2-2 Treasure Hunting

(Monday, April 2, 2018 $\sim$ Saturday, April 7, 2018)

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TREASURE
HUNT
on foot


Hints
&
Tips


Help
Wanted

Problem of the Week (Monday, April 2, 2018 $\sim$ Saturday, April 7, 2018)

(a) Given an array $A[0 \cdots n-1]$, to determine whether there is a value that *occurs more than $\lfloor n/k \rfloor$ times* in $\Theta(n \lg k)$ time and $\Theta(k)$ extra space.

(b) Prove that the *lower bound* of this problem is $\Theta(n \lg k)$.

TIP#1

(Monday, April 2, 2018)

Take $k = 2$.

$\Theta(n)$ time $\&$ $\Theta(1)$ space

TIP#2

(Tuesday, April 3, 2018)

Definition (k-simplified Multiset)

Consider a multiset $\mathcal{M}$. A *k-simplified multiset* for $\mathcal{M}$ is a multiset derived from $\mathcal{M}$ by repeating *deleting k distinct elements* from it until no longer possible.

Theorem

If a value occurs more than $\lfloor n/k \rfloor$ times in $\mathcal{M}$ of n elements, then it is in a k-simplified multiset for $\mathcal{M}$.

Prove this theorem. Take $k = 2$ again. Design an $\Theta(n)$ algorithm for $k = 2$. Generalize it to an algorithm for general k (ignoring $\Theta(n \lg k)$ for now).

TIP#3

(Wednesday, April 4, 2018)

Today, you have an efficient data structure T for a multiset:

We denote a multiset by $\mathcal{M} = \{(v_i, c_i)\}$, where c_i is the number of times v_i occurs in $\mathcal{M}$. The number of distinct values in $\mathcal{M}$ is denoted by $d = |\{v_i\}|$.

The data structure T for $\mathcal{M}$ contains d nodes, each being a pair (v_i, c_i). It supports Insert, Delete, and Search in $\Theta(\lg d)$.

Use this data structure (you are not required to implement it) in your algorithm to achieve the time complexity of $\Theta(n \lg k)$.

TIP#4

(Thursday, April 5, 2018)

Definition (r-multiset)

Let $r = \lfloor n/k \rfloor$. An r-multiset is a multiset of n values such that each of the values $0, 1, \cdots, \lfloor n/r \rfloor - 1$ occurs r times and the value $\lfloor n/r \rfloor$ occurs $(n \mod r)$ times.

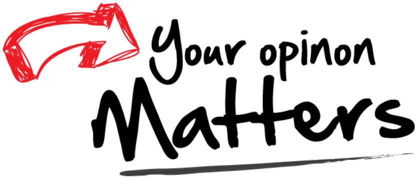
Lemma

There are at least $(k/e)^n$ different r-multisets.

Theorem

Executing these r-multisets on a decision tree will follow different paths.


Thank
You!



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