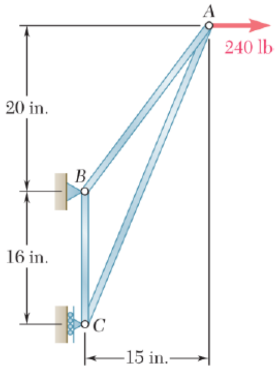
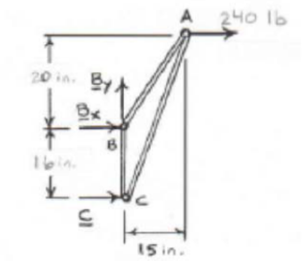
Homework4 Solutions



PROBLEM 6.1

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Entire truss:

$$+\uparrow \Sigma F_y = 0: \quad B_y = 0 \qquad \mathbf{B}_y = 0$$

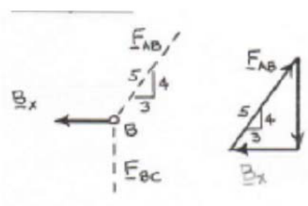
$$+\circlearrowleft \Sigma M_C = 0: \quad -B_x(16 \text{ in.}) - (240 \text{ lb})(36 \text{ in.}) = 0$$

$$B_x = -540 \text{ kN} \qquad \mathbf{B}_x = 540 \text{ lb} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C - 540 \text{ lb} + 240 \text{ lb} = 0$$

$$C = 300 \text{ lb} \qquad \mathbf{C} = 300 \text{ lb} \longrightarrow$$

Free body: Joint B:

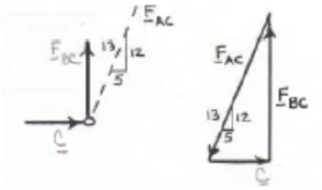


$$\frac{F_{AB}}{5} = \frac{F_{BC}}{4} = \frac{540 \text{ lb}}{3}$$

$$F_{AB} = 900 \text{ lb} \quad T \blacktriangleleft$$

$$F_{BC} = 720 \text{ lb} \quad T \blacktriangleleft$$

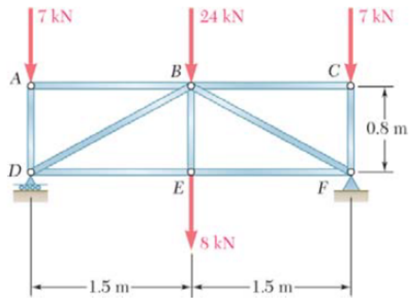
Free body: Joint *C*:



$$\frac{F_{AC}}{13} = \frac{F_{BC}}{12} = \frac{300 \text{ lb}}{5}$$

$F_{AC} = 780$ lb $\quad C$ ◀

$F_{BC} = 720$ lb $\quad$ (checks)

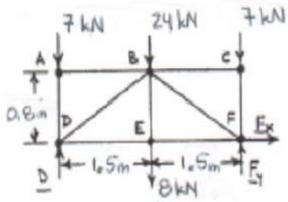


PROBLEM 6.4

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Reactions:



$$+\circlearrowleft \Sigma M_D = 0: \quad F_y(3\text{ m}) - (24\text{ kN} + 8\text{ kN})(1.5\text{ m}) - (7\text{ kN})(3\text{ m}) = 0$$

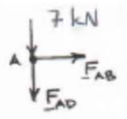
$$\mathbf{F}_y = 23.0\text{ kN}\uparrow$$

$$\Sigma F_x = 0: \quad \mathbf{F}_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad D - (7 + 24 + 8 + 7) + 23 = 0$$

$$\mathbf{D} = 23.0\text{ kN}\uparrow$$

Joint A:

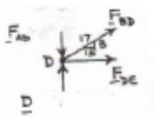


$$\Sigma F_x = 0: \quad F_{AB} = 0 \qquad F_{AB} = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -7 - F_{AD} = 0$$

$$F_{AD} = -7\text{ kN} \qquad F_{AD} = 7.00\text{ kN} \quad C \blacktriangleleft$$

