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Statistical analysis technique on Ad Hoc network topology dynamic characteristics: Markov stochastic process

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Abstract On the basis of analysis on the scene files of mobility models in Ad Hoc network, the paper presents a network topology snapshots capturing method to obtain the Ad Hoc network topology architecture at any moment. Through analyzing on the Ad Hoc network topology snapshots, some dynamic characteristic parameters of Ad Hoc network, such as the number of network topology in steady state or unsteady state appearing during a certain time, as well as the durative time of network topology in steady state or unsteady state, could be obtained statistically. Furthermore, the probability of the network topology invariability and variability event could be predicated by adopting the discrete time and continuous time Markov stochastic process theory. The simulation result shows that the statistical analysis technique on Ad Hoc network topology dynamic characteristic not only is effective, but also has the general attribute, which could be used in the statistical analysis technique on Ad Hoc network topology dynamic characteristic under any mobility models.

Keywords Topology dynamic characteristic · Mobility model · Statistical analysis technique · Ad Hoc network

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1 Introduction

Mobile Ad Hoc network (MANET) is a collection of wireless mobile nodes to compose of a self-configuration, self-organization, multi-hop wireless network. The dynamic characteristic of Ad Hoc network topology is the key cause of influencing the network performance [1, 2]. Therefore, different mobility models are brought forward to simulate the real mobile scenes through describing the mobile rules of wireless mobile nodes in mathematical model manner. The mobility models used in Ad Hoc network could be classified as two types [3–5], one is concerned with the single mobile body, such as Random Waypoint Model (RWP), Random Walk Mobility Model (RW) and Manhattan Mobility Model, etc., the mobile nodes in which could move at random and autonomously without being restricted by other mobile nodes in a certain area. The other type is related with the group of mobile nodes, mainly comprised of Reference Point Group Mobility (RPGM), in which the movement behavior of the member nodes in a group is restricted by whose lead node in the same group. Owing to the movement of nodes in Ad Hoc network, its network topology maybe vary with time going by, which furthermore effects on the Ad Hoc network performance.

Although researches have focused on the dynamic characteristics of mobility models in Ad Hoc network and taken much achievements recently, little attention was paid on

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the link topology dynamic characteristics of mobile models. Narayannan Sadagopan et al. [9] puts forward a statistical method to obtain the dynamic characteristic of MM, which includes how to obtain the probability density distribution of link and path connection time. Nevertheless, the research mainly focus on the viewpoint of the influence of dynamic characteristic on the performance of active network protocols, not on that of the Ad Hoc network measurement. At the same time, its statistical analysis method is only applicable for the certain mobility models with one time to change its' velocity or direction in one second, such as RPGM, Freeway and Manhattan mobility model, not for the other mobility models in NS-2 tool, such as RW and RWP. Although Tian et al. [10] brings forward a link connection time model which could be used to compute the link connection minimum time, and further to obtain the minimum of network topology lifetime. However, the computing model is too complicated for not being simplified. Besides, it is only adaptable for the RWP mobility model, not for the other mobility models in Ad Hoc networks. Wang et al. [11] brings forth a circle mobility model, in which when the initialization position of mobile nodes is known, the network topology architecture of Ad Hoc network could be computed according to the mathematical rules of nodes' movement. Specially, the minimum of network topology lifetime could also be obtained statistically. However, this research on NT measurement technique in Ad Hoc network mainly focus on circle mobility model, it fails to be useful for other mobility models. Therefore, How to put forward an analysis technique on the dynamic characteristic of Ad Hoc network topology, which could be used for all the mobility models supported in NS-2 tool, is an interesting issue to be solved.

It is difficult for the traditional interior network measurement to meet the requirements of Ad Hoc network performance measurement for its weakness, such as, the dependence of the collaboration among internal nodes, the network infrastructure, network protocols and some privacy and security problems, and so on. NT technique (Network tomography, also called as External network measurement technique or End-to-End network measurement technique [12–14]), as a new network measurement method, which uses the measurement data sample of End-to-End to infer internal link performance parameters without the collaboration among internal nodes. Since NT technique is also not concerned with certain network protocols and network infrastructure [15–18], it is very adaptable for link performance inference in Ad Hoc network. The precondition of introducing NT technique to Ad Hoc network measurement is that network topology is yet known, and must not vary during the whole network measurement period [19–22]. It is inconsistent with the truth that the network topology of Ad Hoc network will continuously vary with time going by. Thereby, how to resolve the influence of dynamic characteristic of Ad Hoc network on the result of measurement and link performance inference is one of the key problems. As we all know that movement is absolute, while stillness relative. For example, although two nodes move in different speed, or even different direction in wireless communication, if the distance of the two nodes is within the range of each other's communication coverage area, we consider that the link connect relation between the two mobile nodes does not vary. The rest may be deduced by analogy, that is, if all the link connection relations do not vary during the whole measurement time, that is, the Ad Hoc network topology keep steady, we could safely say that the performance of Ad Hoc network is measurable based on NT technique. This phenomenon is called as measurement time validity. The period during which Ad Hoc network does not vary is called as measurement window time in this paper. Thus it can be seen that it is importance of studying dynamic characteristic analysis technique on the Ad Hoc network topology, which is useful not only for the Ad Hoc network measurement based on NT technique, but also for the performance analysis of Ad Hoc network influenced by the dynamic characteristic of Ad Hoc network topology [6–8].

The contribution of this paper lies in the following points: (1) For being inspired by the concept of "taking photos" in our routine life, the paper brings forth an link topology snapshot capture method to take the photos of Ad Hoc network link topology at any moment by scanning scene files of mobility models in NS-2 tool. (2) Through analyzing on the link topology architectures captured by the above method, the steady and un-steady period of Ad Hoc network topology could be easily found. Furthermore, the statistical characteristic of the number of steady and un-steady period appearing in a certain time, and that of duration time of steady and un-steady period can be obtained by using χ^2 hypothesis testing in probability theory. (3) The continuous time and discrete state Markov stochastic process is adopted to analyzes the probability of the network topology invariable or variable event. The new method could infer the forecasting and warning experimental formula, which could be used to forecast the probability that the state of Ad Hoc network topology keeps invariable or variable after a certain time.

The rest of the paper is organized as followings: Sect. 2 presents the physical topology snapshot capture method. The statistical characteristic of Ad Hoc network topology is presented in Sect. 3. The continuous time and discrete state Markov stochastic process is introduced to analyze on the stochastic characteristic of Ad Hoc network topology invariable or variable event in Sect. 4. The result analysis and discussion is presented in Sect. 5. Finally, the concluding remarks and future works are presented in Sect. 6.

