CSE 250B Winter 2019 - Quiz 5

Feb 22, 2019 - 20 Minutes **Name:** **PID:**

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(1) This is a closed book, closed notes exam. Switch off your cell phone and do not communicate with anyone other than an exam proctor.
(2) Start writing when instructed. Stop writing when your time is up.
(3) Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

Consider the following optimization problem:

$$\begin{aligned} \min \quad & x + 5y \\ \text{subject to:} \quad & xy = 4 \\ & x \geq 0, y \geq 0 \end{aligned}$$

(1) (2 Points) First, solve the optimization problem by using the substitution method (eg, by substituting $y = 4/x$ in the equality constraint.) What is the optimal value of the objective function?

Solution: $x = 2\sqrt{5}$, $y = 2/\sqrt{5}$. Optimal solution value is $2\sqrt{5} + 2\sqrt{5} = 4\sqrt{5}$.

(2) (8 Points) Write down the Lagrangean for the optimization problem. Write down all the KKT conditions. Use the KKT conditions and the optimal solution to solve for the values of the Lagrangean multipliers. Where needed, justify your answer.

The Lagrangean is: $L(x, \lambda, \nu) = x + 5y - \lambda_1 x - \lambda_2 y + \nu(xy - 4)$. The KKT conditions are:

(1) $$1 - \lambda_1 + \nu y = 0$$

(2) $$5 - \lambda_2 + \nu x = 0$$

(3) $$xy = 4$$

(4) $$x \geq 0, \quad y \geq 0$$

(5) $$\lambda_1 \geq 0, \quad \lambda_2 \geq 0$$

(6) $$\lambda_1 x = 0$$

(7) $$\lambda_2 y = 0$$

(8) $$\nu(xy - 4) = 0$$

Due to complementary slackness, and because $x, y \geq 0$ at the optimal solution, $\lambda_1 = \lambda_2 = 0$. Moreover, $\nu = -1/y^* = -\sqrt{5}/2$.