Problem 4.1

[Difficulty: 2]

4.1 An ice-cube tray containing 250 mL of freshwater at 15°C is placed in a freezer at −5°C. Determine the change in internal energy (kJ) and entropy (kJ/K) of the water when it has frozen.

Given: An ice-cube tray with water at 15°C is frozen at –5°C.

Find: Change in internal energy and entropy

Solution: Apply the Tds and internal energy equations

Governing equations: $Tds = du + pdv \qquad du = c_v dT$

Assumption: Neglect volume change
Liquid properties similar to water

The given or available data is:

$$T_1 = (15 + 273)\text{ K} = 288\text{ K} \qquad T_2 = (-5 + 273)\text{ K} = 268\text{ K}$$

$$c_v = 1\frac{\text{kcal}}{\text{kg}\cdot\text{K}} \qquad \rho = 999\frac{\text{kg}}{\text{m}^3}$$

Then with the assumption: $Tds = du + pdv = du = c_v dT$

or $$ds = c_v \frac{dT}{T}$$

Integrating $$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) \quad \text{or} \quad \Delta S = m(s_2 - s_1) = \rho V c_v \ln\left(\frac{T_2}{T_1}\right)$$

$$\Delta S = 999\frac{\text{kg}}{\text{m}^3} \times 250\text{mL} \times \frac{10^{-6}\text{m}^3}{\text{mL}} \times 1\frac{\text{kcal}}{\text{kg}\cdot\text{K}} \times \ln\left(\frac{268}{288}\right) \times 4190\frac{\text{J}}{\text{kcal}}$$

$$\Delta S = -0.0753\frac{\text{kJ}}{\text{K}}$$

Also $$u_2 - u_1 = c_v(T_2 - T_1) \quad \text{or} \quad \Delta U = mc_v(T_2 - T_1) = \rho V c_v \Delta T$$

$$\Delta U = 999\frac{\text{kg}}{\text{m}^3} \times 250\text{mL} \times \frac{10^{-6}\text{m}^3}{\text{mL}} \times 1\frac{\text{kcal}}{\text{kg}\cdot\text{K}} \times (-268 - 288)\text{K} \times 4190\frac{\text{J}}{\text{kcal}}$$

$$\Delta U = -20.9\,\text{kJ}$$

Problem 4.2

(Difficulty: 2)

4.2 A hot air balloon with an initial volume of $2600\ m^3$ rises from sea level to $1000\ m$ elevation. The temperature of the air inside the balloon is 100 ℃ at the start and drops to 90 ℃ at $1000\ m$. What are the net amounts of heat and work transferred between the balloon and the atmosphere?

Find: The heat and work transfers

Assumption: The air in the balloon is an ideal gas

Solution: Apply an energy balance to the air and use the ideal gas relations.

From the first law of thermodynamics for the balloon we have:

$$\delta Q - \delta W = dE$$

Where

$$\delta E = md\left(u + \frac{V^2}{2} + z\right)$$

We need to find the mass of the air in the balloon. The density of the air at sea level at 100 ℃ is:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{101.3\ kPa}{286.9\ \frac{kJ}{kg\ K} \times 373.2\ K} = 0.946\ \frac{kg}{m^3}$$

The mass of the hot air in the balloon is then:

$$m = \rho V = 0.946\ \frac{kg}{m^3} \times 2600\ m^3 = 2460\ kg$$

The change in internal energy is given by

$$mdu = mc_v dT = 2460\ kg \times 717\ \frac{J}{kg\ K} \times (90\ C - 100\ C) = -17{,}650\ kJ$$

The change in kinetic energy is assumed to be zero since the velocities are low. The change in potential energy is

$$m\ g\ dz = 2460\ kg \times 9.8 \frac{m}{s^2} \times (1000\ m - 0\ m) \times \frac{kJ}{1000\ N\ m} = 24{,}120\ kJ$$

The work done by the balloon is given by

$$\delta W = \int p\ dV$$

The pressure varies as the balloon rises, but we will assume that the pressure varies linearly and use the average pressure for the process. The work is then given by

$$\delta W = p_{ave}\,\Delta V$$

The volume at 1000 m depends on the pressure at that elevation. From Appendix A, the pressure at 1000 m is 0.887x101.3 kPa = 89.8 kPa. Using the ideal gas law, the density at 1000 m is

$$\rho_2 = \frac{p_2}{RT_2} = \frac{89.8\ kPa}{286.9\ \frac{kJ}{kg\ K} \times 363.2\ K} = 0.823\ \frac{kg}{m^3}$$

The volume at 1000 m is then

$$V_2 = \frac{m}{\rho_2} = \frac{2460\ kg}{0.823\ \frac{kg}{m^3}} = 2854\ m^3$$

The work is then

$$\delta W = p_{ave}\,\Delta V = 95.55\ kPa \times (2854 - 2460)m^3 = 24{,}301\ kJ$$

The balloon does work on the atmosphere as it increases its volume

The heat transfer is determined using the energy balance

$$\delta Q = \delta W + dE = 24{,}301\ kJ - 17{,}650\ kJ + 24{,}120\ kJ = 30{,}770\ kJ$$

Heat needed to be transferred to the balloon as it rose.

Problem 4.3 [Difficulty: 2]

4.3 A fully loaded Boeing 777-200 jet transport aircraft weighs 325,000 kg. The pilot brings the 2 engines to full takeoff thrust of 450 kN each before releasing the brakes. Neglecting aerodynamic and rolling resistance, estimate the minimum runway length and time needed to reach a takeoff speed of 225 kph. Assume engine thrust remains constant during ground roll.

Given: Data on Boeing 777-200 jet

Find: Minimum runway length for takeoff

Solution:

Basic equation $\Sigma F_x = M \cdot \frac{dV}{dt} = M \cdot V \cdot \frac{dV}{dx} = F_t = \text{constant}$ Note that the "weight" is already in mass units!

Separating variables $M \cdot V \cdot dV = F_t \cdot dx$

Integrating $x = \frac{M \cdot V^2}{2 \cdot F_t}$

$$x = \frac{1}{2} \times 325 \times 10^3 \text{kg} \times \left(225 \frac{\text{km}}{\text{hr}} \times \frac{1 \cdot \text{km}}{1000 \cdot \text{m}} \times \frac{1 \cdot \text{hr}}{3600 \cdot \text{s}}\right)^2 \times \frac{1}{2 \times 425 \times 10^3} \cdot \frac{1}{\text{N}} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \qquad x = 747\,\text{m}$$

For time calculation $M \cdot \frac{dV}{dt} = F_t$ $\qquad dV = \frac{F_t}{M} \cdot dt$

Integrating $t = \frac{M \cdot V}{F_t}$

$$t = 325 \times 10^3 \text{kg} \times 225 \frac{\text{km}}{\text{hr}} \times \frac{1 \cdot \text{km}}{1000 \cdot \text{m}} \times \frac{1 \cdot \text{hr}}{3600 \cdot \text{s}} \times \frac{1}{2 \times 425 \times 10^3} \cdot \frac{1}{\text{N}} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \qquad t = 23.9\,\text{s}$$

Aerodynamic and rolling resistances would significantly increase both these results

Problem 4.4

(Difficulty: 3)

4.4 On the Milford Trek in New Zealand, there is a pass with a cliff known as the "12 second drop" for the time it takes a rock to hit the ground below from the pass. Estimate the height assuming that you throw a 5 cm diameter rock that weighs 200 g rock over the edge for (a) no air resistance and (b) assuming that the drag force is $KV\ \left(\frac{N\cdot s^2}{m^2}\right)$, where V is instantaneous velocity and K = 0.01. Explain why there is a difference in the calculated height.

Find: The height of the pass

Solution:

From the Newton's second law we have:

$$F = m\frac{dV}{dt}$$

(a) If there is no air resistance, we only have:

$$m\frac{dV}{dt} = mg$$

$$\frac{dV}{dt} = g$$

Integrating

$$V = gt$$

Or

$$V = \frac{ds}{dt} = gt$$

Integrating again

$$\int_0^s ds = \int_0^{12} gtdt$$

$$s(t) = [\frac{1}{2}gt^2]_0^{12} = 706\ m$$

(b) From Newton's second law, we now have the additional force of the aerodynamic drag:

$$m\frac{dV}{dt} = mg - KV$$

$$\frac{dV}{dt} = g - \frac{KV}{m} = -\frac{K}{m}\left(V - \frac{m}{K}g\right)$$

Or

$$\frac{d}{dt}\left(V - \frac{m}{K}g\right) = -\frac{K}{m}\left(V - \frac{m}{K}g\right)$$

Separating variables

$$\frac{d\left(V - \frac{m}{K}g\right)}{\left(V - \frac{m}{K}g\right)} = -\,\frac{K}{m}dt$$

Integrating both sides from the start, where *V = 0* at *t = 0*, we have:

$$\int_0^V \frac{d\left(V - \frac{m}{K}g\right)}{\left(V - \frac{m}{K}g\right)} = \int_0^t -\,\frac{K}{m}dt$$

Or

$$\ln\left(V - \frac{m}{K}g\right) - \ln\left(-\frac{m}{K}g\right) = -\,\frac{K}{m}t$$

Or, using the relation for logarithms

$$\ln\left(\frac{V - \frac{m}{K}g}{-\frac{m}{K}g}\right) = ln\left(1 - \frac{KV}{mg}\right) = -\,\frac{K}{m}t$$

Which can be written as

$$1 - \frac{KV}{mg} = e^{-\frac{K}{m}t}$$

Solving for V

$$V = \frac{mg}{K}\left(1 - e^{-\frac{K}{m}t}\right)$$

The distance is then found from

$$V = \frac{ds}{dt} = \frac{mg}{K}\left(1 - e^{-\frac{K}{m}t}\right)$$

Integrating both sides:

$$\int_0^s ds = \frac{mg}{K}\int_0^t dt - \frac{mg}{K}\int_0^t e^{-\frac{K}{m}t}\,dt$$

Then

$$s = \frac{mg}{K}t - \frac{m^2 g}{K^2}\left(1 - e^{-\frac{K}{m}t}\right)$$

With values of $t = 12\ s, m = 0.2\ kg$ and $K = 0.01$, we have

$$s = \frac{0.2 \times 9.81}{0.01} \times 12 - \frac{0.2^2 \times 9.8}{0.01^2}\left(1 - e^{-\frac{0.05}{0.2} \times 12}\right) = 483\ m$$

The distance is less because the drag slows the rock and it takes more time to go the same distance.

Problem 4.5 [Difficulty: 2]

4.5 A high school experiment consists of a block of mass 2 kg sliding across a surface (coefficient of friction $\mu = 0.6$). If it is given an initial velocity of 5 m/s, how far will it slide, and how long will it take to come to rest? The surface is now roughened a little, so with the same initial speed it travels a distance of 2 m. What is the new coefficient of friction, and how long does it now slide?

Given: Block sliding to a stop

Find: Distance and time traveled; new coeeficient of friction

Solution:

Governing equations: $\Sigma F_x = M \cdot a_x$ $\quad F_f = \mu \cdot W$

Assumptions: Dry friction; neglect air resistance

Given data $\quad \mu = 0.6 \quad V_0 = 5 \cdot \frac{m}{s} \quad M = 2 \cdot kg \quad L = 2 \cdot m$

$$\Sigma F_x = -F_f = -\mu \cdot W = M \cdot a_x = \frac{W}{g} \cdot a_x = \frac{W}{g} \cdot \frac{d^2}{dt^2} x \quad \text{or} \quad \frac{d^2}{dt^2} x = -\mu \cdot g$$

Integrating, and using I.C. $V = V_0$ at $t = 0$

Hence
$$\frac{dx}{dt} = -\mu \cdot g \cdot t + c_1 = -\mu \cdot g \cdot t + V_0 \qquad (1)$$

Integrating again
$$x = -\frac{1}{2} \cdot g \cdot t^2 + V_0 \cdot t + c_2 = -\frac{1}{2} \cdot g \cdot t^2 + V_0 \cdot t \quad \text{since } x = 0 \text{ at } t = 0 \qquad (2)$$

We have the final state, at which $\quad x_f = L$ and $\frac{dx}{dt} = 0$ at $t = t_f$

From Eq. 1 $\quad \frac{dx}{dt} = 0 = -\mu \cdot g \cdot t_f + V_0 \quad$ so $\quad t_f = \frac{V_0}{\mu \cdot g} \quad$ Using given data $\quad t_f = 0.850\ s$

Substituting into Eq. 2
$$x = x_f = L = -\frac{1}{2} \cdot g \cdot t^2 + V_0 \cdot t = -\frac{1}{2} \cdot g \cdot t_f^2 + V_0 \cdot t_f = -\frac{1}{2} \cdot g \cdot \left(\frac{V_0}{\mu \cdot g}\right)^2 + V_0 \cdot \frac{V_0}{\mu \cdot g} = \frac{V_0^2}{2 \cdot \mu \cdot g}$$

Solving $\quad x = \frac{V_0^2}{2 \cdot \mu \cdot g} \quad (3) \quad$ Using give data $\quad x = 2.12\ m$

For rough surface, using Eq. 3 with $x = L$ $\quad \mu = \frac{V_0^2}{2 \cdot g \cdot L} \quad \mu = 0.637 \quad t_f = \frac{V_0}{\mu \cdot g} \quad t_f = 0.800\ s$

Problem 4.6

(Difficulty: 3)

4.6 For a small particle of Styrofoam $\left(density = 19.2\ \frac{kg}{m^3}, spherical\ with\ diameter\ d = 1.0\ \ mm\right)$ falling in standard air at speed V, the drag is given by $F_D = 3\pi\mu Vd$, where μ is the air viscosity. Find the maximum speed of the particle starting from rest and the time it takes to reach 95 percent of this speed. Plot the speed $\frac{m}{s}$ as a function of time.

Find: The maximum velocity: V_{max}. The time t to reach $0.95V_{max}$.

Solution:

When the partical reaches the maximum speed, the force is balanced. We have the following force balance equation:

$$F_B = F_D$$

The body force is the weight of the Styrofoam

$$F_B = W = g\rho_{st}Vol$$

The volume of the Styrofoam is calculated as:

$$Vol = \frac{1}{6}\pi d^3 = \frac{\pi}{6} \times (0.001\ m)^3 = 5.24 \times 10^{-10}\ m^3$$

The weight is then

$$W = 9.81\frac{m}{s^2} \times 19.2\ \frac{kg}{m^3} \times 5.24 \times 10^{-10}\ m^3 = 9.86 \times 10^{-8} N$$

The air viscosity is:

$$\mu = 1.827 \times 10^{-5}\ \frac{kg}{m \cdot s}$$

The drag force is given by:

$$F_D = 3\pi\mu Vd$$

The force balance is then

$$W = 3\pi\mu V_{max} d$$

Or the maximum velocity is

$$V_{max} = \frac{W}{3\pi\mu d} = \frac{9.86 \times 10^{-8} N}{3 \times \pi \times 1.827 \times 10^{-5}\ \frac{kg}{m \cdot s} \times 0.0015\ m} = 0.573\ \frac{m}{s}$$

From Newton’s second law, we have:

$$F_B - F_D = m\,a = m\frac{dV}{dt}$$

Or

$$\frac{dV}{dt} = \frac{F_B - F_D}{m} = \frac{W - 3\pi\mu V d}{m}$$

$$\frac{m}{W - 3\pi\mu V d}\,dV = dt$$

Integrating this equation, we have:

$$\int_0^V \frac{m}{W - 3\pi\mu V d}\,dV = -\left(\frac{m}{3\pi\mu d}\right)\int_0^V \frac{d(W - 3\pi\mu V d)}{W - 3\pi\mu V d} = \int_0^t dt$$

$$-\frac{m}{3\pi\mu d}\ln(W - 3\pi\mu V d)_0^V = -\frac{m}{3\pi\mu d}\ln\frac{(W - 3\pi\mu V d)}{W} = t$$

The time as a function of velocity is

$$t = \frac{m}{3\pi\mu d}\ln\left(\frac{W}{W - 3\pi\mu V d}\right)$$

The velocity at 95 % of the maximum is:

$$V = 0.95V_{max} = 0.95 \times 0.573\ \frac{m}{s} = 0.544\ \frac{m}{s}$$

The mass of the Styrofoam sphere is

$$m = \rho_{st} Vol = 19.2\ \frac{kg}{m^3} \times 5.24 \times 10^{-10}\ m^3 = 1.006\ \times 10^{-8}\ kg$$

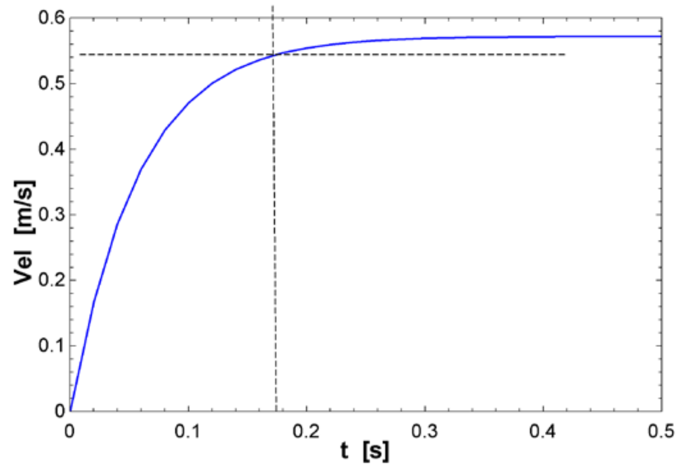
The time to reach this velocity is calculated as follows.

$$t = \frac{1.006\ \times 10^{-8}\ kg}{3 \times \pi \times 1.827 \times 10^{-5}\ \frac{kg}{m \cdot s} \times 0.001\ m} \times$$

$$\ln\left(\frac{9.83 \times 10^{-8} N}{9.83 \times 10^{-8} N - 3 \times \pi \times 1.827 \times 10^{-5}\ \frac{kg}{m \cdot s} \times 0.544\ \frac{m}{s} \times 0.001 m}\right)$$

$$t = 0.17\ s$$

The plot for the velocity as function of time is shown:



0.6
0.5
0.4
0.3
0.2
0.1
0
Vel [m/s]
0
0.1
0.2
0.3
0.4
0.5
t [s]

Problem 4.7 [Difficulty: 2]

4.7 Air at 20°C and an absolute pressure of 1 atm is compressed adiabatically in a piston-cylinder device, without friction, to an absolute pressure of 4 atm in a piston-cylinder device. Find the work done (MJ).

Given: Data on air compression process

Find: Work done

Solution:

Basic equation $\delta w = p \cdot dv$

Assumptions: 1) Adiabatic 2) Frictionless process $\delta w = pdv$

Given data $p_1 = 1 \cdot atm$ $p_2 = 4 \cdot atm$ $T_1 = 20\,°C$ $T_1 = 293\,K$

From Table A.6 $R = 286.9 \cdot \frac{J}{kg \cdot K}$ and $k = 1.4$

Before integrating we need to relate p and v. An adiabatic frictionless (reversible) process is isentropic, which for an ideal gas gives

$$p \cdot v^k = C \qquad \text{where} \qquad k = \frac{c_p}{c_v}$$

$$\delta w = p \cdot dv = C \cdot v^{-k} \cdot dv$$

Integrating

$$w = \frac{C}{k-1} \cdot \left(v_2^{1-k} - v_2^{1-k}\right) = \frac{1}{(k-1)} \cdot \left(p_2 \cdot v_2^k v_2^{1-k} - p_1 \cdot v_1^k \cdot v_2^{1-k}\right)$$

$$w = \frac{R}{(k-1)} \cdot \left(T_2 - T_1\right) = \frac{R \cdot T_1}{(k-1)} \cdot \left(\frac{T_2}{T_1} - 1\right) \qquad (1)$$

But $p \cdot v^k = C$ means $p_1 \cdot v_1^k = p_2 \cdot v_2^k$ or $p_1 \cdot \left(\frac{R \cdot T_1}{p_1}\right)^k = p_2 \cdot \left(\frac{R \cdot T_2}{p_2}\right)^k$

Rearranging

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$$

Combining with Eq. 1

$$w = \frac{R \cdot T_1}{k-1} \cdot \left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right]$$

$$w = \frac{1}{0.4} \times 286.9 \cdot \frac{J}{kg \cdot K} \times (20 + 273)K \times \left[\left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} - 1\right] \qquad w = 102\,\frac{kJ}{kg}$$

Problem 4.8

[Difficulty: 2]

4.8 A block of copper of mass 5 kg is heated to 90°C and then plunged into an insulated container containing 4 L of water at 10°C. Find the final temperature of the system. For copper, the specific heat is 385 J/kg · K, and for water the specific heat is 4186 J/kg · K.

Given: Data on heating and cooling a copper block

Find: Final system temperature

Solution:

Basic equation $Q - W = \Delta E$

Assumptions: 1) Stationary system $\Delta E = \Delta U$ 2) No work $W = 0$ 3) Adiabatic $Q = 0$

Then for the system (water and copper)

$$\Delta U = 0 \qquad \text{or} \qquad M_{copper}\cdot c_{copper}\cdot T_{copper} + M_w\cdot c_w\cdot T_w = \left(M_{copper}\cdot c_{copper} + M_w\cdot c_w\right)\cdot T_f \qquad (1)$$

where T_f is the final temperature of the water ($_w$) and copper ($_{copper}$)

The given data is $M_{copper} = 5\cdot kg$ $\quad c_{copper} = 385\cdot\frac{J}{kg\cdot K}$ $\quad c_w = 4186\cdot\frac{J}{kg\cdot K}$ $\quad V = 4\cdot L$

$T_{copper} = (90 + 273)\cdot K$ $\quad T_w = (10 + 273)\cdot K$

Also, for the water $\rho = 999\cdot\frac{kg}{m^3}$ so $M_w = \rho\cdot V$ $\quad M_w = 4.00\,kg$

Solving Eq. 1 for T_f

$$T_f = \frac{M_{copper}\cdot c_{copper}\cdot T_{copper} + M_w\cdot c_w\cdot T_w}{\left(M_{copper}\cdot c_{copper} + M_w\cdot c_w\right)}$$

$T_f = 291\,K$ $\qquad T_f = 18.1\cdot°C$

Problem 4.9

[Difficulty: 2]

4.9 The average rate of heat loss from a person to the surroundings when not actively working is about 85 W. Suppose that in an auditorium with volume of approximately 3.5×10^5 m^3, containing 6000 people, the ventilation system fails. How much does the internal energy of the air in the auditorium increase during the first 15 min after the ventilation system fails? Considering the auditorium and people as a system, and assuming no heat transfer to the surroundings, how much does the internal energy of the system change? How do you account for the fact that the temperature of the air increases? Estimate the rate of temperature rise under these conditions.

Given: Data on heat loss from persons, and people-filled auditorium

Find: Internal energy change of air and of system; air temperature rise

Solution:

Basic equation $\quad Q - W = \Delta E$

Assumptions: 1) Stationary system $\Delta E = \Delta U$ 2) No work $W = 0$

Then for the air $\quad \Delta U = Q = 85 \cdot \frac{W}{person} \times 6000 \cdot people \times 15 \cdot min \times \frac{60 \cdot s}{min} \qquad \Delta U = 459 \cdot MJ$

For the air and people $\quad \Delta U = Q_{surroundings} = 0$

The increase in air energy is equal and opposite to the loss in people energy

For the air $\quad \Delta U = Q \quad$ but for air (an ideal gas) $\quad \Delta U = M \cdot c_v \cdot \Delta T \quad$ with $\quad M = \rho \cdot V = \frac{p \cdot V}{R_{air} \cdot T}$

Hence $\quad \Delta T = \frac{Q}{M \cdot c_v} = \frac{R_{air} \cdot Q \cdot T}{c_v \cdot p \cdot V}$

From Table A.6 $\quad R_{air} = 286.9 \cdot \frac{J}{kg \cdot K} \quad$ and $\quad c_v = 717.4 \cdot \frac{J}{kg \cdot K}$

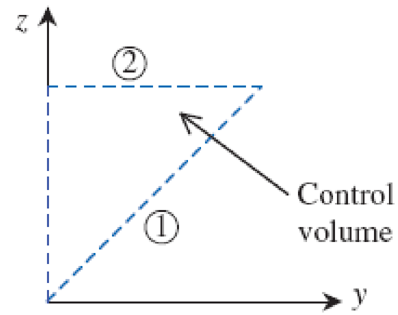
$$\Delta T = \frac{286.9}{717.4} \times 459 \times 10^6 \cdot J \times (20 + 273)K \times \frac{1}{101 \times 10^3} \cdot \frac{m^2}{N} \times \frac{1}{3.5 \times 10^5} \cdot \frac{1}{m^3} \qquad \Delta T = 1.521\ K$$

This is the temperature change in 15 min. The rate of change is then $\quad \frac{\Delta T}{15 \cdot min} = 6.09 \cdot \frac{K}{hr}$

Problem 4.10

[Difficulty: 3]

4.10 The velocity field in the region shown is given by $\vec{V} = (a\hat{j} + by\hat{k})$ where $a = 10$ m/s and $b = 5\text{ s}^{-1}$. For the 1 m $\times$ 1 m triangular control volume (depth $w = 1$ m perpendicular to the diagram), an element of area ① may be represented by $d\vec{A}_1 = wdz\hat{j} - wdy\hat{k}$ and an element of area ② by $d\vec{A}_2 = -wdy\hat{k}$.
(a) Find an expression for $\vec{V} \cdot d\vec{A}_1$.
(b) Evaluate $\int_{A_1} \vec{V} \cdot d\vec{A}_1$.
(c) Find an expression for $\vec{V} \cdot d\vec{A}_2$.
(d) Find an expression for $\vec{V}(\vec{V} \cdot d\vec{A}_2)$.
(e) Evaluate $\int_{A_2} \vec{V}(\vec{V} \cdot d\vec{A}_2)$.



Given: Data on velocity field and control volume geometry

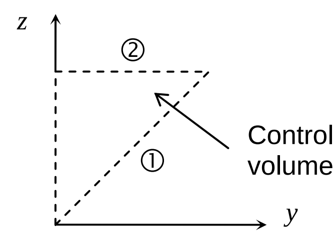
Find: Several surface integrals

Solution:

$$d\vec{A}_1 = wdz\hat{j} - wdy\hat{k} \qquad d\vec{A}_1 = dz\hat{j} - dy\hat{k}$$

$$d\vec{A}_2 = -wdy\hat{k} \qquad d\vec{A}_2 = -dy\hat{k}$$

$$\vec{V} = \left(a\hat{j} + by\hat{k}\right) \qquad \vec{V} = \left(10\hat{j} + 5y\hat{k}\right)$$



(a) $$\vec{V} \cdot dA_1 = \left(10\hat{j} + 5y\hat{k}\right) \cdot \left(dz\hat{j} - dy\hat{k}\right) = 10dz - 5ydy$$

(b) $$\int_{A_1} \vec{V} \cdot dA_1 = \int_0^1 10dz - \int_0^1 5ydy = 10z\Big|_0^1 - \frac{5}{2}y^2\Big|_0^1 = 7.5$$

(c) $$\vec{V} \cdot dA_2 = \left(10\hat{j} + 5y\hat{k}\right) \cdot \left(-dy\hat{k}\right) = -5ydy$$

(d) $$\vec{V}\left(\vec{V} \cdot dA_2\right) = -\left(10\hat{j} + 5y\hat{k}\right)5ydy$$

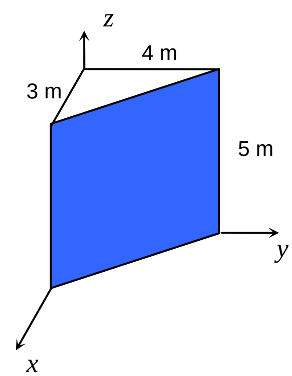
(e) $$\int_{A_2} \vec{V}\left(\vec{V} \cdot dA_2\right) = -\int_0^1 \left(10\hat{j} + 5y\hat{k}\right)5ydy = -25y^2\hat{j}\Big|_0^1 - \frac{25}{3}y^3\hat{k}\Big|_0^1 = -25\hat{j} - 8.33\hat{k}$$

Problem 4.11 [Difficulty: 3]

4.11 The area shown shaded is in a flow where the velocity field is given by $\vec{V} = ax\hat{i} + by\hat{j} + c\hat{k}$; $a = b = 2\ \mathrm{s}^{-1}$ and $c = 1$ m/s. Write a vector expression for an element of the shaded area. Evaluate the integrals $\int_A \vec{V} \cdot d\vec{A}$ and $\int_A \vec{V}(\vec{V} \cdot d\vec{A})$ over the shaded area.

Given: Data on velocity field and control volume geometry

Find: Surface integrals



Solution:

First we define the area and velocity vectors

$$d\vec{A} = dydz\hat{i} + dxdz\hat{j} \qquad \vec{V} = ax\hat{i} + by\hat{j} + c\hat{k} \quad \text{or} \quad \vec{V} = 2x\hat{i} + 2y\hat{j} + \hat{k}$$

We will need the equation of the surface: $y = \frac{3}{2}x$ or $x = \frac{2}{3}y$

Then

$$\int_A \vec{V} \cdot dA = \int_A \left(-ax\hat{i} + by\hat{j} + c\hat{k}\right) \cdot \left(dydz\hat{i} - dxdz\hat{j}\right)$$

$$= \int_0^2 \int_0^3 -axdydz - \int_0^2 \int_0^2 bydxdz = -a\int_0^2 dz \int_0^3 \frac{2}{3}ydy - b\int_0^2 dz \int_0^2 \frac{3}{2}xdx = -2a\frac{1}{3}y^2\Big|_0^3 - 2b\frac{3}{4}x^2\Big|_0^2$$

$$Q = (-6a - 6b) = -24\frac{\mathrm{m}^3}{\mathrm{s}}$$

We will again need the equation of the surface: $y = \frac{3}{2}x$ or $x = \frac{2}{3}y$, and also $dy = \frac{3}{2}dx$ and $a = b$

$$\int_A \vec{V}\left(\vec{V} \cdot d\vec{A}\right) = \int_A \left(-ax\hat{i} + by\hat{j} + c\hat{k}\right)\left(-ax\hat{i} + by\hat{j} + c\hat{k}\right) \cdot \left(dydz\hat{i} - dxdz\hat{j}\right)$$

$$= \int_A \left(-ax\hat{i} + by\hat{j} + c\hat{k}\right)\left(-axdydz - bydxdz\right)$$

$$= \int_A \left(-ax\hat{i} + \frac{3}{2}ax\hat{j} + c\hat{k}\right)\left(-ax\frac{3}{2}dxdz - a\frac{3}{2}xdxdz\right)$$

$$= \int_A \left(-ax\hat{i} + \frac{3}{2}ax\hat{j} + c\hat{k}\right)\left(-3axdxdz\right)$$

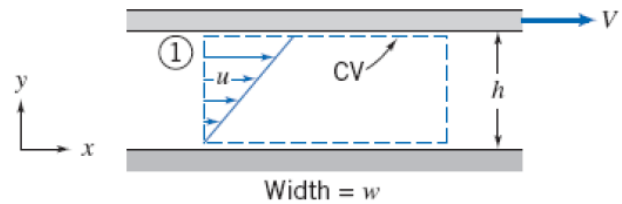
$$= 3\int_0^2 \int_0^2 a^2x^2dxdz\hat{i} - \frac{9}{2}\int_0^2 \int_0^2 a^2x^2dxdz\hat{j} - 3\int_0^2 \int_0^2 acxdxdz\hat{k}$$

$$= (6)\left(a^2\frac{x^3}{3}\Big|_0^2\right)\hat{i} - (9)\left(a^2\frac{x^3}{3}\Big|_0^2\right)\hat{j} - (6)\left(ac\frac{x^2}{2}\Big|_0^2\right) = 16a^2\hat{i} - 24a^2\hat{j} - 12ac\hat{k}$$

$$= 64\hat{i} - 96\hat{j} - 60\hat{k} \quad \frac{\mathrm{m}^4}{\mathrm{s}^2}$$

Problem 4.12 [Difficulty 2]

4.12 Obtain an expression for the kinetic energy flux, $\int (V^2/2)\rho\vec{V}\cdot d\vec{A}$, through cross section ① of the control volume shown.



Given: Control Volume with linear velocity distribution

Find: Kinetic energy flux

Solution: Apply the expression for kinetic energy flux

Governing equation: $kef = \int_A \frac{V^2}{2}\rho\vec{V}\cdot d\vec{A}$

Assumption: (1) Incompressible flow

For a linear velocity profile $\vec{V} = \frac{V}{h}y\hat{i}$ $\quad V(y) = \frac{V}{h}y$ $\quad$ and also $\quad d\vec{A} = -wdy\hat{i}$

The kinetic energy flux is

$$kef = \int_{y=0}^{h} \frac{1}{2}\left(\frac{V}{h}y\right)^2\left(-\rho\frac{Vw}{h}ydy\right) = -\rho\frac{V^3w}{2h^3}\int_{y=0}^{h} y^3dy = -\rho\frac{V^3w}{2h^3}\frac{y^4}{4}\bigg|_0^h$$

$$kef = -\frac{1}{8}\rho V^3wh$$

Problem 4.13

(Difficulty: 2)

4.13 A 0.3 m by 0.5 m rectangular air duct carries a flow of 0.45 $\frac{m^3}{s}$ at a density of 2 $\frac{kg}{m^3}$. Calculate the mean velocity in the duct. If the duct tapers to 0.15 m by 0.5 m size, what is the mean velocity in this section if the density is 1.5 $\frac{kg}{m^3}$ there?

Given: Duct size: $w_1 = 0.3\ m$; $L_1 = 0.5\ m$; $w_2 = 0.3\ m$; $L_2 = 0.5\ m$. Density: $\rho_1 = 2\ \frac{kg}{m^3}$; $\rho_2 = 1.5\ \frac{kg}{m^3}$.

Volumetric flow rate: $Q = 0.45\ \frac{m^3}{s}$.

Find: The mean velocity V_1 and V_2.

Assumption: The density is constant

Solution: Use the continuity equation

$$\dot{m} = \rho \quad V \quad A$$

For the duct entrance, we have for the flow area:

$$A_1 = w_1 L_1 = 0.3\ m \times 0.5\ m = 0.15\ m^2$$

The mean velocity can be calculated:

$$V_1 = \frac{Q}{A_1} = \frac{0.45\ \frac{m^3}{s}}{0.15\ m^2} = 3\ \frac{m}{s}$$

For the tapered section, the mass flow rate is the same as:

$$\dot{m} = \rho_1 V_1 A_1 = 2\ \frac{kg}{m^3} \times 3\ \frac{m}{s} \times 0.15\ m^2 = 0.9\ \frac{kg}{s}$$

The flow area is:

$$A_2 = w_2 L_2 = 0.15\ m \times 0.5\ m = 0.075\ m^2$$

So the mean velocity in the tapered section is:

$$V_2 = \frac{\dot{m}}{\rho_2 A_2} = \frac{0.9\ \frac{kg}{s}}{1.5\ \frac{kg}{m^3} \times 0.075\ m^2} = 8\ \frac{m}{s}$$

Problem 4.14

(Difficulty: 1)

4.14 Across a shock wave in a gas flow there is a great change in gas density ρ. If a shock wave occurs in a duct such that $V = 660\ \frac{m}{s}$ and $\rho = 1.0\ \frac{kg}{m^3}$ before the shock and $V = 250\ \frac{m}{s}$ after the shock, what is ρ after the shock?

Given: The velocity before the shock: $V_1 = 660\ \frac{m}{s}$. The velocity after the shock: $V_2 = 250\ \frac{m}{s}$. The density before the shock: $\rho_1 = 1.0\ \frac{kg}{m^3}$.

Find: The density after the shock ρ_2.

Solution:

From the continuity equation we have:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

For steady flow there is no change with time and we have:

$$0 = \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

or

$$\rho_1 V_1 A = \rho_2 V_2 A$$

$$\rho_2 = \frac{\rho_1 V_1}{V_2} = \frac{1.0\ \frac{kg}{m^3} \times 660\ \frac{m}{s}}{250\ \frac{m}{s}} = 2.64\ \frac{kg}{m^3}$$

Problem 4.15

(Difficulty: 1)

4.15 Water flows in a pipeline composed of 75 mm and 150 mm in pipe. Calculate the mean velocity in the 75 mm pipe when that in the 150 mm pipe is 2.5 $\frac{m}{s}$. What is its ratio to the mean velocity in the 150 mm pipe?

Given: The velocity in the 150 mm: $V_1 = 2.5\ \frac{m}{s}$.

Find: Its ratio γ to the mean velocity in 150 mm pipe.

Solution:

From the continuity equation we have:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

For steady flow there is no change with time and we have:

$$0 = \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

or

$$\rho V_1 A_1 = \rho V_2 A_2$$

Since the density is constant

$$V_2 = \frac{V_1 A_1}{A_2} = V_1 \frac{\frac{\pi D_1^2}{4}}{\frac{\pi D_2^2}{4}} = V_1 \frac{D_1^2}{D_2^2} = 2.5\ \frac{m}{s} \times \frac{(150\ mm)^2}{(75\ mm)^2} = 10\ \frac{m}{s}$$

$$\gamma = \frac{V_2}{V_1} = 4$$

Problem 4.16 [Difficulty 2]

4.16 The velocity distribution for laminar flow in a long circular tube of radius R is given by the one-dimensional expression,

$$\vec{V} = u\hat{i} = u_{\max}\left[1 - \left(\frac{r}{R}\right)^2\right]\hat{i}$$

For this profile obtain expressions for the volume flow rate and the momentum flux through a section normal to the pipe axis. Obtain an expression for the kinetic energy flux, $\int (V^2/2)\rho\vec{V}\cdot d\vec{A}$, through a section normal to the pipe axis.

Given: Control Volume with parabolic velocity distribution

Find: Kinetic energy flux

Solution: Apply the expressions for kinetic energy flux

Governing equation: $kef = \int_A \frac{V^2}{2}\rho\vec{V}\cdot d\vec{A}$

Assumption: (1) Incompressible flow

For a linear velocity profile $\vec{V} = u\hat{i} = u_{\max}\left[1 - \left(\frac{r}{R}\right)^2\right]\hat{i}$ $\quad V = u = u_{\max}\left[1 - \left(\frac{r}{R}\right)^2\right]$ and also $d\vec{A} = 2\pi r dr\hat{i}$

For the volume flow rate:

$$kef = \int_{r=0}^{R} \frac{1}{2}\left\{u_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]\right\}^2 \rho\left\{u_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]\hat{i}\cdot(2\pi r dr\hat{i})\right\}$$

$$= \int_{r=0}^{R} \frac{1}{2}u_{\max}^2\left[1-2\left(\frac{r}{R}\right)^2+\left(\frac{r}{R}\right)^4\right]\rho\left\{2\pi u_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]r dr\right\}$$

$$= \int_{r=0}^{R} \pi\rho u_{\max}^3\left[1-3\left(\frac{r}{R}\right)^2+3\left(\frac{r}{R}\right)^4-\left(\frac{r}{R}\right)^6\right]r dr$$

$$= \int_{r=0}^{R} \pi\rho u_{\max}^3\left[r-3\frac{r^3}{R^2}+3\frac{r^5}{R^4}-\frac{r^7}{R^6}\right]dr$$

$$= \pi\rho u_{\max}^3\left[\frac{r^2}{2}-\frac{3r^4}{4R^2}+\frac{r^6}{2R^4}-\frac{r^8}{8R^6}\right]_0^h$$

$$kef = \frac{1}{8}\pi\rho u_{\max}^3 R^2$$

Problem 4.17

(Difficulty: 1)

4.17 A farmer is spraying a liquid through 10 nozzles, $3-\text{mm}-\text{ID}$, at an average exit velocity of $3\ m/s$. What is the average velocity in the $25-\text{mm}-\text{ID}$ head feeder? What is the system flow rate, in L/min ?

Given: Data on flow through nozzles

Find: Average velocity in the head feeder, flow rate

Solution:

Continuity equation: $\sum_{CS}(\vec{V}\cdot\vec{A}) = 0$

Assumptions: (1) Steady flow (2) Incompressible flow (3) Uniform flow

Then for the nozzle flow:

$$\sum_{CS}(\vec{V}\cdot\vec{A}) = -V_{feeder}\cdot A_{feeder} + 10\cdot V_{nozzle}\cdot A_{nozzle} = 0$$

Hence

$$V_{feeder} = V_{nozzle}\cdot\frac{10\cdot A_{nozzle}}{A_{feeder}} = V_{nozzle}\cdot 10\cdot\left(\frac{D_{nozzle}}{D_{feeder}}\right)^2$$

$$V_{feeder} = 3\ \frac{m}{s}\times 10\times\left(\frac{3\ mm}{25\ mm}\right)^2 = 0.43\ \frac{m}{s}$$

The flow rate is:

$$Q = V_{feeder}\cdot A_{feeder} = V_{feeder}\cdot\frac{\pi\cdot D_{feeder}^2}{4}$$

$$Q = 0.43\ \frac{m}{s}\times\frac{\pi\times\left(\frac{25}{1000}\ m\right)^2}{4}\times\frac{60s}{min} = 0.00403\ \frac{m^3}{min} = 4.03\ \frac{L}{min}$$

Problem 4.18 [Difficulty: 1]

4.18 A university laboratory that generates 15 m^3/s of air flow at design condition wishes to build a wind tunnel with variable speeds. It is proposed to build the tunnel with a sequence of three circular test sections: section 1 will have a diameter of 1.5 m, section 2 a diameter of 1 m, and section 3 a diameter such that the average speed is 75 m/s.

(a) What will be the speeds in sections 1 and 2?

(b) What must the diameter of section 3 be to attain the desired speed at design condition?

Given: Data on wind tunnel geometry

Find: Average speeds in wind tunnel; diameter of section 3

Solution:

Basic equation $Q = V \cdot A$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Given data: $Q = 15 \cdot \frac{m^3}{s}$ $D_1 = 1.5 \cdot m$ $D_2 = 1 \cdot m$ $V_3 = 75 \cdot \frac{m}{s}$

Between sections 1 and 2 $Q = V_1 \cdot A_1 = V_1 \cdot \frac{\pi \cdot D_1^{\ 2}}{4} = V_2 \cdot A_2 = V_2 \cdot \frac{\pi \cdot D_2^{\ 2}}{4}$

Hence $V_1 = \frac{Q}{\frac{\pi}{4} \cdot D_1^{\ 2}}$ $V_1 = 8.49 \frac{m}{s}$ $V_2 = \frac{Q}{\frac{\pi}{4} \cdot D_2^{\ 2}}$ $V_2 = 19.1 \frac{m}{s}$

For section 3 we can use $V_1 \cdot \frac{\pi \cdot D_1^{\ 2}}{4} = V_3 \cdot \frac{\pi \cdot D_3^{\ 2}}{4}$ or $D_3 = D_1 \cdot \sqrt{\frac{V_1}{V_3}}$ $D_3 = 0.505\ m$

Problem 4.19

(Difficulty: 2)

4.19 Hydrogen is being pumped through a pipe system whose temperature is held at $273\ K$. At a section where the pipe diameter is $10\ mm.$, the pressure and average velocity are $200\ kPa$ and 30 $\frac{m}{s}$. Find all possible velocities and pressure at a downstream whose diameter is 20 mm.

Given: Temperature: $T = 273\ K$. The upstream diameter: $D_1 = 10\ mm$. The upstream pressure: $p_1 = 200\ kPa$. The upstream velocity: $V_1 = 30\ \frac{m}{s}$. The downstream diameter: $D_2 = 20\ mm$

Find: The possible downstream velocity V_2 and pressure p_2.

Solution:

From the continuity equation we have:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

For steady flow there is no change with time and we have:

$$0 = \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

or

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

For idea gas, we have:

$$\frac{p}{\rho} = RT$$

$$\rho = \frac{p}{RT}$$

Thus,

$$\frac{p_1}{RT} V_1 A_1 = \frac{p_2}{RT} V_2 A_2$$

$$p_1 V_1 A_1 = p_2 V_2 A_2$$

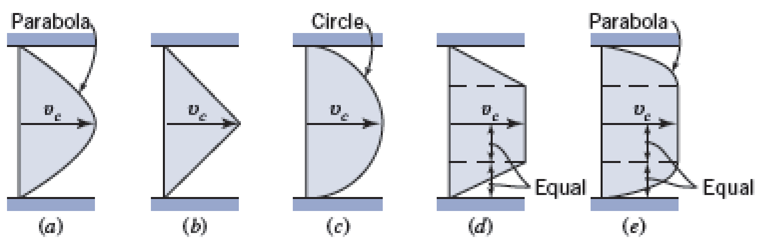
$$p_2 V_2 = \frac{p_1 V_1 A_1}{A_2} = \frac{200 \times 10^3\ Pa \times 30\ \frac{m}{s} \times \frac{\pi}{4} \times (10\ mm)^2}{\frac{\pi}{4} \times (20\ mm)^2} = 1500000\ \frac{Pa \cdot m}{s} = 1.5 \times 10^6\ \frac{Pa \cdot m}{s}$$

Any combination of p_2 and V_2 giving the above result will be acceptable.

Problem 4.20

(Difficulty: 2)

4.20 Calculate the mean velocity for these two-dimensional velocity profiles if $V_c = 3\ \frac{m}{s}$.



Given: All the velocity profiles are shown in the figure with $V_c = 3\frac{m}{s}$.

Find: The mean velocity V_m.

Solution: Use the definition for the mean velocity V_m :

$$V_m = \frac{1}{A}\iint_A V dA$$

(a) Parabola

$$V_m = V_c \int_0^1 (1 - x^2)\, dx = V_c \left(x - \frac{x^3}{3}\right)_0^1 = \frac{2}{3} V_c = 2\ \frac{m}{s}$$

(b) Linear

$$V_m = V_c \int_0^1 (1 - x)\, dx = V_c \left(x - \frac{x^2}{2}\right)_0^1 = \frac{1}{2} V_c = 1.5\ \frac{m}{s}$$

(c) Circle

$$V_m = \frac{1}{2} V_c \frac{A}{2}$$
$$A = \pi R^2$$
$$R = 1$$
$$V_m = \frac{1}{2} \times \frac{\pi}{2} \times V_c = \frac{\pi}{4} V_c = 2.36\ \frac{m}{s}$$

(d) Linear and Flat

$$V_m = \frac{1}{2}\left(\int_0^1 (1 - x) V_c dx + V_c\right) = \frac{1}{2}\left(\frac{1}{2} V_c + V_c\right) = \frac{3}{4} V_c$$
$$V_m = 2.25\ \frac{m}{s}$$

(e) Parabola and Flat

$$V_m = \frac{1}{2}\left(\int_0^1 (1 - x^2)\, V_c dx + V_c\right) = \frac{1}{2}\left(\frac{2}{3} V_c + V_c\right) = \frac{5}{6} V_c$$
$$V_m = 2.5\ \frac{m}{s}$$

Problem 4.21

(Difficulty: 2)

4.21 If the velocity profile in a passage of width $2R$ is given by the equation $\frac{v}{v_c} = \left(\frac{y}{R}\right)^{\frac{1}{n}}$, derive an expression for $\frac{V}{v_c}$ in terms of n: (a) for a two-dimensional passage, and (b) for a cylindrical passage.

Find: The expression for $\frac{V}{v_c}$ in terms of n.

Solution:

We have the equation:

$$v = v_c \left(\frac{y}{R}\right)^{\frac{1}{n}}$$

(a) For the two dimensional passage

$$V = \frac{1}{2R} \cdot 2\int_0^R v_c \left(\frac{y}{R}\right)^{\frac{1}{n}} dy = \frac{v_c}{R}\left(\frac{n}{n+1}\right)\frac{y^{\frac{n+1}{n}}}{R^{\frac{1}{n}}}\Bigg|_0^R = \frac{v_c}{R}\left(\frac{n}{n+1}\right)R = v_c\left(\frac{n}{n+1}\right)$$

So we have:

$$\frac{V}{v_c} = \left(\frac{n}{n+1}\right)$$

(b) For the axisymmetric passage

$$V = \frac{1}{\pi R^2}\int_0^R v_c \left(\frac{y}{R}\right)^{\frac{1}{n}} 2\pi(R-y)dy = \frac{2v_c}{R^{2+\frac{1}{n}}}\int_0^R \left(Ry^{\frac{1}{n}} - y^{\frac{n+1}{n}}\right)dy = \frac{2v_c}{R^{2+\frac{1}{n}}}\left[\frac{Ry^{\frac{n+1}{n}}}{\frac{n+1}{n}} - \frac{y^{\frac{2n+1}{n}}}{\frac{2n+1}{n}}\right]_0^R$$

$$V = v_c\left[\frac{2n}{n+1} - \frac{2n}{2n+1}\right]$$

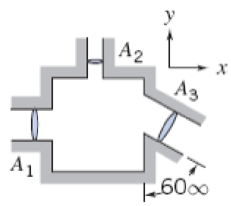
So we have:

$$\frac{V}{v_c} = \frac{2n}{n+1} - \frac{2n}{2n+1}$$

Problem 4.22

(Difficulty: 1)

4.22 Fluid with 1040 $\frac{kg}{m^3}$ density is flowing steadily through the rectangular box shown. Given $A_1 = 0.046\ m^2$, $A_2 = 0.009\ m^2$, $A_3 = 0.056\ m^2$, $\vec{V}_1 = 3\hat{\imath}\ m/s$ and $\vec{V}_2 = 6\hat{\jmath}\ m/s$, determine the velocity $\vec{V}_3$.



Given: Data on flow through box

Find: Velocity at section 3

Solution:

Basic equation: $\sum_{CS}\left(\vec{V}\cdot\vec{A}\right) = 0$

Assumptions: (1) Steady flow (2) Incompressible flow (3) Uniform flow

Then for the box:

$$\sum_{CS}\left(\vec{V}\cdot\vec{A}\right) = -V_1\cdot A_1 + V_2\cdot A_2 + V_3\cdot A_3 = 0$$

Note that the vectors indicate that flow is in at location 1 and out at location 2; we assume out flow at location 3.

Hence

$$V_3 = \frac{V_1\cdot A_1 - V_2\cdot A_2}{A_3} = \frac{3\ \frac{m}{s}\times 0.046\ m^2 - 6\ \frac{m}{s}\times 0.009\ m^2}{0.056\ m^2} = 1.5\ \frac{m}{s}$$

Based on the geometry:

$$V_{3x} = V_3\times\sin 60° = 1.299\ \frac{m}{s}$$

$$V_{3y} = -V_3\times\cos 60° = -0.75\ \frac{m}{s}$$

$$\vec{V}_3 = 1.299\ \hat{\imath}\frac{m}{s} - 0.75\hat{\jmath}\frac{m}{s}$$

Problem 4.23 [Difficulty: 1]

4.23 A rice farmer needs to fill her 150 m × 400 m field with water to a depth of 7.5 cm in 1 hr. How many 37.5-cm-diameter supply pipes are needed if the average velocity in each must be less than 2.5 m/s?

Given: Water needs of farmer

Find: Number of supply pipes needed

Solution:

Basic equation $Q = V \cdot A$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

The given data is: $A = 150 \cdot m \cdot 400 \cdot m$ $\quad A = 6 \times 10^4\, m^2$ $\quad h = 7.5 \cdot cm$ $\quad t = 1 \cdot hr$ $\quad D = 37.5 \cdot cm$ $\quad V = 2.5 \cdot \frac{m}{s}$

Then $Q = \frac{A \cdot h}{t}$ $\quad Q = 1.25 \frac{m^3}{s}$

If n is the number of pipes $Q = V \cdot \frac{\pi}{4} \cdot D^2 \cdot n$ or $n = \frac{4 \cdot Q}{\pi \cdot V \cdot D^2}$ $\quad n = 4.527$

The farmer needs 5 pipes.

Problem 4.24

(Difficulty: 1)

4.24 In your kitchen, the sink is 60 cm by 45.7 cm by 30.5 cm deep. You are filling it with water at the rate of $252 \times 10^{-6}\ m^3/s$. How long will it take (in min) to half fill the sink? After this you turn off the faucet and open the drain slightly so that the tank starts to drain at $63 \times 10^{-6}\ m^3/s$. What is the rate (m/s) at which the water level drops?

Given: Data on filling of a sink

Find: Time to half fill; rate at which level drops

Solution:

This is an unsteady problem if we choose the CS as the entire sink.

Basic equation: $\frac{\partial M_{CV}}{\partial t} + \sum_{CS}(\rho \cdot \vec{V} \cdot \vec{A}) = 0$

Assumptions: (1) Incompressible flow

Given data:

$$L = 60\ cm, w = 45.7\ cm, d = 30.5\ cm$$

$$Q = 252 \times 10^{-6}\ \frac{m^3}{s}$$

$$Q_{drain} = 63 \times 10^{-6}\ \frac{m^3}{s}$$

Hence

$$\frac{\partial M_{CV}}{\partial t} = -\sum_{CS}(\rho \cdot \vec{V} \cdot \vec{A}) = Inflow - Outflow$$

To half fill:

$$V = \frac{L \cdot w \cdot d}{2} = \frac{60\ cm \times 45.7\ cm \times 30.5\ cm}{2} = 0.0418\ m^3$$

Then we have:

$$t = \frac{V}{Q} = \frac{0.0418\ m^3}{252 \times 10^{-6}\ \frac{m^3}{s}} = 165.9\ s$$

After the drain opens, we have the following equation:

$$\frac{dV}{dt} = L \cdot w \cdot V_{level} = -Q_{drain}$$

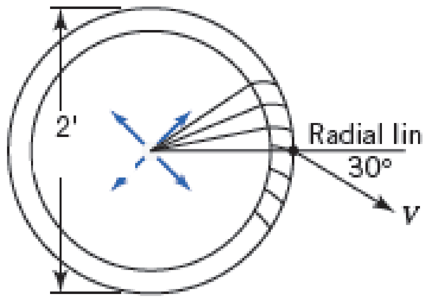
V_{level} is the speed of water level drop.

$$V_{level} = \frac{-Q_{drain}}{L \cdot w} = \frac{-63 \times 10^{-6}\ \frac{m^3}{s}}{60\ cm \times 45.7\ cm} = -2.3 \times 10^{-4}\ \frac{m}{s}$$

Problem 4.25

(Difficulty: 1)

4.25 Fluid passes through this set of thin closely spaced blades. What flow rate q is required for the velocity V to be $10\ \frac{ft}{s}$?



Given: The velocity $V = 10\ \frac{ft}{s}$

Find: The flow rate q.

Solution: Use the continuity equation

The velocity vertical to the blade surface is:

$$V_e = V \cos 30°$$

The lateral surface area is

$$A = 2\pi r h$$

The volumetric flow rate can be calculated by:

$$Q = V_e A = V \cos 30°\, 2\pi r h$$

The volumetric flow rate per unit blade height is:

$$q = \frac{Q}{h} = V \cos 30°\, 2\pi r$$

$$r = 1\ ft = 0.305\ m$$

$$q = 10\ \frac{ft}{s} \times \cos 30° \times 2 \times \pi \times 1\ \ ft = 54.4\ \frac{ft^2}{s}$$

Problem 4.26

(Difficulty: 2)

4.26 A pipeline 0.3 m in diameter divides at a Y into two branches 200 mm and 150 mm in diameter. If the flow rate in the main line is 0.3 $\frac{m^3}{s}$ and the mean velocity in the 200 mm pipe is 2.5 $\frac{m}{s}$, what is the flow rate in the 150 mm pipe?

Given: The diameter for the main line: $D_1 = 0.3\ m$. The diameter for the two branches: $D_2 = 200\ mm$ and $D_3 = 150\ mm$. The flow rate in the main line is: $q_1 = 0.3\ \frac{m^3}{s}$. The mean velocity in the 200 mm pipe: $V_2 = 2.5\ \frac{m}{s}$

Find: The flow rate q_3 in the 150 mm pipe.

Solution:

From the continuity equation we have:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

For steady flow there is no change with time and we have:

$$0 = \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

or

$$\rho V_1 A_1 = \rho V_2 A_2 + \rho V_3 A_3$$

Since the density of water is constant

$$V_1 A_1 = V_2 A_2 + V_3 A_3$$

$$Q_1 = Q_2 + Q_3$$

where

$$Q_1 = V_1 A_1$$

$$Q_2 = V_2 A_2$$

$$Q_3 = V_3 A_3$$

Thus

$$Q_2 = V_2 A_2 = \frac{\pi}{4} D_2^2 V_2 = 2.5\ \frac{m}{s} \times \frac{\pi}{4} \times (0.2\ m)^2 = 0.0785\ \frac{m^3}{s}$$

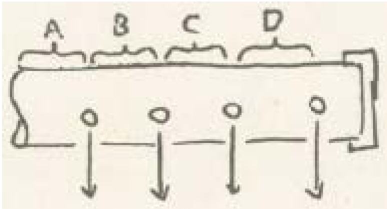
So we can find the flow rate in the 150 mm pipe:

$$Q_3 = Q_1 - Q_2 = 0.3\ \frac{m^3}{s} - 0.0785\ \frac{m^3}{s} = 0.222\ \frac{m^3}{s}$$

Problem 4.27

(Difficulty: 2)

4.27 A manifold pipe of 3 in diameter has four openings in its walls spaced equally along the pipe and is closed at the downstream end. If the discharge from each opening is $0.5\ \frac{ft^3}{s}$, what are the mean velocities in the pipe between the openings?



Given: The pipe diameter $D = 3\ in$. The mass flow rate from each opening is: $q_o = 0.5\ \frac{ft^3}{s}$

Find: The mean velocities in the pipe at A,B,C and D.

Solution:

From the continuity equation we have:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

For steady flow there is no change with time and we have:

$$0 = \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

The density is constant and the volume flow rate in each pipe is

$$Q = AV$$

The area of the main pipe is:

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times \left(\frac{3}{12}\ ft\right)^2 = 0.0491\ ft^2$$

So we have the mass flow rate and mean velocity at cross section A are:

$$Q_A = 4Q_o = 2\ \frac{ft^3}{s}$$

$$V_A = \frac{Q_A}{A} = \frac{2\ \frac{ft^3}{s}}{0.0491\ ft^2} = 40.7\ \frac{ft}{s}$$

The mass flow rate and mean velocity at cross section B are:

$$Q_B = Q_A - Q_o = 2\ \frac{ft^3}{s} - 0.5\ \frac{ft^3}{s} = 1.5\ \frac{ft^3}{s}$$

$$V_B = \frac{Q_B}{A} = \frac{1.5\ \frac{ft^3}{s}}{0.0491\ ft^2} = 30.5\ \frac{ft}{s}$$

The mass flow rate and mean velocity at cross section C are:

$$Q_C = Q_B - Q_o = 1.5\ \frac{ft^3}{s} - 0.5\ \frac{ft^3}{s} = 1.0\ \frac{ft^3}{s}$$

$$V_C = \frac{Q_C}{A} = \frac{1.0\ \frac{ft^3}{s}}{0.0491\ ft^2} = 20.4\ \frac{ft}{s}$$

The mass flow rate and mean velocity at cross section D are:

$$Q_D = Q_C - Q_o = 1.0\ \frac{ft^3}{s} - 0.5\ \frac{ft^3}{s} = 0.5\ \frac{ft^3}{s}$$

$$V_D = \frac{Q}{A} = \frac{0.5\ \frac{ft^3}{s}}{0.0491\ ft^2} = 10.18\ \frac{ft}{s}$$

Problem 4.28

[Difficulty: 1]

4.28 You are trying to pump storm water out of your basement during a storm. The pump can extract 27.5 gpm. The water level in the basement is now sinking by about 4 in./hr. What is the flow rate (gpm) from the storm into the basement? The basement is 30 ft × 20 ft.

