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PORTFOLIO REPLICA

Politecnico di Milano

Mathematical Engineering

Politecnico di Milano

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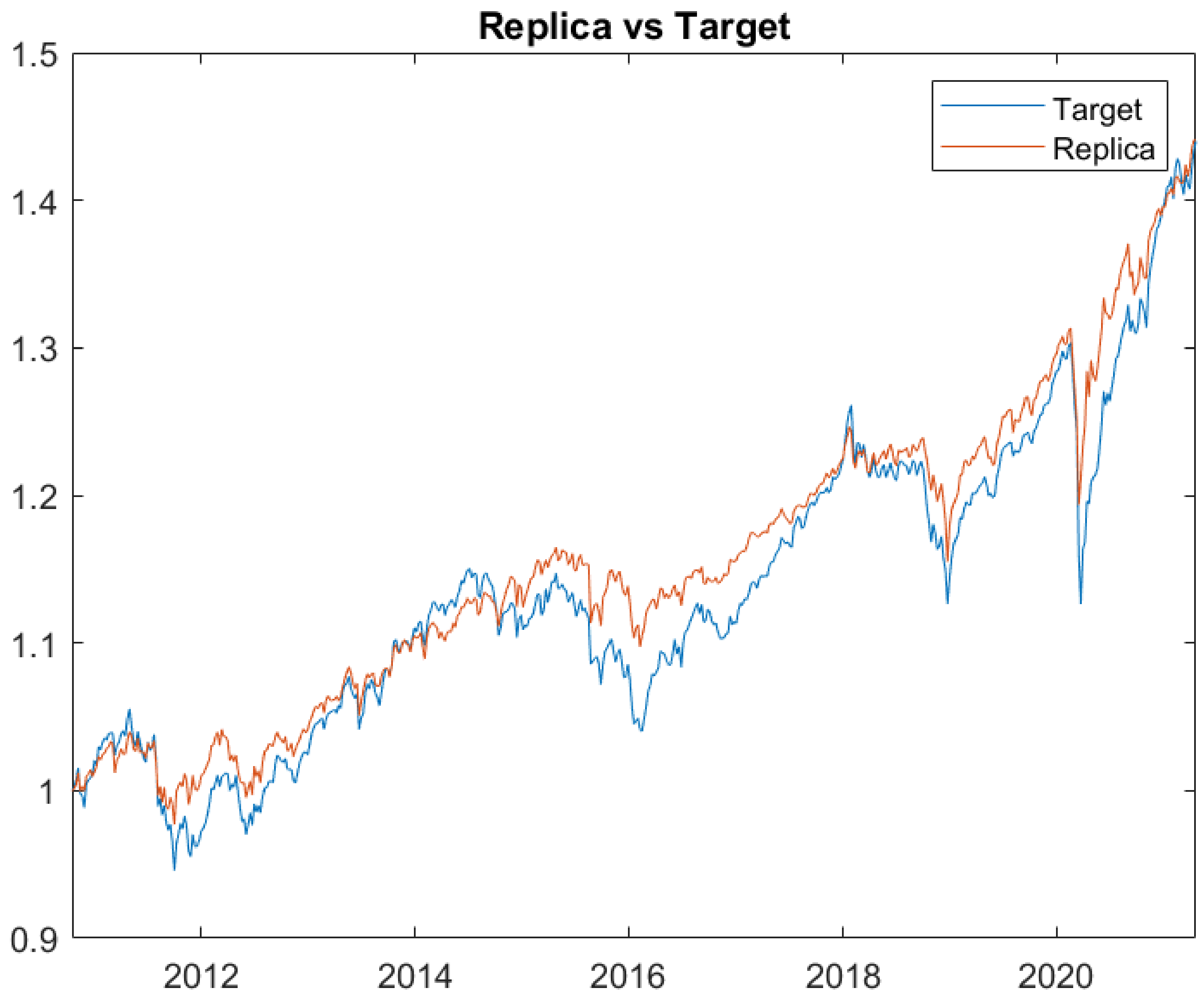
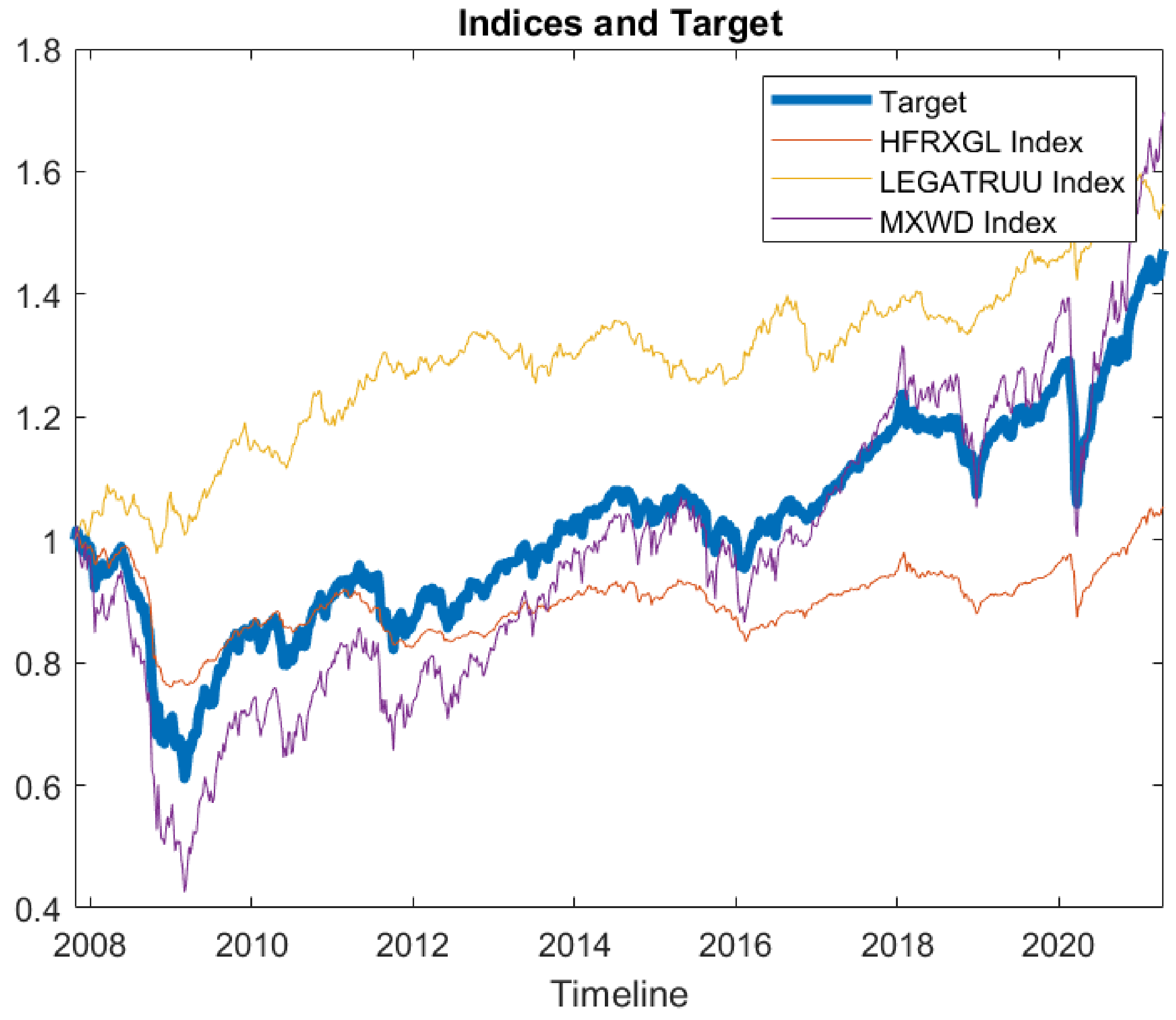
AIM OF THE PROJECT

1. **Replica of a «target index»** by constructing a portfolio of future contracts
2. **Inclusion of fees** to obtain more realistic results
3. **Evaluation of the performances** through VaR and gross exposure computation
4. Comparisons of **different time scales** (monthly vs weekly)
5. **Detection of market anomalies** to improve the performance of the portfolio



THE DATASET: INVESTMENT REPLICA DATA



- Four indexes as possible target variables: «MXWO» , «MXWD» , «LEGATRUU» , «HFRXGL». Our target will be a linear combination of these indexes.
- **Target = 0.25*HFRXGL + 0.5*MXWO + 0.25*LEGATRUU**
- Weekly prices available from Oct. 23rd 2007 to Apr. 20th 2021 (705 observations).

We have the following futures as possible variables:

RX1: Germany Bond 10y
GC1: Gold future
ES1: S&P500 USA future
NQ1: NASDAQ 100 E-Mini
TP1: Japan equity index
TU2: USA Bond 2y

TY1: USA Bond 10y
CO1: Crude oil Europe
VG1: Euro stoxx
LLL1: MSCI Emerging markets
DU1: Euro-Schatz 2y

THE DATASET: FINANCIAL MARKET DATA

- 43 financial indexes on the global financial market
- Binary response variable (1=anomaly, 0=normal)
- 1148 observations from Apr. 27th 1999 to Apr. 20th 2021

