Electric Circuits - Homework 04

Automation Class 1904

(Due date: 2020/12/27)

1. (10 Point) Ans:

a).By the question and figure:

right:

$$-2\frac{di_g}{d_t} + 16\frac{di_2}{d_t} + 32i_2 = 0$$

$$8\frac{di_2}{d_t} + 16i_2 = \frac{di_g}{d_t}$$

b).From the question :

$$i_2 = e^{-t} - e^{-2t}$$

Then we know:

$$8\frac{di_2}{d_t} + 16i_2 = -8e^{-t} + 16e^{-2t} + 16e^{-t} - 16e^{-2t} = 8e^{-t}$$
$$\frac{di_g}{d_t} = 8e^{-t}$$

Thus, satisfied.

c).By question b and a:

$$v_1 = 4\frac{di_g}{d_t} - 2\frac{di_2}{d_t} = 34e^{-t} - 4e^{-2t}$$

d).By the question c:

$$v_1(0) = 30v$$

Yes, it make sense.

2. (10 Points)

a).By the question, $i_1 and i_2$ is clockwise mesh currents in the left and right. Then use Mesh-current method:

$$v_{ab} = L_1 \frac{di_1 - i_2}{dt} + M \frac{di_2}{dt}$$
$$0 = L_2 \frac{di_2}{dt} + M \frac{di_1 - i_2}{dt} - L_1 \frac{di_1 - i_2}{dt} - M \frac{i_2}{dt}$$

Simplified:

$$v_{ab} = L_1 \frac{di_1}{dt} + (M - L_1) \frac{di_2}{dt}$$
$$0 = (L_1 - M) \frac{di_1}{dt} + (L_1 + L_2 - 2M) \frac{di_2}{dt}$$

Thus:

$$v_{ab} = (\frac{L_1 + L_2 - 2M}{L_1 L_2 - M^2}) \frac{di_1}{dt}$$

from which we have:

$$L_{ab} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

b).If the magnetic polarity of coil 2 is reversed, the sign of M reverses,therefore

$$L_{ab} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

3. (20 Points)Ans:

With no finger on the button the circuit and $i = C\frac{dv}{dt}$,then we know:

$$C_1\frac{dv-v_s}{i} + C_2\frac{dv-v_s}{t} = 0$$
$$C_1 = C_2$$

Thus:

$$2C\frac{dv}{dt}$$

With a finger on the button:

$$C_1\frac{dv-v_s}{i} + C_2\frac{dv-v_s}{t} + C_3\frac{dv}{dt} = 0$$

Simplified:

$$(C_1 + C_2 + C_3)\frac{dv}{dt} + C_2\frac{dv_s}{dt} - C1\frac{dv_s}{dt} = 0$$
$$C_1 = C_2 = C_3$$

Thus:

$$3C\frac{dv}{dt} = 0$$

Since,there is no change in the output voltage of this circuit.

4. (10 Points)

Ans: $t < 0$:

$$i_L = \frac{2.5v_0}{v_0} = 2.5A$$

$t > 0$:Find Thevenin resistance seen by inductor:

$$i_T = 2.5A$$
$$R_{Th} = \frac{v_T}{i_T} = 0.4$$

Thus:

$$\tau = \frac{L}{R} = 12.5ms$$
$$\frac{1}{\tau} = 80$$

Then:

$$i_0 = 2.5e^{-80t}A$$
$$v_0 = L\frac{di_0}{dt} = -e^{-80} \qquad t \geq 0$$

5. (10 Points)

Ans:

a).By the question and figure

$$v_0(0^-) = v_0(0^+) = 120V$$

Make $2.5k\Omega$ resistor the right hand to Source Transformation.we will get resistor $37.5k\Omega$ and source 150V series.

$$v_0 = -150V$$
$$R_{eq} = 50k\Omega$$

then:

$$\tau = RC = 1.6ms$$
$$\tfrac{1}{\tau} = 625$$

Thus:

$$v_0 = -150 + (120 - (-150))e^{-625t}$$
$$v_0 = -150 + 270e^{-625t}$$

b).By the question:

$$i_0 = C\tfrac{dv}{dt} = 6.75e^{-625t}mA$$

c).By the question:

$$v_g = v_0 - 2.5 \times 10^3 = -150 + 253.125e^{-625t}$$

d).By the question c:

$$v_g(0) = 103.125V$$

6. (10 Points)

Ans:

When $0 \leq t \leq 200\mu s$:

$$R_{eq} = 150||100 = 60k\Omega$$
$$\tau = RC = 200\mu s$$
$$\frac{1}{\tau} = 5000$$

Thus:

$$v_c = 300e^{-5000t}$$
$$v_c(200\mu s) = 110.36$$

when $200\mu s \leq t \leq \infty$:

$$R_{eq} = 30||60 + 120||40 = 50K\Omega$$
$$\tau = R_{eq}C = 166.67\mu s$$
$$\frac{1}{\tau} = 6000$$

Thus:

$$v_c = 110.36e^{-6000(t-200\mu s)}V$$

When t = $300\mu s$:

$$v_c(300\mu s) = 110.36e^{-6000(100\mu s)} = 60.57V$$
$$i_0(300\mu s) = \frac{u_c}{R_{eq}} = 1.21mA$$

Then:

$$i_1 = \frac{60}{90}i_0 = \frac{2}{3}i_0 = 0.8067mA(\text{direction down});$$
$$i_2 = \frac{40}{160}i_0 = \frac{1}{4}i_0 = 0.3025mA \text{ (direction down);}$$

$$i_{mid} = i_1 - i_2 = \frac{5}{12}i_0 = 0.5mA(\text{direction right})$$

7. (20 Points)

Ans:

By the question and figure:Find Thevenin resistance seen by capacitor

$$v_T = 13 \times 10^4 i_\Delta + (20k\Omega || 80k\Omega) i_T$$
$$i_\Delta = -\frac{20}{100} i_T = -0.2 i_T$$

Then:

$$v_T = -10 \times 10^3 i_T$$

Thus:

$$R_{Th} = \frac{v_T}{i_T} = -10k\Omega$$
$$\tau = R_{Th} C = -0.025$$
$$\frac{1}{\tau} = -40$$

Since:

$$v_c = 20e^{40t} V$$

when $v_c = 20000V$

$$40t = ln1000$$
$$t = 172.69ms$$

8. (10 Points)

Ans:

a).By the question and figure, When $0 \leq t \leq 0.5$:

$$i = \frac{21}{60} + (\frac{30}{60} - \frac{21}{60})e^{-\frac{t}{\tau}}$$
$$\tau = \frac{L}{R}$$

Thus :

$$i = 0.35 + 0.15e^{-\frac{-60t}{L}}$$

When t = 0.5:

$$i(0.5) = 0.35 + 0.15e^{-\frac{-30}{L}} = 0.40$$

So:

$$e^{-\frac{30}{L}} = 3;$$
$$L = \frac{30}{ln3} = 27.31H$$

b).Hypothesized t_r is the time the relay releases: $0 \leq t \leq t_r$

$$i = 0 + (\frac{30}{60} - 0)e^{\frac{-60t}{L}} = 0.5e^{\frac{-60t}{L}}$$

When i = 0.4, the relay released:

$$0.4 = 0.5e^{\frac{-60t_r}{L}}$$

Thus:

$$t_r = \frac{27.31ln1.25}{60} \cong 0.1s$$