Assignment 4 group 11

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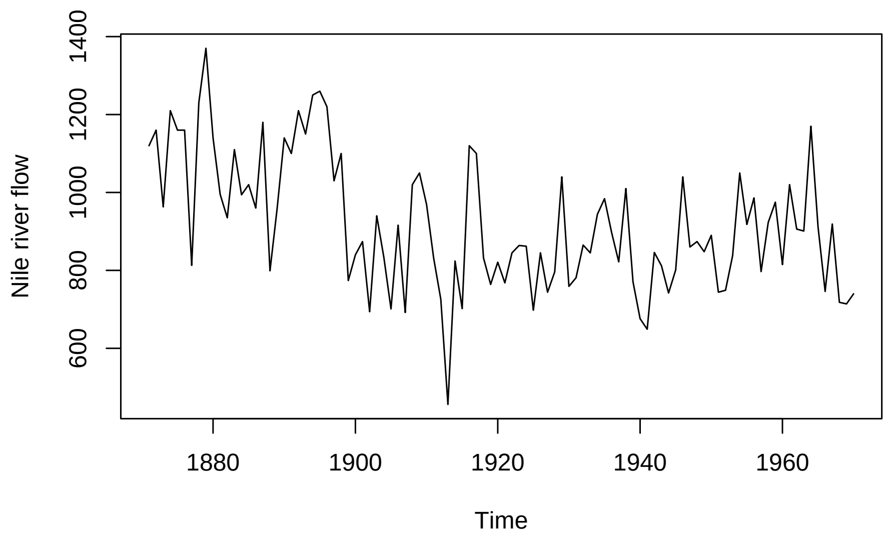
We consider the data on the annual flow of the river Nile at Ashwan between 1871 and 1970, plotted in Figure 1.

Figure 1: Flow of the Nile river at Aswan



We observe that the time series is non-stationary but that it presents an evident change point around 1898. Therefore, we use a *local level* model to capture the main change point and other minor changes. We consider the following model:

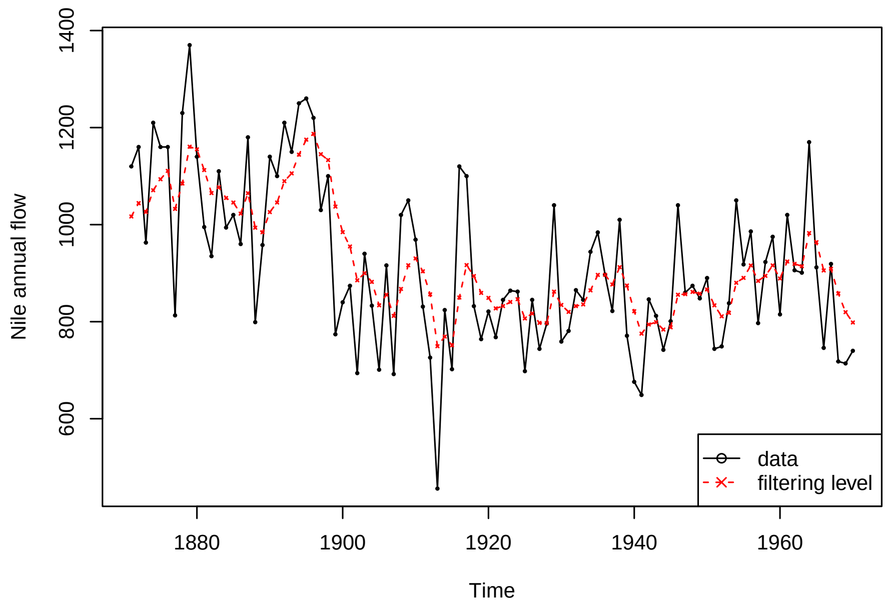
$$\begin{aligned} y_t &= \theta_t + v_t \qquad v_t \sim N(0, V) \\ \theta_t &= \theta_{t-1} + w_t \qquad w_t \sim N(0, W) \end{aligned}$$

To start, we assume that $V = \sigma_v^2 = 15100$ and $W = \sigma_w^2 = 1470$ and we let $\theta_0 \sim N(1000, 1000)$ be the initial distribution.

Exercise 1:

We compute and plot (Figure 2) the filtering states estimates $m_t = E(\theta_0|y_{1:t})$, for **t**=1,2,. . . ,T.

