Course:
ELEC ENG 3110 Electric Power Systems
ELEC ENG 7074 Electric Power Systems PG

(Semester 2, 2021)

Tutorial 4

(Due by 16:10 on Wednesday 13 October 2021)

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T4.1 Transmission line equations

(a) If transmission line losses are neglected show that the voltage and current phasors as a function of distance x km from the sending end of a transmission line of length l km are respectively:

$$\hat{V}(x) = \hat{V}_S\cos(\beta x) - \boxed{j}Z_0\hat{I}_S\sin(\beta x) \quad (1)$$

$$\hat{I}(x) = \hat{I}_S\cos(\beta x) - \boxed{j}\left(\frac{\hat{V}_S}{Z_0}\right)\sin(\beta x) \quad (2)$$

where $\hat{V}_S$ and $\hat{I}_S$ are respectively the sending end voltage (in V) and current (in A) phasors, $Z_0 = \sqrt{\frac{L}{C}}$ Ω is the surge impedance of the line and $\beta = \omega\sqrt{LC}$ rad/km is the imaginary part of the propagation constant.

(b) Let the nominal voltage of the transmission line be V_0 kV (rms, ph-ph) and the three-phase surge impedance load be P_{SIL} MW. Express the transmission voltage and current phasor equations above in per-unit on an MVA base of P_{SIL} and voltage base of V_0.

(c) Suppose the transmission line is terminated with a load of $\hat{S}_R = P_R + jQ_R$ in pu on P_{SIL} and the terminal voltage is $V_R\angle 0°$ in pu on V_0. Derive equations for the sending end voltage magnitude V_S and the real and reactive power, $P_S + jQ_S$ supplied by the source.

(d) Use the equations derived in (c) to plot V_S and Q_S as a function of Q_R in the range from -0.5 to +0.5 for $P_S = 1.0$ pu and $V_R = 1.0$ pu. Suppose the transmission line length is $l = 0.05\lambda$ and recall that $\beta = \frac{2\pi}{\lambda}$ where λ is the wavelength.

(e) Use the equations derived in (c) to plot V_S and Q_S as a function of P_R in the range from 0 to 3 pu for $Q_S = 0$ pu and $V_R = 1.0$ pu and $l = 0.05\lambda$.

T4.2 **<u>Voltage regulating transformers (load voltage control).</u>** This question explores the use of regulating transformers to control voltages at bulk supply load-buses.

An important application of voltage regulating transformers is to control the voltage on the low-voltage (LV) side of transformers in bulk supply substations. Such substations receive power from the transmission system (e.g. at voltages at 132 kV and higher) and transform it to sub-transmission / distribution system voltages (e.g. 33 kV or 11 kV) for distribution to consumers. The transformers in these sub-stations typically employ on-load tap-changing transformers (OLTCs) to regulate the voltage on the LV side of the transformer. By doing so the power and reactive power drawn from the LV winding is approximately constant in the steady-state.

The objective of this question is to obtain insight into the main factors that determine the off-nominal tap position required to achieve a specified voltage at the LV side of the transformer.

Consider the following circuit.

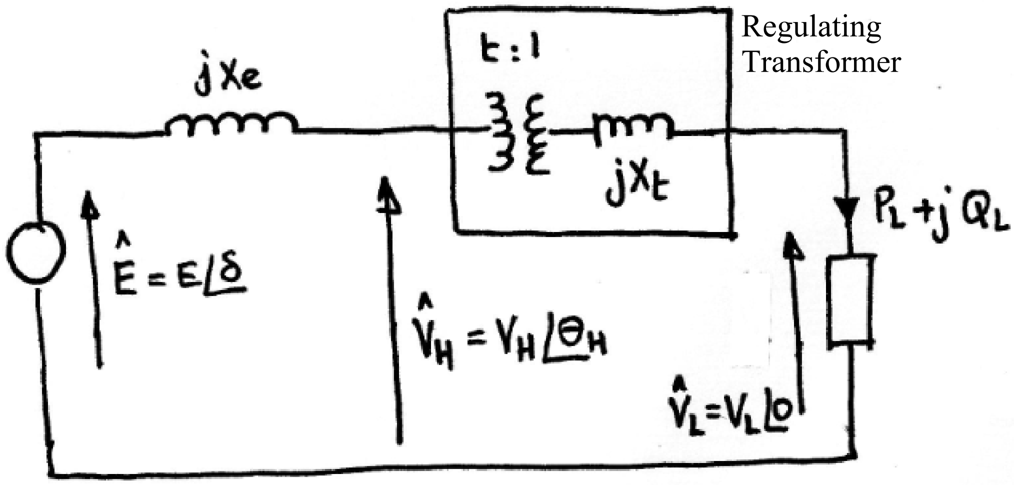


Figure 1: Regulating transformer supplying a load.

