Optimization and Air Supply Management of a Polymer Electrolyte Fuel Cell

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Abstract— A mathematical model of the Polymer electrolyte fuel cell (PEFC), including air compression process and humidification has been developed to study the performances of the fuel cell. The study is focused on the air supply management with the objective to optimize the inlet air pressure and stoechiometry given by the compression system. A simple optimization method is given to maximize the voltage gain including the voltage drop due to the compression process. The optimization has to take into account the outlet relative air humidity (optimization constraint) leaving the fuel cell to avoid drying or flooding problems. The optimization results show that working at fully humidified air at the inlet is not always a good solution especialy for low air mass flow rates because of the high level of air stoechiometry to avoid flooding of the polymer membrane. On the other hand, it is better to work at fully humidified air at the inlet at high air mass flow rates. In all the cases, the optimal pressure is less than 2.5 bar which gives an indication for the design of air compressor for fuel cells.

Index Terms— Fuel Cell, optimization, air management, humidity.

I. Introduction

A FUEL cell (Fig. 1) is an electrochemical "device" that continuously convert chemical energy into electric energy and heat as long as fuel (hydrogen) and oxidant (oxygen) are supplied.
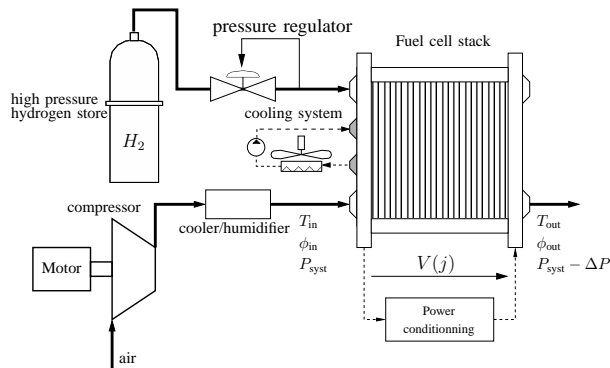

Fig. 1. Air compressor and Polymer electrolyte fuel cell

Most of times, fuel cells are working with pressurized system to reduce the weight, the volume (*i.e.*, the cost) and consequently to augment the power density of the entire system which is very interresting for on board applications like cars or airplanes.

The efficiency of a fuel cell is higher than any combustion engine and its performances are highly dependent on air pressure and temperature. Theoretically, neglecting the air compression process, the higher the pressure is, the higher the performances are.

Unfortunately, the compression process has a cost: working at high pressure levels require a high power. The consumption of the compressor is about 20 % of the power delivered by the fuel cell. The consequence of that is a voltage drop at the output of the fuel cell.
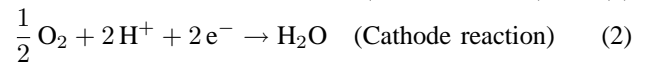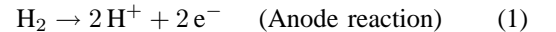
That is why, design, optimization and control of compression system for fuel cells are became one of the main challenge for fuel cells development.

The aim of this paper is to give a simple method to optimize the voltage gain of the fuel cell taking into account the relative air humidity (optimization constraint) in the fuel cell. The optimization parameters will be the air pressure and stoechiometry.

Then, a model of the cell voltage has to be given taking into account the voltage drop due to the compression process. A model of relative humidity (depending on the air pressure and stoechiometry) at the output of the fuel cell has to be given to respect the humidity constraints.

II. Fuel Cell Voltage Formulation

The basic electrochemical reactions which take place in a polymer electrolyte fuel cell are given by the two equation below

$$H_2 \rightarrow 2\,H^+ + 2\,e^- \quad \text{(Anode reaction)} \tag{1}$$

$$\frac{1}{2}\,O_2 + 2\,H^+ + 2\,e^- \rightarrow H_2O \quad \text{(Cathode reaction)} \tag{2}$$

And the global equation is

$$H_2 + \frac{1}{2}\,O_2 \rightarrow H_2O \tag{3}$$

A. Nernst Potential (Open Circuit Voltage)

The ideal performance of a fuel cell is defined by its Nernst potential (E) which is the ideal cell voltage.

The Nernst potential can be deduced with the Nernst equation which provides a relationship between the ideal standard potential (E^o) for the cell reaction and the ideal equilibrium (E) at other temperatures and partial pressures of reactants and

products. For the reaction given by (3) the Nernst equation is [1]:

$$E = E^o + \frac{RT}{2F} \ln\left(\frac{P_{O_2}^{\frac{1}{2}} \cdot P_{H_2}}{P_{H_2O}}\right) \tag{4}$$

The ideal standard potential E^o for a PEFC is 1.229 volts with liquid water product, or 1.18 volts with gaseous water product [2].

