Generation of sequences controlled by their “complexity”

Concrete Problem

We want to generate *sequences* of musical “chords” (a chord is a set of notes basically) with some known constraints (allDiff, etc.) as well as control on the *complexity* of the sequence. This complexity in turn is defined by a dynamic programming algorithm working on the instantiated sequence, which makes the whole problem difficult.

Problem Formalization

Let $x = x_1, \cdots, x_n$ be variables with finite domain X
Let $G = (X, E)$ be a directed graph on X
Let Y be a finite set
Let C be a cost function on $X \times Y$, taking positive numeric values
Let T be a transition cost function between elements of Y
$C: X \times Y \rightarrow [0, +\infty)$
$T: Y \times Y \rightarrow [0, 1]$

For a **path** $x = x_1, \cdots, x_n$ in graph G, we define the sequence

$$h(x) = argmin_{(y_1, \cdots, y_n) \in Y^\wedge n} \sum_{i \leq n} C(x_i, y_i) + \sum_{i < n} T(y_i, y_{i+1})$$

Problems with $T = 0 \; or \; Cte$

Problem 1
Generate paths $x = x_1, \cdots, x_n$ with $h(x) = K$, where K is a constant
Variant 1: x are **cycles**
Variant 2: x are **all different**, $x_i \neq x_j$, $\forall i, j$ (all different constraint on the x’s)

Problem 2
Generate paths $x = x_1, \cdots, x_n$ with a predefined number N of unique values for $h(x)$
This can be seen as posting an **NValue** constraint on $h(x)$
Same variants as problem 1

Note:
This framework may be used to generate chord sequences with control on the harmonic complexity of the sequence (through the constraints K and N). The $x_1, \cdots, x_n$ are the chords, $y_1, \cdots, y_n$ are the scales, obtained by applying an harmonic analysis algorithm to $x_1, \cdots, x_n$.