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**AMS 114**

**Homework #6**

**6.1.8:**

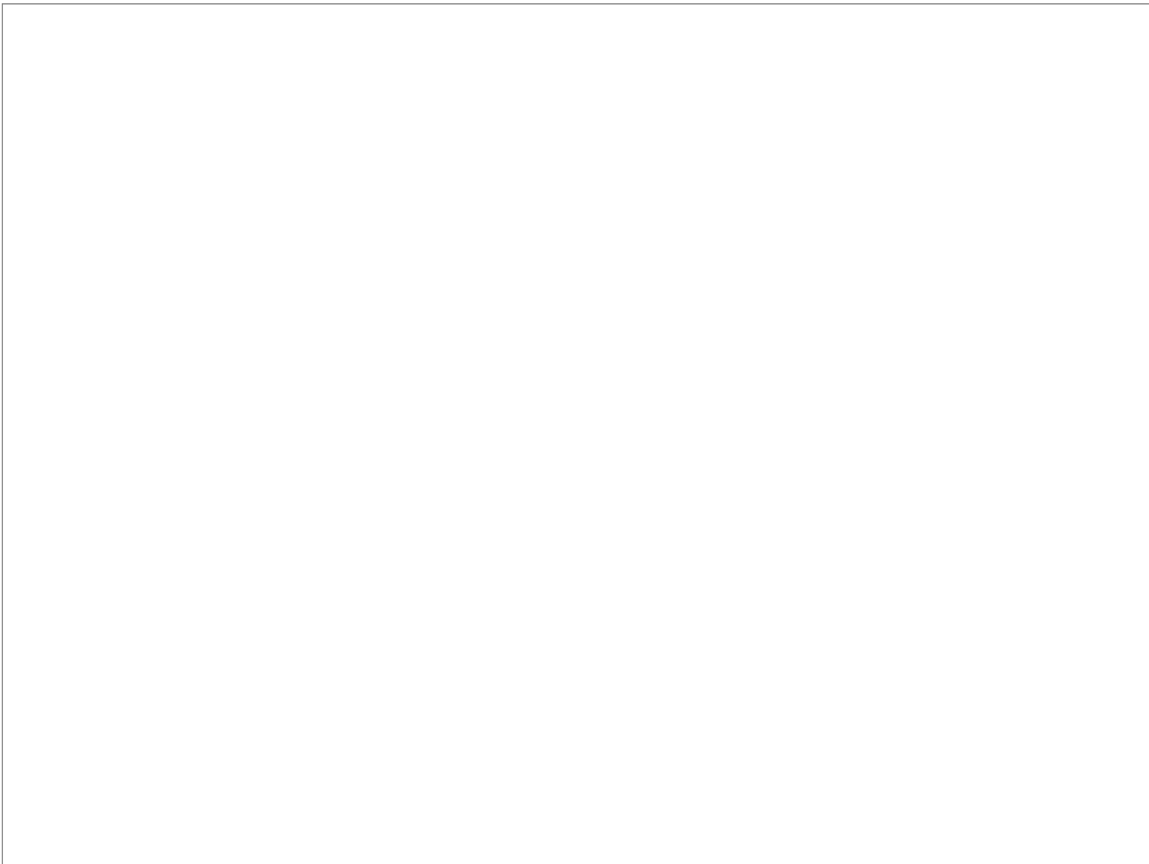
For the given system, plot the phase portrait.

Given system:

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= -x + y(1 - x^2)\end{aligned}$$

Phase portrait:

Phase portrait:

**7.1.1:**

Sketch the phase portrait for the following system.

Given system:

$$\dot{r} = r^3 - 4r$$

$$\dot{\theta} = 1$$

Phase portrait:



### **7.2.7:**

Consider the system:

$$f(x,y) = \dot{x} = y + 2xy$$

$$g(x,y) = \dot{y} = x + x^2 - y^2$$

a) Show that  $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$ .

$$\frac{\partial f}{\partial y} = 1 + 2x$$

$$\frac{\partial g}{\partial x} = 1 + 2x \Rightarrow \frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$$

