Sample Quiz 2.

1.1 Refer to lecture slides F01, slides 22, 23

1.2 Refer to F04 slide 13

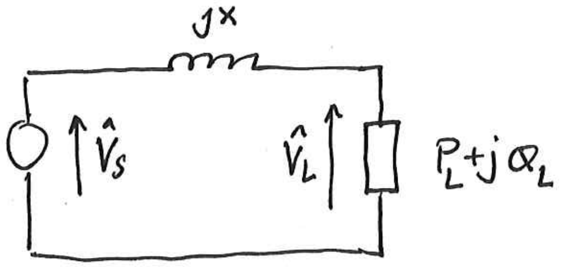
1.3 If load is unity power factor then if

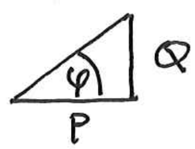
$P_{load} = P_{SIL}$ then transmission line neither generates or consumes reactive power

$P_{load} > P_{SIL}$ then the T/L consumes reactive power

$P_{load} < P_{SIL}$ then the T/L generates reactive power.

2





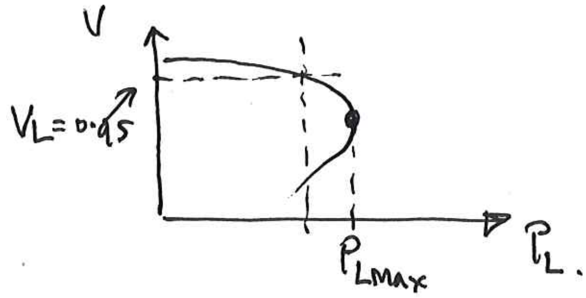
$\varphi = a\cos(0.9)$
$= 25.84°$

$X = 0.3$
$V_S = 1.05$
$V_L = 0.95.$

$$P_L = \frac{V_S V_L}{X} \sin\delta \quad (1)$$

$$Q_L = \frac{V_S V_L}{X} \cos\delta - \frac{V_L^2}{X} = P_L \tan\varphi$$

$$\therefore \Big(\underbrace{P_L \tan\varphi}_{= Q_L} + \frac{V_L^2}{X}\Big) = \frac{V_S V_L}{X} \cos\delta \quad (2)$$



$①^2 + ②^2$

$$P_L^2 + \left(P_L \tan\varphi + \frac{V_L^2}{X}\right)^2 = \left(\frac{V_S V_L}{X}\right)^2 \underbrace{(\sin^2\delta + \cos^2\delta)}_{=1}$$

$$P_L^2 + P_L^2 \tan^2\varphi + 2P_L \tan\varphi \frac{V_L^2}{X} + \left(\frac{V_L^2}{X}\right)^2 = \left(\frac{V_S V_L}{X}\right)^2$$

$$P_L^2 (1 + \tan^2\varphi) + \left(2\tan\varphi \frac{V_L^2}{X}\right) P_L + \left(\left(\frac{V_L^2}{X}\right)^2 - \left(\frac{V_S V_L}{X}\right)^2\right) = 0 \quad (3)$$

Quadratic in P_L of the form

$$aP_L^2 + bP_L + c = 0$$

$$a = 1 + \tan^2\varphi = 1.2346$$

$$b = 2\tan\varphi \frac{V_L^2}{X} = 2.9140$$

$$c = \left(\frac{V_L^2}{X}\right)^2 - \left(\frac{V_S V_L}{X}\right)^2 = -2.0056$$

$$P_L = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 0.5569 \text{ pu}$$

(positive root is required soln).

3 Use equation (3) from 2 in a different way. Now we need to treat V_L in (3) as the unknown so we rearrange it to factor out terms in $(V_L)^2$

$$\frac{1}{X^2}(V_L^2)^2 + \left[\frac{2\tan\varphi\, P_L}{X} - \left(\frac{V_S}{X}\right)^2\right](V_L^2) + P_L^2(1+\tan^2\varphi) = 0$$

$$\Rightarrow (V_L^2)^2 + \underbrace{(2\tan\varphi\, X P_L - V_S^2)}(V_L^2) + \underbrace{P_L^2(1+\tan^2\varphi)\,X^2} = 0 \quad (4)$$

