Modul : Constraint Satisfaction Problem (CSP)

Terminologi Dalam CSP

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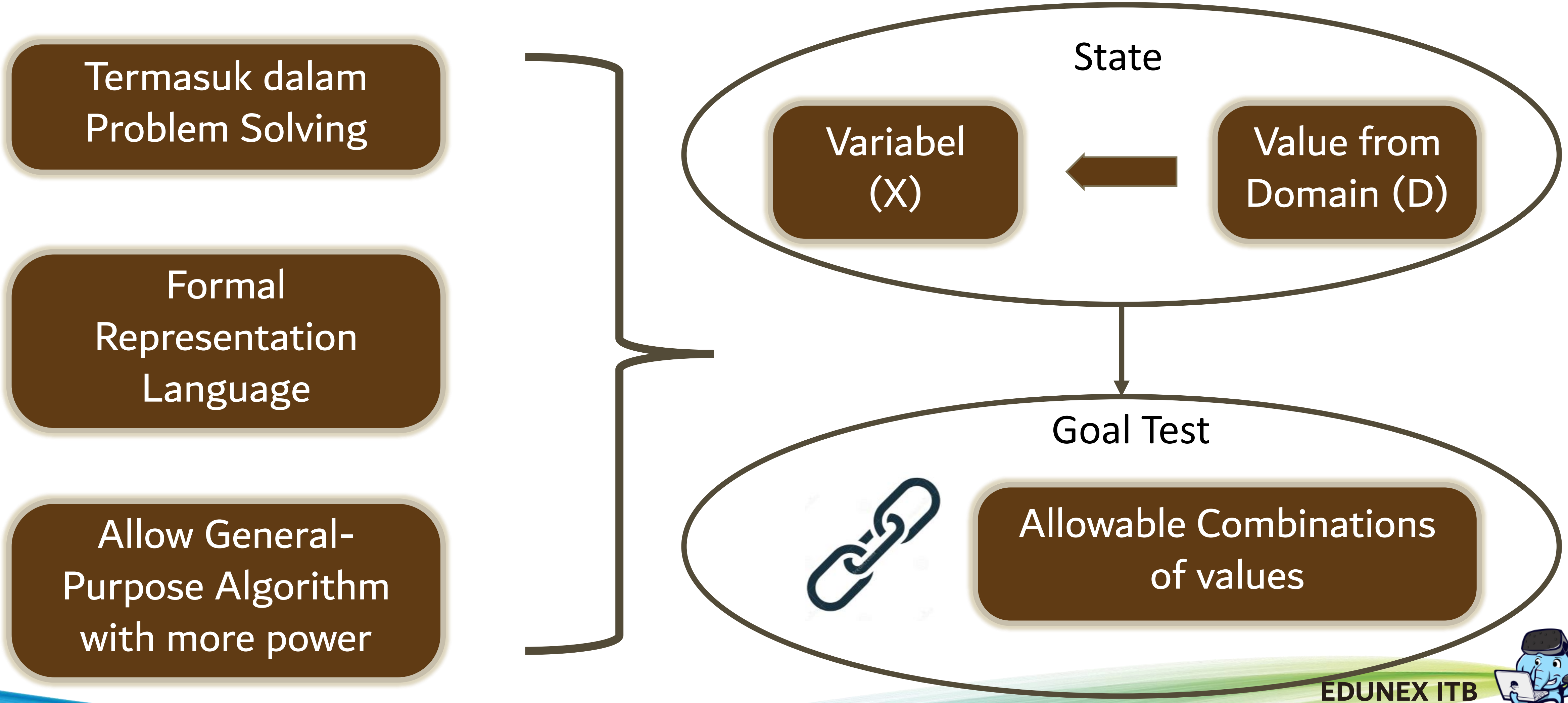
Constraint Satisfaction Problem (CSP)



Termasuk dalam Problem Solving

Formal Representation Language

Allow General-Purpose Algorithm with more power

State

Variabel (X)

Value from Domain (D)

Goal Test

Allowable Combinations of values

Example: Map-Coloring Problem

Variables:

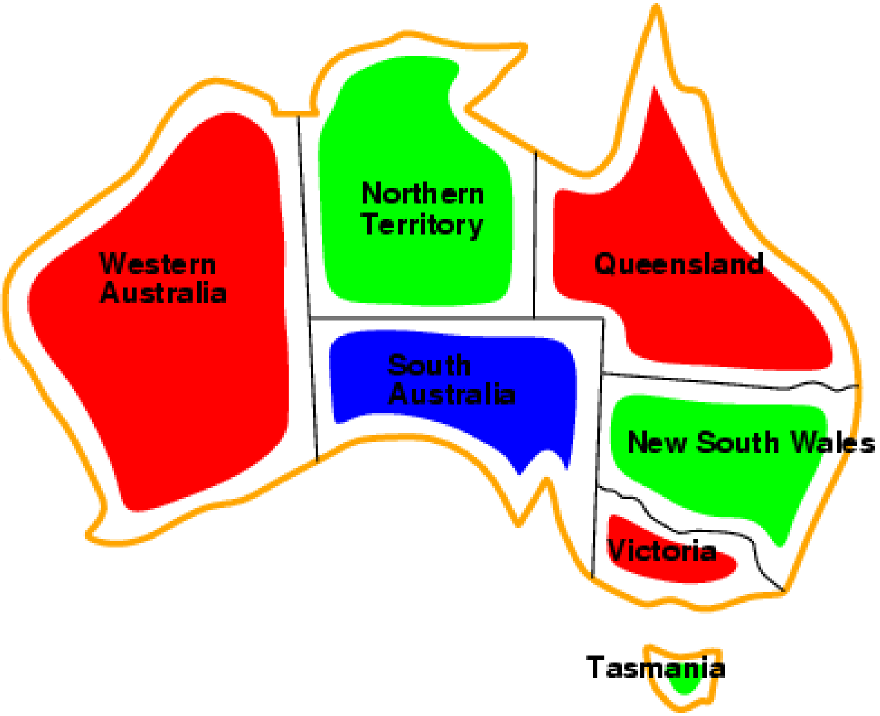
WA, NT, Q, NSW, V, SA, T

Domain:

D_i = {red,green,blue}

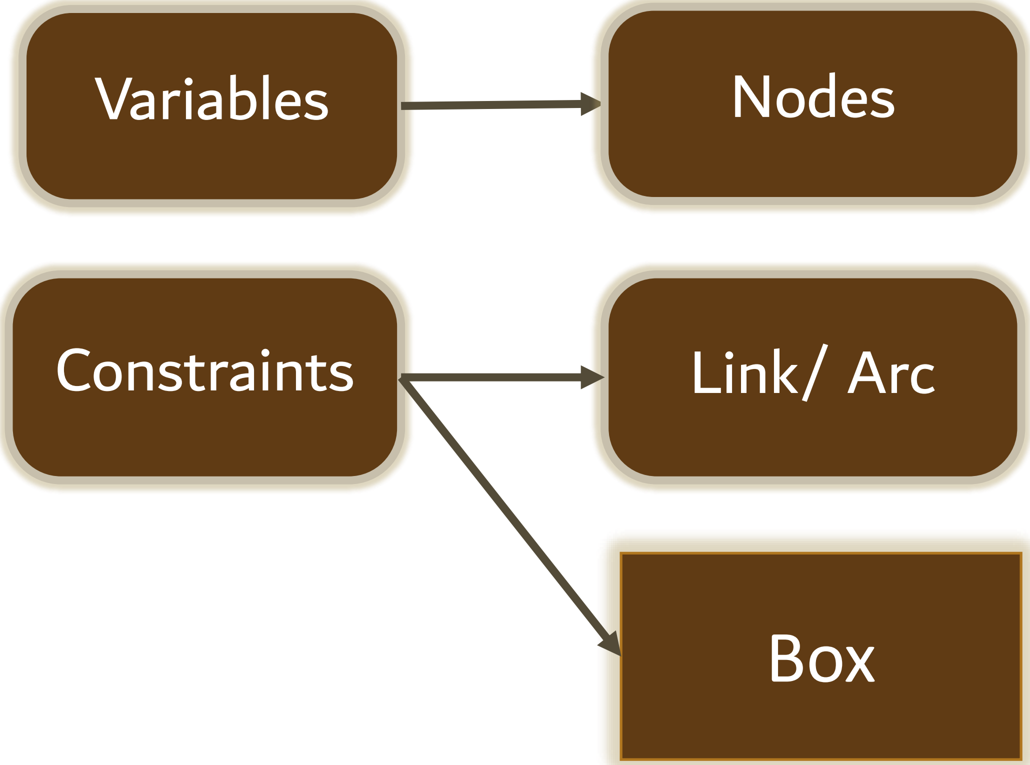
Constraints:

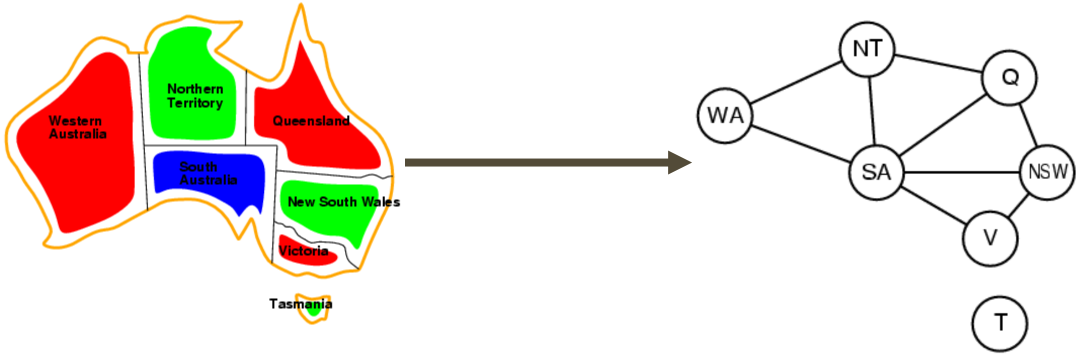
adjacent regions must have different colors



