

Dummy title

¹ **Jane Open Access**  

³ Dummy University Computing Laboratory, [optional: Address], Country

⁴ My second affiliation, Country

⁵ **Joan R. Public¹**  

⁶ Department of Informatics, Dummy College, [optional: Address], Country

⁷ — Abstract —

⁸ Lorem ipsum dolor sit amet, consectetur adipiscing elit. Praesent convallis orci arcu, eu mollis dolor. Aliquam
⁹ eleifend suscipit lacinia. Maecenas quam mi, porta ut lacinia sed, convallis ac dui. Lorem ipsum dolor sit amet,
¹⁰ consectetur adipiscing elit. Suspendisse potenti.

¹¹ **2012 ACM Subject Classification** Replace `ccsdesc` macro with valid one

¹² **Keywords and phrases** Dummy keyword

¹³ **Digital Object Identifier** 10.4230/LIPIcs.CVIT.2016.23

¹⁴ **Funding** *Jane Open Access*: (Optional) author-specific funding acknowledgements

¹⁵ *Joan R. Public*: [funding]

¹⁶ **Acknowledgements** I want to thank ...

¹⁷ 1 Common code: the language

Inductive Tm :=

```
| Tm_Kp      : Kp  -> Tm
| Tm_Kd      : Kd  -> Tm
| Tm_V       : V   -> Tm
| Tm_Comb    : Tm  -> Tm -> Tm.
```

Inductive Callable :=

```
| Callable_Kp    : Kp  -> Callable
| Callable_V     : V   -> Callable
| Callable_Comb : Callable -> Tm -> Callable.
```

Inductive RCallable :=

```
| RCallable_Kp    : Kp  -> RCallable
| RCallable_Comb : RCallable -> Tm -> RCallable.
```

¹⁸ A callable term is a term without a data constructor as functor.

¹⁹ An rcallable is a term with rigid head.

Inductive A := **cut** | **call** : Callable -> A.

²⁰ An atom is the smallest syntactic unit that can be executed in a prolog program \mathcal{P} .

Record R := mkR { head : RCallable; premises : list A }.

²¹ We exploit the typing system to ensure that the head of a "valid" rule is a term with rigid head.

¹ Optional footnote, e.g. to mark corresponding author



© Jane Open Access and Joan R. Public;
licensed under Creative Commons License CC-BY 4.0

42nd Conference on Very Important Topics (CVIT 2016)

Editors: John Q. Open and Joan R. Access; Article No. 23; pp. 23:1–23:5



LIPICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

(*simpler than in the code: signatures of preds are hidden*)
Definition program := seq R.

22 A program is made by a list of rules. Rules in \mathcal{P} are indexed by their position in the list. Given a
 23 list of rules \mathcal{R} and two indexes i and j , s.t. $i \neq j$ then, \mathcal{R}_i has a higher priority than \mathcal{R}_j .
 24 Sigma is a substitution mapping variables to their term instantiation.

Definition Sigma := {fmap V -> Tm} .

25 The backchaining algorithm is the function \mathcal{B} aims to filter only the rules in the program \mathcal{P}
 26 having rules unifying with the current query q in a given substitution σ using the list of modes m .
 27 In particular \mathcal{B} returns for each selected rule r a substitution σ' that is the substitution obtained by
 28 the unification of the query and the head of r .

$$\mathcal{B} : (\mathcal{P}, \sigma, q) \rightarrow \text{seq}(\sigma * R)$$

29 2 Semantics intro

30 We propose two operational semantics for a logic program with cut. The two semantics are based
 31 on different syntaxes, the first syntax (called tree) exploits a tree-like structure and is ideal to have
 32 a graphical view of its evaloution while the progrma is being intepreted. The second syntax is
 33 the elpi's syntax, we call it therefore elpi. We aim to prove the equivalence of the two semantics
 34 together with some interesting lemmas of the cut behavior.

35 2.1 Tree semantics

Inductive tree :=
 | Bot | OK | Dead
 | TA : A -> tree
 | Or : tree -> Sigma -> tree -> tree
 | And : tree -> seq A -> tree -> tree.

36 In the tree we distinguish 6 main cases: Bot and OK are respectively the standard fail \perp and true
 37 \top predicates of prolog. Dead is a special symbol representing a ghost state, that is, a state useful
 38 to keep the structure of a tree from an execution to another but that is completely ignored by the
 39 intepretation of the program.

