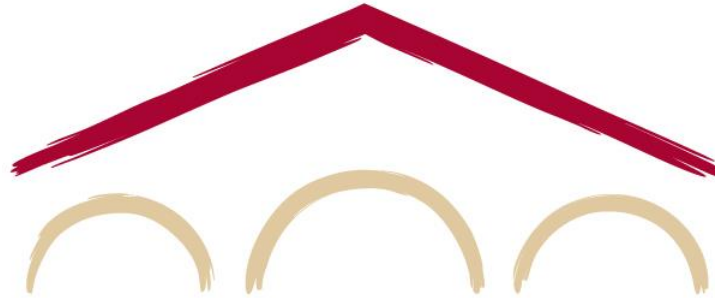


Natural Language Processing with Deep Learning

CS224N/Ling284



Tatsunori Hashimoto

Lecture 8: Self-Attention and Transformers

Lecture Plan

1. From recurrence (RNN) to attention-based NLP models
2. The Transformer model
3. Great results with Transformers
4. Drawbacks and variants of Transformers

Reminders:

See the [2023 lecture notes](#) for some bonus material

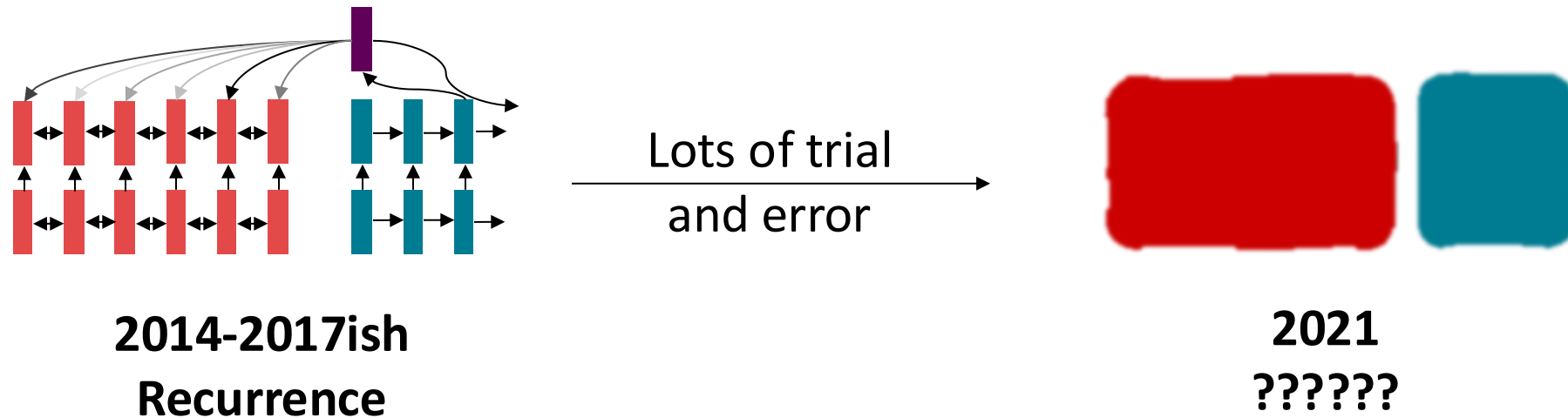
Assignment 4 due Feb 13! Use Colab for the final training if you don't have a GPU.

Final project proposal out tonight, due Tuesday, Feb 11!

Please try to hand in the project proposal on time; we want to get you feedback quickly!

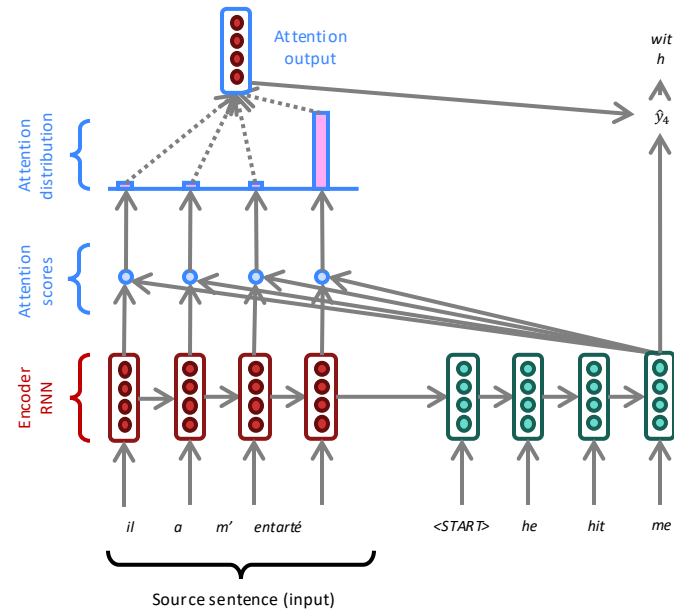
Do we even need recurrence at all?

- Abstractly: Attention is a way to pass information from a sequence (x) to a neural network input. (h_t)
 - This is also *exactly* what RNNs are used for – to pass information!
 - **Can we just get rid of the RNN entirely?** Maybe attention is just a better way to pass information!



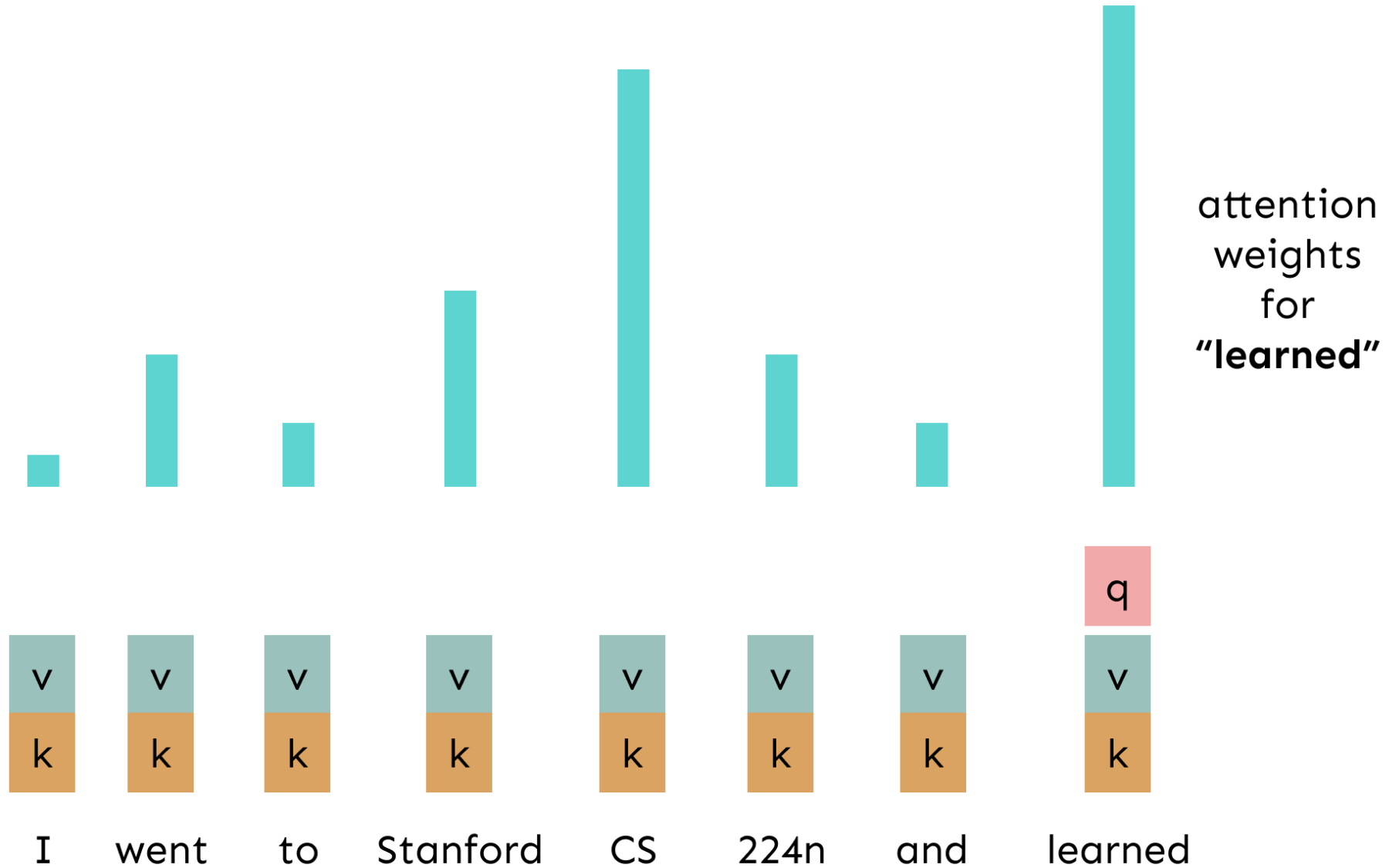
The building block we need: *self* attention

- What we talked about – **Cross** attention: paying attention to the input x to generate y_t



- What we need – **Self** attention: to generate y_t , we need to pay attention to $y_{<t}$

Self-Attention Hypothetical Example



Self-Attention: keys, queries, values from the same sequence

Let $\mathbf{w}_{1:n}$ be a sequence of words in vocabulary V , like *Zuko made his uncle tea*.

For each \mathbf{w}_i , let $\mathbf{x}_i = E\mathbf{w}_i$, where $E \in \mathbb{R}^{d \times |V|}$ is an embedding matrix.

1. Transform each word embedding with weight matrices Q, K, V , each in $\mathbb{R}^{d \times d}$

$$\mathbf{q}_i = Q\mathbf{x}_i \text{ (queries)} \quad \mathbf{k}_i = K\mathbf{x}_i \text{ (keys)} \quad \mathbf{v}_i = V\mathbf{x}_i \text{ (values)}$$

2. Compute pairwise similarities between keys and queries; normalize with softmax

$$\mathbf{e}_{ij} = \mathbf{q}_i^\top \mathbf{k}_j \quad \alpha_{ij} = \frac{\exp(\mathbf{e}_{ij})}{\sum_{j'} \exp(\mathbf{e}_{ij'})}$$

3. Compute output for each word as weighted sum of values

$$\mathbf{o}_i = \sum_j \alpha_{ij} \mathbf{v}_j$$

Barriers and solutions for Self-Attention as a building block

Barriers

- Doesn't have an inherent notion of order!



