THE UNIVERSITY of ADELAIDE

Course:
ELEC ENG 3110 Electric Power Systems
ELEC ENG 7074 Power Systems PG

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Transmission Lines and their Performance

Lecturer and Co-ordinator: David Vowles
david.vowles@adelaide.edu.au

Overhead Transmission Lines[1]

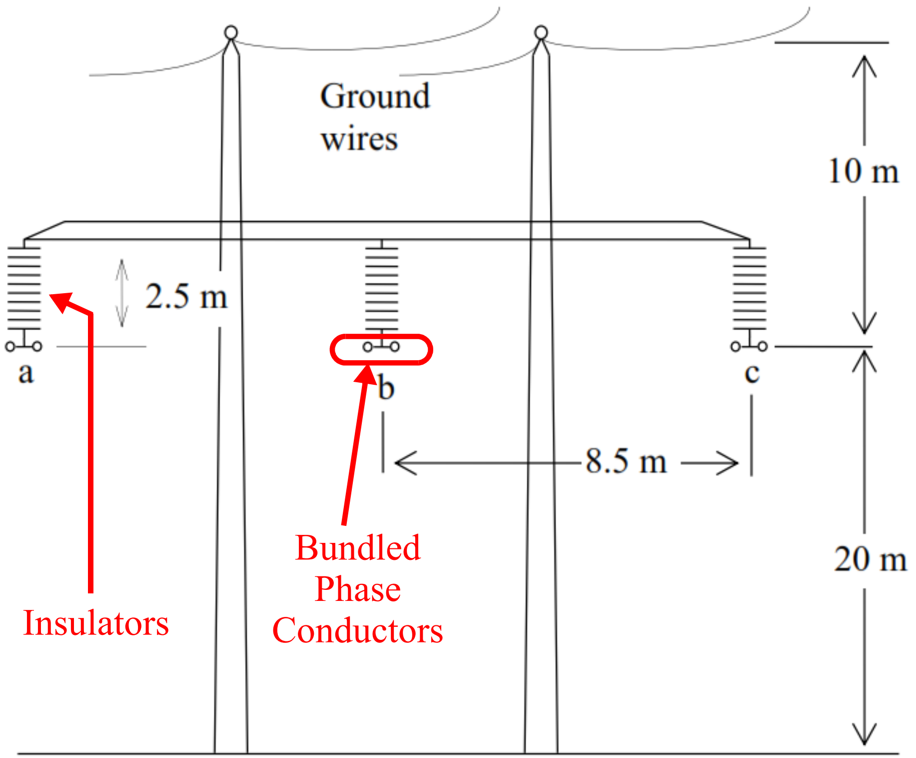


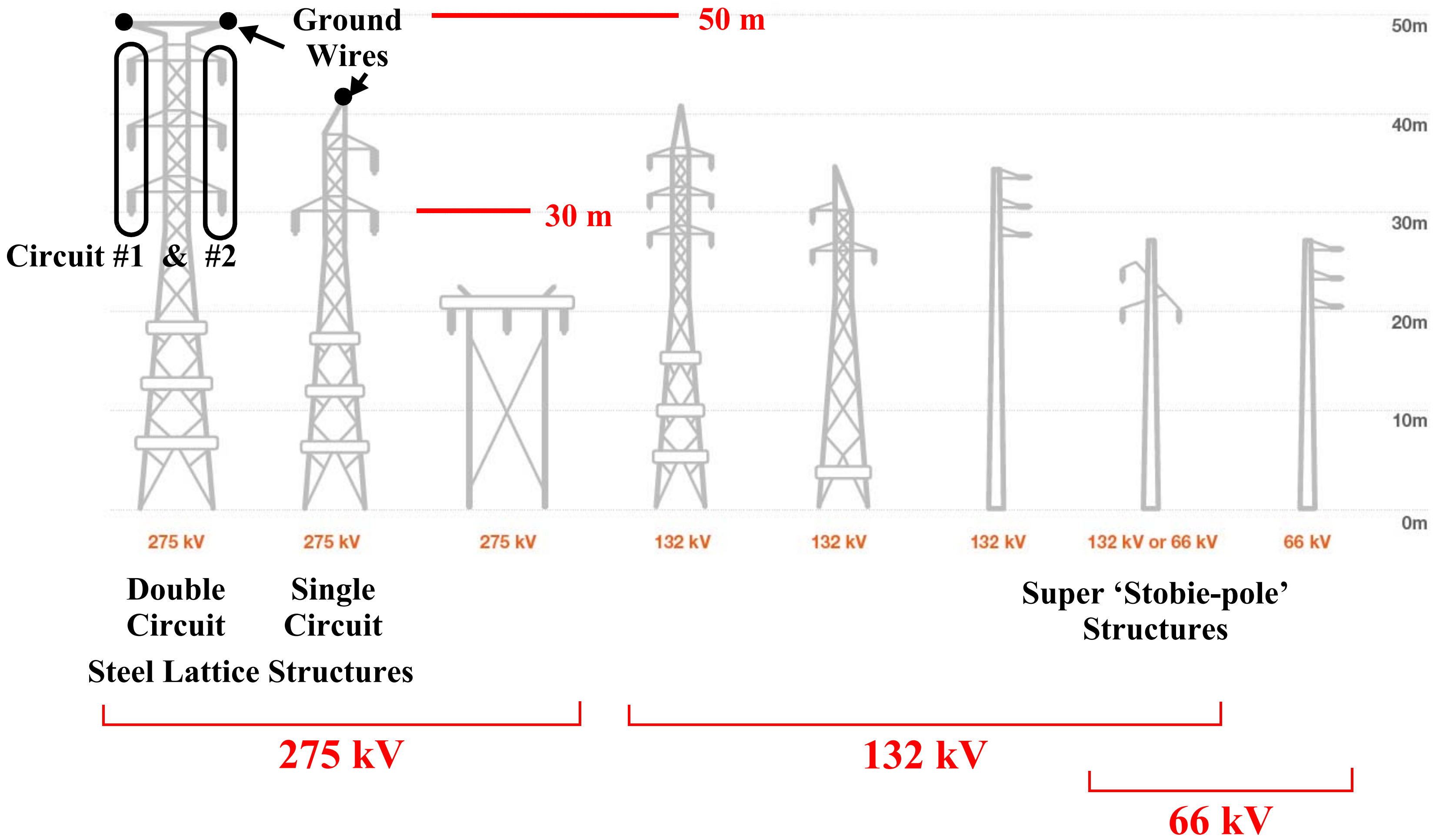
Figure 1: Example of a flat profile tower for a 275–330 kV (l-l) single-circuit transmission line.

Components:

- Transmission towers
  - Wide variety of structures
  - Often two parallel 3-phase circuits on a single tower (double-circuit transmission line)
- Insulators. Ceramic suspension insulators for high voltage lines.
  - Mechanically support conductors
  - Must withstand both normal and surge voltages to ground.
  - Subject to atmospheric contamination that reduces insulation properties (e.g. salt near sea, dust inland)
- Phase conductors.
  - Usually Aluminium Conductor with Steel Reinforcing (ACSR)
  - Typically two or four conductors in a bundle per phase for voltages > 220 kV
    - Reduce corona discharge
- Ground wires
  - Strung above phase conductors and bonded to tower to shield phase circuits from lightning.
  - Parallel ground return path for unbalanced fault currents.

