Cleaning workspace

```
clear;
clc;
```

Loading data:

```
if isfile('data.mat')
     load('data.mat');
end
```

Problem:

```
a = -1; % Left boundary
b = 1;  % Right boundary

u_a = 0; % Value at left boundary
u_b = 0; % Value at right boundary

n = 7; % How many coordinate functions to take
disp([ 'Solving problem with n=' num2str(n) ' coordinate functions']);
```

```
Solving problem with n=7 coordinate functions
```

Matlab solution:

```
h = 0.05; % Step
X = linspace(a, b, (b-a)/h);
init_sol = bvpinit(X, [0 0]);
sol = bvp4c(@odefun, @bcfun, init_sol);
y_matlab = deval(sol, X);
```

Galerkin method:

```
y_galerkin = Galerkin(a, b, n);
```

```
Solving system:
   -2.2622    0.5385    0.4520    0.0227   -0.0007   -0.0010   -0.0003    2.6667
   -1.3457   -3.0292    0.6348    0.4399    0.0074   -0.0033   -0.0014   -0.5333
   -0.0279   -1.7789   -4.2084    0.5475    0.3900   -0.0074   -0.0055         0
    0.0478   -0.1541   -2.1322   -5.4899    0.3980    0.3362   -0.0203         0
    0.0054    0.0387   -0.2602   -2.4434   -6.8165    0.2179    0.2835         0
    0.0004    0.0041    0.0267   -0.3481   -2.7324   -8.1680    0.0203         0
    0.0000    0.0002    0.0025    0.0153   -0.4241   -3.0084   -9.5349         0

Condition number of system:
    5.1900

Decomposition coefficients:
   -1.0808
    0.6146
   -0.2450
    0.0676
   -0.0122
```

```
    0.0007
    0.0004
```

Collocations method:

```
y_collocations = Collocations(a, b, n);
```

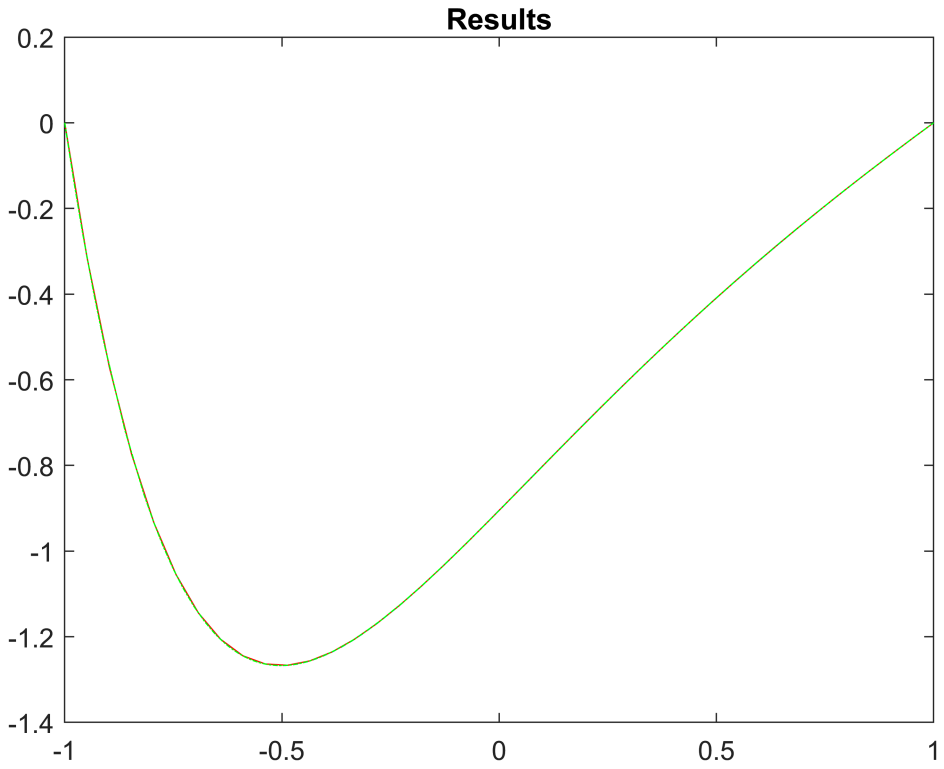
```
Solving system:
    0.4658    1.1047   -5.9548   14.9005  -28.2048   45.5028  -65.7836    2.9749
    0.1606    1.8763   -5.2187    6.9272   -4.0084   -4.0884   14.1606    2.7818
   -0.5564    2.7649   -1.9157   -3.6261    6.9530   -0.8332   -9.8635    2.4339
   -1.6667    2.0000    3.7500   -3.0000   -6.8750    3.7500   10.7552    2.0000
   -2.8438   -1.8447    3.5115    8.1372    4.3982   -7.6460  -15.5208    1.5661
   -3.6513   -7.4477   -9.4132   -6.4454    3.2310   17.5323   29.9329    1.2182
   -3.9686  -11.4423  -24.5277  -44.5454  -71.8024 -105.4577 -143.4963    1.0251

Condition number of system:
   60.2466

Decomposition coefficients:
   -1.0807
    0.6147
   -0.2449
    0.0677
   -0.0123
    0.0004
    0.0005
```

Plots:

```
figure;
plot(X, y_matlab(1,:), '-r');
hold on;
fplot(y_galerkin, [a b], '--b');
fplot(y_collocations, [a b], '-.g');
title('Results');
hold off;
```



Checking values at points:

```
points = [-0.5 0 0.5];
y_matlab_check = deval(sol, points);
for i = 1:length(points)
    point = points(i);
    disp([ 'Difference at x=' num2str(point) ':' ]);

    y_matlab_value = y_matlab_check(1, i);
    syms x;
    y_galerkin_value = vpa(subs(y_galerkin, x, point));
    y_galerkin_diff = y_matlab_value - y_galerkin_value;
    disp('  Galerkin: ');
    disp(y_galerkin_diff);

    y_collocations_value = vpa(subs(y_collocations, x, point));
    y_collocations_diff = y_matlab_value - y_collocations_value;
    disp('  Collocations: ');
    disp(y_collocations_diff);
end
```

```
Difference at x=-0.5:
  Galerkin:
0.00007372468085191603101042764090177 8
  Collocations:
-0.00000499426358088221272823792203337 64
Difference at x=0:
```

```
  Galerkin:
```
$$0.000011235925733355781294176868601031$$
```
  Collocations:
```
$$0.000111999724014976214223244816575 73$$
```
Difference at x=0.5:
  Galerkin:
```
$$-0.000063566791557733587567750250163284$$
```
  Collocations:
```
$$-0.000193495517056976244758261622910 53$$

Saving data:

```
save('data.mat');
```

Functions:

```
function dydx = odefun(x_, y)
    %p = @(x_actual)(subs(P(), x, x_actual));
    syms x;
    p = P();
    q = Q();
    r = R();
    f = F();
    dy_2 = subs((q/p) * y(2) + (r/p) * y(1) - (f/p), x, x_);
    dydx = [
        y(2);
        dy_2
    ];
end

function res = bcfun(ya, yb)
    res = [
        ya(1);
        yb(1)
    ];
end
```