MECH 4012 Test #1, Spring 2022 Name: Yi, Hongrui

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1. A thin, flat plate that is 0.2 m x 0.2 m on a side is oriented parallel to an atmospheric airstream having a velocity of 40 m/s. The air is at a temperature of $T_\infty = 20°C$, while the plate is maintained at $T_s = 120°C$. The airflows over the top and bottom surfaces of the plate, and measurement of the drag force reveals a value of 0.075 N.

For air at 70°C, 1 atm:
$\rho = 1.018\ kg/m^3$
$c_p = 1009\ J/kg \cdot K$
$Pr = 0.70$
$\nu = 20.22 \times 10^{-6} m^2/s$

a. Assume the flow is turbulent over the entire plate. What is the rate of heat transfer from both sides of the plate to the air? [20 points]

$\cancel{Re_L = \frac{\rho U_\infty L}{\nu} = \frac{40 m/s \times 0.2m}{20.22\times10^{-6}} = 3.956\times10^5}$

$\cancel{k = \frac{C_p\mu}{Pr} = \frac{C_p \nu \rho}{Pr} = \frac{1009 \times 20.22\times10^{-6}\times1.018}{0.7} = 2.967\times10^{-2} W/mK}$

$\bar{\tau} = \frac{D}{A} = \frac{0.075N}{(0.2m)^2} = 1.875\ N/m^2$

$\bar{C_f} = \frac{\bar{\tau}}{\rho U_\infty^2/2} = \frac{1.875 N/m^2}{1.018 kg/m^3 \times (40m/s)^2 \times \frac{1}{2}} = \cancel{4.6\times10^{-3}}$ 2.3023

$\therefore$ turbulent

$\therefore \bar{C_f} = \frac{0.072}{Re_L^{1/5}} \Rightarrow Re_L = \left(\frac{0.072}{\bar{C_f}}\right)^5 = \left(\frac{0.072}{2.3\times10^{-3}}\right)^5 = 3\times10^7$

$q = 0.664\ k\ Re_L^{1/2}\ Pr^{1/3}\ b(T_s - T_\infty)$

$= 0.664 \times \cancel{2.967\times10^{-2}}\ 0.0286\ W/mK \cdot (3\times10^7)^{1/2} \times (0.7)^{1/3} \times (0.2m) \times (120-20)K$

$= \cancel{1916.2}\ W$

$\cancel{1847.1}$

b. Is the turbulent boundary layer assumption, correct? Why? [5 points]

$Re_L = \frac{U_\infty L}{\nu} = \frac{40m/s \times 0.2m}{20.22\times10^{-6}} = 3.956\times10^5 < 5\times10^5$

Not correct.

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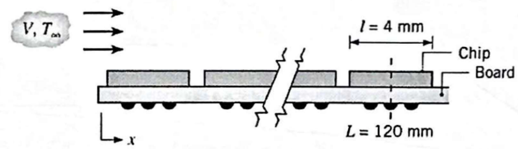
2. Forced air at $T_\infty = 25°C$ and $V = 10$ m/s is used to cool electronic elements on a circuit board. One such element is a chip, 4mm x 4mm, located 120mm from the leading edge of the board. Experiments have revealed that flow over the board is disturbed by the elements and that convection heat transfer is correlated by an expression of the form

$$Nu_x = 0.04\, Re_x^{0.85}\, Pr^{1/3}$$

Estimate the surface temperature of the chip if it is dissipating 30 mW. [25 points]



Air, assume $T_f = (45 + 25)/2 = 35°C = 308K$, 1atm:
$\nu = 16.69 \times 10^{-6}\ m^2/s$
$k = 26.9 \times 10^{-3}$ W/m·K
$Pr = 0.703$

$$Re_L = \frac{\rho U_\infty L}{\nu} = \frac{10 m/s \times 0.12m}{16.69\times10^{-6} m^2/s} = 7.1899\times10^4 < 5\times10^5 \text{ (Laminar)}$$

$$Nu_L = 0.04\, Re_L^{0.85}\, Pr^{1/3} = 0.04 \times (7.1899\times10^4)^{0.85} \times 0.703^{1/3}$$
$$= 477.8147$$

$$Nu_L = \frac{h_c L}{k_f} \Rightarrow h_c = \frac{Nu_L k_f}{L} = \frac{477.8147 \times 26.9\times10^{-3} W/mK}{0.12 m}$$
$$= 107.11\ W/m^2K$$

$$\bar{h}_c = 2h_c = 2\times 107.11\ W/m^2K = 214.22\ W/m^2K$$

$$q = \bar{h}_c A (T_s - T_\infty)$$

$$\Rightarrow T_s = \frac{q}{\bar{h}_c A} + T_\infty = \frac{30\times10^{-3}}{214.22\times(0.004)^2} + 25$$
$$= 33.75°C$$

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3. The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature–time history of a sphere fabricated from pure copper. The sphere, which is 12.7 mm in diameter, is at 66°C before it is inserted into an airstream having a temperature of 27°C. A thermocouple on the outer surface of the sphere indicates 55°C 69s after the sphere is inserted into the airstream. Assume and then justify that the sphere behaves as a spacewise isothermal object and calculate the heat transfer coefficient. [25 points]

Pure copper (333K):
$\rho = 8933\ kg/m^3$
$c_p = 389\ J/kg{\cdot}K$
$k = 398\ W/m{\cdot}K$

