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Self-Organizing Multiagent Approach to Optimization in Positioning Problems

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Abstract. The facility positioning[3] problem concerns the location of facilities such as bus-stops, fire stations, schools, so as to optimize one or several objectives. This paper contributes to research on location problems by proposing a reactive multiagent approach. Particularly, we deal with the p-median problem, where the objective is to minimize the weighted distance between the demand points and the facilities. The proposed model relies on a set of agents (the facilities) situated in a common environment which interact and attempt to reach a global optimization goal: the distance minimization. The interactions between agents and their environment, which is based on the artificial potential fields approach, allow us to locally optimize the agent's location. The optimization of the whole system is then obtained from a self-organization of the agents. The efficiency of the proposed approach is confirmed by computational results based on a set of comparisons with the k-means clustering technique.

1 INTRODUCTION

The facility positioning problems have witnessed an explosive growth in the last four decades. As Krarup and Pruzan [8] point out, this is not at all surprising since location policy is one of the most profitable areas of applied systems analysis. This is due to the importance of location decisions which are often made at all levels of human organization. Then, such decisions are frequently strategic since they have consequential economic effects.

The term facility is used in its broadest sense. It refers to entities such as bus-stops, schools, hospitals, fire stations, etc. The general problem is, then, the location of new facilities to optimize some objectives such as distance, travel time or cost and demand satisfaction.

However, positioning problems are often extremely difficult to solve, at least optimally (often classified as NP-Hard). There have been works based on genetic algorithms, branch and bound, greedy heuristics, etc. These approaches are not easily adapted for dynamic systems where the system constraints or data change. This is a real limitation since most of real problems are subject to change and dynamics. To deal with this lack of flexibility and robustness, we adopt a multiagent approach which is known to be well suited for dynamical problems [5].

This paper proposes a multiagent approach for the facility location problem, which is based on the self-organization of reactive agents. To our knowledge, no reactive agent-based approaches have been already used to deal with this problem. The choice of a multiagent approach provides several advantages. First, multiagent systems are well suited to model distributed problems. In such systems, several entities evolving/moving in a common environment have to cooperate to perform collective and local goals. Second, even if the multiagent approach does not guarantee to find optimal solution, it is, often, able to find satisfying ones without too much computational cost [19]. Through this paper we show that the reactive multiagent approach can be an interesting new way for optimization in positioning problems. Then, it provides satisfying solutions in addition to other assets as flexibility, modularity and adaptability to open systems. In our approach, agent behavior is based on the combination of attractive and repulsive forces. The idea is that the behavior of agents at the microscopic level leads to emergence of solutions at the macroscopic level [12].

This paper is structured as follows: section 2 presents the facility location problems. Then, section 3 details the proposed multiagent approach. Section 4 presents experimental evaluations through comparisons with the k-means approach. In section 5, some aspects of the model are discussed. Then, the last section gives some conclusions and perspectives.

2 THE POSITIONING PROBLEM

2.1 Overview

In the literature, the general facility positioning problem consists in locating new facilities to optimize some objectives such as distance, demand covering, travel time, cost.

There are four components that characterize location problems [14]: (1) a space in which demands and facilities are located, (2) a metric that indicates distance (or other measures as time) between demands and facilities, (3) demands, which must be assigned to facilities, and (4) facilities that have to be located. There exists two types of location problems: continuous and discrete ones. The problem is continuous when the facilities to be sited can generally be placed anywhere on the plane or on the network. In discrete location problems the facilities can be placed only at a limited number of eligible points.

A non-exhaustive list of facilities problems includes: p-center, p-median, set covering, maximal covering, dynamic location, stochastic location and multiobjective location problems. This paper focuses, particularly, on the p-median problem. The mathematical formulations of the previous variants are well known. However, formulating is only one step of analyzing a location problem. The other step and the most challenging one is to find optimal solutions.

Typically, the possible approaches to such a problem and especially to the p-median problem, consist in exact methods which allow to find optimal solutions. A well-known example of methods is

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[3] Deployment, location and siting are used as synonyms

branch and bound [18]. However, these solutions are quickly inefficient for very complex problems, i.e. with hundreds of constraints and variables. Then obtaining optimal solutions for these problems requires colossal computational resources.

