THE UNIVERSITY of ADELAIDE

Course:
ELEC ENG 3110 Electric Power Systems
ELEC ENG 7074 Power Systems PG

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Balanced Three Phase Systems; The Per-unit System

Lecturer & Coordinator: David Vowles
david.vowles@adelaide.edu.au

Summary of Balanced Three Phase Systems

Sources for background and detail

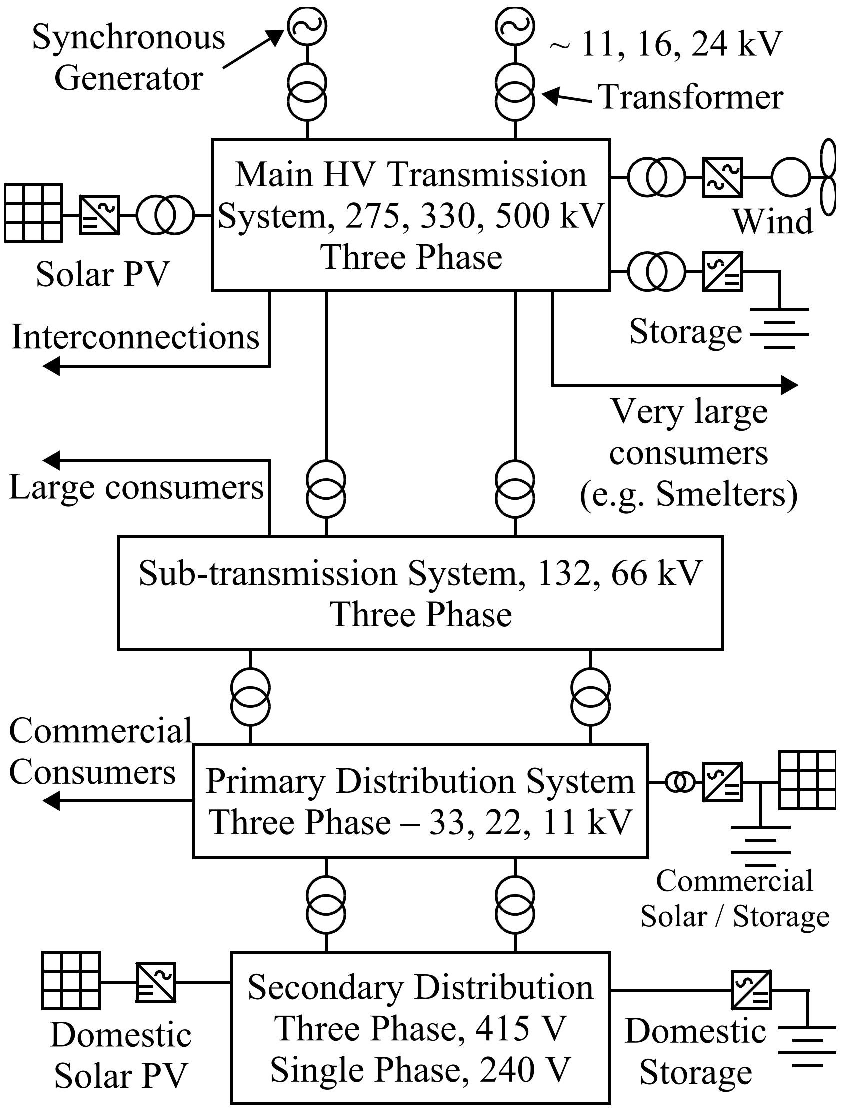
This summary is a review of key points relevant to understanding and analysis of balanced three phase systems. Most introductory texts on power systems will provide the relevant detail and background. Some such sources are listed.

- Section 2.5 of Electrical Energy Systems (EES) lectures.
- Section 2.5 "Balanced Three-Phase Circuits" of J.D. Glover, T.J. Overbye, M.S. Sarma, "Power System Analysis and Design", 6th Edition, Cengage Learning, (c) 2017.
  [Earlier editions also include this material although the section number may be different]
- Section 2.1 "Three-Phase Systems" of B.M. Weedy, J.M. Cory, "Electric Power Systems", 4th Edition, John Wiley & Sons, 1998.
- Section 2.3 "Balanced Three Phase" of A.R. Bergen, "Power Systems Analysis", Prentice Hall, 1986

Introduction

- Power is generated by three-phase generators:
  - Mostly synchronous machines
  - Increasingly asynchronous machines (wind power: induction generators, solar PV: d.c. interfaced to grid with power electronic converters)
- Transformed from low to high voltage by three-phase transformers (e.g. 16 kV to 275 kV)
- Transmitted by three phase transmission lines to load distribution substations (e.g. 132, 275, 330, 500 kV)
- Transformed from high to lower voltages by three phase voltages (e.g. 275 kV to 33 kV)
- Primary distribution of power by three phase distribution lines (e.g. 33kV, 22 kV, 11 kV)
- Secondary distribution: three phase 415V and individual homes at single phase 240 V.
- See schematic representation of power system.

Schematic Representation of a Power System



The Balanced Three Phase Network

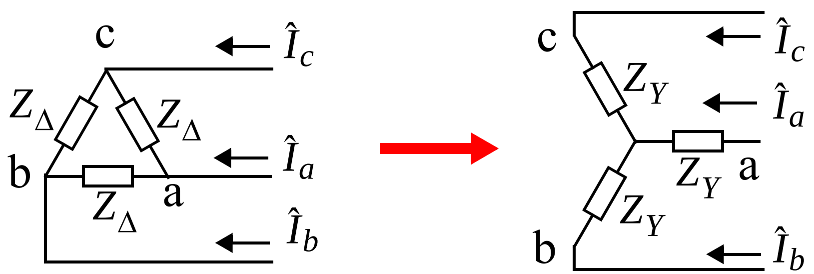
A balanced three-phase voltage source is defined as one in which the three voltages have equal magnitude and are shifted in phase by 120 deg. one with respect to the other.

A balanced set of three-phase currents is similarly defined.

A three-phase network is said to be balanced if all voltage and current sources are balanced, the impedances in all three phases of each network element are equal and all sources (loads) supply (consume) balanced three phase currents.

