重庆大学-辛辛那提大学联合学院

学生实验报告

CQU-UC Joint Co-op Institute (JCI)

Student Experiment Report

实验课程名称 Experiment Course Name Circuit and Sensor Experiment

开课实验室（学院）Laboratory (School) College of Mechanical Engineering

学院 School Joint Co-op Institut 年级 2018 专业 ME01 班 Student Group 02

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成绩 Grade		
教师签名 Signature of Instructor		

批改说明 Marking instructions:

指导老师请用红色水笔批改，在扣分处标明所扣分数并给出相应理由，在封面的平时成绩处注明成绩。

Supervisors should mark the report with a **red ink pen**. Please write down **the points deducted** for each section when errors arise and specify the corresponding reasons. Please write down **the total grade** in the table on the cover page.

University of Cincinnati
College of Engineering & Applied Science
School of Dynamic Systems

Course Code : **20-MECH-3070C**
Course Title : **Circuits & Sensing Lab**

Experiment Number : **3**
Experiment Title : **Dynamic Response of Instruments**

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Group Code : **02**
Group Members : **Yi, Hongrui**
: **Zhuang, Yan**

Instructor : **Hu, Xiaoping**

Experiment Results	=	______/40
Analysis/Discussion	=	______/30
Answers to Questions	=	______/20
Writing	=	______/10

TOTAL SCORE	**=**	**______/100**

ABSTRACT

The objective of this laboratory experiment is to study the dynamic response of different types of test instruments suing different electrical circuits. There are some equations and theories in the experiment. We need to compare the experimental parameters with theoretical parameters. By performing, we can get profound understanding of time response and frequency response of the first order low pass and high pass systems, the time response and the frequency response of the second order lightly damped system.

Generally, there are six parts in the experiment. The first one is to study time response of first order instruments simulated as an RC circuit, get the corresponding differential equation, analyze the relationship between Vi(t) and V0(t) in certain region of frequency, and the amplitudes of output signals with increasing frequency. The second one is about the frequency response of a first order instrument simulated as an RC circuit, get the first order differential equation, analyze the gain and phase on the basis of magnitude and phase spectra of the RC frequency response function. part 3 and 4 are very similar to part 1 and 4, but replace low pass with high pass circuits. As for part 5 and 6, we investigate the time response and frequency response of a second order lightly damped instrument simulated as RLC circuit, get familiar with the data represent the characteristics of step response of an RLC circuit, and study about the response behavior and the natural frequency.

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Glossary of Terms

Terminology for First and Second Order Systems:

Static sensitivity or steady-state gain (k_{ss})

$$k_{ss} = \frac{\text{Vout}}{\text{Vin}}$$

Time constant (τ)

Static sensitivity or steady-state gain (k_{ss})

Terminology for Second Order Systems:

Static sensitivity or steady-state gain (k_{ss})

Break frequency (ω_b)

Undamped natural frequency (ω_n):

Damping ratio (ζ)

Peak time (t_p)

Percent overshoot (PO)

2% settling time (t_s,2%)

Damped period of oscillation (td)

Frequency Response Parameters: $\frac{V_o(\omega)}{V_i(\omega)}$

Frequency response function $\frac{V_o(\omega)}{V_i(\omega)} = \frac{1}{1+j\omega RC}$

Magnitude ratio $\left|\frac{V_o(\omega)}{V_i(\omega)}\right|$

Magnitude ratio function $\left|\frac{V_o(\omega)}{V_i(\omega)}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$

Phase $\Phi(\omega)$

1. Objectives

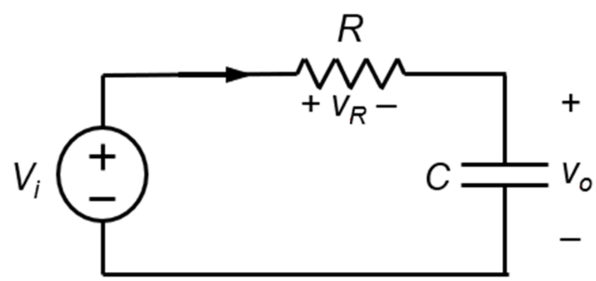
At the end of this experiment, the students are expected to:

- Study the dynamic response of different types of test instruments.
- Learn to use different electrical circuits to simulate the general characteristics of test instruments.
- Learn to represent first and second order dynamic systems by first and second order ordinary differential equations.
- Learn to use the differential equations analysis data and compare the experimental parameters of the circuits to the theoretical parameters.

2. Theoretical Background

3.1 and 3.2 Time and frequency response of low pass instrument simulated as an RC circuit:

The circuits figure is shown:



With Ohm's law $V_R = i * R$ and capacitor equation $i = C\frac{dV_o}{dt}$, the first order differential equation becomes:

$$RC\frac{dV_o}{dt} + V_o = V_i$$

After a charging time of τ, the voltage of the capacitance increases to 63.3% of the voltage of the power supply.

$$\frac{V_o}{V_i} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega CR} = \frac{1}{1 + j\frac{\omega}{\omega_b}} = \frac{1}{1 + j\frac{f}{f_b}}$$

In this function, $\omega_b = \frac{1}{RC}$

Function of magnitude of $\frac{V_o}{V_i}$

$$\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1+(\frac{f}{f_b})^2}}$$

In db form, the magnitude becomes

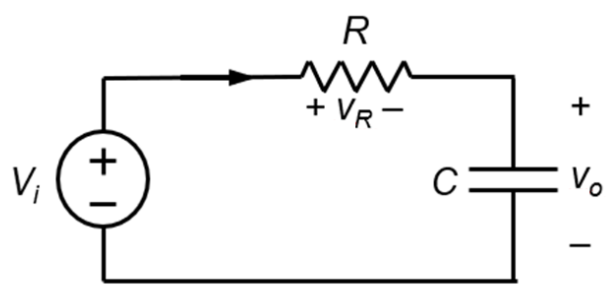
$$20log_{10}\left(\frac{V_o}{Vi}\right) = 20log_{10}(\frac{1}{\sqrt{1+(\omega RC)^2}})$$

Function of phase of $\frac{V_o}{V_i}$

$$\varphi(\omega) = \tan^{-1}\left(\frac{-\frac{j\omega RC}{1+(\omega RC)^2}}{\frac{1}{1+(\omega RC)^2}}\right) = -\tan^{-1}(\omega RC)$$

3.3 and 3.4 Time and frequency response of high pass instrument simulated as an RC circuit:

The circuits figure is shown:



With Ohm's law $V_R = i * R$ and capacitor equation $i = C\frac{dV_o}{dt}$, the first order differential equation becomes:

$$RC\frac{dV_o}{dt} + V_o = V_i$$

After a charging time of τ, the voltage of the capacitance increases to 63.3% of the voltage of the power supply.

$$\frac{V_o}{V_i} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{1+j\omega CR} = \frac{1}{1+j\frac{\omega}{\omega_b}} = \frac{1}{1+j\frac{f}{f_b}}$$

In this function, $\omega_b = \frac{1}{RC}$

Function of magnitude of $\frac{V_o}{V_i}$

$$\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1+(\frac{f}{f_b})^2}}$$

In db form, the magnitude goes like following.

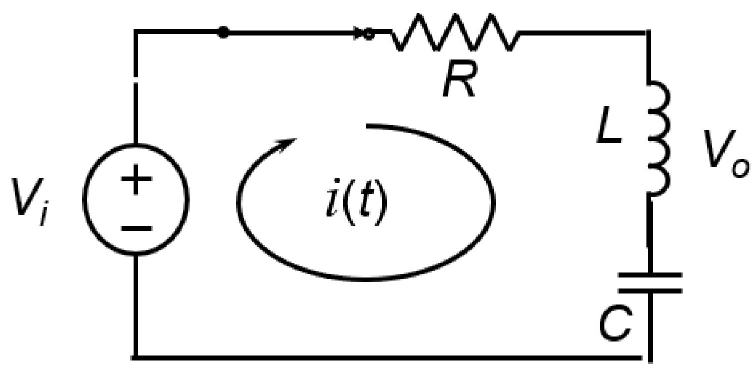
$$20log_{10}\left(\frac{V_o}{Vi}\right) = 20log_{10}(\frac{1}{\sqrt{1+(\omega RC)^2}})$$

Function of phase of $\frac{V_o}{V_i}$

$$\varphi(\omega) = \tan^{-1}\left(\frac{-\frac{j\omega RC}{1+(\omega RC)^2}}{\frac{1}{1+(\omega RC)^2}}\right) = -\tan^{-1}(\omega RC)$$

3.5 and 3.6 Time and frequency response of second order lightly damped instrument-simulated as an RLC circuit:

The circuits figure is shown:



Using KVL, Ohm's law, equation of capacitor and inductor,

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

Roots of the characteristic equation:

$$r_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$r_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Suppose $\alpha = \frac{R}{2L}$ and $\beta = \frac{1}{\sqrt{LC}}$

$$r_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \beta^2}$$

Damping factor:

$$\zeta = \frac{\alpha}{\beta} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

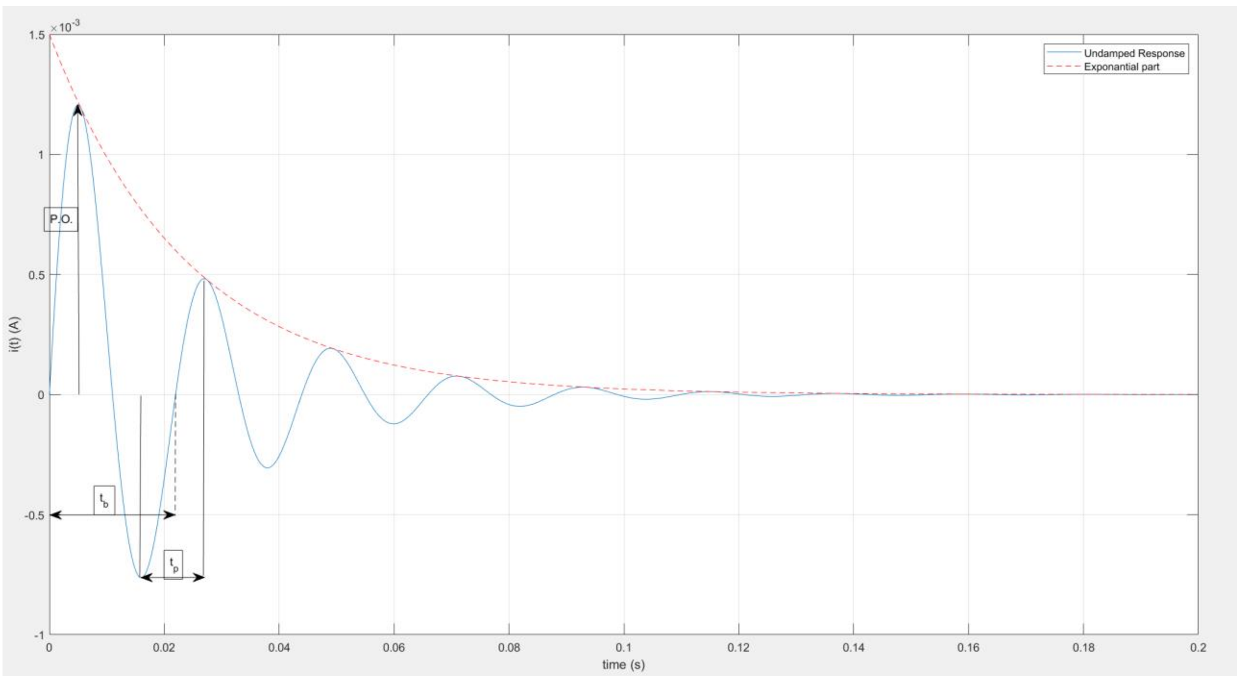
Undamped natural frequency:

$$\omega_n = \beta = \frac{1}{\sqrt{LC}}$$

Oscillation frequency:

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

The ideal graph of the undamped system:



t_p is peak time. $P.O$ is percent overshoot. t_d is damping period.

