Chapter 1

PROBLEM 1.1

On a cold winter day, the outer surface of a 0.2-m-thick concrete wall of a warehouse is exposed to a temperature of –5°C, while the inner surface is kept at 20°C. The thermal conductivity of the concrete is 1.2 W/(m K). Determine the heat loss through the wall, which is 10-m long and 3-m high.

GIVEN

- 10 m long, 3 m high, and 0.2 m thick concrete wall
- Thermal conductivity of the concrete (k) = 1.2 W/(m K)
- Temperature of the inner surface (T_i) = 20°C
- Temperature of the outer surface (T_o) = –5°C

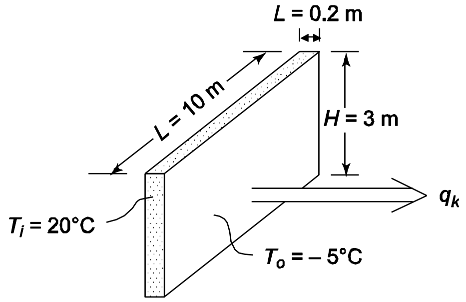
FIND

- The heat loss through the wall (q_k)

ASSUMPTIONS

- One dimensional heat flow
- The system has reached steady state

SKETCH



SOLUTION

The rate of heat loss through the wall is given by Equation (1.3)

$$q_k = (AK/L)\Delta T$$

$$q_k = \frac{(10\,\text{m})(3\,\text{m})\ 1.2\ \text{W/(m K)}}{0.2\,\text{m}}\ (20°\text{C} - (-5°\text{C}))$$

$$q_k = 4500\ \text{W}$$

COMMENTS

Since the inside surface temperature is higher than the outside temperature heat is transferred from the inside of the wall to the outside of the wall.

PROBLEM 1.2

The weight of the insulation in a spacecraft may be more important than the space required. Show analytically that the lightest insulation for a plane wall with a specified thermal resistance is the insulation that has the smallest product of density times thermal conductivity.

GIVEN

- Insulating a plane wall, the weight of insulation is most significant

FIND

- Show that lightest insulation for a given thermal resistance is that insulation which has the smallest product of density (ρ) times thermal conductivity (k)

ASSUMPTIONS

- One dimensional heat transfer through the wall
- Steady state conditions

SOLUTION

The resistance of the wall (R_k), from Equation (1.4) is

$$R_k = \frac{L}{A\,k}$$

where

L = the thickness of the wall

A = the area of the wall

The weight of the wall (w) is

$$w = \rho A L$$

Solving this for L

$$L = w/(\rho A)$$

Substituting this expression for L into the equation for the resistance

$$R_k = \frac{w}{\rho\, k\, A^2}$$

$$\therefore\ w = \rho\, k\, R_k\, A^2$$

Therefore, when the product of $\rho\, k$ for a given resistance is smallest, the weight is also smallest.

COMMENTS

Since ρ and k are physical properties of the insulation material they cannot be varied individually. Hence in this type of design different materials must be tried to minimize the weight.

PROBLEM 1.3

A furnace wall is to be constructed of brick having standard dimensions 22.5 cm* 11 cm * 7.5 cm. Two kinds of material are available. One has a maximum usable temperature of 1040°C and a thermal conductivity of 1.7 W/(m K), and the other has a maximum temperature limit of 870^{0}C and a thermal conductivity of 0.85 W/(m K). The bricks have the same cost and are laid in any manner, but we wish to design the most economical wall for a furnace with a temperature of 1040^{0}C the hot side and 200^{0}C on the cold side. If the maximum amount of heat transfer permissible is 950 W/m^2, determine the most economical arrangement using the available bricks.

GIVEN

- Furnace wall made of 22.5 × 11 × 7.5 cm bricks of two types
  - Type 1 bricks — Maximum useful temperature ($T_{1,max}$) = 1040°C=1313 K; Thermal conductivity (k_1) = 1.7 W/(m K)
  - Type 2 bricks — Maximum useful temperature ($T_{2,max}$) = 870°C= 1143 K; Thermal conductivity (k_2) = 0.85 W/(m K)
- Bricks cost the same
- Wall hot side (T_{hot}) = 1040°C=1313 K and cold side (T_{cold}) = 200°C=473 K
- Maximum heat transfer permissible (q_{max}/A) = 950 W/m^2

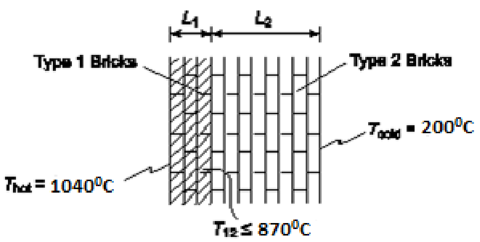
FIND

- The most economical arrangement for the bricks

ASSUMPTIONS

- One dimensional, steady state heat transfer conditions
- Constant thermal conductivities
- The contact resistance between the bricks is negligible

SKETCH



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SOLUTION

Since the type 1 bricks have a higher thermal conductivity at the same cost as the type 2 bricks, the most economical wall would use as few type 1 bricks as possible. However, there must be a thick enough layer of type 1 bricks to keep the type 2 bricks at 1040°C or less.

For one dimensional conduction through the type 1 bricks, from Eq. (1.3),

$$q_k = \frac{k\,A}{L}\,\Delta T$$

$$\frac{q_{max}}{A} = \frac{k_1}{L_1}\,(T_{hot} - T_{12})$$

where L_1 = the minimum thickness of the type 1 bricks

Solving for L_1

$$L_1 = \frac{k_1}{\left(\frac{q_{max}}{A}\right)}\,(T_{hot} - T_{12})$$

$$L_1 = \frac{1.7\ \mathrm{W/(m\ K)}}{950 W/m^2}\,(1313\ \mathrm{K} - 1143\ \mathrm{K}) = 0.3042\ \mathrm{m}$$

This thickness can be achieved with 4 layers of type 1 bricks using the 3 in. dimension.

Similarly, for one dimensional conduction through the type 2 bricks

$$L_2 = \frac{k_2}{\left(\frac{q_{max}}{A}\right)}\,(T_{12} - T_{cold})$$

$$L_2 = \frac{0.85\ \mathrm{W/(m\ K)}}{950\ \mathrm{W/m^2}}\,(1143\ \mathrm{K} - 473\ \mathrm{K}) = 0.60\ \mathrm{m}$$

This thickness can be achieved with 8 layers of type 2 brick using the 3 in. dimension. Therefore, the most economical wall would be built using 4 layers of type 1 bricks and 8 layers of type 2 bricks with three inches dimension of the bricks used as the thickness.

PROBLEM 1.4

To measure thermal conductivity, two similar 1-cm-thick specimens are placed in an apparatus shown in the accompanying sketch. Electric current is supplied to the 6-cm by 6-cm guarded heater, and a wattmeter shows that the power dissipation is 10 watts (W). Thermocouples attached to the warmer and to the cooler surfaces show temperatures of 322 and 300 K, respectively. Calculate the thermal conductivity of the material at the mean temperature in W/(m K).

GIVEN

- Thermal conductivity measurement apparatus with two samples as shown
  - Sample thickness (L) = 1 cm = 0.01 cm
- Area = 6 cm × 6 cm = 36 cm^2 = 0.0036 m^2
- Power dissipation rate of the heater (q_h) = 10 W
- Surface temperatures
  - T_{hot} = 322 K
  - T_{cold} = 300 K

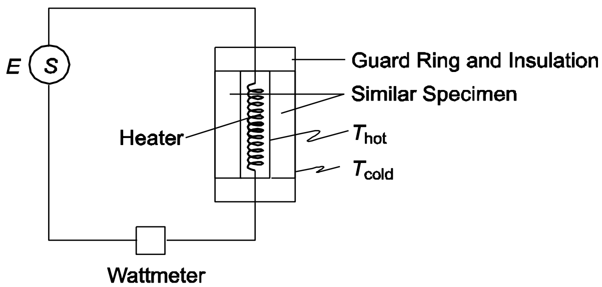
FIND

- The thermal conductivity of the sample at the mean temperature in W/(m K)

ASSUMPTIONS

- One dimensional, steady state conduction
- No heat loss from the edges of the apparatus

SKETCH



SOLUTION

By conservation of energy, the heat loss through the two specimens must equal the power dissipation of the heater. Therefore the heat transfer through one of the specimens is $q_h/2$.

For one dimensional, steady state conduction (from Equation (1.3))

$$q_k = \left[\left(kA/L\right)\Delta T\right] = \left(q_h/2\right)$$

Solving for the thermal conductivity

$$k = \frac{\left(q_h/2\right)}{A}\Delta T = \frac{(5\ \text{W})(0.01\text{m})}{(0.0036\,\text{m}^2)(322\,\text{K} - 300\,\text{K})} = 0.63\ \ \text{W/(m K)}$$

COMMENTS

In the construction of the apparatus care must be taken to avoid edge losses so all the heat generated will be conducted through the two specimens.

PROBLEM 1.5

To determine the thermal conductivity of a structural material, a large 15 cm-thick slab of the material was subjected to a uniform heat flux of 2500 W/m^2, while thermocouples embedded in the wall 2.5 cm. intervals are read over a period of time. After the system had reached equilibrium, an operator recorded the thermocouple readings shown below for two different environmental conditions

Distance from the Surface (cm.)	Temperature (°C)
	Test 1
0	**40**
5	**65**
10	**97**
15	**132**
	Test 2
0	**95**
5	**130**
10	**168**
15	**208**

From these data, determine an approximate expression for the thermal conductivity as a function of temperature between 40 and 208°C.

GIVEN

- Thermal conductivity test on a large, 6-in.-thick slab
- Thermocouples are embedded in the wall 2 in. apart
- Heat flux (q/A) = 2500 W/m^2
- Two equilibrium conditions were recorded (shown above)

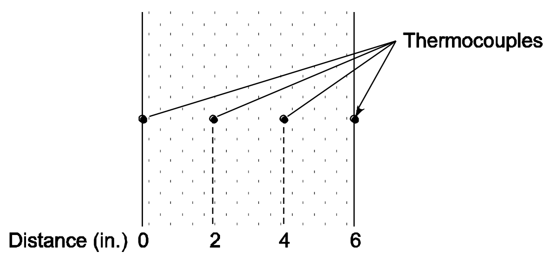
FIND

- An approximate expression for thermal conductivity as a function of temperature between 40 and 208^{0}C

ASSUMPTIONS

- One dimensional conduction

SKETCH



SOLUTION

The thermal conductivity can be calculated for each pair of adjacent thermocouples using the equation for one dimensional conduction, Eq. (1.3),

$$q = kA\,\frac{\Delta T}{L}$$

Solving for thermal conductivity

$$k = \frac{q}{A}\,\frac{L}{\Delta T}$$

This will yield a thermal conductivity for each pair of adjacent thermocouples which will then be assigned to the average temperature for that pair of thermocouples. As an example, for the first pair of thermocouples in Test 1, the thermal conductivity (k_o) is

$$k_o = \left(2500\ \text{W/m}^2\right)\left(\frac{0.05m}{65\,^{o}C - 40\,^{o}C}\right) = 5\ \text{W/(m K)}$$

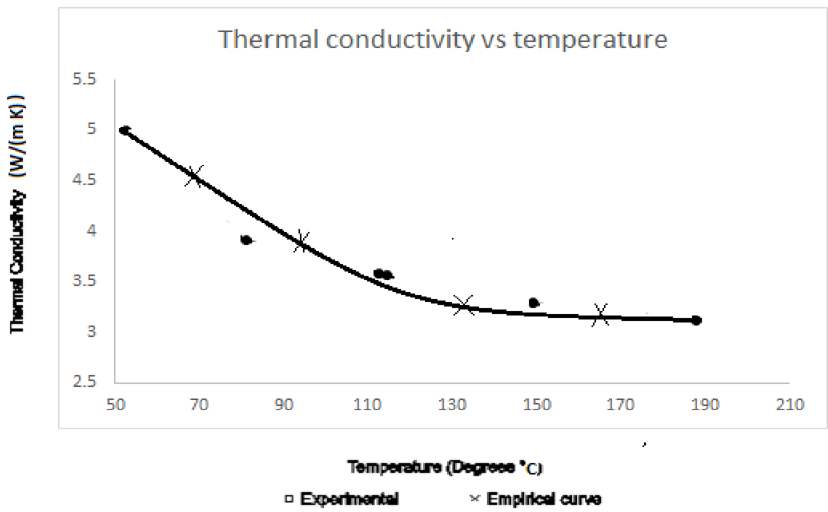
The average temperature for this pair of thermocouples is

$$T_{\text{ave}} = \frac{40\,^{o}C + 65\,^{o}C}{2} = 52.5\ °\text{C}$$

Thermal conductivities and average temperatures for the rest of the data can be calculated in a similar manner

n	Temperature (°C)	Thermal conductivity W/(m K)
1	52.5	5
2	81	3.91
3	114.5	3.57
4	112.5	3.58
5	149	3.29
6	188	3.125

These points are displayed graphically on the following page.



We will use the best fit quadratic function to represent the relationship between thermal conductivity and temperature

$$k(T) = a + b\,T + c\,T^2$$

The constants *a*, *b*, and *c* can be found using a least squares fit.

Let the experimental thermal conductivity at data point *n* be designated as k_n. A least squares fit of the data can be obtained as follows

The sum of the squares of the errors is

$$S = \sum_N [k_n - k(T_n)]^2$$

$$S = \sum k_n^2 - 2a\sum k_n - N a^2 + 2ab\sum T_n - 2b\sum k_n T_n + 2ac\sum T_n^2 + b^2\sum T_n^2 - 2c \sum k_n T_n^2 + 2bc\sum T_n^3 + c^2\sum T_n^4$$

By setting the derivatives of *S* (with respect to *a*, *b*, and *c*) equal to zero, the following equations result

$$N a + \Sigma T_n b + \Sigma T_n^2 c = \Sigma k_n$$

$$\Sigma T_n a + \Sigma T_n^2 b + \Sigma T_n^3 c = \Sigma k_n T_n$$

$$\Sigma T_n^2 a + \Sigma T_n^3 b + \Sigma T_n^4 c = \Sigma k_n T_n^2$$

For this problem

$$\Sigma T_n = 698.5$$

$$\Sigma T_n^2 = 92628.8$$

$$\Sigma T_n^3 = 1.355 \times 10^7$$

$$\Sigma T_n^4 = 2.125 \times 10^9$$

$$\Sigma k_n = 22.48$$

$$\Sigma k_n T_n = 2468$$

$$\Sigma k_n T_n^2 = 3.15 \times 10^5$$

Solving for *a*, *b*, and *c*

$$a = 8.4$$

$$b = -0.07168$$

$$c = 2.39 \times 10^{-4}$$

Therefore the expression for thermal conductivity as a function of temperature between 40 and 200°C is

$$k(T) = 8.4 - 0.07168\,T + 2.39 \times 10^{-4}\,T^2$$

This empirical expression for the thermal conductivity as a function of temperature is plotted with the thermal conductivities derived from the experimental data in the above graph.

COMMENTS

Note that the derived empirical expression is only valid within the temperature range of the experimental data.

PROBLEM 1.6

A square silicone chip 7 mm by 7 mm in size and 0.5 mm thick is mounted on a plastic substrate as shown in the sketch below. The top surface of the chip is cooled by a synthetic liquid flowing over it. Electronic circuits on the bottom of the chip generate heat at a rate of 5 watts that must be transferred through the chip. Estimate the steady state temperature difference between the front and back surfaces of the chip. The thermal conductivity of silicone is 150 W/(m K).

GIVEN

- A 0.007 m by 0.007 m silicone chip
- Thickness of the chip (L) = 0.5 mm = 0.0005 m
- Heat generated at the back of the chip ($\dot{q}_G$) = 5 W
- The thermal conductivity of silicon (k) = 150 W/(m K)

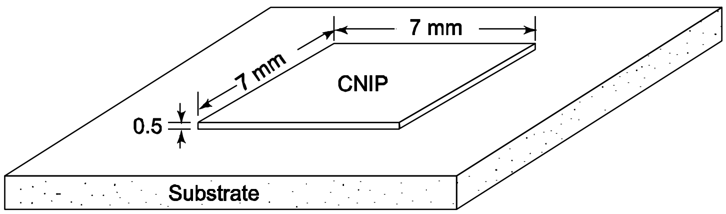
FIND

- The steady state temperature difference (ΔT)

ASSUMPTIONS

- One dimensional conduction (edge effects are negligible)
- The thermal conductivity is constant
- The heat lost through the plastic substrate is negligible

SKETCH



SOLUTION

For steady state the rate of heat loss through the chip, given by Equation (1.3), must equal the rate of heat generation

$$q_k = \frac{A\,k}{L}\,(\Delta T) = \dot{q}_G$$

Solving this for the temperature difference

$$\Delta T = \frac{L\,\dot{q}_G}{k\,A}$$

$$\Delta T = \frac{(0.0005)(5\,\mathrm{W})}{150\ \mathrm{W/(m\,K)}\ (0.007\,\mathrm{m})(0.007\,\mathrm{m})}$$

$$\Delta T = 0.34°\mathrm{C}$$

PROBLEM 1.7

A cooling system is to be designed for a food storage warehouse for keeping perishable foods cool prior to transportation to grocery stores. The warehouse has an effective surface area of 1860 m^2 exposed to an ambient air temperature of 32^{0}C. The warehouse wall insulation (*k* = 0.17 W/(m K)) is 7.5 cm thick. Determine the rate at which heat must be removed (W) from the warehouse to maintain the food at 4°C.

GIVEN

- Cooled warehouse
- Effective area (A) = 1860 m^2
- Temperatures
  - Outside air = 32°C
  - food inside = 4°C
- Thickness of wall insulation (L) = 7.5 cm = 0.075 m
- Thermal conductivity of insulation (k) = 0.17 W/(m K)

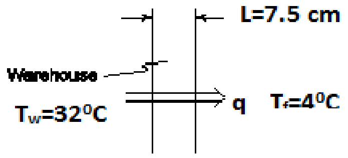
FIND

- Rate at which heat must be removed (q)

ASSUMPTIONS

- One dimensional, steady state heat flow
- The food and the air inside the warehouse are at the same temperature
- The thermal resistance of the wall is approximately equal to the thermal resistance of the wall insulation alone

SKETCH



SOLUTION

The rate at which heat must be removed is equal to the rate at which heat flows into the warehouse. There will be convective resistance to heat flow on the inside and outside of the wall. To estimate the upper limit of the rate at which heat must be removed these convective resistances will be neglected. Therefore the inside and outside wall surfaces are assumed to be at the same temperature as the air inside and outside of the wall. Then the heat flow, from Equation (1.3), is

$$q = \frac{k\,A}{L}\,\Delta T$$

$$q = \frac{(0.17 W/(\mathrm{m\ K}))\,(1860\,m^2)}{0.075\ \mathrm{m}}\ (32°\mathrm{C} - 4°\mathrm{C})$$

$$q = 118048\ \mathrm{W}$$

PROBLEM 1.8

With increasing emphasis on energy conservation, the heat loss from buildings has become a major concern. For a small tract house the typical exterior surface areas and *R*-factors (area × thermal resistance) are listed below

Element	Area (m^2)	*R*-Factors = Area × Thermal Resistance [(m^2 K/W)]
Walls	150	2.0
Ceiling	120	2.8
Floor	120	2.0
Windows	20	0.1
Doors	5	0.5

(a) Calculate the rate of heat loss from the house when the interior temperature is 22°C and the exterior is –5°C.

(b) Suggest ways and means to reduce the heat loss and show quantitatively the effect of doubling the wall insulation and the substituting double glazed windows (thermal resistance = 0.2 m^2 K/W) for the single glazed type in the table above.

GIVEN

- Small house
- Areas and thermal resistances shown in the table above
- Interior temperature = 22°C
- Exterior temperature = –5°C

FIND

(a) Heat loss from the house (q_a)

(b) Heat loss from the house with doubled wall insulation and double glazed windows (q_b). Suggest improvements.

ASSUMPTIONS

- All heat transfer can be treated as one dimensional
- Steady state has been reached
- The temperatures given are wall surface temperatures
- Infiltration is negligible
- The exterior temperature of the floor is the same as the rest of the house

SOLUTION

(a) The rate of heat transfer through each element of the house is given by Equations (1.34) and (1.35)

$$q = \frac{\Delta T}{R_{th}}$$

The total rate of heat loss from the house is simply the sum of the loss through each element

$$q = \Delta T \left(\frac{1}{\left(\frac{AR}{A}\right)_{\text{wall}}} + \frac{1}{\left(\frac{AR}{A}\right)_{\text{ceiling}}} + \frac{1}{\left(\frac{AR}{A}\right)_{\text{floor}}} + \frac{1}{\left(\frac{AR}{A}\right)_{\text{windows}}} + \frac{1}{\left(\frac{AR}{A}\right)_{\text{doors}}} \right)$$

$$q = (22°\text{C} - -5°\text{C})\left(\frac{1}{\left(\frac{2.0\ (\text{m}^2\text{K})/\text{W}}{150\ \text{m}^2}\right)} + \frac{1}{\left(\frac{2.8\ (\text{m}^2\text{K})/\text{W}}{120\ \text{m}^2}\right)} + \frac{1}{\left(\frac{2.0\ (\text{m}^2\text{K})/\text{W}}{120\ \text{m}^2}\right)} + \frac{1}{\left(\frac{0.1\ (\text{m}^2\text{K})/\text{W}}{20\ \text{m}^2}\right)} + \frac{1}{\left(\frac{0.5\ (\text{m}^2\text{K})/\text{W}}{5\ \text{m}^2}\right)}\right)$$

$$q = (22°\text{C} - -5°\text{C})\ (75 + 42.8 + 60 + 200 + 10)\ \text{W/K}$$

$$q = 10{,}500\ \text{W}$$

(b) Doubling the resistance of the walls and windows and recalculating the total heat loss

$$q = (22°\text{C} - -5°\text{C})\left(\frac{1}{\left(\frac{4.0\ (\text{m}^2\text{K})/\text{W}}{150\ \text{m}^2}\right)} + \frac{1}{\left(\frac{2.8\ (\text{m}^2\text{K})/\text{W}}{120\ \text{m}^2}\right)} + \frac{1}{\left(\frac{2.0\ (\text{m}^2\text{K})/\text{W}}{120\ \text{m}^2}\right)} + \frac{1}{\left(\frac{0.2\ (\text{m}^2\text{K})/\text{W}}{20\ \text{m}^2}\right)} + \frac{1}{\left(\frac{0.5\ (\text{m}^2\text{K})/\text{W}}{5\ \text{m}^2}\right)}\right)$$

$$q = (22°\text{C} - -5°\text{C})\ (37.5 + 42.8 + 60 + 100 + 10)\ \text{W/K}$$

$$q = 6800\ \text{W}$$

%Loss reduction= $\dfrac{(10500-6800)}{10500}*100\%$

= 35 %

Doubling the wall and window insulation led to a 35% reduction in the total rate of heat loss.

COMMENTS

Notice that the single glazed windows account for slightly over half of the total heat lost in case (a) and that the majority of the heat loss reduction in case (b) is due to the double glazed windows. Therefore, double glazed windows are strongly suggested.

PROBLEM 1.9

Heat is transferred at a rate of 0.1 kW through glass wool insulation (density = 100 kg/m^3) with a 5 cm thickness and 2 m^2 area. If the hot surface is at 70°C, determine the temperature of the cooler surface.

GIVEN

- Glass wool insulation with a density (ρ) = 100 kg/m^3
- Thickness (L) = 5 cm = 0.05 m
- Area (A) = 2 m^2
- Temperature of the hot surface (T_h) = 70°C
- Rate of heat transfer (q_k) = 0.1 kW = 100 W

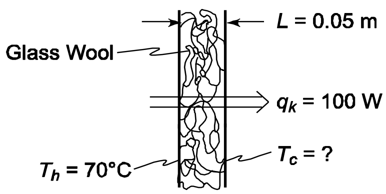
FIND

- The temperature of the cooler surface (T_c)

ASSUMPTIONS

- One dimensional, steady state conduction
- Constant thermal conductivity

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
The thermal conductivity of glass wool at 20°C (k) = 0.036 W/(m K)

SOLUTION

For one dimensional, steady state conduction, the rate of heat transfer, from Equation (1.3), is

$$q_k = \frac{A\,k}{L}\,(T_h - T_c)$$

Solving this for T_c

$$T_c = T_h - \frac{q_k\,L}{A\,k}$$

$$T_c = 70°\text{C} - \frac{(100\,\text{W})(0.05\,\text{m})}{(2\ \text{m}^2)\ 0.036\,\text{W/m K}}$$

$$T_c = 0.6°\text{C}$$

PROBLEM 1.10

> **A heat flux meter at the outer (cold) wall of a concrete building indicates that the heat loss through a wall of 10 cm thickness is 20 W/m^2. If a thermocouple at the inner surface of the wall indicates a temperature of 22°C while another at the outer surface shows 6°C, calculate the thermal conductivity of the concrete and compare your result with the value in Appendix 2, Table 11.**

GIVEN

- Concrete wall
- Thickness (L) = 100 cm = 0.1 m
- Heat loss (q/A) = 20 W/m^2
- Surface temperature
  - Inner (T_i) = 22°C
  - Outer (T_o) = 6°C

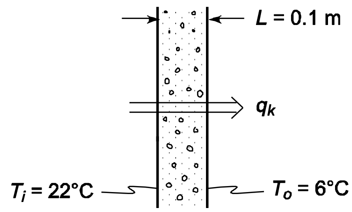
FIND

- The thermal conductivity (k) and compare it to the tabulated value

ASSUMPTIONS

- One dimensional heat flow through the wall
- Steady state conditions exist

SKETCH



SOLUTION

The rate of heat transfer for steady state, one dimensional conduction, from Equation (1.3), is

$$q_k = \frac{k\,A}{L}\,(T_{\text{hot}} - T_{\text{cold}})$$

Solving for the thermal conductivity

$$k = \left(\frac{q_k}{A}\right)\frac{L}{(T_i - T_o)}$$

$$k = (20\ \text{W/m}^2)\left(\frac{0.1\text{m}^2}{22^{\text{o}}\text{C} - 6^{\text{o}}\text{C}}\right) = 0.125\ \text{W/(m K)}$$

This result is very close to the tabulated value in Appendix 2, Table 11 where the thermal conductivity of concrete is given as 0.128 W/(m K).

PROBLEM 1.11

Calculate the heat loss through a 1-m *3-m glass window 7-mm-thick if the inner surface temperature is 20°C and the outer surface temperature is 17°C. Comment on the possible effect of radiation on your answer.

GIVEN

- Window: 1 m by 3 m
- Thickness (L) = 7 mm = 0.007 m
- Surface temperature
  - Inner (T_i) = 20°C
  - outer (T_o) = 17°C

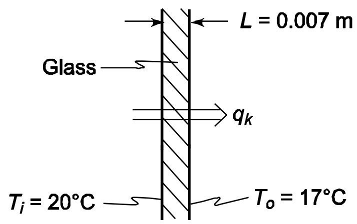
FIND

- The rate of heat loss through the window (q)

ASSUMPTIONS

- One dimensional, steady state conduction through the glass
- Constant thermal conductivity

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 11
Thermal conductivity of glass (k) = 0.81 W/(m K)

SOLUTION

The heat loss by conduction through the window is given by Equation (1.3)

$$q_k = \frac{k\,A}{L}\,(T_{\text{hot}} - T_{\text{cold}})$$

$$q_k = \frac{0.81\,\text{W/(m K)}\;(1\text{m})\,(3\text{m})}{(0.007\,\text{m})}\;(20°\text{C} - 17°\text{C})$$

$$q_k = 1040\text{ W}$$

COMMENTS

- Window glass is transparent to certain wavelengths of radiation, therefore some heat may be lost by radiation through the glass.
- During the day sunlight may pass through the glass creating a net heat gain through the window.

