Radiometry, Reflectance, Lights

Computer Vision I

CSE 252A

Lecture 6

A question from Piazza

Homography Color & Lightening

Hi!

In Homography, we get the projection of the object to another image plane given its projection on one image plane. However, it is just a change of positions of points, but does not consider the colors and lightening problems. Would simply copying the RGB values of points violate the Physics of lightening, i.e. would this cause the image to be unrealistic?

Thanks!

logistics

edit · good question | 0 Updated 3 days ago by Yinglong Miao

Announcements

- HW1 posted
- HWO graded, will be returned today
- If anyone has any registration issues, talk to me.

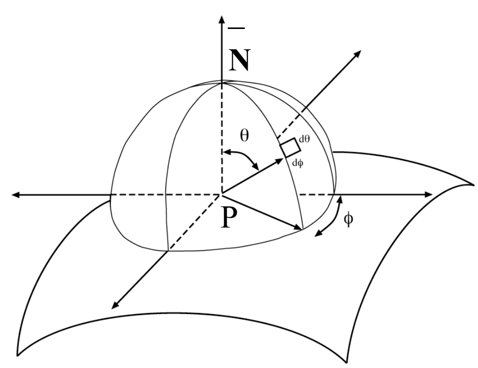
Appearance: lighting, surface reflectance, transmission, camera

Radiometry

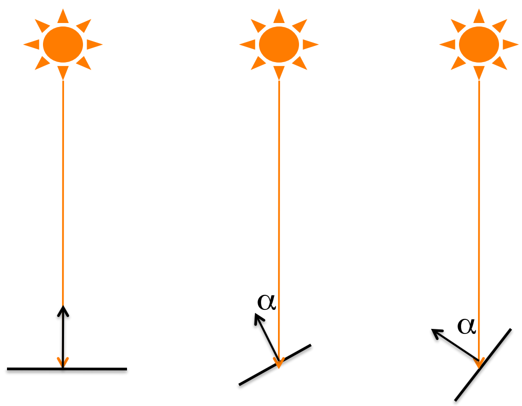
- Read Chapter 4 of Ponce & Forsyth
- Szeliski Sec. 2.2, 2.3
- Solid Angle
- Irradiance
- Radiance
- BRDF
- Lambertian/Phong BRDF

A local coordinate system on a surface

- Consider a point **P** on a surface
- Light arrives at **P** from a hemisphere of directions defined by the surface normal **N**
- We can define a local coordinate system whose origin is **P** and with one axis aligned with **N**
- Convenient to represent in spherical angles.
- θ is well-defined, ϕ isn't

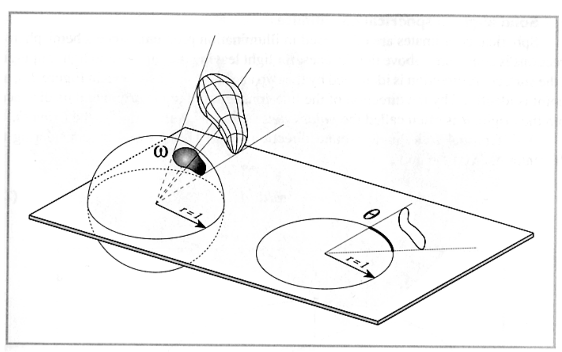


Foreshortening



The surface is "foreshortened" by the cosine of the angle α between the normal and direction to a point (e.g., the light).

Measuring Angle



- The **solid angle** subtended by an object from a point P is the area of the projection of the object onto the unit sphere centered at P
- Definition is analogous to projected angle in 2D
- Measured in *steradians*, sr
- If I'm at P, and I look out, solid angle tells me how much of my view is filled with an object

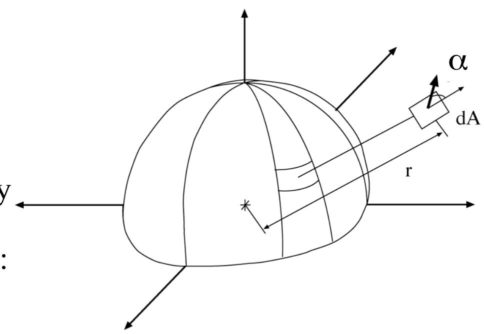
Solid Angle of a patch

- The solid angle subtended by a patch area dA without any foreshortening is

$$d\omega = \frac{dA}{r^2}$$

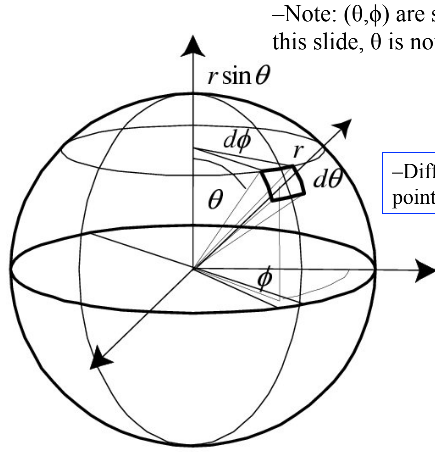
- The solid angle subtended by a patch area dA tilted (foreshortened) at angle α is:

$$d\omega = \frac{dA\cos\alpha}{r^2}$$



Differential Solid Angles

–Note: (θ,ϕ) are spherical angles in this slide, θ is not foreshortening.



$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2\sin\theta\,d\theta\,d\phi$$

–Differential solid angle when representing point on sphere in spherical coordinates

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

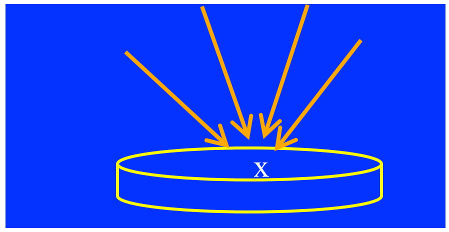
$$S = \int_0^{\pi}\int_0^{2\pi}\sin\theta\,d\theta\,d\phi = 4\pi$$

With foreshortening by α

$$d\omega = \frac{dA}{r^2}\cos\alpha = \sin\theta\cos\alpha\,d\theta\,d\phi$$

CS348B Le Pat Hanrahan, Spring 2002

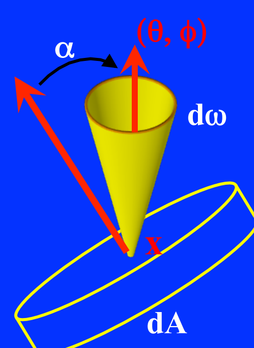
Irradiance



- How much light is arriving at a point x on surface? E(x)
- Units of irradiance: Watts/m^2
- Recall that power is energy per time, and is measured in Watts (Joules/sec)