Joint D:



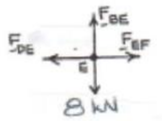
$$+\uparrow \Sigma F_y = 0: \quad -7 + 23.0 + \frac{8}{17}F_{BD} = 0$$

$$F_{BD} = -34.0\text{ kN} \qquad F_{BD} = 34.0\text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{15}{17}(-34.0) + F_{DE} = 0$$

$$F_{DE} = +30.0 \text{ kN} \qquad\qquad F_{DE} = 30.0 \text{ kN} \quad T \blacktriangleleft$$

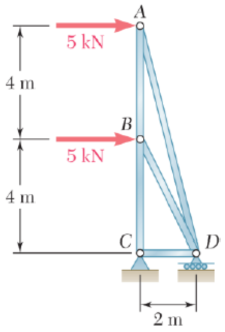
<u>Joint *E*</u>:



$$+\!\uparrow \Sigma F_y = 0: \quad F_{BE} - 8 = 0$$

$$F_{BE} = +8.00 \text{ kN} \qquad\qquad F_{BE} = 8.00 \text{ kN} \quad T \blacktriangleleft$$

Truss and loading symmetrical about ℄.

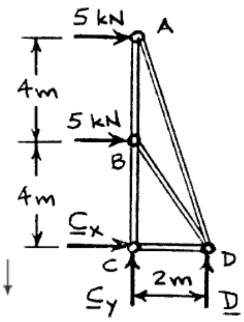


PROBLEM 6.8

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Entire truss:



$\xrightarrow{+} \Sigma F_x = 0\colon \quad C_x + 2(5 \text{ kN}) = 0$

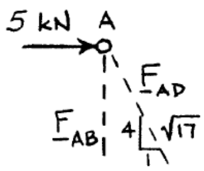
$C_x = -10 \text{ kN} \qquad \mathbf{C}_x = 10 \text{ kN} \longleftarrow$

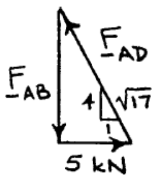
$+\circlearrowleft \Sigma M_C = 0\colon \quad D(2 \text{ m}) - (5 \text{ kN})(8 \text{ m}) - (5 \text{ kN})(4 \text{ m}) = 0$

$D = +30 \text{ kN} \qquad \mathbf{D} = 30 \text{ kN} \uparrow$

$+\uparrow \Sigma F_y = 0\colon \quad C_y + 30 \text{ kN} = 0 \quad C_y = -30 \text{ kN} \quad \mathbf{C}_y = 30 \text{ kN} \downarrow$

Free body: Joint A:



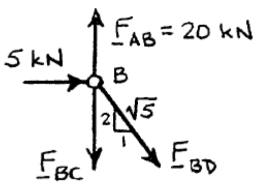


$$\frac{F_{AB}}{4} = \frac{F_{AD}}{\sqrt{17}} = \frac{5 \text{ kN}}{1}$$

$F_{AB} = 20.0 \text{ kN} \quad T$ ◀

$F_{AD} = 20.6 \text{ kN} \quad C$ ◀

Free body: Joint B:



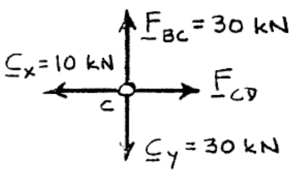
$\xrightarrow{+} \Sigma F_x = 0\colon \quad 5 \text{ kN} + \frac{1}{\sqrt{5}} F_{BD} = 0$

$F_{BD} = -5\sqrt{5} \text{ kN} \qquad F_{BD} = 11.18 \text{ kN} \quad C$ ◀

$+\uparrow \Sigma F_y = 0\colon \quad 20 \text{ kN} - F_{BC} - \frac{2}{\sqrt{5}}(-5\sqrt{5} \text{ kN}) = 0$

