Clusterização das ações do IBRX

March 1, 2021

Nesse exercício vamos clusterizar as ações do índice Brasil 100 (IBrX) com o intuito de encontrar pequenos grupos de ações para identificar pares a serem usados em uma estratégia de negociação de pares. A clusterização pode ser usado para dividir ações e outros tipos de ativos em grupos com características semelhantes para vários outros tipos de estratégias de negociação. Também pode ser eficaz na construção de portfólio, ajudando a garantir que escolhamos um pool de ativos com diversificação suficiente entre eles.

[1]:
```
# Load libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from pandas import read_csv, set_option
from pandas.plotting import scatter_matrix
import seaborn as sns
from sklearn.preprocessing import StandardScaler
import datetime
import pandas_datareader as dr
import yfinance as yf

# Diable the warnings
import warnings
warnings.filterwarnings('ignore')

#Import Model Packages
from sklearn.cluster import KMeans,␣
↪AgglomerativeClustering,AffinityPropagation, DBSCAN
from scipy.cluster.hierarchy import fcluster
from scipy.cluster.hierarchy import dendrogram, linkage, cophenet
from scipy.spatial.distance import pdist
from sklearn.metrics import adjusted_mutual_info_score
from sklearn import cluster, covariance, manifold


#Other Helper Packages and functions
import matplotlib.ticker as ticker
from itertools import cycle
```

1 Acessando e trabalhando os dados

1.1 Baixando as ações listadas no IBRX

[2]:
```
ibovurl = "http://bvmf.bmfbovespa.com.br/indices/ResumoCarteiraTeorica.aspx?
↪Indice=ibrx&idioma=pt-br"
ibrx = pd.read_html(ibovurl)

# Selecionando apenas a columa dos tickers

tickers = ibrx[0][0:]['Código'].tolist()
tickers.remove('Quantidade Teórica Total  Redutor')


# Adicionando .SA em todas as ações

for i in range(0, len(tickers)):
    tickers[i] = tickers[i] + '.SA'

# Buscando no yahoo finance o preço de fechamento ajustado

dataset = yf.download(tickers = tickers, start = '2020-01-01')['Adj Close']

dataset = pd.DataFrame(dataset)
dataset
```

```
[*********************100%***********************]  100 of 100 completed
```

[2]:
```
             ABEV3.SA   ALPA4.SA   ALSO3.SA  AMAR3.SA   AZUL4.SA   B3SA3.SA  \
Date
2020-01-02  18.616690  32.529774  50.009998     13.46  58.799999  43.622253
2020-01-03  18.354891  32.529774  51.419998     13.95  56.759998  42.365135
2020-01-06  18.442158  32.020557  51.070000     13.63  55.000000  41.958981
2020-01-07  18.480942  32.749432  50.470001     13.67  56.820000  43.312805
2020-01-08  18.393677  33.108875  52.000000     13.87  56.919998  43.196762
…                 …          …          …        …          …          …
2021-02-23  14.690000  35.000000  25.110001      5.50  43.099998  55.340000
2021-02-24  14.680000  36.070000  25.070000      5.52  44.950001  55.619999
2021-02-25  14.200000  34.830002  24.930000      5.32  42.419998  54.889999
2021-02-26  14.110000  34.869999  23.530001      5.02  40.730000  55.310001
2021-03-01        NaN        NaN        NaN       NaN        NaN        NaN

             BBAS3.SA   BBDC3.SA   BBDC4.SA   BBSE3.SA  …  TIMS3.SA  \
Date                                                    …
2020-01-02  50.934219  31.287395  33.119057  33.248966  …       NaN
2020-01-03  50.849010  30.799173  33.135822  32.882446  …       NaN
2020-01-06  50.176830  30.622517  32.543949  33.423504  …       NaN
2020-01-07  49.798138  30.101395  31.978584  33.440960  …       NaN
```

```
2020-01-08  49.343704  29.642103  31.483898  33.423504  …        NaN
…                 …          …          …          …  …          …
2021-02-23  30.430000  21.469999  24.180000  26.830000  …      13.22
2021-02-24  30.309999  21.330000  24.010000  26.540001  …      13.30
2021-02-25  29.459999  20.780001  23.490000  25.950001  …      13.13
2021-02-26  28.120001  20.530001  23.080000  25.549999  …      12.86
2021-03-01        NaN        NaN        NaN        NaN  …        NaN

             TOTS3.SA   TRPL4.SA   UGPA3.SA   USIM5.SA   VALE3.SA   VIVT3.SA  \
Date
2020-01-02  22.856628  20.440050  25.279165   9.610517  52.150002  43.943169
2020-01-03  23.342520  19.928377  24.982924   9.511439  51.766048  44.453289
2020-01-06  22.810352  20.045073  24.439817   9.333100  51.458889  44.025158
2020-01-07  22.906208  19.901445  24.982924   9.422270  51.833241  44.999847
2020-01-08  22.598810  19.748840  24.785431   9.303377  51.842838  45.455315
…                 …          …          …          …          …          …
2021-02-23  33.520000  24.848221  21.379999  15.740000  96.949997  45.019234
2021-02-24  33.130001  25.371342  21.280001  17.240000  97.930000  44.300682
2021-02-25  31.940001  24.983845  19.709999  16.490000  95.709999  44.430420
2021-02-26  31.540001  24.139999  19.410000  16.190001  94.660004  44.091103
2021-03-01        NaN        NaN        NaN        NaN        NaN        NaN

            VVAR3.SA   WEGE3.SA   YDUQ3.SA
Date
2020-01-02     11.73  34.803658  45.885044
2020-01-03     11.48  34.359226  44.923176
2020-01-06     11.48  34.448112  45.786896
2020-01-07     11.65  34.714771  44.932987
2020-01-08     11.60  33.401234  44.667984
…                …          …          …
2021-02-23     13.19  83.289383  31.680000
2021-02-24     13.27  86.315903  31.450001
2021-02-25     12.61  79.645592  30.420000
2021-02-26     11.92  78.510643  29.770000
2021-03-01       NaN        NaN        NaN

[287 rows x 100 columns]
```

1.2 Preparando os dados

Nesta etapa, verificamos os NAs nas linhas e os eliminamos.
Vamos nos livrar das colunas com mais de 25% de valores ausentes

[3]:
```python
missing_fractions = dataset.isnull().mean().sort_values(ascending=False)

missing_fractions.head(10)
```

```
drop_list = sorted(list(missing_fractions[missing_fractions > 0.25].index))

dataset.drop(labels=drop_list, axis=1, inplace=True)
dataset.shape
```

[3]:
```
(287, 99)
```

[4]:
```
# Fill the missing values with the last value available in the dataset.
dataset=dataset.fillna(method='ffill')
```

1.3 Transformação dos dados

Para fins de agrupamento, usaremos os retornos anuais e a variância como variáveis, visto que são indicadores primários de desempenho e volatilidade das ações.

