Representing the World

Prof. Fabio Pellacini




[WikimediaCommons]

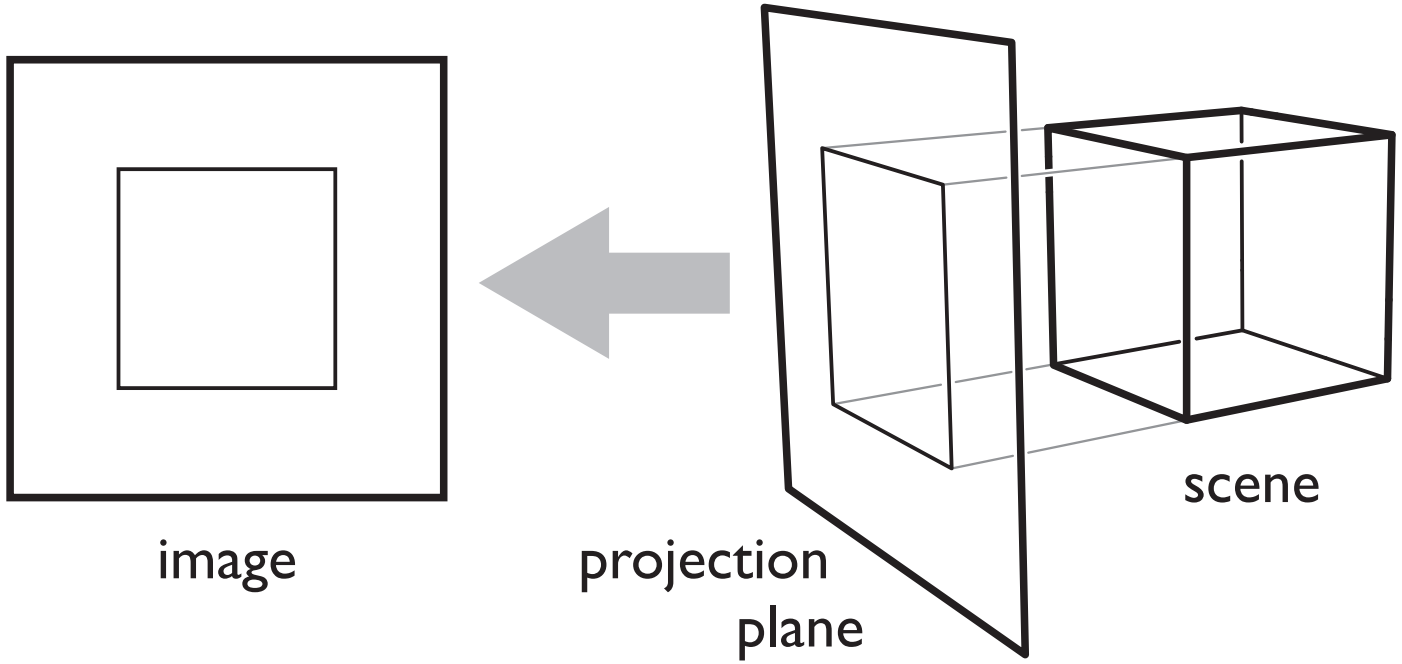
Scene elements

- To make an image, we need to represent
  - cameras
  - shapes
  - materials
- To make a movie, we need to also represent
  - animation (later)

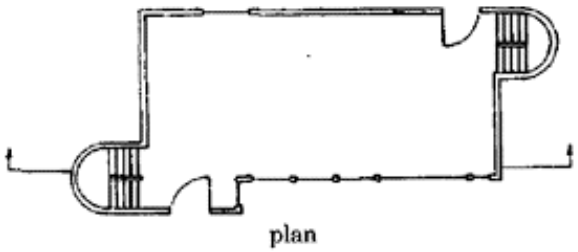
Representing Cameras

Orthographic Projections

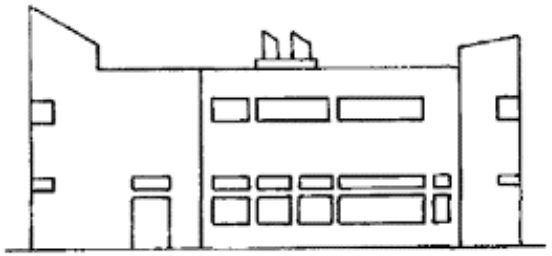
- To render an image of a 3D scene, we project it onto a plane
- Simplest type is parallel projection
- Mostly used in modeling interfaces



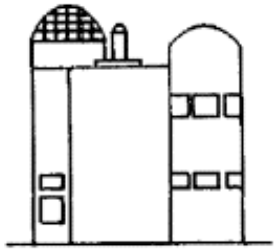
Orthographic Projection



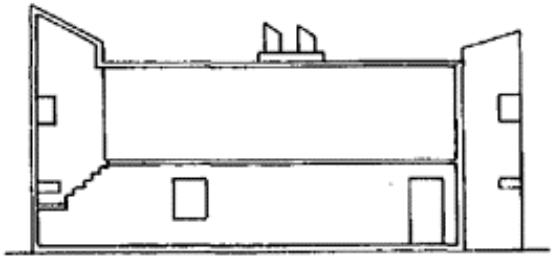
plan



front elevation

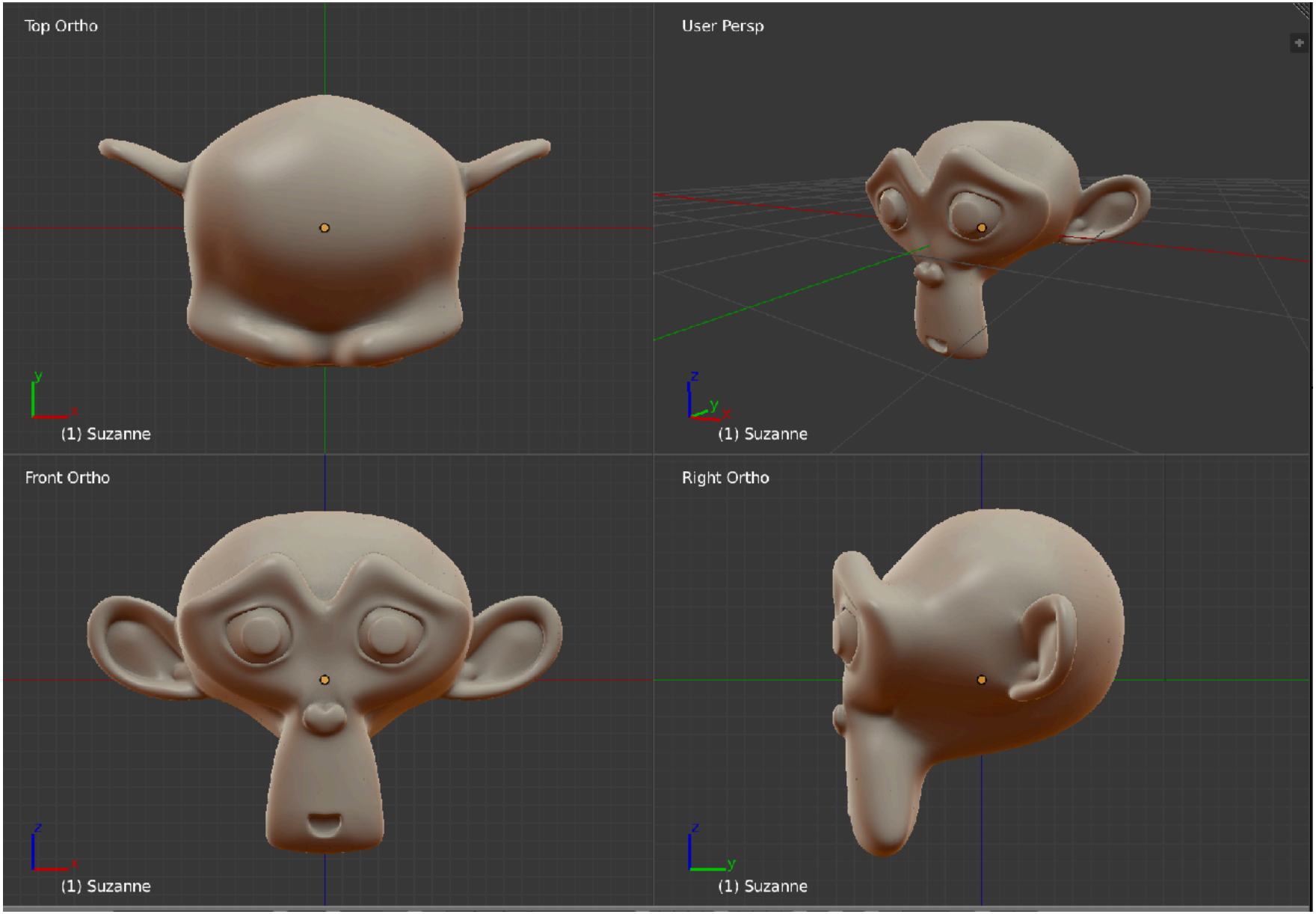


right elevation



section

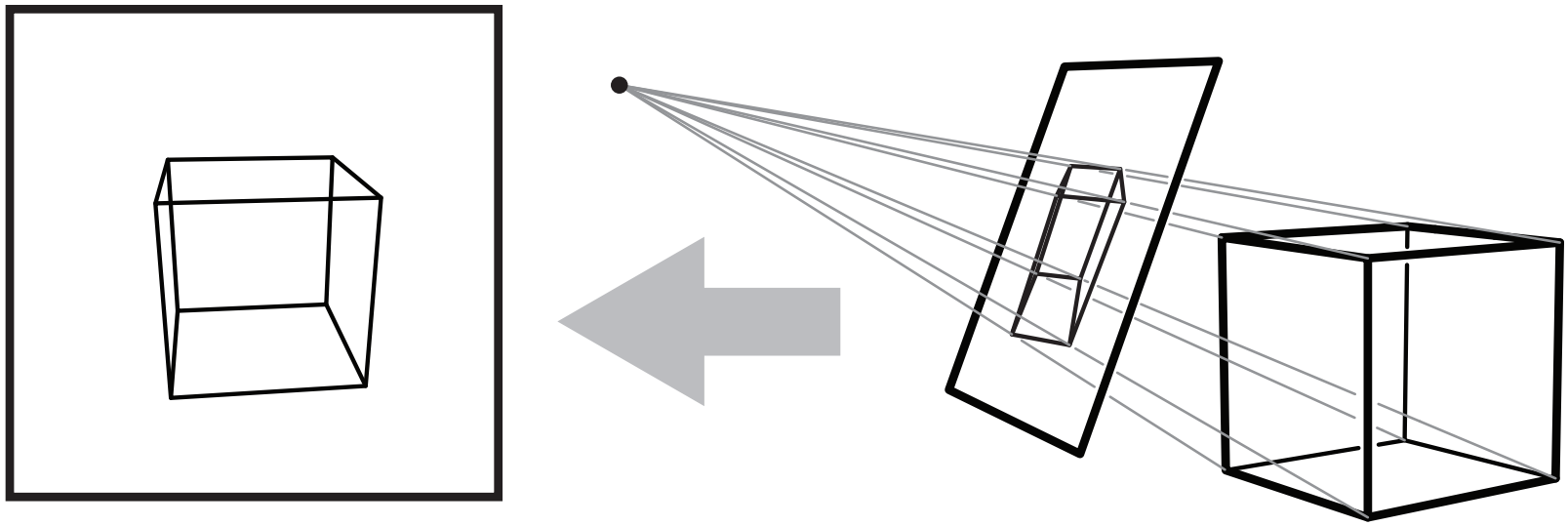
[Carlbom & Paciorek 78]

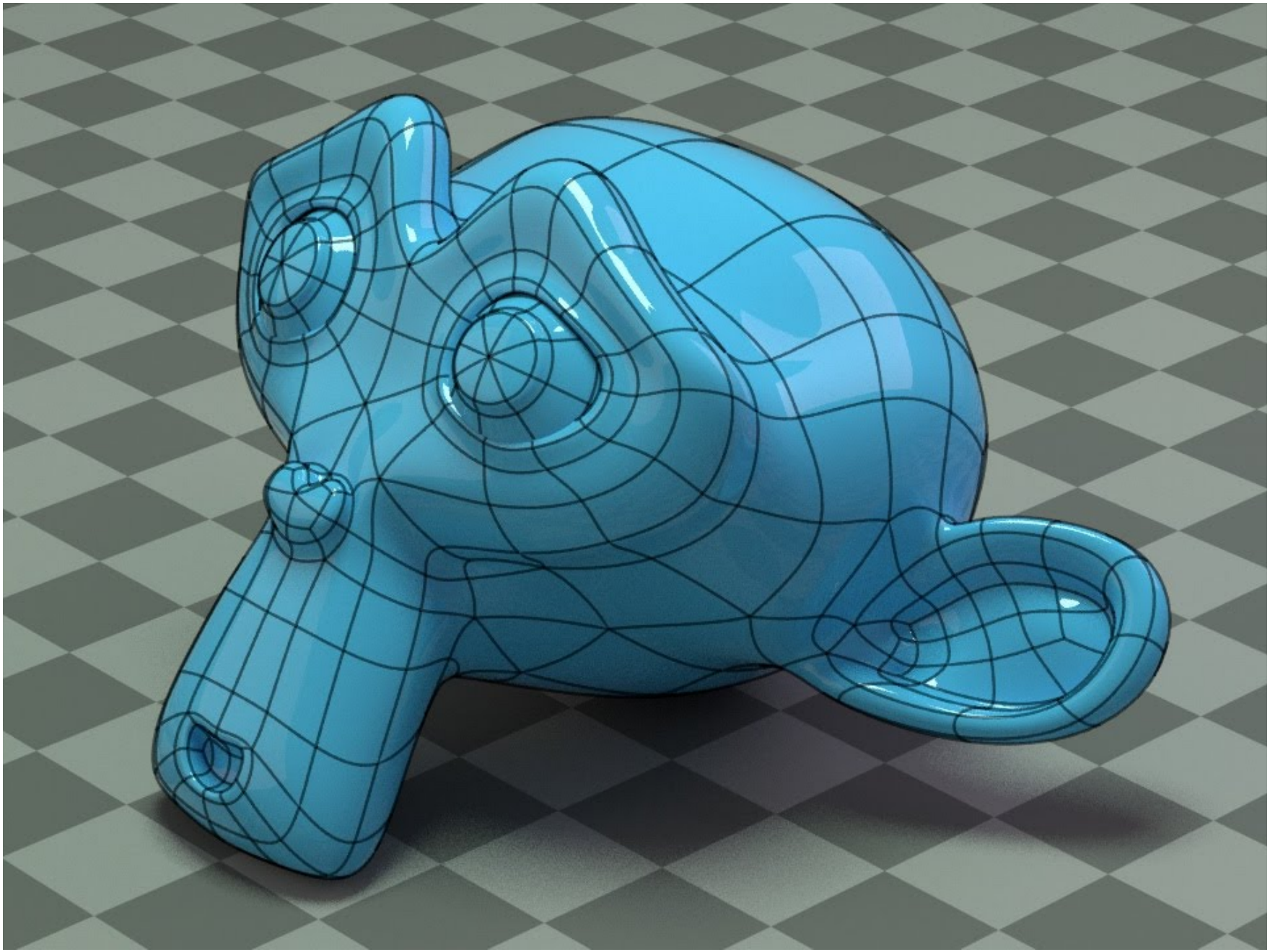


[Blender]

