Course:
ELEC ENG 3110 Electric Power Systems
ELEC ENG 7074 Power Systems PG
(Semester 2, 2021)

Voltage Control

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Principles of Steady-State Voltage and Reactive Power Control

- **Overview**
  - **Steady-state Control Requirements:**
    - **Required to regulate voltages within narrow limits around nominal voltage (e.g. ±10%)**
      - **During both:**
        - **normal operation;**
        - **steady-state following contingency**
      - **In normal operation limit voltages to, e.g. ±5% of nominal, to allow "room" for contingency**
  - **Voltage variation influenced by reactive power variation**
    - **Q consumed by loads supplied from bulk substations varies daily, seasonally**
      - **Q compensation usually provided by distribution companies and large consumers**
    - **Q consumed by inductive elements depends on current flow** $Q_{loss} = I^2 X$
      - **transformers**
      - **distribution feeders and short transmission lines depends on current:**
    - **Q both generated and consumed by medium to long length HV / EHV transmission lines/cables:**

$$Q_{gen} = BV^2 \qquad Q_{loss} = I^2 X$$

      - **Charging varies over a relatively narrow range (e.g. ±20%), Q_{loss} varies over wide range**
      - **Net loss depends on balance between line current and voltage**

Principles of Steady-State Voltage and Reactive Power Control

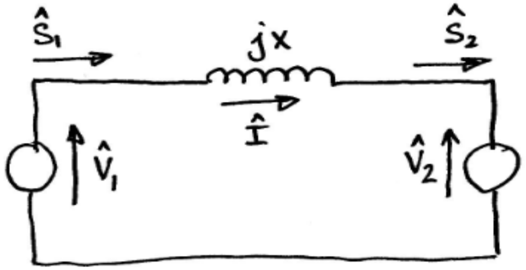
- **Overview (continued)**
  - **Voltage control methods:**
    - **Control production, consumption and flow of reactive power throughout system**
    - **Cannot transmit reactive power long distances**
    - **Compensating equipment:**
      - **Continuously acting automatic controls, for example,**
        - **Synchronous generators -> automatic voltage regulators (AVRs)**
        - **Static VAR Compensators (SVCs)**
        - **Static Synchronous Compensator (STATCOM)**
        - **Grid scale wind- and solar PV farms (many existing controls are discontinuous)**

        Q output automatically adjusted to maintain their bus voltages at specified setpoint
      - **Discontinuous (closed- and open-loop) controls**
        - **Switched shunt capacitors and reactors**
        - **Series capacitors**
        - **On-load tap-changing (OLTC) transformers**
    - **Continuous automatic controls => establish specified voltages at specific system nodes**
    - **Voltages elsewhere determined by P & Q flows through network elements**

Principles of Steady-State Voltage and Reactive Power Control

- **What strategies do we use to**
  - **Specify voltage set-points of voltage-controlling equipment**
  - **Switch reserves of reactive power (both supply and consumption)**
  - **Regulate flows of reactive power**
- **We take as a given power factor correction in distribution systems and industrial plants**

Review of power flow equations (lossless short line / transformer)



$\hat{V}_1 = V_1 e^{j\theta_1}$ $\quad \hat{V}_2 = V_2 e^{j\theta_2}$ $\quad \hat{I} = \dfrac{\hat{V}_1 - \hat{V}_2}{jX}$

$$\boxed{\hat{V}_1 = \hat{V}_2 + j\hat{I}X}$$

$$\begin{aligned}
\hat{S}_1 &= P_1 + jQ_1 = \hat{V}_1 \hat{I}^* \\
&= \hat{V}_1 \left(\frac{\hat{V}_1 - \hat{V}_2}{jX}\right)^* = \frac{V_1^2 - \hat{V}_1 \hat{V}_2^*}{-jX} \\
&= \frac{j}{X}\left(V_1^2 - V_1 V_2 e^{j\delta}\right) \quad , \; \delta = \theta_1 - \theta_2 \\
&= \frac{j}{X}\left(V_1^2 - V_1 V_2 (\cos\delta + j\sin\delta)\right) \\
&= \underbrace{\frac{V_1 V_2}{X} \sin\delta}_{P_1} + j\underbrace{\left(\frac{V_1^2}{X} - \frac{V_1 V_2}{X}\cos\delta\right)}_{Q_1}
\end{aligned}$$

Exercise:

Show $P_2 = \dfrac{V_1 V_2}{X} \sin\delta$

$Q_2 = \dfrac{V_1 V_2}{X} \cos\delta - \dfrac{V_2^2}{X}$

During normal system operation

$\delta < \sim 15^\circ$

$\therefore \sin\delta \rightarrow \delta \text{ (rad)}$

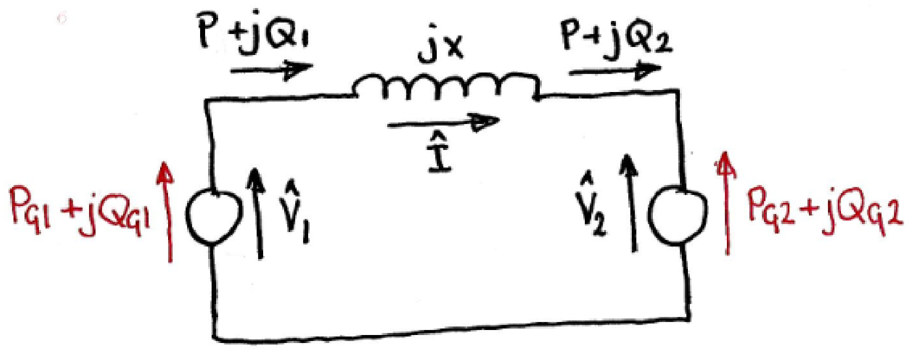
$\cos\delta \rightarrow 1$

$\therefore P_1 = P_2 \simeq \dfrac{V_1 V_2}{X} \cdot \delta$

$Q_1 \simeq \dfrac{V_1 \Delta V}{X}$, $Q_2 \simeq \dfrac{V_2 \Delta V}{X}$

$\Delta V = (V_1 - V_2)$

Control of reactive power flow by adjusting voltages



$P_{G1} + jQ_{G1} = P + jQ_1$ $\quad$ $\underline{P_{G2} + jQ_{G2} = -P - jQ_2}$

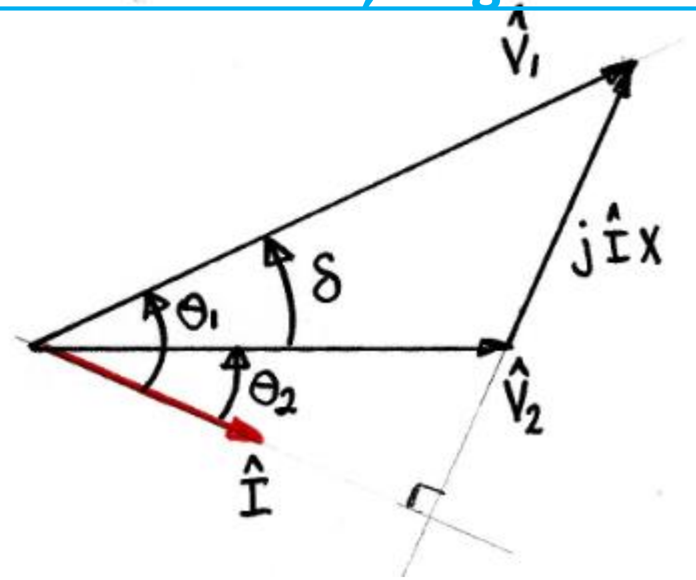
Note that *P + jQ2* is *absorbed* by G2 (i.e. load convention).
G2 *generates* PG2 = -P and QG2 = -Q2

Assume that:

- δ > 0 so power is transmitted from G1 to G2
- V2 magnitude and phase is fixed

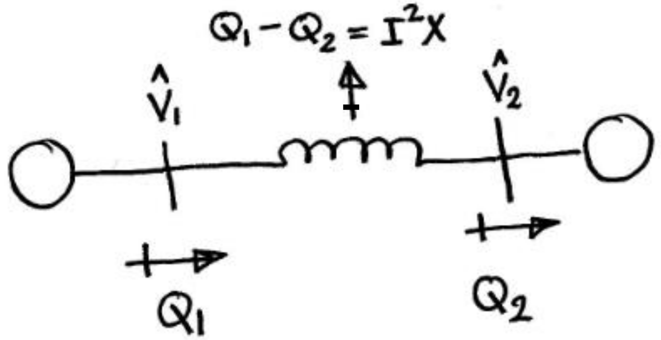
Consider scenarios in which V1 > V2, V1 = V2 and V1 < V2 in terms of phasors

