Statements of Final Task of Computational Fluid Dynamics 1 AE 5011 2025-2026 Semester 1

(The task should be submitted on 4th January 2026 before 23.59 and the presentation will be conducted online on 4th January 2026)

1. Programming

Objective: To obtain numerical solution of Two dimensional Inviscid Burgers equation using finite voleme sith uniform structured grids (each group 2 students)

Inviscid two dimensional Burgers' equation: $\frac{\partial u}{\partial t} + \boldsymbol{a} u \frac{\partial u}{\partial x} + \boldsymbol{b} \frac{\partial u}{\partial y} = 0$

For the domain : $0 \le x \le 1$ and $0 \le y \le 1$ on uniform grid of N x N points, subject to the boundary conditions:

$$u(0, y) = \boldsymbol{c}$$
$$u(1, y) = \boldsymbol{d}$$
$$u(x, 0) = \boldsymbol{c} - (\boldsymbol{c} - \boldsymbol{d})x$$

where ***a,b,c***,and ***d*** are constant

1. Prove the following analytical solution of the above 2D-Burger partial differential equation as case 1 with the coefficients of the equation ***a*** = 1, **b** = 1 and the values of above boundary conditions, **c** = 1,5 and **d** = -0. 5, that is

   For $y \ll 0.5$

   $$u(x, y) = \begin{cases} 1.5 & if\ x \le 1.5y \\ \frac{1.5 - 2.0\,x}{1 - 2y} & if\ 1.5y \le x \ \le (1 - 0.5y) \\ -0.5 & if\ x \ge (1 - 0.5y) \end{cases}$$

   For $y \ge 0.5$

   $$u(x, y) = \begin{cases} 1.5 & if\ x \le (0.5 + 0.5y) \\ -0.5 & if\ x > (0.5 + 0.5y) \end{cases}$$

2. Derive the discrete equation of the 2-Dimensional Burger's equation using finite volume method and its boundary condition
3. Create algorithms for the discrete equation of the Burger's equation using upwind scheme
4. create a program for the domain with grid size (i) N= 21 and (ii) 41 and the numerical solution of the discrete Burger's equation
5. Compute the convergence solution
6. Compare the numerical solution to analytic solution
7. Study for the other cases

Cases	a	b	c	d
2	1	0.5	1.5	- 0.5
3	2	1	1.5	- 0.5
4	1	1	1.0	- 0.6

8. Create the report and presentation