THE UNIVERSITY of ADELAIDE

SUB CRUCE LUMEN

Course:
ELEC ENG 3110 Electric Power Systems
ELEC ENG 7074 Power Systems PG
(Semester 2, 2021)
Powerflow Analysis (Part 1)

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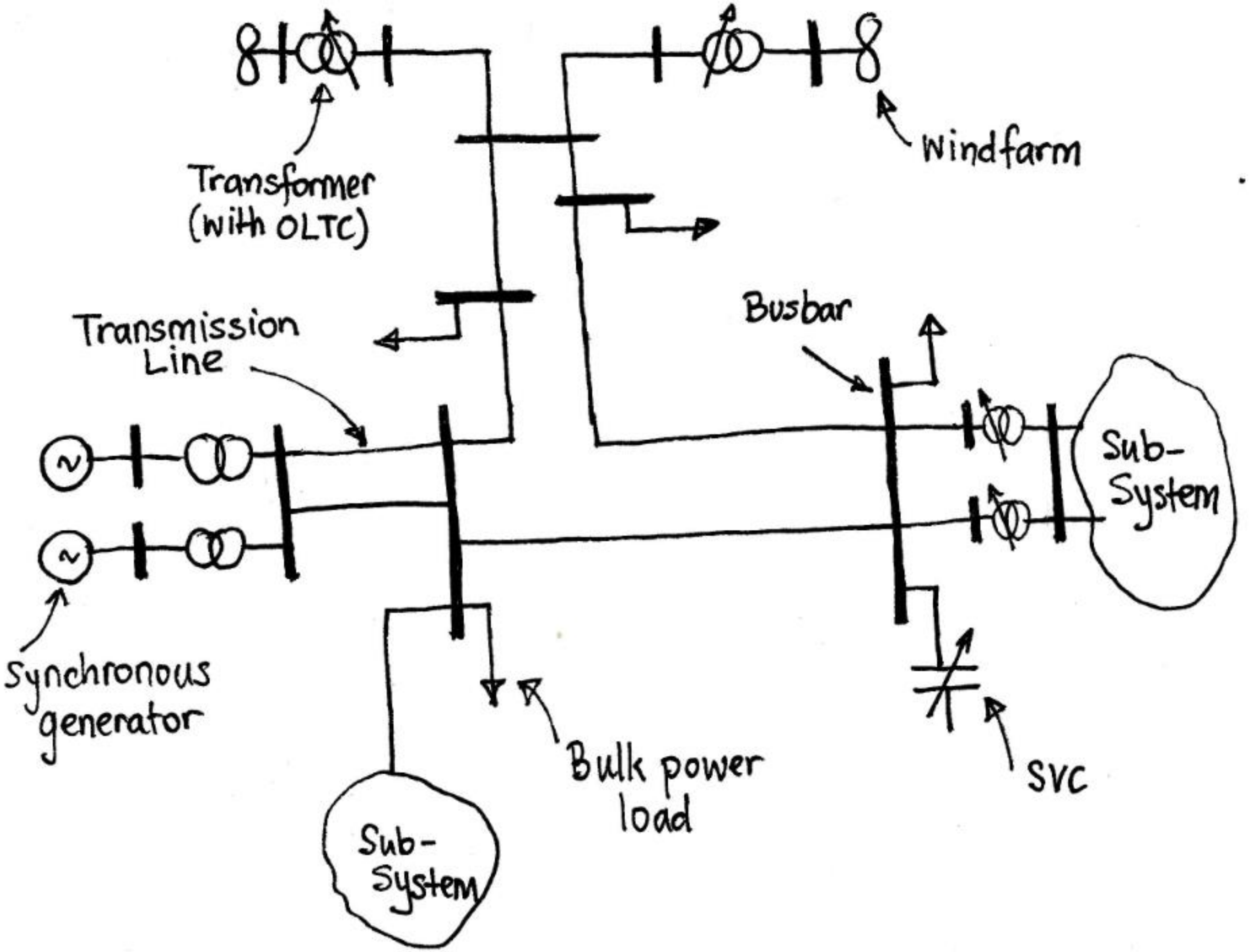
Power-flow Analysis

- **Background**
  - **Studied steady-state behaviour and representation main components. For example:**
    - **Synchronous generators,**
    - **Static VAR Compensators,**
    - **Transmission lines,**
    - **Transformers**
  - **Studied simple radial networks to reveal important concepts**
  - **Identified key voltage control concepts and strategies for managing system voltages**
  - **Require power-flow analysis for steady-state analysis of large interconnected power-systems.**
    - **Power-flow analysis also called load-flow analysis**
  - **Steady-state power-flow analysis concerns:**
    - **calculating the network voltages and power-flows for specified terminal conditions**
    - **analysing and assessing the results for a range of reasons (see later).**

Typical subsection of an interconnected transmission system

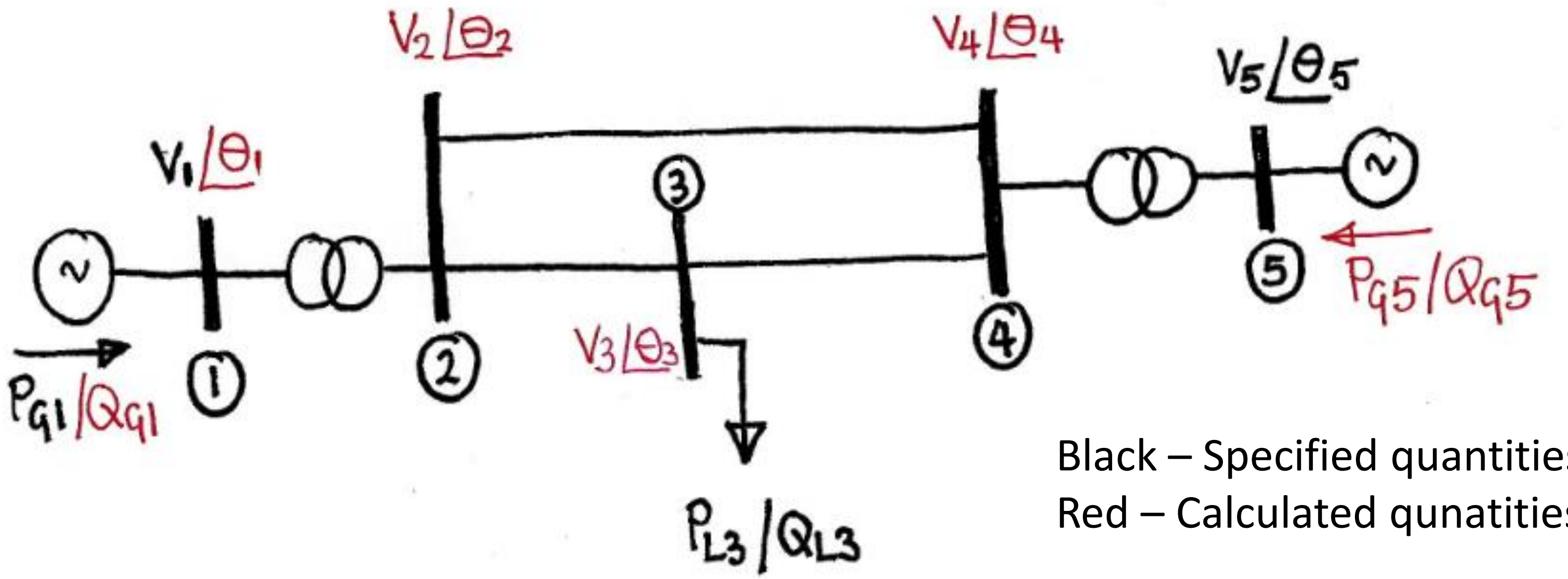
- One-line diagram of a subsection of an interconnected transmission system
- Busbars represent network node
  - voltage at the terminal of all connected elements are equal;
  - the sum of all currents entering a node from the elements connected to it are zero)
- A node typically represents a substation.
  - Details of the switching arrangements with substations are not shown



Why is power-flow analysis necessary?

