



Decision Tree Learning (Russel & Norvig, 2021)

function DECISION-TREE-LEARNING(*examples*, *attributes*, *parent_examples*) **returns**
a tree

if *examples* is empty **then return** PLURALITY-VALUE(*parent_examples*)
else if all *examples* have the same classification **then return** the classification
else if *attributes* is empty **then return** PLURALITY-VALUE(*examples*)
else

$A \leftarrow \operatorname{argmax}_{a \in \text{attributes}} \text{IMPORTANCE}(a, \text{examples})$

tree \leftarrow a new decision tree with root test *A*

for each value v_k of *A* **do**

$\text{exs} \leftarrow \{e : e \in \text{examples} \text{ and } e.A = v_k\}$

subtree \leftarrow DECISION-TREE-LEARNING(*exs*, *attributes* – *A*, *examples*)

add a branch to *tree* with label (*A* = v_k) and subtree *subtree*

return *tree*

The function PLURALITY-VALUE selects the most common output value among a set of examples, breaking ties randomly.



Issues in DTL

**Overfitting
training
data**

**Continuous
-valued
attribute**

**Handling
attributes
with differing
costs**

**Handling
missing
attribute
value**

**Alternative
measures for
selecting
attributes**



Modul : Issues in Decision Tree Learning (DTL)

Overfitting

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Pembelajaran Mesin
(Machine Learning)



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What is Overfit

H : Hypothesis space

A hypothesis: $h \in H$; Alternative hypothesis: $h' \in H$
train: training examples; D : entire distribution of data

$$\text{error}_{\text{train}}(h) < \text{error}_{\text{train}}(h')$$

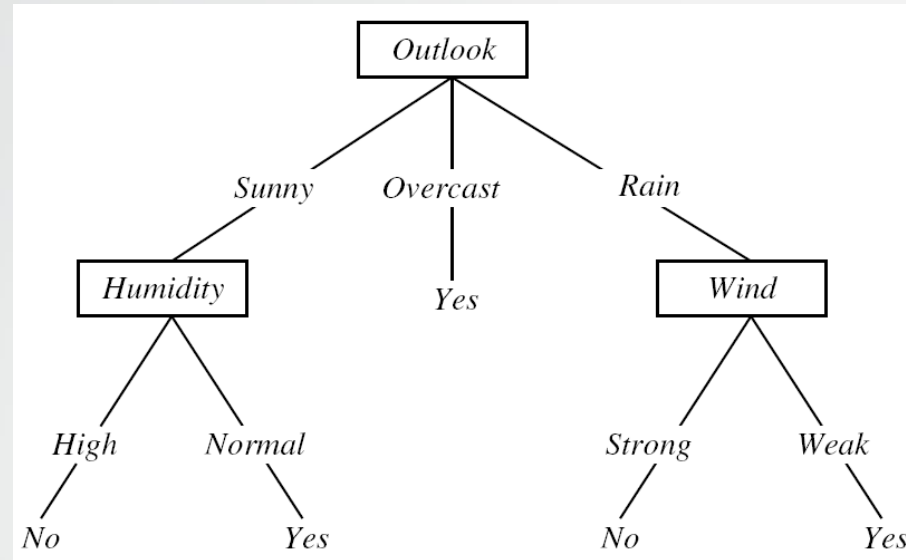
Overfit

$$\text{error}_D(h) > \text{error}_D(h')$$



Illustration

D15 (noisy training examples):
Outlook = Sunny;
Temperature = Hot;
Humidity = Normal;
Wind = Strong;
PlayTennis = No



Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No



Overfitting can happen even training examples is noise-free
(when small numbers of examples are associated with leaf
Nodes) → decrease accuracy 10 – 25% on most problems



Solution Approaches

1. Stop growing the tree earlier, before it reaches the point where it perfectly classifies the training data

2. allow the tree to overfit the data, and then post-prune the tree

Pros

More Direct

More Successfull
in practice

Cons

Difficulty of estimating precisely when to stop growing the tree

Requires more steps (grow until fit, then prune)

what criterion is to be used to determine the correct final tree size



Approaches in Determine the Correct Final Tree Size

1. Use separate examples distinct from training to evaluate the pruning tree

2. Use all available data for training, then conduct a statistical test to check whether expanding (or pruning) a node will produce improvement, example: chi square test

3. Use explicit measure of the complexity for encoding the training examples and decision tree → Minimum Description Length Principle

