Homework6 Solutions

PROBLEM 12.24

An airplane has a mass of 25 Mg and its engines develop a total thrust of 40 kN during take-off. If the drag **D** exerted on the plane has a magnitude $D = 2.25v^2$, where v is expressed in meters per second and D in newtons, and if the plane becomes airborne at a speed of 240 km/h, determine the length of runway required for the plane to take off.

SOLUTION

$$F = ma: \quad 40\times 10^3 \text{ N} - 2.25v^2 = (25\times 10^3 \text{ kg})a$$

Substituting

$$a = v\frac{dv}{dx}: \quad 40\times 10^3 - 2.25\ v^2 = (25\times 10^3)\ v\frac{dv}{dx}$$

$$\int_0^{x_1} dx = \int_0^{v_1} \frac{(25\times 10^3)v\,dv}{40\times 10^3 - 2.25v^2}$$

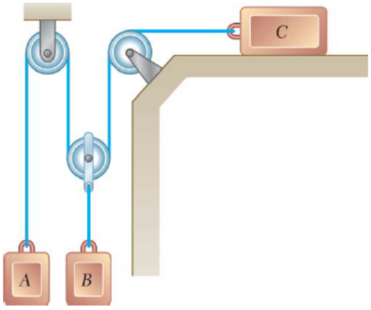
$$x_1 = -\frac{25\times 10^3}{2(2.25)}[\ln(40\times 10^3 - 2.25v^2)]_0^{v_1}$$

$$= \frac{25\times 10^3}{4.5}\ln\frac{40\times 10^3}{40\times 10^3 - 2.25v_1^2}$$

For $v_1 = 240 \text{ km/h} = 66.67 \text{ m/s}$

$$x_1 = \frac{25\times 10^3}{4.5}\ln\frac{40\times 10^3}{40\times 10^3 - 2.25(66.67)^2} = 5.556\ln 1.333$$

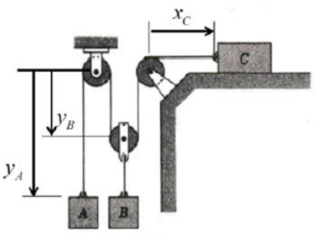
$$= 1.5982\times 10^3 \text{ m} \qquad x_1 = 1.598 \text{ km} \blacktriangleleft$$



PROBLEM 12.33

Knowing that μ = 0.30, determine the acceleration of each block when m_A = 5 kg, m_B = 30 kg, and m_C = 15 kg.

SOLUTION



Given: $\mu = 0.30,\ g = 9.81\ \text{m/s}^2$

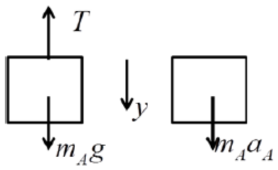
$m_A = 5\ \text{kg},\ m_B = 30\ \text{kg},\ m_C = 15\ \text{kg}$

Kinematics: $y_A + 2y_B + x_C = \text{constant}$

$$v_A + 2v_B + v_C = 0$$

$$a_A + 2a_B + a_C = 0 \qquad (1)$$

Free Body Diagram of Block A:



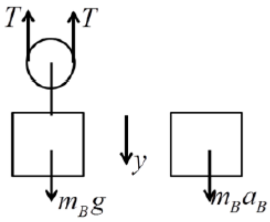
Equation of Motion:

$$\sum F_y = m_A a_A$$

$$m_A g - T = m_A a_A$$

$$a_A = g - \frac{T}{m_A} \qquad (2)$$

Free Body Diagram of Block B:



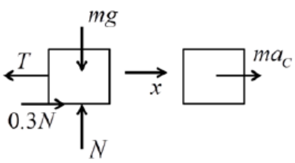
Equation of Motion:

$$\sum F_y = m_B a_B$$

$$m_B g - 2T = m_B a_B$$

$$a_B = g - \frac{2T}{m_B} \qquad (3)$$

Free Body Diagram of Block B:



Equations of Motion:

$$\sum F_y = 0 \qquad \sum F_x = m_C a_C$$

$$N = m_C g \qquad 0.3 m_C g - T = m_C a_C$$

PROBLEM 12.53

Tilting trains, such as the *American Flyer* which will run from Washington to New York and Boston, are designed to travel safely at high speeds on curved sections of track which were built for slower, conventional trains. As it enters a curve, each car is tilted by hydraulic actuators mounted on its trucks. The tilting feature of the cars also increases passenger comfort by eliminating or greatly reducing the side force $\mathbf{F}_s$ (parallel to the floor of the car) to which passengers feel subjected. For a train traveling at 100 mi/h on a curved section of track banked through an angle $\theta = 6°$ and with a rated speed of 60 mi/h, determine (*a*) the magnitude of the side force felt by a passenger of weight W in a standard car with no tilt ($\phi = 0$), (*b*) the required angle of tilt ϕ if the passenger is to feel no side force. (See Sample Problem 12.7 for the definition of rated speed.)

SOLUTION

Rated speed: $\quad v_R = 60 \text{ mi/h} = 88 \text{ ft/s}, \;\; 100 \text{ mi/h} = 146.67 \text{ ft/s}$

From Sample Problem 12.6,

$$v_R^2 = g\rho \tan\theta$$

or

$$\rho = \frac{v_R^2}{g\tan\theta} = \frac{(88)^2}{32.2\tan 6°} = 2288 \text{ ft}$$

Let the x-axis be parallel to the floor of the car.

$$+\swarrow \Sigma F_x = ma_x: \quad F_s + W\sin(\theta+\phi) = ma_n\cos(\theta+\phi)$$

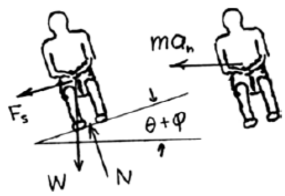
$$= \frac{mv^2}{\rho}\cos(\theta+\phi)$$

(*a*) $\quad \phi = 0.$

$$F_s = W\left[\frac{v^2}{g\rho}\cos(\theta+\phi) - \sin(\theta+\phi)\right]$$

$$= W\left[\frac{(146.67)^2}{(32.2)(2288)}\cos 6° - \sin 6°\right]$$

$$= 0.1858W$$



$F_s = 0.1858W$ ◀

(*b*) $\quad$ For $F_s = 0$,

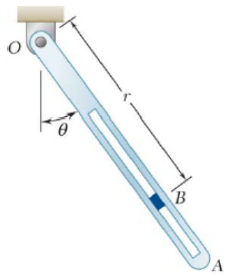
$$\frac{v^2}{g\rho}\cos(\theta+\phi) - \sin(\theta+\phi) = 0$$

$$\tan(\theta+\phi) = \frac{v^2}{g\rho} = \frac{(146.67)^2}{(32.2)(2288)} = 0.29199$$

$$\theta + \phi = 16.28°$$

$$\phi = 16.28° - 6°$$

$\phi = 10.28°$ ◀



PROBLEM 12.69

A 0.5-kg block B slides without friction inside a slot cut in arm OA which rotates in a vertical plane. The rod has a constant angular acceleration $\ddot{\theta} = 10 \text{ rad/s}^2$. Knowing that when $\theta = 45°$ and $r = 0.8$ m the velocity of the block is zero, determine at this instant, (*a*) the force exerted on the block by the arm, (*b*) the relative acceleration of the block with respect to the arm.

SOLUTION

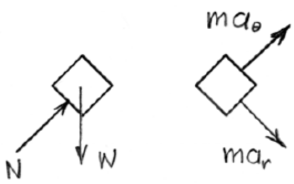
Given: $\theta = 45°, \quad r = 0.8 \text{ m}, \quad \ddot{\theta} = 10 \text{ rad/s}^2$
$m = 0.5 \text{ kg}, \quad W = mg$