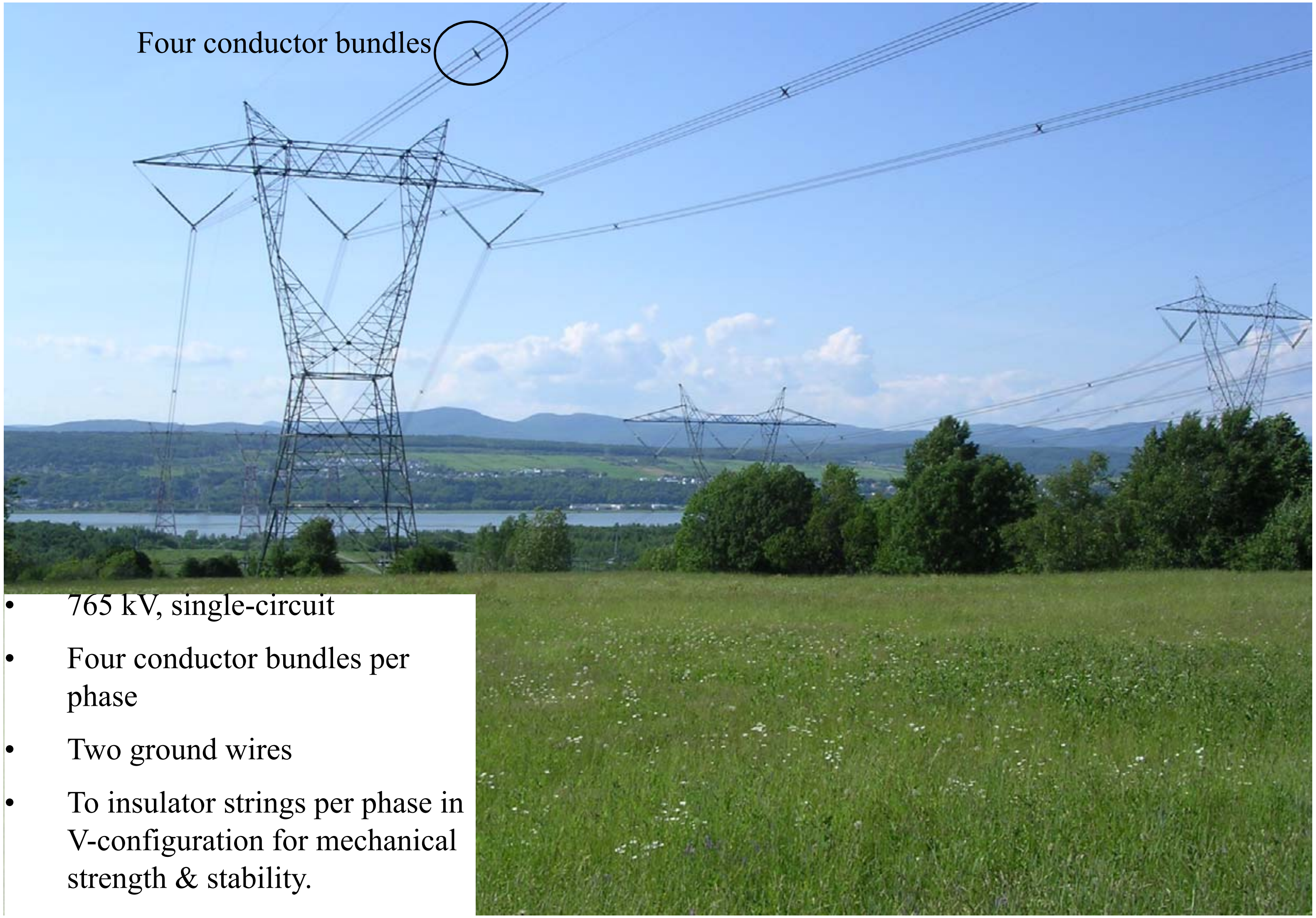
1. This section is based on a set of Power System Notes by M.J. Gibbard

Transmission Tower Profiles[1]



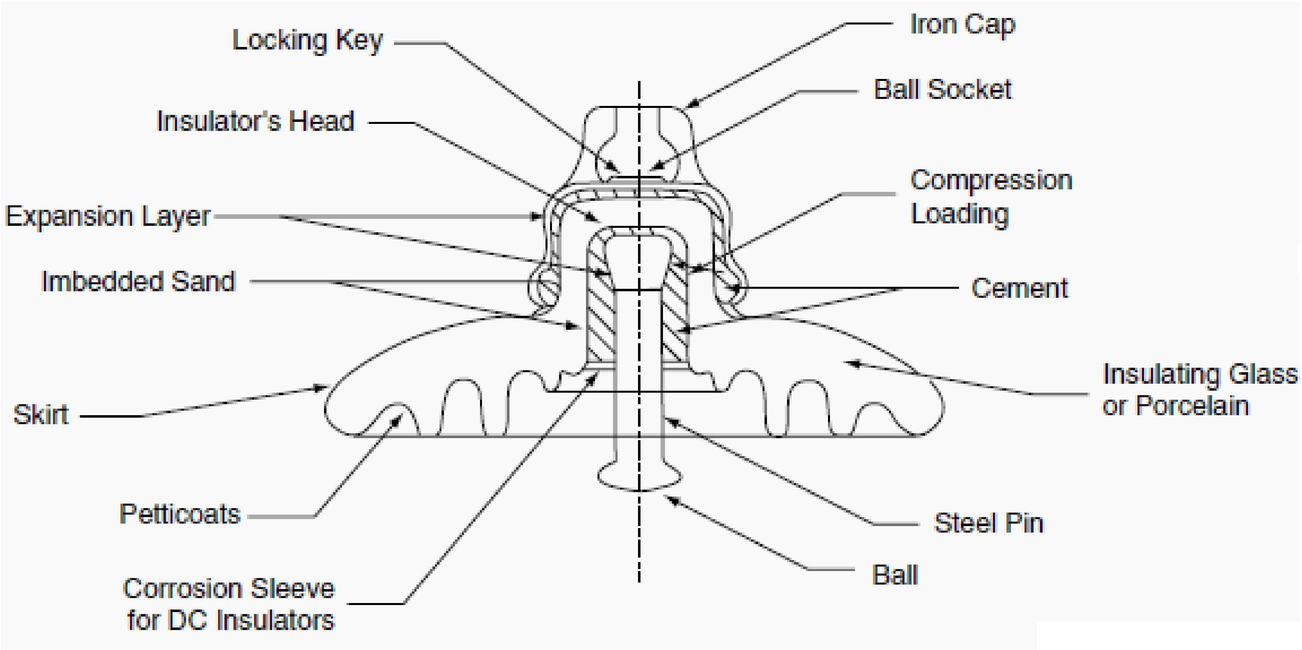
1. ElectraNet image (https://www.electranet.com.au/our-approach/safety/transmission-lines/)

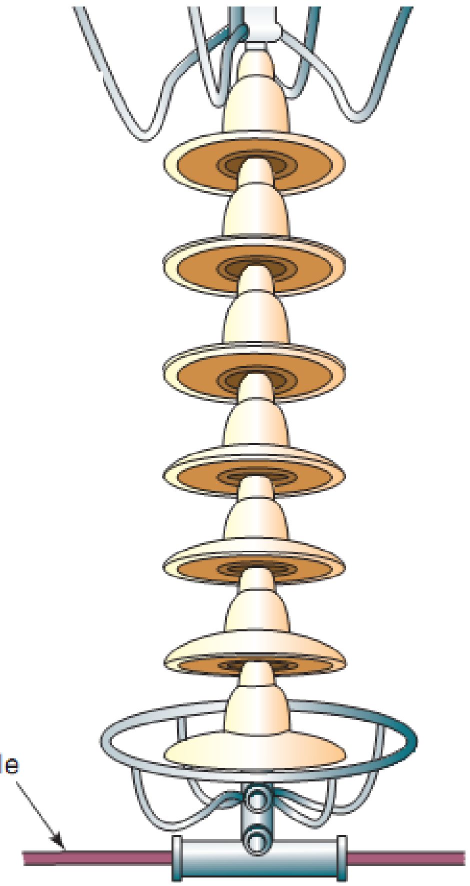
Overhead Transmission Line, 765 kV, Quebec Canada



- 765 kV, single-circuit
- Four conductor bundles per phase
- Two ground wires
- To insulator strings per phase in V-configuration for mechanical strength & stability.

Suspension Insulators





Cross-section of a single suspension type insulator (above)

- Porcelain skirt and petticoats provide elongated insulation path
- Mechanical strength relies on cement bonding between porcelain and iron cap (above) and steel pin below.

Insulator 'string' (right)

- Voltage evenly distributed between the tower (earth) and phase conductor across each insulator disk.

image above: https://electrical-engineering-portal.com/wp-content/uploads/fig-1-cross-section-of-a-standard-ball-and-socket-insulator.gif
image right: https://dc.edu.au/wp-content/uploads/metal-supporting-towers.png

Conductor Transposition

In an equilateral configuration of phase conductors, the inductance and shunt capacitance per phase is the same for all phases.

In a flat configuration, transposition of the conductors is required in order that self and mutual inductances per phase are the same. Typically, transpositions occur at 1/3 and 2/3 of the line length.

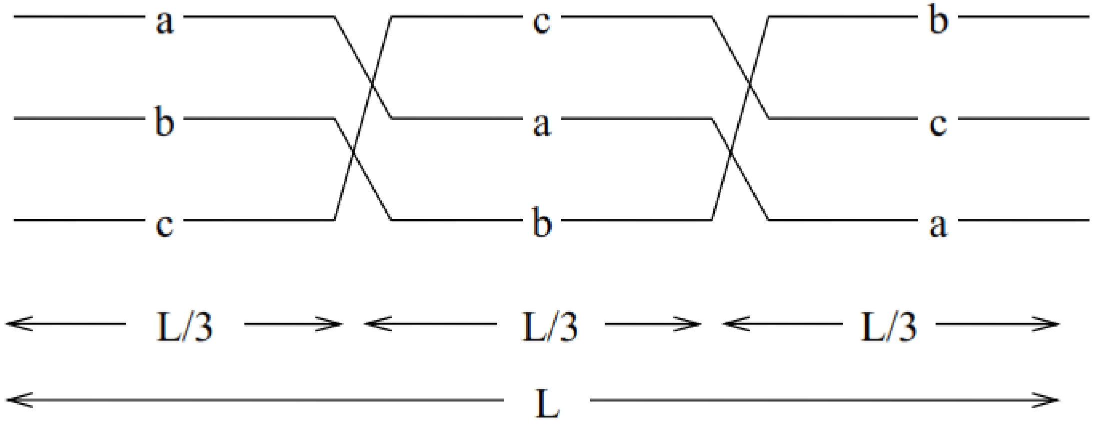


Figure 2: Transposition of phase conductors.

