CHAPTER 8

Section 8-1

8-1

a) The confidence level for $\bar{x} - 2.14\sigma/\sqrt{n} \le \mu \le \bar{x} + 2.14\sigma/\sqrt{n}$ is determined by the value of z_0 which is 2.14. From Table III, Φ(2.14) = P(Z < 2.14) = 0.9838 and the confidence level is 2(0.9838-0.5) = 96.76%.

b) The confidence level for $\bar{x} - 2.49\sigma/\sqrt{n} \le \mu \le \bar{x} + 2.49\sigma/\sqrt{n}$ is determined by the value of z_0 which is 2.14. From Table III, Φ(2.49) = P(Z < 2.49) = 0.9936 and the confidence level is is 2(0.9936-0.5) = 98.72%.

c) The confidence level for $\bar{x} - 1.85\sigma/\sqrt{n} \le \mu \le \bar{x} + 1.85\sigma/\sqrt{n}$ is determined by the by the value of z_0 which is 2.14. From Table III, Φ(1.85) = P(Z < 1.85) = 0.9678 and the confidence level is 93.56%.

d) One-sided confidence interval with z_α = 2. Therefore, α = P(Z > 2) = 0.0228 and confidence = 1 – α = = 0.9772 = 97.72%

e) One-sided confidence interval with z_α = 1.96. Therefore, α = P(Z > 1.96) = 0.0250 and confidence = 1 – α = 0.9750 = 97.50%

8-2 a) A z_α = 2.33 would result in a 98% two-sided confidence interval.
b) A z_α = 1.29 would result in a 80% two-sided confidence interval.
c) A z_α = 1.15 would result in a 75% two-sided confidence interval.

8-3 a) A z_α = 1.29 would result in a 90% one-sided confidence interval.
b) A z_α = 1.65 would result in a 95% one-sided confidence interval.
c) A z_α = 2.33 would result in a 99% one-sided confidence interval.

8-4 a) 95% CI for μ, $n = 10, \quad \sigma = 20 \quad \bar{x} = 1000, \ z = 1.96$

$$\bar{x} - z\sigma/\sqrt{n} \le \mu \le \bar{x} + z\sigma/\sqrt{n}$$

$$1000 - 1.96(20/\sqrt{10}) \le \mu \le 1000 + 1.96(20/\sqrt{10})$$

$$987.6 \le \mu \le 1012.4$$

b) .95% CI for μ, $n = 25, \quad \sigma = 20 \quad \bar{x} = 1000, \ z = 1.96$

$$\bar{x} - z\sigma/\sqrt{n} \le \mu \le \bar{x} + z\sigma/\sqrt{n}$$

$$1000 - 1.96(20/\sqrt{25}) \le \mu \le 1000 + 1.96(20/\sqrt{25})$$

$$992.2 \le \mu \le 1007.8$$

c) 99% CI for μ, $n = 10, \quad \sigma = 20 \quad \bar{x} = 1000, \ z = 2.58$

$$\bar{x} - z\sigma/\sqrt{n} \le \mu \le \bar{x} + z\sigma/\sqrt{n}$$

$$1000 - 2.58(20/\sqrt{10}) \le \mu \le 1000 + 2.58(20/\sqrt{10})$$

$$983.7 \le \mu \le 1016.3$$

d) 99% CI for μ, $n = 25, \quad \sigma = 20 \quad \bar{x} = 1000, \ z = 2.58$

$$\bar{x} - z\sigma / \sqrt{n} \le \mu \le \bar{x} + z\sigma / \sqrt{n}$$
$$1000 - 2.58(20 / \sqrt{25}) \le \mu \le 1000 + 2.58(20 / \sqrt{25})$$
$$989.7 \le \mu \le 1010.3$$

e) When n is larger, the CI is narrower. The higher the confidence level, the wider the CI.

8-5 a) Sample mean from the first confidence interval = 38.02 + (61.98-38.02)/2 = 50
Sample mean from the second confidence interval = 39.95 + (60.05-39.95)/2 = 50

b) The 95% CI is (38.02, 61.98) and the 90% CI is (39.95, 60.05). The higher the confidence level, the wider the CI.

8-6 a) Sample mean from the first confidence interval =37.53 + (49.87-37.53)/2 = 43.7
Sample mean from the second confidence interval =35.59 + (51.81-35.59)/2 = 43.7

b) The 99% CI is (35.59, 51.81) and the 95% CI is (37.53, 49.87). The higher the confidence level, the wider the CI.

8-7 a) Find n for the length of the 95% CI to be 40. $Z_{a/2} = 1.96$
1/2 length $= (1.96)(20) / \sqrt{n} = 20$
$39.2 = 20\sqrt{n}$
$n = \left(\frac{39.2}{20}\right)^2 = 3.84$
Therefore, $n = 4$.

b) Find n for the length of the 99% CI to be 40. $Z_{a/2} = 2.58$
1/2 length $= (2.58)(20) / \sqrt{n} = 20$
$51.6 = 20\sqrt{n}$
$n = \left(\frac{51.6}{20}\right)^2 = 6.66$
Therefore, $n = 7$.

8-8 Interval (1): $3124.9 \le \mu \le 3215.7$ and Interval (2): $3110.5 \le \mu \le 3230.1$
Interval (1): half-length =90.8/2=45.4 and Interval (2): half-length =119.6/2=59.8

a) $\bar{x}_1 = 3124.9 + 45.4 = 3170.3$
$\bar{x}_2 = 3110.5 + 59.8 = 3170.3$ The sample means are the same.

b) Interval (1): $3124.9 \le \mu \le 3215.7$ was calculated with 95% confidence because it has a smaller half-length, and therefore a smaller confidence interval. The 99% confidence level widens the interval.

8-9 a) The 99% CI on the mean calcium concentration would be wider.

b) No, that is not the correct interpretation of a confidence interval. The probability that μ is between 0.49 and 0.82 is either 0 or 1.

c) Yes, this is the correct interpretation of a confidence interval. The upper and lower limits of the confidence limits are random variables.

8-10 95% Two-sided CI on the breaking strength of yarn: where $\bar{x} = 98$, $\sigma = 2$, $n=9$ and $z_{0.025} = 1.96$

$$\bar{x} - z_{0.025}\sigma/\sqrt{n} \le \mu \le \bar{x} + z_{0.025}\sigma/\sqrt{n}$$
$$98 - 1.96(2)/\sqrt{9} \le \mu \le 98 + 1.96(2)/\sqrt{9}$$
$$96.7 \le \mu \le 99.3$$

8-11 95% Two-sided CI on the true mean yield: where $\bar{x} = 90.480$, $\sigma = 3$, $n=5$ and $z_{0.025} = 1.96$

$$\bar{x} - z_{0.025}\sigma/\sqrt{n} \le \mu \le \bar{x} + z_{0.025}\sigma/\sqrt{n}$$
$$90.480 - 1.96(3)/\sqrt{5} \le \mu \le 90.480 + 1.96(3)/\sqrt{5}$$
$$87.85 \le \mu \le 93.11$$

8-12 99% Two-sided CI on the diameter cable harness holes: where $\bar{x} = 1.5045$, $\sigma = 0.01$, $n=10$ and $z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\sigma/\sqrt{n} \le \mu \le \bar{x} + z_{0.005}\sigma/\sqrt{n}$$
$$1.5045 - 2.58(0.01)/\sqrt{10} \le \mu \le 1.5045 + 2.58(0.01)/\sqrt{10}$$
$$1.4963 \le \mu \le 1.5127$$

8-13 a) 99% Two-sided CI on the true mean piston ring diameter

For $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$, and $\bar{x} = 74.036$, $\sigma = 0.001$, $n=15$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$
$$74.036 - 2.58\left(\frac{0.001}{\sqrt{15}}\right) \le \mu \le 74.036 + 2.58\left(\frac{0.001}{\sqrt{15}}\right)$$
$$74.0353 \le \mu \le 74.0367$$

b) 99% One-sided CI on the true mean piston ring diameter

For $\alpha = 0.01$, $z_\alpha = z_{0.01} = 2.33$ and $\bar{x} = 74.036$, $\sigma = 0.001$, $n=15$

$$\bar{x} - z_{0.01}\frac{\sigma}{\sqrt{n}} \le \mu$$
$$74.036 - 2.33\left(\frac{0.001}{\sqrt{15}}\right) \le \mu$$
$$74.0354 \le \mu$$

The lower bound of the one-sided confidence interval is greater than the lower bound of the two-sided interval even though the level of significance is the same. This is because for a one-sided confidence interval the probability in the left tail (α) is greater than the probability in the left tail of the two-sided confidence interval ($\alpha/2$).

8-14 a) 95% Two-sided CI on the true mean life of a 75-watt light bulb

For $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$, and $\bar{x} = 1014$, $\sigma = 25$, $n=20$

$$\bar{x} - z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right)$$
$$1014 - 1.96\left(\frac{25}{\sqrt{20}}\right) \le \mu \le 1014 + 1.96\left(\frac{25}{\sqrt{20}}\right)$$
$$1003 \le \mu \le 1025$$

b) 95% one-sided CI on the true mean piston ring diameter

For $\alpha = 0.05$, $z_\alpha = z_{0.05} = 1.65$ and $\bar{x} = 1014$, $\sigma = 25$, $n=20$

$$\bar{x} - z_{0.05}\frac{\sigma}{\sqrt{n}} \le \mu$$
$$1014 - 1.65\left(\frac{25}{\sqrt{20}}\right) \le \mu$$
$$1005 \le \mu$$

The lower bound of the one-sided confidence interval is greater than the lower bound of the two-sided interval even though the level of significance is the same. This is because for a one-sided confidence interval the probability in the left tail (α) is greater than the probability in the left tail of the two-sided confidence interval ($\alpha/2$).

8-15 a) 95% two sided CI on the mean compressive strength

$z_{\alpha/2} = z_{0.025} = 1.96$, and $\bar{x} = 3250$, $\sigma^2 = 1000$, $n=12$

$$\bar{x} - z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right)$$
$$3250 - 1.96\left(\frac{31.62}{\sqrt{12}}\right) \le \mu \le 3250 + 1.96\left(\frac{31.62}{\sqrt{12}}\right)$$
$$3232.11 \le \mu \le 3267.89$$

b) 99% Two-sided CI on the true mean compressive strength

$z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$
$$3250 - 2.58\left(\frac{31.62}{\sqrt{12}}\right) \le \mu \le 3250 + 2.58\left(\frac{31.62}{\sqrt{12}}\right)$$
$$3226.4 \le \mu \le 3273.6$$

The 99% CI is wider than the 95% CI

8-16 95% Confident that the error of estimating the true mean life of a 75-watt light bulb is less than 5 hours.

For $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$, and $\bar{\sigma} = 25$, $E=5$

$$n = \left(\frac{z_{a/2}\sigma}{E}\right)^2 = \left(\frac{1.96(25)}{5}\right)^2 = 96.04$$

Round up to the next integer. Therefore, $n = 97$

8-17 Set the width to 6 hours with $\sigma = 25$, $z_{0.025} = 1.96$ solve for n.

$1/2 \text{ width} = (1.96)(25)/\sqrt{n} = 3$

$49 = 3\sqrt{n}$

$$n = \left(\frac{49}{3}\right)^2 = 266.78$$

Therefore, n = 267

8-18 99% confidence that the error of estimating the true compressive strength is less than 15 psi

For $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$, and $\bar{\sigma} = 31.62$, E=15

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{2.58(31.62)}{15}\right)^2 = 29.6 \cong 30$$

Therefore, $n = 30$

8-19 To decrease the length of the CI by one half, the sample size must be increased by 4 times (2^2).

$$z_{\alpha/2}\sigma / \sqrt{n} = 0.5l$$

Now, to decrease by half, divide both sides by 2.

$$(z_{\alpha/2}\sigma / \sqrt{n}) / 2 = (l/2)/2$$

$$(z_{\alpha/2}\sigma / 2\sqrt{n}) = l/4$$

$$(z_{\alpha/2}\sigma / \sqrt{2^2 n}) = l/4$$

Therefore, the sample size must be increased by $2^2 = 4$

8-20 If n is doubled in Eq 8-7: $\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

$$\frac{z_{\alpha/2}\sigma}{\sqrt{2n}} = \frac{z_{\alpha/2}\sigma}{1.414\sqrt{n}} = \frac{z_{\alpha/2}\sigma}{1.414\sqrt{n}} = \frac{1}{1.414}\left(\frac{z_{\alpha/2}\sigma}{\sqrt{n}}\right)$$

The interval is reduced by 0.293 or 29.3%

If n is increased by a factor of 4

$$\frac{z_{\alpha/2}\sigma}{\sqrt{4n}} = \frac{z_{\alpha/2}\sigma}{2\sqrt{n}} = \frac{z_{\alpha/2}\sigma}{2\sqrt{n}} = \frac{1}{2}\left(\frac{z_{\alpha/2}\sigma}{\sqrt{n}}\right)$$

The interval is reduced by 0.5.

8-21 a) 99% two sided CI on the mean temperature

$z_{\alpha/2} = z_{0.005} = 2.57$, and $\bar{x} = 13.77$, $\sigma = 0.5$, n=11

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$13.77 - 2.57\left(\frac{0.5}{\sqrt{11}}\right) \le \mu \le 13.77 + 2.57\left(\frac{0.5}{\sqrt{11}}\right)$$

$$13.383 \le \mu \le 14.157$$

b) 95% lower-confidence bound on the mean temperature

For $\alpha = 0.05$, $z_\alpha = z_{0.05} = 1.65$ and $\bar{x} = 13.77$, $\sigma = 0.5$, n =11

$$\bar{x} - z_{0.05}\frac{\sigma}{\sqrt{n}} \le \mu$$

$$13.77 - 1.65\left(\frac{0.5}{\sqrt{11}}\right) \le \mu$$

$$13.521 \le \mu$$

c) 95% confidence that the error of estimating the mean temperature for wheat grown is less than 2 degrees Celsius.