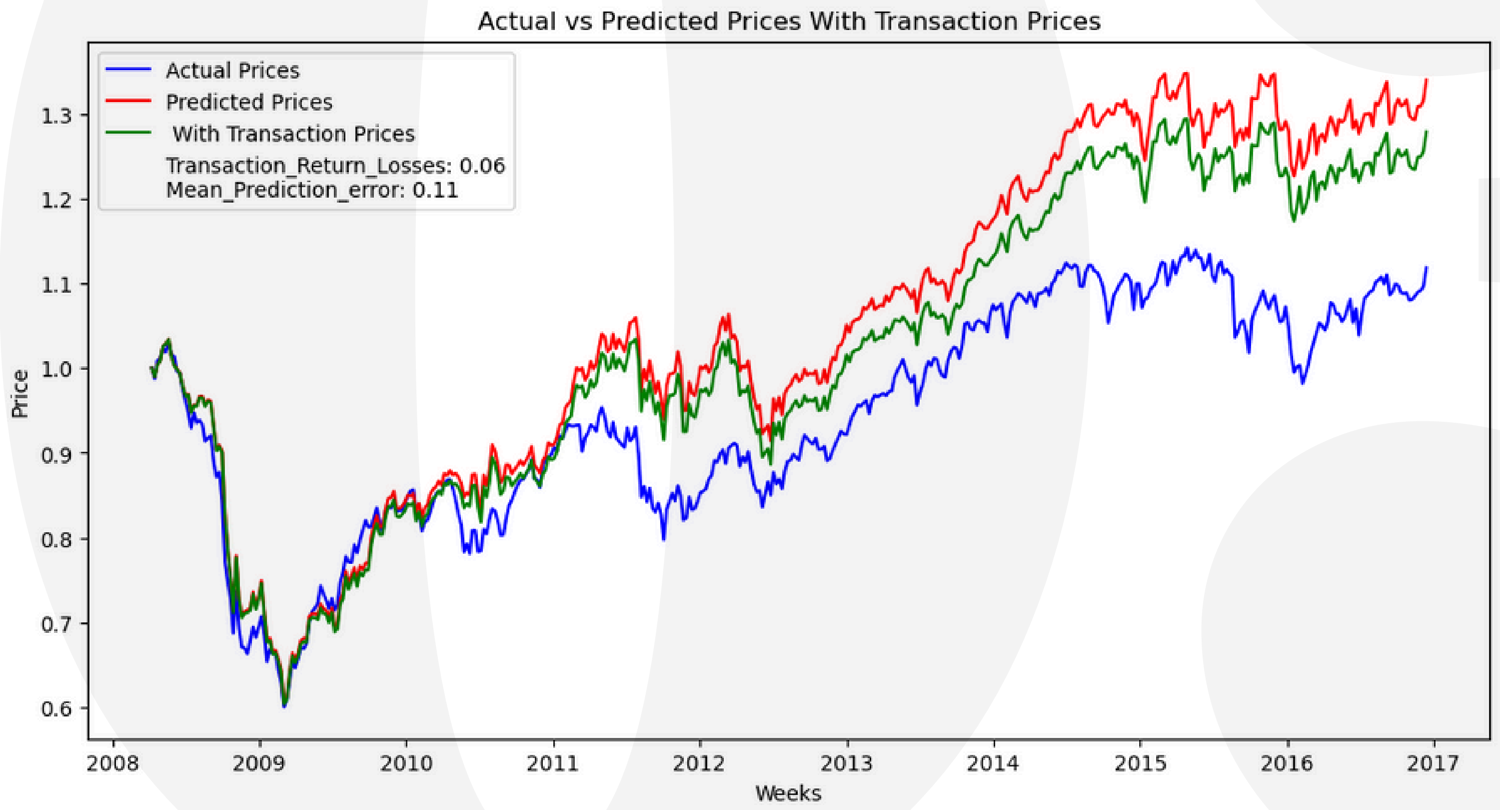
Some features of the dataset:

- VIX = volatility of S&P500
- GTITL2YR = BTP 2y
- USGG3M = USA Bond 3m
- …

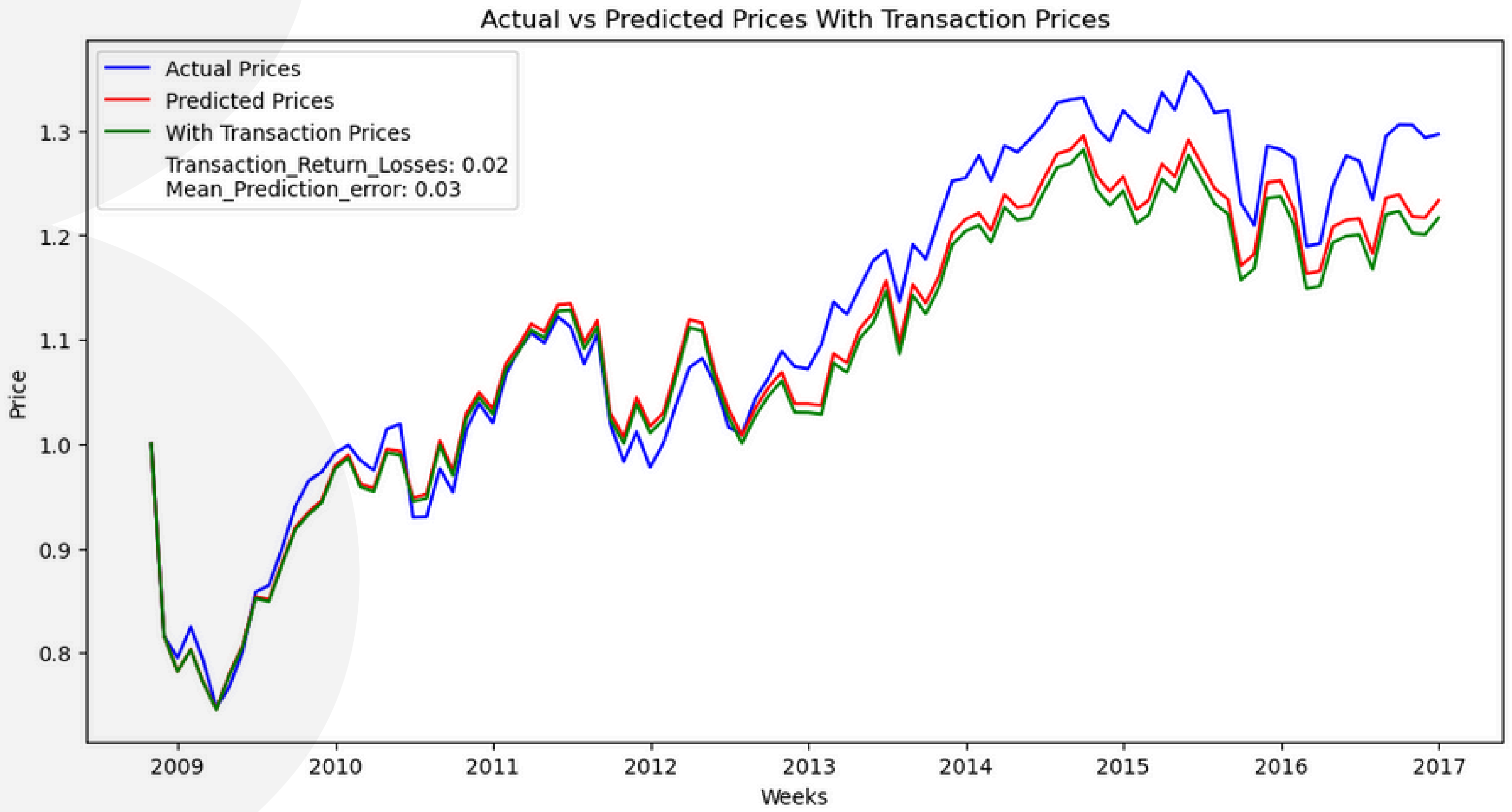
FIRST APPROACH: LASSO REGRESSION, VALIDATION SET

We tried to **replicate** the target index with a **one-step ahead prediction** using a **rolling window linear regression with Lasso penalty**, taking into account **transaction costs**. We also computed the **MAE** between predicted and actual prices.

Weekly rebalancing



Monthly rebalancing

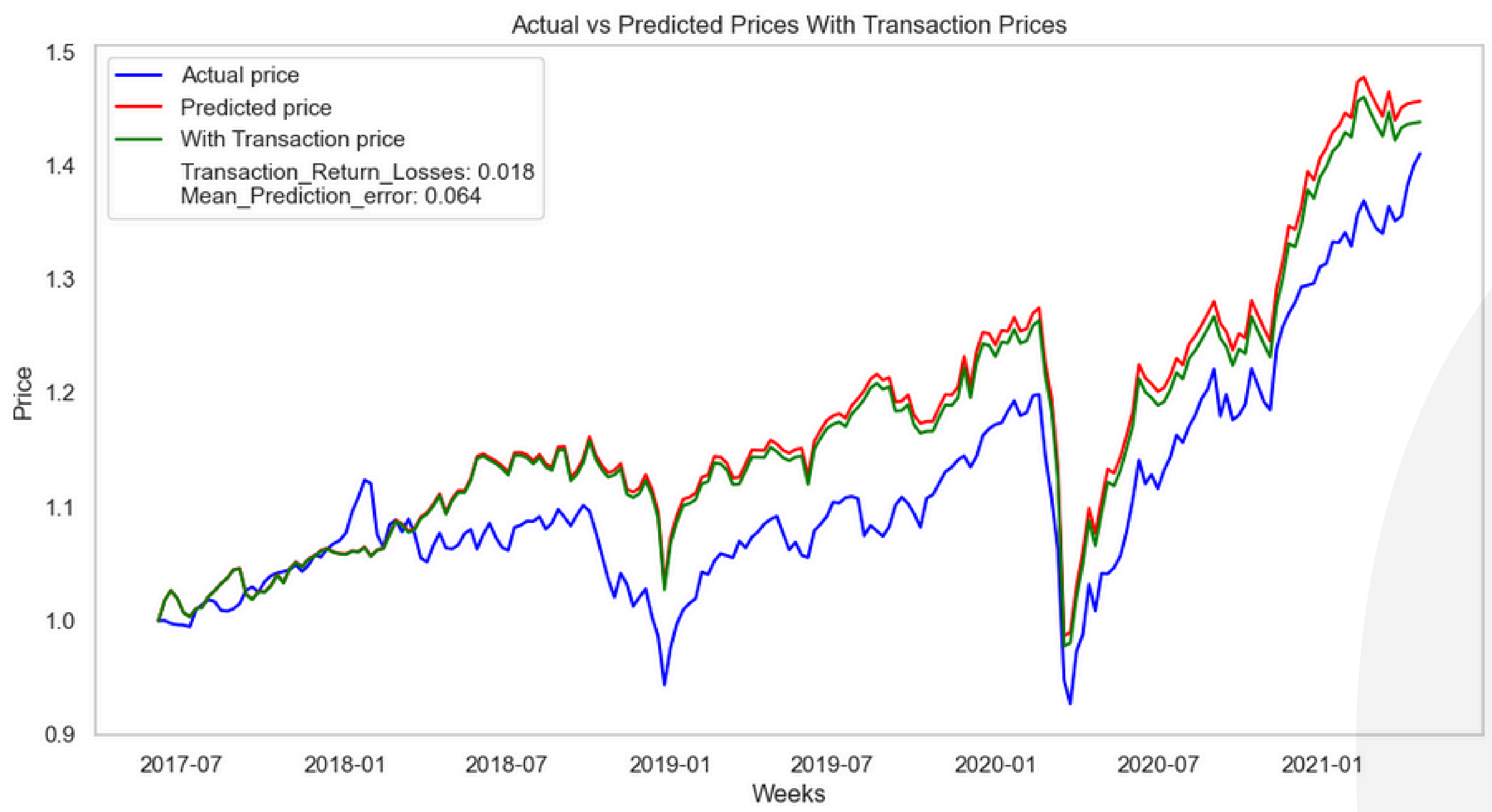


We observe that with a **monthly rebalancing** the **tracking improves** and we have a **reduction in transaction fees**.