[5]:
```
returns = dataset.pct_change().mean() * 252
returns = pd.DataFrame(returns)
returns.columns = ['Returns']
returns['Volatility'] = dataset.pct_change().std() * np.sqrt(252)
data=returns
```

Todas as variáveis devem estar na mesma escala antes de aplicar o clustering, caso contrário, um recurso com grandes valores dominará o resultado. Usamos StandardScaler no sklearn para padronizar os recursos do conjunto de dados em escala unitária (média = 0 e variância = 1).

[6]:
```
from sklearn.preprocessing import StandardScaler
escala = StandardScaler().fit(data)
reescalando = pd.DataFrame(escala.fit_transform(data),
                           columns = data.columns,
                           index = data.index)

X = reescalando
X.head(5)
```

[6]:

	Returns	Volatility
ABEV3.SA	-0.557998	-1.053410
ALPA4.SA	0.402446	0.056583
ALSO3.SA	-1.214463	0.495541
AMAR3.SA	-1.227070	1.833503
AZUL4.SA	0.428813	2.680902

2 Formação e avaliação do modelo

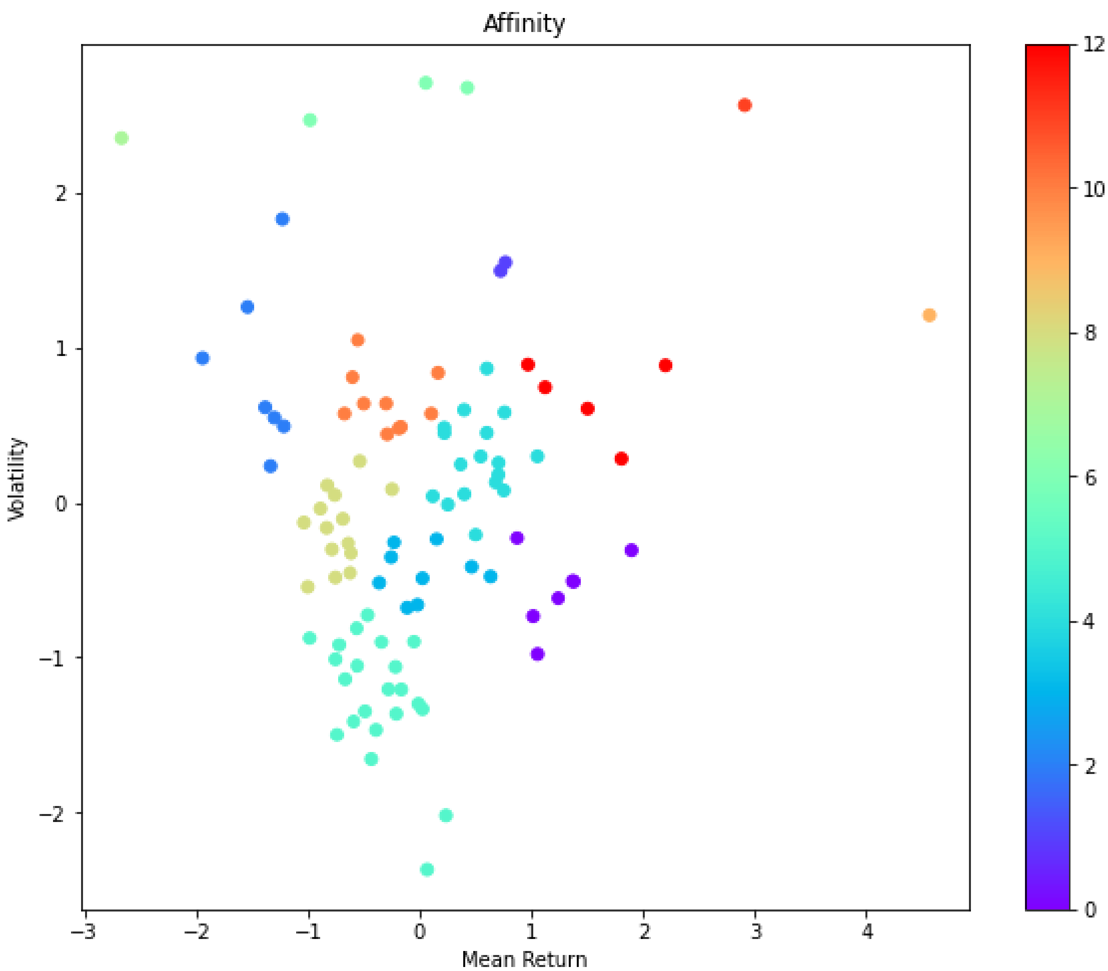
Há algumas formas para avaliar a formação de clusters em ações como, por exemplo, "K-means", "Hierarchical Clustering" e "Affinity Propagation". Como já foi analisado previamente em estudos anteriores, nesse exercício vamos encurtar o código usando apenas a terceira opção, a qual se mostrou com melhores resultados.

2.1 Affinity Propagation

[7]:
```
ap = AffinityPropagation()
ap.fit(X)
clust_labels = ap.predict(X)
```

[8]:
```
fig = plt.figure(figsize=(10,8))
ax = fig.add_subplot(111)
scatter = ax.scatter(X.iloc[:,0],X.iloc[:,1], c =clust_labels, cmap ="rainbow")
ax.set_title('Affinity')
ax.set_xlabel('Mean Return')
ax.set_ylabel('Volatility')
plt.colorbar(scatter)
```