This is a quadratic in $(V_L)^2$

$$a(V_L^2)^2 + b(V_L)^2 + c = 0$$

$$a = 1, \quad b = 2\tan\varphi\, X P_L - V_S^2, \quad c = P_L^2(1+\tan^2\varphi)\,X^2 \quad (5)$$

$$(V_L)^2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad (6)$$

Now, for a real solution we need

$b^2-4ac \geqslant 0$ and $b^2-4ac = 0$ will define the limiting case

$$b^2 - 4ac = 0$$

$$\Rightarrow \left(2\tan\varphi X P_L - V_S^2\right)^2 - 4P_L^2(1+\tan^2\varphi)X^2 = 0$$

$$\Rightarrow \cancel{4\tan^2\varphi X^2 P_L^2} - 4\tan\varphi P_L X V_S^2 + V_S^4 - 4P_L^2 X^2 - \cancel{4P_L^2\tan^2\varphi X^2} = 0$$

$$\Rightarrow -4P_L^2 X^2 - 4\tan\varphi V_S^2 X P_L + V_S^4 = 0$$

$$P_L^2 + \frac{V_S^2\tan\varphi}{X} P_L - \frac{V_S^4}{4X^2} = 0$$

now, the solution of this equation for P_L yields P_{Lmax}, thus

$$P_{Lmax}^2 + \left(\frac{V_S^2}{X}\tan\varphi\right)P_{Lmax} - \frac{1}{4}\left(\frac{V_S^2}{X}\right)^2 = 0$$

$$a = 1, \quad b = \frac{V_S^2}{X}\tan\varphi, \quad c = -\frac{1}{4}\left(\frac{V_S^2}{X}\right)^2$$

$$= 1.7799 \qquad \qquad \doteq -3.3764$$

$$\therefore \quad P_{Lmax} = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$$

$$= 1.1517 \text{ pu} \quad (\text{pos root})$$

To determine the voltage at this level of power transfer we need to substitute $P_L = P_{Lmax}$ in (5) and solve for $(V_L)^2$ using (5) & (6)

From (5) & (6)

$a = 1, \quad b = 2\tan\varphi \times P_{Lmax} - \underline{V_S^2} \qquad c = P_{Lmax}^2 (1+\tan^2\varphi) X^2$

$= -0.7678 \qquad\qquad\qquad\qquad = 0.1474.$

$$(V_L)^2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

verify $b^2 - 4ac = 0$ ✓

$$(V_L)^2 = \frac{-b}{2} = 0.3839$$

$$\Rightarrow \underline{V_L = \sqrt{V_L^2} = 0.6196 \text{ pu}}$$

4.1

$V_S = V_L = 1.05$ pu , $X = 0.4$

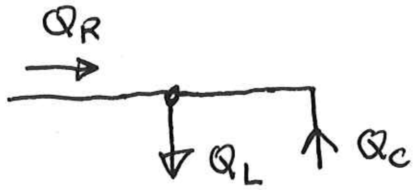
$P_L = 1.2$, $Q_L = 0.6$ pu

$$P_L = \frac{V_S V_L}{X} \sin\delta \Rightarrow \sin\delta = \frac{P_L X}{V_S V_L}$$

$$\cos\delta = \sqrt{1 - \sin^2\delta}$$

$$= \sqrt{1 - \left(\frac{P_L X}{V_S V_L}\right)^2}$$

$$= 0.90025$$



$$Q_c + Q_R = Q_L$$

$$Q_c = Q_L - Q_R$$

$$Q_R = \frac{V_S V_L}{X} \cos\delta - \frac{V_L^2}{X}$$

$$Q_R = Q_L - Q_c = 0.6 - 0.87 = -0.27$$

$$\therefore Q_c = Q_L + \frac{V_L^2}{X} - \frac{V_S V_L}{X} \cos\delta$$

$$= 0.6 + \frac{1.05^2}{0.4} - \frac{1.05^2}{0.4} \cos\delta$$

$$= 0.8749 \text{ pu.}$$

$$Q_c = B_c V_L^2 \Rightarrow B_c = \frac{Q_c}{V_L^2} = 0.7936 \text{ pu.}$$

4.2 Reactive power compensation is required to maintain voltages with the power system within prescribed limits.

Typically loads consume reactive power which must be supplied by a reactive source near the load since reactive power can not be transmitted long distances.

Reactive compensation is also required to supply reactive losses with network elements or, under light load conditions, reactive compensation may be needed to consume excess reactive power generated by transmission lines.