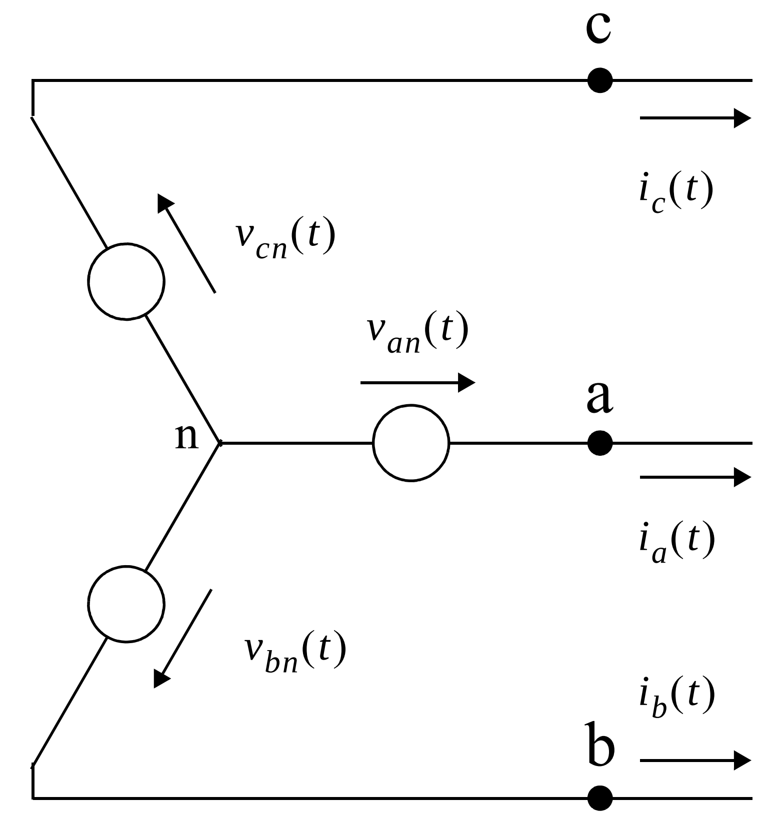
The three voltages may be connected in star or delta. A set of voltages connected in delta can be transformed to an equivalent set of star connected voltages.

Any delta-connected set of impedances can be transformed to an equivalent set of star-connected impedances. For a balanced network the transform is:



$$Z_Y = \frac{Z_\Delta}{3}$$

Balanced Three Phase Source Supplying a Balanced Network



The balanced three phase voltages with phase sequence abc in the time-domain are:

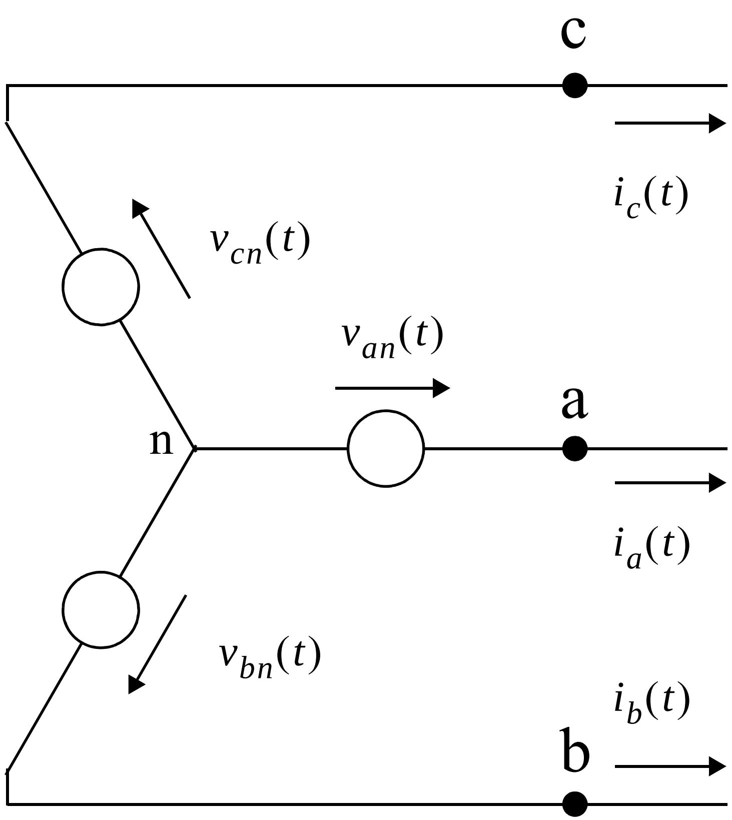
$$\begin{aligned} v_{an}(t) &= V_p\cos(\omega t + \alpha) \\ v_{bn}(t) &= V_p\cos\left(\omega t + \alpha - \frac{2\pi}{3}\right) \\ v_{cn}(t) &= V_p\cos\left(\omega t + \alpha + \frac{2\pi}{3}\right) \end{aligned} \tag{1}$$

The corresponding set of three voltage phasors is:

$$\begin{aligned} \hat{V}_{an} &= V_{nl}e^{j\alpha} \\ \hat{V}_{bn} &= V_{nl}e^{j\left(\alpha - \frac{2\pi}{3}\right)} = a^2\hat{V}_{an} \\ \hat{V}_{cn} &= V_{nl}e^{j\left(\alpha + \frac{2\pi}{3}\right)} = a\hat{V}_{an} \end{aligned} \tag{2}$$

in which $V_{nl} = V_p/\sqrt{2}$ is the line-neutral rms voltage of the source and $a = e^{j\left(\frac{2\pi}{3}\right)}$ is the 120 deg. rotation operator.

Balanced Three Phase Source Supplying a Balanced Network (Cont)



Balanced 3-phase currents

This set of voltages is supplying a balanced set of three-phase currents in which the a-phase current is shifted in phase by an angle of θ in advance of the a-phase voltage. The corresponding set of current phasors are:

$$\hat{I}_a = Ie^{j(\alpha+\theta)},\ \hat{I}_b = a^2\hat{I}_a\ \ \&\ \ \hat{I}_c = a\hat{I}_a \tag{3}$$

Power supplied by a balanced 3-phase source

The complex power supplied by each phase is identical to that of phase-a:

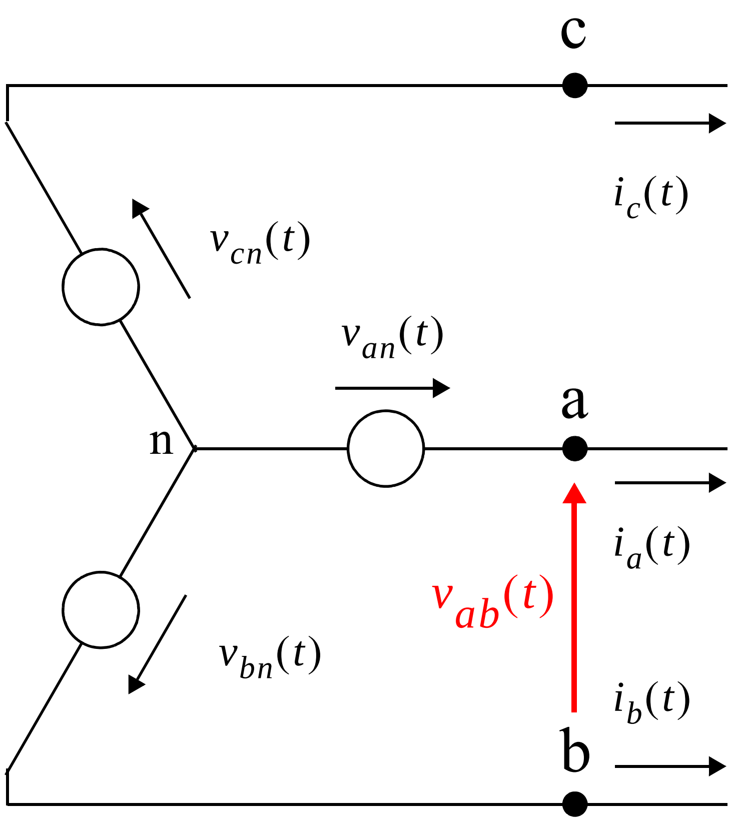
$$\boldsymbol{S}_{1\varphi} = \hat{V}_{an}\hat{I}_a^* = V_{nl}Ie^{-j\theta}$$

The total three-phase complex-power is thus:

$$\boldsymbol{S}_{3\varphi} = \boxed{3}\boldsymbol{S}_{1\varphi} = \boxed{3}V_{nl}Ie^{-j\theta} \tag{4}$$

It is important to note that the instantaneous three-phase power supplied by a balanced source is constant.

Phase-to-Phase Voltages



It is common to express voltages as phase-to-phase (or line-to-line) instead of phase-to-neutral (or line-to-neutral). The voltage of phase a with respect to phase b is:

$$\hat{V}_{ab} = \hat{V}_{an} - \hat{V}_{bn} = V_{ll} e^{j\left(\alpha + \frac{\pi}{6}\right)} \quad \text{where}$$

$$V_{ll} = \sqrt{3} V_{nl} \tag{5}$$

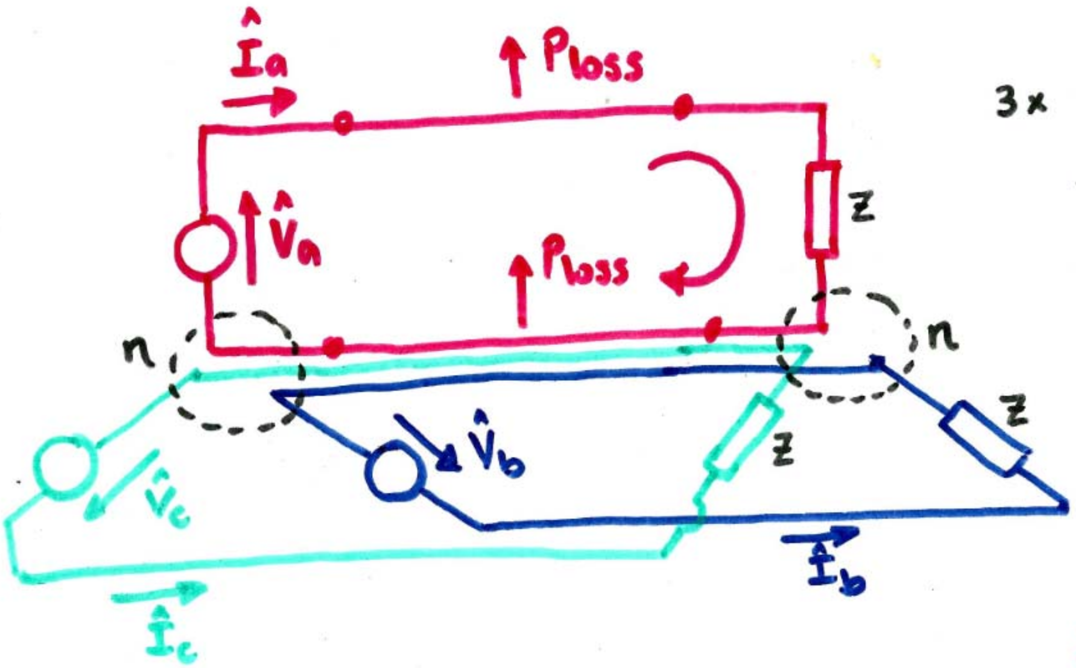
The b-c and c-a voltages are obtained by applying the appropriate 120 deg. rotations:

$$\hat{V}_{bc} = a^2 \hat{V}_{ab} \text{ and } \hat{V}_{ca} = a \hat{V}_{ab} \tag{6}$$

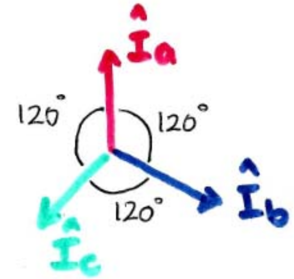
Note that:

$$\boldsymbol{S}_{3\varphi} = \boxed{3V_{nl}} I e^{-j\theta} = \boxed{\sqrt{3}V_{ll}} I e^{-j\theta} \tag{7}$$

