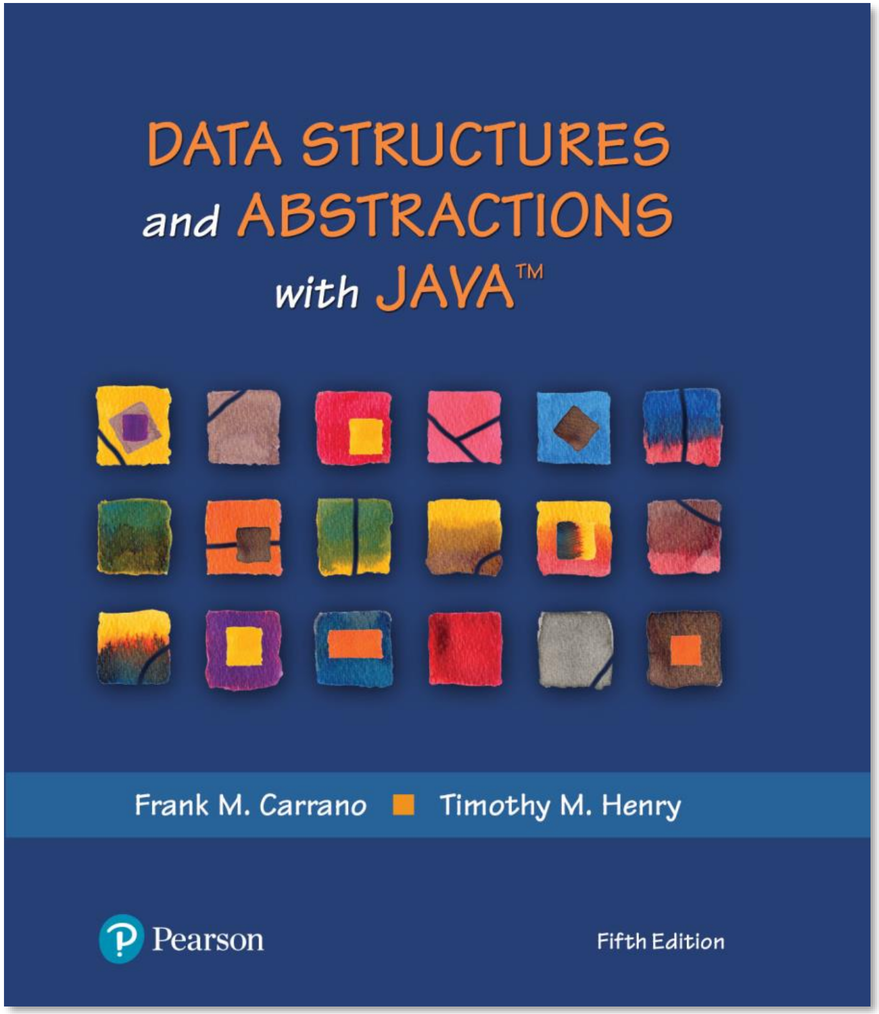
Data Structures and Abstractions with Java™

