

# 重庆大学《Multivariable Calculus》课

☒ A卷  
☐ B卷

2019 — 2020 学年 第 1 学期

开课学院: 数统学院 课程号: MATH20083 考试日期: 2019 12 20

考试方式: ☐ 开卷 ☒ 闭卷 ☐ 其他 考试时间: 120 分钟

题号	一	二	三	四	五	六	七	八	九	十	总分
得分											

## 考试提示

1. 严禁随身携带通讯工具等电子设备参加考试;
2. 考试作弊, 留校察看, 毕业当年不授学位; 请人代考、替他人考试、两次及以上作弊等, 属严重作弊, 开除学籍。

一、(15pts.) Fill in the blanks with correct answers.

1.  $(1, 1, 1) \times (1, 1, 1) = \underline{\hspace{2cm}}$ .
2.  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+y^2}{|x|+|y|} = \underline{\hspace{2cm}}$ .
3. The area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $\underline{\hspace{2cm}}$ ,
4.  $\int_C \nabla f \cdot d\vec{r} = \underline{\hspace{2cm}}$ , where C is a simple closed path and f is a smooth function with 2 variables.
5.  $\text{Curl} \vec{F} = \underline{\hspace{2cm}}$ , where  $\vec{F}(x, y, z)$  is a conservative vector field with continuous second order partial derivatives.

二、(15pts.) Determine whether the following statements are true or false.

1.  $\int_{-C} f(x, y) ds = - \int_C f(x, y) ds$ . ( )
2. The two mixed second order partial derivatives for  $z=f(x, y)$  must equal if they are continuous. ( )
3. Suppose D is a 2-dimensional simple bounded plane region, then  $\iint_D 1 dA = \int_{\partial D} x dy = - \int_{\partial D} y dx$ . ( )
4. If all directional derivatives of  $f(x, y)$  at  $(x_0, y_0)$  exist, then  $f(x, y)$  must be continuous at this point. ( )
5. Given two non-zero vectors  $\alpha = (a_1, \dots, a_n), \beta = (b_1, \dots, b_n)$ , then they are orthogonal if and only if  $\sum_{i=1}^n a_i b_i = 0$ . ( )

三、(15pts.)

$$\text{Suppose } f(x, y) = \begin{cases} (x^2 + y^2) \cos \frac{1}{\sqrt{x^2 + y^2}} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases},$$

1) Is  $f(x, y)$  continuous at  $(0, 0)$ ?

2) Find  $f_x(0, 0)$  and  $f_y(0, 0)$

命题人: 李智强

组题人: 黄辉斥

审题人: 秦越石

命题时间: 2019 12 01

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3) Is  $f(x, y)$  differentiable at  $(0, 0)$ ?

四、(10pts.) Assume the straight line  $\begin{cases} x + y + b = 0 \\ x + ay - z - 3 = 0 \end{cases}$  lies in the plane  $\Pi$ , and this plane is tangent to  $z = x^2 + y^2$  at  $(1, -2, 5)$ , find  $a$  and  $b$ .

五、(10pts.) Find extreme values of  $z = z(x, y)$ , where  $z$  is given by the equation  $x^2 + y^2 + z^2 - 2x + 2y - 4z - 10 = 0$ .

六、(15pts.) Evaluate  $\iint_S 2x^3 dydz + 2y^3 dzdx + 3(z^2 - 1) dxdy$ , where  $S$  is the surface  $z = 1 - x^2 - y^2$  with upwards direction.

七、(10pts.) Suppose  $\vec{F}(x, y) = \frac{-y}{x^2 + y^2} \vec{i} + \frac{x}{x^2 + y^2} \vec{j} = P\vec{i} + Q\vec{j}$ . Prove that  $\int_C \vec{F} \cdot d\vec{r}$  is a constant for any simple closed smooth curve  $C$  which

encloses the origin, with counter clockwise direction.

八、(10pts.) Assume  $D = \{(x, y) \mid x^2 + y^2 \leq 4, x \geq 0, y \geq 0\}$ ,  $f(x)$  is a continuous positive function everywhere, and  $a, b$  are given constants, find the double integral  $\iint_D \frac{a\sqrt{f(x)} + b\sqrt{f(y)}}{\sqrt{f(x)} + \sqrt{f(y)}} dA$ , show your reasons.