Chapter 6 (这一章主要考 exterior flow)

圆柱

只有这个是计算 local 的值

Stagnation point处的local值

$$\mathrm{Nu}_D = \frac{h_c D}{k} = C\sqrt{\frac{\rho U_\infty D}{\mu}}$$

Pr	0.7	0.8	1.0	5.0	10.0
C	1.0	1.05	1.14	2.1	1.7

foward portion的local值，0°是stagntion值

$$\mathrm{Nu}(\theta) = \frac{h_c(\theta)D}{k} = 1.14\left(\frac{\rho U_\infty D}{\mu}\right)^{0.5}\mathrm{Pr}^{0.4}\left[1-\left(\frac{\theta}{90}\right)^3\right]$$

- Constant Surface Temperature
- Portion of surface to which a boundary layer adheres (0 < θ < 80°)

其余所有计算的都是 average 值

（流体垂直打入）

L/D > 4 代表着 long cylinder

Zukauskas

Long circular cylinder normal to gas or liquid flow

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = C\,\mathrm{Re}_D^m \mathrm{Pr}^n (\mathrm{Pr}/\mathrm{Pr}_s)^{1/4}$	$1 < \mathrm{Re}_D < 10^6$

n = 0.37 → Pr < 10

n = 0.36 → Pr > 10

- Uniform surface temp
- Properties @ free-stream

TABLE 6.1 Coefficients and exponents for Eq. (6.3)

Re_D	C	m
1 − 40	0.75	0.4
40 − 1×10^3	0.51	0.5
$1 \times 10^3 - 2 \times 10^5$	0.26	0.6
$2 \times 10^5 - 1 \times 10^6$	0.076	0.7

流体和接触面之间有一定角度

Groehn

Cylinders with angle of inclination (Yaw angle, θ)

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = 0.206\,\mathrm{Re}_N^{0.63}\,\mathrm{Pr}^{0.36}$, $\mathrm{Re}_N = \mathrm{Re}_D \sin\theta$	[ReN = 2500 , Recrit]
$\overline{\mathrm{Nu}}_D = 0.012\,\mathrm{Re}_D^{0.85}\mathrm{Pr}^{0.36}$	$2 \times 10^5 < \mathrm{Re}_D < 10^6$

这一段的适用条件 → [ReN = 2500 , Recrit]

θ	Re_{crit}
15°	2×10^4
30°	8×10^4
45°	2.5×10^5
>45°	$>2.5 \times 10^5$

Re在 [2500/sin， Recrit之间]

当Re较大的时候，便不受角度的影响了

- Uniform surface temp
- Properties @ free-stream

非圆形的柱体

Noncircular cylinder in a gas

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = B\,\mathrm{Re}_D^n$	$2500 < \mathrm{Re}_D < 10^5$

$\mathrm{Re}_D = \frac{\rho U_\infty D}{\mu}$

Jakob

- Properties @ T_{film}

$\frac{T_\infty + T_s}{2}$

TABLE 6.2 Constants in Eq. (6.6) for forced convection perpendicular to noncircular tubes

Flow Direction and Profile	Re_D From	Re_D To	n	B
	5,000	100,000	0.638	0.138
	5,000 19,500	19,500 100,000	0.638 0.782	0.144 0.035
	2,500 5,000	7,500 100,000	0.624 0.588	0.261 0.222
	2,500 5,000	8,000 100,000	0.699 0.675	0.160 0.092
	2,500	15,000	0.612	0.224
	3,000	15,000	0.804	0.085

流体的 Pr 很小

Ishiguro et al.

Circular cylinder in a liquid metal

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = 1.125(\mathrm{Re}_D\mathrm{Pr})^{0.413}$	$1 < \mathrm{Re}_D\mathrm{Pr} < 100$

- Properties @ free-stream

∞

短圆柱

Quarmby and Al-Fakhri 短!

