Algoritmo de Deutsch-Jozsa

Adenilton J. da Silva

@adeniltons

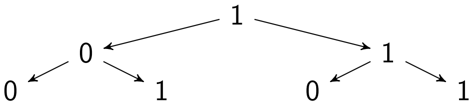
Introdução

- Entrada: Uma função $f : \mathbb{B}^n \to \mathbb{B}$ constante ou balanceada.
- Saída: 0 se a função for constante e um valor diferente de 0 se a função for balanceada.

Qual a complexidade de tempo de um algoritmo clássico para resolução do problema de Deutsch-Jozsa?

1

$$
\begin{tikzcd}
& 1 \arrow[dl] \arrow[dr] & \\
0 & & 1
\end{tikzcd}
$$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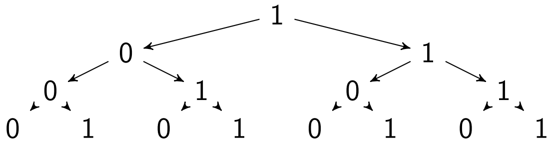
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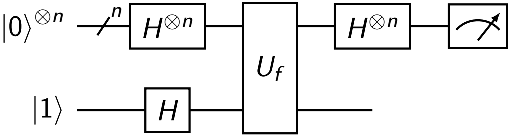
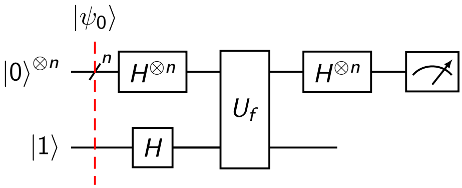
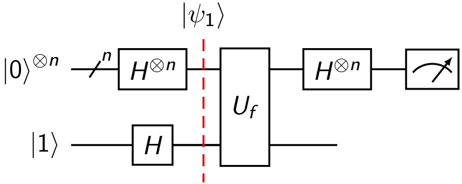




$$|\psi_0\rangle = |0\rangle^{\otimes n} |1\rangle$$



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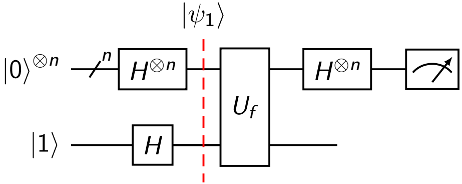

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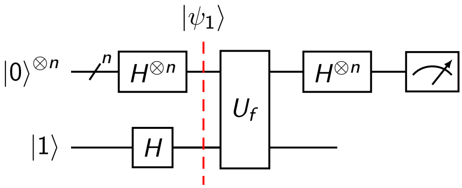
$$|\psi_1\rangle = \left(\frac{|0\rangle+|1\rangle}{\sqrt{(2)}}\right)^{\otimes n} \left(\frac{|0\rangle-|1\rangle}{\sqrt{(2)}}\right)$$



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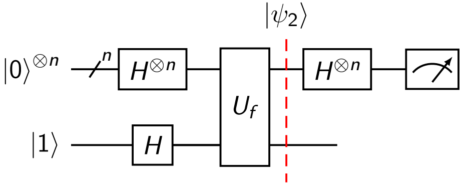

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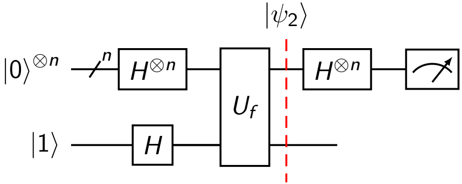

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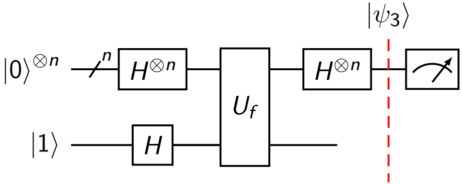

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$$|\psi_3\rangle = \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} (-1)^{f(x)} \left(H^{\otimes n} |x\rangle\right) |-\rangle$$

$H^{\otimes n}|x\rangle$

- $H^{\otimes 3}|010\rangle = \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)$
  $= \frac{1}{\sqrt{8}}(|000\rangle + |001\rangle - |010\rangle - |011\rangle + |100\rangle + |101\rangle - |110\rangle - |111\rangle)$

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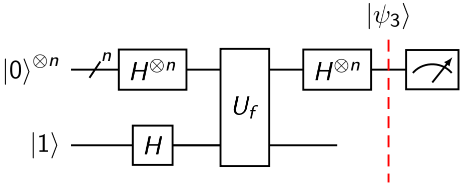

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- Qual a amplitude de $|0\rangle^{\otimes n}$?

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