

# COMSM1201 : Data Structures & Algorithms

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November 17, 2021



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O : Algorithms I - Search

P : Linked Data Structures

Q : ADTs - Collection

R : ADTs - Stacks

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T : ADTs - Trees

U : ADTs - Hashing

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- Let's look at some toy examples to begin with.

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1  #include <stdio.h>
2  #include <string.h>
3
4  #define SWAP(A,B) {char temp; temp=A;A=B;B=temp;}
5
6  void strrev(char* s, int n);
7
8  int main(void)
9  {
10     char str[] = "Hello World!";
11     strrev(str, strlen(str));
12     printf("%s\n", str);
13     return 0;
14 }
15
16 /* Iterative Inplace String Reverse */
17 void strrev(char* s, int n)
18 {
19     for(int i=0, j=n-1; i<j; i++, j--){
20         SWAP(s[i], s[j]);
21     }
22 }
```

Execution :

!dlroW olleH



# Recursion for *strrev()*

```
1  #include <stdio.h>
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6  void strrev(char* s, int start, int end);
7
8  int main(void)
9  {
10     char str[] = "Hello World!";
11     strrev(str, 0, strlen(str)-1);
12     printf("%s\n", str);
13     return 0;
14 }
15
16 /* Recursive : Inplace String Reverse */
17 void strrev(char* s, int start, int end)
18 {
19     if(start >= end){
20         return;
21     }
22     SWAP(s[start], s[end]);
23     strrev(s, start+1, end-1);
24 }
```

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Execution :

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- We need to change the function prototype.
- This allows us to track both the start and the end of the string.

# The Fibonacci Sequence

A well known example of a recursive function is the Fibonacci sequence. The first term is 1, the second term is 1 and each successive term is defined to be the sum of the two previous terms, i.e. :

$\text{fib}(1)$  is 1

$\text{fib}(2)$  is 1

$\text{fib}(n)$  is  $\text{fib}(n-1) + \text{fib}(n-2)$

1, 1, 2, 3, 5, 8, 13, 21, ...

# Iterative & Recursive Fibonacci

```
1  #include <stdio.h>
2
3  #define MAXFIB 24
4
5  int fibonacci(int n);
6
7  int main(void)
8  {
9
10     for(int i=1; i<=MAXFIB; i++){
11         printf("%d = %d\n", i, fibonacci(i));
12     }
13
14     return 0;
15 }
16
17
18 int fibonacci(int n)
19 {
20     if(n <= 2){
21         return 1;
22     }
23     int a = 1;
24     int b = 1;
25     int next;
26     for(int i=3; i<=n; i++){
27         next = a + b;
28         a = b;
29         b = next;
30     }
31     return b;
32 }
```

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11         printf("%d = %d\n", i, fibonacci(i));
12     }
13
14     return 0;
15 }
16
17 int fibonacci(int n)
18 {
19     if(n <= 2){
20         return 1;
21     }
22     int a = 1;
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25     for(int i=3; i<=n; i++){
26         next = a + b;
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28         b = next;
29     }
30     return b;
31 }
32 }
```

Execution :

```
1 = 1
2 = 1
3 = 2
4 = 3
5 = 5
6 = 8
7 = 13
8 = 21
9 = 34
10 = 55
11 = 89
12 = 144
13 = 233
14 = 377
15 = 610
16 = 987
17 = 1597
18 = 2584
19 = 4181
20 = 6765
21 = 10946
22 = 17711
23 = 28657
24 = 46368
```

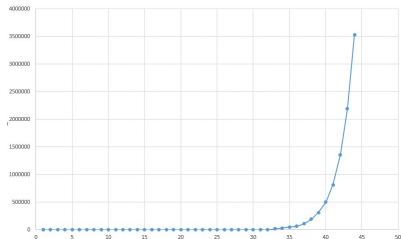
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13
14     return 0;
15
16 }
17
18 int fibonacci(int n)
19 {
20     if(n == 1) return 1;
21     if(n == 2) return 1;
22     return( fibonacci(n-1)+fibonacci(n-2));
23 }
```

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23 }
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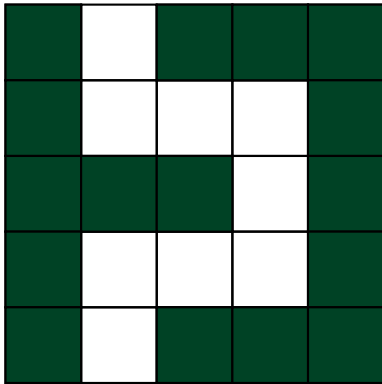
It's interesting to see how run-time increases as the length of the sequence is raised.





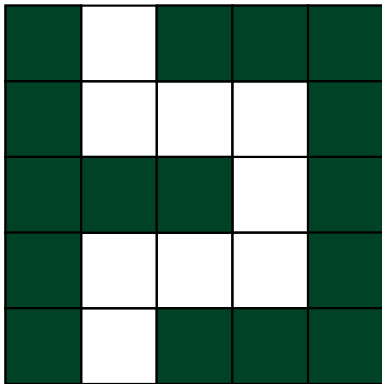
# Maze Escape

The correct route through a maze can be obtained via recursive, rather than iterative, methods.



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```
bool explore(int x, int y, char mz[YS][XS])
{
    if mz[y][x] is exit return true;

    Mark mz[y][x] so we don't return here

    if we can go up :
        if(explore(x, y+1, mz)) return true

    if we can go right :
        if(explore(x+1, y, mz)) return true

    Do left & down in a similar manner

    return false; // Failed to find route
}
```

# Permuting

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- Permutations are all possible ways of rearranging the positions of the characters.

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Execution :

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```
1 // From e.g. http://www.geeksforgeeks.org
2 #include <stdio.h>
3 #include <string.h>
4
5 #define SWAP(A,B) {char temp = *A; *A = *B; *B = temp;}
6
7 void permute(char* a, int s, int e);
8
9 int main()
10 {
11     char str[] = "ABC";
12     int n = strlen(str);
13     permute(str, 0, n-1);
14     return 0;
15 }
16
17 void permute(char* a, int s, int e)
18 {
19     if (s == e){
20         printf("%s\n", a);
21         return;
22     }
23     for (int i = s; i <= e; i++){
24         SWAP((a+s), (a+i)); // Bring one char to the front
25         permute(a, s+1, e);
26         SWAP((a+s), (a+i)); // Backtrack
27     }
28 }
```

# Self-test : Power

- Raising a number to a power  $n = 2^5$  is the same as multiple multiplications  
 $n = 2*2*2*2*2$ .

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```
1  /* Try to write power(a,b) to computer a^b
2  without using any maths functions other than
3  multiplication :
4  Try (1) iterative then (2) recursive
5  (3) Trick that for  $n\%2==0$ ,  $x^n = x^{(n/2)} * x^{(n/2)}$ 
6
7  */
8
9  #include <stdio.h>
10
11 int power(unsigned int a, unsigned int b);
12
13 int main(void)
14 {
15
16     int x = 2;
17     int y = 16;
18
19     printf("%d^%d = %d\n", x, y, power(x,y));
20
21 }
22
23 int power(unsigned int a, unsigned int b)
24 {
25 }
```

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```
1  #include <stdio.h>
2  #include <string.h>
3  #include <assert.h>
4
5  #define NOTFOUND -1
6  #define NUMPEOPLE 6
7  typedef struct person{
8      char* name; int age;
9  } person;
10
11 int findAge(const char* name, const person* p, int n);
12
13 int main(void)
14 {
15     person ppl[NUMPEOPLE] = { {"Ackerby", 21}, {"Bloggs", 25},
16                                {"Chumley", 26}, {"Dalton", 25},
17                                {"Eggson", 22}, {"Fulton", 41} };
18
19     assert(findAge("Eggson", ppl, NUMPEOPLE)==22);
20     assert(findAge("Campbell", ppl, NUMPEOPLE)==NOTFOUND);
21     return 0;
22 }
23
24 int findAge(const char* name, const person* p, int n)
25 {
26     for(int j=0; j<n; j++){
27         if(strcmp(name, p[j].name) == 0){
28             return p[j].age;
29         }
30     }
31     return NOTFOUND;
32 }
```

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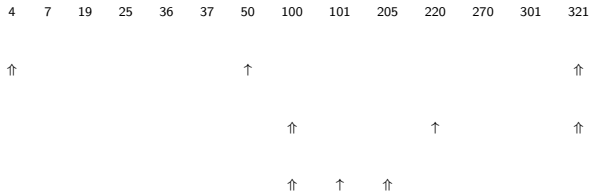
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21     return 0;
22 }
23
24 int findAge(const char* name, const person* p, int n)
25 {
26     for(int j=0; j<n; j++){
27         int m = strcmp(name, p[j].name);
28         if(m == 0) // Braces!
29             return p[j].age;
30         if(m < 0)
31             return NOTFOUND;
32     }
33     return NOTFOUND;
34 }
```

# Binary Search for *101*

- Searching small lists doesn't require much computation time.