Transposition results in a ‘balanced’ transmission line.

Series-inductance and shunt-capacitance per-unit length

Consider an equilateral spacing of conductors (or an equivalent equilateral distribution) as shown below.

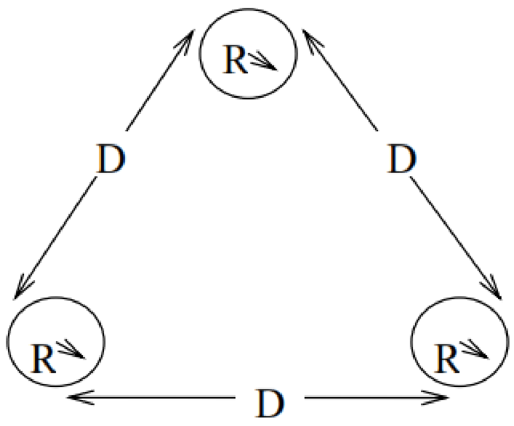


Figure 3: Equilaterally disposed phase conductors.

It can be shown that the series inductance per-phase given by:

$$L = (2 \times 10^{-7}) \ln\left(\frac{D}{R}\right) \text{ H/m} \qquad (1)$$

where D and R are the (equivalent) equilateral spacing between phases and the (equivalent) conductor radius respectively. It is assumed that $D \gg R$.

Likewise it can be shown that the capacitance per phase to a hypothetical neutral point is:

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{R}\right)} \text{ F/m} \qquad (2)$$

where ε is the permittivity of the dielectric medium (i.e. air for an overhead line, for which

$\varepsilon \sim \varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m).

For a ***non-equilateral*** configuration, D becomes the *GMD* (Geometric Mean Distance) between phases, and R is the *GMR* (Geometric Mean Radius) for a stranded conductor or for a ***bundle*** of phase conductors.

Series-inductance and shunt-capacitance per-unit length

For example, for the conductor configuration shown below, the equivalent equilateral spacing is the GMD:

$$D = \sqrt[3]{d_{ab} \times d_{cb} \times d_{ca}} \tag{3}$$

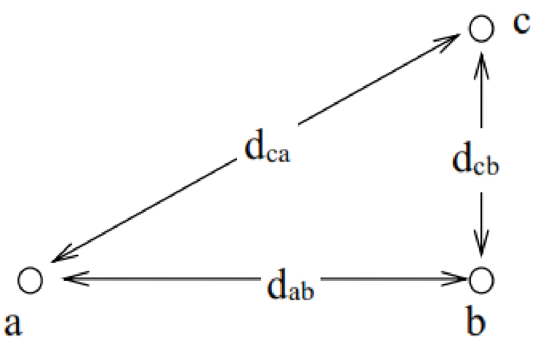


Figure 4: Arbitrary configuration of conductors

The *GMD* is the n^{th} root of the products of all *n* distances between conductors in the phase bundle and all conductors in the other phases.

For a twin-conductor phase bundle, the equivalent radius of a single solid conductor is the *GMR*,

$$R = \sqrt{r \times d_b} \tag{4}$$

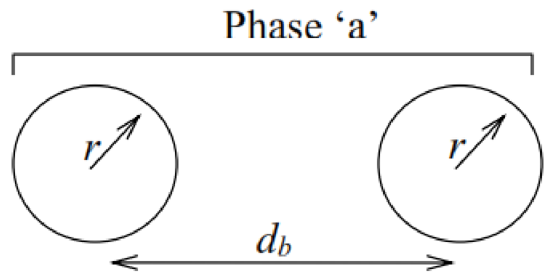


Figure 5: Twin conductor arrangement

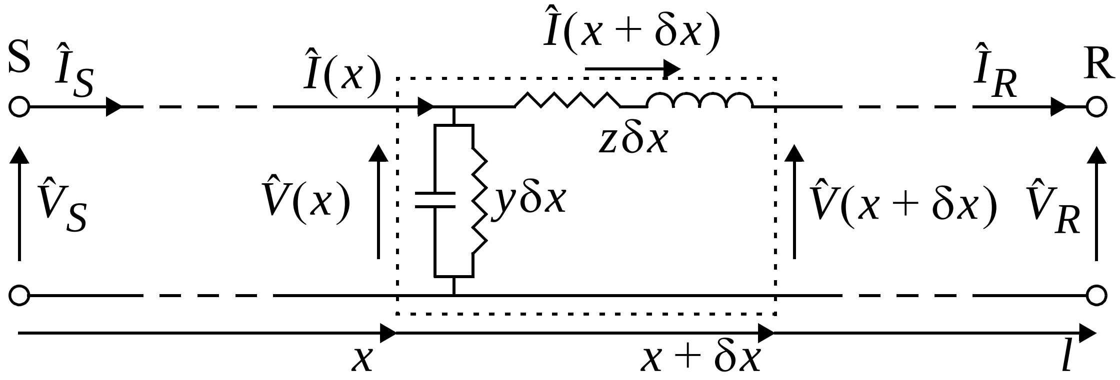
In addition to the series inductance and shunt capacitance a transmission line is characterised by its:

- Series resistance (R, ohm / m) due to conductor resistivity which must account for stranding and skin effect.
- Shunt conductance (G, mho / m) representing losses due to very small leakage currents over the surface of insulator strings and the corona effect. Such losses are very small and G is usually neglected.

Transmission Line Equations[1]

The per-unit length parameters (R, L, G, C) of a transmission line / cable (TL) are (uniformly) distributed along its length. If the line is uniformly transposed the TL equations which describe the variation of the voltage and current phasors along the length of the line can be developed on a per-phase basis.

Distributed circuit representation of TL



- $z = R + j\omega L$ series impedance per-unit length / phase
- $y = G + j\omega C$ shunt admittance per-unit length / phase
- Distance, x, measured from sending end (S) toward receiving end (R)
- The line length is l

- $\hat{V}(x)$ and $\hat{I}(x)$ are respectively the voltage- and current-phasors at an arbitrary position along the line.
- $\hat{V}_S = \hat{V}(0)$ and $\hat{I}_S = \hat{I}(0)$ are respectively the voltage- and current-phasors at the sending end of the line.
- $\hat{V}_R = \hat{V}(l)$ and $\hat{I}_R = \hat{I}(l)$ are the corresponding phasors at the receiving end.

1. This section draws on material in:
P. Kundur, “Power System Stability & Control”, (c) 1994, McGraw-Hill, Inc.

Derivation of distributed TL equations

The voltage phasor at position $x + \delta x$:

$$\hat{V}(x + \delta x) = \hat{V}(x) - (z\delta x)\hat{I}(x + \delta x)$$

Rearranging and letting $\delta x \to 0$ yields:

$$\frac{d\hat{V}(x)}{dx} = -z\hat{I}(x) \tag{5}$$

Similarly, the current phasor at position $x + \delta x$:

$$\hat{I}(x + \delta x) = \hat{I}(x) - (y\delta x)\hat{V}(x)$$

Rearranging and letting $\delta x \to 0$ yields:

$$\frac{d\hat{I}(x)}{dx} = -y\hat{V}(x) \tag{6}$$

Eliminate $\hat{I}(x)$ in (5) by differentiating and substituting for $\frac{d\hat{I}(x)}{dx}$ from (6) to give:

$$\frac{d^2\hat{V}(x)}{dx^2} = -z\left(\frac{d\hat{I}(x)}{dx}\right) = (yz)\hat{V}(x) \tag{7}$$

The sending end voltage ($\hat{V}_S = \hat{V}(0)$) and current ($\hat{I}_S = \hat{I}(0)$) are given boundary conditions. From (5) and (6) it follows that the boundary values of the derivatives of these quantities are:

$$\frac{d\hat{V}_S}{dx} = -z\hat{I}_S \quad \text{and} \quad \frac{d\hat{I}_S}{dx} = -y\hat{V}_S \tag{8}$$

Equation (7) is now solved using the Laplace Transform (LT). Forming the LT of (7) gives:

$$s^2\hat{V}(s) - \left(\frac{d\hat{V}(x)}{dx}\right)_{x=0} - s\hat{V}(0) - (yz)\hat{V}(s) = 0$$

Substitute boundary conditions and rearrange:

$$\hat{V}(s) = \frac{s\hat{V}_S - z\hat{I}_S}{s^2 - yz} \tag{9}$$

Factorize the characteristic equation $s^2 - yz = 0$:

$$(s^2 - yz) = (s - \gamma)(s + \gamma)$$

in which $\gamma = \sqrt{yz}$ is the *propagation constant*. (10)

