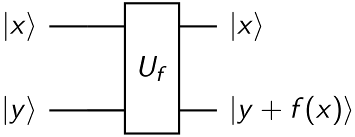
Problema de Deutsch

Adenilton J. da Silva

@adeniltons

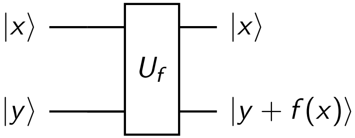
Introdução

- Dada uma função $f : \mathbb{B} \to \mathbb{B}$, podemos determinar um operador quântico U, onde $U|x, y\rangle = |x, y \oplus f(x)\rangle$

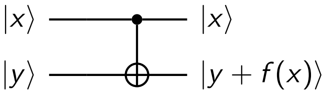


Introdução

- Dada uma função $f : \mathbb{B} \to \mathbb{B}$, podemos determinar um operador quântico U, onde $U\,|x, y\rangle = |x, y \oplus f(x)\rangle$

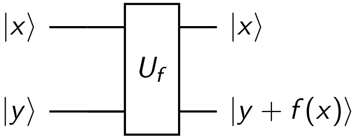


- Por exemplo, se $f(0) = 0$ e $f(1) = 1$, então U_f pode ser escrito como o circuito abaixo.

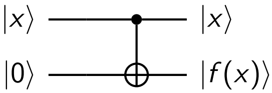


Introdução

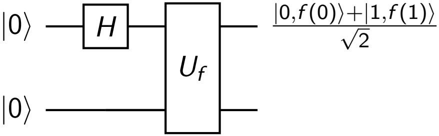
- Dada uma função $f : \mathbb{B} \to \mathbb{B}$, podemos determinar um operador quântico U, onde $U\,|x, y\rangle = |x, y \oplus f(x)\rangle$



- Por exemplo, se $f(0) = 0$ e $f(1) = 1$, então U_f pode ser escrito como o circuito abaixo.



Paralelismo quântico



Problema de Deutsch

- Determinar se uma função binária é constante ou balanceada.

Problema de Deutsch

- Determinar se uma função binária é constante ou balanceada.
- Número de chamadas da função para resolução do problema.

	computador quântico	computador clássico
# chamadas	1	2

Problema de Deutsch

Computador clássico

```python
def classical_dj(f):
    if f(0) == f(1):
        return 'constante'
    else:
        return 'balanceada'
```

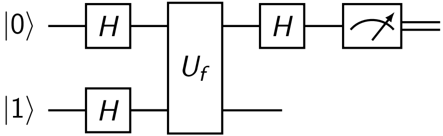
Podemos resolver o problema de Deutsch em um computador clássico com apenas uma chamada da função? [1]

[1] Johansson, Niklas, and Jan-Åke Larsson. "Efficient classical simulation of the Deutsch–Jozsa and Simon's algorithms." Quantum Information Processing 16.9 (2017): 233.

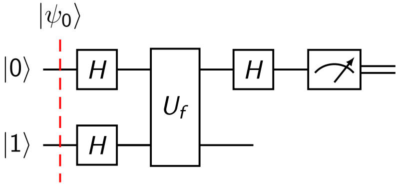
Seção 1

Circuito quântico para o problema de Deutsch

Problema de Deutsch

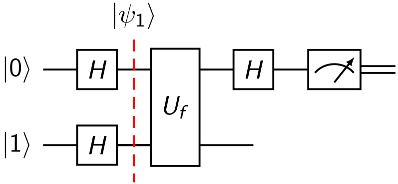


Problema de Deutsch



$$|\psi_0\rangle = |01\rangle$$

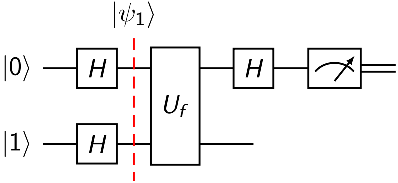
Problema de Deutsch



$$|\psi_0\rangle = |01\rangle$$

$$|\psi_1\rangle = (H \otimes H)\,|01\rangle$$

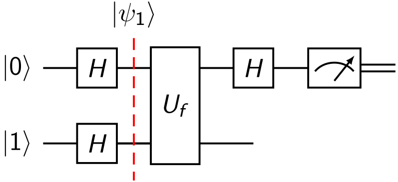
Problema de Deutsch



$$|\psi_0\rangle = |01\rangle$$

$$|\psi_1\rangle = (H \otimes H)\,|01\rangle$$
$$= H\,|0\rangle \otimes H\,|1\rangle$$