# Binary Search for *101*

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- However, as lists get longer (e.g. phone directories), sequential searching becomes extremely inefficient.

4      7      19      25      36      37      50      100      101      205      220      270      301      321

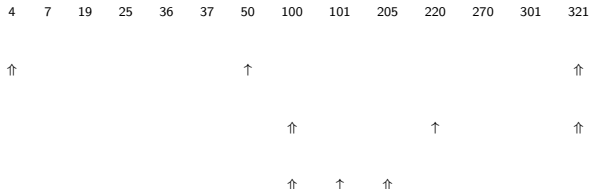
↑↑                  ↑                  ↑↑

[illegible]

↑                    ↑                    ↑

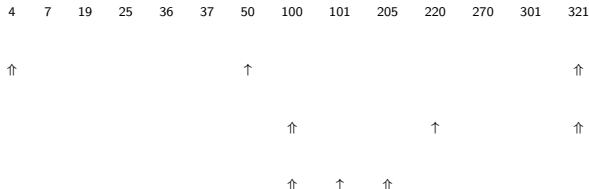
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↑						↑							↑
							↑			↑			↑
								↑					
									↑				

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <assert.h>
4  #include <time.h>
5  #define NMBS 1000000
6
7  int bin_it(int k, const int* a, int l, int r);
8
9  int main(void)
10 {
11     int a[NMBS];
12     srand(time(NULL));
13
14     // Put even numbers into array
15     for(int i=0; i<NMBS; i++){
16         a[i] = 2*i;
17     }
18
19     // Do many searches for a random number
20     for(int i=0; i<10*NMBS; i++){
21         int n = rand()%NMBS;
22         if((n%2) == 0){
23             assert(bin_it(n, a, 0, NMBS-1) == n/2);
24         }
25         else{ // No odd numbers in this list
26             assert(bin_it(n, a, 0, NMBS-1) < 0);
27         }
28     }
29     return 0;
30 }
```

# Iterative v. Recursion Binary Search

```
int bin_it(int k, const int* a, int l, int r)
{
    while(l <= r){
        int m = (l+r)/2;
        if(k == a[m]){
            return m;
        }
        else{
            if (k > a[m]){
                l = m + 1;
            }
            else{
                r = m - 1;
            }
        }
    }
    return -1;
}
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            else{
                r = m - 1;
            }
        }
    }
    return -1;
}
```

```
int bin_rec(int k, const int* a, int l, int r)
{
    if(l > r) return -1;
    int m = (l+r)/2;
    if(k == a[m]){
        return m;
    }
    else{
        if (k > a[m]){
            return bin_rec(k, a, m+1, r);
        }
        else{
            return bin_rec(k, a, l, m-1);
        }
    }
}
```

# Interpolation Search

- When we look for a word in a dictionary, we don't start in the middle. We make an educated guess as to where to start based on the 1st letter of the word being searched for.

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- Now we use an interpolation involving the key, the start of the list and the end.

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- Now we use an interpolation involving the key, the start of the list and the end.

$$i = (k - l[0]) / (l[n - 1] - l[0]) * n$$

- when searching for '15' :

0   4   5   9   10   12   15   20  
                  ↑↑

```
int interp(int k, const int* a, int l, int r)
{
    int m;
    double md;

    while(l <= r){
        md = ((double)(k-a[l])/
              (double)(a[r]-a[l]))*
              (double)(r-l)
              )
            +(double)(l);
        m = 0.5 + md;
        if((m > r) || (m < l)){
            return -1;
        }
        if(k == a[m])
            return m;
        else{
            if (k > a[m]){
                l = m + 1;
            }
            else{
                r = m- 1;
            }
        }
    }
}
```

# Algorithmic Complexity

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5  #define CSEC (double)(CLOCKS_PER_SEC)
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- We'll discuss the dream of a  **$O(1)$**  search later in "Hashing".



# Binary vs. Interpolation Timing

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <assert.h>
4  #include <time.h>
5
6  int bin_it(int k, const int *a, int l, int r);
7  int bin_rec(int k, const int *a, int l, int r);
8  int interp(int k, const int *a, int l, int r);
9  int* parse_args(int argc, char* argv[], int* n, int* srch);
10
11 int main(int argc, char* argv[])
12 {
13
14     int i, n, srch;
15     int* a;
16     int (*p[3])(int k, const int*a, int l, int r) =
17         {bin_it, bin_rec, interp};
18
19     a = parse_args(argc, argv, &n, &srch);
20
21     srand(time(NULL));
22     for(i=0; i<n; i++){
23         a[i] = 2*i;
24     }
25     for(i=0; i<5000000; i++){
26         assert((*p[srch])(a[rand()%n], a, 0, n-1) >= 0);
27     }
28
29     free(a);
30     return 0;
31 }
32 }
```

# Binary vs. Interpolation Timing

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31
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```

Execution :

Binary Search : Iterative

n = 100000 = 0.57

n = 800000 = 0.84

n = 6400000 = 2.20

n = 51200000 = 3.87

Binary Search : Recursive

n = 100000 = 1.23

n = 800000 = 1.79

n = 6400000 = 3.20

n = 51200000 = 4.85

Interpolation

n = 100000 = 0.20

n = 800000 = 0.28

n = 6400000 = 0.50

n = 51200000 = 0.70

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```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include "general.h"
4
5  typedef struct data{
6      int i;
7      struct data* next;
8  } Data;
9
10 Data* allocateData(int i);
11 void printList(Data* l);
12
13 int main(void)
14 {
15     int i;
16     Data* start, *current;
17     start = current = NULL;
18     printf("Enter the first number: ");
19     if(scanf("%i", &i) == 1){
20         start = current = allocateData(i);
21     }
22     else{
23         on_error("Couldn't read an int");
24     }
25
26     printf("Enter more numbers: ");
27     while(scanf("%i", &i) == 1){
28         current->next = allocateData(i);
29         current = current->next;
30     }
31     printList(start);
32     // Should Free List
33     return 0;
34 }
```

# Linked Lists

```
Data* allocateData(int i)
{
    Data* p;
    p = (Data*) calloc(1, sizeof(Data));
    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* l)
{
    printf("\n");
    do{
        printf("Number : %i\n", l->i);
        l = l->next;
    }while(l != NULL);
    printf("END\n");
}
```

# Linked Lists

```
Data* allocateData(int i)
{
    Data* p;
    p = (Data*) malloc(1, sizeof(Data));
    p->i = i;
    // Not really required
    p->next = NULL;
    return p;
}

void printList(Data* l)
{
    printf("\n");
    do{
        printf("Number : %i\n", l->i);
        l = l->next;
    }while(l != NULL);
    printf("END\n");
}
```

## Searching and Recursive printing:

```
Data* inList(Data* n, int i)
{
    do{
        if(n->i==i){
            return n;
        }
        n = n->next;
    }while(n != NULL);
    return NULL;
}

void printList_r(Data* l)
{
    // Recursive Base-Case
    if(l == NULL) return;

    printf("Number: %i\n", l->i);
    printList_r(l->next);
}
```

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- The user of the ADT doesn't need to understand how the data is being stored (e.g. array vs. linked lists etc.)

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```
1  #include "../General/general.h"
2
3  typedef int colltype;
4
5  typedef struct coll coll;
6
7  #include <stdio.h>
8  #include <stdlib.h>
9  #include <assert.h>
10
11 // Create an empty coll
12 coll* coll_init(void);
13 // Add element onto top
14 void coll_add(coll* c, colltype i);
15 // Take element out
16 bool coll_remove(coll* c, colltype d);
17 // Does this exist ?
18 bool coll_isin(coll* c, colltype i);
19 // Return size of coll
20 int coll_size(coll* c);
21 // Clears all space used
22 bool coll_free(coll* c);
```

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Fixed/specific.h:

```
1  #pragma once
2
3  #define COLTYPE "Fixed"
4
5  #define FIXEDSIZE 5000
6  struct coll {
7      // Underlying array
8      colltype a[FIXEDSIZE];
9      int size;
10 };
```

# Collection ADT using a Fixed-size Array

Fixed/fixed.c:

```
1  #include "../coll.h"
2  #include "specific.h"
3
4  coll* coll_init(void)
5  {
6      coll* c = (coll*) nalloc(sizeof(coll), 1);
7      c->size = 0;
8      return c;
9  }
10
11 int coll_size(coll* c)
12 {
13     if(c==NULL){
14         return 0;
15     }
16     return c->size;
17 }
18
19 bool coll_isin(coll* c, colltype d)
20 {
21     for(int i=0; i<coll_size(c); i++){
22         if(c->a[i] == d){
23             return true;
24         }
25     }
26     return false;
27 }
```

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12 {
13     if(c==NULL){
14         return 0;
15     }
16     return c->size;
17 }
18
19 bool coll_isin(coll* c, colltype d)
20 {
21     for(int i=0; i<coll_size(c); i++){
22         if(c->a[i] == d){
23             return true;
24         }
25     }
26     return false;
27 }
```

```
void coll_add(coll* c, colltype d)
{
    if(c){
        if(c->size >= FIXEDSIZE){
            on_error("Collection overflow");
        }
        c->a[c->size] = d;
        c->size = c->size + 1;
    }
}

bool coll_remove(coll* c, colltype d)
{
    for(int i=0; i<coll_size(c); i++){
        if(c->a[i] == d){
            // Shuffle end of array left one
            for(int j=i; j<coll_size(c); j++){
                c->a[j] = c->a[j+1];
            }
            c->size = c->size - 1;
            return true;
        }
    }
    return false;
}

bool coll_free(coll* c)
{
    free(c);
    return true;
}
```

# Collection ADT via an Array (Realloc)

Realloc/specific.h:

```
1  #pragma once
2
3  #define COLTYPE "Realloc"
4
5  #define FIXEDSIZE 16
6  #define SCALEFACTOR 2
7  struct coll {
8      // Underlying array
9      colltype* a;
10     int size;
11     int capacity;
12 };
```

# Collection ADT via an Array (Realloc)

## Realloc/specific.h:

```
1  #pragma once
2
3  #define COLLTTYPE "Realloc"
4
5  #define FIXEDSIZE 16
6  #define SCALEFACTOR 2
7  struct coll {
8      // Underlying array
9      colltype* a;
10     int size;
11     int capacity;
12 };
```

## Realloc/realloc.c:

```
1  #include "../coll.h"
2  #include "specific.h"
3
4  coll* coll_init(void)
5  {
6      coll* c = (coll*) ncalloc(sizeof(coll), 1);
7      c->a = (colltype*) ncalloc(sizeof(colltype), FIXEDSIZE);
8      c->size = 0;
9      c->capacity = FIXEDSIZE;
10     return c;
11 }
12
13 void coll_add(coll* c, colltype d)
14 {
15     if(c){
16         if(c->size >= c->capacity){
17             c->a = (colltype*) nrealloc(c->a,
18                 sizeof(colltype)*c->capacity*SCALEFACTOR);
19             c->capacity = c->capacity*SCALEFACTOR;
20         }
21         c->a[c->size] = d;
22         c->size = c->size + 1;
23     }
```



# Collection ADT via a Linked List

Linked/specific.h:

```
1  #pragma once
2
3  #define COLLTYP "Linked"
4
5  struct dataframe {
6      colltype i;
7      struct dataframe* next;
8  };
9  typedef struct dataframe dataframe;
10
11 struct coll {
12     // Underlying array
13     dataframe* start;
14     int size;
15 };
```

# Collection ADT via a Linked List

## Linked/specific.h:

```
1  #pragma once
2
3  #define COLLYTYPE "Linked"
4
5  struct dataframe {
6      colltype i;
7      struct dataframe* next;
8  };
9  typedef struct dataframe dataframe;
10
11 struct coll {
12     // Underlying array
13     dataframe* start;
14     int size;
15 };
```

## Linked/linked.c:

```
#include "../coll.h"
#include "specific.h"

coll* coll_init(void)
{
    coll* c = (coll*) nalloc(sizeof(coll), 1);
    return c;
}

int coll_size(coll* c)
{
    if(c==NULL){
        return 0;
    }
    return c->size;
}

bool coll_isin(coll* c, colltype d)
{
    if(c == NULL || c->start==NULL){
        return false;
    }
    dataframe* f = c->start;
    do{
        if(f->i == d){
            return true;
        }
        f = f->next;
    }while(f != NULL);
    return false;
}
```

# Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
{
    if(c){
        dataframe* f = nalloc(sizeof(dataframe), 1);
        f->i = d;
        f->next = c->start;
        c->start = f;
        c->size = c->size + 1;
    }
}

bool coll_free(coll* c)
{
    if(c){
        dataframe* tmp;
        dataframe* p = c->start;
        while(p!=NULL){
            tmp = p->next;
            free(p);
            p = tmp;
        }
        free(c);
    }
    return true;
}
```

# Collection ADT via a Linked List II

```
void coll_add(coll* c, colltype d)
{
    if(c){
        dataframe* f = nalloc(sizeof(dataframe), 1);
        f->i = d;
        f->next = c->start;
        c->start = f;
        c->size = c->size + 1;
    }
}

bool coll_free(coll* c)
{
    if(c){
        dataframe* tmp;
        dataframe* p = c->start;
        while(p!=NULL){
            tmp = p->next;
            free(p);
            p = tmp;
        }
        free(c);
    }
    return true;
}
```

```
bool coll_remove(coll* c, colltype d)
{
    dataframe* f1, *f2;
    if((c==NULL) || (c->start==NULL)){
        return false;
    }

    // If Front
    if(c->start->i == d){
        f1 = c->start->next;
        free(c->start);
        c->start = f1;
        c->size = c->size - 1;
        return true;
    }

    f1 = c->start;
    f2 = c->start->next;
    do{
        if(f2->i == d){
            f1->next = f2->next;
            free(f2);
            c->size = c->size - 1;
            return true;
        }
        f1 = f2;
        f2 = f1->next;
    }while(f2 != NULL);
    return false;
}
```

# Collection Summary

- Any code using the ADT can be compiled against any of the implementations, e.g. the test (`testcoll.c`) code.

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# Collection Summary

- Any code using the ADT can be compiled against any of the implementations, e.g. the test (testcoll.c) code.
- The *Collection* interface (coll.h) is never changed.
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  - Fixed Array : Simple to implement - can't avoid the problems of it being a fixed-size. Deletion expensive.



# Collection Summary

- Any code using the ADT can be compiled against any of the implementations, e.g. the test (testcoll.c) code.
- The *Collection* interface (coll.h) is never changed.
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- Any code using the ADT can be compiled against any of the implementations, e.g. the test (testcoll.c) code.
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  - Fixed Array : Simple to implement - can't avoid the problems of it being a fixed-size. Deletion expensive.
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  - Linked : Slightly fiddly implementation - fast to delete an element.

