Robust Adaptive Beamforming via Covariance Matrix Reconstruction and Interference Power Estimation

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***Abstract*— The performance of the traditional Capon beamformer degrades sharply when the signal of interest (SOI) appears in the training data. To reduce the impact of SOI on the Capon beamformer, two methods for the interference-plus-noise covariance matrix (INCM) reconstruction are proposed in this letter. The proposed-1 method is based on the integral of the Capon spectrum without the residual noise power. In the proposed-2 method, the interference power is estimated via the orthogonality between different sparse steering vectors (SVs) to project the sample covariance matrix for the INCM reconstruction. Meanwhile, the inverse of INCM is obtained by eigenvalue decomposition and the SV of SOI is updated by the principal eigenvector of the reconstructed SOI covariance matrix (SCM). Simulation results show that the proposed methods are robust against some mismatch errors.**

***Index Terms*— Robust adaptive beamforming, covariance matrix reconstruction, eigenvalue decomposition, orthogonality.**

I. Introduction

ADAPTIVE beamforming has been widely used in radar, sonar, wireless communication, speech signal processing and so on [1]. However, the performance of Capon beamformer is sensitive to various array errors, such as direction of arrival (DOA) of SOI error and SV random error, especially when the SOI appears in the training data [2], [3]. To solve this problem, some methods based on INCM reconstruction appeared [4]–[12].

INCM reconstruction method was first proposed in [4] to eliminate the impact of SOI appearing in the training data. Utilizing the Capon spectrum to integrate over a region separated from the SOI direction, INCM was reconstructed instead of the sample covariance matrix and the SV of the SOI was obtained through solving a quadratically constrained quadratic programming (QCQP) problem. Later, a more generalized type of the method in [4] was proposed in [5], where the integral region was an annulus uncertainty sector and achieved a better performance. However, the methods based on the Capon spectrum integral were sensitive to the SV mismatch errors [6] and highly depended on the estimated spectra [7]. Via coarray interpolation to increase array aperture and enhance DOA estimation, a novel beamforming method based on coprime arrays was presented in [8] and achieved good performance at high SNRs. Through the analysis of the impact of interference power estimation on the beamformer, a simplified method to estimate the interference powers for robust adaptive beamforming was proposed in [9]. Based on the INCM reconstruction method, a robust orthogonal projection beamformer was proposed in [10] but the SV error of SOI was not considered. Besides, the integral sector contained a redundant sector. Then, a method based on the signal and signal-interference subspace was proposed in [11] to obtain more accurate SVs and an improved projection method to reconstruct INCM was described in [12], where the performance of the beamformer was improved.

To reduce the computation and improve the performance of the beamformer, a new beamformer is proposed in this letter. The initial DOAs of the signal and interferences are obtained by the Capon spectrum and then use the Capon spectrum without the residual noise power to integrate over the signal and interference sector to get the SCM and INCM, not the whole complementary space of the signal, which is called the proposed-1 method. The proposed-2 method reconstructs INCM through the interference powers estimation based on the orthogonality of the sparse SVs. Then, the inverse of INCM is acquired by the eigenvectors of the INCM and the SV of the SOI is updated by the principal eigenvector of the reconstructed SCM to improve the robustness. Simulation results will be provided to demonstrate the effectiveness and robustness of the proposed methods.

II. Signal Model and Background

A uniform linear array (ULA) composed of M omnidirectional sensors spaced half wavelength is considered. Assume that $L+1$ far-field narrowband sources impinge on the array and the array observation at time k can be expressed as

$$\begin{aligned}\mathbf{x}(k) &= \mathbf{x}_s(k) + \mathbf{x}_i(k) + \mathbf{x}_n(k)\\ &= s_0(k)\,\mathbf{a}_0 + \sum_{l=1}^{L} s_l(k)\mathbf{a}_l + \mathbf{x}_n(k)\end{aligned} \tag{1}$$

