Course:
ELEC ENG 3110 Electric Power Systems
ELEC ENG 7074 Electric Power Systems PG

(Semester 2, 2021)

Tutorial 3

(Due by 16:10 on Wednesday 1 September 2021)

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T3.1 Consider the following figure showing a generator supplying a power system. In accordance with the generator sign convention the direction of positive current, $\hat{I}_S$, is in the same direction as the generator voltage $\hat{V}_S$.

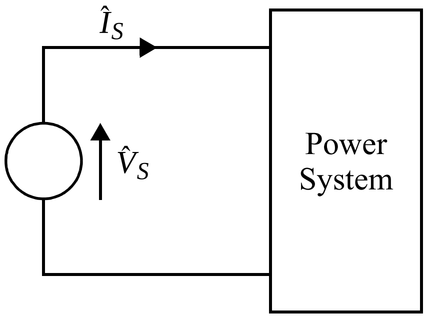


Figure 1: Generator supplying a power system.

(a) Suppose that $\hat{V}_S = 1.05\angle 0°$ pu and $\hat{I}_S = 0.9\angle{-30°}$.

(i) Calculate the real and reactive power supplied by the generator to the power system.

(ii) Calculate the susceptance that supplies the same reactive power as the generator. Is the susceptance capacitive or inductive?

(iii) Describe the generator reactive power output in the form "The generator is supplying xx pu reactive power {leading or lagging}" as appropriate.

(b) Repeat (a) with $\hat{I}_S = 0.9\angle 30°$.

T3.2 Consider the following figure showing a power system supplying a load. In accordance with the load sign convention the direction of positive current, $\hat{I}_L$, is opposite to the direction of the load voltage $\hat{V}_L$.

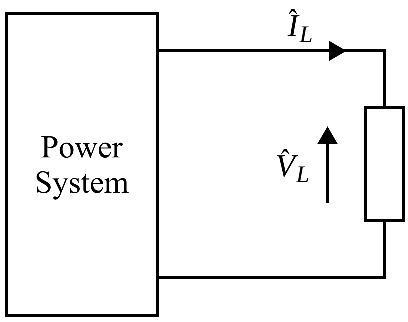


Figure 2: Load supplied from a power system.

(a) Suppose that $\hat{V}_L = 1.05\angle 0°$ pu and $\hat{I}_L = 0.9\angle{-30°}$.

(i) Calculate the real and reactive power supplied to the load from the power system.

(ii) Calculate the susceptance that consumes the same reactive power as the load. Is the susceptance capacitive or inductive?

(iii) Describe the reactive power consumption in the form "The load is consuming xx pu reactive power {leading or lagging}" as appropriate.

(b) Repeat (a) with $\hat{I}_L = 0.9\angle 30°$.

T3.3 Perform the following per-unit calculations.

(a) Convert V = 261.25 kV (rms, l-l) to per-unit on a voltage base of Vb = 275 kV (rms, l-l)

(b) Convert V = 166.71 kV (rms, l-n) to per-unit on Vb = 275 kV (rms, l-l)

(c) Convert I = 500 A (rms, line) to per-unit on a three-phase MVA base of S3b = 100 MVA and a base voltage of Vb = 330 kV (rms, l-l)

(d) A three-phase load is absorbing a total of P + jQ = 50 + j10 MW/MVAr and the voltage at the load bus-bar is 23.25 kV (rms, l-l). What is the per-unit value of load on base values of S3b = 100 MVA and Vb = 22.0 kV (rms,l-l)?

(e) What is the per-unit value of current in the preceding question?

(f) A transformer has an impedance of R + jX = 0.01 + j0.15 pu on a three-phase MVA base of 250 MVA and turns ratio of 16/345 kV/kV. What is the per-unit impedance on a new MVA base value of 100 MVA?

(g) Convert the impedance R + jX = 0.05 + j0.25 pu on a three-phase MVA base of 500 MVA and voltage base of 24 kV to per-unit on 100 MVA and 22 kV.

