Prolog

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Outline

Objectives

You should be able to...

In this lecture we will introduce Prolog.

- Be able to explain the models of data and program for Prolog. (The Two Questions)
- Be able to write some simple programs in Prolog.
- Know how to use Prolog's arithmetic operations.
- Know how to use lists and patterns.

Logic

Question: How do you decide truth?

- Start with some *objects*
  "socrates", "john", "mary"
- Write down some *facts* (true statements) about those objects.
  - Facts express either properies of the object, or
    "socrates is human"
  - relationship to other objects.
    "mary likes john"
- Write down some *rules* (facts that are true if other facts are true).
  "if X is human then X is mortal"
- Facts and Rules can become *predicates*.
  "is socrates mortal?"

First Order Predicate Logic

First Order Predicate Logic is one system for encoding these kinds of questions.

- Predicate means that we have functions that take objects and return "true" or "false".
  human(socrates).
- Logic means that we have *connectives* like and, or, not, and implication.
- First Order means that we have variables (created by "for all" and "there exists"), but that they only work on objects.
  $\forall X \,.\, human(X) \to mortal(X).$

History

- Starting point: First Order Predicate Logic.
- Realization: Computers can reason with this kind of logic.
- Impetus was the study of *mechanical theorem proving*
- Developed in 1970 by Alain Colmerauer and Rober Kowalski and others.
- Uses: databases, expert systems, AI.

The Two Questions

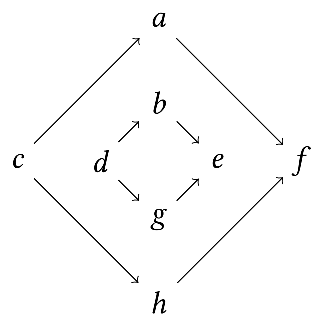
What is the nature of data?

Prolog data consists of *facts* about *objects* and logical *rules*.

What is the nature of a program?

A program in Prolog is a set of facts and rules, followed by a *query*.

The Database




```
human(socrates).
fatherof(socrates,
         jane).
fatherof(zeus,apollo).
```

```
connected(c,a).
connected(c,h).
connected(d,b).
connected(d,g).
connected(a,f).
connected(h,f).
connected(b,e).
connected(g,e).
```

Rules

```
mortal(X) :- human(X).
human(Y) :- fatherof(X,Y), human(X).

pathfrom(X,Y) :- connected(X,Y).
pathfrom(X,Y) :- connected(X,Z),
                 pathfrom(Z,Y).
```

- Capital letters are variables.
  - Appearing left of `:-` means “for all”
  - Appearing right of `:-` means “there exists”

$$\forall x. human(x) \rightarrow mortal(x).$$

$$\forall y. (\exists x. fatherof(x, y) \wedge human(x)) \rightarrow human(y)$$

How it works

Programs are executed by searching the database and attempting to perform unification.

```
?- human(socrates).   -- listed, therefore true
?- mortal(socrates).  -- not listed
```

Relevant rules:

```
human(socrates).
human(Y) :- fatherof(X,Y), human(X).
mortal(X) :- human(X).
```

Socrates is not listed as being mortal, but `mortal(socrates)` unifies with `mortal(X)` if we replace `X` with `socrates`. This gives us a *subgoal*. Replace X with `socrates` and try it....

How it works, next step

Replace `X` with `socrates` in this rule:

```
mortal(X) :- human(X).
```

to get

```
mortal(socrates) :- human(socrates).
```

Since `human(socrates)` is in the database, we know that `mortal(socrates)` is also true.

Another example

```
?- mortal(jane). not in database
but we have: mortal(X) :- human(X).
so we substitute: mortal(jane) :- human(jane).
subgoal: human(jane). -- not there either
  but: human(Y) :- fatherof(X,Y), human(X).
  so we substitute:
     human(jane) :- fatherof(X,jane), human(X).
    subsubgoal1: fatherof(X,jane).
        we find: fatherof(socrates,jane)
                  -- so try the next subgoal
    subsubgoal2: human(socrates).  \emph{yes}
  therefore: human(jane). -- is true
therefore: mortal(jane). -- is true
```

You try...

- Given the connected rules, try to come up with a predicate `exactlybetween(A,B,C)` that is true when `B` is connected to both `A` and `C`.
- Now make a predicate `between(A,B,C)` that is true if there's a path from `A` to `B` to `C`.


```
exactlybetween(A,B,C) :- connected(A,B), connected(B,C).

between(A,B,C) :- pathfrom(A,B), pathfrom(B,C).
```


More than just Yes or No....

- Prolog can also give you a list of elements that make a predicate true. Remember unification.

```
?- fatherof(Who,apollo).
Who = zeus ;

?- pathfrom(c,X).
X = a ;
X = h ;
X = f ;
X = f ;
No
```

The semicolon is entered by the user— it means to keep searching.