Another form of the Nernst equation can be written as a function of the system pressure P_{syst} [1],

$$E = E^o + \frac{RT}{2F} \ln\left(\frac{a \cdot b^{\frac{1}{2}}}{c}\right) + \frac{RT}{4F} \ln(P_{\text{syst}}) \tag{5}$$

where a, b and c are constants depending on the molar masses and concentration of H_2, O_2 and H_2O as following

$$
\begin{aligned}
P_{H_2} &= a\, P_{\text{syst}} \\
P_{O_2} &= b\, P_{\text{syst}} \\
P_{H_2O} &= c\, P_{\text{syst}}
\end{aligned}
$$

According to (5), one can notice that the ideal cell potential at a given temperature can be increased by operating at higher pressures. This will be further studied in section V.

B. Fuel Cell Losses (irreversibilities)

The actual cell potential is decreased from its equilibrium potential because of irreversible losses, as shown in Fig. 2. Multiple phenomena contribute to irreversible losses in an actual fuel cell:

1) *Activation losses.* These are caused by the slowness of the reaction taking place on the surface of the electrodes. The activation losses are directly related to the rates of electrochemical reactions. The consequence is that a proportion of the voltage generated is lost in driving the chemical reaction that transfers the electrons. This voltage drop, is given by the semi-empirical equation, called the Tafel (1905) equation:

$$\Delta V_{\text{act}} = \frac{RT}{2\alpha F} \ln\left(\frac{j}{j_0}\right) \tag{6}$$

where α is called the charge transfer coefficient. Its value depends on the reaction involved and the electrode material, but it must be in the range 0 to 1. At the hydrogen electrode, its value is about 0.5 and at the oxygen electrode between about 0.1 and 0.5 [1]. The current density j_0 is usually called the exchange current density, and can be considered as the current density at which the overvoltage begins to move from zero. The Tafel equation only hold true when $j > j_0$.

2) *Fuel crossover and internal currents.* The electrolyte is ion conductor but unfortunately, it supports very small amount of electron conduction (internal currents). Also, some hydrogen can diffuse from the anode to the cathode through the electrolyte. The hydrogen will react directly with the oxygen, producing no current from the cell. This wasted fuel that migrate through the

electrolyte is called the fuel crossover. These two effects are equivalent [1] but the fuel crossover is probably more important. The two effects — fuel crossover and internal current — are assigned as equivalent to an internal current j_n. Then, (6) can be refined to

$$\Delta V_{\text{act}} = \frac{RT}{2\alpha F} \ln\left(\frac{j + j_n}{j_0}\right) \tag{7}$$

3) *Ohmic losses.* These occur because of resistance to the flow of ions through the electrolyte and electrical resistance of the electrodes. The voltage drop due to the ohmic losses is simply proportional to the current density and is given by

$$\Delta V_{\text{ohm}} = j\,r \tag{8}$$

where, r is called the area specific resistance.

4) *Mass transport or concentration losses.* These results from the change in concentration of the reactants at the surface of the electrodes as the fuel is used. According to (4), the concentration (or partial pressure) affects the voltage. Then, the consequence of a concentration drop will be a voltage drop called concentration losses — also known as mass transport loss because the reduction in concentration can be explained as the result of a failure to transport sufficient reactant to the electrode surface. The most known equation which give the concentration losses is

$$\Delta V_{\text{conc}} = -\mathcal{B} \ln\left(1 - \frac{j}{j_l}\right) \tag{9}$$

where $\mathcal{B}$ is a constant that depends on the fuel cell and its operating state. j_l is the limit current density of the cell.

Another equation (entirely empirical) has become more favoured because of its good results, is given by [3]

$$\Delta V_{\text{conc}} = m \exp(n\,j) \tag{10}$$

where m and n are empirical coefficients.

C. Operating Fuel Cell Voltage

The operating voltage of a fuel cell at a current density j is given by combining (4), (7), (8) and (10):

$$V(j) = E - \Delta V_{\text{ohm}} - \Delta V_{\text{act}} - \Delta V_{\text{conc}}$$

$$V(j) = E - r\,j - \mathcal{A} \ln\left(\frac{j + j_n}{j_0}\right) + m \exp(n\,j) \tag{11}$$

where

$$\mathcal{A} = \frac{RT}{2\alpha F}$$

and

$$E = E^o + \frac{RT}{2F} \ln\left(\frac{P_{O_2}^{\frac{1}{2}} \cdot P_{H_2}}{P_{H_2O}}\right)$$

The last equation is sometimes simplified by neglecting j_n (very small value regarding j) and combining j_0 of the Tafel equation with the Nersnt potential [1]:

$$V(j) = E_{\text{oc}} - r\,j - \mathcal{A} \ln(j) + m \exp(n\,j) \tag{12}$$

where E_{oc} is the practical open circuit voltage,

$$E_{\text{oc}} = E + \mathcal{A} \ln(j_0)$$

Examples of values of the coefficients are given in TABLE. I. These values are for a Ballard Mark V fuel cell given in [4]. The polarization curve $V(j)$ for these coefficients is given on Fig. 2.

