University of Aberdeen
1495

Crowd Modeling

Liam Ramm

Supervisors:

Francesco Ginelli

Silke Henkes

This thesis is presented in partial fulfilment of course PX4011

April 11, 2018

Abstract

Crowds are an example of a complex system that displays emergent behaviour. Such behaviour is a result of the aggregation of interactions between many individuals. Understanding crowd dynamics is very important in a growing society and can have wide reaching benefits in design, safety and transport. Understanding group behaviour has even further potential for applications involving animal behaviour, cell growth and emerging fields such as swarm intelligence. The goal of this project is to create an implementation of a flexible crowd model that is capable of exploring and describing some frequently observed crowd phenomena. I combine this with a review of some of the current approaches to modeling group behaviour, and identify techniques in computation and mathematics that are applicable to the problem. I conclude by applying the model to a common evacuation scenario and identify the role of exits and obstacles to evacuation speed and the danger of crushing due to high pressure zones.

Declaration

This report is entirely my own composition. It has not been accepted in any previous application for a degree. It is a record of my work and all verbatim extracts have been distinguished by quotation marks. My sources of information have been specifically acknowledged.

Signed:
Liam Ramm

March 2016

Acknowledgements

I would like to thank my supervisors Francesco and Silke for their continued help and support throughout the duration of this project. With special thanks to Silke for going above and beyond in my hour of need. I would also like to thank my friends Anna and Zack for keeping me company through the late evenings in the library, and the endless supply of biscuits were certainly a motivational force to be reckoned with.

Contents

Chapter 1

Introduction

An understanding the flow of people is very important in today's society. It has the potential to greatly influence and improve the design of spaces, transport systems and contingency planning. As humans, we are predisposed to seek out each other and interact socially. Whether for religious beliefs, musical experiences or through necessary commuting, we become part of the flow on a regular basis. Large sporting events, concerts or other entertainment attract tens of thousands of people routinely. Even in day to day life we frequently find ourselves in large groups, as passengers gathering at transportation terminals. With the world population surpassing 7.3 billion recently, and projections forecasting a rise to beyond 8.3 billion by 2030 [4] what once where small congregations are evolving into large unwieldy masses. With the new found ease of long-distance travel only accelerating the development, allowing like-minded individuals to gather more frequently, the phenomena of densely packed crowds is moving from being a rare occurrence to more of an everyday situation. Understanding the dynamics of such gatherings can play an important role in planning and catering for them. Better design of buildings and spaces can improve convenience, productivity and safety.

Crowds are large concentrations of people, often moving with more or less common goals and therefore predictable and safe flow. Usually these large gatherings grow and move without incident. But, through flawed design of facilities or poor crowd management, too frequently they can develop into dangerous situations that result in injury and death. When looking into crowd behaviour and dynamics in humans its relevance comes to the fore-front in extreme examples such as disasters. Therefore, It is inevitable that we become focused on understanding how such disasters occur so we can reduce the numbers of lives lost yearly in what are often after the fact deemed preventable accidents. There has been a lot of interest in this field especially in recent years. Since 1999 there have been at least 44 recorded cases of crowd crushing around the world. The Hajj pilgrimage to Mecca is a good example of a large human gathering that happens each year. With the number of pilgrims rising annually the facilities and safety measures have been pushed to the limits on many occasions, leaving a history of serious accidents including serious injuries and fatalities. Due to the frequent nature of the event and the yearly struggle to improve conditions it has offered a good source of empirical evidence for many studies on crowd-dynamics [12]. This is essential in a field that lacks the luxury of experimental data due to the ethical concerns that any potential study would run into, although studies into panic have been carried out on animal test subjects [23].

This project aims to contribute to the study of group movement and collective motion by analysing and developing models that exhibit various aspects of the phenomenon, looking to correlate and isolate the causes of both desired and dangerous behaviour alike, in order to better understand this feature. With the potential implication of improving our world and the living conditions of those who inhabit it. This is an investigation into this fascinating but potentially dangerous occurrence, which is anything but an anomaly.

1.1 Previous Work

Over the years there have been many studies of collective motion. Groups of individuals move through space as a unit only if there is cooperation through co-direction. The focus of these studies ranges from cell and bacteria movement to humans, birds, fish and other animals [11][13]. More recently, similar behaviour has been uncovered and investigated outside the domain of living matter in inorganic materials. Using purely mechanical means to induce movement, the phenomena has been recreated in actuated nonliving components [8][22]. It is not that surprising that many groups have gravitated towards the challenge of improving our explanations of such a ubiquitous phenomena. The study and understanding of human behaviour in particular has potential for high impact in our society, with many applications.

1.1.1 Self Organisation

When people or animals move together they tend to form cohesive groups which adopt some form of general coordination. This coordination emerges as a result of local interactions between individuals and their immediate neighbours. The resulting groups are often able to remain cohesive even through outside perturbation. It is the spontaneous alignment and cooperative movement of groups based on localised influences that is usually considered self organisation [3].

1.1.2 Multi-agent Approach

Discerning the principles and properties of emergent collective motion holds a part of the puzzle of our understanding of biological systems. Through particle models we can build scenarios for deeper study in the field. Borrowing their structure from classical Newtonian mechanics, agent based models provide a way to uncover the governing processes at work by representing individuals with individual attributes interacting with one another. Due to the lack of computer processing power, collective behaviour in multi-particle gas and liquid systems were originally studied almost exclusively through the use of partial differential equations to ultimately simulate the systems on a macroscopic scale. Such an approach has an inherent problem in that it leads to a loss of information about the behaviour of the individuals themselves and their respective influence on the system as a whole. This induced an implicit inflexibility when it came to early models which in turn limited their scenarios and applications. However, today computing and programming techniques have improved to the point where simulating complex multi-particle or multi-agent systems as interacting individuals is no longer infeasible, even without specialist hardware. The more recent approach centers around establishing procedural rules which each of the entities adheres to, thus preserving the individualistic nature. This approach allows for very simple rules to govern the behaviour, usually interpreted more or less directly from real world observations. Another major advantage of such systems is the ability to vary personal attributes allowing more realistic analogue of real world systems or groups [26]. For example, a complex intelligent system can be viewed as a collection of interacting entities cooperating through alignment or avoidance rules and actions. The transmission and adjustment of their personal reactionary adjustments can communicate the presence of obstacles or dangers to the entire group [1].

Multi-agent models do have several disadvantages. Models in general should ideally be defined by simple structures and although these models start very simple, they can become very complex with the introduction of more parameters. These more complex models need a large number of such variables, which may not be easily identifiable or definitely distinguished. This can make analysis and correlation between models and experiments or observations particularly difficult. Also, the increased complexity often directly leads to implementations that approach incomprehensibility in order to avoid the return of computational constraints [26]. The collective dynamics of an overall system cannot be easily ascertained by studying the dynamics and interaction of few agents, this can introduce further issues when scaling the model . One of the main limitations of microscopic scale models emerges from the difficulty in transferring the microscopic information to the macroscopic level as measurable variables [1]. Perhaps the least compromising approach is multi-scale modeling, a marrying of both the macroscopic and microscopic techniques for a more complete picture. Despite these shortcomings such modeling techniques having been used to varying degrees of success, however it is clear that a universal model that is representative in all situations has not yet been formulated. This is not uncommon in the study of similar complex systems and does not imply that no useful results on emergent phenomena can be inferred [1]. Discovering the relationships and influences in this phenomena is likely an incremental process that builds upon many investigations, as with much of science. Given the proven strengths of the multi-agent or microscopic description when applied to crowds, we will be focusing on this technique in our models moving forward in this investigation.

1.1.3 Multi-agent applied to crowds

The application of multi-agent models to authentic crowd simulation requires what are often additional important characteristics to be taken into account. These include the large influence of the geometric element of the environment confining and restricting the movement of pedestrians, and for some investigations, individual choice and variation. The majority of research into crowd behavioural phenomena and similarly flocking and swarms looks at the characteristic self-organising ability of pedestrians in various circumstances as a factor, with a variety of emerging behaviors [14]. If looking into disasters, the identification of the differences that define normal conditions vs panic and the shift between calm and panic circumstances is instrumental in the analysis of a situation. For example, panic induced fluctuations have been shown to cause interrupted flow, excessive pressure or fully jammed conditions [1]. Such factors can be a precursor to catastrophic events, often resulting in injury or death. It has been shown through both observational and experimental evidence that as crowd density increases, so does the risk of accidents [15][12]. The presence of a high density of individuals leads to a reduction in individualistic behaviour especially when coupled with confined or restricted movement, this has be explained somewhat by analogy to a lack of sensory information [7], this in itself increases the potential validity of what are often very simplistic models.

1.1.4 Social Force

An interesting example of the use of the multi-agent paradigm in the field of crowd simulation is the social-force model, introduced by Helbing et al. [16]. Their model is based on the assumption that pedestrians interact using the a social force, the concept introduces the desire to maintain a comfortable amount of personal space around each individual. The model attempts to cover several attributes of pedestrian movement. Pedestrians tend to follow the fastest route heading towards a well defined target. They have a desired speed to reflect the incentive to reach the desired goal and are attributed an individual speed that accounts for their situation and surroundings. They attempt to maintain a certain distance from other pedestrians through introduction of the short range repulsive social force, the distance can depend on the movement speed and local density. An attractive long-range force component can be introduced to account for the tendency for pedestrians to aggregate into groups, a phenomena often observed in empirical data [1]. [16]

The social force concept was later used in a model introduced by Szabo et al. [24] investigating phase transitions in the collective migration of tissue cells. A flocking model of similar basis as Vicsek, their model replaces the influence of the directions of motion of neighbors with a short-range repulsive force. The agents have no explicit short range or long range knowledge of the orientation of their neighbors, but have the tendency to align with their own direction of travel, this leads to similar emergence of collective motion but with different characteristics. The model was shown to display a continuous transition from an unordered phase of chaotic motion to an ordered phase of net transport[24]. This model was investigated further in a later paper by Henkes et al. [17], applied as the Active-Jam Model with a focus on the jamming behaviour of such particles at high densities in confined spaces. Due to its conceptual ability to define basic pedestrian traits such as social interaction and its demonstrated capacity to produce behaviours such as 'laminar', 'stop-and-go' and turbulent flows, this is the model that we have chosen a basis of our investigation. These features were identified as characteristic with regard to the nature of pedestrian flow in an earlier paper by Helbing et al. [15] as part of an investigation into the dynamics of crowd disasters utilising empirical data. A mathematical description of the model is discussed in 2.2.

Chapter 2

Models

2.1 Vicsek

The Vicsek model is a model based on simple interactions, with a focus on describing the nature of flocking behaviour and more generally collective motion of animal, bacterial and cellular systems in noisy environments [25]. The model also displays semantically similar behaviours such as swarming, schooling and herding. Later models building upon Vicsek and/or the core idea of using similarly simple rules to govern agent behaviour were formulated to expand the field of study [26]. The model operates similar to those devised in ferromagnetic studies, whereas ferromagnetic effects consist of the co-alignment of spins due to magnetic fields, Vicsek describes the co-alignment of the directions of motion of the particles introducing transport making the model dynamic in nature [25].

2.1.1 Model

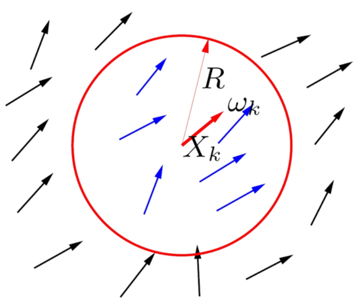


Figure 2.1: Vicsek interaction.[25]

.

The Vicsek model [25] in its original, most simple form is comprised of N interacting particles. Conventionally the particles begin with random, unique, uniformly distributed positions r_i. Each particle is propelled by an initially random directed velocity vector v_i of identical magnitude.
At every time-step $(t+1)$ the particles advance their position according to

$$r_i(t+1) = r_i(t) + v_i(t)\Delta t \tag{2.1}$$

Particles assume the average directions of all neighbouring particles within a range of interaction, R as visualised in 2.1

$$\theta_i(t+1) = \langle \theta_i(t) \rangle_R + \Delta\theta \tag{2.2}$$

Where $\langle \theta_i(t) \rangle_R$ is the mean direction of all particles within the interaction range R of the i_{th} particle. $\Delta\theta$ denotes a random perturbation η applied to the resulting direction representing fluctuations due to biological influence, for example and perhaps most analogous to the Vicsek model, an uncertainty in determining the actual average direction when observing the local neighbourhood. such that... Momentum is not conserved due to the over-damped nature of the propulsion force.

In the original paper and many subsequent models the system is confined to a two-dimensional square domain with periodic boundaries such that it operates in topologically toroidal space. Although this applied boundary is unrealistic on the scale of many of the collectives being modeled and can cause direct implications in behaviour, it is a requirement for cohesion in models that rely only on alignment or repulsion. The particles would otherwise drift apart without the influence of any attractive forces, before any emergent behaviour can take hold [26]. By complicating the space, the simplicity of the model itself can be maintained but the influence must remain in consideration.

2.1.2 Vicsek as a Base Model

We started by implementing a version of the Vicsek model as it is probably the simplest form of a two-dimensional model that shares many of the characteristics of interaction, movement and the emergent properties of collective motion through flocking or swarming types of behaviour. It is important to make sure we had a working model that could be built upon. Once we verified that the Vicsek model was working as intended, we could then add the more complex programming techniques and interactions of the Active-Jam model. The Active-Jam model is also based on the Vicsek model so follows the structure closely. Since we required a high density of particles for our investigation into crowd movement, it was required that we partition the space in order to reduce the overall calculations required. This technique was first implemented and verified as part of the Vicsek model to allow many thousands of simulated particles. This partitioning technique will be discussed further in 2.

2.2 Active Jam Model

An application of a modification to the Vicsek model originally introduced by Szabo et al. [24] in a study of collective migration of tissue cells. The Active Jam model, introduces a repulsive, 'social' force to Vicsek. Particles attempt to maintain a personal space, attempting to preserve comfortable freedom of movement and continual survival. The social force is analogous to soft collision with variable repulsive constant. The particles are given the tendency to align with their velocity. Whereas the collective migration study investigates the movement of these space-occupying particles in both periodic and rectangular confined spaces, the Active Jam study increases the density of the particles, causing them to become jammed in place. [17][1]

2.2.1 Model

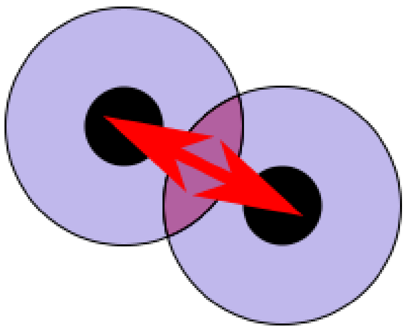


Figure 2.2: Active-Jam interaction.

.

The Active Jam model [17] consists of N interacting particles. The particles begin with random, unique, uniformly distributed positions r_i. Each particle have an initially normal distributed random direction Ψ. Particles are self propelled by a

maximum natural speed of magnitude v_0 analogous to the application of a constant an over-damped force equilibrating with drag on a single free particle.
The equation of motion for the i_{th} particles position is given by

$$\dot{r}_i = v_0 \hat{n}_i + \mu \sum_{j=1}^{Z_i} F_{ij} \tag{2.3}$$

Where $\hat{n}_i$ is the direction Vector of particle i such that $\hat{n}_i = \cos\psi_i \hat{x} + \sin\psi_i \hat{y}$. μ is the mobility of particles*. F_{ij} is the repulsion force between two particles that are in range of interaction $2a$ as shown in 2.2. The force is given by

$$F_{ij} = -k(2a - r)\hat{r}_{ij} \tag{2.4}$$

Particles tend towards alignment with the direction of their velocity vector. This direction change is given by

$$\dot{\psi}_{ii} = \frac{1}{\tau} \sin(\theta_i - \psi_i) + \eta_i \tag{2.5}$$

Where θ_i is the direction of the velocity vector of the i_{th} particle. η_i denotes a random perturbation applied to the resulting direction again, as in the Vicsek model. This represents fluctuations due to biological influence. τ is the lag time in the particles alignment. Similar to Vicsek, momentum is not conserved due to the over-damped nature of the repulsion and propulsion forces.

Chapter 3

Numerical Methods

3.1 Agents

Individuals in our models were designed as objects containing unique personal attributes. These attributes are used to calculate and update the attributes for each subsequent time-step. By treating the agents as objects we can easily perform multiple operations directly relating to their respective personal data.

3.2 Partitioning

Determining whether two bodies are interacting in a simulation can be considered a test of *intersection detection* using their ranges of influence. Being able to test for such interactions is a fundamental requirement for many simulations and as such has an equally wide range of applications.

The very nature of intersection detection introduces the problem of pairwise processing. Here, every particle must be tested against every other particle, pair by pair, for possible interaction. Such tests can be computationally expensive in their own right but with the quadratic increase of calculations as the system increases in size the required calculation time quickly becomes unfeasible. Most solutions to this problem involve partitioning the simulation space in such a way that particles may be tested only against others in their neighbourhood. Some partitioning methods can account for a large amount of processing time, it is therefore important to seek a solution that fits the application while keeping this in mind as it will likely take its place as the new bottleneck in a large interacting system. Figure 3.1 2) and 3) show the effective reduction in calculations. [10]

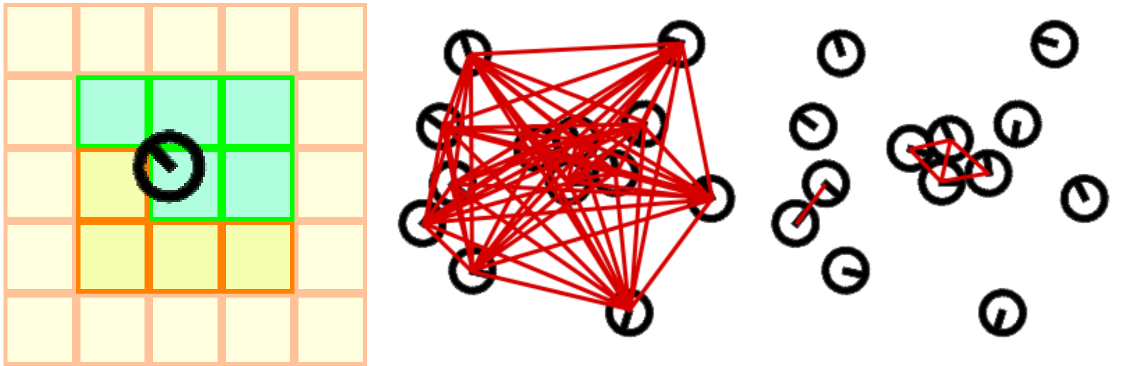


Figure 3.1: Partitioning of an agent 1) Agents intersection with the grid. 2) Interaction tests before partitioning. 3) Interaction tests after partitioning.

For this simulation, where high density and as a result a large number of bodies is a requirement, a uniform partition grid was selected as an effective choice. This requires nothing more than indexing the simulation space and placing agents in bins according to location. This choice is justified by the repulsion forces between particles ensuring a tendency towards maintaining density when unobstructed, with the added bonus of being relatively simple to implement. However, without such a force it would certainly be more efficient to make use of another spatial partitioning solution such as adaptive grids like quad-trees or kd-trees as these deal with varying density much better. With this method the space is divided into a number of cells or bins. Each grid cell contains a list containing the agents in its region. Partitioning is recalculated for every time-step, although this can be approached in more efficient ways. After obtaining these lists we can check agents against only those that are in close proximity, therefore improving the performance greatly. Figure 3.1 1) shows the relevant cells that are checked for colliding neighbours.

The most important aspect when defining a uniform partition grid is to choose an optimal cell size. In this system, which can be equated to soft collision, the maximal range of interaction of the particles is $2r$. Therefore, this is an appropriate dimension for our cells, ensuring every particle pair with the possibility of interaction is tested. This dimension cannot accommodate for the fluctuations in densities due to flow in the system or when we add obstructions to our simulation space but contributes a drastic speedup nonetheless [10]. This is where other spatial partitions offer improvement.

3.3 Walls

For our particular use of the model in analysing crowds, the space through which the agents are moving can be considered a governing influence on their behaviour, it was therefore particularly important to implement a way to define such spaces. This of course required more collision detection. Simplifying the problem by defining linear segment walls, which applied repulsion force in a similar manner to the social force, a flexible tool was created that allowed the definition of complex spaces.

3.3.1 Point-line calculation

For our implementation of a wall analogue we first represent our wall as a line segment defined in 2-dimensional space by two points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. Point $P_3(x_3, y_3)$ defines a point in space to test against. Finding the point $P_4(x_4, y_4)$ lying on the line segment nearest P_3 is achieved using an adaption of the "Minimum distance between a Point and a Line" method documented by Paul Bourke [2].

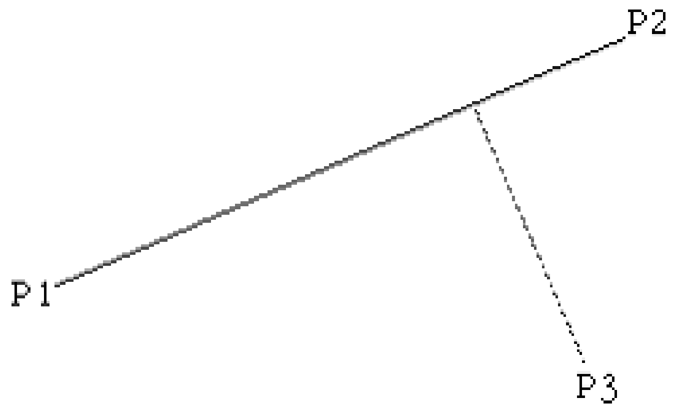


Figure 3.2: Closest point on a line calculation [2].

The equation of the line through the points P_1 and P_2 is given given by

$$P_4 = P_1 + u(P_2 - P_1)$$

Point P_4 is closest to P_3 where the line passing through P_3 and P_4 is orthogonal to the line segment, therefore

$$(P_3 - P_4) \bullet (P_2 - P1) = 0$$

Substituting the equation of the line

$$[P_3 - P_1 - u(P_2 - P_1)] \bullet (P_2 - P_1) = 0$$

Solving for u

$$u = \frac{(x_3 - x_1)(x_2 - x_1) + (y_3 - y_1)(y_2 - y_1)}{|P_2 - P_1|^2} \tag{3.1}$$

If u lies between 0 and 1, P_4 lies on the line segment. Substituting this back into the equation for the line gives the (x, y) coordinates of P_4

$$x_4 = x_1 + u(x_2 - x_1) \tag{3.2}$$

$$y_4 = y_1 + u(y_2 - y_1) \tag{3.3}$$

Using point P_4 we project a repulsion force on particles in range of the line segment. If the point P_4 is found to lie beyond the ends of the line segment i.e u does not lie between 0 and 1 then the ends of the lines are tested for interaction using the same method for agent-agent interaction but with an interaction range of 0.

3.4 Partitioning - Walls

Although not as asymptotically disastrous as the agent-agent interactions, the pair-wise computation problem returns with the introduction of walls to the system. Each time a wall is added, the computation time is increased multiplicatively as each particle must be tested against each wall, this again becomes a problem when many walls are added. To increase the efficiency of our wall-agent interaction we again look to the partition grid with the goal of reducing computation time.

Whereas agents are considered to be confined to one cell (and its neighbouring cells), walls and their influence can span across any number of adjacent cells. To avoid agents being tested against walls more than once, we must handle the wall-agent interaction differently to the agent-agent interaction. It therefore makes sense to store a list of the cells that a wall intersects with for each wall and test the particles in those cells against that wall for interaction. Although the walls are defined as lines, they have a polygon of interaction, it is this that must be projected onto the grid to find the relevant cells.

The problem of finding grid cells that lie within a polygon is well documented in the world of computer graphics where shapes must be “rasterized” to find the pixels or texture coordinates that they occupy and therefore must be drawn to. Many solutions to this problem exist with varying degrees of accuracy, efficiency and complexity. Since we only need to rasterize the polygon every time a wall is added to the simulation space, efficiency is not a big concern. We can therefore select one of the algorithms with less implementation complexity at the cost of run-time. To simplify the process even further we use a rectangular shape projected over the ideal capsule shape of interaction.

3.4.1 Barycentric rasterization algorithm

The barycentric rasterization algorithm works by first computing the bounding box of a triangle. A triangle is defined by three points $\vec{P}_1(x_1, y_1)$, $\vec{P}_2(x_2, y_2)$ and $\vec{P}_3(x_3, y_3)$.

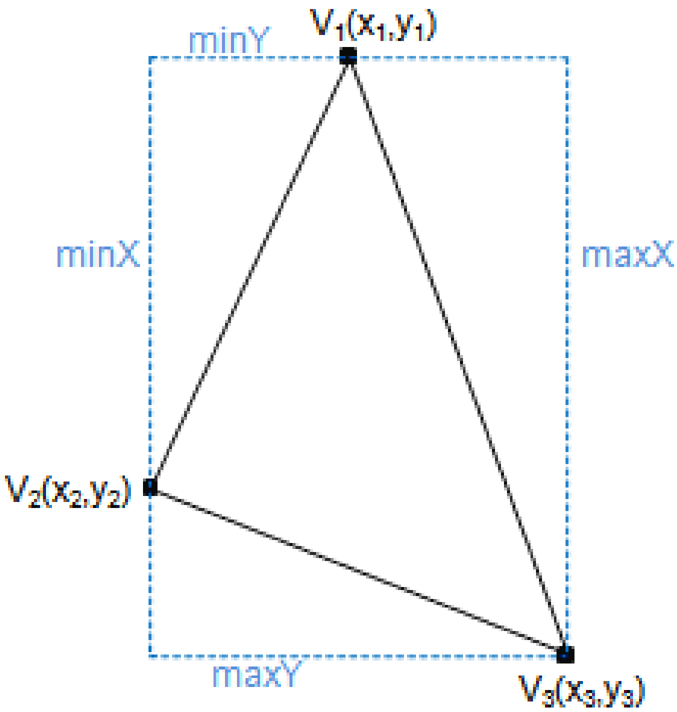


Figure 3.3: Barycentric rasterized bounding box. [21]

Each grid cell that lies within the bounding box is then checked by position to see if it lies within the triangle. This is achieved by constructing vectors from two of the triangle sides

$$\vec{V}_1 = (x_2 - x_1, y_2 - y_1)$$

$$\vec{V}_2 = (x_3 - x_1, y_3 - y_1)$$

A third Vector Q is constructed from vertex V_1 to the grid cell position (X, Y) being checked

$$\vec{Q} = (X - x_1, Y - y_2)$$

Where X and Y are the (x, y) coordinates of the center of the test cell. The cross product is then used to determine whether the vector $\vec{Q}$ is between the two sides

$$s = \frac{\vec{Q} \times \vec{V_2}}{\vec{V_1} \times \vec{V_2}} \tag{3.4}$$

$$t = \frac{\vec{V_1} \times \vec{Q}}{\vec{V_1} \times \vec{V_2}} \tag{3.5}$$

If $s \geq 0$ and $t \geq 0$ then the vector $\vec{Q}$, corresponding to the test cell, lies between vectors V_1 and V_2. If $s + t \leq 1$, the cell lies within triangle $\vec{P_1}\vec{P_2}\vec{P_3}$ and we add it to the list of interacting cells for that wall. This process is repeated for all cells within the bounding box. In our use case we need to rasterize a rectangle, this is of course achieved by running the algorithm for each of two triangles that compose the bounding box.[21]

3.5 Vector Fields

To introduce will of destination to the agents in our model we turn to vector fields. By defining a vector field we introduce a new direction vector that applies to each particle varying with their current position. We do this using the simple rule that all particles attempt to move towards a single destination.

We define points $\vec{P_1}$ as our particle position and $\vec{P_2}$ as the destination position. Similar to the alignment of the particles direction ψ with their respective velocity vector v, the particles additionally now align with the vector $\theta = |\vec{P_2} - \vec{P_1}|$ with direction change given by (needs revising to vector equivalent)

$$\dot{\psi}_{ii} = \frac{1}{\tau_2}(\theta_i - \psi_i) \tag{3.6}$$

Where θ_i is the direction of the velocity vector of the i_{th} particle. τ_2, as before, corresponds to lag time in the particles alignment. This torque is calculated and applied additionally after rotation in the Active-Jam model. Although this definition limits the vector field to that of a point attractor or repeller, the vector fields in our implementation can be generalised to functions dependant on particle position $\vec{P}_1$ for further, more varied scenarios.

3.6 Phase Transitions and Polarisation

Systems consisting of self driven particles are similar to thermodynamic systems in that they undergo a phase transition from one phase-state to another. The observed phase-states can be directly related to their thermodynamic analogues. The gas-like phase - uncorrelated movement with no net transport; the liquid-like phase - localised cohesive groups and the solid-like phase - global cohesive or polarised movement in one direction. The noise introduced in the form of a random perturbation is analogous to thermodynamic temperature It is important to note that while the phases resemble these thermodynamic equivalents there is a major difference in that they are inherently out of equilibrium due to their self propulsion. Sharp phase transitions present in the Vicsek model indicate critical thresh-holds in parameters such as the interaction radius above which the collective motion prevails and below which particles act more like individuals. [26] [25]

Polarisation refers to the alignment of the collective group. Also known as the order parameter, in the case of the Vicsek model it can be calculated as the absolute average normalised velocity using the following formula

$$\phi = \frac{1}{Nv_0}\left|\sum_{i=1}^{N}\vec{v}_i\right|$$

The order parameter can be useful in determining phase state characteristics and transitions. When a group of interacting agents is in a totally unordered state as in our initial conditions, with random positions and more significantly, random directions. We can define this state as having a polarisation/order tending to zero. In the Vicsek model this parameter relates directly to the net transport of the system or analogously, how effectively the particles are able to cooperate towards a goal of common directional movement this is due to the constant propulsion of the particles. [25]

While the order parameter can be applied to the Active Jam model, its direct relation to net transport is lost as particles no longer move with a guaranteed speed due to repulsive forces. Despite this, the correlation with polarisation holds so the order param can still be used as a valuable measurement of order in the system. The formula changes to the following

$$\phi = \frac{1}{N} \left| \sum_{i=1}^{N} \vec{\psi}_i \right|$$

3.7 Density and Pressure

In this investigation it is important to be able to look at how local density and correspondingly local pressure fluctuates, accumulates and dissipates in areas with time under the imposed conditions.
we define Local density for a rectangular area as follows

$$\rho = \frac{1}{A_b ox} \sum_{i}^{Z_i} 1 \tag{3.7}$$

The Virial pressure as defined in [18] for a rectangular area is given by

$$P_{Virial} = \frac{1}{2} \frac{1}{A_b ox} \sum_{i} \sum_{j=1}^{Z_i} \vec{R}_{ij} \vec{F}_{ij} \tag{3.8}$$

Where F_{ij} denotes the force and R_{ij}, the direction between interacting agents i and j. $A_b ox$ is the area of a the rectangular box, the size of which governs the resolution of the pressure data. Forces are summed for all interactions with particles inside the box, this includes particles that lie outside of the box but interact with those inside.

Chapter 4

Implementation

4.1 Implementation

When it comes to designing a tool for the simulation of diverse scenarios it makes sense to prioritise flexibility. For this reason, the object oriented programming paradigm was adopted in the form of c. The Object-oriented programming (OOP) paradigm is formed around the idea of encapsulating data and methods in objects that follow much of the expected traits of real world objects. This gives the paradigm many advantages, including the idea of modularity where parts of the program act as black boxes with inputs and outputs that can be swapped out or reused for the required functionality. By defining objects, the benefits of functions can be expanded to persistent data and interfaces. Objects, interact, affecting and cooperating to build a complete computer program. The individual parts can often be implemented as if in isolation, greatly reducing the complexity of the code-base. This opens up the path of possibility to achieve greater functionality without compromising the readability of what are often critical algorithms in our case, the interaction of agents according to the formulated models. These advantages mean we have been able to build a more useful, usable tool that incorporates complex graphics and user interfaces elements, increasing the potential applications through a wide range of possible scenarios. Of course it is absolutely possible to build such systems as these without the use of OOP. However, OOP is usually the right tool for the job in an application like this. Some other examples of languages that support the OOP paradigm are C#, Python, Ruby, Java, Delphi, Perl, and PHP.

4.1.1 Graphical User Interface

The benefits of utilising a graphical user interface when working with systems that display emergent behaviour cannot be understated. The ability to tweak parameters and geometry on the fly in this way allows for greater discovery possibilities as it is not a requirement to recompile each time a new behaviour requires testing. Throughout the creation of the final simulation engine this idea was kept in mind and expanded through the process of visual debugging. These tools proved invaluable when working in a field of unfamiliarity. Figure 4.1 gives an example of a complicated scenario that can be created and tested in under a minute using the real-time tools that were developed.

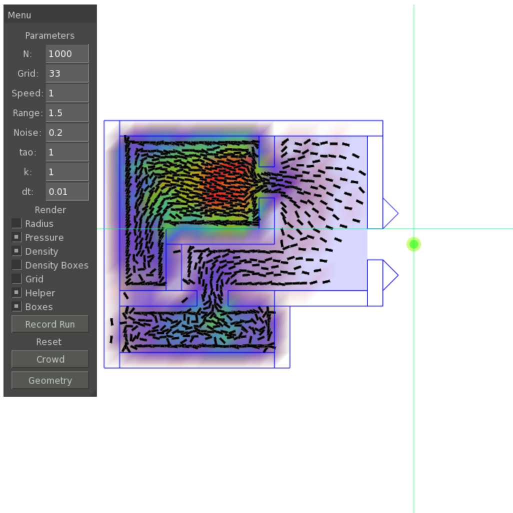


Figure 4.1: An example of the complex structures and scenarios that can be built and defined with our tool.

Another approach for added flexibility that we have not yet touched upon would be the use of scripting through languages such as Lua or Python to define and move data to and from the application allowing tweaking of parameters or even behaviours. A big advantage of taking this approach is the ability to tweak behaviour, define spaces and test them rapidly while retaining reproducibility. This kind of added flexibility affords a huge range of scenarios which could lead to the uncovering of previously undocumented behaviours. Ideally a system would include both the fully real-time approach and scripting techniques exploiting each for their strengths. Indeed we have partially followed this wisdom in our use of Python for graphing and analysis purposes, where it excels. However, currently the implementation does not contain any scripting integration.

Of course we have to mention that this drive for flexibility comes at a cost. That cost manifests itself in development time. With each new parameter and piece of functionality requiring support for tweak-ability, it can quickly become counter-productive adding features for every occasion. Finding a good balance appears to be the key here.
Due to the size of the code-base and the nature of complete C++ projects transferring poorly to paper, an interaction diagram has been included in appendix A to show the general structure of the program in place of the full source code that was used in this investigation. Also known as a collaboration diagram or communication diagram, this is a visualisation of the relationships and interactions between the objects that compose the complete program [9]. The full code repository is available for review at https://Flon@bitbucket.org/Flon/crowddynamics.git, with a readme for building on linux and windows planned shortly.

4.2 Aside On Parallelism

During this project, a brief investigation was carried out on parallel programming with a focus on Microsofts C AMP (Accelerated Massive Parallelism). The purpose of this inquiry was to gain an understanding of the requirements and benefits of parallelism in the context of multi-agent simulation. C AMP is a multi-platform library that aims to accelerate C++ code execution by moving speed-critical calculations to hardware such as graphics processing units which are typically very good at parallel processing. The C++ AMP aims to simplify the process of moving data to and from hardware, making it available for a wide range of applications [20]. Parallelism is the process of splitting processes into smaller, usually isolated calculations. These calculations can be run simultaneously on hardware that supports parallel processing, often resulting in huge run-time reductions. Although this sounds like a universally applicable paradigm, not all applications are a good fit. Good candidates for parallelism are functions that can be divided so that the same task can be run on different data when the data itself must be altered. Or equivalently functions that can be run while accessing the same data for example if the functions only refer to but do not alter the data itself [6]. In this sense with a few tweaks to the code, our model could be parallelised. A naive approach without any clever caching would result in more calculations overall but this tends to be heavily outweighed by the efficiency of parallelised execution. While we chose not to move forward with the parallel implementation due to time constraints this presents potential for future development.

Chapter 5

Crowd Dynamics

The aim of this investigation is to examine the movement of humans through space. We have placed our agents in various scenarios examining the behaviour qualitatively. The active-jam model was implemented by applying all techniques discussed in the 2.2 and 2 sections. Pressure and density were calculated using equations 3.8 and 3.7 respectively The model was run with various starting conditions. Due to the flexibility of a fully real-time solution, tweaking of parameters was straightforward, although not all parameters were exposed to the graphical user interface. Models can generally be validated through comparison with empirical data which can come in two main categories. 1) as qualitative descriptions of emerging behaviors related to the collective self-organizing ability of humans and animals. 2) as quantitative results, for example measures of flow or local density. It is usually necessary to utilise both types in validation while taking into account what are often necessary simplifications carried through into the models. We must kkep in mind that due to frequent differences in the way data is collected between a model and empirical data, direct comparison can be difficult [1]. We first examine the qualitative behaviour of our model.

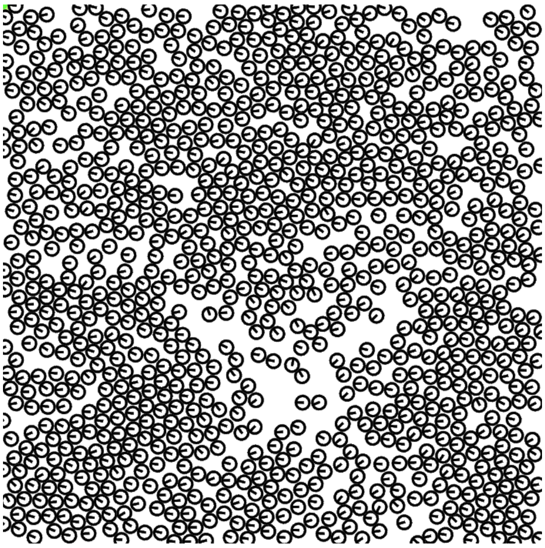
5.1 General Behaviour

Starting from values derived from the application of the Active-Jam model by Henkes et al. [17] we explored the behaviour of the model in both bounded and unbounded, periodic boundary settings. A base state was defined with initial parameters selected through a process of trial and error with the purpose of gaining insight into the models diverse behaviour. Varying the parameters in isolation, we were able to uncover their immediate effect on the nature of the system. Of course this is hardly a comprehensive approach. A more rigorous method would be required to identify the interplay, both through counter and causal relationships in this highly coupled system. It does however give a sense for how we can shape the behavior to our needs, which in this application are to imitate a human counterpart in different environments. For this model to be able to provide an authentic description of crowd behaviour, we expect to be able to identify some of the defining characteristics of dense crowd behaviour such as those identified by Helbing et al. in their 2007 empirical investigation based on footage of mecca pilgrimage routes [15]. While we only applied qualitative analysis at this stage, it allowed realistic representative values to be selected for each parameter.

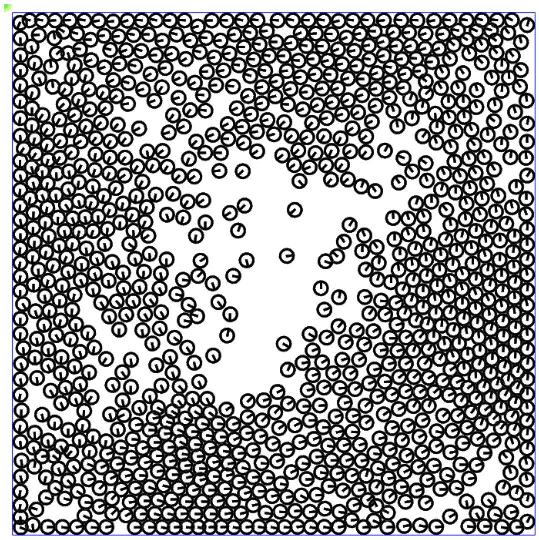
All states were surveyed in a bound and unbound, but otherwise unperturbed state. In the following recordings we list parameters that differ from the base state that we finally arrived at. This state is described in 5.1.1. The time-step throughout our investigations remained at $0.01s$, the simulation area had fixed dimensions of $100 \times 100m$ and partitioned with an extent of 33×33. Visualisation of radius is disabled where appropriate for visual clarity in higher density situations. Walls are given a repulsion factor of $20K$ where K is the repulsion factor of the social force which was kept at a value of 1, this was a value that was found to provide reasonable overlap and repulsion in early testing.

5.1.1 Base State

In the Base State as visualised and defined in 5.1 particles are quick to form cohesive cluster arrangements, this is somewhat counter intuitive due to their purely repulsive forces. However, it can be attributed somewhat to the over-damped nature of the forces acting on the agents due to their interaction. This causes individuals to be forced to the limit of interaction between each other, but no further. The lag time τ ensures that the entities attempt to maintain their initial direction of propulsion ψ. This appears to have the effect of limiting the incurred deviation due to a collision. The constant propulsion v_0 ensures that once agents are aligned they will maintain cohesive movement. If collision occurs between a group and an individual then the individual appears to have little effect on the groups orientation. The individual however, will adjust its orientation greatly, either deflecting away from, or merging



(a) Unbounded



(b) Bounded

Figure 5.1: The Base State with $N = 1000$ as defined by the following parameters: $v_0 = 1$, $Range = 1.5$, $noise = 0.2$, $\tau = 1$, $k = 1$
.

with the group.

When unbounded and with periodic boundaries the agents appear to settle into a highly polarised alignment resulting in net transport in one direction. This process takes approximately $20s$ of simulation time.

Adding a perimeter boundary results in a limiting of the clear alignment behaviour. Particles are no longer induced into full alignment before they are interrupted by contact with a boundary. Upon reaching a boundary their rotation is altered, usually along the boundary in either direction. After some time in the confined arrangement the particles begin moving in a vortex with center in the middle of the bounded space this effect can be seen in 5.1b.

5.1.2 Varying Density

Varying the density of the system has some interesting effects. At higher densities particles form along coherent paths, forcing their way through less coherent individuals and likewise, other groups 5.2 2). In the unbound, periodic context the groups grow larger until one dominates and the whole system moves towards net transport

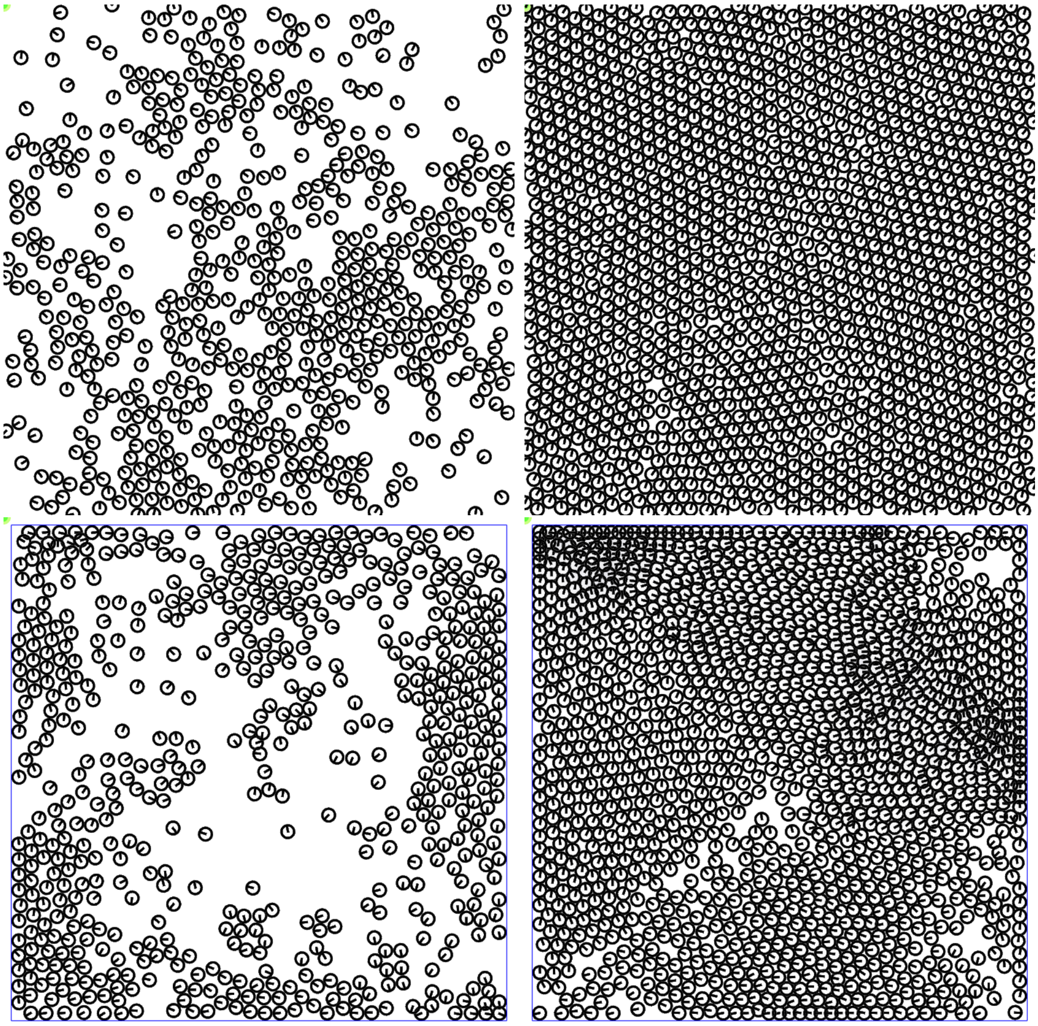


Figure 5.2: Visual output from the program in its base state after extensive settling time. 1) Unbounded low density. 2) Unbounded high density. 3) Bounded low density. 4) Bounded high density

in an arbitrary direction relating to the initially random orientations. This is a commonly observed feature of dense crowd movement referred to as "lanes" by Helbing et al. [15]. It is likely a result of a restriction in the movements individual pedestrians can make, resulting in movement through increasingly ordered arrangements. When moving in this phase the particles fall into hexagonal packing structures that are characteristic of crystal like structure complete with holes where elements of the lattice are missing, this is due to our uniform pedestrian approximation of the pedestrians as perfectly circular and indicates this might be a poor approximation [5]. In the bound context, these groups impact boundaries before they grow very large. Upon impacting the outer walls the groups spread in either direction following the walls. The dominant direction builds up in a similar way to the lane formation in

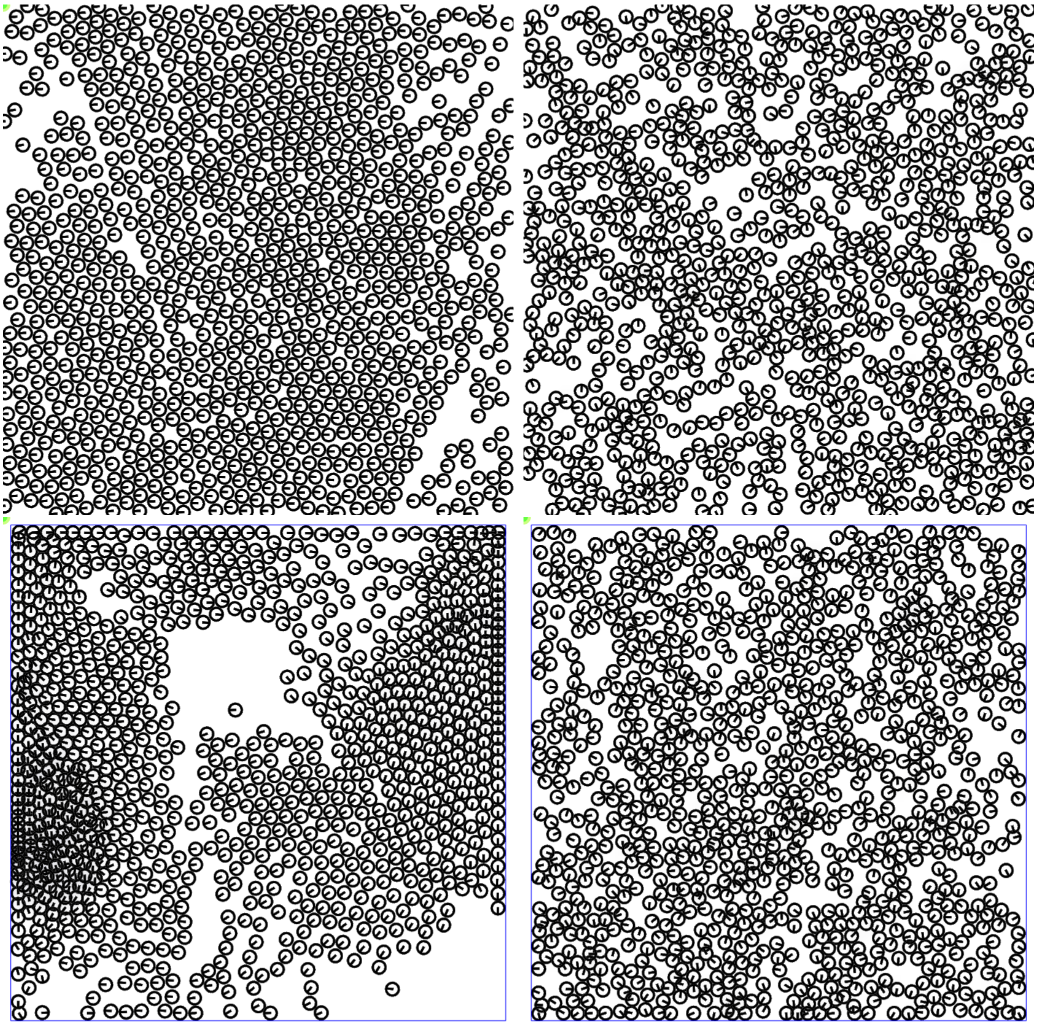


Figure 5.3: Visual output from the program in its base state after extensive settling time. 1) Unbounded $noise = 0.01$. 2) Unbounded $noise = 0.8$. 3) Bounded $noise = 0.01$. 4) Bounded $noise = 0.8$

the unbound setting. However now the lanes of coherent movement follow the outer walls in either clockwise or counter-clockwise rotation. Particles inevitably impact with this lane and are swept in the direction of the motion. Ultimately this results again in a vortex-like phenomenon with particle density concentrating around the boundary and a sparse region in the center 5.2 4).

5.1.3 Varying Noise

At low noise values particles in the unbounded setting tend to align very quickly, their alignment becomes so close that the deviation is indistinguishable 5.3 1). This indicates an interplay between the deviation and repulsion resulting in them canceling each other in an over-damped fashion. the result is fast convergence to high

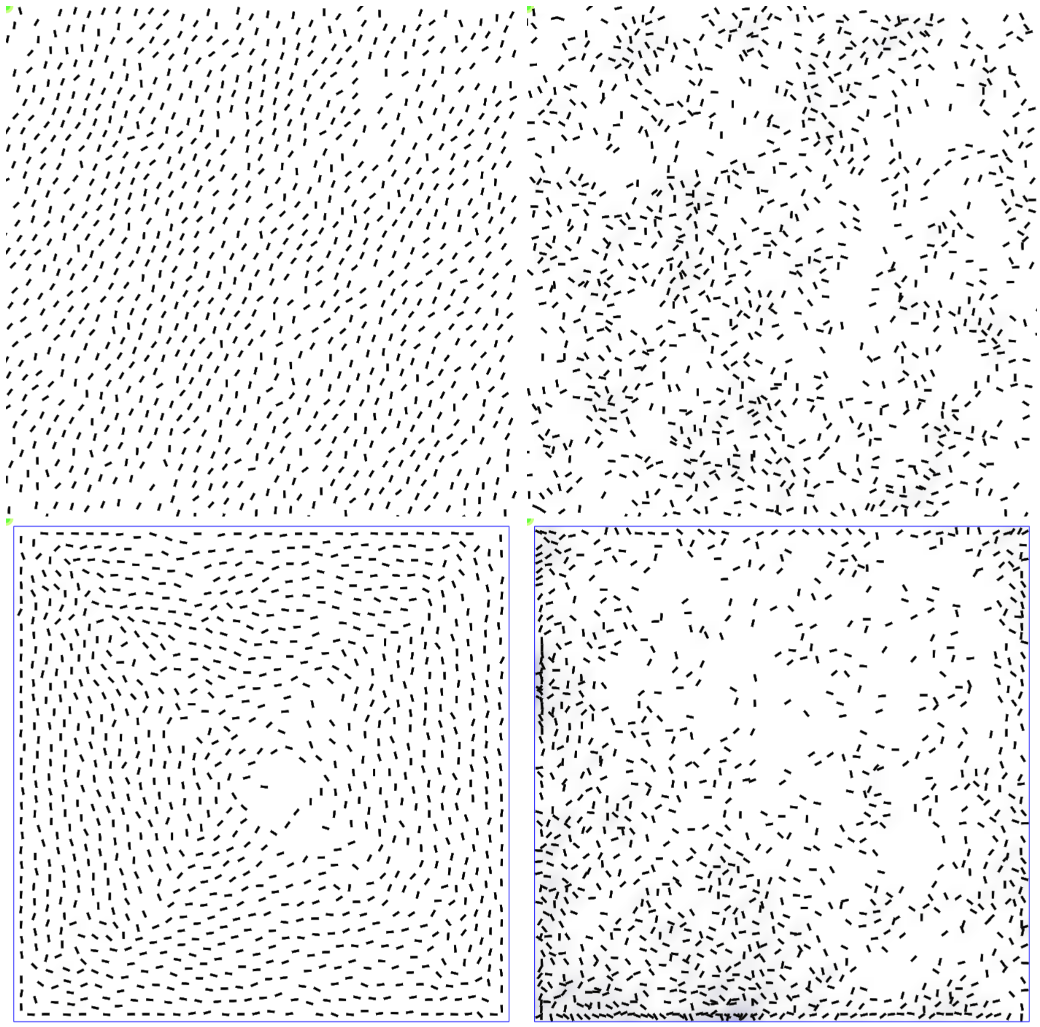


Figure 5.4: Visual output from the program with varied τ after extensive settling time. 1) Unbounded $\tau = 0.1$. 2) Unbounded $\tau = 10$. 3) Bounded $\tau = 0.1$. 4) Bounded $\tau = 10$.

transport in a single direction. Within the bounded setting particles are quick to gather at the edges in dense clusters, eventually shifting to the vortex like structure described and visualised in the base state 5.1b. At high noise, particles remain relatively incoherent and independent, no notable clustering is observed. In fact the opposite is observed, particles force their neighbours away to maintain spacing even beyond their social force boundaries. The system is in high fluctuation and no notable collective cohesive motion can be observed.

5.1.4 Varying τ

For low lag time τ the particles align immediately with their velocity vector. Losing their tendency to push forward, the particles move into uniform spacings similar to the effect of high noise. Interestingly, the particles quickly fall into alignment and net transport as in the high density scenario but through a very different, far more uniform transition.

5.1.5 Bound With Attractor

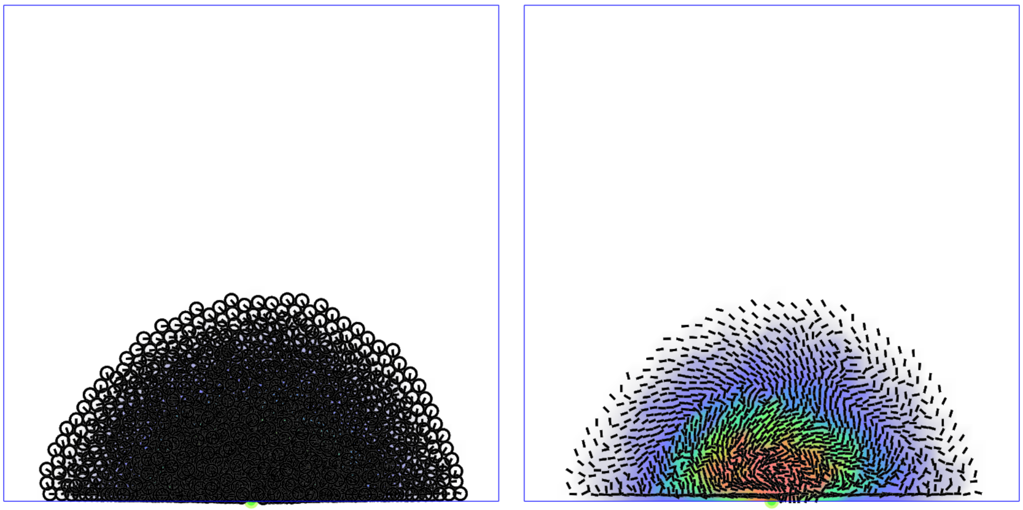


Figure 5.5: Bound Particles with an attractor placed just outside of the walls (green). Left: particles with interaction radius visualised. Right: particles with only orientation visible. Pressure is visualised as a gradient from white through blue, green and finally red as an indicator of problem areas.

As can be seen in 5.5, while under the influence of the point attractor, particles attempt to align their desired direction ψ towards the attractors position. The particles move towards the attractor and build up as a dynamic deposit. Particles close to the centre of the resulting buildup are forced together by incoming force from the outer particles, this results in high central pressure. The particles settle into layers defined by equal pressure and oscillate clockwise and counter clockwise along their contours, moving up a layer toward the attractor when possible. Although not entirely representative of human behaviour, this conduct bears close resemblance to impatience in humans [15].

5.2 Exit Scenario

We elected to examine an classic bottleneck scenario as it is a particularly common scenario with particular relevance when considering evacuation procedure. Bottle-

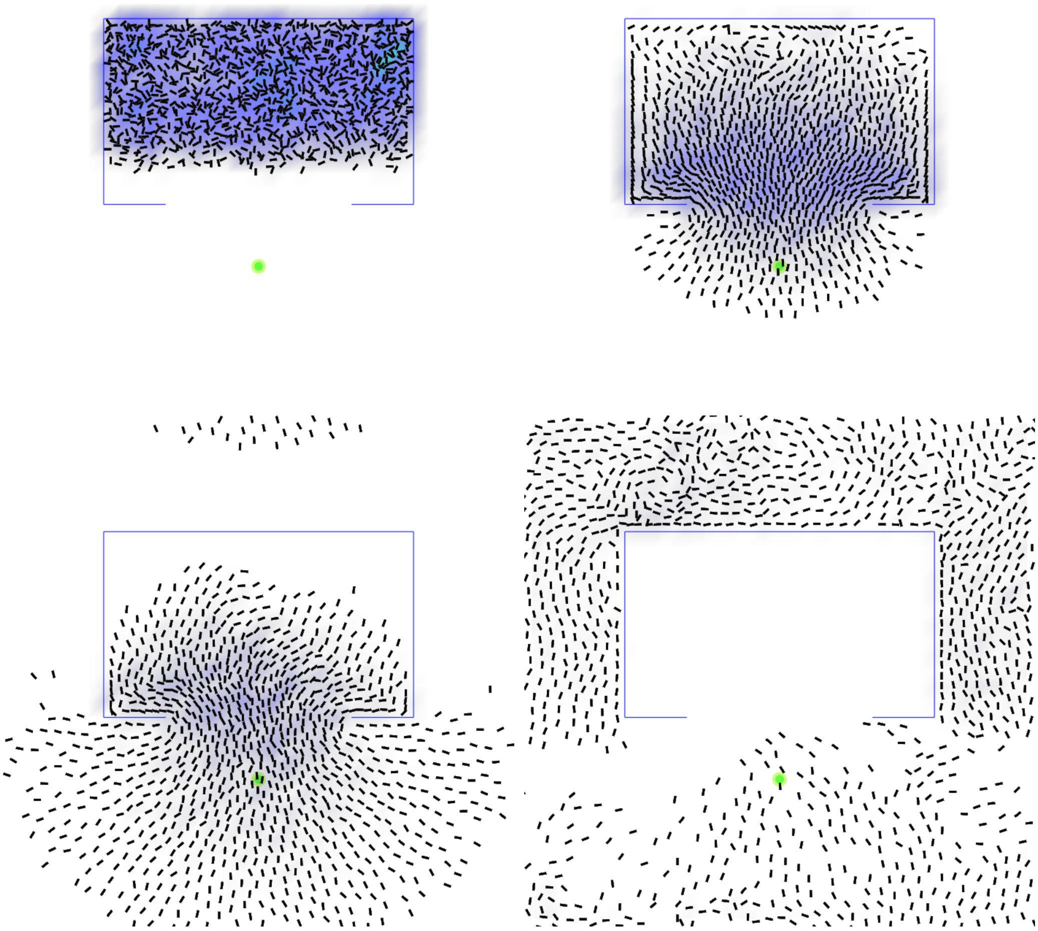


Figure 5.6: 1000 individuals exiting a confined space. Pressure is visualised as a gradient from white to blue, to red as an indicator of problem areas. Actual pressure measurements are performed at the exit aperture

necks such as doorways are a common feature in built up areas that pedestrians tend to accumulate at therefore pedestrian passage and the resulting dynamics is of particular interest.
This scenario is modeled With a simplified situation with a single room and a single exit. Particles are compelled towards the exit via the use of an attractor.

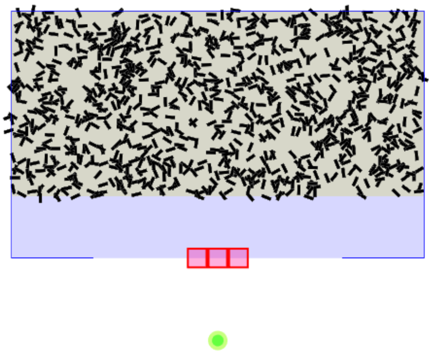


Figure 5.7: Initial conditions and measurement positions for the exit scenario. Red boxes mark the areas used for exit pressure readings. The green marker defines the location of the attractor. The blue area marks the interior. The yellow area marks the starting area, particles are placed within this in a uniform distribution.

Figure 5.6 shows the geometric setup and a complete run-through of our initial exit scenario for one aperture width. Individuals inside the box are compelled to seek the exit by mean of the influence of the attractor marked in green. Attractor strength (read $\frac{1}{strength}$) was kept fixed at 3.0 throughout all experiments. Noise was fixed at 0.2, this produced what was deemed qualitatively to be queue-like lane-formation behaviour in high densities as identified in 5.1.2. The size of the room was fixed at $61 \times 36m$ giving a total area of $2196m^2$, resulting in just over 2 individuals per square metre. The width of the single exit aperture was varied from $0m$ to $36.36m$ in equal increments across 20 runs. Pressure and Density were recorded across 3 boxes located in the span of the aperture, then averaged. The number of particles in the room N_{inside} was recorded, pressure was recorded across 3 boxes at the exit and averaged. It must be noted that all our forces are calculated with an assumed individual mass of $1kg$, therefore direct force comparison with real world data is not possible, relative pressure magnitude does remain relevant though. The simulation was run for a fixed simulated time of $200 seconds$. Each run was repeated 5 times and the mean values taken. Qualitative observations were also made on a single run. We explore the effect that the size of a door has on the evacuation time and pressure at the exit.

5.2.1 Results

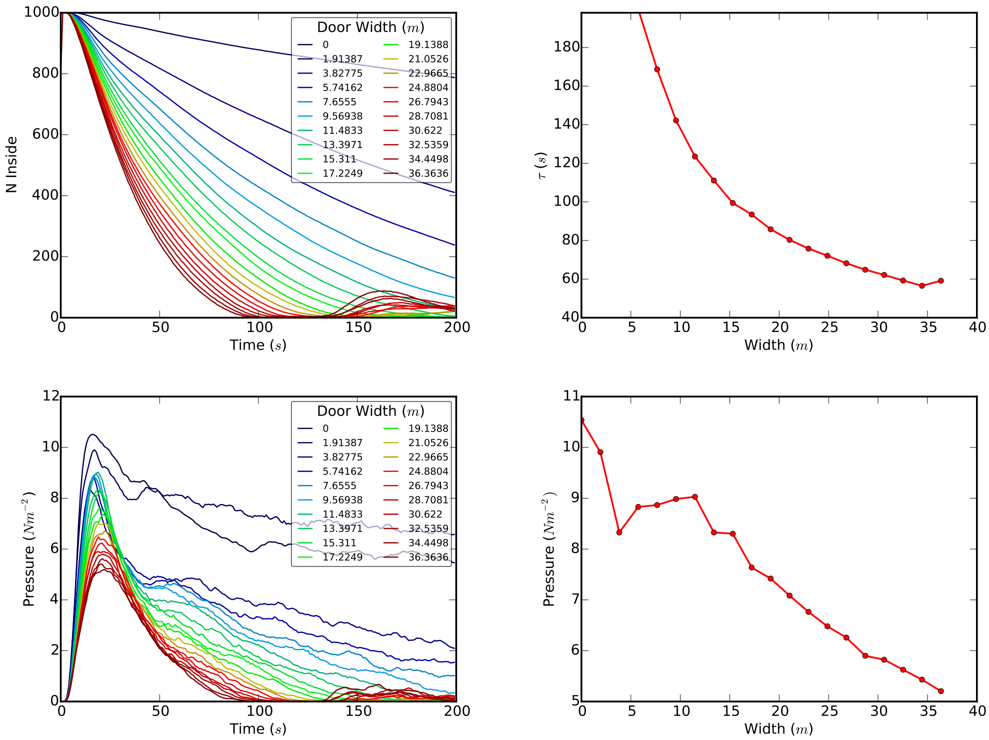


Figure 5.8: Results from exit scenario. Top Left: Number of particles inside vs Time. Top Right: Time taken for 80% of all particles to exit, τ vs door width w. Bottom Left: Pressure vs Time. Bottom Right Maximum pressure vs door width.

Figure 5.8: Top Left and Top Right clearly show that there is a variation in exit time based on the door width. The Number of particles inside appears to follow a close to exponential fall however it was found that this was not the exact trend. The characteristic shape does confirm that the evacuation rate is highly dependent on the number of particles that remain inside the room. The overall evacuation time becomes increase greatly as door width approach the interaction range of the particles. This makes perfect sense as in the context of the wall analogue, the particles interaction range acts as soft collision. Particles are therefore forced into single file through the passage which of course limits their flow greatly. According to Helbing et al. Flow around and through through bottlenecks can be uneven and intermittent causing pressure waves and resulting in oscillations. They state that groups are typically more efficient at passing through bottlenecks as they are

able to follow leaders who successfully move through. In real situations pressure can increase as a result of buildup of unsuccessful pedestrians at the edges of a bottleneck restricting flow further [15][12]. Reviewing the overview of the exit flow 5.2 we observe something closer to laminar flow through the aperture, with some turbulent flow at the rear of the body of the crowd.

In Figure 5.8: Bottom Left we can see that pressure spikes after the initial rush towards the exit, the peak pressure is closely dependant on the width of the door. This dependence is confirmed in Figure 5.9. Interestingly the peak in pressure shifts and reduces its maximum with a door width. The maximal pressure arrives later for increasingly wide doors. The peak shift suggests that for wide doors the clogging effect happens slower. This could also be explained by a funnel-like effect, the attractor affects the outlying individuals differently to inner ones. Outliers are drawn towards the center, compressing the active flow area therefore increasing the pressure but after a delay. This is effect is confirmed in our qualitative analysis 5.5 and is somewhat analogous to the desire to enter the flow similar to the behaviour noted in the empirical data gathered by Helbing et al.

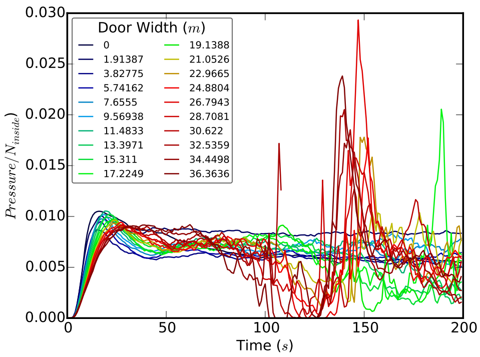


Figure 5.9: Pressure over the number of particles inside the room plotted against time.

Figure 5.9 confirms what seems to be a direct proportional effect of the number of particles vs pressure. What seems to be significant here is that $pressure/N_{inside}$ fluctuates greatly at times around $t = 150$. However, cross checking this result against 5.8 makes it clear that these fluctuations are due to particles reentering the active area.

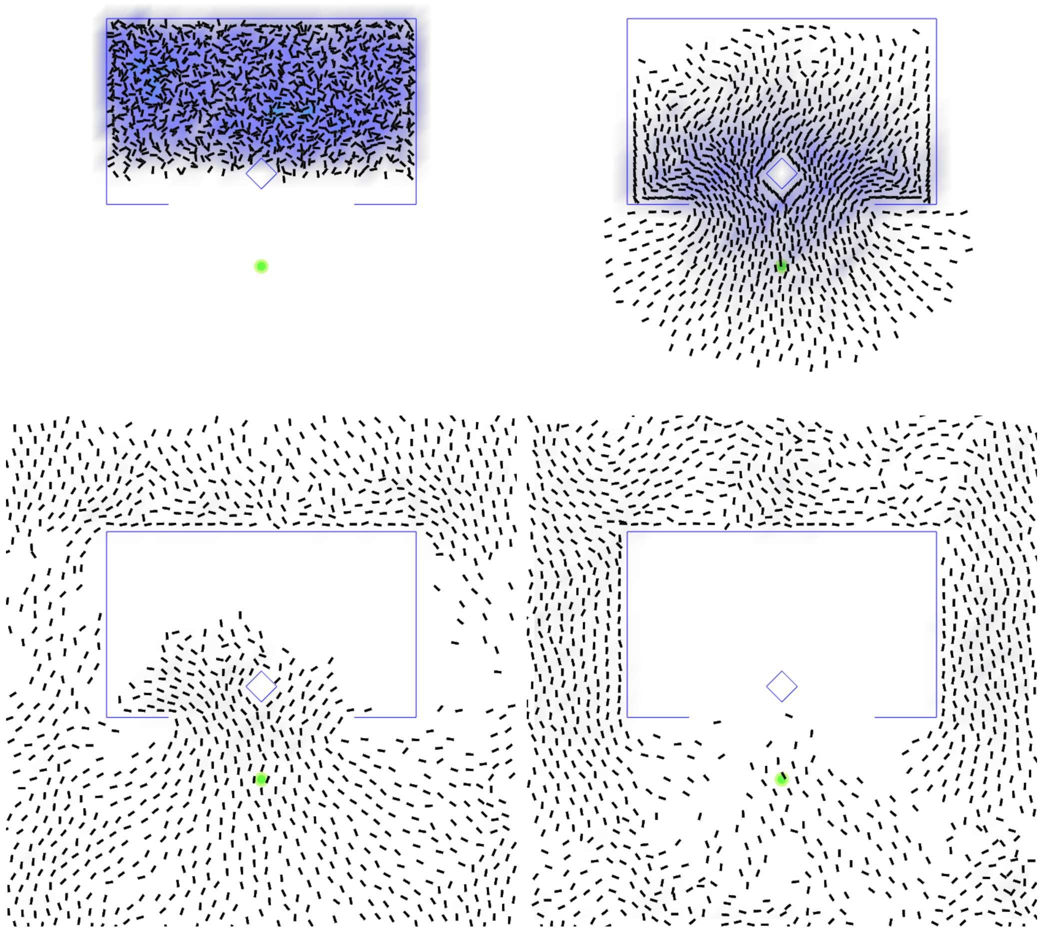


Figure 5.10: 1000 individuals exiting a confined space with obstruction. Pressure is visualised as a gradient from white to blue, to red as an indicator of problem areas. Actual pressure measurements are performed at the exit aperture

5.2.2 Adding an Obstacle

Since we consider areas of high pressure to be directly correlated with increased risk, looking at ways to reduce the overall pressure and excessive buildups in critical areas of the system is our concern. We look into potentially reducing peak pressure identified in 5.2 via the introduction of an obstacle placed in the center, before the doorway in the aperture of the door. 5.10 shows a full run of the model with the obstacle in place. We again vary the size of the door aperture and record the results in an identical procedure to the original exit scenario. All parameters and initial conditions were kept the same.

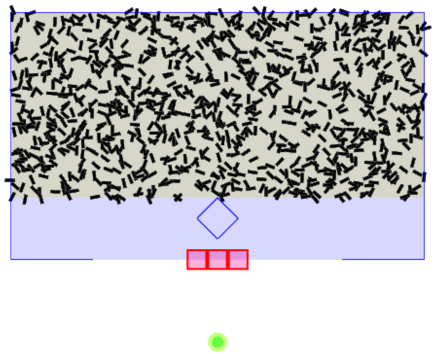


Figure 5.11: Initial conditions and measurement positions for the exit scenario with obstruction. Red boxes mark the areas used for exit pressure readings. The green marker defines the location of the attractor. The blue area marks the interior. The yellow area marks the starting area, particles are placed within this in a uniform distribution.

Figure 5.12: Top Left and Top Right clearly show similar results to the exit scenario. There is again clear variation in exit time based on the door width. The peak pressure for door widths under 4m wide remain relatively similar. This corresponds to zero or near zero flow. However, for door widths above 4m there is a clear drop in peak pressure. We can also note that in general exit time τ has increased slightly when compared to the previous scenario. The introduction of the obstacle seems to have alleviated some of the peak pressure at the cost of overall evacuation time. This result indicates that strategic obstacle placement might be an important design consideration for crowded spaces.

5.2.3 Varying Obstacle Position

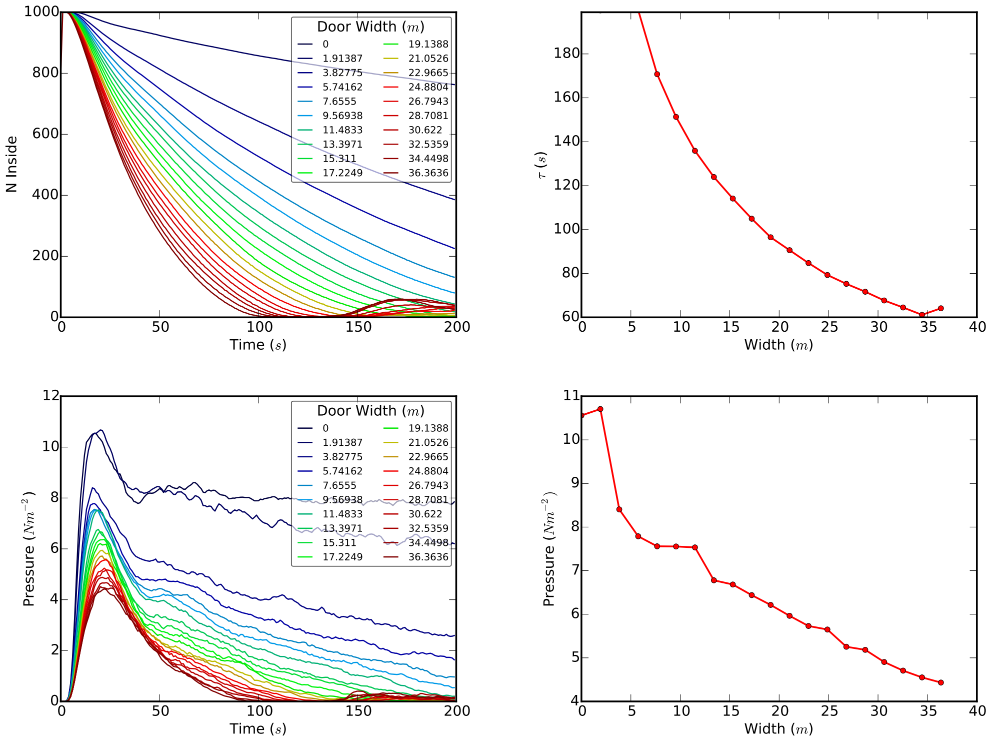


Figure 5.12: Results from exit scenario with obstruction. Top Left: Number of particles inside vs Time. Top Right: Time taken for 80% of all particles to exit, τ vs door width w. Bottom Left: Pressure vs Time. Bottom Right Maximum pressure vs door width.

Chapter 6

Conclusion

The aim of this project has been to approach the field of crowd dynamics by reviewing some current and past techniques with the intention of implementing a flexible model for detailed simulations of pedestrian behaviour. The implementation achieves this somewhat but is definitely a work in progress when it comes to an investigative purpose.

The model itself does appear to produce many of the characteristics documented by Helbing et al. in their empirical study when in a relatively passive state. In the applied scenario however we do not see the fully jammed or stop and go phenomena that has been identified as characteristic of many crowd movement[15]. This indicates a missing element of consideration. It is possible that this relates to the desired velocity that agents in this model hold onto. While the push to move forward is a reasonable approximation when related to humans in a crowd, we have to realise that humans are not mindless. This approximation does not take into account the hesitation, panic and other human nuances that contribute to the overall behaviour. It could be a case of identifying optimal parameters.[26]

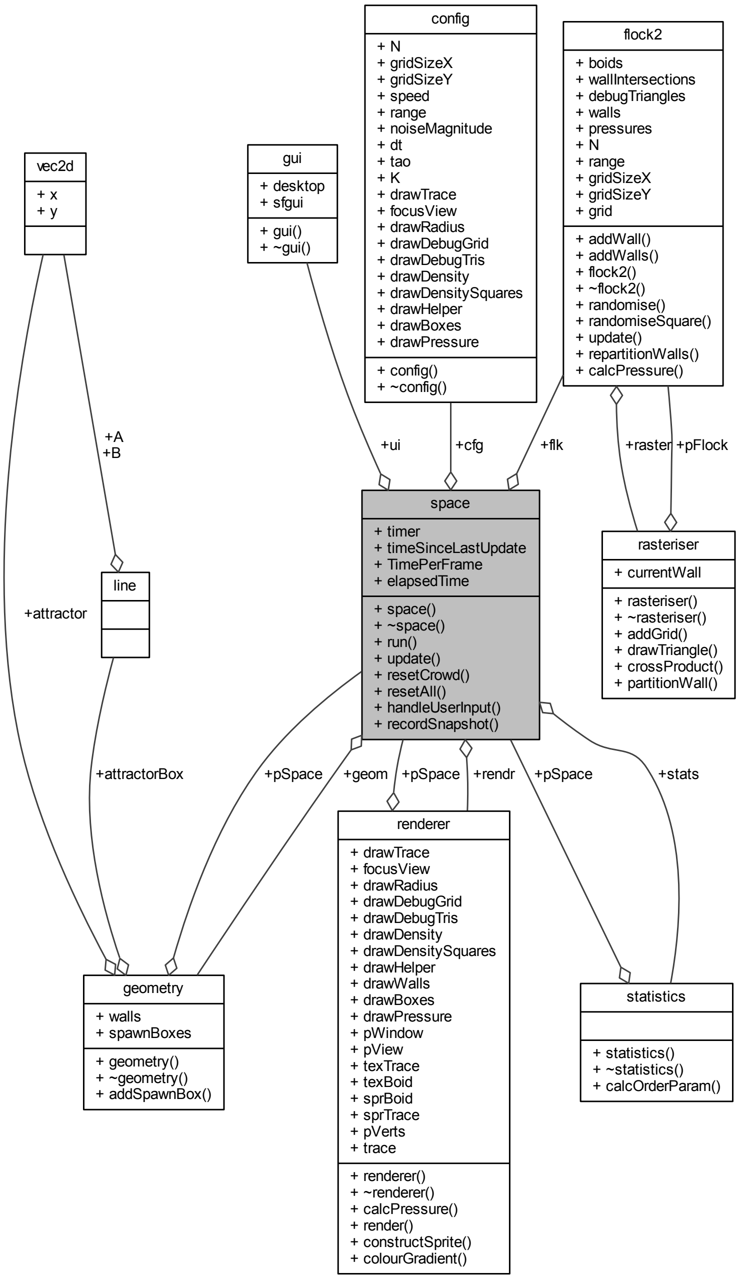
More behaviour artifacts can be observed, on an individual scale. One particular problem that stands out is the lack of any forward reaction of agents responding to others or objects in their path. This appears to be primarily a consequence of the desired destination being implemented as a single point attractor which pedestrians attempt to align with. Although this does not prevent the models ability to display emergent behaviour, it perhaps oversimplifies the system. It is clear that for a wide range of situations the model in its current form will be unable to describe crowd behaviour accurately. Lin et al. suggest the importance of friction forces in accurate crowd turbulence simulation [19]. The tendency towards solely laminar flow in our system seems to support this idea. Further failing can be clearly observed when particles are approaching the attractor with a wall crossing their path at near perpendicular angle. The agent will not attempt to avoid the obstacle but head straight towards it. While many other models completely fail here with particles getting stuck in futile attempts to pass through the wall directly. The Active Jam model offers slight improvement in this situation compared to other reactionary models as the particles tend to move along the wall in what is usually the correct direction, however this behaviour is not a complete or authentic description of human behaviour and it would be naive to think that this does not have a greater impact in the overall group dynamics.

6.0.1 Further Work

Further investigation could focus on applying the model in more complex scenarios. we have covered a very simplified setup here and already noted some failings. However, due to its simplicity the Active-Jam model is a good candidate as a basis for future work. With the addition of some sort of pathfinding and friction interactions I am confident that the authenticity of human-like group behaviour would be greatly improved.

Appendix A

Interaction Diagram



Bibliography

[1] N. Bellomo, C. Bianca, and V. Coscia. "On the modeling of crowd dynamics: An overview and research perspectives". In: *SeMA Journal* 54.1 (Apr. 2011), pp. 25–46. ISSN: 1575-9822. DOI: `10.1007/bf03322586`.

[2] Paul Bourke. *Points, lines, and planes*. 1988. URL: `http://paulbourke.net/geometry/pointlineplane`.

[3] Scott Camazine et al. *Self-organization in biological systems*. 2nd ed. United States: Princeton University Press, Aug. 2003. ISBN: 9780691116242.

[4] Damian Carrington. "World population to hit 11bn in 2100 – with 70% chance of continuous rise". In: *The Guardian* (Sept. 2014). URL: `http://www.theguardian.com/environment/2014/sep/18/world-population-new-study-11bn-2100` (visited on 03/07/2016).

[5] Chemwiki. *Closest packed structures*. chemwiki.ucdavis.edu. Oct. 2013. URL: `http://chemwiki.ucdavis.edu/Core/Physical%5C_Chemistry/Physical%5C_Properties%5C_of%5C_Matter/States%5C_of%5C_Matter/Solids/Crystal%5C_Lattice/Closest%5C_Pack%5C_Structures` (visited on 03/07/2016).

[6] *CMSC 433 programming language technologies and paradigms spring 2013 candidates for parallelism*. Tech. rep. 2013. URL: `https://www.cs.umd.edu/class/spring2013/cmsc433/Notes/17-CMSC433-CandidatesForParallelism.pdf` (visited on 03/07/2016).

[7] Iain D Couzin and Jens Krause. "Self-organization and collective behavior in vertebrates". In: *Advances in the Study of Behavior* 32 (2003), pp. 1–75. DOI: `10.1016/S0065-3454(03)01001-5`. URL: `http://dx.doi.org/10.1016/S0065-3454(03)01001-5` (visited on 02/24/2016).

[8] Julien Deseigne, Olivier Dauchot, and Hugues Chaté. "Collective motion of vibrated polar disks". In: *Physical Review Letters* 105.9 (Aug. 2010). ISSN: 0031-9007. DOI: `10.1103/physrevlett.105.098001`.

[9] *Doxygen: Main page.* URL: http://www.stack.nl/%5Ctextasciitidledimitri/doxygen/ (visited on 02/25/2016).

[10] Christer Ericson. *Real-time collision detection (the Morgan Kaufmann series in interactive 3-D technology).* San Francisco, CA: Morgan Kaufmann Publishers In, 2004.

[11] Peter Giblin, J. K. Parrish, and W. M. Hamner. “Animal groups in Three dimensions”. In: *The Mathematical Gazette* 83.497 (July 1999), p. 379. ISSN: 0025-5572. DOI: 10.2307/3619124.

[12] S. M. V. Gwynne and A. A. Siddiqui. *Traffic and Granular Flow ’11.* Ed. by V. Valery Kozlov et al. Berlin, Heidelberg: Springer Berlin Heidelberg, 2013. Chap. Understanding and Simulating Large Crowds, pp. 217–239. ISBN: 978-3-642-39669-4. DOI: 10.1007/978-3-642-39669-4_22. URL: http://dx.doi.org/10.1007/978-3-642-39669-4_22.

[13] Hisashi Haga et al. “Collective movement of Epithelial cells on a collagen gel substrate”. In: *Biophysical Journal* 88.3 (Mar. 2005), pp. 2250–2256. ISSN: 0006-3495. DOI: 10.1529/biophysj.104.047654.

[14] Dirk Helbing. “Traffic and related self-driven many-particle systems”. In: *Reviews of Modern Physics* 73.4 (Dec. 2001), pp. 1067–1141. ISSN: 0034-6861. DOI: 10.1103/revmodphys.73.1067.

[15] Dirk Helbing, Anders Johansson, and Habib Zein Al-Abideen. “Dynamics of crowd disasters: An empirical study”. In: *Physical Review E* 75.4 (Apr. 2007). ISSN: 1539-3755. DOI: 10.1103/physreve.75.046109.

[16] Dirk Helbing and Péter Molnár. “Social force model for pedestrian dynamics”. In: *Physical Review E* 51.5 (May 1995), pp. 4282–4286. ISSN: 1063-651X. DOI: 10.1103/physreve.51.4282.

[17] Silke Henkes, Yaouen Fily, and Cristina M Marchetti. “Active jamming: Self-propelled soft particles at high density”. In: *Physical Review E* 84.4 (Oct. 2011), p. 40301. DOI: 10.1103/PhysRevE.84.040301. URL: http://dx.doi.org/10.1103/physreve.84.040301 (visited on 02/24/2016).

[18] J. H. Irving and John G. Kirkwood. “The statistical mechanical theory of transport processes. IV. The equations of Hydrodynamics”. In: *The Journal of Chemical Physics* 18.6 (1950), p. 817. ISSN: 0021-9606. DOI: 10.1063/1.1747782.

[19] Ming C Lin, Abhinav Golas, and Rahul Narain. "Continuum modeling of crowd turbulence". In: *Physical Review E* 90.4 (Oct. 2014), p. 42816. DOI: 10.1103/PhysRevE.90.042816. URL: http://dx.doi.org/10.1103/physreve.90.042816 (visited on 02/23/2016).

[20] Microsoft. "C++ AMP (C++ accelerated massive parallelism)". In: (2016). URL: https://msdn.microsoft.com/en-gb/library/hh265137.aspx (visited on 01/08/2016).

[21] Bastian Molkenthin. *Software Rasterization Algorithms for filling triangles.* 2012. URL: http://www.sunshine2k.de/coding/java/TriangleRasterization/TriangleRasterization.html.

[22] J. Palacci et al. "Living crystals of light-activated colloidal surfers". In: *Science* 339.6122 (Jan. 2013), pp. 936–940. ISSN: 0036-8075. DOI: 10.1126/science.1230020.

[23] C. Saloma et al. "Self-organized queuing and scale-free behavior in real escape panic". In: *Proceedings of the National Academy of Sciences* 100.21 (Sept. 2003), pp. 11947–11952. ISSN: 0027-8424. DOI: 10.1073/pnas.2031912100.

[24] B. Szabó et al. "Phase transition in the collective migration of tissue cells: Experiment and model". In: *Physical Review E* 74.6 (Dec. 2006). ISSN: 1539-3755. DOI: 10.1103/physreve.74.061908.

[25] Tamás Vicsek et al. "Novel type of phase transition in a system of self-driven particles". In: *Physical Review Letters* 75.6 (Aug. 1995), pp. 1226–1229. ISSN: 0031-9007. DOI: 10.1103/physrevlett.75.1226.

[26] C. A. Yates et al. "Refining self-propelled particle models for collective behaviour". In: *Canadian Applied Maths Quarterly (CAMQ)* 18.3 (2011). URL: http://people.maths.ox.ac.uk/yatesc/yates2011rsp.pdf.