Lecture 1. Probability of an event

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INTRODUCTION

- Probability theory is a branch of mathematics that has been developed to deal with uncertainty.
- Today, probability theory is recognized as one of the most interesting and also one of the most useful areas of mathematics. It provides the basis for the science of statistical inference through experimentation and data analysis – an area of crucial importance in an increasingly quantitative world.
- Through its applications to problems such as the assessment of system reliability, the interpretation of measurement accuracy, and the maintenance of suitable quality controls, probability theory is particularly relevant to the engineering sciences today.

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Example: The sample space $\mathcal{S}$, of possible outcomes when a coin is flipped, may be written

$$\mathcal{S} = \{H, T\},$$

where H and T correspond to heads and tails, respectively.

EVENTS

Consider the experiment $\mathcal{F}$ with respect to the sample space $\mathcal{S}$.

An **event** A *can be defined as a set of outcomes of an experiment. In other words, an event in probability is a subset of the respective sample space* $\mathcal{S}$ *.*

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Example: Given the sample space $\mathcal{S} = \{t : t \geqslant 0\}$, where t is the life in years of a certain electronic component, then the event A that the component fails before the end of the fifth year is the subset $A = \{t : 0 \leqslant t < 5\}$.

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The **complement** *of an event* A *with respect to* $\mathcal{S}$ *is the subset of all elements of S that are not in* A*, denote by the symbol* A'*.*

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Example: Let R be the event that a red card is selected from an deck of 52 playing cards, and let S be the entire deck. Then R' is the event that the card selected from the deck is not a red card but a black card.

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  Example. 1) Throwing a die once and the die lands on 1 and 6 at the same. 2) Choosing a blue ball from a bag that contains 3 red balls and 5 yellow balls.

OPERATIONS ON EVENTS

Suppose that A and B are two events associated with an experiment. In other words, A and B are subsets of the same sample space $\mathcal{S}$.

(1) The **intersection** of two events A and B, denoted by the symbol $A \cap B$, is the event containing all elements that are common to A and B. The event that occurs, when both A and B occur simultaneously.

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(3) The **union** of the two events A and B, denoted by the symbol $A \cup B$, is the event containing all the elements that belong to A or B or both. This event that occurs, when either A or B or both occur.

Examples.

1) In the tossing of a die, let A be the event that an even number occurs and B the event that a number greater than 3 shows. Then the subsets $A = \{2, 4, 6\}$ and $B = \{4, 5, 6\}$ are subsets of the same sample space $\mathcal{S} = \{1, 2, 3, 4, 5, 6\}$. Note that both A and B will occur on a given toss if the outcome is an element of the subset $\{4, 6\}$, which is just the intersection of A and B.

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If an experiment can result in any one of N *different equally likely outcomes, and if exactly* n *of these outcomes correspond to event* A, *then the probability of event* A *is*

$$P(A) = \frac{n}{N}.$$

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$$\mathcal{S} = \{HH, HT, TH, TT\}.$$

If the coin is balanced, each of these outcomes is equally likely to occur. Therefore, we assign a probability of ω to each sample point. Then $\omega = 1/4$. If A represents the event of at least 1 head occurring, then $A = \{HH, HT, TH\}$ and $P(A) = 1/4 + 1/4 + 1/4 = 3/4$.

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Solution. The sample space is $\mathcal{S} = \{1, 2, 3, 4, 5, 6\}$.
We assign a probability of ω to each odd number and a probability of 2ω to each even number. Since the sum of the probabilities must be 1, we have $\omega = 1/9$. Hence, probabilities of $1/9$ and $2/9$ are assigned to each odd and even number, respectively. Therefore,
$E = \{1, 2, 3\}$ and $P(E) = 1/9 + 2/9 + 1/9 = 4/9$.

Example 3. A statistics class for engineers consists of 25 industrial, 10 mechanical, 10 electrical, and 8 civil engineering students. If a person is randomly selected by the instructor to answer a question, find the probability that the student chosen is (a) an industrial engineering major and (b) a civil engineering or an electrical engineering major.

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Solution. Denote by I, M, E, and C the students majoring in industrial, mechanical, electrical, and civil engineering, respectively. The total number of students in the class is 53, all of whom are equally likely to be selected.

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Example 4. In a poker hand consisting of 5 cards, find the probability of holding 2 aces and 3 jacks.

Ví dụ 5. Tung ngẫu nhiên đồng thời hai con xúc xắc. Tính xác xuất để tổng số chấm xuất hiện của hai con xúc xắc là 7.

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Giải : Gọi A là biến cố tổng số chấm xuất hiện ở hai mặt trên của 2 con súc sắc là 7.