$D = 12.7\,mm = 0.0127\,m \quad T_0 = 66°C \quad T_\infty = 27°C \quad T = 55°C \quad t = 69\,s$

$Bi = \frac{\bar{h} L_c}{k}$

$\tau_t = \frac{1}{\bar{h} A_s}(c\rho V)$

$\Rightarrow \frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\left(\frac{\bar{h} A_s}{c\rho V}\right) t\right]$

$\Rightarrow \ln\left(\frac{T - T_\infty}{T_0 - T_\infty}\right) = -\frac{\bar{h} A_s}{c\rho V} t = -\frac{\bar{h} \cdot \pi D^2}{c\rho \frac{\pi}{6} D^3} t = -\frac{6\bar{h}}{c\rho D} t$

$\Rightarrow \bar{h} = -\frac{c\rho D \ln\left(\frac{T - T_\infty}{T_0 - T_\infty}\right)}{t} = -\frac{389\,J/kg{\cdot}K \times 8933\,kg/m^3 \times \ln\left(\frac{55 - 27}{66 - 27}\right) \times 0.0127}{69\,s}$

$= \cancel{1.6688 \times 10^4}$

$= 211.8327\ W/m^2K$

$Bi = \frac{\bar{h} L_c}{k} = \frac{\bar{h} D}{6k} = \frac{211.8327 \times 0.0127}{6 \times 398} = \cancel{1.1271 \times 10^{-3}} << 0.1$

Hence, it behaves as a spacewise isothermal.

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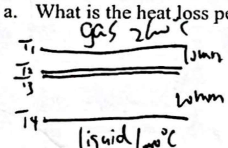
4. A composite wall separates combustion gases at 2600°C from a liquid coolant at 100°C, with gas- and liquid-side convection coefficients of 50 and 1000 W/(m^2.K). The wall is composed of a 10-mm-thick layer of beryllium oxide on the gas side and a 20-mm-thick slab of stainless steel (AISI 304) on the liquid side. The contact resistance between the oxide and the steel is 0.05 m^2.K/W.

Assume the following properties:

St. St. (304) $(\bar{T} \approx 1000\text{K})$: k = 25.4 W/m·K

Beryllium Oxide (T ≈ 1500K): k = 21.5 W/m·K

a. What is the heat loss per unit surface area of the composite? [10 pts] $\frac{q}{A} = ?$ 5



③ $R_{k1} = \frac{L_1}{Ak_B} = \frac{0.01}{21.5A} = \frac{4.6512\times10^{-4}}{A}[K/W]$

④ $R_{k2} = \frac{L_2}{Ak_S} = \frac{0.02}{25.4A} = \frac{7.874\times10^{-4}}{A}[K/W]$

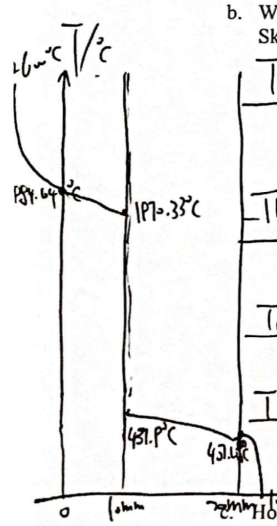
⑤ $R_{contact} = \frac{0.05}{A}[K/W]$

① $R_{c1} = \frac{1}{h_{c1}A} = \frac{1}{50A}[K/W]$

② $R_{c2} = \frac{1}{h_{c2}A} = \frac{1}{1000A}[K/W]$

$\frac{q}{A} = \frac{\Delta T}{R_{total}} = \frac{2600-100}{4.6512\times10^{-4}+7.874\times10^{-4}+0.05+0.02+0.01} = \cancel{4.8778\times10^4 W/m^2}$ $3.0768\times10^4 W/m^2$

b. What are the temperatures on each side of the beryllium oxide and stainless-steel layers? Sketch the temperature distribution from gas to the liquid. [15 pts] 10



$\frac{T_{gas}-T_1}{R_{c1}} = \frac{q}{A} \Rightarrow T_1 = T_{gas} - \frac{q}{A}\cdot R_{c1} = 2600 - \cancel{4.8778\times10^4\times4.6512\times10^{-4}}$
$3.0768\times10^4\times0.02$
$= 1884.64°C$

$\frac{T_{liquid}+T_4}{R_{c2}} = \frac{q}{A} \Rightarrow T_4 = \frac{q}{A}\cdot R_{c2} + T_{liquid} = 3.0768\times10^4\times0.01 + 100$
$= 437.68°C$

$\frac{T_1-T_2}{R_{k1}} = \frac{q}{A} \Rightarrow T_2 = -\frac{q}{A}\cdot R_{k1} + T_1 = -3.0768\times10^4\times4.6512\times10^{-4} + 1884.64$
$= 1870.33°C$

$\frac{T_3-T_4}{R_{k2}} = \frac{q}{A} \Rightarrow T_3 = \frac{q}{A}\cdot R_{k2} + T_4 = 3.0768\times10^4\times7.874\times10^{-4} + 437.68$
$= 431.8°C$

c. How could you improve the accuracy of the results? [E.C.] +2

1° Keep more numbers after the "dot" when do the calculation.

2° Change the value of "k" of the Beryllium Oxide and St.St.(304) according to the estimation of the actual temperature, as $\bar{T}\approx1000K$, $T\approx1500K$ have much uncertainty