Derivation of distributed TL equations (Cont)

Express (9) as a sum of partial fractions:

$$\hat{V}(s) = \frac{s\hat{V}_S - z\hat{I}_S}{s^2 - yz} = \frac{A_1}{(s-\gamma)} + \frac{A_2}{(s+\gamma)} \tag{11}$$

A_1 and A_2 are obtained by multiplying out and equating coefficients as follows.

$$s\hat{V}_S - z\hat{I}_S = (A_1 + A_2)s + (A_1 - A_2)\gamma$$

$(A_1 + A_2) = \hat{V}_S$ and $(A_2 - A_1)\gamma = z\hat{I}_S$ yielding

$A_2 = \hat{V}_S - A_1 \Rightarrow (V_S - 2A_1)\gamma = z\hat{I}_S$ from which

$$A_1 = \frac{\hat{V}_S - Z_C\hat{I}_S}{2},\quad A_2 = \frac{\hat{V}_S + Z_C\hat{I}_S}{2} \text{ where} \tag{12}$$

$$Z_C = \sqrt{\frac{z}{y}} \text{ is the } \underline{\textit{characteristic impedance}}. \tag{13}$$

The voltage phasor, as a function of distance from the sending end, is obtained by forming the inverse LT of (11) and substituting the coefficients in (12) to give:

$$\begin{aligned}
\hat{V}(x) &= A_1 e^{\gamma x} + A_2 e^{-\gamma x} \\
&= \left(\frac{\hat{V}_S - Z_C\hat{I}_S}{2}\right)e^{\gamma x} + \left(\frac{\hat{V}_S + Z_C\hat{I}_S}{2}\right)e^{-\gamma x} \\
&= \hat{V}_S\left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) - Z_C\hat{I}_S\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) \\
&= \hat{V}_S\cosh(\gamma x) - Z_C\hat{I}_S\sinh(\gamma x)
\end{aligned} \tag{14}$$

Substitute $\hat{V}(x)$ from (14) into (5) for $\hat{I}(x)$:

$$\hat{I}(x) = -\frac{1}{z}\frac{d\hat{V}(x)}{dx} = -\frac{\gamma}{z}(A_1 e^{\gamma x} - A_2 e^{-\gamma x})$$

Recognize that $\frac{\gamma}{z} = \frac{\sqrt{yz}}{z} = \sqrt{\frac{y}{z}} = \frac{1}{Z_C}$ and substituting for A_1 and A_2 in the preceding equation yields:

$$\begin{aligned}
\hat{I}(x) &= \left(\frac{(\hat{I}_S - \hat{V}_S/Z_C)}{2}\right)e^{\gamma x} + \left(\frac{\hat{I}_S + \hat{V}_S/Z_C}{2}\right)e^{-\gamma x} \\
&= \hat{I}_S\cosh(\gamma x) - \left(\frac{\hat{V}_S}{Z_C}\right)\sinh(\gamma x)
\end{aligned} \tag{15}$$

Propagation constant and characteristic impedance

The propagation and characteristic impedances are expressed in terms of the TL distributed parameters and approximations of these parameters are made assuming (i) the shunt conductance, G, is negligible; and (ii) the series resistance (R) is much less than the series inductance (ωL).

<u>Propagation Constant</u>

$$\gamma = \sqrt{yz} = \sqrt{(G + j\omega C)(R + j\omega L)}$$
$$= \sqrt{(RG - \omega^2 LC) + j(\omega GL + \omega RC)}$$

The shunt conductance is neglected (i.e. $G = 0$) so,

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L}\right)} \qquad (16)$$

In the case of HV transmission lines usually $R \ll \omega L$ at power system frequency (i.e. 50 Hz) so the following simplification is appropriate for such lines. (It should not be used for distribution lines for which the X/R ratio is near unity or even less than one.)

$$\gamma = \alpha + j\beta \approx j\omega\sqrt{LC}\left(1 - j\frac{R}{2\omega L}\right)$$
$$\alpha \approx \frac{R}{2\sqrt{L/C}} \qquad \beta \approx \omega\sqrt{LC} \qquad \text{if } R \ll \omega L \qquad (17)$$

<u>Characteristic Impedance</u>

$$Z_C = \sqrt{\frac{z}{y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Again neglecting G we have

$$Z_C = \sqrt{\frac{L}{C}}\sqrt{1 - j\left(\frac{R}{\omega L}\right)} \qquad (18)$$

In the case of HV transmission lines in which $R \ll \omega L$ the following approximation is applicable:

$$Z_C = R_C + jX_C \approx \sqrt{\frac{L}{C}}\left(1 - j\frac{R}{2\omega L}\right)$$
$$R_C \approx \sqrt{\frac{L}{C}} \qquad X_C \approx -\left(\frac{R}{2\omega\sqrt{LC}}\right) \qquad \text{if } R \ll \omega L \qquad (19)$$

Surge Impedance Loading

For HV transmission lines G is negligible and $R \ll \omega L$. Consequently when analysing lightening and switching surges it is common to assume the TL is lossless (i.e. $R = G = 0$). The characteristic impedance is then referred to as the surge-impedance:

Surge impedance:

$$Z_0 = \sqrt{\frac{L}{C}} \text{ if } (R = G = 0) \tag{20}$$

The surge impedance has the dimensions of pure resistance.

The power delivered when the line is terminated by its surge impedance is referred to as the surge impedance load (SIL):

Surge Impedance Load:

$$\text{SIL} = \frac{V_0^2\ (\text{kV, l-l})}{Z_0\ (\Omega)} \text{ MW} \tag{21}$$

If a loss-less TL is terminated by its SIL the receiving end current is, from (14)

$$\hat{I}_R = \hat{V}_R / Z_C = \hat{I}_S \cosh(\gamma l) - \left(\frac{\hat{V}_S}{Z_C}\right) \sinh(\gamma l) \text{ and from (15)}$$

$$\hat{V}_R = \hat{V}_S \cosh(\gamma l) - Z_C \hat{I}_S \sinh(\gamma l)$$

From these two equations we have:

$$Z_C \hat{I}_S \cosh(\gamma l) - \hat{V}_S \sinh(\gamma l) = \hat{V}_S \cosh(\gamma l) - Z_C \hat{I}_S \sinh(\gamma l)$$

which, when rearranged yields:

$$(\hat{V}_S - Z_C \hat{I}_S)(\cosh(\gamma l) - \sinh(\gamma l)) = 0$$

$$(\hat{V}_S - Z_C \hat{I}_S) e^{-\gamma l} = 0$$

This equation can only be satisfied for arbitrary length l if

$$\hat{V}_S = Z_C \hat{I}_S, \quad \hat{I}_S = \hat{V}_S / Z_C$$

Substituting these values for $\hat{V}_S$ and $\hat{I}_S$ into the voltage (14) and current (15) equations yields:

$$\hat{V}(x) = \hat{V}_S(\cosh(\gamma x) - \sinh(\gamma x)) = \hat{V}_S e^{-\gamma x}$$

$$\hat{I}(x) = \hat{I}_S(\cosh(\gamma x) - \sinh(\gamma x)) = \hat{I}_S e^{-\gamma x}$$

Surge Impedance Loading (Cont)

For a lossless line the propagation constant is:

$$\gamma = j\beta = j\omega\sqrt{LC}, \qquad \beta = \omega\sqrt{LC} \tag{22}$$

When substituted in the preceding equations the voltage and current phasors are obtained as:

$$\hat{V}(x) = \hat{V}_S e^{-j\beta x},$$
$$\hat{I}(x) = \hat{I}_S e^{-j\beta x} = (\hat{V}_S / Z_C) e^{-j\beta x} \tag{23}$$

It is concluded from the preceding equations that in a loss-less TL terminated by its SIL:

- Voltage and current phasors have **constant amplitude** along the line;
- Voltage and current phasors are **in phase** throughout the length of the line
- Phase difference between sending and receiving end voltage is βl

At natural load (i.e. SIL) the reactive power produced by the line capacitance is equal to that consumed by the line inductance. Voltage and current profiles are flat. The SIL is the optimum loading of a TL for voltage and reactive power control. When evaluating TL performance the SIL is a convenient base quantity.

Transmission line wave length

For a loss-less TL the voltage and current wave lengths at fundamental-frequency (i.e. 50 Hz) are:

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{1}{f_0\sqrt{LC}} \tag{24}$$

For HV transmission lines,

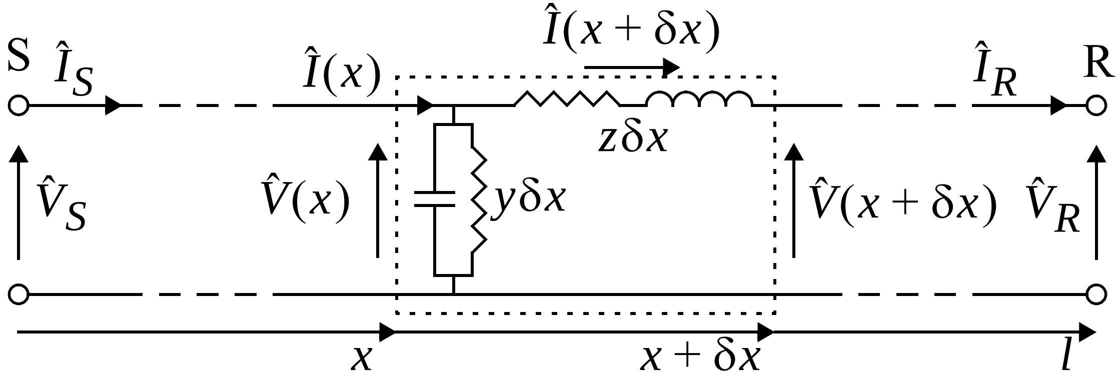
$$\sqrt{LC} \sim \sqrt{(1 \times 10^{-3}) \times (11 \times 10^{-9})} = 3.32 \times 10^{-6}\mathrm{km}^{-1}$$

Thus the TL wave length for a 50 Hz system is of the order of:

$$\lambda = \frac{1}{50 \times 3.32 \times 10^{-6}} = 6025\ \mathrm{km}$$

A quarter wave length is about 1,500 km.

Summary of the distributed TL equations



$$\hat{V}(x) = \left(\frac{\hat{V}_S - Z_C\hat{I}_S}{2}\right)e^{\gamma x} + \left(\frac{\hat{V}_S + Z_C\hat{I}_S}{2}\right)e^{-\gamma x}$$
$$= \hat{V}_S\cosh(\gamma x) - Z_C\hat{I}_S\sinh(\gamma x) \qquad (25)$$

$$\hat{I}(x) = \left(\frac{(\hat{I}_S - \hat{V}_S/Z_C)}{2}\right)e^{\gamma x} + \left(\frac{\hat{I}_S + \hat{V}_S/Z_C}{2}\right)e^{-\gamma x}$$
$$= \hat{I}_S\cosh(\gamma x) - \left(\frac{\hat{V}_S}{Z_C}\right)\sinh(\gamma x) \qquad (26)$$

If G is neglected then the propagation constant γ and characteristic impedance Z_C are respectively:

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L}\right)} \qquad Z_C = \sqrt{\frac{L}{C}}\sqrt{1 - j\left(\frac{R}{\omega L}\right)} \qquad (27)$$

Table 1: Example overhead transmission line parameters (rated frequency 50 Hz)

Parameter		Nominal voltage (kV, rms, l-l)			
		132	275	330	500
R	Ω / ph / km	0.112	0.040	0.037	0.028
L	x 10^{-3} H / ph / km	1.241	1.015	0.973	0.862
C	x 10^{-9} F / ph / km	8.16	11.62	11.98	13.79
α	x 10^{-5} Np / km	14.21	6.754	6.480	5.592
β	x 10^{-3} rad / km	1.010	1.081	1.075	1.085
$X_L = \omega L$	Ω / ph / km	0.390	0.319	0.306	0.271
$B_C = \omega C$	x 10^{-6} S / ph / km	2.564	3.651	3.764	4.332
Z_C	Ω	390	296	285	250
SIL	MW (3 ph)	45	255	382	1000
Q_C	MVAr (3 ph) / km	0.045	0.276	0.410	1.083

Transmission Cables

From the point of view of analysis, 3-phase underground transmission lines can be treated in the same way as overhead lines. They differ in two major ways:

1. The capacitance is much higher than overhead lines, e.g. the line-charging currents for 275 kV overhead line and underground cable are of the order of 0.55 A/km and 16 A/km, respectively. The high value of the latter may require special equipment to absorb the line-charging reactive power (MVAr) at light load.

2. The resistance per kilometre for underground cables must be much lower than for overhead lines having the same current rating. This is due to problems of dissipating the I^2R losses underground.

3. The costs of high voltage cable and its installation are high. Lengths of underground high voltage cable are thus short, and confined mainly to inner city areas, river crossings, etc.

Travelling Wave Interpretation of the Transmission Line Voltage

From (25) the TL voltage phasor has the form:

$$\hat{V}(x) = \hat{V}_B e^{\gamma x} + \hat{V}_F e^{-\gamma x} \tag{28}$$

where $\hat{V}_B = V_B e^{j\theta_B}$, $\hat{V}_F = V_F e^{j\theta_F}$ and $\gamma = \alpha + j\beta$.

Thus,

$$\hat{V}(x) = (V_B e^{\alpha x}) e^{j(\beta x + \theta_B)} + (V_F e^{-\alpha x}) e^{-j(\beta x - \theta_F)}$$

The voltage at location x and time t is thus:

$$v(x, t) = \sqrt{2}(V_B e^{\alpha x})\cos(\omega t + \beta x + \theta_B) + \ldots \quad \sqrt{2}(V_F e^{-\alpha x})\cos(\omega t - \beta x + \theta_F) \tag{29}$$

Let us consider the first term in(29):

$$v_B(x, t) = \sqrt{2}(V_B e^{\alpha x})\cos(\omega t + \beta x + \theta_B)$$

Neglecting amplification ($e^{\alpha x}$), this component of the solution has its peak value at x_{pB} when:

$$\omega t + \beta x_{pB} + \theta_B = 0\text{, or}$$

$$x_{pB} = \frac{-(\omega t + \theta_B)}{\beta} \tag{30}$$

This means that the peak of $v_B(x, t)$ moves in the direction of the source (i.e. decreasing x) as time advances. Thus, $v_B(x, t)$ is a 'backward' travelling wave.

Let us similarly consider the second term in (29):

$$v_F(x, t) = \sqrt{2}(V_F e^{-\alpha x})\cos(\omega t - \beta x + \theta_F).$$

Neglecting the attenuation, $e^{-\alpha x}$, the peak value of this component, x_{pF}, occurs at:

$$x_{pF} = \frac{\omega t - \theta_F}{\beta} \tag{31}$$

which means that the peak of $v_F(x, t)$ moves away from the source (i.e. increasing x) as time advances. Thus, $v_F(x, t)$ is a 'forward' travelling wave.

In (28):

$\hat{V}_B e^{\gamma x}$ represents a <u>backward</u> travelling wave; &

$\hat{V}_F e^{-\gamma x}$ represents a <u>forward</u> travelling wave

Travelling Wave Interpretation of the Transmission Line Voltage (Cont)

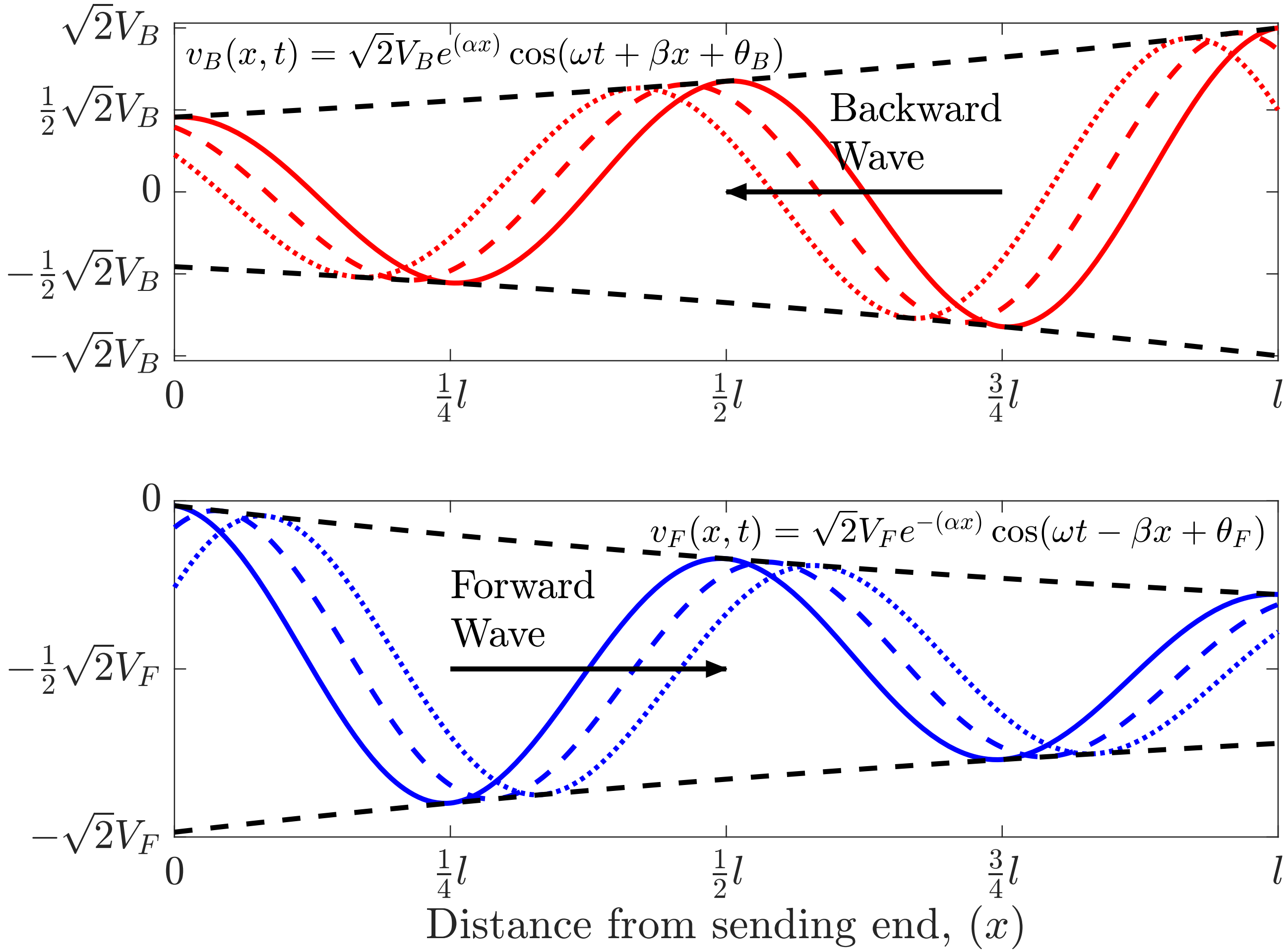
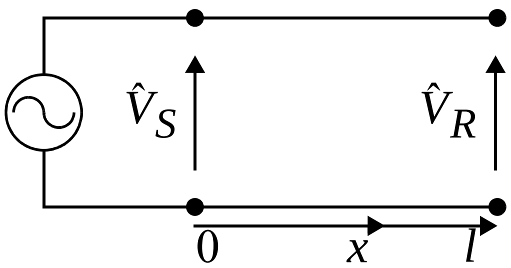


Figure 6: Backward (top) and Forward (bottom) travelling wave components of the transmission line voltage; (solid $\omega t = \varphi_0$, dashed $\omega t = \varphi_0 + 30°$, dotted $\omega t = \varphi_0 + 60°$, unrealistic length for illustrative purposes)

Example: Voltage-Phasor Wave on open-circuit 275 kV TL

Consider a 275 kV (rms, ph-ph) overhead transmission line with the parameters listed in Table 1. The 500 km line is open-circuit at the receiving end so $\hat{I}_R = 0$. The sending end voltage is set to its rated value of 275 kV (rms, ph-ph).



The current phasor at the receiving end of the line is, from (26) with $x = l$:

$$\hat{I}_R = \hat{I}(l) = \hat{I}_S \cosh(\gamma l) - \left(\frac{\hat{V}_S}{Z_C}\right)\sinh(\gamma l) = 0$$

from which the sending end current is derived as:

$$\hat{I}_S = \left(\frac{\hat{V}_S}{Z_C}\right)\left(\frac{\sinh(\gamma l)}{\cosh(\gamma l)}\right) = \left(\frac{\hat{V}_S}{Z_C}\right)\tanh(\gamma l)$$

The receiving end voltage is derived from (25) with $x = l$:

$$\begin{aligned}
\hat{V}_R &= \hat{V}(l) = \hat{V}_S \cosh(\gamma l) - Z_C \hat{I}_S \sinh(\gamma l) \\
&= \hat{V}_S \cosh(\gamma l) - Z_C\left(\frac{\hat{V}_S}{Z_C}\right)\tanh(\gamma l)\sinh(\gamma l) \\
&= \frac{\hat{V}_S}{\cosh(\gamma l)}(\cosh^2(\gamma l) - \sinh^2(\gamma l)) \\
&= \frac{\hat{V}_S}{\cosh(\gamma l)}
\end{aligned}$$

The above expression for the sending-end current is substituted in (25) and (26) to obtain the voltage and current phasors at any point, x, along the line, as follows:

Example: Voltage-Phasor Wave on open-circuit 275 kV TL (Cont)

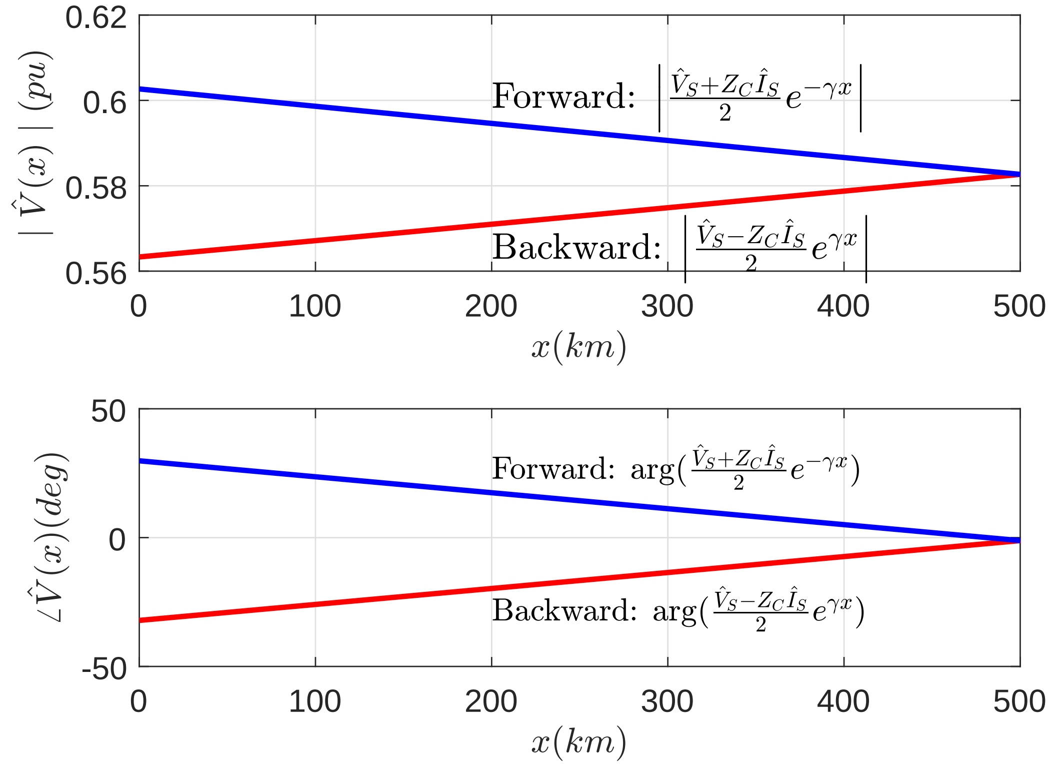
The voltage and current phasors at any point, x, along the line, are as follows:

$$\hat{V}(x) = \hat{V}_S\left(\frac{1-\tanh(\gamma l)}{2}\right)e^{\gamma x} + \hat{V}_S\left(\frac{1+\tanh(\gamma l)}{2}\right)e^{-\gamma x}$$

$$\hat{I}(x) = -\frac{\hat{V}_S}{Z_C}\left(\frac{1-\tanh(\gamma l)}{2}\right)e^{\gamma x} + \frac{\hat{V}_S}{Z_C}\left(\frac{1+\tanh(\gamma l)}{2}\right)e^{-\gamma x}$$

The figure shows the magnitude and phase of the voltage-phasors representing the backward (red) and forward (blue) components of the voltage. Note that:

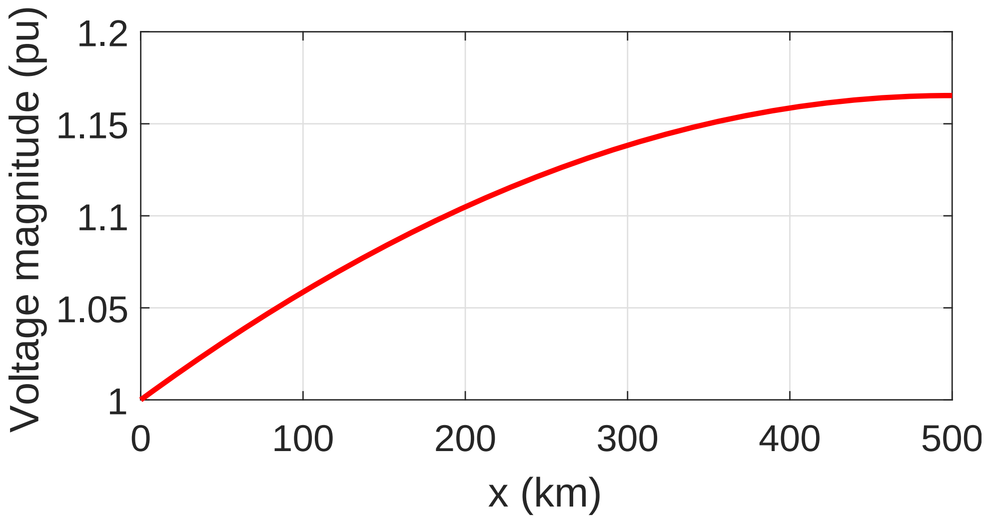
- the backward phasor increases in magnitude and advances in phase as the distance, x, from the sending end increases.
- the forward phasor decreases in magnitude and is retarded in phase as the distance, x, from the sending end increases.

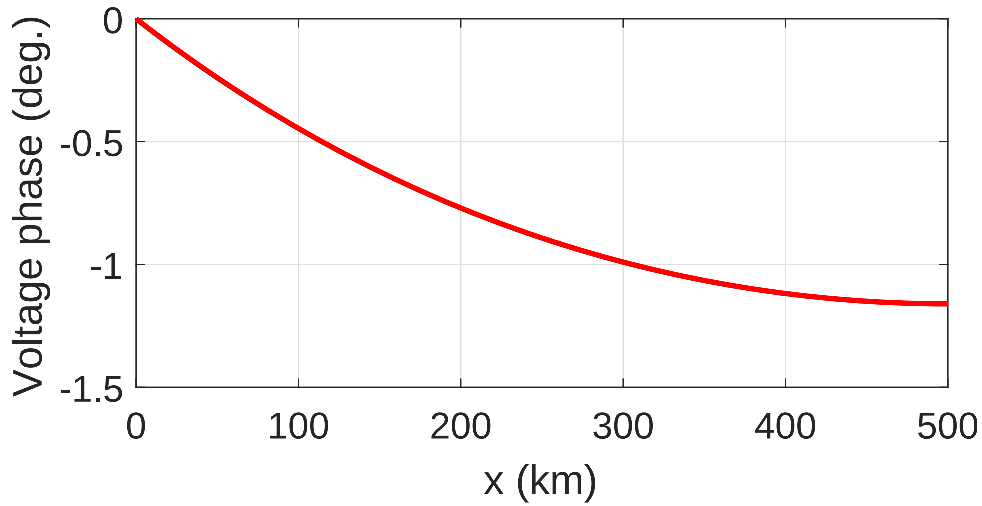


Example: Voltage-Phasor Wave on open-circuit 275 kV TL (Cont)

Voltage Phasor Distribution

- The voltage magnitude increases substantially by 16% to 1.16 pu over the 500 km line length. The phase change, in contrast, is negligible over the line length.
- Voltage rise at the receiving end is due to the flow of line charging (capacitive) current through the line inductance.
- This phenomenon is referred to as the *Ferranti Effect*
- Long HV lines often equipped with reactors (inductance) to limit voltage rise.
- Deviation of phase from zero is due to line resistance.

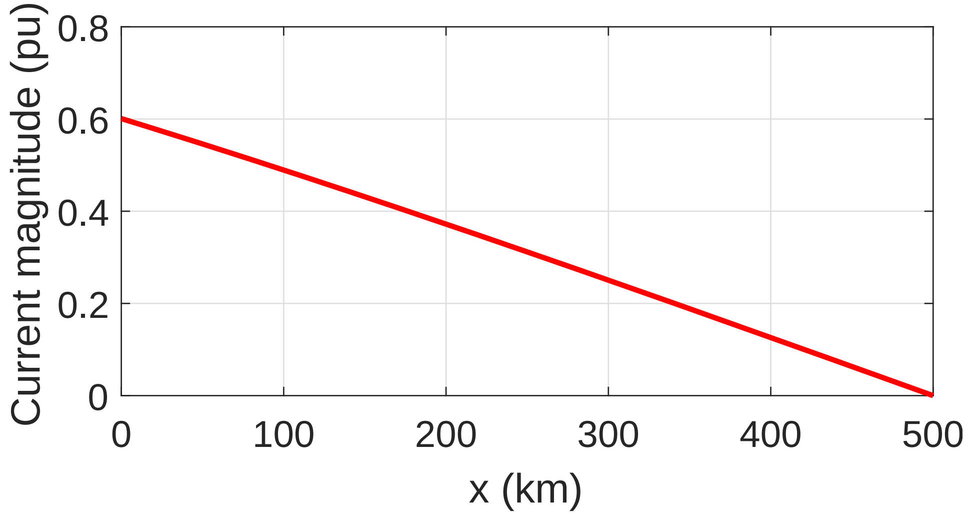


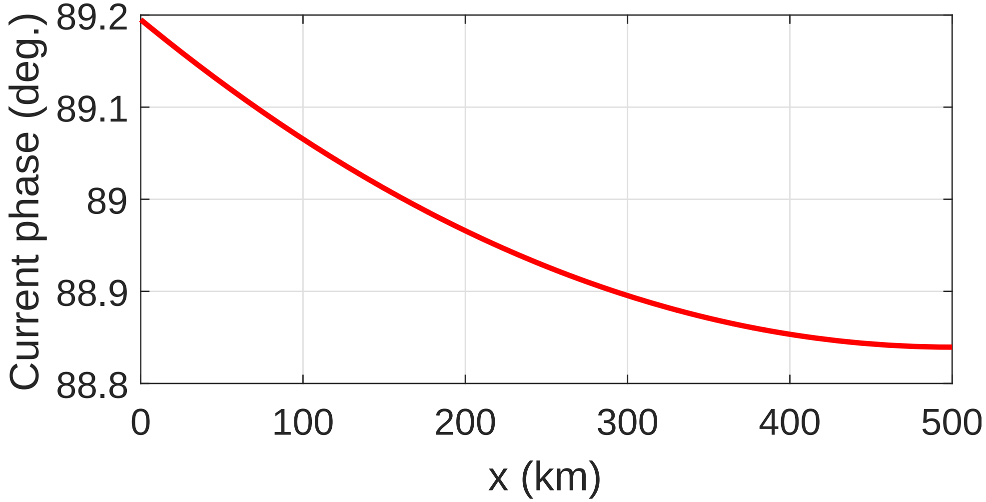


Example: Voltage-Phasor Wave on open-circuit 275 kV TL (Cont)

Current Phasor Distribution

- As the distance, I, from the receiving end of the line increases the magnitude of the current-phasor increases approximately linearly due to the charging of the line capacitance.
- The phase of the current-phasor wave is +90 deg. is consistent with the capacitive nature of the open-circuit TL. (i.e. current leads voltage by 90 deg).
- The slight deviation of the phase from 90 deg is due to the line resistance.





Equivalent pi-circuit model of a TL

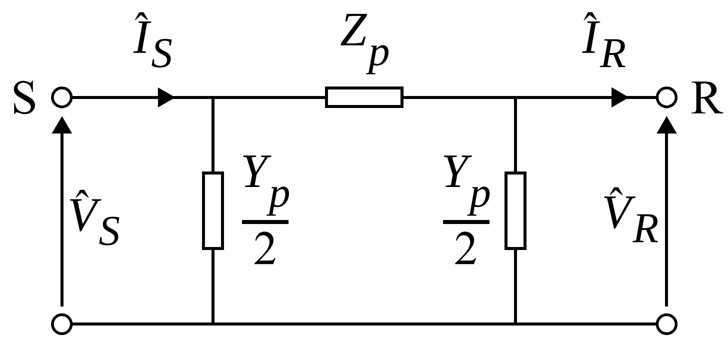
The complete distributed parameter representation of a transmission line connected to a.c. sources was provided earlier.

However, when analysing the performance of an interconnected power system we are concerned with the behaviour of a transmission line at its terminals rather then at intermediate points along its length. Thus, it is convenient to represent the behaviour of a transmission line as seen at its terminals only.

The voltage- and current-phasors at the receiving terminal (R) are expressed in terms of the corresponding quantities at the sending end by setting $x = l$ in equations (25) & (26).

$$\begin{aligned} \hat{V}_R &= \hat{V}_S \cosh(\gamma l) - Z_C \hat{I}_S \sinh(\gamma l) \\ \hat{I}_R &= \hat{I}_S \cosh(\gamma l) - \left(\frac{\hat{V}_S}{Z_C}\right) \sinh(\gamma l) \end{aligned} \qquad (32)$$

The above terminal relationships can be represented by a lumped parameter pi-circuit as shown below.



Equivalent pi-circuit model of a TL (Cont)

The objective now is to determine the relationship between the lumped parameters:

- Z_p, the *series impedance*; and
- Y_p, the *shunt admittance*

and the transmission line characteristic parameters Z_C, γ and l.