Given: Data on filling of a basement during a storm

Find: Flow rate of storm into basement

Solution:

This is an unsteady problem if we choose the CS as the entire basement

Basic equation $$\frac{\partial}{\partial t}M_{CV} + \sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = 0$$

Assumptions: 1) Incompressible flow

Given data: $Q_{pump} = 27.5\cdot gpm$ $\quad \frac{dh}{dt} = 4\cdot\frac{in}{hr}$ $\quad A = 30\cdot ft\cdot 20\cdot ft$

Hence $$\frac{\partial}{\partial t}M_{CV} = \rho\cdot A\cdot\frac{dh}{dt} = \sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = \rho\cdot Q_{storm} - \rho\cdot Q_{pump}$$

where A is the basement area and dh/dt is the rate at which the height of water in the basement changes.

or $$Q_{storm} = Q_{pump} - A\cdot\frac{dh}{dt}$$

$$Q_{storm} = 27.5\cdot\frac{gal}{min} - 30\cdot ft \times 20\cdot ft \times \left(\frac{4}{12}\cdot\frac{ft}{hr}\right) \times \frac{7.48\cdot gal}{ft^3} \times \frac{1\cdot hr}{60\cdot min}$$

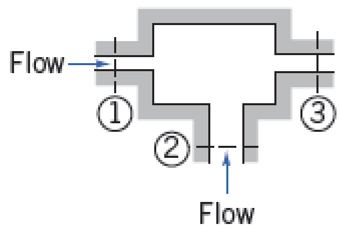
Data on gals from Table G.2

$$Q_{storm} = 2.57\cdot gpm$$

Problem 4.29

(Difficulty: 2)

4.29 In the incompressible flow through the device shown, velocities may be considered uniform over the inlet and outlet sections. The following conditions are known: $A_1 = 0.1\ m^2$, $A_2 = 0.2\ m^2$, $A_3 = 0.6\ m^2$, $V_1 = 10e^{-t/2}\ m/s$, and $V_2 = 2\cos(2\pi t)\ m/s$ (t in seconds). Obtain an expression for the velocity at section (3), and plot V_3 as a function of time. At what instant does V_3 first become zero? What is the total mean volumetric flow at section (3)?



P4.29

Given: Data on flow through device

Find: Velocity V_3; plot V_3 against time; find when V_3 is zero; total mean flow

Solution:

Governing equation: For incompressible flow and uniform flow

$$\int_{CS} \vec{V} \cdot \overrightarrow{dA} = \sum_{CS} \vec{V} \cdot \overrightarrow{dA} = 0$$

Applying to the device (assuming V_3 is out):

$$-V_1 A_1 - V_2 A_2 + V_3 A_3 = 0$$

$$V_3 = \frac{V_1 A_1 + V_2 A_2}{A_3} = \frac{10 \cdot e^{-\frac{t}{2}}\ \frac{m}{s} \times 0.1\ m^2 + 2\cos(2\pi t)\ \frac{m}{s} \times 0.2\ m^2}{0.6\ m^2}$$

The velocity at A_3 is:

$$V_3 = 1.667 e^{-\frac{t}{2}} + 0.667\cos(2\pi t)$$

The plot of V_3 as a function of time is shown below. It is shown that the first time V_3 becomes zero is at:

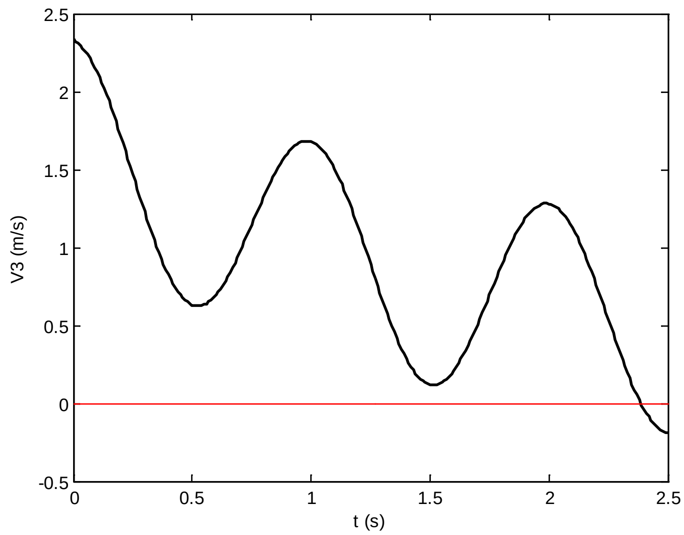
$$t = 2.39\ s$$

The total mean volumetric flow at A_3 is:

$$Q = \int_0^\infty V_3 \cdot A_3 dt = \int_0^\infty \left(1.667e^{-\frac{t}{2}} + 0.667\cos(2\pi t)\right) \cdot 0.6\ dt\ \frac{m^3}{s}$$

$$Q = \lim_{t\to\infty}\left(-2e^{-\frac{t}{2}} + \frac{1}{5\pi}\sin(2\pi t)\right) - (-2)\ m^3 = 2\ m^3$$

$$Q = 2\ m^3$$

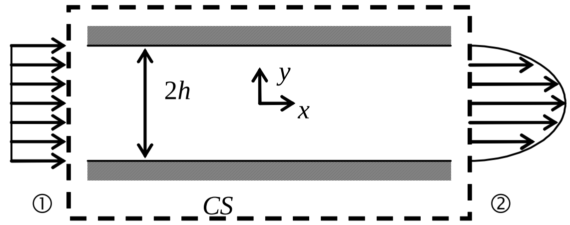


Problem 4.30 [Difficulty: 2]

4.30 Water enters a wide, flat channel of height $2h$ with a uniform velocity of 2.5 m/s. At the channel outlet the velocity distribution is given by

$$\frac{u}{u_{max}} = 1 - \left(\frac{y}{h}\right)^2$$

where y is measured from the centerline of the channel. Determine the exit centerline velocity, u_{max}.



Given: Data on flow at inlet and outlet of channel

Find: Find u_{max}

Solution:

Basic equation $$\int_{CS} \rho\vec{V}\cdot d\vec{A} = 0$$

Assumptions: 1) Steady flow 2) Incompressible flow

Evaluating at 1 and 2 $$-\rho\cdot U\cdot 2\cdot h\cdot w + \int_{-h}^{h} \rho\cdot u(y)\, dy = 0$$

$$\int_{-h}^{h} u_{max}\cdot\left[1 - \left(\frac{y}{h}\right)^2\right] dy = 2\cdot h\cdot U$$

$$u_{max}\cdot\left[[h - (-h)] - \left[\frac{h^3}{3\cdot h^2} - \left(-\frac{h^3}{3\cdot h^2}\right)\right]\right] = 2\cdot h\cdot U$$

$$u_{max}\cdot\frac{4}{3}\cdot h = 2\cdot h\cdot U$$

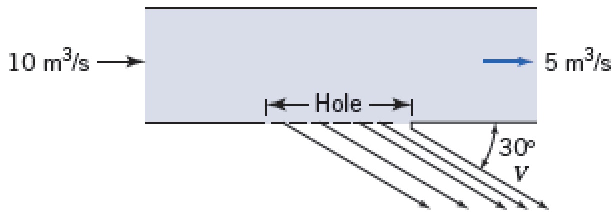
Hence $$u_{max} = \frac{3}{2}\cdot U = \frac{3}{2}\times 2.5\cdot\frac{m}{s}$$

$$u_{max} = 3.75\cdot\frac{m}{s}$$

Problem 4.31

(Difficulty: 1)

4.31 Find the average efflux velocity V if the flow exists from a hole of area $1\ m^2$ in the side of the duct as shown.



Given: The area of the hole $A = 1\ m^2$. All the other parameters are shown in the figure.

Find: The average efflux velocity V.

Solution:

From the continuity equation we have:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

For steady flow there is no change with time and we have:

$$0 = \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Assuming that the density is constant:

$$Q_{in} = Q_{hole} + Q_{out}$$

So we have:

$$Q_{hole} = Q_{in} - Q_{out}$$

$$Q_{hole} = 10\ \frac{m^3}{s} - 5\ \frac{m^3}{s} = 5\ \frac{m^3}{s}$$

For the flux through the hole we have:

$$Q_{hole} = V \sin 30° A$$

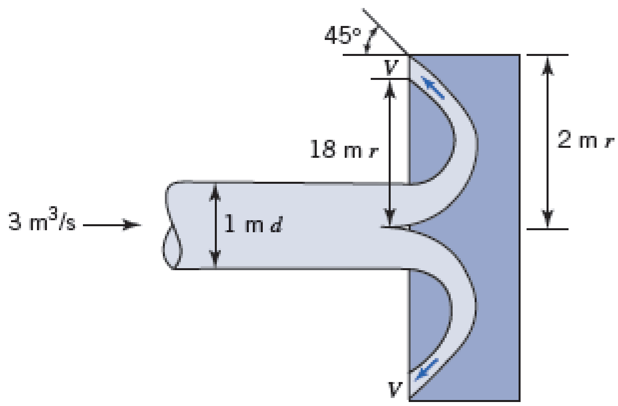
So the average efflux velocity exit from the hole is:

$$V = \frac{Q_{hole}}{A \sin 30°} = \frac{5\ \frac{m^3}{s}}{1\ m^2 \times 0.5} = 10\ \frac{m}{s}$$

Problem 4.32

(Difficulty: 2)

4.32 Find V for this mushroom cap on a pipeline.



Given: All the other parameters are shown in the figure.

Find: The velocity V.

Assume: The density is constant

Solution: From the continuity equation we have:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

For steady flow there is no change with time and we have:

$$0 = \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

The density is constant we have:

$$Q_{in} = Q_{out}$$

Also we can get:

$$Q_{out} = V \cos 45°\, A_{out}$$

The outlet area is:

$$A_{out} = \pi(r_2^2 - r_1^2) = \pi[(2\ m)^2 - (1.8\ m)^2] = 2.39\ m^2$$

The velocity is calculated to be:

$$V = \frac{Q_{out}}{A_{out} \cos 45°} = \frac{3 \ \frac{m^3}{s}}{2.39 \ m^2 \times \cos 45°} = 1.78 \ \frac{m}{s}$$

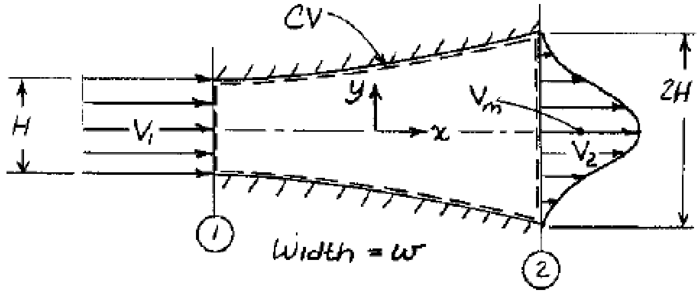
Problem 4.33

[Difficulty: 2]

4.33 Incompressible fluid flows steadily through a plane diverging channel. At the inlet, of height H, the flow is uniform with magnitude V_1. At the outlet, of height $2H$, the velocity profile is

$$V_2 = V_m \cos\left(\frac{\pi y}{2H}\right)$$

where y is measured from the channel centerline. Express V_m in terms of V_1.



Given: Data on flow at inlet and outlet of channel

Find: Find u_{max}

Solution:

Basic equation $$\int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$$

Assumptions: 1) Steady flow 2) Incompressible flow

Evaluating at 1 and 2 $$-\rho \cdot V_1 \cdot H \cdot w + \int_{-H}^{H} \rho \cdot V_2(y) \cdot w \, dy = 0$$

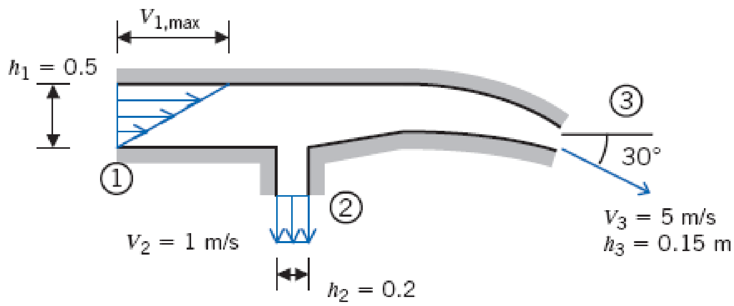
or $$V_1 \cdot H = \int_{-H}^{H} V_m \cdot \cos\left(\frac{\pi \cdot y}{2 \cdot H}\right) dy = 2 \cdot \int_{0}^{H} V_m \cdot \cos\left(\frac{\pi \cdot y}{2 \cdot H}\right) dy = 2 \cdot V_m \cdot \frac{2 \cdot H}{\pi} \cdot \left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right) = \frac{4 \cdot H \cdot V_m}{\pi}$$

Hence $$V_m = \frac{\pi}{4} \cdot V_1$$

Problem 4.34

[Difficulty: 2]

4.34 A two-dimensional reducing bend has a linear velocity profile at section ①. The flow is uniform at sections ② and ③. The fluid is incompressible and the flow is steady. Find the maximum velocity, $V_{1,max}$, at section ①.



Given: Data on flow at inlet and outlet of a reducing elbow

Find: Find the maximum velcoity at section 1

Solution:

Basic equation
$$\int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$$

Assumptions: 1) Steady flow 2) Incompressible flow

Evaluating at 1, 2 and 3
$$-\int_0^{h_1} V_1(y) \cdot w\, dy + V_2 \cdot w \cdot h_2 + V_3 \cdot w \cdot h_3 = 0$$

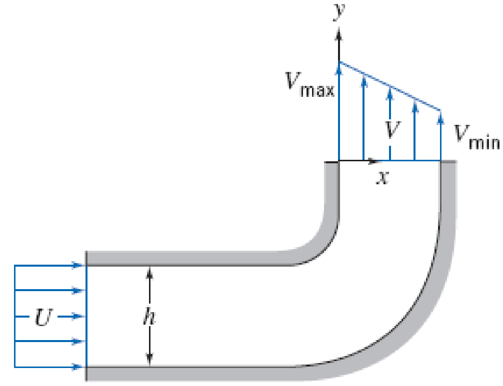
or
$$\frac{V_{1max}}{h_1} \cdot \int_0^{h_1} y\, dy = \frac{V_{1max}}{h_1} \cdot \frac{h_1^{\ 2}}{2} = V_2 \cdot h_2 + V_3 \cdot h_3$$

Hence
$$V_{1max} = \frac{2}{h_1} \cdot \left(V_3 \cdot h_3 + V_2 \cdot h_2\right)$$

$$V_{1max} = \frac{2}{0.5 \cdot m} \cdot \left(5 \cdot \frac{m}{s} \times 0.15 \cdot m + 1 \cdot \frac{m}{s} \times 0.2 \cdot m\right) \qquad V_{1max} = 3.80 \frac{m}{s}$$

Problem 4.35 [Difficulty: 2]

4.35 Water enters a two-dimensional, square channel of constant width, $h = 75.5$ mm, with uniform velocity, U. The channel makes a 90° bend that distorts the flow to produce the linear velocity profile shown at the exit, with $v_{max} = 2\, v_{min}$. Evaluate v_{min}, if $U = 7.5$ m/s.



Given: Data on flow at inlet and outlet of channel

Find: Find u_{max}

Solution:

Basic equation $$\int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$$

Assumptions: 1) Steady flow 2) Incompressible flow

Evaluating at inlet and exit $$-U \cdot w \cdot h + \int_0^h V_{exit}(x) \cdot w\, dx = 0$$

Here we have $$V_{exit} = V_{max} - (V_{max} - V_{min}) \cdot \frac{x}{h}$$ But we also have $$V_{max} = 2 \cdot V_{min}$$

Hence $$V_{exit} = 2 \cdot V_{min} - V_{min} \cdot \frac{x}{h}$$

$$\int_0^h V_{exit}(x) \cdot w\, dx = \int_0^h \left(2 \cdot V_{min} - V_{min} \cdot \frac{x}{h}\right) \cdot w\, dx = \left(2 \cdot V_{min} \cdot h - V_{min} \cdot \frac{h^2}{2 \cdot h}\right) \cdot w = \frac{3}{2} \cdot V_{min} \cdot h \cdot w$$

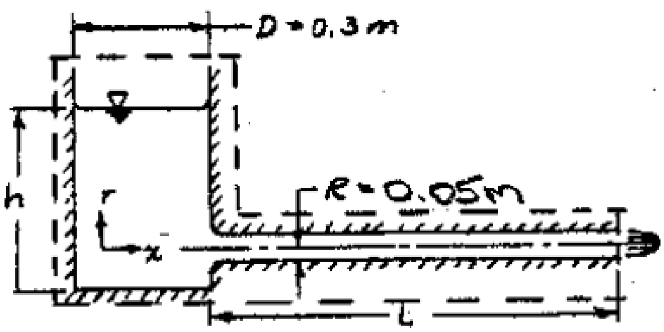
Hence $$\frac{3}{2} \cdot V_{min} \cdot h \cdot w = U \cdot w \cdot h \qquad V_{min} = \frac{2}{3} \cdot U$$

$$V_{min} = \frac{2}{3} \times 7.5 \cdot \frac{m}{s} \qquad V_{min} = 5.00 \cdot \frac{m}{s}$$

Problem 4.36

(Difficulty: 2)

4.36 Viscous liquid from a circular tank, $D = 300\ mm$ in diameter, drains through a long circular tube of radius $R = 50\ mm$. The velocity profile at the tube discharge is $u = u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]$. Show that the average speed of flow in the drain tube is $\bar{V} = \frac{1}{2}u_{max}$. Evaluate the rate of change of liquid level in the tank at the instant when $u_{max} = 0.155\ \frac{m}{s}$.



Given: Tank diameter: $D = 300\ mm$. Tube radius: $R = 50\ mm$.

Velocity profile at the tube discharge: $u = u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]$. Maximum velocity: $u_{max} = 0.155\ \frac{m}{s}$.

Find: Average velocity: $\bar{V} = \frac{1}{2}u_{max}$. Rate of change of liquid level in tank: $\frac{dh}{dt}$.

Assume: The liquid density is constant. The mass flow of air that enters the CV is neglected.

Solution:

a) The average velocity $\bar{V}$ is defined as

$$V = \frac{Q}{A}$$

Since $Q = \int u dA$, $dA = 2\pi r dr$ and $A = \pi R^2$, then

$$\bar{V} = \frac{Q}{A} = \frac{\int_0^R u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]2\pi r dr}{\pi R^2} = \frac{2u_{max}}{R^2}\int_0^R\left[1-\left(\frac{r}{R}\right)^2\right] r dr = \frac{2u_{max}}{R^2}\frac{R^2}{4} = \frac{1}{2}u_{max}$$

b) Apply conservation of mass to the CV shown:

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Then

$$0 = \rho_c \frac{\partial}{\partial t}\forall + \rho_c \bar{V} A = \rho_c \frac{\partial}{\partial t}\left(\frac{\pi D^2}{4} h + L\pi R^2\right) + \rho_c \bar{V} \pi R^2$$

As we have

$$\frac{dL}{dt} = 0$$

$$0 = \frac{\pi D^2}{4}\frac{dh}{dt} + \bar{V}\pi R^2$$

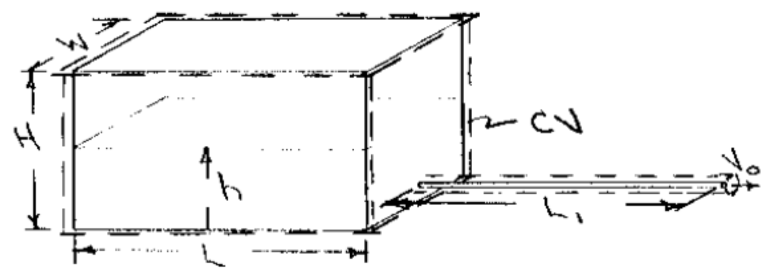
The change rate of the liquid level is:

$$\frac{dh}{dt} = -\frac{4\bar{V}R^2}{D^2} = -\frac{2u_{max}R^2}{D^2} = -2 \times 0.155\ \frac{m}{s} \times \left(\frac{0.05\ m}{0.3\ m}\right)^2 = -0.00861\ \frac{m}{s} = 8.61\ \frac{mm}{s}$$

Problem 4.37

(Difficulty: 2)

4.37 A rectangular tank used to supply water for a Reynolds flow experiment is 230 mm deep. Its width and length are $W = 150\ mm$ and $L = 230\ mm$. Water flows from the outlet tube (inside diameter $D = 6.35\ mm$) at Reynolds number $Re = 2000$, when the tank is half full. The supply valve is closed. Find the rate of change of water level in the tank at this instant.



Given: Tank width: $W = 150\ mm$. Tank length: $L = 230\ mm$. Tube diameter: $D = 6.35\ mm$. Reynolds number $Re = 2000$.

Find: Rate of change of water level in tank: $\frac{dh}{dt}$.

Assumption: (1) uniform flow at exit of tube.

(2) incompressible flow.

(3) neglect mass flow of air entering the control volume.

Solution:

From the continuity equation we have:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

For the control volume shown in the figure:

$$0 = \rho \frac{\partial}{\partial t}\left[wLh + \frac{\pi D^2 L_1}{4}\right] + \rho V \frac{\pi D^2}{4}$$

We have:

$$\frac{dL_1}{dt} = 0$$

$$0 = wL\frac{\partial h}{\partial t} + V\frac{\pi D^2}{4}$$

At exit:

$$Re = \frac{VD}{v} = 2000$$

For water at 20 ℃

$$v = 1 \times 10^{-6}\ \frac{m^2}{s}$$

$$V = \frac{Rev}{D} = \frac{2000 \times 1 \times 10^{-6}\ \frac{m^2}{s}}{6.35 \times 10^{-3}\ m} = 0.315\ \frac{m}{s}$$

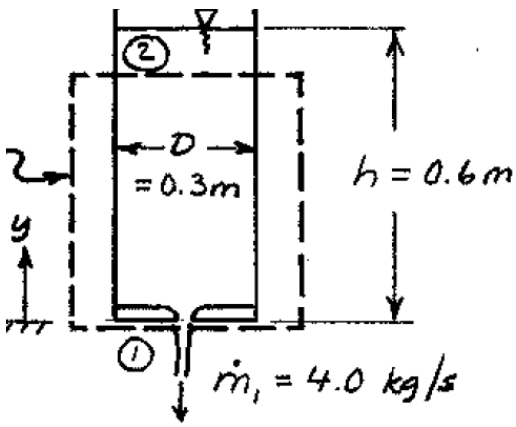
The change rate of the water level in the tank can be calculated by:

$$\frac{\partial h}{\partial t} = -\frac{V\pi D^2}{4wL} = \frac{0.315\ \frac{m}{s} \times \pi \times (6.35 \times 10^{-3}\ m)^2}{4 \times 0.15\ m \times 0.23\ m} = 2.89\ \times 10^{-4}\ \frac{m}{s} = -0.289\ \frac{mm}{s}$$

Problem 4.38

(Difficulty: 2)

4.38 A cylindrical tank, $0.3\ m$ in diameter, drains through a hole in its bottom. At the instant when the depth is $0.6\ m$, the flow rate from the tank is observed to be $4\ \frac{kg}{s}$. Determine the rate of change of water level at this instant.



Given: Tank diameter: $D = 0.3\ m$. Flow rate from the tank: $\dot{m}_1 = 4.0\ \frac{kg}{s}$.

Find: Rate of change of water level in tank: $\frac{dh}{dt}$.

Assumption: (1) uniform flow at exit of tube.

(2) incompressible flow.

(3) the control volume is fixed

Solution:

From the continuity equation we have:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

For the control volume shown in the figure:

$$0 = \rho \frac{\partial}{\partial t}[\forall] + \rho V_2 \frac{\pi D^2}{4} + \dot{m}_1$$

We have:

$$\frac{d\forall}{dt} = 0$$

$$V_2 = -\frac{4\dot{m}_1}{\rho \pi D^2}$$

The density of water is:

$$\rho = 999\ \frac{kg}{m^3}$$

The rate of change of water level is:

$$\frac{dh}{dt} = V_2 = -\frac{4\dot{m}_1}{\rho \pi D^2} = -\frac{4 \times 4.0\ \frac{kg}{s}}{999\ \frac{kg}{m^3} \times \pi \times (0.3\ m)^2} = -0.0566\ \frac{m}{s} = -56.6\ \frac{mm}{s}$$

Problem 4.39

(Difficulty: 2)

4.39 Air enters a tank through an area of $0.018\ m^2$ with a velocity of $4.6\ m/s$ and a density of $15.5\ kg/m^3$. Air leaves with a velocity of $1.5\ m/s$ and a density equal to that in the tank. The initial density of the air in the tank is $10.3\ kg/m^3$. The total tank volume is $0.6\ m^3$ and the exit area is $0.04\ m^2$. Find the initial rate of change of density in the tank.

Given: Tank inlet area: $A_1 = 0.018\ m^2$. Inlet air velocity: $V_1 = 4.6\ \frac{m}{s}$.

Inlet air density: $\rho_1 = 15.5\ kg/m^3$. Tank outlet area: $A_2 = 0.04\ m^2$. Outlet air velocity: $V_2 = 1.5\ \frac{m}{s}$.

Initial air density: $\rho_0 = 10.3\ kg/m^3$. Tank volume: $V = 0.6\ m^3$.

Find: Initial change rate of density in the tank $\frac{\partial \rho_0}{\partial t}$.

Assumption: (1) density is uniform in tha tank.

(2) flow is uniform at inlet and outlet sections.

Solution:

Choose the tank as the control volume we have:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

At the intial time we have:

$$0 = \frac{\partial}{\partial t}(\rho_0 \forall) - \rho_1 V_1 A_1 + \rho_2 V_2 A_2$$

$$0 = \forall \frac{\partial \rho_0}{\partial t} + \rho_0 \frac{\partial \forall}{\partial t} - \rho_1 V_1 A_1 + \rho_2 V_2 A_2$$

So we have:

$$\frac{\partial \forall}{\partial t} = 0$$

$$\rho_2 = \rho_0$$

The initial rate of density change in the tank is:

$$\frac{\partial \rho_0}{\partial t} = \frac{\rho_1 V_1 A_1 - \rho_2 V_2 A_2}{\forall}$$

$$\frac{\partial \rho_0}{\partial t} = \frac{15.5\ \frac{kg}{m^3} \times 4.6\ \frac{m}{s} \times 0.018\ m^2 - 10.3\ \frac{kg}{m^3} \times 1.5\ \frac{m}{s} \times 0.04\ \ m^2}{0.6\ m^3} = 1.109\ \frac{kg}{m^3 \cdot s}$$

Problem 4.40

(Difficulty: 3)

4.40 A cylindrical tank, of diameter $D = 150\ mm$, drains through an opening, $d = 5mm$, in the bottom of the tank. The speed of the liquid leaving the tank is approximately $V = \sqrt{2gy}$ where y is the height from the tank bottom to the free surface. If the tank is initially filled with water to $y_0 = 0.4m$, determine the water depths at $t = 60\ sec$, $t = 120\ sec$, and $t = 180\ sec$. Plot $y(m)$ versus t for the first 180 sec.

Given: Data on draining of a tank

Find: Depth at various times. Plot of depth versus time.

Assumption: (1) Uniform flow

(2) Incompressible flow

(3) Neglect air density

Solution:

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Treaing the tank as the CV the basic equation becomes:

$$\frac{\partial}{\partial t}\int_0^y \rho \cdot A_{tank} dy + \rho \cdot V \cdot A_{opening} = 0$$

$$\rho \cdot \frac{\pi}{4} \cdot D^2 \cdot \frac{dy}{dt} + \rho \cdot \frac{\pi}{4} \cdot d^2 \cdot V = 0$$

Using

$$V = \sqrt{2gy}$$

and simplifying the equation:

$$\frac{dy}{dt} = -\left(\frac{d}{D}\right)^2 \sqrt{2g} y^{\frac{1}{2}}$$

Seperating variables

$$\frac{dy}{y^{\frac{1}{2}}} == -\left(\frac{d}{D}\right)^2 \sqrt{2g} dt$$

Integrating the equation:

$$2\left(y^{\frac{1}{2}} - y_0^{\frac{1}{2}}\right) = -\left(\frac{d}{D}\right)^2 \sqrt{2g}t$$

$$y^{\frac{1}{2}} = -\left(\frac{d}{D}\right)^2 \sqrt{\frac{g}{2}}t + y_0^{\frac{1}{2}}$$

$$y^{\frac{1}{2}} = y_0^{\frac{1}{2}}\left(1 - \left(\frac{d}{D}\right)^2 \sqrt{\frac{g}{2y_0}}t\right)$$

So we have:

$$y = y_0\left(1 - \left(\frac{d}{D}\right)^2 \sqrt{\frac{g}{2y_0}}t\right)^2$$

As we know:

$$y_0 = 0.4\ m$$

$$\frac{d}{D} = \frac{5\ mm}{150\ mm} = \frac{1}{30}$$

At $t = 60\ s$, we have:

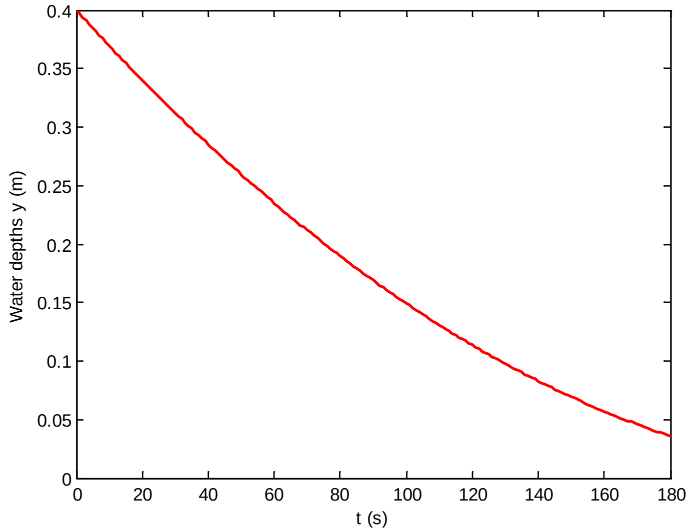
$$y = 0.235\ m$$

At $t = 120\ s$, we have:

$$y = 0.1137\ m$$

At $t = 180\ s$, we have:

$$y = 0.0359\ m$$

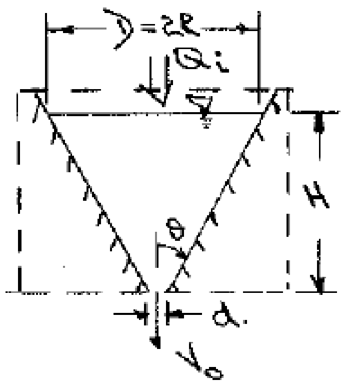


0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Water depths y (m)
0
20
40
60
80
100
120
140
160
180
t (s)

Problem 4.41

(Difficulty: 3)

4.41 A conical flask contains water to height $H = 36.8\ mm$, where the flask diameter is $D = 29.4\ mm$. Water drains out through a smoothly rounded hole of diameter $d = 7.35\ mm$ at the apex of the cone. The flow speed at the exit is $V = \sqrt{2gy}$, where y is the height of the liquid free surface above the hole. A stream of water flows into the top of the flask at constant volume flow rate, $Q = 3.75 \times 10^{-7}\ \frac{m^3}{hr}$. Find the volume flow rate from the bottom of the flask. Evaluate the direction and rate of change of water surface level in the flask at this instant.



Given: Water Height: $H = 36.8\ mm$. Flask diameter: $D = 29.4\ mm$.

Diameter of round hole: $d = 7.35\ mm$. The speed at exit: $V = \sqrt{2gy}$.

Volumetric flow rate into the flask: $Q_{in} = 3.75 \times 10^{-10}\ \frac{m^3}{hr}$.

Find: The volume flow rate from the bottom of the flask Q_{out}. The direction and rate of change of water surface level $\frac{dy}{dt}$.

Assumption: 1) uniform flow at each section.

2) neglect mass of air.

3) Density is constant

Solution:

For the control volume shown in the figure:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Then we have:

$$0 = \rho \frac{d\forall}{dt} + \rho Q_{out} - \rho Q_{in}$$

$$Q_{out} = V_0 \frac{\pi d^2}{4} = \sqrt{2gH}\frac{\pi d^2}{4} = \sqrt{2 \times 9.81\ \frac{m}{s^2} \times 0.0368\ m} \times \frac{\pi \times (0.00735\ m)^2}{4} = 3.61 \times 10^{-5}\ \frac{m^3}{s}$$

$$Q_{out} = 3.61 \times 10^{-5}\ \frac{m^3}{s} = 0.130\ \frac{m^3}{hr}$$

As we know:

$$\frac{d\forall}{dt} = Q_{in} - Q_{out}$$

$$\forall = \frac{1}{3}\pi R^2 y$$

$$R = y \tan\theta$$

So we have the following equation:

$$\frac{d\frac{1}{3}\pi y^3 \tan\theta \tan\theta}{dt} = Q_{in} - Q_{out}$$

$$\pi y^2 \tan\theta \tan\theta \frac{dy}{dt} = \pi R^2 \frac{dy}{dt} = Q_{in} - Q_{out}$$

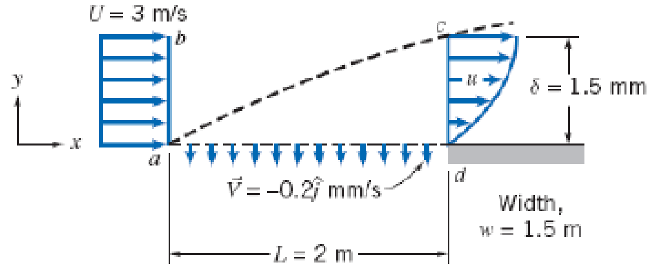
The change rate of the water surface level is:

$$\frac{dy}{dt} = \frac{Q_{in} - Q_{out}}{\pi R^2} = \frac{Q_{in} - Q_{out}}{\pi \frac{D^2}{4}} = \frac{4 \times \left(3.75 \times 10^{-7}\ \frac{m^3}{hr} - 0.130\ \frac{m^3}{hr}\right)}{\pi (0.0294\ m)^2} = -191.5\ \frac{m}{hr} = \ -0.0532\ \frac{m}{s}$$

Problem 4.42

(Difficulty: 2)

4.42 Water flows steadily past a porous flat plate. Constant suction is applied along the porous section. The velocity profile at section cd is $\frac{u}{u_\infty} = 3\left[\frac{y}{\delta}\right] - 2\left[\frac{y}{\delta}\right]^{\frac{3}{2}}$. Evaluate the mass flow rate across the section bc.



Given: The velocity profile at section cd is: $\frac{u}{u_\infty} = 3\left[\frac{y}{\delta}\right] - 2\left[\frac{y}{\delta}\right]^{\frac{3}{2}}$. All the other dimensions and parameters are shown in the figure.

Find: Evaluate the mass flow rate $\dot{m}_{bc}$ across the section bc.

Assumption: (1) steady flow.

(2) incompressible flow.

(3) $\vec{V} = -0.2\,\vec{j}\,\frac{mm}{s}$ along da.

Solution:

Basic Equations: The continuity equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

For steady state we have:

$$0 = \int_{CS} \rho \vec{V} \cdot d\vec{A} = \int_{ab} \rho \vec{V} \cdot d\vec{A} + \dot{m}_{bc} + \int_{cd} \rho \vec{V} \cdot d\vec{A} + \int_{da} \rho \vec{V} \cdot d\vec{A}$$

$$0 = -\rho u_\infty \delta w + \dot{m}_{bc} + \int_0^\delta \rho\, u_\infty \left[3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{\frac{3}{2}}\right] w dy + \rho V_0 w L$$

So we have:

$$\dot{m}_{bc} = \rho u_\infty \delta w - \rho V_0 w L - \rho u_\infty w \delta \int_0^1 \left[3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{\frac{3}{2}} \right] d\left(\frac{y}{\delta}\right)$$

$$\dot{m}_{bc} = \rho w \left\{ u_\infty \delta - u_\infty \delta \left[\frac{3}{2}\left(\frac{y}{\delta}\right)^2 - \frac{2}{2.5}\left(\frac{y}{\delta}\right)^{2.5} \right]_0^1 - V_0 L \right\} = \rho w (0.3 u_\infty \delta - V_0 L)$$

The density for the water is:

$$\rho = 999 \ \frac{kg}{m^3}$$

So the mass flow rate across section bc is:

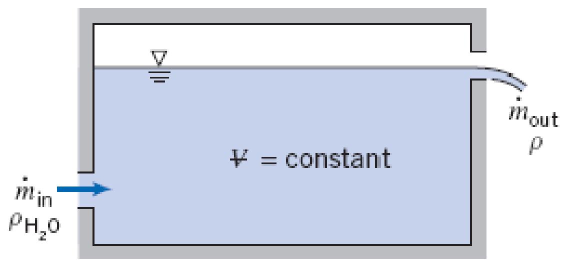
$$\dot{m}_{bc} = 999 \ \frac{kg}{m^3} \times 1.5 \ m \times \left(0.3 \times 3 \frac{m}{s} \times 0.0015 \ m - 0.0002 \ \frac{m}{s} \times 2 \ m \right) = 1.42 \ \frac{kg}{s}$$

The mass flow rate is out of the control volume.

Problem 4.43

(Difficulty: 3)

4.43 A tank of fixed volume contains brine with initial density, ρ_i greater than water. Pure water enters the tank steadily and mixes thoroughly with the brine in the tank. The liquid level in the tank remains constant. Derive expressions for (a) the rate of change of density of the liquid mixture in the tank and (b) the time required for the density to reach the value ρ_f, where $\rho_i > \rho_f > \rho_{H_2O}$.



Given: The initial density: ρ_i.

Find: (a) The rate of density change of liquid mixture. (b) The time required for the density to reach ρ_f.

Assumption: (1) $V_{tank} = constant$.

(2) ρ uniform in the tank.

(3) uniform flows at inlet and outlet sections.

Solution:

(a)

For the control volume shown in the figure, the continuity equation is:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

The tank volume remains constant and so the volume flow rate in and out are related as:

$$V_1 A_1 = V_2 A_2$$

The continuity equation is:

$$0 = \forall \frac{\partial \rho}{\partial t} + \rho V A - \rho_{H_2O} V A$$

Or the rate of change of density with time is

$$\frac{d\rho}{dt} = -\frac{(\rho - \rho_{H_2o})VA}{\forall}$$

(b) We have the relation as:

$$\frac{d\rho}{(\rho - \rho_{H_2o})} = -\frac{VA}{\forall}dt$$

Integrating for both sides from the initial state we have:

$$\int_{\rho_i}^{\rho_f} \frac{d\rho}{(\rho - \rho_{H_2o})} = \int_0^t -\frac{VA}{\forall}dt$$

$$\ln(\rho_f - \rho_{H_2o}) - \ln(\rho_i - \rho_{H_2o}) = -\frac{VA}{\forall}(t - 0)$$

$$\ln\left(\frac{\rho_f - \rho_{H_2o}}{\rho_i - \rho_{H_2o}}\right) = -\frac{VA}{\forall}t$$

Finally we have for the time required for the density to become ρ_f :

$$t = -\frac{\forall}{VA}\ln\left(\frac{\rho_f - \rho_{H_2o}}{\rho_i - \rho_{H_2o}}\right)$$

Note that $\rho_f \rightarrow \rho_{H_2o}$ asymptotically as $t \rightarrow \infty$.

Problem 4.44

(Difficulty: 4)

4.44 A conical funnel of half-angle $\theta = 30°$ drains through a small hole of diameter $d = 6.25\ mm$ at the vertex. The speed of the liquid leaving the funnel is $V = \sqrt{2gy}$, where y is the height of the liquid free surface above the hole. The funnel initially is filled to height $y_0 = 300\ mm$. Obtain an expression for the time, t, for the funnel to completely drain, and evaluate. Find the time to drain from $300\ mm$ to $150\ mm$ (a change in depth of $150\ mm$), and from $150\ mm$ to completely empty (also a change in depth of $150\ mm$). Can you explain the discrepancy in these times? Plot the drain time t as a function diameter d for d ranging from $6.25\ mm$ to $12.5\ mm$.

Given: Data on draining of a funnel

Find: Formula for drain time; time to drain from $12\ in$ to $6\ in$; plot drain time versus hole diameter

Assumption: (1) Uniform flow

(2) Incompressible flow

(3) Neglect air density

Solution:

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Treaing the funnel as the CV the basic equation becomes:

$$\frac{\partial}{\partial t}\int_0^y \rho \cdot A_{funnel} dy + \rho \cdot V \cdot A_{opening} = 0$$

For the funnel

$$A_{funnel} = \pi \cdot r^2 = \pi \cdot (y \cdot \tan(\theta))^2$$

Hence

$$\rho \cdot \pi \cdot \tan(\theta)^2 \cdot \frac{\partial}{\partial t}\int_0^y y^2 dy + \rho \cdot V \cdot \frac{\pi}{4} \cdot d^2 = 0$$

Using

$$V = \sqrt{2gy}$$

$$(\tan(\theta))^2 \cdot \frac{d}{dt}\left(\frac{y^3}{3}\right) = -\sqrt{2gy} \cdot \frac{d^2}{4}$$

Then

$$(\tan(\theta))^2 \cdot y^2 \cdot \frac{dy}{dt} = -\sqrt{2gy} \cdot \frac{d^2}{4}$$

Seperating variables we have:

$$y^{\frac{3}{2}} dy = -\frac{\sqrt{2g}d^2}{4 \cdot (\tan(\theta))^2} dt$$

$$\int_{y_0}^{0} y^{\frac{3}{2}} dy = -\frac{\sqrt{2g}d^2}{4 \cdot (\tan(\theta))^2} t$$

$$\frac{2}{5} y_0^{\frac{5}{2}} = \frac{\sqrt{2g}d^2}{4 \cdot (\tan(\theta))^2} t$$

Then

$$t = \frac{8}{5} \frac{(\tan(\theta))^2 \cdot y_0^{\frac{5}{2}}}{\sqrt{2g}d^2}$$

$$t = \frac{8}{5} \times \frac{(\tan(30°))^2 \cdot (0.3\ m)^{\frac{5}{2}}}{\sqrt{2 \times 9.81\ \frac{m}{s^2}} \times (0.00625\ m)^2} = 152\ s$$

To find the time to drain from 300 mm to 150 mm, we use the time equation with the two depths; this finds the time to drain from 300 mm to 150 mm, the difference is the time we want:

$$\Delta t_1 = \frac{8}{5} \frac{(\tan(\theta))^2 \cdot y_0^{\frac{5}{2}}}{\sqrt{2g}d^2} - \frac{8}{5} \frac{(\tan(\theta))^2 \cdot y_1^{\frac{5}{2}}}{\sqrt{2g}d^2} = 125\ s$$

Similarly from 150 mm to complete empty we have:

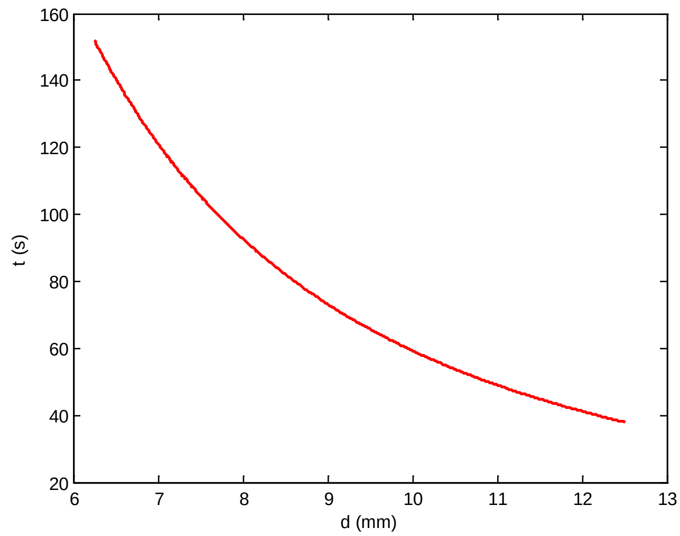
$$\Delta t_2 = \frac{8}{5} \frac{(\tan(\theta))^2 \cdot y_1^{\frac{5}{2}}}{\sqrt{2g}d^2} = 27\ s$$

Note that

$$t = \Delta t_1 + \Delta t_2$$

The second time is a bit shorter because although the flow rate decrease, the area of the funnel does too.

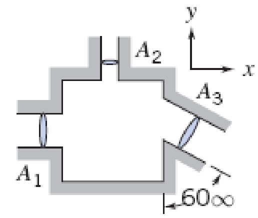
The time t as a function of d ranging from 6.25 mm to 12.5 mm is plotted as:



Problem 4.45* [Difficulty: 3]

*Please Note: this solution shows the solution to the problem with one particular set of values. These values may not match those in the problem you have been given to solve.

4.45 Evaluate the net rate of flux of momentum out through the control surface of Problem 4.22



Given: Data on flow through a control surface

Find: Net rate of momentum flux

Solution:

Basic equation: We need to evaluate $\int_{CS} \vec{V}\rho\vec{V}\cdot dA$

Assumptions: 1) Uniform flow at each section

$V_1 = 10\cdot\frac{ft}{s}$ $A_1 = 0.5\cdot ft^2$ $V_2 = 20\cdot\frac{ft}{s}$ $A_2 = 0.1\cdot ft^2$ $A_3 = 0.6\cdot ft^2$ $V_3 = 5\cdot\frac{ft}{s}$ It is an outlet

Then for the control surface

$$\int_{CS} \vec{V}\rho\vec{V}\cdot dA = \vec{V}_1\rho\vec{V}_1\cdot\vec{A}_1 + \vec{V}_2\rho\vec{V}_2\cdot\vec{A}_2 + \vec{V}_3\rho\vec{V}_3\cdot\vec{A}_3$$
$$= V_1\hat{i}\rho\left(\vec{V}_1\cdot\vec{A}_1\right) + V_2\hat{j}\rho\left(\vec{V}_2\cdot\vec{A}_2\right) + \left[V_3\sin(60)\hat{i} - V_3\cos(60)\hat{j}\right]\rho\left(\vec{V}_3\cdot\vec{A}_3\right)$$
$$= -V_1\hat{i}\rho V_1 A_1 + V_2\hat{j}\rho V_2 A_2 + \left[V_3\sin(60)\hat{i} - V_3\cos(60)\hat{j}\right]\rho V_3 A_3$$
$$= \rho\left[-V_1^2 A_1 + V_3^2 A_3\sin(60)\right]\hat{i} + \rho\left[V_2^2 A_2 - V_3^2 A_3\cos(60)\right]\hat{j}$$

Hence the x component is $\rho\left[-V_1^2 A_1 + V_3^2 A_3\sin(60)\right] =$

$$65\cdot\frac{lbm}{ft^3}\times\left(-10^2\times 0.5 + 5^2\times 0.6\times\sin(60\cdot deg)\right)\cdot\frac{ft^4}{s^2}\times\frac{lbf\cdot s^2}{lbm\cdot ft} = -2406\cdot lbf$$

and the y component is $\rho\left[V_2^2 A_2 - V_3^2 A_3\cos(60)\right] =$

$$65\cdot\frac{lbm}{ft^3}\times\left(20^2\times 0.1 - 5^2\times 0.6\times\cos(60\cdot deg)\right)\cdot\frac{ft^4}{s^2}\times\frac{lbf\cdot s^2}{lbm\cdot ft} = 2113\cdot lbf$$

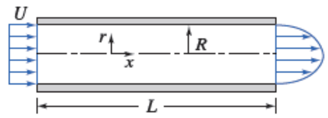
Problem 4.46

(Difficulty: 3)

4.46 Water flows steadily through a pipe of length L and radius $R = 75\ mm$. The velocity distribution across the outlet is given by

$$u = u_{max}\left[1 - \frac{r^2}{R^2}\right]$$

and $u_{max} = 3\ \frac{m}{s}$. Evaluate the ratio of the x-direction momentum flux at the pipe outlet and that at the inlet.



P4.46

Given: Data on flow at inlet and outlet

Find: Ratio of outlet to inlet momentum flux

Assumption: (1) Steady flow

(2) Incompressible flow

Solution:

Continuity equation:

$$0 = \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$-U\pi R^2 + \int_0^R u_{max}\left[1 - \frac{r^2}{R^2}\right] 2\pi r dr = 0$$

$$-U\pi R^2 + 2\pi u_{max}\frac{R^2}{4} = 0$$

Then

$$U = \frac{u_{max}}{2}$$

Momentum flux in x direction at a section:

$$mf_x = \int_A u\rho\vec{V}\cdot d\vec{A}$$

At the inlet we have:

$$mf_{x1} = -\rho U^2\pi R^2$$

$$|mf_{x1}| = \rho U^2\pi R^2$$

At the outlet we have:

$$mf_{x2} = \int_0^R \rho u^2\cdot 2\pi r dr = 2\rho\pi u_{max}^2\int_0^R r\left[1-\frac{r^2}{R^2}\right]^2 dr = 2\rho\pi u_{max}^2\int_0^R \left(r-2\frac{r^3}{R^2}+\frac{r^5}{R^4}\right)dr$$

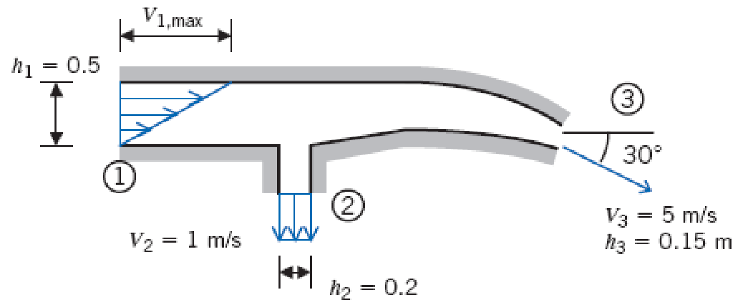
$$mf_{x2} = 2\rho\pi u_{max}^2\left(\frac{R^2}{2}-\frac{R^2}{2}+\frac{R^2}{6}\right) = \frac{\rho\pi u_{max}^2 R^2}{3}$$

Then the ratio of momentum flux is:

$$\frac{|mf_{x2}|}{|mf_{x1}|} = \frac{\frac{\rho\pi u_{max}^2 R^2}{3}}{\rho U^2\pi R^2} = \frac{1}{3}\left(\frac{u_{max}}{U}\right)^2 = \frac{1}{3}\times(2)^2 = 1.33$$

Problem 4.47 [Difficulty: 3]

4.47 Evaluate the net momentum flux through the bend of Problem 4.34 if the depth normal to the diagram is $w = 1$ m.



Given: Data on flow through a bend

Find: Find net momentum flux

Solution:

Basic equations $\int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$ Momentum fluxes: $mf_x = \int_{CS} u \rho \vec{V} \cdot d\vec{A}$ $mf_y = \int_{CS} v \rho \vec{V} \cdot d\vec{A}$

Assumptions: 1) Steady flow 2) Incompressible flow

Evaluating mass flux at 1, 2 and 3 $$-\int_0^{h_1} V_1(y)\cdot w\, dy + V_2\cdot w\cdot h_2 + V_3\cdot w\cdot h_3 = 0$$

or $$V_3\cdot h_3 = \int_0^{h_1} V_1(y)\, dy - V_2\cdot h_2 = \int_0^{h_1} V_{1max}\cdot\frac{y}{h_1}\, dy - V_2\cdot h_2 = \frac{V_{1max}}{h_1}\cdot\frac{h_1^{\,2}}{2} - V_2\cdot h_2$$

Hence $V_{1max} = \frac{2}{h_1}\cdot\left(V_3\cdot h_3 + V_2\cdot h_2\right)$ Using given data $V_{1max} = 3.8\,\frac{m}{s}$

For the x momentum, evaluating at 1, 2 and 3

$$mf_x = -\int_0^{h_1} V_1(y)\cdot\rho\cdot V_1(y)\cdot w\, dy + V_3\cdot\cos(\theta)\cdot\rho\cdot V_3\cdot h_3\cdot w$$

$$mf_x = -\int_0^{h_1} \left(V_{1max}\cdot\frac{y}{h_1}\right)^2\cdot\rho\cdot w\, dy + V_3^{\,2}\cdot\rho\cdot h_3\cdot\cos(\theta)\cdot w = -\frac{V_{1max}^{\,2}}{h_1^{\,2}}\cdot\frac{h_1^{\,3}}{3}\cdot\rho\cdot w + V_3^{\,2}\cdot\rho\cdot h_3\cdot w\cdot\cos(\theta)$$

$$mf_x = \rho\cdot w\cdot\left(-V_{1max}^{\,2}\cdot\frac{h_1}{3} + V_3^{\,2}\cdot\cos(\theta)\cdot h_3\right)$$ Using given data $mf_x = 841\,N$

For the y momentum, evaluating at 1, 2 and 3

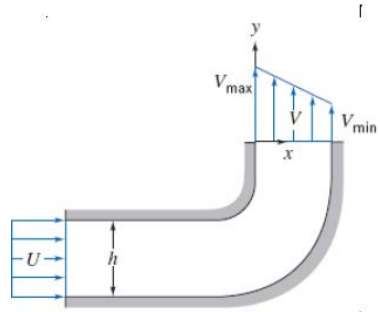
$$mf_y = -V_2\cdot\rho\cdot V_2\cdot h_2\cdot w + V_3\cdot\sin(\theta)\cdot\rho\cdot V_3\cdot h_3\cdot w$$

$$mf_y = \rho\cdot w\cdot\left(-V_2^{\,2}\cdot h_2 - V_3^{\,2}\cdot\sin(\theta)\cdot h_3\right)$$ Using given data $mf_y = -2075\,N$

Problem 4.48

(Difficulty: 2)

4.48 Evaluate the net momentum flux through the channel of Problem 4.35. Would you expect the outlet pressure to be higher, lower, or the same as the inlet pressure? Why?



Find: Would you expect the outlet pressure to be higher, lower or the same as inlet one.

Assumption: (1) incompressible flow

(2) uniform flow at ①

Solution:

The momentum flux is defined as

$$m_1 f = \int \bar{V}(\rho \bar{V} \cdot d\bar{A})$$

The net momentum flux through the CV is

$$m_1 f = \int_{A_1} \bar{V}(\rho \bar{V} \cdot d\bar{A}) + \int_{A_2} \bar{V}(\rho \bar{V} \cdot d\bar{A})$$

where

$$\bar{V}_1 = U\,\hat{\imath}$$

$$\bar{V}_2 = \left\{V_{max} - (V_{max} - V_{min})\frac{x}{h}\right\}\hat{\jmath}$$

$$\bar{V}_2 = \left\{2V_{min} - (V_{min})\frac{x}{h}\right\}\hat{\jmath} = V_{min}\left(2 - \frac{x}{h}\right)\hat{\jmath}$$

$$\int_{A_1} \bar{V}(\rho \bar{V} \cdot d\bar{A}) = \bar{V}_1\{-|\rho V_1 A_1|\} = -\rho U^2 h^2\,\hat{\imath}$$

$$\int_{A_2} \bar{V}(\rho\bar{V}\cdot d\bar{A}) = \int_0^h V_{min}\left(2-\frac{x}{h}\right)\hat{\jmath}\rho V_{min}\left(2-\frac{x}{h}\right)hdx = \hat{\jmath}\rho V_{min}^2 h\int_0^h\left(4-4\frac{x}{h}+\frac{x^2}{h^2}\right)dx$$

$$\int_{A_2} \bar{V}(\rho\bar{V}\cdot d\bar{A}) = \hat{\jmath}\frac{7}{3}\rho V_{min}^2 h^2$$

$$m_1 f = -\rho U^2 h^2\,\hat{\imath} + \frac{7}{3}\rho V_{min}^2 h^2 \hat{\jmath} = \rho h^2\left(-U^2\hat{\imath}+\frac{7}{3}V_{min}^2\hat{\jmath}\right)$$

Evaluating

$$m_1 f = 999\ \frac{kg}{m^3}\times(0.0755\ m)^2\left[-\left(7.5\ \frac{m}{s}\right)^2\hat{\imath}+\frac{7}{3}\left(5\ \frac{m}{s}\right)^2\hat{\jmath}\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$$m_1 f = -320\ \hat{\imath} + 332\ \hat{\jmath}\ N$$

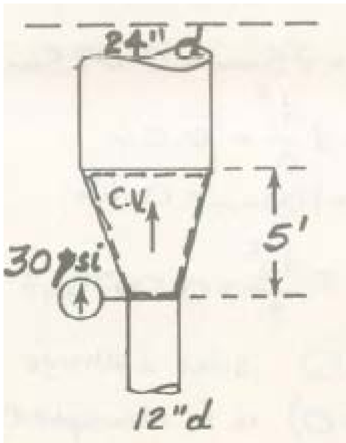
For viscous (real) flow, friction causes a pressure drop in the direction or flow.

For flow in a bend, the streamline curvature results in a pressure gradient normal to the flow.

Problem 4.49

(Difficulty: 3)

4.49 A conical enlargement in a vertical pipeline is 5 ft long and enlarges the pipe from 12 in to 24 in diameter. Calculate the magnitude and direction of the vertical force on this enlargement when 10 $\frac{ft^3}{s}$ of water flow upward through the line and the pressure at the smaller end of the enlargement is 30 psi.



Given: The flow rate: $Q = 10\ \frac{ft^3}{s}$. All the other parameters are shown in the figure.

Find: The magnitude and direction of the vertical force on the enlargement.

Assumptions: The water density is constant.

