March 24, 2021

The results below are generated from an R script.

```
library(rmarkdown)
library(ggplot2)

## Need help getting started?  Try the R Graphics Cookbok:  https://r-graphics.org

library(tidyverse)

## - Attaching packages ------------------------- tidyverse 1.3.0 -
## v tibble  3.1.0    v dplyr  1.0.4
## v tidyr  1.1.2    v stringr 1.4.0
## v readr  1.4.0    v forcats 0.5.1
## v purrr  0.3.4
## - Conflicts ------------------------- tidyverse_conflicts() -
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()

library(rsample)
library(caret)

## Loading required package:  lattice
##
## Attaching package:  'caret'
## The following object is masked from 'package:purrr':
##
##    lift

library(modelr)
library(parallel)
library(foreach)

##
## Attaching package:  'foreach'
## The following objects are masked from 'package:purrr':
##
##    accumulate, when

library(FNN)
library(tseries)

## Registered S3 method overwritten by 'quantmod':
##  method            from
##  as.zoo.data.frame zoo
##
##    'tseries' version:  0.10-48
##
##    'tseries' is a package for time series analysis and computational finance.
##
##    See 'library(help="tseries")' for details.
```

```r
library(stats)
library(urca)
library(AER)

## Loading required package:  car
## Loading required package:  carData
##
## Attaching package:  'car'
## The following object is masked from 'package:dplyr':
##
##     recode
## The following object is masked from 'package:purrr':
##
##     some
## Loading required package:  lmtest
## Loading required package:  zoo
##
## Attaching package:  'zoo'
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
## Loading required package:  sandwich
## Loading required package:  survival
##
## Attaching package:  'survival'
## The following object is masked from 'package:caret':
##
##     cluster

library(dynlm)
library(forecast)

## This is forecast 8.13
##   Want to meet other forecasters?  Join the International Institute of Forecasters:
##   http://forecasters.org/

library(vrtest)
library(readxl)
library(stargazer)

##
## Please cite as:
##   Hlavac, Marek (2018).  stargazer:  Well-Formatted Regression and Summary Statistics Tables.
##   R package version 5.2.2.  https://CRAN.R-project.org/package=stargazer

library(scales)

##
## Attaching package:  'scales'
## The following object is masked from 'package:purrr':
##
##     discard
## The following object is masked from 'package:readr':
##
##     col_factor

library(quantmod)
```

```
## Loading required package:  xts
##
## Attaching package:  'xts'
## The following objects are masked from 'package:dplyr':
##
##     first, last
## Loading required package:  TTR
```

```
library(zoo)
library(fGarch)
```

```
## Loading required package:  timeDate
## Loading required package:  timeSeries
##
## Attaching package:  'timeSeries'
## The following object is masked from 'package:zoo':
##
##     time<-
## Loading required package:  fBasics
##
## Attaching package:  'fBasics'
## The following object is masked from 'package:TTR':
##
##     volatility
## The following object is masked from 'package:car':
##
##     densityPlot
```

```
library(LINselect)
library(aTSA)
```

```
##
## Attaching package:  'aTSA'
## The following object is masked from 'package:forecast':
##
##     forecast
## The following objects are masked from 'package:tseries':
##
##     adf.test, kpss.test, pp.test
## The following object is masked from 'package:graphics':
##
##     identify
```

```
library(egcm)


#Time Series Question 1 Cointegration


# plot both interest series
plot(merge(as.zoo(irates$r1), as.zoo(irates$r3)),
     plot.type = "single",
     lty = c(2, 1),
     lwd = 0.5,
     xlab = "Date",
     ylab = "Percent per annum",
```