Solutions

Fixing the first self-attention problem: **sequence order**

- Since self-attention doesn't build in order information, we need to encode the order of the sentence in our keys, queries, and values.
- Consider representing each **sequence index** as a **vector**

$\mathbf{p}_i \in \mathbb{R}^d$, for $i \in \{1, 2, \dots, n\}$ are position vectors

- Don't worry about what the \mathbf{p}_i are made of yet!
- Easy to incorporate this info into our self-attention block: just add the \mathbf{p}_i to our inputs!
- Recall that \mathbf{x}_i is the embedding of the word at index i . The positioned embedding is:

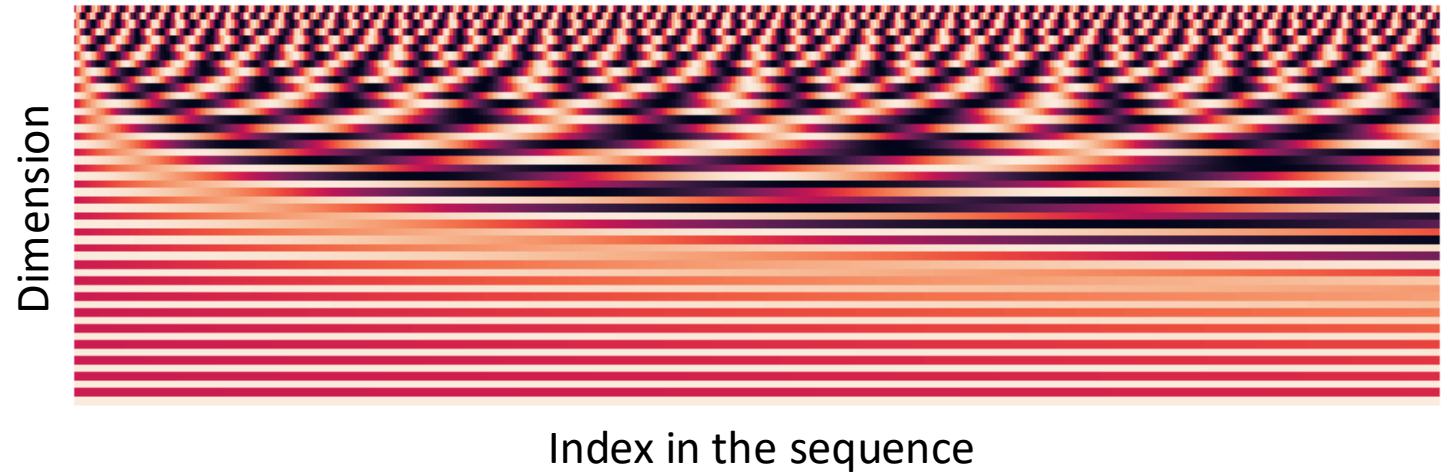
$$\tilde{\mathbf{x}}_i = \mathbf{x}_i + \mathbf{p}_i$$

In deep self-attention networks, we do this at the first layer! You could concatenate them as well, but people mostly just add...

Position representation vectors through sinusoids

- **Sinusoidal position representations:** concatenate sinusoidal functions of varying periods:

$$\mathbf{p}_i = \begin{pmatrix} \sin(i/10000^{2*1/d}) \\ \cos(i/10000^{2*1/d}) \\ \vdots \\ \sin(i/10000^{2*\frac{d}{2}/d}) \\ \cos(i/10000^{2*\frac{d}{2}/d}) \end{pmatrix}$$



- Pros:
 - Periodicity indicates that maybe “absolute position” isn’t as important
 - Maybe can extrapolate to longer sequences as periods restart!
- Cons:
 - Not learnable; also the extrapolation doesn’t really work!

Position representation vectors learned from scratch

- **Learned absolute position representations:** Let all p_i be learnable parameters!
Learn a matrix $\mathbf{p} \in \mathbb{R}^{d \times n}$, and let each \mathbf{p}_i be a column of that matrix!
- Pros:
 - Flexibility: each position gets to be learned to fit the data
- Cons:
 - Definitely can't extrapolate to indices outside $1, \dots, n$.
- Most systems use this!
- Sometimes people try more flexible representations of position:
 - Relative linear position attention [\[Shaw et al., 2018\]](#)
 - Dependency syntax-based position [\[Wang et al., 2019\]](#)

Common, modern position embeddings - RoPE

High level thought process: a *relative* position embedding should be some $f(x, i)$ s.t.

$$\langle f(x, i), f(y, j) \rangle = g(x, y, i - j)$$

That is, the attention function *only* gets to depend on the relative position (i-j). How do existing embeddings not fulfill this goal?

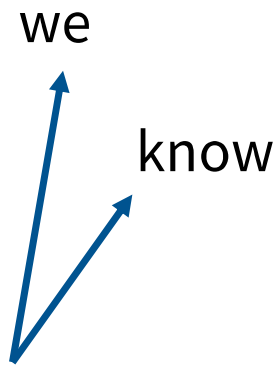
- **Sine:** Has various cross-terms that are not relative
- **Absolute:**

$$e_{ij} = \frac{x_i W^Q (x_j W^K + a_{ij}^K)^T}{\sqrt{d_z}} \quad \text{is not an inner product}$$

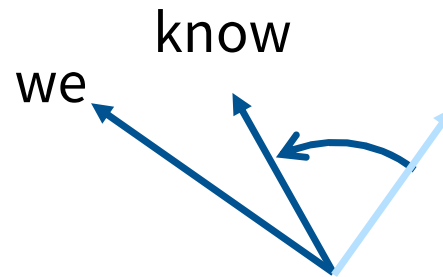
RoPE – Embedding via rotation

How can we solve this problem?

- We want our embeddings to be invariant to absolute position
- We know that inner products are invariant to arbitrary rotation.

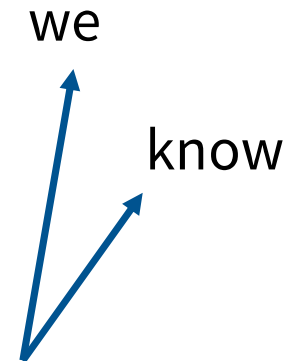


Position independent
embedding



Embedding
“of course we know”

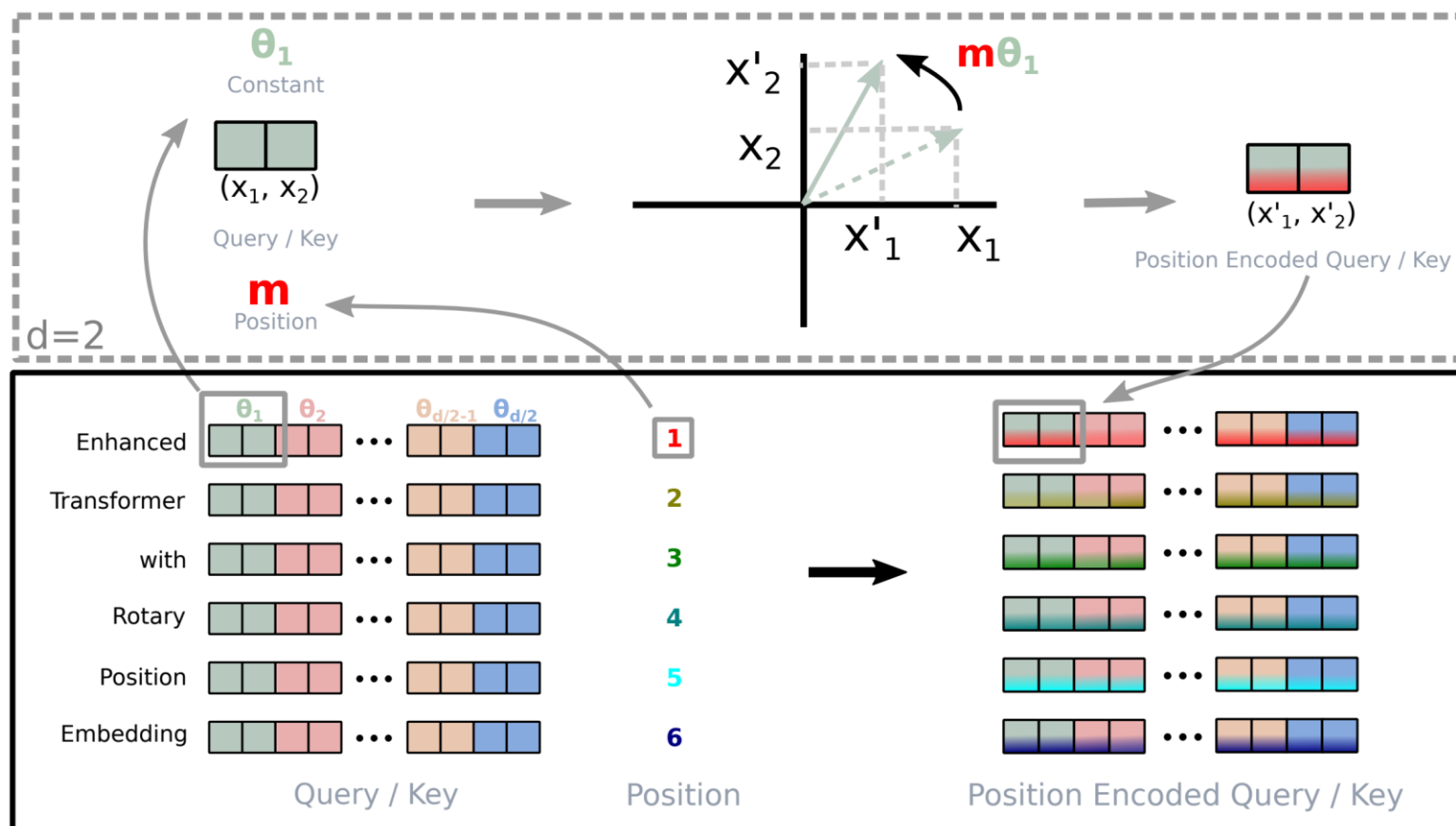
Rotate by ‘2 positions’



Embedding
“we know that”

Rotate by ‘0 positions’

RoPE – From 2 to many dimensions



[Su et al 2021]

Just pair up the coordinates and rotate them in 2d (motivation: complex numbers)

Barriers and solutions for Self-Attention as a building block

Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning! It's all just weighted averages