The flow is steady

Solution:

Basic equations:

Continuity:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation

$$F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho \bar{V} \cdot d\bar{A}$$

Thus for steady, incompressible flow

$$Q = V_1 A_1 = V_2 A_2$$

The area for the inlet section and outlet section are:

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times \left(\frac{12}{12}\ ft\right)^2 = 0.785\ ft^2$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times \left(\frac{24}{12}\ ft\right)^2 = 3.14\ ft^2$$

The velocities for the inlet and outlet section are:

$$V_1 = \frac{Q}{A_1} = \frac{10\ \frac{ft^3}{s}}{0.785\ ft^2} = 12.73\ \frac{ft}{s}$$

$$V_2 = \frac{Q}{A_2} = \frac{10\ \frac{ft^3}{s}}{3.14\ ft^2} = 3.18\ \frac{ft}{s}$$

The pressure at the smaller end is:

$$p_1 = 30\ psi = 4320\ \frac{lbf}{ft^2}$$

From the Bernoulli equation:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g(0) = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gh$$

$$h = 5\ ft$$

$$\rho = 1.94\ \frac{slug}{ft^3} = 1.94\ \frac{lbf \cdot s^2}{ft^4}$$

$$p_2 = p_1 + \frac{\rho}{2}(V_1^2 - V_2^2) - \gamma h = 4320\ \frac{lbf}{ft^2} + \frac{1.94}{2} \frac{lbf \cdot s^2}{ft^4} \times (12.73^2 - 3.18^2) \frac{ft^2}{s^2} - 62.4 \frac{lbf}{ft^3} \times 5ft$$

$$p_2 = 4156\ \frac{lbf}{ft^2}$$

The volume for the enlargement is:

$$V = \frac{1}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right) h = 9.16\ ft^3$$

From the momentum equation, we have:

$$R_y + F_{sy} + F_{By} = -{V_1}^2 \rho A_1 + {V_2}^2 \rho A_2$$

$$R_y + p_1 A_1 - p_2 A_2 - \gamma V = -{V_1}^2 \rho A_1 + {V_2}^2 \rho A_2$$

$$R_y = -{V_1}^2 \rho A_1 + {V_2}^2 \rho A_2 + p_2 A_2 - p_1 A_1 + \gamma V$$

$$R_y = 10060\ lbf$$

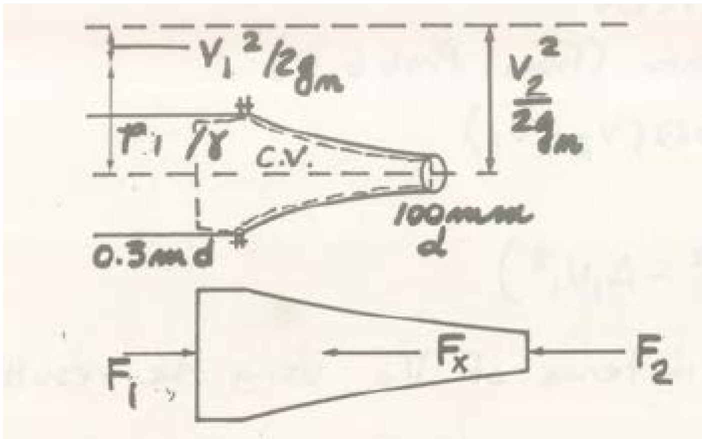
So we have:

$$K_y = -R_y = -10060\ lbf \text{ (direction is going down)}$$

Problem 4.50

(Difficulty: 2)

4.50 A $100\ mm$ nozzle is bolted with 6 bolts to the flange of a $300\ mm$ horizontal pipeline and discharges water into the atmosphere. Calculate the tension load on each bolt when the pressure in the pipe is $600\ kPa$. Neglect vertical forces.



Given: All the parameters are shown in the figure.

Find: The tension load on each bolt.

Assumption: Density is constant

Flow is steady

Solution:

Basic equations:

Continuity:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Thus for steady incompressible flow

$$Q = V_1 A_1 = V_2 A_2$$

The area for the inlet section and outlet section are:

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.3\ m)^2 = 0.071\ m^2$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.1\ m)^2 = 0.0079\ m^2$$

$$V_1 = \frac{V_2 A_2}{A_1} = \frac{1}{9} V_2$$

The pressure at the inlet and outlet is:

$$p_1 = 600\ kPa$$

$$p_2 = 0\ Pa$$

From the Bernoulli equation:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g(0) = \frac{0}{\rho} + \frac{V_2^2}{2} + g(0)$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} = \frac{V_2^2}{2} = \frac{81\ V_1^2}{2}$$

$$\rho = 999\ \frac{kg}{m^3}$$

$$V_1^2 = \frac{p_1}{40\rho}$$

$$V_1 = \sqrt{\frac{p_1}{40\rho}} = \sqrt{\frac{600 \times 10^3 Pa}{40 \times 999\ \frac{kg}{m^3}}} = 3.87\ \frac{m}{s}$$

$$V_2 = 9V_1 = 9 \times 3.87\ \frac{m}{s} = 34.8\ \frac{m}{s}$$

The mass flow rate is

$$\dot{m} = \rho V_1 A_1 = 999\ \frac{kg}{m^3} \times 3.87\ \frac{m}{s} \times 0.071\ m^2 = 275\ \frac{kg}{s}$$

From the momentum equation, we have:

$$R_x + F_{sx} + F_{Bx} = -V_1 \dot{m} + V_2 \dot{m}$$

$$R_x + p_1 A_1 - p_2 A_2 = -V_1 \dot{m} + V_2 \dot{m}$$

$$R_x = \dot{m}(V_2 - V_1) - p_1 A_1$$

$$R_x = -34100\ N$$

So we have:

$$K_x = -R_x = 34100\ N$$

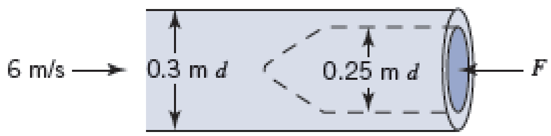
The force on each bolt is (direction to the right):

$$F_x = \frac{K_x}{6} = \frac{34100\ N}{6} = 5680\ N = 5.68\ kN$$

Problem 4.51

(Difficulty: 3)

4.51 The projectile partially fills the end of the $0.3\ m$ pipe. Calculate the force required to hold the projectile in position when the mean velocity in the pipe is $6\ \frac{m}{s}$.



Given: The mean velocity in the pipe $V_1 = 6\ \frac{m}{s}$. All the other parameters are shown in the figure.

Find: The force required to hold the projectile in position.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equations:

Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

For steady incompressible flow

$$Q = V_1 A_1 = V_2 A_2$$

The area for the inlet section and outlet section are:

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.3\ m)^2 = 0.071\ m^2$$

$$A_2 = A_1 - \frac{\pi}{4} D_2^2 = 0.071\ m^2 - \frac{\pi}{4} \times (0.25\ m)^2 = 0.0219\ m^2$$

$$V_1 = 6\ \frac{m}{s}$$

$$V_2 = \frac{V_1 A_1}{A_2} = \frac{6\ \frac{m}{s} \times 0.071\ m^2}{0.0219\ m^2} = 19.45\ \frac{m}{s}$$

The pressure at the outlet is:

$$p_2 = 0\ Pa$$

From the Bernoulli equation:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g(0) = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g(0)$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} = \frac{V_2^2}{2}$$

$$\rho = 999\ \frac{kg}{m^3}$$

$$p_1 = \frac{\rho}{2}(V_2^2 - V_1^2) = \frac{999\ \frac{kg}{m^3}}{2} \times \left(\left(19.45\ \frac{m}{s}\right)^2 - \left(6\ \frac{m}{s}\right)^2\right) = 180\ kPa$$

The mass flow rate is

$$\dot{m} = \rho V_1 A_1 = 999\ \frac{kg}{m^3} \times 6\ \frac{m}{s} \times 0.071\ m^2 = 426\ \frac{kg}{s}$$

From the momentum equation, we have:

$$R_x + F_{sx} + F_{Bx} = -V_1 \dot{m} + V_2 \dot{m}$$

$$R_x + p_1 A_1 - p_2 A_2 = -V_1 \dot{m} + V_2 \dot{m}$$

$$R_x = \dot{m}(V_2 - V_1) - p_1 A_1$$

$$R_x = -7050\ N$$

So we have:

$$F_x = -R_x = 7050\ N$$

Problem 4.52 [Difficulty: 1]

4.52 Considering that in the fully developed region of a pipe, the integral of the axial momentum is the same at all cross sections, explain the reason for the pressure drop along the pipe.

Given: Fully developed flow in pipe

Find: Why pressure drops if momentum is constant

Solution:

Basic equation: Momentum flux in x direction

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!\!/ + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Fully developed flow

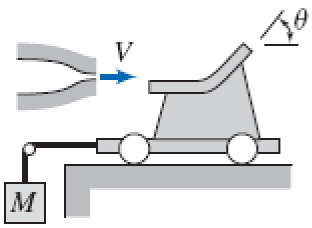
Hence $F_x = \frac{\Delta p}{L} - \tau_w \cdot A_s = 0$ $\Delta p = L \cdot \tau_w \cdot A_s$

where Δp is the pressure drop over length L, τ_w is the wall friction and As is the pipe surface area

The sum of forces in the x direction is zero. The friction force on the fluid is in the negative x direction, so the net pressure force must be in the positive direction. Hence pressure drops in the x direction so that pressure and friction forces balance

Problem 4.53 [Difficulty: 2]

4.53 A jet of water issuing from a stationary nozzle at 10 m/s ($A_j = 0.1\ \text{m}^2$) strikes a turning vane mounted on a cart as shown. The vane turns the jet through angle $\theta = 40°$. Determine the value of M required to hold the cart stationary. If the vane angle θ is adjustable, plot the mass, M, needed to hold the cart stationary versus θ for $0 \le \theta \le 180°$.



Given: Nozzle hitting stationary cart

Find: Value of M to hold stationary; plot M versu θ

Solution:

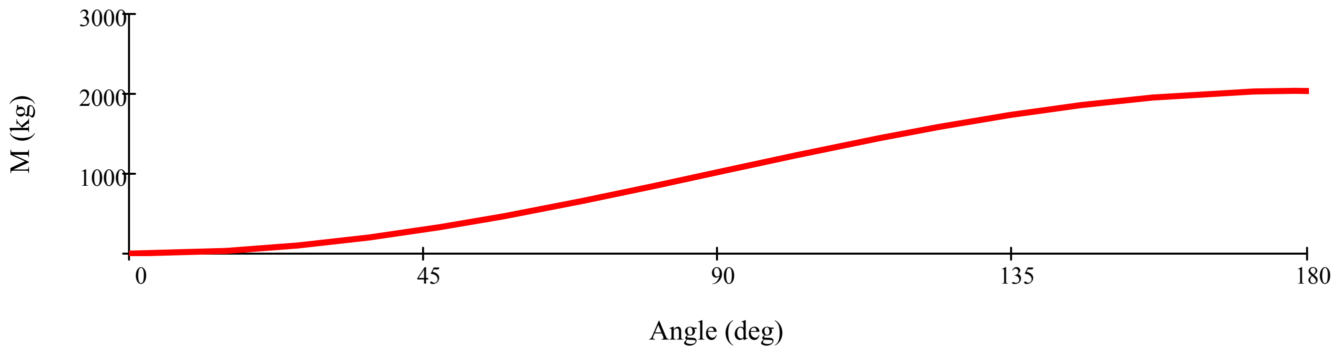
Basic equation: Momentum flux in x direction for the tank

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, dV\!\!\!/ + \sum_{CS} u\,\rho \vec{V}\cdot\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure throughout 4) Uniform flow 5) Exit velocity is V

Hence $\quad R_x = -M\cdot g = V\cdot\rho\cdot(-V\cdot A) + V\cdot\cos(\theta)\cdot(V\cdot A) = \rho\cdot V^2\cdot A\cdot(\cos(\theta) - 1) \qquad M = \frac{\rho\cdot V^2\cdot A}{g}\cdot(1 - \cos(\theta))$

When θ = 40º $\quad M = \frac{s^2}{9.81\cdot m} \times 1000\cdot\frac{kg}{m^3} \times \left(10\cdot\frac{m}{s}\right)^2 \times 0.1\cdot m^2 \times (1 - \cos(40\cdot deg)) \qquad M = 238\ kg$

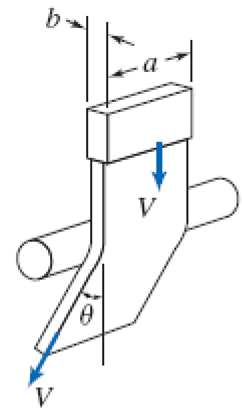


This graph can be plotted in *Excel*

Problem 4.54

[Difficulty: 2]

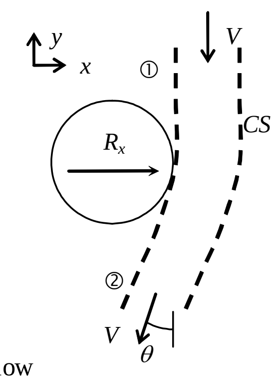
4.54 A circular cylinder inserted across a stream of flowing water deflects the stream through angle θ, as shown. (This is termed the "Coanda effect.") For $a = 12.5$ mm, $b = 2.5$ mm, $V = 3$ m/s, and $\theta = 20°$, determine the horizontal component of the force on the cylinder caused by the flowing water.



Given: Water flowing past cylinder

Find: Horizontal force on cylinder

Solution:



Basic equation: Momentum flux in x direction

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \sum_{CS} u\,\rho \vec{V}\cdot\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure throughout 4) Uniform flow

Hence $\quad R_x = u_1\cdot\rho\cdot(-u_1\cdot A_1) + u_2\cdot\rho\cdot(u_2\cdot A_2) = 0 + \rho\cdot(-V\cdot\sin(\theta))\cdot(V\cdot a\cdot b) \qquad R_x = -\rho\cdot V^2\cdot a\cdot b\cdot\sin(\theta)$

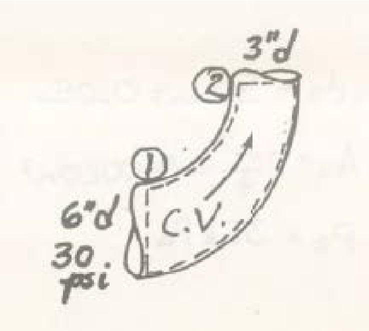
For given data $\quad R_x = -1000\cdot\frac{kg}{m^3}\times\left(3\cdot\frac{m}{s}\right)^2\times 0.0125\cdot m\times 0.0025\cdot m\times\sin(20\cdot deg)\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_x = -0.0962\,N$

This is the force on the fluid (it is to the left). Hence the force on the cylinder is $\quad R_x = -R_x \qquad R_x = 0.0962\,N$

Problem 4.55

(Difficulty: 3)

4.55 A 6 in horizontal pipeline bends through 90° and while bending changes its diameter to 3 in. The pressure in the 6 in pipe is 30 psi. Calculate the magnitude and direction of the force on the bend when 2.0 $\frac{ft^3}{s}$ of water flow therein. Both pipes are in the same horizontal plane.



Given: The pressure at inlet: $p_1 = 30\ psi$. The flow rate: $Q = 2.0\ \frac{ft^3}{s}$. All the other parameters are shown in the figure.

Find: The force on the bend.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equation:

Continuity equation

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation in the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Momentum equation in the y-direction

$$F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho \bar{V} \cdot d\bar{A}$$

For steady incompressible flow we have

$$Q = V_1 A_1 = V_2 A_2$$

The area for the inlet section and outlet section are:

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times \left(\frac{6}{12}\, ft\right)^2 = 0.1963\, ft^2$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times \left(\frac{3}{12}\, ft\right)^2 = 0.0491\, ft^2$$

$$V_1 = \frac{Q}{A_1} = \frac{2.0\, \frac{ft^3}{s}}{0.1963\, ft^2} = 10.19\, \frac{ft}{s}$$

$$V_2 = \frac{Q}{A_2} = \frac{2.0\, \frac{ft^3}{s}}{0.0491\, ft^2} = 40.7\, \frac{ft}{s}$$

The pressure at the inlet is:

$$p_1 = 30\, psi = 4320\, \frac{lbf}{ft^2}$$

From the Bernoulli equation:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g(0) = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g(0)$$

$$p_2 = p_1 + \frac{\rho}{2}(V_1^2 - V_2^2) = 4320\, \frac{lbf}{ft^2} + \frac{1.94\, \frac{slug}{ft^3}}{2} \times \left[\left(10.19\, \frac{ft}{s}\right)^2 - \left(40.7\, \frac{ft}{s}\right)^2\right]$$

$$p_2 = 4320\, \frac{lbf}{ft^2} + \frac{1.94\, \frac{lbf \cdot s^2}{ft^4}}{2} \times \left[\left(10.19\, \frac{ft}{s}\right)^2 - \left(40.7\, \frac{ft}{s}\right)^2\right] = 2810\, \frac{lbf}{ft^2}$$

The mass flow rate is

$$\dot{m} = \rho V_1 A_1 = 1.94\, \frac{lbf \cdot s^2}{ft^4} \times 10.19\, \frac{ft}{s} \times 0.1963\, ft^2 = 3.88\, \frac{lbf \cdot s}{ft}$$

From the x momentum equation, we have:

$$R_x + F_{sx} + F_{Bx} = -V_1 \dot{m}$$

$$R_x + p_1 A_1 = -V_1 \dot{m}$$

$$R_x = -V_1\dot{m} - p_1A_1 = -10.19\ \frac{ft}{s} \times 3.88\ \frac{lbf \cdot s}{ft} - 4320\ \frac{lbf}{ft^2} \times 0.1963\ ft^2 = -888\ \text{lbf}$$

$$F_x = -R_x = 888\ lbf$$

From the y momentum equation, we have:

$$R_y + F_{sy} + F_{By} = V_2\dot{m}$$

$$R_y - p_2A_2 = V_2\dot{m}$$

$$R_y = V_2\dot{m} + p_2A_2 = 40.7\ \frac{ft}{s} \times 3.88\ \frac{lbf \cdot s}{ft} + 2810\ \frac{lbf}{ft^2} \times 0.0491\ ft^2 = 296\ lbf$$

$$F_y = -R_y = -296\ lbf$$

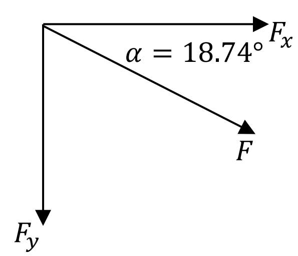
So the force can be computed by:

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(888\ lbf)^2 + (-296\ lbf)^2} = 936\ lbf$$

The direction is calculated by (shown in the figure):

$$\tan\alpha = \frac{F_y}{F_x} = 0.3333$$

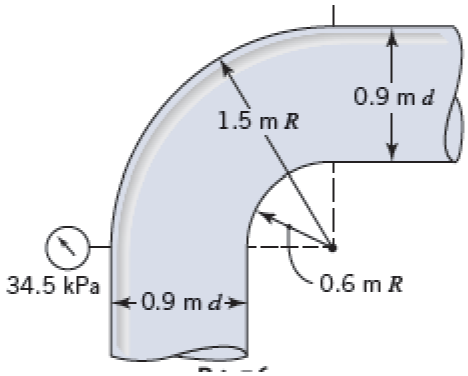
$$\alpha = 18.74°$$



Problem 4.56

(Difficulty: 2)

4.56 The axes of the pipes are in a vertical plane. The flow rate is $2.83 \frac{m^3}{s}$ of water. Calculate the magnitude, direction, and location of the resultant force of the water on the pipe bend.



Given: The flow rate: $Q = 2.83 \frac{m^3}{s}$. All the other parameters are shown in the figure.

Find: The force on the bend.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equations:

Continuity equation

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation in the x-direction

$$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Momentum equation in the y-direction

$$F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho \bar{V} \cdot d\bar{A}$$

For steady incompressible flow

$$Q = V_1 A_1 = V_2 A_2$$

The area for the inlet section and outlet section are:

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.9\ m)^2 = 0.636\ m^2$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.9\ m)^2 = 0.636\ m^2$$

$$V_1 = \frac{Q}{A_1} = \frac{2.83\ \frac{m^3}{s}}{0.636\ m^2} = 4.45\ \frac{m}{s}$$

$$V_2 = \frac{Q}{A_2} = \frac{2.83\ \frac{m^3}{s}}{0.636\ m^2} = 4.45\ \frac{m}{s}$$

The pressure at the inlet is:

$$p_1 = 34.5\ kPa$$

From the Bernoulli equation:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g(0) = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gh$$

$$h = R + \frac{D_2}{2} = 0.6\ m + 0.45\ m = 1.05\ m$$

$$p_2 = p_1 - \gamma h = 34.5\ kPa - 9.81\ \frac{kN}{m^3} \times 1.05m = 24.2\ kPa$$

The mass flow rate is

$$\dot{m} = \rho V_1 A_1 = 999\ \frac{kg}{m^3} \times 4.45\ \frac{m}{s} \times 0.636\ m^2 = 2827\ \frac{kg}{s}$$

From the x momentum equation, we have:

$$R_x + F_{sx} + F_{Bx} = V_2 \dot{m}$$

$$R_x - p_2 A_2 = V_2 \dot{m}$$

$$R_x = V_2 \dot{m} + p_2 A_2 = 4.45\ \frac{m}{s} \times 2827\ \frac{kg}{s} + 24.2 \times 10^3 Pa \times 0.636\ m^2 = 28000\ \mathrm{N}$$

$$F_x = -R_x = -28000\ \mathrm{N}$$

From the y momentum equation, we have:

$$R_y + F_{sy} - F_{By} = -V_1\dot{m}$$

$$R_y + p_1A_1 - F_{By} = -V_1\dot{m}$$

$$F_{By} = \rho g A_1 \frac{2\pi}{4} h = 10290\ N$$

$$R_y = -V_1\dot{m} - p_1A_1 + F_{By} = -4.45\ \frac{m}{s} \times 2827\ \frac{kg}{s} - 34.5 \times 10^3 Pa \times 0.636\ m^2 + 10290\ N$$

$$R_y = -24200\ N$$

$$F_y = 24200\ N$$

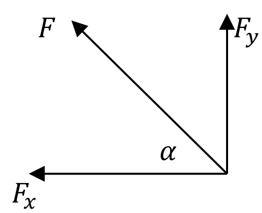
So the force can be computed by:

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-28000\ N)^2 + (24200)^2} = 37000\ N$$

The direction is calculated by (shown in the figure):

$$\tan\alpha = \frac{F_y}{F_x} = 0.8643$$

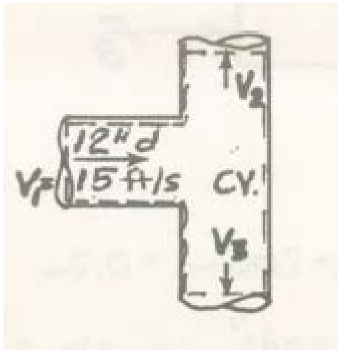
$$\alpha = 49.52°$$



Problem 4.57

(Difficulty: 2)

4.57 Water flows through a tee in a horizontal pipe system. The velocity in the stem of the tee is $15\ \frac{ft}{s}$, and the diameter is $12\ in$. Each branch is of $6\ in$ diameter. If the pressure in the stem is $20\ psi$, calculate magnitude and direction of the force of the water on the tee if the flow rate in the branches are the same.



Given: The diameter: $D_1 = 12\ in.$ $D_2 = D_3 = 6\ in$ The pressure in the stem is: $p_1 = 20\ psi$. All the other parameters are shown in the figure.

Find: The force on the tee.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equation:

Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the y-direction

$$F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho \bar{V} \cdot d\bar{A}$$

The volume flow rates for steady incompressible flow are related as

$$Q = V_2 A_2 + V_3 A_3 = 2V_2 A_2$$

The area for the inlet section and outlet section are:

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times \left(\frac{12}{12}\ ft\right)^2 = 0.785\ ft^2$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times \left(\frac{6}{12}\ ft\right)^2 = 0.196\ ft^2$$

$$A_3 = A_2 = 0.196\ ft^2$$

The volumetric flow rate:

$$Q = V_1 A_1 = 15\ \frac{ft}{s} \times 0.785\ ft^2 = 11.78\ \frac{ft^3}{s}$$

Thus

$$V_2 = \frac{Q}{2A_2} = \frac{11.78\ \frac{ft^3}{s}}{2 \times 0.196\ ft^2} = 30\ \frac{ft}{s}$$

$$V_3 = 30\ \frac{ft}{s}$$

The pressure at the inlet and outlet are:

$$p_1 = 20\ psi = 2880\ \frac{lbf}{ft^2}$$

$$p_2 = 0$$

$$p_3 = 0$$

The mass flow rate is

$$\dot{m}_1 = \rho V_1 A_1 = 1.94\ \frac{lbf \cdot s^2}{ft^4} \times 15\ \frac{ft}{s} \times 0.785\ ft^2 = 22.8\ \frac{lbf \cdot s}{ft}$$

$$\dot{m}_2 = \dot{m}_3 = \frac{1}{2} \dot{m}_1$$

From the x momentum equation, we have:

$$R_x + F_{sx} + F_{Bx} = -V_1 \dot{m}$$

$$R_x + p_1 A_1 = -V_1 \dot{m}$$

$$R_x = -V_1 \dot{m} - p_1 A_1 = -15\ \frac{ft}{s} \times 22.8 \frac{lbf \cdot s}{ft} - 2880\ \frac{lbf}{ft^2} \times 0.785\ ft^2 = -2602\ \text{lbf}$$

$$F_x = -R_x = 2602\ lbf$$

From the y momentum equation, we have:

$$R_y + F_{sy} + F_{By} = V_2\dot{m}_2 - V_3\dot{m}_3$$

$$R_y = 0$$

$$F_y = 0$$

So we have:

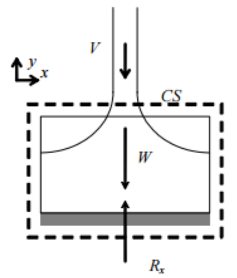
$$F = F_x = 2602\ lbf$$

The direction is to the right.

Problem 4.58

(Difficulty: 4)

4.58 In a laboratory experiment, the water flow rate is to be measured catching the water as it vertically exits a pipe into an empty open tank that is on a zeroed balance. The tank is 10 m directly below the pipe exit, and the pipe diameter is 50 mm. One student obtains a flow rate by noting that after 60 s the volume of water (at 4℃) in the tank was 3 m^3. Another student obtains a flow rate by reading the instantaneous weight accumulated of 3150 kg indicated at the $60-s$ point. Find the mass flow rate each student computes. Why do they disagree? Which one is more accurate? Show that the magnitude of the discrepancy can be explained by any concept you may have.



Given: Water flowing into tank

Find: Mass flow rates estimated by students. Explain discrepancy

Assumption: (1) Steady flow

(2) Incompressible flow

(3) Atmospheric pressure throughout

(4) Uniform flow

Solution:

Basic equation: Momentum flux in y direction:

$$F_y = F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho \vec{V}\cdot d\vec{A}$$

For the first student:

$$m_1 = \frac{\rho V}{t} = 1000\ \frac{kg}{m^3} \times 3\ m^3 \times \frac{1}{60\ s} = 50\ \frac{kg}{s}$$

For the second student:

$$m_2 = \frac{M}{t} = \frac{3150\ kg}{60\ s} = 52.5\ \frac{kg}{s}$$

There is a discrepancy because the second student is measuring instantaneous weight PLUS the force generated as the pipe flow momentum is "killed". To analyse this we first need to find the speed at which the water stream enters the tank, $10\ m$ below the pipe exit. This would be a good place to use the Bernolli equation, but this problem is in the set before the Bernoulli is covered. Instead we use the simple concept that the fluid is failing under gravity (a concluson supported by the Bernoulli equation.) From the equations for fallng under gravity:

$$V_{tank}^2 = V_{pipe}^2 + 2gh$$

Where V_{tank} is the speed entering the tank, V_{pipe} is the speed at the pipe, and $h = 10\ m$ is the distance traveled. V_{pipe} is obtained from :

$$V_{pipe} = \frac{m_1}{\rho \frac{\pi d_{pipe}^2}{4}} = \frac{4v}{\rho \pi d_{pipe}^2}$$

$$V_{pipe} = \frac{4}{\pi} \times 50\ \frac{kg}{s} \times \frac{m^3}{1000\ kg} \times \left(\frac{1}{0.05\ m}\right)^2 = 25.5\ \frac{m}{s}$$

Then

$$V_{tank} = \sqrt{V_{pipe}^2 + 2gh}$$

$$V_{tank} = \sqrt{\left(25.5\ \frac{m}{s}\right)^2 + 2 \times 9.81\ \frac{m}{s^2} \times 10\ m} = 29.1\ \frac{m}{s}$$

We can now use the y momentum equation for the CS shown above:

$$R_y - W = -V_{tank} \cdot \rho(-V_{tank} \cdot A_{tank})$$

Where A_{tank} is the area of the water flow as it enters the tank. But for the water flow

$$V_{tank} \cdot A_{tank} = V_{pipe} \cdot A_{pipe}$$

Hence

$$\Delta W = 1000\ \frac{kg}{m^3} \times 29.1\ \frac{m}{s} \times 25.5\ \frac{m}{s} \times \frac{\pi}{4} \times (0.05\ m)^2 = 1457\ N$$

Inducted as a mass, this is:

$$\Delta m = \frac{\Delta W}{g} = 149\ kg$$

Hence the scale overestimates the weight of water by $1457\ N$, or a mass of $149\ kg$.

For the second student:

$$M = 3150\ kg - 149\ kg = 3001\ kg$$

Hence

$$m_2 = \frac{M}{t}$$

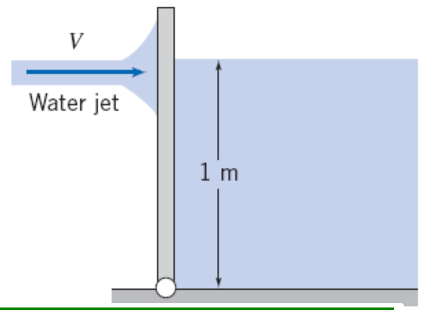
Where m_2 represents mass flow rate,

$$m_2 = \frac{3001\ kg}{60\ s} = 50\ \frac{kg}{s}$$

Comparing with the answer obtained from student 1, we see the students now agree! The discrepancy was entirely caused by the fact that the second student was measuring the weight of tank water PLUS the momentum lost by the water as it entered the tank!

Problem 4.59 [Difficulty: 4]

4.59 A gate is 1 m wide and 1.2 m tall and hinged at the bottom. On one side the gate holds back a 1-m-deep body of water. On the other side, a 5-cm diameter water jet hits the gate at a height of 1 m. What jet speed V is required to hold the gate vertical? What will the required speed be if the body of water is lowered to 0.5 m? What will the required speed be if the water level is lowered to 0.25 m?



Given:
Gate held in place by water jet

Find:
Required jet speed for various water depths

Solution:

Basic equation: Momentum flux in x direction for the wall

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Note: We use this equation ONLY for the jet impacting the wall. For the hydrostatic force and location we use computing equations

$$F_R = p_c \cdot A \qquad y' = y_c + \frac{I_{xx}}{A\cdot y_c}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Hence $$R_x = V\cdot\rho\cdot\left(-V\cdot A_{jet}\right) = -\rho\cdot V^2\cdot\frac{\pi\cdot D^2}{4}$$

This force is the force generated by the wall on the jet; the force of the jet hitting the wall is then

$$F_{jet} = -R_x = \rho\cdot V^2\cdot\frac{\pi\cdot D^2}{4}$$ where D is the jet diameter

For the hydrostatic force $$F_R = p_c\cdot A = \rho\cdot g\cdot\frac{h}{2}\cdot h\cdot w = \frac{1}{2}\cdot\rho\cdot g\cdot w\cdot h^2 \qquad y' = y_c + \frac{I_{xx}}{A\cdot y_c} = \frac{h}{2} + \frac{\frac{w\cdot h^3}{12}}{w\cdot h\cdot\frac{h}{2}} = \frac{2}{3}\cdot h$$

where h is the water depth and w is the gate width

For the gate, we can take moments about the hinge to obtain $$-F_{jet}\cdot h_{jet} + F_R\cdot(h - y') = -F_{jet}\cdot h_{jet} + F_R\cdot\frac{h}{3} = 0$$

where h_{jet} is the height of the jet from the ground

Hence $$F_{jet} = \rho\cdot V^2\cdot\frac{\pi\cdot D^2}{4}\cdot h_{jet} = F_R\cdot\frac{h}{3} = \frac{1}{2}\cdot\rho\cdot g\cdot w\cdot h^2\cdot\frac{h}{3} \qquad V = \sqrt{\frac{2\cdot g\cdot w\cdot h^3}{3\cdot\pi\cdot D^2\cdot h_j}}$$

For the first case (h = 1 m) $$V = \sqrt{\frac{2}{3\cdot\pi}\times 9.81\cdot\frac{m}{s^2}\times 1\cdot m\times(1\cdot m)^3\times\left(\frac{1}{0.05\cdot m}\right)^2\times\frac{1}{1\cdot m}} \qquad V = 28.9\,\frac{m}{s}$$

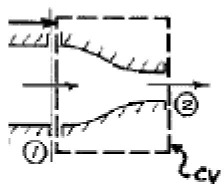
For the second case (h = 0.5 m) $$V = \sqrt{\frac{2}{3\cdot\pi}\times 9.81\cdot\frac{m}{s^2}\times 1\cdot m\times(0.5\cdot m)^3\times\left(\frac{1}{0.05\cdot m}\right)^2\times\frac{1}{1\cdot m}} \qquad V = 10.2\,\frac{m}{s}$$

For the first case (h = 0.25 m) $$V = \sqrt{\frac{2}{3\cdot\pi}\times 9.81\cdot\frac{m}{s^2}\times 1\cdot m\times(0.25\cdot m)^3\times\left(\frac{1}{0.05\cdot m}\right)^2\times\frac{1}{1\cdot m}} \qquad V = 3.61\,\frac{m}{s}$$

Problem 4.60

(Difficulty: 2)

4.60 Water flows steadily through a fire hose and nozzle. The hose is 35-mm-ID and the nozzle tip is 25-mm-ID; water gage pressure in the hose is 510 kPa, and the stream leaving the nozzle is uniform. The exit speed and pressure are 32 $\frac{m}{s}$ and atmospheric, respectively. Find the force transmitted by the coupling between the nozzle and hose. Indicate whether the coupling is in tension or compression.



Given: The diameter of hose: $D_1 = 35\ mm$; Gage pressure in the hose: $p_1 = 510\ kPa$;

The exit speed: $V_2 = 32\ \frac{m}{s}$; The diameter of nozzle tip: $D_2 = 25\ mm$;

The pressure at exit: $p_2 = p_{atm}$

Find: The force transmitted by the coupling between the nozzle and hose.

Assumption: (1) Steady flow

(2) Uniform flow at each section

(3) Incompressible flow

(4) $F_{ex} = 0$

Solution:

Basic equations:

Continuity:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Momentum equation in x-direction:

$$R_x + F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{V} \cdot d\vec{A}$$

The pressure force is:

$$F_{sx} = p_1 A_1$$

$$F_{Bx} = 0$$

For incompressible steady flow we have:

$$0 = -\rho_1 V_1 A_1 + \rho_2 V_2 A_2$$

$$V_1 = V_2 \frac{A_2}{A_1} = V_2 \left(\frac{D_2}{D_1}\right)^2 = 32\ \frac{m}{s} \times \left(\frac{25\ mm}{35\ mm}\right)^2 = 16.33\ \frac{m}{s}$$

$$R_x + p_1 A_1 = V_1(-\rho V_1 A_1) + V_2(\rho V_2 A_2)$$

$$R_x = -p_1 A_1 - V_1(\rho V_1 A_1) + V_2(\rho V_2 A_2) == -p_1 A_1 - V_1(\rho V_2 A_2) + V_2(\rho V_2 A_2)$$

$$R_x = -p_1 A_1 + (\rho V_2 A_2)(V_2 - V_1)$$

The density of water is:

$$\rho = 999\ \frac{kg}{m^3}$$

So we get:

$$R_x = -510000\ \frac{N}{m^2} \times \frac{\pi}{4} \times (0.035\ m)^2 + 999\ \frac{kg}{m^3} \times 32\ \frac{m}{s} \times \frac{\pi}{4} \times (0.025\ m)^2 \times \left(32\ \frac{m}{s} - 16.33\ \frac{m}{s}\right)$$

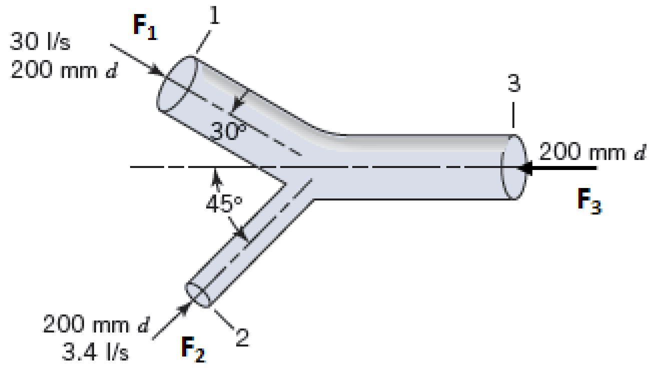
$$R_x = -245\ N$$

The force is on CV to the left, so the coupling must be in tension.

Problem 4.61

(Difficulty: 3)

4.61 Two types of gasoline are blended by passing them through a horizontal "wye" as shown. Calculate the magnitude and direction of the force exerted on the "wye" by the gasoline. The pressure $p_3 = 145\ kPa$.



Given: The pressure $p_3 = 145\ kPa$.All the other parameters are shown in the figure.

Find: The force on the bend.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation;

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the y-direction

$$F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho \bar{V} \cdot d\bar{A}$$

The area for the inlet section and outlet section are:

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.2\ m)^2 = 0.0314\ m^2$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.1\ m)^2 = 0.0079\ m^2$$

$$A_3 = \frac{\pi}{4} D_3^2 = \frac{\pi}{4} \times (0.2\ m)^2 = 0.0314\ m^2$$

The velocity at each section can be calculated by:

$$V_1 = \frac{Q_1}{A_1} = \frac{30\ \frac{L}{s}}{0.0314\ m^2} = \frac{30 \times 10^{-3}\ \frac{m^3}{s}}{0.0314\ m^2} = 0.955\ \frac{m}{s}$$

$$V_2 = \frac{Q_2}{A_2} = \frac{3.4\ \frac{L}{s}}{0.0079\ m^2} = \frac{3.4 \times 10^{-3}\ \frac{m^3}{s}}{0.0079\ m^2} = 0.430\ \frac{m}{s}$$

$$V_3 = \frac{Q_3}{A_3} = \frac{Q_1 + Q_2}{A_3} = \frac{30 \times 10^{-3}\ \frac{m^3}{s} + 3.4 \times 10^{-3}\ \frac{m^3}{s}}{0.0314\ m^2} = 1.064\ \frac{m}{s}$$

The pressure at the outlet is:

$$p_3 = 145\ kPa$$

The density of the gas:

$$\rho = 680.3\ \frac{kg}{m^3}$$

From the Bernoulli equation:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} = \frac{p_2}{\rho} + \frac{V_2^2}{2} = \frac{p_3}{\rho} + \frac{V_3^2}{2}$$

$$p_1 = p_3 + \frac{\rho}{2}(V_3^2 - V_1^2) = 145\ kPa + \frac{680.3\ \frac{kg}{m^3}}{2} \times \left(\left(1.064\ \frac{m}{s}\right)^2 - \left(0.955\ \frac{m}{s}\right)^2\right) = 145.08\ kPa$$

$$p_2 = p_3 + \frac{\rho}{2}(V_3^2 - V_2^2) = 145\ kPa + \frac{680.3\ \frac{kg}{m^3}}{2} \times \left(\left(1.064\ \frac{m}{s}\right)^2 - \left(0.43\ \frac{m}{s}\right)^2\right) = 145.33\ kPa$$

The mass flow rates are

$$\dot{m}_1 = \rho V_1 A_1 = 680.3\ \frac{kg}{m^3} \times 0.955\ \frac{m}{s} \times 0.0314\ m^2 = 20.40\ \frac{kg}{s}$$

$$\dot{m}_2 = \rho V_2 A_2 = 680.3\ \frac{kg}{m^3} \times 0.430\ \frac{m}{s} \times 0.0079\ m^2 = 2.311\ \frac{kg}{s}$$

$$\dot{m}_3 = \rho V_3 A_3 = 680.3 \ \frac{kg}{m^3} \times 1.064 \ \frac{m}{s} \times 0.0314 \ m^2 = 22.72 \ \frac{kg}{s}$$

From the x momentum equation, we have:

$$R_x + F_{sx} + F_{Bx} = -V_1 \cos 30° \, \dot{m}_1 - V_2 \cos 45° \, \dot{m}_2 + V_3 \dot{m}_3$$

$$R_x + p_1 A_1 \cos 30° + p_2 A_2 \cos 45° - p_3 A_3 = -V_1 \cos 30° \, \dot{m}_1 - V_2 \cos 45° \, \dot{m}_2 + V_3 \dot{m}_3$$

$$R_x = -V_1 \cos 30° \, \dot{m}_1 - V_2 \cos 45° \, \dot{m}_2 + V_3 \dot{m}_3 + p_3 A_3 - p_1 A_1 \cos 30° - p_2 A_2 \cos 45°$$

$$R_x = -197.4 \ N$$

$$F_x = 197.4 \ N$$

From the y momentum equation, we have:

$$R_y + F_{sy} - F_{By} = V_1 \dot{m}_1 \sin 30° - V_2 \dot{m}_2 \sin 45°$$

$$R_y - p_1 A_1 \sin 30° + p_2 A_2 \sin 45° = V_1 \dot{m}_1 \sin 30° - V_2 \dot{m}_2 \sin 45°$$

$$R_y = V_1 \dot{m}_1 \sin 30° - V_2 \dot{m}_2 \sin 45° + p_1 A_1 \sin 30° - p_2 A_2 \sin 45°$$

$$R_y = 1475 \ N$$

$$F_y = -1475 \ N$$

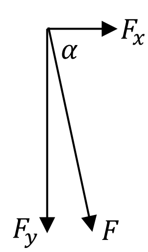
So the force can be computed by:

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(197.4 \ N)^2 + (-1475 \ N)^2} = 1488 \ N$$

The direction is calculated by (shown in the figure):

$$\tan \alpha = \frac{F_y}{F_x} = 7.4721$$

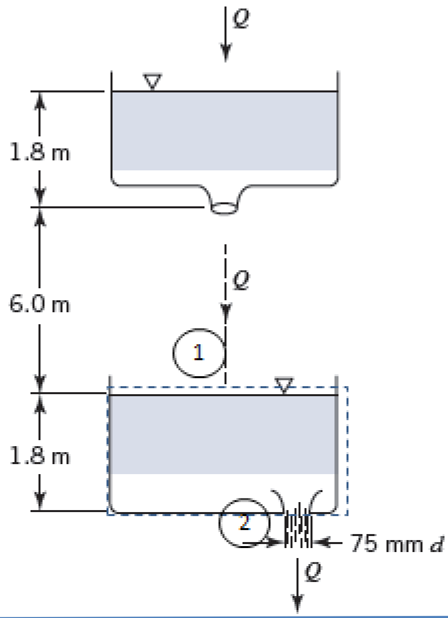
$$\alpha = 82.4°$$



Problem 4.62

(Difficulty: 2)

4.62 The lower tank weighs $224\ N$, and the water in it weighs $897\ N$. If this tank is on a platform scale, what weight will register on the scale beam?



Given: Tank weight: $F_{tank} = 224\ N$. Water weight: $F_{water} = 897\ N$. All the other parameters are shown in the figure.

Find: The weight on the scale beam.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equation:

Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation;

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation for the y-direction

$$F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho\bar{V}\cdot d\bar{A}$$

For the upper surface of the lower tank, from Bernoulli equation we have:

$$\frac{V_1^2}{2} - gh_1 = 0$$

$$h_1 = 7.8\ m$$

$$V_1 = \sqrt{2gh_1} = \sqrt{2\times 9.81\ \frac{m}{s^2}\times 7.8\ m} = 12.36\ \frac{m}{s}$$

For the bottom of the lower tank, we have:

$$\frac{V_2^2}{2} - gh_2 = 0$$

$$h_2 = 1.8\ m$$

$$V_2 = \sqrt{2gh_2} = \sqrt{2\times 9.81\ \frac{m}{s^2}\times 1.8\ m} = 5.94\ \frac{m}{s}$$

The mass flow rate of the lower tank is:

$$\dot{m} = \rho V_2 A_2 = \rho V_2 \frac{\pi}{4} D_2^2 = 999\ \frac{kg}{m^3}\times 5.94\ \frac{m}{s}\times\frac{\pi}{4}\times(0.075\ m)^2 = 26.22\ \frac{kg}{s}$$

Force on scale:

$$F_y = -V_1\dot{m} + V_2\dot{m} = 26.22\ \frac{kg}{s}\times\left(5.94\ \frac{m}{s} - 12.36\ \frac{m}{s}\right) = -168.3\ N$$

Direction is going down.

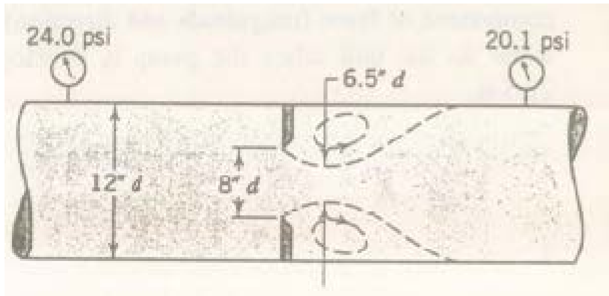
Weight on the scale beam:

$$F_w = F_y + F_{tank} + F_{water} = 168.3\ N + 224\ N + 897\ N = 1289\ N$$

Problem 4.63

(Difficulty: 2)

4.63 The pressure difference results from head loss caused by eddies downstream from the orifice plate. Wall friction is negligible. Calculate the force exerted by the water on the orifice plate. The flow rate is $7.86\ \frac{ft^3}{s}$.



Given: The flow rate: $Q = 7.86\ \frac{ft^3}{s}$. All the other parameters are shown in the figure.

Find: The force exerted by the water.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equation:

Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation;

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

The cross section area is:

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times \left(\frac{12}{12}\ ft\right)^2 = 0.785\ ft^2$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times \left(\frac{12}{12}\ ft\right)^2 = 0.785\ ft^2$$

The pressure at inlet and outlet are:

$$p_1 = 24.0\ psi = 3456\ \frac{lbf}{ft^2}$$

$$p_2 = 20.1\ psi = 2894\ \frac{lbf}{ft^2}$$

The velocity can be calculated by:

$$V_1 = \frac{Q}{A_1} = \frac{7.86\ \frac{ft^3}{s}}{0.785\ ft^2} = 10.01\ \frac{ft}{s}$$

$$V_2 = \frac{Q}{A_2} = \frac{7.86\ \frac{ft^3}{s}}{0.785\ ft^2} = 10.01\ \frac{ft}{s}$$

From the x momentum equation:

$$R_x + p_1 A_1 - p_2 A_2 = -V_1 \dot{m} + V_2 \dot{m} = 0$$

$$R_x = p_2 A_2 - p_1 A_1 = 2894\ \frac{lbf}{ft^2} \times 0.785\ ft^2 - 3456\ \frac{lbf}{ft^2} \times 0.785\ ft^2 = -441\ lbf$$

The force on the orifice place is:

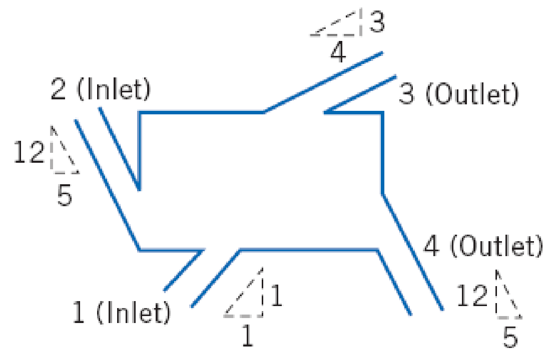
$$F_x = -R_x = 441\ lbf$$

The direction is to the right.

Problem 4.64

[Difficulty: 3]

4.64 Obtain expressions for the rate of change in mass of the control volume shown, as well as the horizontal and vertical forces required to hold it in place, in terms of p_1, A_1, V_1, p_2, A_2, V_2, p_3, A_3, V_3, p_4, A_4, V_4, and the constant density ρ.



Given: Flow into and out of CV

Find: Expressions for rate of change of mass, and force

Solution:

Basic equations: Mass and momentum flux

$$\frac{\partial}{\partial t}\int_{CV} \rho \, dV\!\!\!/ + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$$

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u \, \rho \, dV\!\!\!/ + \int_{CS} u \, \rho \vec{V} \cdot d\vec{A}$$

$$F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v \, \rho \, dV\!\!\!/ + \int_{CS} v \, \rho \vec{V} \cdot d\vec{A}$$

Assumptions: 1) Incompressible flow 2) Uniform flow

For the mass equation

$$\frac{dM_{CV}}{dt} + \sum_{CS} \left(\rho \cdot \vec{V} \cdot \vec{A}\right) = \frac{dM_{CV}}{dt} + \rho \cdot \left(-V_1 \cdot A_1 - V_2 \cdot A_2 + V_3 \cdot A_3 + V_4 \cdot A_4\right) = 0$$

$$\frac{dM_{CV}}{dt} = \rho \cdot \left(V_1 \cdot A_1 + V_2 \cdot A_2 - V_3 \cdot A_3 - V_4 \cdot A_4\right)$$

For the x momentum

$$F_x + \frac{p_1 \cdot A_1}{\sqrt{2}} + \frac{5}{13} \cdot p_2 \cdot A_2 - \frac{4}{5} \cdot p_3 \cdot A_3 - \frac{5}{13} \cdot p_4 \cdot A_4 = 0 + \frac{V_1}{\sqrt{2}} \cdot \left(-\rho \cdot V_1 \cdot A_1\right) + \frac{5}{13} \cdot V_2 \cdot \left(-\rho \cdot V_2 \cdot A_2\right) \ldots$$
$$+ \frac{4}{5} \cdot V_3 \cdot \left(\rho \cdot V_3 \cdot A_3\right) + \frac{5}{13} \cdot V_3 \cdot \left(\rho \cdot V_3 \cdot A_3\right)$$

$$F_x = -\frac{p_1 \cdot A_1}{\sqrt{2}} - \frac{5}{13} \cdot p_2 \cdot A_2 + \frac{4}{5} \cdot p_3 \cdot A_3 + \frac{5}{13} \cdot p_4 \cdot A_4 + \rho \cdot \left(-\frac{1}{\sqrt{2}} \cdot V_1^2 \cdot A_1 - \frac{5}{13} \cdot V_2^2 \cdot A_2 + \frac{4}{5} \cdot V_3^2 \cdot A_3 + \frac{5}{13} \cdot V_3^2 \cdot A_3\right)$$

For the y momentum

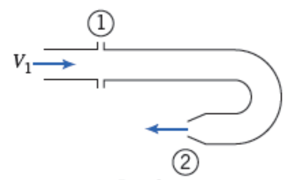
$$F_y + \frac{p_1 \cdot A_1}{\sqrt{2}} - \frac{12}{13} \cdot p_2 \cdot A_2 - \frac{3}{5} \cdot p_3 \cdot A_3 + \frac{12}{13} \cdot p_4 \cdot A_4 = 0 + \frac{V_1}{\sqrt{2}} \cdot \left(-\rho \cdot V_1 \cdot A_1\right) - \frac{12}{13} \cdot V_2 \cdot \left(-\rho \cdot V_2 \cdot A_2\right) \ldots$$
$$+ \frac{3}{5} \cdot V_3 \cdot \left(\rho \cdot V_3 \cdot A_3\right) - \frac{12}{13} \cdot V_3 \cdot \left(\rho \cdot V_3 \cdot A_3\right)$$

$$F_y = -\frac{p_1 \cdot A_1}{\sqrt{2}} + \frac{12}{13} \cdot p_2 \cdot A_2 + \frac{3}{5} \cdot p_3 \cdot A_3 - \frac{12}{13} \cdot p_4 \cdot A_4 + \rho \cdot \left(-\frac{1}{\sqrt{2}} \cdot V_1^2 \cdot A_1 - \frac{12}{13} \cdot V_2^2 \cdot A_2 + \frac{3}{5} \cdot V_3^2 \cdot A_3 - \frac{12}{13} \cdot V_3^2 \cdot A_3\right)$$

Problem 4.65

(Difficulty: 2)

4.65 Water is flowing steadily through the 180° elbow shown. At the inlet to the elbow the gage pressure is 103 kPa. The water discharges to atmospheric pressure. Assume properties are uniform over the inlet and outlet areas: $A_1 = 2500\ mm^2$, $A_2 = 650\ mm^2$, and $V_1 = 3\ \frac{m}{s}$. Find the horizontal component of force required to hold the elbow in place.



P4.65

Given: Water flow through elbow

Find: Force to hold elbow

Assumption: (1) Steady flow

(2) Incompressible flow

(3) Atmospheric pressure at exit

(4) Uniform flow

Solution:

Basic equations:

Momentum equation in x-direction:

$$R_x + F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

The pressure force is:

$$F_{sx} = p_1 A_1$$

$$F_{Bx} = 0$$

Hence:

$$R_x + p_1 A_1 = V_1(-\rho V_1 A_1) - V_2(\rho V_2 A_2)$$

$$R_x = -p_1 A_1 - \rho(V_1^2 \cdot A_1 + V_2^2 \cdot A_2)$$

From continuity equation:

$$0 = -\rho_1 V_1 A_1 + \rho_2 V_2 A_2$$

$$V_2 = V_1 \frac{A_1}{A_2} = 3\ \frac{m}{s} \times \frac{2500\ mm^2}{650\ mm^2} = 11.54\ \frac{m}{s}$$

The density of water is:

$$\rho = 999\ \frac{kg}{m^3}$$

So we get:

$$R_x = -103000\ \frac{N}{m^2} \times 0.0025\ m^2 - 999\ \frac{kg}{m^3} \times \left(\left(3\ \frac{m}{s}\right)^2 \times 0.0025\ m^2 + \left(11.54\ \frac{m}{s}\right)^2 \times 0.000650\ m^2\right)$$

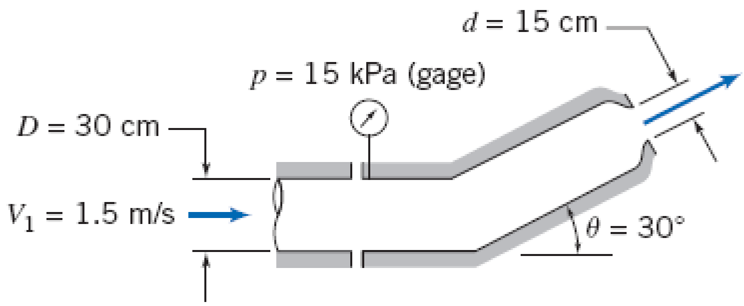
$$R_x = -366\ N$$

The force on the CV is to the left. It is needed to hold the elbow on against the high pressure, plus it generates the large change in x momentum.

Problem 4.66

[Difficulty: 2]

4.66 Water flows steadily through the nozzle shown, discharging to atmosphere. Calculate the horizontal component of force in the flanged joint. Indicate whether the joint is in tension or compression.



Given: Water flow through nozzle

Find: Force to hold nozzle

Solution:

Basic equation: Momentum flux in x direction for the elbow

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Hence $R_x + p_{1g}\cdot A_1 + p_{2g}\cdot A_2 = V_1\cdot(-\rho\cdot V_1\cdot A_1) + V_2\cdot\cos(\theta)\cdot(\rho\cdot V_2\cdot A_2)$ $\qquad R_x = -p_{1g}\cdot A_1 + \rho\cdot\left(V_2^{\,2}\cdot A_2\cdot\cos(\theta) - V_1^{\,2}\cdot A_1\right)$

From continuity $V_2\cdot A_2 = V_1\cdot A_1$ so $\quad V_2 = V_1\cdot\frac{A_1}{A_2} = V_1\cdot\left(\frac{D_1}{D_2}\right)^2 \qquad V_2 = 1.5\cdot\frac{m}{s}\cdot\left(\frac{30}{15}\right)^2 \qquad V_2 = 6\cdot\frac{m}{s}$

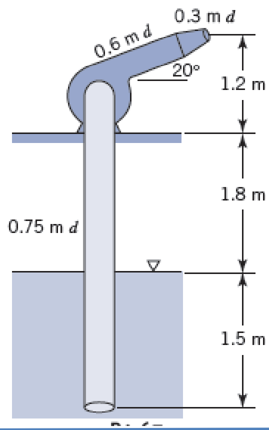
Hence $R_x = -15\times 10^3\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.3\cdot m)^2}{4} + 1000\cdot\frac{kg}{m^3}\times\left[\left(6\cdot\frac{m}{s}\right)^2\times\frac{\pi\cdot(0.15\cdot m)^2}{4}\cdot\cos(30\cdot deg) - \left(1.5\cdot\frac{m}{s}\right)^2\times\frac{\pi\cdot(.3\cdot m)^2}{4}\right]\times\frac{N\cdot s^2}{kg\cdot m}$

$R_x = -668\cdot N$ The joint is in tension: It is needed to hold the elbow on against the high pressure, plus it generates the large change in x momentum

Problem 4.67

(Difficulty: 2)

4.67 The pump, suction pipe, discharge pipe, and nozzle are all welded together as a single unit. Calculate the horizontal component of force (magnitude and direction) exerted by the water on the unit when the pump is developing a head of 22.5 m.



Given: All the parameters are shown in the figure.

Find: The horizontal component force exerted by the water.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equation:

Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation;

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation for the x-direction

$$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

The cross section area is:

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times (0.3\ m)^2 = 0.071\ m^2$$

From the Bernoulli equation we have:

$$\frac{V^2}{2} - gh = 0$$

$$h = 22.5\ m - 1.2\ m - 1.8\ m = 19.5\ m$$

$$V = \sqrt{2gh} = \sqrt{2 \times 9.81\ \frac{m}{s^2} \times 19.5\ m} = 19.55\ \frac{m}{s}$$

The mass flow rate is:

$$\dot{m} = \rho V A = 999\ \frac{kg}{m^3} \times 19.55\ \frac{m}{s} \times 0.071\ m^2 = 1387\ \frac{kg}{s}$$

From the x momentum equation:

$$R_x = V \cos 20°\, \dot{m}$$

$$R_x = 19.55\ \frac{m}{s} \times \cos 20° \times 1387\ \frac{kg}{s} = 25500\ N = 25.5\ kN$$

The horizontal component force exerted by water is :

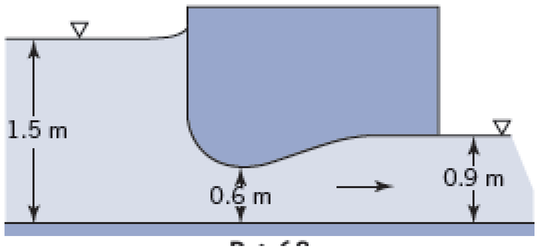
$$F_x = -R_x = -25.5\ kN$$

The direction is to the left.

Problem 4.68

(Difficulty: 2)

4.68 The passage is 1.2 m wide normal to the paper. What will be the horizontal component of force exerted by the water on the structure?



Given: The width of the passage: $w = 1.2\ m$. All the other parameters are shown in the figure.

Find: The horizontal component force exerted by the water.

