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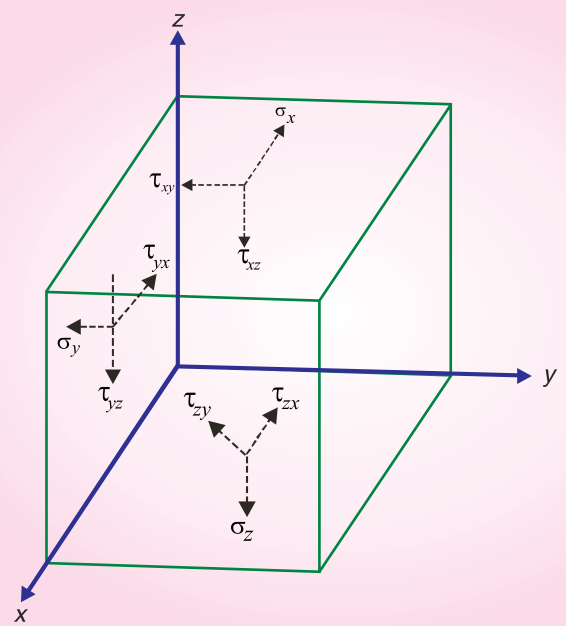
FINITE ELEMENT ANALYSIS

FOURTH EDITION

S S BHAVIKATTI



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FINITE ELEMENT ANALYSIS

FOURTH EDITION

S S BHAVIKATTI
Emeritus Professor
School of Civil and Environmental Engineering
KLE Technological University, Hubli
(Formerly BVBCET, Hubli)
Former Professor and Dean, NITK, Surathkal
SDMCET, Dharwad and Principal, RYMEC
Ballari

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Preface to the Fourth Edition

The Third Edition has been revised by adding summary at the end of each chapter so that important points from each chapter can be remembered by the reader easily. Few print mistakes are also corrected. Suggestions for further improvements are most welcome.

DR S.S. BHAVIKATTI

Preface to the First Edition

Finite Element Analysis was developed as a numerical method of stress analysis, but now it has been extended as a general method of solution to many complex engineering and physical science problems. As it involves lots of calculations, its growth is closely linked with the developments in computer technology. Now-a-days a number of finite element analysis packages are available commercially and number of users is increasing. A user without a basic course on finite element analysis may produce dangerous results. Hence now-a-days in many M.Tech. programmes finite element analysis is a core subject and in undergraduate programmes many universities offer it as an elective subject. The experience of the author in teaching this course to M.Tech (Geotechnical Engineering) and M.Tech. (Industrial Structures) students at National Institute of Technology, Karnataka, Surathkal (formerly, K.R.E.C. Surathkal) and to undergraduate students at SDM College of Engineering and Technology, Dharwad inspired him to write this book. This is intended as a textbook to students and as an introductory course to all users of finite element packages.

The author has developed the finite element concept, element properties and stiffness equations in first nine chapters. In chapter X the various points to be remembered in discritization for producing best results is presented. Isoparametric concept is developed and applications to simple structures like bars, trusses, beams and rigid frames is explained thoroughly taking small problems for hand calculations. Application of this method to complex problems like plates, shells and nonlinear analysis is introduced. Finally a list of commercially available packages is given and the desirable features of such packages is presented.

The author hopes that the students and teachers will find it as a useful textbook. The suggestions for improvements are most welcome.

DR S.S. BHAVIKATTI

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1

Introduction

1.1 GENERAL

The finite element analysis is a numerical technique. In this method all the complexities of the problems, like varying shape, boundary conditions and loads are maintained as they are but the solutions obtained are approximate. Because of its diversity and flexibility as an analysis tool, it is receiving much attention in engineering. The fast improvements in computer hardware technology and slashing of cost of computers have boosted this method, since the computer is the basic need for the application of this method. A number of popular brand of finite element analysis packages are now available commercially. Some of the popular packages are STAAD-PRO, GT-STRUDEL, NASTRAN, NISA and ANSYS. Using these packages one can analyse several complex structures.

