Sample Exam – Power Systems -- 2021

This **sample exam** is intended to provide an indication of the type, style and complexity of questions in the official exam in November 2021. The topics covered in this sample should **NOT** be used as an indication of the topics to be covered in the official exam. That is, revision should include all course material including lectures, tutorials, quizzes, sample quizzes and assignments and should not be restricted to the topics covered in this sample exam.

This sample exam is for both the undergraduate and postgraduate versions of the course. The distinction is that postgraduates will be required to answer all five questions in the allotted time of 150 min whereas undergraduates will be required to answer the first four questions in the allotted time of 120 min.

Question 1. Sinusoids, phasors and ac circuits [20 marks]

1a) Sinusoidal Waveforms and Phasors [8 marks]

A balanced 50 Hz sinusoidal three-phase voltage source with phase sequence abc supplies a balanced three-phase system. The phase ‘a’ to neutral voltage is represented by an anti-clockwise-rotating, cosine-referenced, RMS, phasor

$$\hat{V}_{an} = 158.8e^{-j(\pi/12)} \text{ kV} \tag{1}$$

and the phase ‘a’ line current is represented by the phasor of the same type

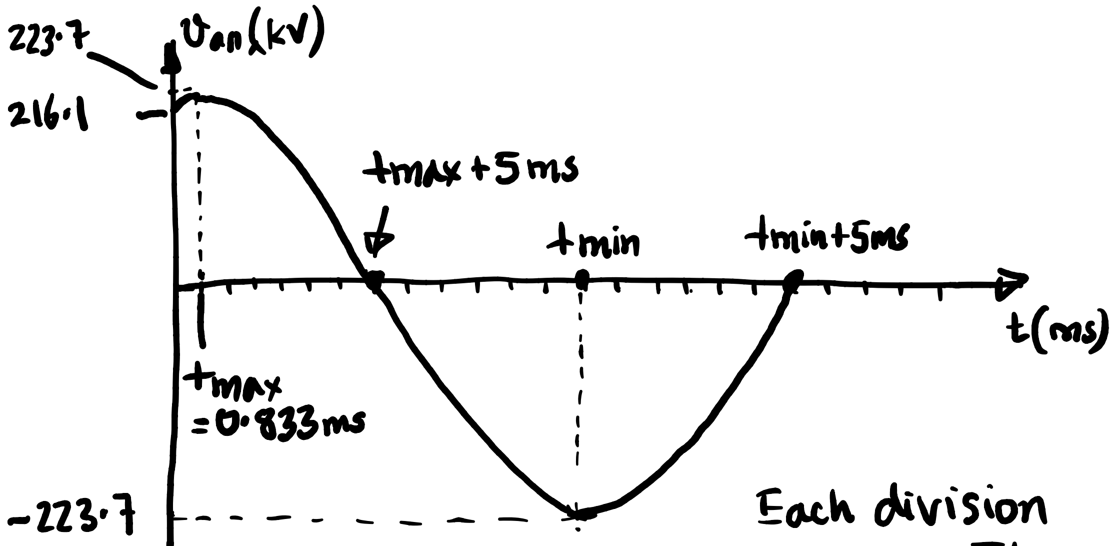
$$\hat{I}_a = 210\angle -30° \text{ A} \tag{2}$$

i) Express the voltage as a sinusoidal function of time. [2 marks]

Period: $T_p = \frac{1}{50} = 20ms$

$$v_{an}(t) = \sqrt{2}\, 158.8 \cos(2\pi \times 50t - \pi/12)$$
$$= 223.7 \cos(100\pi t - \pi/12)$$

ii) Clearly sketch the voltage waveform from time t = 0 marking clearly the value of the voltage at t = 0 and the times and values of the first minimum and maximum voltage. [3 marks]



$v_{an}(0) = 223.7\cos(-\pi/12)$
$= 216.1$ kV

First max
$100\pi t_{max} - \frac{\pi}{12} = 0$

$t_{max} = 0.833ms$

First min:
$t_{min} = t_{max} + T_p/2$
$= 0.833 + 20/2$
$= 10.833$ ms

Each division
$\equiv T_p \times \frac{\pi/12}{2\pi} = \frac{T_p}{24}$

i) Calculate the three-phase real power (P) and reactive power (Q) *generated* by the source. [3 marks]

$$P_{3\varphi} + jQ_{3\varphi} = S_{3\varphi} = 3 \times S_{1\varphi} = 3 \times \hat{V}_{an} \hat{I}_a^*$$

$$= 3 \times \underset{kV}{158.8} \times \underset{kA}{\frac{210}{1000}} \times \angle -15 \times (\angle -30)^* \text{ MVA}$$

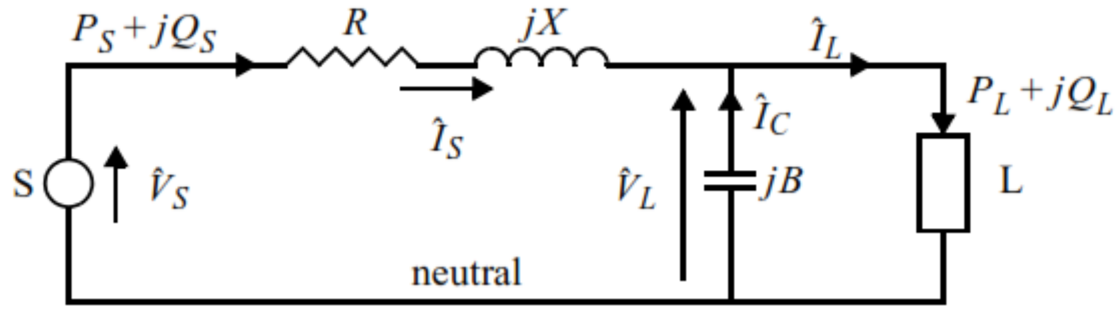
$$= 100.044 \angle 15°$$

$$= 96.64 + j25.89$$

$$P_{3\varphi} = 96.64 \text{ MW}, \quad Q_{3\varphi} = 25.89 \text{ MVAr}$$

1b) Circuit Analysis [12 marks]