2/3 Training set

1/3 Validation set

Source:
Machine Learning by Tom Mitchell chapter 6.6

$$h_{MDL} = \operatorname{argmin}_{h \in H} L_{C_1}(h) + L_{C_2}(D|h)$$

$L_{C_1}(h)$: Length (number of bits) of hypothesis encoding
 $L_{C_2}(D|h)$: Length of data D given hypothesis h encoding



Reduced Error Pruning

Consider decision (attribute) node as candidates for pruning → assign the most common classification affiliated with that node



Grow until fit then prune

Split data into training and validation set

Do until further pruning is harmful:

1. Evaluate impact on validation set of pruning each possible node (plus those below it)
2. Greedily remove the one (node) that most improves validation set accuracy

an effective approach provided a large amount of data is available



Rule Post-Pruning

Improvement of ID3
Algorithm: C4.5

Suitable for
limited data

1. Growing the tree from training set, until the training data is fit as well as possible and allowing overfitting to occur.
2. Convert the learned tree into an equivalent set of rules by creating one rule for each path from the root node to a leaf node.
3. Prune (generalize) each rule by removing any preconditions that result in improving its estimated accuracy.
4. Sort the pruned rules by their estimated accuracy, and consider them in this sequence when classifying subsequent instances.



Example

Decision Rules:

**If Outlook = Sunny and Humidity=High
then No**

Pruning:

**If Outlook = Sunny then No
OR**

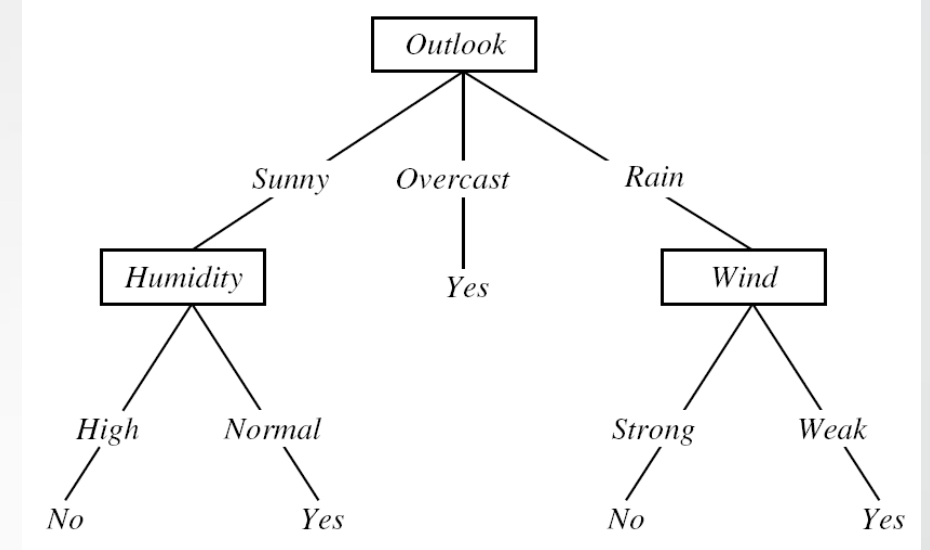
If Humidity=High then No



Increase/ Reduce Accuracy?



**Over validation set/
training set (C4.5)**



Why Decision Tree → Decision Rule ?

1. Distinct path ~ distinct rule:
independent pruning
2. No distinction between
attribute tests
3. Improves readability



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Variable (Attribute) Types

**Source: DataMining Concepts and Techniques
by Jiawei Han, Micheline Kamber, Jian Pei**

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Numeric

Quantitative
(measurable
quantity) → Integer
or Real Values

Interval Scaled
(equal-size units,
have order)