(h) The Thevenin impedance at a network node is R + jX = 0.005 + j0.025 pu on a base of 100 MVA and 132 kV (rms, l-l). What is the impedance in Ohm?

(i) In the preceding question calculate the short-circuit current at the node assuming that the Thevenin equivalent voltage is 1 pu on the base provided. Express the current in per-unit on the provided base values and in Amps.

(j) The defining characteristic of the phase to neutral inductance in a balanced three phase system is $v_{nl} = L\frac{di_l}{dt}$ in which v_{nl} is the phase to neutral voltage (in V), i_l is the inductor current (in A), L is the inductance (in H) and t is time (in s). The principle base quantities for the system are (i) the three phase MVA base S_b MVA; (ii) V_b kV (l-l, rms) is the phase-phase base voltage; and (iii) $t_b = 1\,\text{s}$ is the base value of time. Starting from the defining characteristic of the inductance, derive the base value of inductance (in H) in terms of the principle base quantities.

T3.4 Consider Figure 3 which shows the pi-equivalent representation of a transmission line interconnecting two parts of a power system. It is assumed that the sending (S) and receiving (R) end voltage magnitudes are controllable. The transmission line is assumed to be lossless (i.e. $Z_p = jX$ and $Y_p = jB_C$). The sending end complex power is $\hat{S}_S = P_S + jQ_S$, the receiving end complex power is $\hat{S}_R = P_R + jQ_R$, the transmitted power is $P_R = P_S = P$ and the sending and receiving end voltages are $\hat{V}_S = V_S e^{j\theta_S}$ and $\hat{V}_R = V_R e^{j\theta_R}$ respectively. The transmission angle is $\delta = \theta_S - \theta_R$.

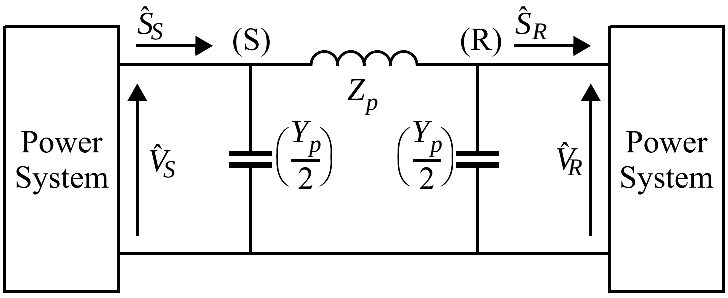


Figure 3: Problem 3.4 – Transmission line

(a) Derive equations for P, Q_S and Q_R in terms of the sending and receiving end voltage magnitudes, the transmission angle and the transmission line parameters. [Hint: the derivation will be similar to that in Tutorial 2, question 2.9 but with modifications to account for line charging capacitance.]

(b) Based on your answer to the preceding question, derive the equation for the reactive power loss in the transmission line.

(c) Assuming that the sending and receiving end voltage magnitudes are equal (i.e. $V_S = V_R = V$ derive an equation for the power transfer at which the reactive power losses are zero.

(d) Consider the system with the following numerical values: $V_S = V_R = 1.05$ pu, $X = 0.27$ pu, $B_C = 0.27$ pu. Produce a plot of the reactive power losses (on the y-axis) as a function of power flow (on the x-axis) for P in the range from 0 to 2.0 pu. Does the powerflow at which the reactive power loss is zero in your plot agree with that which you obtain from the equation that you derived in the previous question?

T3.5 Voltage control using shunt reactive compensation. This question illustrates key concepts and issues associated with the use of shunt reactive compensation for voltage control.

Consider the following network. It is assumed that the source voltage magnitude $E = 1.05$ pu is held constant by a well tuned voltage regulator. The two identical transmission links (which include transformers and transmission lines) are lossless and each have a series reactance of $X_1 = 0.5$ pu. (For simplicity and illustrative purposes, segmentation of the transmission links is not represented). The load is $P_L + jQ_L = 1.0 + j0.3$ pu.

