CHAPTER 9

9.1 The flop counts for the tridiagonal algorithm in Fig. 9.6 can be summarized as

	Mult/Div	Add/Subtr	Total
Forward elimination	$3(n-1)$	$2(n-1)$	$5(n-1)$
Back substitution	$2n-1$	$n-1$	$3n-2$
Total	$5n-4$	$3n-3$	$\mathbf{8n-7}$

Thus, as n increases, the effort is much, much less than for a full matrix solved with Gauss elimination which is proportional to n^3.

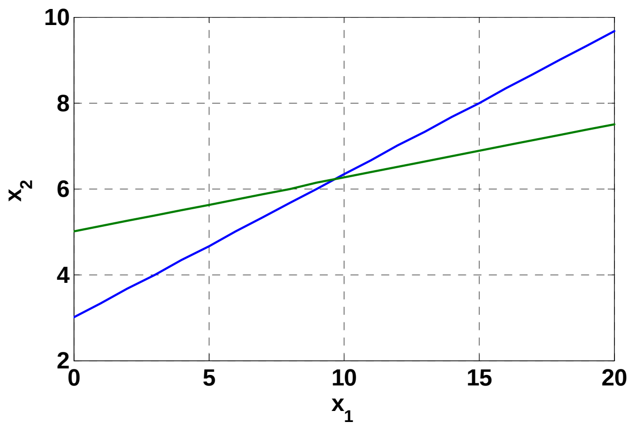
9.2 The equations can be expressed in a format that is compatible with graphing x_2 versus x_1:

$$x_2 = \frac{-18-2x_1}{-6} = 3+0.333333x_1$$

$$x_2 = \frac{40+x_1}{8} = 5+0.125x_1$$

which can be plotted as

```
a11=2;a12=-6;b1=-18;
a21=-1;a22=8;b2=40;
x1=[0:20];x21=(b1-a11*x1)/a12;x22=(b2-a21*x1)/a22;
plot(x1,x21,x1,x22,'--'),grid
xlabel('x_1'),ylabel('x_2')
```



Thus, the solution is $x_1 = 9.6$, $x_2 = 6.2$. The solution can be checked by substituting it back into the equations to give

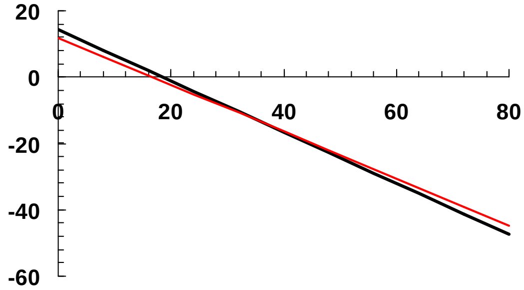
$$2(9.6)-6(6.2) = 19.2-37.2 = -18$$

$$-9.6+8(6.2) = -9.6+49.6 = 40$$

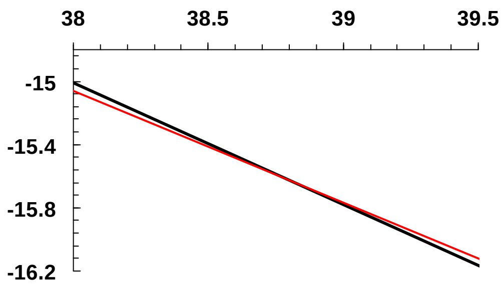
9.3 (a) The equations can be rearranged into a format for plotting x_2 versus x_1:

$$x_2 = 14.25-0.77x_1$$

$$x_2 = 11.76471 - \frac{1.2}{1.7} x_1$$



If you zoom in, it appears that there is a root at about (38.7, –15.6).



The results can be checked by substituting them back into the original equations:

$$0.77(38.7) - 15.6 = 14.2 \cong 14.25$$
$$1.2(38.7) + 1.7(-15.6) = 19.92 \cong 20$$

(b) The plot suggests that the system may be ill-conditioned because the slopes are so similar.