PROBLEM 1.12

A wall of thickness L is made up of material with a thermal conductivity that varies with its thickness x according to the equation k=(ax+b) W/(m K) , where a and b are constants. If the heat flux applied at the surface of one end (x=0) of the wall is 20 W/m^2, derive an expression for the temperature gradient and temperature distribution across the wall thickness (between x=0 and x=L). Use and define appropriate notations for surface temperatures at each end of the wall.

GIVEN

- Thermal conductivity (k) varies according to equation k=(ax+b) W/(m K)
- Heat flux applied (q/A)=20 W/m^2

FIND

- Expression for temperature gradient and temperature distribution across wall thickness.

ASSUMPTIONS

- One dimensional, steady state conduction through the glass
- Constant thermal conductivity

SOLUTION

Let temperature at x=0 be T0 and temperature at x=L be TL.

The rate of heat transfer for steady state, one dimensional conduction, from Equation (1.2), is

$$q_k = -kA\frac{dT}{dx} \qquad \Rightarrow \frac{q_k}{A} = -k\frac{dT}{dx} \qquad \Rightarrow 20 = -(ax+b)\frac{dT}{dx}$$

$\frac{dT}{dx} = \frac{-20}{ax+b}$ is the expression for temperature gradient.

Let temperature at x=0 be T0 and temperature at x=L be TL.

$$dT = -20 * \frac{dx}{ax+b}$$

Integrating the above equation with respect to x we get

$$\int_{T0}^{T} dT = -20 * \int_{0}^{x} \frac{dx}{ax+b} \qquad \Rightarrow \qquad (T - T_0) = -20\frac{\log(ax+b)}{a} + C$$

is required temperature distribution across the wall surface, where C is constant.

PROBLEM 1.13

> **If the outer air temperature in Problem 1.11 is –2°C, calculate the convection heat transfer coefficient between the outer surface of the window and the air assuming radiation is negligible.**

GIVEN

- Window: 1 m by 3 m
- Thickness (L) = 7 mm = 0.007 m
- Surface temperatures
  - Inner (T_i) = 20°C
  - outer (T_o) = 17°C
- The rate of heat loss = 1040 W (from the solution to Problem 1.11)
- The outside air temperature = –2°C

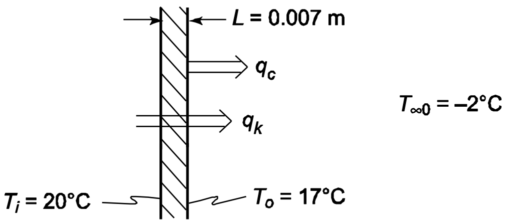
FIND

- The convective heat transfer coefficient at the outer surface of the window ($\bar{h}_c$)

ASSUMPTIONS

- The system is in steady state and radiative loss through the window is negligible

SKETCH



SOLUTION

For steady state the rate of heat transfer by convection (Equation (1.10)) from the outer surface must be the same as the rate of heat transfer by conduction through the glass

$$q_c = \bar{h}_c\ A\ \Delta T = q_k$$

Solving for $\bar{h}_c$

$$\bar{h}_c = \frac{q_k}{A(T_o - T_\infty)}$$

$$\bar{h}_c = \frac{1040\,\text{W}}{(1\text{m})(3\text{m})(17\,^\circ\text{C} - -2\,^\circ\text{C})}$$

$$\bar{h}_c = 18.2\ \text{W/(m}^2\ \text{K)}$$

COMMENTS

- This value for the convective heat transfer coefficient falls within the range given for the free convection of air in Table 1.4.

PROBLEM 1.14

Using Table 1.4 as a guide, prepare a similar table showing the order of magnitudes of the thermal resistances of a unit area for convection between a surface and various fluids.

GIVEN

- Table 1.4— The order of magnitude of convective heat transfer coefficient ($\bar{h}_c$)

FIND

- The order of magnitudes of the thermal resistance of a unit area ($A\ R_c$)

SOLUTION

The thermal resistance for convection is defined by Equation (1.14) as

$$R_c = \frac{1}{\bar{h}_c A}$$

Therefore the thermal resistances of a unit area are simply the reciprocal of the convective heat transfer coefficient

$$A\ R_c = \frac{1}{\bar{h}_c}$$

As an example, the first item in Table 1.4 is 'air, free convection' with a convective heat transfer coefficient of 6–30 W/(m^2 K). Therefore the order of magnitude of the thermal resistances of a unit area for air, free convection is

$$\frac{1}{30\ \text{W/(m}^2\text{K)}} = 0.03\ \ (\text{m}^2\text{K})/\text{W} \ \text{ to } \ \frac{1}{6\ \text{W/(m}^2\text{K)}} = 0.17\ \ (\text{m}^2\text{K})/\text{W}$$

The rest of the table can be calculated in a similar manner

Order of Magnitude of Thermal Resistance of a Unit Area for Convection

Fluid	W/(m^2 K)	Btu/(h ft^2 °F)
Air, free convection	0.03–0.2	0.2–1.0
Superheated steam or air, forced convection	0.003–0.03	0.02–0.2
Oil, forced convection	0.0006–0.02	0.003–0.1
Water, forced convection	0.0002–0.003	0.0005–0.02
Water, boiling	0.00002–0.0003	0.0001–0.002
Steam, condensing	0.000008–0.0002	0.00005–0.001

COMMENTS

The extremely low thermal resistance in boiling and condensation suggests that these resistances can often be neglected in a series thermal network.

PROBLEM 1.15

A thermocouple (0.8-mm-*OD* wire) used to measure the temperature of quiescent gas in a furnace gives a reading of 165°C. It is known, however, that the rate of radiant heat flow per meter length from the hotter furnace walls to the thermocouple wire is 1.1 W/m and the convective heat transfer coefficient between the wire and the gas is 6.8 W/(m² K). With this information, estimate the true gas temperature. State your assumptions and indicate the equations used.

GIVEN

- Thermocouple (0.8 mm *OD* wire) in a furnace
- Thermocouple reading (T_p) = 165°C
- Radiant heat transfer to the wire (q_r/L) = 1.1 W/m
- Heat transfer coefficient ($\bar{h}_c$) = 6.8 W/(m^2 K)

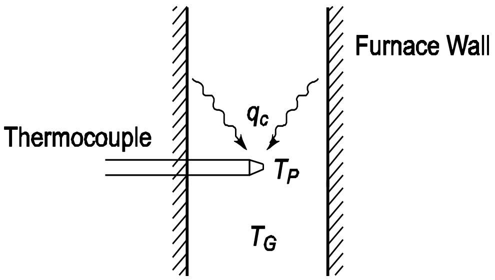
FIND

- Estimate the true gas temperature (T_G)

ASSUMPTIONS

- The system is in equilibrium
- Conduction along the thermocouple is negligible
- Conduction between the thermocouple and the furnace wall is negligible

SKETCH



SOLUTION

Equilibrium and the conservation of energy require that the heat gain of the probe by radiation if equal to the heat lost by convection.

The rate of heat transfer by convection is given by Equation (1.10)

$$q_c = \bar{h}_c \, A \, \Delta T = \bar{h}_c \, \pi D L \, (T_p - T_G)$$

For steady state to exist the rate of heat transfer by convection must equal the rate of heat transfer by radiation

$$q_c = q_r$$

$$\bar{h}_c \, \pi D L \, (T_p - T_G) = \left(\frac{q_r}{L}\right) L$$

$$T_G = T_p - \frac{\left(\frac{q_r}{L}\right) L}{\bar{h}_c \, \pi D L}$$

$$T_G = 165°\text{C} - \frac{(1.1\text{W/m})}{6.8\ \text{W/(m}^2\text{K)}\ \pi(0.0008\,\text{m})}$$

$$T_G = 101°\text{C}$$

COMMENTS

This example illustrates that care must be taken in interpreting experimental measurements. In this case a significant correction must be applied to the thermocouple reading to obtain the true gas temperature. Can you suggest ways to reduce the correction?

PROBLEM 1.16

> **Water at a temperature of 77°C is to be evaporated slowly in a vessel. The water is in a low pressure container surrounded by steam as shown in the sketch below. The steam is condensing at 107°C. The overall heat transfer coefficient between the water and the steam is 1100 W/(m^2 K). Calculate the surface area of the container which would be required to evaporate water at a rate of 0.01 kg/s.**

GIVEN

- Water evaporated slowly in a low pressure vessel surrounded by steam
- Water temperature (T_w) = 77°C
- Steam condensing temperature (T_s) = 107°C
- Overall transfer coefficient between the water and the steam (U) = 1100 W/(m^2 K)
- Evaporation rate $\dot{m}_w$ = 0.01 kg/s

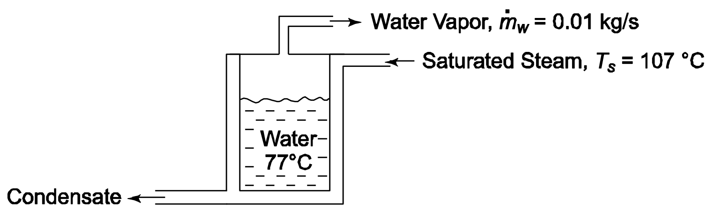
FIND

- The surface area (A) of the container required

ASSUMPTIONS

- Steady state prevails
- Vessel pressure is held constant at the saturation pressure corresponding to 77°C

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 13
The heat of vaporization of water at 77°C (h_{fg}) = 2317 kJ/kg

SOLUTION

The heat transfer required to evaporate water at the given rate is

$$q = \dot{m}_w \, h_{fg}$$

For the heat transfer between the steam and the water

$$q = U A \Delta T = \dot{m}_w \, h_{fg}$$

Solving this for the transfer area

$$A = \frac{\dot{m}_w h_{fg}}{U \, \Delta T}$$

$$A = \frac{(0.01 \text{kg/s})(2317 \text{kJ/kg})(1000 \text{J/kJ})}{1100 \text{W/(m}^2\text{K)} \ (107\,^\text{o}\text{C} - 77^\text{o}\,\text{C})}$$

$$A = 0.70 \text{ m}^2$$

PROBLEM 1.17

The heat transfer rate from hot air by convection at 100°C flowing over one side of a flat plate with dimensions 0.1-m by 0.5-m is determined to be 125 W when the surface of the plate is kept at 30°C. What is the average convective heat transfer coefficient between the plate and the air?

GIVEN

- Flat plate, 0.1-m by 0.5-m, with hot air flowing over it
- Temperature of plate surface (T_s) = 30°C
- Air temperature (T_∞) = 100°C
- Rate of heat transfer (q) = 125 W

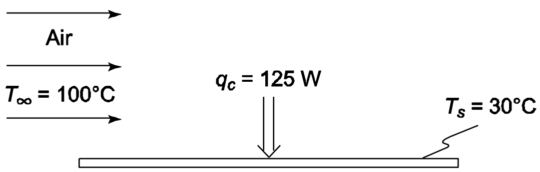
FIND

- The average convective heat transfer coefficient, h_c, between the plate and the air

ASSUMPTION

- Steady state conditions exist

SKETCH



SOLUTION

For convection the rate of heat transfer is given by Equation (1.10)

$$q_c = \bar{h}_c A \Delta T$$

$$q_c = \bar{h}_c \; A \,(T_\infty - T_s)$$

Solving this for the convective heat transfer coefficient yields

$$\bar{h}_c = \frac{q_c}{A(T_\infty - T_s)}$$

$$\bar{h}_c = \frac{125\,\text{W}}{(0.1\,\text{m})(0.5\,\text{m})(100^\circ\text{C} - 30^\circ\text{C})}$$

$$\bar{h}_c = 35.7 \text{ W/(m}^2\text{ K)}$$

COMMENTS

One can see from Table 1.4 (order of magnitudes of convective heat transfer coefficients) that this result is reasonable for free convection in air.

Note that since $T_\infty > T_s$ heat is transferred from the air to the plate.

PROBLEM 1.18

The heat transfer coefficient for a gas flowing over a thin flat plate 3-m-long and 0.3-m-wide varies with distance from the leading edge according to

$$\bar{h}_c\ (x) = 10 \times x^{-\frac{1}{4}}\ \text{W/(m}^2\text{K)}$$

If the plate temperature is 170°C and the gas temperature is 30°C, calculate (a) the average heat transfer coefficient, (b) the rate of heat transfer between the plate and the gas and (c) the local heat flux 2 m from the leading edge.

GIVEN

- Gas flowing over a 3-m-long by 0.3-m-wide flat plate
- Heat transfer coefficient (h_c) is given by the equation above
- The plate temperature (T_P) = 170°C
- The gas temperature (T_G) = 30°C

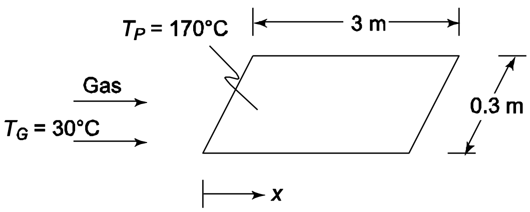
FIND

(a) The average heat transfer coefficient ($\bar{h}_c$)
(b) The rate of heat transfer (q_c)
(c) The local heat flux at x = 2 m (q_c (2)/A)

ASSUMPTIONS

- Steady state prevails

SKETCH



SOLUTION

(a) The average heat transfer coefficient can be calculated by

$$\bar{h}_c = \frac{1}{L}\int_0^L h_c\,(x)\,dx = \frac{1}{L}\int_0^L 10\times{}^{-\frac{1}{4}} = \frac{10}{L}\,\frac{4}{3}\times{}^{\frac{3}{4}}\Big|_0^L = \frac{10}{3}\,\frac{4}{3}\,3^{\frac{3}{4}}$$

$$\bar{h}_c = 10.13\ \text{W/m}^2\ \text{K}$$

(b) The total convective heat transfer is given by Equation (1.10)

$$q_c = \bar{h}_c\ A\ (T_P - T_G)$$

$$q_c = 10.13\ \text{W/(m}^2\text{K)}\quad (3\ \text{m})\ (0.3\ \text{m})\ (170°\text{C} - 30°\text{C})$$

$$q_c = 1273\ \text{W}$$

(c) The heat flux at x = 2 m is

$$\frac{q(x)}{A} = h_c(x)\,(T_P - T_G) = 10 \times {}^{-\frac{1}{4}}\,(T_P - T_G)$$

$$\frac{q(2)}{A} = 10\,(2)^{-\frac{1}{4}}\,(170°\text{C} - 30°\text{C})$$

$$\frac{q(2)}{A} = 1177\ \text{W/m}^2$$

COMMENTS

Note that the equation for h_c does not apply near the leading edge of the plate since h_c approaches infinity as x approaches zero. This behavior is discussed in more detail in Chapter 7.

PROBLEM 1.19

A cryogenic fluid is stored in a 0.3-m-diameter spherical container in still air. If the convection heat transfer coefficient between the outer surface of the container and the

air is 6.8 W/(m² K), the temperature of the air is 27°C and the temperature of the surface of the sphere is –183°C, determine the rate of heat transfer by convection.

GIVEN

- A sphere in still air
- Sphere diameter (D) = 0.3 m
- Convective heat transfer coefficient $\bar{h}_c$ = 6.8 W/(m^2 K)
- Sphere surface temperature (T_s) = –183°C
- Ambient air temperature (T_∞) = 27°C

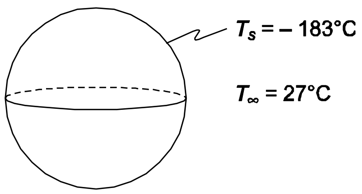
FIND

- Rate of heat transfer by convection (q_c)

ASSUMPTIONS

- Steady state heat flow

SKETCH



SOLUTION

The rate of heat transfer by convection is given by

$$q_c = \bar{h}_c \; A \, \Delta T$$

$$q_c = \bar{h}_c \; (\pi D^2)\,(T_\infty - T_s)$$

$$q_c = \; 6.8\,\mathrm{W/(m^2K)} \;\; \pi\,(0.3 \text{ m})^2 \; [27°\mathrm{C} - (-183°\mathrm{C})]$$

$$q_c = 404 \text{ W}$$

COMMENTS

Condensation would probably occur in this case due to the low surface temperature of the sphere. A calculation of the total rate of heat transfer to the sphere would have to take the rate on condensation and the rate of radiative heat transfer into account.

PROBLEM 1.20

A high-speed computer is located in a temperature controlled room of 26°C. When the machine is operating its internal heat generation rate is estimated to be 800 W. The external surface temperature is to be maintained below 85°C. The heat transfer coefficient for the surface of the computer is estimated to be 10 W/(m^2 K). What surface area would be necessary to assure safe operation of this machine? Comment on ways to reduce this area.

GIVEN

- A high-speed computer in a temperature controlled room
- Temperature of the room (T_∞) = 26°C
- Maximum surface temperature of the computer (T_c) = 85°C
- Heat transfer coefficient (U) = 10 W/(m K)
- Internal heat generation $\dot{q}_G$ = 800 W

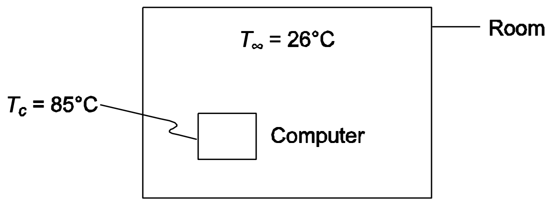
FIND

- The surface area (A) required and comment on ways to reduce this area

ASSUMPTIONS

- The system is in steady state

SKETCH



SOLUTION

For steady state the rate of heat transfer from the computer (given by Equation (1.34)) must equal the rate of internal heat generation

$$q = U A \Delta T = \dot{q}_G$$

Solving this for the surface area

$$A = \frac{\dot{q}_G}{U \Delta T}$$

$$A = \frac{800\,\text{W}}{10\,\text{W/(m}^2\text{K)}\ (85^\circ\text{C} - 26^\circ\text{K})} = 1.4\ \text{m}^2$$

COMMENTS

Possibilities to reduce this surface area include

- Increase the convective heat transfer from the computer by blowing air over it
- Add fins to the outside of the computer

PROBLEM 1.21

> **In an experimental setup in a laboratory, a long cylinder with 5-m diameter, and an electrical resistance heater inside its entire length is cooled with water flowing crosswise over the cylinder at 25ºC and a velocity of 0.8 m/s. For these flow conditions, 20 KW/m of power is required to maintain a uniform temperature of 95ºC at the surface of the cylinder. When water is not available, air at 25ºC is used with the velocity of 10 m/s to maintain the same surface temperature. However, in this case, the cylinder surface heat dissipation rate is reduced to 400 W/m. Calculate the convection heat transfer coefficients for both water and air, and comment on the reason for differences in the value.**

GIVEN

- A long cylinder with diameter d=5 cm=0.05 m
  Case A
- Crosswise water flow at 25ºC and velocity of 0.8 m/s
- Power required(q_c/L)=20,000 W/m
- Surface temperature(Ts)=95 ºC
  Case B
- Crosswise air flow at 25ºC and velocity of 10 m/s
- Power required(q_c/L)=400 W/m
- Surface temperature(Ts)=95 ºC

FIND

- Convection heat transfer coefficient for both air and water.
- Comment on differences in the values.

ASSUMPTIONS

- The system is in steady state

SOLUTION

The rate of heat transfer by convection is given by

$$q_c = \bar{h}_c \, A \, \Delta T$$

$$q_c = \bar{h}_c * \pi d L * \Delta T$$

$$q_c/\mathrm{L} = \bar{h}_c * \pi d * \Delta T$$

$$\bar{h}_c = \frac{q_c}{\mathrm{L}} * \frac{1}{\pi d * (Ts - T\infty)}$$

Case A(for water)

$$\bar{h}_c = 20000 * \frac{1}{\pi * 0.05 * (97 - 25)} \text{ W/(m}^2\text{ K)}$$

$\bar{h}_c$ **=1768** W/(m^2 K)

Case B(for air)

$$\bar{h}_c = 400 * \frac{1}{\pi * 0.05 * (97 - 25)} \text{ W/(m}^2\text{ K)}$$

$\bar{h}_c$ =35.4 W/(m^2 K)

Thus convection heat transfer coefficient for water and air are 1768 and 35.4 W/(m^2 K) respectively.

COMMENTS

The difference in heat transfer coefficients between air and water is due to

- Different physical properties like density, specific heat capacity.
- Different velocity of fluids.

PROBLEM 1.22

In order to prevent frostbite to skiers on chair lifts, the weather report at most ski areas gives both an air temperature and the wind chill temperature. The air temperature is measured with a thermometer that is not affected by the wind. However, the rate of heat loss from the skier increases with wind velocity, and the wind-chill temperature is the temperature that would result in the same rate of heat loss in still air as occurs at the measured air temperature with the existing wind.

Suppose that the inner temperature of a 3-mm-thick layer of skin with a thermal conductivity of 0.35 W/(m K) is 35°C and the ambient air temperature is –20°C. Under calm ambient conditions the heat transfer coefficient at the outer skin surface is about 20 W/(m² K) (see Table 1.4), but in a 40 mph wind it increases to 75 W/(m² K). (a) If frostbite can occur when the skin temperature drops to about 10°C, would you advise the skier to wear a face mask? (b) What is the skin temperature drop due to wind?

GIVEN

- Skier's skin exposed to cold air
- Skin thickness (L) = 3 mm = 0.003 m
- Inner surface temperature of skin (T_{si}) = 35°C
- Thermal conductivity of skin (k) = 0.35 W/(m K)
- Ambient air temperature (T_∞) = –20°C
- Convective heat transfer coefficients
  - Still air (h_{c0}) = 20 W/(m² K)
  - 40 mph air (h_{c40}) = 75 W/(m² K)
- Frostbite occurs at an outer skin surface temperature (T_{so}) = 10°C

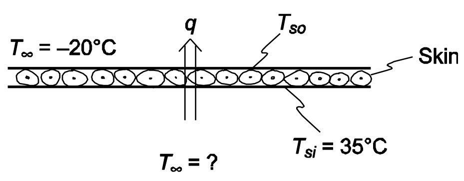
FIND

(a) Will frostbite occur under still or 40 mph wind conditions?
(b) Skin temperature drop due to wind chill.

ASSUMPTIONS

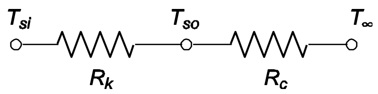
- Steady state conditions prevail
- One dimensional conduction occurs through the skin
- Radiative loss (or gain from sunshine) is negligible

SKETCH



SOLUTION

The thermal circuit for this system is shown below



(a) The rate of heat transfer is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_k + R_c} = \frac{T_{si} - T_\infty}{\left(\dfrac{L}{k\,A}\right) + \left(\dfrac{1}{\bar{h}_c\,A}\right)}$$

$$\therefore \; \frac{q}{A} = \frac{T_{si} - T_\infty}{\dfrac{L}{k} + \dfrac{1}{\bar{h}_c}}$$

The outer surface temperature of the skin in still air can be calculated by examining the conduction through the skin layer

$$q_k = \frac{k\,A}{L}\,(T_{si} - T_{so})$$

Solving for the outer skin surface temperature

$$T_{so} = T_{si} - \frac{q_k}{A}\,\frac{L}{k}$$

The rate of heat transfer by conduction through the skin must be equal to the total rate of heat transfer, therefore

$$T_{so} = T_{si} - \left[\frac{T_{si} - T_\infty}{\dfrac{L}{K} + \dfrac{1}{\bar{h}_c}}\right]\frac{L}{k}$$

Solving this for still air

$$(T_{so})_{\text{still air}} = 35°\text{C} - \left[\frac{35^\text{o}\,\text{C} - (-20^\text{o}\,\text{C})}{\dfrac{0.003\,\text{m}}{0.25\,\text{W/(m K)}} + \dfrac{1}{20\,\text{W/(m}^2\,\text{K)}}}\right]\frac{0.003\,\text{m}}{0.25\,\text{W/(m}^2\text{K)}}$$

$$(T_{so})_{\text{still air}} = 24°\text{C}$$

For a 40 mph wind

$$(T_{so})_{40\text{ mph}} = 35°\text{C} - \left[\frac{35^\text{o}\,\text{C} - (-20^\text{o}\,\text{C})}{\dfrac{0.003\,\text{m}}{0.25\,\text{W/(m K)}} + \dfrac{1}{75\,\text{W/(m}^2\,\text{K)}}}\right]\frac{0.003\,\text{m}}{0.25\,\text{W/(m}^2\text{K)}}$$

$$(T_{so})_{40\text{ mph}} = 9°\text{C}$$

Therefore, frostbite may occur under the windy conditions.

(b) Comparing the above results we see that the skin temperature drop due to the wind chill was 15°C.

PROBLEM 1.23

Using the information in Problem 1.22, estimate the ambient air temperature that could cause frostbite on a calm day on the ski slopes.

GIVEN

- Skier's skin of thickness (L) = 3 mm = 0.003 m exposed to cold air
- Inner surface temperature of skin (T_{si}) = 35°C
- Thermal conductivity of skin (k) = 0.35 W/(m K)
- Convective heat transfer coefficient in still air ($\bar{h}_c$) = 20 W/(m^2 K)
- Frostbite occurs at an outer skin surface temperature (T_{so}) = 10°C

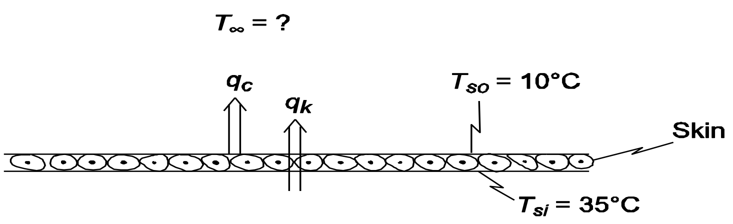
FIND

- The ambient air temperature (T_∞) that could cause frostbite

ASSUMPTIONS

- Steady state conditions prevail
- One dimensional conduction occurs through the skin
- Radiative loss (or gain from sunshine) is negligible

SKETCH



SOLUTION

The rate of conductive heat transfer through the skin at frostbite conditions is given by Equation (1.3)

$$q_k = (kA/L)(T_{si} - T_{so})$$

The rate of convective heat transfer from the surface of the skin, from Equation (1.10), is

$$q_c = \bar{h}_c \; A \, (T_{so} - T_\infty)$$

These heat transfer rates must be equal

$$q_k = q_c \qquad \Rightarrow \qquad (k\,A/L)\;(T_{si} - T_{so}) = \bar{h}_c \; A\,(T_{so} - T_\infty)$$

Solving for the ambient air temperature

$$T_\infty = T_{so}\left(1 + \frac{k}{\bar{h}_c\,L}\right) - T_{si}\left(\frac{k}{\bar{h}_c\,L}\right)$$

$$T_\infty = 10°\text{C}\left[1 + \frac{0.25\,\text{W/(m K)}}{\left[20\,\text{W/(m}^2\text{K)}\right](0.003\,\text{m})}\right] - 35°\text{C}\left[\frac{0.25\,\text{W/(m K)}}{\left[20\,\text{W/(m}^2\text{K)}\right](0.003\,\text{m})}\right]$$

$$T_\infty = -94°\text{C}$$

PROBLEM 1.24

Two large parallel plates with surface conditions approximating those of a blackbody are maintained at 816 and 260°C, respectively. Determine the rate of heat transfer by radiation between the plates in W/m^2 and the radiative heat transfer coefficient in W/(m^2 K).

GIVEN

- Two large parallel plates, approximately black bodies
- Temperatures
  - T_1 = 816°C = 1089 K
  - T_2 = 260°C= 533 K

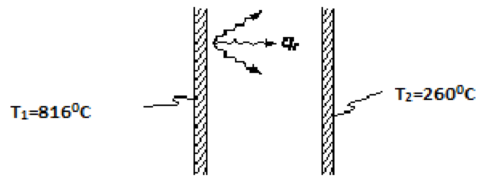
FIND

(a) Rate of radiative heat transfer (q_r/A) in W/m^2
(b) Radiative heat transfer coefficient (h_r) in W/(m^2 K)

ASSUMPTIONS

- Steady state prevails
- Edge effects are negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: Stefan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K)

SOLUTION

(a) The rate of heat transfer is given by Equation (1.16)

$$\frac{q_r}{A} = \sigma(T_1^4 - T_2^4) = \left(5.67\times10^{-8} W/(m^2K^4)\right)\left((1089\,K)^4 - (533\,K)^4\right)$$

$$\frac{q_r}{A} = 75167 \text{ W/m}^2$$

(b) Let h_r represent the radiative heat transfer coefficient

$$q_r = h_r\, A\, \Delta T$$

$$\therefore \qquad h_r = \frac{q_r}{A}\,\frac{1}{\Delta T} = \frac{75167 W/m^2}{1089\,K - 533 K}$$

$$h_r = 135.3 \text{ W/(m}^2\text{ K)}$$

COMMENTS

Note that absolute temperatures must be used in the radiative heat transfer equation, whereas h_r is based on the assumption that the rate of heat transfer is proportional to the temperature difference. Hence h_r cannot be applied to any other temperatures than those specified.