where $\mathbf{x}_s(k)$, $\mathbf{x}_i(k)$ and $\mathbf{x}_n(k)$ are the components of the SOI, interference and noise, respectively. $s_l(k)$ and $\mathbf{a}_l$, $l=0,1,\ldots,L$ are the l-th source and its SV. $\mathbf{x}_n(k)$ is the additive Gaussian noise with zero mean and σ_n^2 variance. Assume that the SOI, interference and noise are statistically independent with each other. The output of the beamformer is obtained as $y(k)=\mathbf{w}^H\mathbf{x}(k)$, where $\mathbf{w}=[w_1,w_2,\ldots,w_M]^T$ is the weight vector. $(\cdot)^H$ and $(\cdot)^T$ denote Hermitian transpose and transpose operation respectively.

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The optimal weight vector can be obtained by solving the following problem

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \quad s.t.\ \mathbf{w}^H \mathbf{a}_0 = 1 \tag{2}$$

where $\mathbf{R}_{i+n} = \sum_{l=1}^{L} \sigma_l^2 \mathbf{a}_l \mathbf{a}_l^H + \sigma_n^2 \mathbf{I}$ is the theoretical INCM and σ_l^2 is the l-th interference power. It can be easily obtained the solution of (2) as

$$\mathbf{w}_{\text{opt}} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{a}_0}{\mathbf{a}_0^H \mathbf{R}_{i+n}^{-1} \mathbf{a}_0} \tag{3}$$

which is famous as the minimum variance distortionless response (MVDR) beamformer. Replacing $\mathbf{R}_{i+n}$ in (3) with $\mathbf{R}_x = E\left\{\mathbf{x}(k)\mathbf{x}^H(k)\right\}$ can realize the same performance, which is known as the Capon beamformer [3]. Since $\mathbf{R}_x$ and $\mathbf{a}_0$ are not available in practice, they are replaced with the sample covariance matrix $\hat{\mathbf{R}}_x = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k)$ with K snapshots and the nominal SV $\bar{\mathbf{a}}_0$. When SNR increases, the performance of the traditional beamformer degrades sharply.

III. Proposed Methods

The noise power is distributed in all directions, which means the interference's Capon power contains the residual noise power. Thus, large errors will be introduced if using the Capon power spectrum to integrate for INCM reconstruction directly because the residual noise power will be large after integral. Different from the INCM reconstruction method in [4], the residual noise power is removed from the power of interference to realize a more precise INCM reconstruction. Meanwhile, to further improve the performance, the estimated interference SVs are used to project the sample covariance matrix for more accurate interference powers estimation and then INCM is reconstructed based on its definition. The details are demonstrated in the following.

A. INCM Reconstruction

The number of sources $L+1$ and the initial directions of sources are obtained by the Capon spectrum search and then the whole space is divided into the SOI sector Θ_s, the interference sector Θ_i and the noise sector Θ_n. The residual noise power can be calculated as

$$\bar{\sigma}_n^2 = \frac{1}{Q}\sum_{q=1}^{Q} \frac{1}{\bar{\mathbf{a}}^H(\theta_q)\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}(\theta_q)} \tag{4}$$

where Q is the number of the sampling points in the noise sector and $\bar{\mathbf{a}}(\theta_q)$ is the nominal SV corresponding to the angle θ_q. The estimated noise power $\hat{\sigma}_n^2$ can be realized by $\hat{\sigma}_n^2 = M\bar{\sigma}_n^2$ [12]. To remove the SOI component from the sample covariance matrix, the proposed-1 method uses the Capon spectrum without the residual noise power to integrate over an angular sector in Θ_i. Then, the INCM can be reconstructed as

$$\hat{\mathbf{R}}_{i+n} = \int_{\Theta_i} \left(\frac{1}{\bar{\mathbf{a}}^H(\theta)\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}(\theta)} - \bar{\sigma}_n^2\right) \bar{\mathbf{a}}(\theta)\bar{\mathbf{a}}^H(\theta)\, d\theta + \hat{\sigma}_n^2 \mathbf{I} \tag{5}$$