Another category of methods are proposed for the p-median problem. These methods, known as heuristics, allow to find good solutions, but do not guarantee finding the optimal one(s): greedy heuristics [2], genetic algorithms [6], lagrangean relaxation [3], etc.
However, these approaches have several drawbacks such as the rigidity, the lack of robustness and flexibility, the computational cost (huge population size and long convergence time, for example in genetic algorithms). Particularly, these approaches are limited in their ability to cope with dynamic problems characterized by the change of problem constraints and optimization criteria.

This paper explores another possible heuristic which is based on multiagent systems. The remainder of the paper will focus on this approach. The next section presents formally the continuous p-median problem.

2.2 Continuous p-median problem statement

In the rest of the paper, the continuous p-median problem is considered. It consists to locate a fixed number of facilities such that the whole environment can be used. The objective is to minimize the distance between demands and facilities.
The problem is expressed as follow [13]:
E = the set of demand points in the plane $\Re^2$ (or more generally $\Re^n$) indexed by e
W_e = a positive weight assigned to each demand
p = the maximum number of facility lo locate
$d(x, e)$= the distance between the facility x and the demand e
The problem is to find a subset X of p facility locations within a feasible region $S \subset \Re^2$, such that:

$$\min_{X \subset S; |X|=p} F_E(X) \tag{1}$$

$$F_E(X) = \sum_{e \in E} W_e . \min_{x \in X} d(x, e)$$

The objective function (1) minimizes the weighted sum of distances of the demand points to their closest facility.

3 A SELF-ORGANIZATION APPROACH FOR THE CONTINUOUS P-MEDIAN PROBLEM

Our model relies on the Artificial Potential Fields (*APF*) approach which is a possible manner to build self-organized systems. This approach is presented in the next section, the proposed model is detailed in section 3.2.

3.1 The artificial potential fields approach

Self-organization exists in many natural systems and especially in insect societies. Such systems are composed of simple entities, for instance ants, which can build tri-dimensional structures or solve complex problems without any global control [11]. Their organization results from the numerous interactions between agents and their environment. It is the environment that guides the agent behaviors and the whole system organization (called stigmergy principle) [12].

Such an approach has been used to define decentralized algorithm to deal with path finding problems (ant algorithm [4]), collective tasks (such as boxpushing [1], navigation [15], foraging with robots), etc. Most of these works are based either on digital pheromones (as inspired by ants) or on artificial potential fields (APF). We adopt this second one because it is well suited to deal with spatial constraints, as it is the case in the p-median problem.

This APF approach has several inspirations (physical, biological, etc). The concept was introduced in Lewin's topological psychology [9]. The basic idea is that human behavior is controlled by a force field generated by objects or situations with positive or negative values or valences. During the past decade, potential field theory has gained popularity among researchers in the field of autonomous robots [7] and especially in robot motion planning thanks to their capability to act in continuous domains in real-time. By assigning repulsive force fields to obstacles and an attractive force field to the desired destination, a robot can follow a collision-free path via the computation of a motion vector from the superposed force fields [1]. In [16], artificial potential fields are used to tackle cooperation and conflict resolution between situated reactive agents.

However, the APF technique is limited by a well known drawback: local minima. Indeed, adding attractive and repulsive fields can produce areas where forces are equilibrated. Then, an agent that uses potential fields to move can be trapped in such places. The originality of our approach relies on the fact that we do not try to avoid such local minima. At the opposite, we exploit them as interesting places where facilities are located at the balance of different influences.

3.2 A self-organizing multiagent model

As facilities are elements to be placed in the environment, we model them as reactive agents. The environment is defined by a finite and continuous space. Demands, which are static data of the problem, are defined as an environment characteristic.

As in the reactive multiagent methodology proposed in [17], our approach consists to, first, define the behavior of a single agent (facility) so as to optimize its position considering the perceived demand. Second, we consider interactions between agents, to obtain the collective problem solving.

3.2.1 Local demand satisfaction

We first define the behavior of an agent which must minimize its distance to the perceived demand. The key idea is that demand induces attraction forces which are applied on the agent. Considering one demand point, an attractive force is defined from the agent towards the demand. It is expressed as a vector the intensity of which is proportional to the demand weight and to the distance between the agent and the demand. Formally, for an agent A perceiving a demand D with weight W_D

$$\vec{F}_{D/A} = W_D \; . \; \overrightarrow{AD} \tag{2}$$

The influence of the attraction decreases when the agent moves towards the demand. Thus, if the agent attains the demand the attraction behavior is inhibited.

