Georg Cantor

Georg Ferdinand Ludwig Philipp Cantor (/ˈkæntɔːr/ *KAN-tor*; German: [ˈɡeːɔʁk ˈfɛʁdinant ˈluːtvɪç ˈfiːlɪp ˈkantɔʁ]; March 3 [O.S. February 19] 1845 – January 6, 1918[1]) was a German mathematician. He invented set theory, which has become a fundamental theory in mathematics. Cantor established the importance of one-to-one correspondence between the members of two sets, defined infinite and well-ordered sets, and proved that the real numbers are more numerous than the natural numbers. In fact, Cantor's method of proof of this theorem implies the existence of an "infinity of infinities". He defined the cardinal and ordinal numbers and their arithmetic. Cantor's work is of great philosophical interest, a fact of which he was well aware.[2]

Cantor's theory of transfinite numbers was originally regarded as so counter-intuitive – even shocking – that it encountered resistance from mathematical contemporaries such as Leopold Kronecker and Henri Poincaré[3] and later from Hermann Weyl and L. E. J. Brouwer, while Ludwig Wittgenstein raised philosophical objections. Cantor, a devout Lutheran,[4] believed the theory had been communicated to him by God.[5] Some Christian theologians (particularly neo-Scholastics) saw Cantor's work as a challenge to the uniqueness of the absolute infinity in the nature of God[6] – on one occasion equating the theory of transfinite numbers with pantheism[7] – a proposition that Cantor vigorously rejected.

The objections to Cantor's work were occasionally fierce: Henri Poincaré referred to his ideas as a "grave disease" infecting the discipline of mathematics,[8] and Leopold Kronecker's public opposition and personal attacks included describing Cantor as a "scientific charlatan", a "renegade" and a "corrupter of youth."[9] Kronecker objected to Cantor's proofs that the algebraic numbers are countable, and that the transcendental numbers are uncountable, results now included in a standard mathematics curriculum. Writing decades after Cantor's death, Wittgenstein lamented that mathematics is "ridden through and through with the pernicious idioms of set theory", which he dismissed as "utter nonsense" that is "laughable" and "wrong".[10] Cantor's recurring bouts of depression from 1884 to the end of his life have been blamed on the hostile attitude of many of his contemporaries,[11] though some have explained these episodes as probable manifestations of a bipolar disorder.[12]

The harsh criticism has been matched by later accolades. In 1904, the Royal Society awarded Cantor its Sylvester Medal, the highest honor it can confer for work in mathematics.[13] David Hilbert defended it from its critics by declaring:[14][15]

> From his paradise that Cantor with us unfolded, we hold our breath in awe; knowing, we shall not be expelled.

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Born	Georg Ferdinand Ludwig Philipp Cantor March 3, 1845 Saint Petersburg, Russian Empire
Died	January 6, 1918 (aged 72) Halle, Province of Saxony, German Empire
Residence	Russian Empire (1845–56) German Empire (1856–1918)
Nationality	German
Alma mater	Swiss Federal Polytechnic University of Berlin
Known for	Set theory
Spouse(s)	Vally Guttmann (1874-1918)
Awards	Sylvester Medal (1904)
Scientific career	
Fields	Mathematics
Institutions	University of Halle
Thesis	*De aequationibus*

	secundi gradus indeterminatis (http://gdz.sub.uni-goettingen.de/index.php?id=11&PPN=PPN237853094&DMDID=DMDLOG_0008&L=1) (1867)
Doctoral advisor	Ernst Kummer Karl Weierstrass

Life of Georg Cantor

Youth and studies

Cantor, around 1870

Georg Cantor was born in 1845 in the western merchant colony of Saint Petersburg, Russia, and brought up in the city until he was eleven. Georg, the oldest of six children, was regarded as an outstanding violinist. His grandfather Franz Böhm (1788–1846) (the violinist Joseph Böhm's brother) was a well-known musician and soloist in a Russian imperial orchestra.[16] Cantor's father had been a member of the Saint Petersburg stock exchange; when he became ill, the family moved to Germany in 1856, first to Wiesbaden, then to Frankfurt, seeking milder winters than those of Saint Petersburg. In 1860, Cantor graduated with distinction from the Realschule in Darmstadt; his exceptional skills in mathematics, trigonometry in particular, were noted. In 1862, Cantor entered the Swiss Federal Polytechnic. After receiving a substantial inheritance upon his father's death in June 1863,[17] Cantor shifted his studies to the University of Berlin, attending lectures by Leopold Kronecker, Karl Weierstrass and Ernst Kummer. He spent the summer of 1866 at the University of Göttingen, then and later a center for mathematical research. Georg Cantor was a good student, so in 1867, he received his doctorate degree.[17][18]

Teacher and researcher

Cantor submitted his dissertation on number theory at the University of Berlin in 1867. After teaching briefly in a Berlin girls' school, Cantor took up a position at the University of Halle, where he spent his entire career. He was awarded the requisite habilitation for his thesis, also on number theory, which he presented in 1869 upon his appointment at Halle University.[18][19]

In 1874, Cantor married Vally Guttmann. They had six children, the last (Rudolph) born in 1886. Cantor was able to support a family despite modest academic pay, thanks to his inheritance from his father. During his honeymoon in the Harz mountains, Cantor spent much time in mathematical discussions with Richard Dedekind, whom he had met two years earlier while on Swiss holiday.

Cantor was promoted to Extraordinary Professor in 1872 and made full Professor in 1879.[18][17] To attain the latter rank at the age of 34 was a notable accomplishment, but Cantor desired a chair at a more prestigious university, in particular at Berlin, at that time the leading German university. However, his work encountered too much opposition for that to be possible.[20] Kronecker, who headed mathematics at Berlin until his death in 1891, became increasingly uncomfortable with the prospect of having Cantor as a colleague,[21] perceiving him as a "corrupter of youth" for teaching his ideas to a younger generation of mathematicians.[22] Worse yet, Kronecker, a well-established figure within the mathematical community and Cantor's former professor, disagreed fundamentally with the thrust of Cantor's work ever since he intentionally delayed the publication of Cantor's first major publication in 1874.[18] Kronecker, now seen as one of the founders of the constructive viewpoint in mathematics, disliked much of Cantor's set theory because it asserted the existence of sets satisfying certain properties, without giving specific examples of sets whose members did indeed satisfy those properties. Whenever Cantor applied for a post in Berlin, he was declined, and it usually involved Kronecker,[18] so Cantor came to believe that Kronecker's stance would make it impossible for him ever to leave Halle.

In 1881, Cantor's Halle colleague Eduard Heine died, creating a vacant chair. Halle accepted Cantor's suggestion that it be offered to Dedekind, Heinrich M. Weber and Franz Mertens, in that order, but each declined the chair after being offered it. Friedrich Wangerin was eventually appointed, but he was never close to Cantor.