(c) The determinant can be computed as

$$D = 0.77(1.7) - 1(1.2) = 0.11$$

which is relatively small. Note that if the system is normalized first by dividing each equation by the largest coefficient,

$$0.77x_1 + x_2 \quad = 14.25$$
$$0.705882x_1 + x_2 = 11.76471$$

the determinant is even smaller

$$D = 0.77(1) - 1(0.705882) = 0.064$$

9.4 (a) The determinant can be computed as:

$$A_1 = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1(0) - 1(1) = -1$$

$$A_2 = \begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} = 2(0) - 1(3) = -3$$

$$A_3 = \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = 2(1) - 1(3) = -1$$

$$D = 0(-1) - 2(-3) + 5(-1) = 1$$

(b) Cramer's rule

$$x_1 = \frac{\begin{vmatrix} 1 & 2 & 5 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix}}{D} = \frac{-2}{1} = -2$$

$$x_2 = \frac{\begin{vmatrix} 0 & 1 & 5 \\ 2 & 1 & 1 \\ 3 & 2 & 0 \end{vmatrix}}{D} = \frac{8}{1} = 8$$

$$x_3 = \frac{\begin{vmatrix} 0 & 2 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{vmatrix}}{D} = \frac{-3}{1} = -3$$

(c) Pivoting is necessary, so switch the first and third rows,

$$\begin{aligned} 3x_1 + x_2 \quad &= 2 \\ 2x_1 + x_2 + x_3 &= 1 \\ 2x_2 + 5x_3 &= 1 \end{aligned}$$

Multiply pivot row 1 by 2/3 and subtract the result from the second row to eliminate the a_{21} term. Note that because $a_{31} = 0$, it does not have to be eliminated

$$\begin{aligned} 3x_1 \quad + x_2 \quad &= 2 \\ 0.33333x_2 + x_3 &= -0.33333 \\ 2x_2 \quad + 5x_3 &= 1 \end{aligned}$$

Pivoting is necessary so switch the second and third row,

$$\begin{aligned} 3x_1 \quad + x_2 \quad &= 2 \\ 2x_2 + 5x_3 &= 1 \\ 0.33333x_2 + \ x_3 &= -0.33333 \end{aligned}$$

Multiply pivot row 2 by 0.33333/2 and subtract the result from the third row to eliminate the a_{32} term.

$$\begin{aligned} 3x_1 \quad + x_2 \quad &= 2 \\ 2x_2 + \quad 5x_3 &= 1 \\ +0.16667x_3 &= -0.5 \end{aligned}$$

Note that, at this point, the determinant can be computed as the product of the diagonal elements

$D = 3 \times 2 \times 0.16667 \times (-1)^2 = 1$

The solution can then be obtained by back substitution

$$x_3 = \frac{-0.5}{0.16667} = -3$$
$$x_2 = \frac{1-5(-3)}{2} = 8$$
$$x_1 = \frac{2-0(-3)-1(8)}{3} = -2$$

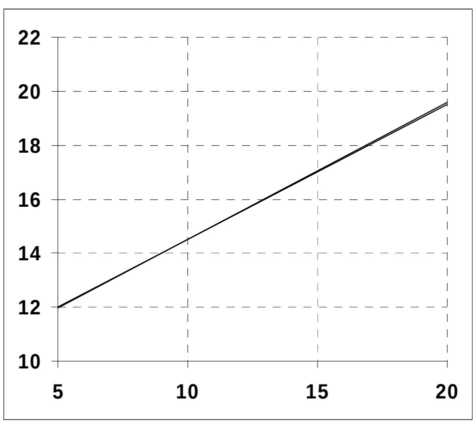
The results can be checked by substituting them back into the original equations:

$$2(8) + 5(-3) = 1$$
$$2(-2) + 8 - 3 = 1$$
$$3(-2) + 8 = 2$$

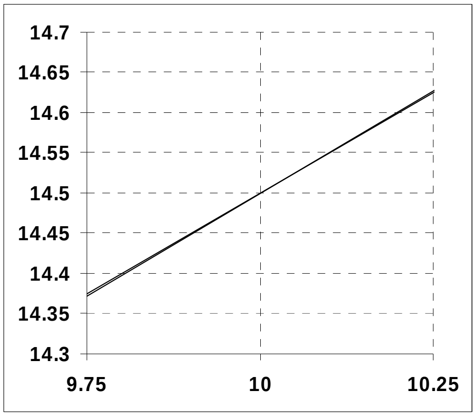
9.5 (a) The equations can be expressed in a format that is compatible with graphing x_2 versus x_1:

$$x_2 = 0.5x_1 + 9.5$$
$$x_2 = 0.51x_1 + 9.4$$

The resulting plot indicates that the intersection of the lines is difficult to detect:



Only when the plot is zoomed is it at all possible to discern that solution seems to lie at about x_1 = 14.5 and x_2 = 10.