PROBLEM 1.25

> **A spherical vessel 0.3-m in diameter is located in a large room whose walls are at 27°C (see sketch). If the vessel is used to store liquid oxygen at –183°C and both the surface of the storage vessel and the walls of the room are black, calculate the rate of heat transfer by radiation to the liquid oxygen in watts and in Btu/h.**

GIVEN

- A black spherical vessel of liquid oxygen in a large black room
- Liquid oxygen temperature (T_o) = –183°C = 90 K
- Sphere diameter (D) = 0.3 m
- Room wall temperature (T_w) = 27°C = 300 K

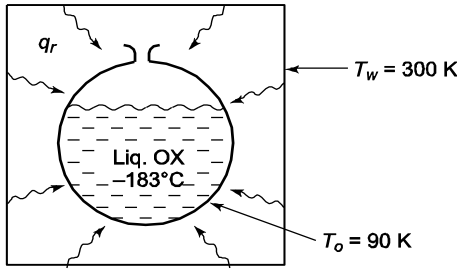
FIND

- The rate of radiative heat transfer to the liquid oxygen in W and Btu/h

ASSUMPTIONS

- Steady state prevails
- The temperature of the vessel wall is the same as the temperature of the oxygen

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The net radiative heat transfer to a black body in a black enclosure is given by Equation (1.16)

$$q_r = A\,\sigma(T_1^4 - T_2^4)$$

$$q_r = \pi D^2\,\sigma(T_w^4 - T_o^4)$$

$$q_r = \pi(0.3\text{ m})^2\left(5.67\times10^{-8}\ W/(\text{m}^2\ K^4)\right)\ (300\text{ K})^4 - (90\text{ K})^4$$

$$q_r = 128.8\text{ W}$$

In Btu/hr $\quad q_r$= 128.8 *3.412 Btu/hr= 439.5 Btu/hr.

COMMENTS

Note that absolute temperatures must be used in the radiative heat transfer equation.

PROBLEM 1.26

Repeat Problem 1.25 but assume that the surface of the storage vessel has an absorptance (equal to the emittance) of 0.1. Then determine the rate of evaporation of the liquid oxygen in kilograms per second and pounds per hour, assuming that convection can be neglected. The heat of vaporization of oxygen at –183°C is 213.3 kJ/kg.

GIVEN

- A spherical vessel of liquid oxygen in a large black room
- Emittance of vessel surface (ε) = 0.1
- Liquid oxygen temperature (T_o) = –183°C = 90 K
- Sphere diameter (D) = 0.3 m
- Room wall temperature (T_w) = 27°C = 300 K
- Heat of vaporization of oxygen (h_{fg}) = 213.3 kJ/kg

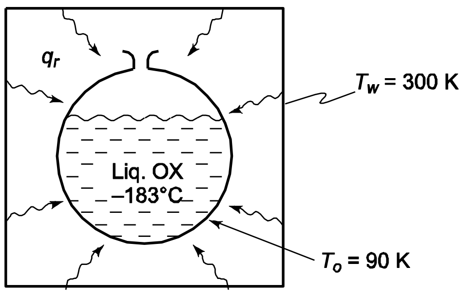
FIND

(a) The rate of radiative heat transfer (q_r) to the liquid oxygen in W
(b) The rate of evaporation of oxygen (m_o) in kg/s

ASSUMPTIONS

- Steady state prevails
- The temperature of the vessel wall is equal to the temperature of the oxygen
- Convective heat transfer is negligible

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

(a) The net radiative heat transfer from a gray body in a black enclosure, from Equation (1.17) is

$$q_r = A_1\, \varepsilon_1\, \sigma (T_1^4 - T_2^4)$$

$$q_r = \pi D^2\, \varepsilon\, \sigma (T_o^4 - T_w^4)$$

$$q_r = \pi (0.3\text{ m})^2\, (0.1)\, \big(5.67 \times 10^{-8}\, [\text{W/(m}^2\text{ K}^4)]\, [(90\text{ K})^4 - (300\text{ K})^4)]$$

$$q_k = -12.9\text{ W}$$

(b) The rate of evaporation of oxygen is given by

$$\dot{m}_o = \frac{q_r}{h_{fg}}$$

$$\dot{m}_o = \frac{(12.9\,\text{W}) \quad \text{J/Ws}}{(213.3\ \text{kJ/kg})(1000\ \text{J/kJ})}$$

$$\dot{m}_o = 6.05 \times 10^{-5}\ \text{kg/s}$$

COMMENTS

Note that absolute temperatures must be used in the radiative heat transfer equation.

The negative sign in the rate of heat transfer indicates that the sphere is gaining heat from the surrounding wall.

Note that the rate of heat transfer by radiation can be substantially reduced (see Problem 1.25) by applying a surface treatment, e.g., applying a metallic coating with low emissivity.

PROBLEM 1.27

Determine the rate of radiant heat emission in watts per square meter from a blackbody at (a) 150°C, (b) 600°C, (c) 5700°C.

GIVEN

- A blackbody

FIND

The rate of radiant heat emission (q_r) in W/m^2 for a temperature of

(a) T = 150°C = 423 K
(b) T = 600°C = 873 K
(c) T = 5700°C = 5973 K

PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The rate of radiant heat emission from a blackbody is given by Equation (1.15)

$$q_r = \sigma A_1 T_1^4 \qquad => \frac{q_r}{A} = \sigma T^4$$

(a) For T = 423 K

$$\frac{q_r}{A} = [5.67 \times 10^{-8} \text{ W/(m}^2 \text{ K}^4)] (423\text{K})^4$$

$$\frac{q_r}{A} = 1820 \text{ W/m}^2$$

(b) For T = 873 K

$$\frac{q_r}{A} = [5.67 \times 10^{-8} \text{ W/(m}^2 \text{ K}^4)] (873 \text{ K})^4$$

$$\frac{q_r}{A} = 32{,}900 \text{ W/m}^2$$

(c) For T = 5973 K

$$\frac{q_r}{A} = [(5.67 \times 10^{-8} \text{ W/(m}^2 \text{ K}^4)] (5974 \text{ K})^4$$

$$\frac{q_r}{A} = 7.2 \times 10^7 \text{ W/m}^2$$

COMMENTS

Note that absolute temperatures must be used in radiative heat transfer equations.

The rate of heat transfer is proportional to the absolute temperature to the fourth power, this results in a rapid increase in the rate of heat transfer with increasing temperature.

PROBLEM 1.28

The sun has a radius of 7×10^8 m and approximates a blackbody with a surface temperature of about 5800 K. Calculate the total rate of radiation from the sun and the emitted radiation flux per square meter of surface area.

GIVEN

- The sun approximates a blackbody
- Surface temperature (T_s) = 5800 K
- Radius (r) = 7×10^8 m

FIND

(a) The total rate of radiation from the sun (q_r)
(b) The radiation flux per square meter of surface area (q_r/A)

PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The rate of radiation from a blackbody, from Equation (1.15), is

$$q_r = \sigma A\, T^4$$

$$q_r = [5.67 \times 10^{-8} \text{ W/(m}^2\text{ K}^4)]\,[4\pi (7 \times 10^8 \text{ m})^2]\,(5800 \text{ K})^4$$

$$q_r = 4.0 \times 10^{26} \text{ W}$$

The flux per square meter is given by

$$\frac{q_r}{A} = \sigma T^4$$

$$\frac{q_r}{A} = [5.67 \times 10^{-8} \text{ W/(m}^2\text{ K}^4)]\,(5800 \text{ K})^4$$

$$\frac{q_r}{A} = 6.4 \times 10^7 \text{ W/m}^2$$

COMMENTS

The solar radiation flux impinging in the earth's atmosphere is only 1400 W/m^2. Most of the radiation from the sun goes into space.

PROBLEM 1.29

A spherical interplanetary probe with 30-cm diameter contains electronic equipment that dissipates 100 W. If the probe surface has emissivity of 0.8, what is its surface temperature in outer space? State your assumptions in calculation.

GIVEN

Interplanetary probe with 30-cm diameter, radius(r) =0.15 m
Dissipation q_r=100 W and emissivity (ε)=0.8

•

FIND

Surface temperature in outer space.

ASSUMPTIONS

- Steady state condition.
- Heat transfer only by radiation.
- Temperatue of outer space is considered 0 Kelvin. (T_2)=0 K

PROPERTIES AND CONSTANTS

- From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The rate of radiation from a body, from Equation (1.17), is

$$q_r = A_1\, \varepsilon_1\, \sigma\, (T_1^4 - T_2^4)$$

$$100=4*\pi*(0.15)^2*0.8*5.67*10^{-8}*(T_1^4-0)$$

$$T_1^4=100/(4*\pi*(0.15)^2*0.8*5.67*10^{-8})$$

$$T_1=297 \text{ K}$$

Thus surface temperature of probe in outer space is 297 K.

COMMENTS

Note that absolute temperatures are determined from radiative heat transfer equations.

PROBLEM 1.30

A small gray sphere having an emissivity of 0.5 and a surface temperature of 537^{0}C is located in a blackbody enclosure having a temperature of 37°C. Calculate for this system: (a) the net rate of heat transfer by radiation per unit of surface area of the sphere, (b) the radiative thermal conductance in W/K if the surface area of the sphere is 95 cm^2, (c) the thermal resistance for radiation between the sphere and its surroundings, (d) the ratio of thermal resistance for radiation to thermal resistance for convection if the convective heat transfer coefficient between the sphere and its surroundings is 11 W/(m^2 K), (e) the total rate of heat transfer from the sphere to the surroundings, and (f) the combined heat transfer coefficient for the sphere.

GIVEN

- Small gray sphere in a blackbody enclosure
- Sphere emissivity (ε_s) = 0.5
- Sphere surface temperature (T_1) = 337^{0}C= 810 K
- Enclosure temperature (T_2) = 37°C = 310 K
- The surface area of the sphere (A) = 95 cm^2=9.5*10^{-3} m^2
- The convective transfer coefficient ($\bar{h}_c$) = 11 W/(m^2 K)

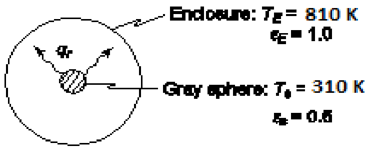
FIND

(a) Rate of heat transfer by radiation per unit surface area
(b) Radiative thermal conductance (K_r) in W/K
(c) Thermal resistance for radiation (R_r)
(d) Ratio of the radiative and conductive resistance
(e) Total rate of heat transfer (q_T) to the surroundings
(f) Combined heat transfer coefficient ($\bar{h}_{cr}$)

ASSUMPTIONS

- Steady state prevails
- The temperature of the fluid in the enclosure is equal to the enclosure temperature

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

(a) For a gray body radiating to a blackbody enclosure the net heat transfer is given by Equation (1.17)

$$q_r = A_1\, \varepsilon_1\, \sigma\,(T_1^4 - T_2^4)$$

$$\frac{q_r}{A} = (0.5)\,[5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)]\,[(810\ \text{K})^4 - (310\ \text{K})^4]$$

$$\frac{q_r}{A} = 11942 \text{ W/m}^2$$

(b) The radiative thermal conductance must be based on some reference temperature. Let the reference temperature be the enclosure temperature. Then, from Equation (1.20), the radiative thermal conductance is

$$K_r = \frac{A_1\, f_{1-2}\, \rho(T_1^4 - T_2^4)}{T_1 - T_2'} \quad \text{where } f_{1-2} = \varepsilon_s$$

$$K_r = \frac{(9.5*10^{-3}\, m^2)(0.5)[5.67\times10^{-8}\, W/(m^2 K^4)](810^4 - 310^4)K^4}{810\,K - 310\,K}$$

$$K_r = 0.227 \text{ W/K}$$

(c) The thermal resistance for radiation is given by

$$R_r = \frac{1}{K_r} = \frac{1}{0.227(W/K)} = 4.4 \text{ K/W}$$

(d) The convective thermal resistance is given by Equation (1.14)

$$R_c = \frac{1}{\bar{h}_c\, A} = \frac{1}{[11W/(m^2\text{K})](9.5*10^{-3} m^2)} = 9.56 \text{ K/W}$$

Therefore the ratio of the radiative to the convective resistance is

$$\frac{R_r}{R_c} = \frac{4.4K/W}{9.56K/W} = 0.46$$

(e) The radiative and convective resistances are in parallel, therefore the total resistance, from Figure 1.18, is

$$R_{\text{total}} = \frac{R_c\, R_r}{R_c + R_r} = \frac{(9.56)(4.4)}{9.56 + 4.4} = 3.01 \text{ K/W}$$

The total heat transfer is given by:

$$q_T = \frac{\Delta T}{R_{\text{total}}} = \frac{810\,K - 310\,K}{3.01K/W} = 166.1 \text{ W}$$

(f) The combined heat transfer coefficient can be calculated from

$$q_T = \overline{h_{cr}}\; A\Delta\, T$$

$$\therefore\ \overline{h_{cr}} = \frac{q_T}{A\Delta T} = \frac{166.1 \text{ W}}{(9.5*10^{-3}\text{ m}^2)\,(810\,K - 310\,K)}$$

$$\overline{h_{cr}} = 34.97 \text{ W/(m}^2\text{ K)}$$

COMMENTS

Note that absolute temperatures must be used in the radiative heat transfer equations.
Both heat transfer mechanisms are of the same order of magnitude in this situation.

PROBLEM 1.31

A spherical communications satellite 2-m in diameter is placed in orbit around the earth. The satellite generates 1000 W of internal power from a small nuclear generator. If the surface of the satellite has an emittance of 0.3 and is shaded from solar radiation by the earth, estimate the surface temperature. What is the temperature if the satellite with an absorptivity of 0.2 is in an orbit in which it is exposed to solar radiation? Assume the sun is a blackbody and irradiation striking the satellite is 1366 W/m^2 and state your assumptions.

GIVEN

- Spherical satellite
- Diameter (D) = 2 m
- Heat generation = 1000 W
- Emittance (ε) = 0.3
- Absorptivity of satellite (α)=0.2
- Solar irradiation(G)=1366 W/m^2

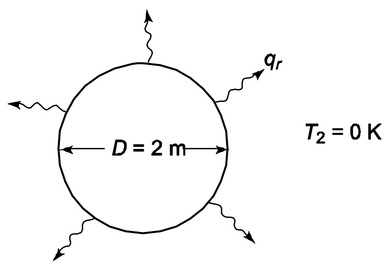
FIND

- The surface temperature (T_s)
- Surface temperature of satellite

ASSUMPTIONS

- The satellite radiates to space which behaves as a blackbody enclosure at 0 K
- The system is in steady state

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

From Equation (1.17), the rate of the heat transfer from a gray body in a blackbody enclosure is

$$q_r = A_1\, \varepsilon_1\, \sigma\, (T_1^4 - T_2^4)$$

Considering single body radiation T_2=0

Solving this for the surface temperature

$$T_1 = \left(\frac{q_r}{A_1 \varepsilon_1 \sigma}\right)^{\frac{1}{4}} = \left(\frac{q_r}{\pi D^2 \varepsilon_1 \sigma}\right)^{\frac{1}{4}}$$

For steady state the rate of heat transfer must equal the rate of internal generation, therefore the surface temperature is

$$T_1 = \left(\frac{1000\,\text{W}}{\pi(2\,\text{m})^2(0.3)5.67\times10^{-8}\,\text{W}/(\text{m}^2\,\text{K}^4)}\right)^{\frac{1}{4}} = 262\text{ K} = -11°\text{C}$$

The amount of radiation absorbed by the satellite is given by
$q_a = \alpha * A * G$
$q_a = 0.2*2*\pi*(1)^2*1366$ W
$q_a = 1716$ W

Total radiation from satellite under steady state is
$Q = q_a + q_r$

$= 1000+1716$ W

$= 2716$ W

From Equation (1.17), the rate of the heat transfer from a gray body in a blackbody enclosure is

$$Q = A_1\, \varepsilon_1\, \sigma (T_1^4 - T_2^4)$$

Considering single body radiation $T_2=0$

Solving this for the surface temperature

$$T_1 = \left(\frac{Q}{A_1\varepsilon_1\sigma}\right)^{\frac{1}{4}} = \left(\frac{Q}{\pi D^2\varepsilon_1\sigma}\right)^{\frac{1}{4}}$$

For steady state the rate of heat transfer must equal the total amount of energy through generation and irradiation, therefore the surface temperature is

$$\boldsymbol{T_1} = \left(\frac{2716\,\text{W}}{\pi(2\,\text{m})^2(0.3)5.67\times10^{-8}\,\text{W}/(\text{m}^2\text{K}^4)}\right)^{\frac{1}{4}} = 335.7\text{ K} = 62.7^0\text{C}$$

PROBLEM 1.32

A long wire 0.7 mm in diameter with an emissivity of 0.9 is placed in a large quiescent air space at 270 K. If the wire is at 800 K, calculate the net rate of heat loss. Discuss your assumptions.

GIVEN

- Long wire in still air
- Wire diameter (D) = 0.7 mm= $7*10^{-4}$ m
- Wire temperature (T_s) = 800 K
- Emissivity (ε) = 0.9
- Air temperature (T_∞) = 270 K

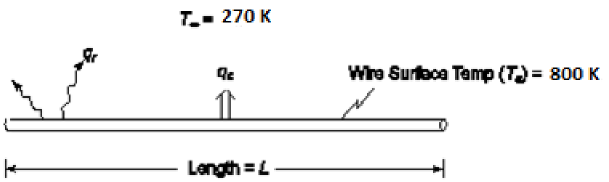
FIND

- The net rate of heat loss

ASSUMPTIONS

- The enclosure around the wire behaves as a blackbody enclosure at the temperature of the air
- The natural convection heat transfer coefficient is 18 W/(m^2 K) (From Table 1.4)
- Steady state conditions prevail

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 2, Table 5: The Stefan-Boltmann constant (σ) = 5.67×10^{-8} W/(m^2 K^4)

SOLUTION

The total rate of heat loss from the wire is the sum of the convective (Equation (1.10)) and radiative (Equation (1.17)) losses

$$q_{\text{total}} = \bar{h}_c\, A\,(T_s - T_\infty) + A\,\varepsilon\,\sigma\,(T_s^4 - T_\infty^4)$$

$$q_{\text{total}} = [18\ \text{W/(m}^2\ \text{K)}]\ \pi\,(7*10^{-4}\ \text{m})\ L\ (800\ \text{K} - 270\ \text{K})$$

$$+ \left(\pi L * 7 * 10^{-4} m\right)\ (0.9)\ [5.67\times 10^{-8}\ \ \text{W/(m}^2\ \text{K)}]\ [(800\ \text{K})^4 - (270\ \text{K})^4]$$

$$\frac{q_{\text{total}}}{L} = 66.35\ \text{W/m} = 66.35\ \text{W per meter of wire length}$$

COMMENTS

The radiative heat transfer is about twice the magnitude of the convective transfer.

The enclosure is more likely a gray body, therefore the actual rate of loss will be smaller than we have calculated.

The convective heat transfer coefficient may differ by a factor of two or three from our assumed value.

PROBLEM 1.33

Wearing layers of clothing in cold weather is often recommended because dead-air spaces between the layers keep the body warm. The explanation for this is that the heat loss from the body is less. Compare the rate of heat loss for single 2 cm thick layer of wool [*k* = 0.04 W/(m K)] with three 0.67 cm layers separated by 1.5 mm air gaps. The thermal conductivity of air is 0.024 W/(m K).

GIVEN

- Wool insulation
- Thermal conductivities
  - Wool (k_w) = 0.04 W/(m K
  - air (k_a) = 0.024 W/(m K)

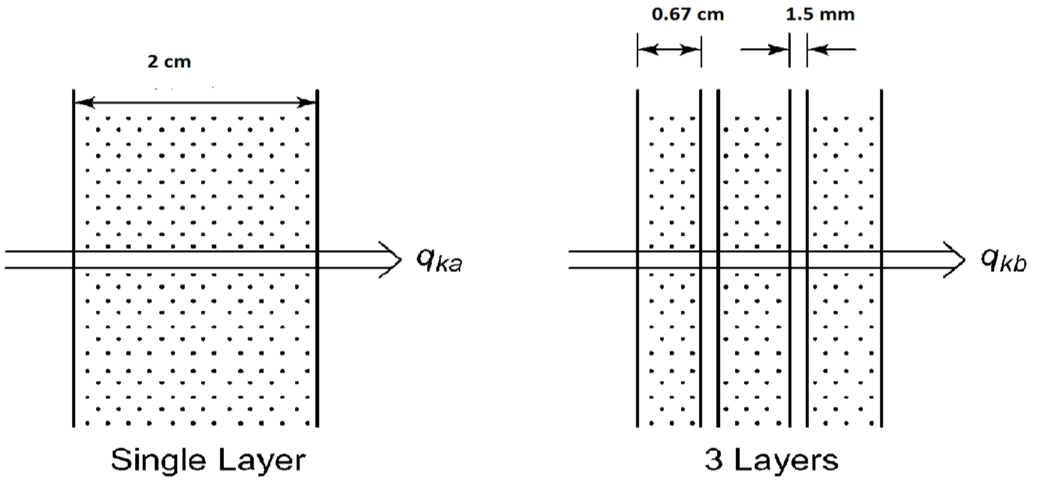
FIND

- Compare the rate of heat loss for a single 2 cm thick layer of wool to that of three 0.67 cm layers separated by 1.5 mm layers of air gaps.

ASSUMPTIONS

- Heat transfer can be approximated as one dimensional, steady state conduction

SKETCH



SOLUTION

The thermal resistance for the single thick layer, from Equation (1.4), is

$$R_{ka} = \frac{L}{K_w A} = \frac{2*10^{-2} m}{[0.04W/(m^2 K)]A} = \frac{1}{A}\ 0.5\ (\text{ m}^2\text{ K})/\text{W}$$

Therefore the rate of conductive heat transfer is

$$q_{ka} = \frac{\Delta T}{R_{ka}} = \frac{\Delta T}{\frac{1}{A}0.5\left(\text{ m}^2\text{ K}\right)/\text{W}} = 2\ A\ \Delta T\ \text{W/(m}^2\text{ K)}$$

The thermal resistance for the three thin layers is the sum of the resistance of the wool and the air between the layers

$$R_{kb} = \frac{L_w}{k_w A} + \frac{L_a}{k_a A} = \frac{(3\,\text{layers})\left(6.7*10^{-3}\,m/\text{layer}\right)}{[0.04W\,/\,(m^2K)]A} + \frac{(2\,\text{layers})\left(1.5*10^{-3}\,m/\text{Layer}\right)}{[0.024W\,/\,(m^2K)]A}$$

$$R_{kb} = \frac{1}{A}\;0.50\ (\text{m}^2\ \text{K})/\text{W} + \frac{1}{A}\;0.125\ (\text{m}^2\ \text{K})/\text{W}$$

$$= \frac{1}{A}\;0.625\ (\text{m}^2\ \text{K})/\text{W}$$

Therefore, the rate of conductive heat transfer for the three layer situation is

$$q_{kb} = \frac{\Delta T}{k_{kb}} = \frac{\Delta T}{\frac{1}{A}0.625(m^2\text{K})/W} = 1.6\,A\,\Delta T\ \ \text{W/(m}^2\ \text{K)}$$

Comparing the rate of heat loss for the two situations

$$\frac{q_{kb}}{q_{ka}} = \frac{1.6}{2} = 0.80$$

Therefore, for the same temperature difference, the heat loss through the three layers of wool is only 80% of the heat loss through the single layer.

PROBLEM 1.34

> **A section of a composite wall with the dimensions shown below has uniform temperatures of 200°C and 50°C over the left and right surfaces, respectively. If the thermal conductivities of the wall materials are: k_A = 70 W/(m K), k_B = 60 W/(m K), k_c = 40 W/(m K) and k_D = 20 W/(m K), determine the rate of heat transfer through this section of the wall and the temperatures at the interfaces.**

GIVEN

- A section of a composite wall
- Thermal conductivities
  - k_A = 70 W/(m K)
  - k_B = 60 W/(m K)
  - k_C = 40 W/(m K)
  - k_D = 20 W/(m K)
- Surface temperatures
  - Left side (T_{As}) = 200°C
  - Right side (T_{Ds}) = 50°C

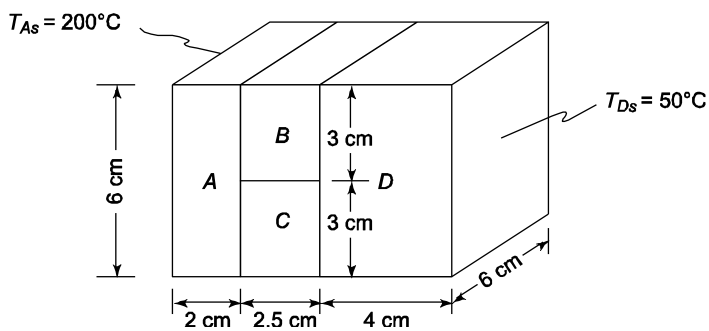
FIND

(a) Rate of heat transfer through the wall (q)
(b) Temperature at the interfaces

ASSUMPTIONS

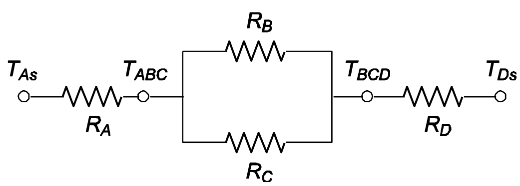
- One dimensional conduction
- The system is in steady state
- The contact resistances between the materials is negligible

SKETCH



SOLUTION

The thermal circuit for the composite wall is shown below



(a) Each of these thermal resistances has a form given by Equation (1.4)

$$R_k = \frac{L}{Ak}$$

Evaluating the thermal resistance for each component of the wall

$$R_A = \frac{L_A}{A_A\,k_A} = \frac{0.02\,\text{m}}{(0.06\,\text{m})(0.06\,\text{m})[70\,\text{W/(m K)}]} = 0.0794\ \text{K/W}$$

$$R_B = \frac{L_B}{A_B\,k_B} = \frac{0.025\,\text{m}}{(0.03\,\text{m})(0.06\,\text{m})[60\,\text{W/(m K)}]} = 0.2315\ \text{K/W}$$

$$R_C = \frac{L_C}{A_C\,k_C} = \frac{0.025\,\text{m}}{(0.03\,\text{m})(0.06\,\text{m})[40\,\text{W/(m K)}]} = 0.3472\ \text{K/W}$$

$$R_D = \frac{L_D}{A_D\,k_D} = \frac{0.04\,\text{m}}{(0.06\,\text{m})(0.06\,\text{m})[20\,\text{W/(m K)}]} = 0.5556\ \text{K/W}$$

The total thermal resistance of the wall section, from Section 1.5.1, is

$$R_{\text{total}} = R_A + \frac{R_B\,R_C}{R_B + R_C} + R_D$$

$$R_{\text{total}} = 0.0794 + \frac{(0.2315)(0.3472)}{0.2315 + 0.3472} + 0.5556\ \text{K/W}$$

$$R_{\text{total}} = 0.7738\ \text{K/W}$$

The total rate of heat transfer through the composite wall is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{200^{\circ}\text{C} - 50^{\circ}\text{C}}{0.7738\,\text{K/W}} = 194\ \text{W}$$

(b) The average temperature at the interface between material *A* and materials *B* and *C* (T_{ABC}) can be determined by examining the conduction through material *A* alone

$$q_{ka} = \frac{T_{As} - T_{ABC}}{R_A} = q$$

Solving for T_{ABC}

$$T_{ABC} = T_{As} - q\,R_A = 200°\text{C} - (194\ \text{W})\ (0.0794\ \text{K/W}) = 185°\text{C}$$

The average temperature at the interface between material *D* and materials *B* and *C* (T_{BCD}) can be determined by examining the conduction through material *D* alone

$$q_{kD} = \frac{T_{BCD} - T_{Ds}}{R_D} = q$$

Solving for T_{BCD}

$$T_{BCD} = T_{Ds} + q\,R_D = 50°\text{C} + (194\ \text{W})\ (0.5556\ \text{K/W}) = 158°\text{C}$$

PROBLEM 1.35

Repeat the Problem 1.34 including a contact resistance of 0.1 K/W at each of the interfaces.

