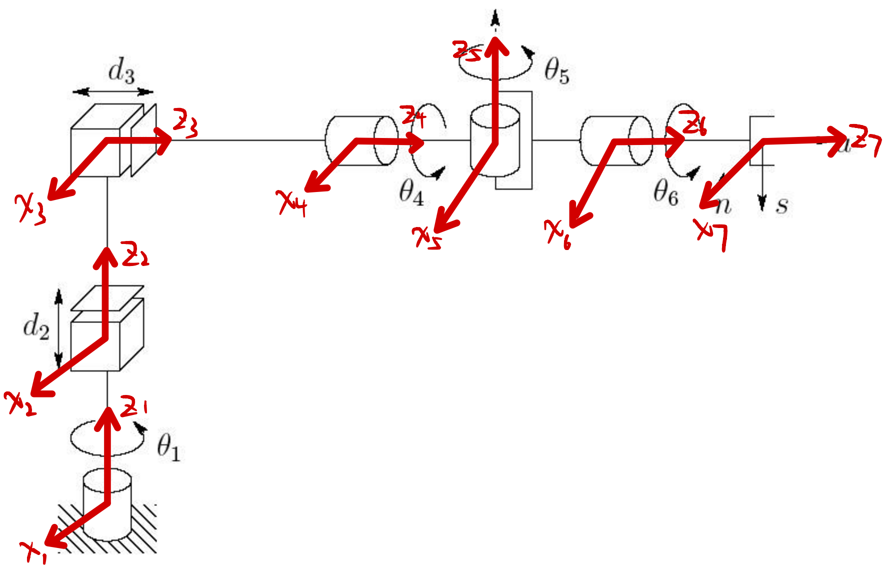
20 MECH5131 Intro to Robotics
HW#9-Forward Kinematics-Position Analysis (80 pts)

Prob 1. (12 pts)
For the given 6 DOF cylindrical robot below, assign appropriate frames for Joint 1 through 6 (assign *x* and *z* axis only, not *y* axis) based on the D-H representation.



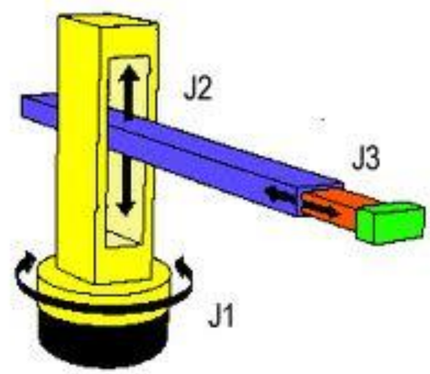


Figure 3.9: Cylindrical robot with spherical wrist.

Prob 2. (24 pts)

Consider the two-link Cartesian manipulator of figure below,

a) Assign the link frames for the two joints and the end effector. (6 pts)

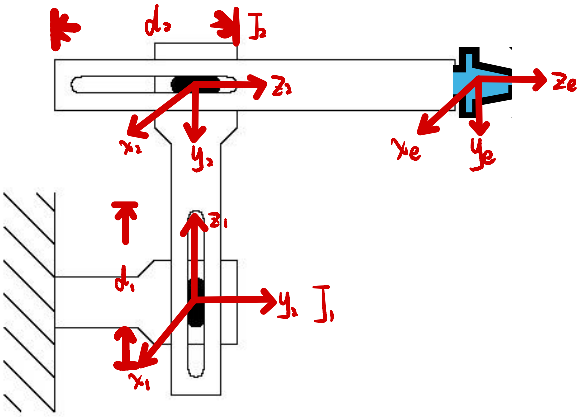
b) Create and fill out D-H parameters table. (4 pts)

c) Find the homogenous transformation matrices (A1 and A2 matrices) for two joints. (6 pts)

d) Find the direct kinematic equation (T matrix). (3 pts)

e) Find the position of the end effector in the base (first) frame when $d_1 = 1$ *ft*, $d_2 = 2$ *ft*, and illustrate this position in the figure. (5 pts)

a)



b)