3. Experimentation

3.1 Time response of a first order (low pass) instrument simulated as an RC circuit

3.1.1 Summary of Procedure

1. After measuring the actual values of R and C, record the values in Table 1, calculate the theoretical values of time constant and static sensitivity with the measured actual values and fill in Table 2.

2. Set RC circuit of figure 1 on the experiment board.

3. Use function generator and oscilloscope to measure step input and response. Determine the time constant τ and static sensitivity Kss according to the step response diagram and record them in Table 2. Draw step response diagram on Figure 2, and indicate the basic information needed to determine τ and Kss on the sketch, such as amplitude, voltage level when τ = 1, τ position, etc.

4. Use function generator to generate square wave and triangle wave in turn, and draw sketch in the table

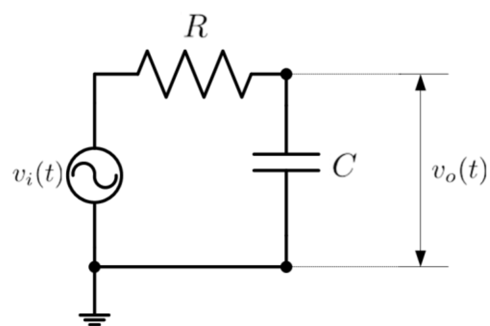


Figure 1. A low pass RC circuit

3.1.2 Results

Put your results here. Use the tables provided and re-encode your data.

Table 1: RC circuit component values

Component	Nominal Value	Measured Value	% Difference
R *(kΩ)*	100	99.91	0.09%

C *(μF)*	0.0333	0.0333	0

Table 2: Low pass circuit parameters

Component	Computed Value	Measured Value	% Error
τ *(ms)*	3.327	3.20	3.8%
k_{ss}	1	0.915	8.5%

Derivation:

The first order differential equation:

$$RC\frac{dV_o}{dt} + V_o = V_i$$

$$\tau\frac{dV_o}{dt} + V_o = V_i$$

Solution:

$$V_o(t) = V_i - V_i e^{-\frac{t}{RC}}$$

Then:

$$\frac{V_o}{V_i} = \frac{1}{1 + (\omega RC)^2} - \frac{j\omega RC}{1 + (\omega RC)^2} \Rightarrow \left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

Sample Computation:

$$\tau = RC = 99910 \times 0.0333 \times 10^{-6} = 3.327$$

$$\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (100 \times 9991 \times 0.0333 \times 10^{-6})^2}} = 0.915$$

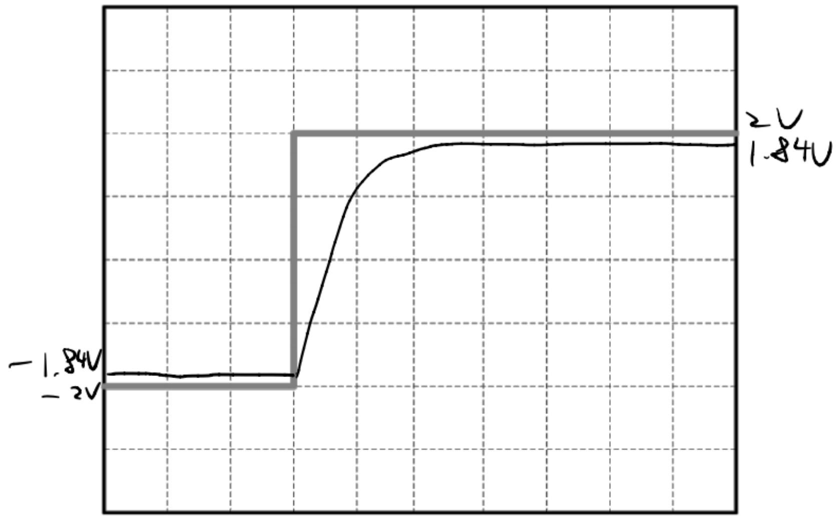


Figure 2. Step response of a low-pass RC circuit

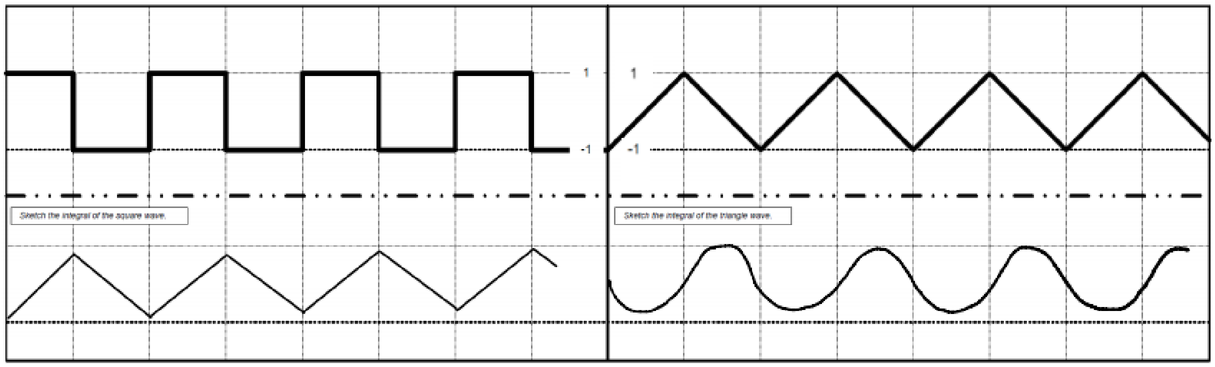


Figure 3. Piece-wise integral of sample periodic functions

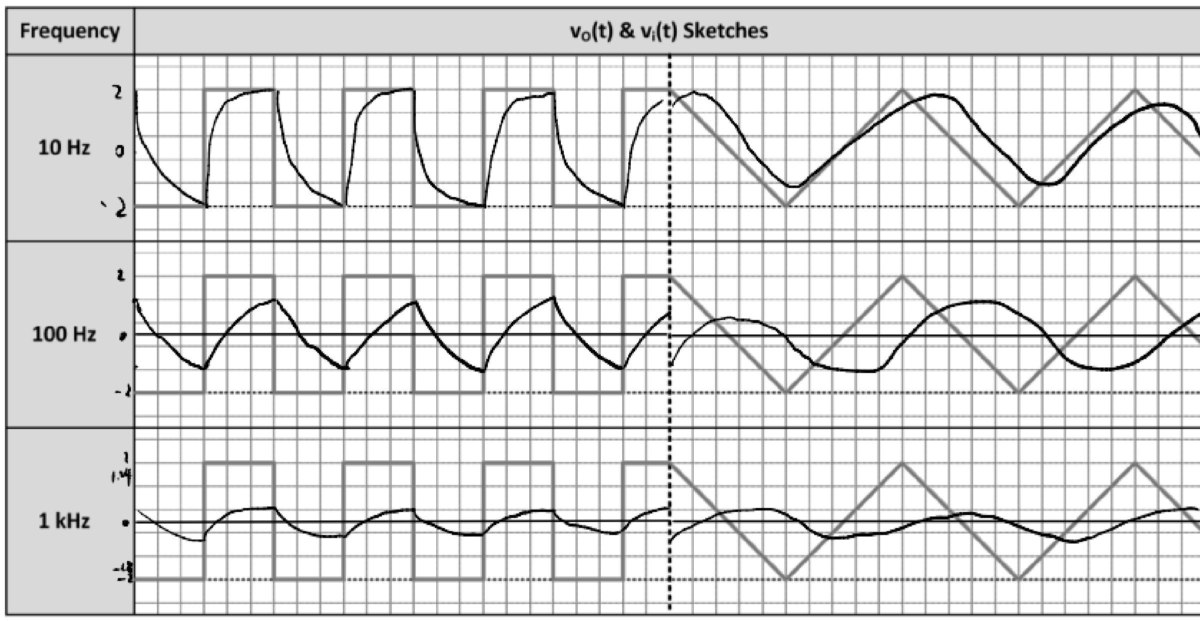


Figure 4. Output waveforms from a low pass RC circuit

3.1.3 Analysis & Discussion

1. The difference between the measured value of the capacitor and the nominal value is 9.09%, and the difference between the measured value of the resistor and the nominal value is 0.200%. In this case, the long-time power on causes the temperature of resistance and capacitance to rise, resulting in small errors.

2. Then in Table 2, we use two methods to approach the time constant τ. The first method is to use τ = RC for the calculated value of 3.00ms, and then measure the actual value of 2.994ms. The data here is very close. The error may be caused by wire resistance, wire aging or even unstable reading of oscilloscope data.

3. Then in the fourth figure, the relationship between the output voltage and the input voltage under two different waveforms at different frequencies is shown. It is easy to find that as the frequency amplitude increases, the output signal amplitude becomes smaller and smaller. Its shape has also become flatter. This is because at high frequencies, the impedance of the capacitor becomes smaller, so if the frequency is very high, the capacitor can be regarded as a wire. Make the output signal almost zero.

3.2 Frequency response of a first order (low pass) instrument simulated as an RC circuit

3.2.1 Summary of Procedure

1. Set the first-order low-pass RC circuit in Section 3.1 (see Figure 1). Record the actual values of R and C in Table 3. Calculate the break frequency (ωb) and the static sensitivity (Kss) with actual value of R and C instead of the nominal value.

2. Write down the expression for the frequency response function for this system in terms of R and C. Use MATLAB to compute the frequency response function $\frac{v_0(w)}{V_i(w)}$ using the actual values of R and C (Table 3) for this experiment, where Vo(ω) is the Fourier transform of the response signal and Vi(ω) is the Fourier transform of the input signal. Generate the magnitude ($|\frac{v_0(w)}{V_i(w)}|$) and phase ($\Phi(\omega)$) spectra of this low pass RC frequency response function.

3. Apply a 4 Vpp, 10, 20, 40, 60, 80, 120, 160, 240, 320, 640Hz sinusoidal input from the function generators, then display the input and output signals versus time on the oscilloscope. Measure and record the magnitude ratio and phase of v0(t) relative to vi(t).

4. Use Matlab to plot the figure of the measured value and theoretical value of gain and phase, then compare these values.

