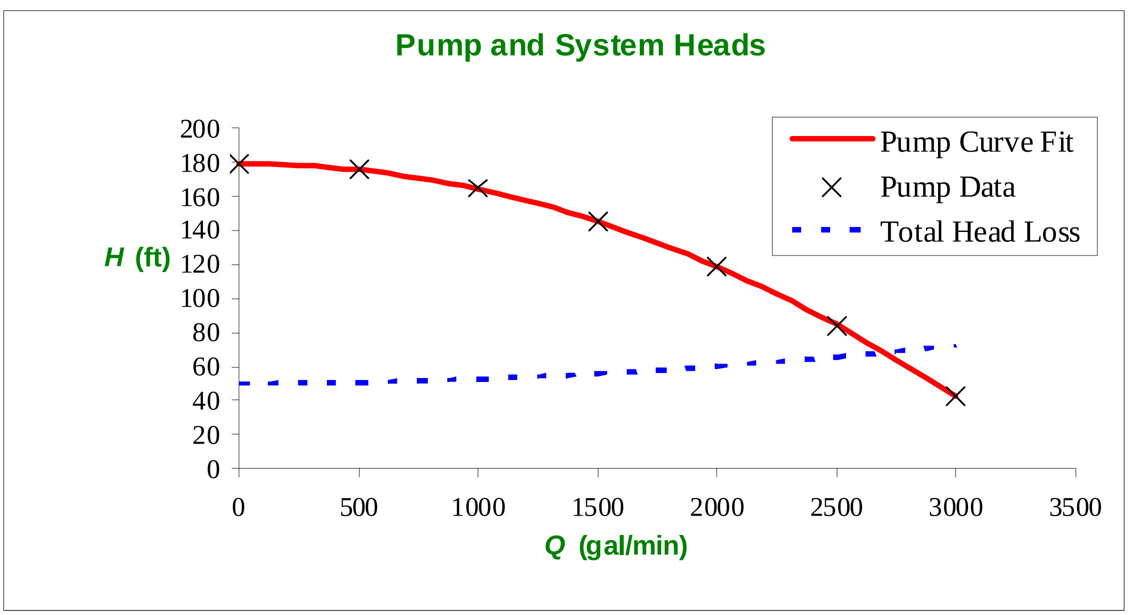
L =	1200	ft	K_{ent} =	0.5	
D =	12	in	K_{exp} =	1	
e =	0.00015	ft	L_e/D_{elbow} =	30	
ν =	1.23E-05	ft^2/s	L_e/D_{valve} =	8	
Hz =	-50	ft			

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

Q (gpm)	Q^2 (gpm)	H_{pump} (ft)	V (ft/s)	Re	f	H_{pump} (fit)	H_{lT} + Hz (ft)
0	0	179	0.00	0	0.0000	180	50.0
500	250000	176	1.42	115325	0.0183	176	50.8
1000	1000000	165	2.84	230649	0.0164	164	52.8
1500	2250000	145	4.26	345974	0.0156	145	56.0
2000	4000000	119	5.67	461299	0.0151	119	60.3
2500	6250000	84	7.09	576623	0.0147	84.5	65.8
3000	9000000	43	8.51	691948	0.0145	42.7	72.4

H_0 = 180 ft
A = 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	H_{lT} + Hz (ft)	Error)
2705	7.67	623829	0.0146	68.3	68.3	0%



For the valve setting to reduce the flow by half, use *Solver* to vary the value below to minimize the error.

L_e/D_{valve} = 26858

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	H_{lT} + Hz (ft)	Error)
1352	3.84	311914	0.0158	151.7	151.7	0%

Problem 10.51

[Difficulty: 3]

10.51 Consider again the pump and piping system of Problem 10.50. Determine the volume flow rate and gate valve loss coefficient for the case of two identical pumps installed in *parallel*.

Given: Data on pump and pipe system

Find: Delivery through parallel pump system; valve position to reduce delivery by half

Solution:

Governing Equations:

For the pumps and system

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} - \Delta h_{\text{pump}}$$

where the total head loss is comprised of major and minor losses

$$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$$

$$h_{l_m} = f \frac{L_e}{D} \frac{\bar{V}^2}{2}$$

$$h_{l_m} = K \frac{\bar{V}^2}{2}$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g \cdot \Delta z = h_{lT} - \Delta h_{\text{pump}}$$

$$h_{lT} + g \cdot \Delta z = g \cdot H_{\text{system}} + g \cdot \Delta z = \Delta h_{\text{pump}} = g \cdot H_{\text{pump}}$$

or $$H_{lT} + \Delta z = H_{\text{pump}}$$

where $$H_{lT} = \left[f \cdot \left(\frac{L}{D} + 2 \cdot \frac{L_e}{D_{\text{elbow}}} + \frac{L_e}{D_{\text{valve}}}\right) + K_{\text{ent}} + K_{\text{exit}}\right] \cdot \frac{V^2}{2 \cdot g}$$

For pumps in parallel $$H_{\text{pump}} = H_0 - \frac{1}{4} \cdot A \cdot Q^2$$

where for a single pump $$H_{\text{pump}} = H_0 - A \cdot Q^2$$

The calculations performed using *Excel* are shown on the next page.

Given or available data **(Note: final results will vary depending on fluid data selected)**:

L =	1200	ft	K_{ent} =	0.5	
D =	12	in	K_{exp} =	1	
e =	0.00015	ft (T	L_e/D_{elbow} =	30	
ν =	1.23E-05	ft^2/s	L_e/D_{valve} =	8	
Δz =	-50	ft			

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

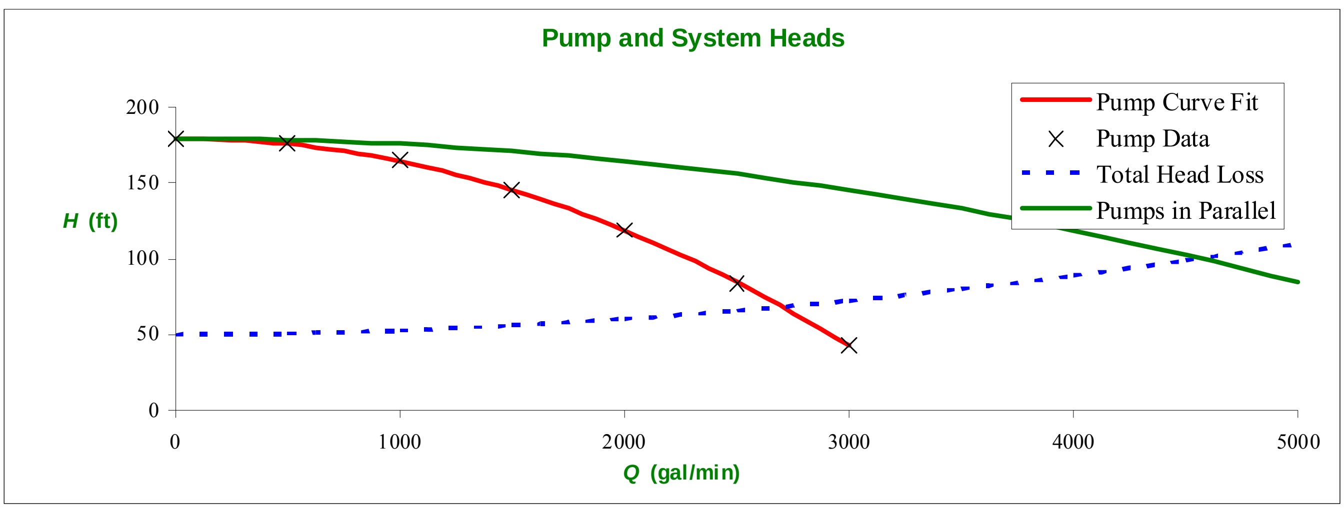
Q (gpm)	Q^2 (gpm)	H_{pump} (ft)	H_{pump} (fit)	V (ft/s)	Re	f
0	0	179	180	0.00	0	0.0000
500	250000	176	176	1.42	115325	0.0183
1000	1000000	165	164	2.84	230649	0.0164
1500	2250000	145	145	4.26	345974	0.0156
2000	4000000	119	119	5.67	461299	0.0151
2500	6250000	84	85	7.09	576623	0.0147
3000	9000000	43	43	8.51	691948	0.0145
3500				9.93	807273	0.0143
4000				11.35	922597	0.0142
4500				12.77	1037922	0.0141
5000				14.18	1153247	0.0140

H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)
180	50.0
179	50.8
176	52.8
171	56.0
164	60.3
156	65.8
145	72.4
133	80.1
119	89.0
103	98.9
85	110.1

H_0 = 180 ft

A = 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	H_{lT} +Δz (ft)	Error)
4565	12.95	1053006	0.0141	100.3	100.3	0%



For the valve setting to reduce the flow by half, use *Solver* to vary the value below to minimize the error.

L_e/D_{valve} = 9965

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	H_{lT} + Δz (ft)	Error)
2283	6.48	526503	0.0149	159.7	159.7	0%

Problem 10.52 [Difficulty: 4]

10.52 Consider again the pump and piping system of Problem 10.50. Estimate the percentage reductions in volume flow rate that occur after (a) 20 years and (b) 40 years of use, if the pump characteristics remain constant. Repeat the calculation if the pump head is reduced 10 percent after 20 years of use and 25 percent after 40 years.

Given: Data on pump and pipe system

Find: Delivery through parallel pump system; reduction in delivery after 20 and 40 years

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)** :

L =	1200	ft	K_{ent} =	0.5
D =	12	in	K_{exp} =	1
e =	0.00015	ft	L_e/D_{elbow} =	30
ν =	1.23E-05	ft^2/s	L_e/D_{valve} =	8
Δz =	-50	ft		

Governing Equations:

For the pumps and system

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{l_T}-\Delta h_{pump}$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2}$$

$$h_{l_m} = K\frac{\bar{V}^2}{2}$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D}+2\cdot\frac{L_e}{D_{elbow}}+\frac{L_e}{D_{valve}}\right)+K_{ent}+K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

For pumps in parallel

$$H_{pump} = H_0 - \frac{1}{4}\cdot A\cdot Q^2$$

where for a single pump

$$H_{pump} = H_0 - A\cdot Q^2$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

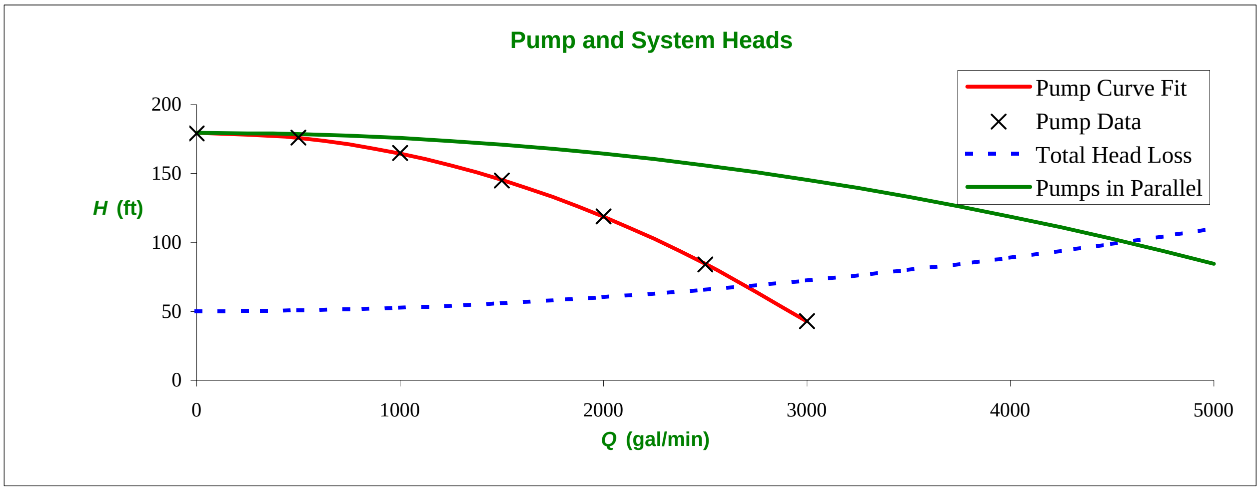
Q (gpm)	Q^2 (gpm)	H_{pump} (ft)	H_{pump} (fit)	V (ft/s)	Re	f
0	0	179	180	0.00	0	0.0000
500	250000	176	176	1.42	115325	0.0183
1000	1000000	165	164	2.84	230649	0.0164
1500	2250000	145	145	4.26	345974	0.0156
2000	4000000	119	119	5.67	461299	0.0151
2500	6250000	84	85	7.09	576623	0.0147
3000	9000000	43	43	8.51	691948	0.0145
3500				9.93	807273	0.0143
4000				11.35	922597	0.0142
4500				12.77	1037922	0.0141
5000				14.18	1153247	0.0140

H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)
180	50.0
179	50.8
176	52.8
171	56.0
164	60.3
156	65.8
145	72.4
133	80.1
119	89.0
103	98.9
85	110.1

H_0 = 180 ft

A = 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
4565	12.95	1053006	0.0141	100.3	100.3	0%



20-Year Old System:

$f = 2.00\, f_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
3906	11.08	900891	0.0284	121.6	121.6	0%

Flow reduction:

660 gpm

14.4% Loss

40-Year Old System:

$f = 2.40\, f_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
3710	10.52	855662	0.0342	127.2	127.2	0%

Flow reduction:

856

18.7%

20-Year Old System and Pumps:

$f = 2.00\, f_{new}$ $H_{pump} = 0.90\, H_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
3705	10.51	854566	0.0285	114.6	114.6	0%

Flow reduction:

860 gpm

18.8% Loss

40-Year Old System and Pumps:

$f = 2.40\, f_{new}$ $H_{pump} = 0.75\, H_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
3150	8.94	726482	0.0347	106.4	106.4	0%

Flow reduction:

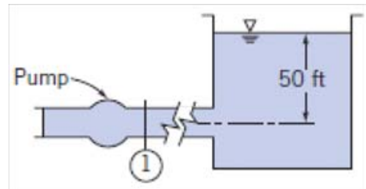
1416

31.0%

Problem 10.53

[Difficulty: 3]

10.53 Consider the flow system shown in Problem 8.94 Assume the minimum *NPSHR* at the pump inlet is 15 ft of water. Select a pump appropriate for this application. Use the data for increase in friction factor with pipe age given in Problem 10.52 to determine and compare the system flow rate after 10 years of operation.



Given: Flow from pump to reservoir

Find: Select a pump to satisfy NPSHR

Solution:

Basic equations $\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT}-h_p \qquad h_{lT}=h_l+h_{lm}=f\cdot\frac{L}{D}\cdot\frac{V_1^2}{2}+K_{exit}\cdot\frac{V_1^2}{2}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 is approximately 1 4) $V_2 << V_1$

Note that we compute head per unit weight, H, not head per unit mass, h, so the energy equation between Point 1 and the free surface (Point 2) becomes $\left(\frac{p_1}{\rho\cdot g}+\frac{V^2}{2\cdot g}\right)-(z_2)=f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}+K_{exit}\cdot\frac{V^2}{2\cdot g}-H_p$

Solving for H_p $\qquad H_p=z_2-\frac{p_1}{\rho\cdot g}-\frac{V^2}{2\cdot g}+f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}+K_{exit}\cdot\frac{V^2}{2\cdot g}$

From Table A.7 (68°F) $\qquad \rho = 1.94\cdot\frac{slug}{ft^3} \qquad \nu = 1.08\times10^{-5}\cdot\frac{ft^2}{s} \qquad Re=\frac{V\cdot D}{\nu} \qquad Re=6.94\times10^5$

For commercial steel pipe $\qquad e = 0.00015\cdot ft \qquad$ so $\qquad \frac{e}{D}=0.0002$

Flow is turbulent: $\qquad$ Given $\qquad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0150$

For the exit $\qquad K_{exit} = 1.0 \qquad$ so we find $\qquad H_p=z_2-\frac{p_1}{\rho\cdot g}+f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}$

Note that for an NPSHR of 15 ft this means $\qquad \frac{p_1}{\rho\cdot g}=15\cdot ft \qquad H_p = z_2-\frac{p_1}{\rho\cdot g}+f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g} \qquad H_p=691\cdot ft$

Note that $\qquad Q=\frac{\pi\cdot D^2}{4}\cdot V \qquad Q=4.42\cdot\frac{ft^3}{s} \qquad Q=1983\cdot gpm$

For this combination of Q and Hp, from Fig. C.11 the best pump appears to be a Peerless two-stage 10TU22C operating at 1750 rpm

After 10 years, the friction factor will have increased by a factor of 2.2 $\qquad f = 2.2\times0.150 \qquad f = 0.330$

We now need to solve $\qquad H_p=z_2-\frac{p_1}{\rho\cdot g}+f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g} \qquad$ for the new velocity V

$V=\sqrt{\frac{2\cdot D\cdot g}{f\cdot L}\cdot\left(H_p-z_2+\frac{p_1}{\rho\cdot g}\right)} \qquad V=2.13\cdot\frac{ft}{s} \qquad$ and f will still be 2.2×0.150

$Q=\frac{\pi\cdot D^2}{4}\cdot V \qquad Q=0.94\cdot\frac{ft^3}{s} \qquad Q=423\cdot gpm \qquad$ Much less!

Problem 10.54

(Difficulty: 3)

10.54 A fire nozzle is supplied through 300 ft of 3-in-diameter canvas hose (with $e = 0.001\ ft$). Water from a hydrant is supplied at 50 psig to a booster pump on board the pumper truck. At design operating conditions, the pressure at the nozzle inlet is 100 psig, and the pressure drop along the hose is 33 psi per 100 ft of length. Calculate the design flow rate and the maximum nozzle exit speed. Select a pump appropriate for this application, determine this efficiency at this operating conditions, and calculate the power required to drive the pump.