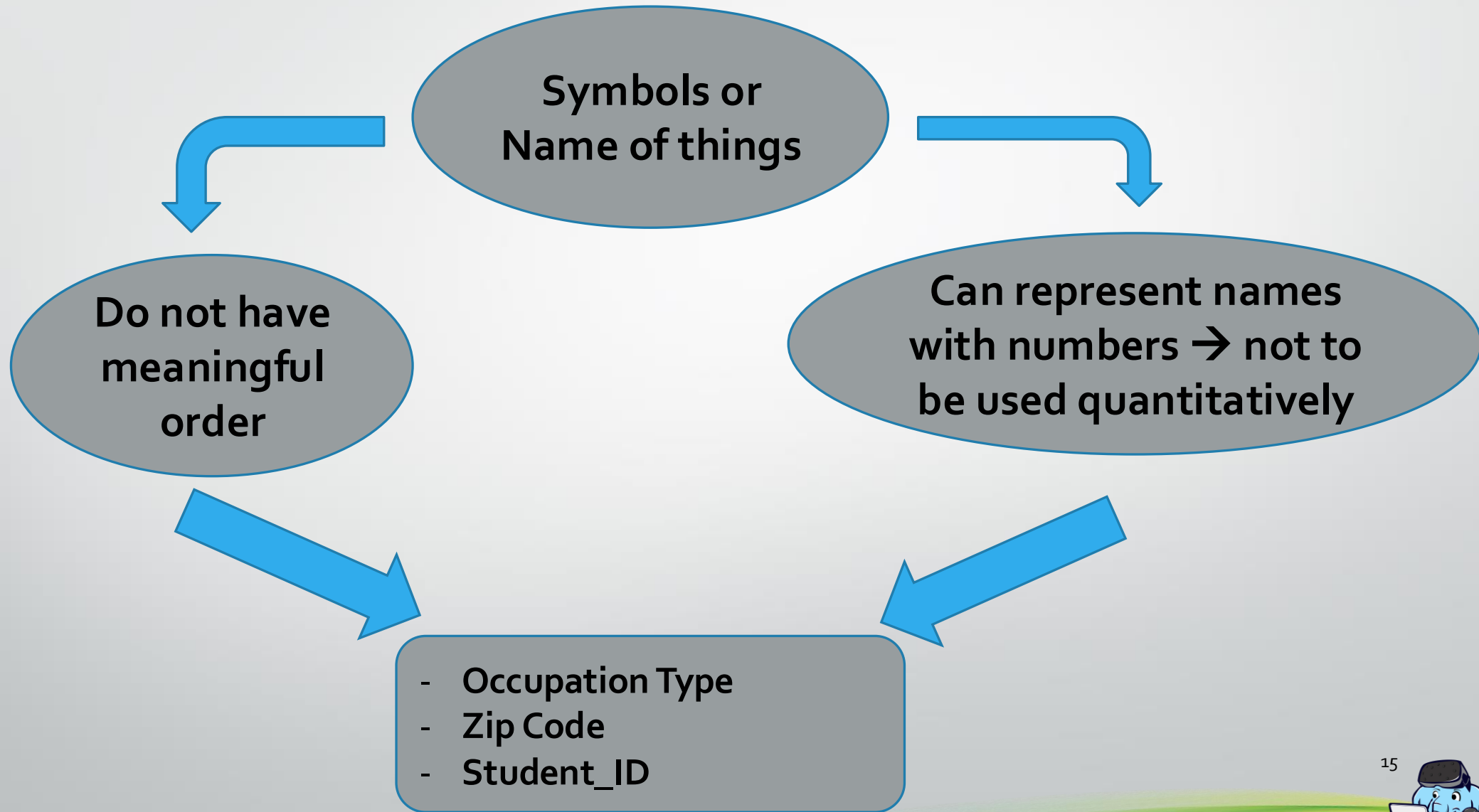
- Temperature
- Calendar dates

Ratio-Scaled
(inherent zero
point, a value can
be multiple of
another value)