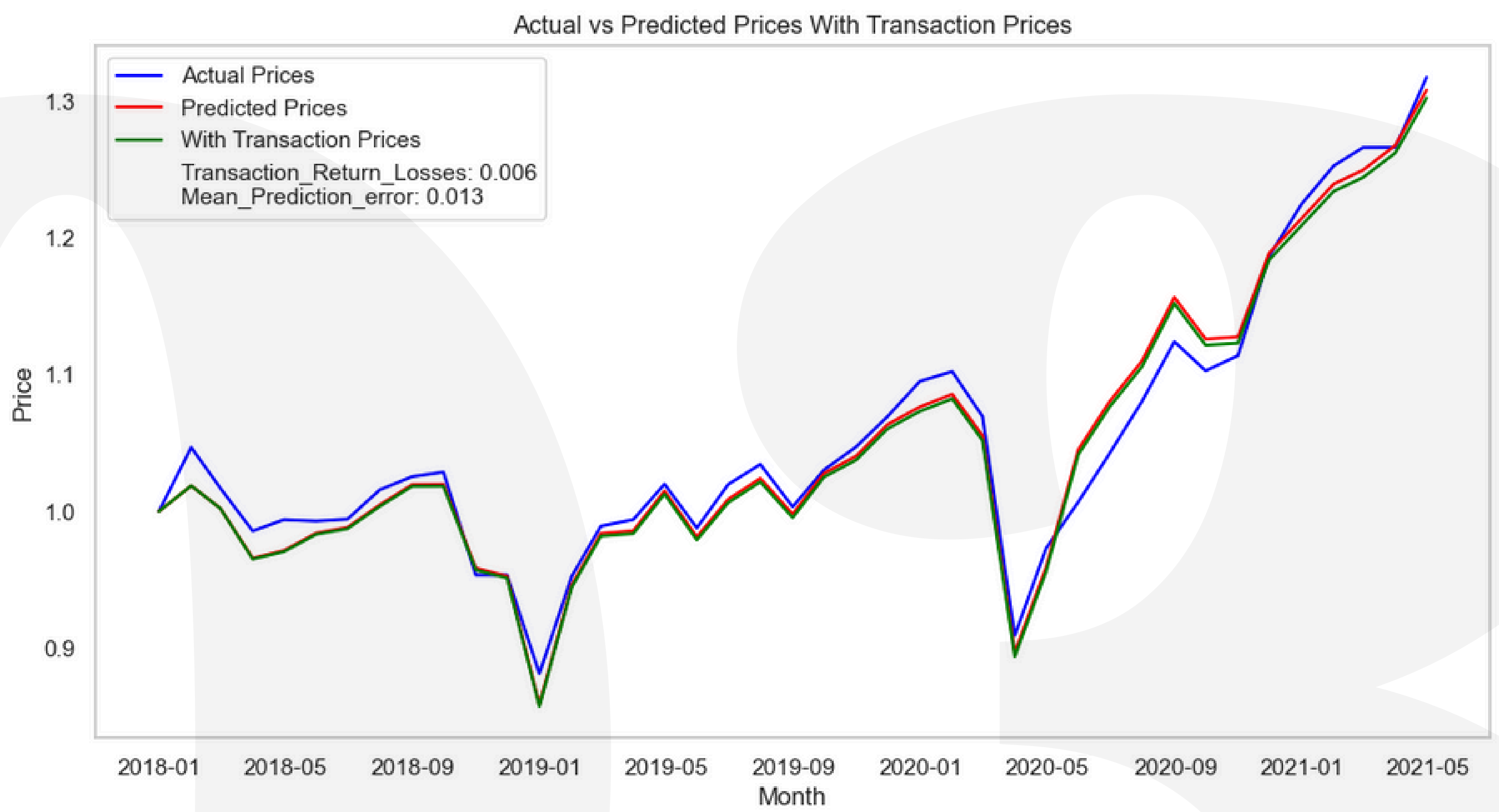
FIRST APPROACH: LASSO REGRESSION, TEST SET

Now we see the performance on the test set

Weekly rebalancing



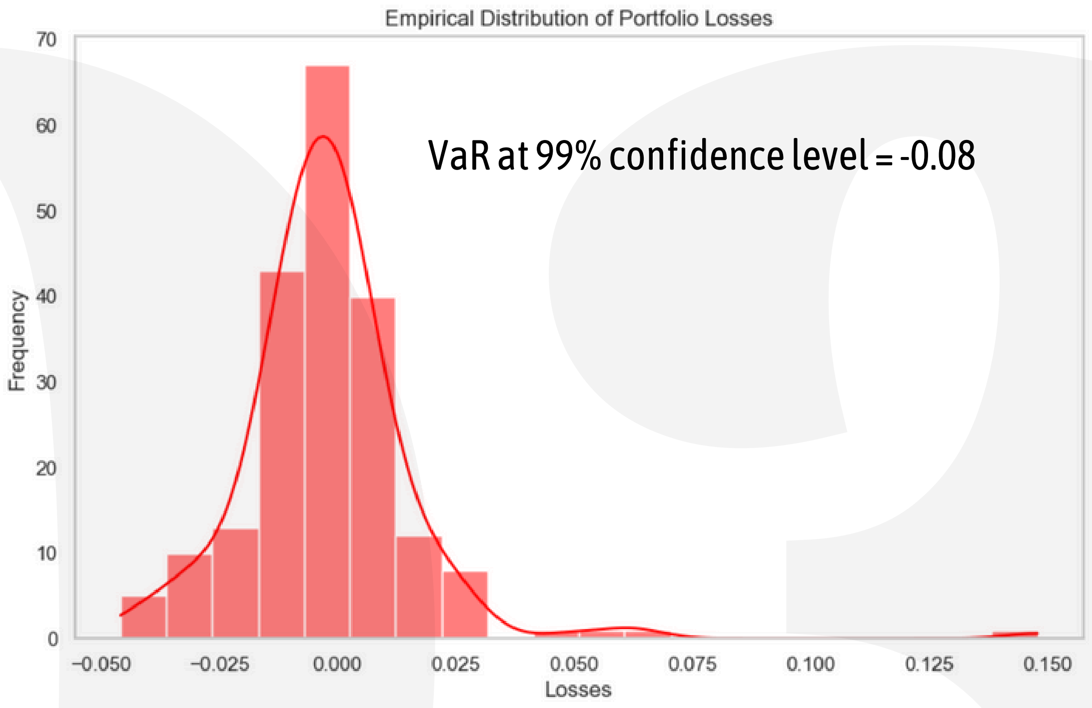
Monthly rebalancing

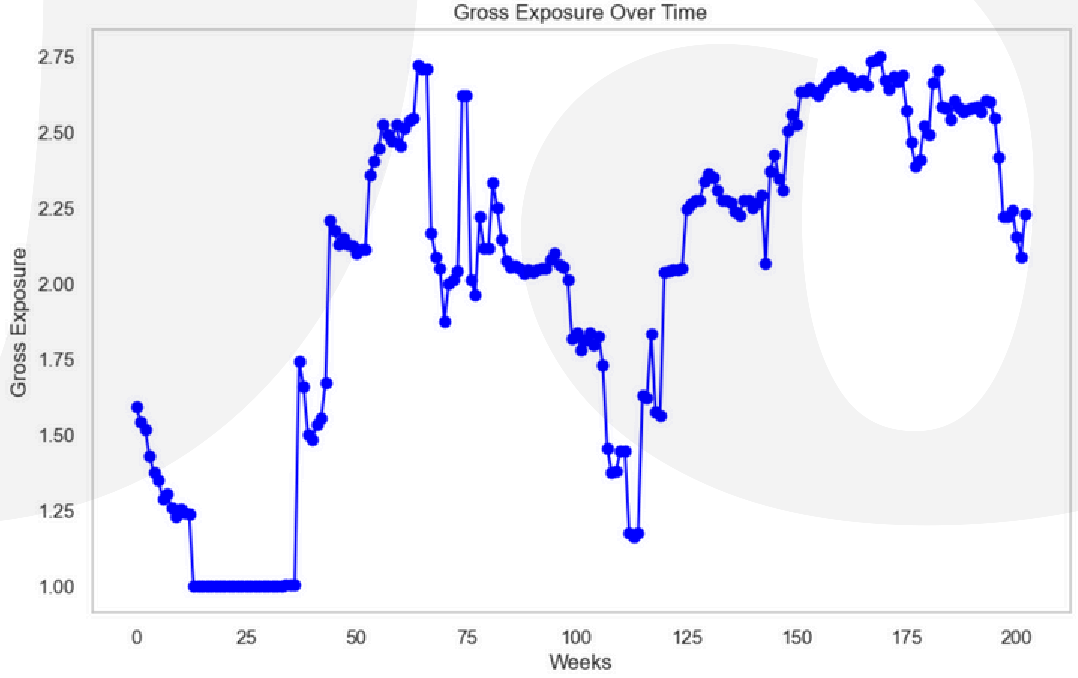


We observe **better tracking results** in the test set and also **lower transaction fees** (due to the lower time span).
The results regarding the frequency of rebalancing are coherent with the ones of the validation set.

