CSE250A HW2

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Section: A

2.1 Probabilistic inference

(a)

$$\begin{aligned}P(A=1) &= P(A=1,E=0,B=0)+P(A=1,E=0,B=1)+P(A=1,E=1,B=0)+P(A=1,E=1,B=1)\\ &= P(A=1|E=0,B=0)P(E=0)P(B=0)+P(A=1|E=0,B=1)P(E=0)P(B=1)\\ &+ P(A=1|E=1,B=0)P(E=1)P(B=0)+P(A=1|E=1,B=1)P(E=1)P(B=1)\\ &= 0.0025\end{aligned}$$

$$P(A=0)=1-P(A=1)=0.9975$$

$$\begin{aligned}P(J=1) &= P(J=1,A=0)+P(J=1,A=1)\\ &= P(J=1|A=0)P(A=0)+P(J=1|A=1)P(A=1)\\ &= 0.0521\end{aligned}$$

$$\begin{aligned}P(A=0|E=1) &= P(A=0|B=0,E=1)P(B=0)+P(A=0|B=1,E=1)P(B=1)\\ &= (1-0.29)(1-0.001)+(1-0.95)0.001\\ &= 0.7093\end{aligned}$$

$$P(A=1|E=1)=0.2907$$

Therefore,

$$\begin{aligned}P(E=1|J=1) &= \frac{0.7093\times 0.002}{0.9975}\cdot\frac{0.05\times 0.9975}{0.0521}+\frac{0.2907\times 0.002}{0.0025}\cdot\frac{0.09\times 0.0025}{0.0521}\\ &= 0.0116\end{aligned}$$

(b)

$$\begin{aligned}P(E=1|J=1,B=1) &= \frac{P(J=1|B=1,E=1)P(E=1|B=1)}{P(J=1|B=1)}\\ &= \frac{\sum_a P(J=1,A=a|B=1,E=1)P(E=1)}{\sum_{a,e} P(J=1,A=a,E=e|B=1)}\ (marginalization)\end{aligned}$$

Numerator:

$$\begin{aligned}&\sum_a P(J=1,A=a|B=1,E=1)P(E=1)\\ &= P(E=1)\times[P(J=1|A=0,B=1,E=1)P(A=0|B=1,E=1)+P(J=1|A=1,B=1,E=1)P(A=1|B=1,E=1)]\\ &= P(E=1)\times[P(J=1|A=0)P(A=0|B=1,E=1)+P(J=1|A=1)P(A=1|B=1,E=1)]\\ &= 0.002\times 0.8575\\ &= 0.0017\end{aligned}$$

Denominator:

$$
\begin{aligned}
&\sum_{a,e} P(J=1, A=a, E=e|B=1) \\
&= P(J=1, A=0, E=0|B=1) + P(J=1, A=0, E=1|B=1) \\
&+ P(J=1, A=1, E=0|B=1) + P(J=1, A=1, E=1|B=1) \\
&= P(J=1|A=0, E=0, B=1)P(A=0|E=0, B=1)P(E=0) \\
&+ P(J=1|A=1, E=0, B=1)P(A=1|E=0, B=1)P(E=0) \\
&+ P(J=1|A=0, E=1, B=1)P(A=0|E=1, B=1)P(E=1) \\
&+ P(J=1|A=1, E=1, B=1)P(A=1|E=1, B=1)P(E=1) \\
&= P(J=1|A=0)P(A=0|E=0, B=1)P(E=0) \\
&+ P(J=1|A=1)P(A=1|E=0, B=1)P(E=0) \\
&+ P(J=1|A=0)P(A=0|E=1, B=1)P(E=1) \\
&+ P(J=1|A=1)P(A=1|E=1, B=1)P(E=1) \\
&= 0.8490
\end{aligned}
$$

Therefore,

$P(E=1|J=1, B=1) = \frac{0.0017}{0.8490} = 0.0020$

(c)

$$
\begin{aligned}
P(A=1|M=0) &= \frac{P(M=0|A=1)P(A=1)}{P(M=0)} \\
&= \frac{(1-P(M=1|A=1))P(A=1)}{P(M=0)} \\
&= \frac{0.3 \times 0.0025}{0.9883} \\
&= 0.00075887
\end{aligned}
$$