(a) Show that the source voltage phasor $\hat{E}$ is:

$$\hat{E} = \left\{tV_L + (t^2X_t + X_e)\left(\frac{Q_L}{tV_L}\right)\right\} + j\left\{(t^2X_t + X_e)\left(\frac{P_L}{tV_L}\right)\right\} = E(\cos\delta + j\sin\delta) \qquad (3)$$

Although it is possible to obtain an accurate closed form solution of the above equation for the off-nominal tap-position t we will instead seek an approximate solution in order to provide conceptual insight. Assume that the transmission angle δ is small and that consequently $\cos\delta \to 1$. Under this assumption it follows that:

$$E = tV_L + (t^2X_t + X_e)\left(\frac{Q_L}{tV_L}\right). \qquad (4)$$

(b) Solve equation (4) for the off-nominal tap-position t.

(c) Using your general solution for t from the previous question calculate t for the following numerical values: $E = 1.05$ pu, $V_L = 1.025$ pu, $Q_L = 0.3$ pu, $X_e = 0.08$ pu and $X_t = 0.16$ pu.

(d) If the substation is supplied from a very strong source (i.e. $X_e \ll X_t$) show that $E \approx V_H$ and that

$$t \approx \left(\frac{V_H}{V_L}\right)\left(1 + \frac{X_tQ_L}{V_L^2}\right)^{-1} \qquad (5)$$

Furthermore, show that if $\dfrac{X_t Q_L}{V_L^2} \ll 1$ then

$$t \approx \left(\frac{V_H}{V_L}\right) \tag{6}$$

(e) Calculate t for the numerical values in (c) using equations (5) and (6). How do these values of t compare with those which you calculated in (c)?

(f) Based on the preceding analysis summarize the main factors that determine the off-nominal tap position required to achieve a specified voltage at the LV side of the transformer.

T4.3 Maximum power transfer.

Consider the power system supplying a radial load through Figure 2. The load is $S_L = P_L + jQ_L$ in which the power factor is assumed to be fixed so that $Q_L = P_L \tan(\varphi)$ in which $\varphi = \operatorname{acos}(pf)$. The source voltage magnitude $V_S = |\hat{V}_S|$ is assumed to be constant.

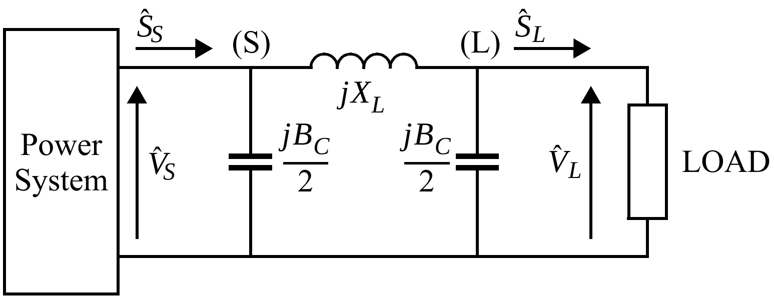


Figure 2: Power system supplying a load through a radial transmission line.

(a) Derive an equation for the load P_L such that the load bus voltage magnitude is $|\hat{V}_L| = V_{\mathrm{Lmin}}$.

(b) Derive an equation for the maximum load that can be supplied from the source and the corresponding load bus voltage magnitude.

(c) If $V_S = 1.05$ pu, $X_L = 0.3$ pu and $B_C = 0.3$ pu calculate:

(i) The load that can be supplied such that $V_L = V_{\mathrm{Lmin}} = 0.9$ pu if the load power factor is 0.98 (lagging).

(ii) The maximum load that can be supplied from the source and the corresponding load bus voltage.

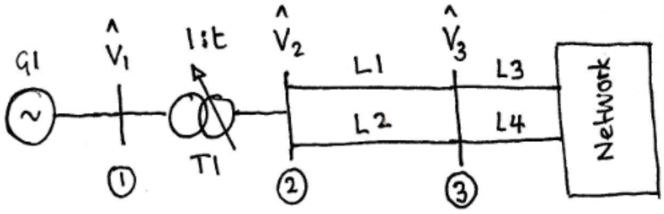
(d) Recalculate the results in (c) if the line susceptance is neglected. Comment on the significance of your findings.

T4.4 **Nodal current equation.** Consider the power system network in Figure 3.

(a) Develop the circuit diagram of the network in which the circuit elements are represented by admittances in per-unit on a base of 100 MVA.