The finite element analysis originated as a method of stress analysis in the design of aircrafts. It started as an extension of matrix method of structural analysis. Today this method is used not only for the analysis in solid mechanics, but even in the analysis of fluid flow, heat transfer, electric and magnetic fields and many others. Civil engineers use this method extensively for the analysis of beams, space frames, plates, shells, folded plates, foundations, rock mechanics problems and seepage analysis of fluid through porous media. Both static and dynamic problems can be handled by finite element analysis. This method is used extensively for the analysis and design of ships, aircrafts, space crafts, electric motors and heat engines.

1.2 GENERAL DESCRIPTION OF THE METHOD

In engineering problems there are some basic unknowns. If they are found, the behaviour of the entire structure can be predicted. The **basic unknowns** or the **Field variables** which are encountered in the engineering problems are displacements in solid mechanics, velocities in fluid mechanics, electric and magnetic potentials in electrical engineering and temperatures in heat flow problems.

In a continuum, these unknowns are infinite. The finite element procedure reduces such unknowns to a finite number by dividing the solution region into small parts called **elements** and by expressing the unknown field variables in terms of assumed **approximating functions** (Interpolating functions/Shape functions) within each element. The approximating functions are defined in terms of field variables of specified points called **nodes** or **nodal points**. Thus in the finite element analysis the unknowns are the field variables of the nodal points. Once these are found the field variables at any point can be found by using interpolation functions.

After selecting elements and nodal unknowns next step in finite element analysis is to assemble **element properties** for each element. For example, in solid mechanics, we have to find the force-displacement i.e. stiffness characteristics of each individual element. Mathematically this relationship is of the form

$$[k]_e \{\delta\}_e = \{F\}_e$$

where $[k]_e$ is element stiffness matrix, $\{\delta\}_e$ is nodal displacement vector of the element and $\{F\}_e$ is nodal force vector. The element of stiffness matrix k_{ij} represent the force in coordinate direction 'i' due to a unit displacement in coordinate direction 'j'. Four methods are available for formulating these element properties viz. direct approach, variational approach, weighted residual approach and energy balance approach. Any one of these methods can be used for assembling element properties. In solid mechanics variational approach is commonly employed to assemble stiffness matrix and nodal force vector (consistant loads).

Element properties are used to assemble global properties/structure properties to get system equations $[k]\{\delta\} = \{F\}$. Then the boundary conditions are imposed. The solution of these simultaneous equations give the nodal unknowns. Using these nodal values additional calculations are made to get the required values e.g. stresses, strains, moments, etc. in solid mechanics problems.

Thus the various steps involved in the finite element analysis are:

(i) Select suitable field variables and the elements.
(ii) Discritise the continua.
(iii) Select interpolation functions.
(iv) Find the element properties.
(v) Assemble element properties to get global properties.
(vi) Impose the boundary conditions.
(vii) Solve the system equations to get the nodal unknowns.
(viii) Make the additional calculations to get the required values.

1.3 A BRIEF EXPLANATION OF FEA FOR A STRESS ANALYSIS PROBLEM

The steps involved in finite element analysis are clarified by taking the stress analysis of a tension strip with fillets (refer Fig.1.1). In this problem stress concentration is to be studies in the fillet zone. Since the problem is having symmetry about both *x* and *y* axes, only one quarter of the tension strip may be considered as shown in Fig.1.2. About the symmetric axes, transverse displacements of all nodes are to be made zero. The various steps involved in the finite element analysis of this problem are discussed below:

Step 1: Four noded isoparametric element (refer Fig 1.3) is selected for the analysis (However note that 8 noded isoparametric element is ideal for this analysis). The four noded isoparametric element can take quadrilateral shape also as required for elements 12, 15, 18, etc. As there is no bending of strip, only displacement continuity is to be ensured but not the slope continuity. Hence displacements of nodes in *x* and *y* directions are taken as basic unknowns in the problem.