40 TA, standing for tree-atom, is a terminal of the tree containg an atom.

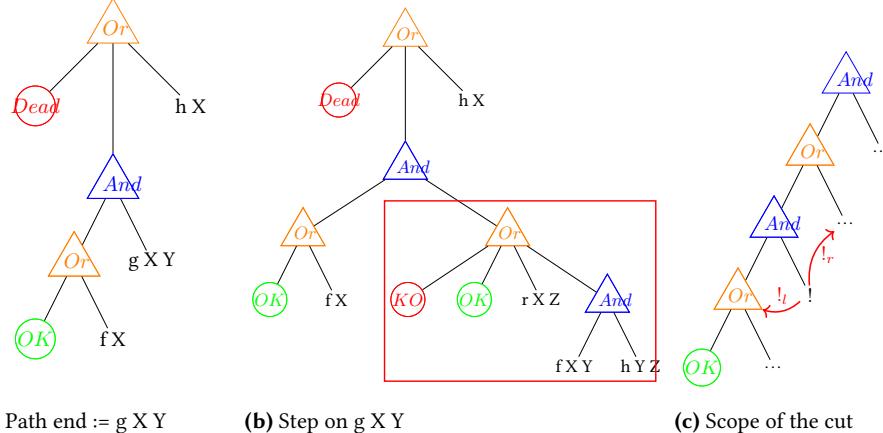
41 The two recursive cases of a tree are the Or and the And non-terinals. The Or non-terminals
 42 $A \vee B_\sigma$ stands for a disjunction between two trees A and B . The second tree branch is decorated
 43 with a suspended substiuion σ so that, when we backtrack to B , we use σ as initial substitution for
 44 B .

45 The And non-terminal $A \wedge_r B$ represents of a conjunction of two trees A and B . We call r the
 46 reset-point and is used to resume the B state in its intial form if some backtracking operation is
 47 performed on A .

48 The interpretation of a tree is performed by two main routines: step and next_alt that
 49 traverse the tree depth-first, left-to-right.

50 We get the first to-be-explored terminal in the tree by getting the end of a path. This path is
 51 created from a tree traversal starting from the roots and immidiately ends if the tree is not neither a
 52 disjunction, nor a conjunction: the to-be-explored terminal is the tree itself. Otherwise, if the tree is
 53 a disjunction, the path continues on the left- or the right-subtree depending of if the path of the lhs
 54 is a dead node. In the case of a conjunction, we look for the path of the lhs. If this path returns a

55 success, we build a path in the rhs, otherwise, we return the lhs. In Figure 1a the first non-explored
 56 node is g X.



(a) Path end := g X Y (b) Step on g X Y (c) Scope of the cut

■ **Figure 1** Tree with first non explored node g X

57 The **s**tep procedure takes a tree and explores it using the path strategy. A success (i.e. a tree
 58 with path ending with OK) and failed tree (i.e. a tree with path ending with KO or Dead) is returned
 59 as it. The two interesting cases are when the path ends with a call or a cut.

60 *Call step* In the former case the call node is replaced with a new subtree made by the rules returned
 61 by the \mathcal{B} function. If \mathcal{B} returns a list l , if l is empty then KO tree is returned, otherwise the call
 62 is replaced by right-skewed tree made of n inner Or nodes, where n is the length of l . The root
 63 Or-node has KO as left child. The lhs of the other nodes is a right-skewed tree of And nodes. The
 64 And nodes are again a right-seked tree containing then atoms (either cut or call) taken from the list
 65 l .

```
g X X. % r1
g X Z :- r X Z. % r2
g X Z :- f X Y, h Y Z. % r3
```

66 A step in the tree in Figure 1a make a backchain operation over the query g X Y and, in the
 67 program above, the new tree would be the one in Figure 1b. We have put a red border aroung
 68 the new generated subtree. It is a disjunction of four subtrees: the first node is the Dead node (by
 69 default), the second is OK, since r1 has no premises, the third and the fourth contains the premises
 70 of respectively r2 and r3.

71 *Cut step* The latter case is more delicate since performing a cut in a tree has

dire che le sostituzioni del backchain sono importanti

72 2.1.1 Valid tree

73 2.2 Elpi semantics

74 The Elpi interpreter is based on an operational semantics close to the one picked by Pusch in [4], in
 75 turn closely related to the one given by Debray and Mishra in [3, Section 4.3]. Push mechanized
 76 the semantics in Isabelle/HOL together with some optimizations that are present in the Warren
 77 Abstract Machine [5, 1].

78 In these operational semantics we need to decorate the cut atom with a list of alternative, morally
 79 a pointer to a sub-list of the overall alternatives. An atom in the elpi semantcis is defined as follows:

```
Inductive G :=
| callE : Callable -> G
| cutE : alts -> G
with alts :=
| no_alt
| more_alt : (Sigma * goals) -> alts -> alts
with goals :=
| no_goals
| more_goals : G -> goals -> goals .
```

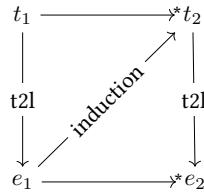
80 We are completely loosing the tree structure. There are no clean reset points. The backtracking
 81 operation is simpler: it is the tail function. The cutr and cctl operations disappears: the alternatives
 82 are stored directly in the cutE terminal.

