Effects on Q-Factor of Scaled-Up CPW Resonator

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Abstract

FIG. 1. A diagram of a CPW resonator. The strip width w, spacing s, relative permittivity ϵ_r of the substrate, thickness of the substrate h, and length of the resonator l all factor into the design.

INTRODUCTION

Transmission line resonators (TLRs) including coplanar waveguide (CPW) resonators are finding novel use in the field of quantum computing. Some useful applications include inter-qubit information transfer and information storage. TLRs are already available in the commercial market as high frequency filters. The goal of this experiment was to construct a scaled up version of a capacitively coupled CPW resonator on an FR-4 substrate and study the effects of liquid nitrogen temperatures on the quality factor. High Q resonators of 10^6 are achievable on silicon substrates using superconducting aluminium or niobium.

DESIGN

A CPW can be designed using four factors, the relative permittivity of the substrate ϵ_r, the thickness of the substrate h, the width of the transmission line s, and the space between the transmission line and ground planes w. Often the latter two are combined into a ratio $k = {}^{w}/_{(2s+w)}$. Additionally, a resonator is designed around the length l of the resonant element and the coupling capacitors' gap width g. The resonator allows standing electromagnetic waves at the resonant frequency. Figure 1 is a diagram of a basic capacitively coupled CPW resonator. A loosely coupled ground plane (not shown) on the back of the substrate helps to reduce interference by confining the electric field to the substrate. It must be loosely coupled to the top ground planes to reduce the capacitance leakage and thus deviation from the existing formulas. These factors are used to find the characteristics of a CPW resonator including resonant frequency ω_n and characteristic impedance Z_0.

NUMERICAL APPROXIMATIONS

Very good approximations of Z_0 can be found using electromagnetic simulation software. Approximations can be made using relatively simple equations found in microwave circuit design handbooks. They tend to be less exact but faster.

For materials that do not have a well known ϵ_r, it can be measured with the capacitance C, surface area A, and the distance between plates d, of a parallel plate capacitor.

$$\epsilon_r = \frac{Cd}{A\epsilon_0}$$

This method does not lend itself well to determining ϵ_r for the simple reason that capacitance is difficult to measure accurately using simple equipment. For handheld LCR meters, the capacitance is measured using the impedance and/or comparing the phase angle between the voltage and current of an AC signal through a capacitor. The capacitance can change significantly for frequency dependent permittivity materials such as the FR-4 used in this experiment. Also, a physically large plate must be used to get an accurate measurement which presents a cost issue. Nevertheless, measuring capacitance is a simple and quick way to estimate the permittivity of a substrate for scaled up CPW resonators.

An alternative method, and the one most often used, is to make a resonator structure on the substrate in question and measure the resonance frequency. This method works well for homogeneous substrates such as silicon with a defined impurity content. The permittivity of various grades of silicon is well documented already. For FR-4 however, the resin content and formula varies widely among manufacturers and even among similar grades from the same manufacturer. In order to accurately predict permittivity using this method, the exact same board would have to be used for the initial estimate and final experiment.

The standing EM wave is present in equal part through the substrate and through the environment surrounding it so the permittivity of the substrate and the environment must be averaged to get the effective permittivity. A more in depth analysis provided by B. Wadell[2] shows that the construction effects ϵ_{eff} by concentrating the field in certain areas. The environment is assumed to be vacuum which results in negligible error if air is used.

FIG. 2. The electric (a) and magnetic (b) field distribution of a CPW. *Note: no ground plane backing is shown.[2]

$$\epsilon_{eff} = \frac{1 + \frac{\epsilon_r K(k') K(kl)}{K(k) K(kl')}}{1 + \frac{K(k') K(kl)}{K(k) K(kl')}} \tag{1}$$

$$kl = \frac{\tanh\left(\frac{\pi}{4}\frac{s}{h}\right)}{\tanh\left(\frac{\pi}{4}\frac{2s+w}{h}\right)}$$

$$k' = \sqrt{1 - k^2}$$

$$kl' = \sqrt{1 - kl^2}$$

Where K is the Elliptical Integral $K(k)$. From the effective permittivity, the characteristic impedance can be calculated.