3.2.2 Results

Table 3: RC circuit component values

Component	Nominal Value	Measured Value	% Difference
R *(kΩ)*	100	99.91	0.09
C *(μF)*	0.0333	0.0333	0

Table 4: Low pass FRF parameters from Matlab

Parameter	Computed Value	Compu Value
ω_b *(rad/sec)*	300.3003	314.9052
k_{ss}	1	0.915

Table 5: Low pass filter FRF parameters for select frequencies

Frequency (Hz)	V_i (V)	V_o (V)	**Gain**, $V_o/V_i\|_{dB}$ (dB)	**Calculated Gain**, (dB)	**Time Delay**, t_d (ms)	**Phase Angle**, θ (deg, °)	**Calculate Phase,** θ (deg, °)
10	*3.96*	*3.56*	*-0.9249*	*-0.1858*	*0.0046*	*-11.2*	*-11.8072*
20	*3.96*	*3.36*	*-1.4271*	*-0.6996*	*0.003*	*-22.0*	*-22.6890*
40	*3.96*	*2.80*	*-3.0107*	*-2.3024*	*0.0028*	*-40.0*	*-39.9013*
50	*3.96*	*2.56*	*-3.7891*	*-3.2066*	*0.0028*	*-45.3*	*-46.2663*
60	*3.96*	*2.28*	*-4.7952*	*-4.1046*	*0.0024*	*-50.5*	*-51.4350*
80	*3.96*	*1.88*	*-6.4707*	*-5.7941*	*0.002*	*-59.9*	*-59.1220*
120	*3.96*	*1.48*	*-9.2476*	*-8.6288*	*0.0014*	*-68*	*-68.2656*
160	*3.96*	*1.08*	*-11.2854*	*-10.8589*	*0.0012*	*-74.0*	*-73.3542*
240	*4.00*	*0.776*	*-14.1567*	*-14.1781*	*960μs*	*-78.1*	*-78.7274*
320	*4.00*	*0.6*	*-16.4782*	*-16.6036*	*740μs*	*-81.1*	*-81.4977*
640	*4.00*	*0.3*	*-22.4988*	*-22.5525*	*375μs*	*-85.1*	*-85.7253*

Derivation:

$$RC\frac{dV_o}{dt} + V_o = V_i$$

$$\tau\frac{dV_o}{dt} + V_o = V_i$$

Solution:

$$V_o(t) = V_i - V_i e^{-\frac{t}{RC}}$$

Then:

$$\frac{V_o}{V_i} = \frac{1}{1+(\omega RC)^2} - \frac{j\omega RC}{1+(\omega RC)^2} \Rightarrow \left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\varphi(\omega) = \tan^{-1}\left(\frac{-\frac{j\omega RC}{1+(\omega RC)^2}}{\frac{1}{1+(\omega RC)^2}}\right) = -\tan^{-1}(\omega RC)$$

The gain is:

$$20log_{10}\left(\frac{V_o}{Vi}\right) = 20log_{10}\left(\frac{1}{\sqrt{1+(\omega RC)^2}}\right)$$

Sample computation:

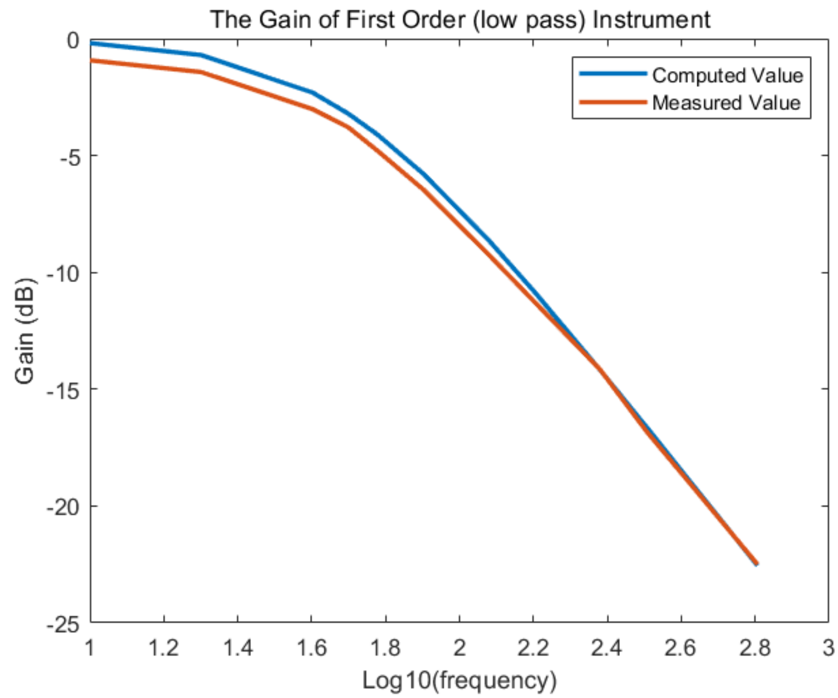
For calculated gain:

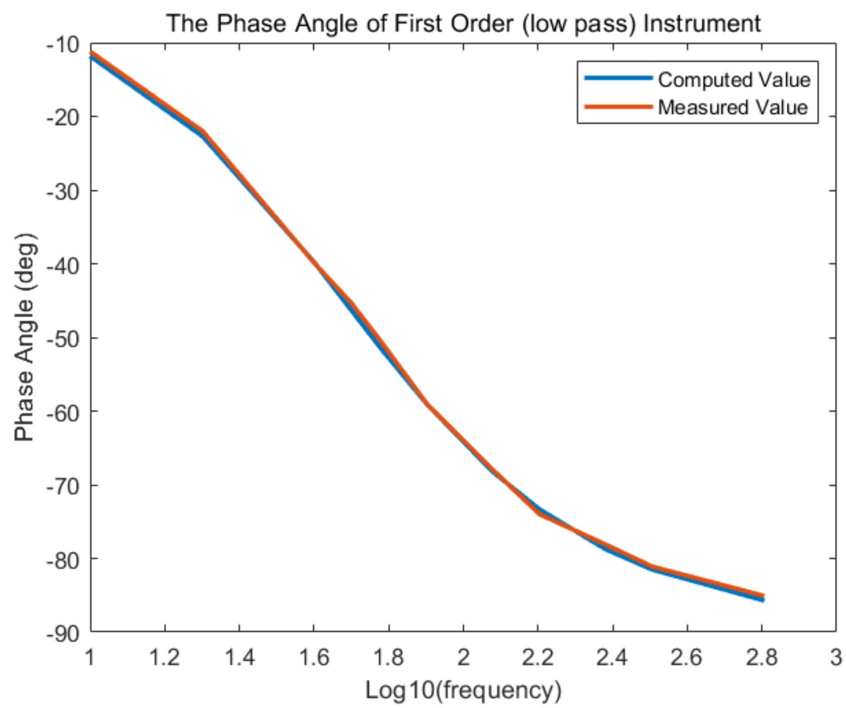
$gain1 = 20log_{10}(\frac{1}{\sqrt{1+(\omega RC)^2}}) = 20log_{10}(\frac{1}{\sqrt{1+(2*\mathrm{pi}*10*99910*0.0333\times10^{-6})^2}}) = -0.1858$

For measured gain:

$$gain1 = 20 \times log_{10}\left(\frac{3.56}{3.96}\right) = -0.9249$$

$$\varphi = -\tan^{-1}(\omega RC) = -\tan^{-1}(2 \times pi \times 10 \times 99910 \times 0.0333 \times 10^{-6}) = -11.2$$


The Gain of First Order (low pass) Instrument
Computed Value
Measured Value
Gain (dB)
Log10(frequency)
0
-5
-10
-15
-20
-25
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3


The Phase Angle of First Order (low pass) Instrument
Computed Value
Measured Value
Phase Angle (deg)
Log10(frequency)
-10
-20
-30
-40
-50
-60
-70
-80
-90
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3

3.2.3 Analysis & Discussion

The results show that the gain and phase angle decrease with the increase of frequency. Moreover, as the frequency increases, the rate of decline slows down. When the frequency increases to infinity, the phase angle converges to - 90 °.

For determining the break frequency (ωb) of experimental value, locate Φ(w)=-45°, because at break frequency the $\frac{V0(t)}{Vi(t)}=\frac{1}{\sqrt{1+(\omega Rc)^2}}=\frac{1}{\sqrt{2}}=0.707$, the ωRc =1, then $\Phi(w)=-tan-1(\omega Rc)=-45°$.

The corresponding break frequency is 334.0. It is close to the theoretical value.

The Kss is 0.97, which is also close to the theoretical value.

For gain value for this part of the experiment has a quite difference from the theoretical value. That may be caused by the long time charging to the power source, which make the temperature of the resistor increase. As the temperature of the resistor increases, the resistance of the resistor would increase. Form the equation which is $gain = 20 \times log_{10}\left(\frac{V_o}{Vi}\right)$, the Vo would increase because of the temperature increasing. Then let the gain will be larger than the theoretical value.

Another reason of this error which is a bit larger than the normal one. The value of the error output signal may due to the unstableness of the oscilloscope output because of some interferences.

3.3 Time response of a first order (high pass) instrument simulated as an RC circuit

3.3.1 Summary of Procedure

1. Measure the actual values of R and C, record the values in Table 6, and set the high pass RC circuit, as shown in Figure 5.

2. Use function generator and oscilloscope to measure step response.

3. Figure 7 shows the four periods of a general square wave. Just below it, draw the derivative of the square wave. Repeat the same procedure for triangular waves.

4. The square wave is applied to the circuit as Vi by using the function generator of Elvis system. Observe the frequency of Vi and Vo with oscilloscope as 10 Hz, 100 Hz and 1 kHz. Draw the observed Vo on figure 8.

5. Repeat step 4 for the triangular wave.

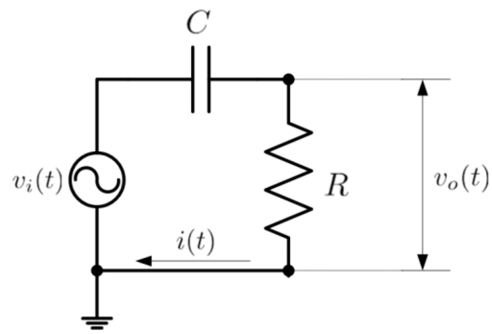


Figure 5. A high pass RC circuit

3.3.2 Results

Table 6: RC circuit component values

Component	Nominal Value	Measured Value	% Difference
R *(kΩ)*	100	99.91	0.09
C *(μF)*	0.0333	0.0333	0

Table 7: High pass circuit parameters

Component	Computed Value	Measured Value	% Error
$\boldsymbol{\tau}$ *(ms)*	3.327	3.4	2.19
$\boldsymbol{k_{ss}}$	0	0.127	∞