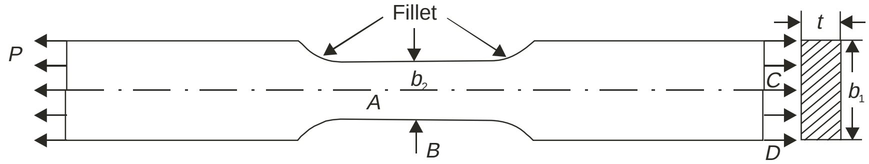


Fig. 1.1 *Typical tension flat*

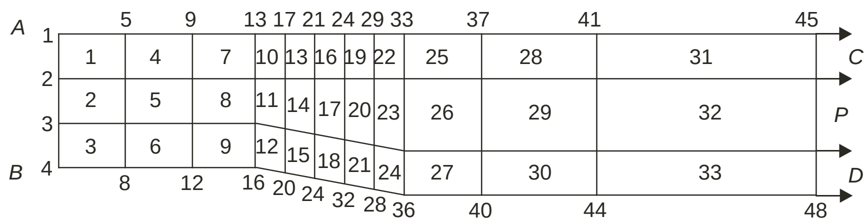
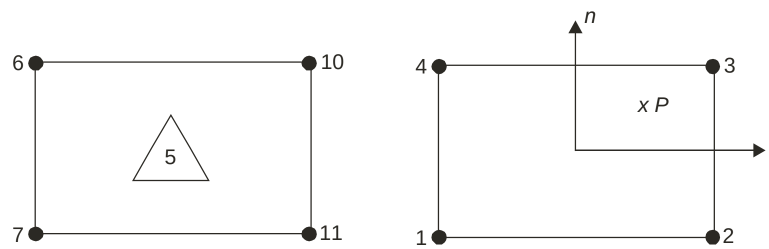


Fig. 1.2 *Discretisation of quarter of tension flat*



(a) Element no. 5 (b) Typical element

Fig. 1.3

Step 2: The portion to be analysed is to be discretised. Fig. 1.2 shows discretised portion. For this 33 elements have been used. There are 48 nodes. At each node unknowns are *x* and *y* components of displacements. Hence in this problem total unknowns (displacements) to be determined are 48 × 2 = 96.

Step 3: The displacement of any point inside the element is approximated by suitable functions in terms of the nodal displacements of the element. For the typical element (Fig. 1.3 b), displacements at *P* are

$$u = \sum N_i u_i = N_1 u_1 + N_2 u_2 + N_3 u_3 + N_4 u_4$$

and $$v = \sum N_i v_i = N_1 v_1 + N_2 v_2 + N_3 v_3 + N_4 v_4 \qquad \ldots(1.2)$$

The approximating functions N_i are called shape functions or interpolation functions. Usually they are derived using polynomials. The methods of deriving these functions for various elements are discussed in this text in latter chapters.

Step 4: Now the stiffness characters and consistant loads are to be found for each element. There are four nodes and at each node degree of freedom is 2. Hence degree of freedom in each element is 4 × 2 = 8. The relationship between the nodal displacements and nodal forces is called element stiffness characteristics. It is of the form

$$[k]_e \{\delta\}_e = \{F\}_e, \text{ as explained earlier.}$$

For the element under consideration, k_e is 8 × 8 matrix and δ_e and F_e are vectors of 8 values. In solid mechanics element stiffness matrix is assembled using variational approach i.e. by minimizing potential energy. If the load is acting in the body of element or on the surface of element, its equivalent at nodal points are to be found using variational approach, so that right hand side of the above expression is assembled. This process is called finding consistant loads.

