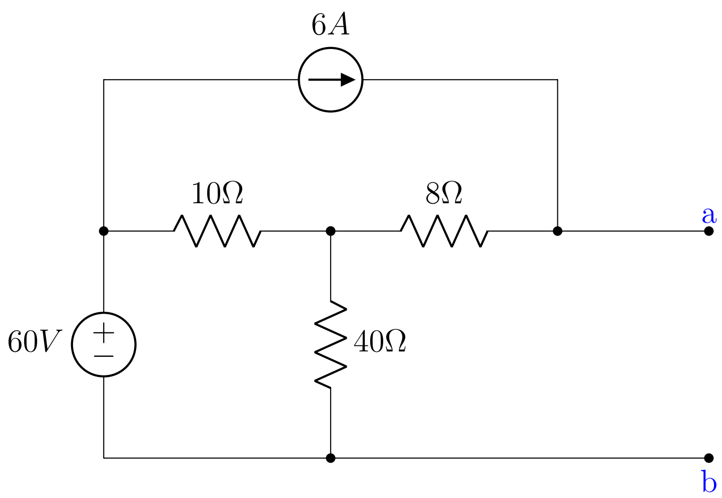
Electric Circuits - Homework 03

Automation Class 1904

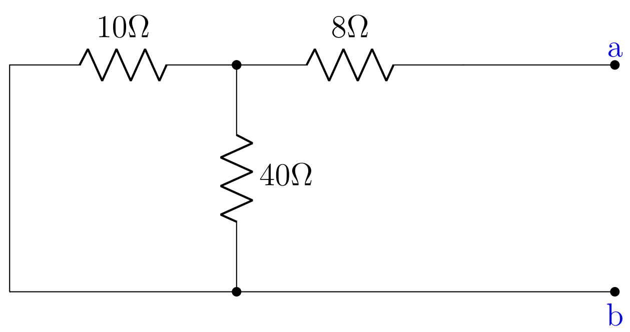
(Due date: 2020/10/5)

1. (10 Point)

   Ans:



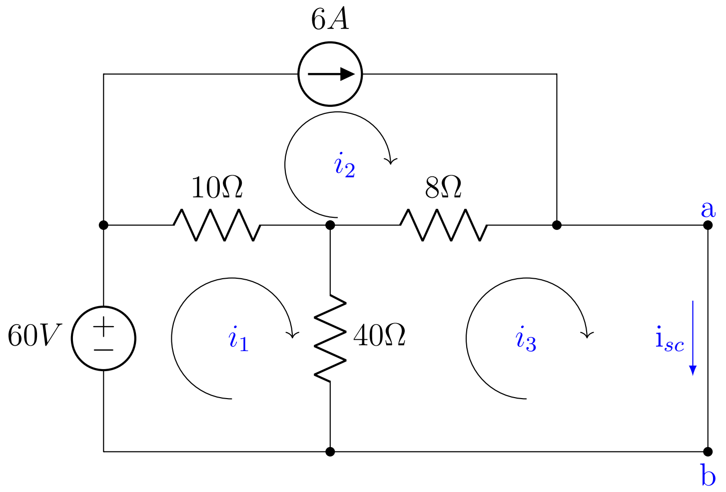
We make the voltage source and the current source deactivated.



Then we can calculate the equivalent resistance R_{eq};

$$R_{eq} = 8 + 10||40 = 16\Omega$$

The next step is to replace a short circuit across the terminals:



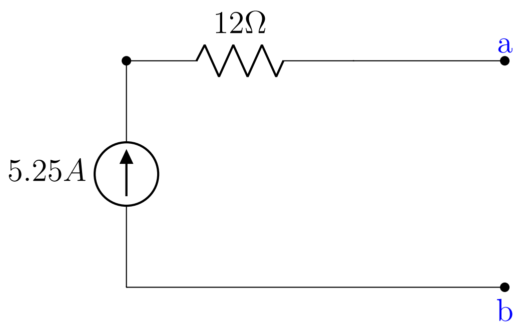
and use the Mesh-Current method calculate the resulting short-curcuit current:

$$10(i_1 - i_2) + 40(i_1 - i_3) - 60 = 0$$
$$10(i_2 - i_1) + 8(i_2 - i_3) = 0$$
$$8(i_3 - i_2) + 40(i_3 - i_2) = 0$$
$$i_2 = 6A$$
$$i_3 = i_{sc}$$