For $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$, and $\sigma = 0.5$, E = 2

$$n = \left(\frac{z_{a/2}\sigma}{E}\right)^2 = \left(\frac{1.96(0.5)}{2}\right)^2 = 0.2401$$

Round up to the next integer. Therefore n = 1.

d) Set the width to 1.5 degrees Celsius with σ = 0.5, $z_{0.025}$ = 1.96 solve for n.

1/2 width = (1.96)(0.5) / $\sqrt{n}$ = 0.75

$0.98 = 0.75\sqrt{n}$

$$n = \left(\frac{0.98}{0.75}\right)^2 = 1.707$$

Therefore, n = 2.

8-22 a) 95% CI for μ, $n = 5 \quad \sigma = 0.66 \quad \bar{x} = 18.56, \; z = 1.96$

$$\bar{x} - z\sigma/\sqrt{n} \le \mu \le \bar{x} + z\sigma/\sqrt{n}$$

$$3.372 - 1.96(0.66/\sqrt{5}) \le \mu \le 3.372 + 1.96(0.66/\sqrt{5})$$

$$2.79 \le \mu \le 3.95$$

b) Width is $2z\sigma/\sqrt{n} = 0.55$, therefore $n = [2z\sigma/0.55]2 = [2(1.96)(0.66)/0.55]2 = 22.13$

Round up to n = 23.

8-23 a) 99% CI for μ, $n = 12 \quad \sigma = 2.25 \quad \bar{x} = 28.0, \; z = 2.576$

$$\bar{x} - z\sigma/\sqrt{n} \le \mu \le \bar{x} + z\sigma/\sqrt{n}$$

$$28.0 - 2.576(2.25/\sqrt{12}) \le \mu \le 28.0 + 2.576(2.25/\sqrt{12})$$

$$26.33 \le \mu \le 29.67$$

b) Width is $2z\sigma/\sqrt{n} = 1.25$

Therefore z = $1.25n^{1/2}/(2\sigma) = 1.25(12^{1/2})/[2(2.25)] = 0.9623$

Therefore P(-0.9623 < Z < 0.9623) = 0.664 = 1 – α so that the confidence is 66.4%

Section 8-2

8-24 $t_{0.025,15} = 2.131$ $t_{0.05,10} = 1.812$ $t_{0.10,20} = 1.325$

$t_{0.005,25} = 2.787$ $t_{0.001,30} = 3.385$

8-25 a) $t_{0.025,12} = 2.179$ b) $t_{0.025,24} = 2.064$ c) $t_{0.005,13} = 3.012$

d) $t_{0.0005,15} = 4.073$

8-26 a) $t_{0.05,14} = 1.761$ b) $t_{0.01,19} = 2.539$ c) $t_{0.001,24} = 3.467$

8-27 a) Mean $= \frac{sum}{N} = \frac{251.848}{10} = 25.1848$

$Variance = (stDev)^2 = 1.605^2 = 2.5760$

b) 95% confidence interval on mean

$n = 10 \quad \bar{x} = 25.1848 \quad s = 1.605 \quad t_{0.025,9} = 2.262$

$$\bar{x} - t_{0.025,9}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,9}\left(\frac{s}{\sqrt{n}}\right)$$

$$25.1848 - 2.262\left(\frac{1.605}{\sqrt{10}}\right) \le \mu \le 25.1848 + 2.262\left(\frac{1.605}{\sqrt{10}}\right)$$

$$24.037 \le \mu \le 26.333$$

8-28 SE Mean $= \frac{stDev}{\sqrt{N}} = \frac{6.11}{\sqrt{N}} = 1.58$, therefore N = 15

Mean $= \frac{sum}{N} = \frac{751.40}{15} = 50.0933$

Variance $= (stDev)^2 = 6.11^2 = 37.3321$

b) 95% confidence interval on mean

$n = 15 \quad \bar{x} = 50.0933 \quad s = 6.11 \quad t_{0.025,14} = 2.145$

$$\bar{x} - t_{0.025,14}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,14}\left(\frac{s}{\sqrt{n}}\right)$$

$$50.0933 - 2.145\left(\frac{6.11}{\sqrt{15}}\right) \le \mu \le 50.0933 + 2.145\left(\frac{6.11}{\sqrt{15}}\right)$$

$$46.709 \le \mu \le 53.477$$

8-29 95% confidence interval on mean tire life

$n = 16 \quad \bar{x} = 60{,}139.7 \quad s = 3645.94 \quad t_{0.025,15} = 2.131$

$$\bar{x} - t_{0.025,15}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,15}\left(\frac{s}{\sqrt{n}}\right)$$

$$60139.7 - 2.131\left(\frac{3645.94}{\sqrt{16}}\right) \le \mu \le 60139.7 + 2.131\left(\frac{3645.94}{\sqrt{16}}\right)$$

$$58197.33 \le \mu \le 62082.07$$

8-30 99% lower confidence bound on mean Izod impact strength

$n = 20 \quad \bar{x} = 1.25 \quad s = 0.25 \quad t_{0.01,19} = 2.539$

$$\bar{x} - t_{0.01,19}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$1.25 - 2.539\left(\frac{0.25}{\sqrt{20}}\right) \le \mu$$

$$1.108 \le \mu$$

8-31 $\bar{x} = 1.10 \quad s = 0.015 \quad n = 25$

95% confidence interval on the mean volume of syrup dispensed

For $\alpha = 0.05$ and n = 25, $t_{\alpha/2,n-1} = t_{0.025,24} = 2.064$

$$\bar{x} - t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right)$$
$$1.10 - 2.064\left(\frac{0.015}{\sqrt{25}}\right) \le \mu \le 1.10 + 2.064\left(\frac{0.015}{\sqrt{25}}\right)$$
$$1.094 \le \mu \le 1.106$$

8-32 95% confidence interval on mean peak power

$n = 7 \quad \bar{x} = 315 \quad s = 16 \quad t_{0.025,6} = 2.447$

$$\bar{x} - t_{0.025,6}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,6}\left(\frac{s}{\sqrt{n}}\right)$$
$$315 - 2.447\left(\frac{16}{\sqrt{7}}\right) \le \mu \le 315 + 2.447\left(\frac{315}{\sqrt{7}}\right)$$
$$300.202 \le \mu \le 329.798$$

8-33 99% upper confidence interval on mean SBP

$n = 14 \quad \bar{x} = 118.3 \quad s = 9.9 \quad t_{0.01,13} = 2.650$

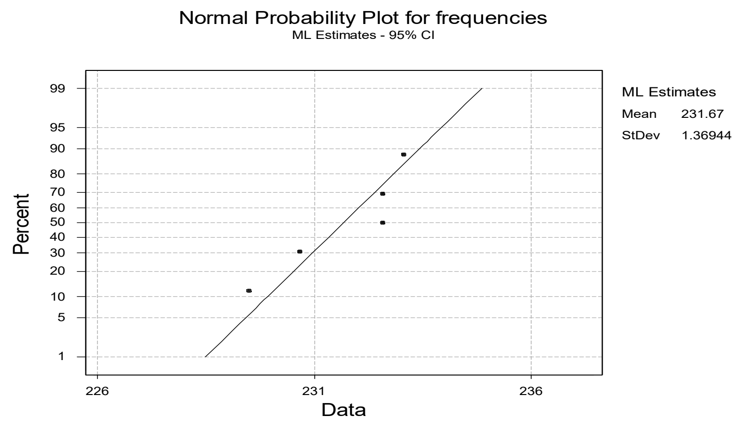
$$\mu \le \bar{x} + t_{0.005,13}\left(\frac{s}{\sqrt{n}}\right)$$
$$\mu \le 118.3 + 2.650\left(\frac{9.9}{\sqrt{14}}\right)$$
$$\mu \le 125.312$$

8-34 90% CI on the mean frequency of a beam subjected to loads

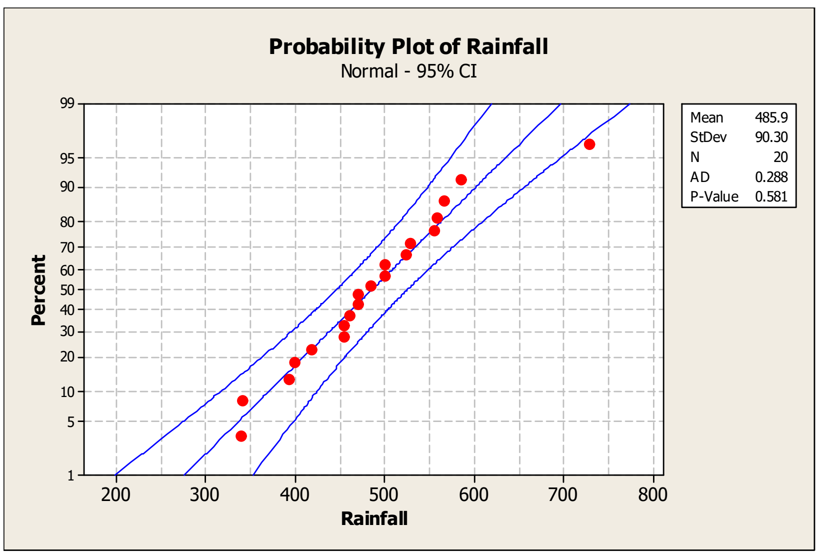
$\bar{x} = 231.67, \quad \mathrm{s} = 1.53, \quad \mathrm{n} = 5, \quad t_{\alpha/2,\mathrm{n}-1} = t_{.05,4} = 2.132$

$$\bar{x} - t_{0.05,4}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.05,4}\left(\frac{s}{\sqrt{n}}\right)$$
$$231.67 - 2.132\left(\frac{1.53}{\sqrt{5}}\right) \le \mu \le 231.67 - 2.132\left(\frac{1.53}{\sqrt{5}}\right)$$
$$230.2 \le \mu \le 233.1$$

By examining the normal probability plot, it appears that the data are normally distributed.



8-35 The data appear to be normally distributed based on the normal probability plot below.



95% confidence interval on mean annual rainfall

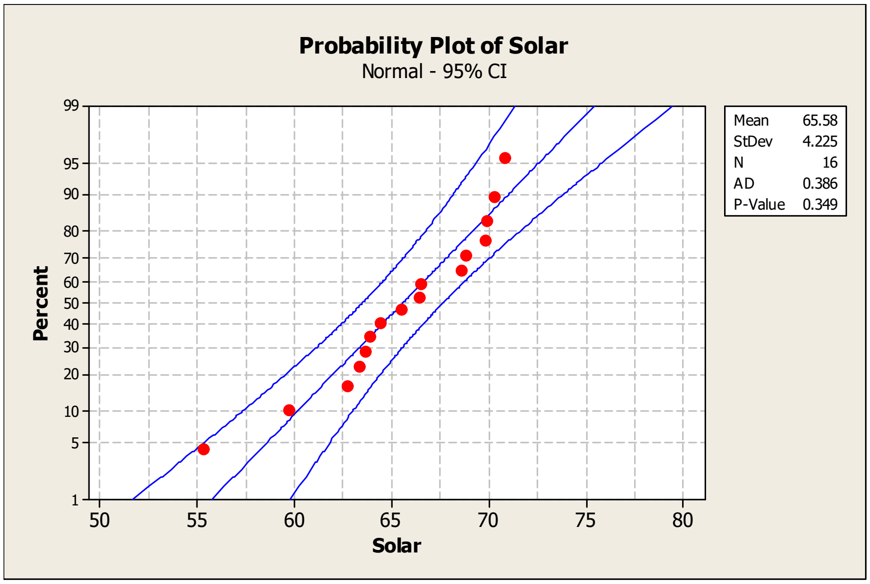
$$n = 20 \quad \bar{x} = 485.8 \quad s = 90.34 \quad t_{0.025,19} = 2.093$$

$$\bar{x} - t_{0.025,19}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,19}\left(\frac{s}{\sqrt{n}}\right)$$

$$485.8 - 2.093\left(\frac{90.34}{\sqrt{20}}\right) \le \mu \le 485.8 + 2.093\left(\frac{90.34}{\sqrt{20}}\right)$$

$$443.520 \le \mu \le 528.080$$

8-36 The data appear to be normally distributed based on the normal probability plot below.



95% confidence interval on mean solar energy consumed

$n = 16 \quad \bar{x} = 65.58 \quad s = 4.225 \quad t_{0.025,15} = 2.131$

$$\bar{x} - t_{0.025,15}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,15}\left(\frac{s}{\sqrt{n}}\right)$$

$$65.58 - 2.131\left(\frac{4.225}{\sqrt{16}}\right) \le \mu \le 65.58 + 2.131\left(\frac{4.225}{\sqrt{16}}\right)$$

$$63.329 \le \mu \le 67.831$$

8-37 99% confidence interval on mean current required
Assume that the data are a random sample from a normal distribution.

