COMSM1201 : Data Structures & Algorithms

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- Let's look at some toy examples to begin with.

```c
#include <stdio.h>
#include <string.h>

#define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}

void strrev(char* s, int n);

int main(void)
{
   char str[] = "Hello World!";
   strrev(str, strlen(str));
   printf("%s\n", str);
   return 0;
}

/* Iterative Inplace String Reverse */
void strrev(char* s, int n)
{
   for(int i=0, j=n-1; i<j; i++, j--){
       SWAP(s[i], s[j]);
   }
}
```

Execution :

```
!dlroW olleH
```

Recursion for *strrev()*

```c
#include <stdio.h>
#include <string.h>

#define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}

void strrev(char* s, int start, int end);

int main(void)
{
   char str[] = "Hello World!";
   strrev(str, 0, strlen(str)-1);
   printf("%s\n", str);
   return 0;
}

/* Recursive : Inplace String Reverse */
void strrev(char* s, int start, int end)
{
   if(start >= end){
      return;
   }
   SWAP(s[start], s[end]);
   strrev(s, start+1, end-1);
}
```

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- We need to change the function prototype.

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   if(start >= end){
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   strrev(s, start+1, end-1);
}
```

Execution :

```
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```

- We need to change the function prototype.
- This allows us to track both the start and the end of the string.

The Fibonacci Sequence

A well known example of a recursive function is the Fibonacci sequence. The first term is 1, the second term is 1 and each successive term is defined to be the sum of the two previous terms, i.e. :

`fib(1)` is 1
`fib(2)` is 1
`fib(n)` is `fib(n-1)`+`fib(n-2)`

1,1,2,3,5,8,13,21, ...

Iterative & Recursive Fibonacci

```c
#include <stdio.h>

#define MAXFIB 24

int fibonacci(int n);

int main(void)
{

   for(int i=1; i<=MAXFIB; i++){
      printf("%d = %d\n", i, fibonacci(i));
   }

   return 0;

}

int fibonacci(int n)
{
   if(n <= 2){
      return 1;
   }
   int a = 1;
   int b = 1;
   int next;
   for(int i=3; i<=n; i++){
      next = a + b;
      a = b;
      b = next;
   }
   return b;
}
```

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int fibonacci(int n);

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{

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      printf("%d = %d\n", i, fibonacci(i));
   }

   return 0;

}

int fibonacci(int n)
{
   if(n <= 2){
      return 1;
   }
   int a = 1;
   int b = 1;
   int next;
   for(int i=3; i<=n; i++){
      next = a + b;
      a = b;
      b = next;
   }
   return b;
}
```

Execution :

```
1 = 1
2 = 1
3 = 2
4 = 3
5 = 5
6 = 8
7 = 13
8 = 21
9 = 34
10 = 55
11 = 89
12 = 144
13 = 233
14 = 377
15 = 610
16 = 987
17 = 1597
18 = 2584
19 = 4181
20 = 6765
21 = 10946
22 = 17711
23 = 28657
24 = 46368
```

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#define MAXFIB 24

int fibonacci(int n);

int main(void)
{

    for(int i=1; i<=MAXFIB; i++){
        printf("%d = %d\n", i, fibonacci(i));
    }

    return 0;

}

int fibonacci(int n)
{
    if(n == 1) return 1;
    if(n == 2) return 1;
    return( fibonacci(n-1)+fibonacci(n-2));
}
```

Iterative & Recursive Fibonacci

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int fibonacci(int n);

int main(void)
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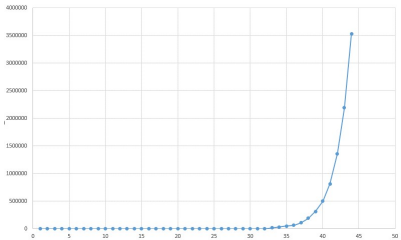
    for(int i=1; i<=MAXFIB; i++){
        printf("%d = %d\n", i, fibonacci(i));
    }

    return 0;