(b) The determinant can be computed as

$$\begin{vmatrix} 0.5 & -1 \\ 1.02 & -2 \end{vmatrix} = 0.5(-2) - (-1)(1.02) = 0.02$$

which is close to zero.

(c) Because the lines have very similar slopes and the determinant is so small, you would expect that the system would be ill-conditioned

(d) Multiply the first equation by 1.02/0.5 and subtract the result from the second equation to eliminate the x_1 term from the second equation,

$$\begin{aligned} 0.5x_1 - x_2 &= -9.5 \\ 0.04x_2 &= 0.58 \end{aligned}$$

The second equation can be solved for

$$x_2 = \frac{0.58}{0.04} = 14.5$$

This result can be substituted into the first equation which can be solved for

$$x_1 = \frac{-9.5 + 14.5}{0.5} = 10$$

(e) Multiply the first equation by 1.02/0.52 and subtract the result from the second equation to eliminate the x_1 term from the second equation,

$$\begin{aligned} 0.52x_1 \quad - x_2 &= -9.5 \\ -0.03846x_2 &= -0.16538 \end{aligned}$$

The second equation can be solved for

$$x_2 = \frac{-0.16538}{-0.03846} = 4.3$$

This result can be substituted into the first equation which can be solved for

$$x_1 = \frac{-9.5 + 4.3}{0.52} = -10$$

Interpretation: The fact that a slight change in one of the coefficients results in a radically different solution illustrates that this system is very ill-conditioned.

9.6 (a) Multiply the first equation by –3/10 and subtract the result from the second equation to eliminate the x_1 term from the second equation. Then, multiply the first equation by 1/10 and subtract the result from the third equation to eliminate the x_1 term from the third equation.

$$\begin{aligned}
10x_1 + 2x_2 \quad - x_3 &= 27 \\
-4.4x_2 + 1.7x_3 &= -53.4 \\
0.8x_2 + 6.1x_3 &= -24.2
\end{aligned}$$

Multiply the second equation by 0.8/(–4.4) and subtract the result from the third equation to eliminate the x_2 term from the third equation,

$$\begin{aligned}
10x_1 + 2x_2 \quad - x_3 &= 27 \\
-4.4x_2 \quad + 1.7x_3 &= -53.4 \\
6.409091x_3 &= -33.9091
\end{aligned}$$

Back substitution can then be used to determine the unknowns

$$x_3 = \frac{-33.9091}{6.409091} = -5.29078$$

$$x_2 = \frac{(-53.4 - 1.7(-5.29078))}{-4.4} = 10.0922$$

$$x_1 = \frac{(27 - 5.29078 - 2(10.0922))}{10} = 0.152482$$

(b) Check:

$$\begin{aligned}
10(0.152482) + 2(10.0922) - (-5.29078) &= 27 \\
-3(0.152482) - 5(10.0922) + 2(-5.29078) &= -61.5 \\
0.152482 + 10.0922 + 5(-5.29078) &= -21.5
\end{aligned}$$

9.7 (a) Pivoting is necessary, so switch the first and third rows,

$$\begin{aligned}
-8x_1 + x_2 - 2x_3 &= -20 \\
-3x_1 - x_2 + 7x_3 &= -34 \\
2x_1 - 6x_2 - x_3 &= -38
\end{aligned}$$

Multiply the first equation by –3/(–8) and subtract the result from the second equation to eliminate the a_{21} term from the second equation. Then, multiply the first equation by 2/(–8) and subtract the result from the third equation to eliminate the a_{31} term from the third equation.

$$\begin{aligned}
-8x_1 + x_2 - 2x_3 &= -20 \\
-1.375x_2 + 7.75x_3 &= -26.5 \\
-5.75x_2 - 1.5x_3 &= -43
\end{aligned}$$

Pivoting is necessary so switch the second and third row,

$$\begin{aligned}
-8x_1 + x_2 - 2x_3 &= -20 \\
-5.75x_2 - 1.5x_3 &= -43 \\
-1.375x_2 + 7.75x_3 &= -26.5
\end{aligned}$$