Combining Three Single Phase Systems to Form a Balanced Three Phase System



3× single phase
2× Ploss
2× conductors



$$\hat{I}_a + \hat{I}_b + \hat{I}_c = 0$$

Connect neutrals

=> zero neutral current

=> no neutral conductors required

=> 1 × Ploss, 3 conductors, not 6

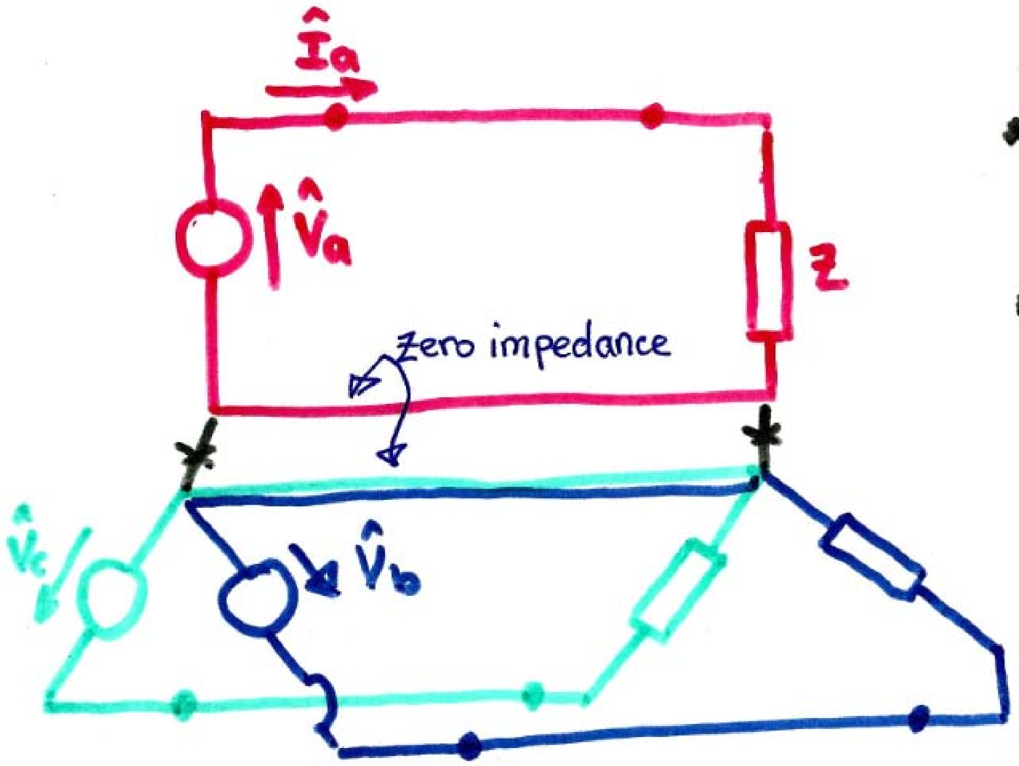
Balanced Three Phase Systems – Per Phase Analysis

The sum of three balanced three-phase currents is zero. Consequently in such a network the neutral current is zero. Thus, in practical three phase systems in which measures are taken to ensure the network is balanced it is often the case that a neutral conductor is not required, or if provided its rating can be significantly less than the phase conductor.

If a balanced three-phase network is transformed such that all components are star-connected then it is possible to connect the neutral points of all components with a zero impedance conductor without changing the results. Then one phase of the system can be analysed in isolation from the others. The total power of the corresponding three-phase components are obtained by multiplying the power from the per-phase solution by three.

Thus, all techniques applicable to the analysis of single-phase networks is applicable to the analysis of balanced three-phase networks.

Balanced Three Phase Systems – Per Phase Analysis



* Connect neutrals with zero impedance line
* Separate a-phase circuit & analyse as a single-phase circuit
* b, c phase voltages & currents inferred from a-phase solution

3 phase power = 3 x single phase power

Advantages of three-phase systems

Some advantages of three-phase systems are:

- Generators and motors have significantly more power capacity per-unit volume.
- Instantaneous three-phase power is constant rather than oscillating at twice the supply frequency as in single phase systems.
- Transmission infrastructure costs are significantly less than for single phase networks.
    - Neutral conductor not required or can have much lower capacity than phase conductors
- Transmission losses are significantly less than for single phase networks.

The Per-Unit System

In power systems analysis it is normal practice to express quantities (e.g. P, Q, V, Z, X, etc.) as fractions of reference or base values of the corresponding quantity.

General Definition

$$\text{per-unit value} = \frac{\text{actual value in any system of units (u)}}{\text{base or reference value in units (u)}}$$

Same equation in a more compact mathematical form:

$$H^{(p)} = \frac{H^{(u)}}{H_B^{(u)}} \text{ where} \tag{8}$$

- $H^{(p)}$ is the per-unit value of the quantity H;
- $H^{(u)}$ is the value of H in a specified system of units u (e.g. the SI system of units);
- $H_B^{(u)}$ is the base or reference value of H in the same system of units (i.e. u)

Example: Suppose the rated line-to-line voltage of a motor is 415 V (rms) is chosen as the base value and the actual supply voltage is measured to be 430 V (rms, l-l) then the per-unit value of the voltage is:

$$V^{(p)} = \frac{V_{\text{measured}}}{V_{\text{rated}}} = \frac{430}{415} = 1.036 \text{ per-unit.}$$

Example: Suppose the rated MVA of a generator is 300 MVA and it is chosen as the base value of power. The generator power output is 240 MW. Then the per-unit value of the power output is:

$$P^{(p)} = \frac{P_{\text{measured}}}{\text{MVA rating}} = \frac{240}{300} = 0.8 \text{ p.u.}$$

Advantages of the per-unit-system

The per-unit system is very useful for the following reasons:

- For a given device (e.g. a transformer, motor), quantities such as voltage drops and power losses at rated output are normally about the same percentage of the rated quantity over a certain range of ratings.
- Where various voltage levels are used in the same system (e.g. 11 kV, 275 kV, 132 kV), calculations can be carried out without having to worry about transformer ratios, etc.
- Parameter values (e.g. impedances) for similar devices are *approximately the same* over a certain range of ratings.
- Factors such as $\sqrt{3}$ associated with the relationship between line-to-line and line-to-neutral voltages, 3 associated with relationship between per-phase and three-phase power disappear from per-unit calculations.
- Use of per-unit quantities assists in ensuring the numerical accuracy and robustness of power simulation models by avoiding numerical values spanning many orders of magnitude.

Changing Base Values – The General Case

Often a quantity is provided in per-unit on a specified base value but it is necessary to change the quantity to be in per-unit on a different base value.

Specifically, let $H^{(p1)}$ be the per-unit value of H on a base value of $H_{B1}^{(u)}$. It is required to express H in per-unit as $H^{(p2)}$ on a new base-value $H_{B2}^{(u)}$.

From (8) it follows that the value of H in the system of units u is $H^{(u)} = H^{(p)} \times H_B^{(u)}$. This equation applies whatever base-value is chosen to represent the quantity in per-unit. Thus, the following equation applies to the original (1) and new (2) per-unit systems:

$$H^{(u)} = H^{(p1)} H_{B1}^{(u)} = H^{(p2)} H_{B2}^{(u)} \tag{9}$$

Rearranging (9) we obtain the expression for the per-unit value of H in the new per-unit system:

$$H^{(p2)} = \frac{H^{(u)}}{H_{B2}^{(u)}} = H^{(p1)} \times \left(\frac{H_{B1}^{(u)}}{H_{B2}^{(u)}}\right) \tag{10}$$

<u>Example:</u> Suppose a generator is supplying $P^{(p1)} = 0.8$ pu to the network on a base value of its MVA rating of 500 MVA. It is required to change this to per-unit on a common system MVA base of 100 MVA.