Assumptions: (1) $\bar{V}_1 = \bar{V}_2$ (2) $z_1 = z_2$ (3) $h_{lm} = 0$

Solution: Apply the energy equation for pipe flow with head loss:

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{lT} = h_l + h_{lm}$$

$$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$$

The velocity and the elevation are the same at both locations and so the energy equation reduces to

$$\frac{p_1}{\rho} - \frac{p_2}{\rho} = \frac{\Delta p}{\rho} = f \frac{L}{D} \frac{\bar{V}^2}{2}$$

$$\frac{e}{D} = \frac{0.001\ ft}{3\ in} \times 12\ \frac{in}{ft} = 0.004$$

Assuming that the flow is in the wholly rough region, from Figure 8.13 the friction factor is

$$f = 0.028$$

The average velocity is then

$$\bar{V} = \left[\frac{2}{0.028} \times 3\ in \times 3 \times (33)\ \frac{lbf}{in^2} \times \frac{ft^3}{1.94\ slug} \times \frac{1}{300\ ft} \times 12\ \frac{in}{ft} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{\frac{1}{2}} = 20.9\ \frac{ft}{s}$$

$$Q = \bar{V} A$$

$$A = \frac{\pi D^2}{4} = \frac{\pi}{4} \times \left(\frac{3}{12}\ ft\right)^2 = 0.0491\ ft^2$$

$$Q = \bar{V}A = 20.9\ \frac{ft}{s} \times 0.0491\ ft^2 \times 7.48\ \frac{gal}{ft^3} \times \frac{60\ s}{min} = 461\ gpm$$

Apply Bernoulli equation to the nozzle,

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_{atm}}{\rho} + \frac{V_n^2}{2} + gz_n$$

$$V_n = \left[\frac{2p_2}{\rho} + V_2^2\right]^{\frac{1}{2}}$$

$$V_n = \left[2 \times 100\ \frac{lbf}{in^2} \times \frac{ft^3}{1.94\ slug} \times 144\ \frac{in^2}{ft^2} + \left(20.9\ \frac{ft}{s}\right)^2\right]^{\frac{1}{2}} = 124\ \frac{ft}{s}$$

The pump head requirement (neglecting the velocity head and elevation difference) will be:

$$H_p = \frac{p_1 - p_0}{\rho g} = [100 + 3 \times (33) - 50]\ \frac{lbf}{in^2} \times \frac{ft^3}{62.4\ lbf} \times 144\ \frac{in^2}{ft^2} = 344\ ft$$

From the pump selector chart (Fig. c.1) choose 3AE98 or 4AE10 pump, 3500 rpm. Based on 4AE12 at 3550 rpm, expect $\eta = 0.75$. The power is then

$$P = \frac{Q\Delta p}{\eta} = \frac{\rho g \Delta h Q}{\eta} = \frac{1}{0.75} \times 20.9\ \frac{ft}{s} \times 0.0491\ ft^2 \times 62.4\ \frac{lbf}{ft^3} \times 344\ ft \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 53.4\ hp$$

10.55 Manufacturer's data for a submersible utility pump are

Discharge height (ft)	0.3	0.7	1.5	3.0	4.5	6.0	8.0
Water flow rate (L/min)	77.2	75	71	61	51	26	0

The owner's manual also states, "Note: These ratings are based on discharge into 25-mm pipe with friction loss neglected. Using 20-mm garden hose adaptor, performance will be reduced approximately 15 percent." Plot a performance curve for the pump. Develop a curve-fit equation for the performance curve; show the curve-fit on the plot. Calculate and plot the pump delivery versus discharge height through a 15-m length of smooth 20-mm garden hose. Compare with the curve for delivery into 25-mm pipe.

Given: Manufacturer data for a pump

Find: (a) Plot performance and develop curve-fit equation.
(b) Calculate pump delivery vs discharge height for length of garden hose

Solution:

Basic equations: $h_{lT} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} + K\cdot\frac{V^2}{2}$ $\quad H = \frac{h}{g}$ $\quad H_p = H_0 - A\cdot Q^2$

For this case, $L_e = K = 0$, therefore: $h_{lT} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$ Here are the results calculated in *Excel*:

Given data:

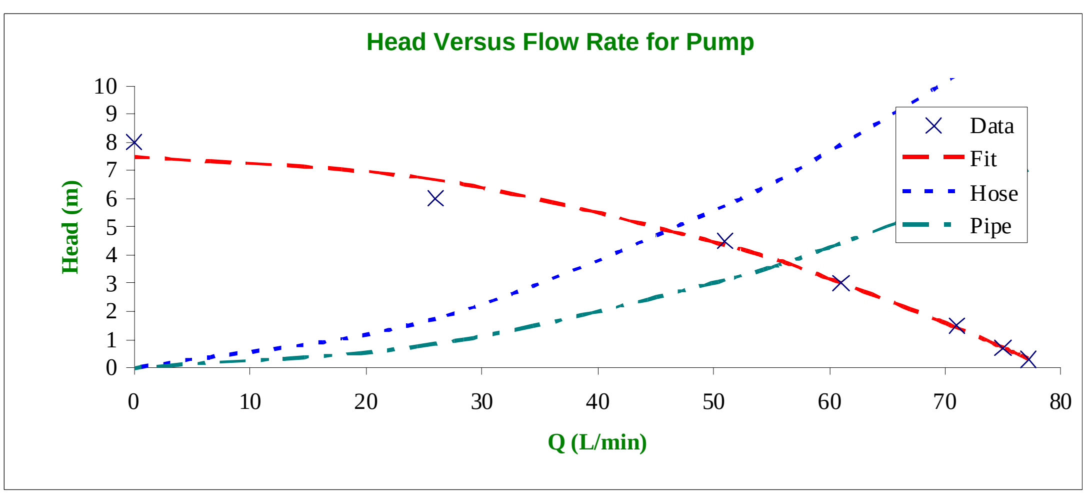
L = 15 m
e = 0 ft
D = 20 mm
ρ = 998 kg/m^3
ν = 1.01E-06 m^2/s

H_0 = 7.48727 m
A = 0.0012 m/(L/min)^2

Here are the data for the head generated by the pump, as well as the head losses for the hose and the pipe:

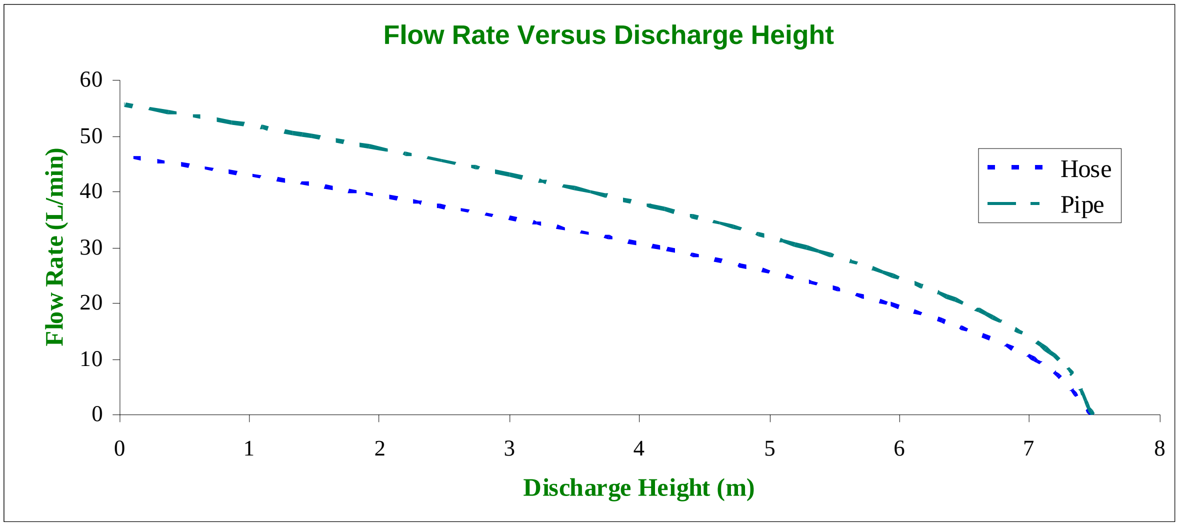
Hose: D = 20 mm, e = 0 mm; Pipe: D = 25 mm, e = 0.15 mm

z (m)	Q (L/min)	Q^2	z_{fit} (m)	V (m/s)	Re_a	f_a	H_L (m)	V (m/s)	Re_a	f_a	H_L (m)
0.3	77.2	5959.840	0.320	4.096	8.11E+04	0.0188	12.1	2.621	6.49E+04	0.0334	7.0
0.7	75.0	5625.000	0.722	3.979	7.88E+04	0.0189	11.4	2.546	6.30E+04	0.0334	6.6
1.5	71.0	5041.000	1.425	3.767	7.46E+04	0.0191	10.4	2.411	5.97E+04	0.0335	6.0
3.0	61.0	3721.000	3.012	3.236	6.41E+04	0.0198	7.9	2.071	5.13E+04	0.0337	4.4
4.5	51.0	2601.000	4.359	2.706	5.36E+04	0.0206	5.8	1.732	4.29E+04	0.0340	3.1
6.0	26.0	676.000	6.674	1.379	2.73E+04	0.0240	1.7	0.883	2.19E+04	0.0356	0.8
8.0	0.0	0.000	7.487	0.000	0.00E+00	0.0000	0.0	0.000	0.00E+00	0.0000	0.0



To determine the discharge heights for the hose and the pipe, we subtract the head loss from the head generated by the pump.

	For the hose:					For the pipe:					
Q (L/min)	**V (m/s)**	**Re_a**	**f_a**	**H_L (m)**	**Disch (m)**	**V (m/s)**	**Re_a**	**f_a**	**H_L (m)**	**Disch (m)**	**% Diff**
0.0	0.000	0.00E+00	0.0000	0.000	7.487	0.000	0.00E+00	0.0000	0.000	7.487	0%
10.0	0.531	1.05E+04	0.0305	0.328	7.039	0.340	8.40E+03	0.0398	0.140	7.227	-3%
20.0	1.061	2.10E+04	0.0256	1.101	5.906	0.679	1.68E+04	0.0364	0.514	6.492	-9%
30.0	1.592	3.15E+04	0.0232	2.248	4.157	1.019	2.52E+04	0.0351	1.115	5.290	-21%
40.0	2.122	4.20E+04	0.0217	3.740	1.823	1.358	3.36E+04	0.0345	1.943	3.620	-50%
50.0	2.653	5.25E+04	0.0207	5.558	-1.077	1.698	4.20E+04	0.0340	2.998	1.483	
60.0	3.183	6.30E+04	0.0199	7.689	-4.531	2.037	5.04E+04	0.0337	4.279	-1.122	

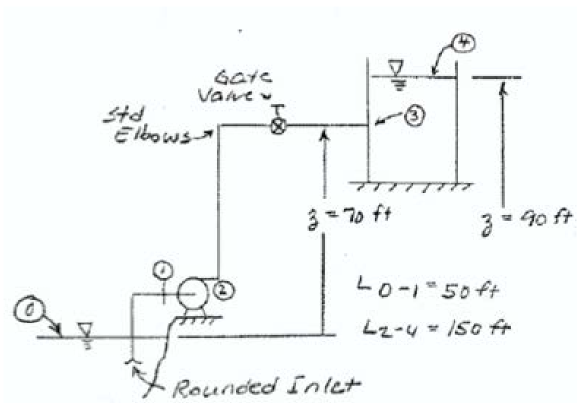


The results show that the 15% performance loss is an okay "ball park" guess at the lower flow rates, but not very good at flow rates above 30 L/min.

Problem 10.56

(Difficulty: 4)

10.56 Water is pumped from a lake (at $z = 0$) to a large storage tank located on a bluff above the lake. The pipe is 3-in-diameter galvanized iron. The inlet section (between the lake and the pump) includes on rounded inlet, one standard 90° elbow, and 50 ft of pipe. The discharge section (between the pump outlet and the discharge to the open tank) includes two standard 90° elbows, one gate valve, and 150 ft of pipe. The pipe discharge (into the side of the tank) is at $z = 70\ ft$. Calculate the system flow curve. Estimate the system operating point. Determine the power input to the pump if its efficiency at the operating point is 80 percent. Sketch the system curve when the water level in the upper tank is at $z = 75\ ft$ and the valve is partially closed to reduce the flow rate to 0.1 ft^3/s, sketch the system curve for this operating condition. Would you expect the pump efficiency to be higher for the first or second operating condition? Why?



Assumptions: (1) Nominal speed is $\bar{V} = 12\ ft/s$, $T = 60$ °F, $\upsilon = 1.21 \times 10^{-5}\ \frac{ft^2}{s}$ (Table A.7) (2) Flow in fully rough zone ($e = 0.0005\ ft$ (Table 8.1), $e/D = 0.002$, $f \approx 0.024$). (3) Cases: (1) Water in tank below (3) (2) Water in tank at $z = 90\ ft$ (3) Valve closed so $Q = 0.1\ ft^3/s$, valve part closed.

Solution: Apply the energy equation with head loss for pipe flow. The pump must overcome the gravity lift plus the head losses in the pipe and fittings.

$$H_{lT} = \frac{h_{lT}}{g} = \left[K_{ent} + f\left(\frac{L}{D} + 3\frac{Le}{D}(elbow) + \frac{Le}{D}(gate\ valve) + K_{exit}\right)\right]\frac{\bar{V}^2}{2g}$$

$$H_{lT} = \left[0.04 + 0.024\left(\frac{200\ ft}{3068\ in} \times 12\ \frac{in}{ft} + 3(30) + 8\right) + 1\right]\frac{\bar{V}^2}{2g} = 22.2\ \frac{\bar{V}^2}{2g}$$

and

$$H_s = z_{end} + 22.2\,\frac{\bar{V}^2}{2g}$$

Assume that the average velocity is $\bar{V} = 12\,\frac{ft}{s}$. Then

$$Q = 276\ gpm$$

$$\frac{\bar{V}^2}{2g} = 2.24\ ft$$

$$H_s = 70 + 22.2\,(2.24) = 120\ ft$$

<u>Case 1:</u>

$$z_{end} = 70\ ft$$

Operating point:

$$Q = 276\ gpm$$

$$H_P = H_s = 120\ ft$$

$$P = \frac{\rho g Q H}{\eta_p} = 62.4\,\frac{lbf}{ft^3} \times 12.0\,\frac{ft}{s} \times 0.0513\ ft^3 \times 120\ ft \times \frac{1}{0.8} \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 10.5\ hp$$

<u>Case 2:</u>

$$z_{end} = 90\ ft$$

$$H_s = 90 + 22.2(2.24) = 140\ ft$$

$$H_s = 90 + 50.0\,\frac{[Q(gpm)]^2}{(276\ gpm)^2} = 90 + 6.56 \times 10^{-4}[Q\ (gpm)]^2$$

<u>Case 3:</u>

$$Q = 0.1\,\frac{ft^3}{s} = 44.9\ gpm$$

$$H_s = H_p$$

Assume that $H_{BEP} = 0.7 H_o$, the pump head is

$$H_p = H_o + \frac{\left(H_o - H_{op}\right)Q^2}{Q_{op}^2} = \frac{120}{0.7} - \frac{\left(\frac{120}{0.7} - 120\right)}{(276)^2}Q^2 = 169 - 6.75 \times 10^{-4} Q^2$$

$$H_p = 168\ ft$$

at $Q = 44.9\ gpm$.

The results for the different cases are given below.

Water pumped from lake to storage tank on bluff:

Input Data:

Friction factor:	f =	0.024	(---)
Pipe diameter:	D =	3.068	in.

Calculated Results:

Pipe area:	A =	0.0513	ft^2

System Curves for Various Conditions:

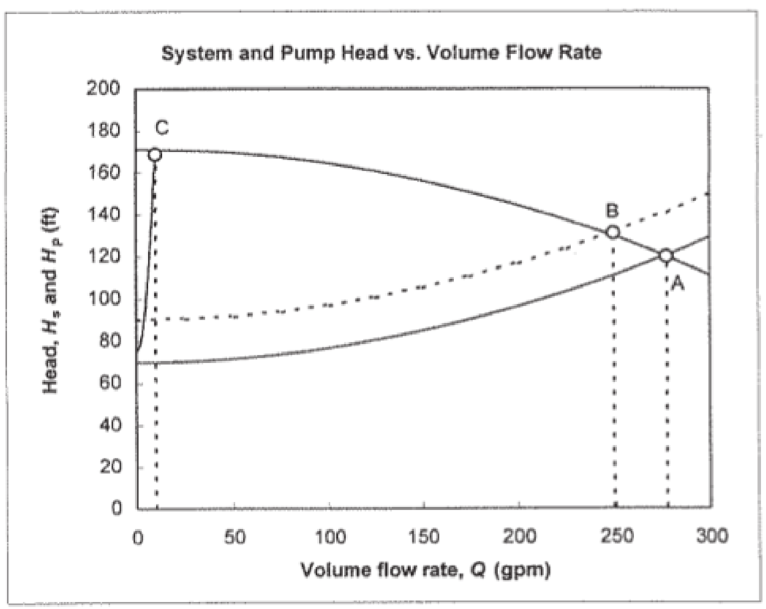
Q (gpm)	V (ft/s)	$V^2/2g$ (ft)	Case 1: H_s (z_3 = 70 ft) (ft)	Case 2: H_s (z_3 = 90 ft) (ft)
0	0	0.00	70.0	90.0
25	1.09	0.02	70.4	90.4
50	2.17	0.07	71.7	91.7
75	3.26	0.16	73.7	93.7
100	4.34	0.29	76.6	96.6
125	5.43	0.46	80.3	100
150	6.51	0.66	84.9	105
175	7.60	0.90	90.2	110
200	8.68	1.17	96.4	116
225	9.77	1.48	103	123
249.3	10.8	1.82	111	131
277	12.0	2.24	121	141
300	13.0	2.63	129	149

Case 3: Valve partially closed

Q (gpm)	H_s (z_3 = 75 ft) (ft)
0	75.0
2	78.8
4	90.0
6	109
8	135
10	169

Pump Head Curve:

Q (gpm)	H_p (ft)
0	171
50	170
100	165
150	156
200	145
250	130
277	120
300	111



Problem 10.57

[Difficulty: 3]

10.57 Performance data for a centrifugal fan of 3-ft diameter, tested at 750 rpm, are

Volume flow rate Q (ft^3/s)	106	141	176	211	246	282
Static pressure rise, Δp (psi)	0.075	0.073	0.064	0.050	0.033	0.016
Power output $\mathscr{P}$ (hp)	2.75	3.18	3.50	3.51	3.50	3.22

Plot the performance data versus volume flow rate. Calculate static efficiency, and show the curve on the plot. Find the best efficiency point, and specify the fan rating at this point.

