Test 1 - CSE 101

October 24, 2024 , 12:30pm-1:50pm

Do not turn the page until you are instructed to do so.

- You may use a one page (both-sided) notesheet. You may not use any electronic devices, or any other form of assistance during this exam except for a calculator (no phones.)
- If you have a question, please remain seated. Raise your hand and wait for assistance.
- Show your work. To receive full credit, your answers must be neatly written and logically organized.

#	Points	Score
1	6	
2	6	
2	13	
Total	25	

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

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1. True and False: (Circle TRUE or FALSE for each statement. No justification necessary.) (each problem is 1 points.)

   (a) (1 points) Let G be a directed acyclic graph with exactly one source and exactly one sink. Then there is a path from the source to the sink.

   True **False**

   (b) (1 points) Let G be a directed graph and consider a vertex s. Let G' be a copy of G but with all the edge-lengths set to 1. If I run BFS on G starting from s and Dijkstra's on G' starting from s then the resulting dist values of the two algorithms will be the same for all vertices.

   True **False**

   (c) (1 point) Circle only one of the choices
   $f(n) = 2^n$, $g(n) = 4^{\sqrt{n}}$

   $f(n) = o(g(n))$ $f(n) = \Theta(g(n))$ $g(n) = o(f(n))$

   (d) (1 point) Circle only one of the choices
   $f(n) = \log_2(2n)$, $g(n) = \log_4(n^2)$

   $f(n) = o(g(n))$ $f(n) = \Theta(g(n))$ $g(n) = o(f(n))$

   (e) (1 points) If G is an undirected graph with distinct positive edge weights and you run Prim's on G starting from a random vertex s, then the last edge to be selected by Prim's must be the heaviest edge of the MST.

   True **False**

   (f) (1 points) If G is an undirected graph with distinct positive edge weights and you run Kruskal's on G, then the last edge to be selected by Kruskal's must be the heaviest edge of the MST.

   True **False**

2. (6 points)

You are given an undirected connected graph G with distinct positive edge lengths along with a starting and ending vertex: (s and t.) You wish to find the length of the shortest path from s to t.

High-level description:
Run Prim's on G starting from s.
Identify the path p from s to t in the output tree.
return the length of p.

Show that this algorithm does not always work by providing a counter-example, showing the result of the algorithm on the counter-example, then identifying a better output.

(2 points for counter-example)
(2 points for correct result of the algorithm on the counter-example.)
(2 points for identifying a better output.)

(scratch paper)

3. (13 points) You are given a *directed graph* $G = (V, E)$ with vertex weights $w(v)$ that are either $0, 1$ or 2.

   Design a reasonably efficient algorithm that returns the length of the shortest path among all paths that start with a vertex of weight 1, end with a vertex of weight 2 and all intermediate vertices in the path have a weight of 0. (Return ∞ if there is no such path.)

   (6 points for algorithm description. High-level is required, implementation-level or pseuodocode is optional.) (3 points for efficiency [contingent on correct algorithm]) (4 points runtime justification.) [NO PROOF OF CORRECTNESS NECESSARY]

Algorithm Description

Runtime Analysis

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