

COMSM1201 : Data Structures & Algorithms

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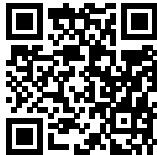


Table of Contents

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P : Linked Data Structures

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X : Algorithms III - Parsing and Grammars

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- Let's look at some toy examples to begin with.

```
1  #include <stdio.h>
2  #include <string.h>
3
4  #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
5
6  void strrev(char* s, int n);
7
8  int main(void)
9  {
10     char str[] = "Hello World!";
11     strrev(str, strlen(str));
12     printf("%s\n", str);
13     return 0;
14 }
15
16 /* Iterative Inplace String Reverse */
17 void strrev(char* s, int n)
18 {
19     for(int i=0, j=n-1; i<j; i++, j--){
20         SWAP(s[i], s[j]);
21     }
22 }
```

Execution :

!dlroW olleH

Recursion for *strrev()*

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1  #include <stdio.h>
2  #include <string.h>
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4  #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
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6  void strrev(char* s, int start, int end);
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8  int main(void)
9  {
10     char str[] = "Hello World!";
11     strrev(str, 0, strlen(str)-1);
12     printf("%s\n", str);
13     return 0;
14 }
15
16 /* Recursive : Inplace String Reverse */
17 void strrev(char* s, int start, int end)
18 {
19     if(start >= end){
20         return;
21     }
22     SWAP(s[start], s[end]);
23     strrev(s, start+1, end-1);
24 }
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- We need to change the function prototype.
- This allows us to track both the start and the end of the string.

The Fibonacci Sequence

A well known example of a recursive function is the Fibonacci sequence. The first term is 1, the second term is 1 and each successive term is defined to be the sum of the two previous terms, i.e. :

$\text{fib}(1)$ is 1

$\text{fib}(2)$ is 1

$\text{fib}(n)$ is $\text{fib}(n-1) + \text{fib}(n-2)$

1, 1, 2, 3, 5, 8, 13, 21, ...

Iterative & Recursive Fibonacci

```
1  #include <stdio.h>
2
3  #define MAXFIB 24
4
5  int fibonacci(int n);
6
7  int main(void)
8  {
9
10     for(int i=1; i<=MAXFIB; i++){
11         printf("%d = %d\n", i, fibonacci(i));
12     }
13
14     return 0;
15 }
16
17
18 int fibonacci(int n)
19 {
20     if(n <= 2){
21         return 1;
22     }
23     int a = 1;
24     int b = 1;
25     int next;
26     for(int i=3; i<=n; i++){
27         next = a + b;
28         a = b;
29         b = next;
30     }
31     return b;
32 }
```

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10     for(int i=1; i<=MAXFIB; i++){
11         printf("%d = %d\n", i, fibonacci(i));
12     }
13
14     return 0;
15 }
16
17 int fibonacci(int n)
18 {
19     if(n <= 2){
20         return 1;
21     }
22     int a = 1;
23     int b = 1;
24     int next;
25     for(int i=3; i<=n; i++){
26         next = a + b;
27         a = b;
28         b = next;
29     }
30     return b;
31 }
32 }
```

Execution :

```
1 = 1
2 = 1
3 = 2
4 = 3
5 = 5
6 = 8
7 = 13
8 = 21
9 = 34
10 = 55
11 = 89
12 = 144
13 = 233
14 = 377
15 = 610
16 = 987
17 = 1597
18 = 2584
19 = 4181
20 = 6765
21 = 10946
22 = 17711
23 = 28657
24 = 46368
```

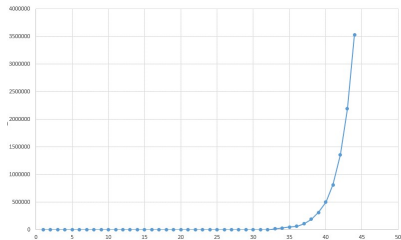
Iterative & Recursive Fibonacci

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11         printf("%d = %d\n", i, fibonacci(i));
12     }
13
14     return 0;
15
16 }
17
18 int fibonacci(int n)
19 {
20     if(n == 1) return 1;
21     if(n == 2) return 1;
22     return( fibonacci(n-1)+fibonacci(n-2));
23 }
```

Iterative & Recursive Fibonacci

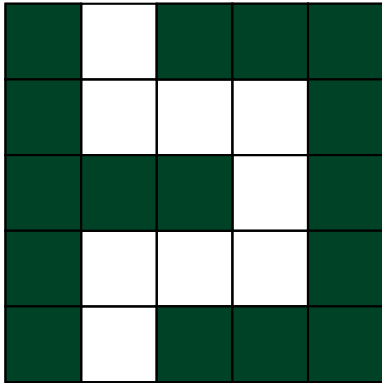
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3  #define MAXFIB 24
4
5  int fibonacci(int n);
6
7  int main(void)
8  {
9
10     for(int i=1; i<=MAXFIB; i++){
11         printf("%d = %d\n", i, fibonacci(i));
12     }
13
14     return 0;
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16
17
18 int fibonacci(int n)
19 {
20     if(n == 1) return 1;
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22     return( fibonacci(n-1)+fibonacci(n-2));
23 }
```

It's interesting to see how run-time increases as the length of the sequence is raised.



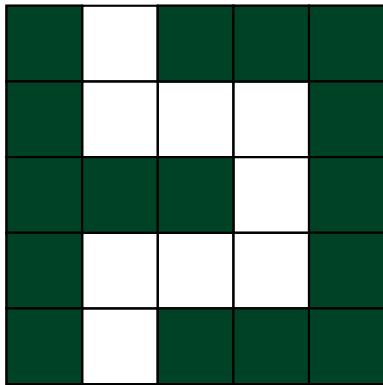
Maze Escape

The correct route through a maze can be obtained via recursive, rather than iterative, methods.



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The correct route through a maze can be obtained via recursive, rather than iterative, methods.



```
bool explore(int x, int y, char mz[YS][XS])
{
    if mz[y][x] is exit return true;

    Mark mz[y][x] so we don't return here

    if we can go up :
        if(explore(x, y+1, mz)) return true

    if we can go right :
        if(explore(x+1, y, mz)) return true

    Do left & down in a similar manner

    return false; // Failed to find route
}
```

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Execution :

ABC
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```
1 // From e.g. http://www.geeksforgeeks.org
2 #include <stdio.h>
3 #include <string.h>
4
5 #define SWAP(A,B) {char temp = *A; *A = *B; *B = temp;}
6
7 void permute(char* a, int s, int e);
8
9 int main()
10 {
11     char str[] = "ABC";
12     int n = strlen(str);
13     permute(str, 0, n-1);
14     return 0;
15 }
16
17 void permute(char* a, int s, int e)
18 {
19     if (s == e){
20         printf("%s\n", a);
21         return;
22     }
23     for (int i = s; i <= e; i++){
24         SWAP((a+s), (a+i)); // Bring one char to the front
25         permute(a, s+1, e);
26         SWAP((a+s), (a+i)); // Backtrack
27     }
28 }
```

Self-test : Power

- Raising a number to a power $n = 2^5$ is the same as multiple multiplications
 $n = 2*2*2*2*2$.

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 $n = 2*2*2*2*2$.
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```
1  /* Try to write power(a,b) to computer a^b
2  without using any maths functions other than
3  multiplication :
4  Try (1) iterative then (2) recursive
5  (3) Trick that for  $n\%2==0$ ,  $x^n = x^{(n/2)} * x^{(n/2)}$ 
6
7  */
8
9  #include <stdio.h>
10
11 int power(unsigned int a, unsigned int b);
12
13 int main(void)
14 {
15
16     int x = 2;
17     int y = 16;
18
19     printf("%d^%d = %d\n", x, y, power(x,y));
20
21 }
22
23 int power(unsigned int a, unsigned int b)
24 {
25 }
```

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```
1  #include <stdio.h>
2  #include <string.h>
3  #include <assert.h>
4
5  #define NOTFOUND -1
6  #define NUMPEOPLE 6
7  typedef struct person{
8      char* name; int age;
9  } person;
10
11 int findAge(const char* name, const person* p, int n);
12
13 int main(void)
14 {
15     person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
16                                {"Chumley", 26}, {"Dalton", 25},
17                                {"Eggson", 22}, {"Fulton", 41} };
18
19     assert(findAge("Eggson", ppl, NUMPEOPLE)==22);
20     assert(findAge("Campbell", ppl, NUMPEOPLE)==NOTFOUND);
21     return 0;
22 }
23
24 int findAge(const char* name, const person* p, int n)
25 {
26     for(int j=0; j<n; j++){
27         if(strcmp(name, p[j].name) == 0){
28             return p[j].age;
29         }
30     }
31     return NOTFOUND;
32 }
```

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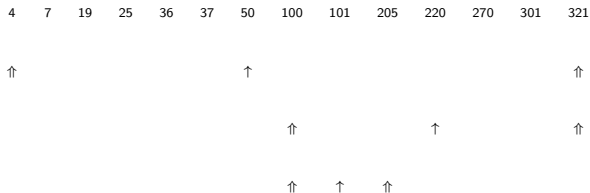
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21     return 0;
22 }
23
24 int findAge(const char* name, const person* p, int n)
25 {
26     for(int j=0; j<n; j++){
27         int m = strcmp(name, p[j].name);
28         if(m == 0) // Braces!
29             return p[j].age;
30         if(m < 0)
31             return NOTFOUND;
32     }
33     return NOTFOUND;
34 }
```

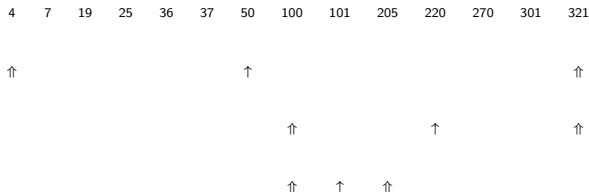
Binary Search for *101*

- Searching small lists doesn't require much computation time.



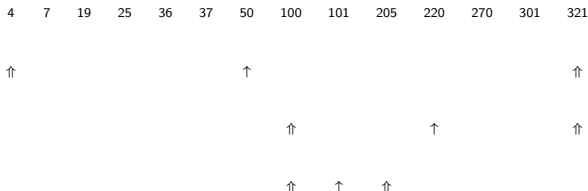
Binary Search for 101

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Binary Search for 101

- Searching small lists doesn't require much computation time.
- However, as lists get longer (e.g. phone directories), sequential searching becomes extremely inefficient.
- A binary search consists of examining the middle element of the array to see if it has the desired value. If not, then half the array may be discarded for the next search.



```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <assert.h>
4  #include <time.h>
5  #define NMBS 1000000
6
7  int bin_it(int k, const int* a, int l, int r);
8
9  int main(void)
10 {
11     int a[NMBS];
12     srand(time(NULL));
13
14     // Put even numbers into array
15     for(int i=0; i<NMBS; i++){
16         a[i] = 2*i;
17     }
18
19     // Do many searches for a random number
20     for(int i=0; i<10*NMBS; i++){
21         int n = rand()%NMBS;
22         if((n%2) == 0){
23             assert(bin_it(n, a, 0, NMBS-1) == n/2);
24         }
25         else{ // No odd numbers in this list
26             assert(bin_it(n, a, 0, NMBS-1) < 0);
27         }
28     }
29     return 0;
30 }
```

Iterative v. Recursion Binary Search

```
int bin_it(int k, const int* a, int l, int r)
{
    while(l <= r){
        int m = (l+r)/2;
        if(k == a[m]){
            return m;
        }
        else{
            if (k > a[m]){
                l = m + 1;
            }
            else{
                r = m - 1;
            }
        }
    }
    return -1;
}
```

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            }
            else{
                r = m - 1;
            }
        }
    }
    return -1;
}
```

```
int bin_rec(int k, const int* a, int l, int r)
{
    if(l > r) return -1;
    int m = (l+r)/2;
    if(k == a[m]){
        return m;
    }
    else{
        if (k > a[m]){
            return bin_rec(k, a, m+1, r);
        }
        else{
            return bin_rec(k, a, l, m-1);
        }
    }
}
```

Interpolation Search

- When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.

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```
int interp(int k, const int* a, int l, int r)
{
    int m;
    double md;

    while(l <= r){
        md = ((double)(k-a[l])/
              (double)(a[r]-a[l]))*
              (double)(r-l)
            )
            +(double)(l);
        m = 0.5 + md;
        if((m > r) || (m < l)){
            return -1;
        }
        if(k == a[m])
            return m;
        else{
            if (k > a[m]){
                l = m + 1;
            }
            else{
                r = m - 1;
            }
        }
    }
}
```

Algorithmic Complexity

```
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- Interpolation Search : **$O(\log \log n)$**
- We'll discuss the dream of a **$O(1)$** search later in "Hashing".