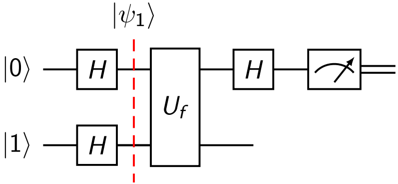
Problema de Deutsch



$$|\psi_0\rangle = |01\rangle$$

$$|\psi_1\rangle = (H \otimes H)\,|01\rangle$$
$$= H\,|0\rangle \otimes H\,|1\rangle$$
$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}}\,\frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

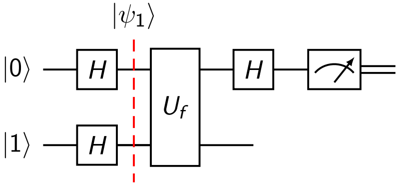
Problema de Deutsch



$$|\psi_0\rangle = |01\rangle$$

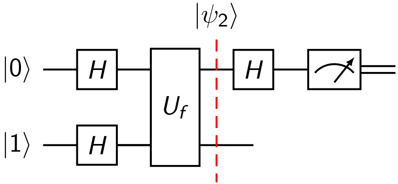
$$|\psi_1\rangle = (H \otimes H)\,|01\rangle$$
$$= H\,|0\rangle \otimes H\,|1\rangle$$
$$= \frac{|0\rangle+|1\rangle}{\sqrt{2}}\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}}\left(|0\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}} + |1\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

Problema de Deutsch



$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} + |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

Problema de Deutsch



$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} + |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$|\psi_2\rangle = U_f\,|\psi_1\rangle$$

Problema de Deutsch

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} + |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$|\psi_2\rangle = U_f\,|\psi_1\rangle$$

Problema de Deutsch

$$|\psi_2\rangle = U_f \left(\frac{1}{\sqrt{2}} \left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} + |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \right)$$

Problema de Deutsch

$$|\psi_2\rangle = U_f\left(\frac{1}{\sqrt{2}}\left(|0\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}} + |1\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(U_f\left(|0\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) + U_f\left(|1\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

Problema de Deutsch

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(U_f\left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) + U_f\left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

	$U_f\left(\|0\rangle \frac{\|0\rangle - \|1\rangle}{\sqrt{2}}\right)$	$U_f\left(\|1\rangle \frac{\|0\rangle - \|1\rangle}{\sqrt{2}}\right)$
$f(x) = 0$		
$f(x) = 1$		

Problema de Deutsch

	$U_f\left(\|0\rangle\,\frac{\|0\rangle-\|1\rangle}{\sqrt{2}}\right)$	$U_f\left(\|1\rangle\,\frac{\|0\rangle-\|1\rangle}{\sqrt{2}}\right)$
$f(x)=0$		
$f(x)=1$		

Problema de Deutsch

	$U_f\left(\|0\rangle \frac{\|0\rangle-\|1\rangle}{\sqrt{2}}\right)$	$U_f\left(\|1\rangle \frac{\|0\rangle-\|1\rangle}{\sqrt{2}}\right)$
$f(x)=0$	$\|0\rangle\left(\|0\rangle-\|1\rangle\right)/\sqrt{2}$	
$f(x)=1$		

Problema de Deutsch

	$U_f\left(\lvert 0\rangle \frac{\lvert 0\rangle-\lvert 1\rangle}{\sqrt{2}}\right)$	$U_f\left(\lvert 1\rangle \frac{\lvert 0\rangle-\lvert 1\rangle}{\sqrt{2}}\right)$
$f(x)=0$	$\lvert 0\rangle\left(\lvert 0\rangle-\lvert 1\rangle\right)/\sqrt{2}$	$\lvert 1\rangle\left(\lvert 0\rangle-\lvert 1\rangle\right)/\sqrt{2}$
$f(x)=1$		

Problema de Deutsch

	$U_f\left(\lvert 0\rangle \frac{\lvert 0\rangle - \lvert 1\rangle}{\sqrt{2}}\right)$	$U_f\left(\lvert 1\rangle \frac{\lvert 0\rangle - \lvert 1\rangle}{\sqrt{2}}\right)$
$f(x) = 0$	$\lvert 0\rangle \left(\lvert 0\rangle - \lvert 1\rangle\right)/\sqrt{2}$	$\lvert 1\rangle \left(\lvert 0\rangle - \lvert 1\rangle\right)/\sqrt{2}$
$f(x) = 1$	$-\lvert 0\rangle \left(\lvert 0\rangle - \lvert 1\rangle\right)/\sqrt{2}$	

Problema de Deutsch

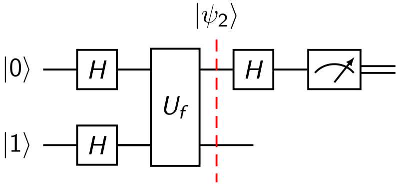
	$U_f\left(\|0\rangle \frac{\|0\rangle-\|1\rangle}{\sqrt{2}}\right)$	$U_f\left(\|1\rangle \frac{\|0\rangle-\|1\rangle}{\sqrt{2}}\right)$
$f(x)=0$	$\|0\rangle\left(\|0\rangle-\|1\rangle\right)/\sqrt{2}$	$\|1\rangle\left(\|0\rangle-\|1\rangle\right)/\sqrt{2}$
$f(x)=1$	$-\|0\rangle\left(\|0\rangle-\|1\rangle\right)/\sqrt{2}$	$-\|1\rangle\left(\|0\rangle-\|1\rangle\right)/\sqrt{2}$

Problema de Deutsch

	$U_f\left(\|0\rangle \frac{\|0\rangle-\|1\rangle}{\sqrt{2}}\right)$	$U_f\left(\|1\rangle \frac{\|0\rangle-\|1\rangle}{\sqrt{2}}\right)$
$f(x)=0$	$\|0\rangle(\|0\rangle-\|1\rangle)/\sqrt{2}$	$\|1\rangle(\|0\rangle-\|1\rangle)/\sqrt{2}$
$f(x)=1$	$-\|0\rangle(\|0\rangle-\|1\rangle)/\sqrt{2}$	$-\|1\rangle(\|0\rangle-\|1\rangle)/\sqrt{2}$

$$U_f\,|x\rangle\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)=(-1)^{f(x)}\,|x\rangle\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

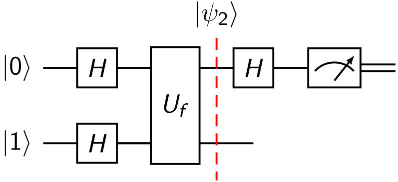
Problema de Deutsch



$$|\psi_2\rangle = U_f(|\psi_1\rangle)$$

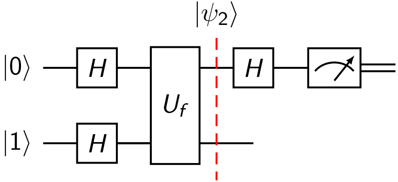
$$= U_f\left(\frac{1}{\sqrt{2}}\left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} + |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

Problema de Deutsch



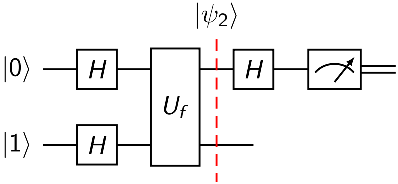
$$
\begin{aligned}
|\psi_2\rangle &= U_f(|\psi_1\rangle) \\
&= U_f\left(\frac{1}{\sqrt{2}}\left(|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}} + |1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right) \\
&= \frac{1}{\sqrt{2}}\left(U_f\left(|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + U_f\left(|1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)
\end{aligned}
$$

Problema de Deutsch



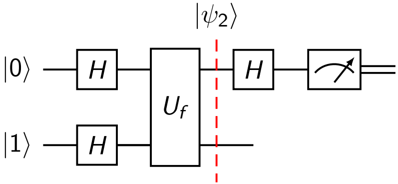
$$|\psi_2\rangle = U_f(|\psi_1\rangle)$$
$$= \frac{1}{\sqrt{2}}\left(U_f\left(|0\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) + U_f\left(|1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)$$

Problema de Deutsch



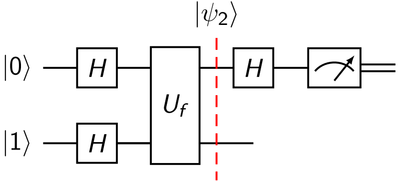
$$
\begin{aligned}
|\psi_2\rangle &= U_f(|\psi_1\rangle) \\
&= \frac{1}{\sqrt{2}}\left(U_f\left(|0\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + U_f\left(|1\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right) \\
&= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)
\end{aligned}
$$

Problema de Deutsch



$$|\psi_2\rangle = U_f(|\psi_1\rangle)$$
$$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)$$