Radiance

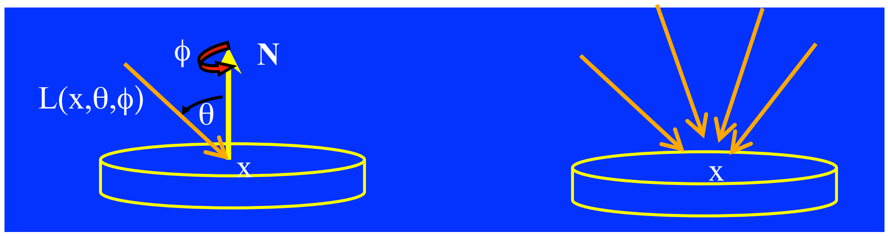
- **Radiance: Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle**
- Symbol: L(**x**,θ,φ)
- Units: watts per square meter per steradian : w/($m^2 sr^1$)



$$L = \frac{P}{(dA \cos\alpha)d\omega}$$

Power emitted from patch, but radiance in direction different from surface normal

Irradiance due to Radiance



A surface experiencing radiance L(x,θ,φ) coming in from solid angle dω experiences **irradiance**:

$$E(x) = L(x,\theta,\phi)\cos\theta d\omega$$

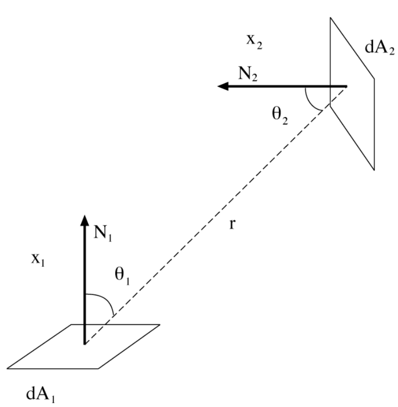
- Total **Irradiance** arriving at the surface is given by integrating incident irradiances over all incoming angles.
- Total irradiance is

$$\int_{hemisphere} L(x,\theta,\phi)\cos\theta \, d\omega =$$

$$\int_{hemisphere} L(x,\theta,\phi)\cos\theta \sin\theta \, d\theta \, d\phi$$

Radiance transfer

What is the power received by a small area dA_2 at distance r from a small area dA_1 emitting radiance L?



From definition of radiance

$$L = \frac{P}{(dA\cos\theta)d\omega}$$

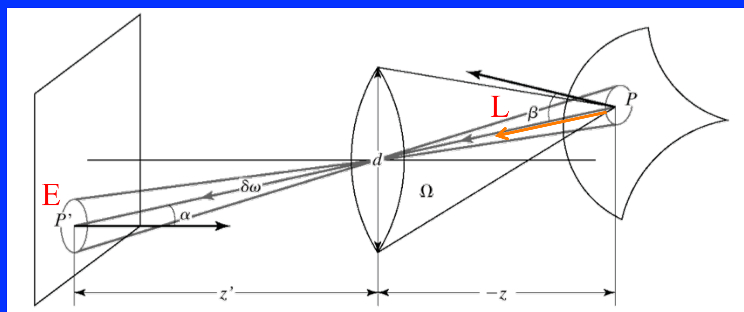
From definition of solid angle

$$d\omega = \frac{dA\cos\theta}{r^2}$$

$$P = L dA_1 \cos\theta_1 d\omega_{1->2}$$

$$= \frac{L}{r^2} dA_1 dA_2 \cos\theta_1 \cos\theta_2$$

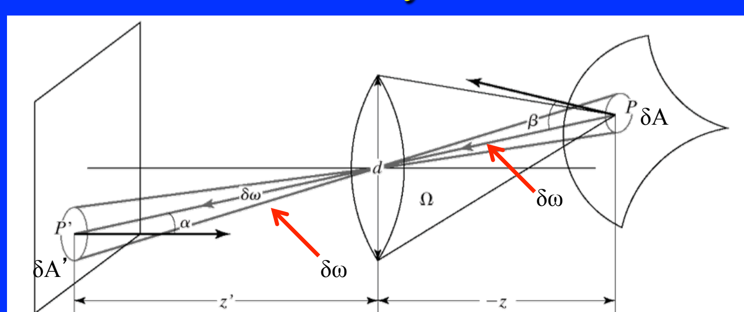
Radiometry of thin lenses



What is image irradiance E for a radiance L emitted from a point P?

Solution is a substantial example problem.

Radiometry of thin lenses



A small patch on the surface δA projects to a small patch on the image δA'.
Let δω be the solid angle subtended by δA (also δA') from the center of the lens

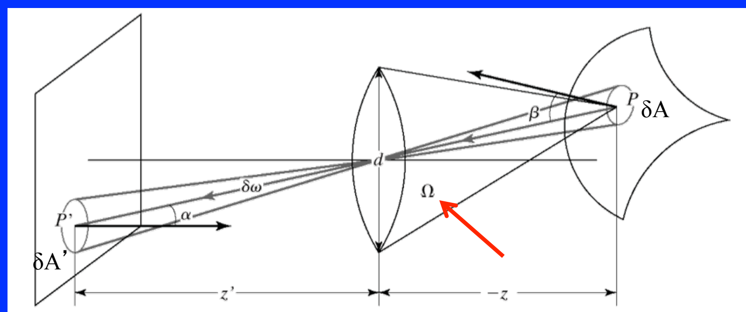
$$d\omega = \frac{\delta A' \cos\alpha}{r^2} = \frac{\delta A' \cos\alpha}{(z'/\cos\alpha)^2} = \frac{\delta A \cos\beta}{(z/\cos\alpha)^2}$$

From solid angle equation

$$\frac{\delta A}{\delta A'} = \frac{\cos\alpha}{\cos\beta}\left(\frac{z}{z'}\right)^2$$

We'll use this later.

