COMSM1201 : Data Structures & Algorithms

Neill.Campbell@bristol.ac.uk

University of Bristol

Built: November 19, 2024



Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

S : ADTs - Queues

T : ADTs - Trees

U : ADTs - Hashing

V : Algorithms II - Sort

W: Algorithms III - Huffman/Strings

Y: Summary

• When a function calls itself, this is known as recursion.

- When a function calls itself, this is known as recursion.
- This is an important theme in Computer Science that crops up time & time again.

- When a function calls itself, this is known as recursion.
- This is an important theme in Computer Science that crops up time & time again.
- Can sometimes lead to very simple and elegant programs.

- When a function calls itself, this is known as recursion.
- This is an important theme in Computer Science that crops up time & time again.
- Can sometimes lead to very simple and elegant programs.
- Let's look at some toy examples to begin with.

- When a function calls itself, this is known as recursion.
- This is an important theme in Computer Science that crops up time & time again.
- Can sometimes lead to very simple and elegant programs.
- Let's look at some toy examples to begin with.

- When a function calls itself, this is known as recursion.
- This is an important theme in Computer Science that crops up time & time again.
- Can sometimes lead to very simple and elegant programs.
- Let's look at some toy examples to begin with.

```
#include <stdio.h>
     #include <string.h>
     #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
     void strrev(char* s, int n);
     int main (void)
        char str[] = "Hello World!":
        strrev(str. strlen(str)):
12
        printf("%s\n", str);
        return 0:
14
15
     /* Iterative Inplace String Reverse */
17
     void strrev(char* s. int n)
18
19
        for(int i=0, j=n-1; i<j; i++, j--){
            SWAP(s[i], s[j]);
20
21
22
```

Execution:

!dlroW olleH

Recursion for *strrev()*

```
#include <stdio.h>
    #include <string.h>
    #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
    void strrev(char* s, int start, int end);
    int main(void)
       char str[] = "Hello World!";
        strrev(str. 0. strlen(str)-1):
        printf("%s\n", str);
13
14
       return 0;
15
    /* Recursive : Inplace String Reverse */
    void strrev(char* s. int start, int end)
19
       if(start >= end){
           return:
       SWAP(s[start], s[end]);
23
24
        strrev(s. start+1, end-1);
```

Execution:

!dlroW olleH

Recursion for *strrev()*

```
#include <stdio.h>
    #include <string.h>
    #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
    void strrev(char* s, int start, int end);
    int main(void)
       char str[] = "Hello World!";
       strrev(str. 0. strlen(str)-1):
       printf("%s\n", str);
13
14
       return 0:
15
    /* Recursive : Inplace String Reverse */
    void strrev(char* s, int start, int end)
19
       if(start >= end){
20
           return:
       SWAP(s[start], s[end]);
23
24
       strrev(s. start+1, end-1);
```

• We need to change the function prototype.

Execution:

!dlroW olleH

Recursion for *strrev()*

```
#include <stdio.h>
    #include <string.h>
    #define SWAP(A.B) {char temp: temp=A:A=B:B=temp:}
    void strrev(char* s, int start, int end);
    int main(void)
       char str[] = "Hello World!";
       strrev(str. 0. strlen(str)-1):
       printf("%s\n", str);
13
14
       return 0:
15
    /* Recursive : Inplace String Reverse */
    void strrev(char* s, int start, int end)
19
       if(start >= end){
20
           return:
       SWAP(s[start], s[end]);
23
24
       strrev(s. start+1, end-1):
```

- We need to change the function prototype.
- This allows us to track both the start and the end of the string.

Execution:

IdlroW olleH

The Fibonacci Sequence

A well known example of a recursive function is the Fibonacci sequence. The first term is 1, the second term is 1 and each successive term is defined to be the sum of the two previous terms, i.e. :

```
fib(1) is 1
fib(2) is 1
fib(n) is fib(n-1)+fib(n-2)
```

1,1,2,3,5,8,13,21, ...

N : Recursion 5 / 8:

```
#include <stdio.h>
    #define MAXFIB 24
    int fibonacci(int n);
     int main(void)
        for(int i=1: i <= MAXFIB: i++){
           printf("%d = %d\n", i, fibonacci(i));
13
14
15
        return 0;
16
17
     int fibonacci(int n)
19
        if(n \le 2)
           return 1;
       int b = 1:
        int next:
        for (int i=3; i \le n; i++){
           next = a + b:
           a = b:
           b = next:
        return b:
32
```

```
#include <stdio.h>
    #define MAXFIB 24
    int fibonacci(int n):
     int main(void)
        for(int i=1: i <= MAXFIB: i++){</pre>
            printf("%d = %d\n", i, fibonacci(i)):
13
14
15
        return 0;
16
17
     int fibonacci(int n)
19
20
        if(n \le 2)
           return 1;
        int b = 1:
        int next:
        for (int i=3; i \le n; i++){
           next = a + b:
           a = b:
29
           b = next:
30
31
        return b:
32
```

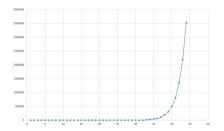
Execution:

```
1 = 1
 = 13
 = 21
9 = 34
10 = 55
11 = 89
12 = 144
13 = 233
14 = 377
15 = 610
16 = 987
17 = 1597
18 = 2584
19 = 4181
20 = 6765
21 = 10946
22 = 17711
23 = 28657
24 = 46368
```

```
#include <stdio.h>
#define MAXFIB 24
int fibonacci(int n);
int main(void)
   for(int i=1; i <= MAXFIB; i++){</pre>
      printf("%d = %d\n", i, fibonacci(i));
   return 0;
int fibonacci(int n)
   if (n == 1) return 1:
   if (n == 2) return 1:
   return (fibonacci(n-1)+fibonacci(n-2));
```

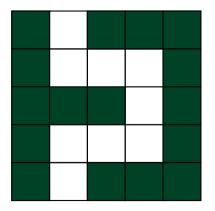
```
#include <stdio.h>
    #define MAXFIB 24
    int fibonacci(int n);
     int main(void)
        for(int i=1; i <= MAXFIB; i++){</pre>
           printf("%d = %d\n", i, fibonacci(i));
       return 0;
    int fibonacci(int n)
20
21
       if (n == 1) return 1:
       if (n == 2) return 1:
        return ( fibonacci(n-1)+fibonacci(n-2));
```

It's interesting to see how run-time increases as the length of the sequence is raised.



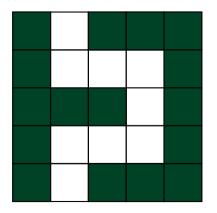
Maze Escape

The correct route through a maze can be obtained via recursive, rather than iterative, methods.



Maze Escape

The correct route through a maze can be obtained via recursive, rather than iterative, methods.



```
bool explore(int x, int y, char mz[YS][XS])
  if mz[y][x] is exit return true;
  Mark mz[y][x] so we don't return here
  if we can go up:
    if(explore(x, y+1, mz)) return true
  if we can go right:
    if(explore(x+1, y, mz)) return true
  Do left & down in a similar manner
  return false: // Failed to find route
```

 Here we consider the ways to permute a string (or more generally an array)

- Here we consider the ways to permute a string (or more generally an array)
- Permutations are all possible ways of rearranging the positions of the characters

Execution:

ABC

ACB

BAC

BCA

CBA CAB

- Here we consider the ways to permute a string (or more generally an array)
- Permutations are all possible ways of rearranging the positions of the characters

Execution:

ABC

ACB

BAC

BCA

CBA CAB

- Here we consider the ways to permute a string (or more generally an array)
- Permutations are all possible ways of rearranging the positions of the characters.

Execution :

ABC

ACB BAC BCA CBA CAB

```
// From e.g. http://www.geeksforgeeks.org
    #include <stdio.h>
    #include <string.h>
    #define SWAP(A,B) {char temp = *A; *A = *B; *B = temp;}
     void permute(char* a, int s, int e);
     int main()
         char str[] = "ABC";
         int n = strlen(str);
         permute(str. 0, n-1);
         return 0:
     void permute(char* a, int s, int e)
18
        if (s == e){
          printf("%s\n", a);
          return:
        for (int i = s: i \le e: i++)
24
           SWAP((a+s), (a+i)); // Bring one char to the front
25
           permute(a, s+1, e);
26
           SWAP((a+s), (a+i)); // Backtrack
27
28
```

N : Recursion

 Raising a number to a power n = 2⁵ is the same as multiple multiplications n = 2*2*2*2*2.

- Raising a number to a power n = 2⁵ is the same as multiple multiplications n = 2*2*2*2*2.
- Or, thinking recursively, $n = 2 * (2^4)$.

- Raising a number to a power n = 2⁵ is the same as multiple multiplications n = 2*2*2*2*2.
- Or, thinking recursively, $n = 2 * (2^4)$.

- Raising a number to a power n = 2⁵ is the same as multiple multiplications n = 2*2*2*2*2
- Or, thinking recursively, $n = 2 * (2^4)$.

```
/* Try to write power(a.b) to computer a^b
        without using any maths functions other than
        multiplication :
        Try (1) iterative then (2) recursive
        (3) Trick that for n\%2==0, x^n = x^(n/2)*x^(n/2)
9
    #include <stdio.h>
11
     int power(unsigned int a, unsigned int b);
12
     int main(void)
16
        int x = 2:
        int v = 16:
19
        printf("%d^%d = %d\n", x, y, power(x,y));
20
21
22
     int power(unsigned int a, unsigned int b)
```

Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

S : ADTs - Queues

T : ADTs - Trees

U : ADTs - Hashing

V : Algorithms II - Sort

W: Algorithms III - Huffman/Strings

Y: Summary

 The need to search an array for a particular value is a common problem.

- The need to search an array for a particular value is a common problem.
- This is used to delete names from a mailing list, or upgrading the salary of an employee etc.

- The need to search an array for a particular value is a common problem.
- This is used to delete names from a mailing list, or upgrading the salary of an employee etc.
- The simplest method for searching is called the sequential search.

- The need to search an array for a particular value is a common problem.
- This is used to delete names from a mailing list, or upgrading the salary of an employee etc.
- The simplest method for searching is called the sequential search.
- Simply move through the array from beginning to end, stopping when you have found the value you require.

- The need to search an array for a particular value is a common problem.
- This is used to delete names from a mailing list, or upgrading the salary of an employee etc.
- The simplest method for searching is called the sequential search.
- Simply move through the array from beginning to end, stopping when you have found the value you require.

- The need to search an array for a particular value is a common problem.
- This is used to delete names from a mailing list, or upgrading the salary of an employee etc.
- The simplest method for searching is called the sequential search.
- Simply move through the array from beginning to end, stopping when you have found the value you require.

```
#include <stdio.h>
     #include <string.h>
     #include <assert.h>
     #define NOTFOUND -1
     #define NUMPEOPLE 6
     typedef struct person {
             char* name; int age;
     } person;
     int findAge(const char* name, const person* p, int n);
     int main (void)
        person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
                   {"Chumley", 26}, {"Dalton", 25},
                   {"Eggson", 22}, {"Fulton", 41} };
        assert(findAge("Eggson",
                                    ppl, NUMPEOPLE) == 22);
        assert (find Age ("Campbell", ppl, NUMPEOPLE) == NOTFOUND);
        return 0:
23
24
     int findAge(const char* name, const person* p, int n)
25
        for (int j=0; j < n; j++){
27
           if (strcmp(name, p[i], name) == 0){
              return p[i].age:
29
30
31
        return NOTFOUND:
32
```

• Sometimes our list of people may not be random.

O: Algorithms I - Search

- Sometimes our list of people may not be random.
- If, for instance, it is sorted, we can use strcmp() in a slightly cleverer manner.

O: Algorithms I - Search

- Sometimes our list of people may not be random.
- If, for instance, it is sorted, we can use strcmp() in a slightly cleverer manner.
- We can stop searching once the search key is alphabetically greater than the item at the current position in the list.

Sequential Search

- Sometimes our list of people may not be random.
- If, for instance, it is sorted, we can use strcmp() in a slightly cleverer manner.
- We can stop searching once the search key is alphabetically greater than the item at the current position in the list.
- This halves, on average, the number of comparisons required.

Sequential Search

- Sometimes our list of people may not be random.
- If, for instance, it is sorted, we can use strcmp() in a slightly cleverer manner.
- We can stop searching once the search key is alphabetically greater than the item at the current position in the list.
- This halves, on average, the number of comparisons required.

Sequential Search

- Sometimes our list of people may not be random.
- If, for instance, it is sorted, we can use strcmp() in a slightly cleverer manner.
- We can stop searching once the search key is alphabetically greater than the item at the current position in the list.
- This halves, on average, the number of comparisons required.

```
#include <stdio h>
     #include <string.h>
     #include <assert.h>
     #define NOTFOUND -1
     #define NUMPFOPLE 6
     typedef struct person{
             char* name; int age;
     } person:
11
     int findAge(const char* name, const person* p, int n):
12
13
     int main (woid)
14
15
        person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
                    {"Chumley", 26}, {"Dalton", 25},
                   {"Eggson", 22}, {"Fulton", 41} }:
        assert (find Age ("Eggson".
                                    ppl NUMPEOPLE) == 22):
        assert (find Age ("Campbell", ppl, NUMPEOPLE) == NOTFOUND):
21
        return 0:
22
23
24
     int findAge(const char* name, const person* p, int n)
25
        for (int j=0; j < n; j++){
27
           int m = strcmp(name, p[i], name);
           if (m == 0) // Braces!
              return p[i].age:
           if(m < 0)
31
              return NOTFOUND:
32
33
        return NOTFOUND:
```

 Searching small lists doesn't require much computation time.

O: Algorithms I - Search

- Searching small lists doesn't require much computation time.
- However, as lists get longer (e.g. phone directories), sequential searching becomes extremely inefficient.

- Searching small lists doesn't require much computation time.
- However, as lists get longer (e.g. phone directories), sequential searching becomes extremely inefficient.
- A binary search consists of examining the middle element of the array to see if it has the desired value. If not, then half the array may be discarded for the next search.

- Searching small lists doesn't require much computation time.
- However, as lists get longer (e.g. phone directories), sequential searching becomes extremely inefficient.
- A binary search consists of examining the middle element of the array to see if it has the desired value. If not, then half the array may be discarded for the next search.

- Searching small lists doesn't require much computation time.
- However, as lists get longer (e.g. phone directories), sequential searching becomes extremely inefficient.
- A binary search consists of examining the middle element of the array to see if it has the desired value. If not, then half the array may be discarded for the next search.

```
    4
    7
    19
    25
    36
    37
    50
    100
    101
    205
    220
    270
    301
    321

    ↑
    ↑
    ↑
    ↑
    ↑
    ↑
    ↑
    ↑
```

```
#include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <time.h>
    #define NMBRS 1000000
    int bin it(int k. const int* a. int 1. int r):
     int main(void)
        int a [NMBRS];
        srand(time(NULL)):
        // Put even numbers into array
        for (int i=0; i < NMBRS; i++){
           a[i] = 2*i:
        // Do many searches for a random number
        for(int i=0; i<10*NMBRS; i++){
           int n = rand()%NMBRS:
           if((n\%2) == 0){
              assert(bin it(n, a, 0, NMBRS-1) = n/2):
25
           else { // No odd numbers in this list
              assert(bin_it(n, a, 0, NMBRS-1) < 0);
        return 0:
```

Iterative v. Recursion Binary Search

```
int bin_it(int k, const int* a, int 1, int r)
{
   while(1 <= r){
      int m = (1+r)/2;
      if(k == a[m]){
            return m;
      }
      else{
        if (k > a[m]) {
            1 = m + 1;
      }
      else {
            r = m - 1;
      }
    }
   return -1;
}
```

Iterative v. Recursion Binary Search

```
int bin_it(int k, const int* a, int 1, int r)
{
  while(1 <= r){
    int m = (1+r)/2;
    if(k = a[m]){
      return m;
    }
    else{
      if (k > a[m]){
            1 = m + 1;
        }
      else{
            r = m - 1;
        }
    }
    return -1;
}
```

```
int bin_rec(int k, const int* a, int l, int r)
{
    if(1 > r) return -1;
    int m = (1+r)/2;
    if(k = a | m |) {
        return m;
    }
    else{
        if (k > a | m |) {
            return bin_rec(k, a, m + 1, r);
        }
        else(
            return bin_rec(k, a, 1, m - 1);
        }
    }
}
```

 When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.

- When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.
- This idea led to the interpolation search.

- When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.
- This idea led to the interpolation search.
- In binary searching, we simply used the middle of an ordered list as a best guess as to where to begin the search.