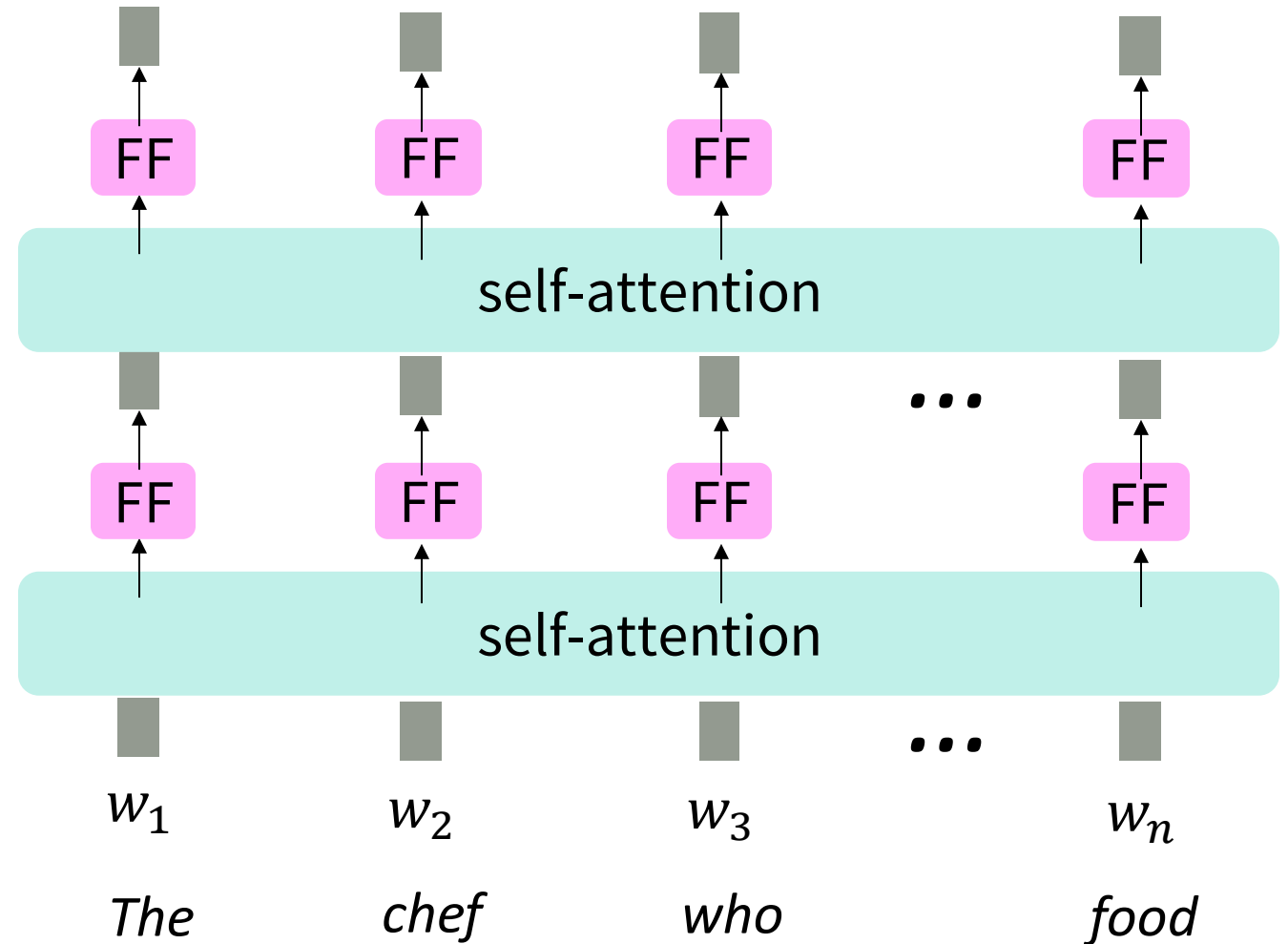
Solutions

- Add position representations to the inputs

Adding nonlinearities in self-attention

- Note that there are no elementwise nonlinearities in self-attention; stacking more self-attention layers just re-averages **value** vectors (Why? Look at the notes!)
- Easy fix: add a **feed-forward network** to post-process each output vector.

$$\begin{aligned} m_i &= MLP(\text{output}_i) \\ &= W_2 * \text{ReLU}(W_1 \text{output}_i + b_1) + b_2 \end{aligned}$$



Intuition: the FF network processes the result of attention

Barriers and solutions for Self-Attention as a building block

Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning magic! It's all just weighted averages
- Need to ensure we don't "look at the future" when predicting a sequence
 - Like in machine translation
 - Or language modeling



Solutions

- Add position representations to the inputs
- Easy fix: apply the same feedforward network to each self-attention output.



Masking the future in self-attention

- To use self-attention in **decoders**, we need to ensure we can't peek at the future.
- At every timestep, we could change the set of **keys and queries** to include only past words. (Inefficient!)
- To enable parallelization, we **mask out attention** to future words by setting attention scores to $-\infty$.

$$e_{ij} = \begin{cases} q_i^\top k_j, j \leq i \\ -\infty, j > i \end{cases}$$

For encoding these words

We can look at these (not greyed out) words

	[START]	The	chef	who
[START]		$-\infty$	$-\infty$	$-\infty$
The			$-\infty$	$-\infty$
chef				$-\infty$
who				

Barriers and solutions for Self-Attention as a building block

Barriers

- Doesn't have an inherent notion of order!
- No nonlinearities for deep learning magic! It's all just weighted averages
- Need to ensure we don't "look at the future" when predicting a sequence
 - Like in machine translation
 - Or language modeling

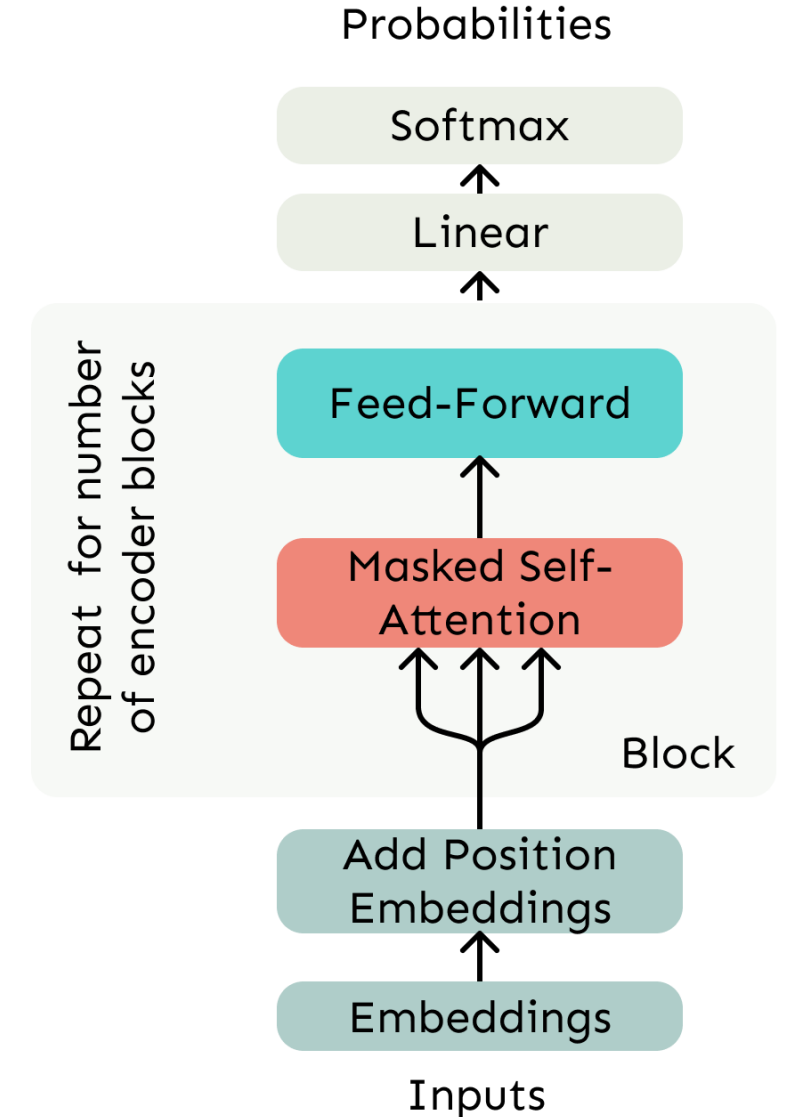


Solutions

- Add position representations to the inputs
- Easy fix: apply the same feedforward network to each self-attention output.
- Mask out the future by artificially setting attention weights to 0!

Necessities for a self-attention building block:

- **Self-attention:**
 - the basis of the method.
- **Position representations:**
 - Specify the sequence order, since self-attention is an unordered function of its inputs.
- **Nonlinearities:**
 - At the output of the self-attention block
 - Frequently implemented as a simple feed-forward network.
- **Masking:**
 - In order to parallelize operations while not looking at the future.
 - Keeps information about the future from “leaking” to the past.

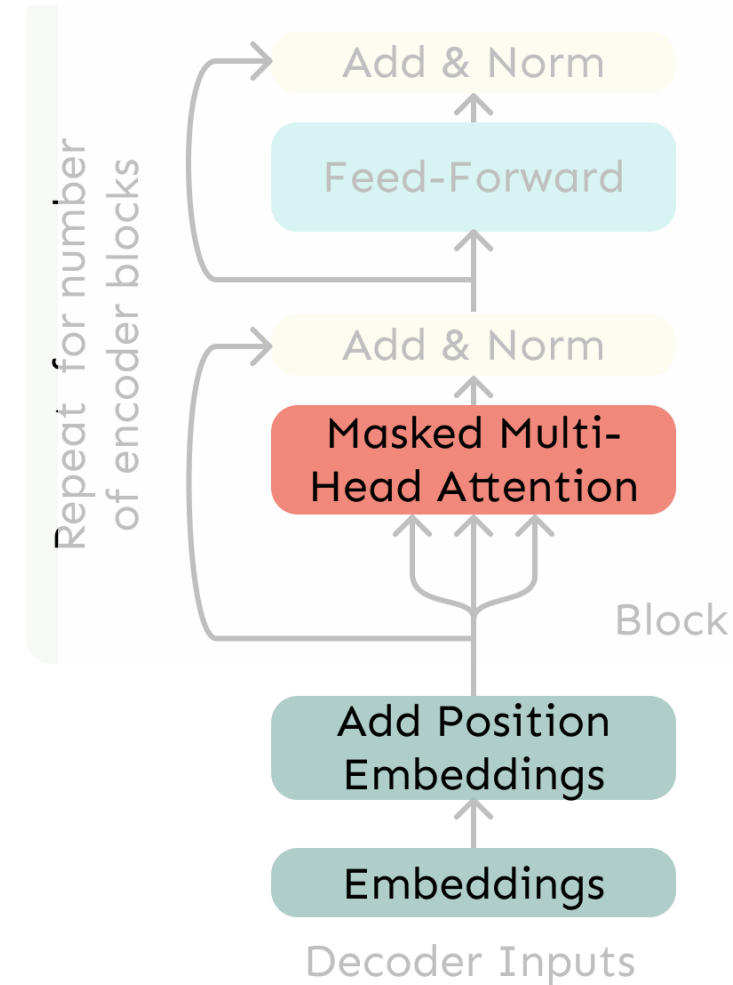


Outline

1. From recurrence (RNN) to attention-based NLP models
2. The Transformer model
3. Great results with Transformers
4. Drawbacks and variants of Transformers

