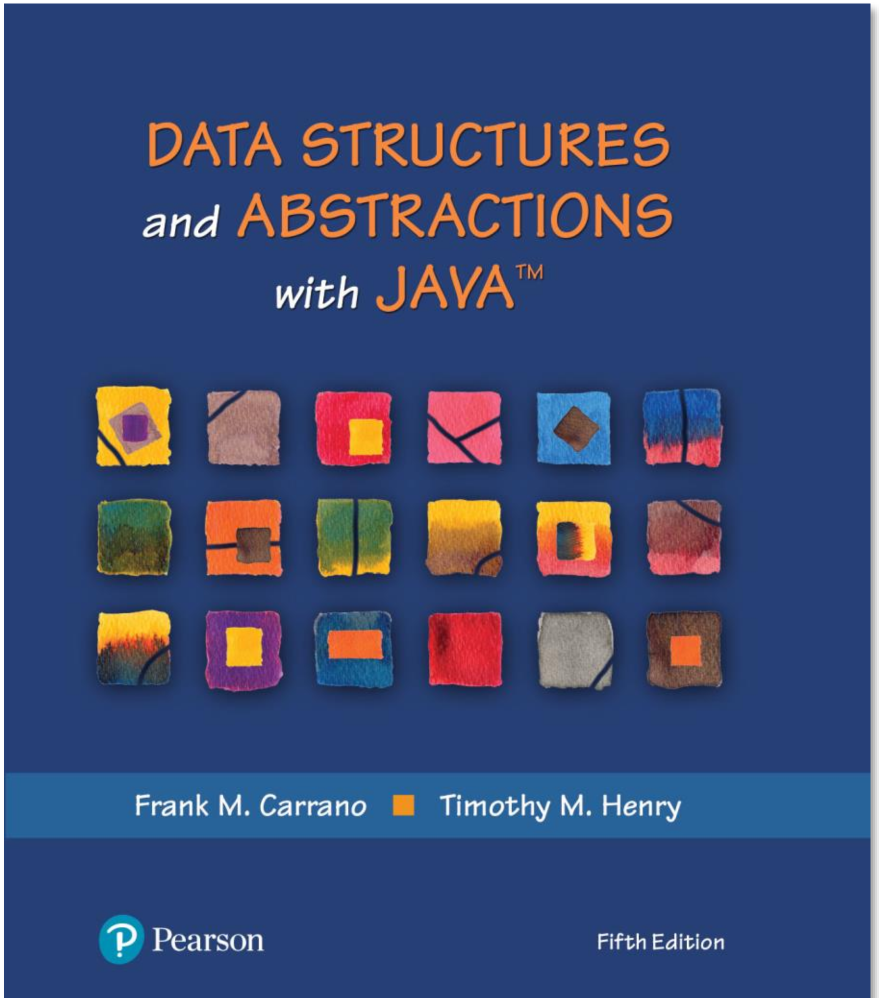
# Data Structures and Abstractions with Java™

5th Edition




# Chapter 4

# The Efficiency of Algorithms

# Why Efficient Code?

- Computers are faster, have larger memories
  - So why worry about efficient code?
- And … how do we measure efficiency?

# Importance of Efficiency

- Consider the problem of summing

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \ldots + n$$

| Algorithm A | Algorithm B | Algorithm C |
|---|---|---|
| `long sum = 0;`<br>`for (long i = 1; i <= n; i++)`<br>`   sum = sum + i;` | `sum = 0;`<br>`for (long i = 1; i <= n; i++)`<br>`{`<br>`   for (long j = 1; j <= i; j++)`<br>`      sum = sum + 1;`<br>`} // end for` | `sum = n * (n + 1) / 2;` |

**FIGURE 4-1**
**Three algorithms for computing the sum 1 + 2 + . . . + n for an integer n > 0**

# What is “best”?

- An algorithm has both time and space constraints – that is complexity
  - Time complexity
  - Space complexity
- This study is called analysis of algorithms

# Counting Basic Operations

- A basic operation of an algorithm
  - Most significant contributor to its total time requirement

| | Algorithm A | Algorithm B | Algorithm C |
|---|---|---|---|
| | `long sum = 0;`<br>`for (long i = 1; i <= n; i++)`<br>`   sum = sum + i;` | `sum = 0;`<br>`for (long i = 1; i <= n; i++)`<br>`{`<br>`   for (long j = 1; j <= i; j++)`<br>`      sum = sum + 1;`<br>`} // end for` | `sum = n * (n + 1) / 2;` |
| Additons | $n$ | $n(n + 1)/2$ | **1** |
| Multiplications | **0** | **0** | **1** |
| Divisions | **0** | **0** | **1** |
| Total Basic Operations | $n$ | $(n^2 + n)/2$ | **3** |