Link	θ	d	a	α
0-1	0	d_1	0	-90°
1-2	0	d_2	0	0

c) $A_1 = Trans(0,0,d_1)\,Rot(x_1, -90^\circ)$

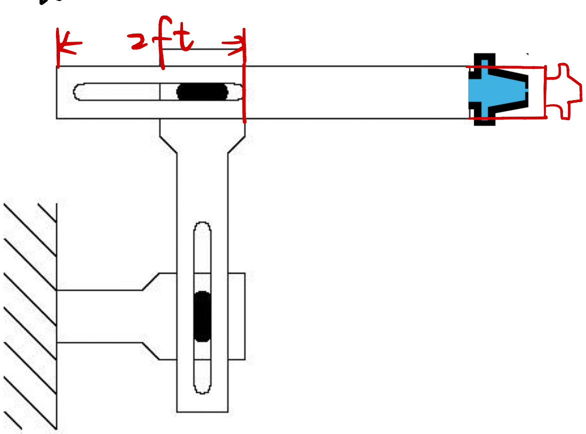
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos -90^\circ & -\sin -90^\circ & 0 \\ 0 & \sin -90^\circ & \cos -90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = Trans(0,0,d_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) $$T_2^0 = A_1 A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

e) $$T_2^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



Prob 3. (22 pts)

Consider the two-link manipulator of figure below, which has joint 1 linear and joint 2 revolute with link lengths $l_1=6''$ and $l_2=12''$.

a) Assign the frames for two joints and end effector. (z_0 is assigned for you) (4 pts)

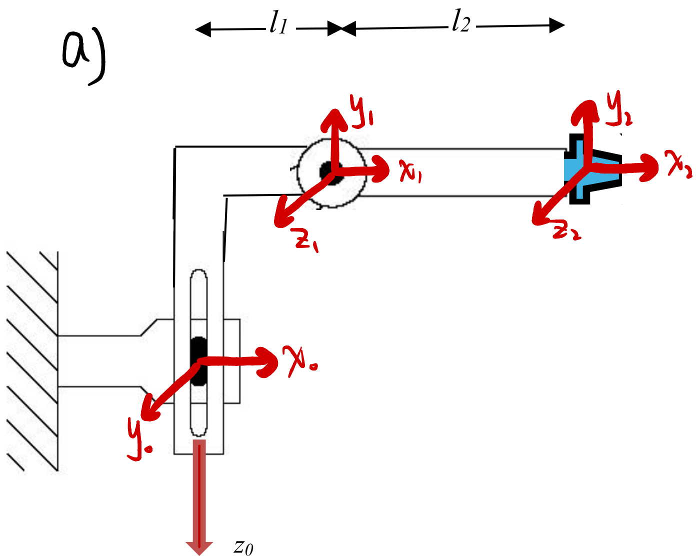
b) Create a D-H parameters table and fill out. (4 pts)

c) Find the homogenous transformation matrices (A_1 and A_2) for two joints. (6 pts)

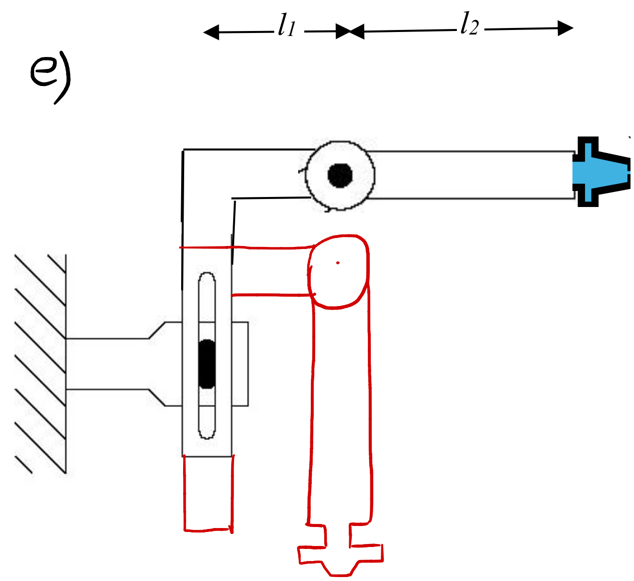
d) Find the direct kinematic equation (T matrix). (3 pts)

e) When $d_1=6''$ and $\theta_2= -90^o$, find the location and orientation of end effector and illustrate them in the figure below to the right. (try to be on scale) (5 pts)

a)



e)



b)

