重庆大学

信号、系统与控制课程报告

2020 至 2021 学年第二学期

项目名称：Project I

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CQU-UC Joint Co-op Institute (JCI)
Student Project Report

Project I of System & Signal Control

Institution CQU-UC Joint Co-op Institute (JCI)

Grade 2018 **Major** Mechanical Engineering **Class** 01

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Course Code: ME40881 **Instructor** Huang, Tao

CQU-UC Joint Co-op Institute

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MECH-4081: Project – Part I (REVISED)
Due Mon., Nov. 30, 6:00 p.m.

Shown is an electro-hydraulic system consisting of the DC motor and fixed-displacement pump, along with mass m coupled via the rod and cylinder to the two interconnected water tanks. The water is incompressible with density ρ and viscosity μ. The cylinder contains a piston which has an (assumed) laminar orifice of length L_1 and diameter d_1. The cross-sectional areas of the tanks and the piston are denoted as A_1, A_2, A_{p1}, and A_{p2}, respectively. Pressures p_1 and p_2 acts uniformly on either side of the piston. The control input $u(t)$ is the voltage to the motor circuit. The output $y(t)$ will be the position x_3.

The water in each tank is governed by conservation of volume. (Refer to in-class discussion and the Appendix for more information.) The laminar orifice flow will be assumed to be governed by:

$$q_1 = \frac{1}{R_1}(p_1 - p_2), \qquad \text{where} \qquad R_1 = \frac{128\,\mu L_1}{\pi\, d_1^4}$$

where q_1 is the volume flow (m^3/s) and R_1 is the laminar resistance.

In addition to the input voltage $u(t)$, independent variables for the problem are i (motor current), ω (motor angular velocity), x_1, x_2, and x_3. Flow q_1 is a dependent variable which should be eliminated by substitution (see Appendix). Nominal parameter values are given in Table 1.

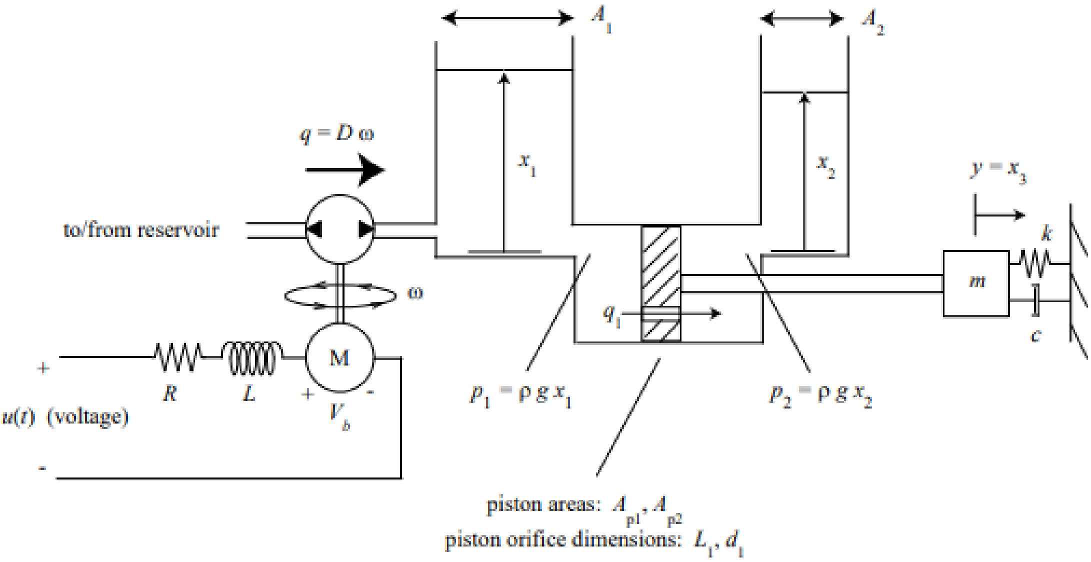

Figure 1: System schematic.