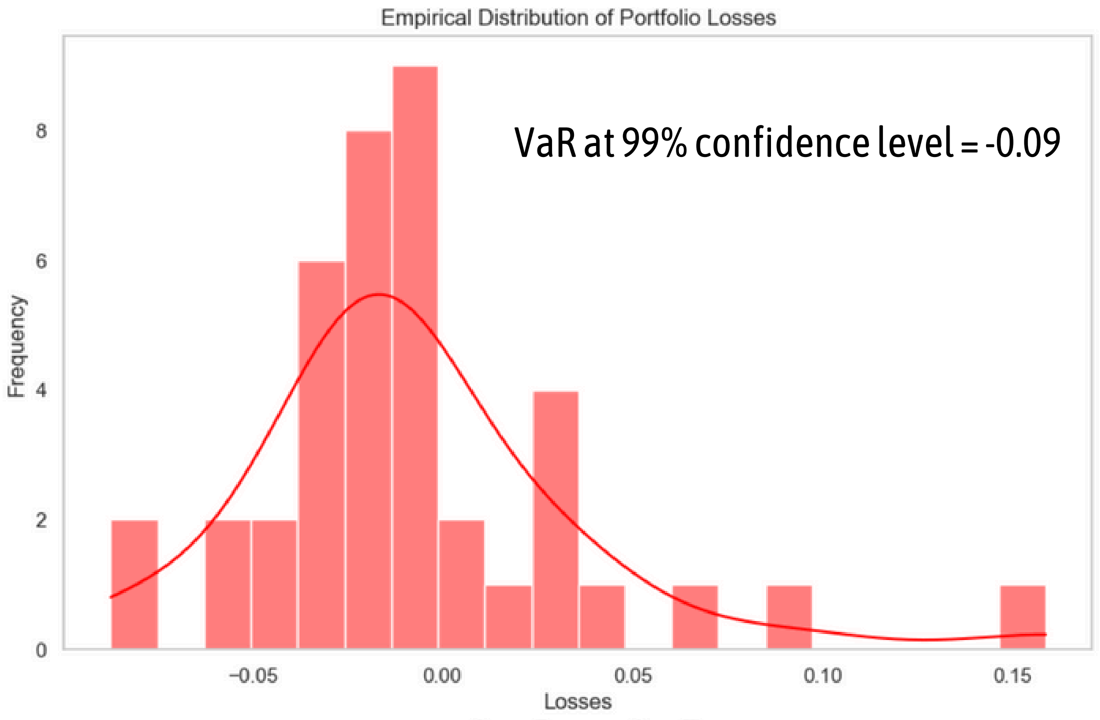
FIRST APPROACH: LASSO REGRESSION, MEASURES OF RISK

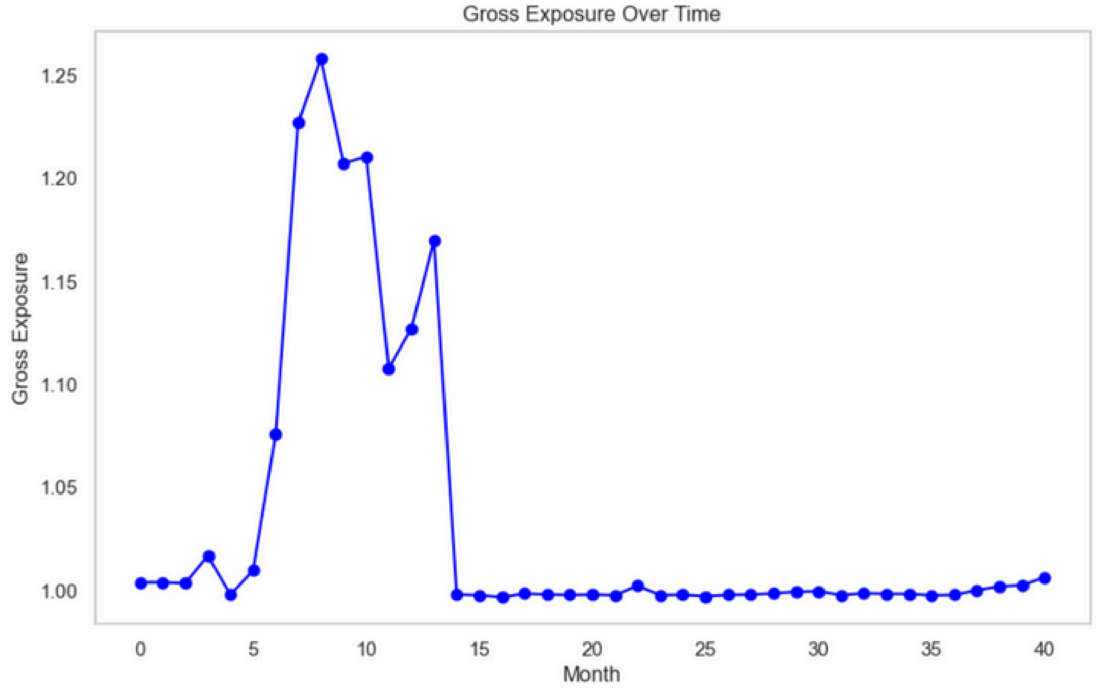
Weekly rebalancing





Monthly rebalancing





FIRST APPROACH: LASSO REGRESSION, RESULTS

- We can see that the **Lasso regression works well** for our problem: we are able to predict and follow the target index with great accuracy.
- **Transaction costs** weigh heavily in the **weekly case** on the total achieved return.
- Switching to a **monthly resample** results in a **lower mean prediction error** since we are more robust to random market movements.
- As we would expect, we manage to **spend way less in transactions** with a monthly rebalancing strategy.
- The monthly rebalancing strategy leads to a more conservative portofolio management with a **lower leverage**.
- Our model performs better in the test set than in the validation set, showing a **good degree of robustness.**

SECOND APPROACH: KALMAN FILTER, INTRODUCTION

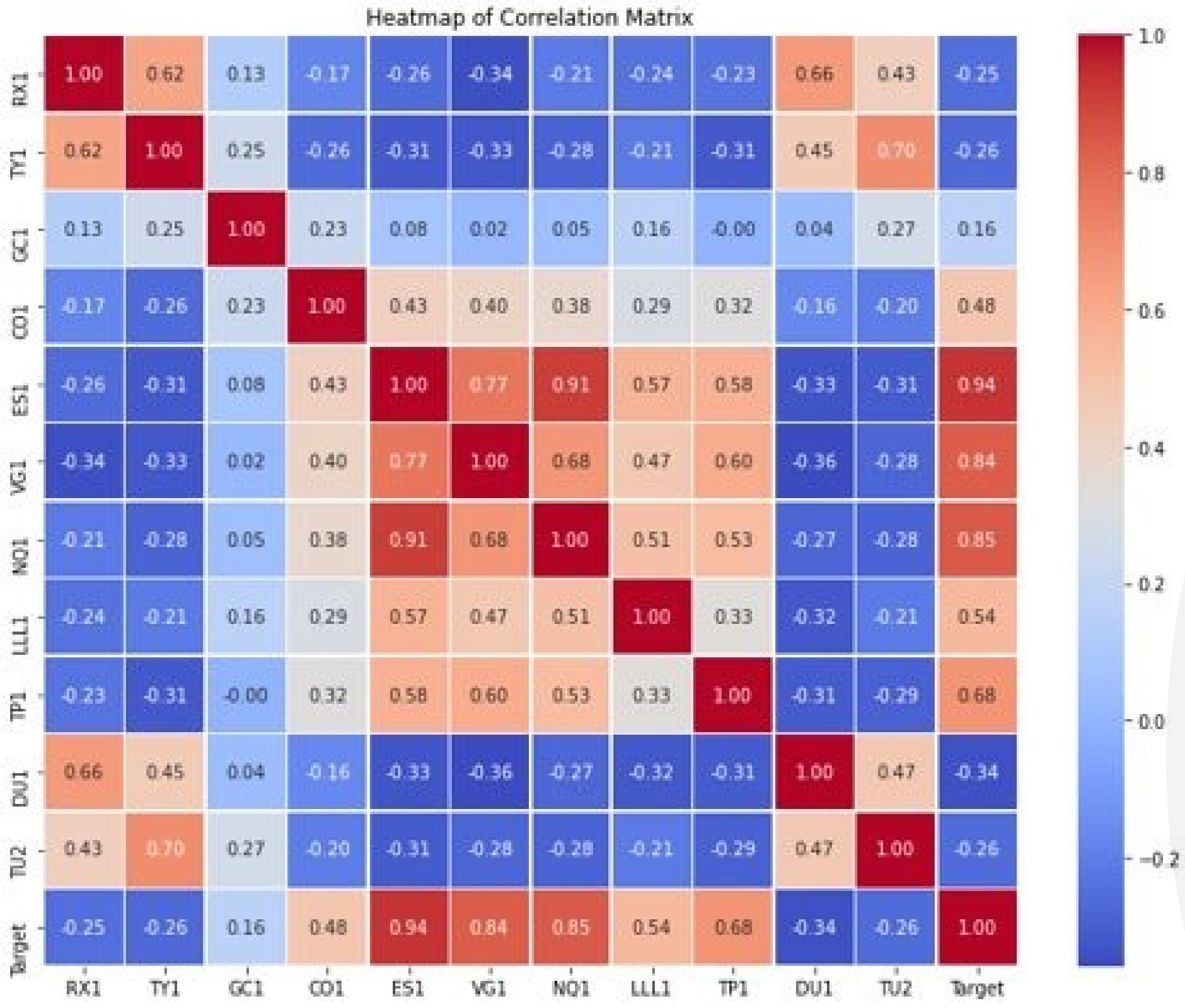
$$\begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = C(t)x(t) + D\epsilon(t) \end{cases}$$

- x(t) is the vector for the weights of the portfolio
- y(t) is the observed return of the portfolio
- $u(t) \sim WN(0,I)$ $\quad \varepsilon(t) \sim WN(0,1)$ $\quad u(t) \perp\!\!\!\perp \varepsilon(t)$

- A is the identity matrix.
- B is a diagonal matrix with the variances of the futures.
- C is a time dependent row vector with the returns of the futures at time t.
- D is a scalar equal to the sample standard deviation of the portfolio we are replicating.

- For this approach we have chosen to **reduce the dimensionality** of our problem with feature selection.
- Moreover we have performed **anomaly detection** to reduce our exposure during big market movements.

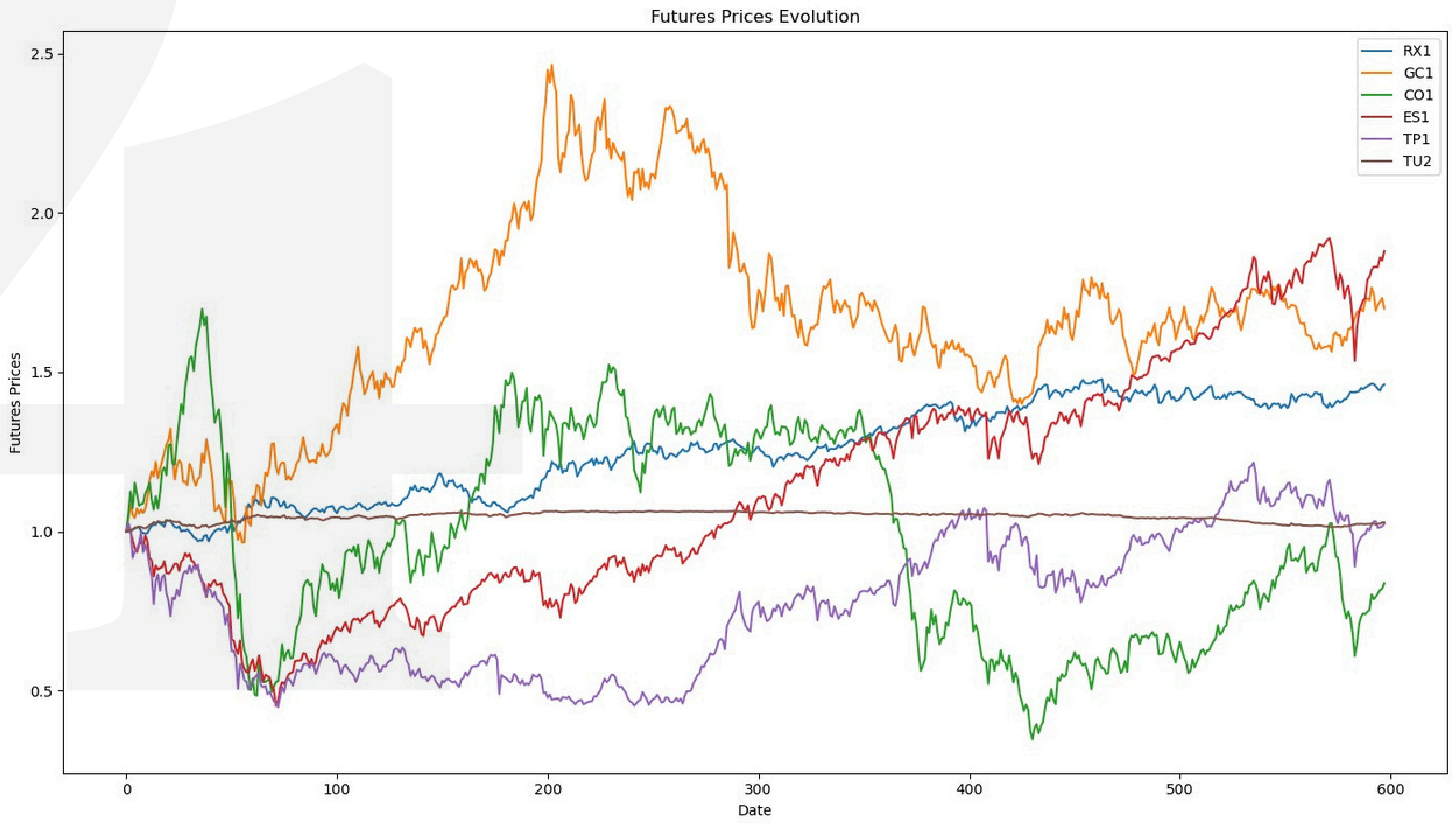
SECOND APPROACH: KALMAN FILTER, FEATURE SELECTION ON INVESTMENT REPLICA DATA



- We have chosen to **eliminate highly correlated features**. Among each subset of correlated features we decided to keep the ones that were more correlated with the target variable. For example, we kept ES1 and dropped VG1 and NQ1.
- The whole procedure of feature selection was also driven by **financial knowledge** on the futures. For example we selected only one of the two USA bonds.
- To confirm the **goodness of our selection** we have finally computed the **VIF** of the remaining features.