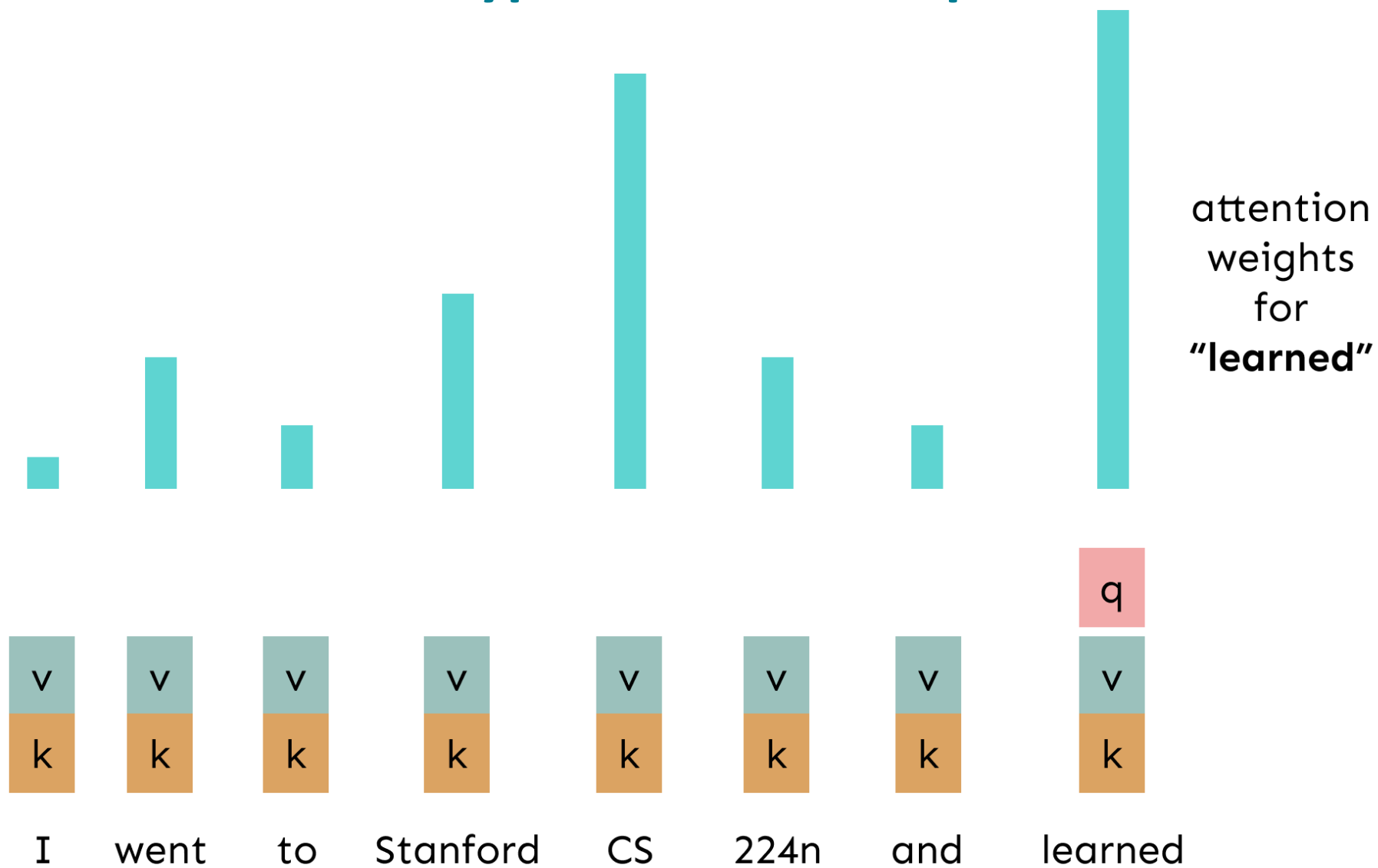
The Transformer Decoder

- A Transformer decoder is how we'll build systems like **language models**.
- It's a lot like our minimal self-attention architecture, but with a few more components.
- The embeddings and position embeddings are identical.
- We'll next replace our self-attention with **multi-head self-attention**.

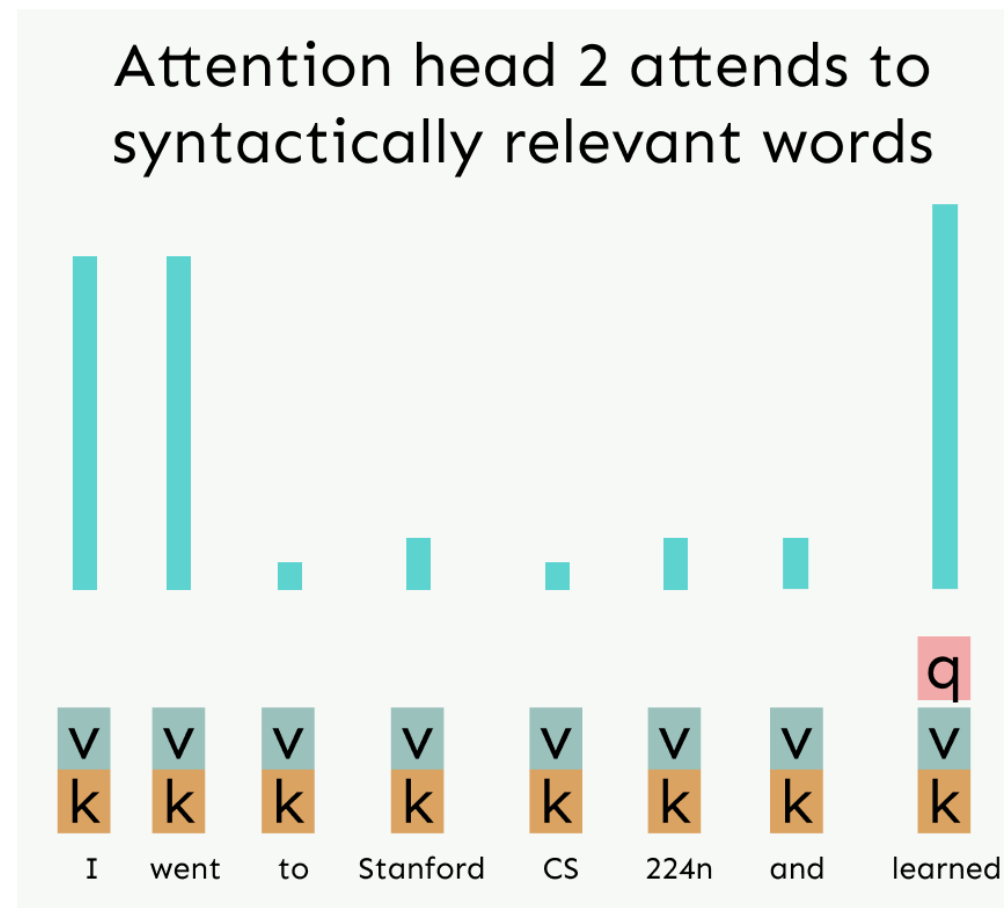
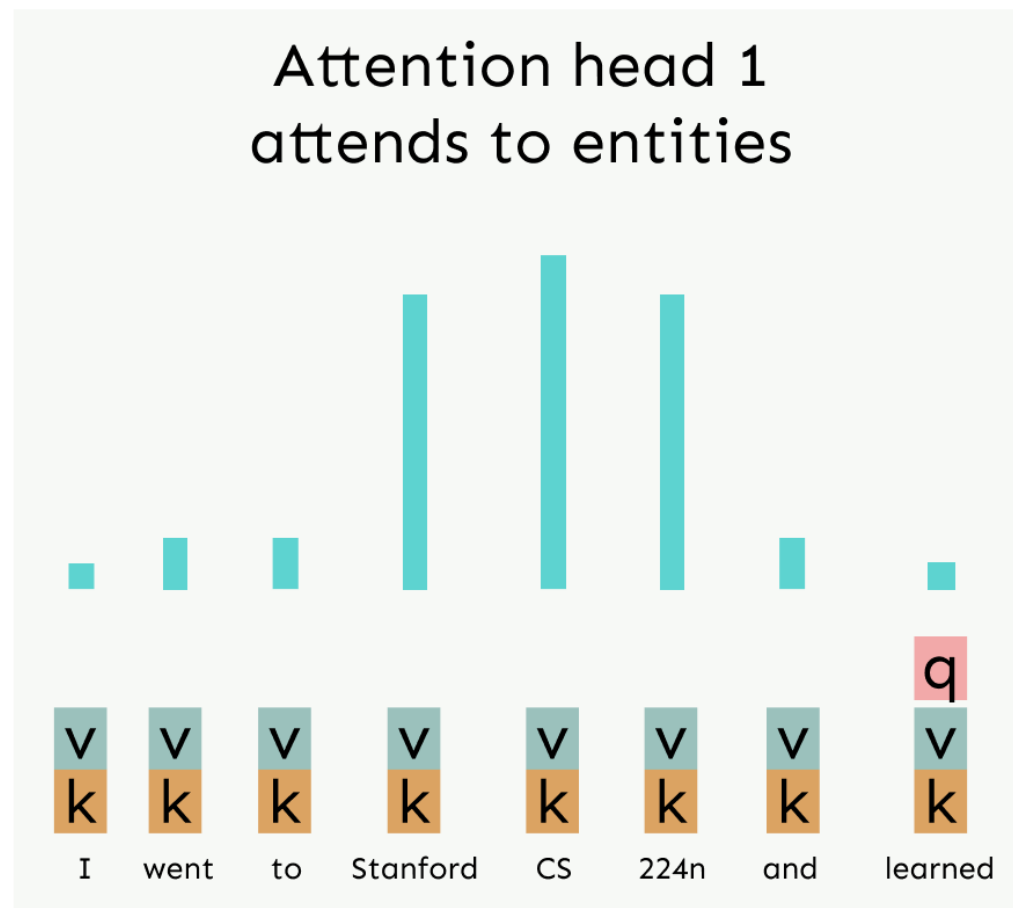


Transformer Decoder

Recall the Self-Attention Hypothetical Example



Hypothetical Example of Multi-Head Attention



I went to Stanford

CS 224n and learned

Sequence-Stacked form of Attention

- Let's look at how key-query-value attention is computed, in matrices.
 - Let $X = [x_1; \dots; x_n] \in \mathbb{R}^{n \times d}$ be the concatenation of input vectors.
 - First, note that $XK \in \mathbb{R}^{n \times d}$, $XQ \in \mathbb{R}^{n \times d}$, $XV \in \mathbb{R}^{n \times d}$.
 - The output is defined as $\text{output} = \text{softmax}(XQ(XK)^T)XV \in \mathbb{R}^{n \times d}$.

First, take the query-key dot products in one matrix multiplication: $XQ(XK)^T$

A diagram illustrating the first step of the attention mechanism. It shows a pink vertical rectangle labeled XQ multiplied by an orange horizontal rectangle labeled $K^T X^T$. The result is a gray rounded rectangle labeled $XQK^T X^T$, which is noted to be in $\mathbb{R}^{n \times n}$. A blue text label "All pairs of attention scores!" points to this result matrix.

$$XQ \cdot K^T X^T = XQK^T X^T \in \mathbb{R}^{n \times n}$$

All pairs of attention scores!

Next, softmax, and compute the weighted average with another matrix multiplication.

A diagram illustrating the second step of the attention mechanism. It shows the matrix $XQK^T X^T$ from the previous step being passed through a "softmax" operation, represented by a large left-facing curly bracket. The result of the softmax is then multiplied by a teal vertical rectangle labeled XV . The final result is a gray vertical rectangle labeled "output", which is noted to be in $\mathbb{R}^{n \times d}$.

$$\text{softmax} \left(XQK^T X^T \right) \cdot XV = \text{output} \in \mathbb{R}^{n \times d}$$

Multi-headed attention

- What if we want to look in multiple places in the sentence at once?
 - For word i , self-attention “looks” where $x_i^\top Q^\top K x_j$ is high, but maybe we want to focus on different j for different reasons?
- We’ll define **multiple attention “heads”** through multiple Q,K,V matrices
- Let, $Q_\ell, K_\ell, V_\ell \in \mathbb{R}^{d \times \frac{d}{h}}$, where h is the number of attention heads, and ℓ ranges from 1 to h .
- Each attention head performs attention independently:
 - $\text{output}_\ell = \text{softmax}(X Q_\ell K_\ell^\top X^\top) * X V_\ell$, where $\text{output}_\ell \in \mathbb{R}^{d/h}$
- Then the outputs of all the heads are combined!
 - $\text{output} = [\text{output}_1; \dots; \text{output}_h] Y$, where $Y \in \mathbb{R}^{d \times d}$
- Each head gets to “look” at different things, and construct value vectors differently.