Some values of the actual interference power without the residual noise power calculated by (5) may be negative because the choice of the interference sector may contain the noise sector in the real application. Thus, only the positive values are chosen to reconstruct the INCM. Considering the practical computation, the integral is substituted with uniform sampling in the interference sector and then a discrete sector $\Theta_i = [\theta_1, \theta_2, \ldots, \theta_P]$, where P is the number of the discrete angle points, and the corresponding SV set $[\bar{\mathbf{a}}(\theta_1), \bar{\mathbf{a}}(\theta_2), \ldots, \bar{\mathbf{a}}(\theta_P)]$ are obtained. Therefore, the INCM becomes

$$\hat{\mathbf{R}}_{i+n} = \sum_{p=1}^{P} \left(\frac{1}{\bar{\mathbf{a}}^H(\theta_p)\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}(\theta_p)} - \bar{\sigma}_n^2\right) \bar{\mathbf{a}}(\theta_p)\bar{\mathbf{a}}^H(\theta_p) + \hat{\sigma}_n^2 \mathbf{I} \tag{6}$$

The proposed-2 method is based on the approximate orthogonality between SVs if SVs are sparse in the space, which is also based on the conventional beamforming [13]. The actual SV of the source can be modeled as [4], [5]

$$\mathbf{a}_l = \bar{\mathbf{a}}_l + \mathbf{e}_l, \quad l = 0, 1, \ldots, L \tag{7}$$

where $\mathbf{e}_l$ is the component of the mismatch error. Bring (7) to (1) and use the nominal interference SV $\bar{\mathbf{a}}_l$, $l = 1, 2, \ldots, L$ based on the estimated directions to project $\mathbf{x}(k)$, which is

$$\begin{aligned} \mathbf{x}_l(k) &= \bar{\mathbf{a}}_l^H \mathbf{x}(k) \\ &= \bar{\mathbf{a}}_l^H \left[s_0(k)(\bar{\mathbf{a}}_0 + \mathbf{e}_0) + \sum_{l=1}^{L} s_l(k)(\bar{\mathbf{a}}_l + \mathbf{e}_l) + \mathbf{x}_n(k) \right] \\ &\approx \bar{\mathbf{a}}_l^H \bar{\mathbf{a}}_l s_l(k) + \bar{\mathbf{a}}_l^H \mathbf{x}_n(k) \end{aligned} \tag{8}$$

Though there are errors between the actual SVs and the nominal SVs, the approximate orthogonality is still valid between different sparse SVs. The sample covariance matrix of (8) is

$$\hat{\mathbf{R}}_{x_l} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}_l(k)\mathbf{x}_l^H(k) = \hat{\sigma}_l^2 \bar{\mathbf{a}}_l^H \bar{\mathbf{a}}_l \bar{\mathbf{a}}_l^H \bar{\mathbf{a}}_l + \hat{\sigma}_n^2 \bar{\mathbf{a}}_l^H \bar{\mathbf{a}}_l \tag{9}$$

The power of the interference $\hat{\sigma}_l^2$ can be obtained as

$$\hat{\sigma}_l^2 = \frac{\hat{\mathbf{R}}_{x_l} - \hat{\sigma}_n^2 \bar{\mathbf{a}}_l^H \bar{\mathbf{a}}_l}{\bar{\mathbf{a}}_l^H \bar{\mathbf{a}}_l \bar{\mathbf{a}}_l^H \bar{\mathbf{a}}_l} \tag{10}$$

Via the powers of the interferences, the INCM can be reconstructed as

$$\hat{\mathbf{R}}_{i+n} = \sum_{l=1}^{L} \hat{\sigma}_l^2 \bar{\mathbf{a}}_l \bar{\mathbf{a}}_l^H + \hat{\sigma}_n^2 \mathbf{I} \tag{11}$$