Assumptions: Flow is steady

Density is constant

Solution: Basic equations are

Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation;

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

The heights of the passage inlet and outlet are:

$$h_1 = 1.5\ m$$

$$h_2 = 0.9\ m$$

The area of inlet and outlet are:

$$A_1 = wh_1 = 1.2\ m \times 1.5\ m = 1.8\ m^2$$

$$A_2 = wh_2 = 1.2\ m \times 0.9\ m = 1.08\ m^2$$

From continuity equation for steady incompressible flow:

$$A_1V_1 = A_2V_2$$

$$V_1 = \frac{A_2}{A_1}V_2 = 0.6V_2$$

From the Bernoulli equation we have:

$$\frac{{V_1}^2}{2} + gh_1 = \frac{{V_2}^2}{2} + gh_2$$

$$\frac{0.36{V_2}^2}{2} + gh_1 = \frac{{V_2}^2}{2} + gh_2$$

$$0.32{V_2}^2 = g(h_1 - h_2)$$

$$V_2 = \sqrt{\frac{g(h_1 - h_2)}{0.32}} = \sqrt{\frac{9.81\ \frac{m}{s^2} \times (1.5\ m - 0.9\ m)}{0.32}} = 4.29\ \frac{m}{s}$$

$$V_1 = 0.6V_2 = 2.57\ \frac{m}{s}$$

The mass flow rate is:

$$\dot{m} = \rho V_1 A_1 = 999\ \frac{kg}{m^3} \times 2.57\ \frac{m}{s} \times 1.8\ m^2 = 4621\ \frac{kg}{s}$$

From the x momentum equation:

$$R_x + p_1A_1 - p_2A_2 = -V_1\dot{m} + V_2\dot{m}$$

$$R_x = \dot{m}(V_2 - V_1) + p_2A_2 - p_1A_1 = \dot{m}(V_2 - V_1) + \frac{\gamma h_2^2 w}{2} - \frac{\gamma h_1^2 w}{2}$$

$$\gamma = 9800\ \frac{N}{m^3}$$

$$R_x = -503\ N$$

The horizontal component force exerted by water is :

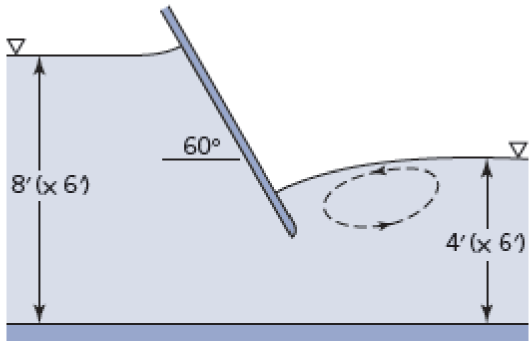
$$F_x = -R_x = 503\ N$$

The direction is to the right.

Problem 4.69

(Difficulty: 2)

4.69 If the two-dimensional flow rate through this sluice gate is 50 $\frac{ft^2}{s}$, calculate the horizontal and vertical components of force on gate, neglecting wall friction.



Given: The flow rate:$q = 50\ \frac{ft^2}{s}$. All the other parameters are shown in the figure.

Find: The horizontal and vertical component force exerted by the water.

Assumptions: Flow is steady

Density is constant

Solution: Basic equations are:

Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

From the continuity equation for steady incompressible flow we have:

$$Q = V_1 h_1 = V_2 h_2$$

$$V_1 = \frac{Q}{h_1} = \frac{50\ \frac{ft^2}{s}}{8\ ft} = 6.25\ \frac{ft}{s}$$

$$V_2 = \frac{Q}{A_2} = \frac{50\ \frac{ft^3}{s}}{4\ ft^2} = 12.5\ \frac{ft}{s}$$

The mass flow rate per width is:

$$\dot{m} = \rho q = 1.94\ \frac{slug}{ft^3} \times 50\ \frac{ft^2}{s} = 1.94\ \frac{lbf \cdot s^2}{ft^4} \times 50\ \frac{ft^2}{s} = 97\ \frac{lbf \cdot s}{ft^2}$$

From the x momentum equation per width:

$$R_x + p_1 h_1 - p_2 h_2 = -V_1 \dot{m} + V_2 \dot{m}$$

$$R_x = \dot{m}(V_2 - V_1) + p_2 h_2 - p_1 h_1 = \dot{m}(V_2 - V_1) + \frac{\gamma h_2 h_2}{2} - \frac{\gamma h_1 h_1}{2}$$

$$\gamma = 62.4\ \frac{lbf}{ft^3}$$

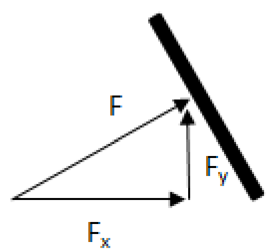
$$R_x = 97\ \frac{lbf \cdot s}{ft^2} \times \left(12.5\ \frac{ft}{s} - 6.25\ \frac{ft}{s}\right) + 62.4\ \frac{lbf}{ft^3} \times \left(\frac{4\ ft \times 4\ ft}{2} - \frac{8\ ft \times 8ft}{2}\right)$$

$$R_x = -891\ \frac{lbf}{ft}$$

Or the force on the gate is

$$F_x = -R_x = 891\ \frac{lbf}{ft}$$

The total force F must be normal to the gate surface. The forces on the gate are then related as:



F_x is the horizontal component force per width, and the y component of the force is then

$$F_y = F_x \tan 30° = 514\ \frac{lbf}{ft}$$

The width for the gate is:

$$w = 6\ ft$$

So the total force can be calculated by:

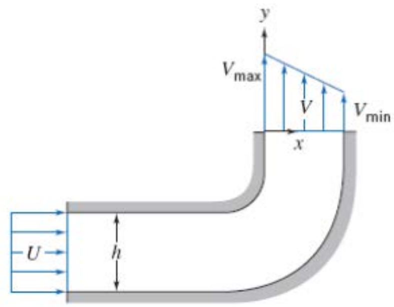
$$F_{tx} = F_x w = 891\ \frac{lbf}{ft} \times 6\ ft = 5350\ lbf$$

$$F_{ty} = F_y w = 514\ \frac{lbf}{ft} \times 6\ ft = 3080\ lbf$$

Problem 4.70

(Difficulty: 2)

4.70 Assume the bend of Problem 4.35 is a segment of a larger channel and lies in a horizontal plane. The inlet pressure is 170 kPa (abs), and the outlet pressure is 130 kPa (abs). Find the force required to hold the bend in place.



Given: The inlet pressure: $p_1 = 170\ kPa$. The outlet pressure: $p_2 = 130\ kPa$.

Find: The force required to hold the bend in place.

Assumption: (1) steady flow.

(2) $F_{Bx} = F_{By} = 0$.

(3) incompressible flow.

(4) atmosphere pressure acts on the outside surfaces.

Solution:

Basic equations: Momentum equation in the x-direction

$$\bar{F}_s + \bar{F}_B = \frac{\partial}{\partial t}\int_{CV} \bar{V}\rho d\forall + \int_{CS} \bar{V}(\rho\bar{V}\cdot d\bar{A})$$

Momentum equation for the y-direction

$$F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho\bar{V}\cdot d\bar{A}$$

The x-momentum equation becomes

$$R_x + p_1 A_1 = \int_{CS} u\,(\rho\bar{V}\cdot d\bar{A}) = U\{-|\rho U A_1|\}$$

$$R_x = -p_1 A_1 - \rho U^2 A_1 = -h^2(p_1 + \rho U^2)$$

$$R_x = -(0.0755\ m)^2 \times \left[(170 - 101) \times 10^3\ \frac{N}{m^2} + 999\ \frac{kg}{m^3} \times \left(7.5\ \frac{m}{s}\right)^2 \times \frac{N \cdot s}{kg \cdot m}\right] = -714\ N$$

The y-momentum equation becomes

$$R_y - p_2 A_2 = \int_{CS} v\ (\rho \bar{V} \cdot d\bar{A})$$

$$U_2 = V_2 = V_{max} - (V_{max} - V_{min}) \frac{x}{h} = 2V_{min} - V_{min} \frac{x}{h} = V_{min} \left(2 - \frac{x}{h}\right)$$

$$R_y - p_2 A_2 = \int_0^h V_{min} \left(2 - \frac{x}{h}\right) \rho V_{min} \left(2 - \frac{x}{h}\right) h dx$$

$$R_y = p_2 A_2 + \rho V_{min}^2 h \int_0^h \left(4 - 4\frac{x}{h} + \frac{x^2}{h^2}\right) dx$$

$$R_y = p_2 A_2 + \rho V_{min}^2 h \left[4h - 2h + \frac{h}{3}\right] = p_2 A_2 + \frac{7}{3} \rho V_{min}^2 h^2$$

$$R_y = h^2 \left(p_2 + \frac{7}{3} \rho V_{min}^2\right) = (0.0755\ m)^2 \left[(130 - 110) \times 10^3\ \frac{N}{m^2} + \frac{7}{3} \times 999\ \frac{kg}{m^3} \times \left(5\ \frac{m}{s}\right)^2 \times \frac{N \cdot s}{kg \cdot m}\right]$$

$$R_y = 498\ N$$

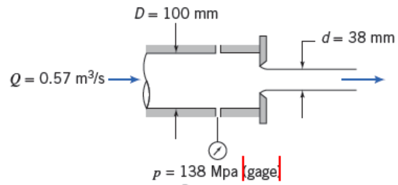
So we get:

$$\bar{R} = -714\hat{\imath} + 498\ \hat{\jmath}\ N$$

Problem 4.71

(Difficulty: 2)

4.71 A flat plate orifice of 50 mm diameter is located at the end of a 100-mm-diamter pipe. Water flows through the pipe and orifice at 57 m^3/s. The diameter of the water jet downstream from the orifice is 38 mm. Calculate the external force required to hold the orifice in place. Neglect friction on the pipe wall.



P4.71

Given: Water flow through orifice plate

Find: Force to hold plate

Assumption: (1) Steady flow

(2) Incompressible flow

(3) Uniform flow

Solution:

Basic equations:

Momentum equation in x-direction:

$$R_x + F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{V} \cdot d\vec{A}$$

The pressure force is:

$$F_{sx} = p_1 A_1$$

$$F_{Bx} = 0$$

Hence:

$$R_x + p_1 A_1 = V_1(-\rho V_1 A_1) + V_2(\rho V_2 A_2)$$

$$R_x = -p_1 A_1 + \rho(V_2^2 \cdot A_2 - V_1^2 \cdot A_1)$$

From continuity equation:

$$0 = -\rho_1 V_1 A_1 + \rho_2 V_2 A_2$$

$$Q = V_1 A_1 = V_2 A_2$$

$$V_1 = \frac{Q}{A_1} = \frac{57 \ \frac{m^3}{s}}{\frac{\pi}{4} \times (0.1 \ m)^2} = 7260 \ \frac{m}{s}$$

$$V_2 = V_1 \frac{A_1}{A_2} = 7260 \ \frac{m}{s} \times \left(\frac{100 \ mm}{38 \ mm}\right)^2 = 50300 \ \frac{m}{s}$$

The density of water is:

$$\rho = 999 \ \frac{kg}{m^3}$$

So we get:

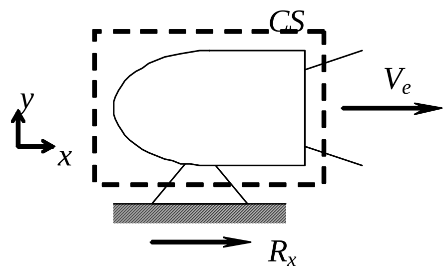
$$R_x = -138 \times 10^6 \ \frac{N}{m^2} \times \frac{\pi}{4} \times (0.1 \ m)^2 + 999 \ \frac{kg}{m^3} \times \left(\left(50300 \ \frac{m}{s}\right)^2 \times \frac{\pi}{4} \times (0.038 \ m)^2 - \left(7260 \ \frac{m}{s}\right)^2 \times \frac{\pi}{4} \times (0.1 \ m)^2 \right)$$

$$R_x = 2.45 \times 10^9 \ N$$

The force on the CV is to the right.

Problem 4.72 [Difficulty: 2]

4.72 At rated thrust, a liquid-fueled rocket motor consumes 80 kg/s of nitric acid as oxidizer and 32 kg/s of aniline as fuel. Flow leaves axially at 180 m/s relative to the nozzle and at 110 kPa. The nozzle exit diameter is $D = 0.6$ m. Calculate the thrust produced by the motor on a test stand at standard sea-level pressure.



Given: Data on rocket motor

Find: Thrust produced

Solution:

Basic equation: Momentum flux in x direction for the elbow $$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho \vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Neglect change of momentum within CV 3) Uniform flow

Hence $$R_x - p_{eg}\cdot A_e = V_e\cdot\left(\rho_e\cdot V_e\cdot A_e\right) = m_e\cdot V_e \qquad R_x = p_{eg}\cdot A_e + m_e\cdot V_e$$

where p_{eg} is the exit pressure (gage), m_e is the mass flow rate at the exit (software cannot render dot over m!) and V_e is the exit velocity

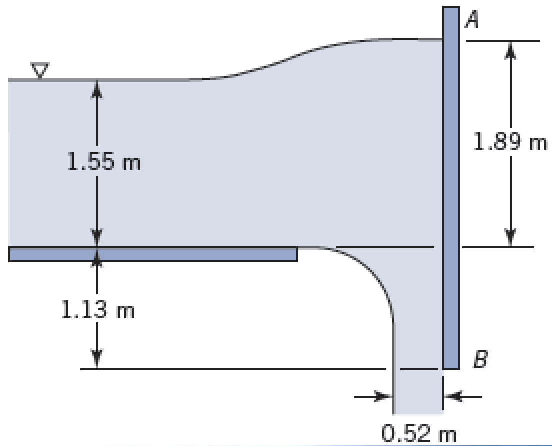
For the mass flow rate $$m_e = m_{nitricacid} + m_{aniline} = 80\cdot\frac{kg}{s} + 32\cdot\frac{kg}{s} \qquad m_e = 112\cdot\frac{kg}{s}$$

Hence $$R_x = (110 - 101)\times 10^3\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.6\cdot m)^2}{4} + 112\cdot\frac{kg}{s}\times 180\cdot\frac{m}{s}\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_x = 22.7\cdot kN$$

Problem 4.73

(Difficulty: 2)

4.73 Flow from the end of a two-dimensional open channel is deflected vertically downward by the gate AB. Calculate the force exerted by the water on the gage. At (and downstream from) B the flow may be considered a free jet.



Given: All the parameters are shown in the figure.

Find: The force exerted by the water on the gage.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equation: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation:

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

From the continuity equation for incompressible flow we have:

$$Q = V_1 h_1 = V_2 h_2$$

$$h_1 = 1.55\ m$$

$$h_2 = 0.52\ m$$

$$V_1 = V_2 \frac{h_2}{h_1} = 0.3355\ V_2$$

From the Bernoulli equation:

$$g z_1 + \frac{V_1^2}{2} = \frac{V_2^2}{2}$$

$$z_1 = 1.55\ m + 1.13\ m = 2.68\ m$$

$$g z_1 + \frac{(0.3355\ V_2)^2}{2} = \frac{V_2^2}{2}$$

$$0.4437 V_2^2 = g z_1$$

$$V_2 = \sqrt{\frac{g z_1}{0.4437}} = \sqrt{\frac{9.81\ \frac{m}{s^2} \times 2.68\ m}{0.4437}} = 7.69\ \frac{m}{s}$$

$$V_1 = 2.58\ \frac{m}{s}$$

The mass flow rate per width is:

$$\dot{m} = \rho V_1 h_1 = 999\ \frac{kg}{m^3} \times 2.58\ \frac{m}{s} \times 1.55\ m = 3995 \frac{kg}{m \cdot s}$$

From the x momentum equation per width:

$$R_x + p_1 h_1 = -V_1 \dot{m}$$

$$R_x + \frac{\rho g h_1}{2} h_1 = -V_1 \dot{m}$$

$$R_x = -V_1 \dot{m} - \frac{\rho g h_1}{2} h_1$$

$$R_x = -2.58\ \frac{m}{s} \times 3995 \frac{kg}{m \cdot s} - \frac{999\ \frac{kg}{m^3} \times 9.81\ \frac{m}{s^2} \times 1.55\ m}{2} \times 1.55\ m = -22080\ \frac{N}{m}$$

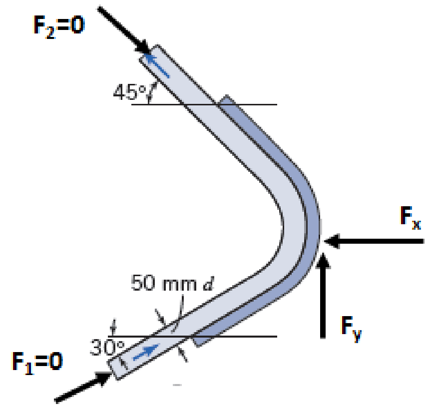
$$F_x = -R_x = 22080\ \frac{N}{m}$$

The direction is to the right.

Problem 4.74

(Difficulty: 2)

4.74 Calculate the magnitude and direction of the vertical and horizontal components and the total force exerted on this stationary blade by a 50 mm jet of water moving at 15 $\frac{m}{s}$.



Given: The diameter is: $D = 50\ mm$. The velocity is: $V = 15\frac{m}{s}$. All the parameters are shown in the figure.

Find: The vertical and horizontal and the total force.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the y-direction

$$F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho \bar{V} \cdot d\bar{A}$$

From the continuity equation for incompressible flow we have:

$$Q = VA$$

$$A = \frac{\pi}{4} D^2$$

$$Q = VA = 15 \frac{m}{s} \times \frac{\pi}{4} \times (0.05\ m)^2 = 0.0295\ \frac{m^3}{s}$$

The mass flow rate is:

$$\dot{m} = \rho Q = 999\ \frac{kg}{m^3} \times 0.0295\ \frac{m^3}{s} = 29.47\ \frac{kg}{s}$$

From the x-momentum equation:

$$R_x = -V \cos 30° \, \dot{m} - V \cos 45° \, \dot{m}$$

$$R_x = -29.47\ \frac{kg}{s} \times 15\ \frac{m}{s} \times \cos 30° - 29.47\ \frac{kg}{s} \times 15\ \frac{m}{s} \times \cos 45° = -695\ N$$

$$F_x = 695\ N$$

From the y-momentum equation:

$$F_y = -V \sin 30° \, \dot{m} + V \sin 45° \, \dot{m} = 91.6\ N$$

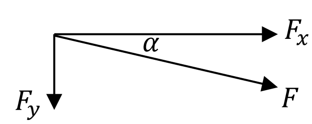
The total force is:

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(695\ N)^2 + (91.6\ N)^2} = 701\ N$$

The direction is:

$$\tan \alpha = \frac{F_y}{F_x} = \frac{91.6\ N}{695\ N} = 0.1318$$

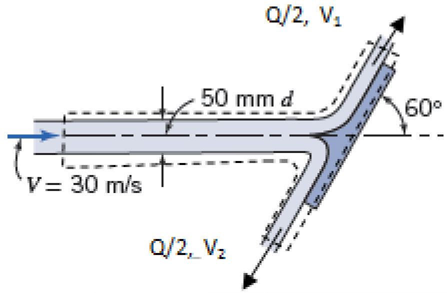
$$\alpha = 7.5°$$



Problem 4.75

(Difficulty: 2)

4.75 This water jet of $50\,mm$ diameter moving at $30\,\frac{m}{s}$ is divided in half by a "splitter" on the stationary flat plate. Calculate the magnitude and direction of the force on the plate. Assume that flow is in a horizontal plane.



Given: The diameter is: $D_1 = 50\,mm$. The velocity is: $V_1 = 30\,\frac{m}{s}$. All the parameters are shown in the figure.

Find: The vertical and horizontal and the total force.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the y-direction

$$F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho \bar{V} \cdot d\bar{A}$$

From the continuity equation for incompressible flow we have:

$$Q = V_1 A_1$$

$$A_1 = \frac{\pi}{4} D_1^2$$

$$Q = V_1 A_1 = 30 \frac{m}{s} \times \frac{\pi}{4} \times (0.05\ m)^2 = 0.0589\ \frac{m^3}{s}$$

The mass flow rate is:

$$\dot{m} = \rho Q = 999\ \frac{kg}{m^3} \times 0.0589\ \frac{m^3}{s} = 58.84\ \frac{kg}{s}$$

From the x-momentum equation, one-half the flow out is in the positive x-direction and one-half in the negative x-direction. The x-momentum equation becomes:

$$R_x = \frac{\dot{m}}{2} V_2 cos(60) - \frac{\dot{m}}{2} V_2 cos(60) - V_1 \dot{m} = -V_1 \dot{m}$$

$$R_x = -30 \frac{m}{s} \times 58.84\ \frac{kg}{s} = -1765\ N$$

$$F_x = 1765\ N$$

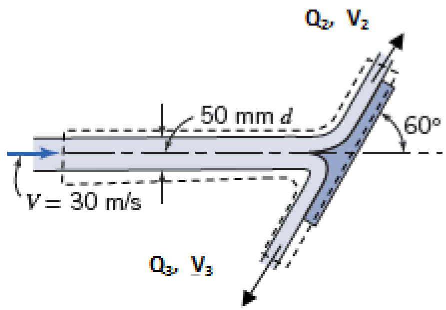
The direction is to the right.

For the y-direction, the momentum flows leaving upwards and downwards are equal. So there is no net force in the y-direction.

Problem 4.76

(Difficulty: 3)

4.76 If the splitter is removed from the plate of the preceding problem and sidewalls are provided on the plate to keep the flow two-dimensional, how will the jet divide after striking the plate?



Given: The diameter is: $D_1 = 50\ mm$. The velocity is: $V_1 = 30\frac{m}{s}$. All the parameters are shown in the figure.

Find: The vertical and horizontal and the total force.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the y-direction

$$F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho \bar{V} \cdot d\bar{A}$$

From the continuity equation for steady incompressible flow we have:

$$Q = V_1 A_1$$

$$A_1 = \frac{\pi}{4} D_1^2$$

$$Q = V_1 A_1 = 30 \frac{m}{s} \times \frac{\pi}{4} \times (0.05\ m)^2 = 0.0589\ \frac{m^3}{s}$$

From the Bernoulli equation, there are no pressure or elevation changes. Therefore the flow velocity remains constant:

$$V_1 = V_2 = V_3$$

From the continuity equation for steady incompressible flow

$$Q = Q_2 + Q_3$$

We also have the velocity components:

$$V_{1x} = 30\ \frac{m}{s}$$

$$V_{2x} = 15\ \frac{m}{s}$$

$$V_{3x} = -15\ \frac{m}{s}$$

$$V_{1y} = 0\ \frac{m}{s}$$

$$V_{2y} = 25.98\ \frac{m}{s}$$

$$V_{3y} = -25.98\ \frac{m}{s}$$

From x-momentum equation:

$$R_x = -V_{1x}\rho Q + V_{2x}\rho Q_2 + V_{3x}\rho Q_3$$

$$F_x = -R_x = V_{1x}\rho Q - V_{2x}\rho Q_2 - V_{3x}\rho Q_3$$

From y-momentum equation:

$$R_y = V_{2y}\rho Q_2 + V_{3y}\rho Q_3$$

$$F_y = -V_{2y}\rho Q_2 - V_{3y}\rho Q_3$$

Because the net force is normal to the plate, we also have:

$$F_y = F_x \tan 30°$$

$$-V_{2y}\rho Q_2 - V_{3y}\rho Q_3 = (V_{1x}\rho Q - V_{2x}\rho Q_2 - V_{3x}\rho Q_3) \tan 30°$$

$$-25.98Q_2 + 25.98Q_3 = 17.32Q - 8.66Q_2 + 8.66Q_3$$

$$-2Q_2 + 2Q_3 = Q$$

$$-Q_2 + Q_3 = 0.5Q$$

$$Q_2 + Q_3 = Q$$

$Q_2 = 0.25Q = 0.015\ \frac{m^3}{s}$ and $Q_3 = 0.75Q = 0.044\,\frac{m^3}{s}$

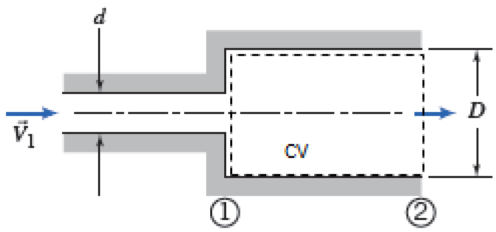
Problem 4.77

(Difficulty: 2)

4.77 Consider flow through the sudden expansion shown. If the flow is incompressible and friction is neglected, show that the pressure rise, $\Delta p = p_2 - p_1$, is given by

$$\frac{\Delta p}{\frac{1}{2}\rho \bar{V}_1^2} = 2\left(\frac{d}{D}\right)^2 \left[1 - \left(\frac{d}{D}\right)^2\right]$$

Plot the nondimensional pressure rise versus diameter ratio to determine the optimum value of $\frac{d}{D}$ and the corresponding value of the nondimensional pressure rise. Hint: Assume the pressure is uniform and equal to p_1 on the vertical surface of the expansion.



Given: The inlet pressure: $p_1 = 170\ kPa$. The outlet pressure: $p_2 = 130\ kPa$.

Find: The nondimensional pressure rise versus diameter ratio.

Assumption: (1) no friction, so surface force due to pressure only.

(2) $F_{Bx} = 0$.

(3) steady flow.

(4) incompressible flow.

(5) uniform flow at section 1 and 2.

(6) uniform pressure p_1 on vertical surface of expansion.

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Applying the x component of momentum equation using fixed CV shown, we have:

$$p_1 A_2 - p_2 A_2 = V_1(-\rho V_1 A_1) + V_2(\rho V_2 A_2)$$

From the continuity for the steady incompressible uniform flow,

$$\rho V_1 A_1 = \rho V_2 A_2$$

$$V_2 = \frac{A_1}{A_2} V_1$$

The pressure difference is then:

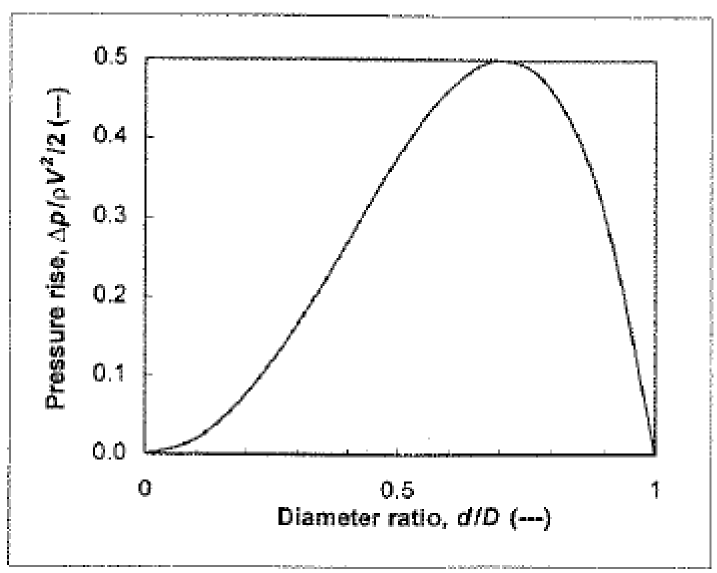
$$\Delta p = p_2 - p_1 = \rho V_1 \frac{A_1}{A_2} V_1 - \rho V_2 V_2 = \rho V_1 \frac{A_1}{A_2} V_1 - \rho V_1 \frac{A_1}{A_2} V_2 = \rho V_1 \frac{A_1}{A_2}(V_1 - V_2)$$

$$\Delta p = \rho {V_1}^2 \frac{A_1}{A_2}\left(1 - \frac{V_2}{V_1}\right) = \rho {V_1}^2 \frac{A_1}{A_2}\left(1 - \frac{A_1}{A_2}\right)$$

Or the nondimensional pressure difference is:

$$\frac{\Delta p}{\frac{1}{2}\rho {V_1}^2} = 2\frac{A_1}{A_2}\left(1 - \frac{A_1}{A_2}\right) = 2\left(\frac{d}{D}\right)^2\left[1 - \left(\frac{d}{D}\right)^2\right]$$

From the plot below we see that $\frac{\Delta p}{\frac{1}{2}\rho {V_1}^2}$ has an optimum value of 0.5 at $\frac{d}{D} = 0.7$.



Note: As expected:

For $d = D$, $\Delta p = 0$ for straight pipe.

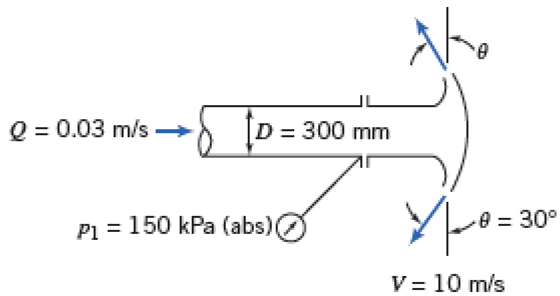
For $\frac{d}{D} \rightarrow 0$, $\Delta p = 0$ for freejet.

Also note that the location of section 2 would have to be chosen with care to make assumption (5) reasonable.

Problem 4.78

(Difficulty: 2)

4.78 A conical spray head is shown. The fluid is water and the exit stream is uniform. Evaluate (a) the thickness of the spray sheet at 400 mm radius and (b) the axial force exerted by the spray head on the supply pipe.



Given: All the parameters are shown in the figure.

Find: The thickness of the spray sheet t at 400 mm. The axial force k_x.

Assumption: (1) $F_{Bx} = 0$.

(2) steady flow.

(3) incompressible flow.

(4) uniform flow at each section.

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

From continuity for incompressible flow:

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4}{\pi} \times 0.03\ \frac{m^3}{s} \times \frac{1}{(0.3\ m)^2} = 0.424\ \frac{m}{s}$$

Assume velocity in the jet sheet is constant at $V = 10\ \frac{m}{s}$. Then

$$Q = 2\pi RtV$$

$$t = \frac{Q}{2\pi RV} = \frac{1}{2\pi} \times 0.03\ \frac{m^3}{s} \times \frac{1}{0.4\ m} \times \frac{1}{10\ \frac{m}{s}} = 0.00119\ m = 1.19\ mm$$

From the x-momentum equation:

$$F_{Bx} = 0$$

$$R_x + p_{1g}A_1 = u_1[-\rho Q] + u_2[-\rho Q]$$

$$u_1 = V_1$$

$$u_2 = -V\sin\theta$$

The density of water is:

$$\rho = 999\ \frac{kg}{m^3}$$

So we have:

$$R_x = -p_{1g}A_1 - (V_1 + V\sin\theta)\rho Q$$

$$R_x = -(150 - 101) \times 10^3\ \frac{N}{m^2} \times \frac{\pi}{4} \times (0.3\ m)^2 - \left(0.424\ \frac{m}{s} + 10\ \frac{m}{s} \times \sin 30°\right) \times 999\ \frac{kg}{m^3} \times 0.03\ \frac{m^3}{s}$$

$$R_x = -3.63\ kN$$

R_x is the force on CV, so the force on supply pipe is:

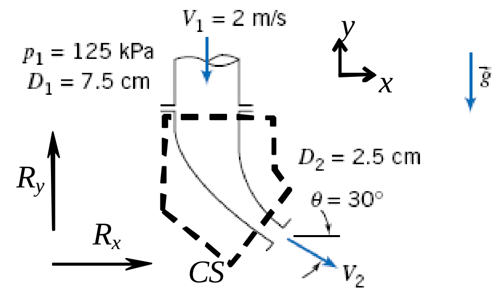
$$k_x = -R_x = 3.63\ kN$$

The direction is to the right.

Problem 4.79

[Difficulty: 2]

4.79 A curved nozzle assembly that discharges to the atmosphere is shown. The nozzle mass is 4.5 kg and its internal volume is 0.002 m^3. The fluid is water. Determine the reaction force exerted by the nozzle on the coupling to the inlet pipe.



Given: Data on nozzle assembly

Find: Reaction force

Solution:

Basic equation: Momentum flux in x and y directions

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, dV + \int_{CS} u\,\rho \vec{V}\cdot d\vec{A}$$

$$F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v\,\rho\, dV + \int_{CS} v\,\rho \vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow CV 3) Uniform flow

For x momentum $$R_x = V_2\cdot\cos(\theta)\cdot\left(\rho\cdot V_2\cdot A_2\right) = \rho\cdot V_2^{\,2}\cdot\frac{\pi\cdot D_2^{\,2}}{4}\cdot\cos(\theta)$$

From continuity $$A_1\cdot V_1 = A_2\cdot V_2 \qquad V_2 = V_1\cdot\frac{A_1}{A_2} = V_1\cdot\left(\frac{D_1}{D_2}\right)^2 \qquad V_2 = 2\cdot\frac{m}{s}\times\left(\frac{7.5}{2.5}\right)^2 \qquad V_2 = 18\,\frac{m}{s}$$

Hence $$R_x = 1000\cdot\frac{kg}{m^3}\times\left(18\cdot\frac{m}{s}\right)^2\times\frac{\pi}{4}\times(0.025\cdot m)^2\times\cos(30\cdot deg)\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_x = 138\cdot N$$

For y momentum $$R_y - p_1\cdot A_1 - W - \rho\cdot Vol\cdot g = -V_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) - V_2\cdot\sin(\theta)\cdot\left(\rho\cdot V_2\cdot A_2\right)$$

$$R_y = p_1\cdot\frac{\pi\cdot D_1^{\,2}}{4} + W + \rho\cdot Vol\cdot g + \frac{\rho\cdot\pi}{4}\cdot\left(V_1^{\,2}\cdot D_1^{\,2} - V_2^{\,2}\cdot D_2^{\,2}\cdot\sin(\theta)\right)$$

where $$W = 4.5\cdot kg\times 9.81\cdot\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} \qquad W = 44.1\,N \qquad Vol = 0.002\cdot m^3$$

Hence $$R_y = 125\times 10^3\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.075\cdot m)^2}{4} + 44.1\cdot N + 1000\cdot\frac{kg}{m^3}\times 0.002\cdot m^3\times 9.81\cdot\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m}\ \ldots$$

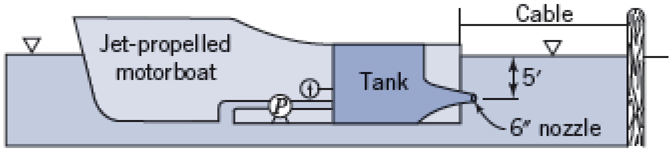
$$+\,1000\cdot\frac{kg}{m^3}\times\frac{\pi}{4}\times\left[\left(2\cdot\frac{m}{s}\right)^2\times(0.075\cdot m)^2 - \left(18\cdot\frac{m}{s}\right)^2\times(0.025\cdot m)^2\times\sin(30\cdot deg)\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$$R_y = 554\cdot N$$

Problem 4.80

(Difficulty: 2)

4.80 The pump maintains a pressure of $10\,psi$ at the gage. The velocity leaving the nozzle is 34 ft/s. Calculate the tension force in the cable.



Given: The pressure at the gage $p_1 = 10\,psi$. All the parameters are shown in the figure.

Find: The tension force in the cable.

Assumptions: Flow is steady and density is constant

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation for the x-direction

$$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

From the Bernoulli equation,

$$\frac{p_1}{\rho} = \frac{V_2^2}{2} + gh_2$$

$$V_2 = \sqrt{2\left(\frac{p_1}{\rho} - gh_2\right)}$$

For water we have:

$$\rho = 1.94\,\frac{slug}{ft^3} = 1.94\,\frac{lbf \cdot s^2}{ft^4}$$

$$g = 32.2\ \frac{ft}{s^2}$$

$$p_1 = 10\ psi = 1440\ \frac{lbf}{ft^2}$$

$$h_2 = 5\ ft$$

$$V_2 = \sqrt{2\left(\frac{p_1}{\rho} - gh_2\right)} = \sqrt{2 \times \left(\frac{1440\ \frac{lbf}{ft^2}}{1.94\ \frac{lbf \cdot s^2}{ft^4}} - 32.2\ \frac{ft}{s^2} \times 5\ ft\right)} = 34.0\ \frac{ft}{s}$$

The flow area is:

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times \left(\frac{6}{12}\ ft\right)^2 = 0.196\ ft^2$$

The mass flow rate is:

$$\dot{m} = \rho V_2 A_2 = 1.94\ \frac{lbf \cdot s^2}{ft^4} \times 34.0\ \frac{ft}{s} \times 0.196\ ft^2 = 12.93\ \frac{lbf \cdot s}{ft}$$

There is no entering x-momentum and the x-momentum equation becomes:

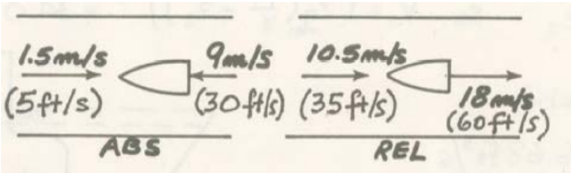
$$R_x = V_2 \dot{m} = 34.0\ \frac{ft}{s} \times 12.93\ \frac{lbf \cdot s}{ft} = 440\ lbf$$

The tension in the cable is 440 lbf.

Problem 4.81

(Difficulty: 2)

4.81 A motorboat moves up a river at a speed of 9 $\frac{m}{s}$ relative to the land. The river flows at a velocity of 1.5 $\frac{m}{s}$. The boat is powered by a jet-propulsion unit which takes in water at the bow and discharges it beneath the surface at the stern. Measurements in the jet show its velocity relative to the boat to be 18 $\frac{m}{s}$. For a flow rate through the unit of 0.15 $\frac{m^3}{s}$, calculate the propulsive force produced.



Given: All the parameters are shown in the figure.

Find: The propulsive force produced.

Assumptions: Flow is steady

Density is constant

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

The boat velocity relative to the river flow is:

$$V_{boat-river} = 9\ \frac{m}{s} + 1.5\ \frac{m}{s} = 10.5\ \frac{m}{s}$$

Now we can assume the river flow is constant and so the jet flow velocity relative to the river flow is:

$$V_{jet-river} = V_{jet-boat} - V_{boat-river} = 18\ \frac{m}{s} - 10.5\ \frac{m}{s} = 7.5\ \frac{m}{s}$$

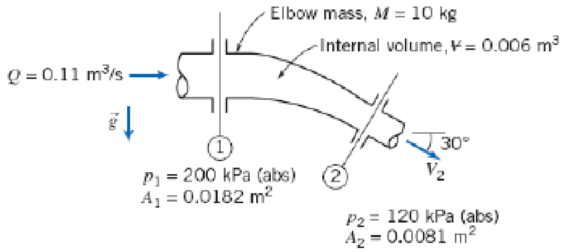
From x momentum equation:

$$R_x = \rho V_{jet-river} Q = 999\ \frac{kg}{m^3} \times 7.5\ \frac{m}{s}\ \times 0.15\ \frac{m^3}{s} = 1124\ N$$

Problem 4.82

(Difficulty: 2)

4.82 A 30° reducing flow elbow is shown. The fluid is water. Evaluate the components of force that must be provided by the adjacent pipes to keep the elbow from moving.



Given: All the parameters are shown in the figure.

Find: The force R_x and R_y must be provided by pipes to keep the elbow from moving.

Assumption: (1) steady flow.

(2) uniform flow at each section

(3) use gage pressures

(4) x horizontal

Solution:

Apply the x and y components of the momentum equation using the CS and CV shown.

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the y-direction

$$F_{sy} + F_{By} = \frac{\partial}{\partial t}\int_{CV} v\rho d\forall + \int_{CS} v\rho \bar{V} \cdot d\bar{A}$$

The density for the water is:

$$\rho = 999\ \frac{kg}{m^3}$$

x component:

$$R_x + p_{1g}A_1 - p_{2g}A_2\cos\theta = u_1[-\rho Q] + u_2[\rho Q]$$

$$u_1 = V_1$$

$$u_2 = V_2\cos\theta$$

The velocities are calculated using the continuity relation

$$V_1 = \frac{Q}{A_1} = \frac{0.11\ \frac{m^3}{s}}{0.0182\ m^2} = 6.04\ \frac{m}{s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.11\ \frac{m^3}{s}}{0.0081\ m^2} = 13.6\ \frac{m}{s}$$

$$R_x = (-V_1 + V_2\cos\theta)\rho Q - p_{1g}A_1 + p_{2g}A_2\cos\theta$$

$$R_x = \left(-6.04\ \frac{m}{s} + 13.6\ \frac{m}{s}\cos 30°\right) \times 999\ \frac{kg}{m^3} \times 0.11\ \frac{m^3}{s} - (200 - 101) \times 1000\frac{N}{m^2} \times 0.0182\ m^2 + (120 - 101) \times 1000\frac{N}{m^2} \times 0.0081 m^2 \times \cos 30°$$

$$R_x = -1040\ N$$

y component:

$$R_y + p_{2g}A_2\sin\theta - Mg - \rho\forall g = -v_1[-\rho Q] + v_2[\rho Q]$$

$$v_1 = 0$$

$$v_2 = -V_2\sin\theta$$

So we have:

$$R_y = -V_2\sin\theta\,\rho Q + Mg + \rho\forall g - p_{2g}A_2\sin\theta$$

$$R_y = -13.6\ \frac{m}{s} \times 0.5 \times 999\ \frac{kg}{m^3} \times 0.11\ \frac{m^3}{s} + 10\ kg \times 9.81\ \frac{m}{s^2} + 999\ \frac{kg}{m^3} \times 0.006\ m^3 \times 9.81\ \frac{m}{s^2} - (120 - 101) \times 1000\ \frac{N}{m^2} \times 0.0081\ m^2 \times \sin 30°$$

$$R_y = -667\ N$$

R_x and R_y are the horizontal and vertical components of force that must be supplied by the adjacent pipes to keep the elbow from moving.

Problem 4.83

(Difficulty: 2)

4.83 A monotube boiler consists of a 6 m length of tubing with 9.5-mm-ID. Water enters at the rate of 0.135 kg/s at 3.45 MPa absolute. Stream leaves at 2.76 MPa gage with 12.4 kg/m^3 density. Find the magnitude and direction of the force exerted by the following fluid on the tube.

Given: The diameter of the tube: $D = 9.5\ mm$. Mass flow rate at inlet: $\dot{m} = 0.135\ \frac{kg}{s}$.

Pressure at inlet: $p_1 = 3.45\ MPa$. Pressure at outlet: $p_2 = 2.76\ MPa$. Density at outlet: $\rho_2 = 12.4\ \frac{kg}{m^3}$.

Find: The magnitude and direction of the force exerted by the fluid on the tube.

Assumption: (1) $F_{Bx} = 0$

(2) steady flow of an incompressible fluid

(3) uniform flow at each section

Solution:

Basic equations:

Continuity:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Momentum equation in x-direction:

$$R_x + F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{V} \cdot d\vec{A}$$

From continuity we have:

$$\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

With constant A, we get:

$$\rho_1 V_1 = \rho_2 V_2$$

The density of water is:

$$\rho_1 = 999\ \frac{kg}{m^3}$$

The density of steam is:

$$\rho_2 = 12.4\ \frac{kg}{m^3}$$

We obtain the velocity by:

$$V_1 = \frac{\dot{m}}{\rho_1 A} = \frac{0.135\ \frac{kg}{s}}{999\ \frac{kg}{m^3} \times \frac{\pi}{4} \times (0.0095\ m)^2} = 1.907\ \frac{m}{s}$$

$$V_2 = \frac{\rho_1}{\rho_2} V_1 = \frac{999\ \frac{kg}{m^3}}{12.4\ \frac{kg}{m^3}} \times 1.907\ \frac{m}{s} = 153.6\ \frac{m}{s}$$

Apply the x component of the momentum equation. The pressures are gage pressures with the atmospheric pressure then zero.

$$R_x + p_1 A - p_2 A = V_1(-\dot{m}) + V_2(\dot{m}) = (V_2 - V_1)\dot{m}$$

$$R_x = (p_2 - p_1)A + (V_2 - V_1)\dot{m}$$

$$R_x = (2.76\ \times 10^6 - 3.45 \times 10^6\ )\ \frac{N}{m^2} \times \frac{\pi}{4} \times (0.0095\ m)^2 + \left(153.6\ \frac{m}{s} - 1.907\ \frac{m}{s}\right) \times 0.135\ \frac{kg}{s}$$

$$R_x = -28.4\ N$$

But R_x is force on CV; force on pipe is F_x.

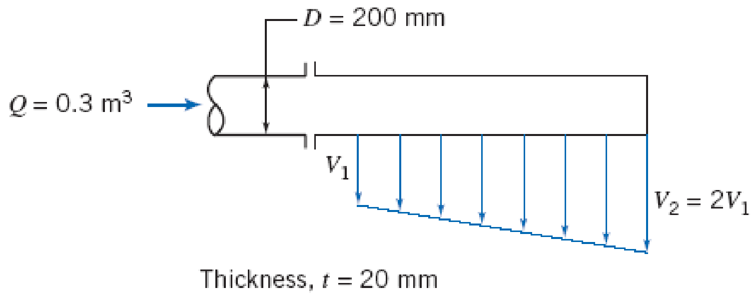
$$F_x = -R_x = 28.4\ N$$

The direction is to right.

Problem 4.84

[Difficulty: 3]

4.84 Water is discharged at a flow rate of 0.3 m^3/s from a narrow slot in a 200-mm-diameter pipe. The resulting horizontal two-dimensional jet is 1 m long and 20 mm thick, but of nonuniform velocity; the velocity at location ② is twice that at location ①. The pressure at the inlet section is 50 kPa (gage). Calculate (a) the velocity in the pipe and at locations ① and ② and (b) the forces required at the coupling to hold the spray pipe in place. Neglect the mass of the pipe and the water it contains.



Given: Data on flow out of pipe device

Find: Velocities at 1 and 2; force on coupling

Solution:

Basic equations (continuity and x and y mom.):

$$\frac{\partial}{\partial t}\int_{CV} \rho\, dV\!\!\!\!/ + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$$

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, dV\!\!\!\!/ + \int_{CS} u\,\rho \vec{V} \cdot d\vec{A} \qquad F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v\,\rho\, dV\!\!\!\!/ + \int_{CS} v\,\rho \vec{V} \cdot d\vec{A}$$

The given data is $\rho = 999\cdot\frac{kg}{m^3}$ $D = 20\cdot cm$ $L = 1\cdot m$ $t = 20\cdot mm$ $p_{3g} = 50\cdot kPa$ $Q = 0.3\cdot\frac{m^3}{s}$

From continuity $Q = A\cdot V_{ave}$ due to linear velocity distribution $V_{ave} = \frac{1}{2}\cdot(V_1 + V_2)$

Note that at the exit $V(x) = V_1 + \frac{(V_2 - V_1)}{L}\cdot x$

Hence $Q = \frac{1}{2}\cdot(V_1 + V_2)\cdot L\cdot t = \frac{1}{2}\cdot(V_1 + 2\cdot V_1)\cdot L\cdot t$

$V_1 = \frac{2\cdot Q}{3\cdot L\cdot t}$ $V_1 = 10\frac{m}{s}$ $V_2 = 2\cdot V_1$ $V_2 = 20\frac{m}{s}$

At the inlet (location 3) $V_3 = \frac{Q}{\frac{\pi}{4}\cdot D^2}$ $V_3 = 9.549\frac{m}{s}$

Applying x momentum $R_x + p_{3g}\cdot\frac{\pi}{4}\cdot D^2 = -V_3\cdot\rho\cdot Q$ $R_x = -p_{3g}\cdot\frac{\pi}{4}\cdot D^2 - V_3\cdot\rho\cdot Q$ $R_x = -4.43\cdot kN$

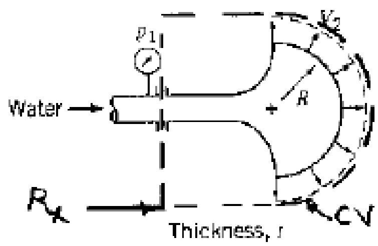
Applying y momentum $R_y = -\int_0^L V(x)\cdot\rho\cdot V(x)\cdot t\,dx = -\rho\cdot t\cdot\int_0^L \left[V_1 + \frac{(V_2 - V_1)}{L}\cdot x\right]^2 dx$

Expanding and integrating $R_y = -\rho\cdot t\cdot\left[V_1^{\,2}\cdot L + 2\cdot V_1\cdot\left(\frac{V_2 - V_1}{L}\right)\cdot\frac{L^2}{2} + \left(\frac{V_2 - V_1}{L}\right)^2\cdot\frac{L^3}{3}\right]$ $R_y = -4.66\cdot kN$

Problem 4.85

(Difficulty: 2)

4.85 A nozzle for a spray system is designed to produce a flat radial sheet of water. The sheet leaves the nozzle at $V_2 = 10\ \frac{m}{s}$, covers 180° of arc, and has thickness $t = 1.5\ mm$. The nozzle discharge radius is $R = 50\ mm$. The water supply pipe is 35 mm in diameter and the inlet pressure is $p_1 = 150\ kPa$ (abs). Evaluate the axial force exerted by the spray nozzle on the coupling.



Given: The sheet leaves nozzle at: $V_2 = 10\ \frac{m}{s}$. Thickness: $t = 1.5\ mm$. Radius: $R = 50\ mm$.

Pipe diameter: $D_1 = 35\ mm$. Inlet pressure: $p_1 = 150\ kPa$.

Find: The axial force exerted by the spray nozzle on the coupling.

Assumption: (1) $F_{Bx} = 0$.

(2) steady flow of an incompressible fluid

(3) uniform flow at each section

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

From continuity we have:

$$Q = V_1 A_1 = V_2 A_2 = V_2 \pi R t = 10\ \frac{m}{s} \times \pi \times 0.05\ m \times 0.0015\ m = 0.00236\ \frac{m^3}{s}$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi}{4}D_1^2} = \frac{0.00236\ \frac{m^3}{s} \times 4}{\pi \times (0.035\ m)^2} = 2.45\ \frac{m}{s}$$

Apply the x component of the momentum equation, using the CV and coordinate shown. The pressures are gage pressures.

$$R_x + p_{1g}A_1 = u_1(-\rho Q) + \int_{A_2} u_2\, \rho V_2 dA_2$$

$$u_1 = V_1$$

$$u_2 = V_2 \cos\theta$$

$$dA_2 = Rtd\theta$$

$$\int_{A_2} u_2\, \rho V_2 dA_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} V_2 \cos\theta \rho V_2\, Rtd\theta = 2\rho V_2^2 Rt \int_0^{\frac{\pi}{2}} \cos\theta d\theta = 2\rho V_2^2 Rt$$

$$R_x = -p_{1g}A_1 - V_1\rho Q + 2\rho V_2^2 Rt$$

The density of the water is:

$$\rho = 999\ \frac{kg}{m^3}$$

Thus

$$R_x = -(150 - 101) \times 1000\ \frac{N}{m^3} \times \frac{\pi \times (0.035\ m)^2}{4} - 2.45\ \frac{m}{s} \times 999\ \frac{kg}{m^3} \times 0.00236\ \frac{m^3}{s} + 2 \times 999\ \frac{kg}{m^3} \times \left(10\ \frac{m}{s}\right)^2 \times 0.05\ m \times 0.0015\ m$$

$$R_x = -37.9\ N$$

But R_x is force on CV. Force on coupling is F_x.

$$F_x = -R_x = 37.9\ N$$

The direction is to the right.

Problem 4.86 [Difficulty: 2]

4.86 The horizontal velocity in the wake behind an object in an air stream of velocity U is given by

$$u(r) = U\left[1 - \cos^2\left(\frac{\pi r}{2}\right)\right] \qquad |r| \le 1$$

$$u(r) = U \qquad |r| > 1$$

where r is the nondimensional radial coordinate, measured perpendicular to the flow. Find an expression for the drag on the object.

Given: Data on wake behind object

Find: An expression for the drag

Solution:

Basic equation:

Momentum
$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Applying this to the horizontal motion

$$-F = U\cdot\left(-\rho\cdot\pi\cdot 1^2\cdot U\right) + \int_0^1 u(r)\cdot\rho\cdot 2\cdot\pi\cdot r\cdot u(r)\,dr \qquad F = \pi\,\rho\cdot\left(U^2 - 2\cdot\int_0^1 r\cdot u(r)^2\,dr\right)$$

$$F = \pi\,\rho\cdot U^2\cdot\left[1 - 2\cdot\int_0^1 r\cdot\left(1 - \cos\left(\frac{\pi\cdot r}{2}\right)^2\right)^2 dr\right]$$

$$F = \pi\,\rho\cdot U^2\cdot\left(1 - 2\cdot\int_0^1 r - 2\cdot r\cdot\cos\left(\frac{\pi\cdot r}{2}\right)^2 + r\cdot\cos\left(\frac{\pi\cdot r}{2}\right)^4 dr\right)$$

Integrating and using the limits
$$F = \pi\,\rho\cdot U^2\left[1 - \left(\frac{3}{8} + \frac{2}{\pi^2}\right)\right] \qquad F = \left(\frac{5\cdot\pi}{8} - \frac{2}{\pi}\right)\cdot\rho\cdot U^2$$

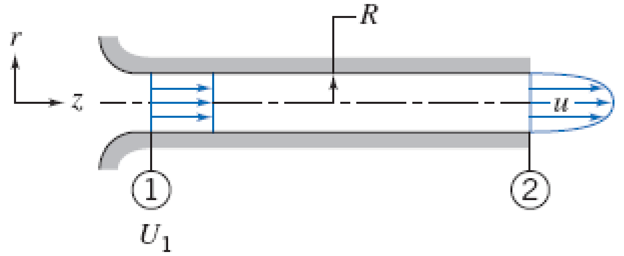
Problem 4.87

[Difficulty: 3]

4.87 An incompressible fluid flows steadily in the entrance region of a circular tube of radius $R = 75$ mm. The flow rate is $Q = 0.1$ m³/s. Find the uniform velocity U_1 at the entrance. The velocity distribution at a section downstream is

$$\frac{u}{u_{max}} = 1 - \left(\frac{r}{R}\right)^2$$

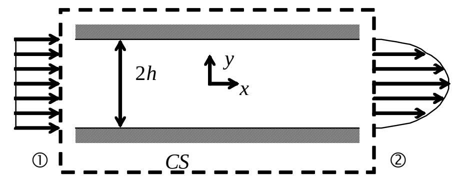
Evaluate the maximum velocity at the downstream section. Calculate the pressure drop that would exist in the channel if viscous friction at the walls could be neglected.



$\rho = 850$ kg/m³

Given: Data on flow in 2D channel

Find: Maximum velocity; Pressure drop



Solution:

Basic equations: Continuity, and momentum flux in x direction

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Neglect friction

Given data $\quad R = 75\cdot mm \quad Q = 0.1\cdot\frac{m^3}{s} \quad \rho = 850\cdot\frac{kg}{m^3}$

From continuity $\quad Q = U_1\cdot\pi\cdot R^2 \quad U_1 = \frac{Q}{\pi\cdot R^2} \quad U_1 = 5.66\frac{m}{s}$

Also $\quad -\rho\cdot U_1\cdot A_1 + \int \rho\cdot u_2\, dA = 0$

$$U_1\cdot\pi\cdot R^2 = \int_0^R u_{max}\cdot\left(1 - \frac{r^2}{R^2}\right)\cdot 2\cdot\pi\cdot r\, dr = 2\cdot\pi\cdot u_{max}\cdot\left(\frac{R^2}{2} - \frac{R^4}{4\cdot R^2}\right) = 2\cdot\pi\cdot u_{max}\cdot\frac{R^2}{4} = \pi\cdot u_{max}\cdot\frac{R^2}{2}$$

Hence $\quad u_{max} = 2\cdot U_1 \quad u_{max} = 11.3\frac{m}{s}$

For x momentum $\quad p_1\cdot A - p_2\cdot A = V_1\cdot\left(-\rho_1\cdot V_1\cdot A\right) + \int \rho_2\cdot u_2\cdot u_2\, dA_2$ $\quad$ Note that there is no R_x (no friction)

$$\left(p_1 - p_2\right)\cdot\pi\cdot R^2 = -\rho\cdot\pi\cdot R^2\cdot U_1^{\,2} + \int_0^R \rho\cdot u_{max}^{\,2}\cdot\left(1 - \frac{r^2}{R^2}\right)^2\cdot 2\cdot\pi\cdot r\, dr = -\rho\cdot\pi\cdot R^2\cdot U_1^{\,2} + 2\cdot\pi\cdot\rho\cdot u_{max}^{\,2}\cdot\left(\frac{R^2}{2} - 2\cdot\frac{R^4}{4\cdot R^2} + \frac{R^6}{6\cdot R^4}\right)$$

$$\Delta p = p_1 - p_2 = -\rho\cdot U_1^{\,2} + \frac{1}{3}\cdot\rho\cdot u_{max}^{\,2} = -\rho\cdot U_1^{\,2} + \frac{1}{3}\cdot\rho\cdot\left(2\cdot U_1\right)^2 = \rho\cdot U_1\cdot\left[\frac{1}{3}\cdot(2)^2 - 1\right] = \frac{1}{3}\cdot\rho\cdot U_1^{\,2}$$

Hence $\quad \Delta p = \frac{1}{3}\times 850\cdot\frac{kg}{m^3}\times\left(5.66\cdot\frac{m}{s}\right)^2\times\frac{N\cdot s^2}{kg\cdot m} \qquad \Delta p = 9.08\cdot kPa$

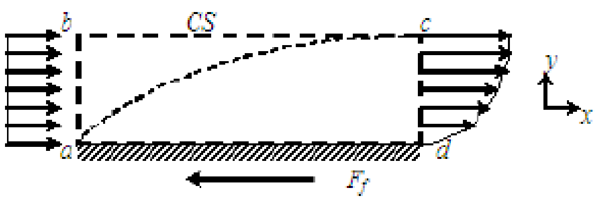
Problem 4.88

(Difficulty: 2)

4.88 Consider the incompressible flow of fluid in a boundary layer as depicted in Example 4.2. Show that the friction drag force of the fluid on the surface is given by

$$F_f = \int_0^\delta \rho u(U-u)\, w dy$$

Evaluate the drag force for the conditions of Example 4.2



Given: All the parameters are shown in the figure.

Find: The drag force $Drag$.

Assumption: (1) steady flow

(2) no net pressure force

(3) $F_{Bx} = 0$

(4) uniform flow at section AB

Solution:

Basic equations: Continuity equation

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Momentum equation in the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Then from continuity:

$$0 = -\rho U w \delta + \int_0^\delta \rho u w dy + \dot{m}_{BC}$$

$$\delta = \int_0^\delta dy$$

The mass flow rate is

$$\dot{m}_{BC} = \rho \int_0^\delta (U-u)wdy$$

Apply the x component of momentumto the controlvolume. The pressure is the same on all surfaces.

$$-F_f = U\{-|\rho U w\delta|\} + \left\{\int_0^\delta \rho u^2 wdy\right\} + U\dot{m}_{BC} = \rho \int_0^\delta [-U^2 + u^2 + U(\sigma - u)]wdy$$

$$Drag = F_f = \int_0^\delta \rho u(U-u)wdy$$

At location CD:

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 = 2\eta - \eta^2$$

$$dy = \delta d\left(\frac{y}{\delta}\right) = \delta d\eta$$

$$Drag = \int_0^\delta \rho U\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\left(U - U\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\right)wdy = \rho U^2 w\delta \int_0^1 (2\eta - \eta^2)(1 - 2\eta + \eta^2)d\eta$$

$$= \rho U^2 w\delta \int_0^1 (2\eta - 5\eta^2 + 4\eta^3 - \eta^4)d\eta = \rho U^2 w\delta \left[\eta^2 - \frac{5}{3}\eta^3 + \eta^4 - \frac{1}{5}\eta^5\right]_0^1$$

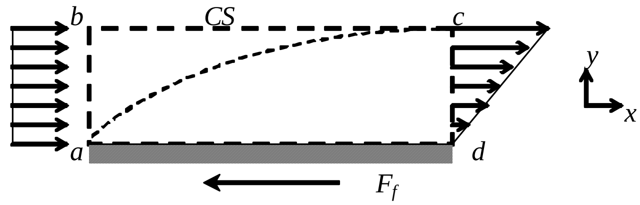
$$Drag = \frac{2}{15}\rho U^2 w\delta$$

The drag force is

$$Drag = \frac{2}{15} \times 1.24\ \frac{kg}{m^3} \times \left(30\ \frac{m}{s}\right)^2 \times 0.6\ m \times 0.005\ m = 0.446\ N$$

Problem 4.89 [Difficulty: 4]

4.89 Air at standard conditions flows along a flat plate. The undisturbed freestream speed is $U_0 = 20$ m/s. At $L = 0.4$ m downstream from the leading edge of the plate, the boundary-layer thickness is $\delta = 2$ mm. The velocity profile at this location is approximated as $u/U_0 = y/\delta$. Calculate the horizontal component of force per unit width required to hold the plate stationary.



