THE UNIVERSITY *of* ADELAIDE

SUB CRUCE LUMEN

Course:

ELEC ENG 3110 Electric Power Systems

ELEC ENG 7074 Power Systems PG

(Semester 2, 2021)

Review of Some Aspects of Single-Phase Systems.
A First Look at Power Transmission.

Co-ordinator & Lecturer: David Vowles
david.vowles@adelaide.edu.au

Key Concepts from Complex Algebra

By definition: $j^2 = -1$ (1)

Complex number:

$$z = x + jy = r\angle\theta = re^{j\theta} \tag{2}$$

rect polar exp

in rectangular, polar and exponential formats

Real and Imaginary parts:

$$x = \Re(z), \quad y = \Im(z) \tag{3}$$

Euler Identity

$$r\left\{e^{j\theta}\right\} = r\{\cos(\theta) + j\sin(\theta)\} \tag{4}$$

Polar to rectangular:

$$x = r\cos(\theta), \quad y = r\sin(\theta) \tag{5}$$

Rectangular to polar:

$$r = |z| = \sqrt{x^2 + y^2}, \quad \theta = \text{atan2}(y, x) \tag{6}$$

Conjugation: $z^* = x - jy = re^{-j\theta}$ (7)

Let $z_1 = x_1 + jy_1 = r_1 e^{j\theta_1}$ and $z_2 = x_2 + jy_2 = r_2 e^{j\theta_2}$. Then the following complex arithmetic operations are defined.

Addition: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$ (8)

Subtraction: $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$ (9)

Multiplication:

$$\begin{aligned} z_1 z_2 &= (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1) \\ &= r_1 r_2 e^{j(\theta_1 + \theta_2)} \end{aligned} \tag{10}$$

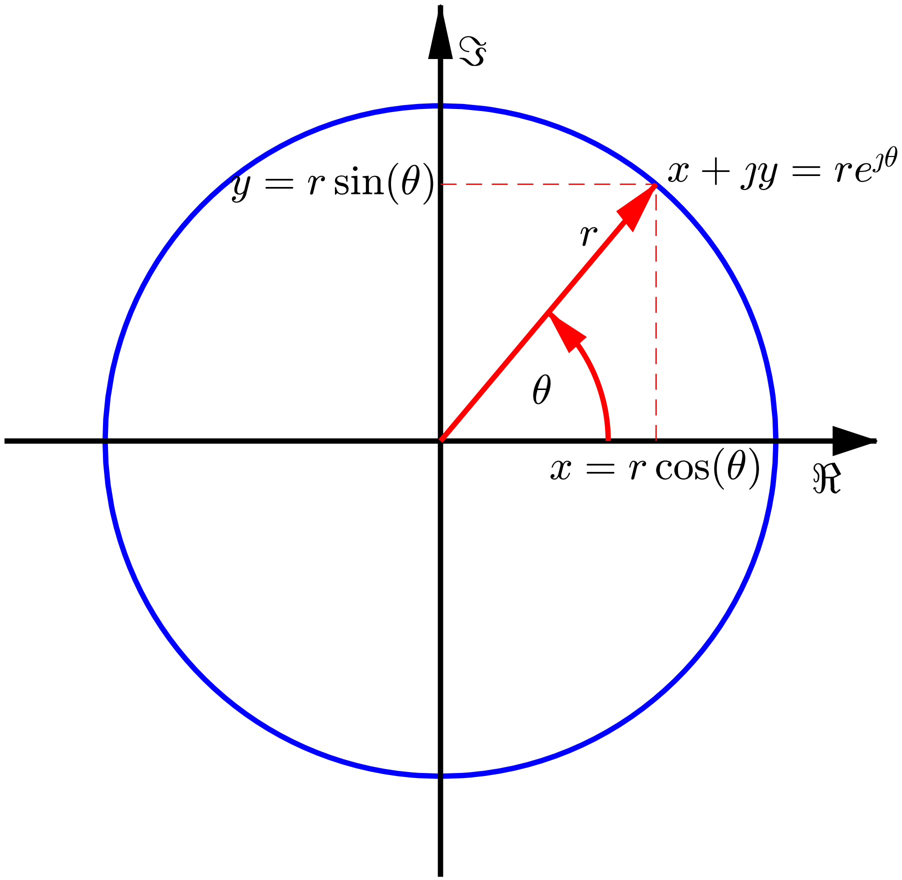
$$z \cdot z^* = x^2 + y^2 = r^2 \tag{11}$$

Division:

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{z_1 \cdot z_2^*}{z_2 \cdot z_2^*} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)} \\ &= \frac{(x_1 x_2 + y_1 y_2)}{x_2^2 + y_2^2} + j\frac{(x_2 y_1 - x_1 y_2)}{x_2^2 + y_2^2} \end{aligned} \tag{12}$$

Euler's Identity

Euler Identity
$r\{e^{\jmath\theta}\} = r\{\cos(\theta) + \jmath\sin(\theta)\}$



Cosine and Sine expressed in terms of complex-exponentials:

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \tag{13}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{j2} \tag{14}$$

Anti-clockwise (ACW) rotation by α (rad): $ze^{j\alpha} = re^{j(\theta+\alpha)}$ (15)

$$\{e^{j(0)} = 1\} \qquad \left\{e^{j\left(\frac{\pi}{2}\right)} = j\right\}$$
$$\{e^{j(\pi)} = -1\} \qquad \left\{e^{-j\left(\frac{\pi}{2}\right)} = -j\right\} \tag{16}$$

The 120 deg. rotation operator

In analysing three-phase networks the 120 deg. rotation operator, a, is very useful. Following is the definition of a and some of its useful properties.

$$a = e^{j\left(\frac{2\pi}{3}\right)} = 1\angle 120° \tag{17}$$

$$a^2 = e^{j\left(\frac{4\pi}{3}\right)} = e^{-j\left(\frac{2\pi}{3}\right)} = 1\angle -120° \tag{18}$$

$$a^3 = 1 \tag{19}$$

$$1 + a + a^2 = 0 \tag{20}$$

$$1 - a = \sqrt{3}e^{\left(-j\frac{\pi}{6}\right)} = \sqrt{3}\angle -30° \tag{21}$$

$$1 - a^2 = \sqrt{3}e^{\left(j\frac{\pi}{6}\right)} = \sqrt{3}\angle 30° \tag{22}$$

$$1 + a = \sqrt{3}e^{\left(j\frac{\pi}{3}\right)} = \sqrt{3}\angle 30° \tag{23}$$

$$1 + a^2 = \sqrt{3}e^{\left(-j\frac{\pi}{3}\right)} = \sqrt{3}\angle -30° \tag{24}$$

$$a - a^2 = \sqrt{3}e^{\left(j\frac{\pi}{2}\right)} = \sqrt{3}\angle 90° = j\sqrt{3} \tag{25}$$