- When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.
- This idea led to the interpolation search.
- In binary searching, we simply used the middle of an ordered list as a best guess as to where to begin the search.
- Now we use an interpolation involving the key, the start of the list and the end.

$$i = (k - I[0])/(I[n-1] - I[0]) * n$$

- When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.
- This idea led to the interpolation search.
- In binary searching, we simply used the middle of an ordered list as a best guess as to where to begin the search.
- Now we use an interpolation involving the key, the start of the list and the end.

$$i = (k - I[0])/(I[n-1] - I[0]) * n$$

• when searching for '15':

0 4 5 9 10 12 15 20

- When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.
- This idea led to the interpolation search.
- In binary searching, we simply used the middle of an ordered list as a best guess as to where to begin the search.
- Now we use an interpolation involving the key, the start of the list and the end.

$$i = (k - I[0])/(I[n-1] - I[0]) * n$$

• when searching for '15':

0 4 5 9 10 12 15 20

- When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.
- This idea led to the interpolation search.
- In binary searching, we simply used the middle of an ordered list as a best guess as to where to begin the search.
- Now we use an interpolation involving the key, the start of the list and the end.

$$i = (k - I[0])/(I[n-1] - I[0]) * n$$

• when searching for '15':

```
0 4 5 9 10 12 15 20
```

```
int interp(int k. const int* a. int l. int r)
   int m:
   double md:
   while(1 \le r)
      md = ((double)(k-a[1])/
            (double)(a[r]-a[1])*
            (double)(r-1)
           +(double)(1):
      m = 0.5 + md:
      if((m > r) | | (m < 1)){
         return -1:
      if (k == a[m])
         return m:
         if (k > a[m]) {
            1 = m + 1:
         elsef
            r = m-1:
```

• This code on an old Dell laptop took:

- This code on an old Dell laptop took:
 - 3.12 seconds using a non-optimzing compiler -O0

- This code on an old Dell laptop took:
 - 3.12 seconds using a non-optimzing compiler -O0
 - 0.00 seconds using an aggressive optimization -O3

- This code on an old Dell laptop took:
 - 3.12 seconds using a non-optimzing compiler -O0
 - 0.00 seconds using an aggressive optimization -O3
- But "wall-clock" time is generally not the thing that excites Computer Scientists.

- This code on an old Dell laptop took:
 - 3.12 seconds using a non-optimzing compiler -O0
 - 0.00 seconds using an aggressive optimization -O3
- But "wall-clock" time is generally not the thing that excites Computer Scientists.

- This code on an old Dell laptop took:
 - 3.12 seconds using a non-optimzing compiler -O0
 - 0.00 seconds using an aggressive optimization -O3
- But "wall-clock" time is generally not the thing that excites Computer Scientists.

 Searching and sorting algorithms have a complexity associated with them, called big-O.

- This code on an old Dell laptop took:
 - 3.12 seconds using a non-optimzing compiler -O0
 - 0.00 seconds using an aggressive optimization -O3
- But "wall-clock" time is generally not the thing that excites Computer Scientists.

- Searching and sorting algorithms have a complexity associated with them, called big-O.
- This complexity indicates how, for n numbers, performance deteriorates when n changes.

```
1  #include <stdio.h>
2  #include <tidib.h>
3  #include <time.h>
4
5  #define CSEC (double)(CLOCKS_PER_SEC)
6  #define BIGLOOP 1000000000
7
8  int main(void)
9  {
10
11    clock_t c1 = clock();
12    for(int i=0; i<EIGLOOP; i++){
13         int j = i * 2;
14    }
15    clock_t c2 = clock();
16    printf("%f\n", (double)(c2-c1)/CSEC);
17    return 0;
18
18
19 }</pre>
```

- This code on an old Dell laptop took:
 - 3.12 seconds using a non-optimzing compiler -O0
 - 0.00 seconds using an aggressive optimization -O3
- But "wall-clock" time is generally not the thing that excites Computer Scientists.

- Searching and sorting algorithms have a complexity associated with them, called big-O.
- This complexity indicates how, for n numbers, performance deteriorates when n changes.
- Sequential Search : O(n)

- This code on an old Dell laptop took:
 - 3.12 seconds using a non-optimzing compiler -O0
 - 0.00 seconds using an aggressive optimization -O3
- But "wall-clock" time is generally not the thing that excites Computer Scientists.

- Searching and sorting algorithms have a complexity associated with them, called big-O.
- This complexity indicates how, for n numbers, performance deteriorates when n changes.
- Sequential Search : O(n)
- Binary Search : O(log n)

- This code on an old Dell laptop took:
 - 3.12 seconds using a non-optimzing compiler -O0
 - 0.00 seconds using an aggressive optimization -O3
- But "wall-clock" time is generally not the thing that excites Computer Scientists.

- Searching and sorting algorithms have a complexity associated with them, called big-O.
- This complexity indicates how, for n numbers, performance deteriorates when n changes.
- Sequential Search : O(n)
- Binary Search : O(log n)
- Interpolation Search : O(log log n)

```
1  #include <stdio.h>
2  #include <tidib.h>
3  #include <time.h>
4
5  #define CSEC (double)(CLOCKS_PER_SEC)
6  #define BIGLOOP 1000000000
7
8  int main(void)
9  {
10
11    clock_t c1 = clock();
12    for(int i=0; i<EIGLOOP; i++){
13         int j = i * 2;
14    }
15    clock_t c2 = clock();
16    printf("%f\n", (double)(c2-c1)/CSEC);
17    return 0;
18
18
19 }</pre>
```

- This code on an old Dell laptop took:
 - 3.12 seconds using a non-optimzing compiler -O0
 - 0.00 seconds using an aggressive optimization -O3
- But "wall-clock" time is generally not the thing that excites Computer Scientists.

- Searching and sorting algorithms have a complexity associated with them, called big-O.
- This complexity indicates how, for n numbers, performance deteriorates when n changes.
- Sequential Search : O(n)
- Binary Search : O(log n)
- Interpolation Search : O(log log n)
- We'll discuss the dream of a O(1) search later in "Hashing".

Binary vs. Interpolation Timing

```
#include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <time.h>
    int bin it(int k, const int *a, int l, int r);
     int bin rec(int k. const int *a. int 1. int r):
     int interp(int k, const int *a, int 1, int r);
     int* parse_args(int argc, char* argv[], int* n, int* srch);
     int main(int argc, char* argv[])
12
        int i, n, srch;
        int* a;
        int (*p[3])(int k, const int*a, int 1, int r) =
            {bin it, bin rec, interp};
        a = parse_args(argc, argv, &n, &srch);
        srand(time(NULL));
22
23
        for (i=0; i < n; i++){
           a[i] = 2*i:
24
25
        for (i=0; i<5000000; i++){}
26
27
           assert ((*p[srch])(a[rand()%n], a, 0, n-1) >= 0);
28
29
        free(a):
30
        return 0;
31
32
```

Binary vs. Interpolation Timing

```
#include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <time.h>
    int bin it(int k, const int *a, int l, int r);
     int bin rec(int k. const int *a. int 1. int r):
     int interp(int k, const int *a, int 1, int r);
     int* parse_args(int argc, char* argv[], int* n, int* srch);
     int main(int argc, char* argv[])
12
13
        int i, n, srch;
        int* a:
        int (*p[3])(int k, const int*a, int 1, int r) =
            {bin it, bin rec, interp};
18
19
20
21
        a = parse_args(argc, argv, &n, &srch);
        srand(time(NULL));
22
23
        for (i=0; i < n; i++){
           a[i] = 2*i:
24
25
        for (i=0; i<5000000; i++){}
26
27
           assert ((*p[srch])(a[rand()%n], a, 0, n-1) >= 0);
28
29
        free(a):
30
        return 0;
31
32
```

Execution:

```
Binary Search : Iterative
       100000 = 0.39
      800000 = 0.57
      6400000 = 1.00
     51200000 = 2.46
Binary Search : Recursive
       100000 = 0.40
       800000 = 0.56
      6400000 = 0.97
n =
     51200000 = 2.42
Interpolation
n =
       100000 = 0.05
       800000 = 0.05
      6400000 = 0.10
n =
     51200000 = 0.13
n =
```

Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

S : ADTs - Queues

T : ADTs - Trees

U : ADTs - Hashing

V : Algorithms II - Sort

W: Algorithms III - Huffman/Strings

Y : Summary

• Linked data representations are useful when:

- Linked data representations are useful when:
 - It is difficult to predict the size and the shape of the data structures in advance.

- Linked data representations are useful when:
 - It is difficult to predict the size and the shape of the data structures in advance.
 - We need to efficiently insert and delete elements.

- Linked data representations are useful when:
 - It is difficult to predict the size and the shape of the data structures in advance.
 - We need to efficiently insert and delete elements.
- To create linked data representations we use pointers to connect separate blocks of storage together. If a given block contains a pointer to a second block, we can follow this pointer there.

- Linked data representations are useful when:
 - It is difficult to predict the size and the shape of the data structures in advance.
 - We need to efficiently insert and delete elements.
- To create linked data representations we use pointers to connect separate blocks of storage together. If a given block contains a pointer to a second block, we can follow this pointer there.
- By following pointers one after another, we can travel right along the structure.

Linked Data Structures

- Linked data representations are useful when:
 - It is difficult to predict the size and the shape of the data structures in advance.
 - We need to efficiently insert and delete elements.
- To create linked data representations we use pointers to connect separate blocks of storage together. If a given block contains a pointer to a second block, we can follow this pointer there.
- By following pointers one after another, we can travel right along the structure.

Linked Data Structures

- Linked data representations are useful when:
 - It is difficult to predict the size and the shape of the data structures in advance.
 - We need to efficiently insert and delete elements.
- To create linked data representations we use pointers to connect separate blocks of storage together. If a given block contains a pointer to a second block, we can follow this pointer there.
- By following pointers one after another, we can travel right along the structure.

```
#include <stdio h>
     #include < stdlih h>
     #include "general.h"
     typedef struct data{
        int i:
        struct data* next:
     } Data;
     Data* allocateData(int i):
11
     void printList(Data* 1):
     int main(void)
        int i:
        Data* start . *current :
        start = current = NULL:
        printf("Enter the first number: "):
19
        if(scanf("%i", &i) == 1){
20
           start = current = allocateData(i):
21
        elsef
           on_error("Couldn't read an int");
        printf("Enter more numbers: ");
27
        while(scanf("%i", &i) == 1){
           current -> next = allocateData(i):
29
           current = current -> next:
30
31
        printList(start):
        return 0: // Should Free List
33
```

Linked Lists

```
Data* allocateData(int i)
{
    Data* p;
    p = (Data*) ncalloc(1, sizeof(Data));
    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* 1)
{
    printf("\n");
    do{
        printf("Number : %i\n", 1->i);
        l = l->next;
    }while(1 != NULL);
    printf("END\n");
}
```

Linked Lists

```
Data* allocateData(int i)
{
    Data* p;
    p = (Data*) ncalloc(1, sizeof(Data));
    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* 1)
{
    printf("\n");
    do{
        printf("Number : %i\n", 1->i);
        1 = 1->next;
    }while(1 != NULL);
    printf("END\n");
}
```

Searching and Recursive printing:

```
Data* inList(Data* n, int i)
{
    do{
        if(n->i==i){
            return n;
        }
        n = n->next;
    } while(n != NULL);
    return NULL;
}

void printList_r(Data* 1)
{
    // Recursive Base-Case
    if(1 == NULL) return;
    printf("Number: %i\n", 1->i);
    printList_r(1->next);
}
```

• But would we really code something like this **every** time we need flexible data storage ?

- But would we really code something like this every time we need flexible data storage?
- This would be horribly error-prone.

- But would we really code something like this **every** time we need flexible data storage ?
- This would be horribly error-prone.
- Build something once, and test it well.

- But would we really code something like this every time we need flexible data storage?
- This would be horribly error-prone.
- Build something once, and test it well.
- One example of this is an **Abstract Data Type (ADT)**.

- But would we really code something like this every time we need flexible data storage?
- This would be horribly error-prone.
- Build something once, and test it well.
- One example of this is an Abstract Data Type (ADT).
- Each ADT exposes its functionality via an interface. The user only accesses the data via this interface.

- But would we really code something like this every time we need flexible data storage?
- This would be horribly error-prone.
- Build something once, and test it well.
- One example of this is an Abstract Data Type (ADT).
- Each ADT exposes its functionality via an interface. The user only accesses the data via this interface.
- The **user** of the ADT doesn't need to understand how the data is being stored (e.g. array vs. linked lists etc.)

- But would we really code something like this every time we need flexible data storage?
- This would be horribly error-prone.
- Build something once, and test it well.
- One example of this is an **Abstract Data Type (ADT)**.
- Each ADT exposes its functionality via an interface. The user only accesses the data via this interface.
- The **user** of the ADT doesn't need to understand how the data is being stored (e.g. array vs. linked lists etc.)
- Actually, I'll sometimes blur the boundaries of Data Structures (e.g. a linked list) with ADTs (e.g. a dictionary) themselves.

Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

S : ADTs - Queues

T · ADTs - Trees

U : ADTs - Hashing

V : Algorithms II - Sort

W: Algorithms III - Huffman/Strings

Y: Summary

• One of the simplest ADTs is the **Collection**.

- One of the simplest ADTs is the **Collection**.
- This is just a simple place to search for/add/delete data elements.

- One of the simplest ADTs is the Collection.
- This is just a simple place to search for/add/delete data elements.
- Some collections allow duplicate elements and others do not (e.g. Sets).

- One of the simplest ADTs is the **Collection**.
- This is just a simple place to search for/add/delete data elements.
- Some collections allow duplicate elements and others do not (e.g. Sets).
- Some are ordered (for faster searching) and others unordered.

- One of the simplest ADTs is the **Collection**.
- This is just a simple place to search for/add/delete data elements.
- Some collections allow duplicate elements and others do not (e.g. Sets).
- Some are ordered (for faster searching) and others unordered.
- Our Collection will be unsorted and will allow duplicates.

- One of the simplest ADTs is the **Collection**.
- This is just a simple place to search for/add/delete data elements.
- Some collections allow duplicate elements and others do not (e.g. Sets).
- Some are ordered (for faster searching) and others unordered.
- Our Collection will be unsorted and will allow duplicates.

- One of the simplest ADTs is the Collection.
- This is just a simple place to search for/add/delete data elements.
- Some collections allow duplicate elements and others do not (e.g. Sets).
- Some are ordered (for faster searching) and others unordered.
- Our Collection will be unsorted and will allow duplicates.

```
#include "../General/general.h"
typedef int colltype:
typedef struct coll coll;
#include <stdio.h>
#include <stdlib h>
#include <assert.h>
// Create an empty coll
coll* coll init(void);
// Add element onto top
void coll add(coll* c, colltype i);
// Take element out
bool coll remove(coll* c. colltype d):
// Does this exist ?
bool coll isin(coll* c. colltype i):
// Return size of coll
int coll size(coll* c):
// Clears all space used
bool coll_free(coll* c);
```

 Note that the interface gives you no hints as to the actual underlying implementation of the ADT.

- Note that the interface gives you no hints as to the actual underlying implementation of the ADT.
- A user of the ADT doesn't really need to know how it's implemented - ideally.

- Note that the interface gives you no hints as to the actual underlying implementation of the ADT.
- A user of the ADT doesn't really need to know how it's implemented - ideally.
- The ADT developer could have several **different** implementations.