Given: Data on flow of boundary layer

Find: Force on plate per unit width

Solution:

Basic equations: Continuity, and momentum flux in x direction

$$\frac{\partial}{\partial t}\int_{CV} \rho \, d\mathcal{V} + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u \, \rho \, d\mathcal{V} + \int_{CS} u \, \rho \vec{V} \cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible 3) No net pressure force

From continuity $$-\rho \cdot U_0 \cdot w \cdot \delta + m_{bc} + \int_0^{\delta} \rho \cdot u \cdot w \, dy = 0$$ where m_{bc} is the mass flow rate across bc (Note: sotware cannot render a dot!)

Hence $$m_{bc} = \int_0^{\delta} \rho \cdot (U_0 - u) \cdot w \, dy$$

For x momentum $$-F_f = U_0 \cdot (-\rho \cdot U_0 \cdot w \cdot \delta) + U_0 \cdot m_{bc} + \int_0^{\delta} u \cdot \rho \cdot u \cdot w \, dy = \int_0^{\delta} \left[-U_0^{\ 2} + u^2 + U_0 \cdot (U_0 - u)\right] \cdot w \, dy$$

Then the drag force is $$F_f = \int_0^{\delta} \rho \cdot u \cdot (U_0 - u) \cdot w \, dy = \int_0^{\delta} \rho \cdot U_0^{\ 2} \cdot \frac{u}{U_0} \cdot \left(1 - \frac{u}{U_0}\right) dy$$

But we have $\frac{u}{U_0} = \frac{y}{\delta}$ where we have used substitution $y = \delta \cdot \eta$

$$\frac{F_f}{w} = \int_0^{\eta=1} \rho \cdot U_0^{\ 2} \cdot \delta \cdot \frac{u}{U_0} \cdot \left(1 - \frac{u}{U_0}\right) d\eta = \rho \cdot U_0^{\ 2} \cdot \delta \cdot \int_0^1 \eta \cdot (1 - \eta) \, d\eta$$

$$\frac{F_f}{w} = \rho \cdot U_0^{\ 2} \cdot \delta \cdot \left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{6} \cdot \rho \cdot U_0^{\ 2} \cdot \delta$$

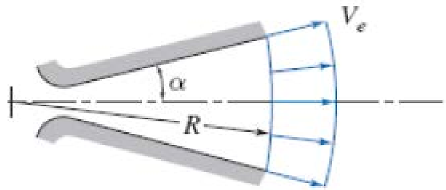
Hence $$\frac{F_f}{w} = \frac{1}{6} \times 1.225 \cdot \frac{kg}{m^3} \times \left(20 \cdot \frac{m}{s}\right)^2 \times \frac{2}{1000} \cdot m \times \frac{N \cdot s^2}{kg \cdot m}$$ (using standard atmosphere density)

$$\frac{F_f}{w} = 0.163 \cdot \frac{N}{m}$$

Problem 4.90

(Difficulty: 2)

4.90 Gases leaving the propulsion nozzle of a rocket are modeled as following radially outward from a point upstream from the nozzle throat. Assume the speed of the exit flow, V_e, has a constant magnitude. Develop an expression for the axial thrust, T_a, developed by flow leaving the nozzle exit plane. Compare your result to the one-dimensional approximation, $T = \dot{m}V_e$. Evaluate the present error for $\alpha = 15°$. Plot the percent error versus α for $0 \leq \alpha \leq 22.5°$.



Given: All the parameters are shown in the figure.

Assumptions: The flow is steady

Find: The expression for the axial thrust T_a. Compare it with the one-dimensional approximation.

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

The mass flow rate is given by

$$\dot{m} = \int_A \rho V dA$$

And the thrust is given by

$$T_a = \int_A u\rho V dA$$

For spherically symmetric flow:

The mass flow rate is $[assuming\ \rho_e \neq f(\theta)]$

$$\dot{m} = \int_A \rho V dA = \int_0^\alpha \rho_e V_e (2\pi R \sin\theta) R d\theta = 2\pi \rho_e V_e R^2 (1 - \cos\alpha)$$

The one-dimensional approximation for thrust is then

$$T = \dot{m} V_e = 2\pi \rho_e V_e^2 R^2 (1 - \cos\alpha)$$

The axial thrust is given by

$$T_a = \int u \rho V dA = \int_0^\alpha V_e \cos\theta\, \rho_e V_e\, (2\pi R \sin\theta) R d\theta = 2\pi \rho_e V_e^2 R^2 \int_0^\alpha \sin\theta \cos\theta\, d\theta$$

$$T_a = \pi p_e V_e^2 R^2 sin^2\alpha$$

The error in the one-dimensional approximation is

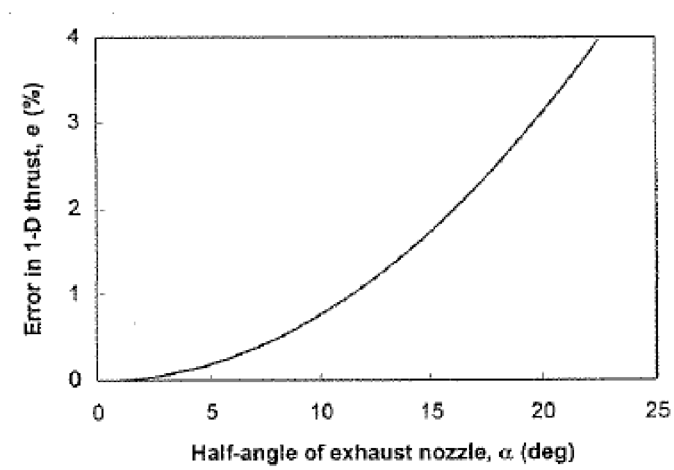
$$e = \frac{T_{1-D} - T_a}{T_a} = \frac{T_{1-D}}{T_a} - 1 = \frac{2\pi \rho_e V_e^2 R^2 (1 - \cos\alpha)}{\pi \rho_e V_e^2 R^2 sin^2\alpha} - 1 = \frac{2(1 - \cos\alpha)}{sin^2\alpha} - 1$$

Eq (1)

The present error is plotted as a function of α.

For $\alpha = 15°$

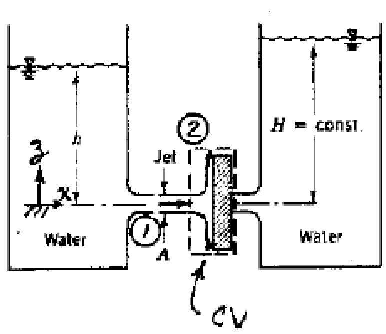
$$e_{15} = \frac{2(1 - \cos 15°)}{sin^2 15°} - 1 = 1.73\%$$



Problem 4.91

(Difficulty: 2)

4.91 Two large tanks containing water have small smoothly contoured orifices of equal area. A jet of liquid issues from the left tank. Assume the flow is uniform and unaffected by friction. The jet impinges on a vertical flat plate covering the opening of the right tank. Determine the minimum value for the height, h, required to keep the plate in place over the opening of the right tank.



Given: All the parameters are shown in the figure.

Find: The minimum value for the height h to keep the plate.

Assumption: (1) steady flow

(2) incompressible flow

(3) No friction

(4) $F_{Bx} = 0$

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Apply Bernoulli equation along a streamsline from the water surface to jet:

$$\frac{p}{\rho} + \frac{V_s^2}{2} + gh = \frac{p}{\rho} + \frac{V^2}{2} + g(0)$$

$$V_s \ll V$$

So that:

$$V = \sqrt{2gh}$$

The pressure is related to depth using the fluid statics relation:

$$p_{3g} = \rho g H$$

Applying the x-momentum equation:

$$-p_{3g}A = -\rho g H A = u_1[-\rho V A] + u_2[\rho V A]$$

$$u_1 = V$$

$$u_2 = 0$$

So we have:

$$-p_{3g}A = -\rho g H A = -\rho V^2 A$$

Thus, using the Bernoulli relation:

$$\rho g H A = \rho V^2 A = \rho 2 g h A$$

$$H = 2h$$

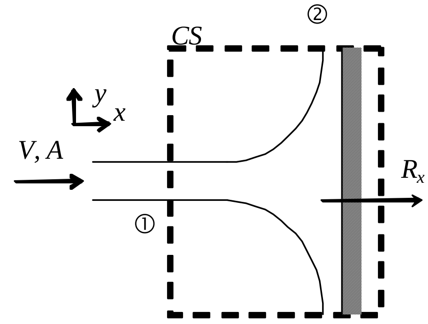
The necessary height to keep the plate in place is

$$h = \frac{H}{2}$$

Problem 4.92

[Difficulty: 3]

4.92 Students are playing around with a water hose. When they point it straight up, the water jet just reaches one of the windows of Professor Pritchard's office, 10 m above. If the hose diameter is 1 cm, estimate the water flow rate (L/min). Professor Pritchard happens to come along and places his hand just above the hose to make the jet spray sideways axisymmetrically. Estimate the maximum pressure, and the total force, he feels. The next day the students again are playing around, and this time aim at Professor Fox's window, 15 m above. Find the flow rate (L/min) and the total force and maximum pressure when he, of course, shows up and blocks the flow.



Given: Water jet shooting upwards; striking surface

Find: Flow rate; maximum pressure; Force on hand

Solution:

Basic equations: Bernoulli and momentum flux in x direction

$$\frac{p}{\rho} + \frac{V^2}{2} + g \cdot z = \text{constant} \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV + \int_{CS} u\,\rho \vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure throughout 4) Uniform flow

Given data $\quad h = 10 \cdot m \quad \rho = 1000 \cdot \frac{kg}{m^3} \quad D = 1 \cdot cm$

Using Bernoulli between the jet exit and its maximum height h $\quad \frac{p_{atm}}{\rho} + \frac{V^2}{2} = \frac{p_{atm}}{\rho} + g \cdot h$

or $\quad V = \sqrt{2 \cdot g \cdot h} \quad V = 14.0\,\frac{m}{s}$

Then $\quad Q = \frac{\pi}{4} \cdot D^2 \cdot V \quad Q = 66.0 \cdot \frac{L}{min}$

For Dr. Pritchard the maximum pressure is obtained from Bernoulli $\quad \frac{p_{atm}}{\rho} + \frac{V^2}{2} = \frac{p_{max}}{\rho} \quad p = \frac{1}{2} \cdot \rho \cdot V^2 \quad p = 98.1 \cdot kPa$ (gage)

For Dr. Pritchard blocking the jet, from x momentum applied to the CV $\quad R_x = u_1 \cdot \left(-\rho \cdot u_1 \cdot A_1\right) = -\rho \cdot V^2 \cdot A$

Hence $\quad F = \rho \cdot V^2 \cdot \frac{\pi}{4} \cdot D^2 \quad F = 15.4\,N$

Repeating for Dr. Fox $\quad h = 15 \cdot m \quad V = \sqrt{2 \cdot g \cdot h} \quad V = 17.2\,\frac{m}{s} \quad Q = \frac{\pi}{4} \cdot D^2 \cdot V \quad Q = 80.8 \cdot \frac{L}{min}$

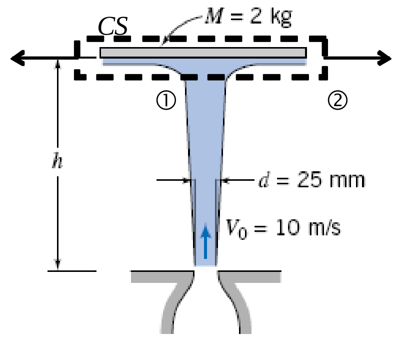
$p = \frac{1}{2} \cdot \rho \cdot V^2 \quad p = 147.1 \cdot kPa$ (gage)

$F = \rho \cdot V^2 \cdot \frac{\pi}{4} \cdot D^2 \quad F = 23.1\,N$

Problem 4.93

[Difficulty: 3]

4.93 A 2-kg disk is constrained horizontally but is free to move vertically. The disk is struck from below by a vertical jet of water. The speed and diameter of the water jet are 10 m/s and 25 mm at the nozzle exit. Obtain a general expression for the speed of the water jet as a function of height, *h*. Find the height to which the disk will rise and remain stationary.



Given: Water jet striking disk

Find: Expression for speed of jet as function of height; Height for stationary disk

Solution:

Basic equations: Bernoulli; Momentum flux in z direction

$$\frac{p}{\rho} + \frac{V^2}{2} + g \cdot z = \text{constant} \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, dV + \int_{CS} u\,\rho \vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow

The Bernoulli equation becomes $\frac{V_0^2}{2} + g\cdot 0 = \frac{V^2}{2} + g\cdot h \qquad V^2 = V_0^2 - 2\cdot g\cdot h \qquad V = \sqrt{V_0^2 - 2\cdot g\cdot h}$

Hence $-M\cdot g = w_1\cdot\left(-\rho\cdot w_1\cdot A_1\right) = -\rho\cdot V^2\cdot A$

But from continuity $\rho\cdot V_0\cdot A_0 = \rho\cdot V\cdot A$ so $V\cdot A = V_0\cdot A_0$

Hence we get $M\cdot g = \rho\cdot V\cdot V\cdot A = \rho\cdot V_0\cdot A_0\cdot\sqrt{V_0^2 - 2\cdot g\cdot h}$

Solving for h $h = \frac{1}{2\cdot g}\cdot\left[V_0^2 - \left(\frac{M\cdot g}{\rho\cdot V_0\cdot A_0}\right)^2\right]$

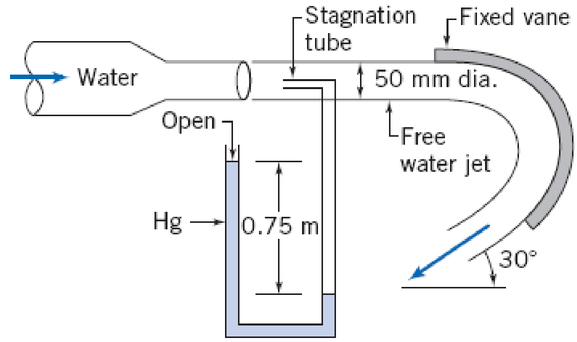
$$h = \frac{1}{2}\times\frac{s^2}{9.81\cdot m}\times\left[\left(10\cdot\frac{m}{s}\right)^2 - \left[2\cdot kg\times\frac{9.81\cdot m}{s^2}\times\frac{m^3}{1000\cdot kg}\times\frac{s}{10\cdot m}\times\frac{4}{\pi\cdot\left(\frac{25}{1000}\cdot m\right)^2}\right]^2\right]$$

$h = 4.28\,m$

Problem 4.94

[Difficulty: 3]

4.94 A stream of water from a 50-mm-diameter nozzle strikes a curved vane, as shown. A stagnation tube connected to a mercury-filled U-tube manometer is located in the nozzle exit plane. Calculate the speed of the water leaving the nozzle. Estimate the horizontal component of force exerted on the vane by the jet. Comment on each assumption used to solve this problem.



Given: Stream of water striking a vane

Find: Water speed; horizontal force on vane

Solution:

Basic equations: Bernoulli; Momentum flux in x direction

$$\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow

Given or available data $\quad D = 50\cdot mm \quad \rho_{water} = 1000\cdot\frac{kg}{m^3} \quad \rho_{Hg} = 13.6\,\rho_{water} \quad \theta = 30\cdot deg \quad \Delta h = 0.75\,m$

From Bernoulli $\quad p_0 = p + \frac{1}{2}\cdot\rho_{water}\cdot V^2 \quad$ and for the manometer $\quad p_0 - p = \rho_{Hg}\cdot g\cdot\Delta h$

Combining $\quad \frac{1}{2}\cdot\rho_{water}\cdot V^2 = \rho_{Hg}\cdot g\cdot\Delta h \quad$ or $\quad V = \sqrt{\frac{2\cdot\rho_{Hg}\cdot g\cdot\Delta h}{\rho_{water}}} \quad V = 14.1\,\frac{m}{s}$

Applying x momentum to the vane

$$R_x = \rho_{water}\cdot V\cdot\left(-V\cdot\frac{\pi}{4}\cdot D^2\right) + \rho_{water}\cdot(-V\cdot\cos(\theta))\cdot\left(V\cdot\frac{\pi}{4}\cdot D^2\right)$$

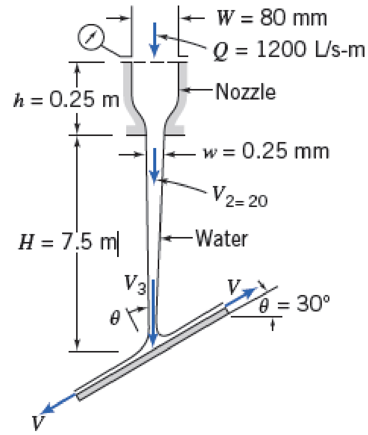
$$R_x = -\rho_{water}\cdot V^2\cdot\frac{\pi}{4}\cdot D^2\cdot(1 + \cos(\theta)) \qquad R_x = -733\,N$$

Assuming frictionless, incompressible flow with no net pressure force is realistic, except along the vane where friction will reduce flow momentum at the exit.

Problem 4.95

(Difficulty: 3)

4.95 A plane nozzle discharges vertically 1200 L/s per unit width downward to atmosphere. The nozzle is supplied with a steady flow of water. A stationary, inclined, flat plate, located beneath the nozzle, is struck by the water stream. The water stream divides and flows along the inclined plate. The two streams leaving the plate are of unequal thickness. Frictional effects are negligible in the nozzle and in the flow along the plate surface. Evaluate the minimum gage pressure required at the nozzle inlet.



P4.95

Given: Nozzle flow striking inclined plate

Find: Minimum gage pressure

Solution:

Basic equations:

Bernoulli equation:

$$\frac{p}{\rho}+\frac{V^2}{2}+gz=const$$

Momentum equation in y-direction:

$$R_y+F_{sy}+F_{By}=\frac{\partial}{\partial t}\int_{CV} v\rho d\forall+\int_{CS} v\rho\vec{V}\cdot d\vec{A}$$

The given data is

$\rho=999\ \frac{kg}{m^3}, q=1200\ \frac{L}{m\cdot s}, W=80\ mm, h=0.25\ m, H=7.5\ m, w=25\ mm, \theta=30°$

For the exit velocity and nozzle velocity:

$$V_1 = \frac{q}{W} = \frac{1200\ \frac{L}{m \cdot s}}{0.08\ m} = 15.0\ \frac{m}{s}$$

$$V_2 = V_1 \frac{W}{w} = 15.0\ \frac{m}{s} \times \frac{80\ mm}{25\ mm} = 48\ \frac{m}{s}$$

Then from Bernoulli:

$$p_1 + \frac{\rho}{2} V_1^2 + \rho g h = p_{atm} + \frac{\rho}{2} V_2^2$$

$$p_1 = \frac{\rho}{2}(V_2^2 - V_1^2) - \rho g h$$

$$p_1 = \frac{999\ \frac{kg}{m^3}}{2} \times \left(\left(48\ \frac{m}{s}\right)^2 - \left(15\ \frac{m}{s}\right)^2\right) - 9810\ \frac{N}{m^3} \times 0.25\ m = 1036\ kPa$$

Applying Bernoulli between 2 and the plate (state 3)

$$p_{atm} + \frac{\rho}{2} V_2^2 = p_{atm} + \frac{\rho}{2} V_3^2 - \rho g H$$

$$V_3 = \sqrt{V_2^2 + 2gH} = \sqrt{\left(48\ \frac{m}{s}\right)^2 + 2 \times 9.81\ \frac{m}{s^2} \times 7.5\ m} = 49.5\ \frac{m}{s}$$

For the plate there is no force along the plate (x momentum) as there is no friction. For the force normal to the plate (y momentum per unit width) we have:

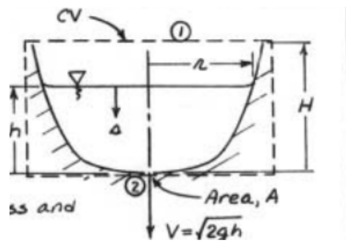
$$R_y = -V_3 \cdot \cos\theta \cdot (-\rho V_3 A_3) = V_3 \cdot \cos\theta \cdot (\rho q)$$

$$R_y = 49.5\ \frac{m}{s} \times \cos 30° \times 999\ \frac{kg}{m^3} \times 1.2\ \frac{m^2}{s} = 98.5\ \frac{kN}{m}$$

Problem 4.96

(Difficulty: 4)

4.96 In ancient Egypt, circular vessels filled with water sometimes were used as crude clocks. The vessels were shaped in such a way that, as water drained from the bottom, the surface level dropped at constant rate, s. Assume that water drains from a small hole of area A. Find an expression for the radius of the vessel, r, as a function of the water level, h. Obtain an expression for the volume of water needed so that the clock will operate for n hours.



Assumptions: (1) Quasi-steady flow; $\frac{\partial}{\partial t}$ small.

(2) Incompressible flow

(3) uniform flow at each cross section

(4) flow along a streamline

(5) no friction

(6) $\rho_{air} \ll \rho_{H_2o}$

Solution:

Apply conservation of mass and the Bernoulli equation.

Basic equations:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = const$$

Writing Bernoulli equation from the liquid surface to the jet exit,

$$\frac{p_{atm}}{\rho} + \frac{v^2}{2} + gh = \frac{p_{atm}}{\rho} + \frac{V^2}{2} + g(0)$$

For $v \ll V$, we have:

$$V = \sqrt{2gh}$$

For the control volume,

$$0 = \frac{\partial}{\partial t}\int_{\forall_{air}} \rho_{air} d\forall + \frac{\partial}{\partial t}\int_{\forall_{H_2o}} \rho_{H_2o} d\forall + \{-|\rho_{air} V_1 A_1|\} + \{|\rho_{H_2o} VA|\}$$

As $\rho_{air} \ll \rho_{H_2o}$ then

$$0 = \rho_{H_2o}\frac{d\forall}{dt} + \rho_{H_2o} VA = \rho_{H_2o}\pi\Omega^2\frac{dh}{dt} + \rho_{H_2o}\sqrt{2gh}A$$

But Ω decreases,

$$\frac{dh}{dt} = -v$$

$$\pi\Omega^2 v = \sqrt{2gh}A$$

$$\Omega = \sqrt[4]{2g}\sqrt{\frac{A}{\pi v}}h^{\frac{1}{4}}$$

For n hours operation, $H = \mathrm{n}v$ and:

$$\forall = \int_0^H \pi\Omega^2\, dh = \int_0^{\mathrm{n}v} \sqrt{2gh}\frac{A}{v}\, dh = \frac{2A}{3v}\sqrt{2g}(nv)^{\frac{3}{2}}$$

$$\forall = \frac{2A\sqrt{2g}(n)^{\frac{3}{2}}(v)^{\frac{1}{2}}}{3}$$

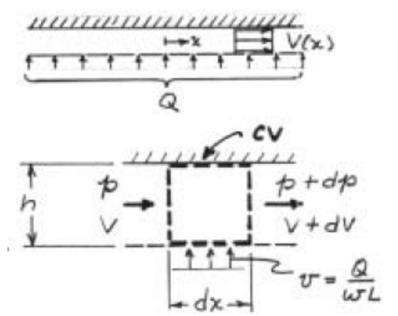
Check dimensions:

$$[\forall] = L^3 = L^2 \times L^{\frac{1}{2}} \times t^{-1} \times t^{\frac{3}{2}} \times L^{\frac{1}{2}} \times t^{-\frac{1}{2}} = L^3$$

Problem 4.97

(Difficulty: 2)

4.97 Incompressible fluid of negligible viscosity is pumped, at total volume flow rate Q, through a porous surface into the small gap between closely spaced parallel plates as shown. The fluid has only horizontal motion in the gap. Assume uniform flow across any vertical section. Obtain an expression for the pressure variation as a function of x. Hint: Apply conservation of mass and the momentum equation to a differential control volume of thickness dx, located at position x.



Given: All the parameters are shown in the figure.

Find: Obtain the pressure variation as function of x.

Assumption: (1) steady flow

(2) incompressible flow

(3) uniform flow at each section

(4) neglect friction

(5) $F_{Bx} = 0$

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Momentum equation for the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

Then from the continuity equation

$$0 = \int_{CS} \bar{V} \cdot d\bar{A} = -Vwh + \left(-\frac{Q}{wL} wdx\right) + (V + dV)wh$$

$$whdV = \frac{Q}{L} dx$$

$$V = \frac{Q}{wh}\frac{x}{L} + c$$

With $V(x = 0) = 0$, we have $c = 0$, so that:

$$V = \frac{Q}{wh}\frac{x}{L}$$

Applying the x- momentum equation:

$$pwh - (p + dp)wh = u_x(-\rho Vwh) + u_{dx}\left(-\rho \frac{Q}{wh} wdx\right) + u_{x+dx}[\rho(V + dV)wh]$$

$$u_x = V$$

The u-component of velocity at the lower surface is zero: $u_{dx} = 0$

$$u_{x+dx} = V + dV$$

From continuity equation, the velocity at the exit of the CV is:

$$(V + dV)wh = Vwh + Q\frac{dx}{L}$$

The momentum equation becomes:

$$-dpwh = -\rho V^2 wh + 0 + (V + dV)\left(Vwh + Q\frac{dx}{L}\right)\rho$$

$$-dpwh = \rho VwhdV + V\rho Q\frac{dx}{L} + \rho QdV\frac{dx}{L}$$

The products of differentials are neglected $(ie. dVdx \ll dx)$, and with the expression for dV

$$dV = \frac{Q}{wh}\frac{dx}{L}$$

The momentum equation becomes

$$-dp = \rho VdV + \frac{V\rho Q}{wh}\frac{dx}{L} = \rho V\frac{Q}{wh}\frac{dx}{L} + \frac{V\rho Q}{wh}\frac{dx}{L} = 2\rho\frac{Q}{wh}\frac{x}{L}\frac{Q}{wh}\frac{dx}{L}$$

$$-dp = 2\rho\left(\frac{Q}{whL}\right)^2 xdx$$

$$p(x) = -\rho\left(\frac{Q}{whL}\right)^2 x^2 + c$$

For $p(0) = p_0$, then

$$p(x) = p_0 - \rho \left(\frac{Q}{wh}\right)^2 \left(\frac{x}{L}\right)^2$$

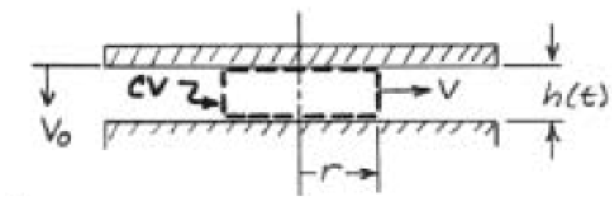
Problem 4.98 [Difficulty: 5]

4.98 The narrow gap between two closely spaced circular plates initially is filled with incompressible liquid. At $t = 0$ the upper plate, initially h_0 above the lower plate, begins to move downward toward the lower plate with constant speed, V_0, causing the liquid to be squeezed from the narrow gap. Neglecting viscous effects and assuming uniform flow in the radial direction, develop an expression for the velocity field between the parallel plates. *Hint*: Apply conservation of mass to a control volume with the outer surface located at radius r. Note that even though the speed of the upper plate is constant, the flow is unsteady. For $V_0 = 0.01$ m/s and $h_0 = 2$ mm, find the velocity at the exit radius $R = 100$ mm at $t = 0$ and $t = 0.1$ s. Plot the exit velocity as a function of time, and explain the trend.

Given: Plates coming together

Find: Expression for velcoity field; exit velocity; plot

Solution: Apply continuity using deformable CV as shown



Basic equation: $\dfrac{\partial}{\partial t}\displaystyle\int_{CV} \rho\, d\mathcal{V} + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$

Assumptions: Incompressible, uniform flow

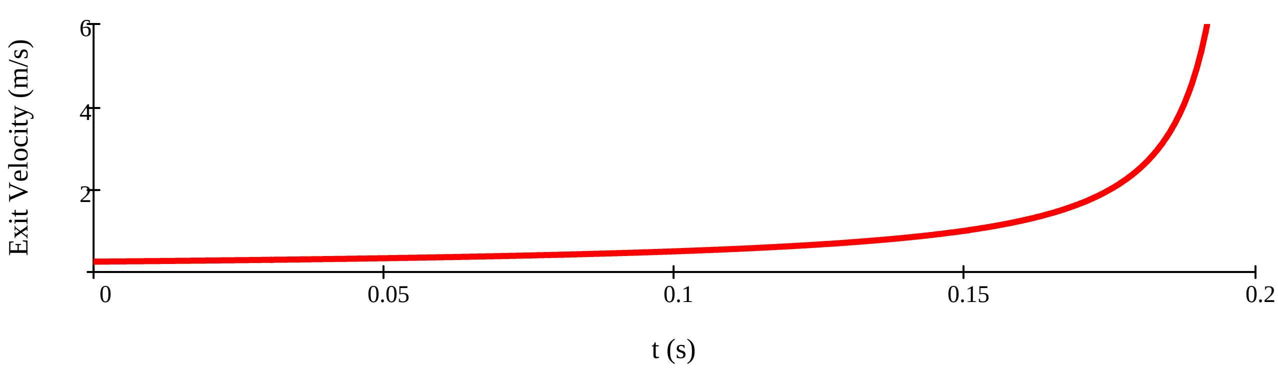
Given data: $V_0 = 0.01 \cdot \dfrac{m}{s}$ $\quad h_0 = 2 \cdot mm$ $\quad R = 100 \cdot mm$

Continuity becomes $\dfrac{\partial \mathcal{V}}{\partial t} + \displaystyle\int_{CS} \vec{V} \cdot d\vec{A} = 0$ or $\dfrac{\partial}{\partial t}\left(\pi \cdot r^2 \cdot h\right) + V \cdot 2 \cdot \pi \cdot r \cdot h = 0$

or $\pi \cdot r^2 \cdot \dfrac{dh}{dt} + V \cdot 2 \cdot \pi \cdot r \cdot h = \pi \cdot r^2 \cdot V_0 + V \cdot 2 \cdot \pi \cdot r \cdot h = 0$ Hence $V(r) = V_0 \cdot \dfrac{r}{2 \cdot h}$

If V_0 is constant $h = h_0 - V_0 \cdot t$ so $V(r,t) = \dfrac{V_0 \cdot r}{2 \cdot \left(h_0 - V_0 \cdot t\right)}$ Note that $t_{max} = \dfrac{h_0}{V_0}$ $\quad t_{max} = 0.200\ s$

Evaluating $V(R,0) = 0.250\ \dfrac{m}{s}$ $\quad V(R, 0.1 \cdot s) = 0.500\ \dfrac{m}{s}$



The velocity greatly increases as the constant flow rate exits through a gap that becomes narrower with time.

Problem 4.99

(Difficulty: 2)

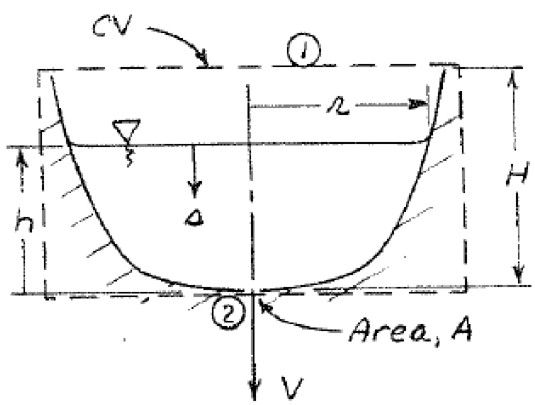
4.99 Design a clepsydra(Egyptian water clock)-a vessel from which water drains by gravity through a hole in the bottom and which indicates time by the level of the remaining water. Specify the dimensions of the vessel and the size of the drain hole; indicate the amount of water needed to fill the vessel and the interval at which it must be filled. Plot the vessel radius as a function of elevation.

Discussion: The original Egyptian water clock was an open water-filled vessel with an orifice in the bottom. The vessel shape was designed so that the water level dropped at a constant rate during use.

Water leaves the orifice at higher speed when the water level within the vessel is high, and at lower speed when the water level within the vessel is low. The size of the orifice is constant. Thus the instantaneous volume flow rate depends on the water level in the vessel.

The rate at which the water level falls in the vessel depends on the volume flow rate and the area of the water surface. The surface area at each water level must be chosen so that the water level within the vessel decreases at a constant rate. The continuity and Bernoulli equations can be applied to determine the required vessel shape so that the water surface level drops at a constant rate.

Use the CV and notation shown.



Find: The vessel radius as a function of elevation.

Assumption: (1) quasi-steady flow

(2) incompressible flow

(3) uniform flow at each cross-section

(4) flow along a streamline

(5) No friction

(6) $\rho_{air} \ll \rho_{H_2o}$

Solution:

Basic equations: Continuity

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

writing Bernoulli equation from the liquid surface to the jet exit.

$$\frac{p_{atm}}{\rho} + \frac{\Delta^2}{2} + gh = \frac{p_{atm}}{\rho} + \frac{V^2}{2} + g(0)$$

For $\Delta \ll V$, then $V = \sqrt{2gh}$.

For the CV,

$$0 = \frac{\partial}{\partial t}\int_{\forall air} \rho_{air} d\forall + \frac{\partial}{\partial t}\int_{\forall H_2o} \rho_{H_2o} d\forall + \{-|\rho_{air} V_1 A_1|\} + \{\rho_{H_2o} VA\}$$

With $\rho_{air} \ll \rho_{H_2o}$ we have:

$$0 = \rho\frac{d\forall}{dt} + \rho VA = \rho\pi r^2 \frac{dh}{dt} + \rho\sqrt{2gh}A$$

But h decreases, so $\frac{dh}{dt} = -\Delta$. Thus

$$\pi\Omega^2\Delta = \sqrt{2gh}A$$

$$\Omega = \sqrt[4]{2g}\sqrt{\frac{A}{\pi\Delta}}h^{\frac{1}{4}}$$

For n hours operation, $H = n\Delta$, and

$$\forall = \int_0^H \pi\Omega^2 dh = \int_0^{n\Delta} \sqrt{2gh}\frac{A}{\Delta}dh$$

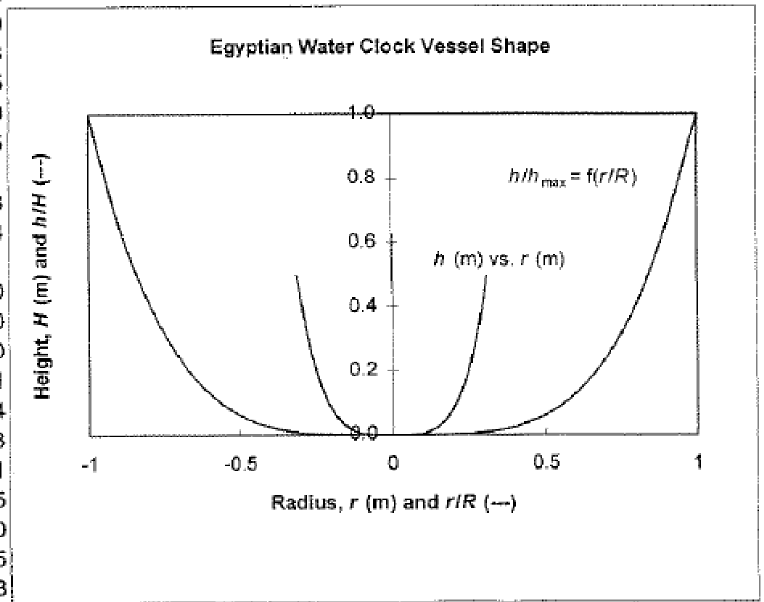
$$\forall = \frac{2A\sqrt{2g}n^{\frac{3}{2}}}{3}\Delta^{\frac{1}{2}}$$

Evaluating and plotting:

Input Parameters:

Maximum water height:	H =	0.5	m
Number of hours' duration:	n =	24	hr

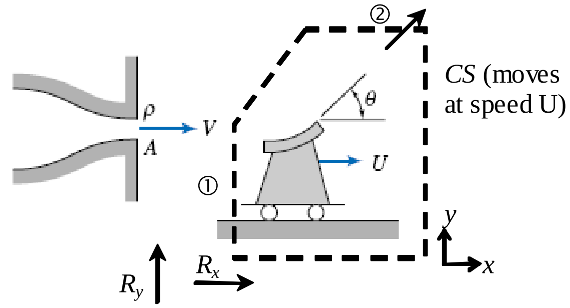
Dimensionless Shape		Actual Shape	
r/R	***h/H***	***r*** **(m)**	***h*** **(m)**
-1	1.00	-0.309	0.500
-0.9	0.656	-0.278	0.328
-0.8	0.410	-0.247	0.205
-0.7	0.240	-0.216	0.120
-0.6	0.130	-0.185	0.065
-0.5	0.063	-0.155	0.031
-0.4	0.026	-0.124	0.013
-0.3	0.008	-0.093	0.004
-0.2	0.002	-0.062	0.001
-0.1	0.000	-0.031	0.000
0	0	0	0
0.1	0.000	0.031	0.000
0.2	0.002	0.062	0.001
0.3	0.008	0.093	0.004
0.4	0.026	0.124	0.013
0.5	0.063	0.155	0.031
0.6	0.130	0.185	0.065
0.7	0.240	0.216	0.120
0.8	0.410	0.247	0.205
0.9	0.656	0.278	0.328
1	1.000	0.309	0.500



Problem 4.100

[Difficulty: 3]

4.100 Water from a stationary nozzle impinges on a moving vane with turning angle $\theta = 120°$. The vane moves away from the nozzle with constant speed, $U = 10$ m/s, and receives a jet that leaves the nozzle with speed $V = 30$ m/s. The nozzle has an exit area of 0.004 m^2. Find the force that must be applied to maintain the vane speed constant.



Given: Water jet striking moving vane

Find: Force needed to hold vane to speed U = 10 m/s

Solution:

Basic equations: Momentum flux in x and y directions

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!\!- + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A} \qquad F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v\,\rho\,dV\!\!\!\!- + \int_{CS} v\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow 5) Jet relative velocity is constant

Then $$R_x = u_1\cdot(-\rho\cdot V_1\cdot A_1) + u_2\cdot(\rho\cdot V_2\cdot A_2) = -(V-U)\cdot[\rho\cdot(V-U)\cdot A] + (V-U)\cdot\cos(\theta)\cdot[\rho\cdot(V-U)\cdot A]$$

$$R_x = \rho(V-U)^2\cdot A\cdot(\cos(\theta)-1)$$

Using given data

$$R_x = 1000\cdot\frac{kg}{m^3}\times\left[(30-10)\cdot\frac{m}{s}\right]^2\times 0.004\cdot m^2\times(\cos(120\cdot deg)-1)\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_x = -2400\,N$$

Then $$R_y = v_1\cdot(-\rho\cdot V_1\cdot A_1) + v_2\cdot(\rho\cdot V_2\cdot A_2) = -0 + (V-U)\cdot\sin(\theta)\cdot[\rho\cdot(V-U)\cdot A]$$

$$R_y = \rho(V-U)^2\cdot A\cdot\sin(\theta) \qquad R_y = 1000\cdot\frac{kg}{m^3}\times\left[(30-10)\cdot\frac{m}{s}\right]^2\times 0.004\cdot m^2\times\sin(120\cdot deg)\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_y = 1386\,N$$

Hence the force required is 2400 N to the left and 1390 N upwards to maintain motion at 10 m/s

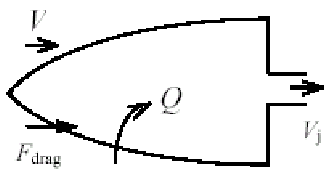
Problem 4.101 [Difficulty: 3]

4.101 A freshwater jet boat takes in water through side vents and ejects it through a nozzle of diameter $D = 75$ mm; the jet speed is V_j. The drag on the boat is given by $F_{drag} \propto kV^2$, where V is the boat speed. Find an expression for the steady speed, V, in terms of water density ρ, flow rate through the system of Q, constant k, and jet speed V_j. A jet speed $V_j = 15$ m/s produces a boat speed of $V = 10$ m/s.

(a) Under these conditions, what is the new flow rate Q?
(b) Find the value of the constant k.
(c) What speed V will be produced if the jet speed is increased to $V_j = 25$ m/s?
(d) What will be the new flow rate?

Given: Data on jet boat

Find: Formula for boat speed; flow rate; value of k; new speed and flow rate



CV in boat coordinates

Solution:

Basic equation:

Momentum
$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\,\rho\, dV\!\!\!\!- + \int_{CS} \vec{V}_{xyz}\,\rho\,\vec{V}_{xyz}\cdot d\vec{A}$$

Given data $D = 75\cdot mm$ $\quad V_j = 15\cdot\frac{m}{s}$ $\quad V = 10\cdot\frac{m}{s}$ $\quad \rho = 1000\cdot\frac{kg}{m^3}$

Applying the horizontal component of momentum

$F_{drag} = V\cdot(-\rho\cdot Q) + V_j\cdot(\rho\cdot Q)$ or, with $F_{drag} = k\cdot V^2$ $\quad k\cdot V^2 = \rho\cdot Q\cdot V_j - \rho\cdot Q\cdot V$

$$k\cdot V^2 + \rho\cdot Q\cdot V - \rho\cdot Q\cdot V_j = 0$$

Solving for V
$$V = -\frac{\rho\cdot Q}{2\cdot k} + \sqrt{\left(\frac{\rho\cdot Q}{2\cdot k}\right)^2 + \frac{\rho\cdot Q\cdot V_j}{k}} \qquad (1)$$

For the flow rate $\quad Q = V_j\cdot\frac{\pi}{4}\cdot D^2 \qquad Q = 0.0663\frac{m^3}{s}$

To find k from Eq 1, let $\quad \alpha = \frac{\rho\cdot Q}{2\cdot k}$ then $\quad V = -\alpha + \sqrt{\alpha^2 + 2\cdot\alpha\cdot V_j}$

$(V+\alpha)^2 = V^2 + 2\cdot\alpha\cdot V + \alpha^2 = \alpha^2 + 2\cdot\alpha\cdot V_j$ or $\quad \alpha = \frac{V^2}{2\cdot\left(V_j - V\right)} \qquad \alpha = 10\frac{m}{s}$

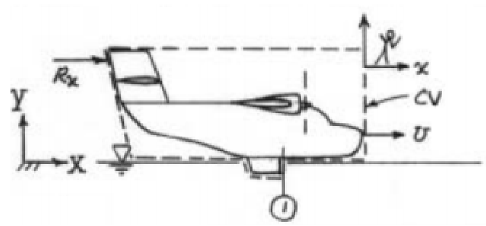
Hence $\quad k = \frac{\rho\cdot Q}{2\cdot\alpha} \qquad k = 3.31\frac{N}{\left(\frac{m}{s}\right)^2}$

For $\quad V_j = 25\cdot\frac{m}{s} \qquad Q = V_j\cdot\frac{\pi}{4}\cdot D^2 \qquad Q = 0.11\frac{m^3}{s} \qquad V = \left[-\frac{\rho\cdot Q}{2\cdot k} + \sqrt{\left(\frac{\rho\cdot Q}{2\cdot k}\right)^2 + \frac{\rho\cdot Q\cdot V_j}{k}}\right] \qquad V = 16.7\frac{m}{s}$

Problem 4.102

(Difficulty: 2)

4.102 The Canadair CL-215T amphibious aircraft is specially designed to fight fires. It is the only production aircraft that can scoop water, at up to 6120 gallons in 12 seconds, from any lake, river, or ocean. Determine the added thrust required during water scooping, as a function of aircraft speed, for a reasonable range of speeds.



Find: The added thrust required during water scooping.

Assumption: (1) horizontal motion, so $F_B = 0$.

(2) neglect u_{xyz} within the CV.

(3) uniform flow at inlet cross-section.

(4) neglect hydrostatic pressure

Solution:

Use a CV that moves with the aircraft, as shown.

Basic equation:

Momentum equation in the x direction:

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\rho d\forall + \int_{CS} u_{xyz}\rho \vec{V}_{xyz} \cdot d\vec{A}$$

Then

$$R_x = u_1\{-|\rho Q|\} = -U(-\rho Q) = U\rho Q$$

$$u_1 = -U$$

From the data given:

$$Q = \frac{\Delta\forall}{\Delta t} = \frac{6120\ gallons}{12\ sec} = 510\ \frac{gallons}{sec} = 68.2\ \frac{ft^3}{s}$$

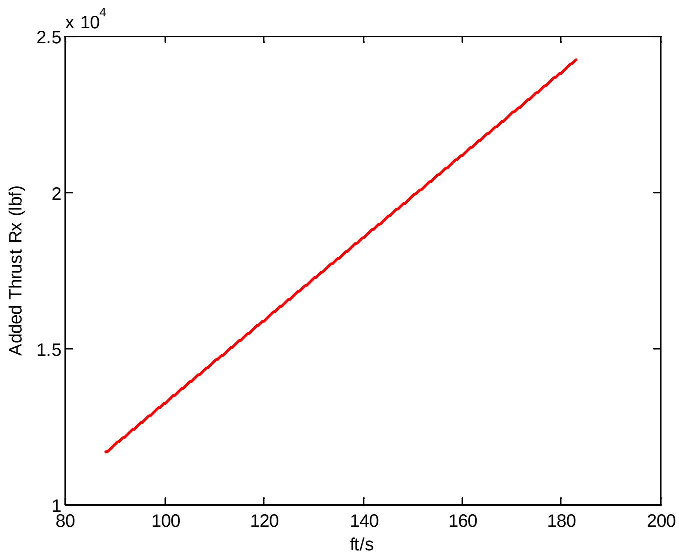
The density of the water is:

$$\rho = 1.94 \, \frac{slug}{ft^3}$$

For an aircraft speed of $U = 75 \, mph = 110 \, \frac{ft}{s}$:

$$R_x = 110 \, \frac{ft}{s} \times 1.94 \, \frac{\frac{lbf \cdot s^2}{ft}}{ft^3} \times 68.2 \, \frac{ft^3}{s} = 14550 \, lbf$$

For a range of aircraft speeds 60 mph-125 mph $\left(88 \, \frac{ft}{s} \sim 188.3 \, \frac{ft}{s}\right)$:

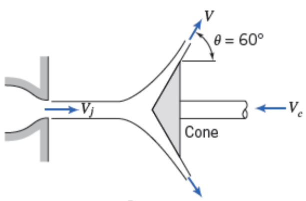


Thus at 60 mph the added thrust is 11640 lbf , while at 125 mph the added thrust is 24200 lbf.

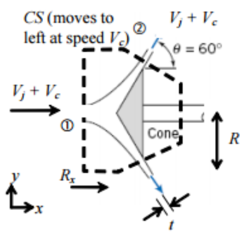
Problem 4.103

(Difficulty: 3)

4.103 Water, in a 100-mm-diameter jet with speed of 30 m/s to the right, is deflected by a cone that moves to the left at 14 m/s. Determine (a) the thickness of the jet sheet at a radius of 230 mm. and (b) the external horizontal force needed to move the cone.



P4.103



Given: Water jet striking moving cone

Find: Thickness of jet sheet. Force needed to move cone.

Assumption: (1) steady flow

(2) incompressible flow

(3) atmospheric pressure in jet

(4) uniform flow

(5) jet relative velocity is constant

Solution:

Use a CV that moves with the aircraft, as shown.

Basic equation:

Continuity equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Momentum equation in the x direction:

$$R_x + F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{V} \cdot d\vec{A}$$

Then

$$-\rho V_1 A_1 + \rho V_2 A_2 = 0$$

$$-\rho(V_j + V_c)\frac{\pi D_j^2}{4} + \rho(V_j + V_c) \cdot 2\pi R t = 0$$

$$t = \frac{D_j^2}{8R} = \frac{(0.1\ m)^2}{8 \times 0.23\ m} = 0.00543\ m$$

Using relative velocities, x momentum is

$$R_x = u_1(-\rho V_1 A_1) + u_2(\rho V_2 A_2) = -(V_j + V_c)[\rho(V_j + V_c)A_j] + (V_j + V_c)\cos\theta[\rho(V_j + V_c)A_j]$$

$$R_x = \rho(V_j + V_c)^2 A_j(\cos\theta - 1)$$

Using the given data we have:

$$R_x = 999\ \frac{kg}{m^3} \times \left(30\ \frac{m}{s} + 14\ \frac{m}{s}\right)^2 \times \frac{\pi \times (0.1\ m)^2}{4} \times (\cos 60° - 1)$$

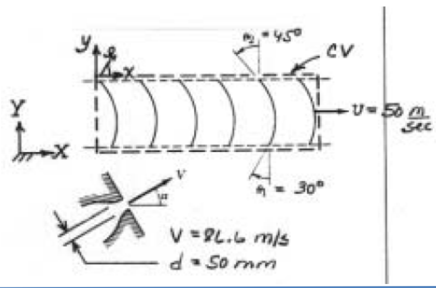
$$R_x = -7.6\ kN$$

Hence the force is 7.6 kN to the left. The upwards equals the weight.

Problem 4.104

(Difficulty: 2)

4.104 Consider a series of turning vanes struck by a continuous jet of water that leaves a 50-mm diameter nozzle at constant speed, $V = 86.6\ \frac{m}{s}$. The vanes move with constant speed, $U = 50\ \frac{m}{s}$. Note that all the mass flow leaving the jet crosses the vanes. The curvature of the vanes is described by angles $\theta_1 = 30°$ and $\theta_2 = 45°$, as shown. Evaluate the nozzle angle, α, required to ensure that the jet enters tangent to the leading edge of each vane. Calculate the force that must be applied to maintain the vane speed constant.



Find: The force must be applied to maintain the vane speed constant.

Assumption: (1) no pressure forces

(2) horizontal $F_{Bx} = 0$

(3) steady flow

(4) uniform flow at each section

(5) no change in relative velocity on vane

(6) flow enters and leaves tangent to vanes.

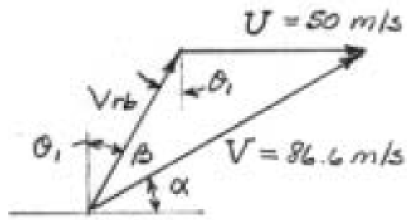
Solution: The nozzle angle may be obtained from trigonometry. inlet velocity relationship is shown in the sketch. Apply momentum equation using CV moving with vanes, as shown.

Basic equation: Momentum equation in the x-direction

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\rho d\forall + \int_{CS} u_{xyz}\rho \bar{V}_{xyz} \cdot d\bar{A}$$

From the relation between the angles for the velocity vectors as shown on the sketch:

$$\frac{\sin\alpha}{V_{rb}} = \frac{\sin(90+\theta_1)}{V} = \frac{sin(\beta)}{U}$$



Where $\beta = sin^{-1}\left[\frac{U}{V}\sin(90+\theta_1)\right] = sin^{-1}\left[\frac{50}{86.6}\sin(120°)\right] = 30°$

From the sketch, $90° = \alpha + \beta + \theta_1$, so $\alpha = 90° - \beta - \theta_1 = 90° - 30° - 30° = 30°$

Also

$$V_{rb}\cos\theta_1 = V\sin\alpha$$

$$V_{rb} = V\frac{\sin\alpha}{\cos\theta_1} = 86.6\ \frac{m}{s} \times \frac{\sin 30°}{\cos 30°} = 50.0\ \frac{m}{s}$$

From momentum equation, as all of $\dot{m}$ flows across vanes

$$R_x = u_1\{-\dot{m}\} + u_2\{\dot{m}\} = V_{rb}\sin\theta_1(-\dot{m}) - V_{rb}\sin\theta_2(\dot{m}) = V_{rb}\dot{m}(-\sin\theta_1 - \sin\theta_2)$$

The velocities are given by.

$$u_1 = V_{rb}\sin\theta_1$$

$$u_2 = -V_{rb}\sin\theta_2$$

$$R_y = \dot{m}V_{rb}(-\cos\theta_1 + \cos\theta_2)$$

Thus, since $\dot{m} = \rho Q$,

$$R_x = V_{rb}\rho Q(-\sin\theta_1 - \sin\theta_2) = 50\ \frac{m}{s} \times 999\ \frac{kg}{m^3} \times 0.170\ \frac{m^3}{s}(-\sin 30° - \sin 45°)\frac{N\cdot s}{kg\cdot m}$$

The net force on the CV in the x-direction is

$$R_x = -10.3\ kN \text{ (to left)}$$

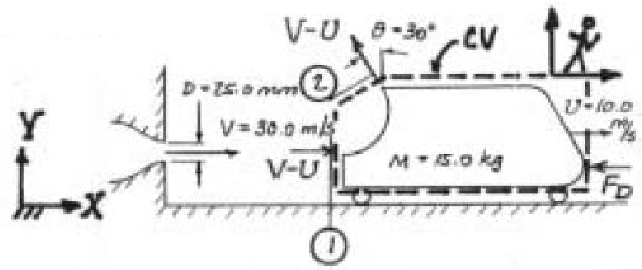
And the net force on the CV in the y-direction

$$R_y = -1.35\ kN.$$

Problem 4.105

(Difficulty: 2)

4.105 A steady jet of water is used to propel as a small cart along a horizontal track as shown. Total resistance to motion of the cart assembly is given by $F_D = kU^2$, where $k = 0.92\ \frac{N \cdot s^2}{m^2}$. Evaluate the acceleration of the cart at the instant when its speed is $U = 10\ \frac{m}{s}$.



Find: The acceleration of the cart when the instant speed $U = 10\ \frac{m}{s}$.

Assumption: (1) Only resistance is F_D

(2) horizontal $F_{Bx} = 0$

(3) neglect $\frac{\partial u}{\partial t}$ of mass of water in CV

(4) no change in speed with respect to vane.

(5) uniform flow at each cross-section

Solution: Apply the momentum equation using control volume and control surface shown.

Basic equation: Momentum equation in x-direction

$$F_{sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t} \int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \bar{V}_{xyz} \cdot d\bar{A}$$

Then

$$-kU^2 - a_{rfx} M_{CV} = -\rho (V-U)^2 A - \rho (V-U)^2 A \sin\theta = -\rho (V-U)^2 A (1 + \sin\theta)$$

So

$$a_{rfx} = \frac{1}{M} [\rho (V-U)^2 A (1 + \sin\theta) - kU^2]$$

$$a_{rfx} = \frac{1}{15\ kg}\left[999\ \frac{kg}{m^3} \times (30 - 10)^2\ \frac{m^2}{s^2} \times \frac{\pi}{4} \times (0.025\ m)^2 \times (1 + \sin 30°) - 0.92\frac{N \cdot s^2}{m^2} \times \left(10\ \frac{m}{s}\right)^2 \times \frac{kg \cdot m}{N \cdot s^2}\right]$$

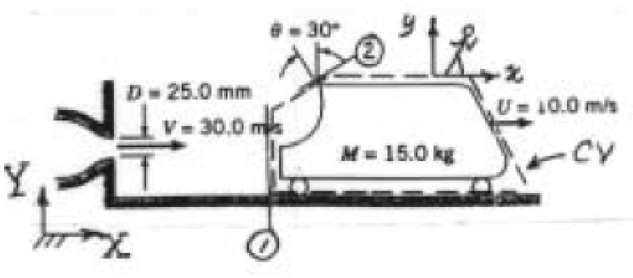
$$a_{rfx} = 13.5\ \frac{m}{s^2}$$

The direction is to the right.

Problem 4.106

(Difficulty: 2)

4.106 The hydraulic catapult of Problem 4.105 is accelerated by a jet of water that strikes the curved vane. The cart moves along a level track with negligible resistance. At any time its speed is U. Calculate the time required to accelerate the cart from rest to $U = \frac{V}{2}$.



Find: The time t required to accelerate the cart from rest to $U = \frac{V}{2}$.

Assumption: (1) $F_{sx} = 0$, since no pressure forces, no resistance.

(2) $F_{Bx} = 0$, since horizontal

(3) neglect mass of water inside control volume

(4) uniform flow in jet

(5) no change in relative velocity on vane

Solution:

Apply x component of momentum equation to accelerating CV.

Basic equation: Momentum equation in x-direction

$$F_{sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t} \int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \bar{V}_{xyz} \cdot d\bar{A}$$

Then

$$-a_{rfx} M_{CV} = u_1\{-\rho(V-U)A\} + u_2\{\rho(V-U)A\}$$

$$u_1 = V - U$$

$$u_2 = -(V-U)\sin\theta$$

So we have:

$$-a_{rfx} M_{CV} = -(1+\sin\theta)\rho(V-U)^2 A$$

$$\frac{dU}{dt} = \frac{\rho A(1 + \sin\theta)}{M}(V - U)^2$$

To integrate, since $V = constant$ we can replace $dU = -d(V - U)$. Separating variables

$$\frac{dU}{(V - U)^2} = -\frac{d(V - U)}{(V - U)^2} = \frac{\rho A(1 + \sin\theta)}{M} dt$$

Or

$$-\int_0^{\frac{V}{2}} \frac{d(V - U)}{(V - U)^2} = \int_0^t \frac{\rho A(1 + \sin\theta)}{M} t$$

Or, integrating and evaluating the integral at the limits

$$\left[\frac{1}{V - U}\right]^{U=\frac{V}{2}} - \left[\frac{1}{V - U}\right]^{U=0} = \frac{1}{V - \frac{V}{2}} - \frac{1}{V} = \frac{1}{V} = \frac{\rho A(1 + \sin\theta)}{M} t$$

Thus the time is

$$t = \frac{M}{\rho V A(1 + \sin\theta)}$$

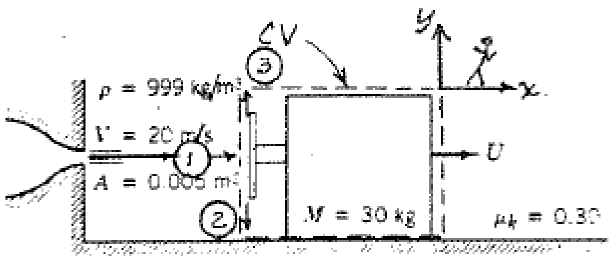
$$t = 15.0\ kg \times \frac{m^3}{999\ kg} \times \frac{s}{30\ m} \times \frac{4}{\pi(0.025\ m)^2} \times \frac{1}{(1 + \sin 30°)}$$

$$t = 0.680\ s$$

Problem 4.107

(Difficulty: 2)

4.107 A vane/slider assembly moves under the influence of a liquid jet as shown. The coefficient of kinetic friction for motion of the slider along the surface is $\mu_k = 0.30$. Calculate the terminal speed of the slider.



