

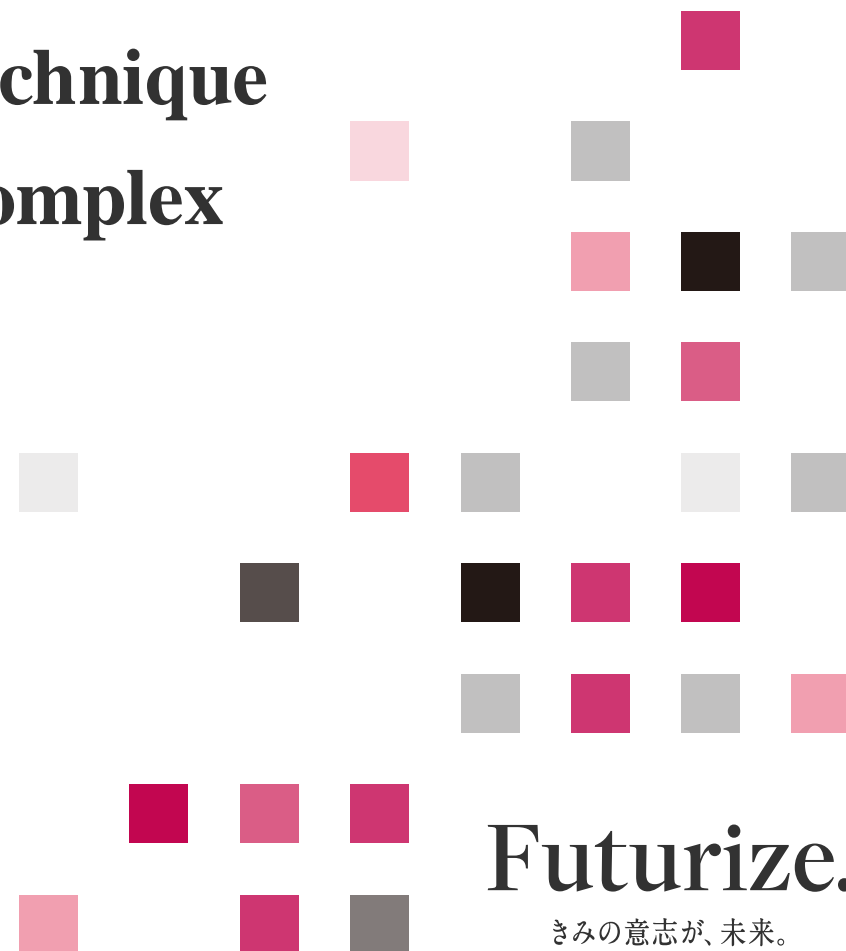
Intelligent Robotic Systems Laboratory

# 「Non-circular Control Barrier Function Technique for Mobile Robots Collision Avoidance in Complex Environments」

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2025/02/03



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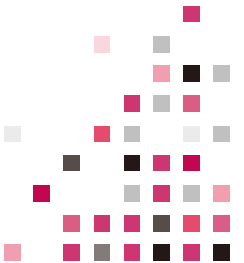
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# Agenda

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- Background
- Related work
- Problem formulation
- Research goal
- Algorithm
- Experiment
- Summary



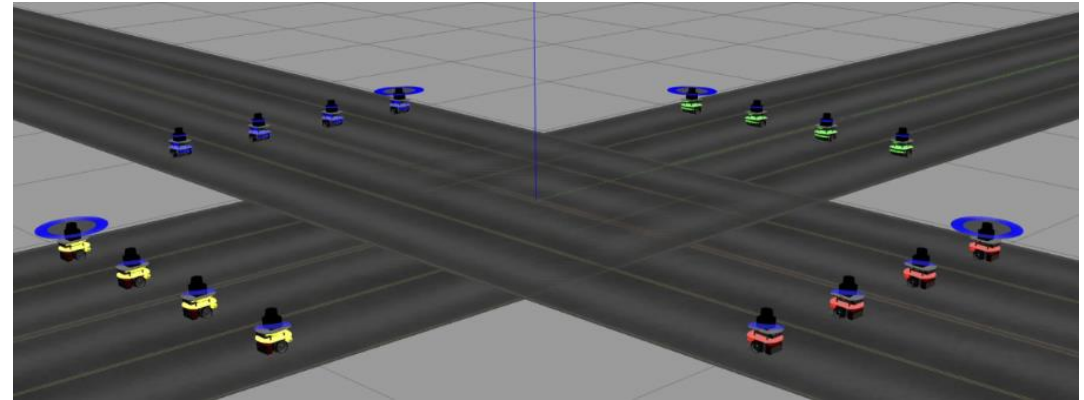
# Background

The development of automation and robotics

- Autonomous vehicles
- Robotic delivery

Key challenges:

- Navigate through crowded environments
- Obstacle avoidance



# Related work

Control barrier functions (CBF): Theory and applications(Ames, A. D,2019)

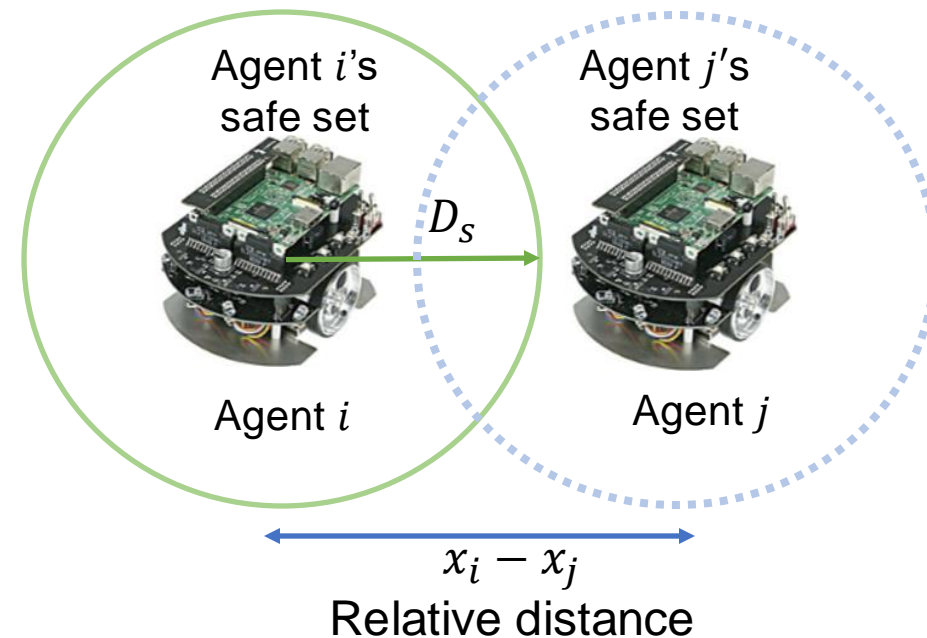
- Mathematical tools ensure system safety
- Ensure agents maintain safety distances
- A 'safe set' represents constraints
- $C_{ij} = \{x_i \in \mathbb{R}^2 \mid h_{ij}(x) = ||x_i - x_j||^2 - D_s^2 \geq 0\}$

$D_s$  -Minimum safety distance between agents

$C_{ij}$ - safe set of states for agents  $i$  and  $j$

$x_i, x_j$ - Position of agent  $i$  and agent  $j$  in 2D space

$h_{ij}$ - Control Barrier Function (CBF)



# Related work

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Hierarchical consensus-based multi-agent reinforcement learning for multi-robot cooperation tasks (Feng, P, 2019)

- CBF's multi-agent reinforcement learning framework
- Safety and cooperation in robotic systems
- Excessive space usage

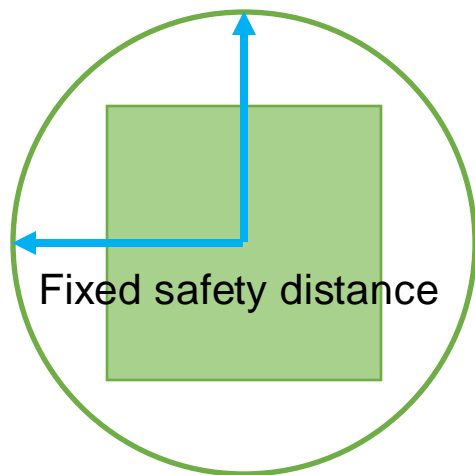
Extent-compatible Control Barrier Functions (Mohit Srinivasan, 2020)

- Physical volume incorporated into safety modeling
- Improved performance in complex environments
- Unable to adapt to irregular shapes

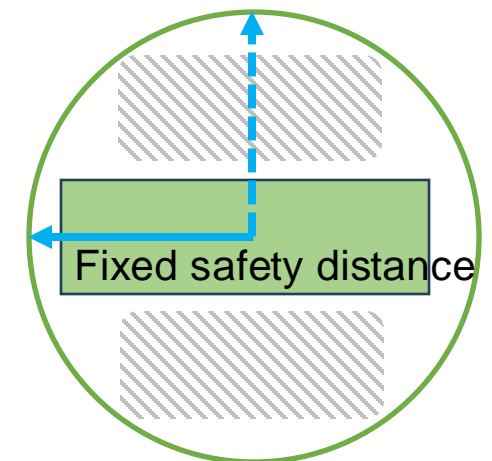
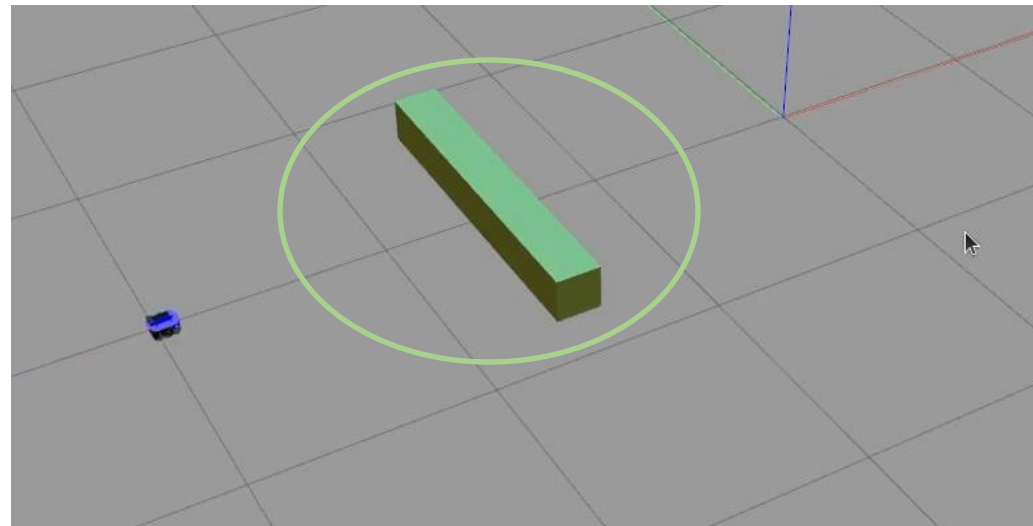
# Problem statement