Given: Data on centrifugal fan

Find: Plot of performance curves; Best effiiciency point

Solution:

Basic equations: $\eta_p = \dfrac{W_h}{W_m}$ $\quad W_h = Q \cdot \Delta p \quad \Delta p = \rho_w \cdot g \cdot \Delta h$ (Note: Software cannot render a dot!)

Here are the results, calculated using *Excel*:

$\rho_w =$ 1.94 slug/ft^3

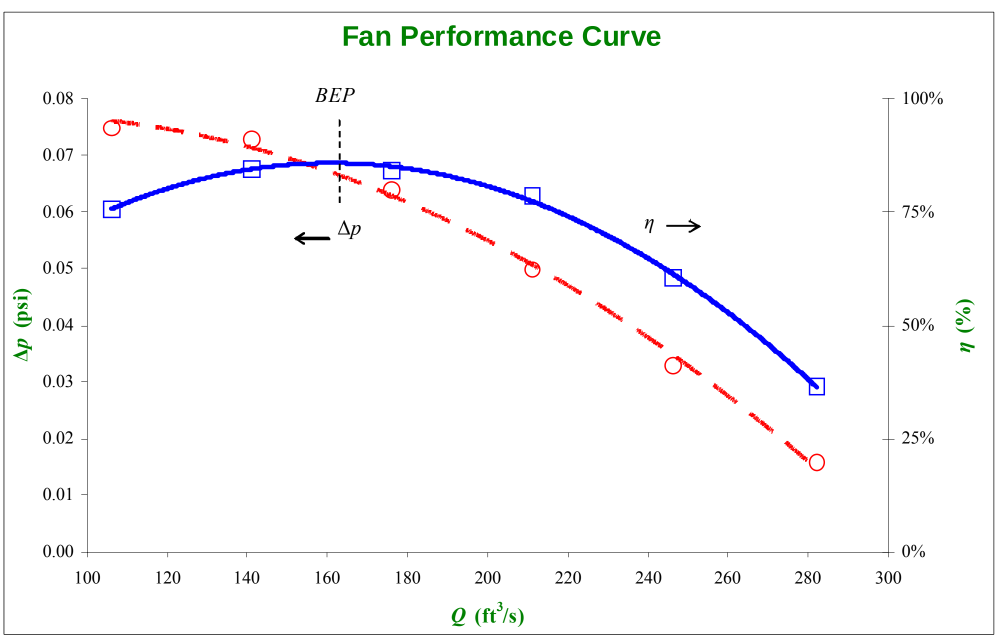
Q (ft^3/s)	Δp (psi)	$\mathscr{P}_m$ (hp)	$\mathscr{P}_h$ (hp)	η (%)
106	0.075	2.75	2.08	75.7%
141	0.073	3.18	2.69	84.7%
176	0.064	3.50	2.95	84.3%
211	0.050	3.51	2.76	78.7%
246	0.033	3.50	2.13	60.7%
282	0.016	3.22	1.18	36.7%

Fitting a 2nd order polynomial to each set of data we find

$\Delta p = -1.51 \times 10^{-6} Q^2 + 2.37 \times 10^{-4} Q + 0.0680$
$\eta = -3.37 \times 10^{-5} Q^2 + 0.0109 Q - 0.0151$

Finally, we use Solver to maximize η by varying Q:

Q (ft^3/s)	Δp (psi)	η (%)
161.72	0.0668	86.6%



Problem 10.58

[Difficulty: 3]

10.58 The performance data of Problem 10.57 are for a 36-in.-diameter fan wheel. The fan also is manufactured with 42-, 48-, 54-, and 60-in. diameter wheels. Pick a standard fan to deliver 600 ft^3/s against a 1-in. H_2O static pressure rise. Determine the required fan speed and input power required.

Given: Data on centrifugal fan and various sizes

Find: Suitable fan; Fan speed and input power

Solution:

Basic equations:
$$\frac{Q'}{Q} = \left(\frac{\omega'}{\omega}\right)\cdot\left(\frac{D'}{D}\right)^3 \qquad \frac{h'}{h} = \left(\frac{\omega'}{\omega}\right)^2\cdot\left(\frac{D'}{D}\right)^2 \qquad \frac{P'}{P} = \left(\frac{\omega'}{\omega}\right)^3\cdot\left(\frac{D'}{D}\right)^5$$

We choose data from the middle of the table above as being in the region of the best efficiency

$$Q = 176\cdot\frac{ft^3}{s} \qquad \Delta p = 0.064\cdot psi \qquad P = 3.50\cdot hp \quad \text{and} \quad \omega = 750\cdot rpm \qquad D = 3\cdot ft \qquad \rho_w = 1.94\cdot\frac{slug}{ft^3}$$

The flow and head are $Q' = 600\cdot\frac{ft^3}{s}$ $\quad h' = 1\cdot in$ $\quad$ At best efficiency point: $h = \frac{\Delta p}{\rho_w\cdot g} = 1.772\cdot in$

These equations are the scaling laws for scaling from the table data to the new fan. Solving for scaled fan speed, and diameter using the first two equations

$$\omega' = \omega\cdot\left(\frac{Q}{Q'}\right)^{\frac{1}{2}}\cdot\left(\frac{h'}{h}\right)^{\frac{3}{4}} \qquad \omega' = 265\cdot rpm \qquad D' = D\cdot\left(\frac{Q'}{Q}\right)^{\frac{1}{2}}\cdot\left(\frac{h}{h'}\right)^{\frac{1}{4}} \qquad D' = 76.69\cdot in$$

This size is too large; choose (by trial and error)

$$Q = 246\cdot\frac{ft^3}{s} \qquad h = \frac{0.033\cdot psi}{\rho_w\cdot g} = 0.914\cdot in \qquad P = 3.50\cdot hp$$

$$\omega' = \omega\cdot\left(\frac{Q}{Q'}\right)^{\frac{1}{2}}\cdot\left(\frac{h'}{h}\right)^{\frac{3}{4}} \qquad \omega' = 514\cdot rpm \qquad D' = D\cdot\left(\frac{Q'}{Q}\right)^{\frac{1}{2}}\cdot\left(\frac{h}{h'}\right)^{\frac{1}{4}} \qquad D' = 54.967\cdot in$$

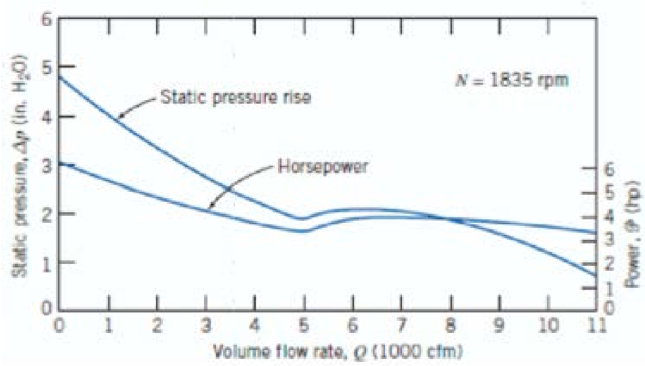
Hence it looks like the 54-inch fan will work; it must run at about 500 rpm. Note that it will NOT be running at best efficiency. The power will be

$$P' = P\cdot\left(\frac{\omega'}{\omega}\right)^3\cdot\left(\frac{D'}{D}\right)^5 \qquad P' = 9.34\cdot hp$$

Problem 10.59

(Difficulty: 4)

10.59 Performance characteristics of a Howden Buffalo axial flow fan are presented below. The fan is used to power a wind tunnel with 1-ft square test section. The tunnel consists of a smooth inlet contraction, two screens (each with loss coefficient K=0.12), the test section, and a diffuser where the cross section is expanded to 24 in. diameter at the fan inlet. Flow from the fan is discharged back to the room. Calculate and plot the system characteristic curve of pressure loss versus volume flow rate. Estimate the maximum air flow speed available in this wind tunnel test section.



Assumptions: (1) $p_0 = p_{atm}$ (2) $V_0 \approx 0$, $\alpha_1 \approx 1$ (3) $z_0 = z_1$ (4) losses in diffuser, screens

Solution: Apply the energy equation with head loss:

$$\frac{p_0}{\rho} + \frac{V_0^2}{2} + gz_0 = \frac{p_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 + h_{lT}$$

$$h_{lT} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K \right] \frac{V^2}{2}$$

The pressure drop is then

$$\frac{\Delta p_{fan}}{\rho} = \frac{p_{atm} - p_1}{\rho} = \frac{V_1^2}{2} + h_{lT} = \frac{V_1^2}{2} + (2K_{screen} + K_{diffuser})\frac{V^2}{2} = \left[2K_s + K_d + \left(\frac{A}{A_1}\right)^2 \right] \frac{Q^2}{2A^2}$$

$$\frac{\Delta p_{fan}}{\rho} = \left[2K_s + K_d + \left(\frac{A}{A_1}\right)^2 \right] \frac{Q^2}{2A^2}$$

From continuity,

$$V_1 A_1 = VA$$

$$V_1^2 = V^2 \left(\frac{A}{A_1}\right)^2$$

$$V = \frac{Q}{A}$$

$$V^2 = \frac{Q^2}{A^2}$$

$$A = 1\ ft^2$$

$$A_1 = \frac{\pi}{4} D_1^2 = 3.14\ ft^2$$

From Fig. 8.19,

$$K_d = C_{pi} - C_p = 1 - \left(\frac{1}{AR}\right)^2 - 0.70 = 1 - \left(\frac{1}{3.14}\right)^2 - 0.70 = 0.199$$

$$\Delta p_{fan} = [2(0.12) + 0.199 + 0.101] \frac{1}{2} \times Q^2 \frac{ft^6}{min^2} \times \frac{1}{(1)^2\ ft^4} \times 0.00238\ \frac{slug}{ft^3} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{min^2}{3600\ s^2}$$

$$\Delta h_{fan} = \frac{\Delta p}{\rho g} = 1.79 \times 10^{-7} [Q(cfm)^2]\ \frac{lbf}{ft^2} \times \frac{ft^3}{62.4\ lbf} \times 12\ \frac{in}{ft} = 3.43 \times 10^{-8}\ [Q(cfm)^2]$$

The resulting curve is plotted above. Computed values are tabulated below.

The system will operate where the fan curve and system curve cross. The approximate operating point is $Q = 7400\ cfm$ at $h = 1.9\ in.\ H_2o$.

The test section speed is:

$$V = \frac{Q}{A} = 7400\ \frac{ft^3}{min} \times \frac{1}{1\ ft} \times \frac{min}{60\ s} = 123\ \frac{ft}{s}$$

$Q\ (1000\ cfm)$	1	2	3	4	5	6	7	8	9	10
$\Delta p\ (in.\ H_2o)$	0.03	0.14	0.31	0.55	0.86	1.24	1.68	2.20	2.78	3.43

Problem 10.60

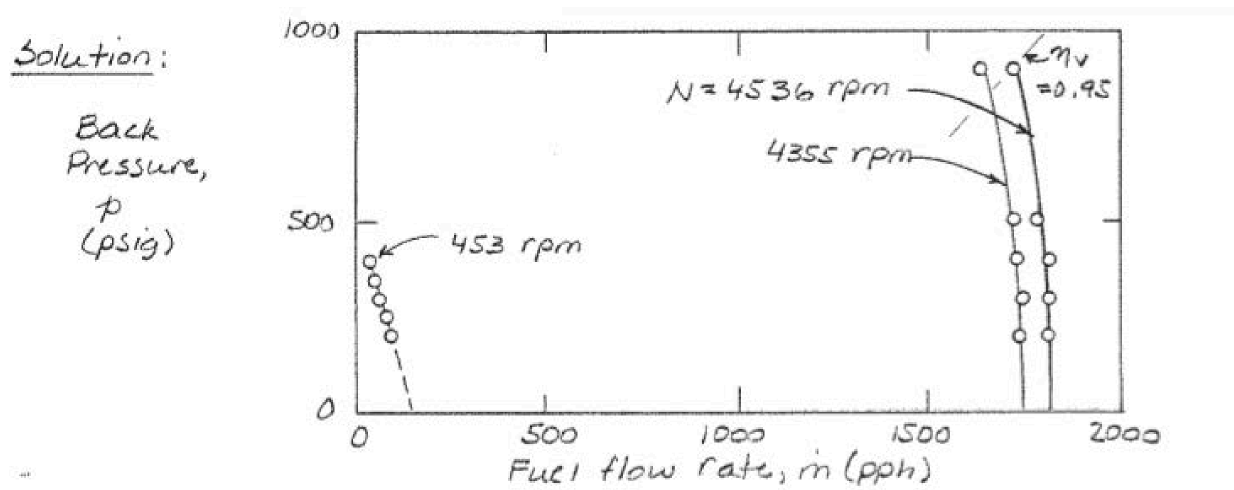
(Difficulty: 3)

10.60 Experimental test data for an aircraft engine fuel pump are presented below. This gear pump is required to supply jet fuel at 450 $pounds$ per hour and 150 $psig$ to the engine fuel controller. Tests were conducted at 10, 96, and 100 percent of the rated pump speed of 4536 rpm. At each constant speed, the back pressure on the pump was set, and the flow rate measured. On one graph, plot curves of pressure versus delivery at three constant speeds. Estimate the pump displacement volume per resolution. Calculate the volumetric efficiency at each test point and sketch contours of constant η_v. Evaluate the energy loss caused by valve throttling at 100 percent speed and full delivery to the engine.

Pump Speed (rpm)	Back Pressure (psig)	Fuel Flow (pph*)	Pump Speed (rpm)	Back Pressure (psig)	Fuel Flow (pph)	Pump Speed (rpm)	Back Pressure (psig)	Fuel Flow (pph)
	200	1810		200	1730		200	89
4536	300	1810	4355	300	1750	453	250	73
(100%)	400	1810	(96%)	400	1735	(10%)	300	58.5
	500	1790		500	1720		350	45
	900	1720		900	1635		400	30

* Fuel flow rate measured in pounds per hour (pph).

Solution: Use the pump characteristics and the relations for displacement volume and flow rate to determine the efficiency:



For the pump, $\dot{m} = \rho \forall N$, so

$$\forall = \frac{\dot{m}}{\rho N}$$

Analyzing the 4536 rpm case,

$$\forall \approx 1810\ \frac{lbm}{hr} \times \frac{gal}{6.8\ lbm} \times \frac{min}{4536\ rev} \times \frac{ft^3}{7.48\ gal} \times 1728\ \frac{in^3}{ft^3} \times \frac{hr}{60\ min} = 0.226\ \frac{in^3}{rev}$$

At constant speed,

$$\eta_v = \frac{\forall_{actual}}{\forall_{geometric}} = \frac{\dot{m}}{\dot{m}\ (p = 0)}$$

Calculation shows η_v decreases as speed is reduced, see below.

Energy loss is:

$$\dot{W}_L = \left(\frac{\dot{m}_p - \dot{m}_L}{\rho}\right) P_L$$
$$= (1810 - 450)\frac{lbm}{hr} \times \frac{gal}{6.8\ lbm} \times 150\ \frac{lbf}{in^2} \times \frac{ft^3}{7.48\ gal} \times 144\ \frac{in^2}{ft^2} \times \frac{hp \cdot s}{550\ ft \cdot lbf} \times \frac{hr}{3600\ s}$$

$$\dot{W}_L = 0.292\ hp$$

At 453 rpm, the best volumetric efficiency is:

$$\eta_v \approx \frac{\dot{m}}{\dot{m}\ (p = 0)} \times \frac{4536}{453} \approx \frac{89\ pph}{1810\ pph} \times \frac{4536}{453} = 0.0492$$

or about 5%.

At 4355 rpm,

$$\eta_v \approx \frac{1730\ pph}{1810\ pph} \times \frac{4536}{4355} = 0.996$$

or more than 99%. (this is doubtful).

Problem 10.61

(Difficulty: 2)

10.61 Preliminary calculations for a hydroelectric power generation site show a net head of 2350 ft is available at a water flow rate of 75 ft^3/s. Compare the geometry and efficiency of Pelton wheels designed to run at (a) 450 rpm and (b) 600 rpm.

Solution: The efficiency of a Pelton wheel at full load is about 0.86. Assuming this value

$$\eta_{max} = 0.86$$

The output power is:

$$P_{out} = \eta\rho Q g H = 0.86 \times 62.4\ \frac{lbf}{ft^3} \times 75\ \frac{ft^3}{s} \times 2350\ ft \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 17200\ hp$$

Neglect nozzle losses and elevation above the tailrace. Then:

$$V_j \approx \sqrt{2gH} = \left[2 \times 32.2\ \frac{ft}{s^2} \times 2350\ ft\right]^{\frac{1}{2}} = 389\ \frac{ft}{s}$$

The blade speed is about 0.47 of the jet speed at maximum power. then

$$U = R\omega \approx 0.47\ V_j = 183\ \frac{ft}{s}$$

Thus

$$D = 2R = \frac{2(0.47\ V_j)}{\omega} = 2 \times 183\ \frac{ft}{s} \times \frac{s}{47.1\ rad} = 7.77\ ft$$

The jet diameter is found from $Q = V_j A_j = \pi V_j D_j^2/4$, so

$$D_j = \sqrt{\frac{4Q}{\pi V_j}} = \left[\frac{4}{\pi} \times 75\ \frac{ft^3}{s} \times \frac{s}{389\ ft}\right]^{\frac{1}{2}} = 0.495\ ft\ (5.95\ in)$$

The ratio of jet diameter to wheel diameter is:

$$r = \frac{D_j}{D} = \frac{0.495\ ft}{7.77\ ft} = 0.0637$$

or 1:15.7 (this is reasonable).

