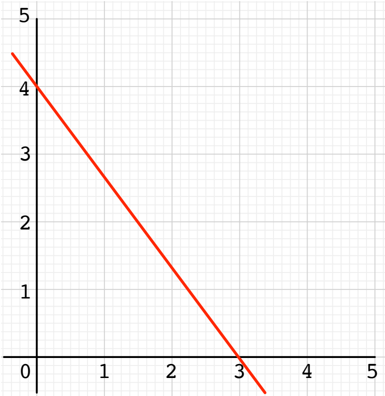
A simple linear classifier

CSE 250B

Linear decision boundary for classification: example



- What is the formula for this boundary?
- What label would we predict for a new point x?

Linear classifiers

Binary classification: data $x \in \mathbb{R}^d$ and labels $y \in \{-1, +1\}$

- Linear classifier:
  - Parameters: $w \in \mathbb{R}^d$ and $b \in \mathbb{R}$
  - Decision boundary $w \cdot x + b = 0$
  - On point x, predict label $\text{sign}(w \cdot x + b)$
- If the true label on point x is y:
  - Classifier correct if $y(w \cdot x + b) > 0$

当真实值y和预测值同号的时候，分类正确

A loss function for classification

What is the **loss** of the linear classifier w, b on a point (x, y)?

One idea for a loss function:

- If $y(w \cdot x + b) > 0$: correct, no loss
- If $y(w \cdot x + b) < 0$: loss $= -y(w \cdot x + b)$

A simple learning algorithm

Fit a linear classifier w, b to the training set using **stochastic gradient descent**.

- Update w, b based on just one data point (x, y) at a time
- If $y(w \cdot x + b) > 0$: zero loss, no update
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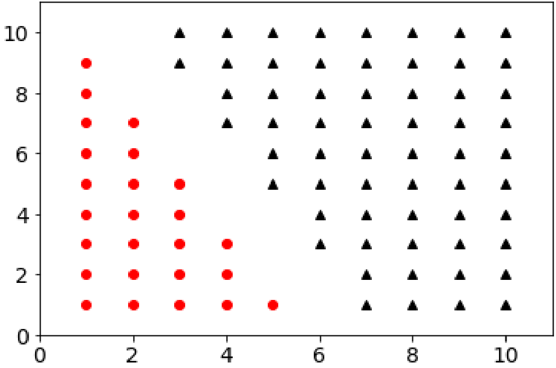
The Perceptron algorithm

- Initialize $w = 0$ and $b = 0$
- Keep cycling through the training data (x, y):
  - If $y(w \cdot x + b) \leq 0$ (i.e. point misclassified):
    - $w = w + yx$ 求导理解一下
    - $b = b + y$ update rule

The Perceptron in action

85 data points, linearly separable.



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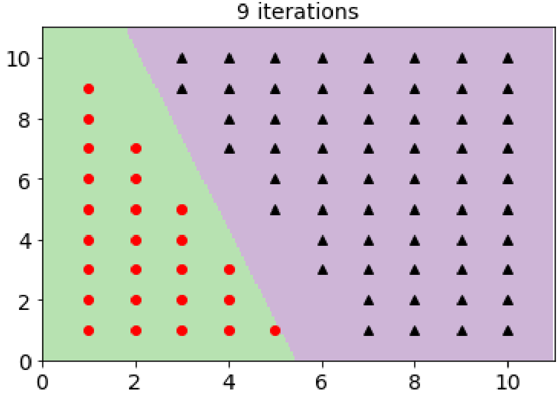

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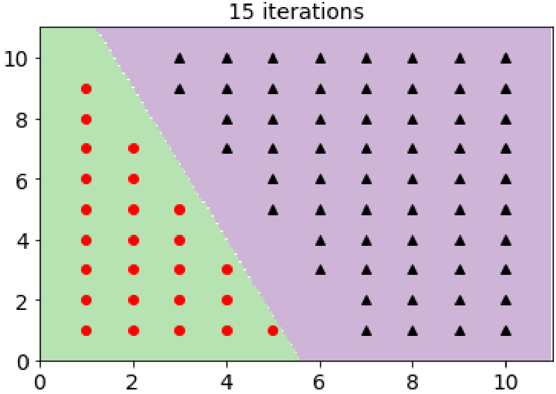

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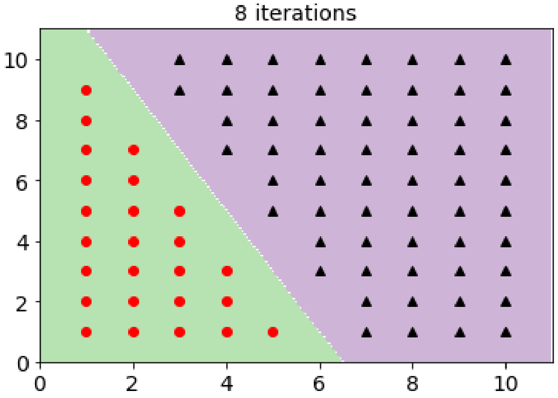

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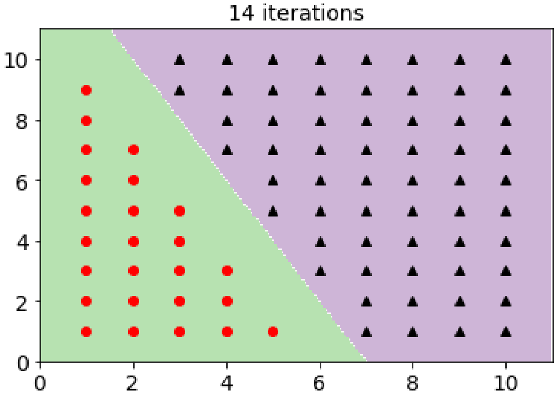

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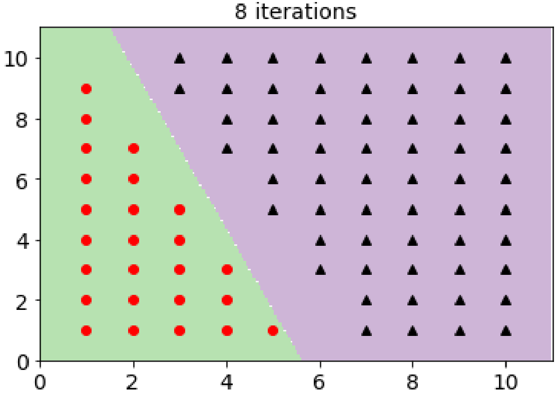

Perceptron: convergence

Theorem: Let $R = \max \|x^{(i)}\|$. Suppose there is a unit vector w^* and some (margin) $\gamma > 0$ such that

$$y^{(i)}(w^* \cdot x^{(i)}) \geq \gamma \text{ for all } i.$$

Then the Perceptron algorithm converges within R^2/γ^2 updates.

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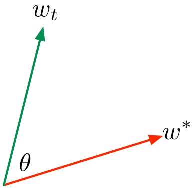
Proof idea. Let w_t be the classifier after t updates.



Track angle between w_t and w^*:

$$\cos(\angle(w_t, w^*)) = \frac{w_t \cdot w^*}{\|w\|}.$$

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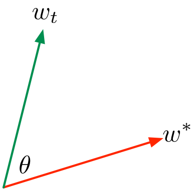

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On each mistake, when w_t is updated to w_{t+1},

- $w_t \cdot w^*$ grows significantly.
- $\|w_t\|$ does not grow much.