

# **Operational semantics for Prolog with Cut in Rocq and its application to determinacy analysis**

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## **Abstract**

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12 **2012 ACM Subject Classification** Replace `ccsdesc` macro with valid one

13 **Keywords and phrases** Dummy keyword

14 **Digital Object Identifier** 10.4230/LIPIcs.CVIT.2016.23

15 **Funding Jane Open Access:** (Optional) author-specific funding acknowledgements

16 **Joan R. Public:** [funding]

17 **Acknowledgements** I want to thank ...

## **1 Introduction**

19 Elpi is a dialect of  $\lambda$ Prolog (see [14, 15, 7, 12]) used as an extension language for the Rocq  
20 prover (formerly the Coq proof assistant). Elpi has become an important infrastructure  
21 component: several projects and libraries depend on it [13, 3, 4, 19, 8, 9]. Examples include  
22 the Hierarchy-Builder library-structuring tool [5] and Derive [17, 18, 11], a program-and-proof  
23 synthesis framework with industrial applications at SkyLabs AI.

24 Starting with version 3, Elpi gained a static analysis for determinacy [10] to help users  
25 tame backtracking. Rocq users are familiar with functional programming but not necessarily  
26 with logic programming and uncontrolled backtracking is a common source of inefficiency  
27 and makes debugging harder. The determinacy checkers identifies predicates that behave  
28 like functions, i.e., predicates that commit to their first solution and leave no *choice points*  
29 (places where backtracking could resume).

30 This paper reports our first steps towards a mechanization, in the Rocq prover, of the  
31 determinacy analysis from [10]. We focus on the control operator *cut*, which is useful to  
32 restrict backtracking but makes the semantic depart from a pure logical reading.

33 We formalize two operational semantics for Prolog with cut. The first is a stack-based  
34 semantics that closely models Elpi's implementation and is similar to the semantics mechanized  
35 by Pusch in Isabelle/HOL [16] and to the model of Debray and Mishra [6, Sec. 4.3].  
36 This stack-based semantics is a good starting point to study further optimizations used  
37 by standard Prolog abstract machines [20, 1], but it makes reasoning about the scope of  
38 *cut* difficult. To address that limitation we introduce a tree-based semantics in which the  
39 branches pruned by *cut* are explicit and we prove the two semantics equivalent. Using the

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<sup>1</sup> Optional footnote, e.g. to mark corresponding author



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42nd Conference on Very Important Topics (CVIT 2016).

Editors: John Q. Open and Joan R. Access; Article No. 23; pp. 23:1–23:9  
Leibniz International Proceedings in Informatics



LIPIcs Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

## 23:2 Operational semantics for Prolog with Cut in Rocq and its application to determinacy analysis

```

Inductive P := IP of nat. Inductive D := ID of nat. Inductive V := IV of nat.

Inductive Tm :=
| Tm_P of P      | Tm_D   of D      | Tm_V of V      | Tm_App of Tm & Tm.

Inductive Callable :=
| Callable_P of P | Callable_App of Callable & Tm.

```

Figure 1 Tm and Callable types

40 tree-based semantics we then show that if every rule of a predicate passes the determinacy  
41 analysis, the call to a deterministic predicate does not leave any choice points.

## 42 2 Common code: the language

43 Before going to the two semanticis, we show the piece of data structure that are shared by  
44 the them. The smallest unit of code that we can use in the langauge is an atom. The atom  
45 inductive (see Type 1) is either a cut or a call. A call carries a callable term (see Figure 1).  
46 A term (Tm) is either a predicate, a datum, a variable or the binary application of a term to  
47 another. A Callable is a term accepting predicates only predicates as functors.

```

48 Inductive A := cut | call : Callable -> A.                               (1)
49 Record R := mkR { head : Callable; premises : list A }.                  (2)
50 Record program := { rules : seq R; sig : sigT }.                         (3)
51 Definition Sigma := {fmap V -> Tm}.                                       (4)
52 Definition bc : Unif -> program -> fvS -> Callable -> Sigma -> (fvS * seq (Sigma(5) R)) :=
```

53 A rule (see Type 2) is made a head of type term and a list of premises, the premises are  
54 atoms. A program (see Type 3) is made by a list of rules and a mapping from predicates to  
55 their signatures. The type sigT is the classic type from the simply typed lambda calculus, i.e.  
56 it is either a base type or an arrow. We decorate arrows to know the mode of the lhs type.

57 A substitution (see Type 4) is a mapping from variables to terms. It is the output of a  
58 successful query and is often called the output of a query.

```

Record Unif := {
  unify : Tm -> Tm -> Sigma -> option Sigma;
  matching : Tm -> Tm -> Sigma -> option Sigma;
}.

```

59 The backchain function (bc, see Type 5) filters the rules in the program that can be  
60 used on a given query. It takes: a unificator  $U$  which explains how to unify terms up to  
61 standard unification (for output terms) or matching (for input terms); a program  $P$  to explore  
62 and filter; a set  $S$  of free variable (fvS) allowing to fresh the program  $P$  by renaming the  
63 its variables; a query  $q$ ; and the substitution  $\sigma$  in which the query  $q$  lives. The result of a  
64 backchain operation is couple made of an extension of  $S$  containing the new variales that  
65 have been allocated during the unification phase and a list of filtered rules  $r$  accompagnate  
66 by their a substition. This substitution is the result of the unification of  $q$  with the head of  
67 each rule in  $r$ .

68 Below we have an example of a simple elpi program which will be used in the following  
69 section of the paper as an example to show how backtracking and the cut operator works  
70 in the semantcis we propose. The translation of these rules in the Rocq representation is  
71 straightforward.

```
f 1 2.    f 2 3.    r 2 4.    r 2 8.
g X X.          % r1
g X Z :- r X Z, !.   % r2
g X Z :- f X Y, f Y Z.   % r3
```

**Figure 2** Small elpi program example

## 2.1 The cut operator

The semantics of the cut operator adopted in the ELPI language corresponds to the *hard cut* operator of standard SWI-PROLOG. This operator has two primary purposes. First, it eliminates all alternatives that are created either simultaneously with, or after, the introduction of the cut into the execution state.

To illustrate this high-level description, consider the program shown in Figure 2 and the query  $q = g 2 Z$ . All three rules for  $g$  can be used on the query  $q$ . They are tried according to their order of appearance in the program: rule  $r_1$  is tried first, followed by  $r_2$ , and  $r_3$ .

The first rule has no premises and immediately returns the assignment  $Z = 2$ . However, the computation does not terminate at this point, since two additional unexplored alternatives remain, corresponding to the premises of rules  $r_2$  and  $r_3$ .

The premises of rule  $r_2$  are  $r 2 Z, !$ . At this stage, the role of the cut becomes apparent. If the premise  $r 2 Z$  succeeds, the cut commits to this choice and removes the premises of rule  $r_3$  from the alternative list, as they were generated at the same point as the cut. Moreover, if the call  $r 2 Z$  itself produces multiple alternatives, only the first one is committed, while the remaining alternatives are discarded. This is because such alternatives have been created at a deeper depth in the search tree than the cut.

Concretely, the call  $r 2 Z$  yields two solutions, assigning  $Z$  the values 4 and 8, respectively. The second solution is eliminated by the cut, and only the first assignment is preserved.

## 3 Semantics intro

We propose two operational semantics for a logic program with cut. The two semantics are based on different syntaxes, the first syntax (called tree) exploits a tree-like structure and is ideal both to have a graphical view of its evolution while the state is being interpreted and to prove lemmas over it. The second syntax, called elpi, is the elpi's syntax and has the advantage of reducing the computational cost of cutting and backtracking alternatives by using shared pointers. We aim to prove the equivalence of the two semantics together with some interesting lemmas of the cut behavior.

### 3.1 Tree semantics

```
Inductive tree :=
| KO | OK | Dead
| TA : A -> tree
| Or  : tree -> Sigma -> tree -> tree
| And : tree -> seq A -> tree -> tree.
```

In the tree we distinguish 6 main cases: *KO*, *OK*, and *Dead* are special meta-symbols representing, respectively, a failed, a successful, and a dead terminal. These symbols are considered meta because they are internal intermediate symbols used to give structure to the

```

Fixpoint path_end A :=
  match A with
  | Dead | OK | KO | TA _ => A
  | Or A _ B =>
    if is_dead A then path_end B
    else path_end A
  | And A BO B =>
    match path_end A with
    | OK => path_end B
    | A => A
    end
  end.

Fixpoint is_dead A :=
  match A with
  | Dead => true
  | OK | KO | TA _ => false
  | And A BO B => is_dead A
  | Or A s B => is_dead A && is_dead B
  end.

```

(a) Defintion of *is\_dead*(b) Defintion of *path\_end*

103 tree. While the first two symbols are of immiediate understanding, we use *Dead* to represent  
 104 ghost state, that is, the *Dead* symbol is always ignored by the tree interpreter.

105 *TA* (acronym for tree-atom) is the constructor of atoms in the tree.

106 The two recursive cases of a tree are the *Or* and *And* non-terminals. The *Or* non-terminal  
 107  $A \vee B_\sigma$  denotes a disjunction between two trees  $A$  and  $B$ . The second branch is annotated  
 108 with a suspended substitution  $\sigma$  so that, upon backtracking to  $B$ ,  $\sigma$  is used as the initial  
 109 substitution for the execution of  $B$ .

110 The *And* non-terminal  $A \wedge_{B_0} B$  represents a conjunction of two trees  $A$  and  $B$ . We  
 111 call  $B_0$  the reset point for  $B$ ; it is used to restore the state of  $B$  to its initial form if a  
 112 backtracking operation occurs on  $A$ . Intuitively in prolog-like syntax, in a tree  $A \wedge_{B_0} B$ , if  
 113  $t2l$  is the function flattening the tree in a list of sequents disjunction and  $t2l(A) = A_1, \dots, A_n$ ,  
 114 then we would have  $(A_1, t2l B); (A_2, B0); \dots; (A_n, B0)$ .

115 A graphical representation of the tree is shown in Figure 4a. To make the graph more  
 116 compact, the *And* and *Or* non-terminals are n-ary rather than binary, with right-binding  
 117 priority. The *KO* and *Dead* terminals act as the neutral elements in the *Or* list, while *OK*  
 118 is the neutral element of the *And* list.

119 The interpretation of a tree is performed by two main routines: *step* and *next\_alt* that  
 120 traverse the tree depth-first, left-to-right. Then, then *run* inductive makes the transitive  
 121 closure of step *step* and *next\_alt*: it iterates the calls to its auxiliary functions. In Types 7–9  
 122 we give the types contrats of these symbols where *fv* is a set of variable names.

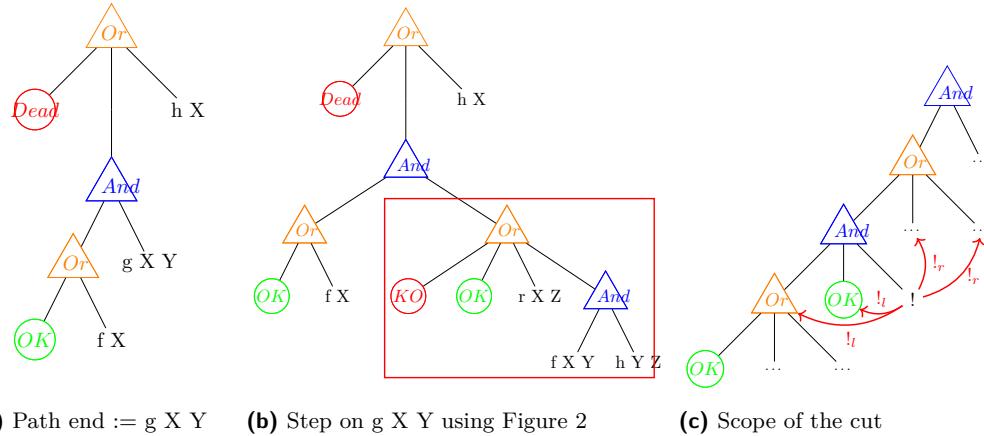
```

123 Inductive step_tag := Expanded | CutBrothers | Failed | Success.           (6)
124 Definition step : program -> fvS -> Sigma -> tree -> (fvS * step_tag * tree) := (7)
125 Definition next_alt : bool -> tree -> option tree :=                      (8)
126 Inductive run (p : program) : fvS -> Sigma -> tree -> option Sigma -> tree -> bool -> Prop := (9)

```

127 A particular tree we want to identify is a *is\_dead* tree (defined in Figure 3a). This tree  
 128 has the property to never produce a solution: it is eiher the *Dead* tree or both branches of  
 129 *Or* are dead, or the lhs of *And* is dead. In the latter case, we note that  $B$  can be non-dead,  
 130 but this is not a problem since the interpreter can run  $B$  only if  $A$  is non-dead.

131 The prolog interpreter explores the state in DFS strategy, it finds the “first-to-be-explored”  
 132 (ftbe) atom of the tree and then interpretes it. In a non-*is\_dead* tree, we get the ftbe node  
 133 via *path\_end*, shown in Figure 3b. The *path\_end* is either the tree itself if the tree is a leaf.  
 134 Otherwise, if the tree is a disjunction, the path continues on the left- or the right-subtree  
 135 depending of if the the lhs is a *is\_dead* tree. In the *Or* case we are clearing ignoring the



**Figure 4** Some tree representations

136 dead (ghost) state.

137 In the case of a conjunction, it is more interesting to see what happens. If the *path\_end*  
 138 *p* of the lhs is a success then we look for the *path\_end* in the rhs, otherwise we return *p*. In  
 139 Figure 4a the *path\_end* of the tree is *g X*.

140 Below we define two special kind of trees depending on their pathend.

141 **Definition** *success* *A* := *path\_end A == OK*. (1)

142 **Definition** *failed* *A* := (*path\_end A == KO*) || (*path\_end A == Dead*). (2)

143 The *step* procedure takes as input a program, a set of free variables (fv), a substitution,  
 144 and a tree, and returns an updated set of free variables, a *step\_tag*, and an updated tree.

145 Free variables are those variables that appear in a tree; they are used in the backchaining  
 146 operation to refresh the variables in the program.

147 The *step\_tag* indicates the type of internal tree step that has been performed. *CutBrothers*  
 148 denotes the interpretation of a superficial cut, i.e., a cut whose parent nodes are all *And*-nodes.  
 149 *Expanded* denotes the interpretation of non-superficial cuts or predicate calls. *Failure* and  
 150 *Success* are returned for, respectively, *failed* and *success* trees.

151 The step procedure is intended to interpretate atoms, that is, it returns the identity for  
 152 *success* and *failed* tree.

153 **Lemma** *success\_step u p fv s A*: *success A -> step u p fv s A = (fv, Success, A)*. (1)  
 154 **Lemma** *failed\_step u p fv s1 A*: *failed A -> step u p fv s1 A = (fv, Failed, A)*. (2)

155 Therefore, the two interesting cases of a tree the interpretation of trees with path-end  
 156 equal to a call or a cut atom.

157 *Call step* The interpretation of a call *c* is performed by replacing the call wrt the result  
 158 of the *B c*, then if *l* is empty then *KO* tree is returned, otherwise the call is replaced by  
 159 right-skewed tree made of *n* inner *Or* nodes, where *n* is the length of *l*. The root has *KO* as  
 160 left child. The lhs of the other nodes is a right-skewed tree of *And* nodes. The *And* nodes  
 161 are again a right-seked tree containing then atoms (either cut or call) taken from the list *l*.

162 A step in the tree of Figure 4a makes a backchain operation over the query *g X Y* and, in  
 163 the program defined in Figure 2, the new tree would be the one in Figure 4b. We have put a  
 164 red border aroung the new generated subtree. It is a disjunction of four subtrees: the first  
 165 node is the *KO* node (by default), the second is *OK*, since *r1* has no premises, the third and  
 166 the fourth contains the premises of respectively *r2* and *r3*.

dire dei reset  
point

dire che le sostituzioni del backchain sono importanti

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## 23:6 Operational semantics for Prolog with Cut in Rocq and its application to determinacy analysis

167        *Cut step* The latter case is delicate since interpreting a cut in a tree has three main  
 168        impacts: at first it is replaced by the *OK* node, then some special subtrees, in the scope  
 169        of the *Cut*, are cut away: in particular we need to soft-kill the left-siblings of the *Cut* and  
 170        hard-kill the right-uncles of the the *Cut*.

171        ► **Definition 1** (Left-siblings (resp. right-sibling)). *Given a node A, the left-siblings (resp.*  
 172        *right-sibling) of A are the list of subtrees sharing the same parent of A and that appear on*  
 173        *its left (resp. right).*

174        ► **Definition 2** (Right-uncles). *Given a node A, the right-uncles of A are the list of right-sibling*  
 175        *of the father of A.*

176        ► **Definition 3** (Soft-kill). *Given a tree t, soft-kill replaces all the leaves of the tree with the*  
 177        *node KO except for the leaves that are part of the path p of t.*

178        ► **Definition 4** (Hard-kill). *Given a tree t, hard-kill replaces all the leaves of the tree with the*  
 179        *node KO*

180        An example of the impact of the cut is show in Figure 4c. The step routine interprets  
 181        the cut if it is at the end of the current path. In the example we have tagged in red the  
 182        arrow  $!_l$  indicating which sub-trees is soft-killed and  $!_r$  indicated which is sub-trees are to be  
 183        hard-killed.

### 184        3.1.1 Execution example

### 185        3.1.2 Valid tree

## 186        3.2 Elpi semantics

187        TODO: dire che la semantica ad albero è più facile per le prove

188        The Elpi interpreter is based on an operational semantics close to the one picked by  
 189        Pusch in [16], in turn closely related to the one given by Debray and Mishra in [6, Section  
 190        4.3]. Push mechanized the semantics in Isabelle/HOL together with some optimizations that  
 191        are present in the Warren Abstract Machine [20, 1].

192        In these operational semantics we need to decorate the cut atom with a list of alternative,  
 193        morally a pointer to a sub-list of the overall alternatives. An atom in the elpi semantics is  
 194        defined as follows:

```
Inductive alts :=
| no_alt
| more_alt : (Sigma * goals) -> alts -> alts
with goals :=
| no_goals
| more_goals : (A * alts) -> goals -> goals .
```

195        We are completely loosing the tree structure. There are no clean reset points. The  
 196        backtracking operation is simpler: it is the tail function. The cutr and cutm operations  
 197        disappears: the alternatives are stored directly in the cutE terminal.

198        The elpi interpreter is as follows:

```
(*TODO: add system of rules*)
Inductive nur : Sigma -> goals -> alts -> Sigma -> alts -> Type :=
| StopE s a : nur s nilC a s a
```

```

| CutE s s1 a ca r gl : nur s gl ca s1 r -> nur s ((cutE ca) :::: gl) a s1 r
| CallE p s s1 a b bs gl r t :
  F u p t s = [:: b & bs ] ->
    nur b.1 (save_goals a gl (a2gs1 p b)) (save_alts a gl ((aa2gs p) bs) ++ a) s1 r ->
    nur s ((callE p t) :::: gl) a s1 r
| FailE p s s1 s2 t gl a al r :
  F u p t s = [::] -> nur s1 a al s2 r -> nur s ((callE p t) :::: gl) ((s1, a) :::: al) s2 r.

```

199 The translation of a tree to a list is as follows:

```

Fixpoint t2l (A: tree) s (bt : alts) : alts :=
match A with
| OK          => [:: (s, [::]) ]
| (KO | Dead) => [::]
| TA a         => [:: (s, [:: (a,[::]) ]) ]
| Or A s1 B   =>
  let 1B := t2l B s1 [::] in
  let 1A := t2l A s1B in
  add_ca_deep bt (1A ++ 1B)
| And A BO B  =>
  let 1BO : goals := r2l BO in
  let 1A  := t2l A s bt in
  if 1A is [:: (s1A, x) & xs] then
    let xz := add_deepG bt 1BO x in
    let xs := add_deep bt 1BO xs in
    let xs := make_1B0 xs 1BO in
    let 1B := t2l B s1A (xs ++ bt) in
    (make_1B01 1B xz) ++ xs
  else [::]
end.

```

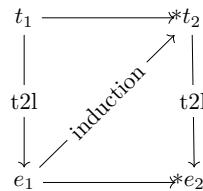
► **Theorem 5** (tree\_to\_elpi).

200  $\forall A \sigma_1 B \sigma_2 b \sigma_0, vt A \rightarrow$   
 201  $run_u \sigma_1 A (\text{Some } \sigma_2) B b \rightarrow$   
 202  $\exists x xs, t2l A \sigma_1 \emptyset = x ::: xs \wedge nur_u x.1 x.2 xs \sigma_2 (t2l B \sigma_0 \emptyset).$

► **Theorem 6** (elpi\_to\_tree).

203  $\forall \sigma_1 \sigma_2 a na g,$   
 204  $nur_u \sigma_1 g a \sigma_2 na \rightarrow$   
 205  $\forall \sigma_0 t, vt t \rightarrow (t2l t \sigma_0 \emptyset) = ((\sigma_1, g) ::: a) \rightarrow$   
 206  $\exists t' n, run_u \sigma_0 t (\text{Some } \sigma_2) t' n \wedge t2l t' \sigma_0 \emptyset = na.$

207 The proof of Theorem 6 is based on the idea explained in [2, Section 3.3]. An ideal  
 208 statement for this lemma would be: given a function  $12t$  transforming an elpi state to a tree,  
 209 we would have have that the the execution of an elpi state  $e$  is the same as executing  $run$  on  
 210 the tree resulting from  $12t(e)$ . However, it is difficult to retrive the strucutre of an elpi state  
 211 and create a tree from it. This is because, in an elpi state, we have no clear information  
 212 about the scope of an atom inside the list and, therefore, no evident clue about where this  
 213 atom should be place in the tree.

**Figure 5** Induction scheme for Theorem 6

Our theorem states that, starting from a valid state  $t$  which translates to a list of alternatives  $(\sigma_1, g) :: a$ . If we run in elpi the list of alternatives, then the execution of the tree  $t$  returns the same result as the execution in elpi. The proof is performed by induction on the derivations of the elpi execution. We have 4 derivations.

We have 4 case to analyse:

---

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