[8]: <matplotlib.colorbar.Colorbar at 0x17def99ecd0>



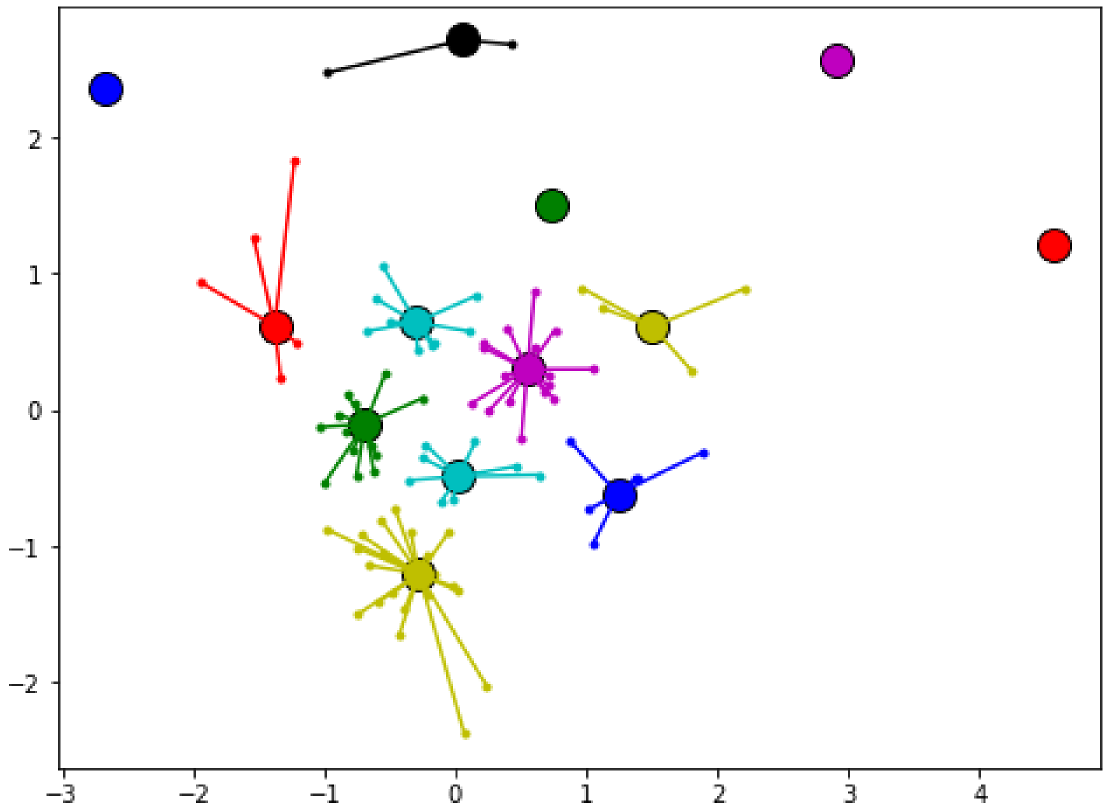
2.2 Vizualização dos Clusters

[9]:
```
cluster_centers_indices = ap.cluster_centers_indices_
labels = ap.labels_
```

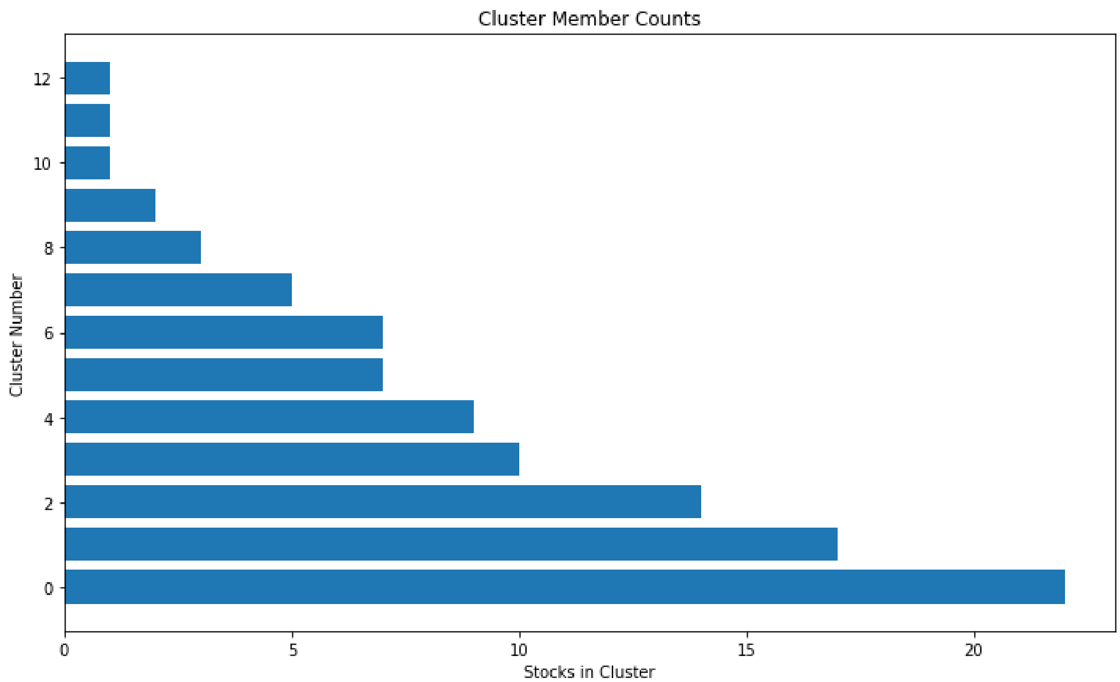
[10]:
```
no_clusters = len(cluster_centers_indices)
print('Estimated number of clusters: %d' % no_clusters)
# Plot exemplars

X_temp=np.asarray(X)
plt.close('all')
plt.figure(1)
plt.clf()

fig = plt.figure(figsize=(8,6))
colors = cycle('bgrcmykbgrcmykbgrcmykbgrcmyk')
for k, col in zip(range(no_clusters), colors):
    class_members = labels == k
    cluster_center = X_temp[cluster_centers_indices[k]]
    plt.plot(X_temp[class_members, 0], X_temp[class_members, 1], col + '.')
    plt.plot(cluster_center[0], cluster_center[1], 'o', markerfacecolor=col,␣
↪markeredgecolor='k', markersize=14)
    for x in X_temp[class_members]:
        plt.plot([cluster_center[0], x[0]], [cluster_center[1], x[1]], col)

plt.show()
```

```
Estimated number of clusters: 13
```

```
<Figure size 432x288 with 0 Axes>
```



[11]:
```
# show number of stocks in each cluster
clustered_series_ap = pd.Series(index=X.index, data=ap.labels_.flatten())
# clustered stock with its cluster label
clustered_series_all_ap = pd.Series(index=X.index, data=ap.labels_.flatten())
clustered_series_ap = clustered_series_ap[clustered_series_ap != -1]

plt.figure(figsize=(12,7))
plt.barh(
    range(len(clustered_series_ap.value_counts())), # cluster labels, y axis
    clustered_series_ap.value_counts()
)
plt.title('Cluster Member Counts')
plt.xlabel('Stocks in Cluster')
plt.ylabel('Cluster Number')
plt.show()
```



2.3 Avaliação dos Clusters

O modelo sugeriu a formação de 13 clusters entre as açoes do IBRX.

2.3.1 Visualizando o retorno dentro de um cluster

Para entender a intuição por trás do clustering, vamos visualizar os resultados dos clusters.

