2-7 Teasure Hunting — Balls into Bins

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There are n bins labelled with the numbers $1, 2, \cdots, n$. Balls are placed in these bins one after the other, with the bin into which a ball is placed being independent random variables that assume the value k with probability p_k . Let X be the number of balls placed so that there is at least one ball in every bin.

- (a) Assume that $p_k = \frac{1}{n}$. What is the expectation of X?
- (b) Assume that $p_k = \frac{1}{n}$. What is the probability distribution of X?
- (c) Prove that $\Pr(X > n \ln n + cn) \le e^{-c}$, $\Pr(X < n \ln n cn) \le e^{-c}$.
- (d) Redo (a) and (b) without the assumption $p_k = \frac{1}{n}$.
- (e) Given a deck of n cards, each time you take the top card from the deck, and insert it into the deck at one of the n distinct possible places, each of them with probability $\frac{1}{n}$. What is the expected times for you to perform the procedure above until the bottom card rises to the top?



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 X_i :

The number of balls needed to fill the i-th bin when i-1 bins have been filled

$$X = \sum_{i=1}^{n} X_i$$











Thank You!



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