## Limitations in control barrier function

- Fixed safety distance and safe set
- Excessive space usage
- Limited applications in high-precision requirements scenarios



Equal sides  
object



Unequal sides  
object

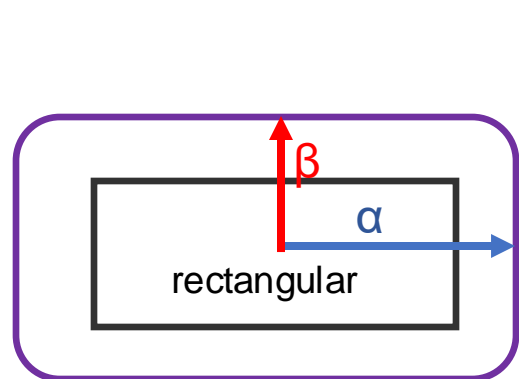
# Research Goal

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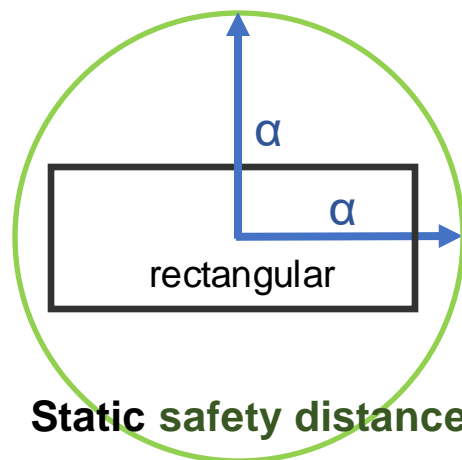
1. Improving barrier function for complex scenarios
  - Flexible safety distance for irregularly shaped
  - Optimize collision avoidance space for efficient execution
  - Achieve greater adaptability for different situations
2. Evaluate method efficacy across scenarios
  - Test the new collision avoidance technique in complex environments

# Super-Ellipse CBF(SE-CBF)

1. Apply Super-Ellipse curve
  - Flexible Mathematical curve
2. Dynamic distance calculation
  - Direction-Dependent safety distance
3. Adaptive various boundary
  - Optimize collision avoidance space