}

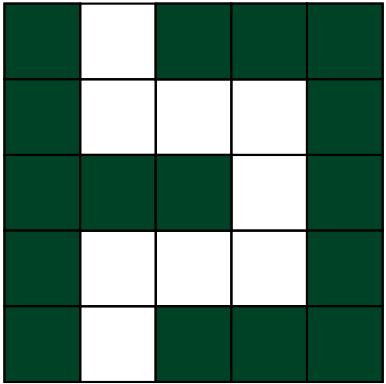
int fibonacci(int n)
{
    if(n == 1) return 1;
    if(n == 2) return 1;
    return( fibonacci(n-1)+fibonacci(n-2));
}
```

It's interesting to see how run-time increases as the length of the sequence is raised.



Maze Escape

The correct route through a maze can be obtained via recursive, rather than iterative, methods.



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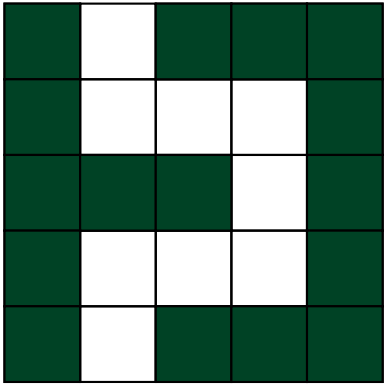

```
bool explore(int x, int y, char mz[YS][XS])
{
  if  mz[y][x] is exit return true;

  Mark mz[y][x] so we don't return here

  if we can go up :
    if(explore(x, y+1, mz)) return true

  if we can go right :
    if(explore(x+1, y, mz)) return true

  Do left & down in a similar manner

  return false; // Failed to find route
}
```

Permuting

- Here we consider the ways to permute a string (or more generally an array)

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- Permutations are all possible ways of rearranging the positions of the characters.

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Execution :

ABC
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```c
// From e.g.  http://www.geeksforgeeks.org
#include <stdio.h>
#include <string.h>

#define SWAP(A,B) {char temp = *A; *A = *B; *B = temp;}

void permute(char* a, int s, int e);

int main()
{
    char str[] = "ABC";
    int n = strlen(str);
    permute(str, 0, n-1);
    return 0;
}

void permute(char* a, int s, int e)
{
   if (s == e){
     printf("%s\n", a);
     return;
   }
   for (int i = s; i <= e; i++){
      SWAP((a+s), (a+i)); // Bring one char to the front
      permute(a, s+1, e);
      SWAP((a+s), (a+i)); // Backtrack
   }
}
```

Self-test : Power

- Raising a number to a power `n = 2^5` is the same as multiple multiplications `n = 2*2*2*2*2`.

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```c
/* Try to write power(a,b) to computer a^b
   without using any maths functions other than
   multiplication :
   Try (1) iterative then (2) recursive
   (3) Trick that for n%2==0, x^n = x^(n/2)*x^(n/2)

*/

#include <stdio.h>

int power(unsigned int a, unsigned int b);

int main(void)
{

   int x = 2;
   int y = 16;

   printf("%d^%d = %d\n", x, y, power(x,y));

}

int power(unsigned int a, unsigned int b)
{
}
```

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- This is used to delete names from a mailing list, or upgrading the salary of an employee etc.
- The simplest method for searching is called the sequential search.
- Simply move through the array from beginning to end, stopping when you have found the value you require.

```c
#include <stdio.h>
#include <string.h>
#include <assert.h>

#define NOTFOUND -1
#define NUMPEOPLE 6
typedef struct person{
        char* name; int age;
} person;

int findAge(const char* name, const person* p, int n);

int main(void)
{
   person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
              {"Chumley", 26}, {"Dalton",   25},
              {"Eggson", 22}, {"Fulton",  41} };

   assert(findAge("Eggson",   ppl, NUMPEOPLE)==22);
   assert(findAge("Campbell", ppl, NUMPEOPLE)==NOTFOUND);
   return 0;
}