Multiply pivot row 2 by –1.375/(–5.75) and subtract the result from the third row to eliminate the a_{32} term.

$$\begin{aligned}
-8x_1 + x_2 - 2x_3 &= -20 \\
-5.75x_2 - 1.5x_3 &= -43 \\
8.108696x_3 &= -16.21739
\end{aligned}$$

At this point, the determinant can be computed as

$$D = -8 \times -5.75 \times 8.108696 \times (-1)^2 = 373$$

The solution can then be obtained by back substitution

$$\begin{aligned}
x_3 &= \frac{-16.21739}{8.108696} = -2 \\
x_2 &= \frac{-43 + 1.5(-2)}{-5.75} = 8 \\
x_1 &= \frac{-20 + 2(-2) - 1(8)}{-8} = 4
\end{aligned}$$

(b) Check:

$$\begin{aligned}
2(4) - 6(8) - (-2) &= -38 \\
-3(4) - (8) + 7(-2) &= -34 \\
-8(4) + (8) - 2(-2) &= -20
\end{aligned}$$

9.8 Multiply the first equation by –0.4/0.8 and subtract the result from the second equation to eliminate the x_1 term from the second equation.

$$\begin{bmatrix} 0.8 & -0.4 & \\ & 0.6 & -0.4 \\ & -0.4 & 0.8 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 41 \\ 45.5 \\ 105 \end{Bmatrix}$$

Multiply pivot row 2 by –0.4/0.6 and subtract the result from the third row to eliminate the x_2 term.

$$\begin{bmatrix} 0.8 & -0.4 & \\ & 0.6 & -0.4 \\ & & 0.533333 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 41 \\ 45.5 \\ 135.3333 \end{Bmatrix}$$

The solution can then be obtained by back substitution

$$x_3 = \frac{135.3333}{0.533333} = 253.75$$
$$x_2 = \frac{45.5 - (-0.4)253.75}{0.6} = 245$$
$$x_1 = \frac{41 - (-0.4)245}{0.8} = 173.75$$

(b) Check:

$$0.8(173.75) - 0.4(245) = 41$$
$$-0.4(173.75) + 0.8(245) - 0.4(253.75) = 25$$
$$-0.4(245) + 0.8(253.75) = 105$$

9.9 Mass balances can be written for each of the reactors as

$$200 - Q_{13}c_1 - Q_{12}c_1 + Q_{21}c_2 = 0$$
$$Q_{12}c_1 - Q_{21}c_2 - Q_{23}c_2 = 0$$
$$500 + Q_{13}c_1 + Q_{23}c_2 - Q_{33}c_3 = 0$$

Values for the flows can be substituted and the system of equations can be written in matrix form as

$$\begin{bmatrix} 130 & -30 & 0 \\ -90 & 90 & 0 \\ -40 & -60 & 120 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} = \begin{Bmatrix} 200 \\ 0 \\ 500 \end{Bmatrix}$$

The solution can then be developed using MATLAB,

```
>> A=[130 -30 0;-90 90 0;-40 -60 120];
>> B=[200;0;500];
>> C=A\B

C =
    2.0000
    2.0000
    5.8333
```

9.10 Let x_i = the volume taken from pit *i*. Therefore, the following system of equations must hold

$$0.52x_1 + 0.20x_2 + 0.25x_3 = 4800$$
$$0.30x_1 + 0.50x_2 + 0.20x_3 = 5800$$
$$0.18x_1 + 0.30x_2 + 0.55x_3 = 5700$$

MATLAB can be used to solve this system of equations for

```
>> A=[0.55 0.2 0.25;0.3 0.5 0.2;0.18 0.3 0.55];
>> b=[4800;5800;5700];
>> x=A\b
```

```
x =
  1.0e+003 *
    4.0058
    7.1314
    5.1628
```

Therefore, we take x_1 = 4005.8, x_2 = 7131.4, and x_3 = 5162.8 m^3 from pits 1, 2 and 3 respectively.

9.11 Let c_i = component i. Therefore, the following system of equations must hold

$$15c_1 + 17c_2 + 19c_3 = 2120$$
$$0.25c_1 + 0.33c_2 + 0.42c_3 = 43.4$$
$$1.0c_1 + 1.2c_2 + 1.6c_3 = 164$$

The solution can be developed with MATLAB:

```
A=[15 17 19;0.25 0.33 0.42;1 1.2 1.6];
b=[2120;43.4;164];
c=A\b

c =
   20.0000
   40.0000
   60.0000
```

Therefore, c_1 = 20, c_2 = 40, and c_3 = 60.