Using Radial and Transverse Components:

$$v_r = \dot{r} = 0, \quad v_\theta = r\dot{\theta} = 0$$

$$\dot{\theta} = 0$$

FBD of Block B



(*a*)
$$\Sigma F_\theta = ma_\theta: \quad N - W\cos 45° = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

$$N = mg\cos 45° + m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

$$= (0.5)(9.81)\cos 45° + 0.5\left[(0.8)(10) + 0\right]$$

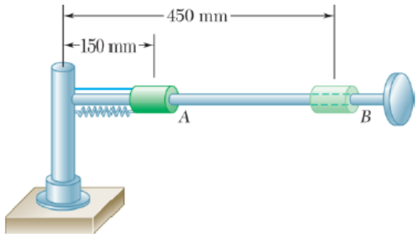
$$= 7.468$$

$N = 7.47 \text{ N} \measuredangle 45°$ ◀

(*b*)
$$\Sigma F_r = ma_r: \quad mg\sin 45° = m\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$\ddot{r} = \frac{mg}{m}\sin 45° + r\dot{\theta}^2$$
$$= g\sin 45° + r\dot{\theta}^2$$
$$= (9.81)\sin 45° + 0$$
$$= 6.937 \text{ m/s}^2$$

$\mathbf{a}_{B/\text{rod}} = 6.94 \text{ m/s}^2 \nwarrow 45°$ ◀

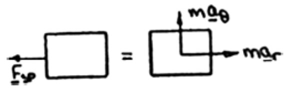


PROBLEM 12.90

A 1 kg collar can slide on a horizontal rod, which is free to rotate about a vertical shaft. The collar is initially held at A by a cord attached to the shaft. A spring of constant 30 N/m is attached to the collar and to the shaft and is undeformed when the collar is at A. As the rod rotates at the rate $\dot{\theta} = 16$ rad/s, the cord is cut and the collar moves out along the rod. Neglecting friction and the mass of the rod, determine (*a*) the radial and transverse components of the acceleration of the collar at A, (*b*) the acceleration of the collar relative to the rod at A, (*c*) the transverse component of the velocity of the collar at B.

SOLUTION

First note $\quad F_{sp} = k(r - r_A)$



(*a*) $\quad F_\theta = 0$ and at A, $\quad F_r = -F_{sp} = 0$

$(a_A)_r = 0$ ◀

$(a_A)_\theta = 0$ ◀

(*b*) $\quad \xrightarrow{+} \Sigma F_r = ma_r$: $\quad -F_{sp} = m(\ddot{r} - r\dot{\theta}^2)$

Noting that $\quad a_{\text{collar/rod}} = \ddot{r}$, $\quad$ we have at A

$$0 = m[a_{\text{collar/rod}} - (150\text{ mm})(16\text{ rad/s})^2]$$

$$a_{\text{collar/rod}} = 38400\text{ mm/s}^2$$

or $\quad (a_{\text{collar/rod}})_A = 38.4\text{ m/s}^2$ ◀

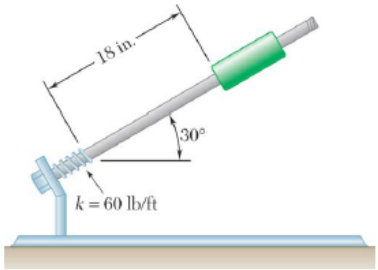
(*c*) After the cord is cut, the only horizontal force acting on the collar is due to the spring. Thus, angular momentum about the shaft is conserved.

$$r_A m(v_A)_\theta = r_B m(v_B)_\theta \quad \text{where} \quad (v_A)_\theta = r_A \dot{\theta}_0$$

Then $\quad (v_B)_\theta = \dfrac{150\text{ mm}}{450\text{ mm}}[(150\text{ mm})(16\text{ rad/s})] = 800\text{ mm/s}$

or $\quad (v_B)_\theta = 0.800\text{ m/s}$ ◀

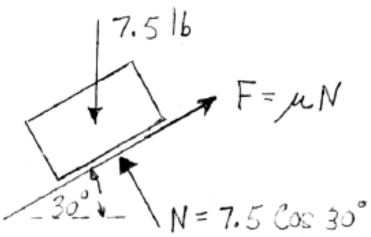
Homework7 Solutions



PROBLEM 13.29

A 7.5-lb collar is released from rest in the position shown, slides down the inclined rod, and compresses the spring. The direction of motion is reversed and the collar slides up the rod. Knowing that the maximum deflection of the spring is 5 in., determine (*a*) the coefficient of kinetic friction between the collar and the rod, (*b*) the maximum speed of the collar.

