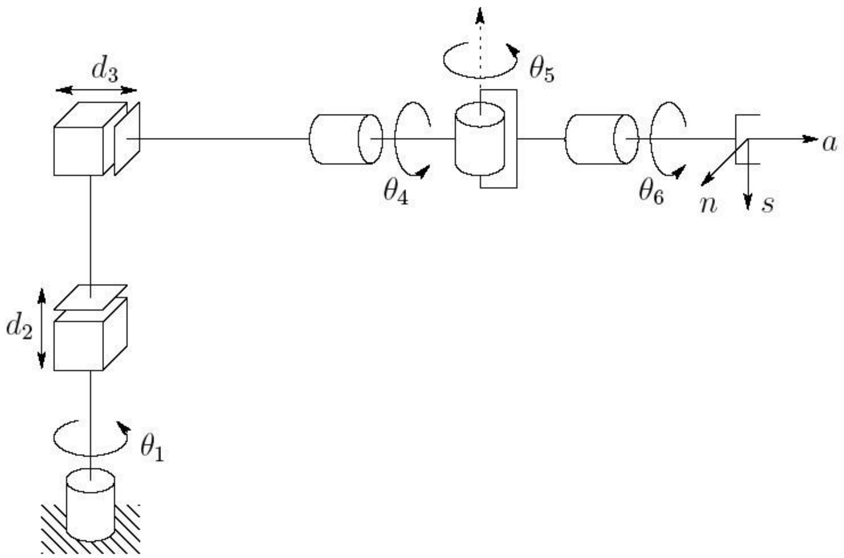
University of Cincinnati
College of Engineering and Applied Science

# 20 MECH5131 Intro to Robotics

## HW#9-Forward Kinematics-Position Analysis (80 pts)

## Solution keys (Courtesy of Tate Mitchell)

**Prob 1**. (12 pts)
For the given 6 DOF cylindrical robot below, assign appropriate frames for Joint 1 through 6 (assign *x* and *z* axis only, not *y* axis) based on the D-H representation.



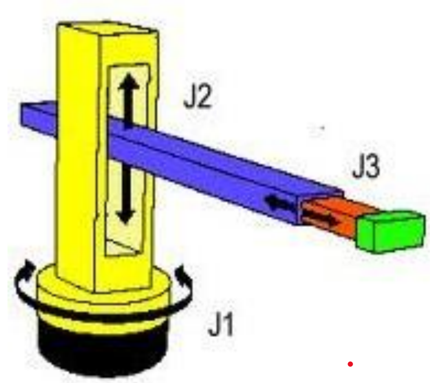
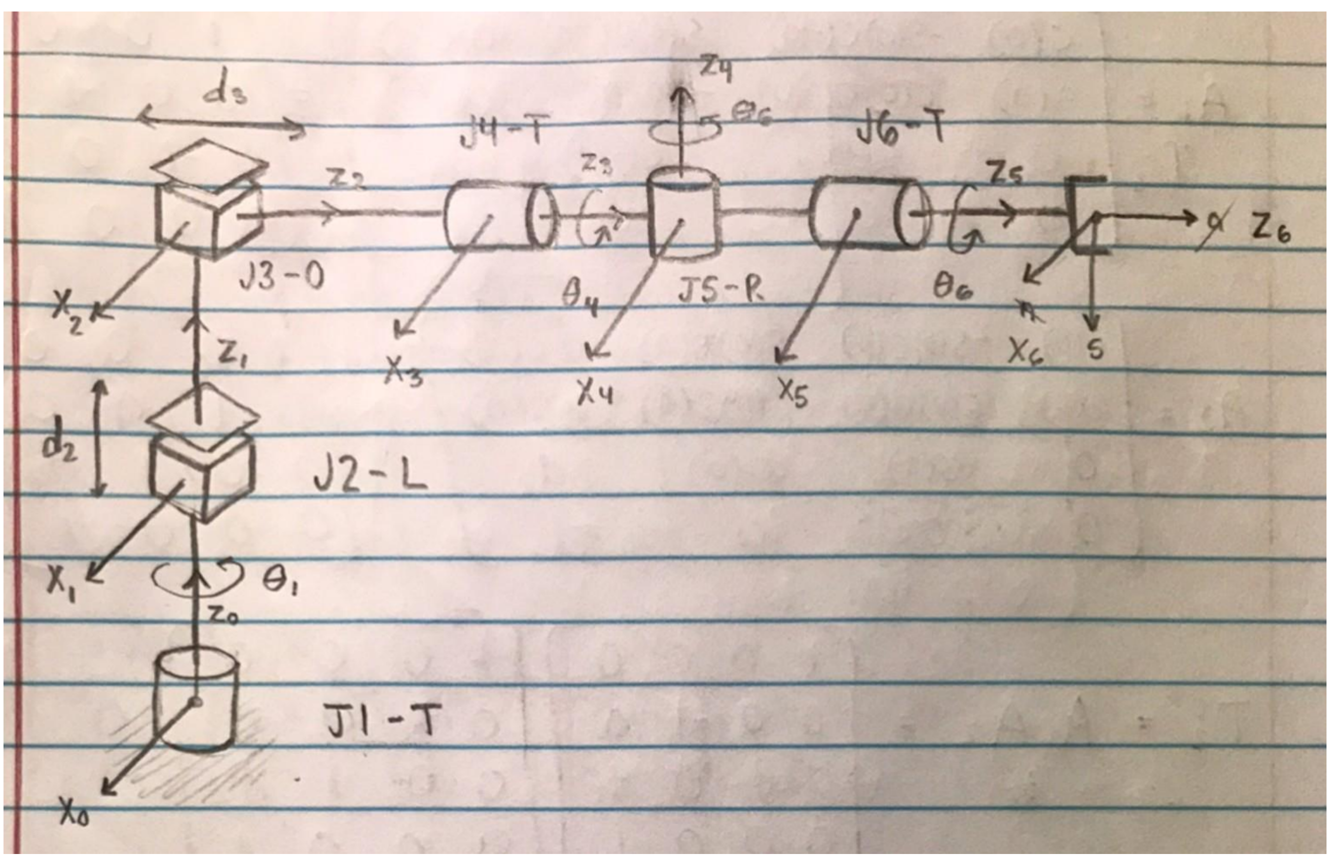



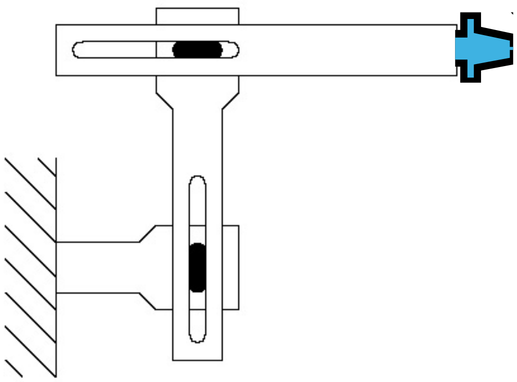

Figure 3.9: Cylindrical robot with spherical wrist.

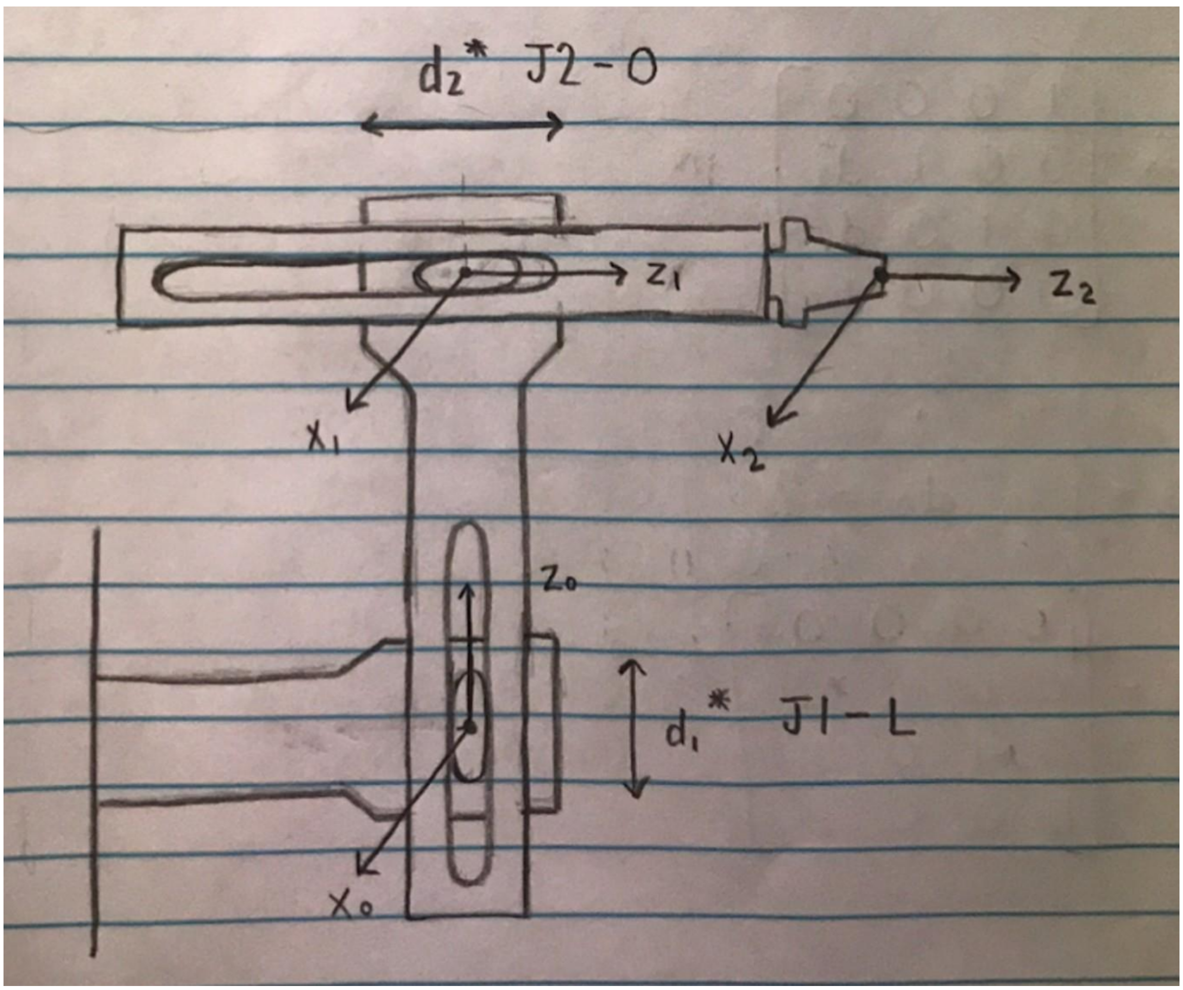


**Prob 2**. (24 pts)

Consider the two-link Cartesian manipulator of figure below,

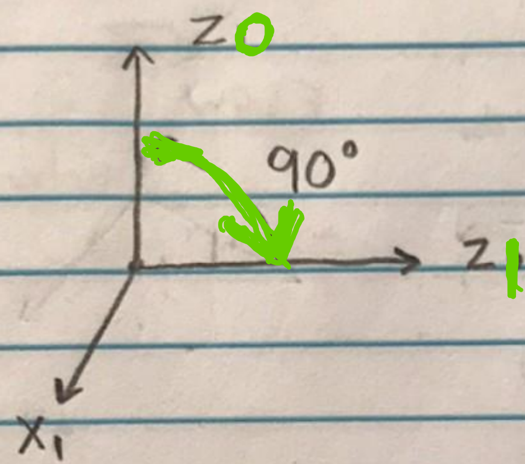
a) Assign the link frames for the two joints and the end effector. (6 pts)
b) Create and fill out D-H parameters table. (4 pts)
c) Find the homogenous transformation matrices (A1 and A2 matrices) for two joints. (6 pts)
d) Find the direct kinematic equation (T matrix). (3 pts)
e) Find the position of the end effector in the base (first) frame when $d_1=1\ ft$, $d_2=2\ ft$, and illustrate this position in the figure. (5 pts)





| # | $\theta$ | $d$ | $a$ | $\alpha$ |
|---|---|---|---|---|
| 0-1 | $\theta_1 = 0$ | $d_1^* =$ | $a_1 = 0$ | $\alpha_1 = -90$ |
| 1-2 | $\theta_2 = 0$ | $d_2^* =$ | $a_2 = 0$ | $\alpha_2 = 0$ |

$\alpha_1$:




$$A_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 C\alpha_1 & S\theta_1 S\alpha_1 & a_1 C\theta_1 \\ S\theta_1 & C\theta_1 C\alpha_1 & -C\theta_1 S\alpha_1 & a_1 S\theta_1 \\ 0 & S\alpha_1 & C\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 C\alpha_2 & S\theta_2 S\alpha_2 & a_2 C\theta_2 \\ S\theta_2 & C\theta_2 C\alpha_2 & -C\theta_2 S\alpha_2 & a_2 S\theta_2 \\ 0 & S\alpha_2 & C\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} C(0) & -S(0)C(-90) & S(0)S(-90) & (0)C(0) \\ S(0) & C(0)C(-90) & -C(0)S(-90) & (0)S(0) \\ 0 & S(-90) & C(-90) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

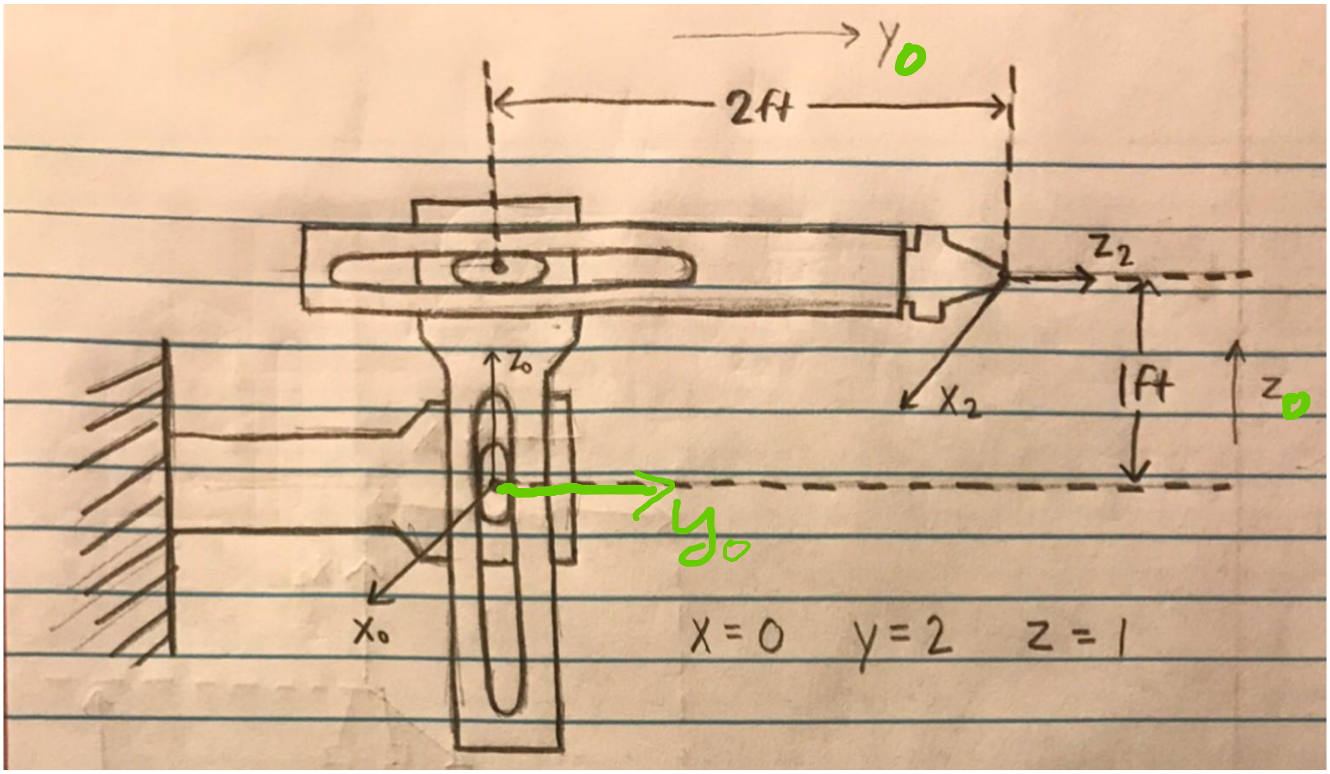
$$A_2 = \begin{bmatrix} C(0) & -S(0)C(0) & S(0)S(0) & 0\,C(0) \\ S(0) & C(0)C(0) & -C(0)S(0) & 0\,S(0) \\ 0 & S(0) & C(0) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$d_1 = 1 \quad d_2 = 2$

$$T_2^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$y_0$
2ft
$z_2$
$z_0$
$x_2$
1ft
$z_0$
$y_0$
$x_0$
$x=0 \quad y=2 \quad z=1$

**Prob 3**. (22 pts)

Consider the two-link manipulator of figure below, which has joint 1 linear and joint 2 revolute with link lengths $l_1$=6" and $l_2$=12".

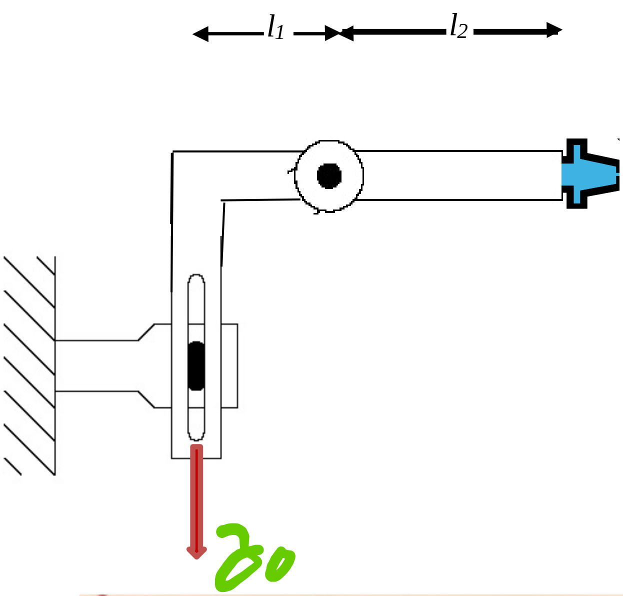
a) Assign the frames for two joints and end effector. ($z_0$ is assigned for you) (4 pts)

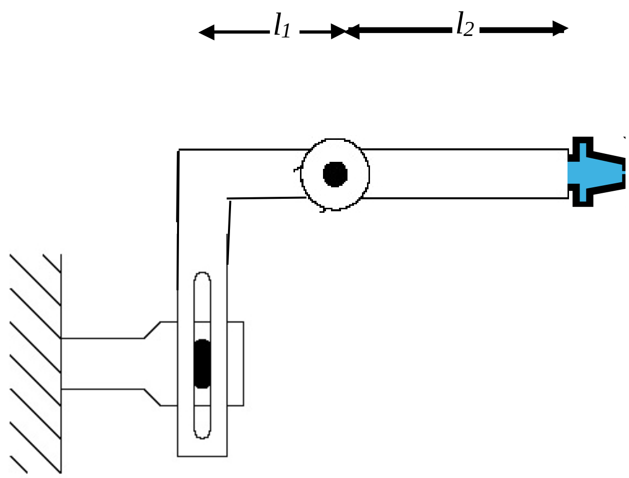
b) Create a D-H parameters table and fill out. (4 pts)

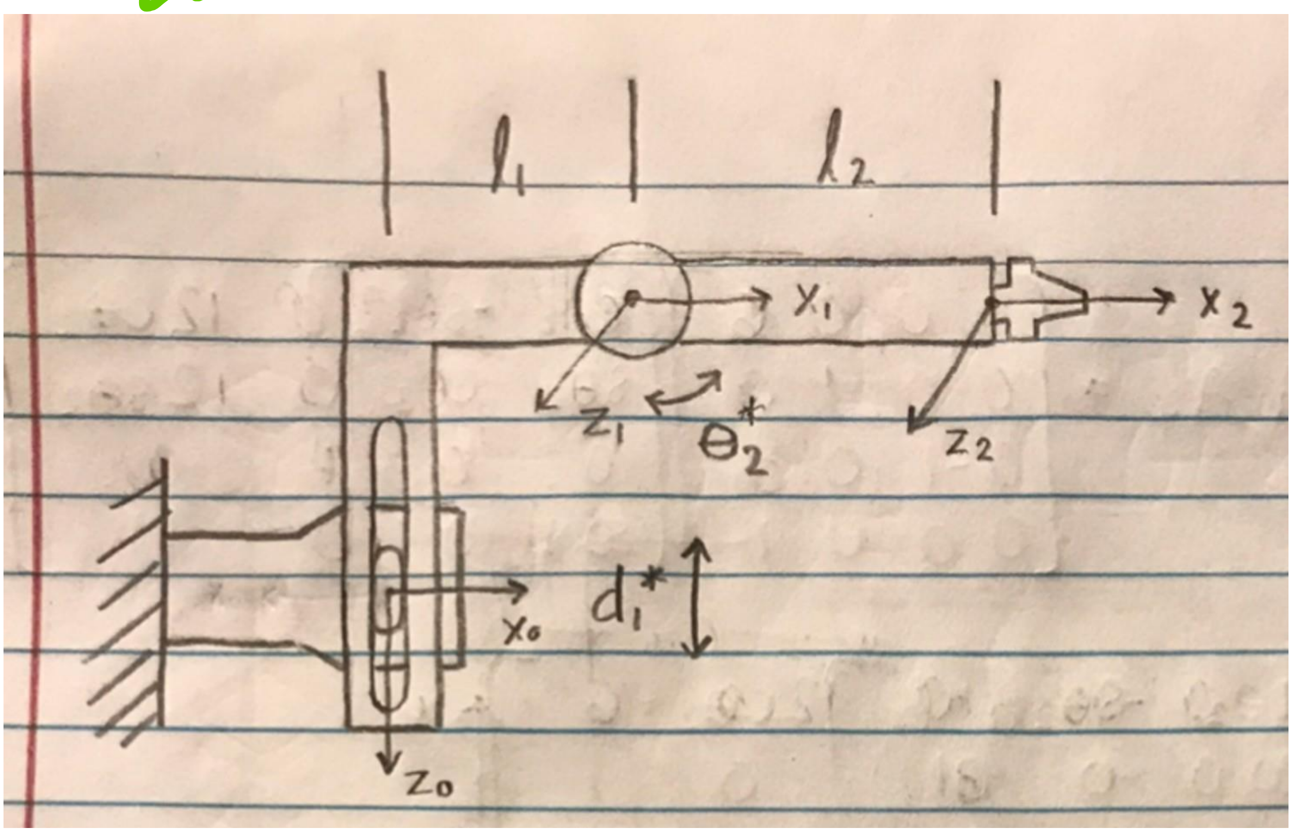
c) Find the homogenous transformation matrices ($A_1$ and $A_2$) for two joints. (6 pts)

d) Find the direct kinematic equation ($T$ matrix). (3 pts)

*e)* When $d_1$=6" and $\theta_2$= - 90°, find the location and orientation of end effector and illustrate them in the figure below to the right. (try to be on scale) (5 pts)

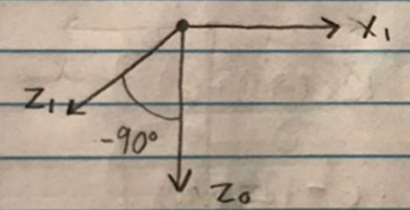


| # | $\theta$ | $d$ | $a$ | $\alpha$ |
|---|---|---|---|---|
| 0-1 | $\theta_1 = 0$ | $d_1^* =$ | $a_1 = l_1$ | $\alpha_1 = -90$ |
| 1-2 | $\theta_2^* =$ | $d_2 = 0$ | $a_2 = l_2$ | $\alpha_2 = 0$ |

$\alpha_1$ :




$$A_1 = \begin{bmatrix} C(0) & -S(0)C(-90) & S(0)S(-90) & l_1 C(0) \\ S(0) & C(0)C(-90) & -C(0)S(-90) & l_1 S(0) \\ 0 & S(-90) & C(-90) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 C(0) & S\theta_2 S(0) & l_2 C\theta_2 \\ S\theta_2 & C\theta_2 C(0) & -C\theta_2 S(0) & l_2 S\theta_2 \\ 0 & S(0) & C(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & l_2 C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & l_2 S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$l_1 = 6''$ $\quad$ $l_2 = 12''$

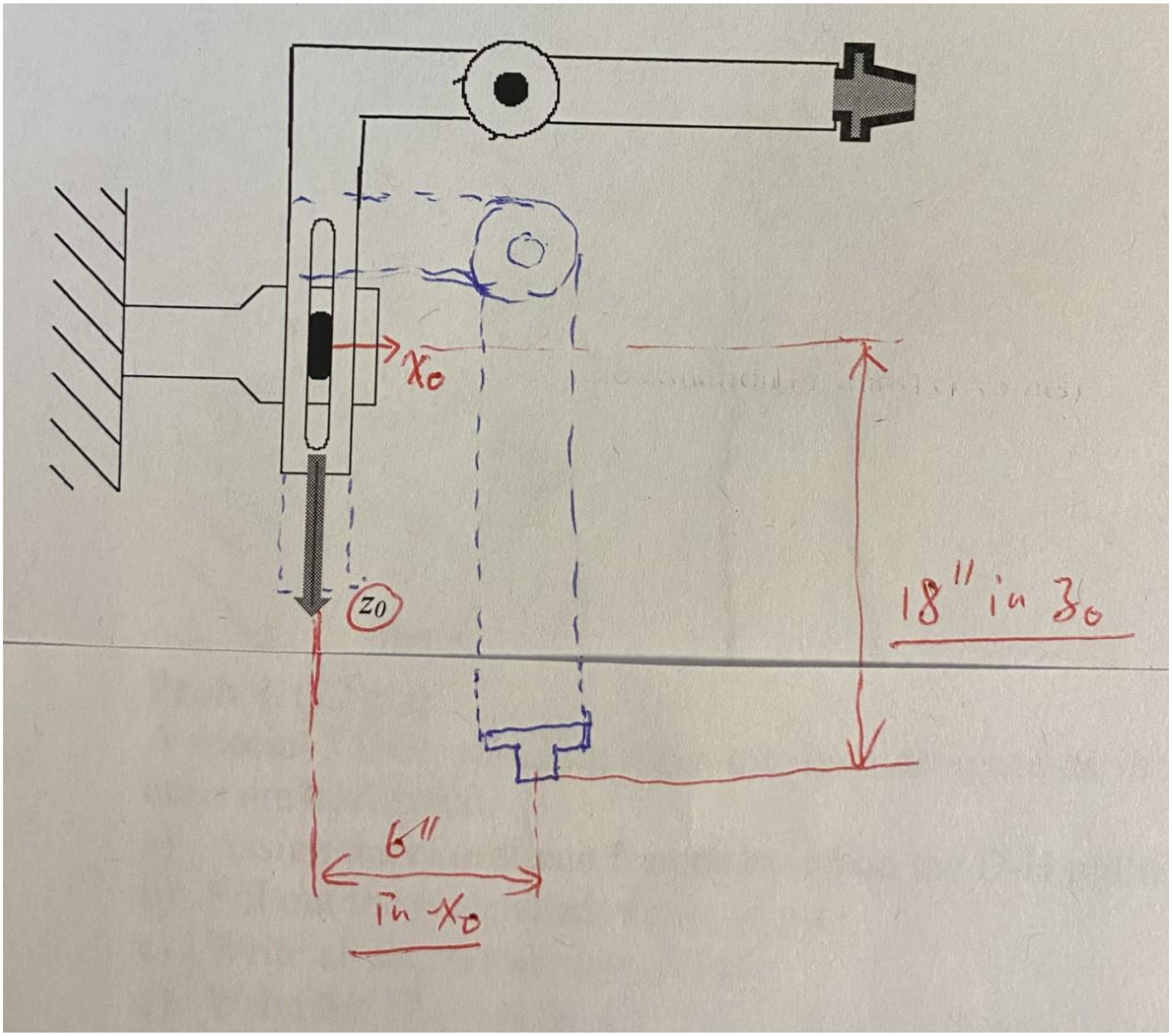
$$T_2^0 = A_1 A_2 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 12C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & 12S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 12C\theta_2 + 6 \\ 0 & 0 & 1 & 0 \\ -S\theta_2 & -C\theta_2 & 0 & d_1 - 12S\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d_1 = 6'' \qquad \theta_2 = -90^\circ$$

$$T_2^0 = \begin{bmatrix} C(-90) & -S(-90) & 0 & 12C(-90) + 6 \\ 0 & 0 & 1 & 0 \\ -S(-90) & -C(-90) & 0 & 6 - 12S(-90) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 18 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

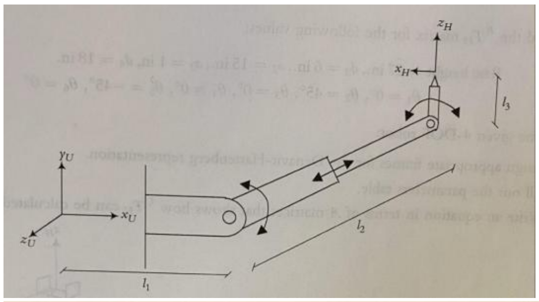


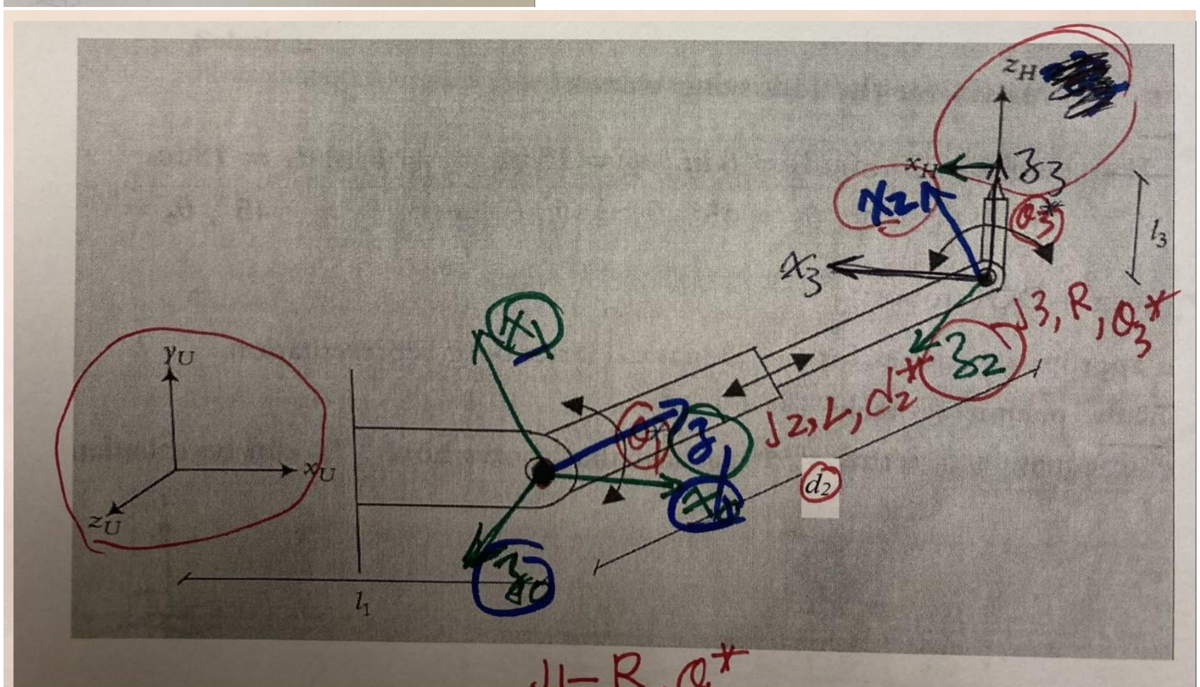
$x_0$
$z_0$
18" in $z_0$
6"
in $x_0$

**Prob 4**. (22 pts)

A special 3 DOF spraying robot has been designed as shown below, and the reset position is when the arms are horizontal.

a) Assign the coordinate frames based on the D-H representation. (8 pts)
b) Fill out the parameters table. (6 pts)
c) Write all the A matrices. (4 pts)
d) Write the ${}^{U}T_{H}$ matrix in terms of the A matrices. (4 pts)




| # | $\theta$ | d | a | $\alpha$ |
|---|---|---|---|---|
| 0-1 | $\theta_1^* =$ | $d_1 = 0$ | $a_1 = 0$ | $\alpha_1 = 90$ |
| 1-2 | $\theta_2 = 0$ | $d_2^* =$ | $a_2 = 0$ | $\alpha_2 = -90$ |
| 2-3 | $\theta_3^* =$ | $d_3 = 0$ | $a_3 = 0$ | $\alpha_3 = 90$ |
| 3-H | $\theta_H = 0$ | $d_H = l_3$ | $a_H = 0$ | $\alpha_H = 0$ |

$$A_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 C(90) & S\theta_1 S(90) & 0\,C\theta_1 \\ S\theta_1 & C\theta_1 C(90) & -C\theta_1 S(90) & 0\,S\theta_1 \\ 0 & S(90) & C(90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C(0) & -S(0)C(-90) & S(0)S(-90) & 0\,C(0) \\ S(0) & C(0)C(-90) & -C(0)S(-90) & 0\,S(0) \\ 0 & S(-90) & C(-90) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 C(90) & S\theta_3 S(90) & 0\,C\theta_3 \\ S\theta_3 & C\theta_3 C(90) & -C\theta_3 S(90) & 0\,S\theta_3 \\ 0 & S(90) & C(90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & 0 \\ S\theta_3 & 0 & -C\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C(0) & -S(0)C(0) & S(0)S(0) & 0\,C(0) \\ S(0) & C(0)C(0) & -C(0)S(0) & 0\,S(0) \\ 0 & S(0) & C(0) & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_H^0 = A_1 A_2 A_3 A_4$$

$$T_H^0 = \begin{bmatrix} C\theta_1 C\theta_3 - S\theta_1 S\theta_3 & 0 & C\theta_1 S\theta_3 + S\theta_1 C\theta_3 & d_2 S\theta_1 + l_3(C\theta_1 S\theta_3 + S\theta_1 C\theta_3) \\ S\theta_1 C\theta_3 + C\theta_1 S\theta_3 & 0 & S\theta_1 S\theta_3 - C\theta_1 C\theta_3 & l_3(S\theta_1 S\theta_3 - C\theta_1 C\theta_3) - d_2 C\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^U_O = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^U_H = T^U_O \times T^O_H$$

$$= \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot A_1 A_2 A_3 A_4$$

$$T^U_H = \begin{bmatrix} C\theta_1 C\theta_3 - S\theta_1 S\theta_3 & 0 & C\theta_1 S\theta_3 + S\theta_1 C\theta_3 \\ S\theta_1 C\theta_3 + C\theta_1 S\theta_3 & 0 & S\theta_1 S\theta_3 - C\theta_1 C\theta_3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow$$

$$\longrightarrow \begin{matrix} d_2 S\theta_1 + l_3 (C\theta_1 S\theta_3 + S\theta_1 C\theta_3) + l_1 \\ l_3 (S\theta_1 S\theta_3 - C\theta_1 C\theta_3) - d_2 C\theta_1 \\ 0 \\ 1 \end{matrix} \Bigg]$$