COMSM1201 : Data Structures & Algorithms

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Simple Recursion

- When a function calls itself, this is known as recursion.
- This is an important theme in Computer Science that crops up time & time again.
- Can sometimes lead to very simple and elegant programs.
- Let's look at some toy examples to begin with.

```
#include <stdio.h>
     #include <string.h>
     #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
     void strrev(char* s, int n);
     int main (void)
        char str[] = "Hello World!":
        strrev(str. strlen(str)):
        printf("%s\n", str);
        return 0:
14
15
     /* Iterative Inplace String Reverse */
17
     void strrev(char* s. int n)
18
19
        for(int i=0, j=n-1; i<j; i++, j--){
            SWAP(s[i], s[j]);
21
22
```

Execution:

!dlroW olleH

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Recursion for *strrev()*

```
#include <stdio.h>
    #include <string.h>
    #define SWAP(A.B) {char temp: temp=A:A=B:B=temp:}
    void strrev(char* s, int start, int end);
    int main(void)
       char str[] = "Hello World!";
       strrev(str. 0. strlen(str)-1):
       printf("%s\n", str);
13
14
       return 0:
15
    /* Recursive : Inplace String Reverse */
    void strrev(char* s, int start, int end)
19
       if(start >= end){
20
           return:
       SWAP(s[start], s[end]);
23
24
       strrev(s. start+1, end-1):
```

- We need to change the function prototype.
- This allows us to track both the start and the end of the string.

Execution:

IdlroW olleH

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The Fibonacci Sequence

A well known example of a recursive function is the Fibonacci sequence. The first term is 1, the second term is 1 and each successive term is defined to be the sum of the two previous terms, i.e. :

```
fib(1) is 1
fib(2) is 1
fib(n) is fib(n-1)+fib(n-2)
```

1,1,2,3,5,8,13,21, ...

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Iterative & Recursive Fibonacci

```
#include <stdio.h>
    #define MAXFIB 24
    int fibonacci(int n):
     int main(void)
        for(int i=1: i <= MAXFIB: i++){</pre>
            printf("%d = %d\n", i, fibonacci(i)):
13
14
15
        return 0;
16
17
     int fibonacci(int n)
19
20
        if(n \le 2)
           return 1;
        int b = 1:
        int next:
        for (int i=3; i \le n; i++){
           next = a + b:
           a = b:
29
           b = next:
30
31
        return b:
32
```

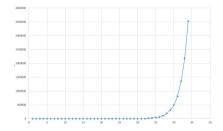
Execution:

```
1 = 1
 = 13
 = 21
9 = 34
10 = 55
11 = 89
12 = 144
13 = 233
14 = 377
15 = 610
16 = 987
17 = 1597
18 = 2584
19 = 4181
20 = 6765
21 = 10946
22 = 17711
23 = 28657
24 = 46368
```

Iterative & Recursive Fibonacci

```
#include <stdio.h>
    #define MAXFIB 24
     int fibonacci(int n);
     int main (void)
        for(int i=1; i <= MAXFIB; i++){</pre>
           printf("%d = %d\n", i, fibonacci(i));
        return 0:
     int fibonacci(int n)
18
19
20
        if (n == 1) return 1:
        if (n == 2) return 1:
        return ( fibonacci (n-1) + fibonacci (n-2));
```

It's interesting to see how run-time increases as the length of the sequence is raised.



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Maze Escape

The correct route through a maze can be obtained via recursive, rather than iterative, methods.



```
bool explore(int x, int y, char mz[YS][XS])
  if mz[y][x] is exit return true;
  Mark mz[y][x] so we don't return here
  if we can go up:
    if(explore(x, y+1, mz)) return true
  if we can go right:
    if(explore(x+1, v, mz)) return true
  Do left & down in a similar manner
  return false: // Failed to find route
```

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Permuting

- Here we consider the ways to permute a string (or more generally an array)
- Permutations are all possible ways of rearranging the positions of the characters.

Execution :

ABC ACB BAC

BAC BCA CBA CAB

```
// From e.g. http://www.geeksforgeeks.org
    #include <stdio.h>
    #include <string.h>
    #define SWAP(A,B) {char temp = *A; *A = *B; *B = temp;}
     void permute(char* a, int s, int e);
     int main()
         char str[] = "ABC";
         int n = strlen(str);
         permute(str. 0, n-1);
         return 0:
     void permute(char* a, int s, int e)
18
        if (s == e){
          printf("%s\n", a);
          return:
        for (int i = s: i \le e: i++)
24
           SWAP((a+s), (a+i)); // Bring one char to the front
25
           permute(a, s+1, e);
26
           SWAP((a+s), (a+i)); // Backtrack
27
28
```

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Self-test: Power

- Raising a number to a power n = 2⁵ is the same as multiple multiplications n = 2*2*2*2*2.
- Or, thinking recursively, $n = 2 * (2^4)$.

```
/* Try to write power(a.b) to computer a^b
        without using any maths functions other than
        multiplication :
        Try (1) iterative then (2) recursive
        (3) Trick that for n\%2==0, x^n = x^(n/2)*x^(n/2)
    #include <stdio.h>
10
11
     int power(unsigned int a, unsigned int b);
12
     int main(void)
16
        int x = 2:
        int v = 16:
19
        printf("%d^%d = %d\n", x, y, power(x,y));
20
21
     int power(unsigned int a, unsigned int b)
```

