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Brief paper

Robust and ultrafast response compensator for unstable invertible plants[☆]

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ABSTRACT

Quasi-linear feedback theory (Kelemen, 2002; Kelemen and Bensoussan, 2004) enabled simultaneously improving time performance and frequency performance of feedback systems, by using compensators whose poles are gain dependent. Although simulation results show theoretical improvements, the gain magnitude of quasi-linear compensators is prohibitive. This paper intends to present a different quasi-linear controller (Bensoussan, 2011) that achieves simultaneous time and frequency optimization: The design is based on gain considerations in order to propose a framework within which phase circuits can be used in order to affect essentially the time response. A practical design methodology is outlined.

1. Introduction

Classical control theory is concerned with improving a controlled system's performance measures in both the frequency and time domains. In particular, improvement objectives of time-domain performances generally involve decreasing rise-times, steady state error, sensitivity to plant uncertainty or external disturbances, and settling-time responses of a given linear time-invariant system. Likewise, improvement objectives in frequency-domain performances generally involve increasing phase and gain stability margins to improve the stability of a given linear time-invariant system. The simplest form of compensation used to improve the transient response of a system is based on high gain feedback. It is well known that increasing gain beneficially results in increased response speeds, decreased steady state error, and the like. However, high gain compensation or classical pole placement require a compromise between the selection of a proper gain and other acceptable performance measures. Indeed, a gain increase to a high enough extent in certain systems can lead to oscillatory behavior and instability. During the 1980s, tremendous interest was given to the problem of robustness, leading to $H\infty$ control theory (Zames, 1981). However, operator optimization of the sensitivity translated essentially into frequency response robustness results. Although the application of high gain feedback in classical control methods sometimes leads to oscillations and instabilities, robustness algorithms showed that it is possible to apply very high gain and still benefit from an improved gain and phase margins while minimizing the sensitivity operator (Bensoussan, 1984). Time response was somehow disregarded for a while, the underlying assumption being that improved time response generally worsens frequency response performance.

As a result of these shortcomings, quasi-linear compensators have been proposed (Kelemen, 2002; Kelemen & Bensoussan, 2004). Quasi-linear compensators eliminate the contradiction between performance and compensator complexity and consequentially achieve arbitrary close to perfect tracking performance when the gain of the compensator tends to infinity (Kelemen & Bensoussan, 2004). Furthermore, quasi-linear feedback compensators have been shown to have non-oscillatory time responses for high compensator gains. These benefits which quasi-linear compensators provide over linear compensators are explained by the automatic adaptation of the closed loop poles to stability and stability margins for higher system gains. However, prior art quasi-linear controllers have yet to comprehensively address all performance considerations, in particular the improvement of system rise times. The present method is a quasi-linear

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controller that is simultaneously optimizable in the time domain and the frequency domain, which achieves arbitrarily fast and robust tracking, improved gain and phase stability margins, improved time domain performances, and improved sensitivity for a variety of stable and unstable systems. Although the design is based on plant inversion, it will be shown that ultrafast non oscillatory time response can cohabit with acceptable cost of feedback.

Robust control methods optimize energy gains. However, the phase distribution of the present compensator can be altered in specific frequency domain ranges in order to get an improved time response or by making a compromise between non ideal optimized energy gains and improved time response.

In the present design, it is proposed to take advantage of the availability of new high bandwidth and high resolution sensors and the high digital processing speed. This is the reason why the design by invertibility methods presented here become realistic.

2. The proposed control method

The method (Bensoussan, 2011) aims at controlling an instable invertible plant transfer function $P(s)$. More specifically, the application of the controller $C(s)$ is restricted to a class of plants $P(s)$ which are linear time invariant, invertible, and strictly proper. More specifically, the plant is invertible if it has no right-half plane zeros, i.e. its reciprocal $P^{-1}(s)$ is holomorphic in $Re(s) \geq 0$. It comprises calculating an open loop transfer function $J(s)$ comprising the product of a high gain filter $J_1(s)$ having a gain (Bensoussan, 2011) sufficient so that $|J(\omega)| > [1 + 1/\varepsilon]$ when $|\omega| \leq \omega_1$, wherein ω_1 is selected to obtain a desired time response; a phase network $J_2(s)$ providing phase lead at intermediate frequencies; a low pass filter $J_3(s)$ selected such that $|1 + J(\omega)| > 1/M$ for all ω and $J(s)$ is strictly proper, wherein $\varepsilon < 1$ and $M > 1$. The values ε and M are selected to meet a desired sensitivity requirement. The compensator of the unity feedback system will have the form $C(s) = H^{-1}(s)J(s)$ and inputting the error signal into the plant. $H(s)$ can be easily shown to be a modified version of $P(s)$ which is identical to $P(s)$ whenever $P(s)$ is minimum phase (Bensoussan, 1984).

Let $u(t)$ be the input signal to the feedback system, $y(t)$ the output signal from the plant, and $e(t)$ the error signal representing the difference between the input signal $u(t)$ and the output signal $y(t)$ as calculated by a subtractor. The closed loop feedback system is described by the group of equations

$$Y(s) = P(s)\,C(s)E(s) \quad \text{and} \quad E(s) = U(s) - Y(s) \tag{1}$$

where the signals $U(s)$, $Y(s)$ and $E(s) = S(s)U(s)$ represent the Laplace transforms of the corresponding time domain functions $u(t)$, $y(t)$ and $e(t)$ respectively. The design objectives of the controller include achieving fast and robust tracking, improved gain and phase stability margins, improved time domain measures and reduced rise times, and improved sensitivity of a variety of stable and unstable systems. The controller achieves these performance measures simultaneously in the time-domain and the frequency-domain and does so without the trade-offs normally associated with linear controllers.

The design objectives of the controller $C(s)$ take into account the time response such that the feedback gain has a dominant pole ω_1 which responds to sufficiently rapid time response objectives while achieving sensitivity objectives. Let $S(\omega)$ be the sensitivity operator defined by $[I + P(\omega)C(\omega)]^{-1}$. Generally, these sensitivity objectives include the minimum sensitivity conditions within a given bandwidth, for instance $|S(\omega)| < \varepsilon < 1$, which does not translate into an excessive maximal sensitivity $M > 1$ over the whole frequency range. In this sense, the sensitivity may be controlled and the feedback system is able to react sufficiently fast while remaining stable. Note that in a unity feedback system, a value of $|S(\omega)|$ close to zero translates into a closed loop transmission close to 1 which indicates excellent tracking of the input by the output. Also note that in a unity feedback system, boundedness of $|S(\omega)|$ ensures the boundedness of the closed loop. Stability therefore follows even when the plant is unstable and invertible (Zames, 1964; Zames & Bensoussan, 1985). The quasi-linear controller presented here is a controller comprising poles which depend in an appropriate way on its gain. Its advantage over linear controllers resides in that it is able to maintain the excess of poles over zeros unaltered and allows for an increase in the loop transmission gain without jeopardizing the phase and gain stability margins. Lead networks can ensure that the intermediate frequency range of the closed loop will also contribute to an improved time response. The phase network $J_2(s)$ allows for design improvement on the transition band between low frequencies and high frequencies. Ideally, a closed loop transfer function equal to unity would allow exact tracking. This could be nearly achieved using high gain in the low frequency band. As the closed loop transfer function is strictly proper, higher cutoff frequencies improve the time response but also increase the invested energy. We will therefore use high frequency cutoff at reduced gains.

The design is based on gain considerations in order to propose a framework within which phase circuits can be used in order to affect essentially the time response. This would lead to a better optimization of the time response and allow the reduction of the controller bandwidth.

3. Construction of the controller

We define

$$C(s) = H^{-1}(s)\,J(s)\,, \tag{2}$$

where $H(s)$ represents a transfer function that behaves like $P(s)$ at high frequencies and $J(s)$ approximates a function in the manner now described. Writing $s = j\omega$, the values of $J(\omega)$ lie in the right-half complex plane over a frequency range given by $|\omega| \leq \omega_1$ *and* $J(\omega)$ will have a high gain so that the values of $P(\omega)C(\omega)$ will be kept outside the sensitivity circle centered at $(-1, 0)$, with radius $1/\varepsilon > 1$. Such a design ensures that the sensitivity on the restricted frequency range $|s| < \omega_1$ is less than ε, i.e.

$$\left\| [1 + P(\omega)\,C(\omega)]^{-1} \right\|_{\omega 1} < \varepsilon. \tag{3}$$

Moreover, the values of $J(s)$ will be kept outside the M sensitivity circle centered at $(-1, 0)$ with radius $1/M < 1$ at all frequencies. Such a design restricts the H_∞ norm of the maximal sensitivity to a value smaller than M at any frequency, i.e.

$$\left\| [1 + P(s)\,C(s)]^{-1} \right\| < M. \tag{4}$$

The sensitivity circle M could be represented on the Nichols chart by representing the graph $20\log\left[\cos\varphi + \alpha\left(\left(\cos^2\varphi\right) - 1 + 1/M^2\right)\right]$ if $\cos^2\varphi > \left(1 - 1/M^2\right)$. Alternatively, the diagram of $J^{-1}(s)$ could be plotted on a Nichols chart in which the classical M-circles (constant magnitudes of the closed loop) now represent the sensitivity circles. The implementation of the compensation $C(s)$ lends itself to a simple graphical conception using classical control tools such as Bode, Nyquist, Evans or Black–Nichols diagrams and charts. The function $J(s)$ can be decomposed in three parts:

$$J(s) = k_1 J_1(s)\,J_2(s)\,J_3(s)\,. \tag{5}$$

Wherein $k_1 J_1(s)$ is a high gain filter with a good time response acting at low frequencies, $J_2(s)$ is a phase network that provides a phase lead in the intermediate frequency range and $J_3(s)$ is a wideband low pass filter that also ensures that the compensator is strictly proper.

We assume the following behavior of the plant at high frequencies: $|P(s)| > c_1/s^q$ for some constant value c_1 and integer q for high frequencies. The unstable plant may be reduced to its minimum phase part $P_1(s)$ and its unstable part $P_2(s)$, such that the cascaded elements represent the single input–single output plant in the following manner:

$$P(s) = P_1(s)P_2(s). \tag{6}$$

A further transfer function $H(s)$ can be defined such that, for some value s_0:

$$H(s) = \left[\frac{c}{(s+s_0)^q}\right] P_2^{-1}(s)\,P(s) = \frac{c}{(s+s_0)^{q'}} P_1(s) \tag{7}$$

such that $H(s)$ has the same behavior as $P(s)$ at high frequency $|s| \geq \omega_c$, i.e.

$$\left\|P(s)H(s)^{-1} - 1\right\| \leq \alpha < 1, \tag{8}$$

where α is a value inferior to unity. Note that $H(s)$ is holomorphic by construction and that its inverse is holomorphically invertible in $Re(s) \geq 0$. The controller $C(s)$ is built as follows:

$$C(s) = H^{-1}(s)\,J(s) = \left(\frac{1}{c}\right)(s+s_0)^{q'} P_1^{-1}(s)\,J(s). \tag{9}$$

Wherein $J(s) = k_1J_1(s)J_2(s)J_3(s)$ and $k_1J_1(s)$ is the transfer function of a high gain filter having an ultra-fast time response, e.g. $J_1(s)$ can have the form:

$$k_1J_1(s) = k_1\frac{\omega_1}{s+\omega_1}.$$

The compensating unit $J_2(s)$ comprises a chain of lead/lag compensator elements as a n phase lead/phase lag compensator acting in the frequency intermediary range, where the poles p_i might be unstable

$$J_2(s) = \prod_{i=1}^{n} \frac{s+z_i}{s+p_i}$$

$J_3(s)$ is the transfer function of a low pass filter acting at a very high frequency such that the transfer function of the controller $C(s)$ remains strictly proper. The low pass filter $J_3(s)$ may be provided for in the general form:

$$J_3(s) = \prod_{i=1}^{k} \frac{\omega_{2i}}{s+\omega_{2i}} \quad k \geq q.$$

Of note, the choice of ω_{2i} could be reduced in various manners to improve implementation, such as a reduction in energy requirements.

As an example, we could choose

$$J(s) = \frac{k_1\omega_1}{(s+\omega_1)}\left[\frac{\omega_2}{(s+\omega_2)}\right]^k \left[\frac{(s+z)}{(s-p)}\right] \quad k \geq q.$$

4. The rationale behind the controller

The compensator $C(s)$ is strictly proper as the order of the denominator of $J_1(s)J_3(s)$ is superior or equal to $q+1$ as $J_1(s)$ has an order superior or equal to 1 and the denominator of $J_3(s)$ has an order $k \geq q$. Indeed, at high frequency, we reformulate Eq. (8) in order to get

$$\left\|P(s)H^{-1}(s)\right\| \leq 1 + \left\|P(s)H^{-1}(s) - 1\right\| \leq 1+\alpha \tag{10}$$

and the order of the numerator of $H^{-1}(s)$ is $q' \leq q$ as

$$\left\|H^{-1}(s)\right\| \leq \left\|P(s)H^{-1}(s)\right\| \cdot \left\|P^{-1}(s)\right\| \leq (1+\alpha)\frac{|s^q|}{c_1}.$$

In fact, it is the right choice of c and s_0 and q' that guarantee the adequate number of encirclements of the critical point by the Nyquist plot. It follows that the order of the denominator of $C(s)$ is at least one.

We now show that $\inf(P(s)H^{-1}(s))$ is non-zero at high frequencies $|s| \geq \omega_c$. $H^{-1}(s)$ is holomorphic in $Re(s) \geq 0$ and $P(s)$ is holomorphic in $|s| \geq \omega_c$. Since $P(s)$ is holomorphically invertible in $Re(s) \geq 0$, $P(s)$ is a nonvanishing function of s in $Re(s) \geq 0$. It follows that $P(s)H^{-1}(s)$ is a continuous nonvanishing function which is bounded below by some constant in $Re(s) \geq 0$. Moreover,

$$\begin{aligned}\sup\left|P(s)H^{-1}(s)\right| &\geq 1 - \sup\left|P(s)H^{-1}(s) - 1\right| \\ &\geq 1-\alpha > 0 \quad \text{for } |s| \geq \omega_c.\end{aligned} \tag{11}$$

Define ω_d as the desired bandwidth for a fast time response, ω_1 is preferably chosen to be superior or equal to ω_d; We select the frequency ω_a such that $P(s)$ and $P^{-1}(s)$ are holomorphic for $|s| > \omega_a$ and such that all the right half plane poles of $P(s)$ are confined in the region $|s| < \omega_a$ and the frequency ω_c above which $P(s)$ and $H(s)$ have the same high frequency behavior, i.e. $\left\|P(s)H^{-1}(s) - 1\right\| \leq \alpha < 1, |s| > \omega_c$ where α is a value inferior to unity and sufficiently small to ensure that $M(1+\alpha) > 1$. We define $\omega_x \geq \max(\omega_1, \omega_a, \omega_c)$.

4.1. Consider the frequency range $|\omega| \leq \omega_1$

The controller is designed so that

$$\sup_{|\omega|<\omega_1} |J(\omega)| > \left|\left[\inf(P(\omega)H^{-1}(\omega))\right]^{-1}\left[\frac{1+\varepsilon}{\varepsilon}\right]\right| \tag{12}$$

wherein $|\omega| \leq \omega_1$.

The gain k_1 which now includes the factor z/p, the frequency ω_2, and the parameter k are adjusted to satisfy conditions (3) and (4). For example:

$$k_1 = 2 \cdot \max\begin{bmatrix}\inf\left[P(\omega)H^{-1}(\omega)\right]\cdot\left(1+\dfrac{1}{\varepsilon}\right) \text{ for } \omega < \omega_1, \\ \inf\left[P(\omega)H^{-1}(\omega)\right]\cdot\left(1+\dfrac{1}{M}\right) \text{ for all } \omega\end{bmatrix}. \tag{13}$$

The frequency ω_2 and the parameter k are adjusted so that the modulus of $J(\omega)$ is higher than a positive value $\gamma < 1$, say $\gamma = 1/2$, i.e. $|J(\omega)| \geq 1/2$ for $|\omega| \leq \omega_1$ while making sure that k is large enough to ensure that $C(s)$ is strictly causal. Note, that for $|k_1J_1(\omega_1)| \geq k_12^{-1/2}$ for $\omega < \omega_1$ and that $|J_3(\omega_1)|$ can be made bigger than $2^{-1/2}$ for ω_2 large enough s_0 that $|J(\omega)| \geq k_1/2$ and so that (3) is satisfied.

It follows that over the limited bandwidth $|\omega| < \omega_1$,

$$\begin{aligned}&\left|(1+P(\omega)C(\omega))^{-l}\right| = \frac{1}{|1+P(\omega)C(\omega)|} \\ &\frac{1}{|1+P(\omega)C(\omega)|} \leq \frac{1}{\inf|P(\omega)C(\omega)-1|} \\ &\frac{1}{\inf|P(\omega)C(\omega)-1|} = \frac{1}{k_l\left|P(\omega)H^{-l}J(\omega)-1\right|} \leq \varepsilon.\end{aligned} \tag{14}$$

Similarly, for $k_1 > \inf\left(P(s)H^{-1}(s)\right)(1+1/M)$ in $Re(s) \geq 0$, it will follow that $\left|1+(P(s)C(s))^{-1}\right|$ is bounded above by M for all $|\omega| < \omega_1$.

4.2. Consider the frequency range $|\omega| > \omega_1$

The design of the controller is carried out with the objective of satisfying the following conditions

$$\sup_\omega |J(\omega)| > \left|\left[\inf\left(P(\omega)H^{-1}(\omega)\right)\right]^{-1}\left[\frac{1+M}{M}\right]\right| \quad (15)$$

for all ω.

We shall use the fact that by construction $k_1J_1(s)$ is positive real, i.e. $Re\{k_1J_1(s)\} \geq 0$. It follows from standard argument about positivity and contractiveness (Kelemen & Bensoussan, 2004; Saeki, 1998) that the function $(1+k_1J_1)(s)$ is holomorphically invertible in $Re(s) \geq 0$ and satisfies the inequalities.

$$\left\|(1+k_1J_1)^{-1}(s)\right\| \leq 1, \qquad \left\|k_1J_1(1+k_1J_1)^{-1}(s)\right\| \leq 1$$

$$\text{and} \quad \|(1+k_1J_1)(s)\| \geq 1 \quad (16)$$

(the validity of which is also clear from the explicit formulas for the frequency responses involved).

The return difference $(1+PC)(s)$ is now expressed in terms of k_1J_1 as follows. For any s in $|s| \geq \omega_1,\ Re(s) \geq 0$,

$$\begin{aligned}
&\|(1+PC)(s)\| = \left\|(1+k_1PH^{-1}J_1J_3)(s)\right\| \\
&= \left\|\left\{1+k_1J_1+(PH^{-1}1)k_1J_1+PH^{-1}k_1J_1(J_3-1)\right\}(s)\right\| \\
&= \|\{[(1+k_1J_1)+[(PH^{-1}-1)k_1J_1 \\
&\quad +PH^{-1}k_1J_1(J_3-1)]](1+k_1J_1)^{-1}(1+k_1J_1)\}(s)\| \\
&\geq \|\{1+[(PH^{-1}-1)k_1J_1 \\
&\quad +PH^{-1}k_1J_1(J_3-1)][(1+k_1J_1)^{-1}]\}(s)\| \\
&\geq 1-\left\|\begin{matrix}\{(PH^{-1}-1)k_1J_1(1+k_1J_1)^{-1} \\ +PH^{-1}k_1J_1(J_3-1)(1+k_1J_1)^{-1}\}(s)\end{matrix}\right\| \\
&= 1-\|\phi(s)\| \quad \text{provided } \|\phi(s)\| < 1. \qquad (17)
\end{aligned}$$

As $P(s)H^{-1}(s)$ is holomorphic for $s \in |s| > \omega_1,\ Re(s) \geq 0$ and we have

$$\begin{aligned}
\|\phi(s)\| \leq{}& \left\|(PH^{-1}-1)(s)\right\| \cdot \left\|k_1J_1(1+k_1J_1)^{-1}(s)\right\| \\
&+ \left\|PH^{-1}(s)\right\| \cdot \left\|\{k_1J_1(1+k_1J_1)^{-1}(1-J_3)\}(s)\right\|. \quad (18)
\end{aligned}$$

Now since H^{-1} is an ultimate right α inverse of P, there exists a value ω_c such that for any

$$\omega > \omega_c, \left\|\left[PH^{-1}-1\right](s)\right\| \leq \alpha.$$

Henceforth, as $\omega > \omega_x \geq \omega_c$ we apply this result, (16), and the identity

$$k_1J_1(1+k_1J_1)^{-1} = \frac{k_1}{1+k_1}J_{1(1+k_1)},$$

where $J_1(s) = \frac{\omega_1}{s+\omega_1}$ and $J_{1(1+k_1)} = \frac{\omega_1(1+k_1)}{s+\omega_1(1+k_1)}$.

To obtain

$$\begin{aligned}
&\|\phi(s)\| \\
&\leq \alpha + \left\{\frac{k_1}{1+k_1}\left\|(PH^{-1}(s))\right\| \cdot \left\|J_{1_{(1+k_1)}}(s)\right\| \cdot \|1-J_3(s)\|\right\}. \quad (19)
\end{aligned}$$

Now, as $J_{1(1+k_1)}(s)$ and $J_3(s)$ are strictly proper, whenever ω_1 is fixed and $n \to \infty$ $\left\|J_{1(1+k_1)}(s)\right\|$ approaches zero uniformly in $|s| \geq \omega_2, Re(s) \geq 0$, and $\|(1-J_3)(s)\|$ approaches zero uniformly in $\omega_1 \leq |s| \leq \omega_2, Re(s) \geq 0$. The $\{\cdot\}$ bracketed term on the right hand side of (19) is made less than $\{(1-\alpha_1)-1/M\}$ for ω_2 large enough. In other words,

$$\begin{aligned}
&\left|\left[\inf(P(s)H^{-1}(s))\right] \cdot \left[k_1\left(\frac{\omega_1}{s+(1+k_1)\omega_1}\right)\right]\right| \cdot \left|1-\left[\frac{\omega_2}{s+\omega_2}\right]^k\right| \\
&\leq \left[1-\alpha-\frac{1}{M}\right]. \qquad (20)
\end{aligned}$$

Note that the condition in Eq. (20) may also be satisfied by increasing the value of ω_2 as needed, i.e. as k_1 is increased, the expression $k_1[\omega_1/(s+(1+k_1)\omega_1)]$ approaches unity, while the expression $[1-[\omega_2/(s+\omega_2)]^k]$ approaches zero as ω_2 increases. This gain-pole dependency is consistent with the quasi-linear property of the feedback compensation.

It follows that

$$\|\phi(s)\| \leq \alpha+1-\alpha-\frac{1}{M} = 1-\frac{1}{M} < 1. \quad (21)$$

From (17) and (21), it follows that

$$\|(1+PC)(s)\| > \frac{1}{M}.$$

Example. Illustratively, the plant is represented by the following transfer function:

$$P(s) = \frac{1}{(s+2)^2}\frac{s+1}{(s-2)}.$$

As part of the controller design, it is desired to have a system time response faster than e^{-4t}, i.e. $\omega_d = 4$ and a sensitivity smaller than 0.01 in the bandwidth $|\omega| < 5$ and smaller than 3 on the whole frequency range, i.e. $\varepsilon = 0.01$ and $M = 3$. Accordingly, $\omega_1 = 5$ is selected. The plant $P(s)$ can be reduced to two transfer functions of the form:

$$P_1(s) = \frac{1}{(s+2)^2} \quad \text{and} \quad P_2(s) = \frac{s+1}{s-2}.$$

The plant $P(s)$ presents a significant obstruction to high performance by a linear feedback. Its excess of poles over zeros limits the increase of the controller gain if the dynamic compensation order is kept at acceptable low levels, affecting performance which calls for increased gain as well as stability. Quasi-linear compensation can be applied to achieve high performance simultaneously in the time and frequency domains. Note that $|P(s)| > c/s^2$ for some constant c and values of s that are superior to 1 and that at high frequencies, $H(s)$ converges towards $c/(s+s_0)^{q'}$, or 1, in this case. Furthermore, consider that $H(s) = 1/(s+2)^2$ and that at high frequencies $\alpha = 0$. In other words, $\omega_\alpha = 1$ and in this particular case, ω_c can take any value e.g. 1.

5. Design methodology

The frequency ω_1 is preferably chosen to be superior or equal to the desired open loop bandwidth ω_d; the gain k_1 and the frequency ω_2 are chosen so that inequality (12) is satisfied.

In practice, ω_2 can be fine-tuned using classical engineering methods as follows. The first objective is to get a good phase margin. A simple analysis on the Nyquist curve shows that the gain margin and the phase margin are improved whenever the value of M approaches unity. Let ω_p be the gain crossover frequency, $\omega_x \leq \omega_p$. At the frequency ω_p, the gain k_1 is a function of $\frac{\omega_p}{\omega_1}$ and $\frac{\omega_p}{\omega_2}$ and the phase of $J(\omega_p)$ is $artg(\frac{\omega_p}{\omega_1}) + kartg(\frac{\omega_p}{\omega_2})$. The gain k_1 and the frequency ω_2 are chosen so as to get a phase margin as close as possible to an ideal phase margin PM_i. This illustrates the quasi-linear dependency of the gain k_1 and the frequency ω_2. Note that the steep decrease of the modulus of $J(\omega)$ at high frequency (which can be accelerated through the exponent k) ensures that the PM_i is maintained to a value as close as possible to the ideal one. The compensator $J_2(s)$ can include a phase circuit which is designed specifically to improve the phase margin. Our second objective is to fine-tune the compensator $J_2(s)$ which is comprised of one or more phase circuits and acts between ω_1 and ω_2. It is designed to align the Nyquist diagram along the real line -0.5 on the widest frequency range to ensure a closed loop gain of unity and therefore a better tracking of the system. The compensator $J_2(s)$ can improve the time response and hardly degrades the stability margins. This will lead to the third objective which is to aim at a wide bandwidth of the closed loop ω_b, $\omega_x \leq \omega_b \leq \omega_p$, as the rise time of the system is inversely proportional to it. The final objective is to further reduce the overshoot which can be handled with a prefilter or a feedforward filter. (Tsai, Schaper, & Kailath, 2004) Constraints due to sensor bandwidth and data processing speed can be taken into consideration when adjusting the parameters of $J(s)$ to get as close as possible to the design stated above.

Note the crucial importance of the phase network: it allows to speed-up the time response. This allows the designer to reduce the high cutoff frequency ω_2, therefore reducing the norm of the compensator (i.e. the control effort), the value of which is related to the inverse of $H(s)$. Indeed, reducing the cutoff frequency allows to reduce the magnitude of the inverse of the plant which appears in the compensator formula. Phase networks can be designed in order to reduce the time of resilience of the time response (Aström & Hägglund, 1995) with little effect on the overall frequency response performances. Lowering of the parameter ω_2 reduces the control effort and slows down the time response. However, this lower time response is offset by the faster response introduced by these phase networks.

To avoid saturations, a limiter can be placed at the plant input. Doing so, it is possible to define its sector/conic relation (Zames, 1966) and also define a complementary inner (outer) conic relation of a frequency response $G(s)$ by its center c, radius γ and gain $(c+\gamma)$, satisfying

$$\|Gx - cx\| \leq r\,\|x\| \qquad (\|Gx - cx\| \geq r\,\|x\|).$$

Such a conic relation could be plotted on a Nichols chart:

$$20\log|G| = c\,\cos\varphi \pm \sqrt{c^2\,\cos^2\varphi - c^2 + r^2}.$$

Provided that $\cos^2\varphi > ab/(a+b)$ where a and b are the corresponding sector bounds, i.e. $a = c - \gamma$, $b = c + \gamma$. This will ensure the stability of the feedback system. In the case of relations which are incrementally conic, the resulting feedback transmission is stable and continuous as well. Moreover, controlling the saturation slope can help avoid reaching the state of full saturation (Saeki, 1998).

Is the invested control effort required for the design of the compensator reasonable? To answer this question, one has to take into consideration the bandwidth of the sensors and the computing power involved. Experimental results on a levitation system (Bensoussan & Boulet, 2015) have confirmed the simulation results. The strength of the compensator resides in its ability to get good sensitivity bounds, good phase margin as well as a closed loop gain close to unity on a greater bandwidth which ensures better tracking. The tuning of the filter can improve the time response with little effect on stability margins. Such a compensator can be further improved with the addition of a prefilter or a feedforward filter.

6. Conclusion

Plant inversion methods are usually discarded as they translate into a prohibitive cost of feedback and bad attenuation of plant input disturbances. The result presented here allows a linear design that results in an acceptable control effort within the thresholds of ultrafast time response and minimal overshoot. Moreover, unstable plants can be controlled to get a time performance which is better than in the case of stable plants. Results could be also improved by inserting a prefilter. The case of a plant with right half plane zeros, plant uncertainty and the multivariable case are challenging but could also benefit from the approach outlined herein.

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