A_i là biến cố súc sắc thứ nhất xuất hiện mặt trên là i chấm $(i=\overline{1,6})$.

B_i là biến cố súc sắc thứ hai xuất hiện mặt trên là i chấm $(i=\overline{1,6})$.

Khi ta tung 2 con súc sắc cùng lúc thì có 36 biến cố sơ cấp đồng khả năng có thể xảy ra, cụ thể:

$$\begin{aligned} W = \{&(A_1,B_1);\ (A_1,B_2);\ \ldots;\ (A_1,B_6) \\ &(A_2,B_1);\ (A_2,B_2);\ \ldots;\ (A_2,B_6) \\ &\ldots \quad \ldots \quad \ldots \quad \ldots \\ &(A_6,B_1);\ (A_6,B_2);\ \ldots;\ (A_6,B_6)\} \end{aligned}$$

Và có 6 biến cố thuận lợi cho biến cố A:

$$(A_1,B_6);\ (A_2,B_5);\ (A_3,B_4);\ (A_4,B_3);\ (A_5,B_2);\ (A_6,B_1)$$

$$\Rightarrow P(A)=\frac{6}{36}=\frac{1}{6}$$

Ví dụ 6. Một người gọi điện thoại nhưng quên 2 số cuối của điện thoại, chỉ nhớ 2 số đó là khác nhau. Tính xác suất để người đó chỉ bấm số 1 lần đúng số cần gọi.

Ví dụ 7. Một hộp gồm 6 bi xanh và 4 bi vàng. Lấy ngẫu nhiên 2 bi từ hộp. Tính xác suất để: a) có 1 bi xanh; b) có 2 bi xanh.

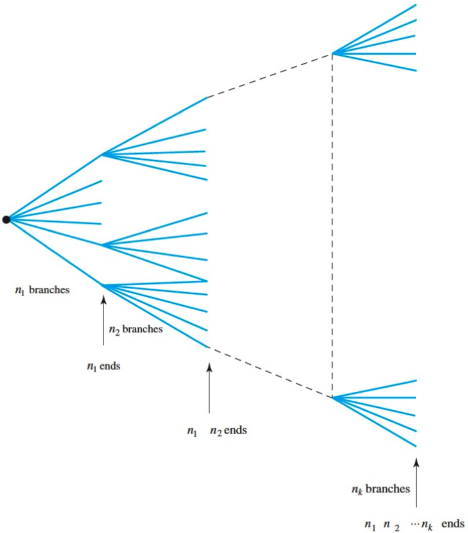
Ví dụ 8. Một hộp đựng 20 quả cầu trong đó có 14 quả màu đỏ và 6 quả màu trắng. Lấy ngẫu nhiên không hoàn lại 5 quả từ trong hộp. Tính xác suất để trong 5 quả lấy ra có 3 quả màu đỏ. Biết các quả cầu cân đối và giống nhau.

COUNTING TECHNIQUES

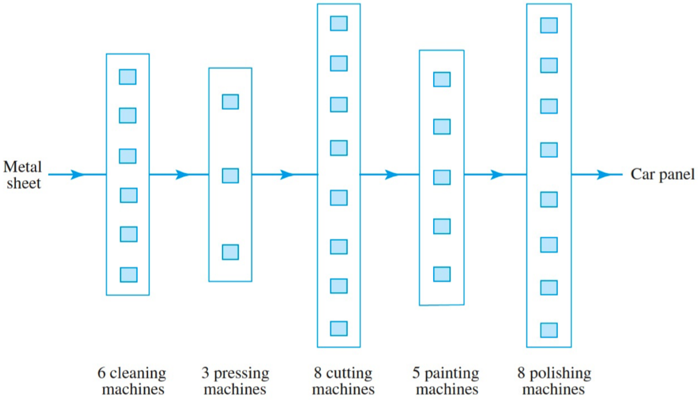
In many cases, we shall be able to solve a probability problem by counting the number of points in the sample space without actually listing each element. The fundamental principle of counting, often referred to as the **multiplication rule**. The generalized multiplication rule covering k operations is stated in the following.

If an operation can be performed in n_1 ways, and if for each of these a second operation can be performed in n_2 ways and for each of the first two a third operation can be performed in n_3 ways, and so forth, then the sequence of k operations can be performed in $n_1 \times n_2 \times \ldots \times n_k$. ways.

Multiplication Rule (contd.)



Example. Experimental configurations for fiber coatings.



Total number of pathways is $6 \times 3 \times 8 \times 5 \times 8 = 5760$.

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Notice that for any non-negative integer n, "n factorial" is defined as $n! = n.(n-1)\ldots 2.1$ with special case $0! = 1$. We have the following theorem.