Sinusoidal Signal in the Time Domain

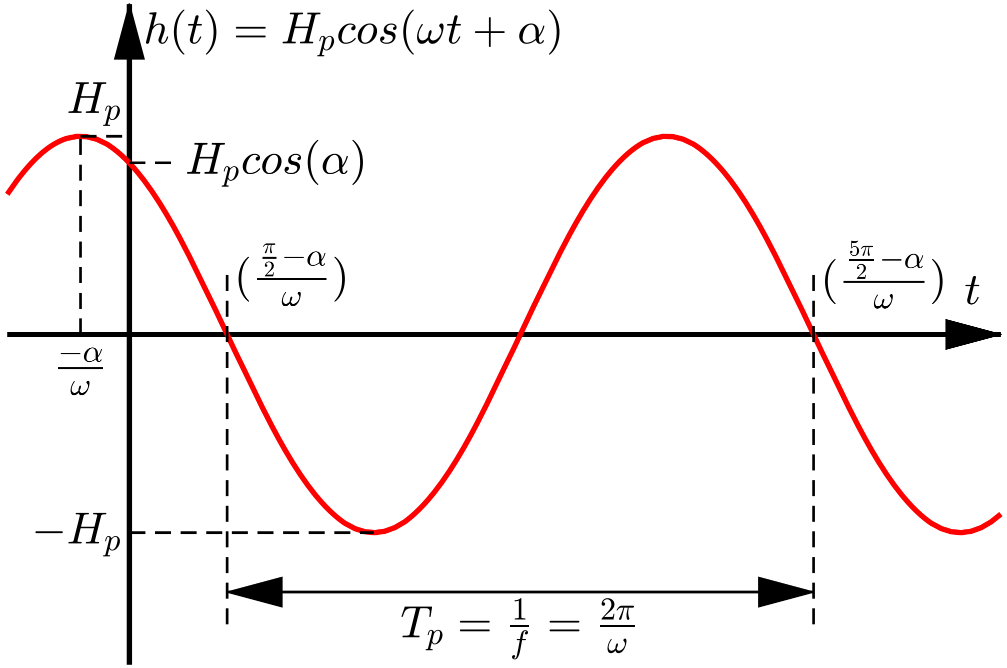


Figure 1: Sinusoidal signal

Power system signals (i.e. voltage and current) are sinusoidal with ***fundamental frequency*** $f = f_0$ Hz (i.e. cycles / sec).

$f_0 = 50$ Hz (Australia, Europe, Africa, most Asia, some South America); $f_0 = 60$ Hz (North America, some South America).

Sinusoidal signal:

$$h(t) = H_p \cos(\omega t + \alpha) \text{ where} \tag{26}$$

t is time (s);

H_p is the maximum value of the signal;

ω is the *angular-frequency* (rad./s); and

α is the phase-shift (rad.) at $t = 0$ w.r.t. a pure cosine wave.

$$\text{Frequency: } f = \omega/(2\pi) \text{ (Hz).} \tag{27}$$

$$\text{Period: } T_p = (2\pi)/\omega = 1/f \text{ (s).} \tag{28}$$

Root Mean Square (RMS) Value of a Signal.

In power system analysis usually refer to the RMS amplitude of a signal rather than its peak or maximum value.

For a *general periodic signal* $h(t)$ with period T_p the RMS value, H_{rms}, is defined as:

RMS definition:

$$H_{rms} = \sqrt{\frac{1}{T_p}\int_{t_0}^{t_0+T_p}\{h(t)\}^2\,dt} \qquad (29)$$

For the sinusoidal signal:

$$h(t) = H_p\cos(\omega t+\alpha)$$

the RMS value is calculated as follows.

Firstly calculate $\{h(t)\}^2$ & use $(\cos\theta)^2 = \dfrac{\cos(2\theta)+1}{2}$

$$\{h(t)\}^2 = H_p^2\cos^2(\omega t+\alpha) = \left(\frac{H_p^2}{2}\right)(1+\cos(2(\omega t+\alpha)))$$

Noting that the integral of a sinusoid over an integral number of cycles is zero it follows that:

$$\int_{t_0}^{t_0+T_p}\{h(t)\}^2\,dt = (H_p^2/2)\int_{t_0}^{t_0+T_p}1+\cos(2(\omega t+\alpha))\,dt$$
$$= H_p^2T_p/2$$

Substitute the previous expression in (29) to give H_{rms} as:

RMS value of sinusoid: $H_{rms} = H_p/\sqrt{2}$ (30)

Phasor Representation of Sinusoidal Signal

By application of Euler's Identity a sinusoidal signal can be represented by the sum of two complex exponentials. Specifically, from (13):

$$
\begin{aligned}
h(t) &= H_p \cos(\omega t + \alpha) \\
&= \frac{1}{2}\left\{H_p e^{j(\omega t + \alpha)} + H_p e^{-j(\omega t + \alpha)}\right\} \\
&= \frac{1}{2}\left\{\left(H_p e^{j\alpha}\right) e^{j\omega t} + \left(H_p e^{-j\alpha}\right) e^{-j\omega t}\right\} \qquad (31) \\
&= \frac{1}{2}\left\{\left(H_p e^{j\alpha}\right) e^{j\omega t} + \left(H_p e^{j\alpha}\right)^* e^{-j\omega t}\right\} \\
&= \frac{1}{2}\left\{\hat{\mathbf{H}}_p e^{j\omega t} + \hat{\mathbf{H}}_p^* e^{-j\omega t}\right\}
\end{aligned}
$$