Solution: complete and consistent assignments

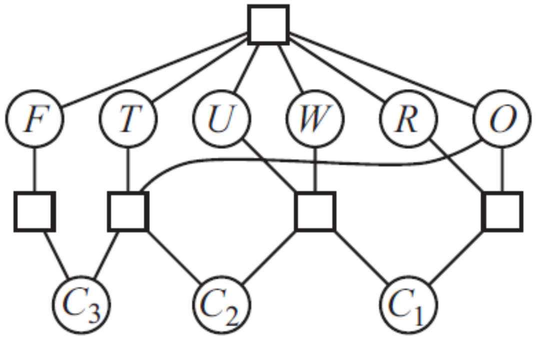
CSP Visualization





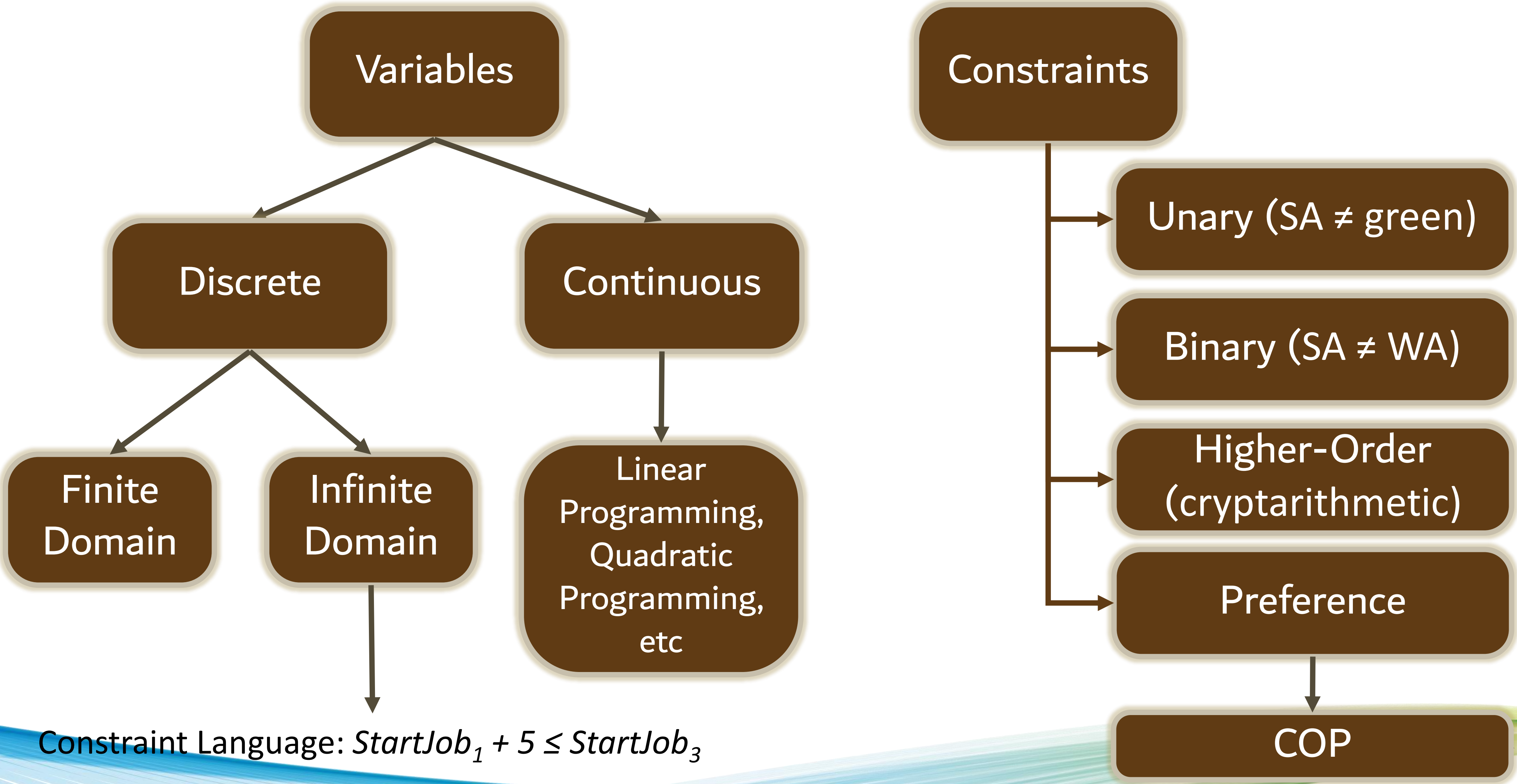
Constraint Graph

$$\begin{array}{r} T\ W\ O \\ +\ T\ W\ O \\ \hline F\ O\ U\ R \end{array}$$



Constraint Hypergraph

Variations of CSP Formalism



Constraint Language: $StartJob_1 + 5 \le StartJob_3$

Example: Cryptarithmetic Puzzle

$$\begin{array}{r} T\ W\ O \\ +\ T\ W\ O \\ \hline F\ O\ U\ R \end{array}$$

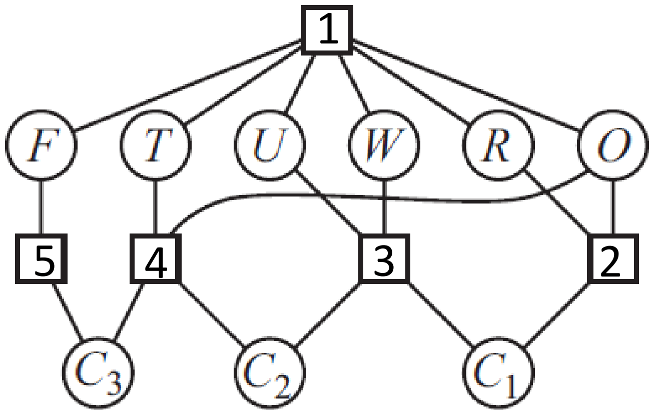
Variables

F,T,U,W,R,O

C_1, C_2, C_3 : auxiliary variables

Domain

F,T,U,W,R,O ={0,1,2,3,4,5,6,7,8,9}



Constraints

1. Alldiff *(F,T,U,W,R,O)*
2. $O + O = R + 10 \cdot C_1$
3. $W + W + C_1 = U + 10 \cdot C_2$
4. $T + T + C_2 = O + 10 \cdot C_3$
5. $F = C_3$, $T \neq 0$, $F \neq 0$

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Inference in CSP

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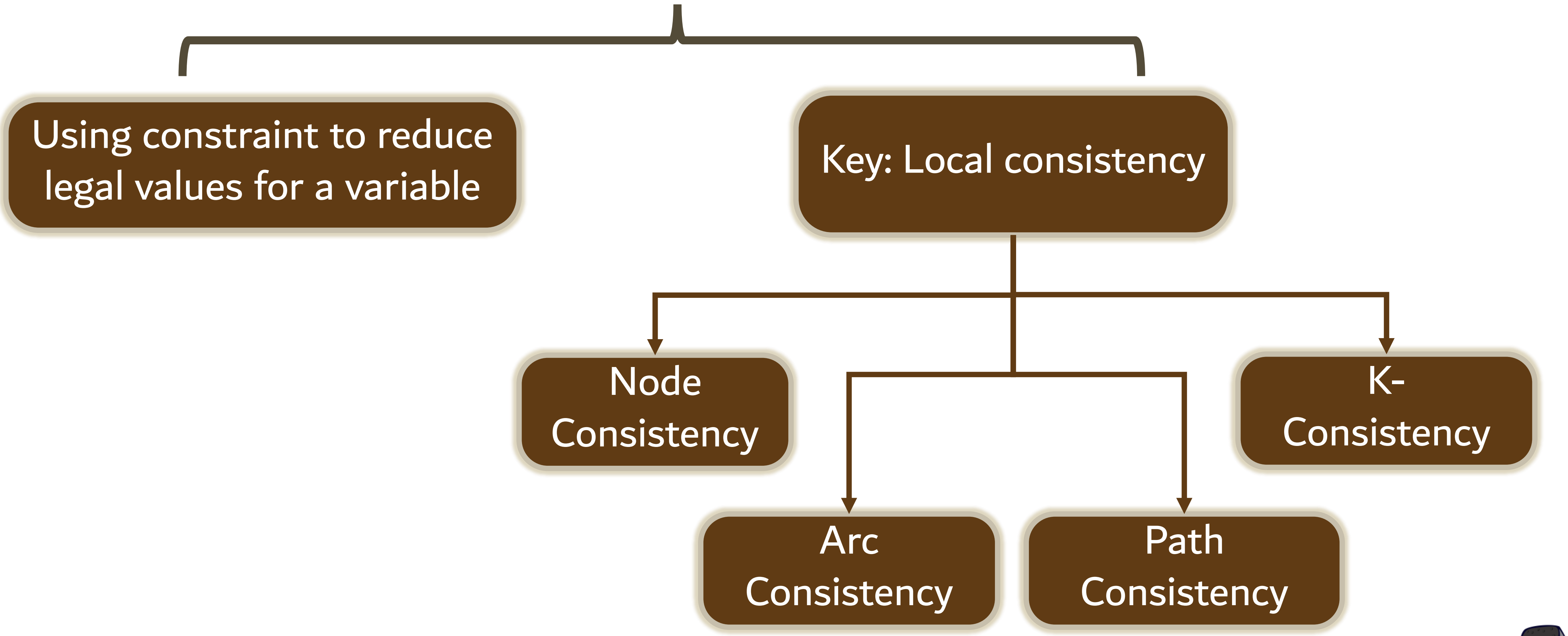
Constraint Propagation



Using constraint to reduce legal values for a variable

Key: Local consistency

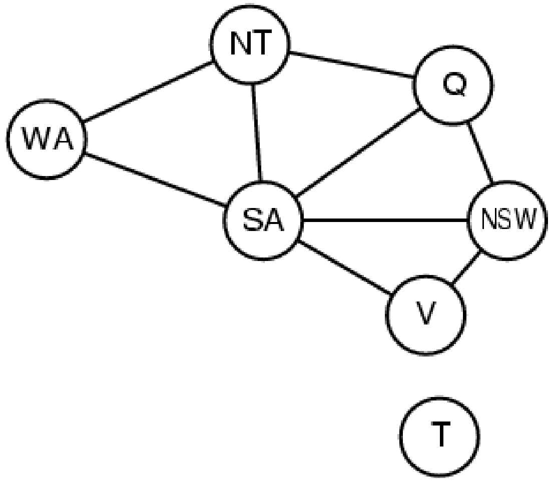
- Node Consistency
- Arc Consistency
- Path Consistency
- K-Consistency