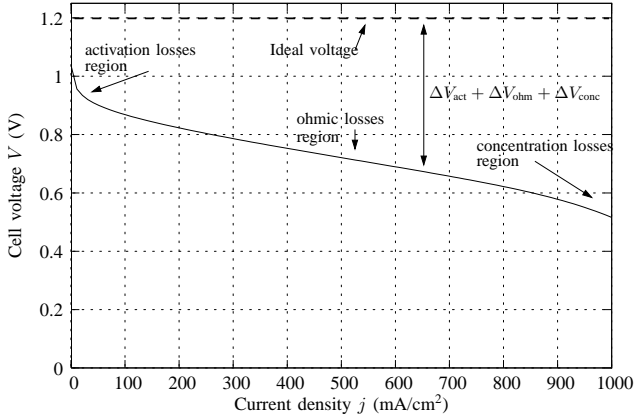| Coefficients	Value (T in $^{\text{o}}$C)
E_{oc} (V)	1.05
$\mathcal{A}$ (V)	$4.01 \times 10^{-2} - 1.40 \times 10^{-4}\,T$
r (k$\Omega\,\text{cm}^2$)	$4.77 \times 10^{-4} - 3.32 \times 10^{-6}\,T$
m (V) : $T \geq 39$ $^{\text{o}}$C	$1.1 \times 10^{-4} - 1.2 \times 10^{-6}\,T$
m (V) : $T < 39$ $^{\text{o}}$C	$3.3 \times 10^{-3} - 8.2 \times 10^{-5}\,T$
n ($\text{cm}^2\,\text{mA}^{-1}$)	8.0×10^{-3}

TABLE I

BALLARD MARK V PEFC COEFFICIENT [4]

Fig. 2. Ideal and actual fuel cell voltage-current characteristic ($T_{\text{fc}} = 80$ $^{\text{o}}$C)

This model does not account for all PEFC mechanisms. It is possible to have a more rigorous theoretical approach like the one given in [5].

III. RELATIVE HUMIDITY OF THE EXIT AIR

The humidity of the air must be carefully controlled. If the air is too dry, it will dry the polymer electrolyte membrane which has to be humidified to conduct the protons. The consequence of the drying is that the resistance of the membrane increases: the ohmic losses increase and the performances of the fuel cell drop (the proton conductivity is directly proportional to the water content). Then, it is essential that the electrolyte membrane retains high water content. On the other hand, if the air is too much humidified, liquid water would collect in the electrodes. The consequence is the flooding of the electrodes, blocking the pores in the electrodes or the gas diffusion layer.

A right balance of water content is consequently needed.

The definition of the relative humidity is given by the equation below,

$$\phi = \frac{P_{\text{H}_2\text{O}}}{P_{\text{sat}}} \quad (13)$$

where $P_{\text{H}_2\text{O}}$ is the partial pressure of water and P_{sat} the saturated vapour pressure.

A. Saturated Vapour Pressure

The saturated vapour pressure varies with temperature in a highly non-linear way (see Fig. 3). In [1], the values of P_{sat} are given for several temperature (see TABLE. II)

T ($^{\text{o}}$C)	Saturated vapour pressure (kPa)
15	1.705
20	2.338
30	4.246
40	7.383
50	12.35
60	19.94
70	31.19
80	47.39
90	70.13

TABLE II

SATURATED VAPOUR PRESSURE OF WATER FOR SEVERAL TEMPERATURES [1]

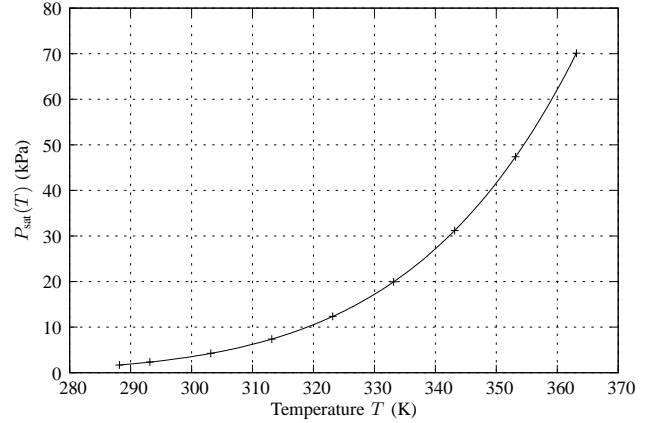Fig. 3. Saturated vapour pressure vs. temperature

