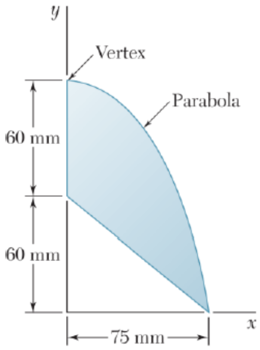
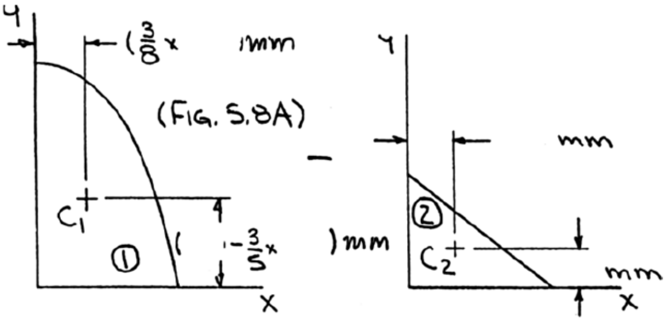
Homework3 Solutions



PROBLEM 5.11

Locate the centroid of the plane area shown.

SOLUTION



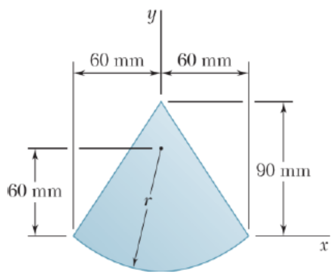
	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{2}{3}(75)(120) = 6000$	28.125	48	168,750	288,000
2	$-\frac{1}{2}(75)(60) = -2250$	25	20	–56,250	–45,000
Σ	3750			112,500	243,000

Then $$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}(3750 \text{ mm}^2) = 112,500 \text{ mm}^3$$ or $\bar{X} = 30.0$ mm ◀

and $$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

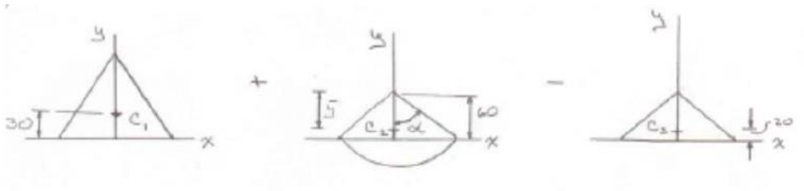
$$\bar{Y}(3750 \text{ mm}^2) = 243,000 \text{ mm}^3$$ or $\bar{Y} = 64.8$ mm ◀



PROBLEM 5.15

Locate the centroid of the plane area shown.

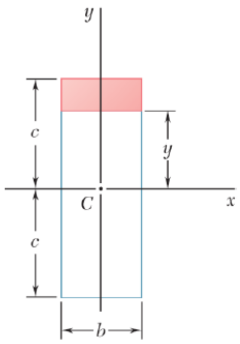
SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{1}{2}(120)(90) = 5400$	0	30	0	162,000
2	$\frac{\pi}{4}(60\sqrt{2})^2 = 5654.9$	0	9.07	0	51,290
3	$-\frac{1}{2}(120)(60) = -3600$	0	20	0	-72,000
Σ	7454.9			0	141,290

Then $\bar{X}A = \Sigma \bar{x}A$ $\qquad \bar{X} = 0 \text{ mm}$ ◀

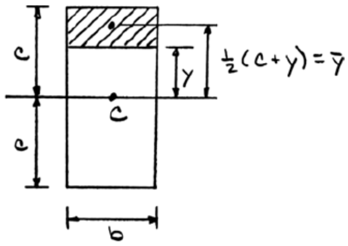
$\bar{Y}A = \Sigma \bar{y}A \qquad \bar{Y}(7454.9 \text{ mm}^2) = 141,290 \text{ mm}^3 \qquad \bar{Y} = 18.95 \text{ mm}$ ◀



PROBLEM 5.23

The first moment of the shaded area with respect to the x-axis is denoted by Q_x. (a) Express Q_x in terms of b, c, and the distance y from the base of the shaded area to the x-axis. (b) For what value of y is Q_x maximum, and what is that maximum value?