Task	Fixed Array	Realloc Array	Linked List
Insert new element	$O(1)$ at end <i>if space</i>	$O(1)$ at end <i>but realloc()</i>	$O(1)$ at front
Search for an element	$O(n)$ <i>brute force</i>	$O(n)$ <i>brute force</i>	$O(n)$ <i>brute force</i>
Search + delete	$O(n) + O(n)$ <i>move left</i>	$O(n) + O(n)$ <i>move left</i>	$O(n) + O(1)$ <i>delete 'free'</i>

- If we had ordered our ADT (ie. the elements were sorted), then the searches could be via a binary / interpolation search, leading to  $O(\log n)$  or  $O(\log \log n)$  search times.

# ADTs Making Coding Simpler

That Linked List code from the previous  
Chapter again:

# ADTs Making Coding Simpler

That Linked List code from the previous Chapter again:

```
1  #include "coll.h"
2  #include "Fixed/specific.h"
3
4  int main(void)
5  {
6      coll* c;
7      int i;
8
9      printf("Please type some numbers :");
10     c = coll_init();
11     while(scanf("%i", &i) == 1){
12         coll_add(c, i);
13     }
14     // Do print etc.
15     coll_free(c);
16     return 0;
17 }
```

# Table of Contents

N : Recursion

O : Algorithms I - Search

P : Linked Data Structures

Q : ADTs - Collection

**R : ADTs - Stacks**

S : ADTs - Queues

T : ADTs - Trees

U : ADTs - Hashing

At the highest level of abstraction, ADTs that we can represent using both dynamic structures (pointers) and also fixed structures (arrays) include:

- Collections (Lists)



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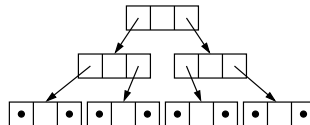
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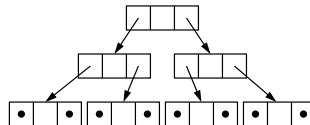
Binary Trees:



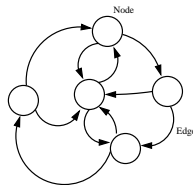
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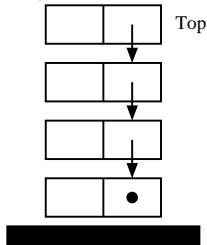
Unidirectional Graph:





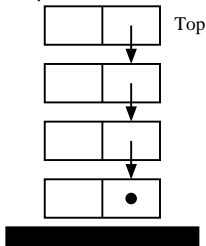
# Stacks

The push-down stack:

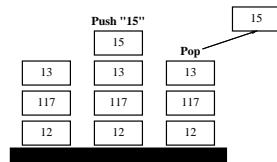


# Stacks

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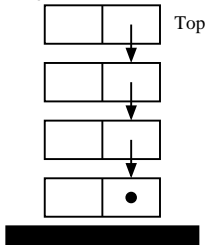
LIFO (Last in, First out):



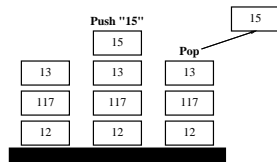
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# Stacks

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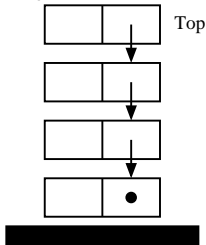
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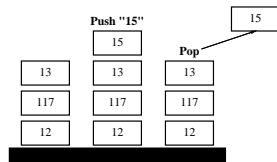
- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.

# Stacks

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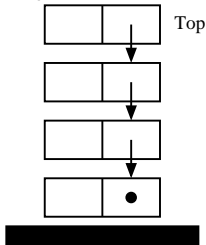


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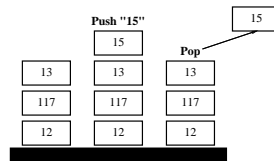


- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.

The push-down stack:



LIFO (Last in, First out):



- Operations include push and pop.
- In the C run-time system, function calls are implemented using stacks.
- Most recursive algorithms can be re-written using stacks instead.
- But, once again, we are faced with the question : How best to implement such a data type ?

# ADT:Stacks Arrays (Realloc) I

stack.h:

```
1  #pragma once
2
3  #include "../General/general.h"
4
5  typedef int stacktype;
6
7  typedef struct stack stack;
8
9  #include <stdio.h>
10 #include <stdlib.h>
11 #include <assert.h>
12 #include <string.h>
13
14 /* Create an empty stack */
15 stack* stack_init(void);
16 /* Add element to top */
17 void stack_push(stack* s, stacktype i);
18 /* Take element from top */
19 bool stack_pop(stack* s, stacktype* d);
20 /* Clears all space used */
21 bool stack_free(stack* s);
22
23 /* Optional? */
24
25 /* Copy top element into d (but don't pop it) */
26 bool stack_peek(stack* s, stacktype* d);
27 /* Make a string version - keep .dot in mind */
28 void stack_tostring(stack* s, char* str);
```

# ADT:Stacks Arrays (Realloc) I

## stack.h:

```
1  #pragma once
2
3  #include "../General/general.h"
4
5  typedef int stacktype;
6
7  typedef struct stack stack;
8
9  #include <stdio.h>
10 #include <stdlib.h>
11 #include <assert.h>
12 #include <string.h>
13
14 /* Create an empty stack */
15 stack* stack_init(void);
16 /* Add element to top */
17 void stack_push(stack* s, stacktype i);
18 /* Take element from top */
19 bool stack_pop(stack* s, stacktype* d);
20 /* Clears all space used */
21 bool stack_free(stack* s);
22
23 /* Optional? */
24
25 /* Copy top element into d (but don't pop it) */
26 bool stack_peek(stack* s, stacktype* d);
27 /* Make a string version - keep .dot in mind */
28 void stack_tostring(stack* s, char* str);
```

## Realloc/specific.h:

```
1  #pragma once
2
3  #define FORMATSTR "%i"
4  #define ELEMSIZE 20
5
6  #define STACKTYPE "Realloc"
7
8  #define FIXEDSIZE 16
9  #define SCALEFACTOR 2
10
11 struct stack {
12     /* Underlying array */
13     stacktype* a;
14     int size;
15     int capacity;
16 };
```

# ADT:Stacks Arrays (Realloc) II

## Realloc/realloc.c

```
1  #include "../stack.h"
2  #include "specific.h"
3
4  #define DOTFILE 5000
5
6  stack* stack_init(void)
7  {
8      stack *s = (stack*) ncalloc(sizeof(stack), 1);
9      /* Some implementations would allow you to pass
10       a hint about the initial size of the stack */
11      s->a = (stacktype*) ncalloc(sizeof(stacktype), FIXESIZE);
12      s->size = 0;
13      s->capacity = FIXESIZE;
14      return s;
15  }
16
17 void stack_push(stack* s, stacktype d)
18 {
19     if(s==NULL){
20         return;
21     }
22     if(s->size >= s->capacity){
23         s->a = (stacktype*) nrealloc(s->a,
24                                     sizeof(stacktype)*s->capacity*SCALEFACTOR);
25         s->capacity = s->capacity*SCALEFACTOR;
26     }
27     s->a[s->size] = d;
28     s->size = s->size + 1;
29 }
```



