**Derek Frank**

**AMS 114**

**Homework #6**

**6.1.8:**

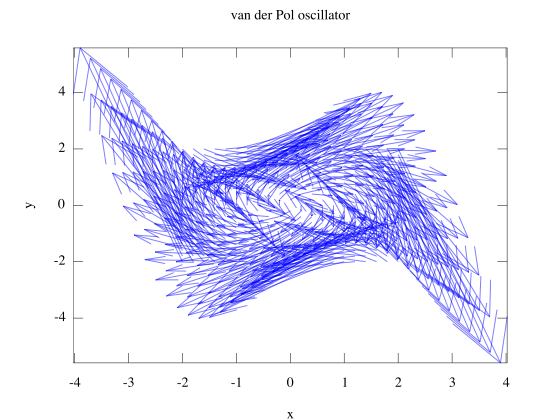
For the given system, plot the phase portrait:

Given system:



Phase portrait:

Phase portrait:



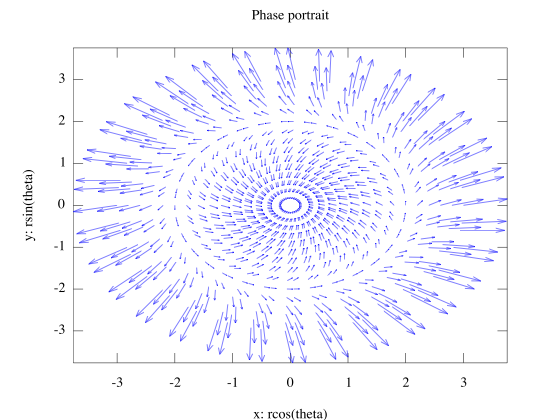
**7.1.1:**

Sketch the phase portrait for the following system:

Given system:



Phase portrait:



**7.2.10:**

Show that the given system has no closed orbits, by constructing a Liapunov funtion V = ax2 + by2 with suitable a,b.:

Given system:

Liapunov function:





Solution:







**7.4.1:**

Show that the given equation has exactly one periodic solution, and classify its stability:

Given system:



Liénard’s equation:





Satisfies Liénard’s Theorem  system has a unique, stable limit cycle.

Fixed points:

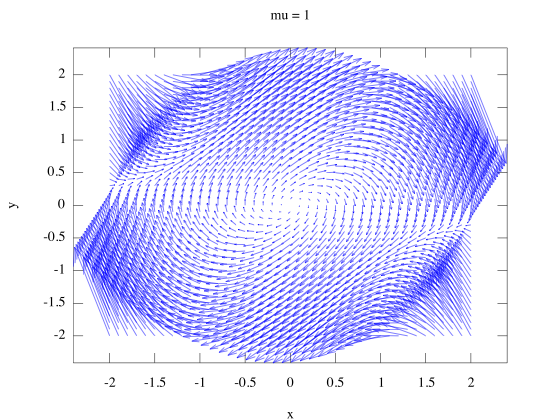


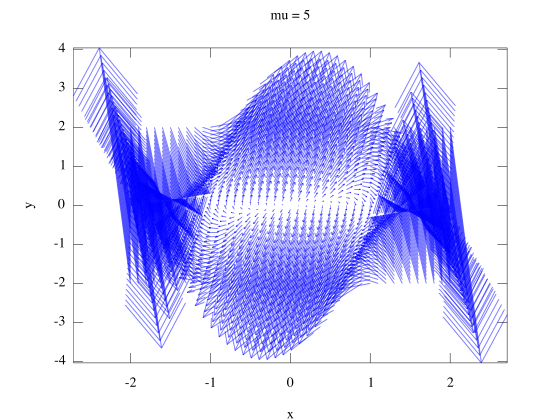
Jacobian matrix:



Jacobian at fixed point:



 Phase portraits:



**7.4.2:**

Consider the sytem:



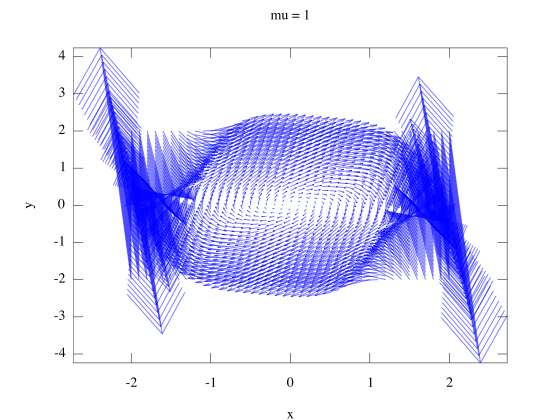
a) Prove that the system has a unique stable limit cycle if 

Liénard’s equation:





Satisfies Liénard’s Theorem  system has a unique, stable limit cycle

b) Phase portraits:

**Octave Code:**

# prob1.m

function prob1

clear

figure(1);

hold off

[x,y] = meshgrid(-2:.2:2);

x\_dot=y;

y\_dot=-x+y.\*(1-x.^2);

h=quiver(x,y,x\_dot,y\_dot);

set(h, "autoscalefactor", 5);

axis("tight");

title("van der Pol oscillator");

xlabel("x");

ylabel("y");

fixAxes;

endfunction

# prob2.m

function prob2

clear

figure(1);

hold off

[r,theta]=meshgrid(-3:.2:3);

r\_dot=(r.^3)-4.\*r;

theta\_dot=1;

x=r.\*cos(theta);

y=r.\*sin(theta);

x\_dot=r\_dot.\*cos(theta)-theta\_dot.\*r.\*sin(theta);

y\_dot=r\_dot.\*sin(theta)+theta\_dot.\*r.\*cos(theta);

h=quiver(x,y,x\_dot,y\_dot);

set(h,"autoscalefactor",2);

axis("tight");

title("Phase portrait");

xlabel("x: rcos(theta)");

ylabel("y: rsin(theta)");

fixAxes;

endfunction;

# prob4.m

function prob4

clear

figure(1);

hold off

[x,y]=meshgrid(-2:.1:2);

mu=5;

x\_dot=y;

y\_dot=((1-(exp(2.\*x)))./(1+(exp(2.\*x))))-mu.\*((x.^2)-1).\*y;

h=quiver(x,y,x\_dot,y\_dot);

set(h,"autoscalefactor",2);

axis("tight");

title("mu = 5");

xlabel("x");

ylabel("y");

fixAxes;

endfunction

# prob5.m

function prob5

clear

figure(1);

hold off

[x,y]=meshgrid(-2:.1:2);

mu=1;

x\_dot=y;

y\_dot=-x-mu.\*((x.^4)-1).\*y;

h=quiver(x,y,x\_dot,y\_dot);

set(h,"autoscalefactor",2);

axis("tight");

title("mu = 1");

xlabel("x");

ylabel("y");

fixAxes;

endfunction