In 1882, the mathematical correspondence between Cantor and Richard Dedekind came to an end, apparently as a result of Dedekind's declining the chair at Halle.[23] Cantor also began another important correspondence, with Gösta Mittag-Leffler in Sweden, and soon began to publish in Mittag-Leffler's journal *Acta Mathematica*. But in 1885, Mittag-Leffler was concerned about the philosophical nature and new terminology in a paper Cantor had submitted to *Acta*.[24] He asked Cantor to withdraw the paper from *Acta* while it was in proof, writing that it was "... about one hundred years too soon." Cantor complied, but then curtailed his relationship and correspondence with Mittag-Leffler, writing to a third party:

> Had Mittag-Leffler had his way, I should have to wait until the year 1984, which to me seemed too great a demand! ... But of course I never want to know anything again about *Acta Mathematica*.[25]

Cantor suffered his first known bout of depression on May 1884.[17][26] Criticism of his work weighed on his mind: every one of the fifty-two letters he wrote to Mittag-Leffler in 1884 mentioned Kronecker. A passage from one of these letters is revealing of the damage to Cantor's self-confidence:

> ... I don't know when I shall return to the continuation of my scientific work. At the moment I can do absolutely nothing with it, and limit myself to the most necessary duty of my lectures; how much happier I would be to be scientifically active, if only I had the necessary mental freshness.[27]

This crisis led him to apply to lecture on philosophy rather than mathematics. He also began an intense study of Elizabethan literature thinking there might be evidence that Francis Bacon wrote the plays attributed to Shakespeare (see Shakespearean authorship question); this ultimately resulted in two pamphlets, published in 1896 and 1897.[28]

Cantor recovered soon thereafter, and subsequently made further important contributions, including his diagonal argument and theorem. However, he never again attained the high level of his remarkable papers of 1874–84, even after Kronecker's death on December 29, 1891.[18] He eventually sought, and achieved, a reconciliation with Kronecker. Nevertheless, the philosophical disagreements and difficulties dividing them persisted.

In 1889, Cantor was instrumental in founding the German Mathematical Society[18] and chaired its first meeting in Halle in 1891, where he first introduced his diagonal argument; his reputation was strong enough, despite Kronecker's opposition to his work, to ensure he was elected as the first president of this society. Setting aside the animosity Kronecker had displayed towards him, Cantor invited him to address the meeting, but Kronecker was unable to do so because his wife was dying from injuries sustained in a skiing accident at the time. Georg Cantor was also instrumental in the establishment of the first International Congress of Mathematicians, which was held in Zürich, Switzerland, in 1897.[18]

Later years and death

After Cantor's 1884 hospitalization, there is no record that he was in any sanatorium again until 1899.[26] Soon after that second hospitalization, Cantor's youngest son Rudolph died suddenly on December 16 (Cantor was delivering a lecture on his views on Baconian theory and William Shakespeare), and this tragedy drained Cantor of much of his passion for mathematics.[29] Cantor was again hospitalized in 1903. One year later, he was outraged and agitated by a paper presented by Julius König at the Third International Congress of Mathematicians. The paper attempted to prove that the basic tenets of transfinite set theory were false. Since the paper had been read in front of his daughters and colleagues, Cantor perceived himself as having been publicly humiliated.[30] Although Ernst Zermelo demonstrated less than a day later that König's proof had failed, Cantor remained shaken, and momentarily questioning God.[13] Cantor suffered from chronic depression for the rest of his life, for which he was excused from teaching on several occasions and repeatedly confined in various sanatoria. The events of 1904 preceded a series of hospitalizations at intervals of two or three years.[31] He did not abandon mathematics completely, however, lecturing on the paradoxes of set theory (Burali-Forti paradox, Cantor's paradox, and Russell's paradox) to a meeting of the *Deutsche Mathematiker–Vereinigung* in 1903, and attending the International Congress of Mathematicians at Heidelberg in 1904.

In 1911, Cantor was one of the distinguished foreign scholars invited to attend the 500th anniversary of the founding of the University of St. Andrews in Scotland. Cantor attended, hoping to meet Bertrand Russell, whose newly published *Principia Mathematica* repeatedly cited Cantor's work, but this did not come about. The following year, St. Andrews awarded Cantor an honorary doctorate, but illness precluded his receiving the degree in person.

Cantor retired in 1913, living in poverty and suffering from malnourishment during World War I.[32] The public celebration of his 70th birthday was canceled because of the war. In June 1917, he entered a sanatorium for the last time and continually wrote to his wife asking to be allowed to go home. Georg Cantor had a fatal heart attack on January 6, 1918, in the sanatorium where he had spent the last year of his life.[17]

Mathematical work

Cantor's work between 1874 and 1884 is the origin of set theory.[33] Prior to this work, the concept of a set was a rather elementary one that had been used implicitly since the beginning of mathematics, dating back to the ideas of Aristotle. No one had realized that set theory had any nontrivial content. Before Cantor, there were only finite sets (which are easy to understand) and "the infinite" (which was considered a topic for philosophical, rather than mathematical, discussion). By proving that there are (infinitely) many possible sizes for infinite sets, Cantor established that set theory was not trivial, and it needed to be studied. Set theory has come to play the role of a foundational theory in modern mathematics, in the sense that it interprets propositions about mathematical objects (for example, numbers and functions) from all the traditional areas of mathematics (such as algebra, analysis and topology) in a single theory, and provides a standard set of axioms to prove or disprove them. The basic concepts of set theory are now used throughout mathematics.[34]

In one of his earliest papers,[35] Cantor proved that the set of real numbers is "more numerous" than the set of natural numbers; this showed, for the first time, that there exist infinite sets of different sizes. He was also the first to appreciate the importance of one-to-one correspondences (hereinafter denoted "1-to-1 correspondence") in set theory. He used this concept to define finite and infinite sets, subdividing the latter into denumerable (or countably infinite) sets and uncountable sets (nondenumerable infinite sets).[36]

Cantor developed important concepts in topology and their relation to cardinality. For example, he showed that the Cantor set is nowhere dense, but has the same cardinality as the set of all real numbers, whereas the rationals are everywhere dense, but countable. He also showed that all countable dense linear orders without end points are order-isomorphic to the rational numbers.

Cantor introduced fundamental constructions in set theory, such as the power set of a set A, which is the set of all possible subsets of A. He later proved that the size of the power set of A is strictly larger than the size of A, even when A is an infinite set; this result soon became known as Cantor's theorem. Cantor developed an entire theory and arithmetic of infinite sets, called cardinals and ordinals, which extended the arithmetic of the natural numbers. His notation for the cardinal numbers was the Hebrew letter ℵ (aleph) with a natural number subscript; for the ordinals he employed the Greek letter ω (omega). This notation is still in use today.

