COMSM1201 : Data Structures & Algorithms

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- Let's look at some toy examples to begin with.

```
#include <stdio.h>
     #include <string.h>
     #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
     void strrev(char* s, int n);
     int main (void)
        char str[] = "Hello World!":
        strrev(str. strlen(str)):
12
        printf("%s\n", str);
        return 0:
14
15
     /* Iterative Inplace String Reverse */
17
     void strrev(char* s. int n)
18
19
        for(int i=0, j=n-1; i<j; i++, j--){
            SWAP(s[i], s[j]);
20
21
22
```

Execution:

!dlroW olleH

Recursion for *strrev()*

```
#include <stdio.h>
    #include <string.h>
    #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
    void strrev(char* s, int start, int end);
    int main(void)
       char str[] = "Hello World!";
        strrev(str. 0. strlen(str)-1):
        printf("%s\n", str);
13
14
       return 0;
15
    /* Recursive : Inplace String Reverse */
    void strrev(char* s. int start, int end)
19
       if(start >= end){
           return:
       SWAP(s[start], s[end]);
23
24
        strrev(s. start+1, end-1);
```

Execution:

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N : Recursion 4 / 8:

Recursion for *strrev()*

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#include <stdio.h>
    #include <string.h>
    #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
    void strrev(char* s, int start, int end);
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```

• We need to change the function prototype.

Execution:

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Recursion for *strrev()*

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#include <stdio.h>
    #include <string.h>
    #define SWAP(A.B) {char temp: temp=A:A=B:B=temp:}
    void strrev(char* s, int start, int end);
    int main(void)
       char str[] = "Hello World!";
       strrev(str. 0. strlen(str)-1):
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    /* Recursive : Inplace String Reverse */
    void strrev(char* s, int start, int end)
19
       if(start >= end){
20
           return:
       SWAP(s[start], s[end]);
23
24
       strrev(s. start+1, end-1):
```

- We need to change the function prototype.
- This allows us to track both the start and the end of the string.

Execution:

IdlroW olleH

The Fibonacci Sequence

A well known example of a recursive function is the Fibonacci sequence. The first term is 1, the second term is 1 and each successive term is defined to be the sum of the two previous terms, i.e. :

```
fib(1) is 1
fib(2) is 1
fib(n) is fib(n-1)+fib(n-2)
```

1,1,2,3,5,8,13,21, ...

N : Recursion 5 / 8:

```
#include <stdio.h>
    #define MAXFIB 24
    int fibonacci(int n);
     int main(void)
        for(int i=1: i <= MAXFIB: i++){
           printf("%d = %d\n", i, fibonacci(i));
13
14
15
        return 0;
16
17
     int fibonacci(int n)
19
        if(n \le 2)
           return 1;
       int b = 1:
        int next:
        for (int i=3; i \le n; i++){
           next = a + b:
           a = b:
           b = next:
        return b:
32
```

```
#include <stdio.h>
    #define MAXFIB 24
    int fibonacci(int n):
     int main(void)
        for(int i=1: i <= MAXFIB: i++){</pre>
            printf("%d = %d\n", i, fibonacci(i)):
13
14
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        return 0;
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     int fibonacci(int n)
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        if(n \le 2)
           return 1;
        int b = 1:
        int next:
        for (int i=3; i \le n; i++){
           next = a + b:
           a = b:
29
           b = next:
30
31
        return b:
32
```

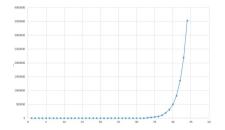
Execution:

```
1 = 1
 = 13
 = 21
9 = 34
10 = 55
11 = 89
12 = 144
13 = 233
14 = 377
15 = 610
16 = 987
17 = 1597
18 = 2584
19 = 4181
20 = 6765
21 = 10946
22 = 17711
23 = 28657
24 = 46368
```

```
#include <stdio.h>
#define MAXFIB 24
int fibonacci(int n);
int main(void)
   for(int i=1; i <= MAXFIB; i++){</pre>
      printf("%d = %d\n", i, fibonacci(i));
   return 0;
int fibonacci(int n)
   if (n == 1) return 1:
   if (n == 2) return 1:
   return (fibonacci(n-1)+fibonacci(n-2));
```

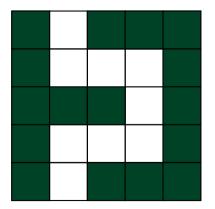
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#include <stdio.h>
    #define MAXFIB 24
    int fibonacci(int n);
     int main(void)
        for(int i=1; i <= MAXFIB; i++){</pre>
           printf("%d = %d\n", i, fibonacci(i));
       return 0;
    int fibonacci(int n)
20
21
       if (n == 1) return 1:
       if (n == 2) return 1:
        return ( fibonacci(n-1)+fibonacci(n-2));
```

It's interesting to see how run-time increases as the length of the sequence is raised.



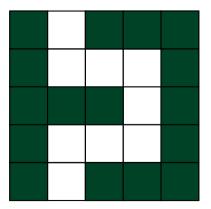
Maze Escape

The correct route through a maze can be obtained via recursive, rather than iterative, methods.



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```
bool explore(int x, int y, char mz[YS][XS])
  if mz[y][x] is exit return true;
  Mark mz[y][x] so we don't return here
  if we can go up:
    if(explore(x, y+1, mz)) return true
  if we can go right:
    if(explore(x+1, y, mz)) return true
  Do left & down in a similar manner
  return false: // Failed to find route
```

 Here we consider the ways to permute a string (or more generally an array)

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- Permutations are all possible ways of rearranging the positions of the characters

Execution:

ABC

ACB

BAC

BCA

CBA CAB

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- Permutations are all possible ways of rearranging the positions of the characters.

Execution :

ABC ACB

BAC BCA CBA CAB

```
// From e.g. http://www.geeksforgeeks.org
    #include <stdio.h>
    #include <string.h>
    #define SWAP(A,B) {char temp = *A; *A = *B; *B = temp;}
     void permute(char* a, int s, int e);
     int main()
         char str[] = "ABC";
         int n = strlen(str);
         permute(str. 0, n-1);
         return 0:
     void permute(char* a, int s, int e)
18
        if (s == e){
          printf("%s\n", a);
          return:
        for (int i = s: i \le e: i++)
24
           SWAP((a+s), (a+i)); // Bring one char to the front
25
           permute(a, s+1, e);
26
           SWAP((a+s), (a+i)); // Backtrack
27
28
```

N : Recursion

 Raising a number to a power n = 2⁵ is the same as multiple multiplications n = 2*2*2*2*2.

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- Or, thinking recursively, $n = 2 * (2^4)$.

```
/* Try to write power(a.b) to computer a^b
        without using any maths functions other than
        multiplication :
        Try (1) iterative then (2) recursive
        (3) Trick that for n\%2==0, x^n = x^(n/2)*x^(n/2)
9
    #include <stdio.h>
11
     int power(unsigned int a, unsigned int b);
12
     int main(void)
16
        int x = 2:
        int v = 16:
19
        printf("%d^%d = %d\n", x, y, power(x,y));
20
21
22
     int power(unsigned int a, unsigned int b)
```

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- The simplest method for searching is called the sequential search.
- Simply move through the array from beginning to end, stopping when you have found the value you require.

```
#include <stdio.h>
     #include <string.h>
     #include <assert.h>
     #define NOTFOUND -1
     #define NUMPEOPLE 6
     typedef struct person {
             char* name; int age;
     } person;
     int findAge(const char* name, const person* p, int n);
     int main (void)
        person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
                   {"Chumley", 26}, {"Dalton", 25},
                   {"Eggson", 22}, {"Fulton", 41} };
        assert(findAge("Eggson",
                                    ppl, NUMPEOPLE) == 22);
        assert (find Age ("Campbell", ppl, NUMPEOPLE) == NOTFOUND);
        return 0:
23
24
     int findAge(const char* name, const person* p, int n)
25
        for (int j=0; j < n; j++){
27
           if (strcmp(name, p[i], name) == 0){
              return p[i].age:
29
30
31
        return NOTFOUND:
32
```

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O: Algorithms I - Search

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Sequential Search

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#include <stdio h>
     #include <string.h>
     #include <assert.h>
     #define NOTFOUND -1
     #define NUMPFOPLE 6
     typedef struct person{
             char* name; int age;
     } person:
11
     int findAge(const char* name, const person* p, int n):
12
13
     int main (woid)
14
15
        person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
                    {"Chumley", 26}, {"Dalton", 25},
                   {"Eggson", 22}, {"Fulton", 41} }:
        assert (find Age ("Eggson".
                                    ppl NUMPEOPLE) == 22):
        assert (find Age ("Campbell", ppl, NUMPEOPLE) == NOTFOUND):
21
        return 0:
22
23
24
     int findAge(const char* name, const person* p, int n)
25
        for (int j=0; j < n; j++){
27
           int m = strcmp(name, p[i], name);
           if (m == 0) // Braces!
              return p[i].age:
           if(m < 0)
31
              return NOTFOUND:
32
33
        return NOTFOUND:
```

 Searching small lists doesn't require much computation time.