83 The elpi interpreter is as follows:

```
(*TODO: add system of rules*)
Inductive nur : Sigma -> goals -> alts -> Sigma -> alts -> Type :=
| StopE s a : nur s nilC a s a
| CutE s s1 a ca r g1 : nur s g1 ca s1 r -> nur s ((cutE ca) :: g1) a s1 r
| CallE p s s1 a b bs g1 r t :
  F u p t s = [:: b & bs] ->
    nur b.1 (save_goals a g1 (a2gs1 p b)) (save_alts a g1 ((aa2gs p) bs) ++
      nur s ((callE p t) :: g1) a s1 r
| FailE p s s1 s2 t g1 a al r :
  F u p t s = [::] -> nur s1 a al s2 r -> nur s ((callE p t) :: g1) ((s1,
```

84 The translation of a tree to a list is as follows:

```
Fixpoint t21 (A: tree) s (bt : alts) : alts :=
match A with
| OK => (s, nilC) :: nilC
| Bot => nilC
| Dead => nilC
| TA cut => (s, ((cutE nilC) :: nilC)) :: nilC
| TA (call t) => (s, ((callE t) :: nilC)) :: nilC
| Or A s1 B =>
  let 1B := t21 B s1 nilC in
  let 1A := t21 A s 1B in
  add_ca_deep bt (1A ++ 1B)
| And A B0 B =>
  let hd := r21 B0 in
  let 1A := t21 A s bt in
  if 1A is more_alt (s1A, x) xs then
    let xz := add_deepG bt hd x in
    let xs := add_deep bt hd xs in
    let xs := make_1B01 xs hd in
    let 1B := t21 B s1A (xs ++ bt) in
    (make_1B01 1B xz) ++ xs
  else nilC
end.
```



■ **Figure 2** Induction scheme for Theorem 2

► **Theorem 1 (tree_to_elpi).**

```

85            $\forall A \sigma_1 B \sigma_2 b \sigma_0, vt A \rightarrow$ 
86            $run_u \sigma_1 A (\text{Some } \sigma_2) B b \rightarrow$ 
87            $\exists x xs, t21 A \sigma_1 \emptyset = x :: xs \wedge nur_u x.1 x.2 xs \sigma_2 (t21 B \sigma_0 \emptyset).$ 
  
```

► **Theorem 2 (elpi_to_tree).**

```

88            $\forall \sigma_1 \sigma_2 a na g,$ 
89            $nur_u \sigma_1 g a \sigma_2 na \rightarrow$ 
90            $\forall \sigma_0 t, vt t \rightarrow (t21 t \sigma_0 \emptyset) = ((\sigma_1, g) :: a) \rightarrow$ 
91            $\exists t' n, run_u \sigma_0 t (\text{Some } \sigma_2) t' n \wedge t21 t' \sigma_0 \emptyset = na.$ 
  
```

92 The proof of Theorem 2 is based on the idea explained in [2, Section 3.3]. An ideal statement for
 93 this lemma would be: given a function $12t$ transforming an elpi state to a tree, we would have
 94 have that the the execution of an elpi state e is the same as executing run on the tree resulting
 95 from $12t(e)$. However, it is difficult to retrive the strucuture of an elpi state and create a tree from
 96 it. This is because, in an elpi state, we have no clear information about the scope of an atom inside
 97 the list and, therefore, no evident clue about where this atom should be place in the tree.

98 Our theorem states that, starting from a valid state t which translates to a list of alternatives
 99 $(\sigma_1, g) :: a$. If we run in elpi the list of alternatives, then the execution of the tree t returns the same
 100 result as the execution in elpi. The proof is performed by induction on the derivations of the elpi
 101 execution. We have 4 derivations.

102 We have 4 case to analyse:

103 — **References** —

- 104 1 Hassan Aït-Kaci. [Warren's Abstract Machine: A Tutorial Reconstruction](#). The MIT Press, 08 1991.
 105 doi:10.7551/mitpress/7160.001.0001.
- 106 2 Yves Bertot. A certified compiler for an imperative language. Technical Report RR-3488, INRIA,
 107 September 1998. URL: <https://inria.hal.science/inria-00073199v1>.
- 108 3 Saumya K. Debray and Prateek Mishra. Denotational and operational semantics for prolog. [J. Log.](#)
 109 [Program.](#), 5(1):61–91, March 1988. doi:10.1016/0743-1066(88)90007-6.
- 110 4 Cornelia Pusch. Verification of compiler correctness for the wam. In Gerhard Goos, Juris Hartmanis, Jan
 111 van Leeuwen, Joakim von Wright, Jim Grundy, and John Harrison, editors, [Theorem Proving in Higher](#)
 112 [Order Logics](#), pages 347–361, Berlin, Heidelberg, 1996. Springer Berlin Heidelberg.
- 113 5 David H.D. Warren. An Abstract Prolog Instruction Set. Technical Report Technical Note 309, SRI
 114 International, Artificial Intelligence Center, Computer Science and Technology Division, Menlo Park,
 115 CA, USA, October 1983. URL: <https://www.sri.com/wp-content/uploads/2021/12/641.pdf>.