$$Z_0 = \frac{60\pi}{\sqrt{\epsilon_{eff}}} \frac{1}{\frac{K(k)}{K(k')} + \frac{K(kl)}{K(kl')}} \tag{2}$$

The more traditional way of defining characteristic impedance is using inductance and capacitance per unit length, L_l and C_l respectively.

$$Z_0 = \sqrt{\frac{L_l}{C_l}} \tag{3}$$

$$L_l = \frac{\mu_r \mu_0}{4} \frac{K(k')}{K(k')}$$

$$C_l = 4\pi \epsilon_{eff} \epsilon_0 \frac{K(k)}{K(k')}$$

Equation (1) is still used to determine ϵ_{eff} for the capacitance per unit length. Equation (3) produces more useful results than Equation (2) when considering higher modes of resonance. Subsequently higher overtones are allowed-characterized by the resonance mode number n. The fundamental mode has $n = 1$ and was designed to establish a standing half-wave (two nodes and one anti-node) at 5 GHz using the capacitance method of estimating ϵ_r.

Figure 3 shows the lumped circuit equivalents in a CPW resonator. From the inductance and capacitance per unit length, the equivalent total inductance L_n and capacitance C can be calculated using the length of the resonator. It is worth noting that only the inductance depends on the mode number.

$$\omega_n = \frac{1}{\sqrt{L_n C}} \tag{4}$$

$$L_n = \frac{2 L_l l}{n^2 \pi^2}$$

$$C = \frac{C_l l}{2}$$

The capacitance C_k for a series gap in a CPW according to W.J. Getsinger[3] is

$$C_k = \frac{2\epsilon_0 \epsilon_{eff} w}{\pi} \left(p - \sqrt{1 + p^2} + \ln\left(\frac{1 + \sqrt{1 + p^2}}{p} \right) \right) \tag{5}$$

Where $p = {}^{g}/_{4w}$ and $g, W \ll \lambda$.

$$R^* = \frac{1 + \omega_n^2 C_k^2 R_L^2}{\omega_n^2 C_k^2 R_L} \tag{6}$$

$$C^* = \frac{C_k}{1 + \omega_n^2 C_k^2 R_L^2} \tag{7}$$

$$Q_L = \frac{C + 2C^*}{\sqrt{L_n (C + 2C^*)} \left(\frac{1}{R_l} + \frac{2}{R^*} \right)} \tag{8}$$

LOSSES

The losses can be split into two groups: conductor loss and dielectric loss. The conductor losses result from real resistance of the conductive material that carries the signal. The dielectric losses are a result of the resistance of the substrate to electromagnetic waves.

Conductor

All conductors have some resistance R. For the simple case of a DC source the resistance is

$$R = \frac{\rho d}{A}$$

Where ρ is the intrinsic resistivity, A is the cross-sectional area, and d is the length of the conductor. In the case of an AC signal, currents begin to crowd toward the outside of a conductor. This is called the skin-effect. The penetration depth δ effectively limits the cross-sectional area that current can travel in, thus increasing the resistance with higher frequency. In the case of a CPW, the total resistance becomes

$$R = \frac{\rho_c d}{w\delta}$$

$$\delta = \sqrt{\frac{2\rho_c}{\omega_n \mu_0}}$$

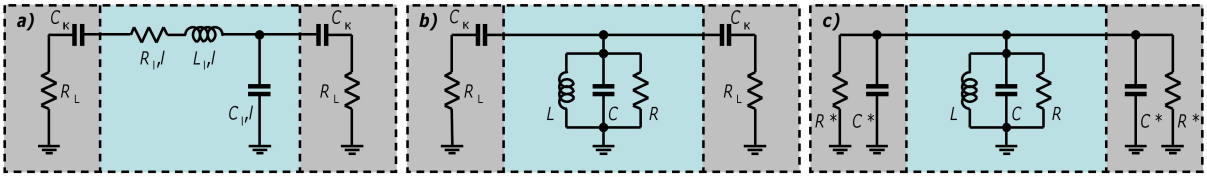

FIG. 3. Lumped circuit equivalents of a CPW resonator. (b) Parallel LCR oscillator representation of TL resonator. (c) Norton equivalent of symmetrically coupled, parallel LCR oscillator. Variables are defined in text. [1]

