CHAPTER 8

8.1

```
>> Aug = [A eye(size(A))]
```

Here's an example session of how it can be employed.

```
>> A = rand(3)

A =
    0.9501    0.4860    0.4565
    0.2311    0.8913    0.0185
    0.6068    0.7621    0.8214

>> Aug = [A eye(size(A))]
Aug =

    0.9501    0.4860    0.4565    1.0000         0         0
    0.2311    0.8913    0.0185         0    1.0000         0
    0.6068    0.7621    0.8214         0         0    1.0000
```

8.2 **(a)** $[A]$: 3×2 $[B]$: 3×3 $\{C\}$: 3×1 $[D]$: 2×4
$[E]$: 3×3 $[F]$: 2×3 $\lfloor G \rfloor$: 1×3

(b) square: $[B]$, $[E]$; column: $\{C\}$, row: $\lfloor G \rfloor$

(c) $a_{12} = 5$, $b_{23} = 6$, d_{32} = undefined, $e_{22} = 1$, $f_{12} = 0$, $g_{12} = 6$

(d)

(1) $[E]+[B]=\begin{bmatrix} 5 & 8 & 13 \\ 8 & 3 & 9 \\ 6 & 0 & 10 \end{bmatrix}$

(2) $[A] + [F]$ = undefined

(3) $[B]-[E]=\begin{bmatrix} 3 & -2 & 1 \\ -6 & 1 & 3 \\ -2 & 0 & -2 \end{bmatrix}$

(4) $7[B]=\begin{bmatrix} 28 & 21 & 49 \\ 7 & 14 & 42 \\ 14 & 0 & 28 \end{bmatrix}$

(5) $[C]^T = \lfloor 2 \quad 6 \quad 1 \rfloor$

(6) $[E][B]=\begin{bmatrix} 21 & 13 & 61 \\ 35 & 23 & 67 \\ 28 & 12 & 52 \end{bmatrix}$

(7) $[B][E]=\begin{bmatrix} 53 & 23 & 75 \\ 39 & 7 & 48 \\ 18 & 10 & 36 \end{bmatrix}$

(8) $[D]^T = \begin{bmatrix} 5 & 2 \\ 4 & 1 \\ 3 & 7 \\ -7 & 5 \end{bmatrix}$

(9) $[G][C] = 56$

(10) $I[B]=\begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 6 \\ 2 & 0 & 4 \end{bmatrix}$

(11) $E^T[E]=\begin{bmatrix} 66 & 12 & 51 \\ 12 & 26 & 33 \\ 51 & 33 & 81 \end{bmatrix}$

(12) $C^T[C] = 41$

8.3 The terms can be collected to give

$$\begin{bmatrix} 0 & -6 & 5 \\ 0 & 2 & 7 \\ -4 & 3 & -7 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 50 \\ -30 \\ 50 \end{Bmatrix}$$

Here is the MATLAB session:

```
>> A = [0 -6 5;0 2 7;-4 3 -7];
>> b = [50;-30;50];
>> x = A\b

x =
  -17.0192
   -9.6154
   -1.5385

>> AT = A'
     0     0    -4
    -6     2     3
     5     7    -7

>> AI = inv(A)
AI =
   -0.1683   -0.1298   -0.2500
   -0.1346    0.0962         0
    0.0385    0.1154         0
```

8.4 (a) Here are all the possible multiplications:

```
>> A=[6 -1;12 7;-5 3];
>> B=[4 0;0.6 8];
>> C=[1 -2;-6 1];
>> A*B
ans =
   23.4000   -8.0000
   52.2000   56.0000
  -18.2000   24.0000
>> A*C
ans =
    12   -13
   -30   -17
   -23    13
>> B*C
ans =
    4.0000   -8.0000
  -47.4000    6.8000
>> C*B
ans =
    2.8000  -16.0000
  -23.4000    8.0000
```

(b) [B][A] and [C][A] are impossible because the inner dimensions do not match:

(2×2) * (3×2)

(c) According to **(a)**, [B][C] ≠ [C][B]

8.5
```
>> A=[3+2*i 4;-i 1]
>> b=[2+i;3]
>> z=A\b

z =
  -0.5333 + 1.4000i
   1.6000 - 0.5333i
```

8.6
```
function X=mmult(Y,Z)
% mmult: matrix multiplication
%   X=mmult(Y,Z)
%     multiplies two matrices
% input:
%   Y = first matrix
%   Z = second matrix
% output:
%   X = product

if nargin<2,error('at least 2 input arguments required'),end
[m,n]=size(Y);[n2,p]=size(Z);
if n~=n2,error('Inner matrix dimensions must agree.'),end
for i=1:m
  for j=1:p
    s=0.;
    for k=1:n
      s=s+Y(i,k)*Z(k,j);
    end
    X(i,j)=s;
  end
end
```

