Assignment 3 - Dynamic Linear Models (Part I)

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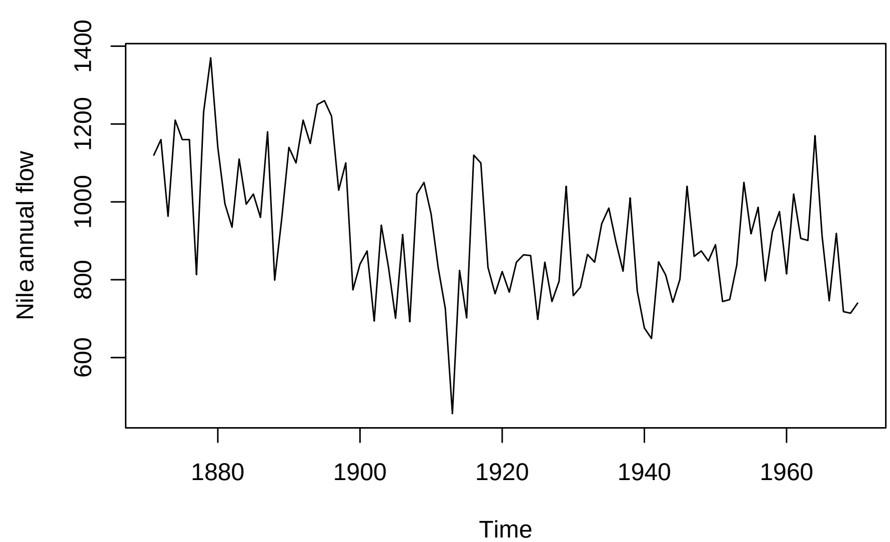
Data and model

We consider the data on the annual flow of the river Nile at Ashwan between 1871 and 1970, plotted in Figure 1.

Figure 1: Annual flow of the river Nile (1871 - 1970)



We observe that the time series is non-stationary but presents an evident change point around 1898. Therefore, we use a *local level* model to capture the main change point and other minor changes. We consider the following random walk plus noise model:

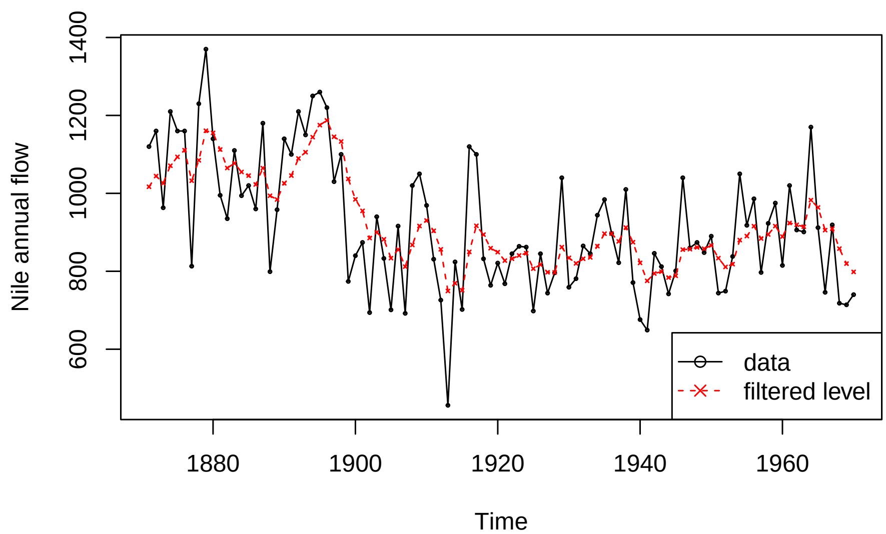
$$\begin{aligned} y_t &= \theta_t + v_t & v_t &\sim N(0, V) \\ \theta_t &= \theta_{t-1} + w_t & w_t &\sim N(0, W) \end{aligned}$$

with the assumption that $\theta_0 \perp (v_t) \perp (w_t)$. We assume that $V = \sigma_v^2 = 15100$ and $W = \sigma_w^2 = 1470$. Let $\theta_0 \sim N(1000, 1000)$ be the initial distribution.

Question 1

We compute and plot (Figure 2) the filtering states estimates $m_t = E(\theta_t|y_{1:t})$, for **t**=1,2,...,T.