Short cylinder in a gas

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = 0.123\,\mathrm{Re}_D^{0.651} + 0.00416(D/L)^{0.85}\,\mathrm{Re}_D^{0.792}$	$7 \times 10^4 < \mathrm{Re}_D < 2.2 \times 10^5$ $L/D < 4$

- Properties @ T_{film}

球型

McAdams

Sphere in a gas

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = \frac{\bar{h}_c D}{k} = \mathrm{Pr}(2.2 + 0.48\,\mathrm{Re}_D^{0.5})$	$1 < \mathrm{Re}_D < 25$
$\overline{\mathrm{Nu}}_D = 0.37\,\mathrm{Re}_D^{0.6}$	$25 < \mathrm{Re}_D < 10^5$

- Properties @ free-stream

Achenbach

Sphere in air

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = 2$	$\mathrm{Re}_D < 1$
$\overline{\mathrm{Nu}}_D = 2 + \left(\frac{\mathrm{Re}_D}{4} + 3 \times 10^{-4}\,\mathrm{Re}_D^{1.6}\right)^{1/2}$	$100 < \mathrm{Re}_D < 2 \times 10^5$
$\overline{\mathrm{Nu}}_D = 430 + 5 \times 10^{-3}\,\mathrm{Re}_D + 0.25 \times 10^{-9}\,\mathrm{Re}_D^2 - 3.1 \times 10^{-17}\,\mathrm{Re}_D^3$	$4 \times 10^5 < \mathrm{Re}_D < 5 \times 10^6$

- Uniform surface temp
- Properties @ free-stream

Sphere in a gas or a liquid

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = 2 + (0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3})\,\mathrm{Pr}^{0.4}(\mu/\mu_s)^{1/4}$	$3.5 < \mathrm{Re}_D < 7.6 \times 10^4$ $0.7 < \mathrm{Pr} < 380$

surface

- Properties @ free-stream

Liquid metal（一般 Pr 很小）

Witte

Sphere in a liquid metal

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = 2 + 0.386(\mathrm{Re}_D\mathrm{Pr})^{1/2}$	$3.6 \times 10^4 < \mathrm{Re}_D < 2 \times 10^5$

- Properties @ T_{film}

特殊形状

下面两个公式都是计算的背面

W → square
D → Disk
L_{eq} → rect

Sogin (Separated wake region)

对于非圆形的截面，用这个公式计算等效diameter $L_{eq} = \sqrt{B \times H}$

Long, flat plate, width D, perpendicular to flow in air

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = 0.20\,\mathrm{Re}_D^{2/3}$	$1 < \mathrm{Re}_D < 4 \times 10^5$

- Properties @ T_{film}

两个计算的都是背面的平均Nu

Half-round cylinder with flat rear surface, in a gas

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = 0.16\,\mathrm{Re}_D^{2/3}$	$1 < \mathrm{Re}_D < 4 \times 10^5$

- Properties @ T_{film}

下面两个公式都计算的正面

方形并且可以容忍倾斜角

Tien and Sparrow

Square plate, dimension, L, flow of a gas or a liquid

用前面提到的等效diameter

Correlation Equation	Restrictions
$(\bar{h}_c/c_p\rho U_\infty)\,\mathrm{Pr}^{2/3} = 0.930\,\mathrm{Re}_L^{-1/2}$	$2 \times 10^4 < \mathrm{Re}_L < 10^5$ angles of pitch and attack from 25° to 90° yaw angle from 0° to 45°

- Properties @ free-stream

圆形

Sparrow and Geiger

Upstream face of a disk with axis aligned with flow, gas, or liquid

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = 1.05\,\mathrm{Re}^{1/2}\mathrm{Pr}^{0.36}$	$5 \times 10^3 < \mathrm{Re}_D < 5 \times 10^4$

- Properties @ free-stream

Wedekind uniform temperature

Isothermal disk with axis perpendicular to flow, gas, or liquid

Correlation Equation	Restrictions
$\overline{\mathrm{Nu}}_D = 0.591\,\mathrm{Re}_D^{0.564}\mathrm{Pr}^{1/3}$	$9 \times 10^2 < \mathrm{Re}_D < 3 \times 10^4$ 0.06 < t/D < 0.16

- Properties @ T_{film}

Chapter 7 (这一章主要考 interior flow)