**FIGURE 4-2 The number of basic operations required by the algorithms**

# Counting Basic Operations

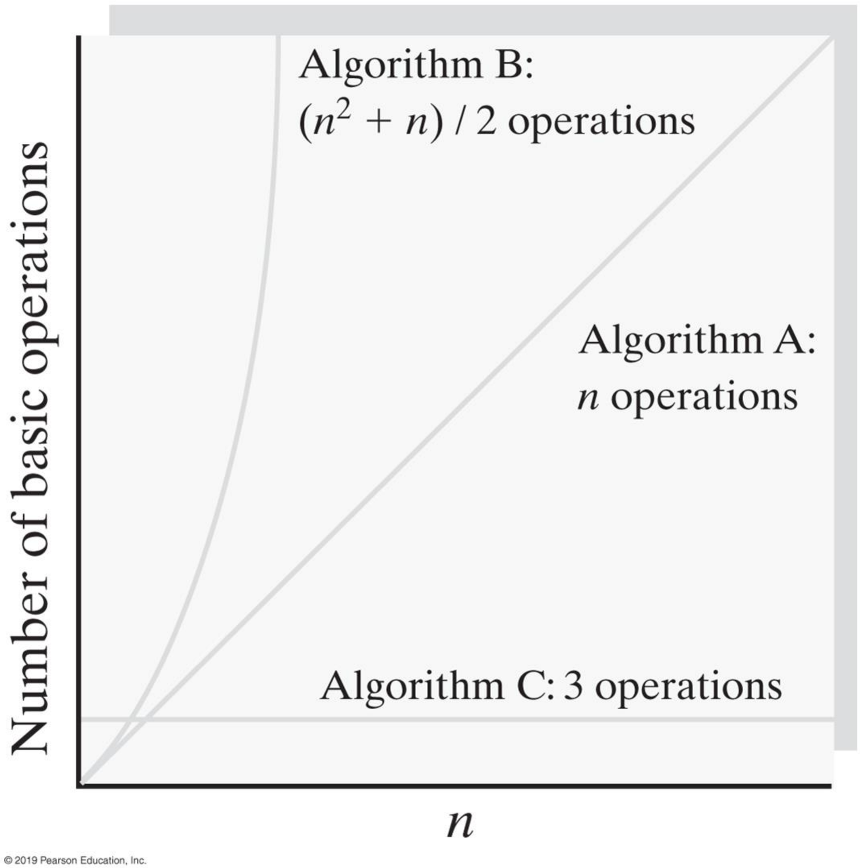



**FIGURE 4-3 Number of basic operations required by the algorithms as a function of *n***

# Counting Basic Operations

| $n$ | (log(log $n$) | log $n$ | $\log^2 n$ | $n$ | $n \log n$ | $n^2$ | $n^3$ | $2^n$ | $n!$ |
|---|---|---|---|---|---|---|---|---|---|
| 10 | 2 | 3 | 11 | 10 | 33 | $10^2$ | $10^3$ | $10^3$ | $10^5$ |
| $10^2$ | 3 | 7 | 44 | 100 | 664 | $10^4$ | $10^6$ | $10^{30}$ | $10^{94}$ |
| $10^3$ | 3 | 10 | 99 | 1,000 | 9,966 | $10^6$ | $10^9$ | $10^{301}$ | $10^{1435}$ |
| $10^4$ | 4 | 13 | 177 | 10,000 | 132,877 | $10^8$ | $10^{12}$ | $10^{3010}$ | $10^{19,335}$ |
| $10^5$ | 4 | 17 | 276 | 100,00 | 1,660,964 | $10^{10}$ | $10^{15}$ | $10^{30,103}$ | $10^{243,338}$ |
| $10^6$ | 4 | 20 | 397 | 1,000,000 | 19,931,569 | $10^{12}$ | $10^{18}$ | $10^{301,301}$ | $10^{2,933,369}$ |

**FIGURE 4-4 Typical growth-rate functions evaluated at increasing values of *n***

# Best, Worst, and Average Cases

- For some algorithms, execution time depends only on size of data set
- Other algorithms depend on the nature of the data itself
  - Goal is to know best case, worst case, average case

# Big Oh Notation

- A function $f(n)$ is of order at most $g(n)$
- That is, $f(n)$ is $O(g(n))$ — if
  - A positive real number $c$ and positive integer $N$ exist …
  - Such that $f(n) \leq c \times g(n)$ for all $n \geq N$
  - That is:
    - $c \times g(n)$ is an upper bound on $f(n)$ when $n$ is sufficiently large

