Quiz 1 - 2021 - Solutions

(1 a)

(i) Period $T_p = 22.5 - 2.5 = 20\,ms = 0.02\,s$

$f = 1/T_p = 1/0.02 = 50\,Hz$

$\omega = 2\pi f = 100\pi$ rad/s. $\boxed{1}$ 1/0

(ii) $V_p = 113.2$ kV - $\boxed{1}$

$\cos(\omega t_p + \alpha) = 1^{(1)} \Rightarrow \omega t_p + \alpha = 0$ (0.5)

$t_p = 2.5/1000$ s $\Rightarrow \alpha = -\omega t_p = -100\pi \times 2.5/1000 = -2.5\pi/10 = -\pi/4$ (0.5)

$\alpha = -\pi/4$ rad, $-45°$ $\boxed{2}$

(iii) $I_p = 1190.1$ A $\boxed{0.5}$

Current <u>lags</u> voltage by $\Delta t = 1.5$ ms (1) for lag

1.5 ms corresponds to $1.5/20 = 0.075$ of a cycle

$\Rightarrow \theta = -360 \times 0.075 = -27° = -0.15\pi$ rad $\boxed{1.5}$

0.5 for angle magnitude, 1 for correct sign.

1b)

(i) $\hat{V} = \frac{V_p}{\sqrt{2}} \angle -30^\circ = \frac{235.8}{\sqrt{2}} \angle -30 = 166.7 \angle -30^\circ$ kV. $\boxed{1}$

(ii) $S = P + jQ = (200 - j65)/3$ (1) MVA/ph

$\hat{I} = (S/\hat{V})^* = \frac{(200 + j65)/3 \times 10^6}{166.7 \angle 30^\circ \times 10^3}$ [VA/ph] / [V (ph-n)] (1)

$= 411.33 - j87.399$ A

$= 420.5 \angle -12^\circ$ A. (1) $\boxed{3}$

(iii) $I_p = \sqrt{2}\,|\hat{I}| = \sqrt{2} \times 420.5 = 594.7$ A (0.5)

$\alpha + \theta = -12^\circ$ (1) $\Rightarrow \theta = -12^\circ - \alpha$

$\theta = -12^\circ - (-30^\circ)$

$= +18^\circ$ (1) $\boxed{2}$

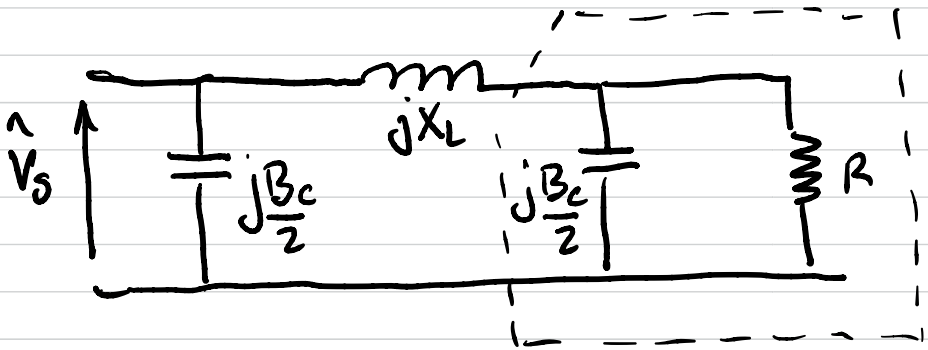
2) $V_p = 408\ kV$

(i) $\hat{V}_S = \frac{V_p}{\sqrt{2}} \angle 0^\circ = \frac{408}{\sqrt{2}} \angle 0^\circ$ (1)

$\hat{V}_S = 288.5 \angle 0^\circ\ kV$ $\boxed{1}$

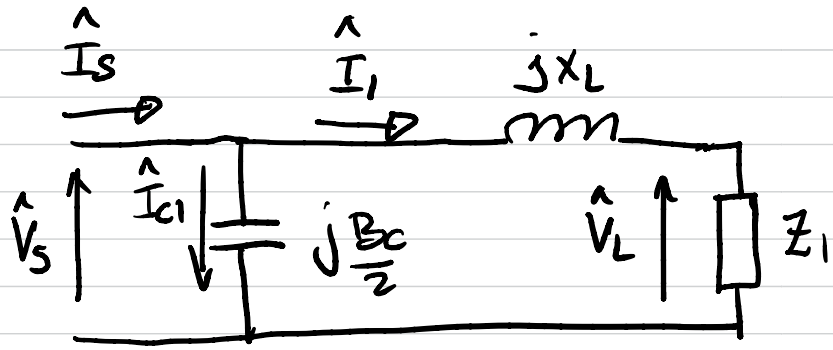
(ii) $B_c = \omega C = 2\pi \times 50 \times 2.76 \times 10^{-6} = 8.6708 \times 10^{-4}\ S$ (1)

