CS 331 — AVL Trees

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Outline

1. Introduction
   - Objectives
2. Balancing Trees
3. Rotations

Objectives

You should be able to...

- Explain why height-balanced trees are necessary.
- Explain how to perform two of the four kinds of rotations:
  - left, right
- Identify the proper kind of rotation for a particular situation.

Motivation

Do you remember how long it takes...

- To insert an element into a BST on average?
- To insert an element into a BST worst case?
- To delete an element from a BST on average?
- To delete an element from a BST worst case?

Motivation

Do you remember how long it takes...

- To insert an element into a BST on average? // $\mathcal{O}(\lg n)$
- To insert an element into a BST worst case? // $\mathcal{O}(n)$
- To delete an element from a BST on average? // $\mathcal{O}(\lg n)$
- To delete an element from a BST worst case? // $\mathcal{O}(n)$

Some Good Insertions

Insert These Nodes

8 6 16 30 7 2 12

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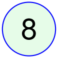

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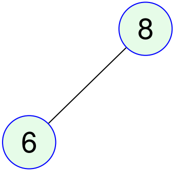

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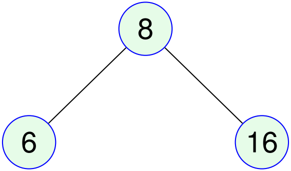

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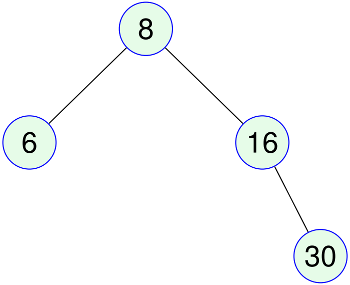

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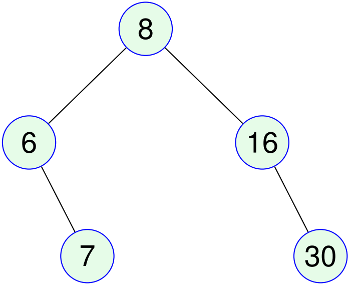

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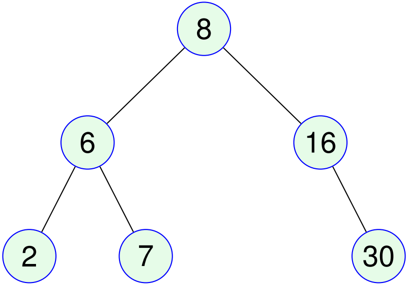

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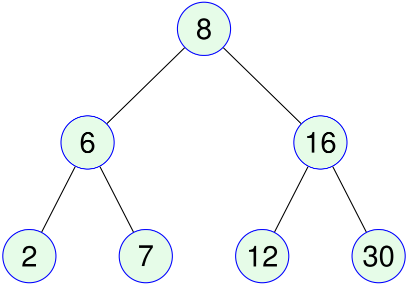

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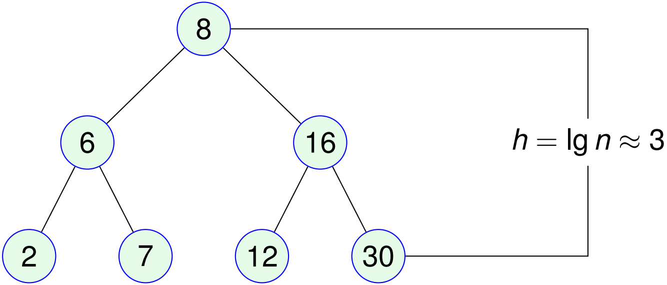

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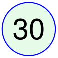

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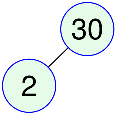

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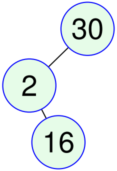

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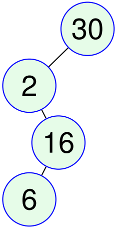

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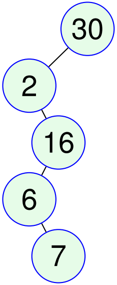

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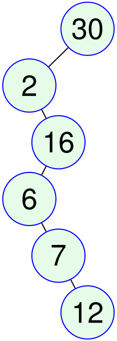

