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HW4

## Exercise 2.1

3.

(g)
$$-0\begin{vmatrix}0&0&1\\6&2&0\\1&-2&3\end{vmatrix}+\begin{vmatrix}2&0&1\\1&2&0\\1&-2&3\end{vmatrix}-0\begin{vmatrix}2&0&1\\1&6&0\\1&1&3\end{vmatrix}+0\begin{vmatrix}2&0&0\\1&6&2\\1&1&-2\end{vmatrix}$$
$$=2\begin{vmatrix}2&0\\-2&3\end{vmatrix}-0\begin{vmatrix}1&0\\1&3\end{vmatrix}+1\begin{vmatrix}1&2\\1&-2\end{vmatrix}$$
$$=2\times 6+(-4)=8$$

(h)
$$-3\begin{vmatrix}1&2&1\\2&-2&1\\2&3&1\end{vmatrix}+0\begin{vmatrix}2&2&1\\-1&-2&1\\-3&3&1\end{vmatrix}-1\begin{vmatrix}2&1&1\\-1&2&1\\-3&2&1\end{vmatrix}+1\begin{vmatrix}2&1&2\\-1&2&-2\\-3&2&3\end{vmatrix}$$
$$=-3\left(\begin{vmatrix}-2&1\\3&1\end{vmatrix}-2\begin{vmatrix}2&1\\2&1\end{vmatrix}+\begin{vmatrix}2&-2\\2&3\end{vmatrix}\right)$$
$$-\left(2\begin{vmatrix}2&1\\2&1\end{vmatrix}-\begin{vmatrix}-1&1\\-3&1\end{vmatrix}+\begin{vmatrix}-1&2\\-3&2\end{vmatrix}\right)$$
$$+\left(2\begin{vmatrix}2&-2\\2&3\end{vmatrix}-\begin{vmatrix}-1&-2\\-3&3\end{vmatrix}+2\begin{vmatrix}-1&2\\-3&2\end{vmatrix}\right)$$
$$=-3\left[(-2-3)-2\times 0+(6+4)\right]$$
$$-\left[2\times 0-2+4\right]+\left[2\times 10+9+2\times 4\right]$$
$$=20$$

4.

(c) 0

(d) 0

11.

(a) $\det(A+B)\neq\det(A)+\det(B)$

(b) $\det(AB)=\det(A)\det(B)$

(c) $\det(AB)=\det(BA)$

13.
$$\det(A)=\begin{vmatrix}a_{11}&a_{12}&&&\\a_{21}&a_{22}&a_{23}&&\\&a_{32}&\ddots&\ddots&\\&&\ddots&\ddots&a_{n-1\,n}\\&&&a_{n\,n-1}&a_{nn}\end{vmatrix}$$
$$=a_{11}\det(M_{11})-a_{12}\begin{vmatrix}a_{21}&a_{23}&&&\\&a_{33}&a_{34}&&\\&a_{43}&\ddots&\ddots&a_{n-1\,n}\\&&&\ddots&\\&&&a_{n\,n-1}&a_{nn}\end{vmatrix}$$
$$\underset{a_{12}=a_{21}}{=}a_{11}\det(M_{11})-a_{11}^{2}\det(B)$$

## Exercise 2.2

1.(b)
$$A=\begin{pmatrix}1&1&1&3\\0&3&1&1\\0&0&2&2\\-1&-1&-1&2\end{pmatrix}\xrightarrow{r_4+r_1}\begin{pmatrix}1&1&1&3\\0&3&1&1\\0&0&2&2\\0&0&0&5\end{pmatrix}=A'$$
$$\det(A')=1\times3\times2\times5=30$$
$$\det(A)=30$$

2.(b) $A \xrightarrow{r_2 \leftrightarrow r_3} \begin{pmatrix} 0 & 1 & 2 & 3 \\ -2 & -2 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & -2 & -3 \end{pmatrix}$

$\xrightarrow{r_3 \leftrightarrow r_4} \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & -2 & -3 \\ -2 & -2 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix} = B$

$A \xrightarrow{r_3 + r_2} \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & 4 & 4 \\ 1 & 2 & -2 & -3 \end{pmatrix}$

$\xrightarrow{r_4 + r_2} \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ -2 & -2 & 3 & 3 \\ 2 & 3 & -1 & -2 \end{pmatrix} = C$