You'll also need the following governing equations:

- In addition to viscous damping torque $b\omega$, the summation of moments on the motor must also include a reaction torque from the pump, equal to $D\,p_1$, where D is the pump displacement.
- The (bidirectional) pump, which is 100% efficient, delivers a flow ouput of $D\omega$ (m^3/s).

Catalog

Question 1: Find the governing differential equations

In the first problem, we need to find the governing differential equation. For the circuit system, motor system, hydraulic piston system, mechanical system, the equations are listed below.

Given

$$q_1 = \frac{\rho g x_1 - \rho g x_2}{R_1}, where\ \ R_1 = \frac{128\mu L_1}{\pi d_1^4}$$

$$q = D\omega$$

$$p_{1,2} = \rho g x_{1,2}$$

$$v_3 = \dot{x}_2$$

We could get

$$\begin{cases} u(t) = Ri + L\dot{i} + k_b\omega \\ k_t i = J\dot{\omega} + b\omega + D\rho g x_1 \\ D\omega = A_1\dot{x}_1 + q_1 + A_{p1}v_3 \\ A_{p2}v_3 + q_1 = A_2\dot{x}_2 \\ p_1 A_{p1} = p_2 A_{p2} + m\ddot{x}_3 + cv_3 + kx_3 \end{cases}$$

Then,

$$D\omega = \frac{\rho g x_1 - \rho g x_2}{R_1} + A_1\dot{x}_1 + A_{p1}v_3$$

$$v_3 = \dot{x}_3$$

$$A_2\dot{x}_2 = \frac{\rho g x_1 - \rho g x_2}{R_1} + A_{p2}v_3$$

Finally,

$$
\Rightarrow \begin{cases}
\dot{i} = -\dfrac{R}{L} i - \dfrac{k_b \omega}{L} \omega + \dfrac{u(t)}{L} \\
\dot{\omega} = \dfrac{k_t}{J} i - \dfrac{b}{J} \omega - \dfrac{D\rho g}{J} x_1 \\
\dot{x}_1 = \dfrac{D}{A_1} \omega - \dfrac{\rho g}{R_1 A_1} x_1 + \dfrac{\rho g}{R_1 A_1} x_2 - \dfrac{A_{p1}}{A_1} \dot{x}_3 \\
\dot{x}_2 = \dfrac{\rho g}{R_1 A_2} x_1 - \dfrac{\rho g}{R_1 A_2} x_2 - \dfrac{A_{p1}}{A_1} \dot{x}_3 \\
\dot{x}_3 = \dot{x}_3 \\
\ddot{x}_3 = \dfrac{A_{p1}\rho g}{m} x_1 + \dfrac{A_{p2}\rho g}{m} x_2 - \dfrac{k}{m} x_3 - \dfrac{c}{m} \dot{x}_3
\end{cases}
$$

Question 2: Convert the differential equations in vector first-order (state variable) form

NOTE: To enable checking of your equations, please group your variables in the following order when you arrange the matrix form of the equations:

$$\mathbf{x} = \begin{bmatrix} i \\ \omega \\ x_1 \\ x_2 \\ x_3 \\ \dot{x}_3 \end{bmatrix}$$

Please refer to the Appendix and other references (provided) for more information. (See also Chap. 8 of the course text (Phillips)).

$$x(t) = \begin{bmatrix} i \\ \omega \\ x_1 \\ x_2 \\ x_3 \\ \dot{x}_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{i} \\ \dot{\omega} \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \ddot{x}_3 \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & -\frac{k_b}{L} & 0 & 0 & 0 & 0 \\ \frac{k_t}{J} & \frac{-b}{J} & \frac{-\rho g D}{J} & 0 & 0 & 0 \\ 0 & \frac{D}{A_1} & \frac{-\rho g}{R_1 A_1} & \frac{\rho g}{R_1 A_1} & 0 & \frac{-A_{p1}}{A_2} \\ 0 & 0 & \frac{\rho g}{R_1 A_2} & \frac{-\rho g}{R_1 A_2} & 0 & \frac{A_{p2}}{A_2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{A_{p1}\rho g}{m} & \frac{-A_{p2}\rho g}{m} & \frac{-k}{m} & \frac{-c}{m} \end{bmatrix} \cdot \begin{bmatrix} i \\ \omega \\ x_1 \\ x_2 \\ x_3 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot u(t)$$

