



## Why is PH in PSPACE?

$PH \subseteq PSPACE$ .

In order to prove it, one has to show that for a language  $A \in \Sigma_k$  (for some  $k \in \mathbb{N}$ ) there exists a turing machine  $M_A$  that decides it in polynomial space.

I am having a hard time understanding why this is true. I'll try to explain:

Let's take for example  $L \in \Sigma_2$ : by definition,  $x \in L \iff \exists y_1 \forall y_2 : V(y_1, y_2) = 1$ . But finding such  $y_1$  and checking it against all possible  $y_2$ s has to take more than polynomial space! The machine has to remember each  $y_2$  that it tried in order to make sure that it indeed tried out all of the options, and the number of possible  $y_2$ s is exponential, which makes it exponential space.

[complexity-theory](#)

[complexity-classes](#)

[space-complexity](#)

asked Aug 3 '16 at 12:06



[ranys](#)

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### 1 Answer

No, it is not necessary to remember all y's tried before. In order to remember that I've tried the numbers  $1, 2, \dots, 200$ , I do not need to remember  $3, 4, 5, 6, \dots, 199$ . If you try them in order, just remembering the last one is enough.

answered Aug 3 '16 at 12:15



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