Binary vs. Interpolation Timing

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <assert.h>
4  #include <time.h>
5
6  int bin_it(int k, const int *a, int l, int r);
7  int bin_rec(int k, const int *a, int l, int r);
8  int interp(int k, const int *a, int l, int r);
9  int* parse_args(int argc, char* argv[], int* n, int* srch);
10
11 int main(int argc, char* argv[])
12 {
13
14     int i, n, srch;
15     int* a;
16     int (*p[3])(int k, const int*a, int l, int r) =
17         {bin_it, bin_rec, interp};
18
19     a = parse_args(argc, argv, &n, &srch);
20
21     srand(time(NULL));
22     for(i=0; i<n; i++){
23         a[i] = 2*i;
24     }
25     for(i=0; i<5000000; i++){
26         assert((*p[srch])(a[rand()%n], a, 0, n-1) >= 0);
27     }
28
29     free(a);
30     return 0;
31 }
32 }
```

Binary vs. Interpolation Timing

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26     }
27
28     free(a);
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```

Execution :

Binary Search : Iterative

n = 100000 = 0.57

n = 800000 = 0.84

n = 6400000 = 2.20

n = 51200000 = 3.87

Binary Search : Recursive

n = 100000 = 1.23

n = 800000 = 1.79

n = 6400000 = 3.20

n = 51200000 = 4.85

Interpolation

n = 100000 = 0.20

n = 800000 = 0.28

n = 6400000 = 0.50

n = 51200000 = 0.70

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```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include "general.h"
4
5  typedef struct data{
6      int i;
7      struct data* next;
8  } Data;
9
10 Data* allocateData(int i);
11 void printList(Data* l);
12
13 int main(void)
14 {
15     int i;
16     Data* start, *current;
17     start = current = NULL;
18     printf("Enter the first number: ");
19     if(scanf("%i", &i) == 1){
20         start = current = allocateData(i);
21     }
22     else{
23         on_error("Couldn't read an int");
24     }
25
26     printf("Enter more numbers: ");
27     while(scanf("%i", &i) == 1){
28         current->next = allocateData(i);
29         current = current->next;
30     }
31     printList(start);
32     // Should Free List
33     return 0;
34 }
```

Linked Lists

```
Data* allocateData(int i)
{
    Data* p;
    p = (Data*) malloc(1, sizeof(Data));
    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* l)
{
    printf("\n");
    do{
        printf("Number : %i\n", l->i);
        l = l->next;
    }while(l != NULL);
    printf("END\n");
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        l = l->next;
    }while(l != NULL);
    printf("END\n");
}
```

Searching and Recursive printing:

```
Data* inList(Data* n, int i)
{
    do{
        if(n->i==i){
            return n;
        }
        n = n->next;
    }while(n != NULL);
    return NULL;
}

void printList_r(Data* l)
{
    // Recursive Base-Case
    if(l == NULL) return;

    printf("Number: %i\n", l->i);
    printList_r(l->next);
}
```

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- The user of the ADT doesn't need to understand how the data is being stored (e.g. array vs. linked lists etc.)

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```
1  #include "../General/general.h"
2
3  typedef int colltype;
4
5  typedef struct coll coll;
6
7  #include <stdio.h>
8  #include <stdlib.h>
9  #include <assert.h>
10
11 // Create an empty coll
12 coll* coll_init(void);
13 // Add element onto top
14 void coll_add(coll* c, colltype i);
15 // Take element out
16 bool coll_remove(coll* c, colltype d);
17 // Does this exist ?
18 bool coll_isin(coll* c, colltype i);
19 // Return size of coll
20 int coll_size(coll* c);
21 // Clears all space used
22 bool coll_free(coll* c);
```

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Fixed/specific.h:

```
1  #pragma once
2
3  #define COLTYPE "Fixed"
4
5  #define FIXEDSIZE 5000
6  struct coll {
7      // Underlying array
8      colltype a[FIXEDSIZE];
9      int size;
10 };
```

Collection ADT using a Fixed-size Array

Fixed/fixed.c:

```
1  #include "../coll.h"
2  #include "specific.h"
3
4  coll* coll_init(void)
5  {
6      coll* c = (coll*) ncalloc(1, sizeof(coll));
7      c->size = 0;
8      return c;
9  }
10
11 int coll_size(coll* c)
12 {
13     if(c==NULL){
14         return 0;
15     }
16     return c->size;
17 }
18
19 bool coll_isin(coll* c, colltype d)
20 {
21     for(int i=0; i<coll_size(c); i++){
22         if(c->a[i] == d){
23             return true;
24         }
25     }
26     return false;
27 }
```

Collection ADT using a Fixed-size Array

Fixed/fixed.c:

```
1  #include "../coll.h"
2  #include "specific.h"
3
4  coll* coll_init(void)
5  {
6      coll* c = (coll*) nalloc(1, sizeof(coll));
7      c->size = 0;
8      return c;
9  }
10
11 int coll_size(coll* c)
12 {
13     if(c==NULL){
14         return 0;
15     }
16     return c->size;
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19 bool coll_isin(coll* c, colltype d)
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25     }
26     return false;
27 }
```

```
void coll_add(coll* c, colltype d)
{
    if(c){
        if(c->size >= FIXEDSIZE){
            on_error("Collection overflow");
        }
        c->a[c->size] = d;
        c->size = c->size + 1;
    }
}

bool coll_remove(coll* c, colltype d)
{
    for(int i=0; i<coll_size(c); i++){
        if(c->a[i] == d){
            // Shuffle end of array left one
            for(int j=i; j<coll_size(c); j++){
                c->a[j] = c->a[j+1];
            }
            c->size = c->size - 1;
            return true;
        }
    }
    return false;
}

bool coll_free(coll* c)
{
    free(c);
    return true;
}
```

Collection ADT via an Array (Realloc)

Realloc/specific.h:

```
1  #pragma once
2
3  #define COLTYPE "Realloc"
4
5  #define FIXEDSIZE 16
6  #define SCALEFACTOR 2
7  struct coll {
8      // Underlying array
9      colltype* a;
10     int size;
11     int capacity;
12 };
```

Collection ADT via an Array (Realloc)

Realloc/specific.h:

```
1  #pragma once
2
3  #define COLLTTYPE "Realloc"
4
5  #define FIXEDSIZE 16
6  #define SCALEFACTOR 2
7  struct coll {
8      // Underlying array
9      colltype* a;
10     int size;
11     int capacity;
12 };
```

Realloc/realloc.c:

```
1  #include "../coll.h"
2  #include "specific.h"
3
4  coll* coll_init(void)
5  {
6      coll* c = (coll*) nalloc(1, sizeof(coll));
7      c->a = (colltype*) nalloc(FIXEDSIZE, sizeof(colltype));
8      c->size = 0;
9      c->capacity = FIXEDSIZE;
10     return c;
11 }
12
13 void coll_add(coll* c, colltype d)
14 {
15     if(c){
16         if(c->size >= c->capacity){
17             c->a = (colltype*) nrealloc(c->a,
18                 sizeof(colltype)*c->capacity*SCALEFACTOR);
19             c->capacity = c->capacity*SCALEFACTOR;
20         }
21         c->a[c->size] = d;
22         c->size = c->size + 1;
23     }
```


Collection ADT via a Linked List

Linked/specific.h:

```
1  #pragma once
2
3  #define COLLTTYPE "Linked"
4
5  struct dataframe {
6      colltype i;
7      struct dataframe* next;
8  };
9  typedef struct dataframe dataframe;
10
11 struct coll {
12     // Underlying array
13     dataframe* start;
14     int size;
15 };
```

Collection ADT via a Linked List

Linked/specific.h:

```
1  #pragma once
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5  struct dataframe {
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8  };
9  typedef struct dataframe dataframe;
10
11 struct coll {
12     // Underlying array
13     dataframe* start;
14     int size;
15 };
```

Linked/linked.c:

```
#include "../coll.h"
#include "specific.h"

coll* coll_init(void)
{
    coll* c = (coll*) nalloc(1, sizeof(coll));
    return c;
}

int coll_size(coll* c)
{
    if(c==NULL){
        return 0;
    }
    return c->size;
}

bool coll_isin(coll* c, colltype d)
{
    if(c == NULL || c->start==NULL){
        return false;
    }
    dataframe* f = c->start;
    do{
        if(f->i == d){
            return true;
        }
        f = f->next;
    }while(f != NULL);
    return false;
}
```

Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
{
    if(c){
        dataframe* f = nalloc(1, sizeof(dataframe));
        f->i = d;
        f->next = c->start;
        c->start = f;
        c->size = c->size + 1;
    }
}

bool coll_free(coll* c)
{
    if(c){
        dataframe* tmp;
        dataframe* p = c->start;
        while(p!=NULL){
            tmp = p->next;
            free(p);
            p = tmp;
        }
        free(c);
    }
    return true;
}
```

Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
{
    if(c){
        dataframe* f = nalloc(1, sizeof(dataframe));
        f->i = d;
        f->next = c->start;
        c->start = f;
        c->size = c->size + 1;
    }
}

bool coll_free(coll* c)
{
    if(c){
        dataframe* tmp;
        dataframe* p = c->start;
        while(p!=NULL){
            tmp = p->next;
            free(p);
            p = tmp;
        }
        free(c);
    }
    return true;
}
```

```
bool coll_remove(coll* c, colltype d)
{
    dataframe* f1, *f2;
    if((c==NULL) || (c->start==NULL)){
        return false;
    }

    // If Front
    if(c->start->i == d){
        f1 = c->start->next;
        free(c->start);
        c->start = f1;
        c->size = c->size - 1;
        return true;
    }

    f1 = c->start;
    f2 = c->start->next;
    do{
        if(f2->i == d){
            f1->next = f2->next;
            free(f2);
            c->size = c->size - 1;
            return true;
        }
        f1 = f2;
        f2 = f1->next;
    }while(f2 != NULL);
    return false;
}
```

Collection Summary

- Any code using the ADT can be compiled against any of the implementations, e.g. the test (`testcoll.c`) code.

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- Any code using the ADT can be compiled against any of the implementations, e.g. the test (testcoll.c) code.
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 - Fixed Array : Simple to implement - can't avoid the problems of it being a fixed-size. Deletion expensive.

Collection Summary

- Any code using the ADT can be compiled against any of the implementations, e.g. the test (testcoll.c) code.
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 - fast to delete an element.

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- Any code using the ADT can be compiled against any of the implementations, e.g. the test (testcoll.c) code.
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 - Fixed Array : Simple to implement - can't avoid the problems of it being a fixed-size. Deletion expensive.
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 - Linked : Slightly fiddly implementation - fast to delete an element.

Task	Fixed Array	Realloc Array	Linked List
Insert new element	$O(1)$ at end <i>if space</i>	$O(1)$ at end <i>but realloc()</i>	$O(1)$ at front
Search for an element	$O(n)$ <i>brute force</i>	$O(n)$ <i>brute force</i>	$O(n)$ <i>brute force</i>
Search + delete	$O(n) + O(n)$ <i>move left</i>	$O(n) + O(n)$ <i>move left</i>	$O(n) + O(1)$ <i>delete 'free'</i>

- If we had ordered our ADT (ie. the elements were sorted), then the searches could be via a binary / interpolation search, leading to $O(\log n)$ or $O(\log \log n)$ search times.

ADTs Making Coding Simpler

That Linked List code from the previous
Chapter again:

ADTs Making Coding Simpler

That Linked List code from the previous Chapter again:

```
1  #include "coll.h"
2  #include "Fixed/specific.h"
3
4  int main(void)
5  {
6      coll* c;
7      int i;
8
9      printf("Please type some numbers :");
10     c = coll_init();
11     while(scanf("%i", &i) == 1){
12         coll_add(c, i);
13     }
14     // Do print etc.
15     coll_free(c);
16     return 0;
17 }
```

Table of Contents

N : Recursion

O : Algorithms I - Search

P : Linked Data Structures

Q : ADTs - Collection

R : ADTs - Stacks

S : ADTs - Queues

T : ADTs - Trees

U : ADTs - Hashing

V : ADTs - Graphs

W : Algorithms II - Sort / Strings / Graphs

X : Algorithms III - Parsing and Grammars

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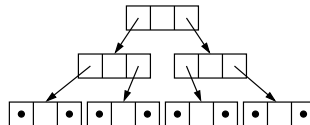
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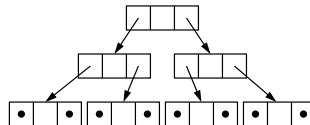
Binary Trees:



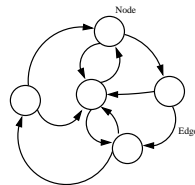
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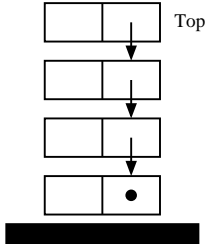


Unidirectional Graph:



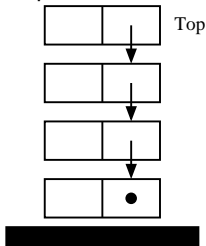
Stacks

The push-down stack:

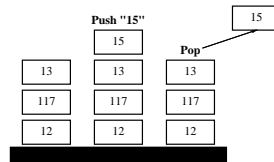


Stacks

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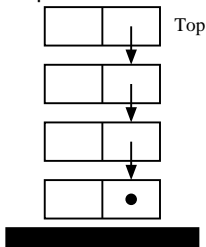
LIFO (Last in, First out):



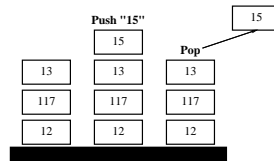
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Stacks

The push-down stack:



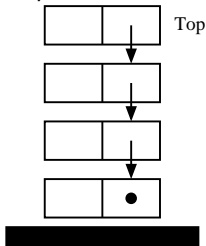
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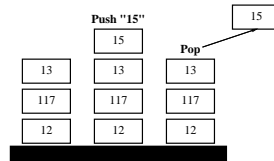
- Operations include push and pop.
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Stacks

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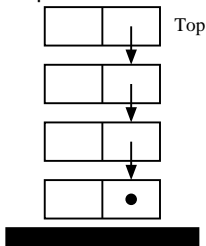


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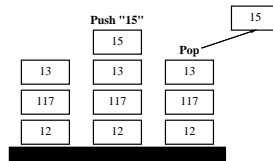


- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.

The push-down stack:



LIFO (Last in, First out):



- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.
- But, once again, we are faced with the question : How best to implement such a data type ?

ADT:Stacks Arrays (Realloc) I

stack.h:

```
1  #pragma once
2
3  #include "../General/general.h"
4
5  typedef int stacktype;
6
7  typedef struct stack stack;
8
9  #include <stdio.h>
10 #include <stdlib.h>
11 #include <assert.h>
12 #include <string.h>
13
14 /* Create an empty stack */
15 stack* stack_init(void);
16 /* Add element to top */
17 void stack_push(stack* s, stacktype i);
18 /* Take element from top */
19 bool stack_pop(stack* s, stacktype* d);
20 /* Clears all space used */
21 bool stack_free(stack* s);
22
23 /* Optional? */
24
25 /* Copy top element into d (but don't pop it) */
26 bool stack_peek(stack* s, stacktype* d);
27 /* Make a string version - keep .dot in mind */
28 void stack_tostring(stack* s, char* str);
```

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9  #include <stdio.h>
10 #include <stdlib.h>
11 #include <assert.h>
12 #include <string.h>
13
14 /* Create an empty stack */
15 stack* stack_init(void);
16 /* Add element to top */
17 void stack_push(stack* s, stacktype i);
18 /* Take element from top */
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```

Realloc/specific.h:

```
1  #pragma once
2
3  #define FORMATSTR "%i"
4  #define ELEMSIZE 20
5
6  #define STACKTYPE "Realloc"
7
8  #define FIXEDSIZE 16
9  #define SCALEFACTOR 2
10
11 struct stack {
12     /* Underlying array */
13     stacktype* a;
14     int size;
15     int capacity;
16 };
```

ADT:Stacks Arrays (Realloc) II

Realloc/realloc.c

```
1  #include "../stack.h"
2  #include "specific.h"
3
4  #define DOTFILE 5000
5
6  stack* stack_init(void)
7  {
8      stack *s = (stack*) ncalloc(1, sizeof(stack));
9      /* Some implementations would allow you to pass
10       a hint about the initial size of the stack */
11      s->a = (stacktype*) ncalloc(FIXEDSIZE, sizeof(stacktype));
12      s->size = 0;
13      s->capacity = FIXEDSIZE;
14      return s;
15  }
16
17  void stack_push(stack* s, stacktype d)
18  {
19      if(s==NULL){
20          return;
21      }
22      if(s->size >= s->capacity){
23          s->a = (stacktype*) nrealloc(s->a,
24                                     sizeof(stacktype)*s->capacity*SCALEFACTOR);
25          s->capacity = s->capacity*SCALEFACTOR;
26      }
27      s->a[s->size] = d;
28      s->size = s->size + 1;
29  }
```


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24                                     sizeof(stacktype)*s->capacity*SCALEFACTOR);
25          s->capacity = s->capacity*SCALEFACTOR;
26      }
27      s->a[s->size] = d;
28      s->size = s->size + 1;
29  }
```

```
1  bool stack_pop(stack* s, stacktype* d)
2  {
3      if((s == NULL) || (s->size < 1)){
4          return false;
5      }
6      s->size = s->size - 1;
7      *d = s->a[s->size];
8      return true;
9  }
10
11  bool stack_peek(stack* s, stacktype* d)
12  {
13      if((s==NULL) || (s->size <= 0)){
14          /* Stack is Empty */
15          return false;
16      }
17      *d = s->a[s->size - 1];
18      return true;
19  }
```

ADT:Stacks Arrays (Realloc) III

Realloc/realloc.c

```
1 void stack_tostring(stack* s, char* str)
2 {
3     char tmp[ELEMSIZE];
4     str[0] = '\0';
5     if((s==NULL) || (s->size <1)){
6         return;
7     }
8     for(int i=s->size-1; i>=0; i--){
9         sprintf(tmp, FORMATSTR, s->a[i]);
10        strcat(str, tmp);
11        strcat(str, "|");
12    }
13    str[strlen(str)-1] = '\0';
14 }
15
16 bool stack_free(stack* s)
17 {
18     if(s==NULL){
19         return true;
20     }
21     free(s->a);
22     free(s);
23     return true;
24 }
```

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```

- We need a thorough testing program
teststack.c

ADT:Stacks Arrays (Realloc) III

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17 {
18     if(s==NULL){
19         return true;
20     }
21     free(s->a);
22     free(s);
23     return true;
24 }
```

- We need a thorough testing program teststack.c
- See also revstr.c : a version of the string reverse code (for which we already seen an iterative (in-place) and a recursive solution).

ADT:Stacks Linked I

Linked/specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%i"
4  #define ELEMSIZE 20
5  #define STACKTYPE "Linked"
6
7  struct dataframe {
8      stacktype i;
9      struct dataframe* next;
10 };
11 typedef struct dataframe dataframe;
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13 struct stack {
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```

Linked/linked.c

```
1  #include "../stack.h"
2  #include "specific.h"
3
4  #define DOTFILE 5000
5
6  stack* stack_init(void)
7  {
8      stack* s = (stack*) nalloc(1, sizeof(stack));
9      return s;
10 }
11
12 void stack_push(stack* s, stacktype d)
13 {
14     if(s){
15         dataframe* f = nalloc(1, sizeof(dataframe));
16         f->i = d;
17         f->next = s->start;
18         s->start = f;
19         s->size = s->size + 1;
20     }
21 }
```

ADT:Stacks Linked II

```
1  bool stack_pop(stack* s, stacktype* d)
2  {
3      if((s==NULL) || (s->start==NULL)){
4          return false;
5      }
6
7      dataframe* f = s->start->next;
8      *d = s->start->i;
9      free(s->start);
10     s->start = f;
11     s->size = s->size - 1;
12     return true;
13 }
14
15 bool stack_peek(stack* s, stacktype* d)
16 {
17     if((s==NULL) || (s->start==NULL)){
18         return false;
19     }
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21     return true;
22 }
```

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1  void stack_tostring(stack* s, char* str)
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3      char tmp[ELEMSIZE];
4      str[0] = '\0';
5      if((s==NULL) || (s->size < 1)){
6          return;
7      }
8      dataframe* p = s->start;
9      while(p){
10         sprintf(tmp, FORMATSIR, p->i);
11         strcat(str, tmp);
12         strcat(str, "|");
13         p = p->next;
14     }
15     str[strlen(str)-1] = '\0';
16 }
17
18 bool stack_free(stack* s)
19 {
20     if(s){
21         dataframe* p = s->start;
22         while(p!=NULL){
23             dataframe* tmp = p->next;
24             free(p);
25             p = tmp;
26         }
27         free(s);
28     }
29     return true;
30 }
```


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ADTs : Queues

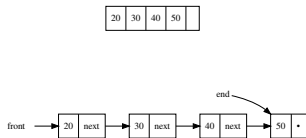
FIFO (First in, First out):



- Intuitively more “useful” than a stack.

ADTs : Queues

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- Intuitively more “useful” than a stack.
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- Operations include enqueue, dequeue and size.

queue.h

```
1  #pragma once
2
3  #include "../General/general.h"
4
5  typedef int queue_type;
6
7  typedef struct queue queue;
8
9  #include <stdio.h>
10 #include <stdlib.h>
11 #include <string.h>
12 #include <assert.h>
13
14 /* Create an empty queue */
15 queue* queue_init(void);
16 /* Add element on end */
17 void queue_enqueue(queue* q, queue_type v);
18 /* Take element off front */
19 bool queue_dequeue(queue* q, queue_type* d);
20 /* Return size of queue */
21 int queue_size(queue* q);
22 /* Clears all space used */
23 bool queue_free(queue* q);
24
25 /* Helps with visualisation & testing */
26 void queue_tostring(queue* q, char* str);
```

ADTs : Queues (Fixed) I

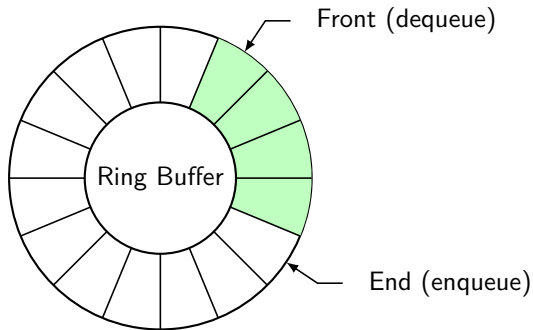
specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
4  #define ELEMSIZE 20
5
6  #define QUEUETYPE "Fixed"
7
8  #define BOUNDED 5000
9
10 struct queue {
11     /* Underlying array */
12     queuetype a[BOUNDED];
13     int front;
14     int end;
15 };
16
17 #define DOTFILE 5000
```

ADTs : Queues (Fixed) I

specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
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10 struct queue {
11     /* Underlying array */
12     queuetype a[BOUNDED];
13     int front;
14     int end;
15 };
16
17 #define DOTFILE 5000
```



ADTs : Queues (Fixed) II

fixed.c

```
1  #include "../queue.h"
2  #include "specific.h"
3
4  void __inc(queuetype* p);
5
6  queue* queue_init(void)
7  {
8      queue* q = (queue*) ncalloc(1, sizeof(queue));
9      return q;
10 }
11
12
13 void queue_enqueue(queue* q, queuetype d)
14 {
15     if(q){
16         q->a[q->end] = d;
17         __inc(&q->end);
18         if(q->end == q->front){
19             on_error("Queue too large");
20         }
21     }
22 }
```

ADTs : Queues (Fixed) II

fixed.c

```
1  #include "../queue.h"
2  #include "specific.h"
3
4  void __inc(queuetype* p);
5
6  queue* queue_init(void)
7  {
8      queue* q = (queue*) ncalloc(1, sizeof(queue));
9      return q;
10 }
11
12
13 void queue_enqueue(queue* q, queuetype d)
14 {
15     if(q){
16         q->a[q->end] = d;
17         __inc(&q->end);
18         if(q->end == q->front){
19             on_error("Queue too large");
20         }
21     }
22 }
```

```
1  bool queue_dequeue(queue* q, queuetype* d)
2  {
3      if((q==NULL) || (q->front==q->end)){
4          return false;
5      }
6      *d = q->a[q->front];
7      __inc(&q->front);
8      return true;
9  }
10
11 void queue_tostring(queue* q, char* str)
12 {
13     char tmp[ELEMSIZE];
14     str[0] = '\0';
15     if((q==NULL) || (queue_size(q)==0)){
16         return;
17     }
18     for(int i=q->front; i != q->end;){
19         sprintf(tmp, FORMATSTR, q->a[i]);
20         strcat(str, tmp);
21         strcat(str, "|");
22         __inc(&i);
23     }
24     str[strlen(str)-1] = '\0';
25 }
```

ADTs : Queues (Fixed) III

```
1  int queue_size(queue* q)
2  {
3      if(q==NULL){
4          return 0;
5      }
6      if(q->end >= q->front){
7          return q->end-q->front;
8      }
9      return q->end + BOUNDED - q->front;
10 }
11
12 bool queue_free(queue* q)
13 {
14     free(q);
15     return true;
16 }
17
18 void _inc(queuetype* p)
19 {
20     *p = (*p + 1) % BOUNDED;
21 }
```

ADTs : Queues (Fixed) III

```
1  int queue_size(queue* q)
2  {
3      if(q==NULL){
4          return 0;
5      }
6      if(q->end >= q->front){
7          return q->end - q->front;
8      }
9      return q->end + BOUNDED - q->front;
10 }
11
12 bool queue_free(queue* q)
13 {
14     free(q);
15     return true;
16 }
17
18 void __inc(queuetype* p)
19 {
20     *p = (*p + 1) % BOUNDED;
21 }
```