5th Edition



Chapter 14

Problem Solving with Recursion

Pearson

Simple Solution to a Difficult Problem

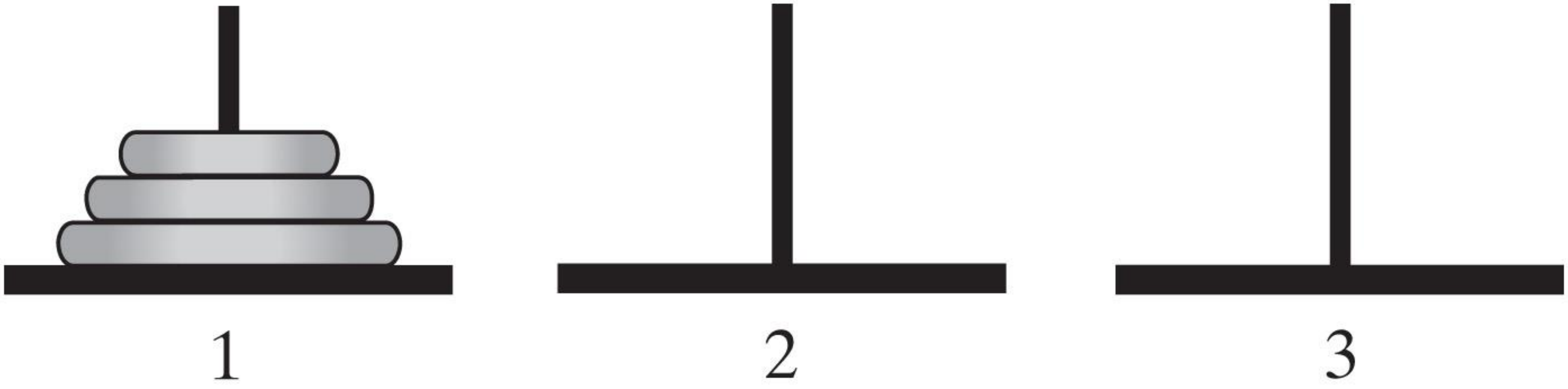
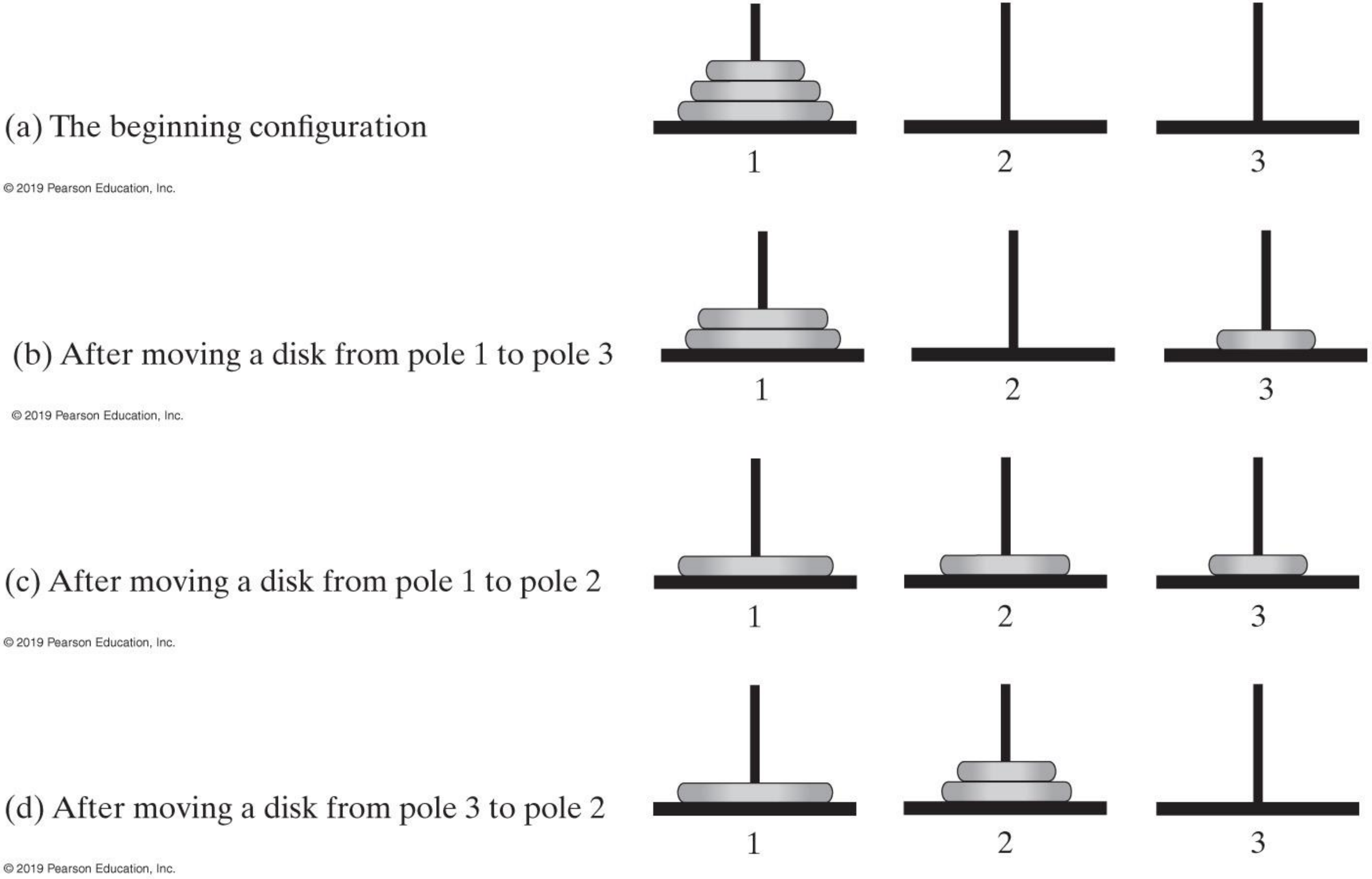


FIGURE 14-1 The initial configuration of the Towers of Hanoi for three disks

Simple Solution to a Difficult Problem

- Rules:
  - Move one disk at a time. Each disk moved must be the topmost disk.
  - No disk may rest on top of a disk smaller than itself.
  - You can store disks on the second (extra) pole temporarily, as long as you observe the previous two rules.

Simple Solution to a Difficult Problem (Part 1)