GIVEN

- Composite wall
- Thermal conductivities:
  - k_A = 70 W/(m K)
  - k_B = 60 W/(m K)
  - k_C = 40 W/(m K)
  - k_D = 20 W/(m K)
- Surface temperatures
  - Left side (T_{As}) = 200°C
  - Right side (T_{Ds}) = 50°C
- Contact resistance at each interface (R_i) = 0.1 K/W

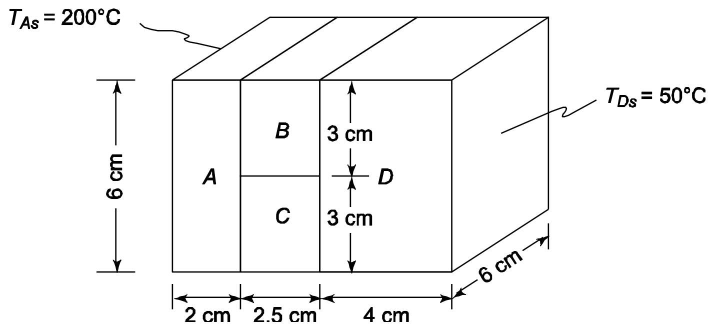
FIND

(a) Rate of heat transfer through the wall (q)
(b) Temperatures at the interfaces

ASSUMPTIONS

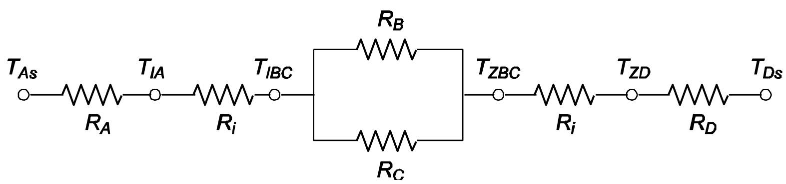
- One dimensional conduction
- The system is in steady state

SKETCH



SOLUTION

The thermal circuit for the composite wall with contact resistances is shown below



The values of the individual resistances, from Problem 1.31, are

R_A = 0.0794 K/W R_B = 0.2315 K/W R_C = 0.3472 K/W R_D = 0.5556 K/W

(a) The total resistance for this system is

$$R_{\text{total}} = R_A + R_i + \frac{R_B R_C}{R_B + R_C} + R_i + R_D$$

$$R_{\text{total}} = 0.0794 + 0.1 + \frac{(0.2315)(0.3472)}{0.2315 + 0.3472} + 0.1 + 0.5556 \text{ K/W}$$

$$R_{\text{total}} = 0.9738 \text{ K/W}$$

The total rate of heat transfer through the composite wall is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{200°\text{C} - 50°\text{C}}{0.9738 \text{ K/W}} = 154 \text{ W}$$

(b) The average temperature on the *A* side of the interface between material *A* and material *B* and *C* (T_{1A}) can be determined by examining the conduction through material *A* alone

$$q = \frac{T_{As} - T_{1A}}{R_A}$$

Solving for T_{1A}

$$T_{1A} = T_{As} - q\,R_A = 200°\text{C} - (154 \text{ W})\,(0.0794 \text{ K/W}) = 188°\text{C}$$

The average temperature on the *B* and *C* side of the interface between material *A* and materials *B* and *C* (T_{1BC}) can be determined by examining the heat transfer through the contact resistance

$$q = \frac{T_{1A} - T_{1BC}}{R_i}$$

Solving for T_{1BC}

$$T_{1BC} = T_{1A} - q\,R_i = 188°\text{C} - (154 \text{ W})\,(0.1 \text{ K/W}) = 172°\text{C}$$

The average temperature on the *D* side of the interface between material *D* and materials *B* and *C* (T_{2D}) can be determined by examining the conduction through material *D* alone

$$q = \frac{T_{2D} - T_{Ds}}{R_D}$$

Solving for T_{2D}

$$T_{2D} = T_{Ds} + q\,R_D = 50°\text{C} + (154 \text{ W})\,(0.5556 \text{ K/W}) = 136°\text{C}$$

The average temperature on the *B* and *C* side of the interface between material *D* and materials *B* and *C* (T_{2BC}) can be determined by examining the heat transfer through the contact resistance

$$q = \frac{T_{2BC} - T_{2D}}{R_i}$$

Solving for T_{2BC}

$$T_{2BC} = T_{2D} + q\,R_i = 136°\text{C} + (154 \text{ W})\,(0.1 \text{ K/W}) = 151°\text{C}$$

COMMENTS

Note that the inclusion of the contact resistance lowers the calculated rate of heat transfer through the wall section by about 20%.

PROBLEM 1.36

> **Repeat the Problem 1.35 but assume that instead of surface temperatures, the given temperatures are those of air on the left and right sides of the wall and that the convective heat transfer coefficients on the left and right surfaces are 6 and 10 W/(m^2 K), respectively.**

GIVEN

- Composite wall
- Thermal conductivities
  - k_A = 70 W/(m K)
  - k_B = 60 W/(m K)
  - k_C = 40 W/(m K)
  - k_D = 20 W/(m K)
- Air temperatures
  - Left side ($T_{A\infty}$) = 200°C
  - Right side ($T_{D\infty}$) = 50°C
- Contact resistance at each interface (R_i) = 0.1 K/W
- Convective heat transfer coefficients
  - Left side ($\bar{h}_{cA}$) = 6 W/(m^2 K)
  - Right side ($\bar{h}_{cD}$) = 10 W/(m^2 K)

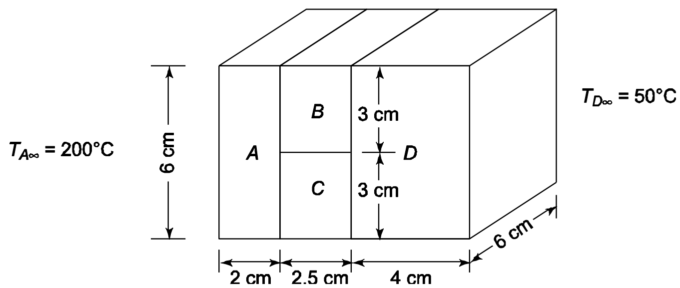
FIND

(a) Rate of heat transfer through the wall (q)
(b) Temperatures at the interfaces

ASSUMPTIONS

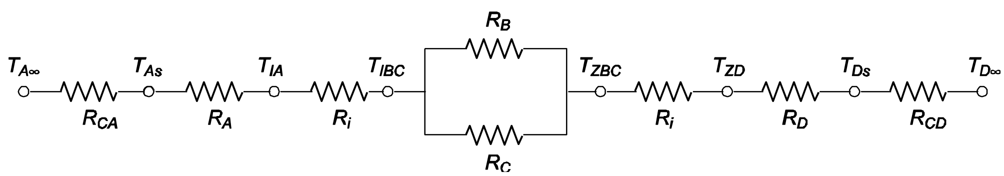
- One dimensional, steady state conduction

SKETCH



SOLUTION

The thermal circuit for the composite wall with contact resistances and convection from the outer surfaces is shown below



The values of the individual conductive resistances, from Problem 1.31, are

R_A = 0.0794 K/W $\quad$ R_B = 0.2315 K/W $\quad$ R_C = 0.3472 K/W $\quad$ R_D = 0.5556 K/W

The values of the convective resistances, using Equation (1.14), are

$$R_{cA} = \frac{1}{\overline{h}_{cA} A} = \frac{1}{[6\,\mathrm{W/(m^2 K)}](0.06\,\mathrm{m})(0.06\,\mathrm{m})} = 46.3\ \mathrm{K/W}$$

$$R_{cD} = \frac{1}{\overline{h}_{cD} A} = \frac{1}{[10\,\mathrm{W/(m^2 K)}](0.06\,\mathrm{m})(0.06\,\mathrm{m})} = 27.8\ \mathrm{K/W}$$

(a) The total resistance for this system is

$$R_{\text{total}} = R_{cA} + R_A + R_i + \frac{R_B R_C}{R_B + R_C} + R_i + R_D + R_{cD}$$

$$R_{\text{total}} = 46.3 + 0.0794 + 0.1 + \frac{(0.2315)(0.3472)}{0.2315 + 0.34472} + 0.1 + 0.5556 + 27.8\ \mathrm{K/W}$$

$$R_{\text{total}} = 75.1\ \mathrm{K/W}$$

The total rate of heat transfer through the composite wall is given by

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{200°C - 50°C}{75.1\mathrm{K/W}} = 2.0\ \mathrm{W}$$

(b) The surface temperature on the left side of material A (T_{As}) can be determined by examining the convection from the surface of material A

$$q = \frac{T_{A\infty} - T_{As}}{R_{cA}}$$

Solving for T_{As}

$$T_{As} = T_{A\infty} - q\,R_{cA} = 200°\mathrm{C} - (2\ \mathrm{W})\,(46.3\ \mathrm{K/W}) = 107.4°\mathrm{C}$$

The average temperature on the A side of the interface between material A and material B and C (T_{1A}) can be determined by examining the conduction through material A alone

$$q = \frac{T_{As} - T_{1A}}{R_A}$$

Solving for T_{1A}

$$T_{1A} = T_{As} - q\,R_A = 107.4°\mathrm{C} - (2\ \mathrm{W})\,(0.0794\ \mathrm{K/W}) = 107.2°\mathrm{C}$$

The average temperature on the B and C side of the interface between material A and material B and C (T_{1BC}) can be determined by examining the heat transfer through the contact resistance

$$q = \frac{T_{1A} - T_{1BC}}{R_i}$$

Solving for T_{1BC}

$$T_{1BC} = T_{1A} - q\,R_i = 107.2°\mathrm{C} - (2\ \mathrm{W})\,(0.1\ \mathrm{K/W}) = 107.0°\mathrm{C}$$

The surface temperature on the D side of the wall (T_{Ds}) can be determined by examining the convection from that side of the wall

$$q = \frac{T_{Ds} - T_{D\infty}}{R_{cD}}$$

Solving for T_{Ds}

$$T_{Ds} = T_{D\infty} + q\,R_{cD} = 50°\mathrm{C} + (2\ \mathrm{W})\,(27.8\ \mathrm{K/W}) = 105.6°\mathrm{C}$$

The average temperature on the D side of the interface between material D and materials B and C (T_{2D}) can be determined by examining the conduction through material D alone

$$q = \frac{T_{2D} - T_{Ds}}{R_D}$$

Solving for T_{2D}

$$T_{2D} = T_{Ds} + q\,R_D = 105.6°C + (2\text{ W})\,(0.5556\text{ K/W}) = 106.7°C$$

The average temperature on the *B* and *C* side of the interface between material *D* and materials *B* and *C* (T_{2BC}) can be determined by examining the heat transfer through the contact resistance

$$q = \frac{T_{2BC} - T_{2D}}{R_i}$$

Solving for T_{2BC}

$$T_{2BC} = T_{2D} + q\,R_i = 106.7°C + (2\text{ W})\,(0.1\text{ K/W}) = 106.9°C$$

COMMENTS

Note that the addition of the convective resistances reduced the rate of heat transfer through the wall section by a factor of 77.

PROBLEM 1.37

Mild steel nails were driven through a solid wood wall consisting of two layers, each 2.5-cm-thick, for reinforcement. If the total cross-sectional area of the nails is 0.5% of the wall area, determine the unit thermal conductance of the composite wall and the per cent of the total heat flow that passes through the nails when the temperature difference across the wall is 25°C. Neglect contact resistance between the wood layers.

GIVEN

- Wood wall
- Two layers 0.025-m-thick each
- Nail cross sectional area of nails = 0.5% of wall area
- Temperature difference (ΔT) = 25°C

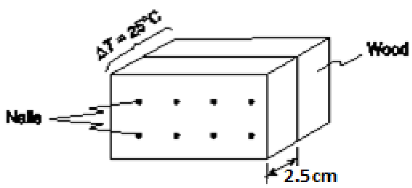
FIND

(a) The unit thermal conductance (k/L) of the wall
(b) Percent of total heat flow that passes through the wall

ASSUMPTIONS

- One dimensional heat transfer through the wall
- Steady state prevails
- Contact resistance between the wall layers is negligible

SKETCH



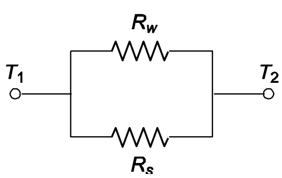
PROPERTIES AND CONSTANTS

From Appendix 2, Tables 10 and 11
Thermal conductivities

- Wood (Pine) (k_w) = 0.15 W/(m K)
- Mild steel (1% C) (k_s) = 43 W/(m K)

SOLUTION

(a) The thermal circuit for the wall is



The individual resistances are

$$R_w = \frac{L_w}{A_w k_w} = \frac{0.05\,\text{m}}{(0.995\,A_{\text{wall}}[0.15\,\text{W/(m K)}]} = \frac{0.335}{A_{\text{wall}}}\ (\text{m}^2\text{K})/\text{W}$$

$$R_s = \frac{L_s}{A_s k_s} = \frac{0.05\,\text{m}}{(0.005A_{\text{wall}}[43\,\text{W/(m K)}]} = \frac{0.233}{A_{\text{wall}}}\ (\text{m}^2\ \text{K})/\text{W}$$

The total resistance of the wood and steel in parallel is

$$R_{\text{total}} = \frac{R_w\,R_s}{R_w + R_s} = \frac{1}{A_{\text{wall}}}\left[\frac{(0.335)(0.233)}{0.335+0.233}\right](\text{m}^2\ \text{K})/\text{W} = \frac{1}{A_{\text{wall}}}\,0.1374\ (\text{m}^2\ \text{K})/\text{W}$$

The unit thermal conductance (k/L) is

$$\frac{k}{L} = \frac{1}{R_{\text{total}}\,A_{\text{wall}}} = \frac{1}{0.1374(\text{m}^2\ \text{K})/\text{W}} = 7.3\ \text{W/(m}^2\ \text{K)}$$

(b) The total heat flow through the wood and nails is given by

$$q_{\text{total}} = \frac{\Delta T}{R_{\text{total}}} = \frac{25°C}{\frac{1}{A_{\text{wall}}}0.1374(\text{m}^2\ \text{K})/\text{W}}$$

$$\therefore\ \frac{q_{\text{total}}}{A_{\text{wall}}} = 182\ \text{W/m}^2$$

The heat flow through the nails alone is

$$q_{\text{nails}} = \frac{\Delta T}{R_{\text{nails}}} = \frac{25°C}{\frac{1}{A_{\text{wall}}}0.233(\text{m}^2\ \text{K})/\text{W}}$$

$$\therefore\ \frac{q_{\text{nails}}}{A_{\text{wall}}} = 107\ \text{W/m}^2$$

Therefore the per cent of the total heat flow that passes through the nails is

$$\text{Percent of heat flow through nails} = \frac{107}{182} \times 100 = 59\%$$

PROBLEM 1.38

Calculate the rate of heat transfer through the composite wall in Problem 1.37 if the temperature difference is 25°C and the contact resistance between the sheets of wood is 0.005 m² K/W.

GIVEN

- Wood wall
  - Two layers 0.025-m-thick each, nailed together
- Nail cross sectional area of nails = 0.5% of wall area
- Temperature difference (ΔT) = 20°C
- Contact resistance ($A\,R_i$) = 0.005 (m² K)/W

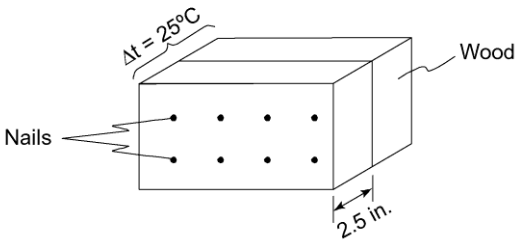
FIND

- The rate of heat transfer through the wall

ASSUMPTIONS

- One dimensional heat transfer through the wall
- Steady state prevails

SKETCH



PROPERTIES AND CONSTANTS

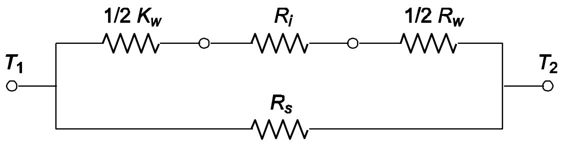
From Appendix 2, Tables 10 and 11

Thermal conductivities

- Wood (Pine) (k_w) = 0.15 W/(m K)
- Mild steel (1% C) (k_s) = 43 W/(m K)

SOLUTION

The thermal circuit for the wall with contact resistance is shown below.



From Problem 1.34, the thermal resistance of the wood and the nails are

$$R_w = \frac{1}{A_{\text{wall}}}\ 0.335\ (\text{K m}^2)/\text{W} \qquad R_s = \frac{1}{A_{\text{wall}}}\ 0.233\ (\text{K m}^2)/\text{W}$$

The combined resistance of the wood and the contact resistance in series is

$$R_{wi} = R_w + R_i = R_w + \frac{1}{A}\ (A\,R_i) = \frac{1}{A_{\text{wall}}}\left[0.355\,(\text{K m}^2)/\text{W} + 0.005\,(\text{K m}^2)/\text{W}\right]$$

$$R_{wi} = \frac{1}{A_{\text{wall}}}\ 0.360\ (\text{K m}^2)/\text{W}$$

The total resistance equals the combined resistance of the wood and the contact resistance in parallel with the resistance of the nails

$$R_{\text{total}} = \frac{R_{wi}\, R_s}{R_{wi} + R_s} = \frac{1}{A_{\text{wall}}} \left[\frac{(0.360)(0.233)}{0.360 + 0.233} \right] (\text{K m}^2)/\text{W} = \frac{1}{A_{\text{wall}}} = 0.1415 \ (\text{K m}^2)/\text{W}$$

Therefore the rate of heat flow through the wall is:

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{25°C}{\frac{1}{A_{\text{wall}}} 0.1415 \ (\text{K m}^2)/\text{W}}$$

$$\therefore \ \frac{q}{A_{\text{wall}}} = 176 \ \text{W/m}^2$$

COMMENTS

In this case the inclusion of the contact resistance lowered the calculated rate of heat transfer by only 3 % because most of the heat is transferred through the nails (see Problem 1.37).

PROBLEM 1.39

On a cold winter day, the outside wall of a home is exposed to air temperature of –2°C when the inside temperature of room is at 22°C. As a result of this temperature gradient, there is heat loss through the wall to the outside. Consider the convective heat transfer coefficients for the air inside the room and at the outside wall surface to be respectively 12.0 and 28.0 W/(m^2 K). If the composite room wall is modelled as a plane wall with thermal resistance per unit area of 0.5 m^2 K/W, determine the temperature at the outer surface of the wall as well as the rate of heat flow through the wall per unit area. If the homeowner were to consider using a fiberglass insulation on the inside wall surface for reducing this heat loss by 50 %, what is the required thickness of this layer and the outside wall temperature for this case?

GIVEN

- Heat transfer through a plane wall
- Air temperature
  - Inside wall (T_i) = 22°C
  - outside wall (T_o) = –2°C
- Heat transfer coefficient
  - Inside wall ($\overline{h_{ci}}$) = 12 W/(m^2 K)
  - Outside wall ($\overline{h_{co}}$) = 28 W/(m^2 K)
- Thermal resistance of a unit area ($A\ R_w$) = 0.5 (m^2 K)/W

FIND

(a) Temperature of the outer surface of the wall (T_{wo})
(b) Rate of heat flow through the wall per unit area (q/A)
(c) Thickness of fiberglass insulation inside wall surface
(d) Outside wall temperature with fiberglass insulation

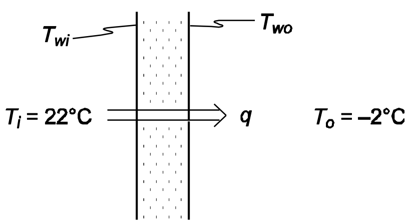
ASSUMPTIONS

- One dimensional heat flow
- Steady state has been reached

PROPERTIES AND CONSTANTS

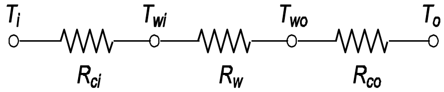
From Appendix 2, Table 11.
Thermal conductivity
Fiber glass (k) =0.035 W/m K

SKETCH



SOLUTION

The thermal circuit for the wall is shown below



The rate of heat transfer can be used to calculate the temperature of the outer surface of the wall, therefore part (b) will be solved first.

(b) The heat transfer situation can be visualized using the thermal circuit shown above. The total heat transfer through the wall, from Equations (1.34) and (1.35), is

$$q = \frac{\Delta T_{\text{total}}}{R_{\text{total}}}$$

The three thermal resistances are in series, therefore

$$R_{\text{total}} = R_{ci} + R_w + R_\infty$$

$$R_{\text{total}} = \frac{1}{A\bar{\bar{h}}_{ci}} + \frac{AR_w}{A} + \frac{1}{A\bar{\bar{h}}_\infty}$$

The heat flow through the wall is

$$q = \frac{T_i - T_o}{\frac{1}{A}\left(\frac{1}{\bar{\bar{h}}_{ci}} + AR_w + \frac{1}{\bar{\bar{h}}_\infty}\right)}$$

$$\therefore \frac{q}{A} = \frac{22\,°\text{C} - (-2\,°\text{C})}{\frac{1}{12\,\text{W/(m}^2\text{K)}} + 0.5\,(\text{m}^2\text{K})\text{/W} + \frac{1}{28\,\text{W/(m}^2\text{K)}}}$$

$$\frac{q}{A} = 38.8\ \text{W/m}^2$$

(a) The temperature of the outer surface of the wall can be calculated by examining the convective heat transfer from the outside of the wall (given by Equation (1.10))

$$\frac{q_c}{A} = \bar{h}_{co}\ (T_{wo} - T_o)$$

Solving for T_{wo}

$$T_{wo} = \frac{q}{A}\frac{1}{\bar{h}_{co}} + T_o = (38.8\ \text{W/m}^2\left(\frac{1}{28\,\text{W/(m}^2\,\text{K)}}\right) + (-2°\text{C}) = -\,0.6°\text{C}$$

With fiber glass included as insulation, total resistance becomes

$$\text{R}_{\text{total}} = R_{ci} + R_w + R_\infty + R_{fg}$$

$$\boldsymbol{R}_{\textbf{TOTAL}} = \frac{1}{A\bar{\bar{h}}_{ci}} + \frac{AR_w}{A} + \frac{1}{A\bar{\bar{h}}_\infty} + \frac{L}{kA}$$

For reducing heat loss by 50 % the overall resistance should be doubled, thus

$$\frac{(22-(-2))}{19.4 * A} = \frac{1}{A\bar{\bar{h}}_{ci}} + \frac{AR_w}{A} + \frac{1}{A\bar{\bar{h}}_\infty} + \frac{L}{kA}$$

$$\frac{L}{k} = 1.24 - \left(\frac{1}{12\,\mathrm{W/(m^2K)}} + 0.5\,(\mathrm{m^2K})/\mathrm{W} + \frac{1}{28\,\mathrm{W/(m^2K)}} \right)$$

$$\frac{L}{k} = 0.62$$

L=0.62*0.035= 0.00217 m =2.2 cm

The temperature of the outer surface of the wall can be calculated by examining the convective heat transfer from the outside of the wall (given by Equation (1.10))

$$\frac{q_c}{A} = \bar{h}_{co}\ (T_{wo} - T_o)$$

Solving for T_{wo}

$$T_{wo} = \frac{q}{A}\frac{1}{\bar{h}_{co}} + T_o = (19.4\ \mathrm{W/m^2}\left(\frac{1}{28\,\mathrm{W/(m^2\,K)}} \right) + (-2°\mathrm{C}) = -\,1.3°\mathrm{C}$$

COMMENTS

Note that the conductive resistance of the wall is dominant compared to the convective resistance.

PROBLEM 1.40

As a designer working for a major electric appliance manufacturer you are required to estimate the amount of fiberglass insulation packing (k=0.035 W/m K) that is needed for a kitchen oven shown in figure below. The fiberglass layer is to be sandwiched between a 2 mm thick aluminum cladding plate on the outside and 5 mm thick stainless steel plate on the inside that forms the core of the oven. The insulation thickness is such that the outside cladding temperature does not exceed 40 °C when the temperature at inside surface of the oven is 300 °C. Also the air temperature in the kitchen varies from 15 °C to 33 °C, and the average heat transfer coefficient between the outer surface of the oven and air is estimated to be 12 W/(m^2 K). Determine the thickness of fiber glass insulation that is required for these conditions. What is the outer surface temperature when the inside surface of the oven is at 475 °C?

How much fiberglass insulation [*k* = 0.035 W/(m K)] is needed to guarantee that the outside temperature of a kitchen oven will not exceed 43°C? The maximum oven temperature to be maintained by the convectional type of thermostatic control is 290°C, the kitchen temperature may vary from 15°C to 33°C and the average heat transfer coefficient between the oven surface and the kitchen is 12 W/(m^2 K).

GIVEN

- Kitchen oven wall insulated with fiberglass sandwiched between aluminum cladding and steel
- Fiberglass thermal conductivity (k) = 0.035 W/(m K)
- Thickness of Al cladding (L_{Al})=0.002 m, thickness of ss plate (L_{ss})=0.005 m
- Convective transfer coefficient on the outside of wall ($\bar{h}_c$) = 12 W/(m^2 K)
- Maximum oven temperature (T_i) = 300°C
- Kitchen temperature (T_∞) may vary: 15°C < T_∞ < 33°C

FIND

- Thickness of fiberglass (L) to keep the temperature of the outer surface of the oven (T_{wo}) at 40°C or less

ASSUMPTIONS

- One dimensional, steady state heat transfer prevails
- The temperature of the inside of the wall (T_{wi}) is the same as the oven temperature
- The thermal resistance of the metal wall of the oven is negligible

PROPERTIES AND CONSTANTS

From Appendix 2, Table 11.
Thermal conductivity
Fiber glass (k_{fg})=0.035 W/m K
Aluminum(k_{Al})=164 W/m K
Stainless steel (k_{ss})=14.4 W/m K

SOLUTION

Total conduction resistance for the wall is

$$R_{cond} = R_{aluminum} + R_{fiberglass} + R_{steel}$$

$$R_{cond} = \frac{L_{Al}}{k_{Al}A} + \frac{L_{fg}}{k_{fg}A} + \frac{L_{ss}}{k_{ss}A}$$

$$R_{cond} = \frac{0.002}{164 * A} + \frac{L}{0.035A} + \frac{0.005}{14.3 * A}$$

For steady state conditions, the heat transfer by conduction through the wall, from Equation (1.3), must be equal to the heat transfer by convection from the outer surface of the wall, from Equation (1.10)

$$q_k = \frac{(T_{wi} - T_{wo})}{\left(\frac{L_{Al}}{k_{Al}A} + \frac{L_{fg}}{k_{fg}A} + \frac{L_{ss}}{k_{ss}A}\right)} = q_c = \bar{h}_c \, A \, (T_{wo} - T_{\infty})$$

By examination of the above equation, the greatest thickness required for a given T_{wo} will occur when T_{wi} and T_{∞} are at their maximum values

Solving for L

$$\frac{0.002}{164} + \frac{L}{0.035} + \frac{0.005}{14.3} = \frac{(300 - 40)}{12 * (40 - 33)}$$

$$\frac{L}{0.035} = 3.095 - 0.00001 - 0.00035$$

L=3.0946*0.035= 0.108 m = 10.8 cm

By examination of the above equation, the greatest thickness required for a given T_{wo} will occur when T_{wi} and T_{∞} are at their maximum values

$$\frac{0.002}{164} + \frac{0.0108}{0.035} + \frac{0.005}{14.3} = \frac{(450 - T)}{12 * (T - 33)}$$

0.309*12*(T-33)=450-T

4.708*T=572.4

T= 121 °C

In a real design a slightly thicker layer of insulation should be chosen to provide a margin of safety in case the convective heat transfer coefficient on the outside of the wall in some circumstances is less than expected due to the location of the oven in the kitchen or other unforeseen factors.