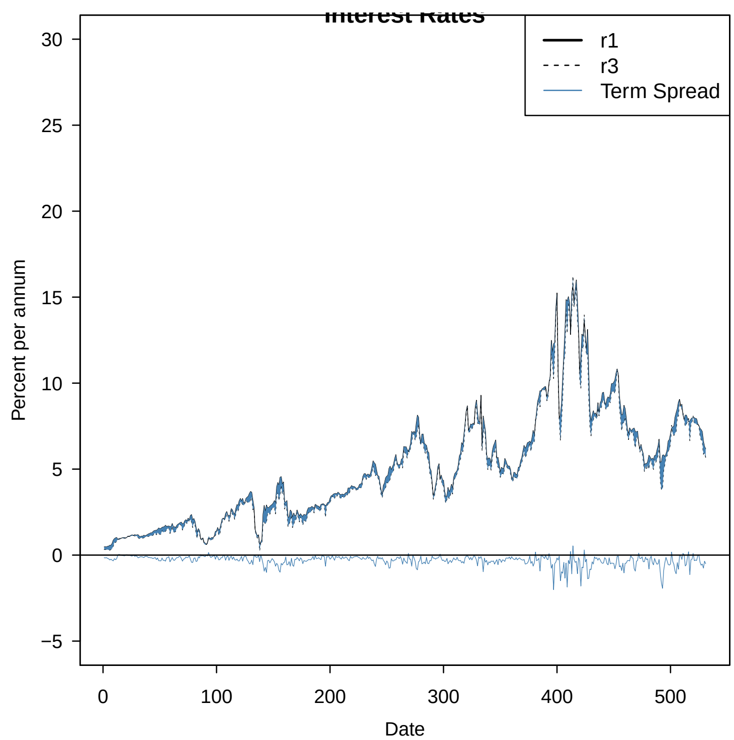
```r
     ylim = c(-5, 30),
     main = "Interest Rates")


TSpread=irates$r1-irates$r3
# add the term spread series
lines(as.zoo(TSpread),
      col = "steelblue",
      lwd = 0.5,
      xlab = "Date",
      ylab = "Percent per annum",
      main = "Term Spread")

# shade the term spread
polygon(c(time(irates$r1), rev(time(irates$r3))),
        c(irates$r1, rev(irates$r3)),
        col = alpha("steelblue", alpha = 7),
        border = NA)

# add horizontal line add 0
abline(0, 0)

# add a legend
legend("topright",
       legend = c("r1", "r3", "Term Spread"),
       col = c("black", "black", "steelblue"),
       lwd = c(2, 1, 1),
       lty = c(1, 2, 1))
```



```r
#The plot suggests that r1 and r3 interest rates are cointegrated:
#That is, both interest series seem to show the same behavior by sharing a
#common stochastic trend.The term spread, which I get by taking the difference
#between r1 and r3 interest rates, seems to be stationary. In fact, the
#expectations theory of the term structure suggests the cointegrating coefficient
#<U+03B8> to be 1. This is consistent with the visual result I obtain.

#(a)

#The null hypothesis is that the data are non-stationary

r1_1=ur.df(irates$r1, type = c("none"), lags = 3)
summary(r1_1)

##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.4371 -0.1534  0.0398  0.2427  2.9574
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1     -0.003874   0.004579  -0.846  0.39797
## z.diff.lag1  0.022573   0.043495   0.519  0.60399
## z.diff.lag2 -0.014892   0.043496  -0.342  0.73220
## z.diff.lag3 -0.114004   0.043573  -2.616  0.00914 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6057 on 523 degrees of freedom
## Multiple R-squared:  0.01572,Adjusted R-squared:  0.008192
## F-statistic: 2.088 on 4 and 523 DF,  p-value: 0.08109
##
##
## Value of test-statistic is: -0.8459
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62

#Ho: tau1: <U+03B3>=0

#For type= "none," tau1 is the null hypothesis for gamma = 0.
#Using the interest rates example,
#I get a test-statistic of -0.8459. The Critical values for
```

```
#test statistics are: tau1 -2.58 -1.95 -1.62.
#Given that the test statistic is within the all 3 regions
#(1%, 5%, 10%) where we fail to reject the null, we should presume
#the data is a random walk, i.e that a unit root is present.


r1_2=ur.df(irates$r1, type = c("trend"), lags = 3)
summary(r1_2)

##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.2262 -0.1855  0.0007  0.1985  3.0382
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0480298  0.0531093   0.904  0.36622
## z.lag.1     -0.0466643  0.0145947  -3.197  0.00147 **
## tt           0.0007070  0.0003021   2.340  0.01965 *
## z.diff.lag1  0.0442049  0.0437827   1.010  0.31314
## z.diff.lag2  0.0068410  0.0437872   0.156  0.87591
## z.diff.lag3 -0.0928695  0.0438271  -2.119  0.03456 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6012 on 521 degrees of freedom
## Multiple R-squared:  0.03388,Adjusted R-squared:  0.02461
## F-statistic: 3.655 on 5 and 521 DF,  p-value: 0.002933
##
##
## Value of test-statistic is: -3.1973 3.5567 5.238
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34

#For the type="trend":

#tau: <U+03B3>=0
#phi3: <U+03B3>=a2=0
#phi2: a0=<U+03B3>=a2=0
```

```
#In this case, the test statistics are -3.1973 3.5567 5.238.
#In each one of these cases, the t-statistics fall within the
#"fail to reject the null" regions. Tau3 implies that we fail
#to reject the null of unit root. Failing to reject phi3 implies
#two things: 1) <U+03B3>=0 (unit root) and 2) there is no time trend term, i.e., a2=0.
#Failing to reject phi2 implies 3 things: 1) <U+03B3>=0 and 2) no time
#trend term and 3) no drift term, i.e. that <U+03B3>=0, that a0=0, and that a2=0.


r1_3=ur.df(irates$r1, type = c( "drift"), lags = 3)
summary(r1_3)

##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.3442 -0.1850 -0.0146  0.2083  3.0055
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.101381   0.048173   2.105   0.0358 *