(d)

$$
\begin{aligned}
P(A=1|J=0, M=0) &= \frac{P(M=0, J=0|A=1)P(A=1)}{P(J=0, M=0)} \\
&= \frac{P(J=0|A=1)P(M=0|A=1)P(A=1)}{\sum_a P(J=0, M=0, A=a)} \\
&= \frac{0.1 \times 0.3 \times 0.0025}{0.95 \times 0.99 \times 0.9975 + 0.1 \times 0.3 \times 0.0025} \\
&= 0.00008
\end{aligned}
$$

(e)

$$
\begin{aligned}
P(M=0) &= P(M=0, A=0) + P(M=0, A=1) \\
&= P(M=0|A=0)P(A=0) + P(M=0|A=1)P(A=1) \\
&= (1-0.01)0.9975 + (1-0.7)0.0025 \\
&= 0.9883
\end{aligned}
$$

$P(M=1) = 0.0117$

$$
\begin{aligned}
P(A=1|M=1) &= \frac{P(M=1|A=1)P(A=1)}{P(M=1)} \\
&= \frac{0.70 \times 0.0025}{0.0117} \\
&= 0.1496
\end{aligned}
$$

(f)

$$
\begin{aligned}
P(A=1|M=1,B=0) &= \frac{P(M=1,B=0|A=1)P(A=1)}{P(M=1,B=0)} \\
&= \frac{P(M=1|A=1)P(B=0|A=1)P(A=1)}{\sum_a P(M=1,B=0|A=a)P(A=a)} \\
&= \frac{P(M=1|A=1)P(B=0|A=1)P(A=1)}{P(M=1|A=0)P(B=0|A=0)P(A=0)+P(M=1|A=1)P(B=0|A=1)P(A=1)}
\end{aligned}
$$

$$
\begin{aligned}
P(B=0|A=0) &= P(B=0,E=0|A=0)+P(B=0,E=1|A=0) \\
&= \frac{P(A=0|B=0,E=0)P(B=0)P(E=0)}{P(A=0)}+\frac{P(A=0|B=0,E=1)P(B=0)P(E=1)}{P(A=0)} \\
&= \frac{(1-0.001)(1-0.001)(1-0.002)}{0.9975}+\frac{(1-0.29)(1-0.001)0.002}{0.9975} \\
&= 0.999923
\end{aligned}
$$

$$
\begin{aligned}
P(B=0|A=1) &= \frac{(0.001)(1-0.001)(1-0.002)}{0.0025}+\frac{(0.29)(1-0.001)0.002}{0.0025} \\
&= 0.6306
\end{aligned}
$$

Therefore,

$$P(A=1|M=1,B=0)=0.09962$$

Therefore,

Compared the result of (b) versus (a), it seems consistent with the commonsense pattern of reasoning, which means the prob will be less when John calls.

Compared the result of (d) versus (c), the result is consistent with the commonsense pattern, which means the prob will be less when John makes calls.

Compared the result of (f) versus (e), the result is consistent with the commonsense patern, which means the prob will be less when actually there's no burglar.

2.2 Probabilistic reasoning

(a)

Numerator of r_k:

$$
\begin{aligned}
P(D=1|S_1=1,S_2=1,\ldots S_k=1) &= \frac{P(S_1=1,S_2=1,\ldots,S_k=1|D=1)}{P(S_1=1,S_2=1,\ldots,S_k=1)}\ (CI2) \\
&= \frac{\prod_{k=1} P(S_k=1|D=1)}{P(S_1=1,S_2=1,\ldots,S_k=1)} \\
&= \frac{1\times\frac{f(1)}{f(2)}\times\frac{f(2)}{f(3)}\times\ldots\times\frac{f(k-1)}{f(k)}}{P(S_1=1,S_2=1,\ldots,S_k=1)} \\
&= \frac{\frac{f(1)}{f(k)}}{P(S_1=1,S_2=1,\ldots,S_k=1)} \\
&= \frac{\frac{1}{2^k+(-1)^k}}{P(S_1=1,S_2=1,\ldots,S_k=1)}
\end{aligned}
$$