$\det(B) + \det(C) = (-1)^2 \det(A) + \det(A)$
$= 20$

3.(a) $\begin{vmatrix} 3 & 1 \\ 6 & 2 \end{vmatrix} = 0$ singular

(f) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 3 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 7 & 3 \end{vmatrix}$

$= -7 \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 0 & 1 & 1 \end{vmatrix} + 3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 0 & 1 & 2 \end{vmatrix}$

$= -7(-3) + 3(-7) = 0$

singular

(f) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 3 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 7 & 3 \end{vmatrix} = -7 \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 0 & 1 & 1 \end{vmatrix}$

$+3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 0 & 1 & 2 \end{vmatrix}$

$= -7\left(-\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}\right)$

$+3\left(-\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}\right)$

$= -7(-1-2) + 3(-1 + 2 \times (-1))$

$= -7 \times (-3) - 7 \times 3$

$= 0$

singular

7.

(c) $\det(2AB) = 2^3 \det(A) \det(B)$
$= 8 \times 4 \times 5 = 160$

(d) $\det(A^{-1}B) = \det(A^{-1}) \det(B)$
$= \frac{1}{\det(A)} \det(B) = \frac{1}{4} \times 5 = \frac{5}{4}$

9.

(d) $\det(AE_1) = \det(A) \det(E_1) = 6(-1) = -6$

(f) $\det(E_1 E_2 E_3) = \det(E_1) \det(E_2) \det(E_3) = -3$

11. $A^2+I=0 \Rightarrow A^2=-I \Rightarrow \det(A^2)=\det(-I)=(-1)^n\det(I)=(-1)^n$

$\Rightarrow (\det(A))^2=(-1)^n$

if $n=2k+1,\ k\in R,\ (\det(A))^2=-1$, not exist

if $n=2k,\ k\in R,\ (\det(A))^2=1,\ \det(A)=\pm 1$, exist.

12.(a) $V \xrightarrow{r_2-r_1} \begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & x_2-x_1 & x_2^2-x_1^2 \\ 1 & x_3 & x_3^2 \end{vmatrix} \xrightarrow{r_3-r_1} \begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & x_2-x_1 & x_2^2-x_1^2 \\ 0 & x_3-x_1 & x_3^2-x_1^2 \end{vmatrix}$

$\det V = \begin{vmatrix} x_2-x_1 & x_2^2-x_1^2 \\ x_3-x_1 & x_3^2-x_1^2 \end{vmatrix} = (x_2-x_1)(x_3+x_1)(x_3-1) - (x_2+x_1)(x_2-x_1)(x_3-x_1)$

$= (x_2-x_1)(x_3-x_1)(x_3-x_2)$

(b) $x_2\neq x_1,\ x_3\neq x_1,\ x_3\neq x_2$

14. $\det(AB)=\det(A)\det(B)$

$AB$ is nonsingular $\Rightarrow \det(AB)\neq 0 \Rightarrow \det(A)\neq 0, \det(B)\neq 0$

$\Rightarrow A, B$ are both nonsingular

18.(a) $\det(E) = \begin{vmatrix} I_k & O_{k\times(n-k)} \\ O_{(n-k)\times k} & B_{(n-k)\times(n-k)} \end{vmatrix}$

$= 1\cdot \begin{vmatrix} I_{k-1} & O_{(k-1)\cdot(n-k)} \\ O_{(n-k)\times(k-1)} & B_{(n-k)\times(n-k)} \end{vmatrix}$

$\vdots$

$= 1^k \left| B_{(n-k)\times(n-k)} \right|$

$= \det(B)$

(b) $\det(F) = \begin{vmatrix} A_{k\times k} & O_{k\times(n-k)} \\ O_{(n-k)\times k} & I_{n-k} \end{vmatrix}$

$= 1 \begin{vmatrix} A_{k\times k} & O_{k\times(n-k-1)} \\ O_{(n-k-1)\times k} & I_{n-k-1} \end{vmatrix}$

$\vdots$

$= 1^{n-k} \left| A_{k\times k} \right|$

$= \det(A)$

(c) $\det(C) = \det\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$

$= \det\left[\begin{pmatrix} A & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & B \end{pmatrix}\right]$

$= \det(FE)$

$= \det(AB)$

$= \det(A)\det(B)$