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- However, as lists get longer (e.g. phone directories), sequential searching becomes extremely inefficient.

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- However, as lists get longer (e.g. phone directories), sequential searching becomes extremely inefficient.
- A binary search consists of examining the middle element of the array to see if it has the desired value. If not, then half the array may be discarded for the next search.

```
#include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <time.h>
    #define NMBRS 1000000
    int bin it(int k. const int* a. int 1. int r):
     int main(void)
        int a [NMBRS];
        srand(time(NULL)):
        // Put even numbers into array
        for (int i=0; i < NMBRS; i++){
           a[i] = 2*i:
        // Do many searches for a random number
        for(int i=0; i<10*NMBRS; i++){
           int n = rand()%NMBRS:
           if((n\%2) == 0){
              assert(bin it(n, a, 0, NMBRS-1) = n/2):
25
           else { // No odd numbers in this list
              assert(bin_it(n, a, 0, NMBRS-1) < 0);
        return 0:
```

Iterative v. Recursion Binary Search

```
int bin_it(int k, const int* a, int 1, int r)
{
   while(1 <= r){
      int m = (1+r)/2;
      if(k == a[m]){
            return m;
      }
      else {
        if (k > a[m]) {
            1 = m + 1;
      }
      else {
            r = m - 1;
        }
    }
   return -1;
}
```

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{
  while(1 <= r){
    int m = (1+r)/2;
    if(k = a[m]){
      return m;
    }
    else{
      if (k > a[m]){
            1 = m + 1;
        }
      else{
            r = m - 1;
        }
    }
    return -1;
}
```

```
int bin_rec(int k, const int* a, int l, int r)
{
    if(1 > r) return -1;
    int m = (1+r)/2;
    if(k = a | m |) {
        return m;
    }
    else{
        if (k > a | m |) {
            return bin_rec(k, a, m + 1, r);
        }
        else(
            return bin_rec(k, a, 1, m - 1);
        }
    }
}
```

 When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.

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- Now we use an interpolation involving the key, the start of the list and the end.

$$i = (k - I[0])/(I[n-1] - I[0]) * n$$

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$$i = (k - I[0])/(I[n-1] - I[0]) * n$$

• when searching for '15':

```
0 4 5 9 10 12 15 20
```

```
int interp(int k. const int* a. int l. int r)
   int m:
   double md:
   while(1 \le r)
      md = ((double)(k-a[1])/
            (double)(a[r]-a[1])*
            (double)(r-1)
           +(double)(1):
      m = 0.5 + md:
      if((m > r) | | (m < 1)){
         return -1:
      if (k == a[m])
         return m:
         if (k > a[m]) {
            1 = m + 1:
         elsef
            r = m-1:
```

• This code on an old Dell laptop took:

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 - 3.12 seconds using a non-optimzing compiler -O0

0 : Algorithms I - Search 17 / 8:

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- This complexity indicates how, for n numbers, performance deteriorates when n changes.

```
1  #include <stdio.h>
2  #include <tddib.h>
3  #include <tdiib.h>
4
5  #define CSEC (double)(CLOCKS_PER_SEC)
6  #define BIGLOOP 1000000000
7
8  int main(void)
9  {
10
11    clock_t c1 = clock();
12    for(int i=0; i<EIGLOOP; i++){
        int j = i * 2;
14    }
15    clock_t c2 = clock();
16    printf(*%f\n", (double)(c2-c1)/CSEC);
17    return 0;
18
18
19 }</pre>
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- Sequential Search : O(n)
- Binary Search : O(log n)

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- This complexity indicates how, for n numbers, performance deteriorates when n changes.
- Sequential Search : O(n)
- Binary Search : O(log n)
- Interpolation Search : O(log log n)

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- Sequential Search : O(n)
- Binary Search : O(log n)
- Interpolation Search : O(log log n)
- We'll discuss the dream of a O(1) search later in "Hashing".

Binary vs. Interpolation Timing

```
#include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <time.h>
    int bin it(int k, const int *a, int 1, int r);
     int bin rec(int k. const int *a. int 1. int r):
     int interp(int k, const int *a, int 1, int r);
     int* parse_args(int argc, char* argv[], int* n, int* srch);
     int main(int argc, char* argv[])
12
        int i, n, srch;
        int* a;
        int (*p[3])(int k, const int*a, int 1, int r) =
            {bin it, bin rec, interp};
        a = parse_args(argc, argv, &n, &srch);
        srand(time(NULL));
22
23
        for (i=0; i < n; i++){
           a[i] = 2*i:
24
25
        for (i=0; i<5000000; i++){}
26
27
           assert ((*p[srch])(a[rand()%n], a, 0, n-1) >= 0);
28
29
        free(a):
30
        return 0;
31
32
```

Binary vs. Interpolation Timing

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           assert ((*p[srch])(a[rand()%n], a, 0, n-1) >= 0);
28
29
        free(a):
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        return 0;
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32
```

Execution:

```
Binary Search : Iterative
       100000 = 0.39
      800000 = 0.57
      6400000 = 1.00
     51200000 = 2.46
Binary Search : Recursive
       100000 = 0.40
       800000 = 0.56
      6400000 = 0.97
n =
     51200000 = 2.42
Interpolation
n =
       100000 = 0.05
       800000 = 0.05
      6400000 = 0.10
n =
     51200000 = 0.13
n =
```

Table of Contents

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- By following pointers one after another, we can travel right along the structure.

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- To create linked data representations we use pointers to connect separate blocks of storage together. If a given block contains a pointer to a second block, we can follow this pointer there.
- By following pointers one after another, we can travel right along the structure.

```
#include <stdio h>
     #include < stdlih h>
     #include "general.h"
     typedef struct data{
        int i:
        struct data* next:
     } Data;
     Data* allocateData(int i):
11
     void printList(Data* 1):
     int main(void)
        int i:
        Data* start . *current :
        start = current = NULL:
        printf("Enter the first number: "):
19
        if(scanf("%i", &i) == 1){
20
           start = current = allocateData(i):
21
        elsef
           on_error("Couldn't read an int");
        printf("Enter more numbers: ");
27
        while(scanf("%i", &i) == 1){
           current -> next = allocateData(i):
29
           current = current -> next:
30
31
        printList(start):
        return 0: // Should Free List
33
```

Linked Lists

```
Data* allocateData(int i)
{
    Data* p;
    p = (Data*) ncalloc(1, sizeof(Data));
    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* 1)
{
    printf("\n");
    do{
        printf("\n");
        il = 1->next;
    } while(1 != NULL);
    printf("END\n");
}
```

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{
    Data* p;
    p = (Data*) ncalloc(1, sizeof(Data));
    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* 1)
{
    printf("\n");
    do{
        printf("Number : %i\n", 1->i);
        1 = 1->next;
    }while(1 != NULL);
    printf("END\n");
}
```

Searching and Recursive printing:

```
Data* inList(Data* n, int i)
{
    do{
        if(n->i==i){
            return n;
    }
        n = n->next;
} while(n != NULL);
return NULL;
}

void printList_r(Data* 1)
{
    // Recursive Base-Case
    if(1 == NULL) return;
    printf(*Number: %i\n*, 1->i);
    printList_r(1->next);
}
```

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- But would we really code something like this every time we need flexible data storage?
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- One example of this is an **Abstract Data Type (ADT)**.
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- The **user** of the ADT doesn't need to understand how the data is being stored (e.g. array vs. linked lists etc.)
- Actually, I'll sometimes blur the boundaries of Data Structures (e.g. a linked list) with ADTs (e.g. a dictionary) themselves.

