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AMS 114

Homework #7

8.2.2:

Show that the given system has pure imaginary eigenvalues when $\mu = 0$:

Given system:

$$\begin{aligned} \dot{x} &= -y + \mu x + xy^2 \\ \dot{y} &= x + \mu y - x^2 \end{aligned} \Rightarrow \begin{aligned} \dot{x} &= -y + xy^2 \\ \dot{y} &= x - x^2 \end{aligned}$$

Fixed points:

$$(x^*, y^*) = (0,0),\ (1,0),\ (1,1)$$

Jacobian:

$$A = \begin{bmatrix} y^2 & 2xy - 1 \\ 1 - 2x & 0 \end{bmatrix}$$

Eigenvalues at fixed points:

$$A(0,0) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \Rightarrow \lambda = \pm\iota$$

$$A(1,0) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \Rightarrow \lambda = \pm\iota$$

$$A(1,1) = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \Rightarrow \lambda = \frac{1 \pm \iota\sqrt{3}}{2}$$

- All have pure imaginary values.

8.2.3:

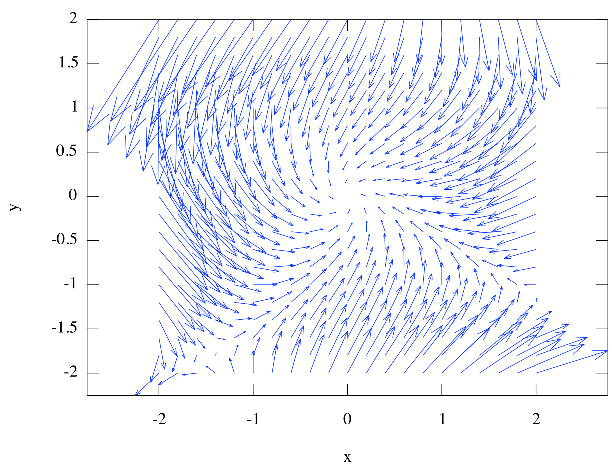
By plotting the phase portraits on the computer, show that the given system undergoes a Hopf bifurcation at $\mu = 0$. Is it subcritical, supercritical, or degenerate?

Given system:

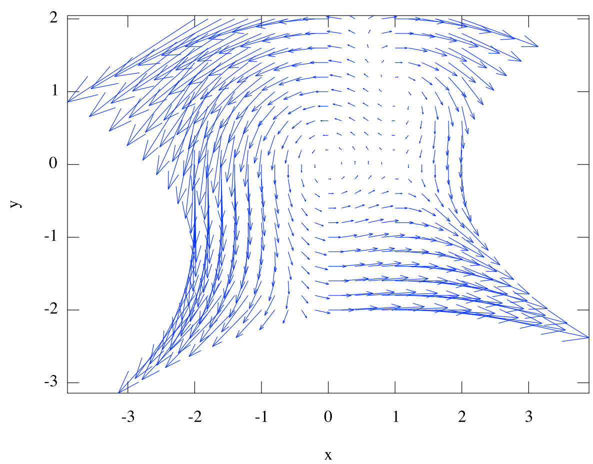
$$\begin{aligned} \dot{x} &= -y + \mu x + xy^2 \\ \dot{y} &= x + \mu y - x^2 \end{aligned}$$

Phase portrait:

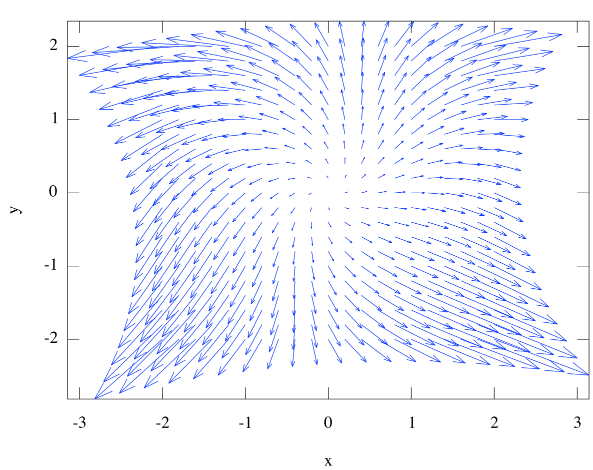
mu = -2



mu = 0



mu = 2



Classification:

- Degenerate Hopf bifurcation

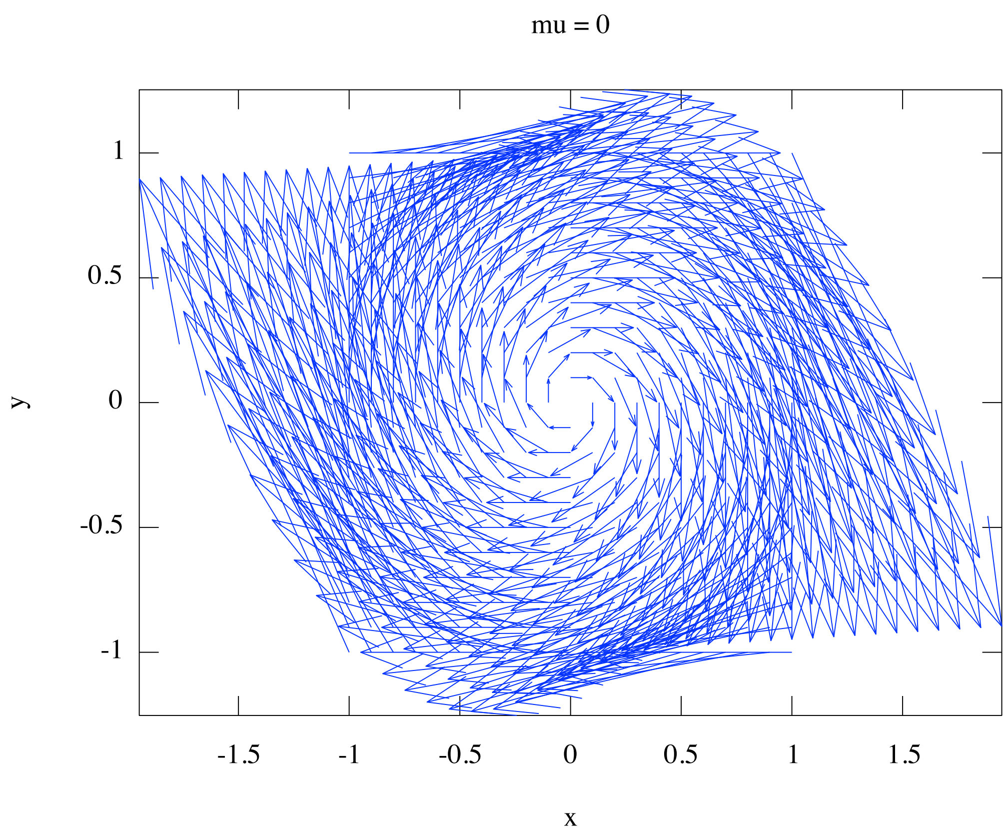
8.2.5:

For the given system, a Hopf bifurcation occurs at the origin when $\mu = 0$. Plot the phase portrait and determine whether the bifuracation is subcritical or supercritical:

Given system:

$$\dot{x} = y + \mu x$$
$$\dot{y} = -x + \mu y - x^2 y$$

Phase portrait:



Classification:

- Subcritical

Octave Code:

```
# prob2a.m
function prob2a
clear
figure(1);
hold off
[x,y] = meshgrid(-2:.2:2);
mu=-2;
x_dot=-y+(mu.*x)+(x.*(y.^2));
y_dot=x+(mu.*y)-(x.^2);
h=quiver(x,y,x_dot,y_dot);
set(h, "autoscalefactor", 4);
axis("tight");
title("mu = -2");
xlabel("x");
ylabel("y");
fixAxes;
endfunction

# prob3.m
function prob3
clear
figure(1);
hold off
[x,y] = meshgrid(-1:.1:1);
mu=0;
x_dot=y+(mu.*x);
y_dot=-x+(mu.*y)-((x.^2).*y);
h=quiver(x,y,x_dot,y_dot);
set(h, "autoscalefactor", 5);
axis("tight");
title("mu = 0");
xlabel("x");
ylabel("y");
fixAxes;
endfunction
```