- Note that the interface gives you no hints as to the actual underlying implementation of the ADT.
- A user of the ADT doesn't really need to know how it's implemented - ideally.
- The ADT developer could have several different implementations.
- Here we'll see *Collection* implemented using:

- Note that the interface gives you no hints as to the actual underlying implementation of the ADT.
- A user of the ADT doesn't really need to know how it's implemented - ideally.
- The ADT developer could have several different implementations.
- Here we'll see *Collection* implemented using:
 - A fixed-size array

- Note that the interface gives you no hints as to the actual underlying implementation of the ADT.
- A user of the ADT doesn't really need to know how it's implemented - ideally.
- The ADT developer could have several different implementations.
- Here we'll see *Collection* implemented using:
 - A fixed-size array
 - A dynamic array

- Note that the interface gives you no hints as to the actual underlying implementation of the ADT.
- A user of the ADT doesn't really need to know how it's implemented - ideally.
- The ADT developer could have several different implementations.
- Here we'll see *Collection* implemented using:
 - A fixed-size array
 - A dynamic array
 - A linked-list

- Note that the interface gives you no hints as to the actual underlying implementation of the ADT.
- A user of the ADT doesn't really need to know how it's implemented - ideally.
- The ADT developer could have several different implementations.
- Here we'll see *Collection* implemented using:
 - A fixed-size array
 - A dynamic array
 - A linked-list

- Note that the interface gives you no hints as to the actual underlying implementation of the ADT.
- A user of the ADT doesn't really need to know how it's implemented - ideally.
- The ADT developer could have several different implementations.
- Here we'll see *Collection* implemented using:
 - A fixed-size array
 - A dynamic array
 - A linked-list

Fixed/specific.h:

```
1  #pragma once
2
3  #define COLLTYPE "Fixed"
4
5  #define FIXEDSIZE 5000
6  struct coll {
7     // Underlying array
8     colltype a[FIXEDSIZE];
9     int size;
10  };
```

Collection ADT using a Fixed-size Array

Fixed/fixed.c:

```
#include " .. / coll . h"
     #include "specific.h"
     coll* coll_init(void)
        coll* c = (coll*) ncalloc(1, sizeof(coll));
        c \rightarrow size = 0;
        return c;
     int coll size(coll* c)
        if (c=NULL){
           return 0:
        return c->size:
19
20
     bool coll_isin(coll* c, colltype d)
        for (int i=0: i < coll size(c): i++){
           if(c\rightarrow a[i] = d)
                return true;
        return false;
```

Collection ADT using a Fixed-size Array

Fixed/fixed.c:

```
#include "../coll.h"
    #include "specific.h"
     coll* coll_init(void)
        coll* c = (coll*) ncalloc(1, sizeof(coll));
        c - > size = 0;
        return c;
     int coll size(coll* c)
13
        if (c=NULL){
           return 0:
16
17
        return c->size;
19
     bool coll_isin(coll* c, colltype d)
20
        for (int i=0: i < coll size(c): i++){
22
           if(c->a[i] == d){}
               return true:
24
        return false;
```

```
void coll add(coll* c. colltype d)
   if(c){
      if(c->size >= FIXEDSIZE){
          on error("Collection overflow"):
      c \rightarrow a[c \rightarrow size] = d:
      c \rightarrow size = c \rightarrow size + 1:
bool coll remove(coll* c. colltype d)
   for (int i=0: i < coll size(c): i++){
      if(c->a[i] == d)f
          // Shuffle end of array left one
          for(int j=i; j < coll_size(c); j++){</pre>
             c - a[i] = c - a[i+1];
          c->size = c->size - 1:
          return true:
   return false:
bool coll_free(coll* c)
   free(c):
   return true:
```

Collection ADT via an Array (Realloc)

Realloc/specific.h:

Collection ADT via an Array (Realloc)

Realloc/specific.h:

```
#pragma once

define COLLTYPE "Realloc"

define INITSIZE 16

define SCALEFACTOR 2

struct coll {

    // Underlying array

    colltype* a;

    int size;

    int capacity;
};
```

Realloc/realloc.c:

```
#include "../coll.h"
     #include "specific.h"
      coll* coll init(void)
         coll* c = (coll*) ncalloc(1, sizeof(coll));
         c->a = (colltype*) ncalloc(INITSIZE, sizeof(colltype));
         c \rightarrow size = 0:
         c->capacity= INITSIZE;
         return c:
11
      void coll add(coll* c. colltype d)
14
         if(c){
             if (c-> size >= c-> capacity) {
17
                 c \rightarrow a = (colltype*) nremalloc(c \rightarrow a.
                         sizeof(colltype)*c->capacity*SCALEFACTOR);
                 c->capacity = c->capacity*SCALEFACTOR;
21
             c \rightarrow a[c \rightarrow size] = d:
             c \rightarrow size = c \rightarrow size + 1:
23
```

Collection ADT via a Linked List

Linked/specific.h:

```
#pragma once

#define COLLTYPE "Linked"

struct dataframe {
    colltype i;
    truct dataframe* next;
};

typedef struct dataframe dataframe;

struct coll {
    // Underlying array
    dataframe* start;
    int size;
};
```

Collection ADT via a Linked List

Linked/specific.h:

```
myragma once

define COLLTYPE "Linked"

define COLLTYPE "Linked"

struct dataframe {
    colltype i;
    struct dataframe* next;
    ;
    struct dataframe dataframe;

struct coll {
    // Underlying array
    dataframe* start;
    int size;
    };
}
```

Linked/linked.c:

```
#include " .. / coll .h"
#include "specific.h"
coll* coll init(void)
   coll* c = (coll*) ncalloc(1, sizeof(coll));
   return c:
int coll size(coll* c)
   if(c==NULL){
      return 0:
   return c->size:
bool coll_isin(coll* c, colltype d)
   if(c == NULL || c->start==NULL){
      return false:
   dataframe* f = c->start:
   dof
      if(f\rightarrow i == d){
          return true:
      f = f - > next;
   } while (f != NULL):
   return false:
```

Q : ADTs - Collection

Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
   if(c){
       dataframe* f = ncalloc(1, sizeof(dataframe));
       f \rightarrow i = d:
       f \rightarrow next = c \rightarrow start:
       c \rightarrow start = f;
       c \rightarrow size = c \rightarrow size + 1;
bool coll_free(coll* c)
   if(c){
       dataframe* tmp;
       dataframe* p = c->start;
       while (p!=NULL) {
           tmp = p->next;
           free(p);
           p = tmp;
       free(c):
   return true;
```

Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
   if(c){
       dataframe* f = ncalloc(1. sizeof(dataframe)):
       f \rightarrow i = d:
       f \rightarrow next = c \rightarrow start:
       c \rightarrow start = f;
       c \rightarrow size = c \rightarrow size + 1:
bool coll free(coll* c)
   if(c){
       dataframe* tmp:
       dataframe* p = c->start:
       while (p!=NULL) {
           tmp = p->next;
           free(p);
           p = tmp;
       free(c):
   return true;
```

```
bool coll_remove(coll* c, colltype d)
   dataframe* f1 . *f2:
   if((c==NULL) || (c->start==NULL)){
      return false:
   // If Front
   if (c->start -> i == d) {
      f1 = c->start->next:
      free(c->start):
      c->start = f1:
      c \rightarrow size = c \rightarrow size - 1;
      return true:
   f1 = c -> start:
   f2 = c->start->next:
   dof
      if(f2->i == d)f
          f1 -> next = f2 -> next:
          free(f2):
          c \rightarrow size = c \rightarrow size - 1:
          return true:
      f1 = f2:
      f2 = f1 -> next:
   } while (f2 != NULL):
   return false;
```

 Any code using the ADT can be compiled against any of the implementations,
 e.g. the test (testcoll.c) code.

- Any code using the ADT can be compiled against any of the implementations,
 e.g. the test (testcoll.c) code.
- The *Collection* interface (coll.h) is never changed.

- Any code using the ADT can be compiled against any of the implementations,
 e.g. the test (testcoll.c) code.
- The *Collection* interface (coll.h) is never changed.
- There are pros and cons of each implementation:

- Any code using the ADT can be compiled against any of the implementations,
 e.g. the test (testcoll.c) code.
- The *Collection* interface (coll.h) is never changed.
- There are pros and cons of each implementation:
 - Fixed Array: Simple to implement can't avoid the problems of it being a fixed-size. Deletion expensive.

- Any code using the ADT can be compiled against any of the implementations,
 e.g. the test (testcoll.c) code.
- The *Collection* interface (coll.h) is never changed.
- There are pros and cons of each implementation:
 - Fixed Array: Simple to implement can't avoid the problems of it being a fixed-size. Deletion expensive.
 - Realloc Array: Implementation fairly simple. Deletion expensive. Every realloc() is very expensive. Need to tune SCALEFACTOR.

- Any code using the ADT can be compiled against any of the implementations,
 e.g. the test (testcoll.c) code.
- The *Collection* interface (coll.h) is never changed.
- There are pros and cons of each implementation:
 - Fixed Array: Simple to implement can't avoid the problems of it being a fixed-size. Deletion expensive.
 - Realloc Array: Implementation fairly simple. Deletion expensive. Every realloc() is very expensive. Need to tune SCALEFACTOR.
 - Linked : Slightly fiddly implementation
 fast to delete an element.

- Any code using the ADT can be compiled against any of the implementations,
 e.g. the test (testcoll.c) code.
- The *Collection* interface (coll.h) is never changed.
- There are pros and cons of each implementation:
 - Fixed Array: Simple to implement can't avoid the problems of it being a fixed-size. Deletion expensive.
 - Realloc Array: Implementation fairly simple. Deletion expensive. Every realloc() is very expensive. Need to tune SCALEFACTOR.
 - Linked : Slightly fiddly implementation
 fast to delete an element.

- Any code using the ADT can be compiled against any of the implementations,
 e.g. the test (testcoll.c) code.
- The *Collection* interface (coll.h) is never changed.
- There are pros and cons of each implementation:
 - Fixed Array: Simple to implement can't avoid the problems of it being a fixed-size. Deletion expensive.
 - Realloc Array: Implementation fairly simple. Deletion expensive. Every realloc() is very expensive. Need to tune SCALEFACTOR.
 - Linked : Slightly fiddly implementation
 fast to delete an element.

Task	Fixed Array	Realloc Array	Linked List
Insert new element	O(1) at end	O(1) at end	O(1) at front
	if space	but realloc()	
Search for an element	O(n)	O(n)	O(n)
	brute force	brute force	brute force
Search + delete	O(n) + O(n)	O(n) + O(n)	O(n) + O(1)
	move left	move left	delete 'free'

 If we had ordered our ADT (ie. the elements were sorted), then the searches could be via a binary / interpolation search, leading to O(log n) or O(log log n) search times.

ADTs Making Coding Simpler

Linked List code from the previous Chapter:

```
#include <stdio.h>
    #include <stdlib.h>
    #include "general.h"
    typedef struct data{
       int i:
       struct data* next:
    } Data;
    Data* allocateData(int i):
    void printList(Data* 1);
13
    int main(void)
14
       int i:
       Data* start, *current;
       start = current = NULL:
        printf("Enter the first number: ");
        if(scanf("%i", &i) == 1){
           start = current = allocateData(i);
        elsef
           on error("Couldn't read an int"):
24
25
        printf("Enter more numbers: ");
        while(scanf("%i", &i) == 1){
           current -> next = allocateData(i):
29
           current = current->next:
30
31
        printList(start):
        return 0: // Should Free List
```

ADTs Making Coding Simpler

Linked List code from the previous Chapter:

```
#include <stdio.h>
    #include <stdlib.h>
    #include "general.h"
    typedef struct data{
       int i:
       struct data* next:
    } Data;
    Data* allocateData(int i):
    void printList(Data* 1);
    int main (void)
14
       int i:
       Data* start, *current;
       start = current = NULL:
       printf("Enter the first number: ");
       if(scanf("%i", &i) == 1){
           start = current = allocateData(i);
       elsef
           on error("Couldn't read an int"):
24
25
       printf("Enter more numbers: ");
       while(scanf("%i", &i) == 1){
           current -> next = allocateData(i):
29
           current = current->next:
30
31
       printList(start):
       return 0: // Should Free List
```

Becomes:

```
1  ##include "coll.h"
2  ##include "Fixed/specific.h"
3
4  int main(void)
5  {
6     printf("Please type some numbers :");
7     coll* c = coll_init();
8     int i;
9     while(scanf("%i", &zi) == 1){
10         coll_add(c, i);
11     }
12     // Do print etc.
13     coll_free(c);
14     return 0;
15 }
```

Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

S : ADTs - Queues

T : ADTs - Trees

U : ADTs - Hashing

V : Algorithms II - Sort

W: Algorithms III - Huffman/Strings

Y: Summary

At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

• Collections (Lists)

At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

- Collections (Lists)
- Stacks

At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

- Collections (Lists)
- Stacks
- Queues

At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

- Collections (Lists)
- Stacks
- Queues
- Sets

At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

- Collections (Lists)
- Stacks
- Queues
- Sets
- Graphs

At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

- Collections (Lists)
- Stacks
- Queues
- Sets
- Graphs
- Trees

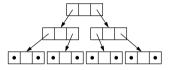
At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

- Collections (Lists)
- Stacks
- Queues
- Sets
- Graphs
- Trees

At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

- Collections (Lists)
- Stacks
- Queues
- Sets
- Graphs
- Trees

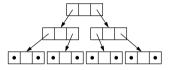
Binary Trees:



At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

- Collections (Lists)
- Stacks
- Queues
- Sets
- Graphs
- Trees

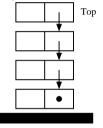
Binary Trees:



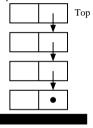
Unidirectional Graph:



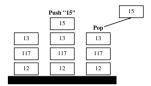
The push-down stack:



The push-down stack:

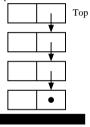


LIFO (Last in, First out):

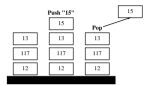


• Operations include push and pop.

The push-down stack:

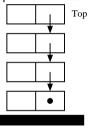


LIFO (Last in, First out):

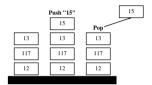


- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.

The push-down stack:

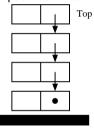


LIFO (Last in, First out):

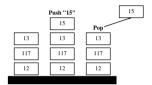


- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.

The push-down stack:



LIFO (Last in, First out):



- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.
- But, once again, we are faced with the question: How best to implement such a data type?

ADT:Stacks Arrays (Realloc) I

stack.h:

```
#pragma once
    #include " .. / General / general . h"
    typedef int stacktype;
    typedef struct stack stack;
    #include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <string.h>
    /* Create an empty stack */
    stack* stack_init(void);
    /* Add element to top */
    void stack push(stack* s, stacktype i);
    /* Take element from top */
    bool stack pop(stack* s. stacktype* d):
    /* Clears all space used */
    bool stack free(stack* s):
23
24
    /* Optional? */
    /* Copy top element into d (but don't pop it) */
    bool stack peek(stack*s. stacktype* d):
    /* Make a string version - keep .dot in mind */
    void stack tostring(stack*. char* str);
```

ADT:Stacks Arrays (Realloc) I

stack.h:

```
#pragma once
    #include " .. / General/general .h"
    typedef int stacktype:
    typedef struct stack stack;
    #include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <string.h>
    /* Create an empty stack */
    stack* stack_init(void);
    /* Add element to top */
    void stack push(stack* s, stacktype i);
    /* Take element from top */
    bool stack pop(stack* s. stacktype* d):
    /* Clears all space used */
    bool stack free(stack* s):
23
24
    /* Optional? */
    /* Copy top element into d (but don't pop it) */
    bool stack peek(stack*s. stacktype* d):
    /* Make a string version - keep .dot in mind */
    void stack tostring(stack*. char* str);
```

Realloc/specific.h:

ADT:Stacks Arrays (Realloc) II

Realloc/realloc.c

```
#include " .. / stack . h"
    #include "specific.h"
    #define DOTFILE 5000
     stack* stack init(void)
        stack *s = (stack*) ncalloc(1, sizeof(stack));
        /* Some implementations would allow you to pass
           a hint about the initial size of the stack */
        s->a = (stacktype*) ncalloc(FIXEDSIZE, sizeof(stacktype));
        s \rightarrow size = 0:
        s->capacity= FIXEDSIZE;
14
15
        return s:
17
     void stack_push(stack* s, stacktype d)
19
        if (s=NULL){
             return:
        if(s->size >= s->capacity){}
23
           s->a = (stacktype*) nremalloc(s->a,
24
                    sizeof(stacktype)*s->capacity*SCALEFACTOR);
25
26
           s->capacity = s->capacity*SCALEFACTOR;
27
        s \rightarrow a[s \rightarrow size] = d:
28
        s \rightarrow size = s \rightarrow size + 1:
```

ADT:Stacks Arrays (Realloc) II

Realloc/realloc.c

```
#include " .. / stack . h"
     #include "specific.h"
     #define DOTFILE 5000
     stack * stack init(void)
         stack *s = (stack*) ncalloc(1, sizeof(stack));
        /* Some implementations would allow you to pass
            a hint about the initial size of the stack */
         s->a = (stacktype*) ncalloc(FIXEDSIZE, sizeof(stacktype));
        s \rightarrow size = 0:
         s->capacity= FIXEDSIZE;
14
         return s:
15
17
     void stack_push(stack* s, stacktype d)
19
         if (s=NULL){
              return:
21
        if(s->size >= s->capacity){}
23
            s \rightarrow a = (stacktvpe*) nremalloc(s \rightarrow a.
24
                     sizeof(stacktype)*s->capacity*SCALEFACTOR);
25
            s->capacity = s->capacity*SCALEFACTOR;
26
27
         s \rightarrow a[s \rightarrow size] = d:
28
         s \rightarrow size = s \rightarrow size + 1:
```

```
bool stack_pop(stack* s, stacktype* d)
{
    if((s == NULL) || (s->size < 1)){
        return false;
    }
    s.>size = s->size - 1;
    *d = s->a(s->size);
    return true;
}

bool stack_peek(stack* s, stacktype* d)

if ((s==NULL) || (s->size <= 0)){
    if((s==NULL) || (s->size <= 0)){
        *d = s->a(s->size);
        return true;
}

*** **True**

**Tru
```