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- Some collections allow duplicate elements and others do not (e.g. Sets).
- Some are ordered (for faster searching) and others unordered.
- Our Collection will be unsorted and will allow duplicates.

```
#include "../General/general.h"
typedef int colltype:
typedef struct coll coll;
#include <stdio.h>
#include <stdlib h>
#include <assert.h>
// Create an empty coll
coll* coll init(void);
// Add element onto top
void coll add(coll* c, colltype i);
// Take element out
bool coll remove(coll* c. colltype d):
// Does this exist ?
bool coll isin(coll* c. colltype i):
// Return size of coll
int coll size(coll* c):
// Clears all space used
bool coll_free(coll* c);
```

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 - A linked-list

Fixed/specific.h:

```
1  #pragma once
2
3  #define COLLTYPE "Fixed"
4
4
4
5  #define FIXEDSIZE 5000
6  struct coll {
7     // Underlying array
6     colltype a[FIXEDSIZE];
9     int size;
10 };
```

Collection ADT using a Fixed-size Array

Fixed/fixed.c:

```
#include " .. / coll . h"
     #include "specific.h"
     coll* coll_init(void)
        coll* c = (coll*) ncalloc(1, sizeof(coll));
        c \rightarrow size = 0;
        return c;
     int coll size(coll* c)
        if (c=NULL){
           return 0:
        return c->size:
19
20
     bool coll_isin(coll* c, colltype d)
        for (int i=0; i < coll size(c); i++){
           if(c\rightarrow a[i] = d)
                return true;
        return false;
```

Collection ADT using a Fixed-size Array

Fixed/fixed.c:

```
#include "../coll.h"
    #include "specific.h"
     coll* coll_init(void)
        coll* c = (coll*) ncalloc(1, sizeof(coll));
        c - > size = 0;
        return c;
     int coll size(coll* c)
13
        if (c=NULL){
           return 0:
16
17
        return c->size;
19
     bool coll_isin(coll* c, colltype d)
20
        for (int i=0: i < coll size(c): i++){
22
           if(c->a[i] == d){}
               return true:
24
        return false;
```

```
void coll add(coll* c. colltype d)
   if(c){
      if(c->size >= FIXEDSIZE){
          on error("Collection overflow"):
      c \rightarrow a[c \rightarrow size] = d:
      c \rightarrow size = c \rightarrow size + 1:
bool coll remove(coll* c. colltype d)
   for (int i=0: i < coll size(c): i++){
      if(c->a[i] == d)f
          // Shuffle end of array left one
          for(int j=i; j < coll_size(c); j++){</pre>
             c - a[i] = c - a[i+1];
          c->size = c->size - 1:
          return true:
   return false:
bool coll_free(coll* c)
   free(c):
   return true:
```

Collection ADT via an Array (Realloc)

Realloc/specific.h:

Q : ADTs - Collection

Collection ADT via an Array (Realloc)

Realloc/specific.h:

```
#pragma once

define COLLTYPE "Realloc"

define INITSIZE 16

define SCALEFACTOR 2

struct coll {
    // Underlying array
    colltype* a;
    int size;
    int capacity;
};
```

Realloc/realloc.c:

```
#include "../coll.h"
     #include "specific.h"
      coll* coll init(void)
         coll* c = (coll*) ncalloc(1, sizeof(coll));
         c->a = (colltype*) ncalloc(INITSIZE, sizeof(colltype));
         c \rightarrow size = 0:
         c->capacity= INITSIZE;
         return c:
11
      void coll add(coll* c. colltype d)
14
         if(c){
             if (c-> size >= c-> capacity) {
17
                 c \rightarrow a = (colltype*) nremalloc(c \rightarrow a.
                         sizeof(colltype)*c->capacity*SCALEFACTOR);
                 c->capacity = c->capacity*SCALEFACTOR;
21
             c \rightarrow a[c \rightarrow size] = d:
             c \rightarrow size = c \rightarrow size + 1:
23
```

Collection ADT via a Linked List

Linked/specific.h:

```
#pragma once

#define COLLTYPE "Linked"

struct dataframe {
    colltype i;
    struct dataframe* next;
    };

typedef struct dataframe dataframe;

struct coll {
    // Underlying array
    dataframe* start;
    int size;
};
```

Collection ADT via a Linked List

Linked/specific.h:

```
#pragma once

#define COLLTYPE "Linked"

struct dataframe {
    colltype i;
    struct dataframe* next;
    };
    typedef struct dataframe dataframe;

truct coll {
    // Underlying array
    dataframe* start;
    int size;
    };
}
```

Linked/linked.c:

```
#include " .. / coll .h"
#include "specific.h"
coll* coll init(void)
   coll* c = (coll*) ncalloc(1, sizeof(coll));
   return c:
int coll size(coll* c)
   if(c==NULL){
      return 0:
   return c->size:
bool coll_isin(coll* c, colltype d)
   if(c == NULL || c->start==NULL){
      return false:
   dataframe* f = c->start:
   dof
      if(f\rightarrow i == d){
          return true:
      f = f - > next;
   } while (f != NULL):
   return false:
```

Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
   if(c){
       dataframe* f = ncalloc(1, sizeof(dataframe));
       f \rightarrow i = d:
       f \rightarrow next = c \rightarrow start:
       c \rightarrow start = f;
       c \rightarrow size = c \rightarrow size + 1;
bool coll_free(coll* c)
   if(c){
       dataframe* tmp;
       dataframe* p = c->start;
       while (p!=NULL) {
           tmp = p->next;
           free(p);
           p = tmp;
       free(c):
   return true;
```

Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
   if(c){
       dataframe* f = ncalloc(1. sizeof(dataframe)):
       f \rightarrow i = d:
       f \rightarrow next = c \rightarrow start:
       c \rightarrow start = f;
       c \rightarrow size = c \rightarrow size + 1:
bool coll free(coll* c)
   if(c){
       dataframe* tmp:
       dataframe* p = c->start:
       while (p!=NULL) {
           tmp = p->next;
           free(p);
           p = tmp;
       free(c):
   return true;
```

```
bool coll_remove(coll* c, colltype d)
   dataframe* f1 . *f2:
   if((c==NULL) || (c->start==NULL)){
      return false:
   // If Front
   if (c->start -> i == d) {
      f1 = c->start->next:
      free(c->start):
      c->start = f1:
      c \rightarrow size = c \rightarrow size - 1;
      return true:
   f1 = c -> start:
   f2 = c->start->next:
   dof
      if(f2->i == d)f
          f1 -> next = f2 -> next:
          free(f2):
          c \rightarrow size = c \rightarrow size - 1:
          return true:
      f1 = f2:
      f2 = f1 -> next:
   } while (f2 != NULL):
   return false;
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Task	Fixed Array	Realloc Array	Linked List
Insert new element	O(1) at end	O(1) at end	O(1) at front
	if space	but realloc()	
Search for an element	O(n)	O(n)	O(n)
	brute force	brute force	brute force
Search + delete	O(n) + O(n)	O(n) + O(n)	O(n) + O(1)
	move left	move left	delete 'free'

 If we had ordered our ADT (ie. the elements were sorted), then the searches could be via a binary / interpolation search, leading to O(log n) or O(log log n) search times.

ADTs Making Coding Simpler

Linked List code from the previous Chapter:

```
#include <stdio.h>
    #include <stdlib.h>
    #include "general.h"
    typedef struct data{
       int i:
       struct data* next:
    } Data;
    Data* allocateData(int i):
    void printList(Data* 1);
13
    int main(void)
14
       int i:
       Data* start, *current;
       start = current = NULL:
        printf("Enter the first number: ");
        if(scanf("%i", &i) == 1){
           start = current = allocateData(i);
        elsef
           on error("Couldn't read an int"):
24
25
        printf("Enter more numbers: ");
        while(scanf("%i", &i) == 1){
           current -> next = allocateData(i):
29
           current = current->next:
30
31
        printList(start):
        return 0: // Should Free List
```

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24
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       printf("Enter more numbers: ");
       while(scanf("%i", &i) == 1){
           current -> next = allocateData(i):
29
           current = current->next:
30
31
       printList(start):
       return 0: // Should Free List
```

Becomes:

```
1  #include "coll.h"
2  #include "Fixed/specific.h"
3
4  int main(void)
5  {
6     printf("Please type some numbers :");
7     coll* c = coll_init();
8     int i;
9     while(scanf("%i", &zi) == 1){
10     coll_add(c, i);
11     }
12     // Do print etc.
13     coll_free(c);
14     return 0;
15 }
```

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At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

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- Sets
- Graphs

At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

- Collections (Lists)
- Stacks
- Queues
- Sets
- Graphs
- Trees

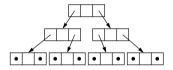
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Binary Trees:



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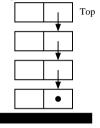
Binary Trees:



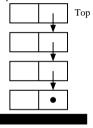
Unidirectional Graph:



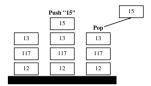
The push-down stack:



The push-down stack:

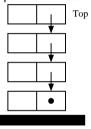


LIFO (Last in, First out):

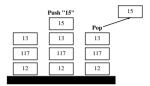


• Operations include push and pop.