$X_L = \omega L = 2\pi \times 50 \times 0.172 = 54.0354\ \Omega$ (1)



$Y_1 = \frac{1}{R} + j\frac{B_c}{2} = \frac{1}{250} + j\frac{B_c}{2} = (4.00 + j0.43354) \times 10^{-3}\ S$ (1)

$Z_1 = 1/Y_1 = 247.10 - j26.782\ \Omega$ (1)



$$\hat{V}_L = \hat{V}_S - jX_L\hat{I}_1 \quad (0.5)$$

$$\hat{I}_1 = \frac{\hat{V}_S}{Z_1 + jX_L} \quad (0.5) \quad \Rightarrow \quad \hat{V}_L = \hat{V}_S\left(1 - \frac{jX_L}{Z_1 + jX_L}\right)$$

$$= \frac{Z_1}{Z_1 + jX_L}\hat{V}_S \quad (1)$$

$$\hat{V}_L = \frac{247.1 - j26.782}{247.1 - j26.782 + j54.0354} \cdot 288.5\angle 0^\circ \text{ kV}$$

$$= (9.7617 - j2.1605) \times 10^{-1} \times 288.5\angle 0^\circ$$

$$= 281.62 - j62.331 \text{ kV}$$

$$\hat{V}_L = 288.44\angle -12.48^\circ \text{ kV} \quad (1)$$

$\boxed{7}$

(iii) $$\hat{I}_S = \hat{I}_1 + \hat{I}_{C1} = \frac{\hat{V}_S}{Z_1 + jX_L} + j\frac{B_C}{2}\hat{V}_S = \left[\frac{1}{Z_1 + jX_L} + j\frac{B_C}{2}\right]\hat{V}_S$$

(1) (1)

$$S_S = P_S + jQ_S = \hat{V}_S \, \hat{I}_S^{*} \qquad (1)$$

$$= V_S \left[\frac{1}{Z_1 + jX_L} + j\frac{B_C}{2} \right]^{*} \hat{V}_S^{*}$$

$$= \left[\frac{1}{Z_1^{*} - jX_L} - j\frac{B_C}{2} \right] V_S^2 \qquad (1)$$

$$= \left[\frac{1}{247.1 + j26.782 - j54.0354} - j\frac{8.6708 \times 10^{-4}}{2} \right] \times \left(\frac{408}{\sqrt{2}} \right)^2 \quad \text{MVA}$$

$$= \left[3.9983 \times 10^{-3} + j7.4609 \times 10^{-6} \right] \times \frac{408^2}{2} \quad \text{MVA}$$

$$= 332.8 + j0.621 \quad \text{MVA}$$

$$\left.\begin{aligned} P_S &= 332.8 \text{ MW} \\ Q_S &= 0.621 \text{ MVAr} \quad (\sim 0) \end{aligned}\right\} (1)$$

$\boxed{5}$

3a)

(i) The T/L current is $\hat{I} = (S_R / \hat{V}_R)^* = \dfrac{(P_R - jQ_R)}{\hat{V}_R^*}$ (1)

The source voltage is

$$\hat{V}_S = \hat{V}_R + jX\hat{I}$$

$$= \hat{V}_R + jX\left[\frac{P_R - jQ_R}{\hat{V}_R^*}\right] \quad (1)$$

$\boxed{2}$

(ii) $\hat{V}_R = 1.05\angle 0$ pu.