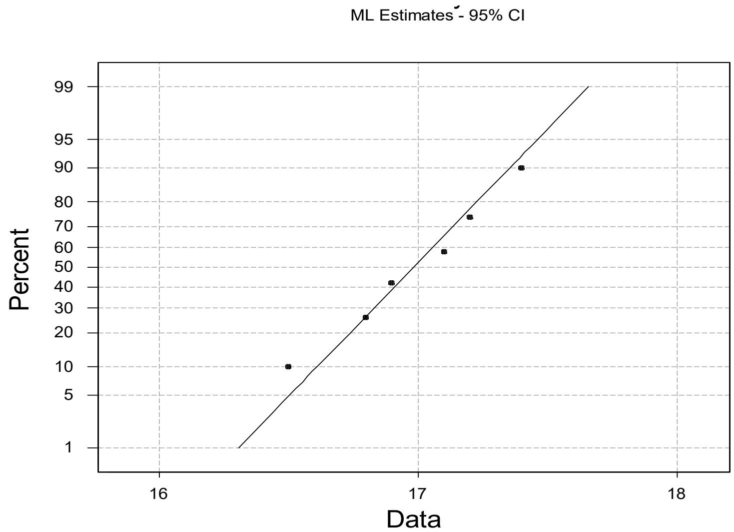
$n = 10 \quad \bar{x} = 317.2 \quad s = 15.7 \quad t_{0.005,9} = 3.250$

$$\bar{x} - t_{0.005,9}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.005,9}\left(\frac{s}{\sqrt{n}}\right)$$

$$317.2 - 3.250\left(\frac{15.7}{\sqrt{10}}\right) \le \mu \le 317.2 + 3.250\left(\frac{15.7}{\sqrt{10}}\right)$$

$$301.06 \le \mu \le 333.34$$

8-38 a) The data appear to be normally distributed based on the normal probability plot below.



b) 99% CI on the mean level of polyunsaturated fatty acid.
For $\alpha = 0.01$, $t_{\alpha/2,n-1} = t_{0.005,5} = 4.032$

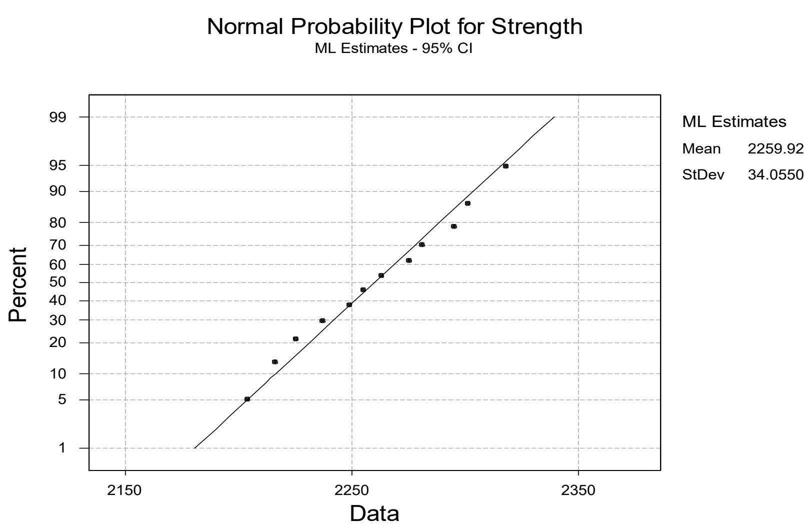
$$\bar{x} - t_{0.005,5}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.005,5}\left(\frac{s}{\sqrt{n}}\right)$$

$$16.98 - 4.032\left(\frac{0.319}{\sqrt{6}}\right) \le \mu \le 16.98 + 4.032\left(\frac{0.319}{\sqrt{6}}\right)$$

$$16.455 \le \mu \le 17.505$$

The 99% confidence for the mean polyunsaturated fat is (16.455, 17.505). There is high confidence that the true mean is in this interval

8-39 a) The data appear to be normally distributed based on examination of the normal probability plot below.



b) 95% two-sided confidence interval on mean comprehensive strength

$$n = 12 \quad \bar{x} = 2259.9 \quad s = 35.6 \quad t_{0.025,11} = 2.201$$

$$\bar{x} - t_{0.025,11}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,11}\left(\frac{s}{\sqrt{n}}\right)$$

$$2259.9 - 2.201\left(\frac{35.6}{\sqrt{12}}\right) \le \mu \le 2259.9 + 2.201\left(\frac{35.6}{\sqrt{12}}\right)$$

$$2237.3 \le \mu \le 2282.5$$

c) 95% lower-confidence bound on mean strength

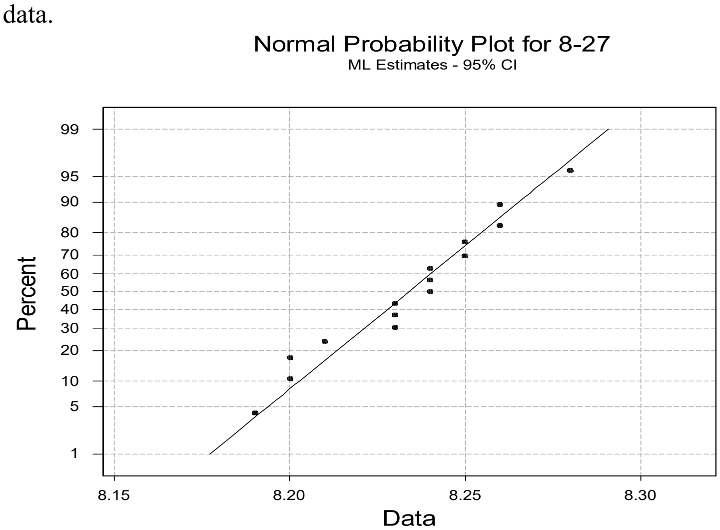
$$\bar{x} - t_{0.05,11}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$2259.9 - 1.796\left(\frac{35.6}{\sqrt{12}}\right) \le \mu$$

$$2241.4 \le \mu$$

8-40

a) According to the normal probability plot, there does not seem to be a severe deviation from normality for this data.



b) 95% two-sided confidence interval on mean rod diameter

For $\alpha = 0.05$ and $n = 15$, $t_{\alpha/2,n-1} = t_{0.025,14} = 2.145$

$$\bar{x} - t_{0.025,14}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,14}\left(\frac{s}{\sqrt{n}}\right)$$

$$8.23 - 2.145\left(\frac{0.025}{\sqrt{15}}\right) \le \mu \le 8.23 + 2.145\left(\frac{0.025}{\sqrt{15}}\right)$$

$$8.216 \le \mu \le 8.244$$

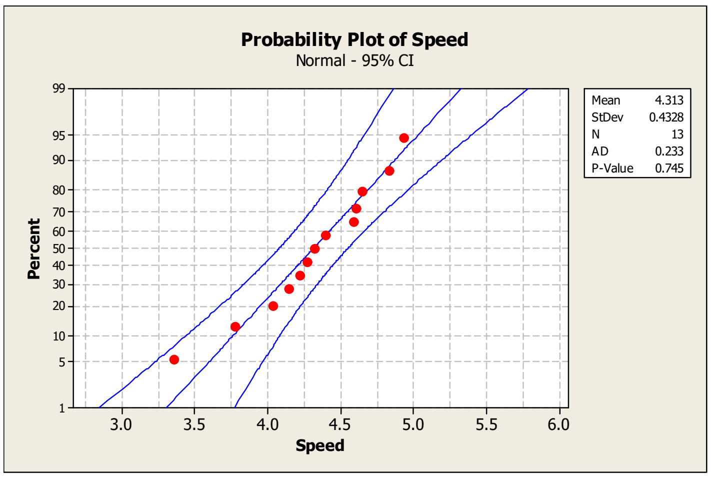
c) 95% upper confidence bound on mean rod diameter $t_{0.05,14} = 1.761$

$$\mu \le \bar{x} + t_{0.025,14}\left(\frac{s}{\sqrt{n}}\right)$$

$$\mu \le 8.23 + 1.761\left(\frac{0.025}{\sqrt{15}}\right)$$

$$\mu \le 8.241$$

8-41 a) The data appear to be normally distributed based on examination of the normal probability plot below.



b) 95% confidence interval on mean speed-up

$n = 13 \quad \bar{x} = 4.313 \quad s = 0.4328 \quad t_{0.025,12} = 2.179$

$$\bar{x} - t_{0.025,12}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,12}\left(\frac{s}{\sqrt{n}}\right)$$

$$4.313 - 2.179\left(\frac{0.4328}{\sqrt{13}}\right) \le \mu \le 4.313 + 2.179\left(\frac{0.4328}{\sqrt{13}}\right)$$

$$4.051 \le \mu \le 4.575$$

c) 95% lower confidence bound on mean speed-up

$n = 13 \quad \bar{x} = 4.313 \quad s = 0.4328 \quad t_{0.05,12} = 1.782$

$$\bar{x} - t_{0.05,12}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$4.313 - 1.782\left(\frac{0.4328}{\sqrt{13}}\right) \le \mu$$

$$4.099 \le \mu$$

8-42 95% lower bound confidence for the mean wall thickness given $\bar{x} = 4.05$, $s = 0.08$, $n = 25$

$t_{\alpha,n-1} = t_{0.05,24} = 1.711$

$$\bar{x} - t_{0.05,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$4.05 - 1.711\left(\frac{0.08}{\sqrt{25}}\right) \le \mu$$

$$4.023 \le \mu$$

There is high confidence that the true mean wall thickness is greater than 4.023 mm.

8-43 a) The data appear to be normally distributed.

b) 99% two-sided confidence interval on mean percentage enrichment

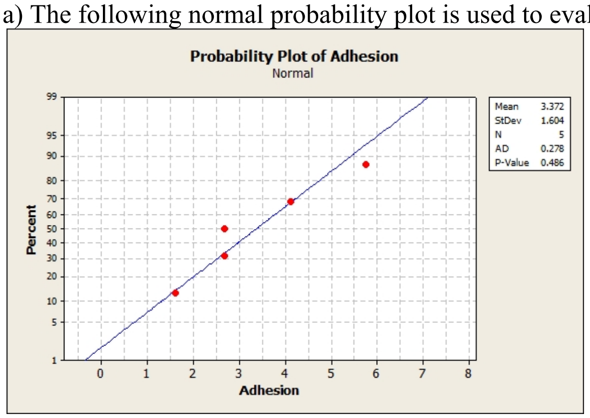
For $\alpha = 0.01$ and $n = 12$, $t_{\alpha/2,n-1} = t_{0.005,11} = 3.106$, $\bar{x} = 2.9017$ $s = 0.0993$

$$\bar{x} - t_{0.005,11}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.005,11}\left(\frac{s}{\sqrt{n}}\right)$$

$$2.902 - 3.106\left(\frac{0.0993}{\sqrt{12}}\right) \le \mu \le 2.902 + 3.106\left(\frac{0.0993}{\sqrt{12}}\right)$$

$$2.813 \le \mu \le 2.991$$

8-44 a) The following normal probability plot is used to evaluate the distribution.



There is no obvious deviation from normality.

b) 95% CI for μ, $n = 5$ $\bar{x} = 18.56$, $s = 1.604$, $t_{0.025,4} = 2.776$

$$\bar{x} - ts/\sqrt{n} \le \mu \le \bar{x} + ts/\sqrt{n}$$

$$3.372 - 2.776(1.604/\sqrt{5}) \le \mu \le 3.372 + 2.776(1.604/\sqrt{5})$$

$$1.38 \le \mu \le 5.36$$

8-45 a) 95% CI for μ, $n = 12$, $\bar{x} = 2.082$, $s = 0.1564$, $t_{0.025,11} = 2.201$

$$\bar{x} - ts/\sqrt{n} \le \mu \le \bar{x} + ts/\sqrt{n}$$

$$2.082 - 2.210(0.1564/\sqrt{12}) \le \mu \le 2.082 + 2.210(0.1564/\sqrt{12})$$

$$1.98 \le \mu \le 2.18$$

b) The lower bound of 95% confidence interval is greater than the historical average of 1.95. Therefore, there is evidence that this clinic performs more CAT scans than usual.

A one-sided confidence interval would be more appropriate to answer this question. The one-sided interval follows.

$t_{0.05,\,11} = 1.7959$

$$\bar{x} - ts/\sqrt{n} \le \mu$$
$$2.082 - 1.7959\,(0.1564 / \sqrt{12}) \le \mu$$
$$2.00 \le \mu$$

and the same conclusion is provided by this interval.

Section 8-3

8-46 $\chi^2_{0.05,10} = 18.31$ $\chi^2_{0.025,15} = 27.49$ $\chi^2_{0.01,12} = 26.22$

$\chi^2_{0.95,20} = 10.85$ $\chi^2_{0.99,18} = 7.01$ $\chi^2_{0.995,16} = 5.14$

$\chi^2_{0.005,25} = 46.93$

8-47 a) 95% upper CI and df = 24 $\chi^2_{1-\alpha,df} = \chi^2_{0.95,24} = 13.85$

b) 99% lower CI and df = 9 $\chi^2_{\alpha,df} = \chi^2_{0.01,9} = 21.67$

c) 90% CI and df = 19

$\chi^2_{\alpha/2,df} = \chi^2_{0.05,19} = 30.14$ and $\chi^2_{1-\alpha/2,df} = \chi^2_{0.95,19} = 10.12$

8-48 99% lower confidence bound for σ^2

For $\alpha = 0.01$ and n = 15, $\chi^2_{\alpha,n-1} = \chi^2_{0.01,14} = 29.14$

$$\frac{14(0.008)^2}{29.14} \le \sigma^2$$
$$0.00003075 \le \sigma^2$$

8-49 99% lower confidence bound for σ from the previous exercise is

$$0.00003075 \le \sigma^2$$
$$0.005545 \le \sigma$$

One may take the square root of the variance bound to obtain the confidence bound for the standard deviation.

