Rocket Fin Design: Mathematics and Considerations

ENGR 112 - SP 2021

Inaugural JMU Rocketry Team

<u>How does rocket stability work?</u>

Rocket stability (S) is the ability of a rocket to keep itself upright while flying. The two points on a rocket that determine stability are the **center of pressure (C_p)** and **center of gravity (C_g).** The center of gravity is the approximated point where the force of gravity acts on the vehicle, while the center of pressure is the approximated point where the sum of the forces of pressure on the vehicle act. Below are diagrams that demonstrate the locations of these points in Figure 1:

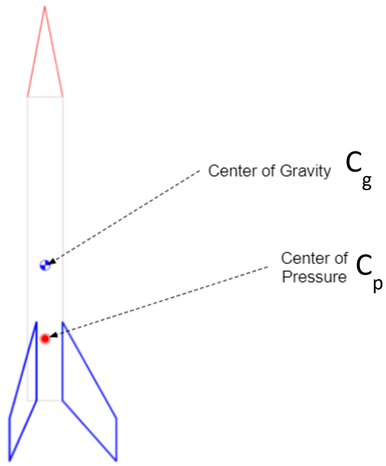

Figure 1. *Center of pressure and center of gravity acting on the rocket.*

For this assignment, the **only** force at the center of pressure that is of concerned is *lift***.** The fins provide a force of “lift” that helps keep the rocket upright. Lift acts as a “restoring force” that keeps the rocket vertical in flight. In addition to lift, rockets experience a *force of thrust* based on the motor within the vehicle and *force of weight* based on the mass of the rocket as illustrated in Figure 2. For this assignment, we will neglect these forces.

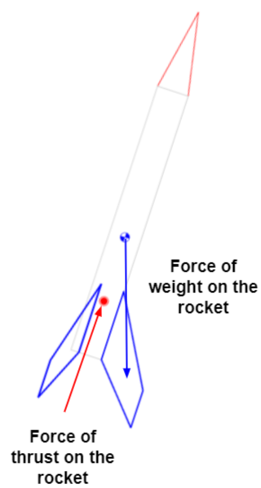

Figure 2. *Force of weight and thrust acting on the rocket.*

Fin size and shape can change the position of the **center of pressure (C_p)**. Think of a rocket as a "lever" or "seesaw". The center of gravity is the point where the rocket pivots. The force of lift works like an individual sitting at one end of the seesaw. The further the points are from one another, the more leverage the force of lift must restore the rocket to a vertical position. If the points are too close, the force of lift does not have enough leverage to restore the rocket to a vertical position. Think about how easy it is to move a seesaw when you are sitting at one end vs sitting near the center; the same thing applies to rocket fins.

Rocket Components

In order to calculate and determine the properties of a desired fin, the equations given in Table 1 must be used to determine the area of the fin.

Table 1: Given equations for fin surface area calculations

Eq. Number	Equation Name	Given Equations
Eq. 1	Stability Margin (S)	$S = (C_p - C_g)/\varnothing$
Eq. 2	Center of Pressure (C_p)	$C_p = \frac{sa_{body} * d_{body} + sa_{fins} * d_{fins} + sa_{cone} * d_{cone}}{sa_{fins} + sa_{body} + sa_{cone}}$
Eq. 3	Cross Sectional Area of 1 Fin (CA_{1fins})	$CA_{1\,fins} = \frac{(sa_{fins})}{(\#\,fins\,*\,2)}$
Eq. 4	Fin Shape Area Equations	$a_{trapezoid} = 1/2 * (t + b) * h$ $a_{rectangle} = l * w$ $a_{triangle} = 1/2\,(b * h)$

It is important to have a good **stability margin (S)** which is defined by the distance between the **center of pressure** (C_p) and the **center of gravity (C_g)**. Based on the desired **stability margin** (**S**) and the **center of gravity (C_g)** of the rocket, the **center of pressure (C_p)** can be determined by using Eq. 1 in Table 1, also outlined below.

