AUMAT/AUPHY/AUCSC 340 – Numerical Methods

Winter 2019

Assignment 3

Submission deadline: Monday, 04 Mar 2019, 8:30am (ideally in class)

1) Lecture notes, p.113: Prove the last equation on the page, i.e. Simpson's rule, in the following way:
   i) Express the coefficients of
   $$P_2(x) = a_0 + a_1 x + a_2 x^2$$
   in terms of f_{-h}, f_0, f_h.
   ii) Replace $f(x)$ by $P_2(x)$ in the integral.
   iii) Integrate the resulting expression.

2) Use Matlab and employ Monte-Carlo integration to approximate the value of π as best as you can.

3) Download the following lecture notes:
   http://www.math.ubc.ca/~feldman/m256/richard.pdf
   In equation (1), identify A with $f'(x)$ and $A(h)$ with the forward derivative formula for $f'(x)$.
   i) What is the value of k?
   ii) Now, approximate A, and hence $f'(x)$, by equation (4), using equation (3).
   iii) Does the result remind you of something in our lecture notes?

4) Consider the function $g(x) = x^2 + 1.0$ and its values at 3 distinct points:
   $$x_0 = 0,\ x_1 = 1.0,\ x_2 = 2.0\ .$$
   Now, try to use least-square minimization to fit the function
   $$f(x) = ae^{bx}$$
   as best as you can to $g(x)$ at those 3 points.
   i) Write down the function $F(a, b)$ to be minimized.
   ii) Define $w \coloneqq e^b$and re-write $F(a, b)$ as $G(a, w)$.
   iii) Write a (short) Matlab script, using `fminsearch`, to determine the values of a and b that minimize F. Also, plot both $g(x)$ and $f(x)$ into the same plot.
   iv) Is this still a linear problem? (Try setting the partial derivatives of F to zero.)