AUMAT/AUPHY/AUCSC 340 – Numerical Methods

Winter 2019

Assignment 5

Submission deadline: Monday, 15 April 2019, 8:30am (via eClass)

1. Newton's Method:
   Solve
   $$F(x_1, x_2) = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix} = \begin{bmatrix} e^{2x_1 - x_2} - x_1 \\ x_1^2 - x_2 \end{bmatrix} = 0$$
   with Matlab, i.e. find x_1, x_2 so that
   $$\varepsilon = |f_1(x_1, x_2)^2 + f_2(x_1, x_2)^2| \leq 10^{-6}.$$

2. Newton's Method: Finding a minimum ("optimization")
   Consider the function
   $$F(x_1, x_2) = -\ln(1 - x_1 - x_2) - \ln(x_1) - \ln(x_2)\ .$$
   Find the minimum of this function "by hand".

   Next, write a Matlab code that finds the solution to this problem.
   Hint: Solve the necessary condition we have for an extremum, just as you did in the first part of this assignment. Again, stop the iterations when $\varepsilon \leq 10^{-6}$.
   Start with an initial guess $x_1 = 0.85, x_2 = 0.05$.

3. Runge-Kutta Method:
   Use 4th-order Runge-Kutta to solve
   $$y'(t) = y^2 + \sin(t)\ ,\ \ t \in [0; 2],\ \ \ y(0) = 0.1\ .$$
   with time steps h increasingly smaller until $y(2)$ changes by less than 10^{-6}.

4. Implicit Euler:
   Assume you want to solve
   $$y'(t) = y\cos(y),\ \ t \in [0; 1],\ \ \ y(0) = 1.0$$
   with the implicit Euler method. Write down the iterative formula you need to apply (with time step h).
   What issue do you encounter?
   How would you go about it? What initial guess would be appropriate?