



# MECH 525

## Mechanics of Microsensors

Project no: #2

Project title:

**FORCE MEASUREMENT IN  $\mu\text{N}$  RANGE**

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### Introduction:

The project is based on a paper that discusses the design, modeling, fabrication, and characterization of triplate differential capacitive sensors used in novel on-chip material testing systems for measuring load or deformation in nanostructures. These sensors have high sensitivity, low temperature drift, and low power dissipation, making them attractive for nanomechanical testing applications. Analytical expressions for stability, linearity, and sensitivity are derived and discussed for the first time, taking into account the electrostatic force generated by the excitation signal in quasi-static applications. The influence of electron beams in electron microscopes on capacitance measurements is also examined.

The devices display high sensitivity of  $0.61 \text{ fF nm}^{-1}$  within a quasi-linear moving range of 2250 nm, offering a displacement resolution of 1 nm and a load resolution of 34 nN. The fabricated sensors are suitable for characterizing one-dimensional nanostructures such as **nanotubes**, **nanowires**, and **nanobelts**, which have unique properties and potential future applications. These sensors offer advantages such as **high sensitivity**, **linearity**, and **real-time measurements**, while also addressing the challenges of high-density electrical interconnections and isolations in capacitive sensors. Experimental results show good agreement with analytical and finite element analysis findings.

### Part 1:

The first part of the project focuses on designing the actuator part. The materials used for this part are mainly Silicon for the solid part and air as the medium for charge permittivity.