9.12 Centered differences (recall Chap. 4) can be substituted for the derivatives to give

$$0 = D\frac{c_{i-1} - 2c_i + c_{i+1}}{\Delta x^2} - U\frac{c_{i+1} - c_{i-1}}{2\Delta x} - kc_i$$

collecting terms yields

$$-(D + 0.5U\Delta x)c_{i-1} + (2D + k\Delta x^2)c_i - (D - 0.5U\Delta x)c_{i+1} = 0$$

Assuming Δx = 1 and substituting the parameters gives

$$-2.5c_{i-1} + 4.2c_i - 1.5c_{i+1} = 0$$

For the first interior node (i = 1),

$$4.2c_1 - 1.5c_2 = 200$$

For the last interior node (i = 9)

$$-2.5c_8 + 4.2c_9 = 15$$

These and the equations for the other interior nodes can be assembled in matrix form as

$$
\begin{bmatrix}
4.2 & -1.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2.5 & 4.2 & -1.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -2.5 & 4.2 & -1.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2.5 & 4.2 & -1.5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2.5 & 4.2 & -1.5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -2.5 & 4.2 & -1.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2.5 & 4.2 & -1.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -2.5 & 4.2 & -1.5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -2.5 & 4.2
\end{bmatrix}
\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \end{Bmatrix}
=
\begin{Bmatrix} 200 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 15 \end{Bmatrix}
$$

The following script generates the solution with the Tridiag function from p. 247 and develops a plot:

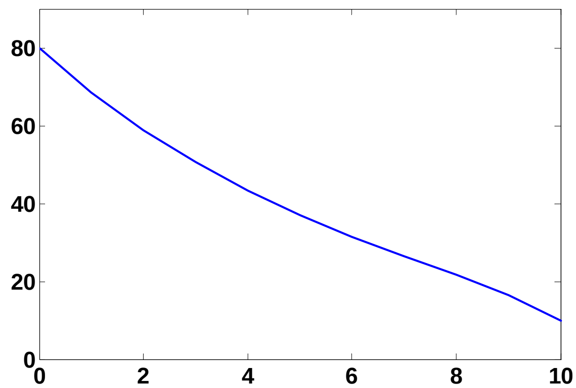
```
clear,clc,clf
D=2;U=1;k=0.2;c0=80;c10=10;dx=1;
diag=(2*D+k*dx^2);
super=-(D-0.5*U*dx);
sub=-(D+0.5*U*dx);
r1=-sub*c0; rn=-super*c10;
n=9;
e=ones(n,1)*sub;f=ones(n,1)*diag;g=ones(n,1)*super;
r=zeros(n,1);r(1)=r1;r(n)=rn;
c=Tridiag(e,f,g,r)
c=[80 c 10]; x=0:1:10;
plot(x,c) ylim([0 90])
```

Alternatively, as in the following script, the solution can be generated directly with MATLAB left division:

```
clear,clc,clf
A=[4.2 -1.5 0 0 0 0 0 0 0
     -2.5 4.2 -1.5 0 0 0 0 0 0
     0 -2.5 4.2 -1.5 0 0 0 0 0
     0 0 -2.5 4.2 -1.5 0 0 0 0
     0 0 0 -2.5 4.2 -1.5 0 0 0
     0 0 0 0 -2.5 4.2 -1.5 0 0
     0 0 0 0 0 -2.5 4.2 -1.5 0
     0 0 0 0 0 0 -2.5 4.2 -1.5
     0 0 0 0 0 0 0 -2.5 4.2];
 b=[200 0 0 0 0 0 0 0 15]';
 c=(A\b)'
 c=[80 c 10];  x=0:1:10;
 plot(x,c), ylim([0 90])
```

In either case, the results are:

```
c =
 68.6613  58.9183  50.5357  43.3029  37.0219  31.4898  26.4684  21.6284 16.4454
```



9.13 For the first stage, the mass balance can be written as

$$F_1 y_{\text{in}} + F_2 x_2 = F_2 x_1 + F_1 x_1$$

Substituting $x = Ky$ and rearranging gives

$$-\left(1 + \frac{F_2}{F_1} K\right) y_1 + \frac{F_2}{F_1} K y_2 = -y_{\text{in}}$$