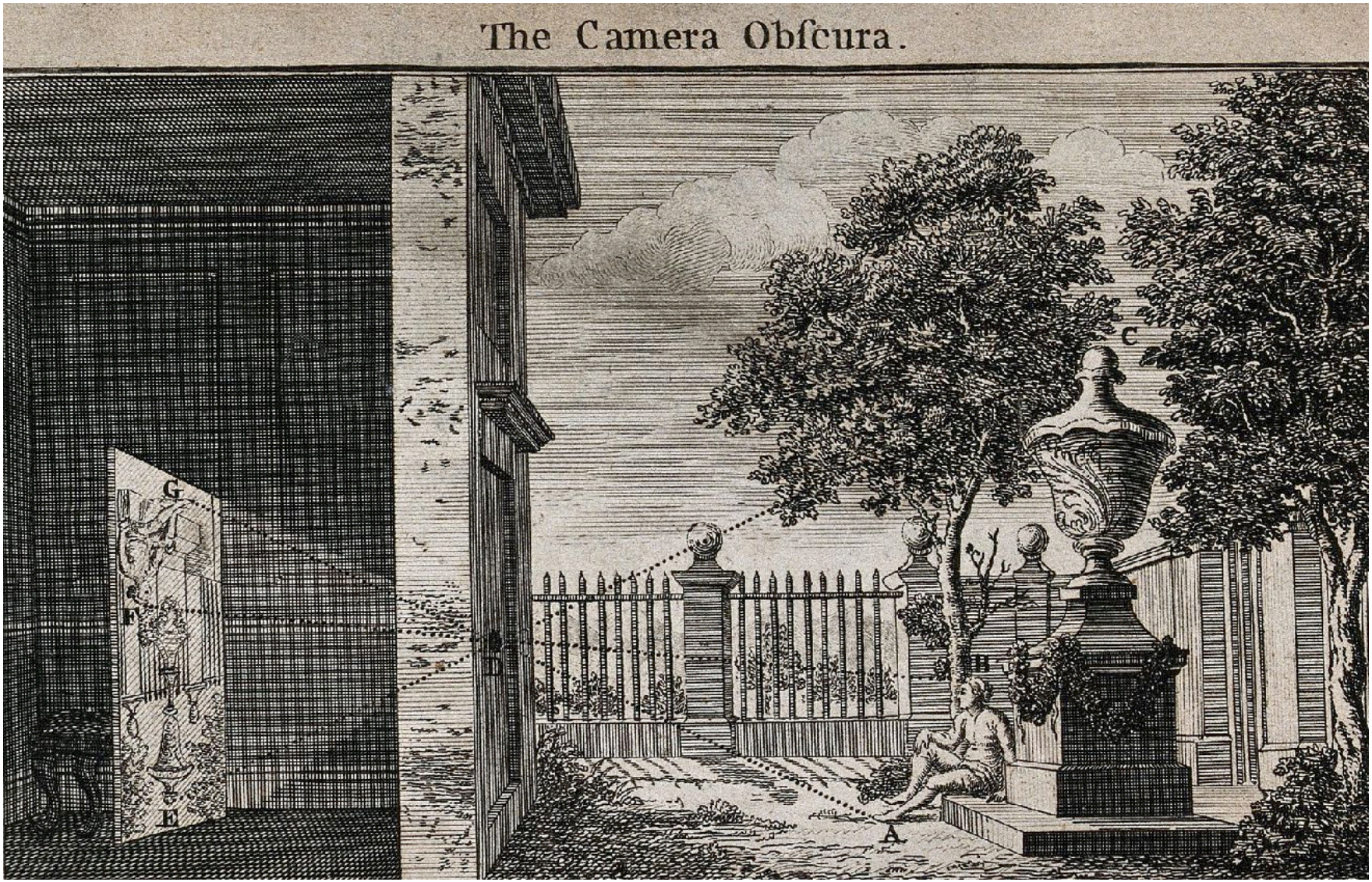
Perspective Projections

- Orthographic projection do not emulate a real camera
- Most common projection type is perspective projection
- Simulates a real pinhole camera

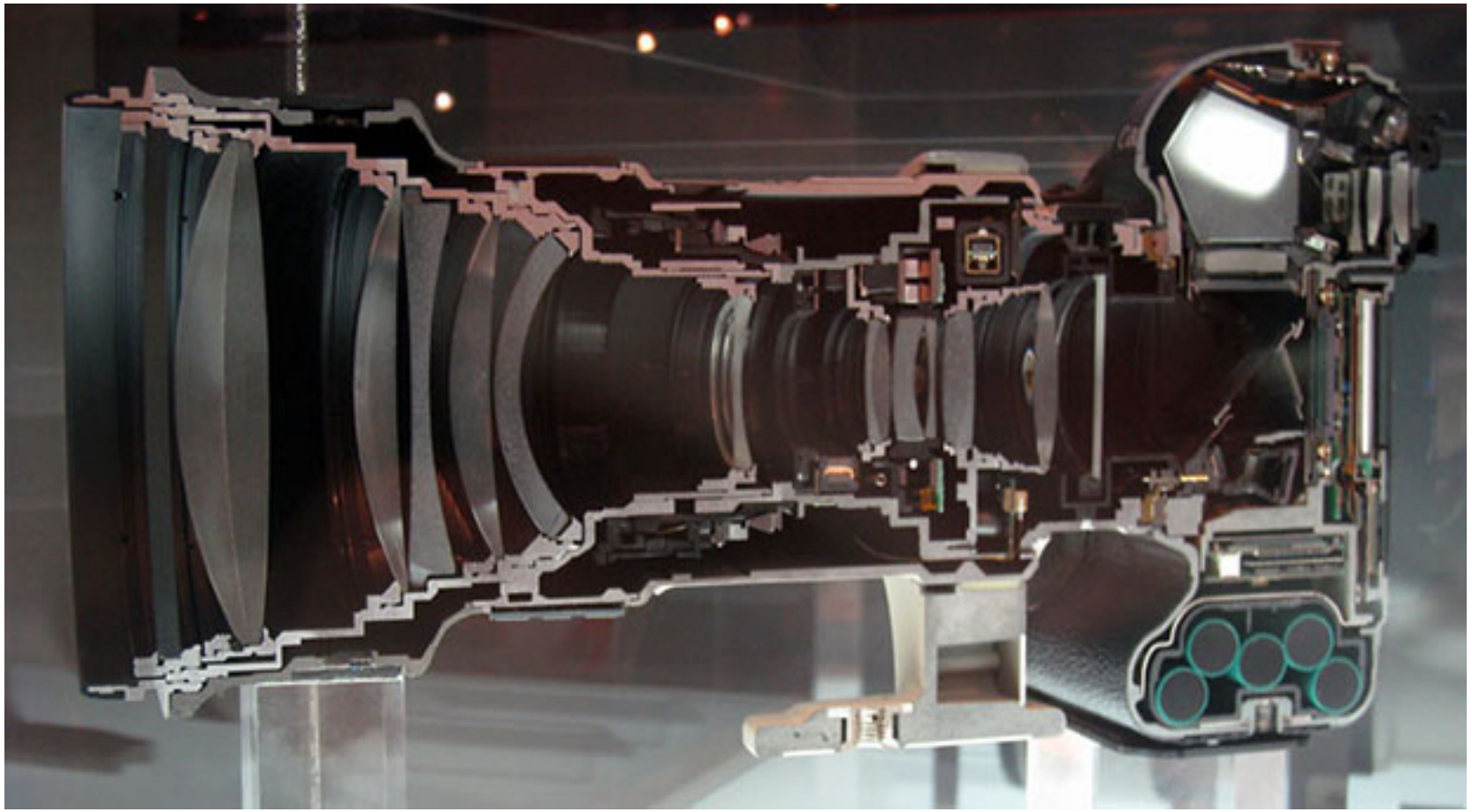




[YouTueb - Blender Nerd]



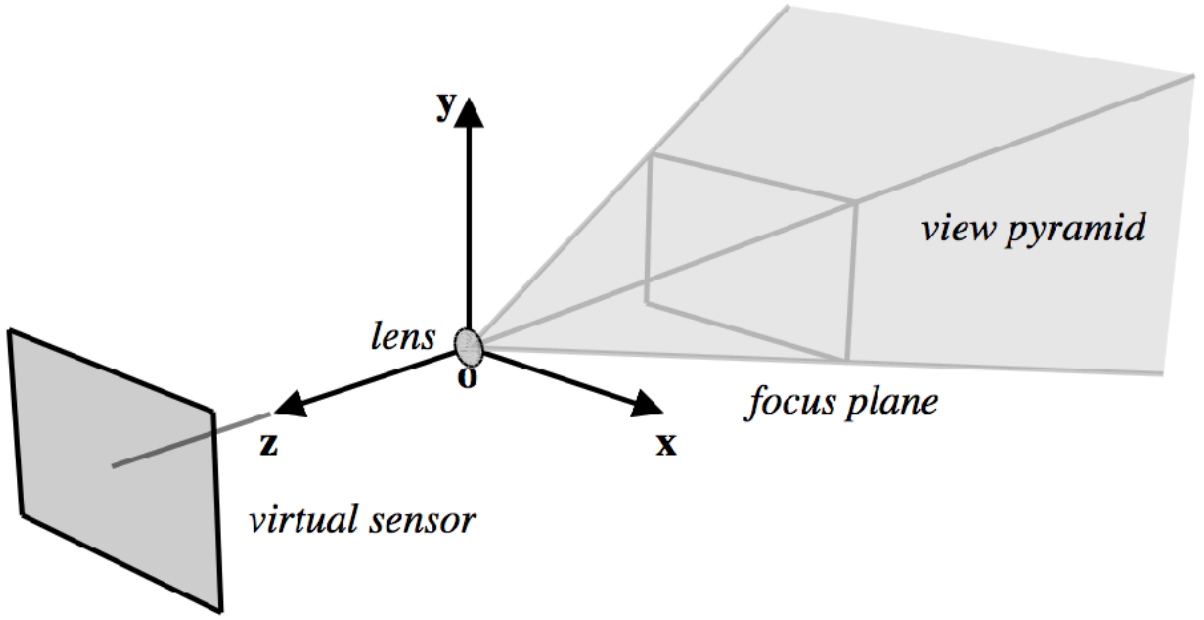
[WikipediaCommons]



[PetaPixel.com]

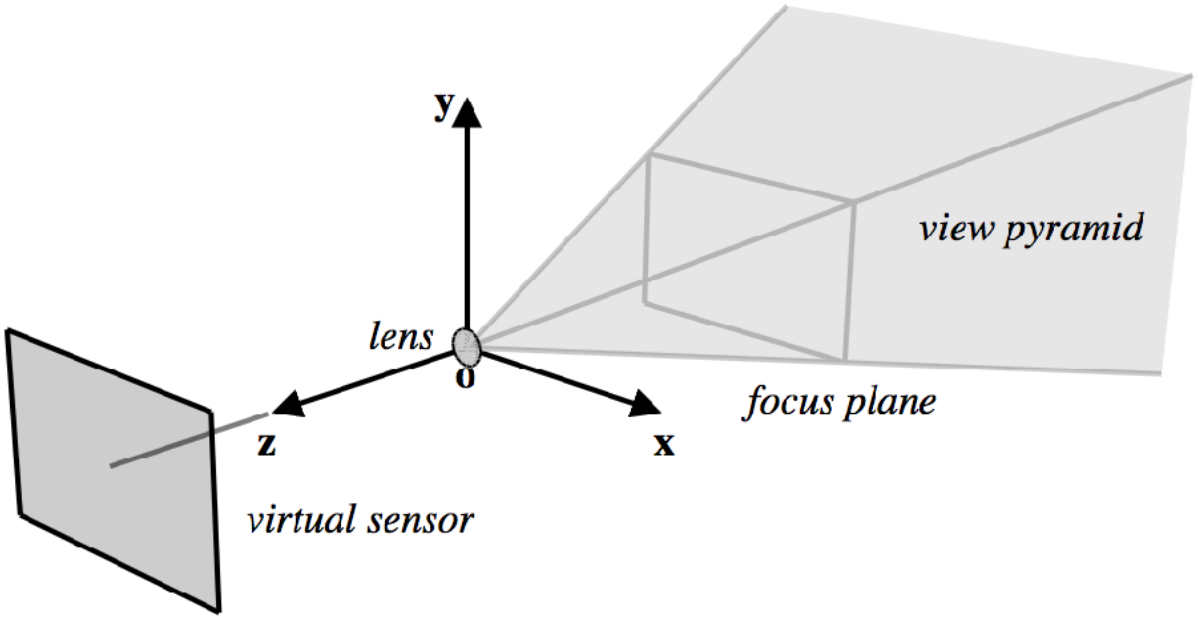
Camera Representation

- The camera position and orientation are defined by a frame
- The camera projection is defined by its lens field of view and the distance to the camera sensor
- The camera focus is defined by the lens aperture and the focus distance



Camera Representation

```
struct camera {
  frame3f frame;
  float lens, aspect, film;
  float aperture, focus;
};
```



Defining Camera Frame

- Providing the frame directly is unintuitive
- Calculate frame camera position and view direction
  - camera position **o,** where the camera is, set as frame origin
  - view direction **-z**: which way the camera is looking
  - up vector **y**: how the camera is oriented, generally world up
- Since view direction and up vector are not orthogonal, use frame building code shown before