### Mechanical Part:

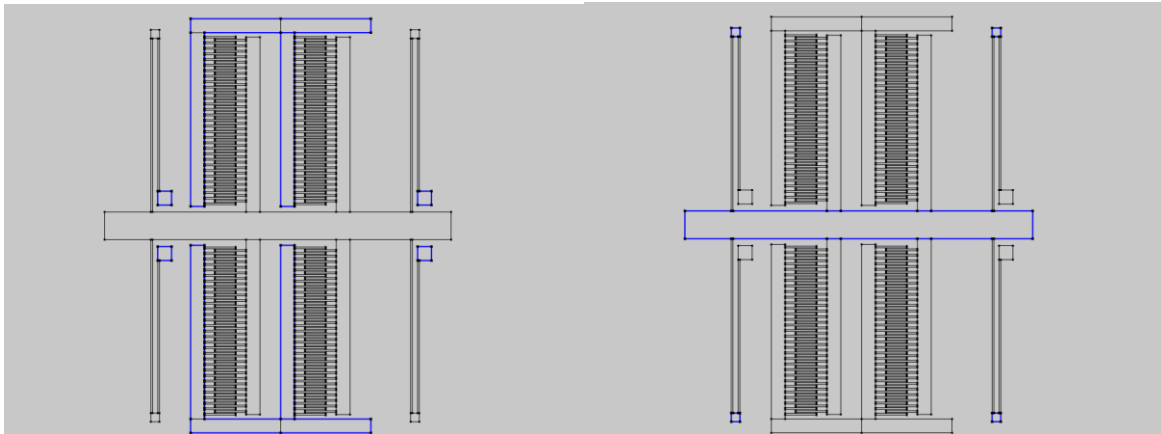
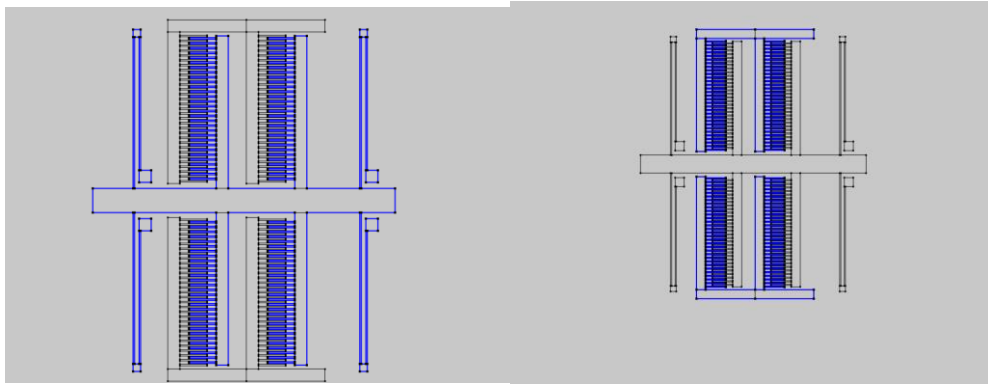
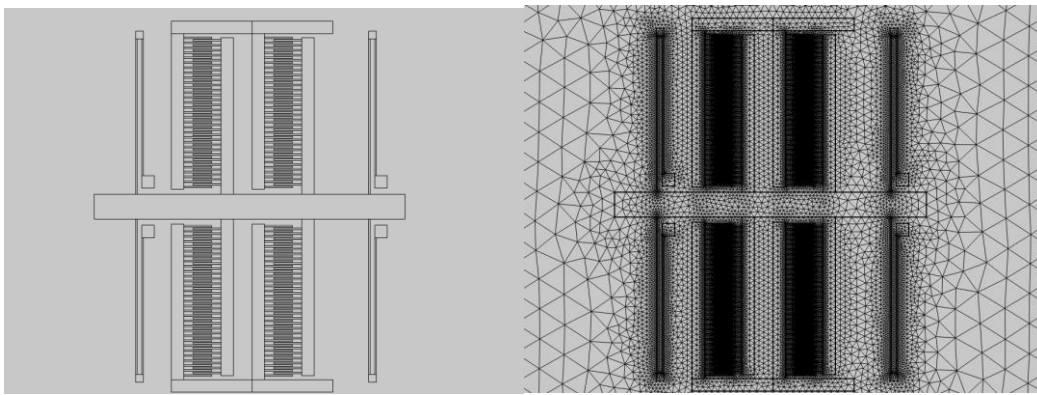
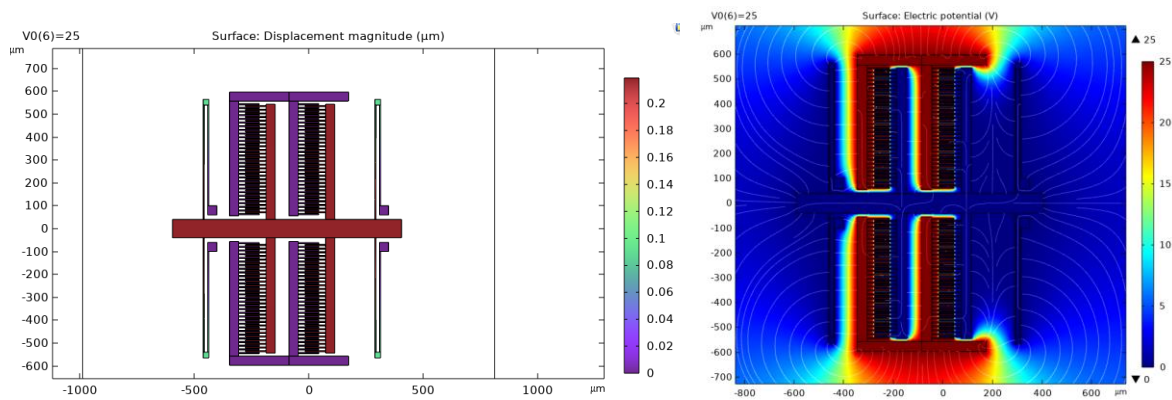


Figure 1 Fixed Constraints (left), Prescribed displacement (right)

*Electrical Part:**Figure 2 Ground (left), Electric Potential (right)**Figure 3 Overall geometry (left), Mesh (right)*