Radiometry of thin lenses

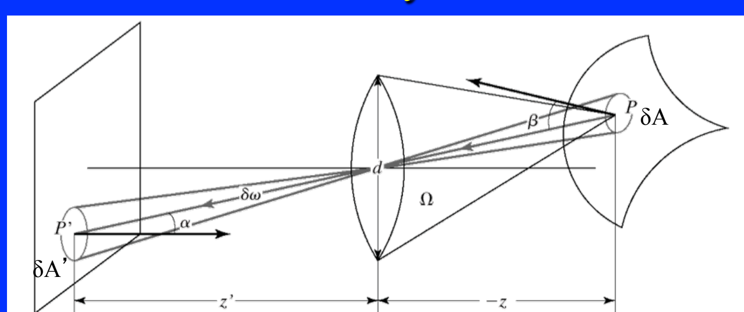


$$d\omega = \frac{\delta A' \cos\alpha}{(z'/\cos\alpha)^2} = \frac{\delta A \cos\beta}{(z'/\cos\alpha)^2} \qquad \frac{\delta A}{\delta A'} = \frac{\cos\alpha}{\cos\beta}\left(\frac{z}{z'}\right)^2$$

Let Ω be the solid angle subtended by the lens from P.

$$\Omega = \frac{\pi}{4}\frac{d^2\cos\alpha}{(z/\cos\alpha)^2} = \frac{\pi}{4}\left(\frac{d}{z}\right)^2\cos^3\alpha$$

Radiometry of thin lenses



$$d\omega = \frac{dA' \cos\alpha}{(z'/\cos\alpha)^2} = \frac{dA \cos\beta}{(z'/\cos\alpha)^2} \qquad \frac{\delta A}{\delta A'} = \frac{\cos\alpha}{\cos\beta}\left(\frac{z}{z'}\right)^2$$

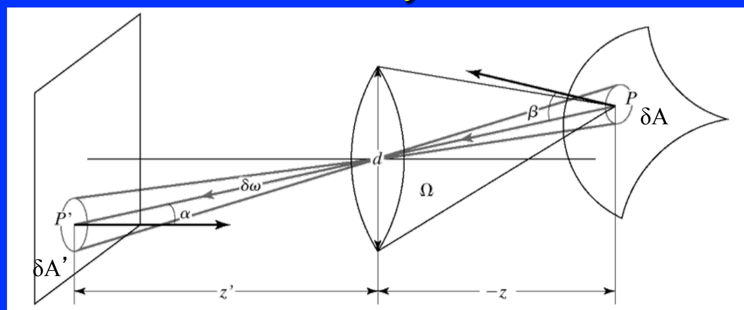
$$\Omega = \frac{\pi}{4}\frac{d^2\cos\alpha}{(z/\cos\alpha)^2} = \frac{\pi}{4}\left(\frac{d}{z}\right)^2\cos^3\alpha$$

The power δP emitted from the patch δA with radiance L and falling on the lens is:

$$\delta p = L\Omega\delta A\cos\beta = \frac{\pi}{4}\left(\frac{d}{z}\right)^2 L\delta A\cos^3\alpha\cos\beta$$

δP ultimately strikes δA', and the irradiance δP/δA' is

Radiometry of thin lenses



$$d\omega = \frac{dA'\cos\alpha}{(z'/\cos\alpha)^2} = \frac{dA\cos\beta}{(z'/\cos\alpha)^2}$$

$$\frac{\delta A}{\delta A'} = \frac{\cos\alpha}{\cos\beta}\left(\frac{z}{z'}\right)^2$$

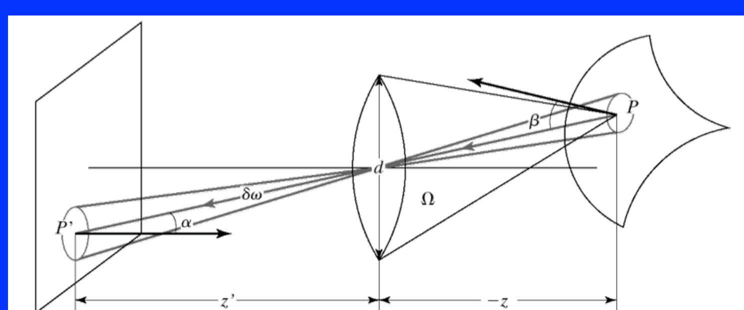
$$\Omega = \frac{\pi}{4}\frac{d^2\cos\alpha}{(z/\cos\alpha)^2} = \frac{\pi}{4}\left(\frac{d}{z}\right)^2\cos^3\alpha$$

$$\delta p = L\Omega\delta A\cos\beta = \frac{\pi}{4}\left(\frac{d}{z}\right)^2 L\delta A\cos^3\alpha\cos\beta$$

Image Irradiance:
Substitute in for δA/δA

$$E = \frac{\delta P}{\delta A'} = \frac{\pi}{4}\left(\frac{d}{z}\right)^2 L\frac{\delta A}{\delta A'}\cos^3\alpha\cos\beta = \left[\frac{\pi}{4}\left(\frac{d}{z'}\right)^2\cos^4\alpha\right]L$$

Image Irradiance Equation

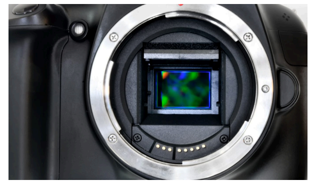


$$E = \left[\frac{\pi}{4}\left(\frac{d}{z'}\right)^2\cos^4\alpha\right]L$$

E: Image irradiance
L: emitted radiance
d : Lens diameter
z' : depth of image plane
α: Angle of patch from optical axis

Camera sensor

- Measured pixel intensity is a function of irradiance integrated over
  – Pixel's area
  – over a range of wavelengths
  – for some period of time



$$I = \int_t \int_\lambda \int_x \int_y E(x,y,\lambda,t)s(x,y)q(\lambda)\,dx\,dy\,d\lambda\,dt$$

- Ideally, it's linear to the radiance, but the camera response C(.) may not be linear

$$I = R\left(\int_t \int_\lambda \int_x \int_y E(x,y,\lambda,t)s(x,y)q(\lambda)\,dx\,dy\,d\lambda\,dt\right)$$

Color Cameras

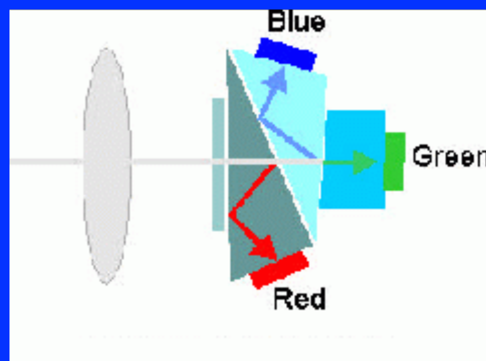
We consider 3 concepts:

1. Prism (with 3 sensors)
2. Filter mosaic
3. Filter wheel

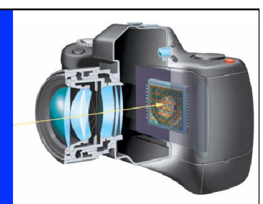
… and X3

Prism color camera

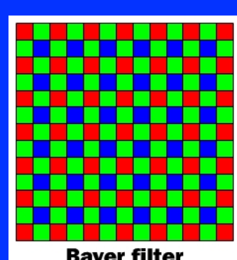
Separate light in 3 beams using dichroic prism
Requires 3 sensors & precise alignment
Good color separation

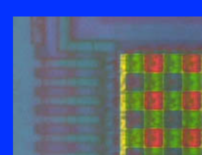


Filter mosaic

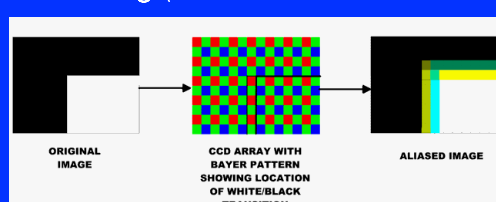


Coat filter directly on sensor





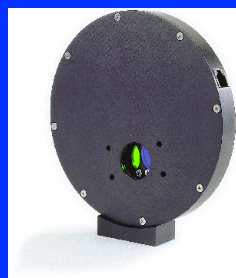
Demosaicing (obtain full colour & full resolution image)



Filter wheel

Rotate multiple filters in front of lens

Allows more than 3 colour bands



Only suitable for static scenes

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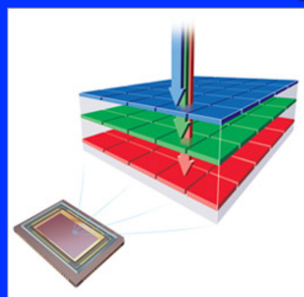
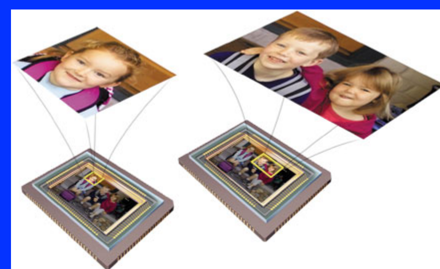
Prism vs. mosaic vs. wheel

approach	Prism	Mosaic	Wheel
# sensors	3	1	1
Separation	High	Average	Good
Cost	High	Low	Average
Framerate	High	High	Low
Artefacts	Low	Aliasing	Motion
Bands	3	3	3 or more
	High-end cameras	Low-end cameras	Scientific applications

–CSE 252A

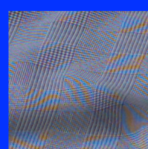
“newer” color CMOS sensor Foveon’s X3

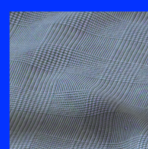


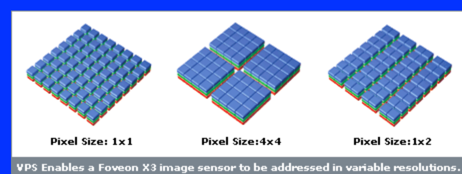


smarter pixels

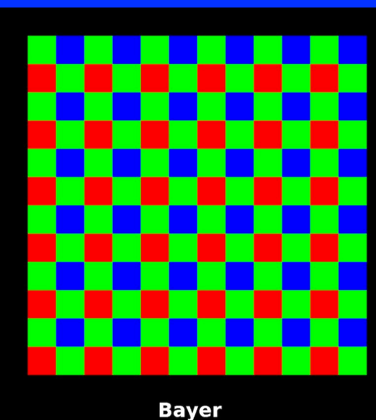
better image quality

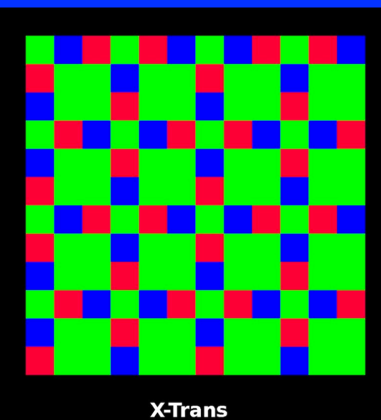






FujiFilm X-Trans





And rumor mill says that Cannon may come out with a 120MP sensor with X3-like technology in 2018.

Light at surfaces

Many effects when light strikes a surface -- could be:

- transmitted
  - Skin, glass
- reflected
  - mirror
- scattered
  - milk
- travel along the surface and leave at some other point
- absorbed
  - sweaty skin

Assume that

- surfaces don't fluoresce
  - e.g. scorpions, detergents
- surfaces don't emit light (i.e. are cool)
- all the light leaving a point is due to that arriving at that point

BRDF

With assumptions in previous slide

- **Bi-directional Reflectance Distribution Function**

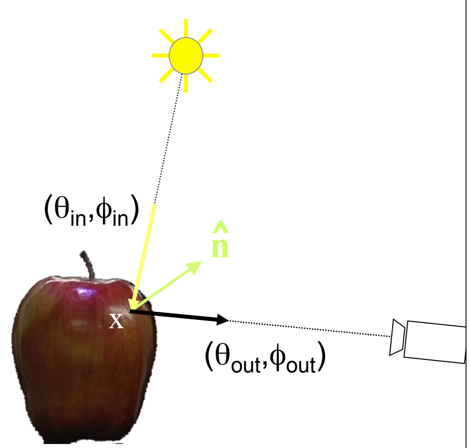
  $\rho(\theta_{in}, \phi_{in}\,;\,\theta_{out}, \phi_{out})$

- **Ratio of emitted radiance to incident irradiance (units: sr^{-1})**

- **In local coordinate system at x**
  - **Incoming light direction:**

    $\theta_{in}\,,\,\phi_{in}$
  - **Outgoing light direction:**

    $\theta_{out}\,,\,\phi_{out}$

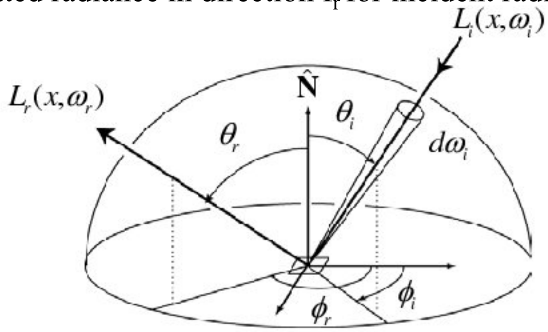


$$\rho\left(x;\theta_{in},\phi_{in};\theta_{out},\phi_{out}\right)=\frac{L_o\left(x;\theta_{out},\phi_{out}\right)}{L_i\left(x;\theta_{in},\phi_{in}\right)\cos\theta_{in}d\omega}$$

Where ρ is sometimes denoted f_r.