[12]:
```
# all stock with its cluster label (including -1)
clustered_series = pd.Series(index=X.index, data=ap.fit_predict(X).flatten())
# clustered stock with its cluster label
clustered_series_all = pd.Series(index=X.index, data=ap.fit_predict(X).
 ↪flatten())
clustered_series = clustered_series[clustered_series != -1]
```

[13]:
```
# get the number of stocks in each cluster
counts = clustered_series_ap.value_counts()

# let's visualize some clusters
cluster_vis_list = list(counts[(counts<25) & (counts>1)].index)[::-1]
cluster_vis_list
```

[13]: [1, 6, 12, 0, 2, 3, 10, 8, 4, 5]

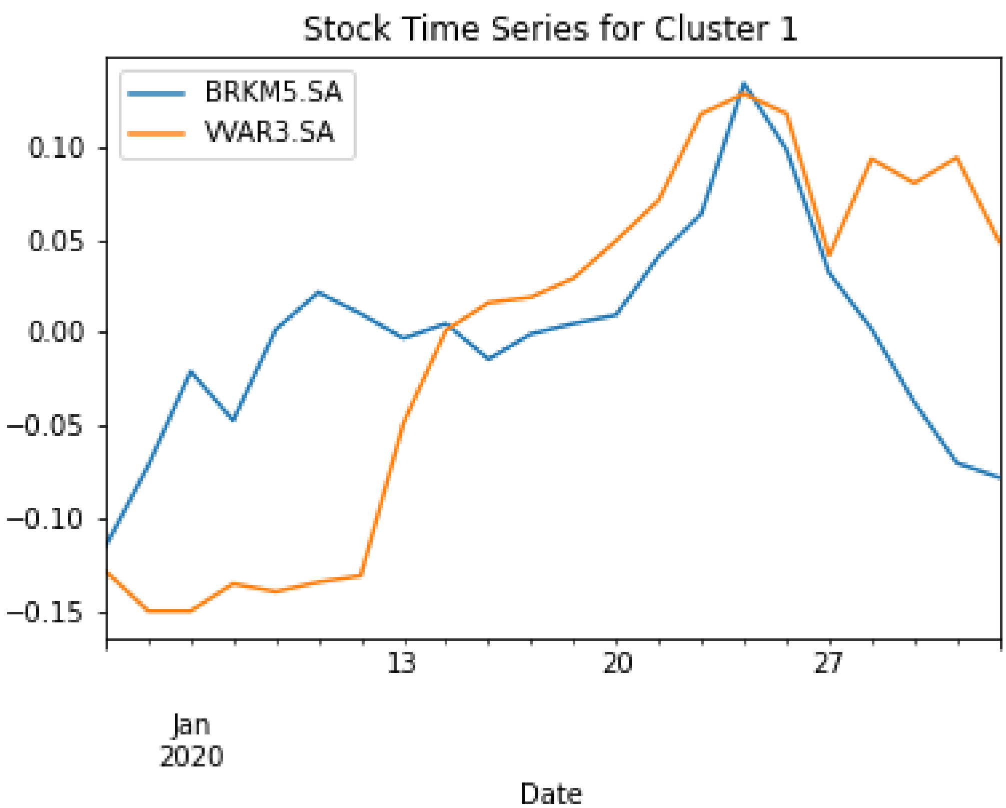
[14]:

```
CLUSTER_SIZE_LIMIT = 9999
counts = clustered_series.value_counts()
ticker_count_reduced = counts[(counts>1) & (counts<=CLUSTER_SIZE_LIMIT)]
print ("Clusters formed: %d" % len(ticker_count_reduced))
print ("Pairs to evaluate: %d" %␣
↪(ticker_count_reduced*(ticker_count_reduced-1)).sum())
```

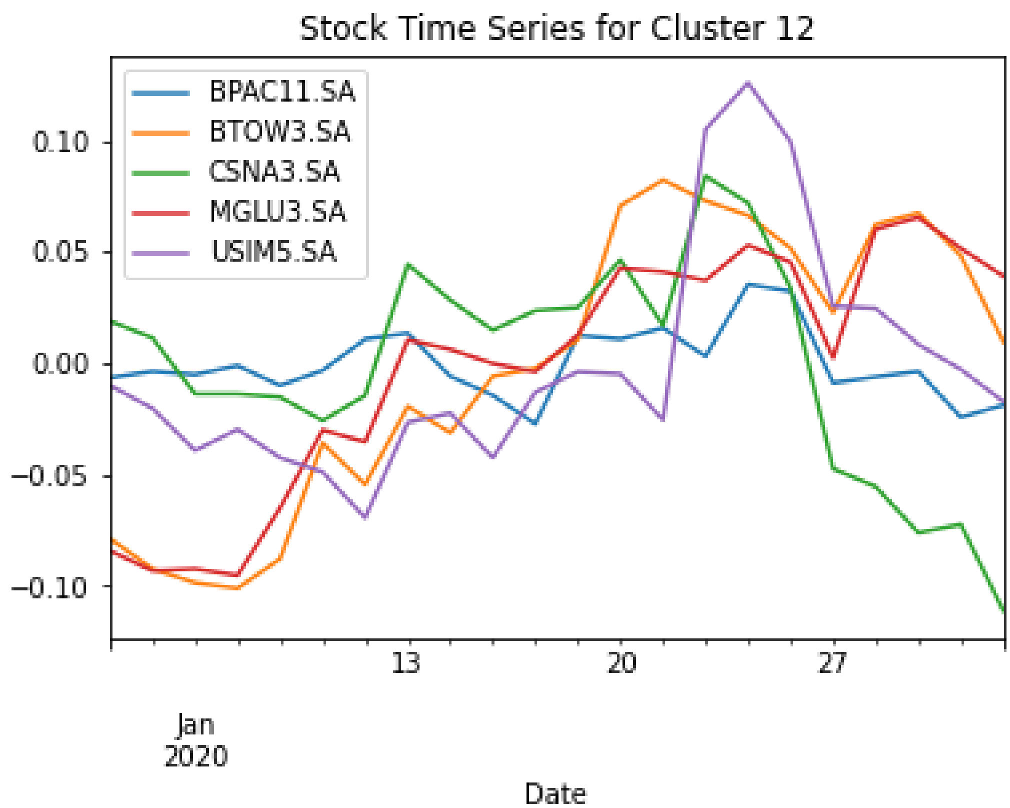
```
Clusters formed: 10
Pairs to evaluate: 1190
```

[15]:

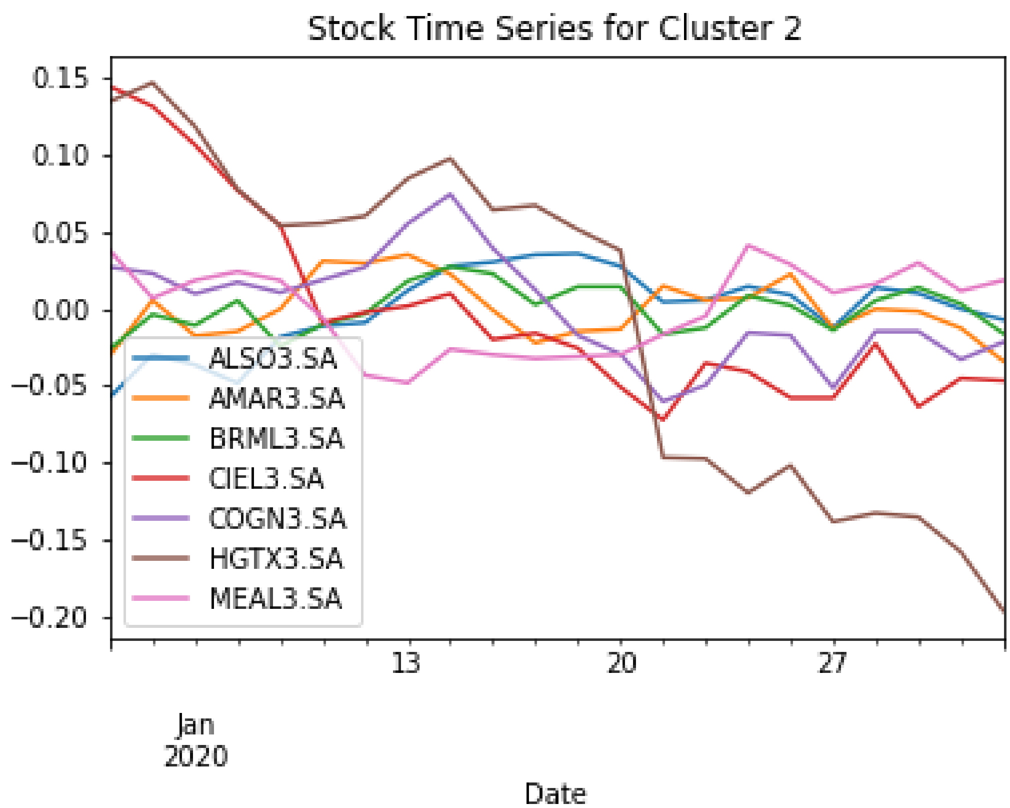
```
# Vizualizando os 10 clusters

for clust in cluster_vis_list[0:min(len(cluster_vis_list),10)]:
    tickers = list(clustered_series[clustered_series==clust].index)
    means = np.log(dataset.loc[:"2020-02-02", tickers].mean())
    data = np.log(dataset.loc[:"2020-02-02", tickers]).sub(means)
    data.plot(title='Stock Time Series for Cluster %d' % clust)
plt.show()
```

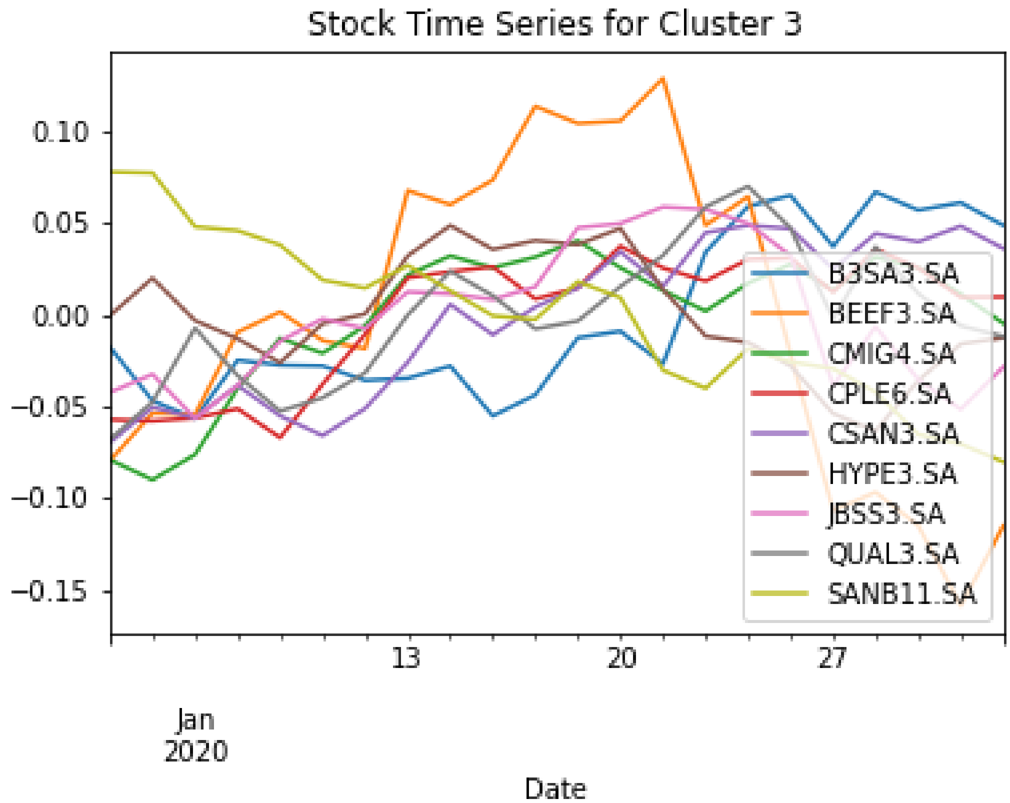



Stock Time Series for Cluster 6
0.05
0.00
−0.05
−0.10
AZUL4.SA
CVCB3.SA
GOLL4.SA
13
20
27
Jan
2020
Date

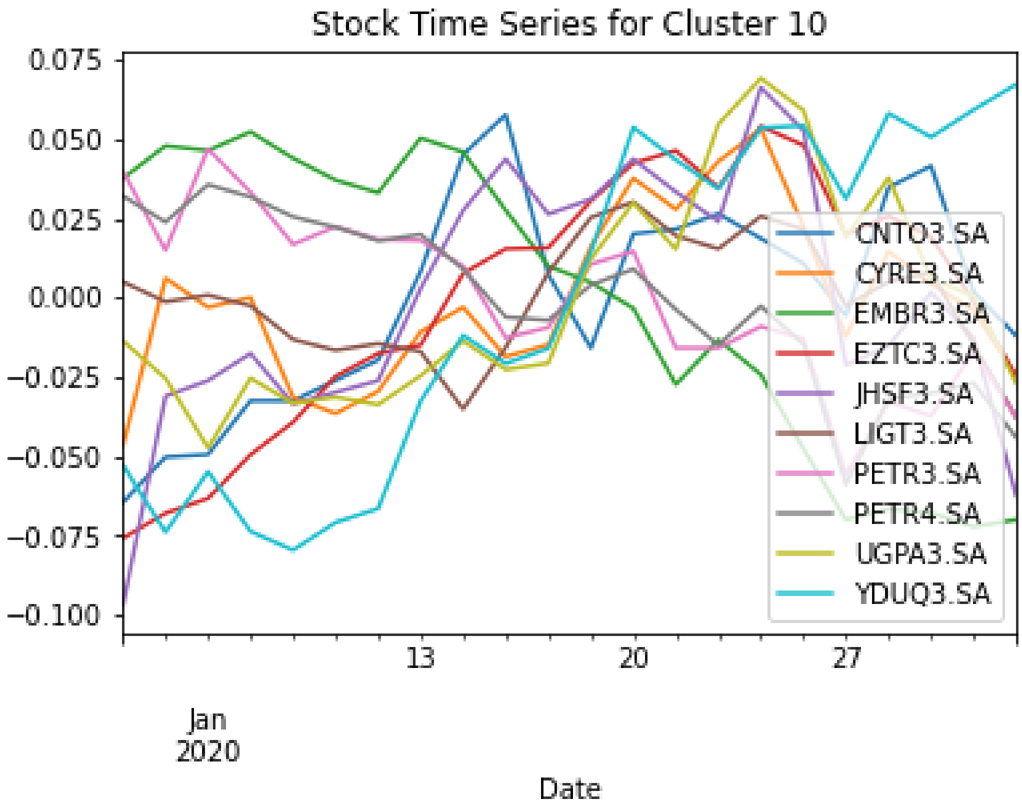

Stock Time Series for Cluster 12
BPAC11.SA
BTOW3.SA
CSNA3.SA
MGLU3.SA
USIM5.SA
0.10
0.05
0.00
−0.05
−0.10
13
20
27
Jan
2020
Date