Permutation (contd.)

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Example. The number of permutations of the four letters a, b, c, d will be $4! = 24$.

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Solution. Since the awards are distinguishable, it is a permutation problem. The total number of sample points is

$$P_2^3 5 = \frac{25!}{(25-3)!} = 13800.$$

Combinations

In many problems, we are interested in the number of ways of selecting r objects from n without regard to order. These selections are called **combinations**. A combination is actually a partition with two cells, the one cell containing the r objects selected and the other cell containing the $(n - r)$ objects that are left.

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Example. A young boy asks his mother to get 5 Game-Boy cartridges from his collection of 10 arcade and 5 sports games. How many ways are there that his mother can get 3 arcade and 2 sports games?
Solution. The number of ways of selecting 3 cartridges from 10 is $C_{10}^3 = 10!/3!.(10-3)! = 120$. The number of ways of selecting 2 cartridges from 5 is $C_5^2 = 5!/2!.3! = 10$.
Using the multiplication rule we have $120.10 = 1200$ ways.

NHẮC LẠI KIẾN THỨC TỔ HỢP

Hoán vị	***Tổ hợp***	***Chỉnh hợp***	***Chỉnh hợp lặp***
Số cách sắp xếp ngẫu nhiên n phần tử $P_n = n!$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử *($k \leq n$)* sao cho k phần tử đó *không lặp và không có phân biệt thứ tự*. $C_n^k = \dfrac{n!}{k!(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử *($k \leq n$)* sao cho k phần tử đó *không lặp và có phân biệt thứ tự*. $A_n^k = \dfrac{n!}{(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử sao cho k phần tử đó *có thể lặp lại và có phân biệt thứ tự*. $B_n^k = n^k$

NHẮC LẠI KIẾN THỨC TỔ HỢP

Hoán vị	***Tổ hợp***	***Chỉnh hợp***	***Chỉnh hợp lặp***
Số cách sắp xếp ngẫu nhiên n phần tử $P_n = n!$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử *($k \le n$)* sao cho k phần tử đó *không lặp và không có phân biệt thứ tự.* $C_n^k = \dfrac{n!}{k!(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử *($k \le n$)* sao cho k phần tử đó *không lặp và có phân biệt thứ tự.* $A_n^k = \dfrac{n!}{(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử sao cho k phần tử đó *có thể lặp lại và có phân biệt thứ tự.* $B_n^k = n^k$

Ví dụ 1. Cho $A = \{1, 2, 3, 4, 5\}$. Từ tập A lập được bao nhiêu số tự nhiên:

- Có 5 chữ số khác nhau?

NHẮC LẠI KIẾN THỨC TỔ HỢP

Hoán vị	***Tổ hợp***	***Chỉnh hợp***	***Chỉnh hợp lặp***
Số cách sắp xếp ngẫu nhiên n phần tử $P_n = n!$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử *($k \le n$)* sao cho k phần tử đó *không lặp và không có phân biệt thứ tự.* $C_n^k = \dfrac{n!}{k!(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử *($k \le n$)* sao cho k phần tử đó *không lặp và có phân biệt thứ tự.* $A_n^k = \dfrac{n!}{(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử sao cho k phần tử đó *có thể lặp lại và có phân biệt thứ tự.* $B_n^k = n^k$

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- Có 5 chữ số khác nhau? $\Longrightarrow P_5 = 5! = 120$ số.

Hoán vị	**Tổ hợp**	**Chỉnh hợp**	**Chỉnh hợp lặp**
Số cách sắp xếp ngẫu nhiên n phần tử $P_n = n!$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử $(k \leq n)$ sao cho k phần tử đó *không lặp* và *không có phân biệt thứ tự.* $C_n^k = \dfrac{n!}{k!(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử $(k \leq n)$ sao cho k phần tử đó *không lặp và có phân biệt thứ tự.* $A_n^k = \dfrac{n!}{(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử sao cho k phần tử đó *có thể lặp lại và có phân biệt thứ tự.* $B_n^k = n^k$

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- Có 5 chữ số khác nhau? $\Longrightarrow P_5 = 5! = 120$ số.
- Có 3 chữ số khác nhau?