2 Link topology snapshots capture method

2.1 Formalization of mobility model

All the mobility models supported by NS-2 [23] have the same format of scene files produced by *setdest* tool. Through analysis on the scene files we could arrive at the conclusion that there is a certain spatial relativity among mobile nodes. That is, the destination position of node j at time t_i is equal to its current position at time t_{i+1} on condition that $v_{t_i}^j$ equals zero, where $v_{t_i}^j$ denotes the velocity of node j from time t_i to t_{i+1}.

$$\begin{cases} C_{t_{i+1}}^j = D_{t_i}^j & (C_{t_i}^j \neq D_{t_i}^j \wedge v_{t_i}^j != 0) \\ C_{t_{i+1}}^j = C_{t_i}^j & (v_{t_i}^j = 0) \end{cases} \tag{1}$$

If let γ denote the snapshot time slot, the relativity between velocity and spatial position could be expressed as formula (2).

$$\begin{cases} c_{x,t_{i+1}}^j = c_{x,t_i}^j + v_{x,t_i}^j \times \gamma \\ c_{y,t_{i+1}}^j = c_{y,t_i}^j + v_{y,t_i}^j \times \gamma \end{cases} \tag{2}$$

where v_{x,t_i}^j and v_{x,t_i}^j denote the x-axis and y-axis value of speed $v_{t_i}^j$ at time t_i, which could be obtained by using position $C_{t_i}^j, D_{t_i}^j$ and $v_{t_i}^j$. Thus it can be seen, the state information of node j at time t_i could be expressed as a three *tuple*$\langle C_{t_i}^j, D_{t_i}^j, v_{t_i}^j \rangle$. Furthermore, position snapshots of mobile nodes at any moment could be derived from formula (2). The method how to get physical topology snapshot is to compute the Euclid distance R between node j and l ($l \in V \backslash \{j\}$) at each time, where V denotes the node set of Ad Hoc network. If R is smaller than the transmission range of mobile node denoted as r, illuminating that there is a chance for the node j and l to build up a wireless connection at link layer, the state of link between node j and l could be set as one, otherwise, as zero. If the same operation is implemented between any mobile nodes at each snapshot time, we could achieve the physical topology snapshot. At last, the steady and un-steady period of Ad Hoc network topology can be obtained by computing all the physically topology snapshots statistically.

2.2 Simulation experiment

Through analyzing on the Ad Hoc network topology snapshots with RW and RWP mobility model, the relation of the link topology in steady or un-steady state and link topology varying ratio varying with time are shown as in Figs. 1(a)–1(d). Next, we will explain the three concepts used in Fig. 1. Link connection ratio is the ratio of the links having a wireless connection with each other to all links in Ad Hoc networks in each one topology snapshot. Topology varying ratio is the ratio of the number of links that the state of which has varied to all links between the two consecutive topology snapshots. Topology lifetime is the time during which the Ad Hoc network topology does not vary. Actually the curve of topology lifetime is equivalent to that of the topology varying ratio in Fig. 1, since when the value of topology varying ratio between the two consecutive topology snapshots is not equal to zero, the topology lifetime is set as two, otherwise set as zero to denote that the Ad Hoc network topology does not vary between the two consecutive topology snapshots. The mobile scene is set as the following parameters in NS-2. There are all 50 mobile nodes, and the stop time is 0 s in RW and 5 s in RWP respectively. The maximum velocity of mobile nodes is 20 m/s, simulation being 900 s, and the scene covers a square area with 1200 m × 1200 m. The wireless communication coverage range is set as a circle with radius being 250 m.

According to the result of analysis on the RW, RWP mobility model as in Figs. 1(a)–1(d) [24], and that on the Freeway, Manhattan and RPGM mobility model in Figs. 1(e)–1(h) [25], we could safely arrive at the conclusion, the steady and un-steady period appear in turn during all simulation time, at the same time, the number of the steady and unsteady state and the duration time in each state vary with different mobility models and the parameters of movement scenes.

3 Stochastic characteristic of network topology

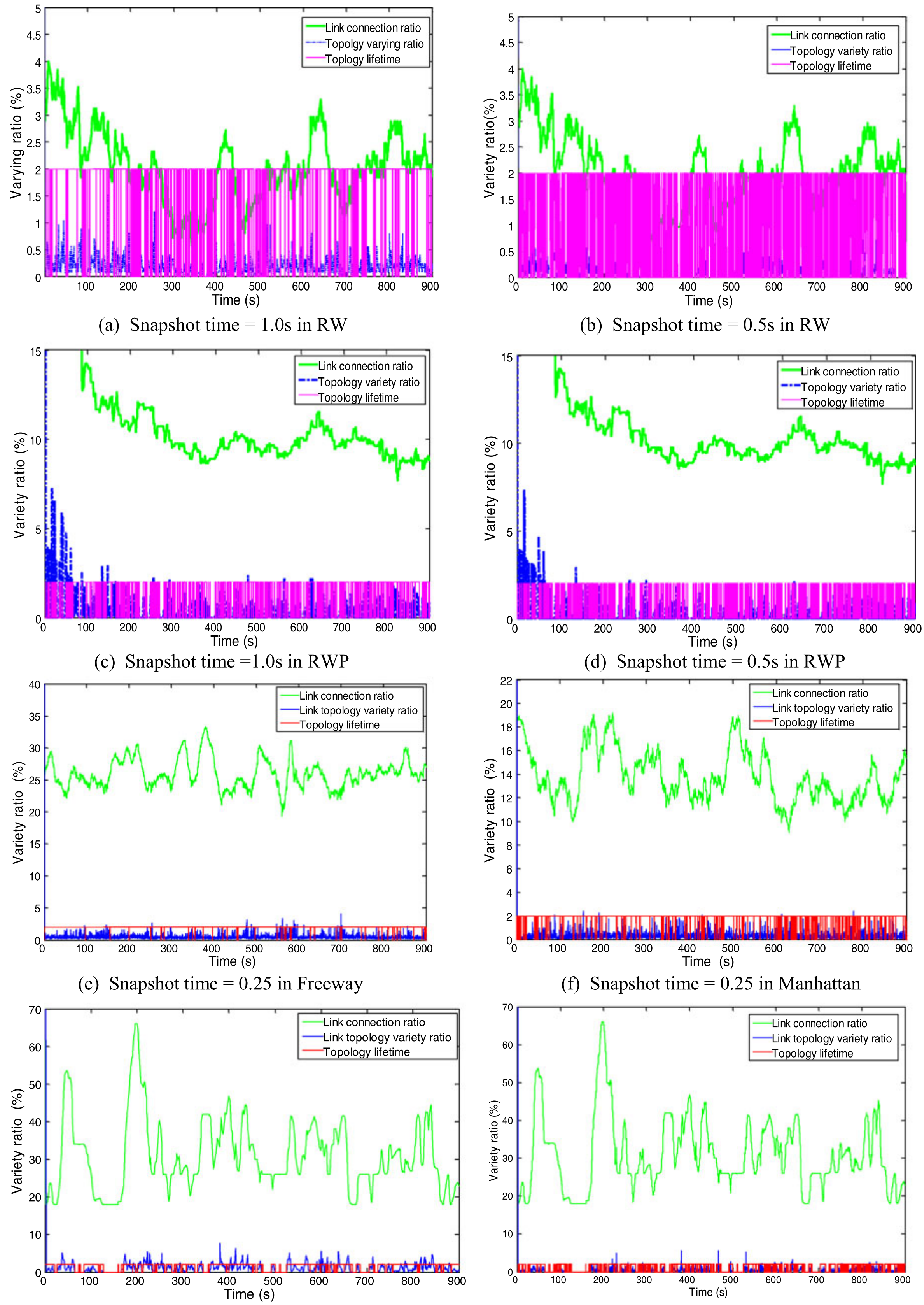
3.1 Statistical characteristic of the number of steady period