# ADT:Stacks Arrays (Realloc) II

## Realloc/realloc.c

```
1  #include "../stack.h"
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6  stack* stack_init(void)
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10       a hint about the initial size of the stack */
11      s->a = (stacktype*) ncalloc(sizeof(stacktype), FIXESIZE);
12      s->size = 0;
13      s->capacity = FIXESIZE;
14      return s;
15  }
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17  void stack_push(stack* s, stacktype d)
18  {
19      if(s==NULL){
20          return;
21      }
22      if(s->size >= s->capacity){
23          s->a = (stacktype*) nrealloc(s->a,
24                                     sizeof(stacktype)*s->capacity*SCALEFACTOR);
25          s->capacity = s->capacity*SCALEFACTOR;
26      }
27      s->a[s->size] = d;
28      s->size = s->size + 1;
29  }
```

```
1  bool stack_pop(stack* s, stacktype* d)
2  {
3      if((s == NULL) || (s->size < 1)){
4          return false;
5      }
6      s->size = s->size - 1;
7      *d = s->a[s->size];
8      return true;
9  }
10
11  bool stack_peek(stack* s, stacktype* d)
12  {
13      if((s==NULL) || (s->size <= 0)){
14          /* Stack is Empty */
15          return false;
16      }
17      *d = s->a[s->size - 1];
18      return true;
19  }
```

# ADT:Stacks Arrays (Realloc) III

## Realloc/realloc.c

```
1 void stack_tostring(stack* s, char* str)
2 {
3     char tmp[ELEMSIZE];
4     str[0] = '\0';
5     if((s==NULL) || (s->size <1)){
6         return;
7     }
8     for(int i=s->size-1; i>=0; i--){
9         sprintf(tmp, FORMATSTR, s->a[i]);
10        strcat(str, tmp);
11        strcat(str, "|");
12    }
13    str[strlen(str)-1] = '\0';
14 }
15
16 bool stack_free(stack* s)
17 {
18     if(s==NULL){
19         return true;
20     }
21     free(s->a);
22     free(s);
23     return true;
24 }
```

# ADT:Stacks Arrays (Realloc) III

## Realloc/realloc.c

```
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10        strcat(str, tmp);
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13    str[strlen(str)-1] = '\0';
14 }
15
16 bool stack_free(stack* s)
17 {
18     if(s==NULL){
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20     }
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22     free(s);
23     return true;
24 }
```

- We need a thorough testing program  
teststack.c

# ADT:Stacks Arrays (Realloc) III

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16 bool stack_free(stack* s)
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18     if(s==NULL){
19         return true;
20     }
21     free(s->a);
22     free(s);
23     return true;
24 }
```

- We need a thorough testing program teststack.c
- See also revstr.c : a version of the string reverse code (for which we already seen an iterative (in-place) and a recursive solution).

# ADT:Stacks Linked I

## Linked/specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%i"
4  #define ELEMSIZE 20
5  #define STACKTYPE "Linked"
6
7  struct dataframe {
8      stacktype i;
9      struct dataframe* next;
10 };
11 typedef struct dataframe dataframe;
12
13 struct stack {
14     /* Underlying array */
15     dataframe* start;
16     int size;
17 };
```

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10 };
11 typedef struct dataframe dataframe;
12
13 struct stack {
14     /* Underlying array */
15     dataframe* start;
16     int size;
17 };
```

## Linked/linked.c

```
1  #include "../stack.h"
2  #include "specific.h"
3
4  #define DOTFILE 5000
5
6  stack* stack_init(void)
7  {
8      stack* s = (stack*) nalloc(sizeof(stack), 1);
9      return s;
10 }
11
12 void stack_push(stack* s, stacktype d)
13 {
14     if(s){
15         dataframe* f = nalloc(sizeof(dataframe), 1);
16         f->i = d;
17         f->next = s->start;
18         s->start = f;
19         s->size = s->size + 1;
20     }
21 }
```

# ADT:Stacks Linked II

```
1  bool stack_pop(stack* s, stacktype* d)
2  {
3      if((s==NULL) || (s->start==NULL)){
4          return false;
5      }
6
7      dataframe* f = s->start->next;
8      *d = s->start->i;
9      free(s->start);
10     s->start = f;
11     s->size = s->size - 1;
12     return true;
13 }
14
15 bool stack_peek(stack* s, stacktype* d)
16 {
17     if((s==NULL) || (s->start==NULL)){
18         return false;
19     }
20     *d = s->start->i;
21     return true;
22 }
```

# ADT:Stacks Linked II

```
1 bool stack_pop(stack* s, stacktype* d)
2 {
3     if((s==NULL) || (s->start==NULL)){
4         return false;
5     }
6
7     dataframe* f = s->start->next;
8     *d = s->start->i;
9     free(s->start);
10    s->start = f;
11    s->size = s->size - 1;
12    return true;
13 }
14
15 bool stack_peek(stack* s, stacktype* d)
16 {
17     if((s==NULL) || (s->start==NULL)){
18         return false;
19     }
20     *d = s->start->i;
21     return true;
22 }
```

```
1 void stack_tostring(stack* s, char* str)
2 {
3     char tmp[ELEMSIZE];
4     str[0] = '\0';
5     if((s==NULL) || (s->size <1)){
6         return;
7     }
8     dataframe* p = s->start;
9     while(p){
10         sprintf(tmp, FORMATSIR, p->i);
11         strcat(str, tmp);
12         strcat(str, "|");
13         p = p->next;
14     }
15     str[strlen(str)-1] = '\0';
16 }
17
18 bool stack_free(stack* s)
19 {
20     if(s){
21         dataframe* p = s->start;
22         while(p!=NULL){
23             dataframe* tmp = p->next;
24             free(p);
25             p = tmp;
26         }
27         free(s);
28     }
29     return true;
30 }
```



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**S : ADTs - Queues**

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# ADTs : Queues

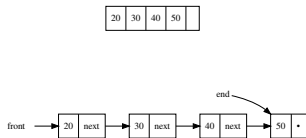
FIFO (First in, First out):



- Intuitively more “useful” than a stack.

# ADTs : Queues

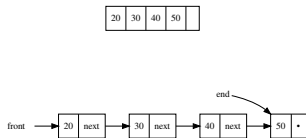
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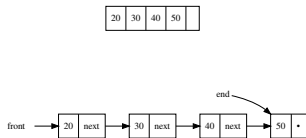
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- Intuitively more “useful” than a stack.
- Think of implementing any kind of service (printer, web etc.)
- Operations include enqueue, dequeue and size.

queue.h

```
1  #pragma once
2
3  #include "../General/general.h"
4
5  typedef int queue_type;
6
7  typedef struct queue queue;
8
9  #include <stdio.h>
10 #include <stdlib.h>
11 #include <string.h>
12 #include <assert.h>
13
14 /* Create an empty queue */
15 queue* queue_init(void);
16 /* Add element on end */
17 void queue_enqueue(queue* q, queue_type v);
18 /* Take element off front */
19 bool queue_dequeue(queue* q, queue_type* d);
20 /* Return size of queue */
21 int queue_size(queue* q);
22 /* Clears all space used */
23 bool queue_free(queue* q);
24
25 /* Helps with visualisation & testing */
26 void queue_tostring(queue* q, char* str);
```

# ADTs : Queues (Fixed) I

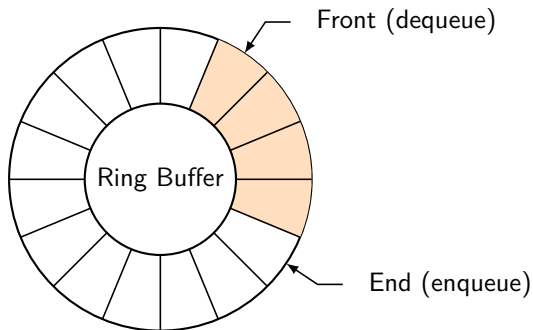
## specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
4  #define ELEMSIZE 20
5
6  #define QUEUETYPE "Fixed"
7
8  #define BOUNDED 5000
9
10 struct queue {
11     /* Underlying array */
12     queuetype a[BOUNDED];
13     int front;
14     int end;
15 };
16
17 #define DOTFILE 5000
```

# ADTs : Queues (Fixed) I

specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
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10 struct queue {
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13     int front;
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15 };
16
17 #define DOTFILE 5000
```





# ADTs : Queues (Fixed) II

## fixed.c

