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Modified projection approach for robust adaptive array beamforming ☆

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ABSTRACT

A novel modified projection method is proposed in this paper, which is robust against the signal steering vector mismatch and covariance matrix uncertainty. First, an enhanced covariance matrix estimate based on a shrinkage method is obtained. Then, the desired signal subspace is estimated from the eigenvectors of the enhanced covariance matrix, and a calibrated steering vector of the desired signal is obtained in sequence by projecting the presumed one onto the new estimated desired signal subspace. Finally, the robust adaptive beamformer weight is obtained from the estimated covariance matrix and the calibrated steering vector. Compared with the traditional projection method, the proposed method can work well even at low signal-to-noise ratio. Also, it is not necessary to estimate the number of sources. Simulation results demonstrate the efficiency of the proposed robust method compared with some of the existing ones.

1. Introduction

Adaptive array processing has been widely used in radar, communications, sonar, acoustics, and medicine in the past decades. In practical array systems, traditional adaptive beamforming algorithms are known to degrade if some of exploited assumptions on the environment, sources, or antenna array become wrong or imprecise. At the same time, as the adaptive weight vector is a function of both the covariance matrix and the signal steering vector, it may be quite sensitive to the perturbation of the covariance matrix and the mismatch of the signal steering vector. Many techniques have been proposed to improve the robustness of adaptive beamformers. Some are aimed at eliminating covariance matrix uncertainty, such as diagonal loading method [1] and eigenspace based method [2]. Some are used to process the desired signal steering vector, such as the method in [3] using the worst-case performance optimization, the method based on the actual steering vector estimation [4], and the projection method based on eigen-decomposition of sample covariance matrix [5]. Some are proposed to deal with joint robustness against covariance matrix uncertainty and the steering vector mismatch, such as the joint robust method [6] in which the mismatch vector between the actual steering vector and the presumed one is estimated iteratively by solving a quadratic convex optimization problem.

By projecting the nominal steering vector onto the signal plus interference subspace to obtain a new steering constraint, projection method in [5] can achieve better performance in alleviating both the perturbation and sample data problems. However, in order to estimate the signal plus interference subspace, determining the source number is a challenging issue in practical applications for projection method. Many methods have been proposed for the source number detection in array processing. The most

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common used techniques are to apply information theoretic criteria, such as Akaike information criterion (AIC) [7] and Rissanen's minimum description length (MDL) principle [8], on the received data. The MDL algorithm is consistent while the AIC principle is not. In most cases, the AIC principle overestimates the number of signals. All the methods mentioned above are based on the eigenvalues of the sample correlation matrix. They only perform reasonably well at medium or high signal-to-noise ratio (SNR), but not at low SNR. Some eigenvector-based techniques have been reported [9]. They perform well even at low SNR. But, a priori knowledge of the steering vector of a true source is required. The eigenvector-based detection methods fail to perform with erroneous steering vector.

In this paper, a modified projection approach for robust adaptive beamforming is proposed. It is a joint robust method. The optimal diagonal loading factor of the sample covariance is estimated based on the minimum mean square error (MMSE) principle, and the calibrated steering constraint is estimated in sequence by projecting the presumed one onto the new estimated desired signal subspace which is obtained from the eigenvectors of the enhanced covariance matrix. An attractive feature of our algorithm is that it does not require knowing the number of sources. Moreover, our algorithm has better performance compared with the traditional projection method and the recently proposed diagonal loading method in [1], and has almost the same performance compared with the recently proposed joint robust method in [6]. Meanwhile the challenging problem of efficiently and exactly solving the quadratic convex optimization problem in [6] is avoided.

The rest of this paper is organized as follows. Section 2 contains background material. In Section 3, the new proposed robust adaptive beamforming algorithm is discussed in detail. Some numerical studies are presented in Section 4 to illustrate the effectiveness of the proposed robust beamforming algorithm. Finally, a brief conclusion is given in Section 5.