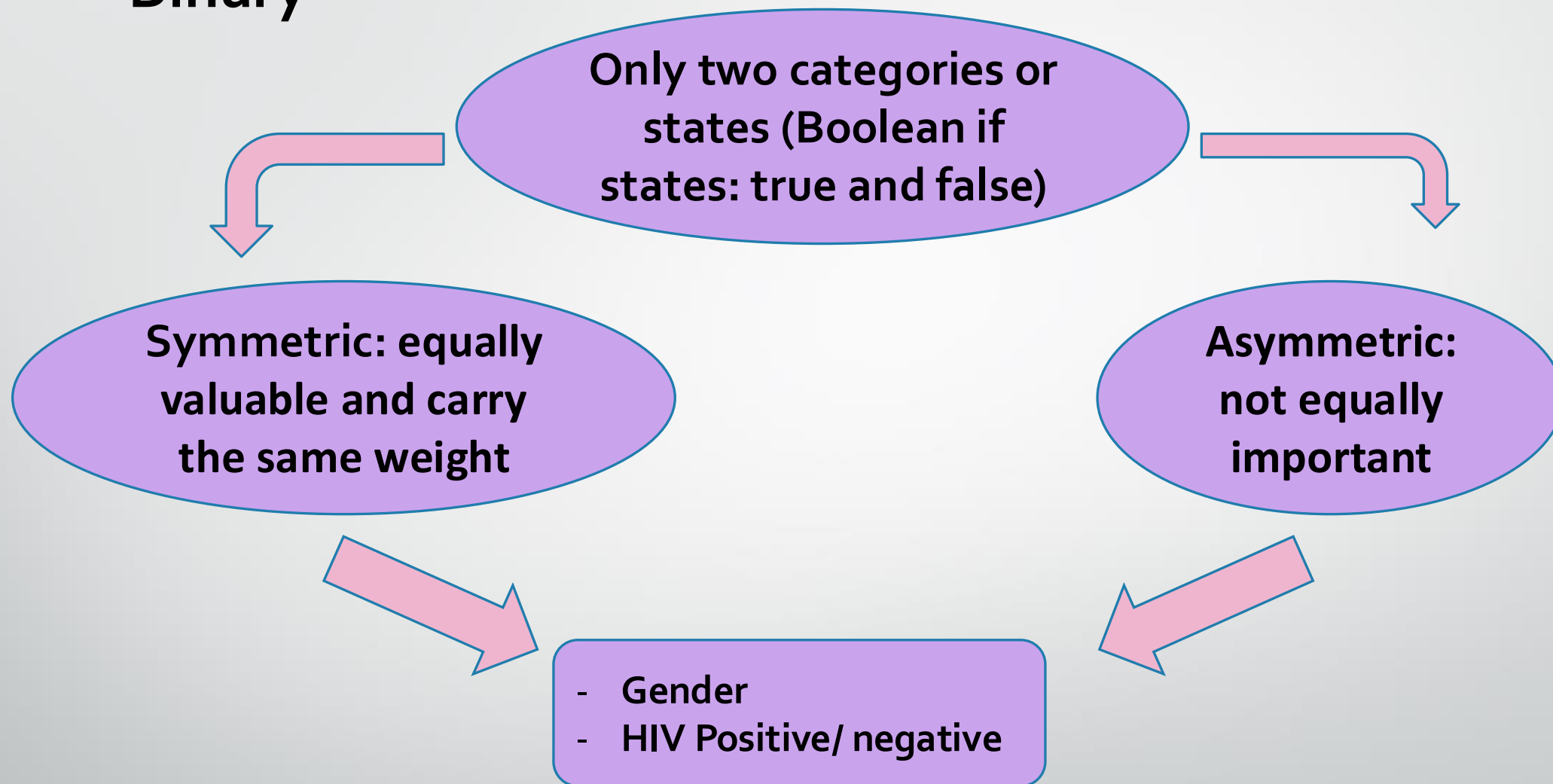
- Years of
experience
- Weight
- Number of words