Problema de Deutsch

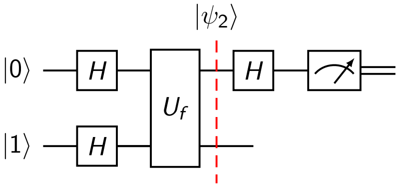


$$|\psi_2\rangle = U_f(|\psi_1\rangle)$$

$$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)$$

Se $f(0) = f(1)$

Problema de Deutsch



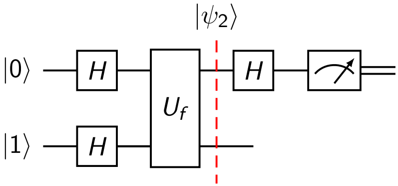
$$|\psi_2\rangle = U_f(|\psi_1\rangle)$$

$$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)$$

Se $f(0) = f(1)$

$$|\psi_2\rangle = \pm\frac{1}{\sqrt{2}}\left(\left(|0\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + \left(|1\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)$$

Problema de Deutsch



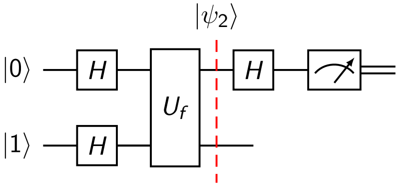
$$|\psi_2\rangle = U_f(|\psi_1\rangle)$$

$$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)$$

Se $f(0) = f(1)$

$$|\psi_2\rangle = \pm\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

Problema de Deutsch



$$|\psi_2\rangle = U_f(|\psi_1\rangle)$$

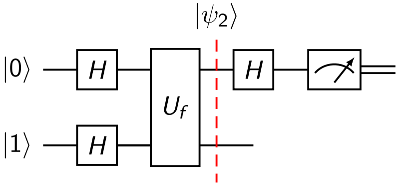
$$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)$$

Se $f(0) = f(1)$

$$|\psi_2\rangle = \pm\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

Se $f(0) \neq f(1)$

Problema de Deutsch



$$|\psi_2\rangle = U_f(|\psi_1\rangle)$$

$$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)$$

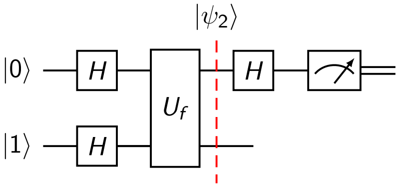
Se $f(0) = f(1)$

$$|\psi_2\rangle = \pm\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

Se $f(0) \neq f(1)$

$$|\psi_2\rangle = \pm\frac{1}{\sqrt{2}}\left(\left(|0\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) - \left(|1\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)$$

Problema de Deutsch



$$|\psi_2\rangle = U_f(|\psi_1\rangle)$$

$$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}\left(|0\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(|1\rangle\,\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right)$$

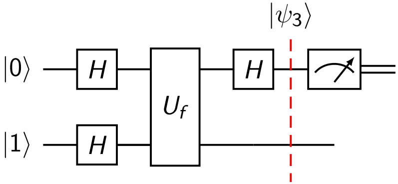
Se $f(0) = f(1)$

$$|\psi_2\rangle = \pm\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

Se $f(0) \neq f(1)$

$$|\psi_2\rangle = \pm\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

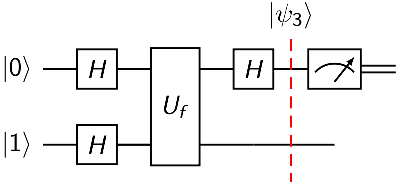
Problema de Deutsch



- Se $f(0) = f(1)$

$$|\psi_2\rangle = \pm \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

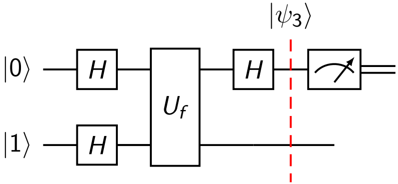
Problema de Deutsch



- Se $f(0) = f(1)$

$$|\psi_3\rangle = \pm |0\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

Problema de Deutsch



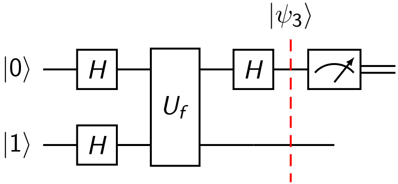
- Se $f(0) = f(1)$

$$|\psi_3\rangle = \pm |0\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

- Se $f(0) \neq f(1)$

$$|\psi_2\rangle = \pm \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

Problema de Deutsch



- Se $f(0) = f(1)$

$$|\psi_3\rangle = \pm |0\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

- Se $f(0) \neq f(1)$

$$|\psi_3\rangle = \pm |1\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

- O algoritmo de Deutsch não possui aplicações práticas.
- Demonstra que circuitos quânticos podem superar circuitos clássicos.
- Mostra uma aplicação do paralelismo quântico.

Problema de Deutsch

Adenilton J. da Silva

@adeniltons