流体和面垂直的

Laminar flow of liquids and gases (Pr > 0.5) *inside circular pipes:*

$$f = \frac{64}{\mathrm{Re}_D}\phi, \text{ where } \phi = \begin{cases}(\mu_s/\mu_b)^{0.14} & \text{for liquids}\\ (T_s/T_b)^{0.14} & \text{for gases}\end{cases}$$

laminar flow

Pipes with $q_s'' =$ uniform:

$$\overline{\mathrm{Nu}}_D = \psi\begin{cases}1.953[L/(D\mathrm{Re}_D\mathrm{Pr})]^{-1/3} & \text{for } [L/(D\mathrm{Re}_D\mathrm{Pr})] < 0.03\\ 4.364 + (0.0722D\mathrm{Re}_D\mathrm{Pr}/L) & \text{for } [L/(D\mathrm{Re}_D\mathrm{Pr})] \geq 0.03\end{cases}$$

Pipes with $T_s =$ uniform:

$$\overline{\mathrm{Nu}}_D = \psi\begin{cases}1.615[L/(D\mathrm{Re}_D\mathrm{Pr})]^{-1/3} - 0.7 & \text{for } [L/(D\mathrm{Re}_D\mathrm{Pr})] \leq 0.005\\ 1.615[L/(D\mathrm{Re}_D\mathrm{Pr})]^{-1/3} - 0.2 & \text{for } 0.005 < [L/(D\mathrm{Re}_D\mathrm{Pr})] < 0.03\\ 3.657 + (0.0499D\mathrm{Re}_D\mathrm{Pr}/L) & \text{for } [L/(D\mathrm{Re}_D\mathrm{Pr})] \geq 0.03\end{cases}$$

$$\psi = \begin{cases}(\mu_b/\mu_s)^{0.14} & \text{for liquid heating/cooling}\\ (T_b/T_s)^n & n = 0.25 \text{ for gas heating; } n = 0.08 \text{ for gas cooling}\end{cases}$$

这个公式一般用不上

Turbulent flow of liquids and gases (Pr > 0.5) *in pipes and ducts*[d]:

$$f = (1.82\log_{10}\mathrm{Re}_{D_h} - 1.64)^{-2}$$

$$\overline{\mathrm{Nu}}_{D_h} = \frac{(f/8)(\mathrm{Re}_{D_h} - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3} - 1)}\left[1 + \left(\frac{D_h}{L}\right)^{2/3}\right]K$$

turbulent flow

$$K = \begin{cases}(\mathrm{Pr}_b/\mathrm{Pr}_s)^{0.11} & \text{for liquids}\\ (T_b/T_s)^{0.45} & \text{for gases}\end{cases}$$

Turbulent flow of liquid metals in smooth pipes ($L/D > 30$):

Pipes with $q_s'' =$ constant:

$$\overline{\mathrm{Nu}}_D = \begin{cases}4.82 + 0.0185(\mathrm{Re}_D\mathrm{Pr})^{0.827} & 100 < \mathrm{Re}_D\mathrm{Pr} < 10^4\\ 3.0\,\mathrm{Re}_D^{0.0833} & \mathrm{Re}_D\,\mathrm{Pr} < 100\end{cases}$$

Pipes with $T_s =$ constant:

$$\overline{\mathrm{Nu}}_D = 5.0 + 0.025(\mathrm{Re}_D\mathrm{Pr})^{0.8} \qquad \mathrm{Re}_D\mathrm{Pr} > 100$$

Re 的计算方式

$$D_H = \frac{4A_c}{P} \qquad \mathrm{Re} = \frac{\rho\bar{U}D_H}{\mu} = \frac{\bar{U}D_H}{\nu} \tag{7.6}$$

In long ducts, where the entrance effects are not important, the flow is laminar when the Reynolds number is below about 2100.* In the range of Reynolds numbers between 2100 and 10,000, a transition from laminar to turbulent flow takes place. The flow in this regime is called transitional. At a Reynolds number of about 10,000, the flow becomes fully turbulent

Fully developed 的计算

$$\left(\frac{x_{\text{fully developed}}}{D}\right)_{\text{lam}} = 0.05\mathrm{Re}_D \tag{7.7}$$

whereas the distance from the inlet at which the temperature profile approaches its fully developed shape is given by the relation [5]

$$\left(\frac{x_{\text{fully developed}}}{D}\right)_{\text{lam,thermal}} = 0.05\mathrm{Re}_D\,\mathrm{Pr} \tag{7.8}$$