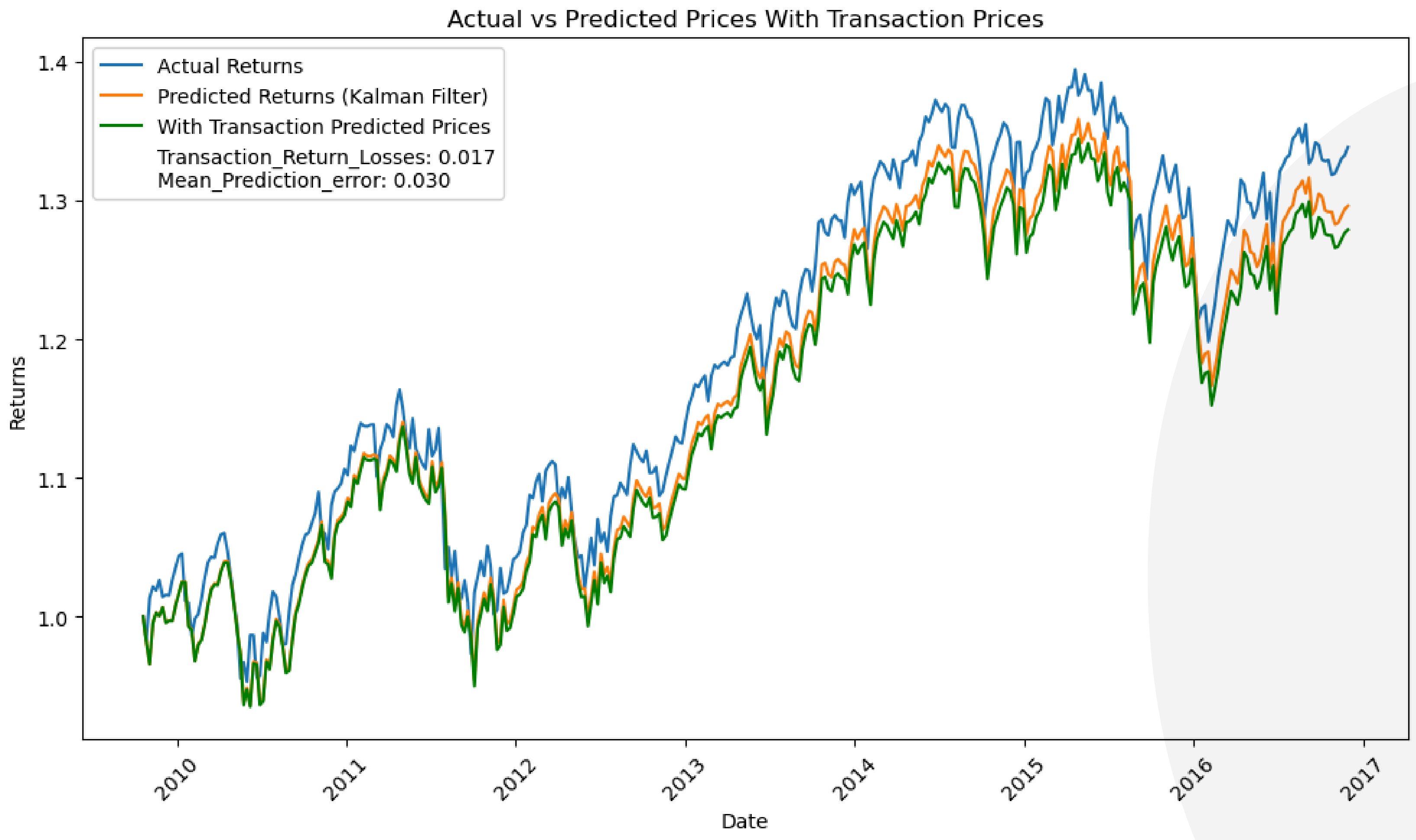
SECOND APPROACH: KALMAN FILTER, FEATURE SELECTION ON INVESTMENT REPLICA DATA

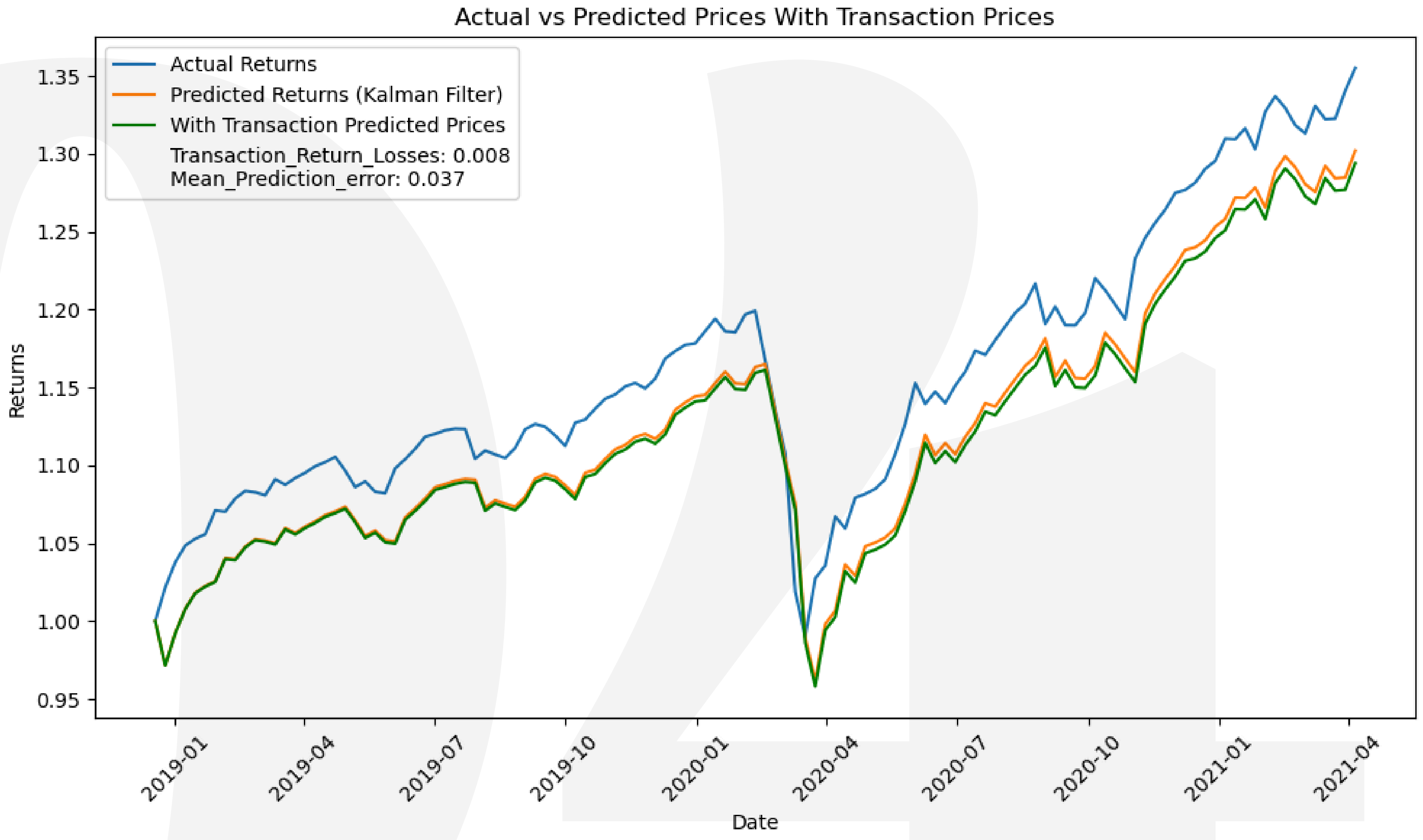
	Description	VIF
RX1	Germany Bond 10y	1.27
GC1	Gold Future	1.20
CO1	Crude Oil Europe	1.34
ES1	S&P500 Future	1.73
TP1	Japan Equity Index	1.56
TU2	USA Bond 2y	1.43



Every selected variable satisfies the «rule of thumb» of VIF < 5, so we have **eliminated** any problem of **collinearity**.