The original (1) and new (2) base values are $S_{B1} = 500$ MVA, $S_{B2} = 100$ MVA. Thus, from (10) the per-unit value of generator power on the new base of S_{B2} is:

$$P^{(p2)} = P^{(p1)}\left(\frac{S_{B1}}{S_{B2}}\right) = 0.8 \times \frac{500}{100} = 4.0 \text{ pu.}$$

Choosing and Deriving Base-Quantities

The usefulness of the per-unit system depends on the form of the equations describing the relationships between various parameters and variables having a similar form in the per-unit system as in the formulation using physical units.

We have freedom to freely choose a principal or canonical set of base quantities. However, to ensure the consistency of analysis based on per-unit values the base-values of all other quantities must be derived from this specified principal set.

It is common in power system analysis to choose as the principle set of base quantities:

- The three-phase apparent power S_{u3b} in which the subscript 'u' denotes this is 'user' selected, '3' denotes that it is a three-phase quantity and 'b' denotes that it is a base value. This quantity is commonly called the "MVA base"
- The line-line rms voltage V_{ullb}.

The nominal system frequency (e.g. 50 Hz) is normally chosen to be the base value of frequency, f_b. The base value of time is normally chosen to be $t_b = 1$ s.

When analysing the performance of a single device such as a motor the rated line-current I_{ulb} might be chosen as a principle base quantity *instead of* three-phase apparent power S_{u3b}.

Derived base values – Examples

To illustrate consider the following examples for which the principle base quantities are chosen to be:

- The MVA base $S_{u3b} = 255\ \text{MVA}$
- RMS line-to-line voltage base $V_{ullb} = 275\ \text{kV}$

<u>Example 1: Derived per-phase base values</u>

<u>Line-neutral base voltage</u>

The line-to-neutral voltage in a balanced three-phase system is obtained from the line-to-line voltage from $|\hat{V}_{nl}| = |\hat{V}_{ll}|/\sqrt{3}$. Thus, the base value of the line-to-neutral voltage is derived from this expression as follows:

$$V_{nlb} = \frac{V_{ullb}}{\sqrt{3}} = \frac{275}{\sqrt{3}} = 158.8\ \text{kV}$$

Suppose that the line-to-line voltage at a certain location in a balanced three-phase network is

$V_{ll} = 288.75\ \text{kV}$ (rms l-l) which corresponds to

$V_{nl} = 288.75/\sqrt{3} = 166.71\ \text{kV}$ (rms l-n).

The per-unit voltage at the specified location is:

$$\begin{aligned} V^{(p)} &= \frac{V_{ll}}{V_{ullb}} = \frac{V_{ll}/\sqrt{3}}{V_{ullb}/\sqrt{3}} = \frac{V_{nl}}{V_{nlb}} \\ &= \frac{288.75}{275} = \frac{166.71}{158.8} \\ &= 1.05\ \text{pu} \end{aligned}$$

Important result:

Since we have specified mutually consistent base-values of the line-to-line and line-neutral rms voltages, numerically, the per-unit value of the voltage (1.05 pu) is the same in both reference frames.

- The $\sqrt{3}$ factor disappears in p.u. voltages

Derived base values – Examples

Single-phase base apparent power

The per-phase or single-phase apparent power in a balanced three-phase network is equal to one-third of the corresponding three-phase apparent power. Thus, the base value of single-phase apparent power is:

$$S_{1b} = \frac{S_{u3b}}{3} = \frac{255}{3} = 85 \text{ MVA / ph}$$

Suppose that the three-phase power consumed by a load in the network is $P_{3\varphi} = 204$ MW and thus the corresponding power consumption per-phase is $P_{1\varphi} = 204/3 = 68$ MW. The per-unit power consumed by the load is:

$$P^{(p)} = \frac{P_{3\varphi}}{S_{u3b}} = \frac{P_{3\varphi}/3}{S_{u3b}/3} = \frac{P_{1\varphi}}{S_{1b}}$$

$$= \frac{204}{255} = \frac{68}{85}$$

$$= 0.8 \text{ pu}$$

Again, due to the consistency of our choice of base values of the three-phase and single-phase apparent power, the per-unit value of power is numerically the same in both cases.

Important: Identity of per-unit three-phase and per-phase quantities

A very convenient feature of the application of the per-unit system in the analysis of balanced three-phase networks illustrated by the preceding examples is that once quantities are transformed into the per-unit world we do not need to distinguish between per-phase and three-phase quantities.

Of particular importance is that once a balanced three-phase system is transformed from its per-phase representation in physical units into its per-unit representation numerical factors such as $\sqrt{2}$, $\sqrt{3}$ and 3 which are associated with relationships between peak and rms quantities, line-to-line and line-to-neutral quantities, three-phase and single-phase power disappear.

Derived Base Values – Examples

Example 2: Base value of line-current

The line-current is related to the three-phase apparent power and line-to-line voltage as follows:

$$S_{3\varphi} = \sqrt{3} \times V_{ll} \times I_l \text{ VA}$$

where V_{ll} is in V (rms, l-l) and I_l is in A (rms, l). Use as basis to define a consistent base-value of line current.

Multiply both sides by $1/(S_{u3b} \times 10^6)$ (VA^{-1}) and multiply the r.h.s by $(V_{ullb} \times 10^3)/(V_{ullb} \times 10^3)$ (V / V):

$$\left(\frac{S_{3\varphi}\text{ (VA)}}{S_{u3b} \times 10^6\text{ (VA)}}\right) = \left(\frac{V_{ll}\text{ (V)}}{V_{ullb} \times 10^3\text{ (V)}}\right) \times \left\{\sqrt{3} \times \left(\frac{V_{ullb} \times 10^3\text{ (V)}}{S_{u3b} \times 10^6\text{ (VA)}}\right)\right\} \times I_l$$

$$\underbrace{\left(\frac{S_{3\varphi} \times 10^{-6}\text{ (MVA)}}{S_{u3b}\text{ (MVA)}}\right)}_{S^{(p)}} = \underbrace{\left(\frac{V_{ll} \times 10^{-3}\text{ (kV)}}{V_{ullb}\text{ (kV)}}\right)}_{V^{(p)}} \times \underbrace{\underbrace{\left\{\sqrt{3} \times \left(\frac{V_{ullb}\text{ (kV)}}{S_{u3b}\text{ (MVA)}}\right) \times 10^{-3}\right\}}_{1/I_{lb}} \times I_l}_{I^{(p)}}$$

Note $S^{(p)} = \left(\frac{S_{3\varphi} \times 10^{-6}}{S_{u3b}}\right)$ pu and $V^{(p)} = \left(\frac{V_{ll} \times 10^{-3}}{V_{ullb}}\right)$ pu are the per-unit MVA and voltage.

