Homework 1 - Error Analysis

Chapter 3

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https://github.com/Fitzy1293/latex-school/blob/main/HW1.tex
https://github.com/Fitzy1293/latex-school/blob/main/HW1.pdf

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1 Problem 1

To measure the activity of a radioactive sample, two students count the alpha particles it emits. Student A watches for 3 minutes and counts 28 particles; Student B watches for 30 minutes and counts 310 particles. (a) What should Student A report for the average number emitted in 3 minutes, with his uncertainty? (b) What should Student B report for the average number emitted in 30 minutes, with her uncertainty? (c) What are the fractional uncertainties in the two measurements? Comment.

Solution 3.1
Section 3.2: The Square-Root Rule for a Counting Experiment

$$student_{A\,Time} = 3 \text{ minutes} \quad student_{A\,emitted\,3\,minutes} = 28 \text{ particles} \tag{1.1}$$

$$student_{B\,Time} = 30 \text{ minutes} \quad student_{B\,emitted\,30\,minutes} = 310 \text{ particles} \tag{1.2}$$

(a) Student A's measurement
Using eq. 3.2 from the textbook: $Avg.\ events\ measurement = \nu \pm \sqrt{\nu}$ where ν - the greek letter *nu* - is the best average.

$$student_{A\,emitted\,3\,minutes} = 28 \text{ particles} \tag{1.3}$$

$$student_{A\,uncertainty} = \sqrt{28} \approx 5.29150262212918 \tag{1.4}$$

$$student_{A\,uncertainty} = \pm 5 \text{ particles} \tag{1.5}$$

$$\boxed{student_{A\,emitted\,3\,minutes} = 28 \pm 5 \text{ particles}} \tag{1.6}$$

(b) Student B's measurement

$$student_{B\,emitted\,30\,minutes} = 310 \text{ particles} \tag{1.7}$$

$$student_{B_{\,uncertainty}} = \sqrt{310} \approx 17.6068168616590 \tag{1.8}$$

$$student_{B_{\,uncertainty}} = \pm 18 \text{ particles} \tag{1.9}$$

$$\boxed{student_{B\,emitted\,30\,minutes} = 310 \pm 18\,particles} \tag{1.10}$$

(c) What are the fractional uncertainties? Comment, i.e interpret, the fractional uncertainties.

Using eq .2.21 from the textbook: $fractional\ uncertainty = \frac{\delta_x}{|x_{best}|}$, we can use eq. (1.11) with Students A and B.

$$fractional\ uncertainty = \frac{\delta_\nu}{|\nu_{best}|} = \frac{\sqrt{student_{\,particle\,count\,best}}}{student_{\,particle\,count\,best}} \tag{1.11}$$

$$student_{A\,fractional\,uncertainty} = \frac{5}{28} \approx 0.178571428571429 \tag{1.12}$$

$$student_{B\,fractional\,uncertainty} = \frac{18}{310} \approx 0.0580645161290323 \tag{1.13}$$

$$student_{A\,fractional\,uncertainty} = 18\% \tag{1.14a}$$

$$student_{B\,fractional\,uncertainty} = 6\% \tag{1.14b}$$

Student B's total uncertainty is higher than student A's, however B has a lower fractional uncertainty. Counting more events will always reduce your fractional uncertainty in these cases. A lower fractional uncertainty implies a more accurate measurement, so B has a better measurement.

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2 Problem 2

Most of the ideas of error analysis have important applications in many different fields. This applicability is especially true for the square-root rule (3.2) for counting experiments, as the following example illustrates. The normal average incidence of a certain kind of cancer has been established as 2 cases per 10,000 people per year. The suspicion has been aired that a certain town (population 20,000) suffers a high incidence of this cancer because of a nearby chemical dump. To test this claim, a reporter investigates the town's records for the past 4 years and finds 20 cases of the cancer. He calculates that the expected number is 16 (check this) and concludes that the observed rate is 25in claiming that this result proves that the town has a higher than normal rate for this cancer?