SOLUTION



Shaded area:

$$A = b(c-y)$$

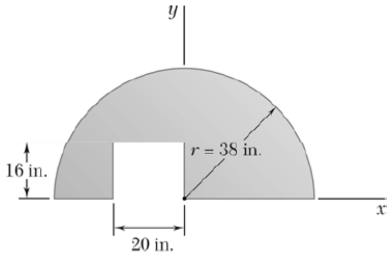
$$Q_x = \bar{y}A$$

$$= \frac{1}{2}(c+y)[b(c-y)]$$

(a) $$Q_x = \frac{1}{2}b(c^2 - y^2) \quad ◀$$

(b) For $Q_{\max}$, $$\frac{dQ}{dy} = 0 \quad \text{or} \quad \frac{1}{2}b(-2y) = 0 \qquad y = 0 \quad ◀$$

For $y = 0$, $$(Q_x) = \frac{1}{2}bc^2 \qquad (Q_x) = \frac{1}{2}bc^2 \quad ◀$$



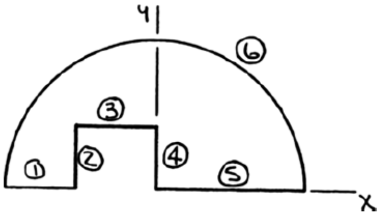
PROBLEM 5.27

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION

First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

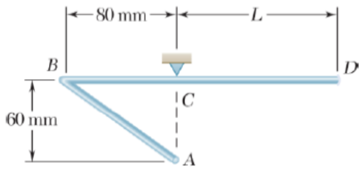
$$\bar{Y}_6 = \frac{2}{\pi}(38 \text{ in.})$$



	L, in.	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}L$, in.^{2}	$\bar{y}L$, in.^{2}
1	18	–29	0	–522	0
2	16	–20	8	–320	128
3	20	–10	16	–200	320
4	16	0	8	0	128
5	38	19	0	722	0
6	$\pi(38) = 119.381$	0	24.192	0	2888.1
Σ	227.38			–320	3464.1

Then $\bar{X} = \dfrac{\Sigma \bar{x} L}{\Sigma L} = \dfrac{-320}{227.38}$ $\bar{X} = -1.407$ in. ◀

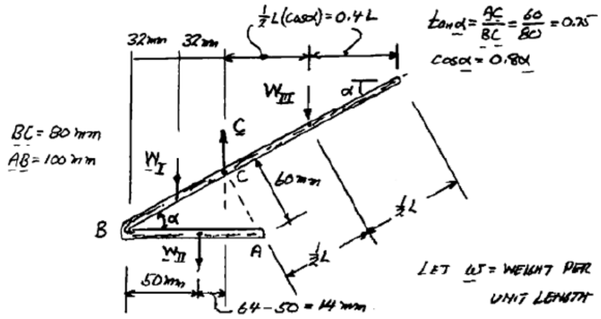
$\bar{Y} = \dfrac{\Sigma \bar{y} L}{\Sigma L} = \dfrac{3464.1}{227.38}$ $\bar{Y} = 15.23$ in. ◀



PROBLEM 5.31

The homogeneous wire *ABCD* is bent as shown and is attached to a hinge at *C*. Determine the length *L* for which portion *AB* of the wire is horizontal.

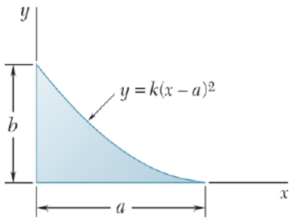
SOLUTION



$$W_{\mathrm{I}} = 80w \qquad W_{\mathrm{II}} = 100w \qquad W_{\mathrm{III}} = Lw$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (80w)(32) + (100w)(14) - (Lw)(0.4L) = 0$$

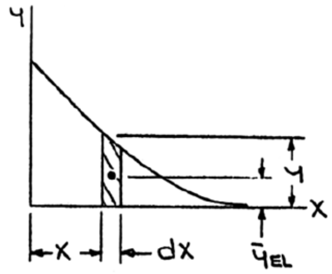
$$L^2 = 9900 \qquad\qquad L = 99.5 \text{ mm} \blacktriangleleft$$



PROBLEM 5.40

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and b.

SOLUTION



At $\qquad x = 0, \quad y = b$

$$b = k(0-a)^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

Then $\qquad y = \frac{b}{a^2}(x-a)^2$

Now $\qquad \bar{x}_{EL} = x$

$$\bar{y}_{EL} = \frac{y}{2} = \frac{b}{2a^2}(x-a)^2$$

and $\qquad dA = y\,dx = \frac{b}{a^2}(x-a)^2\,dx$

Then $\qquad A = \int dA = \int_0^a \frac{b}{a^2}(x-a)^2\,dx = \frac{b}{3a^2}\left[(x-a)^3\right]_0^a = \frac{1}{3}ab$

and $\qquad \int \bar{x}_{EL}\,dA = \int_0^a x\left[\frac{b}{a^2}(x-a)^2\,dx\right] = \frac{b}{a^2}\int_0^a \left(x^3 - 2ax^2 + a^2x\right)dx$