int findAge(const char* name, const person* p, int n)
{
   for(int j=0; j<n; j++){
      if(strcmp(name, p[j].name) == 0){
         return p[j].age;
      }
   }
   return NOTFOUND;
}
```

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} person;

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int main(void)
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    person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
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    assert(findAge("Eggson",   ppl, NUMPEOPLE)==22);
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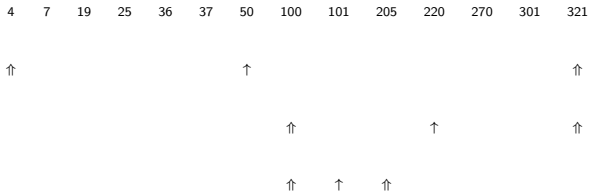
    return 0;
}

int findAge(const char* name, const person* p, int n)
{
    for(int j=0; j<n; j++){
        int m = strcmp(name, p[j].name);
        if(m == 0) // Braces!
            return p[j].age;
        if(m < 0)
            return NOTFOUND;
    }
    return NOTFOUND;
}
```

Binary Search for *101*

- Searching small lists doesn't require much computation time.



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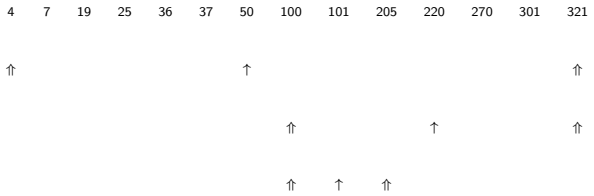
- However, as lists get longer (e.g. phone directories), sequential searching becomes extremely inefficient.



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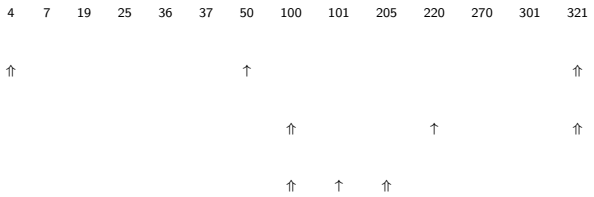
- Searching small lists doesn't require much computation time.
- However, as lists get longer (e.g. phone directories), sequential searching becomes extremely inefficient.
- A binary search consists of examining the middle element of the array to see if it has the desired value. If not, then half the array may be discarded for the next search.



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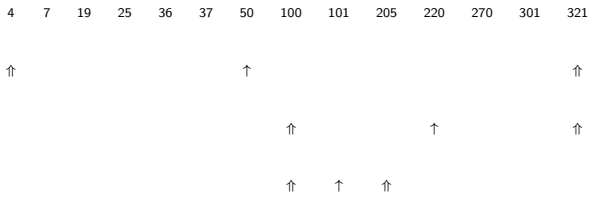

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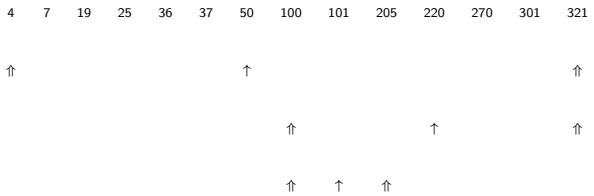


```c
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>
#include <time.h>
#define NMBRS 1000000

int bin_it(int k, const int* a, int l, int r);

int main(void)
{
   int a[NMBRS];
   srand(time(NULL));

   // Put even numbers into array
   for(int i=0; i<NMBRS; i++){
      a[i] = 2*i;
   }

   // Do many searches for a random number
   for(int i=0; i<10*NMBRS; i++){
      int n = rand()%NMBRS;
      if((n%2) == 0){
         assert(bin_it(n, a, 0, NMBRS-1) == n/2);
      }
      else{ // No odd numbers in this list
         assert(bin_it(n, a, 0, NMBRS-1) < 0);
      }
   }
   return 0;
}
```

Iterative v. Recursion Binary Search

```
int bin_it(int k, const int* a, int l, int r)
{
    while(l <= r){
        int m = (l+r)/2;
        if(k == a[m]){
            return m;
        }
        else{
            if (k > a[m]){
                l = m + 1;
            }
            else{
                r = m - 1;
            }
        }
    }
    return -1;
}
```