## z.lag.1     -0.018604   0.008356  -2.226   0.0264 *
## z.diff.lag1  0.028333   0.043439   0.652   0.5145
## z.diff.lag2 -0.009045   0.043443  -0.208   0.8352
## z.diff.lag3 -0.108248   0.043517  -2.487   0.0132 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6038 on 522 degrees of freedom
## Multiple R-squared:  0.02373,Adjusted R-squared:  0.01625
## F-statistic: 3.172 on 4 and 522 DF,  p-value: 0.01364
##
##
## Value of test-statistic is: -2.2264 2.5747
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78

#phi3: <U+03B3>=a2=0

#type = "drift" (where a0 is "a sub-zero" and refers to the constant, or drift term)
#"tau2" is still the gamma=0 null hypothesis. In this case, where the first test
#statistic = -2.2264 is within the region of failing to reject the null, we should
```

```
#presume a unit root, that <U+03B3>=0 at either one of the significance levels of 1%, 5% or
#that of 10%. The phi1 term refers to the second hypothesis, which is a combined null
#hypothesis of a0=gamma=0.  This means that BOTH of the values are tested to be 0
#at the same time. If p<0.05, we reject the null, and presume  that AT LEAST one
#of these is false--i.e. one or both of the terms a0 or gamma are not 0. Failing
#to reject this null implies that BOTH a0 AND gamma=0, implying 1) that <U+03B3>=0 therefore
#a unit root is present, AND 2) a0=0, so there is no drift term. Since a t-statistic
#of 2.5747 is smaller than any of the critical values of phi1, we fail to reject
#the null that one or both terms are not zero. Unit root is present.



r3_1=ur.df(irates$r3, type = c("none"), lags = 3)
summary(r3_1)

##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.4734 -0.0967  0.0372  0.2073  2.6232
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1     -0.003059   0.003873  -0.790   0.4300
## z.diff.lag1  0.120264   0.043762   2.748   0.0062 **
## z.diff.lag2 -0.078709   0.043933  -1.792   0.0738 .
## z.diff.lag3 -0.012009   0.043831  -0.274   0.7842
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5398 on 523 degrees of freedom
## Multiple R-squared:  0.02025,Adjusted R-squared:  0.01276
## F-statistic: 2.702 on 4 and 523 DF,  p-value: 0.02991
##
##
## Value of test-statistic is: -0.7899
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62

#tau: <U+03B3>=0

#For type= "none," tau1 in R output is the null hypothesis for gamma = 0.
#Using the interest rates example,
```

```r
#I get a value of test-statistic is -0.7899. The critical values for test
#statistics are: tau1 -2.58 -1.95 -1.62. The test statistic falls within
#the all 3 regions (1%, 5%, 10%). As such, we fail to reject the null hypothesis.
#We presume that a unit root is present.



r3_2=ur.df(irates$r3, type = c("trend"), lags = 3)
summary(r3_2)

##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.2608 -0.1380  0.0019  0.1626  2.8282
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0481030  0.0475005   1.013  0.31168
## z.lag.1     -0.0405648  0.0127394  -3.184  0.00154 **
## tt           0.0006391  0.0002720   2.350  0.01914 *
## z.diff.lag1  0.1386243  0.0438889   3.159  0.00168 **
## z.diff.lag2 -0.0598721  0.0440638  -1.359  0.17481
## z.diff.lag3  0.0070132  0.0439794   0.159  0.87336
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5358 on 521 degrees of freedom
## Multiple R-squared:  0.03824,Adjusted R-squared:  0.02901
## F-statistic: 4.143 on 5 and 521 DF,  p-value: 0.001064
##
##
## Value of test-statistic is: -3.1842 3.5294 5.1915
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34

#tau: <U+03B3>=0
#phi3: <U+03B3>=a2=0
#phi2: a0=<U+03B3>=a2=0

#The test statistics are -3.1842 3.5294 5.1915.
```

```r
#In this case, although we fail to reject the null of unit root at
#the 1% and 5% significance levels, we are able to reject it at the
#10% significance level.On the other hand, we fail to reject the null
#hypothesis that there is no time trend and unit root in the data.
#Failing to reject phi2 implies 3 things: 1) <U+03B3>=0 AND 2) no time trend
#term AND 3) no drift term, i.e. that <U+03B3>=0, that a0=0, and that a2=0.


r3_3=ur.df(irates$r3, type = c( "drift"), lags = 3)
summary(r3_3)

##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.3846 -0.1297 -0.0150  0.1849  2.7038
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.091960   0.043870   2.096  0.03654 *