(a) The beginning configuration

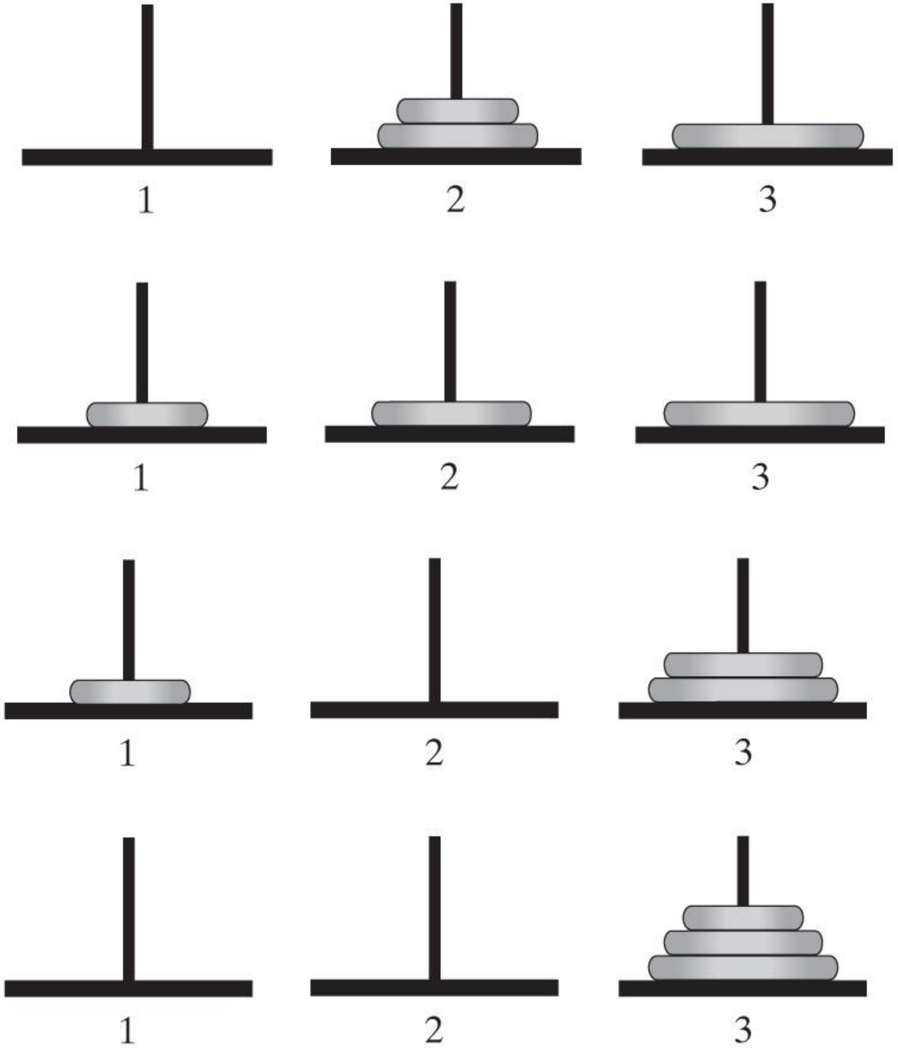
(b) After moving a disk from pole 1 to pole 3

(c) After moving a disk from pole 1 to pole 2

(d) After moving a disk from pole 3 to pole 2

FIGURE 14-2 Sequence of moves for solving Towers of Hanoi problem with 3 disks

Simple Solution to a Difficult Problem (Part 2)



(e) After moving a disk from pole 1 to pole 3

(f) After moving a disk from pole 2 to pole 1

(g) After moving a disk from pole 2 to pole 3

(h) After moving a disk from pole 1 to pole 3

FIGURE 14-2 Sequence of moves for solving Towers of Hanoi problem with 3 disks

A Smaller Problem

solveTowers(number of Disks, startPle, tempPl, endPl)

if (1)

(a) The original configuration

else

(b) After your friend moves three disks from pole 1 to pole 2

(c) After you move one disk from pole 1 to pole 3

(d) After your friend moves three disks from pole 2 to pole 3

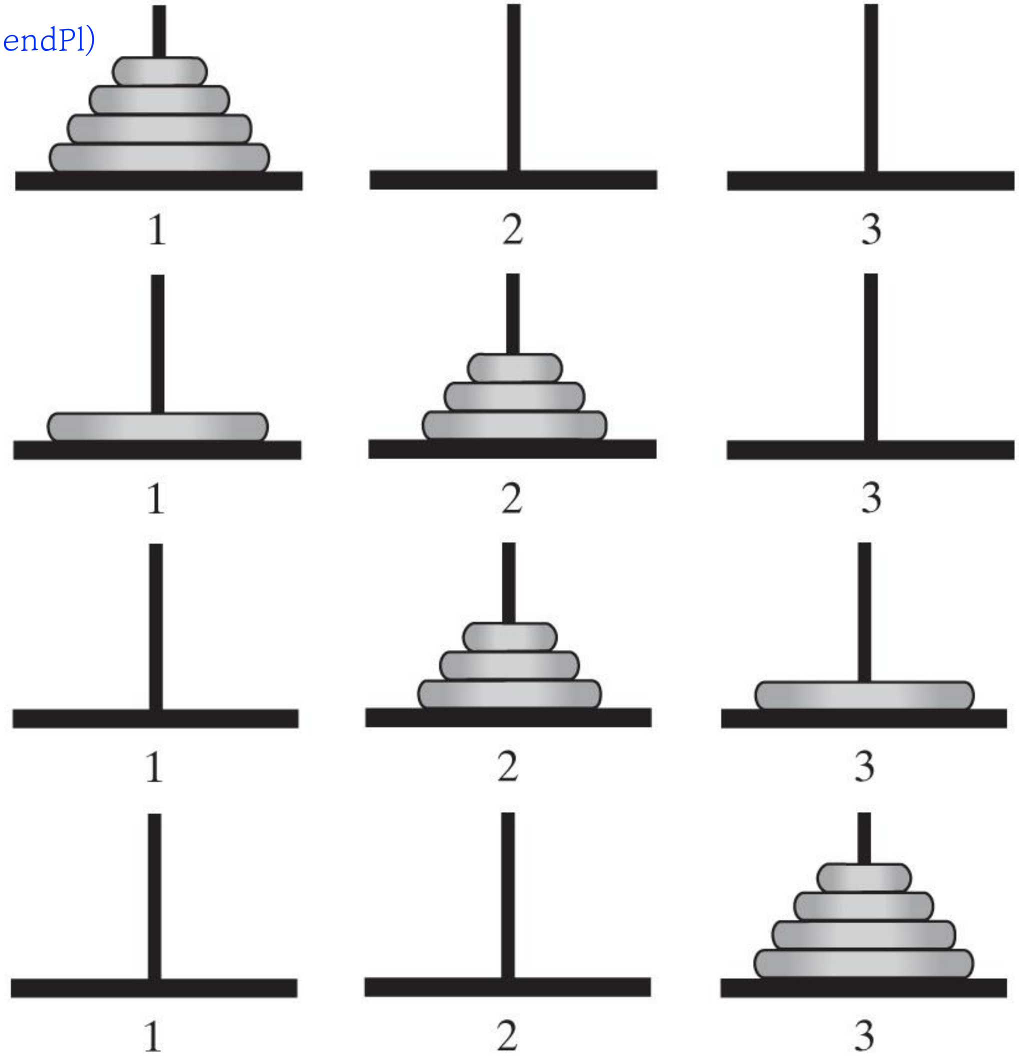


FIGURE 14-3 The smaller problems in a recursive solution for four disks

Solutions

```
Algorithm to move numberOfDisks disks from startPole to endPole using tempPole
   as a spare according to the rules of the Towers of Hanoi problem
if (numberOfDisks == 1)
   Move disk from startPole to endPole
else
{                                                          middle one
    Move all but the bottom disk from startPole to tempPole
    Move disk from startPole to endPole
    Move all disks from tempPole to endPole
}
```

Recursive algorithm to solve any number of disks.

Note: for n disks, solution will be $2^n – 1$ moves

Poor Solution to a Simple Problem

$$F_0 = 1$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2} \text{ when } n \geq 2$$

$1, 1, 2, 3, 5, 8, 13, 21, \cdots$

```
*Algorithm* Fibonacci(n) if (n <= 1)
  return 1
else
  return Fibonacci(n − 1) + Fibonacci(n − 2)
```

Algorithm to generate Fibonacci numbers.
Why is this inefficient?

Pearson

Poor Solution to a Simple Problem

(a) Recursively

F_2 is computed 5 times
F_3 is computed 3 times
F_4 is computed 2 times
F_5 is computed once
F_6 is computed once

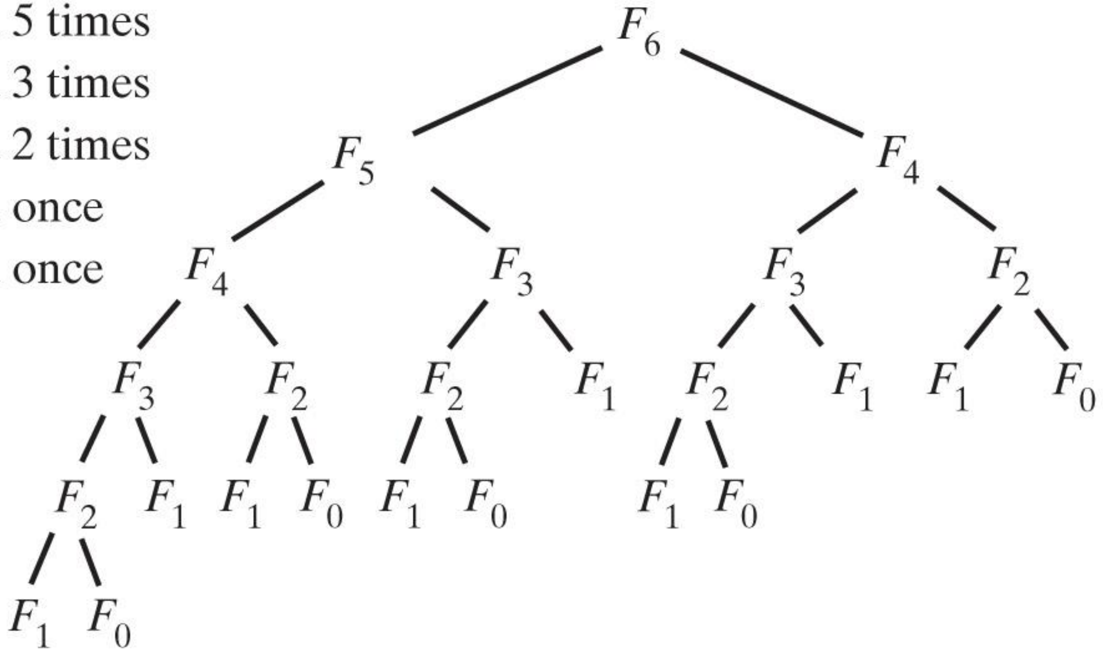
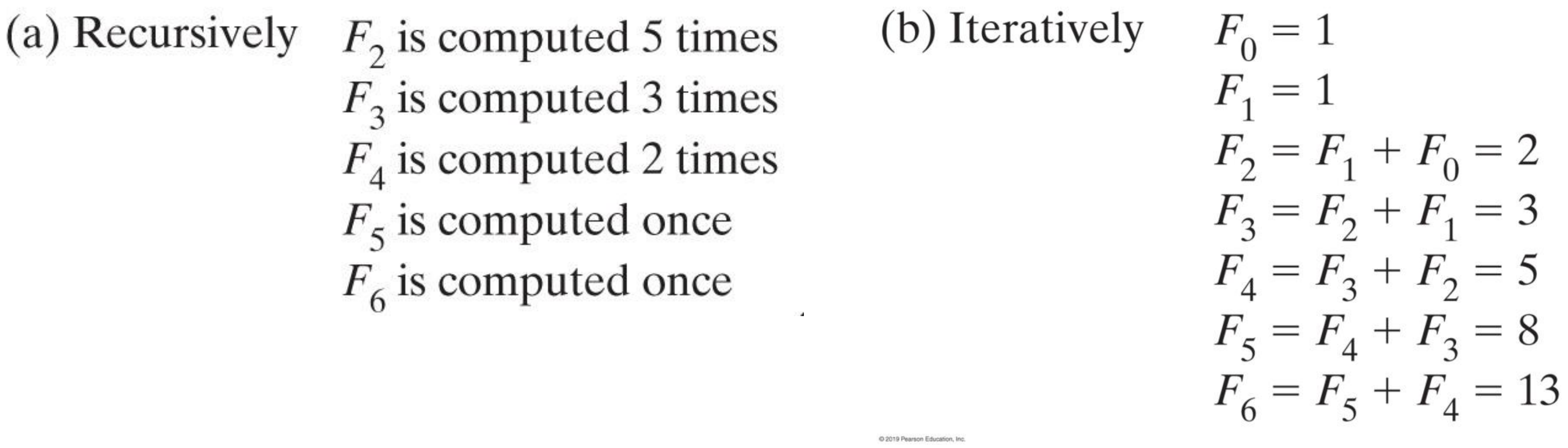

F_6
F_5 F_4
F_4 F_3 F_3 F_2
F_3 F_2 F_2 F_1 F_2 F_1 F_1 F_0
F_2 F_1 F_1 F_0 F_1 F_0 F_1 F_0
F_1 F_0

FIGURE 14-4a The computation of the Fibonacci number F6

Poor Solution to a Simple Problem



(a) Recursively

F_2 is computed 5 times

F_3 is computed 3 times

F_4 is computed 2 times

F_5 is computed once

F_6 is computed once

(b) Iteratively

$$
\begin{aligned}
F_0 &= 1\\
F_1 &= 1\\
F_2 &= F_1 + F_0 = 2\\
F_3 &= F_2 + F_1 = 3\\
F_4 &= F_3 + F_2 = 5\\
F_5 &= F_4 + F_3 = 8\\
F_6 &= F_5 + F_4 = 13
\end{aligned}
$$

FIGURE 14-4b The computation of the Fibonacci number F6

Indirect Recursion

- Example
  - Method A calls Method B
  - Method B calls Method C
  - Method C calls Method A

- Difficult to understand and trace
  - But does happen occasionally

Indirect Recursion

- Consider evaluation of validity of an algebraic expression
  - Algebraic expression is either a term or two terms separated by a + or – operator
  - Term is either a factor or two factors separated by a * or / operator
  - Factor is either a variable or an algebraic expression enclosed in parentheses
  - Variable is a single letter

Indirect Recursion

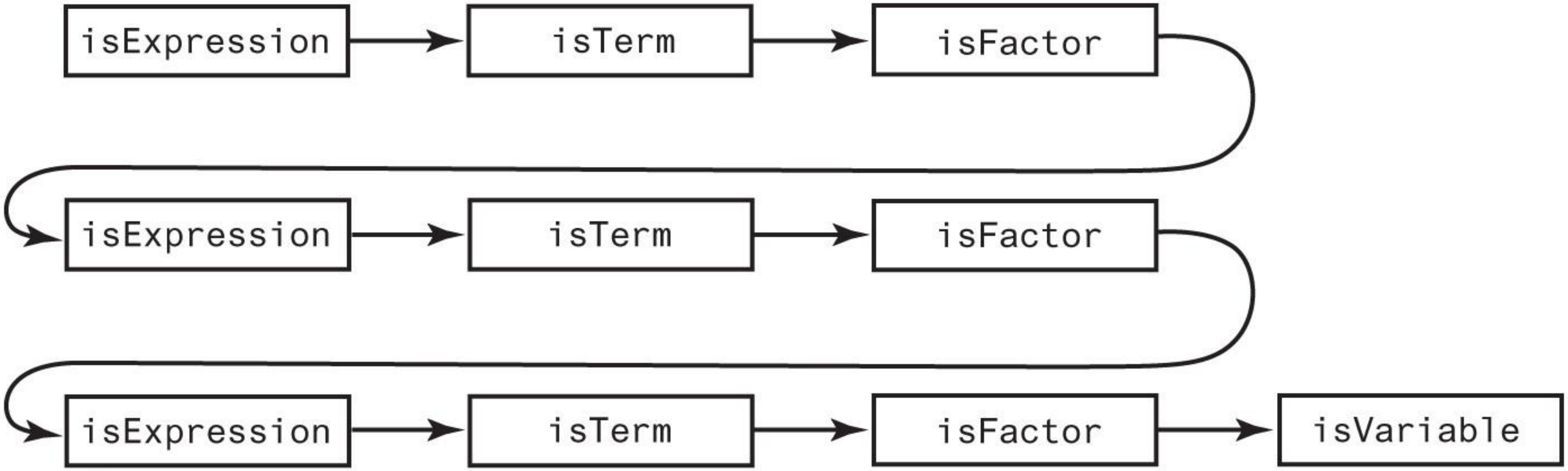


FIGURE 14-5 An example of indirect recursion

Backtracking

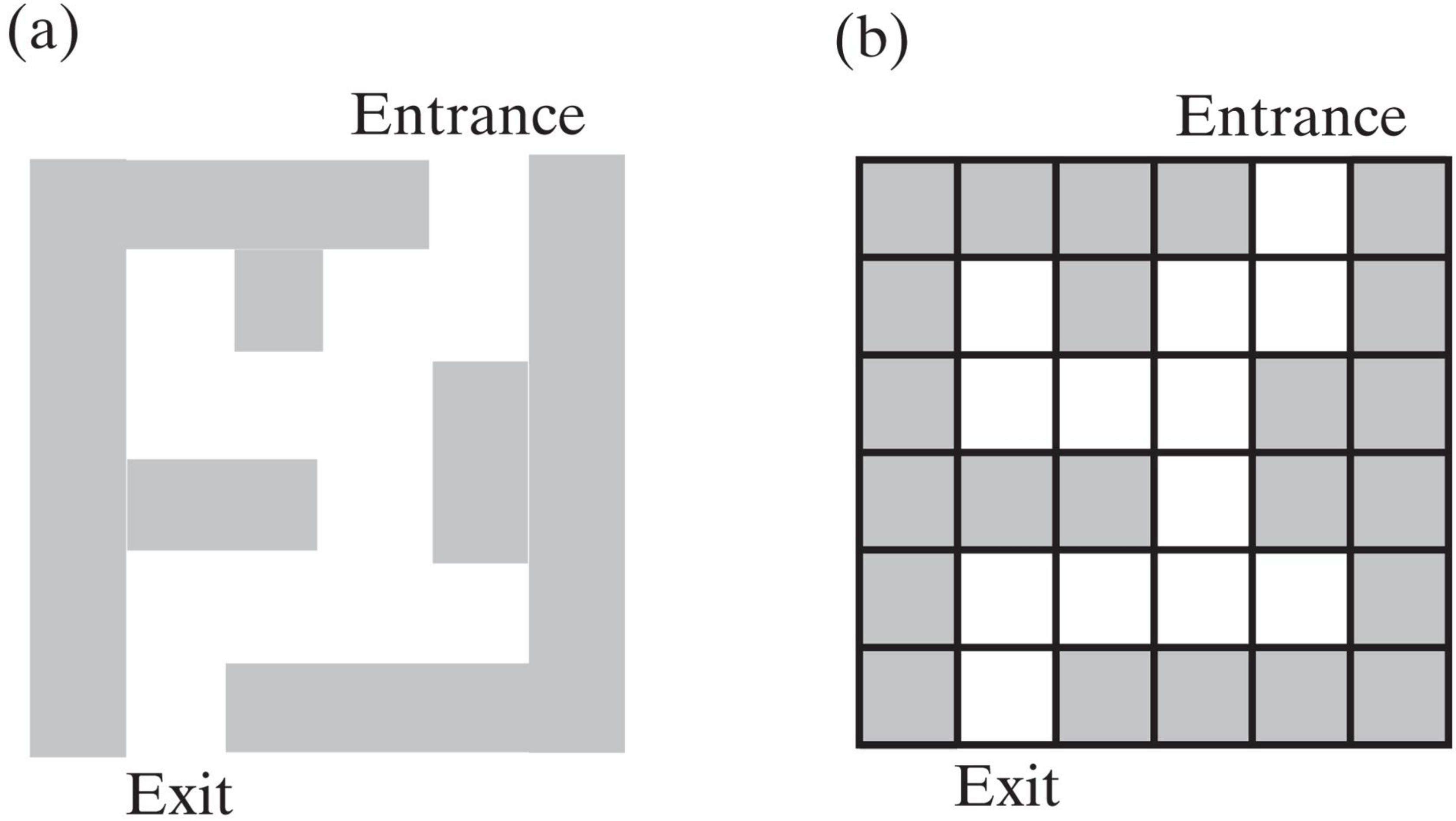


FIGURE 14-6 A two-dimensional maze with one entrance and one exit

Backtracking

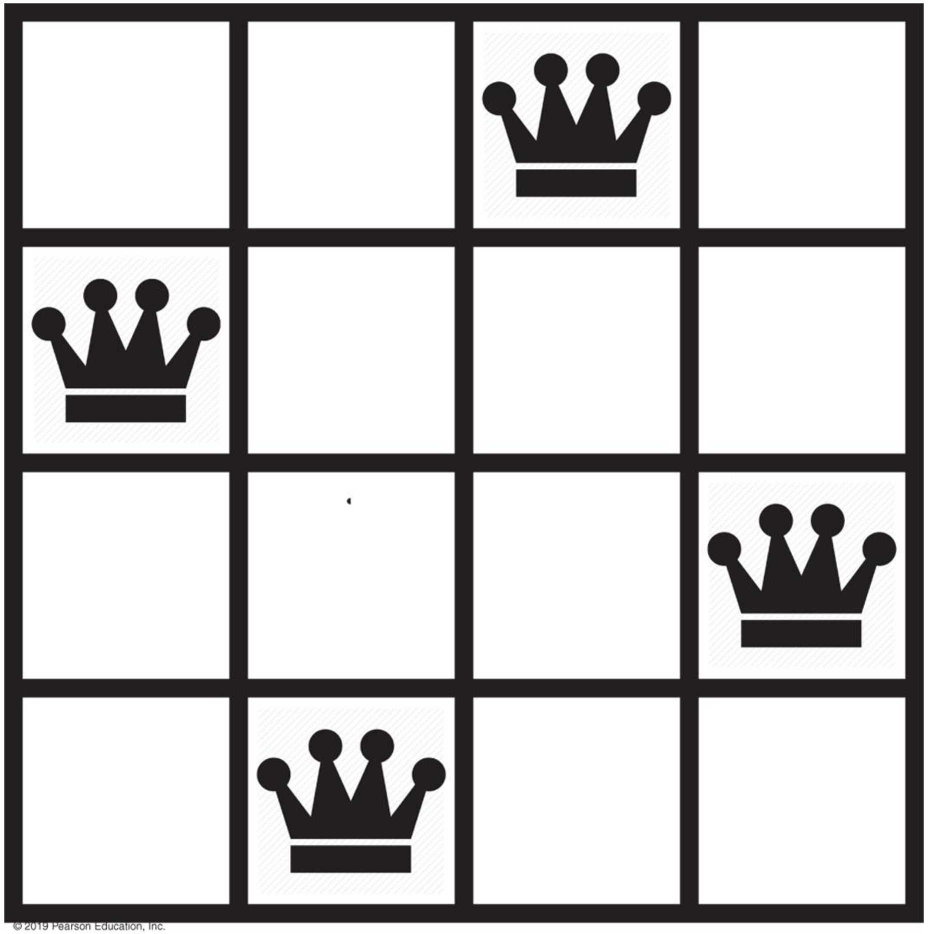
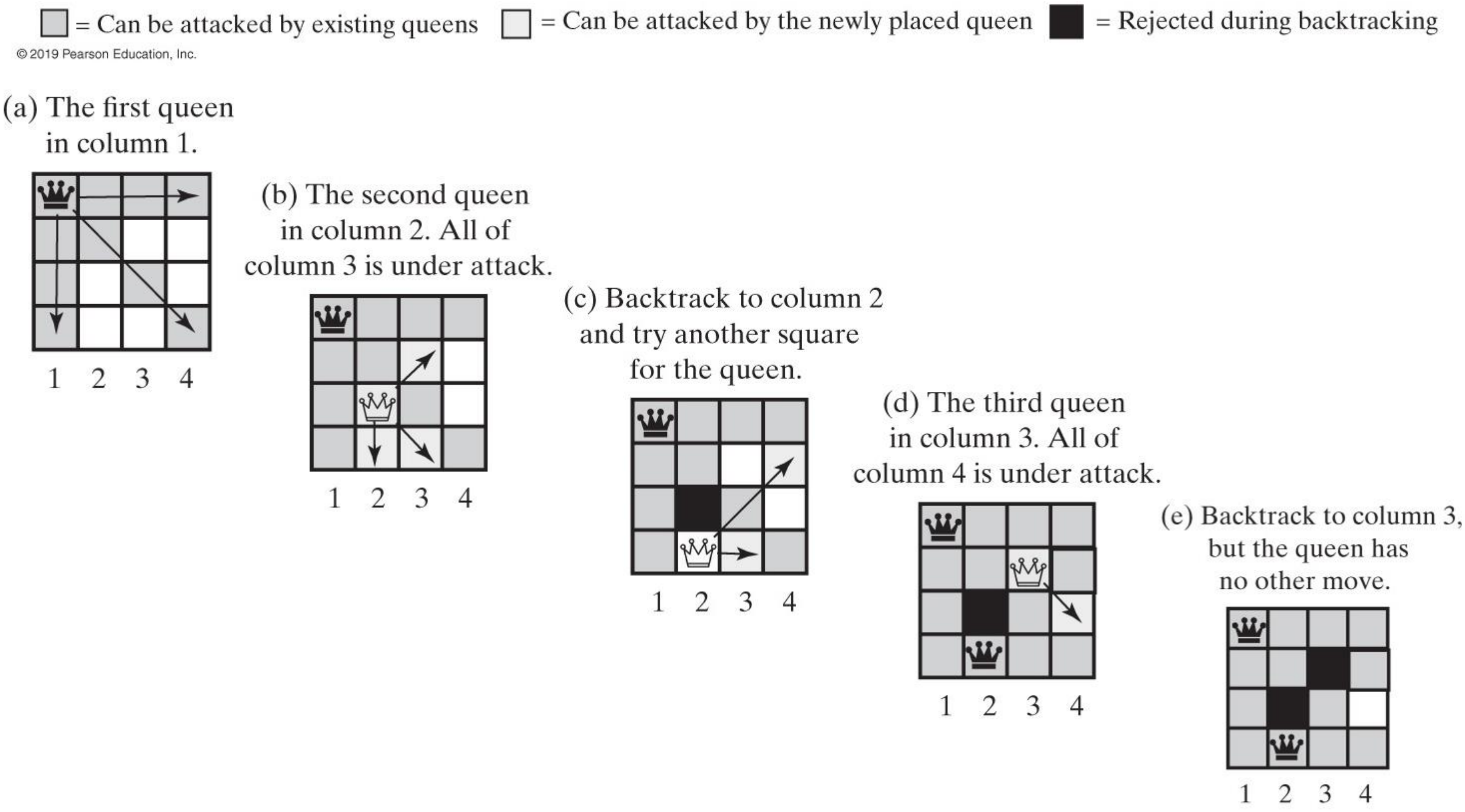


FIGURE 14-7 A solution to the four-queens problem

Backtracking - Queens Solution (Part 1)



▨ = Can be attacked by existing queens □ = Can be attacked by the newly placed queen ■ = Rejected during backtracking

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(a) The first queen in column 1.

(b) The second queen in column 2. All of column 3 is under attack.

(c) Backtrack to column 2 and try another square for the queen.

(d) The third queen in column 3. All of column 4 is under attack.

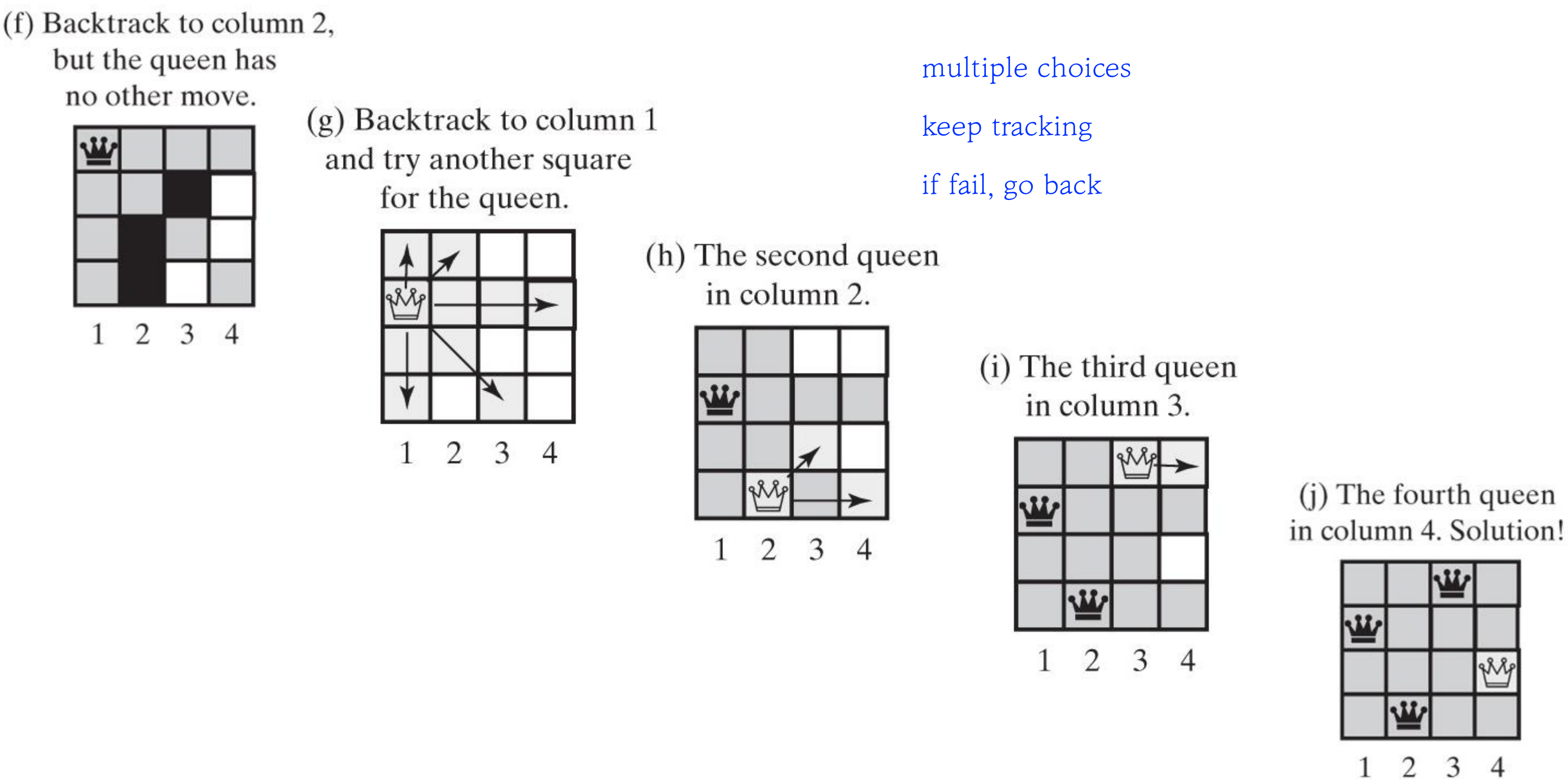
(e) Backtrack to column 3, but the queen has no other move.

FIGURE 14-8 Solving the four-queens problem by placing one queen at a time in each column

Pearson

Backtracking - Queens Solution (Part 2)

▨ = Can be attacked by existing queens □ = Can be attacked by the newly placed queen ■ = Rejected during backtracking



(f) Backtrack to column 2, but the queen has no other move.

(g) Backtrack to column 1 and try another square for the queen.

(h) The second queen in column 2.

(i) The third queen in column 3.

(j) The fourth queen in column 4. Solution!

multiple choices

keep tracking

if fail, go back

FIGURE 14-8 Solving the four-queens problem by placing one queen at a time in each column

Pearson

End

Chapter 14