Nominal/ Categorical



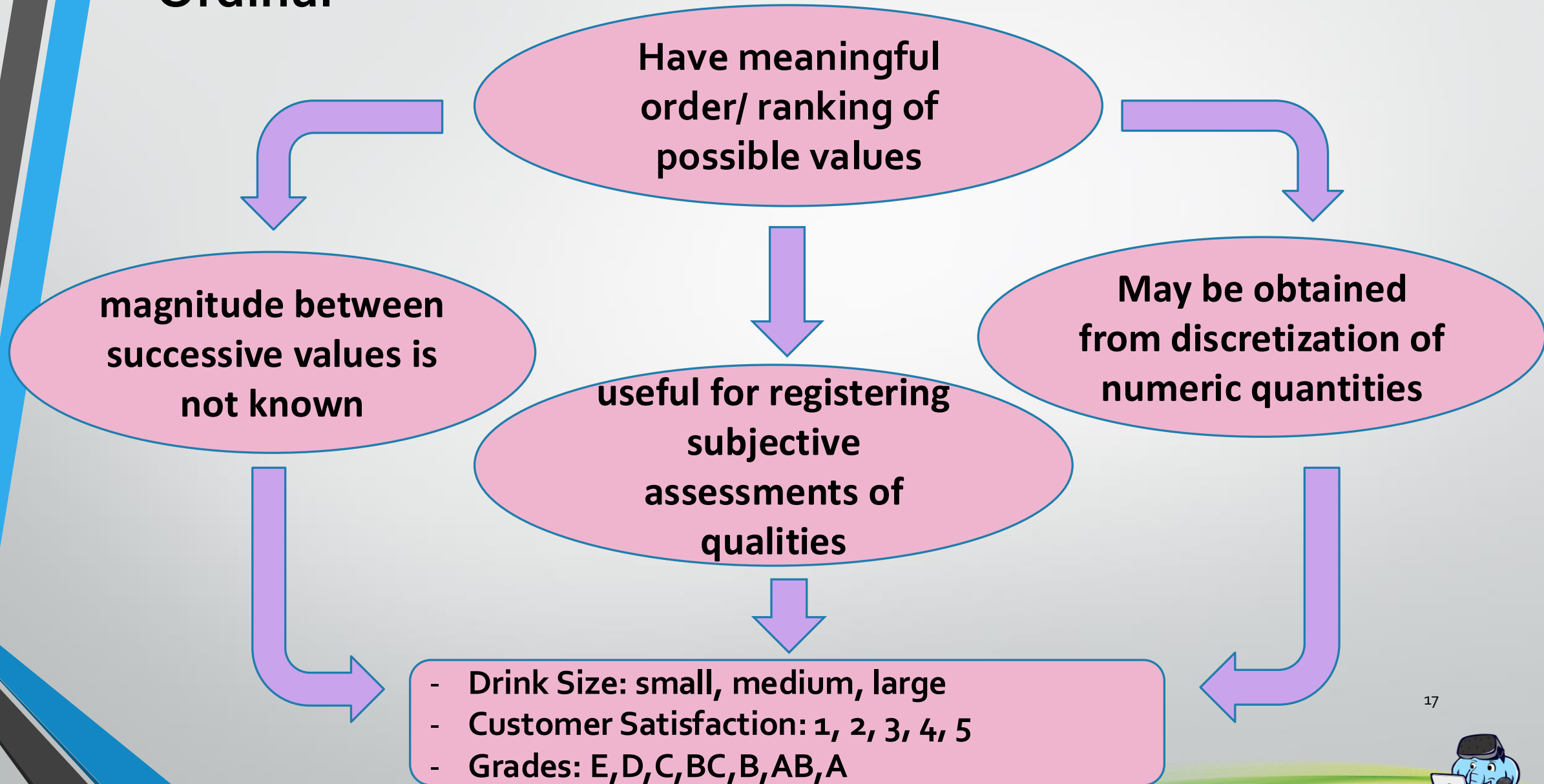
Binary



1	red,	green,	blue
2	1,	0,	0
3	0,	1,	0
4	0,	0,	1



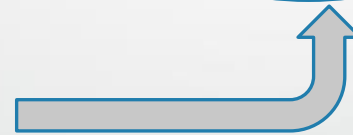
Ordinal



Discrete vs Continuous

has a finite set of values:
Drink size, Age, Medical
test,

has a countably infinite
set of values:
Customer ID, Zip code



Discrete

\neq

Continuous



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Continuous-valued Attribute

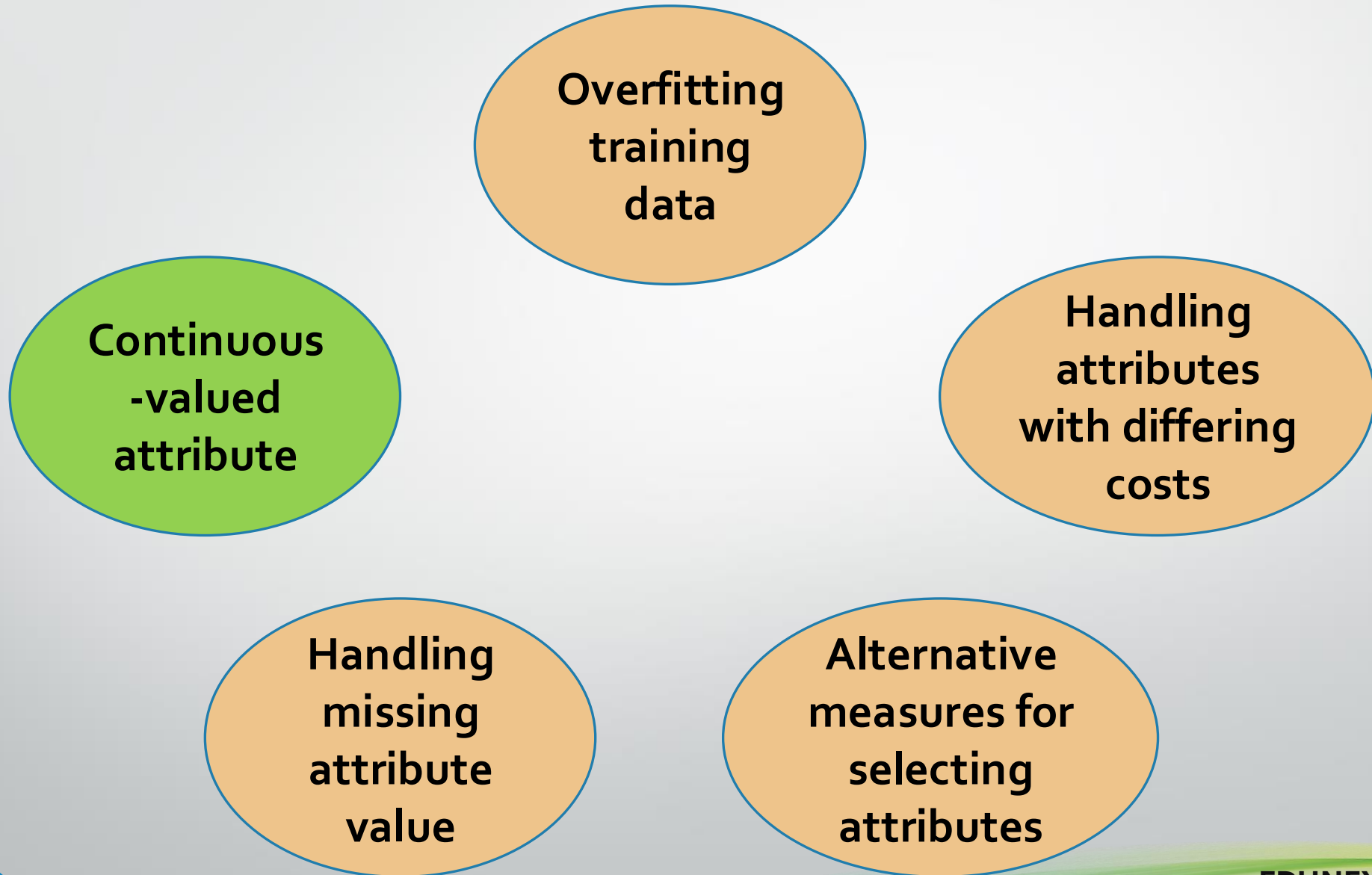
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Issues in DTL



Discretization

Continuous valued attributes →
new discrete valued (boolean)
attribute A_c

True: $A < c$

False: $A < c$

<i>Temperature:</i>	40	48	60	72	80	90
<i>PlayTennis:</i>	No	No	Yes	Yes	Yes	No

Potential optimal breakpoints

$$C = (48+60)/2 = 54$$

Or

$$C = (80+90)/2 = 85$$

What is Best Value
for threshold c ?

Use Information Gain for
each potential breakpoint



Illustration

1. Sort The Continuous-valued attribute

Temperature	40	48	60	72	80	90
Play Tennis	No	No	Yes	Yes	Yes	No

3. Candidates: midway between corresponding values $\rightarrow C : 54$ or $C : 85$

4. Find the greatest Gain from the candidates, and other discrete-valued attributes

Day	Outlook	Temperature	Humidity	Wind	Play Tennis
D1		72			Yes
D2		40			No
D3		90			No
D4		60			Yes
D5		48			No
D6		80			Yes

2. Identify Adjacent examples that differ in their target class

For C: 54

Temperature < 54: 2 examples \rightarrow yes/0, no/2

Temperature \geq 54: 4 examples \rightarrow yes/3, no/1

$$\text{Gain}(S, T_{54}) = \text{Entropi}(S) - [(2/6 * \text{Entropi}(0,2)) + (4/6 * \text{Entropi}(3,1))]$$

For C: 85

Temperature < 85: 5 examples \rightarrow yes/3, no/2

Temperature \geq 85: 1 examples \rightarrow yes/0, no/1

$$\text{Gain}(S, T_{85}) = \text{Entropi}(S) - [(5/6 * \text{Entropi}(3,2)) + (1/6 * \text{Entropi}(0,1))]$$