The Reflection Equation

Emitted radiance in direction f_r for incident radiance L_i.



$$L_r(x,\omega_r) = \int_{H^2} f_r(x,\omega_i \to \omega_r) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

where $\omega_i=(\theta_i, \phi_i)$

CS348B Lecture 10 Pat Hanrahan, Spring 2002

Properties of BRDFs

1. Non-negative: $\rho(\theta_{in}, \phi_{in}\,; \theta_{out}, \phi_{out}) \geq 0$

2. Helmholtz Reciprocity Principle:
   $\rho(\theta_{in}, \phi_{in}\,; \theta_{out}, \phi_{out}) = \rho(\theta_{out}, \phi_{out}\,; \theta_{in}, \phi_{in})$

3. Total energy leaving a surface must be less than total energy arriving at the surface

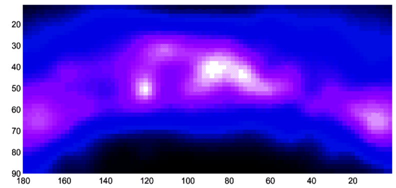
$$\int_{\Omega_i} L_i(\mathbf{x},\theta_i,\phi_i)\cos\theta_i \, d\omega_i \geq \int_{\Omega_o}\left[\int_{\Omega_i} \rho\left(\theta_i,\phi_i;\theta_0,\phi_o\right) L_i(\mathbf{x},\theta_i,\phi_i)\cos\theta_i \, d\omega_i\right]\cos\theta_o d\omega_o$$

Surface Reflectance Models

Common Models

- Lambertian
- Phong
- Physics-based
  - Specular [Blinn 1977], [Cook-Torrance 1982], [Ward 1992]
  - Diffuse [Hanrahan, Kreuger 1993]
  - Generalized Lambertian [Oren, Nayar 1995]
  - Thoroughly Pitted Surfaces [Koenderink et al 1999]
- Phenomenological
  - [Koenderink, Van Doorn 1996]

Arbitrary Reflectance



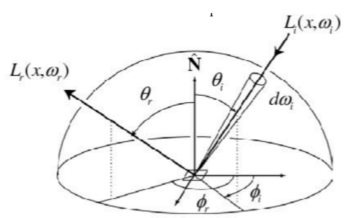
- Non-parametric model
- Anisotropic
- Non-uniform over surface
- BRDF Measurement *[Dana et al, 1999], [Marschner]*

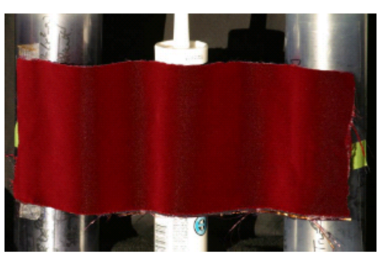
Specialized

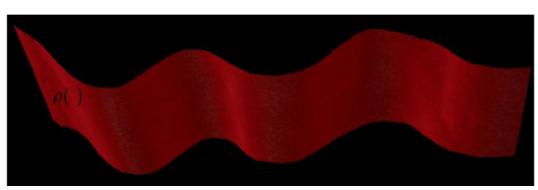
- Hair, skin, threads, paper [Jensen et al]
- Fur

Important class of BRDFs: Isotropic BRDF

$$\rho(\theta_i,\phi_i;\theta_o,\phi_o) = \rho_r(\theta_i,\theta_o,\phi_i - \phi_o)$$

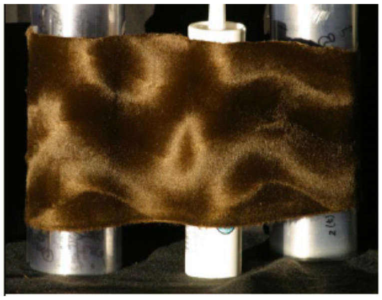






Isotropic BRDF's are symmetric about the surface normal. If the surface is rotated about the normal for the same incident and emitting directions, the value of the BRDF is the same.

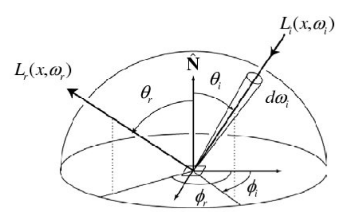
Anisotropic BRDF



Lambertian (Diffuse) Surface

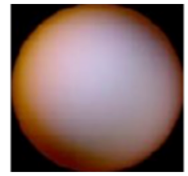
- BRDF is a constant called the albedo. $\rho\left(\underline{x};\theta_{in},\phi_{in};\theta_{out},\phi_{out}\right) = K$
- Emitted radiance is NOT a function of outgoing direction – i.e. constant in all directions.
- For lighting coming in single direction ω_ι, emitted radiance is proportional to cosine of the angle between normal and light direction

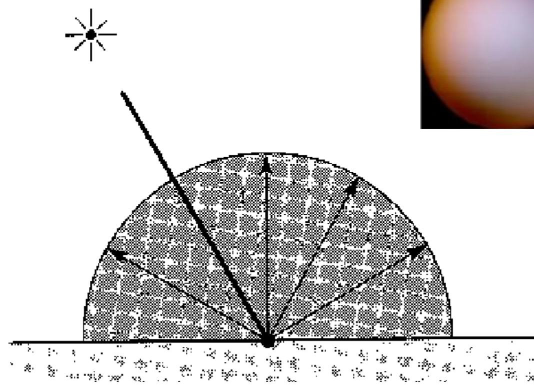
$$L_r = K\hat{N}\cdot\omega_i$$



$$L_r(x,\omega_r) = \int_{H^2} f_r(x,\omega_i \to \omega_r) L_i(x,\omega_i)\cos\theta_i \, d\omega_i$$

Lambertian reflection

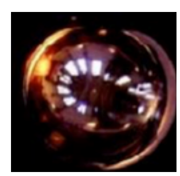


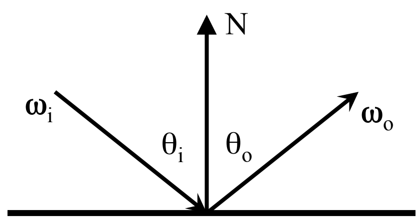


- Lambertian
- Matte
- Diffuse

Light emitted equally in all directions.