Representing Shapes

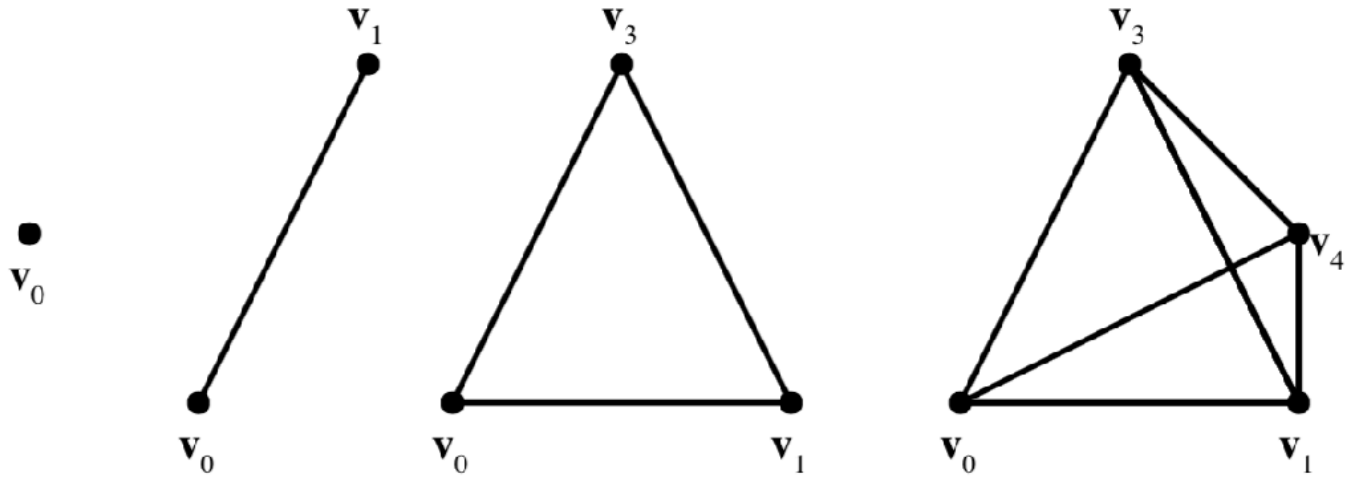
Different Representations

- Different representations are sometimes needed
- Optimize for different shape "types"
- Optimize for different algorithms, e.g. rendering vs simulation
- Support intuitive and efficient editing by artists

- But this implies significant software complexity
  - cannot mix-and-match algorithms
  - hard to exchange shapes
- Trend toward using a fewer representations

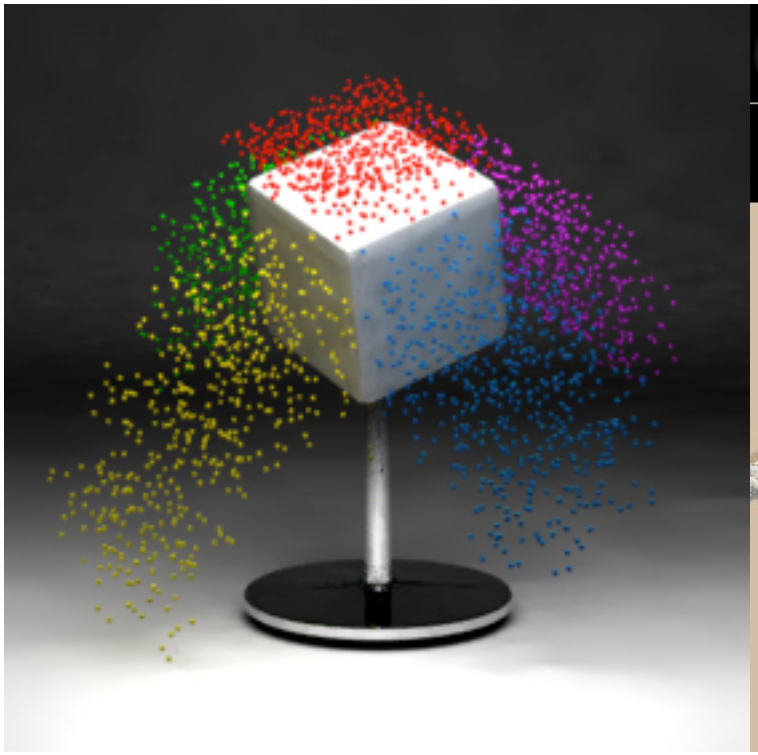
Points, Lines and Triangles

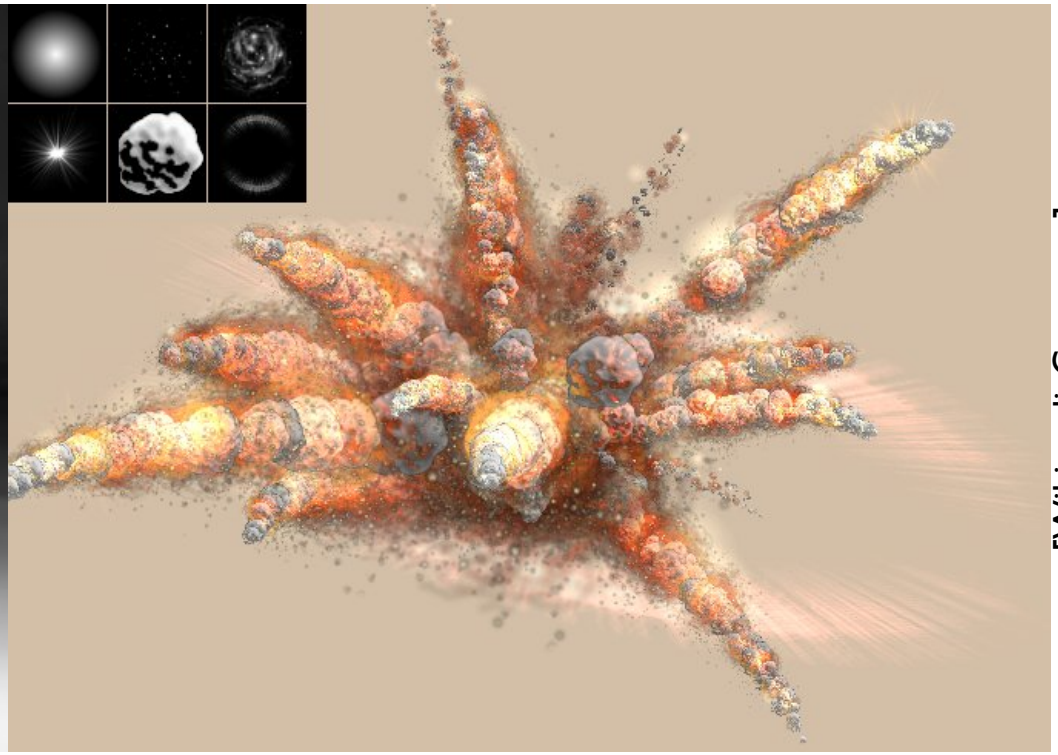
- For many algorithms, we can describe the world as collections of points, lines and triangles
- For simulation, it is sometimes helpful to use tetrahedra to represent the objects' volumes



Points

- Defined by one vertex
- Small enough that their properties do not vary over their "surface"
- Used for small particles (like ember, dust, etc) and simulation





[WikimediaCommons]

Line segments

- Defined by two vertices
- Properties vary along the segment
- Used to represent hair and 2D outlines

[Blender]

Line segments

- Line tangent and length defined as

$$\mathbf{t} = \frac{\mathbf{v}_1 - \mathbf{v}_0}{|\mathbf{v}_1 - \mathbf{v}_0|}$$

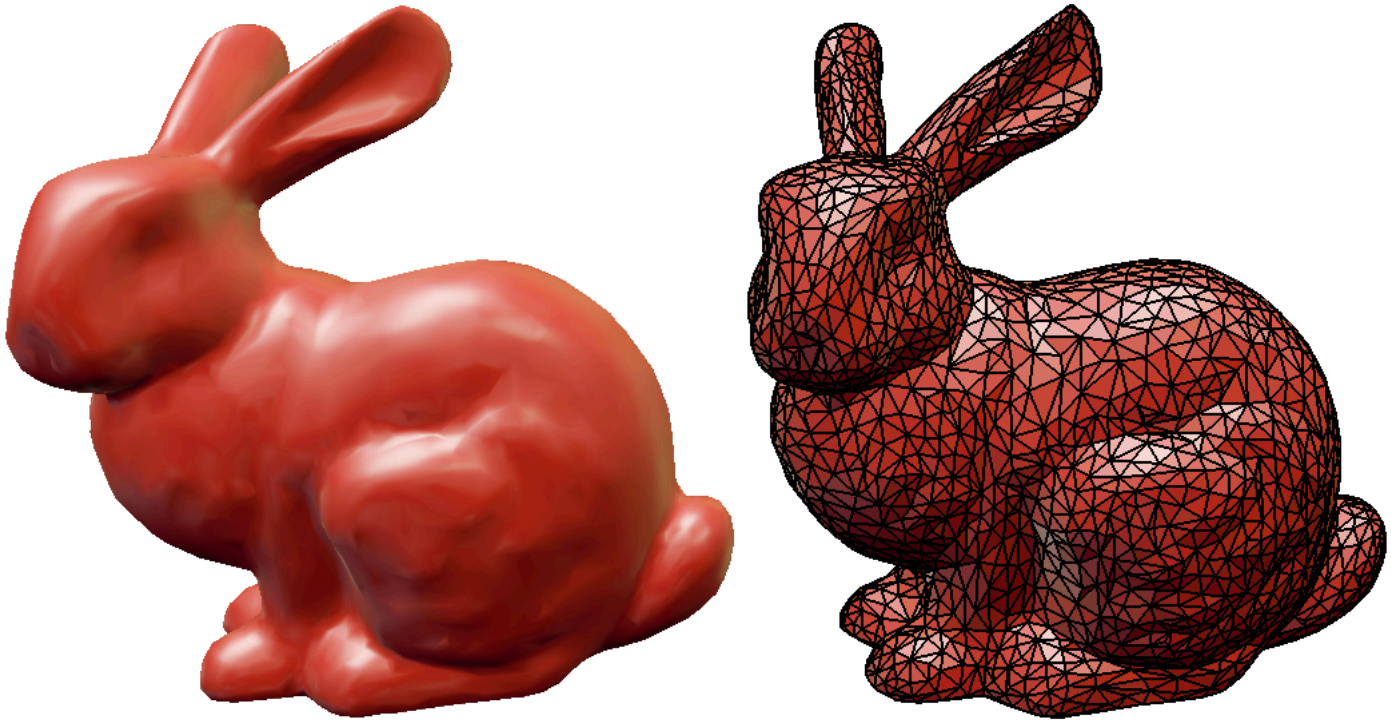
$$l = |\mathbf{v}_1 - \mathbf{v}_0|$$

- Points on a segment can be written as linear combination between the vertices

$$\mathbf{p}(t) = (1-t)\mathbf{v}_0 + t\mathbf{v}_1 \quad t \in [0,1]$$

Triangles

- Defined by three vertices
- Flat shape contained in the plane passing through those vertices
- Used to represent surfaces

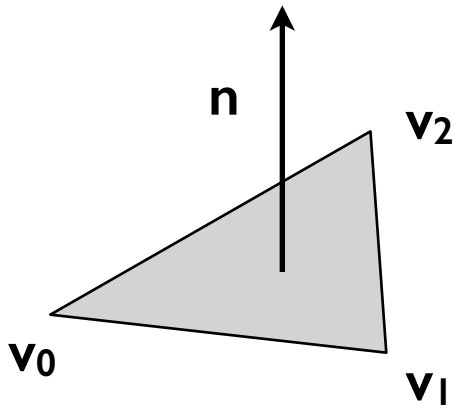


Triangles

- Vector normal to the plane is the triangle's normal, area computed from cross product formulation
- Convention on orientation: vertices are counter-clockwise as seen from "front", surface normal points"outward"

$$\mathbf{n} = \frac{(\mathbf{v}_1 - \mathbf{v}_0) \times (\mathbf{v}_2 - \mathbf{v}_0)}{|(\mathbf{v}_1 - \mathbf{v}_0) \times (\mathbf{v}_2 - \mathbf{v}_0)|}$$

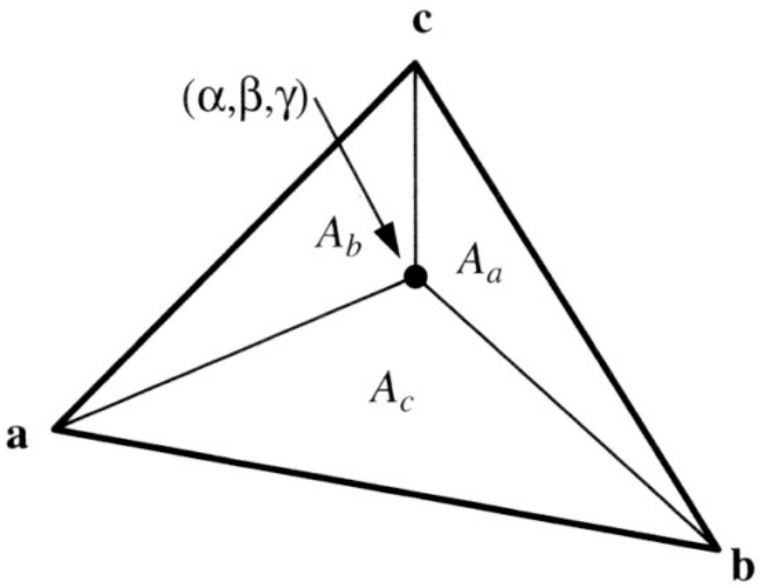
$$A = \frac{|(\mathbf{v}_1 - \mathbf{v}_0) \times (\mathbf{v}_2 - \mathbf{v}_0)|}{2}$$



Barycentric coordinates

- Points on a triangles can be written as linear combination of the vertices — the weights are called *barycentric coordinates*

$$\mathbf{p} = w_0\mathbf{v}_0 + w_1\mathbf{v}_1 + w_2\mathbf{v}_2$$
$$w_0 + w_1 + w_2 = 1$$
$$w_0 \geq 0 \quad w_1 \geq 0 \quad w_2 \geq 0$$



[Shirley 2000]

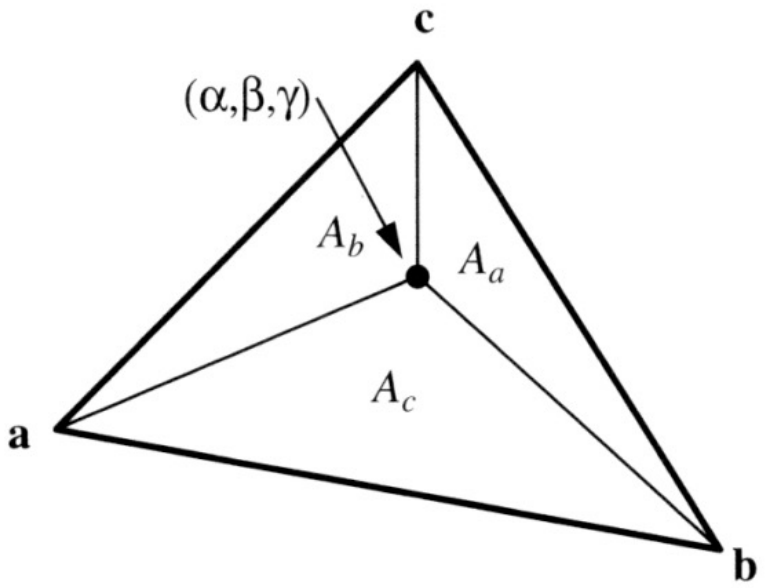
Barycentric coordinates

- Algebraic view: linear combination

$$w_0 = 1 - w_1 - w_2 \Rightarrow$$
$$\mathbf{p}(w_1, w_2) = \mathbf{v}_0 + w_1(\mathbf{v}_1 - \mathbf{v}_0) + w_2(\mathbf{v}_2 - \mathbf{v}_0)$$