$$\text{where } \hat{\mathbf{H}}_p = H_p e^{j\alpha}. \qquad (32)$$

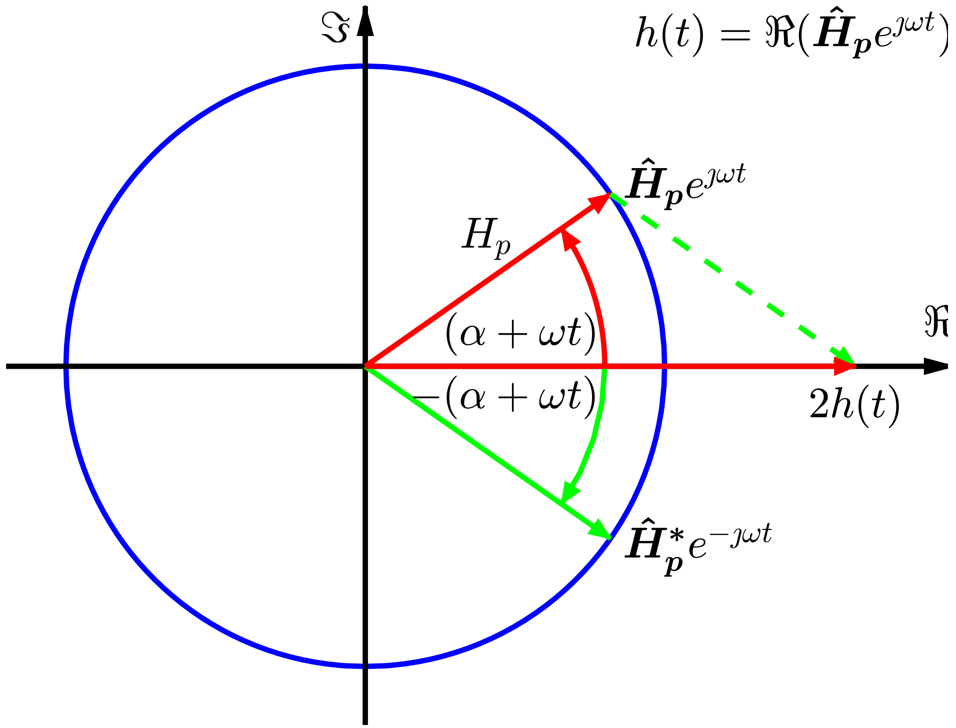
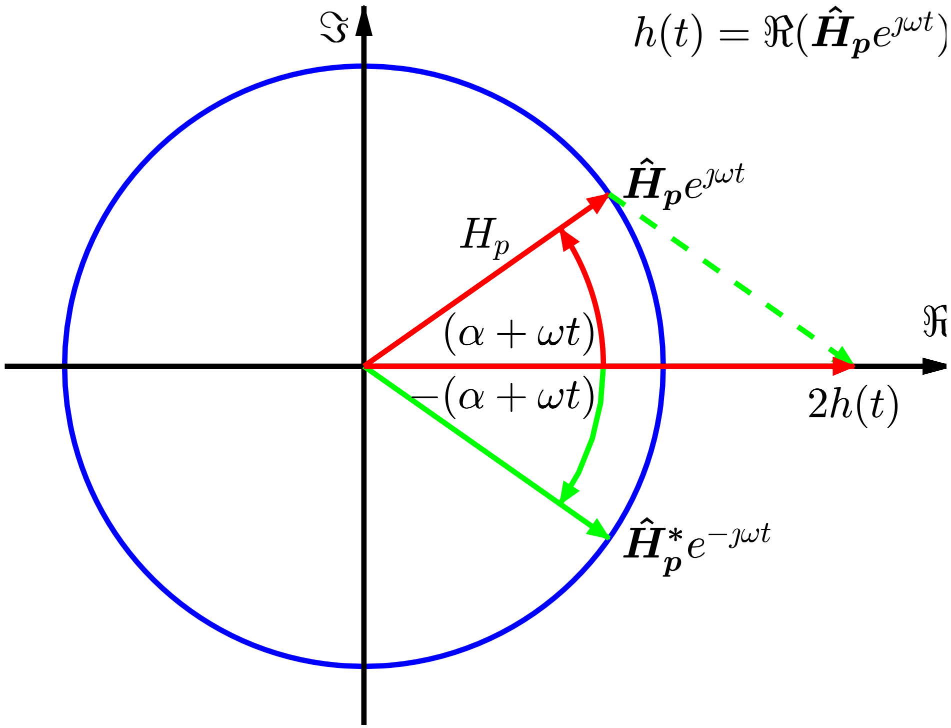


Figure 2: Phasor representation of a sinusoidal signal.

Phasor representation of Sinusoidal Signal (cont)



The complex number $\hat{\mathbf{H}}_p = H_p e^{j\alpha}$ is the value of the CCW phasor at time $t = 0$. It is referred to as the *peak-valued phasor* of the sinusoidal signal $h(t) = H_p \cos(\omega t + \alpha)$ and represents the peak-value of the signal and its **phase shift w.r.t. a pure cosine wave at time $t = 0$.**

Given the peak-valued phasor and the angular-frequency ($\omega = 2\pi f$ rad./s) we can reconstruct the original sinusoidal signal in the time-domain using (31).

Sinusoidal signal is represented by the *(half) sum of two counter-rotating phasors*: $\hat{\mathbf{H}}_p e^{j\omega t}$ rotates anti-clockwise (ACW) and its complex conjugate $\hat{\mathbf{H}}^*_p e^{-j\omega t}$ rotates clockwise (CW).