ADT:Stacks Arrays (Realloc) III

Realloc/realloc.c

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE];
        str[0] = '\0';
        if((s=NULL) || (s->size <1)){
           return:
        for (int i=s-> size-1: i>=0: i--)
           sprintf(tmp, FORMATSTR, s->a[i]);
10
11
12
13
           strcat(str, tmp);
           strcat(str, "|");
        str[strlen(str)-1] = '\0':
14
15
16
17
     bool stack free(stack* s)
18
        if (s=NULL){
           return true:
20
21
        free(s->a):
        free(s):
        return true;
```

ADT:Stacks Arrays (Realloc) III

Realloc/realloc.c

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE];
        str[0] = '\0';
        if((s=NULL) || (s->size <1)){
           return:
        for (int i=s-> size-1: i>=0: i--)
           sprintf(tmp, FORMATSTR, s->a[i]);
           strcat(str, tmp);
10
11
12
13
           strcat(str, "|");
        str[strlen(str)-1] = '\0';
14
15
16
17
     bool stack free(stack* s)
18
        if (s=NULL){
           return true:
20
21
        free(s->a):
        free(s):
        return true;
```

• We need a thorough testing program teststack.c

ADT:Stacks Arrays (Realloc) III

Realloc/realloc.c

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE];
        str[0] = '\0':
        if((s=NULL) || (s->size <1)){
           return:
        for (int i=s->size-1: i>=0: i--) {
           sprintf(tmp, FORMATSTR, s->a[i]);
           strcat(str. tmp):
           strcat(str. "|");
12
13
        str[strlen(str)-1] = '\0':
14
15
16
17
     bool stack free(stack* s)
18
        if (s=NULL){
           return true:
20
21
        free(s->a):
        free(s):
        return true:
```

- We need a thorough testing program teststack c
- See also revstr.c: a version of the string reverse code (for which we already seen an iterative (in-place) and a recursive solution).

ADT:Stacks Linked I

Linked/specific.h

```
#pragma once

#define FORMATSIR "%i"

#define STACKTYPE "Linked"

struct dataframe {
    stacktype i;
    struct dataframe* next;
};

typedef struct dataframe dataframe;

struct stack {
    /* Underlying array */
    dataframe* start;
    int size;
};
```

ADT:Stacks Linked I

Linked/specific.h

```
1  ##pragma once
2
3  ##define FORMAISIR "%i"
4  ##define ELEMSIZE 20
5  ##define STACKTYPE "Linked"
6
7  struct dataframe {
8   stacktype i;
9   struct dataframe* next;
1);
11  typedef struct dataframe dataframe;
12
13  struct stack {
14   /* Underlying array */
15   dataframe* start;
16   int size;
17 };
```

Linked/linked.c

```
#include " .. / stack .h"
     #include "specific.h"
     #define DOTFILE 5000
     stack* stack init(void)
         stack* s = (stack*) ncalloc(1, sizeof(stack));
         return s:
10
11
     void stack push(stack* s. stacktype d)
13
        if(s){
            dataframe* f = ncalloc(1, sizeof(dataframe));
            f \rightarrow i = d:
            f->next = s->start;
            s->start = f:
            s \rightarrow size = s \rightarrow size + 1:
20
```

ADT:Stacks Linked II

```
bool stack_pop(stack* s, stacktype* d)
        if ((s==NULL) || (s->start==NULL)){
            return false;
        dataframe* f = s->start->next;
        *d = s->start->i:
        free(s->start):
        s \rightarrow start = f:
        s \rightarrow size = s \rightarrow size - 1:
12
13
14
        return true;
15
     bool stack_peek(stack* s, stacktype* d)
16
        if((s==NULL) || (s->start==NULL)){
18
            return false;
20
        *d = s->start ->i;
        return true;
```

ADT:Stacks Linked II

```
bool stack_pop(stack* s, stacktype* d)
        if((s==NULL) || (s->start==NULL)){
            return false:
        dataframe* f = s->start->next;
        *d = s->start->i:
        free(s->start):
        s \rightarrow start = f:
        s \rightarrow size = s \rightarrow size - 1:
        return true:
13
14
15
     bool stack peek(stack* s. stacktype* d)
16
17
        if((s==NULL) || (s->start==NULL)){
18
            return false:
19
20
        *d = s->start->i:
        return true;
22
```

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE]:
        str[0] = '\0':
        if((s==NULL) || (s->size <1)){
           return:
        dataframe* p = s->start:
        while (p) f
           sprintf(tmp. FORMATSTR. p->i):
           strcat(str. tmp):
           strcat(str. "|"):
           p = p -> next:
14
        str[strlen(str)-1] = '\0';
16
17
18
     bool stack free(stack* s)
19
20
        if(s){
           dataframe* p = s->start;
           while (p!=NULL){
              dataframe* tmp = p->next;
              free(p):
              p = tmp;
26
27
           free(s):
28
        return true;
30
```

Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

S: ADTs - Queues

T: ADTs - Trees

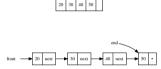
U : ADTs - Hashing

V : Algorithms II - Sort

W: Algorithms III - Huffman/Strings

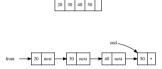
Y: Summary

FIFO (First in, First out):



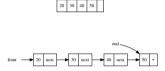
• Intuitively more "useful" than a stack.

FIFO (First in, First out):



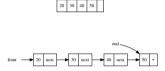
- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)

FIFO (First in, First out):



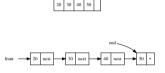
- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)
- Operations include enqueue, dequeue and size.

FIFO (First in, First out):



- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)
- Operations include enqueue, dequeue and size.

FIFO (First in, First out):



- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)
- Operations include enqueue, dequeue and size.

queue.h

```
#pragma once
    #include " .. / General/general .h"
     typedef int queuetype;
     typedef struct queue queue;
     #include <stdio.h>
    #include <stdlib.h>
    #include <string.h>
    Winclude (assert h)
     /* Create an empty queue */
     queue* queue init(void):
     /* Add element on end */
     void queue_enqueue(queue* q, queuetype v);
     /* Take element off front */
     bool queue dequeue(queue* q. queuetype* d):
     /* Return size of queue */
     int queue size(queue* q):
     /* Clears all space used */
     bool queue_free(queue* q);
24
     /* Helps with visualisation & testing */
     void queue tostring(queue* q. char* str):
```

specific.h

```
#pragma once
define FORMATSTR "%d"
define ELEMSIZE 20

define QUEUETYPE "Fixed"

define BOUNDED 5000

struct queue {
    /* Underlying array */
    queuetype a[BOUNDED];
    int front;
    int end;
};

define BOTFILE 5000
```

specific.h

```
#pragma once

#define FORMAISIR "%d"

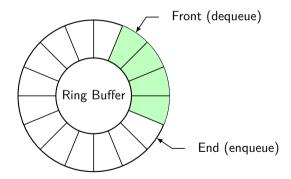
#define ELEMSIZE 20

#define QUEUETYPE "Fixed"

#define BOUNDED 5000

struct queue {
    /* Underlying array */
    queuetype a[BOUNDED];
    int front;
    int end;
};

#define DOTFILE 5000
```



fixed.c

```
#include " .. / queue . h"
     #include "specific.h"
     void inc(int* p);
     queue * queue init(void)
         queue* q = (queue*) ncalloc(1, sizeof(queue));
         return q;
10
11
12
13
14
     void queue_enqueue(queue* q, queuetype d)
15
16
17
18
19
20
21
22
         if (a) {
             q \rightarrow a[q \rightarrow end] = d:
             _inc(&q->end);
             if (q->end == q->front){
                 on_error("Queue too large");
```

fixed.c

```
#include " .. / queue . h"
     #include "specific.h"
     void inc(int* p);
     queue * queue init(void)
         queue* q = (queue*) ncalloc(1, sizeof(queue));
         return q;
     void queue_enqueue(queue* q, queuetype d)
14
15
16
17
         if (a) {
            q \rightarrow a[q \rightarrow end] = d:
            _inc(&q->end);
18
19
20
21
             if (q->end == q->front){
                on_error("Queue too large");
22
```

```
bool queue dequeue(queue* q. queuetype* d)
        if ((a==NULL) || (a->front==a->end)){
           return false:
        *d = q -  a[q -  front]:
        inc(&g->front):
        return true:
9
10
11
     void queue tostring(queue* q. char* str)
12
13
        char tmp[ELEMSIZE];
        str[0] = '\0':
        if((q=NULL) || (queue_size(q)==0)){
16
           return:
17
18
        for(int i=q->front; i != q->end;){
           sprintf(tmp, FORMATSTR, q->a[i]);
20
           strcat(str. tmp):
21
           strcat(str. "|"):
22
           inc(&zi):
23
24
        str[strlen(str)-1] = '\0':
```

```
int queue_size(queue* q)
         if (q==NULL) {
            return 0;
         if(q->end >= q->front){
            return q->end-q->front;
9
10
11
         return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
         free(q):
15
16
         return true;
17
18
19
20
21
     void _inc(int* p)
         *p = (*p + 1) \% BOUNDED;
```

```
int queue_size(queue* q)
         if (q==NULL) {
            return 0:
        if(q-)end = q-)front){
            return q->end-q->front;
9
10
11
        return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
         free(q):
15
16
        return true:
17
18
19
20
     void inc(int* p)
        *p = (*p + 1) \% BOUNDED:
```

 We need a thorough testing program

```
int queue_size(queue* q)
        if (a==NULL) {
            return 0:
        if(q-)end = q-)front)
            return q->end-q->front;
9
10
11
        return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
        free(q);
15
16
        return true:
17
18
19
     void inc(int* p)
20
        *p = (*p + 1) \% BOUNDED;
```

- We need a thorough testing program
- We'll see queues again for traversing trees

```
int queue_size(queue* q)
         if (a==NULL) {
            return 0:
        if(q-)end = q-)front)
            return q->end-q->front;
9
10
11
        return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
         free(q):
15
16
        return true;
17
18
19
     void inc(int* p)
20
        *p = (*p + 1) \% BOUNDED:
```

- We need a thorough testing program
- We'll see queues again for traversing trees
- Simulating a (slow) printer

ADTs: Queues (Linked) I

specific.h

```
#pragma once

define FORMATSIR "%d"

define ELEMSIZE 20

define QUEUETYPE "Linked"

struct dataframe {
    queuetype i;
    struct dataframe* next;
    };

typedef struct dataframe dataframe;

struct queue {
        /* Underlying array */
        dataframe* front;
    dataframe* end;
    int size;
};
```

ADTs: Queues (Linked) I

specific.h

```
#pragma once
    #define FORMATSTR "%d"
    #define ELEMSIZE 20
    #define OUFUETYPE "Linked"
    struct dataframe {
       queuetype i;
        struct dataframe* next;
    }:
12
13
    typedef struct dataframe dataframe;
14
    struct queue {
15
      /* Underlying array */
       dataframe* front:
17
       dataframe* end:
       int size:
19
    }:
```

linked.c

```
#include " .. / queue .h"
      #include "specific.h"
      queue* queue init(void)
          queue* q = (queue*) ncalloc(1, sizeof(queue));
          return q;
      void queue_enqueue(queue* q, queuetype d)
          dataframe* f;
          if (q==NULL) {
             return:
          /* Copy the data */
          f = ncalloc(1, sizeof(dataframe));
          f \rightarrow i = d:
          /* 1st one */
          if (a->front == NULL) {
             a \rightarrow front = f:
24
             a \rightarrow end = f:
             q \rightarrow size = q \rightarrow size + 1;
             return:
28
          /* Not 1st */
          q \rightarrow end \rightarrow next = f:
          a->end = f:
31
          q \rightarrow size = q \rightarrow size + 1;
```

S : ADTs - Queues 45 / 8:

ADTs: Queues (Linked) II

```
bool queue dequeue(queue* q, queuetype* d)
         dataframe* f;
         if((q=NULL) || (q->front=NULL) || (q->end=NULL)){
            return false;
         f = q->front->next;
         *d = q->front->i;
         free(q->front);
         q \rightarrow front = f;
11
12
13
14
15
16
17
18
19
20
         q->size = q->size - 1;
         return true;
     bool queue free (queue * q)
         if (a) {
             dataframe* tmp:
            dataframe* p = q->front;
            while (p!=NULL) {
                tmp = p->next;
22
23
24
25
26
27
28
                free(p);
                p = tmp;
             free(q);
         return true;
```

ADTs: Queues (Linked) II

```
bool queue dequeue(queue* q, queuetype* d)
         dataframe* f:
         if ((q=NULL) || (q->front=NULL) || (q->end=NULL)){
            return false;
         f = q->front->next;
         *d = q-> front -> i;
         free(q->front);
        q \rightarrow front = f;
         q \rightarrow size = q \rightarrow size - 1;
         return true;
13
14
     bool queue free (queue * q)
        if (a) {
18
19
            dataframe* tmp:
            dataframe* p = q->front;
20
            while (p!=NULL) {
                tmp = p -> next:
                free(p);
23
24
                p = tmp:
25
26
            free(q);
         return true;
28
```

```
void queue tostring(queue* q, char* str)
        dataframe *p;
        char tmp[ELEMSIZE];
        str[0] = '\0';
        if ((q=NULL) || (q->front == NULL)){
           return:
        p = q - front;
        while(p){
           sprintf(tmp, FORMATSTR, p->i);
           strcat(str. tmp);
           strcat(str. "|");
           p = p -   next;
16
        str[strlen(str)-1] = '\0';
17
18
     int queue size(queue* q)
20
21
        if ((q=NULL) || (q->front=NULL)){
23
           return 0:
24
25
        return q->size;
```

• There exists a nice package, called Graphviz:

sudo apt install graphviz

• There exists a nice package, called Graphviz:

sudo apt install graphviz

 This allows the visualisation of graphs/dynamic structures using the simple .dot language:

```
digraph {
   a -> b; b -> c; c -> a;
}
```

 There exists a nice package, called Graphviz:

sudo apt install graphviz

 This allows the visualisation of graphs/dynamic structures using the simple .dot language:

```
digraph {
   a -> b; b -> c; c -> a;
}
```

To create a .pdf: dot -Tpdf -o graphviz.pdf examp1.dot

 There exists a nice package, called Graphviz:

sudo apt install graphviz

 This allows the visualisation of graphs/dynamic structures using the simple .dot language:

```
digraph {
   a -> b; b -> c; c -> a;
}
```

To create a .pdf: dot -Tpdf -o graphviz.pdf examp1.dot

 There exists a nice package, called Graphviz:

sudo apt install graphviz

 This allows the visualisation of graphs/dynamic structures using the simple .dot language:

```
digraph {
   a -> b; b -> c; c -> a;
}
```

To create a .pdf: dot -Tpdf -o graphviz.pdf examp1.dot

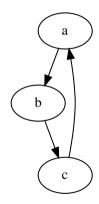


Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

S : ADTs - Queues

T: ADTs - Trees

U : ADTs - Hashing

V : Algorithms II - Sort

W: Algorithms III - Huffman/Strings

Y: Summary

• Binary trees are used extensively in computer science

• Binary trees are used extensively in computer science

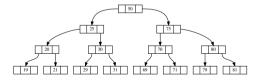
Game Trees

- Binary trees are used extensively in computer science
- Game Trees
- Searching

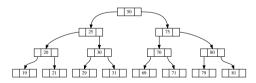
- Binary trees are used extensively in computer science
- Game Trees
- Searching
- Sorting

- Binary trees are used extensively in computer science
- Game Trees
- Searching
- Sorting

- Binary trees are used extensively in computer science
- Game Trees
- Searching
- Sorting

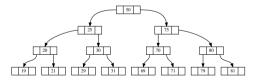


- Binary trees are used extensively in computer science
- Game Trees
- Searching
- Sorting



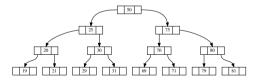
• Trees drawn upside-down!