- Geometric view: relative areas

$$w_0 = A_0/A \quad w_1 = A_1/A \quad w_2 = A_2/A$$

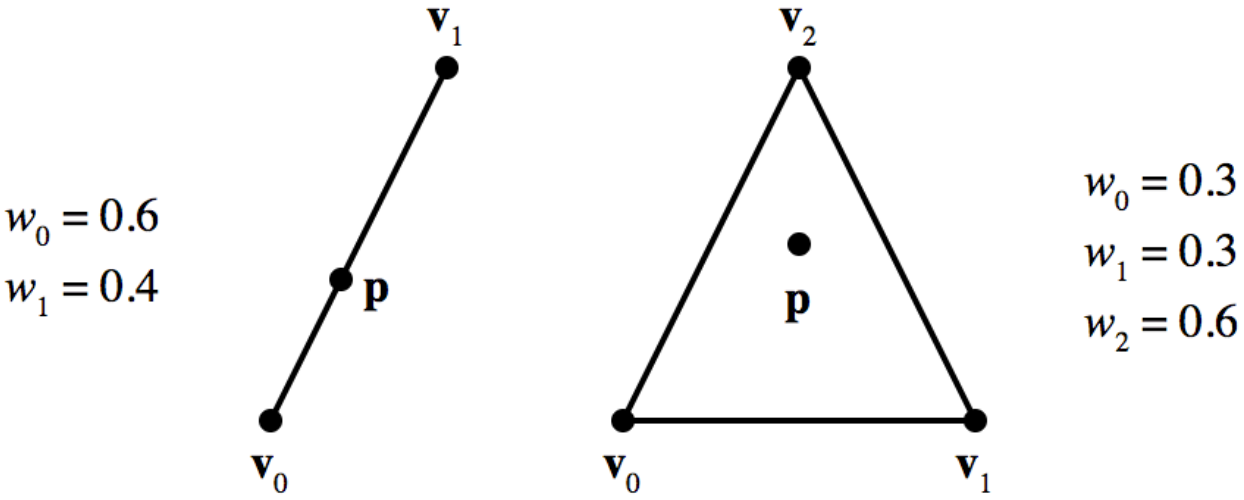


[Shirley 2000]

Barycentric coordinates

- General concept for points, lines, triangles, tetrahedra
- Can write points as linear combination of vertices

$$\mathbf{p} = \sum_{i=0}^{n-1} w_i \mathbf{v}_i \qquad \sum_{I=0}^{n-1} w_i = 1 \qquad w_i \geq 0$$

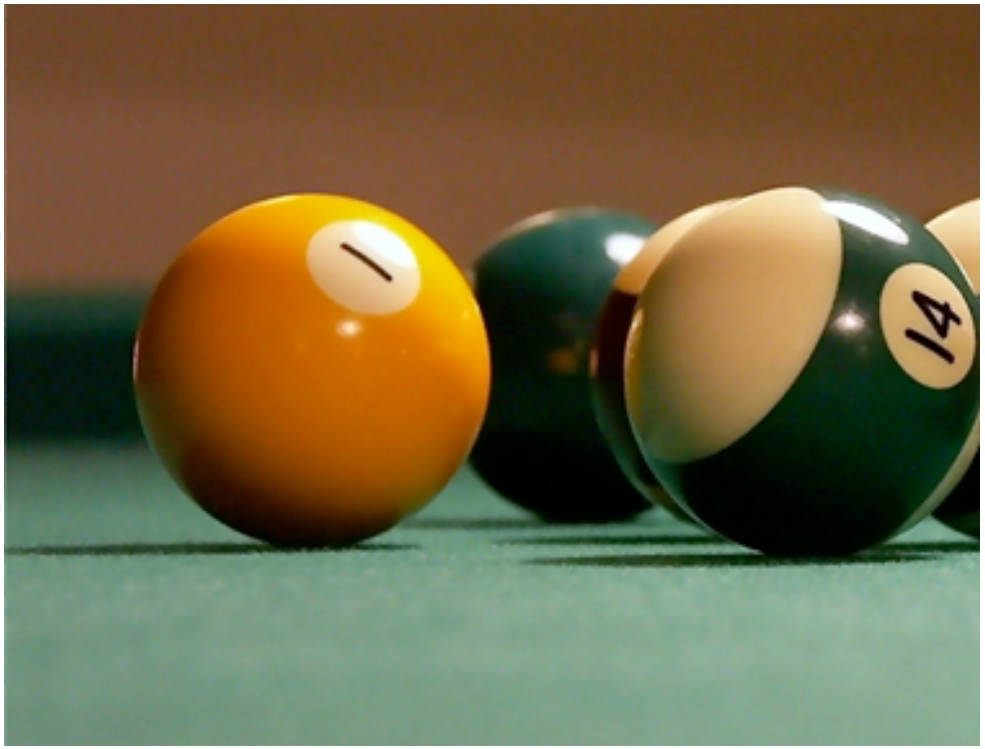


Triangle Meshes

Triangle meshes

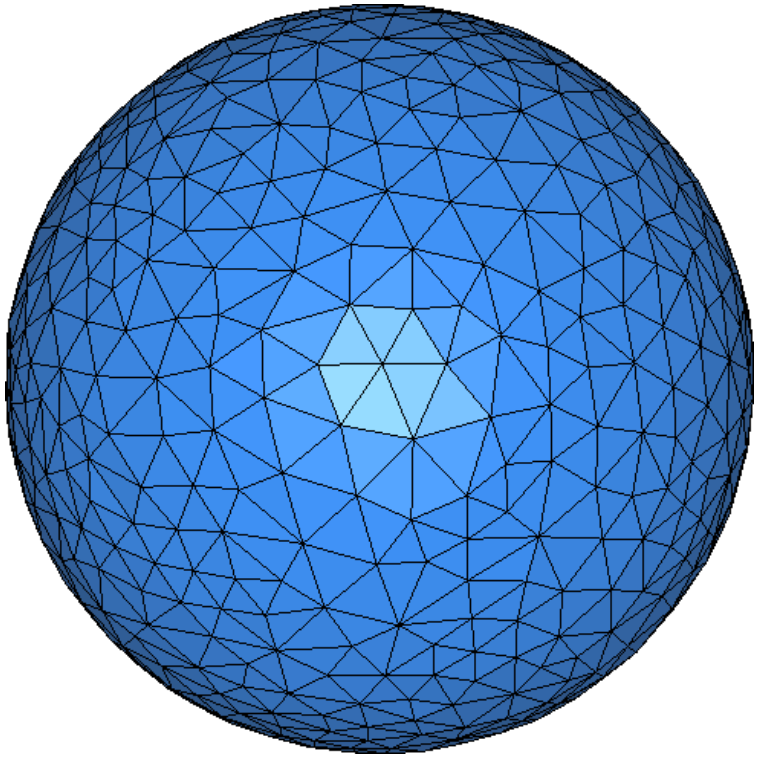
- A set triangles in 3D space connected together to form a surface
- Most common shape used in 3D graphics applications
- Geometrically, a mesh is a piecewise planar surface: it is planar except at the edges where triangles join
- Often, it's a piecewise planar approximation of a smooth surface
  - in this case the creases between triangles are artifacts—we don't want to see them

Approximate geometry



[Andrzej Barabasz]

spheres

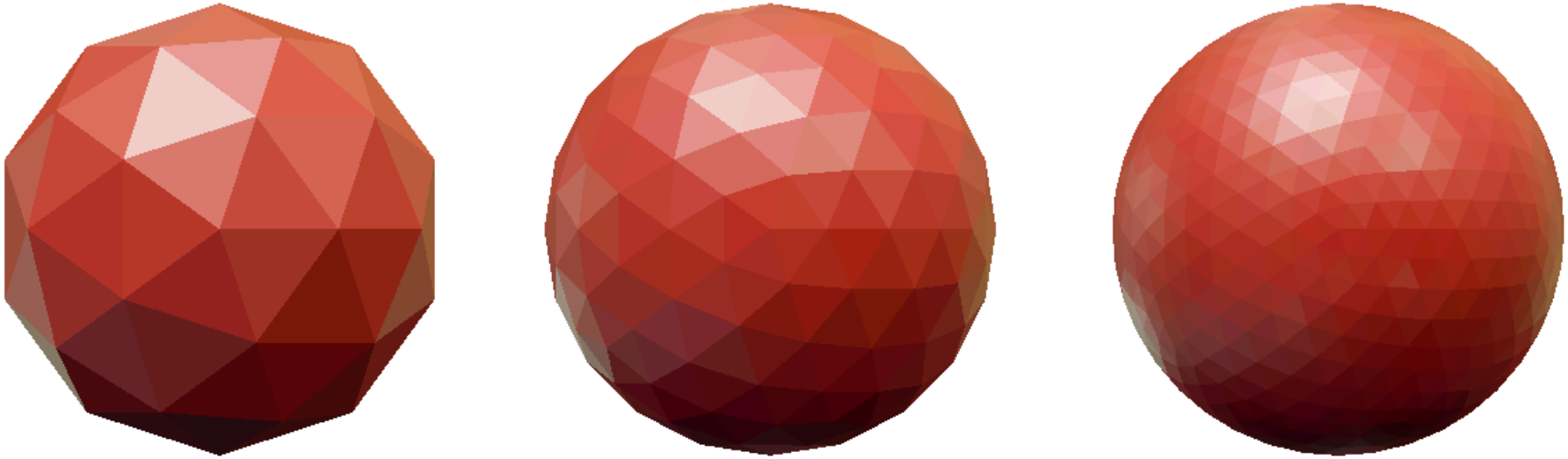


[Rineau & Yvinec CGAL manual]

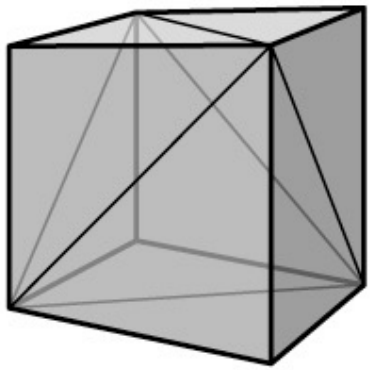
approximate sphere

Approximate geometry

- More triangles, better approximation



A small triangle mesh



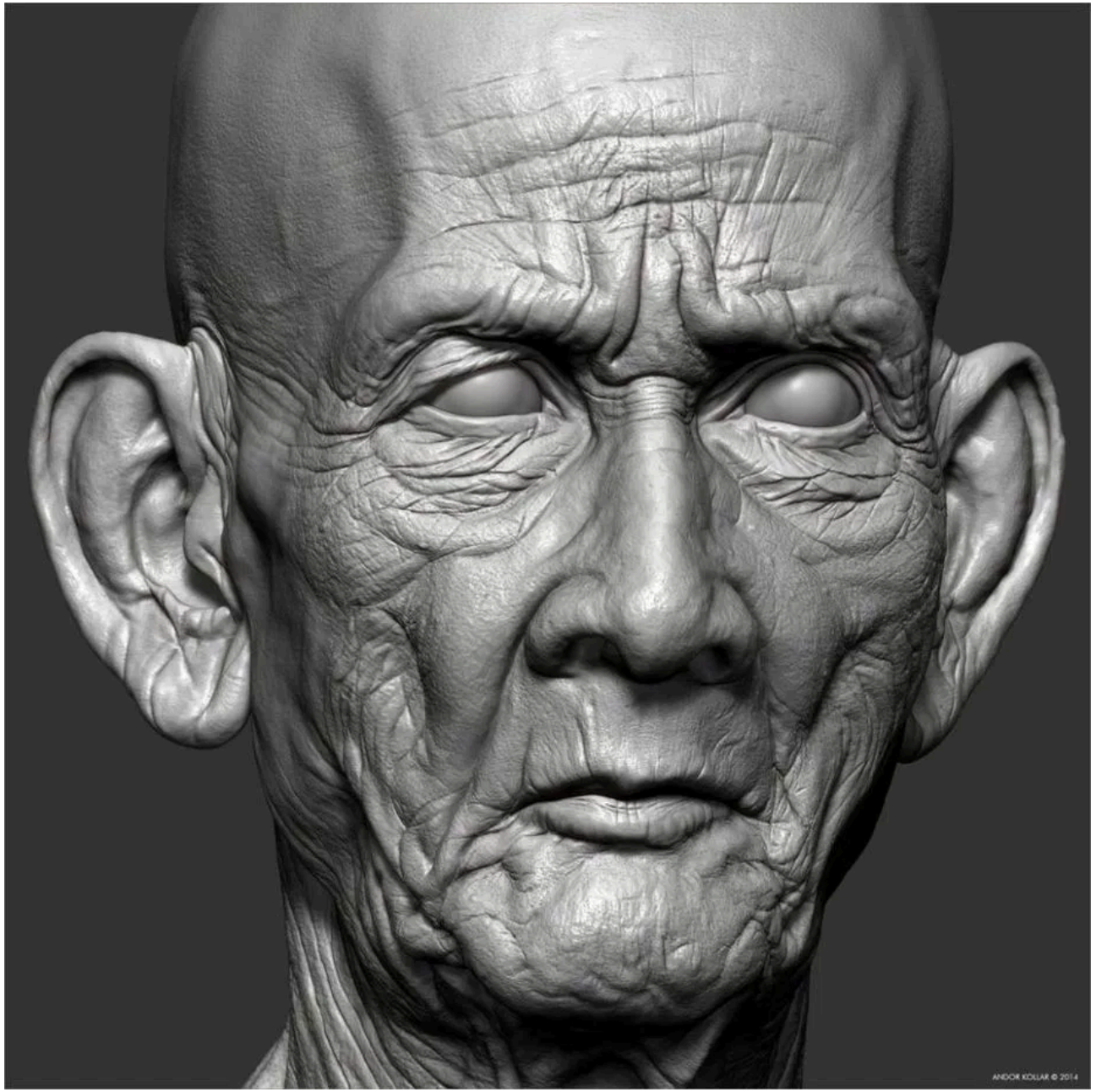
12 triangles, 8 vertices

A large mesh

10 million triangles
from a high-resolution
3D scan

[Traditional Thai sculpture—scan by XYZRGB, inc., image by MeshLab]