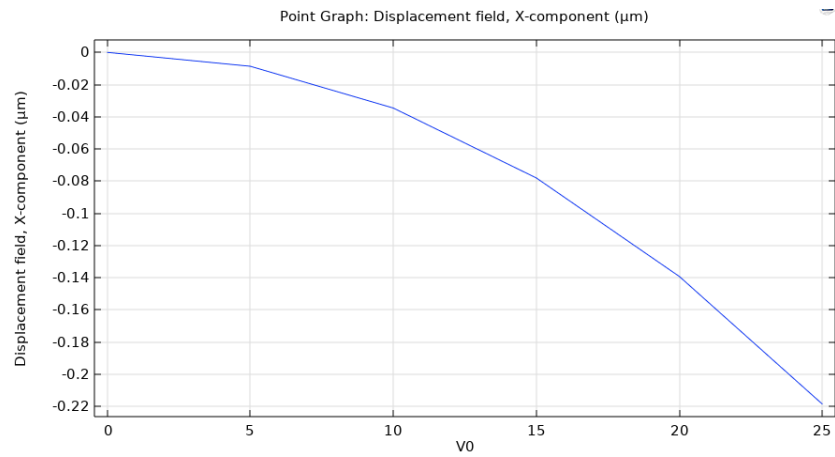


Figure 4 Result graph of Part 1. Displacement vs Voltage

From this graph, we see that the displacement is increasing with voltage. It reaches  $0.22 \mu\text{m}$  when  $V=25$ . The graph is non-linear as can be seen in the figure.

Part 2:

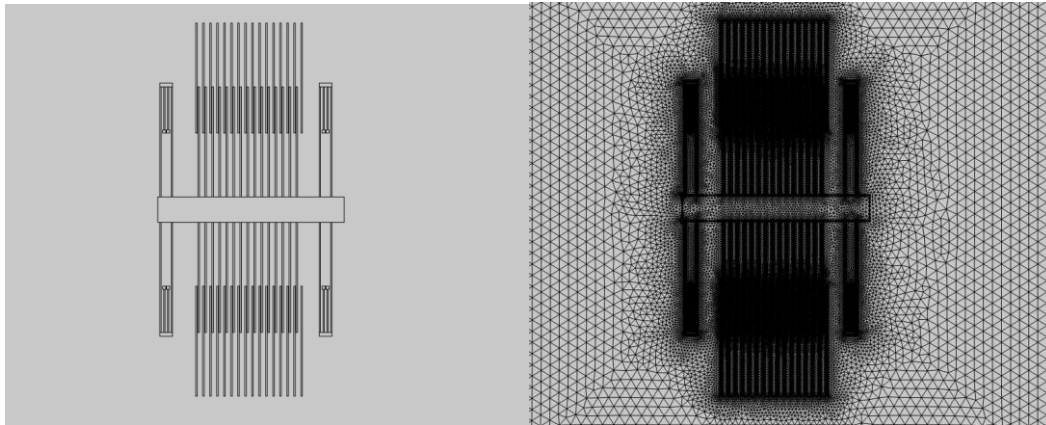


Figure 5 Overall geometry (left), Mesh (right)