## z.lag.1     -0.015850   0.007220  -2.195  0.02859 *
## z.diff.lag1  0.124623   0.043671   2.854  0.00449 **
## z.diff.lag2 -0.073745   0.043855  -1.682  0.09325 .
## z.diff.lag3 -0.007202   0.043750  -0.165  0.86930
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5381 on 522 degrees of freedom
## Multiple R-squared:  0.02805,Adjusted R-squared:  0.0206
## F-statistic: 3.766 on 4 and 522 DF,  p-value: 0.004962
##
##
## Value of test-statistic is: -2.1951 2.511
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78

#phi3: <U+03B3>=a2=0

#In this case, where the first test statistic = -2.1951 is within the region
#of failing to reject the null, we should presume a unit root, that <U+03B3>=0 at
#either the significance level of 1%, 5% or that of 10%.
#The phi1 term refers to the second hypothesis, which is a combined null hypothesis
```

```
#of a0=gamma=0.
#This means that BOTH of the values are tested to be 0 at the same time.
#Since a t-statistic of 2.511 is smaller than any of the critical values of phi1,
#we fail to reject the null that one or both terms
#are not zero.


#In part (a) I conclude that it is plausible to model both interest rate series as I(1).



#(b) Perform a regression by OLS explaining r1 from r3. Test for cointegration.

lm_r1=lm(irates$r1~irates$r3, data=irates)

ur.df(window(irates$r1) - window(irates$r3),
      lags = 3,
      selectlags = "AIC",
      type = "drift")

##
## ###############################################################
## # Augmented Dickey-Fuller Test Unit Root / Cointegration Test #
## ###############################################################
##
## The value of the test statistic is: -8.0431 32.3517

#(c) Perform a regression by OLS explaining r3 from r1. Test for cointegration.

lm_r3=lm(irates$r3~irates$r1, data=irates)

ur.df(window(irates$r3) - window(irates$r1),
      lags = 3,
      selectlags = "AIC",
      type = "drift")

##
## ###############################################################
## # Augmented Dickey-Fuller Test Unit Root / Cointegration Test #
## ###############################################################
##
## The value of the test statistic is: -8.0431 32.3517

#Here I test if term spread is stationairy (cointegration of interest rates) using ADF.
#The test leads me to reject the null hypothesis of non-stationarity of the term spread
#series at any of the standard significance levels. This is strong evidence in favor
#of the alternative hypothesis that the term spread is stationary, implying cointegration
#of r1 and r3 interest rates.

#(d) What is cointegration? Interpret your findings above intuitively.

#When the dependent(Yt) and the independent(Xt) variables are I(1) and if
#there is a <U+03B8> such that Yt<U+623C><U+3E38>Xt is I(0), Xt and Yt are said to be cointegrated.
#In other words, cointegration of Xt and Yt means that Xt and Yt have the same
#or common stochastic trend and that this trend can be eliminated by taking a
```

```
#difference of the series such that the resulting series is stationary.


#Time Series Question 2

#I) Cholesky Decomposition

#(c)

#The Cholesky decomposition of a Hermitian positive-definite matrix A, is a
#decomposition of the form A=LL*, where L  is a lower triangular matrix with
#real and positive diagonal entries, and L* is the conjugate transpose of L.
#Every Hermitian positive-definite matrix (and thus also every real-valued
#symmetric positive-definite matrix) has a unique Cholesky decomposition.
#In other words, Cholesky decomposition reduces a symmetric matrix into a
#lower-triangular matrix. When multiplied by it's transpose this matrix produces
#the original symmetric matrix. In our example we use it for making certain
#assumptions on the variables we are interested in, which allow us to assume
#that the contemporaneous effect(short-run effect) is equal to zero on our
#variables of interest. This is an assumption we make and an assumption we need
#to defend. However, without the Cholesky decomposition, representing this
#assumption into an equation would be more difficult. We know that the cholesky
#decomposition will give us the the parameters we want because we are able to
#represent our parameters in a lower-triangular form where we define as to whether
#or not certain variables are allowed, by assumption, to affect one another.

#(d)

#After imposing the short-run restrictions (i.e.,after we specify if