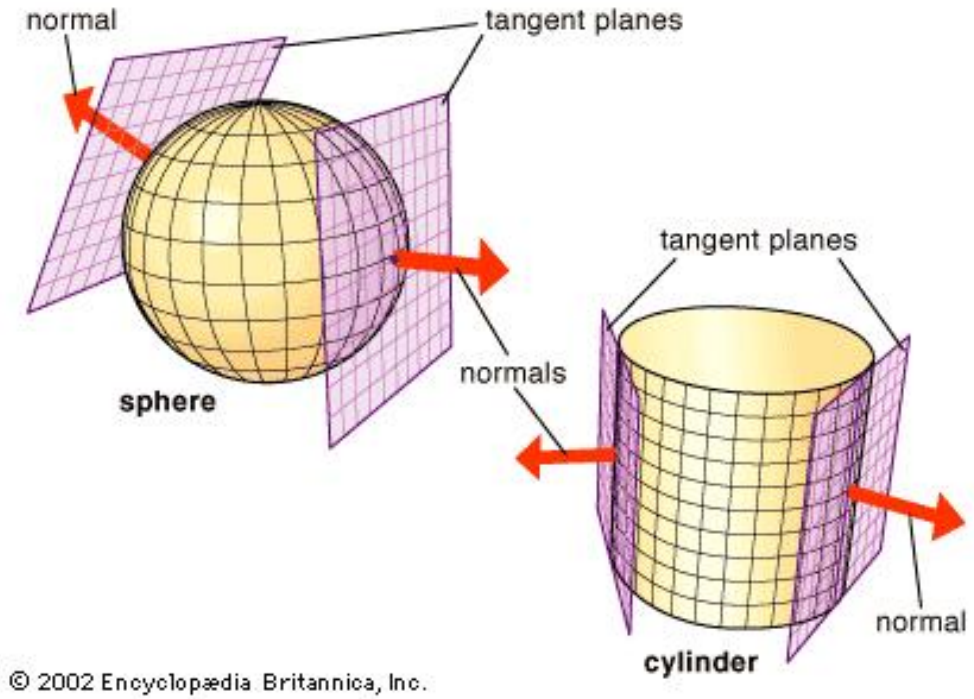
[Andor Kollar]



[Andor Kollar]

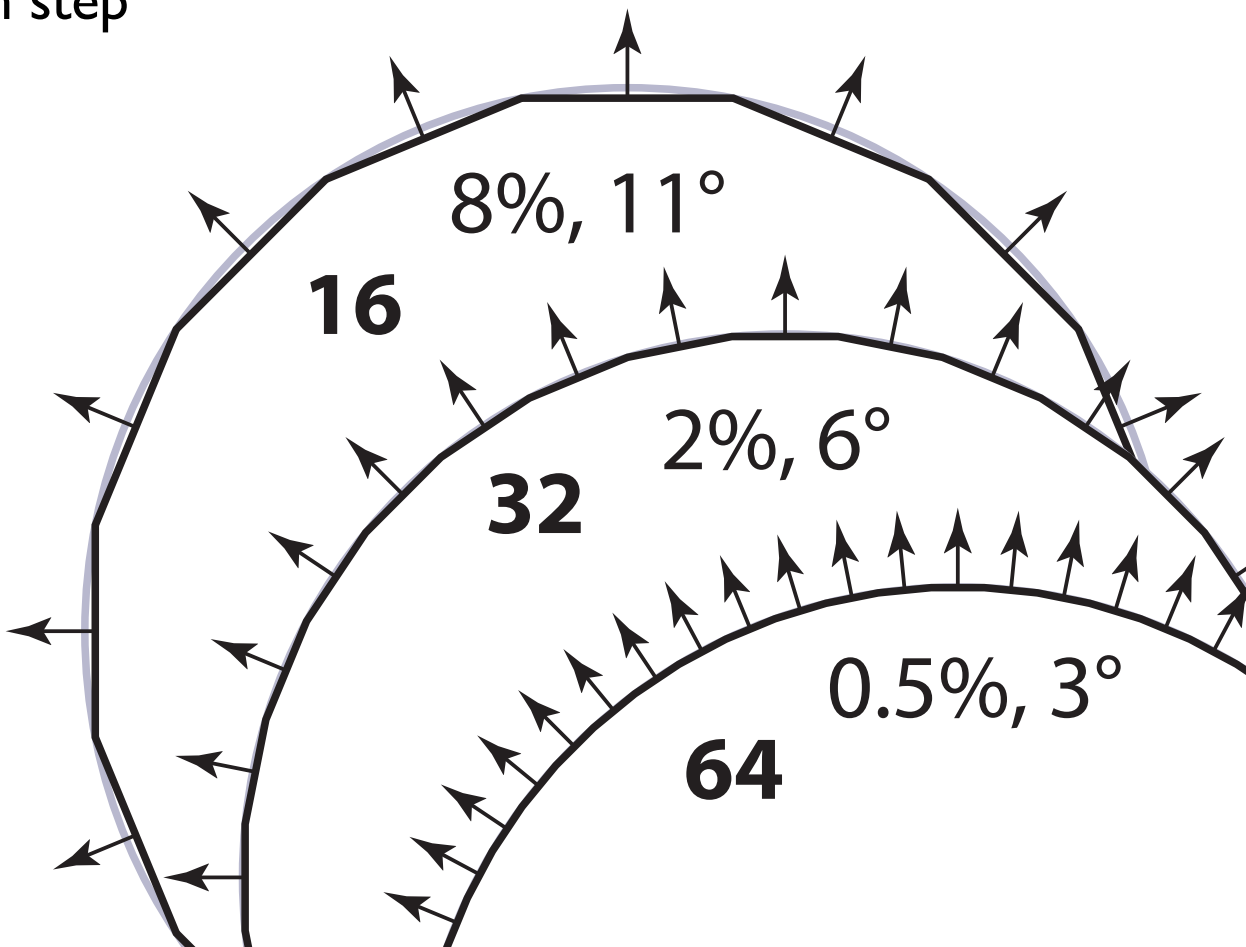
Normals and tangents

- Tangent plane: at a point on a smooth surface in 3D, there is a unique plane tangent to the surface, called the tangent plane
- Normal vector: vector perpendicular to the tangent plane
  - only unique for smooth surfaces (not at corners, edges)



Approximation quality

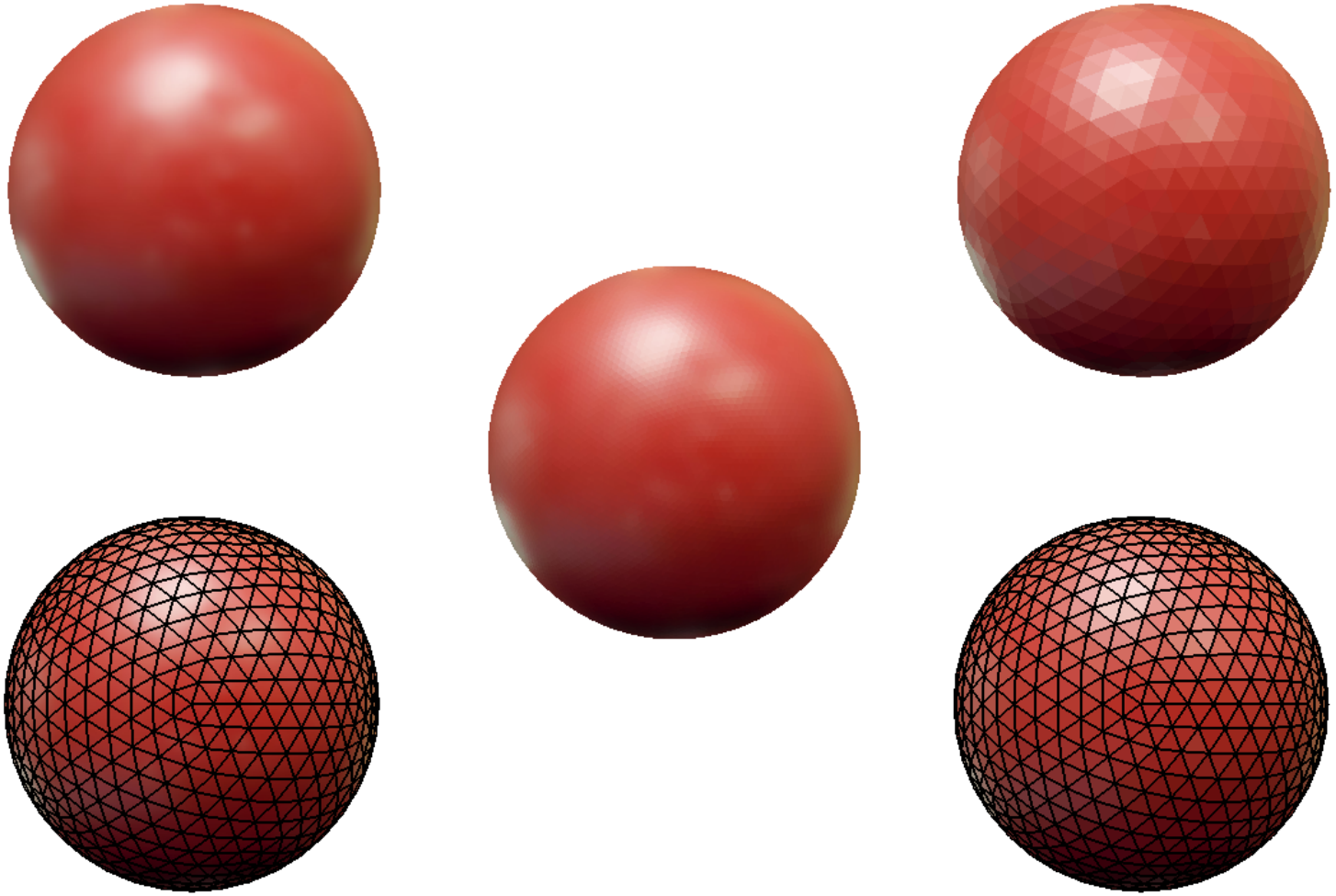
- Approximating circle with increasingly many segments
- Max error in position error drops by factor of 4 at each step
- Max error in normal only drops by factor of 2



Approximation quality

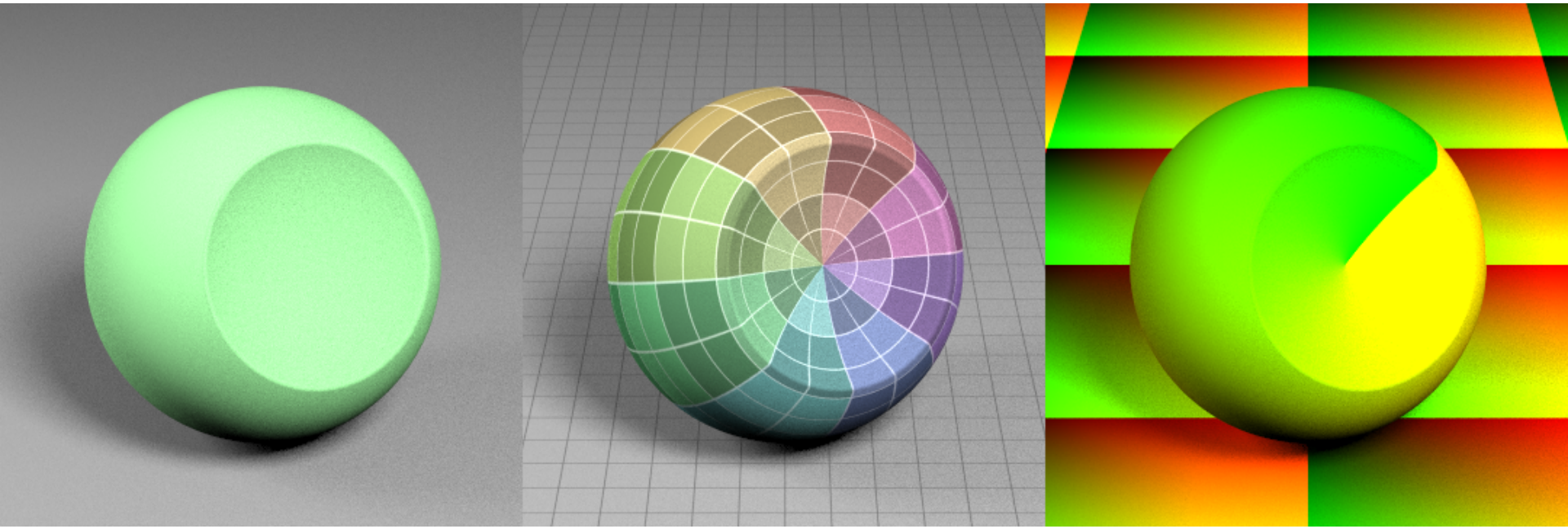
- Piecewise planar approximation converges pretty quickly to the smooth geometry as the number of triangles increases
  - for mathematicians: error is $O(h^2)$
- But the surface normals don't converge so well
  - normal is constant over each triangle, with discontinuous jumps across edges
  - for mathematicians: error is only $O(h)$
- Better: store the "real" normal at each vertex, and interpolate to get normals that vary gradually across triangles

Facet vs. vertex normals



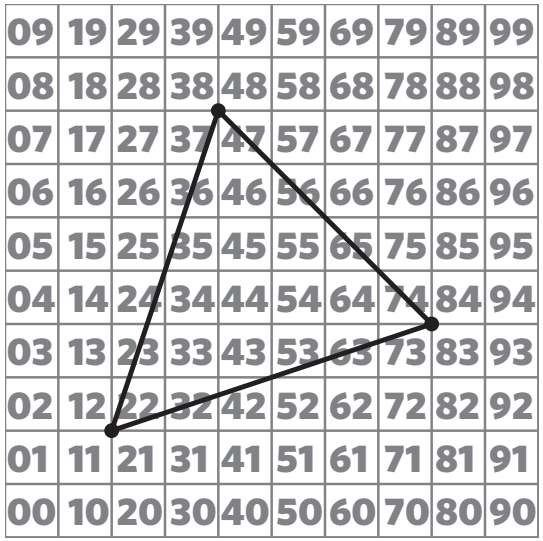
Textures

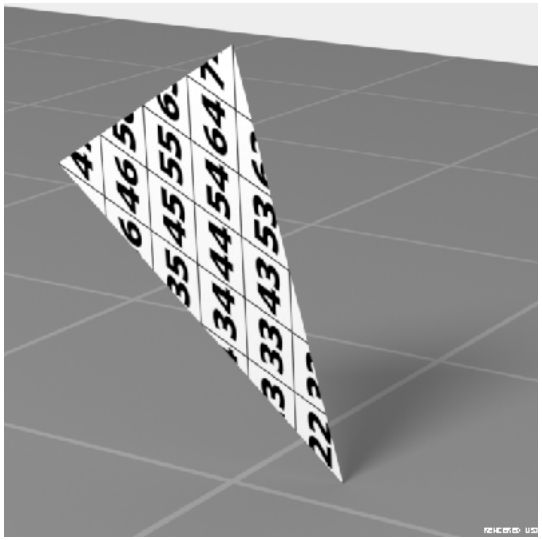
- Surface material parameters can be associated with each triangle
  - so we need ~ 1 triangle/pixel for full color resolution
- Idea: define materials by "pasting" images onto the surface
- We'll discuss in more details later