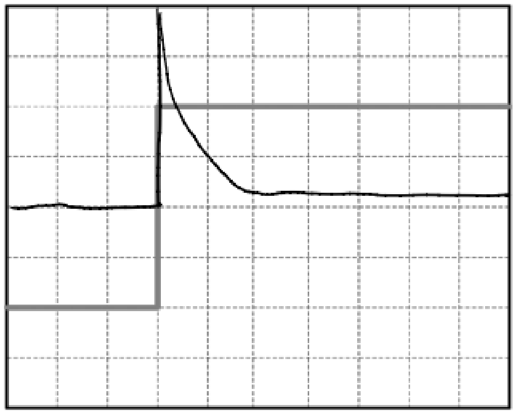


Figure 6. Step response of a high-pass RC circuit

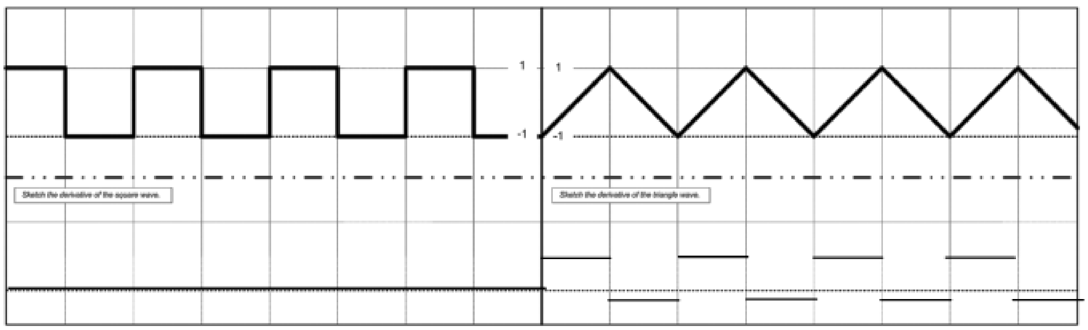


Figure 7. Piece-wise first order derivative of sample periodic functions

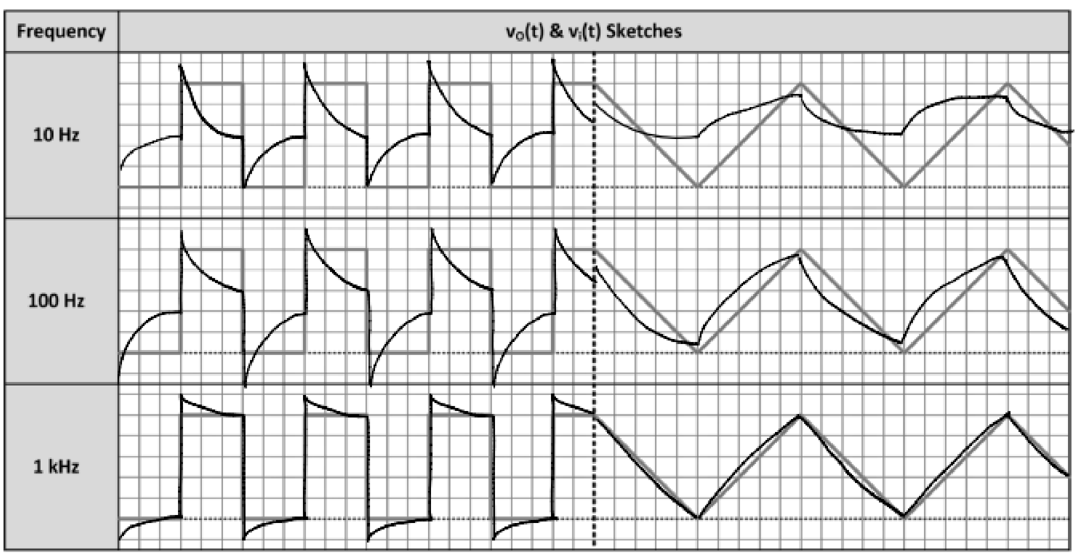


Figure 8. Output waveforms from a high pass RC circuit

Derivation:

$$RC\frac{dV_o}{dt} + V_o = V_i$$

$$V_o(t) = V_i - V_i e^{-\frac{t}{RC}}$$

$$\frac{V_o}{V_i} = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega RC * (1 - j\omega RC)}{(1 + j\omega RC) * (1 - j\omega RC)}$$

$$\frac{V_o}{V_i} = \frac{(\omega RC)^2}{1 + (\omega RC)^2} + \frac{j\omega RC}{1 + (\omega RC)^2}$$

$$\Rightarrow |\frac{V_o}{V_i}| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

Sample computation:

$$\tau = RC = 99910 \times 0.0333 \times 10^{-6} = 3.327ms$$

$$\left|\frac{V_o}{V_i}\right| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}} = \frac{100 \times 99910 \times 0.0333 \times 10^{-6}}{\sqrt{1 + (100 \times 99910 \times 0.0333 \times 10^{-6})^2}} = 0.316$$

3.3.3 Analysis & Discussion

The calculated value of τ is equal to R × C.

The equation of the output voltage signal is:

$$\frac{V_o}{V_i} = \frac{(\omega RC)^2}{1 + (\omega RC)^2} + \frac{j\omega RC}{1 + (\omega RC)^2} \Rightarrow |\frac{V_o}{V_i}| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

$$V_o(t) = V_i(t)e^{\frac{-t}{RC}}$$

It can be seen from the above formula that in the step response of high pass circuit, with the increase of t, Vo finally tends to zero. In theory, the value of Kss is equal to zero.

The higher the frequency, the larger the amplitude of square wave output voltage and triangle wave output voltage. The reason is the time limit. The higher the frequency, the shorter the period of the input signal. If the period is too short for the capacitor to fully complete the discharge process, it will be charged due to the reverse input signal. In other words, due to the charging of the capacitor, the output signal of the resistance starts to drop when it is not close to zero.

3.4 Frequency response of a first order (high pass) instrument simulated as an RC circuit

3.4.1 Summary of Procedure

1. Set the first-order high pass RC circuit of the experiment 3.3 (see Figure 5). Record the actual values of R and C in Table 8. Calculate the break frequency (ω b) and the static sensitivity (Kss).

2. Write down the frequency response function of the system with R and C. Use Matlab to calculate the frequency response function of the experiment, with the actual values of R and C (Table 8) instead of the nominal values.

3. Input 4 Vpp, 10, 20, 40, 50, 60, 80, 120, 160, 240, 320, 640 Hz sine signal in turns from signal function generator.

4. Record the peak to peak voltage of Vo and Vi, time delay (td) and the phase angle (θ)of the input and output signals, which are on the oscilloscope. Calculate the gain value through the equation: $gain = 20log_{10}\left(\frac{V_o}{Vi}\right) = 20log_{10}(\frac{\omega RC}{\sqrt{1+(\omega RC)^2}})$

5. Use Matlab to plot the figure of the measured value and theoretical value of gain and phase, then compare these values.

3.4.2 Results

Table 8: RC circuit component values

Component	Nominal Value	Measured Value	% Difference
R *(kΩ)*	100	99.91	0.09
C *(μF)*	0.0333	0.0333	0

Table 9: High pass FRF parameters from Matlab

Parameter	Computed Value	Experimental Value
ω_b *(rad/sec)*	300.3003	300
k_{ss}	0	0.05

Derivation:

$$RC\frac{dV_o}{dt} + V_o = V_i$$

$$V_o(t) = V_i - V_i e^{-\frac{t}{RC}}$$

$$\frac{V_o}{V_i} = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega RC * (1 - j\omega RC)}{(1 + j\omega RC) * (1 - j\omega RC)}$$

$$\frac{V_o}{V_i} = \frac{(\omega RC)^2}{1 + (\omega RC)^2} + \frac{j\omega RC}{1 + (\omega RC)^2}$$

$$\Rightarrow \left|\frac{V_o}{V_i}\right| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

$$(\omega) = \tan^{-1}\left(\frac{\frac{j\omega RC}{1 + (\omega RC)^2}}{\frac{(\omega RC)^2}{1 + (\omega RC)^2}}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Table 10: High pass filter FRF parameters for select frequencies

Frequency (Hz)	**V_i** (V)	**V_o** (V)	**Gain,** $V_o/V_i\vert_{dB}$ (dB)	**Calculated Gain,** (dB)	**Time Delay,** t_d (ms)	**Phase Angle,** θ (deg, °)	**Calculate Phase,** θ (deg, °)
10	*3.96*	*0.86*	*-13.2639*	*-13.6173*	*21.9*	*79.2*	*78.1825*
20	*3.96*	*1.36*	*-9.2831*	*-8.1291*	*9.78*	*68.4*	*67.2926*
40	*4.00*	*2.32*	*-4.7314*	*-3.7566*	*3.70*	*52.5*	*50.0734*
50	*3.96*	*2.43*	*-4.2418*	*-2.7416*	*3.02*	*46.2*	*43.7079*
60	*3.96*	*2.89*	*-2.7359*	*-2.0712*	*2.47*	*40.5*	*38.5398*
80	*3.82*	*3.10*	*-1.8140*	*-1.2832*	*1.42*	*33.1*	*30.8553*
120	*4.00*	*3.48*	*-1.2096*	*-0.6175*	*0.556*	*22.3*	*21.7167*
160	*3.96*	*3.60*	*-0.8279*	*-0.3580*	*0.274*	*16.9*	*16.6316*
240	*3.96*	*3.72*	*-0.5430*	*-0.1628*	*0.124*	*11.2*	*11.2627*
320	*3.78*	*3.76*	*-0.0461*	*-0.0923*	*0.0845*	*7.9*	*8.4947*
640	*3.82*	*3.80*	*-0.0456*	*-0.0233*	*0.0205*	*4.2*	*4.2708*

Sample computation:

For calculated gain:

$$gain1 = 20log_{10}\left(\frac{\omega RC}{\sqrt{1+(\omega RC)^2}}\right) = 20log_{10}\left(\frac{2*\text{pi}*10*99910*0.0333\times10^{-6}}{\sqrt{1+(2*\text{pi}*10*99910*0.0333\times10^{-6})^2}}\right) = 13.6173$$

For measured gain:

$$gain1 = 20 \times log_{10}\left(\frac{0.86}{3.96}\right) = -13.2639$$

$$\varphi = \tan^{-1}(\frac{1}{\omega RC}) = \tan^{-1}\left(\frac{1}{2 \times pi \times 10 \times 99910 \times 0.0333 \times 10^{-6}}\right) = 78.1825$$

3.4.3 Analysis & Discussion

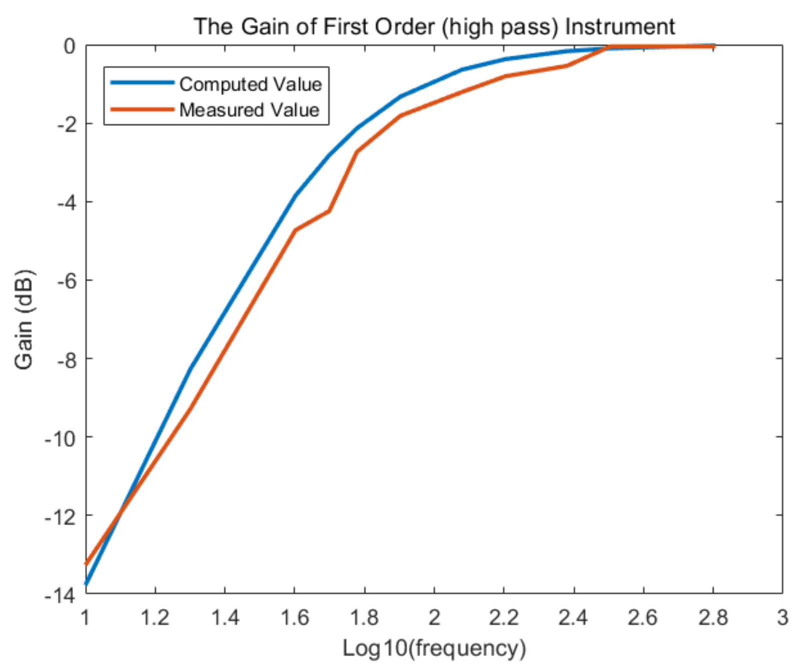
The results show that the gain increase and phase angle decrease with the increase of frequency. Moreover, as the frequency increases, the rate of both the gain and the phase slows down. When the frequency increases to infinity, the phase angle converges to 0 °.