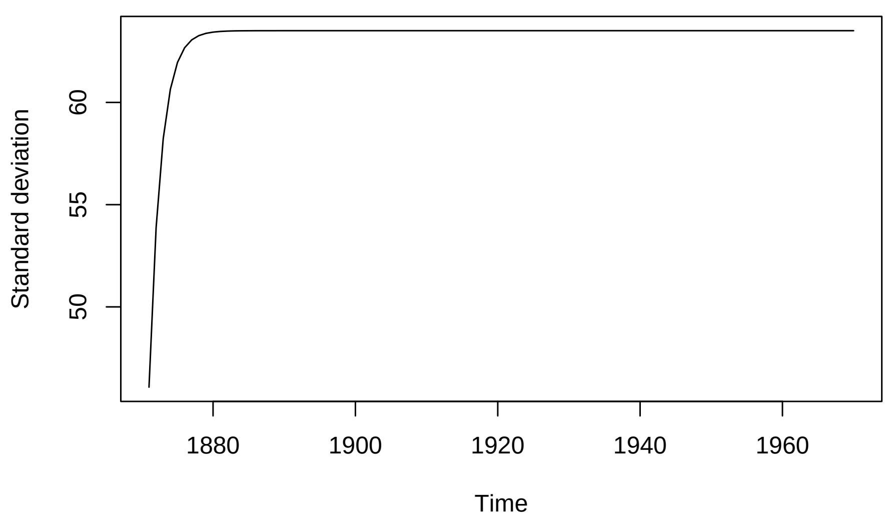
Figure 2: Nile river data and filtering states estimates



We also compute and plot (Figure 3) the corresponding standard deviations:

$$\sqrt{C_t} = V(\theta_t|y_{1:t})^{1/2}$$

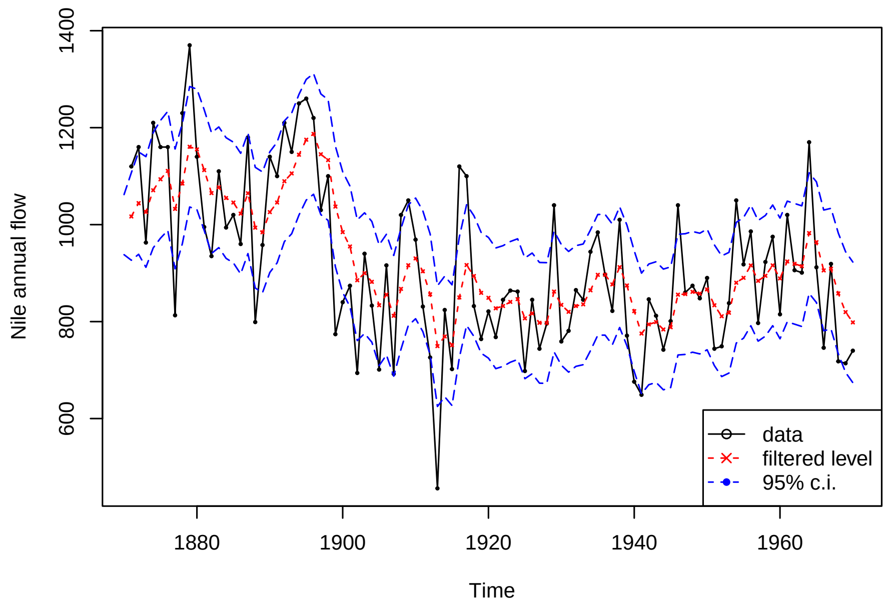
Figure 3: Filetring states Standard Deviations



We find that the standard deviation does not converge to zero but rather to approximately 63.5.

We plot the data together with the filtering state estimates and their 0.95 credible intervals (Figure 4).