- We need a thorough testing program

ADTs : Queues (Fixed) III

```
1  int queue_size(queue* q)
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3      if(q==NULL){
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5      }
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7          return q->end - q->front;
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10 }
11
12 bool queue_free(queue* q)
13 {
14     free(q);
15     return true;
16 }
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18 void __inc(queuetype* p)
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```

- We need a thorough testing program
- We'll see queues again for traversing trees

ADTs : Queues (Fixed) III

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1  int queue_size(queue* q)
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12 bool queue_free(queue* q)
13 {
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15     return true;
16 }
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18 void __inc(queuetype* p)
19 {
20     *p = (*p + 1) % BOUNDED;
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```

- We need a thorough testing program
- We'll see queues again for traversing trees
- Simulating a (slow) printer

ADTs : Queues (Linked) I

specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
4  #define ELEMSIZE 20
5
6  #define QUEUETYPE "Linked"
7
8  struct dataframe {
9      queuetype i;
10     struct dataframe* next;
11 };
12 typedef struct dataframe dataframe;
13
14 struct queue {
15     /* Underlying array */
16     dataframe* front;
17     dataframe* end;
18     int size;
19 };
```

ADTs : Queues (Linked) I

specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
4  #define ELEMSIZE 20
5
6  #define QUEUETYPE "Linked"
7
8  struct dataframe {
9      queuetype i;
10     struct dataframe* next;
11 };
12 typedef struct dataframe dataframe;
13
14 struct queue {
15     /* Underlying array */
16     dataframe* front;
17     dataframe* end;
18     int size;
19 };
```

linked.c

```
1  #include "../queue.h"
2  #include "specific.h"
3
4  queue* queue_init(void)
5  {
6      queue* q = (queue*) nalloc(1, sizeof(queue));
7      return q;
8  }
9
10 void queue_enqueue(queue* q, queuetype d)
11 {
12     dataframe* f;
13     if(q == NULL){
14         return;
15     }
16
17     /* Copy the data */
18     f = nalloc(1, sizeof(dataframe));
19     f->i = d;
20
21     /* 1st one */
22     if(q->front == NULL){
23         q->front = f;
24         q->end = f;
25         q->size = q->size + 1;
26         return;
27     }
28     /* Not 1st */
29     q->end->next = f;
30     q->end = f;
31     q->size = q->size + 1;
32 }
```


ADTs : Queues (Linked) II

```
1  bool queue_dequeue(queue* q, queue_t* d)
2  {
3      dataframe* f;
4      if((q==NULL) || (q->front==NULL) || (q->end==NULL)){
5          return false;
6      }
7      f = q->front->next;
8      *d = q->front->i;
9      free(q->front);
10     q->front = f;
11     q->size = q->size - 1;
12     return true;
13 }
14
15 bool queue_free(queue* q)
16 {
17     if(q){
18         dataframe* tmp;
19         dataframe* p = q->front;
20         while(p!=NULL){
21             tmp = p->next;
22             free(p);
23             p = tmp;
24         }
25         free(q);
26     }
27     return true;
28 }
```

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9      free(q->front);
10     q->front = f;
11     q->size = q->size - 1;
12     return true;
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23             p = tmp;
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26     }
27     return true;
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```

```
1  void queue_tostring(queue* q, char* str)
2  {
3      dataframe *p;
4      char tmp[ELEMSIZE];
5      str[0] = '\0';
6      if((q==NULL) || (q->front == NULL)){
7          return;
8      }
9      p = q->front;
10     while(p){
11         sprintf(tmp, FORMATSTR, p->i);
12         strcat(str, tmp);
13         strcat(str, "|");
14         p = p->next;
15     }
16     str[strlen(str)-1] = '\0';
17 }
18
19 int queue_size(queue* q)
20 {
21     if((q==NULL) || (q->front==NULL)){
22
23         return 0;
24     }
25     return q->size;
26 }
```

Detour : Graphviz

- There exists a nice package, called Graphviz:

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dot -Tpdf -o graphviz.pdf examp1.dot
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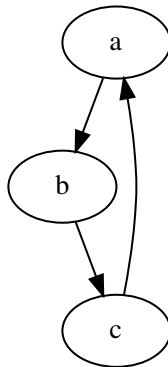


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ADTs : Binary Trees

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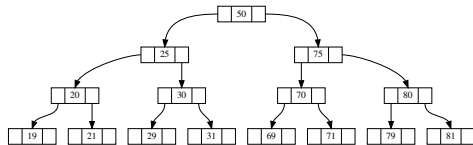
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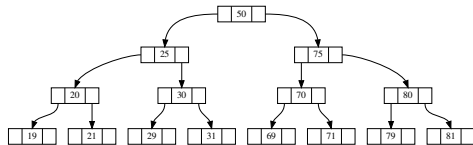
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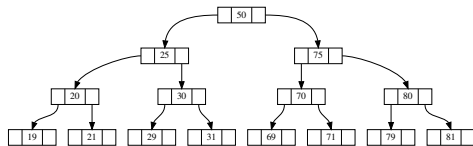
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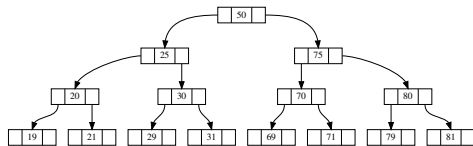
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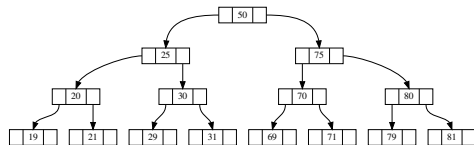
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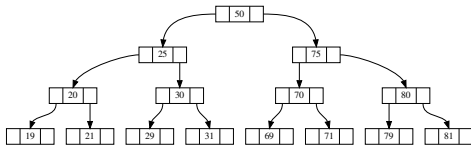
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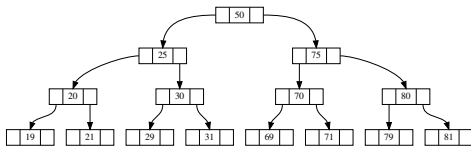
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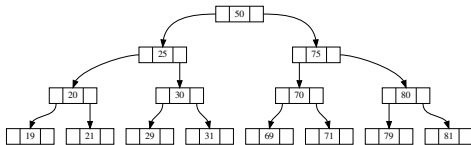
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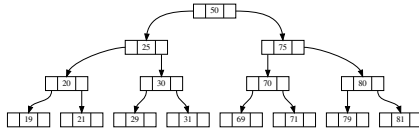
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- Can refer to left and right children
- In a tree, there is only one path from the root to any child
- A node with no children is a leaf
- Most trees need to be created dynamically
- Empty subtrees are set to NULL

Binary Search Trees

In a binary search tree the left-hand tree of a parent contains all keys less than the parent node, and the right-hand side all the keys greater than the parent node.



Binary Search Trees

In a binary search tree the left-hand tree of a parent contains all keys less than the parent node, and the right-hand side all the keys greater than the parent node.



bst.h

```
1  #include "../General/general.h"
2  #include "../Queue/queue.h"
3
4  #include <stdio.h>
5  #include <stdlib.h>
6  #include <assert.h>
7
8  bst* bst_init(void);
9
10 /* Insert 1 item into the tree */
11 bool bst_insert(bst* b, treetype d);
12
13 /* Return number of nodes in tree */
14 int bst_size(bst* b);
15
16 /* Whether the data d is stored in the tree */
17 bool bst_isin(bst* b, treetype d);
18
19 /* Bulk insert n items from an array a into an initialised tree */
20 bool bst_insertarray(bst* b, treetype* a, int n);
21
22 /* Clear all memory associated with tree, & set pointer to NULL */
23 bool bst_free(bst* b);
24
25 /* Optional ? */
26
27 char* bst_preorder(bst* b);
28 void bst_printlevel(bst* b);
29 /* Create string with tree as ((head)(left)(right)) */
30 char* bst_printlisp(bst* b);
31 /* Use Graphviz via a .dot file */
32 void bst_todot(bst* b, char* dotname);
```

Binary Search Trees : Linked I

specific.h

```
1  #include <string.h>
2
3  typedef int treetype;
4  #define FORMATSIR "%i"
5  #define ELEMSIZE 20
6  #define BSTTYPE "Linked"
7
8  struct dataframe {
9      treetype d;
10     struct dataframe* left;
11     struct dataframe* right;
12 };
13 typedef struct dataframe dataframe;
14
15 struct bst {
16     dataframe* top;
17     /* Data element size, in bytes */
18 };
19 typedef struct bst bst;
```


Binary Search Trees : Linked I

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14
15 struct bst {
16     dataframe* top;
17     /* Data element size, in bytes */
18 };
19 typedef struct bst bst;
```

```
/* Based on geekforgeeks.org */
dataframe* __insert(dataframe* t, treetype d)
{
    dataframe* f;
    /* If the tree is empty, return a new frame */
    if (t == NULL){
        f = ncalloc(sizeof(dataframe), 1);
        f->d = d;
        return f;
    }
    /* Otherwise, recurs down the tree */
    if (d < t->d){
        t->left = __insert(t->left, d);
    }
    else if(d > t->d){
        t->right = __insert(t->right, d);
    }
    /* return the (unchanged) dataframe pointer */
    return t;
}
```

Binary Search Trees : Linked II

```
bool __isin(dataframe* t, treetype d)
{
    if(t==NULL){
        return false;
    }
    if(t->d == d){
        return true;
    }
    if(d < t->d){
        return __isin(t->left, d);
    }
    else{
        return __isin(t->right, d);
    }
    return false;
}
```

Binary Search Trees : Linked II

```
bool __isin(dataframe* t, treetype d)
{
    if(t==NULL){
        return false;
    }
    if(t->d == d){
        return true;
    }
    if(d < t->d){
        return __isin(t->left, d);
    }
    else{
        return __isin(t->right, d);
    }
    return false;
}
```

```
char* __printlisp(dataframe* t)
{
    char tmp[ELEMSIZE];
    char *s1, *s2, *p;

    if(t==NULL){
        /* \0 string */
        p = ncalloc(1,1);
        return p;
    }
    sprintf(tmp, FORMATSTR, t->d);
    s1 = __printlisp(t->left);
    s2 = __printlisp(t->right);
    p = ncalloc(strlen(s1)+strlen(s2)+strlen(tmp)+
        strlen(" ( ) "), 1);
    sprintf(p, "%s(%s)(%s)", tmp, s1, s2);
    free(s1);
    free(s2);
    return p;
}
```

Binary Trees using Arrays ?

- Don't rush to assume a linked data structure must be used to implement trees.

Binary Trees using Arrays ?

- Don't rush to assume a linked data structure must be used to implement trees.
- You could use 1 cell of an array for the first node, the next two cells for its children, the next 4 cells for their children and so on.

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Counting from cell 1, for a tree with n nodes:

To find	Use	Iff
The root	$A[1]$	A is nonempty
The left child of $A[i]$	$A[2i]$	$2i \leq n$
The parent of $A[i]$	$A[i/2]$	$i > 1$
Is $A[i]$ a leaf ?	True	$2i > n$

Binary Search Trees : Realloc

specific.h

```
1  #include <stdbool.h>
2
3  typedef int treetype;
4  #define FORMATSTR "%i"
5  #define ELEMSIZE 20
6  #define BSTTYPE "Realloc"
7
8  // Probably (2^n) -1
9  #define INITSIZE 31
10 #define SCALEFACTOR 2
11
12 struct dataframe {
13     treetype d;
14     bool isValid;
15 };
16 typedef struct dataframe dataframe;
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18 struct bst {
19     dataframe* a;
20     int capacity;
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```

Using a queue for Level-Order traversal:

```
void bst_printlevel(bst* b)
{
    treetype n;
    if((b==NULL) || (! __isvalid(b, 0))){
        return;
    }
    /* Make a queue of cell indices */
    queue* q = queue_init();
    queue_enqueue(q, 0);
    while(queue_dequeue(q, &n) && __isvalid(b, (int)n)){
        printf(FORMATSTR, b->a[n].d);
        putchar(' ');
        queue_enqueue(q, __leftchild((int)n));
        queue_enqueue(q, __rightchild((int)n));
    }
}
```

Binary Search Trees : Complexity

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- The tree search performs best when well balanced trees are formed.
- Large body of literature about creating & re-balancing trees - Red-Black trees, Tries, 2-3 trees, AVL trees etc.

Binary Trees : Huffman Compression I

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- To encode the string "BABBAGE":

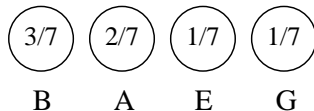
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- Keep a list of characters, ordered by their frequency

Binary Trees : Huffman Compression II

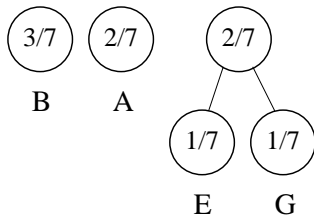
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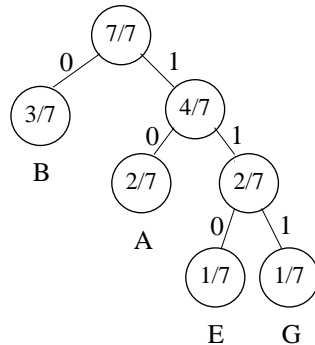
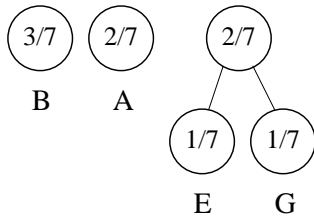
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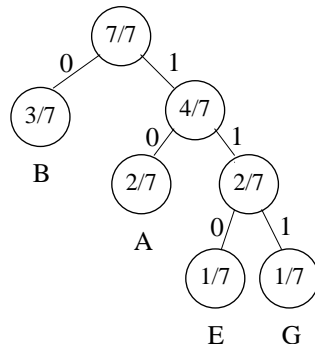
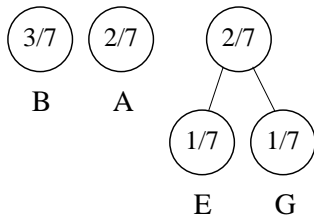
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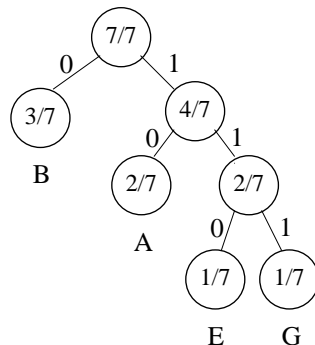
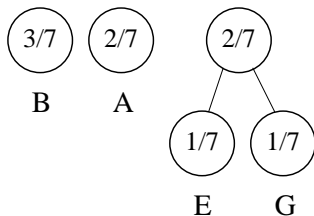
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- Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :



- A = 10, B = 0, E = 110, G = 111
- String stored using 13 bits.

Table of Contents

N : Recursion

O : Algorithms I - Search

P : Linked Data Structures

Q : ADTs - Collection

R : ADTs - Stacks

S : ADTs - Queues

T : ADTs - Trees

U : ADTs - Hashing

V : ADTs - Graphs

W : Algorithms II - Sort / Strings / Graphs

X : Algorithms III - Parsing and Grammars

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- A black-box into which we insert a key (e.g. NI number) and out pops an array index.
- As an example lets use an array of size 11 to store some airport codes, e.g. PHL, DCA, FRA.

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- Inserting "PHL", "ORY" and "GCM":

	0
	1
	2
	3
PHL	4
	5
GCM	6
	7
ORY	8
	9
	10

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	2
	3
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	5
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	7
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	9
	10

- However, inserting "HKG" causes a collision.

	0
	1
	2
	3
PHL	4
GCM	5
	6
ORY	7
	8
	9
	10

HKG ?

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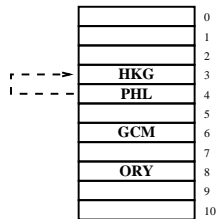
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- The function is chosen so that two keys which collide at the same address will have different probe decrements, e.g. :

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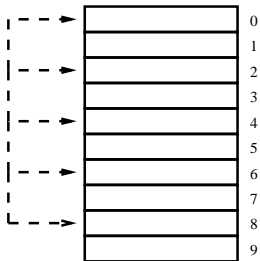
- Although "PHL" and "HKG" share the same primary hash value of $h(K) = 4$, they have different probe decrements:

$$p(\text{"PHL"}) = 4$$

$$p(\text{"HKG"}) = 3$$

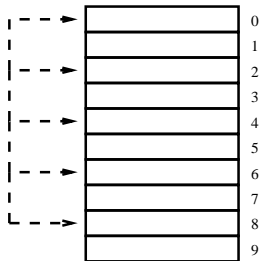
ADTs : Primes and Chaining

- If the size of our array, M , was even and the probe decrement was chosen to be 2, then only half of the locations could be probed.



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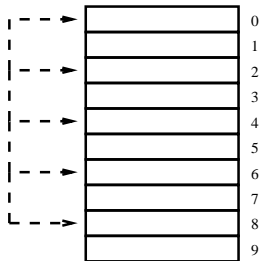
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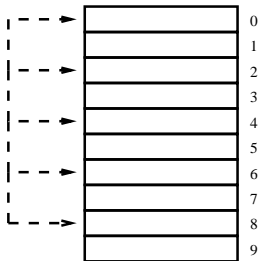
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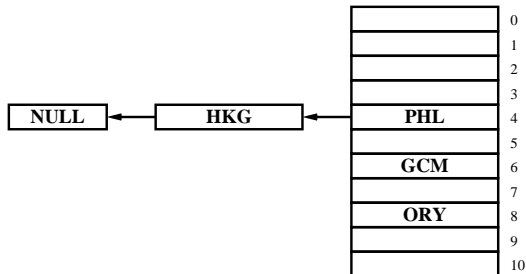
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Open-addressing is not the only method of collision reduction. Another common one is separate chaining.



ADTs : A Practical Hash Function

```
1  #include <stdio.h>
2
3  int hash(unsigned int sz, char *s);
4
5  int main(void)
6  {
7
8      char str[] = "Hello World!";
9      // Hash modulus 7919
10     printf("%d\n", hash(7919, str));
11     return 0;
12 }
13
14 /*
15  Modified Bernstein hashing
16  5381 & 33 are magic numbers required by the algorithm
17  */
18 int hash(unsigned int sz, char *s)
19 {
20     unsigned long hash = 5381;
21     int c;
22     while((c = (*s++))) {
23         hash = 33 * hash ^ c;
24     }
25     return (int)(hash%sz);
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Execution :

5479

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Execution :

5479

Has similarities to the implementation of rand() :

```
#include <stdio.h>

int rand_r(unsigned int* seed);

int main(void)
{
    unsigned int seed = 0;
    printf("%d\n", rand_r(&seed));
    return 0;
}

/* This algorithm is mentioned in the ISO C standard,
   here extended for 32 bits. */
int rand_r(unsigned int* seed)
{
    unsigned int next = *seed;
    int result;

    next *= 1103515245;
    next += 12345;
    result = (unsigned int) (next / 65536) % 2048;

    next *= 1103515245;
    next += 12345;
    result <= 10;
}
```

Execution :

1012484

ADTs : Cuckoo Hashing

- We have two tables, each with their **own** hash function.

Empty: copied farandoles into table 0(4)
Empty: copied bronzine into table 0(12)
Empty: copied auscultatory into table 0(5)
Empty: copied bifer into table 0(13)
Empty: copied steepgrass into table 0(6)
Empty: copied prevised into table 0(7)
Empty: copied oomph into table 0(8)
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Empty: copied auscultatory into table 1(10)
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Empty: copied megalodon into table 0(11)
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Table getting full -> rehashed old sz =16

ADTs : Cuckoo Hashing

- We have two tables, each with their **own** hash function.
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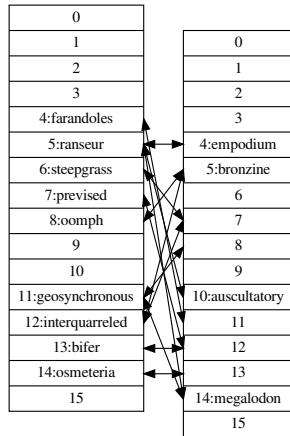


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N : Recursion

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T : ADTs - Trees

U : ADTs - Hashing

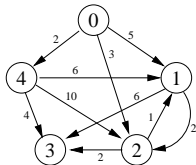
V : ADTs - Graphs

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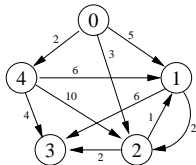
ADTs : Graphs

- A graph, G , consists of a set of vertices (nodes), V , together with a set of edges (links), E , each of which connects two vertices.



ADTs : Graphs

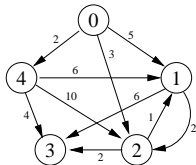
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Vertices are joined to adjacent vertices by these edges.

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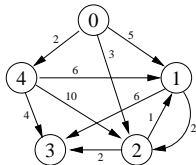
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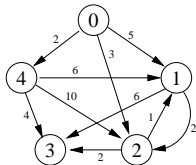
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graph.h (partial)

```
#include <limits.h>
#define INF (INT_MAX)

/* Initialise an empty graph */
graph* graph_init(void);

/* Add new vertex */
int graph_addVert(graph* g, char* label);

/* Add new edge between two Vertices */
bool graph_addEdge(graph* g, int from,
                   int to, edge weight);

/* Returns NO_VERT if not already a vert
   else 0 ... (size-1) */
int graph_getVertNum(graph* g, char* label);

/* Returns label of vertex v */
char* graph_getLabel(graph* g, int v);

/* Returns edge weight - if none = INF */
edge graph_getEdgeWeight(graph* g, int from, int to);

/* Number of verts */
int graph_numVerts(graph* b);

/* Output edge weights e.g. "0->1 200 2->1 100" */
void graph_tostring(graph* g, char* str);

/* Clear all memory associated with graph */
bool graph_free(graph* g);
```

Graph ADT : 2D Realloc I

The graph type could be implemented in a large number of different ways.

- As two sets, one for vertices, one for edges. We haven't looked at an implementation for sets, but one could use lists.

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	0	1	2	3	4
0	0	5	3	∞	2
1	∞	0	2	6	∞
2	∞	1	0	2	∞
3	∞	∞	∞	0	∞
4	∞	6	10	4	0

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3	∞	∞	∞	0	∞
4	∞	6	10	4	0

specific.h

```
1  #define GRAPHTYPE "Realloc"
2
3  #define INITSIZE 8
4  #define SCALEFACTOR 2
5
6  #define TMPSTR 1000
7
8  #define NO_VERT -1
9
10 typedef unsigned int edge;
11
12 struct graph {
13     edge** adjMat;
14     char** labels;
15     /* Actual number of vertices */
16     int size;
17     /* Max vertices before realloc() */
18     int capacity;
19 };
20 typedef struct graph graph;
```


2D Realloc II

```
graph* graph_init(void)
{
    graph* g = (graph*) ncalloc(sizeof(graph), 1);
    int h = INITSIZE;
    int w = h;
    g->capacity = h;
    g->adjMat = (edge**) n2dcalloc(h, w, sizeof(edge));
    g->labels = (char**) n2dcalloc(h, MAXLABEL+1, sizeof(char));
    for(int j=0; j<h; j++){
        for(int i=0; i<w; i++){
            /* It's not clear if weight[j][i] should be 0 or INF */
            g->adjMat[j][i] = INF;
        }
    }
    return g;
}

edge graph_getEdgeWeight(graph* g, int from, int to)
{
    if((g==NULL) || (from >= g->size) || (to >= g->size)){
        return INF;
    }
    return g->adjMat[from][to];
}

int graph_numVerts(graph* g)
{
    if(g==NULL){
        return 0;
    }
    return g->size;
}
```

2D Realloc II

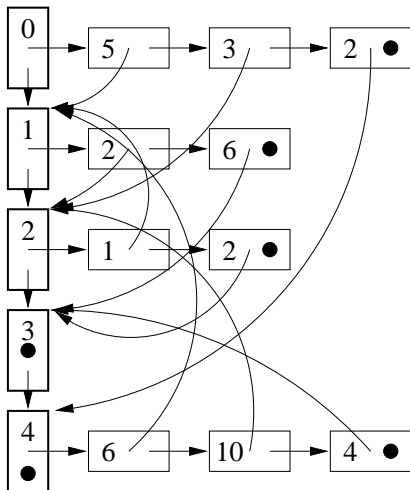
```
graph* graph_init(void)
{
    graph* g = (graph*) ncalloc(sizeof(graph), 1);
    int h = INITSIZE;
    int w = h;
    g->capacity = h;
    g->adjMat = (edge**) n2dcalloc(h, w, sizeof(edge));
    g->labels = (char**) n2dcalloc(h, MAXLABEL+1, sizeof(char));
    for(int j=0; j<h; j++){
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        }
    }
    return g;
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edge graph_getEdgeWeight(graph* g, int from, int to)
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    if((g==NULL) || (from >= g->size) || (to >= g->size)){
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    }
    return g->adjMat[from][to];
}

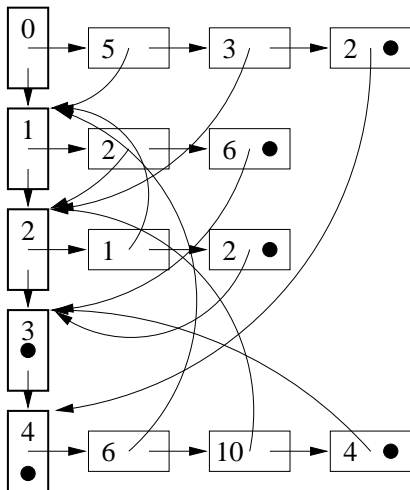
int graph_numVerts(graph* g)
{
    if(g==NULL){
        return 0;
    }
    return g->size;
}
```

```
int graph_addVert(graph* g, char* label)
{
    if(g==NULL){
        return NO_VERT;
    }
    if(graph_getVertNum(g, label) != NO_VERT){
        return NO_VERT;
    }
    /* Resize */
    if(g->size >= g->capacity){
        g->adjMat = (edge**) n2drecalloc((void**)g->adjMat,
            g->capacity, g->capacity*SCALEFACTOR,
            g->capacity, g->capacity*SCALEFACTOR,
            sizeof(edge));
        g->labels = (char**) n2drecalloc((void**)g->labels,
            g->capacity, g->capacity*SCALEFACTOR,
            MAXLABEL+1, MAXLABEL+1, 1);
        for(int j=0; j<g->capacity*SCALEFACTOR; j++){
            for(int i=0; i<g->capacity*SCALEFACTOR; i++){
                if((i>=g->capacity) || (j>=g->capacity)){
                    g->adjMat[j][i] = INF;
                }
            }
        }
        g->capacity = g->capacity*SCALEFACTOR;
    }
    strcpy(g->labels[g->size], label);
    g->size = g->size + 1;
    return g->size - 1;
}
```