# Big Oh Notation

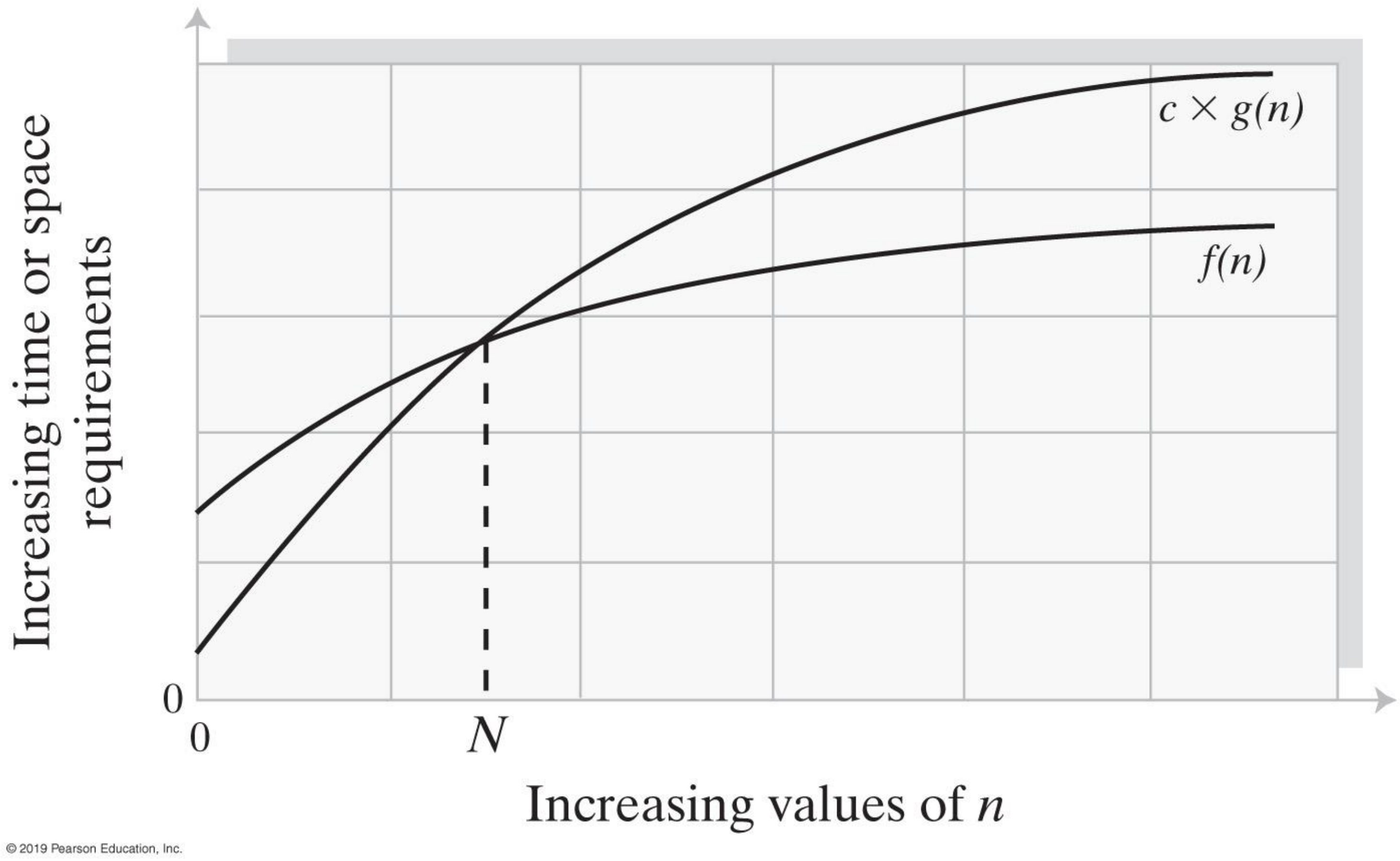



**FIGURE 4-5 An illustration of the values of two growth-rate functions**

# Big Oh Notation

$O(k\, g(n)) = O(g(n))$ for a constant $k$

$O(g_1(n)) + O(g_2(n)) = O(g_1(n) + g_2(n))$

$O(g_1(n)) * O(g_2(n)) = O(g_1(n) * g_2(n))$

$O(g_1(n) + g_2(n) + \ldots + g_m(n)) =$

$$O(max(g_1(n), g_2(n), \ldots, g_m(n))$$

$O(max(g_1(n), g_2(n), \ldots, g_m(n)) =$

