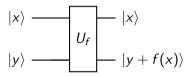
Adenilton J. da Silva

@adeniltons

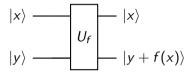
Introdução

▶ Dada uma função $f: \mathbb{B} \to \mathbb{B}$, podemos determinar um operador quântico U, onde $U|x,y\rangle = |x,y \oplus f(x)\rangle$



Introdução

▶ Dada uma função $f: \mathbb{B} \to \mathbb{B}$, podemos determinar um operador quântico U, onde $U|x,y\rangle = |x,y \oplus f(x)\rangle$



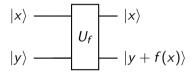
Por exemplo, se f(0) = 0 e f(1) = 1, então U_f pode ser escrito como o circuito abaixo.

$$|x\rangle \longrightarrow |x\rangle$$

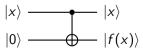
 $|y\rangle \longrightarrow |y+f(x)\rangle$

Introdução

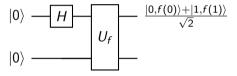
▶ Dada uma função $f: \mathbb{B} \to \mathbb{B}$, podemos determinar um operador quântico U, onde $U|x,y\rangle = |x,y \oplus f(x)\rangle$



Por exemplo, se f(0) = 0 e f(1) = 1, então U_f pode ser escrito como o circuito abaixo.



Paralelismo quântico



Determinar se uma função binária é constante ou balanceada.

- Determinar se uma função binária é constante ou balanceada.
- Número de chamadas da função para resolução do problema.

	computador	computador
	quântico	clássico
# chamadas	1	2

Computador clássico

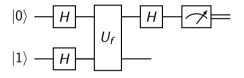
```
def classical_dj(f):
    if f(0) == f(1):
        return 'constante'
    else:
        return 'balanceada'
```

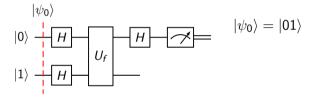
Podemos resolver o problema de Deutsch em um computador clássico com apenas uma chamada da função? ¹

¹Johansson, Niklas, and Jan-Åke Larsson. "Efficient classical simulation of the Deutsch–Jozsa and Simon's algorithms." Quantum Information Processing 16.9 (2017): 233.

Seção 1

Circuito quântico para o problema de Deutsch





$$|\psi_1
angle = |01
angle \ |\psi_0
angle = |01
angle \ |\psi_1
angle = (H\otimes H)|01
angle$$

$$|\psi_1
angle = |01
angle$$
 $|\psi_0
angle = |01
angle$
 $|\psi_1
angle = (H\otimes H)|01
angle$
 $|H| \longrightarrow H$
 $|U_f| \longrightarrow H$
 $|U_f| \longrightarrow H$
 $|\psi_1
angle = H|0
angle \otimes H|1
angle$

$$|\psi_0
angle = |01
angle$$
 $|\psi_1
angle = |01
angle$
 $|\psi_1
angle = (H\otimes H)|01
angle$
 $|H| \longrightarrow H$
 $|U_f| \longrightarrow H |U_f| \longrightarrow H |U_f|$
 $= H|0
angle \otimes H|1
angle$
 $= \frac{|0
angle + |1
angle}{\sqrt{2}} \frac{|0
angle - |1
angle}{\sqrt{2}}$

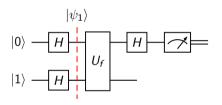
$$|\psi_{0}\rangle = |01\rangle$$

$$|\psi_{1}\rangle = (H \otimes H)|01\rangle$$

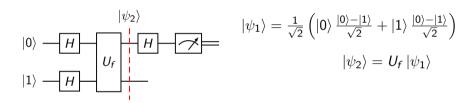
$$= H|0\rangle \otimes H|1\rangle$$

$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} + |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$



$$|\psi_1
angle=rac{1}{\sqrt{2}}\left(|0
angle\,rac{|0
angle-|1
angle}{\sqrt{2}}+|1
angle\,rac{|0
angle-|1
angle}{\sqrt{2}}
ight)$$



$$|\psi_1\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} + |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

$$|\psi_2\rangle = U_f |\psi_1\rangle$$

$$|\psi_2
angle = U_f \left(rac{1}{\sqrt{2}} \left(|0
angle rac{|0
angle - |1
angle}{\sqrt{2}} + |1
angle rac{|0
angle - |1
angle}{\sqrt{2}}
ight)
ight)$$

$$|\psi_2\rangle = U_f \left(\frac{1}{\sqrt{2}} \left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} + |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}} \left(U_f \left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) + U_f \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

$$|\psi_2
angle = rac{1}{\sqrt{2}} \left(\textit{U}_f \left(\ket{0} rac{\ket{0} - \ket{1}}{\sqrt{2}}
ight) + \textit{U}_f \left(\ket{1} rac{\ket{0} - \ket{1}}{\sqrt{2}}
ight)
ight)$$