Step 5: The structure is having 48 × 2 = 96 displacement and load vector components to be determined. Hence global stiffness equation is of the form

$$\underset{96 \times 96}{[k]} \quad \underset{96 \times 1}{\{\delta\}} \quad = \underset{96 \times 1}{\{F\}}$$

Each element stiffness matrix is to be placed in the global stiffness matrix appropriately. This process is called assembling global stiffness matrix. In this problem force vector F is zero at all nodes except at nodes 45, 46, 47 and 48 in x direction. For the given loading nodal equivalent forces are found and the force vector F is assembled.

Step 6: In this problem, due to symmetry transverse displacements along AB and BC are zero. The system equation $[k]\{\delta\} = \{F\}$ is modified to see that the solution for $\{\delta\}$ comes out with the above values. This modification of system equation is called imposing the boundary conditions.

Step 7: The above 96 simultaneous equations are solved using the standard numerical procedures like Gauss-elimination or Choleski's decomposition techniques to get the 96 nodal displacements.

Step 8: Now the interest of the analyst is to study the stresses at various points. In solid mechanics the relationship between the displacements and stresses are well established. The stresses at various points of interest may be found by using shape functions and the nodal displacements and then stresses calculated. The stress concentrations may be studies by comparing the values obtained at various points in the fillet zone with the values at uniform zone, far away from the fillet (which is equal to P/b_2t).

1.4 FINITE ELEMENT METHOD VS CLASSICAL METHODS

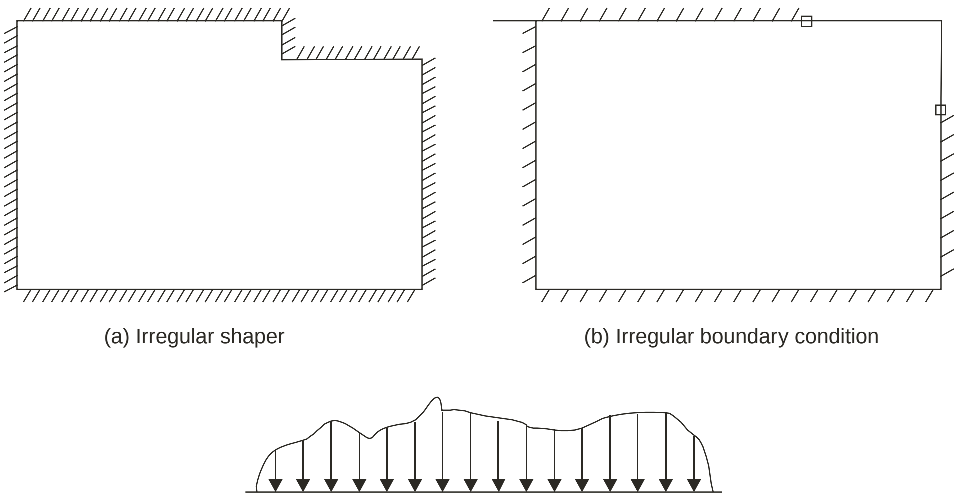
1. In classical methods exact equations are formed and exact solutions are obtained where as in finite element analysis exact equations are formed but approximate solutions are obtained.
2. Solutions have been obtained for few standard cases by classical methods, where as solutions can be obtained for all problems by finite element analysis.
3. Whenever the following complexities are faced, classical method makes the drastic assumptions' and looks for the solutions:

   (a) Shape
   (b) Boundary conditions
   (c) Loading

   Fig. 1.4 shows such cases in the analysis of slabs (plates).

   To get the solution in the above cases, rectangular shapes, same boundary condition along a side and regular equivalent loads are to be assumed. In FEM no such assumptions are made. The problem is treated as it is.
4. When material property is not isotropic, solutions for the problems become very difficult in classical method. Only few simple cases have been tried successfully by researchers. FEM can handle structures with anisotropic properties also without any difficulty.
5. If structure consists of more than one material, it is difficult to use classical method, but finite element can be used without any difficulty.
6. Problems with material and geometric non-linearities can not be handled by classical methods. There is no difficulty in FEM.