Scenario 1: V1 > V2, I lags V1 and V2



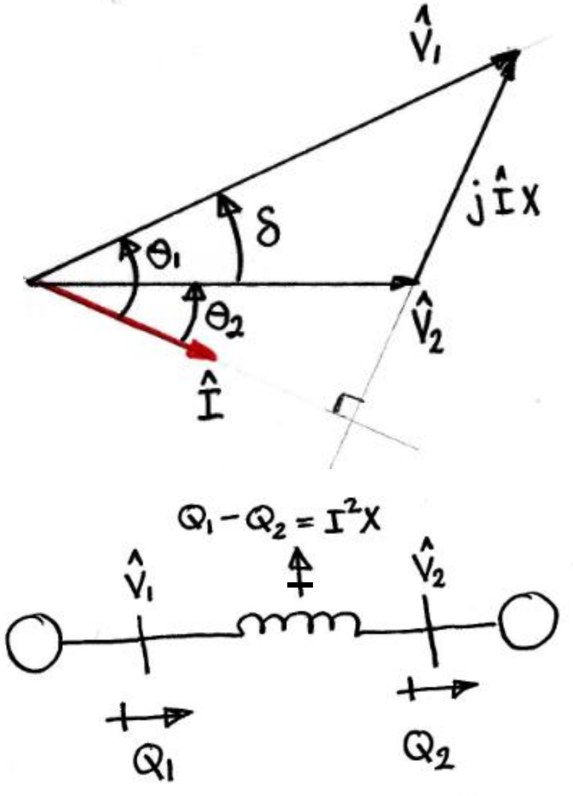
$Q_1 = V_1 I \sin(\theta_1) \qquad Q_2 = V_2 I \sin(\theta_2)$

Since $V_1 > V_2$ and $\theta_1 > \theta_2$ it follows that $Q_1 > Q_2$



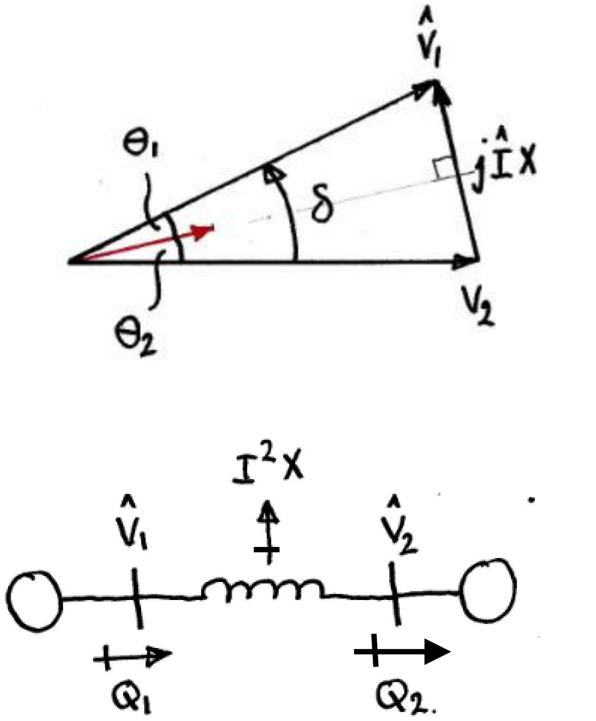
Control of reactive power flow by adjusting voltages

Scenario 1: V1 > V2;
I lags V1 & V2



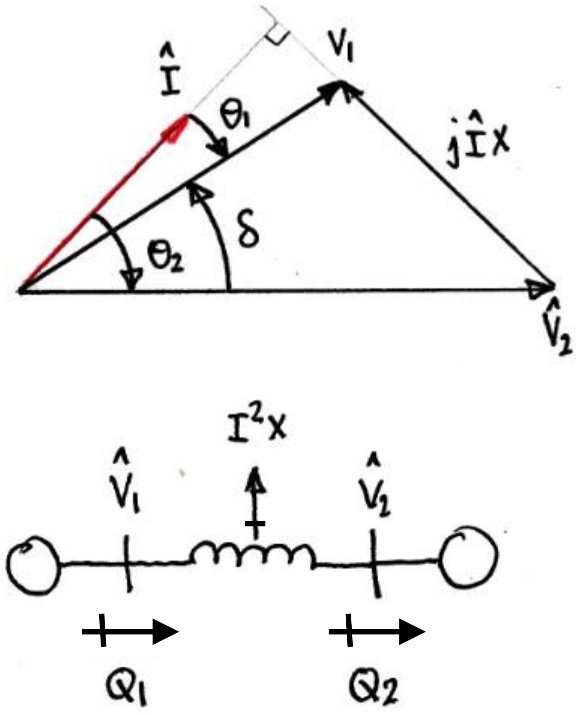
$V_1 > V_2 \quad ; \quad Q_1 > Q_2 > 0$

Scenario 2: V1 = V2; I lags V1 & leads V2
Desirable condition: Reactive losses provided in equal measure by each generator.



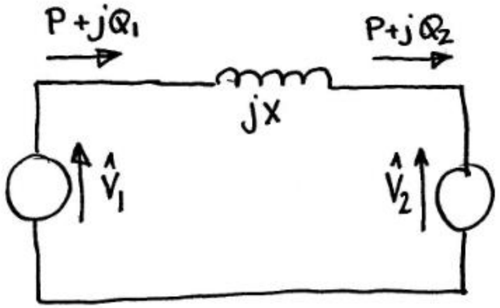
$V_1 = V_2 \quad ; \quad Q_1 = -Q_2 > 0$

Scenario 3: V1 < V2;
I leads V1 & V2



$V_1 < V_2 \quad ; \quad Q_1 < Q_2 < 0$

Reactive power flow and losses: Example



Explore effect of variation of V_1 with $P = P_0 = \text{constant}$, $V_2 = V_{20} = \text{constant}$ on reactive power generation and reactive power losses.

$$P_0 = \frac{V_1 V_{20} \sin\delta}{X} \Rightarrow \sin\delta = \frac{P_0 X}{V_1 V_{20}}$$

$$\cos\delta = \sqrt{1-\sin^2\delta} = \sqrt{1-\left(\frac{P_0 X}{V_1 V_{20}}\right)^2}$$