	$U_f\left(\ket{0}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$	$U_f\left(\ket{1}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$
f(x) = 0		
f(x)=1		

	$U_f\left(\ket{0}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$	$igg U_f\left(\ket{1}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$
f(x) = 0		
f(x) = 1		

	$U_f\left(\ket{0}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$	$U_f\left(\ket{1}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$
f(x)=0	$ 0\rangle(0\rangle- 1\rangle)/\sqrt{2}$	
f(x) = 1		

	$U_f\left(\ket{0}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$	$U_f\left(\ket{1}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$
f(x)=0	$ 0\rangle (0\rangle - 1\rangle)/\sqrt{2}$	$ 1 angle (0 angle - 1 angle)/\sqrt{2}$
f(x)=1		

	$U_f\left(\ket{0}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$	$igg U_f\left(\ket{1}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$
f(x)=0	$ 0 angle (0 angle - 1 angle)/\sqrt{2}$	$ 1 angle (0 angle - 1 angle)/\sqrt{2}$
f(x) = 1	$ - 0 angle (0 angle - 1 angle)/\sqrt{2}$	
	1 / 31 / 1 /// .	

	$U_f\left(\ket{0}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$	$U_f\left(\ket{1}rac{\ket{0}-\ket{1}}{\sqrt{2}} ight)$
f(x)=0	$ 0 angle (0 angle - 1 angle)/\sqrt{2}$	$ 1\rangle(0\rangle- 1\rangle)/\sqrt{2}$
f(x) = 1	$ - 0 angle (0 angle - 1 angle)/\sqrt{2}$	$ - 1 angle (0 angle - 1 angle)/\sqrt{2}$

$$\begin{array}{c|c} & U_f\left(|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) & U_f\left(|1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) \\ \hline f(x) = 0 & |0\rangle\left(|0\rangle-|1\rangle\right)/\sqrt{2} & |1\rangle\left(|0\rangle-|1\rangle\right)/\sqrt{2} \\ \hline f(x) = 1 & -|0\rangle\left(|0\rangle-|1\rangle\right)/\sqrt{2} & -|1\rangle\left(|0\rangle-|1\rangle\right)/\sqrt{2} \end{array}$$

$$U_f\ket{x}\left(rac{\ket{0}-\ket{1}}{\sqrt{2}}
ight)=(-1)^{f(x)}\ket{x}\left(rac{\ket{0}-\ket{1}}{\sqrt{2}}
ight)$$

$$|0\rangle - H - |\psi_2\rangle = U_f(|\psi_1\rangle)$$

$$|1\rangle - H - |U_f| + |\psi_2\rangle = U_f(|\psi_1\rangle)$$

$$= U_f\left(\frac{1}{\sqrt{2}}\left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} + |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$|0\rangle -H - U_{f} -$$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$= \frac{1}{\sqrt{2}} \left(U_{f}\left(|0\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) + U_{f}\left(|1\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$= \frac{1}{\sqrt{2}} \left((-1)^{f(0)} \left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) + (-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \right)$$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$|0\rangle - H - U_{f}$$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$= \frac{1}{\sqrt{2}} \left((-1)^{f(0)} \left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) + (-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \right)$$

$$|1\rangle - H - U_{f}$$

$$|0\rangle - H - U_{$$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

$$|0\rangle - H - U_{f}$$

$$|U_{f}| + H - V_{f}$$

$$|1\rangle - H - V_{f}$$

$$|\psi_{2}\rangle = \pm \frac{1}{\sqrt{2}}\left(\left(|0\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) + \left(|1\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$|0\rangle -H - U_{f}$$

$$|U_{f}|$$

$$|1\rangle -H - U_{f}$$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

$$|0\rangle -H - U_{f}$$

$$|\psi_{2}\rangle = \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$|0\rangle -H - U_{f}$$

$$|\psi_{2}\rangle = \pm\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$= \frac{1}{\sqrt{2}} \left((-1)^{f(0)} \left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) + (-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \right)$$