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Missing Attribute Values

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Alternative Strategies

Assign it with the most common value at node n among other examples

Assign it with the most common value at node n that have classification $c(x)$

Assign probability p_i to each possible value v_i of A (used in C4.5)

The probability
Can be used for
classifying
a new
instance
with
missing
value

Gain(S, A) only consider the fraction of training examples with known value
 $\text{Gain}(S, A) = 10/11 * (\text{Entropy}(S) - [\sum \text{proportion} * \text{entropy_of_known_value}])$

$v_1 = 1$, 6 known examples; $v_2 = 0$, 4 known examples, 1 example with missing value of attr A
 $p_i = 6/10$ added to v_1 ; $p_2 = 4/10$ added to $v_2 \rightarrow$ for splitting



Missing Value as Separate Value

Denoted "?" → Null Value
In C4.X

Not
Appropriate
when:



Values are missing due to different reasons

blood sugar value could be missing when it is very high or very low

field IsPregnant missing for a male patient should be treated differently (no) than for a female patient of age 25 (unknown)



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Alternative Measures for Selecting Attribute

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Attribute with many values (C4.5)

Gain will always select it → example *Date=2021_Jan_31*

Date will perfectly classify training examples,
but very poor predictor for unseen data



GAIN RATIO

$$GainRatio(S, A) \equiv \frac{Gain(S, A)}{SplitInformation(S, A)}$$

$$SplitInformation(S, A) \equiv - \sum_{i=1}^c \frac{|S_i|}{|S|} \log_2 \frac{|S_i|}{|S|}$$

where S_i is subset of S for which A has value v_i

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Illustration

Date	Atr2	Atr3	Class
2021_Jan_01	v1		No
2021_Jan_02	v1		No
2021_Jan_03	v2		Yes
2021_Jan_04	v2		Yes
2021_Jan_05	v1		Yes
2021_Jan_06	v1		No

$$\begin{aligned} \text{SplitInformation}(S, \text{Date}) &= - \sum_{i=1}^6 \frac{|S_i|}{|S|} \log_2 \frac{|S_i|}{|S|} \\ &= - \left(\frac{1}{6} \log_2 \frac{1}{6} + \frac{1}{6} \log_2 \frac{1}{6} + \frac{1}{6} \log_2 \frac{1}{6} + \frac{1}{6} \log_2 \frac{1}{6} + \frac{1}{6} \log_2 \frac{1}{6} + \frac{1}{6} \log_2 \frac{1}{6} \right) \end{aligned}$$

$$\begin{aligned} \text{SplitInformation}(S, \text{Atr2}) &= - \sum_{i=1}^2 \frac{|S_i|}{|S|} \log_2 \frac{|S_i|}{|S|} \\ &= - \left(\frac{4}{6} \log_2 \frac{4}{6} + \frac{2}{6} \log_2 \frac{2}{6} \right) \end{aligned}$$

What if SplitInformation is very small or zero ($|S_i| \approx |S|$)
 \rightarrow GainRatio undefined or very large



Heuristic: Apply GainRatio test only for Attribute with above average Gain



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Attributes with Differing Costs