Test of function for cases from Prob. 8.4:

```
>> A=[6 -1;12 7;-5 3];
>> B=[4 0;0.6 8];
>> C=[1 -2;-6 1];
>> mmult(A,B)
ans =
   23.4000   -8.0000
   52.2000   56.0000
  -18.2000   24.0000

>> mmult(A,C)
ans =
    12   -13
   -30   -17
   -23    13

>> mmult(B,C)
ans =
    4.0000   -8.0000
  -47.4000    6.8000

>> mmult(C,B)
ans =
    2.8000  -16.0000
  -23.4000    8.0000

>> mmult(B,A)
```

```
??? Error using ==> mmult
Inner matrix dimensions must agree.

>> mmult(C,A)
??? Error using ==> mmult
Inner matrix dimensions must agree.
```

8.7

```
function AT=matran(A)
% matran: matrix transpose
%   AT=mtran(A)
%      generates the transpose of a matrix
% input:
%   A = original matrix
% output:
%   AT = transpose

[m,n]=size(A);
for i = 1:m
  for j = 1:n
    AT(j,i) = A(i,j);
  end
end
```

Test of function for cases from Prob. 8.4:

```
>> matran(A)
ans =
     6    12    -5
    -1     7     3

>> matran(B)
    4.0000    0.6000
         0    8.0000

>> matran(C)
ans =
     1    -6
    -2     1
```

8.8

```
function B = permut(A,r1,r2)
% permut: Switch rows of matrix A with a permutation matrix
% B = permut(A,r1,r2)
% input:
% A = original matrix
% r1, r2 = rows to be switched
% output:
% B = matrix with rows switched

[m,n] = size(A);
if m ~= n, error('matrix not square'), end
if r1 == r2 | r1>m | r2>m
  error('row numbers are equal or exceed matrix dimensions')
end
P = zeros(n);
P(r1,r2)=1;P(r2,r1)=1;
for i = 1:m
  if i~=r1 & i~=r2
    P(i,i)=1;
  end
```

```
end
B=P*A;
```

Test script:

```
clc
A=[1 2 3 4;5 6 7 8;9 10 11 12;13 14 15 16]
B = permut(A,3,1)
B = permut(A,3,5)

A =
     1     2     3     4
     5     6     7     8
     9    10    11    12
    13    14    15    16
B =
     9    10    11    12
     5     6     7     8
     1     2     3     4
    13    14    15    16

??? Error using ==> permut
row numbers are equal or exceed matrix dimensions

Error in ==> permutScript at 4
B = permut(A,3,5)
```

8.9 The mass balances can be written as

$$\begin{aligned}
&(Q_{15}+Q_{12})c_1 && && -Q_{31}c_3 && && = Q_{01}c_{01}\\
&-Q_{12}c_1 && +(Q_{23}+Q_{24}+Q_{25})c_2 && && && = 0\\
& && -Q_{23}c_2 && +(Q_{31}+Q_{34})c_3 && && = Q_{03}c_{03}\\
& && -Q_{24}c_2 && -Q_{34}c_3+Q_{44}c_4 && -Q_{54}c_5 && = 0\\
&-Q_{15}c_1 && -Q_{25}c_2 && && +(Q_{54}+Q_{55})c_5 && = 0
\end{aligned}$$

The parameters can be substituted and the result written in matrix form as

$$\begin{bmatrix} 9 & 0 & -3 & 0 & 0 \\ -4 & 4 & 0 & 0 & 0 \\ 0 & -2 & 9 & 0 & 0 \\ 0 & -1 & -6 & 9 & -2 \\ -5 & -1 & 0 & 0 & 6 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{Bmatrix} = \begin{Bmatrix} 120 \\ 0 \\ 350 \\ 0 \\ 0 \end{Bmatrix}$$

The following MATLAB script can then be used to solve for the concentrations

```
clc
Q = [9 0 -3 0 0;
-4 4 0 0 0;
0 -2 9 0 0;
0 -1 -6 9 -2;
-5 -1 0 0 6];
Qc = [120;0;350;0;0];
c = Q\Qc

c =
```

```
   28.4000
   28.4000
   45.2000
   39.6000
   28.4000
```

8.10 The problem can be written in matrix form as

$$\begin{bmatrix} 0.866025 & 0 & -0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0.866025 & 0 & 0 & 0 \\ -0.866025 & -1 & 0 & -1 & 0 & 0 \\ -0.5 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & -0.866025 & 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ H_2 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -2000 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

MATLAB can then be used to solve for the forces and reactions,

```
clc; format short g
A = [0.866025 0 -0.5 0 0 0;
0.5 0 0.866025 0 0 0;
-0.866025 -1 0 -1 0 0;
-0.5 0 0 0 -1 0;
0 1 0.5 0 0 0;
0 0 -0.866025 0 0 -1];
b = [0 -2000 0 0 0 0]';
F = A\b

F =
         -1000
        866.03
       -1732.1
             0
           500
          1500
```