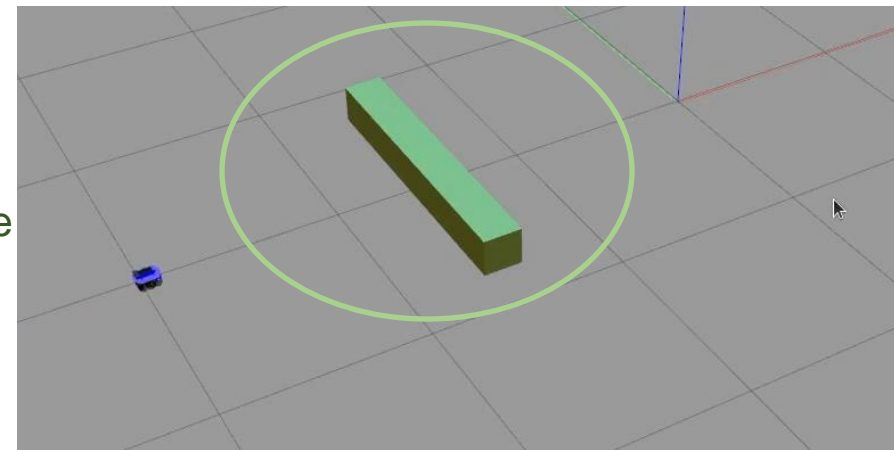


Dynamic safety distance

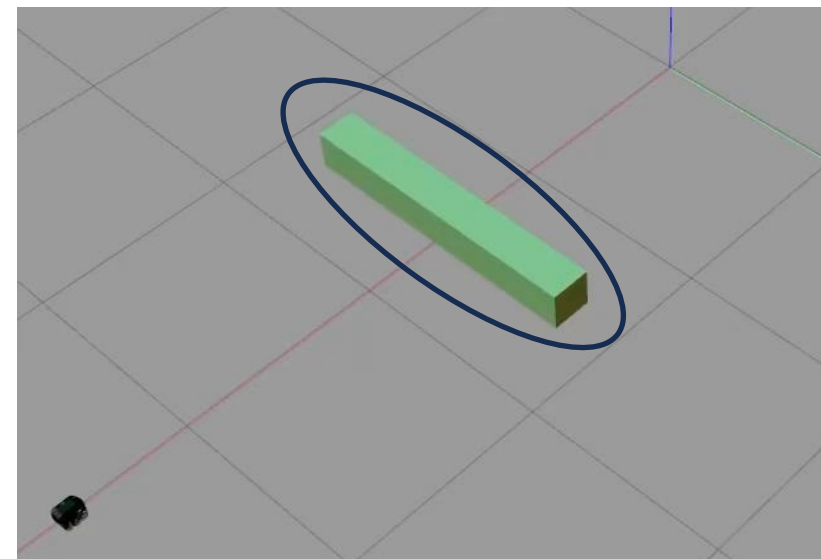


Static safety distance

Static  
safety  
distance

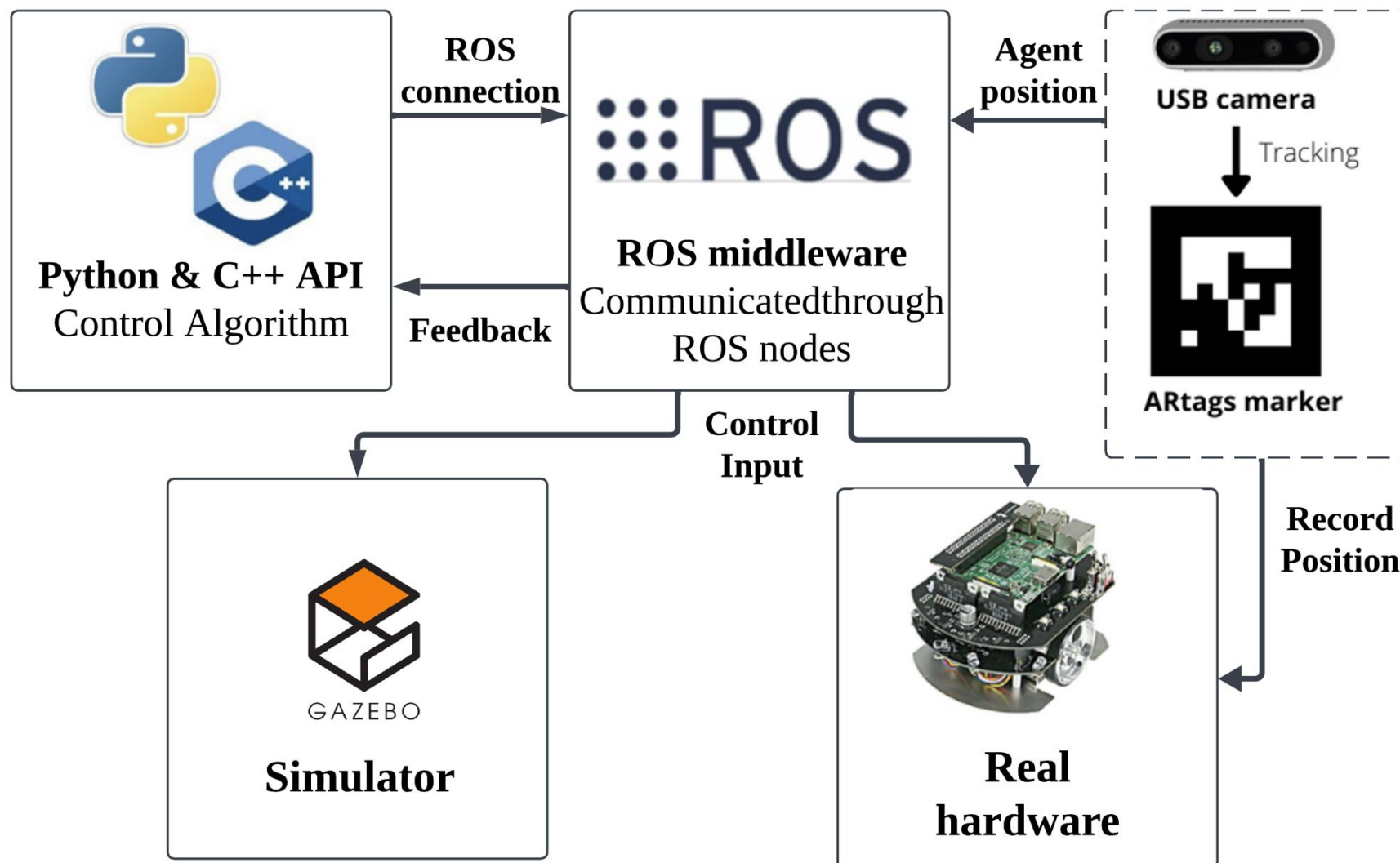


Dynamic  
safety  
distance



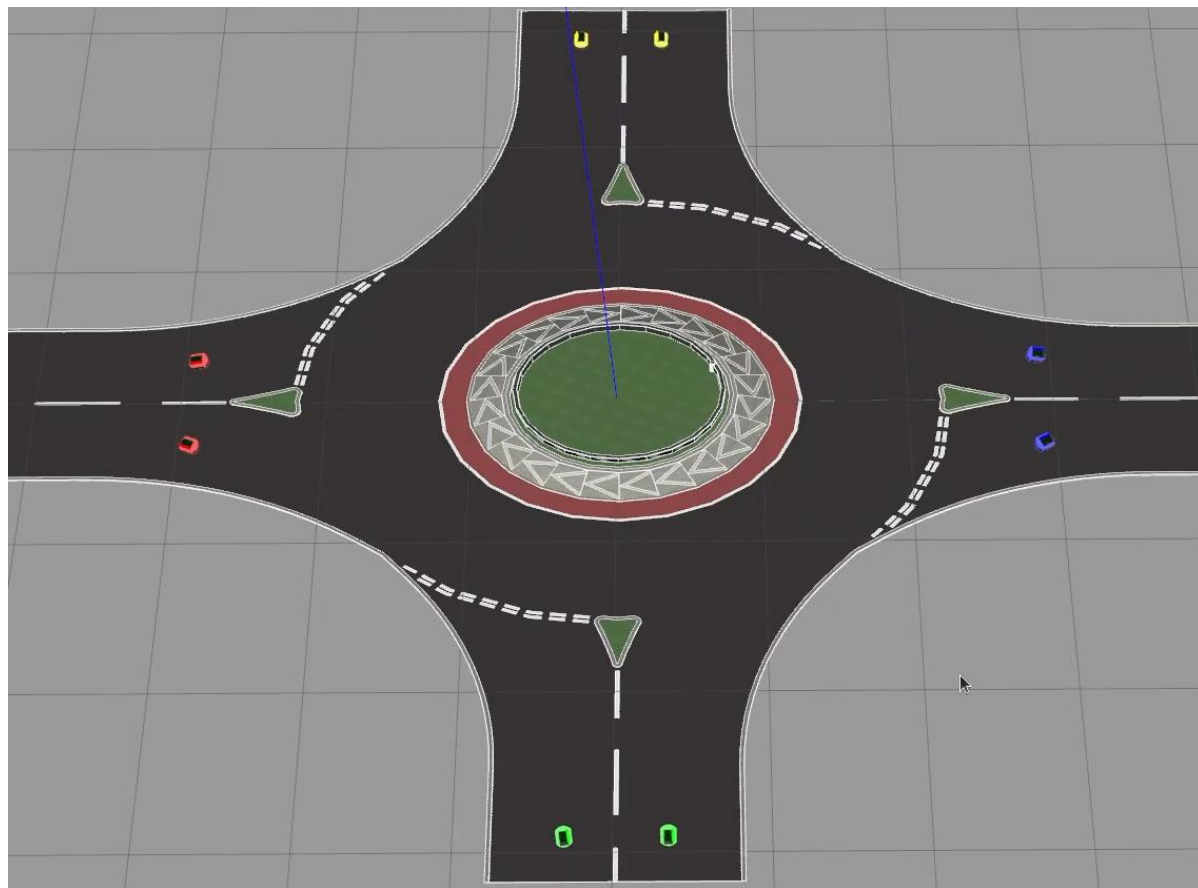


# System Architecture



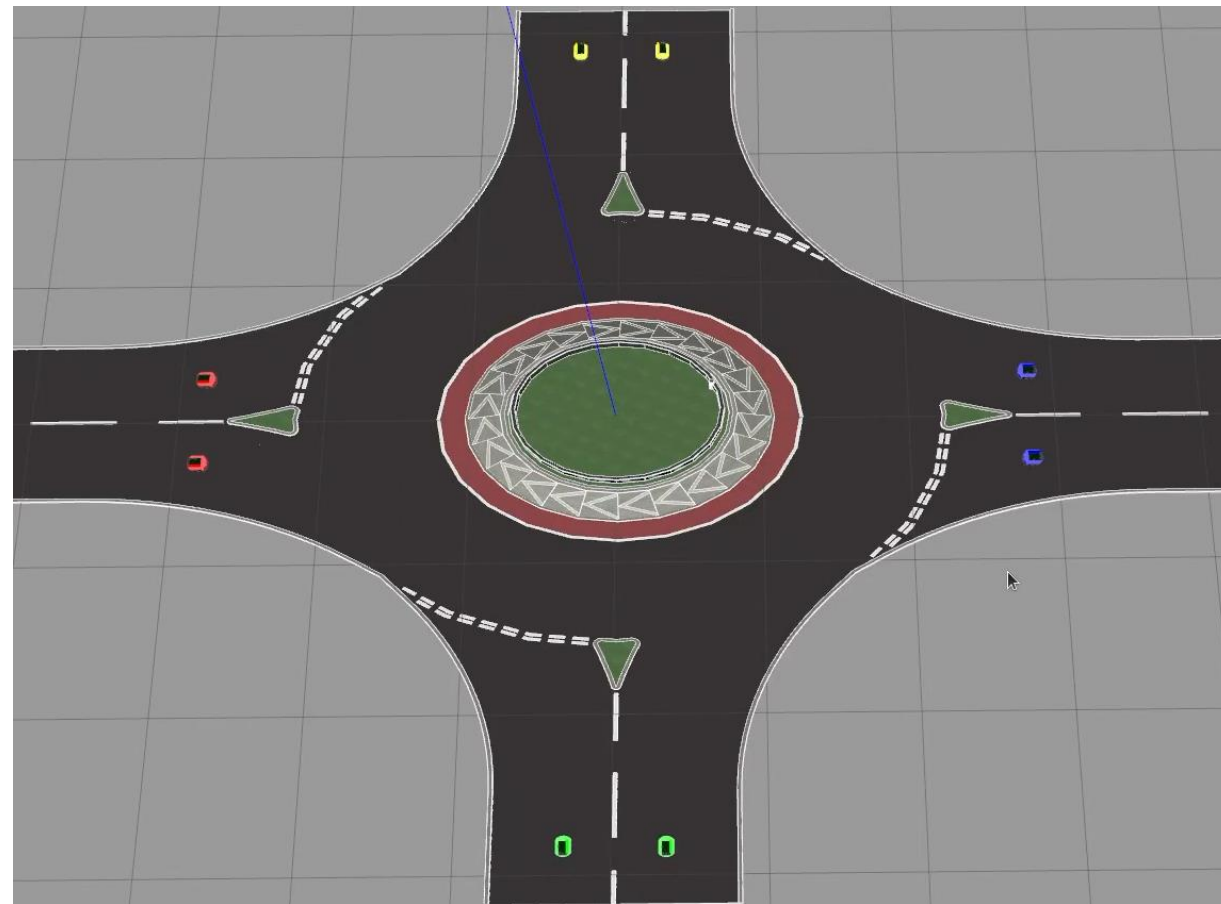
# Intersection Simulation

Dynamic safety distance



Super-Ellipse Control Barrier Function (SE-CBF)

Static safety distance

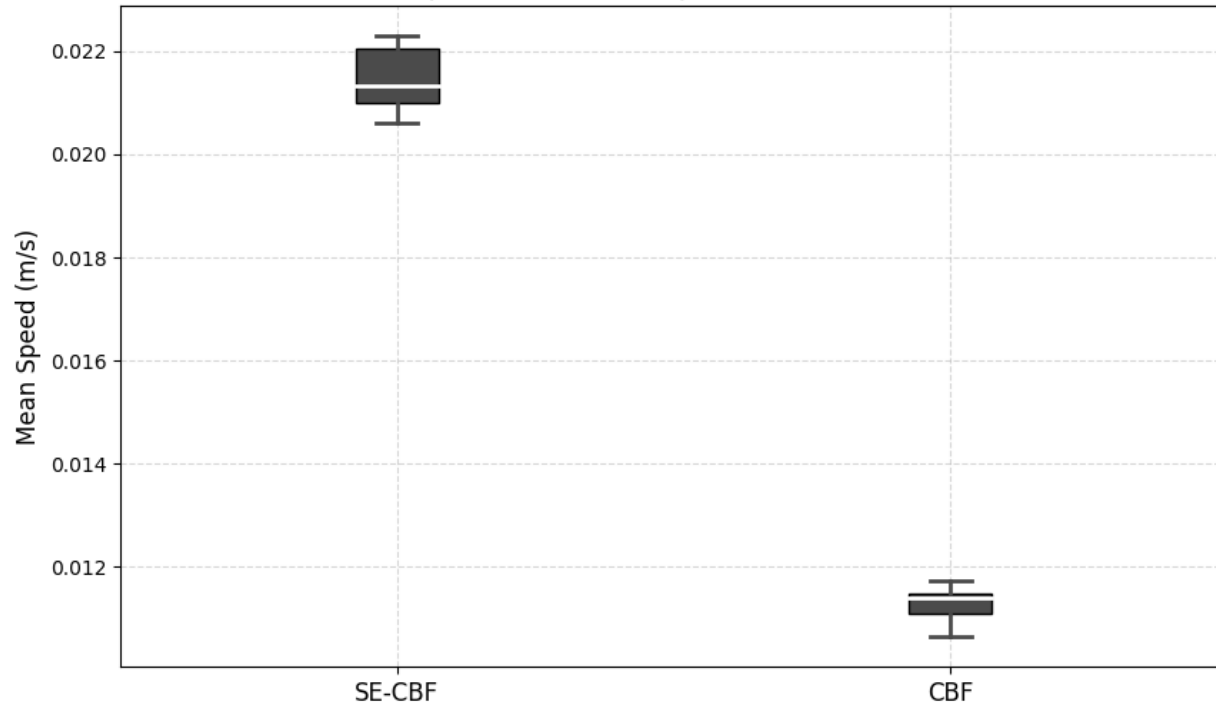


Control Barrier Function (CBF)

