## COMSM1201 : Data Structures & Algorithms

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#### N : Recursion

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P: Linked Data Structures

Q : ADTs - Collection

R : ADTs - Stacks

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- This is an important theme in Computer Science that crops up time & time again.

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- Let's look at some toy examples to begin with.

```
#include <stdio.h>
     #include <string.h>
     #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
     void strrev(char* s, int n);
     int main (void)
        char str[] = "Hello World!":
        strrev(str. strlen(str)):
        printf("%s\n", str);
        return 0:
14
15
     /* Iterative Inplace String Reverse */
17
     void strrev(char* s. int n)
18
19
        for(int i=0, j=n-1; i<j; i++, j--){
            SWAP(s[i], s[j]);
21
22
```

#### Execution:

!dlroW olleH

# Recursion for *strrev()*

```
#include <stdio.h>
    #include <string.h>
    #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
    void strrev(char* s, int start, int end);
    int main(void)
       char str[] = "Hello World!";
        strrev(str. 0. strlen(str)-1):
        printf("%s\n", str);
13
14
       return 0;
15
    /* Recursive : Inplace String Reverse */
    void strrev(char* s. int start, int end)
19
       if(start >= end){
           return:
       SWAP(s[start], s[end]);
23
24
        strrev(s. start+1, end-1);
```

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!dlroW olleH

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#include <stdio.h>
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    #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
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• We need to change the function prototype.

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#include <stdio.h>
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    #define SWAP(A.B) {char temp: temp=A:A=B:B=temp:}
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           return:
       SWAP(s[start], s[end]);
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       strrev(s. start+1, end-1):
```

- We need to change the function prototype.
- This allows us to track both the start and the end of the string.

#### Execution:

IdlroW olleH

## The Fibonacci Sequence

A well known example of a recursive function is the Fibonacci sequence. The first term is 1, the second term is 1 and each successive term is defined to be the sum of the two previous terms, i.e. :

```
fib(1) is 1
fib(2) is 1
fib(n) is fib(n-1)+fib(n-2)
```

1,1,2,3,5,8,13,21, ...

N : Recursion 5 / 5'

```
#include <stdio.h>
    #define MAXFIB 24
    int fibonacci(int n);
     int main(void)
        for(int i=1: i <= MAXFIB: i++){
           printf("%d = %d\n", i, fibonacci(i));
13
14
15
        return 0;
16
17
     int fibonacci(int n)
19
        if(n \le 2)
           return 1;
       int b = 1:
        int next:
        for (int i=3; i \le n; i++){
           next = a + b:
           a = b:
           b = next:
        return b:
32
```

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#include <stdio.h>
    #define MAXFIB 24
    int fibonacci(int n):
     int main(void)
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           next = a + b:
           a = b:
29
           b = next:
30
31
        return b:
32
```

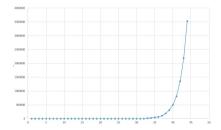
#### Execution:

```
1 = 1
 = 13
 = 21
9 = 34
10 = 55
11 = 89
12 = 144
13 = 233
14 = 377
15 = 610
16 = 987
17 = 1597
18 = 2584
19 = 4181
20 = 6765
21 = 10946
22 = 17711
23 = 28657
24 = 46368
```

```
#include <stdio.h>
    #define MAXFIB 24
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        return 0:
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     int fibonacci(int n)
        if (n == 1) return 1:
        if (n == 2) return 1;
        return ( fibonacci (n-1) + fibonacci (n-2));
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```
#include <stdio.h>
    #define MAXFIB 24
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        if (n == 1) return 1:
        if (n == 2) return 1;
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```

It's interesting to see how run-time increases as the length of the sequence is raised.



# Maze Escape

The correct route through a maze can be obtained via recursive, rather than iterative, methods.



## Maze Escape

The correct route through a maze can be obtained via recursive, rather than iterative, methods.



```
bool explore(int x, int y, char mz[YS][XS])
  if mz[y][x] is exit return true;
  Mark mz[y][x] so we don't return here
  if we can go up:
    if(explore(x, v+1, mz)) return true
  if we can go right:
    if(explore(x+1, v, mz)) return true
  Do left & down in a similar manner
  return false: // Failed to find route
```

 Here we consider the ways to permute a string (or more generally an array)

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- Permutations are all possible ways of rearranging the positions of the characters.

#### Execution:

ABC

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- Permutations are all possible ways of rearranging the positions of the characters.

# Execution :

ABC ACB BAC BCA

CAB

```
// From e.g. http://www.geeksforgeeks.org
    #include <stdio.h>
    #include <string.h>
    #define SWAP(A,B) {char temp = *A; *A = *B; *B = temp;}
     void permute(char* a, int s, int e);
     int main()
         char str[] = "ABC";
         int n = strlen(str);
         permute(str. 0, n-1);
         return 0:
     void permute(char* a, int s, int e)
18
        if (s == e){
          printf("%s\n", a);
          return:
        for (int i = s: i \le e: i++)
24
           SWAP((a+s), (a+i)); // Bring one char to the front
25
           permute(a, s+1, e);
26
           SWAP((a+s), (a+i)); // Backtrack
27
28
```

 Raising a number to a power n = 2<sup>5</sup> is the same as multiple multiplications n = 2\*2\*2\*2\*2.

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```
/* Try to write power(a.b) to computer a^b
        without using any maths functions other than
        multiplication :
        Try (1) iterative then (2) recursive
        (3) Trick that for n\%2==0, x^n = x^(n/2)*x^(n/2)
    #include <stdio.h>
10
11
     int power(unsigned int a, unsigned int b);
12
     int main(void)
16
        int x = 2:
        int v = 16:
19
        printf("%d^%d = %d\n", x, y, power(x,y));
20
21
     int power(unsigned int a, unsigned int b)
```

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- Simply move through the array from beginning to end, stopping when you have found the value you require.

```
#include <stdio.h>
     #include <string.h>
     #include <assert.h>
     #define NOTFOUND -1
     #define NUMPEOPLE 6
     typedef struct person {
             char* name; int age;
     } person;
     int findAge(const char* name, const person* p, int n);
     int main (void)
        person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
                   {"Chumley", 26}, {"Dalton", 25},
                   {"Eggson", 22}, {"Fulton", 41} };
        assert(findAge("Eggson",
                                    ppl, NUMPEOPLE) == 22);
        assert (find Age ("Campbell", ppl, NUMPEOPLE) == NOTFOUND);
        return 0:
23
24
     int findAge(const char* name, const person* p, int n)
25
        for (int j=0; j < n; j++){
27
           if (strcmp(name, p[i], name) == 0){
              return p[i].age:
29
30
31
        return NOTFOUND:
32
```

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# Sequential Search

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- If, for instance, it is sorted, we can use strcmp() in a slightly cleverer manner.
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- This halves, on average, the number of comparisons required.

