The symmetric group on four letters, $S_4$, contains the following permutations:

| permutations | type |
|---|---|
| (12), (13), (14), (23), (24), (34) | 2-cycles |
| (12)(34), (13)(24), (14)(23) | product of 2-cycles |
| (123), (124), (132), (134), (142), (143), (234), (243) | 3-cycles |
| (1234), (1243), (1324), (1342), (1423), (1432) | 4-cycles |

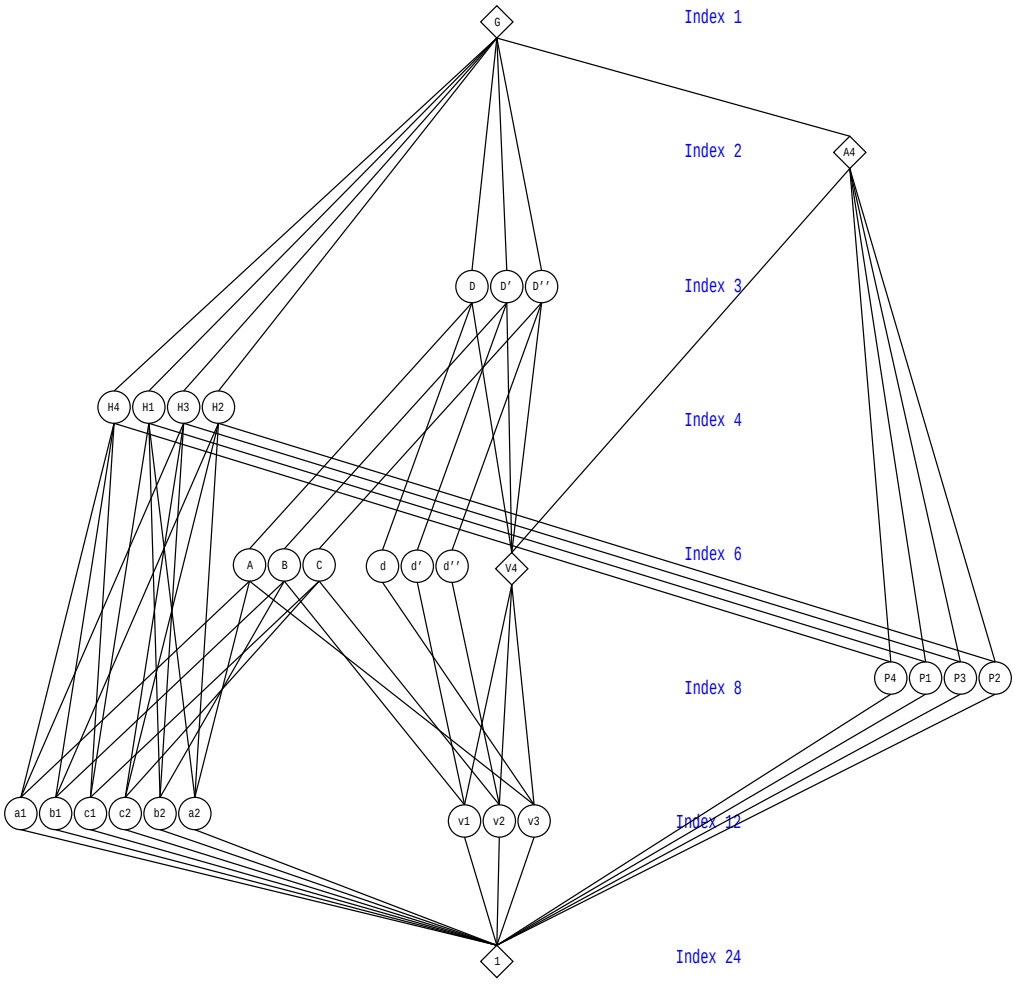


FIGURE 1. Hasse diagram of Sub[$S_4$] drawn by the XGAP program.

There are 30 subgroups of $S_4$, which are displayed in Figure 1. Except for $(e)$ and $S_4$, their elements are given in the following table:

| label | elements | order | isomorphic to |
|---|---|---|---|
| $A_4$ | $\{e, (12)(34), (13)(24), (14)(23), (123), (124), \ldots$ $\ldots, (132), (134), (142), (143), (234), (243)\}$ | 12 | $A_4$ |
| $V_4$ | $\{e, (12)(34), (13)(24), (14)(23)\}$ | 4 | $V_4$ |
| $v_1$, $v_2$, $v_3$ | $\{e, (12)(34)\},\ \{e, (13)(24)\},\ \{e, (14)(23)\}$ | 2, 2, 2 | $Z_2$ |
| $P_1$ | $\{e, (123), (132)\}$ | 3 | $Z_3$ |
| $P_2$ | $\{e, (124), (142)\}$ | 3 | $Z_3$ |
| $P_3$ | $\{e, (134), (143)\}$ | 3 | $Z_3$ |
| $P_4$ | $\{e, (234), (243)\}$ | 3 | $Z_3$ |
| $D$ | $\{e, (12), (12)(34), (13)(24), (14)(23), (34), (1324), (1423)\}$ | 8 | $D_4$ |
| $d$ | $\{e, (12)(34), (1324), (1423)\}$ | 4 | $Z_4$ |
| $D'$ | $\{e, (12)(34), (13), (13)(24), (14)(23), (24), (1234), (1432)\}$ | 8 | $D_4$ |
| $d'$ | $\{e, (13)(24), (1234), (1432)\}$ | 4 | $Z_4$ |
| $D''$ | $\{e, (12)(34), (13)(24), (14), (14)(23), (23), (1243), (1342)\}$ | 8 | $D_4$ |
| $d''$ | $\{e, (14)(23), (1243), (1342)\}$ | 4 | $Z_4$ |
| $H_1$ | $\{e, (12), (13), (23), (123), (132)\}$ | 6 | $S_3$ |
| $H_2$ | $\{e, (12), (14), (24), (124), (142)\}$ | 6 | $S_3$ |
| $H_3$ | $\{e, (13), (14), (34), (134), (143)\}$ | 6 | $S_3$ |
| $H_4$ | $\{e, (23), (24), (34), (234), (243)\}$ | 6 | $S_3$ |
| $A$ | $\{e, (12), (12)(34), (34)\}$ | 4 | $V_4$ |
| $a_1$, $a_2$ | $\{e, (12)\},\ \{e, (34)\}$ | 2, 2 | $Z_2$ |
| $B$ | $\{e, (13), (13)(24), (24)\}$ | 4 | $V_4$ |
| $b_1$, $b_2$ | $\{e, (13)\},\ \{e, (24)\}$ | 2, 2 | $Z_2$ |
| $C$ | $\{e, (14), (14)(23), (23)\}$ | 4 | $V_4$ |
| $c_2$, $c_1$ | $\{e, (14)\},\ \{e, (23)\}$ | 2, 2 | $Z_2$ |