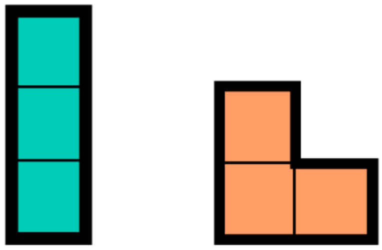
WIKIPEDIA

Tromino

A **tromino** is a polyomino of order 3, that is, a polygon in the plane made of three equal-sized squares connected edge-to-edge.[1]



All possible free trominos

Contents

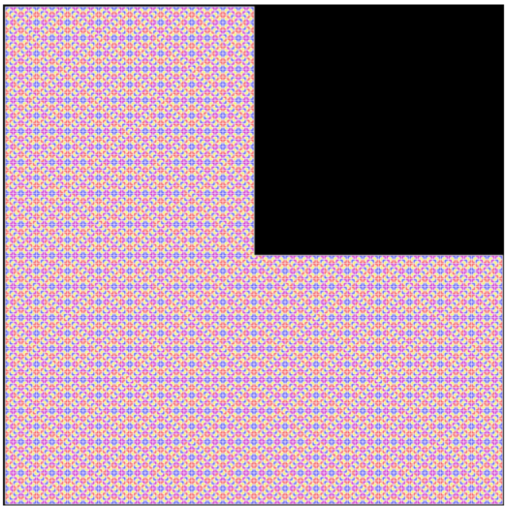
Symmetry and enumeration

When rotations and reflections are not considered to be distinct shapes, there are only two different *free* trominoes: "I" and "L" (the "L" shape is also called "V").

Since both free trominoes have reflection symmetry, they are also the only two *one-sided* trominoes (trominoes with reflections considered distinct). When rotations are also considered distinct, there are six *fixed* trominoes: two I and four L shapes. They can be obtained by rotating the above forms by 90°, 180° and 270°.[2][3]

Rep-tiling and Golomb's tromino theorem

Both types of tromino can be dissected into n^2 smaller trominos of the same type, for any integer $n > 1$. That is, they are rep-tiles.[4] Continuing this dissection recursively leads to a tiling of the plane, which in many cases is an aperiodic tiling. In this context, the L-tromino is called a *chair*, and its tiling by recursive subdivision into four smaller L-trominos is called the chair tiling.[5]



Geometrical dissection of an L-tromino (rep-4)

Motivated by the mutilated chessboard problem, Solomon W. Golomb used this tiling as the basis for what has become known as Golomb's tromino theorem: if any square is removed from a $2^n \times 2^n$ chessboard, the remaining board can be completely covered with L-trominoes. To prove this by mathematical induction, partition the board into a quarter-board of size $2^{n-1} \times 2^{n-1}$ that contains the removed square, and a large tromino formed by the other three quarter-boards. The tromino can be recursively dissected into unit trominoes, and a dissection of the quarter-board with one square removed follows by the induction hypothesis. In contrast, when a chessboard of this size has one square removed, it is not always possible to cover the remaining squares by I-trominoes.[6]

References

1. Golomb, Solomon W. (1994). *Polyominoes* (2nd ed.). Princeton, New Jersey: Princeton University Press. ISBN 0-691-02444-8.
2. Weisstein, Eric W. "Triomino" (http://mathworld.wolfram.com/Triomino.html). *MathWorld*.
3. Redelmeier, D. Hugh (1981). "Counting polyominoes: yet another attack". *Discrete Mathematics*. **36**: 191–203. doi:10.1016/0012-365X(81)90237-5 (https://doi.org/10.1016%2F0012-365X%2881%2990237-5).
4. Niţică, Viorel (2003), "Rep-tiles revisited", *MASS selecta*, Providence, RI: American Mathematical Society, pp. 205–217, MR 2027179 (https://www.ams.org/mathscinet-getitem?mr=2027179).
5. Robinson, E. Arthur, Jr. (1999). "On the table and the chair". *Indagationes Mathematicae*. **10** (4): 581–599. MR 1820555 (https://www.ams.org/mathscinet-getitem?mr=1820555). doi:10.1016/S0019-3577(00)87911-2 (https://doi.org/10.1016%2FS0019-3577%2800%2987911-2)..
6. Golomb, S. W. (1954). "Checker boards and polyominoes". *American Mathematical Monthly*. **61**: 675–682. MR 0067055 (https://www.ams.org/mathscinet-getitem?mr=0067055). doi:10.2307/2307321 (https://doi.org/10.2307%2F2307321)..

External links

- Golomb's inductive proof of a tromino theorem (http://www.cut-the-knot.org/Curriculum/Geometry/Tromino.shtml) at cut-the-knot
- Tromino Puzzle (http://www.cut-the-knot.org/Curriculum/Games/TrominoPuzzle.shtml) at cut-the-knot
- Interactive Tromino Puzzle (http://www.amherst.edu/~nstarr/puzzle.html) at Amherst College