In a certain time t, the number of steady period (or un-steady period) is a discrete stochastic variable X. Through analyzing on the stochastic variable X, we could obtain the frequency of the steady period (or un-steady period) appearing in a certain time. We used the data in Fig. 1(d) as an example to obtain the probability distribution chart of the number of steady period appearing in $t = 10$ s, $t = 15$ s in the Figs. 2(a) and 2(b) respectively.

From the Fig. 2, we could likely arrive at the inconclusive hypothesis that the number of steady period appearing in a certain time approximately follows the poison distribution, and for different time there exists different parameter λ. Next, we will use χ^2 Fit hypothesis testing method to verify this hypothesis. At first, we put forward the following hypothesis test problem:

H_0: The number of steady period follows the poison distribution.

H_1: The number of steady period does not follow the poison distribution.



(a) Snapshot time = 1.0s in RW

(b) Snapshot time = 0.5s in RW

(c) Snapshot time =1.0s in RWP

(d) Snapshot time = 0.5s in RWP

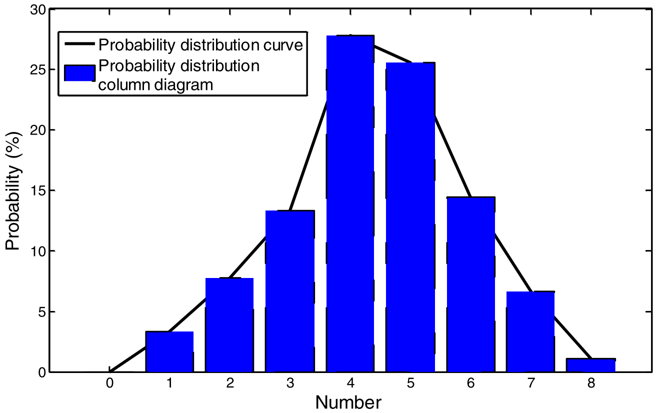
(e) Snapshot time = 0.25 in Freeway

(f) Snapshot time = 0.25 in Manhattan

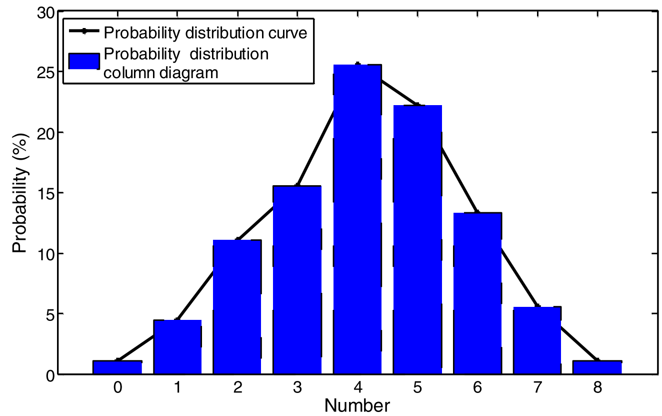
(g) Snapshot time =1.0s in RPGM

(h) Snapshot time =0.25s in RPGM

Fig. 1 Topology dynamic characteristic of Ad Hoc network



(a) t=10s



(b) t=15s

Fig. 2 Probability distribution chart of the number of steady period

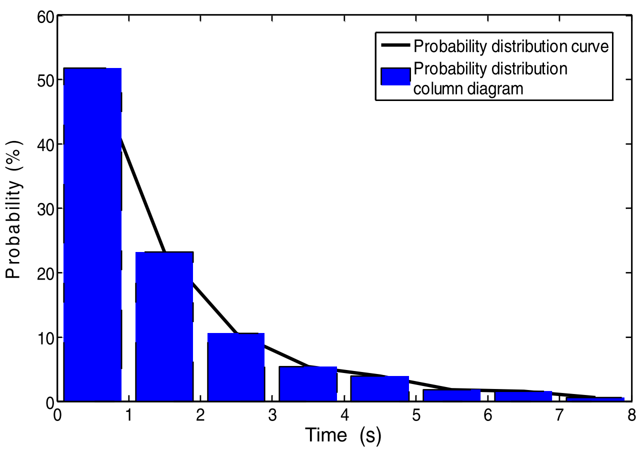


Fig. 3 PDF of steady duration time

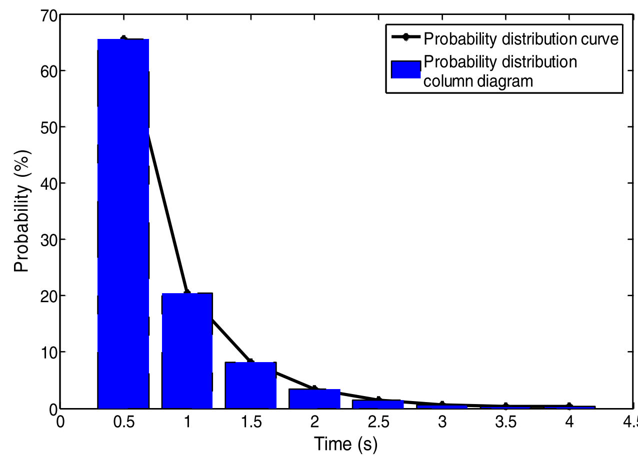


Fig. 4 PDF of un-steady duration time

Under the condition that significance level α equal to 0.05, and free degree r equal to 9, we could get the in-equation relation between the theoretical value of χ^2 distribution function and statistical one as the following:

$$\chi_{\alpha}^{2}(r-l-1)=\chi_{0.05}^{2}(9-1-1)=14.067>7.40 \tag{3}$$

This in-equation relation means that the test statistic variable V does not belong to the rejection range, therefore, we have to accept the hypothesis H_0, and to refuse another hypothesis H_1. It is reasonable for us to believe that the number of steady period appearing in 10 seconds follows the poison distribution with $\lambda = 4.21$, when we choose RWP mobility model in a certain mobile scene as our research object. At the same time, that the number of un-steady period appearing in 10.0 seconds follows the poison distribution with $\lambda = 4.16$ could also be verified as the method above. When the statistical time is equal to different values, such as 15 s, 20 s, and so on, or when we choose other different mobility models, such as RW, Freeway, Manhattan and RPGM, we could also safely arrive at the conclusion that the number of steady or un-steady period appearing in a certain time also follows the poison distribution with different parameter λ. The paper does not discuss these for the limit to its length.