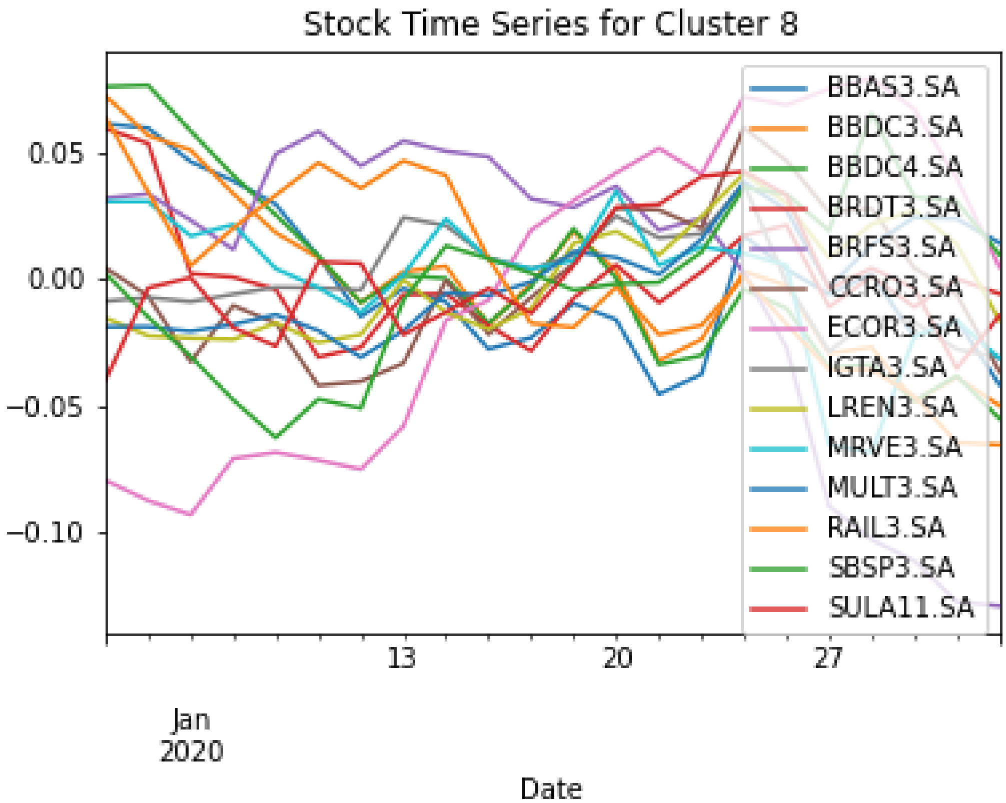
Stock Time Series for Cluster 0
BRAP4.SA
ENEV3.SA
KLBN11.SA
SUZB3.SA
TOTS3.SA
VALE3.SA
WEGE3.SA
0.10
0.05
0.00
−0.05
−0.10
13
20
27
Jan
2020
Date

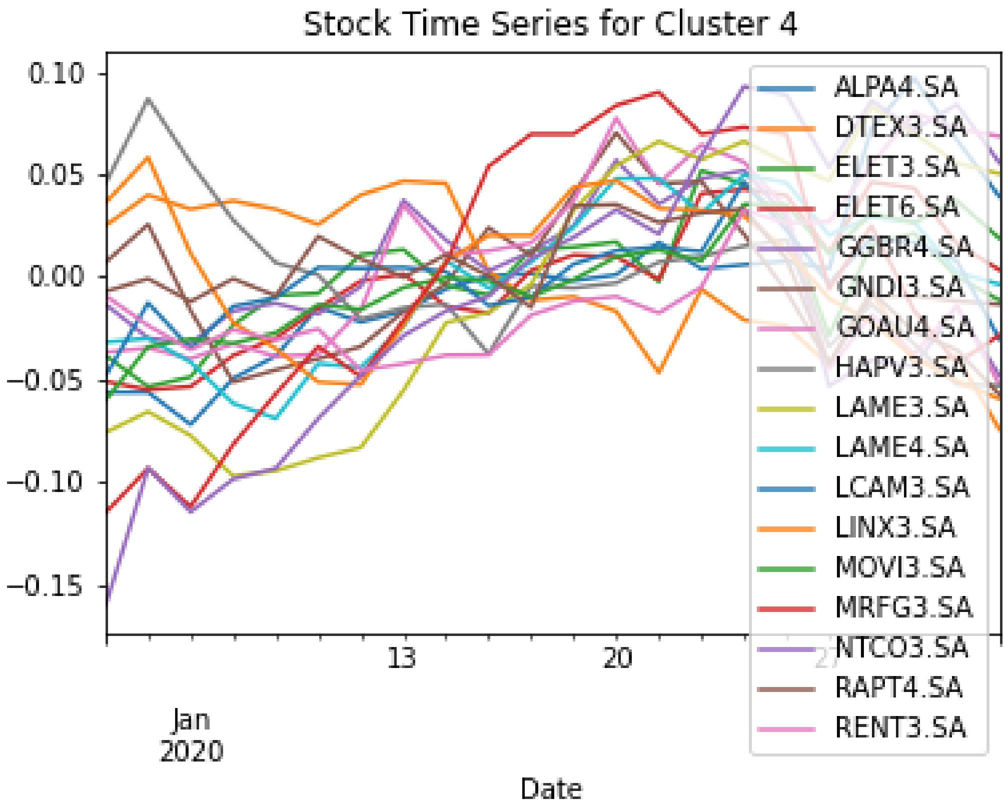

Stock Time Series for Cluster 2
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
ALSO3.SA
AMAR3.SA
BRML3.SA
CIEL3.SA
COGN3.SA
HGTX3.SA
MEAL3.SA
13
20
27
Jan
2020
Date