#any entry has any relationship with any other entry,and/or if any entry is zero or 1),
#the Ao matrix will be a 3 by 3 matrix. The first row will have a value 'of 'a' in the
#first column and values of 0 in the other two columns. This is to represent the fact
#that the first entry cannot depend contemporaneously on the other two entries.
#The second row of the matrix has a value of 'b' in the first column, a value 'c' in
#the second column and a value of '0' in the third column. This is to show that while
#the second entry can depend contemporaneously on the first entry, it cannot depend
#contemporaneously on the third entry. The third row of the matrix takes a value of 'd'
#in the first column, a value of 'e' in the second column, and a value of 'f' in the
#third column. In this row we do not make any restriction assumptions as to what the
#relationship among entries is.


#II) Short Run Restrictions

#(a)

#The intuition behind the restrictions imposed is to move from the parameters of a
#reduced form VAR to the structural VAR parameters of interest. Otherwise, the model
#cannot be identified. That is, because the coefficients in the identification of
#structural parameters is unknown, we need to impose theoretical restrictions to
#reduce the number of unknown structural parameters to be less than or equal to the
#number of estimated parameters of the variance-covariance matrix of the VAR residuals.
#Finding dynamic patterns consistent with the structural model used for the identification
#would provide evidence in support of the theoretical model. Otherwise, the theory
```

```r
#will be invalid or the empirical model somewhat misspecified.
#It is also important to note that we cannot estimate the structural form with OLS because
#one of the main assumptions in time series is violated. The assumption violated is that
#the regressors are correlated with the error term.The Ao matrix is problematic because it
#includes all the contemporaneous relations between endogenous variables.


#(c)

#The effects that are contemporaneously zero as a result of short run restrictions
#are the aggregate demand shock on oil production, the oil-specific demand shock on
#oil production and the oil-specific demand shock on real activity. This is because
#we are assuming that the shock on the real price does not affect contemporaneous
#production and the shock on real economic activity does not affect contemporanous
#production.And then we are also assuming that the real oil price doesn't have a
#contemporaneous effect on real economic activity. This is because we assume that
#all these have lagged effects in the economy, and hence the zero starting value.

#(d)

#Intuitively, the first line of graphs explains the effect of an oil supply shock on
#oil production. Here we have not made any identifying assumptions. Instead, what we
#observe in the first graph is a negative oil supply shock that begins to improve with a lag.
#On the other hand, the oil supply shock seems to have little to no effect on the real
#economic activity. Similarly, it appears that the oil supply shock has no effect on the price
#of oil. The second line of graphs explains the aggregate demand shock on oil production,
#real economic activity and real price of oil. Aggregate demand shock does no seem to
#significantly affect the oil production. On the other hand, we observe a positive aggregate
#demand shock in the real economic activity, which remains fairly stable in the very shock run,
#but starts slightly decreasing the longer run. The aggregate demand shock on the real price
#of oil starts by having no effect, but gradually shows as increase as time passes.
#The third line of graphs shows the oil-specific demand shock on oil production, real
#economic activity and real price of oil. The oil specific demand shock does not seem
#to have a huge impact on oil production and the latter remains fairly stable across time.
#The oil specific demand shock on real economic activity starts from zero, by assumption,
#then increases slightly, then goes back to zero again in the longer run. It is the oil
#specific shock that seems to drive the real price of oil. It begins as a positive shock and
#then gradually declines.
```