# Sequential Search

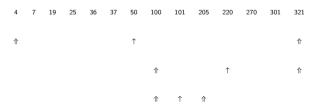
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12
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        person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
                   {"Chumley", 26}, {"Dalton", 25},
                   {"Eggson", 22}, {"Fulton", 41} };
        assert (find Age ("Eggson".
                                    ppl NUMPEOPLE) == 22):
        assert (find Age ("Campbell", ppl, NUMPEOPLE) == NOTFOUND):
21
        return 0:
22
23
24
     int findAge(const char* name, const person* p, int n)
25
        for (int j=0; j < n; j++){
27
           int m = strcmp(name, p[i], name);
           if (m == 0) // Braces!
              return p[i].age:
           if(m < 0)
31
              return NOTFOUND:
32
33
        return NOTFOUND:
```

 Searching small lists doesn't require much computation time.



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- A binary search consists of examining the middle element of the array to see if it has the desired value. If not, then half the array may be discarded for the next search.

4 7 19 25 36 37 50 100 101 205 220 270 301 321 ↑ ↑ ↑ ↑ ↑ ↑ ↑

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↑ ↑ ↑ ↑ ↑
```

```
#include cetdie ha
     #include cetdlib by
     #include <assert h>
     #include <time.h>
     #define NMBBS 1000000
     int bin it(int k, const int* a, int l, int r);
     int main(void)
        int a[NMBBS]:
        srand(time(NULL)):
        // Put even numbers into array
        for (int i=0; i < NMBRS; i++){
           a[i] = 2*i:
        // Do many searches for a random number
20
        for (int i=0: i<10*NMBRS: i++){
21
           int n = rand()%NMBRS:
           if((n\%2) = 0){
23
              assert(bin it(n, a, 0, NMBRS-1) = n/2);
24
25
           else { // No odd numbers in this list
26
              assert(bin it(n, a, 0, NMBRS-1) < 0):
27
28
29
        return 0:
```

#### Iterative v. Recursion Binary Search

```
int bin_it(int k, const int* a, int 1, int r)
{
  while(1 <= r){
    int m = (1+r)/2;
    if(k = a[m]){
      return m;
    }
    else{
      if (k > a[m]){
        1 = m + 1;
      }
      else {
        r = m - 1;
      }
    }
    return -1;
}
```

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      if (k > a[m]){
            1 = m + 1;
        }
      else{
      r = m - 1;
      }
    }
  return -1;
}
```

```
int bin_rec(int k, const int* a, int l, int r)
{
    if(1 > r) return -1;
    int m = (1+r)/2;
    if(k = a | m |) {
        return m;
    }
    else {
        if (k > a | m |) {
            return bin_rec(k, a, m + 1, r);
        }
        else {
            return bin_rec(k, a, l, m - 1);
        }
    }
}
```

 When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.

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- Now we use an interpolation involving the key, the start of the list and the end.

$$i = (k - I[0])/(I[n-1] - I[0]) * n$$

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$$i = (k - I[0])/(I[n-1] - I[0]) * n$$

• when searching for '15':

```
0 4 5 9 10 12 15 20
```

```
int interp(int k. const int* a. int l. int r)
   int m:
   double md:
   while(1 \le r)
      md = ((double)(k-a[1])/
            (double)(a[r]-a[1])*
            (double)(r-1)
           +(double)(1):
      m = 0.5 + md:
      if((m > r) | | (m < 1)){
         return -1:
      if (k == a[m])
         return m:
         if (k > a[m]) {
            1 = m + 1:
         elsef
            r = m-1:
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• This code on an old Dell laptop took:

O: Algorithms I - Search

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- This complexity indicates how, for n numbers, performance deteriorates when n changes.

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- Binary Search : O(log n)

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- Interpolation Search : O(log log n)

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- Searching and sorting algorithms have a complexity associated with them, called big-O.
- This complexity indicates how, for n numbers, performance deteriorates when n changes.
- Sequential Search : O(n)
- Binary Search : O(log n)
- Interpolation Search : O(log log n)
- We'll discuss the dream of a O(1) search later in "Hashing".

#### Binary vs. Interpolation Timing

```
#include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <time.h>
    int bin it(int k, const int *a, int 1, int r);
     int bin rec(int k. const int *a. int 1. int r):
     int interp(int k, const int *a, int 1, int r);
     int* parse_args(int argc, char* argv[], int* n, int* srch);
     int main(int argc, char* argv[])
12
        int i, n, srch;
        int* a;
        int (*p[3])(int k, const int*a, int 1, int r) =
            {bin it, bin rec, interp};
        a = parse_args(argc, argv, &n, &srch);
        srand(time(NULL));
22
23
        for (i=0; i < n; i++){
           a[i] = 2*i:
24
25
        for (i=0; i<5000000; i++){}
26
27
           assert ((*p[srch])(a[rand()%n], a, 0, n-1) >= 0);
28
29
        free(a):
30
        return 0;
31
32
```

#### Binary vs. Interpolation Timing

```
#include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <time.h>
    int bin it(int k, const int *a, int 1, int r);
     int bin rec(int k. const int *a. int 1. int r):
     int interp(int k, const int *a, int 1, int r);
     int* parse_args(int argc, char* argv[], int* n, int* srch);
     int main(int argc, char* argv[])
12
13
        int i, n, srch;
        int* a:
        int (*p[3])(int k, const int*a, int 1, int r) =
            {bin it, bin rec, interp};
18
19
20
21
        a = parse_args(argc, argv, &n, &srch);
        srand(time(NULL));
22
23
        for (i=0; i < n; i++){
           a[i] = 2*i:
24
25
        for (i=0; i<5000000; i++){}
26
27
           assert ((*p[srch])(a[rand()%n], a, 0, n-1) >= 0);
28
29
        free(a):
30
        return 0;
31
32
```