$$Q_1 = \frac{V_1^2}{X} - \frac{V_1 V_2}{X}\cos\delta$$

$$Q_2 = \frac{V_1 V_2}{X}\cos\delta - \frac{V_2^2}{X}$$

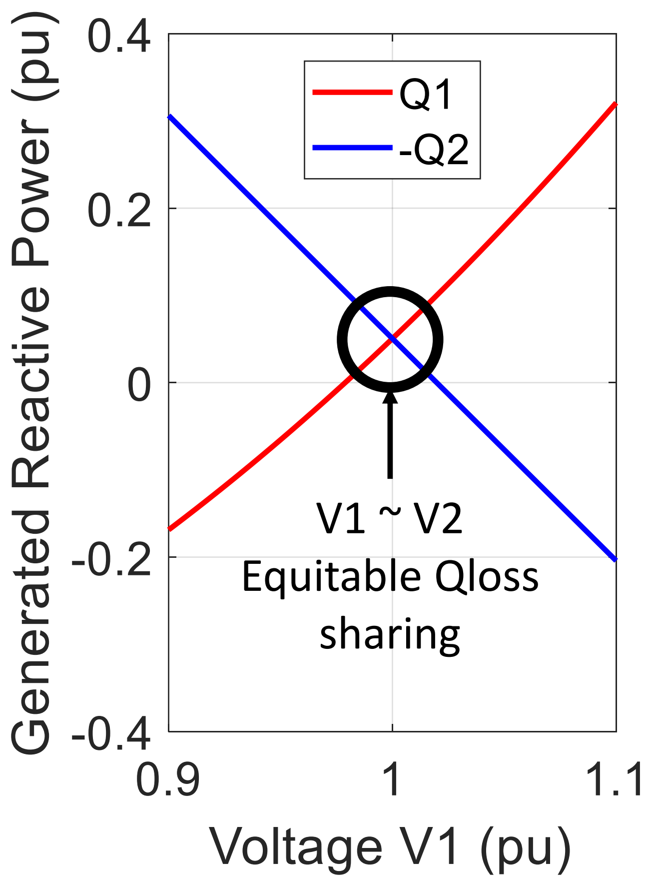
$$Q_{loss} = Q_1 - Q_2$$

Example: X = 0.4 pu, P = 0.5 pu, V2 = 1.0 pu

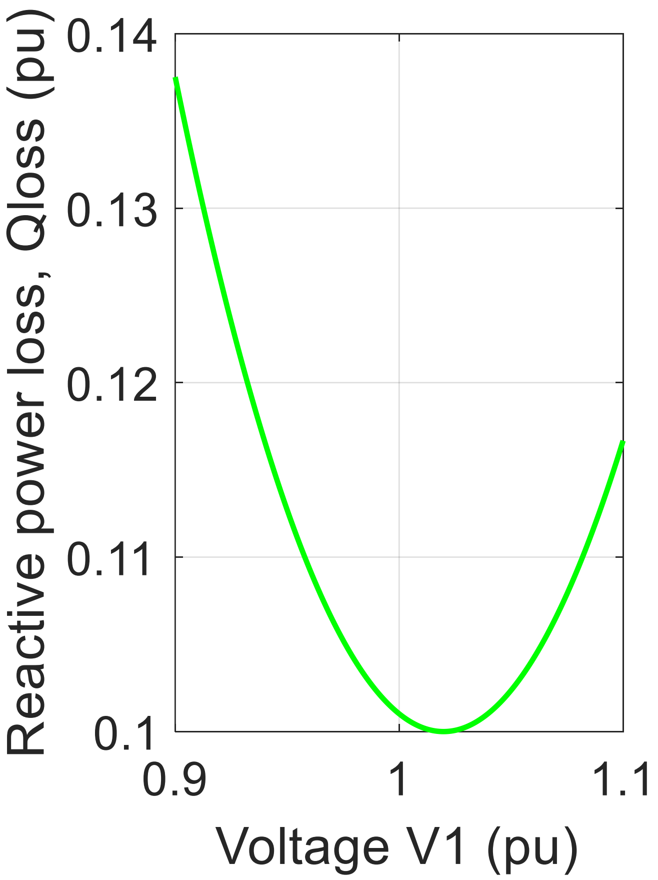
Q1 = -Q2 when V1 = V2 = 1.0 pu.

When V1 < V2, Q1 < (-Q2)

When V1 > V2, Q1 > (-Q2)



Minimum Qloss: V1 ~ V2



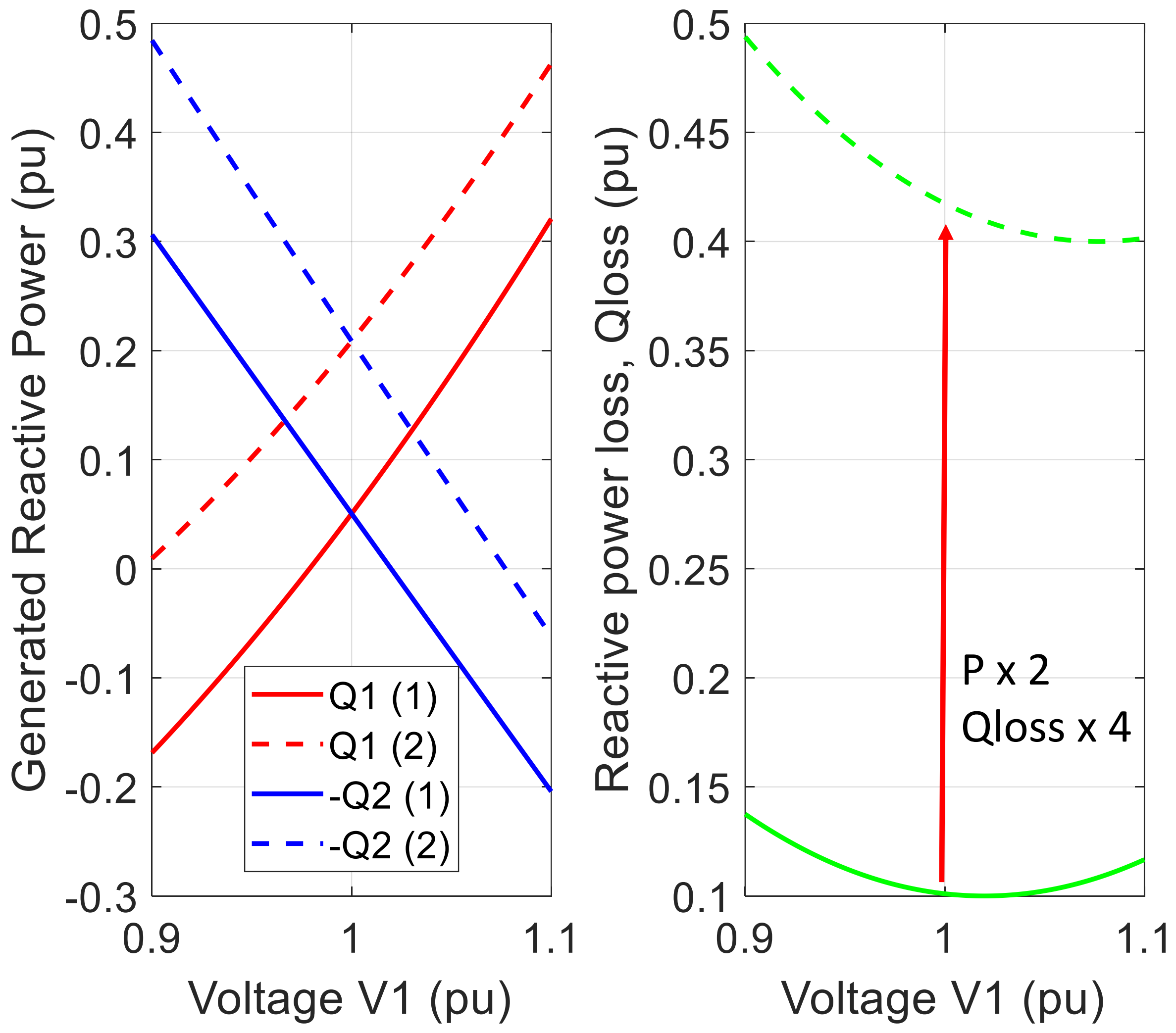
Reactive Power Losses in Reactance: Example

Example: X = 0.4 pu, V2 = 1.0 pu
Compare reactive generation and losses for two levels of power transfer:
P(1) = 0.5 pu (solid lines), P(2) = 1.0 pu (dashed lines)

Qloss = I^2X

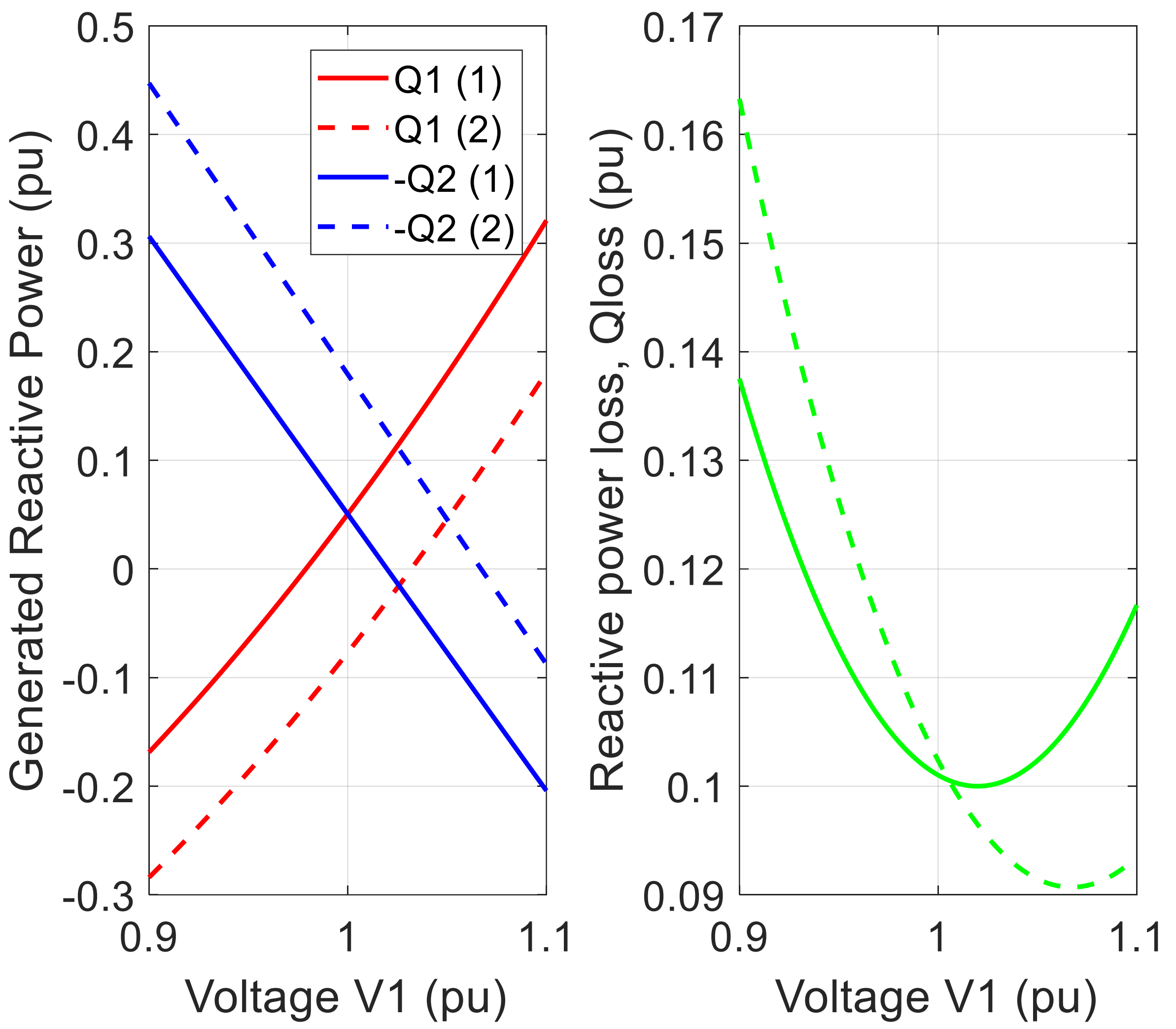
- double current quadruple Qloss!