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Sequential Search

- The need to search an array for a particular value is a common problem.
- This is used to delete names from a mailing list, or upgrading the salary of an employee etc.
- The simplest method for searching is called the sequential search.
- Simply move through the array from beginning to end, stopping when you have found the value you require.

```
#include <stdio.h>
     #include <string.h>
     #include <assert.h>
     #define NOTFOUND -1
     #define NUMPEOPLE 6
     typedef struct person {
             char* name; int age;
     } person;
     int findAge(const char* name, const person* p, int n);
     int main (void)
        person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
                   {"Chumley", 26}, {"Dalton", 25},
                   {"Eggson", 22}, {"Fulton", 41} };
        assert(findAge("Eggson",
                                    ppl, NUMPEOPLE) == 22);
        assert (find Age ("Campbell", ppl, NUMPEOPLE) == NOTFOUND);
        return 0:
23
24
     int findAge(const char* name, const person* p, int n)
25
        for (int j=0; j < n; j++){
27
           if (strcmp(name, p[i], name) == 0){
              return p[i].age:
29
30
31
        return NOTFOUND:
32
```

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Sequential Search

- Sometimes our list of people may not be random.
- If, for instance, it is sorted, we can use strcmp() in a slightly cleverer manner.
- We can stop searching once the search key is alphabetically greater than the item at the current position in the list.
- This halves, on average, the number of comparisons required.

```
#include <stdio h>
     #include <string.h>
     #include <assert.h>
     #define NOTFOUND -1
     #define NUMPEOPLE 6
     typedef struct person{
             char* name; int age;
     } person:
11
     int findAge(const char* name, const person* p, int n):
12
13
     int main (woid)
14
15
        person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
                    {"Chumley", 26}, {"Dalton", 25},
                   {"Eggson", 22}, {"Fulton", 41} };
        assert (find Age ("Eggson".
                                    ppl NUMPEOPLE) == 22):
        assert (find Age ("Campbell", ppl, NUMPEOPLE) == NOTFOUND):
21
        return 0:
22
23
24
     int findAge(const char* name, const person* p, int n)
25
        for (int j=0; j < n; j++){
27
           int m = strcmp(name, p[i], name);
           if (m == 0) // Braces!
              return p[i].age:
           if(m < 0)
31
              return NOTFOUND:
32
33
        return NOTFOUND:
```

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Binary Search for 101

- Searching small lists doesn't require much computation time.
- However, as lists get longer (e.g. phone directories), sequential searching becomes extremely inefficient.
- A binary search consists of examining the middle element of the array to see if it has the desired value. If not, then half the array may be discarded for the next search.

```
#include cetdie ha
     #include cetdlib by
     #include <assert h>
     #include <time.h>
     #define NMBBS 1000000
     int bin it(int k, const int* a, int 1, int r);
     int main(void)
        int a[NMBBS]:
        srand(time(NULL)):
        // Put even numbers into array
        for (int i=0; i < NMBRS; i++){
           a[i] = 2*i:
        // Do many searches for a random number
20
        for (int i=0: i<10*NMBRS: i++){
21
           int n = rand()%NMBRS:
           if((n\%2) = 0){
23
              assert(bin it(n, a, 0, NMBRS-1) = n/2);
24
25
           else { // No odd numbers in this list
26
              assert(bin it(n, a, 0, NMBRS-1) < 0):
27
28
29
        return 0:
```

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Iterative v. Recursion Binary Search

```
int bin_it(int k, const int* a, int 1, int r)
{
  while(1 <= r){
    int m = (1+r)/2;
    if(k = a[m]){
      return m;
    }
    else{
      if (k > a[m]){
            1 = m + 1;
        }
      else{
      r = m - 1;
      }
    }
  return -1;
}
```

```
int bin_rec(int k, const int* a, int 1, int r)
{
    if(1 > r) return -1;
    int m = (1+r)/2;
    if(k = a | m |) {
        return m;
    }
    else {
        if (k > a | m |) {
            return bin_rec(k, a, m + 1, r);
        }
        else {
            return bin_rec(k, a, 1, m - 1);
        }
    }
}
```

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Interpolation Search

- When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.
- This idea led to the interpolation search.
- In binary searching, we simply used the middle of an ordered list as a best guess as to where to begin the search.
- Now we use an interpolation involving the key, the start of the list and the end.

$$i = (k - I[0])/(I[n-1] - I[0]) * n$$

• when searching for '15':

```
0 4 5 9 10 12 15 20
```

```
int interp(int k. const int* a. int l. int r)
   int m:
   double md:
   while(1 \le r)
      md = ((double)(k-a[1])/
            (double)(a[r]-a[1])*
            (double)(r-1)
           +(double)(1):
      m = 0.5 + md:
      if((m > r) | | (m < 1)){
         return -1:
      if (k == a[m])
         return m:
         if (k > a[m]) {
            1 = m + 1:
         elsef
            r = m-1:
```

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Algorithmic Complexity

```
1  #include <stdio.h>
2  #include <tidlib.h>
3  #include <time.h>
4
5  #define CSEC (double)(CLOCKS_PER_SEC)
6  #define BIGLOOP 1000000000
7
7
8  int main(void)
9  {
10
11    clock_t c1 = clock();
12    for(int i=0; i<EIGLOOP; i++){
13         int j = i * 2;
14    }
15    clock_t c2 = clock();
16    printf("%f\n", (double)(c2-c1)/CSEC);
17    return 0;
18
18</pre>
```

- This code on an old Dell laptop took:
 - 3.12 seconds using a non-optimzing compiler -O0
 - 0.00 seconds using an aggressive optimization -O3
- But "wall-clock" time is generally not the thing that excites Computer Scientists.

- Searching and sorting algorithms have a complexity associated with them, called big-O.
- This complexity indicates how, for n numbers, performance deteriorates when n changes.
- Sequential Search : O(n)
- Binary Search : O(log n)
- Interpolation Search : O(log log n)
- We'll discuss the dream of a O(1) search later in "Hashing".