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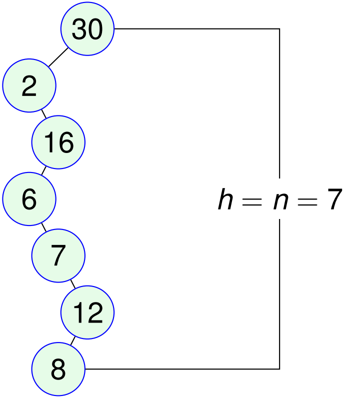
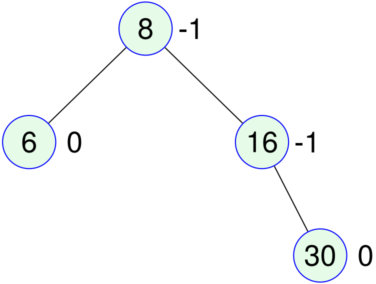


Balance

- The balance of a node is the depth of the left subtree minus the depth of the right subtree.



- Depth is the longest path from the node to a leaf.
- Leaves always have balance of zero.

Some Insertions, with Balances

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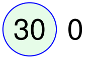

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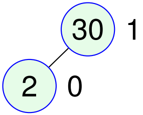

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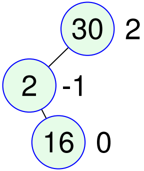

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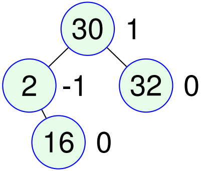

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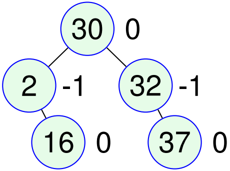

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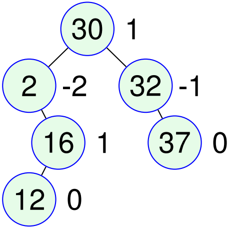


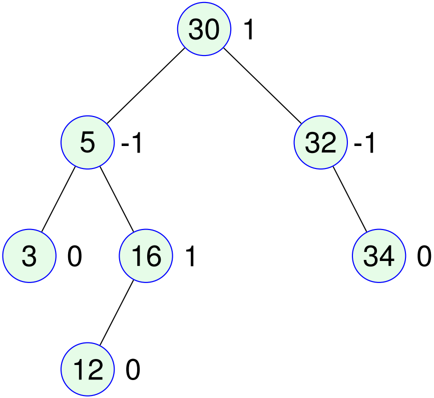
Updating Balance During Add

- Perform an add as normal, using recursion.
- The leaf will have balance zero.
- Upon return:
  - If you went left, increment your balance.
  - If you went right, decrement your balance.
  - If the balance becomes zero, stop updating balances. (Why?)
  - If the balance is $+/-1$, return to the parent.
  - If the balance is $+/-2$, rebalance the node.

Add Example 1

Example

Insert a 1.

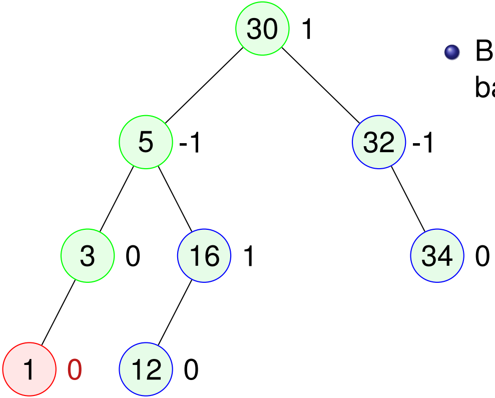


- The node 1 goes to the left of 3.

Add Example 1

Example

Insert a 1.

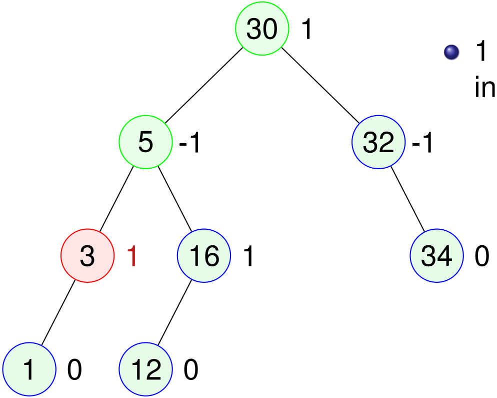


- Because it is a leaf, it’s balance will be zero.

Add Example 1

Example

Insert a 1.

