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- (1) This is a closed book, closed notes exam. Switch off your cell phone and do not communicate with anyone other than an exam proctor.
 - (2) Start writing when instructed. Stop writing when your time is up.
 - (3) Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

State whether the following functions are kernels or not. If they are kernels, write down the corresponding feature space. If they are not, provide a proof. In both cases, x and z are real d -dimensional vectors.

- (1) (5 points) $K(x, z) = \|x\|^2 + \|z\|^2$. Not a kernel. To show this, we pick $x = [0]$, $z = [2]$. The kernel matrix is:

$$M = \begin{bmatrix} 0 & 4 \\ 4 & 4 \end{bmatrix}$$

For a column vector $u = [u_1, u_2]$, $u^\top M u = 8u_1u_2 + 4u_2^2 < 0$ for $u_2 = 1$ and $u_1 = -1$. Thus the kernel matrix is not positive semidefinite and hence K is not a kernel.

- (2) (5 points) $K(x, z) = e^{\|x\|^2 + \|z\|^2}$.

$K(x, z)$ is a kernel, with feature space $\phi(x) = [e^{\|x\|^2}]$.