Specular Reflection: Smooth Surface

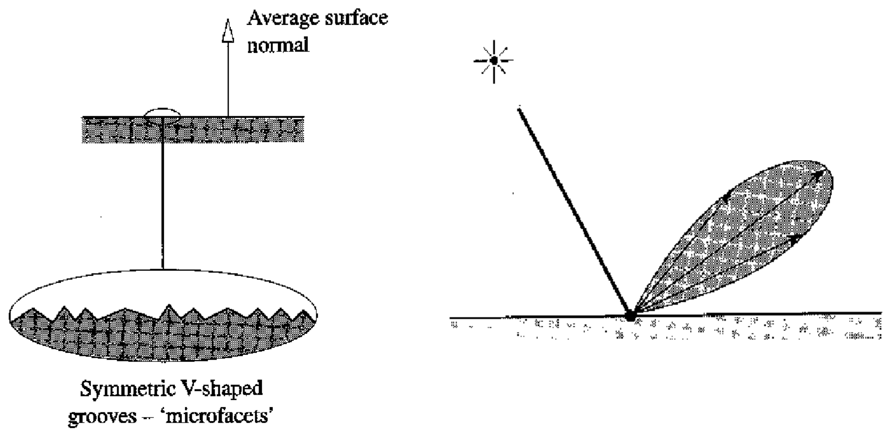




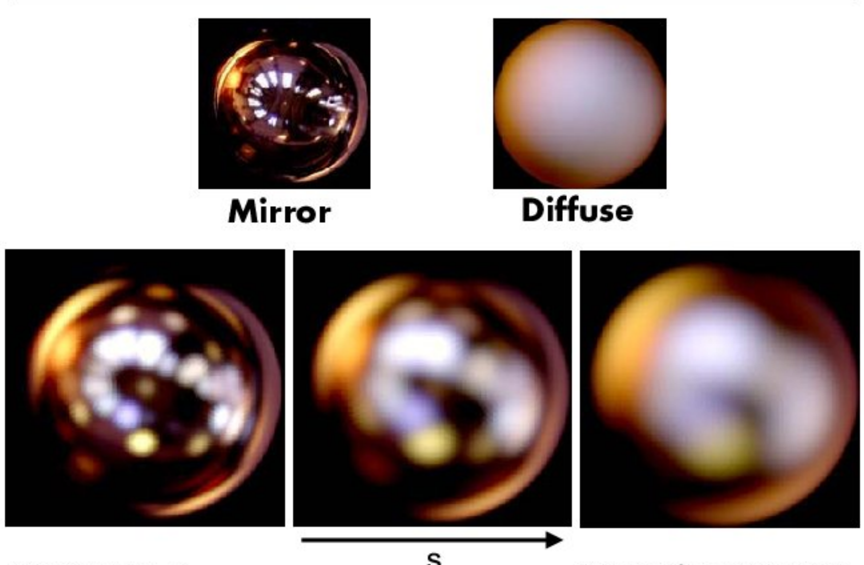
- N, ω_i, ω_o are coplanar
- $\theta_i = \theta_o$

Speculum – Latin for "Mirror"

Rough Specular Surface



Phong Model



Non-Lambertian reflectance

General BRDF: e.g. Velvet

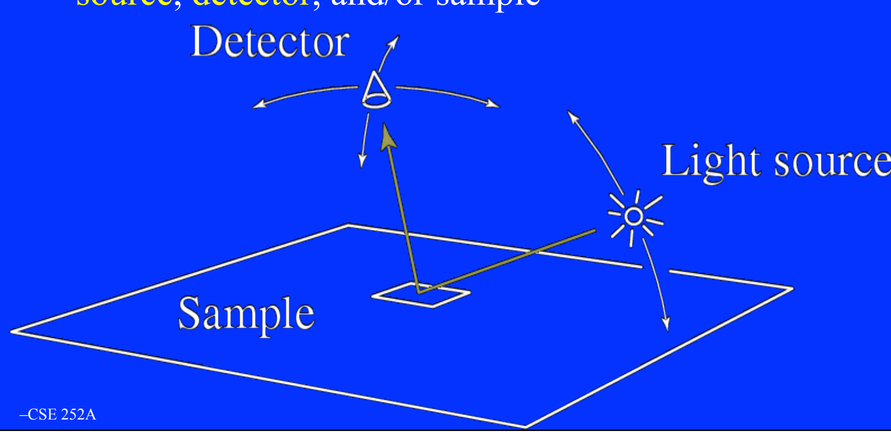
Portrait of Sir Thomas Morre, Hans Holbein the Younger, 1527

[After Koenderink et al, 1998]

Ways to measure BRDFs

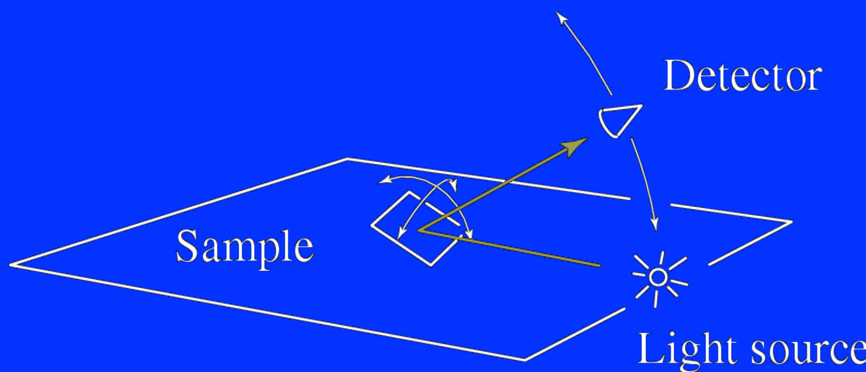
Gonioreflectometers

- Three degrees of freedom spread among light source, detector, and/or sample



Gonioreflectometers

- Three degrees of freedom spread among light source, detector, and/or sample



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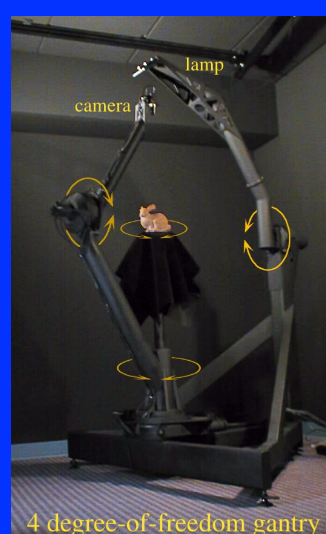
Gonioreflectometers