Relationship between Time and Phasor Domains

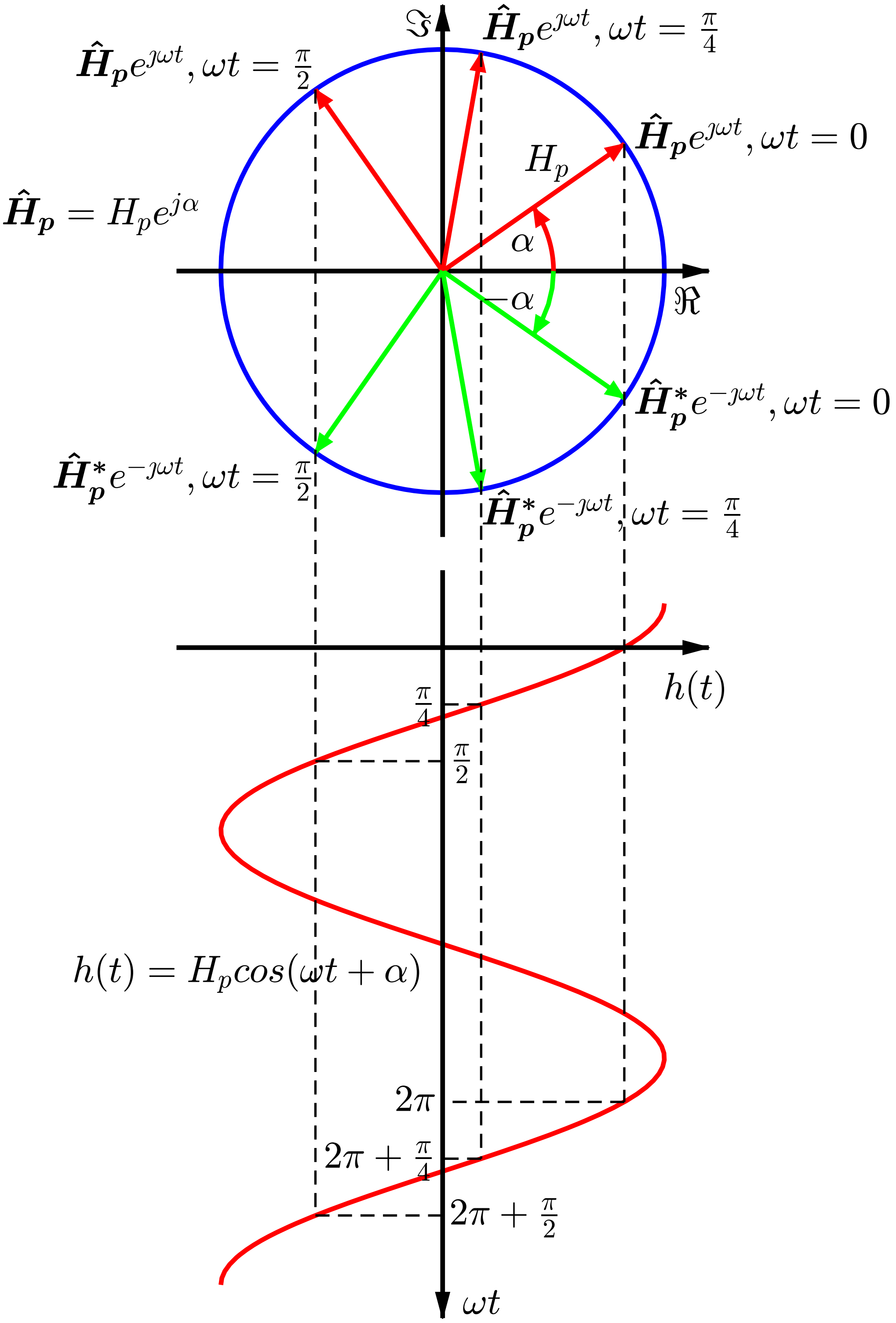


Figure 3: Visualization of the relationship between a sinusoidal signal and its phasor representation.

Phase Reference for Phasors

Recall that in our definition of the peak-valued phasor $\hat{\mathbf{H}}_p = H_p e^{j\alpha}$ the phase shift α is measured *with respect to that of a pure cosine wave at time t = 0.*

For clarity we say the peak-valued phasor is "cosine-referenced". Power engineers conventionally use cosine referenced phasors. We will do the same unless stated otherwise.

In some fields sine-referenced phasors are employed.

In phasor based calculations phase difference between signals is relevant and not the absolute phase of the individual signals. Thus, in most situations if all phasors have the same it is not important to know the phase reference.

RMS Valued Phasor

Natural to develop phasor concept in terms of the peak value of the sinusoid. However, power engineers quantify voltages and currents by their RMS magnitudes, not peak values. Hence, we define the RMS phasor or simply *phasor* of the sinusoidal signal $h(t) = H_p \cos(\omega t + \alpha)$ as follows:

Phasor definition:

$$\hat{\mathbf{H}} = \hat{\mathbf{H}}_{rms} = H_{rms} e^{j\alpha} = \left(\frac{H_p}{\sqrt{2}}\right) e^{j\alpha} \tag{33}$$

Note:

- It is usual to drop the 'rms' subscript in which case it is 'understood' that the phasor is RMS valued.
- When reconstructing the sinusoidal signal $h(t)$ from its RMS valued phasor remember to multiply the RMS amplitude by $\sqrt{2}$ to give the peak value of the signal.

Example

The RMS phasor of a 50 Hz a.c. voltage is $\hat{V} = 240\angle 30°$ V. What is the equation of the voltage waveform?

$$V_p = \sqrt{2} \times 240 = 339.4 \text{ V}; \quad \alpha = 30° \times \frac{\pi}{180} = \frac{\pi}{6} \text{ rad}; \quad \omega = 2\pi \times 50 = 100\pi \text{ rad/s.}$$

$$v(t) = 339.4 \cos\left(100\pi t + \frac{\pi}{6}\right) \text{ V}$$

Instantaneous Power **Supplied** by a Sinusoidal Source

Consider a sinusoidal source in steady-state whose voltage and current are:

$$v(t) = V_p \cos(\omega t + \alpha) = V_p \cos(\varphi) \qquad (34)$$
$$i(t) = I_p \cos(\omega t + \alpha + \theta) = I_p \cos(\varphi + \theta)$$

where $\varphi = \omega t + \alpha$, $V_p = \sqrt{2}V$, $I_p = \sqrt{2}I$ and (35)

θ is the phase angle by which the *current **leads** the voltage*. *(θ < 0 when the current **lags** voltage)*

The ***instantaneous power*** supplied by the source is:

$$\begin{aligned} p(t) &= v(t) \cdot i(t) = V_p I_p \cos(\varphi)\cos(\varphi + \theta) \\ &= (V_p I_p / 2)(\cos(2\varphi + \theta) + \cos(\theta)) \\ &= \left(\frac{(\sqrt{2}V)(\sqrt{2}I)}{2}\right)(\cos(2\varphi)\cos(\theta) - \sin(2\varphi)\sin(\theta) + \cos(\theta)) \qquad (36) \\ &= \{(VI)\cos(\theta)\}[1 + \cos(2(\omega t + \alpha))] + \{-(VI)\sin(\theta)\}\sin(2(\omega t + \alpha)) \\ &= P[1 + \cos(2(\omega t + \alpha))] + Q\sin(2(\omega t + \alpha)) \end{aligned}$$

Instantaneous Power (cont)

$$p(t) = P\left[1+\cos\left(2\left(\omega t+\alpha\right)\right)\right] + Q\sin\left(2\left(\omega t+\alpha\right)\right)$$ where

$P = (VI)\cos(\theta)$ is the ***average power*** supplied by the source and (37)

$Q = -(VI)\sin(\theta)$ is *by definition* the ***reactive power*** supplied by the source (38)

We will return to the meaning of P & Q shortly. Now we will explore $p(t)$.