2. Problem statement

Here a uniform linear array comprising N isotropic antenna elements with half wavelength spacing is considered. It is assumed that K ($K<N$) uncorrelated narrowband and far field signals impinge on the array. The $N\times 1$ dimensional received signal vector present on the antenna elements at time t is denoted by

$$\mathbf{x}(t)=\sum_{i=0}^{K-1}a_i(t)\mathbf{s}(\theta_i)+n(t) \tag{1}$$

where $\mathbf{s}(\theta_i)=[1,e^{j\pi\sin\theta_i},\cdots,e^{j\pi(N-1)\sin\theta_i}]^{\mathrm{T}}$ is the steering vector corresponding to the direction of arrival θ_i, $a_i(t)$ is the complex waveform of the ith signal, and $n(t)$ is an additive white Gaussian noise with covariance $\sigma^2\mathbf{I}$. In this paper, the sources and noise are assumed to be statistically uncorrelated. The $N\times N$ received data covariance matrix $\mathbf{R}$ is given by

$$\mathbf{R}=E[\mathbf{x}(t)\mathbf{x}^H(t)]=P_S\mathbf{s}_d\mathbf{s}_d^H+\sum_{j=1}^{K-1}P_{I_j}\mathbf{s}_{I_j}\mathbf{s}_{I_j}^H+\sigma^2\mathbf{I} \tag{2}$$

where E and H denote expectation and complex conjugate transpose, respectively. P_S and $\mathbf{s}_d$ represent the power and steering vector of the desired signal, respectively. P_{I_j} and $\mathbf{s}_{I_j}$ represent the power and steering vector of the jth interference, respectively. The interference-to-noise ratios (INR) can be expressed as P_{Ij}/σ^2, the signal-to-noise ratio (SNR) can be expressed as P_s/σ^2, and the signal-to-interference ratio (SIR) can be expressed as P_s/P_{Ij}.

In the linearly constrained minimum variance (LCMV) adaptive algorithm, the optimal weight vector $\mathbf{w}$ is obtained by minimizing the array output power subject to linear or derivative constraint. Here, we are concerned with the case in which there is only one constraint on the desired signal steering vector

$$\text{Minimize } \mathbf{w}^H\mathbf{R}\mathbf{w} \text{ Subject to } \mathbf{s}_d^H\mathbf{w}=1 \tag{3}$$

where H denotes Hermitian transpose, $\mathbf{R}$ is the $N\times N$ received data covariance matrix, N is the number of array sensors, the steering constraint $\mathbf{s}_d$ is the $N\times 1$ actual desired signal steering vector. The optimal solution is given by

$$\mathbf{w}=\mathbf{R}^{-1}\mathbf{s}_d(\mathbf{s}_d^H\mathbf{R}^{-1}\mathbf{s}_d)^{-1} \tag{4}$$

The optimum value of SINR can be obtained by Eq. (4)

$$\mathrm{SINR}_{\mathrm{opt}}=P_S\mathbf{s}_d^H\mathbf{Q}^{-1}\mathbf{s}_d \tag{5}$$

where $\mathbf{Q}$ is the actual interference-plus-noise covariance matrix.

In practical application, $\mathbf{R}$ is replaced by a finite sample covariance matrix

$$\hat{\mathbf{R}}=\frac{1}{T}\sum_{t=1}^{T}\mathbf{x}(t)\mathbf{x}^H(t) \tag{6}$$

where T is the training sample size, $\mathbf{x}(t)$ is the observation vector. With limited sample size, the fluctuations in the noise eigenvalues because of the inherent random behavior of noise will cause large errors, but the principle component of $\hat{\mathbf{R}}$ are generally rather robust and tend to remain relatively stable from one data trial to the next, which can be accurately estimated from a small amount of data [10]. When the desired signal exists in the observations, large snapshots are required for Eq. (6) to achieve convergence. Moreover, in practice, due to the existence of array imperfections, some underlying assumptions on adaptive arrays could be invalid which results in a mismatch between the presumed $\hat{\mathbf{s}}_d$ and the exact $\mathbf{s}_d$. The mismatch will degrade the performance of the LCMV beamformer.

The weight vector of LCMV, when using $\hat{\mathbf{R}}$ and $\hat{\mathbf{s}}_d$, is given by

$$\mathbf{w}_{\mathrm{LCMV}}=\hat{\mathbf{R}}^{-1}\hat{\mathbf{s}}_d(\hat{\mathbf{s}}_d^H\hat{\mathbf{R}}^{-1}\hat{\mathbf{s}}_d)^{-1} \tag{7}$$

And the corresponding SINR is

$$\mathrm{SINR}=\frac{P_S\left|\mathbf{w}_{\mathrm{LCMV}}^H\hat{\mathbf{s}}_d\right|^2}{\mathbf{w}_{\mathrm{LCMV}}^H\mathbf{Q}\mathbf{w}_{\mathrm{LCMV}}} \tag{8}$$

In traditional projection method [5], a new steering constraint is obtained by projecting the presumed one onto the signal plus interference subspace.

$$\bar{\mathbf{s}}_d=\mathbf{E}_S\mathbf{E}_S^H\hat{\mathbf{s}}_d \tag{9}$$

where $\mathbf{E}_s$ is the estimated signal plus interference subspace. This method requires knowing the source number to identify the signal plus interference subspace, and determining the source number is a challenging issue in practical applications. Furthermore it cannot perform well at low SNR.