```
1  #include "../queue.h"
2  #include "specific.h"
3
4  void __inc(queuetype* p);
5
6  queue* queue_init(void)
7  {
8      queue* q = (queue*) nalloc(sizeof(queue), 1);
9      return q;
10 }
11
12
13 void queue_enqueue(queue* q, queuetype d)
14 {
15     if(q){
16         q->a[q->end] = d;
17         __inc(&q->end);
18         if(q->end == q->front){
19             on_error("Queue too large");
20         }
21     }
22 }
```

# ADTs : Queues (Fixed) II

## fixed.c

```
1  #include "../queue.h"
2  #include "specific.h"
3
4  void __inc(queuetype* p);
5
6  queue* queue_init(void)
7  {
8      queue* q = (queue*) nalloc(sizeof(queue), 1);
9      return q;
10 }
11
12
13 void queue_enqueue(queue* q, queuetype d)
14 {
15     if(q){
16         q->a[q->end] = d;
17         __inc(&q->end);
18         if(q->end == q->front){
19             on_error("Queue too large");
20         }
21     }
22 }
```

```
1  bool queue_dequeue(queue* q, queuetype* d)
2  {
3      if((q==NULL) || (q->front==q->end)){
4          return false;
5      }
6      *d = q->a[q->front];
7      __inc(&q->front);
8      return true;
9  }
10
11 void queue_tostring(queue* q, char* str)
12 {
13     char tmp[ELEMSIZE];
14     str[0] = '\0';
15     if((q==NULL) || (queue_size(q)==0)){
16         return;
17     }
18     for(int i=q->front; i != q->end;){
19         sprintf(tmp, FORMATSTR, q->a[i]);
20         strcat(str, tmp);
21         strcat(str, "|");
22         __inc(&i);
23     }
24     str[strlen(str)-1] = '\0';
25 }
```

# ADTs : Queues (Fixed) III

```
1  int queue_size(queue* q)
2  {
3      if(q==NULL){
4          return 0;
5      }
6      if(q->end >= q->front){
7          return q->end-q->front;
8      }
9      return q->end + BOUNDED - q->front;
10 }
11
12 bool queue_free(queue* q)
13 {
14     free(q);
15     return true;
16 }
17
18 void _inc(queuetype* p)
19 {
20     *p = (*p + 1) % BOUNDED;
21 }
```

# ADTs : Queues (Fixed) III

```
1  int queue_size(queue* q)
2  {
3      if(q==NULL){
4          return 0;
5      }
6      if(q->end >= q->front){
7          return q->end - q->front;
8      }
9      return q->end + BOUNDED - q->front;
10 }
11
12 bool queue_free(queue* q)
13 {
14     free(q);
15     return true;
16 }
17
18 void __inc(queuetype* p)
19 {
20     *p = (*p + 1) % BOUNDED;
21 }
```

- We need a thorough testing program

# ADTs : Queues (Fixed) III

```
1  int queue_size(queue* q)
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5      }
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12 bool queue_free(queue* q)
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- We need a thorough testing program
- We'll see queues again for traversing trees

# ADTs : Queues (Fixed) III

```
1  int queue_size(queue* q)
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8      }
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10 }
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12 bool queue_free(queue* q)
13 {
14     free(q);
15     return true;
16 }
17
18 void __inc(queuetype* p)
19 {
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```

- We need a thorough testing program
- We'll see queues again for traversing trees
- Simulating a (slow) printer

# ADTs : Queues (Linked) I

## specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
4  #define ELEMSIZE 20
5
6  #define QUEUETYPE "Linked"
7
8  struct dataframe {
9      queuetype i;
10     struct dataframe* next;
11 };
12 typedef struct dataframe dataframe;
13
14 struct queue {
15     /* Underlying array */
16     dataframe* front;
17     dataframe* end;
18     int size;
19 };
```

# ADTs : Queues (Linked) I

## specific.h

```
1  #pragma once
2
3  #define FORMATSTR "%d"
4  #define ELEMSIZE 20
5
6  #define QUEUETYPE "Linked"
7
8  struct dataframe {
9      queuetype i;
10     struct dataframe* next;
11 };
12 typedef struct dataframe dataframe;
13
14 struct queue {
15     /* Underlying array */
16     dataframe* front;
17     dataframe* end;
18     int size;
19 };
```

## linked.c

```
1  #include "../queue.h"
2  #include "specific.h"
3
4  queue* queue_init(void)
5  {
6      queue* q = (queue*) nalloc(sizeof(queue), 1);
7      return q;
8  }
9
10 void queue_enqueue(queue* q, queuetype d)
11 {
12     dataframe* f;
13     if(q == NULL){
14         return;
15     }
16
17     /* Copy the data */
18     f = nalloc(sizeof(dataframe), 1);
19     f->i = d;
20
21     /* 1st one */
22     if(q->front == NULL){
23         q->front = f;
24         q->end = f;
25         q->size = q->size + 1;
26         return;
27     }
28     /* Not 1st */
29     q->end->next = f;
30     q->end = f;
31     q->size = q->size + 1;
32 }
```



# ADTs : Queues (Linked) II

```
1  bool queue_dequeue(queue* q, queue_t* d)
2  {
3      dataframe* f;
4      if((q==NULL) || (q->front==NULL) || (q->end==NULL)){
5          return false;
6      }
7      f = q->front->next;
8      *d = q->front->i;
9      free(q->front);
10     q->front = f;
11     q->size = q->size - 1;
12     return true;
13 }
14
15 bool queue_free(queue* q)
16 {
17     if(q){
18         dataframe* tmp;
19         dataframe* p = q->front;
20         while(p!=NULL){
21             tmp = p->next;
22             free(p);
23             p = tmp;
24         }
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7      f = q->front->next;
8      *d = q->front->i;
9      free(q->front);
10     q->front = f;
11     q->size = q->size - 1;
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14
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16 {
17     if(q){
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25         free(q);
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27     return true;
28 }
```

```
1  void queue_tostring(queue* q, char* str)
2  {
3      dataframe *p;
4      char tmp[ELEMSIZE];
5      str[0] = '\0';
6      if((q==NULL) || (q->front == NULL)){
7          return;
8      }
9      p = q->front;
10     while(p){
11         sprintf(tmp, FORMATSTR, p->i);
12         strcat(str, tmp);
13         strcat(str, "|");
14         p = p->next;
15     }
16     str[strlen(str)-1] = '\0';
17 }
18
19 int queue_size(queue* q)
20 {
21     if((q==NULL) || (q->front==NULL)){
22
23         return 0;
24     }
25     return q->size;
26 }
```

## Detour : Graphviz

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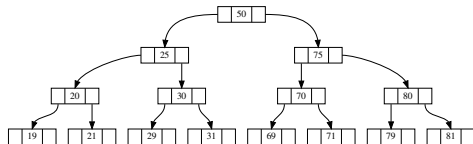
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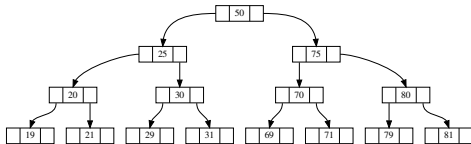
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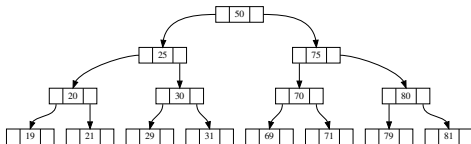
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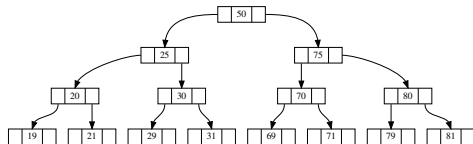




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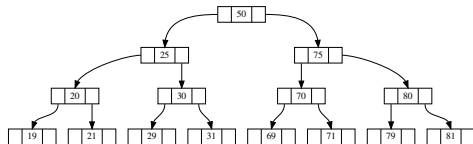
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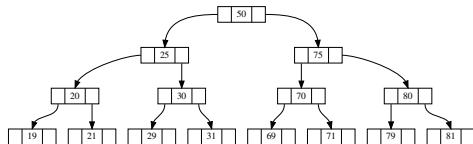
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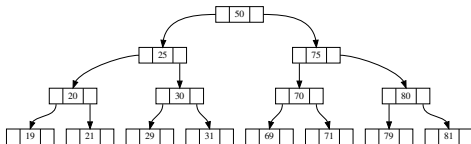
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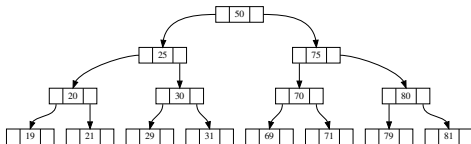
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- Ancestor relationships: A is the parent of E
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- In a tree, there is only one path from the root to any child
- A node with no children is a leaf
- Most trees need to be created dynamically
- Empty subtrees are set to NULL

# Binary Trees using Arrays ?

- Don't rush to assume a linked data structure must be used to implement trees.