4.3. Connect a reactor to reduce voltage.
A reactor consumes reactive power which tends to reduce voltage.

5 Apply KCL at node 2.

$$\hat{I}_2 = 0 = j0.36 \cdot \hat{V}_2 + j0.65 \cdot \hat{V}_2$$
$$+ (\hat{V}_2 - \hat{V}_3)(-j12.3)$$
$$+ (\hat{V}_2 - \hat{V}_1)(-j7.6)$$

$$\therefore \quad 0 = j7.6\hat{V}_1 + j(0.36 + 0.65 - 12.3 - 7.6)\hat{V}_2 + j12.3\hat{V}_3$$

$$= j7.6\hat{V}_1 - j18.89\hat{V}_2 + j12.3\hat{V}_3$$

5.1 $\hat{V}_1 = 1.0\angle 30^\circ, \quad \hat{V}_2 = 1.05\angle 20^\circ$

$$\hat{I}_1 = (-j0.38)\hat{V}_1 + (\hat{V}_1 - \hat{V}_2)(-j7.6)$$

$$= -j(0.38 + 7.6)\hat{V}_1 + j7.6\hat{V}_2$$

$$= -j7.98\hat{V}_1 + j7.6\hat{V}_2$$

$$= -j7.98 \times 1\angle 30^\circ + j7.6 * 1.05\angle 20^\circ$$

$$= 1.2607 + j0.58786$$

$$\hat{S}_1 = \hat{V}_1 \hat{I}_1^* = 1\angle 30^\circ(1.2607 - j0.58786) = P_1 + jQ_1$$

$$Q_1 = 0.1212 \quad pu.$$

5.2 See F08, slide 4.

5.3 A PV bus is one in which the power input to the bus and the voltage magnitude is specified.

6.1 The surge impedance load is the load (in MW) that is supplied by a transmission line when it is terminated by the surge impedance, Z_0

V_0 is the nominal ph-ph rms voltage of the line in kV. Thus, the ph-neutral voltage for a balanced line is $\frac{V_0}{\sqrt{3}} \times 1000$ V

The load supplied by one phase terminated by Z_0 is

$$P_{1\varphi} = \frac{\left(\frac{V_0}{\sqrt{3}} \times 1000\right)^2}{Z_0} = \frac{1}{3}\frac{V_0^2}{Z_0} \times 10^6 \text{ W}$$

$$= \frac{1}{3}\frac{V_0^2}{Z_0} \text{ MW}$$

The 3φ load is $3 \times P_{1\varphi} = \text{S.I.L.} = \frac{V_0^2}{Z_0}$

6·2 $S.I.L. = \dfrac{500^2}{Z_0}$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{wL}{wC}} = \sqrt{\frac{X_L}{B_c}}$$

$$= \sqrt{\frac{0.271}{4.332 \times 10^{-6}}}$$

$$\simeq 250.12\ \Omega$$

$$\therefore \quad SIL = \frac{500^2}{250.12} = 1000\ MW$$

6·3 $\hat{V}_R = 500$ kV ph-ph $\quad \hat{V}_{R\,(ph-n)} = \dfrac{500}{\sqrt{3}}$ kV.

$P_R = 500\,MW, \quad Q_R = 0$

$$\hat{I}_R = \frac{(P_R - j\,Q_R)/3 \times 10^6}{\left(\frac{\hat{V}_R}{\sqrt{3}} \times 10^3\right)^*} \quad \begin{matrix}[\text{W/VAr per phase}] \\ [\text{V, ph-n}]\end{matrix}$$

$$\simeq \frac{1\ (P_R - j\,Q_R) \times 10^3}{\sqrt{3}\ \ \hat{V}_R^*}$$

$$= \frac{1}{\sqrt{3}} \times \frac{500}{500} \times 10^3$$

$$= \frac{1}{\sqrt{3}} \times 10^3 \quad A$$

$$\hat{V}_R = \hat{V}(l) = \hat{V}_S \cos(\beta l) - j Z_0 \sin(\beta l)\hat{I}_S \quad \times \; \frac{j \sin(\beta l)}{Z_0} \qquad (1)$$

$$\hat{I}_R = \hat{I}(l) = \hat{I}_S \cos(\beta l) - j\left(\frac{\hat{V}_S}{Z_0}\right)\sin(\beta l) \quad \times \; \cos(\beta l) \qquad (2)$$

$$\Sigma$$

$$j\frac{\sin(\beta l)}{Z_0} \times \hat{V}_R + \hat{I}_R \cos(\beta l) = \sin^2(\beta l)\hat{I}_S + \cos^2(\beta l)\hat{I}_S$$

$$\therefore \quad \hat{I}_S = \hat{I}_R \cos(\beta l) + j\frac{\sin(\beta l)}{Z_0}\hat{V}_R$$

now, $\beta = \omega\sqrt{LC}$

$$= \sqrt{(\omega L)(\omega C)}$$

$$= \sqrt{X_L B_C}$$

$$= \sqrt{0.271 \times 4.332 \times 10^{-6}}$$

$$= 1.0835 \times 10^{-3} \text{ rad/km}$$

$$\beta l = 0.325$$

$$\hat{I}_S = \underbrace{\frac{1}{\sqrt{3}} \times 10^3}_{I_R(A)} \times \cos(\beta l) + j\frac{\sin(\beta l)}{\underbrace{250.12}_{Z_0(\Omega)}} \times \underbrace{\left(\frac{500 \times 10^3}{\sqrt{3}}\right)}_{V_R(ph-n) \text{ in volts}}$$

$$= 547.12 + j\,368.59 \text{ A}$$

$$= 659.69 \angle 33.97^\circ$$

From (1) $$\hat{V}_S = \frac{\hat{V}_R + jZ_0 \sin(\beta l)\hat{I}_S}{\cos(\beta l)}$$

$$= (2.7356 + j\,0.4616) \times 10^5 \text{ V (ph-n)}$$

$$\hat{V}_S = 277.42 \text{ kV (ph-n)} \angle 9.569^\circ$$

$$|V_S| = \sqrt{3} \times 277.42 = \underline{480.5 \text{ kV (ph-ph)}}$$

$$\hat{S}_{S(1\varphi)} = \hat{V}_S \hat{I}_S^*$$

$$= \underbrace{277.42 \times 10^3}_{V} \angle 9.569^\circ * \underbrace{659.69}_{A} \angle -33.97^\circ \underbrace{\times 10^{-6}}_{W \to MW}$$

$$= 166.67 - j\underbrace{75.6}_{Q_{1\varphi}}$$

Three phase reactive power supplied by source is

$$Q_S = -75.6 \times 3$$

$$\underline{Q_S = -226.8 \text{ MVAR.}}$$

7.1

$P_L = 0.8$ pu, pf $= 0.98$ (lag)

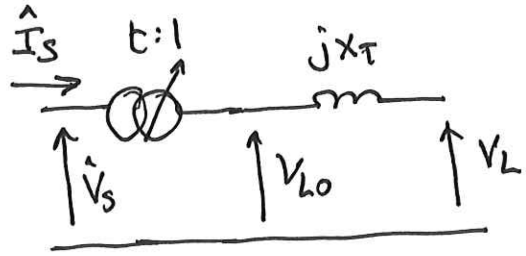
$\Rightarrow Q_L = P_L \tan(\mathrm{acos}(0.98))$

$= 0.1625$ pu.

$X_T = 0.12$ pu.

$V_L = 1.02$ pu. $\angle 0°$ assume without loss of generality.

$$\hat{I}_L = \frac{P_L - jQ_L}{\hat{V}_L^*} = \frac{0.8 - j0.1625}{1.02\angle 0} = 0.7843 - j0.1593$$
$$= 0.8003\angle -11.48$$



$$\hat{V}_{Lo} = \hat{V}_L + jX_T\hat{I}_L$$
$$= 1.02 + j0.12\,\hat{I}_L$$
$$= 1.0391 + j0.09412 = 1.0434\angle 5.18°$$

$$|\hat{V}_s| = t|\hat{V}_{Lo}|$$

$$\Rightarrow t = \frac{|\hat{V}_s|}{|\hat{V}_{Lo}|} = \frac{0.98}{1.0434} = 0.9393 \text{ pu.}$$

7.2 $$\hat{I}_s = \frac{1}{t}\hat{I}_L$$

$$\hat{V}_s = t \cdot \hat{V}_{Lo}$$

$$P_s + jQ_s = \hat{V}_s \hat{I}_s^{*}$$

$$= t\hat{V}_{Lo}\left(\frac{1}{t}\hat{I}_L\right)^{*}$$

$$= \hat{V}_{Lo}\hat{I}_L^{*}$$

as expected since ideal transformer conserves complex power.

$$Q_s = \mathrm{Im}\left(\hat{V}_{Lo}\hat{I}_L^{*}\right)$$

$$\underline{Q_s = 0.2393 \text{ pu.}}$$