International Institute of Information Technology, Bangalore
EG102 Data Structures and Algorithms Theory.
Test 1: February 6, 2018

1. Solve the following recursion.(**4 marks**)

   (a) $T(n) = 2T(n/2) + n^2$.

   (b) $T(n) = T(n/2) + \log n$.

2. Let $F(0) = 0, F(1) = 1, F(2) = 2$ and $F(n) = (F(n-1) + F(n-3) + 1)\%1000$. We would like to have an $O(\log n)$ algorithm to compute $F(n)$ by computing A^n for some matrix. What is the matrix, that we should use here. (**4 marks**)

3. Given a sequence of n numbers, we would like to compute a number which is greater than the median of the given sequence of numbers.(**4 marks**)

   (a) Give an $O(\log n)$ Monte Carlo randomised algorithm to compute a number which is greater than the median . What is the success probabilty of your algorithm ?

   (b) Give an optimum deterministic algorithm to compute a number which is greater than the median . What is the complexity of your algorithm ?

4. You are given a sequence of $10 < n < 10^9$ numbers and a number $10 < k < 10^5$ such that, $1 \leq k \leq n$. Write an efficient algorithm to list the k largest numbers among the given sequence of numbers. What is the complexity of the algorithm ?(**6 marks**)

5. Suppose, you are a manager in a company and your job is to organize, say n meetings in a day. Each meeting has a start time and an end time. Each meeting will be held between the start time and the end time. You may assume, that the these are in minutes, will have integer values between 0 and 1440. Given the, start time and the end time of n meetings, you would like to know the time intervals when exactly k meeting are held, $0 \leq k \leq n$. Write an algorithm to solve this problem. What is the complexity of the algorithm ?(**6 marks**)

6. Does the order in which you write the loops, matter in the following program ? Give reasons for your answer.(**4 marks**)

$for(k = 0; k < n; ++k)$
   $for(i = 0; i < n; ++i)$
      $for(j = 0; j < n; ++j)$
         $if(D[i][j] > D[i][k]+D[k][j])D[i][j] = D[i][k]+D[k][j];$