Though $\hat{\mathbf{R}}_{i+n}$ is obtained based on a ULA in the proposed-2 method, it is also suitable for other complex arrays as long as conventional beamforming is valid. Till now, the INCM has been obtained by the proposed methods. To further remove the interference's impact, a new method to obtain the inverse of INCM is described in this letter. Apply eigenvalue decomposition on $\hat{\mathbf{R}}_{i+n}$ and it can be obtained as

$$\hat{\mathbf{R}}_{i+n} = \sum_{m=1}^{L} \left(\lambda_m + \hat{\sigma}_n^2\right) \mathbf{u}_m \mathbf{u}_m^H + \sum_{m=L+1}^{M} \hat{\sigma}_n^2 \mathbf{u}_m \mathbf{u}_m^H \tag{12}$$

where $(\lambda_m + \hat{\sigma}_n^2)$ and $\mathbf{u}_m$, $m = 1, 2, \ldots, M$ represent the eigenvalue and its corresponding eigenvector of the reconstructed INCM. Besides, $\lambda_m = 0$ for $m > L$. The first L eigenvectors $\mathbf{u}_m$, $m = 1, 2, \ldots, L$ are the orthogonal vectors that span the interference subspace and the left eigenvectors span the noise subspace. The inverse of $\hat{\mathbf{R}}_{i+n}$ can be described as

$$\hat{\mathbf{R}}_{i+n}^{-1} = \sum_{m=1}^{L} \frac{1}{\lambda_m + \hat{\sigma}_n^2} \mathbf{u}_m \mathbf{u}_m^H + \frac{1}{\hat{\sigma}_n^2} \sum_{m=L+1}^{M} \mathbf{u}_m \mathbf{u}_m^H \tag{13}$$

For the strong interferences (i.e. $\lambda_m \gg \hat{\sigma}_n^2$, $m = 1, 2, \ldots, L$), $\hat{\mathbf{R}}_{i+n}^{-1}$ can be approximately equal to

$$\hat{\mathbf{R}}_{i+n}^{-1} \approx \frac{1}{\hat{\sigma}_n^2} \sum_{m=L+1}^{M} \mathbf{u}_m \mathbf{u}_m^H = \frac{1}{\hat{\sigma}_n^2} \mathbf{U}_N \mathbf{U}_N^H = \frac{1}{\hat{\sigma}_n^2} \left(\mathbf{I} - \mathbf{U}_I \mathbf{U}_I^H \right) \tag{14}$$

where $\mathbf{U}_N$ and $\mathbf{U}_I$ are the eigenvector matrix of the noise and the interference respectively. It is distinct that the inverse of INCM can be replaced with the operation of the eigenvectors instead of the direct inverse operation. Though it is under the assumption of strong interferences, it is still effective for weak interferences because the principal component of the INCM's inverse is reserved.

B. SV Estimation

Similar to the INCM reconstruction in (6), the SCM $\hat{\mathbf{R}}_s$ is also reconstructed through the Capon spectrum integral over the signal sector Θ_s. Apply eigenvalue decomposition on $\hat{\mathbf{R}}_s$ and use the eigenvector $\mathbf{v}_1$ associated with the largest eigenvalue α_1 to take the place of the SV of the SOI because α_1 contains the most information of the SOI [12]. Then the estimated SV of the SOI can be described as

$$\hat{\mathbf{a}}_0 = \sqrt{M} \mathbf{v}_1 \tag{15}$$

Based on the inverse of INCM and updated SV, the weight vector of the proposed methods can be described as

$$\mathbf{w}_{\text{proposed}} = \frac{\hat{\mathbf{R}}_{i+n}^{-1} \hat{\mathbf{a}}_0}{\hat{\mathbf{a}}_0^H \hat{\mathbf{R}}_{i+n}^{-1} \hat{\mathbf{a}}_0} \tag{16}$$