8-50 95% two sided confidence interval for σ

$n = 10 \quad s = 4.8$

$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,9} = 19.02$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,9} = 2.70$

$$\frac{9(4.8)^2}{19.02} \le \sigma^2 \le \frac{9(4.8)^2}{2.70}$$
$$10.90 \le \sigma^2 \le 76.80$$
$$3.30 < \sigma < 8.76$$

8-51 95% confidence interval for σ given n = 51, s = 0.37

First find the confidence interval for σ^2

For α = 0.05 and n = 51, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,50} = 71.42$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,50} = 32.36$

$$\frac{50(0.37)^2}{71.42} \le \sigma^2 \le \frac{50(0.37)^2}{32.36}$$

$0.096 \le \sigma^2 \le 0.2115$

Take the square root of the endpoints of this interval to obtain

$0.31 < \sigma < 0.46$

8-52 95% confidence interval for σ

$n = 17 \quad s = 0.09$

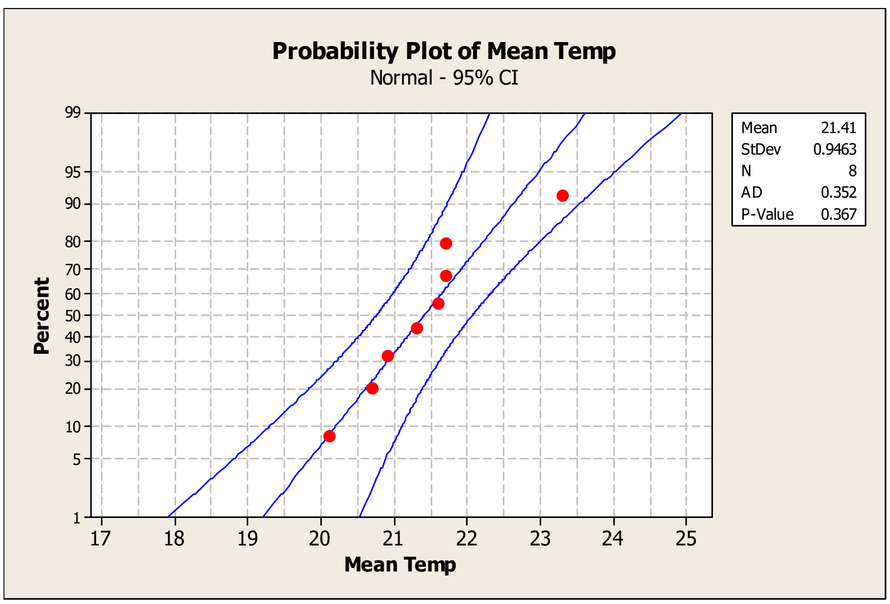
$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,16} = 28.85$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,16} = 6.91$

$$\frac{16(0.09)^2}{28.85} \le \sigma^2 \le \frac{16(0.09)^2}{6.91}$$

$$0.0045 \le \sigma^2 \le 0.0188$$

$$0.067 < \sigma < 0.137$$

8-53 The data appear to be normally distributed based on examination of the normal probability plot below.



95% confidence interval for σ

$n = 8 \quad s = 0.9463$

$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,7} = 16.01$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,7} = 1.69$

$$\frac{7(0.9463)^2}{16.01} \le \sigma^2 \le \frac{7(0.9463)^2}{1.69}$$

$$0.392 \le \sigma^2 \le 3.709$$

$$0.626 < \sigma < 1.926$$

8-54 95% confidence interval for σ

$n = 41 \quad s = 15.99$

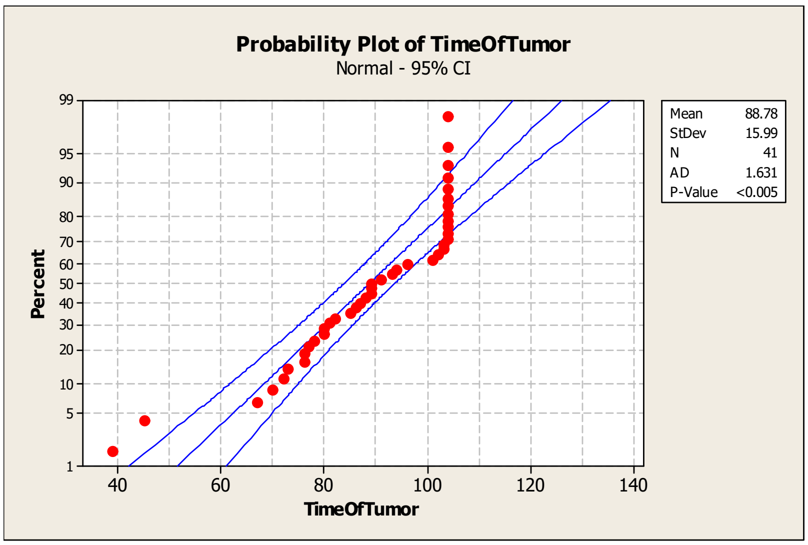
$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,40} = 59.34$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,40} = 24.43$

$$\frac{40(15.99)^2}{59.34} \le \sigma^2 \le \frac{40(15.99)^2}{24.43}$$

$$172.35 \le \sigma^2 \le 418.633$$

$$13.13 < \sigma < 20.46$$

The data do not appear to be normally distributed based on examination of the normal probability plot below. Therefore, the 95% confidence interval for σ is invalid.



8-55 95% confidence interval for σ

$n = 15 \quad s = 0.00831$

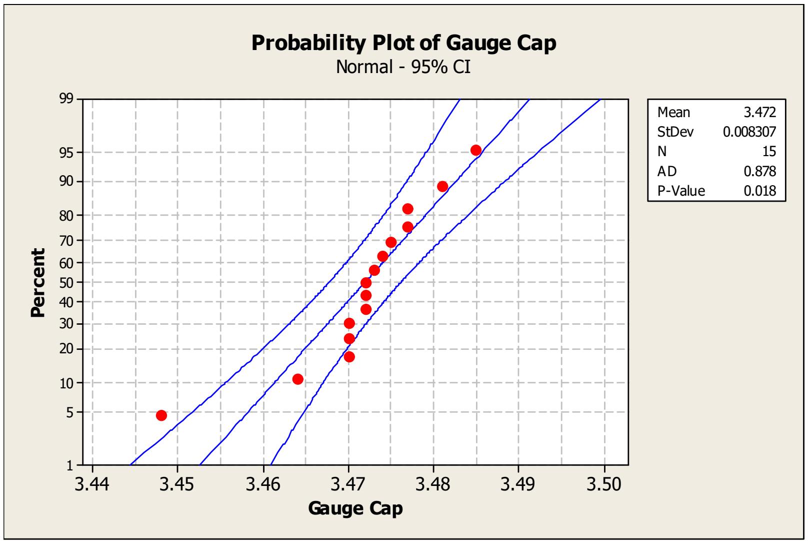
$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,14} = 26.12$ and $\chi^2_{1-\alpha,n-1} = \chi^2_{0.95,14} = 6.53$

$$\sigma^2 \le \frac{14(0.00831)^2}{6.53}$$

$$\sigma^2 \le 0.000148$$

$$\sigma \le 0.0122$$

The data do not appear to be normally distributed based on an examination of the normal probability plot below. Therefore, the 95% confidence interval for σ is not valid.



8-56 a) 99% two-sided confidence interval on σ^2

$$n = 10 \qquad s = 1.913 \qquad \chi^2_{0.005,9} = 23.59 \text{ and } \chi^2_{0.995,9} = 1.73$$

$$\frac{9(1.913)^2}{23.59} \le \sigma^2 \le \frac{9(1.913)^2}{1.73}$$

$$1.396 \le \sigma^2 \le 19.038$$

b) 99% lower confidence bound for σ^2

For $\alpha = 0.01$ and n = 10, $\chi^2_{\alpha,n-1} = \chi^2_{0.01,9} = 21.67$

$$\frac{9(1.913)^2}{21.67} \le \sigma^2$$

$$1.5199 \le \sigma^2$$

c) 90% lower confidence bound for σ^2

For $\alpha = 0.1$ and n = 10, $\chi^2_{\alpha,n-1} = \chi^2_{0.1,9} = 14.68$

$$\frac{9(1.913)^2}{14.68} \le \sigma^2$$

$$2.2436 \le \sigma^2$$

$$1.498 \le \sigma$$

d) The lower confidence bound of the 99% two-sided interval is less than the one-sided interval. The lower confidence bound for σ^2 is in part (c) is greater because the confidence is lower.

8-57 95% two sided confidence interval for σ, $n = 39 \qquad s = 0.6295$

$$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,38} = 55.896 \text{ and } \chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,38} = 22.878$$

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

$$\frac{38(0.6295)^2}{55.896} \le \sigma^2 \le \frac{38(0.6295)^2}{22.878}$$

$$0.265 \le \sigma^2 \le 0.658$$

$$0.514 < \sigma < 0.811$$

8-58 a) 95% two sided confidence interval for σ, $n = 12 \quad s = 0.1564$

$$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,11} = 21.920 \text{ and } \chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,11} = 3.816$$

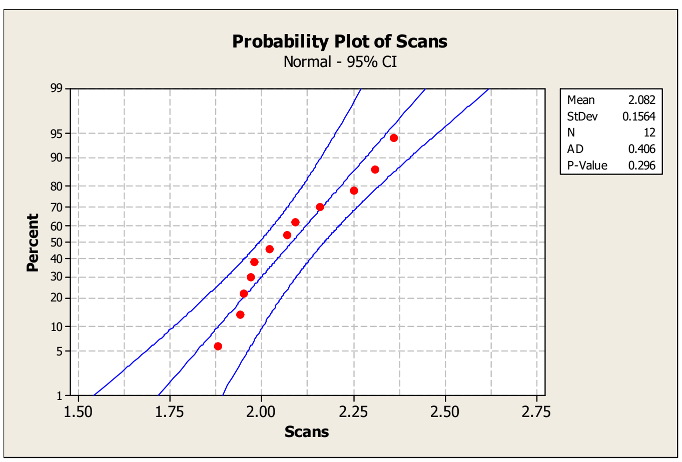
$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

$$\frac{11(0.1564)^2}{21.920} \le \sigma^2 \le \frac{11(0.1564)^2}{3.816}$$

$$0.012 \le \sigma^2 \le 0.070$$

$$0.111 < \sigma < 0.265$$

b) A normal probability plot can be used to check the normality assumption.
The following plot does not indicate serious departures from a normal distribution.



Section 8-4

8-59 a) 95% Confidence Interval on the fraction defective produced with this tool.

$$\hat{p} = \frac{13}{300} = 0.04333 \qquad n = 300 \qquad z_{\alpha/2} = 1.96$$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.04333 - 1.96\sqrt{\frac{0.04333(0.95667)}{300}} \le p \le 0.04333 + 1.96\sqrt{\frac{0.04333(0.95667)}{300}}$$

$$0.02029 \le p \le 0.06637$$

b) 95% upper confidence bound $z_{\alpha} = z_{0.05} = 1.65$

$$p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$p \le 0.04333 + 1.650\sqrt{\frac{0.04333(0.95667)}{300}}$$

$$p \le 0.06273$$

8-60 a) 95% Confidence Interval on the proportion of such tears that will heal.

$\hat{p} = 0.676 \quad n = 37 \; z_{\alpha/2} = 1.96$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.676 - 1.96\sqrt{\frac{0.676(0.324)}{37}} \le p \le 0.676 + 1.96\sqrt{\frac{0.676(0.324)}{37}}$$

$$0.5245 \le p \le 0.827$$

b) 95% lower confidence bound on the proportion of such tears that will heal.

$$\hat{p} - z_{\alpha}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p$$

$$0.676 - 1.64\sqrt{\frac{0.676(0.33)}{37}} \le p$$

$$0.549 \le p$$

8-61 a) 95% confidence interval for the proportion of college graduates in Ohio that voted for George Bush.

$\hat{p} = \frac{412}{768} = 0.536 \quad n = 768 \; z_{\alpha/2} = 1.96$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.536 - 1.96\sqrt{\frac{0.536(0.464)}{768}} \le p \le 0.536 + 1.96\sqrt{\frac{0.536(0.464)}{768}}$$

$$0.501 \le p \le 0.571$$

b) 95% lower confidence bound on the proportion of college graduates in Ohio that voted for George Bush.

$$\hat{p} - z_{\alpha}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p$$
$$0.536 - 1.64\sqrt{\frac{0.536(0.464)}{768}} \le p$$
$$0.506 \le p$$

8-62 a) 95% Confidence Interval on the death rate from lung cancer.

$$\hat{p} = \frac{823}{1000} = 0.823 \quad n = 1000 \; z_{\alpha/2} = 1.96$$
$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$
$$0.823 - 1.96\sqrt{\frac{0.823(0.177)}{1000}} \le p \le 0.823 + 1.96\sqrt{\frac{0.823(0.177)}{1000}}$$
$$0.7993 \le p \le 0.8467$$

b) E = 0.03, α = 0.05, $z_{\alpha/2} = z_{0.025}$ = 1.96 and $\hat{p}$ = 0.823 as the initial estimate of p,

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 \hat{p}(1-\hat{p}) = \left(\frac{1.96}{0.03}\right)^2 0.823(1-0.823) = 621.79,$$

$n \cong 622$.

c) E = 0.03, α = 0.05, $z_{\alpha/2} = z_{0.025}$ = 1.96 at least 95% confident

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 (0.25) = \left(\frac{1.96}{0.03}\right)^2 (0.25) = 1067.11,$$

$n \cong 1068$.