That means, $\dot{x}(t) = Ax(t) + Bu(t)$

$$\text{Where } A = \begin{bmatrix} \frac{-R}{L} & -\frac{k_b}{L} & 0 & 0 & 0 & 0 \\ \frac{k_t}{J} & \frac{-b}{J} & \frac{-\rho g D}{J} & 0 & 0 & 0 \\ 0 & \frac{D}{A_1} & \frac{-\rho g}{R_1 A_1} & \frac{\rho g}{R_1 A_1} & 0 & \frac{-A_{p1}}{A_2} \\ 0 & 0 & \frac{\rho g}{R_1 A_2} & \frac{-\rho g}{R_1 A_2} & 0 & \frac{A_{p2}}{A_2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{A_{p1}\rho g}{m} & \frac{-A_{p2}\rho g}{m} & \frac{-k}{m} & \frac{-c}{m} \end{bmatrix}, \; B = \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Question 3: Find the open loop system transfer function

Find the open loop system transfer function, $Y(s)/U(s)$, from voltage input to the position of mass m_3. Use of Matlab with the state variable version of the governing differential equations will be necessary. Refer to the Appendix of this document for the required procedure.

You should find that the system transfer function is sixth-order. Please,

- List the system poles and zeros, as you find using Matlab.
- Express your transfer function in polynomial form, i.e.,

$$\frac{Y(s)}{U(s)} = \frac{(\ )s + (\ )}{(\ )s^6 + (\ )s^5 + (\ )s^4 + (\ )s^3 + (\ )s^2 + (\ )s + (\ )}$$

NOTE: In problems of this size, the polynomial coefficients are susceptible to roundoff error; be careful to keep an appropriate number of significant figures when using this form, or else use the factored form.

In the third problem, we need to find the open loop system transfer function, Y(s)/U(s), from voltage input to the position of mass m_3. And then find the poles and zeros of it.

$$y(t) = c \cdot x(t)$$

$$c = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0]$$

Use Laplace Translation:

$$\begin{cases} sX(s) = AX(s) + BU(s) \\ Y(s) = CX(s) \end{cases}$$

$$\Rightarrow (sI - A)X(s) = BU(s)$$

$$\Rightarrow X(s) = \frac{BU(s)}{(sI - A)}$$

$$\Rightarrow Y(s) = \frac{CBU(s)}{(sI - A)}$$

$$\Rightarrow H(s) = \frac{Y(s)}{U(s)} = \frac{CB}{(sI - A)}$$

The output result:

the zeros of the system: -18.2609
the poles of the system:
-9.999000976746577e+02 + 0.000000000000000e+00i
-8.285310237815338e+01 + 0.000000000000000e+00i
-5.791945504227670e+01 + 0.000000000000000e+00i
-1.406532994540211e+00 + 1.165437218437696e+01i
-1.406532994540211e+00 - 1.165437218437696e+01i
-5.310068133782269e+00 + 0.000000000000000e+00i

The transfer function can be represented in polynomial form as:

$$H(s) = \frac{(4.1202 \times 10^4)s + (7.5239 \times 10^5)}{s^6 + (1.1488 \times 10^3)s^5 + (1.5498 \times 10^5)s^4 + (6.1557 \times 10^6)s^3 + (6.2045 \times 10^7)s^2 + (8.394 \times 10^8)s + (3.5111 \times 10^9)}$$

Question 4&5 : Plot the system poles, zeros and step response

Plot the system poles and zeros in the complex plane in Matlab using `pzmap`. Ensure that the horizontal and vertical axes of the plot are scaled equally.

Plot the system step response using `step`.

