Subgroups of S_4 of order 6

I have to find the subgroups of S_4 of order 6:

<(12),(123)>={1,(12),(123),(132),(23),(13)}

but how much are?

maybe 4 : <(12),(124)>={1,(12),(124),(142),(24),(14)}

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<(13),(134)>={1,(13),(134),(143),(34),(14)}

<(23),(234)>={1,(23),(234),(243),(34),(24)}
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(abstract-algebra)

edited Jan 28 '16 at 11:43

asked Jan 28 '16 at 11:07
Giulia B.
254 1 9

Do you know all the different types of group of order 6? Then you can work out how each type can sit in S_4, and then you can work out how many of each type there are in S_4. – Gerry Myerson Jan 28 '16 at 11:32

2 Answers

There are two possible groups of order 6: C_6 and S_3.

Since S_4 has no element of order 6, the only possibility is a subgroup isomorphic to S_3, and these are the conjugates of S_3 in S_4.

answered Jan 28 '16 at 11:36
lhf
116k 6 125 287

See also groupprops.subwiki.org/wiki/S3_in_S4 and math.stackexchange.com/questions/379841/…. – lhf Jan 28 '16 at 11:53

why are these subgroups isomorphic to S_3? – Giulia B. Jan 28 '16 at 11:58

@GiuliaB., because they'd cyclic otherwise, as mentioned. – lhf Jan 28 '16 at 12:12

Let us show the 4 subgroups you found are all the subgroups of order 6 of S_4. Let H be a subgroup of order 6. Then H has an element of order 3. It is a 3-cycle. If it is (123), we show $H =< (12), (123) >$. H also contains an element a of order 2. If $a = (12), (13)$, or (23), then we get the subgroup $< (12), (123) >$. If $a = (14)$, we know $(132) \in H$, $(14)(123)(14) = (423) \in H$, H has at least 3 elements of order 3, contradiction. Similarly, $a \neq (24), (34)$. Finally, if $a = (12)(34)$, $(12)(34)(123)(12)(34) = (214) \in H$, contradiction. Similarly, $a \neq (13)(24), (14)(23)$. So the 2 3-cycles of H determine all other elements of H. Since there are 8 3-cycles in S_4, there are 4 subgroups of order 6.

answered Jan 29 '16 at 3:49
stork
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