Path Color Switching

Supervisor: Jean-Charles Régin

Fissore Davide

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MASTER INFORMATIQUE

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Problem Description

We want to generate sequences of musical "chords" with some known constraints as well as control on the complexity of the sequence.

Spotify

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Input An oriented graph whose arcs are colored with a set of colors, two nodes of the graphs s and t.

Output A path $\mathcal{P}$ going from s to t and which minimizes the number of color switch.

Definitions & notations

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$\mathcal{G} = (V, A)$: A directed graph where V is the set of its nodes and A is the set of its arcs.

$\mathcal{C}$: A finite set of colors.

$\mathcal{F}$: The coloring function defined as $\mathcal{F} : A \to 2^{\mathcal{C}}$.

$\mathcal{P} = (v_1, \ldots, v_k)$: A path going from v_1 to v_k.

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$\mathcal{P} = (v_1, \ldots, v_k)$: A path going from v_1 to v_k.

$w(\mathcal{P})$: The cost of the path $\mathcal{P}$ which is given by the sum of its *CS*.

Problem decomposition

The problem can decomposed in small parts:

- Minimize CS on paths;
- Minimize CS on graphs.

Minimize CS on Paths

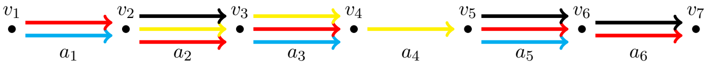

Figure: A path $\mathcal{P}$

What is the color assignation minimizing $w(\mathcal{P})$?

Algorithm

Let $\mathcal{P} = (a_1, \dots, a_k)$ a path
Let $\mathcal{T} : A \rightarrow 2^{\mathcal{C}}$ a function such that:

- $\mathcal{T}(a_1) = \mathcal{F}(a_1)$
- $\mathcal{T}(a_i) = \mathcal{F}(a_i) \cap \mathcal{T}(a_{i-1})$ `if not empty else` $\mathcal{F}(a_i)$[1]

---

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$H : A \to \mathcal{C}$ the function minimizing $w(\mathcal{P})$ such that:

- $H(a_k) =$ `a rnd elt from` $\mathcal{T}(a_k)$
- $H(a_i) = H(a_i + 1)$ `if` $H(a_i + 1) \in \mathcal{T}(a_i)$ `else rnd from` $\mathcal{T}(a_i)$

[1] $\forall i > 1$

Example run

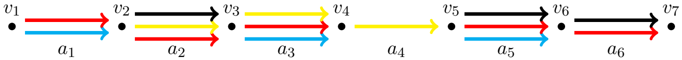


Figure: A path $\mathcal{P}$

Start to compute $\mathcal{T}(\mathcal{P})$

Example run

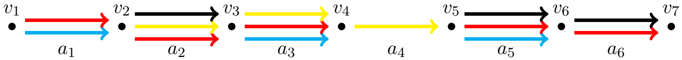


Figure: Computing $\mathcal{T}(\mathcal{P})$

$$\mathcal{T}(a_1) = \mathcal{F}(a_1)$$

Example run

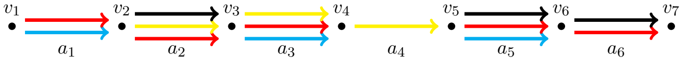
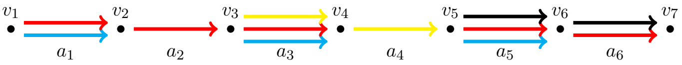


Figure: Computing $\mathcal{T}(\mathcal{P})$

$\mathcal{T}(a_2) = \mathcal{F}(a_2) \cap \mathcal{T}(a_1)$ `since not empty`

Example run



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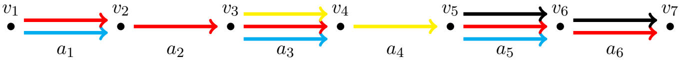

Figure: Computing $\mathcal{T}(\mathcal{P})$

$\mathcal{T}(a_3) = \mathcal{F}(a_3) \cap \mathcal{T}(a_2)$ `since not empty`

Example run

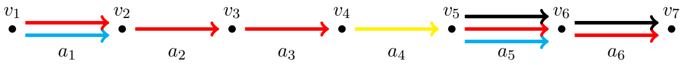

Figure: Computing $\mathcal{T}(\mathcal{P})$

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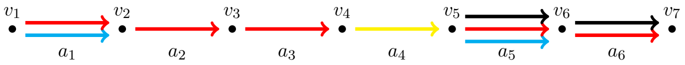

Figure: Computing $\mathcal{T}(\mathcal{P})$

$$\mathcal{T}(a_4) = \mathcal{F}(a_4) \texttt{ since } \mathcal{F}(a_4) \cap \mathcal{T}(a_3) = \varnothing$$

Example run

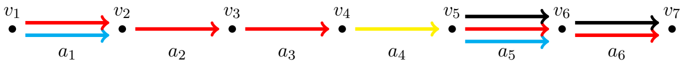

Figure: Computing $\mathcal{T}(\mathcal{P})$

$$\mathcal{T}(a_5) = \mathcal{F}(a_5) \ \texttt{since} \ \mathcal{F}(a_5) \cap \mathcal{T}(a_4) = \varnothing$$

Example run

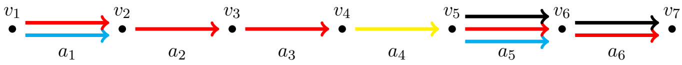

Figure: Computing $\mathcal{T}(\mathcal{P})$

$\mathcal{T}(a_6) = \mathcal{F}(a_6) \cap \mathcal{T}(a_5)$ `since not empty`

Example run

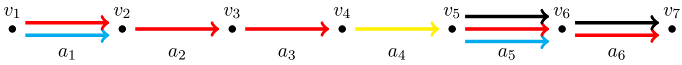

Figure: Computing $\mathcal{T}(\mathcal{P})$

Start to compute $H(\mathcal{P})$

Example run

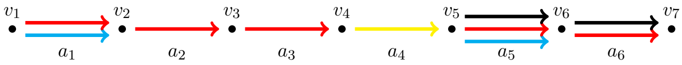

Figure: Computing $H(\mathcal{P})$

$$H(a_6) = black$$

Example run

Figure:

$$H(a_5) = black \ \ \texttt{since} \ \ black \in \mathcal{T}(a_5)$$

Example run

Figure:

$$H(a_5) = black \;\; \texttt{since} \;\; black \in \mathcal{T}(a_5)$$

Example run

Figure:

Nothing to do for a_4, a_3 and a_2 since they only have 1 color

Example run

Figure:

$$H(a_1) = red \;\; \texttt{since} \;\; red \in \mathcal{T}(a_1)$$

Proof

Minimize CS in Graph

- Example

Algorithm with Matrixes

- Complexity

Algo with *MDD*

- Complexity

Proof

The *allDiff* variant

My Implementation

Another representation of the problem

Benchmark

- Sample of Spotify

Conclusion

- Perspective