SOLUTION



Position 1, initial condition

Position 2, spring deflected 5 inches

Position 3, initial contact of spring with collar

$$U_{1\to 2} = \underbrace{-F\left(\frac{18+5}{12}\right)}_{\text{(Friction)}} \underbrace{-\frac{1}{2}(60)\left(\frac{5}{12}\right)^2}_{\text{(Spring)}} + \underbrace{7.5\left(\frac{18+5}{12}\right)\sin 30°}_{\text{(Gravity)}}$$

$$T_1 = T_2 = 0, \therefore U_{1-2} = 0$$

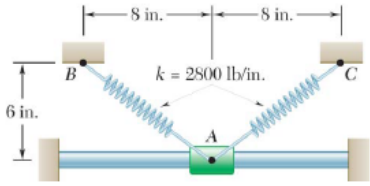
$$0 = -\mu(7.5)(0.866)\left(\frac{23}{12}\right) - \frac{1}{2}(60)\left(\frac{5}{12}\right)^2 + 7.5\left(\frac{23}{12}\right)(0.5)$$

(*a*) $\mu = 0.1590$ ◀

(*b*) Max speed occurs just before contact with the spring

$$U_{1\to 3} = -\mu(7.5)(0.866)\left(\frac{18}{12}\right) + 7.5\left(\frac{18}{12}\right)(0.5) = T_3 = \frac{1}{2}\left(\frac{7.5}{32.2}\right)v_{max}^2$$

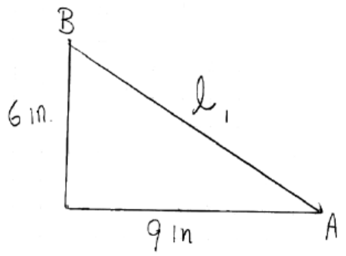
$v_{max} = 5.92$ ft/s ◀



PROBLEM 13.58

A 4-lb collar can slide without friction along a horizontal rod and is in equilibrium at A when it is pushed 1 in. to the right and released from rest. The springs are undeformed when the collar is at A and the constant of each spring is 2800 lb/in. Determine the maximum velocity of the collar.

SOLUTION

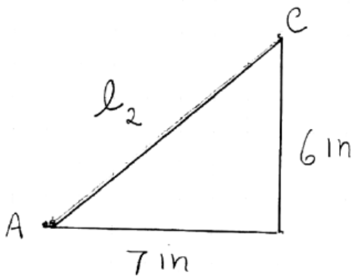


$$\ell_1 = \sqrt{6^2 + 9^2} = 10.817 \text{ in.}$$

$$\ell_0 = \sqrt{(6)^2 + (8)^2} = 10 \text{ in.} = 0.8333 \text{ ft}$$

Stretch = 10.817 − 10 = 0.817 in.

$$S_1 = 0.06805 \text{ ft}$$



$$\ell_2 = \sqrt{(7)^2 + (6)^2} = 9.215 \text{ in.}$$

Stretch = 9.2195 − 10 = −0.7805 in.

$$S_2 = 0.06504 \text{ ft}$$

$$T_1 = 0, \; V_2 = 0$$

$$T_2 = \frac{1}{2} m v_2^2$$

$$= \frac{1}{2}\left(\frac{4}{32.2}\right) v_2^2$$

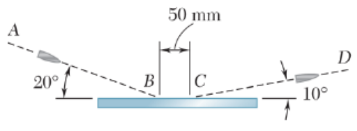
$$V_1 = \frac{1}{2}(33{,}600 \text{ lb/ft})\left(S_1^2 + S_2^2\right)$$

$$V_1 = (16{,}800)(0.008861) = 148.86 \text{ ft} \cdot \text{lb}$$

$$\cancel{T_1} + V_1 = T_2 + \cancel{V_2}$$

$$148.86 = \frac{1}{2}\left(\frac{4}{32.2}\right) v_2^2$$

$$v_2^2 = 2396.7 \qquad v_2 = 49.0 \text{ ft/s} \blacktriangleleft$$



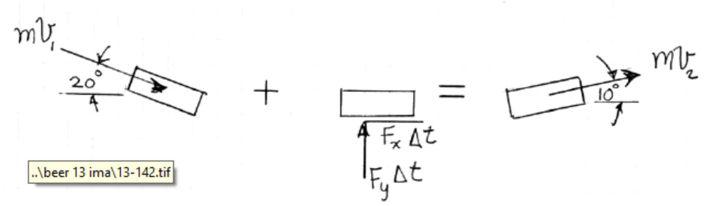
PROBLEM 13.144

A 28-g steel-jacketed bullet is fired with a velocity of 650 m/s toward a steel plate and ricochets along path *CD* with a velocity 500 m/s. Knowing that the bullet leaves a 50-mm scratch on the surface of the plate and assuming that it has an average speed of 600 m/s while in contact with the plate, determine the magnitude and direction of the impulsive force exerted by the plate on the bullet.