3. Proposed method

In the proposed method, first, the theoretical covariance matrix of the array output vector is obtained based on the MMSE principle. Then, the desired signal subspace is estimated from the eigenvectors of the enhanced covariance matrix, and the calibrated steering vector is obtained in sequence by projecting the presumed one onto the new estimated desired signal subspace. Finally, the robust adaptive beamformer weight is obtained from the enhanced covariance matrix and the calibrated steering vector. Our method is with enhanced covariance matrix, and the mismatch steering vector is calibrated by projecting. Then our method can solve the problem of steering angle error in adaptive array beamforming.

3.1. Estimating the covariance matrix [1]

A linear shrinkage estimate of the covariance matrix is given by

$$\tilde{\mathbf{R}} = \beta\hat{\mathbf{R}} + \alpha\mathbf{I} \tag{10}$$

where the shrinkage parameters $\alpha \geq 0$ and $\beta \geq 0$ which can be chosen by minimizing the mean square error (MSE) of the estimator $\tilde{\mathbf{R}}$, and

$$\begin{aligned}\mathrm{MSE}(\tilde{\mathbf{R}}) &= E\{\|\tilde{\mathbf{R}} - \mathbf{R}\|^2\} \\ &= E\{\|\alpha\mathbf{I} - (1-\beta)\mathbf{R} + \beta(\hat{\mathbf{R}} - \mathbf{R})\|^2\} \\ &= \alpha^2 N - 2\alpha(1-\beta)\mathrm{tr}(\mathbf{R}) + (1-\beta)^2\|\mathbf{R}\|^2 + \beta^2 E\{\|\hat{\mathbf{R}} - \mathbf{R}\|^2\}\end{aligned} \tag{11}$$

where $\|\cdot\|$ denotes the Frobenius norm for a matrix (e.g. $\|\mathbf{A}\| = \sqrt{\mathrm{tr}(\mathbf{A}*\mathbf{A})}$) or the Euclidean norm for a vector, and $\hat{\mathbf{R}}$ is the unbiased estimate of $\mathbf{R}$.

The optimal shrinkage parameters can be obtained as [1]

$$\hat{\alpha} = \min\left[\hat{\nu}\frac{\hat{\xi}}{\|\hat{\mathbf{R}} - \hat{\nu}\mathbf{I}\|^2}, \hat{\nu}\right] \tag{12}$$

$$\hat{\beta} = 1 - \frac{\hat{\alpha}}{\hat{\nu}} \tag{13}$$

where $\hat{\nu} = \mathrm{tr}(\hat{\mathbf{R}})/N$,

$$\hat{\xi} = \frac{1}{T^2}\sum_{k=1}^{T}\|\mathbf{x}(k)\|^4 - \frac{1}{T}\|\hat{\mathbf{R}}\|^2.$$

Then the calibrated covariance matrix can be determined by substituting $\hat{\alpha}$ and $\hat{\beta}$ into Eq. (10). The calibrated $\tilde{\mathbf{R}}$ will be used in the following estimation of the actual steering vector.

3.2. Estimating the mismatch steering vector

The ensemble correlation matrix $\mathbf{R}$ defined by Eq. (2) can be eigendecomposed as

$$\mathbf{R} = \sum_{i=1}^{N}\lambda_i e_i e_i^H \tag{14}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{J+2} = \cdots = \lambda_N = \sigma^2$, which are the eigenvalues of $\mathbf{R}$ in descending order, and e_i for $i=1,2,\ldots,N$, are the corresponding eigenvectors.

One may observe from Appendix that the eigenvectors corresponding to the large projections of $\mathbf{s}_d$ on the eigenvector e_i can be used to structure $\mathbf{P}$ which is approximately considered to span the same space as the desired signal subspace. As in practice the mismatch between $\hat{\mathbf{s}}_d$ and $\mathbf{s}_d$ is not too large, the eigenvectors corresponding to the large projections of $\hat{\mathbf{s}}_d$ on the eigenvector e_i can be used to structure $\mathbf{P}$ which spans the new estimated desired signal subspace. First, the projections can be obtained by