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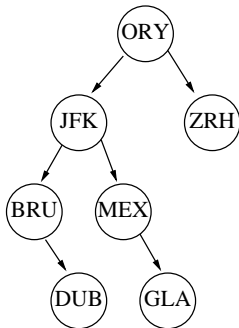
- Don't rush to assume a linked data structure must be used to implement trees.
- You could use 1 cell of an array for the first node, the next two cells for its children, the next 4 cells for their children and so on.
- You need to mark which cells are in use & which aren't ...

Counting from cell 1, for a tree with  $n$  nodes:

To find	Use	Iff
The root	$A[1]$	$A$ is nonempty
The left child of $A[i]$	$A[2i]$	$2i \leq n$
The parent of $A[i]$	$A[i/2]$	$i > 1$
Is $A[i]$ a leaf ?	True	$2i > n$

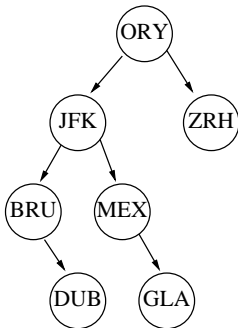
# Binary Search Trees

In a binary search tree the left-hand tree of a parent contains all keys less than the parent node, and the right-hand side all the keys greater than the parent node.



# Binary Search Trees

In a binary search tree the left-hand tree of a parent contains all keys less than the parent node, and the right-hand side all the keys greater than the parent node.



## bst.h

```
1  #include "../General/general.h"
2  #include "../Queue/queue.h"
3
4  #include <stdio.h>
5  #include <stdlib.h>
6  #include <assert.h>
7
8  bst* bst_init(void);
9
10 /* Insert 1 item into the tree */
11 bool bst_insert(bst* b, treetype d);
12
13 /* Return number of nodes in tree */
14 int bst_size(bst* b);
15
16 /* Whether the data d is stored in the tree */
17 bool bst_isin(bst* b, treetype d);
18
19 /* Bulk insert n items from an array a into an initialised tree */
20 bool bst_insertarray(bst* b, treetype* a, int n);
21
22 /* Clear all memory associated with tree, & set pointer to NULL */
23 bool bst_free(bst* b);
24
25 /* Optional ? */
26
27 char* bst_preorder(bst* b);
28 void bst_printlevel(bst* b);
29 /* Create string with tree as ((head)(left)(right)) */
30 char* bst_printlisp(bst* b);
31 /* Use Graphviz via a .dot file */
32 void bst_todot(bst* b, char* dotname);
```

# Binary Search Trees : Linked I

specific.h

```
1  #include <string.h>
2
3  typedef int treetype;
4  #define FORMATSIR "%i"
5  #define ELEMSIZE 20
6  #define BSTTYPE "Linked"
7
8  struct dataframe {
9      treetype d;
10     struct dataframe* left;
11     struct dataframe* right;
12 };
13 typedef struct dataframe dataframe;
14
15 struct bst {
16     dataframe* top;
17     /* Data element size, in bytes */
18 };
19 typedef struct bst bst;
```

# Binary Search Trees : Linked I

## specific.h

```
1  #include <string.h>
2
3  typedef int treetype;
4  #define FORMATSIR "%i"
5  #define ELEMSIZE 20
6  #define BSTTYPE "Linked"
7
8  struct dataframe {
9      treetype d;
10     struct dataframe* left;
11     struct dataframe* right;
12 };
13 typedef struct dataframe dataframe;
14
15 struct bst {
16     dataframe* top;
17     /* Data element size, in bytes */
18 };
19 typedef struct bst bst;
```

```
1  /* Based on geekforgeeks.org */
2  dataframe* __insert(dataframe* t, treetype d)
3  {
4      dataframe* f;
5      /* If the tree is empty, return a new frame */
6      if (t == NULL){
7          f = calloc(sizeof(dataframe), 1);
8          f->d = d;
9          return f;
10     }
11     /* Otherwise, recurs down the tree */
12     if (d < t->d){
13         t->left = __insert(t->left, d);
14     }
15     else if(d > t->d){
16         t->right = __insert(t->right, d);
17     }
18     /* return the (unchanged) dataframe pointer */
19     return t;
20 }
```

# Binary Search Trees : Linked II

```
1  bool __isin(dataframe* t, treetype d)
2  {
3      if(t==NULL){
4          return false;
5      }
6      if(t->d == d){
7          return true;
8      }
9      if(d < t->d){
10         return __isin(t->left, d);
11     }
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13         return __isin(t->right, d);
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16 }
```

```
1  char* __printlisp(dataframe* t)
2  {
3      char tmp[ELEMSIZE];
4      char *s1, *s2, *p;
5
6      if(t==NULL){
7          /* \0 string */
8          p = ncalloc(1,1);
9          return p;
10     }
11     sprintf(tmp, FORMATSTR, t->d);
12     s1 = __printlisp(t->left);
13     s2 = __printlisp(t->right);
14     p = ncalloc(strlen(s1)+strlen(s2)+strlen(tmp)+
15               strlen("( )"), 1);
16     sprintf(p, "%s(%s)(%s)", tmp, s1, s2);
17     free(s1);
18     free(s2);
19     return p;
```



# Binary Search Trees : Realloc

## specific.h

```
1  #include <stdbool.h>
2
3  typedef int treetype;
4  #define FORMATSTR "%i"
5  #define ELEMSIZE 20
6  #define BSTTYPE "Realloc"
7
8  // Probably (2^n) -1
9  #define INITSIZE 31
10 #define SCALEFACTOR 2
11
12 struct dataframe {
13     treetype d;
14     bool isValid;
15 };
16 typedef struct dataframe dataframe;
17
18 struct bst {
19     dataframe* a;
20     int capacity;
21 };
22 typedef struct bst bst;
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17
18 struct bst {
19     dataframe* a;
20     int capacity;
21 };
22 typedef struct bst bst;
```

## Using a queue for Level-Order traversal:

```
1  void bst_printlevel(bst* b)
2  {
3      treetype n;
4      if((b==NULL) || (! __isvalid(b, 0))){
5          return;
6      }
7      /* Make a queue of cell indices */
8      queue* q = queue_init();
9      queue_enqueue(q, 0);
10     while(queue_dequeue(q, &n) && __isvalid(b, (int)n)){
11         printf(FORMATSTR, b->a[n].d);
12         putchar(' ');
13         queue_enqueue(q, __leftchild((int)n));
14         queue_enqueue(q, __rightchild((int)n));
15     }
```

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- In this case, complexity becomes  $O(n)$ .
- The tree search performs best when well balanced trees are formed.
- Large body of literature about creating & re-balancing trees - Red-Black trees, Tries, 2-3 trees, AVL trees etc.