PROBLEM 1.41

A heat exchanger wall consists of a copper plate 2 cm thick. The heat transfer coefficients on the two sides of the plate are 2700 and 7000 W/(m^2 K), corresponding to fluid temperatures of 92 and 32°C, respectively. Assuming that the thermal conductivity of the wall is 375 W/ (m K), (a) compute the surface temperatures in °C, and (b) calculate the heat flux in W/ m^2.

GIVEN

- Heat exchanger wall, thickness (L) = 2 cm = 0.02 m
- Heat transfer coefficients
  - h_{c1} = 2700 W/(m^2 K)
  - h_{c2} = 7000 W/(m^2 K)
- Fluid temperatures
  - T_{f1} = 92°C
  - T_{f2} = 32°C
- Thermal conductivity of the wall (k) = 375 W/(m K)

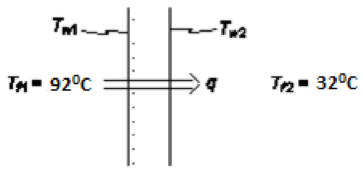
FIND

(a) Surface temperatures (T_{w1}, T_{w1}) in °F
(b) The heat flux (q/A) in W/m^2.

ASSUMPTIONS

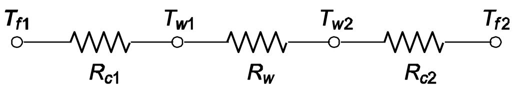
- One dimensional heat transfer prevails
- The system has reached steady state
- Radiative heat transfer is negligible

SKETCH



SOLUTION

The thermal circuit for the wall is shown below



The surface temperatures can only be calculated after the heat flux has been established, therefore part (b) will be solved before part (a).

(b) The resistances are in series, therefore the total resistance is

$$R_{\text{total}} = \sum_{i=1}^{3} R_i = R_{c1} + R_w + R_{c2}$$

The total rate of heat transfer is given by Equation (1.34) and (1.35)

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_{c1} + R_w + R_{c2}} = \frac{T_1 - T_2}{\dfrac{1}{\bar{h}_{c1}A} + \dfrac{L}{kA} + \dfrac{1}{\bar{h}_{c2}A}}$$

Therefore the heat flux (q/A) is

$$\frac{q}{A} = \frac{92\,^{\circ}C - 32\,^{\circ}C}{\dfrac{1}{2700\text{ W}/(m^{2\,\circ}\text{K})} + \dfrac{0.02}{375\text{ W}/(mK)} + \dfrac{1}{7000W/(m^2\text{ K})}} = 1.06\times10^5\text{ W/ m}^2$$

(a) Equation (1.10) can be applied to the convective heat transfer on the fluid 1 side

$$\frac{q_c}{A} = \bar{h}_{c1}\,(T_{f1} - T_{w1})$$

Solving for T_{w1}

$$T_{w1} = T_{f1} - \frac{q}{A}\frac{1}{\bar{h}_{c1}} = 92°\text{C} - [1.06\times10^5\text{ W/ m}^2]\left(\frac{1}{2700W/(m^2\text{ K})}\right) = 52.8°\text{C}$$

Similarly, on the fluid 2 side

$$\frac{q_c}{A} = \bar{h}_{c2}\,(T_{w2} - T_{f2})$$

$$T_{w2} = T_{f2} - \frac{q}{A}\frac{1}{\bar{h}_{c2}} = 32°\text{C} + [1.06\times10^5\text{ W/ m}^2]\left(\frac{1}{7000W/(m^2\text{ K})}\right) = 47.1°\text{C}$$

PROBLEM 1.42

A submarine is to be designed to provide a comfortable temperature of no less than 21°C for the crew. The submarine is idealized by a cylinder 9 m in diameter and 61 m in length, as shown. The combined heat transfer coefficient on the interior is about 14 W/(m^2 K), while on the outside the heat transfer coefficient is estimated to vary from about 57 W/(m^2 K) (not moving) to 847 W/(m^2 K) (top speed). For the following wall constructions, determine the minimum size in kilowatts of the heating unit required if the sea water temperatures vary from 1.1 to 12.8°C during operation. The walls of the submarine are (a) 2 cm aluminum (b) 1.8 cm stainless steel with *a* 2.5 cm thick layer fiberglass insulation on the inside and (c) of sandwich construction with *a* 1.8 cm thick layer of stainless steel, *a* 2.5 cm thick layer of fiberglass insulation, and *a* 0.6 cm thick layer of aluminum on the inside. What conclusions can you draw?

GIVEN

- Submarine
  - Inside temperature (T_i) > 21°C
- Can be idealized as a cylinder
  - Diameter (D) = 9 m length (L) = 61 m
- Combined heat transfer coefficients
  - Inside ($\bar{h}_{ci}$) = 14 W/(m^2 K)
  - Outside ($\bar{h}_{co}$): not moving = 57 W/(m^2 K)
  - top speed: 847 W/(m^2 K)
- Sea water temperature (T_o) varies: 1.1°C < T_o < 12.8°C

FIND

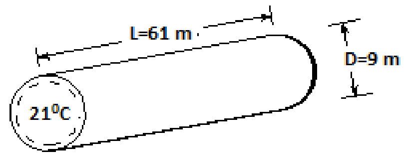
Minimum size of the heating unit (q) in kW for

(a) 2 cm thick aluminum walls
(b) 1.8 cm thick stainless steel with 2.5 cm thick layer of fiberglass insulation
(c) Sandwich of 1.8 cm thick stainless steel, 2.5 cm thick layer of fiberglass insulation, and 0.6 cm layer of aluminum

ASSUMPTIONS

- Steady state prevails
- Heat transfer can be approximated as heat transfer through a flat plate with the surface area of the cylinder
- Constant thermal conductivities
- Contact resistance between the difference materials is negligible

SKETCH



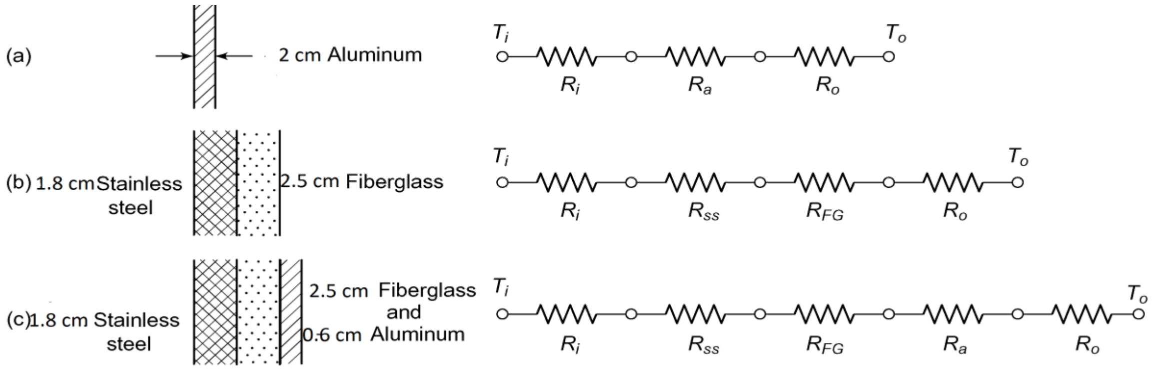
PROPERTIES AND CONSTANTS

From Appendix 2, Tables 10, 11, and 12: The thermal conductivities are

Aluminum (k_a) = 236 W/(m K) at 0°C
Stainless steel (k_s) = 14.36 W/(m K) at 20°C
Fiberglass insulation (k_{fg}) = 0.035 W/(m K) at 20°C

SOLUTION

The thermal circuits for the three cases are shown below



The total surface area of the idealized submarine (A) is

$$A = \pi DL + 2\pi \frac{D^2}{4} = (9 \text{ m})\pi(61 \text{ m}) + \frac{\pi}{2} (9 \text{ m})^2 = 1852 \text{ m}^2$$

(a) For case (a) the total resistance is

$$R_{\text{total}} = \sum_{i-1}^{3} R_i = R_i + R_a + R_o = \frac{1}{\bar{h}_{ci} A} + \frac{L}{k_a A} + \frac{1}{\bar{h}_{co} A}$$

The heat transfer through the wall is

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_i - T_o}{\dfrac{1}{\bar{h}_{ci} A} + \dfrac{L_a}{k_a A} + \dfrac{1}{\bar{h}_{co} A}}$$

By examination of the above equation, the heater requirement will be the largest when T_o is at its minimum value and h_{co} is at its maximum value

$$q = \frac{1852 m^2 (21^0 C - 1.1^0 \text{C})}{\dfrac{1}{14 W/(m^2\, K)} + \dfrac{2 * 10^{-2} m}{236 W/(m K)} + \dfrac{1}{847 W/(m^2\, K)}} = 506985 \text{ W} = 507 \text{ kW}$$

(b) Similarly, for case (b), the total resistance is

$$R_{\text{total}} = \sum_{i=1}^{4} R_i = R_s + R_a + R_{fg} + R_o = \frac{1}{\bar{h}_{ci} A} + \frac{L_s}{k_s A} + \frac{L_{fg}}{k_{fg} A} + \frac{1}{\bar{h}_{co} A}$$

The size of heater needed is

$$q = \frac{1852\,m^2(21^0C - 1.1^0\,\mathrm{C})}{\dfrac{1}{14W/(m^2\,K)} + \dfrac{1.8*10^{-2}}{14.36W/(m\,K)} + \dfrac{2.5*10^{-2}\,m}{0.035W/(m\,K)} + \dfrac{1}{847W/(m^2\,K)}}$$

$$q = 46.7\ \mathrm{kW}$$

(c) The total resistance for case (c) is

$$R_{\text{total}} = \sum_{i=1}^{5} R_i = R_s + R_a + R_{fg} + R_a + R_o = \frac{1}{\bar{h}_{ci}A} + \frac{L_s}{k_s A} + \frac{L_{fg}}{k_{fg}A} + \frac{L_a}{k_a A} + \frac{1}{\bar{h}_{co}A}$$

The size of heater needed is

$$q = \frac{1852\,m^2(21^0C - 1.1^0\,\mathrm{C})}{\dfrac{1}{14W/(m^2\,K)} + \dfrac{1.8*10^{-2}}{14.36W/(m\,K)} + \dfrac{0.6*10^{-2}}{236W/(m\,K)} + \dfrac{2.5*10^{-2}\,m}{0.035W/(m\,K)} + \dfrac{1}{847W/(m^2\,K)}}$$

$$q = 46.7\ \mathrm{kW}$$

COMMENTS

Neither the aluminum nor the stainless steel offers any appreciable resistance to heat loss.

Fiberglass or other low conductivity material is necessary to keep the heat loss down to a reasonable level.

PROBLEM 1.43

A simple solar heater consists of a flat plate of glass below which is located a shallow pan filled with water, so that the water is in contact with the glass plate above it. Solar radiation is passing through the glass at the rate of 490 W/m^2. The water is at 92°C and the surrounding air is 27°C. If the heat transfer coefficients between the water and the glass and the glass and the air are 28 W/(m^2 K), and 7 W/(m^2 K), respectively, determine the time required to transfer 1.1 MJ/m^2 of surface to the water in the pan. The lower surface of the pan may be assumed to be insulated.

GIVEN

- A simple solar heater: shallow pan of water below glass, the water touches the glass
- Solar radiation passing through glass (q_r/A) = 490 W/m^2
- Water temperature (T_w) = 92°C
- Surrounding air temperature (T_∞) = 27°C
- Heat transfer coefficients
  - Between water and glass ($\bar{h}_{cw}$) = 28 W/(m^2 K)
  - Between glass and air ($\bar{h}_{ca}$) = 7 W/(m^2 K)

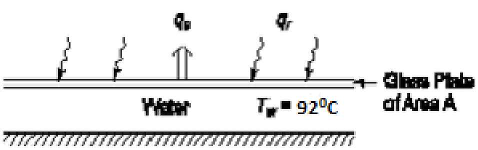
FIND

- The time (t) required to transfer 1.1 MJ/m^2 to the water

ASSUMPTIONS

- One dimensional, steady state heat transfer prevails
- The heat loss from the bottom of the pan is negligible
- The radiative loss from the top of the glass is negligible
- The thermal resistance of the glass is negligible

SKETCH



SOLUTION

The total thermal resistance between the water and the surrounding air is the sum of the two convective thermal resistances

$$R_{\text{total}} = \sum_{i=1}^{2} R_i = R_{cw} + R_{ca} = \frac{1}{\bar{h}_{cw} A} + \frac{1}{\bar{h}_{ca} A}$$

$$R_{\text{total}} = \frac{1}{A[28 W/(\text{m}^2\ \text{K})]} + \frac{1}{A[7\ W/(\text{m}^2\ \text{K})]} = \frac{1}{A}\ 0.1786\ (\text{m}^2\ \text{K})/\text{W}$$

The net rate of heat transfer to the water is

$$\frac{q_{\text{total}}}{A} = \frac{q_r}{A} = \frac{q_c}{A} = \frac{q_r}{A} = \frac{\Delta T}{A R_{\text{total}}}$$

$$\frac{q_{\text{total}}}{A} = 490\ \text{W/m}^2 - \frac{92\,°\text{C} - 27\,°\text{C}}{A\left(\frac{1}{A}\,0.1786\ (\text{m}^2\ \text{K})/\text{W}\right)}$$

$$\frac{q_{\text{Total}}}{A} = 126\ \text{W/m}^2$$

At this rate, the time required to transfer 100 Btu/ft^2 to the water is

$$t = \frac{1.1\ \text{MJ/m}^2}{\frac{q_{\text{Total}}}{A}} = \frac{1.1{*}10^6\ \text{J/m}^2}{126\text{W/m}^2}$$

$$t = 8730\ \text{seconds} = 2.4\ \text{hours.}$$

PROBLEM 1.44

A composite refrigerator wall is composed of 5 cm of corkboard sandwiched between a 1.2 cm thick layer of oak and a 0.8 mm thick layer of aluminum lining on the inner surface. The average convective heat transfer coefficients at the interior and exterior wall are 11 and 8 W/(m^2 K), respectively. (a) Draw the thermal circuit. (b) Calculate the individual resistances of the components of this composite wall and the resistances at the surfaces. (c) Calculate the overall heat transfer coefficient through the wall. (d) For an air temperature of -1^{0}C inside the refrigerator and 32°C outside, calculate the rate of heat transfer per unit area through the wall.

GIVEN

- Refrigerator wall: oak, corkboard, and aluminum
- Thicknesses
  - Oak (L_o) = 1.2 cm= 1.2*10^{-2} m
  - Corkboard (L_c) = 5 cm= 5*10^{-2} m
  - Aluminum (L_a) = 0.8 mm = 8*10^{-4} m
- Convective heat transfer coefficients
  - Interior ($\bar{h}_{ci}$) = 12 W/(m^2 K)
  - Exterior ($\bar{h}_{co}$) = 8 W/(m^2 K)
- Air temperature
  - Inside (T_i) = -1°C
  - Outside (T_o) = 32°C

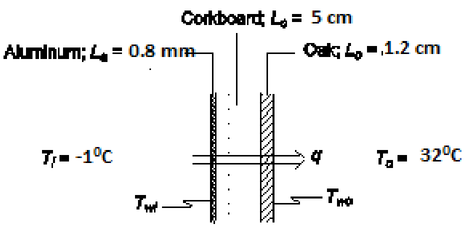
FIND

(a) Draw the thermal circuit
(b) The individual resistances
(c) Overall heat transfer coefficient (U)
(d) Rate of heat transfer per unit area (q/A)

ASSUMPTIONS

- One dimensional, steady state heat transfer
- Constant thermal conductivities
- Contact resistance between the different materials is negligible

SKETCH



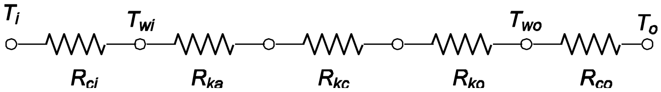
PROPERTIES AND CONSTANTS

From Appendix 2, Tables 11 and 12, the thermal conductivities are

Oak (k_o)= 0.19 W/(m K) at 20°C
Corkboard (k_c) = 0.0415 W/(m K) at 20°C
Aluminum (k_a) = 236 W/(m K) at 0°C

SOLUTION

(a) The thermal circuit for the refrigerator wall is shown below



(b) The resistances to convection from the inner and outer surfaces is given by Equation (1.14)

$$R_c = \frac{1}{\bar{h}_c A}$$

$$R_{ci} = \frac{1}{\bar{h}_{ci} A} = \frac{1}{[12W/(m^2\ \mathrm{K})]A} = \frac{1}{A}\ 0.0833\ \ (\mathrm{m^2\,K})/\mathrm{W}$$

$$R_{\infty} = \frac{1}{\bar{h}_{co} A} = \frac{1}{[8W/(m^2\ \mathrm{K})]A} = \frac{1}{A}\ 0.125\ (\mathrm{m^2\,K})/\mathrm{W}$$

The resistances to conduction through the components of the wall is given by Equation (1.4)

$$R_k = \frac{L}{Ak}$$

$$R_{ka} = \frac{L_a}{Ak_a} = \frac{8*10^{-4}\,m}{A[236W/(\mathrm{m}\,K)]} = \frac{1}{A}\ 3.39 \times 10^{-6}\ (\mathrm{m^2\,K})/\mathrm{W}$$

$$R_{kc} = \frac{L_c}{Ak_c} = \frac{5*10^{-2}}{A[0.0415W/(\mathrm{m}\,K)]} = \frac{1}{A}\ 1.205\ (\mathrm{m^2\,K})/\mathrm{W}$$

$$R_{ko} = \frac{L_o}{Ak_o} = \frac{1.2*10^{-2}}{A[0.19W/(\mathrm{m}\,K)]} = \frac{1}{A}\ 0.063\ (\mathrm{m^2\,K})/\mathrm{W}$$

(c) The overall heat transfer coefficient satisfies Equation (1.35)

$$UA = \frac{1}{R_{\mathrm{total}}}$$

$$\therefore\ U = \frac{1}{AR_{\mathrm{total}}} = \frac{1}{A(R_{ci} + R_{ka} + R_{kc} + R_{ko} + R_{co})}$$

$$U = \frac{1}{(0.0833 + 3.39\times10^{-6} + 1.205 + 0.063 + 0.125)(m^2\ \mathrm{K})/W}$$

$$U = 0.6774\ \mathrm{W/(m^2\ K)}$$

(d) The rate of heat transfer through the wall is given by Equation (1.34)

$$\frac{q}{A} = U\,\Delta T = \left(0.6774\,W/(\mathrm{m^2\ K})\right)\ (32°\mathrm{C} - (\text{-}1)°\mathrm{C}) = 22.35\ \mathrm{W/m^2}$$

COMMENTS

The thermal resistance of the corkboard is more than three times greater than the sum of the other resistances. The thermal resistance of the aluminum is negligible.

PROBLEM 1.45

An electronic device that internally generates 600 mW of heat has a maximum permissible operating temperature of 70°C. It is to be cooled in 25°C air by attaching aluminum fins with a total surface area of 12 cm^2. The convection heat transfer coefficient between the fins and the air is 20 W/(m^2 K). Estimate the operating temperature when the fins are attached in such a way that: (a) there exists a contact resistance between the surface of the device and the fin array of approximately 50 K/W, and (b) there is no contact resistance but the construction of the device is more expensive. Comment on the design options.

GIVEN

- An electronic device with aluminum fin array
- Device generates heat at a rate ($\dot{q}_G$) = 600 mW = 0.6 W
- Surface area (A) = 12 cm^2
- Max temperature of device = 70°C
- Air temperature (T_∞) = 25°C
- Convective heat transfer coefficient ($\bar{h}_c$) = 20 W/(m^2 K)

FIND

Operating temperature (T_o) for

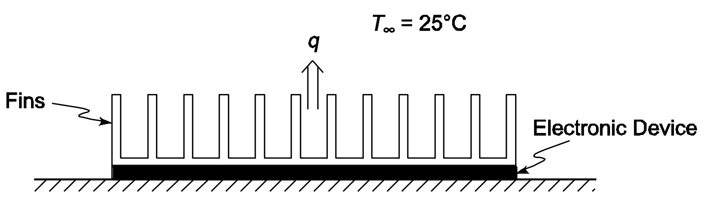
(a) contact resistance (R_i) = 50 K/W

(b) no contact resistance

ASSUMPTIONS

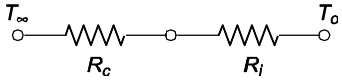
- One dimensional heat transfer
- Steady state has been reached
- The temperature of the device is uniform
- The temperature of the aluminum fins is uniform (the thermal resistance of the aluminum is negligible)
- The heat loss from the edges and back of the device is negligible

SKETCH



SOLUTION

(a) The thermal circuit for the case with contact resistance is shown below



The value of the convective resistance, from Equation (1.14), is

$$R_c = \frac{1}{\bar{h}_c A} = \frac{1}{[20\,\text{W/(m}^2\,\text{K)}](0.0012\,\text{m}^2)} = 41.7 \text{ K/W}$$

For steady state conditions, the heat loss from the device (q) must be equal to the heat generated by the device

$$q = \frac{\Delta T}{R_{\text{total}}} = \frac{T_o - T_\infty}{R_c + R_i} = \dot{q}_G$$

Solving for T_o

$$T_o = T_\infty + \dot{q}_G \, (R_c + R_i) = 25°\text{C} + (0.6 \text{ W}) \, (41.7 \text{ K/W} + 50 \text{ K/W}) = 80°\text{C}$$

(b) Similarly, the operating temperature of the device with no contact resistance is

$$T_o = T_\infty + \dot{q}_G \, R_c = 25°\text{C} + (0.6 \text{ W}) \, (41.7 \text{ K/W}) = 50°\text{C}$$

COMMENTS

The more expensive device with no contact resistance will have to be used to assure that the operating temperature does not exceed 70°C.

PROBLEM 1.46

To reduce the home heating requirements, modern building codes in many parts of the country require the use of double-glazed or double-pane windows, i.e., windows with two panes of glass. Some of these so called thermopane windows have an evacuated space between the two glass panes while others trap stagnant air in the space.

(a) Consider a double-pane window with the dimensions shown in the sketch given below. If this window has stagnant air trapped between the two panes and the convective heat transfer coefficients on the inside and outside surfaces are 4 W/(m² K) and 15 W/(m² K), respectively, calculate the overall heat transfer coefficient for the system.

(b) If the inside air temperature is 22°C and the outside air temperature is –5°C, compare the heat loss through a 4 m² double-pane window with the heat loss through a single-pane window. Comment on the effect of the window frame on this result.

(c) If the total window area of a home heated by electric resistance heaters at a cost of $.10/kWh is 80 m² how much more cost can you justify for the double-pane windows if the average temperature difference during the six winter months when heating is required is about 15°C?

GIVEN

- Double-pane window with stagnant air in gap
- Convective heat transfer coefficients
  - Inside ($\bar{h}_{ci}$) = 4 W/(m² K)
  - Outside ($\bar{h}_{co}$) = 15 W/(m² K)
- Air temperatures
  - Inside (T_i) = 22°C
  - Outside (T_o) = –5°C
- Single window area (A_w) = 4 m²
- During the winter months, (ΔT) = 15°C
- Heating cost = $.1.0/kWh
- Total window area (A_T) = 80 m²

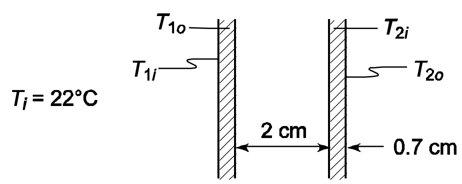
FIND

(a) The overall heat transfer coefficient
(b) Compare heat loss of double- and single-pane window
(c) Cost to justify for the use of double-pane window.

ASSUMPTIONS

- Steady state conditions prevail
- Radiative heat transfer is negligible

SKETCH

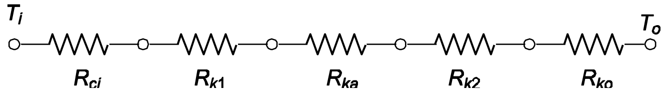


PROPERTIES AND CONSTANTS

From Appendix 2, Tables 11 and 27, the thermal conductivities are
window glass (k_g) = 0.81 W/(m K) at 20°C; dry air (k_a) = 0.0243 W/(m K) at 8.5°C

SOLUTION

The thermal circuit for the system is shown below



The individual resistances are

$$R_{co} = \frac{1}{\bar{h}_{co} A} = \frac{1}{[15\,\mathrm{W/(m^2\,K)}]A} = \frac{1}{A}\ 0.0667\ \mathrm{(m^2\,K)/W}$$

$$R_{k1} = R_{k2} = \frac{L_g}{A k_g} = \frac{0.007\,\mathrm{m}}{A[0.81\,\mathrm{W/(m\,K)}]} = \frac{1}{A}\ 0.00864\ \ \mathrm{(m^2\,K)/W}$$

$$R_{ka} = \frac{L_a}{A k_a} = \frac{0.02\,\mathrm{m}}{A[0.0243\,\mathrm{W/(m\,K)}]} = \frac{1}{A}\ 0.823\ \mathrm{(m^2\,K)/W}$$

$$R_{ci} = \frac{1}{\bar{h}_{ci}\,A} = \frac{1}{[4\,\mathrm{W/(m^2\,K)}]A} = \frac{1}{A}\ 0.25\ \mathrm{(m^2\,K)/W}$$

The total resistance for the double-pane window is

$$R_{\text{total}} = \sum_{i=1}^{5} R_i = R_{co} + R_{k1} + R_{ka} + R_{k2} + R_{ci}$$

$$R_{\text{total}} = \frac{1}{A}\ (0.0667 + 0.00864 + 0.823 + 0.00864 + 0.25)\ \mathrm{(m^2\,K)/W} = \frac{1}{A}\ 1.157\ \mathrm{(m^2\,K)/W}$$

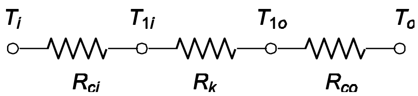
Therefore the overall heat transfer coefficient is

$$U_{\text{double}} = \frac{1}{A R_{\text{total}}} = \frac{1}{1.157\,\mathrm{(m^2K)/W}} = 0.864\ \mathrm{W/(m^2\,K)}$$

(b) The rate of heat loss through the double-pane window is

$$q_{\text{double}} - U\,A\,\Delta T = [0.864\ \mathrm{W/(m^2\,K)}]\ (4\ \mathrm{m^2})\ [22°\mathrm{C} - (-5°\mathrm{C})] = 93\mathrm{W}$$

The thermal circuit for the single-pane window is



The total thermal resistance for the single-pane window is

$$R_{\text{total}} = \sum_{i=1}^{3} R_i = R_{co} + R_{k1} + R_{ci} = \frac{1}{A}\ (0.0667 + 0.00864 + 0.25)\ \mathrm{(m^2\,K)/W}$$

$$R_{\text{total}} = 0.325\ \mathrm{(m^2\,K)/W}$$

The overall heat transfer coefficient for the single-pane window is

$$U_{\text{single}} = \frac{1}{A R_{\text{total}}} = \frac{1}{0.325\,\mathrm{(m^2\,K)/W}} = 3.08\ \mathrm{W/(m^2\,K)}$$

Therefore, the rate of heat loss through the single-pane window is

$$q_{\text{single}} = U\,A\,\Delta T = [3.07\ \mathrm{W/(m^2\,K)}]\ (4\ \mathrm{m^2})\ [22°\mathrm{C} - (-5°\mathrm{C})] = 332\ \mathrm{W}$$

The heat loss through the double-pane window is only 28% of that through the single-pane window.