Tracing pathfrom

```
?- pathfrom(c,X).
---> pathfrom(c,Y) :- connected(c,Y).
X = a ;
```

When we hit semicolon, we tell it to keep searching. So we *backtrack* through our database to try again.

```
pathfrom(c,Y) :- connected(c,Y).
---> X = h ;
```

We tell it to try again with this one, too. At this point, we no longer have any rules that say that c is connected to something.

Tracing pathfrom, II

```
pathfrom(c,Y) :- connected(c,Z), pathfrom(Z,Y).
```

We will first find something in the database that says that `c` is connected to some `Z`, and then check if there is a path between `Z` and `Y`.
We find `a` and `h` as last time. When we check `a`, we check for `pathfrom(a,Y)`, and find that `connected(a,f)` is in the database. The same thing happens for `h`, which is why `f` is reported as an answer twice.

Arithmetic via the is keyword.

```
fact(0,1).
fact(N,X) :- M is N-1, fact(M,Y), X is Y * N.
?- fact(5,X).
```

- Unify `fact(5,X)` with `fact(N,X)`.
  `fact(5,X) :- M is 5-1, fact(M,Y), X is Y * 5.`
- Next compute `M`.
  `fact(5,X) :- 4 is 5-1, fact(4,Y), X is Y * 5.`
- Recursive call sets `Y` to 24.
  `fact(5,X) :- 4 is 5-1, fact(4,24), X is 24 * 5.`
- Compute `X`
  `fact(5,120) :- 4 is 5-1, fact(4,24), 120 is 24 * 5.`

Subgoal ordering

Order of sub-goals is important! Why does this happen?

```
badfact(0,1).
badfact(N,X) :- badfact(M,Y), M is N-1,
                X is Y * N.
?- badfact(5,X).
ERROR: Arguments are not sufficiently
   instantiated
Exception: (8) 0 is _G278-1 ?
```


Lists

Prolog lists are very similar to OCaml lists.

- Empty list: `[]`
- Singleton list: `[x]`
- List with multiple elements: `[x,y,[a,b],c]`
- Head and tail representation `[H|T]`

Differences:

- Prolog lists are *not* monotonic!

List example: mylength

The `length` predicate is built in.

```
mylength([],0).
mylength([H|T],X) :- mylength(T,Y),
                     X is Y + 1.

?- mylength([2,3,4,5],X).
X = 4 ;
No
```

This example looks like `badfact`, in that the `is` clause happens after the recursion. Why is this safe?

List Example: Sum List

```
sumlist([],0).
sumlist([H|T],X) :- sumlist(T,Y),
                    X is Y + H.

?- sumlist([2,3,4,5],X).
X = 14
```

Try writing list product now!

List Example: Append

```
myappend([],X,X).
myappend([H|T],X,[H|Z]) :- myappend(T,X,Z).
?- myappend([2,3,4],[5,6,7],X).
X = [2, 3, 4, 5, 6, 7] ;
No
?- myappend(X,[2,3],[1,2,3,4]).
No
?- myappend(X,[2,3],[1,2,3]).
X = [1] ;
No
```


List Example: Reverse

Accumulator recursion works in Prolog, too!

```
myreverse(X,Y) :- aux(X,Y,[]).
aux([],Y,Y).
aux([HX|TX],Y,Z) :- aux(TX,Y,[HX|Z]).
?- myreverse([2,3,4],Y).
Y = [4, 3, 2]
```

`myreverse([2,3,4],Y)` ⟶ `aux([2,3,4],Y,[])` ⟶ `aux([3,4],Y,[2])` ⟶ `aux([4],Y,[3,2])` ⟶ `aux([],Y,[4,3,2])` ⟶ `aux([],[4,3,2],[4,3,2])` ⟶ `myreverse([2,3,4],[4,3,2])`

Pairs

- The term `socrates` is a pattern. But patterns can have structure....

```
pair((X,Y)).
key((X,Y),X).
value((X,Y),Y).
assoc(X,Y,[H|T]) :- key(H,X), value(H,Y);
                    assoc(X,Y,T).
?- assoc(2,X,[(3,hi),(4,there),(2,guys)]).
X = guys
?- assoc(X,there,[(3,hi),(4,there),
                  (2,guys)]).
X = 4
```


Activity

- Write the Fibonacci predicate. Let $F_0 = 0$ and $F_1 = 1$.
- Make sure you can write it the exponential way.
- Can you write it the linear way?

Solution

- Fibonacci predicate: exponential complexity:

```
fib(0,0).
fib(1,1).
fib(N,X) :- N1 is N - 1,
            fib(N1,X1), N2 is N - 2,
            fib(N2,X2), X is X1 + X2.
```

- Fibonacci predicate: linear complexity:

```
lfibx(0,F1,F2,A) :- A is F2.
lfibx(N,F1,F2,A) :- N1 is N - 1,
                    F3 is F1 + F2,
                    lfibx(N1,F2,F3,A).
lfib(N,A) :- lfibx(N,1,0,A).
```