Using a similar approach, the equation for the last stage is

$$y_4 - \left(1 + \frac{F_2}{F_1} K\right) y_5 = -\frac{F_2}{F_1} x_{\text{in}}$$

For interior stages,

$$y_{\text{i-1}} - \left(1 + \frac{F_2}{F_1} K\right) y_{\text{i}} + \frac{F_2}{F_1} K y_{i+1} = 0$$

These equations can be used to develop the following system,

$$\begin{bmatrix} 11 & -10 & 0 & 0 & 0 \\ -1 & 11 & -10 & 0 & 0 \\ 0 & -1 & 11 & -10 & 0 \\ 0 & 0 & -1 & 11 & -10 \\ 0 & 0 & 0 & -1 & 11 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{Bmatrix} = \begin{Bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

The solution can be developed in a number of ways. For example, using MATLAB,

```
>> format short g
>> A=[11 -10 0 0 0;-1 11 -10 0 0;0 -1 11 -10 0;0 0 -1 11 -10;0 0 0 -1 11];
>> B=[0.1;0;0;0;0];
>> Y=A\B
Y =
    0.0099999
    0.0009999
    9.99e-005
```

```
      9.9e-006
        9e-007
```

Note that the corresponding values of X can be computed as

```
>> X=5*Y
X =
         0.05
    0.0049995
    0.0004995
    4.95e-005
     4.5e-006
```

Therefore, y_{out} = 0.0000009 and x_{out} = 0.05.

9.14 Assuming a unit flow for Q_1, the simultaneous equations can be written in matrix form as

$$\begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & -2 & 3 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix} \begin{Bmatrix} Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$

These equations can then be solved with MATLAB,

```
>> A=[-2 1 2 0 0 0;
0 0 -2 1 2 0;
0 0 0 0 -2 3;
1 1 0 0 0 0;
0 1 -1 -1 0 0;
0 0 0 1 -1 -1];
>> B=[0 0 0 1 0 0 ]';
>> Q=A\B

Q =
    0.5059
    0.4941
    0.2588
    0.2353
    0.1412
    0.0941
```

9.15 The solution can be generated with MATLAB,

```
>> A=[1 0 0 0 0 0 0 0 1 0;
      0 0 1 0 0 0 0 1 0 0;
      0 1 0 3/5 0 0 0 0 0 0;
      -1 0 0 -4/5 0 0 0 0 0 0;
      0 -1 0 0 0 0 3/5 0 0 0;
      0 0 0 0 -1 0 -4/5 0 0 0;
      0 0 -1 -3/5 0 1 0 0 0 0;
      0 0 0 4/5 1 0 0 0 0 0;
      0 0 0 0 0 -1 -3/5 0 0 0;
      0 0 0 0 0 0 4/5 0 0 1];
>> B=[0 0 -74 0 0 24 0 0 0 0]';
>> x=A\B
```

```
x =
   37.3333
  -46.0000
   74.0000
  -46.6667
   37.3333
   46.0000
  -76.6667
  -74.0000
  -37.3333
   61.3333
```

