NP-intermediate

In computational complexity, problems that are in the complexity class NP but are neither in the class P nor NP-complete are called **NP-intermediate**, and the class of such problems is called **NPI**. **Ladner's theorem**, shown in 1975 by Richard E. Ladner,[1] is a result asserting that, if P ≠ NP, then NPI is not empty; that is, NP contains problems that are neither in P nor NP-complete. Since the other direction is trivial, it follows that P = NP if and only if NPI is empty.

Under the assumption that P ≠ NP, Ladner explicitly constructs a problem in NPI, although this problem is artificial and otherwise uninteresting. It is an open question whether any "natural" problem has the same property: Schaefer's dichotomy theorem provides conditions under which classes of constrained Boolean satisfiability problems can not be in NPI.[2] Some problems that are considered good candidates for being NP-intermediate are the graph isomorphism problem, factoring, and computing the discrete logarithm.[3]

1 List of problems that might be NP-intermediate[4]

1.1 Algebra and number theory

- Factoring integers
- Discrete Log Problem and others related to cryptographic assumptions
- Isomorphism problems: Group isomorphism problem, Group automorphism, Ring isomorphism, Ring automorphism
- Determining the result of a comparison between two sums of square roots of integers[5]
- Numbers in boxes problems[6]
- The linear divisibility problem[7]

1.2 Boolean logic

- Intersecting Monotone SAT[8]
- Minimum Circuit Size Problem[9][10]
- Monotone self-duality[11]

1.3 Computational geometry and computational topology

- Computing the rotation distance[12] between two binary trees or the flip distance between two triangulations of the same convex polygon
- The turnpike problem[13] of reconstructing points on line from their distance multiset
- The cutting stock problem with a constant number of object lengths[14]
- Knot triviality[15]
- Deciding whether a given triangulated 3-manifold is a 3-sphere
- Gap version of the closest vector in lattice problem[16]
- Finding a simple closed quasigeodesic on a convex polyhedron[17]

1.4 Game theory

- Determining winner in parity games[18]
- Determining who has the highest chance of winning a stochastic game[18]
- Agenda control for balanced single-elimination tournaments[19]

1.5 Graph algorithms

- Graph isomorphism problem
- Planar minimum bisection[20]
- Deciding whether a graph admits a graceful labeling[21]
- Clustered planarity[22]
- Recognizing leaf powers and k-leaf powers[23]
- Recognizing graphs of bounded clique-width[24]
- Finding a simultaneous embedding with fixed edges[25]

1.6 Miscellaneous

- Assuming NEXP is not equal to EXP, padded versions of NEXP-complete problems
- Problems in TFNP[26]
- Pigeonhole subset sum[27]
- Finding the VC dimension[28]

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3 External links

- *Complexity Zoo*: Class NPI
- Basic structure, Turing reducibility and NP-hardness

- Lance Fortnow (24 March 2003). "Foundations of Complexity, Lesson 16: Ladner's Theorem". Retrieved 1 November 2013.

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