In order to use a function $P_{\text{sat}}(T)$, a non-linear regression has been done using the values given in TABLE. II and the equation below

$$P_{\text{sat}}(T) = a_0 \, \exp\left(\frac{T - 273.16}{T - 273.16 + a_1} \cdot a_2 \right) \quad (14)$$

where P_{sat} is in pascals and T in kelvins.

The coefficient a_0, a_1 and a_2 obtained are the following: $a_0 = 630.388$, $a_1 = 230.173$ and $a_2 = 16.918$.

Then,

$$P_{\text{sat}}(T) = 630.388 \exp\left(\frac{T - 273.16}{T - 42.987} \cdot 16.918 \right) \quad (15)$$

B. Outlet Water Vapour Pressure

The outlet water vapour pressure is given by the following equation [1],

$$P_{\text{H}_2\text{O,out}} = \frac{(0.420 + \psi\,\nu)\,P_{\text{out}}}{(1 + \psi)\,\nu + 0.210} \quad (16)$$

with,

$$\psi = \frac{P_{\text{H}_2\text{O,in}}}{P_{\text{in}} - P_{\text{H}_2\text{O,in}}} \quad (17)$$

where P_{in} is the total inlet air pressure and

$$P_{H_2O,in} = \phi_{in} \cdot P_{sat}(T_{in}) \tag{18}$$

is the inlet water vapour pressure. ν is the air stoechiometry.

C. Exit Air Humidity

The exit air humidity can give an information about the humidity state of the fuel cell. The relative humidity should be above 80 % to prevent excess drying but below 100 %, or liquid water would collect in the electrodes [1].

Using, (13), (15), (16), (17) and (18), we have the equation which gives directly the exit air relative humidity as a function of the air input parameters (T_{in}, P_{in} and ϕ_{in}) and the exit parameters ($P_{out} = P_{in} - \Delta P$ where ΔP is the pressure drop in the fuel cell and T_{out} which is assumed equal to the stack temperature T_{fc})

$$\phi_{out} = \frac{P_{H_2O,out}}{P_{sat}(T_{fc})} \tag{19}$$

With

$$P_{H_2O,out} = \frac{\left(0.420 + \dfrac{\phi_{in} \cdot P_{sat}(T_{in})}{P_{in} - \phi_{in} \cdot P_{sat}(T_{in})} \nu\right)(P_{in} - \Delta P)}{\left(1 + \dfrac{\phi_{in} \cdot P_{sat}(T_{in})}{P_{in} - \phi_{in} \cdot P_{sat}(T_{in})}\right)\nu + 0.210} \tag{20}$$

and

$$P_{sat}(T) = 630.388 \exp\left(\frac{T - 273.16}{T - 42.987} \cdot 16.918\right) \tag{21}$$

IV. PRESSURE DROP IN THE FUEL CELL

The equation giving the pressure drop in a duct is the Darcy-Weisbach equation,

$$\Delta P = \Lambda \frac{L}{D_h} \rho_{air} \frac{U^2}{2} \tag{22}$$

where Λ is the friction coefficient (depends on the Reynolds number $\mathcal{R}_e$), L is the duct length (m), D_h is the hydraulic diameter of the duct (m), ρ_{air} is the fluid density (kg/m^3) and U the average flow velocity (m/s).

The aim of this part is to modify this equation in order to have the pressure drop in the fuel cell as a function of the average air mass flow rate $\langle \dot{q}_{air} \rangle$, and dimensions of the duct.

A. Friction Coefficient

The friction coefficient can be expressed in laminar flow by the following equation [6]:

$$\Lambda = \frac{B}{\mathcal{R}_e} \tag{23}$$

with

$$\mathcal{R}_e = \frac{\rho U D_h}{\mu} \tag{24}$$

where μ is the dynamic or absolute viscosity of the fluid (PI) and B a coefficient depending of the shape of the duct (if the duct is circular $B = 64$). In [6] a graphic (see Fig. 4) gives the value of B for a rectangular duct.