Multi-head self-attention is computationally efficient

- Even though we compute h many attention heads, it's not really more costly.
 - We compute $XQ \in \mathbb{R}^{n \times d}$, and then reshape to $\mathbb{R}^{n \times h \times d/h}$. (Likewise for XK, XV .)
 - Then we transpose to $\mathbb{R}^{h \times n \times d/h}$; now the head axis is like a batch axis.
 - Almost everything else is identical, and the **matrices are the same sizes**.

First, take the query-key dot products in one matrix multiplication: $XQ(XK)^\top$

XQ $K^\top X^\top$ $=$ $XQK^\top X^\top \in \mathbb{R}^{3 \times n \times n}$

3 sets of all pairs of attention scores!

Next, softmax, and compute the weighted average with another matrix multiplication.

$\text{softmax} \left(XQK^\top X^\top \right) XV =$ P mix $=$ output $\in \mathbb{R}^{n \times d}$

Scaled Dot Product [Vaswani et al., 2017]

- “**Scaled Dot Product**” attention aids in training.
- When dimensionality d becomes large, dot products between vectors tend to become large.
 - Because of this, inputs to the softmax function can be large, making the gradients small.

- Instead of the self-attention function we’ve seen:

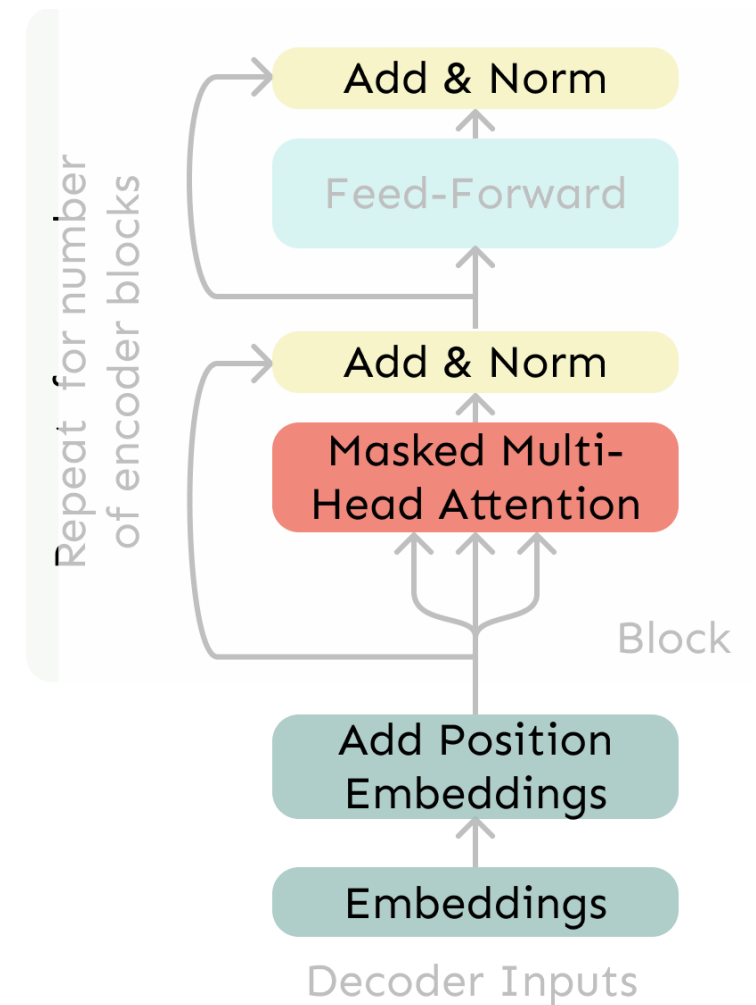
$$\text{output}_\ell = \text{softmax}(XQ_\ell K_\ell^\top X^\top) * XV_\ell$$

- We divide the attention scores by $\sqrt{d/h}$, to stop the scores from becoming large just as a function of d/h (The dimensionality divided by the number of heads.)

$$\text{output}_\ell = \text{softmax}\left(\frac{XQ_\ell K_\ell^\top X^\top}{\sqrt{d/h}}\right) * XV_\ell$$

The Transformer Decoder

- Now that we've replaced self-attention with multi-head self-attention, we'll go through two **optimization tricks** that end up being :
 - **Residual Connections**
 - **Layer Normalization**
- In most Transformer diagrams, these are often written together as “Add & Norm”



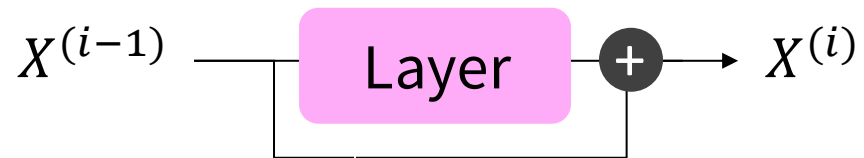
Transformer Decoder

The Transformer Encoder: **Residual connections** [[He et al., 2016](#)]

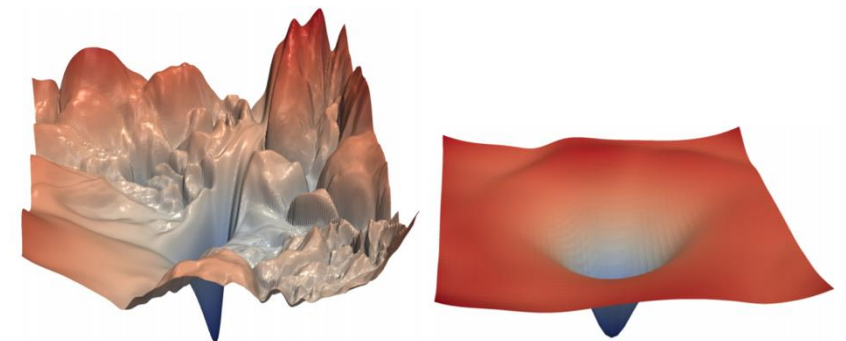
- **Residual connections** are a trick to help models train better.
 - Instead of $X^{(i)} = \text{Layer}(X^{(i-1)})$ (where i represents the layer)



- We let $X^{(i)} = X^{(i-1)} + \text{Layer}(X^{(i-1)})$ (so we only have to learn “the residual” from the previous layer)



- Gradient is **great** through the residual connection; it's 1!
- Bias towards the identity function!



[no residuals]

[residuals]

[Loss landscape visualization,
[Li et al., 2018](#), on a ResNet]

The Transformer Encoder: **Layer normalization** [[Ba et al., 2016](#)]

- **Layer normalization** is a trick to help models train faster.
- Idea: cut down on uninformative variation in hidden vector values by normalizing to unit mean and standard deviation **within each layer**.
 - LayerNorm's success may be due to its normalizing gradients [[Xu et al., 2019](#)]
- Let $x \in \mathbb{R}^d$ be an individual (word) vector in the model.
- Let $\mu = \sum_{j=1}^d x_j$; this is the mean; $\mu \in \mathbb{R}$.
- Let $\sigma = \sqrt{\frac{1}{d} \sum_{j=1}^d (x_j - \mu)^2}$; this is the standard deviation; $\sigma \in \mathbb{R}$.
- Let $\gamma \in \mathbb{R}^d$ and $\beta \in \mathbb{R}^d$ be learned “gain” and “bias” parameters. (Can omit!)
- Then layer normalization computes:

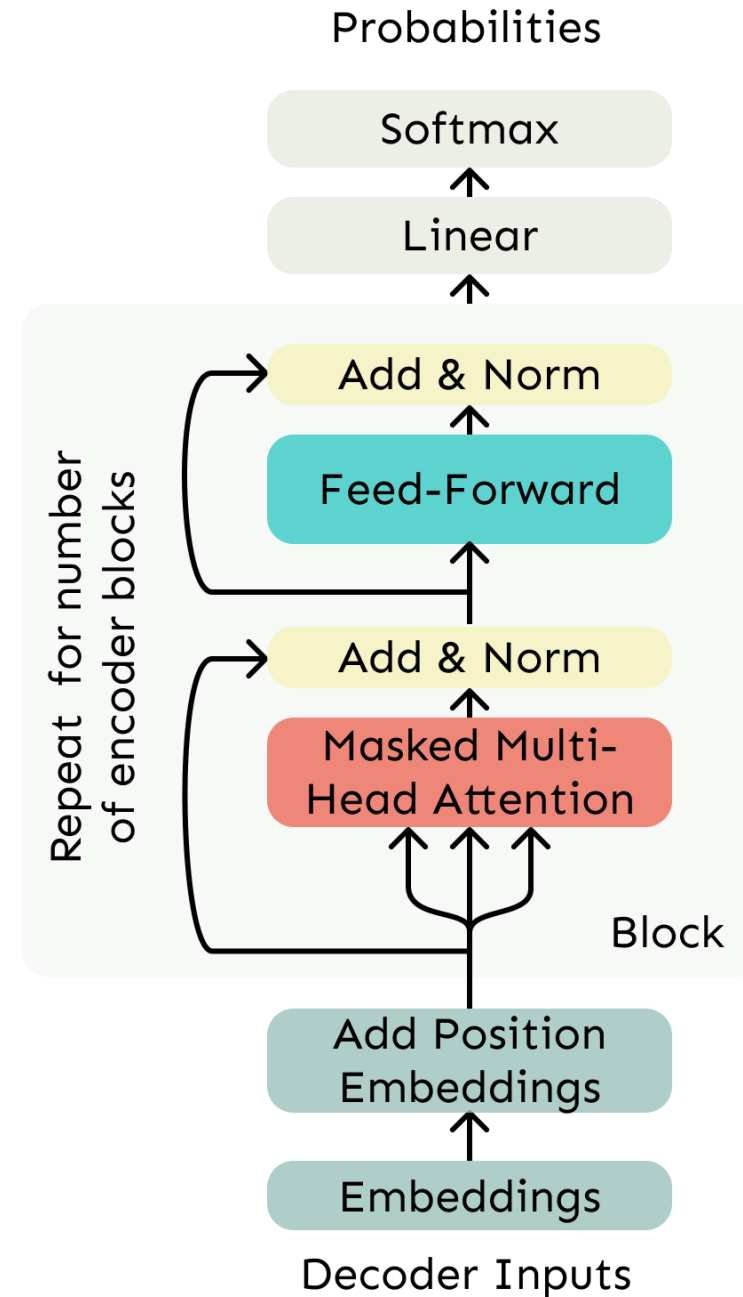
$$\text{output} = \frac{x - \mu}{\sqrt{\sigma} + \epsilon} * \gamma + \beta$$

