Hongrui Yi

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1. Multiple choice questions: [5 points each] **Hint: cos ($A$ + $B$) = cos $A$ cos $B$ – sin $A$ sin $B$**

(Questions i and ii) A machine is subjected to the motion $x(t) = A\cos(50t + \alpha)$ mm. The initial conditions are given by $x(0) = 3$ mm and $\dot{x}(0) = 1$ m/s,

c i) The constant $A$ is:
a) 3.0001 mm
b) 3.9999 mm
c) 20.2237 mm
d) 22.8272 mm

d ii) The constant $\alpha$ is:
a) 35.2312 deg
b) -2.7316 deg
c) 81.4692 rad
d) -1.4219 rad

(Questions iii and iv) If the motion $x(t) = 30\cos(40t + 0.5\text{rad})$ mm is expressed in the form $x(t) = A_1 \cos \omega t + A_2 \sin \omega t$,

a iii) The constant $A_1$ is:
a) 26.33 mm
b) 29.99 mm
c) 22.98 mm
d) 21.84 mm

b iv) The constant $A_2$ is:

a) -0.26 mm
b) -14.38 mm
c) -19.28 mm
d) -22.93 mm

a v) If a harmonic motion has an amplitude of 0.20 cm and a period of 0.15 s, the maximum velocity would be:

a) 8.38 cm/s
b) 350.9 cm/s
c) 8.89 cm/s
d) infinite cm/s

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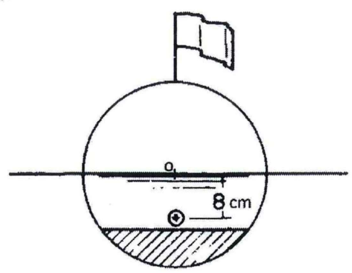
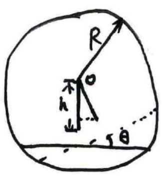
2. A spherical buoy 20 cm in diameter is weighted to float half out of the water, as shown in the figure. The center of gravity of the buoy is 8 cm below its geometric center. If its weight is W and the moment of inertia about the geometrical center is $J_0$, determine the period of the rolling motion (output to the expression should be in seconds). [26pts]




$R = 20cm \quad h = 8cm \quad W \quad J_0 \quad T = ?$




$T = \frac{1}{2} J_0 \cdot \dot{\theta}^2$

$U = W \cdot h(1-\cos\theta) = 8W(1-\cos\theta)$

$D = 0$

$\frac{\partial T}{\partial \dot{\theta}} = J_0 \cdot \dot{\theta} \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = J_0 \cdot \ddot{\theta}$

$\frac{\partial U}{\partial \theta} = 8W \sin\theta$

$\because \theta$ is very small, $\sin\theta \approx \theta$

$\therefore \frac{\partial U}{\partial \theta} = 8W \cdot \theta.$

Hence, $J_0 \cdot \ddot{\theta} + 8W \cdot \theta = 0$ ✓

$\omega_r = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = \sqrt{\frac{k}{m}} = \sqrt{\frac{8W}{J_0}}$ ✓

$T = \frac{2\pi}{\omega_r} = 2\pi \cdot \sqrt{\frac{J_0}{W}} \cdot 2\sqrt{2} = 4\sqrt{2}\pi \sqrt{\frac{J_0}{W}}$

$= \cancel{17.7715} \sqrt{\frac{J_0}{W}}$ (seconds)

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3. Fill-in the blank: A device bought from a surplus store is depicted as a one-degree-of-freedom torsional system. It is not possible to disassemble the device, however, if is found that (1) when the rotor is turned 22.5°, a torque of 176 N·m is needed to maintain this position, (2) when the rotor is held in this position and released, it swings to -18.6° and then back to 15.4°, and (3) the time of the complete swing is 0.42 s. From here: [4pts each]
**Note:** you will get partial credit if the equation is correct - make sure you write them down!