# Simulation Results and Analysis

## Mean Speed Boxplot

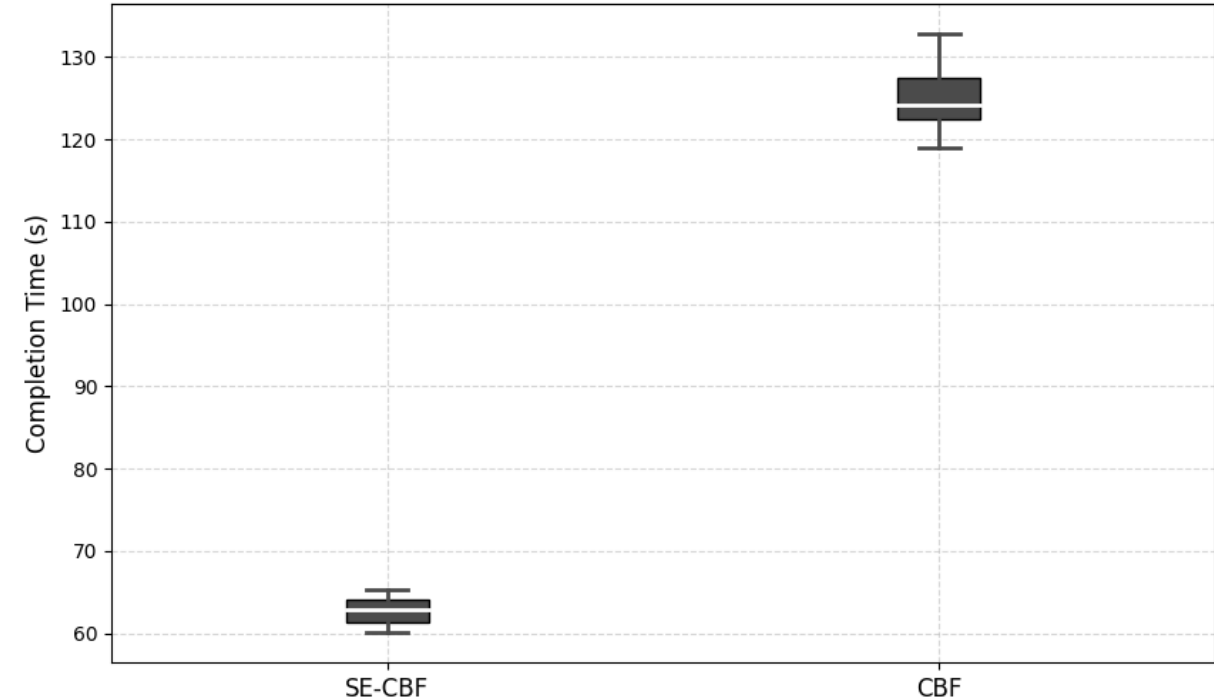
Comparison of Mean Speeds: SE-CBF vs CBF



- SE-CBF robot operates at approximately twice the speed of the CBF robot
- SE-CBF significantly enhances operating speed

## Completion Time Boxplot

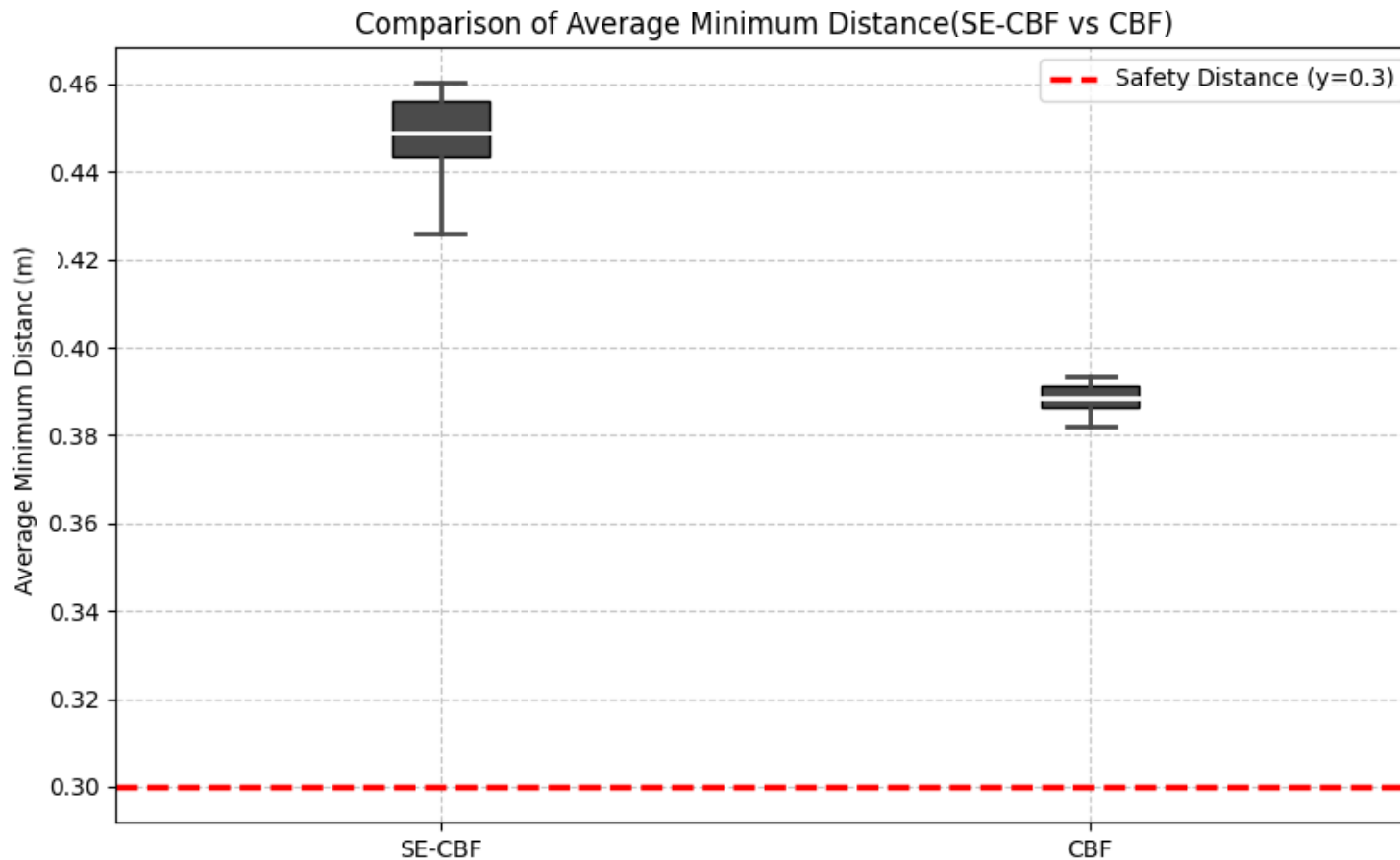
Comparison of Completion Times: SE-CBF vs CBF



- SE-CBF completes tasks in approximately half the time compared to the CBF robot
- SE-CBF method improves task efficiency

# Simulation Results and Analysis

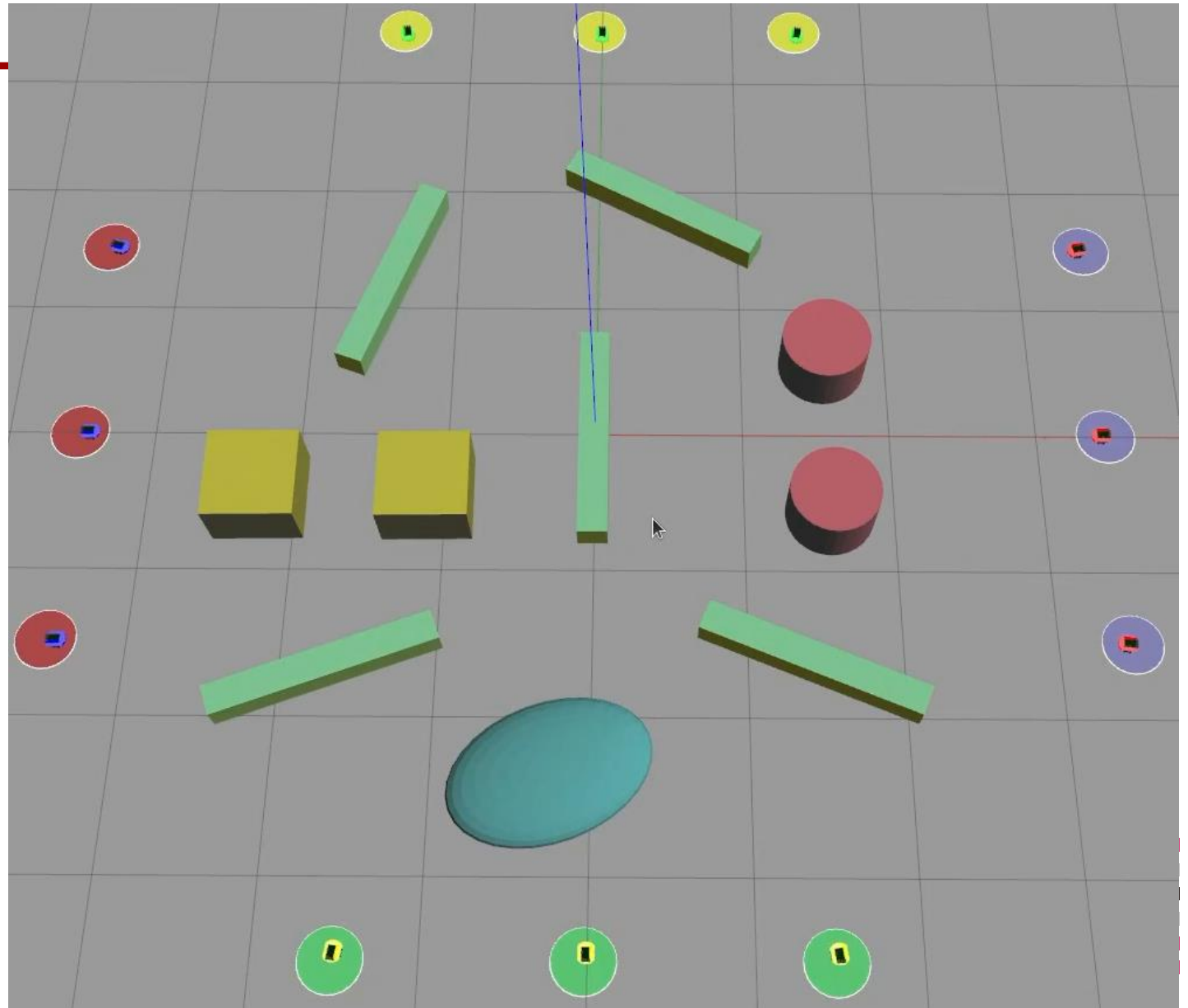
Average Minimum Distance Boxplot



- The SE-CBF method maintains a higher average minimum distance (0.45 m) compared to the CBF method (0.35 m)
- SE-CBF consistently provides a larger safety margin
- SE-CBF method enhanced adaptability and reliability in maintaining safe trajectories