The *Continuum hypothesis*, introduced by Cantor, was presented by David Hilbert as the first of his twenty-three open problems in his address at the 1900 International Congress of Mathematicians in Paris. Cantor's work also attracted favorable notice beyond Hilbert's celebrated encomium.[15] The US philosopher Charles Sanders Peirce praised Cantor's set theory, and, following public lectures delivered by Cantor at the first International Congress of Mathematicians, held in Zurich in 1897, Hurwitz and Hadamard also both expressed their admiration. At that Congress, Cantor renewed his friendship and correspondence with Dedekind. From 1905, Cantor corresponded with his British admirer and translator Philip Jourdain on the history of set theory and on Cantor's religious ideas. This was later published, as were several of his expository works.

Number theory, trigonometric series and ordinals

Cantor's first ten papers were on number theory, his thesis topic. At the suggestion of Eduard Heine, the Professor at Halle, Cantor turned to analysis. Heine proposed that Cantor solve an open problem that had eluded Peter Gustav Lejeune Dirichlet, Rudolf Lipschitz, Bernhard Riemann, and Heine himself: the uniqueness of the representation of a function by trigonometric series. Cantor solved this difficult problem in 1869. It was while working on this problem that he discovered transfinite ordinals, which occurred as indices n in the nth derived set S_n of a set S of zeros of a trigonometric series. Given a trigonometric series f(x) with S as its set of zeros, Cantor had discovered a procedure that produced another trigonometric series that had S_1 as its set of zeros, where S_1 is the set of limit points of S. If S_{k+1} is the set of limit points of S_k, then he could construct a trigonometric series whose zeros are S_{k+1}. Because the sets S_k were closed, they contained their Limit points, and the intersection of the infinite decreasing sequence of sets S, S_1, S_2, S_3,... formed a limit set, which we would now call S_ω, and then he noticed that S_ω would also have to have a set of limit points $S_{\omega+1}$, and so on. He had examples that went on forever, and so here was a naturally occurring infinite sequence of infinite numbers ω, $\omega + 1$, $\omega + 2$, ...[37]

Between 1870 and 1872, Cantor published more papers on trigonometric series, and also a paper defining irrational numbers as convergent sequences of rational numbers. Dedekind, whom Cantor befriended in 1872, cited this paper later that year, in the paper where he first set out his celebrated definition of real numbers by Dedekind cuts. While extending the notion of number by means of his revolutionary concept of infinite cardinality, Cantor was paradoxically opposed to theories of infinitesimals of his contemporaries Otto Stolz and Paul du Bois-Reymond, describing them as both "an abomination" and "a cholera bacillus of mathematics".[38] Cantor also published an erroneous "proof" of the inconsistency of infinitesimals.[39]

Set theory

The beginning of set theory as a branch of mathematics is often marked by the publication of Cantor's 1874 paper,[33] "Ueber eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen" ("On a Property of the Collection of All Real Algebraic Numbers").[41] This paper was the first to provide a rigorous proof that there was more than one kind of infinity. Previously, all infinite collections had been implicitly assumed to be equinumerous (that is, of "the same size" or having the same number of elements).[42] Cantor proved that the collection of real numbers and the collection of positive integers are not equinumerous. In other words, the real numbers are not countable. His proof differs from diagonal argument that he gave in 1891.[43] Cantor's article also contains a new method of constructing transcendental numbers. Transcendental numbers were first constructed by Joseph Liouville in 1844.[44]

Cantor established these results using two constructions. His first construction shows how to write the real algebraic numbers[45] as a sequence a_1, a_2, a_3, ... In other words, the real algebraic numbers are countable. Cantor starts his second construction with any sequence of real numbers. Using this sequence, he constructs nested intervals whose intersection contains a real number not in

An illustration of Cantor's diagonal argument for the existence of uncountable sets.[40] The sequence at the bottom cannot occur anywhere in the infinite list of sequences above.

the sequence. Since every sequence of real numbers can be used to construct a real not in the sequence, the real numbers cannot be written as a sequence – that is, the real numbers are not countable. By applying his construction to the sequence of real algebraic numbers, Cantor produces a transcendental number. Cantor points out that his constructions prove more – namely, they provide a new proof of Liouville's theorem: Every interval contains infinitely many transcendental numbers.[46] Cantor's next article contains a construction that proves the set of transcendental numbers has the same "power" (see below) as the set of real numbers.[47]

Between 1879 and 1884, Cantor published a series of six articles in *Mathematische Annalen* that together formed an introduction to his set theory. At the same time, there was growing opposition to Cantor's ideas, led by Kronecker, who admitted mathematical concepts only if they could be constructed in a finite number of steps from the natural numbers, which he took as intuitively given. For Kronecker, Cantor's hierarchy of infinities was inadmissible, since accepting the concept of actual infinity would open the door to paradoxes which would challenge the validity of mathematics as a whole.[48] Cantor also introduced the Cantor set during this period.

§ 1

The Conception of Power or Cardinal Number

By an "aggregate" (*Menge*) we are to understand any collection into a whole (*Zusammenfassung zu einem Ganzen*) M of definite and separate objects *m* of our intuition or our thought. These objects are called the "elements" of M.

Passage of Georg Cantor's article with his set definition

The fifth paper in this series, "***Grundlagen einer allgemeinen Mannigfaltigkeitslehre***" ("*Foundations of a General Theory of Aggregates*"), published in 1883,[49] was the most important of the six and was also published as a separate monograph. It contained Cantor's reply to his critics and showed how the transfinite numbers were a systematic extension of the natural numbers. It begins by defining well-ordered sets. Ordinal numbers are then introduced as the order types of well-ordered sets. Cantor then defines the addition and multiplication of the cardinal and ordinal numbers. In 1885, Cantor extended his theory of order types so that the ordinal numbers simply became a special case of order types.

In 1891, he published a paper containing his elegant "diagonal argument" for the existence of an uncountable set. He applied the same idea to prove Cantor's theorem: the cardinality of the power set of a set *A* is strictly larger than the cardinality of *A*. This established the richness of the hierarchy of infinite sets, and of the cardinal and ordinal arithmetic that Cantor had defined. His argument is fundamental in the solution of the Halting problem and the proof of Gödel's first incompleteness theorem. Cantor wrote on the Goldbach conjecture in 1894.

In 1895 and 1897, Cantor published a two-part paper in *Mathematische Annalen* under Felix Klein's editorship; these were his last significant papers on set theory.[50] The first paper begins by defining set, subset, etc., in ways that would be largely acceptable now. The cardinal and ordinal arithmetic are reviewed. Cantor wanted the second paper to include a proof of the continuum hypothesis, but had to settle for expositing his theory of well-ordered sets and ordinal numbers. Cantor attempts to prove that if *A* and *B* are sets with *A* equivalent to a subset of *B* and *B* equivalent to a subset of *A*, then *A* and *B* are equivalent. Ernst Schröder had stated this theorem a bit earlier, but his proof, as well as Cantor's, was flawed. Felix Bernstein supplied a correct proof in his 1898 PhD thesis; hence the name Cantor–Bernstein–Schröder theorem.

