International Institute of Information Technology, Bangalore
EG102 Data Structures and Algorithms Problems.

1. Using Masters Theorem, give asymptotic upper and lower bounds for the following recurrences, assuming that $T(n)$ is a constant for all $n < 5$.

   (a) $T(n) = 3T(n/3) + 1$.

   (b) $T(n) = 3T(n/3) + n$.

   (c) $T(n) = T(n/3) + 1$.

   (d) $T(n) = 2T(n/3) + n$.

   (e) $T(n) = T(n/3) + n$.

   (f) $T(n) = T(2n/3) + 1$.

   (g) $T(n) = 2T(n/3) + n \log n$.

   (h) $T(n) = 3T(n/3) + n^2$.

   (i) $T(n) = T(n/3) + n^3$.

   (j) $T(n) = T(n-1) + n$.

   (k) $T(n) = 3T(n/3) + \log n$.

   (l) $T(n) = T(n/3) + \log n$.

2. You are given a sequence which is increasing till say k and then decreasing. That is first k, $0 \leq k \leq n$ elements are increasing. For example $2, 7, 19, 23, 56, 8, 3, 2, 1$ is an increasing decreasing sequence with $k = 5$. Given an increasing decreasing sequence of n numbers, design an $O(\log n)$ algorithm to find the value of k.

3. A sorting algorithm is *stable* if two objects with equal values appear in the same order after sorting. Which of the following Sorting Algorithms are *stable*, and which are not *stable*.

   (a) Bubble Sort.

   (b) Selection Sort.

   (c) Insertion Sort.

   (d) Merge Sort.

(e) Quick Sort.

4. Give a $O(n \log k)$ time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists.

5. Given two arrays A and B, containing m and n integers with $n > m$, design an $O(n \log n)$ algorithm to determine how many points are in common in the given two arrays.

6. Given an array of integers, $a_0, a_1, a_2, \ldots a_{n-1}$, we would like to know, if there are three numbers such that sum of two numbers is equal to the third number. That is if there is i, j, k such that $a_i + a_j = a_k$. Give an $O(n^2)$ algorithm to solve this problem.

7. Let $a_1, a_2, \ldots a_n$ be a sequence of distinct numbers. The pair (i, j) is called a inversion, if $i < j$ and $a_i > a_j$. Write an $O(n \log n)$ algorithm to determine the number of inversions in the given array.

8. You are given a sequence of $10 < n < 10^9$ numbers and a number $10 < k < 10^5$ such that, $1 \leq k \leq n$. Write an efficient algorithm to list the k smallest numbers among the given sequence of numbers. What is the complexity of the algorithm ?

9. Let $a_1, a_2, \ldots a_n$ be the stock prices of a company in the last n days. You are allowed to buy and sell a stock at a later date, you can do it only once. We would like to know what is the maximum profit you can make given the stock prices. Write an algorithm to solve this problem. What is the complexity of the algorithm ?

10. Suppose, you are a manager in a company and your job is to organize, say n meetings in a day. Each meeting has a start time and an end time. Each meeting will be held between the start time and the end time. You may assume, that the these are in minutes, will have integer values between 0 and 1440. Given the, start time and the end time of n meetings, you would to know the time intervals when no meeting is held. Write an algorithm to solve this problem. What is the complexity of the algorithm ?

    For Example, if the input is $[234, 765], [874, 1232], [654, 1440], [1120, 1345]$ then the output is $[0, 234], [765, 654]$.