The R session information (including the OS info, R version and all packages used):

```r
sessionInfo()

## R version 4.0.4 (2021-02-15)
## Platform: x86_64-w64-mingw32/x64 (64-bit)
## Running under: Windows 10 x64 (build 19041)
##
## Matrix products: default
##
## locale:
## [1] LC_COLLATE=English_United Kingdom.1252  LC_CTYPE=English_United Kingdom.1252
## [3] LC_MONETARY=English_United Kingdom.1252 LC_NUMERIC=C
## [5] LC_TIME=English_United Kingdom.1252
##
```

```
## attached base packages:
## [1] parallel  stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
##  [1] egcm_1.0.12         aTSA_3.1.2          LINselect_1.1.3     fGarch_3042.83.2
##  [5] fBasics_3042.89.1   timeSeries_3062.100 timeDate_3043.102   quantmod_0.4.18
##  [9] TTR_0.24.2          xts_0.12.1          scales_1.1.1        stargazer_5.2.2
## [13] readxl_1.3.1        vrtest_0.97         forecast_8.13       dynlm_0.3-6
## [17] AER_1.2-9           survival_3.2-7      sandwich_3.0-0      lmtest_0.9-38
## [21] zoo_1.8-8           car_3.0-10          carData_3.0-4       urca_1.3-0
## [25] tseries_0.10-48     FNN_1.1.3           foreach_1.5.1       modelr_0.1.8
## [29] caret_6.0-86        lattice_0.20-41     rsample_0.0.9       forcats_0.5.1
## [33] stringr_1.4.0       dplyr_1.0.4         purrr_0.3.4         readr_1.4.0
## [37] tidyr_1.1.2         tibble_3.1.0        tidyverse_1.3.0     ggplot2_3.3.3
## [41] rmarkdown_2.7
##
## loaded via a namespace (and not attached):
##  [1] colorspace_2.0-0     ellipsis_0.3.1       class_7.3-18         rio_0.5.26
##  [5] pls_2.7-3            fs_1.5.0             rstudioapi_0.13      farver_2.1.0
##  [9] listenv_0.8.0        furrr_0.2.2          mvtnorm_1.1-1        prodlim_2019.11.13
## [13] fansi_0.4.2          lubridate_1.7.9.2    xml2_1.3.2           codetools_0.2-18
## [17] splines_4.0.4        knitr_1.31           Formula_1.2-4        jsonlite_1.7.2
## [21] pROC_1.17.0.1        broom_0.7.5          dbplyr_2.1.0         compiler_4.0.4
## [25] httr_1.4.2           backports_1.2.1      assertthat_0.2.1     Matrix_1.3-2
## [29] cli_2.3.1            lars_1.2             htmltools_0.5.1.1    tools_4.0.4
## [33] gtable_0.3.0         glue_1.4.2           reshape2_1.4.4       Rcpp_1.0.6
## [37] fracdiff_1.5-1       cellranger_1.1.0     vctrs_0.3.6          nlme_3.1-152
## [41] iterators_1.0.13     gower_0.2.2          xfun_0.22            globals_0.14.0
## [45] spatial_7.3-13       openxlsx_4.2.3       rvest_1.0.0          lifecycle_1.0.0
## [49] gtools_3.8.2         future_1.21.0        MASS_7.3-53          ipred_0.9-9
## [53] hms_1.0.0            elasticnet_1.3       yaml_2.2.1           curl_4.3
## [57] rpart_4.1-15         stringi_1.5.3        highr_0.8            randomForest_4.6-14
## [61] zip_2.1.1            lava_1.6.9           rlang_0.4.10         pkgconfig_2.0.3
## [65] pracma_2.3.3         evaluate_0.14        recipes_0.1.15       tidyselect_1.1.0
## [69] parallelly_1.23.0    plyr_1.8.6           magrittr_2.0.1       R6_2.5.0
## [73] generics_0.1.0       DBI_1.1.1            pillar_1.5.1         haven_2.3.1
## [77] foreign_0.8-81       withr_2.4.1          abind_1.4-5          nnet_7.3-15
## [81] crayon_1.4.1         utf8_1.1.4           grid_4.0.4           data.table_1.14.0
## [85] ModelMetrics_1.2.2.2 reprex_1.0.0         digest_0.6.27        stats4_4.0.4
## [89] munsell_0.5.0        quadprog_1.5-8
```

```
Sys.time()
```

```
## [1] "2021-03-24 16:50:58 CDT"
```