$$max(O(g_1(n)), O(g_2(n)), \ldots, O(g_m(n)))$$

**Identities for Big Oh Notation**

# Picturing Efficiency

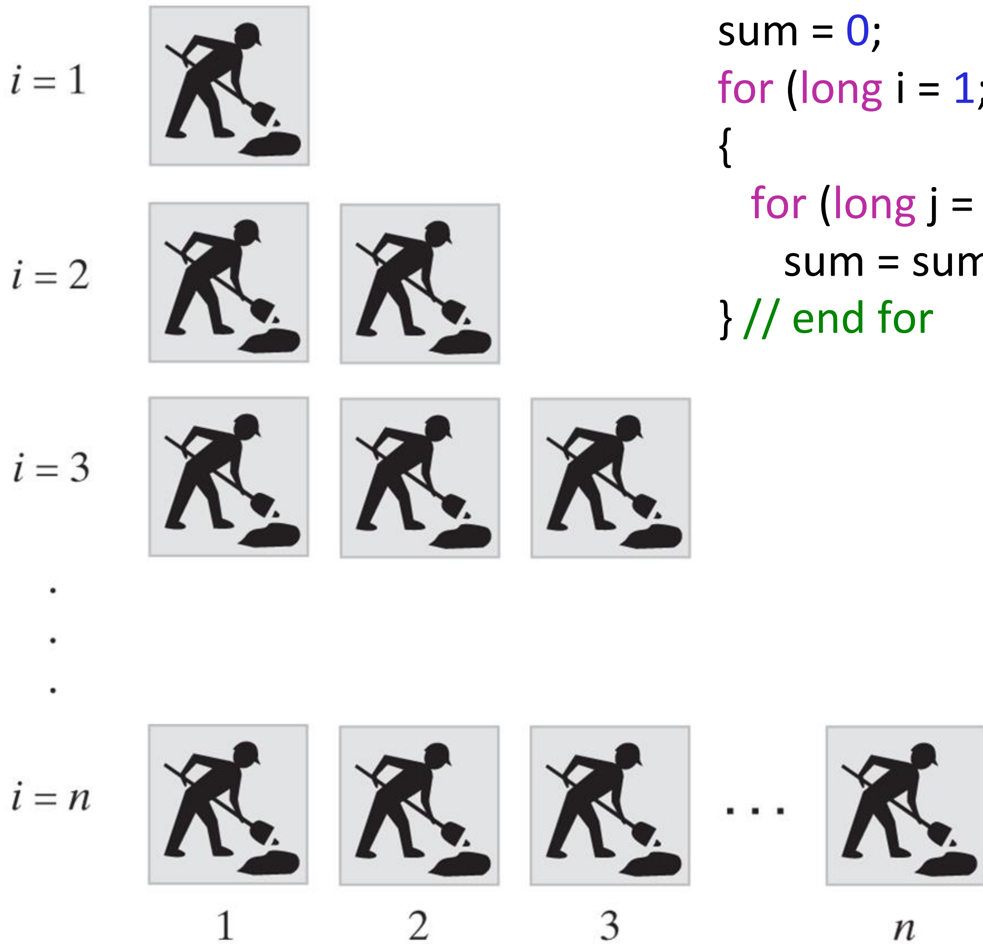
```
long sum = 0;
for (long i = 1; i <= n; i++)
  sum = sum + i;
```


## FIGURE 4-6 An O(n) algorithm

Pearson

# Picturing Efficiency

```
sum = 0;
for (long i = 1; i <= n; i++)
{
   for (long j = 1; j <= i; j++)
      sum = sum + 1;
} // end for
```