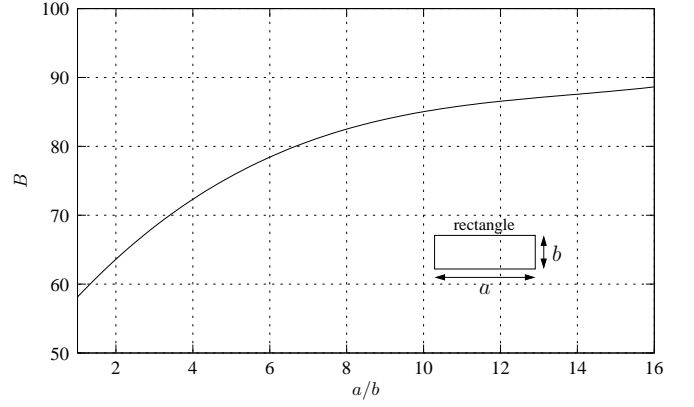Then, if we assume square ducts (*i.e.*, $a = b$ on Fig. 4),

$$B = 57 \tag{25}$$

Fig. 4. Coefficient B for a rectangular duct

B. Hydraulic Diameter

The hydraulic diameter (D_h) of a duct can be calculated using the equation

$$D_h = 4\frac{S}{\chi} \tag{26}$$

where S is the section area (m^2) of the duct and χ the perimeter of the duct (m).

Then for a rectangular duct,

$$D_h = 2\frac{ab}{a+b} \tag{27}$$

and, consequently, for a square duct,

$$D_h = a \tag{28}$$

C. Air Density

The calculation of the air density in the fuel cell is not so easy. It is a function of the pressure, temperature, water pressure and velocity.

The humidified air density can be calculated as following,

$$\rho_{air,hum} = \rho_{air,dry} + \rho_{H_2O} \tag{29}$$

Using the ideal gas law,

$$\rho_{air,dry} = \frac{P_{air,dry}}{\dfrac{R}{\bar{M}_{air}}T} \tag{30}$$

$$\rho_{H_2O} = \frac{P_{H_2O}}{\dfrac{R}{\bar{M}_{H_2O}}T} \tag{31}$$

The total pressure (P_{tot}) is equal to

$$P_{tot} = P_{air,dry} + P_{H_2O} \tag{32}$$

Then,

$$\rho_{air,hum} = \frac{1}{T}\left(P_{tot} - \frac{\bar{M}_{air} - \bar{M}_{H_2O}}{\bar{M}_{H_2O}}P_{H_2O}\right) \tag{33}$$

We will assume that $\rho_{air,hum}$ is constant and equal to the mean value between the input and the output of the fuel cell. Then,

$$\rho_{air,hum} \approx \langle \rho_{air,hum} \rangle = \frac{1}{2}\left(\rho_{air,hum,in} + \rho_{air,hum,out}\right) \tag{34}$$

$$\langle \rho_{\text{air,hum}} \rangle = \frac{1}{2} \left[\frac{1}{T_{\text{in}}} \left(P_{\text{in}} - \frac{\bar{M}_{\text{air}} - \bar{M}_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}} P_{\text{H}_2\text{O,in}} \right) \right.$$
$$\left. + \frac{1}{T_{\text{fc}}} \left((P_{\text{in}} - \Delta P) - \frac{\bar{M}_{\text{air}} - \bar{M}_{\text{H}_2\text{O}}}{\bar{M}_{\text{H}_2\text{O}}} P_{\text{H}_2\text{O,out}} \right) \right] \quad (35)$$

where $P_{\text{H}_2\text{O,in}}$ and $P_{\text{H}_2\text{O,out}}$ are calculated using, respectively, (18) and (16).

D. Average Flow Velocity

The average flow velocity U_{air} can be calculated using the average air mass flow rate $\langle \dot{q}_{\text{air}} \rangle$ by using the equation

$$U_{\text{air}} = \frac{\langle \dot{q}_{\text{air}} \rangle}{\langle \rho_{\text{air,hum}} \rangle S} \quad (36)$$

Then, if we assume a square duct (*i.e.*, $S = a^2$),

$$U_{\text{air}} = \frac{\langle \dot{q}_{\text{air}} \rangle}{\langle \rho_{\text{air,hum}} \rangle a^2} \quad (37)$$

E. Pressure Drop

The pressure drop, using all the equations above assuming square ducts and laminar flow can be written,

$$\Delta P = \frac{1}{2} B \langle \dot{q}_{\text{air}} \rangle \mu \frac{L}{\langle \rho_{\text{air,hum}} \rangle a^4} \quad (38)$$
$$= \frac{57}{2} \langle \dot{q}_{\text{air}} \rangle \mu \frac{L}{\langle \rho_{\text{air,hum}} \rangle a^4} \quad (39)$$

The value of $\langle \rho_{\text{air,hum}} \rangle$ depends on ΔP as seen in (35). Then, knowing $\langle \dot{q}_{\text{air}} \rangle$, we can deduce the pressure drop by resolving (38).

