



Joachim Faulhaber

# An Introduction to the Interval Template Library

**Lecture**  
held at the Boost Library Conference 2009

**2009-05-08**

- Background and Motivation
- Design
- Examples
- Semantics
- Implementation
- Future Works
- Availability



- Interval containers simplified the implementation of date and time related tasks
  - Decomposing “*histories*” of attributed events into segments with constant attributes.
  - Working with time grids, e.g. a grid of months.
  - Aggregations of values associated to date or time intervals.
- ... that occurred frequently in programs like
  - Billing modules
  - Therapy scheduling programs
  - Hospital and controlling statistics

- Background is the date time problem domain ...
- ... but the scope of the **Itl** as a generic library is more general:

*an **interval\_set** is a **set**  
that is implemented as a set of intervals*

*an **interval\_map** is a **map**  
that is implemented as a map of interval value pairs*



- There are two aspects in the design of interval containers
- Conceptual aspect

```
interval set<int> mySet;  
mySet.insert(42);  
bool has_answer = mySet.contains(42);
```

- On the conceptual aspect an interval\_set can be used just as a set of elements
- except for . . .
- . . . *iteration over elements*
- consider `interval set<double>` or `interval set<string>`
- Iterative Aspect
  - **Iteration** is always done over *intervals*

## • Addability and Subtractability

- All of itl's (interval) containers are *Addable* and *Subtractable*
- They implement **operators**  $+=$ ,  $+$ ,  $-=$  and  $-$

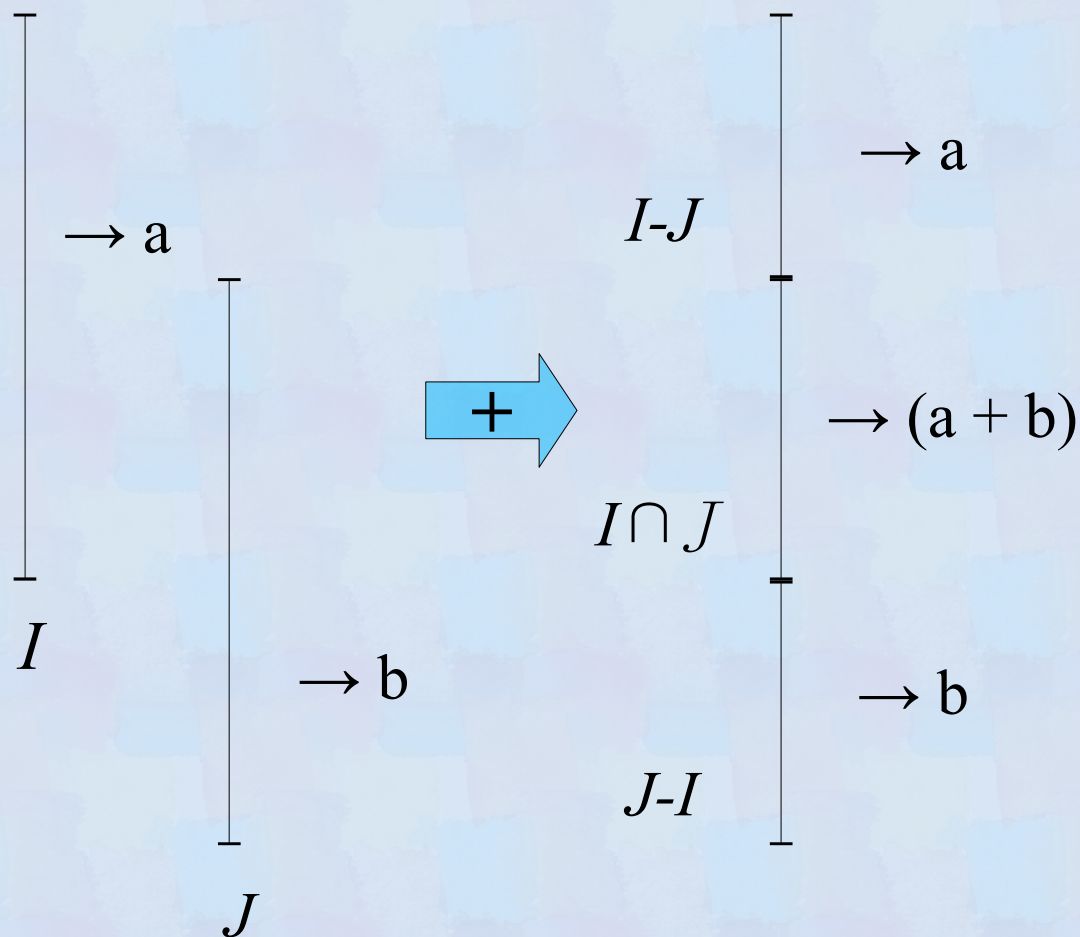
	$+=$	$-=$
sets	set union	set difference
maps	?	?