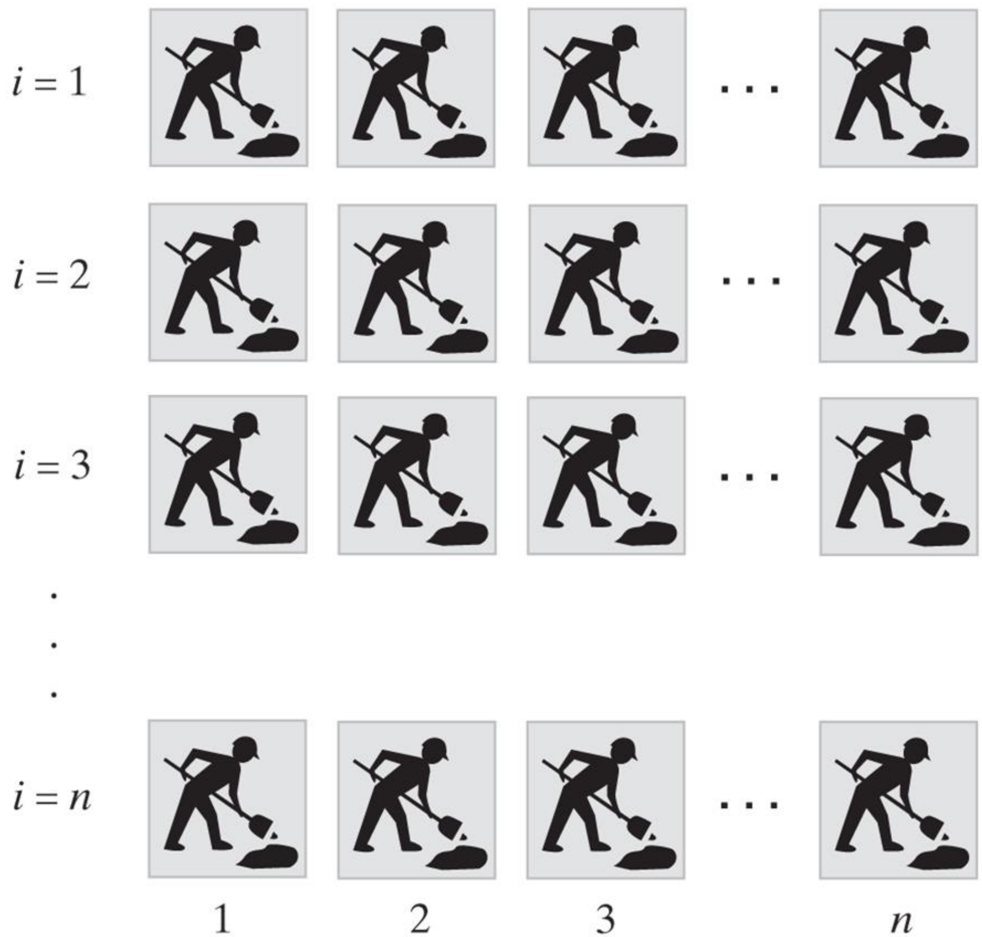



$O(1 + 2 + \ldots + n) = O(n^2)$



**FIGURE 4-7 An $O(n^2)$ algorithm**

# Picturing Efficiency




```
sum = 0;
for (long i = 1; i <= n; i++)
{
   for (long j = 1; j <= n; j++)
      sum = sum + 1;
} // end for
```



**FIGURE 4-8 Another $O(n^2)$ algorithm**

# Picturing Efficiency

| Growth-Rate Function for Size $n$ Problems | Growth-Rate Function for Size $2n$ Problems | Effect on Time Requirement |
|---|---|---|
| 1 | 1 | None |
| $\log n$ | $1 + \log n$ | Negligible |
| $n$ | $2n$ | Doubles |
| $n \log n$ | $2n \log n + 2n$ | Doubles and then adds $2n$ |
| $n^2$ | $(2n)^2$ | Quadruples |
| $n^3$ | $(2n)^3$ | Multiples by 8 |
| $2^n$ | $2^{2n}$ | Squares |

**FIGURE 4-9 The effect of doubling the problem size on an algorithm's time requirement**

# Picturing Efficiency

| Growth-Rate Function $g$ | $g(10^6) / 10^6$ |
|---|---|
| $\log n$ | 0.0000199 seconds |
| $n$ | 1 second |
| $n \log n$ | 19.9 seconds |
| $n^2$ | 11.6 days |
| $n^3$ | 31,709.8 years |
| $2^n$ | $10^{301,016}$ years |

**FIGURE 4-10 The time required to process one million items by algorithms of various orders at the rate of one million operations per second**

# Efficiency of ADT Bag Implementations

| Operation | Fixed-Size Array | Linked |
|---|---|---|
| `add(newEntry)` | O(1) | O(1) |
| `remove()` | O(1) | O(1) |
| `remove(anEntry)` | O(1), O(*n*), O(*n*) | O(1), O(*n*), O(*n*) |
| `clear()` | O(*n*) | O(*n*) |
| `getFrequencyOf(anEntry)` | O(*n*) | O(*n*) |
| `contains(anEntry)` | O(1), O(*n*), O(*n*) | O(1), O(*n*), O(*n*) |
| `toArray()` | O(*n*) | O(*n*) |
| `getCurrentSize(), isEmpty()` | O(1) | O(1) |

**FIGURE 4-11 The time efficiencies of the ADT bag operations for two implementations, expressed in Big Oh notation**

# End

# Chapter 4