1. *p(t)* oscillates at ***twice the frequency*** of *v(t)* and *i(t).*
2. If the *v(t)* & *i(t)* are **in phase** (i.e. $\theta = 0$) $Q = 0$ and the maximum / minimum values of *p(t)* are *2P* = *2(VI)* and 0 respectively.
   - The average power *in this case* is *P* = *VI*
3. If *i(t)* **leads** *v(t)* by **90 deg.** (i.e. $\theta = 90°$) then *P* = *0*, *Q* = *–VI* and the maximum / minimum values of *p(t)* are +*VI* and –*VI* respectively.
   - The source exchanges power with the network at frequency 2ω such that the average power is zero.
4. If *i(t)* **lags** *v(t)* by **90 deg.** (i.e. $\theta = -90°$) then *P* = *0*, *Q* = *VI* and the maximum / minimum values of *p(t)* are (as above) +*VI* and –*VI* respectively.
   - As for the leading case, except that the order of the power peaks (+*VI*) and troughs (–*VI*) is reversed.

Average Power and Reactive Power

From (37) $P = (VI)\cos(\theta)$ is the ***average power*** and

from (38) $Q = -(VI)\sin(\theta)$ is *by definition* the ***reactive power***

where V and I are the RMS values of the voltage and current and θ is the phase angle by which the current ***leads*** the voltage.

Note that P and Q are in quadrature. Thus the "complex power", $\boldsymbol{S}$ is:

$$\underline{\textbf{Complex Power}}\ \mathbf{S} = P + jQ = VI(\cos(\theta) - j\sin(\theta)) = VIe^{-j\theta} \tag{39}$$

$$\underline{\textbf{Apparent Power:}}\ S = |\mathbf{S}| = VI \tag{40}$$

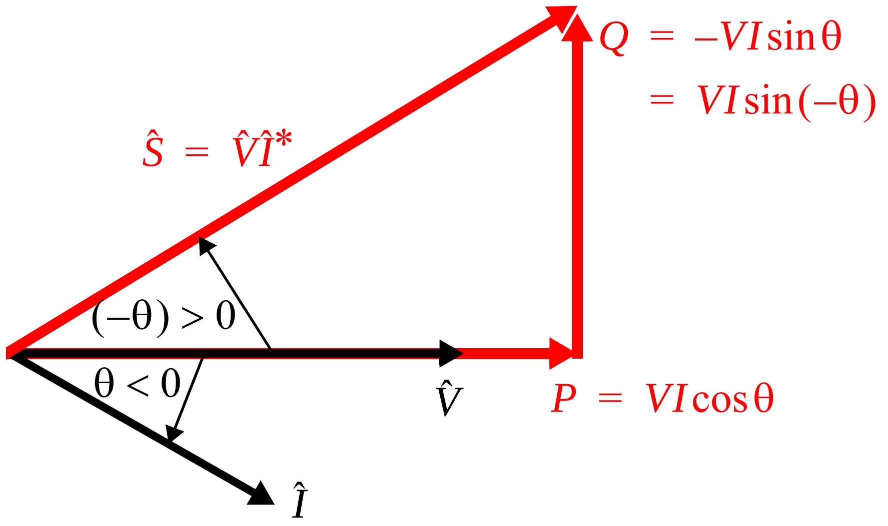
Apply forward and backward rotation by the initial voltage phase shift α, i.e. by $e^{j\alpha}e^{-j\alpha} = e^{j0} = 1$ to obtain:

$$P + jQ = (Ve^{j\alpha})(Ie^{-j(\alpha+\theta)}) = (Ve^{j\alpha})(Ie^{j(\alpha+\theta)})^* = \hat{\mathbf{V}}(\hat{\mathbf{I}})^*$$

This is the familiar equation for average power and reactive power in terms of the voltage and current phasors with which you are familiar:

$$\underline{\textit{\textbf{Complex Power:}}}\ \mathbf{S} = P + jQ = \hat{\mathbf{V}}\hat{\mathbf{I}}^* = VIe^{-j\theta} \tag{41}$$

The Power Triangle



In this example current lags voltage so $\theta < 0$ and consequently $Q = VI\sin(-\theta)$ is positive.

The power factor angle $\alpha = -\theta$ is the angle by which the current **lags** the voltage.

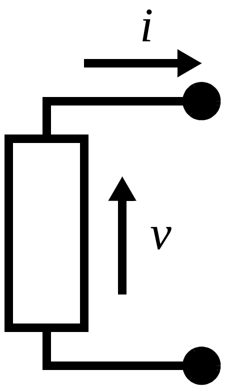
The power factor $PF = \cos(\alpha) = \cos(\theta)$ is lagging if $\theta < 0$ and leading if $\theta > 0$

- If a load is 100 MW with a **lagging** PF of 0.8 => $\theta = -\mathrm{acos}(0.8) = -36.87°$ =>
  $Q = P\tan(\boxed{-\theta}) = 75\ \mathrm{MVAr}$

Sign Conventions

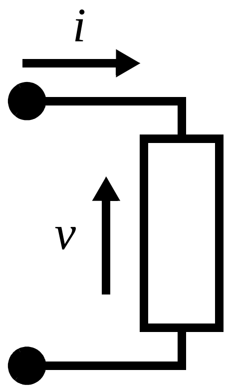
Generator convention normally used for power-sources or for devices that can either supply or consume power (e.g. HVDC links, storage devices)
Load convention normally used for loads.

Generator Convention



- Positive current is in the same direction as positive voltage.
- $P > 0$ means power is supplied to the network.
- $P < 0$ means power is consumed from the network
- $Q > 0$ means *reactive power is supplied* and is *capacitive in nature*. The current lags the voltage so we say the generator is *supplying* “lagging” reactive power.
- $Q < 0$ means *reactive power is consumed* and is *inductive in nature*. The current leads the voltage so we say the generator is *supplying* “leading” reactive power.