Find: The terminal speed of the slider U_t.

Assumption: (1) horizontal motion, so $F_{Bx} = 0$

(2) neglect mass of liquid on vane, $u \approx 0$ on vane

(3) uniform flow at each section

(4) measure velocities relative to CV

Solution:

Apply x momentum equation to linearly accelerating CV.

Basic equation: Momentum equation in x-direction

$$F_{sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t} \int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \bar{V}_{xyz} \cdot d\bar{A}$$

Then

$$-Mg\mu_k - a_{rfx} M = u_1\{-|\rho(V-U)A|\} + u_2\{\dot{m}_2\} + u_3\{\dot{m}_3\}$$

$$u_1 = V - U$$

$$u_2 = 0$$

$$u_3 = 0$$

$$-Mg\mu_k - M\frac{dU}{dt} = -\rho(V-U)^2 A$$

or

$$\frac{dU}{dt} = \frac{\rho(V-U)^2 A}{M} - g\mu_k$$

At terminal speed,

$$\frac{dU}{dt} = 0$$

$$U = U_t$$

So we have:

$$\frac{\rho(V-U_t)^2 A}{M} - g\mu_k = 0$$

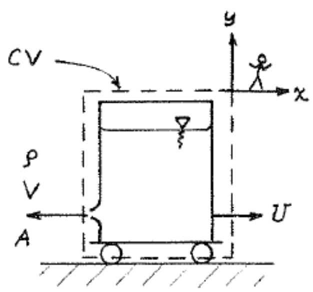
$$(V - U_t) = \sqrt{\frac{Mg\mu_k}{\rho A}}$$

$$U_t = V - \sqrt{\frac{Mg\mu_k}{\rho A}} = 20\ \frac{m}{s} - \left[30\ kg \times 9.81\ \frac{m}{s^2} \times 0.3 \times \frac{m^3}{999\ kg} \times \frac{1}{0.005\ m^2}\right]^{0.5} = 15.8\ \frac{m}{s}$$

Problem 4.108

(Difficulty: 2)

4.108 A cart is propelled by a liquid jet issuing horizontally from a tank as shown. The track is horizontal; resistance to motion may be neglected. The tank is pressurized so that the jet speed may be considered constant. Obtain a general expression for the speed of the cart as it accelerates from rest. If $M_0 = 100\ kg$, $\rho = 999\ \frac{kg}{m^3}$, and $A = 0.005\ m^2$, find the jet speed V required for the cart to reach a speed of $1.5\ \frac{m}{s}$ after 30 seconds. For this condition, plot the cart speed U as a function of time. Plot the cart speed after 30 seconds as a function of jet speed.



Find: Plot the cart speed after 30 seconds as a function of jet speed.

Assumption: (1) no resistance.

(2) $F_{Bx} = 0$ since track is horizontal

(3) neglect change in fluid velocities within CV

(4) uniform flow at jet exit

Solution:

a) Apply x component of momentum equation using linearly accelerating CV shown.

Basic equation: Momentum equation in x-direction

$$F_{sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t} \int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \bar{V}_{xyz} \cdot d\bar{A}$$

Then

$$-a_{rfx} M = u\{|\rho V A|\} = -\rho V^2 A$$

$$u = -V$$

From continuity, the flow rate is given by

$$\dot{m} = \rho V A$$

So the momentum flow is

$$M = M_0 - \rho V A t$$

Using $a_{rfx} = \frac{dU}{dt}$,

$$\frac{dU}{dt} = \frac{\rho V^2 A}{M_0 - \rho V A t}$$

Separating the variables and integrating,

$$\int_0^U dU = U = \int_0^t \frac{\rho V^2 A}{M_0 - \rho V A t} = V \ln\left(\frac{M_0}{M_0 - \rho V A t}\right)$$

or

$$\frac{U}{V} = \ln\left(\frac{M_0}{M_0 - \rho V A t}\right)$$

Check dimensions:

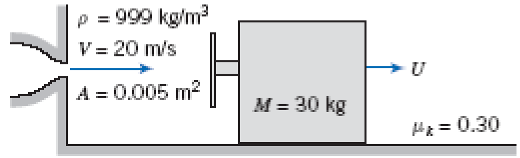
$$[\rho V A t] = \frac{M}{L^3}\frac{L}{t}L^2 t = M$$

b) Using the given data in Excel (with solver) the jet speed for $U = 1.5\ \frac{m}{s}$ at $t = 30\ s$ is

$$V = 0.61\ \frac{m}{s}$$

Problem 4.109 [Difficulty: 4]

4.109 For the vane/slider problem of Problem 4.107 find and plot expressions for the acceleration and speed of the slider as a function of time.



Given: Data on vane/slider

Find: Formula for acceleration and speed; plot

Solution:

The given data is $\rho = 999 \cdot \frac{kg}{m^3}$ $\quad M = 30 \cdot kg$ $\quad A = 0.005 \cdot m^2$ $\quad V = 20 \cdot \frac{m}{s}$ $\quad \mu_k = 0.3$

The equation of motion, from Problem 4.141, is $\frac{dU}{dt} = \frac{\rho \cdot (V - U)^2 \cdot A}{M} - g \cdot \mu_k$

The acceleration is thus $a = \frac{\rho \cdot (V - U)^2 \cdot A}{M} - g \cdot \mu_k$ Separating variables $\frac{dU}{\frac{\rho \cdot (V-U)^2 \cdot A}{M} - g \cdot \mu_k} = dt$

Substitute $u = V - U$ $\quad dU = -du$ $\quad \frac{du}{\frac{\rho \cdot A \cdot u^2}{M} - g \cdot \mu_k} = -dt$

But
$$\int \frac{1}{\left(\frac{\rho \cdot A \cdot u^2}{M} - g \cdot \mu_k\right)} du = -\sqrt{\frac{M}{g \cdot \mu_k \cdot \rho \cdot A}} \cdot \operatorname{atanh}\left(\sqrt{\frac{\rho \cdot A}{g \cdot \mu_k \cdot M}} \cdot u\right)$$

and u = V - U so
$$-\sqrt{\frac{M}{g \cdot \mu_k \cdot \rho \cdot A}} \cdot \operatorname{atanh}\left(\sqrt{\frac{\rho \cdot A}{g \cdot \mu_k \cdot M}} \cdot u\right) = -\sqrt{\frac{M}{g \cdot \mu_k \cdot \rho \cdot A}} \cdot \operatorname{atanh}\left[\sqrt{\frac{\rho \cdot A}{g \cdot \mu_k \cdot M}} \cdot (V - U)\right]$$

Using initial conditions
$$-\sqrt{\frac{M}{g \cdot \mu_k \cdot \rho \cdot A}} \cdot \operatorname{atanh}\left[\sqrt{\frac{\rho \cdot A}{g \cdot \mu_k \cdot M}} \cdot (V - U)\right] + \sqrt{\frac{M}{g \cdot \mu_k \cdot \rho \cdot A}} \cdot \operatorname{atanh}\left(\sqrt{\frac{\rho \cdot A}{g \cdot \mu_k \cdot M}} \cdot V\right) = -t$$

$$V - U = \sqrt{\frac{g \cdot \mu_k \cdot M}{\rho \cdot A}} \cdot \tanh\left(\sqrt{\frac{g \cdot \mu_k \cdot \rho \cdot A}{M}} \cdot t + \operatorname{atanh}\left(\sqrt{\frac{\rho \cdot A}{g \cdot \mu_k \cdot M}} \cdot V\right)\right)$$

$$U = V - \sqrt{\frac{g \cdot \mu_k \cdot M}{\rho \cdot A}} \cdot \tanh\left(\sqrt{\frac{g \cdot \mu_k \cdot \rho \cdot A}{M}} \cdot t + \operatorname{atanh}\left(\sqrt{\frac{\rho \cdot A}{g \cdot \mu_k \cdot M}} \cdot V\right)\right)$$

Note that $\operatorname{atanh}\left(\sqrt{\frac{\rho \cdot A}{g \cdot \mu_k \cdot M}} \cdot V\right) = 0.213 - \frac{\pi}{2} \cdot i$

which is complex and difficult to handle in *Excel*, so we use the identity $\operatorname{atanh}(x) = \operatorname{atanh}\left(\frac{1}{x}\right) - \frac{\pi}{2} \cdot i$ for x > 1 so $$U = V - \sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\frac{1}{\sqrt{\frac{\rho\cdot A}{g\cdot\mu_k\cdot M}}\cdot V}\right) - \frac{\pi}{2}\cdot i\right)$$

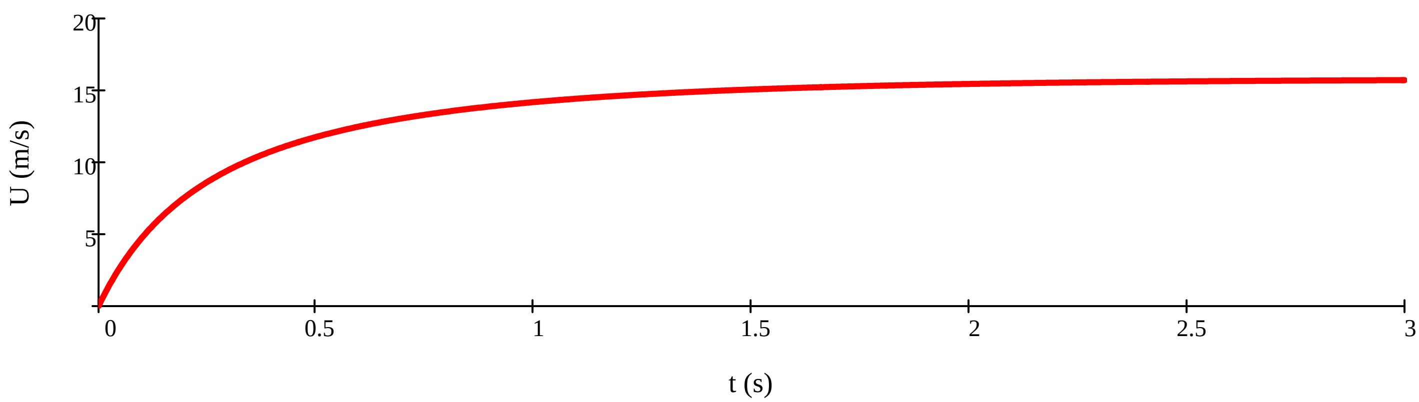
and finally the identity $$\tanh\left(x - \frac{\pi}{2}\cdot i\right) = \frac{1}{\tanh(x)}$$

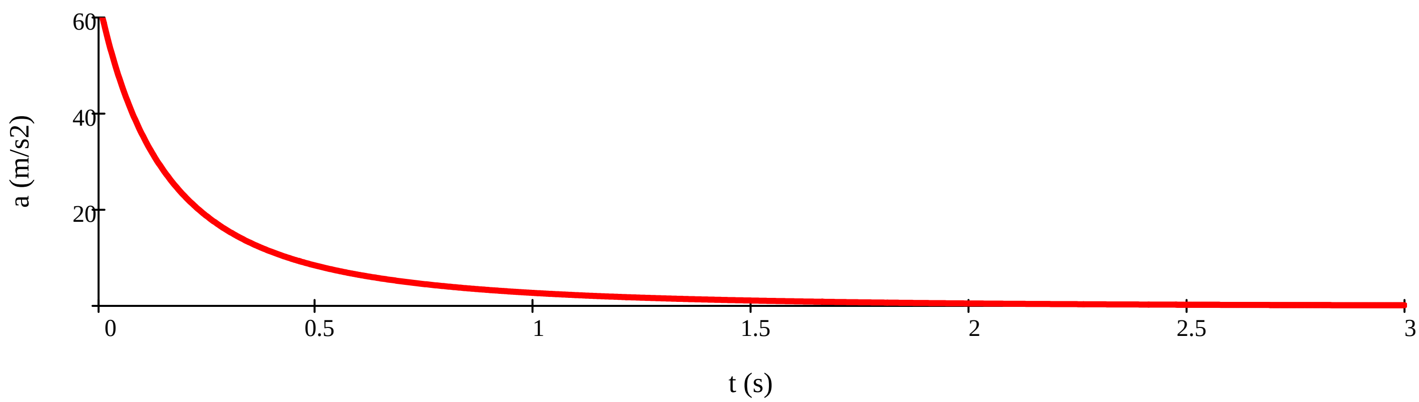
to obtain $$U(t) = V - \frac{\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}}{\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\frac{1}{V}\right)\right)}$$

Note that $$a = \frac{\rho\cdot(V-U)^2\cdot A}{M} - g\cdot\mu_k$$ and $$V - U = \frac{\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}}{\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\frac{1}{V}\right)\right)}$$

Hence $$a(t) = \frac{g\cdot\mu_k}{\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\frac{1}{V}\right)\right)^2} - g\cdot\mu_k$$

The plots are presented below

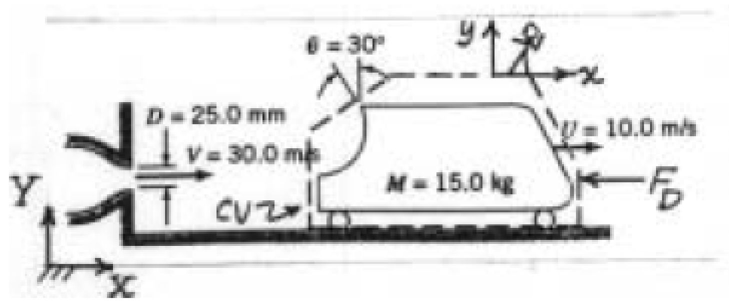




Problem 4.110

(Difficulty: 2)

4.110 If the cart of Problem 4.105 is released at $t = 0$, when would you expect the acceleration to be maximum? Sketch what you would expect for the curve of acceleration versus time. What value of θ would maximize the acceleration at any time? Why? Will the cart speed ever equal the jet speed? Explain briefly.



Find: The value of the angle θ that would maximize the acceleration.

Assumption: (1) $F_{sx} = -F_D = -kU^2$, where $k = 0.92\ \frac{N \cdot s^2}{m^2}$.

(2) $F_{Bx} = 0$, since horizontal

(3) neglect mass of water on vane

(4) uniform flow in jet

(5) no change in relative velocity on vane

Solution:

Apply x component of momentum equation to accelerating CV.

Basic equation: Momentum equation in x-direction

$$F_{sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t} \int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \bar{V}_{xyz} \cdot d\bar{A}$$

Then

$$-kU^2 - a_{rfx} M_{CV} = u_1 \{-\rho(V-U)A\} + u_2 \{\rho(V-U)A\}$$

$$u_1 = V - U$$

$$u_2 = -(V-U)\sin\theta$$

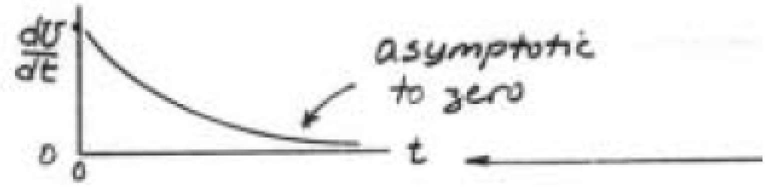
$$-kU^2 - a_{rfx} M_{CV} = -(1+\sin\theta)\rho(V-U)^2 A$$

So we have for the acceleration of the cart

$$\frac{dU}{dt} = \frac{\rho A(1+\sin\theta)}{M}(V-U)^2 - \frac{kU^2}{M}$$

(a) Acceleration is maximum at $t = 0$, when $U = 0$.

(b) Acceleration versus time will be



(c) From the equation for acceleration, $\frac{dU}{dt}$ is maximum when $\sin\theta = 1$, which is $\theta = \frac{\pi}{2} = 90°$

(d) $\frac{dU}{dt}$ will go to zero when $U = V$. This will be the terminal speed for the cart, U_t. From the equation for acceleration $\frac{dU}{dt} = 0$ when

$$\rho A(1+\sin\theta)(V-U)^2 = kU^2$$

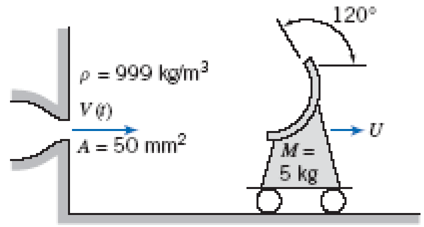
or

$$U = \frac{\left[\frac{\rho A(1+\sin\theta)}{k}\right]^{\frac{1}{2}}}{1+\left[\frac{\rho A(1+\sin\theta)}{k}\right]^{\frac{1}{2}}}V = 0.472V$$

U will be asymptotic to V.

Problem 4.111 [Difficulty: 3]

4.111 The wheeled cart shown rolls with negligible resistance. The cart is to accelerate to the right at a constant rate of 2.5 m/s^2. This is to be accomplished by "programming" the water jet speed, $V(t)$, that hits the cart. The jet area remains constant at 50 mm^2. Find the initial jet speed, and the jet speed and cart speeds after 2.5 s and 5 s. Theoretically, what happens to the value of $(V - U)$ over time?



Given: Vaned cart with negligible resistance

Find: Initial jet speed; jet and cart speeds at 2.5 s and 5 s; what happens to V - U?

Solution: Apply x momentum

$$F_{S_x} + F_{B_x} - \int_{CV} a_{rf_x}\,\rho\,dV = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\,dV + \int_{CS} u_{xyz}\,\rho\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) All changes wrt CV 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow 5) Constant jet area

Given data $\rho = 999\cdot\frac{kg}{m^3}$ $M = 5\cdot kg$ $A = 50\cdot mm^2$ $a = 2.5\cdot\frac{m}{s^2}$ $\theta = 120\cdot deg$

Then $-a\cdot M = u_1\cdot[-\rho\cdot(V-U)\cdot A] + u_1\cdot[\rho\cdot(V-U)\cdot A]$ where $u_1 = V - U$ and $u_2 = (V-U)\cdot\cos(\theta)$

Hence $a\cdot M = \rho\cdot(V-U)^2\cdot(1-\cos(\theta))\cdot A$ From this equation we can see that for constant acceleration V and U must increase at the same rate!

Solving for V $V(t) = a\cdot t + \sqrt{\frac{M\cdot a}{\rho\cdot(1-\cos(\theta))\cdot A}}$

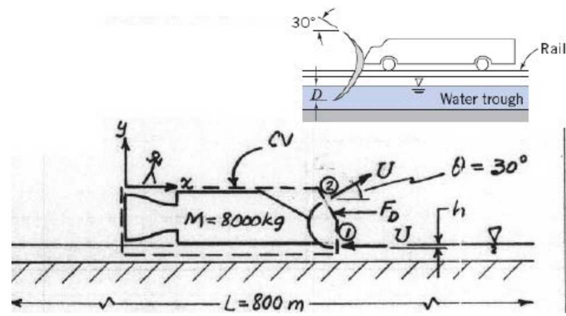
Hence, evaluating $V(0) = 12.9\frac{m}{s}$ $V(2.5\cdot s) = 19.2\frac{m}{s}$ $V(5\cdot s) = 25.4\frac{m}{s}$

Also, for constant acceleration $U(t) = a\cdot t$ so $V - U = \sqrt{\frac{M\cdot a}{\rho\cdot(1-\cos(\theta))\cdot A}} = \text{const!}$

Problem 4.112

(Difficulty: 2)

4.112 A rocket sled is to be slowed from an initial speed of $300\ \frac{m}{s}$ by lowering a scoop into a water trough. The scoop is $0.3\ m$ wide; it deflects the water trough 150°. The trough is $800\ m$ long. The mass of the sled is $8000\ kg$. At the initial speed it experiences an aerodynamic drag force of $90\ kN$. The aerodynamic force is proportional to the square of the sled speed. It is desired to show the sled to $100\ \frac{m}{s}$. Determine the depth D to which the scoop must be lowered into the water.



Find: The depth D to which the scoop must be lowered into the water.

Assumption: (1) $F_{Bx} = 0$

(2) Neglect rate of change of u in CV

(3) uniform flow at each section

(4) no change in relative speed of liquid crossing scoop

Solution:

Apply x component of momentum equation using linearly acceleration CV shown.

Basic equation: Momentum equation for the x-direction

$$F_{sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t} \int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \bar{V}_{xyz} \cdot d\bar{A}$$

Then

$$-F_D - M a_{rfx} = u_1\{-|\rho U w h|\} + u_2\{|\rho U w h|\}$$

Where h = the depth of the scoop immersion. The product of mass and acceleration is

$$M a_{rfx} = M \frac{dU}{dt}$$

We also have

$$u_1 = -U$$

$$u_2 = U \cos\theta$$

The drag force is given by

$$F_D = kU^2$$

The constant k is evaluated as

$$k = \frac{F_{D0}}{U_0^2} = 90kN \times \frac{s^2}{(300\ m)^2} \times \frac{10^3 N}{kN} \times \frac{kg \cdot m}{N \cdot s^2} = 1.00\ \frac{kg}{m}$$

$$-kU^2 - M\frac{dU}{dt} = \rho U^2 wh(1 + \cos\theta)$$

The momentum equation becomes:

$$-M\frac{dU}{dt} = [k + \rho wh(1 + \cos\theta)]U^2$$

The rate of change of velocity with time can be rewritten using the chain rule as

$$\frac{dU}{dt} = \frac{dU}{dx}\frac{dx}{dt} = \frac{dU}{dx}U$$

The momentum can then be re-written and the variables separated as

$$\frac{dU}{U} = -cdx$$

Where the constant c contains the terms

$$c = \frac{k + \rho wh(1 + \cos\theta)}{M}$$

Integrating the equation from the initial velocity where x = 0, we get:

$$\ln\frac{U}{U_0} = -cx$$

So

$$c = -\frac{1}{x}\ln\frac{U}{U_0}$$

The value of c is then

$$c = -\frac{1}{800\ m}\ln\left(\frac{100}{300}\right) = 1.37 \times 10^{-3}\ m^{-1}$$

Solving for h,

$$h = \frac{Mc - k}{\rho w(1 + \cos\theta)}$$

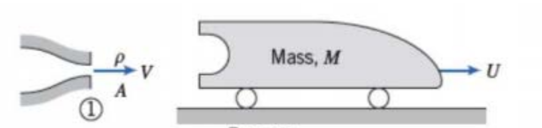
$$h = \left[8000\ kg \times \frac{1.37 \times 10^{-3}}{m} - 1.00\frac{kg}{m}\right]\frac{m^3}{999\ kg} \times \frac{1}{0.3\ m} \times \frac{1}{(1 + \cos 30°)} = 0.0179\ m$$

$$h = 17.9\ mm$$

Problem 4.113

(Difficulty: 3)

4.113 Starting from rest, the cart shown is propelled by a hydraulic catapult (liquid jet). The jet strikes the curved surface and makes 180° turn, leaving horizontally. The mass of the cart is 100 kg and the jet of water leaves the nozzle (of area 0.001 m^2) with a speed of 35 m/s. There is an aerodynamic drag force proportional to the square cart speed, $F_D = kU^2$, with $k = 2.0\ N \cdot s^2/m^2$. Derive an expression for the cart acceleration as a function of cart speed and other given parameters. Evaluate the acceleration of the cart at $U = 10\ m/s$. What fraction is this speed of the terminal speed of cart?



P4.113

Find: The expression for cart acceleration. The fraction of the speed $U = 10\ m/s$ of the terminal final speed.

Assumption: (1) horizontal, $F_{BX} = 0$

(2) neglect mass of liquid in CV (components of u cancel)

(3) uniform flow at each section

(4) measure all velocities relative to CV

(5) no change in stream area or speed on vane

Solution:

Basic equation:

Momentum equation in the x direction:

$$F_{sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t}\int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \vec{V}_{xyz} \cdot d\vec{A}$$

Then

$$-kU^2 - a_{rfx} M = u_1\{-|\rho(V-U)A|\} + u_2\{|\rho(V-U)A|\}$$

$$u_1 = V - U$$

$$u_2 = -(V-U)$$

$$-kU^2 - a_{rfx} M = -2\rho(V-U)^2 A$$

Then

$$a_{rfx} = \frac{dU}{dt} = \frac{2\rho(V-U)^2 A - kU^2}{M}$$

At $U = 10\ \frac{m}{s}$,

$$a_{rfx} = \frac{2 \times 999\ \frac{kg}{m^3} \times (35-10)^2\ \frac{m^2}{s^2} \times 0.001\ m^2 - 2.0\ \frac{N \cdot s^2}{m^2} \times \left(10\ \frac{m}{s}\right)^2}{100\ kg} = 10.49\ \frac{m}{s^2}$$

At terminal speed:

$$a_{rfx} = 0$$

$$2\rho(V-U_t)^2 A = kU_t^2$$

$$(V-U_t)\sqrt{2\rho A} = \sqrt{k}U_t$$

$$U_t = \frac{V}{1+\sqrt{\frac{k}{2\rho A}}}$$

$$U_t = \frac{35\ \frac{m}{s}}{1+\sqrt{\frac{2.0\ \frac{N \cdot s^2}{m^2}}{2 \times 999\ \frac{kg}{m^3} \times 0.001\ m^2}}} = 17.5\ \frac{m}{s}$$

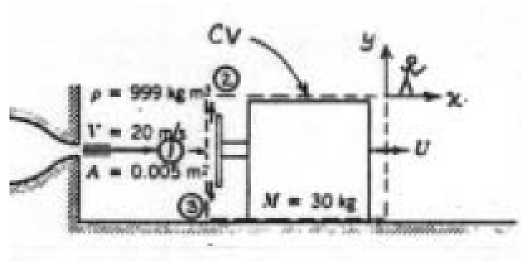
Finally,

$$Fraction = \frac{U}{U_t} = \frac{10\ \frac{m}{s}}{17.5\ \frac{m}{s}} = 57.1\ \%$$

Problem 4.114

(Difficulty: 2)

4.114 Solve Problem 4.107 if the vane and slider ride on a film of oil instead of sliding in contact with the surface. Assume motion resistance is proportional to speed, $F_R = kU$, with $k = 7.5\ \frac{N \cdot S}{m}$.



Assumption: (1) Horizontal, $F_{Bx} = 0$

(2) Neglect mass of liquid in on vane, $u \approx 0$ on vane

(3) uniform flow at each section

(4) measure all velocities relative to the CV

Solution:

Apply x momentum equation to linearly accelerating CV.

Basic equation:

$$F_{Sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t} \int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \bar{V}_{xyz} \cdot d\bar{A}$$

Then

$$-kU - a_{rfx} M = u_1 \{-|\rho(V - U)A|\} + u_2 \{\dot{m}_2\} + u_3 \{\dot{m}_3\}$$

$$u_1 = V - U$$

$$u_2 = 0$$

$$u_3 = 0$$

$$-kU - M \frac{dU}{dt} = -\rho (V - U)^2 A$$

or

$$\frac{dU}{dt} = \frac{\rho(V-U)^2 A}{M} - \frac{kU}{M}$$

$$\frac{dU}{dt} = 999\,\frac{kg}{m^3} \times (20-10)^2\,\frac{m^2}{s^2} \times 0.005\ m^2 \times \frac{1}{30\ kg} - 7.5\ \frac{N\cdot s}{m} \times 10\ \frac{m}{s} \times \frac{1}{30\ kg} \times \frac{kg\cdot m}{N\cdot s^2}$$

$$\frac{dU}{dt} = 14.2\ \frac{m}{s^2}$$

at $U = 10\ \frac{m}{s}$.

At terminal speed, $U = U_t$ and $\frac{dU}{dt} = 0$, so

$$0 = \frac{\rho(V-U)^2 A}{M} - \frac{kU}{M}$$

or

$$V^2 - 2UV + U^2 - \frac{k}{\rho A} U = 0$$

$$U^2 - \left(2V + \frac{k}{\rho A}\right) U + V^2 = 0$$

$$U = \frac{2V + \frac{k}{\rho A} \pm \sqrt{\left(2V + \frac{k}{\rho A}\right)^2 - 4V^2}}{2} = V\left\{\left(1 + \frac{k}{2\rho V A}\right) \pm \sqrt{\left(1 + \frac{k}{2\rho V A}\right)^2 - 1}\right\}$$

$$1 + \frac{k}{2\rho V A} = 1 + \frac{1}{2} \times 7.5\ \frac{N\cdot s}{m} \times \frac{m^3}{999\ kg} \times \frac{s}{20\ m} \times \frac{1}{0.005\ m^2} \times \frac{kg\cdot m}{N\cdot s^2} = 1.0375$$

$$U = V\left\{1.0375 \pm \sqrt{(1.0375)^2 - 1}\right\} = 0.761V = 0.761 \times 20\ \frac{m}{s} = 15.2\ \frac{m}{s}$$

The negative root was chosen so $U_t < V$, as required.

Problem 4.115 [Difficulty: 4]

4.115 For the vane/slider problem of Problem 4.114 plot the acceleration, speed, and position of the slider as functions of time. (Consider numerical integration.)

Given: Data on vane/slider

Find: Formula for acceleration, speed, and position; plot

Solution: Apply x momentum

$$F_{S_x} + F_{B_x} - \int_{CV} a_{rf_x}\,\rho\, dV = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, dV + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) All changes wrt CV 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow 5) Constant jet area

The given data is $\rho = 999\cdot\frac{kg}{m^3}$ $M = 30\cdot kg$ $A = 0.005\cdot m^2$ $V = 20\cdot\frac{m}{s}$ $k = 7.5\cdot\frac{N\cdot s}{m}$

Then $-k\,U - M\cdot a_{rf} = u_1\cdot[-\rho\cdot(V-U)\cdot A] + u_2\cdot m_2 + u_3\cdot m_3$

where $a_{rf} = \frac{dU}{dt}$ $u_1 = V - U$ $u_2 = 0$ $u_3 = 0$

Hence $-k\cdot U - M\cdot\frac{dU}{dt} = -\rho\cdot(V-U)^2\cdot A$

or $\frac{dU}{dt} = \frac{\rho\cdot(V-U)^2\cdot A}{M} - \frac{k\cdot U}{M}$

The acceleration is thus $a = \frac{\rho\cdot(V-U)^2\cdot A}{M} - \frac{k\cdot U}{M}$

The differential equation for *U* can be solved analytically, but is quite messy. Instead we use a simple numerical method - Euler's method

$$U(n+1) = U(n) + \left[\frac{\rho\cdot(V-U(n))^2\cdot A}{M} - \frac{k\cdot U(n)}{M}\right]\cdot\Delta t$$ where Δt is the time step

For the position x $\frac{dx}{dt} = U$

so $x(n+1) = x(n) + U(n)\cdot\Delta t$

The final set of equations is

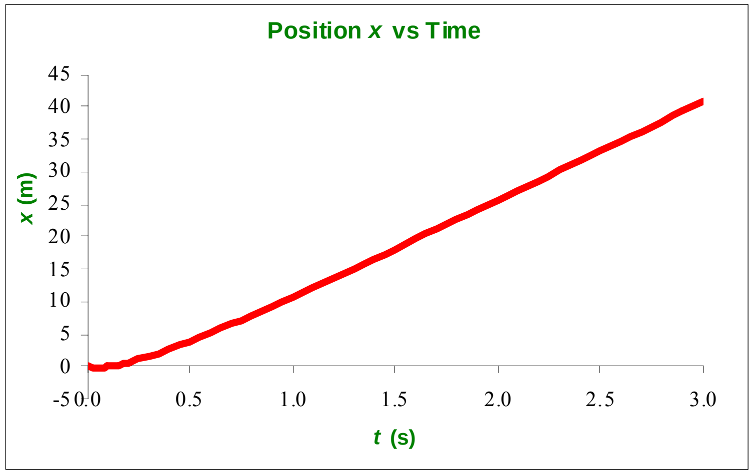
$$U(n+1) = U(n) + \left[\frac{\rho\cdot(V-U(n))^2\cdot A}{M} - \frac{k\cdot U(n)}{M}\right]\cdot\Delta t$$

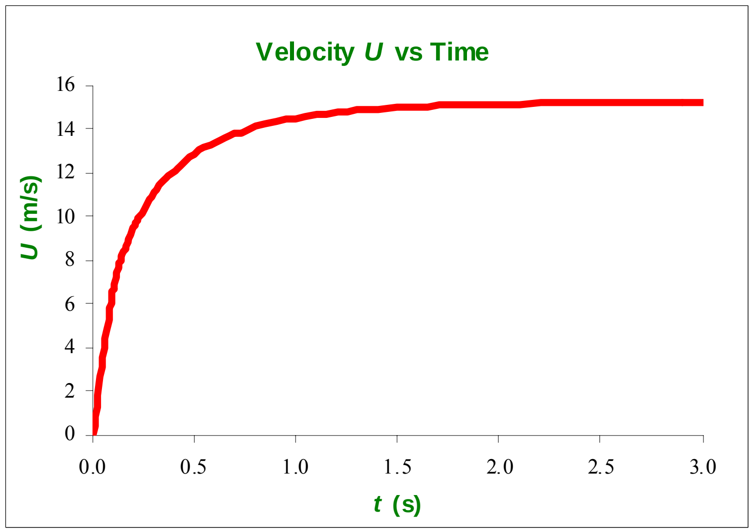
$$a(n) = \frac{\rho\cdot(V-U(n))^2\cdot A}{M} - \frac{k\cdot U(n)}{M}$$

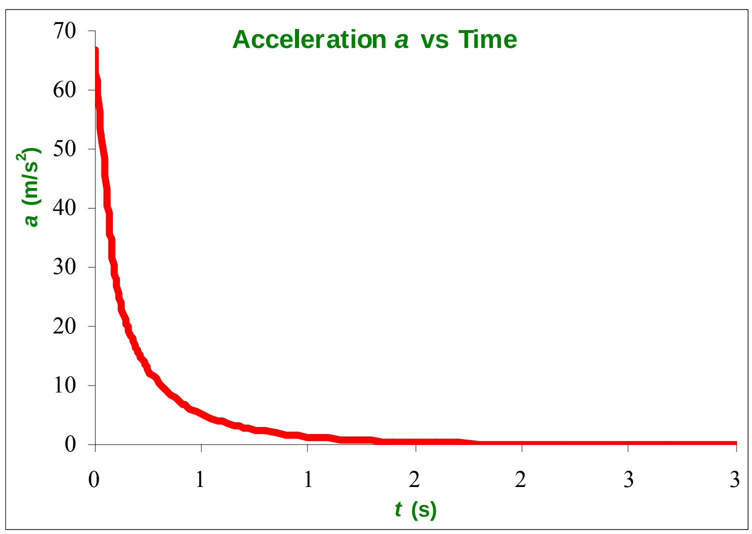
$$x(n+1) = x(n) + U(n)\cdot\Delta t$$

The results can be plotted in *Excel*

t (s)	*x* (m)	*U* (m/s)	*a* (m/s²)
0.0	0.0	0.0	66.6
0.1	0.0	6.7	28.0
0.2	0.7	9.5	16.1
0.3	1.6	11.1	10.5
0.4	2.7	12.1	7.30
0.5	3.9	12.9	5.29
0.6	5.2	13.4	3.95
0.7	6.6	13.8	3.01
0.8	7.9	14.1	2.32
0.9	9.3	14.3	1.82
1.0	10.8	14.5	1.43
1.1	12.2	14.6	1.14
1.2	13.7	14.7	0.907
1.3	15.2	14.8	0.727
1.4	16.6	14.9	0.585
1.5	18.1	15.0	0.472
1.6	19.6	15.0	0.381
1.7	21.1	15.1	0.309
1.8	22.6	15.1	0.250
1.9	24.1	15.1	0.203
2.0	25.7	15.1	0.165
2.1	27.2	15.1	0.134
2.2	28.7	15.2	0.109
2.3	30.2	15.2	0.0889
2.4	31.7	15.2	0.0724
2.5	33.2	15.2	0.0590
2.6	34.8	15.2	0.0481
2.7	36.3	15.2	0.0392
2.8	37.8	15.2	0.0319
2.9	39.3	15.2	0.0260
3.0	40.8	15.2	0.0212



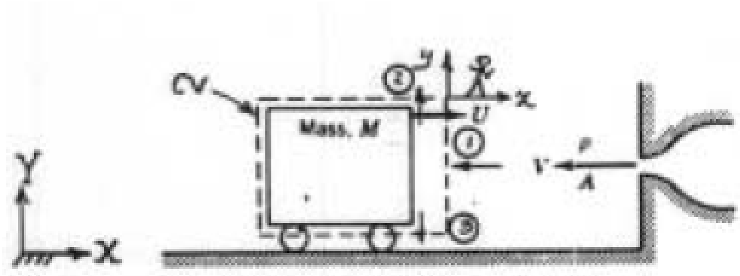




Problem 4.116

(Difficulty: 2)

4.116 A rectangular block of mass M, with vertical faces, rolls without resistance along a smooth horizontal plane as shown. The block travels initially at speed U_0. At $t = 0$ the block is struck by a liquid jet and its speed begins to slow. Obtain an algebraic expression for the acceleration of the block for $t > 0$. Solve the equation to determine the time at which $U = 0$.



Find: The time t at which $U = 0$.

Assumption: (1) no pressure for friction forces, so $F_{sx} = 0$.

(2) horizontal, so $F_{Bx} = 0$.

(3) neglect mass of liquid in CV, $u = 0$ in CV

(4) uniform flow at each section

(5) measure velocities relative to CV

Solution:

Apply x momentum equation to linearly accelerating CV.

Basic equation:

$$F_{sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t}\int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \bar{V}_{xyz} \cdot d\bar{A}$$

Then

$$-Ma_{rfx} = -\mathrm{M}\frac{dU}{dt} = u_1\{-|\rho(V+U)A|\} + u_2\{\dot{m}_2\} + u_3\{\dot{m}_3\}$$

$$u_1 = -(V+U)$$

$$u_2 = 0$$

$$u_3 = 0$$

or

$$\frac{dU}{dt} = -\frac{\rho(V+U)^2 A}{M}$$

But, since $V = constant$, $dU = d(V+U)$, so

$$\frac{d(V+U)}{(V+U)^2} = -\frac{\rho A}{M} dt$$

Integrating from U_0 at $t = 0$ to $U = 0$ at t

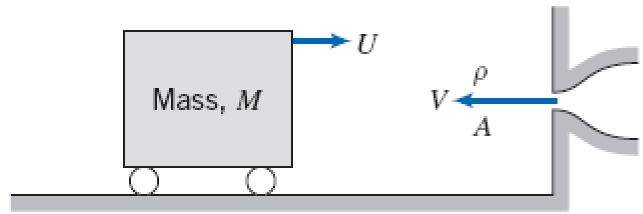
$$\int_{V+U_0}^{V} \frac{d(V+U)}{(V+U)^2} = -\frac{1}{V} + \frac{1}{V+U_0} = \frac{-U_0}{V(V+U_0)} = -\frac{\rho A t}{M}$$

Solving,

$$t = \frac{MU_0}{\rho VA(V+U_0)} = \frac{M}{\rho VA\left(1+\frac{V}{U_0}\right)}$$

Problem 4.117 [Difficulty: 3]

4.117 Consider the diagram of Problem 4.154. If $M = 100$ kg, $\rho = 999$ kg/m^3, and $A = 0.01$ m^2, find the jet speed V required for the cart to be brought to rest after one second if the initial speed of the cart is $U_0 = 5$ m/s. For this condition, plot the speed U and position x of the cart as functions of time. What is the maximum value of x, and how long does the cart take to return to its initial position?



Given: Data on system

Find: Jet speed to stop cart after 1 s; plot speed & position; maximum x; time to return to origin

Solution: Apply x momentum $$F_{S_x} + F_{B_x} - \int_{CV} a_{rf_x}\,\rho\, dV\!\!\!/ = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, dV\!\!\!/ + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) All changes wrt CV 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow 5) Constant jet area

The given data is $\rho = 999\cdot\frac{kg}{m^3}$ $M = 100\cdot kg$ $A = 0.01\cdot m^2$ $U_0 = 5\cdot\frac{m}{s}$

Then $-a_{rf}\cdot M = u_1\cdot[-\rho\cdot(V+U)\cdot A] + u_2\cdot m_2 + u_3\cdot m_3$

where $a_{rf} = \frac{dU}{dt}$ $u_1 = -(V+U)$ and $u_2 = u_3 = 0$

Hence $-\frac{dU}{dt}\cdot M = \rho\cdot(V+U)^2\cdot A$ or $\frac{dU}{dt} = -\frac{\rho\cdot(V+U)^2\cdot A}{M}$ which leads to $\frac{d(V+U)}{(V+U)^2} = -\left(\frac{\rho\cdot A}{M}\cdot dt\right)$

Integrating and using the IC $U = U_0$ at $t = 0$ $$U = -V + \frac{V+U_0}{1 + \frac{\rho\cdot A\cdot(V+U_0)}{M}\cdot t}$$

To find the jet speed V to stop the cart after 1 s, solve the above equation for V, with $U = 0$ and $t = 1$ s. (The equation becomes a quadratic in V). Instead we use *Excel*'s *Goal Seek* in the associated workbook

From *Excel* $V = 5\cdot\frac{m}{s}$

For the position x we need to integrate $$\frac{dx}{dt} = U = -V + \frac{V+U_0}{1 + \frac{\rho\cdot A\cdot(V+U_0)}{M}\cdot t}$$

The result is $$x = -V\cdot t + \frac{M}{\rho\cdot A}\cdot\ln\left[1 + \frac{\rho\cdot A\cdot(V+U_0)}{M}\cdot t\right]$$

This equation (or the one for U with $U = 0$) can be easily used to find the maximum value of x by differentiating, as well as the time for x to be zero again. Instead we use *Excel*'s *Goal Seek* and *Solver* in the associated workbook

From *Excel* $x_{max} = 1.93\cdot m$ $t(x = 0) = 2.51\cdot s$

The complete set of equations is $$U = -V + \frac{V+U_0}{1 + \frac{\rho\cdot A\cdot(V+U_0)}{M}\cdot t} \qquad x = -V\cdot t + \frac{M}{\rho\cdot A}\cdot\ln\left[1 + \frac{\rho\cdot A\cdot(V+U_0)}{M}\cdot t\right]$$

The plots are presented in the *Excel* workbook:

t (s)	*x* (m)	*U* (m/s)
0.0	0.00	5.00
0.2	0.82	3.33
0.4	1.36	2.14
0.6	1.70	1.25
0.8	1.88	0.56
1.0	1.93	0.00
1.2	1.88	-0.45
1.4	1.75	-0.83
1.6	1.56	-1.15
1.8	1.30	-1.43
2.0	0.99	-1.67
2.2	0.63	-1.88
2.4	0.24	-2.06
2.6	-0.19	-2.22
2.8	-0.65	-2.37
3.0	-1.14	-2.50

To find *V* for *U* = 0 in 1 s, use *Goal Seek*

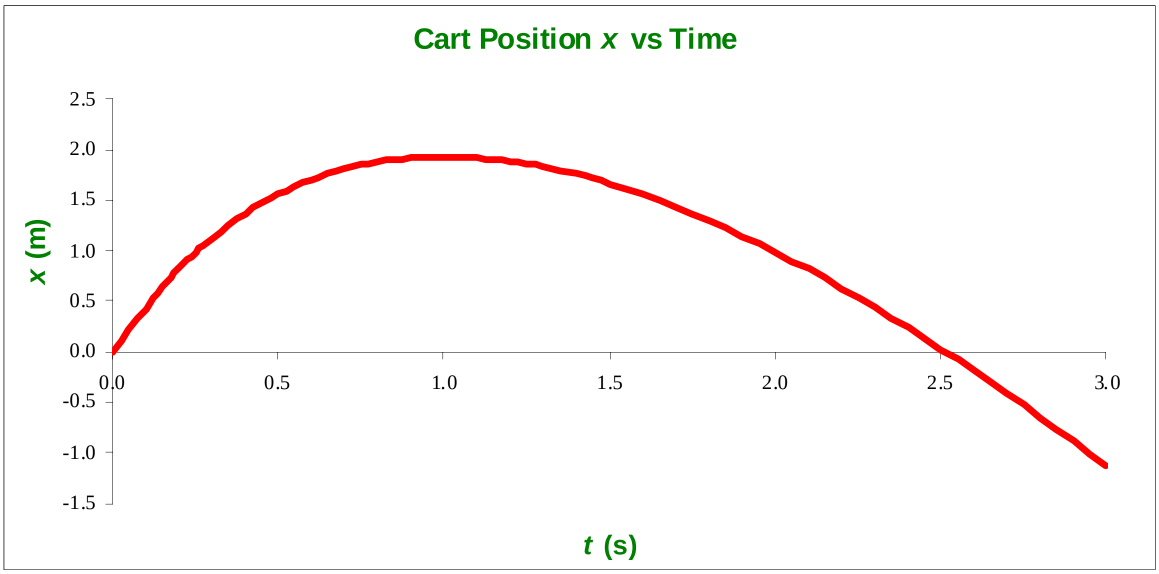
t (s)	*U* (m/s)	*V* (m/s)
1.0	0.00	5.00

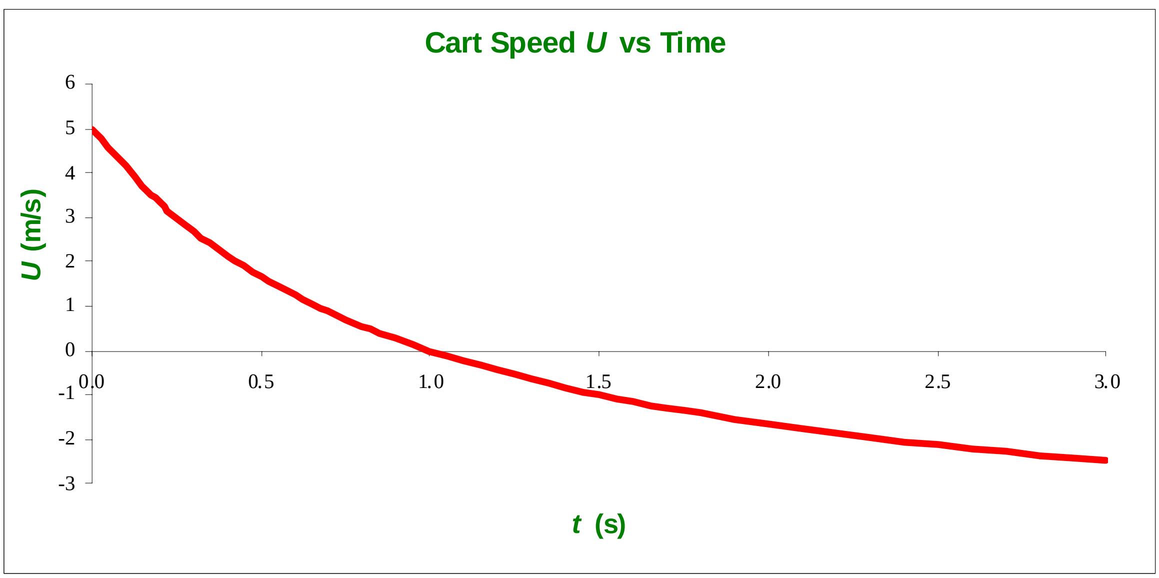
To find the maximum *x*, use *Solver*

t (s)	*x* (m)
1.0	1.93

To find the time at which *x* = 0 use *Goal Seek*

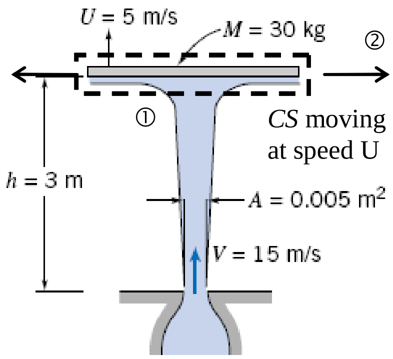
t (s)	*x* (m)
2.51	0.00





Problem 4.118 [Difficulty: 3]

4.118 A vertical jet of water impinges on a horizontal disk as shown. The disk assembly mass is 30 kg. When the disk is 3 m above the nozzle exit, it is moving upward at $U = 5$ m/s. Compute the vertical acceleration of the disk at this instant.



Given:
Water jet striking moving disk

Find:
Acceleration of disk when at a height of 3 m

Solution:

Basic equations: Bernoulli; Momentum flux in z direction (treated as upwards) for linear accelerating CV

$$\frac{p}{\rho} + \frac{V^2}{2} + g \cdot z = \text{constant} \qquad F_{S_z} + F_{B_z} - \int_{CV} a_{rf_z}\,\rho\, dV\!\!\!/ = \frac{\partial}{\partial t}\int_{CV} w_{xyz}\,\rho\, dV\!\!\!/ + \int_{CS} w_{xyz}\,\rho \vec{V}_{xyz} \cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow (All in jet)

The Bernoulli equation becomes $\dfrac{V_0^2}{2} + g \cdot 0 = \dfrac{V_1^2}{2} + g \cdot (z - z_0)$ $\qquad V_1 = \sqrt{V_0^2 + 2 \cdot g \cdot (z_0 - z)}$

$$V_1 = \sqrt{\left(15 \cdot \frac{m}{s}\right)^2 + 2 \times 9.81 \cdot \frac{m}{s^2} \cdot (0 - 3) \cdot m} \qquad V_1 = 12.9 \frac{m}{s}$$

The momentum equation becomes

$$-W - M \cdot a_{rfz} = w_1 \cdot (-\rho \cdot V_1 \cdot A_1) + w_2 \cdot (\rho \cdot V_2 \cdot A_2) = (V_1 - U) \cdot \left[-\rho \cdot (V_1 - U) \cdot A_1\right] + 0$$

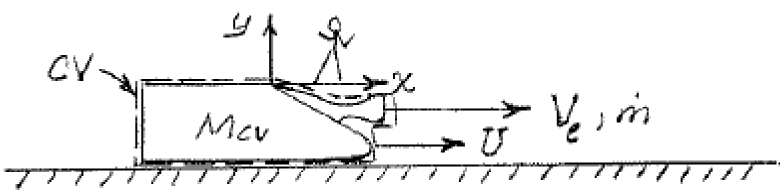
Hence $\quad a_{rfz} = \dfrac{\rho \cdot (V_1 - U)^2 \cdot A_1 - W}{M} = \dfrac{\rho \cdot (V_1 - U)^2 \cdot A_1}{M} - g = \dfrac{\rho \cdot (V_1 - U)^2 \cdot A_0 \cdot \dfrac{V_0}{V_1}}{M} - g \qquad$ using $\qquad V_1 \cdot A_1 = V_0 \cdot A_0$

$$a_{rfz} = 1000 \cdot \frac{kg}{m^3} \times \left[(12.9 - 5) \cdot \frac{m}{s}\right]^2 \times 0.005 \cdot m^2 \times \frac{15}{12.9} \times \frac{1}{30 \cdot kg} - 9.81 \cdot \frac{m}{s^2} \qquad a_{rfz} = 2.28 \frac{m}{s^2}$$

Problem 4.119

(Difficulty: 2)

4.119 A rocket sled traveling on a horizontal track is slowed by a retro-rocket fired in the direction of travel. The initial speed of the sled is $U_0 = 500\ \frac{m}{s}$. The initial mass of the sled is $M_0 = 1500\ kg$. The retro-rocket consumes fuel at the rate of $7.75\ \frac{kg}{s}$, and the exhaust gases leave the nozzle at atmospheric pressure and a speed of $2500\ \frac{m}{s}$ relative to the rocket. The retro-rocket fires for $20\ s$. Neglect aerodynamic drag and rolling resistance. Obtain an plot an algebraic expression for sled speed U as a function of firing time. Calculate the sled speed at the end of retro-rocket firing.



Find: Sled speed $U(t)$. The sled speed at the end of retro-rocket firing $U_{(t\infty)}$.

Assumption: (1) no pressure, drag, or rolling resistance, so $F_{sx} = 0$.

(2) horizontal motion, so $F_{Bx} = 0$.

(3) neglect unsteady effects within CV

(4) uniform flow at nozzle exit plane

(5) $p_e = p_{atm}$

Solution:

Apply x-component of momentum equation to the linearly accelerating CV shown.

From continuity,

$$M_{CV} = M_0 - \dot{m}t, t < t_\infty$$

Basic equation:

$$F_{sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t}\int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \bar{V}_{xyz} \cdot d\bar{A}$$

Then

$$-a_{rfx} M_{CV} = u_e\{\dot{m}\} = \mathrm{V}_e \dot{m}$$

or

$$\frac{dU}{dt} = -\frac{\mathrm{V}_e \dot{m}}{M_{CV}} = -\frac{\mathrm{V}_e \dot{m}}{M_0 - \dot{m}t}$$

Thus

$$dU = \mathrm{V}_e \left(\frac{-\dot{m}dt}{M_0 - \dot{m}t} \right)$$

and

$$U - U_0 = \mathrm{V}_e \ln(M_0 - \dot{m}t)_0^t = \mathrm{V}_e \ln\left(1 - \frac{\dot{m}t}{M_0}\right)$$

$$U(t) = U_0 + \mathrm{V}_e \ln\left(1 - \frac{\dot{m}t}{M_0}\right)$$

for $t < t_\infty$.

At $t = t_\infty$,

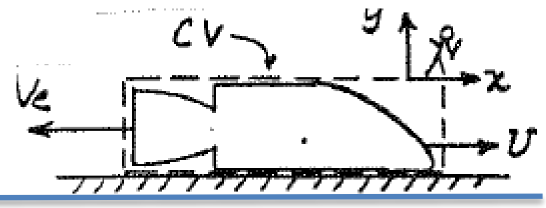
$$U(t_\infty) = 500\ \frac{m}{s} + 2500\ \frac{m}{s} \times \ln\left(1 - 7.75 \frac{kg}{s} \times 20\ s \times \frac{1}{1500\ kg}\right)$$

$$U(t_\infty) = 227\ \frac{m}{s}$$

Problem 4.120

(Difficulty: 2)

4.120 A rocket sled accelerates from rest on a level track with negligible air and rolling resistances. The initial mass of the sled is $M_0 = 600\ kg$. The rocket initially contains $150\ kg$ of fuel. The rocket motor burns fuel at constant rate $\dot{m} = 15\ \frac{kg}{s}$. Exhaust gases leave the rocket nozzle uniformly and axially at $V_e = 2900\ \frac{m}{s}$ relative to the nozzle, and the pressure is atmospheric. Find the maximum speed reached by the rocket sled. Calculate the maximum acceleration of the sled during the run.



Find: The maximum speed U_{max} and the maximum acceleration $\frac{dU}{dt}_{max}$ during the run.

Assumption: (1) no net pressure forces ($p_e = p_{atm}$)

(2) horizontal motion, so $F_{Bx} = 0$

(3) neglect $\frac{\partial}{\partial t}$ in CV

(4) uniform axial jet

Solution:

Apply the momentum equation to linearly accelerating CV shown.

Basic equation:

$$F_{sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t}\int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \bar{V}_{xyz} \cdot d\bar{A}$$

From continuity,

$$M = M_0 - \dot{m}\mathrm{t}$$

$$-a_{rfx}\mathrm{M} = -\frac{dU}{dt}(M_0 - \dot{m}\mathrm{t}) = u_e\{\dot{m}\} = -V_e\dot{m}$$

Eq (1)

Separating variables,

$$dU = V_e \frac{\dot{m}dt}{M_0 - \dot{m}\mathrm{t}}$$

Integrating from $U = 0$ at $t = 0$ to U at t gives

$$U = -V_e \ln(M_0 - \dot{m}\mathrm{t})_0^t = -V_e \ln\frac{(M_0 - \dot{m}\mathrm{t})}{M_0} = V_e \ln\frac{M_0}{(M_0 - \dot{m}\mathrm{t})}$$

Eq (2)

The speed is a maximum at burnout. At burnout $M_f = 0$ and $M = M_0 - \dot{m}\mathrm{t} = 450$ kg.

At burnout,

$$t = \frac{M_{fintial}}{\dot{m}_{fuel}} = 150\ kg \times \frac{s}{15\ kg} = 10\ s$$

Then from Eq (2)

$$U_{max} = 2900\ \frac{m}{s} \times \ln\frac{600\ kg}{450\ kg} = 834\ \frac{m}{s}$$

From Eq (1) the acceleration is:

$$\frac{dU}{dt} = \frac{\dot{m}V_e}{M_0 - \dot{m}\mathrm{t}}$$

The maximum acceleration occurs at the instant prior to burnout

$$\frac{dU}{dt}_{max} = 15\ \frac{kg}{s} \times 2900\ \frac{m}{s} \times \frac{1}{450\ kg} = 96.7\ \frac{m}{s^2}$$

The sled speed as a function of time is

$$U = V_e \ln\frac{M_0}{(M_0 - \dot{m}\mathrm{t})}$$

for $0 \leq t \leq 10\ s$.

$$U = constant = 834\ \frac{m}{s}$$

for $t > 10\ s$ (neglecting resistance).

The sled accelerating is given by

$$\frac{dU}{dt} = \frac{\dot{m}V_e}{M_0 - \dot{m}\mathrm{t}}$$

for $0 \leq t \leq 10\ s$.

$$\frac{dU}{dt} = 0$$

for $t > 10\ s$.

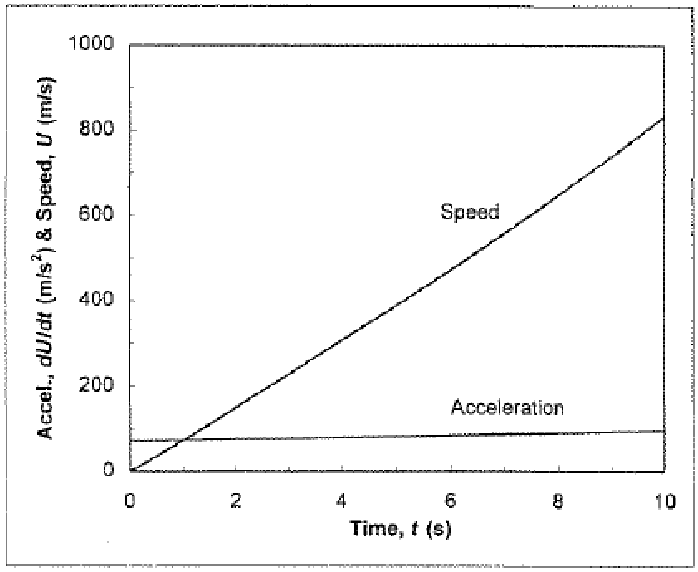
Acceleration and Velocity vs. Time for Rocket Sled:

Input Data:

$M_0 =$	600	kg
$m(\text{dot}) =$	15	kg/s
$V_e =$	2900	m/s

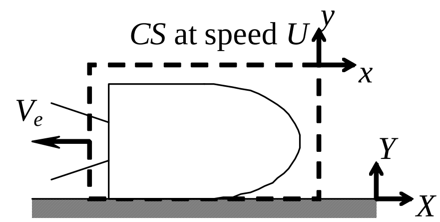
Calculated Results:

Time, t (s)	Acceleration, dU/dt (m/s^2)	Velocity, U (m/s)
0	72.5	0
1	74.4	73.4
2	76.3	149
3	78.4	226
4	80.6	306
5	82.9	387
6	85.3	471
7	87.9	558
8	90.6	647
9	93.5	739
10	96.7	834



Problem 4.121 [Difficulty: 3]

4.121 A rocket sled with initial mass of 900 kg is to be accelerated on a level track. The rocket motor burns fuel at constant rate $\dot{m} = 13.5$ kg/s. The rocket exhaust flow is uniform and axial. Gases leave the nozzle at 2750 m/s relative to the nozzle, and the pressure is atmospheric. Determine the minimum mass of rocket fuel needed to propel the sled to a speed of 265 m/s before burnout occurs. As a first approximation, neglect resistance forces.