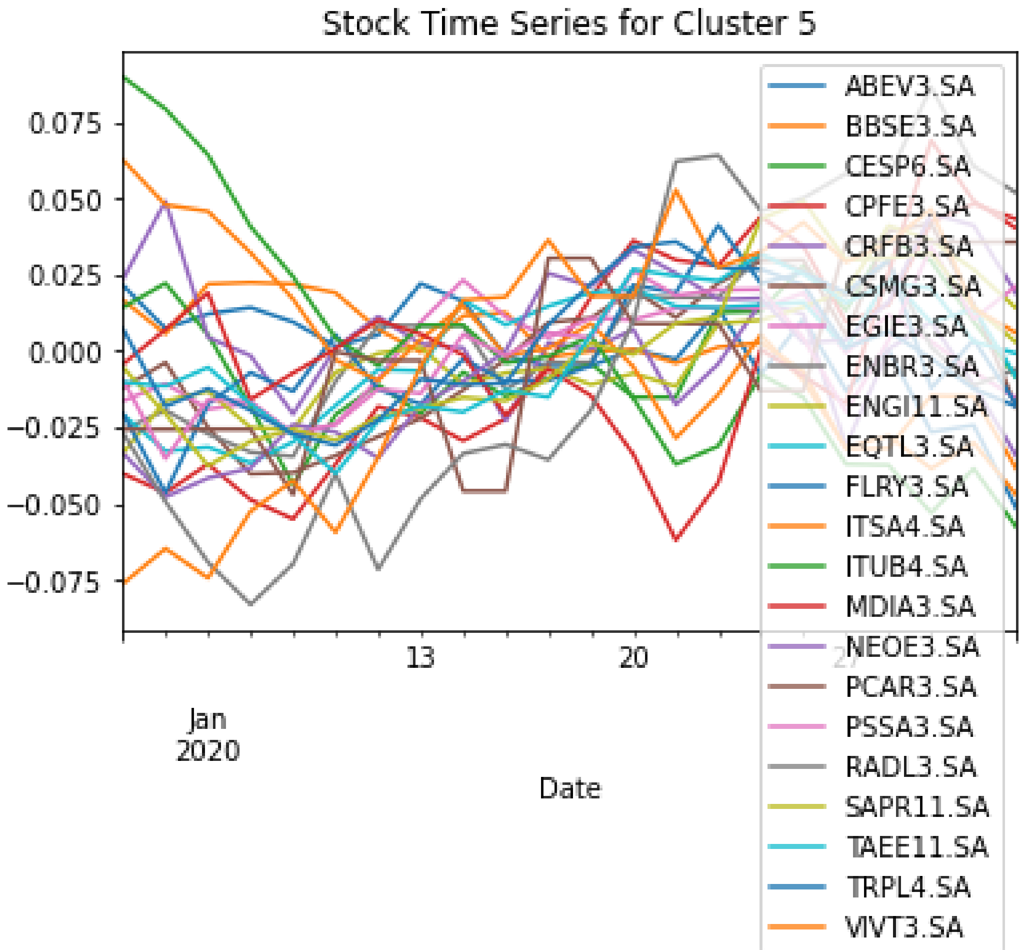


Stock Time Series for Cluster 3
0.10
0.05
0.00
−0.05
−0.10
−0.15
13
20
27
Jan
2020
Date
B3SA3.SA
BEEF3.SA
CMIG4.SA
CPLE6.SA
CSAN3.SA
HYPE3.SA
JBSS3.SA
QUAL3.SA
SANB11.SA


Stock Time Series for Cluster 10
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100
13
20
27
Jan
2020
Date
CNTO3.SA
CYRE3.SA
EMBR3.SA
EZTC3.SA
JHSF3.SA
LIGT3.SA
PETR3.SA
PETR4.SA
UGPA3.SA
YDUQ3.SA


Stock Time Series for Cluster 8
0.05
0.00
−0.05
−0.10
BBAS3.SA
BBDC3.SA
BBDC4.SA
BRDT3.SA
BRFS3.SA
CCRO3.SA
ECOR3.SA
IGTA3.SA
LREN3.SA
MRVE3.SA
MULT3.SA
RAIL3.SA
SBSP3.SA
SULA11.SA
13
20
27
Jan
2020
Date


Stock Time Series for Cluster 4
0.10
0.05
0.00
−0.05
−0.10
−0.15
13
20
27
Jan
2020
Date
ALPA4.SA
DTEX3.SA
ELET3.SA
ELET6.SA
GGBR4.SA
GNDI3.SA
GOAU4.SA
HAPV3.SA
LAME3.SA
LAME4.SA
LCAM3.SA
LINX3.SA
MOVI3.SA
MRFG3.SA
NTCO3.SA
RAPT4.SA
RENT3.SA



3 Seleção de pares

Uma vez que os clusters são criados, várias técnicas estatísticas baseadas em cointegração podem ser aplicadas nos estoques dentro de um cluster para criar os pares. Nesta etapa, examinamos uma lista de títulos dentro de um cluster e testamos a cointegração entre os pares. Primeiro, escrevemos uma função que retorna uma matriz de pontuação de teste de cointegração, uma matriz de valor p e quaisquer pares para os quais o valor p seja menor que 0.05.

[16]:
```
def find_cointegrated_pairs(data, significance=0.05):
    # This function is from https://www.quantopian.com/lectures/
↪introduction-to-pairs-trading
    n = data.shape[1]
    score_matrix = np.zeros((n, n))
    pvalue_matrix = np.ones((n, n))
    keys = data.keys()
    pairs = []
```

```
    for i in range(1):
        for j in range(i+1, n):
            S1 = data[keys[i]]
            S2 = data[keys[j]]
            result = coint(S1, S2)
            score = result[0]
            pvalue = result[1]
            score_matrix[i, j] = score
            pvalue_matrix[i, j] = pvalue
            if pvalue < significance:
                pairs.append((keys[i], keys[j]))
    return score_matrix, pvalue_matrix, pairs
```

[17]:
```
from statsmodels.tsa.stattools import coint
cluster_dict = {}
for i, which_clust in enumerate(ticker_count_reduced.index):
    tickers = clustered_series[clustered_series == which_clust].index
    score_matrix, pvalue_matrix, pairs = find_cointegrated_pairs(
        dataset[tickers]
    )
    cluster_dict[which_clust] = {}
    cluster_dict[which_clust]['score_matrix'] = score_matrix
    cluster_dict[which_clust]['pvalue_matrix'] = pvalue_matrix
    cluster_dict[which_clust]['pairs'] = pairs
```

[18]:
```
pairs = []
for clust in cluster_dict.keys():
    pairs.extend(cluster_dict[clust]['pairs'])
```

[19]:
```
print ("Number of pairs found : %d" % len(pairs))
print ("In those pairs, there are %d unique tickers." % len(np.unique(pairs)))
```

```
Number of pairs found : 15
In those pairs, there are 19 unique tickers.
```

[20]:
```
pairs
```

[20]:
```
[('ABEV3.SA', 'CPFE3.SA'),
 ('ABEV3.SA', 'EGIE3.SA'),
 ('ABEV3.SA', 'ENBR3.SA'),
 ('ABEV3.SA', 'ENGI11.SA'),
 ('ABEV3.SA', 'ITSA4.SA'),
 ('ABEV3.SA', 'ITUB4.SA'),
 ('ALPA4.SA', 'GGBR4.SA'),
 ('ALPA4.SA', 'LINX3.SA'),
 ('ALPA4.SA', 'RAPT4.SA'),
 ('BBAS3.SA', 'ECOR3.SA'),
```

```
 ('BBAS3.SA', 'IGTA3.SA'),
 ('BBAS3.SA', 'LREN3.SA'),
 ('BBAS3.SA', 'MULT3.SA'),
 ('ALSO3.SA', 'BRML3.SA'),
 ('ALSO3.SA', 'MEAL3.SA')]
```