Applying KCL to the sending end node of the pi-circuit we obtain:

$$\hat{V}_R = \left(1 + \frac{Y_p Z_p}{2}\right)\hat{V}_S - Z_p \hat{I}_S \tag{33}$$

Similarly apply KCL to the receiving end node:

$$\hat{I}_R = \frac{\hat{V}_S}{Z_p} - \left(1 + \frac{Y_p Z_p}{2}\right)\frac{\hat{V}_R}{Z_p}$$

$\hat{V}_R$ is eliminated from the preceding equation by substitution from equation (33) to give:

$$\hat{I}_R = \left(1 + \frac{Y_p Z_p}{2}\right)\hat{I}_S - \left(\frac{Y_p}{2}\right)\hat{V}_S \tag{34}$$

Comparing the receiving end voltage equation of the transmission line in (32) with corresponding equation for the pi-circuit (33) we can equate coefficients of $\hat{I}_R$ to obtain:

$$Z_p = Z_C \sinh(\gamma l) \tag{35}$$

Comparing the coefficients of $\hat{V}_R$ in the respective equations gives:

$$1 + \frac{Y_p Z_p}{2} = \cosh(\gamma l)$$

from which Z_p can be eliminated using (35) to yield:

$$\left(\frac{Y_p}{2}\right) = \frac{1}{Z_C}\left(\frac{\cosh(\gamma l) - 1}{\sinh(\gamma l)}\right)$$

Exploit the relationship between hyperbolic functions, $\tanh\left(\frac{x}{2}\right) = \frac{\cosh(x) - 1}{\sinh(x)}$, to give:

$$\left(\frac{Y_p}{2}\right) = \frac{1}{Z_C}\tanh\left(\frac{\gamma l}{2}\right) \tag{36}$$

Nominal pi-Equivalent Circuit

The lumped parameter pi-equivalent circuit with Z_p and $(Y_p/2)$ in equations (35) and (36) provides an exact representation of the transmission line at its terminals. The conditions along the length of the line can not be obtained from the equivalent circuit.

Seek simplifications of the equivalent circuit parameters in situations where the line is relatively short.

Below (17) & (18) are employed, namely, $\gamma = \sqrt{yz}$ and $Z_C = \sqrt{z/y}$, together with the definitions of $z = R + j\omega L$ and $y = G + j\omega C$ with the assumption that G is negligible and is therefore set to zero.

If the line length is such that $|\gamma l| \ll 1$ then the hyperbolic functions in the expressions of the equivalent pi-circuit parameters in equations (35) and (36) can be simplified by approximating their values by the first (i.e. linear) term in their Taylor Series expansions, namely,

$$\begin{aligned} \sinh(\gamma l) &= \gamma l + \frac{(\gamma l)^3}{6} + \text{h.o.t.} \\ \tanh\left(\frac{\gamma l}{2}\right) &= \left(\frac{\gamma l}{2}\right) - \frac{(\gamma l)^3}{24} + \text{h.o.t.} \end{aligned} \tag{37}$$

If the first order approximations are employed the pi-circuit parameters become:

$$\begin{aligned} Z_p &= Z_C \sinh(\gamma l) \approx (Z_C \gamma) l \\ &= zl = (R + j\omega L) l \\ \left(\frac{Y_p}{2}\right) &= \frac{1}{Z_C} \tanh\left(\frac{\gamma l}{2}\right) \approx \frac{1}{2}\left(\frac{\gamma}{Z_C}\right) l \\ \frac{yl}{2} &= j\left(\frac{\omega C}{2}\right) l = j\left(\frac{B_C}{2}\right) l \end{aligned}$$

Thus, if $|\gamma l| \ll 1$, the following pi-equivalent circuit parameters apply:

$$Z_p = (R + j\omega L) l, \quad \left(\frac{Y_p}{2}\right) = j\left(\frac{B_C}{2}\right) l \tag{38}$$

Nominal pi-Equivalent Circuit (Cont)

Now determine an upper-limit on the line-length for which the approximate pi-circuit parameters in (38) yield accurate results.

- For this purpose we assume the line is lossless so that

$$\gamma l = j\beta l = j2\pi\left(\frac{l}{\lambda}\right).$$

- Figure shows the relative error in the linear approximations of $\sinh(\gamma l)$ and $\tanh(\gamma l/2)$ as a function of l/λ.
- To limit the approximation error to less than one percent the line length should be no more than 4% the wavelength.
- For an overhead transmission line this corresponds to a line length of $l < 12000/f$ km (i.e. $l < 240$ km for a 50 Hz system).
- For an underground cable the wavelength is dependent on the permittivity of the dielectric material. Typically the wavelength in a cable will range from about 15 to 30% of that of an overhead line. Thus, for cables nominal pi-equivalent parameter approx. valid for length from about $1800/f$ to $3600/f$ km (i.e. 36 to 72 km for a 50 Hz system).

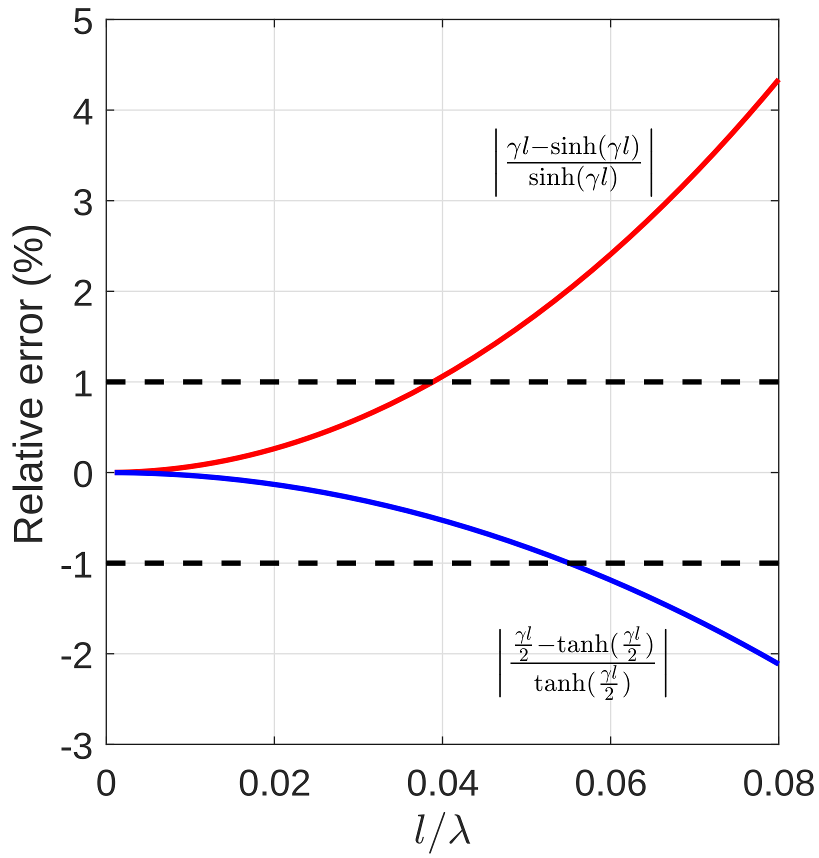


Figure 7: Relative errors in the linear approximations of the hyperbolic functions.

Short lines

For overhead lines shorter than about 25 km (on 50 & 60 Hz systems) the shunt capacitance is very small and can be neglected without significant loss of accuracy. Such short lines are represented by their total series impedance:

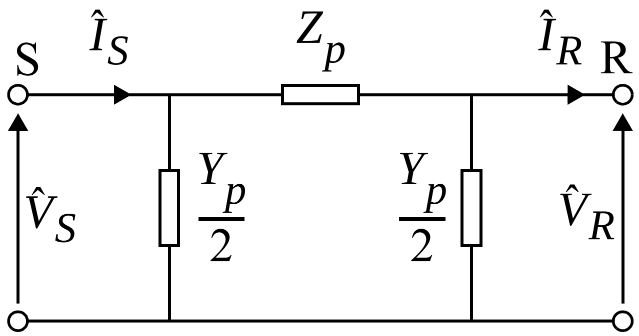
$$Z_p = zl = (R + j\omega L)l$$

Summary of lumped parameter transmission line equivalent circuits

Transmission line distributed parameters: R (ohm / km), L (H / km) , C (F/km), (assume G = 0)

Frequency: $\omega = 2\pi f$ (rad/s) $z = R + j\omega L = R + jX_L$ (Ω/km), $y = j\omega C = jB_C$ (mho/km)

Length: l km, Propagation constant: $\gamma = \sqrt{yz}$, Characteristic Impedance: $Z_C = \sqrt{z/y}$ (ohm)



Model	Applicable Length (km)	Z_p	$Y_p/2$
pi-equivalent (Long line)	All	$Z_C \sinh(\gamma l)$	$\frac{1}{Z_C}\tanh\left(\frac{\gamma l}{2}\right)$
nominal pi-equivalent (Medium line)	< 240 (line) < 36 - 72 (cable)	$(R + j\omega L)l$	$j\left(\frac{B_C}{2}\right)l$
series equivalent (Short line)	< 25 (line) cable N/A.	$(R + j\omega L)l$	0