1) P, Q, and I flows in lines and transformers
   - are their current ratings being exceeded?
2) Voltages at substations and load busbars
   - are they within required limits?
3) P & Q loadings on generators
   - Is the operating point within a desirable region of the operating chart of the machine?
4) Effectiveness of voltage control devices
   - Are synchronous compensators, SVCs, etc. operating within their ratings?
5) Effectiveness of the distribution of switched reactive sources?
   - Would connecting / disconnecting a reactive source improve the voltage profile?
6) Effectiveness of transformer tap-positions
   - Would adjustment of tap-position improve the voltage profile.
7) Effect of outage of lines or generation under steady-state conditions (Review (1) to (5) above).
8) Minimisation of system real and reactive power losses.

Classification of network nodes



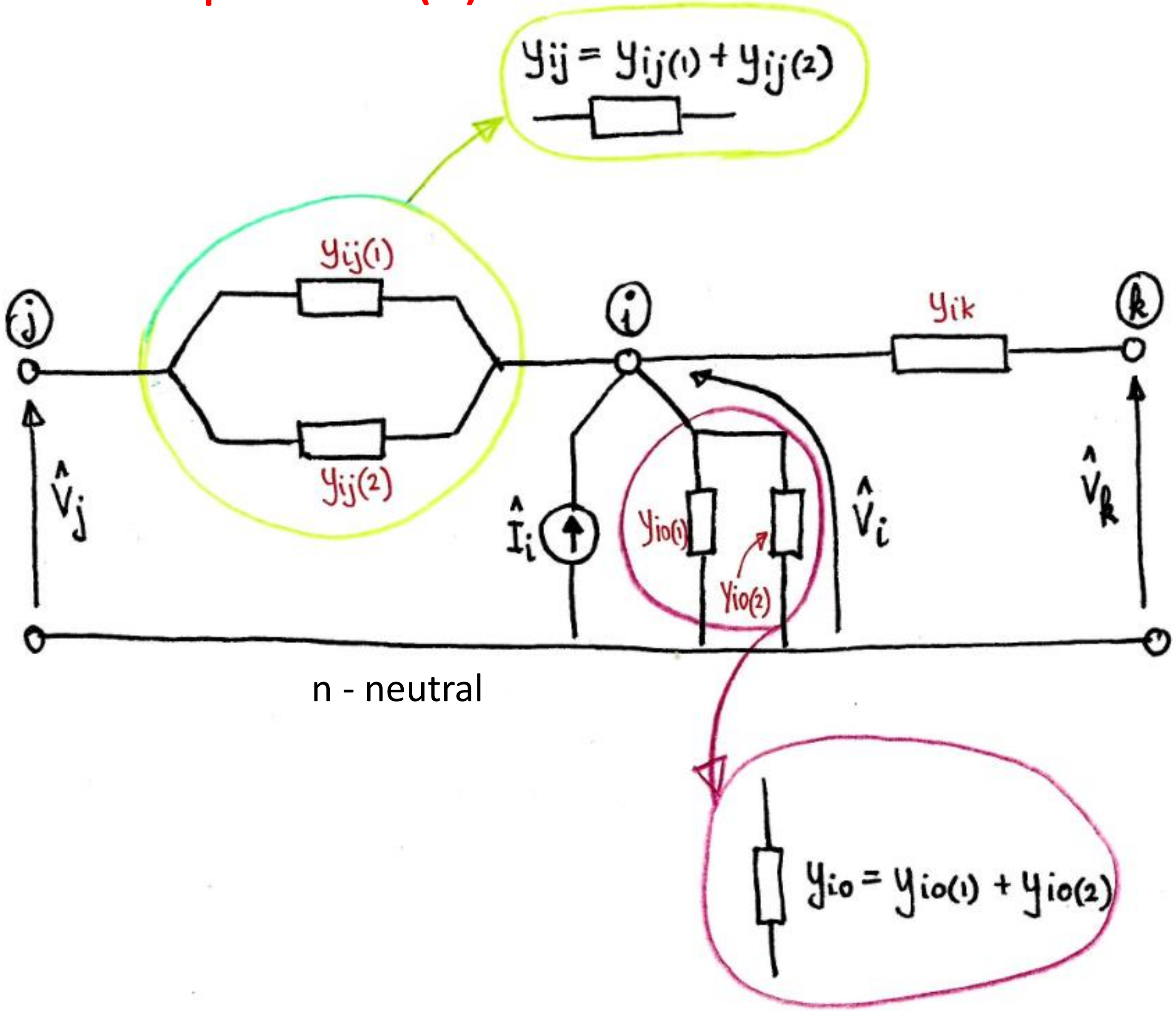
Bus	Type
1	PV
2	PQ
3	PQ
4	PQ
5	VA

Black – Specified quantities
Red – Calculated qunatities

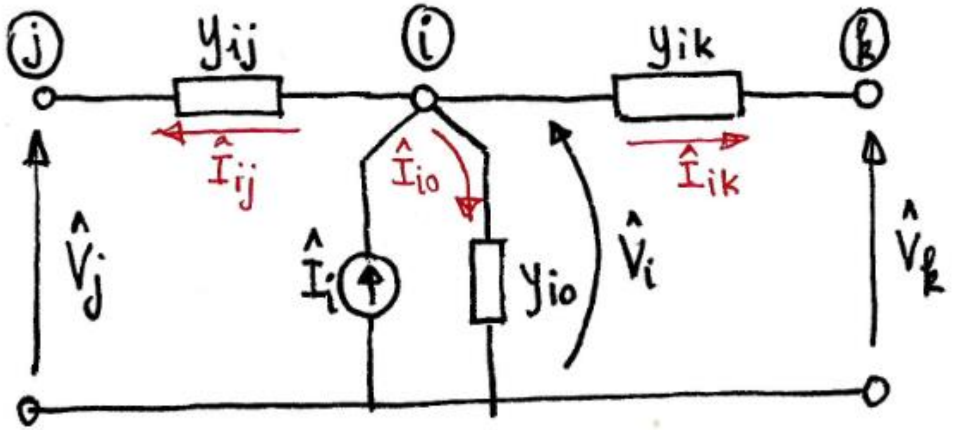
- At each node four quantities are required to completely define the steady-state solution of the network:
  - Power and reactive power injected into the node (i.e. P & Q)
    - For nodes such as 2 & 4 to which no source or load is connected P = Q = 0
  - Voltage magnitude and angle (i.e. V & θ)
- Three bus types are specified (in this introductory course):
  - Voltage-controlled (PV) bus – Power and voltage magnitude (i.e. P & V) are specified, Q & θ to be calculated
  - Load (PQ) bus – Power and reactive power (i.e. P & Q) are specified, V & θ are to be calculated
  - Slack (VA) bus – Voltage and voltage angle (i.e. V & θ) are specified, P & Q are to be calculated (Only one such bus is permitted in each synchronous region. It is required because system losses are unknown a priori.)

Nodal Current Equations (1)

- Our power-flow analysis is restricted to balanced three-phase networks. Therefore per-phase analysis is applicable.
- A per-unit representation of the network is employed.
- Consider a node ***i*** in the per-phase representation of the network. It is connected by admittances to adjacent nodes ***j*** and ***k***.
- A current source injects current into node ***i*** as shown.
- Parallel admittances are added to yield single admittance connecting between adjacent nodes.
- Following this simplification we have …



Nodal Current Equations (2)



Apply KCL to node i

$$\hat{I}_i = \hat{I}_{io} + \hat{I}_{ij} + \hat{I}_{ik}$$

$$= y_{io}\hat{V}_i + y_{ij}(\hat{V}_i - \hat{V}_j) + y_{ik}(\hat{V}_i - \hat{V}_k)$$

$$= \underbrace{(y_{io} + y_{ij} + y_{ik})}_{\text{Sum of } y\text{'s connected to node } i}\hat{V}_i - y_{ij}\hat{V}_j - y_{ik}\hat{V}_k$$

Extend to an n-node network.

$$\hat{I}_i = \underbrace{\left(\sum_{k=0}^{n} y_{ik}\right)}_{\text{Sum of all admittances connected to node } i} \hat{V}_i + \sum_{\substack{k=1 \\ k \neq i}}^{n} \underbrace{(-y_{ik})}_{Y_{ik} = -y_{ik}} \cdot \hat{V}_k$$

Sum of all admittances connected to node i

Node i SELF ADMITTANCE

Y_{ii}

$Y_{ik} = -y_{ik}$

Negative of the sum of all admittances connected between nodes i and k

MUTUAL ADMITTANCE between nodes i & k

Y_{ik}

$$\boxed{\hat{I}_i = \sum_{k=1}^{n} (Y_{ik} \hat{V}_k)}$$

Nodal Current Equations – Matrix Formulation

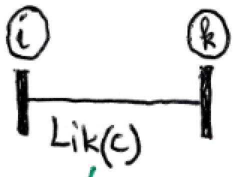
$$
\underbrace{\begin{bmatrix}\hat{I}_1\\ \hat{I}_2\\ \vdots\\ \hat{I}_i\\ \vdots\\ \hat{I}_n\end{bmatrix}}_{\substack{[\hat{I}]\\ \text{Nodal Current}\\ \text{Injection Vector}\\ (n\times 1)}}
=
\underbrace{\begin{array}{c}
\begin{array}{cccccccc} \hat{V}_1 & \hat{V}_2 & & & \hat{V}_i & & & \hat{V}_n \end{array}\\
\begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1i} & \cdots & Y_{1n}\\
Y_{21} & Y_{22} & \cdots & Y_{2i} & \cdots & Y_{2n}\\
 & - & - & - & - & -\\
Y_{i1} & Y_{i2} & \cdots & Y_{ii} & \cdots & Y_{in}\\
 & - & - & - & - & -\\
Y_{n1} & Y_{n2} & \cdots & Y_{ni} & \cdots & Y_{nn}
\end{bmatrix}
\end{array}}_{\substack{[Y] - \text{Nodal Admittance Matrix}\\ (n\times n)}}
\underbrace{\begin{bmatrix}\hat{V}_1\\ \hat{V}_2\\ \vdots\\ \hat{V}_i\\ \vdots\\ \hat{V}_n\end{bmatrix}}_{\substack{[\hat{V}]\\ \text{Nodal Voltage}\\ \text{Vector}\\ (n\times 1)}}
$$

$$\boxed{\underset{\sim}{\hat{I}} = [Y]\,\underset{\sim}{\hat{V}}}$$

Transmission Line — Admittance Formulation.

Transmission line characterised by three parameters

– R, X, B



i – From bus
k – To bus
c – Circuit Identifier (distinguish between parallel circuits)

$$y_{ik(c)} = \frac{1}{Z_{ik(c)}} = \frac{1}{R + jX}$$

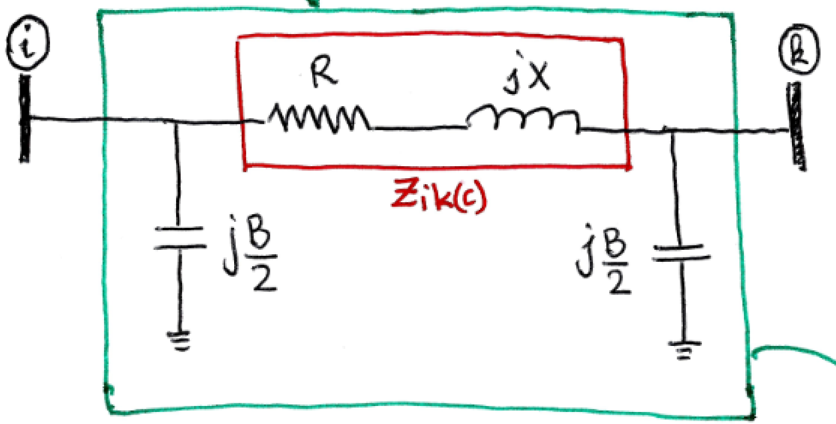
$$= g_{ik(c)} + j\,b_{ik(c)}$$

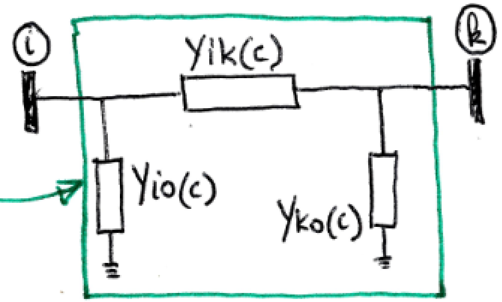
$$= \frac{R}{R^2+X^2} + j\left(\frac{-X}{R^2+X^2}\right)$$

Note Negation.

SERIES → Conductance Susceptance

SHUNT → $y_{io(c)} = y_{ko(c)} = j\frac{B}{2}$

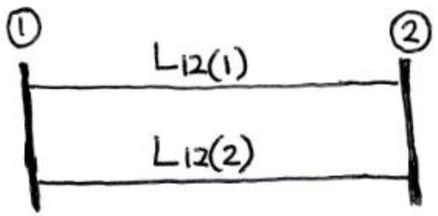




Admittance Formulation

Transmission Line – Admittance Formulation

Example.



All parameters in per-unit on $\underbrace{100\text{MVA}}_{3\phi}$, $\underbrace{275\text{kV}}_{\text{Rms, } \ell\text{-}\ell}$ base.

	PRIMARY PARAMETERS			DERIVED ADMITTANCE PARAMETERS		
	R	X	B	y_{ij}	y_{io}	y_{ko}
IDENTICAL LINES $L_{12(1)}$	0·023	0·167	0·657	$\boxed{0{\cdot}8093 - j5{\cdot}877}$	$\boxed{j0{\cdot}3285}$	$\boxed{j0{\cdot}3285}$
	‖	‖	‖	‖	‖	‖
$L_{12(2)}$	0·023	0·167	0·657	$0{\cdot}8093 - j5{\cdot}877$	$j0{\cdot}3285$	$j0{\cdot}3285$
PARALLEL ADMITTANCES SUM				$1{\cdot}6186 - j11{\cdot}754$	$j0{\cdot}657$	$j0{\cdot}657$

$$y_{12(1)} = g_{12(1)} + j b_{12(1)}$$

$$= \frac{R + j(-X)}{R^2 + X^2}$$

$$= \frac{0{\cdot}023 + j(-0{\cdot}167)}{0{\cdot}023^2 + 0{\cdot}167^2} = \boxed{\underbrace{0{\cdot}8093}_{g_{12(1)}} + j\underbrace{(-5{\cdot}877)}_{b_{12(1)}}} \text{ pu}$$

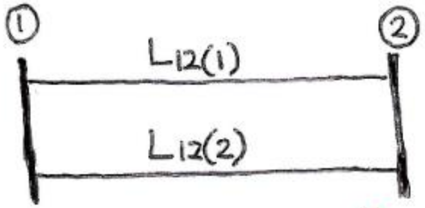
$$y_{10(1)} = y_{20(1)} = j\frac{B}{2}$$

$$= j\,0{\cdot}657/2$$

$$= \boxed{j0{\cdot}3285} \text{ pu.}$$

Transmission Line – Admittance Formulation

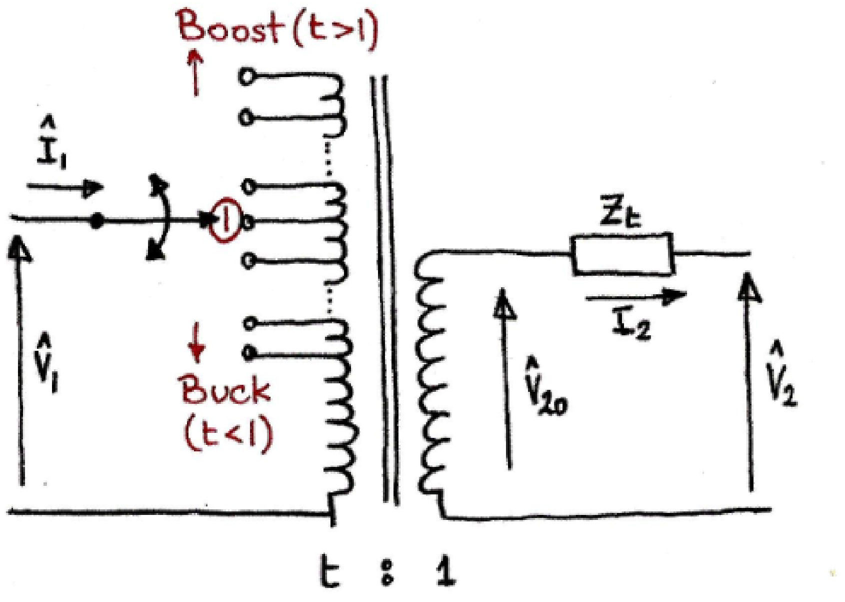
Example.



All parameters in per-unit on $\underbrace{100\,\text{MVA}}_{3\phi}$, $\underbrace{275\,\text{kV}}_{\text{Rms, } \ell\text{-}\ell}$ base.

	PRIMARY PARAMETERS			DERIVED ADMITTANCE PARAMETERS		
	R	X	B	y_{ij}	y_{io}	y_{ko}
$L_{12(1)}$	0·023	0·167	0·657	0·8093 − j5·877	j0·3285	j0·3285
$L_{12(2)}$	0·023	0·167	0·657	0·8093 − j5·877	j0·3285	j0·3285
PARALLEL ADMITTANCES SUM				$\boxed{1{\cdot}6186 - j11{\cdot}754}$	$\boxed{j0{\cdot}657}$	$\boxed{j0{\cdot}657}$

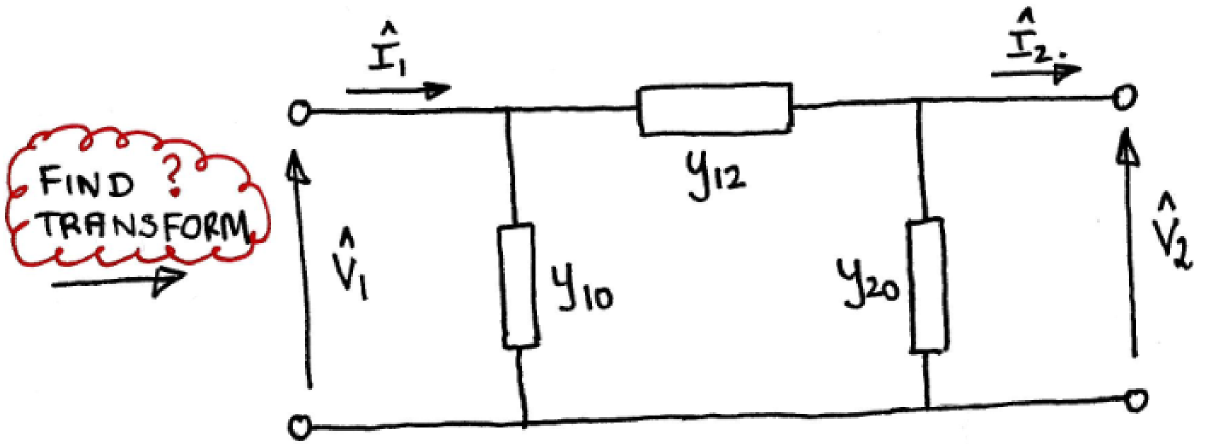
Regulating Transformer – Equivalent π-circuit (1)



$t = 1$ unity tap position
Corresponds to nominal voltage ratio

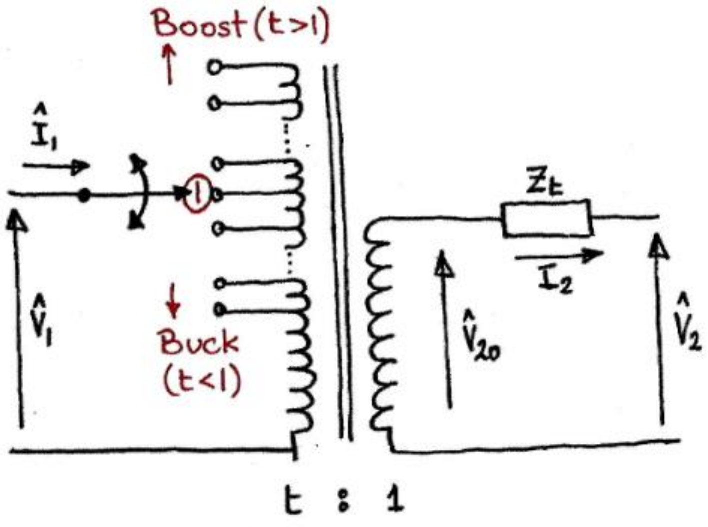
Z_t Per-unit transformer impedance corresponding to unity tap.

$t_{min} < t < t_{max}$



Objective: Find equivalent π-circuit parameters y_{10}, y_{12} and y_{20} such that terminal conditions ($\hat{V}_1, \hat{I}$ & $\hat{V}_2, \hat{I}_2$) are identical to those of the transformer representation at left.

Regulating Transformer – Equivalent π-circuit (2)



$t = 1$ unity tap position
Corresponds to nominal voltage ratio

Z_t Per-unit transformer impedance corresponding to unity tap.

$t_{min} < t < t_{max}$

A. Seek to express $\hat{I}_1$ in terms of $\hat{V}_1$ & $\hat{V}_2$

$$\hat{V}_{20} = \frac{\hat{V}_1}{t} , \quad \hat{I}_2 = t\,\hat{I}_1$$

$$\hat{V}_2 = \hat{V}_{20} - Z_t \hat{I}_2 = \frac{\hat{V}_1}{t} - t Z_t \hat{I}_1$$

$$\therefore \hat{I}_1 = \left(\frac{\hat{V}_1}{t} - \hat{V}_2\right) \cdot \frac{Y_t}{t} \qquad \boxed{Y_t = \frac{1}{Z_t}}$$

$$\hat{I}_1 = \boxed{(\hat{V}_1 - \hat{V}_2)} \boxed{\frac{Y_t}{t}} + \boxed{\hat{V}_1} \boxed{\left(\frac{1}{t} - 1\right) \cdot \frac{Y_t}{t}}$$

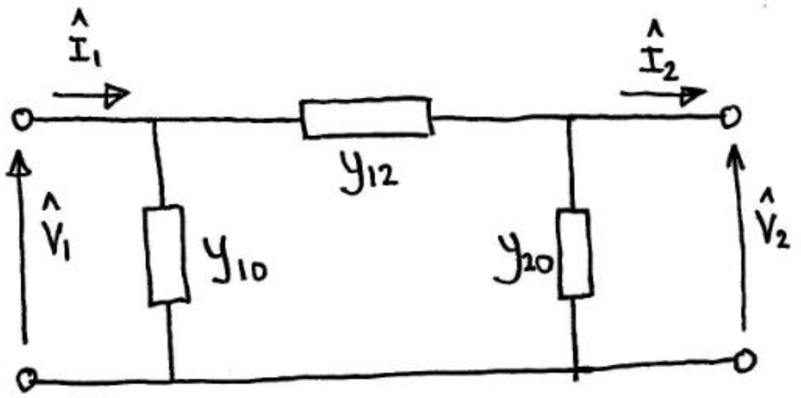
B. Similarly express $\hat{I}_2$ in terms of $\hat{V}_1$ & $\hat{V}_2$

$$\hat{V}_2 = \frac{\hat{V}_1}{t} - Z_t \hat{I}_2$$

$$\therefore \hat{I}_2 = (\hat{V}_1 - t\hat{V}_2) \cdot \frac{Y_t}{t}$$

$$\hat{I}_2 = \boxed{(\hat{V}_1 - \hat{V}_2)} \boxed{\frac{Y_t}{t}} - \boxed{\hat{V}_2} \boxed{\left(1 - \frac{1}{t}\right) Y_t}$$

Regulating Transformer – Equivalent π-circuit (3)



$\hat{I}_1$ & $\hat{I}_2$ of π-circuit in terms of $\hat{V}_1$ & $\hat{V}_2$

$$\hat{I}_1 = \boxed{(\hat{V}_1 - \hat{V}_2)}\,\boxed{y_{12}} + \boxed{\hat{V}_1}\,\boxed{y_{10}}$$

$$\hat{I}_2 = \boxed{(\hat{V}_1 - \hat{V}_2)}\,\boxed{y_{12}} - \boxed{\hat{V}_2}\,\boxed{y_{20}}$$

From previous slide have transformer current equations

$$\hat{I}_1 = \boxed{(\hat{V}_1 - \hat{V}_2)}\,\boxed{\frac{Y_t}{t}} + \hat{V}_1 \boxed{\left(\frac{1}{t} - 1\right)\frac{Y_t}{t}}$$

$$\hat{I}_2 = \boxed{(\hat{V}_1 - \hat{V}_2)}\,\boxed{\frac{Y_t}{t}} - \hat{V}_2 \boxed{\left(1 - \frac{1}{t}\right) Y_t}$$

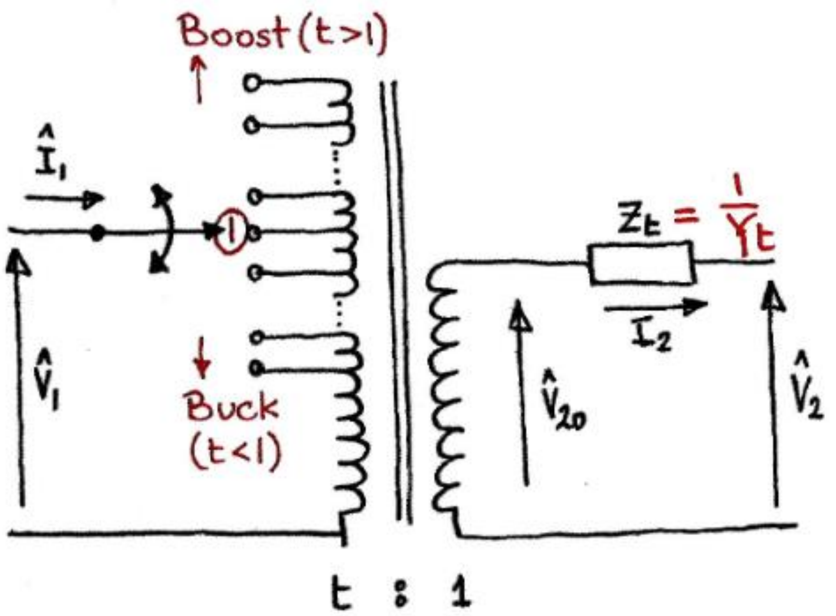
Equate coefficients from π-circuit and transformer current equations

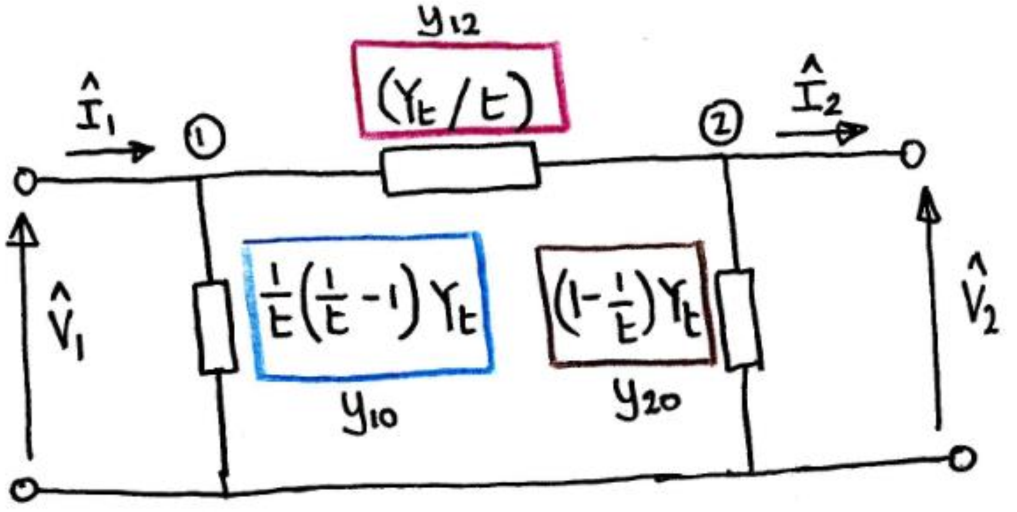
$$\boxed{y_{12} = \frac{Y_t}{t}}$$

$$\boxed{y_{10} = \frac{1}{t}\left(\frac{1}{t} - 1\right) Y_t}$$

$$\boxed{y_{20} = \left(1 - \frac{1}{t}\right) Y_t}$$

Regulating Transformer - Equivalent π-circuit (4)





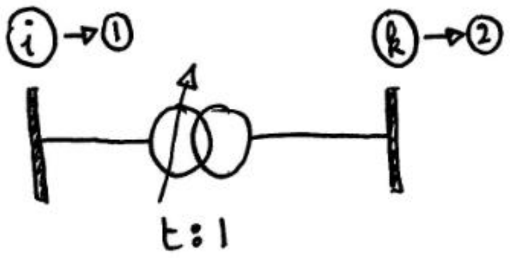
Equivalent π-circuit of Transformer

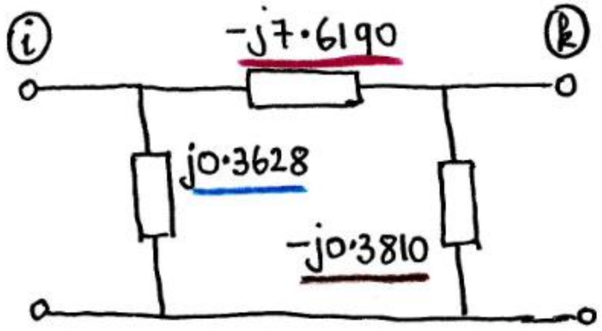
$t = 1$ unity tap position
Corresponds to nominal voltage ratio

Z_t Per-unit transformer impedance corresponding to unity tap.

$t_{min} < t < t_{max}$

Regulating Transformer - Equivalent π-circuit - Example





$Z_t = j0{\cdot}125 \text{ pu} \Rightarrow Y_t = \frac{1}{j0{\cdot}125} = -j8$

$t = 1{\cdot}05 \text{ pu}$

$y_{10} = \frac{1}{t}\left(\frac{1}{t} - 1\right) Y_t$

$\quad = \frac{1}{1{\cdot}05}\left(\frac{1}{1{\cdot}05} - 1\right)(-j8)$

$\quad = -0{\cdot}04535\,(-j8)$

$\boxed{y_{10} = +j0{\cdot}3628 \text{ pu}}$ CAP

$y_{20} = \left(1 - \frac{1}{t}\right) Y_t$

$\quad = \left(1 - \frac{1}{1{\cdot}05}\right)(-j8)$

$\quad = +0{\cdot}04762\,(-j8)$

$\boxed{y_{20} = -j0{\cdot}3810 \text{ pu}}$ IND

$y_{12} = Y_t / t$

$\quad = (-j8)/1{\cdot}05$

$\boxed{y_{12} = -j7{\cdot}6190 \text{ pu}}$

If $t > 1$ then

y_{10} is capacitive

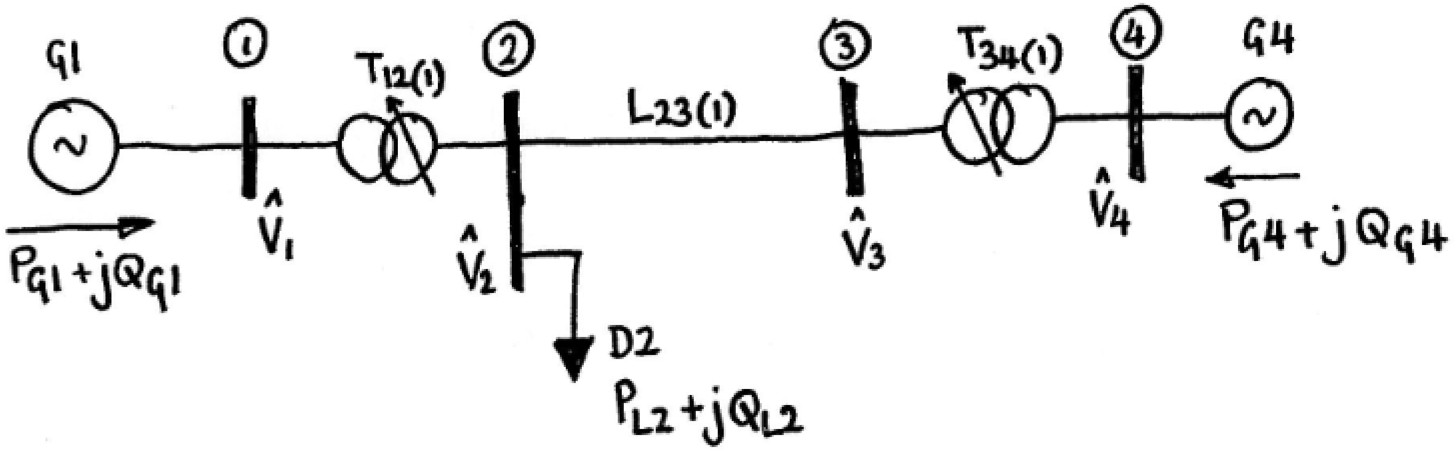
y_{20} is inductive

If $t < 1$ then

y_{10} is inductive

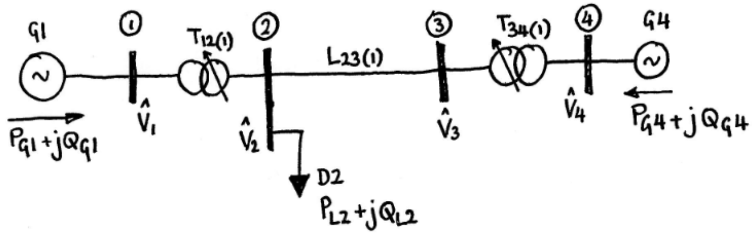
y_{20} is capacitive

Example - Nodal Current Equations



Example -- Nodal Current Equations (2)

Form equivalent pi-circuits for each network element



$T_{12(1)}$

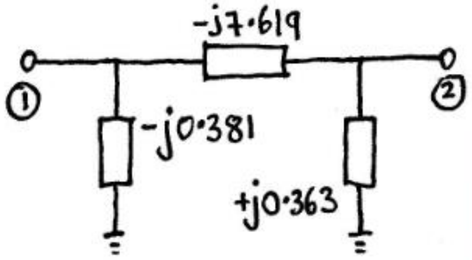
$R = 0, \quad X = 0{\cdot}125$

$t = 1{\cdot}05$

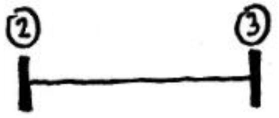
$y_{12} = 0 - j7{\cdot}619$

$y_{10} = 0 - j0{\cdot}381$

$y_{20} = 0 + j0{\cdot}363$



$L_{12(1)}$



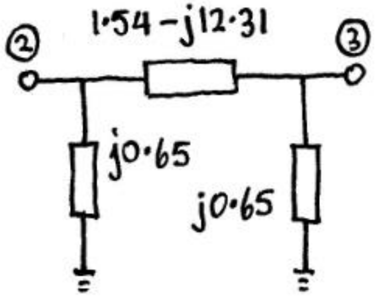
$R = 0{\cdot}01, \quad X = 0{\cdot}08$

$B = 1{\cdot}3$

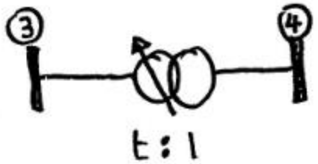
$y_{23} = 1{\cdot}54 - j12{\cdot}31$

$y_{20} = j0{\cdot}65$

$y_{30} = j0{\cdot}65$



$T_{34(1)}$



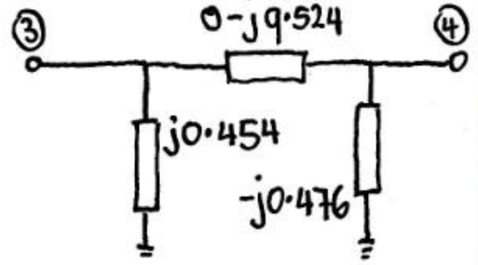
$R = 0, \quad X = 0{\cdot}1$

$t = 1{\cdot}05$

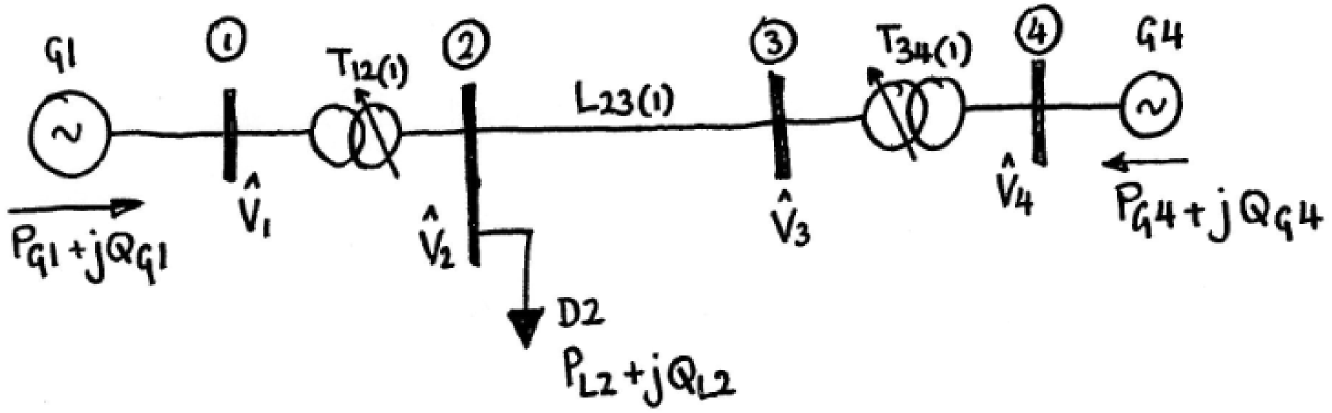
$y_{34} = 0 - j9{\cdot}524$

$y_{30} = 0 + j0{\cdot}454$

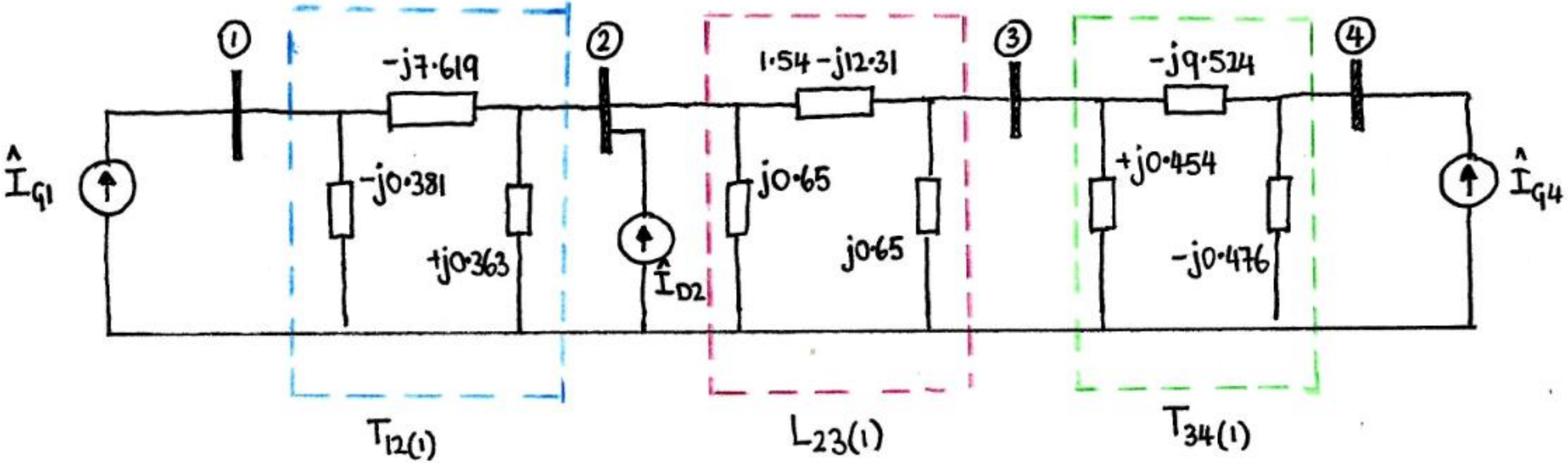
$y_{40} = 0 - j0{\cdot}476$



Example -- Nodal Current Equations (3)

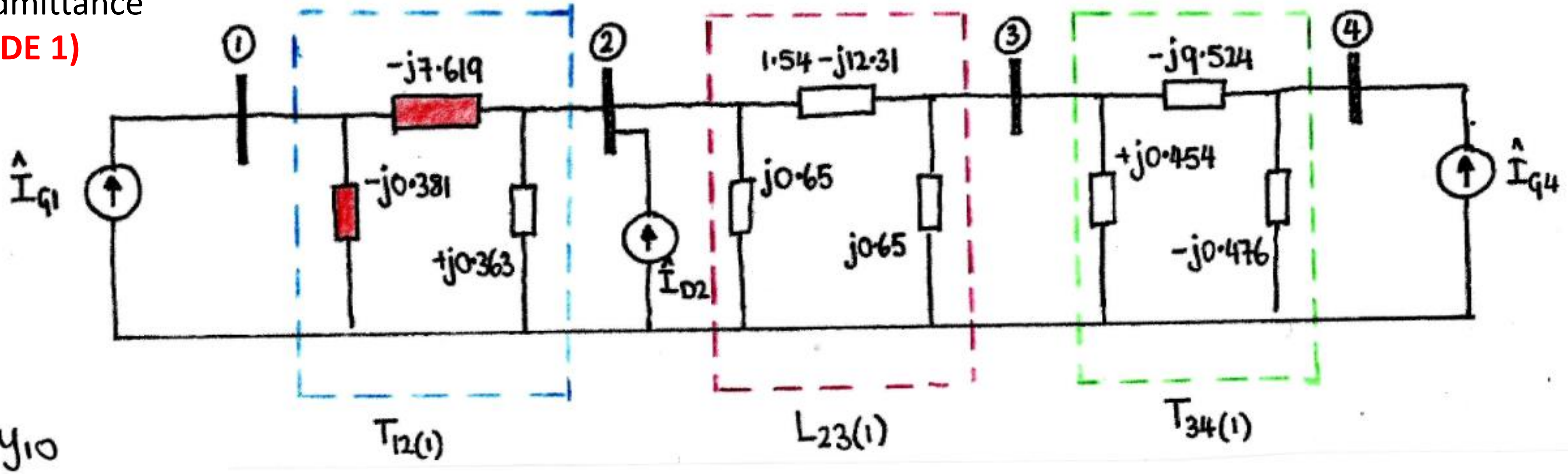


Assemble the network in admittance form



Example -- Nodal Current Equations (4)

Assemble network admittance matrix elements **(NODE 1)**

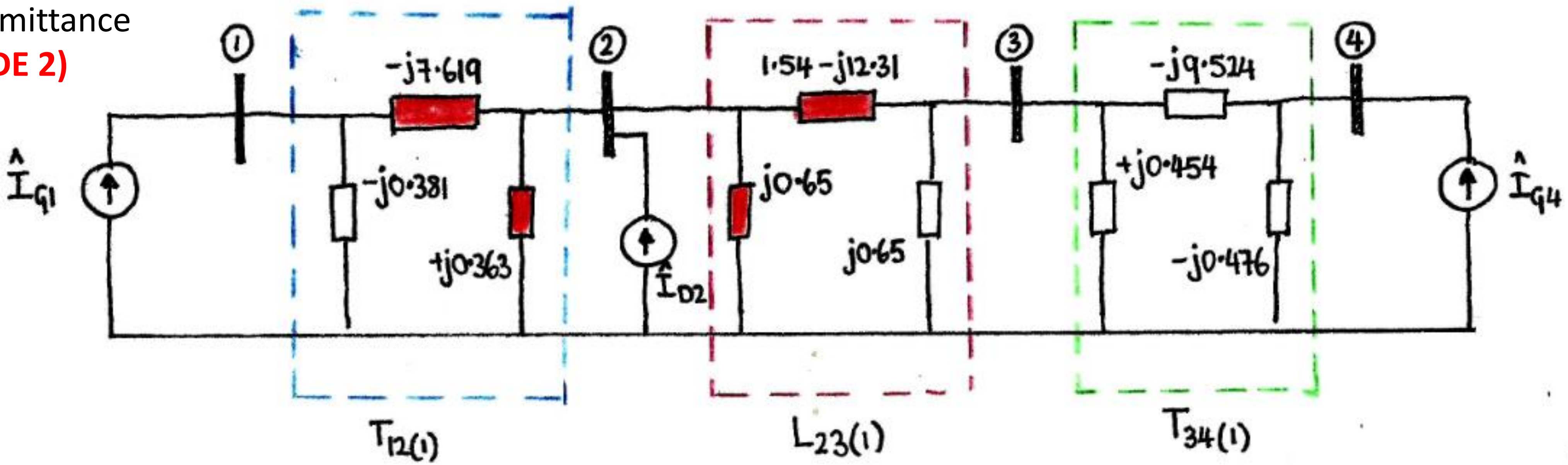


$$Y_{11} = y_{12} + y_{10}$$
$$= -j(7{\cdot}619 + 0{\cdot}381)$$
$$= -j\,8{\cdot}0$$

$$Y_{12} = Y_{21} = -y_{12}$$
$$= +j7{\cdot}619$$

Example -- Nodal Current Equations (5)

Assemble network admittance matrix elements **(NODE 2)**



$$Y_{22} = y_{12} + \underset{(T_{12(1)})}{y_{20}} + \underset{(L_{23(1)})}{y_{20}} + y_{23}$$

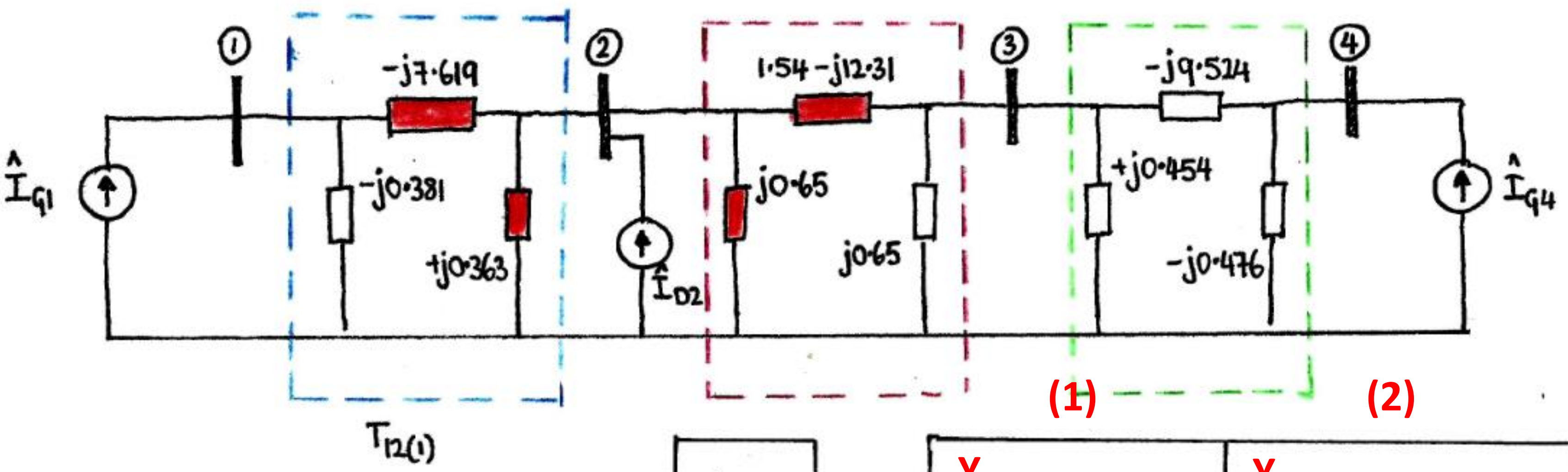
$$= 1{\cdot}54 + j(-7{\cdot}619 + 0{\cdot}363 + 0{\cdot}65 - 12{\cdot}31)$$

$$= 1{\cdot}54 - j(18{\cdot}916)$$

$$Y_{23} = Y_{32} = -y_{23}$$

$$= -1{\cdot}54 + j12{\cdot}31$$

Example -- Nodal Current Equations (6)



Assemble network nodal current equations

$$
\begin{bmatrix} \hat{I}_{g1} \\ \hat{I}_{D2} \\ 0 \\ \hat{I}_{g4} \end{bmatrix}
=
\begin{array}{c|cccc}
 & (1) & (2) & (3) & (4) \\
\hline
(1) & Y_{11}\; 0-j8.0 & Y_{12}\; 0+j7.619 & 0 & 0 \\
(2) & Y_{21}\; 0+j7.619 & Y_{22}\; 1.54-j18.916 & -1.54+j12.31 & 0 \\
(3) & \text{No connection}\; 0 & -1.54+j12.31 & +1.54-j20.73 & 0+j9.524 \\
(4) & 0 & 0 & 0+j9.524 & 0-j10.0
\end{array}
\begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \hat{V}_3 \\ \hat{V}_4 \end{bmatrix}
$$