$$\textbf{(Eq. 1)}\ S = (C_p - C_g)/\emptyset \qquad \textbf{Stability Margin}$$

If the C_p is in front of the C_g, the rocket will spin out of control. If the C_p is not far enough behind the C_g, the restoring force of the fins will not be strong enough to keep the rocket flying straight. Eq. 1 can be manipulated to solve for **center of pressure (C_p)**:

$$\textbf{(Eq. 1a)}\ C_p = C_g + (S * \emptyset) \qquad \textbf{Stability Margin solved for Center of Pressure}$$

The center of pressure equation includes the surface area of each component on the rocket. For simplicity, the rocket has been split into three components: the **nose cone**, **body tube**, and **fins**.

$$\textbf{(Eq. 2)}\ C_p = \frac{sa_{body} * d_{body} + sa_{fins} * d_{fins} + sa_{cone} * d_{cone}}{sa_{fins} + sa_{cone} + sa_{body}} \qquad \textbf{Center of Pressure}$$

The surface area of the fins is the value that is desired from this calculation. In order to solve for the fin surface area, the variable, sa_{fins}, needs to be isolated algebraically from **Eq. 2** in Table 1. The manipulated equation is shown below:

$$\textbf{(Eq. 2a)}\ sa_{fins} = \frac{sa_{body}*d_{body} + sa_{cone}*d_{cone} - C_p*sa_{cone} - C_p*sa_{body}}{C_p - d_{fins}}$$

Once sa_{fins} is solved for, this value can be used in **Eq. 3** from Table 1 to determined what the cross-sectional area of a single fins would be. The cross-sectional area of a fin is the value of the trapezoidal face of the fin. Essentially, it is the thickness multiplied by the height. The cross-sectional area of the fins is dependent upon the number of fins.

$$\textbf{(Eq. 3)}\ CA_{1\,fins} = \frac{(sa_{fins})}{(\#\,fins * 2)}$$

Once the cross-sectional area is determined, the fin area can be determined based on the desired fin shape. Area formulas have been provided above in **Eq. 4** of Table 1.

The fins that are sketched out must have a surface area equivalent to the surface area that you solved for. Note that the fins that have been created are 3 dimensional objects. Both the cross-sectional areas as well as the thickness of the fins must be addressed. Most fins can be broken up into simple geometric shapes such as rectangles, triangles, and trapezoids. While creating your fin, use the formulas in Table 1 to ensure it has the proper surface area.

Rocket Diagram

The diagram below, in Figure 3, is meant to help define the position of some of the rocket components which you will need while working with the surface area formula in this assignment. Each measurement is given as the distance between the **center of gravity (C_g)** of a specific rocket component and a reference point situated at the tip of the nose cone. (Note: These individual **center of gravity (C_g)** values are only used to find the position of the components and are NOT meant to be used in the stability margin formula).

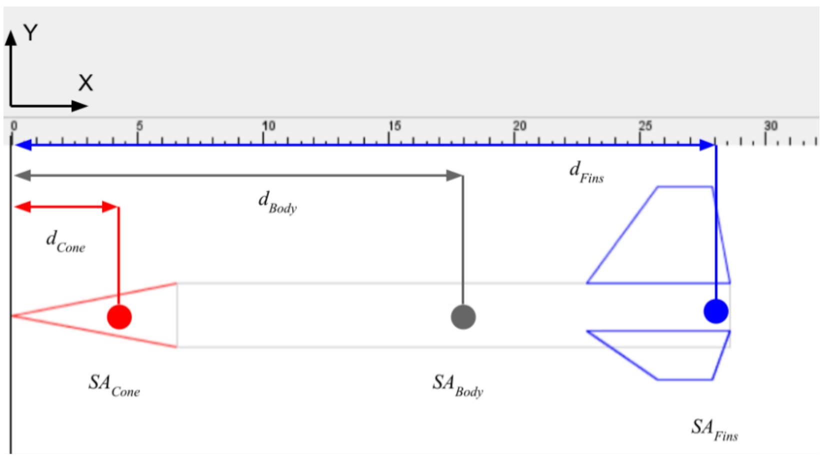

Figure 3. *Rocket Diagram with reference distances and components labeled*

Distances on the rocket are measured from a reference point. Typically, this is the tip of the nose cone. On this diagram, the distances are labeled in Table 2 and are measured from the tip of the nosecone to the specific point on the rocket.

