1,054 8 18 review help

A_n is the only subgroup of S_n of index 2.

How to prove that the only subgroup of the symmetric group S_n of order $n!/2$ is A_n?

Why isn't there other possibility?

Thanks :)

(group-theory) (finite-groups) (permutations) (symmetric-groups)

edited Jan 29 '16 at 10:22 Babak S. 55.5k 6 38 122

asked Mar 14 '11 at 20:57 ShinyaSakai 2,671 15 44

4 Please make the body of your posts self-contained. The title is an indexing feature, and should not be an integral part of the message. Think of it as the title of a book on the spine; it's there to let people know what the post is about, not to impart information without which you cannot understand what is happening. – Mariano Suárez-Álvarez ♦ Mar 14 '11 at 21:00

I am terribly sorry. But I don't know how to reedit it. – ShinyaSakai Mar 14 '11 at 21:07

3 There should be a link below the [abstract algebra] tag that says "edit". Click it, and you can edit. – Arturo Magidin Mar 14 '11 at 21:11

2 Answers

As mentioned by yoyo: if $H \subset S_n$ is of index 2 then it is normal and S_n/H is isomorphic to $C_2 = \{1, -1\}$. We thus have a surjective homomorphism $f : S_n \to C_2$ with kernel H. All transpositions in S_n are conjugate, hence $f(t) \in C_2$ is the same element for every transposition $t \in S_n$ (this uses the fact that C_2 is commutative). S_n is generated by transpositions, therefore C_2 is generated by $f(t)$ (for any transposition $t \in S_n$), therefore $f(t) = -1$, therefore $\ker f = A_n$.

edited Sep 8 '14 at 21:14

answered Mar 14 '11 at 23:01 user8268 15k 1 19 38

Very detailed~ thank you very much~ – ShinyaSakai Mar 15 '11 at 21:28

S_n in the last line, not S_2. Very nice solution. – ReverseFlow Sep 7 '14 at 23:41

@Genomeme: thanks, corrected – user8268 Sep 8 '14 at 21:14

@user8268 Can you explain how $f(t) \in C_2$ is the same element for every transposition $t \in S_n$, and how S_n is generated by transpositions? Thank you. – jstnchng Nov 30 '14 at 18:49

subgroups of index two are normal (exercise). A_n is simple, $n \geq 5$ (exercise). if there were another subgroup H of index two, then $H \cap A_n$ would be normal in A_n, contradiction.

answered Mar 14 '11 at 21:08 yoyo 5,933 14 21

It is really a smart shortcut for the special case of $n \geq 5$~ Thank you very much~ – ShinyaSakai Mar 15 '11 at 21:30

3 How do you know that the intersection is not trivial? {1} is normal in every group and does not contradict simplicity. There is a way around this using conjugacy in S_n...... – Vladhagen Oct 30 '13 at 19:42