Guidelines to reduce general TSP to Triangle TSP

I am looking for the method / correct way to approach to reduce the traveling salesman problem to an instance of traveling salesman problem which satisfies the triangle inequality, ie:

$$D(a,b) \leq D(a,c) + D(c,b)$$

I am not sure how to attack this kind of problem, so any pointers / explanations regarding this would be helpful. Thank you.

cc.complexity-theory np-hardness np tsp hamiltonian-paths

edited Oct 21 '12 at 3:52 Community ♦ 1

asked Oct 10 '12 at 8:23 Dave 36 1 2

You can use a direct reduction from Hamiltonian Cycle problem on planar graphs (NPC). Assign weight 1 if two nodes are connected, weight 2 if the two nodes are not connected; the planar graph has a Hamiltonian cycle iif there is a tour of cost <= |V|. – Marzio De Biasi Oct 10 '12 at 11:26

2 Homework perhaps? – Kristoffer Arnsfelt Hansen Oct 10 '12 at 13:20

@MarzioDeBiasi: The question doesn't ask to prove the metric TSP is NP-hard, but asks for a reduction of the TSP to the metric TSP. I don't see an "easy" reduction right now. – Yoshio Okamoto Oct 11 '12 at 1:20

2 @YoshioOkamoto: There is in fact an easy reduction, and I would classify it as an exercise. – Kristoffer Arnsfelt Hansen Oct 11 '12 at 9:22

1 @KristofferArnsfeltHansen: That's interesting. I'd love to see. – Yoshio Okamoto Oct 12 '12 at 11:53

@KristofferArnsfeltHansen: I didn't think of it too much, but I don't see a quick reduction, too. Can you post it as an answer? – Marzio De Biasi Oct 22 '12 at 10:24

1 Answer

Here is a simple reduction for the TSP problem to the metric TSP problem:

For the given TSP instance with n cities, let $D(i,j) \geq 0$ denote the distance between i and j. Now let $M = \max_{i,j} D(i,j)$. Define the metric TSP instance by the distances $D'(i,j) := D(i,j) + M$. To see that this gives a metric TSP instance, let i,j,k be arbitrary. Then $D'(i,j) + D'(j,k) = D(i,j) + D(j,k) + 2M \geq 2M \geq D(i,k) + M = D'(i,k)$ Since any tour uses exactly n edges, the transformation adds exactly nM to any tour, which shows the correctness of the reduction.

Remark: We can of course also allow for negative distances in the original TSP instance if you prefer by changing the reduction slightly.

edited Oct 23 '12 at 18:24

answered Oct 22 '12 at 15:02 Kristoffer Arnsfelt Hansen 3,681 1 19 34

1 easy ... but only when you know how to do it :-) +1! – Marzio De Biasi Oct 22 '12 at 15:14

Thank you, I like it. – Yoshio Okamoto Oct 23 '12 at 0:59

Can you tell why this reduction is not an *approximation-preserving reduction*? – Ribz Feb 3 at 16:05