Meaning of 'alternating' group ?

What's the meaning of the adjective 'alternating' in the name of the 'alternating group' ?

gr.group-theory terminology

edited Aug 31 '11 at 23:29

asked Aug 31 '11 at 23:18
Ralph
12.7k 21 54

1 Continuing the tradition of people who don't know offering weird guesses (see mathoverflow.net/questions/74004), I've always thought it was because the permutations of even and odd sign alternate under transpositions. However, I have no source for this. – Henry Cohn Aug 31 '11 at 23:35

12 It is the group of permutations of the variables of an "alternating polynomial" which preserve the value of the function. I'm guessing that "alternating polynomial" is an older concept and so this could be the origin. But this is pure speculation. – Brendan McKay Aug 31 '11 at 23:55

Confusingly, "alternating permutation" has been used to mean "zigzag permutation" or "up-down permutation", i.e. $\pi(1) < \pi(2) > \pi(3) < \pi(4) > \pi(5) <> \cdots$. Because this collides with "alternating group" I much prefer the other two terms. – Noam D. Elkies Sep 1 '11 at 0:15

I thought it was because you could generate any element of this group by "alternating" pairs of elements. – Daniel Mansfield Sep 1 '11 at 0:18

1 Answer

Evidence for Brendan McKay's suggestion that it comes from the older concept of an alternating polynomial may be found in Burnside's classic book, *Theory of Groups of Finite Order*. When defining the symmetric and alternating groups, he puts an asterisk next to the word "alternating" and adds the following footnote:

> The symmetric group has been so called because the only functions of the n symbols which are unaltered by all the substitutions of the group are the symmetric functions. All the substitutions of the alternating group leave the square root of the discriminant unaltered.

By "the square root of the discriminant," Burnside means the polynomial

$$\prod_{r=1}^{n-1} \prod_{s=r+1}^{n} (a_r - a_s),$$

which of course is an alternating polynomial.

answered Sep 1 '11 at 2:49
Timothy Chow
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Thanks for the answer. It's also interesting to see that the name of the symmetric group arises from the symmetric functions (formerly I had thought the name comes from the group's relevance in studying symmetries of polyhedra). – Ralph Sep 1 '11 at 7:58