$P_R - jQ_R = 1.2 - j0.2$

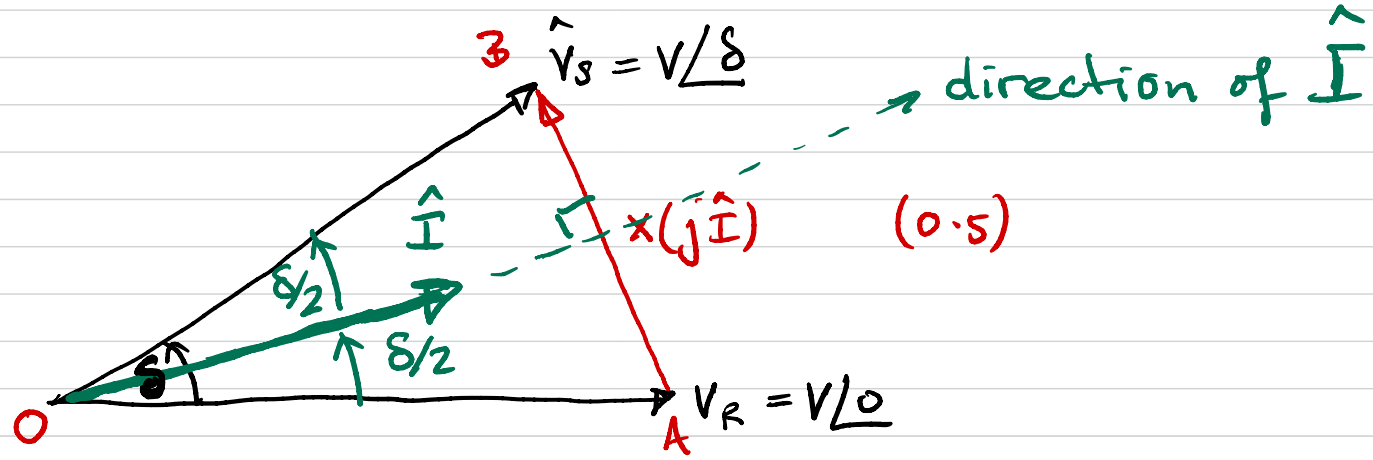
$$\hat{V}_S = 1.05 + j0.3\left[\frac{1.2 - j0.2}{1.05}\right]$$

$$\hat{V}_S = 1.159\angle 17.21^\circ \text{ pu} \quad (1)$$

The source voltage is greater than 1.1 pu and is thus outside of prescribed limits. (1)

$\boxed{2}$

3b)



$$\hat{V}_S = \hat{V}_R + j x \hat{I}$$

$$\Rightarrow \quad = \hat{V}_R + x(j\hat{I}) \qquad (0.5)$$

Since $|\hat{V}_S| = |\hat{V}_R|$ the points A & B lie on a circle. (1)
Thus AB is a chord of the circle.

The current phasor $\hat{I}$ is perpendicular to the chord AB

$$\hat{I} = \underbrace{(-j)}_{\text{clockwise rotation of } 90^\circ} \times (j\hat{I}) \qquad (1)$$

$$|\hat{I}| = \frac{|AB|}{x} \qquad (1)$$

4 a) $$\text{RoCoF} = \frac{d\Delta f}{dt} = \frac{25}{H_s}(P_S - P_D)$$ (1)

$$\therefore H_s = \frac{25}{\left(\frac{d\Delta f}{dt}\right)}(P_S - P_D)$$

[Immediately before the fault $P_S - P_D = 0$. After the fault 2,500MW (1) of generation is lost so immediately after the fault $P_S - P_D = -2500\text{MW}$]
The RoCoF immediately after the fault is -0.5 Hz/s

$$\therefore H_s = \frac{25}{-0.5} \times -2500 = 125{,}000 \text{ MWs.}$$ (1)

$\boxed{3}$

4b) (i) Order generation in price order, omit those generators that can not be dispatched due to security constraints

Generator Owner	Bid Price ($/MWh)	Capacity Offered (MW)	Cumulative Capacity (MW)	
B	-100	600	600	< 1500
D	100	600	1200	< 1500
E	120	400	1600	>1500 stop

(1)

Dispatched generators – B, D and E. (1)

2

(ii) All generators will receive $120 / MWh during the next dispatch cycle.

1

(iii) 1500 MW is to be dispatched. This equates to delivered

energy of $E = \underbrace{1500}_{MW} \times \underbrace{\frac{5}{60}}_{h} = 125$ MWh.

(0.5)

The dispatch price is \$120/MWh so the total cost of the energy supplied in this dispatch cycle is

$$\text{Cost} = \underset{MWh}{125} * \underset{\$/MWh}{120} = \$15{,}000$$

(0.5)

$\boxed{1}$