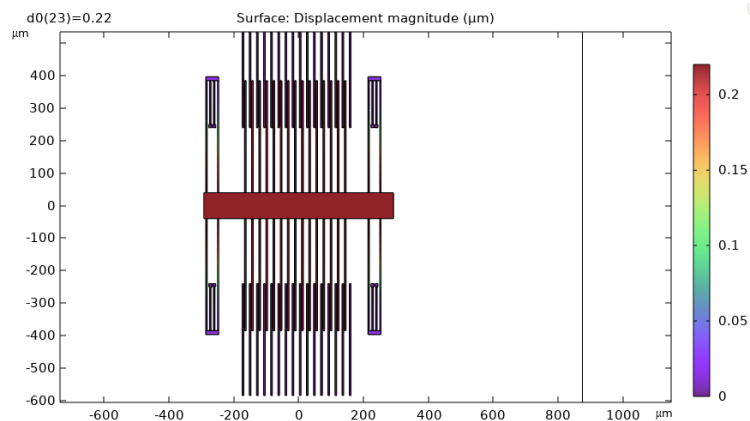


Figure 6 Part-2 Displacement magnitude

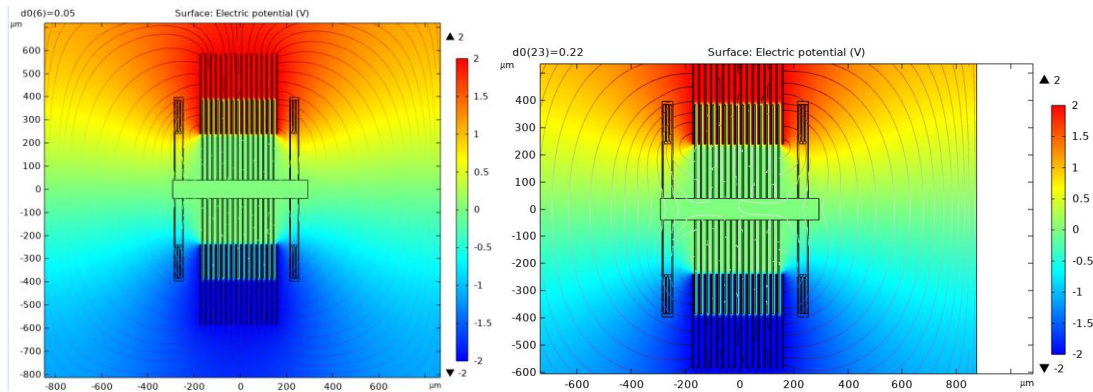


Figure 7 Part-2 Electrical Potential graph

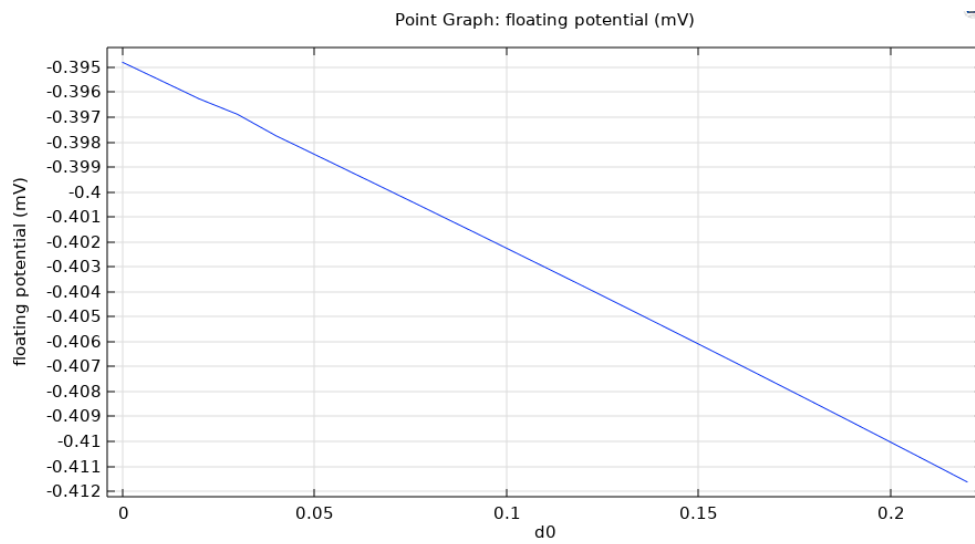


Figure 8 Part-2 Result Floating Potential vs displacement

Part-3 :

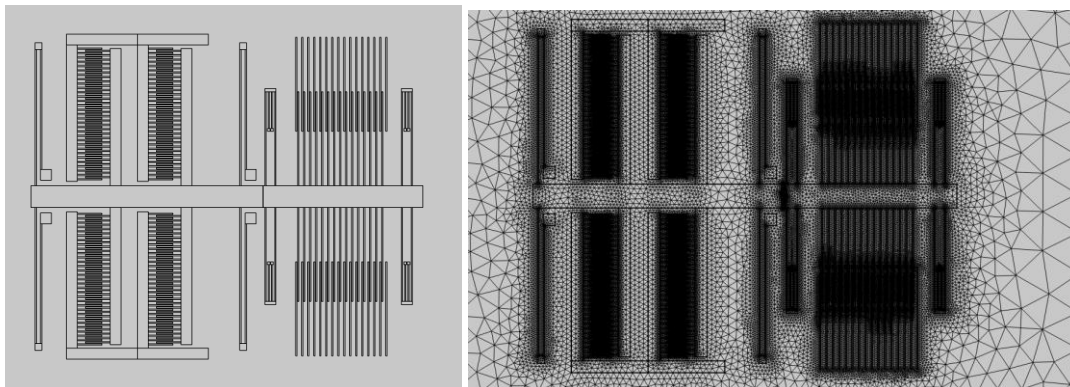


Figure 9 Overall geometry (left), Mesh (right)