(b) Form the nodal current equation for node 2. The nodal current equations for the other network nodes are not required.

(c) For a particular operating condition $\hat{V}_1 = 1.0\angle 18°$ and $\hat{V}_2 = 1.05\angle 9°$. Calculate the real and reactive power supplied by G1.



Item	Parameters
G1	500 MVA, 24 kV
T1	$X_t = 0.12$ pu on 500 MVA, 24 / 275 kV, Off-nominal tap $t = 1.05$ pu
L1 & L2	275 kV, 100 km line with $R = 0.04$ ohm / ph / km; $X_L = 0.319$ ohm / ph / km and $B_C = 3.651 \times 10^{-6}$ S / ph / km.
L3 & L4	275 kV, 200 km parameters the same as for L1& L2

Figure 3: Problem 4.4 – Network for nodal current equation.

T4.5 **Steady-state frequency regulation**.

Power system loads are frequency dependent and can be represented to a good approximation for system studies by:

$$P_L(\Delta f) = P_{L0}(1 + D\Delta f) \tag{7}$$

in which P_{L0} is the load at the system nominal synchronous frequency, f_0, D is the load-damping constant, Δf is the deviation of the frequency from synchronous frequency in per-unit of f_0.

The load-damping constant is expressed as the percentage change in load for a 1% change in frequency.

The steady-state frequency regulation due to frequency controls fitted to power sources (mostly synchronous generators but increasingly asynchronous sources, especially batteries) is represented by a steady-state droop characteristic relating the generator output P_G to the deviation from synchronous frequency:

$$P_G(\Delta f) = P_{G0}\left(1 - \left(\frac{1}{R}\right)\Delta f\right) \tag{8}$$

in which R is the effective 'droop' of the frequency controllers and is defined as the percentage change in frequency due to a percentage change in power output.

(a) Suppose that the system is operating at synchronous frequency and load is P_{L0}. Neglecting losses what is the power output from the power sources?

(b) Suppose that the load is increased from P_{L0} to $P_{LF0} = P_{L0} + \Delta P_{L0}$ where ΔP_{L0} is the load increase that occurs if the system operates at synchronous frequency. That is:

$$P_{LF}(\Delta f) = P_{LF0}(1 + D\Delta f) = (P_{L0} + \Delta P_{L0})(1 + D\Delta f) \tag{9}$$

Taking into account the frequency dependence of loads and the power source frequency regulation characteristic derive equations for:

(i) the frequency deviation Δf; and

(ii) the change in generation $\Delta P_G = P_G - P_{G0}$

due to the increase in load ΔP_{L0} in terms of R and D.

(c) Let $P_{L0} = P_{G0} = 0.8$ pu and let $R = 0.04$ pu and $D = 1.5$ pu. Draw a graph in which the horizontal axis is power (in pu) in the range from 0 to 1 pu and the vertical axis is the frequency deviation Δf (in pu) in the range from -0.05 to 0.05 pu. Plot the load-frequency characteristic (7) and the power source frequency regulation characteristic (8).

Draw on the graph the characteristic corresponding to an increase in load of $\Delta P_{L0} = 0.1$

From your graph determine:

(i) The change in frequency due to the load increase.

(ii) The change in the load and generation due to the increase in load.

(iii) Confirm that your results agree with the equations that you derived in (b).

(d) Comment on the relative importance of R and D in determining the frequency change due to load changes if R is typically of the order of 0.03 to 0.05 pu and D is typically of the order of 1 to 2 pu.

T4.6 <u>Fault current calculations – an introduction.</u>

Fundamental assumptions and principles for calculating the current flows due to three-phase faults on balanced three phase systems are introduced.

We seek to calculate the current phasors due to a short-circuit being applied at a network node. It is found that a very good approximation of fault currents in transmission networks are obtained if we assume:

(a) All loads are assumed to be zero.

(b) All shunt capacitors and reactors and transmission line charging is neglected.

(c) The system is assumed to be loss-less (i.e. all transmission lines, transformers, etc. have zero resistance).

(d) All synchronous generators are represented by an ideal constant voltage source behind a reactance, usually the d-axis sub-transient reactance, X_d''.

(e) Consequently, prior to the fault no current flows in the network and all ideal voltage sources, in per-unit terms, are assumed to be equal to the pre-fault voltage at the faulted bus and all of the voltage sources have a phase-angle of zero.

We begin our exploration with the power system depicted in Figure 4 in which a balanced three-phase fault is applied to bus #2.