The push-down stack:

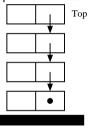


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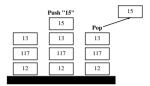


- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.

The push-down stack:

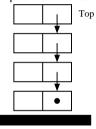


LIFO (Last in, First out):

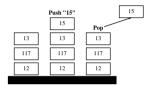


- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.

The push-down stack:



LIFO (Last in, First out):



- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.
- But, once again, we are faced with the question: How best to implement such a data type?

ADT:Stacks Arrays (Realloc) I

stack.h:

```
#pragma once
    #include " .. / General / general . h"
    typedef int stacktype;
    typedef struct stack stack;
    #include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <string.h>
    /* Create an empty stack */
    stack* stack_init(void);
    /* Add element to top */
    void stack push(stack* s, stacktype i);
    /* Take element from top */
    bool stack pop(stack* s. stacktype* d):
    /* Clears all space used */
    bool stack free(stack* s):
23
24
    /* Optional? */
    /* Copy top element into d (but don't pop it) */
    bool stack peek(stack*s. stacktype* d):
    /* Make a string version - keep .dot in mind */
    void stack tostring(stack*. char* str);
```

ADT:Stacks Arrays (Realloc) I

stack.h:

```
#pragma once
    #include "../General/general.h"
    typedef int stacktype:
    typedef struct stack stack;
    #include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <string.h>
    /* Create an empty stack */
    stack* stack_init(void);
    /* Add element to top */
    void stack push(stack* s, stacktype i);
    /* Take element from top */
    bool stack pop(stack* s. stacktype* d):
    /* Clears all space used */
    bool stack free(stack* s):
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    /* Optional? */
    /* Copy top element into d (but don't pop it) */
    bool stack peek(stack*s. stacktype* d):
    /* Make a string version - keep .dot in mind */
    void stack tostring(stack*. char* str);
```

Realloc/specific.h:

```
1  #pragma once
2
3  #define FORMATSIR "%i"
4  #define ELEMSIZE 20
5  #define STACKTYPE "Realloc"
7
7
8  #define FIXEDSIZE 16
9  #define SCALEFACTOR 2
10
11  struct stack {
12     /* Underlying array */
13     stacktype* a;
14     int size;
15     int capacity;
16  };
```

ADT:Stacks Arrays (Realloc) II

Realloc/realloc.c

```
#include " .. / stack . h"
    #include "specific.h"
    #define DOTFILE 5000
     stack* stack init(void)
        stack *s = (stack*) ncalloc(1, sizeof(stack));
        /* Some implementations would allow you to pass
           a hint about the initial size of the stack */
        s->a = (stacktype*) ncalloc(FIXEDSIZE, sizeof(stacktype));
        s \rightarrow size = 0:
        s->capacity= FIXEDSIZE;
14
15
        return s:
17
     void stack_push(stack* s, stacktype d)
19
        if (s=NULL){
             return:
        if(s->size >= s->capacity){}
23
           s->a = (stacktype*) nremalloc(s->a,
24
                    sizeof(stacktype)*s->capacity*SCALEFACTOR);
25
26
           s->capacity = s->capacity*SCALEFACTOR;
27
        s \rightarrow a[s \rightarrow size] = d:
28
        s \rightarrow size = s \rightarrow size + 1:
```

ADT:Stacks Arrays (Realloc) II

Realloc/realloc.c

```
#include " .. / stack . h"
     #include "specific.h"
     #define DOTFILE 5000
     stack * stack init(void)
         stack *s = (stack*) ncalloc(1, sizeof(stack));
        /* Some implementations would allow you to pass
            a hint about the initial size of the stack */
         s->a = (stacktype*) ncalloc(FIXEDSIZE, sizeof(stacktype));
        s \rightarrow size = 0:
         s->capacity= FIXEDSIZE;
14
         return s:
15
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     void stack_push(stack* s, stacktype d)
19
         if (s=NULL){
              return:
21
        if(s->size >= s->capacity){}
23
            s \rightarrow a = (stacktype*) nremalloc(s \rightarrow a.
24
                     sizeof(stacktype)*s->capacity*SCALEFACTOR);
25
            s->capacity = s->capacity*SCALEFACTOR;
26
27
         s \rightarrow a[s \rightarrow size] = d:
28
         s \rightarrow size = s \rightarrow size + 1:
```

```
bool stack_pop(stack* s, stacktype* d)
{
    if((s == NULL) || (s->size < 1)){
        return false;
    }
    s->size = s->size - 1;
    *d = s->a[s->size];
    return true;
}

bool stack_peek(stack* s, stacktype* d)
{
    if((s==NULL) || (s->size <= 0)){
        /* Stack is Empty */
        return false;
}

*d = s->a[s->size-1];
return true;
}
```

ADT:Stacks Arrays (Realloc) III

Realloc/realloc.c

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE];
        str[0] = '\0';
        if((s=NULL) || (s->size <1)){
           return:
        for (int i=s-> size-1: i>=0: i--)
           sprintf(tmp, FORMATSTR, s->a[i]);
10
11
12
13
           strcat(str, tmp);
           strcat(str, "|");
        str[strlen(str)-1] = '\0':
14
15
16
17
     bool stack free(stack* s)
18
        if (s=NULL){
           return true:
20
21
        free(s->a):
        free(s):
        return true;
```

ADT:Stacks Arrays (Realloc) III

Realloc/realloc.c

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE];
        str[0] = '\0';
        if((s=NULL) || (s->size <1)){
           return:
        for (int i=s-> size-1: i>=0: i--)
           sprintf(tmp, FORMATSTR, s->a[i]);
           strcat(str, tmp);
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           strcat(str, "|");
        str[strlen(str)-1] = '\0';
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18
        if (s=NULL){
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20
21
        free(s->a):
        free(s):
        return true;
```

• We need a thorough testing program teststack.c

ADT:Stacks Arrays (Realloc) III

Realloc/realloc.c

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE];
        str[0] = '\0':
        if((s=NULL) || (s->size <1)){
           return:
        for (int i=s->size-1: i>=0: i--) {
           sprintf(tmp, FORMATSTR, s->a[i]);
           strcat(str. tmp):
           strcat(str. "|");
12
13
        str[strlen(str)-1] = '\0':
14
15
16
17
     bool stack free(stack* s)
18
        if (s=NULL){
           return true:
20
21
        free(s->a):
        free(s):
        return true:
```

- We need a thorough testing program teststack c
- See also revstr.c: a version of the string reverse code (for which we already seen an iterative (in-place) and a recursive solution).

ADT:Stacks Linked I

Linked/specific.h

```
#pragma once

define FORMATSIR "%i"

define ELEMSIZE 20

define STACKTYPE "Linked"

struct dataframe {
    stacktype i;
    struct dataframe* next;
    };

typedef struct dataframe dataframe;

struct stack {
    /* Underlying array */
    dataframe* start;
    int size;
};

;
```

ADT:Stacks Linked I

Linked/specific.h

```
#pragma once

define FORMATSIR "%i"

define ELEMSIZE 20
define STACKTYPE "Linked"

struct dataframe {
    stacktype i;
    struct dataframe* next;
};

typedef struct dataframe dataframe;

struct stack {
    /* Underlying array */
    dataframe* start;
    int size;
};
```

Linked/linked.c

```
#include " .. / stack .h"
     #include "specific.h"
     #define DOTFILE 5000
     stack* stack init(void)
         stack* s = (stack*) ncalloc(1, sizeof(stack));
         return s:
10
11
     void stack push(stack* s. stacktype d)
13
        if(s){
            dataframe* f = ncalloc(1, sizeof(dataframe));
            f \rightarrow i = d:
            f->next = s->start;
            s->start = f:
            s \rightarrow size = s \rightarrow size + 1:
20
```

ADT:Stacks Linked II