$F_{BC} = +30 \text{ kN} \qquad F_{BC} = 30.0 \text{ kN} \quad T$ ◀

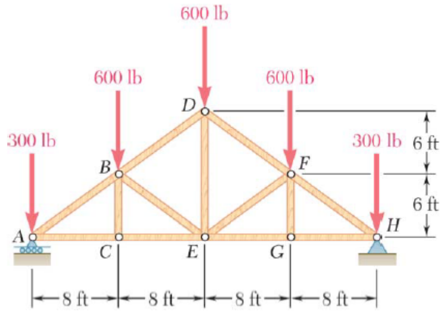
Free body: Joint C:



$\xrightarrow{+} \Sigma F_x = 0\colon \quad F_{CD} - 10 \text{ kN} = 0$

$F_{CD} = +10 \text{ kN} \qquad F_{CD} = 10.00 \text{ kN} \quad T$ ◀

$+\uparrow \Sigma F_y = 0\colon \quad 30 \text{ kN} - 30 \text{ kN} = 0 \quad \text{(checks)}$

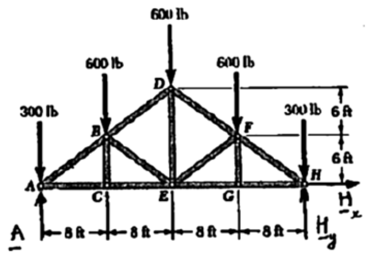


PROBLEM 6.12

Determine the force in each member of the Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



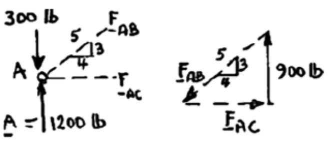
$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

Because of the symmetry of the truss and loading,

$$A = H_y = \frac{1}{2}\text{total load}$$

$$\mathbf{A} = \mathbf{H}_y = 1200 \text{ lb} \uparrow$$

Free body: Joint *A*:



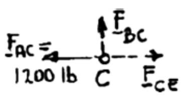
$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3}$$

$\mathbf{F}_{AB} = 1500 \text{ lb} \quad C$ ◀

$\mathbf{F}_{AC} = 1200 \text{ lb} \quad T$ ◀

Free body: Joint *C*:

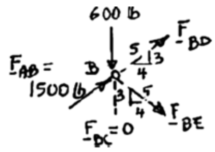
BC is a zero-force member.



$\mathbf{F}_{BC} = 0$

$\mathbf{F}_{CE} = 1200 \text{ lb} \quad T$ ◀

Free body: Joint *B*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{BD} + \frac{4}{5}F_{BC} + \frac{4}{5}(1500 \text{ lb}) = 0$$

or $$F_{BD} + F_{BE} = -1500 \text{ lb} \qquad (1)$$

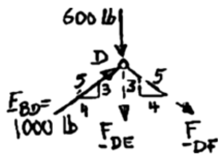
$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - \frac{3}{5}F_{BE} + \frac{3}{5}(1500 \text{ lb}) - 600 \text{ lb} = 0$$

or $$F_{BD} - F_{BE} = -500 \text{ lb} \qquad (2)$$

Add Eqs. (1) and (2): $2F_{BD} = -2000$ lb $\qquad F_{BD} = 1000$ lb C ◀

Subtract Eq. (2) from Eq. (1): $2F_{BE} = -1000$ lb $\qquad F_{BE} = 500$ lb C ◀

Free Body: Joint *D*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}(1000 \text{ lb}) + \frac{4}{5}F_{DF} = 0$$

$F_{DF} = -1000$ lb $\qquad F_{DF} = 1000$ lb C ◀

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}(1000 \text{ lb}) - \frac{3}{5}(-1000 \text{ lb}) - 600 \text{ lb} - F_{DE} = 0$$

$F_{DE} = +600$ lb $\qquad F_{DE} = 600$ lb T ◀

Because of the symmetry of the truss and loading, we deduce that

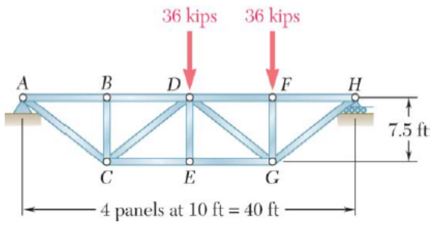
$F_{EF} = F_{BE} \qquad F_{EF} = 500$ lb C ◀