QUESTION 2.

(a) I) CHOLESKY DECOMPOSITION

VECTOR OF STRUCTURAL SHOCKS:

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}$$

By assumption, the structural shocks aren't correlated. As such, the covariance of the diagonals in the structural shocks are zero. The other assumption we make is that the structural shocks have a mean zero & a variance-covariance matrix D of the following form:

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} \sim iid\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \underbrace{\begin{pmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{pmatrix}}_{\substack{\text{variance-covariance} \\ \text{matrix with exogenous} \\ \text{shocks to each variable.}}} \right]$$

(b) The reduced form model:

$$Y_t = B^{-1}\Gamma_0 + B^{-1}\Gamma_1 Y_{t-1} + B^{-1}\varepsilon_t$$

$$Y_t = C_{3\times1} + \Phi_{3\times3} Y_{t-1} + e_t$$

Reduced form shocks:

$$e_{1t} = B^{-1}\varepsilon_{1t}$$

$$e_{2t} = \varepsilon_{2t} + \beta_{21}\varepsilon_{2t}$$

$$\varepsilon_{3t} = \varepsilon_{3t} + \beta_{31}\varepsilon_{3t} + \beta_{32}\varepsilon_{3t}$$

Continuation of I) CHOLESKY DECOMPOSITION (b) :

VARIANCE - COVARIANCE MATRIX :

$$E[e_t] = 0$$

$$E[e_t e_t'] = E[B^{-1}\varepsilon_t \varepsilon_t'(B^{-1})']$$

$$= B^{-1} E[\varepsilon_t \varepsilon_t'] (B^{-1})'$$

$$= B^{-1} D (B^{-1})'$$

$$= \Omega = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{23} & w_{33} \end{bmatrix}$$

The variance - covariance matrix of the reduced - form shocks is not diagonal. This is because the forecast errors are affected by the shocks.

Furthermore, the variance - covariance matrix is a positive definite symmetric matrix.

It depends on B and a diagonal matrix with positive elements.

II) SHORT RUN RESTRICTIONS

(b) Reduced-form VAR:

$$z_t = A_0^{-1} \alpha + \sum_{i=1}^{24} A_0^{-1} A_i z_{t-1} + A_0^{-1} \varepsilon_t$$

Reduced-form shocks:

$$A_0^{-1} \varepsilon_t = e_t = \begin{bmatrix} e_t^{\Delta prod} \\ e_t^{rea} \\ e_t^{rpo} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_t^{\Delta prod} \\ \varepsilon_t^{rea} \\ \varepsilon_t^{rpo} \end{bmatrix}$$

From this we can identify ε_s from e_s.

- $e_t^{\Delta prod} = a_{11} \varepsilon_t^{\Delta prod}$
- $e_t^{rpa} = a_{21} \varepsilon_t^{\Delta prod} + a_{22} \varepsilon_t^{rea}$
- $e_t^{rpo} = a_{31} \varepsilon_t^{\Delta prod} + a_{32} \varepsilon_t^{rea} + a_{33} \varepsilon_t^{rpo}$

For $a_{11} = a_{22} = a_{33} = 1$ due to the Cholesky Decomposition:

$\rightarrow e_t^{\Delta prod} = \varepsilon_t^{\Delta prod}$

$\rightarrow e_t^{rpa} = a_{21} \varepsilon_t^{\Delta prod} + \varepsilon_t^{rea}$

$\rightarrow e_t^{rpo} = a_{31} \varepsilon_t^{\Delta prod} + a_{32} \varepsilon_t^{rea} + \varepsilon_t^{rpo}$

We obtain the structural shocks:

$\Rightarrow \varepsilon_t^{\Delta prod} = e_t^{\Delta prod}$

$\Rightarrow \varepsilon_t^{rea} = e_t^{rpa} - a_{21} \varepsilon_t^{\Delta prod}$

$\Rightarrow \varepsilon_t^{rpo} = e_t^{rpo} - a_{31} \varepsilon_t^{\Delta prod} - a_{32} \varepsilon_t^{rea}$

SHORT-RUN RESTRICTIONS:

1. Solve for the identification problem
2. Use the recursive identification method.
3. Construct a set of uncorrelated structural shocks directly from the reduced-form shocks.
4. Assume certain shocks have effect on only some variables at time t.

- B is the lower triangular in VAR(1).