(c) The average heat loss through double-pane windows during the winter months is

$$q_{\text{double}} = U A_T \Delta T = [0.864 \text{ W/(m}^2 \text{ K)}] (80 \text{ m}^2) 15°\text{C} = 1040 \text{ W}$$

Therefore, the cost of the heat loss from the double-pane windows is

$\text{Cost}_{\text{double}} = q_{\text{double}}$ (heating cost)

$\text{Cost}_{\text{double}} = (1040 \text{ W}) (\$0.10/\text{kWh}) (24 \text{ h/day}) (182 \text{ heating days/year}) (1 \text{ kW}/1000 \text{ W})$

$\text{Cost}_{\text{double}} = \$454/\text{yr}$

The average heat loss through the single-pane windows during the winter months is

$$q_{\text{single}} = U A_T \Delta T = [3.07 \text{ W/(m}^2 \text{ K)}] (80 \text{ m}^2) (15°\text{C}) = 3688 \text{ W}$$

The cost of this heat loss is

$\text{Cost}_{\text{single}} = q_{\text{single}}$ (heat cost)

$\text{Cost}_{\text{single}} = (3688 \text{ W}) (\$0.10/\text{kWh}) (24 \text{ h/day}) (182 \text{ heating days/year}) (1 \text{ kW}/1000 \text{ W})$

$\text{Cost}_{\text{single}} = \$1611/\text{yr}$

The yearly savings of the double-pane windows is \$1157. Therefore if we would like to have a payback period of two years, we would be willing to invest \$2314 in double panes.

PROBLEM 1.47

A flat roof is modeled as a flat plate insulated on the bottom and placed in the sunlight. If the radiant heat that the roof receives from the sun is 600 W/m², the convection heat transfer coefficient between the roof and the air is 12 W/(m² K), and the air temperature is 27°C, determine the roof temperature for the following two cases: (a) Radiative heat loss to space is negligible. (b) The roof is black (ε = 1.0) and radiates to space, which is assumed to be a black-body at 0 K.

GIVEN

- Sunlight striking flat roof.
- Radiant heat received from the sun (q_r/A) = 600 W/m²
- Air temperature (T_∞) = 27°C
- Convective heat transfer coefficient ($\bar{h}_c$) = 12 W/(m² K)

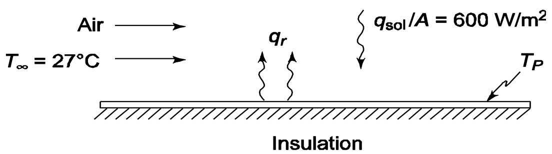
FIND

- The plate temperature (T_p)

ASSUMPTIONS

- Steady state prevails
- No heat is lost from the bottom of the plate

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5: Stefan-Boltzmann constant (σ) = 5.67 × 10⁻⁸ W/(m² K⁴)

SOLUTION

(a) For this case steady state and the conservation of energy require the heat lost by convection, from Equation (1.10), to be equal to the heat gained from the sun

$$q_c = \bar{h}_c \, A \, (T_s - T_\infty) = q_r$$

Solving for T_s,

$$T_s = (q_r/A)(1/\bar{h}_c) + T_\infty$$

$$T_s = (600 \text{ W/m}^2)\left(\frac{1}{12 \text{ W/(m}^2 \text{ K)}}\right) + (27°\text{C})$$

$$= 77°\text{C}$$

(b) In this case, the solar gain must be equal to the sum of the convective loss, from Equation (1.10), and radiative loss, from Equation (1.16)

$$\frac{q_r}{A} = \bar{h}_c \, (T_p - T_\infty) + \sigma (T_p^4 - T_{sp}^4)$$

$$600 \text{ W/m}^2 = 12 \text{ W/(m}^2 \text{ K)} (T_p - 300\text{K}) + 5.67 \times 10^{-8} \text{ W/(m}^2 \text{ K}^4) (T_p^4 - 0)$$

By trial and error

$$T_p = 308 \text{ K} = 35°\text{C}$$

COMMENTS

The addition of a second means of heat transfer from the plate in part (b) allows the plate to operate at a significantly lower temperature.

PROBLEM 1.48

A horizontal 3-mm-thick flat copper plate, 1-m-long and 0.5-m-wide, is exposed in air at 27°C to radiation from the sun. If the total rate of solar radiation absorbed is 300 W and the combined radiative and convective heat transfer coefficients on the upper and lower surfaces are 20 and 15 W/(m² K), respectively, determine the equilibrium temperature of the plate.

GIVEN

- Horizontal, 1-m-long, 0.5-m-wide, and 3-mm-thick copper plate is exposed to air and solar radiation
- Air temperature (T_∞) = 27°C
- Solar radiation absorbed (q_{sol}) = 300 W
- Combined transfer coefficients are
  upper surface ($\bar{h}_u$) = 20 W/(m² K)
  lower surface ($\bar{h}_1$) = 15 W/(m² K)

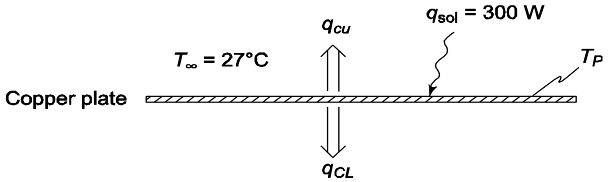
FIND

- The equilibrium temperature of the plate (T_p)

ASSUMPTIONS

- Steady state prevails
- The temperature of the plate is uniform

SKETCH



SOLUTION

For equilibrium the heat gain from the solar radiation must equal the heat lost from the upper and lower surfaces

$$q_{sol} = \bar{h}_u\ A\ (T_p - T_\infty) + \bar{h}_1\ A\ (T_p - T_\infty)$$

Solving for T_p

$$T_p = \frac{q_{sol}}{A}\frac{1}{\bar{h}_u + \bar{h}_1} + T_\infty$$

$$T_p = \left(\frac{300\,\text{W}}{(1\,\text{m})(0.5\,\text{m})}\right)\left(\frac{1}{20\,\text{W/(m}^2\text{K)} + 15\,\text{W/(m}^2\text{K)}}\right) + (27°\text{C})$$

$$T_p = 44°\text{C}$$

PROBLEM 1.49

A small oven with a surface area of 0.28 m^2 is located in a room in which the walls and the air are at a temperature of 27°C. The exterior surface of the oven is at 150°C and the net heat transfer by radiation between the oven's surface and the surroundings is 586 W. If the average convective heat transfer coefficient between the oven and the surrounding air is 11 W/(m^2 K), calculate: (a) the net heat transfer between the oven and the surroundings in W, (b) the thermal resistance at the surface for radiation and convection, respectively, in K/W, and (c) the combined heat transfer coefficient in W/(m^2 K).

GIVEN

- Small oven in a room
- Oven surface area (A) = 0.28 m^2
- Room wall and air temperature (T_∞) = 27^{0}C
- Surface temperature of the exterior of the oven (T_o) = 150°C
- Net radiative heat transfer (q_r) = 586 W
- Convective heat transfer coefficient ($\bar{h}_c$) = 11 W/(m^2 K)

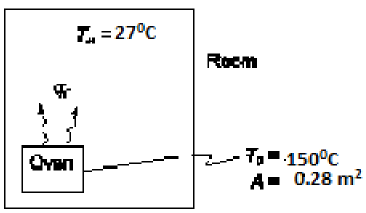
FIND

(a) Net heat transfer (q_T) in W
(b) Thermal resistance for radiation and convection (R_T) in K/W
(c) The combined heat transfer coefficient ($\bar{h}_{cr}$) in W/(m^2 K)

ASSUMPTIONS

- Steady state prevails

SKETCH



SOLUTION

(a) The net heat transfer is the sum of the convective heat transfer, from Equation (1.10), and the net radiative heat transfer

$$q_T = q_c + q_r = \bar{h}_c A (T_o - T_\infty) + q_r$$

$$q_T = 11 \text{ W/(m}^2\text{ K) } (0.28 \text{ m}^2) (150°\text{C} - 27°\text{C}) + 586 \text{ W}$$

$$q_T = 378.8 \text{ W} + 586 \text{ W} = 964.8 \text{ W}$$

(b) The radiative resistance is

$$R_r = \frac{T_o - T_\infty}{q_r} = \frac{150°\text{C} - 27°C}{586W} = 0.210 \text{ K/W}$$

The convective resistance is

$$R_c = \frac{T_o - T_\infty}{q_r} = \frac{150°C - 27°C}{378.8W} = 0.325 \text{ K/W}$$

These two resistances are in parallel, therefore the total resistance is given by

$$R_T = \frac{R_c\, R_r}{R_c + R_r} = \left(\frac{(0.210K / W)(0.325)}{(0.210 + 0.325)\,(K/\text{W})}\right) = 0.1275 \text{ K/W}$$

(c) The combined heat transfer coefficient can be calculated from

$$q_T = \overline{h_{cr}}\, A\, \Delta T$$

$$\therefore\ \overline{h_{cr}} = \frac{q_r}{A\Delta T} = \frac{964.8W}{(0.28m^2)(150°C - 27°\text{C})} = 28.01 \text{ W/(m}^2 \text{ K)}$$

COMMENTS

The thermal resistances for the convection and radiation modes are of the same order of magnitude. Hence, neglecting either one would lead to a considerable error in the rate of heat transfer.

PROBLEM 1.50

A steam pipe 200-mm in diameter passes through a large basement room. The temperature of the pipe wall is 500°C, while that of the ambient air in the room is 20°C. Determine the heat transfer rate by convection and radiation per unit length of steam pipe if the emissivity of the pipe surface is 0.8 and the natural convection heat transfer coefficient has been determined to be 10 W/(m^2 K).

GIVEN

- A steam pipe passing through a large basement room
- Pipe diameter (Δ) = 200 mm = 0.2 m
- The temperature of the pipe wall (T_p) = 500°C = 773 K
- Temperature of ambient air in the room (T_∞) = 20°C = 293 K
- Emissivity of the pipe surface (ε) = 0.8
- Natural convection heat transfer coefficient (h_c) = 10 W/(m^2 K)

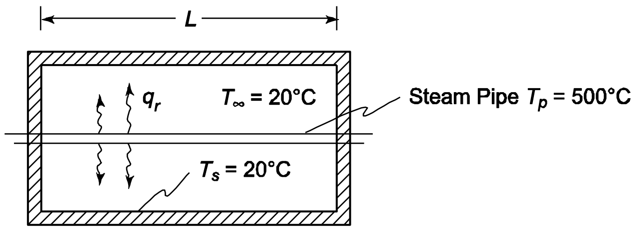
FIND

- Heat transfer rate by convection and radiation per unit length of the steam pipe (q/L)

ASSUMPTIONS

- Steady state prevails
- The walls of the room are at the same temperature as the air in the room
- The walls of the room are black (ε = 1.0)

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stefan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The net radiative heat transfer rate for a gray object in a blackbody enclosure is given by Equation (1.17)

$$q_r = A_1\, \varepsilon_1\, \sigma (T_1^4 - T_2^4) = \pi D\, L\, \varepsilon\, \sigma (T_p^4 - T_s^4)$$

$$\therefore \frac{q_r}{L} = \pi (0.2 \text{ m})\, (0.8)\, [5.67 \times 10^{-8} \text{ W/(m}^2\text{ K}^4)]\, [(773 \text{ K})^4 - (293 \text{ K})^4]$$

$$\frac{q_r}{L} = 9970 \text{ W/m}$$

The convective heat transfer rate is given by

$$q_c = \overline{h_c}\ A\,(T_p - T_\infty) = \overline{h_c}\ (\pi D L)\,(T_p - T_\infty)$$

$$\therefore\ \frac{q_c}{L} = [10\ \text{W/(m}^2\ \text{K)}]\ \pi\,(0.2\ \text{m})\ (500°\text{C} - 20°\text{C})$$

$$\frac{q_c}{L} = 3020\ \text{W/m}$$

COMMENTS

Note that absolute temperatures must be used in the radiative heat transfer equation.

The radiation heat transfer dominates because of the high emissivity of the surface and the high surface temperature which enters to the fourth power in the rate of radiative heat loss.

PROBLEM 1.51

The inner wall of a rocket motor combustion chamber receives 160 kW/m^2 by radiation from a gas at 2760° C. The convection heat transfer coefficient between the gas and the wall is 110 W/(m^2 K). If the inner wall of the combustion chamber is at a temperature of 540°C, determine (a) the total thermal resistance of a unit area of the wall in (m^2 K)/W and (b) the heat flux. Also draw the thermal circuit.

GIVEN

- Wall of a rocket motor combustion chamber
- Radiation to inner surface (q_r/A) = 160 kW/m^2
- Temperature of gas in chamber (T_g) = 2760°C
- Convective heat transfer coefficient on inner wall (h_c) = 110 W/(m^2 K)
- Temperature of inner wall (T_w) = 540°C

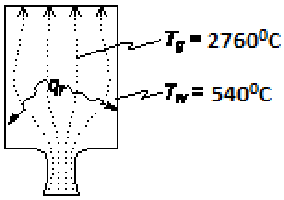
FIND

(a) Draw the thermal circuit
(b) Heat flux
(c) The total thermal resistance of a unit area ($A\ R_{\text{total}}$) in (m^2 K)/W

ASSUMPTIONS

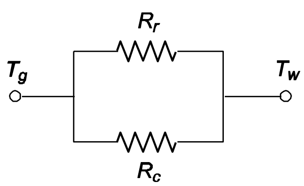
- One dimensional heat transfer through the walls of the combustion chamber
- Steady state heat flow

SKETCH



SOLUTION

(a) The thermal circuit for the chamber wall is shown below



The total thermal resistance can be calculated from the total rate of heat transfer from the pipe

$$q_{\text{total}} = \frac{\Delta T}{R_{\text{total}}}$$

$$\therefore\ A\,R_{\text{total}} = \frac{\Delta T}{\left(\dfrac{q_{\text{total}}}{A}\right)}$$

(b) The total rate of heat transfer is the sum of the radiative and convective heat transfer

$$q_{\text{total}} = q_r + q_c = q_r + \overline{h_c}\ A\,\Delta T$$

$$\therefore\ \frac{q_{\text{total}}}{A} = \frac{q_r}{A} + \overline{h_c}\ \Delta T$$

$$\frac{q_{\text{total}}}{A} = 160{,}000\ \text{W/m}^2 + 110\ \text{W/(m}^2\ \text{K)}\ (2760°\text{C} - 540°\text{C}) = 404{,}200\ \text{W/m}^2$$

is the required heat flux.

(c) Therefore the thermal resistance of a unit area is

$$A\,R_{\text{total}} = \frac{2760\,°\text{C} - 540\,°\text{C}}{404{,}200 W/m^2} = 5.49{*}10^{-3}\ (\text{m}^2\,\text{K})/\ \text{W}$$

An alternate method of solving part (c) is to calculate the radiative and convective resistances separately and then combine them in parallel as illustrated below.
The convective resistance is

$$R_c = \frac{1}{\bar{h}_c\,A} = \frac{1}{A}\left(\frac{1}{110 W/(m^2\ \text{K})}\right) = \frac{1}{A}\ 9.1{*}10^{-3}\ (\text{m}^2\,\text{K})/\text{W}$$

The radiative resistance is

$$R_r = \frac{\Delta T}{q_r} = \frac{\Delta T}{A(q_r/A)} = \frac{1}{A}\ \frac{2760\,^{\text{o}}C - 540\,^{\text{o}}C}{160{,}000 W/m^2} = \frac{1}{A}\ 0.0139\ (\text{m}^2\,\text{K})/\text{W}$$

Combining these two resistances in parallel yields the total resistance

$$R_{\text{total}} = \frac{R_r R_c}{R_r + R_c}$$

$$\therefore\ A\,R_{\text{total}} = \frac{(0.0091(0.0139)}{0.0091 + 0.0139}\,(\text{m}^2\ \text{K})/\ \text{W} = 5.49{*}10^{-3}\ (\text{m}^2\,\text{K})/\ \text{W}$$

PROBLEM 1.52

A flat roof of a house absorbs a solar radiation flux of 600 W/m². The backside of the roof is well insulated, while the outside loses heat by radiation and convection to ambient air at 20°C. If the emittance of the roof is 0.80 and the convective heat transfer coefficient between the roof and the air is 12 W/(m² K), calculate: (a) the equilibrium surface temperature of the roof, and (b) the ratio of convective to radiative heat loss. Can one or the other of these be neglected? Explain your answer.

GIVEN

- Flat roof of a house
- Solar flux absorbed (q_{sol}/A) = 600 W/m²
- Back of roof is well insulated
- Ambient air temperature (T_∞) = 20°C = 293 K
- Emittance of the roof (ε) = 0.80
- Convective heat transfer coefficient ($\bar{h}_c$) = 12 W/(m² K)

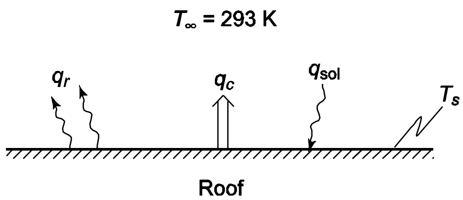
FIND

(a) The equilibrium surface temperature (T_s)
(b) The ratio of the convective to radiative heat loss

ASSUMPTIONS

- The heat transfer from the back surface of the roof is negligible
- Steady state heat flow

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stefan-Boltzmann constant (σ) = 5.67 × 10⁻⁸ W/(m² K⁴)

SOLUTION

(a) For steady state the sum of the convective heat loss, from Equation (1.10), and the radiative heat loss, from Equation (1.15), must equal the solar gain

$$\frac{q_{sol}}{A} = \frac{q_c}{A} + \frac{q_r}{A} = \overline{h_c}\ (T_s - T_\infty) + \varepsilon\,\sigma\,T_s^4$$

$$600\ \text{W/m}^2 = 12\ \text{W/(m}^2\ \text{K)}\ (T_s - 293\text{K}) + (0.8)\ \ 5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\ \ T_s^4$$

$$4.535 \times 10^{-8}\, T_s^4 + 12\, T_s - 4116 = 0$$

By trial and error

$$T_s = 309\ \text{K} = 36°\text{C}$$

(b) The ratio of the convective to radiative loss is

$$\frac{q_c}{q_r} = \frac{\overline{h_c}\,(T_s - T_\infty)}{\varepsilon\,\sigma\,T_s^4} = \frac{12\ \text{W/(m}^2\ \text{K)}\quad 309\,\text{K} - 293\,\text{K}}{(0.8)\ \ 5.67\times 10^{-8}\ \ \text{W/(m}^2\ \text{K}^4)\quad 309\,\text{K}\ ^{4}} = 0.46$$

COMMENTS

Since the radiative and convective terms are of the same order of magnitude, neither one may be neglected without introducing significant error.

PROBLEM 1.53

Determine the power requirement of a soldering iron in which the tip is maintained at 400°C. The tip is a cylinder 3-mm in diameter and 10-mm long. Surrounding air temperature is 20°C and the average convective heat transfer coefficient over the tip is 20 W/(m² K). Initially, the tip is highly polished giving it a very low emittance.

GIVEN

- Soldering iron tip
  - Diameter (D) = 3 mm = 0.003 m
  - Length (D) = 10 mm = 0.01 m
- Temperature of the tip (T_t) = 400°C
- Temperature of the surrounding air (T_∞) = 20°C
- Average convective heat transfer coefficient ($\bar{h}_c$) = 20 W/(m^2 K)
- Emittance is very low ($\varepsilon = 0$)

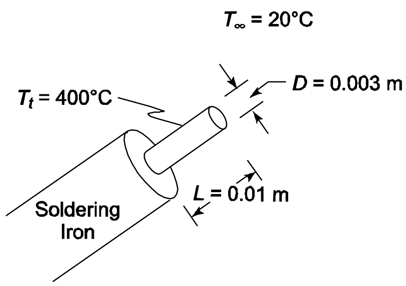
FIND

- The power requirement of the soldering iron ($\dot{q}$)

ASSUMPTIONS

- Steady state conditions exist
- All power used by the soldering iron is used to heat the tip
- Radiative heat transfer from the tip is negligible due to the low emittance
- The end of the tip is flat
- The tip is at a uniform temperature

SKETCH



SOLUTION

The power requirement of the soldering iron, $\dot{q}$, is equal to the heat lost from the tip by convection

$$q_c = \overline{h_{co}}\ A\ \Delta T = \overline{h_c}\ (\pi D^2/4 + \pi D\ L)\ (T_t - T_\infty) = \dot{q}$$

$$\dot{q} = 20\ \mathrm{W/(m^2\ K)}\left[\frac{\pi(0.003\,\mathrm{m})^2}{4} + \pi(0.003\,\mathrm{m})(0.01\,\mathrm{m})\right](400°\mathrm{C} - 20°\mathrm{C})$$

$$\dot{q} = 0.77\ \mathrm{W}$$

PROBLEM 1.54

The soldering iron tip in Problem 1.53 becomes oxidized with age and its gray-body emittance increases to 0.8. Assuming that the surroundings are at 20°C determine the power requirement for the soldering iron.

GIVEN

- Soldering iron tip
  - Diameter (D) = 3 mm = 0.003 m
  - Length (D) = 10 mm = 0.01 m
- Temperature of the tip (T_t) = 400°C
- Temperature of the surrounding air (T_∞) = 20°C
- Average convective heat transfer coefficient ($\bar{h}_c$) = 20 W/(m^2 K)
- Emittance of the tip (ε) = 0.8

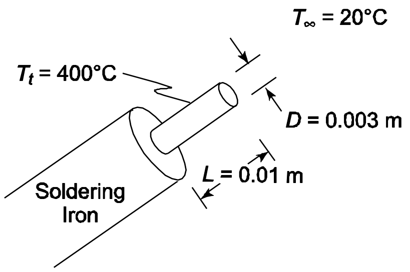
FIND

- The power requirement of the soldering iron ($\dot{q}$)

ASSUMPTIONS

- Steady state conditions exist
- All power used by soldering iron is used to heat the tip
- The surroundings of the soldering iron behave as a blackbody enclosure
- The end of the tip is flat

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stefan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The rate of heat loss by convection, from Problem 1.50, is 0.77 W.
The rate of heat loss by radiation is given by Equation (1.17)

$$q_r = A_1\, \varepsilon_1\, \sigma (T_1^4 - T_2^4) = \left(\frac{\pi D^2}{4} + \pi DL\right) \varepsilon\, \sigma (T_t^4 - T_w^4)$$

$$q_r = \left[\frac{\pi (0.003\,\text{m})^2}{4} + \pi (0.003\,\text{m})(0.01\,\text{m})\right] (0.8)\, [5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)]\, [(673\ \text{K})^4 - (293\ \text{K})^4]$$

$$q_r = 0.91\ \text{W}$$

The power requirement of the soldering iron, $\dot{q}$, is equal to the total rate of heat loss from the tip. The total heat loss is equal to the sum of the convective and radiative losses

$$\dot{q} = q_c + q_r = 0.77\ \text{W} + 0.91\ \text{W} = 1.68\ \text{W}$$

COMMENTS

Note that the inclusion of the radiative term more than doubled the power requirement for the soldering iron.

The power required to maintain the desired temperature could be provided by electric resistance heating.

PROBLEM 1.55

Some automobile manufacturers are currently working on a ceramic engine block that could operate without a cooling system. Idealize such an engine as a rectangular solid, 45 cm by 30 cm by 30 cm. Suppose that under maximum power output the engine consumes 5.7 liters of fuel per hour, the heat released by the fuel is 9.29 kWh per liter and the net engine efficiency (useful work output divided by the total heat input) is 0.33. If the engine block is alumina with a gray-body emissivity of 0.9, the engine compartment operates at 150°C, and the convective heat transfer coefficient is 30 W/(m² K), determine the average surface temperature of the engine block. Comment on the practicality of the concept.

GIVEN

- Ceramic engine block, 0.45m by 0.3m by 0.3m
- Engine gas consumption is 5.7 1/h
- Heat released is 9.29 (kWh)/1
- Net engine efficiency (η) = 0.33
- Emissivity (ε) = 0.9
- Convective heat transfer coefficient (hc) = 30 W/(m² K)
- Engine compartment temperature (T_c) = 150°C = 423 K

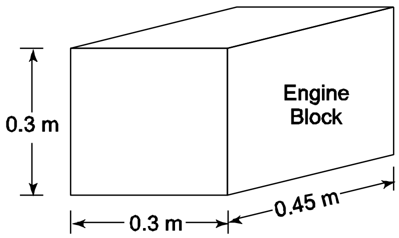
FIND

- The surface temperature of the engine block (T_s)
- Comment on the practicality

ASSUMPTIONS

- Heat transfer has reached steady state
- The engine compartment behaves as a blackbody enclosure

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5, the Stefan-Boltzmann constant (σ) = 5.67×10^{-8} W/(m² K⁴)

SOLUTION

The surface area of the idealized engine block is

$$A = 4\,(0.45\text{m})\,(0.3\text{m}) + 2(0.3\text{m})^2 = 0.72\ \text{m}^2$$

The rate of heat generation within the engine block is equal to the energy from the gasoline that is not transformed into useful work

$$\dot{q}_G = (1 - \eta)\, m_g\, h_g = (1 - 0.33)\,(5.71/\text{h})\,(9.29\ (\text{kWh})/1) = 35.5\ \text{kW}$$

For steady state conditions, the net radiative and convective heat transfer from the engine block must be equal to the heat generation within the engine block

$$q_{\text{total}} = q_r + q_c = \dot{q}_G$$

$$\dot{q}_G = A\,\varepsilon\,\sigma\,(T_s^4 - T_c^4) + \overline{h_c}\;A\,(T_s - T_c)$$

$$35.5\text{ kW} = (0.72\text{ m}^2)\,(0.9)\;\;5.67\times10^{-8}\,\text{W/(m}^2\,\text{K}^4)\;\;[T_s^4 - (423\text{ K})^4] + (0.72\text{ m}^2)\;\;30\,\text{W/(m}^2\,\text{K)}\;\;(T_s - 423)$$

$$3.674 \times 10^{-8}\,T_s^4 + 21.6\,T_s - 45656 = 0$$

By trial and error

$$T_s = 916\text{ K} = 643°\text{C}$$

COMMENTS

The engine operates at a temperature high enough to burn a careless motorist.

Note that absolute temperature must be used in radiation equations.

Hot spots due to the complex geometry of the actual engine may produce local temperatures much higher than 916 K.

PROBLEM 1.56

A pipe carrying superheated steam in a basement at 10°C has a surface temperature of 150°C. Heat loss from the pipe occurs by radiation (ε = 0.6) and natural convection [$\bar{h}_c$ = 25 W/(m^2 K)]. Determine the percentage of the total heat loss by these two mechanisms.

