Lecture 3. Discrete random variable

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Definitions

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In most practical problems, discrete random variables represent count data, such as the number of defectives in a sample of k items or the number of highway fatalities per year in a given state.

Discrete Probability Distributions

A discrete random variable assumes each of its values with a certain probability.

The set of ordered pairs $(x, f(x))$ *is a* **probability function**, **probability mass function**, *or* **probability distribution** *of the discrete random variable* X *if, for each possible outcome* x,

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Example. A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives.
Solution. Let X be a random variable whose values x are the possible numbers of defective computers purchased by the school. Then x can be 0, 1, 2 and

$$f(0) = P(X = 0) = 68/95,\ f(1) = P(X = 1) = 51/190,\ f(2) = F(X = 2) = 3/190.$$

Thus, the probability distribution of X is

x	0	1	2
$f(x)$	$68/95$	$51/190$	$3/190$

Cumulative distribution function

There are many problems where we may wish to compute the probability that the observed value of a random variable X will be less than or equal to some real number x. Writing $F(x) = P(X \leqslant x)$ for every real number x, we define $F(x)$ to be the **cumulative distribution function** of the random variable X.

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Example. A stockroom clerk returns three safety helmets at random to three steel mill employees who had previously checked them. If Smith, Jones, and Brown, in that order, receive one of the three hats, list the sample points for the possible orders of returning the helmets. Let X be the number of correct matches. Find the cumulative distribution function of X

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Solution. The probability distribution of X is

x	0	1	3
$f(x)$	$1/3$	$1/2$	$1/6$

We have $F(2) = P(x \leqslant 2) = f(0) + f(1) = 5/6$.

The cumulative distribution function of X is $F(x) = \begin{cases} 0, & x < 0; \\ 1/3, & 0 \leqslant x < 1; \\ 5/6, & 1 \leqslant x < 3; \\ 1, & x \geqslant 3. \end{cases}$

Mathematical Expectation

Let X be a random variable with probability distribution $f(x)$.

The mean *(or* **expected value***) of a random variable X is a number $\mu = EX$ such that*

$$\mu = EX = \sum x_i\, p_i,$$

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Example. A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

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Solution. Let X represent the number of good components in the sample. Thus,

$$f(x) = \frac{C_4^x . C_3^{3-x}}{C_7^3}, \qquad x = 0, 1, 2, 3.$$

Therefore, the mean $\mu = EX = 0.1/35 + 1.12/35 + 2.18/35 + 3.4/35 = 1,7$. Thus, if a sample of size 3 is selected at random over and over again from a lot of 4 good components and 3 defective components, it will contain, on average, 1.7 good components.

PROPERTIES OF EXPECTATION

- The expectation of a constant is equal to the constant itself

$$EC = C, \qquad C = \text{const}\,.$$

- Constants can be taken out

$$E(C.X) = C.EX.$$

- The expectation of the sum of random variables is equal to the sum of the expectations

$$E(X+Y) = E(X) + E(Y) = \sum_{i=1}^{m} \sum_{j=1}^{n} (x_i + y_i)\, p_{ij}.$$

Consequence: $\quad E(aX + bY) = aEX + bEY, \quad \forall\, a, b = \text{const}.$

- The expectation of the product of two independent random variables is equal to the product of the expectations

$$E(XY) = EX.EY.$$

- $E(XY) - EX.EY = \operatorname{cov}(X, Y).$

Note: If X, Y are two random variables, therefore

$$E(XY) = \sum_{i=1}^{m} \sum_{j=1}^{n} (x_i.y_j)\, p_{ij}, \quad E(X+Y) = \sum_{i=1}^{m} \sum_{j=1}^{n} (x_i + y_j)\, p_{ij}$$

VARIANCE OF THE RANDOM VARIABLE

Let X be a random variable with probability distribution $f(x)$ and mean μ. The quantity is referred to as the **variance of the random variable** X or the **variance of the probability distribution** of X and is denoted by $\mathrm{Var}(X)$ or the symbol σ_x^2 , or simply by σ^2.

The variance of X is $\sigma^2 = E(X-\mu)^2 = \sum_x (x-\mu)^2 f(x)$.

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The positive square root of the variance, σ, is called the **standard deviation** of X.
The quantity $x - \mu$ in definition is called the **deviation of an observation** from its mean.
Since the deviations are squared and then averaged, σ^2 will be much smaller for a set of x values that are close to μ than it will be for a set of values that vary considerably from μ.

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The variance of X *is* $\sigma^2 = EX^2 - \mu^2$.

PROPERTIES OF VARIANCE

- The variance and standard deviation of the constant are equal to 0.

$$\operatorname{Var} C = 0, \qquad C = \text{const}\,.$$

- Take the constant out by the rule

$$\operatorname{Var}(C.X) = C^2.\operatorname{Var} X.$$

- The variance X and $X + C$ are the same

$$\operatorname{Var}(X) = \operatorname{Var}(X + C).$$

- The variance of the sum and difference of independent variables is equal to the sum of the variances of those two random variables.

$$\operatorname{Var}(X + Y) = \operatorname{Var}(X - Y) = \operatorname{Var}(X) + \operatorname{Var}(Y).$$

THANK YOU FOR YOUR ATTENTION!