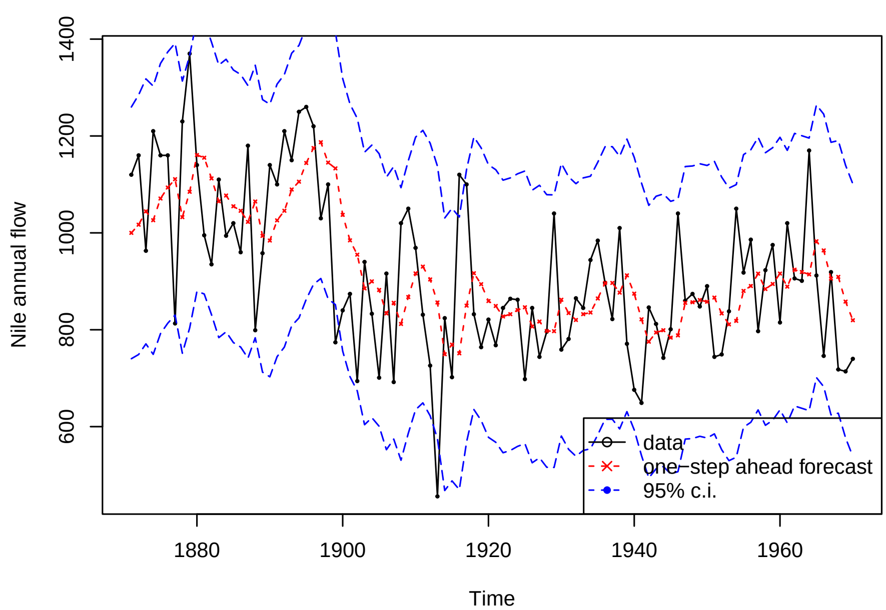
Figure 4: Nile data and filter estimates with credible intervals



Question 2

We compute the one-step ahead forecasts: $f_t = E(Y_t|y_{1:t-1}),\ t = 1, ..., T$ and plot the data, together with the one-step ahead forecasts and their 0.95 credible intervals (Figure 5).

Figure 5: Nile river data and one-step ahead forecasts



Question 3

The signal-to-noise ratio (i.e. the ratio σ_w^2/σ_v^2) affects the forecasts, as it represents the weight given to observed data in the filtering step.

We repeat the excercise with different choices of V (observation variance) and W (evolution variance).

We can use the following command to experiment with different values:

V <- sample(1:20000, 1)

W <- sample(1:20000, 1)

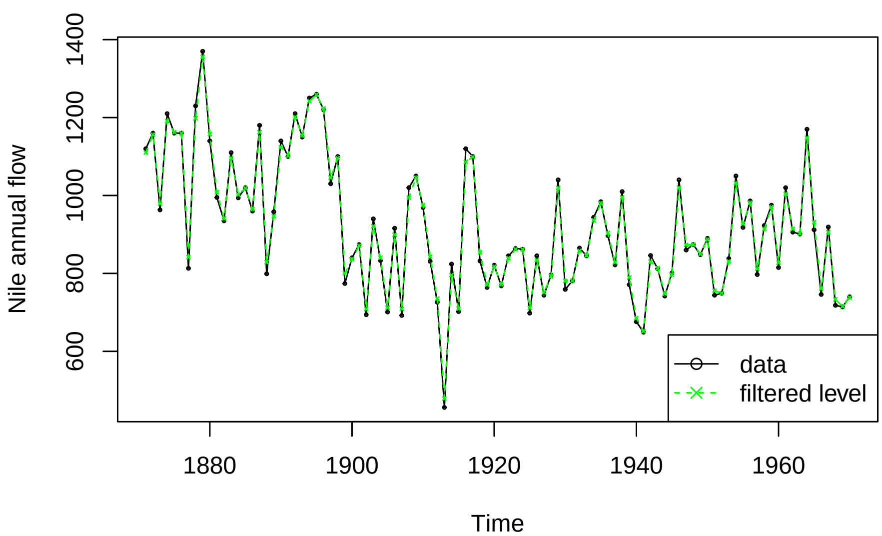
To report our findings, we show an example in which we exchange the two variances and assume the model to be:

$$
\begin{aligned}
y_t &= \theta_t + v_t \qquad & v_t &\sim N(0, V) \\
\theta_t &= \theta_{t-1} + w_t \qquad & w_t &\sim N(0, W)
\end{aligned}
$$

where V = 1470 and W = 15100.