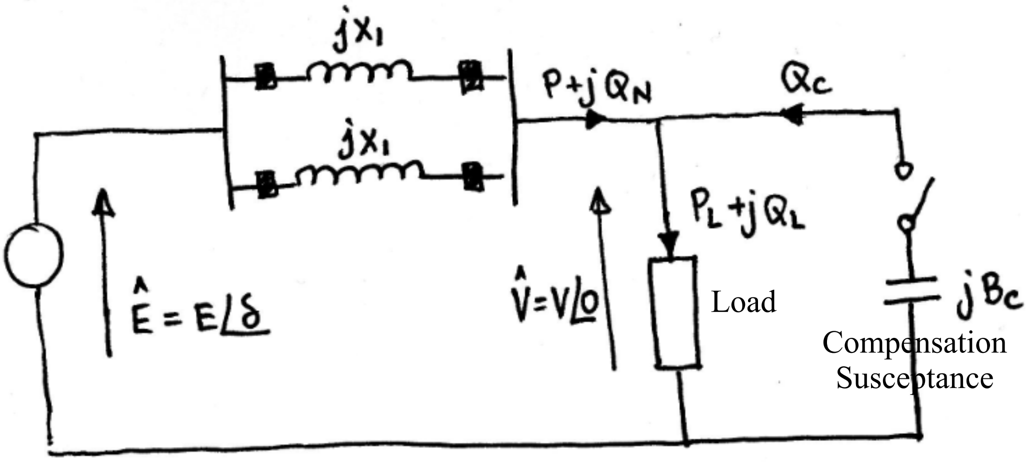


Figure 4: Problem 3.5 – Shunt reactive compensation.

(a) In the absence of load compensation (i.e. $B_C = 0$ or switch open) calculate the load bus voltage.

(b) Show that the compensation susceptance, B_C, required to achieve a specified load bus voltage magnitude of $|\hat{V}| = V_s$ pu is:

$$\text{[Precise] } B_C = \left(\frac{Q_L}{V_s^2}\right) + \frac{1}{X}\left(1 - \frac{E}{V_s}\cos\delta\right) \text{ where } \cos\delta = \sqrt{1 - \left(\frac{PX}{EV_s}\right)^2} \tag{1}$$

where X is the reactance between the source and load.

(c) Furthermore, show that if $\left(\frac{PX}{EV_s}\right)^2 \ll 1$ and $|\hat{V}| = V_s = E$ then the required compensation can be approximated by:

$$\text{[Approx] } B_C \approx \frac{1}{V_s^2}\left(Q_L + \frac{X}{2}\left(\frac{P}{V_s}\right)^2\right). \tag{2}$$

[Hint: Use the Taylors series expansion: $\sqrt{1 - x^2} = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 + \text{h.o.t.}$ and ignore terms in x^4 and higher.]

(d) Calculate the compensation susceptance B_C required to achieve a specified load bus voltage $|\hat{V}| = V_s = E$ using both the precise and approximate equations. What is the percentage error in the approximate calculation?

(e) Assume that the compensation susceptance calculated according to the precise equation in the last question is connected to the load bus. Now, if one of the two identical transmission lines between the source and load is disconnected calculate the load bus voltage. What is the change in voltage? Is this voltage acceptable?

(f) Using (1) calculate the total compensation susceptance required to restore the load bus voltage to the specified load bus voltage $|\hat{V}| = V_s = E$ following the disconnection of the line. What is the additional susceptance as compared to that determined in (d).

(g) With both transmission links in-service what would be the voltage at the load bus if the total shunt compensation determined in the previous question was connected to the load bus?

(h) Discuss the engineering significance of the results in this question. [Hint: Consider whether the use of shunt capacitors results in adequate voltage control in this example. If not, can you explain why?]

T3.6 Series capacitive compensation.

A technique which is sometimes used to improve transient and voltage stability of power systems is to insert capacitors in series with transmission lines. Such series capacitive compensation has the benefit of increasing the effective surge impedance loading of the transmission line and thus for a given power transfer reducing the reactive power losses. To develop a conceptual understanding of the effect of series capacitive compensation consider the following circuit. The reactance of the series capacitive compensation is $-\alpha X$ where X is the line reactance. The compensation factor, α, is never, in practice, more than about 0.75 and usually substantially less. In this tutorial example, the series capacitor is located at the mid-point of the line. For heavily compensated lines the series capacitance is divided between the sending and receiving end of the line.

