Midterm 1A

CSE 101 Spring 2022

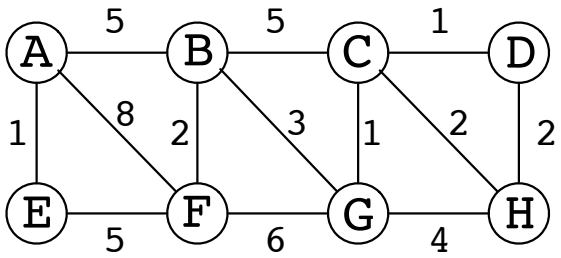
Name: _____

Student ID: _____

Section (Russell/Sanjoy): _____

INSTRUCTIONS: Be clear and concise. Write your answers in the space provided. Use the backs of pages for scratchwork.

1. **(10 points)** Consider the following graph with edge weights.



(a) Give distances from node E to all other nodes.

A	B	C	D	E	F	G	H

(b) Show the corresponding shortest-path tree.

2. **(10 points)** For each of the following statements, say whether it is TRUE or FALSE. No explanation is needed.

   (a) $2^{2n} \in \Theta(2^n)$

   (b) $7n \log n + 20n \in \Theta(n \log n)$

   (c) If $T(1) = 1$ and $T(n) = T(n-1) + O(1)$ for $n \geq 1$, then $T(n) \in O(n)$

   (d) If f and g are functions from positive integers to positive integers, and $f(n) \in O(g(n))$, then $f(n)^2 \in O(g(n)^2)$

   (e) For any directed acyclic graph G, $|E| \in O(|V|)$

3. **(10 points)** A subsequence of a word is one that can be obtained by deleting some characters and listing the remaining characters in the same order. For example, $MATH$ is a subsequence of $AMATEURISH$ by keeping the second, third, fourth and last characters, but is not a subsequence of $ARITHMETIC$ because the only H comes before the only M. Here is an algorithm that, given two words $u_1 \cdots u_n$ and $v_1 \cdots v_m$, decides whether $u_1 \cdots u_n$ is a subsequence of $v_1 \cdots v_m$.

   Subsequence($u[1 \ldots n], v[1 \ldots m]$: words)

   (a) $I \leftarrow 1, J \leftarrow 1$
   (b) While $I \leq n$ and $J \leq m$ do:
   (c) While $J \leq m$ AND $v[J] \neq u[I]$ do: J++
   (d) If $J \leq m$: I++; J++
   (e) IF $I > n$: return $True$
   (f) Return $False$

   Give a time analysis, up to order, for this algorithm. Be sure to explain your answer.

4. **(20 points)** Explain how we can modify or use one of the graph algorithms from class to solve the following problem.

   > Given an undirected graph G, give a minimum sized set of edges $e_1, \ldots, e_k$ so that adding $e_1, \ldots, e_k$ to G causes G to become connected. (If G is already connected, you should return the empty set.)

   (5 points correct algorithm, 5 points correctness proof, 5 points efficiency, 5 points time analysis)

   (a) Give an algorithm for this problem.

   (b) Justify the correctness of your algorithm.

   (c) State, with justification, the running time of the algorithm.