Given: Data on rocket sled

Find: Minimum fuel to get to 265 m/s

Solution:

Basic equation: Momentum flux in x direction
$$F_{S_x} + F_{B_x} - \int_{CV} a_{rf_x}\,\rho\, dV = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, dV + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) No resistance 2) $p_e = p_{atm}$ 3) Uniform flow 4) Use relative velocities

From continuity $\frac{dM}{dt} = m_{rate} = \text{constant}$ so $M = M_0 - m_{rate}\cdot t$ (Note: Software cannot render a dot!)

Hence from momentum
$$-a_{rfx}\cdot M = -\frac{dU}{dt}\cdot\left(M_0 - m_{rate}\cdot t\right) = u_e\cdot\left(\rho_e\cdot V_e\cdot A_e\right) = -V_e\cdot m_{rate}$$

Separating variables
$$dU = \frac{V_e\cdot m_{rate}}{M_0 - m_{rate}\cdot t}\cdot dt$$

Integrating
$$U = V_e\cdot\ln\left(\frac{M_0}{M_0 - m_{rate}\cdot t}\right) = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right) \quad \text{or} \quad t = \frac{M_0}{m_{rate}}\cdot\left(1 - e^{-\frac{U}{V_e}}\right)$$

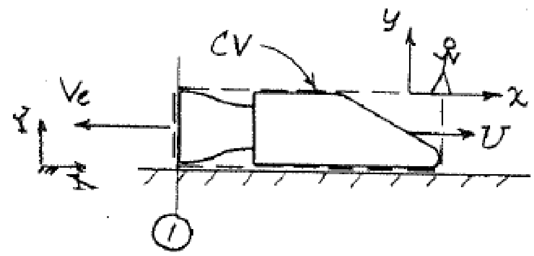
The mass of fuel consumed is
$$m_f = m_{rate}\cdot t = M_0\cdot\left(1 - e^{-\frac{U}{V_e}}\right)$$

Hence
$$m_f = 900\cdot kg \times \left(1 - e^{-\frac{265}{2750}}\right) \qquad m_f = 82.7\,kg$$

Problem 4.122

(Difficulty: 2)

4.122 A rocket sled with initial mass of 3 metric tons, including 1 ton of fuel, rests on a level section of track. At $t = 0$, the solid fuel of the rocket is ignited and the rocket burns fuel at the rate of 75 $\frac{kg}{s}$. The exit speed of the exhaust gas relative to the rocket is $2500\ \frac{m}{s}$, and the pressure is atmospheric. Neglecting friction and air resistance, calculate the acceleration and speed of the sled at $t = 10\ s$.



Find: The acceleration $\frac{dU}{dt}$ and speed of sled U at $t = 10\ s$.

Assumption: (1) $F_{Sx} = 0$, no resistance (given).

(2) $F_{Bx} = 0$, horizontal

(3) neglect $\frac{\partial}{\partial t}$ inside CV

(4) uniform flow at nozzle exit

(5) $p_e = p_{atm}$ (given)

Solution:

Apply the x component of momentum to linearly accelerating CV. Use continuity to find $M(t)$.

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho\, \bar{V}_{xyz} \cdot d\bar{A}$$

$$F_{Sx} + F_{Bx} - \int_{CV} a_{rfx}\rho d\forall = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\rho d\forall + \int_{CS} u_{xyz}\rho \bar{V}_{xyz} \cdot d\bar{A}$$

From continuity,

$$0 = \frac{\partial M}{\partial t} + \{\dot{M}\} = \frac{dM}{dt} + \dot{M}$$

$$dM = -\dot{M}dt$$

Integrating,

$$\int_{M_0}^{M} dM = M - M_0 = \int_0^t -\dot{M}dt = -\dot{M}t$$

$$M = M_0 - \dot{M}t$$

From the momentum equation

$$-a_{rfx}M = -a_{rfx}\left(M_0 - \dot{M}t\right) = u_1\{|\dot{M}|\} = -V_e\dot{M}$$

Thus

$$a_{rfx} = \frac{dU}{dt} = \frac{V_e\dot{M}}{\left(M_0 - \dot{M}t\right)}$$

Eq (1)

At $t = 10\ s$

$$\frac{dU}{dt} = 2500\ \frac{m}{s} \times 75\ \frac{kg}{s} \times \frac{1}{3000\ kg - 75\ \frac{kg}{s} \times 10\ s} = 83.3\ \frac{m}{s^2}$$

From Eq (1),

$$dU = \frac{V_e\dot{M}dt}{\left(M_0 - \dot{M}t\right)}$$

Integrating from $U = 0$ at$t = 0$ to U at t gives

$$U = -V_e \ln\left(M_0 - \dot{M}t\right)_0^t = -V_e \ln\frac{\left(M_0 - \dot{M}t\right)}{M_0}$$

$$U = V_e \ln\frac{M_0}{\left(M_0 - \dot{M}t\right)}$$

Eq (2)

At $t = 10\ s$

$$U = 2500\ \frac{m}{s} \times \ln\frac{3000\ kg}{3000\ kg - 75\frac{kg}{s} \times 10\ s} = 719\ \frac{m}{s}$$

Note that all fuel will be expended at

$$t_{bo} = \frac{M_f}{\dot{M}} = \frac{1000\ kg}{75\frac{kg}{s}} = 13.3\ s$$

The sled speed as a function of time is then

$$U = V_e \ln \frac{M_0}{(M_0 - \dot{M}t)}$$

for $t \leq 13.3\ s$.

$$U = U_{max} = 1010\ \frac{m}{s}$$

for $t \geq 13.3\ s$.

The sled acceleration is given by:

$$\frac{dU}{dt} = \frac{V_e \dot{M}}{(M_0 - \dot{M}t)}$$

for $t \leq 13.3\ s$.

$$\frac{dU}{dt} = 0$$

for $t \geq 13.3\ s$.

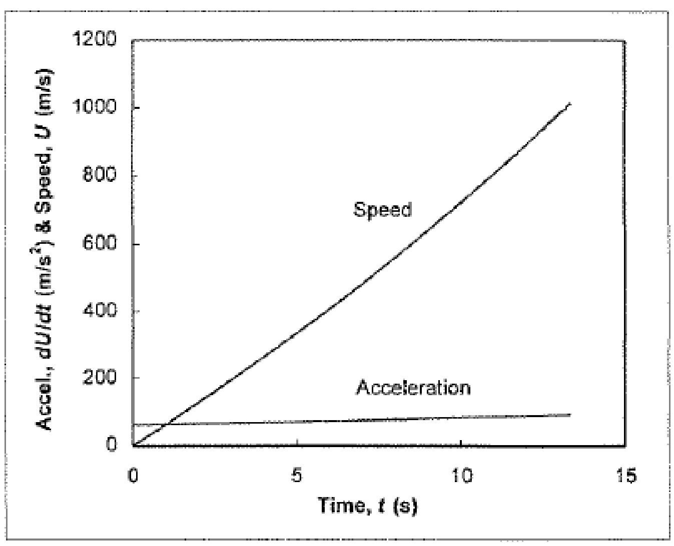
Acceleration and Speed vs. Time for Rocket Sled:

Input Data:

M_0 =	3000	kg
m(dot) =	75	kg/s
V_e =	2500	m/s

Calculated Results:

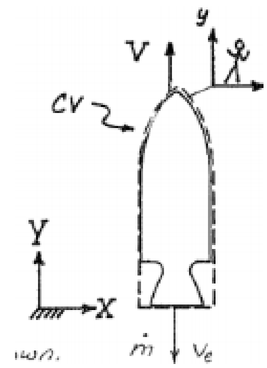
Time, t (s)	Acceleration, dU/dt (m/s^2)	Speed, U (m/s)
0	62.5	0
1	64.1	63.3
2	65.8	128
3	67.6	195
4	69.4	263
5	71.4	334
6	73.5	406
7	75.8	481
8	78.1	558
9	80.6	637
10	83.3	719
11	86.2	804
12	89.3	892
13	92.6	983
13.33	93.8	1014



Problem 4.123

(Difficulty: 2)

4.123 A "home-made" solid propellant rocket has an initial mass of $9\ kg$; $6.8\ kg$ of this is fuel. The rocket is directed vertically upward from rest, burns fuel at a constant rate of $0.225\ \frac{kg}{s}$, and ejects exhaust gas at a speed of $1980\ \frac{m}{s}$ relative to the rocket. Assume that the pressure at the exit is atmospheric and that air resistance may be neglected. Calculate the rocket speed after $20\ s$ and the distance travelled by the rocket in $20\ s$. Plot the rocket speed and the distance travelled as functions of time.



Find: The speed V after $20\ s$. The distance Y travelled in $20\ s$.

Assumption: (1) neglect air resistance; $p_e = p_{atm}$

(2) neglect V_{xyz} and $\frac{\partial}{\partial t}$ within CV

(3) uniform flow at nozzle exit section

Solution:

Apply the y component of momentum equation to accelerating CV using CS shown.

Basic equation:

$$F_{sy} + F_{By} - \int_{CV} a_{rfy} \rho d\forall = \frac{\partial}{\partial t} \int_{CV} V_{xyz} \rho d\forall + \int_{CS} V_{xyz} \rho \vec{V}_{xyz} \cdot d\vec{A}$$

Then

$$F_{By} - a_{rfy} M = -Mg - M a_{rfy} = -V_e \dot{m}$$

And

$$a_{rfy} = \frac{dV}{dt} = \frac{V_e \dot{m}}{M} - g$$

Introducing $M = M_0 - \dot{m}t$ and seperating variables,

$$dV = \left(\frac{V_e \dot{m}}{M_0 - \dot{m}t} - g\right) dt$$

Integrating from rest at $t = 0$

$$V = \int_0^t \left(\frac{V_e \dot{m}}{M_0 - \dot{m}t} - g\right) dt = V_e \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) - gt$$

Eq (1)

At $t = 20\ s$,

$$V = 1980\ \frac{m}{s} \times \ln\left(\frac{9\ kg}{9\ kg - 0.225\ \frac{kg}{s} \times 20\ s}\right) - 9.81\ \frac{m}{s^2} \times 20\ s = 1176\ \frac{m}{s}$$

To find height, note $V = \frac{dY}{dt}$. Substitute into Eq (1) to obtain:

$$\frac{dY}{dt} = -V_e \ln\left(1 - \frac{\dot{m}t}{M_0}\right) - gt$$

Let $\Omega = 1 - \frac{\dot{m}t}{M_0}$, and $d\Omega = -\frac{\dot{m}}{M_0} dt$, then

$$dY = -V_e \ln(\Omega)\, dt - gtdt = \frac{V_e M_0}{\dot{m}} \ln(\Omega)\, d\Omega - gtdt$$

Integrating from $Y = 0$ at $t = 0$:

$$Y = \int_0^t \frac{V_e M_0}{\dot{m}} \ln(\Omega)\, d\Omega - \frac{1}{2} g t^2 = \frac{V_e M_0}{\dot{m}} [\Omega \ln(\Omega) - \Omega]_0^t - \frac{1}{2} g t^2$$

$$Y = \frac{V_e M_0}{\dot{m}} \left\{\left(1 - \frac{\dot{m}t}{M_0}\right)\left[\ln\left(1 - \frac{\dot{m}t}{M_0}\right) - 1\right]\right\}_0^t - \frac{1}{2} g t^2$$

$$Y = \frac{V_e M_0}{\dot{m}} \left\{\left(1 - \frac{\dot{m}t}{M_0}\right)\left[\ln\left(1 - \frac{\dot{m}t}{M_0}\right) - 1\right] + 1\right\} - \frac{1}{2} g t^2$$

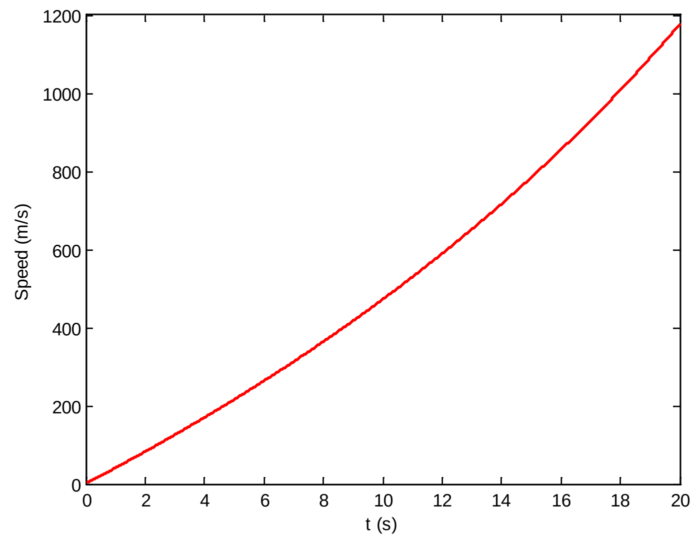
At $t = 20\ s$,

$$1 - \frac{\dot{m}t}{M_0} = 1 - \frac{0.225\ \frac{kg}{s} \times 20\ s}{9\ kg} = 0.5$$

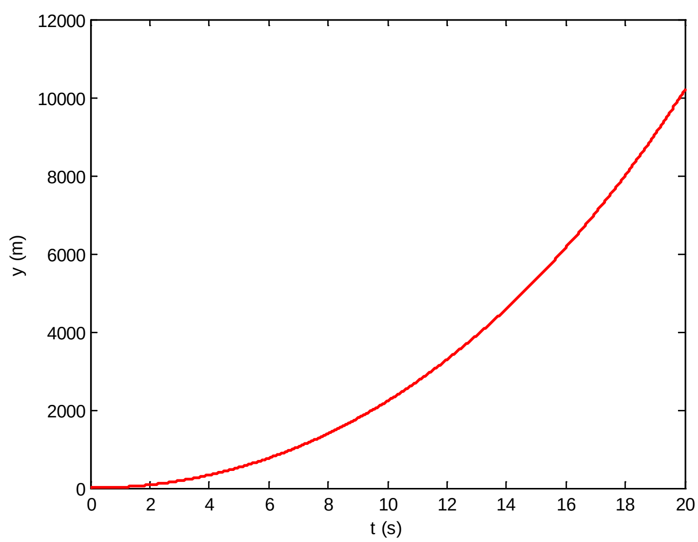
So we have:

$$Y = \frac{1980\ \frac{m}{s} \times 9\ kg}{0.225\ \frac{kg}{s}} \times \{0.5 \times [\ln(0.5) - 1] + 1\} - \frac{1}{2} \times 9.81\ \frac{m}{s^2} \times (20\ s)^2 = 10190\ m$$

The speed as a function of time is shown:

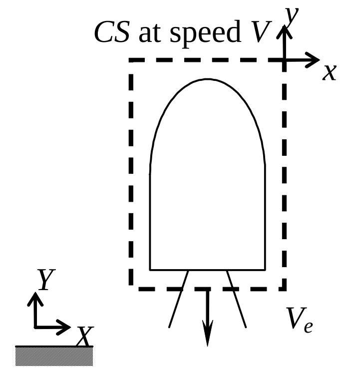


The distance travelled as a function of time is shown:



Problem 4.124 [Difficulty: 3]

4.124 Neglecting air resistance, what speed would a vertically directed rocket attain in 5 s if it starts from rest, has initial mass of 350 kg, burns 10 kg/s, and ejects gas at atmospheric pressure with a speed of 2500 m/s relative to the rocket? What would be the maximum velocity? Plot the rocket speed as a function of time for the first minute of flight.



Given: Data on rocket

Find: Speed after 5 s; Maximum velocity; Plot of speed versus time

Solution:

Basic equation: Momentum flux in y direction $$F_{S_y} + F_{B_y} - \int_{CV} a_{rf_y}\,\rho\, dV\!\!\!\!- = \frac{\partial}{\partial t}\int_{CV} v_{xyz}\,\rho\, dV\!\!\!\!- + \int_{CS} v_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) No resistance 2) $p_e = p_{atm}$ 3) Uniform flow 4) Use relative velocities 5) Constant mass flow rate

From continuity $\quad \frac{dM}{dt} = m_{rate} = \text{constant} \quad$ so $\quad M = M_0 - m_{rate}\cdot t \quad$ (Note: Software cannot render a dot!)

Hence from momentum $\quad -M\cdot g - a_{rfy}\cdot M = u_e\cdot\left(\rho_e\cdot V_e\cdot A_e\right) = -V_e\cdot m_{rate} \quad$ or $\quad a_{rfy} = \frac{dV}{dt} = \frac{V_e\cdot m_{rate}}{M} - g = \frac{V_e\cdot m_{rate}}{M_0 - m_{rate}\cdot t} - g$

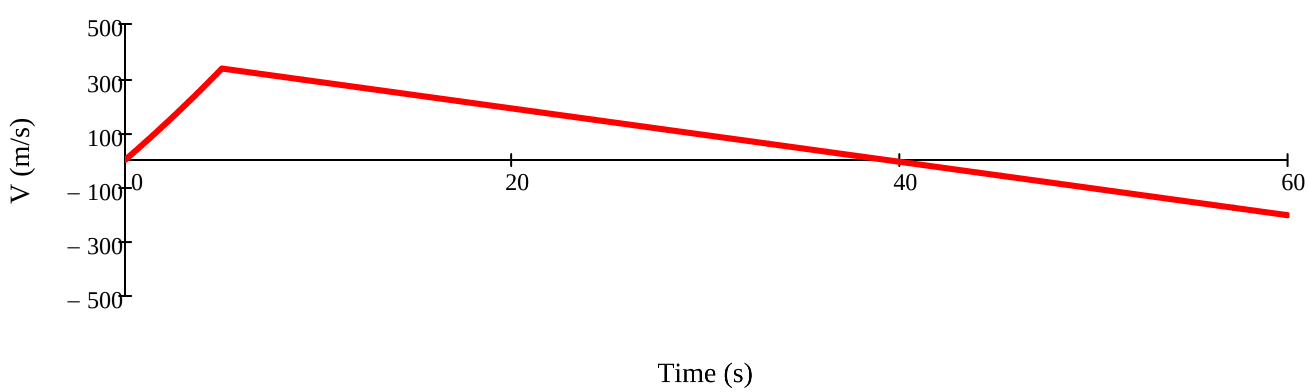
Separating variables $\quad dV = \left(\frac{V_e\cdot m_{rate}}{M_0 - m_{rate}\cdot t} - g\right)\cdot dt$

Integrating from V = at t = 0 to V = V at t = t

$$V = -V_e\cdot\left(\ln\left(M_0 - m_{rate}\cdot t\right) - \ln\left(M_0\right)\right) - g\cdot t = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right) - g\cdot t \qquad V = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right) - g\cdot t$$

At t = 5 s $\quad V_{max} = -2500\cdot\frac{m}{s}\cdot\ln\left(1 - 10\cdot\frac{kg}{s}\times\frac{1}{350\cdot kg}\times 5\cdot s\right) - 9.81\cdot\frac{m}{s^2}\times 5\cdot s \qquad V_{max} = 336\,\frac{m}{s}$

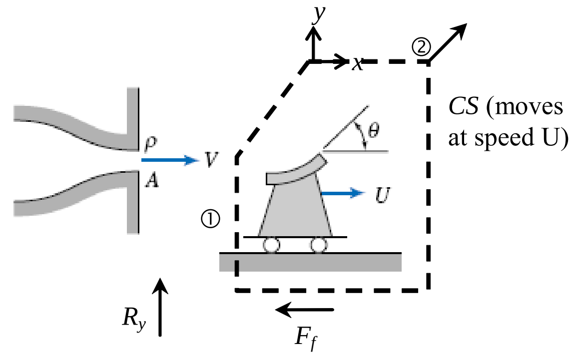
For the motion after 5 s, assuming the fuel is used up, the equation of motion becomes $\quad a = -M\cdot g$



Problem 4.125

[Difficulty: 4]

4.125 The vane/cart assembly of mass $M = 30$ kg, shown in Problem 4.128, is driven by a water jet. The water leaves the stationary nozzle of area $A = 0.02\ \text{m}^2$, with a speed of 20 m/s. The coefficient of kinetic friction between the assembly and the surface is 0.10. Plot the terminal speed of the assembly as a function of vane turning angle, θ, for $0 \le \theta \le \pi/2$. At what angle does the assembly begin to move if the coefficient of static friction is 0.15?



Given: Water jet striking moving vane

Find: Plot of terminal speed versus turning angle; angle to overcome static friction

Solution:

Basic equations: Momentum flux in x and y directions

$$F_{S_x} + F_{B_x} - \int_{CV} a_{rf_x}\,\rho\, dV = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, dV + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

$$F_{S_y} + F_{B_y} - \int_{CV} a_{rf_y}\,\rho\, dV = \frac{\partial}{\partial t}\int_{CV} v_{xyz}\,\rho\, dV + \int_{CS} v_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) Incompressible flow 2) Atmospheric pressure in jet 3) Uniform flow 4) Jet relative velocity is constant

Then $\quad -F_f - M\cdot a_{rfx} = u_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + u_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = -(V-U)\cdot[\rho\cdot(V-U)\cdot A] + (V-U)\cdot\cos(\theta)\cdot[\rho\cdot(V-U)\cdot A]$

$$a_{rfx} = \frac{\rho(V-U)^2\cdot A\cdot(1-\cos(\theta)) - F_f}{M} \qquad (1)$$

Also $\quad R_y - M\cdot g = v_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + v_2\cdot\rho\cdot V_2\cdot A_2 = 0 + (V-U)\cdot\sin(\theta)\cdot[\rho\cdot(V-U)\cdot A]$

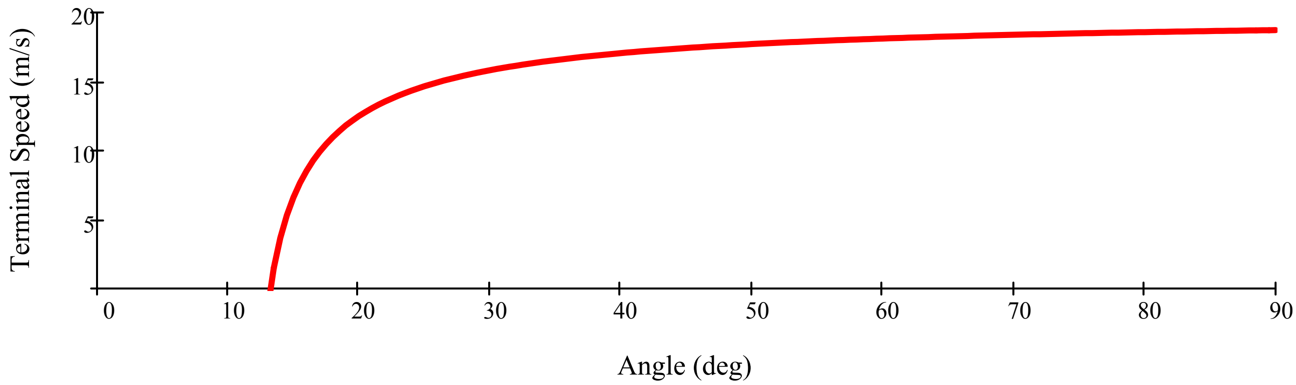
$$R_y = M\cdot g + \rho(V-U)^2\cdot A\cdot\sin(\theta)$$

At terminal speed $a_{rfx} = 0$ and $F_f = \mu_k R_y$. Hence in Eq 1

$$0 = \frac{\rho\cdot\left(V-U_t\right)^2\cdot A\cdot(1-\cos(\theta)) - \mu_k\cdot\left[M\cdot g + \rho\cdot\left(V-U_t\right)^2\cdot A\cdot\sin(\theta)\right]}{M} = \frac{\rho\cdot\left(V-U_t\right)^2\cdot A\cdot\left(1-\cos(\theta)-\mu_k\cdot\sin(\theta)\right)}{M} - \mu_k\cdot g$$

or $\quad V - U_t = \sqrt{\dfrac{\mu_k\cdot M\cdot g}{\rho\cdot A\cdot\left(1-\cos(\theta)-\mu_k\cdot\sin(\theta)\right)}} \qquad U_t = V - \sqrt{\dfrac{\mu_k\cdot M\cdot g}{\rho\cdot A\cdot\left(1-\cos(\theta)-\mu_k\cdot\sin(\theta)\right)}}$

The terminal speed as a function of angle is plotted below; it can be generated in *Excel*



For the static case $F_f = \mu_s \cdot R_y$ and $a_{rfx} = 0$ (the cart is about to move, but hasn't)

Substituting in Eq 1, with U = 0

$$0 = \frac{\rho \cdot V^2 \cdot A \cdot \left[1 - \cos(\theta) - \mu_s \cdot \left(\rho \cdot V^2 \cdot A \cdot \sin(\theta) + M \cdot g\right)\right.}{M}$$

or $$\cos(\theta) + \mu_s \cdot \sin(\theta) = 1 - \frac{\mu_s \cdot M \cdot g}{\rho \cdot V^2 \cdot A}$$

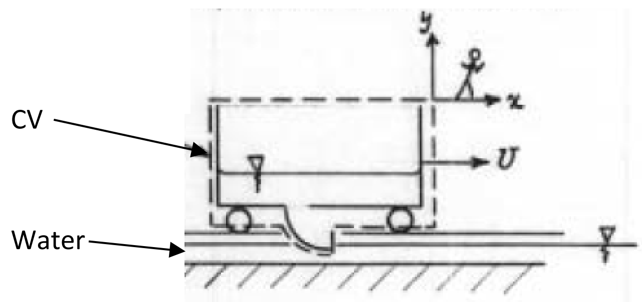
We need to solve this for θ! This can be done by hand or by using Excel's Goal Seek or Solver $\theta = 19.0 \cdot deg$

Note that we need $\theta = 19^{o}$, but once started we can throttle back to about $\theta = 12.5^{o}$ and still keep moving!

Problem 4.126

(Difficulty: 2)

4.126 The moving tank shown is to be slowed by lowering a scoop to pick up water from a trough. The initial mass and speed of the tank and its contents are M_0 and U_0, respectively. Neglect external forces due to pressure or friction and assume that the track is horizontal. Apply the continuity and momentum equations to show that at any instant $U = \frac{U_0 M_0}{M}$. Obtain a general expression for $\frac{U}{U_0}$ as a function of time.



Find: The expression for $\frac{U}{U_0}$ as a function of time.

Assumption: (1) $F_{Sx} = 0$

(2) $F_{Bx} = 0$

(3) neglect u within CV

(4) uniform flow across inlet section

Solution:

Apply continuity and momentum equations to linearly accelerating CV shown.

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho\, \bar{V}_{xyz} \cdot d\bar{A}$$

$$F_{Sx} + F_{Bx} - \int_{CV} a_{rfx} \rho d\forall = \frac{\partial}{\partial t}\int_{CV} u_{xyz} \rho d\forall + \int_{CS} u_{xyz} \rho \bar{V}_{xyz} \cdot d\bar{A}$$

From continuity,

$$0 = \frac{\partial}{\partial t} M_{CV} + \{-|\rho U A|\}$$

or

$$\frac{dM}{dt} = \rho U A$$

From momentum

$$-a_{rfx} M = -\frac{dU}{dt} M = u\{-|\rho U A|\}$$

$$u = -U$$

$$-a_{rfx} M = -\frac{dU}{dt} M = u\{-|\rho U A|\} = U \rho U A$$

But from continuity,

$$\rho U A = \frac{dM}{dt}$$

So we have:

$$M \frac{dU}{dt} + U \frac{dM}{dt} = 0$$

or

$$UM = constant = U_0 M_0$$

$$U = \frac{U_0 M_0}{M}$$

Substituting $M = \frac{U_0 M_0}{U}$ into momentum, $-\frac{dU}{dt}\frac{M_0 U_0}{U} = \rho U^2 A$, or

$$\frac{dU}{U^3} = -\frac{\rho A}{U_0 M_0} dt$$

Integrating,

$$\int_{U_0}^{U} \frac{dU}{U^3} = -\frac{1}{2}\frac{1}{U^2}\bigg|_{U_0}^{U} = -\frac{1}{2}\left(\frac{1}{U^2} - \frac{1}{U_0^2}\right) = -\int_0^t \frac{\rho A}{U_0 M_0} dt = -\frac{\rho A}{U_0 M_0} t$$

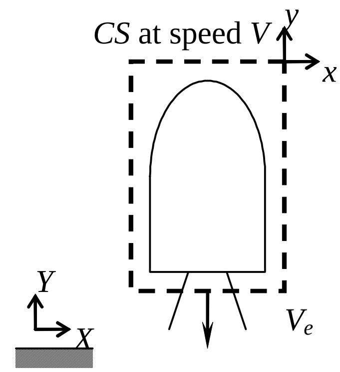
Solving for U,

$$U = \frac{U_0}{\left[1 + \frac{2\rho U_0 A}{M_0} t\right]^{\frac{1}{2}}}$$

Problem 4.127

[Difficulty: 4]

4.127 A model solid propellant rocket has a mass of 69.6 g, of which 12.5 g is fuel. The rocket produces 5.75 N of thrust for a duration of 1.7 s. For these conditions, calculate the maximum speed and height attainable in the absence of air resistance. Plot the rocket speed and the distance traveled as functions of time.



Given: Data on rocket

Find: Maximum speed and height; Plot of speed and distance versus time

Solution:

Basic equation: Momentum flux in y direction

$$F_{S_y} + F_{B_y} - \int_{CV} a_{rf_y}\,\rho\, dV = \frac{\partial}{\partial t}\int_{CV} v_{xyz}\,\rho\, dV + \int_{CS} v_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) No resistance 2) $p_e = p_{atm}$ 3) Uniform flow 4) Use relative velocities 5) Constant mass flow rate

From continuity $\quad \dfrac{dM}{dt} = m_{rate} = \text{constant} \quad$ so $\quad M = M_0 - m_{rate}\cdot t \quad$ (Note: Software cannot render a dot!)

Hence from momentum $\quad -M\cdot g - a_{rfy}\cdot M = u_e\cdot\left(\rho_e\cdot V_e\cdot A_e\right) = -V_e\cdot m_{rate}$

Hence $\quad a_{rfy} = \dfrac{dV}{dt} = \dfrac{V_e\cdot m_{rate}}{M} - g = \dfrac{V_e\cdot m_{rate}}{M_0 - m_{rate}\cdot t} - g$

Separating variables $\quad dV = \left(\dfrac{V_e\cdot m_{rate}}{M_0 - m_{rate}\cdot t} - g\right)\cdot dt$

Integrating from V = at t = 0 to V = V at t = t

$$V = -V_e\cdot\left(\ln\left(M_0 - m_{rate}\cdot t\right) - \ln\left(M_0\right)\right) - g\cdot t = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right) - g\cdot t$$

$$V = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right) - g\cdot t \qquad \text{for} \qquad t \le t_b \qquad \text{(burn time)} \qquad (1)$$

To evaluate at $t_b = 1.7$ s, we need V_e and m_{rate} $\quad m_{rate} = \dfrac{m_f}{t_b} \quad m_{rate} = \dfrac{12.5\cdot gm}{1.7\cdot s} \quad m_{rate} = 7.35\times 10^{-3}\,\dfrac{kg}{s}$

Also note that the thrust F_t is due to momentum flux from the rocket $\quad F_t = m_{rate}\cdot V_e \quad V_e = \dfrac{F_t}{m_{rate}} \quad V_e = \dfrac{5.75\cdot N}{7.35\times 10^{-3}\cdot\dfrac{kg}{s}}\times\dfrac{kg\cdot m}{s^2\cdot N} \quad V_e = 782\,\dfrac{m}{s}$

Hence $\quad V_{max} = -V_e\cdot\ln\left(1 - \dfrac{m_{rate}\cdot t_b}{M_0}\right) - g\cdot t_b$

$$V_{max} = -782\cdot\frac{m}{s}\cdot\ln\left(1 - 7.35\times 10^{-3}\cdot\frac{kg}{s}\times\frac{1}{0.0696\cdot kg}\times 1.7\cdot s\right) - 9.81\cdot\frac{m}{s^2}\times 1.7\cdot s \qquad V_{max} = 138\,\frac{m}{s}$$

To obtain Y(t) we set V = dY/dt in Eq 1, and integrate to find

$$Y = \frac{V_e \cdot M_0}{m_{rate}} \cdot \left[\left(1 - \frac{m_{rate} \cdot t}{M_0}\right) \cdot \left(\ln\left(1 - \frac{m_{rate} \cdot t}{M_0}\right) - 1 \right) + 1 \right] - \frac{1}{2} \cdot g \cdot t^2 \qquad t \le t_b \qquad t_b = 1.7 \cdot s \qquad (2)$$

At $t = t_b$

$$Y_b = 782 \cdot \frac{m}{s} \times 0.0696 \cdot kg \times \frac{s}{7.35 \times 10^{-3} \cdot kg} \cdot \left[\left(1 - \frac{0.00735 \cdot 1.7}{0.0696}\right) \left(\ln\left(1 - \frac{.00735 \cdot 1.7}{.0696}\right) - 1 \right) + 1 \right] \ldots$$

$$+ -\frac{1}{2} \times 9.81 \cdot \frac{m}{s^2} \times (1.7 \cdot s)^2$$

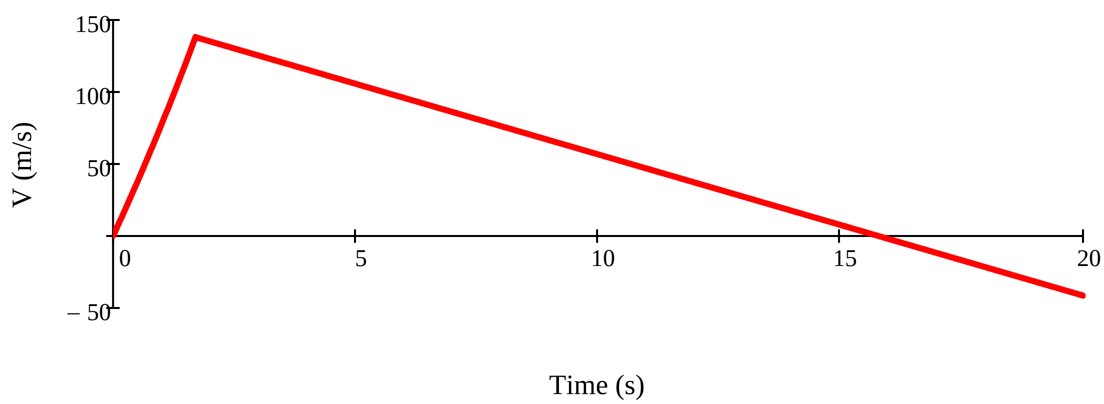
$$Y_b = 113\,m$$

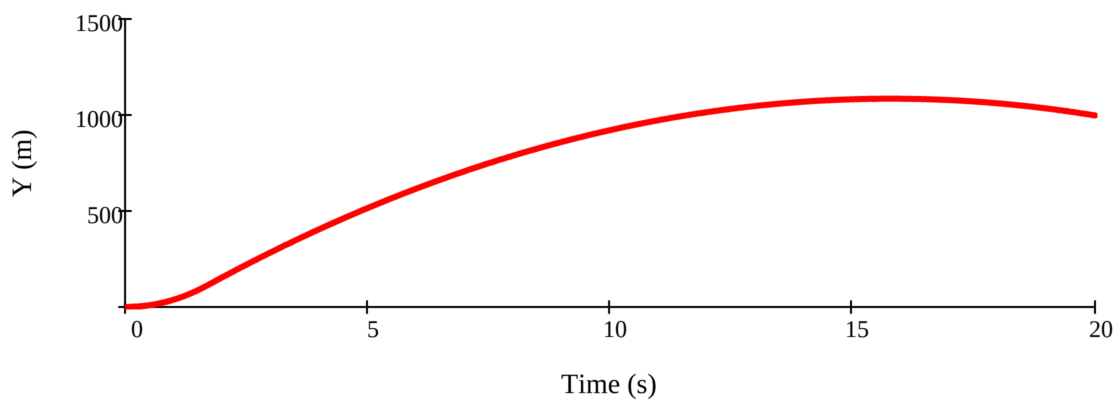
After burnout the rocket is in free assent. Ignoring drag

$$V(t) = V_{max} - g \cdot (t - t_b) \qquad (3)$$

$$Y(t) = Y_b + V_{max} \cdot (t - t_b) - \frac{1}{2} \cdot g \cdot (t - t_b)^2 \qquad t > t_b \qquad (4)$$

The speed and position as functions of time are plotted below. These are obtained from Eqs 1 through 4, and can be plotted in *Excel*





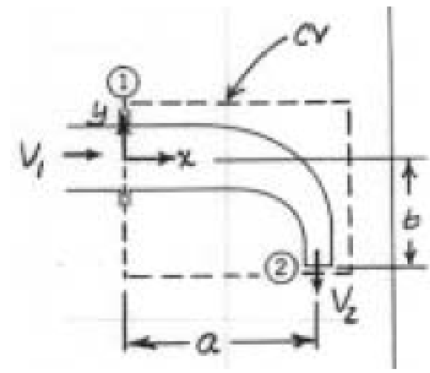
Using Solver, or by differentiating y(t) and setting to zero, or by setting V(t) = 0, we find for the maximum y

$t = 15.8\,s \qquad y_{max} = 1085\,m$

Problem 4.128

(Difficulty: 2)

4.128 The $90°$ reducing elbow of Example 4.6 discharges to atmosphere. Section (2) is located $0.3\ m$ to the right of section (1). Estimate the moment exerted by the flange on the elbow.



Find: The moment $\bar{M}_{flange}$ exerted by the flange on the elbow.

Assumption: (1) neglect body forces

(2) no shafts, so $\bar{T}_{shaft} = 0$

(3) steady flow (given)

(4) uniform flow across each across section

(5) incompressible flow

Solution:

Apply moment of momentum, using the CV and CS shown.

From example problem:

Steady flow,

$$\bar{V}_2 = -16\,\hat{j}\ \frac{m}{s}$$

$$A_1 = 0.01\ m^2$$

$$A_2 = 0.0025\ m^2$$

Basic equation:

$$\bar{r} \times \bar{F}_s + \int_{CV} \bar{r} \times \bar{g}\rho d\forall + \bar{T}_{shaft} = \frac{\partial}{\partial t}\int_{CV} \bar{r} \times \bar{V}\rho d\forall + \int_{CS} \bar{r} \times \bar{V}\rho\, \bar{V} \cdot d\bar{A}$$

Then we have:

$$\bar{M}_{flange} = \bar{r} \times \bar{F}_s)_{flange} = \bar{r}_1 \times \bar{V}_1\{-\rho V_1 A_1\} + \bar{r}_2 \times \bar{V}_2\{\rho V_2 A_2\}$$

Eq (1)

$$\bar{r}_1 = 0$$

$$\bar{r}_2 = a\hat{\imath} + b\hat{\jmath}$$

$$\bar{V}_2 = -V_2\hat{\jmath}$$

$$\bar{r}_2 \times \bar{V}_2 = -aV_2\hat{k} + 0$$

Substituting into equation (1)

$$\bar{M}_{flange} = -aV_2\hat{k}\{\rho V_2 A_2\} = -a\rho V_2^2 A_2\hat{k}$$

$$\bar{M}_{flange} = 0.3m \times 999\, \frac{kg}{m^3} \times (16)^2 \frac{m^2}{s^2} \times 0.0025\, m^2 \times \frac{N \cdot s^2}{kg \cdot m}\, (-\hat{k})$$

$$\bar{M}_{flange} = -192\, \hat{k}\, N \cdot m$$

This is the torque that must be exerted on the CV by the flange.

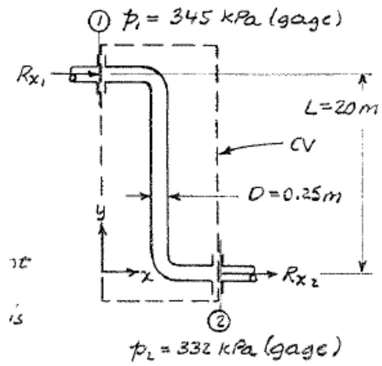
Since $\bar{M}_{flange}$ is in the $-\hat{k}$ direction, it must act CW in the x-y plane.

Problem 4.129

(Difficulty: 2)

4.129 Crude oil $SG = 0.95$ from a tanker dock flows through a pipe of $0.25\ m$ diameter in the configuration shown. The flow rate is $0.58\ \frac{m^3}{s}$, and the gage pressures are shown in the diagram. Determine the force and torque that are exerted by the pipe assembly on its supports.

$$Q = 0.58\ \frac{m^3}{s} \longrightarrow$$



Find: The force and torque exerted by the pipe for support.

Assumption: (1) $F_{Bx} = 0; \bar{g}$ acts in the δ direction

(2) steady flow

(3) uniform flow at each section

(4) no δ component of $\bar{r} \times \bar{g}$

(5) $\bar{T}_{shaft} = 0$

Solution:

No momentum components exist in the y direction. Apply x component of linear momentum and the moment of momentum equations using the CV shown. Location of coordinates is arbitrary. For simplicity, choose as shown.

Basic equation:

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

$$\bar{r} \times \bar{F}_s + \int_{CV} \bar{r} \times \bar{g}\rho d\forall + \bar{T}_{shaft} = \frac{\partial}{\partial t}\int_{CV} \bar{r} \times \bar{V}\rho d\forall + \int_{CS} \bar{r} \times \bar{V}\rho\, \bar{V} \cdot d\bar{A}$$

The area is:

$$A = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.25\ m)^2 = 0.049\ m^2$$

From momentum equation,

$$R_{x1} + R_{x2} + p_1 A - p_2 A = u_1\{-\dot{m}\} + u_2\{\dot{m}\} = 0$$

$$R_{x1} + R_{x2} = (p_2 - p_1)A$$

From momentum of momentum,

$$\bar{r}_1 \times (R_{x1} + p_1 A)\hat{\imath} = \bar{r}_1 \times V_1 \hat{\imath}\{-\dot{m}\}$$

$$\bar{r}_1 = L\hat{\jmath}$$

$$\bar{r}_1 \times \hat{\imath} = -L\hat{k}$$

$$-L(R_{x1} + p_1 A)\hat{k} = -LV_1(-\dot{m})\hat{k} = LV_1\dot{m}\hat{k} = L\frac{Q}{A}(\rho Q)\hat{k} = L\rho\frac{Q^2}{A}\hat{k}$$

$$R_{x1} = -\rho\frac{Q^2}{A} - p_1 A$$

$$R_{x1} = -0.95 \times 999\ \frac{kg}{m^3} \times (0.58)^2\frac{m^6}{s^2} \times \frac{1}{0.049\ m^2} \times \frac{N \cdot s^2}{kg \cdot m} - 3.45 \times 10^5\frac{N}{m^2} \times 0.049\ m^2$$

$$R_{x1} = -23.4\ kN$$

$$R_{x2} = (p_2 - p_1)A - R_{x1} = p_2 A - p_1 A + \rho\frac{Q^2}{A} + p_1 A = p_2 A + \rho\frac{Q^2}{A}$$

$$R_{x2} = 3.32 \times 10^5\frac{N}{m^2} \times 0.049\ m^2 + 0.95 \times 999\ \frac{kg}{m^3} \times (0.58)^2\frac{m^6}{s^2} \times \frac{1}{0.049\ m^2} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$R_{x2} = 22.8\ kN$$

$$\bar{r} \times \bar{F}_s = \bar{r}_1 \times R_{x1}\hat{\imath} = L\hat{\jmath} \times R_{x1}\hat{\imath} = -LR_{x1}\hat{k} = -20\ m \times (-46.0)\ kN\ \hat{k} = 468\ \hat{k}\ kN \cdot m$$

These are forces and torque on CV. The corresponding reactions are:

Force:

$$K_{x1} = -R_{x1} = 23.4\ kN$$

$$K_{x2} = -R_{x2} = -22.8\ kN$$

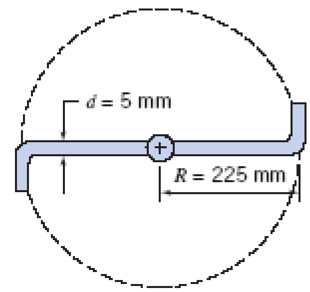
Torque:

$$\bar{M} = -\bar{r} \times \bar{F}_s = -468\ \hat{k}\ kN \cdot m$$

Problem 4.130

[Difficulty: 3]

4.130 The simplified lawn sprinkler shown rotates in the horizontal plane. At the center pivot, $Q = 15$ L/min of water enters vertically. Water discharges in the horizontal plane from each jet. If the pivot is frictionless, calculate the torque needed to keep the sprinkler from rotating. Neglecting the inertia of the sprinkler itself, calculate the angular acceleration that results when the torque is removed.



Given: Data on rotating spray system

Find: Torque required to hold stationary; steady-state speed

Solution:

Basic equation: Rotating CV

$$\vec{r} \times \vec{F}_s + \int_{CV} \vec{r} \times \vec{g}\,\rho\, dV + \vec{T}_{shaft} - \int_{CV} \vec{r} \times [2\vec{\omega} \times \vec{V}_{xyz} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \dot{\vec{\omega}} \times \vec{r}]\,\rho\, dV = \frac{\partial}{\partial t}\int_{CV} \vec{r} \times \vec{V}_{xyz}\,\rho\, dV + \int_{CS} \vec{r} \times \vec{V}_{xyz}\,\rho \vec{V}_{xyz} \cdot d\vec{A} \qquad (4.52)$$

Assumptions: 1) No surface force; 2) Body torques cancel; 3) Sprinkler stationary; 4) Steady flow; 5) Uniform flow; 6) L<<r

The given data is $Q = 15 \cdot \frac{L}{min}$ $\quad R = 225 \cdot mm$ $\quad d = 5 \cdot mm$ $\quad \rho = 999 \cdot \frac{kg}{m^3}$

For each branch $V = \frac{1}{2} \cdot \frac{Q}{\frac{\pi}{4} \cdot d^2}$ $\quad V = 6.37 \frac{m}{s}$

The basic equation reduces to a single scalar equation (FOR EACH BRANCH)

$$T_{shaft} - \int \vec{r} \times (\vec{\alpha} \times \vec{r}) \cdot \rho\, dV = \int \vec{r} \times \overrightarrow{V_{xyz}} \cdot \rho \cdot \overrightarrow{V_{xyz}}\, d\vec{A}$$

where α is the angular acceleration

But $\vec{r} \times (\vec{\alpha} \times \vec{r}) = r^2 \cdot \alpha$ (r and α perpendicular); the volume integral is $\int \vec{r} \times (\vec{\alpha} \times \vec{r}) \cdot \rho\, dV = \int r^2 \cdot \alpha \cdot \rho\, dV = \frac{R^3}{3} \cdot \alpha \cdot \frac{\pi}{4} \cdot d^2$

For the surface integral (FOR EACH BRANCH) $\int \vec{r} \times \overrightarrow{V_{xyz}} \cdot \rho \cdot \overrightarrow{V_{xyz}}\, d\vec{A} = R \cdot V \cdot \rho \cdot \frac{Q}{2}$

Combining $$T_{shaft} - \frac{R^3}{3} \cdot \alpha \cdot \rho \cdot \frac{\pi}{4} \cdot d^2 = R \cdot V \cdot \rho \cdot \frac{Q}{2} \qquad (1)$$

When the sprayer is at rest, $\alpha = 0$, so $T_{shaft} = R \cdot V \cdot \rho \cdot \frac{Q}{2}$ $\quad T_{shaft} = 0.179\ N \cdot m$

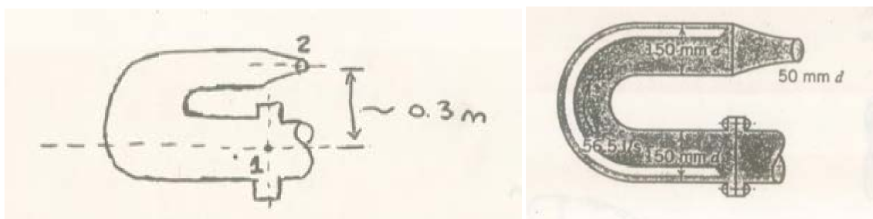
The total torque is then $T_{total} = 2 \cdot T_{shaft}$ $\quad T_{total} = 0.358\ N \cdot m$

When the device is released is released ($T_{shaft} = 0$ in Eq 1), we can solve for α $\quad \alpha = \frac{6 \cdot \rho \cdot Q \cdot V}{\rho \cdot \pi \cdot d^2 \cdot R^2}$ $\quad \alpha = 2.402 \times 10^3 \frac{1}{s^2}$

Problem 4.131

(Difficulty: 2)

4.131 For the configuration below calculate the torque about the pipe's centerline in the plane of the bolted flange that is caused by the flow through the nozzle. The nozzle center line is 0.3 m above the flange centerline. What is the effect of this torque on the force on the bolts? Neglect the effects of the weights of the pipe and the fluid in the pipe.



Given: All the parameters are shown in the figure.

Find: The effect of the torque.

Solution:

Basic equation:

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

$$M = R_x r$$

The flow rate is:

$$Q = 56.5\ \frac{L}{s} = 0.0565\ \frac{m^3}{s}$$

$$A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4} \times (0.05\ m)^2 = 0.00196\ m^2$$

$$V_2 = \frac{Q}{A_2} = \frac{0.0565\ \frac{m^3}{s}}{0.00196\ m^2} = 28.83\ \frac{m}{s}$$

The mass flow rate:

$$\dot{m} = \rho Q = 999\ \frac{kg}{m^3} \times 0.0565\ \frac{m^3}{s} = 56.4\ \frac{kg}{s}$$

From x-momentum equation:

$$R_x = V_2 \dot{m} = 28.83\ \frac{m}{s} \times 56.4\ \frac{kg}{s} = 1626\ N$$

The torque can be calculated by:

$$M = R_x r = 1626\ N \times 0.3\ m = 488\ N \cdot m$$

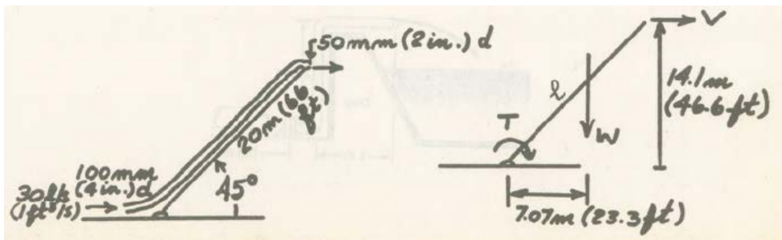
The direction is counter-clockwise.

The momentum increases the force on the upper bolts, the momentum decreases the force on the lower bolts, so the total force is unchanged.

Problem 4.132

(Difficulty: 2)

4.132 A fire truck is equipped with a 66 ft long extension ladder which is attached at a pivot and raised to an angle of 45°. A 4 in. diameter fire hose is laid up the ladder and a 2 in. diameter nozzle is attached to the top of the ladder so that the nozzle directs the stream horizontally into the window of a burning building. If the flow rate is 1 ft^3/s. Compute the torque exerted about the ladder pivot point. The ladder, hose and the water in the hose weigh about 10 lbf/ft.



Given: All the parameters are shown in the figure.

Find: Torque exerted about the ladder pivot point.

Solution:

Basic equations: Conservation of mass

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

Conservation of momentum in x-direction

$$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

The flow rate is:

$$Q = 1\ \frac{ft^3}{s}$$

$$A_N = \frac{\pi}{4} D_N^2 = \frac{\pi}{4} \times \left(\frac{2}{12}\ ft\right)^2 = 0.0218\ ft^2$$

The nozzle velocity is

$$V_N = \frac{Q}{A_N} = \frac{1\ \frac{ft^3}{s}}{0.0218\ ft^2} = 45.8\ \frac{ft}{s}$$

$$A_h = \frac{\pi}{4} D_h^2 = \frac{\pi}{4} \times \left(\frac{4}{12}\ ft\right)^2 = 0.00873\ ft^2$$

The mass flow rate:

$$\dot{m} = \rho Q = 1.94\ \frac{lbf \cdot s^2}{ft^4} \times 1\ \frac{ft^3}{s} = 1.94\ \frac{lbf \cdot s}{ft}$$

From x-momentum equation:

$$F_B = 10\ \frac{lbf}{ft} \times 66\ ft = 660\ lbf$$

$$RM + F_B \frac{L}{2} \cos 45° = V_N \dot{m} L \sin 45°$$

$$RM = V_N \dot{m} L \sin 45° - F_B \frac{L}{2} \cos 45°$$

$$RM = 45.8\ \frac{ft}{s} \times 1.94\ \frac{lbf \cdot s}{ft} \times 66\ ft \times \sin 45° - 660\ lbf \times 33\ ft \times \cos 45°$$

$$RM = -11250\ lbf \cdot ft$$

So the moment on the base by water is:

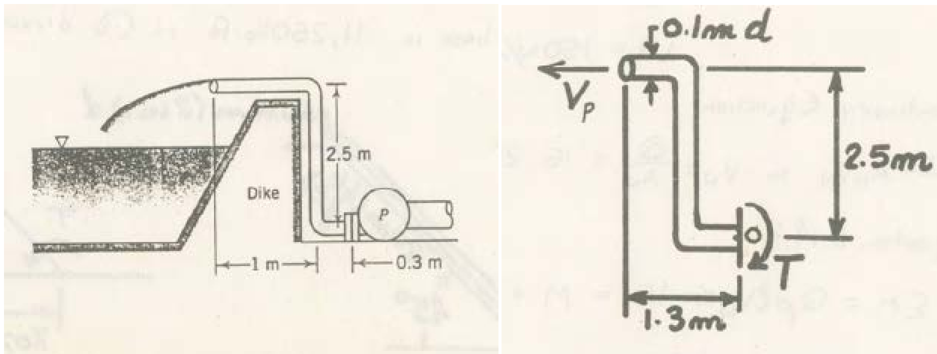
$$M = -RM = 11250\ lbf \cdot ft$$

The direction is clockwise.

Problem 4.133

(Difficulty: 2)

4.133 Calculate the torque exerted on the flange joint by the fluid flow as a function of the pump flow rate. Neglect the weight of the 100 mm diameter pipe and the fluid in the pipe.



Given: All the parameters are shown in the figure.

Find: Torque exerted on the flange.

Solution:

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

The area is:

$$A_p = \frac{\pi}{4} D_p^2 = \frac{\pi}{4} \times (0.1\ m)^2 = 0.0079\ m^2$$

$$V_p = \frac{Q}{A_p} = \frac{Q}{0.0079} \frac{m}{s}$$

The mass flow rate:

$$\dot{m} = \rho Q = 999\ Q\ \frac{kg}{s}$$

From x-momentum equation:

$$R_M = -V_p \dot{m} L$$

$$R_M = -\frac{Q}{0.0079}\frac{m}{s} \times 999\ Q\ \ \frac{kg}{s} \times 2.5\ m = -316000\ Q^2\ N \cdot m = -316Q^2\ kN \cdot m$$

So the torque on the flange is:

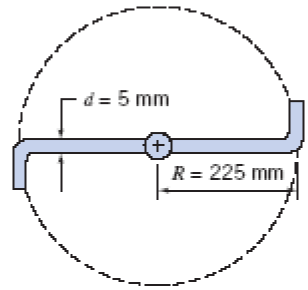
$$F_M = -R_M = 316Q^2\ \ kN \cdot m$$

The direction is clockwise.

Problem 4.134

[Difficulty: 3]

4.134 Consider the sprinkler of Problem 4.130 again. Derive a differential equation for the angular speed of the sprinkler as a function of time. Evaluate its steady-state speed of rotation if there is no friction in the pivot.



Given: Data on rotating spray system

Find: Differential equation for motion; steady speed

Solution:

Basic equation: Rotating CV

$$\vec{r}\times\vec{F}_s + \int_{CV}\vec{r}\times\vec{g}\,\rho\,dV + \vec{T}_{shaft} - \int_{CV}\vec{r}\times[2\vec{\omega}\times\vec{V}_{xyz} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + \dot{\vec{\omega}}\times\vec{r}]\,\rho\,dV = \frac{\partial}{\partial t}\int_{CV}\vec{r}\times\vec{V}_{xyz}\,\rho\,dV + \int_{CS}\vec{r}\times\vec{V}_{xyz}\,\rho\vec{V}_{xyz}\cdot d\vec{A} \qquad (4.52)$$

Assumptions: 1) No surface force; 2) Body torques cancel; 3) Steady flow; 5) Uniform flow; 6) L<<r

The given data is $Q = 15\cdot\frac{L}{min}$ $\quad R = 225\cdot mm$ $\quad d = 5\cdot mm$ $\quad \rho = 999\cdot\frac{kg}{m^3}$

For each branch $V = \frac{1}{2}\cdot\frac{Q}{\frac{\pi}{4}\cdot d^2}$ $\quad V = 6.37\frac{m}{s}$ $\quad A = \frac{\pi}{4}\cdot d^2$ $\quad A = 19.6\,mm^2$

The basic equation reduces to a single scalar equation (FOR EACH BRANCH)

$$-\int \vec{r}\times\left(2\cdot\vec{\omega}\times\vec{V}\times\vec{r} + \vec{\alpha}\times\vec{r}\right)\cdot\rho\,dV = \int \vec{r}\times\overrightarrow{V_{xyz}}\cdot\rho\cdot\overrightarrow{V_{xyz}}\,d\vec{A}$$

where α is the angular acceleration

But $\vec{r}\times\left(2\cdot\vec{\omega}\times\vec{V}\times\vec{r} + \vec{\alpha}\times\vec{r}\right) = 2\cdot\omega\cdot r\cdot V + \alpha\cdot r^2$ (r and α perpendicular)

The volume integral is then

$$-\int \vec{r}\times\left(2\cdot\vec{\omega}\times\vec{V}\times\vec{r} + \vec{\alpha}\times\vec{r}\right)\cdot\rho\,dV = -\left(\omega\cdot R^2\cdot V + \alpha\cdot\frac{R^3}{3}\right)\cdot\rho\cdot A$$

For the surface integral (FOR EACH BRANCH)

$$\int \vec{r}\times\overrightarrow{V_{xyz}}\cdot\rho\cdot\overrightarrow{V_{xyz}}\,d\vec{A} = R\cdot V\cdot\rho\cdot\frac{Q}{2}$$

Combining

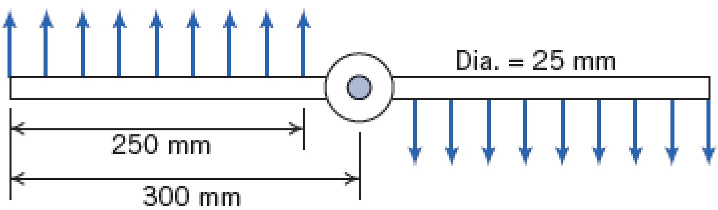
$$-\left(\omega\cdot R^2\cdot V + \alpha\cdot\frac{R^3}{3}\right)\cdot\rho\cdot A = R\cdot V\cdot\rho\cdot\frac{Q}{2} \quad \text{or} \quad \alpha = \frac{3}{A\cdot R^2}\cdot\left(-\omega\cdot V\cdot A\cdot R - \frac{Q\cdot V}{2}\right) \qquad (1)$$

The steady state speed (α = 0 in Eq 1) is then when

$$-\omega_{max}\cdot V\cdot A\cdot R - \frac{Q\cdot V}{2} = 0 \quad \text{or} \quad \omega_{max} = -\frac{Q}{2\cdot A\cdot R}$$

$$\omega_{max} = -28.3\frac{1}{s} \qquad \omega_{max} = -270\,rpm$$

4.135 Water flows out of the 2.5-mm slots of the rotating spray system, as shown. The velocity varies linearly from a maximum at the outer radius to zero at the inner radius. The flow rate is 3 L/s. Find (a) the torque required to hold the system stationary and (b) the steady-state speed of rotation after it is released.