Texture coordinates

- A surface in 3D is a two-dimensional entity
- Sometimes we need 2D coordinates for points on the surface
- Defining these coordinates is *parameterizing* the surface
- For triangle meshes, specify 2D coordinates (in $[0,1]^2$) at the vertices and barycentric interpolate in the triangle

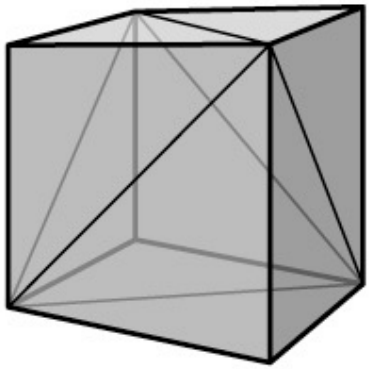




Mesh Representations

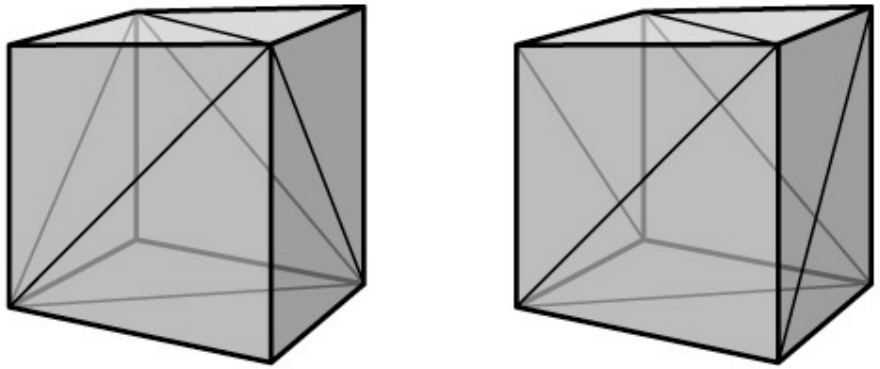
Topology vs. geometry

- Mesh geometry: where the triangles are in 3D space
- Mesh topology: how the triangles are connected
  - ignoring the positions entirely

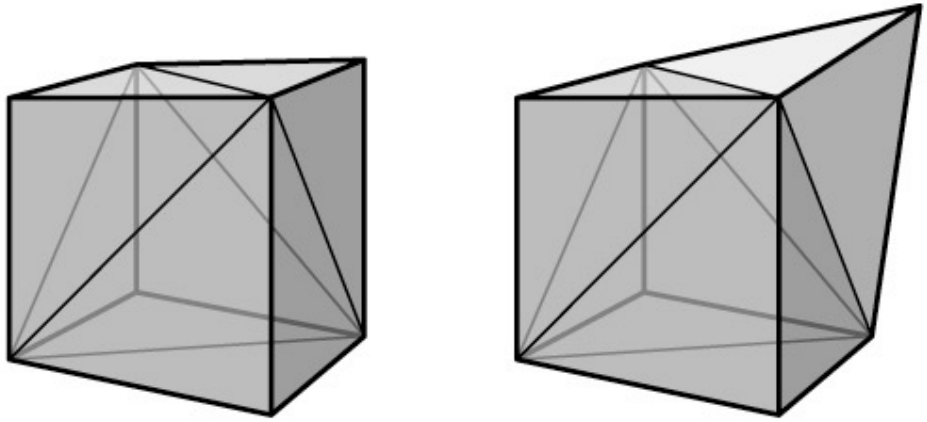


Topology vs. geometry

- same geometry, different mesh topology:



- same mesh topology, different geometry:



Mesh representations

- Compactness
- Efficiency for rendering
  - enumerate all triangles as triples of 3D points
- Efficiency of adjacency queries
  - all vertices of a triangle
  - all triangles around a vertex
  - neighboring triangles of a triangle
  - need depends on application (rendering, subdivs, modeling)

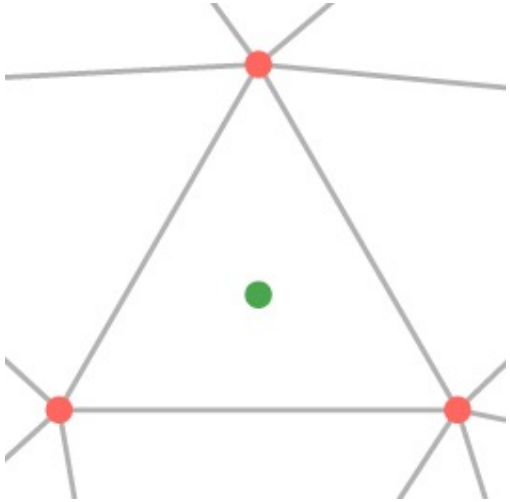
Mesh representations

- Separate triangles
- Indexed triangle set
  - shared vertices
- Triangle strips and triangle fans
  - compression schemes for fast transmission
- Triangle-neighbor data structure
  - supports adjacency queries
- Half-edge data structure
  - supports general polygon meshes

Separate triangles

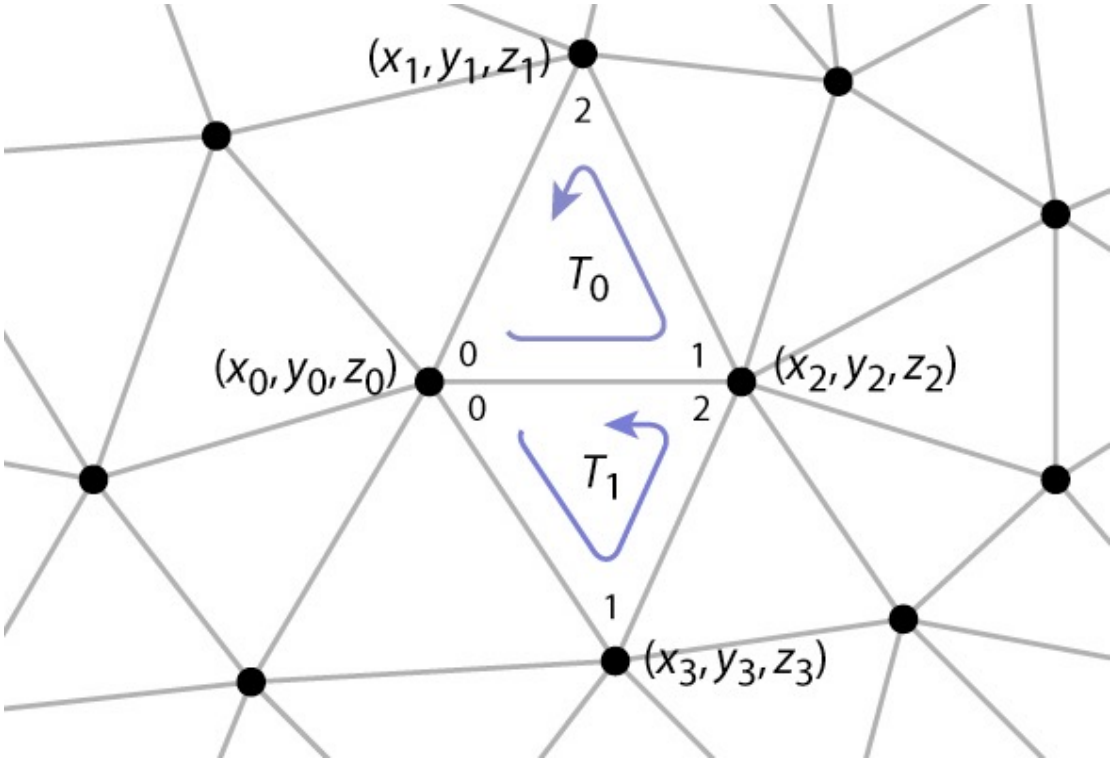
- Store each triangle independently

```
struct mesh {
  vec3f pos[nT][3];
};
```



Separate triangles

	[0]	[1]	[2]
tris[0]	x_0, y_0, z_0	x_2, y_2, z_2	x_1, y_1, z_1
tris[1]	x_0, y_0, z_0	x_3, y_3, z_3	x_2, y_2, z_2
	⋮	⋮	⋮



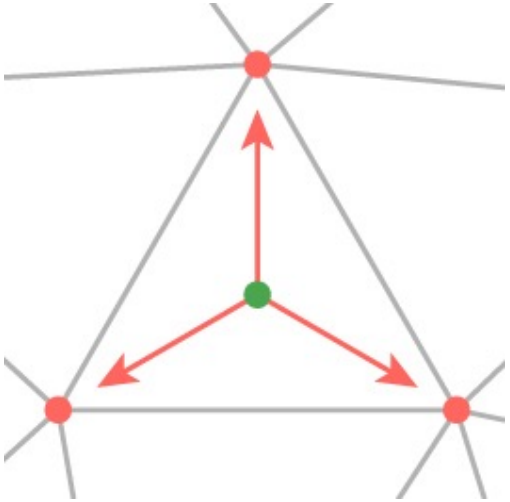
Separate triangles

- Array of triples of points
  - float[n_T][3][3]: about 72 bytes per vertex
  - 2 triangles per vertex (on average)
- Various problems
  - wastes space (each vertex stored 6 times)
  - cracks due to roundoff
  - difficulty of finding neighbors at all

Indexed triangles

- Store each vertex once
- Each triangle points to its three vertices

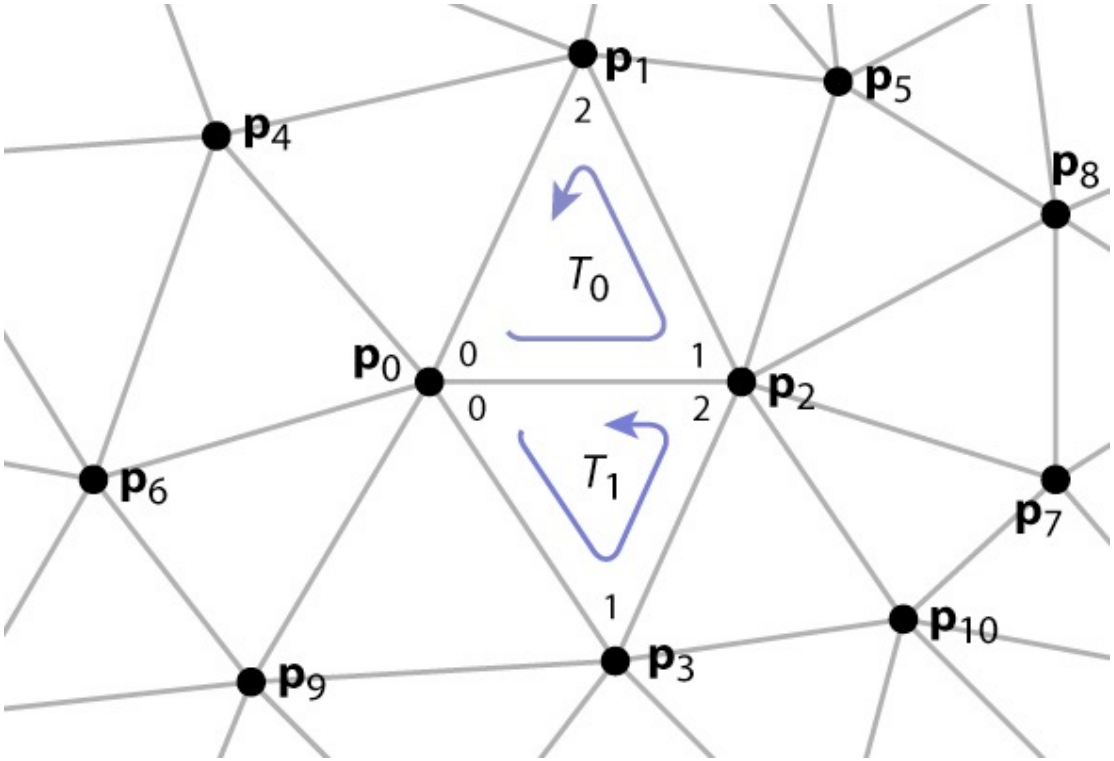
```
struct mesh {
  vec3i tris[nT];
  vec3f pos[nV];
};
```



Indexed triangles

verts[0]	x_0, y_0, z_0
verts[1]	x_1, y_1, z_1
	x_2, y_2, z_2
	x_3, y_3, z_3
	⋮

tInd[0]	0, 2, 1
tInd[1]	0, 3, 2
	⋮



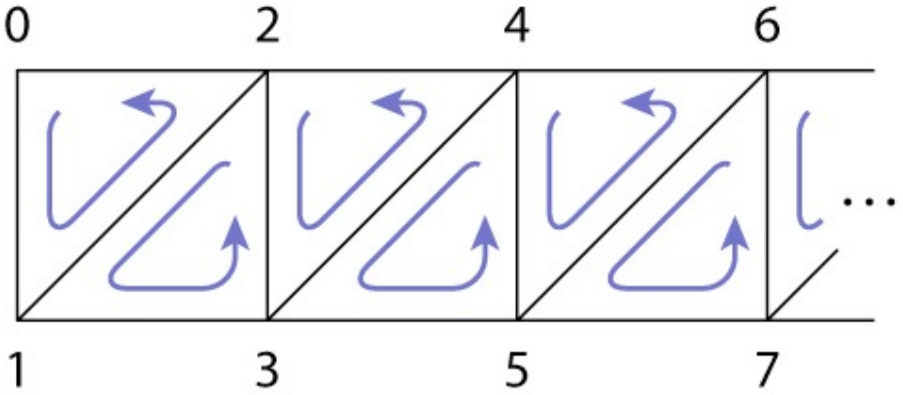
Indexed triangles

- Array of vertex positions
  - float[n_V][3]: 12 bytes per vertex
- Array of triangle indices
  - int[n_T][3]: 12 bytes per triangle
  - 2 triangles per vertex on average: 24 bytes per vertex
- Total storage: 36 bytes per vertex (factor of 2 savings)
- Represents topology and geometry separately
- Finding neighbors is well defined

Triangle strips

- Compress triangle lists by skipping repeated indices
  - each triangle is usually adjacent to the previous
  - let every vertex create a triangle by reusing the second and third vertex of the previous triangle
  - for long strips, this requires about one index per triangle

