ENED 1020: Circuits Lab (Week 2)

Objectives

1. To practice measuring resistance, voltage, and current
2. To explore the properties of resistors connected in parallel
3. To verify experimentally Kirchhoff's Current Law
4. To observe the charging and discharging properties of a capacitor in an RC series circuit
5. To verify experimentally the mathematical equations describing the charging and discharging of capacitors.

F. Parallel Circuits

A circuit diagram with three resistors connected in parallel is shown in Figure 8.

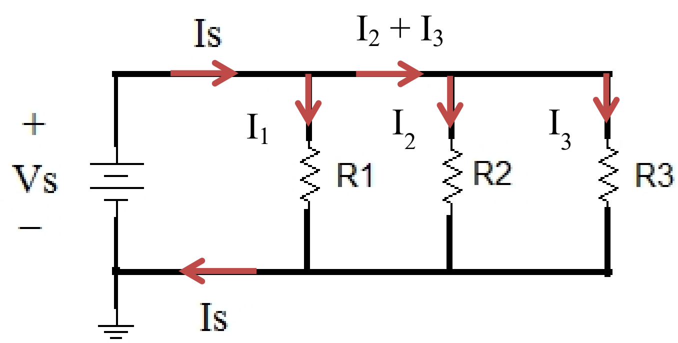


Figure 8: Parallel Circuit with Three Resistors

The total resistance is the inverse of the sum of the inverses of the individual resistors:

$$\textbf{Total Resistance in a Parallel Circuit:}\quad R_{Total} = \frac{1}{\sum \frac{1}{R_i}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

Applying Kirchhoff's Voltage Law, the voltage drop across each resistor will be identical to the battery or source voltage. Resistors in parallel as have the same voltage drop:

$$\textbf{Voltage Drop Across Each Resistor:}\quad V_{R_1} = V_{R_2} = V_{R_3} = V_s$$

Unless the resistors are identical in size, the current through the resistors will not be the same. As shown in Figure 8, current flowing from the positive terminal of the battery can split into two paths; one path is through R1 and the other path is towards resistors R2 and R3. The current flowing toward resistors R2 and R3 will split again with part of the current flowing through R2 and part of the current flowing through R3.

$$\textbf{Current Through Each Resistor (Ohm's Law):}\quad I_{R_i} = \frac{V_s}{R_i}$$

Notice that the branch with the highest resistance will have the smallest current and the branch with the smallest resistance will have the largest current; that is, current takes the path of least resistance. In fact, if R2 and R1 are in parallel and R2 is four times as large as R1, then the current through R2 will be ¼ of the current though R1.

Kirchhoff's Current Law (KCL) is another fundamental law for understanding and analyzing circuits. Kirchhoff's Current Law states that the current flowing into a node (a point in the circuit with multiple paths for current to flow) is exactly equal to the current flowing out of a node. Kirchhoff's Current Law is illustrated in Figure 8:

$$\textbf{KCL for the Parallel Circuit in Figure 8}: \quad \mathbf{I_s} = \mathbf{I_1} + (\mathbf{I_2} + \mathbf{I_3})$$

In this section of the lab, the behavior of parallel circuits will be explored and Kirchhoff's Current Law will be verified experimentally by wiring up a simple parallel circuit with a battery (voltage source) and three resistors.

Directions:

1. From the kit, select a 330 Ω, a 560 Ω, and a 1 kΩ resistor.
2. Measure the resistance of each resistor and enter the measured values into Table 5.
3. Using your three resistors, wire the circuit shown in Figure 9 leaving the wire between the positive terminal of the battery and the first resistor disconnected.
4. Using the measured values for each of the resistors, calculate the expected total resistance for the resistors connected in parallel. Enter your calculated value into Table 5.
   Calculation:

5. Now, ***set up the multi-meter to measure resistance*** and measure the total resistance by connecting the multi-meter across any of the three resistors.
6. Using your measured total resistance and your measured battery voltage, calculate the expected current flow through the parallel circuit by applying Ohm's Law. Enter the calculated value into Table 5.
   Calculation:

7. Now, ***set up the multi-meter to measure current*** and insert the meter into the circuit between the positive terminal of the battery and the unconnected end of the first resistor. Measure the current and record the measured value in Table 5.
8. Remove the multi-meter from the circuit.
9. Connect the top of the first resistor to the positive terminal of the battery to establish current flow.
10. ***Set up the multi-meter to measure voltage.***
11. Measure the voltage drop across each resistor and record the values in Table 5.
12. Using your measured values, apply Ohm's law to calculate the current through each resistor and enter the calculated values into Table 5.
    Sample Calculation:

13. Using your measured currents, verify Kirchhoff's Current Law.
Verification of Kirchhoff's Current Law:

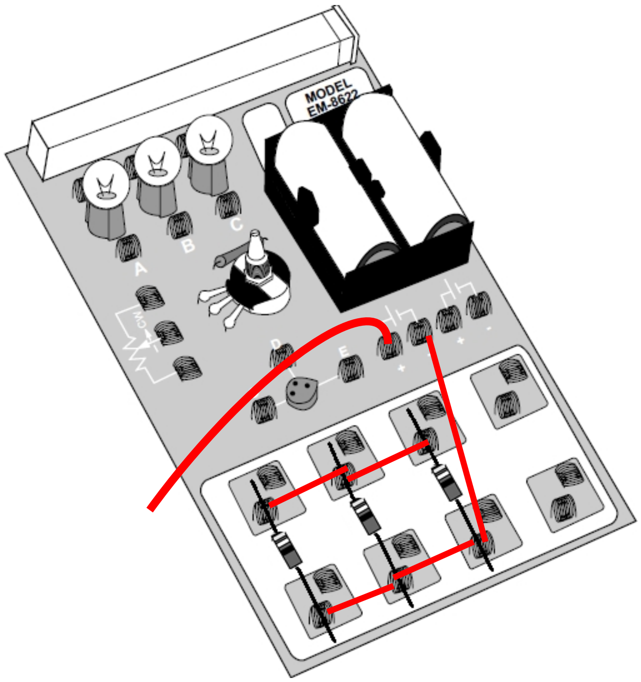


Figure 9: Experimental Set-Up for Parallel Circuit

Table 5: Parallel Circuits

Measured Resistances			Calculated Total Resistance	Measured Total Resistance	Calculated Total Current	Measured Total Current	% Error
R_1	R_2	R_3					
330	547	989	170.37	169.7	8.98mA	8.50mA	3.5%

Resistor	Measured Voltage Drop	Calculated Resistor Currents
R_1	1.498	0.0045
R_2	1.498	0.0027
R_3	1.498	0.0015

Just as we can connect resistors in parallel, we can also connect other circuit elements, like light bulbs, in parallel. Follow the directions below to connect two of the light bulbs on the circuit board in parallel.

Directions:

1. Choose two of the three light bulbs at the top of your circuit board and build the simple circuit shown in Figure 10.
2. Make a note the brightness of the light bulbs compared to how bright the single light bulb was from part D and the series light bulbs from part E you completed last week. What do you think causes this?
   Observations:
   Same bright, because the voltage is the same.
3. Disconnect the wire connected to the positive terminal of the battery from the light bulb. Unscrew one of the light bulbs and reconnect the circuit. What happens? Make a note.
   Observations:
   Brighter.
4. Based on the results of you have witnessed for light bulbs connected in series and parallel, how do you think the electrical outlets in your house are connected? How do you think a light switch is connected?
   Observations:

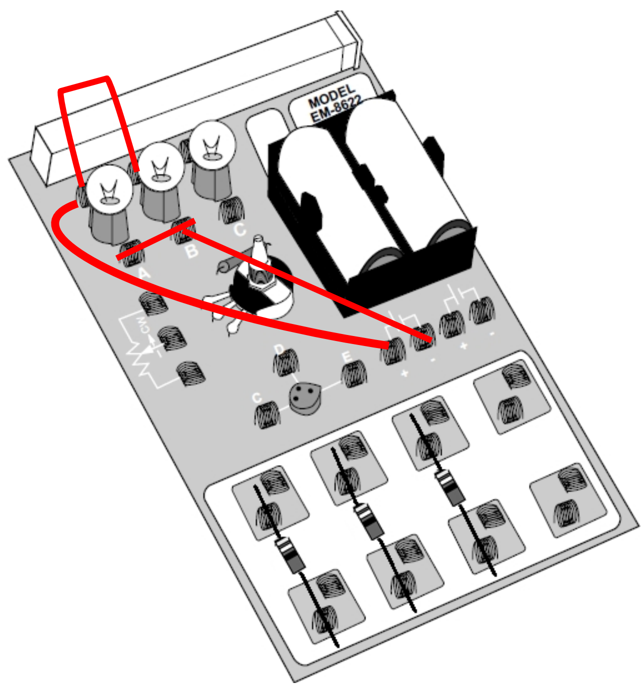


Figure 10: Experimental Set-Up for Parallel Light Bulb Test

G. Charging and Discharging Properties of Capacitors in RC Circuits

Capacitors are extremely useful devices used in a wide range of electronic products. In this section, the behavior of a series RC circuit will be explored and the equations for charging and discharging a capacitor will be verified.

Capacitance
Capacitance (C) is a measure of a capacitor's ability to store an electrical charge and is measured in units of Farads (F). Oftentimes, capacitance is small enough to be measured in microfarads ($1\mu F = 1*10^{-6}$ F), nanofarads ($1nF = 1*10^{-9}$ F), or picofarads ($1pF = 1*10^{-12}$ F).

Consider the RC circuit shown in Figure 11. When the switch is moved (connected) to position, P1, the capacitor will begin to charge exponentially until the voltage across the capacitor is identical to the source voltage. When the capacitor voltage reaches the source voltage, the current becomes zero.

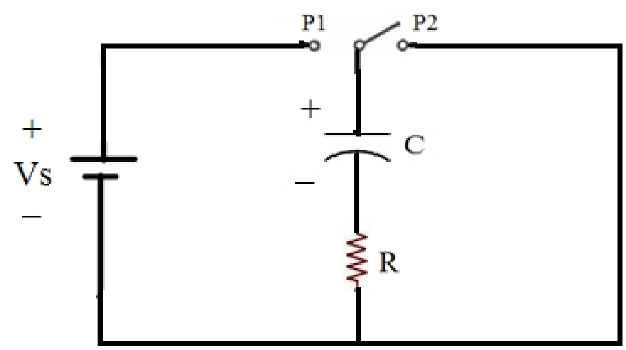


Figure 11: Series RC Circuit

The equation describing how the capacitor voltage varies while the capacitor is charging is:

$$\textbf{Capacitor Charging:}\ \ V_{cap} = V_s\left(1 - e^{-\frac{t}{RC}}\right)$$

If the switch is removed from P1 (but not connected to P2), the capacitor will hold the voltage value it was charged to. Actually, the capacitor will leak charge over time but the voltage drops fairly slowly.

If the switch is moved to position P2, the capacitor will discharge exponentially through the resistor until the capacitor voltage drops to zero. The equation describing how the capacitor voltage varies while the capacitor is discharging is:

$$\textbf{Capacitor Discharging:}\ \ V_{cap} = V_i\, e^{-\frac{t}{RC}}\ \ \textit{where}\ V_i\ \textit{is the initial capacitor voltage}$$

So, a capacitor can be charged to a certain voltage, hold that voltage for a period of time, then act as a temporary battery while discharging.
Looking at the charging and discharging equations, it is apparent that the speed at which the capacitor charges or discharges depends on the product of resistance and capacitance, RC. This product is referred to as a time constant and in fact, has units of seconds.

$$\textbf{Time Constant: } \boldsymbol{\tau = \mathrm{RC}} \qquad \begin{cases} \boldsymbol{\tau \text{ is in seconds } (s)} \\ \boldsymbol{Resistance, R, \text{ is in ohms } (\Omega)} \\ \boldsymbol{Capacitance, C, \text{ is in farads } (F)} \end{cases}$$

To illustrate how the time constant affects rate of charge and discharge of the capacitor, plug two different time values t = τ = RC and t = 5τ = 5RC into the charging and discharging equation.

$$\textbf{Capacitor Charging: } \begin{cases} \boldsymbol{t = RC}: \quad \mathbf{V_{cap}} = \boldsymbol{V_s\left(1 - e^{-\frac{RC}{RC}}\right) = V_s(1 - e^{-1}) = 0.632V_s} \\ \boldsymbol{t = 5RC}: \quad \mathbf{V_{cap}} = \boldsymbol{V_s\left(1 - e^{-\frac{5RC}{RC}}\right) = V_s(1 - e^{-5}) = 0.993V_s} \end{cases}$$

$$\textbf{Capacitor Discharging: } \begin{cases} \boldsymbol{t = RC}: \qquad \mathbf{V_{cap}} = \boldsymbol{V_i e^{-\frac{RC}{RC}} = V_i e^{-1} = 0.368V_i} \\ \boldsymbol{t = 5RC}: \qquad \mathbf{V_{cap}} = \boldsymbol{V_i e^{-\frac{5RC}{RC}} = V_i e^{-5} = 0.007V_i} \end{cases}$$

The time constant, τ = RC, indicates how fast the capacitor will charge or discharge. In one time constant (t = τ) the capacitor will charge to 63.2% of its final voltage and in five time constants (t = 5τ), the capacitor will charge to 99.3% of its final voltage (essentially completely charged.
The discharging profile is very similar to the charging profile. In one time constant (t = τ), the capacitor voltage will drop by 63.2% of its initial voltage and in five time constants (t = 5τ), the capacitor voltage will drop by 99.3% of its initial voltage (essentially discharged).

Directions:

1. Locate the 100 μF capacitor and the 220 kΩ resistor from your kit. The capacitor is electrolytic meaning it has a positive side and a negative side. Make sure you can tell which side is positive and which side is negative.
2. Measure the actual resistance of the resistor and the voltage of your D-cell battery. Enter these values into Table 6.
3. Calculate the time constant, τ = RC and enter this value in Table 6.
4. Wire the charging circuit shown in Figure 11 but ***leave the wire from the positive terminal of the battery to the positive side of the capacitor unconnected***. Make sure the capacitor is oriented correctly.
5. Set the multi-meter up to measure voltage and place the leads of the meter across the capacitor as shown in Figure 12.
6. Now connect the wire between the positive terminal of the battery and the positive side of the capacitor. Observe the capacitor charging – the voltage should reach your battery voltage in about 5 time constants.

7. Disconnect wire from the positive terminal of the battery. What happens to the capacitor voltage?
   Observations:

Stay the same.

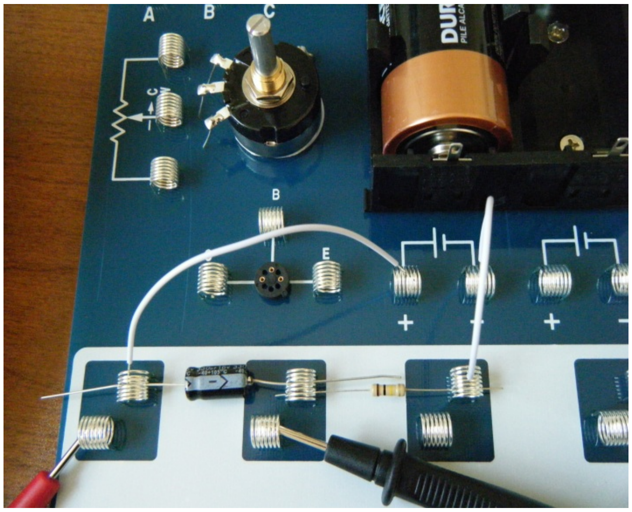
8. Now connect the wire across both the resistor and the capacitor as shown in Figure 12 for the discharging circuit. The capacitor voltage should drop in about 5 time constants to 0 volts.
9. Charge the capacitor back up and this time discharge the capacitor by placing the wire across the capacitor leads only. What happens? Why?
   Observations:

10. Make sure the capacitor is completely discharged, the positive terminal of the battery is not connected to the capacitor, and the multi-meter is set up to measure voltage across the capacitor.
11. Using a stopwatch, allow the capacitor to charge and record the capacitor voltage every 5 seconds. Enter your measurements into Table 6.
12. Using a stopwatch, allow the capacitor to discharge through the resistor and record the capacitor voltage every 10 seconds. Enter your measurements into Table 6.

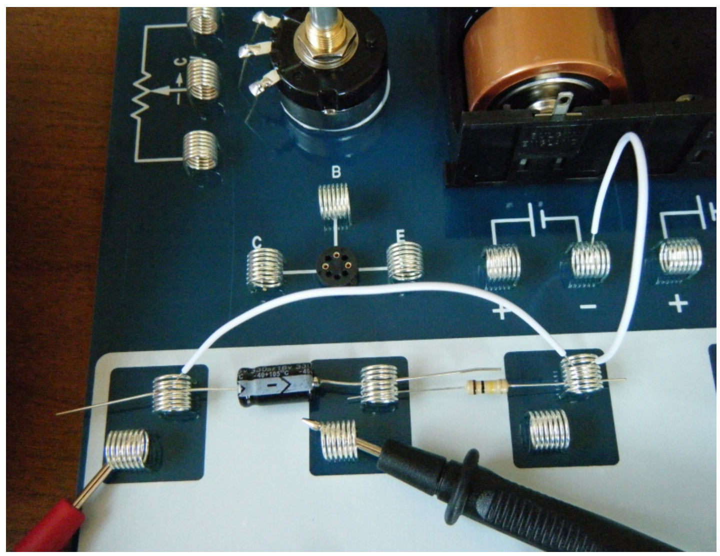
Table 6: Series RC Circuit

Battery Voltage (Measured)	**Resistance (Measured)**	**Capacitance (Labeled Value)**	**Time Constant $\tau = RC$**
1.53 V	210 kOhm	100 µF	21 s

Time (seconds)	**Capacitor Voltage (Charging)**	**Time (seconds)**	**Capacitor Voltage (Discharging)**
0	**0.01**	**0**	**1.344**
10	**0.449**	**10**	**0.928**
20	**0.765**	**20**	**0.634**
30	**0.981**	**30**	**0.436**
40	**1.125**	**40**	**0.305**
50	**1.228**	**50**	**0.211**
60	**1.300**	**60**	**0.146**
70	**1.349**	**70**	**0.101**
80	**1.381**	**80**	**0.070**
90	**1.405**	**90**	**0.049**
100	**1.422**	**100**	**0.035**
110	**1.433**	**110**	**0.024**



Charging a Capacitor and measuring the voltage across the capacitor.



Discharging a Capacitor and measuring voltage across the capacitor.

Figure 12: RC Charging and Discharging Circuits

H. Parallel and Series Sources (Optional: time permitting)

So far we have been looking at how connecting resistors in different configurations affects the performance of a circuit, but what happens if we connect multiple sources? How will connecting the sources in different configurations affect the operation of the circuit?

Using multiple sources is actually very common. Most electronic devices that require batteries often require more than one. You may have come across this when you were little and you opened up that awesome new toy, only to find out you couldn't play with it because it required 4 C batteries and you only had 2! Including multiple sources can provide for increased performance and capabilities in your circuit by changing the available current or voltage.

Just as with resistors, we can connect sources in series and in parallel, and they follow similar rules to parallel and series resistors:

1) Sources in series will have the same current but can have different voltages
2) Sources in parallel will have the same voltage but can have different currents

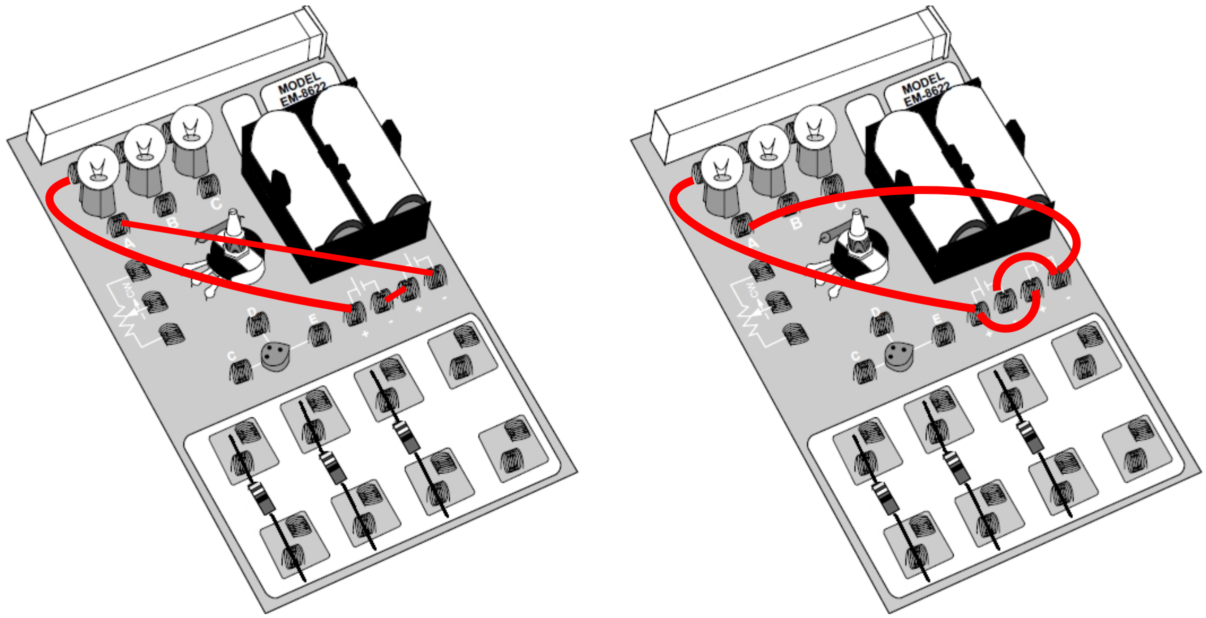
In the two activities below, you will explore what happens when you connect sources in series or parallel.

Directions – Part I:

1. Place a battery in both battery compartments on your board
2. Connect one lightbulb to one battery as you did in Part D of lab during week 1
3. Measure the voltage and current of the lightbulb and record them in the table below
4. Add in a second source as shown in Figure 15a; the batteries are now connected in series
5. Measure the voltage and current of the lightbulb and record them in the table below
6. What do you notice about the intensity of the lightbulb now that you have connected the sources in series?
   Observations:
7. Reconfigure your circuit as shown in Figure 15b; the batteries are now connected in parallel
8. Measure the voltage and current of the lightbulb and record them in the table below
9. What do you notice about the intensity of the lightbulb now that you have connected the sources in parallel?
   Observations:
10. Which property (voltage/current) seems to affect the intensity of the lightbulb? Which configuration (series/parallel) increases this property? Why?

Table 7: Voltage and Current Measurements

	Single Battery:	**Series Batteries:**	**Parallel Batteries:**
Voltage:			
Current:			



(a) **(b)**

Figure 15: Connections for (a) Series and (b) Parallel Sources

Directions – Part II:
For this part, we will be using a new component on the board, the potentiometer. The potentiometer is the large, cylindrical component next to the battery housing (see Figure 16). A potentiometer is a variable resistor, where the resistance between the middle connection and the outer connections changes as the knob of the potentiometer is rotated.

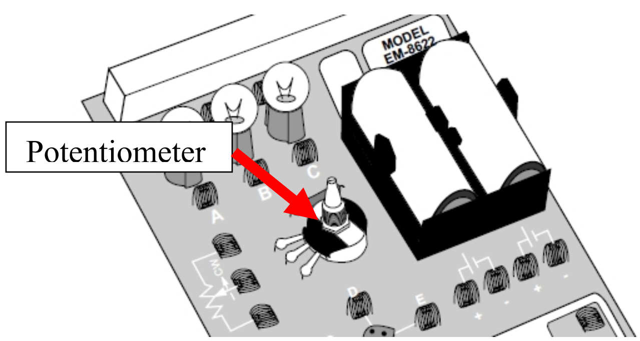


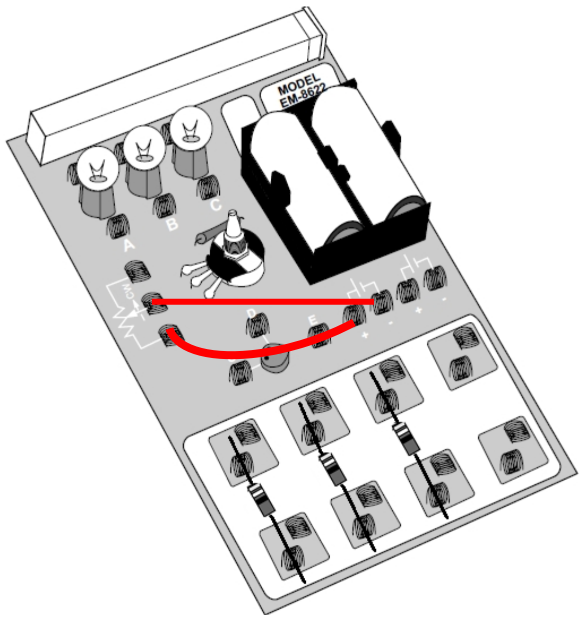
Figure 16: Location of Potentiometer

1. Make sure that your potentiometer is turned the whole way clockwise
2. Connect a single battery to the potentiometer as shown in Figure 17a
3. Measure the voltage of the battery and store the value in the row for Voltage 1
4. Turn the potentiometer the whole way counterclockwise
5. Measure the voltage of the batter and store the value in the row for Voltage 2
6. Turn the potentiometer the whole way clockwise
7. Connect the circuit as shown in Figure 17b and repeat steps 3-6
8. Connect the circuit as shown in Figure 17c and repeat steps 3-6
9. Compute the percent change in the battery voltage

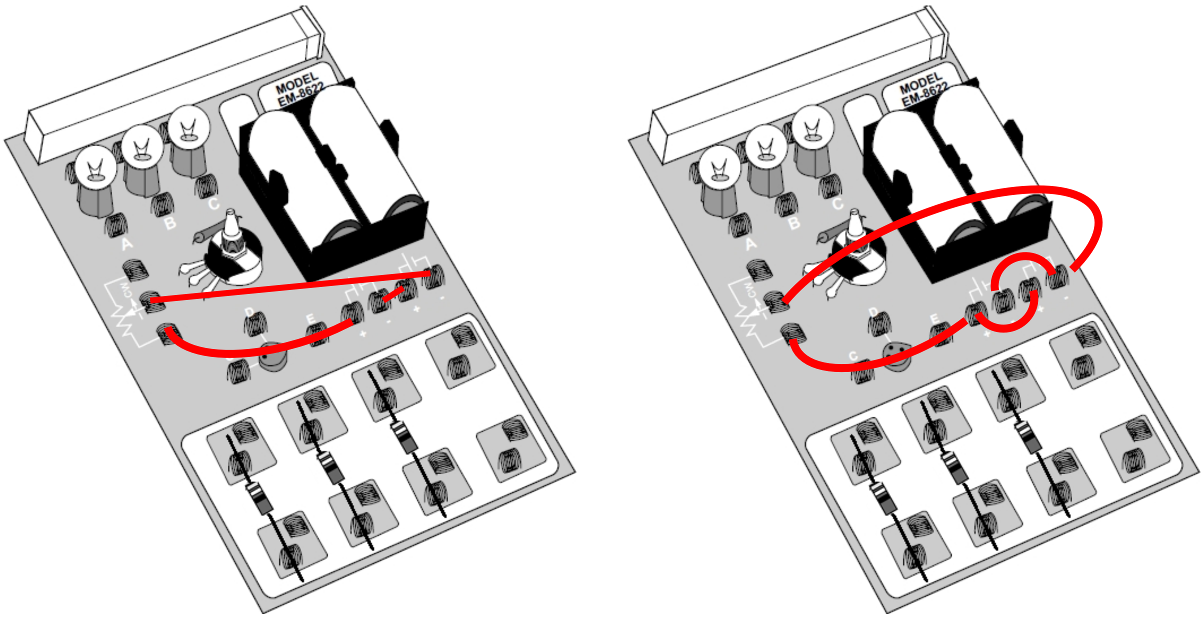
10. As you turn the potentiometer counterclockwise, the resistance decreases, causing an increase in the amount of current drawn. Which configuration does the best job of maintaining the battery voltage at the initial levels? Why?

Table 8: Voltage and Current Measurements

	Single Battery:	**Series Batteries:**	**Parallel Batteries:**
Voltage 1:			
Voltage 2:			
% Change:			



(a)



(b) **(c)**

Figure 17: Connections for (a) Single, (b) Series, and (c) Parallel Sources