SOLUTION

Given: $m = 0.028$ kg, $v_1 = 650$ m/s, $v_2 = 500$ m/s

Impulse-momentum diagram of the bullet:



Impulse momentum in the x-dir $\longrightarrow$

$$mv_1 \cos 20° - F_x \Delta t = mv_2 \cos 10°$$

So,

$$F_x \Delta t = mv_1 \cos 20° - mv_2 \cos 10°$$

$$= 0.028\,(650)\cos 20° - 0.028\,(500)\cos 10° = 3.3151 \text{ N}\cdot\text{s}$$

y-dir $\uparrow$

$$-mv_1 \sin 20° + F_y \Delta t = mv_2 \sin 10°$$

So,

$$F_y \Delta t = mv_2 \sin 10° + mv_1 \sin 20°$$

$$= 0.028\,(500)\sin 10° + 0.028\,(650)\sin 20° = 8.6558 \text{ N}\cdot\text{s}$$

We need Δt. The average velocity is 600 m/s

$$\Delta x = v_{\text{ave}}\,\Delta t;\ \Delta t = \frac{\Delta x}{v_{\text{ave}}} = \frac{0.05 \text{ m}}{600 \text{ m/s}} = 83.33 \times 10^{-6} \text{ s}$$

So

$$\left.\begin{aligned} F_x &= \frac{3.3151}{83.33 \times 10^{-6}} = 39.78 \text{ kN} \\ F_y &= \frac{8.6558}{83.33 \times 10^{-6}} = 103.87 \text{ kN} \end{aligned}\right\}$$

$\bar{F} = 111.2 \text{ kN} \measuredangle 69.0°$ ◀

PROBLEM 13.163

At an amusement park there are 200-kg bumper cars A, B, and C that have riders with masses of 40 kg, 60 kg, and 35 kg respectively. Car A is moving to the right with a velocity $\mathbf{v}_A = 2$ m/s when it hits stationary car B. The coefficient of restitution between each car is 0.8. Determine the velocity of car C so that after car B collides with car C the velocity of car B is zero.

SOLUTION

Assume that each car with its rider may be treated as a particle. The masses are:

$$m_A = 200 + 40 = 240 \text{ kg}$$
$$m_B = 200 + 60 = 260 \text{ kg}$$
$$m_C = 200 + 35 = 235 \text{ kg}$$

Assume velocities are positive to the right. The initial velocities are:

$$v_A = 2 \text{ m/s}, \quad v_B = 0, \quad v_C = ?$$

First impact. Car A hits car B. Let v'_A and v'_B be the velocities after this impact. Conservation of momentum for cars A and B.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$
$$(240)(2) + 0 = 240 v'_A + 260 v'_B \tag{1}$$

Coefficient of restitution for cars A and B.

$$v'_B - v'_A = e(v_A - v_B) = (0.8)(2 - 0) = 1.6 \tag{2}$$

Solving Eqs. (1) and (2) simultaneously,

$$v'_A = 0.128 \text{ m/s}$$
$$v'_B = 1.728 \text{ m/s}$$
$$\mathbf{v}'_A = 0.128 \text{ m/s} \longrightarrow$$
$$\mathbf{v}'_B = 1.728 \text{ m/s} \longrightarrow$$

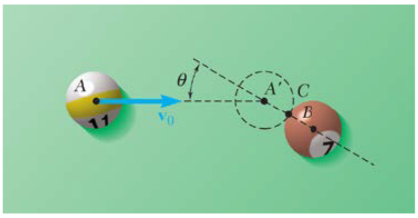
Second impact. Cars B and C hit. Let v''_B and v''_C be the velocities after this impact. $v''_B = 0$. Coefficient of restitution for cars B and C.