Given: Data on rotating spray system

Find: Torque required to hold stationary; steady-state speed

Solution:

Governing equation: Rotating CV

$$\vec{r}\times\vec{F}_s+\int_{CV}\vec{r}\times\vec{g}\,\rho\,dV+\vec{T}_{shaft}-\int_{CV}\vec{r}\times[2\vec{\omega}\times\vec{V}_{xyz}+\vec{\omega}\times(\vec{\omega}\times\vec{r})+\dot{\vec{\omega}}\times\vec{r}]\,\rho\,dV=\frac{\partial}{\partial t}\int_{CV}\vec{r}\times\vec{V}_{xyz}\,\rho\,dV+\int_{CS}\vec{r}\times\vec{V}_{xyz}\,\rho\vec{V}_{xyz}\cdot d\vec{A} \qquad (4.52)$$

The given data is $\rho = 999\cdot\frac{kg}{m^3}$ $\quad\delta = 2.5\cdot mm$ $\quad r_o = 300\cdot mm$ $\quad r_i = (300-250)\cdot mm$ $\quad Q_{in} = 3\cdot\frac{L}{s}$

For no rotation ($\omega = 0$) this equation reduces to a single scalar equation

$$T_{shaft}=\int \vec{r}\times\overrightarrow{V_{xyz}}\cdot\rho\cdot\overrightarrow{V_{xyz}}\,d\vec{A} \qquad \text{or} \qquad T_{shaft}=2\cdot\delta\cdot\int_{r_i}^{r_o} r\cdot V\cdot\rho\cdot V\,dr$$

where V is the exit velocity with respect to the CV. We need to find $V(r)$. To do this we use mass conservation, and the fact that the distribution is linear

$$V(r)=V_{max}\cdot\frac{(r-r_i)}{(r_o-r_i)} \qquad \text{and} \qquad 2\cdot\frac{1}{2}\cdot V_{max}\cdot(r_o-r_i)\cdot\delta=Q_{in}$$

so
$$V(r)=\frac{Q_{in}}{\delta}\cdot\frac{(r-r_i)}{(r_o-r_i)^2}$$

Hence
$$T_{shaft}=2\cdot\rho\cdot\delta\cdot\int_{r_i}^{r_o} r\cdot V^2\,dr=2\cdot\frac{\rho\cdot Q_{in}^2}{\delta}\cdot\int_{r_i}^{r_o} r\cdot\left[\frac{(r-r_i)}{(r_o-r_i)^2}\right]^2 dr \qquad T_{shaft}=\frac{\rho\cdot Q_{in}^2\cdot(r_i+3\cdot r_o)}{6\cdot\delta\cdot(r_o-r_i)}$$

$$T_{shaft}=\frac{1}{6}\times\left(3\cdot\frac{L}{s}\times\frac{10^{-3}\cdot m^3}{L}\right)^2\times\frac{999\cdot kg}{m^3}\times\frac{1}{0.0025\cdot m}\times\frac{(0.05+3\cdot 0.3)}{(0.3-0.05)} \qquad T_{shaft}=2.28\cdot N\cdot m$$

For the steady rotation speed the equation becomes

$$-\int \vec{r}\times\left(2\cdot\vec{\omega}\times\overrightarrow{V_{xyz}}\right)\cdot\rho\, dV = \int \vec{r}\times\overrightarrow{V_{xyz}}\cdot\rho\cdot\overrightarrow{V_{xyz}}\, d\vec{A}$$

The volume integral term $-\int \vec{r}\times\left(2\cdot\vec{\omega}\times\overrightarrow{V_{xyz}}\right)\cdot\rho\, dV$ must be evaluated for the CV. The velocity in the CV varies with r. This variation can be found from mass conservation

For an infinitesmal CV of length dr and cross-section A at radial position r, if the flow in is Q, the flow out is $Q + dQ$, and the loss through the slot is $V\delta dr$. Hence mass conservation leads to

$$(Q + dQ) + V\cdot\delta\cdot dr - Q = 0 \qquad dQ = -V\cdot\delta\cdot dr \qquad Q(r) = Q_i - \delta\cdot\int_{r_i}^{r}\frac{Q_{in}}{\delta}\cdot\frac{(r - r_i)}{(r_o - r_i)^2}\, dr = Q_i - \int_{r_i}^{r} Q_{in}\cdot\frac{(r - r_i)}{(r_o - r_i)^2}\, dr$$

At the inlet ($r = r_i$) $\qquad Q = Q_i = \dfrac{Q_{in}}{2}$

Hence $\qquad Q(r) = \dfrac{Q_{in}}{2}\cdot\left[1 - \dfrac{(r - r_i)^2}{(r_o - r_i)^2}\right]$

and along each rotor the water speed is $\qquad v(r) = \dfrac{Q}{A} = \dfrac{Q_{in}}{2\cdot A}\cdot\left[1 - \dfrac{(r - r_i)^2}{(r_o - r_i)^2}\right]$

Hence the term $\int \vec{r}\times\left(2\cdot\vec{\omega}\times\overrightarrow{V_{xyz}}\right)\cdot\rho\, dV$ becomes $\quad 4\cdot\rho\cdot A\cdot\omega\cdot\left(\int_{r_i}^{r_o} r\cdot v(r)\, dr\right) = 4\cdot\rho\cdot\omega\cdot\int_{r_i}^{r_o}\frac{Q_{in}}{2}\cdot r\cdot\left[1 - \frac{(r - r_i)^2}{(r_o - r_i)^2}\right] dr$

or $\qquad 2\cdot\rho\cdot Q_{in}\cdot\omega\cdot\int_{r_i}^{r_o} r\cdot\left[1 - \dfrac{(r_o - r)^2}{(r_o - r_i)^2}\right] dr = \rho\cdot Q_{in}\cdot\omega\cdot\left(\dfrac{1}{6}\cdot r_o^2 + \dfrac{1}{3}\cdot r_i\cdot r_o - \dfrac{1}{2}\cdot r_i^2\right)$

Recall that $\qquad \int \vec{r}\times\overrightarrow{V_{xyz}}\cdot\rho\cdot\overrightarrow{V_{xyz}}\, d\vec{A} = \dfrac{\rho\cdot Q_{in}^2\cdot(r_i + 3\cdot r_o)}{6\cdot(r_o - r_i)\cdot\delta}$

Hence equation $\qquad -\int \vec{r}\times\left(2\cdot\vec{\omega}\times\overrightarrow{V_{xyz}}\right)\cdot\rho\, dV = \int \vec{r}\times\overrightarrow{V_{xyz}}\cdot\rho\cdot\overrightarrow{V_{xyz}}\, d\vec{A}$

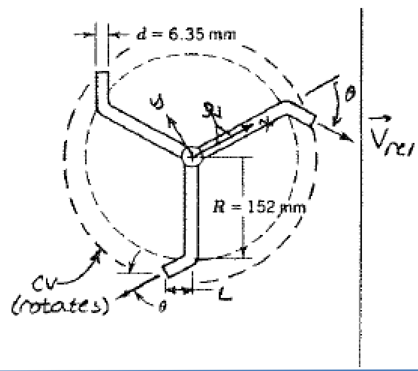
becomes $$\rho \cdot Q_{in} \cdot \omega \cdot \left(\frac{1}{6} \cdot r_o^{\,2} + \frac{1}{3} \cdot r_i \cdot r_o - \frac{1}{2} \cdot r_i^{\,2} \right) = \frac{\rho \cdot Q_{in}^{\,2} \cdot (r_i + 3 \cdot r_o)}{6 \cdot (r_o - r_i) \cdot \delta}$$

Solving for ω $$\omega = \frac{\rho \cdot Q_{in} \cdot (r_i + 3 \cdot r_o)}{\left(r_o^{\,2} + 2 \cdot r_i \cdot r_o - 3 \cdot r_i^{\,2}\right) \cdot (r_o - r_i) \cdot \rho \cdot \delta}$$ $\omega = 387 \cdot \text{rpm}$

Problem 4.136

(Difficulty: 2)

4.136 The lawn sprinkler shown is supplied with water at a rate of $68\ \frac{L}{min}$. Neglecting friction in the pivot, determine the steady-state angular speed for $\theta = 30°$. Plot the steady-state angular speed of the sprinkler for $0 \le \theta \le 90°$.



Find: The angular speed ω of the sprinkler.

Assumption: (1) $F_s = 0$

(2) Body torques cancel

(3) $\bar{T}_{shaft} = 0$

(4) neglect aerodynamic drag

(5) no $\hat{k}$ component of centripetal acceleration

(6) steady flow

(7) $L \ll R$

Solution:

Choose rotating CV. Apply angular momentum principle, Eq.4.53.

Basic equation:

$$\bar{r} \times \bar{F}_s + \int_{CV} \bar{r} \times \bar{g}\rho d\forall + \bar{T}_{shaft} - \int_{CV} \bar{r} \times \left[2\bar{\omega} \times \bar{V}_{xyz} + \bar{\omega} \times (\bar{\omega} \times \bar{r}) + \dot{\bar{\omega}} \times \bar{r}\right]\rho d\forall$$
$$= \frac{\partial}{\partial t}\int_{CV} \bar{r} \times \bar{V}_{xyz}\rho d\forall + \int_{CS} \bar{r} \times \bar{V}_{xyz}\rho\, \bar{V}_{xyz} \cdot d\bar{A}$$

Analyze one arm of sprinkler. From geometry, $\bar{r} = r\hat{\imath}$ in CV. $\bar{r} = R\hat{\imath}$ at jet.

Then

$$-\int_{CV} \bar{r} \times [2\bar{\omega} \times \bar{V}_{xyz}]\rho d\forall = R\hat{\imath} \times (-V \sin\theta \hat{\jmath})\, \rho \frac{Q}{3} = -\rho \frac{QRV}{3} \sin\theta \hat{k}$$

$$r\hat{\imath} \times \left(2\omega\hat{k} \times V\hat{\imath}\right) = 2\omega V r \hat{k}$$

$$-\int_{CV} = -\omega V R^2 \rho A \hat{k}$$

Dropping $\hat{k}$,

$$-\omega V R^2 \rho A = -\frac{\rho Q R V}{3} \sin\theta$$

So with

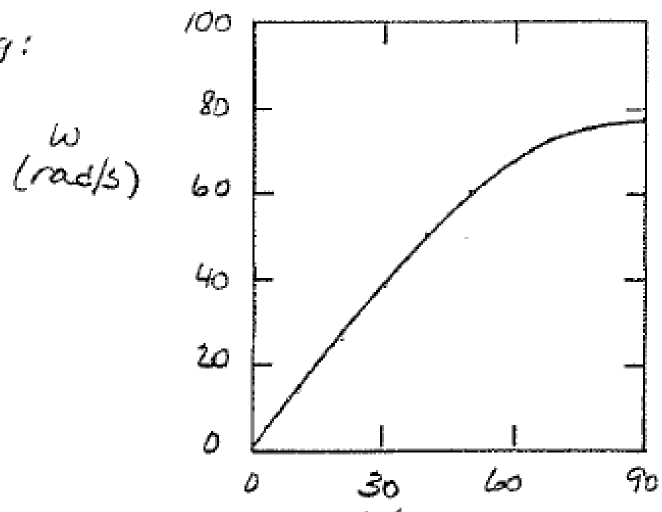
$$VA = \frac{Q}{3}$$

$$\omega = \frac{V}{R} \sin\theta$$

$$V = \frac{Q}{3A} = \frac{4Q}{3\pi d^2} = \frac{4}{3\pi} \times 68 \times 10^{-3} \frac{m^3}{min} \times \frac{1}{(0.00635)^2 m^2} \times \frac{min}{60\ s} = 11.9\ \frac{m}{s}$$

$$\omega = 11.9\ \frac{m}{s} \times \frac{1}{0.152\ m} \times \sin\theta = 78.3 \sin\theta\ \frac{rad}{s}$$

Plotting:



For $\theta = 30°$,

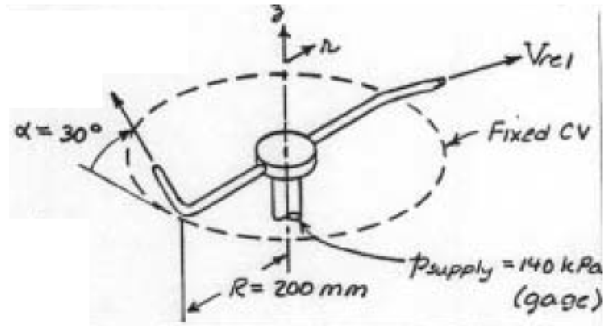
$\omega = 78.3 \sin 30°$

$\omega = 39.1$ rad/s

Problem 4.137

(Difficulty: 2)

4.137 A small lawn sprinkler is shown. The sprinkler operates at a gage pressure of $140\ kPa$. The total flow rate of water through the sprinkler is $\frac{4\ L}{min}$. Each jet discharges at $17\ \frac{m}{s}$ (relative to the sprinkler arm) in a direction inclined $30°$ above the horizontal. The sprinkler rotates about a vertical axis. Friction in the bearing causes a torque of $0.18\ N \cdot m$ opposing rotation. Evaluate the torque required to hold the sprinkler stationary.



Find: The torque required to hold the sprinkler stationary.

Assumption: (1) neglect torque due to surface forces

(2) torques due to body forces cancel by symmetry

(3) steady flow

(4) uniform flow leaving each jet

Solution:

Apply moment of momentum using fixed CV enclosing sprinkler arms.

Basic equation:

$$\bar{r} \times \bar{F}_s + \int_{CV} \bar{r} \times \bar{g} \rho d\forall + \bar{T}_{shaft} = \frac{\partial}{\partial t} \int_{CV} \bar{r} \times \bar{V} \rho d\forall + \int_{CS} \bar{r} \times \bar{V} \rho \bar{V} \cdot d\bar{A}$$

Then

$$-T_f \hat{k} = (\bar{r} \times \bar{V})_{in} \{-\rho Q\} + 2(\bar{r} \times \bar{V})_{jet} \left\{\frac{1}{2} \rho Q\right\}$$

$$(\bar{r} \times \bar{V})_{in} \approx 0$$

$$\bar{V} = (R\omega - V_{rel} \cos \alpha) \hat{\imath}_\theta + V_{rel} \sin \alpha\, \hat{\imath}_z$$

The absolute velocity of the jet leaving sprinkler is

$$\bar{V} = V_{rel}[\cos\alpha\,(-\hat{\imath}_\theta) + \sin\alpha\hat{\imath}_z]$$

Then

$$(\bar{r} \times \bar{V})_z = \{R\hat{\imath}_r \times V_{rel}[\cos\alpha(-\hat{\imath}_\theta) + \sin(\hat{\imath}_z)]\}_z = \{RV_{rel}\cos\alpha(-\hat{\imath}_z) + RV_{rel}\sin\alpha\,(-\hat{\imath}_\theta)\}_z$$

$$(\bar{r} \times \bar{V})_z = -RV_{rel}\cos\alpha$$

Substituting,

$$\bar{T}_{shaft} = T_{ext} - T_f = 2(-RV_{rel}\cos\alpha)\left(\frac{1}{2}\rho Q\right)$$

Thus

$$T_{ext} = T_f - \rho QRV_{rel}\cos\alpha$$

$$T_{ext} = 0.18\ N \cdot m - 999\ \frac{kg}{m^3} \times 4\frac{L}{min} \times 0.2\ m \times 17\ \frac{m}{s} \times 0.866 \times \frac{m^3}{1000\ L} \times \frac{min}{60\ s} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$T_{ext} = -0.0161\ N \cdot m$$

to hold sprinkler stationary.

Since $T_{ext} < 0$, it must be applied in the minus z direction to oppose motion.

Problem 4.138

[Difficulty: 4]

4.138 When a garden hose is used to fill a bucket, water in the bucket may develop a swirling motion. Why does this happen? How could the amount of swirl be calculated approximately?

Discussion: Frequently when filling a bucket the hose is held so that the water stream entering the bucket is not vertical. If, in addition, the water stream is off-center in the bucket, then flow entering the bucket has a tangential component of velocity, a swirl component.

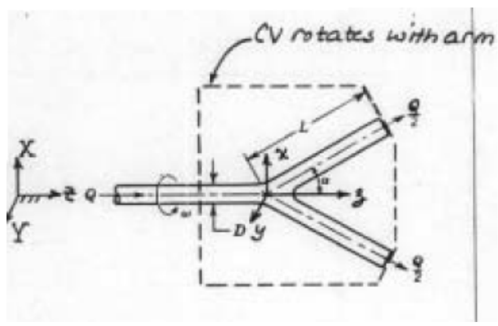
The tangential component of the water velocity entering the bucket has a moment-of-momentum (swirl) with respect to a control volume drawn around the stationary bucket. This entering swirl can only be reduced by a torque acting to oppose it. Viscous forces among fluid layers will tend to transfer swirl to other layers so that eventually all of the water in the bucket has a swirling motion.

Swirl in the bucket may be influenced by viscosity. The swirl may tend to nearly a rigid-body motion to minimize viscous forces between annular layers of water in the bucket. The rigid-body motion assumption may be a reasonable model to calculate the total angular momentum (moment-of-momentum) of the water in the bucket.

Problem 4.139

(Difficulty: 2)

4.139 A pipe branches symmetrically into two legs of length L, and the whole system rotates with angular speed ω around its axis of symmetry. Each branch is inclined at angle α to the axis of rotation. Liquid enters the pipe steadily, with zero angular momentum, at volume flow rate Q. The pipe diameter, D, is much smaller than L. Obtain an expression for the external torque required to turn the pipe. What additional torque would be required to impart angular acceleration $\dot{\omega}$?



Find: The torque required to hold the sprinkler stationary.

Assumption: (1) no surface forces

(2) body forces produce no torque about axis

(3) flow steady in the rotating frame

(4) $\bar{r}$ and $\bar{V}_{xyz}$ are: $\bar{r} \times \bar{V}_{xyz} = 0$.

Solution:

Apply moment of momentum equation using rotating CV.

Basic equation:

$$\bar{r} \times \bar{F}_s + \int_{CV} \bar{r} \times \bar{g}\rho d\forall + \bar{T}_{shaft} - \int_{CV} \bar{r} \times \left[2\bar{\omega} \times \bar{V}_{xyz} + \bar{\omega} \times (\bar{\omega} \times \bar{r}) + \dot{\bar{\omega}} \times \bar{r}\right] \rho d\forall$$
$$= \frac{\partial}{\partial t}\int_{CV} \bar{r} \times \bar{V}_{xyz}\rho d\forall + \int_{CS} \bar{r} \times \bar{V}_{xyz}\rho\, \bar{V}_{xyz} \cdot d\bar{A}$$

Then

$$\bar{T}_{shaft} = \int_{CV} \bar{r} \times \left[2\bar{\omega} \times \bar{V}_{xyz} + \bar{\omega} \times (\bar{\omega} \times \bar{r}) + \dot{\bar{\omega}} \times \bar{r}\right] \rho d\forall$$

Using the coordinates above:

$$\bar{\omega} = \omega\hat{k}$$

$$\dot{\bar{\omega}} = \dot{\omega}\hat{k}$$

$$\bar{r} = r\left(\cos\alpha\hat{k} + \sin\alpha\,\hat{\imath}\right)\ \text{(upper tube)}$$

$$\bar{V}_{xyz} = \frac{Q}{2A}\left(\cos\alpha\hat{k} + \sin\alpha\,\hat{\imath}\right)\ \text{(upper tube)}$$

$$A = \frac{\pi D^2}{4}$$

And

$$\dot{\bar{\omega}} \times \bar{r} = \dot{\omega} r \sin\alpha\,\hat{\jmath}$$

$$\bar{\omega} \times (\bar{\omega} \times \bar{r}) = \omega\hat{k} \times \omega r \sin\alpha\,\hat{\jmath} = -\omega^2 r \sin\alpha\,\hat{\imath}$$

$$2\bar{\omega} \times \bar{V}_{xyz} = 2\omega\frac{Q}{2A}\sin\alpha\hat{\jmath} = \frac{\omega Q}{A}\sin\alpha\,\hat{\jmath}$$

Thus for the upper tube,

$$\bar{T}_{shaft} = \int_0^L \left\{ r\left(\cos\alpha\hat{k} + \sin\alpha\,\hat{\imath}\right) \times \left[\left(\frac{\omega Q}{A} + \dot{\omega} r\right)\sin\alpha\,\hat{\jmath} - \omega^2 r \sin\alpha\,\hat{\imath}\right]\right\} \rho A dr$$

$$\bar{T}_{shaft} = \left[\left(\frac{L^2\omega Q}{2A} + \frac{\dot{\omega} L^3}{3}\right)\sin\alpha\cos\alpha\,\hat{\imath} + \left(\frac{L^2\omega Q}{2A} + \frac{\dot{\omega} L^3}{3}\right)\sin^2\alpha\hat{k} + \frac{\omega^2 L^3}{3}\sin\alpha\cos\alpha\,(-\hat{\jmath})\right]\rho A$$

For the lower tube:

$$\bar{\omega} = \omega\hat{k}$$

$$\dot{\bar{\omega}} = \dot{\omega}\hat{k}$$

$$\bar{r} = r\left(\cos\alpha\hat{k} - \sin\alpha\,\hat{\imath}\right)\ \text{(lower tube)}$$

$$\bar{V}_{xyz} = \frac{Q}{2A}\left(\cos\alpha\hat{k} - \sin\alpha\,\hat{\jmath}\right)\ \text{(lower tube)}$$

And

$$\dot{\bar{\omega}} \times \bar{r} = -\dot{\omega} r \sin\alpha\,\hat{\jmath}$$

$$\bar{\omega} \times (\bar{\omega} \times \bar{r}) = \omega\hat{k} \times (-\omega r \sin\alpha\,\hat{\jmath}) = \omega^2 r \sin\alpha\,\hat{\imath}$$

$$2\bar{\omega} \times \bar{V}_{xyz} = 2\omega\frac{Q}{2A}(-\sin\alpha)\hat{\jmath} = -\frac{\omega Q}{A}\sin\alpha\,\hat{\jmath}$$

So for lower tube:

$$\bar{T}_{shaft} = \int_0^L \left\{ r\left(\cos\alpha\hat{k} - \sin\alpha\,\hat{\imath}\right) \times \left[\left(\frac{\omega Q}{A} + \dot{\omega} r\right)\sin\alpha\,(-\hat{\jmath}) + \omega^2 r \sin\alpha\,\hat{\imath}\right]\right\} \rho A dr$$

$$\bar{T}_{shaft} = \left[\left(\frac{L^2\omega Q}{2A} + \frac{\dot{\omega}L^3}{3}\right)\sin\alpha\cos\alpha\,\hat{\imath} + \left(\frac{L^2\omega Q}{2A} + \frac{\dot{\omega}L^3}{3}\right)\sin^2\alpha\hat{k} + \frac{\omega^2 L^3}{3}\sin\alpha\cos\alpha\,(\hat{\jmath})\right]\rho A$$

Summing these expressions gives:

$$\bar{T}_{shaft} = \left(\frac{L^2\omega Q}{A} + \frac{2\dot{\omega}L^3}{3}\right)\sin^2\alpha\rho A\hat{k}$$

The steady state portion of the torque is:

$$\bar{T}_{shaft} = \frac{L^2\omega Q}{A}\sin^2\alpha\rho A\hat{k} = L^2\rho\omega Q\sin^2\alpha\hat{k}$$

The additional torque need to provide angular acceleration $\dot{\omega}$ is:

$$\bar{T}_{shaft} = \frac{2\dot{\omega}\rho L^3 A}{3}\sin^2\alpha\hat{k}$$

Torques of individual tubes about the x and y axis are reacted internally. They must be considered in design of the tube.

(b) Using fixed CV:

Assumption: (1) no surface forces

(2) body forces symmetric

(3) no change in angular momentum within CV

(4) symmetric in two branches

(5) uniform flow at each cross-section

$$\bar{T}_s = \bar{r}_1 \times \bar{V}_1\{-\rho Q\} + \bar{r}_2 \times \bar{V}_2\left\{\rho\frac{Q}{2}\right\} + \bar{r}_3 \times \bar{V}_3\left\{\rho\frac{Q}{2}\right\} = 2\bar{r}_2 \times \bar{V}_2\left\{\rho\frac{Q}{2}\right\}$$

$$\bar{r}_1 = 0$$

$$\bar{r}_2 = L\sin\alpha\,\hat{\jmath}$$

$$\bar{V}_2 = \omega r_2\hat{k}$$

$$\bar{r}_2 \times \bar{V}_2 = \omega L^2\sin^2\alpha\,\hat{\imath}$$

Or

$$T_s = \rho\omega Q L^2\sin^2\alpha$$

steady state torque.

The torque required for acceleration is:

$$T_{acc} = I\dot{\omega}$$

Where

$$I = \int r^2 dm$$

For one leg of the branch,

$$I = \int r^2 dm = \int_0^L (s \sin\alpha)^2 \rho A ds = \frac{\rho A L^3}{3} \sin^2\alpha$$

(b) Neglect mass of pipe

For both sides,

$$I = \frac{2\rho A L^3}{3} \sin^2\alpha$$

Thus

$$T_{acc} = \frac{2\rho\dot{\omega} A L^3}{3} \sin^2\alpha$$

Torque required for angular acceleration.

The total torque that must be applied is:

$$T = T_s + T_{acc} = \rho\omega Q L^2 \sin^2\alpha + \frac{2\rho\dot{\omega} A L^3}{3} \sin^2\alpha$$

Problem 4.140

(Difficulty: 2)

4.140 For the rotating sprinkler of Example 4.14, what value of α will produce the maximum rotational speed? What angle will provide the maximum area of coverage by the spray? Draw a velocity diagram (using an r, θ, z coordinate system) to indicate the absolute velocity of the water jet leaving the nozzle. What governs the steady rotational speed of the sprinkler? Does the rotational speed of the sprinkler affect the area covered by the spray? How would you estimate the area? For fixed α, what might be done to increase or decrease the area covered by the spray?

Solution:

The results of Example Problem 4.14 were computed assuming steady flow of water and constant frictional retarding torque at the sprinkler pivot.

$$T_f = R(V_{rel}\cos\alpha - \omega R)\rho Q$$

From these results,

$$\omega = \frac{V_{rel}\cos\alpha}{R} - \frac{T_f}{\rho Q R^2}$$

Thus rotational speed of the sprinkler increases as $\cos\alpha$ increases, i.e. as α decreases. The maximum rotational speed occurs when $\alpha = 0$. Then $\cos\alpha = 1$ and the rotational speed is

$$\omega = \frac{V_{rel}}{R} - \frac{T_f}{\rho Q R^2}$$

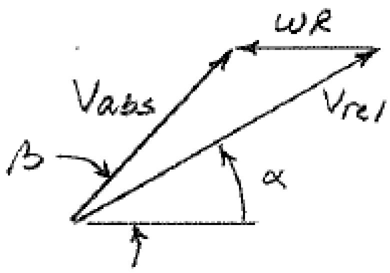
For the conditions of Example Problem 4.14 the maximum rotational speed is

$$\omega = 4.97\ \frac{m}{s} \times \frac{1}{0.150\ m} - 0.0718\ N\cdot m \times \frac{m^3}{999\ kg} \times \frac{min}{7.5\ L} \times \frac{1}{(0.150\ m)^2} \times 1000\frac{L}{m^3} \times 60\frac{s}{min}$$

$$\omega = 7.58\ \frac{rad}{s}$$

The steady rotation speed ω of the sprinkler is governed by torque T_f and angle α.

Maximum coverage by the spray occurs when the "carry" of each jet stream is the longest. When aerodynamic drag on the stream is neglected, maximum carry occurs when the absolute velocity of the stream leaves the sprinkler at $\beta = 45°$, as shown in the velocity diagram below.



Note:

$$\vec{V}_{abs} = \vec{V}_{rel} - \omega R \hat{\imath}_{\theta}$$

Both the magnitude and direction of $\vec{V}_{abs}$ vary with ω.

For $\omega = 0$, the relative velocity angle α and absolute velocity angle β are equal. Therefore maximum carry occurs when $\alpha = 45°$ (see graph on next page).

Any rotation rate ω reduces the magnitude V_{abs} and increases the angle β of the absolute velocity leaving the sprinkler jet. When $\omega > 0$, then $\beta > \alpha$, so for maximum carry α must be less than 45°. Consequently rotation reduces the carry of the stream and the area of coverage; at specified α the area of coverage decreases with increasing ω.

For the conditions of Example Problem 4.14 ($\omega = 30\ rpm$), optimum carry occurs at $\alpha = 42°$, and the coverage area is reduced from approximately $20\ m^2$ with a fixed sprinkler to $15\ m^2$ with $30\ rpm$ rotation. If the rotation speed is increased (by decreasing pivot friction or decreasing nozzle angle α), coverage area may be reduced still further, to 9 m^2 or less.

$$A \approx \pi (x_{max})^2$$

Analysis of Ground Area Covered by Rotating Lawn Sprinkler:

Variables:

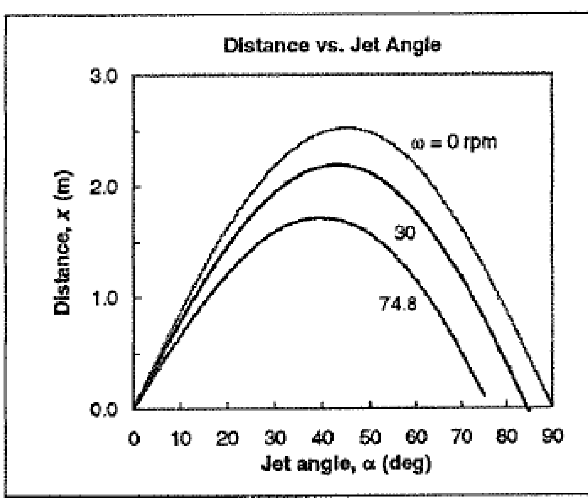
A = ground area covered by spray stream
x = ground distance reached by spray stream
α = angle of jet above ground plane
β = angle of absolute velocity above ground plane

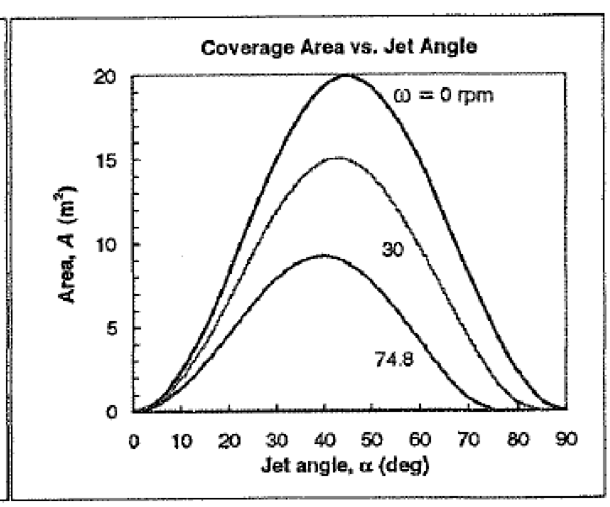
Input Data:

R = 0.150 m
V_{rel} = 4.97 m/s (Q = 7.5 L/min)

Results:

	ω (rpm) =	0		30		74.8
	ωR (m/s) =	0		0.471		1.17
α (deg)	x_{max} (m)	A (m^2)	x_{max} (m)	A (m^2)	x_{max} (m)	A (m^2)
0	0.00	0.00	0.00	0.00	0.00	0.00
5	0.437	0.601	0.396	0.492	0.333	0.349
10	0.861	2.33	0.778	1.90	0.654	1.35
15	1.26	4.98	1.14	4.05	0.951	2.84
20	1.62	8.23	1.46	6.65	1.21	4.61
25	1.93	11.7	1.73	9.37	1.43	6.39
30	2.18	14.9	1.94	11.8	1.59	7.90
35	2.37	17.6	2.09	13.8	1.68	8.90
40	2.48	19.3	2.17	14.8	1.71	9.23
45	2.52	19.9	2.18	14.9	1.68	8.83
50	2.48	19.3	2.11	14.0	1.57	7.72
55	2.37	17.6	1.97	12.3	1.39	6.08
60	2.18	14.9	1.77	9.81	1.15	4.15
65	1.93	11.7	1.50	7.03	0.850	2.269
70	1.62	8.23	1.17	4.30	0.500	0.785
75	1.26	4.98	0.798	2.00	0.109	0.037
78	1.02	3.30	0.557	0.975		
80	0.861	2.33	0.391	0.480		
85	0.437	0.601	-0.04	0.00		
90	0.00	0.00				





Problem 4.141

[Difficulty: 3]

4.141 Compressed air is stored in a pressure bottle with a volume of 100 L, at 500 kPa and 20°C. At a certain instant, a valve is opened and mass flows from the bottle at $\dot{m} = 0.01$ kg/s. Find the rate of change of temperature in the bottle at this instant

Given: Compressed air bottle

Find: Rate of temperature change

Solution:

Basic equations: Continuity; First Law of Thermodynamics for a CV

$$\frac{\partial}{\partial t}\int_{CV} \rho \, dV + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \qquad \dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\,\rho \, dV + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho \vec{V} \cdot d\vec{A}$$

Assumptions: 1) Adiabatic 2) No work 3) Neglect KE 4) Uniform properties at exit 5) Ideal gas

Given data $\quad p = 500 \cdot kPa \qquad T = 20\,°C \qquad T = 293K \qquad V = 100 \cdot L \qquad m_{exit} = 0.01 \cdot \frac{kg}{s}$

Also $\quad R_{air} = \frac{286.9\,N\cdot m}{kg\cdot K} \qquad c_v = 717.4 \cdot \frac{N\cdot m}{kg\cdot K}$

From continuity $\quad \frac{\partial}{\partial t}M_{CV} + m_{exit} = 0 \qquad$ where m_{exit} is the mass flow rate at the exit (Note: Software does not allow a dot!)

$$\frac{\partial}{\partial t}M_{CV} = -m_{exit}$$

From the 1st law $\quad 0 = \frac{\partial}{\partial t}\int u\,dM + \left(u + \frac{p}{\rho}\right)\cdot m_{exit} = u\cdot\left(\frac{\partial}{\partial t}M\right) + M\cdot\left(\frac{\partial}{\partial t}u\right) + \left(u + \frac{p}{\rho}\right)\cdot m_{exit}$

Hence $\quad u\cdot\left(-m_{exit}\right) + M\cdot c_v\cdot\frac{dT}{dt} + u\cdot m_{exit} + \frac{p}{\rho}\cdot m_{exit} = 0 \qquad \frac{dT}{dt} = -\frac{m_{exit}\cdot p}{M\cdot c_v\cdot\rho}$

But $\quad M = \rho\cdot V \qquad$ (where V is volume) so $\qquad \frac{dT}{dt} = -\frac{m_{exit}\cdot p}{V\cdot c_v\cdot\rho^2}$

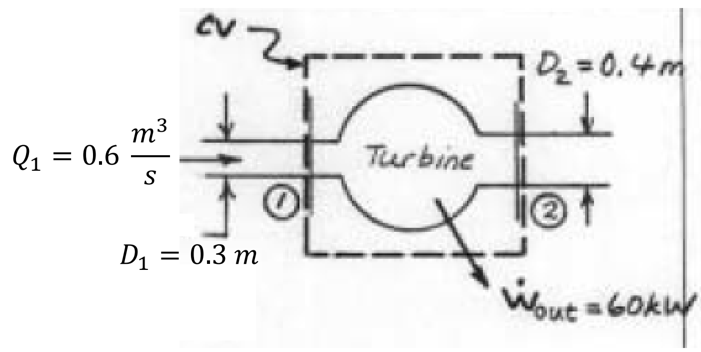
For air $\quad \rho = \frac{p}{R_{air}\cdot T} \qquad \rho = 500\times 10^3\cdot\frac{N}{m^2}\times\frac{kg\cdot K}{286.9\cdot N\cdot m}\times\frac{1}{(20+273)\cdot K} \qquad \rho = 5.95\,\frac{kg}{m^3}$

Hence $\quad \frac{dT}{dt} = -0.01\cdot\frac{kg}{s}\times 500\times 10^3\cdot\frac{N}{m^2}\times\frac{1}{100\cdot L}\times\frac{L}{10^{-3}\cdot m^3}\times\frac{kg\cdot K}{717.4\cdot N\cdot m}\times\left(\frac{m^3}{5.95\cdot kg}\right)^2 = -1.97\cdot\frac{K}{s} = -1.97\cdot\frac{C}{s}$

Problem 4.142

(Difficulty: 2)

4.142 A turbine is supplied with $0.6\ \frac{m^3}{s}$ of water from a $0.3-m-diameter$ pipe; the discharge pipe has a $0.4\ m$ diameter. Determine the pressure drop across the turbine if it delivers $60\ kw$.



Find: The pressure drop $p_1 - p_2$.

Assumption: (1) steady flow

(2) uniform flow at each section

(3) incompressible flow

(4) $\dot{Q} = 0$

(5) $\dot{W}_{shear} = 0$

(6) neglect Δu

(7) neglect $\Delta \delta$

Solution:

Choose rotating CV. Apply angular momentum principle, Eq.4.53.

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + \frac{V^2}{2} + gz + pv\right)\rho \bar{V} \cdot d\bar{A}$$

Then

$$0 = \{-|\rho V_1 A_1|\} + \{|\rho V_2 A_2|\}$$

or

$$V_2 = V_1 \frac{A_1}{A_2} = V_1 \left(\frac{D_1}{D_2}\right)^2$$

and

$$-\dot{W}_s = \left(\frac{V_1^2}{2} + p_1 v\right)\{-|\rho V_1 A_1|\} + \left(\frac{V_2^2}{2} + p_2 v\right)\{|\rho V_2 A_2|\}$$

$$-\dot{W}_s = -\left[\frac{V_1^2 - V_2^2}{2} + (p_1 - p_2)v\right]\rho Q = -\left\{\frac{V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right] + (p_1 - p_2)v\right\}\rho Q$$

or

$$p_1 - p_2 = \frac{1}{v}\left\{\frac{\dot{W}_s}{\rho Q} - \frac{V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right]\right\} = \frac{\dot{W}_s}{Q} - \frac{\rho V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right]$$

But

$$V_1 = \frac{Q}{A_1} = 0.6\ \frac{m^3}{s} \times \frac{4}{\pi} \times \frac{1}{(0.3\ m)^2} = 8.49\ \frac{m}{s}$$

$$\dot{W}_s = \dot{W}_{out} = 60\ kw$$

So we have:

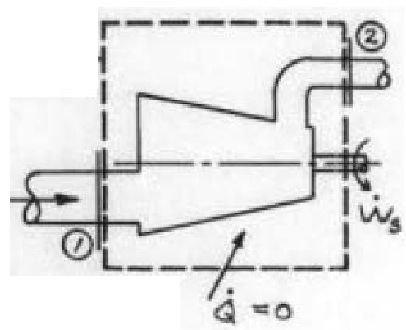
$$p_1 - p_2 = 60 \times 10^3 \frac{N \cdot m}{s} \times \frac{s}{0.6\ m^3} - \frac{1}{2} \times 999\ \frac{kg}{m^3} \times \left(8.49\ \frac{m}{s}\right)^2 \times \left[1 - \left(\frac{0.3\ m}{0.4\ m}\right)^4\right]\frac{N \cdot s^2}{kg \cdot m}$$

$$p_1 - p_2 = 75.4\ kPa$$

Problem 4.143

(Difficulty: 2)

4.143 Air is drawn from atmosphere into a turbo-machine. At the exit, conditions are $500\ kPa$ (gage) and $130°C$. The exit speed is $100\ \frac{m}{s}$ and the mass flow rate is $0.8\ \frac{kg}{s}$. Flow is steady and there is no heat transfer. Compute the shaft work interaction with the surroundings.



Find: The shaft work $\dot{W}_s$ interaction with the surroundings.

Assumption: (1) ideal gas, constant specific heat

(2) $\dot{w}_{shear} = 0$ by choice of CV. $\dot{w}_{other} = 0$

(3) steady flow

(4) uniform flow at each section

(5) neglect $\Delta\delta$

(6) $V_1 = 0$

(7) $\dot{Q} = 0$

Solution:

Apply energy equation, using CV shown.

Basic equation:

$$p = \rho RT$$

$$\Delta h = c_p \Delta T$$

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + \frac{V^2}{2} + gz + pv\right)\rho \bar{V} \cdot d\bar{A}$$

By definition,

$$h = u + pv$$

so we have:

$$-\dot{W}_s = \left(h_1 + \frac{V_1^2}{2}\right)\{-|\dot{m}|\} + \left(h_2 + \frac{V_2^2}{2}\right)\{|\dot{m}|\} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2}{2}\right)$$

Or

$$-\dot{W}_s = \dot{m}\left(h_2 - h_1 + \frac{V_2^2}{2}\right) = \dot{m}\left[c_p(T_2 - T_1) + \frac{V_2^2}{2}\right]$$

$$-\dot{W}_s = 0.8\frac{kg}{s} \times \left[1.0\ \frac{kJ}{kg \cdot K} \times (405 - 288)\ K + \frac{1}{2} \times \left(100\ \frac{m}{s}\right)^2\right] = 96.0\ kW$$

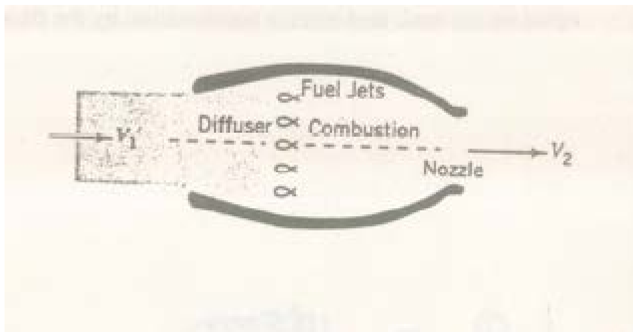
$$\dot{W}_s = -96.0\ kW$$

Power is into CV because $\dot{W}_s < 0$.

Problem 4.144

(Difficulty: 2)

4.144 At high speeds the compressor and turbine of the jet engine may be eliminated entirely. The result is called a ramjet (a subsonic configuration is shown). Here the incoming air is slowed and the pressure increases; the air is heated in the widest part by the burning of injected fuel. The heated air exhausts at high velocity from the converging nozzle. What nozzle area A_2 is needed to deliver a $90\ kN$ thrust at an air speed of $270\ \frac{m}{s}$ if the exhaust velocity is the sonic velocity for the heated air, which is at $1000\ K$. Assume that the jet operates at an altitude of $12\ km$ and neglect the fuel mass and pressure differentials.



Given: All the parameters are shown in the figure.

Find: Nozzle area A_2.

Solution:

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \bar{V} \cdot d\bar{A}$$

The parameters are:

$$V_1 = 270\ \frac{m}{s}$$

$$F_p = 90\ kN$$

$$p = 19.4\ kPa$$

$$T = 1000\ K$$

The mass flow rate:

$$\dot{m} = \rho_2 V_2 A_2$$

From x-momentum equation:

$$F_p = \dot{m}(V_2 - V_1) = \rho_2 V_2 A_2 (V_2 - V_1)$$

$$A_2 = \frac{F_p}{\rho_2 V_2 (V_2 - V_1)}$$

$$V_2 = (kRT)^{\frac{1}{2}} = 633.7\ \frac{m}{s}$$

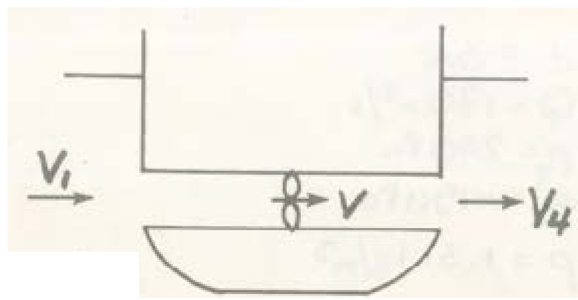
$$\rho_2 = \frac{p}{RT} = 0.0676\ \frac{kg}{m^3}$$

$$A_2 = \frac{90 \times 10^3\ N}{0.0676\ \frac{kg}{m^3} \times 633.7\ \frac{m}{s} \times \left(633.7\ \frac{m}{s} - 270\ \frac{m}{s}\right)} = 5.78\ m^2$$

Problem 4.145

(Difficulty: 2)

4.145 Transverse thrusters are used to make large ships fully maneuverable at low speeds without tugboat assistance. A transverse thruster consists of a propeller mounted in a duct; the unit is then mounted below the waterline in the bow or stern of the ship. The duct runs completely across the ship. Calculate the thrust developed by a 1865 kW unit (supplied to the propeller) if the duct is 2.8 m in diameter and the ship is stationary.



Given: All the parameters are shown in the figure.

Find: The thrust developed.

Solution:

$$P_{unit} = 1865\ kw$$

$$A = \frac{\pi}{4} D^2$$

$$P_{unit} = \frac{AV\rho}{2}(V_4^2 - V_1^2)$$

Since

$$V_1 = 0$$

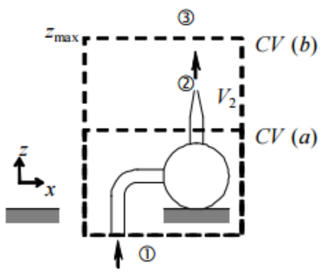
$$V_4 = \sqrt[3]{\frac{2P_{unit}}{A\rho}} = \sqrt[3]{\frac{2 \times 1865 \times 1000\ w}{\frac{\pi}{4} \times (2.8\ m)^2 \times 999\ \frac{kg}{m^3}}} = 8.46\ \frac{m}{s}$$

$$F = \frac{P_{unit}}{V} = \frac{1865\ kw}{8.46\ \frac{m}{s}} = 220000\ N = 220\ kN$$

Problem 4.146

(Difficulty: 3)

4.146 All major harbors are equipped with fire boats for extinguishing ship fires. A 75-mm-diameter hose is attached to the discharge of a 11 kW pump on such a boat. The nozzle attached to the end of hose has a diameter of 25 mm. If the nozzle discharge is held 3 m above the surface of the water, determine the volume flow rate through the nozzle, the maximum height to which the water will rise, and the force on the boat if the water jet is directed horizontally over the stern.



Given: Data on fire boat hose system

Find: Volume flow rate of the nozzle; Maximum water height; Force on boat.

Assumptions: (1) neglect losses

(2) no work

(3) neglect KE at 1

(4) uniform properties at exit

(5) incompressible

(6) p_{atm} at 1 and 2

Solution:

Basic equation:

First law of thermodynamics for a CV:

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

For CV (a):

$$-W_s = \left(\frac{V_2^2}{2} + gz_2\right)m_{exit}$$

$$m_{exit} = \rho V_2 A_2$$

Hence we have:

$$\left(\frac{V_2^2}{2} + gz_2\right)\rho V_2 A_2 = -W_s$$

To solve this we could ignore the gravity term, solve for velocity, and then check that the gravity term is in fact minor. Alternatively we could manually iterate, or use a calculator or excel , to solve. The answer is:

$$V_2 = 35.5\ \frac{m}{s}$$

Hence the volume flow rate is:

$$Q = V_2 A_2 = 35.5\ \frac{m}{s} \times \frac{\pi \times (0.025\ m)^2}{4} = 0.0174\ \frac{m^3}{s}$$

To find the z_{max}, use the first law again to CV (b) to get:

$$-W_s = g z_{max} m_{exit}$$

$$z_{max} = -\frac{W_s}{m_{exit} g} = -\frac{W_s}{\rho Q g}$$

$$z_{max} = \frac{11000\ w}{999\ \frac{kg}{m^3} \times 0.0174\ \frac{m^3}{s} \times 9.81\ \frac{m}{s^2}} = 64.5\ m$$

For the force in the x direction when jet is horizontal we need x momentum equation:

$$R_x + F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho \vec{V} \cdot d\vec{A}$$

Then

$$R_x = u_1(-\rho_1 V_1 A_1) + u_2(\rho_2 V_2 A_2) = V_2 \rho Q$$

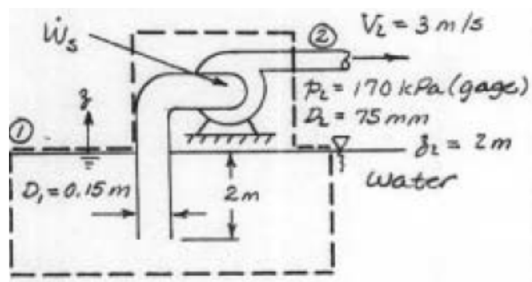
$$R_x = 999\ \frac{kg}{m^3} \times 35.5\ \frac{m}{s} \times 0.0174\ \frac{m^3}{s} = 617\ N$$

This is the force on the CV and the force on the boat is to the left.

Problem 4.147

(Difficulty: 2)

4.147 A pump draws water from a reservoir through a reservoir through a $150-mm-diameter$ suction pipe and delivers it to a $75-mm-diameter$ discharge pipe. The end of the suction pipe is $2\ m$ below the free surface of the reservoir. The pressure gage on the discharge pipe ($2\ m$ above the reservoir surface) reads $170\ kPa$. The average speed in the discharge pipe is $3\ \frac{m}{s}$. If the pump efficiency is 75 percent, determine the power required to drive it.



Find: The power $\dot{W}_{s,actual}$ required to drive it.

Assumption: (1) $\dot{W}_{shear} = \dot{W}_{other} = 0$

(2) steady flow

(3) $V_1 \cong= 0$

(4) $\delta_1 = 0$

(5) $p_1 = 0$ (gage)

(6) uniform flow at each section

(7) incompressible flow; $V_1 A_1 = V_2 A_2$

Solution:

Apply the first law of CV shown. Noting that flow enters with negligible velocity at section 1

Basic equation:

$$\dot{Q} - \dot{W}_s = \left(u_1 + \frac{V_1^2}{2} + g z_1 + \frac{p_1}{\rho}\right)\{-\dot{m}\} + \left(u_2 + \frac{V_2^2}{2} + g z_2 + \frac{p_2}{\rho}\right)\{\dot{m}\}$$

or

$$-\dot{W}_s = \dot{m}\left[\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 + \left(u_2 - u_1 - \frac{\delta Q}{dm}\right)\right]$$

Obtain the ideal or minimum power input by neglecting terminal effects, thus

$$-\dot{W}_{s,ideal} = \dot{m}\left[\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2\right]$$

For the system,

$$\dot{m} = \rho V_2 A_2 = 999\ \frac{kg}{m^3} \times 3\frac{m}{s} \times \frac{\pi}{4} \times (0.075\ m)^2 = 13.2\ \frac{kg}{s}$$

and

$$-\dot{W}_{s,ideal} = 13.2\ \frac{kg}{s} \times \left[1.70 \times 10^8 \frac{N}{m^2} \times \frac{m^3}{999\ kg} + \frac{1}{2} \times \left(3\ \frac{m}{s}\right)^2 + 9.81\ \frac{m}{s^2} \times 2m\right]$$

$$\dot{W}_{s,ideal} = -2.56\ kW$$

Finally

$$\dot{W}_{s,actual} = \frac{\dot{W}_{s,ideal}}{\eta} = \frac{-2.56\ kW}{0.75} = -3.41\ kW$$

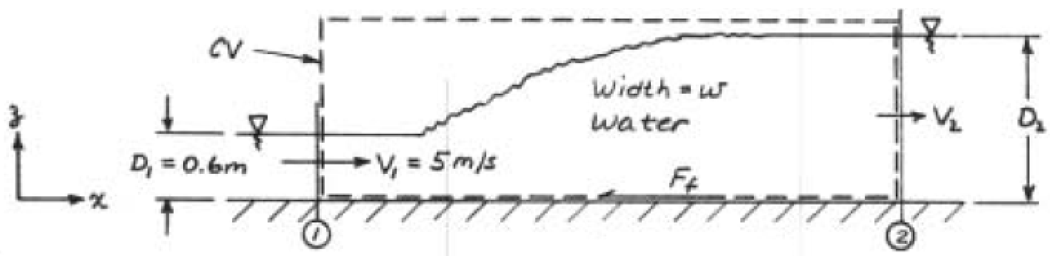
Problem 4.148

(Difficulty: 2)

4.148 Liquid flowing at high speed in a wide, horizontal open channel under some conditions can undergo a hydraulic jump, as shown. For a suitable chosen control volume, the flows entering and leaving the jump may be considered uniform with hydrostatic pressure distributions (see Example 4.7). Consider a channel of width w, with water flow at $D_1 = 6\ mm$ and $V_1 = 5\ \frac{m}{s}$. Show that in general,

$$D_2 = \frac{1}{2} D_1 \left[\sqrt{1 + \frac{8V_1^2}{gD_1}} - 1 \right]$$

Evaluate the change in mechanical energy through the hydraulic jump. If heat transfer to the surroundings is negligible, determine the change in water temperature through the jump.



Find: The change in mechanical energy and water temperature through the pump.

Assumption: (1) steady flow

(2) incompressible flow

(3) uniform flow at each section

(4) hydrostatic pressure distribution at section ① and ②, so $p = \rho g (D - z)$

(5) neglect friction force, F_f, on CV.

(6) $\dot{Q} = 0$

(7) $\dot{W}_s = \dot{W}_{shear} = \dot{W}_{other} = 0$

(8) $F_{Bx} = 0$, since channel is horizontal.

Solution:

Apply continuity, x component of momentum, and energy equations using CV shown.

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \bar{V} \cdot d\bar{A}$$

$$F_{sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} V_x \rho d\forall + \int_{CS} V_x \rho \bar{V} \cdot d\bar{A}$$

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\rho d\forall + \int_{CS} (e + pv)\rho \bar{V} \cdot d\bar{A}$$

$$e = u + \frac{V^2}{2} + gz$$

From continuity,

$$0 = \{-|\rho V_1 A_1|\} + \{|\rho V_2 A_2|\} = -\rho V_1 w D_1 + \rho V_2 w D_2$$

$$V_1 D_1 = V_2 D_2$$

From momentum,

$$F_{sx} = \rho g \frac{D_1}{2} w D_1 - \rho g \frac{D_2}{2} w D_2 = V_{x1}\{-|\rho V_1 w D_1|\} + V_{x2}\{-|\rho V_2 w D_2|\}$$

$$V_{x1} = V_1$$

$$V_{x2} = V_2$$

or

$$\frac{g}{2}(D_1^2 - D_2^2) = V_1 D_1 (V_2 - V_1) = V_1^2 D_1 \left(\frac{V_2}{V_1} - 1\right) = V_1^2 D_1 \left(\frac{D_1}{D_2} - 1\right)$$

or

$$\frac{g}{2}(D_1 + D_2) = V_1^2 \frac{D_1}{D_2}$$

Thus

$$\frac{gD_1}{2}\left(1 + \frac{D_2}{D_1}\right) = V_1^2 \frac{D_1}{D_2}$$

or

$$\frac{D_2}{D_1}\left(1 + \frac{D_2}{D_1}\right) = \frac{2V_1^2}{gD_1}$$

or

$$\left(\frac{D_2}{D_1}\right)^2 + \frac{D_2}{D_1} - \frac{2V_1^2}{gD_1} = 0$$

Using the quadratic equation,

$$\frac{D_2}{D_1} = \frac{1}{2}\left[-1 \pm \sqrt{1 + \frac{8V_1^2}{gD_1}}\right]$$

or

$$D_2 = \frac{D_1}{2}\left[\sqrt{1 + \frac{8V_1^2}{gD_1}} - 1\right]$$

Solving for D_2,

$$D_2 = \frac{1}{2} \times 0.6\ m\left[\sqrt{1 + 8 \times \left(5\ \frac{m}{s}\right)^2 \times \frac{s^2}{9.81\ m} \times \frac{1}{0.6\ m}} - 1\right] = 1.47\ m$$

$$V_2 = \frac{D_1}{D_2}V_1 = \frac{0.6}{1.47} \times 5\frac{m}{s} = 2.04\ \frac{m}{s}$$

From the energy equation with

$$e_{mech} = \frac{V^2}{2} + gz + \frac{p}{\rho}$$

$$dA = wdz$$

The mechanical energy fluxes are

$$mef_1 = \int_0^{D_1}\left[\frac{V_1^2}{2} + gz + \frac{1}{\rho}\rho g(D - z)\right]\rho V_1 wdz = \left(\frac{V_1^2}{2} + gD_1\right)\rho V_1 wD_1$$

$$mef_2 = \int_0^{D_2}\left[\frac{V_2^2}{2} + gz + \frac{1}{\rho}\rho g(D - z)\right]\rho V_2 wdz = \left(\frac{V_2^2}{2} + gD_2\right)\rho V_2 wD_2$$

and

$$\Delta mef = mef_2 - mef_1 = \left[\frac{V_2^2 - V_1^2}{2} + g(D_2 - D_1)\right]\rho V_1 wD_1$$

Since

$$V_1 D_1 = V_2 D_2$$

Thus

$$\frac{\Delta mef}{\dot{m}} = \frac{1}{2}[V_2^2 - V_1^2 + 2g(D_2 - D_1)]$$

$$\frac{\Delta mef}{\dot{m}} = \frac{1}{2}\left[\left(2.04\ \frac{m}{s}\right)^2 - \left(5\ \frac{m}{s}\right)^2 + 2 \times 9.81\ \frac{m}{s^2} \times (1.47\ m - 0.6\ m)\right]\frac{N \cdot s^2}{kg \cdot m} = -1.88\ \frac{N \cdot m}{kg}$$

From the energy equation

$$0 = \left[u_1 + \frac{V_1^2}{2} + gz + \frac{1}{\rho}\rho g(D - z)\right]\{-|\rho V_1 w D_1|\} + \left[u_2 + \frac{V_2^2}{2} + gz + \frac{1}{\rho}\rho g(D - z)\right]\{|\rho V_2 w D_2|\}$$

or

$$0 = (u_2 - u_1)\dot{m} + \Delta mef$$

Thus

$$u_2 - u_1 = c_v(T_2 - T_1) = -\frac{\Delta mef}{\dot{m}}$$

$$\Delta T = T_2 - T_1 = -\frac{\Delta mef}{\dot{m} c_v} = -\left(-1.88\ \frac{N \cdot m}{kg}\right)\frac{kg \cdot K}{1\ kcal} \times \frac{kcal}{4187\ J} = 4.49 \times 10^{-4}\ K$$

This small temperature change would be almost impossible to measure.