How to represent this situation with logic propositions?

The situation:
A, B and C are each either a truth-teller or a liar, truth-tellers can only tell truths, and liars can only lie.
A: I am a truth-teller.
B: A is a truth-teller.
C: A is a liar.

How can I represent these statements such that it will be possible to test permutations of identities? For example: it is possible that A is a liar, but not possible if A is a liar *and* B is a truth-teller.

(logic)

asked Jan 6 at 20:14
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2 Answers

Use A for: A is a truth-teller

What person A says is true if and only if A is a truth-teller, so you get $A \leftrightarrow A$

Likewise you get $B \leftrightarrow A$ and $C \leftrightarrow \neg A$

Using substitution, this also means $C \leftrightarrow \neg B$

No unique assignment of identities exists: it could be that A and B are true, and C is false, or that A and B are false, and C is true.

Here is a more interesting example (Puzzle 102 from http://philosophy.hku.hk/think/logic/knights.php):

"A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

You meet four inhabitants: Bart, Dave, Rex and Zoey. Bart tells you that Rex and Dave are both knights or both knaves. Dave tells you that Zoey is a knave. Rex claims, “Bart is a knave.” Zoey claims, “Rex is a knight and Dave is a knave.”

Symbolize this as:

1. $B \leftrightarrow (R \leftrightarrow D)$
2. $D \leftrightarrow \neg Z$
3. $R \leftrightarrow \neg B$
4. $Z \leftrightarrow (R \land \neg D)$

And now:

5. $B \leftrightarrow (\neg B \leftrightarrow D)$ (substitute 3 in 1)
6. $(B \leftrightarrow \neg B) \leftrightarrow D$ (from 5 by associativity of $\leftrightarrow$)
7. $\bot \leftrightarrow D$ (from 6 since $P \leftrightarrow \neg P \Leftrightarrow \bot$)
8. $\neg D$ (from 7 .. i.e. we now know Dave is a knave)
9. $\bot \leftrightarrow \neg Z$ (substitute 7 in 2)
10. $\top \leftrightarrow Z$ (from 9)
11. Z (from 10 ... so Zoey is a knight)
12. $R \land \neg D$ (from 4 and 11)
13. R (from 12 ... so Rex is a knight)
14. $\neg B$ (from 3 and 13 ... and Bart is a knave)

In other words: with a few simple principles you can solve these kinds of Knights and Knaves puzzles pretty quickly!

answered Jan 6 at 22:17
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The standard approach to solve these "Knights and Knaves" problems is to use a proposition t_A that is true if and only if A is truthful, and likewise for B and C. Then A's statement is encoded thus:

$$t_A \leftrightarrow t_A \quad ,$$

which is tautologous; hence it gives us no information about A, B, and C. In like fashion,

$$t_B \leftrightarrow t_A\,, \quad t_C \leftrightarrow \neg t_A \quad .$$

You can then enumerate the satisfying assignments to the conjunction of the tree sentences. (Of course, you can skip the tautology and just look at $(t_B \leftrightarrow t_A) \wedge (t_C \leftrightarrow \neg t_A)$.) In this case, you can easily see that there are two possible satisfying assignments.

answered Jan 6 at 20:19

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