One-to-one correspondence

Cantor's 1874 Crelle paper was the first to invoke the notion of a 1-to-1 correspondence, though he did not use that phrase. He then began looking for a 1-to-1 correspondence between the points of the unit square and the points of a unit line segment. In an 1877 letter to Richard Dedekind, Cantor proved a far stronger result: for any positive integer *n*, there exists a 1-to-1 correspondence between the points on the unit line segment and all of the points in an *n*-dimensional space. About this discovery Cantor wrote to Dedekind: "*Je le vois, mais je ne le crois pas!*" ("I see it, but I don't believe it!")[51] The result that he found so astonishing has implications for geometry and the notion of dimension.

In 1878, Cantor submitted another paper to Crelle's Journal, in which he defined precisely the concept of a 1-to-1 correspondence and introduced the notion of "power" (a term he took from Jakob Steiner) or "equivalence" of sets: two sets are equivalent (have the same power) if there exists a 1-to-1 correspondence between them. Cantor defined countable sets (or denumerable sets) as sets which can be put into a 1-to-1 correspondence with the natural numbers, and proved that the rational numbers are denumerable. He also proved that *n*-dimensional Euclidean space $\mathbf{R}^n$ has the same power as the real numbers $\mathbf{R}$, as does a countably infinite product of copies of $\mathbf{R}$.

While he made free use of countability as a concept, he did not write the word "countable" until 1883. Cantor also discussed his thinking about dimension, stressing that his mapping between the unit interval and the unit square was not a continuous one.

This paper displeased Kronecker, and Cantor wanted to withdraw it; however, Dedekind persuaded him not to do so and Weierstrass supported its publication.[52] Nevertheless, Cantor never again submitted anything to Crelle.

A bijective function

Continuum hypothesis

Cantor was the first to formulate what later came to be known as the continuum hypothesis or CH: there exists no set whose power is greater than that of the naturals and less than that of the reals (or equivalently, the cardinality of the reals is *exactly* aleph-one, rather than just *at least* aleph-one). Cantor believed the continuum hypothesis to be true and tried for many years to prove it, in vain. His inability to prove the continuum hypothesis caused him considerable anxiety.[11]

The difficulty Cantor had in proving the continuum hypothesis has been underscored by later developments in the field of mathematics: a 1940 result by Gödel and a 1963 one by Paul Cohen together imply that the continuum hypothesis can neither be proved nor disproved using standard Zermelo–Fraenkel set theory plus the axiom of choice (the combination referred to as "ZFC").[53]

Absolute infinite, well-ordering theorem, and paradoxes

In 1883, Cantor divided the infinite into the transfinite and the absolute.[54] The transfinite is increasable in magnitude, while the absolute is unincreasable. For example, an ordinal α is transfinite because it can be increased to α + 1. On the other hand, the ordinals form an absolutely infinite sequence that cannot be increased in magnitude because there are no larger ordinals to add to it.[55] In 1883, Cantor also introduced the well-ordering principle "every set can be well-ordered" and stated that it is a "law of thought."[56]

Cantor extended his work on the absolute infinite by using it in a proof. Around 1895, he began to regard his well-ordering principle as a theorem and attempted to prove it. In 1899, he sent Dedekind a proof of the equivalent aleph theorem: the cardinality of every infinite set is an aleph.[57] First, he defined two types of multiplicities: consistent multiplicities (sets) and inconsistent multiplicities (absolutely infinite multiplicities). Next he assumed that the ordinals form a set, proved that this leads to a contradiction, and concluded that the ordinals form an inconsistent multiplicity. He used this inconsistent multiplicity to prove the aleph theorem.[58] In 1932, Zermelo criticized the construction in Cantor's proof.[59]

Cantor avoided paradoxes by recognizing that there are two types of multiplicities. In his set theory, when it is assumed that the ordinals form a set, the resulting contradiction only implies that the ordinals form an inconsistent multiplicity. On the other hand, Bertrand Russell treated all collections as sets, which leads to paradoxes. In Russell's set theory, the ordinals form a set, so the resulting contradiction implies that the theory is inconsistent. From 1901 to 1903, Russell discovered three paradoxes implying that his set theory is inconsistent: the Burali-Forti paradox (which was just mentioned), Cantor's paradox, and Russell's paradox.[60] Russell named paradoxes after Cesare Burali-Forti and Cantor even though neither of them believed that they had found paradoxes.[61]

In 1908, Zermelo published his axiom system for set theory. He had two motivations for developing the axiom system: eliminating the paradoxes and securing his proof of the well-ordering theorem.[62] Zermelo had proved this theorem in 1904 using the axiom of choice, but his proof was criticized for a variety of reasons.[63] His response to the criticism included his axiom system and a new proof of the well-ordering theorem. His axioms support this new proof, and they eliminate the paradoxes by restricting the formation of sets.[64]
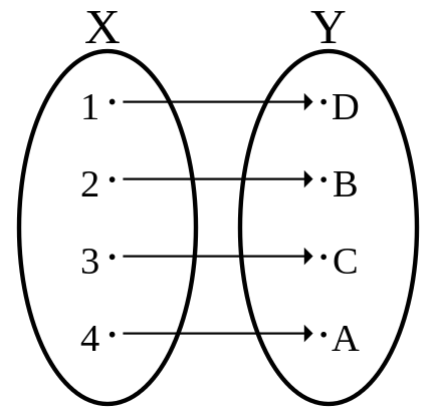
In 1923, John von Neumann developed an axiom system that eliminates the paradoxes by using an approach similar to Cantor's—namely, by identifying collections that are not sets and treating them differently. Von Neumann stated that a class is too big to be a set if it can be put into one-to-one correspondence with the class of all sets. He defined a set as a class that is a member of some class and stated the axiom: A class is not a set if and only if there is a one-to-one correspondence between it and the class of all sets. This axiom implies that these big classes are not sets, which eliminates the paradoxes since they cannot be members of any class.[65] Von Neumann also used his axiom to prove the well-ordering theorem: Like Cantor, he assumed that the ordinals form a set. The resulting contradiction implies that the class of all ordinals is not a set. Then his axiom provides a one-to-one correspondence between this class and the class of all sets. This correspondence well-orders the class of all sets, which implies the well-ordering theorem.[66] In 1930, Zermelo defined models of set theory that satisfy von Neumann's axiom.[67]

Philosophy, religion and Cantor's mathematics

The concept of the existence of an actual infinity was an important shared concern within the realms of mathematics, philosophy and religion. Preserving the orthodoxy of the relationship between God and mathematics, although not in the same form as held by his critics, was long a concern of Cantor's.[68] He directly addressed this intersection between these disciplines in the introduction to his *Grundlagen einer allgemeinen Mannigfaltigkeitslehre,* where he stressed the connection between his view of the infinite and the philosophical one.[69] To Cantor, his mathematical views were intrinsically linked to their philosophical and theological implications – he identified the Absolute Infinite with God,[70] and he considered his work on transfinite numbers to have been directly communicated to him by God, who had chosen Cantor to reveal them to the world.[5]