- Can add fourth degree of freedom to measure anisotropic BRDFs



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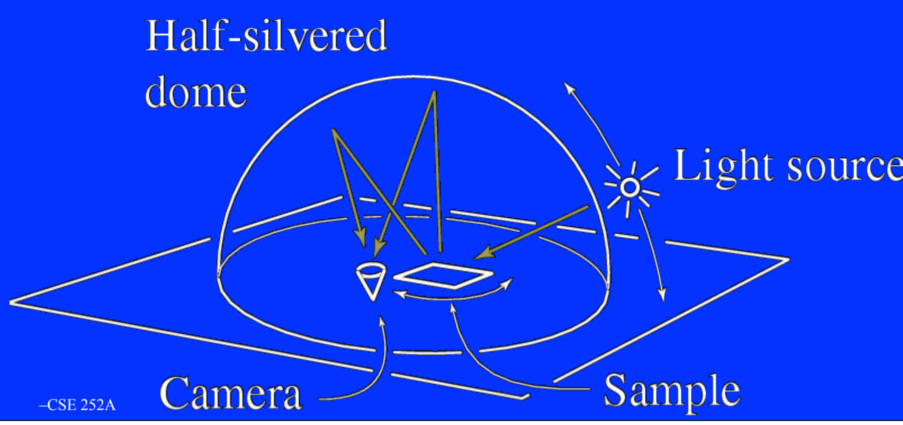
Ward's BRDF Measurement Setup

- Collect reflected light with hemispherical (should be ellipsoidal) mirror [SIGGRAPH 92]



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Ward's BRDF Measurement Setup

- Result: each image captures light at all exitant angles

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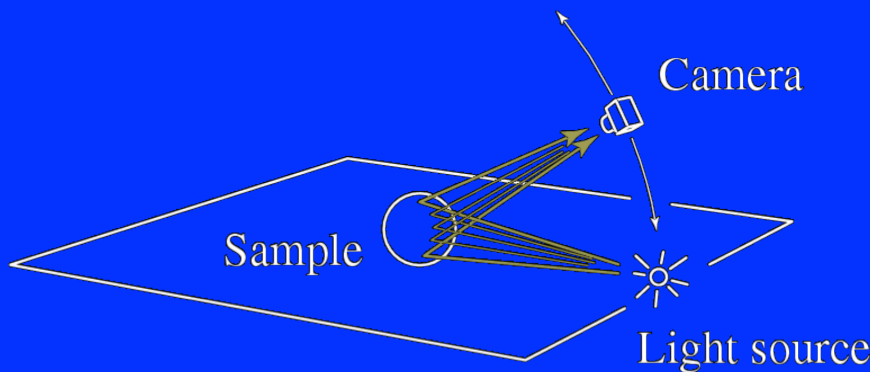
Marschner's Image-Based BRDF Measurement

- For uniform BRDF, capture 2-D slice corresponding to variations in normals



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BRDF Not Always Appropriate

BRDF

BSSRDF

http://graphics.stanford.edu/papers/bssrdf/

(Jensen, Marschner, Levoy, Hanrahan)

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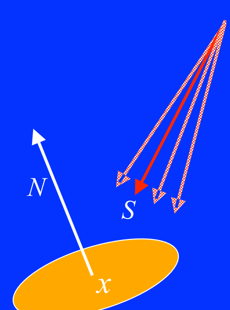
Light sources and shading

- A light source is a power source that emits light (instead of reflecting it)
- Laser – a single ray
- Point source – like a light bulb
- Line source – fluorescent light bulb
- Area source

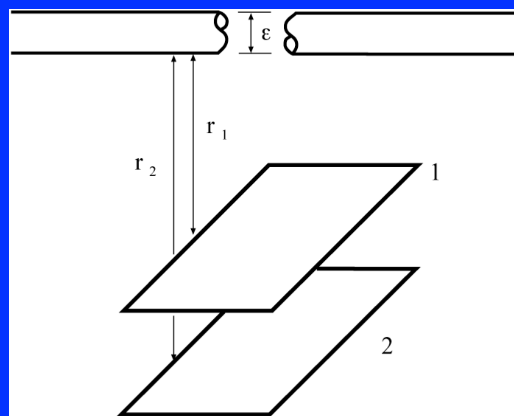
Nearby point source model

$$L = \rho_d(x)\left(\frac{\hat{N}(x) \bullet S(x)}{r(x)^2}\right)$$

- N is the surface normal
- ρ_d is diffuse (Lambertian) albedo
- S is source vector - a vector from x to the source, whose length is the intensity term



Nearby Line Source

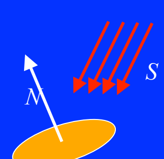


Intensity due to line source varies with inverse distance, if the source is long enough

Distant Point Source Model

- Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn't vary much, and the distance doesn't vary much either, and we can roll the constants together to get:

$$\boxed{L = \rho_d(x) N(x) \bullet S(x)}$$

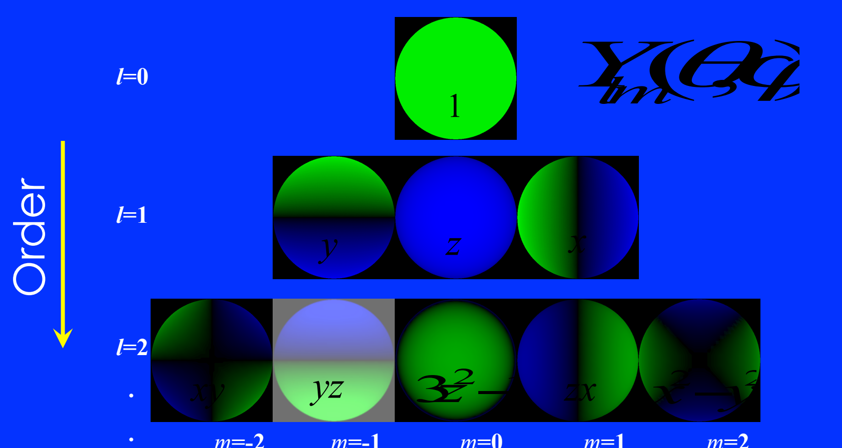


- N is the surface normal
- ρ_d is diffuse (Lambertian) albedo
- S is source vector - a vector in source direction, whose length is the intensity term