$$p(i) = |e_i^H\hat{\mathbf{s}}_d|^2 \quad i = 1,2,\ldots,N. \tag{15}$$

Then $p(i)$ for $i=1,2\ldots,N$ are arranged in descending order, as $p_{[N]} \geq p_{[N-1]} \geq \ldots \geq p_{[1]}$. Corresponding e_i are arranged as $[e_{[N]}, e_{[N-1]},\ldots e_{[1]}]$, where $e_{[i]}$ is the eigenvector corresponding to $d_{[i]}$. Next, the minimum n which satisfies the following expression can be found out:

$$(p_{[N]} + p_{[N-1]} + \ldots + p_{[n]})/N > \rho \tag{16}$$

where N is the sum of projections, and the constant ρ is used to choose the large projections of $\hat{\mathbf{s}}_d$ on the eigenvector e_i, and $0 < \rho < 1$. The eigenvectors corresponding to the large projections can be used to structure $\mathbf{P}$ which spans the new estimated desired signal subspace. It is that the first time the left hand side of Eq. (16) exceeds ρ that we stop, and we obtain the value of n. In this case, the n projections are considered as large projections. Finally, $\mathbf{P}$ can be structured as $[e_{[n]}\, e_{[n+1]}\ldots e_{[N]}]$, and the calibrated steering vector can be obtained by projecting $\hat{\mathbf{s}}_d$ onto the new estimated desired signal subspace which is spanned by $\mathbf{P}$

$$\tilde{\mathbf{s}}_d = \mathbf{P}\mathbf{P}^H\hat{\mathbf{s}}_d \tag{17}$$

To summarize, the proposed robust beamforming approach consists of following steps:

The algorithm

Step 1: Obtain $\tilde{\mathbf{R}}$ in Eq. (10) by calculating $\hat{\alpha}$ and $\hat{\beta}$.
Step 2: Get the calibrated steering vector $\tilde{\mathbf{s}}_d$ in Eq. (17) by estimating $\mathbf{P}$ from Eqs. (15) and (16).
Step 3: Calculate the robust adaptive beamformer weights as

$$\mathbf{w} = \tilde{\mathbf{R}}\tilde{\mathbf{s}}_d(\tilde{\mathbf{s}}_d^H\tilde{\mathbf{R}}^{-1}\tilde{\mathbf{s}}_d)^{-1} \tag{18}$$

4. Simulation results

The studied array is a uniform linear array with 10 elements and with half wavelength inter-element spacing. The non-directional noise is assumed to be a spatially white Gaussian noise with unit covariance. The incident angle of the desired signal is $\theta_p = 5^\circ$. Two interferences are present with incident angle of -50° and -20°. ρ is equal to 0.7 and the sample covariance matrix is computed based on $T=50$ data snapshots. All the simulation results are achieved via 200 Monte-Carlo trails.

The proposed beamforming algorithm is compared to the LCMV beamformer, the diagonal loading method in [1], the traditional projection method in [5] using MDL to estimate the number of sources, and the joint robust beamforming method in which the parameters $\delta = 0.1$, $K=6$, and $\Theta = [\theta_p - 5^\circ, \theta_p + 5^\circ]$ are adopted as the same in [6], where δ is a small number of user choice.

In the first example, the effect of the direction-of-arrival mismatch on array output signal-to-interference plus noise ratio (SINR) is investigated. A look direction mismatch of 3° is assumed, namely, the presumed steering vector is calculated at $\theta_p = 5^\circ$. Interference-to-noise ratios (INR) are both equal to 30 dB. The angle between the estimated steering vector and the actual steering vector at SNR=0 dB, 5 dB, and 15 dB is about 7°, 6°,and 6°, respectively, while the angle between the presumed steering vector and the actual steering vector is about 26.7°. The performances of all methods are shown in Figs. 1 and 2, respectively. They demonstrate that these robust algorithms can solve the problem of steering angle error in adaptive array beamforming. Moreover, figures show that the proposed algorithm has better performance compared with the traditional projection method in [5]. Moreover, the performance of it is better than the robust method in [1] when SNR ≥ -10 dB, and a little worse when SNR < -10 dB. Also, it has almost the same performance as the joint robust method in [6] when SNR ≥ -5 dB, and a little worse when SNR < -5 dB.

In the second example, the influence of both the array geometry error and the direction-of-arrival mismatch on array output SINR is considered. The displacement of each sensor away from its original location is assumed to be drawn uniformly from the set $[-0.05, 0.05]$ measured in wavelength, and the look direction mismatch is assumed to be random and uniformly distributed in $[-4^\circ, 4^\circ]$. Interference-to-noise ratios (INR) are both equal to 30 dB. The performances of all methods are shown in Figs. 3 and 4. It is clear from the figures that these robust algorithms can solve the problem of both the array geometry error and steering angle error in adaptive array beamforming. Figures also show that the proposed algorithm has better performance compared with the traditional projection method and the robust method in [1], and has almost the same performance as the joint robust method in [6].