SECOND APPROACH: KALMAN FILTER,VALIDATION AND TEST SET



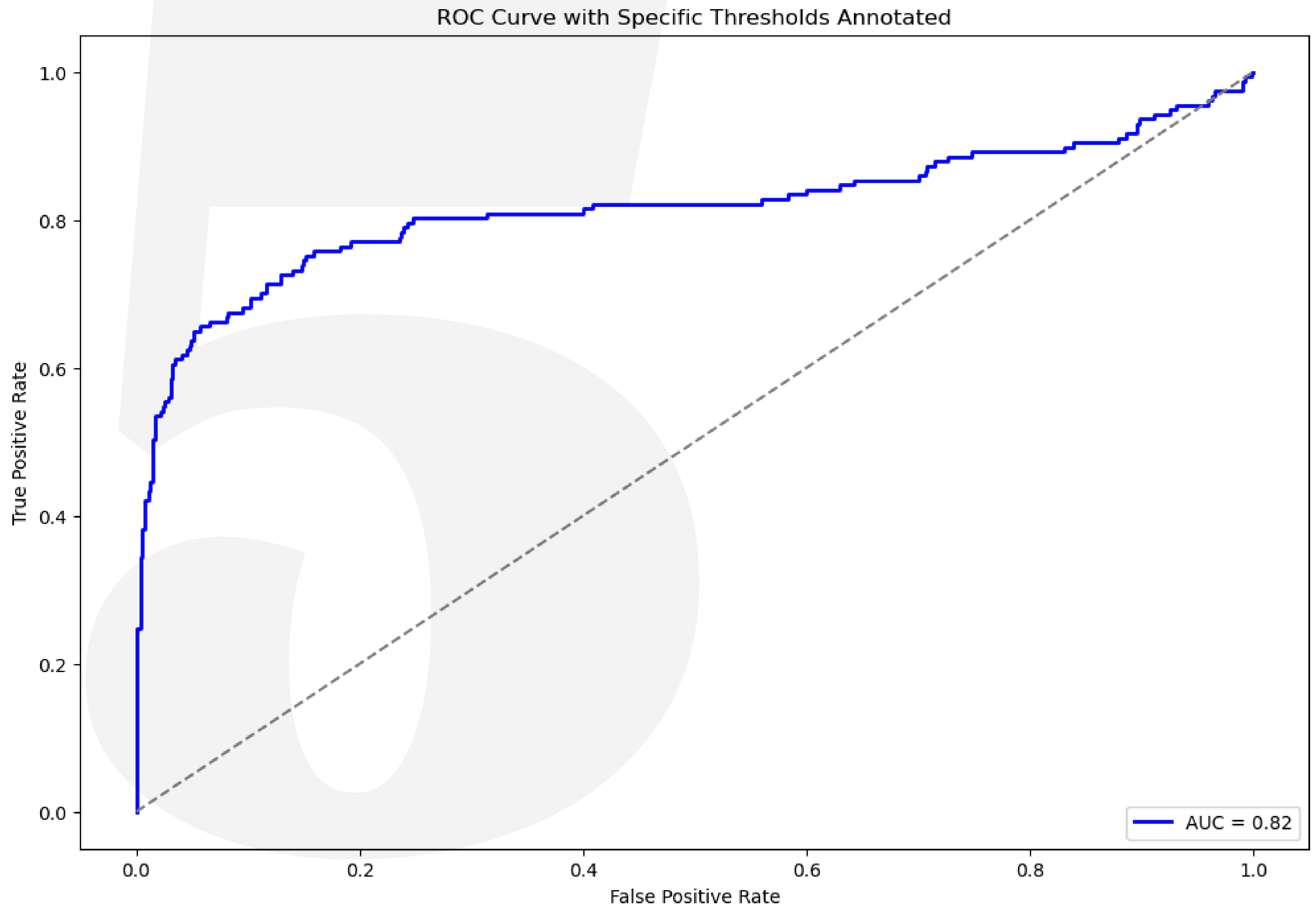


- Again we tried to replicate the target index with a **one step ahed prediction** using a **rolling window** taking into account **transaction costs**. We compute the **MAE** also for this replication.
- We can compare the results obtained with the Lasso regression and the Kalman filter in the table on the right: we can see that the **Kalman filter performs better than the Lasso regression.**

	MAE VALIDATION	MAE TEST
LASSO REGRESSION	0.11	0.064
KALMAN FILTER	0.030	0.037

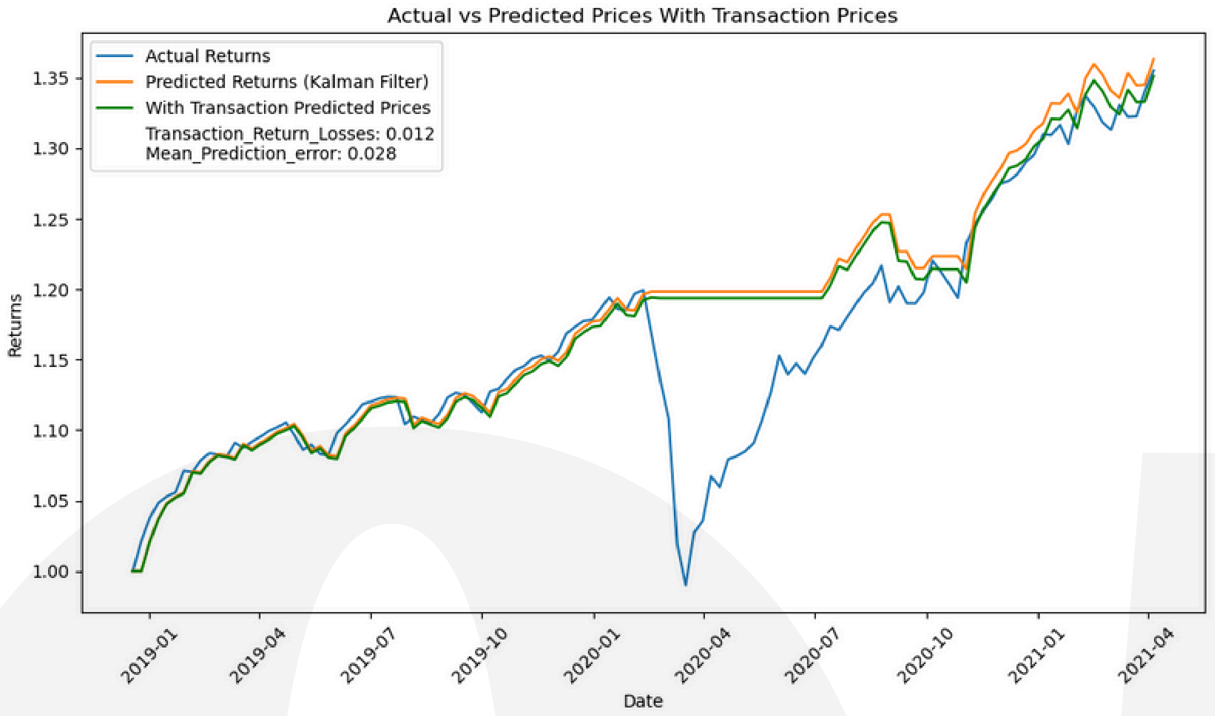
KALMAN FILTER AND ANOMALY DETECTION

- Now we use financial market data to create a neural network classifier to predict future **anomalies** in the market.
- The network that generalizes best on the validation set has one layer and 30 neurons.
- We can choose an **optimal probability threshold** with which we will classify the anomalies: this value is such that we **maximize the positive distance from the target index on the validation set.**
- The idea is that we will use this classification of anomalies to **stop following the target** when there are big market movements and try to slightly outperform the index.
- We will use a parameter (alpha) with which we will **modulate our exposure during market anomalies.**

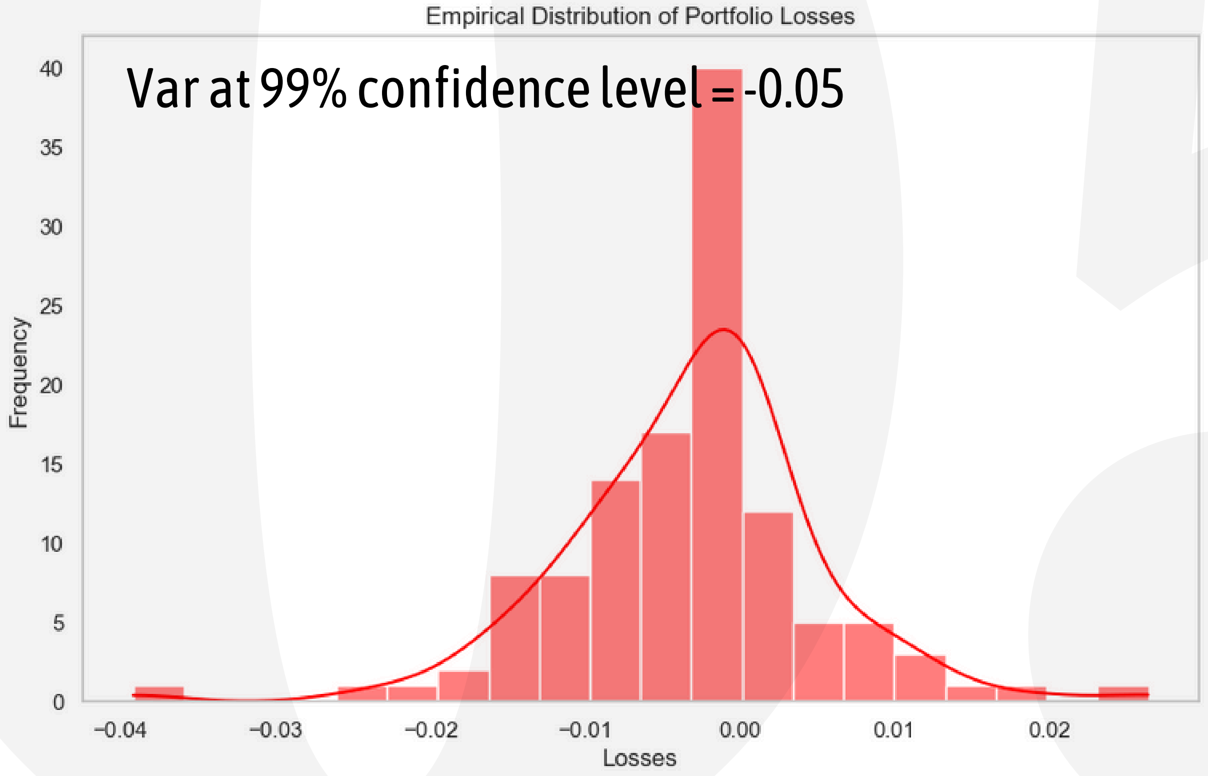


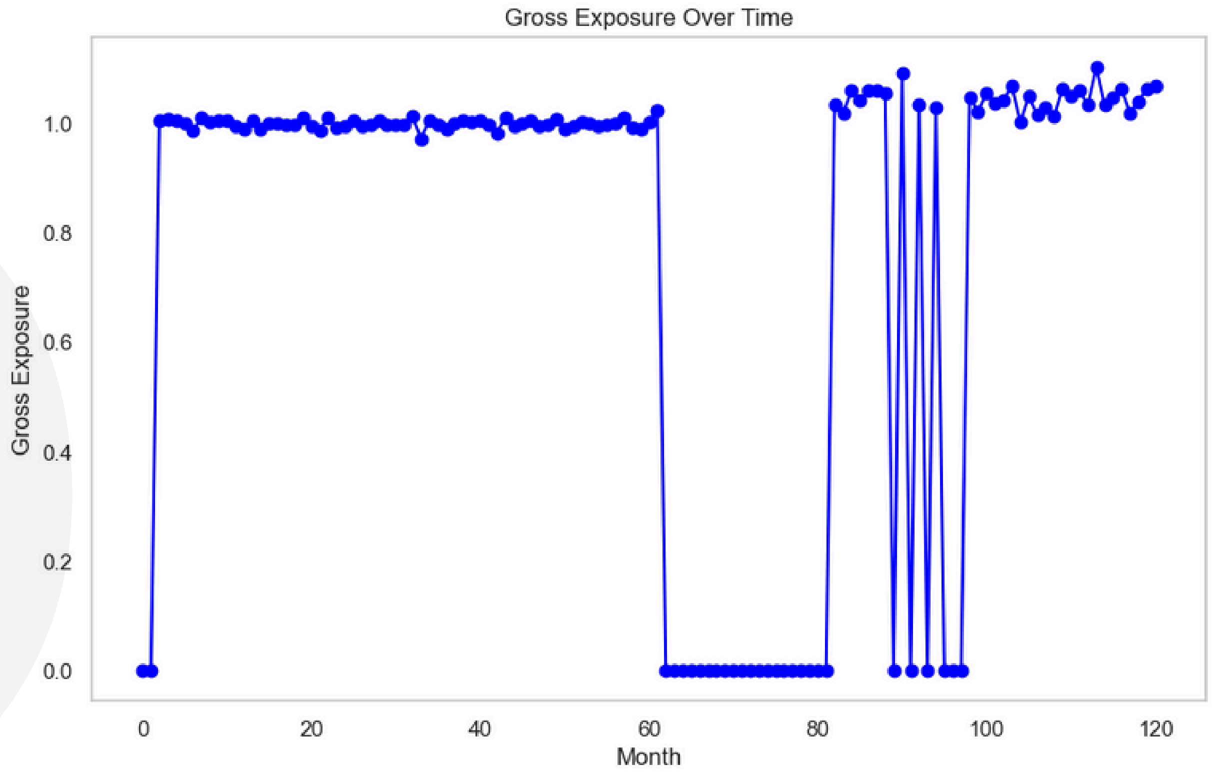
ROC curve and AUC score of the neural network classifier