In this paper we will choose differents values of ΔP and deduce the corresponding value of $\langle \dot{q}_{\text{air}} \rangle$.

V. PRESSURE IMPACT ON THE CELL VOLTAGE

Running a fuel cell at higher pressure will increase the power, but also involves the expenditure of power, and reduces the cost, weight, and space taken up by the compression equipment. The aim of this part is to quantify the benefits (voltage gain), costs (voltage drop) and finally the net voltage gain.

A. Voltage Gain

As seen in (5) in section II-A, the effect of high working pressure is positive. The Nernst equation (5) shows that the voltage of a fuel cell is increased according to the term

$$\frac{\text{R}\, T}{4\, \text{F}} \ln(P_{\text{syst}})$$

.

Then, if the pressure changes from $P_{\text{syst,1}}$ to $P_{\text{syst,2}}$, there will be a *theoretical* change of voltage

$$\Delta V = \underbrace{\frac{\text{R}\, T}{4\, \text{F}}}_{C} \ln \left(\frac{P_{\text{syst,2}}}{P_{\text{syst,1}}} \right) \quad (40)$$

This theoretical agreement is not good for low temperature fuel cells. At lower temperature, the benefits of raising

system pressure are much greater than the Nernst equation predicts [1]. Theoretically, the coefficient C would be 0.01. As seen in [2] the actual values of C are reported ranging from 0.03 to 0.1 (best case). This coefficient depends on the system.

Then, the actual voltage gain is

$$V_{\text{gain}} = C \ln \left(\frac{P_{\text{syst,2}}}{P_{\text{syst,1}}} \right) \quad (41)$$

B. Voltage Drop due to the Compression Process

The increased voltage (or power) has a cost. This power cost is that needed to drive the compressor. The compressor power (*i.e.*, lost power) is given by the equation below

$$p_{\text{comp}} = c_{\text{p}} \frac{T_e}{\eta_{\text{m}}\, \eta_{\text{c}}} \left(\left(\frac{P_{\text{syst,2}}}{P_{\text{syst,1}}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) \dot{q}_{\text{air,in}} \quad (42)$$

We can also write the equivalent voltage drop,

$$V_{\text{drop}} = \frac{1}{4} \frac{\nu}{\text{F}\, \chi_{\text{O}_2}} \bar{M}_{\text{air}}\, T_e \frac{c_{\text{p}}}{\eta_{\text{m}}\, \eta_{\text{c}}} \left(\left(\frac{P_{\text{syst,2}}}{P_{\text{syst,1}}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) \quad (43)$$

The expression $P_{\text{syst,2}}/P_{\text{syst,1}}$ is the compression ratio of the compressor. Then, the voltage drop can be written (the inlet pressure is the atmospheric pressure, 10^5 Pa)

$$V_{\text{drop}} = \frac{1}{4} \frac{\nu}{\text{F}\, \chi_{\text{O}_2}} \bar{M}_{\text{air}}\, T_e \frac{c_{\text{p}}}{\eta_{\text{m}}\, \eta_{\text{c}}} \left(\left(\frac{P_{\text{syst}}}{10^5} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) \quad (44)$$

where P_{syst} is the absolute output pressure of the compressor in pascals (Pa).

C. Net Voltage Gain

From (41) and (44), the net voltage gain can be calculated

$$V_{\text{gain,net}} = V_{\text{gain}} - V_{\text{drop}} \quad (45)$$

Then,

$$V_{\text{gain,net}} = C \ln \left(\frac{P_{\text{syst}}}{10^5} \right)$$
$$- \frac{1}{4} \frac{\nu}{\text{F}\, \chi_{\text{O}_2}} \bar{M}_{\text{air}}\, T_e \frac{c_{\text{p}}}{\eta_{\text{m}}\, \eta_{\text{c}}} \left(\left(\frac{P_{\text{syst}}}{10^5} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) \quad (46)$$

VI. OPTIMIZATION OF THE AIR INPUT PARAMETERS

A. Objective Function

The aim of the optimization is to find the best conditions on the inlet air to maximize the net voltage gain. Then, we have to maximize $V_{\text{gain,net}}$ given by (46).

B. Optimization Parameters

The optimization parameters will be the input air parameters: the system pressure P_{syst} and the air stoichiometry ν.

