Group Homomorphism

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Order of 4

Order of 6

Order of 8 (TJ 9.11)

Group Homomorphism

$D_6 \cong D_3 \times \mathbb{Z}_2$ (TJ 9.16)

(TJ 9.23)

$$G \times K \cong H \times K \;\not\Longrightarrow\; G \cong H$$

$$G = \mathbb{Z}, \quad H = 1, \quad K = \prod_{n \in \mathbb{N}} \mathbb{Z}$$

"On Cancellation in Groups" by R. Hirshon, 1969

$$G \times K \cong H \times K \quad |K| < \infty \implies G \cong H$$

(TJ 11.18)

- $\phi : G_1 \to G_2$
- $H_1 \lhd G_1$
- $\phi(H_1) = H_2$
- $G_1/H_1 \cong G_2/H_2$

$$G_1 = \mathbb{Z}_2 \quad G_2 = \{e\} \quad H_1 = \{0\} \quad H_2 = \{e\}$$

(TJ 11.5)

$$\phi : \mathbb{Z}_{24} \to \mathbb{Z}_{18}$$

$$\begin{aligned} \phi(1) = a &\implies \phi(x) = ax \pmod{18} \\ &\implies Ker(\phi) = \mathbb{Z}_b \cap \mathbb{Z}_{24} : ab \equiv 0 \pmod{18} \end{aligned}$$

$$\phi(1) = ?$$

$$|\phi(1)| | 18 \wedge |\phi(1)| | 24 \implies |\phi(1)| | 6$$

$$\phi(1) = 0, 9, 6, 12, 3, 15$$

$$\phi_3(x) = 6x, \quad Ker(\phi_3) = 3\mathbb{Z}_{24}$$

Normal subgroups

$$\mathbb{Z}/n\mathbb{Z}$$

$$(aH)(bH) = (abH)$$

$$\forall a, b \in G, \forall h_1, h_2 \in H, (ah_1) \in aH, (bh_2) \in bH : (ah_1)(bh_2) \in abH$$

$$\exists h_3 \in H, (ah_1)(bh_2) = abh_3 \iff h_1 b = bh_3 h_2^{-1}$$

$$\forall b \in G, \forall h_1 \in H : \exists h' \in H : h_1 b = bh'$$

$$\forall g \in G, \forall h \in H : \exists h' \in H : hg = gh'$$

Normal subgroups

$$\forall g \in G, \forall h \in H : \exists h' \in H : hg = gh'$$

$$\forall g \in G, Hg \subseteq gH$$

$$\forall g^{-1} \in G, Hg^{-1} \subseteq g^{-1}H$$

$$\forall h \in H, \exists h' \in H : hg^{-1} = g^{-1}h' \iff gh = h'g$$

$$\forall g \in G, Hg \subseteq gH$$

$$\forall g \in G, Hg = gH$$

(TJ 10.13)

$$g \in G, C(g) = \{x \in G : gx = xg\}$$

$$Z(G) = \{x \in G : gx = xg, \forall g \in G\} \implies Z(G) \lhd G$$

$$C(g) \leq G$$

$$\langle g \rangle \lhd G \implies C(g) \lhd G$$

Proof.

$$\forall k \in G, x \in C(g) : k^{-1}xk \in C(g)$$

$$\forall k \in G, x \in C(g) : gk^{-1}xk = k^{-1}xkg$$

$$\langle g \rangle \lhd G \implies k\langle g \rangle = \langle g \rangle k \implies \exists t, gk = kg^t \iff gk^{-1} = k^{-1}g^t$$