8-63 a) 95% Confidence Interval on the proportion of rats that are under-weight.

$$\hat{p} = \frac{12}{30} = 0.4 \quad n = 30 \; z_{\alpha/2} = 1.96$$
$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$
$$0.4 - 1.96\sqrt{\frac{0.4(0.6)}{30}} \le p \le 0.4 + 1.96\sqrt{\frac{0.4(0.6)}{30}}$$
$$0.225 \le p \le 0.575$$

b) E = 0.02, α = 0.05, $z_{\alpha/2} = z_{0.025}$ = 1.96 and $\hat{p}$ = 0.4as the initial estimate of p,

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 \hat{p}(1-\hat{p}) = \left(\frac{1.96}{0.02}\right)^2 0.4(1-0.4) = 2304.96,$$

$n \cong 2305$.

c) E = 0.02, α = 0.05, $z_{\alpha/2} = z_{0.025}$ = 1.96 at least 95% confident

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 (0.25) = \left(\frac{1.96}{0.02}\right)^2 (0.25) = 2401.$$

8-64 a) 95% Confidence Interval on the true proportion of helmets showing damage

$$\hat{p} = \frac{18}{50} = 0.36 \qquad n = 50 \qquad z_{\alpha/2} = 1.96$$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.36 - 1.96\sqrt{\frac{0.36(0.64)}{50}} \le p \le 0.36 + 1.96\sqrt{\frac{0.36(0.64)}{50}}$$

$$0.227 \le p \le 0.493$$

b) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.36(1-0.36) = 2212.76$

$n \cong 2213$

c) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.5(1-0.5) = 2401$

8-65 The worst case would be for $p = 0.5$, thus with E = 0.05 and $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$ we obtain a sample size of:

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{2.58}{0.05}\right)^2 0.5(1-0.5) = 665.64, \quad n \cong 666$$

8-66 E = 0.017, $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{2.58}{0.017}\right)^2 0.5(1-0.5) = 5758.13, \quad n \cong 5759$$

8-67 a) $\hat{p} = \frac{466}{500} = 0.932 \qquad n = 500 \quad z_{\alpha/2} = 1.96$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.932 - 1.96\sqrt{\frac{0.932(0.068)}{500}} \le p \le 0.932 + 1.96\sqrt{\frac{0.932(0.068)}{500}}$$

$$0.910 \le p \le 0.945$$

b) E = 0.01, $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$ and $\hat{p} = 0.932$ as the initial estimate of p,

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 \hat{p}(1-\hat{p}) = \left(\frac{1.96}{0.01}\right)^2 0.932(1-0.932) = 2439.48,$$

$n \cong 2440$

c) $E = 0.01$, $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$
Here we assume the proportion value that generates the greatest variance; namely $p = 0.5$.

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 (0.25) = \left(\frac{1.96}{0.01}\right)^2 (0.25) = 9623.05,$$

$n \cong 9624$

8-68 a) $\hat{p} = \frac{180}{200} = 0.9 \quad n = 200 \; z_{\alpha/2} = 1.96$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.9 - 1.96\sqrt{\frac{0.9(0.1)}{200}} \le p \le 0.9 + 1.96\sqrt{\frac{0.9(0.1)}{200}}$$

$$0.858 \le p \le 0.941$$

b) No, the claim of 93% is within the confidence interval for the true proportion of germinated seeds

8-69 $\hat{p} = \frac{13}{300} = 0.0433 \quad n = 300 \; z_{\alpha/2} = 1.96$

The AC confidence interval follows.

$$\left[\hat{p} + \frac{z^2{}_{\alpha/2}}{2n} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z^2{}_{\alpha/2}}{4n^2}}\right] \Big/ \left[1 + \frac{z^2{}_{\alpha/2}}{n}\right] \le p \le \left[\hat{p} + \frac{z^2{}_{\alpha/2}}{2n} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z^2{}_{\alpha/2}}{4n^2}}\right] \Big/ \left[1 + \frac{z^2{}_{\alpha/2}}{n}\right]$$

$$0.025 \le p \le 0.073$$

The traditional confidence interval follows.

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.04333 - 1.96\sqrt{\frac{0.04333\,(0.95667\,)}{300}} \le p \le 0.04333 + 1.96\sqrt{\frac{0.04333\,(0.95667\,)}{300}}$$

$$0.020 \le p \le 0.066$$

The AC confidence interval is similar to the original one.

8-70 The AC confidence interval follows.

$\hat{p} = \frac{25}{37} = 0.6757 \quad n = 37 \; z_{\alpha/2} = 1.96$

$$\left[\hat{p} + \frac{z^2{}_{\alpha/2}}{2n} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z^2{}_{\alpha/2}}{4n^2}}\right] \Big/ \left[1 + \frac{z^2{}_{\alpha/2}}{n}\right] \le p \le \left[\hat{p} + \frac{z^2{}_{\alpha/2}}{2n} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z^2{}_{\alpha/2}}{4n^2}}\right] \Big/ \left[1 + \frac{z^2{}_{\alpha/2}}{n}\right]$$

$$0.514 \le p \le 0.804$$

The traditional confidence interval follows.

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.676 - 1.96\sqrt{\frac{0.676(0.324)}{37}} \le p \le 0.676 + 1.96\sqrt{\frac{0.676(0.324)}{37}}$$

$$0.5245 \le p \le 0.827$$

The AC confidence interval is shifted to lower values.

8-71 The AC confidence interval follows.

$$\hat{p} = \frac{466}{500} = 0.932 \qquad n = 500 \; z_{\alpha/2} = 1.96$$

$$\left[\hat{p} + \frac{z^2_{\alpha/2}}{2n} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z^2_{\alpha/2}}{4n^2}}\right] / \left[1 + \frac{z^2_{\alpha/2}}{n}\right] \le p \le \left[\hat{p} + \frac{z^2_{\alpha/2}}{2n} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z^2_{\alpha/2}}{4n^2}}\right] / \left[1 + \frac{z^2_{\alpha/2}}{n}\right]$$

$$0.906 \le p \le 0.951$$

The traditional confidence interval follows.

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.932 - 1.96\sqrt{\frac{0.932(0.068)}{500}} \le p \le 0.932 + 1.96\sqrt{\frac{0.932(0.068)}{500}}$$

$$0.910 \le p \le 0.945$$

The AC confidence interval is similar to the original one.

8-72 The AC confidence interval follows.

$$\hat{p} = \frac{180}{200} = 0.9 \qquad n = 200 \; z_{\alpha/2} = 1.96$$

$$\left[\hat{p} + \frac{z^2_{\alpha/2}}{2n} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z^2_{\alpha/2}}{4n^2}}\right] / \left[1 + \frac{z^2_{\alpha/2}}{n}\right] \le p \le \left[\hat{p} + \frac{z^2_{\alpha/2}}{2n} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z^2_{\alpha/2}}{4n^2}}\right] / \left[1 + \frac{z^2_{\alpha/2}}{n}\right]$$

$$0.851 \le p \le 0.934$$

The traditional confidence interval follows.

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.9 - 1.96\sqrt{\frac{0.9(0.1)}{200}} \le p \le 0.9 + 1.96\sqrt{\frac{0.9(0.1)}{200}}$$

$$0.858 \le p \le 0.941$$

The AC confidence interval is slightly narrower than the original one.

<u>Section 8-6</u>

8-73 95% prediction interval on the life of the next tire given $\overline{x} = 60139.7$ $s = 3645.94$ $n = 16$
for $\alpha=0.05$ $t_{\alpha/2,n-1} = t_{0.025,15} = 2.131$

$$\bar{x} - t_{0.025,15}s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.025,15}s\sqrt{1+\frac{1}{n}}$$

$$60139.7 - 2.131(3645.94)\sqrt{1+\frac{1}{16}} \le x_{n+1} \le 60139.7 + 2.131(3645.94)\sqrt{1+\frac{1}{16}}$$

$$52131.1 \le x_{n+1} \le 68148.3$$

The prediction interval is considerably wider than the 95% confidence interval ($58{,}197.3 \le \mu \le 62{,}082.07$). This is expected because the prediction interval includes the variability in the parameter estimates as well as the variability in a future observation.

8-74 99% prediction interval on the Izod impact data

$n = 20 \quad \bar{x} = 1.25 \quad s = 0.25 \quad t_{0.005,19} = 2.861$

$$\bar{x} - t_{0.005,19}s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.005,19}s\sqrt{1+\frac{1}{n}}$$

$$1.25 - 2.861(0.25)\sqrt{1+\frac{1}{20}} \le x_{n+1} \le 1.25 + 2.861(0.25)\sqrt{1+\frac{1}{20}}$$

$$0.517 \le x_{n+1} \le 1.983$$

The lower bound of the 99% prediction interval is considerably lower than the 99% confidence interval ($1.108 \le \mu \le \infty$). This is expected because the prediction interval needs to include the variability in the parameter estimates as well as the variability in a future observation.

8-75 95% prediction Interval on the volume of syrup of the next beverage dispensed

$\bar{x} = 1.10 \quad s = 0.015 \quad n = 25 \quad t_{\alpha/2,n-1} = t_{0.025,24} = 2.064$

$$\bar{x} - t_{0.025,24}s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.025,24}s\sqrt{1+\frac{1}{n}}$$

$$1.10 - 2.064(0.015)\sqrt{1+\frac{1}{25}} \le x_{n+1} \le 1.10 - 2.064(0.015)\sqrt{1+\frac{1}{25}}$$

$$1.068 \le x_{n+1} \le 1.13$$

The prediction interval is wider than the confidence interval: $1.094 \le \mu \le 1.106$

8-76 90% prediction interval the value of the natural frequency of the next beam of this type that will be tested. given $\bar{x} = 231.67$, $s = 1.53$ For $\alpha = 0.10$ and $n = 5$, $t_{\alpha/2,n-1} = t_{0.05,4} = 2.132$

$$\bar{x} - t_{0.05,4}s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.05,4}s\sqrt{1+\frac{1}{n}}$$

$$231.67 - 2.132(1.53)\sqrt{1+\frac{1}{5}} \le x_{n+1} \le 231.67 - 2.132(1.53)\sqrt{1+\frac{1}{5}}$$

$$228.1 \le x_{n+1} \le 235.2$$

The 90% prediction interval is wider than the 90% CI.

8-77 95% Prediction Interval on the volume of syrup of the next beverage dispensed
$n = 20 \quad \bar{x} = 485.8 \quad s = 90.34 \quad t_{\alpha/2,n-1} = t_{0.025,19} = 2.093$

$$\bar{x} - t_{0.025,19}\, s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.025,19}\, s\sqrt{1+\frac{1}{n}}$$
$$485.8 - 2.093(90.34)\sqrt{1+\frac{1}{20}} \le x_{n+1} \le 485.8 - 2.093(90.34)\sqrt{1+\frac{1}{20}}$$
$$292.049 \le x_{n+1} \le 679.551$$

The 95% prediction interval is wider than the 95% confidence interval.

8-78 99% prediction interval on the polyunsaturated fat
$n = 6 \quad \bar{x} = 16.98 \quad s = 0.319 \quad t_{0.005,5} = 4.032$

$$\bar{x} - t_{0.005,5}\, s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.005,5}\, s\sqrt{1+\frac{1}{n}}$$
$$16.98 - 4.032(0.319)\sqrt{1+\frac{1}{6}} \le x_{n+1} \le 16.98 + 4.032(0.319)\sqrt{1+\frac{1}{6}}$$
$$15.59 \le x_{n+1} \le 18.37$$

The prediction interval is much wider than the confidence interval $16.455 \le \mu \le 17.505$.

8-79 Given $\bar{x} = 317.2$ $s = 15.7$ $n = 10$ for $\alpha=0.05$ $t_{\alpha/2,n-1} = t_{0.005,9} = 3.250$

$$\bar{x} - t_{0.005,9}\, s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.005,9}\, s\sqrt{1+\frac{1}{n}}$$
$$317.2 - 3.250(15.7)\sqrt{1+\frac{1}{10}} \le x_{n+1} \le 317.2 - 3.250(15.7)\sqrt{1+\frac{1}{10}}$$
$$263.7 \le x_{n+1} \le 370.7$$

The prediction interval is wider.