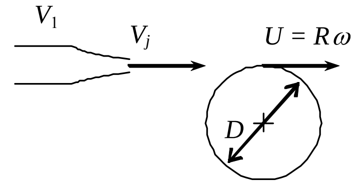
At $N = 600\ rpm$, the jet and wheel speeds are the same for maximum power. The diameter for the higher speed turbine is

$$D = 2R = \frac{2(0.47\ V_j)}{\omega} = 2 \times 183\ \frac{ft}{s} \times \frac{s}{62.8\ rad} = 5.82\ ft$$

Problem 10.62

[Difficulty: 2]

10.62 Conditions at the inlet to the nozzle of a Pelton wheel are $p=700$ psig and $V=15$ mph. The jet diameter is $d=7.5$ in. and the nozzle loss coefficient is $K_{nozzle}=0.04$. The wheel diameter is $D=8$ ft. At this operating condition, $\eta=0.86$. Calculate (a) the power output, (b) the normal operating speed, (c) the approximate runaway speed, (d) the torque at normal operating speed, and (e) the approximate torque at zero speed.



Given: Pelton turbine

Find: 1) Power 2) Operating speed 3) Runaway speed 4) Torque 5) Torque at zero speed

Solution:

Basic equations
$$\left(\frac{p_1}{\rho \cdot g} + \alpha \cdot \frac{V_1^2}{2 \cdot g} + z_1\right) - \left(\frac{p_j}{\rho \cdot g} + \alpha \cdot \frac{V_j^2}{2 \cdot g} + z_j\right) = \frac{h_{lT}}{g} \qquad h_{lT} = h_l + h_{lm} = K \cdot \frac{V^2}{2}$$

and from Example 10.5
$$T_{ideal} = \rho \cdot Q \cdot R \cdot (V_j - U) \cdot (1 - \cos(\theta)) \qquad \theta = 165 \cdot deg$$

Assumptions: 1) $p_j = p_{amt}$ 2) Incompressible flow 3) α at 1 and j is approximately 1 4) Only minor loss at nozzle 5) $z_1 = z_j$

Given data
$$p_{1g} = 700 \cdot psi \qquad V_1 = 15 \cdot mph \qquad V_1 = 22 \cdot \frac{ft}{s} \qquad \eta = 86 \cdot \%$$
$$d = 7.5 \cdot in \qquad D = 8 \cdot ft \qquad R = \frac{D}{2} \qquad K = 0.04 \qquad \rho = 1.94 \cdot \frac{slug}{ft^3}$$

Then
$$\frac{p_{1g}}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} - \frac{V_j^2}{2 \cdot g} = \frac{K}{g} \cdot \frac{V_j^2}{2} \quad \text{or} \quad V_j = \sqrt{\frac{2 \cdot \left(\frac{p_{1g}}{\rho} + \frac{V_1^2}{2}\right)}{1 + K}} \qquad V_j = 317 \cdot \frac{ft}{s}$$

and
$$Q = V_j \cdot \frac{\pi \cdot d^2}{4} \qquad Q = 97.2 \cdot \frac{ft^3}{s} \qquad H = \frac{p_{1g}}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} \qquad H = 1622 \cdot ft$$

Hence
$$P = \eta \cdot \rho \cdot Q \cdot g \cdot H \qquad P = 15392 \cdot hp$$

From Fig. 10.10, normal operating speed is around $U = 0.47 \cdot V_j$
$$U = 149 \cdot \frac{ft}{s} \qquad \omega = \frac{U}{R} \qquad \omega = 37.2 \cdot \frac{rad}{s} \qquad \omega = 356 \cdot rpm$$

At runaway
$$U_{run} = V_j \qquad \omega_{run} = \frac{U_{run}}{\left(\frac{D}{2}\right)} \qquad \omega_{run} = 79.2 \cdot \frac{rad}{s} \qquad \omega_{run} = 756 \cdot rpm$$

From Example 10.5
$$T_{ideal} = \rho \cdot Q \cdot R \cdot (V_j - U) \cdot (1 - \cos(\theta)) \qquad T_{ideal} = 2.49 \times 10^5 \cdot ft \cdot lbf$$

Hence
$$T = \eta \cdot T_{ideal} \qquad T = 2.14 \times 10^5 \cdot ft \cdot lbf$$

Stall occurs when
$$U = 0 \qquad T_{stall} = \eta \cdot \rho \cdot Q \cdot R \cdot V_j \cdot (1 - \cos(\theta)) \qquad T_{stall} = 4.04 \times 10^5 \cdot ft \cdot lbf$$

Problem 10.63

(Difficulty 2)

10.63 A Francis turbine is to operate under a head of $46\ m$ and deliver $18.6\ MW$ while running at $150\ \frac{r}{min}$. The runner diameter is $4\ m$. A $1\ m$ diameter model is operated in a laboratory under the same head. Find the model speed, power, and flow rate.

Find: The physical parameters of the model

Assumption: Water will be the fluid for the model

Solution: Use the relations for geometric and dynamic similitude

For the prototype we have:

$$h_p = 46\ m, \qquad P_p = 18.6\ MW, \qquad D_p = 4\ m, \ \ and\ n_p = 150\ rpm$$

For the model we have

$$h_m = 46\ m, \qquad and\ \ D_m = 1\ m$$

For dynamic similitude, we need the same coefficient relating the head produced

$$\frac{h_p g}{n_p^2 D_p^2} = \frac{h_m g}{n_m^2 D_m^2}$$

$$n_m = \sqrt{\frac{h_m}{D_m^2}\frac{n_p^2 D_p^2}{h_p}} = \sqrt{\frac{46\ m}{(1\ m)^2} \times \frac{(150\ rpm)^2 \times (4\ m)^2}{46\ m}} = 600\ rpm$$

For dynamic similitude, we need equal power coefficients:

$$\frac{\dot{W}_p}{\rho n_p^3 D_p^5} = \frac{\dot{W}_m}{\rho n_m^3 D_m^5}$$

$$\dot{W}_m = \frac{\dot{W}_p n_m^3 D_m^5}{n_p^3 D_p^5} = \frac{18.6\ MW \times (600\ rpm)^3 \times (1\ m)^5}{(150\ rpm)^3 \times (4\ m)^5} = 1.163\ MW$$

For the flow rate, we equate the capacity coefficient:

$$\frac{Q_p}{n_p D_p^3} = \frac{Q_m}{n_m D_m^3}$$

$$Q_m = Q_p \frac{n_m D_m^3}{n_p D_p^3}$$

The prototype flow rate is given in terms of the power produced by

$$Q_p = \frac{\dot{W}_p}{\eta_p \rho g H_p}$$

The model flow rate is then

$$Q_m = \frac{\dot{W}_p}{\eta_p \rho g H_p} \frac{n_m D_m^3}{n_p D_p^3}$$

$$Q_m = \frac{18.6\ MW}{0.924 \times 9800\ \frac{N}{m} \times 46\ m} \times \frac{600\ rpm}{150\ rpm} \times \frac{(1\ m)^3}{(4\ m)^3} = 2.79\ \frac{m^3}{s}$$

Problem 10.64

(Difficulty 2)

10.64 A Kaplan (propeller with variable pitch blades) turbine with a rated capacity of 83 MW at a head of 24 m and 86 rpm was one of 14 units installed at the McNary project on the Columbia River. The characteristic runner diameter is 7 m. If a 6 m head is available in the laboratory, what should be model scale, flow rate , and rpm?

Find: The physical parameters of the model

Assumption: Water will be the fluid for the model

Solution: Use the relations for geometric and dynamic similitude

For the prototype we have:

$$h_p = 24\ m,\ \ P_p = 83\ MW, \qquad D_p = 7\ m, \qquad n_p = 86\ rpm$$

For geometric similitude we have for a 6 m head available in the laboratory:

$$\frac{D_m}{h_m} = \frac{D_p}{h_p}$$

$$D_m = h_m \frac{D_p}{h_p} = 6\ m \times \frac{7\ m}{24\ m} = 1.75\ m$$

For dynamic similitude, we need the same coefficient relating the head produced

$$\frac{h_m}{n_m^2 D_m^2} = \frac{h_p}{n_p^2 D_p^2}$$

$$n_m = \sqrt{\frac{h_m}{D_m^2} \frac{n_p^2 D_p^2}{h_p}}$$

$$n_m = \sqrt{\frac{6\ m}{(1.75\ m)^2} \times \frac{(86\ rpm)^2 \times (7\ m)^2}{24\ m}} = 172\ rpm$$

The model flow rare is determined from the similarity requirement for pump capacity:

$$\frac{Q_m}{n_m D_m^3} = \frac{Q_p}{n_p D_p^3}$$

$$Q_m = \frac{Q_p n_m D_m^3}{n_p D_p^3} = Q_p \frac{172\ rpm}{86\ rpm}\left(\frac{1.75\ m}{7\ m}\right)^3 = 0.03125\ Q_p$$

Assume that $\eta_m = \eta_p = 100\%$, then the prototype power is

$$Q_p = \frac{\dot{W}_p}{\rho_P g h_p} = 352\ \frac{m^3}{s}$$

The model flow rate is then

$$Q_m = 11\ \frac{m^3}{s}$$

Problem 10.65

(Difficulty: 2)

10.65 Francis turbine Units 19, 20, and 21, installed at the Grand Coulee Dam on the Columbia River, are very large [55]. Each runner is 32.6 ft in diameter and contains 550 tons of cast steel. At rated conditions, each turbine develops 820,000 hp at 72 rpm under 285 ft of head. Efficiency is nearly 95 percent at rated conditions. The turbines operate at heads from 220 to 355 ft. Calculate the specific speed at rated operating conditions. Estimate the maximum water flow rate through each turbine.

Solution: Apply definitions of specific speed and efficiency.

$$Ns_{cu} = \frac{NP^{\frac{1}{2}}}{H^{\frac{5}{4}}}$$

$$\eta = \frac{P}{\rho Q g H}$$

Thus

$$Ns_{cu} = \frac{72\ rpm(820{,}000\ hp)^{\frac{1}{2}}}{(285\ ft)^{\frac{5}{4}}} = 55.7$$

From η,

$$Q = \frac{P}{\eta \rho g H}$$

so Q is maximum at minimum head. Assuming $\eta = 0.95$, the

$$Q \approx \frac{1}{0.95} \times 820{,}000\ hp \times \frac{ft^3}{62.4\ lbf} \times \frac{1}{220\ ft} \times 550\ \frac{ft \cdot lbf}{hp \cdot s} = 34{,}600\ \frac{ft^3}{s}$$

{This is an estimate because ηmay not be constant, nor may it be possible develop full power at $H = 220\ ft$.}

Problem 10.66

(Difficulty: 3)

10.66 Measured data for performance of the reaction turbines at Shasta Dam near Redding, California, are shown in Fig. 10.38. Each turbine is rated at 103,000 hp when operating at 138.6 rpm under a net head of 380 ft. Evaluate the specific speed and compute the shaft torque developed by each turbine at rated operating conditions. Calculate and plot the water flow rate per turbine required to produce rated output power as a function of head.

Solution: Apply the definitions of specific speed and efficiency, use data from Fig. 10.38:

$$Ns_{cu} = \frac{NP^{\frac{1}{2}}}{H^{\frac{5}{4}}}$$

$$\eta = \frac{P}{\rho Q g H}$$

$$P = \omega T$$

At rated conditions,

$$Ns_{cu} = \frac{(138.6\,rpm)(103{,}000hp)^{\frac{1}{2}}}{(380\ ft)^{\frac{5}{4}}} = 26.5$$

$$T = \frac{P}{\omega} = 103{,}000\ hp \times \frac{s}{14.5\,rad} \times 550\ \frac{ft \cdot lbf}{hp \cdot s} = 3.91 \times 10^6\ ft \cdot lbf$$

Find Q from definition of η; At rated conditions, $\eta \approx 0.93$ (Fig. 10.38):

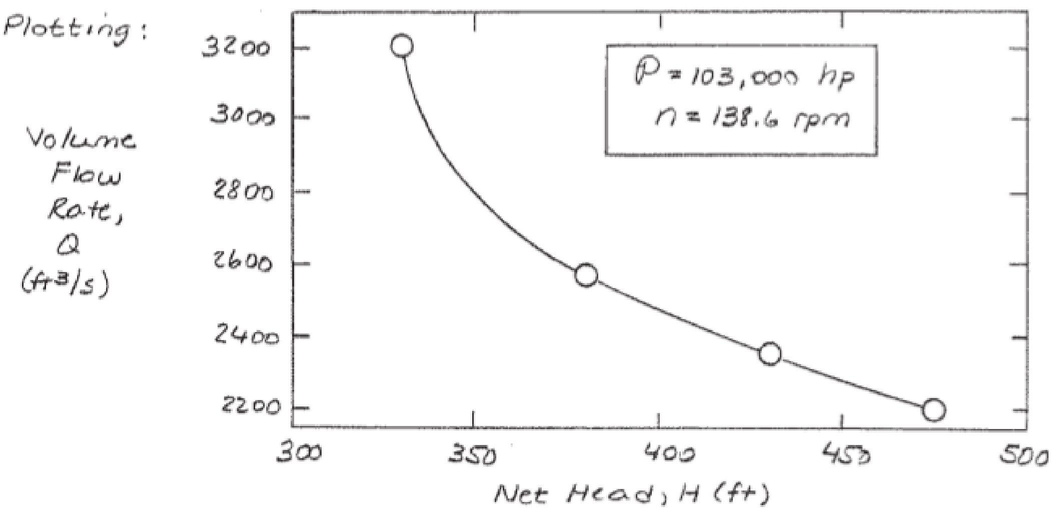
$$Q = \frac{P}{\eta \rho g H} = \frac{1}{0.93} \times 103{,}000\ hp \times \frac{ft^3}{62.4\ lbf} \times \frac{1}{380\ ft} \times 550\ \frac{ft \cdot lbf}{hp \cdot s} = 2570\ \frac{ft^3}{s}$$

Tabulating similar calculations:

H (ft)	$\dot{W}$ (hp)	η (---)	Q (ft^3/s)
238	*	–	–
280	*	–	–
330	103,000	0.86	3200
380	103,000	0.93	2570
430	103,000	0.90	2350
475	103,000	0.87	2200

* cannot produce rated power at this head

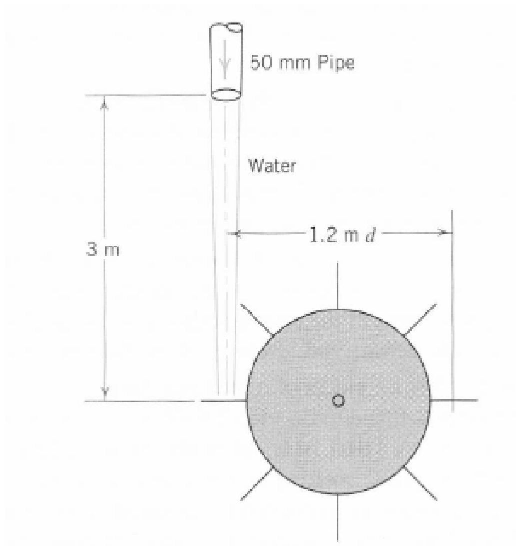
Plotting:



Problem 10.67

(Difficulty 2)

10.67 For a flow rate of 12 $\frac{L}{s}$ and turbine speed of 65 $\frac{r}{min}$, estimate the power transferred from jet to turbine wheel.



Find: The power produced by the turbine.

Assumptions: The water flow is steady and incompressible, and that the flow is ideal from the nozzle to the turbine blade.

Solution: Use the Bernoulli and angular momentum equations to determine the power.

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = constant$$

The angular moment um equation for a rotating system is

$$T = \rho Q (V_{t2} r_2 - V_{t1} r_1)$$

The power is given by

$$\dot{W} = T\omega$$

The initial jet velocity is determined from the flow rate:

$$V_0 = \frac{Q}{A} = \frac{0.012\ \frac{m^3}{s}}{\frac{\pi}{4} \times (0.05\ m)^2} = 6.11\ \frac{m}{s}$$

Applying the Bernoulli equation between the nozzle and the turbine blade yields the absolute velocity of the water hitting the blade

$$\frac{p_0}{\rho g} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1$$

The pressures are atmospheric and equal. We will take the elevation datum as $z_1 = 0$. Then

$$\frac{V_0^2}{2g} + z_0 = \frac{V_1^2}{2g}$$

The velocityV_1 is then

$$V_1 = \sqrt{2g\left(\frac{V_0^2}{2g} + z_0\right)} = \sqrt{2 \times 9.81\ \frac{m}{s^2}\left(\frac{\left(6.11\ \frac{m}{s}\right)^2}{2 \times 9.81\ \frac{m}{s^2}} + 3m\right)} = 9.81\ \frac{m}{s}$$

The turbine blade speed is:

$$U = \frac{2\pi}{60\ \frac{s}{min}} \times 65\ \frac{r}{min} \times \frac{1.2\ m}{2} = 4.08\ \frac{m}{s}$$

The tangential velocity of the water hitting the blade is the difference between the water and blade velocities:

$$V_{r2} = V_1 - U = 9.81\ \frac{m}{s} - 4.08\ \frac{m}{s} = 5.76\frac{m}{s}$$

There is no inlet momentum of the fluid and so the power is

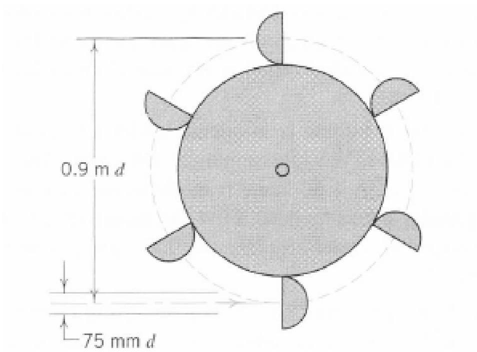
$$\dot{W} = T\omega = \rho Q V_{t2} r_2 \omega = 1000\ \frac{kg}{m^3} \times 0.012\ \frac{m^3}{s} \times 5.76\frac{m}{s} \times \frac{1.2\ m}{2} \times 65 rpm \times \frac{2\pi}{60\ \frac{s}{min}}$$

$$\dot{W} = 281\ W$$

Problem 10.68

(Difficulty 2)

10.68 The velocity of the water jet driving this impulse turbine is 45 $\frac{m}{s}$. The jet has a 75 mm diameter. After leaving the buckets the (absolute) velocity of the water is observed to be 15 $\frac{m}{s}$ in a direction 60° to that of the original jet. Calculate the mean tangential force exerted by jet on turbine wheel and the speed (rpm) of the wheel.



Find: The force on the blades and the speed of the turbine.

Assumptions: The water flow is steady and incompressible.