Debate among mathematicians grew out of opposing views in the philosophy of mathematics regarding the nature of actual infinity. Some held to the view that infinity was an abstraction which was not mathematically legitimate, and denied its existence.[71] Mathematicians from three major schools of thought (constructivism and its two offshoots, intuitionism and finitism) opposed Cantor's theories in this matter. For constructivists such as Kronecker, this rejection of actual infinity stems from fundamental disagreement with the idea that nonconstructive proofs such as Cantor's diagonal argument are sufficient proof that something exists, holding instead that constructive proofs are required. Intuitionism also rejects the idea that actual infinity is an expression of any sort of reality, but arrive at the decision via a different route than constructivism. Firstly, Cantor's argument rests on logic to prove the existence of transfinite numbers as an actual mathematical entity, whereas intuitionists hold that mathematical entities cannot be reduced to logical propositions, originating instead in the intuitions of the mind.[8] Secondly, the notion of infinity as an expression of reality is itself disallowed in intuitionism, since the human mind cannot intuitively construct an infinite set.[72] Mathematicians such as Brouwer and especially Poincaré adopted an intuitionist stance against Cantor's work. Citing the paradoxes of set theory as an example of its fundamentally flawed nature, Poincaré held that "most of the ideas of Cantorian set theory should be banished from mathematics once and for all."[8] Finally, Wittgenstein's attacks were finitist: he believed that Cantor's diagonal argument conflated the intension of a set of cardinal or real numbers with its extension, thus conflating the concept of rules for generating a set with an actual set.[10]

Some Christian theologians saw Cantor's work as a challenge to the uniqueness of the absolute infinity in the nature of God.[6] In particular, Neo-Thomist thinkers saw the existence of an actual infinity that consisted of something other than God as jeopardizing "God's exclusive claim to supreme infinity".[73] Cantor strongly believed that this view was a misinterpretation of infinity, and was convinced that set theory could help correct this mistake:[74]

> *... the transfinite species are just as much at the disposal of the intentions of the Creator and His absolute boundless will as are the finite numbers.*[75]

Cantor also believed that his theory of transfinite numbers ran counter to both materialism and determinism – and was shocked when he realized that he was the only faculty member at Halle who did *not* hold to deterministic philosophical beliefs.[76]

In 1888, Cantor published his correspondence with several philosophers on the philosophical implications of his set theory. In an extensive attempt to persuade other Christian thinkers and authorities to adopt his views, Cantor had corresponded with Christian philosophers such as Tilman Pesch and Joseph Hontheim,[77] as well as theologians such as Cardinal Johannes Franzelin, who once

replied by equating the theory of transfinite numbers with pantheism.[7] Cantor even sent one letter directly to Pope Leo XIII himself, and addressed several pamphlets to him.[74]

Cantor's philosophy on the nature of numbers led him to affirm a belief in the freedom of mathematics to posit and prove concepts apart from the realm of physical phenomena, as expressions within an internal reality. The only restrictions on this metaphysical system are that all mathematical concepts must be devoid of internal contradiction, and that they follow from existing definitions, axioms, and theorems. This belief is summarized in his assertion that "the essence of mathematics is its freedom."[78] These ideas parallel those of Edmund Husserl, whom Cantor had met in Halle.[79]

Meanwhile, Cantor himself was fiercely opposed to infinitesimals, describing them as both an "abomination" and "the cholera bacillus of mathematics".[38]

Cantor's 1883 paper reveals that he was well aware of the opposition his ideas were encountering:

> ... I realize that in this undertaking I place myself in a certain opposition to views widely held concerning the mathematical infinite and to opinions frequently defended on the nature of numbers.[80]

Hence he devotes much space to justifying his earlier work, asserting that mathematical concepts may be freely introduced as long as they are free of contradiction and defined in terms of previously accepted concepts. He also cites Aristotle, Descartes, Berkeley, Leibniz, and Bolzano on infinity.
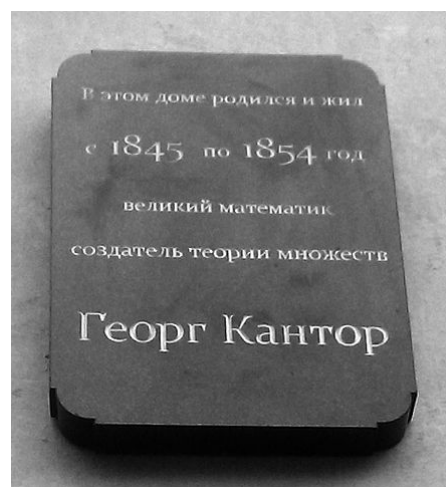
Cantor's ancestry

Cantor's paternal grandparents were from Copenhagen and fled to Russia from the disruption of the Napoleonic Wars. There is very little direct information on his grandparents.[81] Cantor was sometimes called Jewish in his lifetime,[82] but has also variously been called Russian, German, and Danish as well.

Jakob Cantor, Cantor's grandfather, gave his children Christian saints' names. Further, several of his grandmother's relatives were in the Czarist civil service, which would not welcome Jews, unless they converted to Christianity. Cantor's father, Georg Waldemar Cantor, was educated in the Lutheran mission in Saint Petersburg, and his correspondence with his son shows both of them as devout Lutherans. Very little is known for sure about George Woldemar's origin or education.[83] His mother, Maria Anna Böhm, was an Austro-Hungarian born in Saint Petersburg and baptized Roman Catholic; she converted to Protestantism upon marriage. However, there is a letter from Cantor's brother Louis to their mother, stating:

The title on the memorial plaque (in Russian): "In this building was born and lived from 1845 till 1854 the great mathematician and creator of set theory Georg Cantor", Vasilievsky Island, Saint-Petersburg.

> Mögen wir zehnmal von Juden abstammen und ich im Princip noch so sehr für Gleichberechtigung der Hebräer sein, im socialen Leben sind mir Christen lieber ...[83]

("Even if we were descended from Jews ten times over, and even though I may be, in principle, completely in favour of equal rights for Hebrews, in social life I prefer Christians...") which could be read to imply that she was of Jewish ancestry.[84]

There were documented statements, during the 1930s, that called this Jewish ancestry into question:

> More often [i.e., than the ancestry of the mother] the question has been discussed of whether Georg Cantor was of Jewish origin. About this it is reported in a notice of the Danish genealogical Institute in Copenhagen from the year 1937 concerning his father: "It is hereby testified that Georg Woldemar Cantor, born 1809 or 1814, is not present in the registers of the Jewish community, and that he completely without doubt was not a Jew ..."[83]

It is also later said in the same document:

> Also efforts for a long time by the librarian Josef Fischer, one of the best experts on Jewish genealogy in Denmark, charged with identifying Jewish professors, that Georg Cantor was of Jewish descent, finished without result. [Something seems to be wrong with this sentence, but the meaning seems clear enough.] In Cantor's published works and also in his Nachlass there are no statements by himself which relate to a Jewish origin of his ancestors. There is to be sure in the Nachlass a copy of a letter of his brother Ludwig from 18 November 1869 to their mother with some unpleasant antisemitic statements, in which it is said among other things: ...[83]

(the rest of the quote is finished by the very first quote above). In Men of Mathematics, Eric Temple Bell described Cantor as being "of pure Jewish descent on both sides," although both parents were baptized. In a 1971 article entitled "Towards a Biography of Georg Cantor," the British historian of mathematics Ivor Grattan-Guinness mentions (Annals of Science 27, pp. 345–391, 1971) that he was unable to find evidence of Jewish ancestry. (He also states that Cantor's wife, Vally Guttmann, was Jewish).