Node Consistency

All the values in the variable’s domain satisfy the variable’s **unary constraints**

Example: SA ≠ green

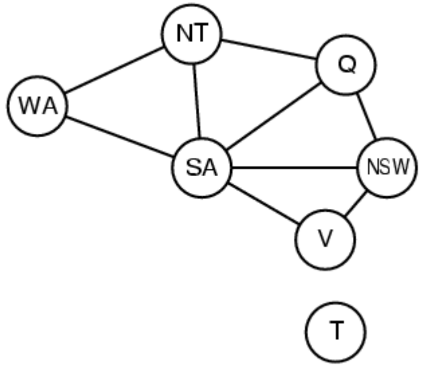
SA = {red, blue}



A network is node-consistent if every variable in the network is node-consistent

Arc Consistency (AC)

A variable in a CSP is **arc-consistent** if every value in its domain satisfies the variable's binary constraints



Example: SA ≠ WA

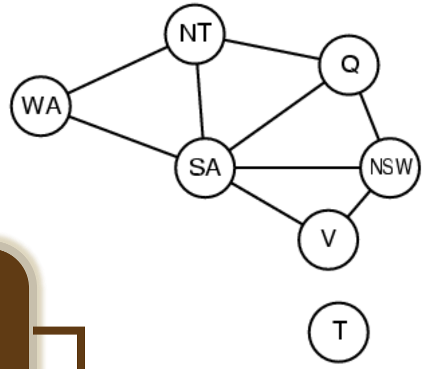
(SA,WA) = {(red , green), (red , blue), (green, red), (green, blue), (blue, red), (blue, green)}

Has no effect in this example (no reduction in the domain)

A network is arc-consistent if every variable is arc consistent with every other variable

Path Consistency (PC)

Arc Consistency: solve the problem if each variable has only 1 value left after the process OR finds that CSP can not be solved



Does not work for map coloring with only 2 values in the domain

(Path Consistency)

{Xi,Xj} is path-consistent to Xm if:

- Assignment {Xi = a,Xj = b} consistent with constraints on {Xi,Xj}
- There is assignment to Xm that satisfies constraints on {Xi,Xm} and {Xm,Xj}.

Example: Coloring Map with 2 colors (red, blue)

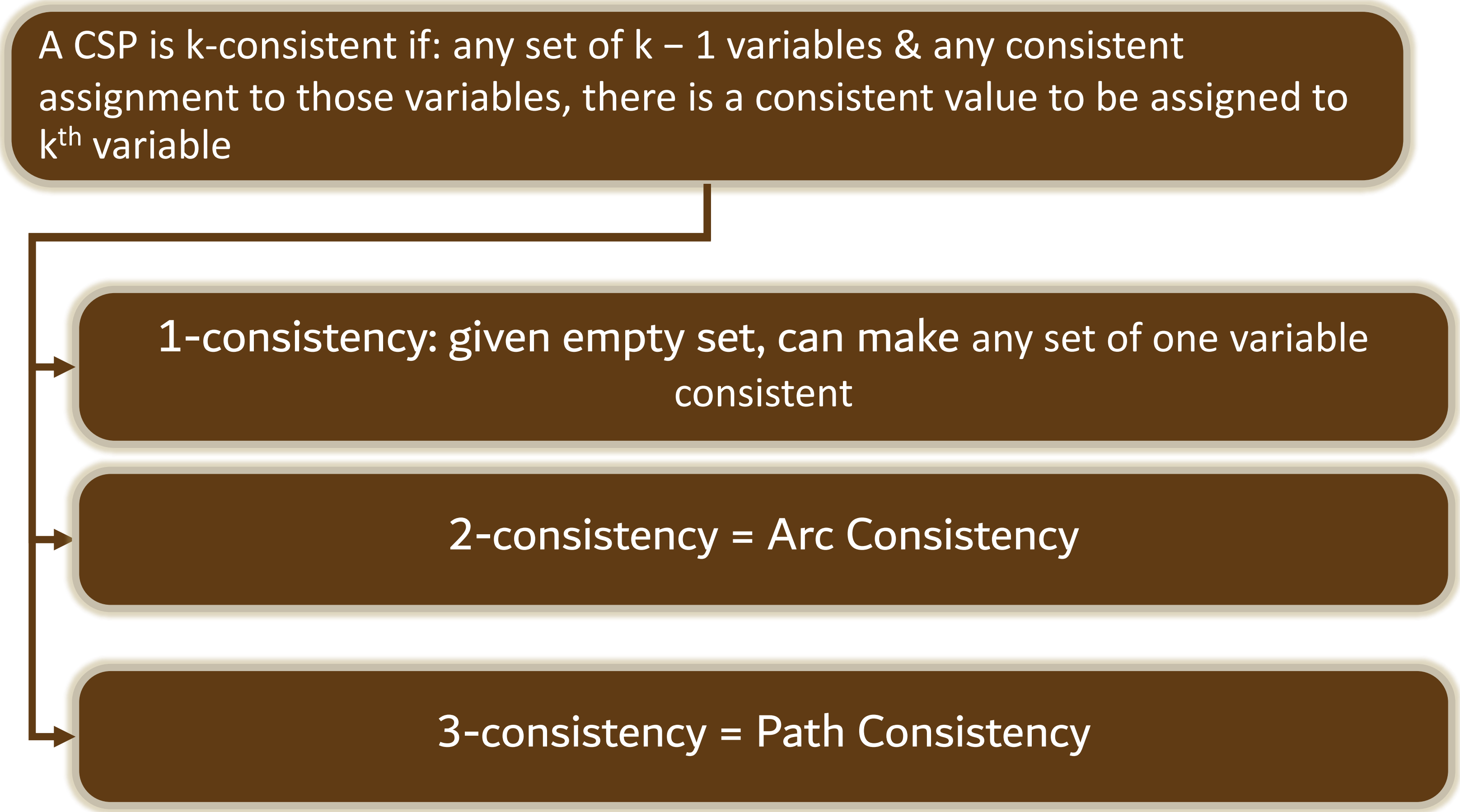
PC: {WA,SA} wrt NT

{WA = red ,SA = blue} or {WA = blue,SA = red}

No valid choice for NT

Eliminate both assignment → No solution

K-Consistency



A CSP is k-consistent if: any set of k – 1 variables & any consistent assignment to those variables, there is a consistent value to be assigned to kth variable

1-consistency: given empty set, can make any set of one variable consistent

2-consistency = Arc Consistency

3-consistency = Path Consistency

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Backtracking Search for CSP

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Backtracking Search

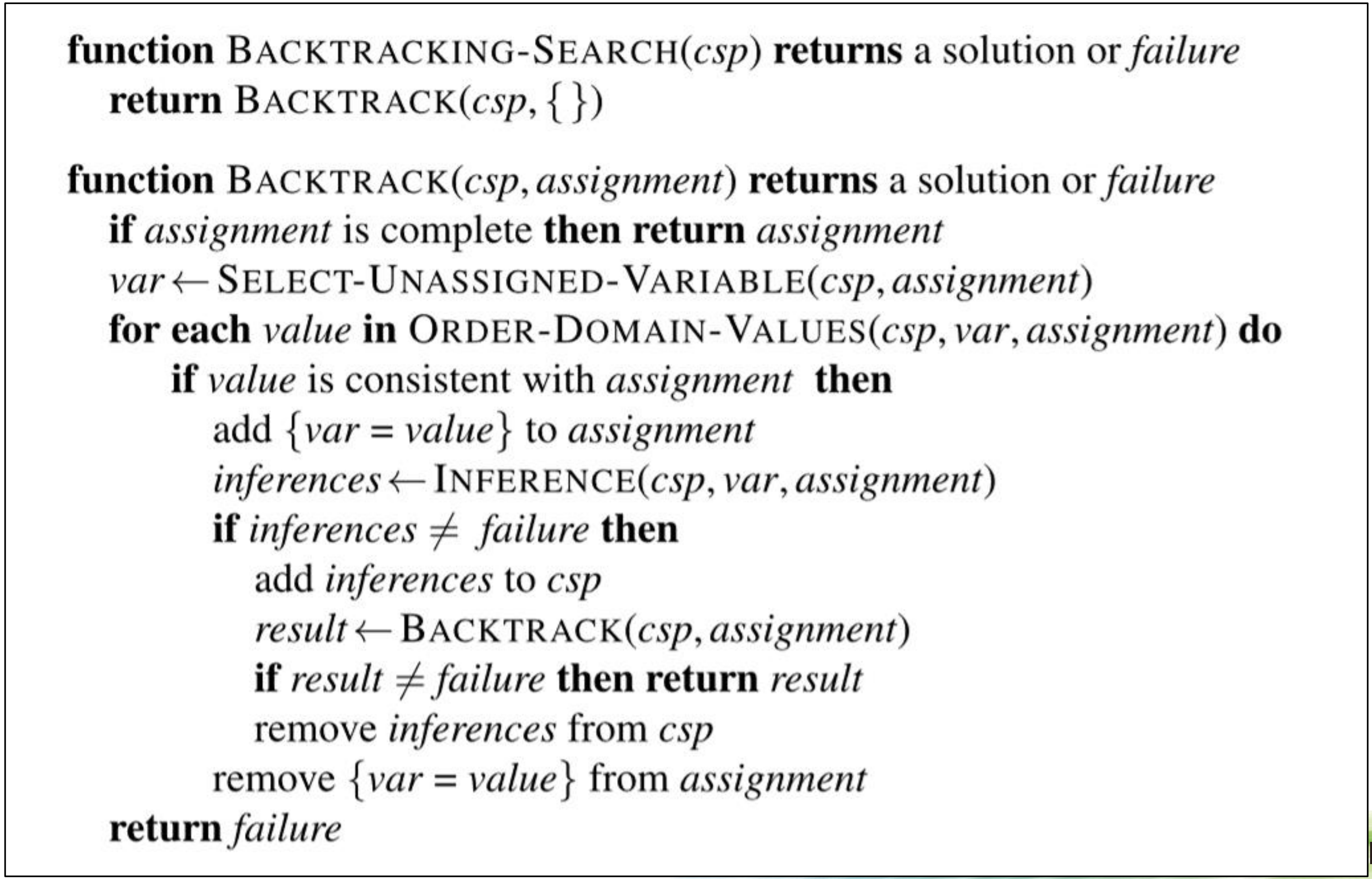
Use Depth First Search → Solution for n variables at depth n

Path is irrelevant → variable assignment commutative

Only consider assignments to a single variable at each node

Basic uninformed algorithm for CSPs

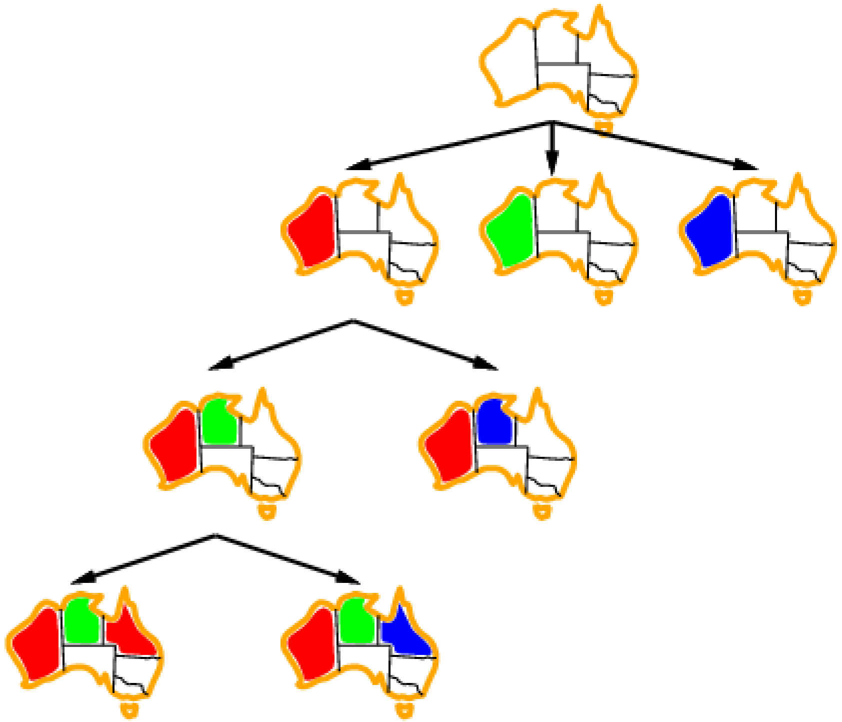
Algorithm

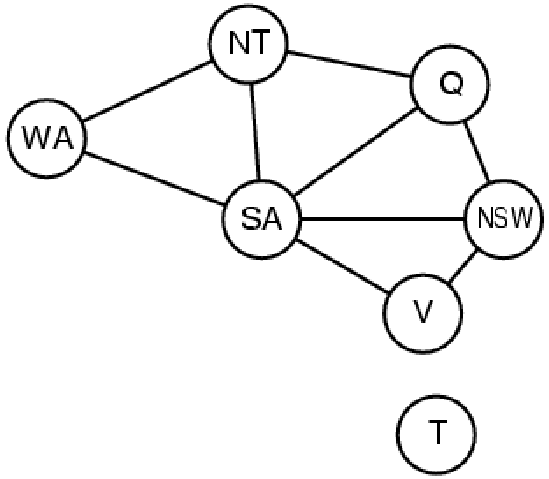


```
function BACKTRACKING-SEARCH(csp) returns a solution or failure
  return BACKTRACK(csp, { })

function BACKTRACK(csp, assignment) returns a solution or failure
  if assignment is complete then return assignment
  var ← SELECT-UNASSIGNED-VARIABLE(csp, assignment)
  for each value in ORDER-DOMAIN-VALUES(csp, var, assignment) do
     if value is consistent with assignment then
        add {var = value} to assignment
        inferences ← INFERENCE(csp, var, assignment)
        if inferences ≠ failure then
           add inferences to csp
           result ← BACKTRACK(csp, assignment)
           if result ≠ failure then return result
           remove inferences from csp
        remove {var = value} from assignment
  return failure
```

Example: Map Coloring Problem





Improving Backtracking Efficiency

Which variable should be assigned next?

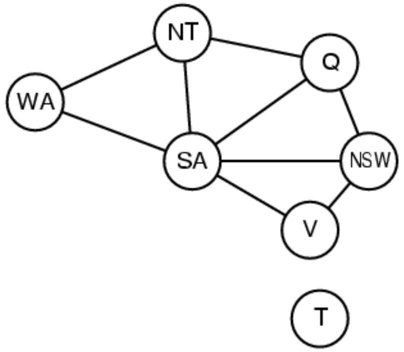
In what order should its values be tried?

Detect inevitable failure early?

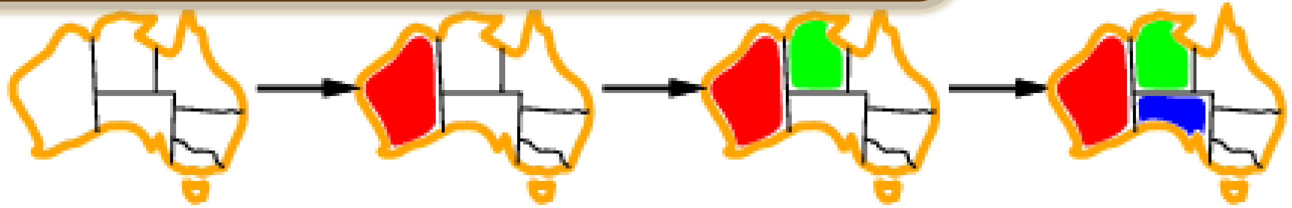
Without Domain Specific Knowledge

Variable Ordering

$var \leftarrow \textsc{Select-Unassigned-Variable}(csp)$



1. Static Variable Ordering: {*WA, NT, SA, Q, NSW, V, T* }



2. Minimum Remaining Values Heuristic

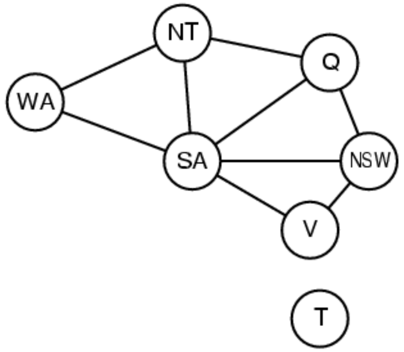
After WA=red and NT=green, SA or Q?

SA={blue}
Q={blue, red}

Choose SA (has minimum remaining value)

Most Constrained Variable

Variable Ordering - 2



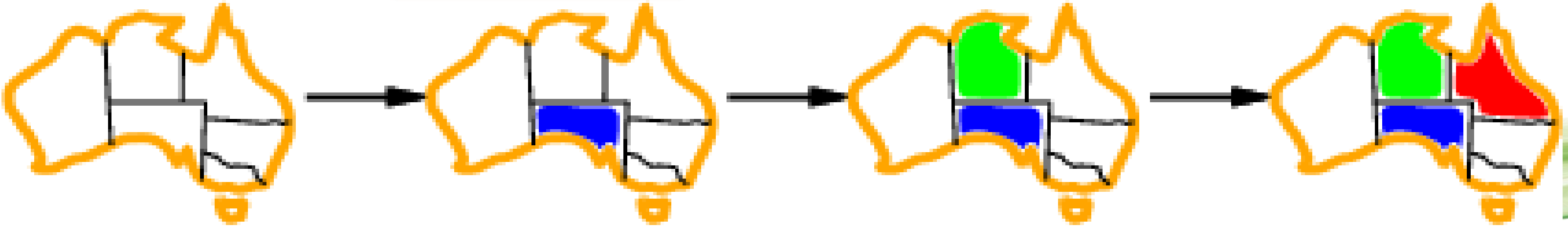
$$var \leftarrow \textsc{Select-Unassigned-Variable}(csp)$$

2. Minimum Remaining Values Heuristic : First Variable to Assign?

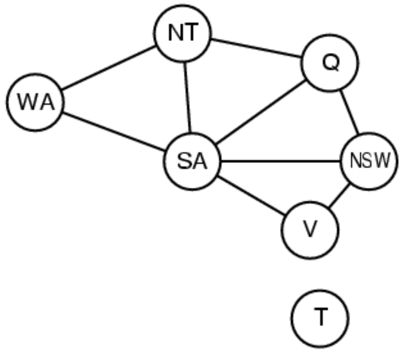
Use degree heuristic → Number of constraints involved in a variable on other unassigned variable

SA: 5
NT, Q, NSW: 3
WA, V: 2
T: 0

→ Choose SA First!!



Value Ordering



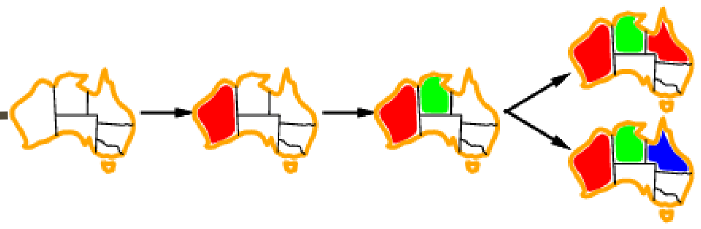
Least Constraining Value Heuristic

Prefer value that rules out fewest choice for neighboring variables

After WA=red and NT=green, What color for Q?

Q=blue → SA = { }
Q=red → SA = {blue}

Choose value red for Q



Value Ordering is irrelevant if we want to have all possible solutions

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Interleaving Search and Inference in CSP

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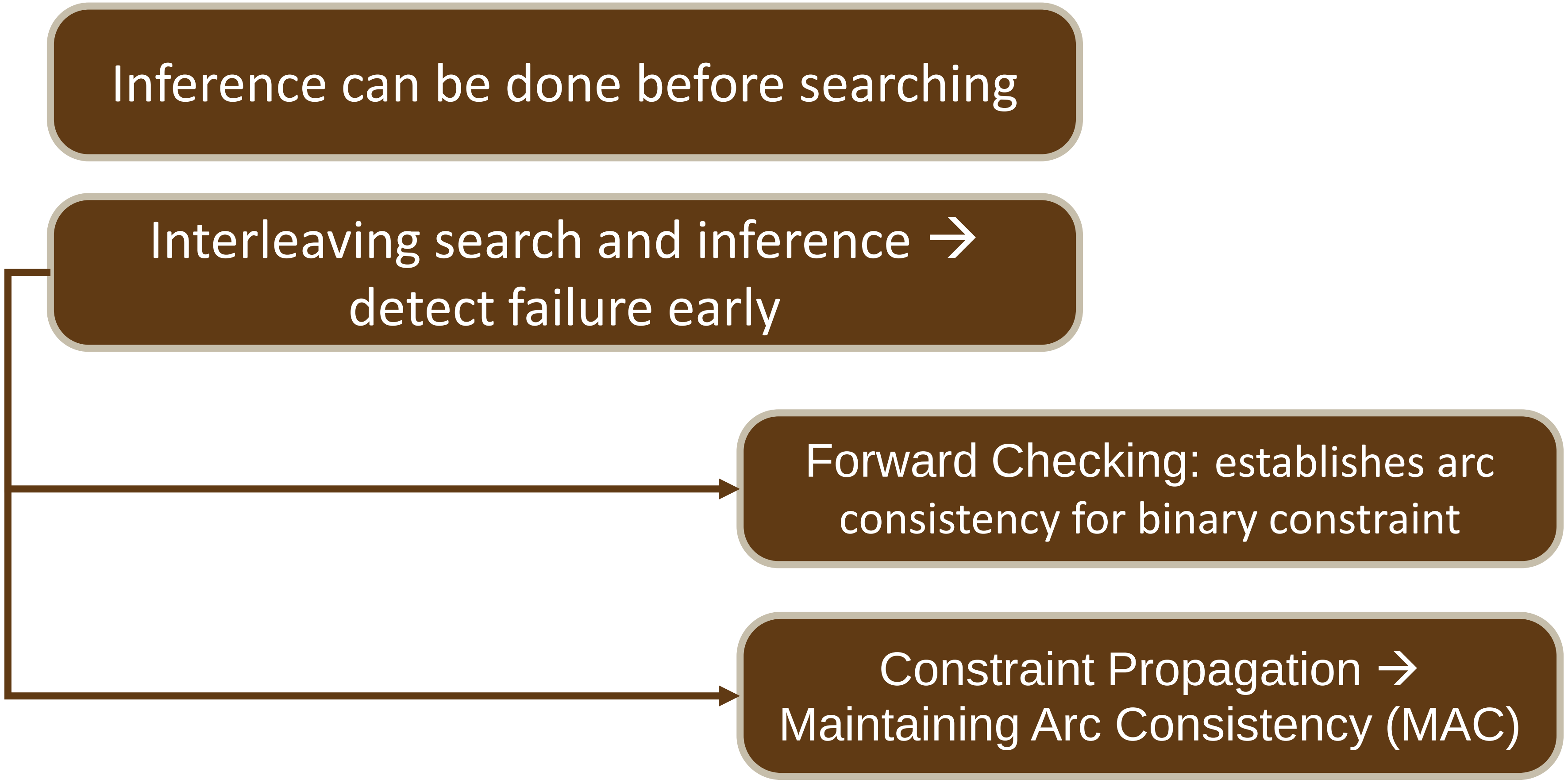
Interleaving search and inference



Inference can be done before searching

Interleaving search and inference → detect failure early

Forward Checking: establishes arc consistency for binary constraint

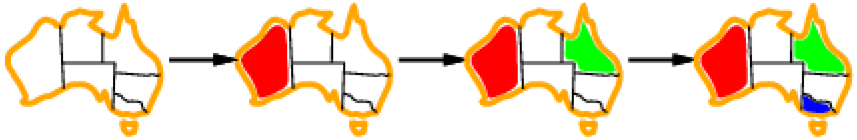
Constraint Propagation → Maintaining Arc Consistency (MAC)

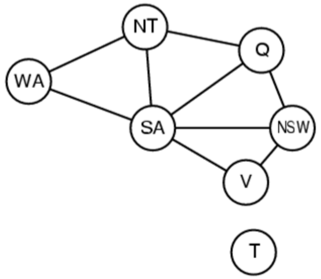
Let’s see an example

Example: Interleave Search and Inference

Keep track of remaining legal values for unassigned variables

Terminate search when any variable has no legal values





WA	NT	Q	NSW	V	SA	T
🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦
🟥	🟩🟦	🟥🟩🟦	🟥🟩🟦	🟥🟩🟦	🟩🟦	🟥🟩🟦
🟥	🟦	🟩	🟥🟦	🟥🟩🟦	🟦	🟥🟩🟦
🟥	🟦	🟩	🟥	🟦		🟥🟩🟦

FAIL!

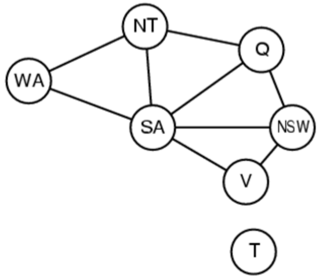
MAC: NT and SA cannot both be blue → MAC: repeatedly enforces constraints locally

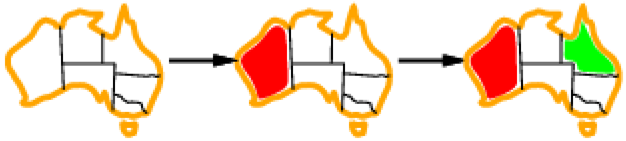
Example: Interleave Search and Inference

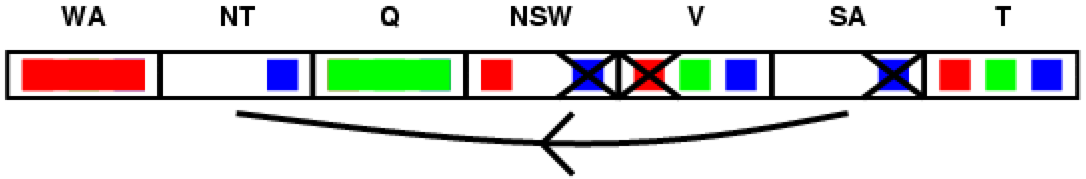
Keep track of remaining legal values for unassigned variables

Terminate search when any variable has no legal values

MAC: repeatedly enforces constraints locally







FAIL!

If variable *X* loses a value, neighbors of *X* need to be rechecked

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Local Search for CSP

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Local Search

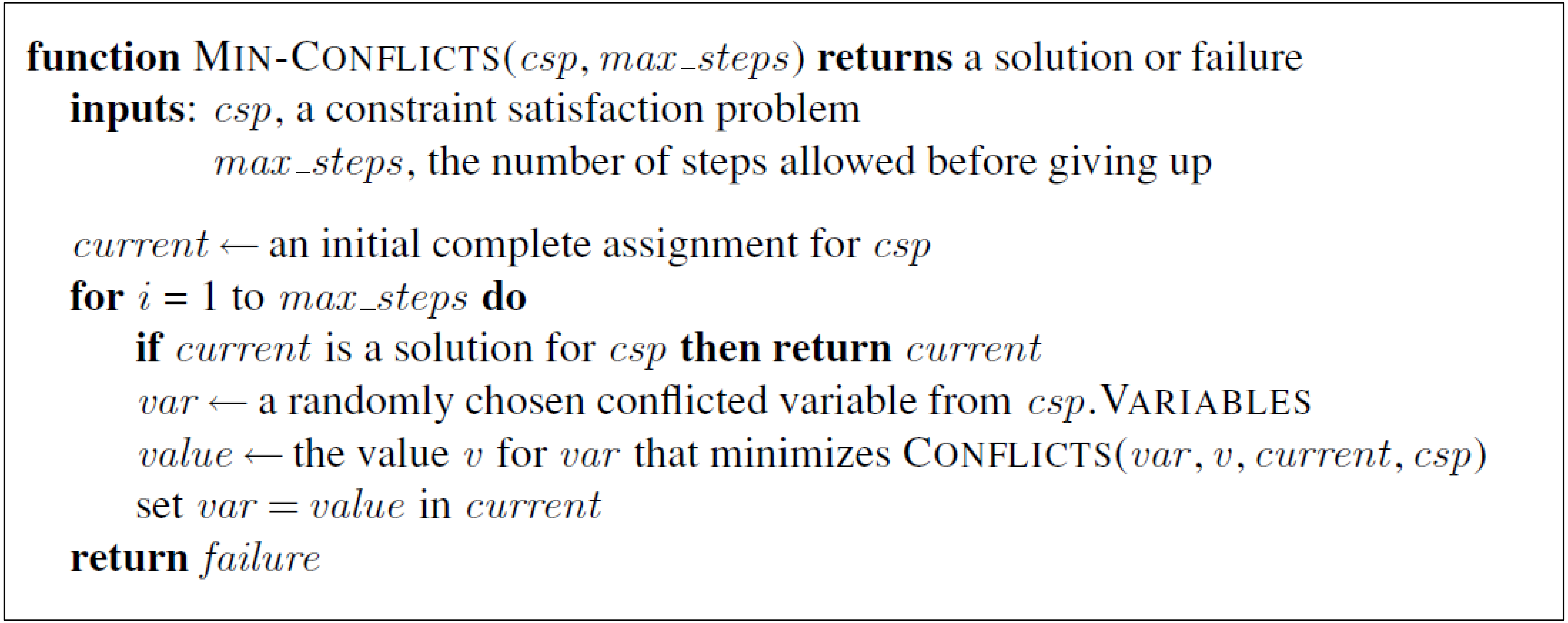
Complete-state formulation → initial state assigns a value to every variable

The search changes the value of one variable at a time

Variable selection: randomly select any conflicted variable

Value selection by min-conflicts heuristic

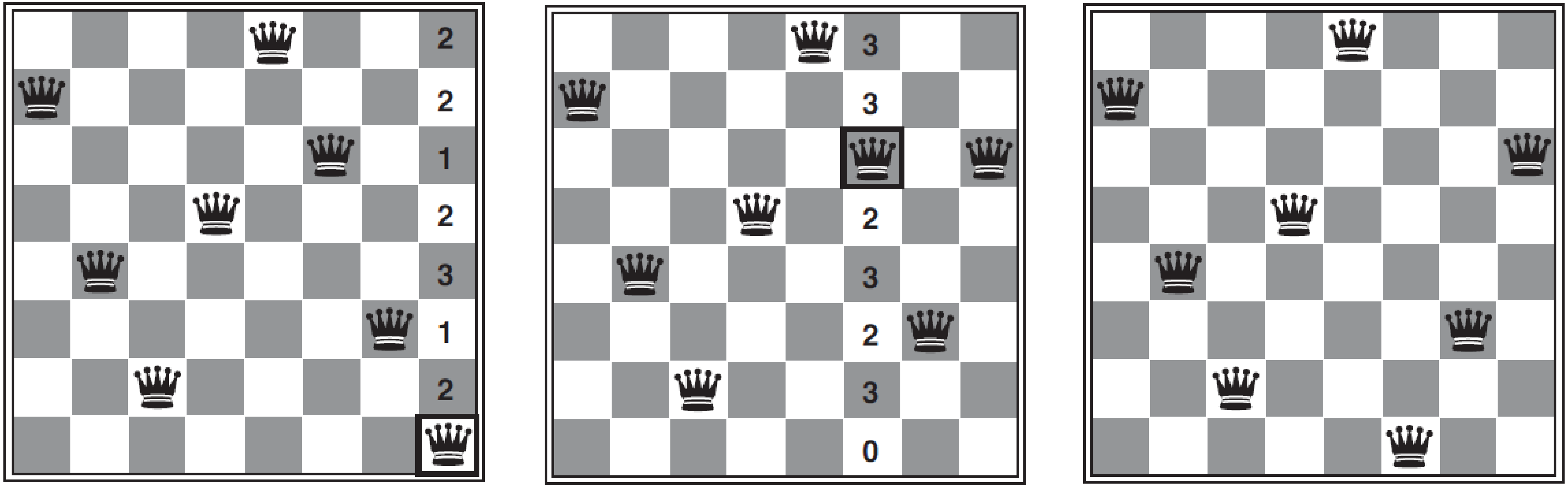
Min-Conflict Heuristic



```
function MIN-CONFLICTS($csp$, $max\_steps$) returns a solution or failure
    inputs: $csp$, a constraint satisfaction problem
            $max\_steps$, the number of steps allowed before giving up

    $current \leftarrow$ an initial complete assignment for $csp$
    for $i$ = 1 to $max\_steps$ do
        if $current$ is a solution for $csp$ then return $current$
        $var \leftarrow$ a randomly chosen conflicted variable from $csp$.VARIABLES
        $value \leftarrow$ the value $v$ for $var$ that minimizes CONFLICTS($var$, $v$, $current$, $csp$)
        set $var = value$ in $current$
    return $failure$
```

Example: n-Queens Problem



Applications

Solve *n*-queens in almost constant time for arbitrary *n* with high probability

Online setting → scheduling

THANK YOU