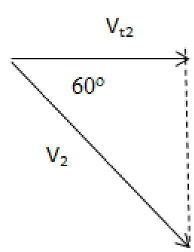
Solution: Use the linear momentum equation to determine the force. For this situation, in which there are no pressure or body forces, the momentum equation is

$$F_x = \rho Q (V_2 - V_1)$$

The volumetric flow rate is calculated by:

$$Q = \frac{\pi}{4} D^2 V_1 = \frac{\pi}{4} \times (0.075\ m)^2 \times 45\ \frac{m}{s} = 0.199\ \frac{m^3}{s}$$

The velocity vector diagram is



The tangential velocity leaving is

$$V_{t2} = V_2 \cos 60° = 15\frac{m}{s}\cos 60° = 7.5\frac{m}{s}$$

The force on the blade is then:

$$F_x = \rho Q(V_1 - V_2 \cos 60°) = 998\ \frac{kg}{m^3} \times 0.199\ \frac{m^3}{s}\left(45\frac{m}{s} - 7.5\frac{m}{s}\right) = 7460\ N$$

To calculate the speed of the wheel, we need to use the energy equation to determine the power and then the speed of the rotor. The power is calculated from the energy equation:

$$\dot{W} = \frac{\rho Q}{2}(V_1^2 - V_2^2) = \frac{998\ \frac{kg}{m^3} \times 0.199\ \frac{m^3}{s}}{2} \times \left(\left(45\ \frac{m}{s}\right)^2 - \left(15\ \frac{m}{s}\right)^2\right) = 179\ kW$$

The power is also given by the product of the force and blade velocity:

$$\dot{W} = F_x\, U$$

The blade velocity is then:

$$U = \frac{\dot{W}}{F_x} = \frac{179\ kW}{7460\ N} = 24\ \frac{m}{s}$$

The speed of the rotor is related to the rotating speed as

$$U = \omega r = \frac{\omega D}{2}$$

Or, the speed is

$$\omega = \frac{2U}{D} = \frac{2 \times 24\ \frac{m}{s}}{0.9\ m} = 53.3\ \frac{rad}{s} = 509\ rpm$$

Problem 10.69 [Difficulty: 3]

10.69 An impulse turbine is to develop 15 MW from a single wheel at a location where the net head is 350 m. Determine the appropriate speed, wheel diameter, and jet diameter for single- and multiple-jet operation. Compare with a double-overhung wheel installation. Estimate the required water consumption.

Given: Impulse turbine requirements

Find: 1) Operating speed 2) Wheel diameter 4) Jet diameter 5) Compare to multiple-jet and double-overhung

Solution:

Basic equations: $V_j = \sqrt{2 \cdot g \cdot H}$ $\quad N_S = \dfrac{\omega \cdot P^{\frac{1}{2}}}{\rho^{\frac{1}{2}} \cdot h^{\frac{5}{4}}}$ $\quad \eta = \dfrac{P}{\rho \cdot Q \cdot g \cdot H}$ $\quad Q = V_j \cdot A_j$

Model as optimum. This means. from Fig. 10.10 $U = 0.47 \cdot V_j$ and from Fig. 10.17 $N_{Scu} = 5$ with $\eta = 89 \cdot \%$

Given or available data $H = 350 \cdot m$ $\quad P = 15 \cdot MW$ $\quad \rho = 1.94 \cdot \dfrac{slug}{ft^3}$

Then $V_j = \sqrt{2 \cdot g \cdot H}$ $\quad V_j = 82.9 \dfrac{m}{s}$ $\quad U = 0.47 \cdot V_j$ $\quad U = 38.9 \dfrac{m}{s}$

We need to convert from N_{Scu} (from Fig. 10.17) to N_S (see discussion after Eq. 10.18b). $N_S = \dfrac{N_{Scu}}{43.46}$ $\quad N_S = 0.115$

The water consumption is $Q = \dfrac{P}{\eta \cdot \rho \cdot g \cdot H}$ $\quad Q = 4.91 \dfrac{m^3}{s}$

For a single jet $\omega = N_S \cdot \dfrac{\rho^{\frac{1}{2}} \cdot (g \cdot H)^{\frac{5}{4}}}{P^{\frac{1}{2}}}$ (1) $\quad \omega = 236 \cdot rpm$ $\quad D_j = \sqrt{\dfrac{4 \cdot Q}{\pi \cdot V_j}}$ (2) $\quad D_j = 0.275\, m$

The wheel radius is $D = \dfrac{2 \cdot U}{\omega}$ (3) $\quad D = 3.16\, m$

For multiple (n) jets, we use the power and flow per jet

From Eq 1 $\omega_n = \omega \cdot \sqrt{n}$ From Eq. 2 $D_{jn} = \dfrac{D_j}{\sqrt{n}}$ an d $D_n = \dfrac{D}{\sqrt{n}}$ from Eq. 3

Results:

$n =$	$\omega_n(n) =$ (rpm)	$D_{jn}(n) =$ (m)	$D_n(n) =$ (m)
1	236	0.275	3.16
2	333	0.194	2.23
3	408	0.159	1.82
4	471	0.137	1.58
5	527	0.123	1.41

A double-hung wheel is equivalent to having a single wheel with two jets

Problem 10.70 [Difficulty: 2]

10.70 An impulse turbine under a net head of 33 ft was tested at a variety of speeds. The flow rate and the brake force needed to set the impeller speed were recorded:

Wheel Speed (rpm)	Flow rate (cfm)	Brake Force (lbf) (R = 0.5 ft)
0	7.74	2.63
1000	7.74	2.40
1500	7.74	2.22
1900	7.44	1.91
2200	7.02	1.45
2350	5.64	0.87
2600	4.62	0.34
2700	4.08	0.09

Calculate and plot the machine power output and efficiency as a function of water turbine speed.

Given: Data on impulse turbine

Find: Plot of power and efficiency curves

Solution:

Basic equations: $T = F \cdot R$ $\quad P = \omega \cdot T$ $\quad \eta = \dfrac{P}{\rho \cdot Q \cdot g \cdot H}$

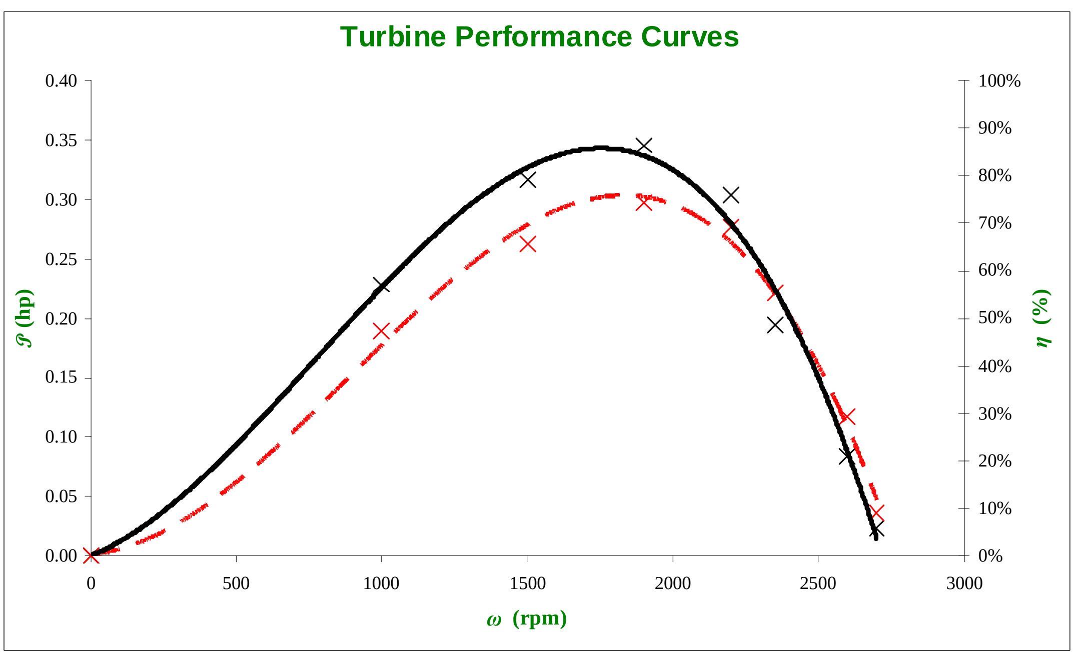
Here are the results calculated in *Excel*:

H = 33 ft

ρ = 1.94 slug/ft^3

R = 0.50 ft

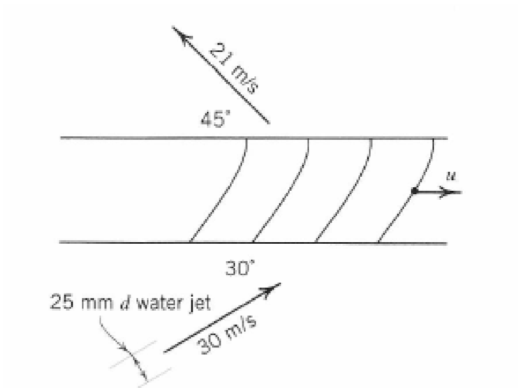
ω (rpm)	Q (cfm)	F (lbf)	T (ft-lbf)	$\mathscr{P}$ (hp)	η (%)
0	7.74	2.63	1.32	0.000	0.0%
1000	7.74	2.40	1.20	0.228	47.3%
1500	7.74	2.22	1.11	0.317	65.6%
1900	7.44	1.91	0.96	0.345	74.4%
2200	7.02	1.45	0.73	0.304	69.3%
2350	5.64	0.87	0.44	0.195	55.3%
2600	4.62	0.34	0.17	0.084	29.2%
2700	4.08	0.09	0.05	0.023	9.1%



Problem 10.71

(Difficulty 3)

10.71 The (absolute) velocities and directions of the jets entering and leaving the blade system are as shown. Calculate the power transferred from jet to blade system and the blade angles required.



Find: The power produced by the blade system and the blade angles.

Assumptions: The water flow is steady and incompressible.

Solution: Use the linear momentum equation to determine the force and the energy equation to determine the power

$$F_x = \rho Q(V_{t2} - V_{t1})$$

The power is given by the energy equation

$$\dot{W} = \frac{\rho Q}{2}(V_1^2 - V_2^2)$$

The volumetric flow rate is calculated as:

$$Q = \frac{\pi}{4}D^2 V_1 = \frac{\pi}{4} \times (0.025\ m)^2 \times 30\ \frac{m}{s} = 0.0147\ \frac{m^3}{s}$$

The power is calculated using the energy equation as:

$$\dot{W} = \frac{\rho Q}{2}(V_1^2 - V_2^2) = \frac{998\ \frac{kg}{m^3} \times 0.0147\ \frac{m^3}{s}}{2} \times \left(\left(30\ \frac{m}{s}\right)^2 - \left(21\ \frac{m}{s}\right)^2\right) = 3370\ W$$

The force on the fluid is calculated using the momentum equation. The tangential velocities are

$$V_{t1} = V_1 \cos 30° = 30\ \frac{m}{s} \times \cos 30° = 26.0\frac{m}{s}$$

$$V_{t2} = V_2 \cos 45° = -21\ \frac{m}{s} \times \cos 45° = -14.8\frac{m}{s}$$

The force is then

$$F_x = \rho Q(V_{t2} - V_{t1}) = 998\ \frac{kg}{m^3} \times 0.0147\ \frac{m^3}{s} \times \left(-14.8\ \frac{m}{s} - 26\ \frac{m}{s}\right) = -600\ N$$

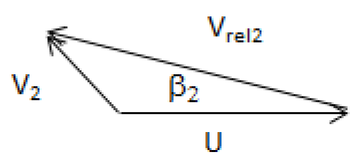
The force and power are related as:

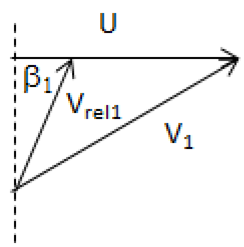
$$\dot{W} = F_x U$$

So the blade velocity is calculated as

$$U = \frac{\dot{W}}{F_x} = \frac{3370\ W}{600\ N} = 5.62\ \frac{m}{s}$$

The blade angles can be calculated using the velocity vector diagram for the inlet and outlet:





For the inlet

$$V_{rel1} \sin \beta_1 = V_1 \sin 30°$$

and

$$V_{rel1} \cos \beta_1 = V_1 \cos 30° - U$$

$$\tan \beta_1 = \frac{V_1 \sin 30°}{V_1 \cos 30° - U} = 0.737$$

The blade angle at the inlet is

$$\beta_1 = 36.4°$$

for the outlet, we have

$$V_{rel2}\cos(\pi - \beta_2) = V_2 \cos 45° + U$$

$$V_{rel2}\sin(\pi - \beta_2) = V_2 \sin 45°$$

So the angle is determined as

$$\tan(\pi - \beta_2) = \frac{V_2 \sin 45°}{V_2 \cos 45° + U} = 0.725$$

The blade angle at the outlet is

$$\beta_2 = 144°$$

Problem 10.72

(Difficulty: 4)

10.72 A small hydraulic impulse turbine is supplied with water through a penstock with diameter D and length L; the jet diameter is d. The elevation difference between the reservoir surface and nozzle centerline is Z. The nozzle head loss coefficient is K_{nozzle} and the loss coefficient from the reservoir to the penstock entrance is $K_{entrance}$. Determine the water jet speed, the volume flow rate, and the hydraulic power of the jet, for the case where Z=300 ft, L=1000 ft, D=6 in, $K_{entrance} = 0.5$, $K_{nozzle} = 0.04$, and $d = 2\ in.$, if the pipe is made from commercial steel. Plot the jet power as a fucntion of jet diameter to determine the optimum jet diameter and the resulting hydraulic power of the jet. Comment on the effects of varying the loss coefficients and pipe roughness.

Assumptions: (1) $p_1 = p_2 = p_{atm}$ (2)$\bar{V}_1 \approx 0, \alpha_2 = 1$ (3) $Le/D = 0$ (4) K_{nozzle} based on V_j^2

Solution: Apply the energy equation with head loss for steady, incompressible pipe flow.

$$\frac{p_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + \frac{h_{lT}}{g}$$

$$h_{lT} = \left[f \left(\frac{L}{D} + \frac{Le}{D} \right) + K \right] \frac{\bar{V}^2}{2}$$

Then with the assumptions for pressure and velocity, the energy equation becomes

$$H = \frac{V_j^2}{2g} + \left(f \frac{L}{D} + K_{entrance} \right) \frac{\bar{V}^2}{2g} + K_{nozzle} \frac{V_j^2}{2g}$$

From continuity,

$$\bar{V} A = V_j A_j$$

$$\bar{V} = \frac{V_j A_j}{A} = V_j \left(\frac{d}{D} \right)^2$$

$$\bar{V}^2 = V_j^2 \left(\frac{d}{D} \right)^4$$

and

$$H = \left[\left(f \frac{L}{D} + K_{ent} \right) \left(\frac{d}{D} \right)^4 + 1 + K_{nozzle} \right] \frac{V_j^2}{2g}$$

Solving for the velocity

$$V_j = \left[\frac{2gH}{\left(f\frac{L}{D} + K_{ent}\right)\left(\frac{d}{D}\right)^4 + 1 + K_n}\right]^{\frac{1}{2}}$$

Assume $e = 0.00015\ ft$ (Table 8.1), so $e/D = 0.0003$. From Fig. 8.13, in the fully rough zone, $f = 0.015$. Then for $d_j = 2\ in$:

$$V_j = \left[2 \times 32.2\ \frac{ft}{s^2} \times 300\ ft \times \frac{1}{\left(0.015\frac{1000\ ft}{0.5\ ft} + 0.5\right)\left(\frac{2}{6}\right)^4 + 1 + 0.04}\right]^{\frac{1}{2}} = 117\ \frac{ft}{s}$$

($\bar{V} = 13.0\ \ ft/s\,, Re = \bar{V}D/\upsilon = 6.05 \times 10^5$, so $f = 0.016$, which makes $V_j = 116\ \ ft/s$).

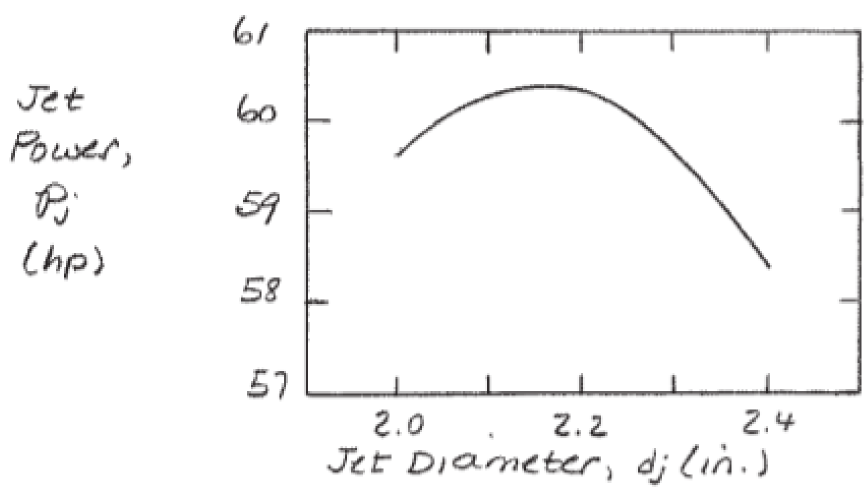
The jet flow rate is:

$$Q = V_j A_j = 116\ \frac{ft}{s} \times \frac{\pi}{4}\left(\frac{2}{12}\ ft\right)^2 = 2.53\ \frac{ft^3}{s}$$

And the jet power is:

$$P_h = \rho Q \frac{V_j^2}{2} = \frac{1}{2} \times 1.94\ \frac{slug}{ft^3} \times \left(116\ \frac{ft}{s}\right)^2 \times 2.53\ \frac{ft^3}{s} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 60.0\ hp$$

Repeating these calculations using a computer program gives:



Peak power, $P_j \approx 60.3\ hp$ occurs for $2.15 < d < 2.20\ in$. Loss coefficients have a minor effect. Making both K_{ent} and K_n increases P_j by 4.8 percent.

Pipe roughness causes larger changes. P_j increased 12.8 percent with $e = 0$ (smooth).