The conductor also suffers from losses due to surface roughness. Most PCB manufacturers purposefully increase the roughness of their conductors to increase adhesion to the substrate. The surface roughness increases the distance the current has to travel through the conductor and thus increases the resistance. For operation in the low frequency range (up to a few hundred MHz) and on typical PCB environments skin depth penetrates the entire conductor and can therefore be neglected. The greater the frequency, however, the greater the effect the surface roughness has on resistance.

$$R_l = \frac{\omega_n Z_0}{\alpha} \tag{9}$$

$$\alpha = \frac{R}{2L_n}$$

Dielectric

The losses due to the dielectric center around the loss tangent $\tan(\delta)$. The loss tangent limits the quality factor.

$$Q \ll \tan(\delta) \tag{10}$$

APPARATUS AND MEASUREMENT SCHEME

The quality factor is determined by measuring the resonant frequency f_0 and width of the peak at half power Δf.

$$Q = \frac{f_0}{\Delta_f} \tag{11}$$

A network analyser was used to scan frequencies from 100 MHz to 20 GHz. The signal power was 25 Dbm and the signal was averaged ten times. Peaks were then identified and centered with a ± 1GHz spread and exported.

RESULTS

Results

DISCUSSION & CONCLUSIONS

Goppl et al. indicates that the Q-factor is highly dependent on power applied to the resonator.[1] The resonator must be driven in a small range of power limited by non-linear effects above and dielectric losses below. This effect should be even more pronounced in non-superconducting materials with real resistance.

A further study is warranted for the effect of water absorption in FR-4 on dielectric losses. Even a small amount of water can vastly effect the dielectric loss by increasing the effective loss tangent. Because the loss tangent of water is well defined at 0.157, all that one needs to find is the water concentration to determine the dielectric loss due to water. Perhaps drying the board in a low temperature oven would be sufficient for a baseline reading of the loss tangent. I am sure that some water was present in my board. The room temperature data was taken after immersion in liquid nitrogen. The cold environment probably saturated the board with water increasing the loss tangent.

Equation (2) was originally used to design the resonator and optimized for $Z_0 = 50\Omega$. Equation (3) produced a characteristic impedance closer to 62Ω. A closer look at the data may reveal which equation is most accurate and useful for further investigation.

Dispersion may account for the small decrease in resonant frequency at the second mode. Goppl et al. also indicates that the gap capacitance is frequency dependent resulting in a frequency shift. The shift comes from a change in the permittivity of the substrate which also effects the characteristic impedance. Source Forge shows that ϵ_r increases asymptotically with frequency resulting in a decrease in characteristic impedance. Any mismatch between input and output impedance will cause reflections and reduce the overall quality.

After completing the analysis I noticed that I did not take into account the change in permittivity of the environment when the circuit was immersed in liquid nitrogen. The permittivity is increased to 1.538 ± 0.025 [**?**] which would increase the effective permittivity by about 25%. It is worth noting the existence of radiation losses in a CPW resonator. The broad spectrum loss due to mismatched impedances is the most damaging. Even in the case of a perfectly designed CPW resonator, half of the signal is lost because the circuit behaves as a simple voltage divider. Even still, there are some losses associated with discontinuities in the microwave circuit. Radiation loss does not deviate much over the range tested so it can be assumed constant. Radiation losses over the range tested in this experiment are negligible compared to dielectric and conductor loss as well.[4] With computer clock speeds in the multi-gigahertz range, it is no wonder why CPU design is such a closely guarded secret. The shear volume of calculations required to ensure proper operation of the hundreds of thousands of transistors in every CPU is astonishing. I barely made it through a single circuit.

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[1] Goppl et. al.
[2] Transmission Line Design Handbook
[3] W. J. Getsinger, "End-Effects in Quasi-TEM Transmission Lines." *IEEE Transactions on Microwave Theory and Techniques*. vol. 41, no. 4, pp. 666-671, Apr. 1993.
[4] "Insertion Loss and Loss Tangent"
[5] "Single Microstrip Line"
[6] http://www.sigcon.com/Pubs/edn/SurfaceRoughness.htm