Table 2: Reference distances based on Figure 3

d_{body}	Distance of the C_g of the body from a reference point	18 cm = 0.18 m
d_{cone}	Distance of the C_g of the nose cone from a reference point	4 cm = 0.04 m
d_{fins}	Distance of the C_g of *all* the rocket's fins from a ref point	28 cm = 0.28 m

Additionally, the surface area of the specific parts on the rocket are labeled on the rocket diagram (Figure 3), as seen in Table 3. Knowing the surface area of each component is important for calculations involving drag and other aerodynamic calculations.

Table 3: Surface area values of rocket components

sa_{body}	Surface area of the body	384.85 cm^2 = 0.038485 m^2
sa_{cone}	Surface area of the cone	75.07cm^2 = 0.007507 m ^2
sa_{fins}	Surface area of the fins	**You will calculate this, as described earlier**

The diameter of the rocket represents the outer diameter of the main body of the rocket and the nose cone. Rocket diameter is important in many aerodynamic calculations, especially in determining the center of pressure. The **stability margin (S)**, the **center of pressure (C_p)**, and the **center of gravity (C_g)** are also important in these calculations and are all represented in Table 4.

Table 4: Stability values and symbol

⌀	Diameter of the Rocket	2.48 cm = 0.0248 m
S	Stability Margin	Based on your JAC
C_g	Center of Gravity	Based on your JAC
C_p	Center of Pressure	**Find (Eq. 1)**

The **stability margin (S)** and **center of gravity (C_g)**, as noted above in Table 4, is based on your JAC ID number for your assignment. The following table, Table 5, indicates what value you will use for these variables.

Table 5: Stability Margin and Center of Gravity Based on JAC

Last # of JAC Student ID	**Stability Margin (S)**	**Center of Gravity (C_g)**
1 - 3	2.5	0.188 m
4 - 6	2.3	0.190 m
7 - 9	2.0	0.200 m

Lastly, the fin cross-sectional area and number of fins will be determined, as seen in **Eq. 4** in Table 1. The following section describes the importance of shape in your fin design.

Fin Design Shape

Shape is also very important. Some shapes are more effective than others. Fins receive a massive amount of stress in flight. Ideal fins have thicker sections towards the body and shorter sections towards the tip of the fin. This provides strength while reducing induced drag. The dimensions of a trapezoidal fin are shown below in Figure 4.

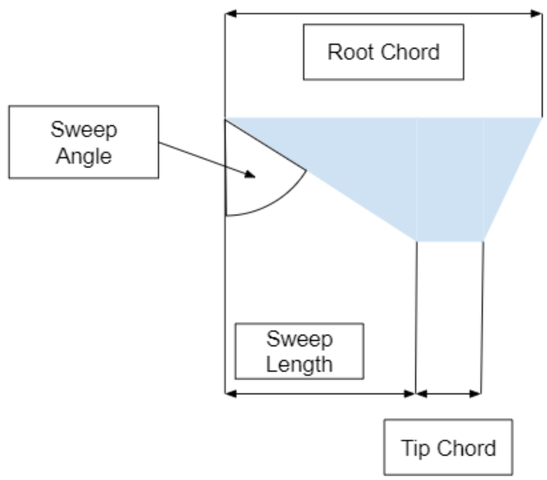

Figure 4. *Rocket Fin Dimensions*

The dimensions do not need to be used to find the surface area but can be used to help model the fin in software like SolidWorks or Open Rocket. The SolidWorks "smart dimension" function can be used to apply these dimensions to your part. Depending on the desired shape of your fin, these dimensions could help speed up the process of creating a sketch to extrude into a full fin. These values can be found using area formulas, or you can verify certain values using the formulas at your disposal. There are a number of different types of trapezoidal fins, but the specific shape is often negligible so long as the tip chord is smaller than the root chord and it has a sweep length greater than 0. Square or axe-head fins are generally not ideal. These shapes usually are not very strong and cause a lot of induced drag. Root chord is never smaller than tip chord and neither have a sweep length greater than zero. Thus, these fins should not be used.