```
bool stack_pop(stack* s, stacktype* d)
        if ((s==NULL) || (s->start==NULL)){
            return false;
        dataframe* f = s->start->next;
        *d = s->start->i:
        free(s->start):
        s \rightarrow start = f:
        s \rightarrow size = s \rightarrow size - 1:
12
13
14
        return true;
15
     bool stack_peek(stack* s, stacktype* d)
16
        if((s==NULL) || (s->start==NULL)){
18
            return false;
20
        *d = s->start ->i;
        return true;
```

ADT:Stacks Linked II

```
bool stack_pop(stack* s, stacktype* d)
        if((s==NULL) || (s->start==NULL)){
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        dataframe* f = s->start->next;
        *d = s->start->i:
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        s \rightarrow start = f:
        s \rightarrow size = s \rightarrow size - 1:
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        if((s==NULL) || (s->start==NULL)){
18
            return false:
19
20
        *d = s->start->i:
        return true;
22
```

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE]:
        str[0] = '\0':
        if((s==NULL) || (s->size <1)){
           return:
        dataframe* p = s->start:
        while (p) f
           sprintf(tmp. FORMATSTR. p->i):
           strcat(str. tmp):
           strcat(str. "|"):
           p = p -> next:
14
        str[strlen(str)-1] = '\0';
16
17
18
     bool stack free(stack* s)
19
20
        if(s){
           dataframe* p = s->start;
           while (p!=NULL){
              dataframe* tmp = p->next;
              free(p):
              p = tmp;
26
27
           free(s):
28
        return true;
30
```

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O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

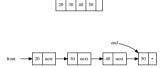
S : ADTs - Queues

T: ADTs - Trees

U: ADTs - Hashing

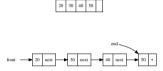
W: Algorithms II - Sort / Strings / Graphs

FIFO (First in, First out):



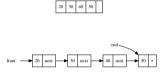
• Intuitively more "useful" than a stack.

FIFO (First in, First out):



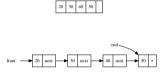
- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)

FIFO (First in, First out):



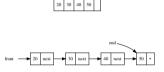
- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)
- Operations include enqueue, dequeue and size.

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FIFO (First in, First out):



- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)
- Operations include enqueue, dequeue and size.

queue.h

```
#pragma once
    #include " .. / General/general .h"
     typedef int queuetype;
     typedef struct queue queue;
     #include <stdio.h>
    #include <stdlib.h>
    #include <string.h>
    Winclude (assert h)
     /* Create an empty queue */
     queue* queue init(void):
     /* Add element on end */
     void queue_enqueue(queue* q, queuetype v);
     /* Take element off front */
     bool queue dequeue(queue* q. queuetype* d):
     /* Return size of queue */
     int queue size(queue* q):
     /* Clears all space used */
     bool queue_free(queue* q);
24
     /* Helps with visualisation & testing */
     void queue tostring(queue* q. char* str):
```

specific.h

```
1  #pragma once
2  #define FORMATSTR "%d"
4  #define ELEMSIZE 20
5  
6  #define QUEUETYPE "Fixed"
7  
8  #define BOUNDED 5000
9  struct queue {
11      /* Underlying array */
12      queuetype a [BOUNDED];
13      int front;
14      int end;
15      };
16  #define DOTFILE 5000
```

specific.h

```
#pragma once

#define FORMAISIR "%d"

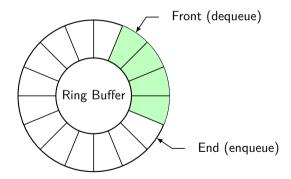
#define ELEMSIZE 20

#define QUEUETYPE "Fixed"

#define BOUNDED 5000

struct queue {
    /* Underlying array */
    queuetype a[BOUNDED];
    int front;
    int end;
};

#define DOTFILE 5000
```



fixed.c

```
#include " .. / queue . h"
     #include "specific.h"
     void inc(queuetype* p);
     queue * queue init(void)
         queue* q = (queue*) ncalloc(1, sizeof(queue));
         return q;
10
11
12
13
14
     void queue_enqueue(queue* q, queuetype d)
15
16
17
18
19
20
21
22
         if (a) {
             q \rightarrow a[q \rightarrow end] = d:
             _inc(&q->end);
             if (q->end == q->front){
                 on_error("Queue too large");
```

fixed.c

```
#include " .. / queue . h"
     #include "specific.h"
     void inc(queuetype* p);
     queue * queue init(void)
         queue* q = (queue*) ncalloc(1, sizeof(queue));
         return q;
     void queue_enqueue(queue* q, queuetype d)
14
15
16
17
         if (a) {
            q \rightarrow a[q \rightarrow end] = d:
            _inc(&q->end);
18
19
20
21
             if (q->end == q->front){
                on_error("Queue too large");
22
```

```
bool queue dequeue(queue* q. queuetype* d)
        if ((a==NULL) || (a->front==a->end)){
           return false:
        *d = q -  a[q -  front]:
        inc(&g->front):
        return true:
9
10
11
     void queue tostring(queue* q. char* str)
12
13
        char tmp[ELEMSIZE];
        str[0] = '\0':
        if ((q==NULL) || (queue_size(q)==0)){
16
           return:
17
18
        for(int i=q->front; i != q->end;){
           sprintf(tmp, FORMATSTR, q->a[i]);
20
           strcat(str. tmp):
21
           strcat(str. "|"):
22
           inc(&zi):
23
24
        str[strlen(str)-1] = '\0':
```

```
int queue_size(queue* q)
         if (q==NULL) {
            return 0;
        if(q->end >= q->front){
            return q->end-q->front;
9
10
11
        return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
         free(q):
15
16
        return true:
17
18
19
20
21
     void _inc(queuetype* p)
        *p = (*p + 1) \% BOUNDED;
```

```
int queue_size(queue* q)
         if (q==NULL) {
            return 0:
        if(q-)end = q-)front){
            return q->end-q->front;
9
10
11
        return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
         free(q):
15
16
        return true:
17
18
19
20
     void inc(queuetvpe* p)
        *p = (*p + 1) \% BOUNDED:
```

 We need a thorough testing program

```
int queue_size(queue* q)
        if (a==NULL) {
            return 0:
        if(q-)end = q-)front)
            return q->end-q->front;
9
10
11
        return q->end + BOUNDED - q->front;
12
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15
16
        return true:
17
18
19
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20
        *p = (*p + 1) \% BOUNDED:
```

- We need a thorough testing program
- We'll see queues again for traversing trees

```
int queue_size(queue* q)
        if (a==NULL) {
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        if(q-)end = q-)front)
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9
10
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        return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
        free(q):
15
16
        return true;
18
19
     void inc(queuetvpe* p)
20
        *p = (*p + 1) \% BOUNDED:
```

- We need a thorough testing program
- We'll see queues again for traversing trees
- Simulating a (slow) printer

ADTs: Queues (Linked) I

specific.h

```
#pragma once

define FORMATSIR "%d"

define ELEMSIZE 20

define QUEUETYPE "Linked"

struct dataframe {
    queuetype i;
    struct dataframe* next;
    };

typedef struct dataframe dataframe;

fine formal struct queue {
    /* Underlying array */
    dataframe* front;
    dataframe* end;
    int size;
};
```

ADTs: Queues (Linked) I

specific.h

```
#pragma once
    #define FORMATSTR "%d"
    #define ELEMSIZE 20
    #define OUFUETYPE "Linked"
    struct dataframe {
       queuetype i;
        struct dataframe* next;
    }:
12
13
    typedef struct dataframe dataframe;
14
    struct queue {
15
      /* Underlying array */
       dataframe* front:
17
       dataframe* end:
       int size:
19
    }:
```

linked.c

```
#include " .. / queue .h"
      #include "specific.h"
      queue* queue init(void)
          queue* q = (queue*) ncalloc(1, sizeof(queue));
          return q;
      void queue_enqueue(queue* q, queuetype d)
          dataframe* f;
          if (q==NULL) {
             return:
          /* Copy the data */
          f = ncalloc(1, sizeof(dataframe));
          f \rightarrow i = d:
          /* 1st one */
          if (a->front == NULL) {
             a \rightarrow front = f:
24
             a \rightarrow end = f:
             q \rightarrow size = q \rightarrow size + 1;
             return:
28
          /* Not 1st */
          q \rightarrow end \rightarrow next = f:
          a->end = f:
31
          q \rightarrow size = q \rightarrow size + 1;
```