#### Execution:

```
Binary Search : Iterative
       100000 = 0.57
      800000 = 0.84
      6400000 = 2.20
     51200000 = 3.87
Binary Search : Recursive
       100000 = 1.23
       800000 = 1.79
      6400000 = 3.20
n =
     51200000 = 4.85
Interpolation
n =
       100000 = 0.20
       800000 = 0.28
      6400000 = 0.50
n =
     51200000 = 0.70
n =
```

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- By following pointers one after another, we can travel right along the structure.

```
#include <stdio h>
     #include < stdlih h>
    #include "general.h"
     typedef struct data{
        int i:
        struct data* next:
     } Data;
     Data* allocateData(int i):
11
     void printList(Data* 1):
     int main(void)
        int i:
        Data* start . *current :
        start = current = NULL:
        printf("Enter the first number: "):
        if(scanf("%i", &i) == 1){
           start = current = allocateData(i):
21
        elsef
           on_error("Couldn't read an int");
        printf("Enter more numbers: ");
27
        while(scanf("%i", &i) == 1){
           current -> next = allocateData(i):
           current = current -> next:
31
        printList(start):
        // Should Free List
        return 0:
```

## Linked Lists

```
Data* allocateData(int i)
{
    Data* p;
    p = (Data*) ncalloc(1, sizeof(Data));
    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* 1)
{
    printf("\n");
    do{
        printf("Number : %i\n", 1->i);
        l = l->next;
    } while(1 != NULL);
    printf("END\n");
}
```

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    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* 1)
{
    printf("\n");
    do{
        printf("Number : %i\n", 1->i);
        1 = 1->next;
    }while(1 != NULL);
    printf("END\n");
}
```

#### Searching and Recursive printing:

```
Data* inList(Data* n, int i)
{
    do{
        if(n->i==i){
            return n;
    }
        n = n->next;
} while(n != NULL);
return NULL;
}

void printList_r(Data* 1)
{
    // Recursive Base-Case
    if(1 == NULL) return;
    printf(*Number: %i\n*, 1->i);
    printList_r(1->next);
}
```

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- Each ADT exposes its functionality via an interface.
- The user only accesses the data via this interface.
- The user of the ADT doesn't need to understand how the data is being stored (e.g. array vs. linked lists etc.)

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- Some collections allow duplicate elements and others do not (e.g. Sets).
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- Our Collection will be unsorted and will allow duplicates.

```
#include "../General/general.h"
typedef int colltype:
typedef struct coll coll;
#include <stdio.h>
#include <stdlib h>
#include <assert.h>
// Create an empty coll
coll* coll init(void);
// Add element onto top
void coll add(coll* c, colltype i);
// Take element out
bool coll remove(coll* c. colltype d):
// Does this exist ?
bool coll isin(coll* c. colltype i):
// Return size of coll
int coll size(coll* c):
// Clears all space used
bool coll_free(coll* c);
```

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  - A linked-list

#### Fixed/specific.h:

```
1  #pragma once
2
3  #define COLLTYPE "Fixed"
4
5  #define FIXEDSIZE 5000
6  struct coll {
7    // Underlying array
8    colltype a[FIXEDSIZE];
9    int size;
10 };
```

# Collection ADT using a Fixed-size Array

#### Fixed/fixed.c:

```
#include " .. / coll . h"
     #include "specific.h"
     coll* coll_init(void)
        coll* c = (coll*) ncalloc(sizeof(coll), 1);
        c \rightarrow size = 0;
        return c;
     int coll size(coll* c)
        if (c=NULL){
           return 0:
        return c->size:
19
20
     bool coll_isin(coll* c, colltype d)
        for (int i=0; i < coll size(c); i++){
           if(c\rightarrow a[i] = d)
                return true;
        return false;
```

# Collection ADT using a Fixed-size Array

#### Fixed/fixed.c:

```
#include "../coll.h"
    #include "specific.h"
     coll* coll_init(void)
        coll* c = (coll*) ncalloc(sizeof(coll), 1);
        c - > size = 0;
        return c;
     int coll size(coll* c)
13
        if (c=NULL){
           return 0:
16
17
        return c->size;
19
     bool coll_isin(coll* c, colltype d)
20
        for (int i=0: i < coll size(c): i++){
22
           if(c\rightarrow a[i] = d){}
                return true:
24
        return false;
```

```
void coll add(coll* c. colltype d)
   if(c){
      if(c->size >= FIXEDSIZE){
          on error("Collection overflow"):
      c \rightarrow a[c \rightarrow size] = d:
      c \rightarrow size = c \rightarrow size + 1:
bool coll remove(coll* c. colltype d)
   for (int i=0: i < coll size(c): i++){
      if(c->a[i] == d)f
          // Shuffle end of array left one
          for(int j=i; j < coll_size(c); j++){</pre>
             c - a[i] = c - a[i+1];
          c->size = c->size - 1:
          return true:
   return false:
bool coll_free(coll* c)
   free(c):
   return true:
```

# Collection ADT via an Array (Realloc)

#### Realloc/specific.h:

# Collection ADT via an Array (Realloc)

#### Realloc/specific.h:

```
#pragma once

define COLLTYPE "Realloc"

define FIXEDSIZE 16
    #define SCALEFACTOR 2

struct coll {
    // Underlying array
    colltype* a;
    int size;
    int capacity;
};
```

#### Realloc/realloc.c:

```
#include "../coll.h"
     #include "specific.h"
      coll* coll init(void)
         coll* c = (coll*) ncalloc(sizeof(coll), 1);
         c->a = (colltype*) ncalloc(sizeof(colltype), FIXEDSIZE);
         c \rightarrow size = 0:
         c->capacity= FIXEDSIZE;
         return c:
      void coll add(coll* c. colltype d)
14
         if(c){
             if (c->size >= c->capacity){
                c \rightarrow a = (colltype*) nremalloc(c \rightarrow a.
                         sizeof(colltype)*c->capacity*SCALEFACTOR);
19
                c->capacity = c->capacity*SCALEFACTOR;
20
21
            c \rightarrow a[c \rightarrow size] = d:
            c \rightarrow size = c \rightarrow size + 1:
23
```

### Collection ADT via a Linked List

#### Linked/specific.h:

```
1  #pragma once
2
3  #define COLLTYPE "Linked"
4
4
5  struct dataframe {
6   colltype i;
7   struct dataframe* next;
8  };
9  typedef struct dataframe dataframe;
10
11  struct coll {
12   // Underlying array
13   dataframe* start;
14   int size;
15 };
```

#### Collection ADT via a Linked List

#### Linked/specific.h:

```
#pragma once

#define COLLTYPE "Linked"

struct dataframe {
    colltype i;
    struct dataframe* next;
    };
    typedef struct dataframe dataframe;

truct coll {
    // Underlying array
    dataframe* start;
    int size;
    };
}
```

#### Linked/linked.c:

```
#include " .. / coll .h"
#include "specific.h"
coll* coll_init(void)
   coll* c = (coll*) ncalloc(sizeof(coll), 1);
   return c:
int coll size(coll* c)
   if(c==NULL){
      return 0:
   return c->size:
bool coll_isin(coll* c, colltype d)
   if(c == NULL || c->start==NULL){
      return false:
   dataframe* f = c->start:
   dof
      if(f\rightarrow i == d){
          return true:
      f = f - > next;
   } while (f != NULL):
   return false:
```

## Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
   if(c){
       dataframe* f = ncalloc(sizeof(dataframe), 1):
       f \rightarrow i = d:
       f \rightarrow next = c \rightarrow start:
       c \rightarrow start = f;
       c \rightarrow size = c \rightarrow size + 1;
bool coll_free(coll* c)
   if(c){
       dataframe* tmp;
       dataframe* p = c->start;
       while (p!=NULL) {
           tmp = p->next;
           free(p);
           p = tmp;
       free(c):
   return true;
```

### Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
   if(c){
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       f \rightarrow next = c \rightarrow start:
       c \rightarrow start = f;
       c \rightarrow size = c \rightarrow size + 1:
bool coll free(coll* c)
   if(c){
       dataframe* tmp:
       dataframe* p = c->start:
       while (p!=NULL) {
           tmp = p->next;
           free(p);
           p = tmp;
       free(c):
   return true;
```

```
bool coll_remove(coll* c, colltype d)
   dataframe* f1 . *f2:
   if((c==NULL) || (c->start==NULL)){
      return false:
   // If Front
   if(c->start->i == d){
      f1 = c->start->next:
      free(c->start):
      c->start = f1:
      c \rightarrow size = c \rightarrow size - 1;
      return true:
   f1 = c -> start:
   f2 = c->start->next:
   dof
      if(f2->i == d)f
          f1 -> next = f2 -> next:
          free(f2):
          c \rightarrow size = c \rightarrow size - 1:
          return true:
      f1 = f2:
      f2 = f1 -> next:
   } while (f2 != NULL):
   return false;
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 Any code using the ADT can be compiled against any of the implementations,
 e.g. the test (testcoll.c) code.

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Task	Fixed Array	Realloc Array	Linked List
Insert new element	O(1) at end	O(1) at end	O(1) at front
	if space	but realloc()	
Search for an element	O(n)	O(n)	O(n)
	brute force	brute force	brute force
Search + delete	O(n) + O(n)	O(n) + O(n)	O(n) + O(1)
	move left	move left	delete 'free'

 If we had ordered our ADT (ie. the elements were sorted), then the searches could be via a binary / interpolation search, leading to O(log n) or O(log log n) search times.

# ADTs Making Coding Simpler

That Linked List code from the previous Chapter again:

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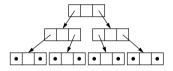
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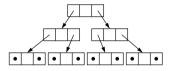
Binary Trees:



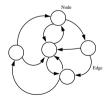
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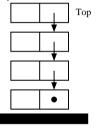
### Binary Trees:



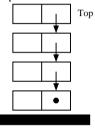
### Unidirectional Graph:



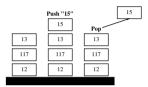
# The push-down stack:



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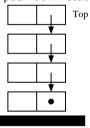


### LIFO (Last in, First out):

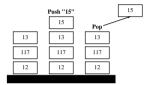


• Operations include push and pop.

The push-down stack:

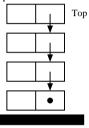


LIFO (Last in, First out):

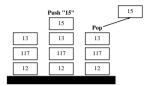


- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.

The push-down stack:

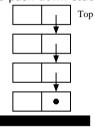


### LIFO (Last in, First out):

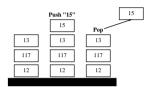


- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.

The push-down stack:



### LIFO (Last in, First out):



- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.
- But, once again, we are faced with the question: How best to implement such a data type?

# ADT:Stacks Arrays (Realloc) I

#### stack.h:

```
#pragma once
    #include " .. / General / general . h"
    typedef int stacktype;
    typedef struct stack stack;
    #include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <string.h>
    /* Create an empty stack */
    stack* stack_init(void);
    /* Add element to top */
    void stack push(stack* s, stacktype i);
    /* Take element from top */
    bool stack pop(stack* s. stacktype* d):
    /* Clears all space used */
    bool stack free(stack* s):
23
24
    /* Optional? */
    /* Copy top element into d (but don't pop it) */
    bool stack peek(stack*s. stacktype* d):
    /* Make a string version - keep .dot in mind */
    void stack tostring(stack*. char* str);
```

# ADT:Stacks Arrays (Realloc) I

#### stack.h:

```
#pragma once
    #include " .. / General/general .h"
    typedef int stacktype:
    typedef struct stack stack;
    #include <stdio.h>
    #include <stdlib.h>
    #include <assert.h>
    #include <string.h>
    /* Create an empty stack */
    stack* stack_init(void);
    /* Add element to top */
    void stack push(stack* s, stacktype i);
    /* Take element from top */
    bool stack pop(stack* s. stacktype* d):
    /* Clears all space used */
    bool stack free(stack* s):
23
24
    /* Optional? */
    /* Copy top element into d (but don't pop it) */
    bool stack peek(stack*s. stacktype* d):
    /* Make a string version - keep .dot in mind */
    void stack tostring(stack*. char* str);
```