- Binary trees are used extensively in computer science
- Game Trees
- Searching
- Sorting



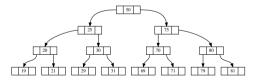
- Trees drawn upside-down!
- Ancestor relationships: '50' is the parent of '25' and '75'.

- Binary trees are used extensively in computer science
- Game Trees
- Searching
- Sorting



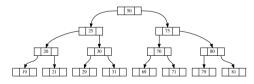
- Trees drawn upside-down!
- Ancestor relationships: '50' is the parent of '25' and '75'.
- Can refer to left and right children

- Binary trees are used extensively in computer science
- Game Trees
- Searching
- Sorting



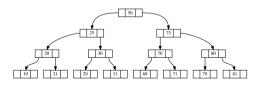
- Trees drawn upside-down!
- Ancestor relationships: '50' is the parent of '25' and '75'.
- Can refer to left and right children
- In a tree, there is only one path from the root to any child

- Binary trees are used extensively in computer science
- Game Trees
- Searching
- Sorting



- Trees drawn upside-down!
- Ancestor relationships: '50' is the parent of '25' and '75'.
- Can refer to left and right children
- In a tree, there is only one path from the root to any child
- A node with no children is a leaf

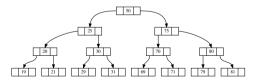
- Binary trees are used extensively in computer science
- Game Trees
- Searching
- Sorting



- Trees drawn upside-down!
- Ancestor relationships: '50' is the parent of '25' and '75'.
- Can refer to left and right children
- In a tree, there is only one path from the root to any child
- A node with no children is a leaf
- Most trees need to be created dynamically

Binary Trees : Data Structures

- Binary trees are used extensively in computer science
- Game Trees
- Searching
- Sorting

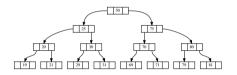


- Trees drawn upside-down!
- Ancestor relationships: '50' is the parent of '25' and '75'.
- Can refer to left and right children
- In a tree, there is only one path from the root to any child
- A node with no children is a leaf
- Most trees need to be created dynamically
- Empty subtrees are set to NULL

「: ADTs - Trees 49 / 83

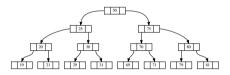
Binary Search Trees

In a binary search tree the left-hand tree of a parent contains all keys less than the parent node, and the right-hand side all the keys greater than the parent node.



Binary Search Trees

In a binary search tree the left-hand tree of a parent contains all keys less than the parent node, and the right-hand side all the keys greater than the parent node.



bst.h

```
#include " .. / General/general . h"
    #include " .. / Queue/queue . h "
     #include <stdio.h>
     #include <stdlib.h>
     #include <assert.h>
     bst* bst_init(void);
     /* Insert 1 item into the tree */
     bool bst insert(bst* b, treetype d);
     /* Return number of nodes in tree */
     int bst size(bst* b);
16
     /* Whether the data d is stored in the tree */
     bool bst isin(bst* b, treetype d);
18
19
     /* Bulk insert n items from an array a into an initialised tree */
20
     bool bst_insertarray(bst* b, treetype* a, int n);
21
     /* Clear all memory associated with tree. & set pointer to NULL */
     bool bst free(bst* b):
24
25
     /* Optional ? */
     char* bst_preorder(bst* b);
     void bst printlevel(bst* b):
     /* Create string with tree as ((head)(left)(right)) */
     char* bst printlisp(bst* b):
     /* Use Graphviz via a .dot file */
     void bst todot(bst* b. char* dotname):
```

Binary Search Trees: Linked I

specific.h

```
#include <string.h>

typedef int treetype;

define FORMATSIR "%i"

define ELEMSIZE 20

define ESTITYPE "Linked"

struct dataframe {
 treetype d;
 struct dataframe* left;
 struct dataframe* right;
 };

typedef struct dataframe dataframe;

dataframe* top;
 /* Data element size, in bytes */
};

typedef struct bst bst;
```

Binary Search Trees: Linked I

specific.h

```
1  ##include <string.h>
2
2
4  #define FORMATSIR "%i"
5  #define ELEMSIZE 20
6  #define BSTTYPE "Linked"
7
8  struct dataframe {
    treetype d;
    struct dataframe* left;
    struct dataframe* right;
    };
12  };
13  typedef struct dataframe dataframe;
14
15  struct bst {
    dataframe* top;
    /* Data element size, in bytes */
    };
19  typedef struct bst bst;
```

```
/* Based on geekforgeeks.org */
dataframe* _insert(dataframe* t, treetype d)
{
    dataframe* f;
    /* If the tree is empty, return a new frame */
    if (t == NULL){
        f = ncalloc(sizeof(dataframe), 1);
        f ->d = d;
        return f;
    }
    /* Otherwise, recurs down the tree */
    if (d < t -> d){
        t -> life = _insert(t -> left, d);
    }
    else if(d > t -> d){
        t -> right = _insert(t -> right, d);
    }
    /* return the (unchanged) dataframe pointer */
    return t;
}
```

Binary Search Trees: Linked II

```
bool __isin(dataframe* t, treetype d)
{
    if(t=NULL){
        return false;
    }
    if(t>d == d){
        return true;
    }
    if(d < t->d){
        return __isin(t->left, d);
    }
    else{
        return __isin(t->right, d);
    }
    return false;
}
```

Binary Search Trees: Linked II

```
bool __isin(dataframe* t, treetype d)
{
   if(t=NULL){
      return false;
   }
   if(t->d == d){
      return true;
   }
   if(d < t->d){
      return __isin(t->left , d);
   }
   else{
      return __isin(t->right , d);
   }
   return false;
}
```

```
char* _printlisp(dataframe* t)
  char tmp[ELEMSIZE];
  char *s1, *s2, *p;
  if(t==NULL){
     /* \0 string */
     p = ncalloc(1,1);
     return p;
  sprintf(tmp, FORMATSTR, t->d);
  s1 = _printlisp(t->left);
  s2 = _printlisp(t->right);
  p = ncalloc(strlen(s1)+strlen(s2)+strlen(tmp)+
       strlen("()() "), 1);
  sprintf(p, "%s(%s)(%s)", tmp, s1, s2);
  free(s1):
  free(s2):
  return p;
```

 Don't rush to assume a linked data structure must be used to implement trees.

- Don't rush to assume a linked data structure must be used to implement trees.
- You could use 1 cell of an array for the first node, the next two cells for its children, the next 4 cells for their children and so on.

- Don't rush to assume a linked data structure must be used to implement trees.
- You could use 1 cell of an array for the first node, the next two cells for its children, the next 4 cells for their children and so on.
- You need to mark which cells are in use & which aren't ...

- Don't rush to assume a linked data structure must be used to implement trees.
- You could use 1 cell of an array for the first node, the next two cells for its children, the next 4 cells for their children and so on.
- You need to mark which cells are in use & which aren't ...

- Don't rush to assume a linked data structure must be used to implement trees.
- You could use 1 cell of an array for the first node, the next two cells for its children, the next 4 cells for their children and so on.
- You need to mark which cells are in use & which aren't ...

Counting from cell 1, for a tree with *n* nodes:

To find	Use	Iff
The root	A[1]	A is nonempty
The left child of $A[i]$	A[2i]	$2i \leq n$
The parent of $A[i]$	A[i/2]	i > 1
Is A[i] a leaf?	True	2 <i>i</i> > <i>n</i>

: ADTs - Trees 53 / 83

Binary Search Trees: Realloc

specific.h

```
#include <stdbool.h>
    typedef int treetype;
    #define FORMATSTR "%i"
    #define ELEMSIZE 20
    #define BSTTYPE "Realloc"
  // Probably (2^n) -1
    #define INITSIZE 31
    #define SCALEFACTOR 2
    struct dataframe {
       treetype d;
       bool isvalid:
15
    typedef struct dataframe dataframe;
17
    struct bst {
19
       dataframe* a:
       int capacity;
    typedef struct bst bst:
```

Binary Search Trees : Realloc

specific.h

```
#include <stdhool h>
    typedef int treetype:
    #define FORMATSTR "%i"
    #define FIFMSIZE 20
    #define BSTTYPE "Realloc"
    // Probably (2^n) -1
    #define INITSIZE 31
    #define SCALEFACTOR 2
    struct dataframe {
        treetype d:
        bool isvalid:
15
    typedef struct dataframe dataframe:
17
    struct bst {
19
       dataframe* a:
       int capacity:
    typedef struct bst bst:
```

Using a queue for Level-Order traversal:

```
void bst_printlevel(bst* b)
{
    treetype n;
    if((b=NULL) || (! _isvalid(b, 0))){
        return;
    }
    /* Make a queue of cell indices */
    queue* q = queue_init();
    queue_enqueue(q, 0);
    while(queue_dequeue(q, &m) && _isvalid(b, (int)n)){
        printf(FORMATSTR, b->a[n].d);
        putchar(' ');
        queue_enqueue(q, _leftchild((int)n));
        queue_enqueue(q, _rightchild((int)n));
}
```

Binary Search Trees : Complexity

ullet So, in a nicely balanced tree, insertion, deletion and search are all $O(\log n)$.

Binary Search Trees: Complexity

- So, in a nicely balanced tree, insertion, deletion and search are all $O(\log n)$.
- But: if the root of the tree is not well chosen, or the keys to be inserted are ordered, the tree can become a linked list!

Binary Search Trees : Complexity

- So, in a nicely balanced tree, insertion, deletion and search are all $O(\log n)$.
- But: if the root of the tree is not well chosen, or the keys to be inserted are ordered, the tree can become a linked list!

• In this case, complexity becomes O(n).

Binary Search Trees : Complexity

- So, in a nicely balanced tree, insertion, deletion and search are all $O(\log n)$.
- But: if the root of the tree is not well chosen, or the keys to be inserted are ordered, the tree can become a linked list!
- In this case, complexity becomes O(n).
- The tree search performs best when well balanced trees are formed.

Binary Search Trees: Complexity

- So, in a nicely balanced tree, insertion, deletion and search are all $O(\log n)$.
- But: if the root of the tree is not well chosen, or the keys to be inserted are ordered, the tree can become a linked list!
- In this case, complexity becomes O(n).
- The tree search performs best when well balanced trees are formed.
- Large body of literature about creating & re-balancing trees Red-Black trees, Tries, 2-3 trees, AVL trees etc.

 Often we wish to compress data, to reduce storage requirements, or to speed transmission.

- Often we wish to compress data, to reduce storage requirements, or to speed transmission.
- Text is particularly suited to compression since using one byte per character is wasteful - some letters occur much more frequently.

- Often we wish to compress data, to reduce storage requirements, or to speed transmission.
- Text is particularly suited to compression since using one byte per character is wasteful - some letters occur much more frequently.
- Need to give frequently occurring letters short codes, typically a few bits. Less common letters can have long bit patterns.

- Often we wish to compress data, to reduce storage requirements, or to speed transmission
- Text is particularly suited to compression since using one byte per character is wasteful - some letters occur much more frequently.
- Need to give frequently occurring letters short codes, typically a few bits. Less common letters can have long bit patterns.

To encode the string "BABBAGE":

- Often we wish to compress data, to reduce storage requirements, or to speed transmission
- Text is particularly suited to compression since using one byte per character is wasteful - some letters occur much more frequently.
- Need to give frequently occurring letters short codes, typically a few bits. Less common letters can have long bit patterns.

To encode the string "BABBAGE":

- Often we wish to compress data, to reduce storage requirements, or to speed transmission
- Text is particularly suited to compression since using one byte per character is wasteful - some letters occur much more frequently.
- Need to give frequently occurring letters short codes, typically a few bits. Less common letters can have long bit patterns.

• To encode the string "BABBAGE":

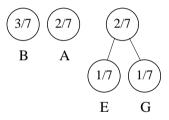


Keep a list of characters, ordered by their frequency

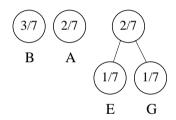
• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :

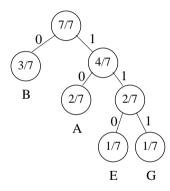
• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :

• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :



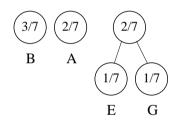
• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :

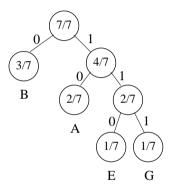




: ADTs - Trees 57 / 83

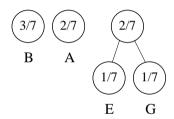
• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :

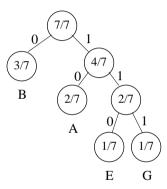




ullet A = 10, B = 0, E = 110, G = 111

• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :





- \bullet A = 10, B = 0, E = 110, G = 111
- String stored using 13 bits.

「: ADTs - Trees 57 / 83

Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

S : ADTs - Queues

T · ADTs - Trees

U : ADTs - Hashing

V : Algorithms II - Sor

W: Algorithms III - Huffman/Strings

Y: Summary

ADTs: Hashing

 To keep records of employees we might index (search) them by using their National Insurance number:

xx-##-##-##-x

ADTs: Hashing

 To keep records of employees we might index (search) them by using their National Insurance number:

```
xx-##-##-##-x
```

• There are 17.6 billion combinations (around 2³⁴).

ADTs : Hashing

 To keep records of employees we might index (search) them by using their National Insurance number:

```
xx-##-##-##-x
```

- There are 17.6 billion combinations (around 2³⁴).
- Could use an array of 17.6 billion entries, which would make searching for a particular entry trivial!

ADTs: Hashing

 To keep records of employees we might index (search) them by using their National Insurance number:

xx-##-##-##-x

- There are 17.6 billion combinations (around 2³⁴).
- Could use an array of 17.6 billion entries, which would make searching for a particular entry trivial!
- Especially wasteful since only our (5000) employees need to be stored.

 To keep records of employees we might index (search) them by using their National Insurance number:

xx-##-##-##-x

- There are 17.6 billion combinations (around 2³⁴).
- Could use an array of 17.6 billion entries, which would make searching for a particular entry trivial!
- Especially wasteful since only our (5000) employees need to be stored.

 To keep records of employees we might index (search) them by using their National Insurance number:

xx-##-##-##-x

- There are 17.6 billion combinations (around 2³⁴).
- Could use an array of 17.6 billion entries, which would make searching for a particular entry trivial!
- Especially wasteful since only our (5000) employees need to be stored.

 Here we examine a method that, using an array of 6000 elements, would require 2.1 comparisons on average.

 To keep records of employees we might index (search) them by using their National Insurance number:

- There are 17.6 billion combinations (around 2³⁴).
- Could use an array of 17.6 billion entries, which would make searching for a particular entry trivial!
- Especially wasteful since only our (5000) employees need to be stored.

- Here we examine a method that, using an array of 6000 elements, would require 2.1 comparisons on average.
- A hash function is a mapping, h(K), that maps from key K, onto the index of an entry.

 To keep records of employees we might index (search) them by using their National Insurance number:

- There are 17.6 billion combinations (around 2³⁴).
- Could use an array of 17.6 billion entries, which would make searching for a particular entry trivial!
- Especially wasteful since only our (5000) employees need to be stored.

- Here we examine a method that, using an array of 6000 elements, would require 2.1 comparisons on average.
- A hash function is a mapping, h(K), that maps from key K, onto the index of an entry.
- A black-box into which we insert a key (e.g. NI number) and out pops an array index.

 To keep records of employees we might index (search) them by using their National Insurance number:

- There are 17.6 billion combinations (around 2³⁴).
- Could use an array of 17.6 billion entries, which would make searching for a particular entry trivial!
- Especially wasteful since only our (5000) employees need to be stored.

- Here we examine a method that, using an array of 6000 elements, would require 2.1 comparisons on average.
- A hash function is a mapping, h(K), that maps from key K, onto the index of an entry.
- A black-box into which we insert a key (e.g. NI number) and out pops an array index.
- As an example lets use an array of size 11 to store some airport codes, e.g. PEK, BRS, FRA.

In a three letter string X₂X₁X₀ the letter 'A' has the value 0,
 'B' has the value 1 etc.