NHẮC LẠI KIẾN THỨC TỔ HỢP

Hoán vị	***Tổ hợp***	***Chỉnh hợp***	***Chỉnh hợp lặp***
Số cách sắp xếp ngẫu nhiên *n* phần tử $P_n = n!$	Số cách chọn ngẫu nhiên *k* phần tử từ *n* phần tử *(k≤n)* sao cho *k* phần tử đó *không lặp và không có phân biệt thứ tự.* $C_n^k = \dfrac{n!}{k!(n-k)!}$	Số cách chọn ngẫu nhiên *k* phần tử từ *n* phần tử *(k≤ n)* sao cho *k* phần tử đó *không lặp và có phân biệt thứ tự.* $A_n^k = \dfrac{n!}{(n-k)!}$	Số cách chọn ngẫu nhiên *k* phần tử từ *n* phần tử sao cho *k* phần tử đó *có thể lặp lại và có phân biệt thứ tự.* $B_n^k = n^k$

Ví dụ 1. Cho $A = \{1, 2, 3, 4, 5\}$. Từ tập A lập được bao nhiêu số tự nhiên:

- Có 5 chữ số khác nhau? $\implies P_5 = 5! = 120$ số.
- Có 3 chữ số khác nhau? $\implies A_5^3 = \dfrac{5!}{(5-3)!} = 60$ số.

NHẮC LẠI KIẾN THỨC TỔ HỢP

Hoán vị	***Tổ hợp***	***Chỉnh hợp***	***Chỉnh hợp lặp***
Số cách sắp xếp ngẫu nhiên n phần tử	Số cách chọn ngẫu nhiên k phần tử từ n phần tử *($k \le n$)* sao cho k phần tử đó *không lặp và không có phân biệt thứ tự.*	Số cách chọn ngẫu nhiên k phần tử từ n phần tử *($k \le n$)* sao cho k phần tử đó *không lặp và có phân biệt thứ tự.*	Số cách chọn ngẫu nhiên k phần tử từ n phần tử sao cho k phần tử đó *có thể lặp lại và có phân biệt thứ tự.*
$P_n = n!$	$C_n^k = \dfrac{n!}{k!(n-k)!}$	$A_n^k = \dfrac{n!}{(n-k)!}$	$B_n^k = n^k$

Ví dụ 1. Cho $A = \{1, 2, 3, 4, 5\}$. Từ tập A lập được bao nhiêu số tự nhiên:

- Có 5 chữ số khác nhau? $\implies P_5 = 5! = 120$ số.
- Có 3 chữ số khác nhau? $\implies A_5^3 = \dfrac{5!}{(5-3)!} = 60$ số.
- Có 3 chữ số ?

NHẮC LẠI KIẾN THỨC TỔ HỢP

Hoán vị	***Tổ hợp***	***Chỉnh hợp***	***Chỉnh hợp lặp***
Số cách sắp xếp ngẫu nhiên n phần tử $P_n = n!$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử $(k \leq n)$ sao cho k phần tử đó *không lặp và không có phân biệt thứ tự*. $C_n^k = \dfrac{n!}{k!(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử $(k \leq n)$ sao cho k phần tử đó *không lặp và có phân biệt thứ tự*. $A_n^k = \dfrac{n!}{(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử sao cho k phần tử đó *có thể lặp lại và có phân biệt thứ tự*. $B_n^k = n^k$

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- Có 5 chữ số khác nhau? $\Longrightarrow P_5 = 5! = 120$ số.
- Có 3 chữ số khác nhau? $\Longrightarrow A_5^3 = \dfrac{5!}{(5-3)!} = 60$ số.
- Có 3 chữ số ? $\Longrightarrow B_5^3 = 5^3 = 125$ số.

NHẮC LẠI KIẾN THỨC TỔ HỢP

Hoán vị	***Tổ hợp***	***Chỉnh hợp***	***Chỉnh hợp lặp***
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Ví dụ 2. Một tổ có 5 học sinh. Có ? cách phân công 3 học sinh đi trực nhật.

NHẮC LẠI KIẾN THỨC TỔ HỢP

Hoán vị	***Tổ hợp***	***Chỉnh hợp***	***Chỉnh hợp lặp***
Số cách sắp xếp ngẫu nhiên n phần tử $P_n = n!$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử *($k \leq n$)* sao cho k phần tử đó *không lặp và không có phân biệt thứ tự.* $C_n^k = \dfrac{n!}{k!(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử *($k \leq n$)* sao cho k phần tử đó *không lặp và có phân biệt thứ tự.* $A_n^k = \dfrac{n!}{(n-k)!}$	Số cách chọn ngẫu nhiên k phần tử từ n phần tử sao cho k phần tử đó *có thể lặp lại và có phân biệt thứ tự.* $B_n^k = n^k$

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Ví dụ 2. Một tổ có 5 học sinh. Có ? cách phân công 3 học sinh đi trực nhật.

$\Longrightarrow C_5^3 = \dfrac{5!}{3!(5-3)!} = 10$ cách.

THANK YOU FOR YOUR ATTENTION!