PARAMETRIZACION DENAVIT-HARTENBERG

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>> robot1

Código:

```
syms theta1
syms theta2
syms theta3
T1=[cos(theta1),-sin(theta1),0,0;0,0,1,0;-sin(theta1),-
cos(theta1),0,0;0,0,0,1]
syms L1
syms L2
T2=[cos(theta2),-
sin(theta2),0,L1;sin(theta2),cos(theta2),0,0;0,cos(theta2),1,0;0,0,0,1]
syms L2
T3=[cos(theta3),-
sin(theta3),0,L2;sin(theta3),cos(theta3),0,0;0,cos(theta3),1,0;0,0,0,1]
syms ans
ans =T1*T2*T3
```

$$T_1^0 = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_1) & -\cos(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{2}^{1} = \begin{bmatrix} \cos(\theta_{2}) & -\sin(\theta_{2}) & 0 & L_{1} \\ \sin(\theta_{2}) & \cos(\theta_{2}) & 0 & 0 \\ 0 & \cos(\theta_{2}) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{3}^{2} = \begin{bmatrix} \cos(\theta_{3}) & -\sin(\theta_{3}) & 0 & L_{2} \\ \sin(\theta_{3}) & \cos(\theta_{3}) & 0 & 0 \\ 0 & \cos(\theta_{3}) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos(\theta_{3}) \sigma_{2} - \sin(\theta_{3}) \sigma_{3} \qquad \sigma_{1} \qquad 0 \quad L_{2} \sigma_{2} + L_{1} \cos(\theta_{1})$$

$$\cos(\theta_{2}) \sin(\theta_{3}) \qquad \cos(\theta_{3}) + \cos(\theta_{2}) \cos(\theta_{3}) \qquad 1 \qquad 0$$

$$\sigma_{1} \qquad \sin(\theta_{3}) \sigma_{3} - \cos(\theta_{3}) \sigma_{2} \qquad 0 \quad -L_{2} \sigma_{3} - L_{1} \sin(\theta_{1})$$

$$0 \qquad 0 \qquad 0 \qquad 1$$

 T_{3}^{0}

where

$$\begin{split} &\sigma_1 = -\text{cos}(\theta_3) \; \sigma_3 - \sin(\theta_3) \; \sigma_2 \\ &\sigma_2 = \text{cos}(\theta_1) \; \text{cos}(\theta_2) - \sin(\theta_1) \; \text{sin}(\theta_2) \end{split}$$

 $\sigma_3 = \cos(\theta_1)\sin(\theta_2) + \cos(\theta_2)\sin(\theta_1)$



>> Robot2

Código:

```
syms theta1
syms theta2
syms theta3
T1=[cos(theta1),-
sin(theta1),0,0;sin(theta1),cos(theta1),0,0;0,0,1,0;0,0,0,1]
syms L1
T2=[cos(theta2),-sin(theta2),0,L1;0,0,1,0;-sin(theta2),-
cos(theta2),0,0;0,0,0,1]
syms L2
T3=[cos(theta3),-
sin(theta3),0,L2;sin(theta3),cos(theta3),0,0;0,0,1,0;0,0,0,1]
syms ans
ans =T1*T2*T3
```

$$T_1^0 = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_2^1 = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_1 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_3^2 = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

 T_3^0

```
 \begin{pmatrix} \cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) & -\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_3)\sin(\theta_2) & -\sin(\theta_1) & L_1\cos(\theta_1) + L_2\cos(\theta_1)\cos(\theta_2) \\ \cos(\theta_2)\cos(\theta_3)\sin(\theta_1) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) & -\cos(\theta_2)\sin(\theta_1)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) & L_1\sin(\theta_1) + L_2\cos(\theta_2)\sin(\theta_1) \\ -\cos(\theta_2)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_2) & \sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3) & 0 & -L_2\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}
```

>> robot3

Código:

```
syms theta1
syms theta2
syms theta3
syms d2
T1=[cos(theta1),-sin(theta1),0,0;0,0,1,0;-sin(theta1),-
cos(theta1),0,0;0,0,0,1]
syms L1
```



```
 T2 = [\cos(\text{theta2}), -\sin(\text{theta2}), 0, L1; 0, 0, -1, -d2; -\sin(\text{theta2}), -\cos(\text{theta2}), 0, 0; 0, 0, 0, 1] \\ \text{syms } L2 \\ T3 = [\cos(\text{theta3}), -\sin(\text{theta3}), 0, L2; 0, 0, 1, 0; -\sin(\text{theta3}), -\cos(\text{theta3}), 0, 0; 0, 0, 0, 1] \\ \text{syms } ans \\ \text{ans } = T1*T2*T3
```

$$T_1^0 = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_1) & -\cos(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_2^1 = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_1 \\ 0 & 0 & -1 & -d_2 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_3^2 = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_2 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_3) & -\cos(\theta_3) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_3) & -\cos(\theta_3)\sin(\theta_1) - \cos(\theta_1)\cos(\theta_2)\sin(\theta_3) & -\cos(\theta_1)\sin(\theta_2) & L_1\cos(\theta_1) + d_2\sin(\theta_1) + L_2\cos(\theta_1)\cos(\theta_2) \\ -\cos(\theta_3)\sin(\theta_2) & \sin(\theta_3)\sin(\theta_3) & -\cos(\theta_3)\sin(\theta_3) & -\cos(\theta_3)\sin(\theta_2) & L_2\sin(\theta_3) & -L_2\sin(\theta_3) \end{pmatrix}$$

0

 $-\cos(\theta_1)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)\sin(\theta_1) \\ \quad \cos(\theta_2)\sin(\theta_1)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_3) \\ \quad \sin(\theta_1)\sin(\theta_2) \\ \quad d_2\cos(\theta_1) - L_1\sin(\theta_1) - L_2\cos(\theta_2)\sin(\theta_1)\cos(\theta_2) \\ \quad d_1\cos(\theta_1)\sin(\theta_2) \\ \quad d_2\cos(\theta_1)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\sin(\theta_1)\cos(\theta_2) \\ \quad \sin(\theta_1)\cos(\theta_2)\cos(\theta_2)\sin(\theta_1)\cos(\theta_2)\cos(\theta_2) \\ \quad \sin(\theta_1)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \sin(\theta_1)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \sin(\theta_1)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \cos(\theta_1)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_2) \\ \quad \cos(\theta_2)\cos(\theta$

0

 T_3^0

Código:

>> robot4

```
syms theta1
syms theta2
syms theta3
syms d1
syms d2
T1=[cos(theta1),-
sin(theta1),0,0;sin(theta1),cos(theta1),0,0;0,0,1,0;0,0,0,1]
syms L1
T2=[cos(theta2),-
sin(theta2),0,3/4L1;sin(theta2),cos(theta2),0,0;0,0,1,d1;0,0,0,1]
syms L2
T3=[cos(theta3),-
sin(theta3),0,L2;sin(theta3),cos(theta3),0,0;0,0,1,0;0,0,0,1]
syms ans
ans =T1*T2*T3
```



$$T_1^0 = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_2^1 = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_1 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_3^2 = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_3^0 = \begin{pmatrix} \sigma_1 & -\cos(\theta_3) & \sigma_3 - \sin(\theta_3) & \sigma_2 & 0 & L_2 & \sigma_2 + L_1 & \cos(\theta_1) \\ \cos(\theta_3) & \sigma_3 + \sin(\theta_3) & \sigma_2 & \sigma_1 & 0 & L_2 & \sigma_3 + L_1 & \sin(\theta_1) \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

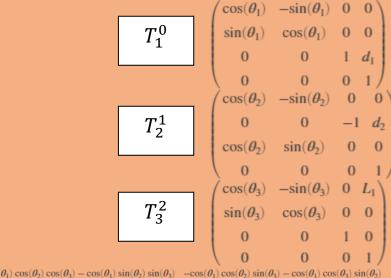
$$\begin{split} &\sigma_1 = \cos(\theta_3) \; \sigma_2 - \sin(\theta_3) \; \sigma_3 \\ &\sigma_2 = \cos(\theta_1) \; \cos(\theta_2) - \sin(\theta_1) \; \sin(\theta_2) \\ &\sigma_3 = \cos(\theta_1) \; \sin(\theta_2) + \cos(\theta_2) \; \sin(\theta_1) \end{split}$$

>> robot5

Código:

```
syms theta1
syms theta2
syms theta3
syms d1
syms d2
T1=[cos(theta1),-
sin(theta1),0,0;sin(theta1),cos(theta1),0,0;0,0,1,d1;0,0,0,1]
syms L1
T2=[cos(theta2),-sin(theta2),0,0;0,0,-
1,d2;cos(theta2),sin(theta2),0,0;0,0,0,1]
T3=[cos(theta3),-
sin(theta3),0,L1;sin(theta3),cos(theta3),0,0;0,0,1,0;0,0,0,1]
syms ans
ans =T1*T2*T3
```





 T_3^0

```
 \begin{pmatrix} \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) & -\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) -\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) & \sin(\theta_1) & \ln(\cos(\theta_1)\cos(\theta_2)-d_2\sin(\theta_1)\cos(\theta_2)\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) & -\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) & -\cos(\theta_2)\sin(\theta_1)\sin(\theta_2) & -\cos(\theta_1)\sin(\theta_2) & -\cos(\theta_1)\sin(\theta_2)\sin(\theta_2)\sin(\theta_3) & \cos(\theta_2)\sin(\theta_3) & 0 & d_1+L_1\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\sin(\theta_3) & 0 & d_1+L_1\cos(\theta_2)\cos(\theta_2)\cos(\theta_2)\cos(\theta_3)\sin(\theta_2)\cos(\theta_2)\sin(\theta_3) & 0 & 0 & 0 & 1 \end{pmatrix}
```

>> robot6

Código:

```
syms theta1
syms theta2
syms theta3
syms d1
T1=[cos(theta1),-
sin(theta1),0,0;sin(theta1),cos(theta1),0,0;0,0,1,0;0,0,0,1]
syms L1
T2=[cos(theta2),-
sin(theta2),0,L1;sin(theta2),cos(theta2),0,0;0,0,1,0;0,0,0,1]
T3=[cos(theta3),-
sin(theta3),0,L2;sin(theta3),cos(theta3),0,0;0,0,1,d1;0,0,0,1]
syms ans
ans =T1*T2*T3
```

$$T_1^0 egin{array}{c} \cos(heta_1) & -\sin(heta_1) & 0 & 0 \ \sin(heta_1) & \cos(heta_1) & 0 & 0 \ 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 1 \ \end{array} \ egin{array}{c} \cos(heta_2) & -\sin(heta_2) & 0 & L_1 \ \sin(heta_2) & \cos(heta_2) & 0 & 0 \ 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 1 \ \end{array} \ egin{array}{c} \cos(heta_2) & \cos(heta_2) & 0 & 0 \ 0 & 0 & 0 & 1 \ \end{array} \ egin{array}{c} \cos(heta_2) & \cos(heta_2) & 0 & 0 \ 0 & 0 & 0 & 1 \ \end{array} \ egin{array}{c} \cos(heta_2) & \cos(heta_2) & 0 & 0 \ 0 & 0 & 0 & 1 \ \end{array} \ egin{array}{c} \cos(heta_2) & \cos(heta_2) & 0 & 0 \ 0 & 0 & 0 & 1 \ \end{array} \ egin{array}{c} \cos(heta_2) & \cos(heta_2) & 0 & 0 \ 0 & 0 & 0 & 1 \ \end{array} \ egin{array}{c} \cos(heta_2) & \cos(heta_2) & 0 & 0 \ 0 & 0 & 0 & 1 \ \end{array} \ egin{array}{c} \cos(heta_2) & \cos(heta_2) & 0 & 0 \ 0 & 0 & 0 & 1 \ \end{array} \ egin{array}{c} \cos(heta_2) & \cos(heta_2) & 0 & 0 \ 0 & 0 & 0 & 1 \ \end{array} \ egin{array}{c} \cos(heta_2) & \cos(heta_2) & \cos(heta_2) & \cos(heta_2) \ \end{array} \ egin{array}{c} \cos(heta_2) & \cos(heta_2) & \cos(heta_2) & \cos(heta_2) \ \end{array} \ egin{array}{c} \cos(heta_2) & \cos(heta_$$



$$T_3^2 = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_3^0 = \begin{pmatrix} \sigma_1 & -\cos(\theta_3) & \sigma_3 - \sin(\theta_3) & \sigma_2 & 0 & L_2 & \sigma_2 + L_1 & \cos(\theta_1) \\ \cos(\theta_3) & \sigma_3 + \sin(\theta_3) & \sigma_2 & \sigma_1 & 0 & L_2 & \sigma_3 + L_1 & \sin(\theta_1) \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \cos(\theta_3) \ \sigma_2 - \sin(\theta_3) \ \sigma_3$$

$$\sigma_2 = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$$

$$\sigma_3 = \cos(\theta_1)\sin(\theta_2) + \cos(\theta_2)\sin(\theta_1)$$

