Computational Astrophysics and Cosmology, SN exercise

I. PROBLEM 1: DISCOVER COSMIC ACCELERATION

The files attached to this exercise contain the measurements and covariance of Supernovae luminosity distance measurements from arXiv:1710.00845.

These measure the (log) flux of many Supernovae:

$$D_L(z) = \sqrt{\frac{L}{4\pi F}} \tag{1}$$

as a function of redshift. Note that we do not really know L but we do know that it is the same for all objects (i.e. Supernovae are standard candles).

To conform to strange astronomy conventions (magnitudes) we consider:

$$\mu = 5\log_{10} D_L(z) \tag{2}$$

This makes the following sense: we are measuring fluxes to high signal to noise ratio (SNR). The distribution of measured fluxes is going to be Gaussian because of the central limit theorem. The distribution of a ratio of Gaussian variables is not Gaussian. When considering the log, on the other hand, close-to-Gaussianity is guaranteed by high SNR and then everything is a linear combination of Gaussian variables, hence Gaussian.

We start with a Gaussian likelihood for the data x:

$$\ln \mathcal{L} = -\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu) \tag{3}$$

Note that we neglect all parameter independent normalization factors.

Since we do not really know the true intrinsic luminosity of Supernovae we are not interested in D_0, which depends on the value of the Hubble constant. Then we marginalize analytically over a zero point additive constant:

$$\mathcal{L}_m = \int \mathcal{L}\, d\alpha = \int \exp\left(-\frac{1}{2}(d-\mu+\alpha\,(\vec{1}))\Sigma^{-1}(d-\mu+\alpha\,(\vec{1}))\right) d\alpha \tag{4}$$

where $\alpha \sim \log_{10} D_0$ and $(\vec{1})$ is a vector of ones. In this case α is an additive parameter that is the same for all data. **Show that**, after some algebra:

$$\ln \mathcal{L}_m = -\frac{1}{2}(d-\mu)\Sigma^{-1}(d-\mu) + \frac{1}{2}\frac{\left[(\vec{1})\Sigma^{-1}(d-\mu)\right]^2}{(\vec{1})\Sigma^{-1}\,(\vec{1})}\,. \tag{5}$$

Calculate D_L. Analytic marginalization means that D_L will not depend on H_0. The redshift range that we are considering implies that radiation is negligible. We can neglect curvature. The only free parameter in the standard model is then Ω_m. The abundance of Dark Energy $\Omega_{\rm DE} = 1-\Omega_m$ is fixed by the value of Ω_m.

Write the likelihood in terms of its only input parameter Ω_m. **Grid sample** the likelihood and plot it. Compute the mean and variance of the distribution. You can check your results against arXiv:1710.00845 (part of the exercise is to fish out the correct number). Have you discovered cosmic acceleration (i.e. $\Omega_m < 1$)?

Finish off by calculating everything we have seen during the lecture (exactly on a grid) for the Ω_m distribution.

II. PROBLEM 2: FUN WITH SUPERNOVAE

Can you think of some modification of the luminosity distance formula, perhaps with an extra parameter, and estimate that from data in the same way? What is the physical meaning of your modification? What are the physical implications of your results?