$F_{EG} = F_{CE} \qquad F_{EG} = 1200$ lb T ◀

$F_{FG} = F_{BC} \qquad F_{FG} = 0$ ◀

$F_{FH} = F_{AB} \qquad F_{FH} = 1500$ lb C ◀

$F_{GH} = F_{AC} \qquad F_{GH} = 1200$ lb T ◀



PROBLEM 6.45

Determine the force in members *BD* and *CD* of the truss shown.

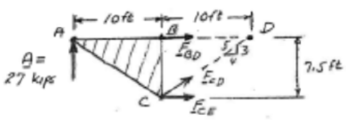
SOLUTION

<u>Reactions from Free body of entire truss</u>:

$$\mathbf{A} = \mathbf{A}_y = 27 \text{ kips}\uparrow \triangleleft$$

$$\mathbf{H} = 45 \text{ kips}\uparrow \triangleleft$$

We pass a section through members *BD*, *CD*, and *CE* and use the <u>free body shown</u>.

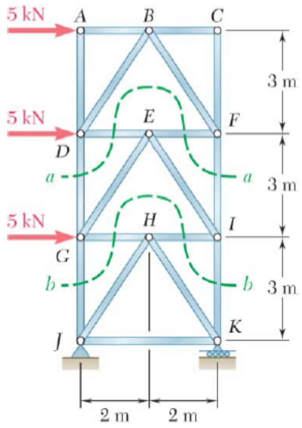


$$+\circlearrowleft \Sigma M_C = 0: \quad -F_{BD}(7.5 \text{ ft}) - (27 \text{ kips})(10 \text{ ft}) = 0$$

$$F_{BD} = -36.0 \text{ kips} \qquad F_{BD} = 36.0 \text{ kips} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 27 \text{ kips} + \left(\frac{3}{5}\right) F_{CD} = 0$$

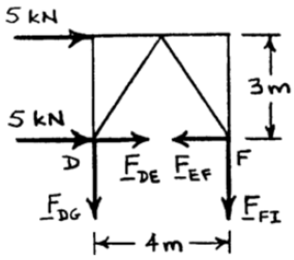
$$F_{CD} = -45.0 \text{ kips} \qquad F_{CD} = 45.0 \text{ kips} \quad C \blacktriangleleft$$



PROBLEM 6.61

Determine the force in members DG and FI of the truss shown. (*Hint:* Use section aa.)

SOLUTION

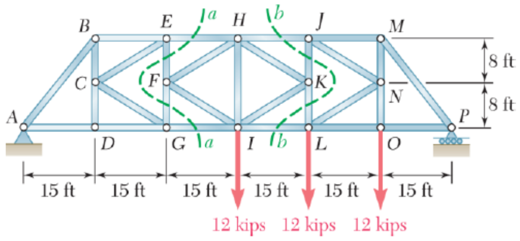


$$+\circlearrowleft \Sigma M_F = 0: \quad F_{DG}(4\text{ m}) - (5\text{ kN})(3\text{ m}) = 0$$

$$F_{DG} = +3.75\text{ kN} \qquad\qquad F_{DG} = 3.75\text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -3.75\text{ kN} - F_{FI} = 0$$

$$F_{FI} = -3.75\text{ kN} \qquad F_{FI} = 3.75\text{ kN} \quad C \blacktriangleleft$$

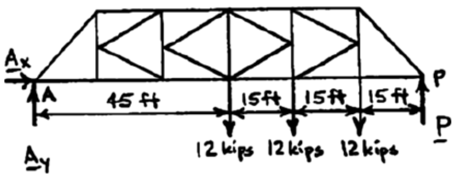


PROBLEM 6.63

Determine the force in members *EH* and *GI* of the truss shown. (*Hint:* Use section *aa*.)

SOLUTION

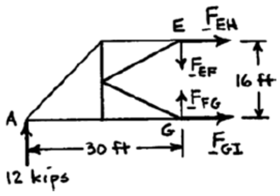
<u>Reactions</u>:



$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_P = 0: \quad 12 \text{ kips}(45 \text{ ft}) + 12 \text{ kips}(30 \text{ ft}) + 12 \text{ kips}(15 \text{ ft}) - A_y(90 \text{ ft}) = 0$$

$$\mathbf{A}_y = 12 \text{ kips} \uparrow$$

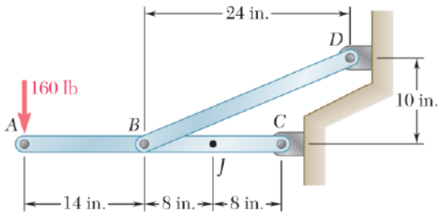


$$+\uparrow \Sigma F_y = 0: \quad 12 \text{ kips} - 12 \text{ kips} - 12 \text{ kips} - 12 \text{ kips} + P = 0 \qquad \mathbf{P} = 24 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_G = 0: \quad -(12 \text{ kips})(30 \text{ ft}) - F_{EH}(16 \text{ ft}) = 0$$

$$F_{EH} = -22.5 \text{ kips} \qquad F_{EH} = 22.5 \text{ kips} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{GI} - 22.5 \text{ kips} = 0 \qquad F_{GI} = 22.5 \text{ kips} \quad T \blacktriangleleft$$



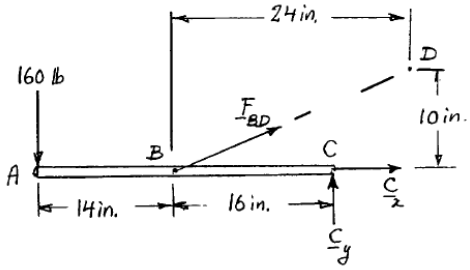
PROBLEM 6.76

Determine the force in member BD and the components of the reaction at C.

SOLUTION

We note that BD is a two-force member. The force it exerts on ABC, therefore, is directed along line BD.

Free body: ABC:



$$BD = \sqrt{(24)^2 + (10)^2} = 26 \text{ in.}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (160 \text{ lb})(30 \text{ in.}) - \left(\frac{10}{26} F_{BD}\right)(16 \text{ in.}) = 0$$

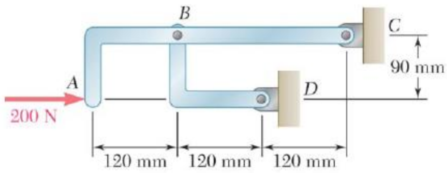
$$F_{BD} = +780 \text{ lb} \qquad F_{BD} = 780 \text{ lb} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma M_x = 0: \quad C_x + \frac{24}{26}(780 \text{ lb}) = 0$$

$$C_x = -720 \text{ lb} \qquad \mathbf{C}_x = 720 \text{ lb} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 160 \text{ lb} + \frac{10}{26}(780 \text{ lb}) = 0$$

$$C_y = -140.0 \text{ lb} \qquad \mathbf{C}_y = 140.0 \text{ lb} \downarrow \blacktriangleleft$$

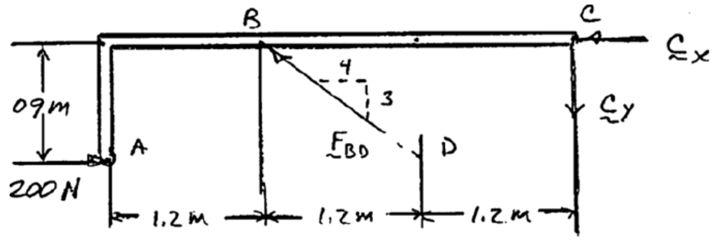


PROBLEM 6.77

For the frame and loading shown, determine the force acting on member *ABC* (*a*) at *B*, (*b*) at *C*.

