

重庆大学《Multivariable Calculus》课程

☒ A卷

☐ B卷

2017 — 2018 学年 第 1 学期

开课学院: 数统学院 课程号: MATH20083 考试日期: 2017 12 15

考试方式: ☐ 开卷 ☒ 闭卷 ☐ 其他 考试时间: 120 分钟

题号	一	二	三	四	五	六	七	八	九	十	总分
得分											

考试提示

1. 严禁随身携带通讯工具等电子设备参加考试;
2. 考试作弊, 留校察看, 毕业当年不授学位; 请人代考、替他人考试、两次及以上作弊等, 属严重作弊, 开除学籍。

一、(15pts.) Fill in the blanks with correct answers.

1. The curvature of the parabola $y=x^2$ at $(0,0)$ is _____.
2. The area of the surface $\begin{cases} x^2 + y^2 = 4 \\ 0 \leq z \leq 1 \end{cases}$ is _____.
3. $\iint_D x dA =$ _____, where D is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$.
4. $\int_C \nabla f \cdot d\vec{r} =$ _____, where C is a simple closed path and f is a smooth function with 2 variables.
5. $\text{Curl } \vec{F} =$ _____, where $\vec{F}(x,y,z) = xz\vec{i} + xyz\vec{j} - y^2\vec{k}$.

二、(15pts.) Determine whether the following statements are true or false.

1. If $\text{curl } \vec{F} = 0$ for a smooth 3-dimensional vector field \vec{F} , then it must be conservative. ()
2. The two mixed second order partial derivatives for $z=f(x,y)$ must be equal. ()
3. Suppose D is a 2-dimensional simple bounded plane region, then $\iint_D 1 dA = \int_{\partial D} x dy = -\int_{\partial D} y dx$. ()
4. If the two partial derivatives f_x and f_y at (x_0, y_0) exist, then $f(x,y)$ must be continuous at this point. ()
5. Given two vectors $\alpha = (a_1, \dots, a_n), \beta = (b_1, \dots, b_n)$, then they are orthogonal if and only if $\sum_{i=1}^n a_i b_i = 0$. ()

三、(10pts.)

$$\text{Suppose } f(x,y) = \begin{cases} (x^2 + y^2) \cos \frac{1}{\sqrt{x^2 + y^2}} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases},$$

- 1) Is $f(x,y)$ continuous at $(0,0)$?

2) Find $f_x(0,0)$ and $f_y(0,0)$

3) Is $f(x,y)$ differentiable at $(0,0)$?

四、(10pts.) 1) Assume $f(x,y,z) = z\sqrt{\frac{x}{y}}$, find the total differential of f at $(1,1,1)$

2) $F(x,y,z) = x^3 + y^3 + z^3 + 6xyz - 1 = 0$ defines implicitly a function $z = f(x,y)$, find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$.

五、(10pts.) Suppose $f(x,y) = (y + \frac{x^3}{3})e^{x+y}$, find the maximal and minimal values of $f(x,y)$.

六、(10pts.) Use the Divergence theorem to evaluate $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F}(x,y,z) = 3xy^2\vec{i} + (xe^z)\vec{j} + z^3\vec{k}$, and S is the surface of the region bounded by $y^2 + z^2 = 1$ and $x = -1, x = 2$.

七、(30pts.) Suppose $\vec{F}(x,y) = \frac{-y}{x^2+y^2}\vec{i} + \frac{x}{x^2+y^2}\vec{j} = P\vec{i} + Q\vec{j}$.

1) (5pts.) Find the domain D of this vector field, and show that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ for all points in the domain D of this vector field.

2) (5pts.) Calculate $\int_C \vec{F} \cdot d\vec{r}$, where C is any positive oriented simple closed path which doesn't enclose the origin.

3) (8pts.) Calculate $\int_C \vec{F} \cdot d\vec{r}$, where C is any positive oriented simple closed path which enclose the origin.

4) (7pts.) Prove that $\int_C \vec{F} \cdot d\vec{r}$ is not independent of paths in the domain D by showing that this line integral differs along two different paths in D.

5) (5pts.) Is $\vec{F}(x,y)$ conservative on D? On what kind of regions is $\vec{F}(x,y)$ conservative?