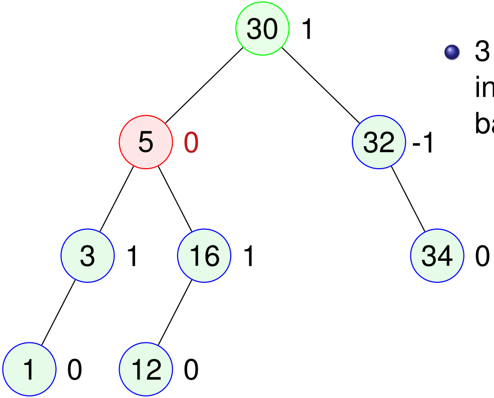


- 1 is the left child of 3, so increment 3's balance.

Add Example 1

Example

Insert a 1.

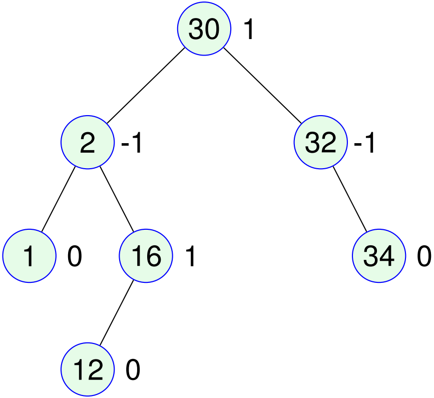


- 3 is the left child of 5, so increment 5's balance. The balance of 5 is 0, so we stop.

Examples

Example

Insert a 33.

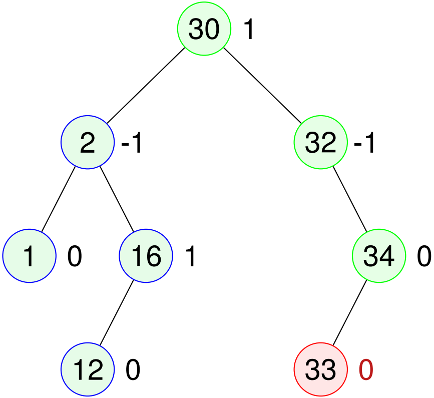


- The node 33 goes to the left of 34.

Examples

Example

Insert a 33.

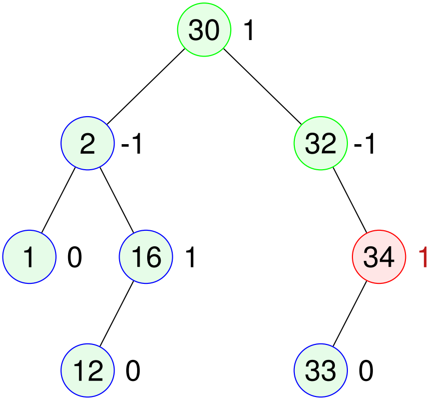


- Because it is a leaf, it's balance will be zero.

Examples

Example

Insert a 33.

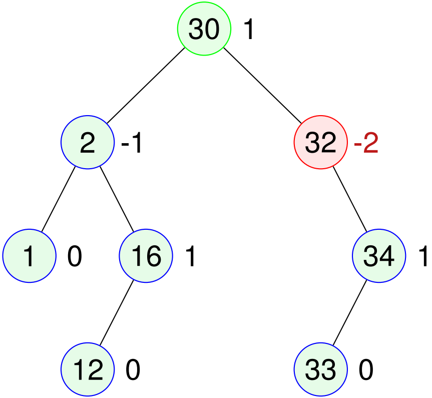


- 33 is the left child of 34, so increment 34's balance.

Examples

Example

Insert a 33.

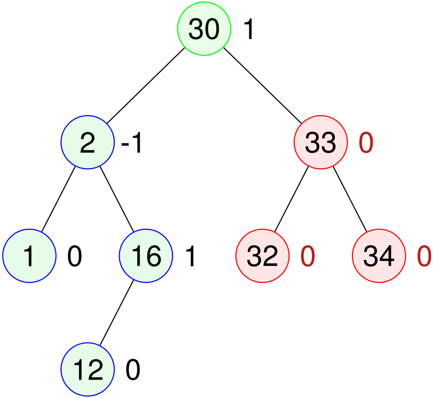


- 34 is the right child of 32, so decrement 32's balance. This node is out of balance, so we will rebalance here.