Graph ADT - Linked



Graph ADT - Linked



specific.h

```
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3  #define INITSIZE 8
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6  #define TMPSTR 1000
7
8  #define NO_VERT -1
9
10 typedef unsigned int edge;
11
12 struct vertex {
13     char* label;
14     struct vertex* nextv;
15     void* firste;
16     int num;
17 };
18 typedef struct vertex vertex;
19
20 struct edge {
21     edge weight;
22     vertex* v;
23     struct edge* nexte;
24 };
25 typedef struct edge edgel;
26
27 struct graph {
28     vertex* firstv;
29     vertex* endv;
30     int size;
31 };
32 typedef struct graph graph;
```

Linked II

```
graph* graph_init(void)
{
    graph* g = (graph*) ncalloc(1, sizeof(graph));
    return g;
}

edge graph_getEdgeWeight(graph* g, int from, int to)
{
    if((g==NULL) || (from >= g->size) || (to >= g->size)){
        return INF;
    }
    vertex* v = g->firstv;
    for(int i=0; i<from; i++){
        v = v->nextv;
    }
    if((v==NULL) || (v->num != from)){
        return INF;
    }
    edge* e = v->firste;
    while(e != NULL){
        if(e->v->num == to){
            return e->weight;
        }
        e = e->nexte;
    }
    return INF;
}
```

Linked II

```
graph* graph_init(void)
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    graph* g = (graph*) nalloc(1, sizeof(graph));
    return g;
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    if((g==NULL) || (from >= g->size) || (to >= g->size)){
        return INF;
    }
    vertex* v = g->firstv;
    for(int i=0; i<from; i++){
        v = v->nextv;
    }
    if((v==NULL) || (v->num != from)){
        return INF;
    }
    edge* e = v->firste;
    while(e != NULL){
        if(e->v->num == to){
            return e->weight;
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        e = e->nexte;
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```

```
bool graph_addEdge(graph* g, int from, int to, edge w)
{
    if((g==NULL) || (g->size == 0)){
        return false;
    }
    if((from >= g->size) || (to >= g->size)){
        return false;
    }
    vertex* f = g->firstv;
    for(int i=0; i<from; i++){
        f = f->nextv;
    }
    vertex* t = g->firstv;
    for(int i=0; i<to; i++){
        t = t->nextv;
    }
    return _addEdge(f, t, w);
}
```

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Algorithms : Sorting

```
#define NUMS 6

void bubble_sort(int b[], int s);

int main(void)
{
    int a[] = {3, 4, 1, 2, 9, 0};
    bubble_sort(a, NUMS);
    for(int i=0; i<NUMS; i++){
        printf("%i ", a[i]);
    }
    printf("\n");
    return 0;
}

void bubble_sort(int b[], int s)
{
    bool changes;
    do{
        changes = false;
        for(int i=0; i<s-1; i++){
            if(b[i] > b[i+1]){
                SWAP(b[i], b[i+1]);
                changes = true;
            }
        }
    }while(changes);
}
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Execution :

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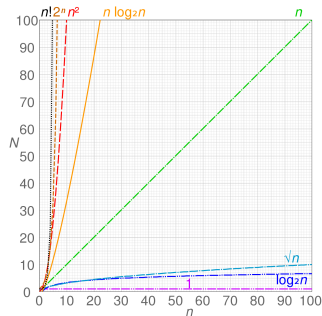
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{
    int a[] = {3, 4, 1, 2, 9, 0};
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```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <string.h>
4
5  void mergesort(int *src, int *spare, int l, int r);
6  void merge(int *src, int *spare, int l, int m, int r);
7
8  #define NUM 5000
9
10 int main(void)
11 {
12     int a[NUM];
13     int spare[NUM];
14
15     for(int i=0; i<NUM; i++){
16         a[i] = rand()%100;
17     }
18
19     mergesort(a, spare, 0, NUM-1);
20
21     for(int i=0; i<NUM; i++){
22         printf("%4d ==> %d\n", i, a[i]);
23     }
24
25     return 0;
26 }
```

Merge Sort II

```
void mergesort(int *src, int *spare, int l, int r)
{
    int m = (l+r)/2;
    if(l != r){
        mergesort(src, spare, l, m);
        mergesort(src, spare, m+1, r);
        merge(src, spare, l, m, r);
    }
}

void merge(int *src, int *spare, int l, int m, int r)
{
    int s1 = l;
    int s2 = m+1;
    int d = l;

    do{
        if(src[s1] < src[s2]){
            spare[d++] = src[s1++];
        }
        else{
            spare[d++] = src[s2++];
        }
    }while((s1 <= m) && (s2 <= r));

    if(s1 > m){
        memcpy(&spare[d], &src[s2], sizeof(spare[0])*(r-s2+1));
    }
    else{
        memcpy(&spare[d], &src[s1], sizeof(spare[0])*(m-s1+1));
    }
    memcpy(&src[l], &spare[l], (r-l+1)*sizeof(spare[0]));
}
```

Merge Sort II

```
void mergesort(int *src, int *spare, int l, int r)
{
    int m = (l+r)/2;
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- Once again, the sort is then applied recursively.

Algorithms : Quicksort

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <math.h>
4
5  int partition(int *a, int l, int r);
6  void quicksort(int *a, int l, int r);
7
8  #define NUM 100000
9
10 int main(void)
11 {
12     int a[NUM];
13
14     for(int i=0; i<NUM; i++){
15         a[i] = rand()%100;
16     }
17     quicksort(a, 0, NUM-1);
18
19     return 0;
20 }
21
22 void quicksort(int *a, int l, int r)
23 {
24     int pivpoint = partition(a, l, r);
25     if(l < pivpoint){
26         quicksort(a, l, pivpoint-1);
27     }
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29         quicksort(a, pivpoint+1, r);
30     }
31 }
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27     }
28     if(r > pivpoint){
29         quicksort(a, pivpoint+1, r);
30     }
31 }
```

```
int partition(int *a, int l, int r)
{
    int piv = a[l];
    while(l<r){
        /* Right -> Left Scan */
        while(piv < a[r] && l<r) r--;
        if(r!=l){
            a[l] = a[r];
            l++;
        }
        /* Left -> Right Scan */
        while(piv > a[l] && l<r) l++;
        if(r!=l){
            a[r] = a[l];
            r--;
        }
    }
    a[r] = piv;
    return r;
}
```

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```
1  #include <stdio.h>
2  #include <stdlib.h>
3
4  int intcompare(const void *a, const void *b);
5
6  int main(void)
7  {
8      int a[10];
9
10     for(int i=0; i<10; i++){
11         a[i] = 9 - i;
12     }
13
14     qsort(a, 10, sizeof(int), intcompare);
15
16     for (int i=0; i<10; i++){
17         printf(" %d",a[i]);
18     }
19     printf("\n");
20     return 0;
21 }
22
23
24 int intcompare(const void *a, const void *b)
25 {
26     const int *ia = (const int *)a;
27     const int *ib = (const int *)b;
28     return *ia - *ib;
29 }
```

Algorithms : The Radix Sort

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459 254 472 534 649 239 432 654 477

0

1

2 472 432

3

4 254 534 654

5

6

7 477

8

9 459 649 239

Read out the new list:

472 432 254 534 654 477 459 649 239

Radix Sort II

472 432 254 534 654 477 459 649 239

0

1

2

3 432 534 239

4 649

5 254 654 459

6

7 472 477

8

9

432 534 239 649 254 654 459 472 477

Radix Sort II

472 432 254 534 654 477 459 649 239

0

1

2

3 432 534 239

4 649

5 254 654 459

6

7 472 477

8

9

432 534 239 649 254 654 459 472 477

432 534 239 649 254 654 459 472 477

0

1

2 239 254

3

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5 534

6 649 654

7

8

9

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Radix Sort Discussion and gprof

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- Now: `gprof ./executable gmon.out` shows the function-call profile of your code.

Algorithm : Rabin-Karp String Searching

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$$h("NEILL") =$$

$$(13 \times 26^4 + 4 \times 26^3 + 8 \times 26^2 + 11 \times 26 + 11) \% P$$

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- This can be expanded by Horner's method to:

$$(((((((13 \times 26) + 4) \times 26) + 8) \times 26) + 11) \times 26 + 11) \% P$$

Rabin-Karp II

- For a large search string, overflow can occur. We therefore move the *mod* operation inside the brackets:

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```
1  #include <string.h>
2  #include <assert.h>
3
4  #define Q 33554393
5  #define D 26
6  #define index(C) (C - 'A')
7
8  int rk(char *p, char *a);
9
10 int main(void)
11 {
12     assert(rk("STING",
13              "A STRING EXAMPLE CONSISTING OF ...") == 22);
14     return 0;
15 }
16
17 int rk(char *p, char *a)
18 {
19     int i, dM = 1, h1=0, h2=0;
20     int m = strlen(p);
21     int n = strlen(a);
22     for(i=1; i<m; i++) dM = (D*dM)%Q;
23     for(i=0; i<m; i++){
24         h1 = (h1*D+index(p[i]))%Q;
25         h2 = (h2*D+index(a[i]))%Q;
26     }
27     // h1 = search string hash, h2 = master string hash
28     for(i=0; h1!=h2; i++){
29         h2 = (h2+D*Q-index(a[i])*dM) % Q;
30         h2 = (h2*D+index(a[i+m])) % Q;
31         if(i>n-m) return n;
32     }
33     return i;
34 }
```


Algorithm : Boyer-Moore String Searching

The Boyer-Moore algorithm uses (in part) an array flagging which characters form part of the search string and an array telling us how far to slide right if that character appears in the master and causes a mismatch.

Execution :

```
A STRING SEARCHING EXAMPLE CONSISTING OF ...  
  |      |  
STING    |  
    STING  
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- The next comparison is between the G and the S. We can slide the search string right until it matches the S in the master.

Boyer-Moore II

Execution :

A STRING SEARCHING EXAMPLE CONSISTING OF ...

STING							
	STING						
		STING					
			STING				
				STING			

- Now the C doesn't appear in the master and once again we can slide a full 5 places to the right.

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      |      |      |      |      |  
STING  |      |      |      |      |  
      |      |      |      |      |  
      STING  |      |      |      |  
            |      |      |      |  
            STING  |      |      |  
                  |      |      |  
                  STING  |  
                        |  
                        STING
```

- Now the C doesn't appear in the master and once again we can slide a full 5 places to the right.
- After 3 more full slides right we arrive at the T in CONSISTING.
- We align the T's, and have found our match using 7 compares (plus 5 to verify the match).

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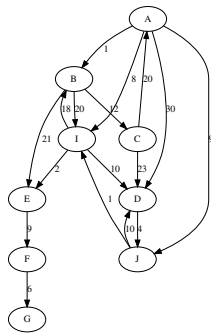
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- Typically gives results within 25% of the optimal solution, but sometimes give a worst-case solution ...
- A -> B -> C -> D -> J -> I -> E -> F -> G



```
edge graph_salesman(graph* g, int from, char* str)
{
    bool* unvis;
    int curr, ncurr, nvs;
    edge cst, bcst, e;

    nvs = graph_numVerts(g);
    if((g==NULL) || (from >= nvs) || (str==NULL)){
        return INF;
    }
    unvis = (bool*)ncalloc(nvs, sizeof(bool));
    for(int v=0; v<nvs; v++){
        unvis[v] = true;
    }
    curr = from;
    bcst = 0;
    strcpy(str, graph_getLabel(g, from));
    do{
        unvis[curr] = false;
        cst = INF;
        ncurr = NO_VERT;
        /* Look at neighbours of curr */
```



```

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        /* Look at neighbours of curr */

```

```

        for(int v=0; v<nvs; v++){
            e = graph_getEdgeWeight(g, curr, v);
            if((v!=curr) && unvis[v] && (e!=INF)){
                if(e < cst){
                    cst = e;
                    ncurr = v;
                }
            }
        }
        /* Add in cost to go to closest */
        if(cst < INF){
            bcst += cst;
            curr = ncurr;
            strcat(str, " ");
            strcat(str, graph_getLabel(g, ncurr));
        }
    }while((cst < INF) && (curr != NO_VERT));
    free(unvis);
    return bcst;
}

```

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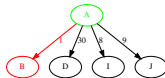
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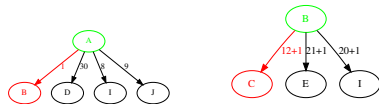
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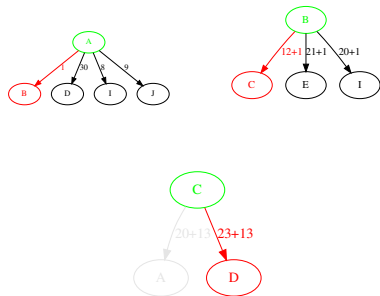


Table of Contents

N : Recursion

O : Algorithms I - Search

P : Linked Data Structures

Q : ADTs - Collection

R : ADTs - Stacks

S : ADTs - Queues

T : ADTs - Trees

U : ADTs - Hashing

V : ADTs - Graphs

W : Algorithms II - Sort / Strings / Graphs

X : Algorithms III - Parsing and Grammars

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- You need to worry about the $/$ first since that has precedence.

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Infix :

$A+B*C$

$A*B+C$

$((A+B)*C+D)/(E+F+G)$

Reverse Polish :

$ABC*+$

$AB*C+$

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Notice that no brackets are required in Reverse Polish. Therefore, for simple applications, we could require the user to enter expressions in postfix form.

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8 2 5 * + 1 3 2 * + 4 - /

	*	+		*	+	-		/
5			2					
2	10		3	6		4		
8	8	18	1	1	7	7	3	
			18	18	18	18	18	6

Coding a Postfix Parser

```
while(fscanf(stdin, "%s", input) == 1){
    // If number: push
    if(sscanf(input, FORMATSTR, &d)==1){
        stack_push(s, d);
    }
    else{
        // Must be an operator ?
        assert(stack_pop(s, &g2));
        assert(stack_pop(s, &g1));
        switch(input[0]){
            case '+':
                d = g1 + g2; break;
            case '-':
                d = g1 - g2; break;
            case '*':
                d = g1 * g2; break;
            case '/':
                d = g1 / g2; break;
            case '^':
                d = (int)pow(g1,g2); break;
            default:
                fprintf(stderr, "Can't understand ? %d\n", input[0]);
                exit(EXIT_FAILURE);
        }
        stack_push(s, d);
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}
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Execution :

12 4 5 7 6 * +
Answer = 34
Stack still had items on it (12) ?

General Infix to Postfix

www.geeksforgeeks.org/stack-set-2-infix-to-postfix

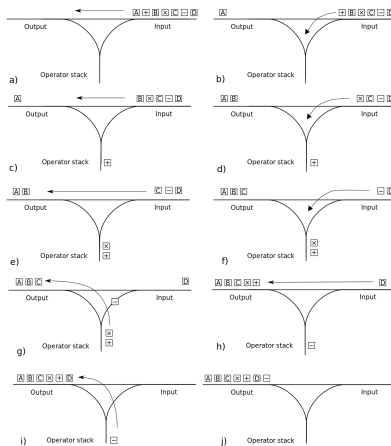
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4. If the scanned character is an '(', push it to the stack.
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https://en.wikipedia.org/wiki/Shunting-yard_algorithm



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- You don't need to understand the input to check if it's valid or not.

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- You don't need to understand the input to check if it's valid or not.

Formal Grammars

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Based on :

web.stanford.edu/class/archive/cs/cs143/cs143.1128

```
<SENTENCE>      ::=      <SUBJECT>      <VERB-  
PHRASE> <OBJECT>  
<SUBJECT> ::= "This" | "Computers" | "I"  
<VERB-PHRASE> ::= <ADVERB> <VERB> | <VERB>  
<ADVERB> ::= "never"  
<VERB> ::= "is" | "run" | "am" | "tell"  
<OBJECT> ::= "the" <NOUN> | "a" <NOUN> | <NOUN>  
<NOUN> ::= "university" | "world" | "cheese" | "lies"
```

Formal Grammars

- Parsing a program is the process of grammatically analysing how it is composed into parts.
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<ADVERB> ::= "never"  
<VERB> ::= "is" | "run" | "am" | "tell"  
<OBJECT> ::= "the" <NOUN> | "a" <NOUN> | <NOUN>  
<NOUN> ::= "university" | "world" | "cheese" | "lies"
```

This is a university.

Computers run the world.

I am the cheese.

I never tell lies.

A Formal Grammer : 0&1s

- Say we wish to create a new computer language whose sole purpose is to print out noughts and ones onto the screen :

A Formal Grammer : 0&1s

- Say we wish to create a new computer language whose sole purpose is to print out noughts and ones onto the screen :

A Formal Grammer : 0&1s

- Say we wish to create a new computer language whose sole purpose is to print out noughts and ones onto the screen :

```
BEGIN  
  ONE  
  NOUGHT  
  ONE  
END
```

A Formal Grammer : 0&1s

- Say we wish to create a new computer language whose sole purpose is to print out noughts and ones onto the screen :

BEGIN

ONE

NOUGHT

ONE

END

- The formal grammar : Backus-Naur form (BNF) :

`<PROG> ::= "BEGIN" <CODE>`

`<CODE> ::= "END" | <STATEMENT> <CODE>`

`<STATEMENT> ::= "ONE" | "NOUGHT"`

A Formal Grammer : 0&1s

- Say we wish to create a new computer language whose sole purpose is to print out noughts and ones onto the screen :

BEGIN

ONE

NOUGHT

ONE

END

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A Formal Grammer : 0&1s

- Say we wish to create a new computer language whose sole purpose is to print out noughts and ones onto the screen :

```
BEGIN
  ONE
  NOUGHT
  ONE
END
```

- The formal grammar : Backus-Naur form (BNF) :

```
<PROG>      ::= "BEGIN" <CODE>
<CODE>      ::= "END" | <STATEMENT> <CODE>
<STATEMENT> ::= "ONE" | "NOUGHT"
```

- The '|' means OR.

A Formal Grammer : 0&1s

- Say we wish to create a new computer language whose sole purpose is to print out noughts and ones onto the screen :

```
BEGIN
  ONE
  NOUGHT
  ONE
END
```

- The formal grammar : Backus-Naur form (BNF) :

```
<PROG>      ::= "BEGIN" <CODE>
<CODE>      ::= "END" | <STATEMENT> <CODE>
<STATEMENT> ::= "ONE" | "NOUGHT"
```

- The '|' means OR.
- "BEGIN", "ONE", "NOUGHT" and "END" are string constants.

A Formal Grammer : 0&1s

- Say we wish to create a new computer language whose sole purpose is to print out noughts and ones onto the screen :

```
BEGIN
  ONE
  NOUGHT
  ONE
END
```

- The formal grammar : Backus-Naur form (BNF) :

```
<PROG>      ::= "BEGIN" <CODE>
<CODE>      ::= "END" | <STATEMENT> <CODE>
<STATEMENT> ::= "ONE" | "NOUGHT"
```

- The '|' means OR.
- "BEGIN", "ONE", "NOUGHT" and "END" are string constants.
- <CODE> is described recursively.

A Formal Grammer : 0&1s

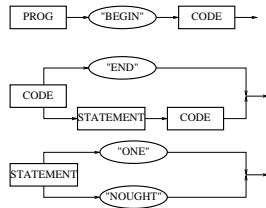
- Say we wish to create a new computer language whose sole purpose is to print out noughts and ones onto the screen :

```
BEGIN
  ONE
  NOUGHT
  ONE
END
```

- The formal grammar : Backus-Naur form (BNF) :

```
<PROG>      ::= "BEGIN" <CODE>
<CODE>      ::= "END" | <STATEMENT> <CODE>
<STATEMENT> ::= "ONE" | "NOUGHT"
```

- The '|' means OR.
- "BEGIN", "ONE", "NOUGHT" and "END" are string constants.
- <CODE> is described recursively.
- You could also think of this grammar in terms of a *railroad diagram*:



Coding a 0 & 1s Parser

```
1  #include <stdio.h>
2  #include <string.h>
3  #include <stdlib.h>
4  #include <assert.h>
5
6  #define MAXNUMTOKENS 100
7  #define MAXTOKENSIZE 20
8  #define strsame(A,B) (strcmp(A, B)==0)
9  #define ERROR(PHRASE) { fprintf(stderr, \
10     "Fatal Error %s occurred in %s, line %d\n", PHRASE, \
11     __FILE__, __LINE__); \
12     exit(EXIT_FAILURE); }
13
14  struct prog{
15     char wds[MAXNUMTOKENS][MAXTOKENSIZE];
16     int cw; // Current Word
17 };
18 typedef struct prog Program;
19
20 void Prog(Program *p);
21 void Code(Program *p);
22 void Statement(Program *p);
23
24 int main(void)
25 {
26     Program* prog = calloc(1, sizeof(Program));
27     int i=0;
28     while(scanf("%s", prog->wds[i++])!=1 && i<MAXNUMTOKENS);
29     assert(i<MAXNUMTOKENS);
30     Prog(prog);
31     printf("Parsed OK\n");
32     return 0;
33 }
```

Coding a 0 & 1s Parser

```
1  #include <stdio.h>
2  #include <string.h>
3  #include <stdlib.h>
4  #include <assert.h>
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14  struct prog{
15     char wds[MAXNUMTOKENS][MAXTOKENSIZE];
16     int cw; // Current Word
17 };
18 typedef struct prog Program;
19
20 void Prog(Program *p);
21 void Code(Program *p);
22 void Statement(Program *p);
23
24 int main(void)
25 {
26     Program* prog = calloc(1, sizeof(Program));
27     int i=0;
28     while(scanf("%s", prog->wds[i++])!=1 && i<MAXNUMTOKENS);
29     assert(i<MAXNUMTOKENS);
30     Prog(prog);
31     printf("Parsed OK\n");
32     return 0;
33 }
```

```
void Prog(Program *p)
{
    if (!strsame(p->wds[p->cw], "BEGIN")){
        ERROR("No BEGIN statement ?");
    }
    p->cw = p->cw + 1;
    Code(p);
}

void Code(Program *p)
{
    if (strsame(p->wds[p->cw], "END")){
        return;
    }
    Statement(p);
    p->cw = p->cw + 1;
    Code(p);
}

void Statement(Program *p)
{
    if (strsame(p->wds[p->cw], "ONE")){
        return;
    }
    if (strsame(p->wds[p->cw], "NOUGHT")){
        return;
    }
    ERROR("Expecting a ONE or NOUGHT ?");
}
```

Running the Parser

Running the Parser

```
BEGIN  
  ONE  
  NOUGHT  
  ONE  
END
```

Parsed OK

Running the Parser

```
BEGIN  
  ONE  
  NOUGHT  
  ONE  
END
```

Parsed OK

```
BEGIN ONE NOUGHT NOUGHT END
```

Parsed OK

Running the Parser

```
BEGIN  
  ONE  
  NOUGHT  
  ONE  
END
```

Parsed OK

```
BEGIN ONE NOUGHT NOUGHT END
```

Parsed OK

```
BEGIN END
```

Parsed OK

Running the Parser

```
BEGIN  
  ONE  
  NOUGHT  
  ONE  
END
```

Parsed OK

```
BEGIN ONE NOUGHT NOUGHT END
```

Parsed OK

```
BEGIN END
```

Parsed OK

```
BEGIN  
  ONE  
  TWO  
END
```

Fatal Error Expecting a ONE or NOUGHT ? occurred in p01a.c, line 79

Running the Parser

```
BEGIN  
  ONE  
  NOUGHT  
  ONE  
END
```

Parsed OK

```
BEGIN ONE NOUGHT NOUGHT END
```

Parsed OK

```
BEGIN END
```

Parsed OK

```
BEGIN  
  ONE  
  TWO  
END
```

Fatal Error Expecting a ONE or NOUGHT ? occurred in p01a.c, line 79

```
BEGIN  
  ONE  
  NOUGHT
```

Fatal Error Expecting a ONE or NOUGHT ? occurred in p01a.c, line 79

Running the Parser