主要的计算步骤

$$q_c = \dot{m}c_p[(T_s - T_{b,\text{in}}) - (T_s - T_{b,\text{out}})] = \dot{m}c_p(\Delta T_{\text{in}} - \Delta T_{\text{out}})$$

Constant Temperature:

$$\Delta T = T_s - T_b$$

$$q_c = \bar{h}_c A_s\left[\frac{\Delta T_{\text{out}} - \Delta T_{\text{in}}}{\ln(\Delta T_{\text{out}}/\Delta T_{\text{in}})}\right]$$

Constant flux:

$$\bar{h}_c = \frac{q_c}{A(T_s - T_b)}$$

特殊横截面积的 Nu(需要达到 fully developed 区域) H1 是 flux 不变， T 是温度不变

TABLE 7.1 Nusselt number and friction factor for fully developed laminar flow ($L/D_h > 100$) of a Newtonian fluid through a circular and some non-circular cross-section ducts[a]

Geometry	$\alpha = (2b/2a)$	$\overline{\mathrm{Nu}}_{H1}$	$\overline{\mathrm{Nu}}_T$	$f\,\mathrm{Re}_{D_h}$	$\overline{\mathrm{Nu}}_{H1}/\overline{\mathrm{Nu}}_T$
2a, 2b	0	8.235	7.541	94.00	1.092
2a, 2b, Insulation	0	5.385	4.861	96.00	1.108
2a, 2b	0.125	6.490	5.597	82.34	1.160
	0.25	5.331	4.439	72.93	1.201
	0.5	4.123	3.391	62.19	1.216
	1	3.608	2.976	56.91	1.212
	–	4.364	3.658	64.00	1.193
2a, 2b	0.8	4.387	3.669	64.39	1.196
	0.5	4.558	3.742	67.29	1.218
	0.25	4.880	3.792	72.96	1.287
a		4.011	3.436[b]	60.22	1.167
60°, 2b, 2a	$\sqrt{3}/2$	3.111	2.470	53.33	1.260

Very short tubes and Rectangular Ducts

Flow-over-flat-plate approximation

- Initial uniform velocity and temp Distribution
- 0.7 < Pr < 15
- L/D < 0.0048 Re_D for tubes
- L/D_H < 0.0021 Re_{D_H} for flat ducts

$$\overline{\mathrm{Nu}}_{D_H} = \frac{\mathrm{Re}_{D_H}\mathrm{Pr}D_H}{4L}\ln\left[\frac{1}{1 - (2.654/\mathrm{Pr}^{0.167})(\mathrm{Re}_{D_H}\mathrm{Pr}D_H/L)^{-0.5}}\right] \tag{7.38}$$

泊肃叶

$\boxed{\text{Poiseuille Flow}}$ — Long Tube (F.D) — Forced Convection
— Laminar — S.S.

$$u = \frac{r^2 - r_s^2}{4\mu}\cdot\frac{dp}{dx} \qquad \frac{u}{u_{max}} = \frac{r^2 - r_s^2}{-r_s^2} = 1 - \left(\frac{r}{r_s}\right)^2$$

Pressure Loss $\Delta p = f\frac{L}{D}\frac{1}{2}\rho\bar{U}^2$ $\quad f = 4C_F = \frac{4\tau_s}{\frac{1}{2}\rho\bar{U}^2}$

Mass Flow $\dot{m} = \rho\bar{U}A = \frac{\Delta p\pi r_s^4\rho}{8\mu L}$

Laminar $f = \frac{64}{\mathrm{Re}_D}$ turb $f \propto$ roughness

Pumping Power $P_p = \frac{\Delta p\dot{Q}}{\eta_p}$ $\quad \dot{Q} = \frac{\dot{m}}{\rho}$