Denominator of r_k:

$$
\begin{aligned}
P(D=0|S_1=1,S_2=1,\dots S_k=1) &= \frac{P(S_1=1,S_2=1,\dots,S_k=1|D=0)}{P(S_1=1,S_2=1,\dots,S_k=1)}(CI2) \\
&= \frac{P(S_1=1|D=0)P(S_2=1|D=0)\dots P(S_k=1|D=0)}{P(S_1=1,S_2=1,\dots,S_k=1)} \\
&= \frac{\frac{1}{2^k}}{P(S_1=1,S_2=1,\dots,S_k=1)}
\end{aligned}
$$

Then

$$
\begin{aligned}
r_k &= \frac{\frac{1}{2^k+(-1)^k}}{\frac{1}{2^k}} \\
&= \frac{2^k}{2^k+(-1)^k}
\end{aligned}
$$

Therefore, when k is an odd integer, $r_k = \frac{2^k}{2^k-1} > 1$, and the patient will be diagnose as $D=1$ form.

When k is an even integer, $r_k = \frac{2^k}{2^k+1} < 1$,and the patient will be diagnose as $D=0$ form.,

(b)

As k values becomes larger, it means $k \to \infty$. Then

When k is and odd number, $r_k = \frac{1}{1-\frac{1}{2^k}}$.

Then r_k will be 1.

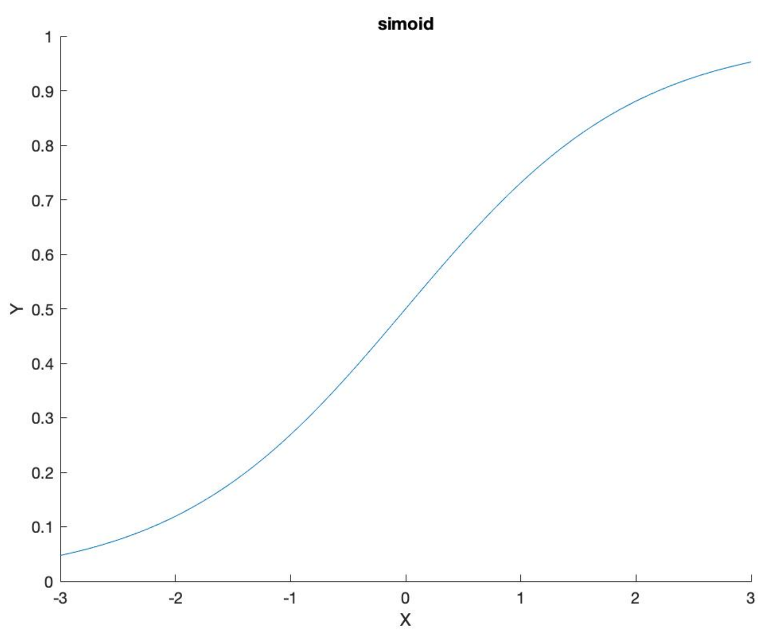
Likewise,

r_k will also be 1 when k is an even number.

Therefore, the result will be less certain when the k goes up.

2.3 Sigmoid function

(a)



$$\begin{aligned}\sigma^{-1}(z) &= \frac{0 \times (1+e^{-z}) + 1 \times (e^{-z})}{(1+e^{-z})^2} \\ &= \frac{e^{-z}}{(1+e^{-z})^2}\end{aligned}$$

At the same time

$$\begin{aligned}\sigma(z) \times \sigma(-z) &= \frac{1}{1+e^{-z}} \times \frac{1}{1+e^{z}} \\ &= \frac{1}{1+e^{-z}} \times \frac{e^{-z}}{e^{-z}+1} \\ &= \frac{e^{-z}}{(1+e^{-z})^2}\end{aligned}$$

Therefore, it concludes that $\sigma^{-1}(z) = \sigma(z) \times \sigma(-z)$

(b)

$$\begin{aligned}\sigma(z) + \sigma(-z) &= \frac{1}{1+e^{-z}} + \frac{1}{1+e^{z}} \\ &= \frac{1}{1+e^{-z}} + \frac{e^{-z}}{e^{-z}+1} \\ &= \frac{1+e^{-z}}{1+e^{-z}} = 1\end{aligned}$$

(c)

$$\begin{aligned}L(\sigma(z)) &= log(\frac{\sigma(z)}{1-\sigma(z)}) \\ &= log(\frac{\frac{1}{1+e^{-z}}}{1-\frac{1}{1+e^{-z}}}) \\ &= log(\frac{1}{1+e^{-z}-1}) \\ &= log(\frac{1}{e^{-z}}) \\ &= log(e^{z}) = z\end{aligned}$$

(d)

Since $p_i = P(Y=1|X_i, X_j = 0 \ for \ all \ j \neq i)$,

Therefore $p_i = \sigma(w_i)$.