8-80 95% prediction interval on the next rod diameter tested
$n = 15 \quad \bar{x} = 8.23 \quad s = 0.025 \quad t_{0.025,14} = 2.145$

$$\bar{x} - t_{0.025,14}\, s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.025,14}\, s\sqrt{1+\frac{1}{n}}$$
$$8.23 - 2.145(0.025)\sqrt{1+\frac{1}{15}} \le x_{n+1} \le 8.23 - 2.145(0.025)\sqrt{1+\frac{1}{15}}$$
$$8.17 \le x_{n+1} \le 8.29$$

95% two-sided confidence interval on mean rod diameter is $8.216 \le \mu \le 8.244$

8-81 90% prediction interval on the next specimen of concrete tested
given $\bar{x} = 2260$ $s = 35.57$ $n = 12$ for $\alpha = 0.05$ and $n = 12$, $t_{\alpha/2,n-1} = t_{0.05,11} = 1.796$

$$\bar{x} - t_{0.05,11} s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.05,11} s\sqrt{1+\frac{1}{n}}$$

$$2260 - 1.796(35.57)\sqrt{1+\frac{1}{12}} \le x_{n+1} \le 2260 + 1.796(35.57)\sqrt{1+\frac{1}{12}}$$

$$2193.5 \le x_{n+1} \le 2326.5$$

8-82 90% prediction interval on wall thickness on the next bottle tested.
Given $\bar{x} = 4.05$ $s = 0.08$ $n = 25$ for $t_{\alpha/2,n-1} = t_{0.05,24} = 1.711$

$$\bar{x} - t_{0.05,24} s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.05,24} s\sqrt{1+\frac{1}{n}}$$

$$4.05 - 1.711(0.08)\sqrt{1+\frac{1}{25}} \le x_{n+1} \le 4.05 - 1.711(0.08)\sqrt{1+\frac{1}{25}}$$

$$3.91 \le x_{n+1} \le 4.19$$

8-83 90% prediction interval for enrichment data given $\bar{x} = 2.9$ $s = 0.099$ $n = 12$ for $\alpha = 0.10$ and $n = 12$, $t_{\alpha/2,n-1} = t_{0.05,11} = 1.796$

$$\bar{x} - t_{0.05,12} s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.05,12} s\sqrt{1+\frac{1}{n}}$$

$$2.9 - 1.796(0.099)\sqrt{1+\frac{1}{12}} \le x_{n+1} \le 2.9 + 1.796(0.099)\sqrt{1+\frac{1}{12}}$$

$$2.71 \le x_{n+1} \le 3.09$$

The 90% confidence interval is

$$\bar{x} - t_{0.05,12} s\sqrt{\frac{1}{n}} \le \mu \le \bar{x} + t_{0.05,12} s\sqrt{\frac{1}{n}}$$

$$2.9 - 1.796(0.099)\sqrt{\frac{1}{12}} \le \mu \le 2.9 - 1.796(0.099)\sqrt{\frac{1}{12}}$$

$$2.85 \le \mu \le 2.95$$

The prediction interval is wider than the CI on the population mean with the same confidence.

The 99% confidence interval is

$$\bar{x} - t_{0.005,12} s\sqrt{\frac{1}{n}} \le \mu \le \bar{x} + t_{0.005,12} s\sqrt{\frac{1}{n}}$$

$$2.9 - 3.106(0.099)\sqrt{\frac{1}{12}} \le \mu \le 2.9 + 3.106(0.099)\sqrt{\frac{1}{12}}$$

$$2.81 \le \mu \le 2.99$$

The prediction interval is even wider than the CI on the population mean with greater confidence.

8-84 To obtain a one sided prediction interval, use $t_{\alpha,n-1}$ instead of $t_{\alpha/2,n-1}$
Because we want a 95% one sided prediction interval,
$t_{\alpha/2,n-1} = t_{0.05,24} = 1.711$ and $\bar{x} = 4.05$ $s = 0.08$ $n = 25$

$$\bar{x} - t_{0.05,24} s\sqrt{1+\frac{1}{n}} \le x_{n+1}$$

$$4.05 - 1.711(0.08)\sqrt{1+\frac{1}{25}} \le x_{n+1}$$

$$3.91 \le x_{n+1}$$

The prediction interval bound is lower than the confidence interval bound of 4.023 mm

8-85 95% tolerance interval on the life of the tires that has a 95% CL
Given $\bar{x}$ = 60139.7 s = 3645.94 n = 16 we find *k*=2.903

$$\bar{x} - ks,\ \bar{x} + ks$$

$$60139.7 - 2.903(3645.94),\ 60139.7 + 2.903(3645.94)$$

$$(49555.54,\ 70723.86)$$

95% confidence interval ($58{,}197.3 \le \mu \le 62{,}082.07$) is narrower than the 95% tolerance interval.

8-86 99% tolerance interval on the Izod impact strength PVC pipe that has a 90% CL
Given $\bar{x}$=1.25, s=0.25 and n=20 we find *k*=3.368

$$\bar{x} - ks,\ \bar{x} + ks$$

$$1.25 - 3.368(0.25),\ 1.25 + 3.368(0.25)$$

$$(0.408,\ 2.092)$$

The 99% tolerance interval is much wider than the 99% confidence interval on the population mean ($1.090 \le \mu \le 1.410$).

8-87 95% tolerance interval on the syrup volume that has 90% confidence level
$\bar{x}$ = 1.10 s = 0.015 n = 25 and k=2.474

$$\bar{x} - ks,\ \bar{x} + ks$$

$$1.10 - 2.474(0.015),\ 1.10 + 2.474(0.015)$$

$$(1.06,\ 1.14)$$

8-88 99% tolerance interval on the polyunsaturated fatty acid in this type of margarine that has a confidence level of 95% $\bar{x}$ = 16.98 s = 0.319 n=6 and k = 5.775

$$\bar{x} - ks,\ \bar{x} + ks$$

$$16.98 - 5.775(0.319),\ 16.98 + 5.775(0.319)$$

$$(15.14,\ 18.82)$$

The 99% tolerance interval is much wider than the 99% confidence interval on the population mean ($16.46 \le \mu \le 17.51$).

8-89 95% tolerance interval on the rainfall that has a confidence level of 95%

$$n = 20 \quad \bar{x} = 485.8 \quad s = 90.34 \quad k = 2.752$$

$$\bar{x} - ks, \bar{x} + ks$$
$$485.8 - 2.752(90.34), 485.8 + 2.752(90.34)$$
$$(237.184, 734.416)$$

The 95% tolerance interval is much wider than the 95% confidence interval on the population mean ($443.52 \le \mu \le 528.08$).

8-90 95% tolerance interval on the diameter of the rods in exercise 8-27 that has a 90% confidence level
$\bar{x}$ = 8.23 s = 0.0.25 n=15 and k=2.713

$$\bar{x} - ks, \bar{x} + ks$$
$$8.23 - 2.713(0.025), 8.23 + 2.713(0.025)$$
$$(8.16, 8.30)$$

The 95% tolerance interval is wider than the 95% confidence interval on the population mean ($8.216 \le \mu \le 8.244$).

8-91 99% tolerance interval on the brightness of television tubes that has a 95% CL
Given $\bar{x}$ = 317.2 s = 15.7 n = 10 we find k = 4.433

$$\bar{x} - ks, \bar{x} + ks$$
$$317.2 - 4.433(15.7), 317.2 + 4.433(15.7)$$
$$(247.60, 386.80)$$

The 99% tolerance interval is much wider than the 95% confidence interval on the population mean

$$301.06 \le \mu \le 333.34$$

8-92 90% tolerance interval on the comprehensive strength of concrete that has a 90% CL
Given $\bar{x}$ = 2260 s = 35.57 n = 12 we find k=2.404

$$\bar{x} - ks, \bar{x} + ks$$
$$2260 - 2.404(35.57), 2260 + 2.404(35.57)$$
$$(2174.5, 2345.5)$$

The 90% tolerance interval is much wider than the 95% confidence interval on the population mean $2237.3 \le \mu \le 2282.5$

8-93 99% tolerance interval on rod enrichment data that have a 95% CL
Given $\bar{x}$ = 2.9 s = 0.099 n = 12 we find k=4.150

$$\bar{x} - ks, \bar{x} + ks$$
$$2.9 - 4.150(0.099), 2.9 + 4.150(0.099)$$
$$(2.49, 3.31)$$

The 99% tolerance interval is much wider than the 95% CI on the population mean ($2.84 \le \mu \le 2.96$)

8-94 a) 90% tolerance interval on wall thickness measurements that have a 90% CL
Given $\bar{x}$ = 4.05 s = 0.08 n = 25 we find k=2.077

$$\bar{x} - ks,\ \bar{x} + ks$$
$$4.05 - 2.077(0.08),\ 4.05 + 2.077(0.08)$$
$$(3.88,\ 4.22)$$

The lower bound of the 90% tolerance interval is much lower than the lower bound on the 95% confidence interval on the population mean ($4.023 \le \mu \le \infty$)

b) 90% lower tolerance bound on bottle wall thickness that has confidence level 90%.
given $\bar{x} = 4.05$ $s = 0.08$ $n = 25$ and $k = 1.702$

$$\bar{x} - ks = 4.05 - 1.702(0.08) = 3.91$$

The lower tolerance bound is of interest if we want the wall thickness to be greater than a certain value so that a bottle will not break.

Supplemental Exercises

8-95 Where $\alpha_1 + \alpha_2 = \alpha$. Let $\alpha = 0.05$
Interval for $\alpha_1 = \alpha_2 = \alpha / 2 = 0.025$
The confidence level for $\bar{x} - 1.96\sigma / \sqrt{n} \le \mu \le \bar{x} + 1.96\sigma / \sqrt{n}$ is determined by the by the value of z_0 which is 1.96.
From Table III, we find $\Phi(1.96) = P(Z<1.96) = 0.975$ and the confidence level is 95%.
Interval for $\alpha_1 = 0.01,\quad \alpha_2 = 0.04$
The confidence interval is $\bar{x} - 2.33\sigma / \sqrt{n} \le \mu \le \bar{x} + 1.75\sigma / \sqrt{n}$, the confidence level is the same because $\alpha = 0.05$. The symmetric interval does not affect the level of significance; however, it does affect the width. The symmetric interval is narrower.

8-96 $\mu = 50$ σ unknown
a) $n = 16$ $\bar{x} = 52$ $s = 1.5$

$$t_o = \frac{52 - 50}{8 / \sqrt{16}} = 1$$

The *P*-value for $t_0 = 1$, degrees of freedom = 15, is between 0.1 and 0.25. Thus, we conclude that the results are not very unusual.

b) $n = 30$

$$t_o = \frac{52 - 50}{8 / \sqrt{30}} = 1.37$$

The P-value for $t_0 = 1.37$, degrees of freedom = 29, is between 0.05 and 0.1. Thus, we conclude that the results are somewhat unusual.

c) $n = 100$ (with $n > 30$, the standard normal table can be used for this problem)

$$z_o = \frac{52 - 50}{8 / \sqrt{100}} = 2.5$$

The P-value for $z_0 = 2.5$, is 0.00621. Thus we conclude that the results are very unusual.

d) For constant values of $\bar{x}$ and s, increasing only the sample size, we see that the standard error of $\bar{X}$ decreases and consequently a sample mean value of 52 when the true mean is 50 is more unusual for the larger sample sizes.

8-97 $\mu = 50, \quad \sigma^2 = 5$

a) For $n = 16$ find $P(s^2 \geq 7.44)$ or $P(s^2 \leq 2.56)$

$$P(S^2 \geq 7.44) = P\left(\chi^2_{15} \geq \frac{15(7.44)}{5^2}\right) = 0.05 \leq P\left(\chi^2_{15} \geq 22.32\right) \leq 0.10$$

Using computer software $P(S^2 \geq 7.44) = 0.0997$

$$P(S^2 \leq 2.56) = P\left(\chi^2_{15} \leq \frac{15(2.56)}{5}\right) = 0.05 \leq P\left(\chi^2_{15} \leq 7.68\right) \leq 0.10$$

Using computer software $P(S^2 \leq 2.56) = 0.064$

b) For $n = 30$ find $P(S^2 \geq 7.44)$ or $P(S^2 \leq 2.56)$

$$P(S^2 \geq 7.44) = P\left(\chi^2_{29} \geq \frac{29(7.44)}{5}\right) = 0.025 \leq P\left(\chi^2_{29} \geq 43.15\right) \leq 0.05$$

Using computer software $P(S^2 \geq 7.44) = 0.044$

$$P(S^2 \leq 2.56) = P\left(\chi^2_{29} \leq \frac{29(2.56)}{5}\right) = 0.01 \leq P\left(\chi^2_{29} \leq 14.85\right) \leq 0.025$$

Using computer software $P(S^2 \leq 2.56) = 0.014$.

c) For $n = 71$ $P(s^2 \geq 7.44)$ or $P(s^2 \leq 2.56)$

$$P(S^2 \geq 7.44) = P\left(\chi^2_{70} \geq \frac{70(7.44)}{5}\right) = 0.005 \leq P\left(\chi^2_{70} \geq 104.16\right) \leq 0.01$$

Using computer software $P(S^2 \geq 7.44) = 0.0051$

$$P(S^2 \leq 2.56) = P\left(\chi^2_{70} \leq \frac{70(2.56)}{5}\right) = P\left(\chi^2_{70} \leq 35.84\right) \leq 0.005$$

Using computer software $P(S^2 \leq 2.56) < 0.001$

d) The probabilities decrease as n increases. As n increases, the sample variance should approach the population variance; therefore, the likelihood of obtaining a sample variance much larger than the population variance decreases.

e) The probabilities decrease as n increases. As n increases, the sample variance should approach the population variance; therefore, the likelihood of obtaining a sample variance much smaller than the population variance decreases.

8-98 a) The data appear to follow a normal distribution based on the normal probability plot because the data fall along a straight line.

b) It is important to check for normality of the distribution underlying the sample data because the confidence intervals to be constructed have the assumption of normality (especially since the sample size is less than 30 and the central limit theorem does not apply).

c) No, with 95% confidence, we cannot infer that the true mean if14.05 because this value is not contained within the given 95% confidence interval.

d) As with part b, to construct a confidence interval on the variance, the normality assumption must hold for the results to be reliable.

e) Yes, it is reasonable to infer that the variance could be 0.35 since the 95% confidence interval on the variance contains this value.

f) i) & ii) No, doctors and children would represent two completely different populations not represented by the population of Canadian Olympic hockey players. Because neither doctors nor children were the target of this study or part of the sample taken, the results should not be extended to these groups.