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            else{
                r = m - 1;
            }
        }
    }
    return -1;
}
```

```
int bin_rec(int k, const int* a, int l, int r)
{
    if(l > r) return -1;
    int m = (l+r)/2;
    if(k == a[m]){
        return m;
    }
    else{
        if (k > a[m]){
            return bin_rec(k, a, m+1, r);
        }
        else{
            return bin_rec(k, a, l, m-1);
        }
    }
}
```

Interpolation Search

- When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.
- This idea led to the interpolation search.
- In binary searching, we simply used the middle of an ordered list as a best guess as to where to begin the search.
- Now we use an interpolation involving the key, the start of the list and the end.

$$i = (k - l[0])/(l[n-1] - l[0]) * n$$

- when searching for '15' :

0 4 5 9 10 12 15 20

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```
int interp(int k, const int* a, int l, int r)
{

   int m;
   double md;

   while(l <= r){
      md = ((double)(k-a[l])/
            (double)(a[r]-a[l])*
            (double)(r-l)
           )
           +(double)(l);
      m  = 0.5 + md;
      if((m > r) || (m < l)){
         return -1;
      }
      if(k == a[m])
         return m;
      else{
         if (k > a[m]){
            l = m + 1;
         }
         else{
            r = m- 1;
         }
      }
   }
```

Algorithmic Complexity

```c
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

#define CSEC (double)(CLOCKS_PER_SEC)
#define BIGLOOP 1000000000

int main(void)
{

    clock_t c1 = clock();
    for(int i=0; i<BIGLOOP; i++){
        int j = i * 2;
    }
    clock_t c2 = clock();
    printf("%f\n", (double)(c2-c1)/CSEC);
    return 0;

}
```

- This code on an old Dell laptop took:
  - 3.12 seconds using a non-optimzing compiler -O0
  - 0.00 seconds using an aggressive optimization -O3
- But "wall-clock" time is generally not the thing that excites Computer Scientists.

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```

- This code on an old Dell laptop took:
  - 3.12 seconds using a non-optimzing compiler -O0
  - 0.00 seconds using an aggressive optimization -O3
- But "wall-clock" time is generally not the thing that excites Computer Scientists.

- Searching and sorting algorithms have a complexity associated with them, called big-O.
- This complexity indicates how, for *n* numbers, performance deteriorates when *n* changes.
- Sequential Search : **O(n)**
- Binary Search : **O(log n)**
- Interpolation Search : **O(log log n)**
- We'll discuss the dream of a **O(1)** search later in "Hashing".

Binary vs. Interpolation Timing

```c
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>
#include <time.h>

int bin_it(int k, const int *a, int l, int r);
int bin_rec(int k, const int *a, int l, int r);
int interp(int k, const int *a, int l, int r);
int* parse_args(int argc, char* argv[], int* n, int* srch);

int main(int argc, char* argv[])
{

   int i, n, srch;
   int* a;
   int (*p[3])(int k, const int*a, int l, int r) =
       {bin_it, bin_rec, interp};

   a = parse_args(argc, argv, &n, &srch);

   srand(time(NULL));
   for(i=0; i<n; i++){
      a[i] = 2*i;
   }
   for(i=0; i<5000000; i++){
      assert((*p[srch])(a[rand()%n], a, 0, n-1) >= 0);
   }

   free(a);
   return 0;

}
```

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   free(a);
   return 0;

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```

Execution :

```
Binary Search : Iterative
n =     100000 = 0.57
n =     800000 = 0.84
n =    6400000 = 2.20
n =   51200000 = 3.87
Binary Search : Recursive
n =     100000 = 1.23
n =     800000 = 1.79
n =    6400000 = 3.20
n =   51200000 = 4.85
Interpolation
n =     100000 = 0.20
n =     800000 = 0.28
n =    6400000 = 0.50
n =   51200000 = 0.70
```