## • A possible implementation for maps

- Propagate addition/subtraction to the associated values
- . . . or aggregate on overlap
- . . . or aggregate on collision



## Aggregate on overlap



- Decompositional effect on Intervals
- Accumulative effect on associated values

$I, J$ : intervals,  $a, b$ : associated values

## Aggregate on overlap, a minimal example

```
typedef itl::set<strin > !ests;
interval map<time, !ests> party;

party += ma"e_pair(
    interval<time>::rightopen(2#$##, 22$##), !ests(%&ary%));

party += ma"e_pair(
    interval<time>::rightopen(2'$##, 2($##), !ests(%)arry%));

** party now contains
+2#$##, 2'$##),>-%&ary%.
+2'$##, 22$##),>-%)arry%,%&ary%. ** !est sets a re ated
+22$##, 2($##),>-%)arry%.
```



## • The Itl's class templates

Granularity	Style	Sets	Maps
interval		<code>interval</code>	
	joining	<code>interval set</code>	<code>interval map</code>
	separating	<code>separate interval set</code>	
	splitting	<code>split interval set</code>	<code>split interval map</code>
element		<code>set</code>	<code>map</code>

## Interval Combining Styles: *Joining*

- Intervals are joined on overlap or on touch
- ... *for maps*, if associated values are equal
- Keeps interval\_maps and sets in a minimal form

### interval set

```

{ [1      3)
+   [2      4)
+   [4  5)

= { [1      4)
= { [1      5)

```

### interval map

```

{ [1      3) ->1
+   [2      4) ->1
+   [4  5) ->1

={ [1  2) [2  3) [3  4)
   ->1   ->2   ->1
={ [1  2) [2  3) [3      5)
   ->1   ->2           ->1

```



## Interval Combining Styles: *Splitting*

- Intervals are split on overlap and kept separate on touch
- All interval borders are preserved (insertion memory)

### split interval set

```

{ [1      3)
+   [2      4)
+           [4 5)

= { [1 2) [2 3) [3 4)
= { [1 2) [2 3) [3 4) [4 5) }
```

### split interval map

```

{ [1      3) ->1
+   [2      4) ->1
+           [4 5) ->1

={ [1 2) [2 3) [3 4)
  ->1   ->2   ->1
={ [1 2) [2 3) [3 4) [4 5)
  ->1   ->2   ->1   ->1 }
```

## Interval Combining Styles: *Separating*

- Intervals are joined on overlap but kept separate on touch
- Preserves borders that are never crossed (preserves a hidden grid).

*separate interval set*

$$\begin{aligned}
 & \{ [1 \quad 3) \\
 + & \quad [2 \quad 4) \\
 + & \quad \quad [4 \quad 5) \\
 = & \{ [1 \quad 4) \\
 = & \{ [1 \quad 4) [4 \quad 5) \}
 \end{aligned}$$



## • A few instances of intervals (interval.cpp)

```
interval<int> int_interval = interval<int>::closed(0, 0);

interval<double> s1rt_interval
    = interval<double>::rightopen(*s1rt(2.0), s1rt(2.0));

interval<std::string> city_interval
    = interval<std::string>::leftopen("Barcelona", "Boston");

interval<boost::ptime> time_interval
    = interval<boost::ptime>::open(
        time_from_string("2003-04-25 05:00"),
        time_from_string("2003-04-25 02:00")
    );
```

- A way to iterate over months and weeks  
(month\_and\_wee"\_\_ rid.cpp)

```
6include <boost*itl* re orian.hpp> **boost$$ re orian pl!s adapter code
6include <boost*itl*split_inter/al_set.hpp>

** 7 split_inter/al_set of re orian dates as date_ rid.
typedef split_inter/al_set<boost$$ re orian$$date> date_ rid;

** 8omp!te a date_ rid of months !sin boost$$ re orian.
date_ rid month_ rid(const inter/al<date>9 scope)
-
    date_ rid month_ rid;
    ** 8omp!te a date_ rid of months !sin boost$$ re orian.
    . . .
    ret!rn month_ rid;
.

** 8omp!te a date_ rid of wee"s !sin boost$$ re orian.
date_ rid wee"_ rid(const inter/al<date>9 scope)
-
    date_ rid wee"_ rid;
    ** 8omp!te a date_ rid of wee"s !sin boost$$ re orian.
    . . .
    ret!rn wee"_ rid;
.
```



## ● A way to iterate over months and weeks

```

/oid month_and_time_ rid()
-
date someday = day_cloc "$$local_day();
date thenday = someday : months(2);
inter/al<date> scope = inter/al<date>$$ri htopen(someday, thenday);

** 7n intersection of the month and wee" rids ...
date_ rid month_and_wee"_ rid
    = month_ rid(scope) 9 wee"_ rid(scope);

** ... allows to iterate months and wee"s. ;hene/er a month
** or a wee" chan es there is a new inter/al.
for(date_ rid$$iterator it = month_and_wee"_ rid.be in();
    it <= month_and_wee"_ rid.end(); it::)
-    . . . .

** ;e can also intersect the rid into an inter/al_map to ma"e
** sh!re that all inter/als are within months and wee" bounds.
inter/al_map<boost$$ re orian$$date, some_type> accr!al;
comp!te_some_res!lt(accr!al, scope);
accr!al 9= month_and_wee"_ rid;