Problem 10.73

[Difficulty: 3]

10.73 A fanboat in the Florida Everglades is powered by a propeller, with $D = 1.5$ m, driven at maximum speed, $N = 1800$ rpm, by a 125 kW engine. Estimate the maximum thrust produced by the propeller at (a) standstill and (b) $V = 12.5$ m/s.

Given: Data on fanboat and propeller

Find: Thrust at rest; Thrust at 12.5 m/s

Solution:

Assume the aircraft propeller coefficients in Fi.g 10.40 are applicable to this propeller.

At V = 0, J = 0. Extrapolating from Fig. 10.40b $C_F = 0.16$

We also have $D = 1.5 \cdot m$ $n = 1800 \cdot rpm$ $n = 30 \cdot \frac{rev}{s}$ and $\rho = 1.225 \cdot \frac{kg}{m^3}$

The thrust at standstill (J = 0) is found from $F_T = C_F \cdot \rho \cdot n^2 \cdot D^4$ (Note: n is in rev/s) $F_T = 893 \cdot N$

At a speed $V = 12.5 \cdot \frac{m}{s}$ $J = \frac{V}{n \cdot D}$ $J = 0.278$ and so from Fig. 10.40b $C_P = 0.44$ and $C_F = 0.145$

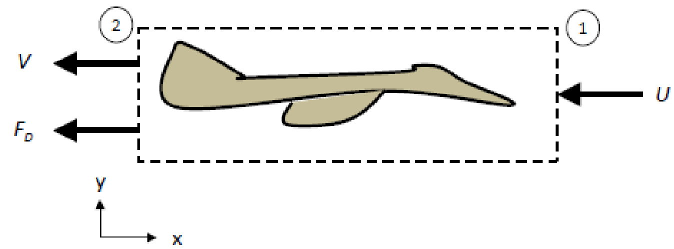
The thrust and power at this speed can be found $F_T = C_F \cdot \rho \cdot n^2 \cdot D^4$ $F_T = 809 \cdot N$ $P = C_P \cdot \rho \cdot n^3 \cdot D^5$ $P = 111 \cdot kW$

Problem 10.74 [Difficulty: 3]

10.74 A jet-propelled aircraft traveling at 225 m/s takes in 50 kg/s of air. If the propulsive efficiency (defined as the ratio of the useful work output to the mechanical energy input to the fluid) of the aircraft is 45 percent, determine the speed at which the exhaust is discharged relative to the aircraft.

Given: Data on jet-propelled aircraft

Find: Propulsive efficiency



Solution:

Basic equation:

$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\,\rho\,dV + \int_{CS} \vec{V}_{xyz}\,\rho\vec{V}_{xyz}\cdot d\vec{A}$$

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\,\rho\,dV + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

Assumption: 1) Atmospheric pressure on CS 2) Horizontal 3) Steady w.r.t. the CV 4) Use velocities relative to CV

The x-momentum is then $-F_D = u_1\cdot(-m_{rate}) + u_2\cdot(m_{rate}) = (-U)\cdot(-m_{rate}) + (-V)\cdot(m_{rate})$

or $F_D = m_{rate}\cdot(V - U)$ where $m_{rate} = 50\cdot\frac{kg}{s}$ is the mass flow rate

The useful work is then $F_D\cdot U = m_{rate}\cdot(V - U)\cdot U$

The energy equation simplifies to $-W = \left(\frac{U^2}{2}\right)\cdot(-m_{rate}) + \left(\frac{V^2}{2}\right)\cdot(m_{rate}) = \frac{m_{rate}}{2}\cdot\left(V^2 - U^2\right)$

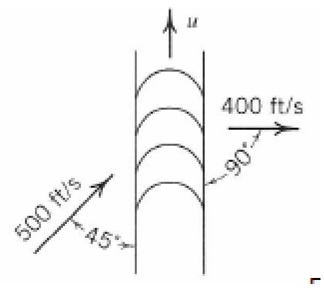
Hence $\eta = \dfrac{m_{rate}\cdot(V-U)\cdot U}{\frac{m_{rate}}{2}\cdot\left(V^2 - U^2\right)} = \dfrac{2\cdot(V-U)\cdot U}{\left(V^2 - U^2\right)} = \dfrac{2}{1 + \frac{V}{U}}$

With $U = 225\cdot\frac{m}{s}$ and $\eta = 45\%$ $V = U\cdot\left(\frac{2}{\eta} - 1\right)$ $V = 775\,\frac{m}{s}$

Problem 10.75

(Difficulty 2)

10.75 When an air jet of $1\ in$ diameter strikes a series of blades on a turbine rotor, the (absolute) velocities are as shown. If the air is assumed to have a constant specific weight of $0.08\ \frac{lbf}{ft^3}$, what is the force on the turbine rotor? How much horsepower is transferred to the rotor? What must be the velocity of the blade system?



Find: The power produced by the blade system and the blade velocity.

Assumptions: The air flow is steady and incompressible.

Solution: Use the linear momentum equation to determine the force and the energy equation to determine the power. For the y-direction

$$F_y = \rho Q(V_{t2} - V_{t1})$$

The power is given by the energy equation

$$\dot{W} = \frac{\rho Q}{2}(V_1^2 - V_2^2)$$

The mass flow rate is:

$$Q = \rho V A = \frac{0.08\ \frac{\frac{slug\cdot ft}{s^2}}{ft^3}}{32.2\ \frac{ft}{s^2}} \times 500\ \frac{ft}{s} \times \frac{\pi}{4} \times \left(\frac{1}{12}\ ft\right)^2 = 0.00677\ \frac{slug}{s}$$

The force in the y-direction is determined as follows. There is no component of velocity in the y-direction of the leaving fluid.

$$F_y = Q(V_{2y} - V_{1y}) = Q(0 - V_1 \sin 45°) = 0.00677\ \frac{slug}{s} \times \left(-500\ \frac{ft}{s} \times \sin 45°\right) = -2.37\ lbf$$

The velocity of the blade system is determined from the relation between the force and the power

$$\dot{W} = F_x U$$

So the blade velocity is calculated as

$$U = \frac{\dot{W}}{F_y} = \frac{305\ \frac{lbf \cdot ft}{s}}{2.39\ lbf} = 127.6\ \frac{ft}{s}$$

There is also a force on the rotor in the x direction. This force is

$$F_x = Q(V_{1x} - V_{2x}) = Q(V_1 \cos 45° - V_2) = 0.00677\ \frac{slug}{s} \times \left(500\ \frac{ft}{s} \times \cos 45° - 400\ \frac{ft}{s}\right)$$

$$F_x = 0.314\ lbf$$

The total force on the blade is the vector sum of the forces and is calculated as:

$$F = \sqrt{(-F_x)^2 + \left(-F_y\right)^2} = 2.39\ lbf$$

Problem 10.76

(Difficulty 2)

10.76 The volume flow rate through the propeller of an airboat (a boat driven by a propeller moving air) is $50 m^3/s$. When the boat is docked, the speed of the slipstream behind the propeller a location where the flow has returned atmospheric pressure is 40 m/s. Determine (a) the propeller diameter, (b) the thrust produced when the boat is docked, (c) the thrust produced at the same flow rate when the airboat is moving ahead at 15 m/s, and (d) the maximum speed of the boat.

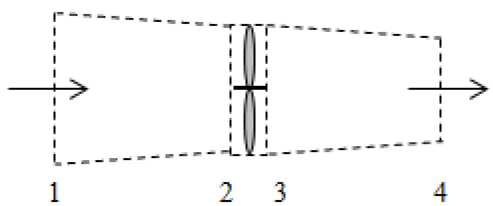
Find The propeller diameter and the thrust of the docked and moving boat.

Assumptions: The flow through the propeller is steady and uniform

Solution: Use the momentum relation for flow through the engine.

$$\vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho\bar{V}\cdot d\bar{A}$$

The control volume for the propeller is



For steady horizontal flow, the momentum equation applied to the control volume between locations 1 and 4 becomes

$$F_x = \dot{m}(V_4 - V_1)$$

The relation between the velocities for a propeller (Example 10.13) is

$$V_2 = V_3 = \frac{V_1 + V_4}{2}$$

When the boat is docked, the velocity V_1 is zero and the velocity V_4 is 40 m/s. Thus

$$V_2 = V_3 = \frac{0 + 40\frac{m}{s}}{2} = 20\frac{m}{s}$$

From the continuity equation, the flow rate at locations 2 and 3 is given by

$$Q = A_2V_2 = \frac{\pi}{4}D^2V_2$$

Or the diameter is

$$D = \sqrt{\frac{4}{\pi}\frac{Q}{V_2}} = \sqrt{\frac{4}{\pi} \times \frac{50\,\frac{m^3}{s}}{20\,\frac{m}{s}}} = 1.78\ m$$

The mass flow rate is

$$\dot{m} = \rho A_2 V_2 = 1.2\,\frac{kg}{m^3} \times \frac{\pi}{4}(1.78m)^2 \times 20\,\frac{m}{s} = 60.2\,\frac{kg}{s}$$

From the momentum equation, the thrust at rest is

$$F_x = \dot{m}(V_4 - V_1) = 60.2\,\frac{kg}{s}\left(40\,\frac{m}{s} - 0\right) = 2410\ N$$

When the boat is moving at 15 m/s, the velocity V_1 is 15 m/s and the velocity V_4 is

$$V_4 = 45\,\frac{m}{s} - V_1 = 45\,\frac{m}{s} - 15\,\frac{m}{s} = 30\,\frac{m}{s}$$

The thrust is then

$$F_x = \dot{m}(V_4 - V_1) = 60.2\,\frac{kg}{s}\left(30\,\frac{m}{s} - 15\right) = 903\ N$$

The maximum speed of the boat is when the thrust is zero, or

$$F_x = \dot{m}(V_4 - V_1) = 0$$

Or

$$V_4 - V_1 = \left(45\,\frac{m}{s} - V_1\right) - V_1 = 0$$

Solving for the maximum speed V_1

$$V_1 = 22.5\,\frac{m}{s}$$

Problem 10.77

(Difficulty: 4)

10.77 The propeller for the Gossamer Condor human-powered aircraft has *D*=12 ft and rotates at *N* =107 rpm. The wing loading is 0:4 lbf/ft^2 of wing area, the drag is approximately 6 lbf at 12 mph, the total weight is 200 lbf, and the effective aspect ratio is 17. Estimate the dimensionless performance characteristics and efficiency of this propeller at cruise conditions. Assume the pilot expends 70 percent of maximum power at cruise.

Assumptions: Assume that the speed is steady and at constant altitude.

Solution: From the solution to Problem 9.174, minimum power to propel the aircraft occurs at $V = 10.7\ mph\ (16.0\ ft/s)$. Assume this is the cruise condition.

From the given data, at 12 mph (17.6 ft/s), $F_o = 6\ lbf$. The dynamic head is

$$\frac{1}{2}\rho V^2 = \frac{1}{2} \times 0.00238\ \frac{slug}{ft^3} \times \left(17.6\ \frac{ft}{s}\right)^2 \times \frac{lbf \cdot s^2}{slug \cdot ft} = 0.369\ \frac{lbf}{ft^2}$$

The lift coefficient is then

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{W}{gA} = \frac{W/A}{g} = \frac{0.4\ \frac{lbf}{ft^2}}{0.369\ \frac{lbf}{ft^2}} = 1.08$$

The drag coefficient is then

$$C_D = C_L \frac{F_D}{F_L} = 1.08 \times \frac{6\ lbf}{200\ lbf} = 0.0324$$

The drag coefficient at zero angle of attack is estimated as

$$C_{D,o} = C_D - C_{D,i} = C_D - \frac{C_L^2}{\pi ar} = 0.0324 - \frac{(1.08)^2}{\pi(17)} = 0.0106$$

For the propeller,

$$J = \frac{V}{nD} = 17.6\ \frac{ft}{s} \times \left(\frac{60}{107}\right) \frac{s}{rev} \times \frac{1}{12\ ft} = 0.834$$

$$C_F = \frac{F_D}{\rho n^2 D^4} = 6.0\ lbf \times \frac{ft^3}{0.00238\ slug} \times \left(\frac{60}{107}\ \frac{s}{rev}\right)^2 \times \frac{1}{(12.0\ ft)^4} \times \frac{slug \cdot ft}{lbf \cdot s^2} = 0.0382$$

The power requiement is

$$P = F_D V = 6\ lbf \cdot 17.6 \frac{ft}{s} = 105.6\ \frac{ft\ lbf}{s} = 0.19\ hp$$

The pilot expends

$$P_{in} = \frac{0.19\ hp}{0.7} = 0.274\ hp$$

The propeller power is related to the torque as

$$P_{prop} = \omega T$$

The torque is then

$$T = \frac{0.19\ hp \cdot \frac{550\ ft\ lbf}{hp}}{\frac{107}{60}\ \frac{rev}{s} \cdot \frac{2\pi\ rad}{rev}} = 9.3\ ft \cdot lbf$$

The torque coefficient is then

$$C_T = \frac{T}{\rho n^2 D^5} = 9.3\ ft \cdot lbf \times \frac{ft^3}{0.00238\ slug} \times \left(\frac{60}{107}\ \frac{s}{rev}\right)^2 \times \frac{1}{(12.0\ ft)^5} \times \frac{slug \cdot ft}{lbf \cdot s^2} = 0.00493$$

And

$$C_p = \frac{C_T}{n} = 0.00642 \times \frac{60}{107} = 0.0036$$

Problem 10.78

(Difficulty: 3)

10.78 A typical American multiblade farm windmill has D=7 ft and is designed to produce maximum power in winds with V=15 mph. Estimate the rate of water delivery, as a function of the height to which the water is pumped, for this windmill.

Assumptions: Assume that the wind is steady at 15 mph, and that it represents typical wind conditions. Assume the efficiency trends shown in Fig. 10.44 are representative.

Solution: use the definition of the power coefficient

$$C_p = \frac{P}{\frac{1}{2}\rho V^3 \pi R^2}$$

$$X = \frac{\omega R}{V}$$

From Fig. 10.44, $C_{p\,max} \approx 0.3$ at $X = 0.8$. $V = 15\ mph\ (22.0\ ft/s)$. Then the power developed is:

$$P = \frac{\pi}{2} \times 0.3 \times 0.00238\ \frac{slug}{ft^3} \times \left(22\ \frac{ft}{s}\right)^3 \times \left(\frac{7}{2}\ ft\right)^2 \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 0.266\ hp$$

Converting this mechanical power to pumping gives hydraulic power as:

$$P_h = \rho Q g h = \eta P_m$$

Thus

$$Qh = \frac{\eta P_m}{\rho g} = 0.7 \times 0.266\ hp \times \frac{ft^3}{62.4\ lbf} \times 550\ \frac{ft \cdot lbf}{hp \cdot s} \times 7.48\ \frac{gal}{ft^3} \times 60\ \frac{s}{min}$$

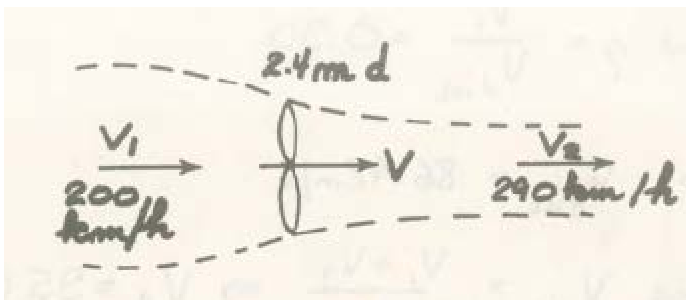
$$Qh = 737\ gpm \cdot ft$$

Q varis inversely with the distance lifted, h. The volume flow rate actually delivered would be less, due to suction lift, pipe friction, and minor losses.

Problem 10.79

(Difficulty 2)

10.79 An airplane flies at 200 $\frac{km}{h}$ through still air of specific weight 12 $\frac{N}{m^3}$. The propeller is 2.4 m in diameter, and its slipstream has a velocity of 290 $\frac{km}{h}$ relative to the fuselage. Calculate: (a) the propeller efficiency. (b) the velocity through the plane of the propeller. (c) the power input. (d) the power output. (e) the thrust of the propeller. (f) the pressure difference across the propeller disk.



Find The propeller characteristics

Assumptions: The flow through the propeller is steady and uniform

Solution: Use the continuity and momentum relations for flow through the propeller.

The thrust of propeller is given by applying the momentum equation to the propeller as a control volume:

$$F_P = \rho Q(V_2 - V_1) = \rho A_{disk} V_{disk}(V_2 - V_1)$$

The velocities V_1 and V_2 are

$$V_1 = 200\ \frac{km}{h} = 55.6\ \frac{m}{s}\ \ and\ \ V_2 = 290\ \frac{km}{h} = 80.6\ \frac{m}{s}$$

Using the Bernoulli equation, the velocity of the air through the propeller is (see Example 10.15):

$$V_{prop} = \frac{V_1 + V_2}{2} = 68.1\ \frac{m}{s}$$

The efficiency is calculated by:

$$\eta = \frac{V_1}{V_{prop}} = \frac{55.6\ \frac{m}{s}}{68.1\ \frac{m}{s}} = 81.6\%$$

The thrust of the propeller is:

$$F_P = \rho Q(V_2 - V_1) = \rho A_{prop} V_{prop}(V_2 - V_1)$$

$$F_P = \frac{12\ \frac{N}{m^3}}{9.81\ \frac{m}{s^2}} \times \frac{\pi}{4} \times (2.4\ m)^2 \times 68.1\ \frac{m}{s} \times \left(80.6\ \frac{m}{s} - 55.6\ \frac{m}{s}\right) = 9420\ N$$

The input power is:

$$P_{in} = F_P V_{prop} = 9420\ N \times 68.1\ \frac{m}{s} = 642\ kW$$

The output power is:

$$P_{out} = P_{in}\eta = 642\ kW \times 0.816 = 524\ kW$$

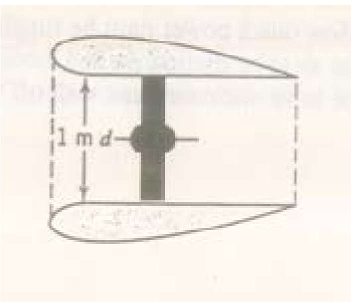
The pressure difference across the propeller disk is:

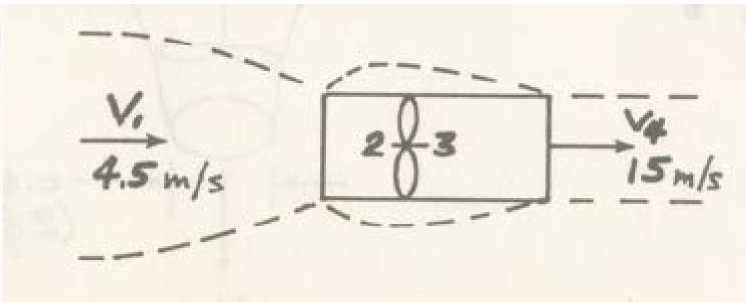
$$\Delta p = \frac{F_P}{A_{prop}} = \frac{9420\ N}{\frac{\pi}{4} \times (2.4\ m)^2} = 2080\ Pa$$

Problem 10.80

(Difficulty 2)

10.80 This ducted propeller unit drives a ship through still water at a speed of 4.5 $\frac{m}{s}$. Within the duct the mean velocity of the water (relative to the unit) is 15 $\frac{m}{s}$. Calculate the propulsive force produced by the unit. Calculate the force exerted on the fluid by the propeller. Account for the difference between these forces.