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- To encode the string "BABBAGE":

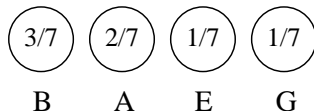
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- Keep a list of characters, ordered by their frequency

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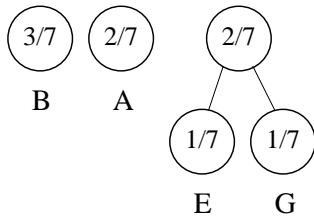
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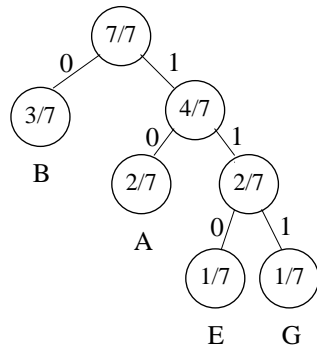
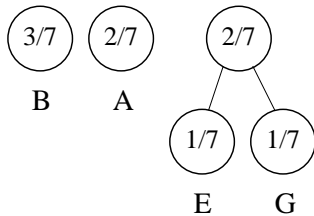
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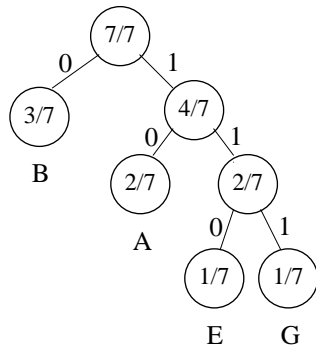
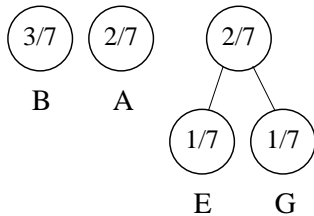
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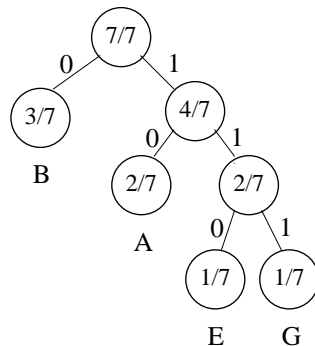
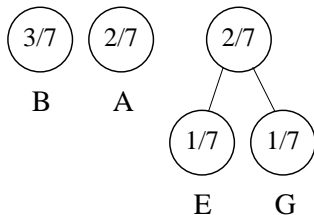
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- Use the two least frequent to form a sub-tree, and re-order (sort) the nodes :



- A = 10, B = 0, E = 110, G = 111
- String stored using 13 bits.

# Table of Contents

N : Recursion

O : Algorithms I - Search

P : Linked Data Structures

Q : ADTs - Collection

R : ADTs - Stacks

S : ADTs - Queues

T : ADTs - Trees

**U : ADTs - Hashing**

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- As an example lets use an array of size 11 to store some airport codes, e.g. PHL, DCA, FRA.

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- Inserting "PHL", "ORY" and "GCM":

	0
	1
	2
	3
PHL	4
	5
GCM	6
	7
ORY	8
	9
	10

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	9
	10

- However, inserting "HKG" causes a collision.

	0
	1
	2
	3
?	4
PHL	4
	5
GCM	6
	7
ORY	8
	9
	10

HKG

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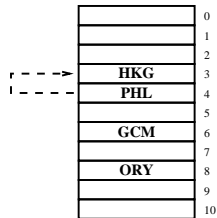
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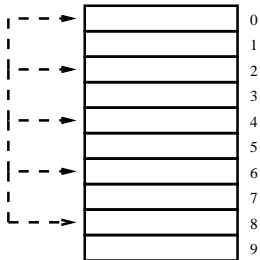
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- Although "PHL" and "HKG" share the same primary hash value of  $h(K) = 4$ , they have different probe decrements:  
 $p(\text{"PHL"}) = 4$   
 $p(\text{"HKG"}) = 3$

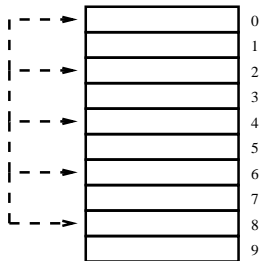
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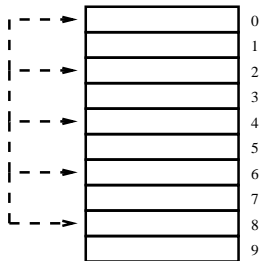
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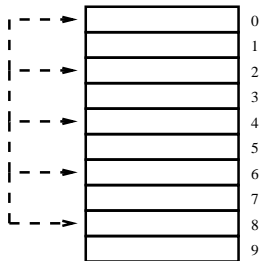
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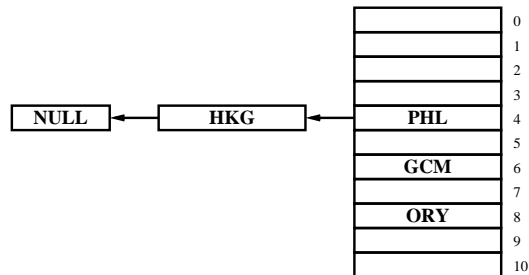
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Open-addressing is not the only method of collision reduction. Another common one is separate chaining.



# ADTs : A Practical Hash Function

```
1  #include <stdio.h>
2
3  int hash(unsigned int sz, char *s);
4
5  int main(void)
6  {
7
8      char str[] = "Hello World!";
9      // Hash modulus 7919
10     printf("%d\n", hash(7919, str));
11     return 0;
12 }
13
14 /*
15  Modified Bernstein hashing
16  5381 & 33 are magic numbers required by the algorithm
17  */
18 int hash(unsigned int sz, char *s)
19 {
20     unsigned long hash = 5381;
21     int c;
22     while((c = (*s++))) {
23         hash = 33 * hash ^ c;
24     }
25     return (int)(hash%sz);
26 }
27 }
```

Execution :

5479



# ADTs : A Practical Hash Function

```
1  #include <stdio.h>
2
3  int hash(unsigned int sz, char *s);
4
5  int main(void)
6  {
7
8      char str[] = "Hello World!";
9      // Hash modulus 7919
10     printf("%d\n", hash(7919, str));
11     return 0;
12 }
13
14 /*
15  Modified Bernstein hashing
16  5381 & 33 are magic numbers required by the algorithm
17  */
18 int hash(unsigned int sz, char *s)
19 {
20     unsigned long hash = 5381;
21     int c;
22     while((c = (*s++))) {
23         hash = 33 * hash ^ c;
24     }
25     return (int)(hash%sz);
26 }
27 }
```

Execution :

5479

Has similarities to the implementation of rand() :

```
#include <stdio.h>

int rand_r(unsigned int* seed);

int main(void)
{
    unsigned int seed = 0;
    printf("%d\n", rand_r(&seed));
    return 0;
}

/* This algorithm is mentioned in the ISO C standard,
   here extended for 32 bits. */
int rand_r(unsigned int* seed)
{
    unsigned int next = *seed;
    int result;

    next *= 1103515245;
    next += 12345;
    result = (unsigned int) (next / 65536) % 2048;

    next *= 1103515245;
    next += 12345;
    result <= 10;
}
```

Execution :

1012484

# ADTs : Cuckoo Hashing

- We have two tables, each with their **own** hash function.
- We only need to check two cells when searching.
- On collision, the existing item is 'cuckooed' out of it's cell into the other table.

Empty: copied farandoles into table 0(4)  
Empty: copied bronzine into table 0(12)  
Empty: copied auscultatory into table 0(5)  
Empty: copied bifer into table 0(13)  
Empty: copied steepgrass into table 0(6)  
Empty: copied prevised into table 0(7)  
Empty: copied oomph into table 0(8)  
empodium, so cuckooed out auscultatory from table 0(5)  
Empty: copied auscultatory into table 1(10)  
interquarreled, so cuckooed out bronzine from table 0(12)  
Empty: copied bronzine into table 1(5)  
ranseur, so cuckooed out empodium from table 0(5)  
Empty: copied empodium into table 1(4)  
Empty: copied megalodon into table 0(11)  
geosynchronous, so cuckooed out megalodon from table 0(11)  
Empty: copied megalodon into table 1(14)  
Empty: copied osmeteria into table 0(14)  
Table getting full -> rehashed old sz =16

# ADTs : Cuckoo Hashing

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