Therefore, in kN

AB = 37.3333	BC = –46	AD = 74	BD = –46.6667	CD = 37.3333
DE = 46	CE = –76.6667	A_x = –74	A_y = –37.33333	E_y = 61.3333

9.16

```
function x=pentasol(A,b)
% pentasol: pentadiagonal system solver banded system
%   x=pentasol(A,b):
%      Solve a pentadiagonal system Ax=b
% input:
%   A = pentadiagonal matrix
%   b = right hand side vector
% output:
%   x = solution vector

% Error checks
[m,n]=size(A);
if m~=n,error('Matrix must be square');end
if length(b)~=m,error('Matrix and vector must have the same number of
rows');end
x=zeros(n,1);

% Extract bands
d=[0;0;diag(A,-2)];
e=[0;diag(A,-1)];
f=diag(A);
g=diag(A,1);
h=diag(A,2);
delta=zeros(n,1);
epsilon=zeros(n-1,1);
gamma=zeros(n-2,1);
alpha=zeros(n,1);
c=zeros(n,1);
z=zeros(n,1);

% Decomposition
delta(1)=f(1);
epsilon(1)=g(1)/delta(1);
gamma(1)=h(1)/delta(1);
alpha(2)=e(2);
delta(2)=f(2)-alpha(2)*epsilon(1);
epsilon(2)=(g(2)-alpha(2)*gamma(1))/delta(2);
gamma(2)=h(2)/delta(2);
for k=3:n-2
  alpha(k)=e(k)-d(k)*epsilon(k-2);
  delta(k)=f(k)-d(k)*gamma(k-2)-alpha(k)*epsilon(k-1);
```

```
  epsilon(k)=(g(k)-alpha(k)*gamma(k-1))/delta(k);
  gamma(k)=h(k)/delta(k);
end
alpha(n-1)=e(n-1)-d(n-1)*epsilon(n-3);
delta(n-1)=f(n-1)-d(n-1)*gamma(n-3)-alpha(n-1)*epsilon(n-2);
epsilon(n-1)=(g(n-1)-alpha(n-1)*gamma(n-2))/delta(n-1);
alpha(n)=e(n)-d(n)*epsilon(n-2);
delta(n)=f(n)-d(n)*gamma(n-2)-alpha(n)*epsilon(n-1);
% Forward substitution
c(1)=b(1)/delta(1);
c(2)=(b(2)-alpha(2)*c(1))/delta(2);
for k=3:n
  c(k)=(b(k)-d(k)*c(k-2)-alpha(k)*c(k-1))/delta(k);
end
% Back substitution
x(n)=c(n);
x(n-1)=c(n-1)-epsilon(n-1)*x(n);
for k=n-2:-1:1
  x(k)=c(k)-epsilon(k)*x(k+1)-gamma(k)*x(k+2);
end
```

A script to test the function can be developed as:

```
clear,clc
A=[8 -2 -1 0 0;-2 9 -4 -1 0;-1 -3 7 -1 -2;0 -4 -2 12 -5;0 0 -7 -3 15];
b=[5 2 1 1 5]';
x=pentasol(A,b)'

x =
    1.0825    1.1759    1.3082    1.1854    1.1809
```

9.17 Here is the M-file function based on Fig. 9.5 to implement Gauss elimination with partial pivoting

```
function [x, D] = GaussPivotNew(A, b, tol)
% GaussPivotNew: Gauss elimination pivoting
%   [x, D] = GaussPivotNew(A,b,tol): Gauss elimination with pivoting.
% input:
%   A = coefficient matrix
%   b = right hand side vector
%   tol = tolerance for detecting "near zero"
% output:
%   x = solution vector
%   D = determinant

[m,n]=size(A);
if m~=n, error('Matrix A must be square'); end
nb=n+1;
Aug=[A b];
npiv=0;
% forward elimination
for k = 1:n-1
  % partial pivoting
  [big,i]=max(abs(Aug(k:n,k)));
  ipr=i+k-1;
  if ipr~=k
    npiv=npiv+1;
    Aug([k,ipr],:)=Aug([ipr,k],:);
  end
  absakk=abs(Aug(k,k));
  if abs(Aug(k,k))<=tol
    D=0;
```

```
    error('Singular or near singular system')
  end
  for i = k+1:n
    factor=Aug(i,k)/Aug(k,k);
    Aug(i,k:nb)=Aug(i,k:nb)-factor*Aug(k,k:nb);
  end
end
for i = 1:n
  if abs(Aug(i,i))<=tol
    D=0;
    error('Singular or near singular system')
  end
end
% back substitution
x=zeros(n,1);
x(n)=Aug(n,nb)/Aug(n,n);
for i = n-1:-1:1
  x(i)=(Aug(i,nb)-Aug(i,i+1:n)*x(i+1:n))/Aug(i,i);
end
D=(-1)^npiv;
for i=1:n
  D=D*Aug(i,i);
end
```

Here is a script to solve Prob. 9.5 for the two cases of tol:

```
clear; clc; format short g
A=[0.5 -1;1.02 -2];
b=[-9.5;-18.8];
disp('Solution and determinant calculated with GaussPivotNew:')
[x, D] = GaussPivotNew(A,b,1e-5)
disp('Determinant calculated with det:')
D=det(A)
```

The resulting output is

```
Solution and determinant calculated with GaussPivotNew:
x =
           10
         14.5
D =
         0.02

Determinant calculated with det:
D =
         0.02
```