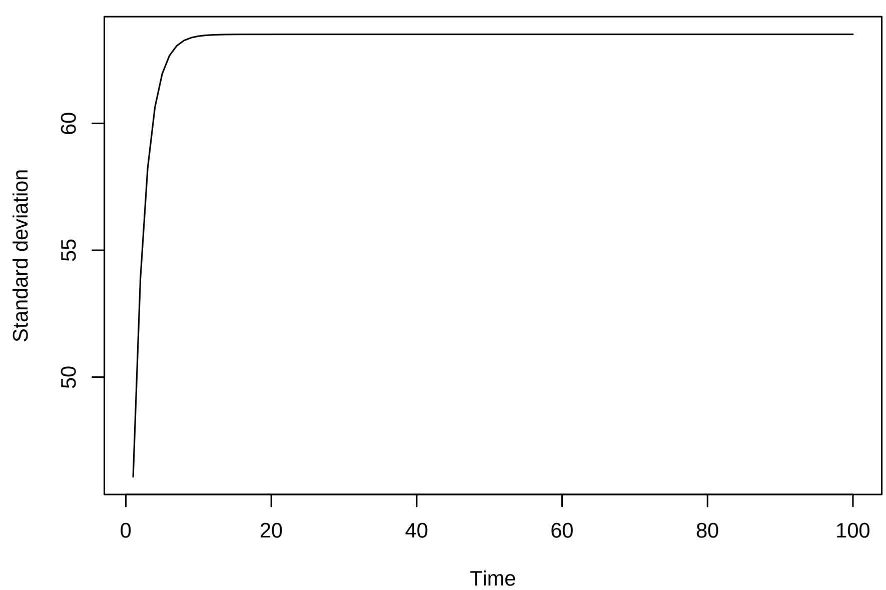
Figure 2: Estimated states



We also compute ane plot (Figure 3) the corresponding standard deviations:

$$\sqrt{C_t} = V(\theta_t|y_{1:t})^{1/2}$$

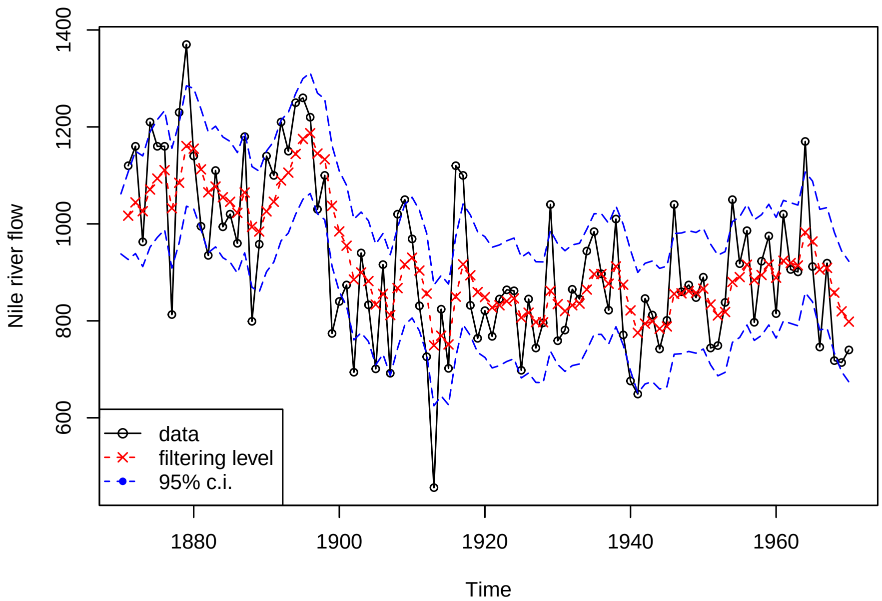
Figure 3: Standard deviation on the filtering estimates



We find that the standard deviation does not converge to zero but rather to approximately 63.5.

We plot the data together with the filtering state estimates and their 0.95 credible intervals (Figure 4):