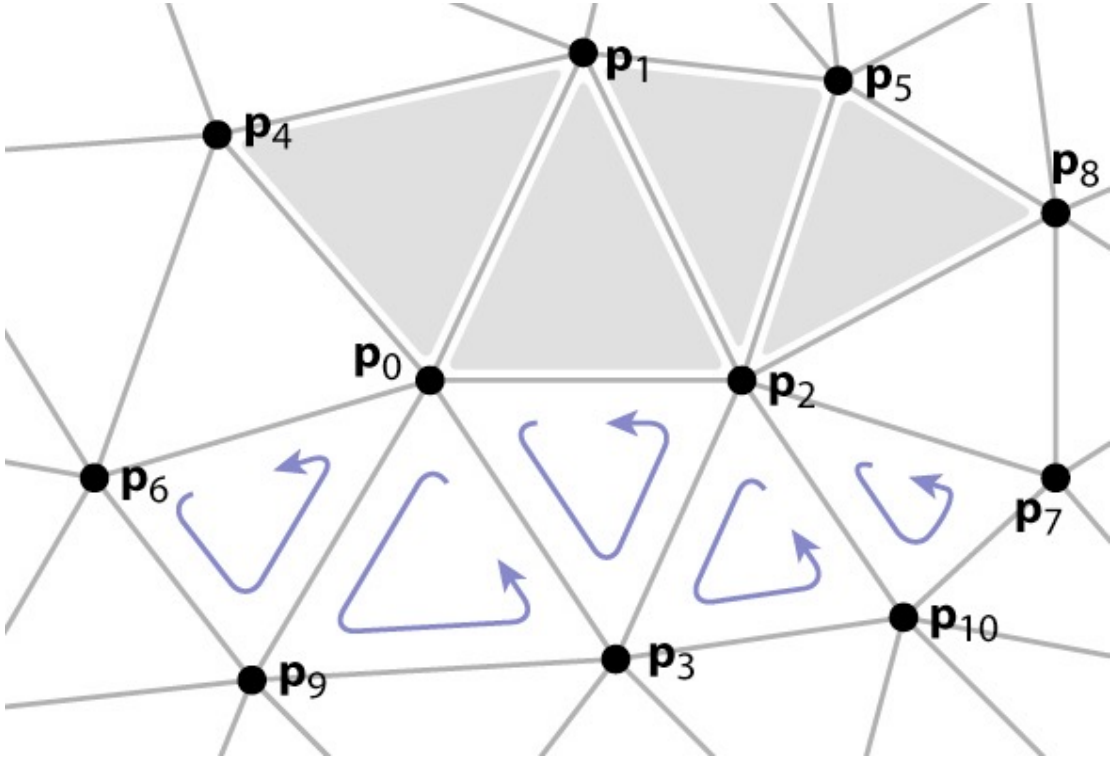
```
struct mesh {
  int strips[nT][];
  float pos[nV][3];
};
```



Triangle strips

verts[0]	x_0, y_0, z_0
verts[1]	x_1, y_1, z_1
	x_2, y_2, z_2
	x_3, y_3, z_3
	⋮

tStrip[0]	6, 0, 4, 1, 2, 5, 8
tStrip[1]	6, 9, 0, 3, 2, 10, 7
	⋮



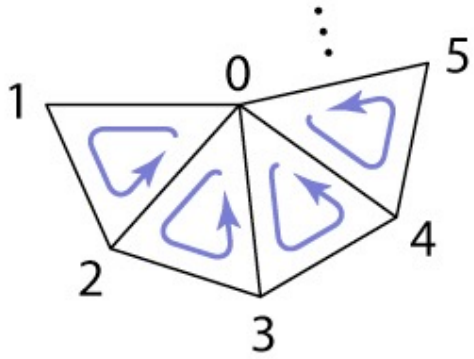
Triangle strips

- Array of vertex positions
  - float[n_V][3]: 12 bytes per vertex
- Array of index lists
  - int[n_S][variable]: (2 + n)x4 bytes per strip
  - on average, (1 + ε) indices per triangle (assuming long strips)
- Total is 20 bytes per vertex (limiting best case)
  - factor of 3.6 over separate triangles; 1.8 over indexed mesh

Triangle fans

- Same idea as triangle strips, but keep oldest rather than newest
  - every sequence of three vertices produces a triangle
  - for long fans, this requires about one index per triangle
- Memory considerations exactly the same as triangle strip

```
struct mesh {
  int fans[nT][];
  float pos[nV][3];
};
```

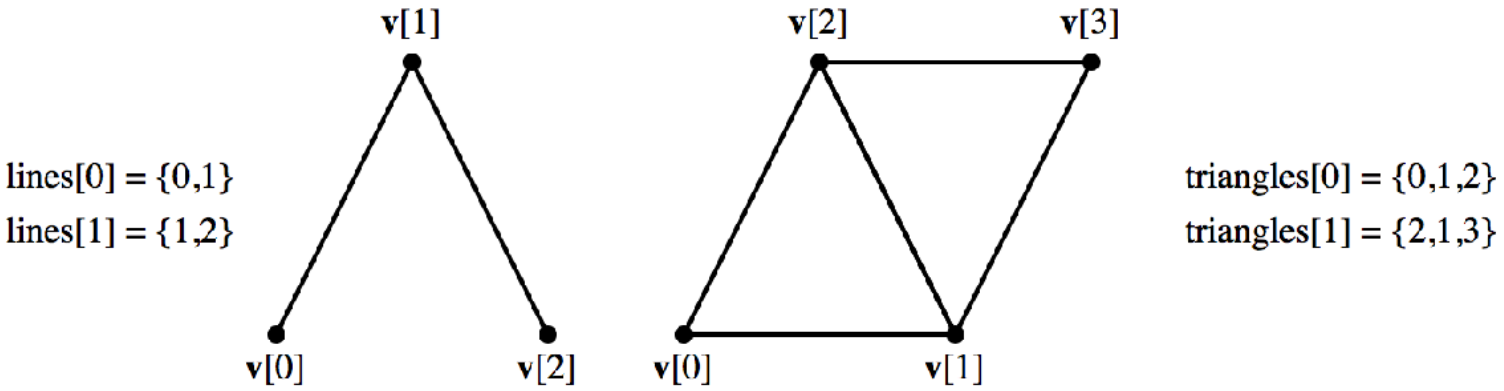


Data on meshes

- Often need to store additional information besides just the geometry
- Can store additional data at faces, vertices, or edges
- Store on faces for discontinuous data, e.g. facet colors
- Information about creases stored at edges
- Quantities that vary continuously (without sudden changes, or discontinuities) gets stored at vertices

Indexed Mesh Data Structure

- Indexed triangles is the most used representation
  - efficient for rendering and many geometry operations
  - can easily add/remove triangles and vertices
- For GPU execution: store data on vertices
- In modeling applications: store data on vertices and faces
- Indexed representation used also for segments and tetrahedra



Key types of vertex data

- Positions
  - at some level this is just another piece of data
  - position varies continuously between vertices
- Surface normals
  - when a mesh is approximating a curved surface, store normals at vertices
- Texture coordinates
  - 2D coordinates that tell you how to paste images on the surface

Indexed Mesh Data Structure

- A typical implementation uses separate arrays for each vertex data
  - this allows to only store data if present
- Element indices are held in either an integer array or a typed array (as here) for easier accessing

```
struct shape {
  vector<vec3f> pos;       // vertex positions
  vector<vec3f> norm;      // vertex normals
  vector<vec2f> texcoord;  // vertex texture coordinates

  vector<int>   points;    // point indices
  vector<vec2i> lines;     // line indices
  vector<vec3i> triangles; // triangle indices
}
```


Computing vertex normals

- Ideally, store normals from the "true" geometry
- Practically, "true" geometry is not available most of the time, e.g. in hand modeling or 3D scanning
- Compute the normal $\mathbf{n}_i$ of vertex *i* as the average the normals $\mathbf{n}_f$ of the triangles *f* that are adjacent to *i*, weighted by the triangles areas A_f

$$\mathbf{n}_i = \frac{\sum_{f \in N_i} A_f \mathbf{n}_f}{\sum_{f \in N_i} A_f}$$

- Implement this by looping over faces and updating vertex normals, which is *O(n)*, instead of a trivial implementation of the equation above which is $O(n^2)$

Computing vertex normals

```
vector<vec3f> compute_normals(const vector<vec3f>& positions,
                              const vector<vec3i>& triangles) {

  // initialize normals to 0
  auto normals = vector<vec3f>(pos.size(), {0,0,0});
  // accumulate triangle normals
  for(auto t : triangles) {
    auto n = triangle_normal(positions[t.x],
      positions[t.y], positions[t.z]);
    auto a = triangle_area(positions[t.x],
      positions[t.y], positions[t.z]);
    normals[t.x] += a * n; normals[t.y] += a * n;
    normals[t.z] += a * n;
  }
  // normalize result
  for(auto& n : norm) n = normalize(n);
  return norm;
}
```


Interpolating vertex data

- Interpolation of vertex data follows barycentric coordinates
  - for normals we normalize at the end

$$\mathbf{p} = w_0\mathbf{p}_0 + w_1\mathbf{p}_1 + w_2\mathbf{p}_2 \quad \mathbf{n} = \frac{w_0\mathbf{n}_0 + w_1\mathbf{n}_1 + w_2\mathbf{n}_2}{|w_0\mathbf{n}_0 + w_1\mathbf{n}_1 + w_2\mathbf{n}_2|}$$

```
vec3f normal(shape* shape, int element, vec2f uv) {
  auto t = shape->triangles[element];
  return normalize((1 - uv.x - uv.y) * shape->normals[t.x] +
          uv.x * shape->normals[t.y] +
          uv.y * shape->normals[t.z]);
}
vec3f position(shape* shape, int element, vec3f uv) {
  auto t = shape->triangles[element];
  return (1 - uv.x - uv.y) * shape->positions[t.x] +
         uv.x * shape->positions[t.y] +
         uv.y * shape->positions[t.z]);
}
```

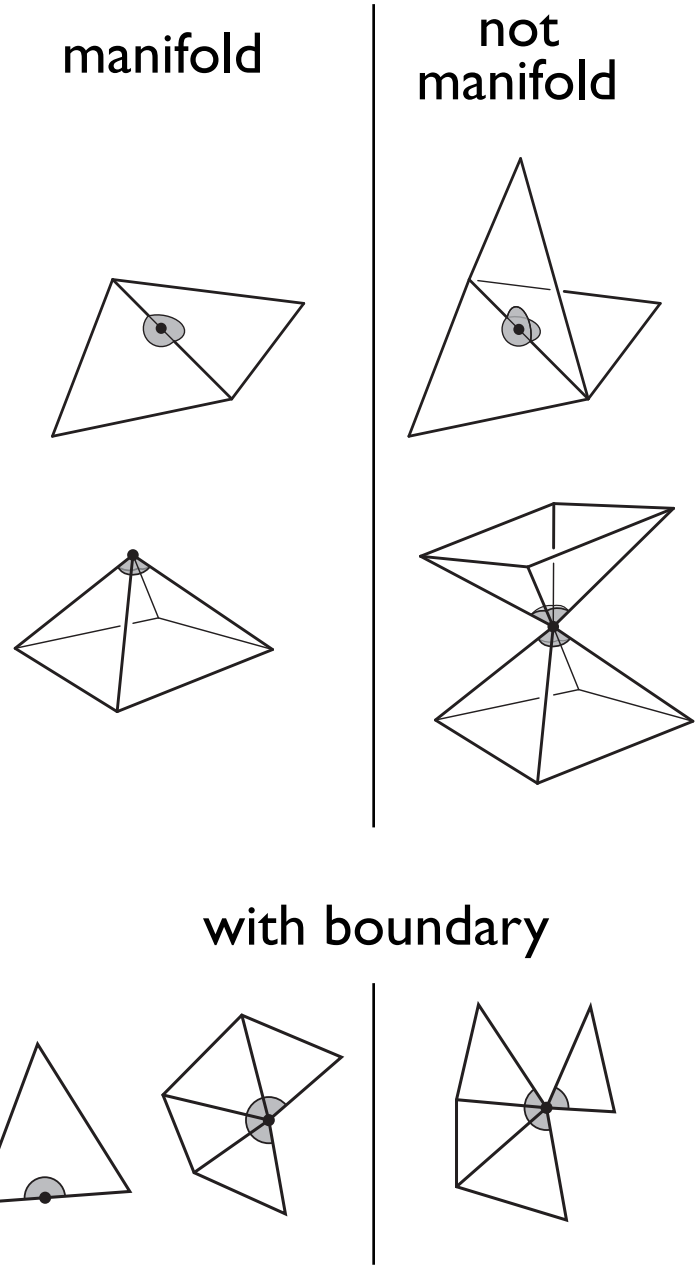
Mesh Validity

Validity of triangle meshes

- In many cases we care about the mesh being able to bound a region of space nicely
- In other cases we want triangle meshes to fulfill assumptions of algorithms that will operate on them (and may fail on malformed input)
- Two completely separate issues:
  - topology: how the triangles are connected
  - geometry: where the triangles are in 3D space

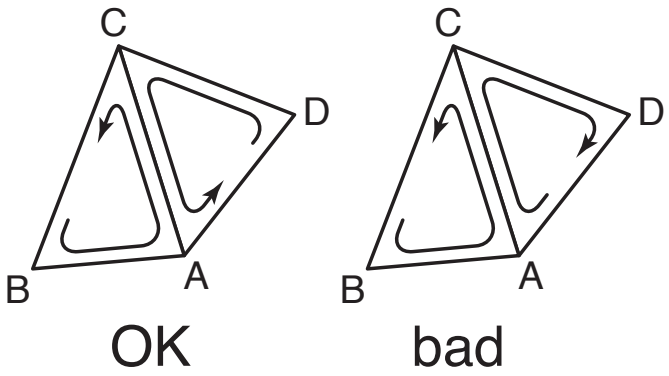
Topological validity

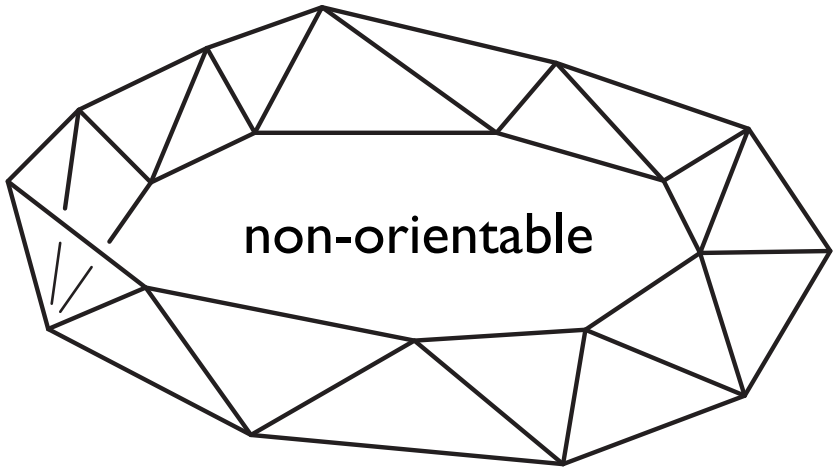
- Strongest property: be a manifold
  - this means that no points should be "special"
  - interior points are fine
  - edge points: each edge must have exactly 2 triangles
  - vertex points: each vertex must have one loop of triangles
- Slightly looser: manifold with boundary
  - weaken rules to allow boundaries