Examples

Example

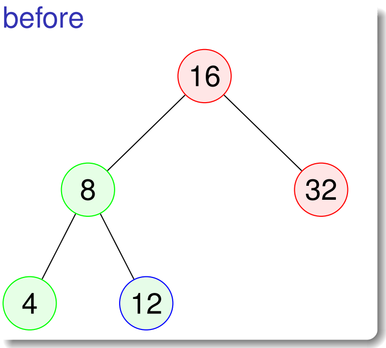
Insert a 33.

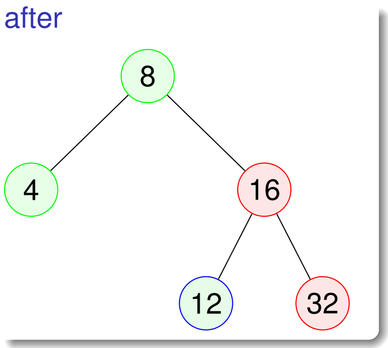


- Here is the result of rebalancing. Let’s talk about that next....

Effect of a rotation

- This is a Right Rotation.



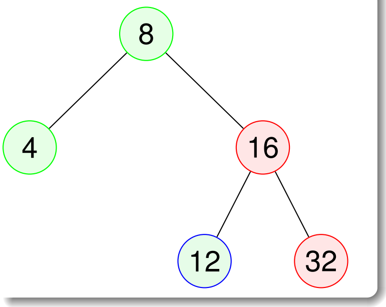


- What happens to the balance of the root?

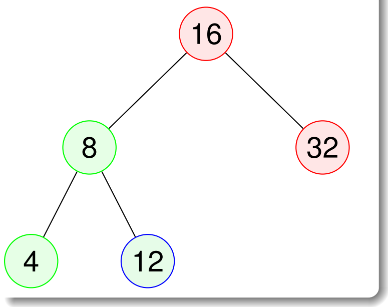
Effect of a rotation

- This is a Left Rotation. It should look familiar.

before



after

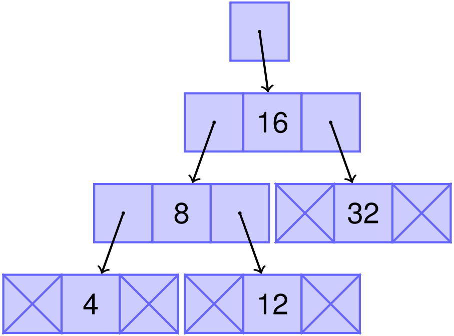


- What happens to the balance of the root?

Effects of Rotations

- A Left Rotation adds 2 to the balance of the node. Use it when the balance is -2.
- A Right Rotation subtracts 2 from the balance of the node. Use it when the balance is 2.
- The “heavy” part of the tree needs to be on the *outer* side for this to work.

Implementing a Right Rotation



```
curr.balance += 2;
curr.left.balance++;
tmp = curr.left;
curr.left =
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tmp.right = curr;
curr.parent = tmp;
```

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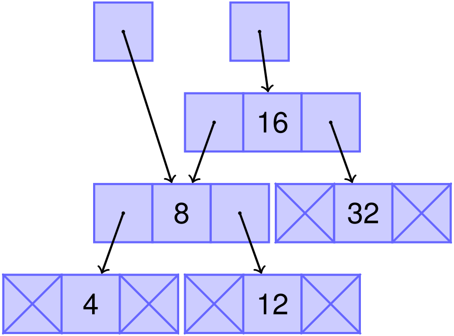

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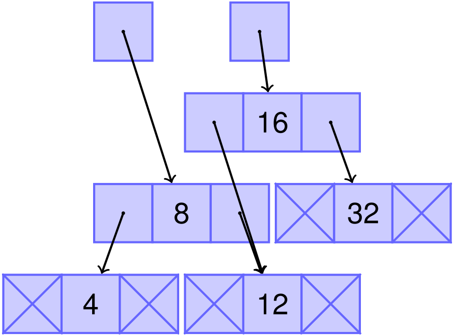

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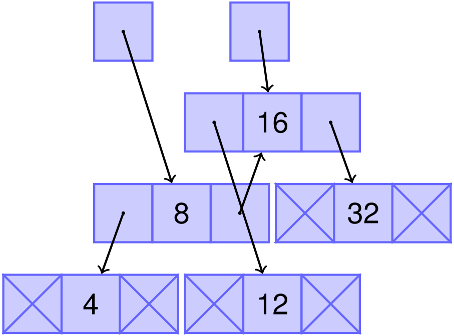

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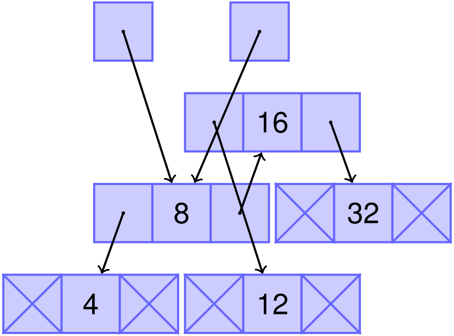

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- You have to update the parent's link also.