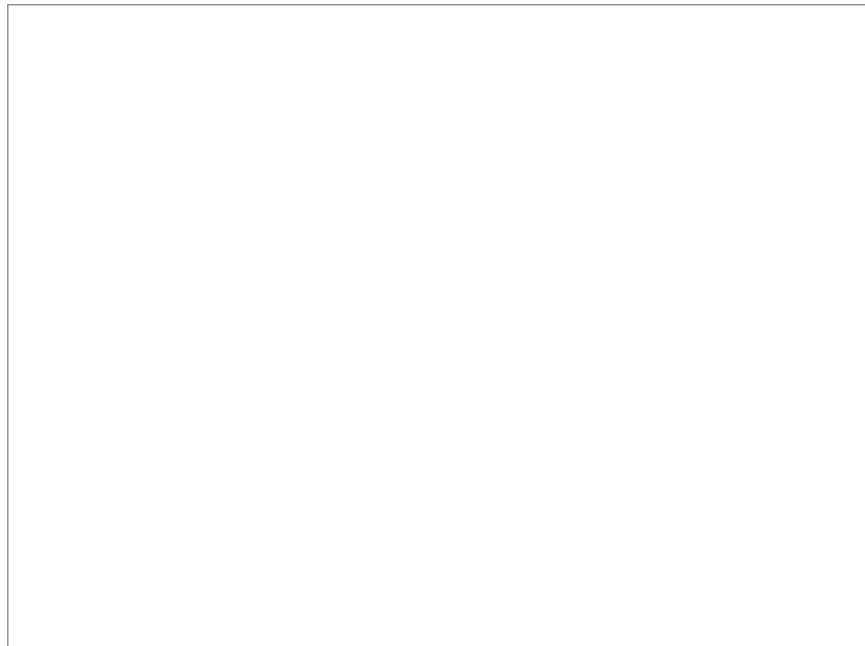
b) Find V.

$$-\frac{\partial V}{\partial x} = y + 2xy \Rightarrow -\int \partial V = \int (y + 2xy) \partial x \Rightarrow V = -xy - x^2 y$$

$$-\frac{\partial V}{\partial y} = x + x^2 - y^2 \Rightarrow -\int \partial V = \int (x + x^2 - y^2) \partial y \Rightarrow V = -xy - x^2 y + \frac{y^3}{3}$$

Since the both results of  $V$  are not equal,  $V$  is not a gradient system.

c) Sketch the phase portrait.



### **Midterm Problem 1:**

Figures (a) and (b) show the neighborhood of two different fixed points.

The eigendirections 1 and 2 are indicated in both figures and these

directions correspond to the eigenvalues  $\lambda_1$  and  $\lambda_2$ , respectively. What is

the correct statement about the relation between  $\lambda_1$  and  $\lambda_2$ :

4. Figure a:  $\lambda_1 > \lambda_2$  and Figure b:  $\lambda_1 > \lambda_2$

It is obvious from Figure (a) that both eigenvalues are

negative since the fixed point is a stable node. The reason  $\lambda_2$  is

smaller is because the lines move in that direction more quickly than in the direction concerned with  $\lambda_1$ .

It is obvious from Figure (b) that  $\lambda_1 > 0$  and  $\lambda_2 < 0$  since the eigendirections corresponding to these eigenvalues are unstable and stable respectively. This results in a saddle node and ensures that  $\lambda_2 < \lambda_1$  since  $\lambda_2$  is negative and  $\lambda_1$  is positive.

### **7.3.2:**

Using numerical integration, compute the limit cycle of the given system and verify that it lies in the trapping region you constructed.

Given system:

$$\begin{aligned}\dot{x} &= x - y - x(x^2 + 5y^2) \\ \dot{y} &= x + y - y(x^2 + y^2)\end{aligned}$$

### **Octave Code:**

```
# prob1.m
function prob1
clear
figure(1);
hold off
[x,y] = meshgrid(0:.1:2.5);
f1=x.*(3-2.*x-2.*y);
f2=y.*(2-x-y);
#[dx1,dy1] = gradient(f1,.1,.1);
#[dx2,dy2] = gradient(f2,.1,.1);
```

```

###
# Not likely correct
#quiver(x,y,dx1,dy1)
#hold on
#quiver(x,y,dx2,dy2)
###
h=quiver(x,y,f1,f2);
set (h, "autoscalefactor", 5);
axis("tight");
title("Rabbits versus Sheep");
xlabel("x: Rabbits");
ylabel("y: Sheep");
fixAxes;
endfunction

```

```

# prob2.m
function prob2
clear
figure(1);
hold off
[r,th] = meshgrid(0:.1:5);
f1=r.*(1-r.^2).*(9-r.^2);
f2=1;
h=quiver(r,th,f1,f2);
set (h, "autoscalefactor", 5);
axis("tight");
title("Phase Portrait: Polar Coordinates");
xlabel("r");
ylabel("theta");
fixAxes;
clear
figure(2);
hold off
[x,y] = meshgrid(-3.5:.1:3.5);
r=sqrt((x.^2)+(y.^2));
f1=r.*(1-r.^2).*(9-r.^2);
g1=(r*f1)./(x+(y.^2)./x);
g2=((r.^2)./x)+(y.*r*f1)./(r.^2);
h=quiver(x,y,g1,g2);
set (h, "autoscalefactor", 5);
axis("tight");
title("Phase Portrait: Cartesian Coordinates");
xlabel("x");
ylabel("y");
fixAxes;
endfunction

```

```
# prob3.m
function prob3
clear
figure(1);
hold off
[x,y] = meshgrid(-8:.4:8);
f1=y+2.*x.*y;
f2=x+(x.^2)-(y.^2);
h=quiver(x,y,f1,f2);
set(h,"autoscalefactor",5);
axis("tight");
title("Phase Portrait");
xlabel("x");
ylabel("y");
fixAxes;
endfunction
```