GIVEN

- Pipe in a basement
- Pipe surface temperature (T_s) = 150°C = 423 K
- Basement temperature (T_∞) = 10°C = 283 K
- Pipe surface emissivity (ε) = 0.6
- Convective heat transfer coefficient ($\bar{h}_c$) = 25 W/(m^2 K)

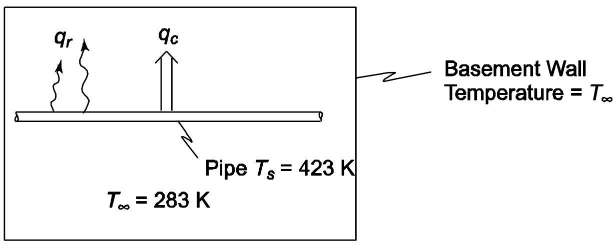
FIND

- The percentage of the total heat loss due to radiation and convection

ASSUMPTIONS

- The system is in steady state
- The basement behaves as a blackbody enclosure at 10°C

SKETCH



PROPERTIES AND CONSTANTS

From Appendix 1, Table 5: the Stefan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/(m^2 K^4)

SOLUTION

The rate of heat transfer from a gray-body to a blackbody enclosure, from Equation (1.17), is

$$q_r = A_1\, \varepsilon_1\, \sigma\,(T_1^4 - T_2^4) = A\, \varepsilon\, \sigma\,(T_s^4 - T_\infty^4)$$

$$\therefore\ \frac{q_r}{A} = (0.6)\,[5.67 \times 10^{-8}\ \text{W/(m}^2\ \text{K}^4)]\ [(423\ \text{K})^4 - (283\ \text{K})^4]$$

$$\frac{q_r}{L} = 870\ \text{W/m}$$

The rate of heat transfer by convection, from Equation (1.10), is

$$q_c = \bar{h}_c\ A\,(T_s - T_\infty)$$

$$\therefore\ \frac{q_c}{A} = 25\ \text{W/(m}^2\ \text{K)}\ (423\ \text{K} - 283\ \text{K}) = 3500\ \text{W/m}^2$$

The total rate of heat transfer is the sum of the radiative and convective rates

$$\frac{q_{\text{total}}}{A} = \frac{q_r}{A} + \frac{q_c}{A} = 870\ \text{W/m}^2 + 3500\ \text{W/m}^2 = 4370\ \text{W/m}^2$$

The percentage of the total heat transfer due to radiation is

$$\frac{q_r / A}{q_{\text{total}} / A} \times 100 = \frac{870}{4370} \times 100 = 20\%$$

The percentage of the total heat transfer due to convection is

$$\frac{q_c / A}{q_{\text{total}} / A} \times 100 = \frac{3500}{4370} \times 100 = 80\%$$

COMMENTS

This pipe surface temperature and rate of heat loss are much too high to be acceptable. In practice, a layer of mineral wool insulation would be wrapped around the pipe. This would reduce the surface temperature as well as the rate of heat loss.

PROBLEM 1.57

The walls of a typical industrial furnace used for the heat treating of the metal are usually a composite structure made up of layers of fire brick, high temperature insulation, steel plates, and outer surface cladding. Radiant tubes/beams and/or combustion burners inside the furnace provide the requisite heat. Consider plain surface of such furnace wall where the inner wall surface is heated to 850°C by radiant heating from hot gases at 2000 °C with a heat flux of 45,000W/m^2. The convection heat transfer coefficient of hot gases inside the furnace is estimated to be is 15 W/(m^2 K), the thermal conductance per unit area of the composite wall is 250 W/(m^2 K), and there is convection at the outer surface of the furnace to the ambient air at T_∞. Sketch the thermal resistance circuit for the heat transfer in this system, and calculate the total heat flow per unit area through the composite wall and the exterior wall surface temperature. Also if outside air temperature is T_∞=30°C, estimate the average convection heat transfer coefficient at the exterior surface.

GIVEN

- A furnace wall
- Convective heat transfer coefficient ($\bar{h}_c$) = 15 W/(m^2 K)
- Temperature of hot gases inside furnace (T_g) = 2000°C
- Rate of radiative heat flow to the interior of the wall (q_r/A) = 45,000 W/m^2
- Unit thermal conductance of the wall (k/L) = 250 W/(m^2 K)
- Interior surface temperature (T_{wi}) is about 850°C
- Convection occurs from outer surface of the wall

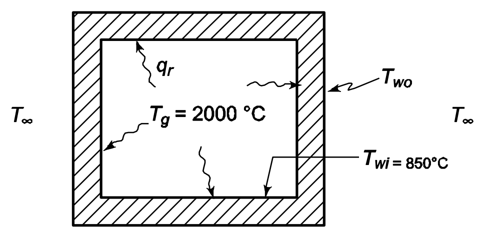
FIND

(a) Draw the thermal circuit
(b) Rate of heat flow per unit area (q/A)
(c) The exterior surface temperature (T_{wo})
(d) Average convection heat transfer coefficient at exterior surface.

ASSUMPTIONS

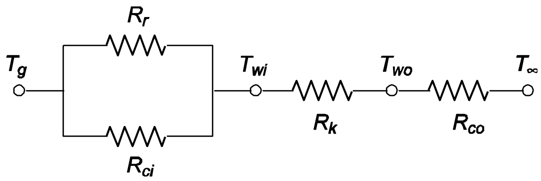
- Heat flow through the wall is one dimensional
- Steady state prevails

SKETCH



SOLUTION

The thermal circuit for the furnace wall is shown below



The rate of heat flow per unit area through the wall is equal to the rate of convective and radiative heat flow to the interior wall

$$\frac{q}{A} = \frac{q_r}{A} + \frac{q_c}{A} = \frac{q_r}{A} + \overline{h_c}\ (T_g - T_{wi})$$

$$\frac{q}{A} = 45{,}000 \text{ W/m}^2 + 15 \text{ W/(m}^2\text{ K) } (2000°\text{C} - 850°\text{C}) = 62{,}250 \text{ W/m}^2$$

We can calculate the outer surface temperature of the wall by examining the conductive heat transfer through the wall given by Equation (1.3)

$$q_k = \frac{K\,A}{L}\ (T_{wi} - T_{wo})$$

$$\therefore\ T_{wo} = T_{wi} - \frac{q_k}{A}\,\frac{1}{k/L} = 850°\text{C} - (62{,}250 \text{ W/m}^2)\left(\frac{1}{250\,\text{W/(m}^2\text{K)}}\right) = 601°\text{C}$$

$q_k/A = \overline{h_0}\ (T_0 - T_\infty)$

$62{,}250 \text{ W/m}^2 = \overline{h_0}\ (601\text{-}30)$

Solving for $\overline{h_0}$ we get

$\overline{h_0} = 109 \text{ W/(m}^2\text{ K)}$

COMMENTS

The corner sections should be analyzed separately since the heat flow there is not one dimensional.

PROBLEM 1.58

Draw the thermal circuit for heat transfer through a double-glazed window. Identify each of the circuit elements. Include solar radiation to the window and interior space.

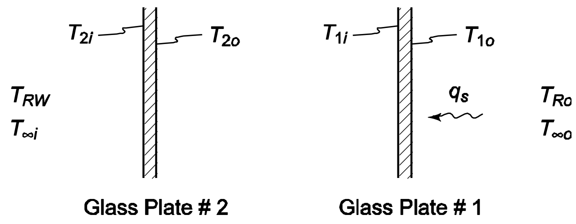
GIVEN

- Double-glazed window

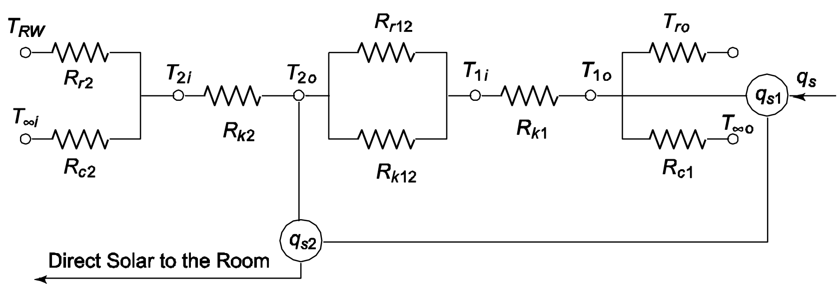
FIND

- The thermal circuit

SKETCH



SOLUTION



where	R_{r1}, R_{r12}, R_{r2}	= Radiative thermal resistances
	R_{k1}, R_{k2}, R_{k12}	= Conductive thermal resistances
	R_{c1}, R_{c2}	= Convective thermal resistances
	T_{rw}, T_{ro}	= Effective temperatures for radiative heat transfer
	T_∞	= Air temperatures
	T_{1i}, T_{1o}, T_{2i}	= Surface temperatures of the glass
	q_{s1}, q_{s2}	= Solar energy incident on the window panes

PROBLEM 1.59

The ceiling of a tract house is constructed of wooden studs with fiberglass insulation between them. On the interior of the ceiling is plaster and on the exterior is a thin layer of sheet metal. A cross section of the ceiling with dimensions is shown below.

(a) The *R*-factor describes the thermal resistance of insulation and is defined by:

$$\boldsymbol{R\text{-factor} = L/k_{\mathrm{eff}} = \Delta T/(q/A)}$$

Calculate the *R*-factor for this type of ceiling and compare the value of this *R*-factor with that for a similar thickness of fiberglass. Why are the two different?

(b) Estimate the rate of heat transfer per square meter through the ceiling if the interior temperature is 22°C and the exterior temperature is –5°C.

GIVEN

- Ceiling of a tract house, construction shown below
- Inside temperature (T_i) = 22°C
- Outside temperature (T_o) = –5°C

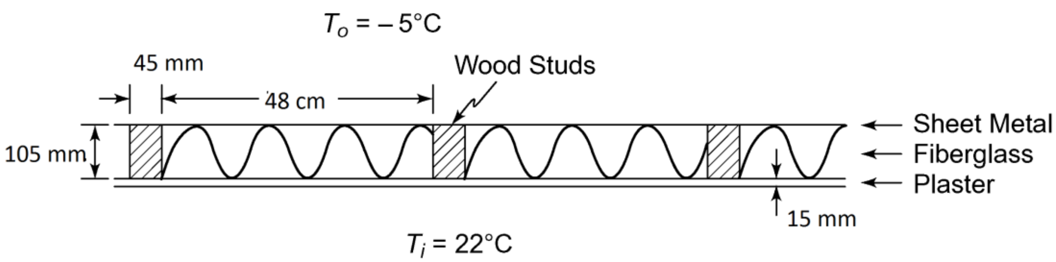
FIND

(a) *R*-factor for the ceiling (RF_c). Compare this to the *R*-factor for the same thickness of fiberglass (RF_{fg}). Why do they differ?
(b) Rate of heat transfer (q/A)

ASSUMPTIONS

- Steady state heat transfer
- One dimensional conduction through the ceiling
- Thermal resistance of the sheet metal is negligible

SKETCH



PROPERTIES AND CONSTANTS

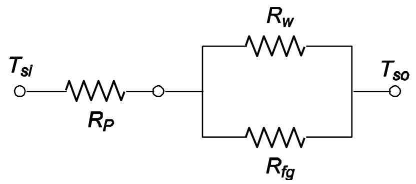
From Appendix 2, Table 11, the thermal conductivities of the ceiling materials are

Pine or fir wood studs (k_w) = 0.15 W/(m K) at 20°C
Fiberglass (k_{fg}) = 0.035 W/(m K) at 20°C

Plaster (k_p) = 0.814 W/(m K) at 20°C

SOLUTION

The thermal circuit for the ceiling with studs is shown below



where R_p = thermal resistance of the plaster
R_w = thermal resistance of the wood
R_{fg} = thermal resistance of the fiberglass

Each of these resistances can be evaluated using Equation (1.4)

$$R_p = \frac{L_P}{A_{\text{wall}}\, k_P} = \frac{15*10^{-3}\,m}{(A_{\text{wall}})[0.814\,\text{W/(m K)}]} = \frac{1}{A_{\text{wall}}}\ 0.0184\ \text{m}^2\ \text{K/W}$$

$$R_w = \frac{L_w}{A_w\, k_w} = \frac{105*10^{-3}\,\text{m}}{(A_w)[0.15\,\text{W/(m K)}]} = \frac{1}{A_{\text{wall}}}\ 0.71\ \text{m}^2\ \text{K/W}$$

$$R_{fg} = \frac{L_{fg}}{A_{fg}\, k_{fg}} = \frac{105*10^{-3}\,\text{m}}{(A_{\text{fg}})[0.035\,\text{W/(m K)}]} = \frac{1}{A_{\text{wall}}}\ 3\ \text{m}^2\ \text{K/W}$$

To convert these all to a wall area basis the fraction of the total wall area taken by the wood studs and the fiberglass must be calculated

$$\text{wood studs} = \frac{A_w}{A_{\text{wall}}} = \frac{4.5\,\text{cm.}}{48\text{cm}} = 0.094$$

$$\text{fiberglass} = \frac{A_{fg}}{A_{\text{wall}}} = \frac{43.5\,\text{cm}}{48\,\text{cm}} = 0.906$$

Therefore the resistances of the studs and the fiberglass based on the wall area are

$$R_w = \frac{1}{0.094\,A_{\text{wall}}}\ 0.71\ \text{m}^2\ \text{K/W} = \frac{1}{A_{\text{wall}}}\ 7.55\ \text{m}^2\ \text{K/W}$$

$$R_{fg} = \frac{1}{0.906\,A_{\text{wall}}}\ 3\ \text{m}^2\text{K/W} = \frac{1}{A_{\text{wall}}}\ 3.31\ \text{m}^2\ \text{K/W}$$

The R-Factor of the wall is related to the total thermal resistance of the wall by

$$RF_c = A_{\text{wall}}\, R_{\text{total}} = A_{\text{wall}} \left[R_p + \frac{R_w\, R_{fg}}{R_w + R_{fg}} \right] =$$

$$0.0184 + \frac{(7.55)\,(3.31)}{7.55 + 3.31}\ \text{m}^2\text{K/W} \quad = 2.33\ \text{m}^2\ \text{K/W}$$

For 120 mm. of fiberglass alone, the R-factor is

$$RF_{fg} = \frac{L}{k_{fg}} = \frac{120 * 10^{-3} m}{0.035\, \mathrm{W/(m\,K)}} = 3.42\ \mathrm{m^2\ K/W}$$

The R-factor of the ceiling is only 68% that of the same thickness of fiberglass. This is mainly due to the fact that the wood studs act as a 'thermal short' conducting heat through the ceiling more quickly than the surrounding fiberglass.

(b) The rate of heat transfer through the ceiling is

$$\frac{q}{A} = \frac{\Delta T}{RF_c} = \frac{22\,°\mathrm{C} - (-5\,°\mathrm{C})}{2.33\ \mathrm{m^2 K/W}} = 11.6\ \mathrm{W/m^2}$$

COMMENTS

R-factors are given in handbooks. For example, *Mark's Standard Handbook for Mechanical Engineers* lists the R-factor of a multi-layer masonry wall as 6.36 Btu/(h ft^2) = 20 W/m^2.

PROBLEM 1.60

Two electric resistance heaters with a 20 cm length and a 2 cm diameter are inserted into a well insulated 10-gallon tank of water that is initially at 300 K. If each heater dissipates 500 W, what is the time required for bringing the water temperature in the tank to 340 K? State your assumption for analysis.

GIVEN

- Two electric resistance heaters with l=0.2 m and d=0.02 m
- V=10 gallon= 37.85 liter of water at T_0 =300 K
- Each heater power= 500 W

FIND

(a) Time required for bringing water temperature in the tank to T_f =340 K.
(b) State the assumption for analysis.

ASSUMPTIONS

- Steady state heat transfer
- All the heat dissipated through the heater is gained by water.
- There is uniform mixing of fluid maintaining same temperature at each point at given time.

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the thermodynamic properties of water at 320 K are

Specific heat(c_p)= 4177 J/(kg K)
Density(ρ) = 989 kg/m^3

SOLUTION

Total energy required for heating 10 gallons of water from 300 to 340 K is

$$Q= V* \rho* c_p*(T_f-T_0)$$

$$Q=0.0375*4117*989*(340-300) \text{ J}$$

$$Q=6107.57 \text{ KJ}$$

Power generated through two electric resistance heaters P=2*500 W= 1 KW.
Thus,
Total time required to heat the water =Q/P sec
=6107.57/1 sec
=6107 seconds
= 1.7 hours.

COMMENTS

It is considered that all the heat dissipated by heaters is used to heat the water.

PROBLEM 1.61

In problem 1.60, instead of diffusive heating (or by conduction), consider that the water is heated by natural convection from the surface of the heaters. The heat transfer coefficient $\overline{h_c}$ in W/(m² K) is expressed in the form $\overline{h_c}=350\left(T_s-T_w\right)^{1/3}$, where T_s is surface temperature of the heaters and T_w is water temperature in K. Assuming the tank to be well mixed(water is at uniform temperature at any instant of time), plot the variation of water temperature as a function of time, for the first two hours of the heating.

GIVEN

- Two electric resistance heaters with h=0.2 m and d=0.02 m
- V=10 gallon= 37.85 liter of water at T_0 =300 K
- Each heater power= 500 W
- Heat transfer coefficient ($\overline{h_c}$)=$350\left(T_s-T_w\right)^{1/3}$ W/(m² K)

FIND

(a) Plot variation of temperature as function of time

ASSUMPTIONS

- Steady state heat transfer
- All the heat dissipated through the heater is gained by water.
- There is uniform mixing of fluid maintaining same temperature at each point at given time.

PROPERTIES AND CONSTANTS

From Appendix 2, Table 13, the thermodynamic properties of water at 320 K are

Specific heat(c_p)= 4177 J/(kg K)
Density(ρ) = 989 kg/m³

SOLUTION

Heat dissipated by two heaters (Q)= 1000 W

A= $2(2\pi rh)$

$=4\pi rh$

$=4*\pi*0.01*0.2$ m²

=0.025 m²

Q= $\overline{h_c}$ *A*($\left(T_s-T_w\right)$

Q= 350*$\left(T_s-T_w\right)^{4/3}$*A*($\left(T_s-T_w\right)$

T_s= T_w+ $\left(\dfrac{1000}{0.025*350}\right)^{3/4}$ = (T_w+34) K

Thus the surface temperature at the start of heating $T_{s,i}$=334 K and surface temperature at end of the heating is $T_{s,f}$=374 K.

$$V^* \rho^* c_p^*(T_w-300) = 350^* 0.025^*((T_s - T_w)^{4/3} {}^*t$$

$$t = \frac{V^* \ \rho^* \ c_p^*(T_w - 300)}{350^* 0.025^* (T_s - T_w)^{4/3}}$$

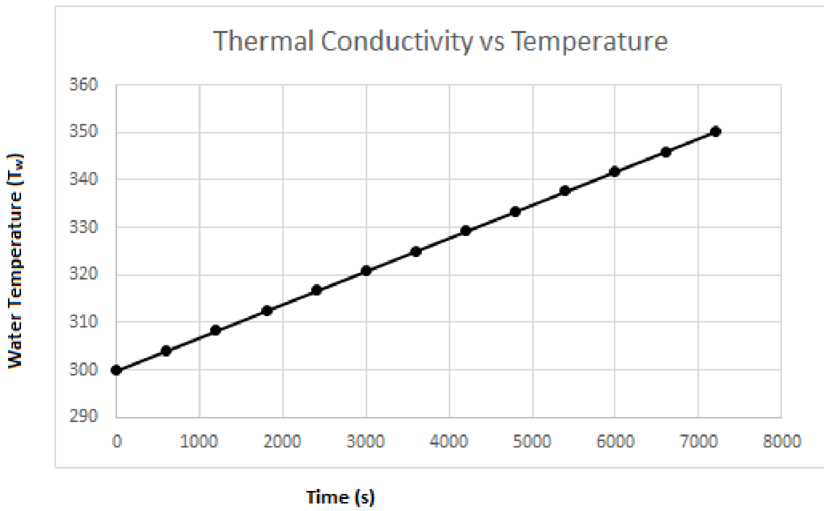
$$t = \frac{0.0375^* 4117^* 989^* (T_w - 300)}{350^* 0.025^* (T_s - T_w)^{4/3}}$$

$$t = \frac{0.0375^* 4117^* 989^* (T_w - 300)}{350^* 0.025^* (T_s - T_w)^{4/3}}$$

$$t = \frac{15783(T_w - 300)}{(T_s - T_w)^{4/3}}$$

$$t = \frac{15783(T_w - 300)}{(34)^{4/3}} = 143.3^* (T_w-300)$$

Water temperature at each 600 sec duration for duration of 2 hrs is plotted below



PROBLEM 1.62

A homeowner wants to replace an electric hot-water heater. There are two models in the store. The inexpensive model costs \$280 and has no insulation between the inner and outer walls. Due to natural convection, the space between the inner and outer walls has an effective conductivity of 3 times that of air. The more expensive model costs \$310 and has fiberglass insulation in the gap between the walls. Both models are 3.0 m tall and have a cylindrical shape with an inner wall diameter of 0.60 m and a 5 cm gap. The surrounding air is at 25°C, and the convective heat transfer coefficient on the outside is 15 W/(m^2 K). The hot water inside the tank results in an inside wall temperature of 60°C.

If energy costs 6 cents per kilowatt-hour, estimate how long it will take to pay back the extra investment in the more expensive hot-water heater. State your assumptions.

GIVEN

- Two hot-water heaters
  - Height (H) = 3.0 m
  - Inner wall diameter (D_i) = 0.60 m
  - Gap between walls (L) = 0.05 m
- Water heater #1
  - Cost = \$280.00
  - Insulation: none
  - Effective Conductivity between wall (k_{eff}) = 3(k_a)
- Water heater #2
  - Cost = \$310.00
  - Insulation: Fiberglass
- Surrounding air temperature (T_∞) = 25°C
- Convective heat transfer coefficient (h_c) = 15 W/(m^2 K)
- Inside wall temperature (T_{wi}) = 60°C
- Energy cost = \$0.06/kWh

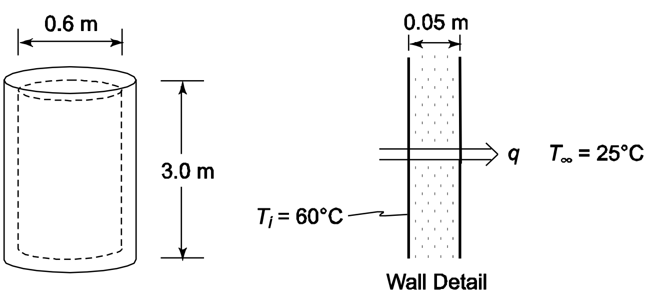
FIND

- The time it will take to pay back the extra investment in the more expensive hot-water heater

ASSUMPTIONS

- Since the diameter is large compared to the wall thickness, one-dimensional heat transfer is assumed
- To simplify the analysis, we will assume there is no water drawn from the heater, therefore the inside wall is always at 60°C
- Steady state conditions prevail

SKETCH



PROPERTIES AND CONSTANTS

From Appendix, Table 11 and 27: The thermal conductivities are

fiberglass (k_i) = 0.035 W/(m K) at 20°C

dry air (k_a) = 0.0279 W/(m K) at 60°C

SOLUTION

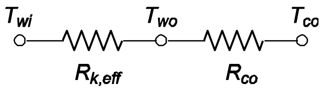
The areas of the inner and outer walls are

$$A_i = 2\,\frac{\pi D_i^2}{4} + \pi D_i H = 2\,\frac{\pi (0.6\,\text{m})^2}{4} + \pi (0.6\text{ m})\,(3\text{ m}) = 6.22\text{ m}^2$$

$$A_o = 2\,\frac{\pi D_o^2}{4} + \pi D_o H = 2\,\frac{\pi (0.7\,\text{m})^2}{4} + \pi (0.7\text{ m})\,(3\text{ m}) = 7.37\text{ m}^2$$

The average area for the air or insulation between the walls (A_a) = 6.8 m^2.

The thermal circuit for water heater #1 is



The rate of heat loss for water heater #1 is

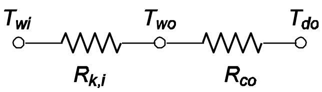
$$q_1 = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_{k,\text{eff}} + R_{co}} = \frac{T_{wi} - T_\infty}{\dfrac{L}{k_{\text{eff}}\,A_a} + \dfrac{1}{\bar{h}_\infty A_{co}}}$$

$$q_1 = \frac{60\,°\text{C} - 25\,°\text{C}}{\dfrac{0.05\,\text{m}}{3[0.0279\,\text{W/(m K)}](6.8\,\text{m}^2)} + \dfrac{1}{[15\,\text{W/(m}^2\,\text{K)}](7.37\,\text{m}^2)}} = 361\text{ W} = 0.361\text{ kW}$$

Therefore the cost to operate water heater #1 is

$$\text{Cost}_1 = q_1\,(\text{energy cost}) = 0.361\text{ kW }(\$0.06/\text{kWh})\,(24\text{ h/day}) = \$0.52/\text{day}$$

The thermal circuit for water heater #2 is



The rate of heat loss from water heater #2 is

$$q_2 = \frac{60\,°\text{C} - 25\,°\text{C}}{\dfrac{0.05\,\text{m}}{[0.035\,\text{W/(m K)}](6.8\,\text{m}^2)} + \dfrac{1}{[15\,\text{W/(m}^2\,\text{K)}](7.37\,\text{m}^2)}} = 160\text{ W} = 0.16\text{ kW}$$

Therefore the cost of operating water heater #2 is

$$\text{Cost}_2 = q_2\,(\text{energy cost}) = 0.16\text{ kW }(\$0.06/\text{kWh})\,(24\text{ h/day}) = \$0.23/\text{day}$$

The time to pay back the additional investment is the additional investment divided by the difference in operating costs

$$\text{Payback time} = \frac{\$310 - \$280}{\$0.52/\text{day} - \$0.23/\text{day}}$$

$$\text{Payback time} = 103\text{ days}$$

COMMENTS

When water is periodically drawn from the water heater, energy must be supplied to heat the cold water entering the water heater. This would be the same for both water heaters. However, drawing water from the heater also temporarily lowers the temperature of the water in the heater thereby lowering the heat loss and lowering the cost savings of water heater #2. Therefore, the payback time calculated here is somewhat shorter than the actual payback time.

A more accurate, but much more complex estimate could be made by assuming a typical daily hot water usage pattern and power output of heaters. But since the payback time is so short, the increased complexity is not justified since it will not change the bottom line—buy the more expensive model and save money as well as energy!

PROBLEM 1.63

Liquid oxygen (LOX) for the Space Shuttle can be stored at 90 K prior to launch in a spherical container 4 m in diameter. To reduce the loss of oxygen, the sphere is insulated with superinsulation developed at the U.S. Institute of Standards and Technology's Cryogenic Division that has an effective thermal conductivity of 0.00012 W/(m K). If the outside temperature is 20°C on the average and the LOX has a heat of vaporization of 213 J/g, calculate the thickness of insulation required to keep the LOX evaporation rate below 200 g/h.

GIVEN

- Spherical LOX tank with superinsulation with diameter (D) = 4 m
- LOX temperature (T_{LOX}) = 90 K
- Ambient temperature (T_∞) = 20°C = 293 K
- Thermal conductivity of insulation (k) = 0.00012 W/(m K)
- Heat of vaporization of LOX (h_{fg}) = 213 kJ/kg
- Maximum evaporation rate ($\dot{m}_{Lox}$) = 0.2 kg/h

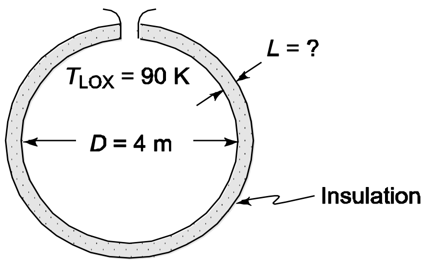
FIND

- The minimum thickness of the insulation (L) to keep evaporation rate below 0.2 kg/h

ASSUMPTIONS

- The thickness is small compared to the sphere diameter so the problem can be considered one dimensional
- Steady state conditions prevail
- Radiative heat loss is negligible

SKETCH



SOLUTION

The maximum permissible rate of heat transfer is the rate that will evaporate 0.2 kg/h of LOX

$$q = \dot{m}_{Lox}\, h_{fg}$$

$$q = (0.2 \text{ kg/h})\ (213 \text{ kJ/kg}) \left(\frac{\text{h}}{3600\,\text{s}}\right) \left(\frac{1000\,\text{J}}{\text{kJ}}\right) \ \text{Ws/J}$$

$$= 11.8 \text{ W}$$

An upper limit can be put on the rate of heat transfer by assuming that the convective resistance on the outside of the insulation is negligible and therefore the outer surface temperature is the same as the ambient air temperature. With this assumption, heat transfer can be calculated using Equation (1.3), one dimensional steady state conduction

$$q_k = \frac{k\,A}{L}\,(T_{\text{hot}} - T_{\text{cold}}) = \frac{k\,\pi\,D^2}{L}\,(T_\infty - T_{\text{LOX}})$$

Solving for the thickness of the insulation (L)

$$L = \frac{k\,\pi\,D^2}{q_k}\,(T_\infty - T_{\text{LOX}}) = \frac{[0.00012\,\text{W/(m K)}]\,\pi\,(4\,\text{m})^2}{11.8\,\text{W}}\,(293\text{ K} - 90\text{ K}) = 0.10\text{ m} = 10\text{ cm}$$

COMMENTS

The insulation thickness is small compared to the diameter of the tank. Therefore, the assumption of one dimensional conduction is reasonable.