3.2 Statistical characteristic of the steady or un- steady duration time

When Ad Hoc network topology is in the steady state, the duration of which is called as steady duration time, otherwise, called as un-steady duration time. Because the steady duration time is a continuous stochastic variable, the statistical analysis method on the data about steady duration time in Fig. 1(d) is different from that on the number of steady period appearing in a certain time (Figs. 3 and 4).

Under the condition that significance level α is equal to 0.05, and free degree r equal to 10, we could get the in-equation relation between the theoretical value of χ^2 distribution function and statistical one as the following:

$$\chi_{\alpha}^{2}(r-1)=\chi_{0.05}^{2}(10-1)=15.507>V=11.12 \tag{4}$$

The above in-equation relation means that the test statistic variable V does not belong to the rejection range, therefore, we have to refuse the hypothesis H_1, and accept another hypothesis H_0. It is reasonable for us to believe that the steady duration time follows the exponential distribution with the $\lambda' = 0.584$, when we choose RWP mobility model in a certain mobile scene as our research object. At the same time, we could also prove that the un-steady duration time in the whole simulation time follows the exponential distribution with $\lambda' = 1.276$. In the same way, when the statistical time is equal to different values, such as 15 s, 20 s, and so on, or when we choose other different mobility models, such as RW, Freeway, Manhattan and RPGM, we could also safely arrive at the conclusion that the steady or un-steady duration time follows the exponential distribution with different parameter λ.

4 Markov stochastic process analysis method

4.1 Discrete Markov chain statistical analysis

According to the analysis result above, the dynamic characteristic of Ad Hoc network topology mainly embodies the following two points: one is that there is two states about Ad Hoc network topology, that is, the steady state and the un-steady state. Specially, the number of steady state or un-steady state appearing in a certain time follows the poison distribution with parameter λ. Another is that the steady or un-steady duration time follows the exponential distribution with parameter λ'. Therefore, we could easily arrive at the Theorem 1.

Theorem 1 *The dynamic varying process of Ad Hoc network topology is actually a continuous time and discrete state Markov stochastic one.*

Since the state variety of Ad Hoc network topology is independent of its historical state, if X_n denotes the state of Ad Hoc network at time n, stochastic variable $X_n\{n \geq 0\}$ accords with Markov chain, and its state transfer matrix with one step P could be denoted as the follow.

$$P = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} = \begin{pmatrix} \alpha & 1-\alpha \\ 1-\beta & \beta \end{pmatrix} \tag{5}$$

where α, β denotes the probability that when the current state Ad Hoc network topology snapshot is steady or non-steady, and that of the next snapshot is also the steady or non-steady one respectively. Since the state transfer matrix with one step P of Markov chain has been defined above, the probability of Ad Hoc network topology variety with m step $p_{ij}^{(n+m)}$ is defined as the follow.

$$p_{ij}^{(n+m)} = p(X_{n+m} = j | X_n = i) \tag{6}$$

where $p_{ij}^{(n+m)}$ expresses the conditional probability of Ad Hoc network topology being state j ($j \in \{0, 1\}$) at the snapshot time $n + m$, when that is i ($i \in \{0, 1\}$) at snapshot time n. On the condition that state transfer matrix with one step P has been known, it is easy to obtain $p_{ij}^{(n+m)}$ as the following formula according to Chapman–Kolmogorov equation.

$$\begin{aligned} &p_{ij}^{(n+m)} = p_{ij}^{(m)} \times p_{ij}^{(n)}, \quad \text{and} \\ &p_{ij}^{(m)} = p_{ij} \times p_{ij}^{(m-1)}, \qquad p_{ij}^{(n)} = p_{ij} \times p_{ij}^{(n-1)} \end{aligned} \tag{7}$$

4.2 Continuous Markov chain statistical analysis

In order to solve the limitation of Markov chain with discrete time and state, the Markov stochastic process with discrete state and continuous time could be used to analyze the Ad Hoc network topology dynamic characteristics through decreasing the snapshot time Δt and increasing the number of snapshots. The following results could be obtained according to Sect. 3.

(1) If the state of Ad Hoc network topology enters i, the duration of state i accords with exponent distribution with parameter γ_i.
(2) If the state of Ad Hoc network topology enters i, no matter how long duration of state i is, the probability P_{ij} that changes into another state j is one (or 100 %).

If the state of Ad Hoc network is i at time t, and during the period from t to $t + s$ the state is also i, then the conditional probability could be denoted as $P(z > s + \Delta t | z > s)$ during the period from $t + s$ to $t + s + \Delta t$, where z denotes the duration of Ad Hoc network topology being state i. Since exponent distribution has the non-memorial attribution, the formula (8) could be easily obtained.

$$P(z > s + \Delta t | z > s) = P(z > \Delta t) \tag{8}$$

The process of Ad Hoc network topology changing dynamically could be considered as a Markov stochastic one. At different time $t_0, t_1, \ldots, t_n$ $(0 \leq t_0 < t_1 < \cdots < t_n)$, the state of Ad Hoc network topology is $i_0, i_1, \ldots, i_n \in E = \{0, 1\}$ in turn. Since the state of Ad Hoc network topology is obtained through comparing the two successive snapshots of topology, which is independent of its historical snapshots, then formula(9) could be obtained.

$$\begin{aligned} &P(X_t = j | X_{t_0} = i_0, X_{t_1} = i_1, \ldots, X_{t_n} = i_n) \\ &\quad = P(X_t = j | X_{t_n} = i_n) \end{aligned} \tag{9}$$

According to formula (11), the stochastic process of Ad Hoc network topology has the Markov attribution, that is, conditional shift probability of Markov stochastic process with continuous time is only concerned with the current state, which is irrespective of any historical state. The above results could also be denoted as formula (10).

$$P(X_{s+\Delta t} = j | X_s = i) = P_{ij(\Delta t)} \tag{10}$$

Formula (10) indicates that the conditional shift probability of the stochastic process about Ad Hoc network topology changing dynamically is independent of start time s, which is only correlative with time slice Δt, that is, the stochastic process of Ad Hoc network topology has the homogeneous attribution.

The homogeneous shift probability of stochastic process about Ad Hoc network topology changing dynamically, $\{X_t, t \geq 0\}$, is corresponding to a shift probability matrix, $P(t) = \{(P_{ij}(t))\}$, $t \geq 0$, where $P(t)$ is called as the shift probability matrix of Markov stochastic process.