Thus the i$_{sc}$ is equal:

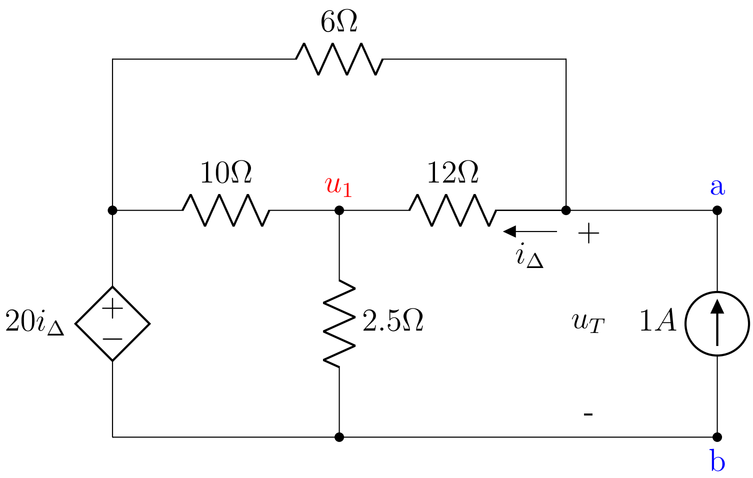
$$i_{sc} = 5.25A$$

The circuit shown in:



2. (10 Points)

Ans:



Because there no independent source, so $v_{Th} = 0$;Add a independent current source for 1A at port ab;

According the Node-Voltage method:

$$\frac{u_1 - 20i_\Delta}{10} + \frac{u_1 - u_T}{12} + \frac{u_1}{2.5} = 0$$
$$\frac{u_T - 20i_\Delta}{6} + \frac{u_T - u_1}{12} - 1 = 0$$
$$\frac{u_T - u_1}{12} = i_\Delta$$

We can get :

$$i_\Delta = 1.5A \qquad u_1 = 9V \qquad u_T = 27V$$

if we add a independent current cource for 2A at port ab:

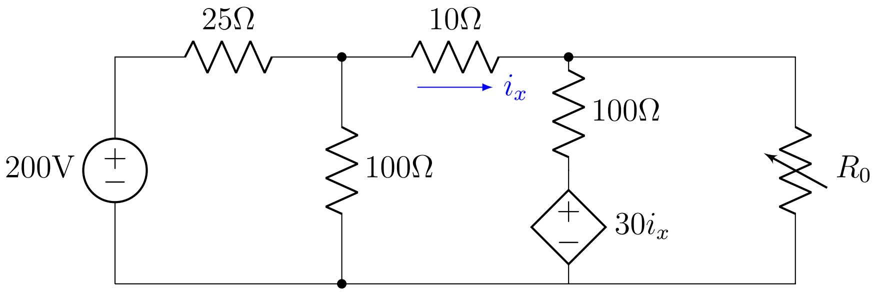
$$i_\Delta = 3A \qquad u_1 = 18V \qquad u_T = 54V$$

Thus when we add a independent current cource for nA at port ab:

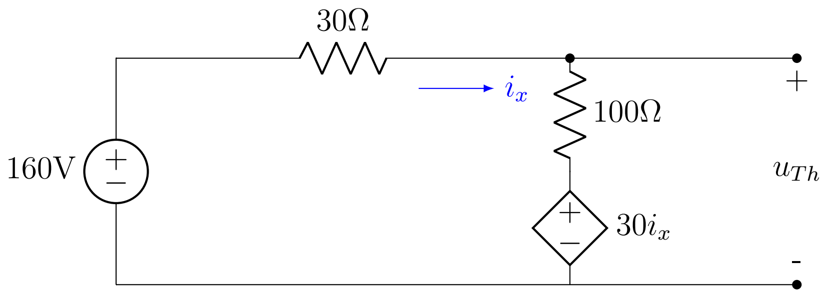
$$i_\Delta = 1.5nA \qquad u_1 = 9nV \qquad u_T = 27nV$$

3. (20 Points)

Ans:



Using Source Transformation to be:



then we can calculate i_x :

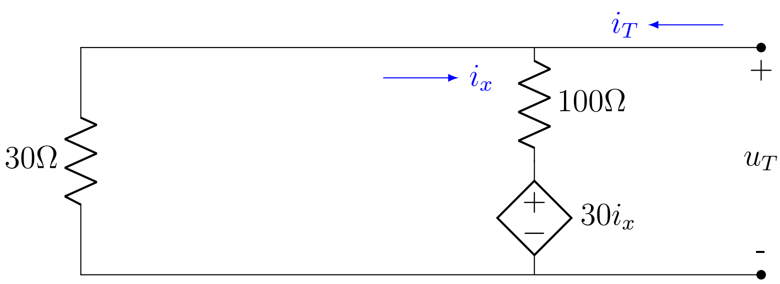
$$\frac{160-30i_x}{130} = i_x$$

$$i_x = 1A$$

since

$$u_{Th} = 100i_x + 30i_x = 130V$$

Using the test-source method to find the Th´evenin resistance gives:



Thus use KCL:

$$i_x = -u_T/30$$
$$i_T = -i_x + \frac{u_T - 30i_x}{100}$$

$$R_{eq} = \frac{u_T}{i_T} = 18.75\Omega$$

Then :

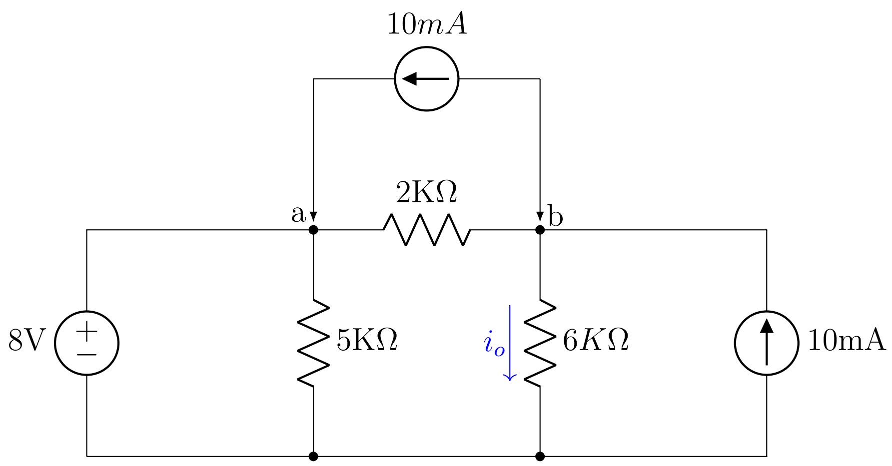
$$P = i^2 \times R = (\frac{u_{Th}}{R_{eq}+R_0})^2 \times R_0 = 225W$$

Thus :

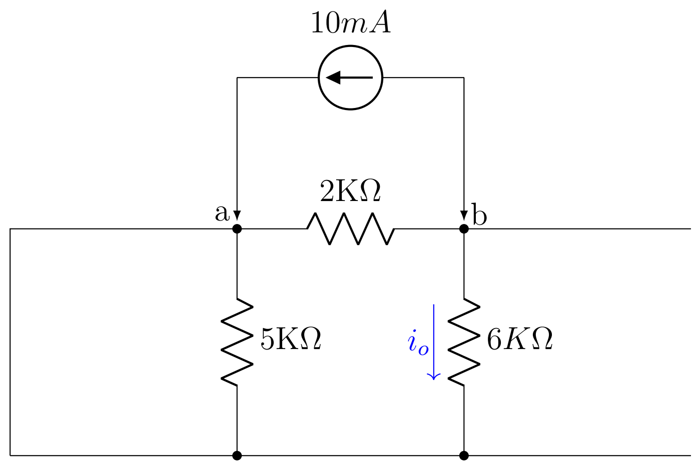
$$R_0 = \frac{625}{36}\Omega \qquad R_0 = \frac{81}{4}\Omega$$