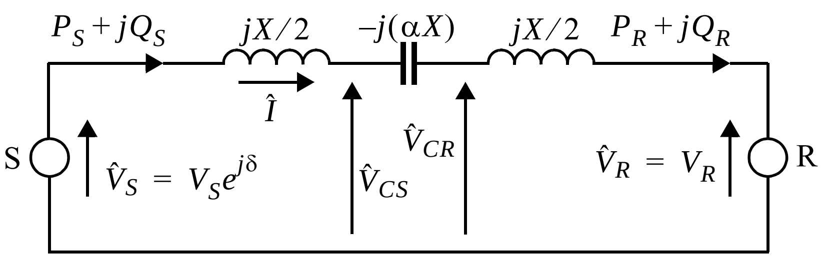


Figure 5: Series capacitive compensated transmission line.

Assume the sending and receiving end voltages are controlled to have the same magnitude V.

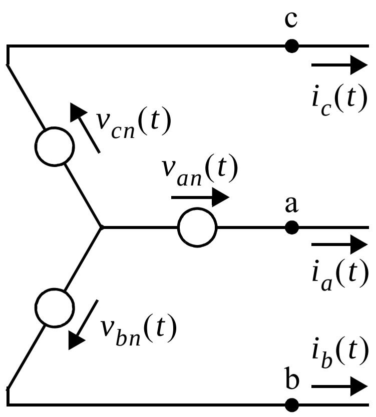
(a) For a given power transfer $P_S = P_R = P$ derive an equation for the current $\hat{I}$ in terms of P, V, X and α.

(b) Let $P = 1.0$ pu, $X = 0.4$ pu and $V = 1.05$ pu. Plot the magnitude of the current as a function of the compensation factor in the range from 0 (i.e. no compensation) to 0.75. Discuss the significance of the plot.

(c) Derive an equation for the reactive power generated by the series capacitor in terms of the current magnitude.

(d) For $\alpha = 0.5$, plot the reactive power generated by the series capacitor and the reactive power generated by the sending and receiving end sources as the power transfer is increased from zero to 3.0 pu. As in (b) let $X = 0.4$ pu and $V = 1.05$ pu. Repeat for $\alpha = 0$ and 0.25. Discuss the significance of the plots.

T3.7 A balanced set of three-phase voltages is depicted in Figure 6.



$$v_{an}(t) = V_p\cos(\omega t + \alpha)$$
$$v_{bn}(t) = V_p\cos\left(\omega t + \alpha - \frac{2\pi}{3}\right)$$
$$v_{cn}(t) = V_p\cos\left(\omega t + \alpha + \frac{2\pi}{3}\right)$$

Figure 6: Problem 3.7 – Balanced three-phase voltage source.

(a) What is the phase-sequence of the source?

(b) Represent this set of three-phase voltages as three RMS cosine-referenced phasors. In doing so it is convenient to use the rotation operator $a = e^{j\left(\frac{2\pi}{3}\right)} = 1\angle 120°$. Depict the phasors on a phasor diagram.

(c) What is the sum of the three three-phase voltages?

(d) Calculate the line-to-line voltage phasor $\hat{V}_{ab} = \hat{V}_{an} - \hat{V}_{bn}$ based on geometrical analysis on a phasor diagram.

(e) What is the RMS value of the line-to-line voltage compared to the phase-neutral value? What are the line-to-line voltage phasors $\hat{V}_{bc} = \hat{V}_{bn} - \hat{V}_{cn}$ and $\hat{V}_{ca} = \hat{V}_{cn} - \hat{V}_{an}$

(f) This three-phase source supplies a balanced three phase load. The RMS current phasor for the ‘a’ phase line-current is $\hat{I}_a = Ie^{j(\alpha + \theta)}$. Write down the current phasors for the other two line currents.

