Propositional logic problem about truth tellers and liars

I'm not sure if I'm right about this, so I'd appreciate a little guidance.

> In a remote village, there exists two types of people
>
> - Truth tellers who will always tell the truth
> - Liars who will always lie
>
> One day, a stranger visited the village. He met two of the inhabitants, Jack and Emily. The stranger asked them: "Is anyone of you a liar?". Jack replied: "At least one of us is a liar."
>
> What are Jack and Emily? Truth tellers or liars?

I know I can get the answer with the help of truth tables, but I'm not sure what the intial formula should be for Jack's reply. What I came up with for "At least one of us is a liar" was:

J: Jack is a truth teller

E: Emily is a truth teller

$$\neg(J \vee E)$$

Would you agree?

(discrete-mathematics)

asked Sep 27 '16 at 22:50

Steve
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More simply...If Jack is a liar then his statement is True, a contradiction! Hence he must be a truth teller. But then... – lulu Sep 27 '16 at 22:55

your inititial formula is wrong (you don't mention that Jack is the one who is speaking) the right formula is J <-> ~(J v E) – Willemien Sep 27 '16 at 22:58

Are you sure? That formula translated would be something in the region of "Jack is a truth teller if and only if Jack or Emily are lying." @Willemien – Steve Sep 27 '16 at 23:08

1 @Steve the formula ~(J v E) does not mean "Jack or Emily are lying", it's saying "it is not the case that at least one of them is a truth teller", which is equivalent to "both of them are lying" and so the formula written by Willemien is saying "Jack is a truth teller if and only if Jack and Emily are both liers", wich is a contradiction (anyway that formula doest't seem to help) – la flaca Sep 27 '16 at 23:33

@Eliana and Steve, sorry my fault the right translation is J <-> (~J v ~E) (and this is equivalent to J & ~E) (I just was thinking it must start with J <-> and that bit was missing) just start this way and then go simplify ing – Willemien Sep 28 '16 at 18:34

1 Answer

I don't know if it was a typo and you tryed to write $\neg(J \wedge E)$ or your intention was the formula you wrote, since it is equivalent to $\neg J \wedge \neg E$ (both are liars). I will assume it was a typo and in that case (as Willemien pointed out) your formula $\neg(J \wedge E)$ is a translation of what Jack said but in principle it could be false (he could be a liar). If it were false then Jack would be a liar so (as lulu pointed out) what he said would be true and that's a contradiction and because of that it must be the case that Jack is not a liar and that validates your initial formula, now you can add the formula

$$J$$

to derive from them

$$\neg(J \wedge E) \wedge J$$

and from that conclude

$$\neg E$$

So Jack is a truth teller and Emily is a liar

edited Sep 28 '16 at 0:59

answered Sep 27 '16 at 23:24

la flaca
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