Derek Frank
dmfrank@ucsc.edu
AMS 147
Due: 1/27/10

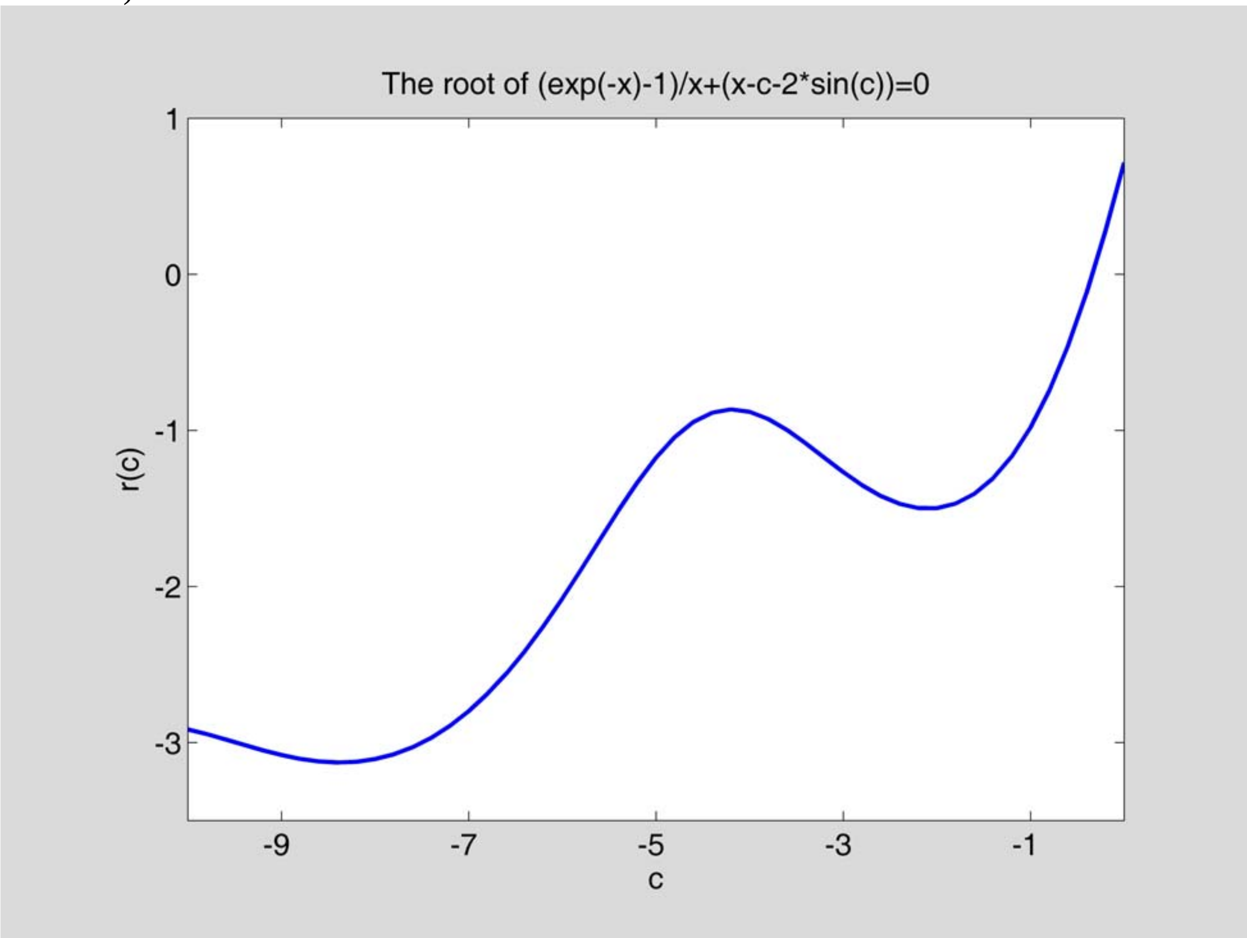
Homework #2

Part B:
(1):

i) The problem I am to solve is the non-linear equation *(exp(-x)-1)/x+(x-c-2*sin(c))=0*. I am also to plot the function's root as a function of *c* contained in [-10,0].

ii) Using Matlab, I am to use Newton's method to solve the given equation. Newton's method will produce a root as a function of *c*.

iii)



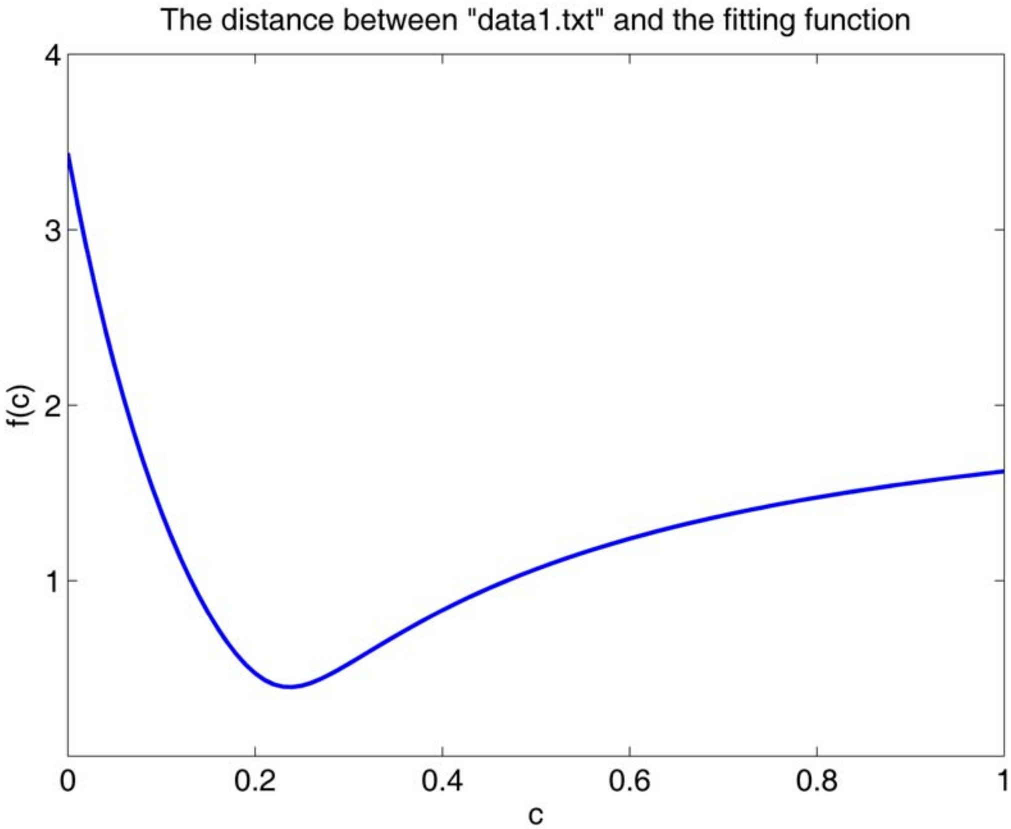
iv) The root of the given function is shown above contained on an interval where *c* is between [-10,0]. The root, on this interval, does not exceed a value above 1 or below -4. Although the root is wave-like and slightly decreases after a steady increase, it overall becomes increasingly larger as *c* gets bigger.

(2):

i) The problem I am to solve is to find the distance between data points found in "data1.txt" and the fitting function *f(x)=exp(-c*x)*cos(2*x)*. I am to plot this distance function, *dd(c)=sqrt(sum of (f(x(i))-y(i))^2)* (*i=0,1,…,50*), as a function of *c* in [0,1].

ii) Using Matlab, I am to graph the distance between the data and fitting function with a varying *c* value. Using a for loops to increment the *c* vector and its corresponding *d* (distance) vector. Each time around the loop I send to “dd.m” a *c* and all *x* and y values. The funtion calculates the fitting function *y* value and sends back a distance which I store in the *d* vector according to its corresponding *c* value. I am able to save information in the form of vectors and use these vectors to plot the curve.

iii)



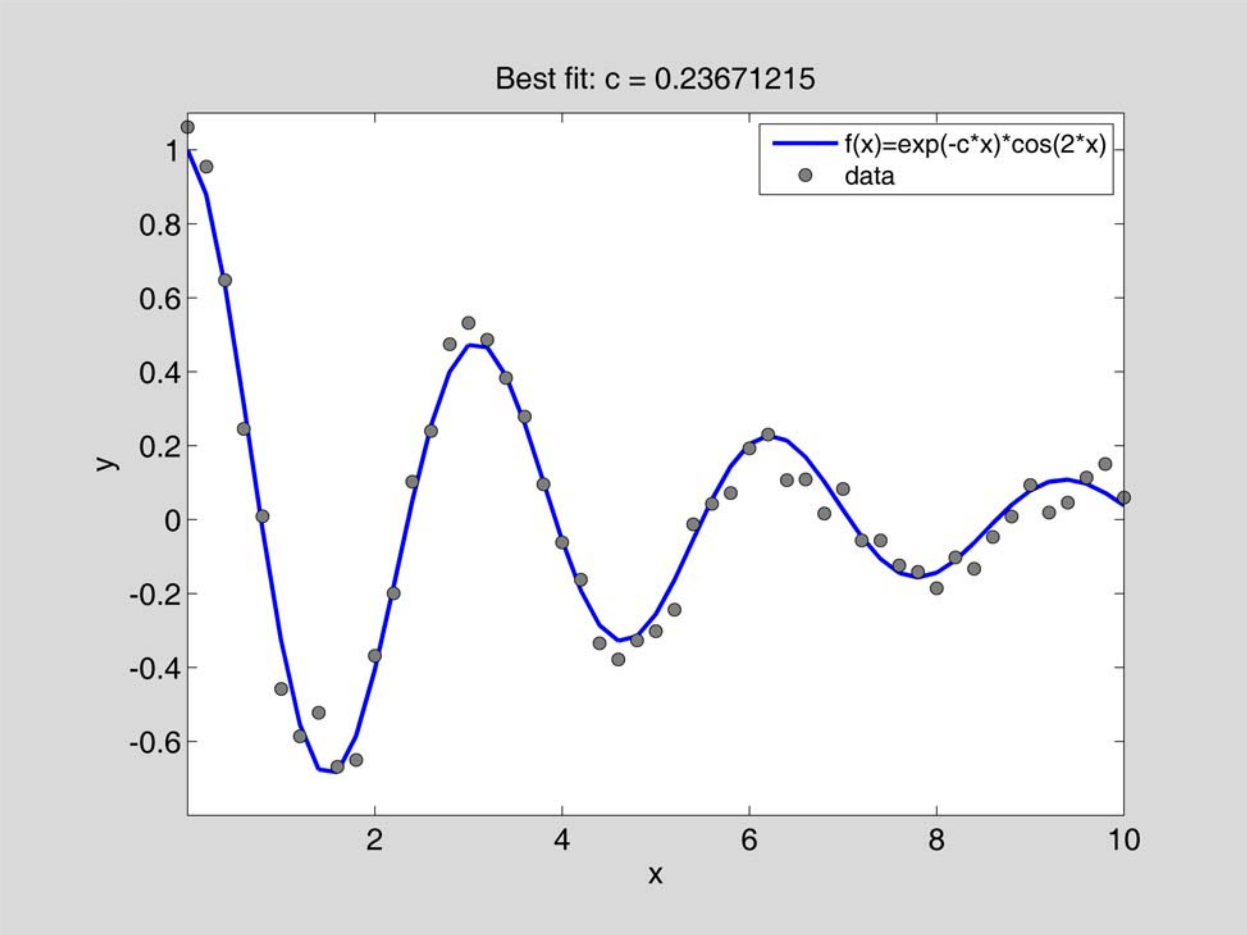
iii) These results show the distance (*f(c)*) between the fitting funcion and the given data points. This graph only shows the values of *f(c)* when *c* is between [0,1]. The greatest distance, on this graph, between the data points and the fitting function is greatest when c is 0 and quadratically decreases to a minimum (optimal value) at a little after *c*=.2. After the distance reaches a minimum and *c* keeps increasing, it does not grow like a quadratic funtion, but much more slowly and even appears to possibly be asymptotic at around *y*=1.8.

(3):

i) The problem I am to solve is find the optimal fitting function from the previous problem. That is find the best value of *c* that minimizes the distance between the given data in "data1.txt" and the fitting function *f(x)=exp(-c*x)*cos(2*x).*

ii) Using the golden search method to find the best value for c, I enter code in Matlab to calculate and graph my problems.

iii)



iv) The results show that the best value for *c* to optimize the distance is $c = .23671215$ (this value is very close to the average for *c* in homework assignment #1).

<u>APPENDIX</u>

(Part B1):

"f.m"

```
function [y]=f(x,c)
%
% This function calculates f(x) for a given x and a parameter c.
%
y=(exp(-x)-1)./x+(x-c-2.*sin(c));
%
```

"fp.m"

```
function [y]=fp(x,c)
%
% This function calculates df(x)/dx for a given x and a parameter
% c.
%
y=-1.*(exp(-x)./(x.^2))-1.*(exp(-x)./x)+1./(x.^2)+2;
%
```

"fp_2.m"

```
function [y]=fp_2(x,c)
%
```

```
% This function calculates df(x)/dx for a given x and a parameter
% c.
%
h=1.0e-5;
y=(f(x+h,c)-f(x-h,c))/(2*h);
%
```

"newton.m"

```
function [r, n]=newton(funct_name, deriv_name, c, x0, tol)
%
% This function finds a root of f(x) = 0 using Newton's method.
%
% Input:
%   funct_name: the name of the .m file for calculating the
%     function f(x)
%   deriv_name: the name of the .m file for calculating df(x)/dx
%   c:  a parameter in functions "f" and "fp"
%   x0: the starting point for Newton's method
%   tol: the error tolerance
% Output:
%   r:  the root found
%   n:  the number of iterations
%
err=1.0;
n=0;
%
while(err > tol),
  n=n+1;
  f_x0=feval(funct_name,x0,c);
  fp_x0=feval(deriv_name,x0,c);
  x1=x0-f_x0/fp_x0;
  err=abs(x1-x0);
  x0=x1;
end
%
r=x0;
%
```

"calc_data.m"

```
%
% Consider the non-linear equation (exp(-x)-1)/x+(x-c-2*sin(c))=0.
% Here "c" is a parameter in the equation. The root of the
% equation varies with "c" and thus the root is a function of "c".
% This code calculates the root for "c" in [-10,0] and stores the
% data in data1.mat. Later on the data is used in "plot_curve.m
% to plot the root as a function of "c".
%
clear
%
c_v=[-10:0.2:0];
nc=size(c_v,2);
r_v=zeros(1,nc);
tol=1.0e-10;
r=1;
%
for i=1:nc,
  c=c_v(i);
  [r, n]=newton('f', 'fp_2', c, r, tol);
  r_v(i)=r;
end
```

```
%
save data1 c_v r_v
%
```

"plot_curve.m"

```
%
% Consider the non-linear equation (exp(-x)-1)/x+(x-c-2*sin(c))=0.
% Here "c" is a parameter in the equation. The root of the
% equation varies with "c" and thus the root is a function of "c".
% The code "calc_data.m" calculates the root for "c" in [-10,0]
% and stores the data in data1.mat.
% This code loads in data1.mat and plots the root as a function
% of "c".
%
clear
clf
axes('position',[0.15,0.13,0.75,0.75])
%
load data1.mat
plot(c_v, r_v,'linewidth',2.0)
axis([-10,0,-3.5,1.0])
set(gca,'xtick',[-9:2:0])
set(gca,'ytick',[-3:1:1])
set(gca,'fontsize',14)
xlabel('c')
ylabel('r(c)')
title('The root of (exp(-x)-1)/x+(x-c-2*sin(c))=0')
%
```

(Part B2):

"data1.txt"

```
%       x               y
%
   0.0000000e+00   1.0616627e+00
   2.0000000e-01   9.5438313e-01
   4.0000000e-01   6.4726669e-01
   6.0000000e-01   2.4543142e-01
   8.0000000e-01   8.6474234e-03
   1.0000000e+00  -4.5847332e-01
   1.2000000e+00  -5.8629116e-01
   1.4000000e+00  -5.2270060e-01
   1.6000000e+00  -6.6897708e-01
   1.8000000e+00  -6.5025323e-01
   2.0000000e+00  -3.6829629e-01
   2.2000000e+00  -1.9946104e-01
   2.4000000e+00   1.0214847e-01
   2.6000000e+00   2.3939828e-01
   2.8000000e+00   4.7441685e-01
   3.0000000e+00   5.3204202e-01
   3.2000000e+00   4.8645531e-01
   3.4000000e+00   3.8277756e-01
   3.6000000e+00   2.7859469e-01
   3.8000000e+00   9.5304196e-02
   4.0000000e+00  -6.1937557e-02
   4.2000000e+00  -1.6264995e-01
   4.4000000e+00  -3.3465224e-01
   4.6000000e+00  -3.7848057e-01
   4.8000000e+00  -3.2684557e-01
   5.0000000e+00  -3.0202142e-01
```

```
   5.2000000e+00  -2.4418751e-01
   5.4000000e+00  -1.2606921e-02
   5.6000000e+00   4.2709432e-02
   5.8000000e+00   7.1364914e-02
   6.0000000e+00   1.9256373e-01
   6.2000000e+00   2.2975779e-01
   6.4000000e+00   1.0651973e-01
   6.6000000e+00   1.0873230e-01
   6.8000000e+00   1.6313790e-02
   7.0000000e+00   8.2753168e-02
   7.2000000e+00  -5.6705147e-02
   7.4000000e+00  -5.6552321e-02
   7.6000000e+00  -1.2417349e-01
   7.8000000e+00  -1.4140695e-01
   8.0000000e+00  -1.8606224e-01
   8.2000000e+00  -1.0253530e-01
   8.4000000e+00  -1.3337982e-01
   8.6000000e+00  -4.7083796e-02
   8.8000000e+00   8.5379099e-03
   9.0000000e+00   9.3244738e-02
   9.2000000e+00   1.8795176e-02
   9.4000000e+00   4.5569780e-02
   9.6000000e+00   1.1312777e-01
   9.8000000e+00   1.5024618e-01
   1.0000000e+01   5.9238496e-02
%
```

"dd.m"

```
function [dist]=dd(c,x,y)
%
% This function calculates the distance between the data
% and the function f(x)=exp(-c*x)*cos(2x)
%
fx=exp(-c*x).*cos(2*x);
dist=norm(fx-y);
```

"calc_data.m”

```
%
% Consider the non-linear equation sqrt(sum of (f(x(i))-y(i))^2),
% where, depending on c, x(i) is an array of values produced by
% the fitting funtion found in "dd.m" and y(i) is an array of
% values found in "data1.txt".  Here "c" is a parameter in the
% equation. The distance of the equation varies with "c" and thus
% the distance is a function of "c".
% This code calculates the distance for "c" in [0,1] and stores
% the data in data1.mat. Later on the data is used "plot_curve.m"
% to plot he distance as a function of "c".
%
load -ascii data1.txt

% vector of x values found in "data1.txt"
x=data1(:,1);
% array/vector of y values found in "data1.txt"
y1=data1(:,2);
% array/vector of 100 c values between [0,1]
c_v=[0:.01:1];
% scalar number of c values in array/vector c_v
nc=size(c_v,2);
% initialization for array/vector of distance values
```

```
d_v=zeros(1,nc);

for i=1:nc,
  c=c_v(i);
  d(i)=dd(c,x,y1);
end
%
save data1 c_v y_v
```

“plot_curve.m”

```
%
% Consider the non-linear equation fitting function
% f(x)=exp(-c*x)*cos(2*x).  With a given x, "c" is a parameter in
% the equation.  The code "calc_data.m" calculates
% sqrt(sum of (f(x(i))-y(i))^2), the distance between the data
% given in "datal.txt" and the fitting function for "c" in [0,1]
% and stores the data in data1.mat.  This code loads in data1.mat
% and plots the distance as a function of "c".
%
clear
figure(3)
clf
axes('position',[0.15,0.13,0.75,0.75])
%
load data1.mat
plot(c_v, y_v,'linewidth',2.0)
axis([0,1,0,4])
set(gca,'xtick',[0:.2:1])
set(gca,'ytick',[1:1:4])
set(gca,'fontsize',14)
xlabel('c')
ylabel('f(c)')
title('The distance between "data1.txt" and the fitting function')
%
```

(Part B3):

"dd.m"

```
function [dist]=dd(c,x,y)
%
% This function calculates the distance between the data
% and the function f(x)=exp(-c*x)*cos(2x)
%
fx=exp(-c*x).*cos(2*x);
dist=norm(fx-y);
```

"golden.m"

```
%
% This code first reads in data (x, y) from "data1.txt."
% Then it uses the golden search method to find the value of c
% such that the distance between the data and the function
% f(x)=exp(-c*x)*cos(2*x) is minimized.
% Finally, it plots the data along with the best fit.
%
clear
clf reset
axes('position',[0.15,0.13,0.75,0.75])
%
load -ascii data1.txt
x=data1(:,1);
y=data1(:,2);
```

```
%
a=0;
b=2;
tol=1.0e-10;
n=0;
%
g=(sqrt(5)-1)/2;
r1=a+(b-a)*(1-g);
f1=dd(r1,x,y);
r2=a+(b-a)*g;
f2=dd(r2,x,y);
%
while (b-a) > tol,
  n=n+1;
  if f1 < f2,
    b=r2;
    r2=r1;
    f2=f1;
    r1=a+(b-a)*(1-g);
    f1=dd(r1,x,y);
  else
    a=r1;
    r1=r2;
    f1=f2;
    r2=a+(b-a)*g;
    f2=dd(r2,x,y);
  end
end
c0=(a+b)/2;
%
fx=exp(-c0*x).*cos(2*x);
plot(x,fx,'b-','linewidth',2.0)
hold on
plot(x,y,'ko','markerfacecolor',[0.5,0.5,0.5])
axis([0,10,-.8,1.1])
set(gca,'xtick',[2:2:10])
set(gca,'ytick',[-.6:0.2:1.1])
set(gca,'fontsize',14)
xlabel('x')
ylabel('y')
title(['Best fit: c = ',num2str(c0,8)])
h1=legend('f(x)=exp(-c*x)*cos(2*x)','data');
set(h1,'fontsize',12)
%
disp('   ')
disp(['  The function attains a minimum at c0 =
',num2str(c0,'%24.16e'),'.'])
disp(['  It takes n = ',num2str(n),' iterations to reach err <=
',num2str(tol),'.'])
disp('   ')
```