To show the advantage of the proposed methods, the followings give a computational complexity analysis between the proposed methods and some other methods based on INCM reconstruction. A convex optimization problem needs to be solved in [4], [5], and [12] with the computational complexity $\mathcal{O}(M^{3.5})$. It can be seen that the main computational cost of the proposed methods lies in the eigenvalue decomposition with $\mathcal{O}(M^3)$ and the covariance matrix reconstruction. Let J stand for the number of sampling points in the whole space and the proposed methods have an overall $\mathcal{O}\left(JM^2\right)$ computational complexity, where J is generally bigger than M for a better result. However, $\mathcal{O}(SM^2)$ computational cost lies in the volume integral operation in [5], where $S \gg J$ is the number of annulus sampling points. Though the proposed methods have the same computational cost as the method in [11], the proposed methods can achieve a better performance than the latter. Therefore, the proposed methods are more efficient than some compared methods based on INCM reconstruction.

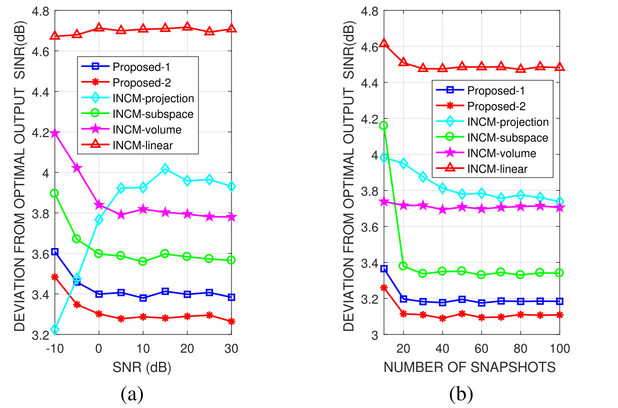


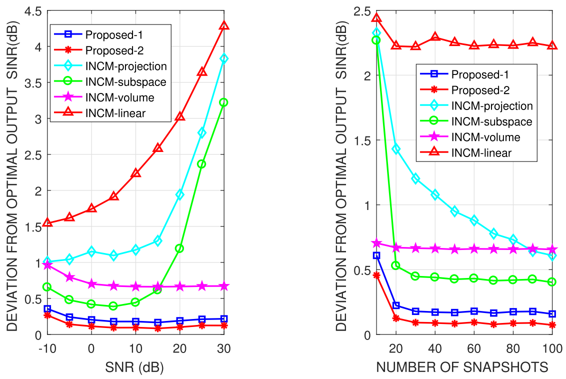
Fig. 1. Deviation from optimal output SINR versus (a) input SNR and (b) number of snapshots in the case of SV random error.

IV. Simulation Results

In this section, assume that three far-field sources impinge on a ULA with $M = 10$ omnidirectional sensors spaced half wavelength from the directions $\theta_0 = -5°$, $\theta_1 = -25°$ and $\theta_2 = 25°$ and the estimated directions are $\hat{\theta}_0 = -8°$, $\hat{\theta}_1 = -28°$ and $\hat{\theta}_2 = 22°$. The first source is assumed to be the SOI and the left are the two interferences with 20dB interference-to-noise ratio (INR) in all the simulations. The number of snapshots is fixed at 30 when the performance is versus input SNR. When considering the performance versus the number of snapshots, SNR is set as 10dB. The additive Gaussian noise with zero mean and unit variance is considered. The sector of SOI and interference are set as $\Theta_s = \left[\hat{\theta}_0 - 8°, \hat{\theta}_0 + 8°\right]$ and $\Theta_i = \Theta_{i1} \bigcup \Theta_{i2} = \left[\hat{\theta}_1 - 8°, \hat{\theta}_1 + 8°\right] \bigcup \left[\hat{\theta}_2 - 8°, \hat{\theta}_2 + 8°\right]$, respectively. The sectors are uniformly sampled with $0.1°$ interval in the simulations. To highlight the performance of the proposed methods, some compared algorithms are chosen. They are covariance matrix reconstruction based on projection (INCM-projection) [12], INCM reconstruction based on subspace (INCM-subspace) [11], INCM based on volume integration (INCM-volume) [5], covariance matrix reconstruction based on linear integration (INCM-linear) [4]. Seven dominant eigenvectors are set in [12] and the threshold ρ is 0.9 in [11]. Besides, $\epsilon = \sqrt{0.1}$ is employed in [5] and the degree interval is $0.1°$ in the simulations. The CVX toolbox is used to solve the optimization problem in other methods [14]. 200 Monte-Carlo simulations are performed for each case.

