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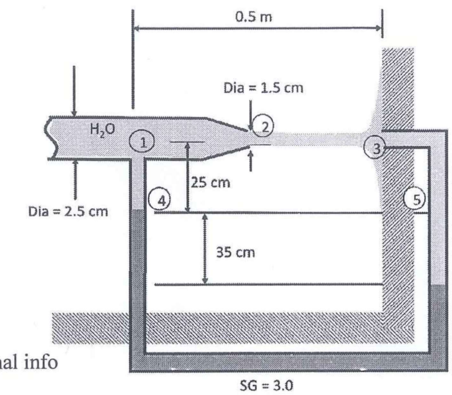
MECH3011 Test #1, Summer 2021 Name: Yi, Hongmi

1. Multiple choice questions: [5pts each]

Consider the diagram showing a jet impinging on a wall. The pressure tap in the wall and the pressure tap in the pipe are connected with a manometer as shown.

- Fluid in the pipe is water, $\rho_{water} = 1000$ kg/m^3
- All pipes have circular cross section



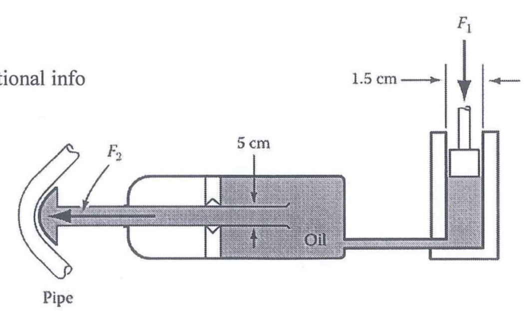
a i. Circle the correct statement
a) $P_4 - P_1 = P_5 - P_3$
b) $P_4 - P_1 > P_5 - P_3$
c) $P_4 - P_1 < P_5 - P_3$
d) cannot be determined w/o additional info

b ii. Circle the correct statement
a) $P_3 > P_2 > P_1$
b) $P_3 > P_1 > P_2$
c) $P_3 = P_2 < P_1$
d) $P_1 > P_2$, P_3 cannot be determined w/o additional info

A hydraulic jack is used to bend pipe as shown. Assume the height variation within the system is negligible.

X a iii. Circle the correct statement
a) $P_2 < P_1$
b) $P_2 > P_1$
c) $P_2 = P_1$
d) cannot be determined w/o additional info

X C iv. Circle the correct statement
a) $F_2 < F_1$
b) $F_2 > F_1$
c) $F_2 = F_1$
d) cannot be determined w/o additional info



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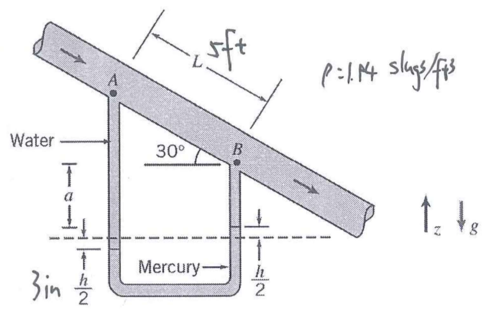
MECH3011 Test #1, Summer 2021 Name: [illegible] Yi, Hongmei
M131[illegible]6P

2. Water flows downward along a pipe that is inclined at 30° below the horizontal, as shown. Pressure difference $p_A - p_B$ is due partly to gravity and partly to friction. Derive an algebraic expression for the pressure difference in psi. Evaluate the pressure difference if L = 5 ft and h = 6 in. ρ_{water} = 1.94 slugs/ft³, SG_{Hg} = 13.55, g = 32.2 ft/s². [25pts]

· Bernoulli:

$$\frac{p_A}{\rho_{H_2O}} + \frac{\cancel{V_A^2}}{2} + gz_A = \frac{p_B}{\rho_{H_2O}} + \frac{\cancel{V_B^2}}{2} + gz_B$$

$$\frac{1}{\rho_{H_2O}}(p_A - p_B) = g z_B - z_A$$



$\rho = 1.94$ slugs/ft³

$$p_A + \rho_{water}\, g(L\sin 30° + a + h) = p_B + \rho_{water}\, g \cdot a + SG_{Hg} \cdot \rho_{water}\, g\, h$$

$$\Rightarrow p_A - p_B = \rho_{water}\, g(a + SG_{Hg}\, h - L\sin 30° - a - h)$$

$$= \rho_{water}\, g\,(SG_{Hg} \cdot h - L\sin 30° - h)$$

$$= 1.94\ \text{slugs/ft}^3 \cdot 32.2\ \text{lbf/slug} \cdot \left(13.55 \times \left(6\text{in} \cdot \frac{1\text{ft}}{12\text{in}}\right) - 5\text{ft} \times \frac{1}{2} - 6\text{in} \cdot \frac{1\text{ft}}{12\text{in}}\right)$$

$$= \boxed{235.8167\ \cancel{\text{psi}}}\ \text{lbf/ft}^2$$

$$= 235.8167\ \text{lbf/ft}^2 \left[\frac{1\text{ft}}{12\text{in}}\right]^2$$

$$= \boxed{1.6376\ \text{psi}}$$

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MECH3011 Test #1, Summer 2021 Name: 易弘青 Yi, Hongmi