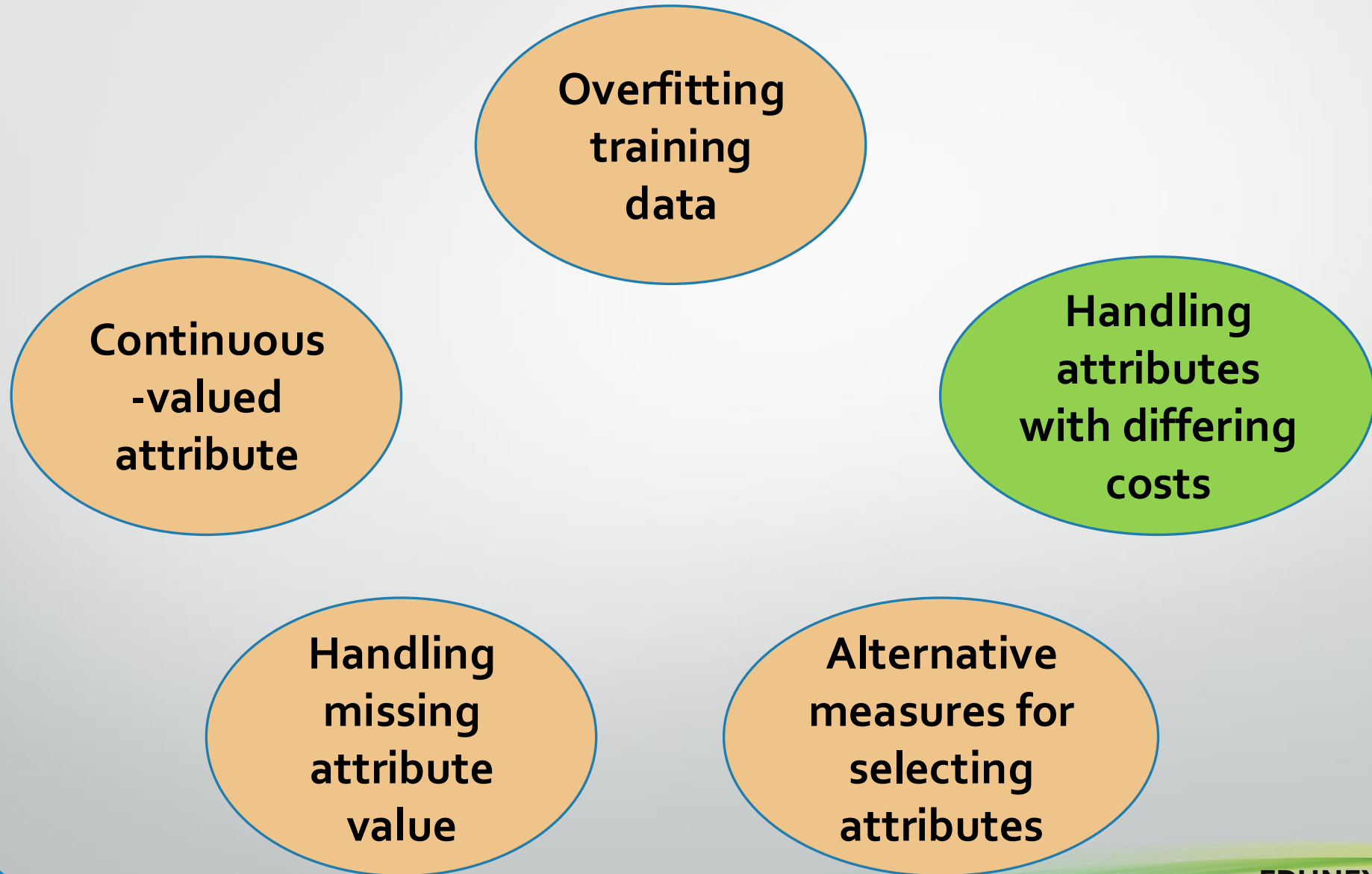
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Issues in DTL



Attribute with Different Cost

Attributes: Temperature, BiopsyResult, Pulse, BloodTestResults

Have Different Cost (monetary and patient comfort)



Use low cost attribute where possible, high cost only when required to produce reliable classification

Cost is considered in calculating Gain of each attribute



Approaches

Tan and Schlimmer (1990)
and Tan (1993):

$$\frac{Gain^2(S, A)}{Cost(A)}$$

Nunez (1988):

$$\frac{2^{Gain(S, A)} - 1}{(Cost(A) + 1)^w}$$

Where $w \in [0, 1]$ determine
importance of cost



Exercise

Outlook	Temp	Humidity	Windy	Class
sunny	75	70	TRUE	Play
sunny	80	90	TRUE	Don't Play
sunny	85	85	FALSE	Don't Play
sunny	72	95	FALSE	Don't Play
sunny	69	70	FALSE	Play
?	72	90	TRUE	Play
overcast	83	78	FALSE	Play
overcast	64	65	TRUE	Play
overcast	81	75	FALSE	Play
rain	71	80	TRUE	Don't Play
rain	65	70	TRUE	Don't Play
rain	75	80	FALSE	Play
rain	68	80	FALSE	Play
rain	70	96	FALSE	Play

1. What is GainRatio for Outlook?
2. What are Examples (instances) for Outlook = sunny and what is the weight for instance with outlook=? related with outlook=sunny
3. Based on the result of question (2), define the threshold in "Humidity" discretization, and the leaf node for each branch.





THANK YOU