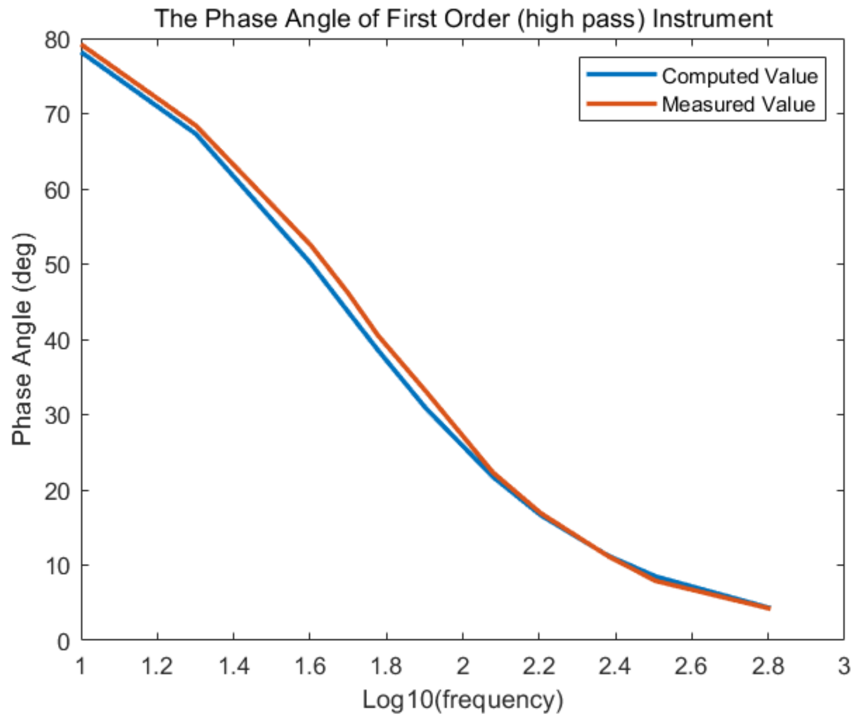
For determining the break frequency (ωb) of experimental value, locate Φ(w)=-45°, because at break frequency the $\frac{V0(t)}{Vi(t)}$=$\frac{\omega RC}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{2}} = 0.707$, the ωRc =1, then $\Phi(w)$=tan−1($\frac{1}{\omega Rc}$)=45°.

The corresponding break frequency is 334.0. It is close to the theoretical value.

The Kss is 0.057, which is also close to the theoretical value.

For phase value for this part of the experiment has a quite difference from the theoretical value when the frequency close to the end of the experiment. That may be caused by the long time charging to the power source, which make the temperature of the resistor increase, then the unstableness of the oscilloscope output because of some interferences occur.





3.5 Time response of a second order lightly damped instrument simulated as an RLC circuit

3.5.1 Summary of Procedure

1. Measure the actual inductance L of the coil (nominal value is 0.47 H), then the trimming resistor is connected in series with the inductance to form RL + R. Adjust the trimmer until the series RL + R reads 1 k or the value RL + R of around 1 k. Measure the actual value of the capacitor (nominal value C = 0.01 μ f). Record the nominal and measured values of RLC circuit elements in Table 11.

2. Set RLC circuit on the experimental board, as shown in Figure 9. Second order differential equations related to V0 and Vi are written in standard form, which represent output (or response) and input signals, respectively. The nominal values of RL + R, L and C are used to estimate the undamped natural frequency ωn, damping ratio ζ and static sensitivity Kss. Record the calculated values in Table 12.

3. Use function generator and oscilloscope (Elvis oscilloscope) to measure step response. Sketch Vo in Figure 10. According to the time response diagram, the peak time Tp, overshoot percentage

P.O, 2% stable time Ts, 2%, oscillation damping period TD and steady-state gain Kss are determined. Record the measurements in table 13.

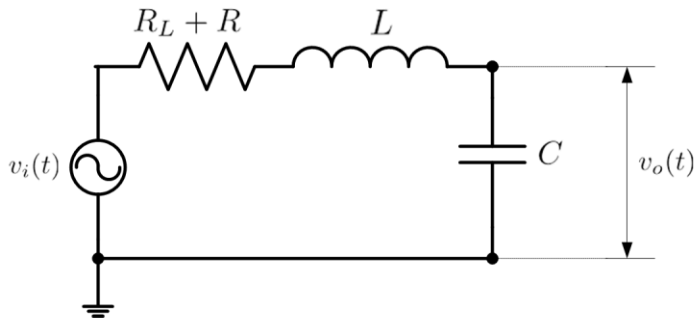


Figure 9. An RLC circuit

3.5.2 Results

Table 11: RLC circuit component values

Component	Nominal Value	Measured Value	% Difference
RL + R *(kΩ)*	*1*	*1*	*0*
L *(H)*	*0.47*	*0.464*	*1.28%*
C *(μF)*	*0.01*	*0.01*	*0*

Table 12: RLC circuit parameters - Theoretical

Parameter	Computed Value
ω_n *(rad/sec)*	14680.5
ω_d *(rad/sec)*	14640.9
ζ	0.0734
k_{ss}	1

Table 13: High pass FRF parameters from Matlab

Parameter	Computed Value
ω_n *(rad/sec)*	14680.5
ω_d *(rad/sec)*	14640.9
ζ	0.1565
k_{ss}	1.1

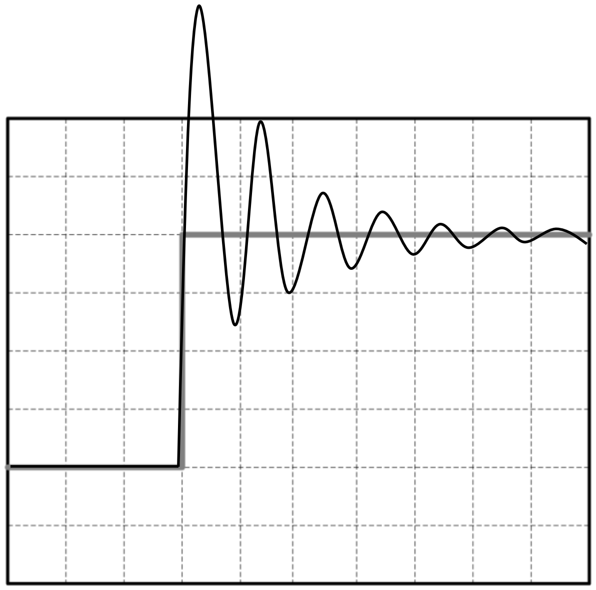


Figure 10. Step response of an RLC circuit

Table 14: RLC circuit time response parameters - Measured

Parameter	Computed Value
t_p (μs)	230
PO	75.2%
$t_{s,2\%}$ (ms)	3.33
t_d (μs)	443
k_{ss}	1.13

Derivation:

$$Ri + L\frac{di}{dt} + V_C = V_i$$

$$\Rightarrow R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{dV_C}{dt} = \frac{dV_i}{dt}$$

$$\Rightarrow \frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

$$r_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$r_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$i(t) = A_1e^{r_1t} + A_2e^{r_2t}$$

Sample computation:

$$\omega_n = \beta = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.464 \times 0.01 \times 10^{-6}}} = 14680.5$$

$$\omega_d(Computed) = \sqrt{\alpha - \beta} = \beta\sqrt{1 - \left(\frac{\alpha}{\beta}\right)^2} = \frac{1}{\sqrt{LC}}\frac{R^2C}{2^2L}$$

$$= \frac{1}{\sqrt{0.464 \times 0.01 \times 10^{-6}}}\frac{1000^2 \times 0.01 \times 10^{-6}}{2^2 \times 0.464} = 14640.9$$

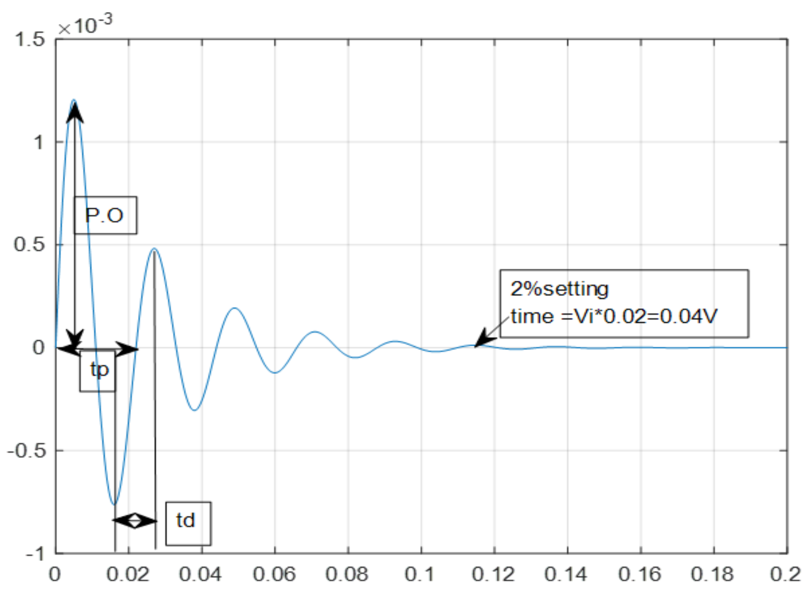
$$\zeta(Theoretical) = \frac{\alpha}{\beta} = \frac{R}{2}\sqrt{\frac{C}{L}} = \frac{1000}{2} \times \sqrt{\frac{0.01 \times 10^{-6}}{0.464}} = 0.0734$$

$$\omega_d(Theoretical) = \omega_n\sqrt{1-\zeta^2} = 14680.5 \times \sqrt{1 - 0.0734^2} = 14640.9$$

$$\zeta(Computed) = \sqrt{1 - \left(\frac{\omega_d}{\omega_n}\right)^2} = 0.0742$$

3.5.3 Analysis & Discussion

The ωd equation of this experiment is: $\omega_d = \omega_n\sqrt{1-\zeta^2}$. The ωn equation is: $\omega_n = \frac{1}{\sqrt{LC}}$.



The ζ equation is: $\zeta = \frac{\alpha}{\beta} = \frac{R}{2}\sqrt{\frac{C}{L}}$. The value of the ωd must use ω_n to calculate.

Difference if the ω_d, ζ and k_{ss} all may due to the waveform vibration of oscilloscope. The unstableness of the display input and output voltage signals can cause a big error in the experiment. In the experiment, we try to calculate the computed values then base on the computed values to confirm the measured values. Or the error just because of the loss of the trimming resistor due to the time passing.

t_p is peak time.

$P.O$ is percent overshoot.

t_d is damping period.

3.6 Frequency response of a second order lightly damped instrument simulated as an RLC circuit

3.6.1 Summary of Procedure

1. Measure the magnitude of the ***R_L* + *R*, *L*, *C*** and calculate the difference between the nominal value and the measured value, then put the values into the table14.

2. Set up the RLC circuit of part 2.5 (the graph circuit in part 3.4and 3.6section of the Theoretical Background).

3. Set the sinusoidal input signal with frequency of 50, 400, 800, 1200, 1600, 2000, 2322, 3000, 4000, 5000Hz in turn, check the image of the power input signal and the output signal at both ends of the inductor on the oscilloscope.