$$|0\rangle - H - U_{f}$$

$$|U_{f}| + H - V$$

$$|0\rangle - H - V_{f}$$

$$|U_{f}| + V_{f}$$

$$|\psi_{2}\rangle = \pm \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

$$|\psi_{2}\rangle = \pm \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$
Se $f(0) \neq f(1)$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)$$
 $|0\rangle$
 $|H|$
 $|\psi_{2}\rangle = f(1)$
 $|\psi_{2}\rangle = \pm\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$
Se $f(0) \neq f(1)$
 $|\psi_{2}\rangle = \pm\frac{1}{\sqrt{2}}\left(\left(|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) - \left(|1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)$

$$|\psi_{2}\rangle = U_{f}(|\psi_{1}\rangle)$$

$$= \frac{1}{\sqrt{2}} \left((-1)^{f(0)} \left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) + (-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \right)$$

$$|0\rangle - H - |H - |F|$$

$$|\psi_{2}\rangle = \pm \left((-1)^{f(0)} \left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \left((-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \right)$$

$$|0\rangle - H - |H - |F|$$

$$|\psi_{2}\rangle = \pm \left((-1)^{f(0)} \left(|0\rangle - |1\rangle \right) \left((-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \right)$$

$$|0\rangle - H - |H - |F|$$

$$|\psi_{2}\rangle = \pm \left((-1)^{f(0)} \left(|0\rangle - |1\rangle \right) \left((-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \right)$$

$$|0\rangle - H - |H - |F|$$

$$|\psi_{2}\rangle = \pm \left((-1)^{f(0)} \left(|0\rangle - |1\rangle \right) \left((-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \right)$$

$$|0\rangle - H - |H - |F|$$

$$|\psi_{2}\rangle = \pm \left((-1)^{f(0)} \left(|0\rangle - |1\rangle \right) \left((-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

$$|0\rangle - |H - |F|$$

$$|\psi_{2}\rangle = \pm \left((-1)^{f(0)} \left(|0\rangle - |1\rangle \right) \left((-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

$$|0\rangle - |H - |F|$$

$$|\psi_{2}\rangle = \pm \left((-1)^{f(0)} \left(|0\rangle - |1\rangle \right) \left((-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

$$|0\rangle - |H - |F|$$

$$|\psi_{2}\rangle = \pm \left((-1)^{f(0)} \left(|0\rangle - |1\rangle \right) \left((-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

$$|0\rangle - |H - |F|$$

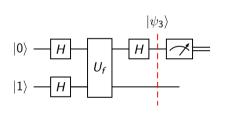
$$|\psi_{2}\rangle = \pm \left((-1)^{f(0)} \left(|0\rangle - |1\rangle \right) \left((-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

$$|0\rangle - |H - |F|$$

$$|\psi_{2}\rangle = \pm \left((-1)^{f(0)} \left(|0\rangle - |1\rangle \right) \left((-1)^{f(1)} \left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

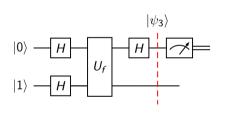
$$|0\rangle - |H - |F|$$

$$|0\rangle - |H$$



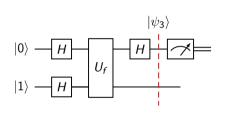
Se
$$f(0) = f(1)$$

$$|\psi_2\rangle = \pm \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$



Se
$$f(0)=f(1)$$

$$|\psi_3\rangle=\pm\,|0\rangle\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

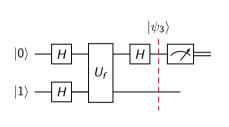


▶ Se f(0) = f(1)

$$|\psi_3
angle=\pm\,|0
angle\left(rac{|0
angle-|1
angle}{\sqrt{2}}
ight)$$

▶ Se $f(0) \neq f(1)$

$$|\psi_2
angle = \pm \left(rac{|0
angle - |1
angle}{\sqrt{2}}
ight) \left(rac{|0
angle - |1
angle}{\sqrt{2}}
ight)$$



▶ Se f(0) = f(1)

$$|\psi_3
angle=\pm\,|0
angle\left(rac{|0
angle-|1
angle}{\sqrt{2}}
ight)$$

▶ Se $f(0) \neq f(1)$

$$|\psi_3
angle=\pm\ket{1}\left(rac{\ket{0}-\ket{1}}{\sqrt{2}}
ight)$$

- O algoritmo de Deutsch não possui aplicações práticas.
- Demonstra que circuitos quânticos podem superar circuitos clássicos.
- Mostra uma aplicação do paralelismo quântico.

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