In the third example, the influence of ρ on array output SINR is considered. The mismatch error of this example is the same as that in the second example. Interference-to-noise ratios (INR) are both equal to 30 dB. Fig. 5 shows that the larger ρ is, the better performance is achieved at high SNR, and the smaller ρ is, the better performance is achieved at low SNR. And they all have better performance

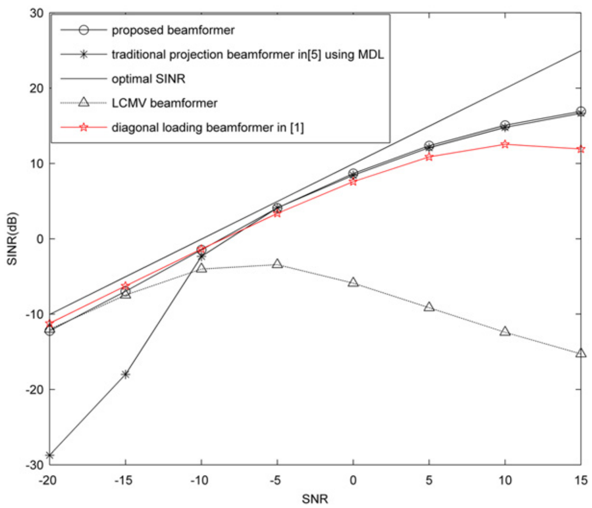

Fig. 1. Output SINR versus SNR while the steering direction error exists.

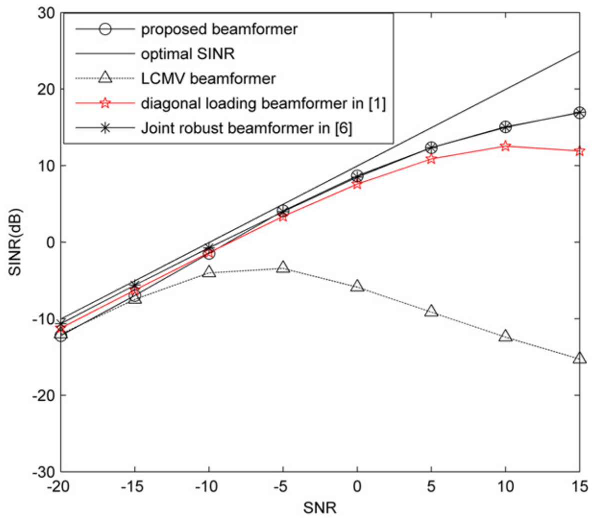

Fig. 2. Output SINR versus SNR while the steering direction error exists.

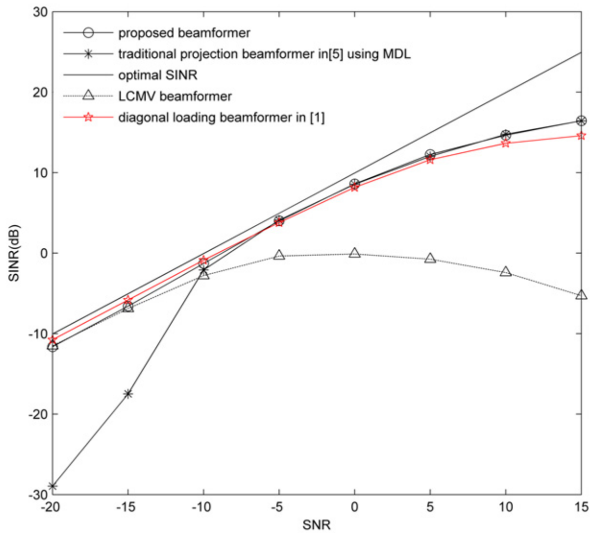

Fig. 3. Output SINR versus SNR while both the steering direction error and array geometry error is present.