8-99 a) The probability plot shows that the data appear to be normally distributed.

b) 99% lower confidence bound on the mean $\vec{x} = 25.12, \quad s = 8.42, \quad n = 9 \qquad t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$25.12 - 2.896\left(\frac{8.42}{\sqrt{9}}\right) \le \mu$$

$$16.99 \le \mu$$

The lower bound on the 99% confidence interval shows that the mean comprehensive strength is greater than 16.99 Megapascals with high confidence.

c) 98% two-sided confidence interval on the mean $\vec{x} = 25.12, \quad s = 8.42, \quad n = 9 \quad t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right)$$

$$25.12 - 2.896\left(\frac{8.42}{\sqrt{9}}\right) \le \mu \le 25.12 + 2.896\left(\frac{8.42}{\sqrt{9}}\right)$$

$$16.99 \le \mu \le 33.25$$

The 98% two-sided confidence interval shows that the mean comprehensive strength is greater than 16.99 Megapascals and less than 33.25 Megapascals with high confidence.

The lower bound of the 99% one sided CI is the same as the lower bound of the 98% two-sided CI because the value of α for the one-sided example is one-half the value for the two-sided example.

d) 99% one-sided upper bound on the confidence interval on σ^2 comprehensive strength

$$s = 8.42, \quad s^2 = 70.90 \quad \chi^2_{0.99,8} = 1.65$$

$$\sigma^2 \le \frac{8(8.42)^2}{1.65}$$

$$\sigma^2 \le 343.74$$

The upper bound on the 99% confidence interval on the variance shows that the variance of the comprehensive strength is less than 343.74 Megapascals^2 with high confidence.

e) 98% two-sided confidence interval on σ^2 of comprehensive strength

$$s = 8.42, \quad s^2 = 70.90 \quad \chi^2_{0.01,9} = 20.09 \quad \chi^2_{0.99,8} = 1.65$$

$$\frac{8(8.42)^2}{20.09} \leq \sigma^2 \leq \frac{8(8.42)^2}{1.65}$$
$$28.23 \leq \sigma^2 \leq 343.74$$

The 98% two-sided confidence-interval on the variance shows that the variance of the comprehensive strength is less than 343.74 Megapascals^2 and greater than 28.23 Megapascals^2 with high confidence.

The upper bound of the 99% one-sided CI is the same as the upper bound of the 98% two-sided CI because the value of α for the one-sided example is one-half the value for the two-sided example.

f) 98% two-sided confidence interval on the mean $\vec{x} = 23$, $s = 6.31$, $n = 9$ $t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right)$$
$$23 - 2.896\left(\frac{6.31}{\sqrt{9}}\right) \leq \mu \leq 23 + 2.896\left(\frac{6.31}{\sqrt{9}}\right)$$
$$16.91 \leq \mu \leq 29.09$$

98% two-sided confidence interval on σ^2 comprehensive strength

$$s = 6.31, \quad s^2 = 39.8 \quad \chi^2_{0.01,9} = 20.09 \quad \chi^2_{0.99,8} = 1.65$$
$$\frac{8(39.8)}{20.09} \leq \sigma^2 \leq \frac{8(39.8)}{1.65}$$
$$15.85 \leq \sigma^2 \leq 192.97$$

Fixing the mistake decreased the values of the sample mean and the sample standard deviation. Because the sample standard deviation was decreased, the widths of the confidence intervals were also decreased.

g) A 98% two-sided confidence interval on the mean $\vec{x} = 25$, $s = 8.41$, $n = 9$ $t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right)$$
$$25 - 2.896\left(\frac{8.41}{\sqrt{9}}\right) \leq \mu \leq 25 + 2.896\left(\frac{8.41}{\sqrt{9}}\right)$$
$$16.88 \leq \mu \leq 33.12$$

98% two-sided confidence interval on σ^2 of comprehensive strength

$$s = 8.41, \quad s^2 = 70.73 \quad \chi^2_{0.01,9} = 20.09 \quad \chi^2_{0.99,8} = 1.65$$
$$\frac{8(8.41)^2}{20.09} \leq \sigma^2 \leq \frac{8(8.41)^2}{1.65}$$
$$28.16 \leq \sigma^2 \leq 342.94$$

Fixing the mistake did not affect the sample mean or the sample standard deviation. They are very close to the original values. The widths of the confidence intervals are also very similar.

h) When a mistaken value is near the sample mean, the mistake does not affect the sample mean, standard deviation or confidence intervals greatly. However, when the mistake is not near the sample mean, the value can greatly affect the sample mean, standard deviation and confidence intervals. The farther from the mean, the greater is the effect.

8-100

With $\sigma = 8$, the 95% confidence interval on the mean has length of at most 5; the error is then $E = 2.5$.

a) $n = \left(\frac{z_{0.025}}{2.5}\right)^2 8^2 = \left(\frac{1.96}{2.5}\right)^2 64 = 39.34 = 40$

b) $n = \left(\frac{z_{0.025}}{2.5}\right)^2 6^2 = \left(\frac{1.96}{2.5}\right)^2 36 = 22.13 = 23$

As the standard deviation decreases, with all other values held constant, the sample size necessary to maintain the acceptable level of confidence, and the width of the interval, decreases.

8-101 $\bar{x} = 15.33 \quad s = 0.62 \quad n = 20 \quad k = 2.564$

a) 95% Tolerance Interval of hemoglobin values with 90% confidence

$$\bar{x} - ks,\ \bar{x} + ks$$
$$15.33 - 2.564(0.62),\ 15.33 + 2.564(0.62)$$
$$(13.74, \quad 16.92)$$

b) 99% Tolerance Interval of hemoglobin values with 90% confidence $k = 3.368$

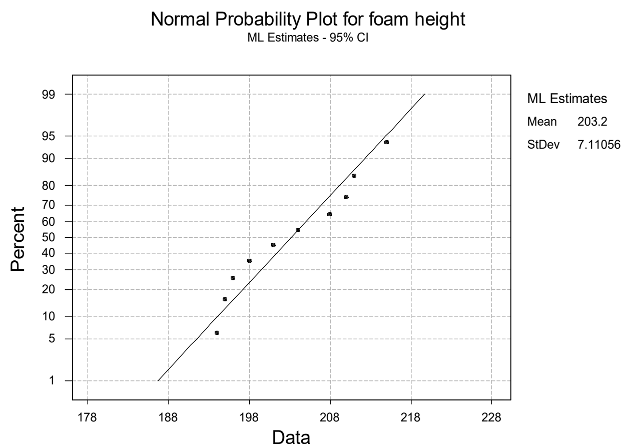
$$\bar{x} - ks,\ \bar{x} + ks$$
$$15.33 - 3.368(0.62),\ 15.33 + 3.368(0.62)$$
$$(13.24, 17.42)$$

8-102 95% prediction interval for the next sample of concrete that will be tested.

Given $\bar{x} = 25.12$ $s = 8.42$ $n = 9$ for $\alpha = 0.05$ and $n = 9$, $t_{\alpha/2,n-1} = t_{0.025,8} = 2.306$

$$\bar{x} - t_{0.025,8}\, s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.025,8}\, s\sqrt{1+\frac{1}{n}}$$
$$25.12 - 2.306(8.42)\sqrt{1+\frac{1}{9}} \le x_{n+1} \le 25.12 + 2.306(8.42)\sqrt{1+\frac{1}{9}}$$
$$4.65 \le x_{n+1} \le 45.59$$

8-103 a) The data appear to be normally distributed.



b) 95% confidence interval on the mean $\vec{x} = 203.20, \quad s = 7.5, \quad n = 10 \quad t_{0.025,9} = 2.262$

$$\bar{x} - t_{0.025,9}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} - t_{0.025,9}\left(\frac{s}{\sqrt{n}}\right)$$

$$203.2 - 2.262\left(\frac{7.50}{\sqrt{10}}\right) \le \mu \le 203.2 + 2.262\left(\frac{7.50}{\sqrt{10}}\right)$$

$$197.84 \le \mu \le 208.56$$

c) 95% prediction interval on a future sample

$$\bar{x} - t_{0.025,9}s\sqrt{1+\frac{1}{n}} \le \mu \le \bar{x} - t_{0.025,9}s\sqrt{1+\frac{1}{n}}$$

$$203.2 - 2.262(7.50)\sqrt{1+\frac{1}{10}} \le \mu \le 203.2 + 2.262(7.50)\sqrt{1+\frac{1}{10}}$$

$$185.41 \le \mu \le 220.99$$

d) 95% tolerance interval on foam height with 99% confidence $k = 4.265$

$$\bar{x} - ks,\ \bar{x} + ks$$

$$203.2 - 4.265(7.5),\ 203.2 + 4.265(7.5)$$

$$(171.21,\ 235.19)$$

e) The 95% CI on the population mean is the narrowest interval. For the CI, 95% of such intervals contain the population mean. For the prediction interval, 95% of such intervals will cover a future data value. This interval is wider than the CI on the mean. The tolerance interval is the widest interval of all. For the tolerance interval, 99% of such intervals will include 95% of the true distribution of foam height.

8-104

a) Normal probability plot for the coefficient of restitution.

b) 99% CI on the true mean coefficient of restitution

$\bar{x} = 0.624$, $s = 0.013$, $n = 40$ $t_{a/2,\, n-1} = t_{0.005,\, 39} = 2.7079$

$$\bar{x} - t_{0.005,39}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{0.005,39}\frac{s}{\sqrt{n}}$$

$$0.624 - 2.7079\frac{0.013}{\sqrt{40}} \le \mu \le 0.624 + 2.7079\frac{0.013}{\sqrt{40}}$$

$$0.618 \le \mu \le 0.630$$

c) 99% prediction interval on the coefficient of restitution for the next baseball that will be tested.

$$\bar{x} - t_{0.005,39}s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.005,39}s\sqrt{1+\frac{1}{n}}$$

$$0.624 - 2.7079(0.013)\sqrt{1+\frac{1}{40}} \le x_{n+1} \le 0.624 + 2.7079(0.013)\sqrt{1+\frac{1}{40}}$$

$$0.588 \le x_{n+1} \le 0.660$$

d) 99% tolerance interval on the coefficient of restitution with a 95% level of confidence

$$(\bar{x} - ks, \bar{x} + ks)$$

$$(0.624 - 3.213(0.013), 0.624 + 3.213(0.013))$$

$$(0.582, 0.666)$$

e) The confidence interval in part (b) is for the population mean and we may interpret this to imply that 99% of such intervals will cover the true population mean. For the prediction interval, 99% of such intervals will cover a future baseball's coefficient of restitution. For the tolerance interval, 95% of such intervals will cover 99% of the true distribution.

8-105 95% Confidence Interval on the proportion of baseballs with a coefficient of restitution that exceeds 0.635.

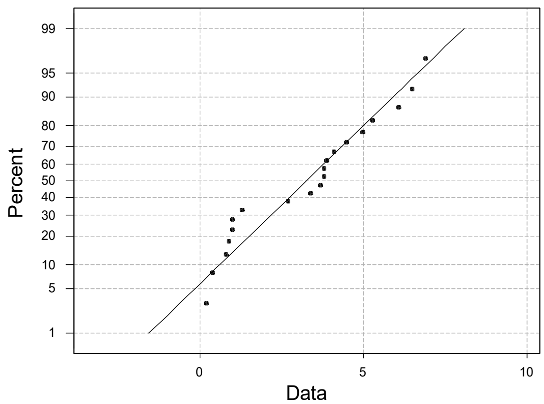
$$\hat{p} = \frac{8}{40} = 0.2 \qquad n = 40 \qquad z_\alpha = 1.65$$

$$\hat{p} - z_\alpha\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p$$

$$0.2 - 1.65\sqrt{\frac{0.2(0.8)}{40}} \le p$$

$$0.0956 \le p$$

8-106 a) The normal probability shows that the data are mostly follow the straight line, however, there are some points that deviate from the line near the middle.



b) 95% CI on the mean dissolved oxygen concentration

$\bar{x} = 3.265$, s = 2.127, n = 20 $t_{a/2,\, n-1} = t_{0.025,\, 19} = 2.093$

$$\bar{x} - t_{0.025,19}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{0.025,19}\frac{s}{\sqrt{n}}$$

$$3.265 - 2.093\frac{2.127}{\sqrt{20}} \le \mu \le 3.265 + 2.093\frac{2.127}{\sqrt{20}}$$

$$2.270 \le \mu \le 4.260$$

c) 95% prediction interval on the oxygen concentration for the next stream in the system that will be tested

$$\bar{x} - t_{0.025,19}s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.025,19}s\sqrt{1+\frac{1}{n}}$$

$$3.265 - 2.093(2.127)\sqrt{1+\frac{1}{20}} \le x_{n+1} \le 3.265 + 2.093(2.127)\sqrt{1+\frac{1}{20}}$$

$$-1.297 \le x_{n+1} \le 7.827$$

d) 95% tolerance interval on the values of the dissolved oxygen concentration with a 99% level of confidence

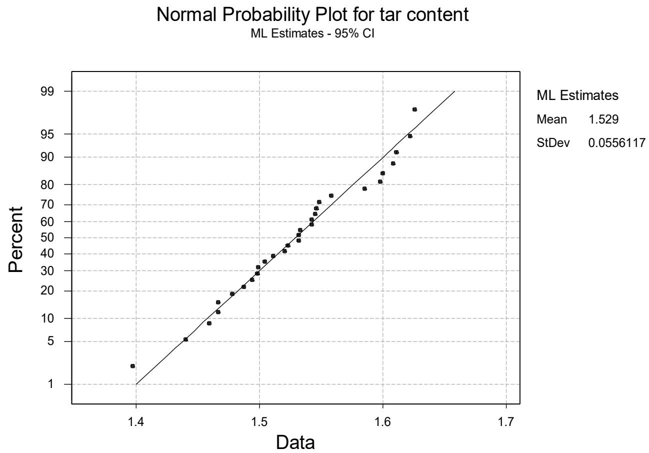
$$(\bar{x} - ks,\ \bar{x} + ks)$$

$$(3.265 - 3.168(2.127),\ 3.265 + 3.168(2.127))$$

$$(-3.473,\ 10.003)$$

e) The confidence interval in part (b) is for the population mean and we may interpret this to imply that 95% of such intervals will cover the true population mean. For the prediction interval, 95% of such intervals will cover a future oxygen concentration. For the tolerance interval, 99% of such intervals will cover 95% of the true distribution

8-107 a) The data appear normally distributed. The data points appear to fall along the normal probability line.



b) 99% CI on the mean tar content

$\bar{x} = 1.529$, s = 0.0566, n = 30 $t_{a/2,\, n-1} = t_{0.005,\, 29} = 2.756$

$$\bar{x} - t_{0.005,29}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{0.005,29}\frac{s}{\sqrt{n}}$$

$$1.529 - 2.756\frac{0.0566}{\sqrt{30}} \le \mu \le 1.529 + 2.756\frac{0.0566}{\sqrt{30}}$$

$$1.501 \le \mu \le 1.557$$

c) 99% prediction interval on the tar content for the next sample that will be tested..

$$\bar{x} - t_{0.005,19}s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.005,19}s\sqrt{1+\frac{1}{n}}$$

$$1.529 - 2.756(0.0566)\sqrt{1+\frac{1}{30}} \le x_{n+1} \le 1.529 + 2.756(0.0566)\sqrt{1+\frac{1}{30}}$$

$$1.370 \le x_{n+1} \le 1.688$$

d) 99% tolerance interval on the values of the tar content with a 95% level of confidence

$$(\bar{x} - ks,\ \bar{x} + ks)$$

$$(1.529 - 3.350(0.0566),\ 1.529 + 3.350(0.0566))$$

$$(1.339,\ 1.719)$$

e) The confidence interval in part (b) is for the population mean and we may interpret this to imply that 95% of such intervals will cover the true population mean. For the prediction interval, 95% of such intervals will cover a future observed tar content. For the tolerance interval, 99% of such intervals will cover 95% of the true distribution.

8-108 a) 95% Confidence Interval on the population proportion

n=1200 x=8 $\hat{p} = 0.0067$ $z_{\alpha/2} = z_{0.025} = 1.96$

$$\hat{p} - z_{a/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{a/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.0067 - 1.96\sqrt{\frac{0.0067(1-0.0067)}{1200}} \le p \le 0.0067 + 1.96\sqrt{\frac{0.0067(1-0.0067)}{1200}}$$

$$0.0021 \le p \le 0.0113$$

b) No, there is not sufficient evidence to support the claim that the fraction of defective units produced is one percent or less at $\alpha = 0.05$. This is because the upper limit of the control limit is greater than 0.01.

8-109 99% Confidence Interval on the population proportion

n=1600 x=8 $\hat{p} = 0.005$ $z_{\alpha/2}=z_{0.005}=2.58$

$$\hat{p} - z_{a/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{a/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.005 - 2.58\sqrt{\frac{0.005(1-0.005)}{1600}} \le p \le 0.005 + 2.58\sqrt{\frac{0.005(1-0.005)}{1600}}$$

$$0.0004505 \le p \le 0.009549$$

b) E = 0.008, $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{2.58}{0.008}\right)^2 0.005(1-0.005) = 517.43, \quad n \cong 518$$

c) E = 0.008, $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{2.58}{0.008}\right)^2 0.5(1-0.5) = 26001.56, \quad n \cong 26002$$

d) A bound on the true population proportion reduces the required sample size by a substantial amount. A sample size of 518 is much smaller than a sample size of over 26,000.

8-110 $\hat{p} = \frac{117}{484} = 0.242$

a) 90% confidence interval; $z_{\alpha/2} = 1.645$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.210 \le p \le 0.274$$

With 90% confidence, the true proportion of new engineering graduates who were planning to continue studying for an advanced degree is between 0.210 and 0.274.

b) 95% confidence interval; $z_{\alpha/2} = 1.96$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.204 \le p \le 0.280$$

With 95% confidence, the true proportion of new engineering graduates who were planning to continue studying for an advanced degree lies between 0.204 and 0.280.

c) Comparison of parts (a) and (b):

The 95% confidence interval is wider than the 90% confidence interval. Higher confidence produces wider intervals, all other values held constant.

d) Yes, since both intervals contain the value 0.25, thus there in not enough evidence to conclude that the true proportion differs from 0.25.

8-111

a) The data appear to follow a normal distribution based on the normal probability plot. The data fall along a straight line.

b) It is important to check for normality of the distribution underlying the sample data because the confidence intervals have the assumption of normality (especially since the sample size is less than 30 and the central limit theorem does not apply).

c) 95% confidence interval for the mean

$n = 11 \quad \bar{x} = 22.73 \quad s = 6.33 \quad t_{0.025,10} = 2.228$

$$\bar{x} - t_{0.025,10}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,10}\left(\frac{s}{\sqrt{n}}\right)$$

$$22.73 - 2.228\left(\frac{6.33}{\sqrt{11}}\right) \le \mu \le 22.73 + 2.228\left(\frac{6.33}{\sqrt{11}}\right)$$

$$18.478 \le \mu \le 26.982$$

d) As with part b, to construct a confidence interval on the variance, the normality assumption must hold for the results to be reliable.

e) 95% confidence interval for variance

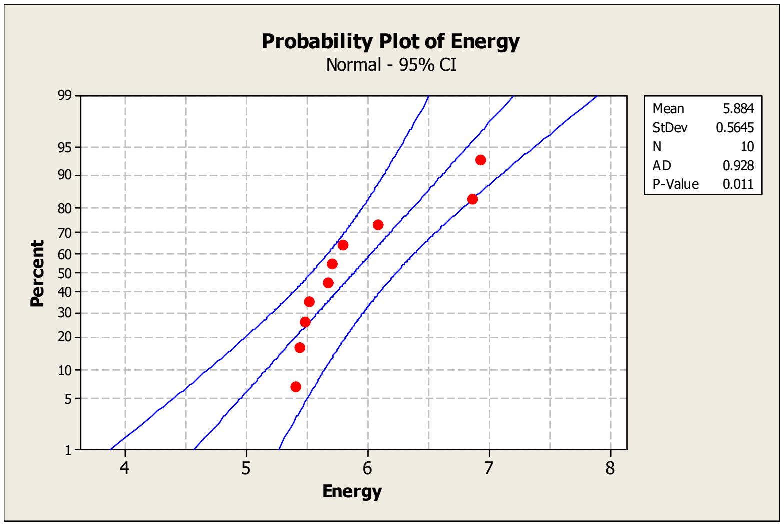
$n = 11 \quad s = 6.33$

$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,10} = 20.48$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,10} = 3.25$

$$\frac{10(6.33)^2}{20.48} \le \sigma^2 \le \frac{10(6.33)^2}{3.25}$$

$$19.565 \le \sigma^2 \le 123.289$$

8-112 a) The data appear to be normally distributed based on examination of the normal probability plot below.



b) 99% upper confidence interval on mean energy (BMR)

$n = 10 \quad \bar{x} = 5.884 \quad s = 0.5645 \quad t_{0.005,9} = 3.250$

$$\bar{x} - t_{0.005,9}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.005,9}\left(\frac{s}{\sqrt{n}}\right)$$

$$5.884 - 3.250\left(\frac{0.5645}{\sqrt{10}}\right) \le \mu \le 5.884 + 3.250\left(\frac{0.5645}{\sqrt{10}}\right)$$

$$5.304 \le \mu \le 6.464$$

Mind Expanding Exercises

8-113 a) $P(\chi^2_{1-\frac{\alpha}{2},2r} < 2\lambda T_r < \chi^2_{\frac{\alpha}{2},2r}) = 1-\alpha$

$$= P\left(\frac{\chi^2_{1-\frac{\alpha}{2},2r}}{2T_r} < \lambda < \frac{\chi^2_{\frac{\alpha}{2},2r}}{2T_r}\right)$$

Then a confidence interval for $\mu = \dfrac{1}{\lambda}$ is $\left(\dfrac{2T_r}{\chi^2_{\frac{\alpha}{2},2r}}, \dfrac{2T_r}{\chi^2_{1-\frac{\alpha}{2},2r}}\right)$

b) n = 20 , r = 10 , and the observed value of T_r is 199 + 10(29) = 489.

A 95% confidence interval for $\dfrac{1}{\lambda}$ is $\left(\dfrac{2(489)}{34.17}, \dfrac{2(489)}{9.59}\right) = (28.62, 101.98)$

8-114 $\alpha_1 = \int_{z_{\alpha_1}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 1 - \int_{-\infty}^{z_{\alpha_1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

Therefore, $1-\alpha_1 = \Phi(z_{\alpha_1})$.

To minimize L we need to minimize $\Phi^{-1}(1-\alpha_1) + \Phi(1-\alpha_2)$ subject to $\alpha_1 + \alpha_2 = \alpha$.

Therefore, we need to minimize $\Phi^{-1}(1-\alpha_1)+\Phi(1-\alpha+\alpha_1)$.

$$\frac{\partial}{\partial\alpha_1}\Phi^{-1}(1-\alpha_1) = -\sqrt{2\pi}\ e^{\frac{z_{\alpha_1}^2}{2}}$$

$$\frac{\partial}{\partial\alpha_1}\Phi^{-1}(1-\alpha+\alpha_1) = \sqrt{2\pi}\ e^{\frac{z_{\alpha-\alpha_1}^2}{2}}$$

Upon setting the sum of the two derivatives equal to zero, we obtain $e^{\frac{z_{\alpha-\alpha_1}^2}{2}} = e^{\frac{z_{\alpha_1}^2}{2}}$. This is solved by $z_{\alpha_1} = z_{\alpha-\alpha_1}$. Consequently, $\alpha_1 = \alpha-\alpha_1$, $2\alpha_1 = \alpha$ and $\alpha_1 = \alpha_2 = \frac{\alpha}{2}$.

8-115 a) n = ½ + (1.9/.1)(9.4877/4), then $n = 46$

b) (10 - 0.5)/(9.4877/4) = (1 + p)/(1 - p)

p = 0.6004 between 10.19 and 10.41.

8-116 a)

$$P(X_i \le \tilde{\mu}) = 1/2$$
$$P(allX_i \le \tilde{\mu}) = (1/2)^n$$
$$P(allX_i \ge \tilde{\mu}) = (1/2)^n$$

$$P(A\cup B) = P(A)+P(B)-P(A\cap B)$$
$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n = 2\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{n-1}$$
$$1-P(A\cup B) = P(\min(X_i) < \tilde{\mu} < \max(X_i)) = 1-\left(\frac{1}{2}\right)^n$$

b) $P(\min(X_i) < \tilde{\mu} < \max(X_i)) = 1-\alpha$

The confidence interval is min(X_i), max(X_i)

8-117

From the definition of a confidence interval we expect 950 of the confidence intervals to include the value of μ. Let X be the number of intervals that contain the true mean (μ). We can use the large sample approximation to determine the probability that P(930 < X < 970).

Let $p = \frac{950}{1000} = 0.950$ $p_1 = \frac{930}{1000} = 0.930$ and $p_2 = \frac{970}{1000} = 0.970$

The variance is estimated by $\frac{p(1-p)}{n} = \frac{0.950(0.050)}{1000}$

$$P(0.930 < p < 0.970) = P\left(Z < \frac{(0.970 - 0.950)}{\sqrt{\frac{0.950(0.050)}{1000}}}\right) - P\left(Z < \frac{(0.930 - 0.950)}{\sqrt{\frac{0.950(0.050)}{1000}}}\right)$$

$$= P\left(Z < \frac{0.02}{0.006892}\right) - P\left(Z < \frac{-0.02}{0.006892}\right) = P(Z < 2.90) - P(Z < -2.90) = 0.9963$$

8-118 $\tilde{p} = \frac{2}{54} = 0.0370 \qquad n = 50 \;\; z_{\alpha/2} = 1.96$

$$\tilde{p} - z_{\alpha/2}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n}} \le p \le \tilde{p} + z_{\alpha/2}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n}}$$

$$0.0370 - 1.96\sqrt{\frac{0.0370(1-0.0370)}{50}} \le p \le 0.0370 + 1.96\sqrt{\frac{0.0370(1-0.0370)}{50}}$$

$$-0.015 \le p \le 0.089$$

$$p \le 0.089$$

8-119 a) $\hat{p} = \frac{2}{35} = 0.0571 \qquad n = 35 \;\; z_{\alpha/2} = 1.96$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.0571 - 1.96\sqrt{\frac{0.0571(1-0.0571)}{35}} \le p \le 0.0571 + 1.96\sqrt{\frac{0.0571(1-0.0571)}{35}}$$

$$-0.020 \le p \le 0.134$$

$$p \le 0.134$$

b) $\tilde{p} = \frac{4}{39} = 0.1026 \qquad n = 35 \;\; z_{\alpha/2} = 1.96$

$$\tilde{p} - z_{\alpha/2}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n}} \le p \le \tilde{p} + z_{\alpha/2}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n}}$$

$$0.1026 - 1.96\sqrt{\frac{0.1026(1-0.1026)}{35}} \le p \le 0.1026 + 1.96\sqrt{\frac{0.1026(1-0.1026)}{35}}$$

$$0.002 \le p \le 0.203$$

This confidence interval is much wider than the interval in part (a). Because of the small sample size and small estimated proportion, the interval in part (a) can be overly optimistic and the modified interval in part (b) is probably a better choice.