ADTs: Queues (Linked) II

```
bool queue dequeue(queue* q, queuetype* d)
         dataframe* f;
         if((q=NULL) || (q->front=NULL) || (q->end=NULL)){
            return false;
         f = q - front - next;
         *d = q->front->i;
         free(q->front);
         q \rightarrow front = f;
11
12
13
14
15
16
17
18
19
20
         q->size = q->size - 1;
         return true;
     bool queue free (queue * q)
         if (a) {
             dataframe* tmp:
            dataframe* p = q->front;
            while (p!=NULL) {
                tmp = p->next;
22
23
24
25
26
27
28
                free(p);
                p = tmp;
             free(q);
         return true;
```

ADTs: Queues (Linked) II

```
bool queue dequeue(queue* q, queuetype* d)
         dataframe* f:
         if ((q=NULL) || (q->front=NULL) || (q->end=NULL)){
            return false;
         f = q - front - next;
         *d = q - front - i;
         free(q->front);
        q \rightarrow front = f;
         q \rightarrow size = q \rightarrow size - 1;
         return true;
13
14
     bool queue free (queue * q)
        if (a) {
18
19
            dataframe* tmp:
            dataframe* p = q->front;
20
            while (p!=NULL) {
                tmp = p -> next:
                free(p);
23
24
                p = tmp:
25
26
            free(q);
         return true;
28
```

```
void queue tostring(queue* q, char* str)
        dataframe *p;
        char tmp[ELEMSIZE];
        str[0] = '\0';
        if ((q=NULL) || (q->front == NULL)){
           return:
        p = q - front;
        while(p){
           sprintf(tmp, FORMATSTR, p->i);
           strcat(str. tmp);
           strcat(str. "|");
           p = p -   next;
16
        str[strlen(str)-1] = '\0';
17
18
     int queue size(queue* q)
20
21
        if ((q=NULL) || (q->front=NULL)){
23
           return 0:
24
25
        return q->size;
```

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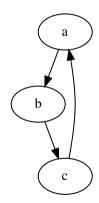


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R: ADTs - Stacks

S : ADTs - Queues

T: ADTs - Trees

U: ADTs - Hashing

W: Algorithms II - Sort / Strings / Graphs

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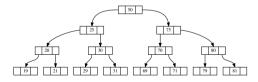
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- Searching

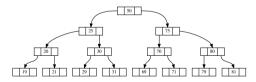
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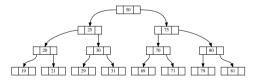


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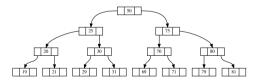
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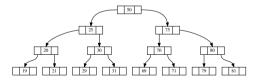
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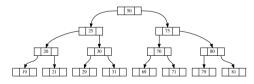
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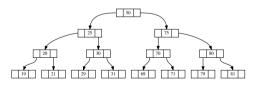
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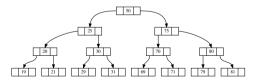
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Binary Trees : Data Structures

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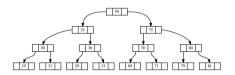


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- Can refer to left and right children
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- A node with no children is a leaf
- Most trees need to be created dynamically
- Empty subtrees are set to NULL

「: ADTs - Trees 49 / 81

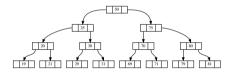
Binary Search Trees

In a binary search tree the left-hand tree of a parent contains all keys less than the parent node, and the right-hand side all the keys greater than the parent node.



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bst.h

```
#include " .. / General/general . h"
    #include " .. / Queue/queue . h "
     #include <stdio.h>
     #include <stdlib.h>
     #include <assert.h>
     bst* bst_init(void);
     /* Insert 1 item into the tree */
     bool bst insert(bst* b, treetype d);
     /* Return number of nodes in tree */
     int bst size(bst* b);
16
     /* Whether the data d is stored in the tree */
     bool bst isin(bst* b, treetype d);
18
19
     /* Bulk insert n items from an array a into an initialised tree */
20
     bool bst_insertarray(bst* b, treetype* a, int n);
21
     /* Clear all memory associated with tree. & set pointer to NULL */
     bool bst free(bst* b):
24
25
     /* Optional ? */
     char* bst_preorder(bst* b);
     void bst printlevel(bst* b):
     /* Create string with tree as ((head)(left)(right)) */
     char* bst printlisp(bst* b):
     /* Use Graphviz via a .dot file */
     void bst todot(bst* b. char* dotname):
```

Binary Search Trees: Linked I

specific.h

```
#include <string.h>

typedef int treetype;

define FORMATSIR "%1"

define ELEMSIZE 20

define ESTITYPE "Linked"

struct dataframe {
 treetype d;
 struct dataframe* left;
 struct dataframe* right;
 };

typedef struct dataframe dataframe;

struct dataframe in the struct dataframe dataframe;

// * Data element size, in bytes */
};

typedef struct bst bst;
```

Binary Search Trees: Linked I

specific.h

```
typedef int treetype;
define FORMATSIR "%i"
tydefine ELEMSIZE 20
define ESTTYPE 'Linked'

struct dataframe {
  treetype d;
  struct dataframe* left;
  struct dataframe* right;
};
typedef struct dataframe dataframe;

struct dataframe top;
/* Data element size, in bytes */
};
typedef struct bst bst;
```

```
/* Based on geekforgeeks.org */
dataframe* __insert(dataframe* t, treetype d)
{
    dataframe* f;
    /* If the tree is empty, return a new frame */
    if (t == NULL){
        f = ncalloc(sizeof(dataframe), 1);
        f > 2d = d;
        return f;
    }
    /* Otherwise, recurs down the tree */
    if (d < t->d) {
        t -> lift = __insert(t->left, d);
    }
    else if(d > t->d){
        t -> right = __insert(t->right, d);
    }
    /* return the (unchanged) dataframe pointer */
    return t;
}
```

Binary Search Trees: Linked II

```
bool __isin(dataframe* t, treetype d)
{
   if(t=NULL){
      return false;
   }
   if(t>d == d){
      return true;
   }
   if(d < t->d){
      return __isin(t->left , d);
   }
   else{
      return __isin(t->right , d);
   }
   return false;
}
```

Binary Search Trees: Linked II

```
bool __isin(dataframe* t, treetype d)
{
   if(t=NULL){
      return false;
   }
   if(t>d == d){
      return true;
   }
   if(d < t>d){
      return __isin(t->left, d);
   }
   else{
      return __isin(t->right, d);
   }
   return false;
}
```

```
char* _printlisp(dataframe* t)
  char tmp[ELEMSIZE];
  char *s1, *s2, *p;
  if(t==NULL){
     /* \0 string */
     p = ncalloc(1,1);
     return p;
  sprintf(tmp, FORMATSTR, t->d);
  s1 = _printlisp(t->left);
  s2 = _printlisp(t->right);
  p = ncalloc(strlen(s1)+strlen(s2)+strlen(tmp)+
       strlen("()() "), 1);
  sprintf(p, "%s(%s)(%s)", tmp, s1, s2);
  free(s1):
  free(s2):
  return p;
```

 Don't rush to assume a linked data structure must be used to implement trees.