3. The radial component of velocity in an incompressible two-dimensional flow is given by $V_r = 3r - 2r^2 \cos(\theta)$. Determine the general expression for the θ component of velocity. [25pts]

Assumption: incompressible : $\rho =$ constant.

2D : $z = 0$

$V_r = 3r - 2r^2 \cos(\theta)$

Continuity Equation:

$$\frac{1}{r}\frac{\partial(r\cancel{\rho} V_r)}{\partial r} + \frac{1}{r}\frac{\partial(\cancel{\rho} V_\theta)}{\partial \theta} + \cancel{\frac{\partial(\rho V_z)}{\partial z}} + \cancel{\frac{\partial \rho}{\partial t}} = 0$$

$$\Rightarrow \frac{1}{r}\cdot\frac{\partial(r \cdot V_r)}{\partial r} + \frac{1}{r}\frac{\partial(V_\theta)}{\partial \theta} = 0$$

$$\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}(3r^2 - 2r^3\cos\theta) + \frac{1}{r}\frac{\partial}{\partial\theta}V_\theta = 0$$

$$\Rightarrow \frac{\partial}{\partial\theta}V_\theta = -(6r - 6r^2\cos\theta) = 6r^2\cos\theta - 6r$$

$$\Rightarrow V_\theta = \int(6r^2\cos\theta - 6r)\,d\theta$$

$$= \boxed{6r^2\sin\theta - 6r\theta + C}$$, where C is constant.

$\because r = 0, V_\theta = 0$

$\therefore C = 0$

$\therefore V_\theta = \boxed{6r^2\sin\theta - 6r\theta}$ + f(r)

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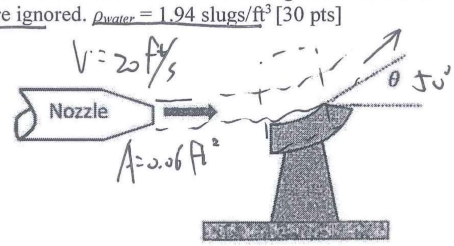
4. A horizontal jet of water from a stationary nozzle with a speed of 20 ft/sec, strikes a vane and is turned through an angle of $\theta = 50°$. The nozzle has an exit area of 0.06 ft^2. Find the anchoring force needed to hold the vane stationary if gravity and viscous effects are ignored. $\rho_{water} = 1.94$ slugs/ft^3 [30 pts]



Assumption: incompressible, S.S.

Conservation of mass:

$$\cancel{\frac{\partial}{\partial t}\int_{cv}\rho\, d\forall} + \int_{cs}\rho\vec{V}\cdot d\vec{A} = 0$$

$\Rightarrow \sum V\cdot A = 0$

$\Rightarrow -V\cdot A + V_{out}\cdot A_{out} = 0$

$\Rightarrow V_{out}\cdot A_{out} = V\cdot A = 20\,ft/s \cdot 0.06\,ft^2 = 1.2\,ft^3/s = \dot{Q}$, $V_{out} = V$.

Momentum Equation:

$$F_x = \cancel{F_{sx}} + \cancel{F_{Bx}} + R_x = \cancel{\frac{\partial}{\partial t}\int_{cv}\vec{V}\rho\, d\forall} + \int_{cs}\vec{V}\rho\vec{V}\cdot d\vec{A}$$

$$= (V_{out}(\cos\theta - 1))\cdot\dot{m}$$

$\dot{m} = \rho_{water}\dot{Q} = 1.94\,slugs/ft^3 \cdot 1.2\,ft^3/s \cdot \frac{32.2\,lbm}{1\,slug} = 74.9616\,lbm/s = 2.328\,slugs/s$

$R_x = 20\,ft/s \times (\cos 50° - 1) \times \cancel{74.9616\,lbm/s}$ $2.328\,slugs/s$

$= \cancel{29.928\,lbf} \rightarrow$

~~$R_y = 20\,ft/s\,\times$~~

$= -16.6318\,lbf \leftarrow$

$R_y = 20\,ft/s \times \sin 50° \times 2.328\,slugs/s$

$= 35.667\,lbf \uparrow$

$R = \sqrt{R_x^2 + R_y^2} = \boxed{39.3542\,lbf}$

$\theta = \arctan\frac{35.667}{16.6318} = \boxed{65°}$