Consider the following circuit showing a single phase of a balanced three-phase network. The network parameters are in per-unit on a three-phase MVA base of 100 MVA and voltage base of 275 kV (rms, l-l).



Parameter	Value (per-unit on 100 MVA, 275 kV (rms, l-l))
R (resistance)	0.05
X (reactance)	0.15
B (susceptance)	0.50

1b)(I) Circuit Analysis (Part I) [6 marks]

The load bus voltage phasor is $\hat{V}_L = 1.05\angle 0°$ pu and the load is $P_L + jQ_L = 120 + j40$ MW/MVAr.

i) Calculate the current phasor $\hat{I}_C$ in per-unit. [1 mark]

$$\hat{I}_c = (0 - \hat{V}_L) \times jB = -jB\hat{V}_L = -j0.5 \times 1.05 = -j0.525 \text{ pu.}$$

ii) Calculate the reactive power generated by the shunt capacitance in per-unit and MVAr? [1 mark]

$$S = \hat{V}_L(\hat{I}_c)^* = \hat{V}_L(-jB\hat{V}_L)^* = jB\hat{V}_L\hat{V}_L^* = jBV_L^2$$

$$\Rightarrow Q = BV_L^2 = 0.5 \times 1.05^2 = 0.5513 \text{ pu} = 55.13 \text{ MVAr } (3\phi)$$

iii) Calculate the per-unit load current phasor $(\hat{I}_L)$? [2 marks]

$$\hat{I}_L = \left(\frac{S_L}{\hat{V}_L}\right)^* = \frac{\frac{120 - j40}{100}}{1.05} = \frac{1.2 - j0.4}{1.05} \frac{(\text{pu})}{(\text{pu})}$$

$$= 1.1429 - j0.3810$$

$$= 1.205\angle -18.43°$$

iv) Calculate the per-unit source current phasor $\left(\hat{I}_S\right)$? [2 marks]

$$\hat{I}_S = \hat{I}_L + (-\hat{I}_c) = 1\cdot1429 - j0\cdot3810 + j0\cdot525$$

$$= 1\cdot1429 + j0\cdot1440$$

$$= 1\cdot152\angle 7\cdot184^\circ$$

1b)(II) Circuit Analysis (Part II) [6 marks]

For a different operating condition than that in 1b(I) the load bus voltage phasor is found to be $\hat{V}_L = 0.98\angle 0°$ pu and the source current is $\hat{I}_S = 1.4 - j0.4$ pu.

i) Calculate the real power loss in transmitting the power from the source to the load in per-unit and MW? [1 mark]

$R = 0\cdot05$ pu

$$P_{Loss} = R|I|^2$$
$$= 0\cdot05 \times 2\cdot12$$
$$= 0\cdot106 \text{ pu}$$
$$= 10\cdot6 \text{ MW}$$

Losses from fundamentals.

$$\hat{I} = \frac{\hat{V}_S - \hat{V}_R}{R}$$

$$\Rightarrow \hat{V}_S - \hat{V}_R = R\hat{I}$$

$$S_{Loss} = \hat{V}_S\hat{I}^* - \hat{V}_R\hat{I}^*$$
$$= (\hat{V}_S - \hat{V}_R)\hat{I}^*$$
$$= R\hat{I}\hat{I}^*$$
$$= R|I|^2 = P_{Loss}$$

ii) Calculate the per-unit source voltage phasor $\left(\hat{V}_S\right)$ magnitude and phase angle? [2 marks]

$$\hat{V}_S = \hat{V}_L + (R + jX).\hat{I}_S$$

$$= 0\cdot98 + (0\cdot05 + j0\cdot15) \times (1\cdot4 - j0\cdot4)$$

$$= 1\cdot11 + j0\cdot19$$

$$= 1\cdot126\angle 9\cdot713^\circ$$

iii) Calculate the real and reactive power $\left(P_S + jQ_S\right)$ supplied by the source in per-unit and in MW / MVAr? [3 marks]

$$S_S = P_S + jQ_S = \hat{V}_S\hat{I}_S^* = (1\cdot11 + j0\cdot19) * (1\cdot4 + j0\cdot4)$$

$$= 1\cdot478 + j0\cdot71$$

$P_S = 1\cdot1478$ pu $= 114\cdot8$ MW

$Q_S = 0\cdot71$ pu $= 71\cdot0$ MVAr

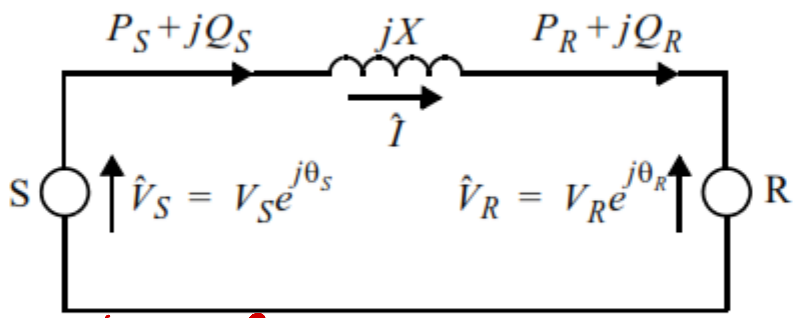
Question 2. Power System Analysis [20 marks]

