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AMS 114

Homework #6

6.1.8:

For the given system, plot the phase portrait.

Given system:

$$\dot{x} = y$$
$$\dot{y} = -x + y\left(1 - x^2\right)$$

Phase portrait:

Phase portrait:

7.1.1:

Sketch the phase portrait for the following system.

Given system:

$$\dot{r} = r^3 - 4r$$
$$\dot{\theta} = 1$$

Phase portrait:

7.2.7:

Consider the system:

$$f(x,y) = \dot{x} = y + 2xy$$
$$g(x,y) = \dot{y} = x + x^2 - y^2$$

a) Show that $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$.

$$\frac{\partial f}{\partial y} = 1 + 2x$$
$$\frac{\partial g}{\partial x} = 1 + 2x \Rightarrow \frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$$

b) Find V.

$$-\frac{\partial V}{\partial x} = y + 2xy \Rightarrow -\int \partial V = \int (y + 2xy)\partial x \Rightarrow \mathrm{V} = \text{-xy} - \mathrm{x}^2\mathrm{y}$$
$$-\frac{\partial V}{\partial y} = x + x^2 - y^2 \Rightarrow -\int \partial V = \int (x + x^2 - y^2)\partial y \Rightarrow \mathrm{V} = \text{-xy} - \mathrm{x}^2\mathrm{y} + \frac{\mathrm{y}^3}{3}$$

Since the both results of V are not equal, V is not a gradient system.

c) Sketch the phase portrait.

Midterm Problem 1:

Figures (a) and (b) show the neighborhood of two different fixed points. The eigendirections 1 and 2 are indicated in both figures and these directions correspond to the eigenvalues λ_1 and λ_2, respectively. What is the correct statement about the relation between λ_1 and λ_2:

4. Figure a: $\lambda_1 > \lambda_2$ and Figure b: $\lambda_1 > \lambda_2$

It is obvious from Figure (a) that both eigenvalues are negative since the fixed point is a stable node. The reason λ_2 is smaller is because the lines move in that direction more quickly than in the direction concerned with λ_1.

It is obvious from Figure (b) that $\lambda_1 > 0$ and $\lambda_2 < 0$ since the eigendirections corresponding to these eigenvalues are unstable and stable respectively. This results in a saddle node and ensures that $\lambda_2 < \lambda_1$ since λ_2 is negative and λ_1 is positive.

<u>7.3.2:</u>

Using numerical integration, compute the limit cycle of the given system and verify that it lies in the trapping region you constructed.

Given system:

$$\dot{x} = x - y - x\left(x^2 + 5y^2\right)$$
$$\dot{y} = x + y - y\left(x^2 + y^2\right)$$

<u>Octave Code:</u>

```
# prob1.m
function prob1
clear
figure(1);
hold off
[x,y] = meshgrid(0:.1:2.5);
f1=x.*(3-2.*x-2.*y);
f2=y.*(2-x-y);
#[dx1,dy1] = gradient(f1,.1,.1);
#[dx2,dy2] = gradient(f2,.1,.1);
```

```
###
# Not likely correct
#quiver(x,y,dx1,dy1)
#hold on
#quiver(x,y,dx2,dy2)
###
h=quiver(x,y,f1,f2);
set (h, "autoscalefactor", 5);
axis("tight");
title("Rabbits versus Sheep");
xlabel("x: Rabbits");
ylabel("y: Sheep");
fixAxes;
endfunction

# prob2.m
function prob2
clear
figure(1);
hold off
[r,th] = meshgrid(0:.1:5);
f1=r.*(1-r.^2).*(9-r.^2);
f2=1;
h=quiver(r,th,f1,f2);
set (h, "autoscalefactor", 5);
axis("tight");
title("Phase Portrait: Polar Coordinates");
xlabel("r");
ylabel("theta");
fixAxes;
clear
figure(2);
hold off
[x,y] = meshgrid(-3.5:.1:3.5);
r=sqrt((x.^2)+(y.^2));
f1=r.*(1-r.^2).*(9-r.^2);
g1=(r.*f1)./(x+(y.^2)./x);
g2=((r.^2)./x)+(y.*r.*f1)./(r.^2);
h=quiver(x,y,g1,g2);
set (h, "autoscalefactor", 5);
axis("tight");
title("Phase Portrait: Cartesian Coordinates");
xlabel("x");
ylabel("y");
fixAxes;
endfunction
```

```
# prob3.m
function prob3
clear
figure(1);
hold off
[x,y] = meshgrid(-8:.4:8);
f1=y+2.*x.*y;
f2=x+(x.^2)-(y.^2);
h=quiver(x,y,f1,f2);
set(h,"autoscalefactor",5);
axis("tight");
title("Phase Portrait");
xlabel("x");
ylabel("y");
fixAxes;
endfunction
```