### Realloc/specific.h:

```
1  #pragma once
2
3  #define FORMATSIR "%i"
4  #define ELEMSIZE 20
5  #define STACKTYPE "Realloc"
7
7
8  #define FIXEDSIZE 16
9  #define SCALEFACTOR 2
10
11  struct stack {
12     /* Underlying array */
13     stacktype* a;
14     int size;
15     int capacity;
16  };
```

# ADT:Stacks Arrays (Realloc) II

### Realloc/realloc.c

```
#include " .. / stack . h"
    #include "specific.h"
    #define DOTFILE 5000
     stack* stack init(void)
        stack *s = (stack*) ncalloc(sizeof(stack), 1):
        /* Some implementations would allow you to pass
            a hint about the initial size of the stack */
        s->a = (stacktype*) ncalloc(sizeof(stacktype), FIXEDSIZE);
        s \rightarrow size = 0:
        s->capacity= FIXEDSIZE;
14
15
        return s:
17
     void stack_push(stack* s, stacktype d)
19
        if (s=NULL){
             return:
        if(s->size >= s->capacity){}
23
            s->a = (stacktype*) nremalloc(s->a,
24
                    sizeof(stacktype)*s->capacity*SCALEFACTOR);
25
26
            s->capacity = s->capacity*SCALEFACTOR;
27
        s \rightarrow a[s \rightarrow size] = d:
28
        s \rightarrow size = s \rightarrow size + 1:
```

# ADT:Stacks Arrays (Realloc) II

### Realloc/realloc.c

```
#include " .. / stack . h"
     #include "specific.h"
     #define DOTFILE 5000
     stack * stack init(void)
         stack *s = (stack*) ncalloc(sizeof(stack), 1):
        /* Some implementations would allow you to pass
            a hint about the initial size of the stack */
         s->a = (stacktype*) ncalloc(sizeof(stacktype), FIXEDSIZE);
        s \rightarrow size = 0:
         s->capacity= FIXEDSIZE;
14
         return s:
15
17
     void stack_push(stack* s, stacktype d)
19
         if (s=NULL){
              return:
21
        if(s->size >= s->capacity){}
23
            s \rightarrow a = (stacktvpe*) nremalloc(s \rightarrow a.
24
                     sizeof(stacktype)*s->capacity*SCALEFACTOR);
25
            s->capacity = s->capacity*SCALEFACTOR;
26
27
         s \rightarrow a[s \rightarrow size] = d:
28
         s \rightarrow size = s \rightarrow size + 1:
```

```
bool stack_pop(stack* s, stacktype* d)
{
    if((s == NULL) || (s->size < 1)){
        return false;
    }
    s.>size = s->size - 1;
    *d = s->a(s->size);
    return true;
}

bool stack_peek(stack* s, stacktype* d)

if ((s==NULL) || (s->size <= 0)){
    if((s==NULL) || (s->size <= 0)){
        *d = s->a(s->size);
        return true;
}

*** **Teven **Teven
```

# ADT:Stacks Arrays (Realloc) III

#### Realloc/realloc.c

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE];
        str[0] = '\0';
        if((s=NULL) || (s->size <1)){
           return:
        for (int i=s-> size-1: i>=0: i--)
           sprintf(tmp, FORMATSTR, s->a[i]);
10
11
12
13
           strcat(str, tmp);
           strcat(str, "|");
        str[strlen(str)-1] = '\0':
14
15
16
17
     bool stack free(stack* s)
18
        if (s=NULL){
           return true:
20
21
        free(s->a):
        free(s):
        return true;
```

# ADT:Stacks Arrays (Realloc) III

### Realloc/realloc.c

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE];
        str[0] = '\0';
        if((s=NULL) || (s->size <1)){
           return:
        for (int i=s-> size-1: i>=0: i--)
           sprintf(tmp, FORMATSTR, s->a[i]);
10
11
12
13
           strcat(str, tmp);
           strcat(str, "|");
        str[strlen(str)-1] = '\0':
14
15
16
17
     bool stack free(stack* s)
18
        if (s=NULL){
           return true:
20
21
        free(s->a):
        free(s):
        return true;
```

We need a thorough testing program teststack.c

# ADT:Stacks Arrays (Realloc) III

### Realloc/realloc.c

```
void stack tostring(stack* s, char* str)
        char tmp[ELEMSIZE];
        str[0] = '\0':
        if((s=NULL) || (s->size <1)){
            return:
        for (int i=s->size-1: i>=0: i--) {
            sprintf(tmp, FORMATSTR, s->a[i]);
           strcat(str. tmp):
10
11
12
13
            strcat(str. "|");
        str[strlen(str)-1] = '\0':
14
15
16
17
     bool stack free(stack* s)
18
        if (s=NULL){
19
            return true:
20
21
        free(s->a):
        free(s):
        return true:
```

- We need a thorough testing program teststack.c
- See also revstr.c: a version of the string reverse code (for which we already seen an iterative (in-place) and a recursive solution).

# ADT:Stacks Linked I

### Linked/specific.h

```
#pragma once

define FORMATSIR "%i"

define ELEMSIZE 20

define STACKTYPE "Linked"

struct dataframe {
    stacktype i;
    struct dataframe* next;
};

typedef struct dataframe dataframe;

struct stack {
    /* Underlying array */
    dataframe* start;
    int size;
};
```

## ADT:Stacks Linked I

### Linked/specific.h

```
1  #pragma once
2
3  #define FORMATSIR "%i"
4  #define ELEMSIZE 20
5  #define STACKTYPE "Linked"
6
7  struct dataframe {
8   stacktype i;
9   struct dataframe* next;
);
11  typedef struct dataframe dataframe;
12
13  struct stack {
14   /* Underlying array */
15   dataframe* start;
16  int size;
17 };
```

### Linked/linked.c

```
#include " .. / stack .h"
     #include "specific.h"
     #define DOTFILE 5000
     stack* stack init(void)
         stack* s = (stack*) ncalloc(sizeof(stack), 1);
         return s:
10
11
     void stack push(stack* s. stacktype d)
13
        if(s){
            dataframe* f = ncalloc(sizeof(dataframe), 1):
            f \rightarrow i = d:
            f->next = s->start;
            s->start = f:
            s \rightarrow size = s \rightarrow size + 1:
20
```

