

Generation of sequences controlled by their “complexity”

Concrete Problem

We want to generate *sequences* of musical “chords” (a chord is a set of notes basically) with some known constraints (allDiff, etc.) as well as control on the *complexity* of the sequence. This complexity in turn is defined by a dynamic programming algorithm working on the instantiated sequence, which makes the whole problem difficult.

Problem Formalization

Let $x = x_1, \dots, x_n$ be variables with finite domain X

Let $G = (X, E)$ be a directed graph on X

Let Y be a finite set

Let C be a cost function on $X \times Y$, taking positive numeric values

Let T be a transition cost function between elements of Y

$C: X \times Y \rightarrow [0, +\infty)$

$T: Y \times Y \rightarrow [0, 1]$

For a **path** $x = x_1, \dots, x_n$ in graph G , we define the sequence

$$h(x) = \operatorname{argmin}_{(y_1, \dots, y_n) \in Y^n} \sum_{i \leq n} C(x_i, y_i) + \sum_{i < n} T(y_i, y_{i+1})$$

Problems with $T = 0$ or Cte

Problem 1

Generate paths $x = x_1, \dots, x_n$ with $h(x) = K$, where K is a constant

Variant 1: x are **cycles**

Variant 2: x are **all different**, $x_i \neq x_j, \forall i, j$ (all different constraint on the x 's)

Problem 2

Generate paths $x = x_1, \dots, x_n$ with a predefined number N of unique values for $h(x)$

This can be seen as posting an **NValue** constraint on $h(x)$

Same variants as problem 1

Note:

This framework may be used to generate chord sequences with control on the harmonic complexity of the sequence (through the constraints K and N). The x_1, \dots, x_n are the chords, y_1, \dots, y_n are the scales, obtained by applying an harmonic analysis algorithm to x_1, \dots, x_n .