

Cleaning workspace

```
clear;  
clc;
```

Loading data:

```
if isfile('data.mat')  
    load('data.mat');  
end
```

Problem:

```
a = -1; % Left boundary  
b = 1; % Right boundary  
  
u_a = 0; % Value at left boundary  
u_b = 0; % Value at right boundary  
  
n = 6; % How many coordinate functions to take  
disp(['Solving problem with n=' num2str(n) ' coordinate functions']);
```

Solving problem with n=6 coordinate functions

Matlab solution:

```
h = 0.05; % Step  
X = linspace(a, b, (b-a)/h);  
init_sol = bvpinit(X, [0 0]);  
sol = bvp4c(@odefun, @bcfun, init_sol);  
y_matlab = deval(sol, X);
```

Galerkin method:

```
y_galerkin = Galerkin(a, b, n);
```

Solving system:

-2.2622	0.5385	0.4520	0.0227	-0.0007	-0.0010	2.6667
-1.3457	-3.0292	0.6348	0.4399	0.0074	-0.0033	-0.5333
-0.0279	-1.7789	-4.2084	0.5475	0.3900	-0.0074	0
0.0478	-0.1541	-2.1322	-5.4899	0.3980	0.3362	0
0.0054	0.0387	-0.2602	-2.4434	-6.8165	0.2179	0
0.0004	0.0041	0.0267	-0.3481	-2.7324	-8.1680	0

Condition number of system:

4.4102

Decomposition coefficients:

-1.0808
0.6146
-0.2450
0.0676
-0.0122

Collocations method:

```
y_collocations = Collocations(a, b, n);
```

Solving system:

0.4532	1.1422	-5.9362	14.5092	-26.7319	41.7113	2.9659
0.0236	2.1390	-4.7223	4.1325	1.5296	-9.6891	2.7071
-0.9828	2.7687	0.4441	-5.5564	2.0037	7.9008	2.2588
-2.3762	0.1222	5.0503	4.1794	-5.0445	-10.5655	1.7412
-3.4985	-6.0612	-5.2532	1.0634	11.5515	20.0719	1.2929
-3.9567	-11.2440	-23.6702	-42.0297	-65.8583	-93.3492	1.0341

Condition number of system:

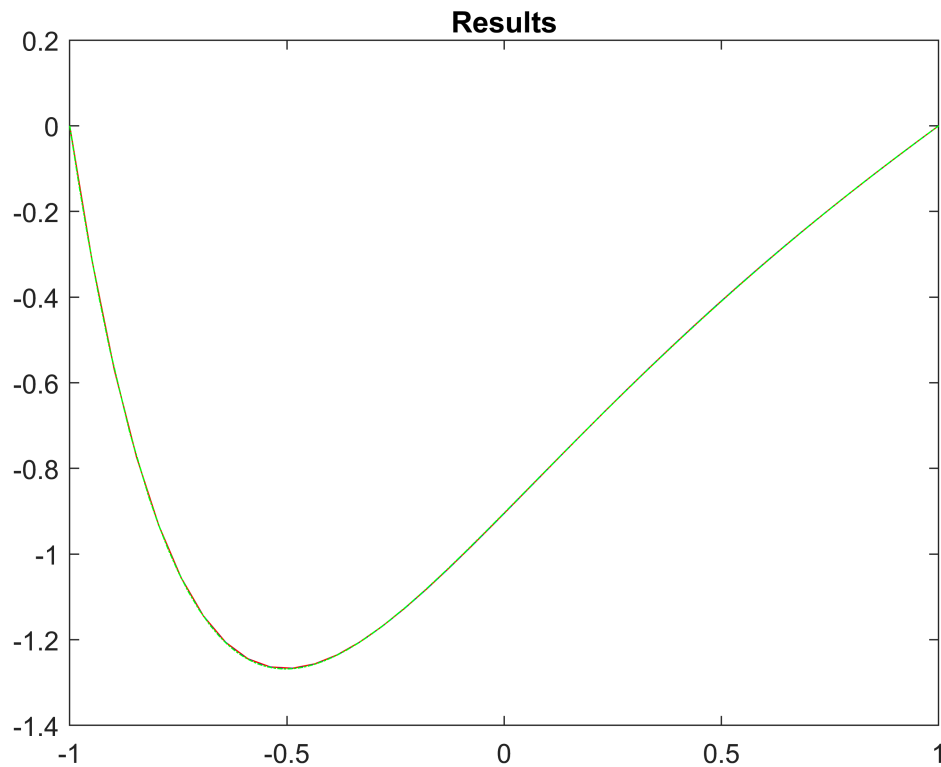
40.9228

Decomposition coefficients:

-1.0809
0.6147
-0.2453
0.0678
-0.0113
0.0003

Plots:

```
figure;
plot(X, y_matlab(1,:), '-r');
hold on;
fplot(y_galerkin, [a b], '--b');
fplot(y_collocations, [a b], '-.g');
title('Results');
hold off;
```



Checking values at points:

```
points = [-0.5 0 0.5];
y_matlab_check = deval(sol, points);
for i = 1:length(points)
    point = points(i);
    disp(['Difference at x=' num2str(point) ':' ]);

    y_matlab_value = y_matlab_check(1, i);
    syms x;
    y_galerkin_value = vpa(subs(y_galerkin, x, point));
    y_galerkin_diff = y_matlab_value - y_galerkin_value;
    disp(' Galerkin: ');
    disp(y_galerkin_diff);

    y_collocations_value = vpa(subs(y_collocations, x, point));
    y_collocations_diff = y_matlab_value - y_collocations_value;
    disp(' Collocations: ');
    disp(y_collocations_diff);
end
```

Difference at x=-0.5:

Galerkin:

0.0002081304869137975942103508178449

Collocations:

0.00090125969877796992263605511432056

Difference at x=0:

```
Galerkin:
-0.00019201002034250334339626764545983
Collocations:
-0.00090643384760365490644684616228233
Difference at x=0.5:
Galerkin:
0.00007260126877771426764484729438065
Collocations:
0.00074221997688271852417285306090233
```

Saving data:

```
save('data.mat');
```

Functions:

```
function dydx = odefun(x_, y)
    %p = @(x_actual)(subs(P(), x, x_actual));
    syms x;
    p = P();
    q = Q();
    r = R();
    f = F();
    dy_2 = subs((q/p) * y(2) + (r/p) * y(1) - (f/p), x, x_);
    dydx = [
        y(2);
        dy_2
    ];
end

function res = bcfun(ya, yb)
    res = [
        ya(1);
        yb(1)
    ];
end
```