The above observation suggests an important operating principle …



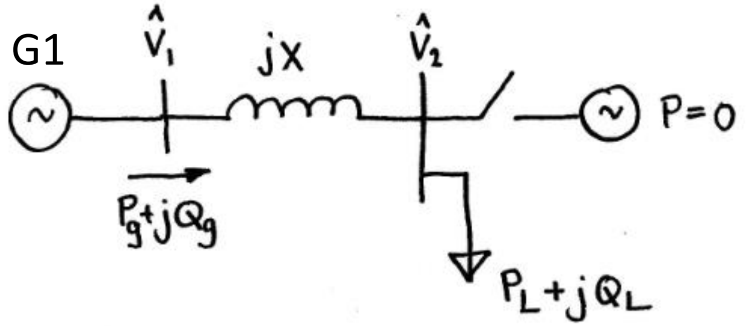
Reactive Power Losses in Reactance: Example

<u>Example: X = 0.4 pu, P = 0.5 pu</u>
Compare reactive generation and losses for V2(1) = 1.0 (solid) and V2(2) = 1.05 pu (dashed)



<u>Operate with highest possible voltages to minimize current and hence reactive (& real) losses.</u>

Supplying a load through a radial line from a generator



Switch open – Uncompensated load

G1 regulates terminal voltage to V_1 &
Power output to $P_g = P_L$
What is Q_g, V_2?

$$P_L = \frac{V_1 V_2}{X} \sin\delta \quad , \quad Q_L = \frac{V_1 V_2}{X} \cos\delta - \frac{V_2^2}{X}$$

δ = transmission angle = $\angle\hat{V}_1 - \angle\hat{V}_2$

$$P_L^2 + \left(Q_L + \frac{V_2^2}{X}\right)^2 = \left(\frac{V_1 V_2}{X}\right)^2$$

Use: $\sin^2\delta + \cos^2\delta = 1$

$$\therefore \; V_2^4 - (V_1^2 - 2Q_L X) V_2^2 + (P_L^2 + Q_L^2) X^2 = 0$$

Solve for V_2^2

$$V_2^2 = \frac{b \pm \sqrt{b^2 - 4c}}{2}$$

where $b = V_1^2 - 2Q_L X$, $c = (P_L^2 + Q_L^2) X^2$

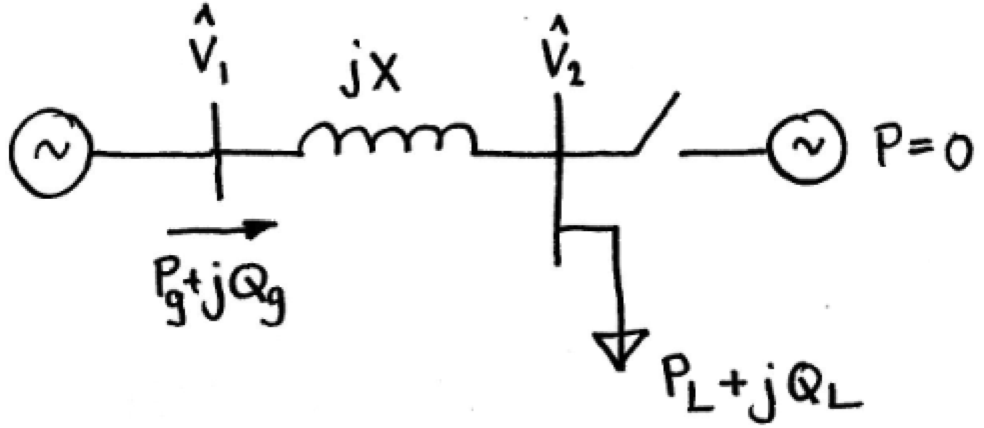
$$V_2 = \sqrt{\frac{b \pm \sqrt{b^2 - 4c}}{2}}$$

Require $b^2 \geq 4c$ otherwise load cannot be supplied. That is:

$$-P_L^2 - \frac{V_1^2 Q_L}{X} + \left(\frac{V_1^2}{2X}\right)^2 \geq 0$$

Note that if $b^2 > 4c$ there are two solutions for the load bus voltage.
If $b^2 = 4c$ there is a single solution.

Example: Maximum uncompensated demand supplied through a radial line



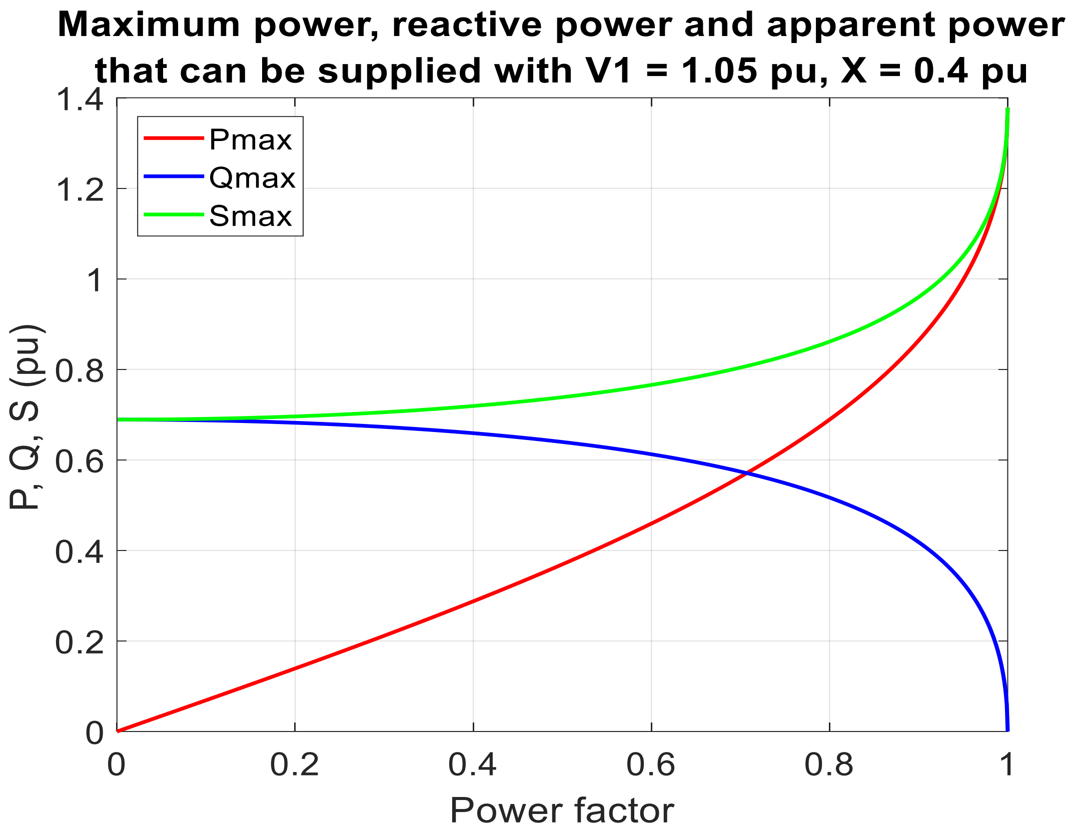
Find maximum load, PL + jQL, that can be supplied from a generator through a radial line. Assume that:

$$Q_L = P_L \tan(\varphi) \text{ where } \varphi = \arccos(\text{pf})$$

Using $b^2 - 4c = 0$ from the previous slide obtain following equation for determining Pmax.

$$P_{\max}^2 + \left(\frac{V_1^2 \tan(\varphi)}{X}\right) P_{\max} - \left(\frac{V_1^2}{2X}\right) = 0$$

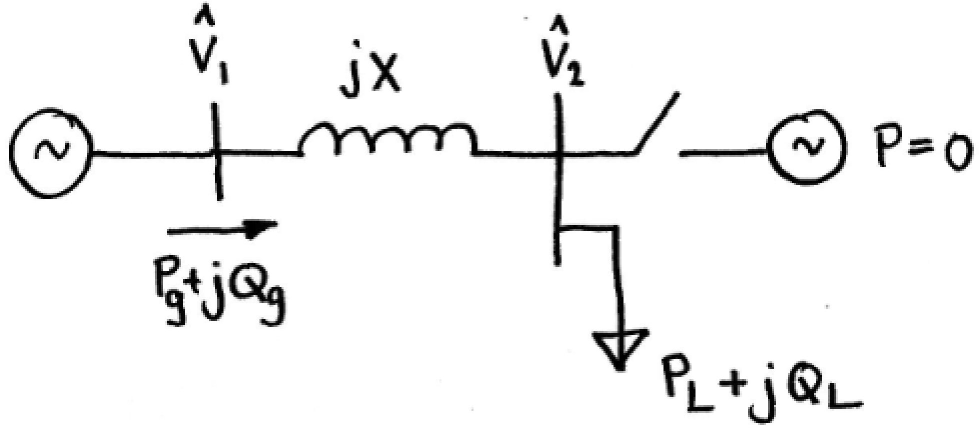
Example parameters: V1 = 1.05 pu, X = 0.4 pu used in solution of Pmax with lagging power factors (pf) from 0 to 1.



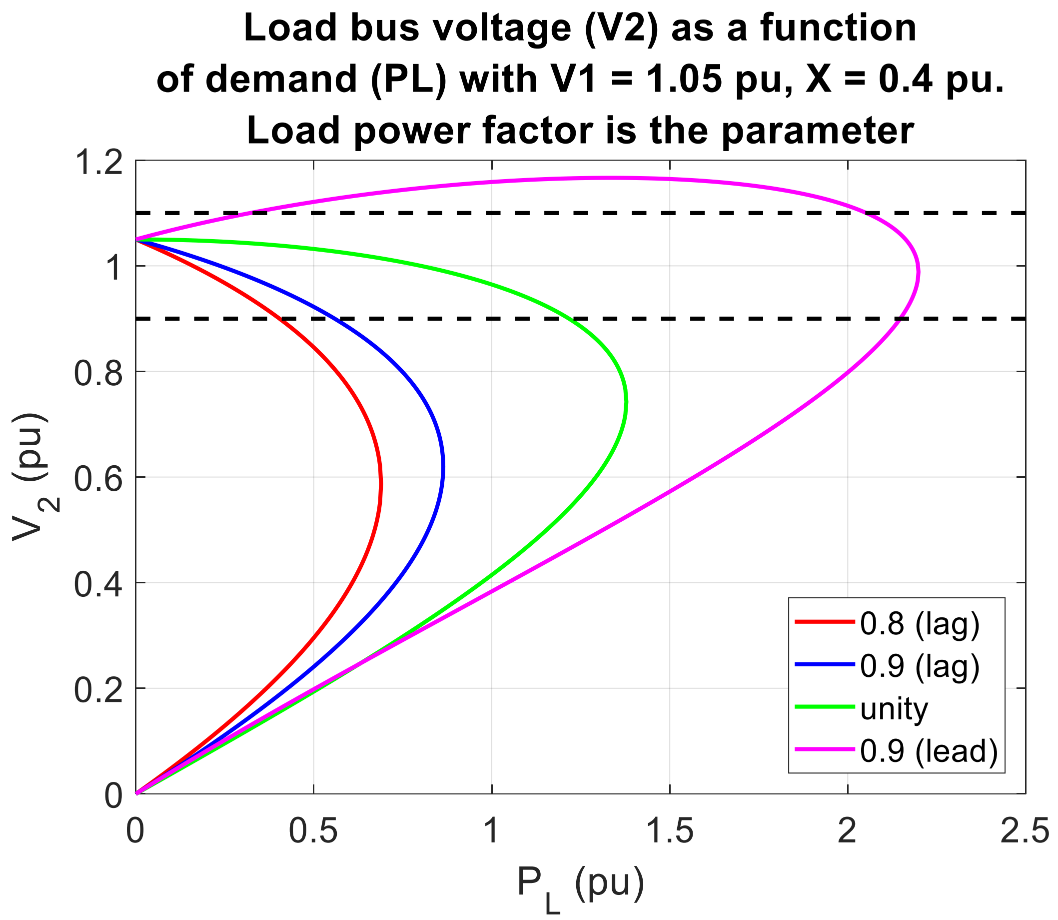
Note: With pf = 0, (i.e. P = 0) $Q_{\max} = \left(\frac{V_1^2}{4X}\right) = 0.69$ pu

and pf =1 (i.e. Q = 0) $P_{\max} = \left(\frac{V_1^2}{2X}\right) = 1.38$ pu

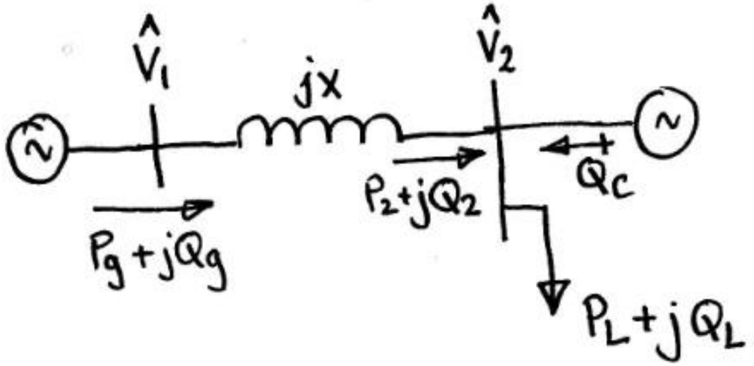
Example: PV characteristics for radially fed uncompensated load



- PV curves calculated for uncompensated load, PL + jQL supplied from a generator through a radial line.
- The parameters in the previous example are used and the formula for V1 derived in slide 11 is used to compute P(V).
- For each curve the load power factor is maintained constant as PL is increased.
- Curves are plotted for four power factors.



Example: Find Q compensation to achieve specified load voltage



$$Q_2 + Q_c = Q_L$$

$$\therefore Q_c = Q_L - Q_2$$

$$Q_2 = \frac{V_1 V_2}{X}\cos\delta - \frac{V_2^2}{X}$$

$$\therefore Q_c = Q_L - \frac{V_1 V_2}{X}\cos\delta + \frac{V_2^2}{X} \qquad (1)$$

$$P_L = \frac{V_1 V_2}{X}\sin\delta \Rightarrow \sin\delta = \frac{P_L X}{V_1 V_2}$$

$$\therefore \cos\delta = \sqrt{1 - \left(\frac{P_L X}{V_1 V_2}\right)^2} \qquad (2)$$

Substitute (2) in (1)

$$Q_c = Q_L - \frac{V_1 V_2}{X}\sqrt{1 - \left(\frac{P_L X}{V_1 V_2}\right)^2} + \frac{V_2^2}{X}$$

Example: $V_1 = 1{\cdot}05$ pu, $X = 0{\cdot}4$ pu

$P_L = 1{\cdot}5$ pu @ 0·9 pf lag

Require $V_2 = 1{\cdot}05$ pu.

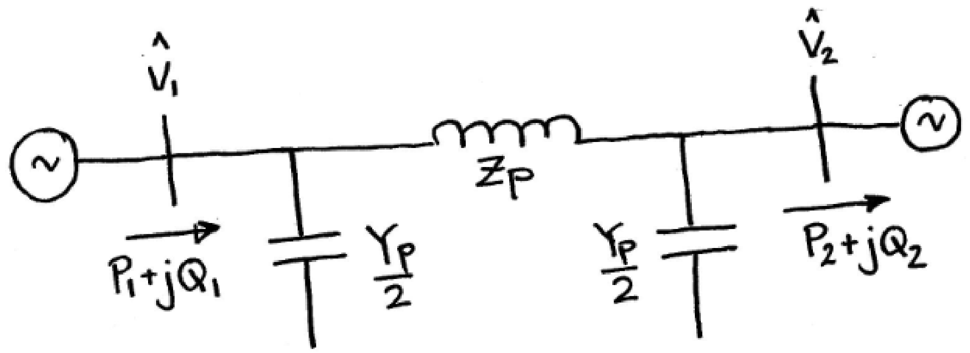
$$Q_L = 1{\cdot}5 \times \tan(\text{acos}(0{\cdot}9)) = 0{\cdot}7265 \text{ pu}$$

$$\therefore Q_c = 0{\cdot}7265 - \frac{1{\cdot}05^2}{0{\cdot}4}\sqrt{1 - \left(\frac{1{\cdot}5 \times 0{\cdot}4}{(1{\cdot}05)^2}\right)^2} + \frac{1{\cdot}05^2}{0{\cdot}4}$$

$$Q_c = 1{\cdot}1704 \text{ pu}$$

Effect of line charging on reactive power flows

- Medium – long HV transmission lines have significant shunt capacitance
- Lightly loaded lines (relative to SIL) tend to be net generators of reactive power
- Heavily loaded lines (relative to SIL) tend to be net consumers of reactive power
- Terminals of such lines must have the capability to both **generate** and **absorb** reactive power depending on line loading.
- Radially fed load may require inductive compensation under light load conditions.



Example: 200 km line, 330 kV, 50 Hz

$X_L = 0{\cdot}306\ \Omega/ph/km$, $B_c = 3{\cdot}764 \times 10^{-6}\ \mho/ph/km$

Assume loss less line

$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{X_L}{B_c}} = \sqrt{\frac{0{\cdot}306}{3{\cdot}764\times10^{-6}}} = 285{\cdot}1\ \Omega$$

$$\therefore\ SIL = \frac{V_0^2}{Z_c} = \frac{330^2}{285{\cdot}1} = 382\ MW$$

$$Z_P = jX_L \times 200\,km = j0{\cdot}306 \times 200 = 61{\cdot}2\ \Omega$$

$$\frac{Y_P}{2} = \frac{jB_c \times 200\,km}{2} = j\,3{\cdot}764\times10^{-6} \times 100 = 3{\cdot}764\times10^{-4}\ \mho$$

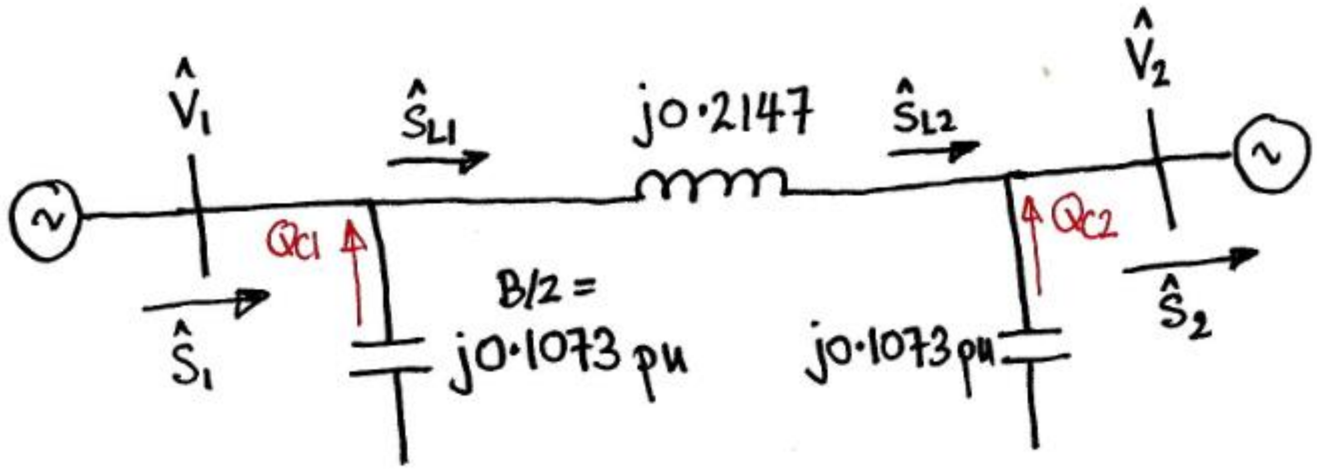
Effect of line charging on reactive power flows

Use SIL as MVA base and $V_{base} = 330$ kV (l-l, rms)

$$Z_{base} = \frac{330^2}{382} = 285.08 \ \Omega/\text{ph.}$$

$$Z_p = j\frac{61.2}{285.08} = j0.2147 \text{ pu}$$

$$\frac{Y_p}{2} = j3.764 \times 10^{-4} \times Z_{base} = j0.1073 \text{ pu}$$



- Series reactance X = 0.2147 pu absorbs reactive power
- Shunt susceptance B/2 = 0.1073 at each end of the line generates reactive power ($Q_C = (B/2)*V^2$)
- Voltages are near 1.0 pu. Therefore reactive power generation (Q_C) by line approximately $B_C/2$ pu at each end.
- Series reactive power consumption I^2X varies depending on loading.

Effect of line charging on reactive power flows

Suppose $P_2 = 100\text{ MW} = \frac{100}{382} = 0.2618$ pu

and $\hat{V}_1 = 1.05\angle\delta$, $\hat{V}_2 = 1.05\angle 0$ pu.

Find Q_1 and Q_2 delivered by the respective generators.

$$P = \frac{V_1 V_2}{X}\sin\delta \Rightarrow \delta = \text{asin}\left(\frac{PX}{V_1 V_2}\right) =$$

$$= \text{asin}\left(\frac{0.2618 \times 0.2147}{1.05 \times 1.05}\right)$$

$$\underline{\delta = 2.92^\circ}$$

$$Q_{L1} = \frac{V_1^2}{X} - \frac{V_1 V_2}{X}\cos\delta$$

$$= \frac{1.05^2}{0.2147} - \frac{1.05 \times 1.05}{0.2147}\cdot\cos(2.92^\circ)$$

$$= 0.0067 \text{ pu}$$

$$Q_{L2} = \frac{V_1 V_2}{X}\cos\delta - \frac{V_2^2}{X} = -0.0067 \text{ pu}$$

$$Q_{C1} = \frac{B}{2}V_1^2 = 0.1073 \times 1.05^2$$

$$= 0.1183 \text{ pu}$$

$$Q_{C2} = \frac{B}{2}\cdot V_2^2 = 0.1183 \text{ pu}$$

$$Q_1 = Q_{L1} - Q_{C1}$$

$$= 0.0067 - 0.1183$$

$$Q_1 = -0.1116 \text{ pu}$$

$$Q_2 = Q_{L2} + Q_{C2}$$

$$Q_2 = +0.1116 \text{ pu}$$

Both generators must absorb line charging. There is negligible Q_{loss} in the line reactance

Effect of line charging on reactive power flows

For high line flow (>SIL) of

$P_2 = 500\ \text{MW} = 500/382 = 1{\cdot}309\ \text{pu},$

$\hat{V}_1 = 1{\cdot}05\angle\delta$ and $\hat{V}_2 = 1{\cdot}05\angle 0$ pu.

$Q_{L1} = 0{\cdot}1696$ pu $\quad Q_{C1} = 0{\cdot}1183$ pu

$Q_{L2} = -0{\cdot}1696$ pu $\quad Q_{C2} = 0{\cdot}1183$ pu

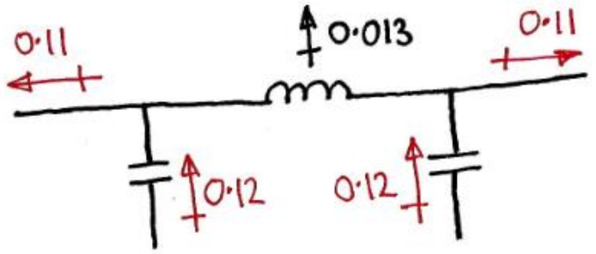
$Q_1 = Q_{L1} - Q_{C1} = +0{\cdot}0513$ pu

$Q_2 = Q_{L2} + Q_{C2} = -0{\cdot}0513$ pu

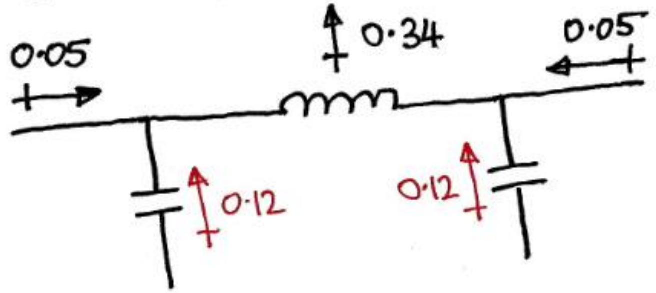
Both generators must supply series reactive losses that exceed line charging

Reactive Power Audit

Light load case $P = 0{\cdot}26$ pu of SIL



High load case $P = 1{\cdot}309$ pu of SIL



Capacitive

Inductive

Sources and sinks of reactive power

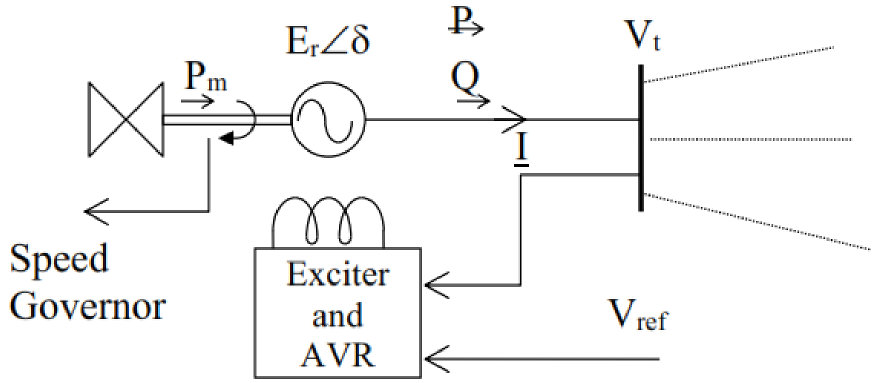
- Continuous control
  - Synchronous generators
  - Synchronous compensators (or condensors)
  - Static VAR compensators (SVCs)
  - Wind / solar PV farm centralized voltage control systems
  - Q output automatically adjusted to maintain bus voltages at specified setpoint

- Discontinuous control
  - Fixed and switched capacitors and reactors
  - Series capacitors
  - Regulating transformers (on-load tap-changing – OLTC – transformers)

- Overview some of these technologies …

Sources and sinks of reactive power

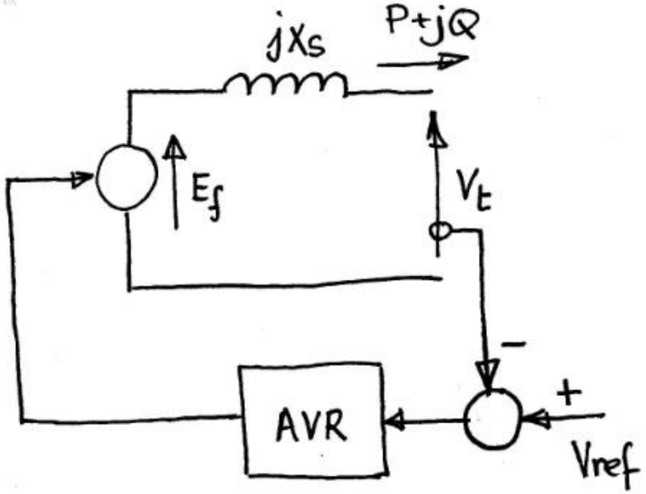
Synchronous Generators



- Turbine adjusts Pm to keep P = Pref (constant)
- AVR adjusts field voltage to keep Vt = Vref (constant)
- Q > 0 if external voltage < Vref
- Q < 0 if external voltage > Vref
- Must operate within generator PQ capability
    - Turbine power limit
    - Stator & field current limits
    - Stability and end winding heating limits

Synchronous Compensators

- Synchronous motor with no load (P = 0)
- Fitted with AVR
- Used to regulate voltage similarly to synchronous generators.



$V_t > V_{ref}$ then decrease E_f

$\Rightarrow$ decrease Q

$V_t < V_{ref}$ then increase E_f

$\Rightarrow$ increase Q

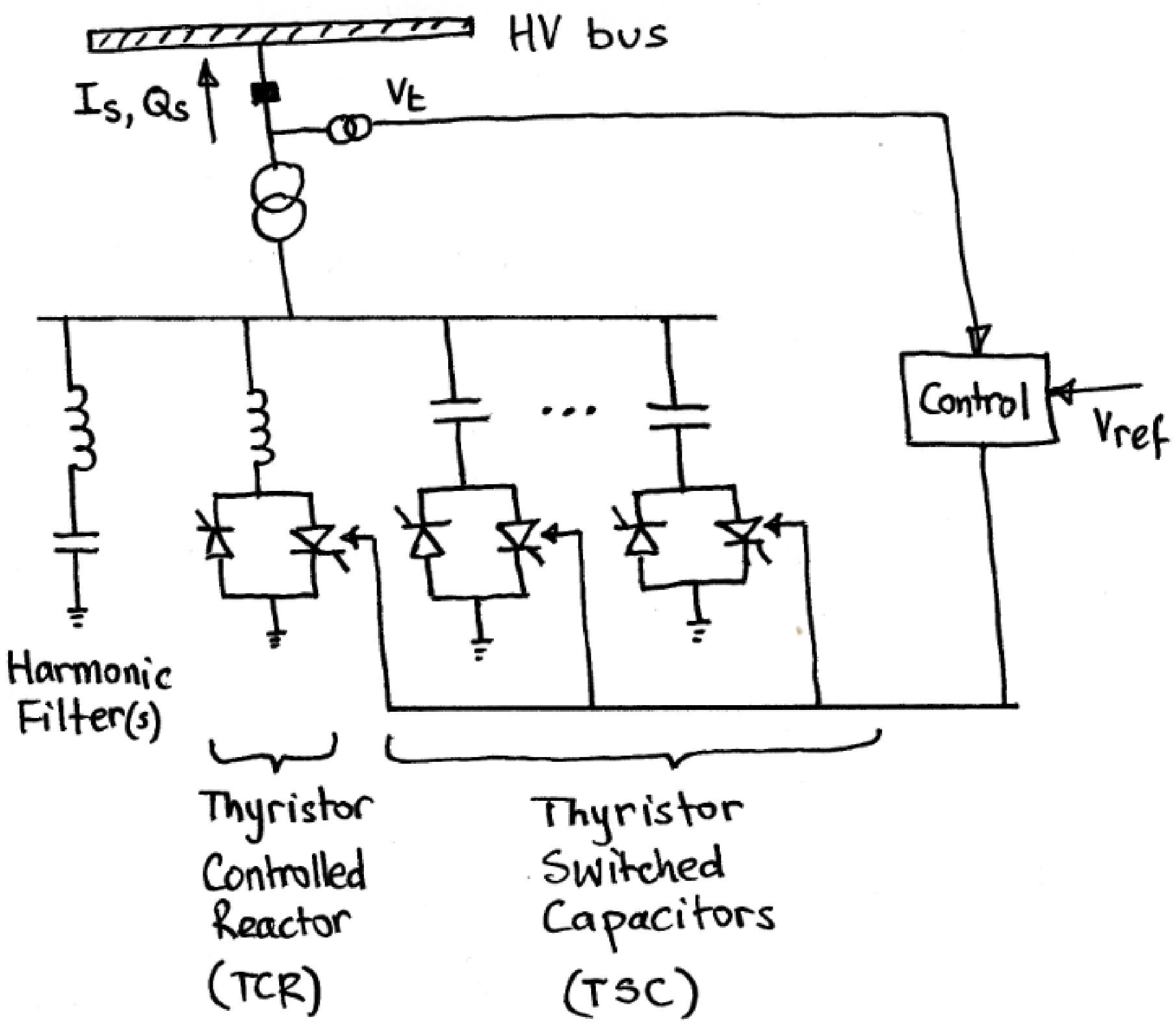
In steady-state control is

$E_f = K(V_{ref} - V_t)$

Sources and sinks of reactive power

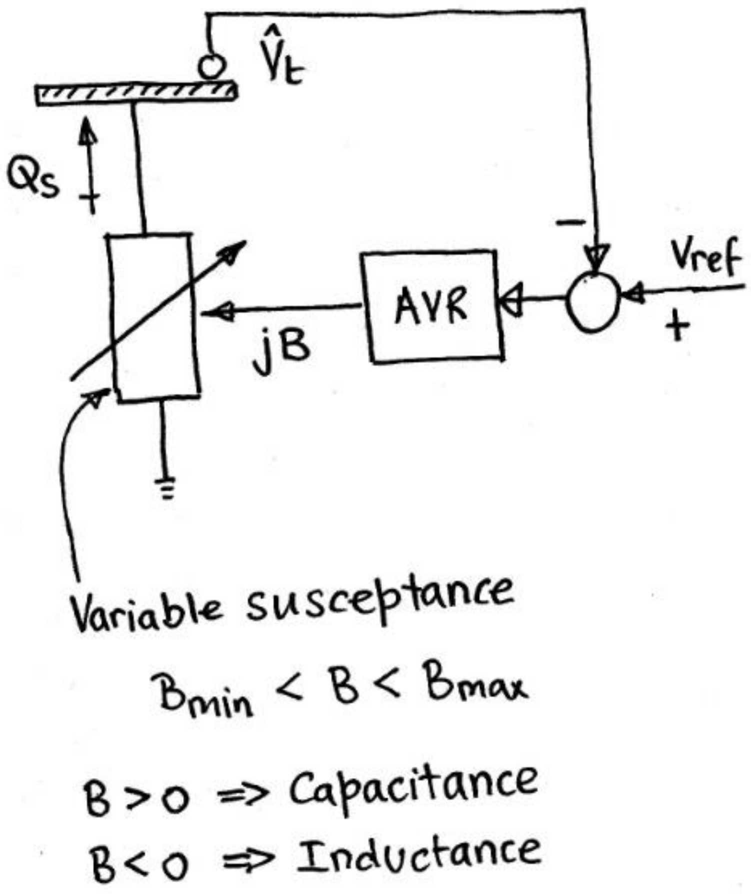
Static VAR Compensators (SVCs)