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Counting from cell 1, for a tree with *n* nodes:

To find	Use	Iff
The root	A[1]	A is nonempty
The left child of $A[i]$	A[2i]	$2i \leq n$
The parent of $A[i]$	A[i/2]	i > 1
Is A[i] a leaf?	True	2 <i>i</i> > <i>n</i>

Binary Search Trees : Realloc

specific.h

```
#include <stdbool.h>
    typedef int treetype;
    #define FORMATSTR "%i"
    #define ELEMSIZE 20
    #define BSTTYPE "Realloc"
  // Probably (2^n) -1
    #define INITSIZE 31
    #define SCALEFACTOR 2
    struct dataframe {
       treetype d;
       bool isvalid:
15
    typedef struct dataframe dataframe;
17
    struct bst {
19
       dataframe* a:
       int capacity;
    typedef struct bst bst:
```

Binary Search Trees : Realloc

specific.h

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#include <stdhool h>
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    #define FORMATSTR "%i"
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    typedef struct dataframe dataframe:
17
    struct bst {
19
       dataframe* a:
       int capacity:
    typedef struct bst bst:
```

Using a queue for Level-Order traversal:

```
void bst_printlevel(bst* b)
{
    treetype n;
    if((b=NULL) || (! _isvalid(b, 0))){
        return;
    }
    /* Make a queue of cell indices */
    queue* q = queue_init();
    queue_enqueue(q, 0);
    while (queue_dequeue(q, &n) && _isvalid(b, (int)n)){
        printr(FORMATSIR, b->a[n].d);
        putchar(' ');
        queue_enqueue(q, _leftchild((int)n));
        queue_enqueue(q, _rightchild((int)n));
}
```

Binary Search Trees : Complexity

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- The tree search performs best when well balanced trees are formed.
- Large body of literature about creating & re-balancing trees Red-Black trees, Tries, 2-3 trees, AVL trees etc.

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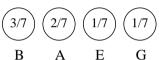
• To encode the string "BABBAGE":

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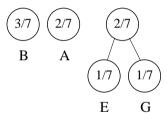
Keep a list of characters, ordered by their frequency

: ADTs - Trees 56 / 81

• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :

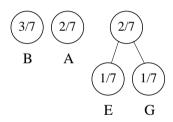
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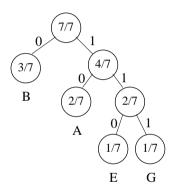
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「: ADTs - Trees 57 / 81

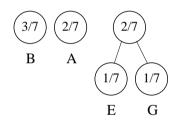
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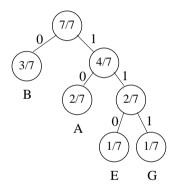




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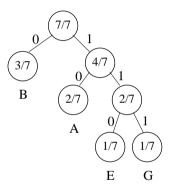




ullet A = 10, B = 0, E = 110, G = 111

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- \bullet A = 10, B = 0, E = 110, G = 111
- String stored using 13 bits.

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U : ADTs - Hashing 58 / 81

ADTs: Hashing

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- A black-box into which we insert a key (e.g. NI number) and out pops an array index.
- As an example lets use an array of size 11 to store some airport codes, e.g. PEK, BRS, FRA.

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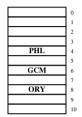
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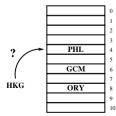
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• However, inserting "HKG" causes a collision.



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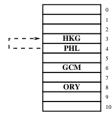
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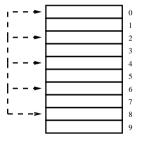
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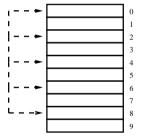
• Although "PHL" and "HKG" share the same primary hash value of h(K) = 4, they have different probe decrements:

$$p("PHL") = 4$$
$$p("HKG") = 3$$

• If the size of our array, M, was even and the probe decrement was chosen to be 2, then only half of the locations could be probed.

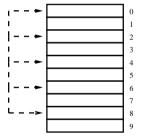


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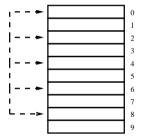
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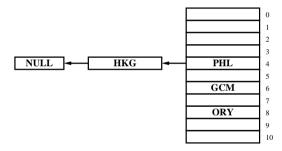
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Open-addressing is not the only method of collision reduction. Another common one is separate chaining.



A Practical Hash Function

```
#include <stdio.h>
    int hash(unsigned int sz, char *s);
    int main(void)
       char str[] = "Hello World!";
       // Hash modulus 7919
        printf("%d\n", hash(7919, str));
        return 0:
12
13
14
15
    Modified Bernstein hashing
    5381 & 33 are magic numbers required by the algorithm
19
    int hash(unsigned int sz, char *s)
       unsigned long hash = 5381;
       int c;
       while ((c = (*s++))){
           hash = 33 * hash ^ c:
        return (int)(hash%sz);
```

Execution:

5479

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    int hash(unsigned int sz. char *s)
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        unsigned long hash = 5381;
        int c:
        while ((c = (*s++))){
           hash = 33 * hash ^ c:
        return (int)(hash%sz);
```

Execution:

5479

Has similarities to the implementation of rand():

```
int rand r(unsigned int* seed):
int main (void)
  unsigned int seed = 0:
   printf("%d\n", rand r(&seed)):
  return 0:
/* This algorithm is mentioned in the ISO C standard,
   here extended for 32 bits. */
int rand_r(unsigned int* seed)
 unsigned int next = *seed;
  int result:
 next *= 1103515245;
  next += 12345:
 result = (unsigned int) (next / 65536) % 2048;
 next *= 1103515245;
 next += 12345;
 result <<= 10:
 result ^= (unsigned int) (next / 65536) % 1024:
  next *= 1103515245:
 next += 12345:
 result <<= 10:
 result ^= (unsigned int) (next / 65536) % 1024:
 *seed = next;
 return result:
```

Execution:

Cuckoo Hashing

 We have two tables, each with their own hash function.

```
Empty: copied farandoles into table 0(4)
Empty: copied bronzine into table 0(12)
Empty: copied auscultatory into table 0(5)
Empty: copied bifer into table 0(13)
Empty: copied steepgrass into table 0(6)
Empty: copied prevised into table 0(7)
Empty: copied oomph into table 0(8)
empodium, so cuckooed out auscultatory from table 0(5)
Empty: copied auscultatory into table 1(10)
interquarreled, so cuckooed out bronzine from table 0(12)
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ranseur, so cuckooed out empodium from table 0(5)
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Empty: copied megalodon into table 0(11)
geosynchronous, so cuckooed out megalodon from table 0(11)
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U : ADTs - Hashing 65 / 81

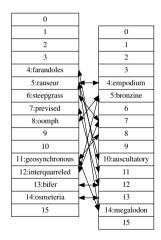
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: ADTs - Hashing 65 / 81

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Q: ADTs - Collection

R: ADTs - Stacks

S : ADTs - Queues

T : ADTs - Trees

U : ADTs - Hashing

 $W: \mathsf{Algorithms}\;\mathsf{II}\;\mathsf{-}\;\mathsf{Sort}\;/\;\mathsf{Strings}\;/\;\mathsf{Graphs}$

```
#define NUMS 6
void bubble_sort(int b[], int s);
int main(void)
   int a[] = \{3, 4, 1, 2, 9, 0\};
   bubble_sort(a, NUMS);
   for (int i=0: i < NUMS: i++){
      printf("%i ", a[i]);
   printf("\n");
   return 0;
void bubble sort(int b[], int s)
   bool changes:
   dof
      changes = false:
      for(int i=0; i <s-1; i++){
         if(b[i] > b[i+1])
            SWAP(b[i], b[i+1]);
            changes = true:
   } while (changes);
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• Bubblesort has complexity $O(n^2)$, therefore very inefficient.

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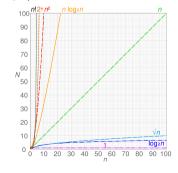
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```
#include <stdio.h>
     #include <stdlib.h>
     #include <string.h>
     void mergesort(int *src, int *spare, int 1, int r);
     void merge(int *src. int *spare. int 1. int m. int r):
     #define NUM 5000
10
     int main(void)
11
        int a[NUM]:
        int spare[NUM]:
        for (int i=0: i < NUM: i++){
            a[i] = rand()\%100:
        mergesort(a, spare, 0, NUM-1);
20
21
        for(int i=0: i <NUM: i++){</pre>
22
            printf("%4d \Rightarrow %d\n", i, a[i]):
23
24
25
        return 0;
26
```

```
void mergesort(int *src, int *spare, int 1, int r)
  int m = (1+r)/2:
  if(1 != r){
      mergesort(src. spare. 1. m):
     mergesort(src, spare, m+1, r);
     merge(src, spare, 1, m, r);
void merge(int *src, int *spare, int 1, int m, int r)
  int s1 = 1:
  int s2 = m+1:
  int d = 1:
  dof
     if(src[s1] < src[s2])
         spare[d++] = src[s1++];
      elsef
         spare[d++] = src[s2++]:
  while((s1 \le m) && (s2 \le r));
  if(s1 > m){
     memcpy(&spare[d], &src[s2], sizeof(spare[0])*(r-s2+1));
  else {
     memcpy(&spare[d], &src[s1], sizeof(spare[0])*(m-s1+1));
  memcpv(\&src[1], \&spare[1], (r-1+1)*sizeof(spare[0])):
```

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  else {
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• Quicksort is also divide-and-conquer.

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- Quicksort is also divide-and-conquer.
- Choose some value in the array as the *pivot* key.

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```

- Quicksort is also divide-and-conquer.
- Choose some value in the array as the *pivot* key.
- This key is used to divide the array into two partitions. The left partition contains keys ≤ pivot key, the right partition contains keys > pivot.

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      elsef
         spare[d++] = src[s2++]:
  while((s1 \le m) && (s2 \le r));
  if(s1 > m){}
     memcpv(&spare[d], &src[s2], sizeof(spare[0])*(r-s2+1));
  else {
     memcpy(&spare[d], &src[s1], sizeof(spare[0])*(m-s1+1));
  memcpv(\&src[1], \&spare[1], (r-1+1)*sizeof(spare[0]));
```

- Quicksort is also divide-and-conquer.
- Choose some value in the array as the *pivot* key.
- This key is used to divide the array into two partitions. The left partition contains keys ≤ pivot key, the right partition contains keys > pivot.
- Once again, the sort is then applied recursively.

