Problem 1

Write the expression for Gamma factorial function.
Solution
The gamma function is an extension of the factorial function,

$$\Gamma(z) = \int_0^1 (\ln \frac{1}{t})^{z-1} dt$$

Problem 2

For which values is this function defined?
Solution
The gamma function is defined for all complex numbers (including all real positive numbers) except the non-positive integers.

Problem 3

Give the recurrence relation which it satisfies.
Solution
If n is a positive integer, the recurrence relation can be expressed as:

$$\Gamma(n) = n\Gamma(n-1)!$$

Problem 4

Using Pascal's triangle, calculate 11^8
Solution

$n = 0$: 1

$n = 1$: 1 1

$n = 2$: 1 2 1

$n = 3$: 1 3 3 1

$n = 4$: 1 4 6 4 1

$n = 5$: 1 5 10 10 5 1

$n = 6$: 1 6 15 20 15 6 1

$n = 7$: 1 7 21 35 35 21 7 1

$n = 8$: 1 8 28 56 70 56 28 8 1

Thus, $11^8 = 214358881$

Problem 5

Write the seventh Tetrahedron number. Show calculation.
Solution
$1+(1+2)+(1+2+3)+(1+2+3+4)+(1+2+3+4+5)+(1+2+3+4+5+6)+(1+2+3+4+5+6+7) = 84.$

Problem 6

Write the statement of Binomial Theorem.
Solution
Binomial is a sum or difference of two terms, eg $a-b$ or $a+b$. The binomial theorem describes the algebraic expansion of powers of a binomial:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Problem 7

Give it's generalisation.
Solution
Let, for an arbitrary n, factorial be defined as:

$$\binom{r}{k} = \frac{r(r-1)\dots(r-k+1)}{k!} = \frac{(r)_k}{k!}$$

Using this, Newton's generalisation can be expressed as:

$$\begin{aligned}(x+y)^r &= \sum_{k=0}^{\infty} \binom{r}{k} x^{r-k} y^k \\ &= x^r + rx^{r-1}y + \frac{r(r-1)}{2!}x^{r-2}y^2 + \dots\end{aligned}$$

Problem 8

Write the expansion of $(1+x)^{-1/2}$
Solution
It can be expressed as an infinite sum series using Newton's expansion:

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \dots$$

Problem 9

Describe the golden angle, and write its value.
Solution
In geometry, the golden angle is the smaller of the two angles created by sectioning the circumference of a circle according to the golden ratio. [Figure 1]

$$GoldenAngle = 360(1 - \frac{1}{\phi}) = 360(1 + (1 - 1) - \frac{1}{\phi})) = 360(2 - \phi) = \frac{360}{\phi^2} = 137.508^\circ$$

Problem 10

Write the Fibonacci coding for:
(i) : 96
Solution : 01010000011
(ii) : 45
Solution : 001010011

Problem 11

Write the Fibonacci decoding for:
(i) : 1001001011
Solution : 82
(ii) : 101010101011
Solution : 232