Poles and zeros of the transfer function:

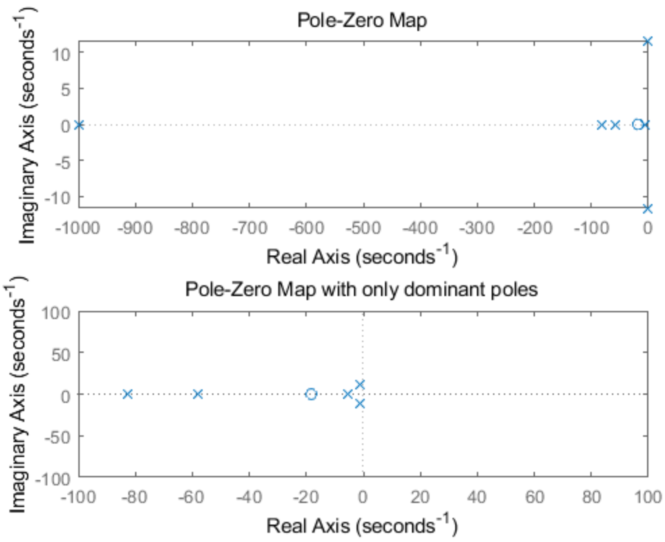

Fig 1. Pole-Zero Map

Step response for open loop system:

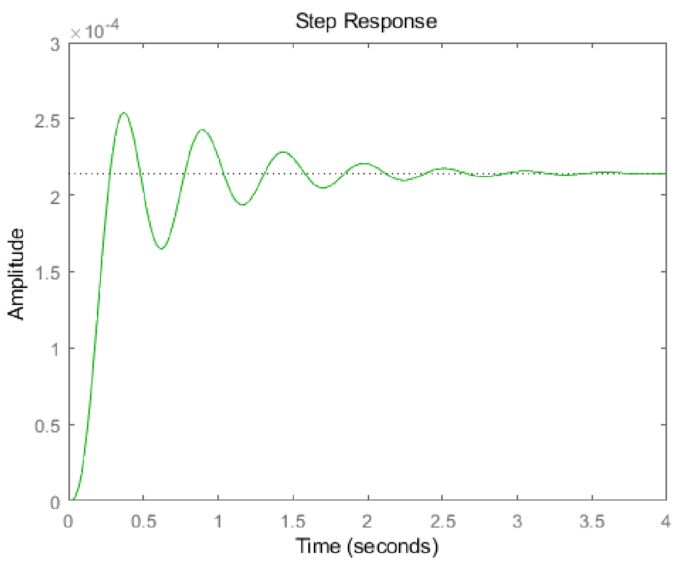

Fig 2. Step Response

Question 6: Evaluate the effect of the parameter variations

With all other system parameters at their nominal values (Table 1), evaluate the effect of the following parameter variations (one at a time):

Orifice diameter:	$0.010 \le d_1 \le 0.020$ m
Tank #1 cross-sectional area:	$0.010 \le A_1 \le 0.075$ m^2
Piston (#1 side) area:	$0.015 \le A_{p_1} \le 0.050$ m^2

Using multiple PZMAP and STEP plots at different values, evaluate the effect of the parameter change on the dominant poles. Summarize results in a table. Discuss how each of these parameter changes affects the system response. Can first-order and/or second-order transient performance criteria be improved over the nominal case?

For the effect of the parameter change on the dominant poles, Table 1 represents the results.

Table 1 The effect of the parameter change on the dominant poles

d1			
	0.01	0.015	0.02
Distance to imaginary axis	3.15(2),3.79	1.41(2),5.31	0.755(2),5.47
Distance to real axis	13.4(2),0	11.7(2),0	11.7(2),0
A1			
	0.01	0.025	0.075
Distance to imaginary axis	1.9(2),7.84	1.41(2),5.31	0.787(2),2.52
Distance to real axis	11.4(2),0	11.7(2),0	11.5(2),0
Ap1			
	0.015	0.025	0.05
Distance to imaginary axis	0.636(2)	1.41(2),5.31	4.68(2),2.76
Distance to real axis	11.1(2)	11.7(2),0	16.3(2),0

For the effect of the system response:

1. The system response change of orifice diameter (d1):

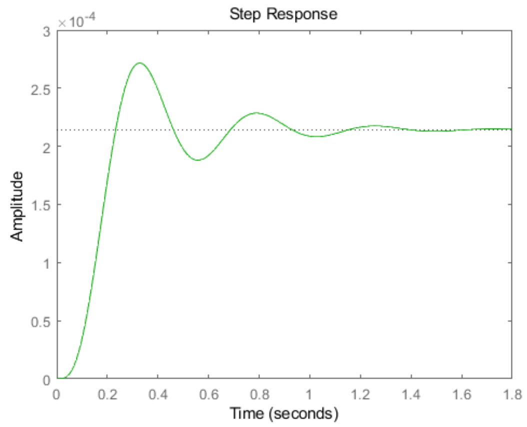

Fig 3. Step Response when d1=0.01

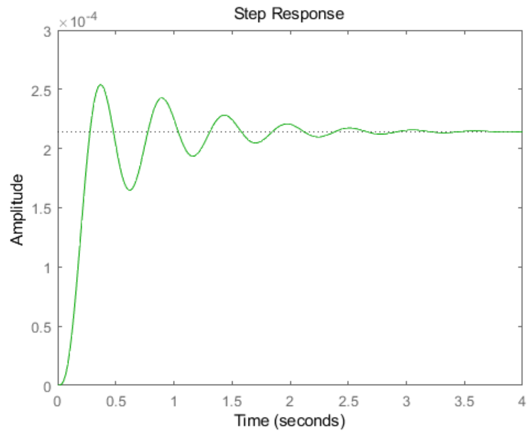

Fig 4. Step Response when d1=0.015

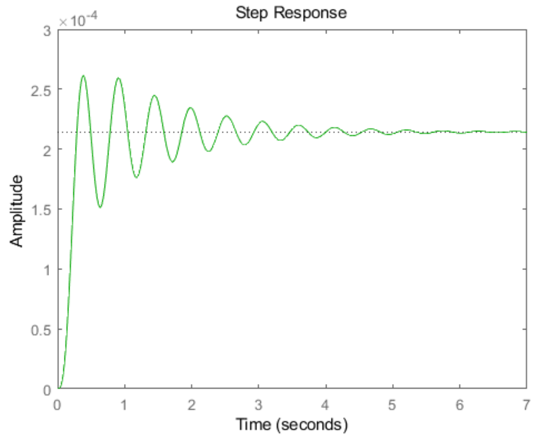

Fig 5. Step Response when d1=0.02

2. The system response change of Tank #1 cross-sectional area (A1):

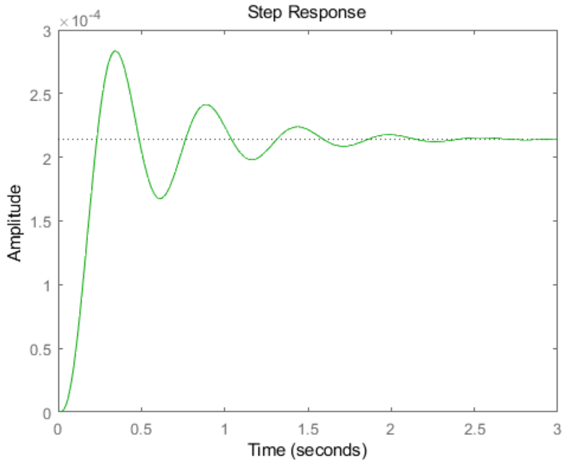

Fig 6. Step Response when A1=0.01

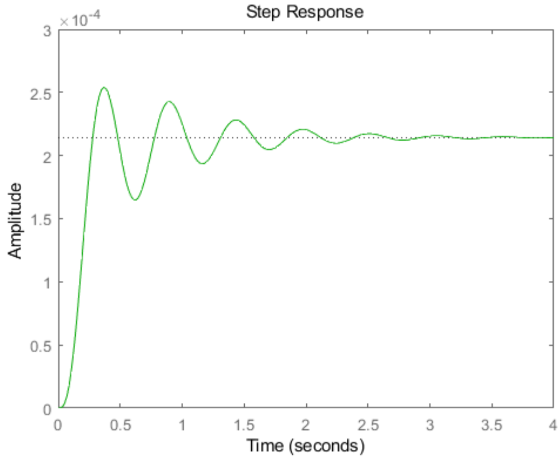

Fig 7. Step Response when A1=0.025

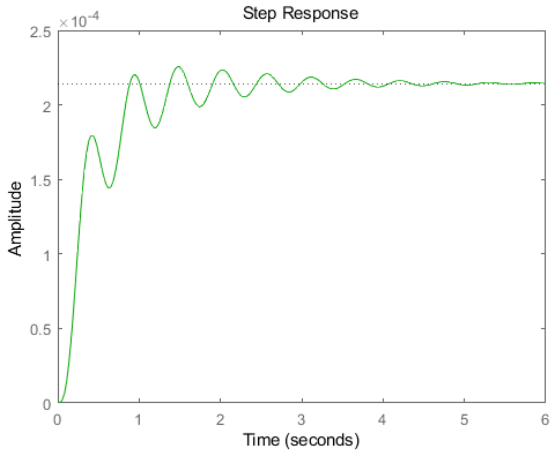

Fig 8. Step Response when A1=0.075

3. The system response change of Piston (#1 side) area (Ap1):

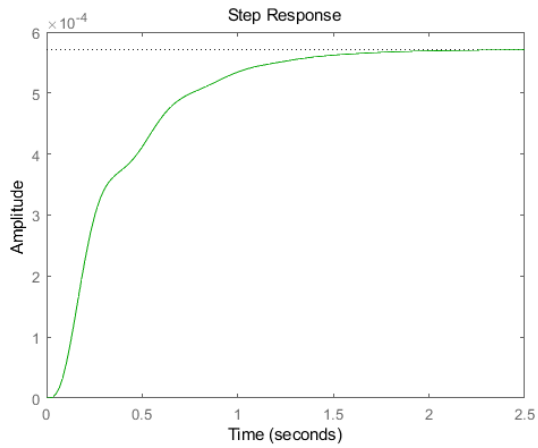

Fig 9. Step Response when Ap1=0.015

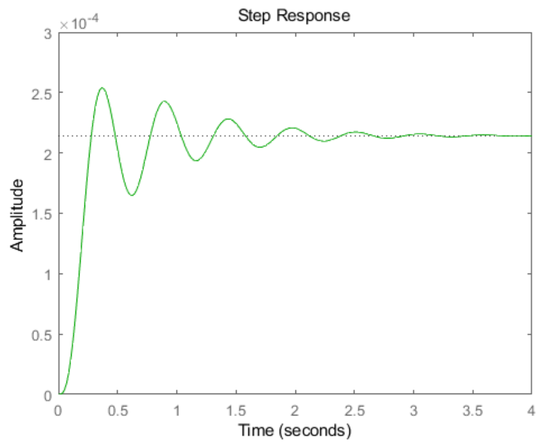

Fig 10. Step Response when A1=0.025

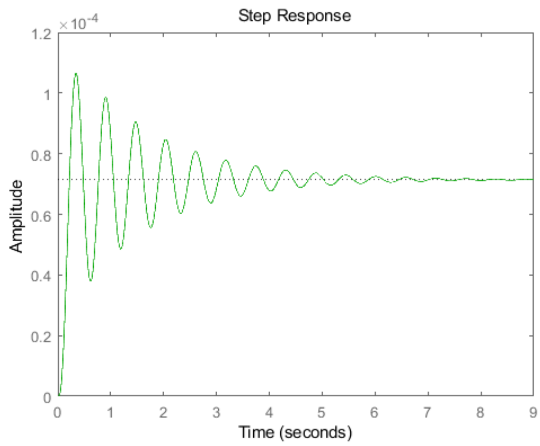

Fig 11. Step Response when A1=0.05

Question 7: Short summary and discussion

This report studies an electro-hydraulic system composed of DC motor and fixed displacement pump.

Overall, this MATLAB project is essentially a simulation of system response based on an almost real engineering scenario.