# ADT:Stacks Linked II

```
bool stack_pop(stack* s, stacktype* d)
        if ((s==NULL) || (s->start==NULL)){
            return false;
        dataframe* f = s->start->next;
        *d = s->start->i:
        free(s->start):
        s \rightarrow start = f:
        s \rightarrow size = s \rightarrow size - 1:
12
13
14
        return true;
15
     bool stack_peek(stack* s, stacktype* d)
16
        if((s==NULL) || (s->start==NULL)){
18
            return false;
20
        *d = s->start ->i;
        return true;
```

## ADT:Stacks Linked II

```
bool stack_pop(stack* s, stacktype* d)
         if((s==NULL) || (s->start==NULL)){
            return false:
        dataframe* f = s->start->next;
        *d = s->start->i:
         free(s->start):
        s \rightarrow start = f:
        s \rightarrow size = s \rightarrow size - 1:
12
13
         return true:
14
15
     bool stack peek(stack* s. stacktype* d)
16
17
         if((s==NULL) || (s->start==NULL)){
18
            return false:
19
20
        *d = s->start->i:
        return true;
22
```

```
void stack_tostring(stack* s, char* str)
        char tmp[ELEMSIZE]:
        str[0] = '\0':
        if((s==NULL) || (s->size <1)){
           return:
        dataframe* p = s->start:
        while (p) f
           sprintf(tmp. FORMATSTR. p->i):
           strcat(str. tmp):
           strcat(str. "|"):
           p = p -> next:
14
        str[strlen(str)-1] = '\0';
16
17
18
     bool stack free(stack* s)
19
20
        if(s){
           dataframe* p = s->start;
           while (p!=NULL){
              dataframe* tmp = p->next;
              free(p):
              p = tmp;
26
27
           free(s):
28
        return true;
30
```

### Table of Contents

N: Recursion

O: Algorithms I - Search

P: Linked Data Structures

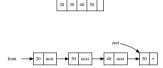
Q: ADTs - Collection

R : ADTs - Stacks

S: ADTs - Queues

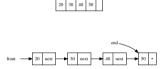
T: ADTs - Queues

FIFO (First in, First out):



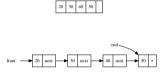
• Intuitively more "useful" than a stack.

FIFO (First in, First out):



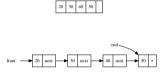
- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)

FIFO (First in, First out):



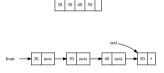
- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)
- Operations include enqueue, dequeue and size.

FIFO (First in, First out):



- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)
- Operations include enqueue, dequeue and size.

### FIFO (First in, First out):



- Intuitively more "useful" than a stack.
- Think of implementing any kind of service (printer, web etc.)
- Operations include enqueue, dequeue and size.

### queue.h

```
#pragma once
    #include " .. / General/general .h"
     typedef int queuetype;
     typedef struct queue queue;
     #include <stdio.h>
    #include <stdlib.h>
    #include <string.h>
    Winclude (assert h)
     /* Create an empty queue */
     queue* queue init(void):
     /* Add element on end */
     void queue_enqueue(queue* q, queuetype v);
     /* Take element off front */
     bool queue dequeue(queue* q. queuetype* d):
     /* Return size of queue */
     int queue size(queue* q):
     /* Clears all space used */
     bool queue_free(queue* q);
24
     /* Helps with visualisation & testing */
     void queue tostring(queue* q. char* str):
```

### specific.h

### specific.h

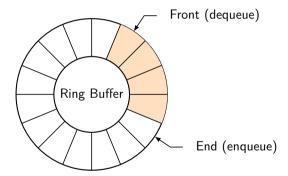
```
#pragma once
define FORMATSIR "%d"
#define ELEMSIZE 20

define QUEUETYPE "Fixed"

define BOUNDED 5000

struct queue {
    /* Underlying array */
    queuetype a[BOUNDED];
    int front;
    int end;
};

define DOTFILE 5000
```



#### fixed.c

```
#include " .. / queue . h"
     #include "specific.h"
     void inc(queuetype* p);
     queue * queue init(void)
         queue* q = (queue*) ncalloc(sizeof(queue), 1);
         return q;
10
11
12
13
14
     void queue_enqueue(queue* q, queuetype d)
15
16
17
18
19
20
21
22
         if (a) {
             q \rightarrow a[q \rightarrow end] = d:
             _inc(&q->end);
             if (q->end == q->front){
                 on_error("Queue too large");
```

#### fixed.c

```
#include " .. / queue . h"
     #include "specific.h"
     void inc(queuetype* p);
     queue * queue init(void)
         queue* q = (queue*) ncalloc(sizeof(queue), 1);
         return q;
     void queue_enqueue(queue* q, queuetype d)
14
15
16
17
         if (a) {
            q \rightarrow a[q \rightarrow end] = d:
            _inc(&q->end);
18
19
20
21
             if (q->end == q->front){
                on_error("Queue too large");
22
```

```
bool queue dequeue(queue* q. queuetype* d)
        if ((a==NULL) || (a->front==a->end)){
           return false:
        *d = q -  a[q -  front]:
        inc(&g->front):
        return true:
9
10
11
     void queue tostring(queue* q. char* str)
12
13
        char tmp[ELEMSIZE];
        str[0] = '\0':
        if ((q==NULL) || (queue_size(q)==0)){
16
           return:
17
18
        for(int i=q->front; i != q->end;){
           sprintf(tmp, FORMATSTR, q->a[i]);
20
           strcat(str. tmp):
21
           strcat(str. "|"):
22
           inc(&zi):
23
24
        str[strlen(str)-1] = '\0':
```

```
int queue_size(queue* q)
         if (q==NULL) {
            return 0;
        if(q->end >= q->front){
            return q->end-q->front;
9
10
11
        return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
         free(q):
15
16
        return true;
17
18
19
20
21
     void _inc(queuetype* p)
        *p = (*p + 1) \% BOUNDED;
```

