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1. For each of the periodic waveforms below, all having period $T_0 = 2$ sec.,

(a) $x(t)=e^{-t}$, $0 \le t < 1$; $x(t)=0$, $1 \le t < 2$

(b) $x(t)=e^{-t}$, $0 \le t \le 1$; $x(t)=-e^{-t}$, $1 \le t \le 2$

(c) $x(t) = e^{t} - 1$, $0 \le t < 1$; $x(t) = 0$, $1 \le t < 2$

(d) $x(t) = 1$, $0 \le t < 0.5$; $x(t) = 0.5$, $0.5 \le t < 1$; $x(t) = 0$, $1 \le t < 2$

(b) $C_k = \frac{1}{T_0}\int_{T_0} x(t)\cdot e^{-jk\omega_0 t}dt$

$= \frac{1}{2}\int_0^1 e^{-t}e^{-jk\frac{2\pi}{2}t}dt + \frac{1}{2}\int_1^2 -e^{-t}e^{-jk\pi t}$

$= \frac{1}{2}\int_0^1 e^{(-1-jk\pi)t}dt - \frac{1}{2}\int_1^2 e^{(-1-jk\pi)t}dt$

$= -\frac{1}{2(1+jk\pi)}\cdot e^{(-1-jk\pi)t}\Big]_0^1 + \frac{1}{2(1+jk\pi)}e^{(-1-jk\pi)t}\Big]_1^2$

$= -\frac{1}{2(1+jk\pi)}(e^{-1-jk\pi}-1) + \frac{1}{2(1+jk\pi)}[e^{(-1-jk\pi)}-e^{(-1-jk\pi)}]$

$= \frac{e^{2(-1-jk\pi)}-2e^{-1-jk\pi}+1}{2(1+jk\pi)}$

(d) $C_k = \frac{1}{T_0}\int_{T_0} x(t)\cdot e^{-jk\omega_0 t}dt$

$= \frac{1}{2}\int_0^{0.5} e^{-jk\pi t}dt + \frac{1}{2}\int_{0.5}^1 \frac{1}{2}e^{-jk\pi t}dt$

$= -\frac{1}{2jk\pi}e^{-jk\pi t}\Big]_0^{0.5} - \frac{1}{4jk\pi}e^{-jk\pi t}\Big]_{0.5}^1$

$= -\frac{1}{2jk\pi}(e^{-\frac{1}{2}jk\pi}-1) - \frac{1}{4jk\pi}(e^{-jk\pi}-e^{-\frac{1}{2}jk\pi})$

$= \frac{-e^{-jk\pi}-e^{-\frac{1}{2}jk\pi}+2}{4jk\pi}$

2. For each of the of the aperiodic pulse signals shown, find the Fourier transform by using the integral formula. In each case, $x(t) = 0$ for all time t outside the interval $-1 \le t \le 1$ sec. Simplify if possible.

(a) $x(t) = \cos\left(\frac{\pi}{2}t\right) = \frac{1}{2}(e^{j\frac{\pi}{2}t} + e^{-j\frac{\pi}{2}t})$, $-1 \le t \le 1$

(b) $x(t) = e^{2t}$, $-1 \le t \le 0$; $x(t) = e^{-2t}$, $0 \le t \le 1$

(c) $x(t) = \cosh(t) = \frac{1}{2}(e^{t} + e^{-t})$, $-1 \le t \le 1$

(d) $x(t) = 0.5$, $-1 \le t < -0.5$ and $0.5 \le t \le 1$; $x(t) = 1$, $-0.5 \le t < 0.5$

(b) $H(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}dt$

$= \int_{-1}^0 e^{2t}e^{-j\omega t}dt + \int_0^1 e^{-2t}e^{-j\omega t}dt$

$= \int_{-1}^0 e^{(2-j\omega)t}dt + \int_0^1 e^{(-2-j\omega)t}dt$

$= \frac{1}{2-j\omega}e^{(2-j\omega)t}\Big]_{-1}^0 - \frac{1}{2+j\omega}e^{(-2-j\omega)t}\Big]_0^1$

$= \frac{1}{2-j\omega}(1-e^{j\omega-2}) - \frac{1}{2+j\omega}(e^{(-2-j\omega)}-1)$

(d) $H(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}dt$

$= \int_{-1}^{-0.5} 0.5e^{-j\omega t}dt + \int_{-0.5}^{0.5} e^{-j\omega t}dt + \int_{0.5}^1 0.5e^{-j\omega t}dt$

$= \frac{1}{-2j\omega}e^{-j\omega t}\Big]_{-1}^{-0.5} - \frac{1}{j\omega}e^{-j\omega t}\Big]_{-0.5}^{0.5} - \frac{1}{2j\omega}e^{-j\omega t}\Big]_{0.5}^1$

$= -\frac{1}{2j\omega}(e^{\frac{1}{2}j\omega}-e^{j\omega}) - \frac{1}{j\omega}(e^{-\frac{1}{2}j\omega}-e^{\frac{1}{2}j\omega}) - \frac{1}{2j\omega}(e^{-j\omega}-e^{-\frac{1}{2}j\omega})$