For any state $i, j \in E$ and time $s, t \geq 0$, $P_{ij}(t)$ is satisfied with the following rules:

(1) $P_{ij}(t) \geq 0$ and $\sum_{j \in E} P_{ij(t)=1}$
(2) $P_{ij}(t+s) = \sum_{k \in E} P_{ik}(t) P_{kj}(s)$
(3) For any state $i, j \in E$, $P_{ij}(t)$ is a uniformly continuous function with parameter t, where t is time.

When the state of Ad Hoc network topology is i, if time Δt is small enough, the probability of Ad Hoc network topology being state i at time $t + \Delta t$ could be denoted as the formula (11).

$$\begin{aligned} P_{ii}(\Delta t) &= P\big(X(t+\Delta t) = i | X(t) = i\big) \\ &= 1 - \gamma_i \Delta t + o(\Delta t) \end{aligned} \tag{11}$$

At the same time, Formula (11) shows that the probability of the Ad Hoc network topology leaving state i at time $t + \Delta t$ is equal to $\gamma_i \Delta t + o(\Delta t)$. If $P_{ij}(\Delta t)$ denotes the probability of Ad Hoc network topology changes into state j $(i \neq j)$ after time Δt when the current state is i, for any state i, j $(i \neq j)$, formula (12) could be obtained as the follow.

$$\begin{aligned} P_{ij}(\Delta t) &= P\big(X(t+\Delta t) = j | X(t) = i\big) \\ &= \big(\gamma_i \Delta t + o(\Delta t)\big) P_{ij} \\ &= \gamma_i P_{ij} \Delta t + o(\Delta t) \end{aligned} \tag{12}$$

If $\gamma_i P_{ij}$ is denoted as γ_{ij}, where γ_{ij} expresses the speed of Ad Hoc network topology changing from state i to j as the formula (13).

$$\gamma_{ij} = P'_{ij}(0) = \lim_{t \to 0^+} \frac{P_{ij(t)}}{t} \tag{13}$$

γ_{ij} in formula (15) is also called as shift speed of state changing from i to j, and matrix $Q = (\gamma_{ij})$ is called as state shift intensity matrix, which satisfied the following rule.

$$\gamma_{ii} \leq 0 \quad \text{and} \quad \gamma_{ij} \geq 0 \quad (i, j \in E, i \neq j) \tag{14}$$

According to the analysis results in Sect. 3, if the Ad Hoc network topology is in steady state (denoted as "0") now, after a steady duration time in this state, it transfers to the non-steady state (denoted as "1"), and the non-steady duration time accords with the exponential distribution with parameter λ_1. However, the steady duration time accords with the same distribution with parameter λ_2. Therefore, the density matrix of this Markov stochastic process could be denoted as the following Q.

$$Q = \begin{pmatrix} -\lambda_1 & \lambda_1 \\ \lambda_2 & -\lambda_2 \end{pmatrix}$$

Theorem 2 *The sum of each row in the density matrix Q equals to zero.*

Proof Supposed that 0 and 1 denotes steady and non-steady state respectively. The duration of Ad Hoc network topology steady and non-steady duration time accords with exponent distribution with parameter λ_1 and λ_2. $P_{i,j}(t)$ denotes the probability of the state about Ad Hoc network topology changing from state i to j after time t. According to the exponent distribution, the following formula could be obtained.

$$\begin{aligned} &P_{0,1}(\Delta t) = \lambda_1 \Delta t + o(\Delta t) \\ &P_{1,0}(\Delta t) = \lambda_2 \Delta t + o(\Delta t) \end{aligned} \tag{15}$$

If $P_{i,j}(t)$ is successive at time t equal to zero, for any state i and j $(i, j \in E = \{0, 1\})$, There exists the following formula.

$$\lim_{t \to 0} P_{ij(t)} = 0 (i \neq j), \quad \text{or} \quad = 1 (i = j)$$

According to the formula $P_{0,1}(\Delta t)$, $P_{1,0}(\Delta t)$, and the exponent distribution of Ad Hoc network topology steady and non-steady duration time, each element of the density matrix Q could be obtained as the following formula.

$$\begin{aligned} q_{00} &= \lim_{\Delta t \to 0^+} \frac{P_{00}(\Delta t) - P_{00}(0)}{\Delta t} = \lim_{\Delta t \to 0^+} \frac{[1 - P_{01}(\Delta t)] - 1}{\Delta t} \\ &= \lim_{\Delta t \to 0^+} \frac{-\lambda_1 \Delta t + o(\Delta t)}{\Delta t} = -\lambda_1 \\ q_{01} &= \lim_{\Delta t \to 0^+} \frac{P_{01}(\Delta t) - P_{01}(0)}{\Delta t} \\ &= \lim_{\Delta t \to 0^+} \frac{\lambda_1 \Delta t + o(\Delta t) - 0}{\Delta t} = \lambda_1 \\ q_{10} &= \lim_{\Delta t \to 0^+} \frac{P_{10}(\Delta t) - P_{10}(0)}{\Delta t} \\ &= \lim_{\Delta t \to 0^+} \frac{\lambda_2 \Delta t + o(\Delta t) - 0}{\Delta t} = \lambda_2 \end{aligned}$$

$$q_{11} = \lim_{\Delta t \to 0^+} \frac{P_{11}(\Delta t) - P_{11}(0)}{\Delta t} = \lim_{\Delta t \to 0^+} \frac{[1 - P_{11}(\Delta t)] - 1}{\Delta t}$$
$$= \lim_{\Delta t \to 0^+} \frac{-\lambda_2 \Delta t + o(\Delta t)}{\Delta t} = -\lambda_2$$

Next, the sum of the first row in the density matrix Q is expressed as the following formula which equals to zero according to conditional probability in complete set. In the same way, that of the second row in the density matrix Q also equals to zero.

$$q_{00} + q_{01}$$
$$= \lim_{\Delta t \to 0^+} \frac{(P_{00}(\Delta t) - P_{00}(0)) + (P_{01}(\Delta t) - P_{01}(0))}{\Delta t}$$
$$= \lim_{\Delta t \to 0^+} \frac{(P_{00}(\Delta t) + P_{01}(\Delta t)) - (P_{00}(0) + P_{01}(0))}{\Delta t}$$

Therefore, the Theorem 2 is proved. □

According to the forward differential equation of continuous time Markov stochastic process [18–20], $P'(t) = P(t)Q$, the following differential equations (16) could be obtained.

$$\begin{cases} p'_{00}(t) = \sum_{k=0}^{1} p_{0k}(t).q_{k0} = -\lambda_1 p_{00}(t) + \lambda_2 p_{01}(t) \\ p'_{01}(t) = \sum_{k=0}^{1} p_{0k}(t).q_{k1} = \lambda_1 p_{00}(t) - \lambda_2 p_{01}(t) \\ p'_{10}(t) = \sum_{k=0}^{1} p_{1k}(t).q_{k0} = -\lambda_1 p_{10}(t) + \lambda_2 p_{11}(t) \\ p'_{11}(t) = \sum_{k=0}^{1} p_{1k}(t).q_{k1} = \lambda_1 p_{10}(t) - \lambda_2 p_{11}(t) \end{cases} \tag{16}$$