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Binary vs. Interpolation Timing

```
#include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <time.h>
    int bin it(int k, const int *a, int l, int r);
     int bin rec(int k. const int *a. int 1. int r):
     int interp(int k, const int *a, int 1, int r);
     int* parse_args(int argc, char* argv[], int* n, int* srch);
     int main(int argc, char* argv[])
12
13
        int i, n, srch;
        int* a:
        int (*p[3])(int k, const int*a, int 1, int r) =
            {bin it, bin rec, interp};
18
19
20
21
        a = parse_args(argc, argv, &n, &srch);
        srand(time(NULL));
22
23
        for (i=0; i < n; i++){
           a[i] = 2*i:
24
25
        for (i=0; i<5000000; i++){}
26
27
           assert ((*p[srch])(a[rand()%n], a, 0, n-1) >= 0);
28
29
        free(a):
30
        return 0;
31
32
```

Execution:

```
Binary Search : Iterative
       100000 = 0.57
      800000 = 0.84
      6400000 = 2.20
     51200000 = 3.87
Binary Search : Recursive
       100000 = 1.23
       800000 = 1.79
      6400000 = 3.20
n =
     51200000 = 4.85
Interpolation
n =
       100000 = 0.20
       800000 = 0.28
      6400000 = 0.50
n =
     51200000 = 0.70
n =
```

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Linked Data Structures

- Linked data representations are useful when:
 - It is difficult to predict the size and the shape of the data structures in advance.
 - We need to efficiently insert and delete elements
- To create linked data representations we use pointers to connect separate blocks of storage together. If a given block contains a pointer to a second block, we can follow this pointer there.
- By following pointers one after another, we can travel right along the structure.

```
#include <stdio h>
     #include <stdlib b>
    #include "general.h"
     typedef struct data{
        int i:
        struct data* next:
     } Data;
     Data* allocateData(int i):
11
     void printList(Data* 1):
     int main(void)
        int i:
        Data* start . *current :
        start = current = NULL:
        printf("Enter the first number: "):
        if(scanf("%i", &i) == 1){
           start = current = allocateData(i):
21
        elsef
           on_error("Couldn't read an int");
        printf("Enter more numbers: ");
27
        while(scanf("%i", &i) == 1){
           current -> next = allocateData(i):
           current = current -> next:
31
        printList(start):
        // Should Free List
        return 0:
```

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Linked Lists

```
Data* allocateData(int i)
{
    Data* p;
    p = (Data*) ncalloc(1, sizeof(Data));
    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* 1)
{
    printf("\n");
    do{
        printf("Number : %i\n", 1->i);
        1 = 1->next;
    }while(1 != NULL);
    printf("END\n");
}
```

Searching and Recursive printing:

```
Data* inList(Data* n, int i)
{
    do{
        if(n->i==i){
            return n;
        }
        n = n-Next;
    }while(n != NULL);
    return NULL;
}

void printList_r(Data* 1)
{
    // Recursive Base-Case
    if(1 == NULL) return;
    printf(*Number: %i\n*, 1->i);
    printList_r(1->next);
}
```

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Abstract Data Types

- But would we really code something like this every time we need flexible data storage?
- This would be horribly error-prone.
- Build something once, and test it well.
- One example of this is an Abstract Data Type (ADT).
- Each ADT exposes its functionality via an interface.
- The user only accesses the data via this interface.
- The user of the ADT doesn't need to understand how the data is being stored (e.g. array vs. linked lists etc.)

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Collections

- One of the simplest ADTs is the **Collection**.
- This is just a simple place to search for/add/delete data elements.
- Some collections allow duplicate elements and others do not (e.g. Sets).
- Some are ordered (for faster searching) and others unordered.
- Our Collection will be unsorted and will allow duplicates.

```
#include "../General/general.h"
typedef int colltype:
typedef struct coll coll;
#include <stdio.h>
#include <stdlib h>
#include <assert.h>
// Create an empty coll
coll* coll init(void);
// Add element onto top
void coll add(coll* c, colltype i);
// Take element out
bool coll remove(coll* c. colltype d):
// Does this exist ?
bool coll isin(coll* c. colltype i):
// Return size of coll
int coll size(coll* c):
// Clears all space used
bool coll_free(coll* c);
```

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Collection ADT

- Note that the interface gives you no hints as to the actual underlying implementation of the ADT.
- A user of the ADT doesn't really need to know how it's implemented - ideally.
- The ADT developer could have several different implementations.
- Here we'll see *Collection* implemented using:
 - A fixed-size array
 - A dynamic array
 - A linked-list

Fixed/specific.h:

```
1  #pragma once
2
3  #define COLLTYPE "Fixed"
4
5  #define FIXEDSIZE 5000
6  struct coll {
7    // Underlying array
8    colltype a[FIXEDSIZE];
9   int size;
10 };
```

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Collection ADT using a Fixed-size Array

Fixed/fixed.c:

```
#include "../coll.h"
    #include "specific.h"
     coll* coll_init(void)
        coll* c = (coll*) ncalloc(sizeof(coll), 1);
        c - > size = 0;
        return c;
     int coll size(coll* c)
13
        if (c=NULL){
           return 0:
16
17
        return c->size;
19
     bool coll_isin(coll* c, colltype d)
20
        for (int i=0: i < coll size(c): i++){
22
           if(c->a[i] == d){}
               return true:
24
        return false;
```

```
void coll add(coll* c. colltype d)
   if(c){
      if(c->size >= FIXEDSIZE){
          on error("Collection overflow"):
      c \rightarrow a[c \rightarrow size] = d:
      c \rightarrow size = c \rightarrow size + 1:
bool coll remove(coll* c. colltype d)
   for (int i=0: i < coll size(c): i++){
      if(c->a[i] == d)f
          // Shuffle end of array left one
          for(int j=i; j < coll_size(c); j++){</pre>
             c - a[i] = c - a[i+1];
          c->size = c->size - 1:
          return true:
   return false:
bool coll_free(coll* c)
   free(c):
   return true:
```

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Collection ADT via an Array (Realloc)

Realloc/specific.h:

Realloc/realloc.c:

```
#include "../coll.h"
     #include "specific.h"
      coll* coll init(void)
         coll* c = (coll*) ncalloc(sizeof(coll), 1);
         c->a = (colltype*) ncalloc(sizeof(colltype), FIXEDSIZE);
         c \rightarrow size = 0:
         c->capacity= FIXEDSIZE;
         return c:
      void coll add(coll* c. colltype d)
14
         if(c){
             if (c->size >= c->capacity){
                c \rightarrow a = (colltype*) nremalloc(c \rightarrow a.
                         sizeof(colltype)*c->capacity*SCALEFACTOR);
19
                c->capacity = c->capacity*SCALEFACTOR;
20
21
            c \rightarrow a[c \rightarrow size] = d:
            c \rightarrow size = c \rightarrow size + 1:
23
```

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Collection ADT via a Linked List

Linked/specific.h:

```
#pragma once

#define COLLTYPE "Linked"

struct dataframe {
    colltype i;
    struct dataframe* next;
    };
    typedef struct dataframe dataframe;

truct coll {
    // Underlying array
    dataframe* start;
    int size;
    };
}
```

Linked/linked.c:

```
#include " .. / coll .h"
#include "specific.h"
coll* coll_init(void)
   coll* c = (coll*) ncalloc(sizeof(coll), 1);
   return c:
int coll size(coll* c)
   if (c==NULL){
      return 0:
   return c->size:
bool coll_isin(coll* c, colltype d)
   if(c == NULL || c->start==NULL){
      return false:
   dataframe* f = c->start:
   dof
      if(f\rightarrow i == d){
          return true:
      f = f - > next;
   } while (f != NULL):
   return false:
```

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Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
   if(c){
       dataframe* f = ncalloc(sizeof(dataframe), 1):
       f \rightarrow i = d:
       f \rightarrow next = c \rightarrow start:
       c \rightarrow start = f;
       c \rightarrow size = c \rightarrow size + 1:
bool coll free(coll* c)
   if(c){
       dataframe* tmp:
       dataframe* p = c->start:
       while (p!=NULL) {
           tmp = p->next;
           free(p);
           p = tmp;
       free(c):
   return true;
```

```
bool coll_remove(coll* c, colltype d)
   dataframe* f1 . *f2:
   if((c==NULL) || (c->start==NULL)){
      return false:
   // If Front
   if(c->start->i == d){
      f1 = c->start->next:
      free(c->start):
      c->start = f1:
      c \rightarrow size = c \rightarrow size - 1:
      return true:
   f1 = c -> start:
   f2 = c->start->next:
   dof
      if(f2->i == d)f
          f1 -> next = f2 -> next:
          free(f2):
          c \rightarrow size = c \rightarrow size - 1:
          return true:
      f1 = f2:
      f2 = f1 -> next:
   } while (f2 != NULL):
   return false;
```

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Collection Summary

- Any code using the ADT can be compiled against any of the implementations,
 e.g. the test (testcoll.c) code.
- The Collection interface (coll.h) is never changed.
- There are pros and cons of each implementation:
 - Fixed Array: Simple to implement can't avoid the problems of it being a fixed-size. Deletion expensive.
 - Realloc Array: Implementation fairly simple. Deletion expensive. Every realloc() is very expensive. Need to tune SCALEFACTOR.
 - Linked : Slightly fiddly implementation
 fast to delete an element.

Task	Fixed Array	Realloc Array	Linked List
Insert new element	O(1) at end	O(1) at end	O(1) at front
	if space	but realloc()	
Search for an element	O(n)	O(n)	O(n)
	brute force	brute force	brute force
Search + delete	O(n) + O(n)	O(n) + O(n)	O(n) + O(1)
	move left	move left	delete 'free'

 If we had ordered our ADT (ie. the elements were sorted), then the searches could be via a binary / interpolation search, leading to O(log n) or O(log log n) search times.

Q : ADTs - Collection 30 / 65

ADTs Making Coding Simpler

That Linked List code from the previous Chapter again:

```
1  #include "coll.h"
2  #include "Fixed/specific.h"
3  int main(void)
5  {
6    coll* c;
7    int i;
8    printf("Please type some numbers :");
10    c = coll_init();
11    while(scanf("%i", &i) == 1){
12     coll_add(c, i);
13  }
14    // Do print etc.
15    coll_free(c);
16    return 0;
17 }
```

Q : ADTs - Collection 31 / 65

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R : ADTs - Stacks 32 / 65

ADTs

At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

- Collections (Lists)
- Stacks
- Queues
- Sets
- Graphs
- Trees

Binary Trees:



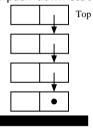
Unidirectional Graph:



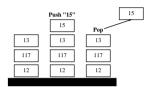
R : ADTs - Stacks 33 / 65

Stacks

The push-down stack:



LIFO (Last in, First out):



- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.
- But, once again, we are faced with the question: How best to implement such a data type?

R : ADTs - Stacks 34 / 65

ADT:Stacks Arrays (Realloc) I

stack.h:

```
#pragma once
    #include " .. / General/general .h"
    typedef int stacktype:
    typedef struct stack stack;
    #include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <string.h>
    /* Create an empty stack */
    stack* stack_init(void);
    /* Add element to top */
    void stack push(stack* s, stacktype i);
    /* Take element from top */
    bool stack pop(stack* s. stacktype* d):
    /* Clears all space used */
    bool stack free(stack* s):
23
24
    /* Optional? */
    /* Copy top element into d (but don't pop it) */
    bool stack peek(stack*s. stacktype* d):
    /* Make a string version - keep .dot in mind */
    void stack tostring(stack*. char* str);
```

Realloc/specific.h:

```
1  #pragma once
2
3  #define FORMATSIR "%i"
4  #define ELEMSIZE 20
5  #define STACKTYPE "Realloc"
7
7
8  #define FIXEDSIZE 16
9  #define SCALEFACTOR 2
10
11  struct stack {
12     /* Underlying array */
13     stacktype* a;
14     int size;
15     int capacity;
16  };
```

R : ADTs - Stacks 35 / 65

ADT:Stacks Arrays (Realloc) II

Realloc/realloc.c

```
#include " .. / stack . h"
     #include "specific.h"
     #define DOTFILE 5000
     stack * stack init(void)
         stack *s = (stack*) ncalloc(sizeof(stack), 1):
        /* Some implementations would allow you to pass
            a hint about the initial size of the stack */
         s->a = (stacktype*) ncalloc(sizeof(stacktype), FIXEDSIZE);
        s \rightarrow size = 0:
         s->capacity= FIXEDSIZE;
14
         return s:
15
17
     void stack_push(stack* s, stacktype d)
19
         if (s=NULL){
              return:
21
        if(s->size >= s->capacity){}
23
            s \rightarrow a = (stacktype*) nremalloc(s \rightarrow a.
24
                     sizeof(stacktype)*s->capacity*SCALEFACTOR);
25
            s->capacity = s->capacity*SCALEFACTOR;
27
         s \rightarrow a[s \rightarrow size] = d:
28
         s \rightarrow size = s \rightarrow size + 1:
```

```
bool stack_pop(stack* s, stacktype* d)
{
    if((s == NULL) || (s->size < 1)){
        return false;
    }
    s -> size = s-> size - 1;
    *d = s->a[s->size];
    return true;
}

bool stack_peek(stack* s, stacktype* d)
{
    if((s==NULL) || (s->size <= 0)){
        /* Stack is Empty */
        return false;
}

* d = s->a[s->size-1];
return true;
}
```

R : ADTs - Stacks 36 / 65

ADT:Stacks Arrays (Realloc) III

Realloc/realloc.c

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE];
        str[0] = '\0':
        if((s=NULL) || (s->size <1)){
            return:
        for (int i=s->size-1: i>=0: i--) {
            sprintf(tmp, FORMATSTR, s->a[i]);
           strcat(str. tmp):
10
11
12
13
            strcat(str. "|");
        str[strlen(str)-1] = '\0':
14
15
16
17
     bool stack free(stack* s)
18
        if (s=NULL){
19
            return true:
20
21
        free(s->a):
        free(s):
        return true:
```

- We need a thorough testing program teststack.c
- See also revstr.c: a version of the string reverse code (for which we already seen an iterative (in-place) and a recursive solution).

R : ADTs - Stacks 37 / 65

ADT:Stacks Linked I

Linked/specific.h

```
1  #pragma once
2
3  #define FORMATSIR "%i"
4  #define ELEMSIZE 20
5  #define STACKTYPE "Linked"
6
7  struct dataframe {
8   stacktype i;
9   struct dataframe* next;
);
11  typedef struct dataframe dataframe;
12
13  struct stack {
14   /* Underlying array */
15   dataframe* start;
16  int size;
17 };
```

Linked/linked.c

```
#include " .. / stack .h"
     #include "specific.h"
     #define DOTFILE 5000
     stack* stack init(void)
         stack* s = (stack*) ncalloc(sizeof(stack), 1);
         return s:
10
11
     void stack push(stack* s. stacktype d)
13
        if(s){
            dataframe* f = ncalloc(sizeof(dataframe), 1):
            f \rightarrow i = d:
            f->next = s->start;
            s->start = f:
            s \rightarrow size = s \rightarrow size + 1:
20
```

R : ADTs - Stacks 38 / 65

ADT:Stacks Linked II

```
bool stack_pop(stack* s, stacktype* d)
        if((s==NULL) || (s->start==NULL)){
            return false:
        dataframe* f = s->start->next;
        *d = s->start->i:
        free(s->start):
        s \rightarrow start = f:
        s \rightarrow size = s \rightarrow size - 1:
        return true:
13
14
15
     bool stack peek(stack* s. stacktype* d)
16
17
        if((s==NULL) || (s->start==NULL)){
18
            return false:
19
20
        *d = s->start->i:
        return true;
22
```

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE]:
        str[0] = '\0':
        if((s==NULL) || (s->size <1)){
           return:
        dataframe* p = s->start:
        while (p) f
           sprintf(tmp. FORMATSTR. p->i):
           strcat(str. tmp):
           strcat(str. "|"):
           p = p -> next:
14
        str[strlen(str)-1] = '\0';
16
17
18
     bool stack free(stack* s)
19
20
        if(s){
           dataframe* p = s->start;
           while (p!=NULL){
              dataframe* tmp = p->next;
              free(p):
              p = tmp;
26
27
           free(s):
28
        return true;
30
```

R : ADTs - Stacks 39 / 65

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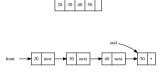
T: ADTs - Trees

U: ADTs - Hashing

S : ADTs - Queues 40 / 65

ADTs: Queues

FIFO (First in, First out):



- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)
- Operations include enqueue, dequeue and size.

queue.h

```
#pragma once
    #include " .. / General/general .h"
     typedef int queuetype;
     typedef struct queue queue;
     #include <stdio.h>
    #include <stdlib.h>
    #include <string.h>
    Winclude (assert h)
     /* Create an empty queue */
     queue* queue init(void):
     /* Add element on end */
     void queue_enqueue(queue* q, queuetype v);
     /* Take element off front */
     bool queue dequeue(queue* q. queuetype* d):
     /* Return size of queue */
     int queue size(queue* q):
     /* Clears all space used */
     bool queue_free(queue* q);
24
     /* Helps with visualisation & testing */
     void queue tostring(queue* q. char* str):
```

S : ADTs - Queues 41 / 65

ADTs: Queues (Fixed) I

specific.h

```
#pragma once

#define FORMATSTR "%d"

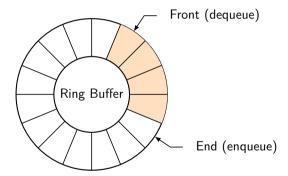
#define ELEMSIZE 20

#define QUEUETYPE "Fixed"

#define BOUNDED 5000

struct queue {
    /* Underlying array */
    queuetype a[BOUNDED];
    int front;
    int end;
};

#define DOTFILE 5000
```



S : ADTs - Queues 42 / 65

ADTs: Queues (Fixed) II

fixed.c

```
#include " .. / queue . h"
     #include "specific.h"
     void inc(queuetype* p);
     queue * queue init(void)
         queue* q = (queue*) ncalloc(sizeof(queue), 1);
         return q;
     void queue_enqueue(queue* q, queuetype d)
14
15
16
17
         if (a) {
            q \rightarrow a[q \rightarrow end] = d:
            _inc(&q->end);
18
19
20
21
             if (q->end == q->front){
                on_error("Queue too large");
22
```

```
bool queue dequeue(queue* q. queuetype* d)
        if ((a==NULL) || (a->front==a->end)){
           return false:
        *d = q -  a[q -  front]:
        inc(&g->front):
        return true:
9
10
11
     void queue tostring(queue* q. char* str)
12
13
        char tmp[ELEMSIZE];
        str[0] = '\0':
        if ((q==NULL) || (queue_size(q)==0)){
16
           return:
17
18
        for(int i=q->front; i != q->end;){
           sprintf(tmp, FORMATSTR, q->a[i]);
20
           strcat(str. tmp):
21
           strcat(str. "|"):
22
           inc(&zi):
23
24
        str[strlen(str)-1] = '\0':
```

S : ADTs - Queues 43 / 65

ADTs: Queues (Fixed) III

```
int queue_size(queue* q)
         if (a==NULL) {
            return 0:
        if(q-)end = q-)front)
            return q->end-q->front;
9
10
11
        return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
         free(q):
15
16
        return true;
17
18
19
     void inc(queuetvpe* p)
20
        *p = (*p + 1) \% BOUNDED:
```

- We need a thorough testing program
- We'll see queues again for traversing trees
- Simulating a (slow) printer

S : ADTs - Queues 44 / 65

ADTs: Queues (Linked) I

specific.h

```
#pragma once
    #define FORMATSTR "%d"
    #define ELEMSIZE 20
    #define OUFUETYPE "Linked"
    struct dataframe {
       queuetype i;
        struct dataframe* next;
    }:
12
13
    typedef struct dataframe dataframe;
14
    struct queue {
15
      /* Underlying array */
       dataframe* front:
17
       dataframe* end:
       int size:
19
    }:
```

linked.c

```
#include " .. / queue .h"
      #include "specific.h"
      queue* queue init(void)
          queue* q = (queue*) ncalloc(sizeof(queue), 1);
          return q;
      void queue_enqueue(queue* q, queuetype d)
          dataframe* f;
          if (q==NULL) {
             return:
          /* Copy the data */
          f = ncalloc(sizeof(dataframe), 1);
          f \rightarrow i = d:
          /* 1st one */
          if (a->front == NULL) {
             a \rightarrow front = f:
24
             a \rightarrow end = f:
             q \rightarrow size = q \rightarrow size + 1;
             return:
28
          /* Not 1st */
          q \rightarrow end \rightarrow next = f:
          a->end = f:
31
          q \rightarrow size = q \rightarrow size + 1;
```

S : ADTs - Queues 45 / 65

ADTs: Queues (Linked) II

```
bool queue dequeue(queue* q, queuetype* d)
         dataframe* f:
         if ((q=NULL) || (q->front=NULL) || (q->end=NULL)){
            return false;
         f = q - front - next;
         *d = q-> front -> i;
         free(q->front);
        q \rightarrow front = f;
         q \rightarrow size = q \rightarrow size - 1;
         return true;
13
14
     bool queue free (queue * q)
        if (a) {
18
19
            dataframe* tmp:
            dataframe* p = q->front;
20
            while (p!=NULL) {
                tmp = p -> next:
                free(p);
23
24
                p = tmp:
25
26
            free(q);
         return true;
28
```

```
void queue tostring(queue* q, char* str)
        dataframe *p;
        char tmp[ELEMSIZE];
        str[0] = '\0';
        if ((q=NULL) || (q->front == NULL)){
           return:
        p = q - front;
        while(p){
           sprintf(tmp, FORMATSTR, p->i);
           strcat(str. tmp);
           strcat(str. "|");
           p = p -   next;
16
        str[strlen(str)-1] = '\0';
17
18
     int queue size(queue* q)
20
21
        if ((q=NULL) || (q->front=NULL)){
23
           return 0:
24
25
        return q->size;
```

S : ADTs - Queues 46 / 65

Detour : Graphviz

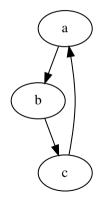
 There exists a nice package, called Graphviz:

sudo apt install graphviz

 This allows the visualisation of graphs/dynamic structures using the simple .dot language:

```
digraph {
   a -> b; b -> c; c -> a;
}
```

To create a .pdf: dot -Tpdf -o graphviz.pdf examp1.dot



S : ADTs - Queues 47 / 65

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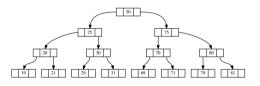
T : ADTs - Trees

U: ADTs - Hashing

T : ADTs - Trees 48 / 65

ADTs: Binary Trees

- Binary trees are used extensively in computer science
- Game Trees
- Searching
- Sorting



- Trees drawn upside-down!
- Ancestor relationships: A is the parent of E
- Can refer to left and right children
- In a tree, there is only one path from the root to any child
- A node with no children is a leaf
- Most trees need to be created dynamically
- Empty subtrees are set to NULL

「: ADTs - Trees 49 / 65

Binary Trees using Arrays?

- Don't rush to assume a linked data structure must be used to implement trees.
- You could use 1 cell of an array for the first node, the next two cells for its children, the next 4 cells for their children and so on.
- You need to mark which cells are in use & which aren't ...

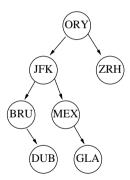
Counting from cell 1, for a tree with *n* nodes:

To find	Use	Iff
The root	A[1]	A is nonempty
The left child of $A[i]$	A[2i]	$2i \leq n$
The parent of $A[i]$	A[i/2]	i > 1
Is A[i] a leaf?	True	2 <i>i</i> > <i>n</i>

: ADTs - Trees 50 / 65

Binary Search Trees

In a binary search tree the left-hand tree of a parent contains all keys less than the parent node, and the right-hand side all the keys greater than the parent node.



bst. h

```
#include " .. / General/general . h"
    #include " .. / Queue/queue . h "
     #include <stdio.h>
     #include <stdlib.h>
     #include <assert.h>
     bst* bst_init(void);
     /* Insert 1 item into the tree */
     bool bst insert(bst* b, treetype d);
     /* Return number of nodes in tree */
     int bst size(bst* b);
     /* Whether the data d is stored in the tree */
     bool bst isin(bst* b, treetype d);
     /* Bulk insert n items from an array a into an initialised tree */
20
     bool bst_insertarray(bst* b, treetype* a, int n);
     /* Clear all memory associated with tree. & set pointer to NULL */
     bool bst free(bst* b):
25
     /* Optional ? */
     char* bst_preorder(bst* b);
     void bst printlevel(bst* b):
     /* Create string with tree as ((head)(left)(right)) */
     char* bst printlisp(bst* b):
     /* Use Graphviz via a .dot file */
     void bst todot(bst* b. char* dotname):
```

T : ADTs - Trees 51 / 65

Binary Search Trees: Linked I

specific.h

```
#include <string.h>
    typedef int treetype;
    #define FORMATSTR "%i"
    #define ELEMSIZE 20
    #define BSTTYPE "Linked"
    struct dataframe {
        treetype d;
       struct dataframe* left;
        struct dataframe* right;
    }:
    typedef struct dataframe dataframe:
15
    struct bst {
       dataframe* top:
       /* Data element size . in bytes */
    typedef struct bst bst:
```

```
/* Based on geekforgeeks.org */
     dataframe* insert(dataframe* t, treetype d)
         dataframe* f;
         /* If the tree is empty, return a new frame */
          if (t == NULL){
             f = ncalloc(sizeof(dataframe), 1);
             f \rightarrow d = d:
             return f;
         /* Otherwise, recurs down the tree */
          if (d < t > d)
              t \rightarrow left = insert(t \rightarrow left . d):
         else if (d > t - >d)
16
             t->right = _insert(t->right, d);
17
18
         /* return the (unchanged) dataframe pointer */
19
         return t:
20
```

T : ADTs - Trees 52 / 65

Binary Search Trees: Linked II

```
1 bool _isin(dataframe* t, treetype d)
2 {
3    if(t=NULL){
4       return false;
5    }
6    if(t->d == d){
7       return true;
8    }
9    if(d < t->d){
10       return _isin(t->left, d);
11    }
12    else{
13       return _isin(t->right, d);
14    }
15    return false;
16 }
```

```
char* printlisp(dataframe* t)
        char tmp[ELEMSIZE];
        char *s1, *s2, *p;
        if (t==NULL) {
           /* \0 string */
           p = ncalloc(1.1);
           return p;
11
        sprintf(tmp. FORMATSTR. t->d):
        s1 = _printlisp(t->left);
13
        s2 = _printlisp(t->right);
        p = ncalloc(strlen(s1)+strlen(s2)+strlen(tmp)+
            strlen("()() "). 1):
        sprintf(p, "%s(%s)(%s)", tmp, s1, s2);
17
        free(s1):
        free(s2):
19
        return p;
```

T : ADTs - Trees 53 / 65

Binary Search Trees : Realloc

specific.h

```
#include <stdbool b>
    typedef int treetype:
    #define FORMATSTR "%i"
    #define FIFMSIZE 20
    #define BSTTYPE "Realloc"
    // Probably (2^n) -1
    #define INITSIZE 31
    #define SCALEFACTOR 2
    struct dataframe {
        treetype d:
        bool isvalid:
15
    typedef struct dataframe dataframe:
17
    struct bst {
19
       dataframe* a:
       int capacity:
21
    typedef struct bst bst:
```

Using a queue for Level-Order traversal:

```
void bst printlevel(bst* b)
        treetype n:
        if ((b=NULL) || (! isvalid(b, 0))){
          return:
        /* Make a queue of cell indices */
        queue* q = queue init();
        queue enqueue(q. 0):
        while (queue_dequeue(q, &m) && _isvalid(b, (int)n)){
11
           printf(FORMATSTR. b->a[n].d):
12
           putchar(' '):
13
          queue_enqueue(q, _leftchild((int)n));
          queue_enqueue(q, _rightchild((int)n));
15
```

T : ADTs - Trees 54 / 65

Binary Search Trees: Complexity

- So, in a nicely balanced tree, insertion, deletion and search are all $O(\log n)$.
- But: if the root of the tree is not well chosen, or the keys to be inserted are ordered, the tree can become a linked list!
- In this case, complexity becomes O(n).
- The tree search performs best when well balanced trees are formed.
- Large body of literature about creating & re-balancing trees Red-Black trees, Tries, 2-3 trees, AVL trees etc.

T : ADTs - Trees 55 / 65

Binary Trees: Huffman Compression I

- Often we wish to compress data, to reduce storage requirements, or to speed transmission.
- Text is particularly suited to compression since using one byte per character is wasteful - some letters occur much more frequently.
- Need to give frequently occurring letters short codes, typically a few bits. Less common letters can have long bit patterns.

• To encode the string "BABBAGE":

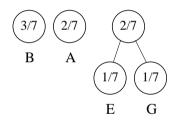


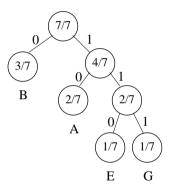
Keep a list of characters, ordered by their frequency

: ADTs - Trees 56 / 65

Binary Trees: Huffman Compression II

• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :





- \bullet A = 10. B = 0. E = 110. G = 111
- String stored using 13 bits.

「: ADTs - Trees 57 / 65

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ADTs: Hashing

 To keep records of employees we might index (search) them by using their National Insurance number:

- There are 17.6 billion combinations (around 2³⁴).
- Could use an array of 17.6 billion entries, which would make searching for a particular entry trivial!
- Especially wasteful since only our (5000) employees need to be stored.

- Here we examine a method that, using an array of 6000 elements, would require 2.1 comparisons on average.
- A hash function is a mapping, h(K), that maps from key K, onto the index of an entry.
- A black-box into which we insert a key (e.g. NI number) and out pops an array index.
- As an example lets use an array of size 11 to store some airport codes, e.g. PHL, DCA, FRA.

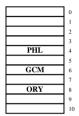
U : ADTs - Hashing 59 / 65

ADTs: Aiport Codes

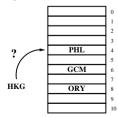
- In a three letter string X₂X₁X₀ the letter 'A' has the value 0, 'B' has the value 1 etc.
- One hash function is:

$$h(K) = (X_2 * 26^2 + X_1 * 26 + X_0)\%11$$

• Applying this to "DCA": h("DCA") = $(3*26^2 + 2*26 + 0)\%11$ h("DCA") = (2080)%11h("DCA") = 1 Inserting "PHL", "ORY" and "GCM":



• However, inserting "HKG" causes a collision.



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ADTs: Collisions

- An ideal hashing function maps keys into the array in a uniform and random manner.
- Collisions occur when a hash function maps two different keys onto the same address
- It's very difficult to choose 'good' hashing functions.
- Collisions are common the von Mises paradox. When 23 keys are randomly mapped onto 365 addresses there is a 50% chance of a collision.

- The policy of finding another free location if a collision occurs is called open-addressing.
- If a collision occurs then keep stepping backwards (with wrap-around) until a free location is encountered.



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ADTs: Double Hashing

- This simple method of open-addressing is linear-probing.
- The step taken each time (probe decrement) need not be 1.
- Open-addressing through use of linear-probing is a very simple technique, double-hashing is generally much more successful.
- A second function p(K) decides the size of the probe decrement.

 The function is chosen so that two keys which collide at the same address will have different probe decrements, e.g.:

$$p(K) = MAX(1, ((X_2*26^2 + X_1*26 + X_0)/11)\%11)$$

• Although "PHL" and "HKG" share the same primary hash value of h(K) = 4, they have different probe decrements:

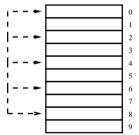
$$p("PHL") = 4$$

 $p("HKG") = 3$

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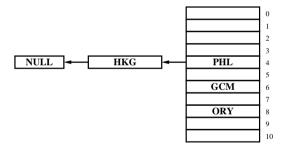
ADTs: Primes and Chaining

 If the size of our array, M, was even and the probe decrement was chosen to be 2, then only half of the locations could be probed.



• Often we choose our table size to be a prime number and our probe decrement to be a number in the range $1 \dots M - 1$.

Open-addressing is not the only method of collision reduction. Another common one is separate chaining.



U : ADTs - Hashing 63 / 65

ADTs: A Practical Hash Function

```
#include <stdio.h>
     int hash(unsigned int sz, char *s);
     int main (void)
        char str[] = "Hello World!";
        // Hash modulus 7919
        printf("%d\n", hash(7919, str));
        return 0:
12
13
15
     Modified Bernstein hashing
     5381 & 33 are magic numbers required by the algorithm
19
     int hash(unsigned int sz. char *s)
20
21
        unsigned long hash = 5381;
        int c:
23
        while ((c = (*s++))){
24
           hash = 33 * hash ^ c:
25
        return (int)(hash%sz):
```

Execution:

5479

Has similarities to the implementation of rand():

```
#include <stdio.h>
int rand r(unsigned int* seed);
int main(void)
   unsigned int seed = 0:
   printf("%d\n", rand r(&seed));
   return 0:
/* This algorithm is mentioned in the ISO C standard.
   here extended for 32 bits. */
int rand r(unsigned int * seed)
  unsigned int next = *seed;
  int result:
  next *= 1103515245:
  next += 12345:
  result = (unsigned int) (next / 65536) % 2048;
  next *= 1103515245;
  next += 12345:
  result <<= 10:
```

Execution:

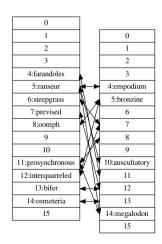
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ADTs: Cuckoo Hashing

- We have two tables, each with their own hash function.
- We only need to check two cells when searching.
- On collision, the existing item is 'cuckooed' out of it's cell into the other table.

Empty: copied bronzine into table 0(12) Empty: copied auscultatory into table 0(5) Empty: copied bifer into table 0(13) Empty: copied steepgrass into table 0(6) Empty: copied prevised into table 0(7) Empty: copied oomph into table 0(8) empodium, so cuckooed out auscultatory from table 0(5) Empty: copied auscultatory into table 1(10) interquarreled, so cuckooed out bronzine from table 0(12) Empty: copied bronzine into table 1(5) ranseur, so cuckooed out empodium from table 0(5) Empty: copied empodium into table 1(4) Empty: copied megalodon into table 0(11) geosynchronous, so cuckooed out megalodon from table 0(11) Empty: copied megalodon into table 1(14) Empty: copied osmeteria into table 0(14) Table getting full -> rehashed old sz =16

Empty: copied farandoles into table 0(4)



: ADTs - Hashing 65 / 65