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AMS 114

Homework #6

6.1.8:

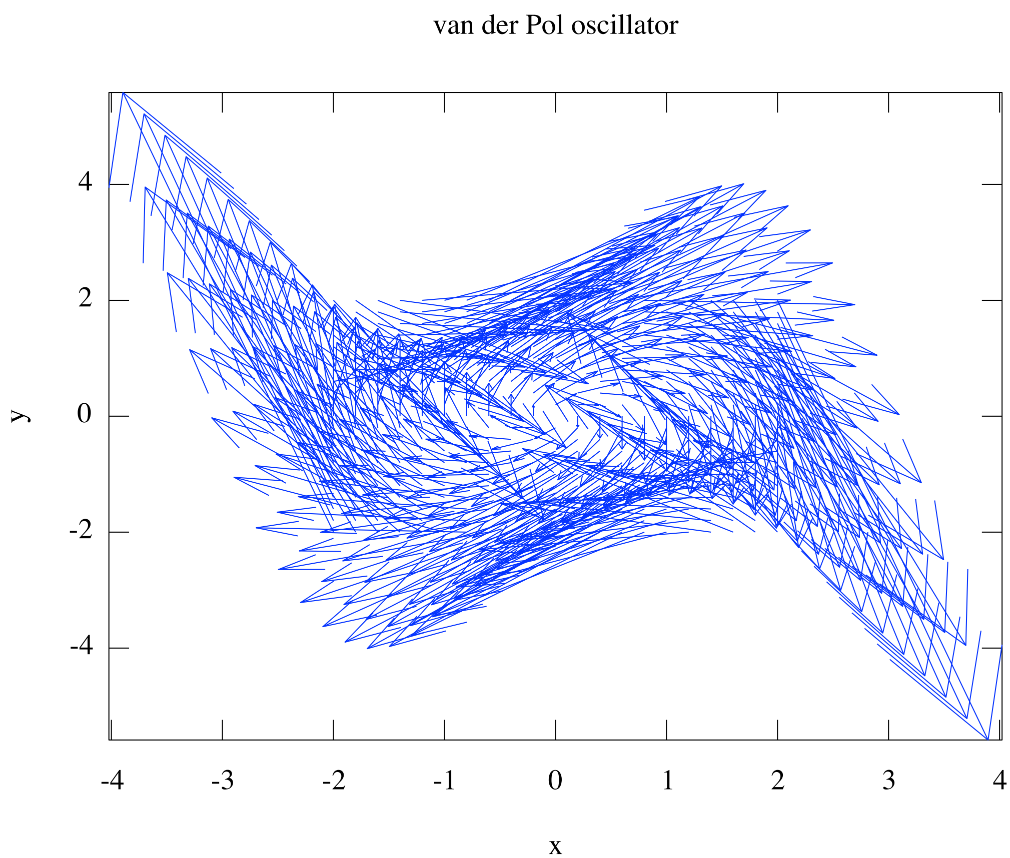
For the given system, plot the phase portrait:

Given system:

$$\dot{x} = y$$
$$\dot{y} = -x + y\left(1 - x^2\right)$$

Phase portrait:

Phase portrait:



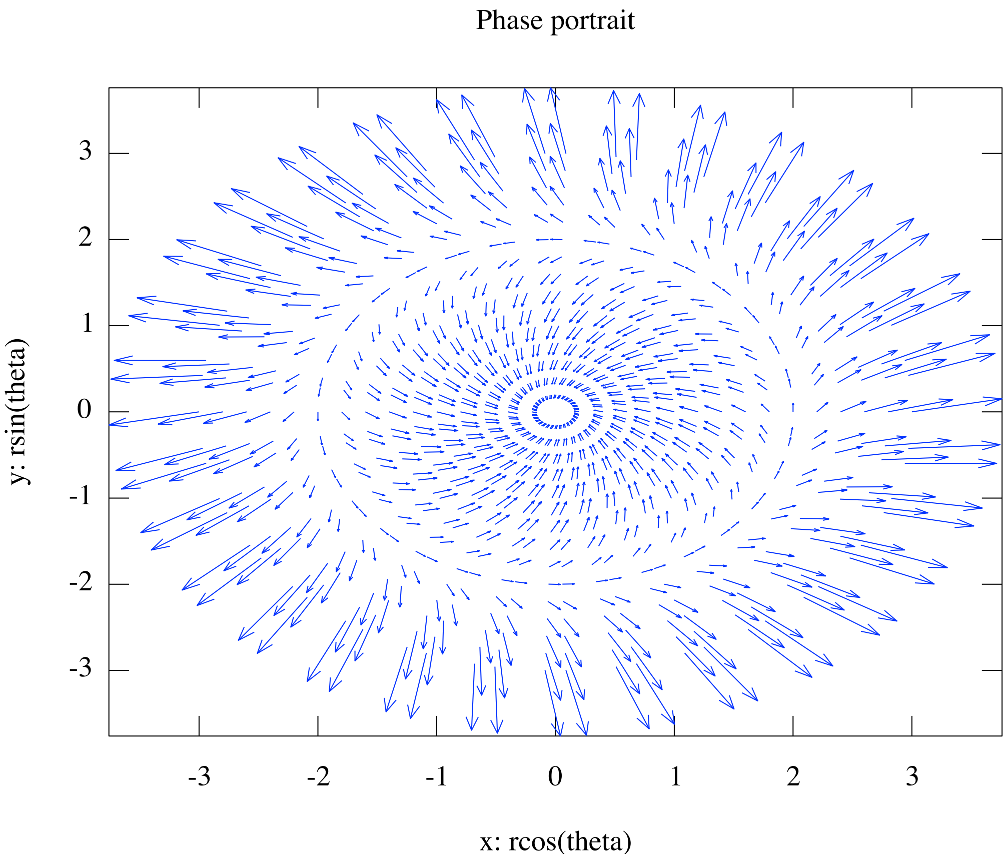
7.1.1:

Sketch the phase portrait for the following system:

Given system:

$$\dot{r} = r^3 - 4r$$
$$\dot{\theta} = 1$$

Phase portrait:



<u>7.2.10:</u>

Show that the given system has no closed orbits, by constructing a Liapunov funtion

V = $ax^2 + by^2$ with suitable a,b.:

Given system:

$$\begin{aligned}\dot{x} &= y - x^3 \\ \dot{y} &= -x - y^3\end{aligned} \Rightarrow \left(\mathrm{x}^*, y^*\right) = (0,0)$$

Liapunov function:

$$\mathrm{V}(\mathrm{x},\mathrm{y}) = ax^2 + by^2 \qquad \text{a,b to be decided}$$

$$\dot{V}(x,y) = 2ax\dot{x} + 2by\dot{y} = 2\left(xy(a-b) - ax^4 - by^4\right)$$

Solution:

$$a = b = 1$$

$$\dot{V}(x,y) = -2x^4 - 2y^4 < 0,\ (x,y) \neq (x^*,y^*)$$

$$V(x,y) = x^2 + y^2 > 0,\ (x,y) \neq (x^*,y^*)$$

7.4.1:

Show that the given equation has exactly one periodic solution, and classify its stability:

Given system:

$$\ddot{x} + \mu(x^2 - 1)\dot{x} + \tanh x = 0,\quad \forall \mu > 0$$

Liénard's equation:

$$\dot{x} = y$$
$$\dot{y} = -\tanh x - \mu(x^2 - 1)y$$

$$f(x) = \tanh x \ \ \&\ \ g(x) = \mu(x^2 - 1)$$

Satisfies Liénard's Theorem $\therefore$ system has a unique, stable limit cycle.

Fixed points:

$$(x^*,y^*) = (0,0)$$

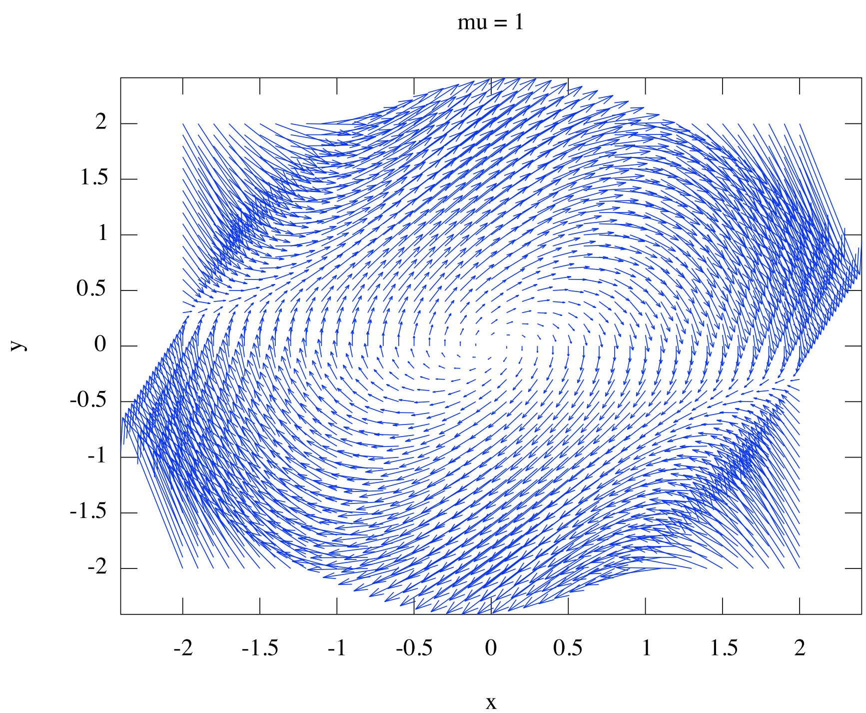
Jacobian matrix:

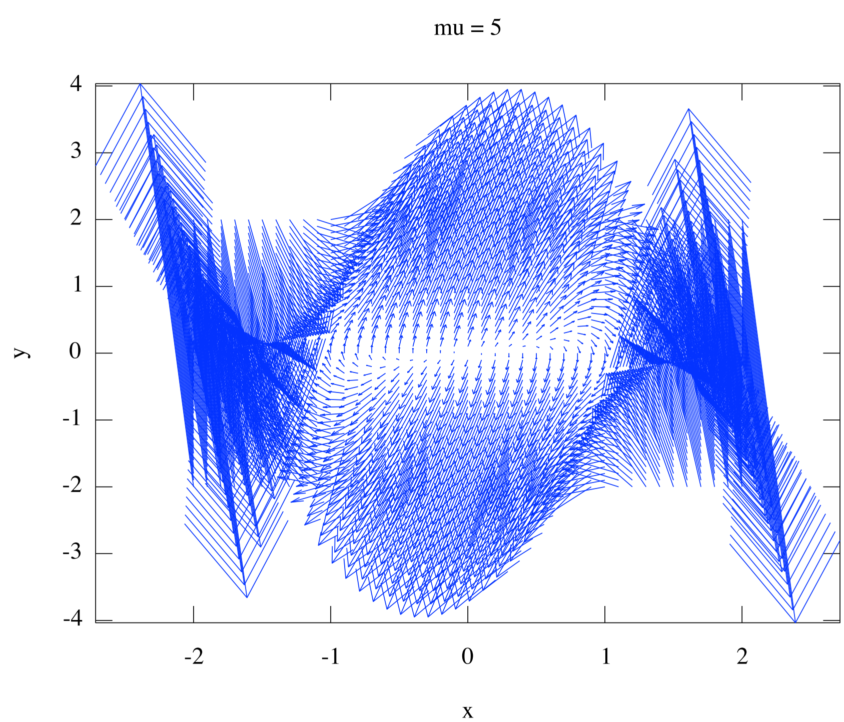
$$A = \begin{bmatrix} 0 & 1 \\ -\operatorname{sech}^2 x - 4\mu x^3 y & -\mu(x^2 - 1) \end{bmatrix}$$

Jacobian at fixed point:

$$A(x^*,y^*) = \begin{bmatrix} 0 & 1 \\ -1 & \mu \end{bmatrix} \Rightarrow \lambda = \frac{\mu \pm \sqrt{\mu^2 - 4}}{2}$$

Phase portraits:





7.4.2:

Consider the sytem:

$$\ddot{x} + \mu\left(x^4 - 1\right)\dot{x} + x = 0, \;\; \forall \mu > 0$$

a) Prove that the system has a unique stable limit cycle if $\mu > 0$

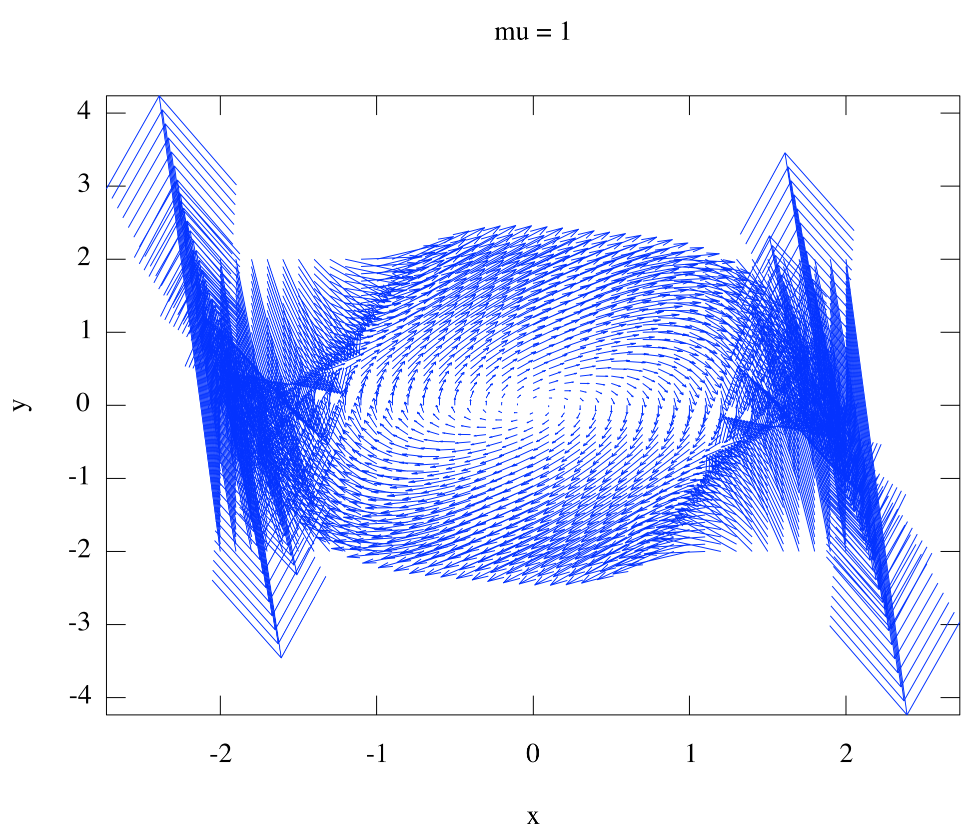
Liénard's equation:

$$\dot{x} = y$$
$$\dot{y} = -x - \mu\left(x^4 - 1\right)y$$

$$f(x) = x \;\;\&\;\; g(x) = \mu\left(x^4 - 1\right)$$

Satisfies Liénard's Theorem $\therefore$ system has a unique, stable limit cycle

b) Phase portraits:



Octave Code:

```
# prob1.m
function prob1
clear
figure(1);
hold off
[x,y] = meshgrid(-2:.2:2);
x_dot=y;
y_dot=-x+y.*(1-x.^2);
h=quiver(x,y,x_dot,y_dot);
set(h, "autoscalefactor", 5);
axis("tight");
title("van der Pol oscillator");
xlabel("x");
ylabel("y");
fixAxes;
endfunction

# prob2.m
function prob2
clear
figure(1);
hold off
[r,theta]=meshgrid(-3:.2:3);
r_dot=(r.^3)-4.*r;
theta_dot=1;
x=r.*cos(theta);
y=r.*sin(theta);
x_dot=r_dot.*cos(theta)-theta_dot.*r.*sin(theta);
y_dot=r_dot.*sin(theta)+theta_dot.*r.*cos(theta);
h=quiver(x,y,x_dot,y_dot);
set(h,"autoscalefactor",2);
axis("tight");
title("Phase portrait");
xlabel("x: rcos(theta)");
ylabel("y: rsin(theta)");
fixAxes;
endfunction;

# prob4.m
function prob4
clear
figure(1);
hold off
[x,y]=meshgrid(-2:.1:2);
```

```
mu=5;
x_dot=y;
y_dot=((1-(exp(2.*x)))./(1+(exp(2.*x))))-mu.*((x.^2)-1).*y;
h=quiver(x,y,x_dot,y_dot);
set(h,"autoscalefactor",2);
axis("tight");
title("mu = 5");
xlabel("x");
ylabel("y");
fixAxes;
endfunction

# prob5.m
function prob5
clear
figure(1);
hold off
[x,y]=meshgrid(-2:.1:2);
mu=1;
x_dot=y;
y_dot=-x-mu.*((x.^4)-1).*y;
h=quiver(x,y,x_dot,y_dot);
set(h,"autoscalefactor",2);
axis("tight");
title("mu = 1");
xlabel("x");
ylabel("y");
fixAxes;
endfunction
```