1-4 基本的算法结构

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Longest Monotone Subsequence

ES 24.8： Longest Monotone Subsequence

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Monotone increasing *vs.* decreasing   strictly *vs.* non-strictly

Longest existence? uniqueness?

ES 24.8: Longest (Strictly) Increasing Subsequence (LIS)

- Given an integer array $A[0 \ldots n-1]$
- To find (the length L of) an LIS

$$0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15 \implies 0, 2, 6, 9, 11, 15$$

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Theorem (Erdős-Szekeres Theorem)

Let n be a positive integer. Every sequence of n^2+1 distinct integers must contain a monotone subsequence of length $n+1$.

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```
P(0) = 1;

for (int i = 1; i < n; ++i)
  P(i) = max_{0 ≤ j < i, A[j] < A[i]} {P(j) + 1}

return L = max_{0 ≤ i < n} P(i);
```

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$$\texttt{return } L = \max_{0 \le i < n} P(i);$$

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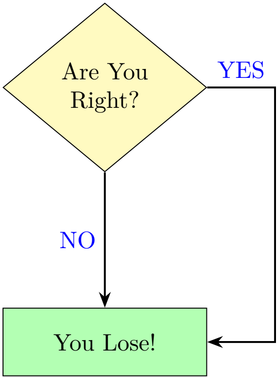
return $L = \max\limits_{0 \le i < n} P(i);$

Flowcharts

How to Argue with Your Girlfriend?



Simulations

DH 2.5: Simulations

Show how to perform the following simulations of some control constructs by others.

(a) "`for-do`" by "`while-do`"

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for (init; cond; inc)
  statement
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  statement
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init;
while (cond)
  statement
  inc
```

Whether to use "while" or "for" is largely a matter of personal preference.

— K&R C Bible

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(b) "`if-then` & `if-then-else`" by "`while-do`"

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(b) "`if-then` & `if-then-else`" by "`while-do`"

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```

```
flag = 1
while (A && flag)
  B
  flag = 0
```

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(b) "`if-then` & `if-then-else`" by "`while-do`"

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flag = 1
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```
if (A)
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  C
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flag = 1
while (A && flag)
  B
  flag = 0
```

```
if (A)
  B
else
  C
```

```
flag = 1
while (A && flag)
  B
  flag = 0
while (¬ A && flag)
  B
  flag = 0
```

DH 2.5: Simulations

Simulate the following control constructs by others.

(c) "`while-do`" by "`if-then & goto`"

(d) "`while-do`" by "`repeat-until & if-then`"

```
while (A)
  B
```

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Simulate the following control constructs by others.

(c) "while-do" by "if-then & goto"

(d) "while-do" by "repeat-until & if-then"

```
while (A)
    B
```

```
L: if (A)
      B
      goto loop
```

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Simulate the following control constructs by others.

(c) "while-do" by "if-then & goto"

(d) "while-do" by "repeat-until & if-then"

```
while (A)
  B
```

```
L: if (A)
      B
      goto loop
```

```
if (A)
  repeat
    B
  until (¬ A)
```

DH 2.8: Simulations

Simulate "`while-do`" by "`if-then-else & recursive`".

```
while (A)
  B
```

DH 2.8: Simulations

Simulate “`while-do`” by “`if-then-else & recursive`”.

```
while (A)
  B
```

```
simulateWhile() {
  if (A)
    B
    simulateWhile();

  return;
}
```

(1) `A;B`
(2) `if-then`
(3) `if-then-else`
(4) `for-do`
(5) `while-do`
(6) `repeat-until`

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  B
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  B
until (¬ A)
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while (A)
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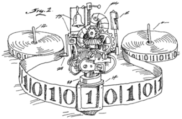
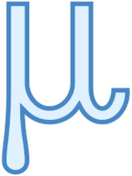
```

Theorem ("On Folk Theorems" (David Harel, 1980))

Any computable function can be computed by a "while-do" (and ";") program (with additional Boolean variables).





Simulations

Bounded iteration vs. Unbounded iteration

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Given a list L of N integers, to produce in S and P the sum of the even numbers in L and the product of the odd ones, respectively.

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```
int S = 0, P = 1;
for (int i = 0; i < N; ++i) {
  if (L(i) % 2 == 0)
    S += L(i);
  else
    P *= L(i);
}
```

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DH 2.1: Salary Summation

$N - 1$ vs. N iterations

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$N-1$ vs. N iterations

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Write algorithms that compute $n!$, given a non-negative integer n.

(a) Using iteration statements.

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```
int P = 1;
for (int i = 1; i <= n; ++i) {
  P *= i;
}
```

```
int recursive-factorial(int n) {
  if (n == 1)
    return 1;
  else n * recursive-factorial(n-1);
}
```


Thank
You!