For the set of perceived demands, the influence on an agent is defined as the sum of all induced forces. Formally, the local attraction force undergone by an agent A is computed as follows:

$$\vec{F}_{demands/A} = \frac{\sum_{i=1}^{n} \vec{F}_{i/A}}{n} \tag{3}$$

n is the number of demands perceived by the agent A through its attraction radius r_a (n = 5 in Fig.1). The demand is indexed by i.

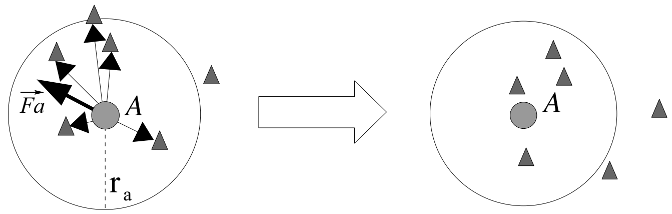


Figure 1. Attractions lead the agent to the weighted barycenter of demands

As a consequence the agent moves to the weighted barycenter of the demands, which is known to approach the minimum average distance to several close weighted points [10, 13]. For example, if an agent is subject to two attractive forces (from two different demands), it will be more attracted towards the biggest demand. Then, it will move towards a balance point. This point is defined as the place where the two attraction forces are equilibrated.

Now, we have to consider several agents applying such a behavior. Then, some of them could move to the same locations. In such a case the process is sub-optimal since several agents cover the same demand. To prevent such a process repulsive forces are introduced to the model.

3.2.2 Local coordination

In order to avoid that agents have the same locations, we introduce repulsive forces between them. It concerns close agents, i.e. situated under a particular distance, defined as the repulsion radius (r_r in Fig.2).

The force intensity is defined as inversely proportional to the inter-agent distance (see Fig.2).

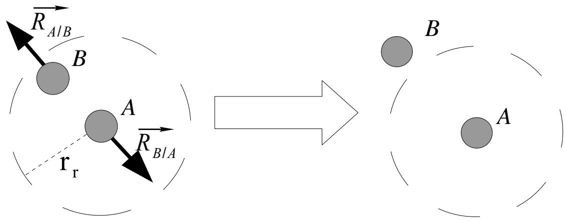


Figure 2. Repulsions between agents *A* and *B* lead them to keep away

Formally the repulsive force induced by an agent *B* on an agent *A* is expressed as follow:

$$\vec{R}_{B/A} = \frac{\overrightarrow{BA}}{\left\|\overrightarrow{AB}\right\|^2} \tag{4}$$

Then, the local repulsive force undergone by an agent *A* is computed as follows:

$$\vec{R}_{agents/A} = \frac{\sum_{j=1}^{m} \vec{R}_{j/A}}{m} \tag{5}$$

m is the number of agents perceived by the agent *A*. These agents are indexed by *j*. Fig.2 illustrates this repulsive process between two agents.

This repulsion process allows the coordination of agents while moving to the demand (see next section). Moreover, such repulsive forces can allow to respect constraints on minimal distances separating facilities (constraint present is many facility location applications).

3.2.3 Collective solving

The agent behavior is defined as the weighted sum of both local attraction and repulsion forces. Formally, for an agent *A*, it is expressed as follows:

$$\overrightarrow{Move} = \alpha\, \vec{F}_{demands/A} + (1-\alpha)\, \vec{R}_{agents/A} \tag{6}$$

The coefficient α allows us to favour either the attraction or the repulsion.

We now consider the whole system, where several facilities must optimize their positioning to cover numerous demands. In the self-organizing approach, no global control is used. Agents are created and distributed in the environment and act following the defined individual behavior.

To implement the proposed multiagent model, we can (i) assign a thread to each agent or (ii) define a scheduler that simulates the parallel agents computation. We adopt the second solution which is generally used for reactive agents implementation. Finally, the collective solving process is presented in Algorithm 1. The initialization (step 1) and the fitness computation (step 9) are detailed in the next section.