PROBLEM 1.64

> **The interior wall of a large commercial, walk-in type meat freezer is covered under normal operating conditions with a 2 cm thick layer of ice. One day, a power outage cuts electricity to the refrigeration system of the freezer. Estimate the time required to melt this layer of ice if it has mass density of 700 kg/m^3 and latent heat of fusion of 334 kJ/kg. Consider the air temperature inside the freezer to be 20^{0}C with a heat transfer coefficient of 2 W/(m^2 K) for convection from freezer surface to air, and clearly state the assumptions made in the calculations.**

GIVEN

- Freezer covered with L=2 cm=0.02 m thick layer of ice of mass density ρ=700 kg/m^3
- Latent heat of fusion h_{sf}= 334 kJ/kg.
- Temperature inside freezer (T_∞) = 20°C = 293 K
- Heat of vaporization of LOX (h_{fg}) = 213 kJ/kg
- Convection heat transfer coefficient $\overline{h_c}$ =2 W/(m^2 K)

FIND

- Time required to melt the layer of ice

ASSUMPTIONS

- Steady state conditions prevail
- Radiative heat loss is negligible

SOLUTION

Heat transferred from surrounding to the ice surface through convection is given by

$$q_k/A=\overline{h_c}\ (T_0 - T_\infty)$$

$$q_k/A= 2*(20\text{-}0)= 40\ W/m^2=0.04\ kW/\text{m}^2$$

$$\text{P}=0.04\ kW/\text{m}^2$$

Total heat energy required for melting of ice is given by Q= m* h_{sf}

$$\text{Q}=\rho*\text{A}*\text{L}*\ h_{sf}$$

$$\text{Q/A} = 700*0.02*213\ \text{kJ}/\text{m}^2$$

$$=2982\ \text{kJ/ m}^2$$

Thus the time required for melting ice of thickness 2 cm = $\dfrac{\text{Q}/\text{A}}{P}$ sec

$$=\frac{2982}{0.04}\ \text{sec}== 74550\ \text{seconds}$$

$$= 20.7\ \text{hrs}$$

COMMENTS

Thus the temperature of meat freezer will start melting after 20.7 hrs when all the ice melts down. High time required is due to low convective heat transfer coefficient.

PROBLEM 1.65

Many specialized applications (ranging from advanced gas-turbine blades to medical devices and implants) require metal components that are coated with protective material layer. In a manufacturing plant for such coatings, infrared lamp is used for curing the coating on metal plates. The lamp provides uniform irradiation of which a flux of 1600 W/m^2 is absorbed by the metal plate surface, which has an emissivity of 0.5 for the radiation at the temperature of the coating. Air is used to cool the plate, which is kept in an enclosed environment, and it enters the enclosure at 20^{0}C and leaves from the coating process at 30^{0}C. (a) If the convective heat transfer coefficient between the coated plate surface and cooling air is 15 W/ (m^2 K), estimate the temperature of coating on plate surface. (b) If the engineer who invented believes that the cure temperature (or surface temperature) of 50^{0}C provides a coating with better durability, estimate the infrared lamp irradiation necessary to attain the required average cure temperature for the process.

GIVEN

- Infrared lamp of which flux of Q=1600 W/m^2 is absorbed by metal surface of emissivity ε=0.5
- Cooling air enters enclosure at (T_∞) = 20°C = 293 K and leaves the coating process at (T_f) = 30°C = 303 K
- Convection heat transfer coefficient $\overline{h_c}$ =15 W/(m^2 K)
- Cure temperature of 50^{0}C provides a coating with better durability

FIND

- Temperature of coating on plate surface
- Infrared lamp irradiation necessary to attain required cure temperature.

ASSUMPTIONS

- Steady state conditions prevail
- Other heat loss to surrounding is negligible

SOLUTION

(a) The flux irradiated to the surface is dissipated by convection through convective heat transfer through air and radiation from the surface. Thus

$Q=q_r+q_c$

$1600= \varepsilon*\sigma*(T_s^4-T_\infty^4)+ h_c*(T_s-T_\infty)$

$1600=0.5*5.67*10^{-8}*(T_s^4-293^4)+15*(T_s-T_\infty)$

$1600=2.835*10^{-8}*T_s^4-208.9+15 T_s-4395$

$2.835*10^{-8}*T_s^4+15 T_s-6203.9=0$

Solving by trial and error method we get

T_s=413.6 K=140.6^{0}C

(b) If the surface temperature of 50^{0}C provides a coating with better durability, the irradiation required on the surface for **T_s=323 K is given by**

$Q=\varepsilon*\sigma*(T_s^4-T_\infty^4)+ h_c*(T_s-T_\infty)$

$Q=0.5*5.67*10^{-8}*(323^4-293^4)+15*(323-293)$ W/m^2

Q= 550 W/ m^2

PROBLEM 1.66

At present time, radioactive waste from nuclear power plants placed in long, thin-walled cylindrical containers that are submerged in a water bath for cooling. The radioactive waste generates heat non-uniformly across the radial direction in the cylinder according to the following expression $q_G(r) = q_0\left[1-(r/r_0)^2\right]$ where $q_G(r)$ is the local rate of heat generation per unit volume q_0 is a constant and r_0 is the radius of the cylindrical container. Assuming that the heat transfer coefficient between the external surface of the container and the surrounding water is the uniform value of $\overline{h_c}$ W/ (m^2 K), obtain an expression for the total rate at which heat is generated per unit length of a container and an expression to obtain the temperature T_s at the container wall.[Students should note that this procedure does not provide permanent method of dealing with radioactive waste and different ways are currently being investigated to permanently dispose of radioactive waste produced form hundreds of nuclear power plants in operation].

GIVEN

- Heat generation non uniformly according to expression $q_G(r) = q_0\left[1-(r/r_0)^2\right]$

FIND

- Expression for the total rate at which heat is generated per unit length of a container
- Expression to obtain the temperature T_s at the container wall.

ASSUMPTIONS

- Steady state conditions prevail
- Heat transfer coefficient between the external surface of the container and the surrounding water is the uniform value of $\overline{h_c}$

SOLUTION

(a)Total heat generated from the cylindrical container is given by

$$Q=\int q_G(r) * rdr * L$$

$$Q/L=\int_0^{r_0} q_0\left[1-(r/r_0)^2\right] * rdr$$

$$Q/L=\frac{q_0 * r_0^2}{4}$$

This is the total rate at which heat is generated per unit length of a container.

(b) If T_∞ is the temperature of the surrounding and $\overline{h_c}$ is heat transfer coefficient

$$Q= 2*\pi*r_0*\overline{h_c}\, L*(T_s- T_\infty)$$

$$Q/L=2*\pi*r_0*\overline{h_c}\,(T_s- T_\infty)$$

$$\frac{q_0 * r_0^2}{4} = 2*\pi*r_0*h_c\ (T_s - T_\infty)$$

$T_s = T_\infty + \dfrac{q_0 * r_0}{8\pi h_c}$ is expression to obtain temperature at container wall.

COMMENTS

Heat transfer in axial direction is neglected considering the container of long length i.e. $L >> r_0$

PROBLEM 1.67

In beauty salons and in homes, a ubiquitous device is a hair dryer. The front end of the typical hair dryer is idealized as a thin walled cylindrical duct with a 6-cm diameter with a fan at the inlet that blows air over an electric heating coil as schematically shown in the figure. The design of the appliance requires two power settings, with which the air blown over the electric heating coil is heated from the ambient temperature of 22^{0}C to an outlet temperature of 44^{0}C and with exit air velocities of 1.0 m/s and 1.5 m/s. Estimate the electric power required for the heating coil to meet these conditions, assuming that the heat loss from the outside of the dryer duct is neglected.

GIVEN

- Dryer idealized as cylindrical duct of d= 6 cm =0.06 m with 2 power settings
- Air blown over coil is heated from T_∞=22^{0}C to outlet temp T_f=44^{0}C with exit air velocities (a) 1.0 m/s and (b) 1.5 m/s.

FIND

- Electric power required for heating coil to meet these condition.

ASSUMPTIONS

- Steady state conditions prevail
- Heat loss from outside of dryer duct is neglected.

PROPERTIES AND CONSTANTS

From Appendix 2, Table 28 : The air properties at 300 K are

Density of air (ρ)=1.177 kg/m^3

Specific heat capacity(Cp)= 1.0049 kJ/kgK

SOLUTION

Mass flow rate of air in each case is given by

$$m= A*v*\rho$$
$$=\pi*d^2/4*v*\rho$$

(a) For velocity of 1.0 m/s we have

$$m=3.33*10^{-3} \text{ kg/s}$$

Power required for heating the air is given by formula

$Q=m*Cp*(T_f- T_\infty)$

$Q=3.33*10^{-3}*1.0049*(44-22)$ kW

Q=0.0736 kW= 73.6 W

(b) For velocity of 1.0 m/s we have

$$m=5*10^{-3} \text{ kg/s}$$

Power required for heating the air is given by formula

$Q=m*Cp*(T_f- T_\infty)$

$Q=5*10^{-3}*1.0049*(44-22)$ kW

Q=0.1105 kW= 110.5 W

PROBLEM 1.68

In problem 1.67, the heat loss from the hairdryer duct enclosure was neglected. To a designer, while this may appear to be reasonable assumption, it should really be checked by order of magnitude calculations even if all the parameters are not known. In order to determine whether the heat loss from the exterior of the duct enclosure is neglected or not, assume a high surface emissivity of 0.8, an average surface temperature of 40^0C, and from Table 1.4, consider an estimate for natural convection coefficient at a high reasonable value. If the length of hair dryer is 20 cm, calculate the expected heat loss per unit length of the dryer duct, compare it with heat dissipation per unit length from the internal electrical heating, and comment on validity of negligible heat loss assumption.

GIVEN

- High surface emissivity of 0.8, an average surface temperature of 40^0C
- Length of the hear drier L=20 cm =0.2 m

FIND

- Expected heat loss per unit length of dryer duct
- Compare it with heat dissipation per unit length from the internal electrical heating.

ASSUMPTIONS

- Steady state conditions prevail
- Heat loss from outside of dryer duct is neglected.

PROPERTIES AND CONSTANTS

From table 1.4 and appendix 2, table 28, the air properties are

Convection heat transfer coefficient for air (considering higher value) (h_c)=20 W/(m^2 K)
Specific heat capacity(Cp)= 1.0049 kJ/kgK

SOLUTION

Heat radiation from body grey body of average temperature of 40^0C=313 K to ambiance of 22^0C =295 K is given by

$q_r = \varepsilon * \sigma * A * (T_a^4 - T_\infty^4)$

$q_r / L = 0.8 * 5.67 * 10^{-8} * 2 * \pi * 0.03 * (313^4 - 295^4)$ W/m

$q_r / L = 17.3$ W/m

Heat transferred by natural convection is given by

$q_c = h_c A \left(T_a - T_\infty \right)$

$q_c / L = 20 * 2 * \pi * 0.03 * (40 - 22)$ W/m

$q_c / L = 67.8$ W/m

Heat dissipation per unit length of dryer from internal electrical heating is given by

Q/L=110.5/0.2= 552.2 W/m

COMMENT

Thus heat transferred by natural convection and radiation which is total heat loss is negligible as compared to heat dissipation per unit length from internal electrical heating.

PROBLEM 1.69

The heat transfer coefficient between a surface and a liquid is 57 W/(m^2 K). How many watts per square meter will be transferred in this system if the temperature difference is 10°C?

GIVEN

- The heat transfer coefficient between a surface and a liquid (h_c) = 57 W/(m^2 K)
- Temperature difference (ΔT) = 10°C

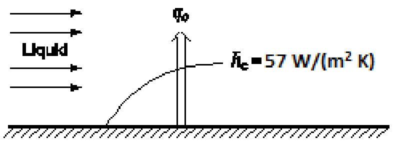
FIND

- The rate of heat transfer in watts per square meter

ASSUMPTIONS

- Steady state conditions
- Surface temperature is higher than the liquid temperature

SKETCH



SOLUTION

The rate of convective heat transfer per unit area (q_c/A) is

$$\frac{q_c}{A} = \overline{h_c}\ \Delta T = 57\ \text{W/(m}^2\ \text{K)} * 10^0\ \text{C}$$

$$\frac{q_c}{A} = 570\ \text{W/m}^2$$

PROBLEM 1.70

The thermal conductivity of fiberglass insulation at 68°F is 0.02 Btu/(h ft °F). What is its value in SI units?

GIVEN

- Thermal conductivity (k) = 0.02 Btu/(h ft °F)

FIND

- Thermal conductivity is SI units: W/(m K)

SOLUTION

$$k = 0.02\ \text{Btu/(h ft}^2\,{}^\circ\text{F)}\left(\frac{1055\,\text{J}}{\text{Btu}}\right)\left(\frac{\text{h}}{3600\,\text{s}}\right)\ \text{Ws/J}\ \left(\frac{3.281\,\text{ft}}{\text{m}}\right)(1.8\,{}^\circ\text{F}/{}^\circ\text{C})$$

$$k = 0.035\ \text{W/ m}\,{}^\circ\text{C} = 0.035\ \text{W/(m K)}$$

COMMENTS

Note that 1°C = 1 K if a temperature difference is involved as in the units for thermal conductivity.

PROBLEM 1.71

The thermal conductivity of silver at 212°F is 238 Btu/(h ft °F). What is the conductivity in SI units?

GIVEN

- Thermal conductivity of silver (k) = 238 Btu/(h ft °F)

FIND

- Thermal conductivity of silver in SI units: W/(m K)

SOLUTION

The conversion can be done one unit at a time

$$k = 238 \ \mathrm{Btu/(h\,ft^2\,{}^{o}F)} \left(\frac{1055\,\mathrm{J}}{\mathrm{Btu}}\right)\left(\frac{\mathrm{h}}{3600\,\mathrm{s}}\right)(\mathrm{W\,s})/\mathrm{J} \left(\frac{\mathrm{ft}}{0.3048\,\mathrm{m}}\right) \ 1.8\,{}^{\circ}\mathrm{F/K}$$

$$k = 412 \ \mathrm{W/(m\,K)}$$

If the appropriate conversion factor is available, the whole group of units can be converted in one step

$$k = 238 \ \mathrm{Btu/ \ h\,ft\,{}^{\circ}F} \left(\frac{1.731\,\mathrm{W/(m\,K)}}{\mathrm{Btu/(h\,ft\,{}^{\circ}F)}}\right)$$

$$k = 412 \ \mathrm{W/(m\,K)}$$

COMMENTS

Although the single step conversion may be faster, it is important to understand the relationship between units of power, energy, length, etc. in the two systems. This understanding may be more effectively developed by converting each unit separately at first.

Also note that the names of the units can be canceled as a check on the final result.

PROBLEM 1.72

An ice chest (see sketch) is to be constructed from Styrofoam [*k* = 0.033 W/(m K)]. If the wall of the chest is 5-cm-thick, calculate its *R*-value in (m^2 K)/(W cm).

GIVEN

- Ice chest constructed of Styrofoam, k = 0.0333 W/(m K)
- Wall thickness 5 cm

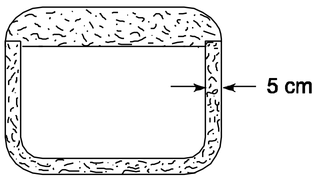
FIND

(a) *R*-value of the ice chest wall

ASSUMPTIONS

(a) One-dimensional, steady conduction

SKETCH



SOLUTION

From Section 1.6 the *R*-value is defined as

$$R\text{-value} = \frac{\text{thickness}}{\text{thermal conductivity}}$$

The thermal conductivity in engineering units is

$$k = 0.033\ \text{W/(m K)}\ \frac{[1\,\text{Btu/(hr ft °F)}]}{[1.731\,\text{W/(m K)}]} = 0.019\ \text{Btu/(hr ft °F)}$$

and the thickness is

$$t = 5\ \text{cm}\ \frac{(1\text{in.})}{(2.54\,\text{cm})} = 1.97\ \text{in.} = 0.05\ \text{m}$$

so

$$R\text{-value} = \frac{0.05m}{(0.033W/(mK))} = 1.51\ (\text{m}^2\ \text{K})/\text{W}$$

From the problem statement, it is clear that we are asked to determine the *R*-value on a 'per-icm' basis. Dividing the above *R*-value by the thickness in inches, we get

$$R\text{-value} = \frac{1.51}{5} = 0.302\ (\text{m}^2\ K)/(\text{W cm})$$

PROBLEM 1.73

> **Estimate the *R*-values for a 5 cm-thick fiberglass board and a 2.5 cm-thick polyurethane foam layer. Then compare their respective conductivity-times-density products if the density for fiberglass is 50 kg/m^3 and the density of polyurethane is 30 kg/m^3. Use the units given in Figure 1.27.**

GIVEN

- 5 cm thick fiberglass board, density = 50 kg/m^3
- 2.5 cm thick polyurethane, density = 30 kg/m^3

FIND

(a) *R*-values for both
(b) Conductivity-times-density products for both

ASSUMPTIONS

(a) One-dimensional, steady conduction

SOLUTION

Ranges of conductivity for both of these materials are given in Fig. 1.28. Using mean values we find:

fiberglass board k = 0.04 W/(m K)
polyurethane foam k = 0.025 W/(m K)

For the 2 in. fiberglass we have

t = 0.05 m
k = 0.04 W/(m K)

From section 1.6 the *R*-value is given by

$$R\text{-value} = \frac{\text{thickness}}{\text{thermal conductivity}} = \frac{0.05\,\text{m}}{0.04\ \text{W/(m K)}} = 1.25\ (\text{m}^2\ \text{K})/\text{W}$$

and

$$\text{conductivity} \times \text{density} = 0.04\ \text{W/(m K)}\ (50\ \text{kg/m}^3) = 2\ (\text{kg W})/(\text{m}^4\text{K})$$

For the 2.5 cm polyurethane we have

t = 0.025 m
k = 0.025 W/(m K)

$$R\text{-value} = \frac{t}{k} = 1\ (\text{m}^2\ \text{K})/\text{W}$$

$$\text{conductivity} \times \text{density} = 0.025\ \text{W/(m K)}\ \ 30\ \text{kg/m}^3 = 0.75\ (\text{kgW})/(\text{m}^4\text{K})$$

Summarizing, we have

	R-value [(m^2 K)/W]	conductivity × density [(kg W)/(m^4 K)]
2-in. fiberglass board	1.27	2
1-in. polyurethane foam	1	0.75

PROBLEM 1.74

A manufacturer in the U.S. wants to sell a refrigeration system to a customer in Germany. The standard measure of refrigeration capacity used in the United States is the 'ton'; a one-ton capacity means that the unit is capable of making about one ton of ice per day or has a heat removal rate of 12,000 Btu/hr. The capacity of the American system is to be guaranteed at 3 tons. What would this guarantee be in SI units?

GIVEN

- A three-ton refrigeration unit to be sold in Germany

FIND

(a) The rating in SI units

SOLUTION

Converting the refrigeration capacity to SI units we have

$$3 \quad 12{,}000\ \text{Btu/hr}\ \left(\frac{\text{Whr}}{3.413\,\text{Btu}}\right) = 10{,}548\ \text{W}$$

Although the watt is a *derived* unit in the SI system, it would be used to express the capacity of the system rather than Newton meters per second.

PROBLEM 1.75

Referring to Problem 1.74, how many kilograms of ice can a 3-ton refrigeration unit produce in a 24-hour period? The heat of fusion of water is 330 kJ/kg.

GIVEN

- A three-ton refrigeration unit
- Heat of fusion of ice is 330 kJ/kg

FIND

(a) Kilograms of ice produced by the unit per 24 hour period
(b) The refrigeration unit capacity is the net value, i.e., it includes heat losses

ASSUMPTIONS

(a) Water is cooled to just above the freezing point before entering the unit

SOLUTION

The mass of ice produced in a given period of time Δt is given by

$$m_{\text{ice}} = \frac{q\Delta T}{h_f}$$

where h_f is the heat of fusion and q is the rate of heat removal by the refrigeration unit. From Problem 1.65 we have q = 10,548 W. Inserting the given values we have

$$m_{\text{ice}} = \frac{(10{,}548\,\text{W})\,(24\,\text{hr})}{3.30\times10^5\ \text{J/kg}\ \ (\text{Ws})/\text{J}\left(\dfrac{\text{hr}}{3600\text{s}}\right)} = 2762\ \text{kg}$$

PROBLEM 1.76

Explain a fundamental characteristic that differentiates conduction from convection and radiation.

SOLUTION

Conduction is the only heat transfer mechanism that dominates in solid materials. Convection and radiation play important roles in fluids or, for radiation, in a vacuum. Under certain conditions, e.g., a transparent solid, radiation could be important in a solid.

PROBLEM 1.77

> **Explain in your own words: (a) what is the mode of heat transfer through a large steel plate that has its surfaces at specified temperatures? (b) what are the modes when the temperature on one surface of the steel plate is not specified, but the surface is exposed to a fluid at a specified temperature.**

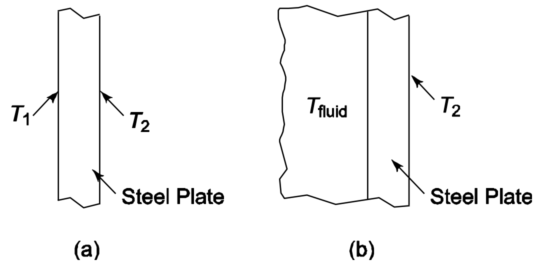
GIVEN

(a) Steel plate with specified surface temperatures
(b) Steel plate with one specified temperature and another surface exposed to a fluid

FIND

(a) Modes of heat transfer

SKETCH



SOLUTION

(a) Since the surface temperatures are specified, the only mode of heat transfer of importance is conduction through the steel plate
(b) In addition to conduction to the steel plate, convection at the surface exposed to the fluid must be considered

PROBLEM 1.78

What are the important modes of heat transfer for a person sitting quietly in a room? What if the person is sitting near a roaring fireplace?

GIVEN

- Person sitting quietly in a room
- Person sitting in a room with a fireplace

FIND

(a) Modes of heat transfer for each situation

ASSUMPTIONS

- The person is clothed

SOLUTION

(a) Since the person is clothed, we would need to consider conduction through the clothing, and convection and radiation from the exposed surface of the clothing.
(b) In addition to the modes identified in (a), we would need to consider that surfaces of the person oriented towards the fire would be absorbing radiation from the flames.

PROBLEM 1.79

Consider the cooling of (a) a personal computer with a separate CPU, and (b) a laptop computer. The reliable functioning of these machines depends upon their effective cooling. Identify and briefly explain all modes of heat transfer that are involved in the cooling process.

GIVEN

- A personal computer with a separate CPU (the monitor, keyboard, and mouse are separate and not considered).
- A laptop computer.

FIND

Identify and describe modes of heat transfer involved in their cooling.

ASSUMPTIONS

- The computers are turned on and in normal operation.

SOLUTION

(a) The cooling would first involve conduction from microchips to heat sinks (finned structures) mounted on them as well as conduction to the surface of printed-circuit boards, and convection from heat sinks and printed-circuit boards to air flowing over them (most PCs have a fan that blows air through the computer compartment). From the printed circuit boards, which are mounted to the casing of the computer, heat would also be conducted to the casing. Furthermore, there would be some radiation from heat sinks and printed-circuit boards to the casing, and then from outer computer casing to the surroundings; from the outer casing there will also be convection (natural convection) heat loss to the room's atmosphere.

(b) In a laptop computer, the heat produced in the microchips and other electrical circuitry would be conducted to the heat sinks mounted on them as well as through the circuit boards to the casing of the computer. From the outer casing the heat would then be dissipated by natural convection and radiation to the room's atmosphere. Some laptop computers come mounted with a small fan, in which case heat removal by internal forced convection would also be part of the total thermal management (cooling strategy) of the device. Furthermore, some makers install heat pipes in the casing for heat removal. A heat pipe is a "wicking" device that involves evaporation of a thin liquid film inside the device and the condensation of vapor so generated (the student can learn more about a heat pipe in Chapter 10)

PROBLEM 1.80

Describe and compare the modes of heat loss through the single-pane and double-pane window assemblies shown in the sketch below.

GIVEN

- A single-pane and a double-pane window assembly

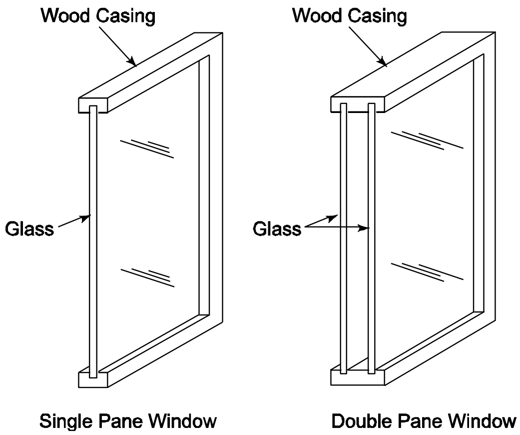
FIND

(a) The modes of heat transfer for each
(b) Compare the modes of heat transfer for each

ASSUMPTIONS

- The window assembly wood casing is a good insulator

SKETCH



SOLUTION

The thermal network for both cases is shown above and summarizes the situation. For the single-pane window, we have convection on both exterior surfaces of the glass, radiation from both exterior surfaces of the glass, and conduction through the glass. For the double-pane window, we would have these modes in addition to radiation and convection exchange between the facing surfaces of the glass panes. Since the overall thermal network for the double-pane assembly replaces the pane-conduction with two-pane conductions plus the convection/radiation between the two panes, the overall thermal resistance of the double-pane assembly should be larger. Therefore, we would expect lower heat loss through the double-pane window.

PROBLEM 1.81

> **A person wearing a heavy parka is standing in a cold wind. Describe the modes of heat transfer determining heat loss from the person's body.**

GIVEN

- Person standing in a cold wind, wearing a heavy parka

FIND

(a) The modes of heat transfer

SKETCH

SOLUTION

The thermal circuit for the situation is shown above. Assume that the person is wearing one other garment, i.e., a shirt, under the parka. The modes of heat transfer include conduction through the shirt and the parka and convection from the outer surface of the parka to the cold wind. We expect that the largest thermal resistance will be the parka insulation. We have neglected radiation from the parka outer surface because its influence on the overall heat transfer will be small compared to the other terms.

PROBLEM 1.82

Discuss the modes of heat transfer that determine the equilibrium temperature of the space shuttle Endeavor when it is in orbit. What happens when it reenters the earth's atmosphere?

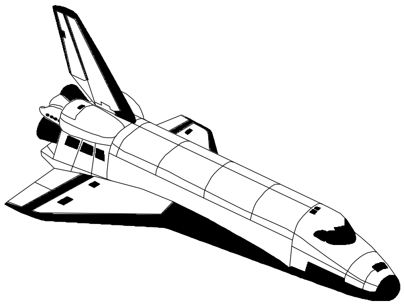
GIVEN

- Space shuttle Endeavor in orbit
- Space shuttle Endeavor during reentry

FIND

(a) Modes of heat transfer

SKETCH



SOLUTION

Heat generated internally will have to be rejected to the skin of the shuttle or to some type of radiator heat exchanger exposed to space. The internal loads that are not rejected actively, i.e., by a heat exchanger, will be transferred to the internal surface of the shuttle by radiation and convection, transferred by conduction through the skin, then radiated to space. These two paths of heat transfer must be sufficient to ensure that the interior is maintained at a comfortable working temperature.

During reentry, the exterior surface of the shuttle will be exposed to a heat flux that results from frictional heating by the atmosphere. In this case, it is likely that the net heat flow will be into the space shuttle. The thermal design must be such that during reentry the interior temperature does not exceed some safe value