Figure 4: Filtering state estimates

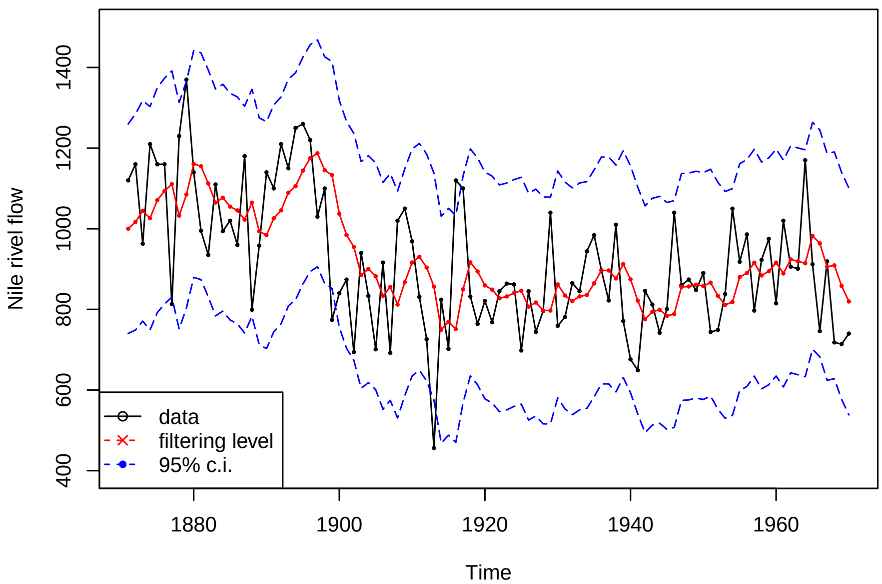


Exercise 2:

We compute the one-step ahead forecasts: $f_t = E(Y_t|y_{1:t-1}),\ t = 1, ..., T.$

We plot the data, together with the one-step ahead forecasts and their 0.95 credible intervals (Figure 5).

Figure 5: One-step ahead forecasts



Exercise 3

The signal-to-noise ratio (i.e. the ratio σ_w^2/σ_v^2) affects the forecasts, as it represents the weight given to observed data in the filtering step.

We repeat the excercise with different choices of V (observation variance) and W (evolution variance).

We use the following command to experiment with different values:

V <- sample(1:15100, 1)

W <- sample(1:1470, 1)

For example

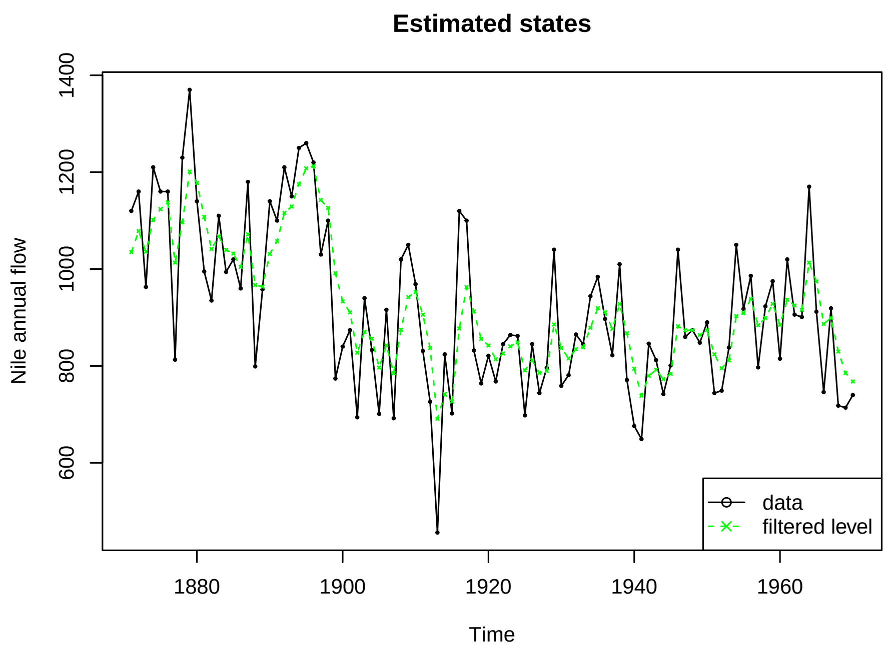
If V is:

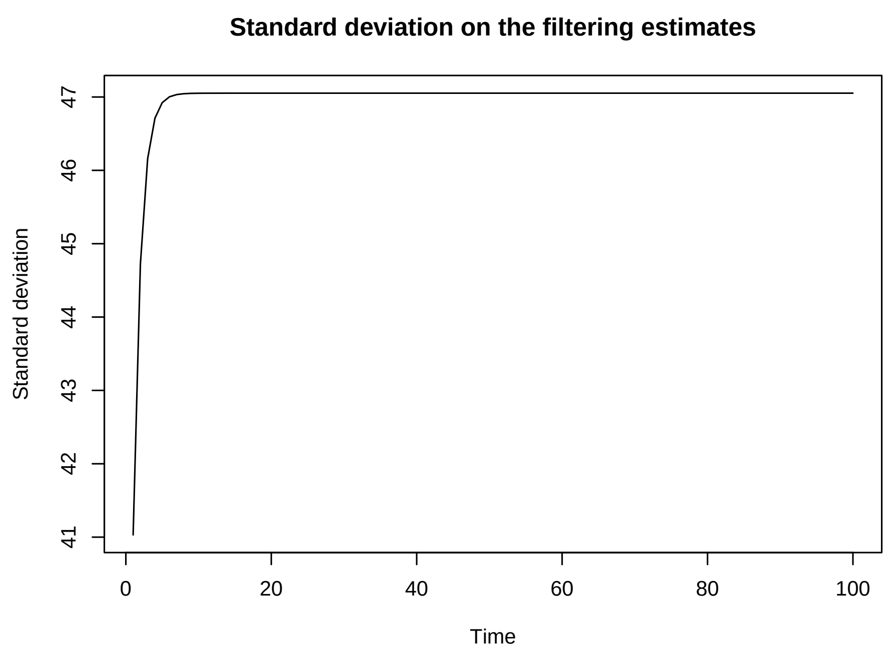
```
## [1] 5776
```

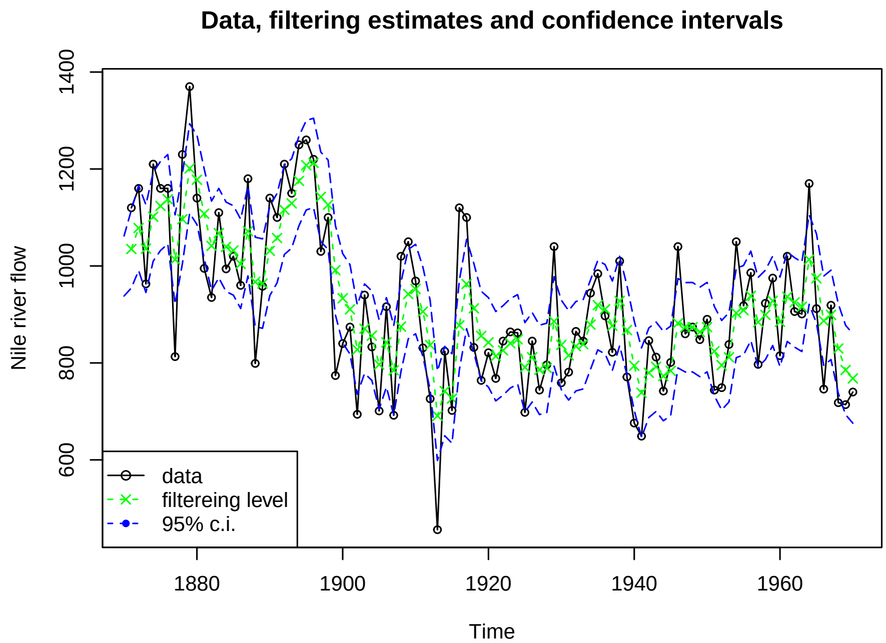
and W is:

```
## [1] 1376
```

We obtain the following:






Data, filtering estimates and confidence intervals
Nile river flow
Time
1880
1900
1920
1940
1960
600
800
1000
1200
1400
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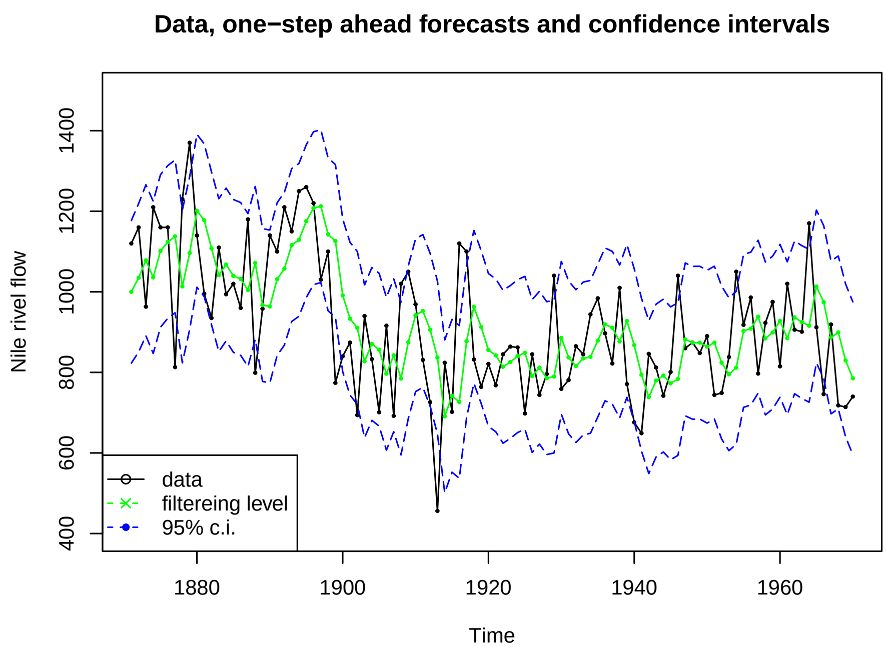
Data, one−step ahead forecasts and confidence intervals
Nile rivel flow
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Comment: