

Dummy title

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⁷ — Abstract —

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¹⁷ 1 Common code: the language

Inductive Tm :=

```
| Tm_Kp      : Kp -> Tm
| Tm_Kd      : Kd -> Tm
| Tm_V       : V   -> Tm
| Tm_Comb    : Tm -> Tm -> Tm.
```

Inductive Callable :=

```
| Callable_Kp    : Kp -> Callable
| Callable_V     : V   -> Callable
| Callable_Comb : Callable -> Tm -> Callable.
```

Inductive RCallable :=

```
| RCallable_Kp    : Kp -> RCallable
| RCallable_Comb : RCallable -> Tm -> RCallable.
```

¹⁸ A callable term is a term without a data constructor as functor.

¹⁹ An rcallable is a term with rigid head.

Inductive A := `cut` | `call` : Callable -> A.

²⁰ An atom is the smallest syntactic unit that can be executed in a prolog program \mathcal{P} . The execution of an atom, inside a program and a substitution either succeeds returning an output substitution, or it fails. In both cases it returns a list of choice points, representing suspending states that can be resumed for backtracking.

¹ Optional footnote, e.g. to mark corresponding author

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```
Record R := mkR { head : RCallable; premises : list A }.
```

24 We exploit the typing system to ensure that the head of a "valid" rule is a term with rigid
25 head.

(*simpler than in the code: signatures of preds are hidden*)

```
Definition program := seq R.
```

26 A program is made by a list of rules. Rules in \mathcal{P} are indexed by their position in the list.
27 Given a list of rules \mathcal{R} and two indexes i and j , s.t. $i \neq j$ then, \mathcal{R}_i has a higher priority than
28 \mathcal{R}_j .

29 Sigma is a substitution mapping variables to their term instantiation.

```
Definition Sigma := {fmap V -> Tm}.
```

30 The backchaining algorithm is the function \mathcal{B} aims to filter only the rules in the program
31 \mathcal{P} having rules unifying with the current query q in a given substitution σ using the list
32 of modes m . In particular \mathcal{B} returns for each selected rule r a substitution σ' that is the
33 substitution obtained by the unification of the query and the head of r .

$$\mathcal{B} : (\mathcal{P}, \sigma, q) \rightarrow \text{seq}(\sigma * R)$$

34 2 Semantics intro

35 We propose two operational semantics for a logic program with cut. The two semantics are
36 based on different syntaxes, the first syntax (called tree) exploits a tree-like structure and is
37 ideal to have a graphical view of its evaloution while the prorma is being intepreted. The
38 second syntax is the elpi's syntax, we call it therefore elpi. We aim to prove the equivalence
39 of the two semantics together with some interesting lemmas of the cut behavior.

40 2.1 Tree semantics

```
Inductive tree :=
| Bot | OK | Dead
| TA : program -> A -> tree
| Or : tree -> Sigma -> tree -> tree
| And : tree -> (program * seq A) -> tree -> tree.
```

41 In the tree we distinguish 6 main cases: Bot and OK are respectively the standard fail \perp
42 and true \top predicates of prolog. Dead is a special symbol representing a ghost state, that
43 is, a state useful to keep the structure of a tree from an execution to another but that is
44 completely ignored by the interpretation of the program.

45 TA, standing for tree-atom, is a terminal of the tree containing an atom and a program.

46 The two recursive cases of a tree are the Or and the And non-terminals. The Or non-
47 terminals $A \vee B_\sigma$ stands for a disjunction between two trees A and B . The second tree branch
48 is decorated with a suspended substitution σ so that, when we backtrack to B , we use σ as
49 initial substitution for B .

50 The And non-terminal $A \wedge_r B$ represents of a conjunction of two trees A and B . We call
51 r the reset-point and is used to resume the B state in its intial form if some backtracking
52 operation is performed on A .

53 The main

```
Inductive step_tag := Expanded | CutBrothers| Failure | Success.
```

```
Fixpoint step s A : (step_tag * tree) :=
  match A with
  | OK          => (Success, OK)
  | Bot | Dead   => (Failure, A)

  | TA _ cut     => (CutBrothers, OK)
  | TA pr (call t) => (Expanded, (big_or pr s t))

  | Or A sB B =>
    if is_dead A then
      let rB := (step sB B) in
      (if is_cb rB.1 then Expanded else rB.1, Or A sB rB.2)
    else
      let rA := step s A in
      (if is_cb rA.1 then Expanded else rA.1, Or rA.2 sB (if is_cb rA.1 then cutr B else B))
  | And A B0 B =>
    let rA := step s A in
    if is_sc rA.1 then
      let rB := (step (get_substS s rA.2) B) in
      (rB.1, And (if is_cb rB.1 then cutl A else A) B0 rB.2)
    else (rA.1, And rA.2 B0 B)
  end.
```

Figure 1 Step for tree semantics

```
Fixpoint next_alt b (A : tree) : option (tree) :=
  match A with
  | Bot | Dead => None
  | OK => if b then None else Some OK
  | TA _ _ => Some A
  | And A (pr, B0) B =>
    let build_B0 A := Some (And A (pr, B0) (big_and pr B0)) in
    let reset := obind build_B0 (next_alt (success A) A) in
    if success A then
      match next_alt b B with
      | None => reset
      | Some B => Some (And A (pr, B0) B)
      end
    else if failed A then reset
    else Some (And A (pr, B0) B)
  | Or A sB B =>
    if is_dead A then omap (fun x => (Or A sB x)) (next_alt b B)
    else match next_alt b A with
      | None => obind (fun x => Some (Or (dead A) sB x)) (next_alt false B)
      | Some nA => Some (Or nA sB B)
      end
  end.
```

Figure 2 backtracking operation

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```

Fixpoint valid_tree s :=
  match s with
  | TA _ _ | OK | Bot => true
  | Dead => false
  | Or A _ B =>
    if is_dead A then valid_tree B
    else valid_tree A && (bbOr B)
  | And A BO B =>
    valid_tree A &&
    if success A then valid_tree B
    else
      let B' := big_and BO.1 BO.2 in
      B == B'
  end.

```

■ **Figure 3** Valid tree

TODO: define path 55 The tree interpreter is made by two fixpoints and an inductive.
 We make the distinction between some kind of particular trees:
 56 1. success is a tree with a successfull path
 57 2. failed is a tree with a failed path
 58 3. dead is a tree with deads states

59 2.2 Elpi semantics

60 The Elpi interpreter is based on an operational semantics close to the one picked by Pusch in
 61 [?], in turn closely related to the one given by Debray and Mishra in [?, Section 4.3]. Push
 62 mechanized the semantics in Isabelle/HOL together with some optimizations that are present
 63 in the Warren Abstract Machine [?, ?].

64 In these operational semantics we need to decorate the cut atom with a list of alternative,
 65 morally a pointer to a sub-list of the overall alternatives. An atom in the elpi semantcis is
 66 defined as follows:

```

Inductive G :=
| callE : program -> Callable -> G
| cutE : alts -> G
with alts := Al of seq (seq G)
with goals := Gl of seq goals.

```

67 We are completely loosing the tree structure. There are no clean reset points. The
 68 backtracking operation is simpler: it is the tail function. The cutr and ccutl operations
 69 disappears: the alternatives are stored directly in the cutE terminal.

70 The elpi interpreter is as follows:

```

(*TODO: add system of rules*)
Inductive nur : Sigma -> goals -> alts -> Sigma -> alts -> Type :=
| StopE s a : nur s nilC a s a
| CutE s s1 a ca r gl : nur s gl ca s1 r -> nur s ((cutE ca) :: gl) a s1 r
| CallE p s s1 a b bs gl r t :

```

```

F u p t s = [:: b & bs ] ->
  nur b.1 (save_goals a gl (a2gs1 p b)) (save_alts a gl ((aa2gs p) bs) ++ a) s1 r ->
    nur s ((callE p t) :::: gl) a s1 r
| FailE p s s1 s2 t gl a al r :
F u p t s = [:::] -> nur s1 a al s2 r -> nur s ((callE p t) :::: gl) ((s1, a) :::: al) s2 r.

```

71 The translation of a tree to a list is as follows:

```

(*TODO: add notation for nilC which is [::]*)
Fixpoint t21 (A: tree) s (bt : alts) : alts :=
match A with
| OK => (s, nilC) :: nilC
| Bot => nilC
| Dead => nilC
| TA _ cut => (s, ((cuteE nilC) :: nilC)) :: nilC
| TA pr (call t) => (s, ((callE pr t) :: nilC)) :: nilC
| Or A s1 B =>
  let lB := t21 B s1 nilC in
  let lA := t21 A s lB in
  add_ca_deep bt (lA ++ lB)
| And A B0 B =>
  let hd := r21 B0.1 B0.2 in
  let lA := t21 A s bt in
  if lA is more_alt (s1A, x) xs then
    (* the reset point exists, it has to be added to all cut-to alternatives *)
    let xz := add_deepG bt hd x in
    let xs := add_deep bt hd xs in
    (* each alt in xs must have hd as rightmost conjunct *)
    let xs := make_lB0 xs hd in
    (* xs are alternatives that should be added in the deep cuts in B *)
    let lB := t21 B s1A (xs ++ bt) in
    (* lB are alternatives, each of them have x as head *)
    (make_lB01 lB xz) ++ xs
  else nilC
end.

```

► **Theorem 1** (`tree_to_elpi`).

72 $\forall A \sigma_1 B \sigma_2 b \sigma_0, vt A \rightarrow$
 73 $run_u \sigma_1 A (Some \sigma_2) B b \rightarrow$
 74 $\exists x xs, t2l A \sigma_1 \emptyset = x :: xs \wedge nur_u x.1 x.2 xs \sigma_2 (t2l B \sigma_0 \emptyset).$

► **Theorem 2** (`elpi_to_tree`).

75	$\forall \sigma_1 \sigma_2 a na g,$
76	$nur_u \sigma_1 g a \sigma_2 na \rightarrow$
77	$\forall \sigma_0 t, vt t \rightarrow (t2l t \sigma_0 \emptyset) = ((\sigma_1, g) :: a) \rightarrow$
78	$\exists t' n, run_u \sigma_0 t (Some \sigma_2) t' n \wedge t2l t' \sigma_0 \emptyset = na.$