Furthur,

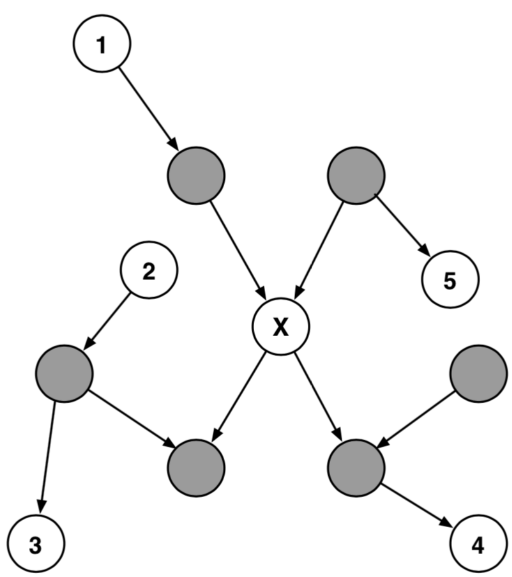
$L(p_i) = L(\sigma(w_i)) = w_i$ (according to the conclusion in problem (c))

2.4 Conditional independence

Proof:

X	Y	Z
month	water	{sprinkler,rain}
month	water	{sprinkler,rain,accidenct}
month	accident	{sprinkler,water}
month	accident	{rain,water}
month	accident	{sprinkler,rain}
month	accident	{water}
month	accident	{sprinkler,rain,water}
sprinkler	rain	{month}
sprinkler	accident	{water}
sprinkler	accident	{water, month}
sprinkler	accident	{water, rain}
sprinkler	accident	{water, month, rain}
rain	accident	{water}
rain	accident	{water, month}
rain	accident	{water, rain}
rain	accident	{water, month, rain}

2.5 Markov blanket



Proof:

There're 5 cases for node Y.

Case1: Node Y is the parent of X's parent

For example, node 1 meets this case. The right top gray node is the evidence. Therefore, d-seperation case 1 is satisfied, where left top gray node is flowed through.(node1=> evidence => nodeX)

Case2: Node Y is the child of X's parent

For example, node 5 meets this case. The right top gray node is the evidence. Therefore, d-seperation case 2 is satisfied, where right top gray node is diversed.(node5 <= evidence => nodeX)

Case3: Node Y is the parent of X's child's parent

For example, node 2 meets this case. There's a path satisfying conditional independence case1, where an evidence node is flowed through.(node2 => **nodex's child's parent** => nodex's child => nodeX)

Case4: Node Y is the child of X's child's parent

On the contrary, node 3 is the child of node x's child's parent. So there's a path satisfying conditional independence case2, where an evidence node is diversed.(node2 <= **nodex's child's parent** => nodex's child => nodeX)

Case5: Node Y is the child of X's child

Node 4 is the child of node x's child. So there's a path satisfying conditional independence case1, where an evidence node is flowed through.(nodex => **nodex's child** => node4)

All in all, no matter node Y meets which case, it must have $P(X,Y|BX) = P(X|B_X)P(Y|B_X)$.

2.6 True or false

Value	Formula
True	$P(C,D|A) = P(A|S)$
False	$P(A|D) = P(A|B,D)$
True	$P(C,E) = P(C)P(E)$
False	$P(C,D,E) = P(C)P(D)P(E)$
False	$P(F,G) = P(F)P(G)$
True	$P(F,G|D) = P(F|D)P(G|D)$
False	$P(A,D,G) = P(A)P(D|A)P(G|D)$
False	$P(B|E) = P(B|E,G)$
False	$P(C|E) = P(C|E,G)$

2.7 Subsets

(a)

$P(A|C) = P(A|C,E,F,B,D,G)$ $S = \{C,E,F,B,D,G\}$

(b)

$P(C) = P(C|B,D,G)$ $S = \{B,D,G\}$

(c)

$P(C|A) = P(C|A, B, D, G)$ $S = \{A, B, D, G\}$

(d)

$P(C|A, E) = P(C|A, E, D, B, G)$ $S = \{A, E, D, B, G\}$

(e)

$P(C|A, E, F) = P(C|A, E, F)$ $S = \{A, E, F\}$

(f)

$P(C|A, D, E, F) = P(C|A, D, E, F, G)$ $S = \{A, D, E, F, G\}$

(g)

$P(F) = P(F)$ $S = \{\}$

(h)

$P(F|C) = P(F|C, E)$ $S = \{C, E\}$

(i)

$P(F|C, D) = P(F|C, D, E, G)$ $S = \{C, D, E, G\}$

(j)

$P(B, G) = P(B, G|A, C, E)$ $S = \{A, C, E\}$

2.8 Noisy-OR

(a)

$left = 1 - 1 = 0$

$right = 1 - (1 - p_y) = p_y$

$\therefore left < right$

(b)

$left = 1 - (1 - p_x) = p_x$

$right = 1 - (1 - p_y) = p_y$

$\therefore left < right$

(c)

$left = 1 - (1 - p_x) = p_x$

$right = 1 - (1 - p_x)(1 - p_y) = (p_x + p_y - p_x p_y)$

$right - left = p_y - p_x p_y = p_y(1 - p_x) > 0$

$\therefore left < right$

(d)

$left = right$ (d-seperation III)

(e)

$left < right$

(f)

$left > right$

(g)

$$\begin{aligned}right &= P(X=1,Y=1,Z=1)\\ &= P(Z=1|X=1,Y=1)P(X=1,Y=1)\\ &= P(Z=1|X=1,Y=1)P(X=1)P(Y=1)\end{aligned}$$

$$left = P(Z=1)P(X=1)P(Y=1)$$

Compared to no evidence, it is more likely to make $Z=1$ when $X=1,Y=1$.

Therefore, $left < right$.

2.9 Polytree inference

(a)

$$P(B) = \sum_{a} P(B|A=a)P(A=a)$$

(b)

$$\begin{aligned}P(D|C) &= \sum_{b} P(D|B=b,C)P(B=b|C)\\ &= \sum_{b} P(D|B=b,C)P(B=b)\end{aligned}$$

Insert the result from (a).

$P(D|C)=\sum_{b}{P(D|B=b,C)[\sum_{a}{P(B|A=a)P(A=a)}]}$

(c)

$$\begin{aligned}P(F|C,E) &= \sum_{d} P(F,D=d|C,E)\\ &= \sum_{d} P(F|C,D=d,E)P(D=d|C,E)\\ &= \sum_{d} P(F|D=d,E)\{\sum_{b} P(D|B=b,C)[\sum_{a} P(B|A=a)P(A=a)]\}\end{aligned}$$

(d)

$$\begin{aligned}P(E|C,F) &= \frac{P(F|C,E)P(E|C)}{P(F|C)}\\ &= \frac{P(F|C,E)P(E)}{P(F|C)}\end{aligned}$$

$$
\begin{aligned}
P(F|C) &= \sum_e P(F, E=e|C) \\
&= \sum_e P(F|C, E=e)P(E=e|C) \\
&= \sum_e P(F|C, E=e)P(E=e)
\end{aligned}
$$

Therefore,

$P(E|C,F)=\frac{\{\sum_d{P(F|D=d,E)\{\sum_{b}{P(D|B=b,C)[\sum_{a}{P(B|A=a)P(A=a)}]\}\}}}\times P(E)}{\sum_{e}\ {{\sum_d{P(F|D=d,E)\{\sum_{b}{P(D|B=b,C)[\sum_{a}{P(B|A=a)P(A=a)}]\}}}\}P(E=e)}}$

(e)

In order to calculate the result of $P(E|C,F)$, we need to traverse n possible values for variable E to calculate $P(F|C)$. Then in order to calcualte $P(F|C)$, we need to traverse n possible values for variable D in term $P(F|C,E)$. Further, we need to traverse n possible values for variable B in calculating $P(D=d|C)$. Finally, we need to traverse n possibilities for variable A. The whole process is recursive from the top to the bottom.

All in all, the calculation complexity is polynomial and the degree is 4.