FINITE AUTOMATA WITH E-TRANSITIONS

IF 2124 TEORI BAHASA FORMAL OTOMATA

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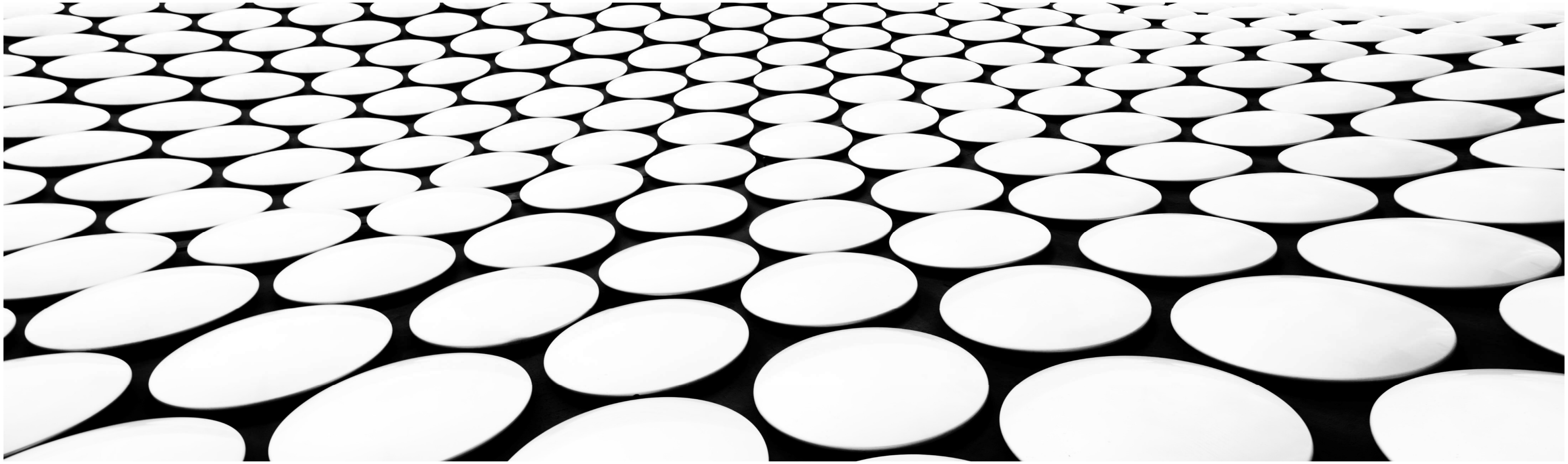
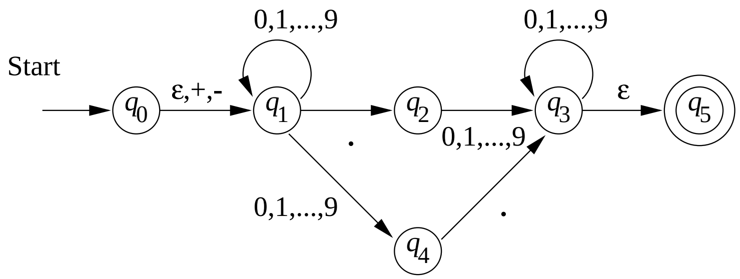


FA's with Epsilon-Transitions

An ϵ-NFA accepting decimal numbers consisting of:

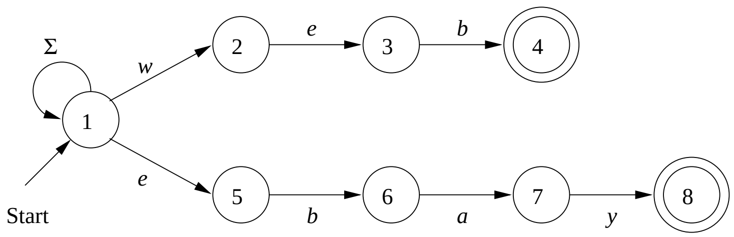
1. An optional + or - sign
2. A string of digits
3. a decimal point
4. another string of digits

One of the strings (2) are (4) are optional



Example:

ϵ-NFA accepting the set of keywords {ebay, web}



An ϵ-NFA is a quintuple $(Q, \Sigma, \delta, q_0, F)$ where δ is a function from $Q \times \Sigma \cup \{\epsilon\}$ to the powerset of Q.

Example: The ϵ-NFA from the previous slide

$$E = (\{q_0, q_1, \ldots, q_5\}, \{., +, -, 0, 1, \ldots, 9\}\ \delta, q_0, \{q_5\})$$

where the transition table for δ is

	ϵ	+,-	.	$0, \ldots, 9$
$\rightarrow q_0$	$\{q_1\}$	$\{q_1\}$	$\emptyset$	$\emptyset$
q_1	$\emptyset$	$\emptyset$	$\{q_2\}$	$\{q_1, q_4\}$
q_2	$\emptyset$	$\emptyset$	$\emptyset$	$\{q_3\}$
q_3	$\{q_5\}$	$\emptyset$	$\emptyset$	$\{q_3\}$
q_4	$\emptyset$	$\emptyset$	$\{q_3\}$	$\emptyset$
$\star q_5$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