4. (10 Points)

Ans:



By hypothesis $i_o' + i_o'' = i_o$:



$$i_o'' = 10 \times \frac{2}{2+6} = 2.5mA$$

$$i_o' = i_o - i_o'' = 1mA$$

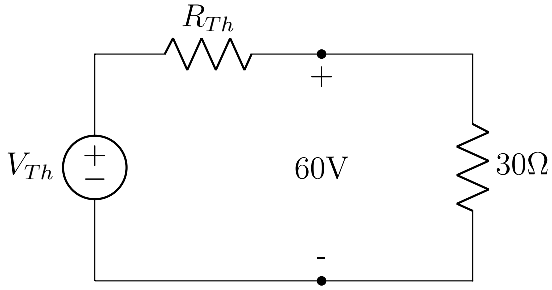
Using KCL to find the value of i_o after the current source is attached:

$$\frac{U_b'-8}{2000}+\frac{U_b'}{6000}+0.01-0.01=0$$

$$u_b'=6V \qquad i_o'=U_b/6000=1mA$$

5. (10 Points)

Ans:



According to the question :

$$V_{Th}=75V \qquad i=\frac{v_0}{R_L}=\frac{60}{30}=2A$$

Thus :

$$R_{Th}=\frac{75-60}{2}=7.5\Omega$$

and:

$$\frac{V_{Th}}{R_{Th}+R_L}=\frac{v_0}{R_L}$$

Since:

$$R_{Th}=(\frac{V_{Th}}{v_0}-1)R_L$$

6. (10 Points)

Ans:

a).We known from the question:

$$t:0\rightarrow 250\mu s \quad (250\times 10^{-6}s) \qquad C:0.2\mu F \quad (0.2\times 10^{-6}F)$$
$$v_0:-100V \qquad i:100e^{-1000t}mA \quad (0.1e^{-1000t}A)$$

because of :

$$i = C\frac{dV}{dt};$$

Thus:

$$v = \frac{1}{0.2\times10^{-6}}\int_0^{250\times10^{-6}} 0.1e^{-1000t}\,dt - 100$$

Then we can calculate V:

$$v = 500(1-e^{-0.25}) - 100 = 10.6V$$

because of:

$$w = \frac{1}{2}Cv^2$$

Thus :

$$w = 0.5\times0.2\times10^{-6}\times10.6^2 = 1.1236\times10^{-5}J = 11.236\mu J$$

b).Because of t $\rightarrow\infty$;

Thus :

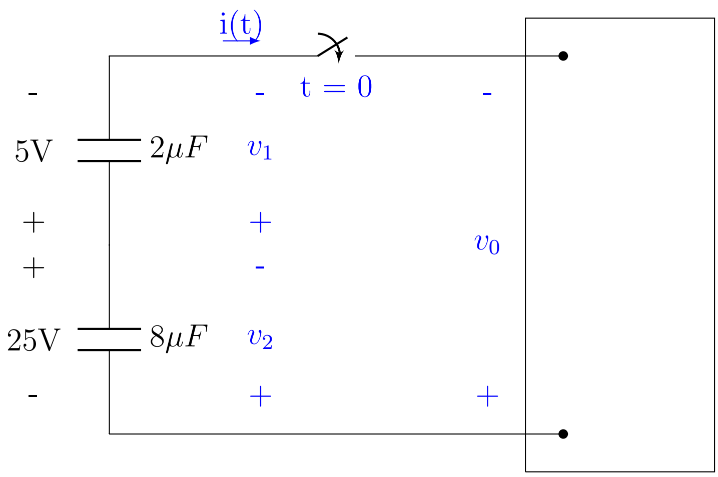
$$u_\infty = 500 - 100 = 400V$$

Since :