Load Convention

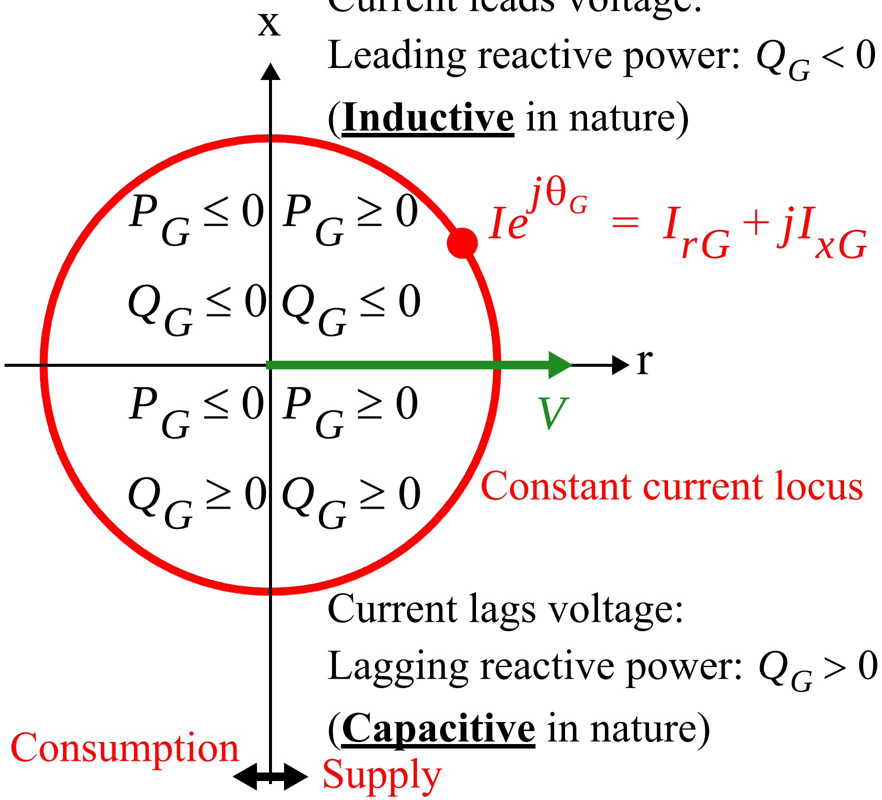


- Positive current is in the opposite direction to positive voltage.
- $P > 0$ means power is consumed from the network.
- $P < 0$ means power is supplied to the network
- $Q > 0$ means *reactive power is consumed* and is *inductive in nature*. The current lags the voltage so we say the load is *consuming* “lagging” reactive power.
- $Q < 0$ means *reactive power is supplied* and is *capacitive in nature*. The current leads the voltage so we say the load is *consuming* “leading” reactive power.

P and Q over the Constant Current Locus

Generator Convention

Current leads voltage:
Leading reactive power: $Q_G < 0$
(**Inductive** in nature)

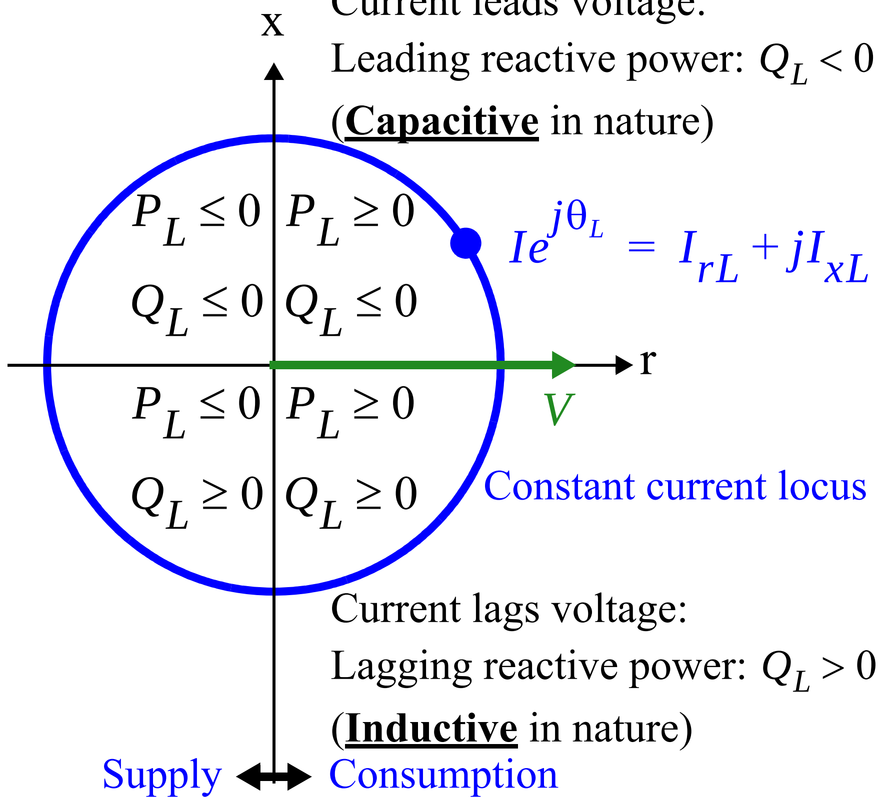


Current lags voltage:
Lagging reactive power: $Q_G > 0$
(**Capacitive** in nature)

$$P_G + jQ_G = \hat{V}\hat{I}_G^{\,*} = V(I_{rG} + jI_{xG})^*$$
$$= V(I_{rG} - jI_{xG})$$
$$= VI\cos\theta_G - jVI\sin\theta_G$$

Load Convention

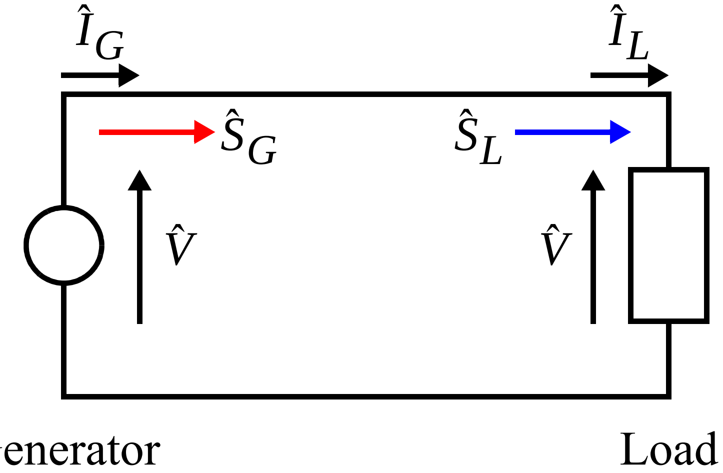
Current leads voltage:
Leading reactive power: $Q_L < 0$
(**Capacitive** in nature)



Current lags voltage:
Lagging reactive power: $Q_L > 0$
(**Inductive** in nature)

$$P_L + jQ_L = \hat{V}\hat{I}_L^{\,*} = V(I_{rL} + jI_{xL})^*$$
$$= V(I_{rL} - jI_{xL})$$
$$= VI\cos\theta_L - jVI\sin\theta_L$$