Example 1 (SV Random Error): Some common errors like sensor location error, look direction error and so on can be modeled with a general expression as (7). The random error $\mathbf{e}_l$ is described as [5] $\mathbf{e}_l = \frac{\varepsilon_l}{\sqrt{M}} \left[e^{j\phi_0^l}, e^{j\phi_1^l}, \ldots, e^{j\phi_{M-1}^l} \right]^T$, where ε_l is subject to uniform distribution in $\left[0, \sqrt{0.3}\right]$ and ϕ_m^l, $m = 0, 1, \ldots, M-1$ is the phase error which is uniformly distributed in $[0, 2\pi)$.

As it depicts in Fig.1(a), the proposed-1 method has a better performance than INCM-subspace because it reduces the impact of residual noise power on the INCM reconstruction. Besides, the interferences are suppressed via the eigenvalue



(a) (b)

Fig. 2. Deviation from optimal output SINR versus (a) input SNR and (b) number of snapshots in the case of incoherent local scattering error.

decomposition approximation. The proposed-2 method shows the best performance among the tested beamformers, which benefits from a more precise INCM reconstruction through more precise interference powers estimation and the corrected SVs. Fig.1(b) illustrates the deviation from optimal output SINR versus the number of snapshots. All the tested beamformers show a stable performance because more information can be obtained when the number of snapshots increases. Meanwhile, it is obvious that the proposed methods still outperform other methods.

Example 2 (Incoherent Local Scattering Error): Consider the SOI is modeled as $\tilde{\mathbf{x}}_s(k) = s_0(k)\,\mathbf{a}_0(\theta_0) + \sum_{p=1}^{4} s_p(k)\,\bar{\mathbf{a}}(\theta'_p)$, where $\mathbf{a}_0$ represents the direct path corresponding to the direction θ_0 and $\bar{\mathbf{a}}(\theta'_p)$ is the incoherent local scattering SV. θ'_p, $p = 1,2,3,4$ is subject to Gaussian distribution with θ_0 mean and 2° standard deviation. $s_0(k)$ and $s_p(k)$, $p = 1,2,3,4$ are independent and identically subject to the complex Gaussian distribution with zero mean and unit variance.

As input SNR increases in Fig.2(a), there is a close gap between the proposed beamformers and the optimal beamformer. Moreover, the proposed beamformers keep an outstanding performance in the wide range of the input SNR. When input SNR increases, the performance of most beamformers degrades, which is because the actual SV is magnified owing to the incoherent local scattering error, which leads to the increase of deviations from optimal output SINR in Fig.2(a). Benefiting from more precise INCM reconstruction, the proposed methods show a better performance when input SNR increases. Meanwhile, it is distinct in Fig.2(b) that all the tested beamformers have a stable performance when the snapshots change and the proposed-2 method has the lowest deviation from the optimal beamformer.

V. Conclusion

In this letter, robust adaptive beamforming methods based on covariance matrix reconstruction and interference power estimation have been proposed to reduce the impact on the Capon beamformer's performance when the SOI is present in the training data. The proposed-1 method has used the Capon spectrum integral without the residual noise power in the interference sector for the INCM reconstruction. The interference power has been estimated based on the orthogonality between sparse SVs in the proposed-2 method. Then, the inverse of the INCM has been obtained through eigenvalue decomposition and a more accurate SV of the SOI has been estimated at the same time. Simulation results have shown the effectiveness and robustness of the proposed methods.

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