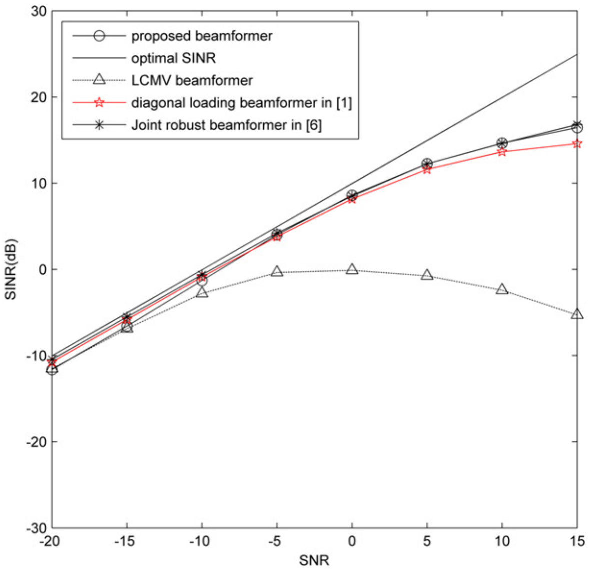

Fig. 4. Output SINR versus SNR while both the steering direction error and array geometry error is present.

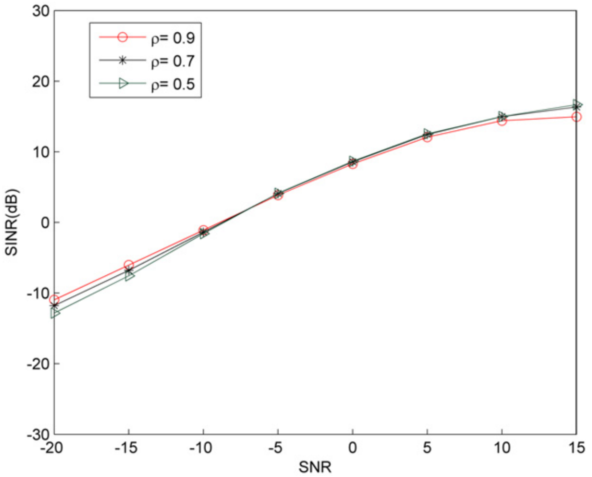

Fig. 5. Output SINR versus SNR with different ρ while both the steering direction error and array geometry error is present.

than the tradition projection method at low SNR. In this case, we choose ρ to be 0.7 in this paper to obtain both well performances at low and high SNR.

In the fourth example, the performance of the algorithm in the absence of these mismatch error is studied. Fig. 6 shows that the proposed method works well in the absence of mismatch, and has better performance compared with the traditional projection method and the robust method in [1].

In the fifth example, the influence of SIR on array output SINR is considered. The mismatch error of this example is the same as that in the second example. Interference-to-noise ratios (INR) are both equal to 10 dB. The performances of all methods are shown in Fig. 7. Fig. 7 shows that the robust algorithms can solve the problem of both the array geometry error and steering

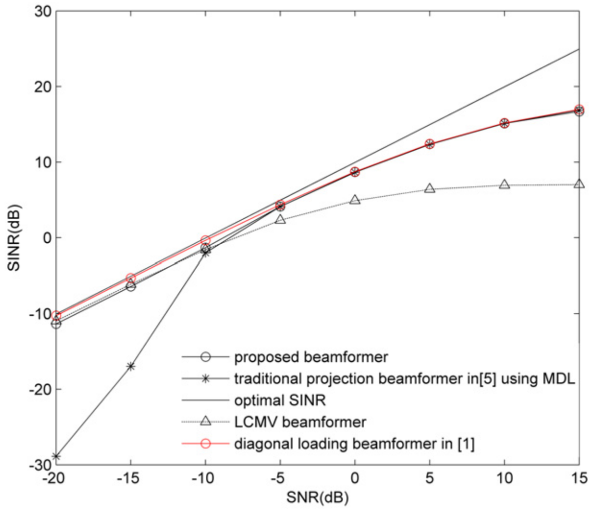

Fig. 6. Output SINR versus SNR while no mismatch error is present.

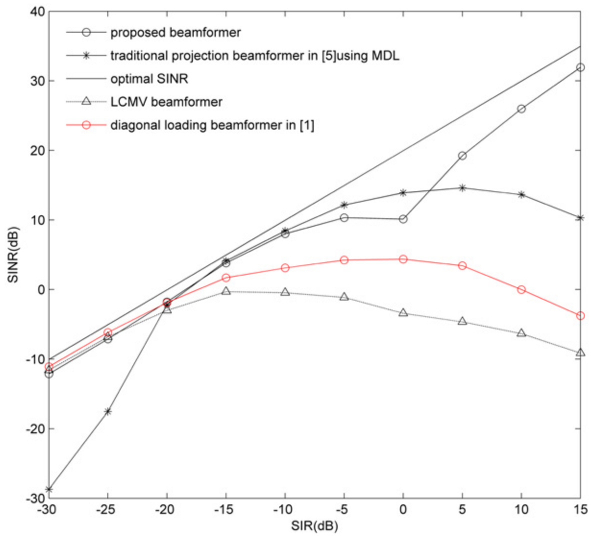

Fig. 7. Output SINR versus SIR while both the steering direction error and array geometry error is present.

angle error in adaptive array beamforming. Fig. 7 also shows that the proposed algorithm has better performance compared with the traditional projection method in [5] when SIR < -15 dB or SIR > 5 dB, and a litter worse performance when -15 dB $<$ SIR < 5 dB. While compared with the recently proposed robust method in [1], our proposed method has better performance when SIR > -20 dB and a little worse when SIR < -20 dB.

5. Conclusion

A new modified projection technique is proposed for robust adaptive array processing. A present algorithm against the covariance matrix uncertainty is used to obtain enhanced covariance matrix estimate. Then the desired signal subspace is estimated from the eigenvectors of the enhanced covariance matrix. Furthermore, a calibrated steering constraint is calculated in sequence by projecting the presumed one onto the estimated desired signal subspace. As the proposed method performs without estimating the number of sources, it is easier to be implemented compared with traditional projection method. Moreover, it has better performance than the traditional projection method at low SNR. Simulation results demonstrate the validity of the modified projection method compared with some of the existing ones.

Appendix A

In the following analysis, the projections of the exact desired signal steering vector $\mathbf{s}_d$ on the eigenvector e_i are examined, and the case of one interference uncorrelated with the desired signal is considered

$$\begin{aligned}\mathbf{R} &= [\mathbf{s}_d, \mathbf{s}_I]\begin{pmatrix} P_s & 0 \\ 0 & P_I \end{pmatrix}[\mathbf{s}_d, \mathbf{s}_I]^H + \sigma^2 \mathbf{I} \\ &= [e_1, e_2]\begin{pmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{pmatrix}[e_1, e_2]^H + \sigma^2 \mathbf{I}\end{aligned} \tag{A-1}$$

where σ^2 is the noise power, $\mathbf{s}_I$ is the steering vector of interference, P_s and P_I represent the power of the desired signal and interference, respectively. μ_i for $i=1,2$, are the eigenvalues of the noise free correlation matrix, and $\mu_1 \geq \mu_2$. Since $\mathbf{s}_d$ is orthogonal to e_i for $i=3,\ldots,N$, $|\mathbf{s}_d^H e_i|^2 = 0$ $(i=3,\ldots,N)$. From [2] we can obtain that

$$\mu_1 = \frac{(N(P_S+P_I) + \sqrt{N^2(P_S+P_I)^2 - 4N^2 P_S P_I (1-|d|^2)})}{2}, \tag{A-2}$$

$$\mu_2 = \frac{(N(P_S+P_I) - \sqrt{N^2(P_S+P_I)^2 - 4N^2 P_S P_I (1-|d|^2)})}{2} \tag{A-3}$$

and

$$|\mathbf{s}_d^H e_i|^2 = N(-1)^i \frac{-\mu_i + NP_I(1-|d|^2)}{\mu_1 - \mu_2}, \quad i=1,2 \tag{A-4}$$

where $|d| = (1/N)\mathbf{s}_d^H \mathbf{s}_I$. Substituting Eqs. (A-2) and (A-3) into Eq. (A-4), we can get

$$|\mathbf{s}_d^H e_1|^2 = N \frac{(((P_S+P_I) + \sqrt{(P_S+P_I)^2 - 4P_S P_I(1-|d|^2)}))/2 - P_I(1-|d|^2)}{\sqrt{(P_S+P_I)^2 - 4P_S P_I (1-|d|^2)}} \tag{A-5}$$

$$|\mathbf{s}_d^H e_2|^2 = N \frac{P_I(1-|d|^2) - ((P_S+P_I) - \sqrt{(P_S+P_I)^2 - 4P_S P_I(1-|d|^2)}))/2}{\sqrt{(P_S+P_I)^2 - 4P_S P_I (1-|d|^2)}} \tag{A-6}$$

For angular separation larger than a beamwidth, $|d|^2 \ll 1$ [2]. Therefore, Eqs. (A-5) and (A-6) can be approximated to be

$$\begin{cases} |\mathbf{s}_d^H e_1|^2 \approx N \frac{P_I |d|^2}{P_I - P_S} \\ |\mathbf{s}_d^H e_2|^2 \approx N\left(1 - \frac{P_I |d|^2}{P_I - P_S}\right) \end{cases} \text{for} \quad P_I > P_S. \tag{A-7}$$