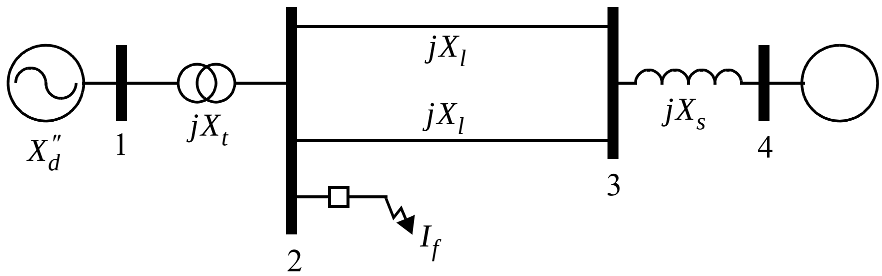


Figure 4: Power system to illustrate symmetrical fault current calculations.

The generator connected to bus #1 is represented as an ideal voltage source behind its sub-transient reactance as shown below:

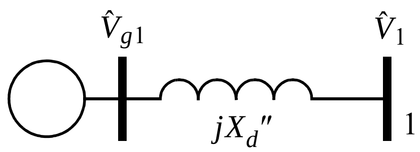


Figure 5: Equivalent circuit for synchronous generator used in symmetrical fault current calculations.

A pre-fault electrical network diagram is developed in which all reactances are expressed in per-unit on a single system MVA base as shown in Figure 6. The effect of the balanced three-phase fault is represented by closing the switch SF.

If, prior to the fault, the voltage at the fault bus (i.e. bus 2) is $V_F\angle 0°$, what is the voltage of the two voltage sources. Recall that we assume that prior to the fault current flow in the network is zero.

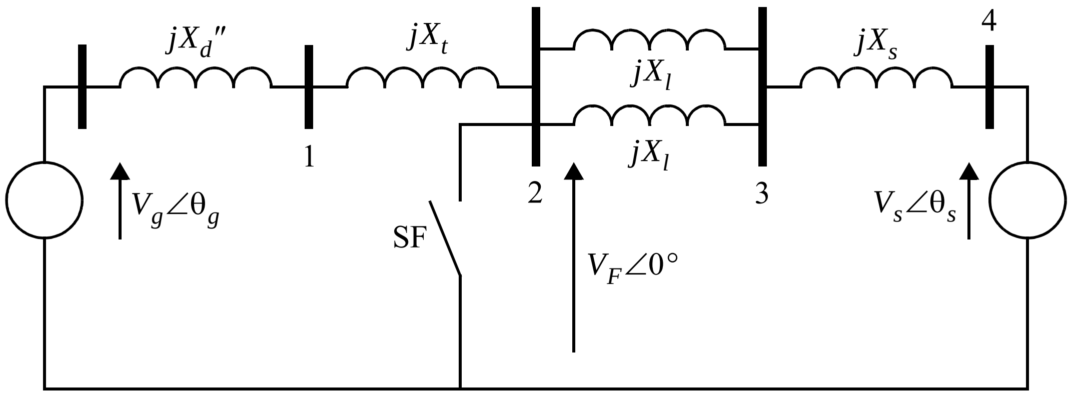


Figure 6: Electrical network diagram – pre-fault.

The fault current is the 'steady-state' current which flows through the switch SF when it is closed. We neglect transients in the calculation of the fault current. In engineering practice standards have been developed to add correction factors to account for transient effects and d.c. current offsets.

There are a variety of methods for calculating the fault current. One method that has general application is to realize that the short circuit can be represented by two equal and opposite voltage sources: $\hat{V}_{\mathrm{SCn}} = -\hat{V}_{\mathrm{SCp}}$ and in which $\hat{V}_{\mathrm{SCp}} = V_F\angle 0°$. The post-fault electrical network diagram is shown in Figure 7. Our aim is to calculate the fault current $\hat{I}_F$ which flows in the steady state when the switch is closed.