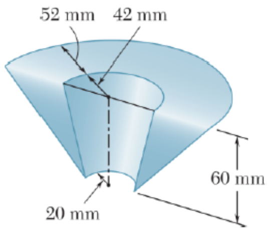
$$= \frac{b}{a^2}\left(\frac{x^4}{4} - \frac{2}{3}ax^3 + \frac{a^2}{2}x^2\right) = \frac{1}{12}a^2b$$

$$\int \bar{y}_{EL}\,dA = \int_0^a \frac{b}{2a^2}(x-a)^2\left[\frac{b}{a^2}(x-a)^2\,dx\right] = \frac{b^2}{2a^4}\left[\frac{1}{5}(x-a)^5\right]_0^a$$

$$= \frac{1}{10}ab^2$$

Hence $\qquad \bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left(\frac{1}{3}ab\right) = \frac{1}{12}a^2b \qquad \bar{x} = \frac{1}{4}a$ ◀

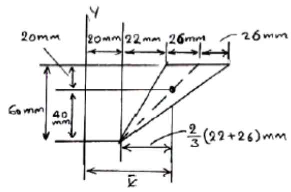
$$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left(\frac{1}{3}ab\right) = \frac{1}{10}ab^2 \qquad \bar{y} = \frac{3}{10}b \text{ ◀}$$



PROBLEM 5.61

Determine the volume and total surface area of the bushing shown.

SOLUTION



Volume:

The volume can be obtained by rotating the triangular area shown through π radians about the y axis.

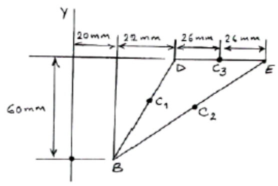
The area of the triangle is:

$$A = \frac{1}{2}(52)(60) = 1560 \text{ mm}^2$$

Applying the theorems of Pappus-Guldinus, we have

$$V = \pi\bar{x}A = \pi(52 \text{ mm})\left(1560 \text{ mm}^2\right) \qquad \text{or} \qquad V = 255 \times 10^3 \text{ mm}^3 \blacktriangleleft$$

The surface area can be obtained by rotating the triangle shown through an angle of π radians about the y axis.



Considering each line BD, DE, and BE separately:

Line $BD: L_1 = \sqrt{22^2 + 60^2} = 63.906 \text{ mm}$ $\qquad$ $\bar{x}_1 = 20 + \dfrac{22}{2} = 31 \text{ mm}$

Line $DE: L_2 = 52 \text{ mm}$ $\qquad$ $\bar{x}_2 = 20 + 22 + 26 = 68 \text{ mm}$

Line $BE: L_3 = \sqrt{74^2 + 60^2} = 95.268 \text{ mm}$ $\qquad$ $\bar{x}_1 = 20 + \dfrac{74}{2} = 57 \text{ mm}$

Then applying the theorems of Pappus-Guldinus for the part of the surface area generated by the lines:

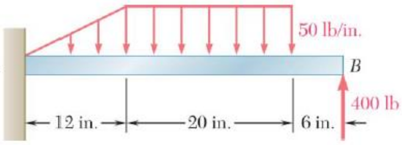
$$A_L = \pi\Sigma\bar{x}A = \pi\left[(31)(63.906) + (68)(52) + (57)(95.268)\right] = \pi\left[10947.6\right] = 34.392 \times 10^3 \text{mm}^2$$

The area of the "end triangles":

$$A_E = 2\left[\frac{1}{2}(52)(60)\right] = 3.12 \times 10^3 \text{ mm}^2$$

Total surface area is therefore:

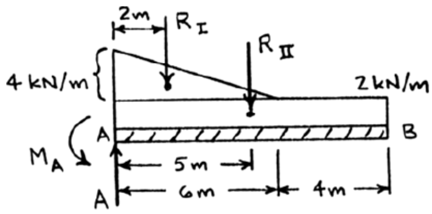
$$A = A_L + A_E = (34.392 + 3.12) \times 10^3 \text{ mm}^2 \qquad \text{or } A = 37.5 \times 10^3 \text{ mm}^2 \blacktriangleleft$$



PROBLEM 5.69

Determine the reactions at the beam supports for the given loading.