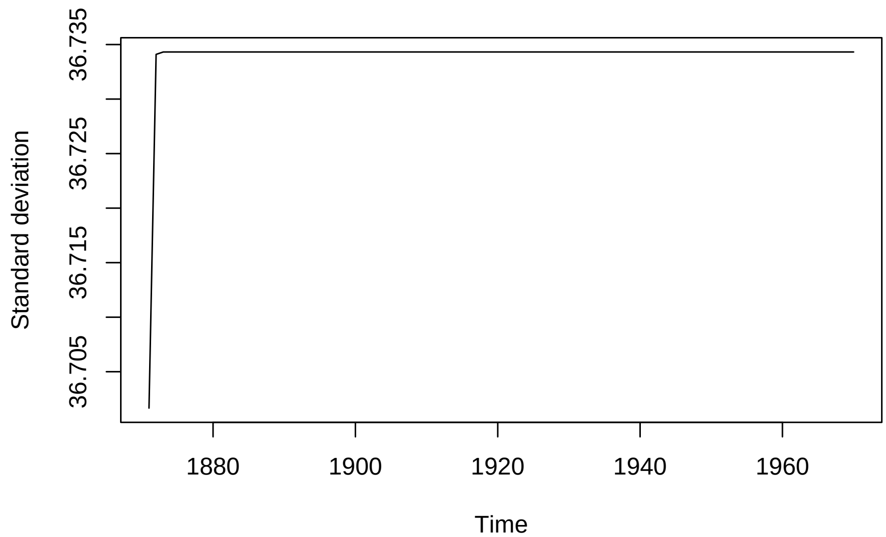
The signal-to-noise ratio of the model 1 was $\sigma_w^2/\sigma_v^2 = 1470/15100 = 0.097$. The signal-to-noise ratio of the model 2 is $\sigma_w^2/\sigma_v^2 = 15100/1470 = 10.272$, way larger than before.

Figure 6: Nile river data and filtering states estimates



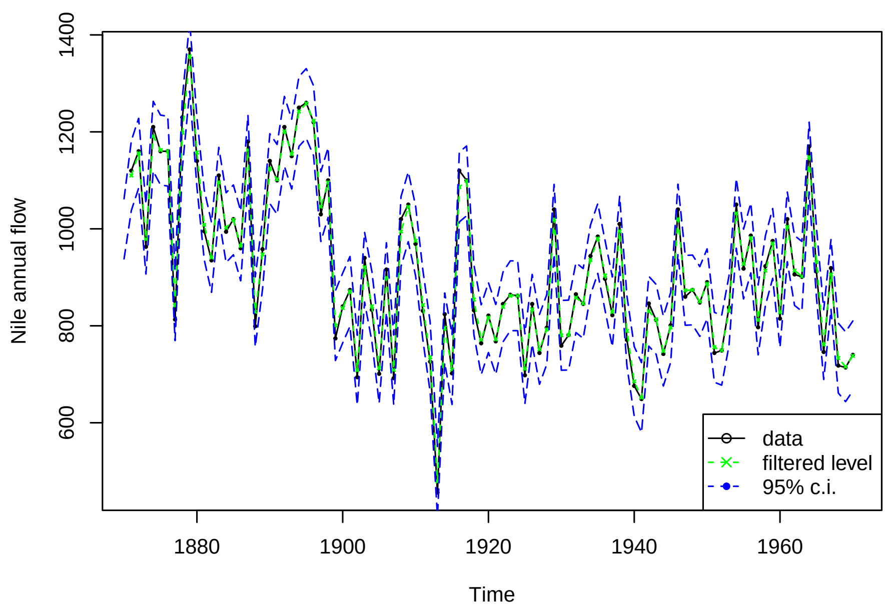
As expected, in model 2, with a higher signal-to-noise ratio, the filter estimates better tracks the actual observed data, as observed data are given more weight in the filtering step (Figure 6).

Figure 7: Filetring states Standard Deviations



Also the standard deviation of the estimated model (Figure 7) converges to a lower level (36.735) than before (63.5), indicating a better precision of the estimates (Figure 8).

Figure 8: Nile data and filter estimates with credible intervals



A higher signal-to-noise ratio determines a higher reliance of the last observation. Hence, the one-step ahead forecast in this case is close to be a one-step rightward shift of the observed data (Figure 9).

Figure 9: Nile river data and one-step ahead forecasts