Hence FEM is superior to the classical methods only for the problems involving a number of complexities which cannot be handled by classical methods without making drastic assumptions. For all regular problems, the solutions by classical methods are the best solutions. Infact, to check the validity of the FEM programs developed, the FEM solutions are compared with the solutions by classical methods for standard problems.



(a) Irregular shaper

(b) Irregular boundary condition

(c) Irregular loading

Fig. 1.4

1.5 FEM VS FINITE DIFFERENCE METHOD (FDM)

1. FDM makes **pointwise approximation** to the governing equations i.e. it ensures continuity only at the node points. Continuity along the sides of grid lines are not ensured.

   FEM make piecewise approximation i.e. it ensures the continuity at node points as well as along the sides of the element.
2. FDM do not give the values at any point except at node points. It do not give any approximating function to evaluate the basic values (deflections, in case of solid mechanics) using the nodal values.

   FEM can give the values at any point. However the values obtained at points other than nodes are by using suitable interpolation formulae.
3. FDM makes stair type approximation to sloping and curved boundaries as shown in Fig. 1.5.

   FEM can consider the sloping boundaries exactly. If curved elements are used, even the curved boundaries can be handled exactly.
4. FDM needs larger number of nodes to get good results while FEM needs fewer nodes.
5. With FDM fairly complicated problems can be handled where as FEM can handle all complicated problems.

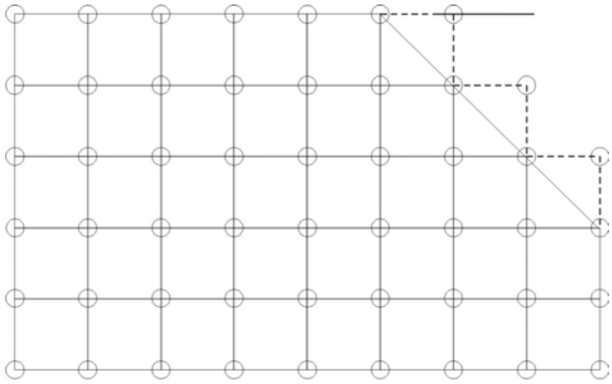


Fig. 1.5 *FDM approximation of shape*

1.6 A BRIEF HISTORY OF FEM

Engineers, physicists and mathematicians have developed finite element method independently. In 1943 Courant [1] made an effort to use piecewise continuous functions defined over triangular domain.

After that it took nearly a decade to use this distribution idea. In fifties renewed interest in this field was shown by Polya [2], Hersh [3] and Weinberger [4]. Argyris and Kelsey [5] introduced the concept of applying energy principles to the formation of structural analysis problems in 1960. In the same year Clough [6] introduced the word **'Finite Element Method'**.

In sixties convergence aspect of the finite element method was pursued more rigorously. One such study by Melesh [7] led to the formulation of the finite element method based on the principles of minimum potential energy. Soon after that de Veubeke [8] introduced equilibrium elements based on the principles of minimum potential energy, Pion [9] introduced the concept of hybrid element using the duel principle of minimum potential energy and minimum complementary energy.

In Late 1960's and 1970's, considerable progress was made in the field of finite element analysis. The improvements in the speed and memory capacity of computers largely contributed to the progress and success of this method. In the field of solid mechanics from the initial attention focused on the elastic analysis of plane stress and plane strain problems, the method has been successfully extended to the cases of the analysis of three dimensional problems, stability and vibration problems, non-linear analysis. A number of books [10 – 20] have appeared and made this field interesting.

1.7 NEED FOR STUDYING FEM

Now, a number of users friendly packages are available in the market. Hence one may ask the question 'What is the need to study FEA?'.

The above argument is not sound. The finite element knowledge makes a good engineer better while just user without the knowledge of FEA may produce more dangerous results. To use the FEA packages properly, the user must know the following points clearly:

1. Which elements are to be used for solving the problem in hand.
2. How to discritise to get good results.
3. How to introduce boundary conditions properly.