Normalize by scalar mean and variance

Modulate by learned elementwise gain and bias

The Transformer Decoder

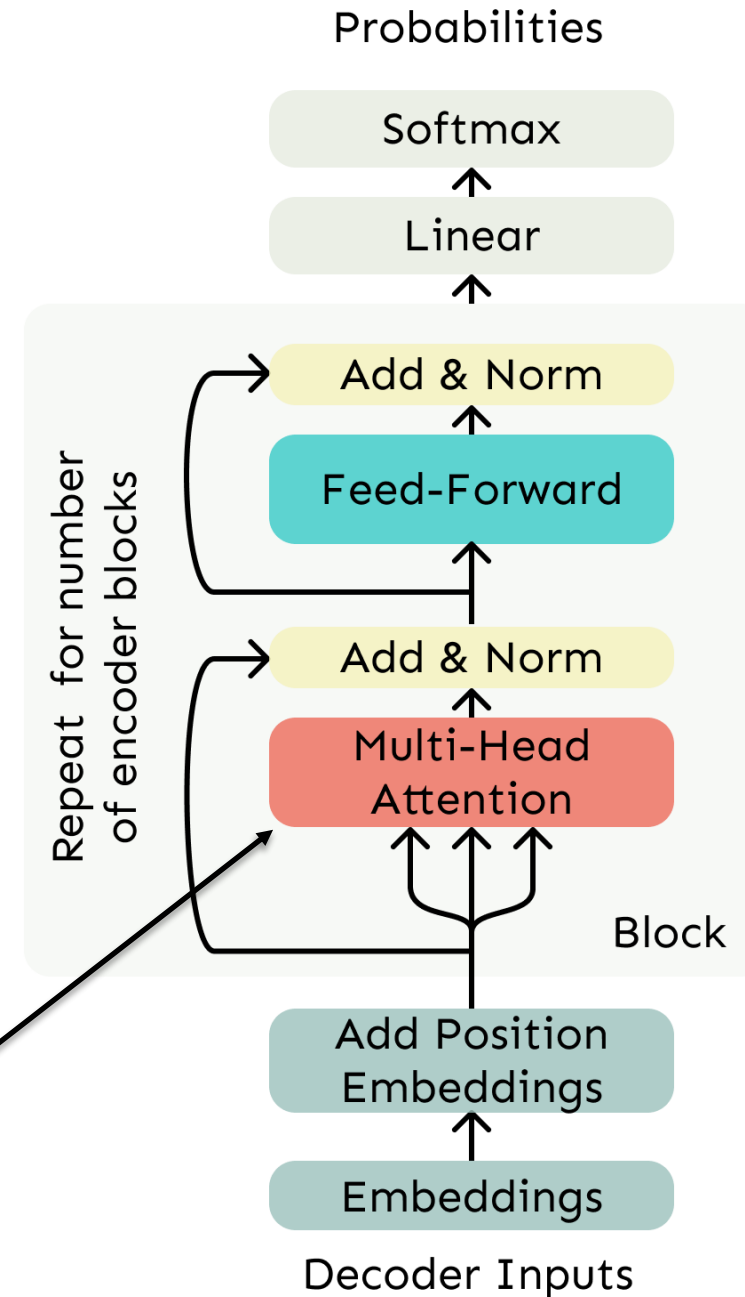
- The Transformer Decoder is a stack of Transformer Decoder **Blocks**.
- Each Block consists of:
 - Self-attention
 - Add & Norm
 - Feed-Forward
 - Add & Norm
- That's it! We've gone through the Transformer Decoder.



The Transformer Encoder

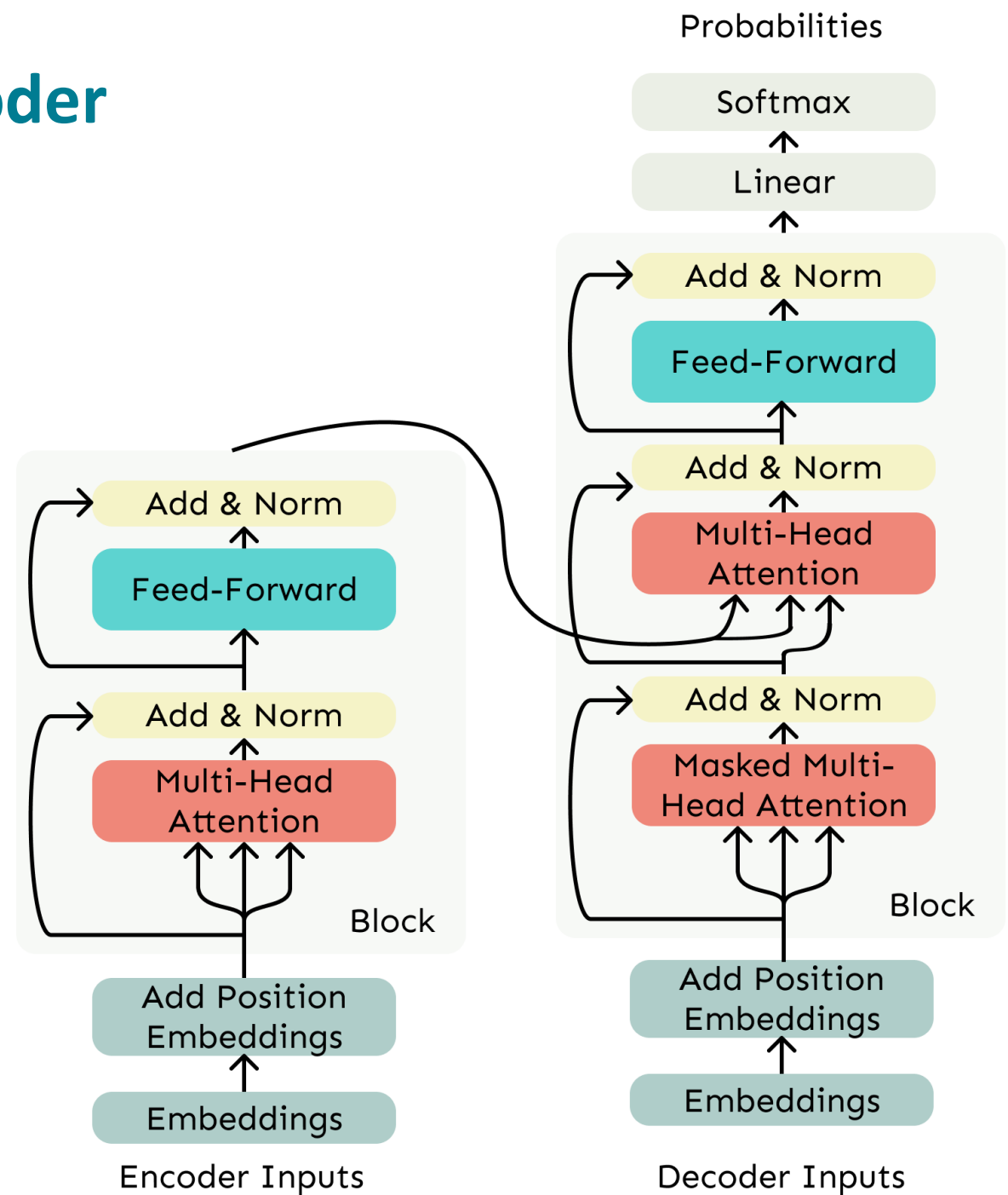
- The Transformer Decoder constrains to **unidirectional context**, as for **language models**.
- What if we want **bidirectional context**, like in a bidirectional RNN?
- This is the Transformer Encoder. The only difference is that we **remove the masking** in the self-attention.

No Masking!



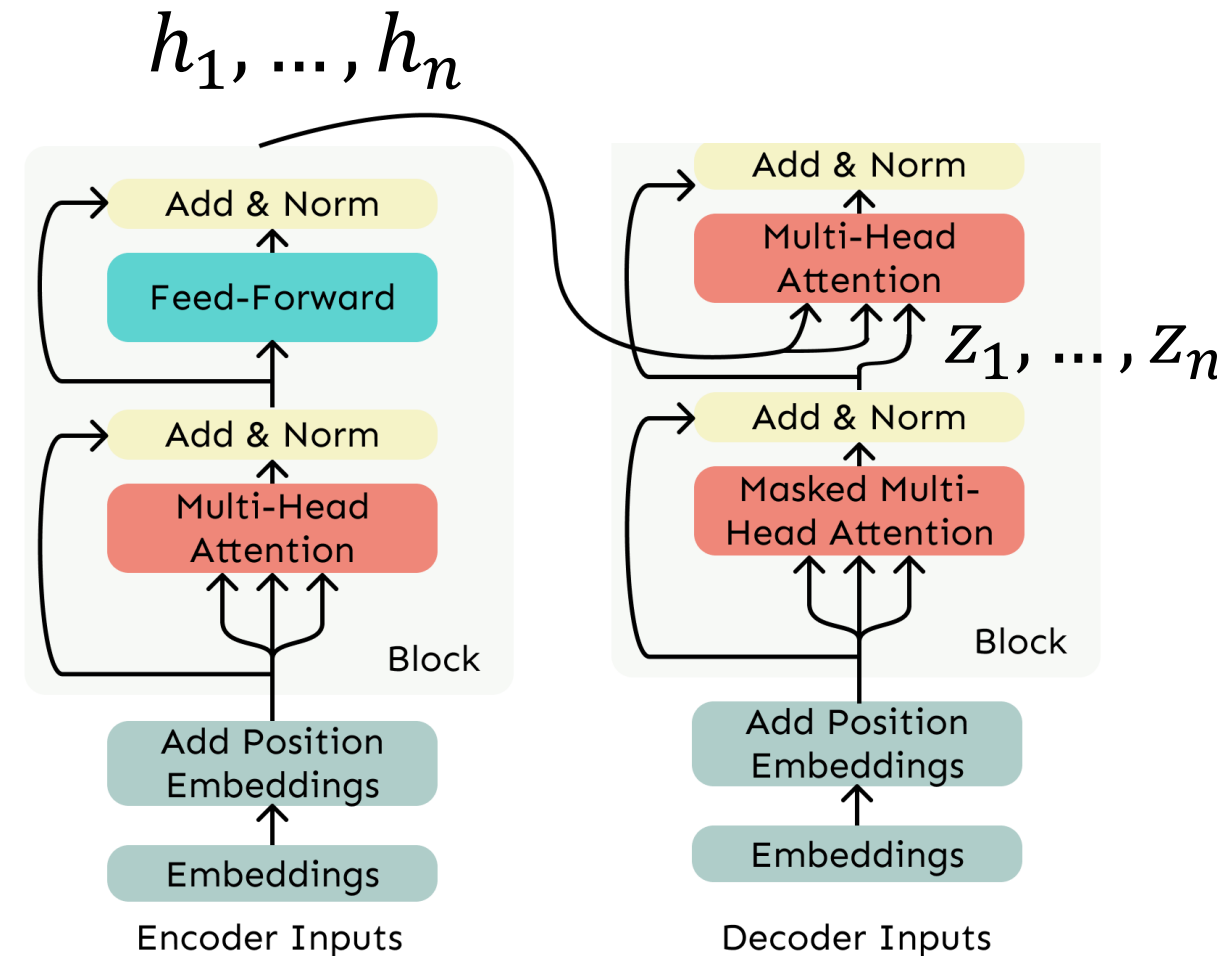
The Transformer Encoder-Decoder

- Recall that in machine translation, we processed the source sentence with a **bidirectional** model and generated the target with a **unidirectional model**.
- For this kind of seq2seq format, we often use a Transformer Encoder-Decoder.
- We use a normal Transformer Encoder.
- Our Transformer Decoder is modified to perform **cross-attention** to the output of the Encoder.