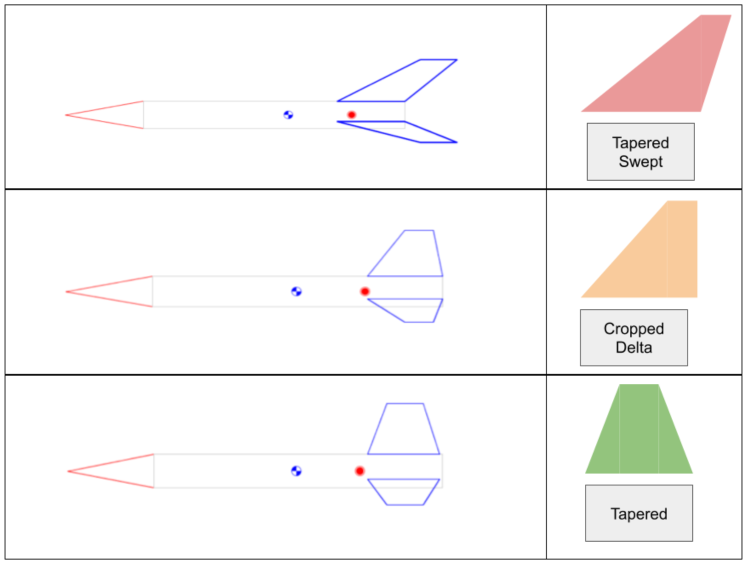

Figure 5: *Trapezoidal Fin Shapes*

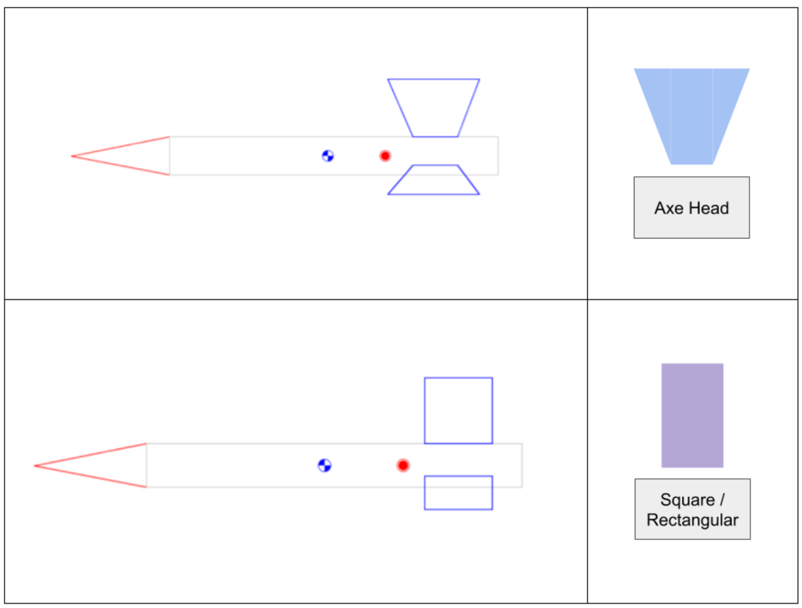

Figure 6: *Other Fin Shapes*

Choosing the number of fins to put on a rocket affects the stability of the rocket. For your design, you can choose **any number of fins you would like**, but keep in mind that the design should be **feasible** to create and manufacture. It is up to you as a designer to determine what "feasible" is. You can see in your code how the number of fins affects other values in the MATLAB code.

Summary

Below, in Table 5, is a summary of all the given variables and their values.

Table 5: Given variables for fin surface area calculation

Variables and Givens for Model Rocket		
Code	**Description**	**Value**
$f_{thickness}$	Fin thickness	0.3 cm = 0.003 m
S	Stability	**Based on JAC**
C_p	Center of Pressure	**You will calculate this**
C_g	Center of Gravity	**Based on JAC**
⌀	Diameter of the Rocket	2.48 cm = 0.0248 m
sa_{body}	Surface area of the body	384.85 cm^2 = 0.038485 m^2
sa_{cone}	Surface area of the cone	75.07 cm^2 = 0.007507 m^2
sa_{fins}	Surface area of the fins	**You will calculate this**
d_{body}	Distance of the body CG from ref point	18 cm = 0.18 m
d_{cone}	Distance of the cone CG from ref point	4 cm = 0.04 m
d_{fins}	Distance of fins CG from ref point	28 cm = 0.28 m
#fins	Number of fins	**You will select how many fins**
CA1fin	Cross sectional area of a fin	**You will calculate this**