```

## Aggregating with interval\_maps

- Computing averages via implementing **operator +=**  
(partys\_!est\_a/era e.cpp)

```
class counted_sum
-
public$
    counted_sum()$_s!m(#),_co!nt(#)-.
    co!nted_s!m(int s!m)$_s!m(s!m),_co!nt(')-.

    int s!m()const -ret!rn _s!m;.
    int co!nt()const-ret!rn _co!nt;.
    do!ble a/era e()const
    - ret!rn _co!nt==# = #.# $_s!m*static_cast<do!ble>(_co!nt); .

    co!nted_s!m9 operator += (const co!nted_s!m9 ri ht)
    - _s!m := ri ht.s!m(); _co!nt := ri ht.co!nt(); ret!rn >this; .

private$
    int _s!m;
    int _co!nt;
    .;

bool operator == (const co!nted_s!m9 left, const co!nted_s!m9 ri ht)
- ret!rn left.s!m()==ri ht.s!m() 99 left.co!nt()==ri ht.co!nt(); .
```



## Aggregating with interval\_maps

### Computing averages via implementing **operator +=**

```

void partys_hei ht_a/era e()
-
    interval_map<ptime, counted sum> hei ht_s!ms;

    hei ht_s!ms += (
        make_pair(
            interval<ptime>$$ri htopen(
                time_from_strin (%2##3, #4, 2# '5$(#%),
                time_from_strin (%2##3, #4, 2# 2($##%)),
                counted sum(165)) ** &ary is ',?4 m tall.
        );

    ** 7dd hei ht of more pary !ests . . .

    interval_map<ptime, counted_s!m>$$iterator hei ht_s!m_ =
        hei ht_s!ms.begin();
    while(hei ht_s!m_ <= hei ht_s!ms.end())
    -
        interval<ptime> when = hei ht_s!m_,>first;
        double hei ht_a/era e = (>hei ht_s!m_::).second.average();

        cout << %+% << when.first() << % , % << when.upper() << %)%
            << %$ % << hei ht_a/era e << % cm% << endl;
    .
    .

```

- Interval containers allow to express a variety of date and time operations in an easy way.
  - Example `man power.cpp` ...
  - Subtract weekends and holidays from an `interval_set`  
`worktime -= weekends(scope)`  
`worktime -= german reunification day`
  - Intersect an `interval_map` with an `interval_set`  
`claudias working hours &= worktime`
  - Subtract an `interval_set` from an `interval map`  
`claudias working hours -= claudias absense times`
  - Adding `interval_maps`  
`interval map<date,int> manpower;`  
`manpower += claudias working hours;`  
`manpower += bodos working hours;`



## Interval\_maps can also be intersected Example `user_groups.cpp`

```
typedef boost::interval_map<date, &emberSet@> &embership@;

void user_groups()
{
    . . .

    &embership@ med_!sers;
    /** 8omp!te membership of medical staff
    med_!sers := make_pair(member_interval, &emberSet@(%Ar.Be"yll%));
    med_!sers := . . .

    &embership@ admin_!sers;
    /** 8omp!te membership of administration staff
    med_!sers := make_pair(member_interval_2, &emberSet@(%&r.)yde%));
    . . .

    &embership@ all_!sers = med_!sers + admin_!sers;

    &embership@ super_!sers = med_!sers & admin_!sers;
    . . .
}
```

- The semantics of *itl sets* is based on a concept **itl::Set**
  - **itl::set**, **interval set**, **split interval set** and **separate interval set** are models of concept **itl::Set**

```

** 7bstrat part
empty set$      Set$Set()
s!bset relation$ bool Set$contained_in(const Set9 s2) const
e1!ality$      bool is_element_e1!al(const Set9 s', const Set9 s2)
set !nion$      Set9 operator := (Set9 s', const Set9 s2)
                Set operator : (const Set9 s', const Set9 s2)
set difference$ Set9 operator ,= (Set9 s', const Set9 s2)
                Set operator , (const Set9 s', const Set9 s2)
set intersection$ Set9 operator 9= (Set9 s', const Set9 s2)
                Set operator 9 (const Set9 s', const Set9 s2)

** cart related to se1!ential orderin
sortin order$   bool operator < (const Set9 s', const Set9 s2)
ledico raphical e1!ality$
                bool operator == (const Set9 s', const Set9 s2)
    
```



- The semantics of *itl maps* is based on a concept **itl::Map**
  - **itl::map**, **interval map** and **split interval map** are models of concept **itl::Map**