Derived Base Values – Examples

The reciprocal of the term in {} is therefore derived to be the base value of line current in A:

$$I_{lb} = \frac{S_{u3b}\ (\mathrm{MVA})}{\sqrt{3}V_{llb}(\mathrm{kV})} \times 1000\ \ \mathrm{A} \tag{11}$$

Thus, the per-unit value of line current is:

$$I^{(p)} = \frac{I_l}{I_{lb}}\ \mathrm{pu} \tag{12}$$

and, moreover, the per-unit equation for apparent power is:

$$S^{(p)} = V^{(p)} I^{(p)}\ \ \mathrm{pu} \tag{13}$$

Important result:

Note that due to per-unitization the factor of $\sqrt{3}$ disappears from the per-unit expression. Furthermore, since the numerical values of (i) the line-line and line-to-neutral voltages; and (ii) the three-phase and per-phase apparent power in the per-unit system are identical the subscripts such as ll, nl, etc. can be omitted.

Derived Base Values – Examples

Example:
The three-phase real and reactive power consumed by a load is $\boldsymbol{S} = P + jQ = 204 + j153$ MW/MVAr and the line-to-line voltage at the load terminals is $V = 261.25$ kV. What are the per-unit values of P, Q, V and the load-current I?

Recall that the principle base values chosen for this series of examples are: $S_{u3b} = 255$ MVA (3 ph) and $V_{ullb} = 275$ kV (rms, l-l).

$$\boldsymbol{S}^{(p)} = \frac{\boldsymbol{S}}{S_{u3b}} = P^{(p)} + jQ^{(p)}$$

$$= \frac{204}{255} + j\frac{153}{255} = 0.8 + j0.6 \text{ pu}$$

Important result:
This example shows that the base value of 3-phase apparent power (i.e. S_{u3b}) is also the base value of real and reactive-power.

Per-unit voltage:

$$V^{(p)} = \frac{V}{V_{ullb}} = \frac{261.25}{275} = 0.95 \text{ pu}$$

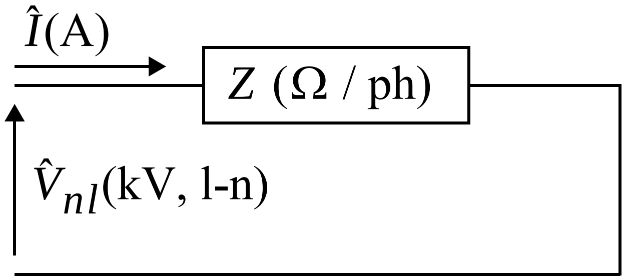
Use (13) to calculate the per-unit load current.

$$I^{(p)} = \left|\boldsymbol{S}^{(p)}\right| / V^{(p)} = \frac{\sqrt{0.8^2 + 0.6^2}}{0.95} = 1.053 \text{ pu}$$

Important: We did not need to compute the load current in physical units (i.e. A) because we could use (13) to directly compute the per-unit value from the previously determined per-unit values of P, Q and V.

Derived Base Values – Base Value of Per Phase Impedance

In balanced three-phase networks delta connected components are transformed to equivalent star-connected components prior to conducting per-phase analysis. Therefore, in per-unit calculations we assume that devices are star-connected. The following circuit forms the basis for the derivation of the base value of (per-phase) impedance.



$$\hat{V}_{nl}\ (\mathrm{kV}) \times 10^3 = Z\hat{I}\ \mathrm{V}$$

Divide both sides of this equation by the base value of line-neutral voltage $V_{nlb}\ (\mathrm{kV}) \times 10^3$ V and multiply the r.h.s. by I_{lb}/I_{lb} so that we obtain:

$$\underbrace{\left(\frac{\hat{V}_{nl}}{V_{nlb}}\right)}_{\hat{V}^{(p)}} = \underbrace{Z\underbrace{\left(\frac{I_{lb}}{V_{nlb} \times 10^3}\right)}_{1/Z_{Yb}}}_{Z^{(p)}}\underbrace{\left(\frac{\hat{I}}{I_{lb}}\right)}_{\hat{I}^{(p)}}$$

We note that $\hat{V}^{(p)} = \left(\frac{\hat{V}_{nl}}{V_{nlb}}\right)$ and $\hat{I}^{(p)} = \left(\frac{\hat{I}}{I_{lb}}\right)$. The base value of impedance is therefore defined as:

$$Z_{Yb} = \frac{V_{nlb}\ (\mathrm{kV}) \times 10^3}{I_{lb}\ (\mathrm{A})}\ \Omega\ /\ \mathrm{ph\ and}$$

$$Z^{(p)} = Z/Z_{Yb}\ \mathrm{pu} \tag{14}$$

so that the following per-unit equation relating voltage, current and impedance is defined:

$$\hat{V}^{(p)} = Z^{(p)}\hat{I}^{(p)}$$

Derived Base Values – Base Value of Per Phase Impedance

It is convenient to express the base value of impedance directly in terms of the principle base quantities. Recall that:

$V_{nlb} = \dfrac{V_{ullb}}{\sqrt{3}}$ and from (11) $I_{lb} = \dfrac{S_{u3b}\,(\mathrm{MVA})}{\sqrt{3}V_{llb}(\mathrm{kV})} \times 1000$ which when substituted in (14) yields:

$$Z_{Yb} = \frac{V_{ullb}^2}{S_{u3b}}\ \Omega\,/\,\mathrm{ph},\ (V_{ullb}\text{ in kV and }S_{u3b}\text{ in MVA}) \tag{15}$$

Suppose that the source impedance is $Z^{(p)} = 0.02 + j0.1$ pu on the principle base values chosen for this series of examples (i.e. $S_{u3b} = 255$ MVA (3 ph) and $V_{ullb} = 275$ kV (rms, l-l)). What is the value of the impedance in ohms?

$$\begin{aligned} Z &= Z^{(p)} \times Z_{Yb} = Z^{(p)} \times \frac{V_{ullb}^2}{S_{u3b}} = (0.02 + j0.1) \times \frac{275^2}{255}\ \Omega\,/\,\mathrm{ph} \\ &= 5.93 + j296.57 \end{aligned}$$

Summary of commonly used base values

In power systems analysis a commonly employed principal set of base quantities are listed in Table 1. This set is not unique and other choices are possible. In addition, the table lists for convenience a number of commonly needed derived base quantities. It is straight-forward to derive necessary base quantities from first principles as illustrated in the preceding examples.