Therefore,

$F_1 = -1000$ $F_2 = 866.025$ $F_3 = -1732.1$ $H_2 = 0$ $V_2 = 500$ $V_3 = 1500$

8.11

```
clc; format short g
k1=10;k2=40;k3=40;k4=10;
m1=1;m2=1;m3=1;
km=[(1/m1)*(k2+k1),-(k2/m1),0
    -(k2/m2),(1/m2)*(k2+k3),-(k3/m2)
    0,-(k3/m3),(1/m3)*(k3+k4)]
x=[0.05;0.04;0.03];
kmx=-km*x

km =
    50   -40     0
   -40    80   -40
     0   -40    50
kmx =
          -0.9
 -2.2204e-016
           0.1
```

Therefore, $\ddot{x}_1 = -0.9$, $\ddot{x}_2 = 0$, and $\ddot{x}_3 = 0.1$ m/s^2.

8.12 Vertical force balances can be written to give the following system of equations,

$$
\begin{aligned}
&m_1 g + k_2(x_2 - x_1) - k_1 x_1 = 0 \\
&m_2 g + k_3(x_3 - x_2) - k_2(x_2 - x_1) = 0 \\
&m_3 g + k_4(x_4 - x_3) - k_3(x_3 - x_2) = 0 \\
&m_4 g + k_5(x_5 - x_4) - k_4(x_4 - x_3) = 0 \\
&m_5 g - k_5(x_5 - x_4) = 0
\end{aligned}
$$

Collecting terms,

$$
\begin{bmatrix}
k_1 + k_2 & -k_2 & & & \\
-k_2 & k_2 + k_3 & -k_3 & & \\
 & -k_3 & k_3 + k_4 & -k_4 & \\
 & & -k_4 & k_4 + k_5 & -k_5 \\
 & & & -k_5 & k_5
\end{bmatrix}
\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix}
=
\begin{Bmatrix} m_1 g \\ m_2 g \\ m_3 g \\ m_4 g \\ m_5 g \end{Bmatrix}
$$

After substituting the parameters, the equations can be expressed as (g = 9.81),

$$
\begin{bmatrix}
120 & -40 & & & \\
-40 & 110 & -70 & & \\
 & -70 & 170 & -100 & \\
 & & -100 & 120 & -20 \\
 & & & -20 & 20
\end{bmatrix}
\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix}
=
\begin{Bmatrix} 637.65 \\ 735.75 \\ 588.60 \\ 735.75 \\ 882.90 \end{Bmatrix}
$$

The solution can then be obtained with the following MATLAB script:

```
clc; format short g
g=9.81;
m1=65;m2=75;m3=60;m4=75;m5=90;
k1=80;k2=40;k3=70;k4=100;k5=20;
A=[k1+k2 -k2 0 0 0
-k2 k2+k3 -k3 0 0
0 -k3 k3+k4 -k4 0
0 0 -k4 k4+k5 -k5
0 0 0 -k5 k5]
b=[m1*g m2*g m3*g m4*g m5*g]'
x=A\b

A =
   120   -40     0     0     0
   -40   110   -70     0     0
     0   -70   170  -100     0
     0     0  -100   120   -20
     0     0     0   -20    20
b =
       637.65
       735.75
        588.6
```

```
        735.75
         882.9
x =
        44.758
        118.33
        149.87
        166.05
         210.2
```

8.13 The position of the three masses can be modeled by the following steady-state force balances

$$0 = k(x_2 - x_1) + m_1 g - kx_1$$
$$0 = k(x_3 - x_2) + m_2 g - k(x_2 - x_1)$$
$$0 = m_3 g - k(x_3 - x_2)$$

Terms can be combined to yield

$$\begin{aligned} 2kx_1 - kx_2 \quad\quad &= m_1 g \\ -kx_1 + 2kx_2 - kx_3 &= m_2 g \\ -kx_2 + kx_3 &= m_3 g \end{aligned}$$

Substituting the parameter values

$$\begin{bmatrix} 30 & -15 & 0 \\ -15 & 30 & -15 \\ 0 & -15 & 15 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 19.62 \\ 24.525 \\ 29.43 \end{Bmatrix}$$

A MATLAB script can be used to obtain the solution for the displacements

```
clc; format short g
g=9.81;k=15;
K=[2*k -k 0;-k 2*k -k;0 -k k]
m=[2;2.5;3];
mg=m*g
x=K\mg

K =
    30   -15     0
   -15    30   -15
     0   -15    15
mg =
         19.62
        24.525
         29.43
x =
         4.905
         8.502
        10.464
```