KALMAN FILTER AND ANOMALY DETECTION: ALPHA = 0

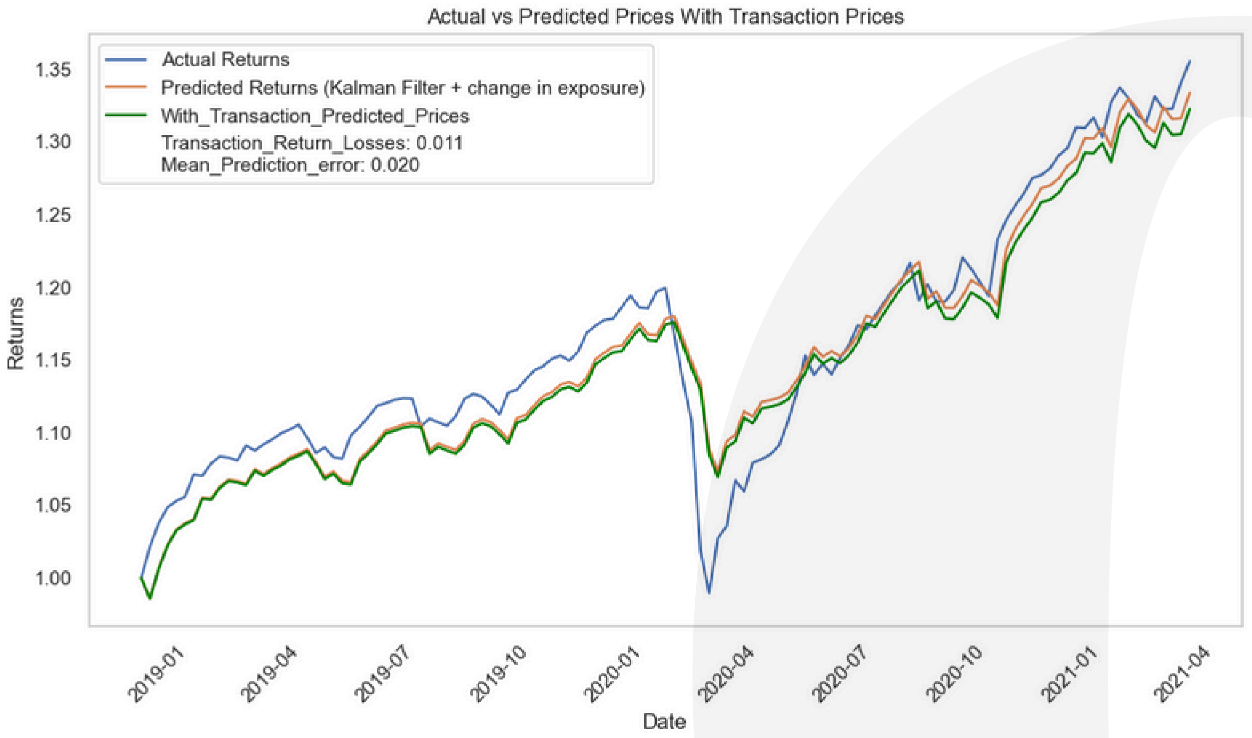


- Now our goal is not just to track the index, but also to **reduce our portfolio volatility.**
- We managed to **avoid the big 2020 dip** as well as other minor market anomalies.
- We see that this value of alpha makes us **stay out of the market for a long period of time**, which is not ideal. During the last period we frequently **jump from zero to one exposure** which makes us lose quite a lot of returns on transaction fees.

