Tail Recursion

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Objectives

- Understand what makes a function tail recursive.
- Explain how the compiler makes tail recursion efficient.

Tail Calls

Tail Position A subexpression *s* of expressions *e*, if it is evaluated, will be taken as the value of *e*.

- `if x > 3 then x + 2 else x - 4`
- `f (x * 3)` — no (proper) tail position here.

Tail Call A function call that occurs in tail position.

- `if h x then h x else x + g x`

Your Turn

Find the tail calls!

Example Code

```
calc n i | n==2 = i
         | odd n = calc (n*3+1) (i+1)
         | otherwise = calc (n `div` 2) (i+1)

fib 0 = 0
fib 1 = 1
fib n = fib (n-1) + fib (n-2)
```

Tail Call Example

- If one function calls another in tail position, we get a special behavior.

Example

```
foo x = bar (x+1)
bar y = baz (y+1)
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- What happens when we call `foo 1`?

Accumulating Recursion | Tail vs. Non-Tail

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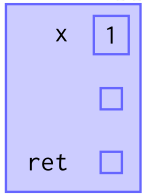

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Accumulating Recursion | Tail vs. Non-Tail

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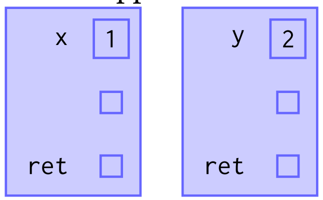

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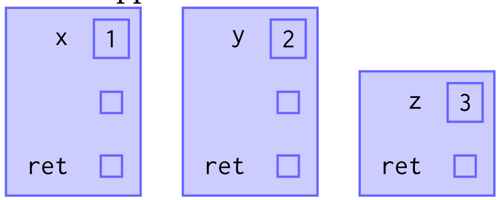

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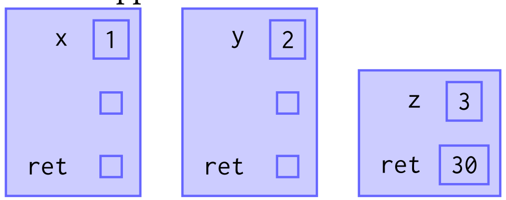

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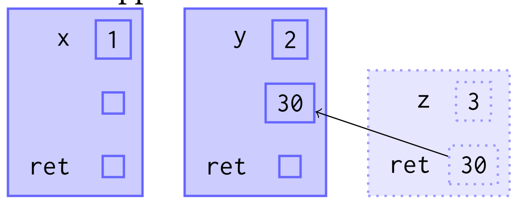

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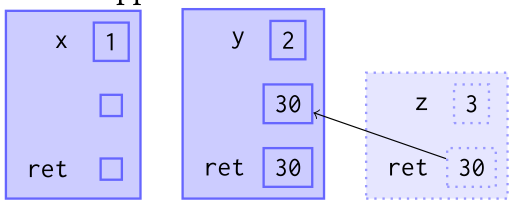

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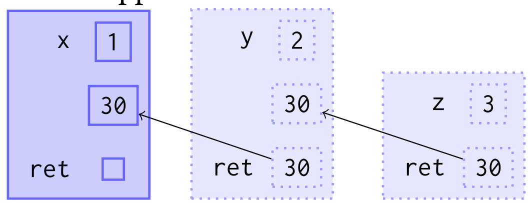

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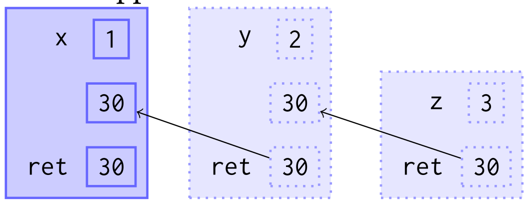

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- If that's the case, we can cut out the middle man...

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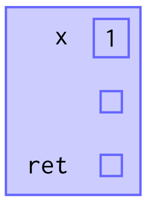

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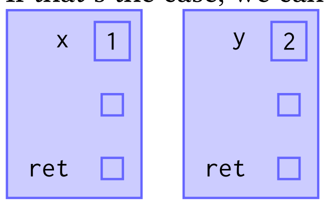

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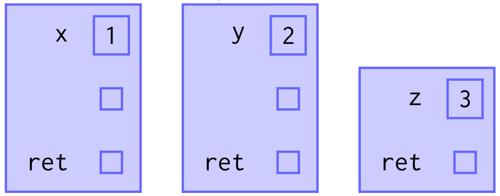

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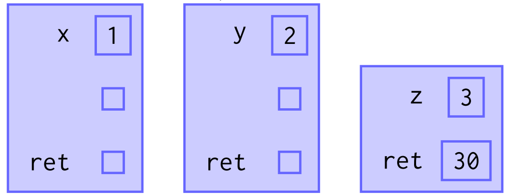

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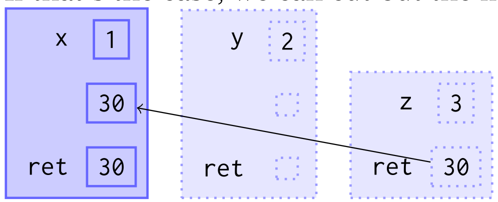

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- If that's the case, we can cut out the middle man...
- Actually, we can do even better than that.

The optimization

- When a function is in tail position, the compiler will *recycle the activation record*!

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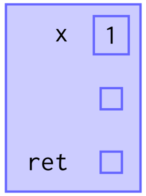

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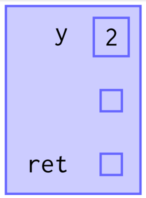

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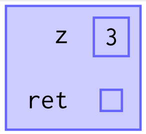

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z	3
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- This allows recursive functions to be written as loops internally.

Direct-Style Recursion

- In recursion, you split the input into the "first piece" and the "rest of the input".
- In direct-style recursion: the recursive call computes the result for the rest of the input, and then the function combines the result with the first piece.
- In other words, you wait until the recursive call is done to generate your result.

Direct Style Summation

```
sum [] = 0
sum (x:xs) = x + sum xs
```


Accumulating Recursion

- In accumulating recursion: generate an intermediate result *now*, and give that to the recursive call.
- Usually this requires an auxiliary function.

Tail Recursive Summation

```
sum xx = aux xx 0
  where aux [] a = a
        aux (x:xs) a = aux xs (a+x)
```

Further Reading

- Forward recursion can be made to traverse a list at return time rather than call time, forming a pattern called "There and Back Again," which can do some interesting things....
- Example: write a function `convolve` which takes two lists $(x_1 \quad x_2 \quad \cdots \quad x_n)$ and $(y_1 \quad y_2 \quad \cdots \quad y_n)$ and produces an output list $(x_1 y_n \quad x_2 y_{n-2} \quad \cdots \quad x_n y_1)$ where n is unknown. Use only $\mathcal{O}(n)$ recursive calls, and no temporary lists.
- For the solution, see Olivier Danvy's paper There and Back Again.