Chapter 8

Isothermal Vertical Plate and Cylinders
(analytical solution, as compared to Fig 8.5)

$$h_{cx} = 0.508\mathrm{Pr}^{1/2}\frac{\mathrm{Gr}_x^{1/4}}{(0.952 + \mathrm{Pr})^{1/4}}\frac{k}{x}$$

$$\bar{h}_c = \frac{1}{L}\int_0^{x=L} h_{cx}\,dx = 0.68\mathrm{Pr}^{1/2}\frac{\mathrm{Gr}_L^{1/4}}{(0.952 + \mathrm{Pr})^{1/4}}\frac{k}{L}$$

$$\delta(x) = 4.3x\left[\frac{\mathrm{Pr} + 0.56}{\mathrm{Pr}^2\mathrm{Gr}_x}\right]^{1/4}$$

Isothermal Vertical Plate and Cylinders

LAM

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}_c L}{k} = 0.68\mathrm{Pr}^{1/2}\frac{\mathrm{Gr}_L^{1/4}}{(0.952 + \mathrm{Pr})^{1/4}} \qquad 10 < \mathrm{Gr}_L\mathrm{Pr} < 10^8$$

TRANS-TURB

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}_c L}{k} = 0.13(\mathrm{Gr}_L\mathrm{Pr})^{1/3} \qquad \mathrm{Gr}_L > 10^9$$

Liquid Metal

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}_c L}{k} = 0.68(\mathrm{Gr}_L\mathrm{Pr}^2)^{1/3} \qquad \mathrm{Pr} < 0.03$$

Laminar Flow

Horizontal Plates

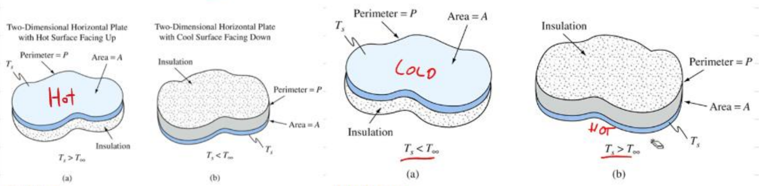

FIGURE 5.9 FIGURE 5.10

Upper surface hot or lower surface cool [Figs. 8.9(a) and (b)]:

$$\overline{\mathrm{Nu}}_L = 0.54\mathrm{Ra}_L^{1/4} \quad (10^5 \lesssim \mathrm{Ra}_L \lesssim 10^7)$$
$$\overline{\mathrm{Nu}}_L = 0.15\mathrm{Ra}_L^{1/3} \quad (10^7 \lesssim \mathrm{Ra}_L \lesssim 10^{10})$$

Lower surface hot or upper surface cool [Figs. 8.10(a) and (b)]:

$$\overline{\mathrm{Nu}}_L = 0.27\mathrm{Ra}_L^{1/4} \quad (10^5 \lesssim \mathrm{Ra}_L \lesssim 10^{10})$$

Cooled Circular Horizontal Plate Facing down
Liquid Metal

COLD

$$\overline{\mathrm{Nu}}_D = \frac{\bar{h}_c D}{k} = 0.26(\mathrm{Gr}_D\mathrm{Pr}^2)^{0.35}$$

Long plate with θ from vertical (insulated the other)

Heated plate facing down Cooled plate facing upward

$$\overline{\mathrm{Nu}}_L = 0.56(\mathrm{Gr}_L\mathrm{Pr}\cos\theta)^{1/4}$$

$10^5 < \mathrm{Gr}_L\mathrm{Pr}\cos\theta < 10^{11}$ $\quad 0 \leq \theta \leq 89°$

Horizontal Plate $L = \frac{A_s}{P}$

Upper surface hot / lower surface cool

$$\overline{\mathrm{Nu}}_L = 0.54\mathrm{Ra}_L^{1/4} \quad 10^5 \leq \mathrm{Ra}_L \leq 10^7$$
$$\overline{\mathrm{Nu}}_L = 0.15\mathrm{Ra}_L^{1/3} \quad 10^7 \leq \mathrm{Ra}_L \leq 10^{10}$$

Upper surface cool / lower surface hot

$$\overline{\mathrm{Nu}}_L = 0.27\mathrm{Ra}_L^{1/4} \quad 10^5 \leq \mathrm{Ra}_L \leq 10^{10}$$

Cooled circular horizontal Plate facing down in

$$\overline{\mathrm{Nu}}_D = \frac{\bar{h}_c D}{k} = 0.26(\mathrm{Gr}_D\mathrm{Pr}^2)^{0.35}$$ liquid metal.