# Unknown area exploration

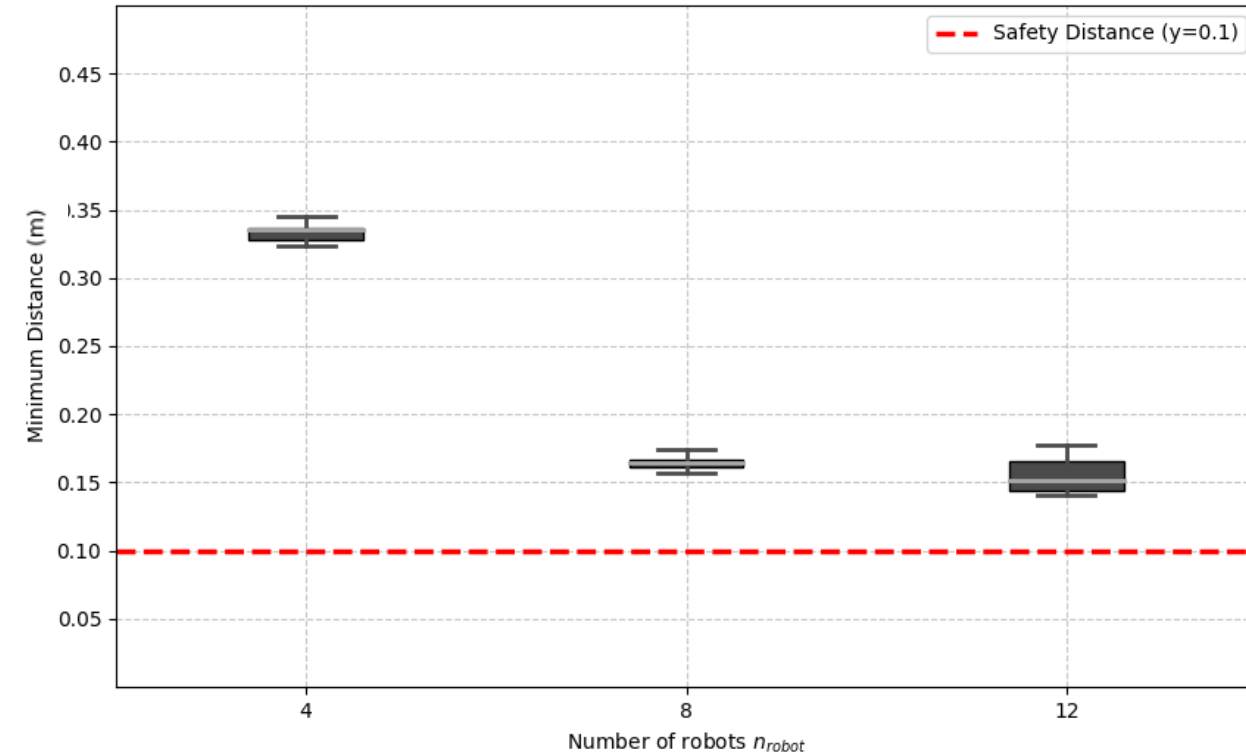
- Obstacles are randomly placed in the central area
- Test the performance of path-planning and collision avoidance



# Simulation Results and Analysis

## Minimum distance

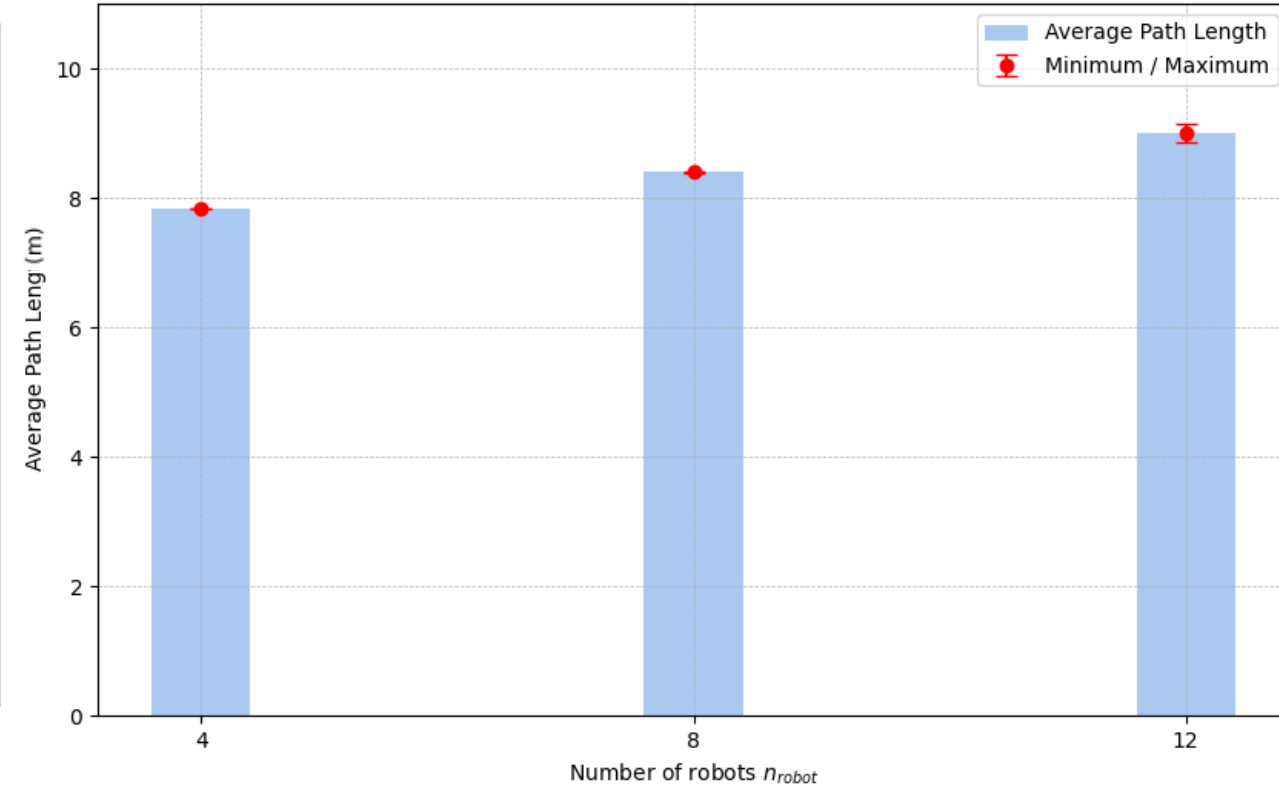
Comparison of Minimum Distance (4robots, 8robots, 12robots)



- SE-CBF demonstrates excellent adaptability while maintaining distances above the safety threshold.

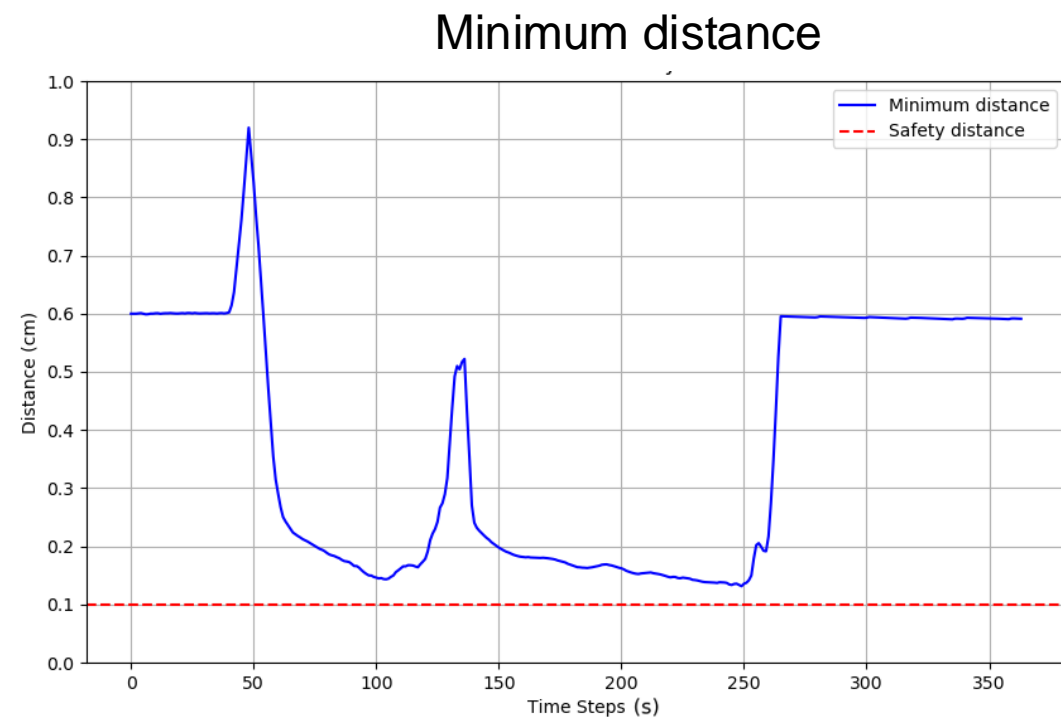
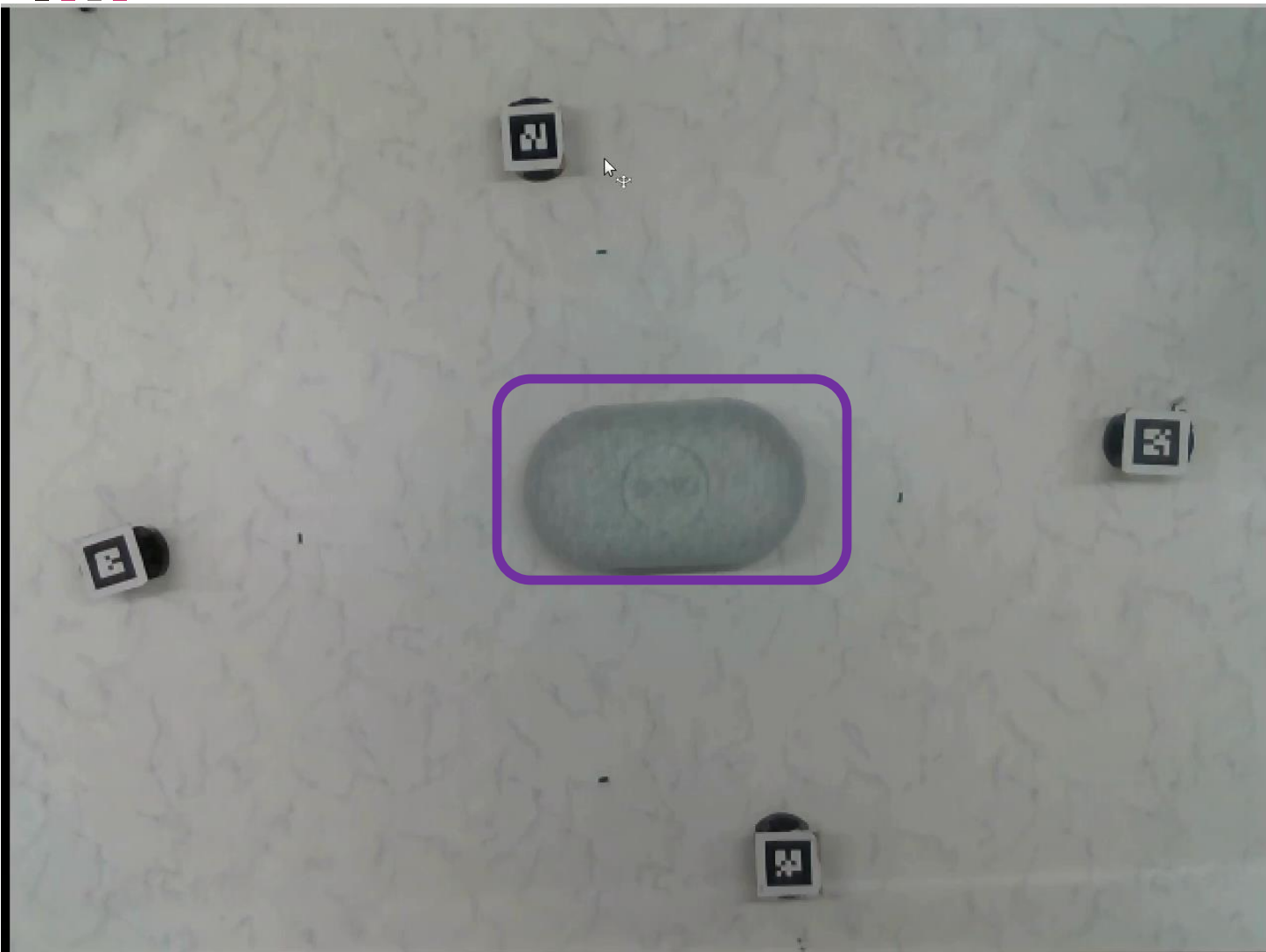
## Average Path Length

Average Path Length vs. Number of Robots



- SE-CBF maintains a consistent average path length and demonstrates higher adaptability and efficient path planning

# Real-world Experiment



# Summary

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- Introduced Super-Ellipse CBF collision avoidance technique
- Developed a dynamic safety distance framework
- Improved collision avoidance efficiency and system performance
- Enhanced the system applicability in various scenarios



# Reference

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## Reference:

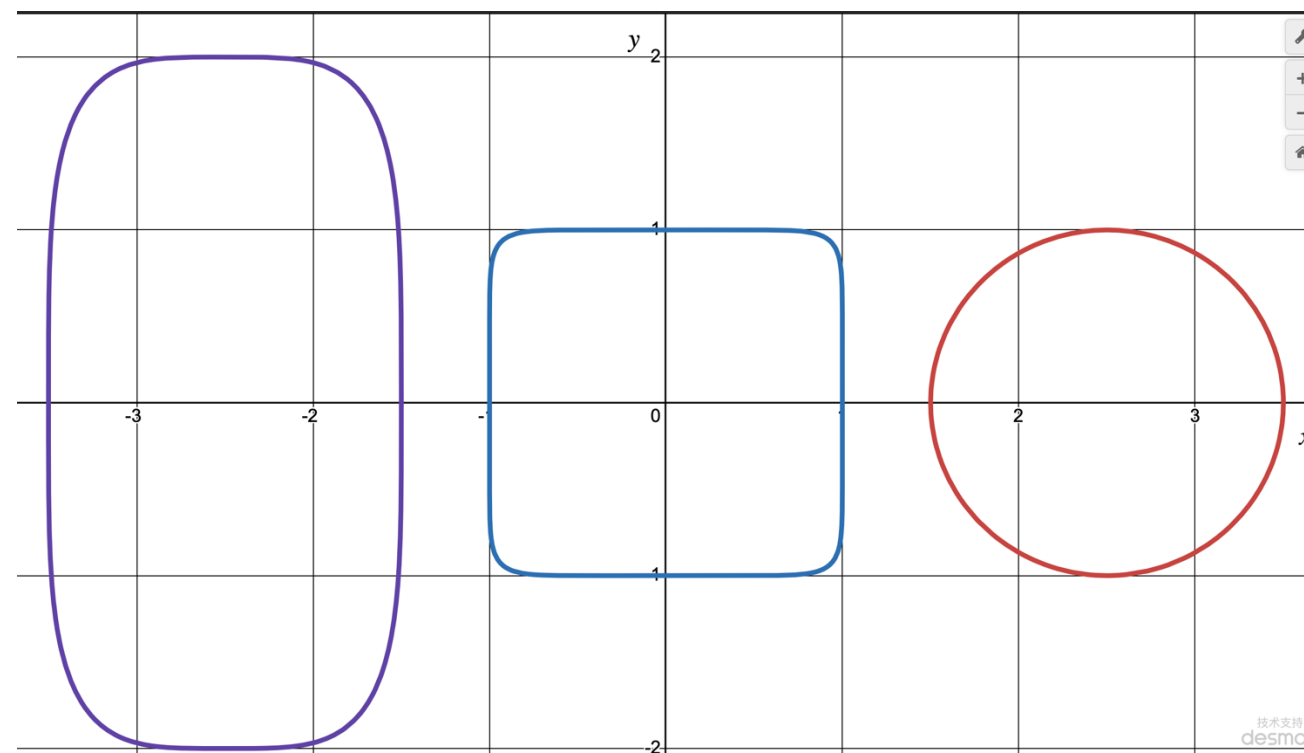
1. Ames, A. D., Coogan, S., Egerstedt, M., Notomista, G., Sreenath, K., & Tabuada, P. (2019). Control barrier functions: Theory and applications. arXiv. <https://arxiv.org/abs/1903.11199>
2. Feng, P., Liang, J., Wang, S., Yu, X., Ji, X., Chen, Y., Zhang, K., Shi, R., & Wu, W. (2024). Hierarchical consensus-based multi-agent reinforcement learning for multi-robot cooperation tasks. arXiv. <https://arxiv.org/abs/2407.08164>
3. Srinivasan, M., Abate, M., Nilsson, G., & Coogan, S. (2021). Extent-compatible control barrier functions. *Systems & Control Letters*, 150, 104895. <https://doi.org/10.1016/j.sysconle.2021.104895>

# Algorithm

## Super-ellipse :

- Mathematical curve
- Flexible in shape circle, oval, rectangular, square
- Respond to varying circumstances

$$\left(\frac{x}{\alpha}\right)^n + \left(\frac{y}{\beta}\right)^n = 1$$



$$\begin{aligned} n &= 4 \\ \alpha &= 1 \\ \beta &= 2 \end{aligned}$$

$$\begin{aligned} n &= 10 \\ \alpha &= 1 \\ \beta &= 1 \end{aligned}$$

$$\begin{aligned} n &= 2 \\ \alpha &= 1 \\ \beta &= 1 \end{aligned}$$

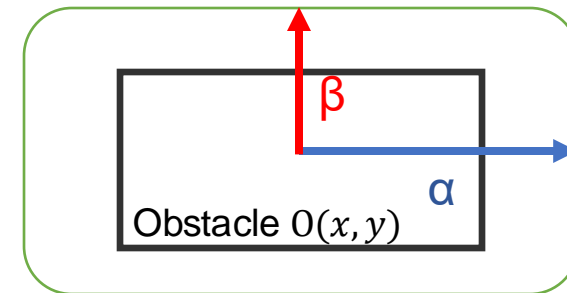
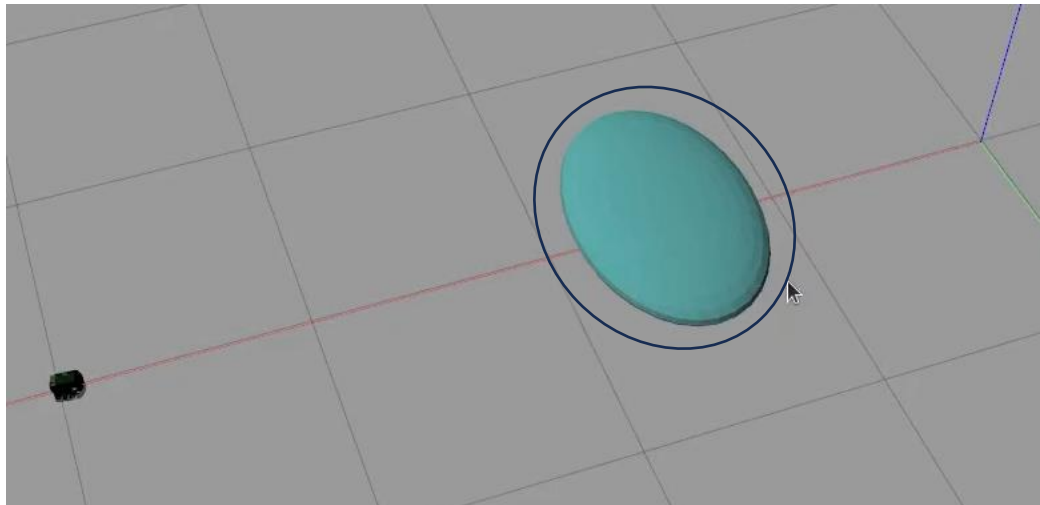
# Appendix Super-Ellipse CBF:

$$\left(\frac{P_x - O_x}{\alpha}\right)^n + \left(\frac{P_y - O_y}{\beta}\right)^n = 1$$

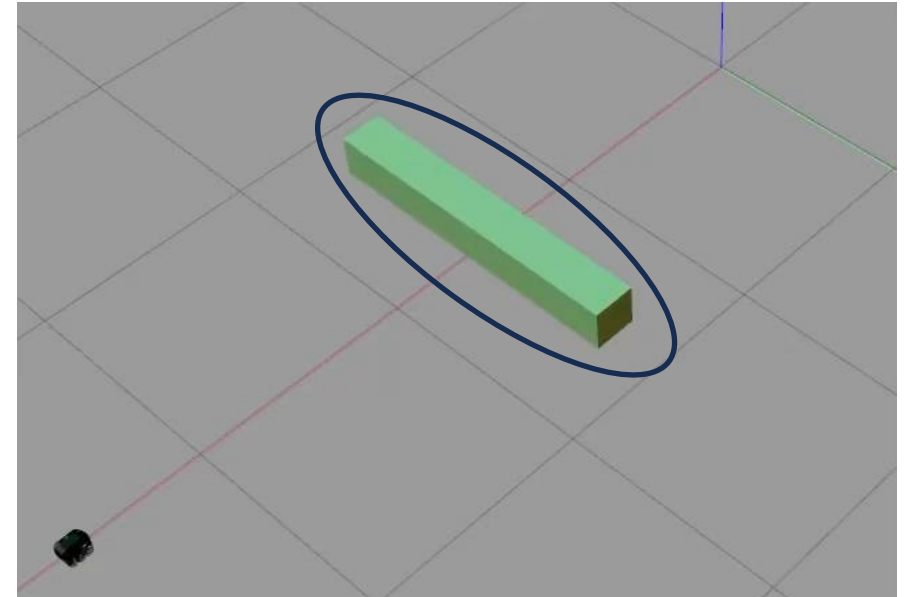
$P_x$  and  $P_y$ : The position of robots

$O_x$  and  $O_y$ : The position of obstacle

$\alpha$  and  $\beta$ : Scaling factors in the x and y directions

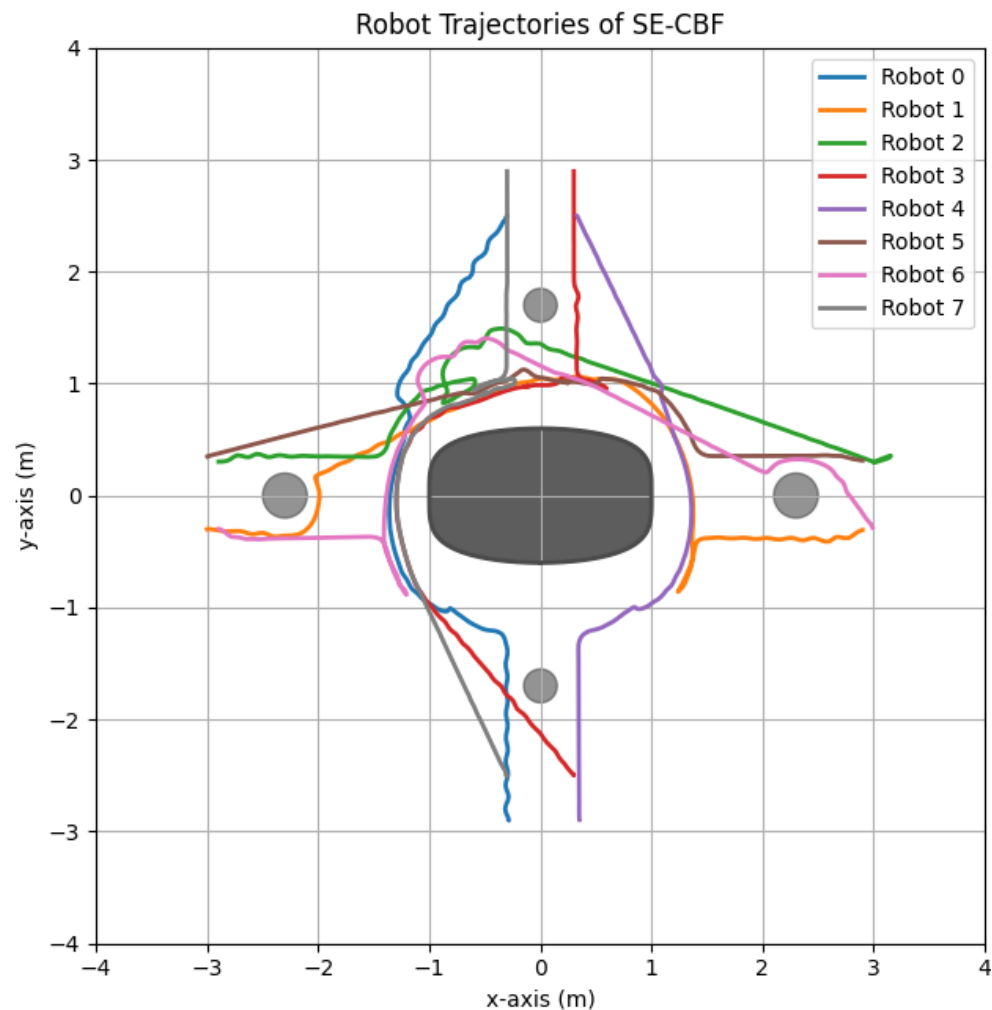


Agent  
 $P(x, y)$

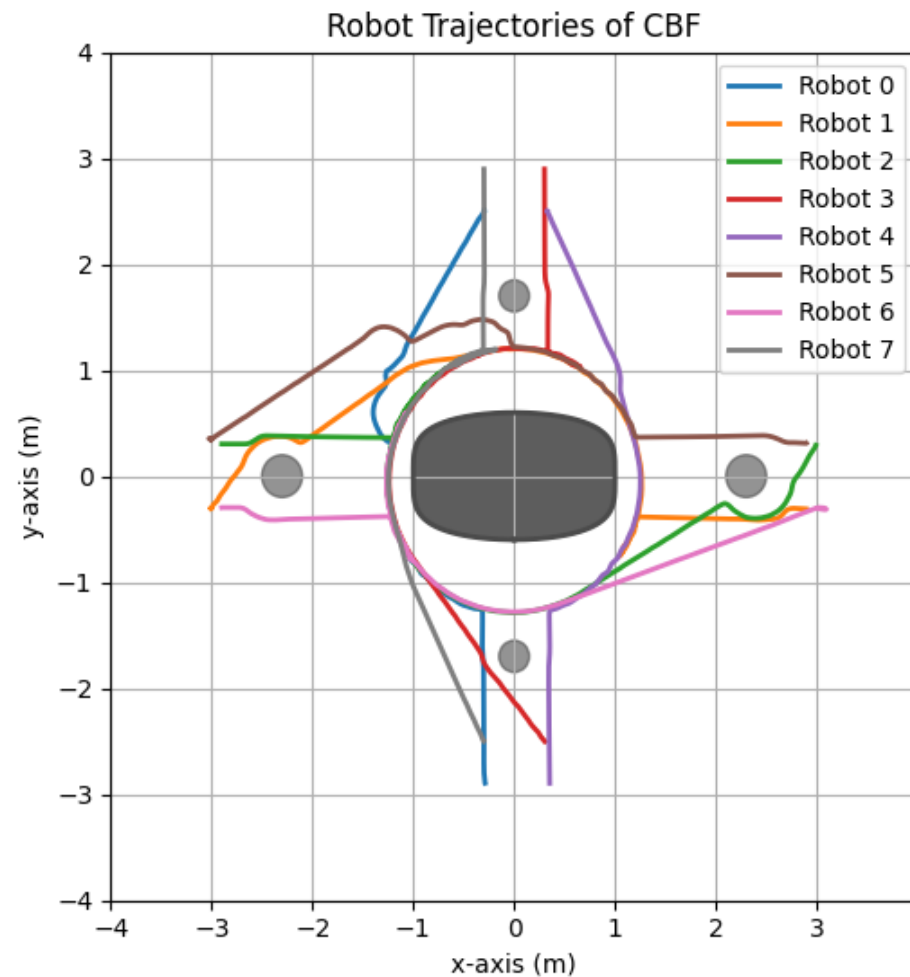


# Simulation of Intersection

## Path diagram of SE-CBF

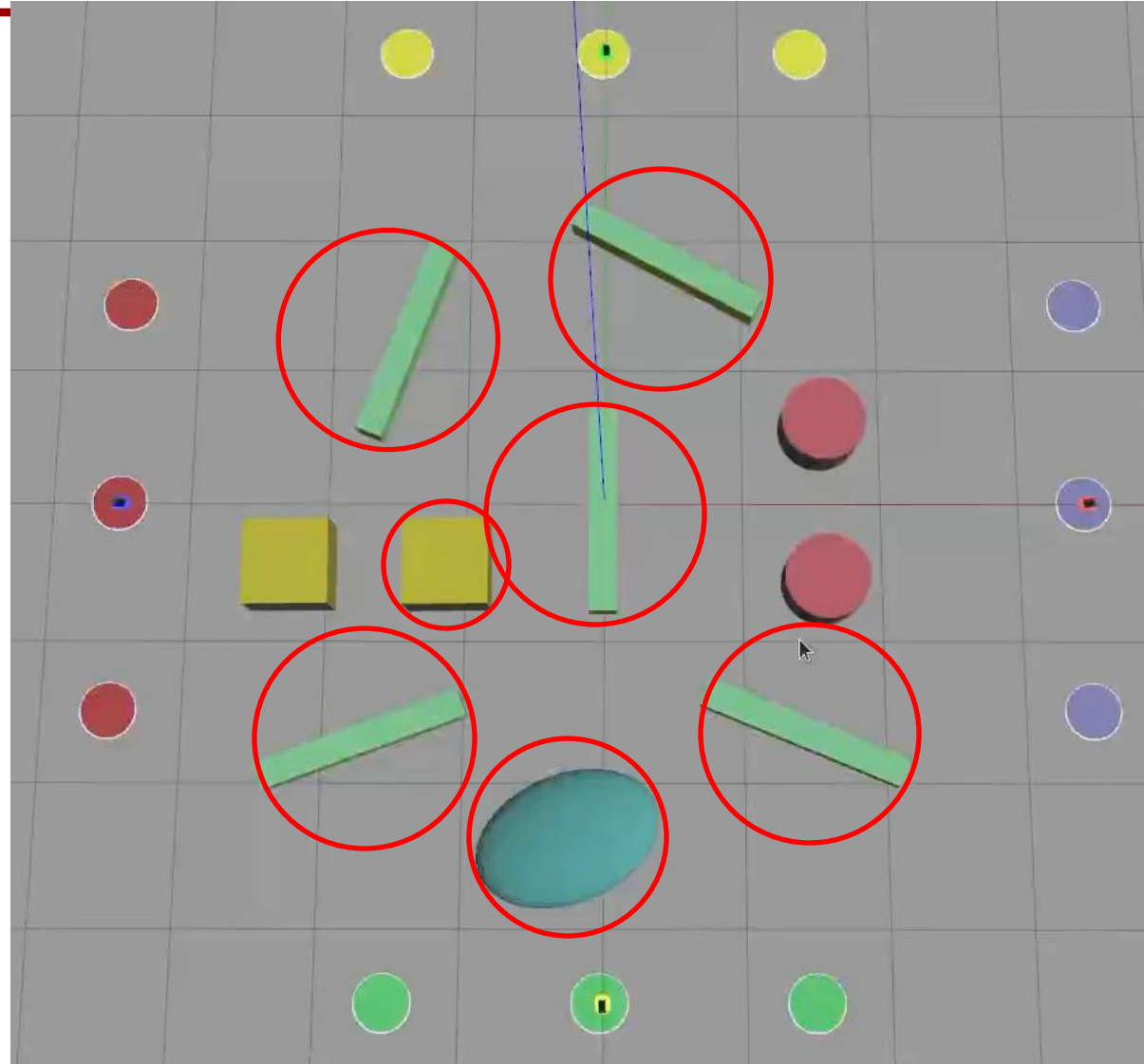


## Path diagram of CBF



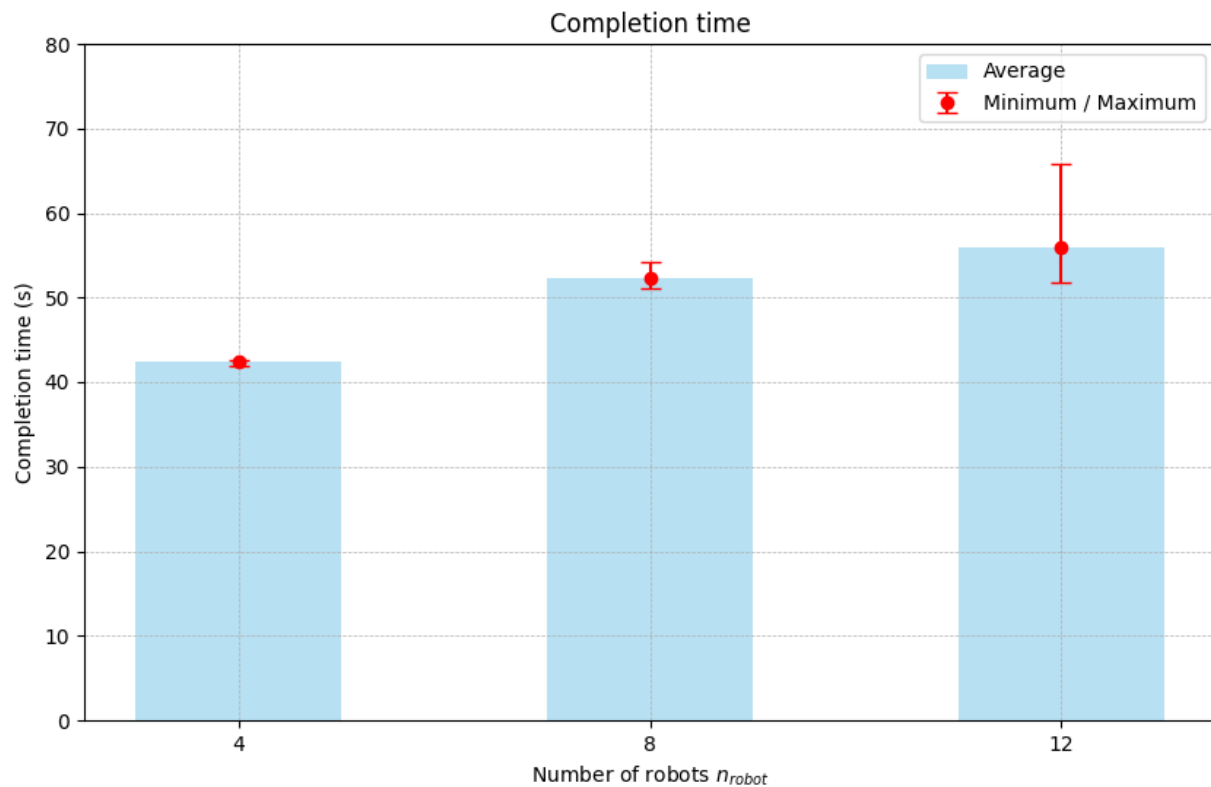
# Unknown area exploration

- CBF can not work due to space limitation
- Robots do not have adequate space for safe movement

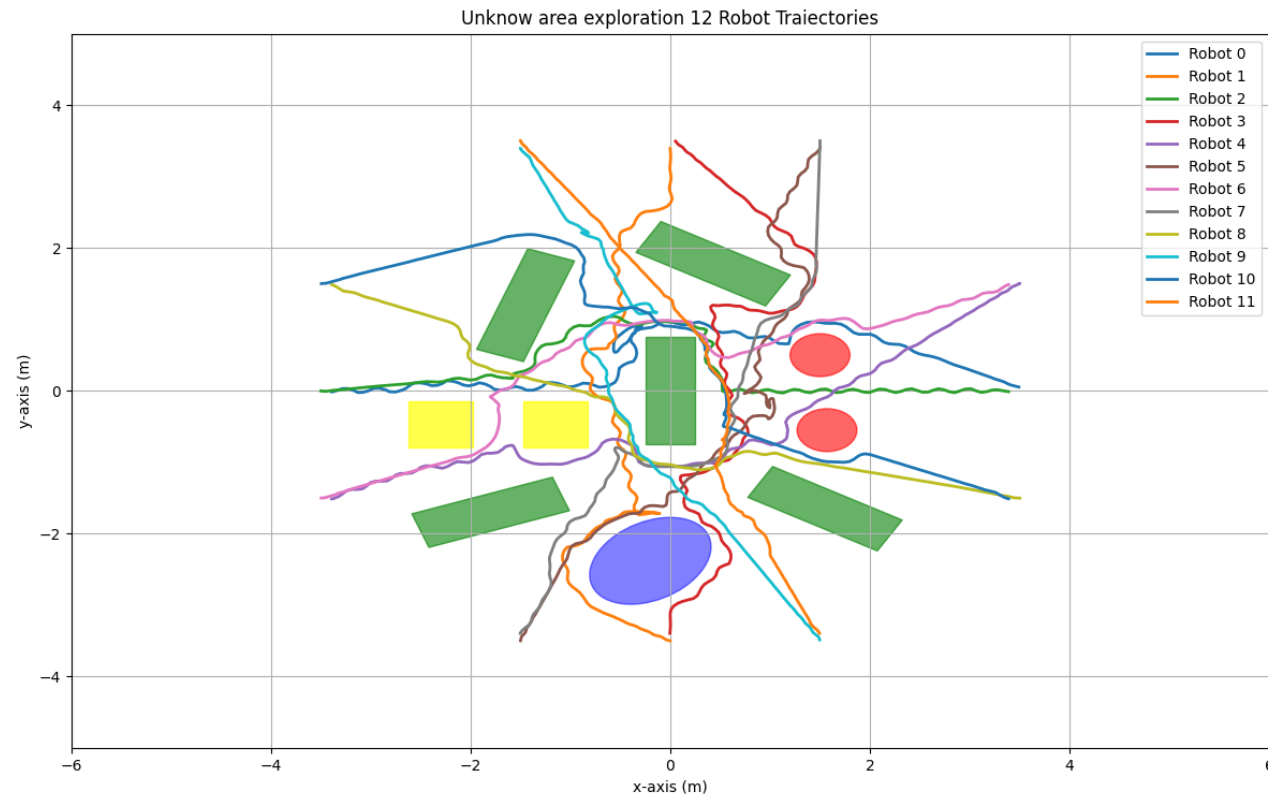


# Simulation Results and Analysis

## Task Completion Time



## Path diagram



# Appendix

Experiment setup:

