

- (1) This is a closed book, closed notes exam. Switch off your cell phone and do not communicate with anyone other than an exam proctor.
- (2) Start writing when instructed. Stop writing when your time is up.
- (3) Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

Let v_1, \dots, v_k be k vectors in \mathbb{R}^d . Recall that the subspace spanned by v_1, \dots, v_k is defined as the set of all vectors $\sum_{i=1}^k c_i v_i$ where c_i 's are *any* scalars. The cone spanned by v_1, \dots, v_k is defined as the set of all vectors $\sum_{i=1}^k c_i v_i$ where the c_i 's are *positive* scalars.

Suppose we are given training data with binary ± 1 labels, and we are using the Perceptron algorithm to train a classifier $\text{sign}(w^\top x + b)$ on training data $(x^{(1)}, y^{(1)}), \dots, (x^{(n)}, y^{(n)})$.

State whether following statements are true or false, and justify your answer in each case.

- (1) (4 points) Does w lie in the subspace spanned by $x^{(1)}, \dots, x^{(n)}$?

True. From the perceptron algorithm, note that we begin with $w_1 = 0$, and update w_t as $w_t = w_t + y^{(t)} x^{(t)}$; this means that the w at the end can be written as:

$$w = \sum_{j \in I} y^{(j)} x^{(j)}$$

where I is the set of indices where updates are made. Thus w is a linear combination of the $x^{(i)}$'s and therefore it lies in the subspace spanned by the $x^{(i)}$'s.

- (2) (3 points) Does w always lie in the cone spanned by $x^{(1)}, \dots, x^{(n)}$?

False. This is because some of the $y^{(i)}$'s may be -1 .

- (3) (3 points) Does w lie in the cone spanned by $y^{(1)} x^{(1)}, \dots, y^{(n)} x^{(n)}$?

True – for this, $c_i = 1$.