4. Record the peak value, time delay and phase difference of the two signals.

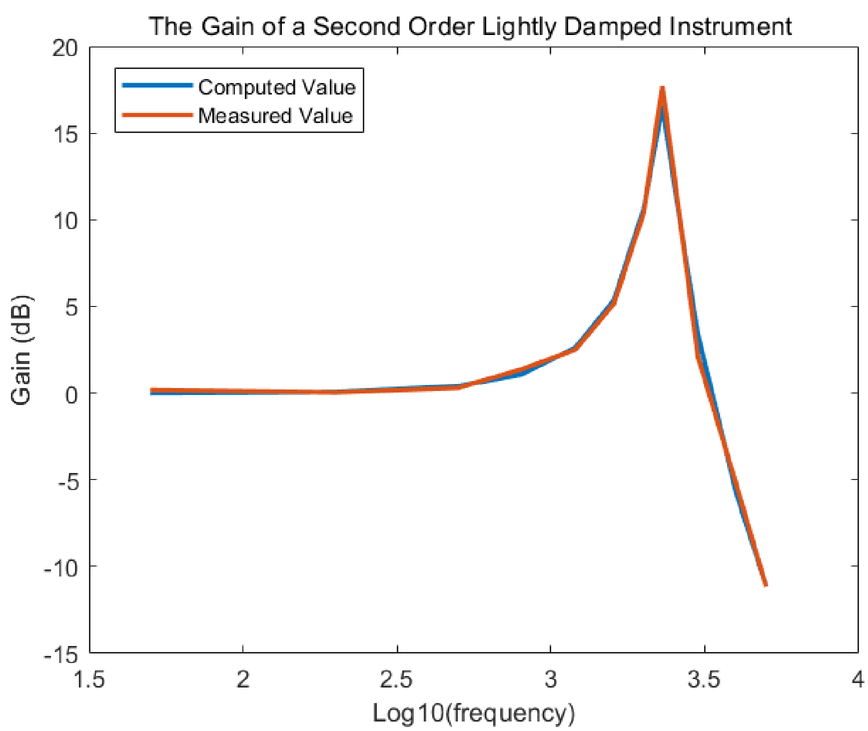
3.6.2 Results

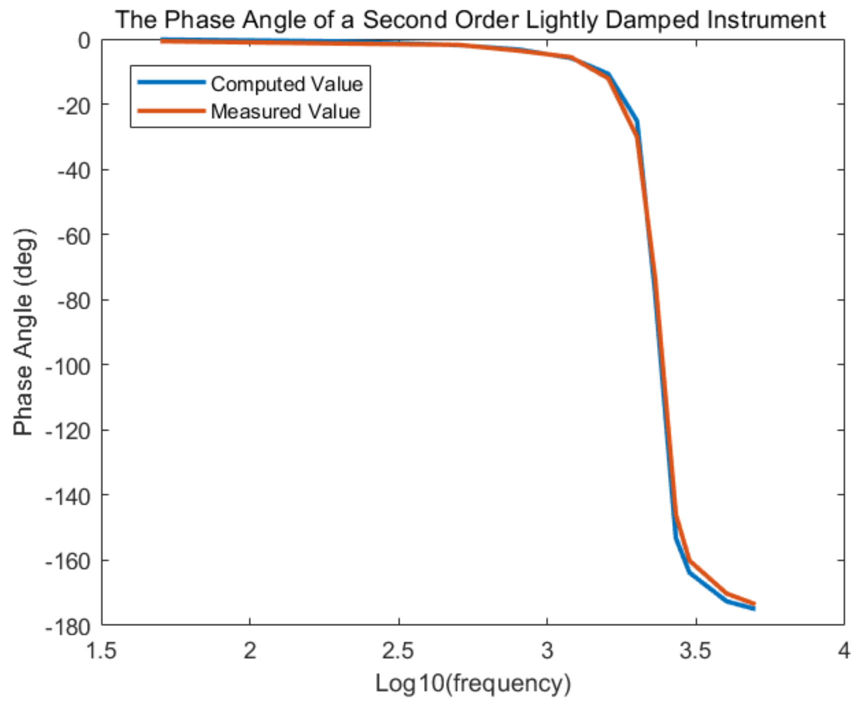
Table 15: RLC circuit component values

Component	Nominal Value	Measured Value	% Difference
R_L* + *R *(kΩ)*	1	1	0
L *(H)*	0.47	0.464	1.28%
C *(μF)*	0.01	0.01	*0*

Table 15 Second Order Lightly Damped Instrument FRF parameters for select frequencies

Frequency (Hz)	V_i (V)	V_o (V)	**Gain,** $V_o/V_i\vert_{dB}$ (dB)	**Time Delay,** t_d (msec)	**Phase Angle,** θ (deg, °)
50	*2*	*2.02*	*0.086*	3.889×10^{-5}	*-0.7*
200	*2*	*2.08*	*0.0632*	6.000×10^{-5}	*-1.4*
500	*2*	*2.16*	*0.4025*	8.000×10^{-5}	*-1.8*
800	*2*	*2.26*	*1.061*	1.250×10^{-5}	*-3.6*
1200	*2*	*2.70*	*2.607*	1.296×10^{-5}	*-5.6*
1600	*2*	*3.66*	*5.249*	2.101×10^{-5}	*-12.1*
2000	*2*	*6.60*	*10.370*	4.181×10^{-5}	*-30.1*
2300	*2*	*11.6*	*16.6067*	0.920×10^{-4}	*-73.5*
2700	*2*	*5.72*	*8.4998*	1.400×10^{-4}	*-140*
3000	*2*	*2.90*	*3.227*	1.482×10^{-4}	*-160.1*
4000	*2*	*1.04*	*-5.680*	1.182×10^{-4}	*-170.2*
5000	*2*	*0.56*	*-11.057*	9.639×10^{-5}	*-173.5*





3.6.3 Analysis & Discussion

Sample computation:

For measured gain:

$$gain1 = 20 \times log_{10}\left(\frac{2.02}{2.00}\right) = 0.086$$

4. Answers/Solutions to Questions

1. Use the data from part 2.5 step (c) to determine the undamped natural frequency ωn and the damping ratio ζ for the RLC circuit. Compare these results with the analytic solution.

Parameter	Experimental Value
t_p (μs)	230
P.O	75.2%
$t_{s.2\%}$ (ms)	3.33
t_d (μs)	443
k_{ss}	1.13

Thus,

$$f = \frac{1}{t_d} = 2257.33Hz$$

$$\omega = 2\pi f = 14183.26rad/sec$$

$$\zeta = \frac{2\pi RC}{2t_d} = 0.0725$$

Analytic:

$$\omega_n = \sqrt{\frac{1}{LC}} = 14586.5rad/sec$$

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}} = 0.0729$$

2. Why do we care about the frequency response of a system? That is, what useful information is obtained from the frequency response?

Frequency response refers to the relationship between output signals and input signals of different frequencies in the system. It is used to characterize the dynamics of the system, which will reflect special features of the system. Thus, it is important.

The frequency of the output signal is equal to the input signal. Moreover, for a linear system, doubling the amplitude of the input will double the amplitude of the output. In addition, if the system is time-invariant, then the frequency response also will not vary with time.

3. Compare the frequency response measured in part 2.6 with the MATLAB prediction.

5. Conclusions

Having performed the experiment, and after a thorough analysis of the data, the following points are therefore concluded:

1) Part 3.1 is the time response of the first order (low pass) system. The differential equation is $V_0 = V_i(1 - e^{\frac{-t}{Rc}})$. The relationship between V_i and V_0 is a good approximation of the relative high frequency signal. The higher the frequency is, the smaller the amplitude of the output voltage is.

2) Part 3.2 is the frequency response of a first order (low pass) system, the differential equation is $\frac{\mathrm{V0(w)}}{\mathrm{Vi(w)}} = \frac{1}{\sqrt{1+(wRC)^2}}$. The gain and phase decrease with the increasing frequency. The process for determining the break frequency the $\frac{\mathrm{V0(t)}}{\mathrm{Vi(t)}} = \frac{1}{\sqrt{1+(\omega\mathrm{Rc})^2}} = \frac{1}{\sqrt{2}} = 0.707$,then $\Phi(w)=-tan^{-1}(\omega\mathrm{Rc})=-45°$. Computing the break frequency, $\omega_b=\frac{1}{\mathrm{RC}}$. The gain and phase decrease with the increasing frequency, which verified the characteristic of the low pass characteristic.

3) Part 3.3 is the time response of the first order (high pass) system. The differential equation is $V_0=V_i\, e^{\frac{-t}{Rc}}$. The higher the frequency, the higher the amplitude of the output voltage. The relationship between Vi and Vo is a good approximation to the relative low frequency signal differentiator. The higher the frequency, the higher the amplitude of the output voltage.

4) Part 3.4 is experiment of frequency response of a first order (high pass) system, the differential equation is $\frac{\mathrm{V0(w)}}{\mathrm{Vi(w)}} = \frac{wRC}{\sqrt{1+(wRC)^2}}$, With the increasing frequency, the gain increase while the phase decrease. As the frequency increases, the gain increases while the phase decreases, which verified the characteristic the high pass characteristic.

5) Part 3.5 is experiment of time response of a second order lightly damped system, many data from the figure illustrates the step response of an under damped system.

6) Part 3.6 is experiment of the frequency response of a second order lightly damped system, study about the resonance behavior and get the natural frequency from the figure when the output reaches the peak value. We can see the resonance behavior and learn how to calculate the natural frequency from the figure when the output reaches the peak value.