8.14 Just as in Sec. 8.3, the simultaneous equations can be expressed in matrix form as

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & R_{52} & -R_{32} & 0 & -R_{54} & -R_{43} \\ R_{12} & -R_{52} & 0 & -R_{65} & 0 & 0 \end{bmatrix} \begin{Bmatrix} i_{12} \\ i_{52} \\ i_{32} \\ i_{65} \\ i_{54} \\ i_{43} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_1 - V_6 \end{Bmatrix}$$

or substituting the resistances

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 10 & -10 & 0 & -15 & -5 \\ 5 & -10 & 0 & -20 & 0 & 0 \end{bmatrix} \begin{Bmatrix} i_{12} \\ i_{52} \\ i_{32} \\ i_{65} \\ i_{54} \\ i_{43} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 200 \end{Bmatrix}$$

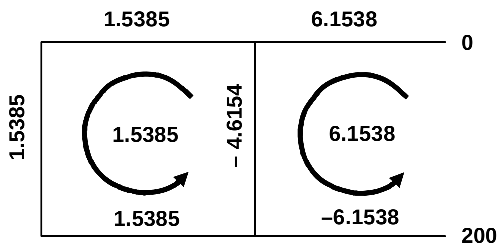
This system can be solved with MATLAB,

```
clc; format short g
R12=5;R52=10;R32=10;R65=20;R54=15;R43=5;
V1=200;V6=0;
A=[1 1 1 0 0 0;
0 -1 0 1 -1 0;
0 0 -1 0 0 1;
0 0 0 0 1 -1;
0 R52 -R32 0 -R54 -R43;
R12 -R52 0 -R65 0 0]
B=[0 0 0 0 0 V1-V6]'
I=A\B

A =
     1     1     1     0     0     0
     0    -1     0     1    -1     0
     0     0    -1     0     0     1
     0     0     0     0     1    -1
     0    10   -10     0   -15    -5
     5   -10     0   -20     0     0
B =
     0
     0
     0
     0
     0
   200
I =
       6.1538
      -4.6154
      -1.5385
      -6.1538
      -1.5385
      -1.5385
```

$i_{21} = 6.1538$ $i_{52} = -4.6154$ $i_{32} = -1.5385$ $i_{65} = -6.1538$ $i_{54} = -1.5385$ $i_{43} = -1.5385$

Here are the resulting currents superimposed on the circuit:



8.15 The current equations can be written as

$$-i_{21} - i_{23} + i_{52} = 0$$
$$i_{23} - i_{35} + i_{43} = 0$$
$$-i_{43} + i_{54} = 0$$
$$i_{35} - i_{52} + i_{65} - i_{54} = 0$$

Voltage equations:

$$i_{21} = \frac{V_2 - 20}{35} \qquad i_{54} = \frac{V_5 - V_4}{15}$$
$$i_{23} = \frac{V_2 - V_3}{30} \qquad i_{35} = \frac{V_3 - V_5}{7}$$
$$i_{43} = \frac{V_4 - V_3}{8} \qquad i_{52} = \frac{V_5 - V_2}{10}$$
$$i_{65} = \frac{140 - V_5}{5}$$

$$\begin{bmatrix} -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 35 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 10 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} i_{21} \\ i_{23} \\ i_{52} \\ i_{35} \\ i_{43} \\ i_{54} \\ i_{65} \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 140 \end{Bmatrix}$$

A MATLAB script can be developed to generate the solution,

```
clc; format short g
R12=35;R52=10;R32=30;R34=8;R45=15;R35=7;R25=10;R65=5;
V1=20;V6=140;
A=[-1   -1  1   0   0   0   0   0  0  0  0;
    0   1   0   -1  1   0   0   0  0  0  0;
```

```
    0   0   0   0   -1  1   0   0  0  0  0;
    0   0   -1  1   0   -1  1   0  0  0  0;
    R12 0   0   0   0   0   0   -1 0  0  0;
    0   R32 0   0   0   0   0   -1 1  0  0;
    0   0   0   0   R34 0   0   0  1  -1 0;
    0   0   0   0   0   R45 0   0  0  1  -1;
    0   0   0   R35 0   0   0   0  -1 0  1;
    0   0   R25 0   0   0   0   1  0  0  -1;
    0   0   0   0   0   0   R65 0  0  0  1]
B=[0 0 0 0 -V1 0 0 0 0 0 V6]'
I=A\B

I =
       2.5107
     -0.55342
       1.9573
     -0.42429
      0.12913
      0.12913
       2.5107
       107.87
       124.48
       125.51
       127.45
```

Thus, the solution is

$i_{21} = 2.5107$ $i_{23} = -0.55342$ $i_{52} = 1.9573$ $i_{35} = -0.42429$ $i_{43} = 0.12913$
$i_{54} = 0.12913$ $i_{65} = 2.5107$ $V_2 = 107.87$ $V_3 = 124.48$ $V_4 = 125.51$
$V_5 = 127.45$