Cross-attention (details)

- We saw that self-attention is when keys, queries, and values come from the same source.
- In the decoder, we have attention that looks more like what we saw last week.
- Let h_1, \dots, h_n be **output** vectors **from** the Transformer **encoder**; $x_i \in \mathbb{R}^d$
- Let z_1, \dots, z_n be input vectors from the Transformer **decoder**, $z_i \in \mathbb{R}^d$
- Then keys and values are drawn from the **encoder** (like a memory):
 - $k_i = Kh_i, v_i = Vh_i$.
- And the queries are drawn from the **decoder**, $q_i = Qz_i$.



Cross-attention (details)

- Let's look at how cross-attention is computed, in matrices.
 - Let $H = [h_1; \dots; h_T] \in \mathbb{R}^{T \times d}$ be the concatenation of encoder vectors.
 - Let $Z = [z_1; \dots; z_T] \in \mathbb{R}^{T \times d}$ be the concatenation of decoder vectors.
 - The output is defined as $\text{output} = \text{softmax}(ZQ(HK)^\top) \times HV$.

First, take the query-key dot products in one matrix multiplication: $ZQ(HK)^\top$

The diagram illustrates the first step of cross-attention. It shows a light blue vertical rectangle labeled ZQ followed by an orange rounded rectangle labeled $K^\top H^\top$. An equals sign follows, then a light gray rounded rectangle labeled $ZQK^\top H^\top$. To the right of this is the text $\in \mathbb{R}^{T \times T}$. Further to the right, in blue text, is the phrase "All pairs of attention scores!".

$$ZQ \quad K^\top H^\top = ZQK^\top H^\top \in \mathbb{R}^{T \times T}$$

All pairs of attention scores!

Next, softmax, and compute the weighted average with another matrix multiplication.

The diagram illustrates the second step of cross-attention. It shows the word "softmax" to the left of a large left square bracket. Inside the bracket is a light gray rounded rectangle labeled $ZQK^\top H^\top$. To the right of the bracket is a pink vertical rectangle labeled HV . An equals sign follows, then a light gray vertical rectangle. To the right of this rectangle is the text "output $\in \mathbb{R}^{T \times d}$ ". A curved arrow points from the $ZQK^\top H^\top$ box in the previous diagram to the $ZQK^\top H^\top$ box in this diagram.

$$\text{softmax} \left(ZQK^\top H^\top \right) HV = \text{output} \in \mathbb{R}^{T \times d}$$

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Great Results with Transformers

First, Machine Translation from the original Transformers paper!

Model	BLEU		Training Cost (FLOPs)	
	EN-DE	EN-FR	EN-DE	EN-FR
ByteNet [18]	23.75			
Deep-Att + PosUnk [39]		39.2		$1.0 \cdot 10^{20}$
GNMT + RL [38]	24.6	39.92	$2.3 \cdot 10^{19}$	$1.4 \cdot 10^{20}$
ConvS2S [9]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5 \cdot 10^{20}$
MoE [32]	26.03	40.56	$2.0 \cdot 10^{19}$	$1.2 \cdot 10^{20}$
Deep-Att + PosUnk Ensemble [39]		40.4		$8.0 \cdot 10^{20}$
GNMT + RL Ensemble [38]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1 \cdot 10^{21}$
ConvS2S Ensemble [9]	26.36	41.29	$7.7 \cdot 10^{19}$	$1.2 \cdot 10^{21}$

Great Results with Transformers

Next, document generation!

Model	Test perplexity	ROUGE-L
<i>seq2seq-attention, $L = 500$</i>	5.04952	12.7
<i>Transformer-ED, $L = 500$</i>	2.46645	34.2
<i>Transformer-D, $L = 4000$</i>	2.22216	33.6
<i>Transformer-DMCA, no MoE-layer, $L = 11000$</i>	2.05159	36.2
<i>Transformer-DMCA, MoE-128, $L = 11000$</i>	1.92871	37.9
<i>Transformer-DMCA, MoE-256, $L = 7500$</i>	1.90325	38.8

The old standard



Transformers all the way down.



Great Results with Transformers

Before too long, most Transformers results also included **pretraining**, a method we'll go over next.

Transformers' parallelizability allows for efficient pretraining, and have made them the de-facto standard.

On this popular aggregate benchmark, for example:



All top models are Transformer (and pretraining)-based.

Rank Name		Model	URL	Score
1	DeBERTa Team - Microsoft	DeBERTa / TuringNLRv4	↗	90.8
2	HFL iFLYTEK	MacALBERT + DKM		90.7
+	3	Alibaba DAMO NLP	StructBERT + TAPT	↗ 90.6
+	4	PING-AN Omni-Sinitic	ALBERT + DAAF + NAS	90.6
	5	ERNIE Team - Baidu	ERNIE	↗ 90.4
	6	T5 Team - Google	T5	↗ 90.3

More results Thursday when we discuss pretraining.

[[Liu et al., 2018](#)]

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What would we like to fix about the Transformer?

- **Training instabilities (Pre vs Post norm)**
- **Quadratic compute in self-attention :**
 - Computing all pairs of interactions means our computation grows **quadratically** with the sequence length!
 - For recurrent models, it only grew linearly!

Pre vs Post norm

The one thing everyone agrees on (in 2024)

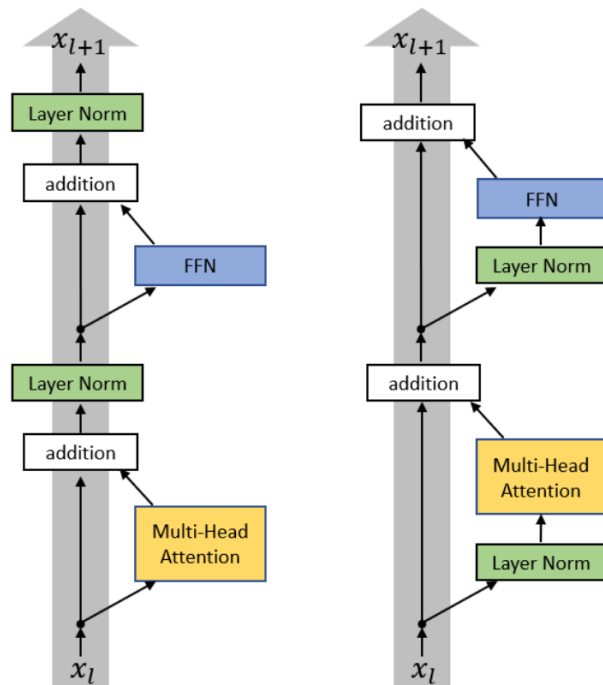


Figure from Xiong 2020

Post-LN Transformer	Pre-LN Transformer
$x_{l,i}^{post,1} = \text{MultiHeadAtt}(x_{l,i}^{post}, [x_{l,1}^{post}, \dots, x_{l,n}^{post}])$	$x_{l,i}^{pre,1} = \text{LayerNorm}(x_{l,i}^{pre})$
$x_{l,i}^{post,2} = x_{l,i}^{post} + x_{l,i}^{post,1}$	$x_{l,i}^{pre,2} = \text{MultiHeadAtt}(x_{l,i}^{pre,1}, [x_{l,1}^{pre,1}, \dots, x_{l,n}^{pre,1}])$
$x_{l,i}^{post,3} = \text{LayerNorm}(x_{l,i}^{post,2})$	$x_{l,i}^{pre,3} = x_{l,i}^{pre} + x_{l,i}^{pre,2}$
$x_{l,i}^{post,4} = \text{ReLU}(x_{l,i}^{post,3} W^{1,l} + b^{1,l}) W^{2,l} + b^{2,l}$	$x_{l,i}^{pre,4} = \text{LayerNorm}(x_{l,i}^{pre,3})$
$x_{l,i}^{post,5} = x_{l,i}^{post,3} + x_{l,i}^{post,4}$	$x_{l,i}^{pre,5} = \text{ReLU}(x_{l,i}^{pre,4} W^{1,l} + b^{1,l}) W^{2,l} + b^{2,l}$
$x_{l+1,i}^{post} = \text{LayerNorm}(x_{l,i}^{post,5})$	$x_{l+1,i}^{pre} = x_{l,i}^{pre,5} + x_{l,i}^{pre,4}$
Final LayerNorm: $x_{Final,i}^{pre} \leftarrow \text{LayerNorm}(x_{L+1,i}^{pre})$	

Set up LayerNorm so that it doesn't affect the main residual signal path (on the left)

Almost all modern LMs use pre-norm (but BERT was post-norm)

(One somewhat funny exception – OPT350M. I don't know why this is post-norm)

Quadratic computation as a function of sequence length

- One of the benefits of self-attention over recurrence was that it's highly parallelizable.
- However, its total number of operations grows as $O(n^2d)$, where n is the sequence length, and d is the dimensionality.

$$\begin{matrix} \text{teal box} \\ XQ \end{matrix} \begin{matrix} \text{orange box} \\ K^T X^T \end{matrix} = \begin{matrix} \text{grey box} \\ XQK^T X^T \end{matrix} \in \mathbb{R}^{n \times n}$$

Need to compute all pairs of interactions!
 $O(n^2d)$

- Think of d as around **1,000** (though for large language models it's much larger!).
 - So, for a single (shortish) sentence, $n \leq 30$; $n^2 \leq \mathbf{900}$.
 - In practice, we set a bound like $n = 512$.
 - **But what if we'd like $n \geq 50,000$?** For example, to work on long documents?

Back to the future – RNNs are back!

