Chapter 7

PROBLEM 7.1

To measure the mass flow rate of a fluid in a laminar flow through a circular pipe, a hot wire type velocity meter is placed in the center of the pipe. Assuming that the measuring station is far from the entrance of the pipe, the velocity distribution is parabolic

$$\frac{u(r)}{U_{max}} = \left[1 - \frac{2r}{D}^{\,2}\right]$$

where U_{max} is the centerline velocity (r = 0), r is the radial distance from the pipe centerline, D is the pipe diameter.

(a) Derive an expression for the average fluid velocity at the cross-section in terms of U_{max} and D.

(b) Obtain an expression for the mass flow rate.

(c) If the fluid is mercury at 30°C, D = 10 cm, and the measured value of U_{max} is 0.2 cm/s, calculate the mass flow rate from the measurement.

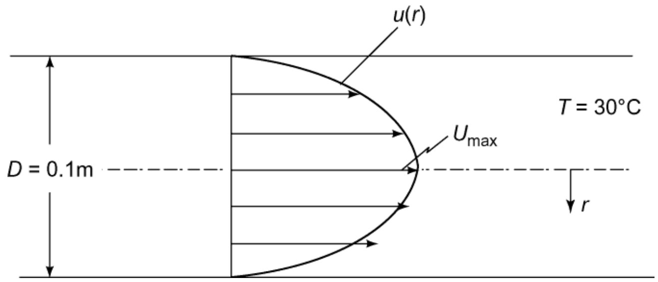
GIVEN

- Fully developed flow of mercury through a circular pipe
- Parabolic velocity distribution: $u(r)/U_{max} = 1 - (2r/D)^2$
- Mercury temperature (T) = 30°C
- Pipe diameter (D) = 10 cm = 0.1 m
- Measured center velocity (U_{max}) = 0.2 cm/s = 0.002 m/s

FIND

(a) An expression for the average fluid velocity ($\bar{u}$)
(b) An expression for the mass flow rate ($\dot{m}$)
(c) The value of the mass flow rate ($\dot{m}$)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 26, for Mercury at 30°C: Density (ρ) = 13,555 kg/m^3

SOLUTION

(a) The average fluid velocity is calculated as follows

$$\bar{u} = \frac{1}{r_o}\int_o^{r_o} u(r)dr \qquad \text{where } r_o = \frac{D}{2}$$

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$$\bar{u} = \frac{U_{max}}{r_o}\int_o^{r_o}\left[1-\left(\frac{r}{r_o}\right)^2\right]dr = \frac{U_{max}}{r_o}\left(r-\frac{1}{3}\frac{r^3}{r_o^2}\right)\Bigg|_0^{r_o}$$

$$\bar{u} = U_{max}\left(1-\frac{1}{3}\right) = \frac{2}{3}\ U_{max}$$

(b) The mass flow rate is given by

$$\dot{m} = \bar{u}\,A_c\,\rho = \frac{2}{3}\ U_{max}(\pi\, r_o^2)\rho$$

$$\dot{m} = \frac{2}{3}\,\pi\, U_{max}\, r_o^2\,\rho$$

(c) Inserting the values of these quantities into this expression

$$\dot{m} = \frac{2}{3}\,\pi\quad 0.002\ \text{m/s}\quad (0.05\ \text{m})^2\quad 13{,}555\ \text{kg/m}^3\quad = 0.14\ \text{kg/s}$$

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PROBLEM 7.2

Consider fully developed laminar flow of a fluid inside a wide rectangular duct with both the upper and lower surface at uniform surface temperature, as schematically shown in the figure below. The effect of the two sides of the duct is neglected (W>> H), and at a location far from the entrance the velocity distribution is assumed to be uniform, u(x) = C_u whereas the temperature distribution has a parabolic profile given by T(x)= Ts + C_T (1 –(2x/H)^2). From the given velocity and temperature distributions, calculate the Nusselt number, which is given by its definition as

$$N\overline{u}_H = \frac{\overline{h}_c H}{k} = \frac{q_s''(H/k)}{(T_s - T_b)}$$

where Tb is the average bulk temperature of the flowing fluid at the specified location, and k is the thermal conductivity of the fluid.

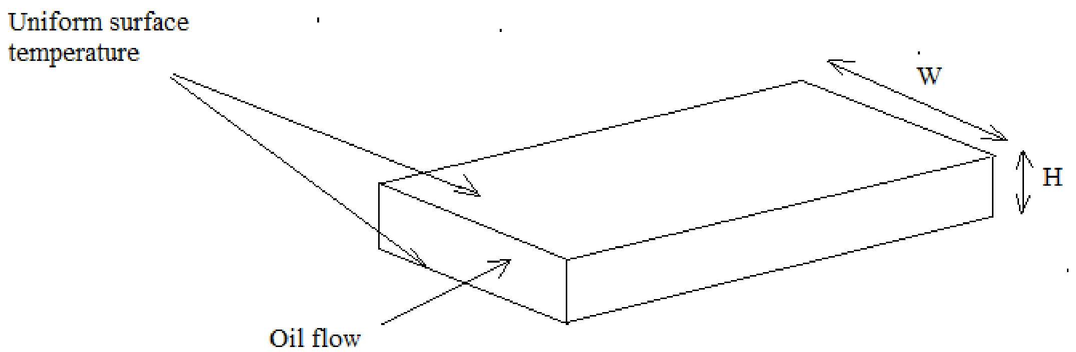
GIVEN

- Fully developed laminar flow of fluid inside rectangular duct
- Uniform upper and lower surface temperature
- Uniform velocity distribution u(x) = C_u
- Parabolic temperature distribution T(x)= Ts + C_T (1 –(2x/H)^2)

FIND

Nusselt number

SKETCH



SOLUTION

The bulk temperature of a fluid is given by formula

$$T_b = \frac{\int_{-H/2}^{H/2} u(x)T(x)\,dx}{\int_{-H/2}^{H/2} u(x)dx}$$

Substituting velocity and temperature distribution in above equation we get

$$T_b = \frac{\int_{-H/2}^{H/2} C_u * (T_s + C_T(1-(2x/H)^2)\,dx}{\int_{-H/2}^{H/2} C_u dx}$$

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$$T_b = \frac{T_s[x]_{-H/2}^{H/2} + C_T[x]_{-H/2}^{H/2} - C_T\left[2x^3/(3H^2)\right]_{-H/2}^{H/2}}{[x]_{-H/2}^{H/2}}$$

$$T_b = \frac{T_s * H + C_T * H - C_T\left[2H^3/(24H^2) + 2H^3/(24H^2)\right]}{H}$$

$$T_b = T_s + C_T - C_T[1/12 + 1/12]$$

$$T_b = T_s + \frac{5}{6}C_T$$

Now, Nu is given by definition as

$$N\bar{u}_H = \frac{\bar{h}_c H}{k} = \frac{q_s''(H/k)}{\left(T_s - T_b\right)}$$

$$= \frac{q_s''(H/k)}{\left(T_s - T_s - \frac{5}{6}C_T\right)}$$

$$N\bar{u}_H = \frac{\bar{h}_c H}{k} = \frac{-6q_s''H}{5C_T k}$$

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PROBLEM 7.3

A miniature heat sink heat exchanger is constructed with circular cross section channels (similar to that in Example 7.4), each of which has a diameter of 3.0 mm, drilled through a rectangular block. Cooling water at 20°C flows at the rate of 0.5 kg/h through each circular pipe, which has a constant surface temperature of 80°C. Assuming fully developed velocity and temperature distribution conditions, determine the length of the drilled pipe if the outlet water temperature is 60°C. Also, based on your calculations, verify the fully developed condition assumption.

GIVEN

- Heat exchanger constructed with circular cross section channels.
- Diameter of channel (D_i)= 3 mm = $3*10^{-3}$ m
- Cooling water inlet temperature (T_{in})= 20^0C
- Cooling water flow rate ($\dot{m}$)=0.5 kg/h = $1.39*10^{-4}$ kg/s
- Cooling water outlet temperature (T_{out})= 60^0C
- Surface temperature(T_s)=80^0C

FIND

(a) Length of the drilled pipe.
(b) Verify fully developed condition assumption

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for Water at 60°C:

Density (ρ) = 982.8 kg/m^3
Specific heat (c_p) = 4182.8 J/(kg K)
Thermal conductivity (k) = 0.657 W/(m K)
Absolute viscosity (μ) = 483.7 × 10^{-6} (Ns)/m^2
Prandtl Number (Pr) = 3.02

SOLUTION

The Reynold's Number for given flow is

$$\text{Re} = \frac{4\dot{m}}{\mu\pi D_i} = \frac{4*1.39*10^{-4}}{483.7*10^{-6}*\pi*3*10^{-3}} = 122$$

Thus the flowrate is in laminar region.

Thus, for a fully developed flow at constant wall temperature Nu=3.657

$$\text{Nu} = \frac{\bar{h}_c D_i}{k} = 3.657$$

$$\Rightarrow \quad \bar{h}_c = \frac{3.657*0.657}{3*10^{-3}} \text{ W/(m}^2\text{ K)}$$

$$\Rightarrow \quad \bar{h}_c = 800 \text{ W/(m}^2\text{ K)}$$

The rate of heat transfer to the cooling water is given by

$$q_c = \dot{m} c_p (T_{out} - T_{in})$$

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$$q_c = 1.39*10^{-4} * 4182.8*(60-20)W$$

$$q_c = 23.25 \text{ W}$$

For T_s=80^{0}C, finding LMTD given in equation (7.37) we have

$$LMTD = \frac{\Delta T_{out} - \Delta T_{in}}{\ln\left(\Delta T_{out} / \Delta T_{in}\right)}$$

$$LMTD = \frac{20-40}{\ln\left(20/40\right)}$$

$$LMTD = \frac{20-40}{\ln\left(20/40\right)}$$

$$LMTD = 28.85\,^0\text{C}$$

Now, we have from equation (7.37)

$$q_c = \bar{h}_c A_s * LMTD \text{ where } A_s = \pi D_i L$$

⇨ $$L = \frac{q_c}{\pi D_i \bar{h}_c * LMTD}$$

⇨ $$L = \frac{23.25}{\pi 3*10^{-3}*800.8*28.85} \text{ m}$$

⇨ $$L = \frac{23.25}{\pi 3*10^{-3}*800.8*28.85}$$

⇨ $$L = 0.107 \text{ m}$$

⇨ L=10.7 cm

We have,

$$\left(\frac{x_{fd}}{D_i}\right) = 0.05\,\text{Re}_D$$

$$x_{fd} = 0.05\,\text{Re}_D * D_i$$

$$x_{fd} = 0.05*122*3*10^{-3} = 0.018 \text{ m} = 1.8 \text{ cm}$$

$$\left(x_{fd}\right)_{th} = 1.8*\text{Pr} = 5.5 \text{ cm}$$

Since, the length of the drilled pipe is 10.7 cm and the velocity profile is fully developed at 1.8 cm, and thermal profile is fully developed in 5.5 cm, the fully developed assumption is valid.

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PROBLEM 7.4

Water enters a small copper tube, with an inner diameter of 2.5 cm, at the rate of 0.025 kg/s and a temperature of 15°C. Steam is condensing on the outer surface of the tube at atmospheric pressure so that tube-surface temperature is uniformly at 100°C. If the tube length is 5 meters, determine (a) the outlet temperature, and (b) the average convection heat transfer coefficient between the water and the pipe.

GIVEN

- Diameter of copper tube (D_i)= 2.5 cm = 0.025 m
- Cooling water inlet temperature (T_{in})= 20^0C
- Cooling water flow rate ($\dot{m}$)=0.025 kg/s
- Tube length (L)= 5 m
- Surface temperature(T_s)=100^0C

FIND

(a) Outlet temperature
(b) Average convection heat transfer coefficient between water and pipe.

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for Water at 35°C: (guess bulk temperature)

Density (ρ) = 994.1 kg/m^3
Specific heat (c_p) = 4175 J/(kg K)
Thermal conductivity (k) = 0.624 W/(m K)
Absolute viscosity (μ) = 719.8 × 10^{-6} (Ns)/m^2
Prandtl Number (Pr) = 4.8

SOLUTION

The Reynold's Number for given flow is $\mathrm{Re} = 4\dot{m}\big/\mu\pi D_i = \dfrac{4*0.025}{719.8*10^{-6}*\pi*0.025}$ =1768

Thus the flowrate is in laminar region.

Thus, for a fully developed flow at constant wall temperature Nu= $\bar{h}_c D_i \big/ k$ =3.657

$$\mathrm{Nu}=\bar{h}_c = \frac{3.657*0.624}{3*10^{-3}} \text{ W/(m}^2\text{ K)}$$

⇨ $\bar{h}_c = 760.6$ W/(m^2 K)

Thus the average heat transfer coefficient between water and pipe is 760.6 W/(m^2 K).

The rate of heat transfer to the cooling water is given by

$$q_c = \dot{m}\,c_p(T_{out} - T_{in}) = \bar{h}_c A_s * LMTD$$

$$760.6*\pi*0.025*5*\frac{\Delta T_{out} - \Delta T_{in}}{\ln\left(\Delta T_{out}/\Delta T_{in}\right)} = 0.025*4182.8*(T_{out} - 20)W$$

$$\frac{(20 - T_{out})}{\ln\left((100 - T_{out})/80\right)} = 0.35*(T_{out} - 20)$$

$$T_{out} = 95.4^0 C$$

Thus the cooling water outlet temperature is 95.4^0C.

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PROBLEM 7.5

> **In a biomedical processing plant, a newly developed liquid drug or medication that leaves a mixing chamber at 20°C needs to be heated to 70°C in a thermal curing process. This is achieved by heating the liquid media at the rate of 10 kg/h in a thin metallic circular pipe with a 10 mm inner diameter that is electrically heated by wrapping it in resistance wire and insulation. If the tube surface is heated with a constant heat flux of 5,100 W/m^2 , calculate the length of tubing needed for this heat exchange device and the inner surface temperature of the pipe at its exit. Assume fully developed convection, and verify this assumption after calculating the length. The physical properties of the fluid media are considered as follows: C_p= 4.0 kJ/(kg K), k= 0.5 W/(m K), μ= 0.002 Ns/m^2 , and ρ=1000 kg/m^3.**

GIVEN

- Diameter of copper tube (D_i)= 10 mm = 0.01 m
- Cooling water inlet temperature (T_{in})= 20^0C
- Cooling water flow rate ($\dot{m}$)=10 kg/hr = 2.78*10^{-3} kg/s
- Cooling water outlet temperature (T_{out})= 70^0C
- Surface heat flux(q'') = 5100 W/m^2

FIND

(a) Length of tubing needed for this heat exchange device.
(b) Inner surface temperature of pipe at exit.

PROPERTIES AND CONSTANTS

Given, for the fluid

Density (ρ) = 1000 kg/m^3
Specific heat (c_p) = 4000 J/(kg K)
Thermal conductivity (k) = 0.5 W/(m K)
Absolute viscosity (μ) = 0.002 (Ns)/m^2

SOLUTION

The Reynold's Number for given flow is

$$\mathrm{Re} = \frac{4\dot{m}}{\mu\pi D_i} = \frac{4*2.78*10^{-3}}{0.002*\pi*0.01} = 177$$

Thus the flowrate is in laminar region.

Thus, for a fully developed flow at constant wall temperature Nu=4.364

$$\mathrm{Nu} = \frac{\bar{h}_c D_i}{k} = 4.364$$

$$\Rightarrow \quad \bar{h}_c = \frac{4.364*0.5}{0.01}\ \text{W/(m}^2\text{ K)}$$

$$\Rightarrow \quad \bar{h}_c = 218.5\ \text{W/(m}^2\text{ K)}$$

Thus the average heat transfer coefficient between water and pipe is 218.5 W/(m^2 K).

The rate of heat transfer to the cooling water is given by

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$$q_c = \dot{m} c_p (T_{out} - T_{in}) = \pi D_i L * q''$$

$$L = \frac{\dot{m} c_p (T_{out} - T_{in})}{q'' * \pi D_i} \text{ m}$$

$$L = \frac{2.78 * 10^{-3} * 4000 * (70 - 20)}{5100 * \pi * 0.01} \text{ m}$$

$$L = 3.47 \text{ m}$$

Thus the length of device required for this heat exchanger is 3.47 m.

At, pipe exit for constant heat flux

$$q'' = \bar{h}_c * (T_w - T_{out})$$

$$T_w = q'' / \bar{h}_c + T_{out}$$

$$T_w = 5100 / 218.5 + 70\ ^{0}\text{C}$$

$$T_w = 93.3\ ^{0}\text{C}$$

Thus inner surface temperature of pipe at exit is 93.3^{0}C.

The length at which flow becomes fully developed is

$$\left(\frac{x_{fd}}{D_i} \right) = 0.05 \text{Re}_D$$

$$x_{fd} = 0.05 \text{Re}_D * D_i$$

$$x_{fd} = 0.05 * 177 * 0.01 = 0.0885 \text{ m} = 8.5 \text{ cm}$$

Since, the pipe length is 3.47 m the fully developed flow assumption is correct.

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PROBLEM 7.6

Purified water, flowing at the rate of 45 kg/h is to be heated from 25°C to 75°C in a food processing plant before it is mixed with thickened tomato puree for sauce production. A thin-walled, 1.25 cm diameter, stainless steel tube is used, which is maintained at a constant surface temperature of 175°C by immersing it in a hot mineral bath. Calculate the length of tubing required and determine whether the forced convection inside the tube is fully developed or has entrance effects. Also, what is the frictional loss?

GIVEN

- Diameter of steel tube (D_i)= 1.25 cm = 0.0125 m
- Cooling water inlet temperature (T_{in})= 25^0C
- Cooling water flow rate ($\dot{m}$)=45 kg/h=0.0125 kg/h
- Water outlet temperature (T_{out})= 75^0C
- Surface temperature(T_s)=175^0C

FIND

(a) Length of tubing required.
(b) whether the forced convection inside tube is fully developed or has entrance effects.

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for Water at 50°C:

Density (ρ) = 988.1 kg/m^3
Specific heat (c_p) = 4178 J/(kg K)
Thermal conductivity (k) = 0.647 W/(m K)
Absolute viscosity (μ) = 555.1 × 10^{-6} (Ns)/m^2
Prandtl Number (Pr) = 3.55

SOLUTION

The Reynold's Number for given flow is

$$\mathrm{Re} = \frac{4\dot{m}}{\mu \pi D_i} = \frac{4*0.0125}{555.1*10^{-6}*\pi*0.0125} = 2293$$

Thus the flowrate is in transition region. For the problem we can consider it as laminar flow regime.

For T_s=175^0C, finding LMTD given in equation (7.37) we have

$$LMTD = \frac{\Delta T_{out} - \Delta T_{in}}{\ln\left(\Delta T_{out}/\Delta T_{in}\right)} \quad => \quad LMTD = \frac{100-150}{\ln\left(100/150\right)}$$

$$LMTD = 123.3\,^0\mathrm{C}$$

Now, we have from equation (7.37)

$$q_c = \dot{m} c_p (T_{out} - T_{in}) = \bar{h}_c A_s * LMTD \text{ where } A_s = \pi D_i L$$

$$\Rightarrow \quad \bar{h}_c L = \frac{\dot{m} c_p (T_{out} - T_{in})}{\pi D_i * LMTD}$$

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$\Rightarrow \quad \bar{h}_c L = \dfrac{0.0125 * 4178 * (75-25)}{\pi * 0.0125 * 123.3}$ m

$\Rightarrow \quad \bar{h}_c L = 539.2$ W/(m K) ………………………….. (I)

We have,

$\left(\dfrac{x_{fd}}{D_i}\right) = 0.05\,\mathrm{Re}_D$ => $x_{fd} = 0.05\,\mathrm{Re}_D * D_i$

$x_{fd} = 0.05 * 2293 * 0.0125$ = 1.43 m

$\left(x_{fd}\right)_{th} = 1.8 * \mathrm{Pr}$ =5 m

For L=5 m $\dfrac{L}{D\,\mathrm{Re}_D\,\mathrm{Pr}} = 0.05$, using Equation (7.40) we have

$N\bar{u}_D = 3.657 + \left(0.0499 D\,\mathrm{Re}_D\,\mathrm{Pr}/L\right)$

$\bar{h}_c = \dfrac{k}{D}\left(3.657 + \left(0.0499 D\,\mathrm{Re}_D\,\mathrm{Pr}/L\right)\right)$ …………………………….. (II)

From which $\bar{h}_c = 13.637 * \dfrac{0.647}{0.0125}$ =705.9 W/(m^2 K)

Now substituting value of $\bar{h}_c$ in equation (I) we get

L= 0.76 m

Now $\dfrac{L}{D\,\mathrm{Re}_D\,\mathrm{Pr}} = 0.007$

Thus from second correlation of equation (7.40) we have

$N\bar{u}_D = 1.615\left(L / D\,\mathrm{Re}_D\,\mathrm{Pr}\right)^{-1/3} - 0.2$

$\bar{h}_c = \dfrac{k}{D}\left(1.615\left(L / D\,\mathrm{Re}_D\,\mathrm{Pr}\right)^{-1/3} - 0.2\right)$ …………………………… (III)

From which $\bar{h}_c = 8.06 * \dfrac{0.647}{0.0125}$ =417.2 W/(m^2 K)

Now substituting this value of $\bar{h}_c$ in (I) we get

L=1.29 m

Now, solving the problem iteratively we get

3rd iteration => $\bar{h}_c$ =348.2 W/(m^2 K) L=1.54 m

On further iterating we reach the final solution of L=1.70 m which is the required length of the pipe.

and $\bar{h}_c$ = 316.35 W/(m^2 K).

COMMENTS

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Since, the length of the drilled pipe is 1.70 m and the velocity profile is fully developed at 1.43 m, and thermal profile is fully developed in 5 m, the forced convection still has thermal entrance effects at outlet.

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PROBLEM 7.7

Nitrogen at 30°C and atmospheric pressure enter a triangular duct 0.02 m on each side at a rate of 4 × 10^{-4} kg/s. If the duct temperature is uniform at 200°C, estimate the bulk temperature of the nitrogen 2 m and 5 m from the inlet.

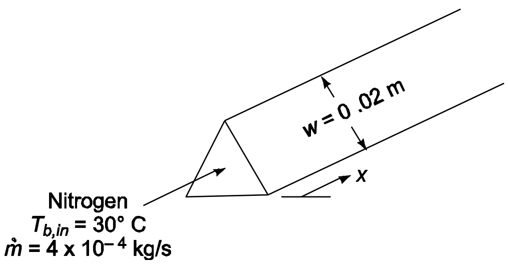
GIVEN

- Atmospheric nitrogen flowing through a triangular duct
- Bulk inlet temperature ($T_{b,\text{in}}$) = 30°C
- Width of each side of the duct (w) = 0.02 m
- Mass flow rate ($\dot{m}$) = 4 × 1^{-4} kg/s
- Duct temperature (T_s) = 200°C (uniform)

FIND

- The bulk temperature (a) 2 m from the inlet and, (b) 5 m from the inlet

SKETCH



SOLUTION

(a) Assuming the outlet temperature is 70°C, then the average bulk temperature is 50°C
From Appendix 2, Table 33, for nitrogen

Specific heat (c_p) = 1042 J/(kg K)
Thermal conductivity (k) = 0.0278 W/(m K)
Absolute viscosity (μ) = 18.79 × 10^{-6} (Ns)/m^2
Prandtl Number (Pr) = 0.71

The hydraulic diameter of the duct is

$$D_H = \frac{4A_c}{P} = \frac{4\left[\frac{1}{2}\mathrm{W}\sqrt{\mathrm{W}^2 - \left(\frac{\mathrm{W}}{2}\right)^2}\right]}{3\mathrm{W}} = \frac{4(1.73\times10^{-4}\ \mathrm{m}^2)}{3(0.02\,\mathrm{m})} = 0.0115\ \mathrm{m}$$

$$\therefore\ Re_D = \frac{U_\infty D\rho}{\mu} = \frac{\dot{m}D_H}{A_c\,\mu} = \frac{4\times10^{-4}\ \mathrm{kg/s}\ \ (0.0115\,\mathrm{m})}{1.73\times10^{-4}\mathrm{m}^2\ \ 18.79\times10^{-6}\ \mathrm{Ns/m}^2\quad (\mathrm{kg\,m})/(\mathrm{Ns}^2)} = 1415$$

The length from the entrance at which the velocity and temperature profiles become fully developed can be obtained from Equations (7.7) and (7.8)

$$x_{fd} = 0.05\ D_H\ Re_D = 0.05\ (0.0115\ \mathrm{m})(1415) = 0.81\ \mathrm{m}$$

$$x_{ft,T} = 0.05\ D_H\ Re_D\ \mathrm{Pr} = 0.05\ (0.0115\ \mathrm{m})\ (1415)(0.71) = 0.58\ \mathrm{m}$$

Therefore, the flow is fully developed over most of the duct length.

From Table 7.1, for fully developed flow in triangular cross-section duct: $\overline{Nu}_D$ = 2.47

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$$\therefore\ \bar{h}_c = \overline{Nu}_D \frac{k}{D_H} = 2.47 \frac{0.0278\ \text{W/(m K)}}{0.0115\ \text{m}} = 5.98\ \text{W/(m}^2\ \text{K)}$$

Rearranging Equation (7.36)

$$T_{b,\text{out}} = T_s + (T_{b,\text{in}} - T_s)\ \exp\left(-\frac{P L h_c}{\dot{m} c_p}\right)$$

$$T_{b,\text{out}} = 200°\text{C} + (30°\text{C} - 200°\text{C})\ \exp\left(-\frac{3(0.02\ \text{m})(2\ \text{m})\ \ 5.98\ \text{W/(m}^2\text{K)}}{4\times10^{-4}\ \text{kg/s}\ \ \ 1042\ \text{J/(kg K)}\ \ \ \text{Ws/J}}\right) = 170°\text{C}$$

With this outlet temperature, the average bulk temperature will be 100°C. This is far enough from the initial guess that another iteration is warranted

c_p = 1045 J/(kg K) $\qquad$ $\bar{h}_c$ = 6.75 W/(m^2 K)

k = 0.0314 W/(m K) $\qquad$ $T_{b,\text{out}}$ = 176°C

The bulk temperature at x = 2 m is 176°C.

(b) The same procedure can be used to find the bulk temperature at x = 5 m. Let $T_{b,\text{out}}$ = 190°C. Average bulk temperature = 110°C

c_p = 1045

k = 0.0321 W/(m K)

$\bar{h}_c$ = 6.90 W/(m^2 K)

$T_{b,\text{out}}$ = 199°C

The bulk temperature a x = 5 m is about 199°C.

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PROBLEM 7.8

Air at 30°C enters a rectangular duct 1-m-long and 4 mm by 16 mm in cross-section at a rate of 0.0004 kg/s. If a uniform heat flux of 500 W/m² is imposed on both of the long sides of the duct, calculate (a) the air outlet temperature (b) the average duct surface temperature, and (c) the pressure drop.

GIVEN

- Air flowing through a rectangular duct
- Inlet bulk air temperature ($T_{b,\text{in}}$) = 30°C
- Duct length (L) = 1 m
- Duct height (H) 4 mm = 0.004 m
- Duct width (w) = 16 mm = 0.016 m
- Air mass flow rate ($\dot{m}$) = 0.0004 kg/s
- Uniform heat flux (q/A) = 500 W/m² on the long sides

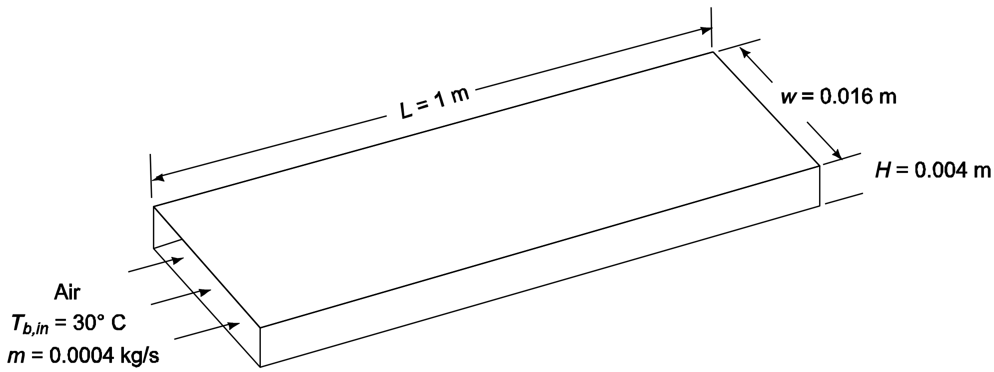
FIND

(a) Air outlet temperature ($T_{b,\text{out}}$)
(b) The average duct surface temperature (T_s)
(c) The pressure drop ($\Delta\, p$)

ASSUMPTIONS

- The short sides of the duct are insulated
- Entrance effects are negligible

SKETCH



SOLUTION

(a) The total rate of heat transfer to the air

$$q = \left(\frac{q}{A}\right) A = \left(\frac{q}{A}\right) 2\ L\ w = \ 500\ \text{W/(m}^2\text{K)}\ \ (2)(1\ \text{m})(0.016\ \text{m}) = 16\ \text{W}$$

$$q = \dot{m}\ c_p\, \Delta T = \dot{m}\, c_p\, (T_{b,\text{out}} - T_{b,\text{in}}) \Rightarrow T_{b,\text{out}} = T_{b,\text{in}} + \frac{q}{\dot{m}\, c_p}$$

The specific heat (c_p) ≈ 1000 J/(kg K), therefore, $T_{b,\text{out}}$ ≈ 70°C. From Appendix 2, Table 28, the specific heat at the approximate average bulk temperature of 50°C is 1016 J/(kg K).

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$$\therefore\ T_{b,\text{out}} = 30°\text{C} + \frac{16\,\text{W}}{0.0004\,\text{kg/s}\ \ 1016\,\text{J/(kg K)}\ \ (\text{Ws})/\text{J}} = 69.4°\text{C}$$

(b) The average duct surface temperature is given by

$$\frac{q}{A} = h_c\,(T_s - T_{b,\text{ave}}) \Rightarrow T_s = T_{b,\text{ave}} + \frac{q}{A h_c} = \frac{T_{b,\text{in}} + T_{b,\text{out}}}{2} + \frac{q}{A h_c}$$

The heat transfer coefficient can be obtained from the proper correlation.
The hydraulic diameter of the duct is

$$D_H = \frac{4A}{P} = \frac{4\,\text{w}\,\text{H}}{2(L+H)} = \frac{4(0.016\,\text{m})(0.004\,\text{m})}{2(0.02\,\text{m})} = 0.0064\ \text{m}$$

$$\frac{L}{D_H} = \frac{1\,\text{m}}{0.0064\,\text{m}} = 156$$

Therefore, entrance effects will be neglected.
From Appendix 2, Table 28, for dry air at the average bulk temperature of 49.7°C

Thermal conductivity (k) = 0.0272 W/(m K)
Absolute viscosity (μ) = 19.503×10^{-6} (Ns)/m^2
Density (ρ) = 1.015 kg/m^3

The Reynolds number is

$$Re_D = \frac{U_\infty D \rho}{\mu} = \frac{\dot{m} D_H}{\text{w}\,H\,\mu} = \frac{0.0004\,\text{kg/s}}{(0.016\,\text{m})(0.004\,\text{m})\ 19.503\times 10^{-6}\,(\text{Ns})/\text{m}^2\ \ (\text{kg m})/(\text{Ns}^2)} = 2051 < 2100$$

Therefore, the flow is laminar.

The Nusselt number for this geometry is given in Table 7.1

For $\frac{2b}{2a} = \frac{0.008}{0.032} = 0.25$, $\overline{Nu}_D = \overline{Nu}_{H_2} = 2.93$

$$\therefore\ h_c = Nu_D \frac{k}{D_H} = 2.93\ \frac{0.0272\,\text{W/(m K)}}{0.0064\,\text{m}} = 12.5\ \text{W/(m}^2\ \text{K)}$$

The average surface temperature is

$$T_s = \frac{30°\text{C} + 69.4°\text{C}}{2} + \frac{500\,\text{W/m}^2}{12.5\,\text{W/(m}^2\ \text{K)}} = 90°\text{C}$$

From Table 7.1 for $2b/2a = 1/4$, $fRe_D = 72.93$

$$\therefore\ f = \frac{72.93}{\text{Re}_D} = \frac{72.93}{2051} = 0.0356$$

The pressure drop is given by Equation (7.13)

$$\Delta p = f\,\frac{L}{D_H}\,\frac{\rho U^2}{2g_c} = f\,\frac{L}{D_H}\,\frac{1}{2g_c\rho}\left(\frac{\dot{m}}{wH}\right)^2$$

$$\Delta p = 0.0356\ \frac{1\,\text{m}}{0.0064\,\text{m}}\ \frac{1}{2\ (\text{kg m})/(\text{Ns}^2)\ \ 1.059\,\text{kg/m}^3}\left(\frac{0.0004\,\text{kg/s}}{(0.016\,\text{m})(0.004\,\text{m})}\right)^2 = 102\ \text{Pa}$$

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PROBLEM 7.9

Air at an average temperature of 150°C flows through a short square duct 10 × 10 × 2.25 cm at a rate of 15 kg/h. The duct wall temperature is 430°C. Determine the average heat transfer coefficient, using the duct equation with appropriate *L*/*D* correction. Compare your results with flow-over-flat-plate relations.

GIVEN

- Air flowing through a short square duct
- Average air temperature (T_a) = 150°C
- Duct dimensions = 10 × 10 × 2.25 cm = 0.1 × 0.1× 0.0225 m
- Duct wall surface temperature (T_s) = 430°C
- Mass flow rate ($\dot{m}$) = 15 kg/h

FIND

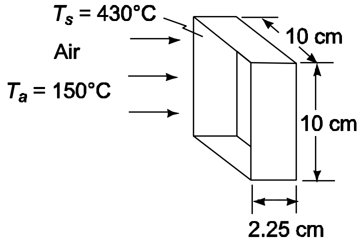
The average heat transfer coefficient ($\bar{h}_c$) using

(a) The duct equation with appropriate L/D correction
(b) The flow-over-flat-plate relation

ASSUMPTIONS

- Constant and uniform duct wall temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average temperature of 150°C

Thermal conductivity (k) = 0.0339 W/(m K)
Absolute viscosity (μ_b) = 23.683 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71

At the surface temperature of 430°C, the absolute viscosity (μ_s) = 33.66 × 10^{-6} (Ns)/m^2.

SOLUTION

The hydraulic diameter of the duct is

$$D_H = \frac{4A_c}{P} = \frac{4(0.1\text{m})(0.1\text{m})}{4(0.1\text{m})} = 0.1 \text{ m}$$

The Reynolds number is

$$Re_{D_H} = \frac{VD_H\rho}{\mu} = \frac{\dot{m}D_H}{A_c\,\mu} = \frac{15\text{kg/h}\ (0.1\text{m})}{(0.1\text{m})(0.1\text{m})\ 23.683\times10^{-6}\,(\text{Ns})/\text{m}^2\ \ 3600\text{s/h}\ \ (\text{kg m})/(\text{N s}^2)} = 1760$$

Therefore, the flow is laminar.

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(a) Using the Hausen correlation, Equation (7.41) to estimate the Reynolds number with $D/L = D_H/L = 10/2.25 = 4.44$

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = \left[3.66 + \frac{0.0668 Re_{D_H} Pr\left(\frac{D}{L}\right)}{1 + 0.045\left[Re_{D_H} Pr\left(\frac{D}{L}\right)\right]^{0.66}}\right]\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\overline{Nu}_{D_H} = \left[3.66 + \frac{0.0668(1760)(0.71)(4.44)}{1 + 0.045[(1760)(0.71)(4.44)]^{0.66}}\right]\left(\frac{23.683}{33.666}\right)^{0.14} = 28.3$$

$$\bar{h}_c = \overline{Nu}_{D_H} \frac{k}{D_H} = 28.3 \frac{0.0339\,\text{W/(m K)}}{0.1\text{m}} = 9.59\ \text{W/(m}^2\ \text{K)}$$

(b) Applying the flow-over-flat-plate relation of Equation (7.38)

$$\overline{Nu}_{D_H} = \frac{Re_{D_H} Pr}{4}\frac{D_H}{L}\ln\left[\frac{1}{1 - \frac{2.654}{Pr^{0.167}\left[Re_{D_H} Pr\ \frac{D_H}{L}\right]^{0.5}}}\right]$$

$$\overline{Nu}_{D_H} = \frac{(1760)(0.71)}{4}(4.44)\ln\left[\frac{1}{1 - \frac{2.654}{(0.71)^{0.167}[1760(0.71)(4.44)]^{0.5}}}\right] = 53.3$$

$$\bar{h}_c = \overline{Nu}_{D_H} \frac{k}{D_H} = 53.3 \frac{0.0339\,\text{W/(m K)}}{0.1\text{m}} = 18.1\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

The flat plate estimate is almost twice the previous estimate based on flow through a short duct. It should be noted that the flow-over-flat-plate relation is only applicable in the following range: $[Re_D\ Pr\ (D/L)]$ from 100 to 1500. For this problem, $Re_D\ Pr\ D/L = 5548$.

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PROBLEM 7.10

Show that for fully developed laminar flow between two flat plates spaced 2*a* apart, the Nusselt number based on the 'bulk mean' temperature and the passage spacing is 4.12 if the temperature of both walls varies linearly with the distance *x*, i.e., $\partial T/\partial x$ = C. The 'bulk mean' temperature is defined as

$$T_b = \frac{\int_{-a}^{a} u(y)T(y)\,dy}{\int_{-a}^{a} u(y)\,dy}$$

GIVEN

- Fully developed laminar flow between two flat plates
- Spacing = $2a$ & $\partial T/\partial x$ = C
- Bulk mean temperature as defined above

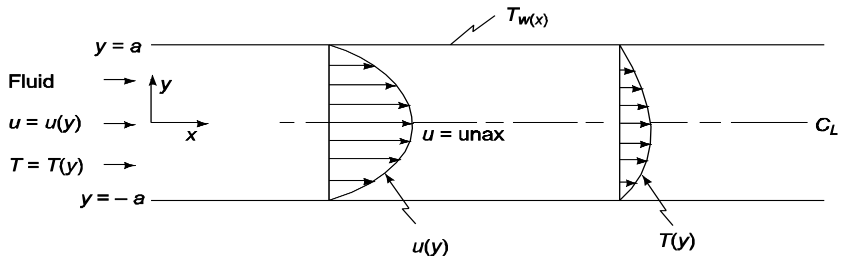
FIND

- Show that the Nusselt number based on the bulk mean temperature = 4.12

ASSUMPTION

- Steady state
- Constant and uniform property values
- Fluid temperature varies linearly with x
  (This corresponds to a constant heat flux boundary)

SKETCH



SOLUTION

The solution will progress as follows

1. Derive the temperature distribution in the fluid.
2. Use the temperature distribution to obtain an expression for the bulk mean temperature.
3. Use the bulk mean temperature to derive the Nusselt number.

Beginning with the laminar flow energy equation of Equation (5.7b)

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial^2 T}{\partial y^2}$$

v = component of the velocity in the Y direction = 0

$$\therefore\; u\,\frac{\partial T}{\partial x} = \alpha\,\frac{\partial^2 T}{\partial y^2}$$

Note that $\partial T/\partial x$ = constant by assumption.

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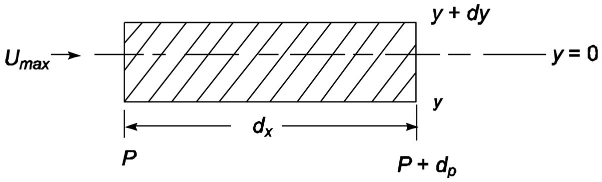
The velocity profile $u(y)$ must be substituted into this equation before the equation can be solved for the temperature distribution. The velocity profile can be derived by considering a differential element of fluid of width w as shown below



A force balance on this element yields

$$2\,w y\;[p-(p+dp)] = 2\,\tau\,\mathrm{w}\,dx = \mu\,\frac{\partial u}{\partial y} - \mu\left(\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y}dy\right) w\,dx$$

Since the flow is fully developed $$\frac{dp}{dx} = \mu\,\frac{d^2 u}{dy^2}$$

Integrating with respect to y $$u = \frac{1}{\mu}\,\frac{dp}{dx}\,\frac{y^2}{2} + C$$

This is subject to the following boundary conditions

$u = u_{\max}$ at $y = 0$ therefore, $C = u_{\max}$

$u = 0$ at $y = +a$ therefore, $u_{\max} = -\dfrac{a^2}{2\mu}\,\dfrac{dp}{dx}$

Therefore, the velocity distribution is $$u = u_{\max}\left[1-\left(\frac{y}{a}\right)^2\right]$$

Substituting this into the energy equation

$$\frac{\partial^2 T}{\partial y^2} = \frac{u_{\max}}{\alpha}\,\frac{\partial T}{\partial x}\left[1-\left(\frac{y}{a}\right)^2\right]$$

Let $z = \dfrac{u_{\max}}{\alpha}\,\dfrac{\partial T}{\partial x}$ (a constant)

Subject to the boundary conditions $\dfrac{\partial T}{\partial y} = 0$ at $y = 0$ (by symmetry)

$T = T_w$ at $y = +a$

Integrating once

$$\frac{\partial T}{\partial y} = z\left(y - \frac{1}{3}\,\frac{y^3}{a^2}\right) + C_1$$

Applying the first boundary condition, $C_1 = 0$

Integrating again

$$T = z\left(\frac{1}{2}y^2 - \frac{1}{12}\,\frac{y^4}{a^2}\right) + C_2$$

Applying the second boundary condition

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$$T_w = z\left(\frac{1}{2}a^2 - \frac{1}{12}a^2\right) + C_2 \Rightarrow C_2 = T_w - \frac{5}{12}\, z\, a^2$$

Therefore, the temperature distribution is

$$T(x,y) = T_w(x) - \frac{5}{12}\, z\, a^2 + \frac{z}{2}\, y^2 - \frac{z}{12}\frac{y^4}{a^2}$$

The bulk mean temperature is defined as $T_b = \dfrac{\int_{-a}^{a} u(y)\,T(y)\,dy}{\int_{-a}^{a} u(y)\,dy}$

Solving the numerator of this expression

$$\int_{-a}^{a} u(y)T(x,y)dy = \int_{-a}^{a} u_{\max}\left[1-\left(\frac{y}{a}\right)^2\right]\left[T_w(x) - \frac{5}{12}za^2 + \frac{z}{2}y^2 - \frac{z}{12a^2}\,y^4\right]dy$$

$$\int_{-a}^{a} u(y)T(x,y)dy = u_{\max}\left[2a\left(T_w(x) - \frac{5}{12}za^2\right) + \frac{z}{3}a^3 - \frac{z}{30}a^3 - \frac{2}{3}\,a\left(T_w(x) - \frac{5}{12}\,za^2\right) - \frac{z}{5}a^3 + \frac{z}{42}a^3\right]$$

$$\int_{-a}^{a} u(y)T(x,y)dy = u_{\max}\left[\frac{4}{3}a\left(T_w(x) - \frac{5}{12}za^2\right) + \frac{13}{105}za^3\right]$$

The denominator is

$$\int_{-a}^{a} u_{\max}\left[1-\left(\frac{y}{a}\right)^2\right]dy = u_{\max}\left[2a - \frac{2}{3}a\right] = \frac{4}{3}\,u_{\max}\,a$$

$$\therefore\ T_b = \frac{\frac{4}{3}a\left(T_w(x) - \frac{5}{12}za^2\right) + \frac{13}{105}za^3}{\frac{4}{3}a}$$

$$T_b - T_w = \frac{13}{105}\left(\frac{3}{4}\right) za^2 - \frac{5}{12}\, za^2 = -\frac{34}{105}\, za^2$$

The rate of heat transfer is given by $\dfrac{q}{A} = \bar{h}_c\,(T_b - T_w) = -k\left.\dfrac{\partial T}{\partial y}\right|_{y=a}$

$$\text{where } \left.\frac{\partial T}{\partial y}\right|_{y=a} = z\,a - \frac{z}{3}\frac{a^3}{a^2} = \frac{2}{3}\,a\,z$$

$$\therefore\ \bar{h}_c = \frac{-k\left.\frac{\partial T}{\partial y}\right|_{y=a}}{T_b - T_w} = \frac{-k\left(\frac{2}{3}\,az\right)}{-\frac{34}{105}\,za^2} = \frac{210}{51}\left(\frac{k}{2}\,a\right)$$

$$\overline{Nu} = \frac{\bar{h}_c\,L}{k} = \frac{\bar{h}_c\,2a}{k} = \frac{210}{51} = 4.12$$

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PROBLEM 7.11

Repeat Problem 7.10 but assume that one wall is insulated while the temperature of the other walls increases linearly with *x*.

GIVEN

- Fully developed laminar flow between two flat plates
- Spacing = 2a & $\partial T/\partial x$ = C
- Bulk mean temperature as defined above
- One wall is insulated
- The temperature of the other wall increases linearly with x

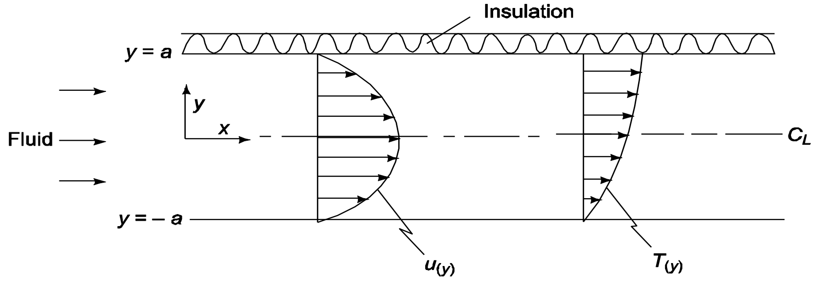
FIND

- The Nusselt number based on the bulk mean temperature (Nu)

ASSUMPTIONS

- Steady state
- Constant and uniform property values
- Fluid temperature varies linearly with x
  (This corresponds to a constant heat flux boundary)

SKETCH



SOLUTION

The velocity profile derived in the solution to Problem 7.10 remains unchanged

$$u = u_{\max}\left[1-\left(\frac{y}{a}\right)^2\right]$$

As does the energy equation

$$\frac{\partial^2 T}{\partial y^2} = z\left[1-\left(\frac{y}{a}\right)^2\right] \qquad \text{where } z= \frac{u_{\max}}{\alpha}\frac{\partial T}{\partial x} \text{ (a constant)}$$

The new boundary conditions are $\dfrac{\partial T}{\partial y} = 0$ at $y = a$ (due to the insulation)

$$T = T_w(x) \text{ at } y = -a$$

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Integrating the energy equation once

$$\frac{\partial T}{\partial y} = z\left(y - \frac{1}{3}\frac{y^3}{a^2}\right) + C_1$$

Applying the first boundary condition

$$0 = za - \frac{za^3}{3a^2} + C_1 \Rightarrow C_1 = -\frac{2}{3}\, za$$

Integrating the energy equation again

$$T = -\frac{2}{3}\, zay + \frac{1}{2}\, z y^2 - \frac{z}{12a^2}\, y^4 + C_2$$

Applying the second boundary condition

$$T_w(x) = -\frac{2}{3}\, z a^2 + \frac{1}{2}\, z a^2 - \frac{z}{12a^2}\, a^4 + C_2 \Rightarrow C_2 = T_w(x) + \frac{1}{4}\, za^2$$

Therefore, the temperature distribution is

$$T(x,y) = T_w(x) + \frac{1}{4}\, za^2 - \frac{2}{3}\, zay + \frac{1}{2}\, zy^2 - \frac{z}{12a^2}\, y^4$$

The numerator of the bulk mean temperature expression is

$$\int_{-a}^{a} u(y)T(x,y)\,dy = \int_{-a}^{a} u_{max}\left[1 - \left(\frac{y}{a}\right)^2\right]\left[\left(T_w(x) + \frac{1}{4}za^2\right) - \frac{2}{3}zay + \frac{1}{2}zy^2 - \frac{z}{12a^2}y^4\right]dy$$

$$= u_{max}\left[2a\left(T_w(x) + \frac{1}{4}za^2\right) + \frac{1}{3}za^3 - \frac{1}{30}za^3 - \frac{2}{3}a\left(T_w(x) + \frac{1}{4}za^2\right) - \frac{1}{5}za^3 + \frac{1}{42}za^3\right]$$

$$\int_{-a}^{a} u(y)T(x,y)\,dy = u_{max}\left[\frac{4}{3}a\left(T_w(x) + \frac{1}{4}za^2\right) + \frac{13}{105}za^3\right]$$

The denominator of the bulk mean temperature is

$$\int_{-a}^{a} u_{max}\left[1 - \left(\frac{y}{a}\right)^2\right]dy = u_{max} = \left[2a - \frac{2}{3}a\right]\frac{4}{3}\, u_{max}\, a$$

$$\therefore\ T_b = \left(T_w(x) + \frac{1}{4}za^2\right) + \left(\frac{3}{4}\right)\frac{13}{105}\, za^2 = T_w(x) + \frac{57}{210}\, za^2$$

$$T_w(x) - T_b = -\frac{57}{210}\, za^2$$

$$\text{At } z = -a:\ \frac{\partial T}{\partial y} = z(-a) - \frac{z}{3a^2}\,(-a)^3 - \frac{2}{3}\, za = -\frac{4}{3}\, za$$

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$$\therefore\ \bar{h}_c = \frac{-k\left.\dfrac{\partial T}{\partial y}\right|_{y=-a}}{T_b - T_w} = \frac{-k\left.\left(-\dfrac{4}{3}a\,z\right)\right|}{\dfrac{57}{210}z\,a^2} = \frac{560}{57}\left(\frac{k}{2}a\right)$$

By definition $\quad Nu = \dfrac{\bar{h}_c L}{k} = \dfrac{\bar{h}_c\, 2a}{k} = \dfrac{560}{57} = 9.82$

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PROBLEM 7.12

Engine oil flows at a rate of 0.5 kg/s through a 2.5-cm-*ID* tube. The oil enters 25°C while the tube wall is at 100°C. (a) If the tube is 4-m-long. Determine whether the flow is fully developed. (b) Calculate the heat transfer coefficient.

GIVEN

- Engine oil flows through a tube
- Mass flow rate ($\dot{m}$) = 0.5 kg/s
- Inside diameter (D) = 2.5 cm = 0.025 m
- Oil temperature at entrance (T_i) = 25°C
- Tube surface temperature (T_s) = 100°C
- Tube length (L) = 4 m

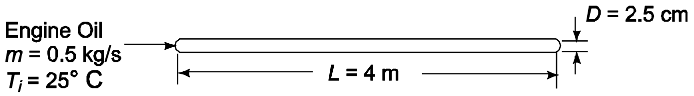
FIND

(a) Is flow fully developed?
(b) The heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 17, for unused engine oil at the initial temperature of 25°C

Density (ρ) = 885.2 kg/m^3
Thermal conductivity (k) = 0.145 W/(m K)
Absolute viscosity (μ) = 0.652 (Ns)/m^2
Prandtl number (Pr) = 85.20
Specific heat (c) = 1091 J/(kg K)

SOLUTION

The Reynolds number is

$$Re_D = \frac{V D \rho}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \;\; 0.5\,\text{kg/s}}{\pi(0.025\,\text{m}) \;\; 0.652\,\text{Ns/m}^2 \quad \text{kg m/(Ns}^2)} = 39.1$$

Therefore, the flow is laminar.

(a) The entrance length at which the velocity profile approaches its fully developed shape is given by Equation (7.7)

$$\frac{x_{fd}}{D} = 0.05\, Re_D \Rightarrow x_{fd} = 0.05\, D\, Re_D = 0.05\ (0.025\ \text{m})\ (39.1) = 0.049\ \text{m} = 4.9\ \text{cm}$$

Therefore, the velocity profile is fully developed for 98.8% of the tube length.

The entrance length at which the temperature profile approaches its fully developed shape is given by Equation (7.8)

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$$\frac{x_{fd}}{D} = 0.05\ Re_D\ Pr \Rightarrow x_{fd} = 0.05\ D\ Re_D\ Pr = 0.05(0.025\text{ m})\ (39.1)\ (8520) = 416\text{ m}$$

Therefore, the temperature profile is not fully developed.

(b) Since the velocity profile is fully developed but the temperature profile is not, Figure 7.12 will be used to estimate the Nusselt number

$$\frac{Re_D Pr D}{L} \times 10^{-2} = \frac{(39.1)(85.20)(0.025\text{ m})}{4\text{ m}} \times 10^{-2} = 0.208$$

Using the 'parabolic velocity' curve of Figure 7.12, $Nu_D \approx 4.8$

$$h_c = Nu_D \frac{k}{D} = 4.8 \frac{0.145\text{ W/(m K)}}{0.025\text{ m}} = 27.8\text{ W/(m}^2\text{ K)}$$

COMMENTS

The rate of heat transfer calculated with the heat transfer coefficient at the inlet is

$$q_{\max} = h_c\ \pi D\ L\ (T_s - T_b) = 27.8\text{ W/(m}^2\text{K)} \quad \pi(0.025\text{ m})\ (4\text{ m})\ (100°\text{C} - 25°\text{C}) = 656\text{ W}$$

The outer temperature (T_o) is given by

$$q_{\max} = \dot{m}\ c\ (T_{o,\max} - T_i)$$

$$T_o - T_i \le \frac{q_{\max}}{\dot{m} c} = \frac{656\text{ W}\ \ \text{J/(Ws)}}{0.5\text{ kg/s}\ \ 1091\text{ J/(kg K)}} = 1.2°\text{C}$$

This small temperature change does not warrant another iteration. If the temperature change was larger, the fluid properties would need to be re-evaluated at the average bulk temperature and a new heat transfer coefficient calculate.

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PROBLEM 7.13

The equation

$$\overline{Nu} = \frac{\bar{h}_c D}{k} = \left[3.65 + \frac{0.0668\left(\frac{D}{L}\right) RePr}{1 + 0.04\left[\left(\frac{D}{L}\right) RePr\right]^{\frac{2}{3}}}\right] = \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

was recommended by H. Hausen (Zeitschr. Ver. Deut. Ing., Belherft No. 4, 1943)

for forced-convection heat transfer in fully developed laminar flow through tubes. Compare the values of the Nusselt number predicted by Hausen's equation for *Re* = 1000, *Pr* = 1, and *L*/*D* = 2, 10 and 100, respectively, with those obtained from two other appropriate equations or graphs in the text.

GIVEN

- Fully developed laminar flow through a tube
- The Nusselt number correlation shown above
- Reynolds number (*Re*) = 1000
- Prandtl number (*Pr*) = 1
- Length divided by diameter (*L*/*D*) = 2, 10, or 100

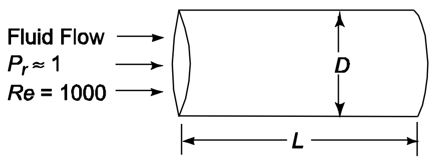
FIND

- The Nusselt number (*Nu*) from the above correlation and two others from the text

ASSUMPTIONS

- $\mu_b / \mu_s \approx 1.0$
- Constant wall temperature

SKETCH



SOLUTION

Using the Hausen correlation and *L*/*D* = 2

$$\overline{Nu} = \frac{\bar{h}_c D}{k} = \left[3.65 + \frac{0.0668\left(\frac{1}{2}\right)(1000)(1)}{1 + 0.04\left[\left(\frac{1}{2}\right)(1000)(1)\right]^{\frac{2}{3}}}\right]\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 13.1\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \approx 13.1$$

Similarly for the other cases   For $\frac{L}{D} = 10 \rightarrow \overline{Nu} \approx 7.2$

For $\frac{L}{D} = 100 \rightarrow \overline{Nu} \approx 4.2$

Figure 7.12 can also be used to estimate the Nusselt number. The velocity entrance region for this calculated for Equation (7.7)

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$$\frac{x_{fd}}{D} = 0.05\ Re_D = 0.05\ (1000) = 50$$

The thermal entrance for this problem can be calculated from Equation (7.8)

$$\frac{x_{fd,T}}{D} = 0.05\ Re_D\ Pr = 0.05\ (1000)(1) = 5$$

Therefore, in the first case, the temperature and velocity profiles are not fully developed and the 'short duct approximation' curve will be used

$$Re_D\ Pr\ \frac{D}{L} \times 10^{-2} = 1000\ (1)\left(\frac{1}{2}\right) \times 10^{-2} = 5$$

From Figure 7.12, $Nu \approx 14$

For $\frac{L}{D} = 10$

$$Re_D\ Pr\ \frac{D}{L} \times 10^{-2} = 1000\ (1)\left(\frac{1}{10}\right) \times 10^{-2} = 0.1$$

From Figure 7.12, for a parabolic velocity distribution, $Nu \approx 7.5$

For $\frac{L}{D} = 100$

$$Re_D\ Pr\ \frac{D}{L} \times 10^{-2} = 1000\ (1)\ \frac{1}{100} \times 10^{-2} = 0.1$$

From Figure 7.12 for a parabolic velocity distribution, $Nu \approx 4.1$

Finally, the Sieder and Tate correlations contained in Equation (7.42) can be applied (since $Pr = 1$ implies that the fluid is a liquid)

$$Nu_{D_H} = 1.86\left(Re_D Pr \frac{D}{L}\right)^{0.33}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

For $\frac{L}{D} = 2$ $\quad Nu_{D_H} = 1.86\left[1000\ (1)\left(\frac{1}{2}\right)\right]^{0.33}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 14.8\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \approx 14.8$

Similarly for $\quad \frac{L}{D} = 10 \rightarrow Nu \approx 8.6 \qquad \frac{L}{D} = 100 \rightarrow Nu \approx 4.0$

Tabulating the results

	Nusselt Numbers, *Nu*		
L/*D*	2	10	100
Hausen Correlation	13.1	7.2	4.2
Figure 7.10	14	7.5	4.1
Sieder and Tate Correlation	14.8	8.6	4.0
Average	14.0	7.8	4.1
Maximum % Variation from Average	6%	10%	2%

COMMENTS

The agreement among the three correlations is within the accuracy of empirical correlations.

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PROBLEM 7.14

> **Water enters a double pipe heat-exchanger at 60°C. The water flows on the inside through a copper tube 2.54 cm *ID* at a velocity of 2 cm/s. Steam flows in the annulus and condenses on the outside of the copper tube at a temperature of 80°C. Calculate the outlet temperature of the water if the heat exchanger is 3-m-long.**

GIVEN

- Water flow through a tube in a double pipe heat-exchanger
- Water entrance temperature ($T_{b,\text{in.}}$) = 60°C
- Inside tube diameter (D) = 2.54 cm = 0.0254 m
- Water velocity (V) = 2 cm/s = 0.02 m/s
- Steam condenses at (T_s) = 0.80°C on the outside of the pipe
- Length of heat exchanger (L) = 3 m

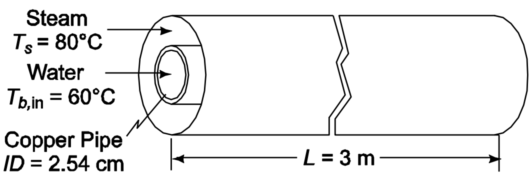
FIND

- Outlet temperature of the water ($T_{b,\text{out}}$)

ASSUMPTIONS

- Steady state
- Thermal resistance of the copper pipe is negligible
- Pressure in the annulus is uniform therefore, T_s is uniform
- Heat transfer coefficient of the condensing steam is large (see Table 1.4) so its thermal resistance can be neglected
- Outside surface of the heat exchanger is insulated

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the inlet temperature of 60°C

Specific heat (c) = 4182 J/(kg K)
Thermal conductivity (k) = 0.657 W/(m K)
Kinematic viscosity (ν) = 0.480×10^{-6} m²/s
Prandtl number (Pr) = 3.02
Density (ρ) = 982.8 kg/m³

The absolute viscosity is $\mu_b = 484 \times 10^{-6}$ (Ns)/m² at 60°C

$\mu_s = 357 \times 10^{-6}$ (Ns)/m² at 80°C

SOLUTION

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{(0.02\,\text{m/s})\,(0.0254\,\text{m})}{0.480 \times 10^{-6}\,\text{m}^2/\text{s}} = 1058 \text{ (Laminar)}$$

The thermal entrance length is given by Equation (7.8)

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$$\frac{x_{fd}}{D} = 0.05\, Re_D\, \Pr = 0.05\,(1058)\,(3.02) = 159.8 \rightarrow x_{fd} = 159.8\,(0.0254\text{m}) = 4.06\text{m} > L$$

Therefore, the flow is not fully developed and the Sieder and Tale correlation, Equation (7.42) will be used

$$\overline{Nu}_{D_H} = 1.86\left(Re_D Pr\,\frac{D}{L}\right)^{0.33}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\overline{Nu}_{D_H} = 1.86\left[1058\,(3.02)\left(\frac{0.0254}{3\text{m}}\right)\right]^{0.33}\left(\frac{484}{357}\right)^{0.14} = 5.76$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 5.76\,\frac{0.657\,\text{W/(m K)}}{0.0254\,\text{m}} = 149.1\ \text{W/(m}^2\ \text{K)}$$

The outlet temperature is given by Equation (7.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s} = \exp\left(-\frac{PL\bar{h}_c}{\dot{m}c_p}\right) = \exp\left(-\frac{(\pi D)L\bar{h}_c}{\rho V\left(\frac{\pi}{4}D^2\right)c_p}\right)$$

Solving for the bulk water outlet temperature

$$T_{b,\text{out}} = T_s + (T_{b,\text{in}} - T_s)\exp\left(-\frac{P\bar{h}_c L}{\rho V D c_p}\right)$$

$$T_{b,\text{out}} = 80°\text{C} + (60°\text{C} - 80°\text{C})\exp$$

$$\left(-\frac{4\ 149.1\,\text{W/(m}^2\,\text{K)}\ (3\text{m})}{982.8\,\text{kg/m}^3\ \ 0.02\,\text{m/s}\ (0.0254\,\text{m})\ 4182\,\text{J/(kg K)}\ \ \text{Ws/J}}\right) = 71.5°\text{C}$$

Performing a second iteration using the water properties at the average temperature of 66°C

$c = 4186$ J/(kg K)	$\nu = 0.434 \times 10^{-6}$ m²/s	$Re_D = 1171$
$\rho = 988.1$ kg/m³	$Pr = 2.71$	$\overline{Nu}_D = 5.68$
$k = 0.662$ W/(m K)	$\mu_b = 440.9 \times 10^{-6}$ (Ns)/m²	$\bar{h}_c = 147.9$ W/(m² K)
		$T_{b,\text{out}} = 71.4°\text{C}$

COMMENTS

The negligible change of $T_{b,\text{out}}$ in the second iteration could be expected because the changes in the water properties are small.

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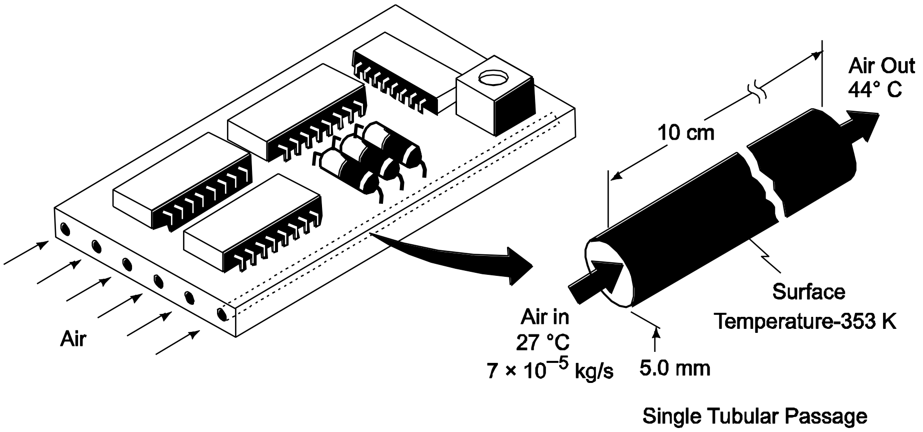
PROBLEM 7.15

An electronic device is cooled by passing air at 27°C through six small tubular passages in parallel drilled through the bottom of the device in parallel as shown. The mass flow rate per tube is 7 × 10^{-5} kg/s.



Heat is generated in the device resulting in approximately uniform heat flux to the air in the cooling passage. To determine the heat flux, the air outlet temperature is measured and found to be 77°C. Calculate the rate of heat generation, the average heat transfer coefficient, and the surface temperature of the cooling channel at the center and at the outlet.

GIVEN

- Air flow through small tubular passages as shown above
- Air temperature
  - Entrance ($T_{b,\text{in}}$) = 27°C
  - Exit ($T_{b,\text{out}}$) = 77°C
- Mass flow rate per passage ($\dot{m}$)= 7 × 10^{-5} kg/s
- Number of passages (N) = 6

FIND

(a) The rate of heat generation ($\dot{Q}_G$)
(b) The average heat transfer coefficient ($\bar{h}_c$)
(c) Cooling channel surface temperature at the center ($T_{s,c}$)
(d) Cooling channel surface temperature at the outlet ($T_{s,\text{out}}$)

ASSUMPTIONS

- Steady state
- Uniform heat generation
- Uniform heat flux to the air
- Viscosity variation is negligible
- Heat transfer coefficient is approximately constant axially

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the average bulk temperature of 52°C

Specific heat (c) = 1016 J/(kg K)
Thermal conductivity (k) = 0.0273 W/(m K)

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Absolute viscosity (μ) = 19.593×10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71

SOLUTION

The Reynolds number is

$$Re_D = \frac{V D \rho}{\mu} = \frac{4 \dot{m}}{\pi D \mu} = \frac{4\ 7 \times 10^{-5}\ \text{kg/s}}{\pi\ 0.005\,\text{m}\ \ 19.593 \times 10^{-6}\ (\text{Ns})/\text{m}^2\ \ (\text{kg m})/(\text{Ns}^2)} = 910 \text{ (Laminar)}$$

The thermal entrance length is given by Equation (7.8)

$$\frac{x_{fd}}{D} = 0.05\ Re_D\ Pr = 0.05\ (910)\ (0.71) = 32.3 \rightarrow x_{fd} = 32.3\ (0.005\text{ m}) = 0.16\text{ m} > L$$

Therefore, the temperature profile is not fully developed.

(a) The total rate of heat generation can be obtained by an energy balance

$$\dot{q}_G = N \dot{m}_{\text{total}}\ c(T_{b,\text{out}} - T_{b,\text{in}}) = 6\ \ 7 \times 10^{-5}\ \text{kg/s}\ \ \ 1016\ \text{J/(kg K)}\ \ \ \text{W s/J}\ \ (77°\text{C} - 27°\text{C}) = 21.3\text{ W}$$

(b) The Nusselt number for this geometry with uniform heat flux and fully developed flow is given Table 7.1 as $\overline{Nu} = 4.364$. Since no correction for entrance effect in a tube with uniform heat flux boundary is given in the text, the fully developed value will be used.

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 4.36\ \frac{0.0273\,\text{W/(m K)}}{0.005\,\text{m}} = 23.8\text{ W/(m}^2\text{ K)}$$

(c) The surface temperature at the center is the average surface temperature (T_s) given by

$$q = \bar{h}_c\ 6\ \pi D L\ (T_s - T_{b,\text{ave}}) = \dot{q}_G$$

Solving for the duct surface temperature

$$T_s = \frac{\dot{q}_G}{\bar{h}_c 6 \pi D L} + \frac{T_{b,\text{in}} + T_{b,\text{out}}}{2} = \frac{21.3\,\text{W}}{23.8\,\text{W/(m}^2\text{K)}\ \ 6\pi\,(0.005\,\text{m})\,(0.1\,\text{m})} + \frac{27\,°\text{C} + 77\,°\text{C}}{2} = 147°\text{C}$$

(d) The heat flux to be air is

$$\frac{q}{A} = \frac{\dot{q}_G}{A} = \frac{\dot{q}_G}{6\pi DL} = \frac{21.3\,\text{W}}{6\pi\,(0.005\,\text{m})\,(0.1\,\text{m})} = 2260\text{ W/m}^2$$

The surface temperature at the outlet is given be

$$\frac{q}{A} = h_{cL}\ (T_{s,\text{out}} - T_{b,\text{out}}) \rightarrow T_{s,\text{out}} = \frac{q}{A}\,\frac{1}{h_{cL}} = T_{b,\text{out}}$$

$$\therefore\ (T_{s,\text{out}})_{\max} = \frac{2260\ \text{W/m}^2}{23.8\ \text{W/(m}^2\text{K)}} + 77°\text{C} = 172°\text{Cs}$$

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PROBLEM 7.16

Unused engine oil with a 100°C inlet temperature flows at a rate of 0.25 kg/sec through a 5.1-cm-*ID* pipe that is enclosed by a jacket containing condensing steam at 150°C. If the pipe is 9 m long, determine the outlet temperature of the oil. Also at what length of the pipe would the oil temperature be at 110^{0}C?

GIVEN

- Unused engine oil flows through a pipe enclosed by a jacket containing condensing steam.
- Oil flow rate, $\dot{m}$ = 0.25 kg/s.
- Oil inlet temperature, $T_{b,\text{in}}$ = 100°C.
- Inner or inside diameter of pipe in which oil flows, D = 5.1 cm = 0.051 m.
- Length of heated pipe (heated by condensing steam) in which oil flows, L = 9 m.
- Temperature of condensing steam, T_s = 150°C.

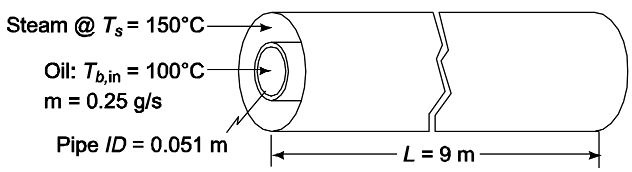
FIND

- Temperature of oil, $T_{b,\text{out}}$, at the outlet of the 9-m-long heated pipe.
- Length of pipe at which oil temperature will be 110^{0}C.

ASSUMPTIONS

- Steady-state flow of oil and its heating by the condensing steam in the outer jacket.
- The temperature of condensing steam is constant and uniform across the length of pipe.
- The thermal resistance of the pipe is negligible, and hence the inside surface temperature of the pipe is $T_w = T_s$, this represents a uniform pipe surface temperature condition.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 17, for unused engine oil at $T_{b,\text{in}}$ = 100°C

Density, ρ = 840.0 kg/m^3

Thermal conductivity, k = 0.137 W/(m K)

Absolute viscosity, μ_b = 17.1 × 10^{-3} (Ns)/m^2

Prandtl number, Pr = 276

Specific heat, c_p = 2219 J/(kg K)

At the pipe surface temperature of 150°C, the absolute viscosity μ_s = 5.52 × 10^{-3} (Ns)/m^2

SOLUTION

The Reynolds number for oil flow inside the pipe is

$$\text{Re}_D = \frac{\rho VD}{\mu_b} = \frac{4\dot{m}}{\pi D \mu_b} = \frac{4 \times 0.25}{\pi \times 0.051 \times 0.017} = 367.1 \Rightarrow \text{Laminar flow}$$

The thermal entrance length is given by Equation (7.8) for laminar flow, and it can be calculated as

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$$x_{fd} = 0.05D\,\text{Re}_D\,\text{Pr} = 0.05 \times 0.051 \times 367 \times 276 = 258 \text{ m} \gg L = 9 \text{ m}$$

Hence, the temperature profile is NOT fully developed, or the flow is thermally developing.

Because there is a large variation in the oil viscosity at the pipe wall temperature and the bulk temperature, the effect of property (viscosity) variation has to be considered. From Section 7.3.3 either the Hausen correlation of Equation (7.43) or the Sieder and Tate correlation of Equation (7.44) could be used because (μ_b/μ_s) = 3.1 (< 9.75; the limit for Equation (7.44) to calculate the Nusselt number. Thus, using the more simpler Sieder and Tate correlation

$$\overline{\text{Nu}}_D = 1.86\left(\frac{\text{Re}_D\,\text{Pr}\,D}{L}\right)\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 1.86\left(\frac{367 \times 276 \times 0.051}{9}\right)\left(\frac{0.0171}{0.00552}\right)^{0.14} = 1251$$

$$\Rightarrow \qquad \bar{h}_c = \overline{\text{Nu}}_D \frac{k}{D} = 1251\frac{0.137}{0.051} = 3361 \text{ W/(m}^2\text{ K)}$$

The outlet temperature can now be calculated by Equation (7.36) as

$$\frac{\Delta T_{out}}{\Delta T_{in}} = \exp\left(-\frac{\bar{h}_c PL}{\dot{m}c_p}\right) \qquad \Rightarrow \qquad T_{b,out} = T_s + \left(T_{b,in} - T_s\right)\exp\left(-\frac{\bar{h}_c PL}{\dot{m}c_p}\right)$$

$$\therefore \quad T_{b,\,out} = 150 + \left(100 - 150\right)\exp\left(-\frac{3361 \times \pi \times 0.051 \times 9}{0.25 \times 2219}\right) = 149.9 \approx 150°\text{C}$$

When the oil temperature reaches 110^0C,

$$\frac{\Delta T_{out}}{\Delta T_{in}} = \exp\left(-\frac{\bar{h}_c PL}{\dot{m}c_p}\right)$$

$$\frac{40}{50} = \exp\left(-\frac{3361 \times \pi \times 0.051 \times L}{0.25 \times 2219}\right)$$

$$\text{L} = -\frac{\ln(40/50) * 0.25 * 2219}{3361 * \pi * 0.051} \text{ m}$$

$$\text{L} = 0.19 \text{ m}$$

Thus oil temperature reaches 110^0C at 0.19 m.

COMMENTS

The oil flow attains the tube wall (or the condensing steam) temperature at the outlet of the 9-m-long pipe. Also, because of the 50°C temperature difference between the inlet and the outlet, the above calculation should be repeated after evaluating the properties at the average temperature between the inlet and outlet.

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PROBLEM 7.17

Determine the rate of heat transfer per meter length to a light oil flowing through a 2.5 cm-*ID*, 60 cm-long copper tube at a velocity of 0.03 m/s. The oil enters the tube at 16°C and the tube is heated by steam condensing on its outer surface at atmospheric pressure with a heat transfer coefficient of 11.3 kW/(m^2 K). The properties of the oil at various temperatures are listed in the accompanying tabulation

T(°C)	15	30	40	65	100
ρ (kg/m^3)	912	912	896	880	864
$c\,(kJ/(\text{kg }K))$	1.8	1.84	1.925	2	2.135
$k\,(W/(\text{m }K))$	0.133	0.133	0.131	0.129	0.128
μ (kg/m s)	0.089	0.0414	0.023	0.00786	0.0033
Pr	1204	573	338	122	55

GIVEN

- Oil flowing through a copper tube with atmospheric pressure steam condensing on the outer surface
- Oil properties listed above
- Inside diameter (D) = 2.5 cm = 0.025 m
- Tube length (L) = 60 cm = 0.6 m
- Oil velocity (V) = 0.03 m/s
- Inlet oil temperature ($T_{b,\text{in}}$) = 16°C
- Heat transfer coefficient on outside of pipe ($\bar{h}_{c,o}$)= 11.3 kW/(m^2 K)

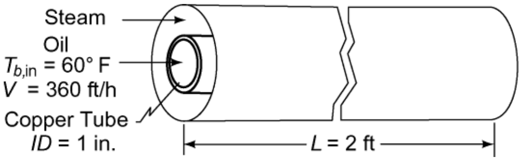
FIND

- The rate of heat transfer (q) to the oil

ASSUMPTIONS

- Steady state
- The thermal resistance of the copper tube is negligible
- Constant wall temperature
- The tube wall is thin

SKETCH



PROPERTIES AND CONSTANTS

At atmospheric pressure, steam condenses at a temperature (T_s) of 100°C.

SOLUTION

The Reynolds number for the oil flowing through the pipe is

$$Re_D = \frac{V\,D\,\rho}{\mu}$$

Using the oil properties at the inlet temperature of 16°C

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$$Re_D = \frac{(0.03 m/s)\,(0.025)\left(912 kg/m^3\right)}{(0.089\, kg/(ms))} = 7.68 \text{ (Laminar)}$$

The thermal entrance length is given by Equation (7.8)

$$\frac{x_{fd}}{D} = 0.05\, Re_D\, Pr = 0.05\,(7.68)\,(1210) = 465 \rightarrow x_{fd} = 465\,(0.025 \text{ m}) = 11.6 \text{ m} >> \text{L}$$

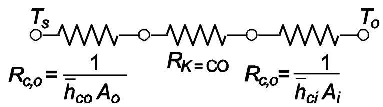
Therefore, the temperature profile is not fully developed and the Hausen correlation of Equation (7.41) will be used (assuming the wall temperature ≈ T_s for μ_s)

$$\overline{Nu} = \left[3.66 + \frac{0.0668 Re_{D_H} Pr\left(\frac{D}{L}\right)}{1 + 0.045\left[Re_{D_H} Pr\left(\frac{D}{L}\right)\right]^{0.66}}\right]\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\overline{Nu} = \left[3.66 + \frac{0.0668\,(7.68)\,(1204)\left(\frac{0.025}{0.6}\right)}{1 + 0.045\left[(7.68)\,(1204)\left(\frac{0.025}{0.6}\right)\right]^{0.66}}\right]\left(\frac{0.089}{0.0033}\right)^{0.14} = 18.5$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 18.5\, \frac{(0.133 W/(m K))}{0.025 \text{ m}} = 98.4 \text{ W/ (m}^2\text{ K)}$$

The thermal circuit for heat flow from the steam to the oil is shown below



If the tube wall is thin, $A_o \approx A_i = \pi DL = \pi(0.025 \text{ m})(0.6 \text{ m}) = 0.047 \text{ m}^2$ and the thermal resistance is

$$A\,R_{co} = \frac{1}{\left(11300\, W/(\text{m}^2\, K)\right)} = 8.85{*}10^{-5} \text{ (m}^2\text{ K)/W}$$

$$A\,R_{co} = \frac{1}{\left(98.4 W/(\text{m}^2\, K)\right)} = 0.01016 \text{ (m}^2\text{ K)/W}$$

$$A\,R_{\text{total}} = A\,R_{co} + A\,R_{ci} = (\,8.85{*}10^{-5} + 0.01016)\ \left((m^2\, K)/\text{W}\right) = 0.01025 \text{ (m}^2\text{ K)/W}$$

The outlet temperature can be calculated by replacing $h_c A$ by $1/AR_{\text{total}}$ in Equation (7.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s} = \exp\left(-\frac{P\,L}{(A\,R_{\text{total}})\,\dot{m}\,c_p}\right) = \exp\left(-\frac{\pi DL}{(A\,R_{\text{total}})\,(\rho VA)\,c_p}\right)$$

$$T_{b,\text{out}} = T_s + (T_{b,\text{in}} - T_s)\,\exp\left(-\frac{4L}{(A\,R_{\text{total}})\,D\rho VA\,c_p}\right)$$

$$T_{b,\text{out}} = 100°\text{C} + (15°\text{C} - 100°\text{C})$$

$$\exp\left(-\frac{4(0.6m)}{\left(0.01025(m^2\,K)/W\right)(0.025\,m)\left(912\,kg/m^3\right)(0.03 m/s)(1800 W/(\text{kg K}))}\right) = 29.7°\text{C}$$

This is a significant change in the oil temperature and warrants another iteration using the properties of the oil at the average bulk temperature of 22.4°C. Interpolating the oil properties from the given data

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$$\rho = 913 \text{ kg/m}^3 \qquad Re = 12.1$$

$$c = 1842 \text{ W/(kg K)} \qquad \overline{Nu}_D = 17.3$$

$$k = 0.133 \text{ W/(m K)} \qquad A\,R_{\text{total}} = 0.0105 \ \left((m^2\,K)/W\right)$$

$$\mu_b = 0.0653 \text{ kg/(s m)} \qquad T_{b,\text{out}} = 30°\text{C}$$

$$Pr = 802$$

The rate of heat transfer is given by Equation (7.37) substituting $1/A\,R_{\text{total}}$ for h_c

$$q_c = \frac{A}{A\,R_{\text{total}}}\left[\frac{\Delta T_{\text{out}} - \Delta T_{\text{in}}}{\ln \dfrac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}}}\right]$$

$$q_c = \frac{0.0471\,m^2}{0.0105\,(m^2\,K)/W}\left[\frac{(100-30)°\text{F} - (100-15)°\text{F}}{\ln\left(\dfrac{100-30}{100-15}\right)}\right] = 346.5 \text{ W}$$

COMMENTS

Note that 99% of the thermal resistance is on the inside of the pipe.

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PROBLEM 7.18

Lubricating oil is cooled in a tubular heat exchanger to maintain its viscosity and effectiveness in the journal bearings used in a large steam turbine of an electric power plant. Oil flows at the rate of 0.1 kg/s inside a 12.5-mm-diameter circular tube, which is maintained at a uniform surface temperature of 25°C, and the oil is cooled from 80°C to 40°C. Calculate the length of tube required for this heat exchanger, the total heat flux dissipated from its surface in the cooling process, and the pressure drop across the tube length.

GIVEN

- Lubricating oil cooled in tubular heat exchanger.
- Oil flow rate, $\dot{m}$ = 0.1kg/s.
- Oil inlet temperature, $T_{b,\mathrm{in}}$ = 80°C.
- Oil outlet temperature $T_{b,out}=40^0C$
- Inner or inside diameter of pipe in which oil flows, D = 12.5 mm = 0.0125 m.
- Temperature of pipe surfaceT_s = 25°C.

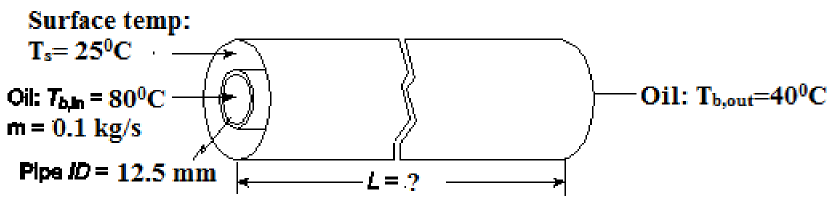
FIND

- Length of tube required for heat exchanger
- Total heat flux dissipated from its surface in cooling process.
- Pressure drop across the tube length.

ASSUMPTIONS

- The temperature of wall is constant and uniform across the length of pipe.
- The thermal resistance of the pipe is negligible, and hence the inside surface temperature of the pipe is $T_w = T_s$, this represents a uniform pipe surface temperature condition.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 17, for unused engine oil at T_b= 60°C

Density, ρ = 864.0 kg/m^3

Thermal conductivity, k = 0.140 W/(m K)

Absolute viscosity, $\mu_b = 72.5 \times 10^{-3}$ (Ns)/m^2

Prandtl number, Pr = 1050

Specific heat, c_p = 2047 J/(kg K)

At the pipe surface temperature of 25°C, the absolute viscosity $\mu_s = 652 \times 10^{-3}$ (Ns)/m^2

SOLUTION

The Reynolds number for oil flow inside the pipe is

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$$\text{Re}_D = \frac{\rho VD}{\mu_b} = \frac{4\dot{m}}{\pi D \mu_b} = \frac{4\times 0.1}{\pi \times 0.0125\times 0.0725} = 140.5 \Rightarrow \text{Laminar flow}$$

x_{fd}= $0.05D\,\text{Re}_D\,\text{Pr} = 0.05\times 0.0125\times 135.5\times 1050$ = 92.2 m

Total heat dissipated from the tube is given by

$$q_a = \dot{m}\,c_p(T_{b,in} - T_{b,out})$$

$$q_a = 0.1*2047*(80-40)\ \text{W}$$

$$q_a = 0.1*2047*(80-40)\,\text{W}$$

$$q_a = 8188\ \text{W}$$

Also we have

$$q_a = \bar{h}_c * \pi DL * \frac{\Delta T_{out} - \Delta T_{in}}{\ln\left(\Delta T_{out}/\Delta T_{in}\right)}$$

$$8188 = \bar{h}_c * \pi * 0.0125 * L * \frac{15-55}{\ln\left(15/55\right)}$$

$$\bar{h}_c L = 6772\ \text{W/m} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ (\text{I})$$

Calculating thermal entrance length we have

For, laminar flow we have entrance length given as

$$\left(\frac{x_{fd}}{D_i}\right) = 0.05\,\text{Re}_D\,\text{Pr}$$

$$x_{fd} = 0.05\,\text{Re}_D * D_i * \text{Pr}$$

$$x_{fd} = 0.05*140.5*1.25*10^{-2}*1050 = 92.2\ \text{m}$$

Considering this length and we have $\dfrac{L}{D\,\text{Re}_D\,\text{Pr}}$ =0.05 for which we use Nu correlation as

using Equation (7.40) we have

$$Nu = 3.657 + \left(0.0499\,\frac{D\,\text{Re}_D\,\text{Pr}}{L}\right) = 4.655$$

$$\bar{h}_c = \frac{k}{D}\left(3.657 + \left(0.0499 D\,\text{Re}_D\,\text{Pr}/L\right)\right) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ (\text{II})$$

From which $\bar{h}_c = 4.655*\dfrac{0.140}{0.0125}$ =52.1 W/(m^2 K)

Susbtituting this value of $\bar{h}_c$ in equation (I) we get

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L= 129.9 m

Now $\frac{L}{D\,\mathrm{Re}_D\,\mathrm{Pr}} = 0.07$

Considering L=129.9 m we have $\frac{L}{D\,\mathrm{Re}_D\,\mathrm{Pr}}$ =0.07

Substituting this value of L in equation (II) above we get

$\bar{h}_c = 4.4 * \frac{0.140}{0.0125}$ =49.3 W/(m^2 K)

Now substituting this value of $\bar{h}_c$ in (I) we get

L=137.4 m

Now, solving the problem iteratively further we get to

$\bar{h}_c$ =48.3 W/(m^2 K)   L=140.2 m

On further iterating we reach the final solution of L=140.2 m which is the required length of the pipe.

Total heat flux dissipated is given by $q_a = 8188$ W

The pressure drop in pipe flow is given by

$$\Delta p = f\,\frac{L}{D}\left(\frac{\rho \bar{U}^2}{2g_c}\right)$$ where f is given by equation (7.18) as

$$f = \frac{64}{\mathrm{Re}_D}\left(\frac{\mu_s}{\mu_b}\right)^{0.14} = \frac{64}{140.5}\left(\frac{652}{72.5}\right)^{0.14} = 0.62$$

Thus

$$\Delta p = 0.62\,\frac{140.2}{0.0125}\left(\frac{864*(0.943)^2}{2*1}\right) \text{ Pa} = 2.6*10^6 \text{ Pa}$$

COMMENTS

We can see that the length required for the pipe is extremely large. This is due to the high temperature difference and high mass flow rate. In practical cases, the flowrate is distributed among the bundles of tubes arranged in rows and columns in parallel. As the thermal entrance length is quite high, the high heat transfer rate can be achieved by doing so due to high heat transfer coefficient.

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PROBLEM 7.19

A large high-power transformer is installed and operated in an electric-power distribution station. To maintain transform efficiency and prevent its failure (burnout), the transformer oil is cooled via a water-cooled, coiled tube heat exchanger and circulated through the transformer winding module. For one such application, transformer oil, with a flow rate of 0.3 kg/s, is to be cooled to 30°C while flowing through the coiled thin walled metallic tubing. The tube has an inner diameter of 5.0 cm, and its surface is maintained uniformly at a temperature of 20°C. If the oil enters the heat exchanger tube at a temperature of 50°C, and if the coil effects can be ignored and tube modeled as a straight circular pipe, what is the tube length required for this heat exchanger? What is the expected pressure drop across the tube length? Also, if the tube surface temperature increases to 25°C, determine the outlet temperature of transformer oil for the calculated pipe length.

GIVEN

- Transformer oil cooled via water-cooled, coiled tube heat exchanger.
- Oil flow rate, $\dot{m}$ = 0.3 kg/s.
- Oil inlet temperature, $T_{b,\text{in}}$ = 50°C.
- Oil outlet temperature $T_{b,\text{out}}=30^0$C
- Inner or inside diameter of pipe in which oil flows, D = 5 cm = 0.05 m.
- Temperature of pipe surfaceT_s = 20°C.

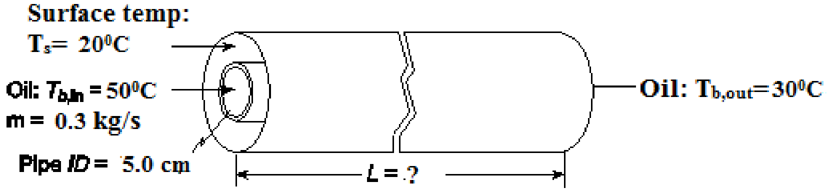
FIND

- Length of tube required for heat exchanger
- Oulet temperature is surface temperature reaches to 25^0C for given length.

ASSUMPTIONS

- The temperature of wall is constant and uniform across the length of pipe.
- The thermal resistance of the pipe is negligible, and hence the inside surface temperature of the pipe is $T_w = T_s$, this represents a uniform pipe surface temperature condition.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 18, for Transformer oil at T_b= 40°C

Density, ρ = 867.0 kg/m^3

Thermal conductivity, k = 0.109 W/(m K)

Absolute viscosity, μ_b = 9.3 × 10^{-3} (Ns)/m^2

Prandtl number, Pr = 157

Specific heat, c_p = 1830 J/(kg K)

At the pipe surface temperature of 25°C, the absolute viscosity μ_s = 21.1 × 10^{-3} (Ns)/m^2

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SOLUTION

The Reynolds number for oil flow inside the pipe is

$$\mathrm{Re}_D = \frac{\rho VD}{\mu_b} = \frac{4\dot{m}}{\pi D \mu_b} = \frac{4\times 0.3}{\pi\times 0.05\times 0.0093} = 821.5 \Rightarrow \text{Laminar flow}$$

$x_{fd}= 0.05D\,\mathrm{Re}_D\,\mathrm{Pr} = 0.05\times 0.05\times 821.2\times 1.57*10^2 = 322$ m

Total heat dissipated from the tube is given by

$$q_a = \dot{m} c_p (T_{b,in} - T_{b,out})$$

$$q_a = 0.3*1830*(50-30)\ \text{W}$$

$$q_a = 10980\ \text{W}$$

Also we have

$$q_a = \bar{h}_c * \pi DL * \frac{\Delta T_{out} - \Delta T_{in}}{\ln\left(\Delta T_{out}/\Delta T_{in}\right)}$$

$$10980 = \bar{h}_c * \pi * 0.05 * L * \frac{10-30}{\ln(10/30)}$$

$$\bar{h}_c L = 3839\ \text{W/(m K)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(I)}$$

Calculating thermal entrance length we have

For, laminar flow we have entrance length given as

$$\left(\frac{x_{fd}}{D_i}\right) = 0.05\,\mathrm{Re}_D\,\mathrm{Pr}$$

$$x_{fd} = 0.05\,\mathrm{Re}_D * D_i * \mathrm{Pr}$$

$$x_{fd} = 0.05*821.5*0.05*157 = 322\ \text{m}$$

Considering this length and we have $\dfrac{L}{D\,\mathrm{Re}_D\,\mathrm{Pr}}$ =0.05 for which we use Nu correlation in Equation (7.40)

$$Nu = 3.657 + \left(0.0499\frac{D\,\mathrm{Re}_D\,\mathrm{Pr}}{L}\right) = 4.655$$

$$\frac{\bar{h}_c D}{k} = 4.655$$

$$\bar{h}_c = \frac{k}{D}\left(3.657 + \left(0.0499 D\,\mathrm{Re}_D\,\mathrm{Pr}/\,L\right)\right)$$

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From which $\bar{h}_c = 4.655 * \frac{0.109}{0.05}$ =10 W/(m^2 K)

Susbtituting this value of $\bar{h}_c$ in equation (I) we get

L= 383.9 m

Now $\frac{L}{D \operatorname{Re}_D \operatorname{Pr}} = 0.06$

Considering L=383.9 m we have $\frac{L}{D \operatorname{Re}_D \operatorname{Pr}}$ =0.06

Thus, using Equation (7.40) we have

$\overline{Nu}_D = 3.657 + \left(0.0499 D \operatorname{Re}_D \operatorname{Pr}/ L\right)$

$\bar{h}_c = \frac{k}{D}\left(3.657 + \left(0.0499 D \operatorname{Re}_D \operatorname{Pr}/ L\right)\right)$ …………………………….. (II)

From which $\bar{h}_c = 4.50 * \frac{0.109}{0.05}$ =9.8 W/(m^2 K)

Now substituting value of $\bar{h}_c$ in equation (I) we get

L= 391.7

Now $\frac{L}{D \operatorname{Re}_D \operatorname{Pr}} = 0.061$

Now, solving the problem iteratively further we get to

$\bar{h}_c$ =9.75 W/(m^2 K)   L=393.7 m

On further iterating we reach the final solution of L=393.7 m which is the required length of the pipe.

The pressure drop in pipe flow is given by

$$\Delta p = f \frac{L}{D}\left(\frac{\rho \bar{U}^2}{2 g_c}\right)$$ where f is given by equation (7.18) as

$$f = \frac{64}{\operatorname{Re}_D}\left(\frac{\mu_s}{\mu_b}\right)^{0.14} = \frac{64}{821.5}\left(\frac{21.1}{9.3}\right)^{0.14} = 0.087$$

Thus

$$\Delta p = 0.087 * \frac{393.7}{0.05}\left(\frac{867 * (0.176)^2}{2 * 1}\right) \text{ Pa} = 9.2*10^3 \text{ Pa}$$

If the temperature increases to T_s =25^{0}C

From Equation (7.36) we have

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$$\frac{\Delta T_{out}}{\Delta T_{in}} = \exp\left(\frac{-\bar{h}_c PL}{\dot{m}c_p}\right) = \exp\left(\frac{-9.75*\pi*0.05*393.7}{0.3*1830}\right)$$

$$\frac{T_{out} - T_s}{T_{in} - T_s} = \exp(-1.096) = 0.334$$

$$\frac{T_{out} - 25}{50 - 25} = 0.334$$

$T_{out} = 33.4^0C$

Thus outlet temperature increases to 33.4^0C if surface temperature is increased to 25^0C.

COMMENTS

We can see that the length required for the pipe is extremely large. This is due to the high temperature difference and high mass flow rate. In practical cases, the flowrate is distributed among the bundles of tubes arranged in rows and columns in parallel. As the thermal entrance length is quite high, the high heat transfer rate can be achieved by doing so due to high heat transfer coefficient.

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PROBLEM 7.20

Water at 20°C enters a 1.91-cm-*ID*, 57-cm-long tube at a flow rate of 3 gm/s. The tube wall is maintained at 30°C. Determine the water outlet temperature. What error in the water temperature results if natural convection effects are neglected?

GIVEN

- Water flowing through a tube
- Entering water temperature ($T_{b,\text{in}}$) = 20°C
- Tube inside diameter (D) = 1.91 cm = 0.0191 m
- Tube length (L) = 57 cm = 0.57 m
- Mass flow rate (m) = 3 gm/s = 0.003 kg/s
- Tube wall surface temperature (T_s) = 30°C

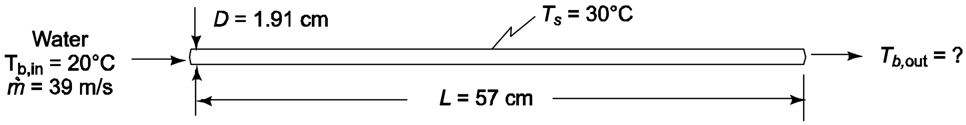
FIND

(a) The water outlet temperature ($T_{b,\text{out}}$)
(b) Percent error in water temperature rise if natural convection is neglected

ASSUMPTIONS

- Steady state
- Tube temperature is uniform and constant
- The tube is horizontal

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 20°C
Specific heat (c) = 4182 J/(kg K)
Thermal conductivity (k) = 0.597 W/(m K)
Kinematic viscosity (ν) = 1.006 × 10^{-6} m^2/s
Prandtl number (Pr) = 7.0
Absolute viscosity (μ_b) = 993 × 10^{-6} (Ns)/m^2
Thermal expansion coefficient (β) = 2.1 × 10^{-4} 1/K
At 30°C μ_s = 792 × 10^{-6} (Ns)/m^2

SOLUTION

The Reynolds number is

$$Re_D = \frac{VD\rho}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4\ 0.003\,\text{kg/s}}{\pi\ 0.0191\,\text{m}\ \ 993\times10^{-6}\,(\text{N s})/\text{m}^2\ \ (\text{kg m})/(\text{Ns}^2)} = 210.4 \text{ (Laminar)}$$

The Graetz number is

$$Gz = \frac{\pi}{4} Re_D\, Pr \frac{D}{L} = \frac{\pi}{4}\ (201.4)\ (7.0) \left(\frac{0.0191\,\text{m}}{0.57\,\text{m}}\right) = 37.10$$

The Grashof number (from Table 8.3) will be based on the diameter since the tube is horizontal

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$$Gr_D = \frac{g\beta(T_s - T_b)D^3}{\nu^2} = \frac{9.8\,\text{m/s}^2 \; 2.1\times10^{-4}\,1/\text{K} \; (30°\text{C} - 20°\text{C})(0.0191\,\text{m})^3}{1.006\times10^{-6}\,(\text{Ns})/\text{m}^2 \;\; ^2} = 1.42 \times 10^5$$

$$Gr_D\, Pr \frac{D}{L} = 1.42 \times 10^5\,(7.0)\left(\frac{0.0191\,\text{m}}{0.57\,\text{m}}\right) = 3.3 \times 10^4$$

For this value and $Re_D = 200$, Figure 7.12a indicates that the flow is in the 'mixed convection laminar flow' region.

$$\overline{Nu}_D = 1.75\left(\frac{\mu_b}{\mu_s}\right)^{0.14}\left[Gz + 0.12\,(Gz\; Gr_D^{\frac{1}{3}} Pr^{0.36})^{0.88}\right]^{\frac{1}{3}}$$

(a) The Nusselt number can be estimated using Equation (7.47)

$$\overline{Nu}_D = 1.75\left(\frac{\mu_b}{\mu_s}\right)^{0.14}\left[Gz + 0.12\,(Gz\; Gr_D^{\frac{1}{3}} Pr^{0.36})^{0.88}\right]^{\frac{1}{3}}$$

$$\overline{Nu}_D = 1.75\left(\frac{933}{792}\right)^{0.14}\left(37.1 + 0.12\left[(37.1)\,(1.42 \times 10^5)^{\frac{1}{3}}\,(7)^{0.36}\right]^{0.88}\right)^{\frac{1}{3}} = 10.7$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 10.7\,\frac{0.597\,\text{W/(m K)}}{0.0191\,\text{m}} = 334\ \text{W/(m}^2\ \text{K)}$$

The outlet temperature can be calculated from Equation (7.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}} = \exp\left(-\frac{PLh_c}{\dot{m}c_p}\right) = \exp\left(-\frac{h_c \pi DL}{\dot{m}c}\right)$$

$$T_{b,\text{out}} = T_s - (T_s - T_{b,\text{in}})\exp\left(-\frac{h_c \pi DL}{\dot{m}c}\right)$$

$$T_{b,\text{out}} = 30°\text{C} - (30°\text{C} - 20°\text{C})\exp\left(-\frac{334\,\text{W/(m}^2\text{K)}\;\pi\;0.0191\,\text{m}\;\;0.57\,\text{m}}{0.003\,\text{kg/s}\;\;4182\,\text{J/(kg K)}\;\;(\text{Ws})/\text{J}}\right)$$

$$T_{b,\text{out}} = 26°\text{C}$$

The average bulk temperature is 23°C. Another iteration is therefore not warranted because the change in property values will not affect the result appreciably.

(b) Natural convection can be neglected by applying Equation (7.42) for the Nusselt number

$$Nu_D = 1.86\left(Re_D\; Pr \frac{D}{L}\right)^{0.33}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$Nu_D = 1.86\left[201.4\;\; 7.0\;\left(\frac{0.0191\,\text{m}}{0.57\,\text{m}}\right)\right]^{0.33}\left(\frac{933}{792}\right)^{0.14} = 6.8$$

$$h_c = Nu_D \frac{k}{D} = 6.8\,\frac{0.597\,\text{W/(m K)}}{0.0191\,\text{m}} = 212.3\ \text{W/(m}^2\ \text{K)}$$

$$T_{b,\text{out}} = 30° - (30°\text{C} - 20°)\exp\left(-\frac{212.3\,\text{W/(m}^2\text{K)}\;\pi\;0.0191\,\text{m}\;\;0.57\,\text{m}}{0.003\,\text{kg/s}\;\;4182\,\text{J/(kg K)}\;\;(\text{Ws})/\text{J}}\right)$$

$$T_{b,\text{out}} = 24.4°\text{C}$$

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The error in outlet temperature is

$$\text{Error} = 26 - 24.4 = 1.6°\text{C}$$

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PROBLEM 7.21

A solar thermal central receiver generates heat by focusing sunlight with a field of mirrors on a bank of tubes through which a coolant flows. Solar energy absorbed by the tubes is transferred to the coolant which can then deliver useful heat to a load. Consider a receiver fabricated from multiple horizontal tubes in parallel. Each tube is 1-cm-*ID* and 1-m-long. The coolant is molten salt which enters the tubes at 370°C. Under start-up conditions, the salt flow is 10 gm/s in each tube and the net solar flux absorbed by the tubes is 10^4 W/m^2. The tube wall material will tolerate temperatures up to 600°C. Will the tubes survive start-up? What is the salt outlet temperature?

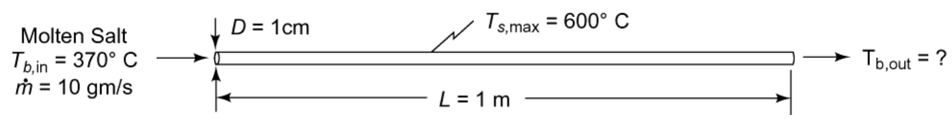
GIVEN

- Molten salt flowing through a horizontal tube that is absorbing solar energy
- Tube inside diameter (D) = 1 cm = 0.01 m
- Tube length (L) = 1 m
- Entering salt temperature ($T_{b,\text{in}}$) = 370°C
- Start-up mass flow rate (m) = 10 gm/s = 0.01 kg/s
- Net solar energy absorbed by the tube (q_s) = 10^4 W/m^2
- Maximum tube wall temperature (T_s) = 600°C

FIND

(a) Salt outlet temperature ($T_{b,\text{out}}$)
(b) Will the tubes survive start-up?

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 24, for molten salt at 370°C

Specific heat (c) = 1629 J/(kg K)

SOLUTION

(a) By the conservation of energy

$$q_s A = \dot{m}\, c(T_{b,\text{out}} - T_{b,\text{in}})$$

$$\therefore \qquad T_{b,\text{out}} = T_{b,\text{in}} + \frac{q_s(\pi DL)}{\dot{m}c} = 370°\text{C} + \frac{10^4\ \text{W/m}^2\ \ \pi\ \ 0.01\,\text{m}\ \ 1.0\,\text{m}}{0.01\,\text{kg/s}\ \ 1629\,\text{J/(kg K)}\ \ (\text{Ws})/\text{J}} = 389°\text{C}$$

Evaluating the molten salt properties at the average bulk temperature of 380°C

Density (ρ) = 1849 kg/m^3
Absolute viscosity (μ) = 1970×10^{-6} (Ns)/m^2
Thermal expansion coefficient (β) = 3.55×10^{-4} 1/K
Thermal conductivity (k) = 0.516 W/(m K)
Kinematic viscosity (ν) = 1.065×10^{-6} m^2/s
Prandtl number (Pr) = 6.18

(b) The Reynolds number is

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$$Re_D = \frac{VD\rho}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4\ 0.01\,\text{kg/s}}{\pi(0.01\,\text{m})\ 1970\times10^{-6}\,(\text{Ns})/\text{m}^2\quad(\text{kg m})/(\text{Ns}^2)} = 646 \text{ (Laminar)}$$

With laminar flow and the high temperature differences possible, natural convection may be important. Since we do not know the tube wall temperature needed to evaluate the Grashof number, an iterative procedure must be used. For the first iteration, let the average tube wall temperature be 10°C above the average bulk salt temperature (T_s = 390°C).

From Appendix 2, Table 24, at the tube temperature of 390°C $\mu_s = 1882 \times 10^{-6}$ (Ns)/m^2

The Graetz number is $$Gz = \frac{\pi}{4} Re_D\, Pr \frac{D}{L} = \frac{\pi}{4}\ (646.3)(6.18) \left(\frac{0.01\,\text{m}}{1\,\text{m}}\right) = 31.37$$

The Grashof number based on the diameter, from Table 5.3 is

$$Gr_D = \frac{g\beta(T_s - T_b)D^3}{\nu^2} = \frac{9.8\,\text{m}^2/\text{s}\quad 3.55\times10^{-4}\,1/\text{K}\ \ (390°\text{C} - 380°\text{C})(0.01\,\text{m})^3}{1.065\times10^{-6}\,(\text{Ns})/\text{m}^2\ \ ^2} = 3.07 \times 10^4$$

$$Gr_D\, Pr \frac{D}{L} = 3.07 \times 10^4\ (6.18)\ (0.01) = 1.9 \times 10^3$$

For this value and Re_D = 6.5 × 102, Figure 7.12a indicates the flow is in the mixed convection regime, therefore, Equation (7.47) will be used to estimate the Nusselt number. Note that this will be a rough estimate since Equation (7.47) is technically only for isothermal tubes.

$$\overline{Nu}_D = 1.75\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \left[Gz + 0.12\ (Gz\ Gr_D^{\frac{1}{3}} Pr^{0.36})^{0.88}\right]^{\frac{1}{3}}$$

$$\overline{Nu}_D = 1.75\ \left(\frac{1970}{1882}\right)^{0.14} \left(31.37.1 + 0.12\left[(31.37)\ (3.07\times10^4)^{\frac{1}{3}}(6.18)^{0.36}\right]^{0.88}\right)^{\frac{1}{3}} = 8.76$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 8.76 \frac{0.516\,\text{W/(m K)}}{0.01\,\text{m}} = 452 \text{ W/(m}^2\text{ K)}$$

The rate of heat transfer to the molten salt is

$$q_c = \bar{h}_c A_t\,(T_s - T_b) = q_s''\, A_t$$

$$\therefore \qquad T_s - T_b = \frac{q_s''}{h_c} = \frac{10^4\,\text{W/m}^2}{452\,\text{W/(m}^2\text{K)}} = 22.1°\text{C}$$

Further iterations are necessary. However, the fluid properties will not change appreciably. Therefore, Re_D, Pr, and Gz will not change.

Iteration #	2	3
T_s (°C)	402	401
μ_s × 106	1791	1798
$T_s - T_b$ (°C)	22.1	20.7
$Gr_D \times 10^{-4}$	6.78	6.35
$\overline{Nu}_D$	9.36	9.31
$\bar{h}_c$ W/(m^2 K)	483	481
$T_s - T_b$ (°C)	20.7	20.8

The maximum tube wall temperature is therefore

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$$T_{b,\text{out}} + (T_s - T_b) = 389°\text{C} + 21°\text{C} = 410°\text{C}$$

which is well below the tube melting point. The tube will have no problems surviving the start-up in good shape.

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PROBLEM 7.22

Calculate the Nusselt number and the convection heat transfer coefficient for water at a bulk temperature of 32°C flowing at a velocity of 1.5 m/s through a 2.54-cm-*ID* duct with a wall temperature of 43°C. Use the Gnielinski correlation, Eq. (7.66), and two other correlations given in te text and cmpare the results.

GIVEN

- Water flowing through a duct
- Bulk water temperature (T_b) = 32°C
- Water velocity (V) = 1.5 m/s
- Inside diameter of duct (D) = 2.54 cm = 0.0254 m
- Duct wall surface temperature (T_s) = 43°C

FIND

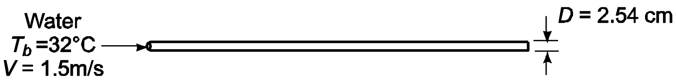
Use three different methods to find

(a) The Nusselt number (Nu_D)
(b) The convective heat transfer coefficient (h_c)

ASSUMPTIONS

- Steady state
- Fully developed, incompressible flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at a reference temperature equal to the bulk temperature (T_{ref} = 32°C)

Thermal conductivity (k) = 0.619 W/(m K)
Kinematic viscosity (ν) = 0.773×10^{-6} m^2/s
Prandtl number (Pr_b) = 5.16
Absolute viscosity (μ_b) = 763×10^{-6} (Ns)/m^2
Prandtl number at surface temperatue(Pr_s) = 4.06

At the surface temperature of 43°C: Absolute viscosity (μ_s) = 626.3×10^{-6} (Ns)/m^2

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_{co}\ D}{\nu} = \frac{1.5\,\text{m/s}\ \ (0.0254\,\text{m})}{0.773\times 10^{-6}\ \text{m}^2/\text{s}} = 4.93 \times 10^4 > 2000$$

Therefore, the flow is turbulent.

(a) Therefore, the flow is turbulent. Three different correlations that can be used to calculate the Nusselt number are contained in Table 7.3

1. The Gnielinski correlation Equation (7.66)
2. The Sieder and Tate Equation (7.62)
3. The Petukhov-Popov Equation (7.64a)

1. $$\overline{Nu}_D = \frac{(f/8)(\text{Re}_D - 100)\,\text{Pr}}{1 + 12.7(f/8)^{1/2}\left(\text{Pr}^{2/3} - 1\right)}\left[1 + (D/L)^{2/3}\right]K$$

Where K=(Pr_b/Pr_s) for liquid

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Thus, K=5.16/4.06=1.27

Considering D<<<L, D/L=0

$f = (1.82 \log Re_D - 1.64)^{-2} = [1.82 \log(4.93 \times 10^4) - 1.64]^{-2} = 0.0210$

Thus, $\overline{Nu}_D = \dfrac{(0.021/8)(49300-100)*5.16}{1+12.7(0.021/8)^{1/2}(5.16^{2/3}-1)}\left[1+(0)^{2/3}\right]*1.27$

$\overline{Nu}_D = 369$

2. $\overline{Nu}_D = 0.027 Re_D^{0.8} Pr^{0.3}\left(\dfrac{\mu_b}{\mu_s}\right)^{0.14} = 0.027\,(4.93 \times 10^4)^{0.8}\,(5.16)^{0.3}\left(\dfrac{763}{626.3}\right)^{0.14} = 257$

3. $\overline{Nu}_D = \dfrac{\left(\dfrac{f}{8}\right) Re_D Pr}{K_1 + K_2\left(\dfrac{f}{8}\right)^{\frac{1}{2}}(Pr^{\frac{2}{3}} - 1)}$

where $f = (1.82 \log Re_D - 1.64)^{-2} = [1.82 \log(4.93 \times 10^4) - 1.64]^{-2} = 0.0210$

$K_1 = 1 + 3.4\,f = 1 + 3.4(0.0210) = 1.071$

$K_2 = 11.7 + \dfrac{1.8}{Pr^{\frac{2}{3}}} = 11.7 + \dfrac{1.8}{\left[(5.16)^{\frac{2}{3}}\right]} = 12.30$

$$\overline{Nu}_D = \frac{\left(\dfrac{0.0210}{8}\right)(4.93\times 10^4)\,(5.16)}{1.071 + 12.30\left(\dfrac{0.0210}{8}\right)^{\frac{1}{2}}\left[(5.16)^{\frac{2}{3}} - 1\right]} = 288$$

(b) The heat transfer coefficient is given by

1. $\bar{h}_c = 369 * \dfrac{0.619\ \text{W/(m K)}}{0.0254\ \text{m}} = 8992\ \text{W/(m}^2\ \text{K)}$

2. $\bar{h}_c = 257\ \dfrac{0.619\ \text{W/(m K)}}{0.0254\ \text{m}} = 6263\ \text{W/(m}^2\ \text{K)}$

3. $\bar{h}_c = 288\ \dfrac{0.619\ \text{W/(m K)}}{0.0254\ \text{m}} = 7019\ \text{W/(m}^2\ \text{K)}$

COMMENTS

The Nusselt numbers vary by about 8% around the average value of 266. This is within the accuracy of empirical correlations.

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PROBLEM 7.23

Compute the average heat transfer coefficient h_c for 10°C water flowing at 4 m/s in a long, 2.5-cm-*ID* pipe (surface temperature 40°C) by three different equations and compare your results. Also determine the pressure drop per meter length of pipe.

GIVEN

- Water flowing through a pipe
- Water temperature (T_b) = 10°C
- Water velocity (V) = 4 m/s
- Inside diameter of pipe (D) = 2.5 cm = 0.025 m
- Pipe surface temperature (T_s) = 40°C

FIND

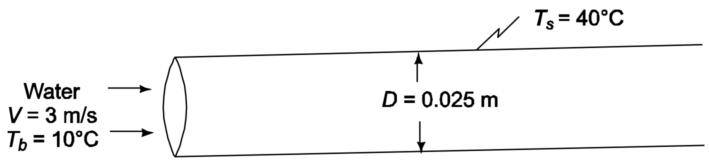
(a) The average heat transfer coefficient ($\bar{h}_c$) by 3 different equations.

(b) The pressure drop per meter length ($\Delta p/L$)

ASSUMPTIONS

- Steady state
- Uniform and constant wall surface temperature
- Pipe wall is smooth
- Fully developed flow (L/D > 60)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 10°C

Density (ρ) = 999.7 kg/m^3
Thermal conductivity (k) = 0.577 W/(m K)
Kinematic viscosity (ν) = 1.300 × 10^{-6} m^2/s
Prandtl number (Pr) = 9.5
Absolute viscosity (μ_b) = 1296 × 10^{-6} (Ns)/m^2

At the surface temperature of 40°C μ_s = 658 × 10^{-6} (Ns)/m^2

SOLUTION

The Reynolds number for this problem is

$$Re_D = \frac{V\,D}{\nu} = \frac{(4\text{ m/s})\,(0.025\text{m})}{1.3\times10^{-6}\text{ m}^2/\text{s}} = 7.69\times10^4 \text{ (Turbulent)}$$

(a)

1. Using the Dittus-Boelter correlation of Equation (7.61)

$$Nu_D = 0.023\, Re_D^{0.8}\, Pr^n \text{ where } n = 0.4 \text{ for heating}$$

$$Nu_D = 0.023\,(7.69\times10^4)^{0.8}\,(9.5)^{0.4} = 458.8$$

$$h_c = Nu_D\,\frac{k}{D} = 458.8\,\frac{0.577\text{ W/(m K)}}{0.025\text{m}} = 10{,}590 \text{ W/(m}^2\text{ K)}$$

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2. Using the Sieder-Tale correlation of Equation (7.62)

$$Nu_D = 0.027\, Re_D^{0.8}\, Pr^{0.3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 0.027\,(7.69 \times 10^4)^{0.8}\,(9.5)^{0.3} \left(\frac{1296}{658}\right)^{0.14} = 472.9$$

$$h_c = Nu_D \frac{k}{D} = 472.9 \frac{0.577\ \text{W/(mK)}}{0.025\,\text{m}} = 10{,}914\ \text{W/(m}^2\ \text{K)}$$

3. Using the Petukhov-Popov correlation of Equation (7.64a)

$$Nu_D = \frac{\left(\frac{f}{8}\right) Re_D Pr}{K_1 + K_2 \left(\frac{f}{8}\right)^{\frac{1}{2}} (Pr^{\frac{2}{3}} - 1)}$$

where $f = (1.82 \log(Re_D) - 1.64)^{-2} = (1.82 \log(7.69 \times 10^4) - 1.64)^{-2} = 0.0190$

$$K_1 = 1 + 3.4\, f = 1 + 3.4(0.019) = 1.065$$

$$K_2 = 11.7 + \frac{1.8}{Pr^{\frac{1}{3}}} = 11.7 + \frac{1.8}{9.5^{\frac{1}{3}}} = 12.55$$

$$Nu_D = \frac{\left(\frac{0.019}{8}\right)(7.69 \times 10^4)\,(9.5)}{1.065 + 12.55\left(\frac{0.019}{8}\right)^{\frac{1}{2}}\left[(9.5)^{\frac{2}{3}} - 1\right]} = 543$$

$$h_c = Nu_D \frac{k}{D} = 543 \frac{0.577\ \text{W/(mK)}}{0.025\,\text{m}} = 12{,}530\ \text{W/(m}^2\ \text{K)}$$

(b) The friction factor correlation of Equation (7.52) is good only for $1 \times 10^5 < Re_D$. Therefore, the friction factor will be estimated from the bottom curve of Figure 7.7: For $Re = 7.69 \times 10^4$, $f \approx 0.0188$ (Note that this is in good agreement with the friction factor, f in the Petukhov-Popov correlation).

The pressure drop per unit length can be calculated from Equation (7.13)

COMMENTS

The heat transfer coefficients vary around the average of 11,345 W/(m^2 K) by a maximum of 10%. This is within the accuracy of empirical correlations.

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PROBLEM 7.24

Water at 80°C is flowing through a thin copper tube (15.2-cm-*ID*) at a velocity of 7.6 m/s. The duct is located in a room at 15°C and the heat transfer coefficient at the outer surface of the duct is 14.1 W/(m^2 K). (a) Determine the heat transfer coefficient at the inner surface. (b) Estimate the length of duct in which the water temperature drops 1°C.

GIVEN

- Water flowing through a thin copper tube in a room
- Water temperature (T_b) = 80°C
- Inside diameter of tube (D) = 15.2 cm = 0.152 cm
- Water velocity (V) = 7.6 m/s
- Room air temperature (T_∞) = 15°C
- Outer surface heat transfer coefficient ($\bar{h}_{co}$) = 14.1 W/(m^2 K)

FIND

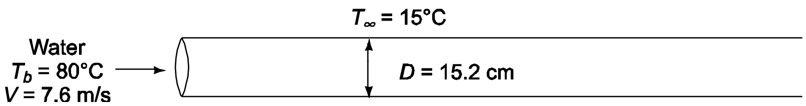
(a) The heat transfer coefficient at the inner surface ($\bar{h}_{ci}$)

(b) Length of duct (L) for temperature drop of 1°C

ASSUMPTIONS

- Steady state
- Thermal resistance of the copper tube is negligible
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 80°C

Density (ρ) 971.6 kg/m^3
Thermal conductivity (k) = 0.673 W/(m K)
Absolute viscosity (μ) = 356.7 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 2.13
Specific heat (c) = 4194 J/(kg K)

SOLUTION

The Reynolds number is

$$Re_D = \frac{V D \rho}{\mu} = \frac{(7.6\,\text{m/s})\,(0.152\,\text{m})\;\;971.6\,\text{kg/m}^3}{356.7\times 10^{-6}\;\;(\text{N s})/\text{m}^2\;\;\;(\text{kg m})/(\text{s}^2\,\text{N})} = 3.15 \times 10^6 \text{ (Turbulent)}$$

(a) Applying the Dittus-Boelter correlation of Equation (7.61)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \text{ where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\,(3.15 \times 10^6)^{0.8}\,(2.13)^{0.3} = 4555$$

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$$\bar{h}_{ci} = \overline{Nu}_D \frac{k}{D} = 4555 \frac{0.673\,\text{W/(m K)}}{0.152\,\text{m}} = 20{,}170 \text{ W/(m}^2\text{ K)}$$

(b) Since the pipe wall is thin, $A_o = A_i$ and the overall heat transfer coefficient is

$$\frac{1}{U} = \frac{1}{\bar{h}_{ci}} + \frac{1}{\bar{h}_{co}} = \left(\frac{1}{20{,}170} + \frac{1}{14.1}\right) \text{(m}^2\text{ K)/W} = 0.071 \text{ (m}^2\text{ K)/W} \Rightarrow U = 14.1 \text{ W/(m}^2\text{ K)} = \bar{h}_{co}$$

The length can be calculated using Equation (7.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_{b,\text{out}} - T_{co}}{T_{b,\text{in}} - T_{co}} = \exp\left(\frac{-P\,L\,\bar{h}_{co}}{\dot{m}\,c_p}\right) = \exp\left[\frac{-\pi\,D\,L\,\bar{h}_{co}}{\frac{\pi}{4}D^2 \rho V c_p}\right]$$

Solving for the length

$$L = -\frac{D\,\rho V\,c_p}{4\bar{h}_c} \ln\left[\frac{T_{b,\text{out}} - T_{co}}{T_{b,\text{in}} - T_{co}}\right]$$

$$L = -\frac{(0.152\,\text{m})\;\;971.6\,\text{kg/m}^3\;\;7.6\,\text{m/s}\;\;4194\,\text{J/(kg K)}\;\;\text{(Ws)/J}}{4\;\;14.11\,\text{W/(m}^2\text{ K)}} \ln\left(\frac{79\,°\text{C} - 15\,°\text{C}}{80\,°\text{C} - 15\,°\text{C}}\right) = 1294 \text{ m}$$

For these conditions, it would take over a kilometer for a 1°C temperature drop. This is largely the result of the small natural convection heat transfer coefficient over the outer surface.

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PROBLEM 7.25

Mercury at an inlet bulk temperature of 90°C flows through a 1.2-cm-*ID* tube at a flow rate of 4535 kg/h. This tube is part of a nuclear reactor in which heat can be generated uniformly at any desired rate by adjusting the neutron flux level. Determine the length of tube required to raise the bulk temperature of the mercury to 230°C without generating any mercury vapor, and determine the corresponding heat flux. The boiling point of mercury is 355°C.

GIVEN

- Mercury flow in a tube
- Inlet bulk temperature ($T_{b,\mathrm{in}}$) = 90°C
- Inside tube diameter (D) = 1.2 cm = 0.012 m
- Flow rate ($\dot{m}$) = 4535 kg/h = 1.26 kg/s
- Outlet bulk temperature ($T_{b,\mathrm{out}}$) = 230°C
- Boiling point of mercury = 355°C

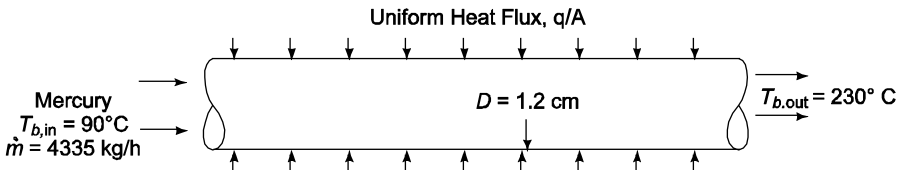
FIND

(a) The length of tube (L) required to obtain $T_{b,\mathrm{out}}$ without generating mercury vapor
(b) The corresponding heat flux (q/A)

ASSUMPTIONS

- Steady state
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 26, for mercury at the average bulk temperature of 160°C

Density (ρ) = 13,240 kg/m^3
Thermal conductivity (k) 11.66 W/(m K)
Absolute viscosity (μ) = 11.16 × 10^{-4} (Ns)/m^2
Prandtl number (Pr) = 0.0130
Specific heat (c) = 140.6 J/(kg K)

SOLUTION

The Reynolds number is

$$Re_D = \frac{V D \rho}{\pi} = \frac{4\dot{m}}{\pi D \mu} = \frac{4\ 1.26\,\mathrm{kg/s}}{\pi(0.012\,\mathrm{m})\ 11.16\times10^{-4}\,(\mathrm{Ns})/\mathrm{m}^2\quad(\mathrm{kg\,m})/(\mathrm{Ns}^2)} = 1.2\times10^2\ \text{(Turbulent)}$$

$$Re_D\,Pr = 1.2\times10^5\,(0.013) = 1557 > 100$$

Therefore, Equation (7.69) can be applied to calculate the Nusselt Number

$$\overline{Nu}_D = 4.82 + 0.0185(Re_D\,Pr)^{0.827} = 4.82 + 0.0185\ (1557)0.827 = 12.9$$

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$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 12.9 \frac{11.66\,\text{W/(mK)}}{0.012\,\text{m}} = 1.25 \times 10^4\ \text{W/(m}^2\ \text{K)}$$

(b) The maximum allowable heat flux is determined by the outlet conditions. The outlet wall temperature must not be higher than the mercury boiling point

$$\frac{q}{A} = (T_{\text{wall,max}} - T_{b,\text{out}})\ \bar{h}_c = (355°\text{C} - 230°\text{C})\ \ 1.25 \times 10^4\ \text{W/(m}^2\ \text{K)}\ \ = 1.57 \times 10^2\ \text{W/(m}^2\ \text{K)}$$

(a) The length of the tube required can be calculated from the following

$$q = \dot{m}\,c\,(T_{b,\text{out}} - T_{b,\text{in}}) = \frac{q}{A}\ (\pi D L)$$

Solving for the length

$$L = \frac{\dot{m}\,c\,(T_{b,\text{out}} - T_{b,\text{in}})}{\frac{q}{A}\pi D} = \frac{1.26\,\text{kg/s}\ \ 140.6\,\text{J/(kg K)}\ \ (230\,°\text{C} - 90\,°\text{C})}{1.57 \times 10^6\ \text{W/(m}^2\text{K)}\ \ \text{J/(Ws)}\ \ \pi\,(0.012\,\text{m})} = 0.419\ \text{m}$$

COMMENTS

Note that L/D = 0.419 m/0.012 m = 35 > 30, therefore, the assumption of fully developed flow and use of Equation (7.69) is valid.

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PROBLEM 7.26

> **Exhaust gases having properties similar to dry air enter a thin-walled cylindrical exhaust stack at 800 K. The stack is made of steel and is 8-m-tall and 0.5 m inside diameter. If the gas flow rate is 0.5 kg/s and the heat transfer coefficient at the outer surface is 16 W/(m^2 K), estimate the outlet temperature of the exhaust gas if the ambient temperature is 280 K.**

GIVEN

- Gas properties are similar to dry air
- Gas entrance temperature ($T_{b,\text{in}}$) = 800 K
- Length of stack (L) = 8 m
- Diameter of stack (D) = 0.5 m
- Mass flow rate ($\dot{m}$) = 0.5 kg/s
- Heat transfer coefficient on the outer surface ($\bar{h}_{c,o}$) = 16 W/(m^2 K)
- Ambient temperature (T_∞) 280 K

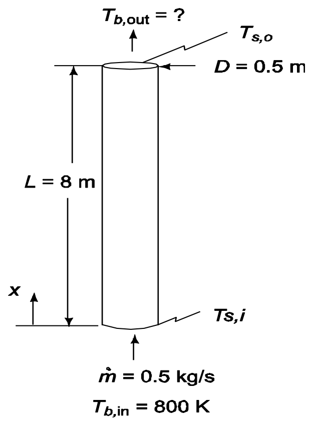
FIND

- The outlet temperature of the exhaust gas ($T_{b,\text{out}}$)

ASSUMPTIONS

- Radiation heat transfer is negligible
- Natural convection can be neglected
- The inlet to the stack is sharp-edged
- Thermal resistance of the stack wall is negligible

SKETCH



SOLUTION

For this problem, neither the heat flux nor the surface temperature will be constant. However, the ambient temperature will be constant, therefore, Equation (7.33) can be applied by replacing the surface temperature (T_s) with the constant ambient temperature (T_∞) and replacing h_c with U where

$$U = \text{Overall heat transfer coefficient} = 1\Bigg/\left(\frac{1}{\bar{h}_{co}} + \frac{1}{\bar{h}_{ci}}\right)$$

This results in the following version of Equation (7.36)

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$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_{b,\text{out}} - T_{co}}{T_{b,\text{in}} - T_{co}} = \exp\left(-\frac{P\,L\,U}{\dot{m}\,c_p}\right) = \exp\left(-\frac{U\,\pi\,D\,L}{\dot{m}\,c_p}\right)$$

$$\therefore\ T_{b,\text{out}} = T_\infty + (T_{b,\text{in}} - T_\infty)\exp\left(-\frac{U\,\pi\,D\,L}{\dot{m}\,c_p}\right)$$

The internal heat transfer coefficient and the average fluid properties will depend on the outlet bulk fluid temperature, therefore, an iterative procedure is required. For the first iteration, let $T_{b,\text{out}}$ = 500 K. From Appendix 2, Table 28, for dry air at the average bulk temperature of 650 K

Specific Heat (c_p) = 1056 J/(kg K)
Thermal conductivity (k) = 0.0472 W/(m K)
Absolute viscosity (μ) = 31.965 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71

The Reynolds number for flow in the stack is

$$Re_D = \frac{U_{co}\,D\,\rho}{\mu} = \frac{4\,\dot{m}}{\pi\,D\,\mu} = \frac{4\ \ 0.5\,\text{kg/s}}{\pi(0.5\,\text{m})\ \ 31.965\times10^{-6}\ (\text{Ns})/\text{m}^2\ \ \ \text{kg m/(Ns}^2)} = 3.98\times10^4\ \text{(Turbulent)}$$

L/D = (8 m)/(0.5) = 16

Since 2 < L/D < 20, the flow will not be fully developed, therefore, the correlation of Molki and Sparrow for sharp-edged inlets, will be used to correct the correlation of Dittus-Boelter, Equation (7.61)

$$\overline{Nu}_{fd} = 0.023\ Re_D^{0.8}\ Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

and

$$\overline{Nu} = \overline{Nu}_{fd}\left[1 + a\left(\frac{L}{D}\right)^b\right]$$

where $a = 24/Re_D^{0.23} = 24/(3.98\times10^4)^{0.23} = 2.10$

$b = 2.08\times10^{-6}\ Re_D - 0.815 = 2.08\times10^{-6}\ (3.98\times10^4) - 0.815 = -0.732$

$$\overline{Nu} = 0.023\ (3.98\times10^4)^{0.8}\ (0.71)^{0.4}\ [1 + 2.10(16)^{-0.732}] = 122.5$$

$$\bar{h}_c = \overline{Nu}\ \frac{k}{D} = 122.5\ \frac{0.0472\,\text{W/(m K)}}{0.5\,\text{m}} = 11.6\ \text{W/(m}^2\ \text{K)}$$

$$U = \frac{1}{\left(\frac{1}{16} + \frac{1}{11.6}\right)(\text{m}^2\text{K})/\text{W}} = 6.7\ \text{W/(m}^2\ \text{K)}$$

$$\therefore\ T_{b,\text{out}} = 280\ \text{K} + (800\ \text{K} - 280\ \text{K})\exp\left(-\frac{6.7\,\text{W/(m}^2\text{K)}\ \ \pi(0.5\,\text{m})(8\,\text{m})}{0.5\,\text{kg/s}\ \ \ 1056\,\text{J/(kg K)}\ \ \ (\text{Ws})/\text{J}}\right) = 723\ \text{K}$$

Another iteration using the same procedure yields

Average bulk temperature = 762 K
Specific Heat (c_p) = 1074 J/(kg K)
Thermal conductivity (k) = 0.0534 W/(m K)
Absolute viscosity (μ) = 35.460 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.72
Reynolds number (Re_D) = 3.59 × 10^4
Heat transfer coefficient ($\bar{h}_{ci}$) = 12.1 W/(m^2 K)
Outlet temperature ($T_{b,\text{out}}$) = 722 K

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The outlet gas temperature = 722 K

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PROBLEM 7.27

Water at an average temperature of 27°C is flowing through a smooth 5.08-cm-*ID* pipe at a velocity of 0.91 m/s. If the temperature at the inner surface of the pipe is 49°C, determine (a) the heat transfer coefficient, (b) the rate of heat flow per meter of pipe, (c) the bulk temperature rise per meter, and (d) the pressure drop per meter.

GIVEN

- Water flowing through a smooth pipe
- Average water temperature (T_w) = 27°C
- Pipe inside diameter (D) = 5.08 cm = 0.0508 m
- Water velocity (V) = 0.91 m/s
- Inner surface temperature of pipe (T_s) = 49°C

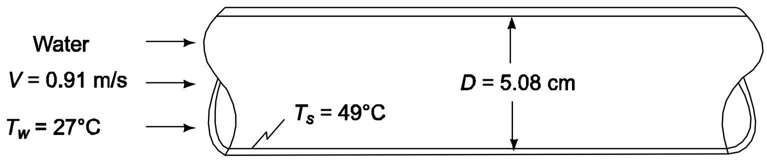
FIND

(a) The heat transfer coefficient ($\bar{h}_c$)
(b) The rate of heat flow per meter of pipe (q/L)
(c) The bulk temperature rise per meter of pipe ($\Delta T_w/L$)
(d) The pressure drop per meter of pipe ($\Delta p/L$)

ASSUMPTIONS

- Steady state
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 27°C

Density (ρ) = 996.5 kg/m^3
Specific Heat (c_p) = 4178 J/(kg K)
Thermal conductivity (k) = 0.608 W/(m K)
Absolute viscosity (μ) = 845.3 × 10^{-6} (Ns)/m^2
Kinematic viscosity (ν) = 0.852 × 10^{-6} m^2/s
Prandtl number (Pr) = 5.8

At the surface temperature of 49°C
Absolute viscosity (μ_s) = 565.1 × 10^{-6} (Ns)/m^2

SOLUTION

The Reynolds number for this flow is

$$Re_D = \frac{V\,D}{\nu} = \frac{(0.91\,\text{m/s})(0.0508\,\text{m})}{0.852\times10^{-6}\,\text{m}^2/\text{s}} = 5.42\times10^4 > 2000$$

Therefore, the flow is turbulent.

The variation in property values is accounted for by using Equation (7.62) to calculate the Nusselt number

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$$\overline{Nu}_D = 0.027 Re_D^{0.8} Pr^{0.3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 0.027(5.42 \times 10^4)^{0.8} (5.8)^{0.3} \left(\frac{845.3}{565.1}\right)^{0.14} = 296$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 296 \; \frac{0.608\,\text{W/(m K)}}{0.0508\,\text{m}} = 3543 \text{ W/(m}^2 \text{ K)}$$

(b) The rate of convective heat transfer is given by

$$q = \bar{h}_c A_t (T_s - T_w) = h_c\, \pi D\, L\, (T_s - T_w)$$

$$\frac{q}{L} = 3543\,\text{W/(m}^2\text{K)} \quad \pi (0.0508 \text{ m}) (49°\text{C} - 27°\text{C}) = 12{,}438 \text{ W/m}$$

(c) This rate of heat transfer will lead to a temperature rise in the water given by

$$q = \dot{m} c_p\, \Delta T_w = \left(\rho V \frac{\pi}{4} D^2\right) c_p\, \Delta T_w$$

$$\therefore \; \frac{\Delta T_w}{L} = \frac{4}{\rho V \pi D^2 c_p} \left(\frac{q}{L}\right)$$

$$\frac{\Delta T_w}{L} = \frac{4}{996.5\,\text{kg/m}^3 \quad 0.91\,\text{m/s} \;\; \pi \quad 0.0508\,\text{m}^{\;2} \quad 4178\,\text{J/(kg K)} \quad (\text{Ws})/\text{J}} \; 12{,}438\,\text{W/m} = 1.6 \text{ K/m}$$

(d) From Table 7.4, the friction factor for fully developed turbulent flow through smooth tubes is given by Equation (7.57)

$$f = 0.184\, Re_D^{-0.2} = 0.184\, (5.42 \times 10^4)^{-0.2} = 0.0208$$

The pressure drop is given by Equation (7.13)

$$\frac{\Delta p}{L} = \frac{f \rho V^2}{2D} = \frac{0.0208 \;\; 996.5\,\text{kg/m}^3 \quad 0.91\,\text{m/s}^{\;\;2}}{2\,(0.0508\,\text{m}) \;\; (\text{kg m})/(\text{s}^2\text{N}) \quad \text{N/(Pa m}^2)} = 169 \text{ Pa/m}$$

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PROBLEM 7.28

An aniline-alcohol solution is flowing at a velocity of 3 m/s through a long, 2.5 cm-*ID* thin-wall tube. Steam is condensing at atmospheric pressure, on the outer surface of the tube, and the tube-wall temperature is 100°C. The tube is clean, and there is no thermal resistance due to a scale deposit on the inner surface. Using the physical properties tabulated below, estimate the heat transfer coefficient between the fluid and the pipe using Equations (7.61) and (7.66), and compare the results. Assume that the bulk temperature of the aniline solution is 20°C and neglect entrance effects.

Physical properties of the aniline solution

Temperature (°C)	Viscosity (kg/(m s))	Thermal Conductivity $(\mathbf{W/(m}\ K))$	Specific Gravity	Specific Heat (kJ/(kg K))
20	0.0051	0.173	1.03	2.09
60	0.0014	0.169	0.98	2.22
100	0.0006	0.164		2.34

GIVEN

- An aniline-alcohol solution flowing through a thin-walled tube
- Tube is clean with no scaling on inner surface
- Velocity (V) =3 m/s
- Inside diameter of tube (D) = 2.5 cm = 0.025 m
- Tube wall surface temperature (T_s) = 100°C
- Solution has the properties listed above
- Solution bulk temperature (T_b)= 20°C

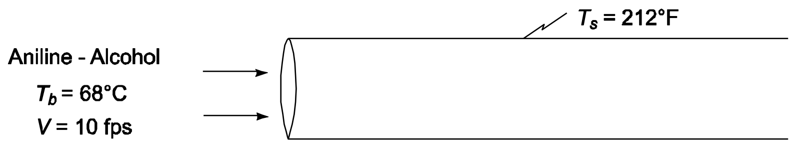
FIND

- The heat transfer coefficient ($\bar{h}_c$) using: (a) Equation (7.61) (b) Equation (7.66)

ASSUMPTIONS

- Steady state
- Entrance effects are negligible
- Thermal resistance of the tube is negligible
- Tube wall temperature is constant and uniform
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

The density of water ≈ 1000 kg/m^3

SOLUTION

The kinematic viscosity (ν) of the solution at the bulk temperature is

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$$\nu = \frac{\mu}{\rho} = \frac{\mu}{(s.g.)\rho_{\text{water}}} = \frac{0.0051}{(1.03)(1000)} = 4.95*10^{-6} \text{ m}^2/\text{s}$$

The Prandtl number is

$$Pr = \frac{c_p \mu}{k} = \frac{(2090 J / kgK)[0.0051 kg / (ms)]}{(0.173 \text{ W/(m K)})} = 61.61$$

The Reynolds number is

$$Re_D = \frac{V D}{\nu} = \frac{(3 m / s)(0.025 m)}{(4.95*10^{-6} m^2/s)} = 15{,}151 \text{ (Turbulent)}$$

(a) Applying the Dittus-Boelter correlation of Equation (7.61)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\, (15{,}151)^{0.8}\, (61.61)^{0.4} = 264.0$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 264.0 \; \frac{(0.173 W/(\text{m K}))}{0.025 \text{ m}} = 1828 \text{ W/(m}^2\text{ K)}$$

(b) Using the Sieder-Tate correlation of Equation (7.62)

$$\overline{Nu}_D = 0.027 Re_D^{0.8}\, Pr^{0.3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 0.027(15{,}151)^{0.8}\, (61.69)^{0.3} \left(\frac{5.1}{0.6}\right)^{0.14} = 277$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 277 \; \frac{(0.173 W/(\text{m K}))}{0.025 \text{ m}} = 1918 \text{ W/(m}^2\text{ K)}$$

COMMENTS

These estimates vary by about 3% around an average value of 1828 W/(m^2 K). But the Sieder-Tate correlation is more applicable in this case because it takes the large variation of the viscosity with temperature into account.

Note that the above correlations require that all properties (except μ_s) be evaluated at the bulk temperature.

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PROBLEM 7.29

> **In an industrial refrigeration system, brine (10 % NaCl by weight) having a viscosity of 0.0016 (Ns)/m^2 and a thermal conductivity of 0.85 W/(m K) is flowing through a long 2.5-cm-*ID* pipe at 6.1 m/s. Under these conditions, the heat transfer coefficient was found to be 16,500 W/(m^2 K). For a brine temperature of –1°C and a pipe temperature of 18.3°C, determine the temperature rise of the brine per meter length of pipe if the velocity of the brine is doubled. Assume that the specific heat of the brine is 3768 J/(kg K) and that its density is equal to that of water.**

GIVEN

- Brine flowing through a pipe
- Brine properties
  - Viscosity (μ) = 0.0016 Ns/m^2
  - Thermal conductivity (k) = 0.85 W/(m K)
  - 10% NaCl by weight
  - Specific heat (c) = 3768 J/(kg K)
- Pipe inside diameter (D) = 2.5 cm = 0.025 m
- Brine velocity (V) = 6.1 m/s
- Heat transfer coefficient ($\bar{h}_c$) = 16,500 W/(m^2 K)
- Brine temperature (T_b) = –1°C
- Pipe temperature (T_s) = 18.3°C

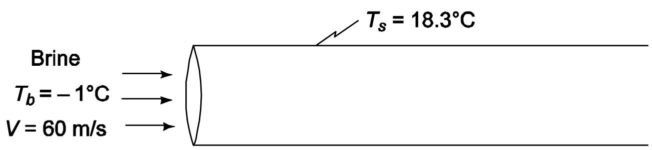
FIND

- Temperature rise of the brine per meter length ($\Delta T_b/m$) if the velocity is doubled (V = 12.2 m/s)

ASSUMPTIONS

- Steady state
- Fully developed flow
- Constant and uniform pipe wall temperature
- Density of the brine is the same as water density

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, density (ρ) of water ≈ 1000 kg/m^3

SOLUTION

The Reynolds number at the original velocity is

$$Re_D = \frac{V D \rho}{\mu} = \frac{(6.1\,\text{m/s})(0.025\,\text{m})\ 1000\,\text{kg/m}^3}{0.0016\,(\text{Ns})/\text{m}^2\ \ (\text{kg m})/(\text{s}^2\text{N})} = 95{,}313 \text{ (Turbulent)}$$

The thermal conductivity of the fluid can be calculated from the given heat transfer coefficient using the Dittus-Boelter correlation of Equation (7.61)

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

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$$k = \frac{\bar{h}_c D}{0.023 Re_D^{0.8}\left(\frac{c\,\mu}{k}\right)^{0.4}}$$

$$k = \left[\frac{\bar{h}_c D}{0.023 Re_D^{0.8}(c\,\mu)^{0.4}}\right]^{\frac{1}{0.6}}$$

$$k = \left[\frac{0.025\,\text{m}\ \ 16{,}500\,\text{W/(m}^2\text{K)}}{0.023(95{,}313)^{0.8}\left[\ 3768\,\text{J/(kg K)}\ \ 0.0016\,\text{(Ns)/m}^2\ \ \text{(kg m)/(s}^2\text{N)}\ \ \text{(Ws)/J}\ ^{0.4}\right]}\right]^{\frac{1}{0.6}}$$

$$= 0.852\ \text{W/(m K)}$$

The Prandtl number is

$$Pr = \frac{c\,\mu}{k} = \frac{3768\,\text{J/(kg K)}\ \ 0.0016\,\text{(Ns)/m}^2\ \ \text{(kg m)/(s}^2\text{N)}}{0.852\,\text{W/(m K)}\ \ \text{J/(Ws)}} = 7.08$$

The Reynolds number for the new velocity is twice the original Reynolds number: Re_D = 190,626. From Equation (7.61): For fully developed flow

$$\overline{Nu}_D = 0.023\ (190{,}626)^{0.8}\ (7.08)^{0.4} = 843$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 843\ \frac{0.852\,\text{W/(m K)}}{0.025\,\text{m}} = 28{,}733\ \text{W/(m}^2\ \text{K)}$$

The temperature after one meter is given by Equation (7.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_{b,\text{out}} - T_{co}}{T_{b,\text{in}} - T_{co}} = \exp\left(-\frac{P\,L\,\bar{h}_c}{\dot{m}\,c}\right) = \exp\left(-\frac{4\,\bar{h}_c\,L}{\rho V\,D\,c}\right)$$

$$\frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s} = \exp\left(-\frac{4\ \ 38{,}733\,\text{W/(m}^2\text{K)}\ \ (1\text{m})}{1000\,\text{kg/m}^3\ \ 12.2\,\text{m/s}\ \ (0.025\,\text{m})\ \ 3768\,\text{J/(kg K)}\ \ \text{(Ws)/J}}\right)$$

$$= 0.9048\ \text{(per m length)}$$

$$\Delta T_b = T_{b,\text{out}} - T_{b,\text{in}} = \left[\left(\frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s}\right)\ T_{b,\text{in}} - T_s\ \ + T_s\right] - T_{b,\text{in}}$$

$$\Delta T_b = [0.9043\ (-1°\text{C} - 18.3°\text{C}) + 18.3°\text{C}] + 1°\text{C} = 1.84°\text{C per meter length}$$

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PROBLEM 7.30

Derive an equation of the form $h_c = f(T, D, U)$ for turbulent flow of water through a long tube in the temperature range between 20° and 100°C.

GIVEN

- Turbulent water flow through a long tube
- Water temperature range (T_b) = 20°C to 100°C

FIND

- An expression of the form $\bar{h}_c = f(T, D, U)$

ASSUMPTIONS

- Steady state
- Variation of properties with temperature can be approximated with a power law
- Fully developed flow
- Water is being heated

SKETCH

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water

Temperature (°C)	20	100
Temperature (K)	293	373
Density, ρ (kg/m^3)	998.2	958.4
Thermal conductivity, k W/(m K)	0.597	0.682
Absolute viscosity, (Ns)/m^2	993×10^{-6}	277.5×10^{-6}
Prandtl number, Pr	7.0	1.75

SOLUTION

Applying the Dittus-Boelter expression of Equation (7.61) for the Nusselt number

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023 \left(\frac{\rho D U}{\mu}\right)^{0.8} Pr^n$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 0.023\, \frac{\rho^{0.8} Pr^{0.4}\, \mathrm{k}}{\mu^{0.8}} D^{-0.2}\, U^{0.8}$$

To put this in the required form, the fluid properties must be expressed as a function of temperature. Assuming the power law variation

$$\text{Property} = A T^R$$

where A and n are constant evaluated from the property values.

For density $\quad \rho(293) = 998.2 \text{ kg/m}^3 = A(293)n$

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$$\rho(373) = 958.2 \text{ kg/m}^3 = A(373)n$$

Solving these simultaneously

$$A = 2613 \qquad n = -0.1694$$

Therefore, $\rho(T) = 2613\ T^{-0.1694}$

Applying a similar analysis for the remaining properties yields the following relationships

$$k(T) = 0.02605\ T^{0.5514}$$
$$\mu(T) = 1.058 \times 1010\ T^{-5.281}$$
$$Pr(T) = 1.026 \times 1015\ T^{-5.7426}$$

Substituting these into the expression for the heat transfer coefficient

$$\bar{h}_c = 0.023 \frac{(2612 T^{-0.1694})^{0.8}(1.026 \times 10^{15} T^{-5.7426})^{0.4}(0.02605 T^{0.5514})}{(1.058 \times 10^{10}\ T^{-5.281})^{0.8}}\ D^{-0.2}\,\mathrm{U}^{0.8}$$

$$\bar{h}_c = 0.0031\ T^{2.34} D^{-0.2}\ U^{0.8}$$

COMMENTS

Note that in equations of the type derived, the coefficient has definite dimensions. Hence, the use of such equations is limited to the conditions specified and are not recommended.

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PROBLEM 7.31

The intake manifold of an automobile engine can be approximated as a 4-cm-*ID* tube, 30 cm in length. Air at a bulk temperature of 20°C enters the manifold at a flow rate of 0.01 kg/s. The manifold is a heavy aluminum casting and is at a uniform temperature of 40°C. Determine the temperature of the air at the end of the manifold.

GIVEN

- Air flow through a tube
- Tube inside diameter (D) = 4 cm = 0.04 m
- Tube length (L) = 30 cm = 0.30 m
- Inlet bulk temperature ($T_{b,\text{in}}$) = 20°C
- Air flow rate ($\dot{m}$) = 0.01 kg/s
- Tube surface temperature (T_s) = 40°C

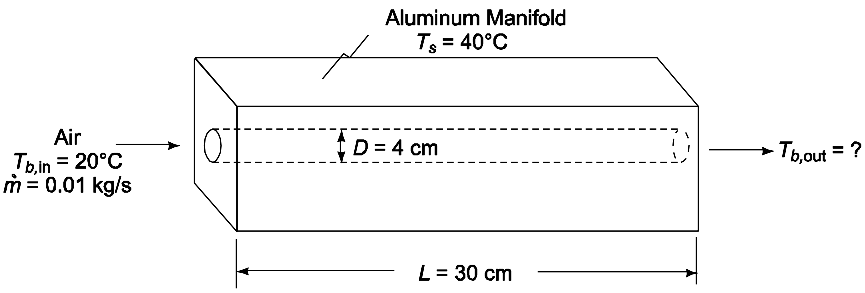
FIND

- Outlet bulk temperature ($T_{b,\text{out}}$)

ASSUMPTIONS

- Steady state
- Constant and uniform tube surface temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the inlet bulk temperature of 20°C

Thermal conductivity (k) = 0.0251 W/(m K)
Absolute viscosity (μ) = 18,240 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71
Specific heat (c) = 1012 J/(kg K)

SOLUTION

The Reynolds number is

$$Re_D = \frac{V D \rho}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.01\,\text{kg/s})}{\pi(0.04\,\text{m})\ 18.240\times 10^{-6}\,(\text{Ns})/\text{m}^2 \quad (\text{kg m})/(\text{Ns}^2)}$$

$$= 17{,}451 \text{ (Turbulent)}$$

The Nusselt number for fully developed flow can be estimated from the Dittus-Boelter correlation of Equation (7.61)

$$Nu_{fd} = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

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$$Nu_{fd} = 0.023\ (17{,}451)^{0.8}\ (0.71)^{0.4} = 49.62$$

$$h_{c,fd} = Nu_{fd}\ \frac{k}{D} = 49.62\ \frac{0.0251\,\mathrm{W/(m\,K)}}{0.04\,\mathrm{m}} = 31.14\ \mathrm{W/(m^2\ K)}$$

Since L/D = 30cm/4 cm = 7.5 < 60, the flow is not fully developed and the fully developed heat transfer coefficient must be

$$\frac{Nu}{Nu_{fd}} = \frac{h_{c,L}}{h_{c,\,fd}} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24/Re_D^{0.23} = 24/(17{,}451)^{0.23} = 2.538$

$b = 2.08 \times 10^{-6}\ Re_D - 0.815 = 2.08 \times 10^{-6}\ (17{,}451) - 0.815 = -\,0.7787$

$$h_{c,L} = 31.14\ \mathrm{W/(m^2K)}\ \left[1 + 2.538\left(\frac{30\,\mathrm{cm}}{4\,\mathrm{cm}}\right)^{-0.7787}\right] = 47.60\ \mathrm{W/(m^2\ K)}$$

Applying Equation (7.36) to determine the outlet air temperature

$$\frac{\Delta T_{\mathrm{out}}}{\Delta T_{\mathrm{in}}} = \frac{T_{b,\mathrm{out}} - T_s}{T_{b,\mathrm{in}} - T_s} = \exp\left(-\frac{P\,L\,h_c}{\dot{m}\,c}\right) = \exp\left(-\frac{h_c\,\pi\,D\,L}{\dot{m}\,c}\right)$$

$$T_{b,\mathrm{out}} = T_s - (T_s - T_{b,\mathrm{in}})\ \exp\left(-\frac{h_c\,\pi\,D\,L}{\dot{m}\,c}\right)$$

$$T_{b,\mathrm{out}} = 40°\mathrm{C} - (40°\mathrm{C} - 20°\mathrm{C})\ \exp\left(-\frac{47.60\,\mathrm{W/(m^2K)}\ \ \pi\,(0.04\,\mathrm{m})\,(0.3\,\mathrm{m})}{0.01\,\mathrm{kg/s}\ \ \ 1012\,\mathrm{J/(kg\,K)}\ \ \ (\mathrm{Ws})/\mathrm{J}}\right) = 23.2°\mathrm{C}$$

COMMENTS

The rise in air temperature is not large enough to require another iteration using new air properties at the average bulk air temperature.

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PROBLEM 7.32

High-pressure water at a bulk inlet temperature of 93°C is flowing with a velocity of 1.5 m/s through a 0.015-m-diameter tube, 0.3-m-long. If the tube wall temperature is 204°C, determine the average heat transfer coefficient and estimate the bulk temperature rise of the water.

GIVEN

- Water flowing through a tube
- Bulk inlet water temperature ($T_{b,\text{in}}$) = 93°C
- Water velocity (V) = 1.5 m/s
- Tube diameter (D) = 0.015 m
- Tube length (L) = 0.3 m
- Tube surface temperature (T_s) = 204°C

FIND

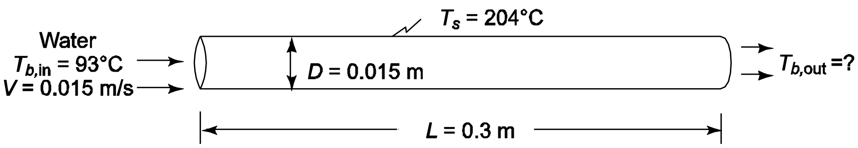
(a) The average heat transfer coefficient ($\bar{h}_{c,L}$)

(b) The bulk temperature rise of the water (ΔT_b)

ASSUMPTIONS

- Steady state
- Constant and uniform tube temperature
- Pressure is high enough to supress vapor generation.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at the inlet bulk temperature of 93°C

Density (ρ) = 963.0 kg/m^3
Thermal conductivity (k) = 0.679 W/(m K)
Kinematic viscosity (ν) = 0.314 × 10^{-6} m^2/s
Prandtl number (Pr) = 1.88
Specific heat (c) = 4205 J/(kg K)

SOLUTION

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{(1.5\,\text{m/s})(0.015\,\text{m})}{0.314 \times 10^{-6}\,\text{m}^2/\text{s}} = 71{,}656 \text{ (Turbulent)}$$

(a) The Nusselt number for fully developed flow can be estimated from the Dittus-Boelter correlation of Equation (7.61)

$$\overline{Nu}_{fd} = 0.023\, Re_D^{0.8}\, Pr^n \quad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_{fd} = 0.023\,(71{,}656)^{0.8}\,(1.88)^{0.4} = 226.8$$

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$$\bar{h}_{c,\,fd} = \overline{Nu}_{fd}\,\frac{k}{D} = 226.8\,\frac{0.679\,\text{W/(m K)}}{0.015\,\text{m}} = 10{,}265\ \text{W/(m}^2\ \text{K)}$$

Since L/D = 0.3/0.015 = 20 < 60, we get

$$\frac{\overline{Nu}}{\overline{Nu}_{fd}} = \frac{\bar{h}_{c,L}}{\bar{h}_c} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24\ Re_D^{0.23} = 24/(71{,}656)^{0.23} = 1.834$

$b = 2.08 \times 10^{-6}\ Re_D - 0.815 = 2.08 \times 10^{-6}\,(71{,}656) - 0.815 = -\,00.666$

$$\frac{\bar{h}_{c,L}}{\bar{h}_{c,\,fd}} = 1 + 1.834\left(\frac{0.3}{0.015}\right)^{-0.666} = 1.249$$

and

$$\frac{\overline{Nu}}{\overline{Nu}_{fd}} = \frac{\bar{h}_{c,L}}{\bar{h}_{c,\,fd}} = 1 + \left(\frac{6D}{L}\right) = 1 + \left(\frac{6(0.015)}{0.3}\right) = 1.300$$

The average of the two values is $\bar{h}_c, \bar{L} / \bar{h}_{c,\,fd} = 1.27$

$$\therefore\quad \bar{h}_{c,L} = 1.27\ 10{,}265\,\text{W/(m}^2\text{K)} = 13{,}037\ \text{W/(m}^2\ \text{K)}$$

(b) The bulk temperature can be calculated from Equations (7.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}} = \exp\left(-\frac{P\,L\,\bar{h}_c}{\dot{m}\,c_p}\right) = \exp\left(-\frac{4\,\bar{h}_c\,L}{\rho V\,D\,c}\right)$$

$$T_{b,\text{out}} = T_s - (T_s - T_{b,\text{in}})\ \exp = \left(-\frac{4\,\bar{h}_c\,L}{\rho V\,D\,c}\right)$$

$$T_{b,\text{out}} = 204\ ^\circ\text{C} - (204^\circ\text{C} - 93^\circ\text{C})$$

$$\exp\left(-\frac{4\ 13{,}037\,\text{W/(m}^2\text{K}\ \ (0.3\text{m})}{963.0\,\text{kg/m}^3\ \ 1.5\,\text{m/s}\ \ 0.015\,\text{m}\ \ 4205\,\text{J/(kg K)}\ \ (\text{Ws})/\text{J}}\right) = 111^\circ\text{C}$$

The bulk temperature rise is

$$\Delta T_b = T_{b,\text{out}} - T_{b,\text{in}} = 111^\circ\text{C} - 93^\circ\text{C} = 18^\circ\text{C}$$

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PROBLEM 7.33

Suppose an engineer suggests that air instead of water could flow through the tube of Problem 7.32 and the velocity of the air could be increased until the heat transfer coefficient with the air equals that obtained with water at 1.5 m/s. Determine the velocity required and comment on the feasibility of the engineer's suggestion. Note that the speed of sound in air at 100°C is 387 m/s.

GIVEN

- Air flow through a tube
- Bulk inlet air temperature ($T_{b,\text{in}}$) = 93°C
- Tube diameter (D) = 0.015 m
- Tube length (L) = 0.3 m
- Tube surface temperature (T_s) = 204°C
- From Problem 7.32: $\bar{h}_{c,L}$ = 13,037 W/(m^2 K)

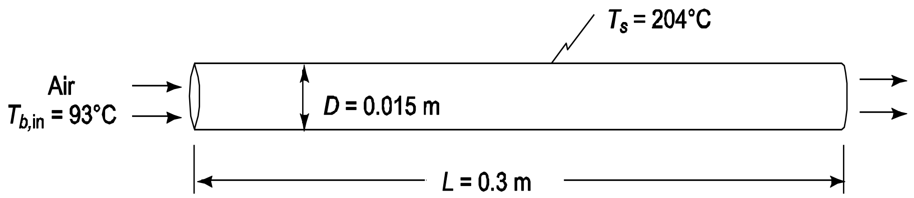
FIND

- The velocity (V) required to obtain $\bar{h}_{c,L}$ = 13,037 W/(m^2 K)

ASSUMPTIONS

- Steady state
- Constant and uniform tube temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the inlet bulk temperature of 93°C

Thermal conductivity (k) = 0.0302 W/(m K)

Kinematic viscosity (ν) = 22.9 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

SOLUTION

The flow must be turbulent, therefore, the heat transfer coefficient of the fully developed case must be 13,037 W/(m^2 K) as shown in Problem 7.24. Therefore, the Nusselt number is

$$\overline{Nu}_{fd} = \frac{\bar{h}_{c,fd} D}{k} = \frac{13{,}037\,\text{W/(m}^2\,\text{K)}\;\;0.015\,\text{m}}{0.0302\,\text{W/(m K)}} = 6475$$

Applying the Dittus-Boelter correlation of Equation (7.61)

$$\overline{Nu}_{fd} = 0.023\, Re_D^{0.8}\, Pr^n = 5099 \qquad \text{where } n = 0.4 \text{ for heating}$$

Solving for the Reynolds number

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$$Re_D = \frac{V\,D}{\nu} = \left(\frac{\overline{Nu}_{fd}}{0.023Pr^{0.4}}\right)^{1.25} = \left(\frac{6475}{0.023(0.71)^{0.4}}\right)^{1.25} = 7.70 \times 10^6$$

Solving for the velocity

$$V = Re_D\,\frac{\nu}{D} = 7.70 \times 10^6\,\frac{22.9\times10^{-6}\,\text{m}^2/\text{s}}{0.015\,\text{m}} = 11{,}749 \text{ m/s}$$

This velocity is obviously unrealistic because it corresponds to a Mach number of 30. Under such conditions when the speed of sound is reached, a shock wave will form and choke the flow.

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PROBLEM 7.34

Air at 16°C and atmospheric pressure enters a 1.25-cm-*ID* tube at 30 m/s. For an average wall temperature of 100°C, determine the discharge temperature of the air and the pressure drop if the pipe is (a) 10-cm-long and (b) 102-cm-long.

GIVEN

- Atmospheric air flowing through a tube
- Entering air temperature ($T_{b,\text{in}}$) = 16°C
- Tube inside diameter (D) = 1.25 cm = 0.0125 m
- Air velocity (V) = 30 m/s
- Average wall surface temperature (T_s) = 100°C

FIND

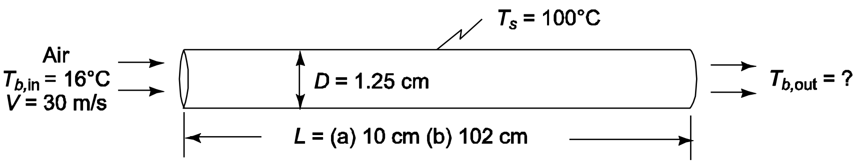
The discharge temperature ($T_{b,\text{out}}$) and the pressure drop (Δp) if the pipe length (L) is

(a) 10 cm (0.1 m)
(b) 102 cm (1.02 m)

ASSUMPTIONS

- Steady state
- The tube is smooth

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the entering bulk temperature of 16°C

Density (ρ) = 1.182 kg/m^3
Thermal conductivity (k) = 0.0248 W/(m K)
Kinematic viscosity (ν) = 15.3×10^{-6} m^2/s
Prandtl number (Pr) = 0.71
Specific heat (c) = 1012 J/(kg K)

SOLUTION

The discharge temperature will first be calculated using air properties evaluated at the entering temperature and will then be recalculated using the average bulk air temperature of the first iteration to evaluate the air properties.

The Reynolds number is

$$Re_D = \frac{V\,D}{\nu} = \frac{30\,\text{m/s}\quad 0.0125\,\text{m}}{15.3\times10^{-6}\,\text{m}^2/\text{s}} = 24{,}510 \text{ (Turbulent)}$$

(a) L/D = 0.1 m/0.0125 m = 8 < 20. The Dittus-Boelter correlation of Equation (7.61) will be used to calculate the fully developed Nusselt number

$$\overline{Nu}_{fd} = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_{fd} = 0.023\,(24{,}510)^{0.8}\,(0.71)^{0.4} = 65.11$$

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$$\bar{h}_{c,fd} = \overline{Nu}_{fd}\frac{k}{D} = 65.11\frac{0.0248\,\text{W/(m K)}}{0.0125\,\text{m}} = 129.2\ \text{W/(m}^2\ \text{K)}$$

Now

$$\frac{\overline{Nu}}{\overline{Nu}_{fd}} = \frac{\bar{h}_{c,L}}{\bar{h}_c} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24/Re_D^{0.23} = 24/(24{,}510)^{0.23} = 2.347$

$b = 2.08 \times 10^{-6}\, Re_D - 0.815 = 2.08 \times 10^{-6}\,(24{,}510) - 0.815 = -0.7640$

$$\frac{\bar{h}_{c,L}}{\bar{h}_{c,fd}} = 1 + 2.347\left(\frac{0.1}{0.0125}\right)^{-0.7640} = 1.480$$

$$\therefore \quad \bar{h}_{c,L} = 1.480\ (129.2\,\text{W/(m}^2\text{K)}) = 191.1\ \text{W/(m}^2\ \text{K)}$$

The outlet temperature is given be Equation (7.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}} = \exp\left(-\frac{PL\bar{h}_c}{\dot{m}c_p}\right) = \exp\left(-\frac{4\bar{h}_c L}{\rho V D c}\right)$$

$$T_{b,\text{out}} = T_s - (T_s - T_{b,\text{in}})\exp\left(-\frac{4\bar{h}_c L}{\rho V D c}\right)$$

$$T_{b,\text{out}} = 100°\text{C} - (100°\text{C} - 16°\text{C})\exp\left(-\frac{4\ (191.1\,\text{W/(m}^2\ \text{K)})\ (0.1\,\text{m})}{(1.182\,\text{kg/m}^3)\ (30\,\text{m/s})\ (0.0125\,\text{m})\ (1012\,\text{J/(kg K)})\ (\text{(Ws)/J})}\right)$$

$$= 29.2°\text{C}$$

Performing another iteration

$T_{b,\text{avg}} = 22.6°\text{C}$	$c = 1012$ J/(kg K)
$\rho = 1.155$ kg/m^3	$Re = 23{,}584$
$k = 0.0253$ W/(m K)	$\overline{Nu}_{fd} = 63.1$
$\nu = 15.9 \times 10^{-6}$ m^2/s	$\bar{h}_{c,L} = 189.4$
$Pr = 0.71$	$T_{b,\text{out}} = 29.3°\text{C}$ ($T_{b,\text{avg}} = 22.7°\text{C}$)

(b) The friction factor, from Equation (7.57) is

$$f = \frac{0.184}{Re_D^{0.2}} = \frac{0.184}{(23{,}584)^{0.2}} = 0.0246$$

The pressure drop is given by Equation (7.13)

$$\Delta p = f\,\frac{L}{D_H}\,\frac{\rho V^2}{2g_c} = 0.0246\left(\frac{0.1}{0.0125}\right)\frac{(1.155\,\text{kg/m}^3)\ (30\,\text{m/s})^2}{2\ (\text{N/(m}^2\text{Pa)})\ (\text{(kg m)/(s}^2\text{N)})} = 102.1$$

Pa

For L = 1.02 m, L/D = 1.02m/0.0125 m = 81.6 > 60. Therefore, the analysis is the same as above. From the first iteration, the heat transfer coefficient ($h_{c,fd}$) = 129.2 W/(m^2 K).

$$\therefore\ T_{b,\text{out}} = 100°\text{C} - (100°\text{C} - 16°\text{C})\exp\left(-\frac{4\ (129.2\,\text{W/(m}^2\text{K)})\ (1.02\,\text{m})}{(1.182\,\text{kg/m}^3)\ (30\,\text{m/s})\ (0.0125\,\text{m})\ (1012\,\text{J/(kg K)})\ (\text{(Ws)/J})}\right)$$

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$$T_{b,\text{out}} = 74.1°\text{C}$$

Performing another iteration

$T_{b,\text{avg}} = 45.0°\text{C}$	$c = 1015$ J/(kg K)
$\rho = 1.075$ kg/m^3	$Re_D = 20{,}718$
$k = 0.0270$ W/(m K)	$\overline{Nu}_D = 56.9$
$\nu = 18.1 \times 10^{-6}$ m^2/s	$\bar{h}_c = 123.0$ W/(m^2 K)
$Pr = 0.71$	$T_{b,\text{out}} = 75.4°\text{C}$ ($T_{b,\text{avg}} = 45.7°\text{C}$)

From Equation (7.57) $$f = \frac{0.184}{(20{,}178)^{0.2}} = 0.0252$$

From Equation (7.13) $$\Delta p = 0.0252\left(\frac{0.2}{0.0125}\right)\frac{1.075\,\text{kg/m}^3 \quad 30\,\text{m/s}^{\;2}}{2\;\;\text{N/(m}^2\text{Pa)} \quad (\text{kg m})/(\text{s}^2\text{N})} = 995.0\text{ Pa}$$

COMMENTS

Note that by increasing the length of the pipe by a factor of 10 leads to a temperature rise increase of about 350% and a pressure drop increase of about 875%

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PROBLEM 7.35

The equation $$Nu = 0.116\,(Re^{\frac{2}{3}} - 125)\,Pr^{\frac{1}{3}}\left[1+\left(\frac{D}{L}\right)^{\frac{2}{3}}\right]\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

has been proposed by Hausen for the transition range (2300 < *Re* < 8000) as well as for higher Reynolds numbers. Compare the values of Nu predicated by Hausen's equation for *Re* = 3000 and *Re* = 20,000 at *D*/*L* = 0.1 and 0.01 with those obtained from Gnielinski correlation, or Eq. (7.66), given in the text and Table 7.3. Assume the fluid is water at 15°C flowing through a pipe at 100°C.

GIVEN

- Water flowing through a pipe
- The Hausen correlation given above
- Water temperature = 15°C
- Pipe temperature = 100°C

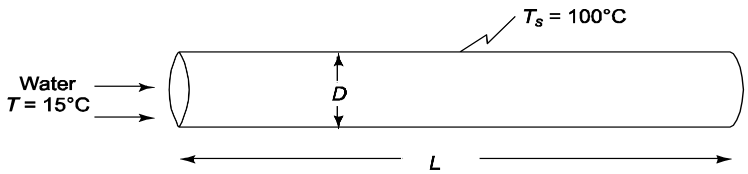
FIND

- The Nusselt number using the Hausen correlation and appropriate equations and charts in the text for Re = 3000 and 20,000 and D/L = 0.1 and 0.01

ASSUMPTIONS

- Steady state
- Constant and uniform pipe temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 15°C

Absolute viscosity (μ_b) = 1136 × 10^{-6} (Ns)/m^2

Prandtl number (Pr) = 8.1

At 100°C μ_s = 277.5 × 10^{-6} (Ns)/m^2

Pr_s= 1.75

SOLUTION

For Re = 3000, D/L = 0.1 the flow is in the transition region. In addition, L/D = 10. Therefore, the flow is not fully developed. The "short duct approximation" curve of Figure 7.12 in the text will be used to estimate the Nusselt number

$$Re_D\, Pr\frac{D}{L} \times 10^{-2} = 3000(8.1)\,(0.1) \times 10^{-2} = 24.3$$

From Figure 7.12, Nu_D= 23.

For Re = 3000, D/L = 0.01, the flow is fully developed and the Nusselt number will be estimated by Gnielinski correlation, or Eq. (7.66)

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$$\overline{Nu}_D = \frac{(f/8)(\mathrm{Re}_D - 100)\,\mathrm{Pr}}{1 + 12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3} - 1)}\left[1 + (D/L)^{2/3}\right]K$$

Where $K = (Pr_b/Pr_s)^{0.11}$ for liquid

Thus, $K = (8.1/1.75)^{0.11} = 1.18$

$D/L = 0.01$

$f = (1.82 \log Re_D - 1.64)^{-2} = [1.82 \log(3000) - 1.64]^{-2} = 0.045$

$$\overline{Nu}_D = \frac{(0.045/8)(3000 - 100)8.1}{1 + 12.7(0.045/8)^{1/2}(8.1^{2/3} - 1)}\left[1 + (0.01)^{2/3}\right]1.18$$

$$Nu_D = 41.95$$

$\overline{Nu}_D$ is calculated for other Re and L/D ratio as per above equation and tabulated below.

For Re = 20,000, D/L = 0.1, the flow is turbulent, not fully developed. The fully developed Nusselt number can be estimated from Equation (7.61)

$$Nu_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$Nu_D = 0.023\,(20{,}000)^{0.8}\,(8.1)^{0.4} = 146.5$$

Correcting this for the entrance effect

$$\frac{Nu}{Nu_{fd}} = \frac{h_{c,L}}{h_c} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24/Re_D^{0.23} = 24/(20{,}000)^{0.23} = 2.459$

$b = 2.08 \times 10^{-6}\, Re - 0.815 = 2.08 \times 10^{-6}\,(20{,}000) - 0.815 = -0.7734$

$$\frac{Nu}{Nu_{fd}} = 1 + 2.459\,(10)^{-0.7734} = 1.41$$

$$\therefore \quad Nu = 1.41\,(146.5) = 206.6$$

For Re = 20,000, D/L = 0.01, the entrance effect can be neglected Nu_D = 146.5

The Hausen correlation yields

$$\overline{Nu} = 0.116\ (3000)^{\frac{2}{3}} - 125)\ (8.1)^{\frac{1}{3}}\left[1 + (0.1)^{\frac{2}{3}}\right]\left(\frac{1136}{277.5}\right)^{0.14} = 28.63$$

Applying the Hausen correlation to the remaining cases and comparing them to the results from the text yields the following

Case	1	2	3	4
Re	3000	3000	20,000	20,000
D/L	0.1	0.01	0.1	0.01
$\overline{Nu}$ from text	23	13.88	206.6	146.5
$\overline{Nu}$ from Hausen	28.63	24.65	211.0	181.7
$\overline{Nu}$ from Gnielneski	48.72	41.95	235.0	202.4
Percent Difference	20%	44%	12%	14%

COMMENTS

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Note that the large difference in Case 2 is probably due to the use of a laminar correlation from the text when the flow is transitional. There are large variations in flow and heat transfer in this regime and it is usually avoided by good designers.

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PROBLEM 7.36

Atmospheric pressure air is heated in a long annulus (25-cm-*ID*, 38-cm-*OD*) by steam condensing at 149°C on the inner surface. If the velocity of the air is 6 m/s and its bulk temperature is 38°C, calculate the heat transfer coefficient.

GIVEN

- Atmospheric flow through an annulus with steam condensing in inner tube
- Inside diameter (D_i) = 25 cm = 0.25 m
- Outside diameter (D_o) = 38 cm = 0.38 m
- Steam temperature (T_s) = 149°C
- Air velocity (V) = 6 m/s
- Air bulk temperature (T_b) = 38°C

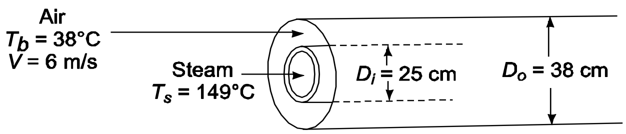
FIND

- The heat transfer coefficient ($\bar{h}_c$)

ASSUMPTIONS

- Steady state
- Steam temperature is constant and uniform
- Heat transfer to the outer surface is negligible
- Air temperature given is the average air temperature
- Thermal resistance of inner tube wall and condensing steam is negligible (Inner tube wall surface temperature = T_s)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 38°C

Density (ρ) = 1.099 kg/m^3
Thermal conductivity (k) = 0.0264 W/(m K)
Absolute viscosity (μ_b) = 19.0 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71

At the surface temperature of 149°C μ_s = 23.7 × 10^{-6} (Ns)/m^2

SOLUTION

As shown in Equation (7.3), the hydraulic diameter of the annulus is given by

$$D_H = D_o - D_i = 0.38 \text{ m} - 0.25 \text{ m} = 0.13 \text{ m}$$

The Reynolds number based on this diameter is

$$Re_D = \frac{V\, D_H\, \rho}{\mu} = \frac{(6 \text{ m/s})\,(0.13\text{m})\;1.099\,\text{kg/m}^3}{19.035 \times 10^{-6}\;(\text{Ns})/\text{m}^2\quad(\text{kg m})/(\text{s}^2\,\text{N})} = 4.50 \times 10^4 \text{ (Turbulent)}$$

Applying the Sieder-Tale correlation of Equation (7.62)

$$\overline{Nu}_D = 0.027\, Re_D^{0.8}\, Pr^{0.3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 0.027\,(4.50 \times 10^4)^{0.8}\,(0.71)^{0.3} \left(\frac{19.0}{23.7}\right)^{0.14} = 125$$

$$\bar{h}_c = \overline{Nu}_D\, \frac{k}{D} = 125\, \frac{0.0264\,\text{W/(m K)}}{0.13\,\text{m}} = 25.4 \text{ W/(m}^2\text{ K)}$$

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PROBLEM 7.37

If the total resistance between the steam and the air (including the pipe wall and scale on the steam side) in Problem 7.36 is 0.05 m² K/W, calculate the temperature difference between the outer surface of the inner pipe and the air. Show the thermal circuit.

GIVEN

- Atmospheric flow through an annulus with steam condensing in inner tube
- Diameters Inside
  - (D_i) = 25 cm = 0.25 m
  - Outside (D_o) = 38 cm = 0.38 m
- Steam temperature (T_s) = 149°C
- Air velocity (V) = 6 m/s
- Total resistance between the steam and air ($A_t\, R_{\text{tot}}$) = 0.05 (m² K)/W
- Air bulk temperature (T_b) = 38°C
- From Problem 7.36 heat transfer coefficient on the outer surface of the inner pipe ($\bar{h}_c$) = 25.4 W/(m² K)

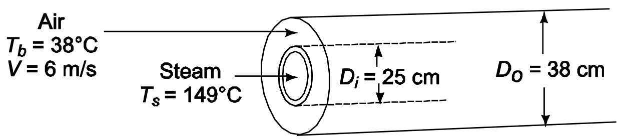
FIND

- The temperature difference between the outer surface of the inner pipe and the air (ΔT)

ASSUMPTIONS

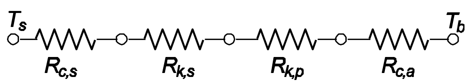
- Steady state
- Steam temperature is constant and uniform
- Heat transfer to the outer surface is negligible
- Air temperature given is the average air temperature
- Thermal resistance of inner tube wall and condensing steam is negligible (Inner tube wall surface temperature = T_s)

SKETCH



SOLUTION

The thermal circuit for the heat transfer between the steam and the air is shown below



where $R_{c,s}$ = Convective thermal resistance on the steam side

$R_{k,s}$ = Conductive thermal resistance of scaling on the steam side

$R_{k,p}$ = Conductive thermal resistance of the pipe wall

$R_{c,a}$ = Convective thermal resistance on the air side = $1/A_t\,\bar{h}_c$

$R_{\text{Tot}} = R_{c,s} + R_{k,s} + R_{k,p} + R_{c,a}$

$$R_{ca} = \frac{1}{A_t\,\bar{h}_c} \rightarrow A_t\, R_{ca} = \frac{1}{\bar{h}_c} = \frac{1}{25.4\ \text{W/(m}^2\text{K)}} = 0.0394\ (\text{m}^2\ \text{K})/\text{W}$$

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The total rate of heat transfer must equal the rate of convective heat transfer from the pipe wall to the air

$$\frac{T_s - T_b}{R_{\text{total}}} = \frac{\Delta T}{R_{ca}}$$

$$\Delta T = \frac{R_{ca}}{R_{\text{total}}}(T_s - T_b) = \frac{A_t R_{ca}}{A_t R_{\text{total}}}(T_s - T_b) = \left(\frac{0.0394}{0.05}\right)(149°\text{C} - 38°\text{C}) = 87.4°\text{C}$$

COMMENTS

Note that 79% of the thermal resistance is the convective resistance on the air side.

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PROBLEM 7.38

Atmospheric air at a velocity of 61 m/s and a temperature of 16°C enters a 0.61-m-long square metal duct of 20 × 20 cm cross section. If the duct wall is at 149°C, determine the average heat transfer coefficient. Comment briefly on the *L*/*Dₕ* effect.

GIVEN

- Atmospheric air flow through a square metal duct
- Air velocity (V) = 61 m/s
- Inlet air temperature ($T_{b,\text{in}}$) = 16°C
- Duct dimensions: 20 cm × 10 cm × 0.61 m = 0.2 m × 0.2 m × 0.61 m
- Duct wall surface temperature (T_s) = 149°C

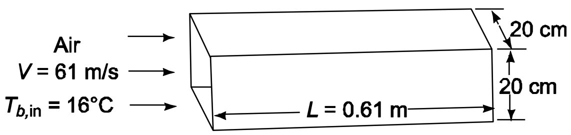
FIND

- The average heat transfer coefficient ($\bar{h}_c$)

ASSUMPTIONS

- Steady state
- Constant and uniform wall surface temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at the inlet temperature of 16°C

Density (ρ) = 1.182 kg/m^3
Thermal conductivity (k) = 0.0248 W/(m K)
Absolute viscosity (μ_b) = 18.08 × 10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.71
Specific heat (c) = 1012 J/(kg K)

At the wall temperature of 149°C μ_s = 23.8 × 10^{-6} (Ns)/m^2

SOLUTION

The hydraulic diameter of the duct is given by Equation (7.2)

$$D_H = \frac{4A_c}{P} = \frac{4(0.2\,\text{m})(0.2\,\text{m})}{4(0.2\,\text{m})} = 0.2\text{ m} \Rightarrow = \frac{L}{D_H} = \frac{0.61\,\text{m}}{0.2\,\text{m}} = 3.05$$

The Reynolds number based on the hydraulic diameter is

$$Re_D = \frac{VD_H\rho}{\mu} = \frac{(61\,\text{m/s})\,(0.2\,\text{m})\;1.182\,\text{kg/m}^3}{18.08\times10^{-6}\;(\text{Ns})/\text{m}^2\;\;(\text{kg m})/(\text{Ns}^2)} = 7.97\times10^5 \text{ (Turbulent)}$$

Using the Sieder-Tate correlation of Equation (7.62) with the hydraulic diameter

$$\overline{Nu}_{D_H} = 0.027\,Re_{D_H}^{\;0.8}\,Pr^{0.3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

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$$= 0.027(7.97 \times 10^5)^{0.8}\,(0.71)^{0.3}\left(\frac{18.08}{23.8}\right)^{0.14} = 1235$$

$$\bar{h}_c = \overline{Nu}_{D_H}\,\frac{k}{D_H} = 1235\,\frac{0.0248\,\text{W/(m K)}}{0.2\,\text{m}} = 153\ \text{W/(m}^2\ \text{K)}$$

Note that since $2 < L/D_H < 20$, the heat transfer coefficient will be

$$\frac{\overline{Nu}}{\overline{Nu}_{fd}} = \frac{\bar{h}_{c,L}}{\bar{h}_c} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24/Re_D^{0.23} = 24/(7.97 \times 10^5)0.23 = 1.054$

$b = 2.08 \times 10^{-6}\ Re_D - 0.815 = 2.08 \times 10^{-6}\ (7.97 \times 10^5) - 0.815 = 0.843$

$$\bar{h}_{c,L} = 153\,\text{W/(m}^2\text{K)}\ \ [1 + 1.054\,(3.05)^{0.843} = 3.70] = 566\ \text{W/(m}^2\ \text{K)}$$

The air properties at the inlet temperature were used in the calculation. This may lead to significant errors if the air temperature rises appreciably within the duct, therefore, the outlet air temperature will be calculated. The outlet temperature can be calculated using Equation (7.36)

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_s - T_{b,\text{out}}}{Ts - T_{b,\text{in}}} = \exp\left(-\frac{P\,L\,\overline{h}_{c,L}}{\dot{m}\,c_p}\right) = \exp\left(-\frac{\overline{h}_{c,L}\,P\,L}{A_c\,\rho V\,c}\right)$$

$$T_{b,\text{out}} = T_s - (T_s - T_{b,\text{in}})\,\exp\left(-\frac{\overline{h}_{c,L}\,P\,L}{A_c\,\rho V\,c}\right)$$

$$T_{b,\text{out}} = 149°\text{C} - (149°\text{C} - 16°\text{C})$$

$$\exp\left(-\frac{566\,\text{W/(m}^2\,\text{K)}\ \ 4(0.2\,\text{m})(0.61\,\text{m})}{(0.2\,\text{m})^2\ \ 1.182\,\text{kg/m}^3\ \ \ 61\,\text{m/s}\ \ \ 1012\,\text{J/(kg K)}\ \ \ (\text{Ws})/\text{J}}\right) = 28°\text{C}$$

Therefore, the average air temperature is about 22°C. The difference in air properties at 22°C and 16°C is not great enough to justify another iteration.

COMMENTS

Note that the average heat transfer coefficient in the duct is greater than that in a long duct due to the L/D effect. The heat transfer coefficient is largest at the entrance. This is analogous to flow over a flat plate as discussed in Chapter 5.

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PROBLEM 7.39

Atmospheric air at 10°C enters a 2 m long smooth rectangular duct with a 7.5 cm × 15 cm cross-section. The mass flow rate of the air is 0.1 kg/s. If the sides are at 150°C, estimate (a) the heat transfer coefficient, (b) the air outlet temperature, (c) the rate of heat transfer, and (d) the pressure drop.

GIVEN

- Atmospheric air flow through a rectangular duct
- Inlet bulk temperature ($T_{b,\text{in}}$) = 10°C
- Duct length (L) = 2 m
- Cross-section = 7.5 cm × 15 cm = 0.075 m × 0.15 m
- Mass flow rate (m) = 0.1 kg/s
- Duct surface temperature (T_s) = 150°C

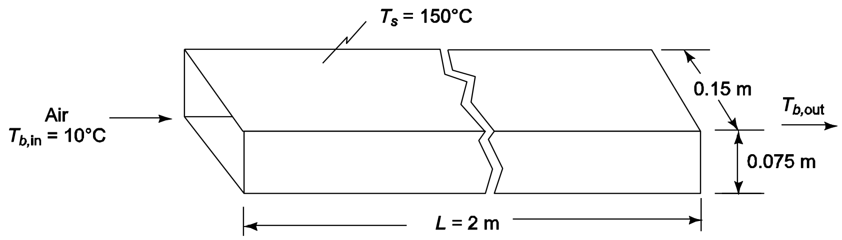
FIND

(a) The heat transfer coefficient $\bar{h}_c$
(b) The air outlet temperature ($T_{b,\text{out}}$)
(c) The rate of heat transfer (q)
(d) The pressure drop (Δp)

ASSUMPTIONS

- Steady state
- The duct is smooth

SKETCH



SOLUTION

The hydraulic diameter of the duct is

$$D_H = \frac{4A_c}{P} = \frac{4(0.15\,\text{m})(0.075\,\text{m})}{2(0.15\,\text{m})(0.075\,\text{m})} = 0.10\text{m}$$

For the first iteration, let $T_{b,\text{out}}$ = 50°C. For dry air at the average bulk temperature of 30°C

Density (ρ) = 1.128 kg/m^3

Thermal conductivity (k) = 0.0258 W/(m K)

Absolute viscosity (μ) = 18.68 × 10^{-6} (Ns)/m^2

Prandtl number (Pr) = 0.71

Specific heat (c_p) = 1013 J/(kg K)

The Reynolds number is

$$Re_D = \frac{VD_H}{\nu} = \frac{\dot{m}D_H}{A\mu} = \frac{0.1\text{kg/s}\quad 0.10\,\text{m}}{0.15\text{m}\quad 0.075\text{m}\quad 18.68\times 10^{-6}\,(\text{Ns})/\text{m}^2\quad (\text{kg}\,\text{m})/(\text{Ns}^2)}$$

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$= 47{,}585 > 10{,}000$ (Turbulent)

$$\frac{L}{D_H} = \frac{2\,\text{m}}{0.1\,\text{m}} = 20$$

(a) Therefore, entrance effects may be significant.
From Equation (7.61)

$$\overline{Nu}_{fd} = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_{fd} = 0.023(47{,}585)^{0.8}\,(0.71)^{0.4} = 111$$

$$\bar{h}_{c,fd} = \overline{Nu}_{fd}\,\frac{k}{D} = 111\,\frac{0.0258\,\text{W/(m K)}}{0.1\,\text{m}} = 28.56\ \text{W/(m}^2\ \text{K)}$$

Since $L/D = 20$, we get

$$\frac{\bar{h}_{c,L}}{\bar{h}_{c,fd}} = 1 + a\left(\frac{L}{D}\right)^b$$

where $a = 24\, Re^{-0.23} = 24(47{,}585)^{-0.23} = 2.02$

$b = 2.08 \times 10^{-6}\, Re - 0.815 = 2.08 \times 10^{-6}\,(47{,}585) - 0.815 = -0.716$

$$\frac{\bar{h}_{c,L}}{\bar{h}_{c,fd}} = 1 + 2.02\left(\frac{2}{0.1}\right)^{-0.716} = 1.24$$

and

$$\frac{\bar{h}_{c,L}}{\bar{h}_{c,fd}} = 1 + \left(6\,\frac{D}{L}\right) = 1 + \frac{6(0.1)}{2} = 1.3$$

The average of the two values is $\bar{h}_{c,L}/\bar{h}_{c,fd} = 1.27$

$$\therefore \quad \bar{h}_{c,L} = 1.27\ 28.56\,\text{W/(m}^2\text{K)} = 36.22\ \text{W/(m}^2\ \text{K)}$$

(b) The outlet temperature is found by rearranging Equation (7.36)

$$T_{b,\text{out}} = T_s - (T_{b,\text{in}} - T_s)\exp\left(-\frac{PL\bar{h}_c}{\dot{m}c}\right)$$

$$T_{b,\text{out}} = 150°\text{C} - (10°\text{C} - 150°\text{C})\exp\left(-\frac{2\ 0.075\,\text{m}+0.15\,\text{m}\quad 2\,\text{m}\quad 36.32\,\text{W/(m}^2\text{K)}}{0.1\,\text{kg/s}\quad 1013\,\text{J/(kg K)}\quad (\text{Ws})/\text{J}}\right) = 48°\text{C}$$

No further iteration is needed since the result is close to the initial guess.

(c) The rate of heat transfer is given by

$$q = \dot{m}\,c\Delta T_b = 0.1\,\text{kg/s}\quad 1013\,\text{J/(kg K)}\ (48°\text{C} - 10°\text{C}) = 3849\ \text{W}$$

(d) The friction can be estimated from the lowest line for Figure 7.7: $Re = 47{,}585 \rightarrow f \approx 0.021$. The pressure drop is given by Equation (7.13)

$$\text{o}\Delta p = f\,\frac{L}{D_H}\,\frac{\rho V^2}{2} = f\,\frac{L}{D_H}\,\frac{\rho\left(\frac{4\dot{m}}{\rho\pi D_H^2}\right)^2}{2} = f\,\frac{L}{D_H}\,\frac{8}{\rho}\left(\frac{\dot{m}}{\pi D_H^2}\right)^2$$

So

$$\Delta p = 0.021\left(\frac{2\,\text{m}}{0.1\,\text{m}}\right)\left(\frac{8}{1.128\,\text{kg/m}^3}\right)\left(\frac{0.1\,\text{kg/s}}{\pi\ 0.1\,\text{m}^{\ 2}}\right)^2\ (\text{Pa m}^2)/\text{N}\quad (\text{s}^2\text{N})/(\text{kg m}) = 30.2\ \text{Pa}$$

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PROBLEM 7.40

Determine the heat transfer coefficient for liquid bismuth flowing through an annulus (5-cm-*ID*, 6.1-cm-*OD*) at a velocity of 4.5 m/s. The wall temperature of the inner surface is 427°C and the bismuth is at 316°C. It may be assumed that heat losses from the outer surface are negligible.

GIVEN

- Liquid bismuth flowing through an annulus
- Annulus diameters
  - D_i = 5 cm = 0.05 m
  - D_o = 6.1 cm = 0.061 m
- Bismuth velocity (V) = 4.5 m/s
- Temperature
  - Inner wall surface ($T_s i$) = 427°C
  - Bismuth (T_b) = 316°C

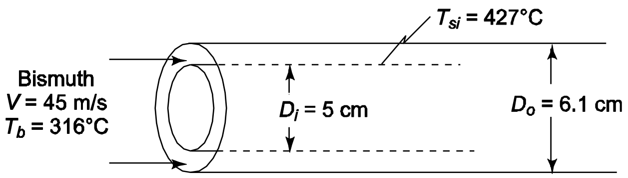
FIND

- The heat transfer coefficient

ASSUMPTIONS

- Steady state
- Heat losses from outer surfaces are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 25, for bismuth at the bulk temperature of 316°C

Thermal conductivity (k) = 16.44 W/(m K)
Kinematic viscosity (ν) = 1.57 × 10^{-7} m^2/s
Prandtl number (Pr) = 0.014

SOLUTION

The hydraulic diameter for the annual is given by Equation (7.3)

$$D_H = D_o - D_i = 0.061 \text{ m} - 0.05 \text{ m} = 0.011 \text{ m}$$

The Reynolds number based on the hydraulic diameter is

$$Re_{D_H} = \frac{VD_H}{\nu} = \frac{4.5 \text{m/s} \quad 0.011 \text{m}}{1.57 \times 10^{-7} \text{ m}^2/\text{s}} = 3.15 \times 10^5$$

For liquid metals, the Nusselt number is given by Equation (7.68)

$$\overline{Nu}_{D_H} = 0.625\,(Re_{D_H}\; Pr)^{0.4} = 0.625\,[3.15 \times 10^5\,(0.014)]^{0.4} = 17.9$$

$$\bar{h}_c = \overline{Nu}_{D_H} \frac{k}{D_H} = 17.9 \frac{16.44 \text{W/(m K)}}{0.011 \text{m}} = 26{,}800 \text{ W/(m}^2\text{ K)}$$

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PROBLEM 7.41

Mercury flows inside a copper tube 9 m long with a 5.1 cm inside diameter at an average velocity of 7 m/s. The temperature at the inside surface of the tube is 38°C uniformly throughout the tube, and the arithmetic mean bulk temperature of the mercury is 66°C. Assuming the velocity and temperature profiles are fully developed, calculate the rate of heat transfer by convection for the 9 m length by considering the mercury as (a) an ordinary liquid and (b) liquid metal. Compare the results.

GIVEN

- Mercury flows inside a copper tube
- Tube length (L) = 9 m
- Inside diameter (D) = 5.1 cm = 0.051 m
- Average mercury velocity (V) = 7 m/s
- Tube inside surface temperature (T_s) = 38°C (uniform)
- Bulk temperature of mercury (T_b) = 66°C

FIND

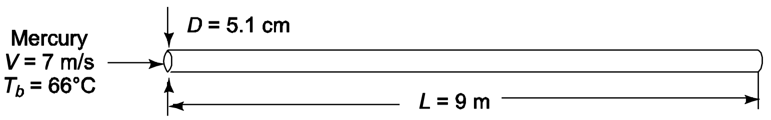
The rate of heat transfer for the 9 m length considering mercury as

(a) an ordinary liquid, and
(b) a liquid metal

ASSUMPTIONS

- Steady state
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 26, for mercury at the bulk temperature of 66°C

Thermal conductivity (k) = 9.76 W/(m K)

Kinematic viscosity (ν) = 0.1004×10^{-6} m²/s

Prandtl number (Pr) = 0.0193

SOLUTION

The Reynolds number is

$$Re_D = \frac{VD}{\nu} = \frac{(7\,\text{m/s})\ 0.051\,\text{m}}{0.1004 \times 10^{-6}\,\text{m}^2/\text{s}} = 3.6 \times 10^6$$

(a) The mercury will be treated as an ordinary liquid by applying the Dittus-Boelter Equation (7.64)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\, (3.6 \times 10^6)^{0.8}\, (0.0193)^{0.3} = 1229$$

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$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 1229 \frac{9.76\,\text{W/(m K)}}{0.051\,\text{m}} = 2.35 \times 10^5 \text{ W/(m}^2\text{ K)}$$

The rate of heat transfer is

$$q = \bar{h}_c A\,(T_b - T_s) = \bar{h}_c\, \pi D L\,(T_b - T_s)$$

$$q = \left(2.35\times10^5\ \text{W/(m}^2\text{K)}\right) \pi\,(0.051\text{ m})\,(9\text{ m})\,(66°\text{C} - 38°\text{C}) = 9.5 \times 10^6\text{ W}$$

(b) For liquid metals and a constant surface temperature boundary, the Nusselt number is given by Equation (7.71)

$$\overline{Nu}_D = 5.0 + 0.025\,(Re_D\,Pr)^{0.8} = 5.0 + 0.025\,[(3.6 \times 10^6)\,(0.0193)]^{0.8} = 191$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 191 \frac{9.76\,\text{W/(m K)}}{0.051\,\text{m}} = 3.65 \times 10^4 \text{ W/(m}^2\text{ K)}$$

$$q = \left(3.65\times10^4\ \text{W/(m}^2\text{K)}\right) \pi\,(0.051\text{ m})\,(9\text{ m})\,(66°\text{C} - 38°\text{C}) = 1.47 \times 10^6\text{ W}$$

COMMENTS

Applying the Dittus-Boelter equation, which is valid for $Pr > 0.5$ only, to mercury ($Pr \approx 0.02$) leads to a 648% overestimation in the rate of heat transfer to the pipe. This shows that application of empirical outside the limits of experimental verification can lead to serious errors.

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PROBLEM 7.42

A heat exchanger is to be designed to heat a flow of molten bismuth from 377°C to 477°C. The heat exchanger consists of a 50-mm-*ID* tube with surface temperature maintained uniformly at 500°C by an electrical heater. Find the length of the tube and the power required to heat 4 kg/s and 8 kg/s of bismuth.

GIVENS

- Molten Bismuth flows through a tube
- Bismuth temperature
  - Inlet ($T_{b,\text{in}}$) = 377°C
  - Outlet ($T_{b,\text{out}}$) = 477°C
- Tube inside diameter (D) = 50 mm = 0.05 m
- Surface temperature (T_s) = 500°C

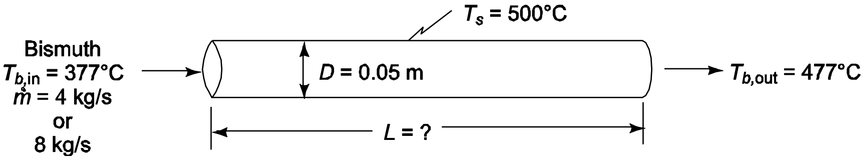
FIND

- The length (L) tube and power required (q) to heat 4 kg/s and 8 kg/s of bismuth

ASSUMPTIONS

- Steady state
- Uniform and constant surface temperature
- Losses from the heater are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 25, for Bismuth at the average bulk temperature of 427°C

Specific heat (c) = 150 J/(kg K)
Thermal conductivity (k) = 15.58 W/(m K)
Absolute viscosity (μ) = 13.39 × 10^{-4} (Ns)/m^2
Prandtl number (Pr) = 0.013

SOLUTION

At $\dot{m}$ = 4 kg/s the Reynolds number is

$$Re = \frac{VD\rho}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4\ \ 4\,\text{kg/s}}{\pi\ \ 0.05\,\text{m}\ \ 13.39\times10^{-4}\,(\text{Ns})/\text{m}^2\ \ \ (\text{kg}\,\text{m})/(\text{Ns}^2)} = 76{,}070$$

$Re\,Pr$ = 76,070 (0.013) = 989

Therefore, Equation (7.71) can be used. The resulting L/D should be greater than 30

$$\overline{Nu}_D = 5.0 + 0.025\,(Re_D\,Pr)^{0.8} = 5.0 + 0.025\,(989)^{0.8} = 11.23$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 11.23\,\frac{15.58\,\text{W/(m K)}}{0.05\,\text{m}} = 3498\ \text{W/(m}^2\ \text{K)}$$

Equation (7.36) can be used to find the length required

$$\frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}} = \exp\left(-\frac{PL\bar{h}_c}{\dot{m}c_p}\right) = \exp\left(-\frac{\bar{h}_c\pi DL}{\dot{m}c}\right)$$

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$$L = -\frac{\dot{m}c}{\bar{h}_c \pi D} \ln\left(\frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}}\right) = -\frac{4\,\text{kg/s} \quad 150\,\text{J/(kg K)} \quad \text{(Ws)/J}}{3498\,\text{W/(m}^2\text{K)} \;\; \pi \;\; 0.05\,\text{m}} \ln\left(\frac{500°\text{C} - 477°\text{C}}{500°\text{C} - 377°\text{C}}\right) = 1.83 \text{ m}$$

L/D = (1.83 m)/(0.05 m) = 37

Repeating the analysis for $\dot{m}$ = 8 kg/s yields the following

Re = 152,140

$RePr$ = 1978

$\overline{Nu}_D$ = 15.84

$\bar{h}_c$ = 4935 W/(m^2 K)

L = 2.60 m

$\frac{L}{D}$ = 52

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PROBLEM 7.43

Liquid sodium is to be heated from 500 K to 600 K by passing it at a flow rate of 5.0 kg/s through a 5-cm-*ID* tube whose surface is maintained at 620 K. What length of tube is required?

GIVEN

- Liquid sodium flow in a tube
- Inlet bulk temperature ($T_{b,\text{in}}$) = 500 K
- Outlet bulk temperature ($T_{b,\text{out}}$) = 600 K
- Inside tube diameter (D) = 5 cm = 0.05 m
- Tube surface temperature (T_s) = 620 K
- Mass flow rate (m) = 5.0 kg/s

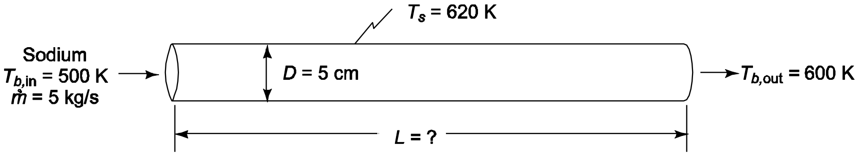
FIND

- The length of tube (L) required

ASSUMPTIONS

- Surface temperature is constant and uniform

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for liquid sodium at the average bulk temperature of 550 K

Specific heat (c) = 1322 J/(kg K)
Thermal conductivity (k) = 76.9 W/(m K)
Absolute viscosity (μ) = 3.67 × 10^{-4} (Ns)/m^2
Prandtl number (Pr) = 0.0063

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D\rho}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4\ \ 5\,\text{kg/s}}{\pi\ \ 0.05\,\text{m}\ \ 3.67\times10^{-4}\,(\text{Ns})/\text{m}^2\ \ \ (\text{kg m})/(\text{Ns}^2)} = 3.47\times10^5$$

$$Re_D\,Pr = 3.47\times10^5\,(0.0063) = 2186$$

This is within the range of Equation (7.71)

$$\overline{Nu}_D = 5.0 + 0.025(Re_D\,Pr)^{0.8} = 5.0 + 0.025(2186)^{0.8} = 16.7$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 16.7\,\frac{76.9\,\text{W/(m K)}}{0.05\,\text{m}} = 2.57\times10^4\ \text{W/(m}^2\ \text{K)}$$

Solving Equation (7.36) for the length

$$L = \frac{\dot{m}c_p}{\bar{h}_c\pi D}\ln\left(\frac{T_s - T_{b,\text{out}}}{T_s - T_{b,\text{in}}}\right) = \frac{5\,\text{kg/s}\ \ 1322\,\text{J/(kg K)}}{2.57\times10^4\ \text{W/(m}^2\text{K)}\ \ \pi\ \ 0.05\,\text{m}\ \ \ \text{J/(Ws)}}\ln\left(\frac{620\,\text{K} - 600\,\text{K}}{620\,\text{K} - 500\,\text{K}}\right) = 2.93\ \text{m}$$

Note that L/D = 2.93 m/0.05 m = 58.6 > 30. Therefore, use of Equation (7.71) is valid.

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PROBLEM 7.44

For fully turbulent flow in a long tube of diameter *D*, develop a relation between the ratio (*L*/Δ*T*)/*D* in terms of flow and heat transfer parameters, where *L*/Δ*T* is the tube length required to raise the bulk temperature of the fluid by Δ*T*. Use Equation (7.61) for fluids with Prandtl number of the order of unity or larger and Equation (7.68) for liquid metals.

GIVEN

- Fully developed turbulent flow in a long tube
- Diameter = D
- $L/\Delta T$ = Tube length required to raise the bulk temperature by ΔT

FIND

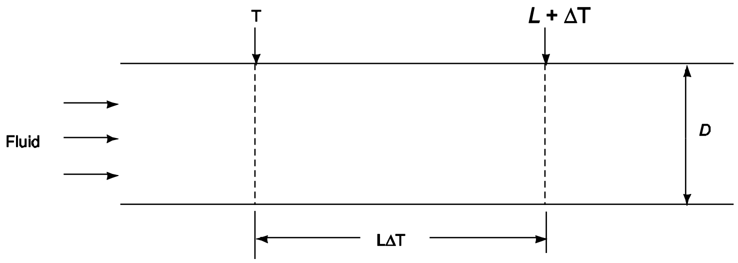
A relationship for $(L/\Delta T)/D$ in terms of flow and heat transfer parameters using

(a) Equation 7.61 for fluids with $Pr \approx 1$
(b) Equation 7.68 for liquid metals

ASSUMPTIONS

- Steady state
- Constant fluid properties
- Uniform wall temperatures

SKETCH



SOLUTION

Let

k = the thermal conductivity of the fluid
μ = the absolute viscosity of the fluid
c = the specific heat of the fluid
V = the velocity of the fluid
ρ = the density of the fluid
T_b = Average bulk fluid temperature
T_w = wall temperature

(a) Using Equation (7.61) for the Nusselt number

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\bar{h}_c = \overline{Nu}_D \frac{k}{D} = 0.023 \frac{k}{D} Re_D^{0.8}\, Pr^{0.4}$$

The rate of heat transfer to the fluid must equal the energy needed to raise the temperature of the fluid by ΔT

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$$q = \bar{h}_c \; \pi D L\,(T_b - T_w) = \dot{m}\, c\, \Delta T$$

$$\frac{L}{\Delta T} = \frac{\dot{m} c}{\bar{h}_c \pi D (T_b - T_w)} = \frac{\rho V \frac{\pi}{4} D^2 C}{0.023 \frac{k}{D} Re_D^{0.8} Pr^{0.4} \pi D (T_b - T_w)}$$

But $Re_D = \dfrac{V D \rho}{\mu}$ and $Pr = \dfrac{c\,\mu}{k}$

$$\therefore \; \frac{L}{\Delta T} = 10.87 \; \frac{\rho V D^2 C}{k \left(\frac{V \rho D}{\mu}\right)^{0.8} \left(\frac{c\,\mu}{k}\right)^{0.4} (T_b - T_w)}$$

$$\frac{\frac{L}{D}}{\Delta T} \quad 10.87\, \rho^{0.2}\, V^{0.2}\, D^{0.2}\, c^{0.6}\, \mu^{0.4}\, k^{-0.6}\, (T_b - T_w)^{-1}$$

Checking the units

$$\left[\frac{\frac{L}{D}}{\Delta T}\right] = \left[\mathrm{kg/m^3}\right]^{0.2} \; \mathrm{m/s}^{\;0.2}\, [\mathrm{m}]^{0.2} \;\; \mathrm{J/(kgK)}^{\;0.6} \left[(\mathrm{Ns})/\mathrm{m}^2\right]^{0.4} \;\; \mathrm{W/(mK)}^{\;-0.6} \;\; [\mathrm{K}]^{-1}$$

$$(\mathrm{Ws})/\mathrm{J}^{\;0.6} \left[\mathrm{kg\,m/(s^2 N)}\right]^{0.4} = 1/\mathrm{K}$$

(b) From Equation (7.68)

$$\bar{h}_c = 0.625 \; \frac{k}{D} \; Re_D^{0.4}\, Pr^{0.4}$$

$$\frac{L}{\Delta T} = \frac{\rho V \frac{\pi}{4} D^2 c}{0.625 \frac{k}{D} Re_D^{0.4} Pr^{0.4} \pi D (T_b - T_w)}$$

$$\therefore \; \frac{L}{\Delta T} = 0.40 \; \frac{\rho V D^2 c}{k \left(\frac{V \rho D}{\mu}\right)^{0.4} \left(\frac{c\,\mu}{k}\right)^{0.4} (T_b - T_w)}$$

$$\frac{\frac{L}{D}}{\Delta T} = 0.40\, \rho^{0.6}\, V^{0.6}\, D^{0.6}\, c^{0.6}\, k^{-0.6}\, (T_b - T_w)^{-1}$$

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PROBLEM 7.45

A 2.54-cm-*OD*, 1.9-cm-*ID* steel pipe carries dry air at a velocity of 7.6 m/s and a temperature of –7°C. Ambient air is at 21°C and has a dew point of 10°C. How much insulation with a conductivity of 0.18 W/(m K) is needed to prevent condensation on the exterior of the insulation if *h* = 2.4 W/(m^2 K) on the outside?

GIVEN

- Dry air flowing through an insulated steel pipe
- Pipe diameters
  - Inside (D_i) = 1.9 cm = 0.019 m
  - Outside (D_o) = 2.54 cm = 0.0254 m
- Air velocity (V) = 7.6 m/s
- Air temperature (T_a) = –7°C
- Ambient temperature (T_∞) = 21°C
- Ambient dew point (T_{dp}) = 10°C
- Thermal conductivity of insulation (k_I) = 0.18 W/(m K)
- Heat transfer coefficient on exterior $\bar{h}_{c\infty}$ = 2.4 W/(m^2 K)

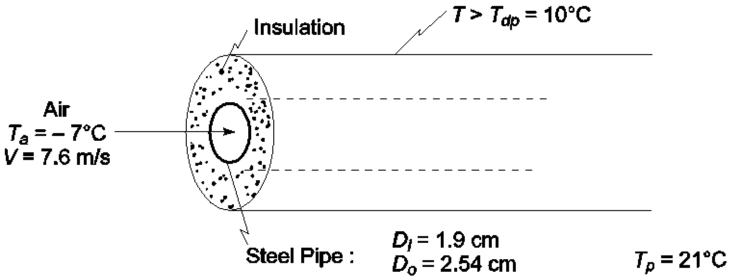
FIND

- Thickness of insulation (t) required to prevent condensation

ASSUMPTIONS

- Steady state
- Flow is fully developed
- Pipe surface temperature can be considered uniform and constant
- Radiation heat transfer to the insulation is negligible or included in $\bar{h}_{c\infty}$
- Pipe is 1% carbon steel

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at –7°C by extrapolation

Thermal conductivity (k) = 0.0232 W/(m K)

Kinematic viscosity (ν) = 13.3 × 10^{-6} m^2/s

Prandtl number (Pr) = 0.71

From Appendix 2, Table 10, the thermal conductivity of 1% carbon steel (ks) = 52 W/(m K)

SOLUTION

Interior heat transfer coefficient $\bar{h}_{ca}$:

The Reynolds number for the air flow is

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$$Re_D = \frac{VD_i}{\nu} = \frac{7.6\,\text{m/s} \quad 0.019\,\text{m}}{13.3\times10^{-6}\,\text{m}^2/\text{s}} = 10{,}860 \text{ (Turbulent)}$$

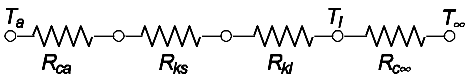
Applying Equation (7.61)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(10{,}860)^{0.8}\,(0.71)^{0.4} = 33.95$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D_i} = 33.95\,\frac{0.0232\,\text{W/(m K)}}{0.019\,\text{m}} = 41.45 \text{ W/(m}^2\text{ K)}$$

Thermal Circuit:



where

$$R_{ca} = \frac{1}{\bar{h}_{ca}A_i} = \frac{1}{\bar{h}_{ca}\pi D_i L} = \frac{1}{41.45\,\text{W/(m}^2\text{K)}\ \ \pi(0.019\,\text{m})L} = \left(0.404\frac{1}{L}\right) \text{ (m K)/W}$$

$$R_{ks} = \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi L k_s} = \frac{\ln\left(\frac{2.54}{1.9}\right)}{2\pi L\ \ 52\,\text{W/(m K)}} = \left(0.00089\frac{1}{L}\right) \text{ (m K)/W}$$

$$R_{kl} = \frac{\ln\left(\frac{D_I}{D_o}\right)}{2\pi L k_I} = \frac{\ln\left(\frac{D_I}{D_o}\right)}{2\pi L\ \ 0.18\,\text{W/(m K)}} = \left(0.884\frac{\ln\left(\frac{D_I}{D_o}\right)}{L}\right) \text{ (m K)/W}$$

$$R_{c\infty} = \frac{1}{\bar{h}_{c\infty}A_I} = \frac{1}{\bar{h}_{c\infty}\pi D_I L} = \frac{1}{2.4\,\text{W/(m}^2\text{K)}\ \ \pi D_I L} = \left(0.1326\frac{1}{D_I L}\right) \text{ (m}^2\text{ K)/W}$$

The heat transfer from T_∞ to T_I and from T_I to T_a will be equated

$$\frac{T_\infty - T_I}{R_{c\infty}} = \frac{T_I - T_a}{R_{kI} + R_{ks} + R_{ca}}$$

$$\frac{D_I\ \ T_\infty - T_I}{0.1326\,(\text{m}^2\text{K})/\text{W}} = \frac{T_I - T_a}{\left(0.00089 + 0.884\ln\left(\frac{D_I}{D_o} + 0.404\right)\right)(\text{m}^2\text{K})/\text{W}}$$

$$D_I\left(6.67\,1/\text{m}\ln\left[\frac{D_I}{(0.0254\text{ m})}\right] + 3.05\,1/\text{m}\right) = \frac{T_I - T_a}{T_\infty - T_I} = \frac{10°\text{C} + 7°\text{C}}{21°\text{C} - 10°\text{C}} = 1.545$$

By trial and error: D_I = 0.117 m = 11.7 cm

Therefore, the insulation thickness must be greater than

$$t > \frac{D_I - D_o}{2} = \frac{11.7\,\text{cm} - 2.54\,\text{cm}}{2} = 4.6 \text{ cm}$$

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PROBLEM 7.46

A double-pipe heat exchanger is used to condense steam at 7370 N/m². Water at an average bulk temperature of 10°C flows at 3.0 m/s through the inner pipe, which is made of copper and has a 2.54-cm-*ID* and a 3.05-cm-*OD*. Steam at its saturation temperature flows in the annulus formed between the outer surface of the inner pipe and an outer pipe of 5.08-cm-*ID*. The average heat transfer coefficient of the condensing steam is 5700 W/(m² K), and the thermal resistance of a surface scale on the outer surface of the copper pipe is 0.000118 (m² K)/W. (a) Determine the overall heat transfer coefficient between the steam and the water based on the outer area of the copper pipe and sketch the thermal circuit. (b) Evaluate the temperature at the inner surface of the pipe. (c) Estimate the length required to condense 45 gm/s of steam. (d) Determine the water inlet and outlet temperatures.

GIVEN

- Double-pipe heat exchanger, steam in annulus, and water in inner pipe.
- Steam is condensing at a pressure of 7370 N/m².
- Average bulk water temperature, T_b = 10°C, and water velocity, V = 3.0 m/s.
- Copper inner pipe
  - Inside diameter, $D_{p,i}$ = 0.0254 m
  - Outside diameter, $D_{p,o}$ = 0.0305 m
- Outer pipe inside diameter, D_o = 0.0508 m
- Heat transfer coefficient of condensing steam, $\bar{h}_{c,s}$ = 5700 W/(m² K)
- Thermal resistance of scale on outside of copper pipe, $(AR_{k,s})$ = 0.000118 (m² K)/W

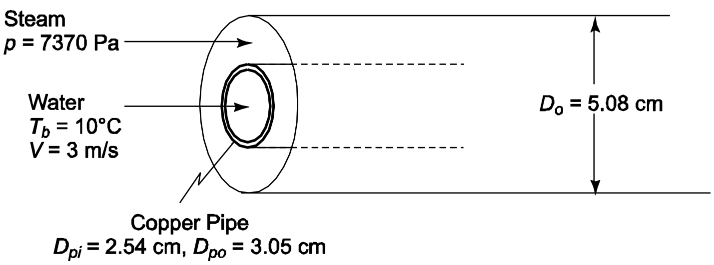
FIND

(a) Overall heat transfer coefficient, U_o.
(b) Temperature of inner surface of the pipe, T_{wi}.
(c) The length, L, required to condense 0.45 kg/s of steam.
(d) Water inlet and outlet temperatures, $T_{w,in}$ and $T_{w,out}$.

ASSUMPTIONS

- Steady-state.
- Constant steam temperature during condensation.
- The flow is fully developed, and copper tube is made of pure copper.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for steam at 7370 N/m², the saturation temperature T_s = 40°C, and the heat of vaporization, h_{fg} = 2406 kJ/kg.

From Appendix 2, Table 12, for copper at ~ 40°C (~ steam temperature) the thermal conductivity k_c = 398 W/(m K).

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From Appendix 2, Table 13, for water at average bulk temperature of 10°C

Density, ρ = 999.7 kg/m^3
Thermal conductivity, k = 0.577 W/(m K)
Absolute viscosity, μ_b = 1.296 × 10^{-3} (Ns)/m^2
Prandtl number, Pr = 9.5
Specific heat, c_p = 4195 J/(kg K)

SOLUTION

The Reynolds number for water flow inside the pipe is

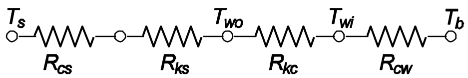
$$Re_D = \frac{\rho V D_{p,i}}{\mu_b} = \frac{999.7 \times 3.0 \times 0.0254}{0.001296} = 58{,}779 \Rightarrow \text{turbulent flow}$$

Using the simpler Dittus-Boelter correlation for turbulent pipe flow, Equation (7.61), the average Nusselt number and hence the heat transfer coefficient for water flow can be calculated as

$$\bar{N}u_D = 0.023 Re_D^{0.8} Pr^{0.4} = 0.023(58{,}779)^{0.8}(9.5)^{0.4} = 370$$

$$\Rightarrow \ \bar{h}_{c,w} = \bar{N}u_D \frac{k}{D_{p,i}} = 370 \frac{0.577}{0.0254} = 8405 \ \text{W/(m}^2\text{ K)}$$

The thermal circuit for heat flow from the steam to the water can be sketched as follows



Here, considering the pipe length to by L, each of the four resistances can be calculated (see Chapter 1, Section 1.6.3, and Chapter 2, Section 2.3.2, for respective definitions) as follows

$$R_{c,s} = \frac{1}{\bar{h}_{c,s} \pi D_{p,o} L} = \frac{1}{5700 \times \pi \times 0.0305 \times L} = \left(\frac{0.00183}{L}\right) \ \text{(m K)/W}$$

$$R_{k,s} = \frac{A R_{k,s}}{\pi D_{p,o} L} = \frac{0.000118}{\pi \times 0.0305 \times L} = \left(\frac{0.00123}{L}\right) \ \text{(m K)/W}$$

$$R_{k,c} = \frac{\ln\left(\dfrac{D_{p,o}}{D_{p,i}}\right)}{2\pi k_c L} = \frac{\ln\left(\dfrac{0.0305}{0.0254}\right)}{2\pi \times 398 \times L} = \left(\frac{0.000073}{L}\right) \ \text{(m K)/W}$$

$$R_{c,w} = \frac{1}{\bar{h}_{c,w} \pi D_{p,i} L} = \frac{1}{8405 \times \pi \times 0.0254 \times L} = \left(\frac{0.00149}{L}\right) \ \text{(m K)/W}$$

(a) The overall heat transfer coefficient based on the outer area of the copper pipe is

$$U_o = \frac{1}{A_o R_{total}} = \frac{1}{\pi D_{p,o} L \ \ R_{c,s} + R_{k,s} + R_{k,c} + R_{c,w}}$$

$$\therefore \qquad U_o = \frac{1}{\pi \ \ 0.0305 \ \ \ 0.00183 + 0.00123 + 0.000073 + 0.00149} = 2257 \ \text{W/(m}^2\text{ K)}$$

(b) The temperature of the inner surface of the pipe can be calculated by equating the rate of heat transfer between steam and water to the rate of convection to the water

$$q = U_o \ \ \pi D_{p,o} L \ \ \ T_s - T_b \ \ = \bar{h}_{c,w} \ \ \pi D_{p,i} L \ \ \ T_{wi} - T_b$$

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$$\therefore \qquad T_{wi} = T_b + \frac{U_o D_{p,o} \;\; T_s - T_b}{\bar{h}_{c,w} D_{p,i}} = 10 + \frac{2257 \times 0.0305 \;\; 40 - 10}{8405 \times 0.0254} = 19.7°\text{C}$$

(c) The length L can now be determined from the rate of heat transfer needed to condense 0.45 kg/s of steam as follows

$$q = \dot{m} h_{fg} = U_o \;\; \pi D_{p,o} L \;\;\; T_s - T_b$$

$$\therefore \qquad L = \frac{\dot{m} h_{fg}}{U_o \pi D_{p,o} \;\; T_s - T_b} = \frac{0.45 \times 2406 \times 1000}{2257 \times \pi \times 0.0305 \;\; 40 - 10} = 167 \text{ m}$$

(d) Recognizing that with steam condensation on the outside of the copper pipe its surface temperature would be nearly constant and uniform, and hence the inlet and outlet temperatures for water flow can be calculated from Equation (7.36) as follows

$$\frac{T_{wi} - T_{w,out}}{T_{w,i} - T_{w,in}} = \exp\left(-\frac{\bar{h}_{c,w} \pi D_{p.i} L}{\dot{m}_w c_p} \right)$$

$$\text{and} \;\; T_b = \frac{T_{w,in} + T_{w,out}}{2} \;\; \Rightarrow \;\; T_{w,in} = 2T_b - T_{w,out}$$

Thus, if the water mass flow rate, $\dot{m}_w$ is known then both $T_{w,in}$ and $T_{w,out}$ can be calculated.

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PROBLEM 7.47

Assume that the inner cylinder in Problem 7.40 is a heat source consisting of an aluminum-clad rod of uranium, 5-cm-*OD* and 2-m-long. Estimate the heat flux that raises the temperature of the bismuth 40°C and the maximum center and surface temperatures necessary to transfer heat at this rate.

GIVEN

- Liquid bismuth flowing through an annulus
- Annulus inside diameter (D_i) = 5 cm = 0.05 m
- Annulus outside diameter (D_o) = 6.1 cm = 0.061 m
- Bismuth velocity (V) = 4.5 m/s
- Bismuth temperature (T_b) = 316°C
- Inner cylinder is an aluminum clad uranium heat source
- Cylinder length (L) = 2 m
- From Problem 7.40: $\bar{h}_c$ = 26,800 W/(m^2 K)

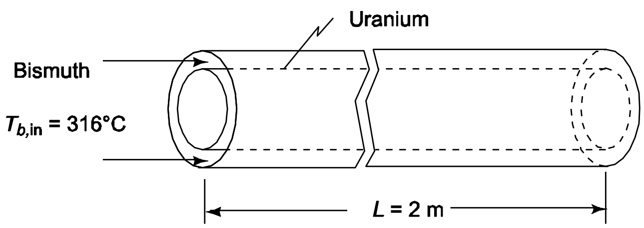
FIND

(a) The heat flux (Q_G/A_t) necessary to raise the bismuth temperature 40°C, and
(b) The maximum center ($T_{u,o}$) and surface ($T_{u,ro}$) temperatures of the uranium

ASSUMPTIONS

- Steady state
- The Bismuth temperature given in Problem 7.40 is the bulk Bismuth temperature
- Thermal resistance of the aluminum is negligible
- Thickness of the aluminum is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 12, for uranium

Thermal conductivity (k_u) = 36.4 W/(m K) at 427°C

From Appendix 2, Table 25, for Bismuth at 316°C

Specific heat (c_b) = 144.5 J/(kg K)

Density (ρ) = 10,011 kg/m^3

SOLUTION

(a) The rate of heat transfer required to raise the Bismuth by 40°C is

$$q = \dot{m}\, c_b \Delta T_b = \rho\, V A_c\, c_b\, \Delta T_b = \rho\, V \frac{\pi}{4} (D_o^2 - D_i^2)\, c_b\, \Delta T_b$$

$$q = 10{,}011 \text{ kg/m}^3 \;\; 4.5 \text{ m/s} \;\; \frac{\pi}{4} [(0.061 \text{ m})^2 - (0.05 \text{ m})^2] \;\; 144.5 \text{ J/(kg K)} \;\; (40°\text{C}) \;\;\; (\text{Ws})/\text{J}$$

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$$= 2.50 \times 10^5 \text{ W}$$

Therefore, the average heat flux is

$$\frac{\dot{Q}_G}{A_t} = \frac{q}{A_t} = \frac{q}{\pi D_i L} = \frac{2.50\times 10^5 \text{ W}}{\pi\ 0.05\,\text{m}\quad 2\,\text{m}} = 7.95 \times 10^5 \text{ W/m}^2$$

The temperature difference between the uranium and bismuth (ΔT_{ub}) required to transfer this heat can be calculated from

$$\frac{q}{A_t} = \bar{h}_c\,\Delta T_{ub} \;\Rightarrow \Delta T_{ub} = \frac{q}{A_t \bar{h}_c} = \frac{7.95\times 10^5 \text{W/m}^2}{26{,}500\,\text{W/(m}^2\text{K)}} = 29.7 \text{ K}$$

The maximum uranium surface temperature will occur at the outlet where the bismuth temperature is $T_{b,\max} = 316°\text{C} + 0.5(\Delta T_b) = 336°\text{C}$

$$T_{u,ro,\max} = T_{b,\max} + \Delta T_{ub} = 336°\text{C} + 29.7 \text{ K} \approx 366°\text{C}$$

The rate of internal heat generation per unit volume is

$$\dot{q}_G = \frac{\dot{Q}_G}{\text{Volume}} = \frac{q}{\frac{\pi}{4} D_i^2 L} = \frac{2.50\times 10^5 \text{ W}}{\frac{\pi}{4}\ 0.05\,\text{m}^{\ 2}\ \ 2\,\text{m}} = 6.37 \times 10^7 \text{ W/m}^3$$

The maximum temperature at the center of the uranium is given by Equation (2.60)

$$T_{u,o,\max} = T_{u,ro,\max} + \frac{\dot{q}_G r_o^{\,2}}{4 k_u} = 366°\text{C} + \frac{6.37\times 10^7 \text{ W/m}^3\quad 0.05\,\text{m}/2^{\ 2}}{4\ \ 36.4\,\text{W/(m K)}} = 639°\text{C}$$

At the inlet

$$T_{u,ro} = (T_b - 0.5\ \Delta T_b) + \Delta T_{ub} + \frac{\dot{q}_G r_o^{\,2}}{4 k_u}$$

$$T_{u,ro} = [316°\text{C} - 0.5(40°\text{C})] + 29.7°\text{C} + \frac{6.37\times 10^7 \text{ W/m}^3\quad 0.05\,\text{m}/2^{\ 2}}{4\ \ 36.4\,\text{W/(m K)}} = 599°\text{C}$$

Therefore, the average uranium temperature is approximately

$$T_{u,\text{ave}} = \frac{\left(\frac{366+639}{2} + \frac{329+599}{2}\right)}{2} = 483°\text{C}$$

Repeating the calculation using the thermal conductivity of uranium evaluated at this temperature yields the following result

$T_{u,\text{ave}} = 483°\text{C}$

$k_u = 37.7$ W/(m K)

$T_{u,o,\max} = 630°\text{C}$

$T_{u,ro}$ (inlet) $= 590°\text{C}$

$T_{u,\text{ave}} = 478°\text{C}$ (Convergence)

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PROBLEM 7.48

Evaluate the rate of heat loss per meter from pressurized water flowing at 200°C through a 10-cm-*ID* pipe at a velocity of 3 m/s. The pipe is covered with a 5-cm-thick layer of 85% magnesia wool which has an emissivity of 0.5. Heat is transferred to the surroundings at 20°C by natural convection and radiation. Draw the thermal circuit and state all assumptions.

GIVEN

- Pressurized water flowing through an insulated pipe
- Water temperature (T_w) = 200°C = 493 K
- Pipe inside diameter (D_i) = 10 cm = 0.1 m
- Water velocity (V) = 3 m/s
- Magnesia wool insulation thickness (t) = 5 cm = 0.05 m
- Emissivity of the wool insulation (ε) = 0.5
- Temperature of the surroundings (T_∞) = 20°C = 293 K

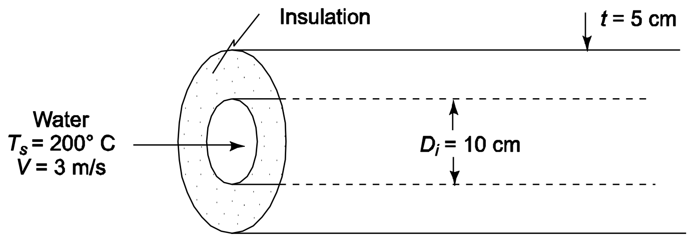
FIND

- The rate of heat loss per meter (q/L)

ASSUMPTIONS

- Steady state
- Pipe surface temperature can be considered constant and uniform
- Surroundings behave as a black body
- Pipe is horizontal
- Thermal resistance of the pipe is negligible
- Ambient air is still
- Pipe thickness is negligible
- Fully developed flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11, the thermal conductivity of 85% magnesia (k_I) = 0.059 W/(m K) at 20°C.

From Appendix 2, Table 13, for water at 200°C

Thermal conductivity (k_w) = 0.665 W/(m K)
Kinematic viscosity (ν_w) = 0.160×10^{-6} m^2/s
Prandtl number (Pr_w) = 0.95

From Appendix 1, Table 5, the Stephan Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4).

SOLUTION

Heat transfer coefficient on the water side:

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The Reynolds number of the water flow is

$$Re_D = \frac{VD}{\nu_w} = \frac{(3\,\mathrm{m/s})(0.1\,\mathrm{m})}{0.16 \times 10^{-6}\,\mathrm{m^2/s}} = 1.875 \times 10^6 \text{ (Turbulent)}$$

Applying Equation (7.61)

$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\,(1.875 \times 10^6)^{0.8}\,(0.95)^{0.3} = 2363$$

$$\bar{h}_{cw} = \overline{Nu}_D \frac{k_w}{D_i} = 2363\, \frac{0.665\,\mathrm{W/(m\,K)}}{0.1\,\mathrm{m}} = 15{,}713\ \mathrm{W/(m^2\,K)}$$

Heat transfer coefficient on the air side:

The natural convection heat transfer coefficient on the outside of the insulation is a function of the exterior temperature of the insulation (T_I). For a first iteration, let $T_I = T_\infty + 20° = 40°C$. Evaluating the air properties from Appendix 2, Table 28, at the film temperature of 30°C

Thermal expansion coefficient (β) = 0.0033 1/K
Thermal conductivity (k_a) = 0.0258 W/(m K)
Kinematic viscosity (ν_a) = 16.7×10^{-6} m²/s
Prandtl number (Pr_a) = 0.71

The Grashof number is

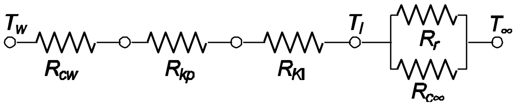
$$Gr_D = \frac{g\beta(T_I - T_\infty)\,D_I^3}{\nu_a^2} = \frac{9.8\,\mathrm{m/s^2}\;\; 0.0033\,1/\mathrm{K}\;\; (20°\mathrm{C})(0.2\,\mathrm{m})^3}{16.7 \times 10^{-6}\,\mathrm{m^2/s}} = 1.855 \times 10^7$$

Applying Equation (8.20)

$$\overline{Nu}_D = 0.53\,(Gr_D Pr)^{\frac{1}{4}} = 0.53\left[1.855 \times 10^7\;(0.71)\right]^{\frac{1}{4}} = 31.93$$

$$\bar{h}_{ca} = \overline{Nu}_D \frac{k_a}{D_I} = 31.93\, \frac{0.0258\,\mathrm{W/(m\,K)}}{0.2\,\mathrm{m}} = 4.12\ \mathrm{W/(m^2\,K)}$$

The thermal circuit for this problem is shown below



where

$$R_{cw} = \frac{1}{\bar{h}_{cw} A_w} = \frac{1}{\bar{h}_{cw}\,\pi D_w L} = \frac{1}{15{,}713\,\mathrm{W/(m^2K)}\;\pi\,(0.1\,\mathrm{m})L} = \left(0.000203\,\frac{1}{L}\right)\ \mathrm{(m\,K)/W}$$

R_{kp} = Thermal résistance of the pipe wall ≈ 0

$$R_{kI} = \frac{\ln\left(\frac{D_I}{D_i}\right)}{2\pi L k_I} = \frac{\ln\left(\frac{0.2}{0.1}\right)}{2\pi L\;\, 0.059\,\mathrm{W/(m\,K)}} = \left(1.870\,\frac{1}{L}\right)\ \mathrm{(m\,K)/W} \quad \text{From Equation (2.41)}$$

R_r = Radiative resistance

R_{cw} = Natural convective resistance

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The insulation temperature (T_I) can be determined by equating the heat transfer between T_w and T_I to that from T_I to T_∞

$$\frac{T_w - T_I}{R_{cw} - R_{kI}} = q_{ca} + q_{ra} = A_I\,[\,\bar{h}_{ca}\,(T_I - T_\infty) + \varepsilon\,\sigma\,(T_I^4 - T_\infty^4)]$$

$$\frac{473\,\text{K} - T_I}{(0.000203+1.87)\left(\frac{1}{L}\right)\ (\text{m K})/\text{W}} = \pi\,(0.2\text{ m})\,L$$

$$\left[\ 4.12\,\text{W}/(\text{m}^2\text{K})\ (T_I - 293\text{K}) + 0.5\ 5.67\times10^{-8}\ \text{W}/(\text{m}^2\text{K}^4)\ [T_I^4 - (293\text{K})^4]\right]$$

Checking the units then eliminating them for clarity

$$1.79 \times 10^{-8}\ T_I^4 + 3.124\ T_I - 1142.7 = 0$$

By trial and error: T_I = 312 K = 39°C

Further iterations are not required. The rate of heat loss can be calculated from

$$q = \frac{T_w - T_I}{R_{cw} - R_{kI}} = \frac{473\text{K} - 312\text{K}}{1.87\left(\frac{1}{L}\right)\ (\text{m K})/\text{W}} = 86.1\text{ W/m}$$

COMMENTS

Note that the convective resistance in the turbulent water is negligible compared to that of the insulation.

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PROBLEM 7.49

In a pipe-within-a-pipe heat exchanger, water is flowing in the annulus and an aniline-alcohol solution having the properties listed in Problem 7.28 flows in the central pipe. The inner pipe has a 1.3 cm-*ID* and a 1.6 cm-*OD*, and the *ID* of the outer pipe is 1.9 cm. For a water bulk temperature of 27°C and an aniline bulk temperature of 60°C, determine the overall heat transfer coefficient based on the outer diameter of the central pipe and the frictional pressure drop per unit length of the water and the aniline for the following volumetric flow rates, (a) water rate 0.06 L/s, aniline rate 0.06 L/s, (b) water rate 0.6 L/s, aniline rate 0.06 L/s, (c) water rate 0.06 L/s, aniline rate 0.6 L/s, and (d) water rate 0.6 L/s, aniline rate 0.6 L/s (*L*/*D* = 400). Physical properties of aniline solution

Temperature (°C)	Viscosity (kg/(m s))	Thermal Conductivity $(W/(\mathbf{m\,K}))$	Specific Gravity	Specific Heat $(kJ/(\mathbf{kg\,K}))$
20	**0.0051**	**0.173**	**1.03**	**2.09**
60	**0.0014**	**0.169**	**0.98**	**2.22**
100	**0.0006**	**0.164**		**2.34**

GIVEN

- Pipe-within-a-pipe heat exchanger with water in the annulus and aniline-alcohol solution in the inner pipe
- Solution properties listed above
- Pipe diameters
  - Inner pipe D_{ii} = 1.3 cm= 0.013 m
    D_i = 1.6 cm = 0.016 m
  - Inside of outer pipe D_o = 1.9 cm= 0.019 m
- Bulk temperatures
  - Water (T_w) = 27°C
  - Aniline (T_a) = 60°C
- L/D = 400

FIND

- The overall heat transfer coefficient (U) based on D_i and the pressure drop (Δp) for the following volumetric flow rates ($\dot{V}$)

Case	(a)	(b)	(c)	(d)
Water flow rate (L/s)	0.06	0.6	0.06	0.6
Aniline flow rate (L/s)	0.06	0.06	0.6	0.6

ASSUMPTIONS

- Steady state
- Thermal resistance of the pipe is negligible
- Nusselt number can be estimated from correlations for constant and uniform surface temperature
- The effect of viscosity variation is negligible
- The tubes are smooth
- Fully developed flow (L/D = 400)

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 27°C

Density (ρ) = 996.3 kg/m^3
Thermal conductivity (k_w) = 0.609 W/(m K)
Kinematic viscosity (ν_w) = 0.857*10^{-6} m^2/s

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Prandtl number (Pr_w) = 5.87

SOLUTION

Water side heat transfer coefficient:

The Reynolds number on the water side is

$$Re_D = \frac{V\,D_{ii}}{\nu_w} = \frac{\dot{V}\,D_{ii}}{A_c\,\nu_w} = \frac{4\dot{V}}{\pi\,D_{ii}\,\nu_w}$$

For $\dot{V}$ = 0.06 L/s

$$Re_D = \frac{4\,(0.06\text{l}/s)\left(1/1000\,m^3/\text{l}\right)}{\pi\,(0.013m)\left(0.857{*}10^{-6}\,m^2/s\right)} = 6857 \text{ (Turbulent)}$$

Applying Equation (7.61)

$$\overline{Nu}_D = 0.023\,Re_D^{0.8}\,Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(6857)^{0.8}\,(5.87)^{0.4} = 54.7$$

$$\bar{h}_{cw,1} = \overline{Nu}_D\,\frac{k}{D_{ii}} = 55.7\,\frac{(0.609\,W/(\text{m}\,K))}{0.013m} = 2562.5 \text{ W/(m}^2\text{ K)}$$

For V = 0.6 l/s, Re = 0.6857 × 10^5 (Turbulent)

$$\overline{Nu}_D = 0.023\,(0.6857 \times 10^5)^{0.8}\,(5.87)^{0.4} = 345.2$$

$$\bar{h}_{cw,10} = 345.2\,\frac{(0.609\,W/(\text{m}\,K))}{0.013m} = 16171\text{ W/(m}^2\,K)$$

Aniline side heat transfer coefficient:

The hydraulic diameter of the annulus, from Equation (7.3) is

$$D_H = D_o - D_i = 1.9\text{ cm} - 1.6\text{ cm} = 0.3\text{ cm} = 0.003\text{ m}$$

From the given properties

Density, $\rho = \rho_{H2O}$(s.g.) = 996.3 kg/m^3 (0.98) = 976.4 kg/m^3

Kinematic viscosity, $\nu_a = \dfrac{\mu}{\rho} = \dfrac{0.0014\text{ kg}/(ms)}{976.4\,kg/m^3} = 1.43{*}10^{-6}\text{ m}^2/\text{s}$

Prandtl number, $Pr = \dfrac{c\,\mu}{k} = \dfrac{(2220\,J/(\text{kg K}))\,(0.0014)(kg/(ms))}{(0.169W/(\text{m}K))} = 18.39$

for ($\dot{V}$) = 0.06 l/s

$$Re_{D_H} = \frac{V\,D_H}{\nu_a} = \frac{\dot{V}\,D_H}{A_c\,\nu_a} = \frac{4\dot{V}\,D_H}{\pi\,(D_o^2 - D_i^2)\,\nu_a}$$

$$Re_{D_H} = \frac{4(0.06\text{l}/s)\left(1/1000m^3/\text{l}\right)*0.03m}{\pi[(0.019m)^2 - (0.016m)^2]\left(1.43{*}10^{-6}\,m^2/s\right)} = 1526.4 \text{ (Laminar)}$$

From Table (7.2): For D_i/D_o = (0.016)/(0.019) = 0.842: $\overline{Nu}_D \approx 5.15$

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$$\bar{h}_{ca,1} = \overline{Nu}_{D_H} \frac{k}{D_H} = 5.15 \frac{(0.169\,\mathrm{W/(m}\,K))}{0.003\,m} = 290.1 \text{ W/(m}^2\text{ K)}$$

For $\dot{V}$ = 0.6 l/s Re = 15,264 (Turbulent)

Applying Equation (7.61)

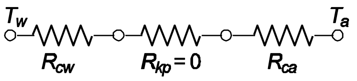
$$\overline{Nu}_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\,(16{,}120)^{0.8}\,(18.32)^{0.4} = 163.8$$

$$\bar{h}_{ca,10} = 163.8 \frac{(0.169\,\mathrm{W/(m}\,K))}{0.003\,m} = 9231 \text{ W/(m}^2\text{ K)}$$

Overall heat transfer coefficient:

The thermal circuit for the problem is shown below



where

$$R_{cw} = \frac{1}{\bar{h}_{cw} A_w} = \frac{1}{\bar{h}_{cw}\, \pi\, D_{ii}\, L}$$

$$R_{kp} \approx 0$$

$$R_{ca} = \frac{1}{\bar{h}_{ca} A_a} = \frac{1}{\bar{h}_{ca}\, \pi\, D_i\, L}$$

The overall heat transfer coefficient is

$$U A_{\text{ref}} = \frac{1}{R_{cw} + R_{ca}} \quad \text{where } A_{\text{ref}} = \pi\, D_i\, L$$

$$\therefore\ U = \frac{1}{D_i \left(\dfrac{1}{\bar{h}_{cw,1}\, D_{ii}} + \dfrac{1}{\bar{h}_{ca,1}\, D_i} \right)}$$

For case (a)

$$U = \frac{1}{0.016\, m \left(\dfrac{1}{(2562\,W/(m^2\mathrm{K})(0.013\,m))} + \dfrac{1}{(290.1\,W/(\mathrm{m}^2\mathrm{K})(0.016\,m))} \right)}$$

$$= 286.1 \text{ W/(m}^2\text{ K)}$$

Substituting the appropriate convective heat transfer coefficients into the above equations yields the following overall heat transfer coefficient for the remaining cases

(b) U = 283.7 W/(m^2 K)

(c) U = 1698.8 W/(m^2 K)

(d) U = 5421 W/ (m^2 K)

Friction factors and pressure drop:

For the turbulent cases, the friction factor is given by Equation (7.57)

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$$f = 0.184\, Re_p^{-0.2}$$

For water with $\dot{V}$ = 0.06 l/s: $f_{w,1}$ = 0.184 (6857)$^{-0.2}$ = 0.0314

For water with $\dot{V}$ = 0.6 l/s : $f_{w,10}$ = 0.184 (68570)$^{-0.2}$ = 0.0198

For the aniline solution with $\dot{V}$ = 0.6 l/s: $f_{a,10}$ = 0.184 (15264)$^{-0.2}$ = 0.0268

For the aniline solution with $\dot{V}$ = 1 gpm, the flow is laminar and the friction factor is given by Table (7.2): $f\, Re_{D_H} = 95.7 \rightarrow f_{a,1} = 0.0594$

The pressure drop is given by Equation (7.13)

$$\Delta p = f \frac{L}{D} \frac{\rho V^2}{2 g_c} = \frac{f \rho V^2}{2 g_c A_c^2} \frac{L}{D}$$

For the water, $A_c = (\pi/4) D_{ii}^2$

For the aniline solution, $A_c = (\pi/4)(D_o^2 - D_i^2)$

For the water with $\dot{V}$ = 1 gpm

$$\Delta p = \frac{0.0314\,(400)\left(996.3\, kg/m^3\right)(0.06\, l/s)^2 \left(1/1000\, m^3/l\right)^2}{2(1))\left[\frac{1}{4}(0.013\, m)^2\right]^2} = 0.533 \text{ Pa}$$

Similarly for the other cases

For water, $\dot{V}$ = 0.6 l/s: Δp = 54.9 Pa

For aniline solution $\dot{V}$ = 0.06 l/s: Δp = 0.98 Pa

For aniline solution $\dot{V}$ = 0.6 l/s: Δp = 74.32 Pa

Tabulating all the results

Case	(a)	(b)	(c)	(d)
Water flow rate (l/s)	0.06	0.6	0.06	0.6
Aniline flow rate (l/s)	0.06	0.06	0.6	0.6
Overall heat transfer coef. $\left(\text{W/(m}^2 K)\right)$	286.1	283.7	1698.8	5421
Water pressure drop (Pa)	0.533	54.9	0.533	54.9
Aniline pressure drop (Pa)	0.98	0.98	74.32	74.32

COMMENTS

Note that the flow rate of the aniline solution has a greater effect on the overall heat transfer coefficient than that of the water because the aniline flow changes from laminar to turbulent, whereas the water flow is turbulent at both flow rates.

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PROBLEM 7.50

A plastic tube of 7.6-cm-*ID* and 1.27 cm wall thickness having a thermal conductivity of 1.7 W/(m K), a density of 2400 kg/m^3, and a specific heat of 1675 J/(kg K) is cooled from an initial temperature of 77°C by passing air at 20°C inside and outside the tube parallel to its axis. The velocities of the two air streams are such that the coefficients of heat transfer are the same on the interior and exterior surfaces. Measurements show that at the end of 50 min, the temperature difference between the tube surfaces and the air is 10 per cent of the initial temperature difference. A second experiment is proposed in which a tube of a similar material having an inside diameter of 15 cm and a wall thickness of 2.5 cm is cooled from the same initial temperature, again using air at 20°C and feeding it to the inside of the tube the same number of kilograms of air per hour that was used in the first experiment. The air-flow rate over the exterior surfaces is adjusted to give the same heat transfer coefficient on the outside as on the inside of the tube. It is assumed that the air-flow rate is so high that the temperature rise along the axis of the tube is neglected. Using the experience gained initially with the 4.5-cm tube, estimate how long it takes to cool the surface of the larger tube to 27°C under the conditions described. Indicate all assumptions and approximations in your solution.

GIVEN

- Air flow inside and outside a plastic tube

Case 1

- Tube 1 inside diameter (D_{1i}) = 7.6 cm = 0.076 m
- Tube 1 wall thickness (S_1) = 1.27 cm = 0.0127 m
- Plastic properties
    - Thermal conductivity (k_p) = 1.7 W/(m K)
    - Density (ρ) = 2400 kg/m^3
    - Specific heat (c) = 1675 J/(kg K)
- Tube initial temperature (T_{ti}) = 77°C
- Air temperature (T_a) = 20°C
- After 10 min: ($T_t - T_a$) = 10% of initial ($T_t - T_a$)

Case 2

- Tube 2 inside diameter (D_{2i}) = 15 cm = 0.15 m
- Tube 2 wall thickness (S_2) = 2.5 cm = 0.025 m
- Same initial temperature and air temperature as Case 1
- Same interior air flow rate as Case 1
- Air velocities are such that heat transfer coefficients inside and outside are equal

FIND

- Time for T_t to reach 27°C in Case 2

ASSUMPTIONS

- Temperature rise along the tube is negligible
- Tube may be treated as a lumped capacitance (This will be checked)

SKETCH

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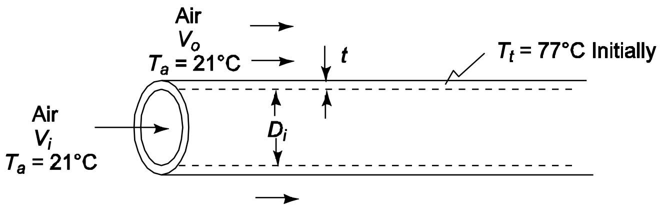

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 20°C

Density (ρ) = 1.164 kg/m^3
Thermal conductivity (k) = 0.0251 W/(m K)
Kinematic viscosity (ν) = 15.7 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71
Absolute viscosity (μ) = 18.24 × 10^{-6} (Ns)/m^2

SOLUTION

Case 1

Assuming the tube can be treated as a lumped capacitance: Equation (3.3) can be applied

$$\ln\left(\frac{T_{tf}-T_a}{T_{ti}-T_a}\right) = -\frac{\bar{h}_c A_s}{c\,\rho V}\,t = -\frac{\bar{h}_c\,\pi\,(D_i+D_o)L}{c\,\rho\left[\frac{\pi}{4}(D_o^2-D_i^2)\,L\right]}\;t$$

where $D_o = D_i + 2s$ = 0.076 m + 2(0.0127 m) = 0.1014 m

Solving for the heat transfer coefficient

$$\bar{h}_c = -\frac{c\rho(D_o^2-D_i^2)}{4\,(D_i+D_o)t}\,\ln\left(\frac{T_{tf}-T_a}{T_{ti}-T_a}\right)$$

$$\bar{h}_c = -\frac{1675\,\text{J/(kg K)}\;\;2400\,\text{kg/m}^3\;\;[(0.1014\,\text{m})^2-(0.076\,\text{m})^2]}{4(0.076\,\text{m}+0.1014\,\text{m})(50\,\text{min})\;\;60\,\text{s/min}\;\;\text{J/(Ws)}}\;\ln(0.10) = 19.6\ \text{W/(m}^2\ \text{K)}$$

Checking the lumped capacity assumption, the Biot number should be based on half of the tube wall thickness since convection occurs equally on the inside and outside of the tube

$$B_i = \frac{\bar{h}_c\,s}{2\,k_s} = \frac{19.6\,\text{W/(m}^2\text{K)}\;\;(0.0127\,\text{m})}{2\;\;1.7\,\text{W/(m K)}} = 0.07 < 0.1$$

Therefore, the lumped capacity assumption is valid. Assuming the air flow is turbulent and applying Equation (7.61) to determine the Reynolds number for the interior air flow

$$\overline{Nu}_D = 0.023\,Re_D^{0.8}\,Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\therefore\ Re_D = \left[\frac{\bar{h}_c\,D_i}{0.023Pr^{0.4}\text{K}}\right]^{1.25} = \left[\frac{19.6\,\text{W/(m}^2\text{K)}\;\;(0.076\,\text{m})}{0.023(0.71)^{0.4}\;\;0.0251\,\text{W/(m K)}}\right]^{1.25} = 21{,}825\ \text{(Turbulent)}$$

Therefore, the air velocity is

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$$V = \frac{Re_D \nu}{D_i} = \frac{21{,}825 \; 15.7\times10^{-6}\,\text{m}^2/\text{s}}{0.076\,\text{m}} = 4.51 \text{ m/s}$$

The mass flow rate is

$$\dot{m} = V\rho A_c = V\rho\frac{\pi}{4}D_i^2 = 4.51 \text{ m/s} \; 1.1641\,\text{kg/m}^3 \; \frac{\pi}{4}(0.076 \text{ m})^2 = 0.024 \text{ kg/s}$$

Case 2

Applying the mass flow rate to Case 2

$$Re_D = \frac{VD\rho}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4 \; 0.024\,\text{kg/s}}{\pi(0.15\,\text{m}) \; 18.24\times10^{-6}\,(\text{Ns})/\text{m}^2 \quad \text{kg m/(Ns}^2)} = 11{,}169(\text{Turbulent})$$

Applying Equation (7.61)

$$\overline{Nu}_D = 0.023\,(11{,}169)^{0.8}\,(0.71)^{0.4} = 34.72$$

$$\bar{h}_c = \overline{Nu}_D\,\frac{k}{D} = 34.72\;\frac{0.0251\,\text{W/(m K)}}{0.15\,\text{m}} = 5.81 \text{ W/(m}^2\text{ K)}$$

The Biot number is

$$Bi = \frac{\bar{h}_c\,s}{2k_s} = \frac{5.81\,\text{W/(m}^2\text{K)} \; (0.025\,\text{m})}{2 \; 1.7\,\text{W/(m K)}} = 0.04 < 0.1$$

Therefore, the internal thermal resistance can be neglected and Equation (3.3) can be applied. Solving for the time

$$t = -\frac{c\rho(D_o^2 - D_i^2)}{4(D_i + D_o)\bar{h}_c}\ln\left(\frac{T_{tf} - T_a}{T_{ti} - T_a}\right) = -\frac{c\rho(D_o - D_i)}{4\,\bar{h}_c}\ln\left(\frac{T_{tf} - T_a}{T_{ti} - T_a}\right) = -\frac{c\rho(2s)}{4\,\bar{h}_c}\ln\left(\frac{T_{tf} - T_a}{T_{ti} - T_a}\right)$$

$$t = -\frac{1675\,\text{J/(kg K)} \quad 2400\,\text{kg/m}^3 \;\; 2(0.025\,\text{m})}{4 \; 5.81\,\text{W/(m}^2\text{K)} \quad \text{J/(Ws)}}\ln\left(\frac{27\,°\text{C} - 20\,°\text{C}}{77\,°\text{C} - 20\,°\text{C}}\right)$$

$$t = 18{,}137 \text{ s} = 302 \text{ min} \approx 5 \text{ hours}$$

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PROBLEM 7.51

Exhaust gases having properties similar to dry air enter an exhaust stack at 800 K. The stack is made of steel and is 8-m-tall and 0.5-m-*ID*. The gas flow rate is 0.5 kg/s and the ambient temperature is 280 K. The outside of the stack has an emissivity of 0.9. If heat loss from the outside is by radiation and natural convection, calculate the gas outlet temperature.

GIVEN

- Exhaust gas flow through a steel stack
- Exhaust gas has the properties of dry air
- Entering exhaust temperature ($T_{b,\text{in}}$) = 800 K
- Stack height (L) = 8 m
- Stack diameter (D) = 0.5 m
- Gas flow rate ($\dot{m}$)= 0.5 kg/s
- Ambient temperature (T_∞) = 280 K
- Stack emissivity (ε) = 0.9

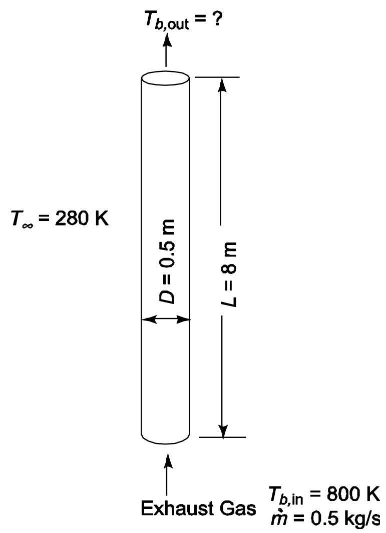
FIND

- The outlet gas temperature ($T_{b,\text{out}}$)

ASSUMPTIONS

- Steady state
- The surrounding behave as a black body enclosure at the ambient temperature
- Thermal resistance of the duct is negligible
- Duct thickness is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 28, for dry air at 800 K

Density (ρ) = 0.433 kg/m^3
Thermal conductivity (k) = 0.0552 W/(m K)
Kinematic viscosity (ν) = 86.4×10^{-6} (Ns)/m^2
Prandtl number (Pr) = 0.72
Specific heat (c) = 1079 J/(kg K)

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From Appendix 1, Table 5, the Stephan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

Interior convection

The Reynolds number is

$$Re_D = \frac{VD\rho}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4\ 0.5\,\text{kg/s}}{\pi(0.5\,\text{m})\ 86.4\times10^{-6}\,(\text{Ns})/\text{m}^2\ \ (\text{kg m})/(\text{Ns}^2)} = 14{,}740$$

Applying Equation (7.61)

$$\overline{Nu}_D = 0.023\ Re_D^{0.8}\ Pr^n \qquad \text{where } n = 0.3 \text{ for cooling}$$

$$\overline{Nu}_D = 0.023\ (14{,}740)^{0.8}\ (0.72)^{0.3} = 45.0$$

$$\bar{h}_{\text{cfi}} = \overline{Nu}_D\ \frac{k}{D} = 45.0\ \frac{0.0552\,\text{W/(m K)}}{0.5\,\text{m}} = 4.97 \text{ W/(m}^2\text{ K)}$$

For the first iteration, let the duct temperature (T_d) equal the average of the exhaust and ambient temperatures = 540 K. Then the interior film temperature is 670 K. From Appendix 2, Table 28, for dry air at 670 K

Thermal expansion coefficient (β) = 0.00149 1/K
Thermal conductivity (k) = 0.0485 W/(m K)
Kinematic viscosity (ν) 64.6 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.72

The Grashof number is

$$Gr_L = \frac{g\beta(T_I - T_\infty)L^3}{\nu_a^2} = \frac{9.8\,\text{m/s}^2\ \ 0.00149\,(1/\text{K})\ \ (800\,\text{K} - 540\,\text{K})(8\,\text{m})^3}{64.6\times10^{-6}\,\text{m}^2/\text{s}\ ^2} = 4.66\times10^{11}$$

$$\frac{Gr}{Re^2} = \frac{4.66\times10^{11}}{(14{,}740)^2} = 2143$$

Therefore, natural convection cannot be neglected.

The interior natural convection heat transfer coefficient will be estimated using the vertical plant correlation of Equation (8.13)

$$\overline{Nu}_L = 0.13\ (Gr_L Pr)^{\frac{1}{3}} = 0.13\left[4.66\times10^{11}\ (0.72)\right]^{\frac{1}{3}} = 903$$

$$\bar{h}_{\text{cni}} = \overline{Nu}_L\ \frac{k}{L} = 903\ \frac{0.0485\,\text{W/(m K)}}{8\,\text{m}} = 5.47 \text{ W/(m}^2\text{ K)}$$

Combining the natural forced coefficients using Equation (8.57)

$$\bar{h}_{ci} = \ \bar{h}_{\text{cfi}}^{\ 3} + \bar{h}_{\text{cfn}}^{\ 3}\ ^{\frac{1}{3}} = \left[(4.97)^3 + (5.47)^3\right]^{\frac{1}{3}} = 6.59 \text{ W/(m}^2\text{ K)}$$

Exterior convection:

Air properties at the exterior film temperature of 410 K are

Thermal expansion coefficient (β) = 0.00247 1/K
Thermal conductivity (k) = 0.033 W/(m K)

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Absolute viscosity (ν) = 28.0×10^{-6} m^2/s
Prandtl number (Pr) = 0.71

$$Gr_L = \frac{9.8\,\text{m/s}^2 \;\; 0.00247\,(1/\text{K}) \;\; (540\,\text{K} - 280\,\text{K})(8\,\text{m})^3}{28.0 \times 10^{-6}\,\text{m}^2/\text{s}} = 4.11 \times 10^{12}$$

$$\overline{Nu}_L = 0.13 \left[4.11 \times 10^{12} \;\; (0.71)\right]^{\frac{1}{3}} = 1857$$

$$\bar{h}_{co} = 1857 \; \frac{0.033\,\text{W/(m K)}}{8\,\text{m}} = 7.66\ \text{W/(m}^2\ \text{K)}$$

Duct temperature:

The rate of convection to the duct interior must equal the sum of convection and radiation from the exterior

$$\bar{h}_{ci}\, A_t\, (T_b - T_d) = \bar{h}_{co}\, A_t\, (T_d - T_\infty) + A_t\, \varepsilon\, \sigma\, (T_d^4 - T_\infty^4)$$

$$6.59\,\text{W/(m}^2\text{K)} \;\; (800\ \text{K} - T_d) = \;\; 7.66\,\text{W/(m}^2\text{K)} \;\; (T_d - 280\ \text{K}) + 0.9 \;\; 5.67 \times 10^{-8}\ \text{W/(m}^2\text{K}^4) \;\; [T_d^4 - (280\ \text{K})^4]$$

$$5.13 \times 10^{-8}\;\; T_d^4 + 14.25\, T_d - 7730 = 0$$

By trial and error: T_d = 425 K

Using the duct temperature to estimate the rate of heat transfer

$$q = \bar{h}_{ci}\;\; \pi D\, L\, (T_b - T_d) = \;\; 6.59\,\text{W/(m}^2\text{K)} \;\; \pi\,(0.5\ \text{m})\ (8\ \text{m})\ (800\ \text{K} - 425\ \text{K}) = 3.11 \times 10^4\ \text{W}$$

The temperature rise of the exhaust gas is

$$\Delta T_b = \frac{q}{\dot{m}c} = \frac{3.11 \times 10^4\,\text{W}}{0.5\,\text{kg/s} \;\; 1079\,\text{J/(kg K)} \;\; (\text{Ws})/\text{J}} = 57.5\ \text{K}$$

$$T_{b,\text{out}} = T_{b,\text{in}} - \Delta T_b = 800\ \text{K} - 57.5\ \text{K} = 742\ \text{K}$$

The average bulk temperature is 721 K. This is close enough to the first iteration value that another iteration is not necessary.

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PROBLEM 7.52

A 3.05 m long vertical cylindrical exhaust duct from a commercial laundry has an *ID* of 15.2 cm. Exhaust gases having physical properties approximating those of dry air enter at 316°C. The duct is insulated with 10.2 cm of rock wool having a thermal conductivity of: *k* = 0.7 + 0.016 *T* (where *T* is in °C and *k* in W/(m K).

If the gases enter at a velocity of 0.61 m/s, calculate

(a) The rate of heat transfer to quiescent ambient air at 15.6 °C.

(b) The outlet temperature of the exhaust gas.

Show your assumptions and approximations.

GIVEN

- Exhaust gases flowing through an insulated long vertical cylindrical duct
- Exhaust gases have the physical properties of dry air
- Duct length (L) = 3.05 m
- Duct inside diameter (D) = 15.2 cm = 0.152 m
- Entering exhaust temperature ($T_{b,\text{in}}$)
- Rock wool insulation thickness (s) = 10.2 cm = 0.102 m
- Thermal conductivity of insulation (k) = 0.7 + 0.016 T $\left(\text{k in } W/(m^2\ K), \text{T in } °C\right)$
- Exhaust velocity (V) = 0.61 m/s
- Gas inlet temperature ($T_{b,\text{in}}$) = 316°C

FIND

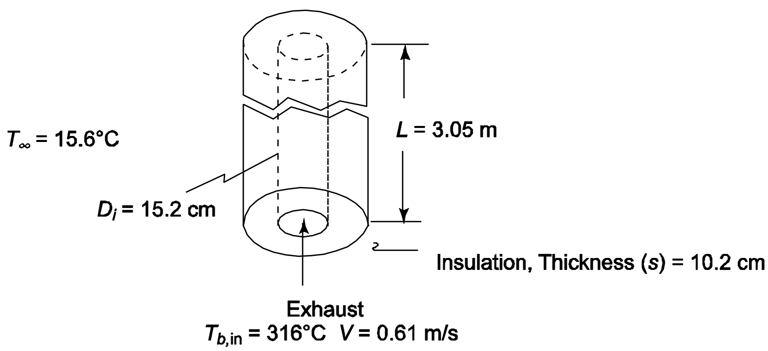
(a) Rate of heat transfer to ambient air at (T_∞) = 15.6

(b) Outlet exhaust gas temperature ($T_{b,\text{out}}$)

ASSUMPTIONS

- Steady state
- Thermal resistance of the duct wall is negligible
- Heat transfer by radiation is negligible
- Natural convection on the inside of the duct can be approximated by natural convection from a vertical plate
- The interior heat transfer coefficient can be accurately estimated using uniform surface temperature correlations
- The ambient air is still

SKETCH



PROPERTIES AND CONSTANTS

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From Appendix 2, Table 28, for dry air at the inlet temperature of 316°C

Thermal expansion coefficient (β) = 0.00175 1/K
Thermal conductivity (k) = 0.0438 W/(m K)
Kinematic viscosity (ν) = 51.7 × 10^{-6} m^2/s
Prandtl number (Pr) = 0.71
Density (ρ) = 0.582 kg/m^3
Specific heat (c) = 1049 J/(kg K)
Absolute viscosity (μ_b) = 28.869 × 10^{-6} Ns/m^2

SOLUTION

Interior convection:

The Reynolds number for the exhaust flow is

$$Re_D = \frac{V\,D}{\nu} = \frac{(0.61\,\text{m/s})(0.152\,\text{m})}{51.7\times10^{-6}\,\text{m}^2/\text{s}} = 1793 \text{ (Laminar)}$$

For the first iteration, let the duct wall temperature (T_d) = 300°C and the insulation surface temperature (T_I) = 20°C. From Appendix, Table 27, the absolute viscosity at T_d = 300°C is μ_s = 29.332 × 10^{-6} (Ns)/m^2. Applying Equation (7.41)

$$\overline{Nu}_D = \left[3.66 + \frac{0.0668 Re_D Pr\left(\frac{D}{L}\right)}{1+0.045\left[Re_D Pr\left(\frac{D}{L}\right)\right]^{0.66}}\right]\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\overline{Nu}_D = \left[3.66 + \frac{0.0668(1793)(0.71)\left(\frac{0.152}{3.05}\right)}{1+0.045\left[(1793)(0.71)\left(\frac{0.152}{3.05}\right)\right]^{0.66}}\right]\left(\frac{28.869}{29.332}\right)^{0.14} = 6.17$$

$$\bar{h}_{c,\text{forced}} = \overline{Nu}_D\,\frac{k}{D_i} = 6.17\,\frac{0.0438\,\text{W/(m K)}}{0.152\,\text{m}} = 1.77 \text{ W/(m}^2\text{ K)}$$

The interior Grashof number based on the duct length is

$$Gr_L = \frac{g\,\beta(T_{b,\text{in}} - T_d)L^3}{\nu_a^2} = \frac{9.8\,\text{m/s}^2\ \ 0.00171(1/\text{K})\ \ (316°\text{C} - 300°\text{C})(3.05\,\text{m})^3}{51.7\times10^{-6}\,\text{m}^2/\text{s}\ ^2} = 2.85\times10^9$$

$$\frac{Gr_L}{Re_D^2} = \frac{2.85\times10^9}{(1793)^2} = 885$$

Therefore, natural convection on the inside of the duct cannot be ignored. The natural convection Nusselt number will be estimated with Equation (5.13)

$$\overline{Nu}_L = 0.13\ (Gr_L Pr)^{\frac{1}{3}} = 0.13\left[2.85\times10^9\,(0.71)\right]^{\frac{1}{3}} = 164.4$$

$$\bar{h}_{c,\text{natural}} = \overline{Nu}_L\,\frac{k}{L} = 164.4\,\frac{0.0438\,\text{W/(m K)}}{3.05\,\text{m}} = 2.36 \text{ W/(m}^2\text{ K)}$$

Combining the free and forced convection using Equation (5.49)

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$$\bar{h}_{ci} = \left(\bar{h}_{ci}^3 + \bar{h}_{cn}^3\right)^{\frac{1}{3}} = \left[(1.77)^3 + (2.36)^3\right]^{\frac{1}{3}} = 2.65 \text{ W/(m}^2\text{ K)}$$

Exterior convection:

The Grashof number on the exterior of the insulation is

$$Gr_L = \frac{g\beta(T_1 - T_\infty)L^3}{\nu_a^2}$$

For the film temperature of 17.8°C

$$\beta = 0.00344 \text{ 1/K}$$

$$\nu = 15.5 \times 10^{-6} \text{ m}^2\text{/s}$$
$$Pr = 0.71$$
$$k = 0.0249 \text{ W/(m K)}$$

$$Gr_L = \frac{\left(9.8\,\text{m/s}^2\right)\left(0.00344\,(1/\text{K})\right)(20°\text{C} - 15.6°\text{C})(3.05\,\text{m})^3}{\left(15.5\times 10^{-6}\,\text{m}^2/\text{s}\right)^2} = 1.75 \times 10^{10}$$

The Nusselt number is given by Equation (5.13)

$$\overline{Nu}_L = 0.13\,(Gr_L Pr)^{\frac{1}{3}} = 0.13\left[1.75\times 10^{10}(0.71)\right]^{\frac{1}{3}} = 301.2$$

$$\bar{h}_{co} = \overline{Nu}_L \frac{k}{L} = 301.2 \left(\frac{0.0249\,\text{W/(m K)}}{3.05\,\text{m}}\right) = 2.46 \text{ W/(m}^2\text{ K)}$$

Conduction through the insulation:

The rate of heat transfer through the insulation is

$$q = -k A \frac{dT}{dr}$$

$$\frac{q}{2\pi L}\int_{r_i}^{r_o}\frac{dr}{r} = \int_{T_d}^{T_i} k dT = -\int_{T_d}^{T_i}(0.7 + 0.016T)\,dt$$

where T_I = exterior insulation temperature

T_d = duct wall temperature = interior insulation temperature

$$\frac{q}{2\pi L}\ln\left(\frac{r_o}{r_i}\right) = -0.7\,(T_I - T_d) - \frac{0.016}{2}\,(T_I^2 - T_d^2)$$

$$q = \frac{2\pi L}{\ln\left(\frac{r_o}{r_i}\right)}\,[0.7\,(T_d - T_I) + 0.008\,(T_d^2 - T_I^2)] = \frac{T_d - T_I}{R_k}$$

$$\therefore \quad \frac{1}{R_k} = \frac{2\pi L}{\ln\left(\frac{r_o}{r_i}\right)}\left[0.7 + 0.008\left(\frac{T_d^2 - T_I^2}{T_d - T_I}\right)\right] = \frac{2\pi L}{\ln\left(\frac{r_o}{r_i}\right)}\,[0.7 + 0.008\,(T_D + T_I)]$$

where $r_i = D_i/2 = (0.0152 \text{ m})/2 = 0.076 \text{ m}$

$r_o = r_i + s = 0.076 \text{ m} + 0.102 \text{ m} = 0.178 \text{ m}$

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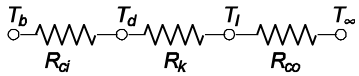
$$\frac{1}{R_k} = \frac{2\pi(3.05\,\text{m})}{\ln\left(\dfrac{0.178}{0.076}\right)} \text{ [0.7 + 0.008 (300°C + 20°)] = 73.4 W/m}$$

$$R_k = 0.0136 \text{ K/W}$$

The thermal circuit for the problem is shown below



$$R_{ci} = \frac{1}{\bar{h}_{ci} A_i} = \frac{1}{\bar{h}_{ci}\,\pi D_i L} = \frac{1}{2.65\,\text{W/(m}^2\text{K)}\ \pi(0.152\,\text{m})(3.05\,\text{m})} = 0.259 \text{ K/W}$$

$$R_{co} = \frac{1}{\bar{h}_{co}\,\pi D_o L} = \frac{1}{2.46\,\text{W/(m}^2\text{K)}\ \pi[0.152\,\text{m} + 2(0.102\,\text{m})](3.05\,\text{m})} = 0.119 \text{ K/W}$$

The total rate of heat transfer is

$$q = \frac{T_b - T_\infty}{R_{ci} + R_k + R_{co}} = \frac{316\,°\text{C} - 15.6\,°\text{C}}{(0.259 + 0.0136 + 0.119)\,\text{K/W}} = 767 \text{ W}$$

Calculating a new duct wall temperature and insulation temperature

$$q = \frac{T_b - T_d}{R_{ci}} \Rightarrow T_d = T_b - q\,R_{ci} = 316°\text{C} - 767\text{ W}\ \ 0.259\,\text{K/W} = 117.3°\text{C}$$

$$q = \frac{T_I - T_\infty}{R_{co}} \Rightarrow T_I = T_\infty + q\,R_{co} = 15.6°\text{C} + 767\text{ W}\ \ 0.119\,\text{K/W} = 106.9°\text{C}$$

$$\Delta T_{\text{bulk}} = \frac{q}{\dot{m}c} = \frac{q}{\frac{\pi}{4} D_i^2 \rho V c} = \frac{767\,\text{W}}{\frac{\pi}{4}(0.152\,\text{m})^2\left(0.582\,\text{kg/m}^3\right)(0.61\,\text{m/s})(1049\,\text{J/(kg K)})((\text{Ws})/\text{J})}$$

$$= 113°\text{C}$$

Therefore, the rate of heat transfer is about 767 watts.

The outlet exhaust gas temperature = $T_{b,\text{in}} - \Delta T_b$ = 316°C – 113°C = 202°C

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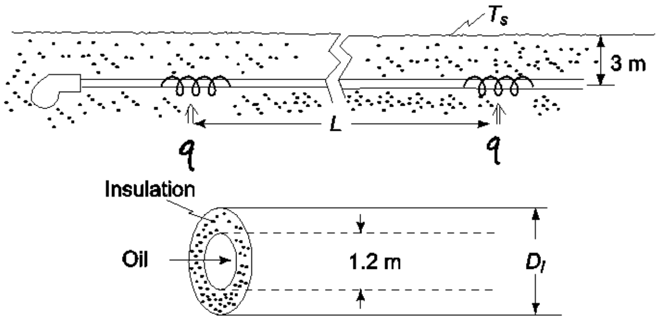
PROBLEM 7.53

A long 1.2-m-*OD* pipeline carrying oil is to be installed in Alaska. To prevent the oil from becoming too viscous for pumping, the pipeline is buried 3 m below ground. The oil is also heated periodically at pumping stations as shown schematically in the figure that follows.



The oil pipe is to be covered with insulation having a thickness *t* and a thermal conductivity of 0.05 W/(m K). It is specified by the engineer installing the pumping station that the temperature drop of the oil in a distance of 100 km should not exceed 5°C when the soil surface temperature T_s = – 40°C. The temperature of the pipe after each heating is to be 120°C and the flow rate is 500 kg/s. The properties of the oil being pumped are given below

Density (ρ_{oil}) = 900 kg/m^3

Thermal conductivity (k_{oil}) = 0.14 W/(m K)

Kinematic viscosity (ν_{oil}) = 8.5 × 10^{-4} m^2/s

Specific heat (c_{oil}) = 2000 J/(kg K)

The soil under arctic conditions is dry $\left(\text{From Appendix 2, Table 11, } k_s = 0.35\ \text{W/(m K)}\right)$.

(a) Estimate the thickness of insulation necessary to meet the specifications of the engineer. (b) Calculate the required rate of heat transfer to the oil at each heating point. (c) Calculate the pumping power required to move the oil between two adjacent heating stations.

GIVEN

- An insulated underground oil pipeline
- Pipe outside diameter (D_{po}) = 1.2 m
- Depth to centerline (Z) = 3 m
- Insulation thickness = t
- Insulation thermal conductivity (k_i) = 0.05 W/(m K)
- For L = 100 km = 100,000 m, Maximum ΔT_b = 5°C when ground surface temp. (T_s) = – 40°C
- Oil temperature after heating ($T_{b,\text{in}}$) = 120°C
- Mass flow rate ($\dot{m}$) = 500 kg/s
- Fluid properties listed above
- Soil thermal conductivity (k_s) = 0.35 W/(m K)

FIND

(a) The thickness of insulation (t) required
(b) The required rate of heat transfer to the oil at each heating point (q_h)
(c) The pumping power required

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ASSUMPTIONS

- Constant thermal properties
- Uniform ground surface temperature
- Flow is fully developed
- The thermal resistance of the pipe is negligible
- The thickness of the pipe is negligible compared to the diameter

SOLUTION

The interior heat transfer coefficient can be evaluated from correlations. The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{4\dot{m}}{\pi D \nu \rho} = \frac{4\ \ 500\,\text{kg/s}}{\pi(1.2\,\text{m})\ \ 900\,\text{kg/m}^3\ \ \ 8.5\times 10^{-4}\ \text{m}^2/\text{s}} = 693 \text{ (Laminar)}$$

Since the oil bulk temperature is to drop only 5°C, for practical purposes, the pipe is isothermal. Therefore, for fully developed flow: $\overline{Nu}_D = 3.66$

$$\bar{h}_c = \overline{Nu}_D \frac{k_{\text{oil}}}{D} = 3.66\ \frac{0.14\,\text{W/(m K)}}{1.2\,\text{m}} = 0.427 \text{ W/(m}^2\text{ K)}$$

(a) A heat balance on an element of the oil yields

$$dq = \dot{m}\, c_p dT_b$$

The rate of heat flow from the element is

$$dq = U\,(T_b - T_s) \quad \text{where } U = \frac{1}{R_{\text{total}}} = \frac{1}{R_c + R_{ki} + R_{ks}}$$

where R_c = interior convective resistance = $\dfrac{1}{\bar{h}_c\, A} = \dfrac{1}{\bar{h}_c\, \pi D_{po}\, dx}$

$$R_{ki} = \text{conductive resistance of the insulation} = \frac{\ln\left(\dfrac{D_i}{D_{po}}\right)}{2\,\pi\, k_i\, dx}$$

$$R_{ks} = \text{conductive resistance of the soil} = \frac{1}{k_s\, S}$$

The shape factor (S) is given in Table 2.2

$$S = \frac{2\pi\, dx}{\cosh^{-1}\left(\dfrac{2Z}{D_i}\right)}$$

$$\therefore\ R_{\text{total}} = \frac{1}{dx}\left[\frac{1}{\bar{h}_c\, \pi D_{po}} + \frac{\ln\left(\dfrac{D_i}{D_{po}}\right)}{2\pi k_i} + \frac{\cos^{-1}\left(\dfrac{2Z}{D_i}\right)}{2\pi k_s}\right]$$

$$\text{Let } U' = \frac{1}{dx R_{\text{total}}} = \frac{U}{dx} \ \text{ then } dq = U'(T_b - T_s)\, dx = \frac{U}{dx}$$

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$$\frac{dT_b}{T_b - T_s} = \frac{U'}{\dot{m}c_p}\,dx$$

Integrating

$$\int_{T_{b,\text{in}}}^{T_{b,\text{out}}} \frac{1}{T_b - T_s}\,dT_b = -\int_0^L \frac{U'}{\dot{m}\,c_p}\,dx$$

$$\ln\left(\frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s}\right) = -\frac{U'L}{\dot{m}\,c_p}$$

Solving for the overall heat transfer coefficient

$$U' = \frac{\dot{m}\,c_p}{L}\ \ln\left(\frac{T_{b,\text{out}} - T_s}{T_{b,\text{in}} - T_s}\right) = -\frac{500\,\text{kg/s}\quad 2000\,\text{J/(kg K)}\quad (\text{Ws})/\text{J}}{100{,}000\,\text{m}}\ \ln = \left(\frac{115\,°\text{C} + 40\,°\text{C}}{120\,°\text{C} + 40\,°\text{C}}\right)$$

$$= 0.317\ \text{W/(m K)}$$

$$\frac{1}{U'} = \frac{1}{\bar{h}_c\,\pi D_{po}} + \frac{\ln\left(\frac{D_i}{D_{po}}\right)}{2\pi k_i} + \frac{\cosh^{-1}\left(\frac{2Z}{D_i}\right)}{2\pi k_s}$$

$$\frac{1}{0.317\,\text{W/(m K)}} = \frac{1}{0.427\,\text{W/(m}^2\text{K)}\ \pi\,(1.2\,\text{m})} + \frac{\ln\left(\frac{D_i}{1.2\text{m}}\right)}{2\pi\ 0.05\,\text{W/(m K)}} + \frac{\cosh^{-1}\left(\frac{2\,(3\,\text{m})}{D_i}\right)}{2\pi\ 0.35\,\text{m W/(m K)}}$$

checking the units then eliminating them for clarity

$$5.571 = 7.01\,n\left(\frac{D_i}{1.2\ \text{m}}\right) + \cosh^{-1}\left(\frac{6}{D_i}\right)$$

by trial and error: D_i = 2.06 m

$$t = \frac{(D_i - D)}{2} = \frac{(2.06\ \text{m}) - (1.2\ \text{m})}{2} = 0.43\ \text{m} = 43\ \text{cm}$$

(b) The rate of heating required at each pumping station is

$$q = \dot{m}c_p\,\Delta T = (500\ \text{kg/s})\ \ 2000\,\text{J/(kgK)}\ \ (5°\text{C})\ \ (\text{Ws})/\text{J} = 5 \times 10^6\ \text{W} = 5\ \text{MW}$$

(c) The pumping power P, equals the product of the volumetric flow rate and the pressure drop, or

$$P = \dot{m}_{\Delta p}$$

Incorporating Equation (7.13) for the pressure drop and Equation (7.18) for the friction factor

$$P = \left(\frac{\dot{m}}{\rho}\right)\frac{64}{Re_d}\,\frac{L}{D}\,\frac{\rho U^2}{2g_c} = 32\ \frac{\dot{m}\,L}{Dg_c Re_D}\left(\frac{4\dot{m}}{\rho\pi D^2}\right)^2 = \frac{512}{\pi^2}\,\frac{L\,\dot{m}^3}{g_c Re_D \rho^2 D^5}$$

$$P = \frac{512}{\pi^2}\,\frac{100{,}000\,\text{m}\ \ 500\,\text{kg/s}\ ^3}{(\text{kg m})/(\text{s}^2\text{N})\ \ (693)\ \ 900\,\text{kg/m}^3\ ^2\ (1.2\,\text{m})^5}\ (\text{Ws})/(\text{Nm})$$

$$= 1.46 \times 10^6\ \text{W} = 1.46\ \text{MW}$$

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PROBLEM 7.54

Water in turbulent flow is to be heated in a single-pass tubular heat exchanger by steam condensing on the outside of the tubes. The flow rate of the water, its inlet and outlet temperatures, and the steam pressure are fixed. Assuming that the tube wall temperature remains constant, determine the dependence of the total required heat exchanger area on the inside diameter of the tubes.

GIVEN

- Water in turbulent flow in tubes with steam condensing on the outside
- Water flow rate, inlet and outlet temperatures, and steam pressure are fixed

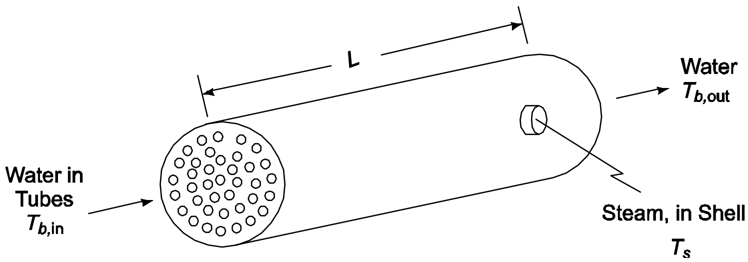
FIND

- At = $f(D)$ where A_t = Total heat exchanger area
  D = Inside diameter of the tubes

ASSUMPTIONS

- Steady state
- Fully developed flow
- Tube wall temperature remains constant
- The heat exchanger is designed such that the flow is fully developed turbulent flow
- Thermal resistance of the condensing steam is negligible
- Thermal resistance of the water pipe is negligible

SKETCH



SOLUTION

Let

N = The number of tubes
$\dot{m}$ = Mass flow rate of the water
$T_{b,in}$ = Water inlet bulk temperature
$T_{b,out}$ = Water outlet bulk temperature
$T_{b,avg}$ = Average of water inlet and outlet bulk temperatures
T_s = Saturation temperature of the steam
k = Thermal conductivity of water evaluated at T_b
ρ = Density of water evaluated at T_b
μ = Absolute viscosity of water evaluated at T_b
Pr = Prandtl number of water evaluated at T_b
c = Specific heat of water evaluated at T_b

The Nusselt number on the inside of the tubes is given by Equation (7.60)

$$Nu_D = 0.023\, Re_D^{0.8}\, Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

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$$h_c = Nu_D \frac{k}{D} = 0.023 \frac{k}{D} Re_D^{0.8} Pr^{0.4} = 0.023 \frac{k}{D} \left(\frac{V D \rho}{\mu}\right)^{0.8} Pr^{0.4}$$

$$h_c = 0.023 \frac{k}{D} \left(\frac{4\left(\frac{\dot{m}}{N}\right)}{\pi D \mu}\right)^{0.8} Pr^{0.4} = 0.0279\, k\, D^{-1.8} \left(\frac{\dot{m}}{N \mu}\right)^{0.8} Pr^{0.4}$$

The heat transfer by convection to the water must equal the energy required to raise the water temperature by the given amount

$$h_c A_t (T_s - T_{b,\text{ave}}) = \dot{m}\, c\, (T_{b,\text{out}} - T_{b,\text{in}})$$

$$A_t = \frac{\dot{m} c}{h_c} \frac{T_{b,\text{out}} - T_{b,\text{in}}}{T_s - T_{b,\text{ave}}} = \frac{\dot{m} c}{0.0279 k\, D^{-1.8} \left(\frac{\dot{m}}{N \mu}\right)^{0.8} Pr^{0.4}} \frac{T_{b,\text{out}} - T_{b,\text{in}}}{T_s - T_{b,\text{ave}}}$$

$$A_t = 35.8 \frac{\dot{m}^{0.2} \mu^{0.8} N^{0.8}}{k\, Pr^{0.4}} \left(\frac{T_{b,\text{out}} - T_{b,\text{in}}}{T_s - T_{b,\text{ave}}}\right) D^{1.8}$$

$$A_t \propto D^{1.8}$$

Checking the units

$$[A_t] = \text{kg/s}^{\,0.2} \left[(\text{Ns})/\text{m}^2\right]^{0.8} \ \text{J/(kg K)} \ \ \text{W/(m K)}^{\,-1} \ \ (\text{Ws})/\text{J}^{\,0.6} \ [\text{m}]^{1.8} = [\text{m}]^2$$

COMMENTS

The tube diameter must not become so large that the water flow becomes laminar.

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PROBLEM 7.55

A 5000 m^2 condenser is constructed with 2.5-cm-OD brass tubes that are 2.7 m long and have a 1.2 mm wall thickness. The following thermal resistance data were obtained at various water velocities inside the tubes (*Trans. ASME*, Vol. 58, p. 672, 1936).

$1/U_o \times 10^3$ $((\text{m}^2\,K)/W)$	Water Velocity (m/s)	$1/U_o \times 10^3$ $((\text{m}^2\,K)/W)$	Water Velocity (m/s)
0.364	2.11	0.544	0.90
0.373	1.94	0.485	1.26
0.391	1.73	0.442	2.06
0.420	1.50	0.593	0.87
0.531	0.89	0.391	1.91
0.368	2.14		

Assuming that the heat transfer coefficient on the steam side is 11.3 kW/(m^2 K) and the mean bulk water temperature is 50°C, determine the scale resistance.

GIVEN

- Water flowing inside a brass tube condenser
- Total transfer are (A_t) = 5,000 m^2
- Tube outside diameter (D) = 2.5 cm= 0.025 m
- Tube length (L) = 2.7 m
- Tube wall thickness (t) = 1.2 mm = 0.0012 m
- Heat transfer coefficient on the steam side ($\bar{h}_{cs}$) = 11300 W/(m^2 K)
- Mean bulk water temperature = 50°C
- Thermal resistance data shown above

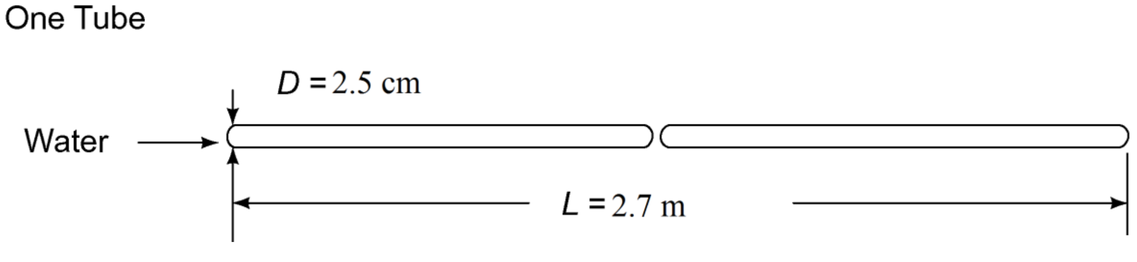
FIND

- The scale resistance ($A_t R_{ks}$)

ASSUMPTIONS

- Data were taken at steady state
- The tube temperature can be considered uniform and constant
- Condenser surface area is based on the tube outside diameter

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, for water at 50°C

Thermal conductivity (k) = 0.647 W/(m K)

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Kinematic viscosity (ν) = 0.556*10^{-6} m^2/s
Prandtl number (Pr) = 3.55

From Appendix 2, Table 10, the thermal conductivity of brass (k_b) = 111 W/(m K)

SOLUTION

The inside tube diameter is $D_i = D_o - 2t = 0.025 - 2(0.0012\text{ m}) = 0.902\text{ in.} = 0.0226\text{ m}$

The maximum and minimum velocities in the given data are 0.87 m/s and 2.11 m/s. These correspond to the following Reynolds numbers

$$Re_{\min} = \frac{V\,D}{\nu} = \frac{(0.87\,m/\text{s})(0.0226)}{\left(0.566*10^{-6}\,m^2/s\right)} = 34{,}738$$

$$Re_{\max} = \frac{V\,D}{\nu} = \frac{(2.11\,m/\text{s})(0.0226)}{\left(0.566*10^{-6}\,m^2/s\right)} = 84{,}260$$

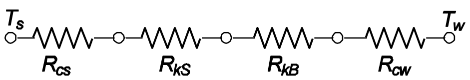
Therefore, the flow is turbulent in all cases. Applying Equation (7.61) to the minimum Re case

$$\overline{Nu}_D = 0.023\,Re_D^{0.8}\,Pr^n \qquad \text{where } n = 0.4 \text{ for heating}$$

$$\overline{Nu}_D = 0.023\,(34{,}738)^{0.8}\,(3.55)^{0.4} = 162.9$$

$$\bar{h}_{cw} = \overline{Nu}_D\,\frac{k}{D} = 162.9\,\frac{\left(0.647\,W/(\text{m}\,K)\right)}{0.0226\ m} = 4652\text{ W/(m}^2\text{ K)}$$

The thermal circuit for this problem is shown below



$$A_t\,R_{cs} = \frac{1}{\bar{h}_{cs}} = \frac{1}{\left(11300\,W/(\text{m}^2\,K)\right)} = 8.85\times 10^{-5}\ (\text{m}^2\text{ K})/\text{W}$$

$$A_t\,R_{ks} = \text{scaling resistance}$$

For one tube

$$A_t\,R_{kB} = \pi\,D\,L\,\frac{\ln\left(\dfrac{D_o}{D_i}\right)}{2\,\pi\,L k_s}$$

$$A_t\,R_{kB} = \pi\,(0.025\text{ m})\,(2.7\text{ m})\,\frac{\ln\left(\dfrac{0.025}{0.0226}\right)}{2\,\pi(2.7\,m)(111\text{W}/(\text{m}\,K))} = 1.14\times 10^{-5}\ (\text{m}^2\text{ K})/\text{W}$$

For the minimum Re case

$$A_r\,R_{cw} = \frac{1}{\bar{h}_{cw}} = \frac{1}{\left(4652\,W/(\text{m}^2\,K)\right)} = 2.15\times 10^{-4}\ (\text{m}^2\text{ K})/\text{W}$$

These resistances are in series, therefore

$$A_r\,R_{\text{total}} = \frac{1}{U_o} = A_t\,(R_{cs} + R_{ks} + R_{kB} + R_{cw})$$

$$\therefore\ A_t\,R_{ks} = \frac{1}{U_o} - A_t\,(R_{cs} + R_{kB} + R_{cw})$$

For the minimum Re case

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$$A_t\, R_{ks} = \left(0.593 \times 10^{-3}\, (m^2\, K)/W\right) - (8.855 \times 10^{-5} + 1.14 \times 10^{-5} + 2.15 \times 10^{-4})\ (\mathrm{m^2\ K})/\mathrm{W}$$

$$A_t\, R_{ks} = 0.2781 \times 10^{-3}\ (\mathrm{m^2\ K})/\mathrm{W}$$

Repeating this method for the rest of the data given

Water Velocity (m/s)	$\bar{h}_{cw}\ \left(W/(m^2\, K)\right)$	$A_t R_{ks} \times 10^{-4}\ \left((m^2\, K)/W\right)$
2.11	9601	1.585
1.94	8977	1.603
1.73	8210	1.673
1.50	7296	1.814
0.89	4832	1.26
2.14	9715	2.22
0.90	4860	2.36
1.26	6348	2.25
2.06	9437	2.34
0.87	4741	2.8
1.91	8880	1.76
	Average: 2.0×10^{-4} (m^2 K)/W	

COMMENTS

The standard deviation in the scale resistance is 24%.

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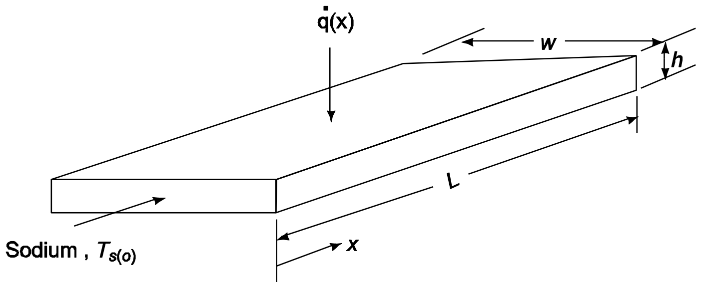
PROBLEM 7.56

A nuclear reactor has rectangular flow channels with a large aspect ratio (w/h)>>1



Heat generation is equal from the upper and lower surface and uniform at any value of *x*. However, the rate varies along the flow path of the sodium coolant according to

$$q''(x) = q_o'' \sin(\pi x/L)$$

Assuming that entrance effects are negligible so that the convection heat transfer coefficient is uniform

(a) Obtain an expression for the variation of the mean temperature of the sodium, $T_m(x)$.

(b) Derive a relation for the surface temperature of the upper and lower portion of the channel, $T_s(x)$.

(c) Determine the distance x_{max} at which $T_s(x)$ is maximum.

GIVEN

- Sodium flow through a rectangular flow channel with a large aspect ratio
- Heat generation from each surface (upper and lower): $q''(x) = q_o'' \sin(\pi x/L)$

FIND

(a) An expression for the variation of the mean sodium temperature, $T_m(x)$
(b) A relationship for the upper and lower surface temperature $T_s(x)$
(c) The distance x_{max} at which $T_s(x_{max})$ is maximum

ASSUMPTIONS

- Entrance effects are negligible
- The convective heat transfer coefficient is uniform
- Steady state

SOLUTION

The hydraulic diameter for the duct is

$$D_H = \frac{4A_c}{P} = \frac{4wh}{2w+2h} = 2h$$

(a) In steady state, all of the heat generation must be removed by the sodium, Therefore, the heat transfer to an element of sodium in the duct is

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$$dq = 2\,q''\,w\,dx = 2\,q''_o \sin\left(\frac{\pi x}{L}\right)\,w\,dx$$

This will lead to a rise in temperature in the sodium according to

$$dq = \dot{m}\,c\,dT_m = 2\,q''_o \sin\left(\frac{\pi x}{L}\right)\,w\,dx$$

$$\frac{dT_m}{dx} = \frac{2q''_o w}{\dot{m}c}\,\sin\left(\frac{\pi x}{L}\right)$$

Integrating

$$\int_{T_{m,\mathrm{in}}}^{T_m(x)} dT_m = T_m(x) - T_{m,\mathrm{in}} = \frac{2q''_o w}{\dot{m}c}\int_0^x \sin\left(\frac{\pi x}{L}\right)dx$$

$$T_m(x) = T_{m,\mathrm{in}} + \frac{2q''_o wL}{\pi \dot{m} c}\left[1 - \cos\left(\frac{\pi x}{L}\right)\right]$$

(b) The rate of heat transfer from both surfaces must equal the rate of heat generation

$$q_{cx} = \dot{q}(x) \Rightarrow \quad 2\,h_c\,w\,dx\,(T_s - T_m) = 2q''_o\,\sin\left(\frac{\pi x}{L}\right)w\,dx$$

Solving for the surface temperature

$$T_s = T_m + \frac{q''_o}{h_c}\sin\left(\frac{\pi x}{L}\right)$$

$$T_s = T_{m,\mathrm{in}} + \frac{2q''_o wL}{\pi \dot{m} c}\left[1 - \cos\left(\frac{\pi x}{L}\right)\right] + \frac{q''_o}{h_c}\,\sin\left(\frac{\pi x}{L}\right)$$

Assuming the flow is fully developed and approximating the heat flux as uniform, the Nusselt number, from Table 7.1, is 8.235. Therefore, $h_c = 8.235\ k/D_H$.

$$T_s = T_{m,\mathrm{in}} + \frac{2q''_o wL}{\pi \dot{m} c}\left[1 - \cos\left(\frac{\pi x}{L}\right)\right] + \frac{q''_o D_H}{16.47k}\,\sin\left(\frac{\pi x}{L}\right)$$

(c) The maximum occurs when the first derivative of the expression for T_s is zero

$$\frac{dT_s}{dx} = \frac{2q''_o\,w}{\dot{m}c}\,\sin\left(\frac{\pi x}{L}\right) - \frac{q''_o\,\pi D_H}{16.47\,k\,L}\,\cos\left(\frac{\pi x}{L}\right) = 0$$

$$\frac{\sin\left(\frac{\pi x}{L}\right)}{\cos\left(\frac{\pi x}{L}\right)} = \frac{\pi \dot{m} c D_H}{16.47k\,2wL}$$

$$x_{\max} = \frac{L}{\pi}\ \mathrm{Arctan}\left(\frac{\pi \dot{m} c h}{16.47\,k w L}\right)$$

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PROBLEM 7.57

Reconsider Problem 7.19, where the tube-to-coil effects were ignored. By considering a helical coiled tube heat exchanger where the tube length (straight length of an unwound coil) is the same as calculated previously, recommend a coil diameter, d_c, for meeting the same heat load required for cooling the transformer oil to 30°C with the tube surface maintained at 20°C. Again, based on your coil design, what is the outlet oil temperature when the tube surface temperature increases to 25°C? What is the difference in pressure drop between that for the coiled tube and the straight tube?

GIVEN

- Transformer oil cooled via water-cooled, coiled tube heat exchanger.
- Oil flow rate, $\dot{m}$ = 0.3 kg/s.
- Oil inlet temperature, $T_{b,\text{in}}$ = 50°C.
- Oil outlet temperature $T_{b,\text{out}}=30^0$C
- Inner or inside diameter of pipe in which oil flows, D = 5 cm = 0.05 m.
- Temperature of pipe surfaceT_s = 20°C.

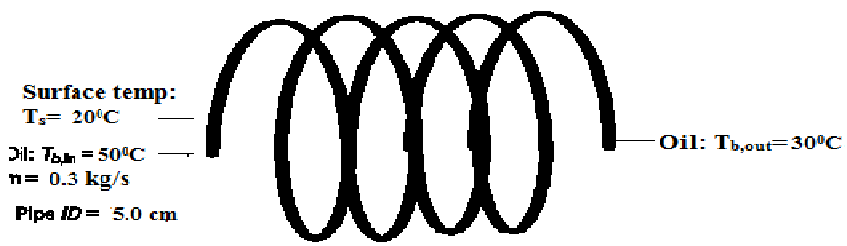
FIND

- Required coil diameter d_c for length calculated in Problem 7.19 for same heat load.
- Oulet temperature if surface temperature reaches to 25^0C for given length.
- Difference in pressure drop for coiled tube and straight tube.

ASSUMPTIONS

- The temperature of wall is constant and uniform across the length of pipe.
- The thermal resistance of the pipe is negligible, and hence the inside surface temperature of the pipe is $T_w = T_s$, this represents a uniform pipe surface temperature condition.

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 18, for Transformer oil at T_b= 40°C

Density, ρ = 867.0 kg/m^3

Thermal conductivity, k = 0.109 W/(m K)

Absolute viscosity, $\mu_b = 9.3 \times 10^{-3}$ (Ns)/m^2

Prandtl number, Pr = 157

Specific heat, c_p = 1830 J/(kg K)

At the pipe surface temperature of 25°C, the absolute viscosity $\mu_s = 21.1 \times 10^{-3}$ (Ns)/m^2

SOLUTION

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The Reynolds number for oil flow inside the pipe is

$$\mathrm{Re}_D = \frac{\rho VD}{\mu_b} = \frac{4\dot{m}}{\pi D\mu_b} = \frac{4\times 0.3}{\pi\times 0.05\times 0.0093} = 821.5 \Rightarrow \text{Laminar flow}$$

From Problem (7.19) we have

$\bar{h}_c$ =9.75 W/(m^2 K)   L=393.7 m

The Nusselt number is given by

$$\mathrm{Nu} = \bar{h}_c \frac{D}{k} = 9.75 * \frac{0.05}{0.109} = 4.47$$

Nusselt number for coiled tube is given by formula

$$\overline{Nu}_D = \left[\left\{3.657 + \frac{4.43}{[1+(957/\Pr .He^2)]^2}\right\}^3 + 1.158\left\{\frac{He}{[1+(0.477/\Pr)]}\right\}^{3/2}\right]^{1/3}$$

Substituting the value of Nu=4.47 and Pr=157 and solving for He in mathematica ® we get

He=107.15

Considering H<<d_c we have

He= De where De is given by

$$De = \mathrm{Re}_D\left(D/d_c\right)^{1/2}$$

$$107.15 = 821\left(0.05/d_c\right)^{1/2}$$

$$\frac{d_c}{0.05} = \left(7.66\right)^2$$

$$\frac{d_c}{0.05} = \left(7.66\right)^2$$

$$d_c = 2.93m$$

The pressure drop in pipe flow is given by

$$\Delta p = f\,\frac{L}{D}\left(\frac{\rho\bar{U}^2}{2g_c}\right)$$ where f is given by equation (7.18) as

$$f = \frac{64}{\mathrm{Re}_D}\left[\left(1 - \frac{0.18}{\{1+(35/He)^2\}^{0.5}}\right)^m + \left(1+\frac{D}{d_c}\right)^2\left(\frac{He}{88.33}\right)\right]^{0.5}$$ where m=2 for

De<20.

Thus, $$f = \frac{64}{821.5}\left[0.89 + 0.121\right]^{0.5} = 0.078$$

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$$\Delta p = 0.078 * \frac{393.7}{0.05}\left(\frac{867*(0.176)^2}{2*1}\right) \text{ Pa} = 8283 \text{ Pa}$$

The difference in pressure drop is therefore 920 Pa.

If the temperature increases to $T_s = 25^0$C

From Equation (7.36) we have

$$\frac{\Delta T_{out}}{\Delta T_{in}} = \exp\left(\frac{-\bar{h}_c PL}{\dot{m}c_p}\right) = \exp\left(\frac{-9.75*\pi*0.05*393.7}{0.3*1830}\right)$$

$$\frac{T_{out} - T_s}{T_{in} - T_s} = \exp(-1.096) = 0.334$$

$$\frac{T_{out} - 25}{50 - 25} = 0.334$$

$T_{out} = 33.4^0$C

Thus outlet temperature increases to 33.4^0C if surface temperature is increased to 25^0C.

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