Table 1: Typical Principal and Derived Base Quantities in Power Systems Analysis

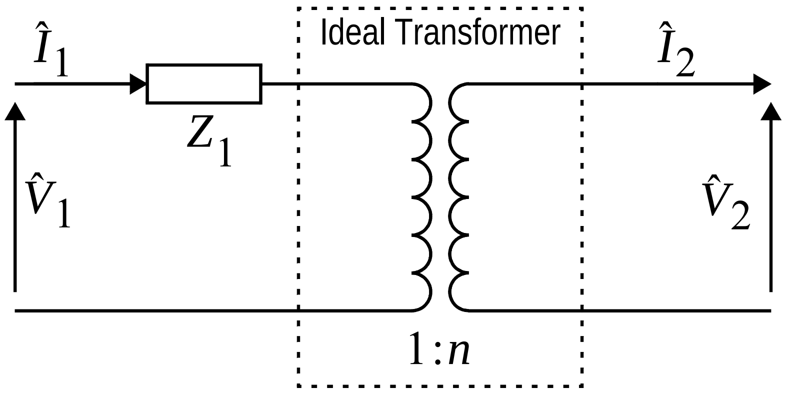
Base Quantity	Units	Description
Principal set of base quantities from which other base quantities are derived		
S_{3ub}	MVA (3ph)	Apparent three-phase power base (MVA) • Equipment apparent power rating; • Network wide apparent power base
V_{llub}	kV (rms, l-l)	RMS line-to-line voltage base (kV) • Rated equipment voltage
f_{ub}	Hz	Frequency base • System synchronous frequency • Rated equipment frequency
t_b	s	Base value of time chosen to be 1 second

Base Quantity	Units	Description
Derived base quantities		
S_{3b}	VA (3ph)	Apparent three-phase power base (VA) $S_{3b} = S_{3ub} \times 10^6$
S_{1b}	VA (1ph)	Apparent single-phase power base (VA) $S_{1b} = S_{3b}/3$
V_{llb}	V (rms, l-l)	RMS line-to-line voltage base (V) $V_{llb} = V_{llub} \times 10^3$
V_{nlb}	V (rms, l-n)	RMS line-to-neutral voltage base (V) $V_{nlb} = V_{llb}/\sqrt{3} = \dfrac{V_{llub} \times 10^3}{\sqrt{3}}$
I_{lb}	A (rms, l)	RMS line current base (A) $I_{lb} = \dfrac{S_{1b}}{V_{nlb}} = \dfrac{S_{3ub}}{\sqrt{3}V_{llub}} \times 10^3$
Z_{YB}	Ohm / ph	Per-phase impedance base for a star-connected impedance $Z_{YB} = \dfrac{V_{nlb}}{I_{lb}} = \dfrac{V_{llub}^2}{S_{3ub}}$
L_{YB}	H / ph	Per-phase inductance (star-connected) $L_{YB} = \dfrac{Z_{YB}}{2\pi f_b} = \left(\dfrac{1}{2\pi f_b}\right)\left(\dfrac{V_{llub}^2}{S_{3ub}}\right)$
C_{YB}	F / ph	Per-phase capacitance (star-connected) $C_{YB} = \dfrac{1}{(2\pi f_b)Z_{YB}} = \left(\dfrac{1}{2\pi f_b}\right)\left(\dfrac{S_{3ub}}{V_{llub}^2}\right)$

Per-unit system for transformers

There is an advantage to choosing different bases for the two sides of a transformer.

For simplicity consider a single-phase transformer with a winding ratio of 1:n in which the total series impedance of the transformer has been referred to the winding 1 side ($Z_1\ (\Omega)$).



On winding 1 assume that V_{1b} (V, l-n) and I_{1b} (A) have been chosen as the base values of voltage and current. The winding 1 per-unit impedance is then:

$$Z_1^{(p)} = \frac{Z_1}{(V_{1b}/I_{1b})} = Z_1\left(\frac{I_{1b}}{V_{1b}}\right) \text{ pu}$$

Refer the transformer impedance to winding 2 instead of winding 1 to obtain:

$$Z_2\ (\Omega) = n^2 \times Z_1\ (\Omega)$$

On the secondary side we now **choose** the base values of voltage and current in terms of the corresponding winding 1 values and in accordance with the winding ratio as follows:

$$V_{2b} = nV_{1b} \text{ (V, l-n)}, \quad I_{2b} = I_{1b}/n \text{ (A)}$$

With this choice of winding 2 base values the base values of impedance on the winding 2 side of the transformer are:

$$Z_{2b} = \frac{V_{2b}}{I_{2b}} = n^2\left(\frac{V_{1b}}{I_{1b}}\right) = n^2 Z_{1b} \qquad (16)$$

Thus, the per-unit value of Z_2 is given by:

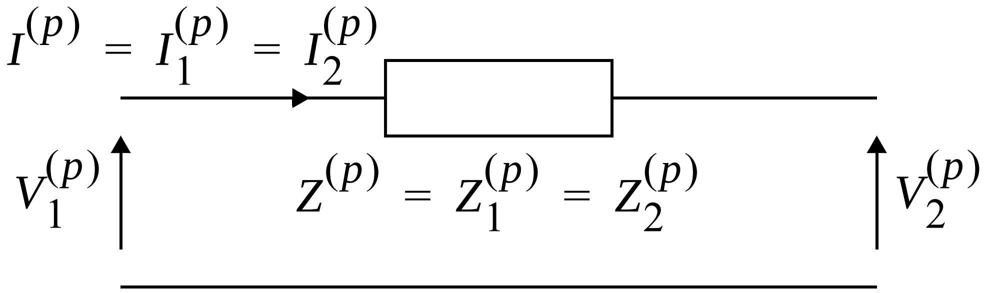
$$Z_2^{(p)} = \frac{(n^2 Z_1)}{Z_{2b}} = \frac{n^2 Z_1}{n^2 Z_{1b}} = \frac{Z_1}{Z_{1b}} = Z_1^{(p)}$$

Per-unit system for transformers

$$Z_2^{(p)} = \frac{(n^2 Z_1)}{Z_{2b}} = \frac{n^2 Z_1}{n^2 Z_{1b}} = \frac{Z_1}{Z_{1b}} = Z_1^{(p)}$$

Consequently, using the per-unit system, the per-unit series impedance referred to winding 1 has the same numerical value as the series impedance referred to winding 2, <u>providing base values of voltage are chosen in accordance with the transformer turns ratio.</u>

Following is the per-unit representation of a two-winding transformer (shunt elements neglected).