Algorithm 1 Collective solving process

```
1:  Initialization of Agent positions
2:  while Fitness in progress do
3:      for all Agents do
4:          Attraction computation
5:          Repulsion computation
6:          Move computation
7:          Move execution
8:      end for
9:      Fitness computation
10: end while
```

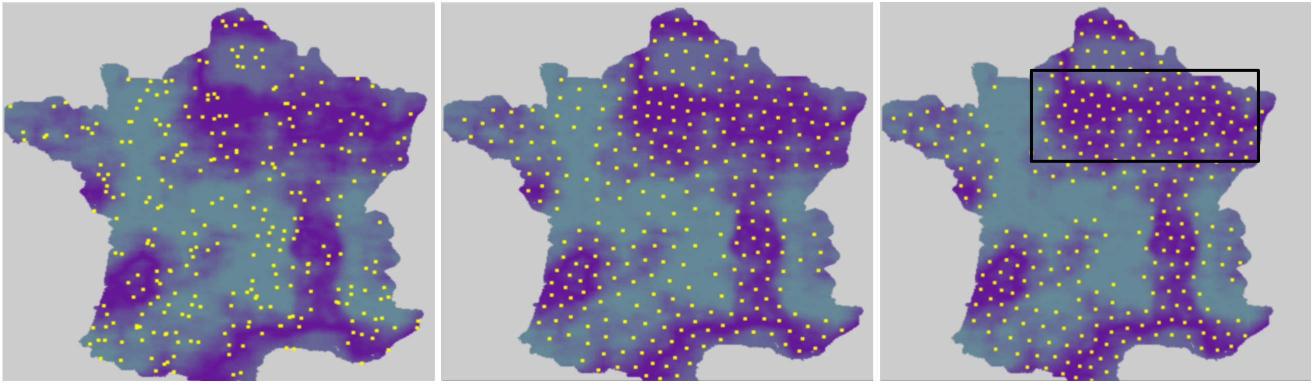
4 EXPERIMENTATIONS

After exposing the principle of the approach, the model is evaluated on a case study. It consists in positioning facilities (bus-stops, restaurants, etc) on a continuous environment corresponding to the map of France presented in Fig.3 (400x400 size). It contains the demand weights which are values between 0 and 255 (randomly generated). These weights are represented as a gradation from black color (255) to white color (0).

The initial positioning of facilities is performed with a random computation (Fig.3 (a)). Parameters values are: $\alpha = 0.5$, $r_a = 25$, $r_r = 20$.

When the algorithm starts, facility agents (the white points in Fig.3) move towards demands while avoiding other agents (Fig.3 (b)). In the first iterations we can observe important moves to the highest demands while the repulsive forces globally stabilize this tendency. The system iterates until it attains a global equilibrium state (convergence to a stable state).

Fig.3 (c) shows the final state to which the system converges. The facilities repartition is characterized by an intensification of the



(a) Demand representation (dark areas) and random initialization of facility locations

(b) Facilities motion (iteration 20)

(c) Final result (since the iteration 41)

Figure 3. The evolution of facilities positioning for the case study with 400 agents

agents in areas where demands is high. This result is visible inside the rectangular area. It is also clear that all facilities respect a minimal distance between them.

The performance of the multiagent model has been compared with the k-means clustering technique. The k-means algorithm is a well known technique that computes very good solutions to the facility location problem [10]. It allows us to classify or to group objects based on attributes/features into *K* number of groups. The grouping is done by minimizing the sum of distances between data and the corresponding cluster centroid (Algorithm 2).

Algorithm 2 The k-means clustering

1: **repeat**
2: Place k points into the space represented by the objects that are being clustered.
3: Assign each object to the group that has the closest centroid. When all objects have been assigned, recalculate the positions of the k centroids as weighted barycenters.
4: **until** The centroids no longer move.

Comparisons are made according to a global fitness index expressed by the formula (7) and corresponding to the mean distance between each demand and the nearest facility:

$$Fitness = \frac{\sum_{ij} D_{ij} * d(C_{ij}, x_{ij})}{\sum_{ij} D_{ij}} \quad (7)$$

D_{ij} = the demand at point x_{ij}
$d(C_{ij}, x_{ij})$ = the distance between the point x_{ij} and the nearest facility C_{ij}

Comparisons are carried out on different number of facilities, as shown in Table 1. For each facility number, 50 tests have been executed. The fitness values obtained by applying the multiagent approach are very close to the k-means ones. The difference is small and it is inversely proportional to the number of facilities.

A second comparison has been performed considering another criterion: the time required to converge to a solution. Results are presented in Table 2, they show that the multiagent model converges to a solution more rapidly than the k-mean, e.g. for 400 facilities the multiagent approach is 3 times faster than the k-means. It is particularly interesting to note that the multiagent approach is more efficient than the k-means while the number of agents increases.