According to the probability theory, there exists the following restriction condition.

$$\begin{cases} p_{00}(t) = 1 - p_{01}(t) \\ p_{11}(t) = 1 - p_{10}(t) \end{cases}$$

If we use the equation $p_{01}(t) = 1 - p_{00}(t)$ to replace the $p_{01}(t)$ in the first differential equation of formula (16), then the following equation could be obtained.

$$p'_{00}(t) = \lambda_2 - (\lambda_1 + \lambda_2) p_{00}(t)$$

Let $Q_{00}(t)$ be equal to $e^{(\lambda_1+\lambda_2)t} p_{00}(t)$, that is, $Q_{00}(t) = e^{(\lambda_1+\lambda_2)t} p_{00}(t)$. Then to implement the differential coefficient operation on both sides of this equation for the parameter t, we could get the formula (17).

$$Q'_{00}(t) = (\lambda_1 + \lambda_2) e^{(\lambda_1+\lambda_2)t} p_{00}(t) + e^{(\lambda_1+\lambda_2)t} p'_{00}(t) \tag{17}$$

To multiply the first equation of the formula (16) by $e^{(\lambda_1+\lambda_2)t}$ on its both sides, the formula (17) could be simplified as the following formula (18).

$$Q'_{00}(t) = \lambda_2 e^{(\lambda_1+\lambda_2)t} \tag{18}$$

Through implementing the integral operation on the both sides of the formula (18) and adopting the initial condition: $p_{00}(0) = 1$, we could finally obtain the following forecast experimental formula (19) and (20).

$$p_{00}(t) = \frac{\lambda_2}{\lambda_1 + \lambda_2} \left(1 + e^{-(\lambda_1+\lambda_2)t}\right) \tag{19}$$

$$p_{11}(t) = \frac{1}{\lambda_1 + \lambda_2} \left(\lambda_1 + \lambda_2 e^{-(\lambda_1+\lambda_2)t}\right) \tag{20}$$

Formula (19) means that if the Ad Hoc network topology is in the steady state now, after time t, the probability that it is still in steady state is $p_{00}(t)$. Formula (20) means that if the Ad Hoc network topology is in the un-steady state now, after time t, the probability that it is still in un-steady state is $p_{11}(t)$. Therefore, formula (19) and (20) are called as the Ad Hoc network topology steady and un-steady state keeping experimental formula respectively. Next, we use the concept of opposite events in probability theory to obtain warning experimental formal (21) and (22).

$$p_{01}(t) = 1 - p_{00}(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2} \left(1 - e^{-(\lambda_1+\lambda_2)t}\right) \tag{21}$$

$$p_{10}(t) = 1 - p_{11}(t) = \frac{\lambda_2}{\lambda_1 + \lambda_2} \left(1 - e^{-(\lambda_1+\lambda_2)t}\right) \tag{22}$$

Formula (21) means that if the Ad Hoc network topology is in the steady state now, after time t, the probability that its state varies as un-steady one is $p_{01}(t)$. While formula (22) means that if the Ad Hoc network topology is in the un-steady state now, after time t, the probability that its state varies as steady one is $p_{10}(t)$. Therefore, formula (21) and (22) are called as the Ad Hoc network topology steady and un-steady state varying warning formula respectively. When time is set as 4 s, 8 s and 10 s respectively, the experimental probability about state keeping invariable and varying is shown as the Figs. 5, 6 and 7 according to the forecast formula (19), (20) and the warning formula (21), (22), where x axis denotes the parameter of exponential distribution λ_1, y axis the parameter λ_2, and z axis denotes the probability value.

In order to understand Ad Hoc network topology dynamic characteristic, we set λ_2 as 1.276 and λ_1 as 0.584 to obtain the error between theoretical value p_i according to the forecast formula (19), (20), and the warning formula (21), (22), and real value $\overline{p}_i$. The maximum error and mean error is defined as the Table 1.

Table 2 shows that the error between the theoretical value according to steady and un-steady state keeping experimental formula and state varying warning formula, and real value. The maximum error of the former is 0.120, and that of the rear is 0.095, and the mean error is no more than 0.075. this result shows that the Ad Hoc network topology steady and un-steady state keeping experimental formula and state varying warning formula is effective.

Table 1 Error computing method

Conception	Computing method
Maximum error	$\max \lvert p_i - \overline{p}_i \rvert$
Mean error	$\frac{1}{n-1} \sum \lvert p_i - \overline{p}_i \rvert$

Table 2 The error between the theoretical value and real value

Probability	Mean error	Maximum error
p_{00}	0.048	0.084
p_{11}	0.074	0.120
p_{01}	0.058	0.076
p_{10}	0.061	0.095