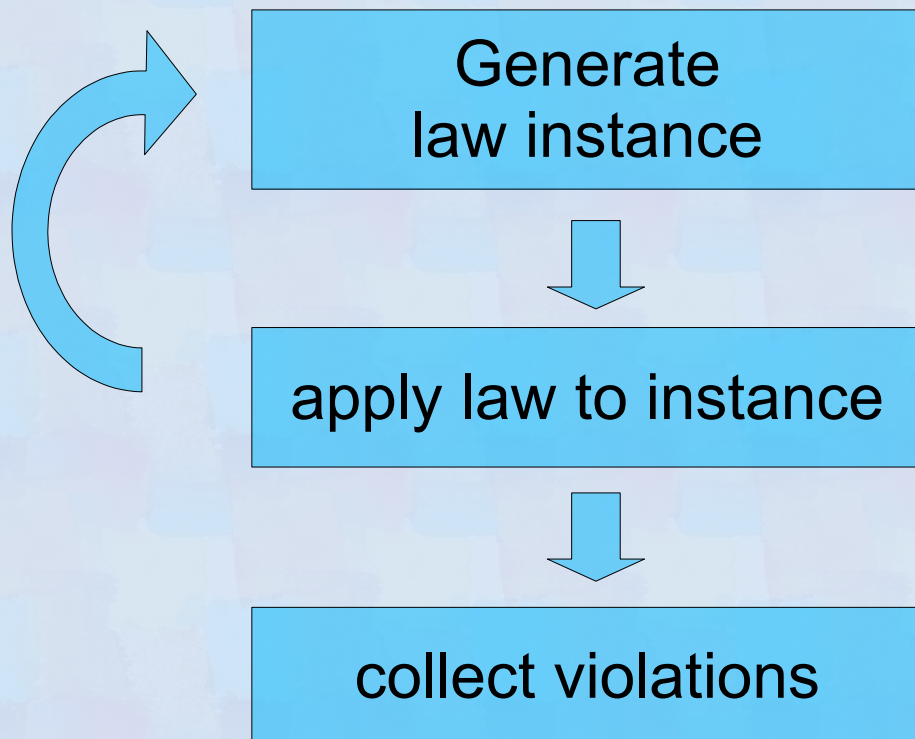
```

** 7bstruct part
empty map$          &ap$$&ap$()
s!bmap relation$    bool &ap$$contained_in(const &ap$ m2) const
e!ality$           bool is_element_e!al(const &ap$ m', const &ap$ m2)
map !nion$          &ap$ operator := (&ap$ m', const &ap$ m2)
                    &ap$ operator : (const &ap$ m', const &ap$ m2)
map difference$     &ap$ operator ,= (&ap$ m', const &ap$ m2)
                    &ap$ operator , (const &ap$ m', const &ap$ m2)
map intersection$   &ap$ operator 9= (&ap$ m', const &ap$ m2)
                    &ap$ operator 9 (const &ap$ m', const &ap$ m2)

** cart related to se!ential orderin
sortin order$       bool operator < (const &ap$ m', const &ap$ m2)
ledico raphical e!ality$
                    bool operator == (const &ap$ m', const &ap$ m2)

```

- Defining semantics of itl concepts via sets of laws
  - aka c++0x axioms
- Checking law sets via automatic testing:
  - A **Law Based Test Automaton** **LaBatea**



Commutativity<T a, U b, +>:  
 $a + b = b + a;$



## • Lexicographical Ordering and Equality

- For all itl containers `operator <` implements a ***strict weak ordering***.
- The ***induced equivalence*** of this ordering is ***lexicographical equality*** which is implemented as `operator ==`
- This is in line with the semantics of `SortedAssociativeContainers`

## • Subset Ordering and Element Equality

- For all itl containers function **contained in** implements a ***partial ordering***.
- The ***induced equivalence*** of this ordering is ***equality of elements*** which is implemented as function **is element equal**.



- itl::Sets
- **All** itl sets implement a **Set Algebra**, which is to say satisfy a “*classical*” set of laws . . .
  - . . . using **is element equal** as equality
  - Associativity, Neutrality, Commutativity (for **+** and **&**)
  - Distributivity, DeMorgan, Symmetric Difference
- **Most of** the itl sets satisfy the classical set of laws even if . . .
  - . . . lexicographical equality: **operator ==** is used
  - The differences reflect proper inequalities in sequence that occur for **separate interval set** and **split interval set**.

- Concepts induction / concept transition
  - The semantics of itl Maps appears to be *determined* by the *codomain type* of the map
  - Itl Maps are *mapping* the semantics of the *codomain type* on themselves.

*is model of example*

Map<D, Monoid>	Monoid	interval map<int, string>
Map<D, CommutMonoid>	CommutMonoid	interval map<int, unsigned>
Map<D, AbelianGroup>	AbelianGroup	interval map<int, int>
Map<D, Set>	Set	interval map<int, set<int>>



- Itl containers are implemented simply based on `std::set` and `std::map`
  - Basic operations like *adding* and *subtracting* intervals have a *best case complexity of  $O(\lg n)$* , if the added or subtracted intervals are *relatively small*.
  - Worst case complexity of *adding* or *subtracting* intervals *for `interval_set`* is  $O(n)$ .
  - For all other interval containers *adding* or *subtracting* intervals has a *worst case performance of  $O(n \lg(n))$* .
  - There is a *potential* for optimization . . .

- A **segment\_tree** implementation: A balanced tree, where . . .
  - an interval represents a perfectly balanced subtree
  - large intervals are rotated towards the root
- First results
  - much better worst case performance  $O(n)$  instead of  $O(n \lg(n))$
  - but slower for best case due to heavier bookkeeping and recursive algorithms.



- Completing and optimizing the segment\_tree implementation of interval containers
- Implementing interval\_maps of sets more efficiently
- Revision of features of the extended itl (itl\_plus.zip)
  - **Decomposition of histories**:  $k$  histories  $h_k$  with attribute types  $A_1, \dots, A_k$  are “*decomposed*” to a product history of tuples of attribute sets:  
$$(h_1\langle T, A_1 \rangle, \dots, h\langle T, A_k \rangle) \rightarrow h\langle T, (\text{set}\langle A_1 \rangle, \dots, \text{set}\langle A_k \rangle) \rangle$$
  - **Cubes** (generalized crosstables): Applying *aggregate on collision* to **maps of tuple value pairs** in order to organize hierarchical data and their aggregates.

