Existence of subgroup of order six in A_4

> Show that the alternating group A_4 of all even permutations of S_4 does not contain a subgroup of order 6.

For me am thinking to write all elements of A_4 and trying to find every cyclic subgroup generated by each element of A_4, then I have to check whether there exist such a subgroup or not! This is a long procedure for me, I ask if there is a short way to do this.

(abstract-algebra) (group-theory) (finite-groups) (permutations)

edited Dec 25 '11 at 23:51
Srivatsan
19.5k 3 62 118

asked Dec 25 '11 at 23:44
Junior II
73 1 5

If the subgroup has order 6, then what can you say about the order of the quotient group? What can you conclude after that? – mathmath8128 Dec 25 '11 at 23:48

The quotient group has order 2 – Junior II Dec 25 '11 at 23:57

Here is the ML link. – Ehsan M. Kermani Dec 26 '11 at 0:36

Three proofs of this fact are given in Keith Conrad's notes here.. Highly recommended! – Prism Aug 24 '13 at 14:09

3 Answers

Assume $H \leq A_4$ is a subgroup of order 6. Then H contains a unique subgroup C of order 3. So C is characteristic in H. And as H is normal in A_4, we obtain C is normal in A_4. On the other hand, if (abc) is a generator of C, conjugating (abc) by $(ab)(cd) \in A_4$ we obtain $(bad) \notin C$, from which we obtain not only a contradiction, but a potential pun - not too shabby.

edited Dec 26 '11 at 0:12

answered Dec 26 '11 at 0:03
jspecter
6,748 1 13 35

i get u,thanks a lot – Junior II Dec 26 '11 at 0:13

@JuniorII It would be nice of you to upvote and accept jspecter's answer since you found it helpful. – Alex Becker Dec 26 '11 at 5:39

There is a proof in the QUESTION here.

Note that There are twelve elements in A_4 :
$(1),\ (12)(34),\ (13)(24),\ (14)(23),\ (123),\ (132),\ (124),\ (142),\ (134),\ (143),\ (234),\ (243)$
.

edited Jan 11 '15 at 14:57

answered Jan 11 '15 at 13:05
L.G.
1,534 5 31

There are only two groups of order 6: the cyclic group of order 6, and a group isomorphic to S_3 . But the maximum order of permutations in S_4 is 4, which excludes a cyclic subgroup of order 6, and S_3 includes simple interchanges, which are not in A_4.

answered Mar 1 '15 at 21:07
Bill Kleinhans
1,192 4 7