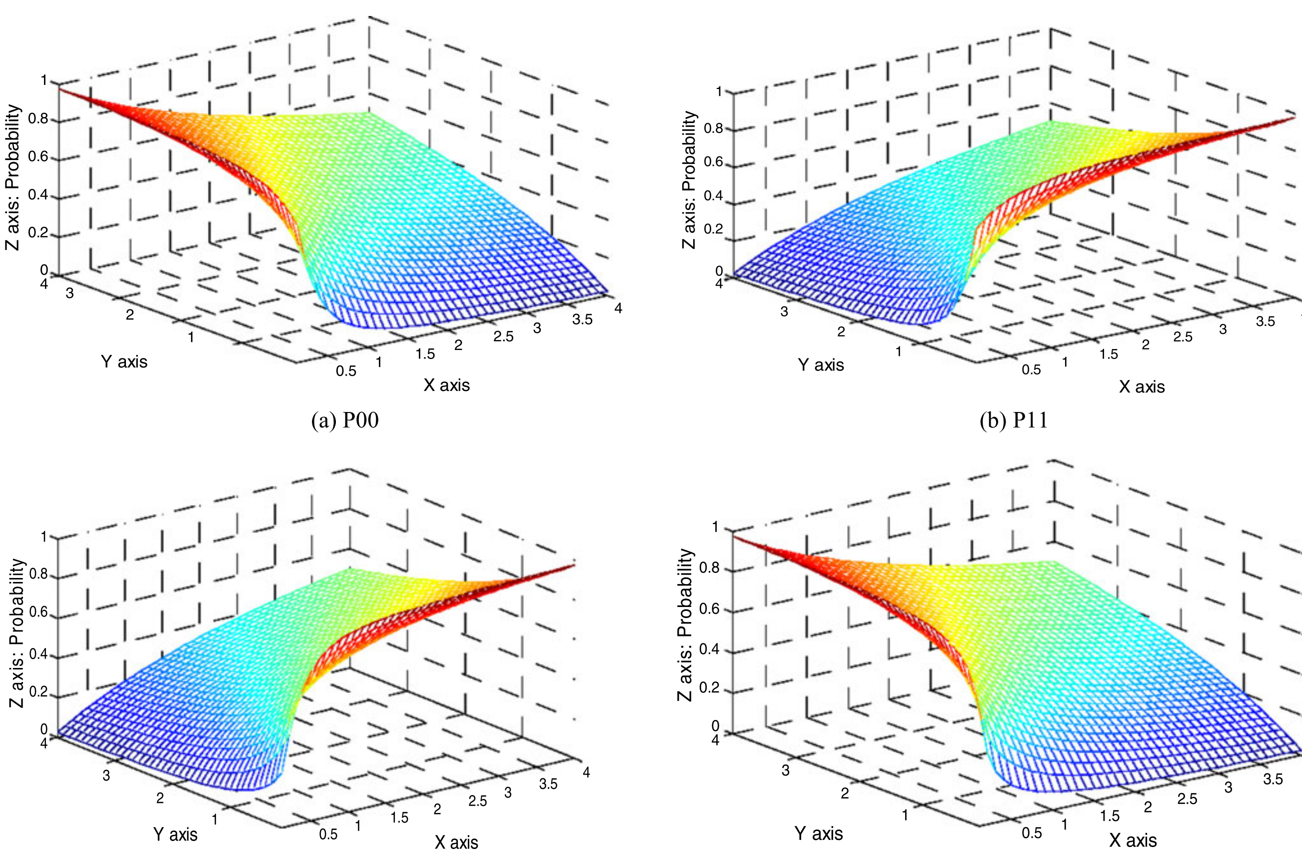
5 Result analysis and discussion

As shown in Figs. 5, 6 and 7, we could safely arrive at the following conclusion: (1) P_{01} and P_{11} increases, while P_{00} and P_{10} decrease with the increment of parameter λ_1. (2) P_{00} and P_{10} increase, while P_{01} and P_{11} decrease with the increment of parameter λ_2. (3) P_{01} and P_{10} increases, while P_{00} and P_{11} decrease with the increment of time t. The results are shown as in Table 3.

If the number of steady period appearing in a certain time is larger than 2, with the increment of parameter λ_1 in passion distribution, the number of steady period appearing will becomes smaller according to the progression theory, that is, the probability of Ad Hoc network topology keeping steady period will decrease. Therefore, P_{01} and P_{11} in-

Table 3 Probability varies with parameters λ_1, λ_2, and time t

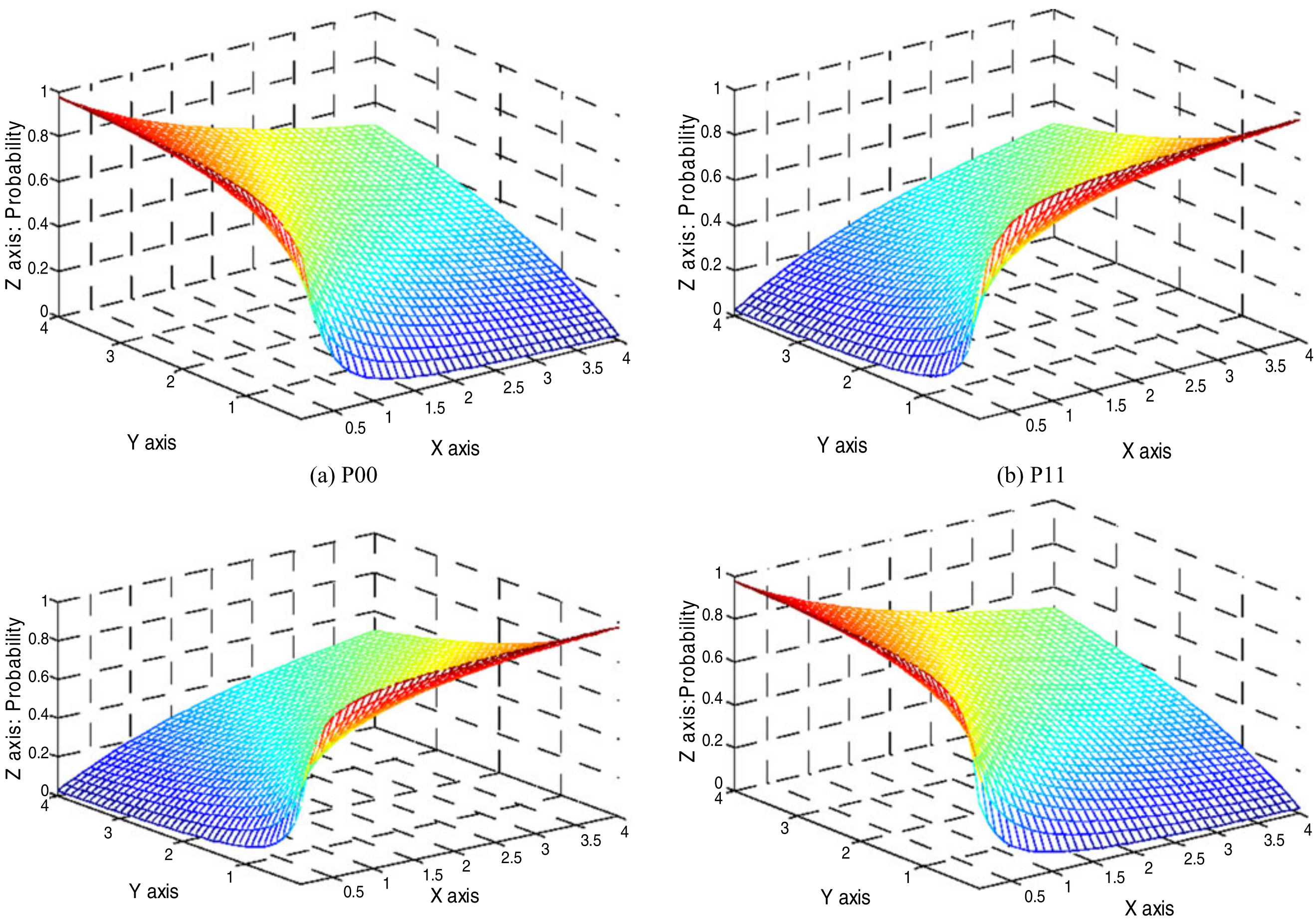
Probability	P_{00}	P_{01}	P_{11}	P_{10}
$\lambda_1 \uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
$\lambda_2 \uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$t \uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$



(a) P00 (b) P11

(c) P01 (d) P10

Fig. 5 Experimental probability about forecast and warning formula with $t = 4$ s



(a) P00

(b) P11

(c) P01

(d) P10

Fig. 6 Experimental probability about forecast and warning formula with $t = 8$ s

creases, while P_{00} and P_{10} decrease with the increment of λ_1 in a certain time. If steady duration time is lager than 1.0 s, with the increment of parameter λ_2 in exponential distribution, the steady duration time will become larger according to the progression theory, that is, the probability of Ad Hoc network topology keeping steady period will increase. Therefore, P_{01} and P_{11} decreases, while P_{00} and P_{10} increase with the increment of parameter λ_2 in a certain time. With the increment of time t, the probability of Ad Hoc network topology keeping its former state(i.e., steady state or un-steady state) will become smaller. Therefore, P_{01} and P_{10} increase, while P_{00} and P_{11} decrease with the increment of time t with a certain parameters λ_1 and λ_2.