4. How the element properties are developed and what are their limitations.
5. How the displays are developed in pre and post processor to understand their limitations.
6. To understand the difficulties involved in the development of FEA programs and hence the need for checking the commercially available packages with the results of standard cases.

Unless user has the background of FEA, he may produce worst results and may go with overconfidence. Hence it is necessary that the users of FEA package should have sound knowledge of FEA.

1.8 WARNING TO FEA PACKAGE USERS

When hand calculations are made, the designer always gets the feel of the structure and get rough idea about the expected results. This aspect cannot be ignored by any designer, whatever be the reliability of the program, a complex problem may be simplified with drastic assumptions and FEA results obtained. Check whether expected trend of the result is obtained. Then avoid drastic assumptions and get more refined results with FEA package. User must remember that structural behaviour is not dictated by the computer programs. Hence the designer should develop feel of the structure and make use of the programs to get numerical results which are close to structural behaviour.

Summary

1. FEM is a numerical technique. Field variables of nodal points are the unknowns. Approximating/shape functions are used to define field variable at any point in the element in terms of field variables of nodal points of the element.

2. The system equation is of the form $[k]\{\delta\} = \{F\}$ where $[k]$ is global stiffness matrix, $\{\delta\}$ is displacement of nodal points and $\{F\}$ is nodal force vector.

3. After imposing boundary conditions, system equations are solved to get nodal unknowns. Using these values additional calculations are made to get required quantities.

4. FEM is superior to classical methods only for the problems involving a number of complexities which cannot be handled by classical methods.

5. FEM is superior to FDM.

6. FEM knowledge makes a good engineer better while just user of packages without the knowledge of FEA may produce more dangerous results.

7. A designer should develop feel of the structure and make use of FEA packages to get numerical results which are close to structural behaviour.

QUESTIONS

1. Explain the concept of FEM briefly and outline the procedure.
2. Discuss the advantages and disadvantages of FEM over
   (i) Classical method
   (ii) Finite difference method.
3. Clearly point out the situations in which FEM is preferred over other methods.
4. When there are several FEM packages are available is there need to study this method? Discuss.

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FINITE ELEMENT ANALYSIS

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The method is made clear by solving many problems by hand calculations. The application of finite element method to plates, shells and nonlinear analysis is presented. After listing some of the commercially available finite element analysis packages, the structure of a finite element programme and the desired features of commercial packages are discussed.

Dr. S S Bhavikatti studied at BVB College of Engineering and Technology, Hubli, Karnataka University, Dharwad in 1963 for his BE (Civil) degree, sharing first rank with another candidate. In the same year he joined Karnataka Regional Engineering College (presently NITK), Surathkal, as a lecturer. He secured ME degree in Structural Engineering in 1967 from University of Roorkee, Roorkee (presently IIT Roorkee) under Technical Teachers Training Programme and PhD in 1977 from IIT Delhi under Quality Improvement Programme. He served at NITK, Surathkal in different capacities like Head of Civil Engineering Department; Chairman, Centre for Continuing Education; Chairman, Central Computer Centre and Dean (Administration). From November 2001 to November 2005, he served at SDM College of Engineering and Technology, Dharwad. He then served at RYMEC, Ballari as a Principal upto September 2007. Presently he is working as an Emeritus Professor at KLE Technological University, Hubli. He has published 66 technical papers and 66 books. He has guided dissertation of 69 students of M.Tech. and 6 students of PhD. He was a member of organizing scientific committee of international seminars organized by IASS held at Taegue (South Korea) in 1990, Tokyo (Japan) in 1993 and Taipei (Taiwan) in 1997. He served as planning board member of Indira Gandhi National Open University (IGNOU), New Delhi for the calendar years 1998, 1999 and 2000. He is also a member of several Indian and international professional bodies and several technical committees.

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