Rotational Symmetries

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$C \cong S_4$

- Order of 1: id $(\# = 1)$
- Order of 4: face-to-face

$$
\begin{aligned}
f_{td} &= (1\ 2\ 3\ 4) & f_{td}^2 &= (1\ 3)(2\ 4) & f_{td}^3 &= (1\ 4\ 3\ 2) \\
f_{lr} &= (1\ 2\ 4\ 3) & f_{lr}^2 &= (1\ 4)(2\ 3) & f_{lr}^3 &= (1\ 3\ 4\ 2) \\
f_{fb} &= (1\ 4\ 2\ 3) & f_{fb}^2 &= (1\ 2)(3\ 4) & f_{fb}^3 &= (1\ 3\ 2\ 4)
\end{aligned}
$$

$C \cong S_4$

- Order of 3: vertex-to-vertex

$$
\begin{aligned}
v_1 &= (2\ 3\ 4) & v_1^2 &= (2\ 4\ 3) \\
v_2 &= (1\ 4\ 3) & v_2^2 &= (1\ 3\ 4) \\
v_3 &= (1\ 2\ 4) & v_3^2 &= (1\ 4\ 2) \\
v_4 &= (1\ 2\ 3) & v_4^2 &= (1\ 3\ 2)
\end{aligned}
$$

- Order of 2: edge-to-edge

$$
\begin{aligned}
e_{12} &= (1\ 2) & e_{13} &= (1\ 3) & e_{14} &= (1\ 4) \\
e_{23} &= (2\ 3) & e_{24} &= (2\ 4) & e_{34} &= (3\ 4)
\end{aligned}
$$

Subgroups of S_4

Order of 6:

$$H \cong S_3 = \{1, r, r^2, s, rs, r^2 s\}$$

Subgroups of S_4

Order of 8:

$$H \cong D_4 = \{1, r, r^2, r^3, s, rs, r^2 s, r^3 s\}$$

Subgroups of S_4

Order of 12:

$$H \cong A_4$$

Theorem

There is only one subgroup of order 12 in S_4.

Proof.

$\square$