In a practical Ad Hoc network application system, we could use GPS or other position location technology to obtain the position of mobile nodes in any moment, instead of analyzing on the scene file. Next we could use the computing and analysis method in the paper to obtain the dynamic characteristic of Ad Hoc network topology, which could be used for performance evaluation and optimization of Ad Hoc network.

6 Conclusion

The paper introduces an Ad Hoc network topology snapshot capture method to obtain the network topology architecture at any moment under any mobility models in Ad Hoc network. Through analysis on the data about the link topology snapshot, we could arrive on the following conclusion: (1) The dynamic varying process of Ad Hoc network topology is actually a discrete state and continuous time Markov stochastic one. The discrete state means that there exists two states in Ad Hoc network topology, one is steady state, the other is un-steady state. Furthermore, the consecutive time slices during which the Ad Hoc network topology keeps in steady state compose of the steady period, otherwise, of the un-steady period. The continuous time means that the steady or un-steady duration time is a continuous time stochastic variable. (2) The Ad Hoc network topology steady period and un-steady period appear in turn periodically. (3) The number of steady or un-steady period appearing in a certain time follows the poison distribution, and the steady or un-steady duration time follows the exponential distribution.

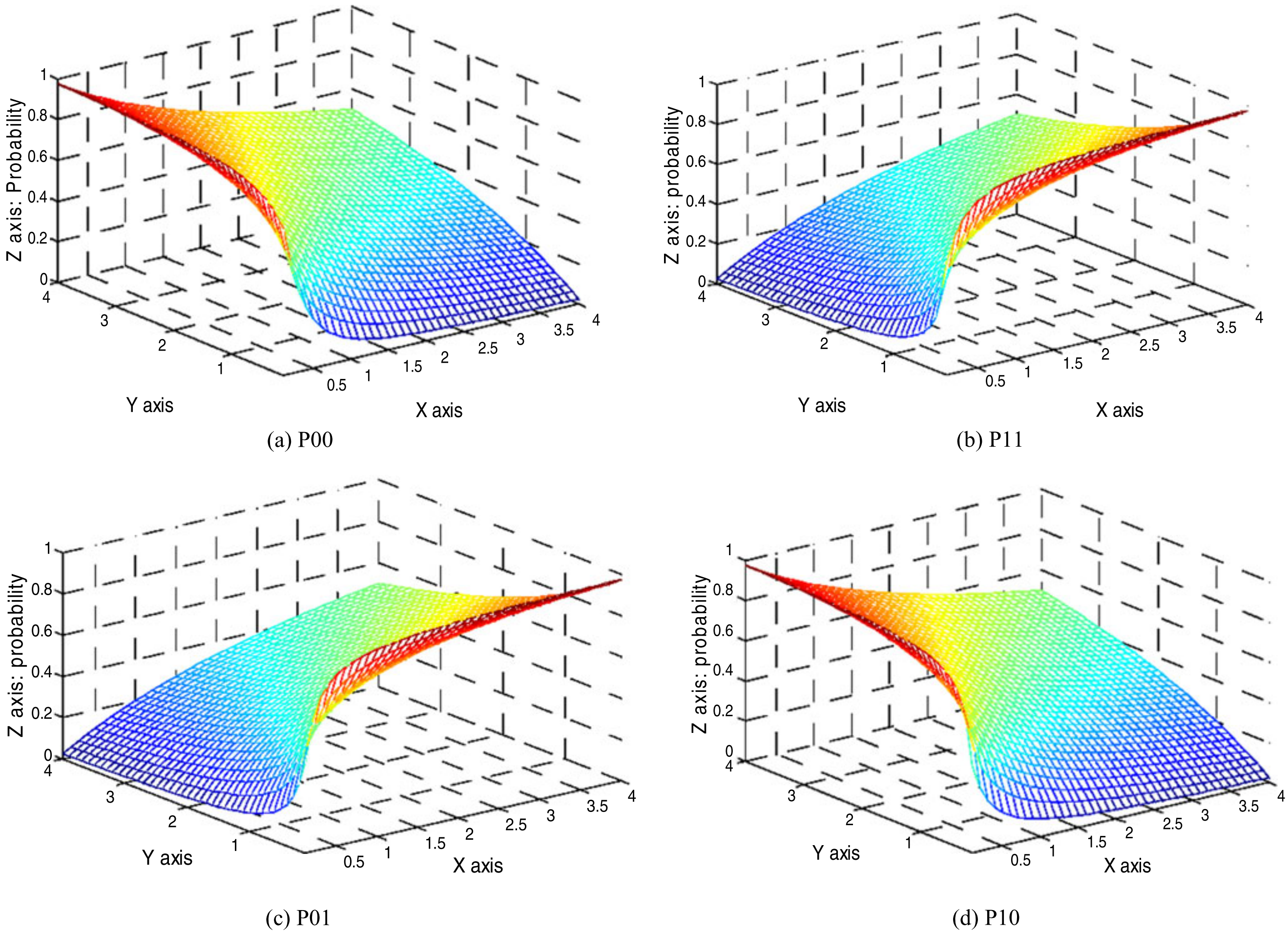


Fig. 7 Experimental probability about forecast and warning formula with $t = 10$ s

Since the statistical analysis method in the paper could be used for all the mobility models supported by NS-2 tool, it has the attribute of universality. However, all data and results in this paper comes from the simulation, which are not tested in the practical Ad Hoc network. Therefore, our future work will focus on how to use measurement data to analyze the Ad Hoc network topology dynamic characteristic. Next, we will use the analysis technique in the paper to achieve some rules about the dynamic characteristic of practical Ad Hoc network application system.

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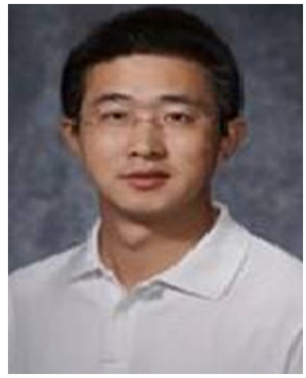

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