SOLUTION

FBD ABC:



Note: *BD* is two-force member

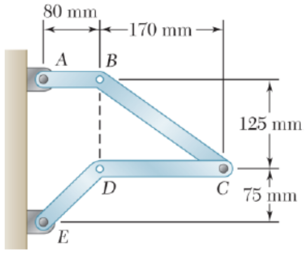
(*a*) $$\circlearrowleft \Sigma M_C = 0: \quad (0.09\text{ m})(200\text{ N}) - (2.4\text{ m})\left(\frac{3}{5}F_{BD}\right) = 0$$

$$\mathbf{F}_{BD} = 125.0\text{ N} \searrow 36.9^\circ$$ ◀

(*b*) $$\xrightarrow{} \Sigma F_x = 0: \quad 200\text{ N} - \frac{4}{5}(125\text{ N}) - C_x = 0 \qquad \mathbf{C}_x = 100\text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - C_y = 0 \qquad \mathbf{C}_y = \frac{3}{5}(125\text{ N}) = 75\text{ N} \downarrow$$

$$\mathbf{C} = 125.0\text{ N} \swarrow 36.9^\circ$$ ◀



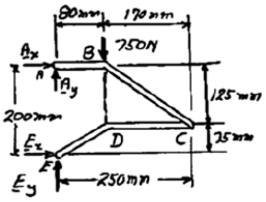
PROBLEM 6.85

Determine the components of the reactions at A and E if a 750-N force directed vertically downward is applied (*a*) at B, (*b*) at D.

SOLUTION

<u>Free body: Entire frame</u>:

The following analysis is valid for both parts (*a*) and (*b*) since position of load on its line of action is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -(750\text{ N})(80\text{ mm}) - A_x(200\text{ mm}) = 0$$

$$A_x = -300\text{ N} \quad \mathbf{A}_x = 300\text{ N} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x - 300\text{ N} = 0 \quad E_x = 300\text{ N} \quad \mathbf{E}_x = 300\text{ N} \longrightarrow$$

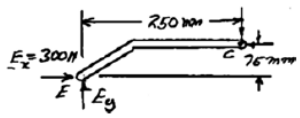
$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y - 750\text{ N} = 0 \qquad (1)$$

(*a*) <u>Load applied at *B*</u>.

<u>Free body: Member *CE*</u>:

CE is a two-force member. Thus, the reaction at E must be directed along CE.

$$\frac{E_y}{300\text{ N}} = \frac{75\text{ mm}}{250\text{ mm}} \quad E_y = 90\text{ N}\uparrow$$



From Eq. (1): $A_y + 90\text{ N} - 750\text{ N} = 0 \qquad A_y = 660\text{ N}\uparrow$

Thus, reactions are

$$\mathbf{A}_x = 300\text{ N} \longleftarrow, \ \mathbf{A}_y = 660\text{ N}\uparrow \blacktriangleleft$$

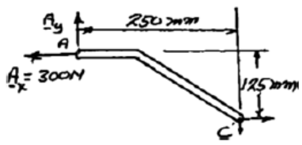
$$\mathbf{E}_x = 300\text{ N} \longrightarrow, \ \mathbf{E}_y = 90.0\text{ N}\uparrow \blacktriangleleft$$

(*b*) <u>Load applied at *D*</u>.

<u>Free body: Member *AC*</u>:

AC is a two-force member. Thus, the reaction at A must be directed along AC.

$$\frac{A_y}{300\text{ N}} = \frac{125\text{ mm}}{250\text{ mm}} \quad A_y = 150\text{ N}\uparrow$$



From Eq. (1):

$$A_y + E_y - 750\text{ N} = 0$$
$$150\text{ N} + E_y - 750\text{ N} = 0$$

$$E_y = 600\text{ N} \quad \mathbf{E}_y = 600\text{ N}\uparrow$$

Thus, reactions are

$$\mathbf{A}_x = 300\text{ N} \longleftarrow, \quad \mathbf{A}_y = 150.0\text{ N}\uparrow \blacktriangleleft$$

$$\mathbf{E}_x = 300\text{ N} \longrightarrow, \quad \mathbf{E}_y = 600\text{ N}\uparrow \blacktriangleleft$$