Find The force produced by the propeller and that on the fluid

Assumptions: The flow through the propeller is steady and uniform

Solution: Use the continuity, momentum, and Bernoulli relations for flow through the propeller.

Applying the Bernoulli equation from section 1 to section 2:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

Applying the Bernoulli equation from section 3 to section 4:

$$\frac{p_3}{\gamma} + \frac{V_3^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$$

Combing these two equations gives the pressure difference across the propeller as:

$$p_3 - p_2 = \frac{\rho}{2}(V_4^2 - V_1^2)$$

The pressure difference is then

$$p_3 - p_2 = \frac{1000\ \frac{kg}{m^3}}{2} \times \left(15^2\ - 4.5^2\right)\frac{m^2}{s^2} \times \frac{1\ N \cdot s^2}{kg \cdot m} = 102.4\ kPa$$

The force produced by the propeller is the pressure difference times the swept area:

$$F_P = (p_3 - p_2)A_d = 102.4\ kPa \times \frac{\pi}{4} \times 1\ m^2 = 80.4\ kN$$

The force produced by the entire ducted propeller is found from the momentum equation to be:

$$F_T = \rho A_d V_2 (V_4 - V_1) = 1000\ \frac{kg}{m^3} \times \frac{\pi}{4} \times 1\ m^2 \times 15 \frac{m}{s} \times (15 - 4.5) \frac{m}{s} \times \frac{1\ N \cdot s^2}{kg \cdot m} = 123.7\ kN$$

So the duct produces a force on the fluid equal to the difference between the total and that by the propeller:

$$F_D = F_T - F_P = 43.3\ kN$$

The design of the duct is important to the power of the engine.

Problem 10.81

[Difficulty: 2]

10.81 A model of an American multiblade farm windmill is to be built for display. The model, with $D = 1$ m, is to develop full power at $V = 10$ m/s wind speed. Calculate the angular speed of the model for optimum power generation. Estimate the power output.

Given: Model of farm windmill

Find: Angular speed for optimum power; Power output

Solution:

Basic equations: $C_P = \dfrac{P}{\frac{1}{2}\cdot\rho\cdot V^3\cdot\pi\cdot R^2}$ $\quad X = \dfrac{\omega\cdot R}{V}$ $\quad$ and we have $\quad \rho = 1.225\cdot\dfrac{kg}{m^3}$

From Fig. 10.45 $\quad C_{Pmax} = 0.3 \quad$ at $\quad X = 0.8 \quad$ and $\quad D = 1\cdot m \quad R = \dfrac{D}{2} \quad R = 0.5\,m$

Hence, for $\quad V = 10\cdot\dfrac{m}{s} \quad \omega = \dfrac{X\cdot V}{R} \quad \omega = 16\cdot\dfrac{rad}{s} \quad \omega = 153\cdot rpm$

Also $\quad P = C_{Pmax}\cdot\dfrac{1}{2}\cdot\rho\cdot V^3\cdot\pi\cdot R^2 \quad P = 144\,W$

Problem 10.82 [Difficulty: 2]

10.82 A large Darrieus vertical axis wind turbine was built by the U.S. Department of Energy near Sandia, New Mexico [48]. This machine is 18 m tall and has a 5-m radius; the area swept by the rotor is over 110 m^2. If the rotor is constrained to rotate at 70 rpm, plot the power this wind turbine can produce in kilowatts for wind speeds between 5 and 50 knots.

Given:
NASA-DOE wind turbine generator

Find:
Estimate rotor tip speed and power coefficient at maximum power condition

Solution:

Basic equations: $C_P = \dfrac{P_m}{\frac{1}{2}\cdot\rho\cdot V^3\cdot\pi\cdot R^2}$ $\quad X = \dfrac{\omega\cdot R}{V}$ $\quad U = \omega\cdot R$ $\quad \eta = \dfrac{P_m}{P_{ideal}}$

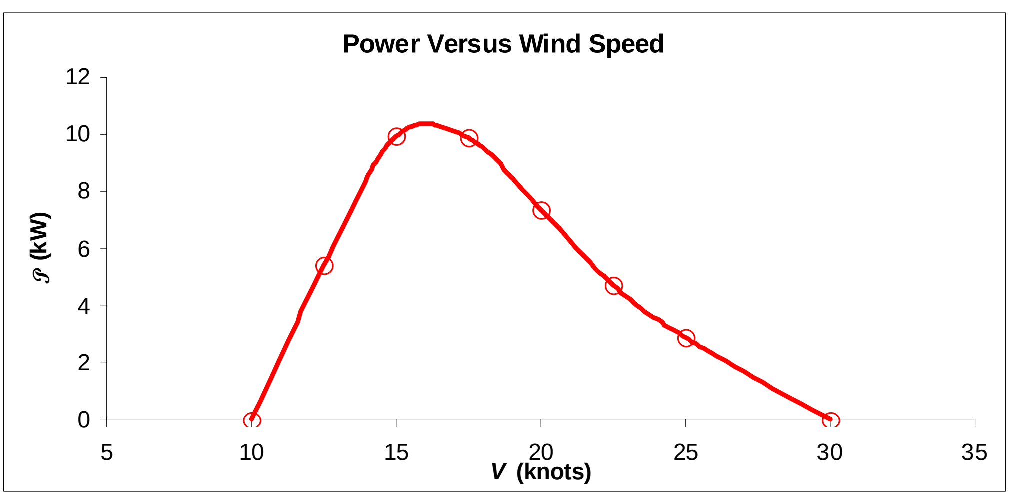
and we have $\rho = 1.23\cdot\dfrac{kg}{m^3}$ $\quad \omega = 70\cdot rpm$ $\quad R = 5\cdot m$ $\quad H = 18\cdot m$ $\quad A = 110\cdot m^2$ $\quad U = \omega\cdot R = 36.652\dfrac{m}{s}$

From Fig. 10.45: $C_P = 0.34$ when $X = 5.3$ (maximum power condition) If we replace the $\pi\cdot R^2$ term in the power coefficient with the swept area we will get: $P = \dfrac{1}{2}\cdot C_P\cdot\rho\cdot V^3\cdot A$

Here are the results, calculated using *Excel*:

A = 110.00 m^2 Power coefficient data were taken from Fig. 10.45
ρ = 1.23 kg/m^3
U = 36.65 m/s

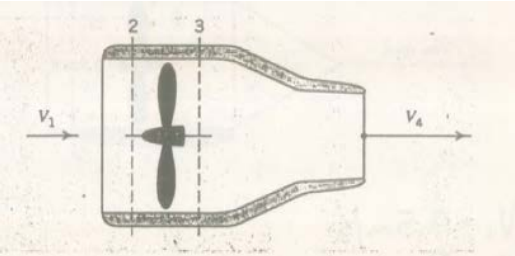
V (kt)	V (m/s)	X	C_P	$\mathcal{P}$ (kW)
10.0	5.14	7.125	0.00	0.00
12.5	6.43	5.700	0.30	5.40
15.0	7.72	4.750	0.32	9.95
17.5	9.00	4.071	0.20	9.87
20.0	10.29	3.562	0.10	7.37
22.5	11.57	3.167	0.05	4.72
25.0	12.86	2.850	0.02	2.88
30.0	15.43	2.375	0.00	0.00



Problem 10.83

(Difficulty 3)

10.83 Show that this ducted propeller system when moving forward at velocity V_1 will have an efficiency given by $\frac{2V_1}{(V_4+V_1)}$. If for a specific design and point of operation, $\frac{V_2}{V_1}=\frac{9}{4}$ and $\frac{V_4}{V_2}=\frac{5}{4}$, what fraction of the propulsive force will be contributed: (a) by the propeller, and (b) by the duct?



Find The efficiency produced by the propeller system and the forces produced by the propeller and the duct.

Assumptions: The flow through the propeller is steady and uniform

Solution: Use the continuity, momentum, and Bernoulli relations for flow through the propeller.

For the propeller we have the input power as the product of the force and velocity:

$$\dot{W}_{in} = F_T V = \frac{\rho Q}{2}(V_4^2 - V_1^2)$$

The output power is obtained from the momentum equation, assuming that the pressures at the inlet plane and the outlet plane are equal, and equal to the local fluid pressure.

$$\dot{W}_{out} = \rho Q V_1 (V_4 - V_1)$$

The efficiency is the ratio of output to input power:

$$\eta = \frac{\dot{W}_{out}}{\dot{W}_{in}} = \frac{\rho Q V_1 (V_4 - V_1)}{\frac{\rho Q}{2}(V_4^2 - V_1^2)} = \frac{2V_1(V_4 - V_1)}{(V_4 + V_1)(V_4 - V_1)} = \frac{2V_1}{(V_4 + V_1)}$$

To find the ratio of the forces produced by the propeller and the system, we first apply the Bernoulli equation from 1 to 2 and 3 to 4:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\frac{p_3}{\gamma}+\frac{V_3^2}{2g}=\frac{p_4}{\gamma}+\frac{V_4^2}{2g}$$

The velocities V_2 and V_3 are equal and the pressures p_1 and p_4 are equal. The pressure difference across the propeller is then:

$$p_3-p_2=\frac{\rho}{2}(V_4^2-V_1^2)$$

The force produced by the propeller is found from the momentum equation, as V_2 and V_3 are equal:

$$F_{prop}=(p_3-p_2)A_2=\frac{\rho}{2}(V_4^2-V_1^2)A_2$$

And that for the entire system is also from the momentum equation, as p_1 and p_4 are equal:

$$F_{sys}=\rho Q(V_4-V_1)$$

So we have for the ratio of the propeller force to the system force:

$$\frac{F_{prop}}{F_{sys}}=\frac{\frac{\rho}{2}(V_4^2-V_1^2)A_2}{\rho V_2A_2(V_4-V_1)}=\frac{(V_4+V_1)(V_4-V_1)}{2V_2(V_4-V_1)}=\frac{1}{2}\left(\frac{V_4}{V_2}+\frac{V_1}{V_2}\right)$$

For the specific situation we have:

$$\frac{V_2}{V_1}=\frac{9}{4}\quad and\quad \frac{V_4}{V_2}=\frac{5}{4}$$

$$\frac{F_{prop}}{F_{sys}}=\frac{1}{2}\left(\frac{V_4}{V_2}+\frac{V_1}{V_2}\right)=\frac{1}{2}\left(\frac{5}{4}+\frac{4}{9}\right)=0.847=84.7\%$$

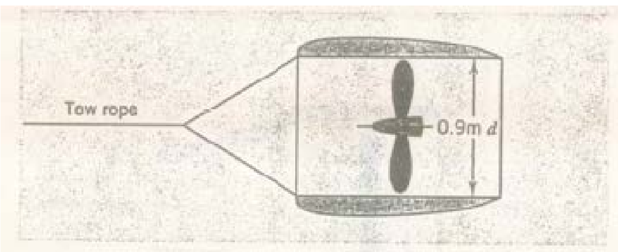
The ratio of the duct power to the system power is then the differenc

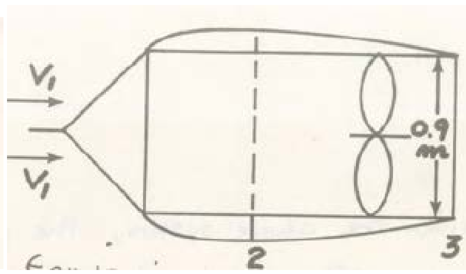
$$\frac{F_{duct}}{F_{sys}}=1-\frac{F_{prop}}{F_{sys}}=0.153=15.3\%$$

Problem 10.84

(Difficulty 2)

10.84 This ducted propeller unit (now operating as a turbine) is towed through still water at a speed of $7.5\ \frac{m}{s}$. Calculate the maximum power that the propeller can develop. Neglect all friction effects.





Solution:

We have the following:

$$V_1 = 7.5\ \frac{m}{s}$$

$$p_1 = p_3$$

$$D_2 = D_3 = 0.9\ m$$

Apply the Bernoulli equation from 1 to 2 we have:

$$p_1 + \frac{\rho V_1^2}{2} = p_2 + \frac{\rho V_2^2}{2}$$

$$p_2 - p_1 = \frac{\rho}{2}(V_1^2 - V_2^2)$$

Thus

$$p_3 - p_2 = \frac{\rho}{2}(V_2^2 - V_1^2)$$

The power produced by the unit is:

$$p = -(p_3 - p_2)AV_2 = -\frac{\rho}{2}(V_2^2 - V_1^2)AV_2$$

For the maximum power we have:

$$\frac{dp}{dV_2} = -\frac{\rho}{2}A(3V_2^2 - V_1^2) = 0$$

$$V_2 = \sqrt{\frac{V_1^2}{3}} = \sqrt{\frac{\left(7.5\ \frac{m}{s}\right)^2}{3}} = 4.33\ \frac{m}{s}$$

$$A = \frac{\pi}{4} D_2^2 = 0.636\ m^2$$

So the maximum power can be calculated as:

$$p = -\frac{\rho}{2}(V_2^2 - V_1^2)AV_2 = -\frac{998\ \frac{kg}{m^3}}{2} \times \left(\left(4.33\ \frac{m}{s}\right)^2 - \left(7.5\ \frac{m}{s}\right)^2\right) \times 0.636\ m^2 \times 4.33\ \frac{m}{s} = 51.5\ kW$$

Problem 10.85

(Difficulty: 5)

10.85 Aluminum extrusions, patterned after NACA symmetric airfoil sections, frequently are used to form Darrieus wind turbine "blades." Below are section lift and drag coefficient data [57] for a NACA 0012 section, tested at $Re = 6 \times 10^6$ with standard roughness (the section stalled for $\alpha > 12°$):

Angle of attack, α (deg)	0	2	4	6	8	10	12
Lift coefficient, C_L (—)	0	0.23	0.45	0.68	0.82	0.94	1.02
Drag coefficient, C_D (—)	0.0098	0.0100	0.0119	0.0147	0.0194	—	—

Analyze the air flow relative to a blade element of a Darrieus wind turbine rotating about its troposkien axis. Develop a numerical model for the blade element. Calculate the power coefficient developed by the blade element as a function of tip-speed ratio. Compare your result with the general trend of power output for Darrieus rotors shown in Fig. 10.49.

Solution: Use the relations for lift and drag on an airfoil

$$F_L = C_L \frac{1}{2} \rho V_r^2 A_p$$

$$V_r = relative\ velocity$$

$$F_D = C_D \frac{1}{2} \rho V_r^2 A_p$$

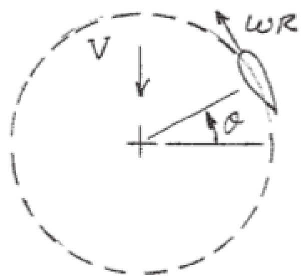
$$A_p = planform$$

$$A_s = swept$$

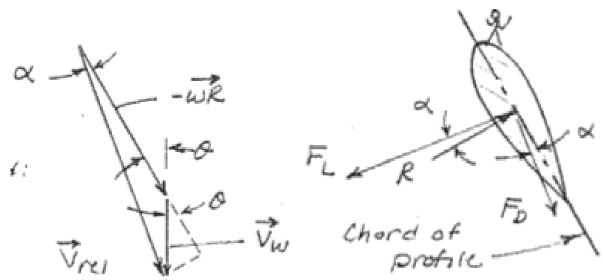
$$C_p = \frac{P}{\frac{1}{2} \rho V^3 A_s}$$

$$V = wind\ velocity$$

Consider plan view of rotor element showing absolute velocities:



Resolve to relative velocity for the positioin shown:



$$\bar{V}_{abs} = \bar{V}_{blade} + \bar{V}_{rel}$$

$$\bar{V}_{rel} = \bar{V}_{abs} - \bar{V}_{blade}$$

To compare $\bar{V}_{rel}$ resolve into components along (a) and transverse (t) to the airfoil chord:

$$\bar{V}_{rel)\,a} = \omega R + V_\omega \cos\theta$$

$$\bar{V}_{rel)\,t} = V_\omega \sin\theta$$

$$\bar{V}_{rel} = \left[\bar{V}_{rel)\,a}^2 + \bar{V}_{rel)\,t}^2\right]^{\frac{1}{2}}$$

$$\alpha = \tan^{-1}\left[\frac{\bar{V}_{rel)\,t}}{\bar{V}_{rel)\,a}}\right]$$

Lift force (F_L) is normal to $\bar{V}_{rel}$ and drag force F_D is parallel to $\bar{V}_{rel}$. Thus

$$T = R(F_L \sin\alpha - F_D \cos\alpha)$$

(torque, $T > 0$ when $F_L/F_D > \cot\alpha$).

Both C_L and C_D must be modeled as a functions of angle of attack, α. Curve fitting the relations from a graph of C_L and C_D verses α, as in Figure 9.17a, a satisfactory representation is:

$C_L = 0.12\alpha - 0.002b|\alpha|\alpha$, $-12 < \alpha < 12$ degrees.

$C_L = 0$, $|\alpha| > 12\ degrees$.