- Typical SVC structure depicted
- Controllable susceptance used to regulate HV bus voltage to specified set point Vref
- TCR provides for continuous adjustment of susceptance within its range
- TSCs provide means of providing leading (capacitive) reactive power (i.e. ability to boost voltage)



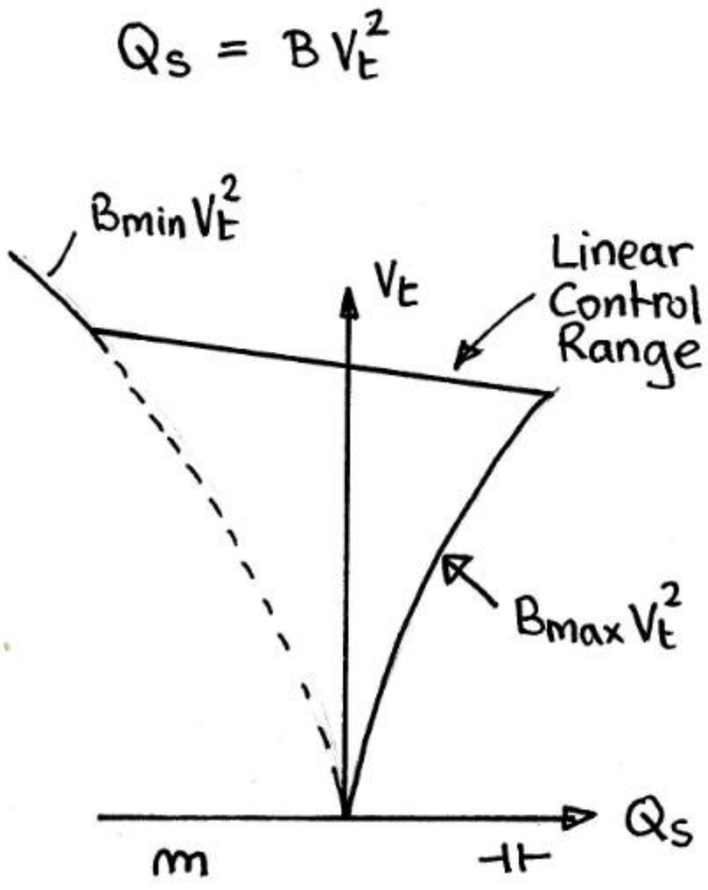
Sources and sinks of reactive power

Static VAR Compensators (SVCs)

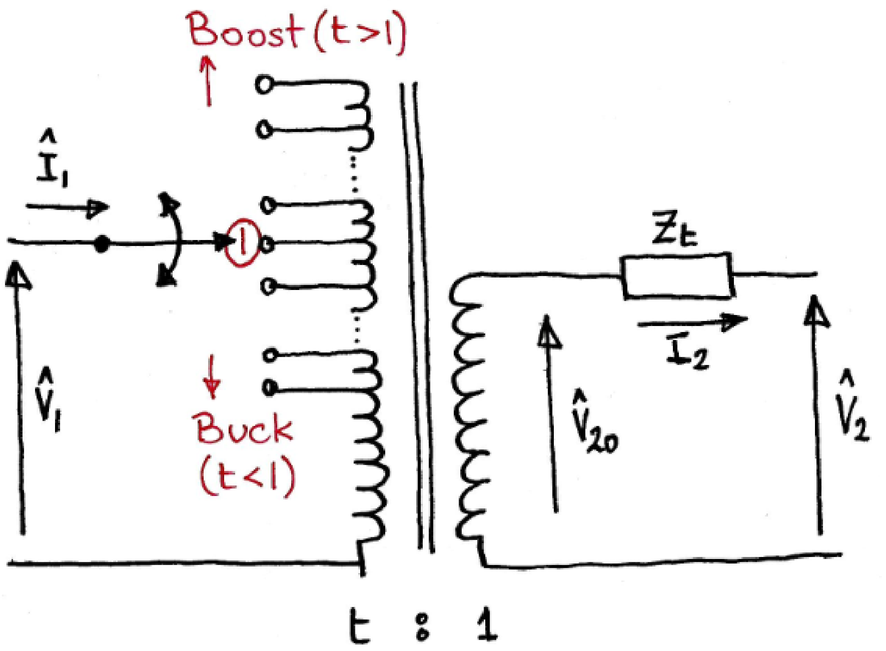


Overall SVC control schematic (left) and steady-state control characteristic (right).

Note that reactive support from SVC decreases quadratically with voltage once upper capacitive susceptance limit is reached.



Regulating transformer – Controlling reactive power flow



$$\hat{V}_{20} = \left(\frac{1}{t}\right)\cdot \hat{V}_1$$

$$\hat{I}_2 = t \cdot \hat{I}_1$$

$$\hat{V}_2 = \hat{V}_{20} - Z_t \hat{I}_2 = \frac{1}{t}\hat{V}_1 - Z_t (t \cdot \hat{I}_1)$$

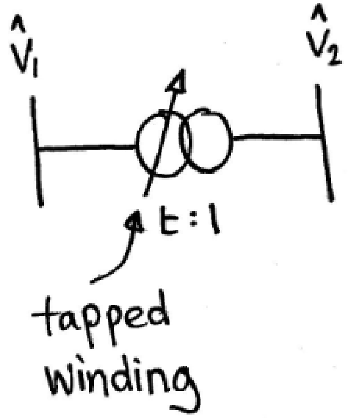
$$\therefore \hat{V}_1 = t\hat{V}_2 + t^2 Z_t \hat{I}_1$$

Voltage V_1 is boosted w.r.t. that of V_2 if $t > 1$ and conversely it is reduced if $t < 1$

$t = 1$ unity tap position
Corresponds to nominal voltage ratio

Z_t Per-unit transformer impedance corresponding to unity tap.

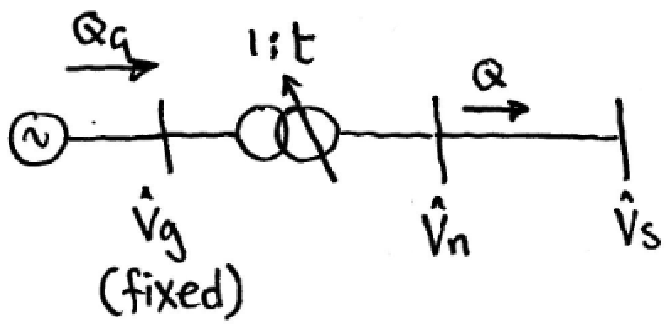
$t_{min} < t < t_{max}$



Circuit Symbol

Regulating transformer -- Controlling reactive power flow

Application – Adjust reactive power flow from generator



Assume operating at $t = 1$ with V_g fixed due to AVR action.

Suppose t increased (by say +0·01 pu)

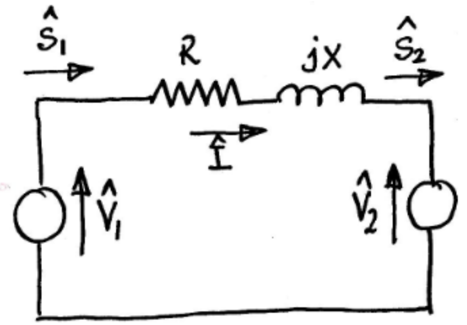
V_n will increase w.r.t. V_s.

$\Rightarrow$ Q will increase and Q_G will increase

Raise taps to increase Q output from generator

Lower taps to reduce Q output from generator

Geometry of the Powerflow Equations - Power Circles



$$Z = R + jX = |Z|e^{j\beta}, \quad \hat{V}_1 = V_1 e^{j\theta_1}, \quad \hat{V}_2 = V_2 e^{j\theta_2}$$

$$\hat{S}_1 = \hat{V}_1 \hat{I}^* = \hat{V}_1 \left(\frac{\hat{V}_1 - \hat{V}_2}{Z}\right)^*$$

$$\boxed{\hat{S}_1 = \frac{V_1^2}{|Z|} e^{j\beta} - \frac{V_1 V_2}{|Z|} e^{j\beta} \cdot e^{j\delta}, \quad \delta = \theta_1 - \theta_2}$$

$$\boxed{\hat{S}_2 = \frac{V_1 V_2}{|Z|} e^{j\beta} e^{j\delta} - \frac{V_2^2}{|Z|} e^{j\beta}}$$

Define $C_1 = \frac{V_1^2}{|Z|} e^{j\beta}$, $r = \frac{V_1 V_2}{|Z|} e^{j\beta}$, $C_2 = -\frac{V_2^2}{|Z|} e^{j\beta}$

$$\left.\begin{aligned} \hat{S}_1 &= C_1 - r e^{j\delta} \\ \hat{S}_2 &= C_2 + r e^{j\delta} \end{aligned}\right\} \text{Complex Power Circles}$$

Constructed for extreme case:
VS = 1.1 pu, VR = 0.9 pu