The input (constant) parameters are the fuel cell temperature T_{fc}, the ambient temperature T_e, the pressure drop ΔP (constant for a given mass flow rate), the inlet temperature T_{in} and the inlet humidity ratio ϕ_{in}. We could have considered T_{fc}, T_{in} and ϕ_{in} as optimization parameters, but here, we assume that these parameters are constant when the fuel cell is working.

C. Optimization Constraints

The optimization constraints are the following:

1) *Air stoechiometry.* The air stoechiometry will be at least 2 to avoid concentration losses (if the stoechiometry is 1, the exit air would be completely depleted of oxygen). There is no need of maximum limit. Then,

$$\nu \geq 2 \tag{47}$$

2) *Pressure constraint.* The compressor can not give a pressure under atmospheric pressure and the output pressure of the fuel cell can not be under the atmospheric pressure (stronger constraint). Then,

$$P_{\text{syst}} - \Delta P \geq 10^5 \text{ Pa} \tag{48}$$

3) *Outlet humidity constraint.* This constraint is the main constraint. It is not a physical constraint but an "objective constraint". This constraint has been given in the section III-C,

$$0.85 \leq \phi_{\text{out}} \leq 0.95 \tag{49}$$

VII. RESULTS AND DISCUSSIONS

All the values of the different variables are defined below:

Variable	Value
T_e	288 K (15 °C)
T_{fc}	353 K (80 °C)
$\bar{M}_{\text{air}}$	28.5×10^{-3} kg/mol
c_{p}	$1,004 \text{ J K}^{-1} \text{ kg}^{-1}$
χ_{O_2}	0.21
γ	1.4
F	96, 485 C/mol
η_{c}	0.75
η_{m}	0.85
C	0.1 (optimistic value)
a	0.002 m from [7]
L	0.1 m from [7]
μ	1.3×10^{-5} PI

Several optimizations have been done for different values of ΔP (*i.e.*, *average* mass flow rate) and inlet humidity ratio. The results are given on Fig. 7.
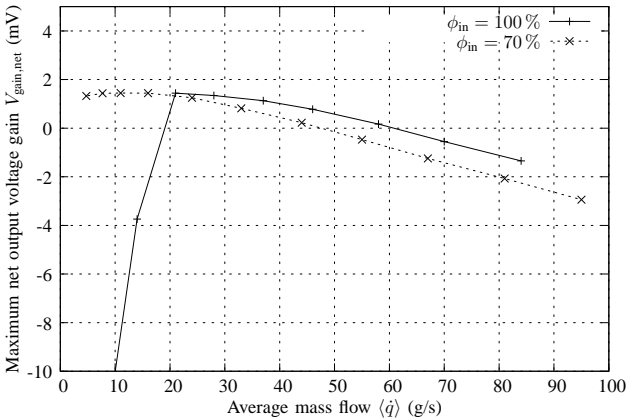
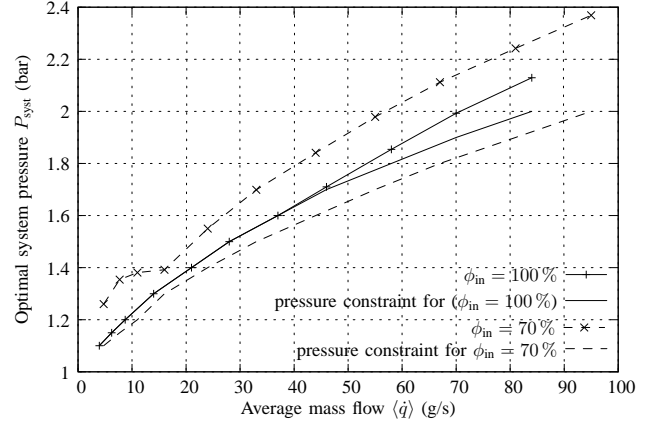Fig. 5. Maximum net voltage gain for two inlet relative humidity values

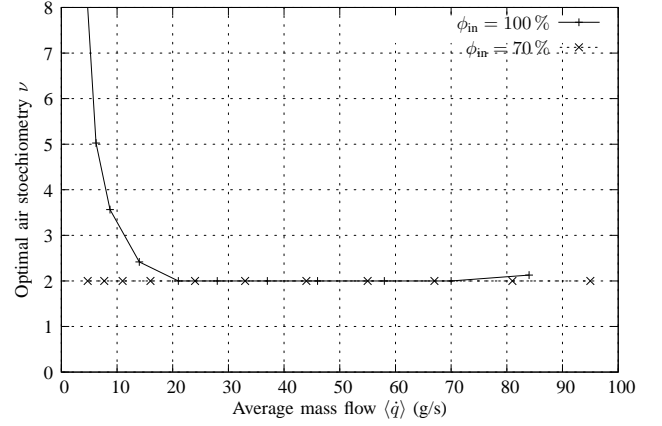Fig. 6. Optimal system pressure for two inlet relative humidity values

Fig. 7. Results of optimizations for two inlet relative humidity values

Working with fully humidified gases is not a good solution at low flow rates. In this case, the air stoechiometry has to be increased to achieve proper humidification (humidity constraint). It is then better, for a voltage gain (see Fig. 5), to work at a higher pressure ratio with an air stoechiometry at 2 and an inlet relative humidity of 70 %.