Table 1. Comparison with k-means clustering

Fitness: minimal values					
Facilities	50	100	150	200	400
Multiagent	16,592	11,187	9,164	7,945	5,696
k-means	15,556	10,965	9,010	7,820	5,593
Difference	1,036	0,222	0,154	0,125	0,095

Table 2. Comparison of computation time

Computation time (in second)			
Facilities	100	150	400
Multiagent	34.289	24.925	33.019
k-means	37.678	73.306	118.251

Fig.4 plots the evolution of the fitness values for 400 facilities. We can show that the fitness decreases until the convergence to a constant value. Here, the convergence is attained rapidly: since the 41 th iteration.

All the experimentations have shown that the agents systematically converge to a stable location. It corresponds to a global balance between attraction to demands and inter-agents repulsive forces.

5 DISCUSSION

The previous experimentations allow to point up some observations on the proposed model. The obtained solutions are globally satisfying considering the fitness values. We have shown that these solutions are quickly obtained.

For each specific application, the multiagent approach needs a parameter setting stage. However, the proposed model depends only on three parameters: attraction and repulsion radius, and the weight combination of influences (α in formula (6)). Attraction and repulsion radius depend on the considered application. Generally, the attraction radius is defined by the coverage distance (for a demand, the next facility must be within a specified distance, the covering radius).

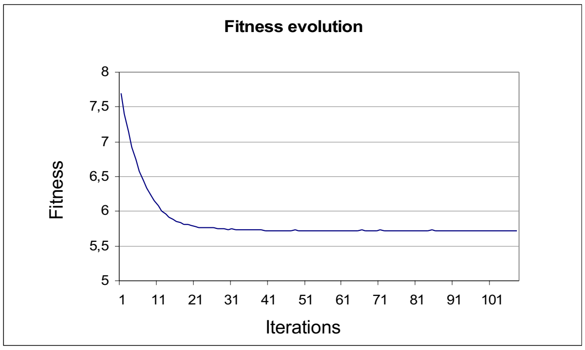


Figure 4. The fitness evolution for the case study with 400 agents

The repulsion radius is defined by the minimal distance allowed between two facilities. α is defined following the designer objectives.

Generally, the existing solutions for facility positioning [2, 18, 6] are not easily adaptable when the problem constraints change, particularly, for dynamic perturbations. It can concern the environment structure (e.g. demands), the facilities number, etc. For instance, when facilities have been located, any change in the demand will generally needs a new execution of the employed algorithm (it is the case with genetic algorithms, branch and bound, etc). At the opposite, our self-organizing approach will immediately adapt the locations to the perturbation. The algorithm can compute this adaptation from its last state. Moreover, it is interesting to not stop the algorithm in order to observe new solutions which are dynamically obtained while perturbing the system.

The proposed model can be also adapted to variations in the problem statement. For instance, we have applied our model to bus-stops positioning in a real bus-line network. The considered network serves a 60000 inhabitants city. Demands correspond to real values of inhabitants density per quarter. In this problem the positioning of facilities is limited to the lines. The model has been adapted to this new constraint without changing the agent behaviors. We have just constrained the agents to stay on lines (the move vector is transformed so as the agents move along lines). Experimentations have shown that the fitness value decreases until its convergence to a constant value, which corresponds to an optimization of the bus-stops location.

6 CONCLUSIONS

This paper has presented a self-organizing multiagent approach for the continuous p-median problem. Facilities, which are modeled as reactive agents, move according to their local perception. Demands induce attractive forces and the proximity between agents generates repulsive ones. The agent behavior is defined as a combination of these two kind of opposite influences.

Local minima, which must be avoided in the artificial potential fields approach are exploited in our model as balance points between demands.

At a collective level, we have shown that the system, i.e. the whole set of agents, interact to minimize distances to demands. Then, the system converges to a global stable state.

The relevance of our approach has been shown through its application to location of facilities on a continuous environment. In particular, it has been compared to the k-means clustering algorithm. A first evaluation criterion concerns the variation of the final fitness value. The multiagent results tend to the k-means values when the number of agents increases. The second evaluation concerns the computation time to obtain a stable solution. In this case, the multiagent approach is clearly faster and this advantage grows with the number of agents. These different evaluations show that a self-organizing multiagent approach can be an interesting perspective to optimization in positioning problems.

Future works deal, first, with a more formal evaluation of the global system convergence. Then, we seek to apply our approach to another problematic in location problems: the dimensioning problem. It consists to optimize the number of facilities to locate, since each new facility increases the location cost. We obtain a multicriteria problem. We then propose to add two behaviors allowing the creation and the removing of agents in order to optimize facilities location and number.

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