2-4 Teasure Hunting — Mergesort

(Wednesday, April 11, 2018 $\sim$ Thursday, April 19, 2018)

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April 11, 2018


TREASURE
HUNT
on foot


Hints
&
Tips


Help
Wanted

Analysis of Mergesort in CLRS (# of Comparisions; $a_i : \infty$ not Counted)

(a) Analyze the worst case $W(n)$ and the best case $B(n)$ time complexity of mergesort *as accurately as possible*.
Explore the relation between them and the binary representations of numbers.
Plot $W(n)$ and $B(n)$ and explain what you observe.

(b) Analyze the average case $A(n)$ time complexity of mergesort.
Plot $A(n)$ and explain what you observe.

(c) Prove that: The minimum number of comparisons needed to merge two sorted arrays of equal size m is $2m - 1$.

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(WED., April 11, 2018)

$W(n)$: Consider $W(n+1)$

(THU, April 12, 2018)

$$W(n) = \begin{cases} 0, & n = 1 \\ W(\lfloor \frac{n}{2} \rfloor) + W(\lceil \frac{n}{2} \rceil) + (n-1), & \textit{o.w.} \end{cases}$$

$$W(n+1) - W(n)$$

Plot the coefficient of the linear time $\frac{1}{n}(W(n) - n \log n)$

The total number of bits in the binary representations of *all the numbers less than n*.

(FRI, April 13, 2018)

$$B(n) = \begin{cases} 0, & n = 1 \\ B(\lfloor \frac{n}{2} \rfloor) + B(\lceil \frac{n}{2} \rceil) + \lfloor \frac{n}{2} \rfloor, & \textit{o.w.} \end{cases}$$

The total number of 1s in the binary representations of *all the numbers less than n*.

(MON, April 16, 2018)

$\textsc{Merge}(A, B)$: keeps comparing the smallest remaining elements in A and B (and removing the minimum), *until* one of them is empty.

Definition (R)

R is the number of elements left in the non-empty array by $\textsc{Merge}(A, B)$ on A of size a and B of size b.

Theorem

$$Pr(R \geq r) = \frac{\binom{a+b-r}{a}}{\binom{a+b}{a}} + \frac{\binom{a+b-r}{b}}{\binom{a+b}{a}}, \; \mathbb{E}(R) = \sum_r Pr(R \geq r) = \frac{a}{b+1} + \frac{b}{a+1}$$

(TUE, April 17, 2018)

$$A(n) = A(\lfloor \frac{n}{2} \rfloor) + A(\lceil \frac{n}{2} \rceil) + \Big(n - (\underbrace{\frac{\lfloor \frac{n}{2} \rfloor}{\lceil \frac{n}{2} \rceil + 1} + \frac{\lceil \frac{n}{2} \rceil}{\lfloor \frac{n}{2} \rfloor + 1}}_{\text{Remember Tip } 4})\Big)$$

You are *not* required to solve this recurrence.

(WED, April 18, 2018)

$$A : A_1 < \cdots < A_m \qquad B : B_1 < \cdots < B_m$$

$$\boxed{B_1 < A_1 < B_2 < A_2 < \cdots < B_m < A_m}$$

Theorem

Each of the $2m-1$ comparisons $B_1 : A_1, A_1 : B_2, B_2 : A_2, \cdots, B_m : A_m$ must be made.

(THU, April 19, 2018)

Paper (Clickable):

Mellin Transforms and Asymptotics: The Mergesort Recurrence.pdf

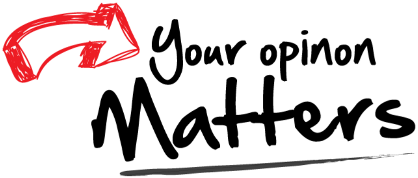
Video (Clickable):

Mergesort recurrences and the binary representations of numbers

Reference (Not Clickable):

The Art of Computer Programming, Volume 3: Sorting and Searching.
Section 5.3.2: Minimum-Comparison Merging (Theorem M)


Thank
You!



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