In a letter written by Georg Cantor to Paul Tannery in 1896 (Paul Tannery, Memoires Scientifique 13 Correspondence, Gauthier-Villars, Paris, 1934, p. 306), Cantor states that his paternal grandparents were members of the Sephardic Jewish community of Copenhagen. Specifically, Cantor states in describing his father: "Er ist aber in Kopenhagen geboren, von israelitischen Eltern, die der dortigen portugisischen Judengemeinde..." ("He was born in Copenhagen of Jewish (lit: "Israelite") parents from the local Portuguese-Jewish community.")[85]

In addition, Cantor's maternal great uncle,[86] a Hungarian violinist Josef Böhm, has been described as Jewish,[87] which may imply that Cantor's mother was at least partly descended from the Hungarian Jewish community.[88]

In a letter to Bertrand Russell, Cantor described his ancestry and self-perception as follows:

> Neither my father nor my mother were of German blood, the first being a Dane, borne in Kopenhagen, my mother of Austrian Hungar descension. You must know, Sir, that I am not a *regular just Germain*, for I am born 3 March 1845 at Saint Peterborough, Capital of Russia, but I went with my father and mother and brothers and sister, eleven years old in the year 1856, into Germany.[89]

Historiography

Until the 1970s, the chief academic publications on Cantor were two short monographs by Schönflies (1927) – largely the correspondence with Mittag-Leffler – and Fraenkel (1930). Both were at second and third hand; neither had much on his personal life. The gap was largely filled by Eric Temple Bell's *Men of Mathematics* (1937), which one of Cantor's modern biographers describes as "perhaps the most widely read modern book on the history of mathematics"; and as "one of the worst".[90] Bell presents Cantor's relationship with his father as Oedipal, Cantor's differences with Kronecker as a quarrel between two Jews, and Cantor's madness as Romantic despair over his failure to win acceptance for his mathematics, and fills the picture with stereotypes. Grattan-Guinness (1971) found that none of these claims were true, but they may be found in many books of the intervening period, owing to the absence of any other narrative. There are other legends, independent of Bell – including one that labels Cantor's father a foundling, shipped to Saint Petersburg by unknown parents.[91] A critique of Bell's book is contained in Joseph Dauben's biography.[92] Writes Dauben:

> Cantor devoted some of his most vituperative correspondence, as well as a portion of the *Beiträge*, to attacking what he described at one point as the 'infinitesimal Cholera bacillus of

mathematics', which had spread from Germany through the work of Thomae, du Bois Reymond and Stolz, to infect Italian mathematics ... Any acceptance of infinitesimals necessarily meant that his own theory of number was incomplete. Thus to accept the work of Thomae, du Bois-Reymond, Stolz and Veronese was to deny the perfection of Cantor's own creation. Understandably, Cantor launched a thorough campaign to discredit Veronese's work in every way possible.[93]

See also

- Cantor algebra
- Cantor cube
- Cantor function
- Cantor medal – award by the Deutsche Mathematiker-Vereinigung in honor of Georg Cantor.
- Cantor set
- Cantor space
- Cantor's back-and-forth method
- Cantor–Bernstein theorem
- Controversy over Cantor's theory
- Heine–Cantor theorem
- Infinity
- List of German inventors and discoverers
- Pairing function

Notes

1. Grattan-Guinness 2000, p. 351.
2. The biographical material in this article is mostly drawn from Dauben 1979. Grattan-Guinness 1971, and Purkert and Ilgauds 1985 are useful additional sources.
3. Dauben 2004, p. 1.
4. Dauben, Joseph Warren (1979). *Georg Cantor His Mathematics and Philosophy of the Infinite*. princeton university press. pp. introduction. ISBN 9780691024479.
5. Dauben 2004, pp. 8, 11, 12–13.
6. Dauben 1977, p. 86; Dauben 1979, pp. 120, 143.
7. Dauben 1977, p. 102.
8. Dauben 1979, p. 266.
9. Dauben 2004, p. 1; Dauben 1977, p. 89 *15n*.
10. Rodych 2007.
11. Dauben 1979, p. 280: "...the tradition made popular by Arthur Moritz Schönflies blamed Kronecker's persistent criticism and Cantor's inability to confirm his continuum hypothesis" for Cantor's recurring bouts of depression.
12. Dauben 2004, p. 1. Text includes a 1964 quote from psychiatrist Karl Pollitt, one of Cantor's examining physicians at Halle Nervenklinik, referring to Cantor's mental illness as "cyclic manic-depression".
13. Dauben 1979, p. 248.
14. Hilbert (1926, p. 170): "Aus dem Paradies, das Cantor uns geschaffen, soll uns niemand vertreiben können." (Literally: "Out of the Paradise that Cantor created for us, no one must be able to expel us.")
15. Reid, Constance (1996), *Hilbert*, New York: Springer-Verlag, p. 177, ISBN 0-387-04999-1
16. ru: The musical encyclopedia (Музыкальная энциклопедия) (http://dic.academic.ru/dic.nsf/enc_music/924/Бём).
17. "Cantor biography" (http://www-history.mcs.st-andrews.ac.uk/Biographies/Cantor.html). *www-history.mcs.st-andrews.ac.uk*. Retrieved 2017-10-06.
18. Bruno, Leonard C.; Baker, Lawrence W. (1999). *Math and mathematicians: the history of math discoveries around the world*. Detroit, Mich.: U X L. p. 54. ISBN 0787638137. OCLC 41497065 (https://www.worldcat.org/oclc/41497065).

19. O'Connor, John J; Robertson, Edmund F (1998). "Georg Ferdinand Ludwig Philipp Cantor" (http://www-history.mcs.st-andrews.ac.uk/Biographies/Cantor.html). MacTutor History of Mathematics.