Horizontal Cylinder

Local Nu. $\mathrm{Nu}_{D\alpha} = 0.604\mathrm{Gr}_D^{1/4}\phi(\alpha)$ $\quad$ for air ($\mathrm{Pr} = 0.71$)

	Bottom Half			Top Half				
α	−90	−60	−30	0	30	60	75	90
$\phi(\alpha)$	0.76	0.75	0.72	0.66	0.58	0.46	0.36	0

Average $\overline{\mathrm{Nu}}$ $\quad \overline{\mathrm{Nu}}_D = 0.53(\mathrm{Gr}_D\mathrm{Pr})^{1/4}$ $\quad$ valid for $\mathrm{Pr} > 0.5$, $10^3 < \mathrm{Gr} < 10^9$

$\overline{\mathrm{Nu}}_D = 0.53(\mathrm{Gr}_D\mathrm{Pr}^2)^{1/4}$ — liquid metal

Chapter 11

$$\lambda_{max}T = 2.898 \times 10^{-3}\text{ m K }(5216.4\ \mu\ °\text{R})$$

$$\frac{E_{b\lambda}(T)}{E_{b\lambda max}(T)} = \left(\frac{2.898 \times 10^{-3}}{\lambda T}\right)^5\left(\frac{e^{4.965} - 1}{e^{0.014388/\lambda T} - 1}\right)$$

$$E_b(T) = \frac{q_r}{A} = \sigma T^4$$

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5(e^{C_2/\lambda T} - 1)} \tag{11.1}$$

q_r = total rate of radiant heat emitted, W (Btu/h)
A = area of the blackbody emitting the radiation, m² (ft²)
T = absolute temperature of the area A in K (°R)
σ = Stefan-Boltzmann constant
= 5.60×10^{-8} W/m²K⁴ (0.1714×10^{-8} Btu/ft²°R⁴)

$E_{b\lambda}$ = monochromatic emissive power of a blackbody at absolute temperature T, W/m²μ (Btu/h ft²μ)
λ = wavelength, m (μ)
T = absolute temperature of the body, K (degrees °R = 460 + °F)
C_1 = first radiation constant
= 3.7415×10^{-16} W m² (1.1870×10^8 Btu μ⁴/h ft²)
C_2 = second radiation constant
= 1.4388×10^{-2} m K (2.5896×10^4 μ °R)

- Laminar: Ra < 10^8
- Transition range: 10^8 < Ra < 10^{10}
- Turbulent: Ra > 10^{10}

$$\pi_3 = \frac{g\beta(T - T_\infty)L^3}{\nu^2}$$

Radiation function

$$E_{b\lambda max}(T) = \frac{C_1 T^5}{(0.002898)^5(e^{C_2/0.002898} - 1)} = 12.87 \times 10^{-6}T^5\ \text{W}/\mu\ \text{m}^2$$

$$\frac{E_{b\lambda}(T)}{E_{b\lambda max}(T)} = \left(\frac{2.898 \times 10^{-3}}{\lambda T}\right)^5\left(\frac{e^{4.965} - 1}{e^{0.014388/\lambda T} - 1}\right)$$

Intensity of Radiation $d\omega = \frac{dA_n}{r^2}$

$$I_b(\theta, \phi) = \frac{dq_r}{dA_1\cos\theta\,d\omega}\ (\text{W/m}^2\ \text{sr})$$

$$dE_b = \frac{dq_r}{dA_1} = I_b(\theta, \phi)\cos\theta\,d\omega$$

$$\left(\frac{q}{A}\right)_r = \int_0^{2\pi}\int_0^{\pi/2} I_b(\theta, \phi)\cos\theta\sin\theta\,d\theta\,d\phi$$

Lambert cosine Law

$$\left(\frac{q}{A}\right)_r = E_b = \pi I_b\ [\text{W/m}^2]$$

Irradiation

$$G_\lambda = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda, \theta, \phi)\cos\theta\sin\theta\,d\theta\,d\phi$$

$$G = \int_0^\infty G_\lambda(\lambda)d\lambda = \int_0^\infty\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda, \theta, \phi)\cos\theta\sin\theta\,d\theta\,d\phi\,d\lambda$$