S : ADTs - Queues 44 / 5ī

```
int queue_size(queue* q)
         if (q==NULL) {
            return 0:
        if(q-)end = q-)front){
            return q->end-q->front;
9
10
11
        return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
         free(q):
15
16
        return true:
17
18
19
20
     void inc(queuetvpe* p)
        *p = (*p + 1) \% BOUNDED:
```

 We need a thorough testing program

```
int queue_size(queue* q)
        if (a==NULL) {
            return 0:
        if(q-)end = q-)front)
            return q->end-q->front;
9
10
11
        return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
        free(q);
15
16
        return true:
17
18
19
     void inc(queuetvpe* p)
20
        *p = (*p + 1) \% BOUNDED:
```

- We need a thorough testing program
- We'll see queues again for traversing trees

```
int queue_size(queue* q)
         if (a==NULL) {
            return 0:
        if(q-)end = q-)front)
            return q->end-q->front;
9
10
11
        return q->end + BOUNDED - q->front;
12
13
     bool queue_free(queue* q)
14
         free(q):
15
16
        return true;
17
18
19
     void inc(queuetvpe* p)
20
        *p = (*p + 1) \% BOUNDED:
```

- We need a thorough testing program
- We'll see queues again for traversing trees
- Simulating a (slow) printer

## ADTs: Queues (Linked) I

### specific.h

```
#pragma once

define FORMATSIR "%d"

define ELEMSIZE 20

define QUEUETYPE "Linked"

struct dataframe {
    queuetype i;
    struct dataframe* next;
    };

typedef struct dataframe dataframe;

struct queue {
    /* Underlying array */
    dataframe* front;
    dataframe* end;
    int size;
};
```

### ADTs: Queues (Linked) I

### specific.h

```
#pragma once
    #define FORMATSTR "%d"
    #define ELEMSIZE 20
    #define OUFUETYPE "Linked"
    struct dataframe {
       queuetype i;
        struct dataframe* next;
    }:
12
13
    typedef struct dataframe dataframe;
14
    struct queue {
15
      /* Underlying array */
       dataframe* front:
17
       dataframe* end:
       int size:
19
    }:
```

#### linked.c

```
#include " .. / queue .h"
      #include "specific.h"
      queue* queue init(void)
          queue* q = (queue*) ncalloc(sizeof(queue), 1);
          return q;
      void queue_enqueue(queue* q, queuetype d)
          dataframe* f;
          if (q==NULL) {
             return:
          /* Copy the data */
          f = ncalloc(sizeof(dataframe), 1);
          f \rightarrow i = d:
          /* 1st one */
          if (a->front == NULL) {
             a \rightarrow front = f:
24
             a \rightarrow end = f:
             q \rightarrow size = q \rightarrow size + 1;
             return:
28
          /* Not 1st */
          q \rightarrow end \rightarrow next = f:
          a->end = f:
31
          q \rightarrow size = q \rightarrow size + 1;
```

S : ADTs - Queues 45 / 5'

### ADTs: Queues (Linked) II

```
bool queue dequeue(queue* q, queuetype* d)
         dataframe* f;
         if((q=NULL) || (q->front=NULL) || (q->end=NULL)){
            return false;
         f = q - front - next;
         *d = q->front->i;
         free(q->front);
         q \rightarrow front = f;
11
12
13
14
15
16
17
18
19
20
         q->size = q->size - 1;
         return true;
     bool queue free (queue * q)
         if (a) {
             dataframe* tmp:
            dataframe* p = q->front;
            while (p!=NULL) {
                tmp = p->next;
22
23
24
25
26
27
28
                free(p);
                p = tmp;
             free(q);
         return true;
```

### ADTs: Queues (Linked) II

```
bool queue dequeue(queue* q, queuetype* d)
         dataframe* f:
         if ((q=NULL) || (q->front=NULL) || (q->end=NULL)){
            return false;
         f = q - front - next;
         *d = q-> front -> i;
         free(q->front);
        q \rightarrow front = f;
         q \rightarrow size = q \rightarrow size - 1;
         return true;
13
14
     bool queue free (queue * q)
        if (a) {
18
19
            dataframe* tmp:
            dataframe* p = q->front;
20
            while (p!=NULL) {
                tmp = p -> next:
                free(p);
23
24
                p = tmp:
25
26
            free(q);
         return true;
28
```

```
void queue tostring(queue* q, char* str)
        dataframe *p;
        char tmp[ELEMSIZE];
        str[0] = '\0';
        if ((q=NULL) || (q->front == NULL)){
           return:
        p = q - front;
        while(p){
           sprintf(tmp, FORMATSTR, p->i);
           strcat(str. tmp);
           strcat(str. "|");
           p = p -   next;
16
        str[strlen(str)-1] = '\0';
17
18
     int queue size(queue* q)
20
21
        if ((q=NULL) || (q->front=NULL)){
23
           return 0:
24
25
        return q->size;
```

• There exists a nice package, called Graphviz:

sudo apt install graphviz

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 This allows the visualisation of graphs/dynamic structures using the simple .dot language:

```
digraph {
   a -> b; b -> c; c -> a;
}
```

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To create a .pdf: dot -Tpdf -o graphviz.pdf examp1.dot

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```
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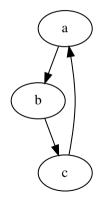
• There exists a nice package, called Graphviz:

sudo apt install graphviz

 This allows the visualisation of graphs/dynamic structures using the simple .dot language:

```
digraph {
   a -> b; b -> c; c -> a;
}
```

To create a .pdf: dot -Tpdf -o graphviz.pdf examp1.dot



### Table of Contents

N : Recursion

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Binary trees are used extensively in computer science

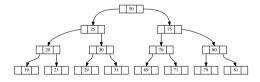
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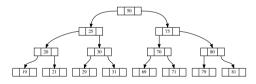
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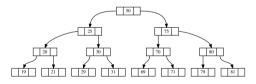


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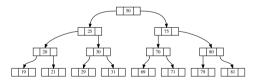
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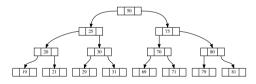
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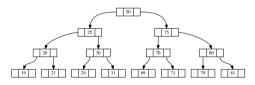
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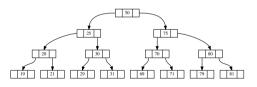
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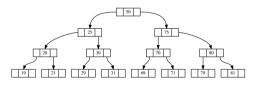
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- Empty subtrees are set to NULL

「: ADTs - Queues 49 / 57

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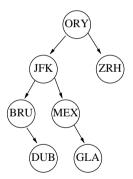
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Counting from cell 1, for a tree with *n* nodes:

To find	Use	Iff
The root	A[1]	A is nonempty
The left child of $A[i]$	A[2i]	$2i \leq n$
The parent of $A[i]$	A[i/2]	i > 1
Is $A[i]$ a leaf?	True	2 <i>i</i> > <i>n</i>

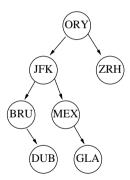
# Binary Search Trees

In a binary search tree the left-hand tree of a parent contains all keys less than the parent node, and the right-hand side all the keys greater than the parent node.