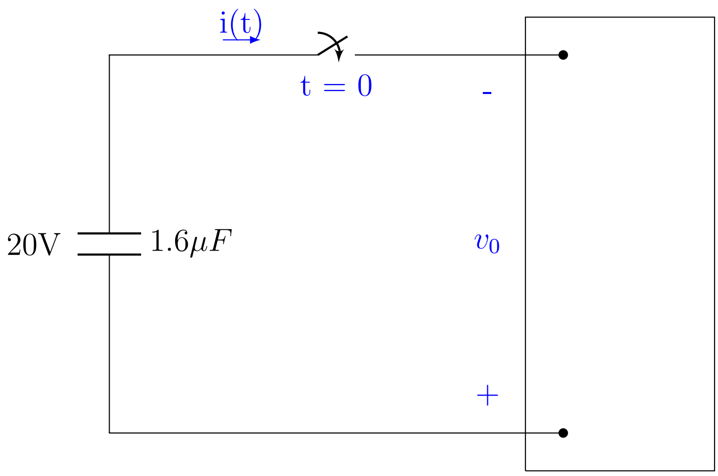
$$w_\infty = 0.5\times0.2\times10^{-6}\times400^2 = 0.016J = 1.6\times10^4\mu J$$

7. (20 Points)

Ans:



a).



Because of :

$$i = C\frac{dV}{dt};$$

Thus:

$$v_0 = \frac{1}{1.6\times10^{-6}}\int_0^t 960\times10^{-6}e^{-30t}\ dt - 20$$

$$v_0 = -20e^{-30t}V, \quad t \geq 0$$

b).In a similar way:

$$v_1 = \frac{1}{2\times10^{-6}}\int_0^t 960\times10^{-6}e^{-30t}\ dt + 5$$

$$v_1 = -16e^{-30t} + 21(V), \quad t \geq 0$$

c).In a similar way:

$$v_2 = \frac{1}{8\times10^{-6}}\int_0^t 960\times10^{-6}e^{-30t}\ dt - 25$$

$$v_2 = -4e^{-30t} - 21(V), \quad t \geq 0$$

d).From a). we known v_0, then we can calculate p and w:

$$p = -vi = -(-20e^{-30t})(960\times10^{-6}e^{-30t}) = 1.92\times10^{-2}e^{-60t}$$

$$w_\infty = \int_0^\infty 1.92\times10^{-2}e^{-60t}\,dt = 3.2\times10^{-4}J = 320\mu J$$

e).Because of $w = \frac{1}{2}Cv^2$

$$w = 0.5\times(2\times10^{-6})\times5^2 + 0.5\times(8\times10^{-6})\times25^2 = 2525\mu J$$

f).

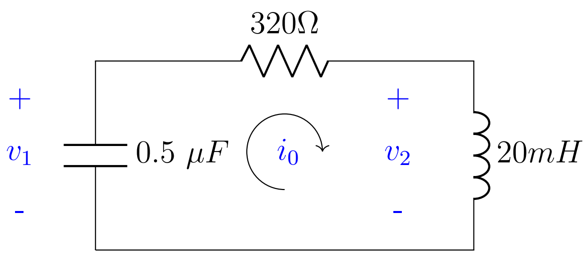
$$w_{trapped} = w_{initial} - w_{delivered} = 2525 - 320 = 2205\mu J$$

g).When t $\to \infty$:

$$v_1 = 21V \qquad v_2 = -21V$$

$$w = 0.5 \times (2 \times 10^{-6}) \times 21^2 + 0.5 \times (8 \times 10^{-6}) \times (-21)^2 = 2205\mu J$$

Ans:



From the question :

$i_0 = 50e^{-8000t}(cos6000t+2sin6000t)mA = 0.05e^{-8000t}(cos6000t+2sin6000t)A$

Then we can calculate:

$$\frac{di_0}{dt} = e^{-8000t}(200cos6000t - 1100sin6000t)$$

$$\frac{di_0}{dt}(0^+) = 1 \times (200 - 0) = 200$$

So $v_2(0^+) = L\frac{di_0}{dt}(0^+)$:

$$v_2(0^+) = 20 \times 10^{-3}\frac{di_0}{dt}(0^+) = 4V$$

And $v_C = v_R + v_L$:

$$i_0(0^+) = 0.05A$$

$$v_1(0^+) = 320i_0(0^+) + v_2(0^+) = 20V$$