1-4 基本的算法结构

魏恒峰

hfwei@nju.edu.cn

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Longest Monotone Subsequence

ES 24.8: Longest Monotone Subsequence

Write a computer program that takes as its input a sequence of distinct integers and returns as its output the length of a longest monotone subsequence.

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Longest existence? uniqueness?

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Understanding this problem:

Subsequence	*vs.* substring	
Monotone	increasing *vs.* decreasing	strictly *vs.* non-strictly
Longest	existence? uniqueness?	
The Length	*vs.* the subsequence itself	

ES 24.8: Longest (Strictly) Increasing Subsequence (LIS)

- Given an integer array $A[0 \dots n-1]$
- To find the length L of an LIS

$$0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15 \implies 0, 2, 6, 9, 11, 15$$

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Theorem (Erdős-Szekeres Theorem)

Let n be a positive integer. Every sequence of $n^2 + 1$ distinct integers must contain a monotone subsequence of length $n + 1$.

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$P(i)$ 是什么?

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P(0) = 1;

for (int i = 1; i < n; ++i)
  P(i) = max_{0≤j<i, A[j]<A[i]} {P(j) + 1}

  return L = max_{0≤i<n} P(i);
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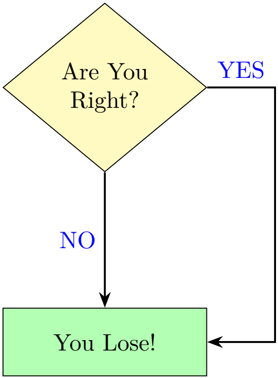
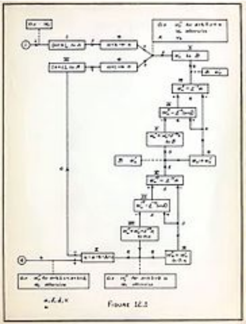
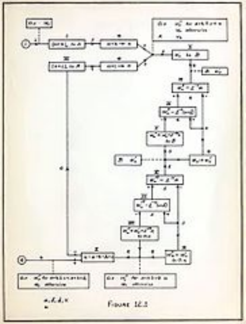
Flowcharts

How to Argue with Your Girlfriend?







We feel certain that a moderate amount of experience with this stage of coding suffices to remove from it all difficulties, and to make it a perfectly routine operation.

— John von Neumann and Herman Goldstine, late 1940s


我的内心几
乎是崩溃的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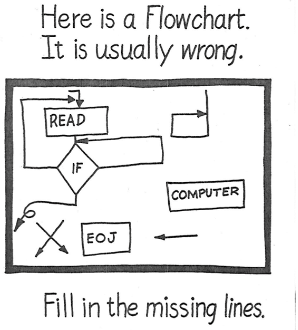
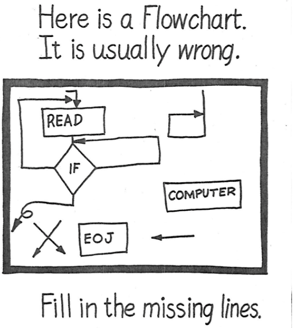
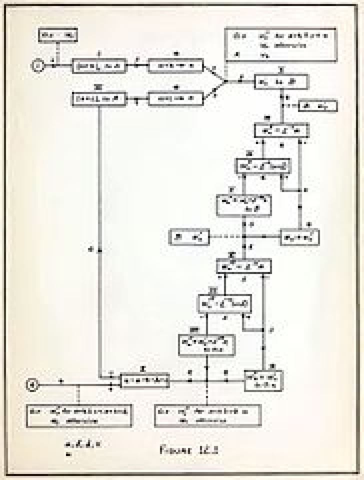

Here is a Flowchart.
It is usually wrong.
READ
IF
COMPUTER
EOJ
Fill in the missing lines.



Any resemblance between our flow charts and the present program is purely coincidental.

— Donald Knuth, 1963



Flowcharts Considered Harmful.

Just my opinion...

Just my opinion...

Draw it when it does help

Just my opinion...

Draw it when it does help

OR you have to.

Simulations

DH 2.5: Simulations

Show how to perform the following simulations of some control constructs by others.

(a) "`for-do`" by "`while-do`"

```
for (init; cond; inc)
    statement
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init;
while (cond)
  statement
  inc
```

Whether to use "while" or "for" is largely a matter of personal preference.

— *K&R C Bible*

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(b) "`if-then` & `if-then-else`" by "`while-do`"

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  B
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(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
```

```
flag = 1
while (A && flag)
  B
  flag = 0
```

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```

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  B
  flag = 0
```

```
if (A)
  B
else
  C
```

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  flag = 0
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if (A)
  B
else
  C
```

```
flag = 1
while (A && flag)
  B
  flag = 0
while (¬ A && flag)
  B
  flag = 0
```

DH 2.5: Simulations

Simulate the following control constructs by others.

(c) "`while-do`" by "`if-then & goto`"

(d) "`while-do`" by "`repeat-until & if-then`"

```
while (A)
  B
```

DH 2.5: Simulations

Simulate the following control constructs by others.

(c) "while-do" by "if-then & goto"

(d) "while-do" by "repeat-until & if-then"

```
while (A)
  B
```

```
L: if (A)
     B
     goto loop
```

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Simulate the following control constructs by others.

(c) "while-do" by "if-then & goto"

(d) "while-do" by "repeat-until & if-then"

```
while (A)
  B
```

```
L: if (A)
     B
     goto loop
```

```
if (A)
  repeat
    B
  until (¬ A)
```

DH 2.8: Simulations

Simulate "`while-do`" by "`if-then-else & recursive`".

```
while (A)
  B
```

DH 2.8: Simulations

Simulate "`while-do`" by "`if-then-else & recursive`".

```
while (A)
  B
```

```
simulateWhile() {
  if (A)
    B
    simulateWhile();

  return;
}
```

(1) `A;B`

(2) `if-then`

(3) `if-then-else`

(4) `for-do`

(5) `while-do`

(6) `repeat-until`

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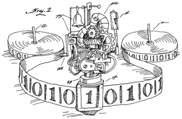
while (A)
  B
```

Theorem ("On Folk Theorems" (David Harel, 1980))

Any computable function can be computed by a "while-do" (and ";") program (with additional Boolean variables).



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Simulations

Bounded iteration vs. Unbounded iteration

DH 2.4: Bounded Iteration

Given a list L of N integers, to produce in S and P the sum of the even numbers in L and the product of the odd ones, respectively.

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```
int S = 0, P = 1;
for (int i = 0; i < N; ++i) {
  if (L(i) % 2 == 0)
    S += L(i);
  else
    P *= L(i);
}
```

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```

DH 2.1: Salary Summation

$N - 1$ vs. N iterations

DH 2.4: Bounded Iteration

Given a list L of N integers, to produce in S and P the sum of the even numbers in L and the product of the odd ones, respectively.

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DH 2.1: Salary Summation

$N - 1$ vs. N iterations

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Write algorithms that compute $n!$, given a non-negative integer n.

(a) Using iteration statements.

(b) Using recursion.

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(a) Using iteration statements.

(b) Using recursion.

```
int P = 1;
for (int i = 1; i <= n; ++i) {
  P *= i;
}
```

```
int recursive-factorial(int n) {
  if (n == 1)
    return 1;
  else n * recursive-factorial(n-1);
}
```


Thank
You!