20. Dauben 1979, p. 163.

21. Dauben 1979, p. 34.

22. Dauben 1977, p. 89 *15n.*

23. Dauben 1979, pp. 2–3; Grattan-Guinness 1971, pp. 354–355.

24. Dauben 1979, p. 138.

25. Dauben 1979, p. 139.

26. Dauben 1979, p. 282.

27. Dauben 1979, p. 136; Grattan-Guinness 1971, pp. 376–377. Letter dated June 21, 1884.

28. Dauben 1979, pp. 281–283.

29. Dauben 1979, p. 283.

30. For a discussion of König's paper see Dauben 1979, pp. 248–250. For Cantor's reaction, see Dauben 1979, pp. 248, 283.

31. Dauben 1979, pp. 283–284.

32. Dauben 1979, p. 284.

33. Johnson, Phillip E. (1972), "The Genesis and Development of Set Theory", *The Two-Year College Mathematics Journal*, **3** (1): 55, JSTOR 3026799 (https://www.jstor.org/stable/3026799), doi:10.2307/3026799 (https://doi.org/10.2307%2F3026799)

34. Suppes, Patrick (1972), *Axiomatic Set Theory* (https://books.google.com/?id=sxr4LrgJGeAC&pg=PA1), Dover, p. 1, ISBN 9780486616308, "With a few rare exceptions the entities which are studied and analyzed in mathematics may be regarded as certain particular sets or classes of objects. ... As a consequence, many fundamental questions about the nature of mathematics may be reduced to questions about set theory."

35. Cantor 1874

36. A countable set is a set which is either finite or denumerable; the denumerable sets are therefore the infinite countable sets. However, this terminology is not universally followed, and sometimes "denumerable" is used as a synonym for "countable".

37. Cooke, Roger (1993), "Uniqueness of trigonometric series and descriptive set theory, 1870–1985", *Archive for History of Exact Sciences*, **45** (4): 281, doi:10.1007/BF01886630 (https://doi.org/10.1007%2FBF01886630).

38. Katz, Karin Usadi and Katz, Mikhail G. (2012), "A Burgessian Critique of Nominalistic Tendencies in Contemporary Mathematics and its Historiography" (http://arxiv.org/pdf/1104.0375), *Foundations of Science*, **17** (1): 51–89, doi:10.1007/s10699-011-9223-1 (https://doi.org/10.1007%2Fs10699-011-9223-1)

39. Ehrlich, P. (2006), "The rise of non-Archimedean mathematics and the roots of a misconception. I. The emergence of non-Archimedean systems of magnitudes" (http://www.ohio.edu/people/ehrlich/AHES.pdf) (PDF), *Arch. Hist. Exact Sci.*, **60** (1): 1–121, doi:10.1007/s00407-005-0102-4 (https://doi.org/10.1007%2Fs00407-005-0102-4)

40. This follows closely the first part of Cantor's 1891 paper.

41. Cantor 1874. English translation: Ewald 1996, pp. 840–843.

42. For example, geometric problems posed by Galileo and John Duns Scotus suggested that all infinite sets were equinumerous – see Moore, A.W. (April 1995), "A brief history of infinity" (http://math123.net/hchs/MathDept/MathTalks/2012-13/b-h-inf.pdf) (PDF), *Scientific American*, **272** (4): 112–116 (114), doi:10.1038/scientificamerican0495-112 (https://doi.org/10.1038%2Fscientificamerican0495-112)

43. For this, and more information on the mathematical importance of Cantor's work on set theory, see e.g., Suppes 1972.

44. Liouville, Joseph (May 13, 1844). A propos de l'existence des nombres transcendants (http://bibnum.education.fr/mathematiques/theorie-des-nombres/propos-de-lexistence-des-nombres-transcendants).

45. The real algebraic numbers are the real roots of polynomial equations with integer coefficients.

46. For more details on Cantor's article, see Georg Cantor's first set theory article and Gray, Robert (1994), "Georg Cantor and Transcendental Numbers" (http://www.maa.org/sites/default/files/pdf/upload_library/22/Ford/Gray819-832.pdf) (PDF), *American Mathematical Monthly*, **101**: 819–832, doi:10.2307/2975129 (https://doi.org/10.2307%2F2975129). Gray (pp. 821–822) describes a computer program that uses Cantor's constructions to generate a transcendental number.

47. Cantor's construction starts with the set of transcendentals T and removes a countable subset $\{t_n\}$ (for example, $t_n = e / n$). Call this set T_0. Then $T = T_0 \cup \{t_n\} = T_0 \cup \{t_{2n-1}\} \cup \{t_{2n}\}$. The set of reals $\mathbf{R} = T \cup \{a_n\} = T_0 \cup \{t_n\} \cup \{a_n\}$ where a_n is the sequence of real algebraic numbers. So both T and $\mathbf{R}$ are the union of three pairwise disjoint sets: T_0 and two countable sets. A one-to-one correspondence between T and $\mathbf{R}$ is given by the function: $f(t) = t$ if $t \in T_0$, $f(t_{2n-1}) = t_n$, and $f(t_{2n}) = a_n$. Cantor actually applies his construction to the irrationals rather than the transcendentals, but he knew that it applies to any set formed by removing countably many numbers from the set of reals (Cantor 1879, p. 4).

48. Dauben 1977, p. 89.

49. Cantor 1883.

50. Cantor (1895), Cantor (1897). The English translation is Cantor 1955.

51. Wallace, David Foster (2003), *Everything and More: A Compact History of Infinity*, New York: W. W. Norton and Company, p. 259, ISBN 0-393-00338-8

52. Dauben 1979, pp. 69, 324 *63n*. The paper had been submitted in July 1877. Dedekind supported it, but delayed its publication due to Kronecker's opposition. Weierstrass actively supported it.

53. Some mathematicians consider these results to have settled the issue, and, at most, allow that it is possible to examine the formal consequences of CH or of its negation, or of axioms that imply one of those. Others continue to look for "natural" or "plausible" axioms that, when added to ZFC, will permit either a proof or refutation of CH, or even for direct evidence for or against CH itself; among the most prominent of these is W. Hugh Woodin. One of Gödel's last papers argues that the CH is false, and the continuum has cardinality Aleph-2.

54. Cantor 1883, pp. 587–588; English translation: Ewald 1996, pp. 916–917.

55. Hallett 1986, pp. 41–42.

56. Moore 1982, p. 42.

57. Moore 1982, p. 51. Proof of equivalence: If a set is well-ordered, then its cardinality is an aleph since the alephs are the cardinals of well-ordered sets. If a set's cardinality is an aleph, then it can be well-ordered since there is a one-to-one correspondence between it and the well-ordered set defining the aleph.

58. Hallett 1986, pp. 166–169.

59. Cantor's proof, which is a proof by contradiction, starts by assuming there is a set S whose cardinality is not an aleph. A function from the ordinals to S is constructed by successively choosing different elements of S for each ordinal. If this construction runs out of elements, then the function well-orders the set S. This implies that the cardinality of S is an aleph, contradicting the assumption about S. Therefore, the function maps all the ordinals one-to-one into S. The function's image is an inconsistent submultiplicity contained in S, so the set S is an inconsistent multiplicity, which is a contradiction. Zermelo criticized Cantor's construction: "the intuition of time is applied here to a process that goes beyond all intuition, and a fictitious entity is posited of which it is assumed that it could make *successive* arbitrary choices." (Hallett 1986, pp. 169–170.)