Lighting at infinity

- Direction is a three vector **s**, with $|s| = 1$.
- Described as function on a sphere: radiance as a function of direction r(**s**)
- Single point source is a delta function at some direction
- Multiple point sources: sum of delta functions

Diffuse lighting at infinity: Spherical Harmonics

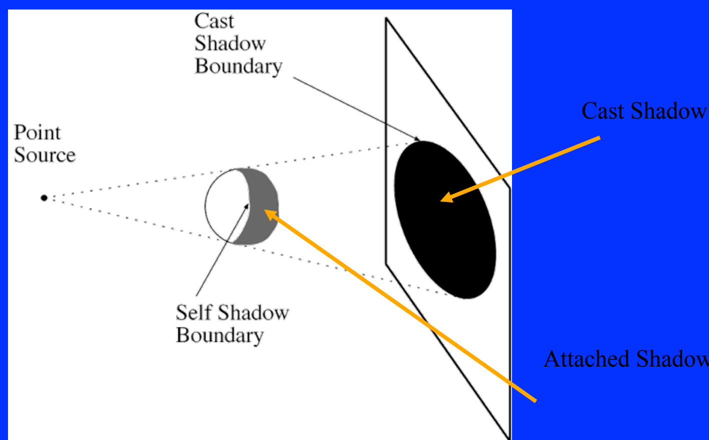


(Borrowed from: Ramamoorthi, Hanrahan, SIGGRAPH'01)

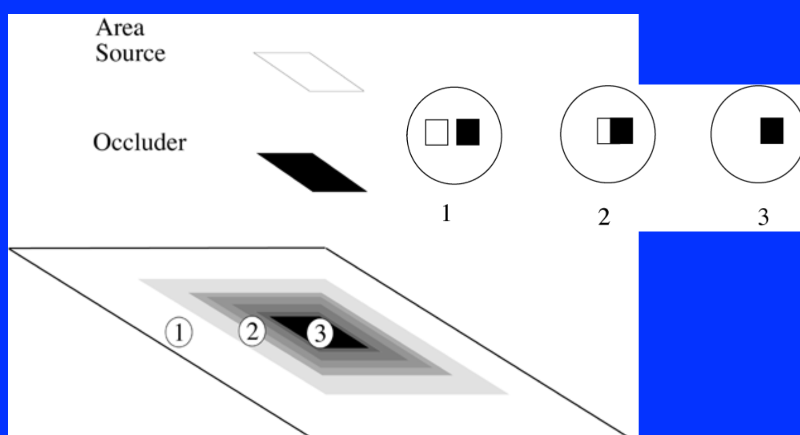
Green: Positive
Blue: Negative

Shadows cast by a point source

- A point that can't see the source is in shadow
- For point sources, the geometry is simple



Area Source Shadows



1. Fully illuminated
2. Penumbra
3. Umbra (shadow)

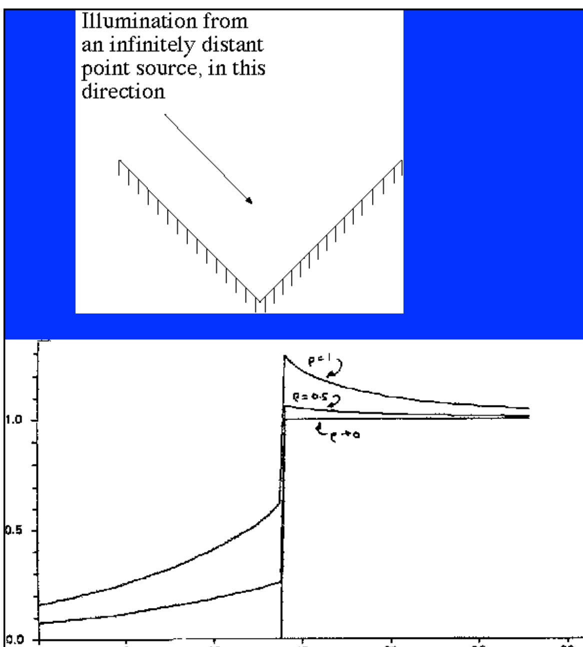
Shading models

Local shading model

- Surface has incident radiance due only to sources visible at each point
- Advantages:
  - often easy to manipulate, expressions easy
  - supports quite simple theories of how shape information can be extracted from shading
- Used in vision & real time graphics

Global shading model

- surface radiosity is due to radiance reflected from other surfaces as well as from surfaces
- Advantages:
  - usually very accurate
- Disadvantage:
  - extremely difficult to infer anything from shading values
- Rarely used in vision, often in photorealistic graphics



At the top, geometry of a gutter with triangular cross-section; below, predicted radiosity solutions, scaled to lie on top of each other, for different albedos of the geometry. When albedo is close to zero, shading follows a local model; when it is close to one, there are substantial reflexes.

Figure from "Mutual Illumination," by D.A. Forsyth and A.P. Zisserman, Proc. CVPR, 1989, copyright 1989 IEEE

Irradiance observed in an image of this geometry for a real white gutter.

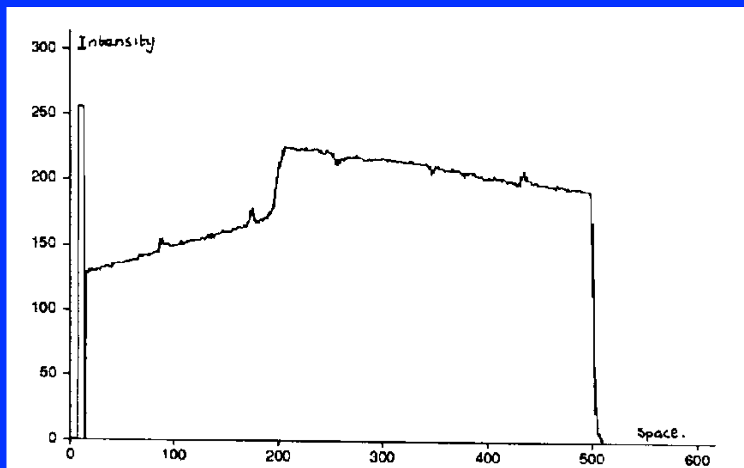


Figure from "Mutual Illumination," by D.A. Forsyth and A.P. Zisserman, Proc. CVPR, 1989, copyright 1989 IEEE

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