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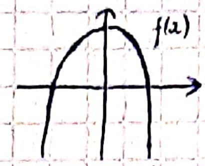
1) a) $D_f \Rightarrow -4x^2 + 36 > 0 \Leftrightarrow -4x^2 > -36 \Leftrightarrow$

$\Leftrightarrow x^2 < 9 \Leftrightarrow$

$\Leftrightarrow x > -3 \vee x < 3$

$D_f = \,]-3, 3[$



Logo, não é injetiva.

b) $y = \log(-4x^2 + 36) \Leftrightarrow e^y = -4x^2 + 36 \Leftrightarrow$

$\Leftrightarrow e^y - 36 = -4x^2 \Leftrightarrow \frac{-e^y}{4} + 9 = x^2 \Leftrightarrow x = \pm\sqrt{\frac{-e^y}{4} + 9}$

$D_{f^{-1}} = \mathbb{R} \longrightarrow D'_{f^{-1}} = \,]-3, 3[$

$y \longrightarrow x = \pm\sqrt{\frac{-e^y}{4} + 9}$

c) $tg(\arccos(-1/2) + x) = 1 \Leftrightarrow$

$\Leftrightarrow tg(2\pi/3 + x) = 1 \Leftrightarrow$

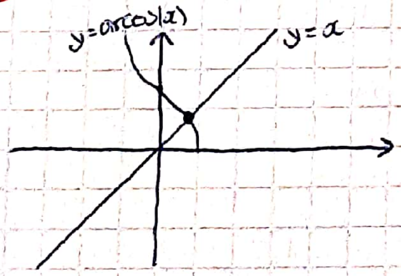
$\Leftrightarrow tg(2\pi/3 + x) = tg(\pi/4) \Leftrightarrow$

$\Leftrightarrow \frac{2\pi}{3} + x = \frac{\pi}{4} + k\pi, \; k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4} - \frac{2\pi}{3} + k\pi, \; k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \frac{3\pi}{12} - \frac{8\pi}{12} + k\pi, \; k \in \mathbb{Z} \Leftrightarrow x = -\frac{5\pi}{12} + k\pi, \; k \in \mathbb{Z}$

2) a) $x - \arccos(x) = 0 \Leftrightarrow x = \arccos(x)$



O intervalo, de amplitude 1, que contém a solução da equação é $[0, 1]$

b) $x_0 = 1$

$x_1 = \frac{0 + \frac{1}{3}}{2} = \frac{1}{6}$ erro $\leq |\frac{1}{6} - 1| \Leftrightarrow$ erro $\leq \frac{5}{6}$

$x_2 = \frac{\frac{1}{3} + \frac{2}{3}}{2} = \frac{1}{2}$ erro $\leq |\frac{1}{2} - \frac{1}{6}| \Leftrightarrow$ erro $\leq \frac{2}{6} \Leftrightarrow$ erro $\leq \frac{1}{3}$

$x_3 = \frac{\frac{2}{3} + \frac{3}{3}}{2} = \frac{\frac{5}{3}}{2} = \frac{5}{6}$ erro $\leq |\frac{5}{6} - \frac{1}{2}| \Leftrightarrow$ erro $\leq \frac{2}{6} \Leftrightarrow$ erro $\leq \frac{1}{3}$

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3.

a) $\int x \frac{1-x^2}{\sqrt[3]{1-x^2}}\, dx =$ $\qquad t = \sqrt[3]{1-x^2}$

$= \int -\frac{3t^4}{2}\, dt = -\frac{3}{2}\int t^4\, dt =$

$= -\frac{3}{2} \times \frac{t^5}{5} + c,\ c \in \mathbb{R} =$

$= -\frac{3}{2} \times \frac{(\sqrt[3]{1-x^2})^5}{5} + c,\ c \in \mathbb{R} =$

$= -\frac{3\sqrt[3]{(1-x^2)^2}(1-x^2)}{10} + c,\ c \in \mathbb{R}$

b) $\int \frac{3x}{4+x^4}\, dx =$ $\qquad t = x^2$

$= \int \frac{3}{2(4+t^2)}\, dt = \frac{3}{2}\int \frac{1}{4+t^2}\, dt =$

$= \frac{3}{2} \times \frac{1}{2} \times \operatorname{arctg}(t/2) + c,\ c \in \mathbb{R} =$

$= \frac{3}{4}\operatorname{arctg}\left(\frac{x^2}{2}\right) + c,\ c \in \mathbb{R}$

4. $\int_{-1}^{1} \arccos(x)\, dx \approx \frac{0,5}{3} \times [f(-1) + 4f(-0,5) + 2f(0) + 4f(0,5) + f(1)] =$

$= \frac{0,5}{3} \times [3,14 + 4 \times 2,09 + 2 \times 1,57 + 4 \times 1,05 + 0] =$

$= \frac{0,5}{3} \times 18,84 = \frac{1}{6} \times 18,84 = \frac{18,84}{6} = 3,14$

5.

a) $\int_{-1}^{1} (4 - e^{-x}) - \sqrt{1-x^2}\, dx$

$x^2 + y^2 = 1 \Leftrightarrow y = \pm\sqrt{1-x^2}$

$x = -\ln(-y+4) \Leftrightarrow$

$\Leftrightarrow e^{-x} = -y + 4 \Leftrightarrow e^{-x} - 4 = -y \Leftrightarrow$

$\Leftrightarrow y = 4 - e^{-x}$

b) $\int_0^1 \pi\left((-\ln(-y+4))^2 - (\sqrt{1-y^2})^2\right) dy =$

$x^2 + y^2 = 1 \Leftrightarrow x = \pm\sqrt{1-y^2}$

$= \pi\int_0^1 (-\ln(-y+4))^2 - 1 + y^2\, dy$

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c) $x = -1 \rightarrow 4 - e^{-(-1)} = 4 - e$

$x = 1 \rightarrow 4 - e^{-1} = 4 - \frac{1}{e}$

$$\text{Perímetro} = (4-e) + (4 - \frac{1}{e}) + \frac{2\pi \times 1^2}{2} + \int_{-1}^{1} \sqrt{1 + [(4-e^{-x})']^2}\, dx =$$

$$= 8 - e - \frac{1}{e} + \pi + \int_{-1}^{1} \sqrt{1 + (-e^{-x})^2}\, dx =$$

$$= \frac{8e - e^2 - 1 + \pi e}{e} + \int_{-1}^{1} \sqrt{1 + e^{-2x}}\, dx$$