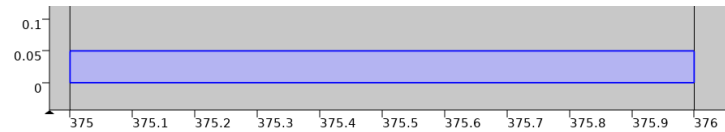


Figure 10 Nanowire

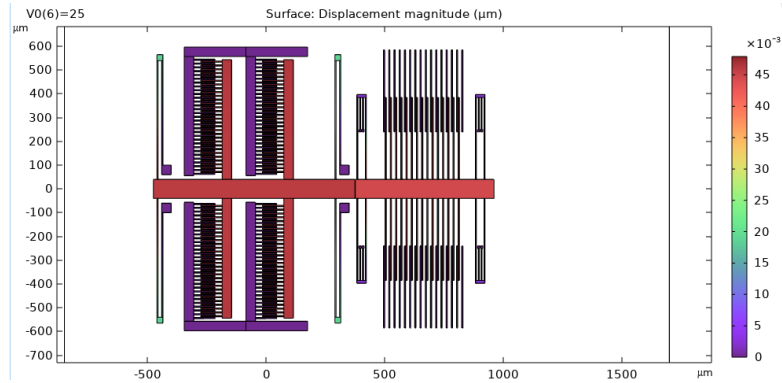


Figure 11 Displacement magnitude

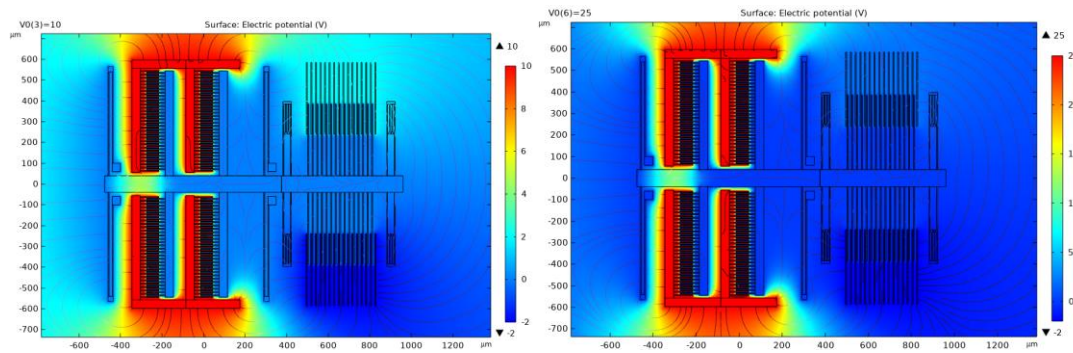
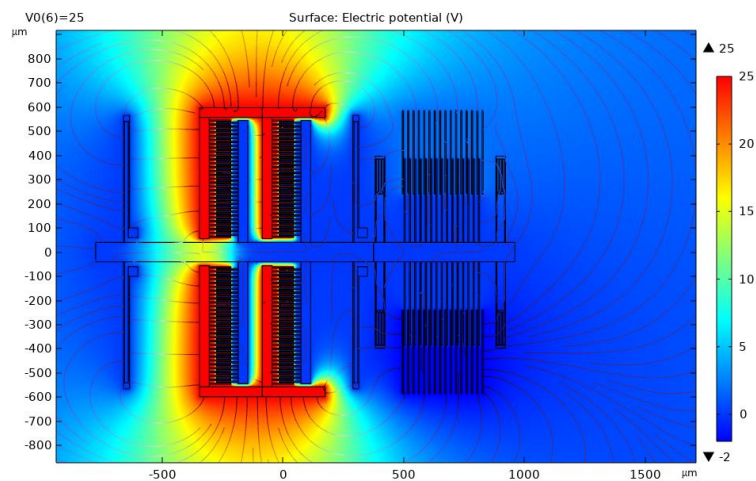


Figure 12 Electric potential surface plot.