Per-unit representation of networks

When representing a power system network in the per-unit system it is necessary to ensure that a **consistent set of base values are employed**. It is conventional to:

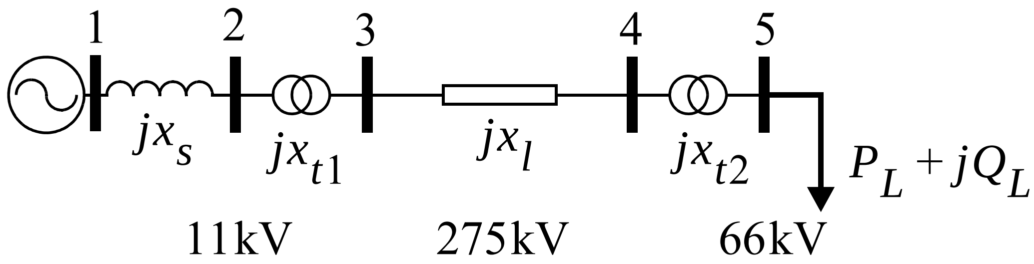
- choose a single value of three-phase apparent power base (S_{u3b}) for the entire network.
- choose voltage bases at each node in the network in accordance with the nominal voltage at that bus.

If these rules are used the per-unit impedance of a transformer does not change whether referred to one winding or the other. (There are *complications if a transformer has off-nominal tap*)

The example network in the following figure illustrates the method. Some parameters are provided in physical units and must be converted to per-unit values on the appropriate base values. Some other parameters are provided in per-unit on one set of base values and must be converted to per-unit on the chosen system base values.

In general, the conversion of per-unit values from one base to another is handled by equation (10). A useful specific example is the conversion of a per-unit impedance from an old base value (1) to a new base value (2) when the specified base values are the apparent power and voltage.

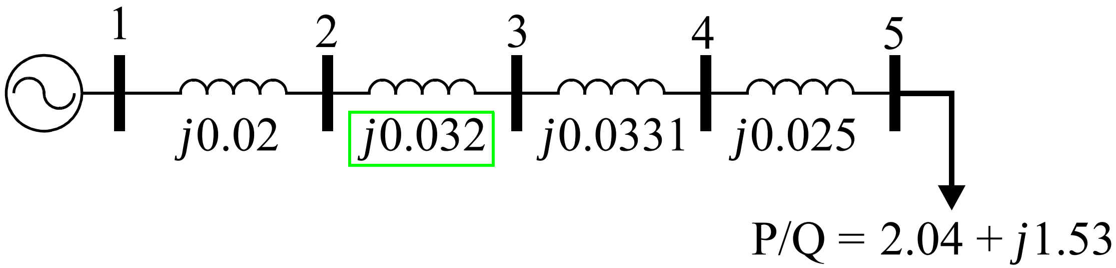
$$Z^{(p2)} = Z^{(p1)} \times \left(\frac{S_{b2}}{S_{b1}}\right) \times \left(\frac{V_{b1}}{V_{b2}}\right)^2 \tag{17}$$



Element	Input Value	Input units / base values	pu on 100 MVA & nominal node voltages
x_s	0.1	pu on 500 MVA, 11 kV	$0.1 \times \frac{100}{500} = 0.02$
x_{t1}	0.16	pu on 500 MVA, 11 / 275 kV	$0.16\left(\frac{100}{500}\right) = 0.032$
x_l	25.0	Ohm / ph	$25 / \left(\frac{275^2}{100}\right) = 0.0331$
x_{t2}	0.12	pu on 480 MVA, 275 / 66 kV	$0.12 \times \frac{100}{480} = 0.025$
P_L, Q_L	204, 153	MW, MVAr	$\frac{204}{100} = 2.04$, $\frac{153}{100} = 1.53$

Per-unit representation of networks – Example (Cont)

Following is the per-unit representation of the network on the previous page in which all quantities are in per-unit on 100 MVA and 11 kV for nodes 1 & 2, 275 kV for nodes 3 & 4 and 66 kV for node 5.



Suppose that the voltage at bus 5 is 62.7 kV. Determine the internal voltage of the generator (i.e. the voltage at node 1).

$$\hat{V}_5 = \frac{62.7}{66}\angle 0 = 0.95\angle 0 \text{ pu}$$

$$\hat{I}_5 = \frac{P - jQ}{\hat{V}_5^*} = \frac{2.04 - j1.53}{0.95} = 2.147 - j1.611 \text{ pu}$$

The series impedance between nodes 1 & 5 is $\boxed{jx = j0.1101}$ pu and thus,

$$\begin{aligned} \hat{V}_1 &= \hat{V}_5 + jx\hat{I}_5 \\ &= 0.95 + \boxed{j0.1101} \times (2.147 - j1.611) \\ &= \boxed{1.1519\angle 11.84°} \end{aligned}$$

Per-unitization – A note of caution

Per-unitization is very simple in principle but is frequently a source of easily avoidable error.

- Follow general principles
- Pay meticulous attention to:
    - units and their consistency (take care when mixing kV, A, MVA, etc.);
    - whether voltages and currents are rms or peak
        - being out by a factor of $\sqrt{2}$ is a symptom of incorrect accounting for the distinction between rms and peak values
    - whether voltages are line-to-line or line-to-neutral
        - being out by a factor of $\sqrt{3}$ is a symptom of incorrect accounting for the distinction l-l and l-n values
    - whether apparent power is single or three-phase
        - being out by a factor of 3 is a symptom of incorrect accounting for the distinction between single and three-phase apparent power
- Work through the conversion between quantities in physical-units and per-unit step-by-step.