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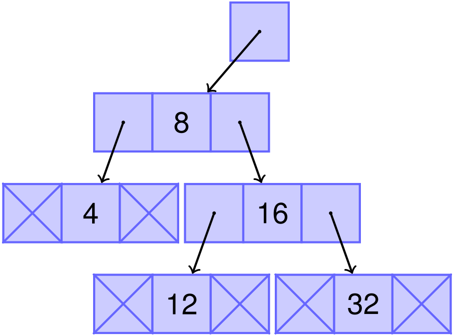

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Bad Insertions with Rotations

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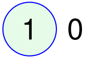

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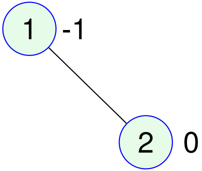

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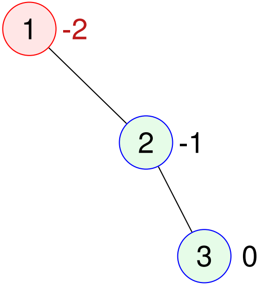

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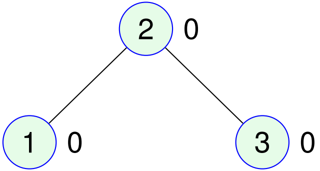

- Try inserting the next two yourself before looking ahead!

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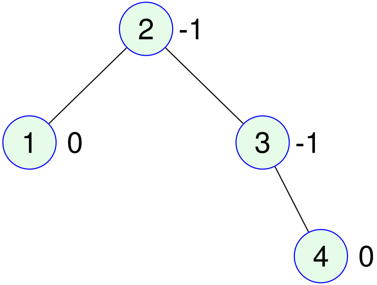

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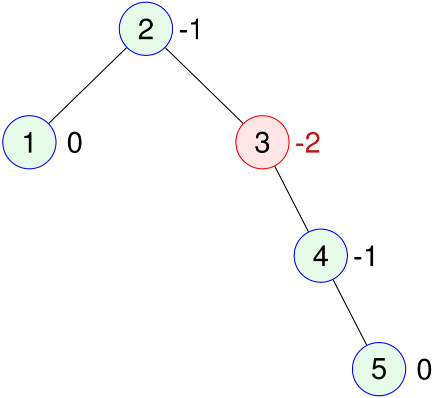

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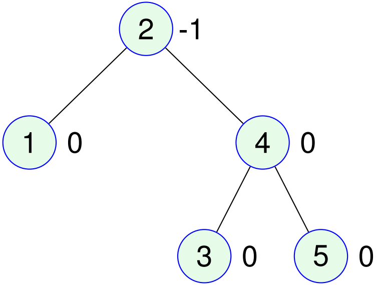

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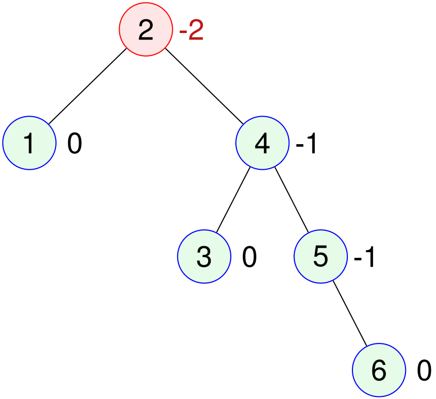

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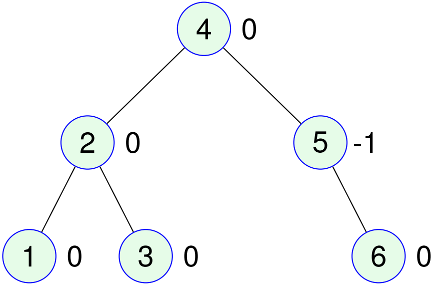
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