$C_D = 0.00952 + 1.52 \times 10^{-4}\ \alpha^2$, $-12 < \alpha < 12$ degrees.

$C_D = 0.0314$, $|\alpha| > 12\ degrees$.

(The model is in the stalled region, $|\alpha| > 12\ degrees$).

Sample calculation:

Choose $R = 10\ ft$, $C = 0.5\ ft$, $w = 1\ ft$, $X = 5$, $V_w = 20\ mph$.

At $\theta = 30°$, with $V_w = 20\ mph$ ($29.3\ \ ft/s$):

$$X = \frac{\omega R}{V_w}$$

$$\omega = \frac{XV_w}{R} = 5 \times 29.3\ \frac{ft}{s} \times \frac{1}{10\ ft} = 14.7\ \frac{rad}{s}\ \ (N = 140\ rpm)$$

$$\omega R = 14.7\ \frac{rad}{s} \times 10\ ft = 147\ \frac{ft}{s}$$

$$\bar{V}_{rel)\ a} = \omega R + V_w \cos\theta = 147 + 29.3\cos 30° = 172\ \frac{ft}{s}$$

$$\bar{V}_{rel)\ t} = V_\omega \sin\theta = 29.3\sin\theta = 14.7\ \frac{ft}{s}$$

$$\bar{V}_{rel} = \left[\bar{V}^2_{rel)\ a} + \bar{V}^2_{rel)\ t}\right]^{\frac{1}{2}} = \left[(172\ \frac{ft}{s})^2 + (14.7\ \frac{ft}{s})^2\right]^{\frac{1}{2}} = 173\ \frac{ft}{s}$$

$$\alpha = \tan^{-1}\left[\frac{\bar{V}_{rel)\ t}}{\bar{V}_{rel)\ a}}\right] = \tan^{-1}\left(\frac{14.7}{172}\right) = 4.88\ degrees$$

$$q = \frac{1}{2}\rho\bar{V}^2_{rel} = \frac{1}{2} \times 0.00238\ \frac{slug}{ft^3} \times \left(173\ \frac{ft}{s}\right)^2 \times \frac{lbf \cdot s^2}{slug \cdot ft} = 35.6\ \frac{lbf}{ft^2}$$

A_p (projected area of airfoil sections)$= c\omega = 0.5\ ft \times 1\ ft = 0.5\ ft^2$.

$$C_L = 0.12\alpha - 0.0026|\alpha|\alpha = 0.12 \times 4.88 - 0.0026|4.88|4.88 = 0.524$$

$$C_D = 0.00952 + 1.52 \times 10^{-4}\alpha^2 = 0.00952 + 1.52 \times 10^{-4}(4.88)^2 = 0.0131$$

$$F_L = C_L q A_p = 0.524 \times 35.6\ \frac{lbf}{ft^2} \times 0.5\ ft^2 = 9.33\ lbf$$

$$F_D = C_D q A_p = 0.0131 \times 35.6\ \frac{lbf}{ft^2} \times 0.5\ ft^2 = 0.233\ lbf$$

$$\frac{F_L}{F_D} = 40.0$$

$$T = R(F_L \sin\alpha - F_D \cos\alpha) = 10\ ft(9.33 \sin(4.88°) - 0.233 \cos(4.88°))lbf = 5.62\ lbf \cdot ft$$

$$P = \omega T = 14.7\ \frac{rad}{s} \times 5.62\ lbf \cdot ft = 82.6\ \frac{lbf \cdot ft}{s}\ (0.150\ hp)$$

$$C_p = \frac{P}{\frac{1}{2}\rho V_w^3 A_s}$$

$A_s = area$ swept by coefficient$= 2R\omega = 2 \times 10\ ft \times 1\ ft = 20\ ft^2$.

$$C_p = 82.6\ \frac{lbf \cdot ft}{s} \times \frac{ft^3}{\frac{1}{2} \times 0.00238\ slug} \times \frac{1}{\left(29.3\ \frac{ft}{s}\right)^3} \times \frac{1}{20\ ft^2} \times \frac{slug \cdot ft}{lbf \cdot s^2} = 0.138\ (at\ \theta = 15°)$$

Obtain $\bar{C}_p$ for a complete rotor revolution by integrating numerically. Such results are presented on the next page, and plotted versus tip speed ratio, $X = \omega R / V_w$.

From the plot, $\bar{C}_p$ is small at low X. It increases as X is raised, then peaks and decreases again. Comparison with Fig. 10.45 shows the trends are similar, but the model predicts useful power at larger X than observed experimentally. Blade elements at smaller radius on the rotor would produce less power, since $\omega = constant$ along rotor. $\bar{C}_p$ at large X is also sensitive to C_D. Low $\bar{C}_p$ at small X occurs becuases the airfoil is stalled.

Computed results: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*                                                       *
```


Airfoil: NACA 0012 Section; Chord, c = 6 in.

Blade element: Span, w = 1 ft; Radius, R = 10 ft

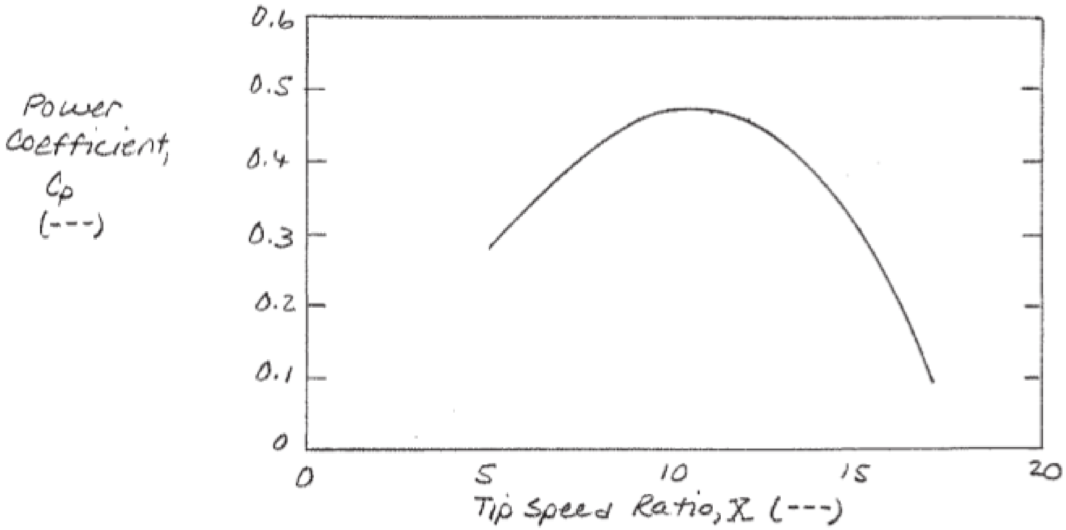
Input data: Tip speed ratio, X = 5.0 (---)
Wind speed, Vw = 20 mph (29.3 ft/sec)

Calculated: Rotor speed, omega = 140.4 rpm

theta (deg)	Vrel (ft/s)	alpha (deg)	Cl (--)	Cd (--)	Fl (lbf)	Fd (lbf)	T (ft-lbf)	Cp (--)
0	176	0.00	0.00	0.010	0.0	0.176	-1.8	-0.043
30	173	4.87	0.52	0.013	9.3	0.233	5.6	0.136
60	164	8.95	0.87	0.022	13.8	0.343	18.1	0.439
90	150	11.31	1.02	0.029	13.7	0.384	23.1	0.562
120	135	10.89	1.00	0.027	10.8	0.295	17.5	0.425
150	122	6.90	0.70	0.017	6.3	0.148	6.1	0.147
180	118	-0.00	-0.00	0.010	-0.0	0.078	-0.8	-0.019
210	122	-6.90	-0.70	0.017	-6.3	0.148	6.1	0.147
240	135	-10.89	-1.00	0.027	-10.8	0.295	17.5	0.425
270	150	-11.31	-1.02	0.029	-13.7	0.384	23.1	0.562
300	164	-8.95	-0.87	0.022	-13.8	0.343	18.1	0.439
330	173	-4.87	-0.52	0.013	-9.3	0.233	5.6	0.136
360	176	0.00	0.00	0.010	0.0	0.176	-1.8	-0.043

Average power coefficient for complete revolution: Cp,bar = 0.280

Plotting results of similar calculations at various tip speed ratios give:



Problem 10.86

(Difficulty 1)

10.86 What is the maximum power that can be expected from a windmill 30 m in diameter in a wind of 50 km/h? Assume air density 1:225 kg/m^3.

Find: The maximum power of a windmill

Assumptions: The wind is steady

Solution: Determine the power available through the wind to the windmill and use the maximum efficiency.

The velocity 50 km/hr corresponds to 13.9 m/s. We will first find the mass flow through the swept area of the windmill:

$$\dot{m} = \rho A V = 1.225\frac{kg}{m^3}\cdot\frac{\pi}{4}\cdot(30\ m)^2\cdot 13.9\frac{m}{s} = 12040\ \frac{kg}{s}$$

The available power of the wind is the kinetic energy of the wind

$$\dot{W}_{available} = \dot{m}\frac{V^2}{2} = 12040\ \frac{kg}{s}\cdot\frac{\left(13.9\frac{m}{s}\right)^2}{2} = 1160\ kW$$

The maximum efficiency of a windmill is 0.593, so the maximum power is

$$\dot{W}_{max} = \eta \dot{W}_{available} = 0.593\cdot 1160kW = 690\ kW$$

Problem 10.87

(Difficulty 2)

10.87 If an ideal windmill is operating at best efficiency in a wind of 48 km/h, what is the velocity through the disk and at some distance behind the windmill? What is the thrust on this windmill, assuming a diameter of 60 m and an air density of 1:23 kg/m^3? What are the mean pressures just ahead of and directly behind the windmill disk?

Find: Velocity, thrust, and pressure for an ideal windmill

Assumptions: The flow through the windmill is steady and uniform.

Solution: Use the ideal relations for flow through a windmill. For the velocities, using the interference factor a, the velocities at the blade and downstream of the windmill are given by

$$V_{blade} = (1-a)V \ \ and \ \ V_{downstream} = (1-2a)V$$

For an ideal windmill, the interference factor is a = 1/3. The wind velocity of 48 km/hr corresponds to 13.3 m/s. We have then for the velocities

$$V_{blade} = \left(1-\frac{1}{3}\right)13.3\frac{m}{s} = 8.86\ \frac{m}{s} \ \ and \ \ V_{downstream} = \left(1-\frac{2}{3}\right)13.3\frac{m}{s} = 4.43\ \frac{m}{s}$$

The thrust is evaluated from the linear momentum relation, eq. 10.42

$$F_T = 2\pi R^2 \rho a(1-a) = \pi\frac{D^2}{2}\rho a(1-a)$$

With a = 1/3, we have for the thrust

$$F_T = \pi\cdot\frac{(60m)^2}{2}\cdot 1.23\frac{kg}{m^3}\cdot\frac{1}{3}\left(1-\frac{1}{3}\right) = 273\ kN$$

To determine the pressure on the upstream side of the blade, we apply the Bernoulli equation to the flow from the wind to the blade. In terms of gage pressures, we have

$$\frac{V^2}{2g} = \frac{p_{blade}}{\gamma} + \frac{V_{blade}^2}{2g}$$

Or

$$p_{blade} = \frac{\rho\left(V^2 - V_{blade}^2\right)}{2} = 1.23\frac{kg}{m^3}\frac{(13.3^2 - 8.86^2)}{2}\ \left(\frac{m}{s}\right)^2 = 60.5\ Pa$$

For the pressure on the downstream side of the blade, we make a momentum balance on the blade. The velocity of the air on the upstream and downstream side of the blades is equal and the momentum flows cancel out

$$p_{blade}A - p_{down}A - F_T = 0$$

Or

$$p_{down} = p_{blade} - \frac{F_T}{A} = 60.5\ Pa - \frac{273000N}{\pi\frac{60m^2}{4}} = -36.0\ Pa$$

Problem 10.88

[Difficulty: 2]

10.88 A prototype air compressor with a compression ratio of 7 is designed to take 8.9 kg/s air at 1 atmosphere and 20°C. The design point speed, power requirement, and efficiency are 600 rpm, 5.6 MW, and 80 percent, respectively. A 1:5-scale model of the prototype is built to help determine operability for the prototype. If the model takes in air at identical conditions to the prototype design point, what will the mass flow and power requirement be for operation at 80 percent efficiency?

Given: Prototype air compressor, 1/5 scale model to be built

Find: Mass flow rate and power requirements for operation at equivalent efficiency

Solution:

Basic equations:
$$\eta = f_1\left(\frac{M\cdot\sqrt{R\cdot T_{01}}}{p_{01}\cdot D^2}, \frac{\omega\cdot D}{c_{01}}\right) \qquad \frac{W_c}{\rho_{01}\cdot\omega^3\cdot D^5} = f_2\left(\frac{M\cdot\sqrt{R\cdot T_{01}}}{p_{01}\cdot D^2}, \frac{\omega\cdot D}{c_{01}}\right) \qquad \frac{D_m}{D_p} = \frac{1}{5}$$

Given data: $M_p = 8.9\cdot\frac{kg}{s}$ $\quad \omega_p = 600\cdot rpm$ $\quad W_{cp} = 5.6\cdot MW$

Since the efficiencies are the same for the prototype and the model, it follows that:

$$\frac{M_m\cdot\sqrt{R_m\cdot T_{01m}}}{p_{01m}\cdot D_m^2} = \frac{M_p\cdot\sqrt{R_p\cdot T_{01p}}}{p_{01p}\cdot D_p^2} \qquad \frac{\omega_m\cdot D_m}{c_{01m}} = \frac{\omega_p\cdot D_p}{c_{01p}} \qquad \frac{W_{cm}}{\rho_{01m}\cdot\omega_m^3\cdot D_m^5} = \frac{W_{cp}}{\rho_{01p}\cdot\omega_p^3\cdot D_p^5}$$

Given identical entrance conditions for model and prototype and since the working fluid for both is air:

$$\frac{M_m}{D_m^2} = \frac{M_p}{D_p^2}$$ Solving for the mass flow rate of the model: $$M_m = M_p\cdot\left(\frac{D_m}{D_p}\right)^2 \qquad M_m = 0.356\,\frac{kg}{s}$$

$$\omega_m\cdot D_m = \omega_p\cdot D_p$$ Solving for the speed of the model: $$\omega_m = \omega_p\cdot\frac{D_p}{D_m} = 3000\cdot rpm$$

$$\frac{W_{cm}}{\omega_m^3\cdot D_m^5} = \frac{W_{cp}}{\omega_p^3\cdot D_p^5}$$ Solving for the power requirement for the model: $$W_{cm} = W_{cp}\cdot\left(\frac{\omega_m}{\omega_p}\right)^3\cdot\left(\frac{D_m}{D_p}\right)^5$$

$$W_{cm} = 0.224\cdot MW$$

Problem 10.89

[Difficulty: 3]

10.89 A compressor has been designed for entrance conditions of 14.7 psia and 70°F. To economize on the power required, it is being tested with a throttle in the entry duct to reduce the entry pressure. The characteristic curve for its normal design speed of 3200 rpm is being obtained on a day when the ambient temperature is 58°F. At what speed should the compressor be run? At the point on the characteristic curve at which the mass flow would normally be 125 lbm/s, the entry pressure is 8.0 psia. Calculate the actual mass flow rate during the test.

Given: Prototype air compressor equipped with throttle to control entry pressure

Find: Speed and mass flow rate of compressor at off-design entrance conditions

Solution:

Basic equations: $\eta = f_1\left(\frac{M\cdot\sqrt{T_{01}}}{p_{01}}, \frac{\omega}{\sqrt{T_{01}}}\right)$ $\qquad$ $\frac{\Delta T_{01}}{T_{01}} = f_2\left(\frac{M\cdot\sqrt{T_{01}}}{p_{01}}, \frac{\omega}{\sqrt{T_{01}}}\right)$

Given data: $p_{01d} = 14.7\cdot psi$ $\quad T_{01d} = 70\,°F$ $\quad \omega_d = 3200\cdot rpm$ $\quad T_{01} = 58\,°F$ $\quad M_d = 125\cdot\frac{lbm}{s}$ $\quad p_{01} = 8.0\cdot psi$

Since the normalized speed is equal to that of the design point, it follows that: $\frac{\omega}{\sqrt{T_{01}}} = \frac{\omega_d}{\sqrt{T_{01d}}}$

Solving for the required speed: $\omega = \omega_d\cdot\sqrt{\frac{T_{01}}{T_{01d}}}$ $\qquad \omega = 3164\cdot rpm$

At similar conditions: $\frac{M\cdot\sqrt{T_{01}}}{p_{01}} = \frac{M_d\cdot\sqrt{T_{01d}}}{p_{01d}}$ Solving for the actual mass flow rate: $M = M_d\cdot\sqrt{\frac{T_{01d}}{T_{01}}}\cdot\frac{p_{01}}{p_{01d}}$ $\qquad M = 68.8\cdot\frac{lbm}{s}$

Problem 10.90 [Difficulty: 4]

10.90 We have seen many examples in Chapter 7 of replacing working fluids in order to more easily achieve similitude between models and prototypes. Describe the effects of testing an air compressor using helium as the working fluid on the dimensionless and dimensional parameters we have discussed for compressible flow machines.

Discussion: When we change the working fluid, we need to be sure that we use the correct similitude relationships. Specifically, we would need to keep fluid-specific parameters (gas constant and specific heat ratio) in the relationships. The functional relationships are:

$$\frac{\Delta h_{0s}}{(ND)^2}, \eta, \frac{P}{\rho_{01} N^3 D^5} = f_1\left(\frac{\dot{m}}{\rho_{01} N D^3}, \frac{\rho_{01} N D^2}{\mu}, \frac{ND}{c_{01}}, k\right)$$

So these dimensionless groups need to be considered. When we replace air with helium, both the gas constant *R* and the specific heat ratio *k* will increase. Given a fixed inflow pressure and temperature and a fixed geometry, the effect would be to decrease density and increase sound speed. Therefore, replacing air with helium should result in decreased mass flow rate and power, and an increased operating speed.

When considering dimensional parameters, the important thing to remember is that the operability maps for compressors and/or turbines were constructed for a single working fluid. Therefore, to be safe, an engineer should reconstruct an operability map for a new working fluid.