APPENDICES

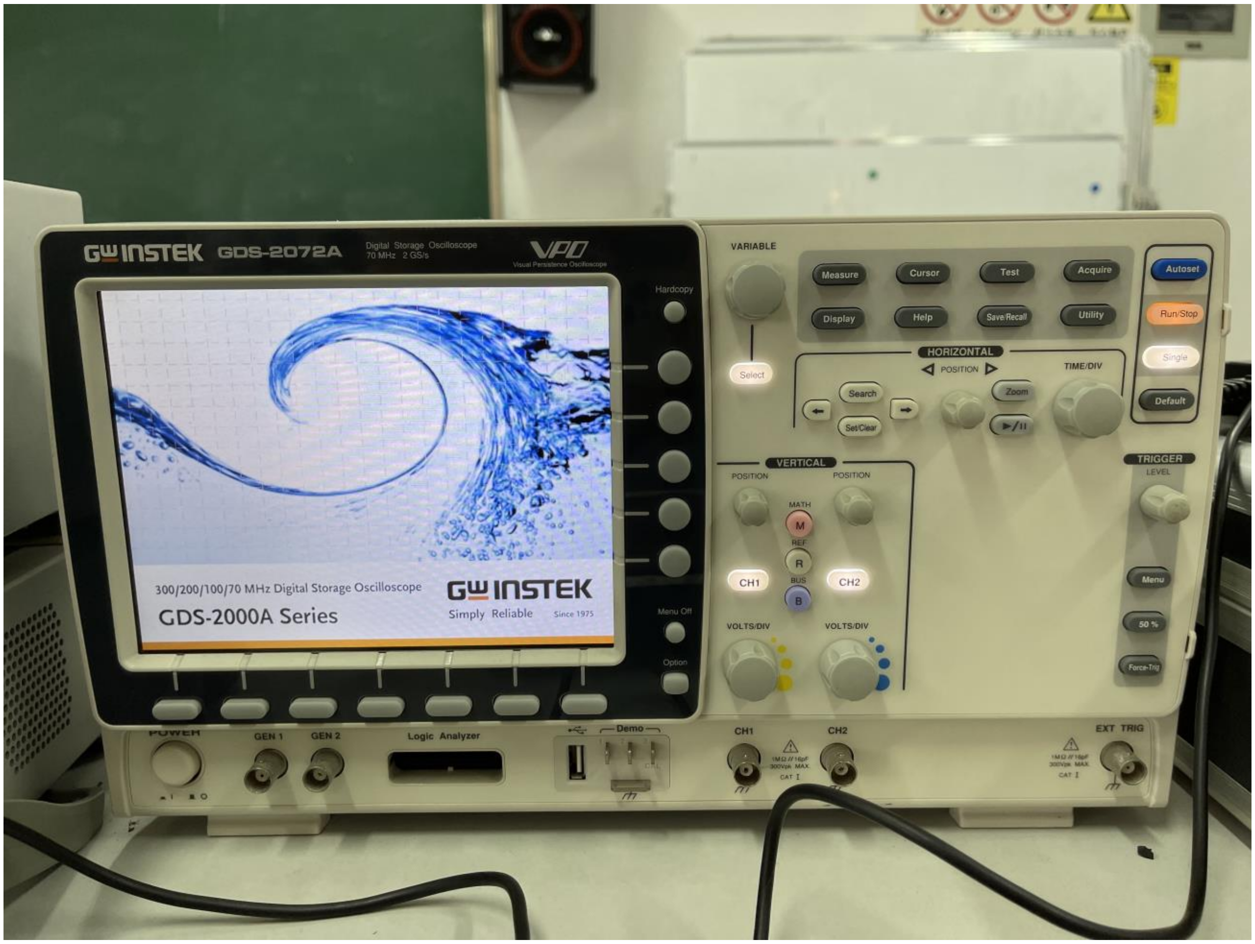
A – EQUIPMENT LIST

Table 12. Equipment List

Equipment Description
LINI-T Multimeter
VICI 6243 Digital Capacitance Inductance Multimeter
RIGOL DG1022U Function Waveform Generator
GWINSTEK GDS-2072A Digital Storage Oscilloscope(70 MHZ 2Gs/s)

VICI 6243 Digital Capacitance Inductance Multimeter   LINI-T Multimeter

GDS-2072A Digital Storage Oscilloscope



RIGOL DG1022U Function Waveform Generator

B – Lab Notes

Group Number			Names			Date	
Expt. Number	3		Expt. Title	Dynamic Response of Instruments			

Part 1: Time response of a first order (low pass) instrument simulated as an RC circuit

Table 1: RC circuit component values

Component	Nominal Value	Measured Value	% Difference
R ($k\Omega$)	100	99.91	0.09
C (μF)	0.0333	0.0333	0

Table 2: Low pass circuit parameters

Component	Computed Value	Measured Value	% Error
τ (ms)	RC = 3.327	3.20	3.8
k_{ss}	1	0.915	8.5

Derivations:

First order differential equation in standard form relating $v_o(t)$ and $v_i(t)$:

$$RC \cdot \frac{dV_c}{dt} + V_o(t) = V_i(t)$$

$$V_o(t) = V_i\left(1 - e^{-\frac{t}{\tau}}\right)$$

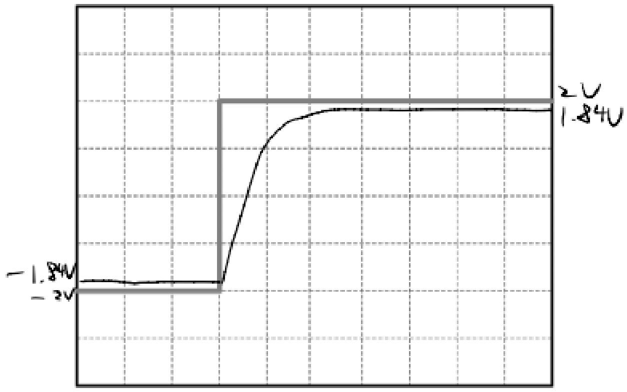


Figure 2. Step response of a low-pass RC circuit

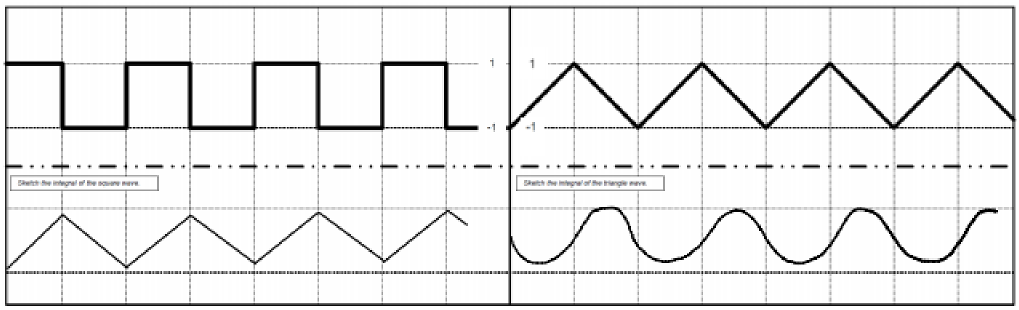


Figure 3. Piece-wise integral of sample periodic functions

Figure 4. Output waveforms from a low pass RC circuit

Expt. Number	3	Expt. Title	Basic Instruments

Part 2: Frequency response of a first order (low pass) instrument simulated as an RC circuit

Table 3: RC circuit component values

Component	Nominal Value	Measured Value	% Difference
R ($k\Omega$)	100	99.91	0.09
C (μF)	0.0333	0.0333	0

Table 4: Low pass FRF parameters from MATLAB

Parameter	Computed Value	Experimental Value
ω_b (rad/sec)	300.3003	314.9052
k_{ss}	1	0.915

Derivations:

Frequency response function $\frac{V_o(\omega)}{V_i(\omega)}$:

$$\frac{V_o(\omega)}{V_i(\omega)} = \frac{1}{\sqrt{1+(\omega CR)^2}}$$

Table 5: Low pass filter FRF parameters for select frequencies

Frequency (Hz)	V_i (V)	V_o (V)	Gain, $V_o/V_i\|_{dB}$ (dB)	Time Delay, t_d (sec)	Phase Angle, θ (deg, °)	Cal. G	Cal θ
10	3.96	3.56	−0.9249	4.6×10^{-3}	−11.2	−0.1858	−11.8072
20	3.96	3.36	−1.4271	3×10^{-3}	−22°	−0.6996	−21.6890
40	3.96	2.80	−3.0107	2.8×10^{-3}	−40.0°	−2.3024	−39.9013
80	3.96	1.88	−6.4707	2.0×10^{-3}	−59.1°	−5.7941	−59.1260
160	3.96	1.08	−11.2854	1.2×10^{-3}	−74.0°	−10.8699	−73.3844
320	4.00	0.6	−16.4782	740×10^{-6}	−81.1°	−16.6836	−81.0911
640	4.00	0.3	−22.4988	375×10^{-6}	−85.1°	−22.6625	−85.7953

Sample Computations:

50	3.96	2.56	−3.7891	2.8×10^{-3}	−45.3°	−3.2066	−46.663
60	3.96	2.28	−4.7952	2.4×10^{-3}	−50.5°	−4.1346	−51.4550
120	3.96	1.48	−9.2476	1.4×10^{-3}	−68	−8.6488	−68.5636
240	3.96	0.776	−14.1567	960×10^{-6}	−78.1°	−14.1781	−78.7314

Part 3: Time response of a first order (high pass) instrument simulated as an RC circuit

Table 6: RC circuit component values

Component	Nominal Value	Measured Value	% Difference
R *(kΩ)*	100	99.91	0.09
C *(μF)*	0.0333	0.0333	0

Table 7: High pass circuit parameters

Component	Computed Value	Measured Value	% Error
τ *(ms)*	3.327	3.4	2.19
k_{ss}	0	0.107	∞

Derivations:

First order differential equation in standard form relating $v_o(t)$ and $v_i(t)$:

$$RC \cdot \frac{dV_c}{dt} + V_o(t) = V_i(t)$$

$$V_o(t) = V_i e^{-\frac{t}{RC}}$$

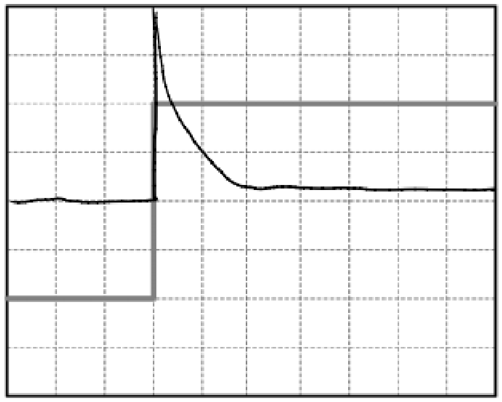


Figure 6. Step response of a high-pass RC circuit

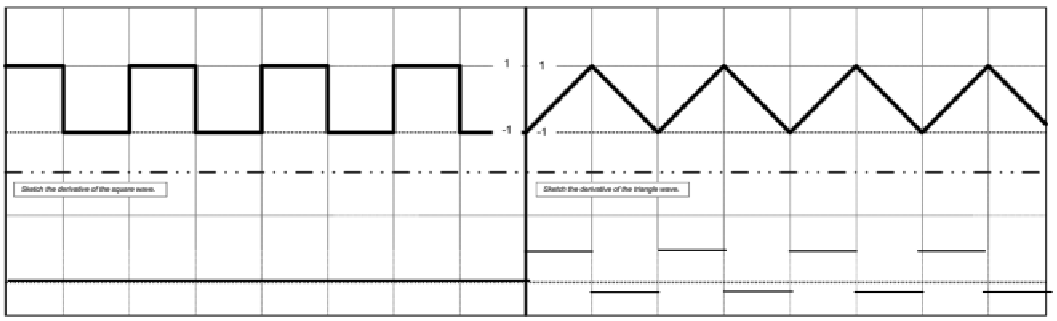


Figure 7. Piece-wise first order derivative of sample periodic functions

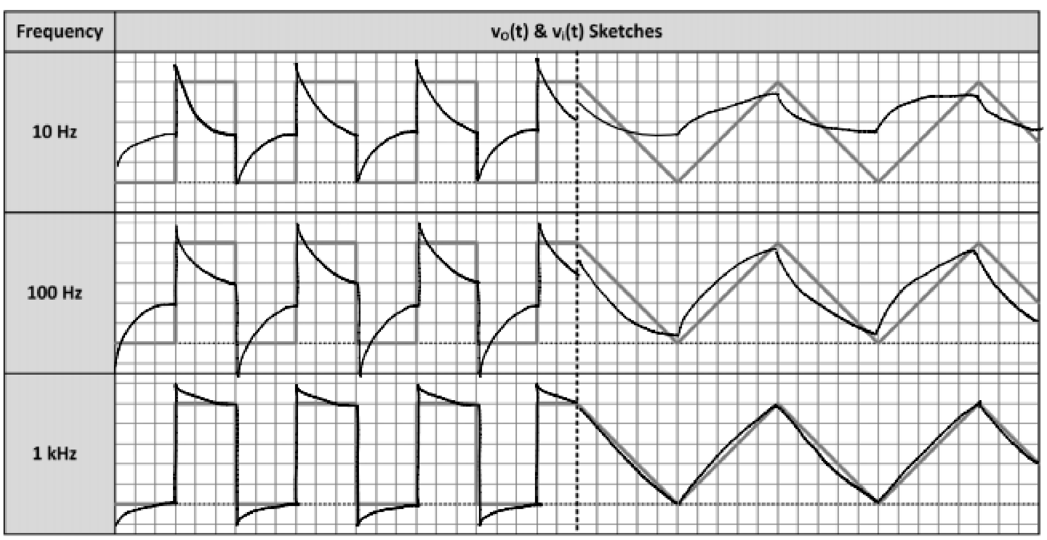


Figure 8. Output waveforms from a high pass RC circuit

Expt. Number	3	Expt. Title	Basic Instruments

Part 4: Frequency response of a first order (high pass) instrument simulated as an RC circuit