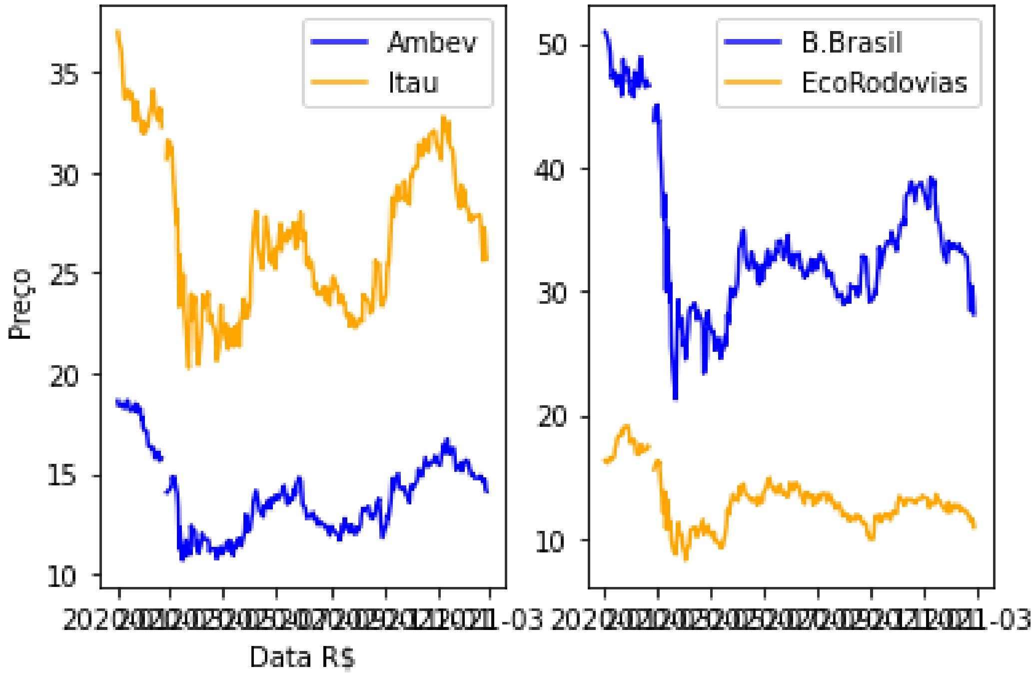
4 Vizualização das ações em pares

Vamos analisar o gráfico de 4 pares dos 16 formados.

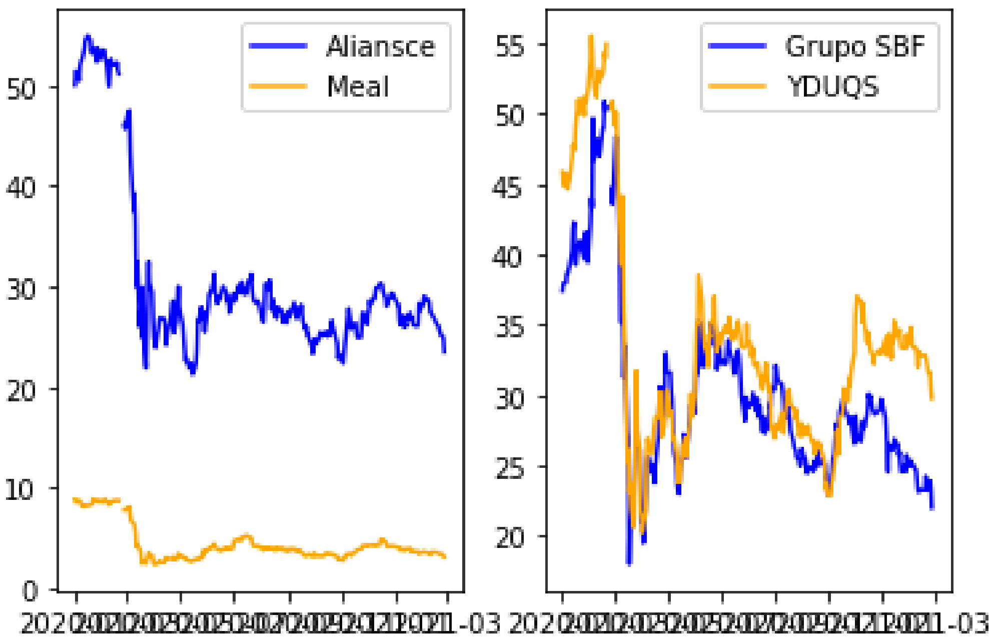
[21]:
```
tickers = ['ABEV3.SA', 'ALPA4.SA', 'BBAS3.SA', 'CNTO3.SA', 'ALSO3.SA', 'CPFE3.
 ↪SA', 'EGIE3.SA',
           'ENBR3.SA', 'ENGI11.SA', 'ITSA4.SA','ITUB4.SA', 'GGBR4.SA','LINX3.
 ↪SA', 'RAPT4.SA',
           'ECOR3.SA', 'IGTA3.SA', 'LREN3.SA', 'MULT3.SA', 'YDUQ3.SA', 'BRML3.
 ↪SA', 'MEAL3.SA']

stocks = yf.download(tickers = tickers, start = '2020-01-01')['Adj Close']
stocks = pd.DataFrame(stocks)
```

```
[*********************100%***********************]  21 of 21 completed
```

[22]:
```
plt.subplot(1,2,1)
plt.plot(stocks['ABEV3.SA'], color = 'blue', label = 'Ambev')
plt.plot(stocks['ITUB4.SA'], color = 'orange', label = 'Itau')
plt.legend(loc = 'best')
plt.xlabel('Data R$')
plt.ylabel('Preço')

plt.subplot(1,2,2)
plt.plot(stocks['BBAS3.SA'], color = 'blue', label = 'B.Brasil')
plt.plot(stocks['ECOR3.SA'], color = 'orange', label = 'EcoRodovias')
plt.legend(loc = 'best')
fig.autofmt_xdate()

plt.show()
```



[23]:
```
plt.subplot(1,2,1)
plt.plot(stocks['ALSO3.SA'], color = 'blue', label = 'Aliansce')
plt.plot(stocks['MEAL3.SA'], color = 'orange', label = 'Meal')
plt.legend(loc = 'best')
fig.autofmt_xdate()


plt.subplot(1,2,2)
plt.plot(stocks['CNTO3.SA'], color = 'blue', label = 'Grupo SBF')
plt.plot(stocks['YDUQ3.SA'], color = 'orange', label = 'YDUQS')
plt.legend(loc = 'best')
fig.autofmt_xdate()
```


Aliansce
Meal
Grupo SBF
YDUQS
0
10
20
30
40
50
20
25
30
35
40
45
50
55