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a) its damped natural frequency is ~~0.9934~~ Hz $\omega_r$ 2.38

$\omega_r = \frac{1}{T} = \frac{1}{0.42s} = 2.38 Hz$

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b) the damping factor $\zeta$ of the system is 0.0603

$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \ln\left(\frac{\theta_1}{\theta_2}\right) = \ln\left(\frac{22.5}{15.4}\right) = 0.3791$

$\Rightarrow \zeta \approx 0.0603$

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c) its undamped natural frequency is ~~0.8434~~ Hz $\Omega_r$ 2.3843

$\omega_r = \pm\Omega_r\sqrt{1-\zeta^2}$

$\Rightarrow \Omega_r = \frac{\omega_r}{\sqrt{1-\zeta^2}} = \frac{2.38}{\sqrt{1-0.0603^2}} = \boxed{2.3843}$

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d) the torsional stiffness $K_T$ for the system is ~~224.09 N·m/rad~~ 7.8222 N·m/deg

$J_{cg}\cdot\ddot{\theta} + k_T\cdot\theta + G\dot{\theta} = 0.$

$k_T = \frac{F}{\theta} = \frac{176\,N\cdot m}{22.5\cdot\frac{2\pi}{180}} = \boxed{224.09\ N\cdot m/rad}$

2

e) the mass moment of inertia $J_{cg}$ of the rotor is ~~38.4186 N·m²~~

$\Omega_r = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_T}{J_{cg}}} = 2.3843$

$\Rightarrow J_{cg} = \frac{224.09}{2.3843^2} = \boxed{39.4186\ N\cdot m^2}$

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f) the damping required for it to be critically damped is 16.583 , and the torsional damping coefficient $C_T$ for the system is 11.3347 N·s/rad

$C_c = 2\sqrt{J_{cg}\cdot k_T}$

$= 2\sqrt{39.4186\times 224.09}$

$= 187.9714\ N\cdot s/rad$

$\zeta = \frac{C_T}{C_c}$

$\Rightarrow C_T = \zeta\cdot C_c$

$= 0.0603\times 187.9714$

$= 11.3347\ N\cdot s/rad$

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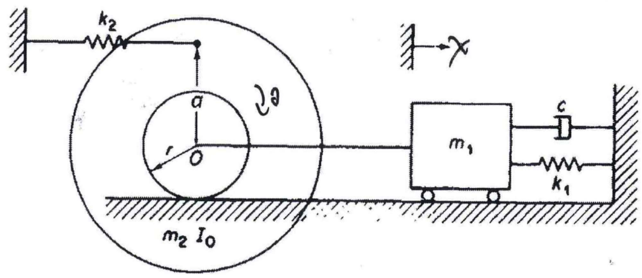
4. For the following dynamic system (*Io* is the moment of inertia):




a) using Lagrange's Method, determine the equation of motion of the dynamic system in terms of the given coordinate x. [21pts]

$+\frac{1}{2}m_2$

$$T = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}I_0\dot{\theta}^2 = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}I_0\left(\frac{\dot{x}}{r}\right)^2 \qquad \boxed{\theta = \frac{x}{r}} \Rightarrow \dot{\theta} = \frac{\dot{x}}{r},\ \ddot{\theta} = \frac{\ddot{x}}{r}$$

$$U = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2\cdot(\theta\cdot a)^2 = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2\left(\frac{a}{r}x\right)^2$$

$$D = \frac{1}{2}c\dot{x}^2$$

$$\frac{\partial T}{\partial \dot{x}} = m_1\dot{x} + I_0\cdot\frac{1}{r^2}\cdot\dot{x} \qquad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = m_1\ddot{x} + \frac{1}{r^2}I_0\cdot\ddot{x}$$

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$$\frac{\partial U}{\partial x} = k_1x + k_2\cdot\frac{a^2}{r^2}\cdot x$$

$$\frac{\partial D}{\partial \dot{x}} = c\dot{x}$$

Hence, $\left(m_1 + \frac{I_0}{r^2}\right)\ddot{x} + c\dot{x} + \left(k_1 + \frac{a^2}{r^2}k_2\right)x = 0$

b) establish the critical damping for the system. [4pts]

$$C_c = 2\sqrt{mk} = 2\sqrt{\left(m_1 + \frac{I_0}{r^2}\right)\left(k_1 + \frac{a^2}{r^2}k_2\right)}$$

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