```
BEGIN
  ONE
  NOUGHT
  ONE
END
```

Parsed OK

```
BEGIN ONE NOUGHT NOUGHT END
```

Parsed OK

```
BEGIN END
```

Parsed OK

```
BEGIN
  ONE
  TWO
END
```

Fatal Error Expecting a ONE or NOUGHT ? occurred in p01a.c, line 79

```
BEGIN
  ONE
  NOUGHT
```

Fatal Error Expecting a ONE or NOUGHT ? occurred in p01a.c, line 79

```
ONE
NOUGHT
END
```

Fatal Error No BEGIN statement ? occurred in p01a.c, line 55

- Notice that the END statement is actually used as the recursive base-case in the formal grammar in the function Code().

Running the Parser

```
BEGIN
  ONE
  NOUGHT
  ONE
END
```

Parsed OK

```
BEGIN ONE NOUGHT NOUGHT END
```

Parsed OK

```
BEGIN END
```

Parsed OK

```
BEGIN
  ONE
  TWO
END
```

Fatal Error Expecting a ONE or NOUGHT ? occurred in p01a.c, line 79

```
BEGIN
  ONE
  NOUGHT
```

Fatal Error Expecting a ONE or NOUGHT ? occurred in p01a.c, line 79

```
ONE
NOUGHT
END
```

Fatal Error No BEGIN statement ? occurred in p01a.c, line 55

- Notice that the END statement is actually used as the recursive base-case in the formal grammar in the function Code().
- The parser doesn't actually **do** anything other than check that the input is **valid** or not.

Running the Parser

```
BEGIN
  ONE
  NOUGHT
  ONE
END
```

Parsed OK

```
BEGIN ONE NOUGHT NOUGHT END
```

Parsed OK

```
BEGIN END
```

Parsed OK

```
BEGIN
  ONE
  TWO
END
```

Fatal Error Expecting a ONE or NOUGHT ? occurred in p01a.c, line 79

```
BEGIN
  ONE
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```

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```
ONE
NOUGHT
END
```

Fatal Error No BEGIN statement ? occurred in p01a.c, line 55

- Notice that the END statement is actually used as the recursive base-case in the formal grammar in the function Code().
- The parser doesn't actually **do** anything other than check that the input is **valid** or not.
- An interpreter performs the required operations (e.g. printing to the screen in this case) alongside the parser checking the syntax.

Running the Parser

```
BEGIN
  ONE
  NOUGHT
  ONE
END
```

Parsed OK

```
BEGIN ONE NOUGHT NOUGHT END
```

Parsed OK

```
BEGIN END
```

Parsed OK

```
BEGIN
  ONE
  TWO
END
```

Fatal Error Expecting a ONE or NOUGHT ? occurred in p01a.c, line 79

```
BEGIN
  ONE
  NOUGHT
```

Fatal Error Expecting a ONE or NOUGHT ? occurred in p01a.c, line 79

```
ONE
NOUGHT
END
```

Fatal Error No BEGIN statement ? occurred in p01a.c, line 55

- Notice that the END statement is actually used as the recursive base-case in the formal grammar in the function Code().
- The parser doesn't actually **do** anything other than check that the input is **valid** or not.
- An interpreter performs the required operations (e.g. printing to the screen in this case) alongside the parser checking the syntax.
- A slight modification to the code is required to produce an interpreter.

Interpreters are Modified Parsers

```
void Statement(Program *p)
{
    if(strsame(p->wds[p->cw], "ONE")){
        printf("1\n");
        return;
    }
    if(strsame(p->wds[p->cw], "NOUGHT")){
        printf("0\n");
        return;
    }
    ERROR("Expecting a ONE or NOUGHT ?");
}
```

Interpreters are Modified Parsers

```
void Statement(Program *p)
{
    if(strsame(p->wds[p->cw], "ONE")){
        printf("1\n");
        return;
    }
    if(strsame(p->wds[p->cw], "NOUGHT")){
        printf("0\n");
        return;
    }
    ERROR("Expecting a ONE or NOUGHT ?");
}
```

Execution :

```
BEGIN
ONE NOUGHT ONE NOUGHT
END
1
0
1
0
```

Interpreters are Modified Parsers

```
void Statement(Program *p)
{
    if(strsame(p->wds[p->cw], "ONE")){
        printf("1\n");
        return;
    }
    if(strsame(p->wds[p->cw], "NOUGHT")){
        printf("0\n");
        return;
    }
    ERROR("Expecting a ONE or NOUGHT ?");
}
```

Execution :

```
BEGIN
ONE NOUGHT ONE NOUGHT
END
1
0
1
0
```

- I've also taken out the "Parsed OK" message.

Interpreters are Modified Parsers

```
void Statement(Program *p)
{
    if(strsame(p->wds[p->cw], "ONE")){
        printf("1\n");
        return;
    }
    if(strsame(p->wds[p->cw], "NOUGHT")){
        printf("0\n");
        return;
    }
    ERROR("Expecting a ONE or NOUGHT ?");
}
```

Execution :

```
BEGIN
ONE NOUGHT ONE NOUGHT
END
1
0
1
0
```

- I've also taken out the "Parsed OK" message.
- To extend the parser to be an interpreter you might now need to 'understand' what the input means - the context-free requirement is removed somewhat.

Formal Grammar for Parsing Maths Expressions

To parse a string such as:

"A+B*C"

"A*(B+C)" or

"-(B*F)"

we could invent our own grammar :

Formal Grammar for Parsing Maths Expressions

To parse a string such as:

"A+B*C"

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we could invent our own grammar :

$\langle \text{EXPR} \rangle ::= \langle \text{EXPR} \rangle \langle \text{OP} \rangle \langle \text{EXPR} \rangle \mid$

$\text{"("} \langle \text{EXPR} \rangle \text{"}" \mid$

$\text{"-"} \langle \text{EXPR} \rangle \mid \text{Letter}$

$\langle \text{OP} \rangle ::= \text{"+"} \mid \text{"-"} \mid \text{"*"} \mid \text{"/"}$

Formal Grammar for Parsing Maths Expressions

To parse a string such as:

"A+B*C"

"A*(B+C)" or

"-(B*F)"

we could invent our own grammar :

$\langle \text{EXPR} \rangle ::= \langle \text{EXPR} \rangle \langle \text{OP} \rangle \langle \text{EXPR} \rangle \mid$

"(" $\langle \text{EXPR} \rangle$ ")" |

"-" $\langle \text{EXPR} \rangle$ | Letter

$\langle \text{OP} \rangle ::= "+" \mid "-" \mid "*" \mid "/"$

```
1  #include <stdio.h>
2  #include <ctype.h>
3  #include <stdlib.h>
4  #define MAXEXPR 400
5  struct prog{
6      char str[MAXEXPR];
7      int count;
8  };
9  typedef struct prog Prog;
10 void Op(Prog *p);
11 int isop(char c);
12 void Expr(Prog *p);
13 #define ON_ERROR(S) {fprintf(stderr, "%s", S);\
14                     exit(EXIT_FAILURE);}
15 int main(void)
16 {
17     Prog p;
18     p.count = 0;
19     if (scanf("%[A-Z-+()]s", p.str) != 1){
20         ON_ERROR("Couldn't read your expression ?\n");
21     }
22     Expr(&p);
23     printf("Parsed OK !\n");
24     return 0;
25 }
26
27 int isop(char c)
28 {
29     if (c=='+' || c=='-' || c=='*' || c=='/'){
30         return 1;
31     }
32     return 0;
33 }
```

Running the Maths Parser

```
void Op(Prog *p)
{
    if(!isop(p->str[p->count]))
        ON_ERROR("I was expecting a letter ?\n");
}
void Expr(Prog *p)
{
    if(p->str[p->count] == '('){
        p->count = p->count + 1;
        Expr(p);
        p->count = p->count + 1;
        if(p->str[p->count] != ')'){
            ON_ERROR("I was expecting a ) ?\n");
        }
    }
    else if(p->str[p->count] == '-'){
        p->count = p->count + 1;
        Expr(p);
    }
    // Note Look-Ahead
    else if(isop(p->str[p->count+1])){
        if(isupper(p->str[p->count])){
            p->count = p->count + 1;
            Op(p);
            p->count = p->count + 1;
            Expr(p);
        }
    }
    else{
        if(!isupper(p->str[p->count]) ||
            isupper(p->str[p->count+1])){
            ON_ERROR("Expected a single letter ?\n");
        }
    }
}
```

Running the Maths Parser

```
void Op(Prog *p)
{
    if (!isop(p->str[p->count]))
        ON_ERROR("I was expecting a letter ?\n");
}
void Expr(Prog *p)
{
    if (p->str[p->count] == '('){
        p->count = p->count + 1;
        Expr(p);
        p->count = p->count + 1;
        if (p->str[p->count] != ')'){
            ON_ERROR("I was expecting a ) ?\n");
        }
    }
    else if (p->str[p->count] == '-'){
        p->count = p->count + 1;
        Expr(p);
    }
    // Note Look-Ahead
    else if (isop(p->str[p->count+1])){
        if (isupper(p->str[p->count])){
            p->count = p->count + 1;
            Op(p);
            p->count = p->count + 1;
            Expr(p);
        }
    }
    else{
        if (!isupper(p->str[p->count]) ||
            isupper(p->str[p->count+1])){
            ON_ERROR("Expected a single letter ?\n");
        }
    }
}
```

Execution :

A+(B*C)

Parsed OK !

Running the Maths Parser

```
void Op(Prog *p)
{
    if (!isop(p->str[p->count]))
        ON_ERROR("I was expecting a letter ?\n");
}

void Expr(Prog *p)
{
    if (p->str[p->count] == '('){
        p->count = p->count + 1;
        Expr(p);
        p->count = p->count + 1;
        if (p->str[p->count] != ')'){
            ON_ERROR("I was expecting a ) ?\n");
        }
    }
    else if (p->str[p->count] == '-'){
        p->count = p->count + 1;
        Expr(p);
    }
    // Note Look-Ahead
    else if (isop(p->str[p->count+1])){
        if (isupper(p->str[p->count])){
            p->count = p->count + 1;
            Op(p);
            p->count = p->count + 1;
            Expr(p);
        }
    }
    else{
        if (!isupper(p->str[p->count]) ||
            isupper(p->str[p->count+1])){
            ON_ERROR("Expected a single letter ?\n");
        }
    }
}
```

Execution :

A+(B*C)
Parsed OK !

Execution :

-(B*C+D)
Parsed OK !

Running the Maths Parser

```
void Op(Prog *p)
{
    if (!isop(p->str[p->count]))
        ON_ERROR("I was expecting a letter ?\n");
}
void Expr(Prog *p)
{
    if (p->str[p->count] == '('){
        p->count = p->count + 1;
        Expr(p);
        p->count = p->count + 1;
        if (p->str[p->count] != '){
            ON_ERROR("I was expecting a ) ?\n");
        }
    }
    else if (p->str[p->count] == '-'){
        p->count = p->count + 1;
        Expr(p);
    }
    // Note Look-Ahead
    else if (isop(p->str[p->count+1])){
        if (isupper(p->str[p->count])){
            p->count = p->count + 1;
            Op(p);
            p->count = p->count + 1;
            Expr(p);
        }
    }
    else{
        if (!isupper(p->str[p->count]) ||
            isupper(p->str[p->count+1])){
            ON_ERROR("Expected a single letter ?\n");
        }
    }
}
```

Execution :

A+(B*C)
Parsed OK !

Execution :

-(B*C+D)
Parsed OK !

Execution :

A
Parsed OK !

Running the Maths Parser

Execution :

A+(C*

I was expecting a single letter ?

Running the Maths Parser

Execution :

A+(C*

I was expecting a single letter ?

Execution :

a+c

Couldn't read your expression ?

Running the Maths Parser

Execution :

A+(C*

I was expecting a single letter ?

Execution :

a+c

Couldn't read your expression ?

Execution :

A*B+(C*D

I was expecting a) ?

Running the Maths Parser

Execution :

A+(C*

I was expecting a single letter ?

Execution :

a+c

Couldn't read your expression ?

Execution :

A*B+(C*D

I was expecting a) ?

- The formal grammar doesn't explain everything that the programmer needs to know.

Running the Maths Parser

Execution :

A+(C*

I was expecting a single letter ?

Execution :

a+c

Couldn't read your expression ?

Execution :

A*B+(C*D

I was expecting a) ?

- The formal grammar doesn't explain everything that the programmer needs to know.
- It is not clear whether the $a+c$ example is invalid or not.

Running the Maths Parser

Execution :

A+(C*

I was expecting a single letter ?

Execution :

a+c

Couldn't read your expression ?

Execution :

A*B+(C*D

I was expecting a) ?

- The formal grammar doesn't explain everything that the programmer needs to know.
- It is not clear whether the $a+c$ example is invalid or not.
- It is not clear how spaces should be dealt with.