#### Binary Search Trees

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#### bst. h

```
#include " .. / General/general . h"
    #include " .. / Queue/queue . h "
     #include <stdio.h>
     #include <stdlib.h>
     #include <assert.h>
     bst* bst_init(void);
     /* Insert 1 item into the tree */
     bool bst insert(bst* b, treetype d);
     /* Return number of nodes in tree */
     int bst size(bst* b);
     /* Whether the data d is stored in the tree */
     bool bst isin(bst* b, treetype d);
     /* Bulk insert n items from an array a into an initialised tree */
20
     bool bst_insertarray(bst* b, treetype* a, int n);
     /* Clear all memory associated with tree. & set pointer to NULL */
     bool bst free(bst* b):
25
     /* Optional ? */
     char* bst_preorder(bst* b);
     void bst printlevel(bst* b):
     /* Create string with tree as ((head)(left)(right)) */
     char* bst_printlisp(bst* b);
     /* Use Graphviz via a .dot file */
     void bst todot(bst* b. char* dotname):
```

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T : ADTs - Queues

#### Binary Search Trees: Linked I

#### specific.h

```
#include <string.h>
    typedef int treetype;
    #define FORMATSTR "%i"
    #define ELEMSIZE 20
    #define BSTTYPE "Linked"
    struct dataframe {
        treetype d;
       struct dataframe* left;
        struct dataframe* right;
    }:
    typedef struct dataframe dataframe:
15
    struct bst {
       dataframe* top:
       /* Data element size . in bytes */
    typedef struct bst bst:
```

```
/* Based on geekforgeeks.org */
     dataframe* insert(dataframe* t, treetype d)
         dataframe* f;
         /* If the tree is empty, return a new frame */
          if (t == NULL){
             f = ncalloc(sizeof(dataframe), 1);
             f \rightarrow d = d:
             return f;
         /* Otherwise, recurs down the tree */
          if (d < t > d)
              t \rightarrow left = insert(t \rightarrow left . d):
         else if (d > t - >d)
16
             t->right = _insert(t->right, d);
17
18
         /* return the (unchanged) dataframe pointer */
19
         return t:
20
```

#### Binary Search Trees: Linked II

```
1  bool _isin(dataframe* t, treetype d)
2  {
3    if(t=NULL){
4       return false;
5    }
6    if(t>d=d){
7       return true;
8    }
9    if(d < t->d){
10       return _isin(t->left, d);
11    }
12    else{
13       return _isin(t->right, d);
14    }
15    return false;
16 }
```

```
char* printlisp(dataframe* t)
       char tmp[ELEMSIZE];
       char *s1, *s2, *p;
       if (t==NULL) {
          /* \0 string */
          p = ncalloc(1.1);
          return p;
       sprintf(tmp, FORMATSTR, t->d);
       s1 = _printlisp(t->left);
       s2 = _printlisp(t->right);
       p = ncalloc(strlen(s1)+strlen(s2)+strlen(tmp)+strlen("()() ")
       sprintf(p, "%s(%s)(%s)", tmp, s1, s2);
       free(s1):
17
       free(s2):
       return p;
19
```

#### Binary Search Trees : Realloc

#### specific.h

```
#include <stdbool b>
    typedef int treetype:
    #define FORMATSTR "%i"
    #define FIFMSIZE 20
    #define BSTTYPE "Realloc"
    // Probably (2^n) -1
    #define INITSIZE 31
    #define SCALEFACTOR 2
    struct dataframe {
        treetype d:
        bool isvalid:
15
    typedef struct dataframe dataframe:
17
    struct bst {
19
       dataframe* a:
       int capacity:
    typedef struct bst bst:
```

#### Using a queue for Level-Order traversal:

```
void bst printlevel(bst* b)
        treetype n:
        if ((b=NULL) || (! isvalid(b, 0))){
          return:
        /* Make a queue of cell indices */
        queue* q = queue init();
        queue enqueue(q. 0):
        while (queue_dequeue(q, &m) && _isvalid(b, (int)n)){
11
           printf(FORMATSTR. b->a[n].d):
12
           putchar(' '):
13
          queue_enqueue(q, _leftchild((int)n));
          queue_enqueue(q, _rightchild((int)n));
15
```

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- In this case, complexity becomes O(n).
- The tree search performs best when well balanced trees are formed.
- Large body of literature about creating & re-balancing trees Red-Black trees, Tries, 2-3 trees, AVL trees etc.

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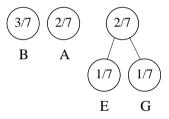


Keep a list of characters, ordered by their frequency

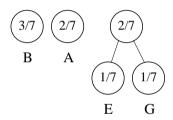
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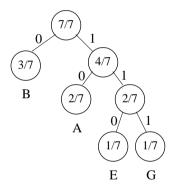
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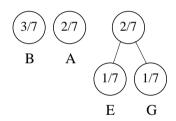


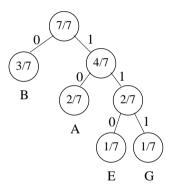
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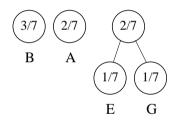
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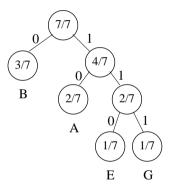




 $\bullet$  A = 10, B = 0, E = 110, G = 111

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- A = 10. B = 0. E = 110. G = 111
- String stored using 13 bits.