$V=10$  (left),  $V=25$  (right)

Another plot with changing distance for anchors on the actuator:





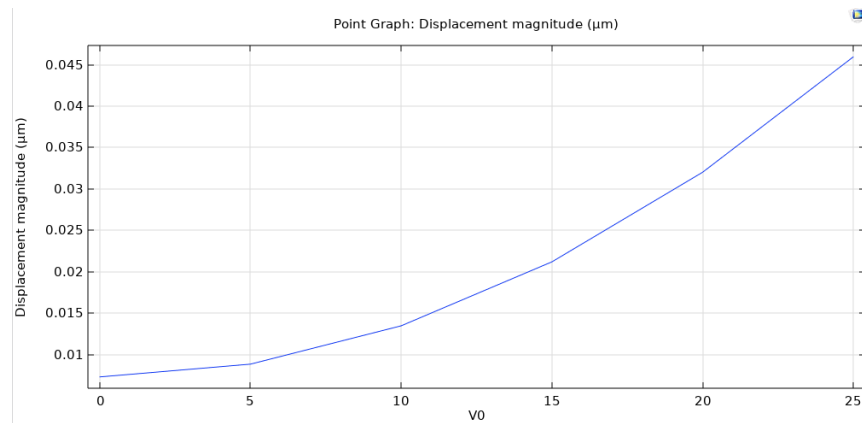


Figure 13 Plotting displacement as function of voltage

Parts a, b, c :

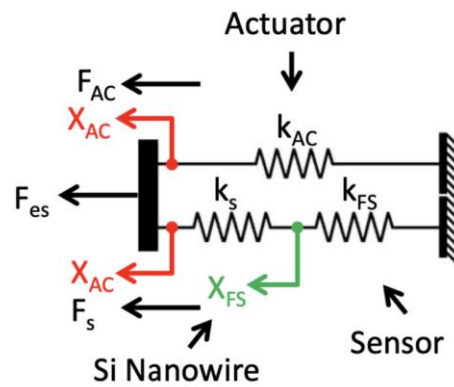


Figure 14 mechanical diagram for the device (G. Nadar, M.S. Thesis, Koc University, 2016)

$$W = \frac{1}{2} CV^2$$

$$C = \frac{\epsilon A}{d} = 2C_x + 2C_y = 2N_{AC} \epsilon_0 t \left[ \frac{w_{AC}}{g_x - x} - \frac{(h+x)}{g_y} \right]$$

$$F_{es} = \frac{\partial W}{\partial x} = \frac{1}{2} \frac{\partial C}{\partial x} V^2 = N_{AC} \epsilon_0 t \left[ \frac{w_{AC}}{(g_x - x)^2} + \frac{1}{g_y} \right] V^2$$

Fixed free:

$$K = \frac{3EI}{L^3}$$

Fixed-guided:

$$K = \frac{12EI}{L^3}$$

Folded Cantilever:

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\frac{1}{K} = \frac{L_1^3 + L_2^3}{12EI}$$

Double folded Cantilever

$$K = 2 * \frac{12EI}{L_1^3 + L_2^3}$$

$$K_1 = \frac{12EI}{L_1^3} \quad K_2 = \frac{12EI}{L_2^3}$$

$$K = \frac{12EI}{L_1^3 + L_2^3}$$

Thus, the stiffness of the actuator is:

$$k_{AC} = 4 k_{sf} = 4 \frac{12EI}{L_1^3 + L_2^3}$$

Change of capacitance with respect to displacement:

$$\Delta C = N(C_2 - C_1) = 2 N \varepsilon_0 A x \left[ \frac{1}{(d_1^2 - x^2)} + \frac{1}{(d_2^2 - x^2)} \right]$$

Four double folded cantilevers are used with each one modeled as two single folded cantilevers connected in series.

$$k_{df} = \frac{24EI}{L_1^3 + L_2^3}$$

$$k_{fs} = 4 * k_{df} = \frac{96EI}{L_1^3 + L_2^3}$$

Silicon nano wire is experiencing only axial forces. The wire is basically a long cylinder. Thus, the stiffness can be determined by the following equation:

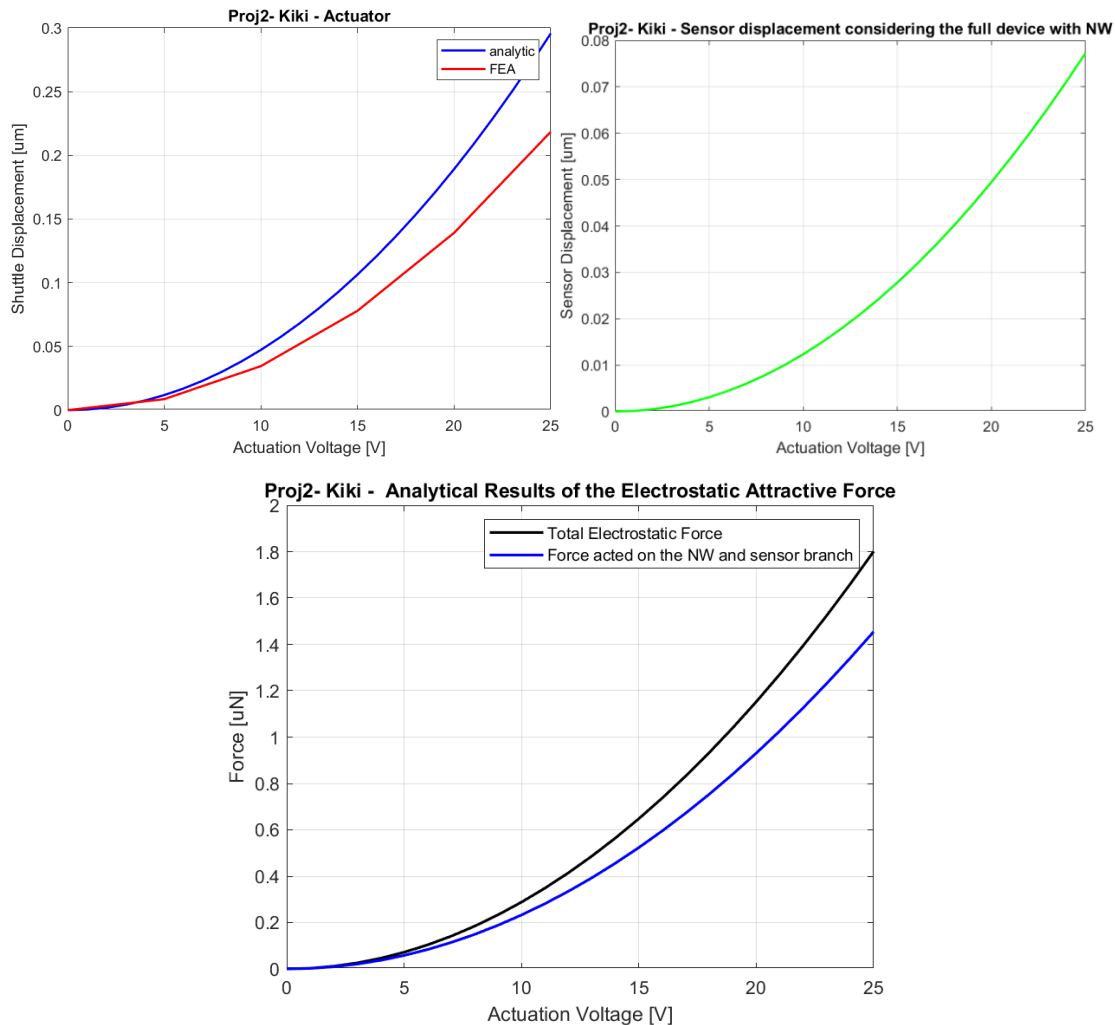
$$k_s = \frac{EA}{L} = \frac{E * \pi * D^2}{L 4} = \frac{E\pi r^2}{L}$$

The force that the sample and the sensor support can be found using the theory of series and parallel spring network and is determined by:

$$F_1 = k_1 \frac{F_{es}}{k_{AC} + k_1}$$

$$k_1 = \frac{k_s k_{FS}}{k_s + k_{FS}}$$





We see that there is minor deviation initially between the analytical and FEA graph. The difference increases with higher voltage as the simplification of the model doesn't hold as strong. We see that the maximum error reaches near 34% at the highest voltage ( $V=25$ ), which can be attributed to the model simplification. Still, the approximation is close enough for lower voltage values.

Since force is a function of actuation voltage, there is definitely an upper limit for it for the device. Having extreme voltage and thus extreme forces will break the sensor or might cause plastic deformations and render the sensor useless. This limit of course is dependent also on the geometry and material selection.