We start by writing a system of differential equations in terms of the transfer of different physical quantities, specifically the transform of different forms of energy, and then the system of differential equations is written in state variable form. Besides, by specifying the independent and dependent variables, we can get an open loop transfer function. By this stage, the main part of the question has been answered.

In fact, we think the hard part is how to get the exact differential equation and how to reduce the equation system to state variable form.

Appendix

```
%% Project
% Group Member: Xiangyu Han 20186102
%               Hongrui Yi    20186103
%               Yan Zhuang    20186105
% Initialize
clear;clc;close all; format long e
%% Constant parameters
R=2;
L=0.002;
J=0.005;
b=0.5;
kb=0.03;
kt=0.03;
D=0.0035;
m=2.5;
k=300;
c=0.25;
A1=0.025;
A2=0.015;
Ap1=0.025;
Ap2=0.010;
```

```
density=1000;

viscosity=0.00089;

d1=0.015;

L1=0.03;

g=9.81;

R1=128*viscosity*L1/pi/d1^4;


%% first-order (state variable) form

A = [-R/L, -kb/L, 0, 0, 0, 0;

kt/J, -b/J,-D*density*g/J, 0, 0, 0;

0, D/A1,-density*g/A1/R1, density*g/A1/R1, 0, - Ap1/A1;

0, 0, density*g/A2/R1, -density*g/A2/R1, 0, - Ap2/A2;

0, 0, 0, 0, 0, 1;

0, 0, Ap1*density*g/m, -Ap2*density*g/m, -k/m, - c/m;];


%% Find the open loop system transfer function

%denote y(t) = x3

% so y(t) = [0 0 0 0 1 0]*[Unknowns]';

b = [1/L; 0; 0; 0; 0; 0];

c = [0, 0, 0, 0, 1, 0];

sys1 = ss(A,b,c,0);

[num,den] = tfdata(sys1,'v'); root_zeros = roots(num); root_poles =
```

```
roots(den); tf_func = tf(num,den)


%% Plots
figure(1)
pzmap(sys1);
axis equal
figure(2)
step(sys1);
title('Step response for open loop system')
figure(3)
rlocus(sys1);
axis([-1500 500 -1000 1000])
axis equal
zoom on
figure(4)
rlocus(sys1);
axis([-120 20 -70 70])
axis equal zoom on


%% Part II prob1
% Select k0 making gain = 1.5e04;
value_num = polyval(num,-77.4);
```

```
value_den = polyval(den,-77.4); k0 = 100;
sys2 = k0;
sys3 = series(sys2,sys1);
sys4 = feedback(sys3,1);
figure(5)
step(sys4);
title('Step response for close loop system')

%% Part II prob2
% set z1 to z4
z1 = 4.50210380482392;
z2 = 59.7138897620699;
z3 = 82.5247594166687;
z4 = 5;
alpha = 1;% set alpha
s = tf('s');
sys = 3.87e+9 * ((s+z1)*(s+z2)*(s+z3)*(s+z4)) /
((s+alpha*z1)*(s+alpha*z2)*(s+alpha*z3)*(s+alpha*z4))
*       ((4.12*10^4*s+7.524*10^5) /
(s^6+1149*s^5+1.549e+5*s^4+6.1e+6*s^3+5.833e+7*s^2+9.
325e+8*s+3.511e+9));
% plot the step response and root locus
```

```
figure(6)
step(sys)
figure(7)
rlocus(sys)

%% Part II prob3
% set z1 to z4
z1 = 4.50210380482392;
z2 = 59.7138897620699;
z3 = 82.5247594166687;
z4 = 5;
% seperate Gc1 and Gc2
s = tf('s');
Gc1=k0*(s+z3)/((s+20*z3));
Gc2=(s+z1)*(s+z4)*(s+z2)/((s+20*z1)*(s+20*z4)*(s+20*z2))
;
sys = Gc1*sys1/(1+Gc1*sys1*Gc2);
% plot the step response and root locus
figure(8)
step(sys)
figure(9)
rlocus(sys)
```