Link	θ	d	a	α
0 − 1	0	d^*	6	$-90°$
1 − 2	θ_2^*	0	12	0

c) $$A_1 = Trans(0,0,d_1) \times Trans(l_1,0,0) \times Rot(x_1,-90°) = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = Rot(z,\theta_2)\,Trans(l_1,0,0) = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 12\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 12\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

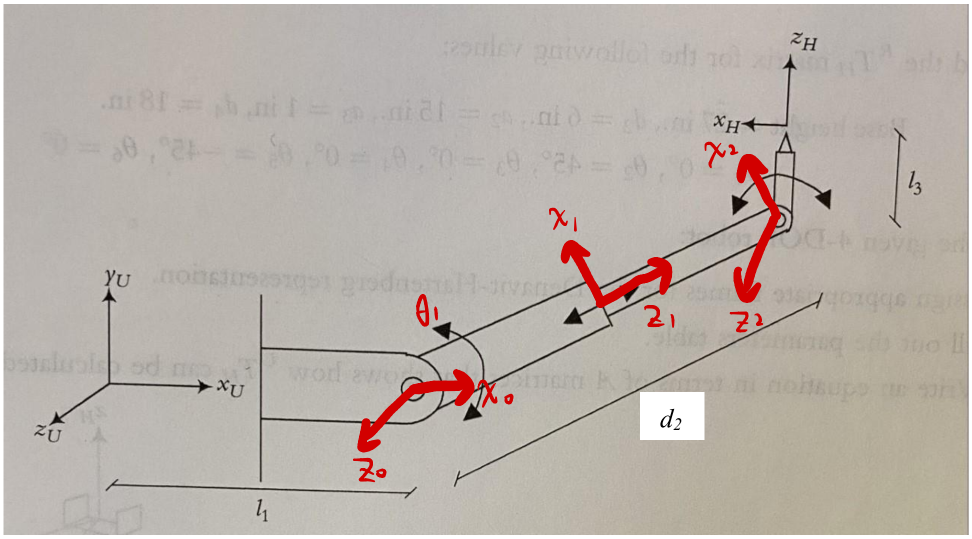
d) $$T_2^0 = A_1A_2 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 12\cos\theta_2 \\ \sin\theta_1 & \cos\theta_2 & 0 & 12\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 6+12\cos\theta_2 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & d_1-12\sin\theta_2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

e) $$T_2^0 = \begin{bmatrix} 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 18 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Prob 4. (22 pts)
A special 3 DOF spraying robot has been designed as shown below, and the reset position is when the arms are horizontal.

a) Assign the coordinate frames based on the D-H representation. (8 pts)
b) Fill out the parameters table. (6 pts)
c) Write all the A matrices. (4 pts)
d) Write the ${}^{U}T_{H}$ matrix in terms of the A matrices. (4 pts)

a)



b)

Link	θ	d	a	α
0-1	θ_1^*	0	0	90°
1-2	0	d_2^*	0	−90°
2-3	θ_3^*	0	d_3	0

c) $A_1 = Rot(z,\theta_1)\,Rot(x,-90°)$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90° & -\sin 90° & 0 \\ 0 & \sin 90° & \cos 90° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = Trans(0,0,d_2)\,Rot(x,-90°) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos -90° & -\sin -90° & 0 \\ 0 & \sin -90° & \cos -90° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = Rot(z,\theta_3)\,Trans(l_3,0,0) = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_3 & 0 & l_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & l_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) ${}^{U}T_{H} = Trans(l_1,0,0)\,A_1 A_2 A_3$

$$= \begin{bmatrix} \cos\theta_1\cos\theta_3 - \sin\theta_1\theta_3 & -\cos\theta_1\sin\theta_3 - \sin\theta_1\cos\theta_3 & 0 & l_1 + l_3\cos\theta_1\cos\theta_3 + d_2\sin\theta_1 - l_3\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_3 + \cos\theta_1\sin\theta_3 & -\sin\theta_1\sin\theta_3 + \cos\theta_1\cos\theta_2 & 0 & l_3\sin\theta_1\cos\theta_3 - d_2\cos\theta_2 + l_3\cos\theta_1\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$