At higher mass flow rates, it become more interesting to work with fully humidified air because the pressure ratio needed to achieve the humidity constraint is lower than for a 70 % inlet humidity. In a high mass flow rate case, the pressure ratio has to be increased to avoid drying of the membranes, the outlet relative humidity is for the both at its lower constraint, (*i.e.*, $\phi_{\text{out}} = 0.85$).

Concerning the pressure system, we can see that there is no need of a high compression ratio: the value of 2.5 would be the maximum pressure to design a compressor for fuel cell.

VIII. CONCLUSION

A model of the PEFC, including air compression process and humidification has been developed and focused on the study of air supply management. The objective was to optimize the inlet air pressure and stoechiometry given by the compression system. A simple optimization method has been given to maximize the voltage gain including the voltage drop due to the compression process and taking into account the

outlet relative air humidity (optimization constraint) leaving the fuel cell to avoid drying or flooding problems.

The optimization results show that working at fully humidified air at the inlet is not always a good solution especially for low air mass flow rates because of the high level of air stoechiometry to avoid the flooding of the polymer membrane. On the other hand, it is better to work at fully humidified air at the inlet at high air mass flow rates. In all the cases, the optimal pressure is less than 2.5 bar which gives an indication for the design of air compressor for fuel cells.

NOMENCLATURE

Greek letters

α	Charge transfer coefficient
γ	Ratio of the specific capacities of a gas (for air $\gamma = 1.4$)
ΔP	Pressure drop in the fuel cell
ΔV_{act}	Activation losses (Tafel equation) (V)
ΔV_{conc}	Concentration losses (V)
ΔV_{ohm}	Ohmic losses (V)
η_{c}	Efficiency of the compressor
η_{m}	Efficiency of the electrical motor driving the compressor
Λ	Friction coefficient
μ	Dynamic or absolute viscosity (PI)
ν	Air stoechiometry
ρ	Fluid density (kg/m^3)
ϕ	Relative humidity
χ	Perimeter of the duct (m)
χ_{O_2}	Molar fraction of oxygen in air
ψ	Coefficient function of water and total input pressures

Roman letters

$\mathcal{A}$	Slope of the Tafel line (V)
a	Height of the duct (m)
b	Weight of the duct (m)
$\mathcal{B}$	Constant (concentration losses) (V)
C	Coefficient of the voltage gain
c_{p}	Specific heat capacity of air, $c_{\text{p}} = 1,004\ \text{J K}^{-1}\,\text{kg}^{-1}$
D_{h}	Hydraulic diameter (m)
E	Nernst potential (ideal voltage of the cell) (V)
E^o	Nernst potential at standard conditions (V)
E_{oc}	Practical open circuit voltage
F	Faraday constant, $96,485$ C
j	Current density (A/cm^2)
j_0	Exchange current density (A/cm^2)
j_l	Current density limit (A/cm^2)
j_n	Internal and fuel crossover equivalent current density (A/cm^2)
L	Duct length (m)
$\bar{M}$	Molar mass (kg/mol)
m	constant in the concentration losses (V)
n	constant in the concentration losses (cm^2 A^{-1})
P_x	If x is a gas, it is the partial pressure of the gaz x. If the subscript x is in or out, it means total pressure at the input or output of the fuel cell
P_{sat}	Saturated vapour pressure
P_{syst}	System pressure

$\dot{q}$	Masse flow rate (kg/s)
r	Area specific resistance ($\Omega\,\text{cm}^2$)
R	Universal gaz constant, 8.134 J·K^{-1}· mol^{-1}
$\mathcal{R}_{\text{e}}$	Reynolds number
S	Area section of the duct (m^2)
T	Temperature (K)
T_e	Entry temperature of the air (compressor)
V	Fuel cell voltage (V)
V_{drop}	Voltage drop due to the compressor
V_{gain}	Voltage gain due to the compression process
$V_{\text{gain,net}}$	Net voltage gain (compression process)

Subscripts

air	Air
dry	The gas is dry
fc	Fuel cell
hum	The gas is humidified
in	Input of the fuel cell
out	Output of the fuel cell
tot	Total pressure of the gas

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