$$v''_C - v''_B = e(v'_B - v_C) = (0.8)(1.728 - v_C)$$
$$v''_C = 1.3824 - 0.8 v_C$$

Conservation of momentum for cars B and C.

$$m_B v'_B + m_C v_C = m_B v''_B + m_C v''_C$$
$$(260)(1.728) + 235 v_C = (260)(0) + (235)(1.3824 - 0.8 v_C)$$
$$(235)(1.8) v_C = (235)(1.3824) - (260)(1.728)$$
$$v_C = -0.294 \text{ m/s} \qquad \mathbf{v}_C = 0.294 \text{ m/s} \longleftarrow \blacktriangleleft$$

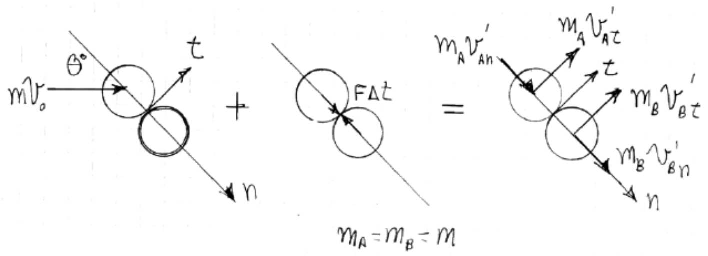
Note: There will be another impact between cars A and B.



PROBLEM 13.164

Two identical billiard balls can move freely on a horizontal table. Ball A has a velocity $\mathbf{v}_0$ as shown and hits ball B, which is at rest, at a Point C defined by $\theta = 45°$. Knowing that the coefficient of restitution between the two balls is $e = 0.8$ and assuming no friction, determine the velocity of each ball after impact.

SOLUTION



Ball A: *t-dir*

$$\cancel{m} v_0 \sin\theta = \cancel{m} v'_{At} \Rightarrow v'_{At} = v_0 \sin\theta$$

Ball B: *t-dir*

$$0 = m_B v'_{Bt} \Rightarrow v'_{Bt} = 0$$

Balls $A + B$: *n-dir*

$$\cancel{m} v_0 \cos\theta + 0 = \cancel{m} v'_{An} + \cancel{m} v'_{Bn} \qquad (1)$$

Coefficient of restitution

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$

$$v'_{Bn} - v'_{An} = e(v_0 \cos\theta - 0) \qquad (2)$$

Solve (1) and (2)

$$v'_{An} = v_0\left(\frac{1-e}{2}\cos\theta\right); \quad v'_{Bn} = v_0\left(\frac{1+e}{2}\right)\cos\theta$$

With numbers

$$e = 0.8; \quad \theta = 45°$$

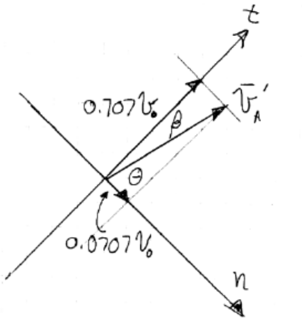
$$v'_{At} = v_0 \sin 45° = 0.707 v_0$$

$$v'_{An} = v_0\left(\frac{1-0.8}{2}\cos 45°\right) = 0.0707\, v_0$$

$$v'_{Bt} = 0$$

$$v'_{Bn} = v_0\left(\frac{1+0.8}{2}\right)\cos 45° = 0.6364 v_0$$

(*A*)

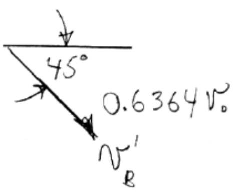


$$|v''_A| = [(0.707v_0)^2 + (0.0707v_0)^2]^{\frac{1}{2}} = 0.711v_0$$

$$\beta = \tan^{-1}\left(\frac{0.0707}{0.707}\right) = 5.7106°$$

So $\theta = 45 - 5.7106 = 39.3°$

(*B*)



$\vec{v}'_A = 0.711v_0$ ⦣39.3° ◀

$\vec{v}'_B = 0.636v_0$ ⦪45° ◀