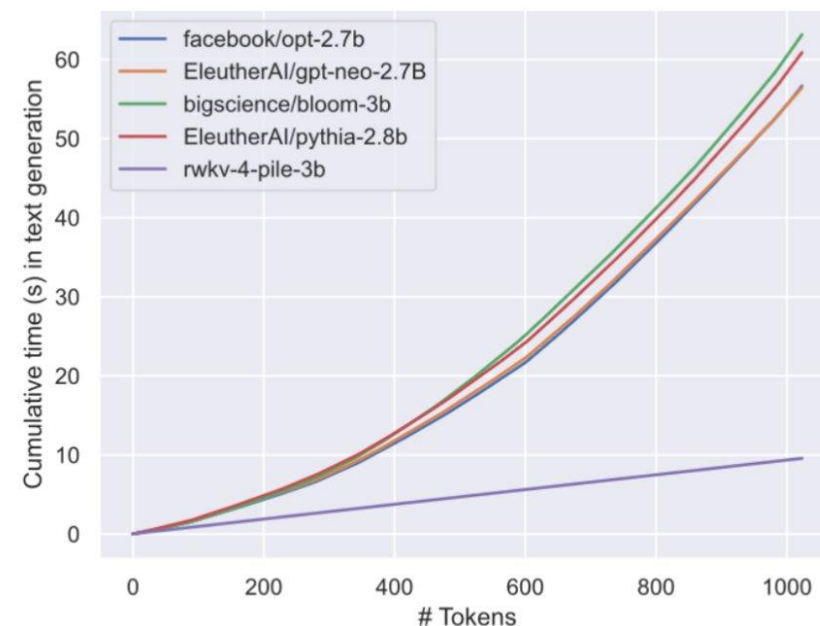
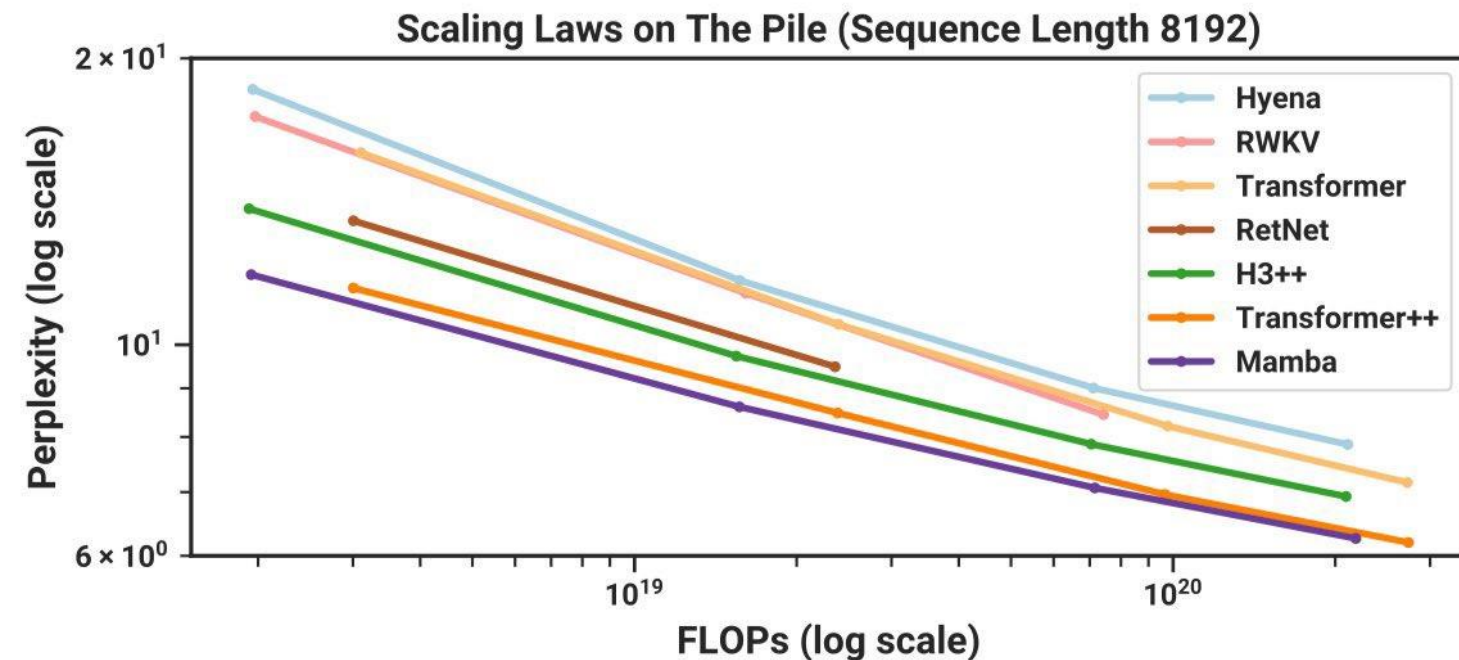


Figure 7: Cumulative time on text generation for LLM
Unlike transformers, RWKV exhibits linear scaling.

If you want *really* long context, RNNs provide this (linear complexity).
Modern RNNs (RWKV, Mamba, etc) are getting better!

Do we even need to remove the quadratic cost of attention?

- As Transformers grow larger, a larger and larger percent of compute is **outside** the self-attention portion, despite the quadratic cost.
- In practice, **production Transformer language models use quadratic cost attention**
 - The cheaper methods tend not to work as well at scale.
 - Systems optimizations work well (Flash attention – Jun 2022)

Foundation Model Context Length



Do Transformer Modifications Transfer?

- "Surprisingly, we find that most modifications do not meaningfully improve performance."

Model	Params	Ops	Step/s	Early loss	Final loss	SGLUE	XSum	WebQ	WMT EnDe
Vanilla Transformer	223M	11.1T	3.50	2.182 ± 0.005	1.838	71.66	17.78	23.02	26.62
GeLU	223M	11.1T	3.58	2.179 ± 0.003	1.838	75.79	17.86	25.13	26.47
Swish	223M	11.1T	3.62	2.186 ± 0.003	1.847	73.77	17.74	24.34	26.75
ELU	223M	11.1T	3.56	2.270 ± 0.007	1.932	67.83	16.73	23.02	26.08
GLU	223M	11.1T	3.59	2.174 ± 0.003	1.814	74.20	17.42	24.34	27.12
GoGLU	223M	11.1T	3.55	2.130 ± 0.006	1.792	75.96	18.27	24.87	26.87
ReGLU	223M	11.1T	3.57	2.145 ± 0.004	1.803	76.17	18.36	24.87	27.02
SeLU	223M	11.1T	3.55	2.315 ± 0.004	1.948	68.76	16.76	22.75	25.99
SwiGLU	223M	11.1T	3.53	2.127 ± 0.003	1.789	76.00	18.20	24.34	27.02
LaGLU	223M	11.1T	3.59	2.149 ± 0.005	1.798	75.34	17.97	24.34	26.53
Sigmoid	223M	11.1T	3.63	2.291 ± 0.019	1.867	74.31	17.51	23.02	26.30
Softplus	223M	11.1T	3.47	2.207 ± 0.011	1.850	72.45	17.65	24.34	26.89
RMS Norm	223M	11.1T	3.68	2.167 ± 0.008	1.821	75.45	17.94	24.07	27.14
Resero	223M	11.1T	3.51	2.262 ± 0.003	1.939	61.69	15.64	20.90	26.37
Resero + LayerNorm	223M	11.1T	3.26	2.223 ± 0.006	1.858	70.42	17.58	23.02	26.29
Resero + RMS Norm	223M	11.1T	3.34	2.221 ± 0.009	1.875	70.33	17.32	23.02	26.19
Fixup	223M	11.1T	2.95	2.382 ± 0.012	2.067	58.56	14.42	23.02	26.31
24 layers, $d_k = 1536, H = 6$	224M	11.1T	3.33	2.200 ± 0.007	1.843	74.89	17.75	25.13	26.89
18 layers, $d_k = 2048, H = 8$	223M	11.1T	3.38	2.185 ± 0.005	1.831	76.45	16.83	24.34	27.10
8 layers, $d_k = 4096, H = 18$	223M	11.1T	3.69	2.190 ± 0.005	1.847	74.58	17.69	23.28	26.85
6 layers, $d_k = 6144, H = 24$	223M	11.1T	3.70	2.201 ± 0.010	1.857	73.55	17.59	24.60	26.66
Block sharing	65M	11.1T	3.91	2.407 ± 0.037	2.164	64.50	14.53	21.96	25.48
+ Factorized embeddings	45M	9.4T	4.21	2.631 ± 0.305	2.183	60.84	14.00	19.84	25.27
+ Factorized & shared embeddings	20M	9.1T	4.37	2.907 ± 0.313	2.385	53.95	11.37	19.84	25.19
Encoder only block sharing	170M	11.1T	3.68	2.298 ± 0.023	1.929	69.60	16.23	23.02	26.23
Decoder only block sharing	144M	11.1T	3.70	2.352 ± 0.029	2.082	67.93	16.13	23.81	26.08
Factorized Embedding	227M	9.4T	3.80	2.208 ± 0.006	1.855	70.41	15.92	22.75	26.50
Factorized & shared embeddings	202M	9.1T	3.92	2.320 ± 0.010	1.952	68.69	16.33	22.22	26.44
Tied encoder/decoder input embeddings	248M	11.1T	3.55	2.192 ± 0.002	1.840	71.70	17.72	24.34	26.49
Tied decoder input and output embeddings	248M	11.1T	3.57	2.187 ± 0.007	1.827	74.86	17.74	24.87	26.67
Unified embeddings	273M	11.1T	3.53	2.195 ± 0.005	1.834	72.99	17.58	23.28	26.48
Adaptive input embeddings	204M	9.2T	3.55	2.250 ± 0.002	1.899	66.57	16.21	24.07	26.66
Adaptive softmax	204M	9.2T	3.60	2.364 ± 0.005	1.982	72.91	16.67	21.16	25.56
Adaptive softmax without projection	223M	10.8T	3.43	2.229 ± 0.009	1.914	71.82	17.10	23.02	25.72
Mixture of softmaxes	232M	16.3T	2.24	2.227 ± 0.017	1.821	76.77	17.62	22.75	26.82
Transparent attention	223M	11.1T	3.33	2.181 ± 0.014	1.874	54.31	10.40	21.16	26.80
Lightweight convolution	257M	11.8T	2.65	2.403 ± 0.009	2.047	58.30	12.67	21.16	17.03
Envelop Transformer	224M	10.4T	4.07	2.370 ± 0.010	1.989	63.07	14.86	23.02	24.73
Synthesizer (dense)	217M	9.9T	3.69	2.220 ± 0.003	1.863	73.47	10.76	24.07	26.58
Synthesizer (dense plus)	224M	11.4T	3.47	2.334 ± 0.021	1.962	61.03	14.27	16.14	26.63
Synthesizer (dense plus alpha)	243M	12.6T	3.22	2.191 ± 0.010	1.840	73.98	16.96	23.81	26.71
Synthesizer (dense plus alpha)	243M	12.6T	3.01	2.180 ± 0.007	1.828	74.25	17.02	23.28	26.61
Synthesizer (factorized)	207M	10.1T	3.94	2.341 ± 0.017	1.968	62.78	15.39	23.55	26.42
Synthesizer (random)	254M	10.1T	4.08	2.326 ± 0.012	2.009	54.27	10.35	19.56	26.44
Synthesizer (random plus)	292M	12.0T	3.63	2.189 ± 0.004	1.842	73.32	17.04	24.87	26.43
Synthesizer (random plus alpha)	292M	12.0T	3.42	2.186 ± 0.007	1.828	75.24	17.08	24.08	26.39
Universal Transformer	84M	40.0T	0.88	2.406 ± 0.036	2.053	70.13	14.09	19.05	23.91
Mixture of experts	648M	11.7T	3.20	2.146 ± 0.006	1.785	74.55	18.13	24.08	26.84
Switch Transformer	1100M	11.7T	3.18	2.135 ± 0.007	1.758	75.38	18.02	26.19	26.81
Funnel Transformer	223M	1.9T	4.30	2.288 ± 0.008	1.918	67.34	16.26	22.75	23.20
Weighted Transformer	280M	71.0T	0.59	2.378 ± 0.021	1.989	69.04	16.98	23.02	26.30
Product key memory	421M	386.6T	0.25	2.155 ± 0.003	1.798	75.16	17.04	23.55	26.73

Do Transformer Modifications Transfer Across Implementations and Applications?

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Parting remarks

- Pretraining next!
- Good luck on assignment 4!
- Remember to work on your project proposal!