Power Calculation – Numerical Example



$$\hat{V} = 1.0\angle 0°$$

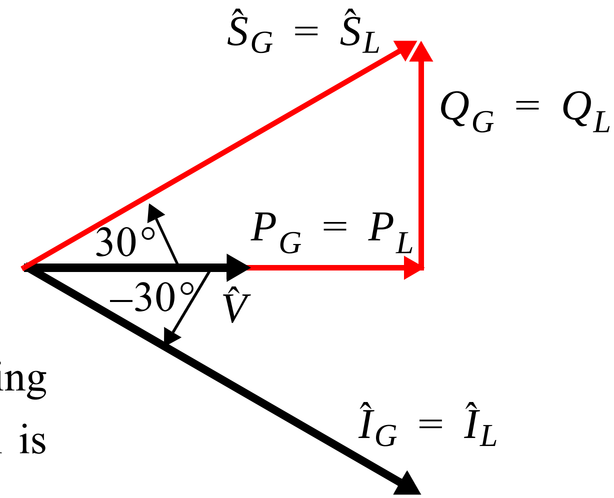
$$\hat{I}_G = 2.0\angle{-30°}$$

$$\begin{aligned}\hat{S}_G &= P_G + jQ_G \\ &= \hat{V}\hat{I}_G{}^* \\ &= (1\times 2)\angle 30° \\ &= 2(\cos(30°) + j\sin(30°)) \\ &= 2\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = \sqrt{3} + j1\end{aligned}$$

$$P_G = \sqrt{3}$$

$$Q_G = 1$$

$Q_G > 0$ => Generator is supplying "lagging reactive power" which is **capacitive** in nature.



$$\hat{I}_L = \hat{I}_G = 2.0\angle{-30°}$$

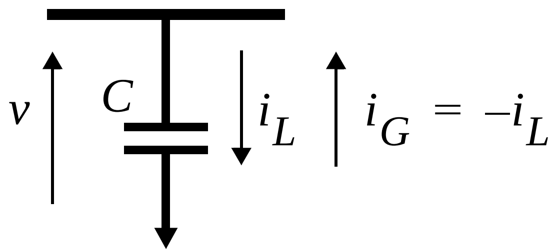
$$\begin{aligned}\hat{S}_L &= P_L + jQ_L \\ &= \hat{V}\hat{I}_L{}^* \\ &= \hat{V}\hat{I}_G{}^* \\ &= \hat{S}_G \\ &= \sqrt{3} + j1\end{aligned}$$

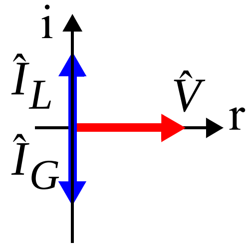
$$P_L = \sqrt{3}$$

$$Q_L = 1$$

$Q_L > 0$ => Load is consuming "lagging reactive power" which is **inductive** in nature.

Capacitor in the generator and load conventions





$$v(t) = \hat{V}e^{j\omega t} \quad i_L(t) = \hat{I}_L e^{j\omega t} = C\frac{dv}{dt} = j(\omega C)\hat{V}e^{j\omega t}$$

$\hat{I}_L = jB\hat{V}$, Susceptance, $B = \omega C$

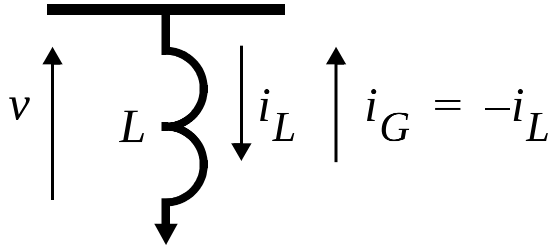
$$Q_L = Im(\hat{V}\hat{I}_L^*) = -BV^2 < 0$$

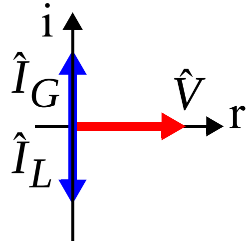
(Cap. leading Q in load convention)

$$Q_G = Im(\hat{V}\hat{I}_G^*) = Im(-\hat{V}\hat{I}_L^*) = -Q_L = BV^2 > 0$$

(Cap. lagging Q in generator convention)

Reactor (inductance) in the generator and load conventions





$$i_L(t) = \hat{I}_L e^{j\omega t} \quad v(t) = \hat{V}e^{j\omega t} = L\frac{di_L}{dt} = j(\omega L)\hat{I}_L e^{j\omega t}$$

$$\hat{I}_L = -j\frac{\hat{V}}{X},\ \hat{V} = jX\hat{I}_L \ \text{ Reactance, } X = \omega L$$

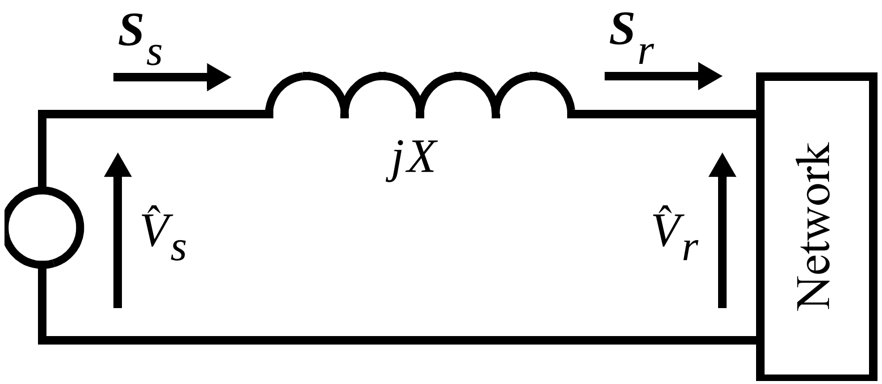
$$Q_L = Im(\hat{V}\hat{I}_L^*) = \frac{V^2}{X} = XI_L^2 > 0$$

(Ind. lagging Q in load convention)

$$Q_G = Im(\hat{V}\hat{I}_G^*) = Im(-\hat{V}\hat{I}_L^*) = -Q_L = -XI_G^2 < 0$$

(Ind. leading Q in generator convention)

Conceptual Overview of Power Transmission (Lossless, R = 0)



Derive sending end power and reactive power equations.