60. Moore 1988, pp. 52–53; Moore and Garciadiego 1981, p. 330.

61. Moore and Garciadiego 1981, pp. 331, 343; Purkert 1989, p. 56.

62. Moore 1982, pp. 158–160. Moore argues that the latter was his primary motivation.

63. Moore devotes a chapter to this criticism: "Zermelo and His Critics (1904–1908)", Moore 1982, pp. 85–141.

64. Moore 1982, pp. 158–160. Zermelo 1908, pp. 263–264; English translation: van Heijenoort 1967, p. 202.

65. Hallett 1986, pp. 288, 290–291. Cantor had pointed out that inconsistent multiplicities face the same restriction: they cannot be members of any multiplicity. (Hallett 1986, p. 286.)

66. Hallett 1986, pp. 291–292.

67. Zermelo 1930; English translation: Ewald 1996, pp. 1208–1233.

68. Dauben 1979, p. 295.

69. Dauben 1979, p. 120.

70. Hallett 1986, p. 13. Compare to the writings of Thomas Aquinas.

71. Dauben 1979, p. 225

72. Snapper, Ernst (1979), "The Three Crises in Mathematics: Logicism, Intuitionism and Formalism" (http://www2.gsu.edu/~matgtc/three%20crises%20in%20mathematics.pdf) (PDF), *Mathematics Magazine*, **524**: 207–216

73. Davenport, Anne A. (1997), "The Catholics, the Cathars, and the Concept of Infinity in the Thirteenth Century", *Isis*, **88** (2): 263–295, JSTOR 236574 (https://www.jstor.org/stable/236574), doi:10.1086/383692 (https://doi.org/10.1086%2F383692)

74. Dauben 1977, p. 85.

75. Cantor 1932, p. 404. Translation in Dauben 1977, p. 95.

76. Dauben 1979, p. 296.

77. Dauben 1979, p. 144.

78. Dauben 1977, pp. 91–93.

79. On Cantor, Husserl, and Gottlob Frege, see Hill and Rosado Haddock (2000).

80. Dauben 1979, p. 96.

81. *E.g.*, Grattan-Guinness's only evidence on the grandfather's date of death is that he signed papers at his son's engagement.

82. For example, Jewish Encyclopedia, art. "Cantor, Georg"; Jewish Year Book 1896–97, "List of Jewish Celebrities of the Nineteenth Century", p. 119; this list has a star against people with one Jewish parent, but Cantor is not starred.

83. Purkert and Ilgauds 1985, p. 15.

84. For more information, see: Dauben 1979, p. 1 and notes; Grattan-Guinness 1971, pp. 350–352 and notes; Purkert and Ilgauds 1985; the letter is from Aczel 2000, pp. 93–94, from Louis' trip to Chicago in 1863. It is ambiguous in German, as in English, whether the recipient is included.

85. Tannery, Paul (1934) *Memoires Scientifique 13 Correspondance*, Gauthier-Villars, Paris, p. 306.

86. Dauben 1979, p. 274.

87. Mendelsohn, Ezra (ed.) (1993) *Modern Jews and their musical agendas* (http://trove.nla.gov.au/work/11440061?versionId=13425381), Oxford University Press, p. 9.

88. *Ismerjük*oket?: zsidó származású nevezetes magyarok arcképcsarnoka*, István Reményi Gyenes Ex Libris, (Budapest 1997), pages 132–133*

89. Russell, Bertrand. *Autobiography*, vol. I, p. 229. In English in the original; italics also as in the original.

90. Grattan-Guinness 1971, p. 350.

91. Grattan-Guinness 1971 (quotation from p. 350, note), Dauben 1979, p. 1 and notes. (Bell's Jewish stereotypes appear to have been removed from some postwar editions.)

92. Dauben 1979

93. Dauben, J.: The development of the Cantorian set theory, pp.~181–219. See pp.216–217. In Bos, H.; Bunn, R.; Dauben, J.; Grattan-Guinness, I.; Hawkins, T.; Pedersen, K. From the calculus to set theory, 1630–1910. An introductory history. Edited by I. Grattan-Guinness. Gerald Duckworth & Co. Ltd., London, 1980.

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External links

- Works by or about Georg Cantor (https://archive.org/search.php?query=%28%28subject%3A%22Cantor%2C%20Georg%22%20OR%20subject%3A%22Georg%20Cantor%22%20OR%20creator%3A%22Cantor%2C%20Georg%22%20OR%20creator%3A%22Georg%20Cantor%22%20OR%20creator%3A%22Cantor%2C%20G%2E%22%20OR%20title%3A%22Georg%20Cantor%22%20OR%20description%3A%22Cantor%2C%20Georg%22%20OR%20description%3A%22Georg%20Cantor%22%29%20OR%20%28%221845-1918%22%20AND%20Cantor%29%29%20AND%20%28-mediatype:software%29) at Internet Archive
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- O'Connor, John J.; Robertson, Edmund F., "A history of set theory" (http://www-history.mcs.st-andrews.ac.uk/HistTopics/Beginnings_of_set_theory.html), *MacTutor History of Mathematics archive*, University of St Andrews. Mainly devoted to Cantor's accomplishment.
- Georg Cantor (https://www.genealogy.math.ndsu.nodak.edu/id.php?id=29561) at the Mathematics Genealogy Project
- Stanford Encyclopedia of Philosophy: Set theory (http://plato.stanford.edu/entries/set-theory/) by Thomas Jech.
- Grammar school Georg-Cantor Halle (Saale): Georg-Cantor-Gymnasium Halle (http://www.cantor-gymnasium.de/)
- Poem about Georg Cantor (http://www.readmypoems.co.uk/2014/02/a-pentameter-of-questions-about-infinity.html)
- "Cantor infinities", analysis of Cantor's 1874 article, *BibNum (http://www.bibnum.education.fr/mathematiques/theorie-des-nombres/cantor-et-les-infinis)* (for English version, click 'à télécharger'). There is an error in this analysis. It states Cantor's Theorem 1 correctly: Algebraic numbers can be counted. However, it states his Theorem 2 incorrectly: Real numbers cannot be counted. It then says: "Cantor notes that, taken together, Theorems 1 and 2 allow for the redemonstration of the existence of non-algebraic real numbers …" This existence demonstration is non-constructive. Theorem 2 stated correctly is: Given a sequence of real numbers, one can determine a real number that is not in the sequence. Taken together, Theorem 1 and this Theorem 2 produce a non-algebraic number. Cantor also used Theorem 2 to prove that the real numbers cannot be counted. See Cantor's first set theory article or Georg Cantor and Transcendental Numbers (http://www.maa.org/sites/default/files/pdf/upload_library/22/Ford/Gray819-832.pdf).

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