- In a three letter string X₂X₁X₀ the letter 'A' has the value 0,
 'B' has the value 1 etc.
- One hash function is:

$$h(K) = (X_2 * 26^2 + X_1 * 26 + X_0)\%11$$

- In a three letter string X₂X₁X₀ the letter 'A' has the value 0, 'B' has the value 1 etc.
- One hash function is:

$$h(K) = (X_2 * 26^2 + X_1 * 26 + X_0)\%11$$

• Applying this to "DCA": h("DCA") = $(3*26^2 + 2*26 + 0)\%11$ h("DCA") = (2080)%11h("DCA") = 1

- In a three letter string X₂X₁X₀ the letter 'A' has the value 0, 'B' has the value 1 etc.
- One hash function is:

$$h(K) = (X_2 * 26^2 + X_1 * 26 + X_0)\%11$$

• Applying this to "DCA": h("DCA") = $(3*26^2 + 2*26 + 0)\%11$ h("DCA") = (2080)%11h("DCA") = 1

- In a three letter string X₂X₁X₀ the letter 'A' has the value 0,
 'B' has the value 1 etc.
- One hash function is:

$$h(K) = (X_2 * 26^2 + X_1 * 26 + X_0)\%11$$

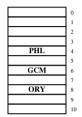
• Applying this to "DCA": h("DCA") = $(3*26^2 + 2*26 + 0)\%11$ h("DCA") = (2080)%11h("DCA") = 1 Inserting "PHL", "ORY" and "GCM":



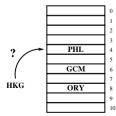
- In a three letter string X₂X₁X₀ the letter 'A' has the value 0, 'B' has the value 1 etc.
- One hash function is:

$$h(K) = (X_2 * 26^2 + X_1 * 26 + X_0)\%11$$

• Applying this to "DCA": h("DCA") = $(3*26^2 + 2*26 + 0)\%11$ h("DCA") = (2080)%11h("DCA") = 1 • Inserting "PHL", "ORY" and "GCM":



• However, inserting "HKG" causes a collision.



 An ideal hashing function maps keys into the array in a *uniform* and *random* manner.

- An ideal hashing function maps keys into the array in a uniform and random manner.
- Collisions occur when a hash function maps two different keys onto the same address.

- An ideal hashing function maps keys into the array in a uniform and random manner.
- Collisions occur when a hash function maps two different keys onto the same address.
- It's very difficult to choose 'good' hashing functions.

- An ideal hashing function maps keys into the array in a uniform and random manner.
- Collisions occur when a hash function maps two different keys onto the same address.
- It's very difficult to choose 'good' hashing functions.
- Collisions are common the von Mises paradox. When 23 keys are randomly mapped onto 365 addresses there is a 50% chance of a collision.

- An ideal hashing function maps keys into the array in a uniform and random manner.
- Collisions occur when a hash function maps two different keys onto the same address.
- It's very difficult to choose 'good' hashing functions.
- Collisions are common the von Mises paradox. When 23 keys are randomly mapped onto 365 addresses there is a 50% chance of a collision.

- An ideal hashing function maps keys into the array in a uniform and random manner.
- Collisions occur when a hash function maps two different keys onto the same address.
- It's very difficult to choose 'good' hashing functions.
- Collisions are common the von Mises paradox. When 23 keys are randomly mapped onto 365 addresses there is a 50% chance of a collision.

 The policy of finding another free location if a collision occurs is called open-addressing.

- An ideal hashing function maps keys into the array in a uniform and random manner.
- Collisions occur when a hash function maps two different keys onto the same address.
- It's very difficult to choose 'good' hashing functions.
- Collisions are common the von Mises paradox. When 23 keys are randomly mapped onto 365 addresses there is a 50% chance of a collision.

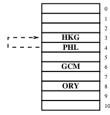
- The policy of finding another free location if a collision occurs is called open-addressing.
- If a collision occurs then keep stepping backwards (with wrap-around) until a free location is encountered.

- An ideal hashing function maps keys into the array in a uniform and random manner.
- Collisions occur when a hash function maps two different keys onto the same address.
- It's very difficult to choose 'good' hashing functions.
- Collisions are common the von Mises paradox. When 23 keys are randomly mapped onto 365 addresses there is a 50% chance of a collision.

- The policy of finding another free location if a collision occurs is called open-addressing.
- If a collision occurs then keep stepping backwards (with wrap-around) until a free location is encountered.

- An ideal hashing function maps keys into the array in a *uniform* and *random* manner.
- Collisions occur when a hash function maps two different keys onto the same address.
- It's very difficult to choose 'good' hashing functions.
- Collisions are common the von Mises paradox. When 23 keys are randomly mapped onto 365 addresses there is a 50% chance of a collision.

- The policy of finding another free location if a collision occurs is called open-addressing.
- If a collision occurs then keep stepping backwards (with wrap-around) until a free location is encountered.



 This simple method of open-addressing is linear-probing.

- This simple method of open-addressing is linear-probing.
- The step taken each time (probe decrement) need not be 1.

- This simple method of open-addressing is linear-probing.
- The step taken each time (probe decrement) need not be 1.
- Open-addressing through use of linear-probing is a very simple technique, double-hashing is generally much more successful.

- This simple method of open-addressing is linear-probing.
- The step taken each time (probe decrement) need not be 1.
- Open-addressing through use of linear-probing is a very simple technique, double-hashing is generally much more successful.
- A second function p(K) decides the size of the probe decrement.

- This simple method of open-addressing is linear-probing.
- The step taken each time (probe decrement) need not be 1.
- Open-addressing through use of linear-probing is a very simple technique, double-hashing is generally much more successful.
- A second function p(K) decides the size of the probe decrement.

- This simple method of open-addressing is linear-probing.
- The step taken each time (probe decrement) need not be 1.
- Open-addressing through use of linear-probing is a very simple technique, double-hashing is generally much more successful.
- A second function p(K) decides the size of the probe decrement.

 The function is chosen so that two keys which collide at the same address will have different probe decrements, e.g.:

$$p(K) = MAX(1, ((X_2 * 26^2 + X_1 * 26 + X_0)/11)\%11)$$

- This simple method of open-addressing is linear-probing.
- The step taken each time (probe decrement) need not be 1.
- Open-addressing through use of linear-probing is a very simple technique, double-hashing is generally much more successful.
- A second function p(K) decides the size of the probe decrement

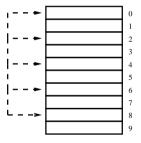
 The function is chosen so that two keys which collide at the same address will have different probe decrements, e.g.:

$$p(K) = MAX(1, ((X_2 * 26^2 + X_1 * 26 + X_0)/11)\%11)$$

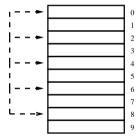
• Although "PHL" and "HKG" share the same primary hash value of h(K) = 4, they have different probe decrements:

$$p("PHL") = 4$$
$$p("HKG") = 3$$

• If the size of our array, M, was even and the probe decrement was chosen to be 2, then only half of the locations could be probed.

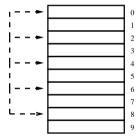


 If the size of our array, M, was even and the probe decrement was chosen to be 2, then only half of the locations could be probed.



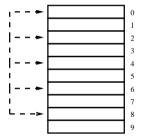
• Often we choose our table size to be a prime number and our probe decrement to be a number in the range $1 \dots M - 1$.

 If the size of our array, M, was even and the probe decrement was chosen to be 2, then only half of the locations could be probed.



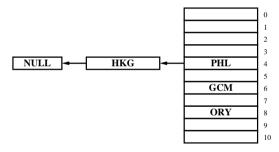
• Often we choose our table size to be a prime number and our probe decrement to be a number in the range $1 \dots M - 1$.

 If the size of our array, M, was even and the probe decrement was chosen to be 2, then only half of the locations could be probed.



• Often we choose our table size to be a prime number and our probe decrement to be a number in the range $1 \dots M - 1$.

Open-addressing is not the only method of collision reduction. Another common one is separate chaining.



A Practical Hash Function

```
#include <stdio.h>
    int hash(unsigned int sz, char *s);
    int main(void)
       char str[] = "Hello World!";
       // Hash modulus 7919
        printf("%d\n", hash(7919, str));
        return 0:
12
13
14
15
    Modified Bernstein hashing
    5381 & 33 are magic numbers required by the algorithm
19
    int hash(unsigned int sz, char *s)
       unsigned long hash = 5381;
       int c;
       while ((c = (*s++))){
           hash = 33 * hash ^ c:
        return (int)(hash%sz);
```

Execution:

5479

A Practical Hash Function

```
#include <stdio h>
    int hash(unsigned int sz. char *s):
     int main (woid)
       char str[] = "Hello World!";
       // Hash modulus 7919
        printf("%d\n", hash(7919, str));
        return 0:
12
13
15
    Modified Bernstein hashing
17
    5381 & 33 are magic numbers required by the algorithm
19
    int hash(unsigned int sz. char *s)
20
21
        unsigned long hash = 5381;
        int c:
        while ((c = (*s++))){
           hash = 33 * hash ^ c:
        return (int)(hash%sz);
```

Execution:

5479

Has similarities to the implementation of rand():

```
int rand r(unsigned int* seed):
int main (void)
  unsigned int seed = 0:
   printf("%d\n", rand r(&seed)):
  return 0:
/* This algorithm is mentioned in the ISO C standard,
   here extended for 32 bits. */
int rand_r(unsigned int* seed)
 unsigned int next = *seed;
  int result:
 next *= 1103515245;
  next += 12345:
 result = (unsigned int) (next / 65536) % 2048;
 next *= 1103515245;
 next += 12345;
 result <<= 10:
 result ^= (unsigned int) (next / 65536) % 1024:
  next *= 1103515245:
 next += 12345:
 result <<= 10:
 result ^= (unsigned int) (next / 65536) % 1024:
 *seed = next;
 return result:
```

Execution:

Cuckoo Hashing

 We have two tables, each with their own hash function.

```
Empty: copied farandoles into table 0(4)
Empty: copied bronzine into table 0(12)
Empty: copied auscultatory into table 0(5)
Empty: copied bifer into table 0(13)
Empty: copied steepgrass into table 0(6)
Empty: copied prevised into table 0(7)
Empty: copied oomph into table 0(8)
empodium, so cuckooed out auscultatory from table 0(5)
Empty: copied auscultatory into table 1(10)
interquarreled, so cuckooed out bronzine from table 0(12)
Empty: copied bronzine into table 1(5)
ranseur, so cuckooed out empodium from table 0(5)
Empty: copied empodium into table 1(4)
Empty: copied megalodon into table 0(11)
geosynchronous, so cuckooed out megalodon from table 0(11)
Empty: copied megalodon into table 1(14)
Empty: copied osmeteria into table 0(14)
Table getting full -> rehashed old sz =16
```

Cuckoo Hashing

- We have two tables, each with their own hash function.
- We only need to check two cells when searching.

Empty: copied farandoles into table 0(4) Empty: copied bronzine into table 0(12) Empty: copied auscultatory into table 0(5) Empty: copied bifer into table 0(13) Empty: copied steepgrass into table 0(6) Empty: copied prevised into table 0(7) Empty: copied oomph into table 0(8) empodium, so cuckooed out auscultatory from table 0(5) Empty: copied auscultatory into table 1(10) interquarreled, so cuckooed out bronzine from table 0(12) Empty: copied bronzine into table 1(5) ranseur, so cuckooed out empodium from table 0(5) Empty: copied empodium into table 1(4) Empty: copied megalodon into table 0(11) geosynchronous, so cuckooed out megalodon from table 0(11) Empty: copied megalodon into table 1(14) Empty: copied osmeteria into table 0(14) Table getting full -> rehashed old sz =16

Cuckoo Hashing

- We have two tables, each with their own hash function.
- We only need to check two cells when searching.
- On collision, the existing item is 'cuckooed' out of it's cell into the other table.

```
Empty: copied farandoles into table 0(4)
Empty: copied bronzine into table 0(12)
Empty: copied auscultatory into table 0(5)
Empty: copied bifer into table 0(13)
Empty: copied steepgrass into table 0(6)
Empty: copied prevised into table 0(7)
Empty: copied oomph into table 0(8)
empodium, so cuckooed out auscultatory from table 0(5)
Empty: copied auscultatory into table 1(10)
interquarreled, so cuckooed out bronzine from table 0(12)
Empty: copied bronzine into table 1(5)
ranseur, so cuckooed out empodium from table 0(5)
Empty: copied empodium into table 1(4)
Empty: copied megalodon into table 0(11)
geosynchronous, so cuckooed out megalodon from table 0(11)
Empty: copied megalodon into table 1(14)
Empty: copied osmeteria into table 0(14)
Table getting full -> rehashed old sz =16
```

Cuckoo Hashing

- We have two tables, each with their own hash function.
- We only need to check two cells when searching.
- On collision, the existing item is 'cuckooed' out of it's cell into the other table.

```
Empty: copied farandoles into table 0(4)
Empty: copied bronzine into table 0(12)
Empty: copied auscultatory into table 0(5)
Empty: copied bifer into table 0(13)
Empty: copied steepgrass into table 0(6)
Empty: copied prevised into table 0(7)
Empty: copied oomph into table 0(8)
empodium, so cuckooed out auscultatory from table 0(5)
Empty: copied auscultatory into table 1(10)
interquarreled, so cuckooed out bronzine from table 0(12)
Empty: copied bronzine into table 1(5)
ranseur, so cuckooed out empodium from table 0(5)
Empty: copied empodium into table 1(4)
Empty: copied megalodon into table 0(11)
geosynchronous, so cuckooed out megalodon from table 0(11)
Empty: copied megalodon into table 1(14)
Empty: copied osmeteria into table 0(14)
Table getting full -> rehashed old sz =16
```

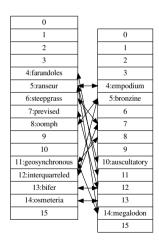
U : ADTs - Hashing 65 / 83

Cuckoo Hashing

- We have two tables, each with their own hash function.
- We only need to check two cells when searching.
- On collision, the existing item is 'cuckooed' out of it's cell into the other table.

Empty: copied bronzine into table 0(12) Empty: copied auscultatory into table 0(5) Empty: copied bifer into table 0(13) Empty: copied steepgrass into table 0(6) Empty: copied prevised into table 0(7) Empty: copied comph into table 0(8) empodium, so cuckooed out auscultatory from table 0(5) Empty: copied auscultatory into table 1(10) interquarreled, so cuckooed out bronzine from table 0(12) Empty: copied bronzine into table 1(5) ranseur, so cuckooed out empodium from table 0(5) Empty: copied empodium into table 1(4) Empty: copied megalodon into table 0(11) geosynchronous, so cuckooed out megalodon from table 0(11) Empty: copied megalodon into table 1(14) Empty: copied osmeteria into table 0(14) Table getting full -> rehashed old sz =16

Empty: copied farandoles into table 0(4)



: ADTs - Hashing 65 / 83

Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

S : ADTs - Queues

T: ADTs - Trees

U : ADTs - Hashing

V : Algorithms II - Sort

W: Algorithms III - Huffman/Strings

Y : Summary

```
#define NUMS 6
void bubble_sort(int b[], int s);
int main(void)
   int a[] = \{3, 4, 1, 2, 9, 0\};
   bubble_sort(a, NUMS);
   for (int i=0: i < NUMS: i++){
      printf("%i ", a[i]);
   printf("\n");
   return 0;
void bubble sort(int b[], int s)
   bool changes:
   dof
      changes = false;
      for(int i=0; i <s-1; i++){
         if(b[i] > b[i+1])
            SWAP(b[i], b[i+1]);
            changes = true:
   } while (changes);
```

Execution:

0 1 2 3 4 9

```
#define NUMS 6
void bubble sort(int b[]. int s):
int main(void)
   int a[] = \{3, 4, 1, 2, 9, 0\};
   bubble sort(a, NUMS);
   for (int i=0: i < NUMS: i++){
      printf("%i ". a[i]):
   printf("\n");
   return 0:
void bubble sort(int b[], int s)
   bool changes:
   dof
      changes = false:
      for (int i=0; i < s-1; i++){
         if(b[i] > b[i+1]){
            SWAP(b[i], b[i+1]);
            changes = true:
   } while (changes):
```

• Bubblesort has complexity $O(n^2)$, therefore very inefficient.

Execution:

0 1 2 3 4 9

```
#define NIIMS 6
void bubble sort(int b[]. int s):
int main(void)
   int a[] = \{3, 4, 1, 2, 9, 0\};
   bubble sort(a, NUMS);
   for (int i=0: i < NUMS: i++){
      printf("%i ". a[i]):
   printf("\n"):
   return 0:
void bubble sort(int b[], int s)
   bool changes:
   dof
      changes = false;
      for (int i=0; i < s-1; i++){
         if(b[i] > b[i+1]){
            SWAP(b[i], b[i+1]);
            changes = true:
   } while (changes):
```

- Bubblesort has complexity $O(n^2)$, therefore very inefficient.
- If an algorithm uses comparison keys to decide the correct order then the theoretical lower bound on complexity is $O(n \log n)$. From wiki:

Execution:

0 1 2 3 4 9

```
#define NIIMS 6
void bubble sort(int b[]. int s):
int main(void)
   int a[] = \{3, 4, 1, 2, 9, 0\};
   bubble sort(a, NUMS);
   for (int i=0: i < NUMS: i++){
      printf("%i ". a[i]):
   printf("\n"):
   return 0:
void bubble sort(int b[], int s)
   bool changes:
   dof
      changes = false;
      for (int i=0; i < s-1; i++){
         if(b[i] > b[i+1]){
            SWAP(b[i], b[i+1]);
            changes = true:
   } while (changes):
```

- Bubblesort has complexity $O(n^2)$, therefore very inefficient.
- If an algorithm uses comparison keys to decide the correct order then the theoretical lower bound on complexity is $O(n \log n)$. From wiki:

Execution:

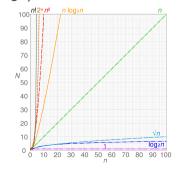
0 1 2 3 4 9

```
#define NIIMS 6
void bubble sort(int b[]. int s):
int main(void)
   int a[] = \{3, 4, 1, 2, 9, 0\};
   bubble_sort(a, NUMS);
   for (int i=0: i < NUMS: i++){
      printf("%i ". a[i]):
   printf("\n"):
   return 0:
void bubble sort(int b[], int s)
   bool changes:
   dof
      changes = false:
      for (int i=0: i < s-1: i++){}
         if(b[i] > b[i+1]){
            SWAP(b[i], b[i+1]);
            changes = true:
   } while (changes):
```

Execution:

0 1 2 3 4 9

- Bubblesort has complexity $O(n^2)$, therefore very inefficient.
- If an algorithm uses comparison keys to decide the correct order then the theoretical lower bound on complexity is O(n log n). From wiki:



• Transposition (Bubblesort)

- Transposition (Bubblesort)
- Insertion Sort (Lab Work)

- Transposition (Bubblesort)
- Insertion Sort (Lab Work)
- Priority Queue (Selection sort, Heap sort)

- Transposition (Bubblesort)
- Insertion Sort (Lab Work)
- Priority Queue (Selection sort, Heap sort)
- Divide & Conquer (Merge & Quick sorts)

- Transposition (Bubblesort)
- Insertion Sort (Lab Work)
- Priority Queue (Selection sort, Heap sort)
- Divide & Conquer (Merge & Quick sorts)
- Address Calculation (Proxmap)

- Transposition (Bubblesort)
- Insertion Sort (Lab Work)
- Priority Queue (Selection sort, Heap sort)
- Divide & Conquer (Merge & Quick sorts)
- Address Calculation (Proxmap)

- Transposition (Bubblesort)
- Insertion Sort (Lab Work)
- Priority Queue (Selection sort, Heap sort)
- Divide & Conquer (Merge & Quick sorts)
- Address Calculation (Proxmap)

 Merge sort is divide-and-conquer in that you divide the array into two halves, mergesort each half and then merge the two halves into order.

- Transposition (Bubblesort)
- Insertion Sort (Lab Work)
- Priority Queue (Selection sort, Heap sort)
- Divide & Conquer (Merge & Quick sorts)
- Address Calculation (Proxmap)

 Merge sort is divide-and-conquer in that you divide the array into two halves, mergesort each half and then merge the two halves into order.

- Transposition (Bubblesort)
- Insertion Sort (Lab Work)
- Priority Queue (Selection sort, Heap sort)
- Divide & Conquer (Merge & Quick sorts)
- Address Calculation (Proxmap)

 Merge sort is divide-and-conquer in that you divide the array into two halves, mergesort each half and then merge the two halves into order.

```
#include <stdio.h>
     #include <stdlib.h>
     #include <string.h>
     void mergesort(int *src, int *spare, int 1, int r);
     void merge(int *src. int *spare. int 1. int m. int r):
     #define NUM 5000
10
     int main(void)
11
        int a[NUM]:
        int spare[NUM]:
        for (int i=0: i < NUM: i++){
            a[i] = rand()\%100:
        mergesort(a, spare, 0, NUM-1);
20
21
        for(int i=0: i <NUM: i++){</pre>
22
            printf("%4d \Rightarrow %d\n", i, a[i]):
23
24
25
        return 0;
26
```

```
void mergesort(int *src, int *spare, int 1, int r)
  int m = (1+r)/2:
  if(1 != r){
      mergesort(src. spare. 1. m):
     mergesort(src, spare, m+1, r);
     merge(src, spare, 1, m, r);
void merge(int *src, int *spare, int 1, int m, int r)
  int s1 = 1:
  int s2 = m+1:
  int d = 1:
  dof
     if(src[s1] < src[s2])
         spare[d++] = src[s1++];
      elsef
         spare[d++] = src[s2++]:
  while((s1 \le m) && (s2 \le r));
  if(s1 > m){
     memcpy(&spare[d], &src[s2], sizeof(spare[0])*(r-s2+1));
  else {
     memcpy(&spare[d], &src[s1], sizeof(spare[0])*(m-s1+1));
  memcpv(\&src[1], \&spare[1], (r-1+1)*sizeof(spare[0])):
```

```
void mergesort(int *src, int *spare, int 1, int r)
  int m = (1+r)/2:
  if(1 != r){
      mergesort(src. spare. 1. m):
      mergesort(src, spare, m+1, r);
     merge(src, spare, 1, m, r);
void merge(int *src, int *spare, int 1, int m, int r)
  int s1 = 1:
  int s2 = m+1:
  int d = 1:
  dof
     if(src[s1] < src[s2])
         spare[d++] = src[s1++]:
      elsef
         spare[d++] = src[s2++]:
  while((s1 \le m) && (s2 \le r));
  if(s1 > m){}
     memcpv(\&spare[d], \&src[s2], sizeof(spare[0])*(r-s2+1));
  else {
     memcpy(&spare[d], &src[s1], sizeof(spare[0])*(m-s1+1));
  memcpv(\&src[1], \&spare[1], (r-1+1)*sizeof(spare[0])):
```

• Quicksort is also divide-and-conquer.

```
void mergesort(int *src, int *spare, int 1, int r)
  int m = (1+r)/2:
  if(1 != r){
      mergesort(src. spare. 1. m):
      mergesort(src. spare. m+1, r):
     merge(src, spare, 1, m, r);
void merge(int *src, int *spare, int 1, int m, int r)
  int s1 = 1:
  int s2 = m+1:
  int d = 1:
  dof
     if(src[s1] < src[s2])
         spare[d++] = src[s1++]:
      elsef
         spare[d++] = src[s2++]:
  while((s1 \le m) && (s2 \le r));
  if(s1 > m){}
     memcpv(&spare[d], &src[s2], sizeof(spare[0])*(r-s2+1));
  else {
     memcpy(&spare[d], &src[s1], sizeof(spare[0])*(m-s1+1));
  memcpv(\&src[1], \&spare[1], (r-1+1)*sizeof(spare[0])):
```

- Quicksort is also divide-and-conquer.
- Choose some value in the array as the *pivot* key.

```
void mergesort(int *src. int *spare. int 1. int r)
  int m = (1+r)/2:
  if(1 != r){
      mergesort(src. spare. 1. m):
      mergesort(src. spare. m+1. r):
     merge(src. spare. 1. m. r):
void merge(int *src. int *spare. int 1. int m. int r)
  int s1 = 1:
  int s2 = m+1:
  int d = 1:
  dof
      if(src[s1] < src[s2]){
         spare[d++] = src[s1++];
      elsef
         spare[d++] = src[s2++]:
  while((s1 \le m) && (s2 \le r));
  if(s1 > m){}
     memcpv(&spare[d], &src[s2], sizeof(spare[0])*(r-s2+1));
  else {
     memcpy(&spare[d], &src[s1], sizeof(spare[0])*(m-s1+1));
  memcpv(\&src[1], \&spare[1], (r-1+1)*sizeof(spare[0]));
```

- Quicksort is also divide-and-conquer.
- Choose some value in the array as the *pivot* key.
- This key is used to divide the array into two partitions. The left partition contains keys ≤ pivot key, the right partition contains keys > pivot.

```
void mergesort(int *src. int *spare. int 1. int r)
  int m = (1+r)/2:
  if(1 \mid = r){
      mergesort(src. spare. 1. m):
      mergesort(src. spare. m+1, r):
     merge(src. spare. 1. m. r):
void merge(int *src. int *spare. int 1. int m. int r)
  int s1 = 1:
  int s2 = m+1:
  int d = 1:
  dof
      if(src[s1] < src[s2]){
         spare[d++] = src[s1++]:
      elsef
         spare[d++] = src[s2++]:
  while((s1 \le m) && (s2 \le r));
  if(s1 > m){
     memcpv(&spare[d], &src[s2], sizeof(spare[0])*(r-s2+1));
  else {
     memcpy(&spare[d], &src[s1], sizeof(spare[0])*(m-s1+1));
  memcpv(\&src[1], \&spare[1], (r-1+1)*sizeof(spare[0]));
```

- Quicksort is also divide-and-conquer.
- Choose some value in the array as the *pivot* key.
- This key is used to divide the array into two partitions. The left partition contains keys ≤ pivot key, the right partition contains keys > pivot.
- Once again, the sort is then applied recursively.

Algorithms: Quicksort

```
#include <stdio.h>
     #include <stdlib.h>
     #include <math.h>
     int partition(int *a, int 1, int r);
     void quicksort(int *a. int 1. int r):
     #define NUM 100000
10
     int main(void)
11
12
        int a[NUM]:
13
        for (int i=0: i < NUM: i++) f
15
           a[i] = rand()\%100;
16
17
18
19
20
21
22
        quicksort(a, 0, NUM-1);
        return 0:
     void quicksort(int *a, int 1, int r)
23
24
        int pivpoint = partition(a, 1, r);
25
26
27
        if(1 < pivpoint){
            quicksort(a, 1, pivpoint-1);
28
        if (r > pivpoint) {
29
30
           quicksort(a, pivpoint+1, r);
31
```

Algorithms: Quicksort

```
#include <stdio.h>
     #include <stdlib.h>
     #include <math.h>
     int partition(int *a, int 1, int r);
     void quicksort(int *a, int 1, int r);
     #define NUM 100000
10
     int main(void)
11
12
        int a[NUM]:
13
14
        for (int i=0: i < NUM: i++) f
15
           a[i] = rand()%100:
16
17
18
19
20
        quicksort(a, 0, NUM-1);
        return 0:
21
22
     void quicksort(int *a, int 1, int r)
23
24
        int pivpoint = partition(a, 1, r);
25
26
27
        if(1 < pivpoint){
            quicksort(a, 1, pivpoint-1);
28
        if (r > pivpoint) {
29
30
           quicksort(a. pivpoint+1, r);
31
```

```
int partition(int *a, int 1, int r)
{
  int piv = a[1];
  while(1<r){
     /* Right -> Left Scan */
     while(piv < a[r] && 1<r) r--;
     if(r!=1){
        a[1] = a[r];
        1++;
     }
     /* Left -> Right Scan */
     while(piv > a[1] && 1<r) 1++;
     if(r!=1){
        a[r] = a[1];
        r--;
     }
}
a[r] = piv;
    return r;
}</pre>
```

 Theoretically both methods have a complexity O(n log n)

- Theoretically both methods have a complexity O(n log n)
- Quicksort is preferred because it requires less memory and is generally faster.

- Theoretically both methods have a complexity $O(n \log n)$
- Quicksort is preferred because it requires less memory and is generally faster.
- Quicksort can go badly wrong if the pivot key chosen is either the maximum or minimum value in the array.

- Theoretically both methods have a complexity $O(n \log n)$
- Quicksort is preferred because it requires less memory and is generally faster.
- Quicksort can go badly wrong if the pivot key chosen is either the maximum or minimum value in the array.
- Quicksort is so loved by programmers that a library version of it exists in ANSI C.

- Theoretically both methods have a complexity $O(n \log n)$
- Quicksort is preferred because it requires less memory and is generally faster.
- Quicksort can go badly wrong if the pivot key chosen is either the maximum or minimum value in the array.
- Quicksort is so loved by programmers that a library version of it exists in ANSI C.
- If you need an off-the-shelf sort, this is often a good option.

- Theoretically both methods have a complexity $O(n \log n)$
- Quicksort is preferred because it requires less memory and is generally faster.
- Quicksort can go badly wrong if the pivot key chosen is either the maximum or minimum value in the array.
- Quicksort is so loved by programmers that a library version of it exists in ANSI C.
- If you need an off-the-shelf sort, this is often a good option.

- Theoretically both methods have a complexity $O(n \log n)$
- Quicksort is preferred because it requires less memory and is generally faster.
- Quicksort can go badly wrong if the pivot key chosen is either the maximum or minimum value in the array.
- Quicksort is so loved by programmers that a library version of it exists in ANSI C.
- If you need an off-the-shelf sort, this is often a good option.

```
#include <stdio.h>
     #include <stdlib.h>
     int intcompare(const void *a, const void *b);
     int main(void)
        int a[10]:
        for (int i=0; i<10; i++){
           a[i] = 9 - i:
        gsort(a, 10, sizeof(int), intcompare):
16
        for (int i=0; i<10; i++){
           printf(" %d".a[i]):
        printf("\n"):
        return 0:
21
     int intcompare(const void *a, const void *b)
25
         const int *ia = (const int *)a:
         const int *ib = (const int *)b:
         return *ia - *ib:
```

 The radix sort is also know as the bin sort, a name derived from its origin as a technique used on (now obsolete) card sorters.

- The radix sort is also know as the bin sort, a name derived from its origin as a technique used on (now obsolete) card sorters.
- For integer data, repeated passes of radix sort focus on the right digit (the units), then the second digit (the tens) and so on.

- The radix sort is also know as the bin sort, a name derived from its origin as a technique used on (now obsolete) card sorters.
- For integer data, repeated passes of radix sort focus on the right digit (the units), then the second digit (the tens) and so on.
- Strings could be sorted in a similar manner.

- The radix sort is also know as the bin sort, a name derived from its origin as a technique used on (now obsolete) card sorters.
- For integer data, repeated passes of radix sort focus on the right digit (the units), then the second digit (the tens) and so on.
- Strings could be sorted in a similar manner.

- The radix sort is also know as the bin sort, a name derived from its origin as a technique used on (now obsolete) card sorters.
- For integer data, repeated passes of radix sort focus on the right digit (the units), then the second digit (the tens) and so on.
- Strings could be sorted in a similar manner

```
459 254 472 534 649 239 432 654 477
```

```
0
1
2 472 432
3
4 254 534 654
5
6
7 477
8
9 459 649 239
```

Read out the new list: 472 432 254 534 654 477 459 649 239

Radix Sort II

```
472 432 254 534 654 477 459 649 239
3 432 534 239
4 649
5 254 654 459
7 472 477
432 534 239 649 254 654 459 472 477
```

Radix Sort II

```
472 432 254 534 654 477 459 649 239
                                               432 534 239 649 254 654 459 472 477
                                               2 239 254
3 432 534 239
4 649
                                               4 432 459 472 477
5 254 654 459
                                               5 534
                                               6 649 654
7 472 477
432 534 239 649 254 654 459 472 477
                                               239 254 432 459 472 477 534 649 654
```

• This has a theoretical complexity of O(n).

- This has a theoretical complexity of O(n).
- It is difficult to write an all-purpose radix sort - you need a different one for doubles, integers, strings etc.

- This has a theoretical complexity of O(n).
- It is difficult to write an all-purpose radix sort - you need a different one for doubles, integers, strings etc.
- O(n) simply means that the number of operations can be bounded by k.n, for some constant k.

- This has a theoretical complexity of O(n).
- It is difficult to write an all-purpose radix sort - you need a different one for doubles, integers, strings etc.
- O(n) simply means that the number of operations can be bounded by k.n, for some constant k.
- With the radix sort, *k* is often very large.

- This has a theoretical complexity of O(n).
- It is difficult to write an all-purpose radix sort - you need a different one for doubles, integers, strings etc.
- O(n) simply means that the number of operations can be bounded by k.n, for some constant k.
- With the radix sort, *k* is often very large.
- For many lists this may be less efficient than more traditional $O(n \log n)$ algorithms.

- This has a theoretical complexity of O(n).
- It is difficult to write an all-purpose radix sort - you need a different one for doubles, integers, strings etc.
- O(n) simply means that the number of operations can be bounded by k.n, for some constant k.
- With the radix sort, *k* is often very large.
- For many lists this may be less efficient than more traditional $O(n \log n)$ algorithms.

 Sometimes you'll want to profile your code.

- This has a theoretical complexity of O(n).
- It is difficult to write an all-purpose radix sort - you need a different one for doubles, integers, strings etc.
- O(n) simply means that the number of operations can be bounded by k.n, for some constant k.
- With the radix sort, *k* is often very large.
- For many lists this may be less efficient than more traditional $O(n \log n)$ algorithms.

- Sometimes you'll want to profile your code.
- Compile with the -pg flag.

- This has a theoretical complexity of O(n).
- It is difficult to write an all-purpose radix sort - you need a different one for doubles, integers, strings etc.
- O(n) simply means that the number of operations can be bounded by k.n, for some constant k.
- With the radix sort, *k* is often very large.
- For many lists this may be less efficient than more traditional $O(n \log n)$ algorithms.

- Sometimes you'll want to profile your code.
- Compile with the -pg flag.
- Executing your code produces a gmon.out file.

- This has a theoretical complexity of O(n).
- It is difficult to write an all-purpose radix sort - you need a different one for doubles, integers, strings etc.
- O(n) simply means that the number of operations can be bounded by k.n, for some constant k.
- With the radix sort, *k* is often very large.
- For many lists this may be less efficient than more traditional $O(n \log n)$ algorithms.

- Sometimes you'll want to profile your code.
- Compile with the -pg flag.
- Executing your code produces a gmon.out file.
- Now: gprof ./executable gmon.out shows the function-call profile of your code

Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

S: ADTs - Queues

T · ADTs - Trees

U : ADTs - Hashing

V : Algorithms II - Sort

W: Algorithms III - Huffman/Strings

Y: Summary

W : Algorithms III - Huffman/Strings

75 / 83

 Often we wish to compress data, to reduce storage requirements, or to speed transmission.

- Often we wish to compress data, to reduce storage requirements, or to speed transmission.
- Text is particularly suited to compression since using one byte per character is wasteful - some letters occur much more frequently.

- Often we wish to compress data, to reduce storage requirements, or to speed transmission.
- Text is particularly suited to compression since using one byte per character is wasteful - some letters occur much more frequently.
- Need to give frequently occurring letters short codes, typically a few bits. Less common letters can have long bit patterns.

- Often we wish to compress data, to reduce storage requirements, or to speed transmission.
- Text is particularly suited to compression since using one byte per character is wasteful - some letters occur much more frequently.
- Need to give frequently occurring letters short codes, typically a few bits. Less common letters can have long bit patterns.

To encode the string "BABBAGE":

- Often we wish to compress data, to reduce storage requirements, or to speed transmission.
- Text is particularly suited to compression since using one byte per character is wasteful - some letters occur much more frequently.
- Need to give frequently occurring letters short codes, typically a few bits. Less common letters can have long bit patterns.

To encode the string "BABBAGE":

- Often we wish to compress data, to reduce storage requirements, or to speed transmission
- Text is particularly suited to compression since using one byte per character is wasteful - some letters occur much more frequently.
- Need to give frequently occurring letters short codes, typically a few bits. Less common letters can have long bit patterns.

• To encode the string "BABBAGE":

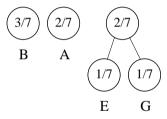


Keep a list of characters, ordered by their frequency

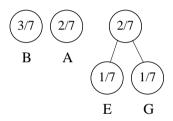
• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :

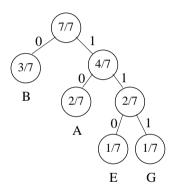
• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :

• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :

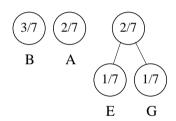


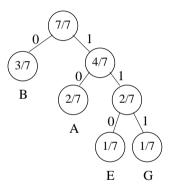
• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :





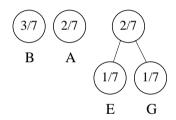
• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :

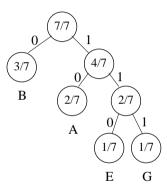




ullet A = 10, B = 0, E = 110, G = 111

• Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :





- \bullet A = 10, B = 0, E = 110, G = 111
- String stored using 13 bits.

 The task of searching for a string amongst a large amount of text is commonly required in word-processors, but more interestingly in massive Biological Databases e.g. searching for amino acids in protein sequences.

- The task of searching for a string amongst a large amount of text is commonly required in word-processors, but more interestingly in massive Biological Databases e.g. searching for amino acids in protein sequences.
- How difficult can it be ? Don't you just do a character by character brute-force search ?

 ${\bf Master\ String: AAAAAAAAAAAA}$

Substring : AAAAAAH
Substring : AAAAAAH
Substring : AAAAAAH

- The task of searching for a string amongst a large amount of text is commonly required in word-processors, but more interestingly in massive Biological Databases e.g. searching for amino acids in protein sequences.
- How difficult can it be ? Don't you just do a character by character brute-force search ?

 ${\bf Master\ String: AAAAAAAAAAAAA}$

Substring : AAAAAAH
Substring : AAAAAAH
Substring : AAAAAAH

- The task of searching for a string amongst a large amount of text is commonly required in word-processors, but more interestingly in massive Biological Databases e.g. searching for amino acids in protein sequences.
- How difficult can it be ? Don't you just do a character by character brute-force search ?

 ${\bf Master\ String: AAAAAAAAAAAAA}$

Substring : AAAAAH
Substring : AAAAAH
Substring : AAAAAH

• If the master string has m characters, and the search string has n characters then this search has complexity: O(mn)

- The task of searching for a string amongst a large amount of text is commonly required in word-processors, but more interestingly in massive Biological Databases e.g. searching for amino acids in protein sequences.
- How difficult can it be ? Don't you just do a character by character brute-force search ?

 ${\bf Master\ String: AAAAAAAAAAAA}$

Substring : AAAAAH
Substring : AAAAAH
Substring : AAAAAH

- If the master string has m characters, and the search string has n characters then this search has complexity: O(mn)
- Recall that to compute a hash function on a word we did something like:

$$h("NEILL") =$$

$$(13\times 26^4 + 4\times 26^3 + 8\times 26^2 + 11\times 26 + 11)\% \textit{P}$$

where P is a big prime number.

- The task of searching for a string amongst a large amount of text is commonly required in word-processors, but more interestingly in massive Biological Databases e.g. searching for amino acids in protein sequences.
- How difficult can it be ? Don't you just do a character by character brute-force search ?

 ${\bf Master\ String: AAAAAAAAAAAAA}$

Substring : AAAAAH
Substring : AAAAAH
Substring : AAAAAH

- If the master string has m characters, and the search string has n characters then this search has complexity: O(mn)
- Recall that to compute a hash function on a word we did something like:

$$h("NEILL") =$$

$$(13\times 26^4 + 4\times 26^3 + 8\times 26^2 + 11\times 26 + 11)\%\textit{P}$$

where P is a big prime number.

• This can be expanded by Horner's method to:

 For a large search string, overflow can occur. We therefore move the mod operation inside the brackets:

$$((((((((13 \times 26) + 4)\%P \times 26) + 8)\%P \times 26) + 11)\%P \times 26 + 11)\%P$$

 For a large search string, overflow can occur. We therefore move the mod operation inside the brackets:

$$(((((((13 \times 26) + 4)\%P \times 26) + 8)\%P \times 26) + 11)\%P \times 26 + 11)\%P$$

 We can compute a hash number for the search string, and for the initial part of the master string.

 For a large search string, overflow can occur. We therefore move the mod operation inside the brackets:

$$(((((((13 \times 26) + 4)\%P \times 26) + 8)\%P \times 26) + 11)\%P \times 26 + 11)\%P$$

- We can compute a hash number for the search string, and for the initial part of the master string.
- When we compute the hash number for the next part of the master, most of the computation is common, we just need to take out the effect of the first letter and add in the effect of the new one.

 For a large search string, overflow can occur. We therefore move the mod operation inside the brackets:

$$(((((((13 \times 26) + 4)\%P \times 26) + 8)\%P \times 26) + 11)\%P \times 26 + 11)\%P$$

- We can compute a hash number for the search string, and for the initial part of the master string.
- When we compute the hash number for the next part of the master, most of the computation is common, we just need to take out the effect of the first letter and add in the effect of the new one.
- One small calculation each time we move one place right in the master.

79 / 83

 For a large search string, overflow can occur. We therefore move the mod operation inside the brackets:

$$(((((((13 \times 26) + 4)\%P \times 26) + 8)\%P \times 26) + 11)\%P \times 26 + 11)\%P$$

- We can compute a hash number for the search string, and for the initial part of the master string.
- When we compute the hash number for the next part of the master, most of the computation is common, we just need to take out the effect of the first letter and add in the effect of the new one.
- One small calculation each time we move one place right in the master.
- Complexity O(m+n) roughly, but need to check that two identical hash numbers really has identified two identical strings.

 For a large search string, overflow can occur. We therefore move the mod operation inside the brackets:

$$(((((((13 \times 26) + 4)\%P \times 26) + 8)\%P \times 26) + 11)\%P \times 26 + 11)\%P$$

- We can compute a hash number for the search string, and for the initial part of the master string.
- When we compute the hash number for the next part of the master, most of the computation is common, we just need to take out the effect of the first letter and add in the effect of the new one.
- One small calculation each time we move one place right in the master.
- Complexity O(m+n) roughly, but need to check that two identical hash numbers really has identified two identical strings.

Rabin-Karp II

 For a large search string, overflow can occur. We therefore move the mod operation inside the brackets:

```
(((((((13\times 26)+4)\%P\times 26)+8)\%P\times 26)+11)\%P\times 26+11)\%P
```

- We can compute a hash number for the search string, and for the initial part of the master string.
- When we compute the hash number for the next part of the master, most of the computation is common, we just need to take out the effect of the first letter and add in the effect of the new one.
- One small calculation each time we move one place right in the master.
- Complexity O(m+n) roughly, but need to check that two identical hash numbers really has identified two identical strings.

```
#include <string.h>
     #include <assert h>
     #define Q 33554393
     #define D 26
     #define index(C) (C-'A')
     int rk(char *p, char *a);
     int main(void)
        assert (rk("STING".
               "A STRING EXAMPLE CONSISTING OF ...")==22):
        return 0:
15
17
     int rk(char *p. char *a)
19
        int i. dM = 1. h1=0. h2=0:
        int m = strlen(p):
21
        int n = strlen(a):
        for (i=1: i \le m: i++) dM = (D*dM)\%O:
23
        for (i=0; i \le m; i++){
           h1 = (h1*D+index(p[i]))%Q:
           h2 = (h2*D+index(a[i]))%Q:
27
        // h1 = search string hash, h2 = master string hash
28
        for(i=0; h1!=h2: i++){}
           h2 = (h2+D*Q-index(a[i])*dM) % Q:
           h2 = (h2*D+index(a[i+m])) % Q;
31
           if(i>n-m) return n:
32
        return i:
```

The Boyer-Moore algorithm uses (in part) an array flagging which characters form part of the search string and an array telling us how far to slide right if that character appears in the master and causes a mismatch.

```
Execution:
```

```
A STRING SEARCHING EXAMPLE CONSISTING OF ...

STING |
STING
STING
```

The Boyer-Moore algorithm uses (in part) an array flagging which characters form part of the search string and an array telling us how far to slide right if that character appears in the master and causes a mismatch.

Execution:

 With a right-to-left walk through the search string we see that the G and the R mismatch on the first comparison.

The Boyer-Moore algorithm uses (in part) an array flagging which characters form part of the search string and an array telling us how far to slide right if that character appears in the master and causes a mismatch.

Execution:

```
A STRING SEARCHING EXAMPLE CONSISTING OF ...

STING

STING

STING
```

- With a right-to-left walk through the search string we see that the G and the R mismatch on the first comparison.
- Since R doesn't appear in the search string, we can take 5 steps to the right.

The Boyer-Moore algorithm uses (in part) an array flagging which characters form part of the search string and an array telling us how far to slide right if that character appears in the master and causes a mismatch.

Execution:

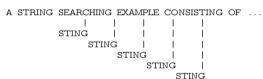
```
A STRING SEARCHING EXAMPLE CONSISTING OF ...

| | |
STING |
STING
STING
```

- With a right-to-left walk through the search string we see that the G and the R mismatch on the first comparison.
- Since R doesn't appear in the search string, we can take 5 steps to the right.
- The next comparison is between the G and the S. We can slide the search string right until it matches the S in the master.

Boyer-Moore II

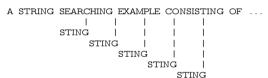
Execution:



 Now the C doesn't appear in the master and once again we can slide a full 5 places to the right.

Boyer-Moore II

Execution:

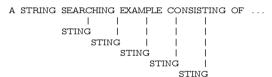


- Now the C doesn't appear in the master and once again we can slide a full 5 places to the right.
- After 3 more full slides right we arrive at the T in CONSISTING.

W : Algorithms III - Huffman/Strings 81 / 83

Boyer-Moore II

Execution:



- Now the C doesn't appear in the master and once again we can slide a full 5 places to the right.
- After 3 more full slides right we arrive at the T in CONSISTING.
- We align the T's, and have found our match using 7 compares (plus 5 to verify the match).

W : Algorithms III - Huffman/Strings

Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

Q: ADTs - Collection

R: ADTs - Stacks

S : ADTs - Queues

T: ADTs - Trees

U : ADTs - Hashing

V : Algorithms II - Sort

W: Algorithms III - Huffman/Strings

Y : Summary

• TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)

- TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)
- $current = current \rightarrow next$

- TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)
- $current = current \rightarrow next$
- The beautiful Binary Tree Search. A real need for recursion.

- TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)
- $current = current \rightarrow next$
- The beautiful Binary Tree Search. A real need for recursion.
- Understanding why qsort() is $O(n \log n)$.

- TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)
- $current = current \rightarrow next$
- The beautiful Binary Tree Search. A real need for recursion.
- Understanding why qsort() is $O(n \log n)$.
- von Mises paradox. The joy of hashing.

- TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)
- current = current \rightarrow next
- The beautiful Binary Tree Search. A real need for recursion.
- Understanding why qsort() is $O(n \log n)$.
- von Mises paradox. The joy of hashing.
- Abstract Data Types a head-start on OOP/Java

- TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)
- current = current \rightarrow next
- The beautiful Binary Tree Search. A real need for recursion.
- Understanding why qsort() is $O(n \log n)$.
- von Mises paradox. The joy of hashing.
- Abstract Data Types a head-start on OOP/Java

- TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)
- $current = current \rightarrow next$
- The beautiful Binary Tree Search. A real need for recursion.
- Understanding why qsort() is $O(n \log n)$.
- von Mises paradox. The joy of hashing.
- Abstract Data Types a head-start on OOP/Java

CotW

- TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)
- current = current \rightarrow next
- The beautiful Binary Tree Search. A real need for recursion.
- Understanding why qsort() is $O(n \log n)$.
- von Mises paradox. The joy of hashing.
- Abstract Data Types a head-start on OOP/Java

- CotW
- Things we skipped in C : varargs, ##, unions, bit fields

- TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)
- current = current \rightarrow next
- The beautiful Binary Tree Search. A real need for recursion.
- Understanding why qsort() is $O(n \log n)$.
- von Mises paradox. The joy of hashing.
- Abstract Data Types a head-start on OOP/Java

- CotW
- Things we skipped in C: varargs, ##, unions, bit fields
- Things we skipped in Algorithms: Skip lists, Self-Balancing Trees, MinMax Heaps

- TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)
- current = current \rightarrow next
- The beautiful Binary Tree Search. A real need for recursion.
- Understanding why qsort() is $O(n \log n)$.
- von Mises paradox. The joy of hashing.
- Abstract Data Types a head-start on OOP/Java

- CotW
- Things we skipped in C : varargs, ##, unions, bit fields
- Things we skipped in Algorithms: Skip lists, Self-Balancing Trees, MinMax Heaps
- In 69 days $Hello\ World \rightarrow Recursive$ Walking of a Tree

- TAs (Phoenix, Harry, Andy, Alex, Liz and Ali)
- current = current \rightarrow next
- The beautiful Binary Tree Search. A real need for recursion.
- Understanding why qsort() is $O(n \log n)$.
- von Mises paradox. The joy of hashing.
- Abstract Data Types a head-start on OOP/Java

- CotW
- Things we skipped in C: varargs, ##, unions, bit fields
- Things we skipped in Algorithms: Skip lists, Self-Balancing Trees, MinMax Heaps
- In 69 days $Hello\ World o Recursive$ Walking of a Tree
- Why I teach this unit