- Itl project on **sourceforge** (version 2.0.1)  
<http://sourceforge.net/projects/itl>
- Latest version on **boost vault/Containers** (3.0.0)  
<http://www.boostpro.com/vault/> → containers
  - itl.zip : Core itl in preparation for boost
  - itl\_plus.zip : Extended itl including product histories, cubes and automatic validation (LaBatea).
- **Online documentation** at  
<http://www.herold-faulhaber.de/>
  - Doxygen generated docs for (version 2.0.1)  
<http://www.herold-faulhaber.de/itl/>
  - Latest boost style documentation (version 3.0.0)  
[http://www.herold-faulhaber.de/boost\\_itl/doc/libs/itl/doc/html/](http://www.herold-faulhaber.de/boost_itl/doc/libs/itl/doc/html/)



## ■ Boost sandbox

<https://svn.boost.org/svn/boost/sandbox/itl/>

- Core itl: Interval containers preparing for boost

<https://svn.boost.org/svn/boost/sandbox/itl/boost/itl/>

<https://svn.boost.org/svn/boost/sandbox/itl/libs/itl/>

- Extended itl\_xl: “histories” and cubes


[https://svn.boost.org/svn/boost/sandbox/itl/boost/itl\\_xt/](https://svn.boost.org/svn/boost/sandbox/itl/boost/itl_xt/)

[https://svn.boost.org/svn/boost/sandbox/itl/libs/itl\\_xt/](https://svn.boost.org/svn/boost/sandbox/itl/libs/itl_xt/)

- Validator LaBatea: Currently only vc8 or newer

<https://svn.boost.org/svn/boost/sandbox/itl/boost/validate/>

<https://svn.boost.org/svn/boost/sandbox/itl/libs/validate/>



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## Lecture Outline

- Background and Motivation
- Design
- Examples
- Semantics
- Implementation
- Future Works
- Availability

## Background and Motivation

- Interval containers simplified the implementation of date and time related tasks
  - Decomposing “*histories*” of attributed events into segments with constant attributes.
  - Working with time grids, e.g. a grid of months.
  - Aggregations of values associated to date or time intervals.
- ... that occurred frequently in programs like
  - Billing modules
  - Therapy scheduling programs
  - Hospital and controlling statistics



- Background is the date time problem domain ...
- ... but the scope of the **ItI** as a generic library is more general:

an **interval\_set** is a **set**  
*that is implemented as a set of intervals*

an **interval\_map** is a **map**  
*that is implemented as a map of interval value pairs*

- There are two aspects in the design of interval containers
- Conceptual aspect

```
interval set<int> mySet;  
mySet.insert(42);  
bool has_answer = mySet.contains(42);
```

- On the conceptual aspect an `interval_set` can be used just as a set of elements
- except for . . .
- . . . **iteration** over *elements*
- consider `interval set<double>` or `interval set<string>`
- Iterative Aspect
  - **Iteration** is always done over *intervals*



- Addability and Subtractability

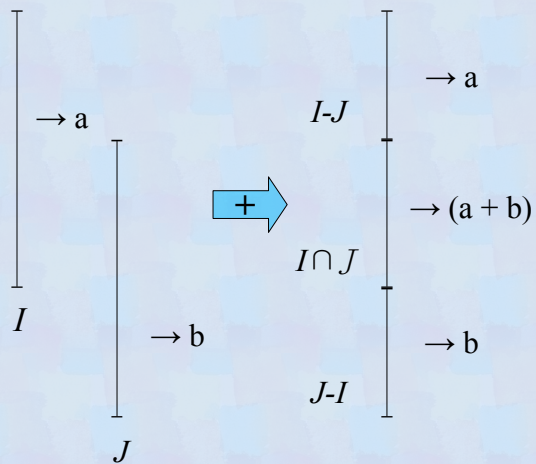
- All of itl's (interval) containers are *Addable* and *Subtractable*
- They implement **operators** `+=`, `+`, `-=` and `-`

	<code>+=</code>	<code>-=</code>
sets	set union	set difference
maps	?	?

- A possible implementation for maps

- Propagate addition/subtraction to the associated values
- . . . or aggregate on overlap
- . . . or aggregate on collision

● Aggregate on overlap



- Decompositional effect on Intervals
- Accumulative effect on associated values

$I, J$ : intervals,  $a, b$ : associated values



- Aggregate on overlap, a minimal example

```
typedef
```

- The Itl's class templates

Granularity	Style	Sets	Maps
interval		<code>interval</code>	
	joining	<code>interval set</code>	<code>interval map</code>
	separating	<code>separate interval set</code>	
	splitting	<code>split interval set</code>	<code>split interval map</code>
element		<code>set</code>	<code>map</code>



Interval Combining Styles: *Joining*

- Intervals are joined on overlap or on touch
- ... *for maps*, if associated values are equal
- Keeps interval\_maps and sets in a minimal form

interval set

```
{ [1      3)      ]
+   [2      4)
+           [4 5)
= { [1      4)      ]
= { [1      5) }
```

interval map

```
{ [1      3) ->1      ]
+   [2      4) ->1
+           [4 5) ->1
= { [1 2) [2 3) [3 4)      ]
    ->1  ->2  ->1
= { [1 2) [2 3) [3      5)      ]
    ->1  ->2      ->1
```

● Interval Combining Styles: *Splitting*

- Intervals are split on overlap and kept separate on touch
- All interval borders are preserved (insertion memory)

split interval set

```
{ [1      3)      ]
+      [2      4)
+              [4 5)

= { [1 2) [2 3) [3 4)      ]
= { [1 2) [2 3) [3 4) [4 5) }
```

split interval map

```
{ [1      3) ->1      ]
+      [2      4) ->1
+              [4 5) ->1

= { [1 2) [2 3) [3 4)      ]
   ->1 ->2 ->1
= { [1 2) [2 3) [3 4) [4 5) }
   ->1 ->2 ->1 ->1
```



- Interval Combining Styles: *Separating*

- Intervals are joined on overlap but kept separate on touch
- Preserves borders that are never crossed (preserves a hidden grid).

separate interval set

$$\begin{aligned}
 & \{ [1 \quad 3) \quad ] \\
 + & \quad [2 \quad 4) \\
 + & \quad [4 \quad 5) \\
 = & \{ [1 \quad 4) \quad ] \\
 = & \{ [1 \quad 4) [4 \quad 5) \}
 \end{aligned}$$

● A few instances of intervals (interval.cpp)

```

inter/al<int> int_inter/al = inter/al<int>$$closed((,0);
inter/al<double> s1rt_inter/al
  = inter/al<double>$$rightopen('*s1rt(2.#), s1rt(2.#));
inter/al<std$ssstrin > city_inter/al
  = inter/al<std$ssstrin >$$leftopen(%2arcelona%, %2oston%);
inter/al<boost$stime> time_inter/al
  = inter/al<boost$stime>$$open(
    time_from_strin (%2##3,#4,2# '5$(#%),
    time_from_strin (%2##3,#4,2# 2($##%)
  );

```



- A way to iterate over months and weeks  
(month\_and\_wee" \_ rid.cpp)

```

6#include <boost*itl* re orian.hpp> **boost$$ re orian pl!s adapter code
6#include <boost*itl*split_inter/al_set.hpp>

** 7 split_inter/al_set of re orian dates as date_ rid.
typedef split_inter/al_set<boost$$ re orian$$date> date_ rid;

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date_ rid month_ rid(const inter/al<date>9 scope)
-
    date_ rid month_ rid;
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    .
    .
    .
    ret!rn month_ rid;
    .

** 8omp!te a date_ rid of wee"s !sin boost$$ re orian.
date_ rid wee" _ rid(const inter/al<date>9 scope)
-
    date_ rid wee" _ rid;
    ** 8omp!te a date_ rid of wee"s !sin boost$$ re orian.
    .
    .
    .
    ret!rn wee" _ rid;
    .

```

● A way to iterate over months and weeks

```

/oid month_and_time_ rid()
-
    date someday = day_cloc"$$local_day();
    date thenday = someday : months(2);
    inter/al<date> scope = inter/al<date>$$ri htopen(someday, thenday);

    ** 7n intersection of the month and wee" rids ...
    date_ rid month_and_wee"_ rid
        = month_ rid(scope) 9 wee"_ rid(scope);

    ** ... allows to iterate months and wee"s. ;hene/er a month
    ** or a wee" chan es there is a new inter/al.
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        it <= month_and_wee"_ rid.end(); it::)
    - . . . .

    ** ;e can also intersect the rid into an inter/al_map to ma"e
    ** sh!re that all inter/als are within months and wee" bounds.
    inter/al_map<boost$$ re orian$$date, some_type> accr!al;
    comp!te_some_res!lt(accr!al, scope);
    accr!al 9= month_and_wee"_ rid;
    .

```

## ● Aggregating with interval\_maps

- Computing averages via implementing **operator +=**  
(partys\_!est\_a/era e.cpp)

```
class counted_sum
{
public:
    counted_sum()$_s!m(#),_co!nt(#)-.
    co!nted_s!m(int s!m)$_s!m(s!m),_co!nt(')-.

    int s!m()const -ret!rn _s!m;.
    int co!nt()const-ret!rn _co!nt;.
    do!ble a/era e()const
    - ret!rn _co!nt==# = #.# $_s!m*static_cast<do!ble>(_co!nt); .

    co!nted_s!m9 operator += (const co!nted_s!m9 ri ht)
    - _s!m := ri ht.s!m(); _co!nt := ri ht.co!nt(); ret!rn >this; .

private:
    int _s!m;
    int _co!nt;
    .;

    bool operator == (const co!nted_s!m9 left, const co!nted_s!m9 ri ht)
    - ret!rn left.s!m()==ri ht.s!m() 99 left.co!nt()==ri ht.co!nt(); .
}
```



## ● Aggregating with interval\_maps

- Computing averages via implementing **operator +=**

```
void partys_hei ht_a/era e()
{
    interval_map<ptime, counted_sum> hei ht_s!ms;

    hei ht_s!ms += (
        make_pair(
            interval<ptime>$$ri htopen(
                time_from_strin (%2##3, #4, 2# '5$(#%),
                time_from_strin (%2##3, #4, 2# 2($##%)),
                counted_sum(165)) ** &ary is ',?4 m tall.
        );

        ** 7dd hei ht of more pary !ests . . .

    interval_map<ptime, counted_s!m>$$iterator hei ht_s!m_ =
        hei ht_s!ms.be in();
    while(hei ht_s!m_ <= hei ht_s!ms.end())
    {
        interval<ptime> when = hei ht_s!m_,>first;
        double hei ht_a/era e = (>hei ht_s!m_::).second.average();

        cout << %+% << when.first() << % , % << when.upper() << %)%
            << %$ % << hei ht_a/era e << % cm% << endl;
    }
}
```

- Interval containers allow to express a variety of date and time operations in an easy way.
  - Example `man power.cpp` ...
  - Subtract weekends and holidays from an `interval_set`  
`worktime -= weekends(scope)`  
`worktime -= german reunification day`
  - Intersect an `interval_map` with an `interval_set`  
`claudias working hours &= worktime`
  - Subtract an `interval_set` from an `interval map`  
`claudias working hours -= claudias absense times`
  - Adding `interval_maps`  
`interval map<date,int> manpower;`  
`manpower += claudias working hours;`  
`manpower += bodos working hours;`

- Interval\_maps can also be intersected  
Example `user_groups.cpp`

```
typedef boost::interval_map<date, memberSet> memberSet;
typedef interval_map<date, memberSet> membership;

void user_roles()
{
    . . .

    membership med_users;
    // compute membership of medical staff
    med_users := make_pair(member_interval, memberSet("Ar.Beyl"));
    med_users := . . .

    membership admin_users;
    // compute membership of administration staff
    med_users := make_pair(member_interval_2, memberSet("r.yde"));
    . . .

    membership all_users = med_users + admin_users;

    membership super_users = med_users & admin_users;
    . . .
}
```



- The semantics of *itl sets* is based on a concept `itl::Set`
  - `itl::set`, `interval set`, `split interval set` and `separate interval set` are models of concept `itl::Set`

```

** 7abstract part
empty set$      Set$Set()
subset relation$ bool Set$contained_in(const Set9 s2) const
equality$       bool is_element_equal(const Set9 s', const Set9 s2)
set union$      Set9 operator := (Set9 s', const Set9 s2)
                Set operator : (const Set9 s', const Set9 s2)
set difference$ Set9 operator ,= (Set9 s', const Set9 s2)
                Set operator , (const Set9 s', const Set9 s2)
set intersection$ Set9 operator &= (Set9 s', const Set9 s2)
                 Set operator & (const Set9 s', const Set9 s2)

** part related to sequential ordering
sort in order$   bool operator < (const Set9 s', const Set9 s2)
lexicographical equality$
                 bool operator == (const Set9 s', const Set9 s2)

```

- The semantics of *itl maps* is based on a concept `itl::Map`
  - `itl::map`, `interval map` and `split interval map` are models of concept `itl::Map`

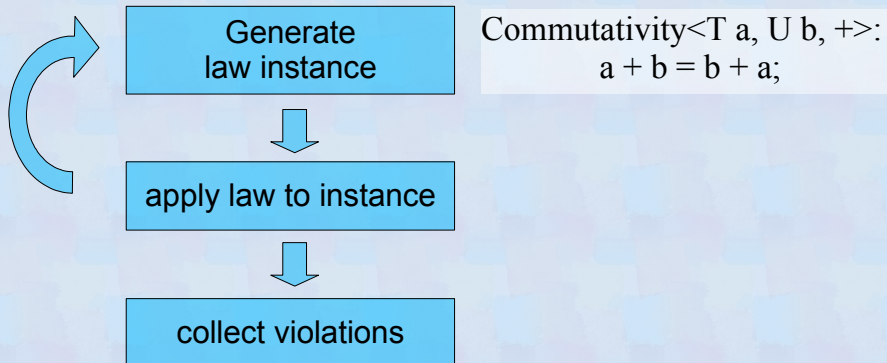
```

** 7bstrack part
empty map$      &ap$&ap$()
sibmap relation$ bool &ap$&ap$contained_in(const &ap$ m2) const
equality$      bool is_element_e1!al(const &ap$ m', const &ap$ m2)
map !nion$      &ap$ operator := (&ap$ m', const &ap$ m2)
               &ap$ operator : (const &ap$ m', const &ap$ m2)
map difference$ &ap$ operator ,= (&ap$ m', const &ap$ m2)
               &ap$ operator , (const &ap$ m', const &ap$ m2)
map intersection$ &ap$ operator 9= (&ap$ m', const &ap$ m2)
               &ap$ operator 9 (const &ap$ m', const &ap$ m2)

** Cart related to sequential orderin
sortin order$   bool operator < (const &ap$ m', const &ap$ m2)
ledicographical equality$
               bool operator == (const &ap$ m', const &ap$ m2)

```

- Defining semantics of itl concepts via sets of laws
  - aka c++0x axioms
- Checking law sets via automatic testing:
  - A **Law Based Test Automaton** *LaBatea*





- Lexicographical Ordering and Equality

- For all itl containers `operator <` implements a ***strict weak ordering***.
- The ***induced equivalence*** of this ordering is ***lexicographical equality*** which is implemented as `operator ==`
- This is in line with the semantics of SortedAssociativeContainers

- Subset Ordering and Element Equality
  - For all itl containers function **contained in** implements a ***partial ordering***.
  - The ***induced equivalence*** of this ordering is ***equality of elements*** which is implemented as function **is element equal**.

- itl::Sets
- All itl sets implement a **Set Algebra**, which is to say satisfy a “*classical*” set of laws . . .
  - . . . using **is element equal** as equality
  - Associativity, Neutrality, Commutativity (for + and &)
  - Distributivity, DeMorgan, Symmetric Difference
- Most of the itl sets satisfy the classical set of laws even if . . .
  - . . . lexicographical equality: **operator ==** is used
  - The differences reflect proper inequalities in sequence that occur for **separate interval set** and **split interval set**.



- Concepts induction / concept transition
  - The semantics of Itl Maps appears to be *determined* by the *codomain type* of the map
  - Itl Maps are *mapping* the semantics of the *codomain type on themselves*.

*is model of example*

<code>Map&lt;D, Monoid&gt;</code>	<code>Monoid</code>	<code>interval map&lt;int, string&gt;</code>
<code>Map&lt;D, CommutMonoid&gt;</code>	<code>CommutMonoid</code>	<code>interval map&lt;int, unsigned&gt;</code>
<code>Map&lt;D, AbelianGroup&gt;</code>	<code>AbelianGroup</code>	<code>interval map&lt;int, int&gt;</code>
<code>Map&lt;D, Set&gt;</code>	<code>Set</code>	<code>interval map&lt;int, set&lt;int&gt;&gt;</code>

- Itl containers are implemented simply based on `std::set` and `std::map`
  - Basic operations like *adding* and *subtracting* intervals have a *best case complexity of  $O(\lg n)$* , if the added or subtracted intervals are *relatively small*.
  - Worst case complexity of *adding* or *subtracting* intervals *for `interval_set`* is  $O(n)$ .
  - For all other interval containers *adding* or *subtracting* intervals has a *worst case performance of  $O(n \lg(n))$* .
  - There is a *potential* for optimization . . .

- A **segment\_tree** implementation: A balanced tree, where . . .
  - an interval represents a perfectly balanced subtree
  - large intervals are rotated towards the root
- First results
  - much better worst case performance  $O(n)$  instead of  $O(n \lg(n))$
  - but slower for best case due to heavier bookkeeping and recursive algorithms.



- Completing and optimizing the segment\_tree implementation of interval containers
- Implementing interval\_maps of sets more efficiently
- Revision of features of the extended itl (itl\_plus.zip)
  - **Decomposition of histories**:  $k$  histories  $h_k$  with attribute types  $A_1, \dots, A_k$  are “*decomposed*” to a product history of tuples of attribute sets:  
 $(h_1 \langle T, A_1 \rangle, \dots, h_k \langle T, A_k \rangle) \rightarrow h \langle T, (\text{set} \langle A_1 \rangle, \dots, \text{set} \langle A_k \rangle) \rangle$
  - **Cubes** (generalized crosstables): Applying *aggregate on collision* to *maps of tuple value pairs* in order to organize hierarchical data and their aggregates.

- Itl project on **sourceforge** (version 2.0.1)  
<http://sourceforge.net/projects/itl>
- Latest version on **boost vault/Containers** (3.0.0)  
<http://www.boostpro.com/vault/> → containers
  - itl.zip : Core itl in preparation for boost
  - itl\_plus.zip : Extended itl including product histories, cubes and automatic validation (LaBatea).
- **Online documentation** at  
<http://www.herold-faulhaber.de/>
  - Doxygen generated docs for (version 2.0.1)  
<http://www.herold-faulhaber.de/itl/>
  - Latest boost style documentation (version 3.0.0)  
[http://www.herold-faulhaber.de/boost\\_itl/doc/libs/itl/doc/html/](http://www.herold-faulhaber.de/boost_itl/doc/libs/itl/doc/html/)

- Boost sandbox

<https://svn.boost.org/svn/boost/sandbox/itl/>

- Core itl: Interval containers preparing for boost  
<https://svn.boost.org/svn/boost/sandbox/itl/boost/itl/>  
<https://svn.boost.org/svn/boost/sandbox/itl/libs/itl/>
- Extended itl\_xl: “histories” and cubes  
[https://svn.boost.org/svn/boost/sandbox/itl/boost/itl\\_xl/](https://svn.boost.org/svn/boost/sandbox/itl/boost/itl_xl/)  
[https://svn.boost.org/svn/boost/sandbox/itl/libs/itl\\_xl/](https://svn.boost.org/svn/boost/sandbox/itl/libs/itl_xl/)
- Validator LaBatea: Currently only vc8 or newer  
<https://svn.boost.org/svn/boost/sandbox/itl/boost/validate/>  
<https://svn.boost.org/svn/boost/sandbox/itl/libs/validate/>