Table 8: RC circuit component values

Component	Nominal Value	Measured Value	% Difference
R *(kΩ)*	100	99.91	0.09
C *(μF)*	0.0333	0.0333	0

Table 9: High pass FRF parameters from MATLAB

Parameter	Computed Value	Experimental Value
ω_b *(rad/sec)*	300.3003	300
k_{ss}	0	0.05

Derivations:

Frequency response function $\frac{V_o(\omega)}{V_i(\omega)}$:

$$\frac{V_o(\omega)}{V_i(\omega)} = \frac{\omega c R}{\sqrt{1+(\omega c R)^2}}$$

Table 10: High pass filter FRF parameters for select frequencies

Frequency (Hz)	V_i (V)	V_o (V)	Gain, $V_o/V_i\vert_{dB}$ (dB)	Time Delay, t_d (sec)	Phase Angle, θ (deg, °)	Cal. G	Cal θ.
10	3.96	0.86	-13.2639	21.9	79.2	-13.6173	78.1925
20	3.96	1.36	-9.2831	9.78	68.4	-8.1391	67.2696
40	4.00	2.32	-4.7314	3.70	52.5	-3.7566	50.0734
80	3.82	3.10	-1.8140	1.42	33.1	-1.2832	30.8555
160	3.96	3.60	-0.8279	0.274	16.9	-0.3589	16.6116
320	3.78	3.76	-0.0461	0.0845	7.9	-0.0923	8.4897
640	3.82	3.80	-0.0456	0.0205	4.2	-0.0233	4.2703
50	3.96	2.43	-4.2418	3.02	46.2	-2.7416	43.7779
60	3.96	2.89	-2.7359	2.47	40.5	-2.2712	38.0398
120	4.00	3.48	-1.2096	0.586	22.3	-0.6175	21.717
240	3.96	3.72	-0.5430	0.124	11.2	-0.1628	11.261

Sample Computations:

Expt. Number	3	Expt. Title	**Basic Instruments**

Part 5: Time response of a second order lightly damped instrument – simulated as an RLC circuit

Table 11: RLC circuit component values

Component	Nominal Value	Measured Value	% Difference
$R_L + R$ $(k\Omega)$	1	1	0
L (H)	0.47	0.464	1.28%
C (μF)	0.01	0.01	0%

Derivations:

Second order differential equation in standard form relating $v_o(t)$ and $v_i(t)$:

$$V_i(t) = LC\frac{d^2V}{dt^2} + RC\frac{dV_o}{dt} + V_o$$

Table 12: RLC circuit parameters - Theoretical

Parameter	Computed Value
ω_n (rad/sec)	1468.5
ζ	0.0734
k_{ss}	1
ω_d	14640.9

Sample Computations:

Table 15: RLC circuit parameters - Measured

Parameter	Measured Value
ω_n (rad/sec)	1468.5
ζ	0.0742
k_{ss}	1.1
ω_d	14640.P

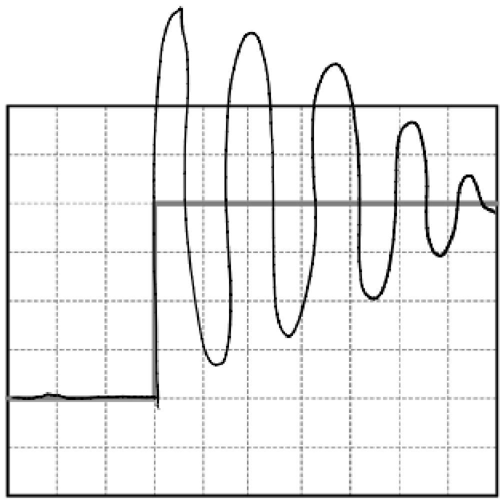


Figure 10. Step response of an RLC circuit

Table 13: RLC circuit time response parameters - Measured

Parameter	Experimental Value
t_p (μs)	230
PO	75.2%
$t_{s,2\%}$ (ms)	3.33
t_d (μs)	443
k_{ss}	1.13

Expt. Number	3		Expt. Title	Dynamic Response of Instruments

Part 6: Frequency response of a second order lightly damped instrument simulated as an RLC circuit

Table 14: RLC circuit component values

Component	Nominal Value	Measured Value	% Difference
$R_L + R$ ($k\Omega$)	1	1	0
L (H)	0.47	0.464	1.28%
C (μF)	0.01	0.01	0%

Derivations:

Frequency response function $\frac{V_o(\omega)}{V_i(\omega)}$:

$$\frac{V_o}{V_i} = \frac{1}{-LC\omega^2 + j\omega RC + 1}$$

Calculated θ: −0.1801 −0.7253 −1.8857 −3.2580 −5.8474 −10.7184
−25.1898 −77.8008 −133.1683 −163.7856 −171.5838 −174.8842

Calculated G: 0.0058 0.0632 0.4325 1.0680 2.6145 5.3442
10.5833 16.6067 8.4888 3.4081 −5.7881 −11.

Sample Computations:

Frequency (Hz)	V_i (V)	V_o (V)	Gain, $V_o/V_i\|_{dB}$ (dB)	Time Delay, t_d (sec)	Phase Angle, θ (deg, °)
10 50	2	2.02	0.176	3.888×10^{-5}	−0.7
20 200	2	2.08	0.042	6×10^{-5}	−1.0
40 500	2	2.16	0.374	8×10^{-5}	−1.8
80 800	2	2.26	1.361	1.250×10^{-5}	−3.6
160 1200	2	2.70	2.537	1.286×10^{-5}	−5.6
320 1600	2	3.66	5.149	2.101×10^{-5}	−12.1
640 2000	2	6.60	10.370	4.18×10^{-5}	−30.1
2300	2	11.6	17.34	2.92×10^{-4}	−73.8°
[illegible]	2	[illegible]	[illegible]	4×10^{-4}	−146
3000	2	2.80	2.127	1.482×10^{-4}	−160.1
4000	2	1.34	−3.280	1.180×10^{-4}	−170.2
5000	2	0.56	−11.157	9.638×10^{-5}	−173.5

TA initials: ____ Date: ____

Lab Notes for Experiment 3 – **Dynamic Response of Instruments** (revised by ____) Page 9 of

C – Matlab Code

```
%3.2
clc; clear all;close all;
f=[10 20 40 50 60 80 120 160 240 320 640];
G=[-0.9249 -1.4271 -3.0107 -3.7891 -4.7952 -6.4707 -9.2476 -11.2854 -
14.1567 -16.7891 -22.4988];
theta=[-11.2 -22 -40 -45.3 -50.5 -59.1 -68 -74 -78.1 -81.1 -85.1];
R=99910;
C=0.0333*10^-6;
Vr=1./(1+(2*pi*f*R*C).^2).^0.5;
Vdb=20*log10(Vr);
phi=-atan(2*pi*f*R*C)*180/pi;
Wc=1/R/C;
f=log10(f);
figure;

plot(f,Vdb, "LineWidth", 2);
hold on;
title('The Gain of First Order (low pass) Instrument')
plot(f,G, "LineWidth", 2);
legend("Computed Value", "Measured Value")
xlabel("Log10(frequency)")
ylabel("Gain (dB)")
figure;

plot(f,phi, "LineWidth", 2);
hold on;
title('The Phase Angle of First Order (low pass) Instrument')
plot(f,theta, "LineWidth", 2);
legend("Computed Value", "Measured Value")
xlabel("Log10(frequency)")
ylabel("Phase Angle (deg)")
disp(Vdb);
disp(Wc);
disp(phi);

%3.4
```

```
clc; clear all;close all;
f=[10 20 40 50 60 80 120 160 240 320 640];
G=[-13.2639 -9.2831 -4.7314 -4.2418 -2.7359 -1.8140 -1.2096 -0.8079 -
0.5430 -0.0461 -0.0456];
theta=[79.2 68.4 52.5 46.2 40.5 33.1 22.3 16.9 11.2 7.9 4.2];
Vdb=20*log10((2*pi*f*0.00333)./(1+(2*pi*f*0.00333).^2).^0.5);
phi=atan(1./(2*pi*f*0.00333))*180/pi;
f=log10(f);
plot(f,Vdb, "LineWidth", 2);
hold on;
title('The Gain of First Order (high pass) Instrument')
plot(f,G, "LineWidth", 2);
legend("Computed Value", "Measured Value")
xlabel("Log10(frequency)")
ylabel("Gain (dB)")
figure;
plot(f,phi, "LineWidth", 2);
hold on;
title('The Phase Angle of First Order (high pass) Instrument')
plot(f,theta, "LineWidth", 2);
legend("Computed Value", "Measured Value")
xlabel("Log10(frequency)")
ylabel("Phase Angle (deg)")
disp(Vdb);
disp(phi);

%3.6
clc; clear all;close all;
f = [50 200 500 800 1200 1600 2000 2300 2700 3000 4000 5000];
R = 1000; C=0.01e-6; L=0.464;
vovir = 1./(-L*C*(2*pi*f).^2+(1j)*2*pi*f*R*C+1);
mag = abs(vovir);
vovirdB = 20*log10(abs(vovir));
vovirdB_ = [0.176 0.0422 0.3074 1.361 2.507 5.149 10.37 17.7043 8.3998
2.127 -5.280 -11.157];
aglvovir = angle(vovir)*180/pi;
f = log10(f);
```

```
figure;
plot(f, vovirdB, "LineWidth", 2);
hold on
title('The Gain of a Second Order Lightly Damped Instrument ')
plot(f, vovirdB_, "LineWidth", 2)
legend("Computed Value", "Measured Value")
xlabel("Log10(frequency)")
ylabel("Gain (dB)")

f_ = [50 200 500 800 1200 1600 2000 2300 2700 3000 4000 5000];
w_r = 2*pi*f_;
phase_r = 1./(1+((R+w_r*L*1i).*(w_r*C*1i)));
phase_r = angle(phase_r)/pi*180;
f_ = log10(f_);
theta=-[0.7 1.4 1.8 3.6 5.6 12.1 30.1 73.5 146 160.1 170.2 173.5];

figure;
plot(f_,phase_r, "LineWidth", 2);
hold on;
title('The Phase Angle of a Second Order Lightly Damped Instrument ')
plot(f,theta, "LineWidth", 2);
legend("Computed Value", "Measured Value")
xlabel("Log10(frequency)")
ylabel("Phase Angle (deg)")
```