Algorithms: Quicksort

```
#include <stdio.h>
     #include <stdlib.h>
     #include <math.h>
     int partition(int *a, int 1, int r);
     void quicksort(int *a. int 1. int r):
     #define NUM 100000
10
     int main(void)
11
12
        int a[NUM]:
13
        for (int i=0: i < NUM: i++) f
15
           a[i] = rand()\%100;
16
17
18
19
20
21
22
        quicksort(a, 0, NUM-1);
        return 0:
     void quicksort(int *a, int 1, int r)
23
24
        int pivpoint = partition(a, 1, r);
25
26
27
        if(1 < pivpoint){
            quicksort(a, 1, pivpoint-1);
28
        if (r > pivpoint) {
29
30
           quicksort(a, pivpoint+1, r);
31
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```

```
int partition(int *a, int 1, int r)
{
  int piv = a[1];
  while(1<r){
     /* Right -> Left Scan */
     while(piv < a[r] && 1<r) r--;
     if(r!=1){
        a[1] = a[r];
        1++;
     }
     /* Left -> Right Scan */
     while(piv > a[1] && 1++;
     if(r!=1){
        a[r] = a[1];
        r--;
        }
     a[r] = piv;
     return r;
}
```

 Theoretically both methods have a complexity O(n log n)

W : Algorithms II - Sort / Strings / Graphs

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$\mathsf{qsort}()$

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```
#include <stdio.h>
     #include <stdlib.h>
     int intcompare(const void *a, const void *b);
     int main(void)
        int a[10]:
        for (int i=0; i<10; i++){
           a[i] = 9 - i:
        gsort(a, 10, sizeof(int), intcompare):
16
        for (int i=0; i<10; i++){
           printf(" %d".a[i]):
        printf("\n"):
        return 0:
21
22
23
     int intcompare(const void *a, const void *b)
25
         const int *ia = (const int *)a:
         const int *ib = (const int *)b:
         return *ia - *ib:
```

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```
459 254 472 534 649 239 432 654 477
```

```
0
1
2 472 432
3
4 254 534 654
5
6
7 477
8
9 459 649 239
```

Read out the new list: 472 432 254 534 654 477 459 649 239

Radix Sort II

```
472 432 254 534 654 477 459 649 239
3 432 534 239
4 649
5 254 654 459
7 472 477
432 534 239 649 254 654 459 472 477
```

Radix Sort II

```
472 432 254 534 654 477 459 649 239
                                               432 534 239 649 254 654 459 472 477
                                               2 239 254
3 432 534 239
4 649
                                               4 432 459 472 477
5 254 654 459
                                               5 534
                                               6 649 654
7 472 477
432 534 239 649 254 654 459 472 477
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W : Algorithms II - Sort / Strings / Graphs

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- Now: gprof ./executable gmon.out shows the function-call profile of your code.

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Substring : AAAAAAH
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- Recall that to compute a hash function on a word we did something like:

$$h("NEILL") =$$

$$(13\times 26^4 + 4\times 26^3 + 8\times 26^2 + 11\times 26 + 11)\%\textit{P}$$

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where P is a big prime number.

• This can be expanded by Horner's method to:

$$(((((((13 \times 26) + 4)\%P \times 26) + 8)\%P \times 26) + 11)\%P \times 26 + 11)\%P$$

 For a large search string, overflow can occur. We therefore move the mod operation inside the brackets:

$$(((((((13 \times 26) + 4)\%P \times 26) + 8)\%P \times 26) + 11)\%P \times 26 + 11)\%P$$

 We can compute a hash number for the search string, and for the initial part of the master string.

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```
#include <string.h>
     #include <assert h>
     #define Q 33554393
     #define D 26
     #define index(C) (C-'A')
     int rk(char *p, char *a);
     int main(void)
        assert (rk("STING".
                "A STRING EXAMPLE CONSISTING OF ...")==22):
        return 0:
15
17
     int rk(char *p. char *a)
19
        int i. dM = 1. h1=0. h2=0:
        int m = strlen(p):
21
        int n = strlen(a):
        for (i=1: i \le m: i++) dM = (D*dM)\%O:
23
        for (i=0; i \le m; i++){
           h1 = (h1*D+index(p[i]))%Q:
           h2 = (h2*D+index(a[i]))%Q:
27
        // h1 = search string hash, h2 = master string hash
28
        for(i=0; h1!=h2: i++){}
           h2 = (h2+D*Q-index(a[i])*dM) \% Q:
           h2 = (h2*D+index(a[i+m])) % Q;
31
           if(i>n-m) return n:
32
        return i:
```

The Boyer-Moore algorithm uses (in part) an array flagging which characters form part of the search string and an array telling us how far to slide right if that character appears in the master and causes a mismatch.

Execution:

```
A STRING SEARCHING EXAMPLE CONSISTING OF ...

STING |
STING
STING
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- Since R doesn't appear in the search string, we can take 5 steps to the right.

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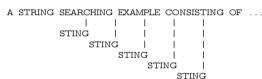
```
A STRING SEARCHING EXAMPLE CONSISTING OF ...

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STING |
STING
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- With a right-to-left walk through the search string we see that the G and the R mismatch on the first comparison.
- Since R doesn't appear in the search string, we can take 5 steps to the right.
- The next comparison is between the G and the S. We can slide the search string right until it matches the S in the master.

Boyer-Moore II

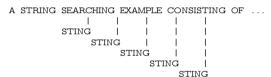
Execution:



 Now the C doesn't appear in the master and once again we can slide a full 5 places to the right.

Boyer-Moore II

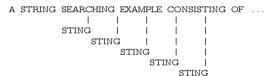
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Boyer-Moore II

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- Now the C doesn't appear in the master and once again we can slide a full 5 places to the right.
- After 3 more full slides right we arrive at the T in CONSISTING.
- We align the T's, and have found our match using 7 compares (plus 5 to verify the match).

Algorithms : TSP on Graphs

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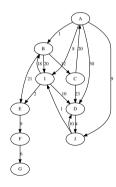
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- A -> B -> C -> D -> J -> I -> E -> F -> G



TSP II

```
edge graph_salesman(graph* g, int from, char* str)
  bool* unvis:
  int curr, ncurr, nvs;
  edge cst, bcst, e;
  nvs = graph_numVerts(g);
  if ((g=NULL) || (from >= nvs) || (str=NULL)){
      return INF;
  unvis = (bool*)ncalloc(nvs, sizeof(bool));
   for(int v=0; v<nvs; v++){
      unvis[v] = true;
  curr = from;
  bcst = 0:
  strcpy(str, graph_getLabel(g, from));
  do{
      unvis[curr] = false:
      cst = INF:
      ncurr = NO VERT:
      /* Look at neighbours of curr */
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  bool* unvis:
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  unvis = (bool*)ncalloc(nvs, sizeof(bool));
  for(int v=0; v<nvs; v++){
     unvis[v] = true;
  curr = from :
  bcst = 0:
  strcpv(str. graph getLabel(g. from));
  dof
     unvis[curr] = false:
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- Mark visited when done with neighbors.





- It's often important to find the shortest path through a graph from one vertex to another.
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- Picks the unvisited vertex with the lowest distance, & calculate the distance through it to each unvisited neighbor, updating the neighbour's distance if smaller.
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