SOLUTION



$$R_{\mathrm{I}} = \frac{1}{2}(50\text{ lb/in.})(12\text{ in.})$$
$$= 300\text{ lb}$$
$$R_{\mathrm{II}} = (50\text{ lb/in.})(20\text{ in.})$$
$$= 1000\text{ lb}$$

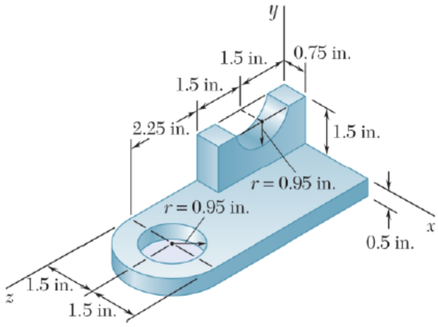
$$+\!\uparrow \Sigma F_y = 0: \quad A_y - 300\text{ lb} - 1000\text{ lb} + 400\text{ lb} = 0$$

$$\mathbf{A} = 900\text{ lb}\uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (300\text{ lb})(8\text{ in.}) - (1000\text{ lb})(22\text{ in.}) + (400\text{ lb})(38\text{ in.}) = 0$$

$$\mathbf{M}_A = 9200\text{ lb}\cdot\text{in.}\circlearrowleft \blacktriangleleft$$

PROBLEM 5.102



For the machine element shown, locate the y coordinate of the center of gravity.

SOLUTION

For half-cylindrical hole,

$$r = 0.95 \text{ in.}$$

$$\bar{y}_{\text{III}} = 1.5 - \frac{4(0.95)}{3\pi}$$

$$= 1.097 \text{ in.}$$

For half-cylindrical plate, $r = 1.5$ in.

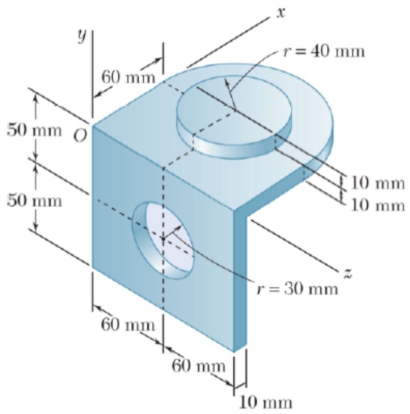
$$\bar{z}_{\text{IV}} = 5.25 + \frac{4(1.5)}{3\pi} = 5.887 \text{ in.}$$

		V, in^3	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{y}V$, in^4	$\bar{z}V$, in^4
I	Rectangular plate	$(5.25)(3)(0.5) = 7.875$	-0.25	2.625	-1.9688	20.672
II	Rectangular plate	$(3)(1.5)(0.75) = 3.375$	0.75	1.5	2.5313	5.0625
III	–(Half cylinder)	$-\frac{\pi}{2}(0.95)^2(0.75) = -1.063$	1.097	1.5	-1.1664	-1.595
IV	Half cylinder	$\frac{\pi}{2}(1.5)^2(0.5) = 1.767$	-0.25	5.887	-0.4418	10.403
V	–(Cylinder)	$-\pi(0.95)^2(0.5) = -1.418$	-0.25	5.25	0.3544	-7.443
	Σ	10.536			-0.6910	27.10

$$\bar{Y}\Sigma V = \Sigma \bar{y} V$$

$$\bar{Y}(10.536 \text{ in}^3) = -0.6910 \text{ in}^4$$

$\bar{Y} = -0.0656$ in. ◀

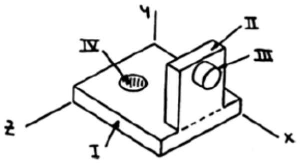


PROBLEM 5.105

For the machine element shown, locate the x coordinate of the center of gravity.

SOLUTION

First assume that the machine element is homogeneous so that its center of gravity will coincide with the centroid of the corresponding volume.



	V, mm^3	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}V$, mm^4	$\bar{y}V$, mm^4
I	$(120)(100)(10) = 120\times10^3$	5	-50	0.60×10^6	-6.00×10^6
II	$(120)(50)(10) = 60\times10^3$	35	-5	2.10×10^6	-0.30×10^6
III	$\frac{\pi}{2}(60)^2(10) = 56.549\times10^3$	85.5	-5	4.8349×10^6	-0.28274×10^6
IV	$\pi(40)^2(10) = 50.266\times10^3$	60	5	3.0160×10^6	0.25133×10^6
V	$-\pi(30)^2(10) = -28.274\times10^3$	5	-50	-0.141370×10^6	1.41370×10^6
Σ	258.54×10^3			10.4095×10^6	-4.9177×10^6

We have $$\bar{X}\Sigma V = \Sigma\bar{x}V$$

$$\bar{Y}(258.54\times10^3\text{ mm}^3) = 10.4095\times10^6\text{ mm}^4$$ or $\bar{X} = 40.3$ mm ◀