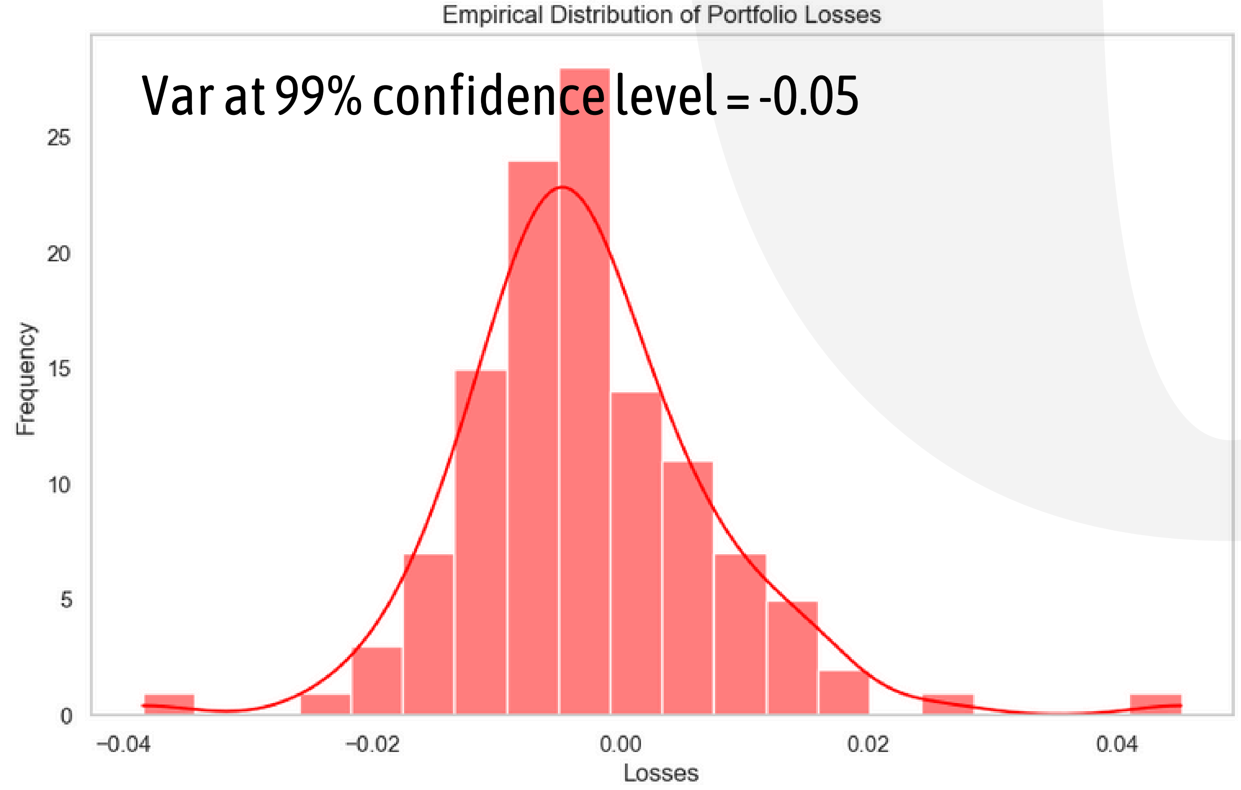


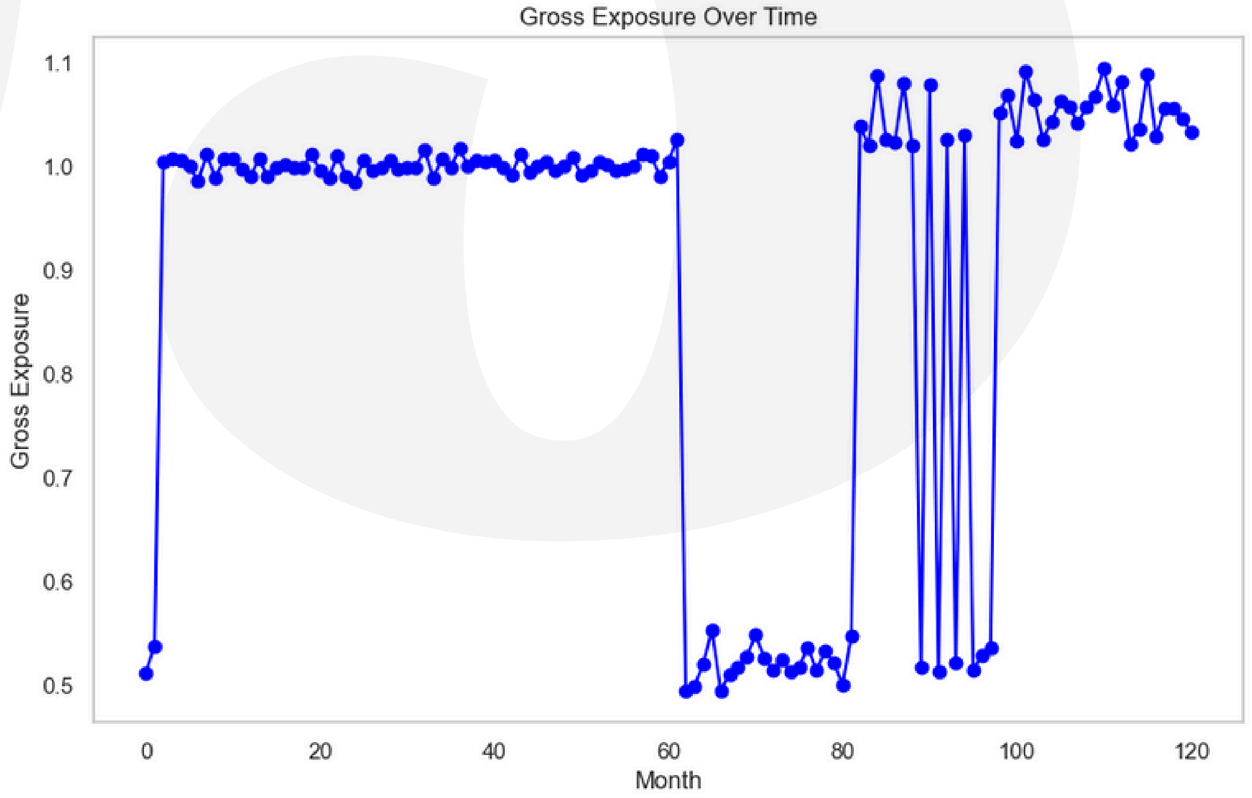


KALMAN FILTER AND ANOMALY DETECTION: ALPHA = 0.5

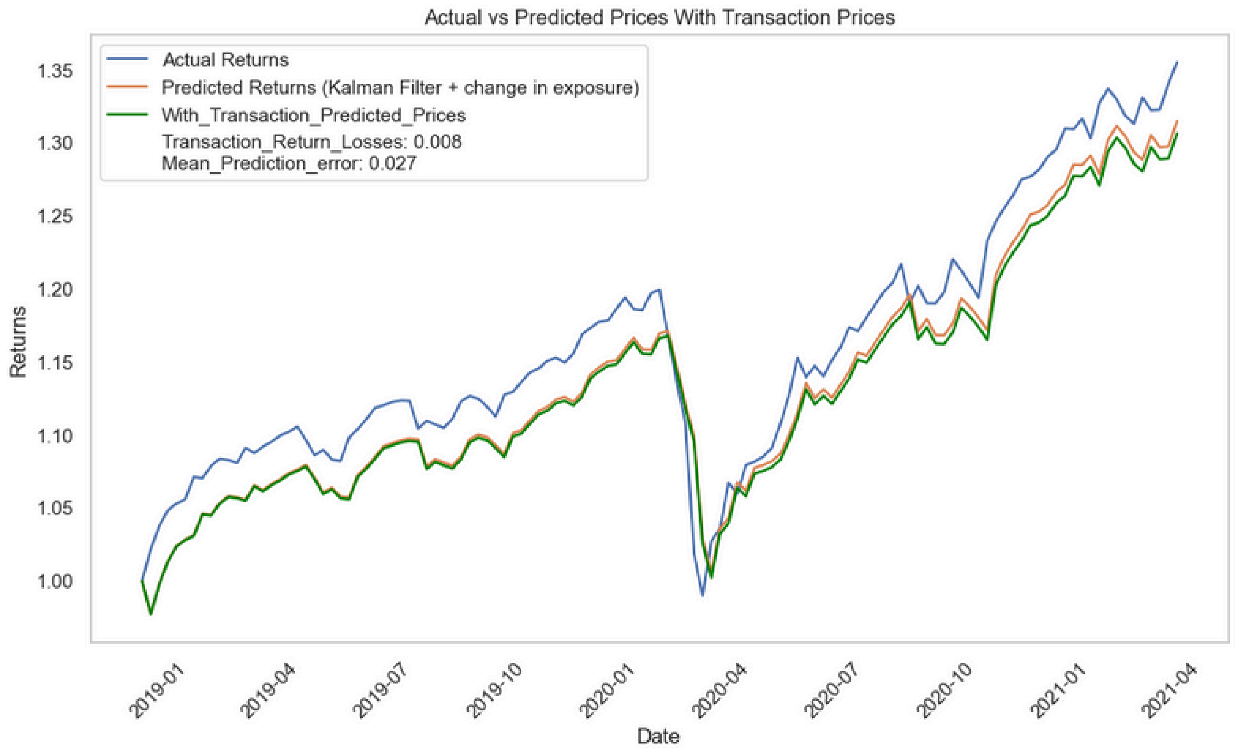


- In this case we follow more the target index and we **accurately reduce our exposure during the 2020**. Our portfolio variance is increased.
- Differently from the previous case we never get to 0 leverage which **reduces our transaction costs** but **lowers our total realized return.**

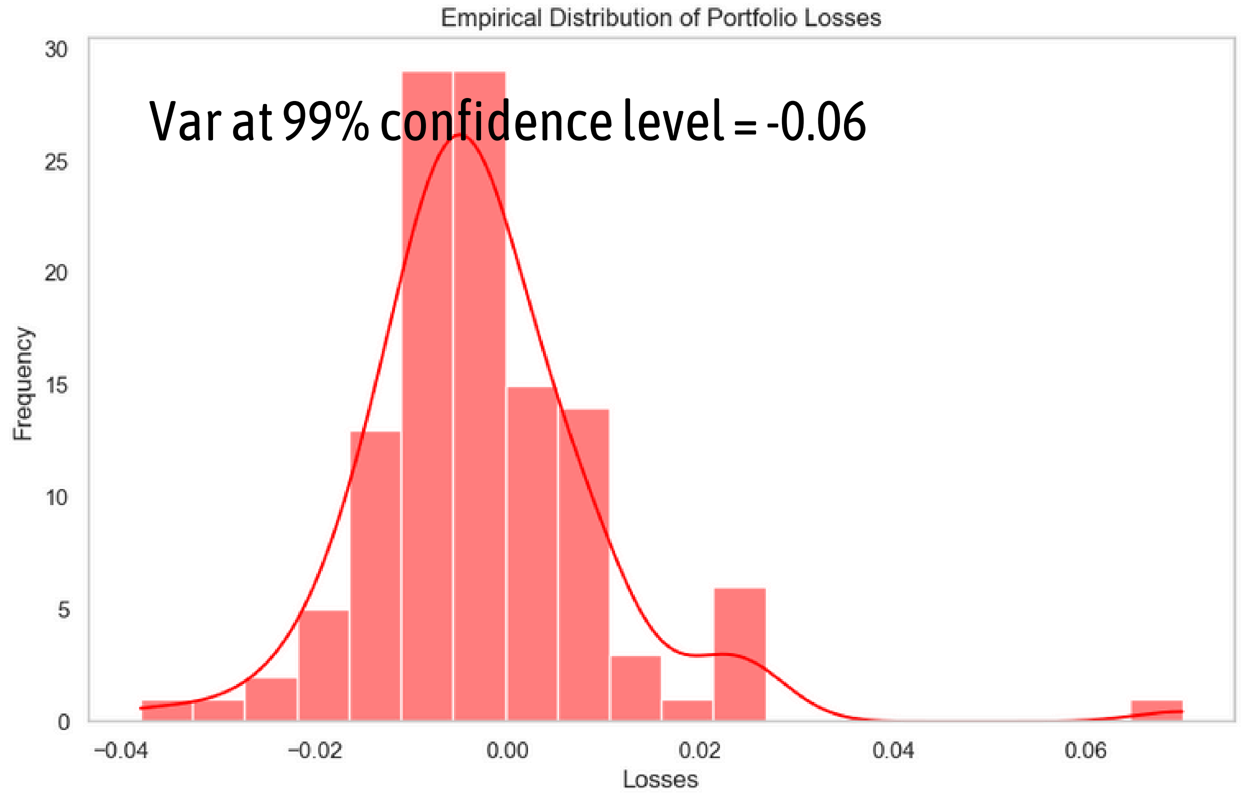


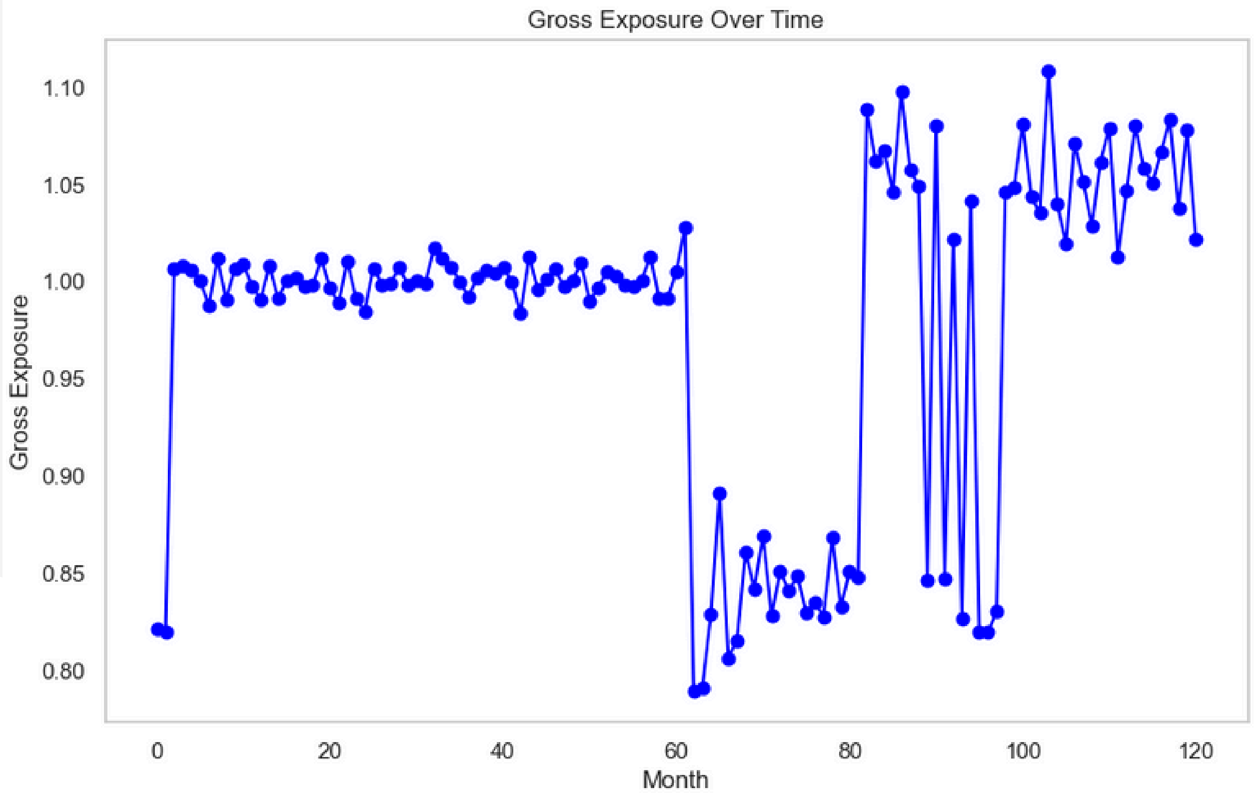


KALMAN FILTER AND ANOMALY DETECTION: ALPHA = 0.8



- With this value of alpha we almost come back to the **starting situation** of tracking perfectly the index
- As expected, the **overall exposure increases**. We would not recommend using such a high value for alpha because it just **increases transaction costs without avoiding any big losses**





KALMAN FILTER AND ANOMALY DETECTION, RESULTS

- The Kalman filter manages to **replicate the target index** very accurately **without having to frequently change the portfolio weights** and therefore losing too much returns on transaction fees.
- We can accurately **predict future market anomalies** using a neural network classifier and adjusting our market exposition to have a **more steady portfolio with less volatility.**
- We believe that the choice of the exposure parameter alpha is business oriented and can be adjusted to meet client's needs.