$\boxed{\text{ECLOSE}}$

We close a state by adding all states reachable by a sequence $\epsilon\epsilon\cdots\epsilon$

Inductive definition of $\text{ECLOSE}(q)$

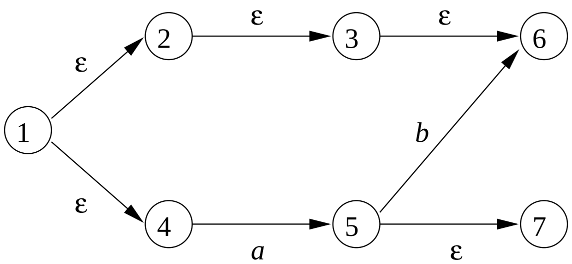
Basis:

$q \in \text{ECLOSE}(q)$

Induction:

$p \in \text{ECLOSE}(q)$ and $r \in \delta(p, \epsilon) \quad \Rightarrow$
$r \in \text{ECLOSE}(q)$

Example of ϵ-closure



For instance,

$$\text{ECLOSE}(1) = \{1, 2, 3, 4, 6\}$$

- Inductive definition of $\widehat{\delta}$ for ϵ-NFA's

Basis:

$$\widehat{\delta}(q, \epsilon) = \text{ECLOSE}(q)$$

Induction:

$$\widehat{\delta}(q, xa) = \bigcup_{p \in \delta(\widehat{\delta}(q,x),a)} \text{ECLOSE}(p)$$

Let's compute on the blackboard in class $\widehat{\delta}(q_0, 5.6)$ for the NFA on slide 43

Given an ϵ-NFA

$$E = (Q_E, \Sigma, \delta_E, q_0, F_E)$$

we will construct a DFA

$$D = (Q_D, \Sigma, \delta_D, q_D, F_D)$$

such that

$$L(D) = L(E)$$

Details of the construction:

- $Q_D = \{S : S \subseteq Q_E \text{ and } S = \text{ECLOSE}(S)\}$

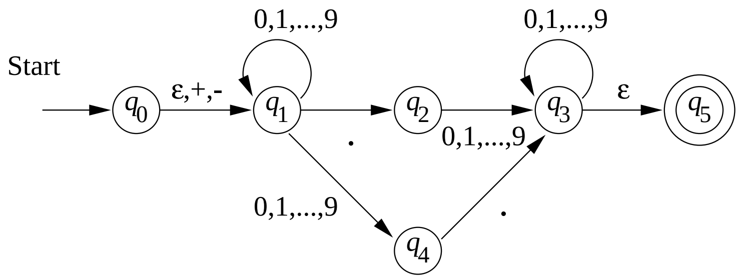
- $q_D = \text{ECLOSE}(q_0)$

- $F_D = \{S : S \in Q_D \text{ and } S \cap F_E \neq \emptyset\}$

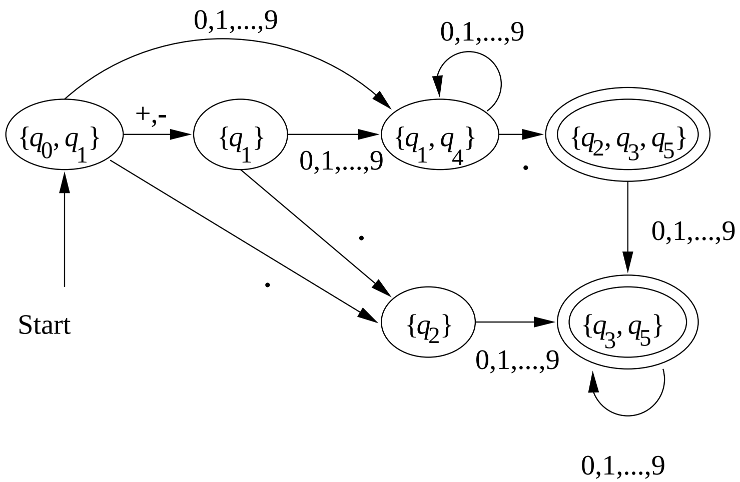
- $\delta_D(S, a) =$

  $\bigcup\{\text{ECLOSE}(p) : p \in \delta(t, a) \text{ for some } t \in S\}$

Example: ϵ-NFA E



DFA D corresponding to E



Theorem 2.22: A language L is accepted by some ϵ-NFA E if and only if L is accepted by some DFA.

Proof: We use D constructed as above and show by induction that $\widehat{\delta}_D(q_0, w) = \widehat{\delta}_E(q_D, w)$

Basis: $\widehat{\delta}_E(q_0, \epsilon) = \text{ECLOSE}(q_0) = q_D = \widehat{\delta}(q_D, \epsilon)$

Induction:

$$\hat{\delta}_E(q_0, xa) = \bigcup_{p \in \delta_E(\hat{\delta}_E(q_0,x),a)} \text{ECLOSE}(p)$$

$$= \bigcup_{p \in \delta_D(\hat{\delta}_D(q_D,x),a)} \text{ECLOSE}(p)$$

$$= \bigcup_{p \in \hat{\delta}_D(q_D,xa)} \text{ECLOSE}(p)$$

$$= \hat{\delta}_D(q_D, xa)$$