#### d) Sensor sensitivity:

Sensor sensitivity is defined as the sensor output change per unit displacement (for displacement measurement) or per unit external force (for force measurement)[1]. From the 2 equations below, we see that the capacitive sensor output is not a linear function of the displacement or the external force;

thus, the sensitivity changes as the sensor moves [1]. Therefore, when an average sensitivity is calculated or measured, the corresponding working range should be indicated [1].

$$V_{out} = a V_e \Delta C = 2a N \varepsilon_0 A V_e x \left[ \frac{1}{d_1^2 - x^2} - \frac{1}{d_2^2 - x^2} \right]$$

$$F_t = \left\{ k_0 - 2 N \varepsilon_0 A V_e^2 \left[ \frac{d_2}{(d_2^2 - x^2)} + \frac{d_1}{(d_1^2 - x^2)} \right] \right\} x$$

To get higher sensitivity, we need more displacement per unit force. One way to achieve this is to change the parameters of the geometrical design of the device to make them more compliant. Another approach is change the material. We could use a material that has a lower value of Young's Modulus, which will mean the material has less resistance to the force and we will see more strain or displacement and ultimately be more sensitive.

The force  $F_{es}$  is given by the equation below

$$F_{es} = \frac{\partial W}{\partial x} = \frac{1}{2} \frac{\partial C}{\partial x} V^2 = N_{AC} \varepsilon_0 t \left[ \frac{w_{AC}}{(g_x - x)^2} + \frac{1}{g_y} \right] V^2$$

We see from this equation that the force value will change depending on geometrical factors like  $w_{AC}$ ,  $g_x$  and  $g_y$ , etc. As is the case with any capacitor, when the distance between the plates decreases, the force increases. Additionally, when the surface area increases, the force increases too. Thus, decreasing the values of  $g_x$ ,  $g_y$  and increasing the value of  $w_{AC}$  will result in increasing the force range for any given displacement.

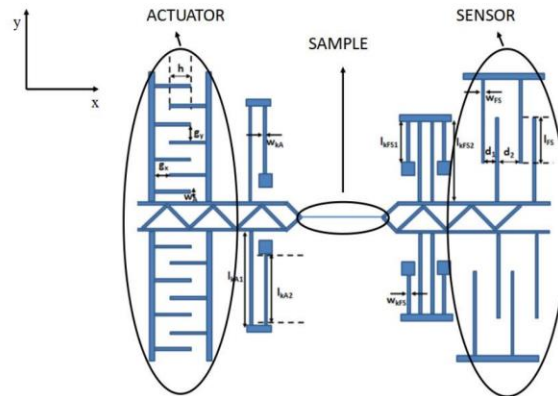


Figure 1. A schematic representation of the nanoscale testing setup, housing actuator and sensor bridged via NW sample.

Table 1 Simulation parameters for the actuator and sensor

Actuator			Force Sensor		
Design Parameter	Representation	Value	Design Parameter	Representation	Value
Finger width	$W_A$	$5\ \mu m$	Finger width	$W_A$	$5\ \mu m$
Vertical gap between fingers	$g_y$	$3\ \mu m$	Small gap between fingers	$d_1$	$2\ \mu m$
Horizontal gap between fingers	$g_x$	$30\ \mu m$	Larger gap between fingers	$d_2$	$10\ \mu m$
Overlap	$h$	$60\ \mu m$	Overlap	$l_{FS}$	$200\ \mu m$
Spring beam width	$W_{kA}$	$5\ \mu m$	Spring beam width	$W_{kFS}$	$4\ \mu m$
Spring beam lengths	$l_{kA1}, l_{kA2}$	$500, 440\ \mu m$	Spring beam lengths	$l_{kFS1}, l_{kFS2}$	$345, 135\ \mu m$
Number of single folded springs	$N_{kA}$	4	Number of single folded springs	$N_{kFS}$	8
Number of fingers	$N_{act.}$	120	Number of fingers	$N_{FS}$	30
Actuation voltage	$V_A$	25 V	Excitation voltage	$V_{FS}$	2 V
Device thickness	$t$	$8\ \mu m$	Device thickness	$t$	$8\ \mu m$

All used values can be seen also in COMOSL Parameter section for each file.

#### References:

[1] Zhang, D., Drissen, W., Breguet, J. M., Clavel, R., and Michler, J., 2009, "A High-Sensitivity and Quasi-Linear Capacitive Sensor for Nanomechanical Testing Applications," J. Micromechanics Microengineering, 19(7), p. 075003.