$$\hat{V}_s = V_s e^{j\theta_s},\ \hat{V}_r = V_r e^{j\theta_r},\ \delta = \theta_s - \theta_r$$

$$\hat{I}_{sr} = -j(\hat{V}_s - \hat{V}_r)/X$$

(Note: $\boldsymbol{S}_r = P_r + jQ_r$ derived similarly)

$$\boldsymbol{S}_s = P_s + jQ_s = \hat{V}_s \hat{I}_{sr}^*$$

$$= \frac{j\hat{V}_s(\hat{V}_s^* - \hat{V}_r^*)}{X} = j\left(\frac{V_s^2 - V_s e^{j\theta_s}(V_r e^{j\theta_r})^*}{X}\right) = j\left(\frac{V_s^2 - V_s V_r e^{j(\theta_s - \theta_r)}}{X}\right) = j\left(\frac{V_s^2 - V_s V_r e^{j\delta}}{X}\right)$$

$$= j\left(\frac{V_s^2 - V_s V_r(\cos(\delta) + j\sin(\delta))}{X}\right) = \underbrace{\left(\frac{V_s V_r}{X}\right)\sin(\theta_{sr})}_{P_s} + j\underbrace{\left(\frac{V_s^2 - V_s V_r \cos(\theta_{sr})}{X}\right)}_{Q_s}$$

$$P_s = P_r = \left(\frac{V_s V_r}{X}\right)\sin(\delta),\quad Q_s = \frac{V_s^2 - V_s V_r \cos(\delta)}{X},\quad Q_r = \frac{V_s V_r \cos(\delta) - V_r^2}{X}$$

Real and Reactive Power Transmission Equations (R = 0)

Sending and receiving end real and reactive power:

$$P_s = P_r = \left(\frac{V_s V_r}{X}\right)\sin(\delta) \tag{42}$$

$$Q_s = \frac{V_s^2 - V_s V_r \cos(\delta)}{X} \tag{43}$$

$$Q_r = \frac{V_s V_r \cos(\delta) - V_r^2}{X} \tag{44}$$

- Transmission line shunt capacitance not explicitly represented. Assume that series reactance is a Thevenin equivalent of the transmission line.
- In HV transmission systems X/R is high so neglecting losses (i.e. R) is reasonable in an initial conceptual analysis. In practice R is included.
- In distribution systems X/R is low (can be less than one) so R should be included then.

Approximate P, Q relationships for small δ

- if $\delta \to 0$ then $\sin(\delta) \to \delta, \quad \cos(\delta) \to 1$
  - $P_s = P_r \approx \left(\frac{V_s V_r}{X}\right)\delta$
  - $Q_s = \frac{V_s \Delta V_{sr}}{X}, \quad Q_r = \left(\frac{V_r \Delta V_{sr}}{X}\right)$
- Stable system operation requires voltages near 1 pu and relatively small transmission angles, thus observe that:
  - P and θ are closely coupled
  - Q and ΔV are closely coupled
- P transmission depends primarily on the transmission angle
- Q transmission depends primarily on voltage magnitudes and flows from high to low voltage.

Example: Reactive power transmission

Consider a transmission line with X = 0.3 pu sending P = 0.8 pu, and a reactive power load of Qr = 0.6 pu (i.e. 0.8 pf lag). The receiving end voltage is Vr = 1.0 pu. What is the required sending end voltage and reactive power? What is the loss of reactive power in the transmission line?

- Rearrange (42) & (43) to give:

$$V_s \sin(\delta) = \frac{XP_r}{V_r}, \quad V_s \cos(\delta) = V_r + \frac{XQ_r}{V_r} \qquad (45)$$

- Square both equations and sum them to give:

$$V_s^2 = \left(V_r + \frac{XQ_r}{V_r}\right)^2 + \left(\frac{XP_r}{V_r}\right)^2$$

which yields:

$$V_s = \sqrt{\left(V_r + \frac{XQ_r}{V_r}\right)^2 + \left(\frac{XP_r}{V_r}\right)^2} \qquad (46)$$

- Substituting numerical values gives:

$$V_s = \sqrt{\left(1 + \frac{0.3 \times 0.6}{1}\right)^2 + \left(\frac{0.3 \times 0.8}{1}\right)^2} = 1.204\,\text{pu}$$

This is well above 1.05 pu which is a typical maximum voltage for normal operation in a power system. (Operating at a voltage less than the upper continuous limit of 1.1 pu because it allows headroom in the event of a contingency.)

- From (45) $\delta = \operatorname{asin}\left(\frac{XP_r}{V_sV_r}\right) \times \frac{180}{\pi}$. Substituting numerical values gives:

$$\delta = \operatorname{asin}\left(\frac{0.3 \times 0.8}{1.204 \times 1}\right) \times \frac{180}{\pi} = 11.5^\circ$$

- The sending end reactive power is obtained from (43):

$$Q_s = \frac{V_s^2 - V_sV_r\cos(\theta_{sr})}{X} = \frac{1.204}{0.3}(1.204 - \cos(11.5^\circ)) = 0.9 \text{ pu}$$

Example: Reactive power transmission (Cont)

The reactive power lost in transmission is: $Q_{loss} = Q_s - Q_r$ which in this example is $Q_{loss} = 0.9 - 0.6 = 0.3$ pu.

- Cross check:

$$Q_{loss} = I^2 X = \left(\frac{P_r^2 + Q_r^2}{V_r^2}\right) X \quad \text{pu}$$

$$= \left(\frac{0.8^2 + 0.6^2}{1^2}\right) 0.3 = 0.3 \text{ pu}$$

Validity of approximate formula for Q

$$Q_r = \frac{V_r \Delta V_{sr}}{X},$$

$$\Delta V_{sr} = \left(\frac{Q_r X}{V_r}\right) = \left(\frac{0.6 \times 0.3}{1}\right) = 0.18 \text{ pu}$$

compared with exact value:

$\Delta V_{sr} = 1.204 - 1 = 0.204$ pu

Error in estimated voltage rise is about –12%

Significance

- Unable to transmit reactive power long distances.
- Significant constraint on reactive power transmission is the limited voltage range within which power systems must operate.
- High temporary overvoltages can occur when loads with high reactive consumption are rejected (disconnected).
- Minimizing reactive power transmission reduces losses:
  - $P_{loss} = I^2 R = \left(\dfrac{P^2 + Q^2}{V^2}\right) R$
  - By keeping transmission system voltages high (e.g. ~ 1.05 pu) contributes to lowering losses.
- In steady-state power system analysis (i.e. load-flow analysis) loads are usually represented as P + jQ and **not** by impedances. Consequently network equations are non-linear. (Employ loadflow analysis programs).