For $P_I \gg P_S$, we can obtain that $|\mathbf{s}_d^H e_1|^2 \approx 0$ and $|\mathbf{s}_d^H e_2|^2 \approx N$, which indicate that the angle between e_2 and $\mathbf{s}_d$ is approximately 0°. In this case, $|\mathbf{s}_d^H e_2|^2 \approx N$ is considered as the large projection. It means that the space spanned by e_2 can be approximately considered as the desired signal subspace and the calibrated steering constraint $\tilde{\mathbf{s}}_d$ can be obtained by projecting the presumed one $\hat{\mathbf{s}}_d$ onto the space spanned by e_2.

Next, the general case in which $P_I \gg P_S$ is not satisfied is considered. In this case, $|\mathbf{s}_d^H e_1|^2 \neq 0$, and the space spanned by e_2 cannot be simply approximately considered as the space spanned by $\mathbf{s}_d$. For example, when $P_I = P_S = P$, from Eqs. (A-2) and (A-3) we can get

$$\mu_1 = NP(1+|d|), \tag{A-8}$$

$$\mu_2 = NP(1-|d|). \tag{A-9}$$

Substituting Eqs. (A-8) and (A-9) into Eq. (A-4), we can obtain

$$|\mathbf{s}_d^H e_1|^2 = \frac{N}{2}(1+|d|) \approx \frac{N}{2}, \tag{A-10}$$

$$|\mathbf{s}_d^H e_2|^2 = \frac{N}{2}(1-|d|) \approx \frac{N}{2}. \tag{A-11}$$

In this case, both $|\mathbf{s}_d^H e_1|^2$ and $|\mathbf{s}_d^H e_2|^2$ are considered as large projections. It means that $\mathbf{s}_d$ not only lies in the space spanned by e_1 but also lies in the space spanned by e_2 in this case. Then the exact desired signal lies in the space spanned by the new structured matrix $\mathbf{P} = [e_1 \quad e_2]$. The calibrated steering constraint $\tilde{\mathbf{s}}_d$ can be obtained by projecting the presumed one $\hat{\mathbf{s}}_d$ onto the space spanned by $\mathbf{P}$.

Moreover, as in practice $\mathbf{R}$ is replaced by a finite sample covariance matrix, $|\mathbf{s}_d^H e_i|^2 \neq 0$ for $i=3,\ldots,N$. The eigenvectors corresponding to the large projections of $\mathbf{s}_d$ on the eigenvector e_i can be used to structure $\mathbf{P}$ which is approximately considered to span the same space as the desired signal subspace.

References

[1] L. Du, J. Li, P. Stoica, Fully automatic computation of diagonal loading levels for robust adptive beamforming, IEEE Transactions on Aerospace and Electronic Systems 46 (1) (2010) 449–458.

[2] L. Chang, C.C. Yeh, Performance of DMI and eigenspace-based beamformers, IEEE Transactions on Antennas and Propagation 40 (1992) 1336–1347.

[3] S.A. Vorobyov, A.B. Gershman, Z.Q. Luo, Robust adaptive beamforming using worst-case performance optimisation: a solution to the signal mismatch problem, IEEE Transactions on Signal Processing 51 (2003) 313–324.

[4] F. Huang, W.X. Sheng, X.F. Ma, W. Wang, Robust adaptive beamforming for large-scale arrays, Signal Processing 90 (2010) 165–172.

[5] D.D. Feldman, L.J. Griffiths, A constraint projection approach for robust adaptive beamforming, in: Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing, 1991, pp. 1381–1384.

[6] Y.J. Gu, W.P. Zhu, M.N.S. Swamy, Adaptive beamforming with joint robustness against covariance matrix uncertainty and signal steering vector mismatch, IET Electronics Letters 46 (1) (2010) 86–88.

[7] H. Akaike, A new look at the statistical model identification, IEEE Transactions on Automatic Control 19 (1974) 716–723.

[8] M. Wax, T. Kailath, Detection of signals by information theoretic criteria, IEEE Transactions on Acoustics, Speech and Signal Processing 33 (1985) 387–392.

[9] H. Lee, F. Li, An eigenvector technique for detecting the number of emitters in a cluster, IEEE Transactions on Signal Processing 42 (1994) 2380–2388.

[10] F.G. William, Using spectral estimation techniques in adaptive processing antenna systems, IEEE Transactions on Antennas and Propagation 34 (1986) 291–300.