(g) Calculate the instantaneous power, p(t), delivered by the balanced three-phase source. [Hint: Take advantage of the fact that three sinusoids that are displaced in phase by 120 deg. from each other sum to zero.]

T3.8 Consider the ideal three-phase transformer in Figure 7. The low voltage windings have 28 turns per phase and the high voltage windings have 404 turns per phase. For the purposes of this exercise we assume the transformer is ideal (i.e. leakage flux and losses are neglected).

Suppose that the high voltage phasors are $\hat{V}_{AN} = V_H e^{j\theta}$, $\hat{V}_{BN} = \hat{V}_{AN} e^{-j\left(\frac{2\pi}{3}\right)}$, $\hat{V}_{CN} = \hat{V}_{AN} e^{j\left(\frac{2\pi}{3}\right)}$, $V_H = \frac{275}{\sqrt{3}} \times 1.05$ kV (rms) and $\theta = 0$ rad. and the line currents on the low voltage side of the transformer are $\hat{I}_a = I_l e^{j\beta}$, $\hat{I}_b = \hat{I}_a e^{-j\left(\frac{2\pi}{3}\right)}$, $\hat{I}_c = \hat{I}_a e^{j\left(\frac{2\pi}{3}\right)}$, $I_l = 8.658$ kA (rms) and $\beta = \theta - \frac{\pi}{6}$ rad.

(a) Calculate the voltage phasor $\hat{V}_{ab}$.

(b) Calculate the line current phasor $\hat{I}_B$.

(c) Calculate the total three-phase power supplied to the low-voltage side of the transformer in MW.

(d) Calculate the total three-phase reactive power supplied by the high-voltage side of the transformer in MVAr.

(e) Calculate the voltage ratio $\frac{\hat{V}_{AB}}{\hat{V}_{ab}}$ where $\hat{V}_{AB} = \hat{V}_{AN} - \hat{V}_{BN}$.

(f) Calculate the line current ratio $\frac{\hat{I}_a}{\hat{I}_A}$.

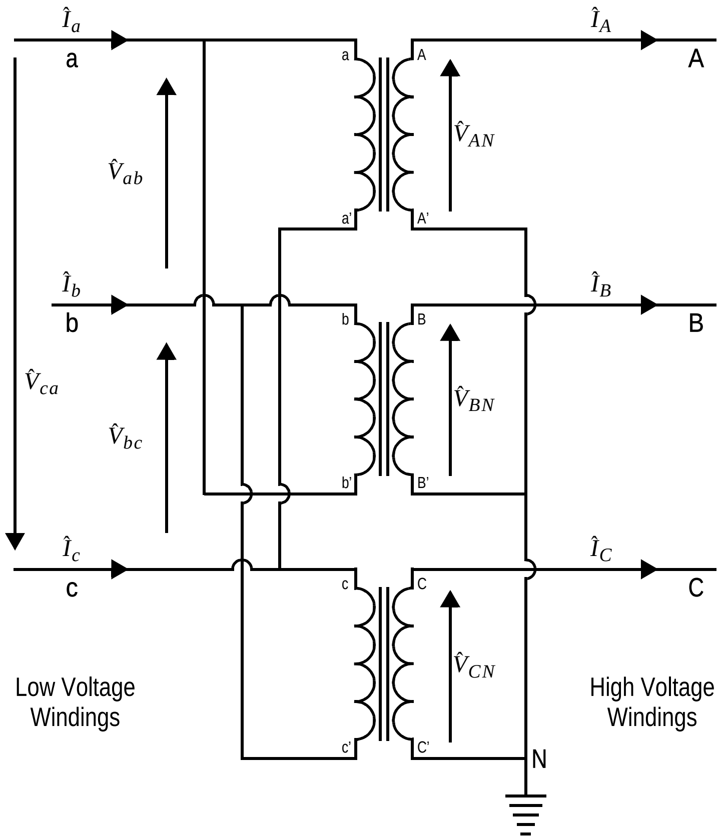


Figure 7: Problem 3.8 – Ideal three-phase transformer.