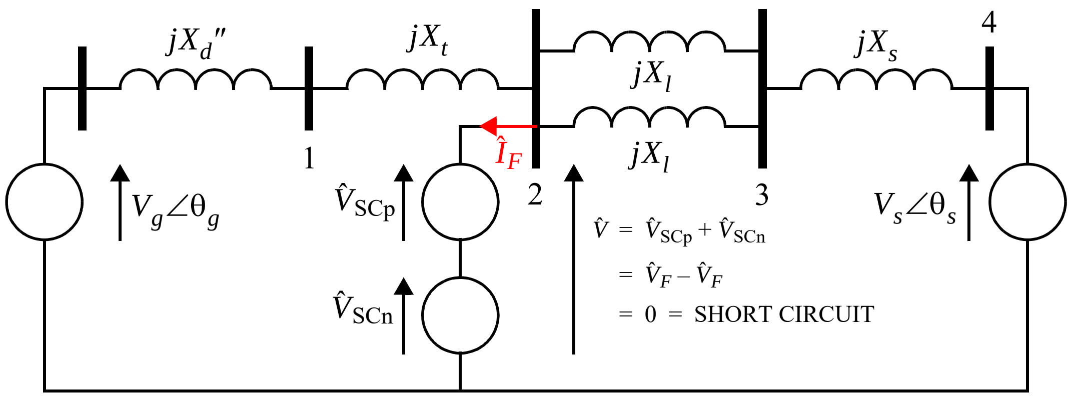


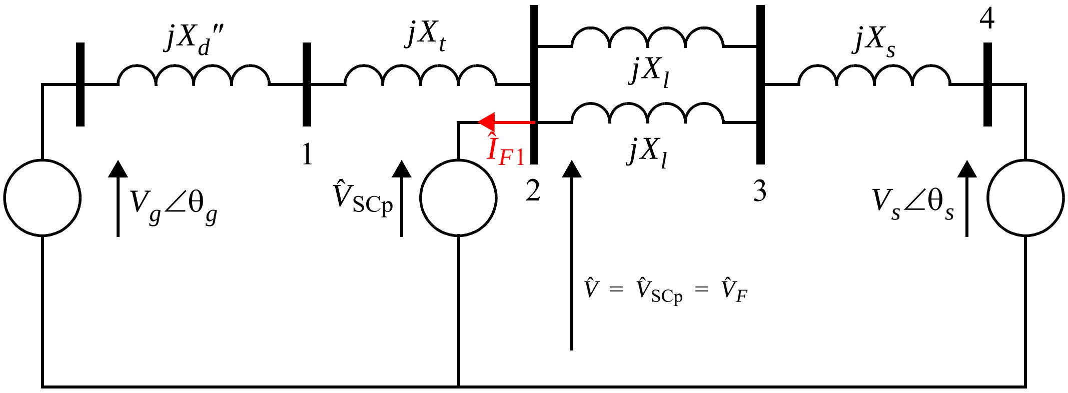
Figure 7: Electrical network diagram – post-fault network in which the short circuit is represented by two equal and opposite voltage sources.

The calculation of I_F can be made by applying the principle of superposition. We superimpose the contributions to the fault current from the addition of the two current components $\hat{I}_{F1}$ and $\hat{I}_{F2}$ obtained from the superposition of circuits 1 & 2 in Figure 8.

In the first circuit, in which the source SCn is replaced by a short circuit, the current $\hat{I}_{F1} = 0$. Explain why.

In the second circuit all voltage sources, except for the source SCn, are replaced by short circuits. *Derive an expression for the current $\hat{I}_{F2}$.*

CIRCUIT 1



CIRCUIT 2

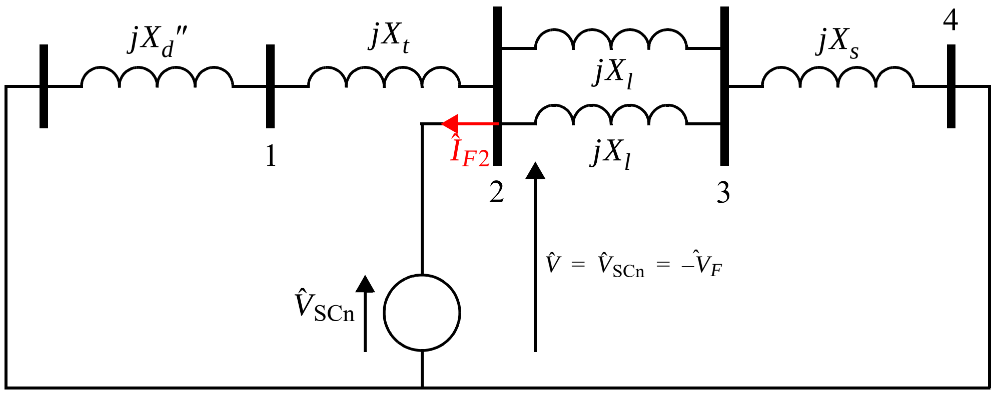


Figure 8: Electrical network diagram – post-fault network decomposed into the superposition of two circuits.

Calculate the fault current if $X_d'' = 0.25$ pu on 500 MVA, $X_t = 0.2$ pu on 550 MVA, $X_l = 0.06$ pu on 100 MVA and $X_s = 0.01$ pu on 100 MVA. The pre-fault voltage of bus #2 is 1.0 pu with a phase angle of zero.

Calculate the current supplied by the generator connected to bus #1 during the fault.