

## 2-4 Treasure Hunting — Mergesort

(Wednesday, April 11, 2018 ~ Thursday, April 19, 2018)

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## Analysis of Mergesort in CLRS (# of Comparisons; $a_i : \infty$ not Counted)

- (a) Analyze the **worst case**  $W(n)$  and the **best case**  $B(n)$  time complexity of mergesort *as accurately as possible*.

Explore the relation between them and the binary representations of numbers.

Plot  $W(n)$  and  $B(n)$  and explain what you observe.

- (b) Analyze the **average case**  $A(n)$  time complexity of mergesort.

Plot  $A(n)$  and explain what you observe.

- (c) **Prove that:** The minimum number of comparisons needed to merge two sorted arrays of equal size  $m$  is  $2m - 1$ .

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(WED., April 11, 2018)

$W(n)$  : Consider  $W(n + 1)$



(THU, April 12, 2018)

$$W(n) = \begin{cases} 0, & n = 1 \\ W(\lfloor \frac{n}{2} \rfloor) + W(\lceil \frac{n}{2} \rceil) + (n - 1), & \text{o.w.} \end{cases}$$

$$W(n+1) - W(n)$$

Plot the coefficient of the linear time  $\frac{1}{n}(W(n) - n \log n)$

The total number of bits in the binary representations  
of *all the numbers less than  $n$* .



(FRI, April 13, 2018)

$$B(n) = \begin{cases} 0, & n = 1 \\ B(\lfloor \frac{n}{2} \rfloor) + B(\lceil \frac{n}{2} \rceil) + \lfloor \frac{n}{2} \rfloor, & o.w. \end{cases}$$

The total number of **1s** in the binary representations  
of *all the numbers less than  $n$* .



(MON, April 16, 2018)

**MERGE**( $A, B$ ): keeps comparing the smallest remaining elements in  $A$  and  $B$  (and removing the minimum), *until* one of them is empty.

**Definition** ( $R$ )

$R$  is the number of elements left in the non-empty array by **MERGE**( $A, B$ ) on  $A$  of size  $a$  and  $B$  of size  $b$ .

**Theorem**

$$Pr(R \geq r) = \frac{\binom{a+b-r}{a}}{\binom{a+b}{a}} + \frac{\binom{a+b-r}{b}}{\binom{a+b}{b}}, \quad \mathbb{E}(R) = \sum_r Pr(R \geq r) = \frac{a}{b+1} + \frac{b}{a+1}$$



(TUE, April 17, 2018)

$$A(n) = A(\lfloor \frac{n}{2} \rfloor) + A(\lceil \frac{n}{2} \rceil) + \left( n - \underbrace{\left( \frac{\lfloor \frac{n}{2} \rfloor}{\lceil \frac{n}{2} \rceil + 1} + \frac{\lceil \frac{n}{2} \rceil}{\lfloor \frac{n}{2} \rfloor + 1} \right)}_{\text{Remember Tip 4}} \right)$$

You are *not* required to solve this recurrence.





(WED, April 18, 2018)



**KEEP  
CALM  
AND  
STAY  
TUNED**



(THU, April 19, 2018)



**KEEP  
CALM  
AND  
STAY  
TUNED**

Thank  
You!



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