Solution 3.3
Section 3.2: Abnormal number of cancer cases?

$$town_{expected\,cases} = 16 \quad town_{actual\,cases} = 20 \tag{2.1}$$

$$\sqrt{town_{actual\,cases}} = \sqrt{20} \approx 4.47213595499958 \tag{2.2}$$

$$\sqrt{town_{actual\,cases}} = \pm 4 \text{ cases} \tag{2.3}$$

The fewest possible cases are seen by substracting the uncertainty $20 - 4 = 16$. This is within the expected range, so he is not justified in his claim.

3 Problem 3

Binomial theorem exploration.

Solution 3.8
Section 3.3: Explore how the binomial theorem works.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\cdot 2}x^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^3 + ... \tag{3.1}$$

(a)
$n = 2$ case

$$(1+x)^2 = 1 + 2x + \frac{2(2-1)}{1\cdot 2}x^2 + \frac{2(2-1)(2-2)}{1\cdot 2\cdot 3}x^3 + ... \tag{3.2}$$

All terms that include $(n-2)$ will cancel; RHS terms to a higher power of x than 2 will go to 0.

$$(1+x)^2 = 1 + 2x + \frac{2(2-1)}{1\cdot 2}x^2 \tag{3.3}$$

$$(1+x)^2 = x^2 + 2x + 1 \tag{3.4}$$

$$(1+x)^2 = (x+1)(x+1) \tag{3.5}$$

$$\boxed{(1+x)^2 = (1+x)^2} \tag{3.6}$$

$n = 3$ case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\cdot 2}x^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^3 + \frac{n(n-1)(n-2)(n-3)}{1\cdot 2\cdot 3\cdot 4}x^4 + ... \tag{3.7}$$

All terms that include $(n-3)$ will cancel; RHS terms to a higher power of x than 3 will go to 0.

$$(1+x)^3 = 1 + 3x + \frac{3(3-1)}{1\cdot 2}x^2 + \frac{3(3-1)(3-2)}{1\cdot 2\cdot 3}x^3 + \frac{3(3-1)(3-2)(3-3)}{1\cdot 2\cdot 3\cdot 4}x^4 + ... \tag{3.8}$$

$$(1+x)^3 = 1 + 3x + 3x^2 + x^3 \tag{3.9}$$

$$\boxed{(1+x)^3 = (1+x)^3} \tag{3.10}$$

(b)
$n = -1$ case

$$(1+x)^{-1} = 1 + -1x + \frac{-1(-1-1)}{1\cdot 2}x^2 + \frac{-1(-1-1)(-1-2)}{1\cdot 2\cdot 3}x^3 + ... \tag{3.11}$$

$$\boxed{(1+x)^{-1} = 1 - x + x^2 - x^3 + ...} \tag{3.12}$$

Now use $(1+x)^{-1} \approx 1 - x$ for $x = 0.5, 0.1, 0.01$ and find how much it differs from the exact value.

$$(1+.5)^{-1} \approx 1 - .5 \tag{3.13}$$

$$.67 \approx .5 \tag{3.14}$$

$$dif. = .67 - .5 = .17 \tag{3.15}$$

$$\boxed{percent\ dif = .17/.67 * 100\% = 25\%} \tag{3.16}$$

$$(1+.1)^{-1} \approx 1 - .1 \tag{3.17}$$

$$.91 \approx .9 \tag{3.18}$$

$$dif = .91 - .9 = .01 \tag{3.19}$$

$$\boxed{percent\ dif. = .01/.91 * 100\% = 1\%} \tag{3.20}$$

$$(1+.01)^{-1} \approx 1 - .01 \tag{3.21}$$

$$.9901 \approx .99 \tag{3.22}$$

$$dif = .9901 - .99 = .0001 \tag{3.23}$$

$$\boxed{percent\ dif. = .0001/.9901 * 100\% = .01\%} \tag{3.24}$$