2a) Derivation of complex power transmission equations [6 marks]

In the loss-less transmission system shown in the following figure recall that:

$$P_S = P_R = \left(\frac{V_S V_R}{X}\right)\sin(\delta),\quad Q_S = \frac{V_S^2}{X} - \left(\frac{V_S V_R}{X}\right)\cos(\delta),\quad Q_R = \left(\frac{V_S V_R}{X}\right)\cos(\delta) - \frac{V_R^2}{X}$$

in which $\delta = \theta_S - \theta_R$ is the transmission angle.



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Referring to ~~**Error! Reference source not found.**~~ derive the equations for the real and reactive power delivered to the receiving end (R) (i.e. P_R and Q_R ~~in~~ equation ~~**Error! Reference source not found.**~~). *You must clearly show the steps in your derivation.*

$$S_R = \hat{V}_R \hat{I}^*, \quad \hat{I} = \frac{\hat{V}_S - \hat{V}_R}{jX} = -\frac{j}{X}(\hat{V}_S - \hat{V}_R)$$

$$\hat{I}^* = \frac{j}{X}(\hat{V}_S^* - \hat{V}_R^*)$$

$$= \frac{j\hat{V}_R}{X}(\hat{V}_S^* - \hat{V}_R^*)$$

$$= j\frac{\hat{V}_R\hat{V}_S^*}{X} - j\frac{V_R^2}{X}$$

$$= j\left(\frac{V_R e^{j\theta_R} V_S e^{-j\theta_S}}{X} - \frac{V_R^2}{X}\right) = j\left(\frac{V_S V_R e^{-j\overbrace{(\theta_S - \theta_R)}^{=\delta}}}{X} - \frac{V_R^2}{X}\right)$$

$$= j\left(\frac{V_S V_R(\cos(-\delta) + j\sin(-\delta)) - V_R^2}{X}\right)$$

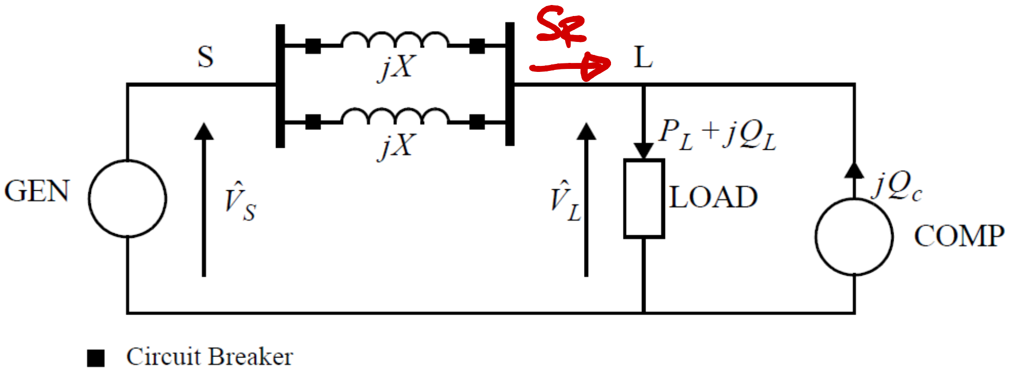
$$= j\left(\frac{V_S V_R(\cos(\delta) - j\sin(\delta)) - V_R^2}{X}\right) = \frac{V_S V_R}{X}\sin\delta + j\left(\frac{V_S V_R\cos\delta - V_R^2}{X}\right)$$

$$P_R = \mathrm{Re}(S_R) = \frac{V_S V_R}{X}\sin\delta$$

$$Q_R = \mathrm{Im}(S_R) = \frac{V_S V_R}{X}\cos\delta - \frac{V_R^2}{X}$$

2b) Reactive power compensation and voltage control [14 marks]

Consider the network in the following figure in which two identical transmission lines, each with reactance $X = 0.3$ pu, supply a load from a generating source connected to node S. A static VAR compensator (SVC) is connected to the load bus, L, to control the load bus voltage to a specified value.

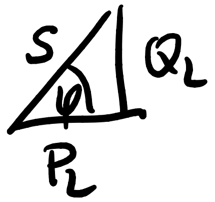


The *maximum load* to be *securely* supplied by the network is $P_{L\max} = 2.0$ pu at a power factor of 0.95 (lag) and the source voltage magnitude is $V_S = 1.05$ pu. The compensator is required to control the load bus voltage to $V_L = 1.05$ pu.

a) For the purpose of determining the required reactive power capacity of the SVC do you assume that there are one or two transmission lines operating? Explain why. [2 marks]

Assume that the load is supplied by a single circuit because in this case the reactive power losses in the T/L are higher by a factor of about 2 as compared to supplying the load over two T/L. Thus the SVC will need to compensate for higher Q losses if the load is supplied by a single line

$$Q_L = P_L \tan\varphi = P_L \tan(\mathrm{acos}(0.95))$$
$$= 2 \times \tan(\mathrm{acos}(0.95))$$
$$= 0.6574 \text{ pu}$$



$pf = \cos\varphi =$

b) Calculate the minimum reactive power capacity of the SVC required to securely supply the load. [6 marks]

Assume load supplied by single circuit and assume compensator regulates load voltage to 1.05 pu.

Conserve complex power at the load bus

$$\Rightarrow P_R = \frac{V_S V_L}{X} \sin\delta = P_{Lmax} = 2.0 \quad (1)$$

$$Q_R + Q_C = \frac{V_S V_L}{X}\cos\delta - \frac{V_L^2}{X} + Q_C = Q_L$$

$$\Rightarrow Q_C = Q_L + \frac{V_L^2}{X} - \frac{V_S V_L}{X}\cos\delta \quad (2)$$

From (1) $\sin\delta = \frac{P_{Lmax} * X}{V_S V_L} \Rightarrow \delta = \text{asin}\left(\frac{2 \times 0.3}{1.05^2}\right) = 0.5755$ rad.

From (2) $Q_C = 0.6574 + \frac{1.05^2}{0.3}\left(1 - \cos(0.5755)\right)$

$= 1.2492$ pu is the min SVC capacity.

c) Give two reasons for conducting power-flow analysis of power systems. [2 marks]

Refer to F08, slide 4 for several reasons

d) Explain why it is not possible to transmit reactive power long distances in a power system. [2 marks]

* Long transmission lines => high impedance
  high impedance => large voltage drop and high reactive losses if Q is high.
* Since voltages must be regulated close to 1.0 pu we can not accept large voltage drops that would result from transmitting high Q
* Supplying high reactive losses is expensive

e) If it is necessary to increase the voltage at a network bus would you connect a capacitor or inductor? Explain why. [2 marks]

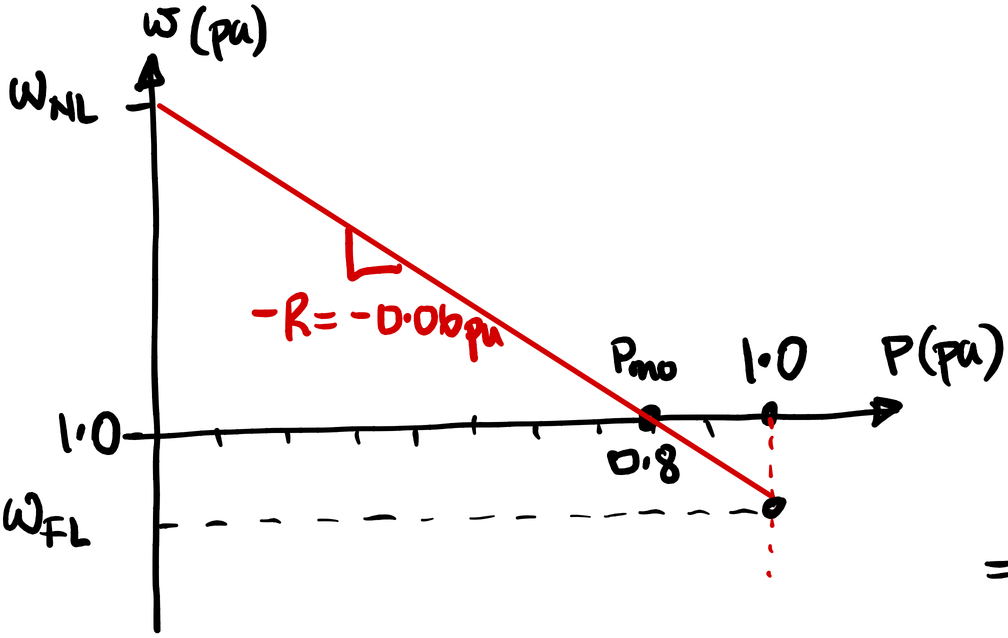
Connect a capacitor

- Power networks are inherently inductive and thus consume reactive power.
- By connecting a capacitor we generate reactive power thus reducing reactive losses
- Reducing reactive losses tends to increase voltages

Question 3. Frequency and power control [20 marks]

3a) Steady-state governor droop characteristic [4 marks]

A generating unit with a power rating of 250 MW is operating steadily at synchronous speed with a power output of 200 MW. The steady-state governor droop is 0.06 per-unit on the power rating of the unit. Sketch the per-unit governor steady-state characteristic representing this operating condition. Mark clearly on the characteristic the value of the generator rotor speed when the power output is (a) zero; and (b) one per-unit.



$P_{mo} = \frac{200}{250} = 0{\cdot}8$ pu

$\frac{\omega_{NL} - 1}{0 - P_{mo}} = -R = -0{\cdot}06$pu

$\omega_{NL} = 1 + 0{\cdot}06 * 0{\cdot}8$

$= 1{\cdot}048$ pu

$\frac{\omega_{NL} - \omega_{FL}}{0 - 1} = -R$

$\Rightarrow \omega_{FL} = \omega_{NL} - R$

$= 1{\cdot}048 - 0{\cdot}06$

$= 0{\cdot}988$pu.

3b) Parallel operation of generating units [9 marks]

Two generators are operating in parallel to supply an isolated load. The generators are fitted with governors with a steady-state droop characteristic. The power ratings and droop settings of the generators are listed in the following table.

Gen	Power Rating (MW)	Governor Steady-State Droop (pu on generator power rating)
1	500	0.03
2	800	0.05

The system load is frequency dependent and is found to increase by 1.5% due to a frequency increase of 1%. Initially the network frequency is 50 Hz and generators 1 & 2 are supplying 350 MW and 500 MW respectively. The load is increased abruptly by 30 MW (as determined at 50 Hz) from 850 to 880 MW. Calculate the resulting steady-state change in frequency (in Hz), and the steady-state changes in the power output of each of the two generating units (in MW).

The load increase must be balanced by an increase in output from the generators. Thus

$P_{R1} = 500$ MW
$P_{R2} = 800$ MW

$$\Delta P_L = \Delta P_{M1} + \Delta P_{M2} \qquad (1)$$

Now, $\Delta P_{M1} = -\left(\frac{\Delta\omega}{R_1}\right) * P_{R1}$, $\Delta P_{M2} = -\left(\frac{\Delta\omega}{R_2}\right) P_{R2}$ (2)

ΔP_L is frequency dependent.

Now, by definition $D = \dfrac{\left(\dfrac{\Delta P_L}{P_{L0}}\right)}{\Delta f}$ ← per unit $\Rightarrow \Delta P_L(f) = D P_{L0} \Delta f$

Thus, $P_L(f) = \underbrace{P_{L0}}_{\text{load @ 50 Hz}} + D P_{L0} \Delta f = P_{L0}(1 + D \Delta f)$

Now, $\Delta P_L = P_{L2} - P_{L10} = P_{L20}(1 + D\Delta f) - P_{L10}$

$$= \underbrace{(P_{L20} - P_{L10})}_{30\,MW} + \underset{880\,MW}{P_{L20}}\ \underset{1.5}{D}\ \Delta f$$

$$\Delta P_L = 30 + 1320\, \Delta f \quad (3)$$

$\Delta\omega = \Delta f$

$\therefore$ Combining (1), (2) and (3) we have

$$\Delta P_L = 30 + 1320\, \Delta\omega = -\Delta\omega\left(\frac{P_{R1}}{R_1} + \frac{P_{R2}}{R_2}\right)$$

$$\therefore\ 30 = -\Delta\omega\left(1320 + \frac{500}{0.03} + \frac{800}{0.05}\right) = -\Delta\omega(1320 + 32667)$$

$$\therefore\ \Delta\omega = -\frac{30}{33{,}987} = -8.827 \times 10^{-4}\ pu$$

The changes in generator output are from ②

$$\Delta P_{M1} = -\frac{P_{R1}}{R_1}\Delta\omega = \frac{500}{0.03} \times 8.827 \times 10^{-4} = 14.71\ MW$$

$$\Delta P_{M2} = -\frac{P_{R2}}{R_2}\Delta\omega = \frac{800}{0.05} \times 8.827 \times 10^{-4} = 14.12\ MW$$

3d) Load and frequency control of interconnected power systems [7 marks]

i) A test is conducted on a 50 Hz system to determine its frequency "stiffness". Immediately before the test the system frequency is 50 Hz and the Automatic Generation Control (AGC) is disabled temporarily. A load of 400 MW is connected to the system. It is found that the system frequency settles to a value of 49.95 Hz. What is the frequency stiffness of the system in MW / Hz? [2 marks]

$\Delta f = 49.95 - 50 = -0.05$ Hz, $\Delta P = 400$ MW

$$k = -\frac{\Delta P}{\Delta f} = +\frac{400}{0.05} = 8000 \text{ MW/Hz}.$$

ii) Explain why isochronous (i.e. constant speed) governors cannot be used to control frequency when two or more generators are connected in parallel. [2 marks]

If two generators try to regulate frequency with zero error (i.e. integral control) then a small difference in frequency measurement by the two units will mean that one unit will try and increase output and the other will try and decrease output continuously until one unit has ramped to maximum output and the other to minimum output. This "fighting" results from measurement errors.

iii) Summarize the purpose of the primary, secondary and tertiary power / frequency controls of a power system. [3 marks]

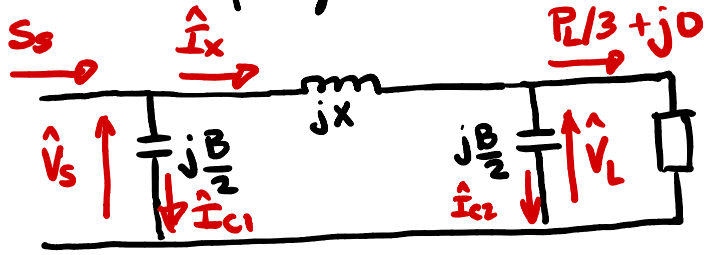
Refer, for example, to F11 pages 27 to 32.

Question 4. Power-flow analysis and fault calculations [20 marks]

4a) Transmission Lines [7 marks]

Consider a 500 kV (rms, l-l) transmission line of length 250 km connected between constant voltage sources at the sending (S) and receiving (R) ends of the line. The line is assumed to be lossless and with an inductance of $L = 0.862 \times 10^{-3}$ H / phase / km and capacitance of $C = 13.79 \times 10^{-9}$ F / phase / km. The system frequency is 50 Hz. The magnitude of the voltage at ~~each end~~ (the receiving end) of the line is 525 kV (rms l-l) and the ~~voltage source at the~~ receiving end of the line is absorbing 2000 MW of power from the line at unity power factor. Calculate the reactive power supplied to the sending end of the line.

Use a nominal pi-equivalent circuit representation of the T/L



Per-phase representation of T/L

$$X = \omega L \times l = 2\pi \times 50 \times 0.862 \times 10^{-3} \times 250 = 67.701 \ \Omega/ph$$

$$\frac{B}{2} = \omega C \times l = \frac{100\pi \times 13.79 \times 10^{-9} \times 250}{2} = 5.4153 \times 10^{-4} \ S/ph$$

$$\hat{V}_L = \frac{525}{\sqrt{3}} = 303.11\angle 0^\circ \ kV \ (l-n)$$

$$\hat{V}_S = 303.11\angle\delta \ kV \ (l-n)$$

$$\hat{I}_L = \frac{2000/3 \times 10^6}{303.11 \times 10^3} = 2199.4 \ A$$

$$\hat{I}_{c2} = j\frac{B}{2} \times \hat{V}_L = j164.14 \ A$$

$$\hat{I}_x = \hat{I}_L + \hat{I}_{c2} = 2199.4 + j164.14 \ A$$

$$\hat{V}_S = \hat{V}_L + jX I_x = 303.11 + j67.701 \times (2199.4 + j164.14) \times 10^{-3}$$

$$= 292 + j148.9 \ kV$$

$$\hat{I}_S = \hat{I}_x + \hat{I}_{c1} = \hat{I}_x + j\frac{B}{2}\hat{V}_S = 2118.8 + j322.27$$

$$S_S = 3 \times \hat{V}_S \hat{I}_S^* = 2000 + j664.19 \ MVA$$

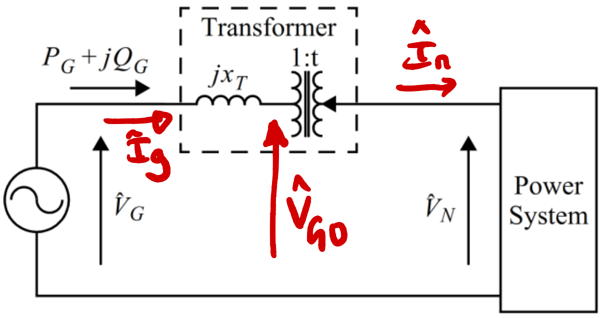
$$\therefore \underline{Q_S = 664.19} \ MVAr \ (3 \text{ phase})$$

4b) Voltage control with transformer taps [4 marks]

The generator step-up transformer in the following circuit has a leakage reactance of $x_T = 0.15$ pu on the transformer MVA rating of 500 MVA. The generator is supplying 350 MW at a lagging power factor of 0.97 and its automatic voltage regulator is regulating its terminal voltage to $|\hat{V}_G| = 1.0$ pu.

$$P_g = \frac{350}{500} = 0.7 \text{ pu}$$

$$Q_g = P_g \tan\varphi = P_g \tan(\text{acos}(0.97)) = 0.25062 P_g = 0.1754 \text{ pu}$$



Calculate the tap position t such that the magnitude of the network voltage $|\hat{V}_N| = 1.05$ pu.

MMF balance $\Rightarrow$ $\hat{I}_g \times 1 = \hat{I}_n \times t$

Complex power conservation $\Rightarrow$ $\hat{V}_{g0} \hat{I}_g^* = \hat{V}_N \hat{I}_n^* = \hat{V}_N \left(\frac{\hat{I}_g}{t}\right)^*$

$\Rightarrow$ $\hat{V}_{g0} = \frac{\hat{V}_N}{t}$ (Note t is real)

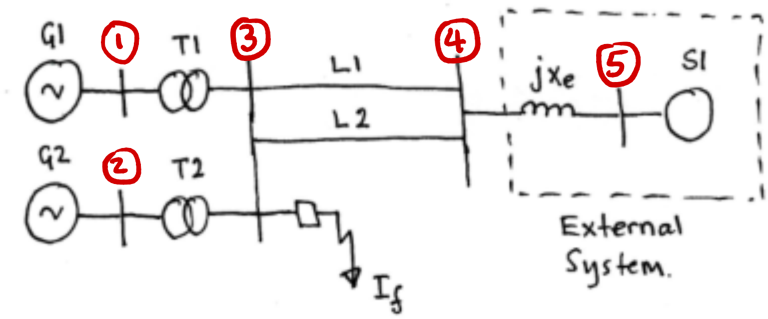
$$\hat{V}_{g0} = \hat{V}_g - jx_T \hat{I}_g = \hat{V}_g - jx_T \left(\frac{P_g + jQ_g}{\hat{V}_g}\right)^*$$

$$= 1.0 - j0.15\left(\frac{0.7 - j0.1754}{1}\right) = 0.9793 \angle -6.155^\circ$$

$$t = \frac{|\hat{V}_N|}{|\hat{V}_{g0}|} = \frac{1.05}{0.9793} = 1.0722 \text{ pu.}$$

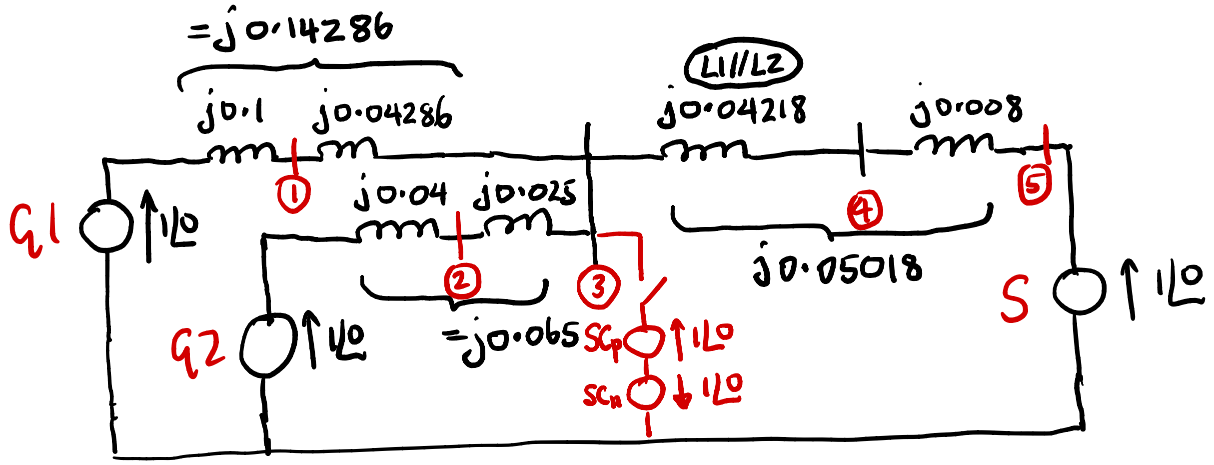
4c) Fault current calculation. [8 marks]

Consider the following power system one-line diagram.



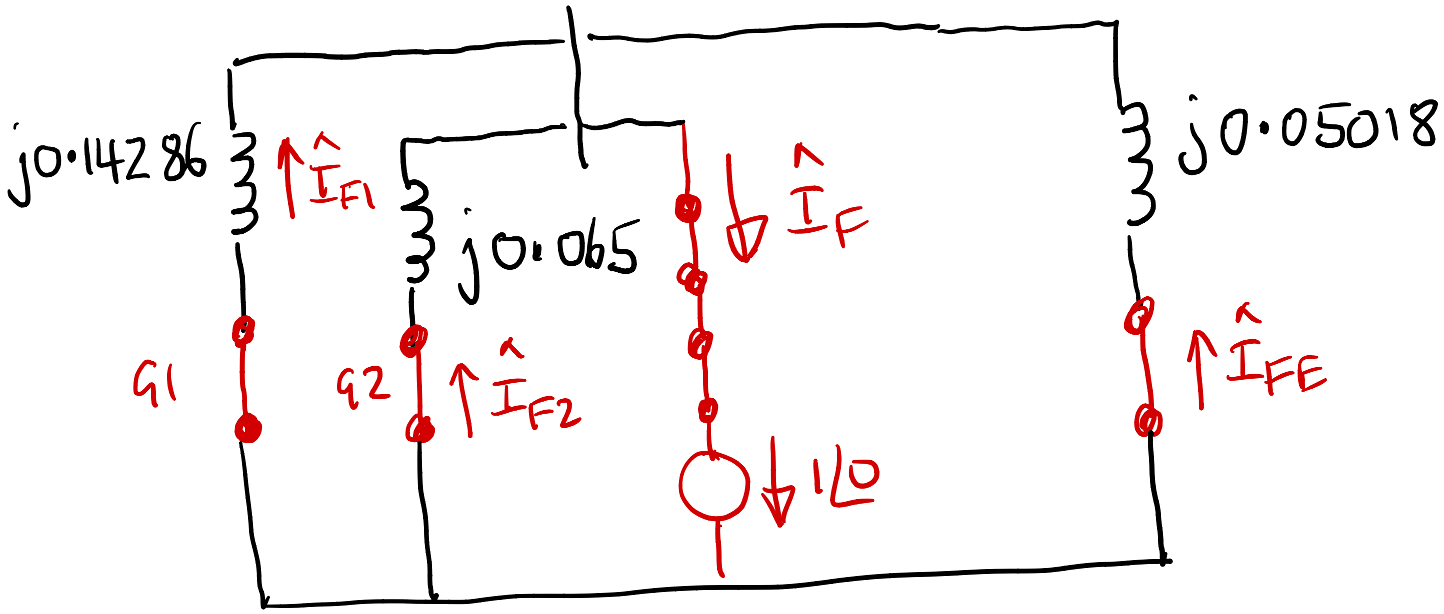
Item	Parameters	pu on 100MVA
G1	Two identical units each with $X_d'' = 0.25$ pu on 250 MVA, 16 kV	$\frac{0.25}{2.5} = 0.10$
T1	Two identical units each with $X_t = 0.12$ pu on 280 MVA, 16 / 275 kV	$\frac{0.12}{2.8} = 0.04286$
G2	$X_d'' = 0.2$ pu on 500 MVA, 24 kV	$\frac{0.2}{5} = 0.04$
T2	$X_t = 0.15$ pu on 600 MVA, 24 / 275 kV	$\frac{0.15}{6} = 0.025$
L1	275 kV, 200 km line with $R = 0.04$ ohm / ph / km; $X_L = 0.319$ ohm / ph / km and $B_C = 3.651 \times 10^{-6}$ S / ph /km. $Z_b = \frac{V_b^2}{S_b} = \frac{275^2}{100} = 756.25\Omega$	$\frac{63.8}{756.25} = 0.08436$
L2	Identical to line L1. Ignore R, B $X = 0.319 \times 200 = 63.8\Omega$	0.08436
Xe	0.008 pu on 100 MVA, 275 kV	0.008
S1	Ideal source (no internal impedance)	

i) Draw the network diagram to be used for the purpose of calculating balanced three phase fault currents. Show all network impedances in per-unit on 100 MVA. [4 marks]



ii) Calculate the fault current I_f in per-unit due to a balanced three phase fault as indicated in the power-circuit one-line diagram. [4 marks]

Close switch, apply superposition. Circuit 1 short circuit SCn only. Resulting circuit is equivalent to pre-fault network since SCp can be removed without changing current flows. All current flows in circuit 1 are zero. Circuit 2 short circuit all sources except SCn. Obtain following



The fault current $\hat{I}_F$ is obtained as

$$\hat{I}_F = \hat{I}_{F1} + \hat{I}_{F2} + \hat{I}_{FE}$$

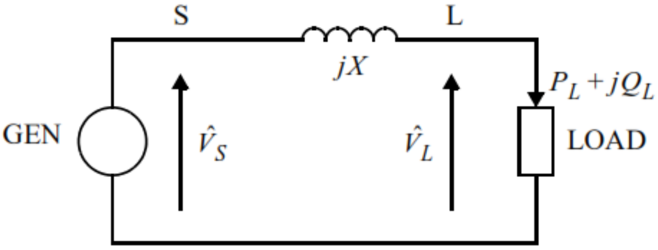
$$= 1.0\angle 0 \left(\frac{1}{j0.14286} + \frac{1}{j0.065} + \frac{1}{j0.05018} \right)$$

$$\underline{\hat{I}_F = -j\,42.313 \text{ pu}}$$

Question 5. Powerflow limits and short answer questions [20 marks]

5a) Powerflow limits [10 marks]

Consider the following circuit in which all quantities are in per-unit on a consistent set of base values.



Quantity	Value (per-unit)
X	0.3
$V_S = \lvert\hat{V}_S\rvert$	1.0

i) If the reactive power consumed by the load (Q_L) is zero, calculate the load power (P_L) at which the load bus voltage (V_L) is 0.9 pu. [3 marks]

$$P_L = \frac{V_S V_L}{X}\sin\delta = \frac{1\times 0.9}{0.3}\sin\delta = 3\sin\delta \quad (1)$$

$$Q_L = 0 = \frac{V_S V_L}{X}\cos\delta - \frac{V_L^2}{X} = 3\cos\delta - 2.7$$

$$\therefore 2.7 = 3\cos\delta \quad (2)$$

$$(1)^2 + (2)^2 \Rightarrow P_L^2 + 2.7^2 = 9(\sin^2\delta + \cos^2\delta)$$

$$P_L^2 = 9 - 2.7^2 = 1.71 \Rightarrow \underline{P_L = 1.308\text{ pu}}$$

ii) Calculate the maximum load that can be supplied at unity power factor. [7 marks]

[Hint: solution of $ax^2 + bx + c = 0$ is $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$]

$$P_L = \frac{V_S V_L}{X}\sin\delta \quad (1)$$

$$Q_L = 0 = \frac{V_S V_L}{X}\cos\delta - \frac{V_L^2}{X} \Rightarrow \frac{V_L^2}{X} = \frac{V_S V_L}{X}\cos\delta \quad (2)$$

$$①^2 + ②^2 \Rightarrow \quad P_L^2 + \left(\frac{V_L^2}{X}\right)^2 = \left(\frac{V_S V_L}{X}\right)^2 \underbrace{\left[\sin^2\delta + \cos^2\delta\right]}_{=1}$$

$$(V_L^2)^2 - (V_S^2)\, V_L^2 + (X P_L)^2 = 0$$

We have a quadratic in V_L^2 for which the solution is

$$V_L^2 = \frac{V_S^2 \pm \sqrt{(V_S^2)^2 - 4(X P_L)^2}}{2}$$

The maximum power transmission occurs when

$$(V_S^2)^2 - 4(X P_L)^2 = 0$$

$$\Rightarrow P_{Lmax} = \sqrt{\frac{V_S^4}{4X^2}} = \frac{1}{2}\frac{V_S^2}{X} = \frac{1}{2 \times 0.3} = 3.333 \text{ pu}$$

5c) Short answer questions [10 marks]

Answer the following questions using words only (that is, without any sketches, drawings or equations).

i) Summarize the mission of a power system. [3 marks]

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ii) Briefly describe how the voltage of a power system is controlled. [3 marks]

Key points:

- Aim to regulate voltages to slightly higher than 1.0 pu, say 1.05 pu.
- Ideally regulate voltages at each end of a T/L to the same value to minimize reactive power losses.
- Aim to supply reactive power where it is consumed to avoid transmitting reactive power long distances.
- Use automatic voltage regulators on generators and SVC's to continuously regulate voltages
- Switched reactors and capacitors provide coarse voltage regulation
- Transformer OLTCS useful for regulating HV bus voltages in generating stations and distribution network voltages in bulk supply substations.

iii) Explain the significance of the surge impedance load of a transmission line. [2 marks]

If a T/L is loaded with its SIL then the reactive losses are exactly balanced by line charging. This means that the source does not need to supply any reactive power.

If the load is > SIL then the T/L is a net consumer of reactive power

If the load is < SIL then the T/L is a net generator of reactive power.

iv) Explain the purpose of power system stabilizers fitted to synchronous generators.

[2 marks]

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