易弘睿
ME01
HW2

Chap 1

Exercise 1.3

1.

(a)
$$2A = 2\begin{pmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 2 & 8 \\ -4 & 0 & 2 \\ 2 & 4 & 4 \end{pmatrix}$$

(b)
$$A+B = \begin{pmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 1 & 6 \\ -5 & 1 & 2 \\ 3 & -2 & 3 \end{pmatrix}$$

(c)
$$2A-3B = \begin{pmatrix} 6 & 2 & 8 \\ -4 & 0 & 2 \\ 2 & 4 & 4 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 6 \\ -9 & 3 & 3 \\ 6 & -12 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 2 \\ 5 & -3 & -1 \\ -4 & 16 & 1 \end{pmatrix}$$

(d)
$$(2A)^T-(3B)^T = \begin{pmatrix} 6 & -4 & 2 \\ 2 & 0 & 4 \\ 8 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 3 & -9 & 6 \\ 0 & 3 & -12 \\ 6 & 3 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 5 & -4 \\ 2 & -3 & 16 \\ 2 & -1 & 1 \end{pmatrix}$$

(e)
$$AB = \begin{pmatrix} 3-3+8 & 0+1-16 & 6+1+4 \\ -2+0+2 & 0+0-4 & -4+0+1 \\ 1-6+4 & 0+2-8 & 2+2+2 \end{pmatrix} = \begin{pmatrix} 8 & -15 & 11 \\ 0 & -4 & -3 \\ -1 & -6 & 6 \end{pmatrix}$$

(f)
$$BA = \begin{pmatrix} 3+0+2 & 1+0+4 & 4+0+4 \\ -9-2+1 & -3+0+2 & -12+1+2 \\ 6+8+1 & 2+0+2 & 8-4+2 \end{pmatrix} = \begin{pmatrix} 5 & 5 & 8 \\ -10 & -1 & -9 \\ 15 & 4 & 6 \end{pmatrix}$$

(g)
$$A^T B^T = \begin{pmatrix} 3 & -2 & 1 \\ 1 & 0 & 2 \\ 4 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3+0+2 & -9-2+1 & 6+8+1 \\ 1+0+4 & -3+0+2 & 2+0+2 \\ 4+0+4 & -12+1+2 & 8-4+2 \end{pmatrix} = \begin{pmatrix} 5 & -10 & 15 \\ 5 & -1 & 4 \\ 8 & -9 & 6 \end{pmatrix}$$

(h)
$$(BA)^T = \begin{pmatrix} 5 & 5 & 8 \\ -10 & -1 & -9 \\ 15 & 4 & 6 \end{pmatrix}^T = \begin{pmatrix} 5 & -10 & 15 \\ 5 & -1 & 4 \\ 8 & -9 & 6 \end{pmatrix}$$

2. (a) $\begin{pmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{pmatrix}$

$= \begin{pmatrix} 6+5+4 & 3+15+1 \\ -4+0+8 & -2+0+2 \end{pmatrix}$

$= \begin{pmatrix} 15 & 19 \\ 4 & 0 \end{pmatrix}$

(b) Impossible

(c) $\begin{pmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{pmatrix} \times \begin{pmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{pmatrix}$

$= \begin{pmatrix} 3+4+12 & 2+4+15 \\ 0+1+16 & 0+1+20 \\ 0+0+8 & 0+0+10 \end{pmatrix}$

$= \begin{pmatrix} 19 & 21 \\ 17 & 21 \\ 8 & 10 \end{pmatrix}$

(d) $\begin{pmatrix} 4 & 6 \\ 2 & 1 \end{pmatrix} \times \begin{pmatrix} 3 & 15 \\ 4 & 16 \end{pmatrix}$

$= \begin{pmatrix} 12+24 & 60+96 \\ 6+4 & 30+16 \end{pmatrix}$

$= \begin{pmatrix} 36 & 156 \\ 10 & 46 \end{pmatrix}$

(e) Impossible

(f) $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 3 & 2 & 4 & 5 \end{pmatrix}$

$= \begin{pmatrix} 6 & 4 & 8 & 10 \\ -3 & -2 & -4 & -5 \\ 9 & 6 & 12 & 15 \end{pmatrix}$

4. (a) $\begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ 3 & -2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$

9. (a) $x_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{cases} x_1 + 2x_2 = 4 \\ x_1 - 2x_2 = 0 \end{cases}$

$\Rightarrow \left(\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & -2 & 0 \end{array}\right)$

$\Rightarrow \left(\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -4 & -4 \end{array}\right)$

$\Rightarrow -4x_2 = -4, \quad x_2 = 1$

$x_1 + 2 = 4, \quad x_1 = 2$

$\Rightarrow 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

(b) According to (a), there is no free variable in the echlon form of $x_1 a_1 + x_2 a_2 = b$.
In $Ax = b$, $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$,

So $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is the only solution.

(c) $x_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

$$\Rightarrow \begin{array}{l} x_1 + 2x_2 = -3 \\ x_1 - 2x_2 = -2 \end{array}$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 2 & -3 \\ 1 & -2 & -2 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & -4 & 1 \end{array}\right)$$

$$\Rightarrow \begin{array}{l} -4x_2 = 1,\ x_2 = -\frac{1}{4} \\ x_1 - \frac{1}{2} = -3,\ x_1 = -\frac{5}{2} \end{array}$$

$$\Rightarrow -\frac{5}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{1}{4}\begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

11. $b = a_1 + a_2 = a_2 + a_3$

$\Rightarrow a_1 = a_3$

$\Rightarrow b = a_1 + a_2 + x(a_1 - a_3)$

$= (1+x)a_1 + a_2 - x$

Hence, for $Ax = b$,

$x = (1+x,\ 1,\ -x)^T,\ x \in \mathbb{R}$,

there are infinitely many solutions.