Topological validity

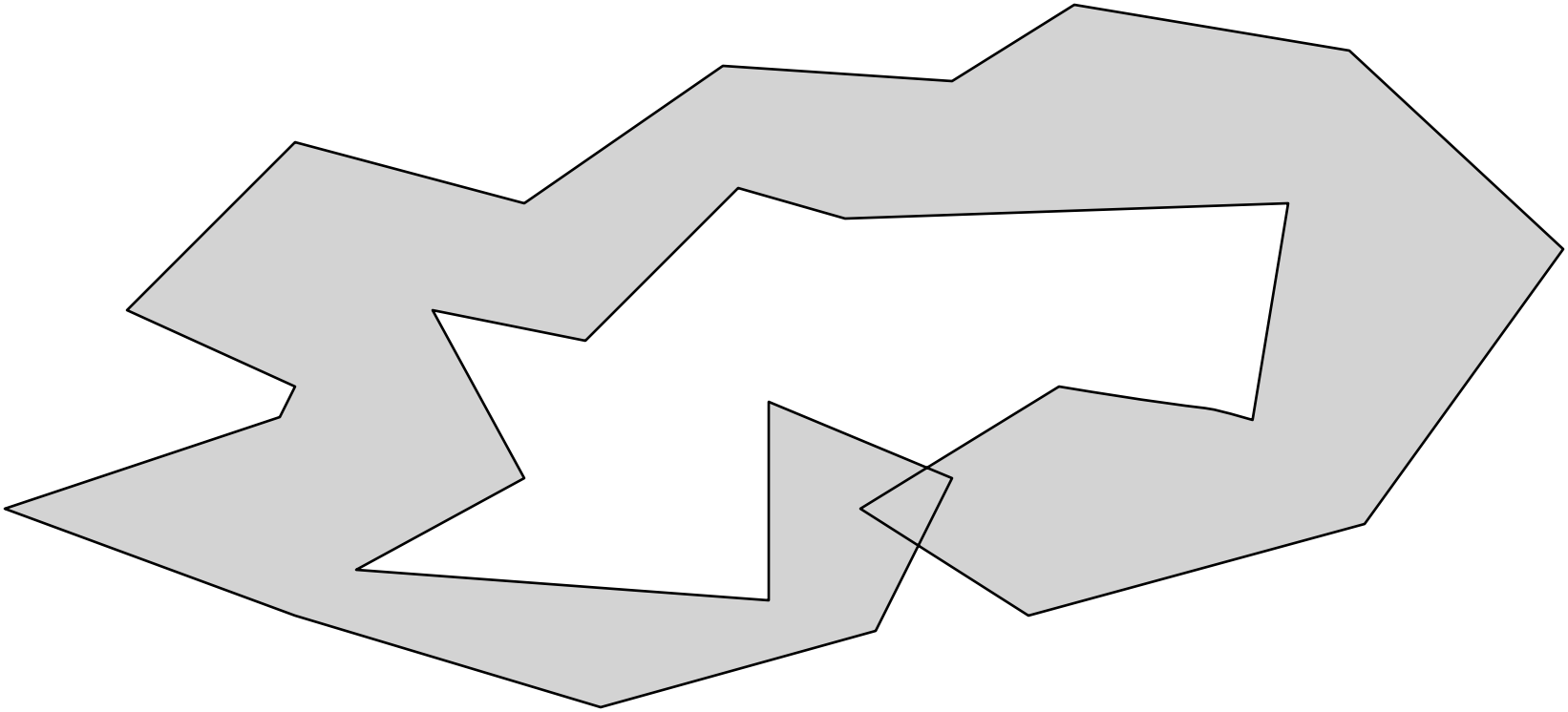
- Consistent orientation
  - which side is the "front" or "outside" of the surface and which is the "back" or "inside?"
  - rule: you are on the outside when you see the vertices in counter-clockwise order
  - in mesh, neighboring triangles should agree about which side is the front!
  - caution: not always possible





Geometric validity

- Generally want non-self-intersecting surface
- Hard to guarantee in general
  - because far-apart parts of mesh might intersect



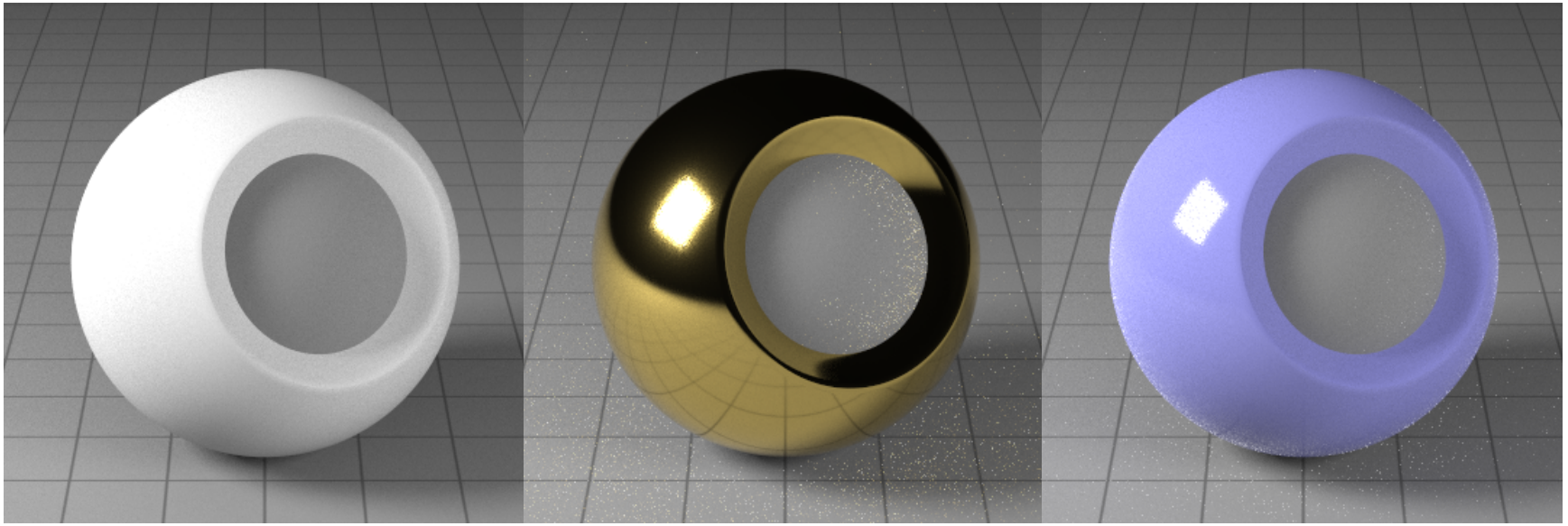
Representing Materials


corona
Brooklyn NYC Loft by Yarko Kushta | www.behance.net/YarkoKushta
rendered with Corona Renderer | www.corona-renderer.com

[Activision]

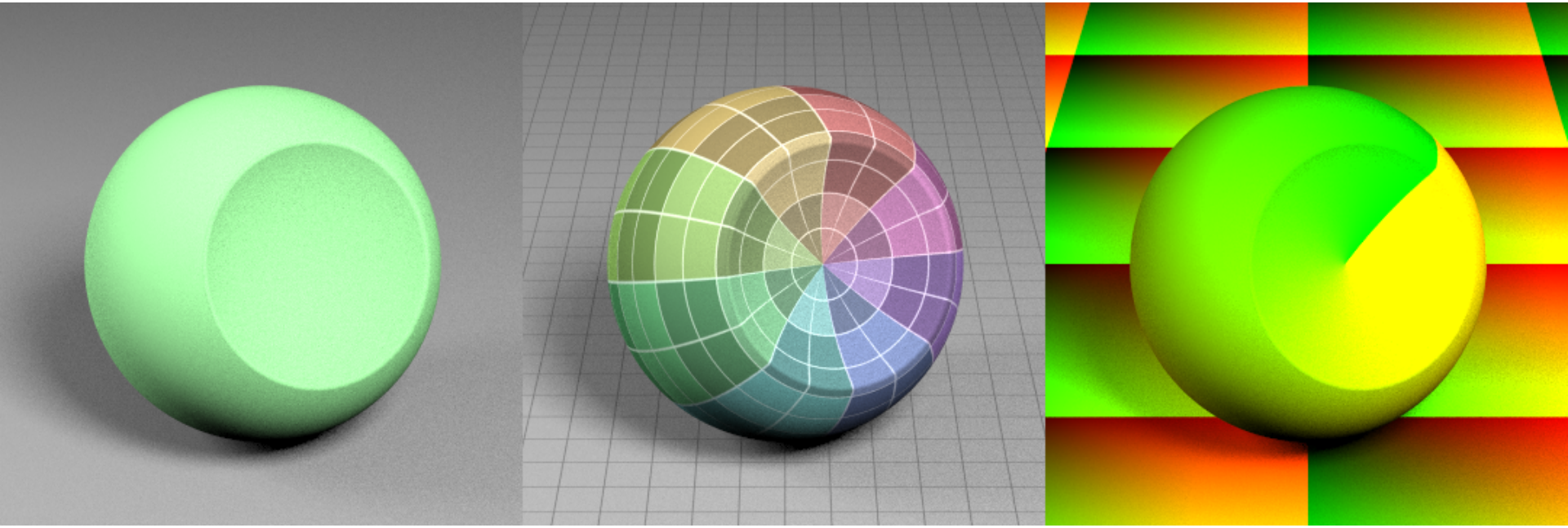
Representing Materials

- Start with a simple model that covers plastics and metals
  - augment the model later for other material types
- Diffuse color: controls color for plastics
- Specular color : controls color for metals
- Specular roughness: controls highlight/reflection size and blur



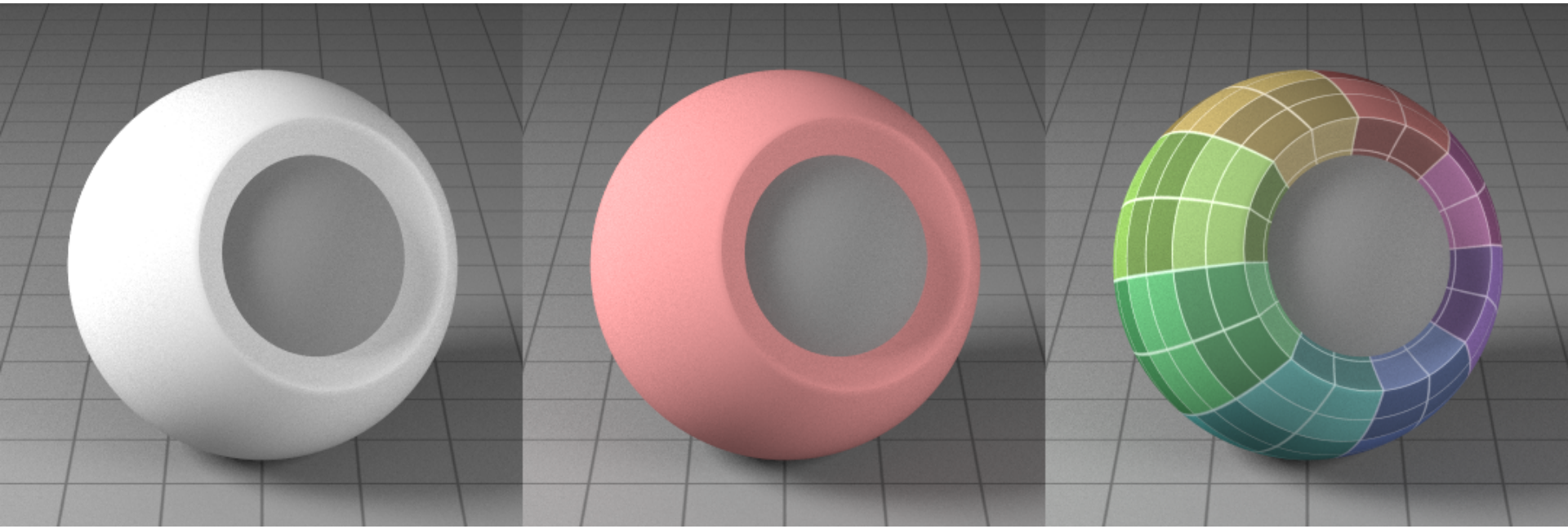
Representing Materials

- Common to specify parameters both as constant and textures
  - take the product of the two



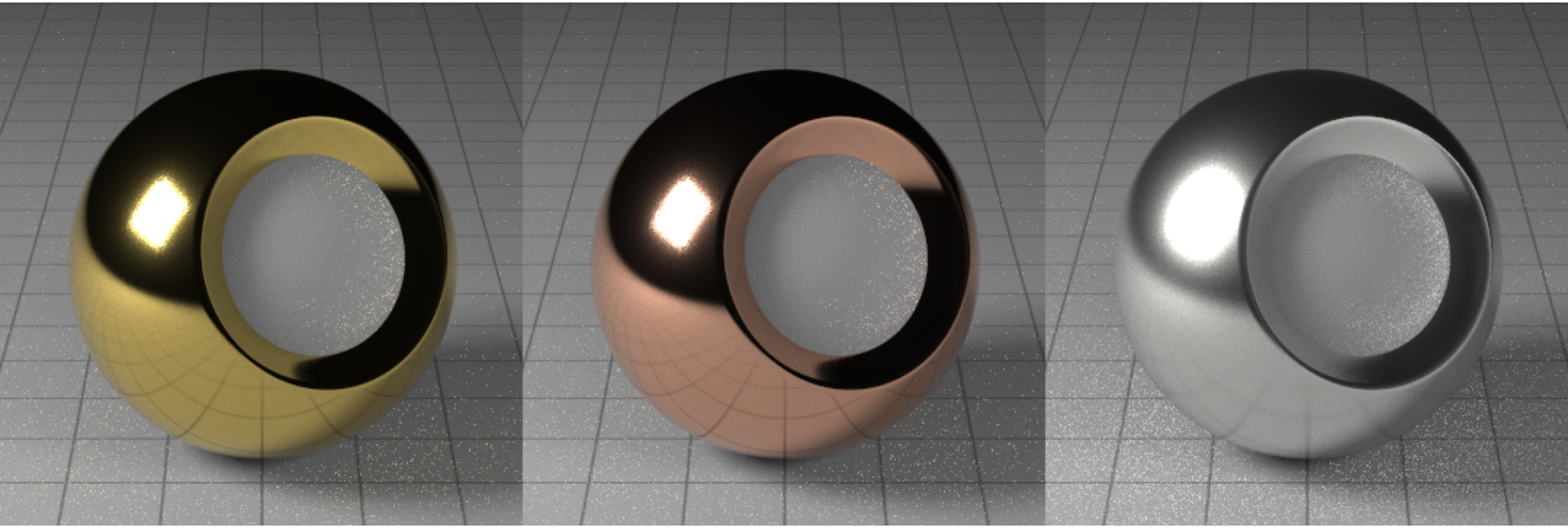
Matte surfaces

- Change color by changing the diffuse coefficient
- Apply textures to specify variations



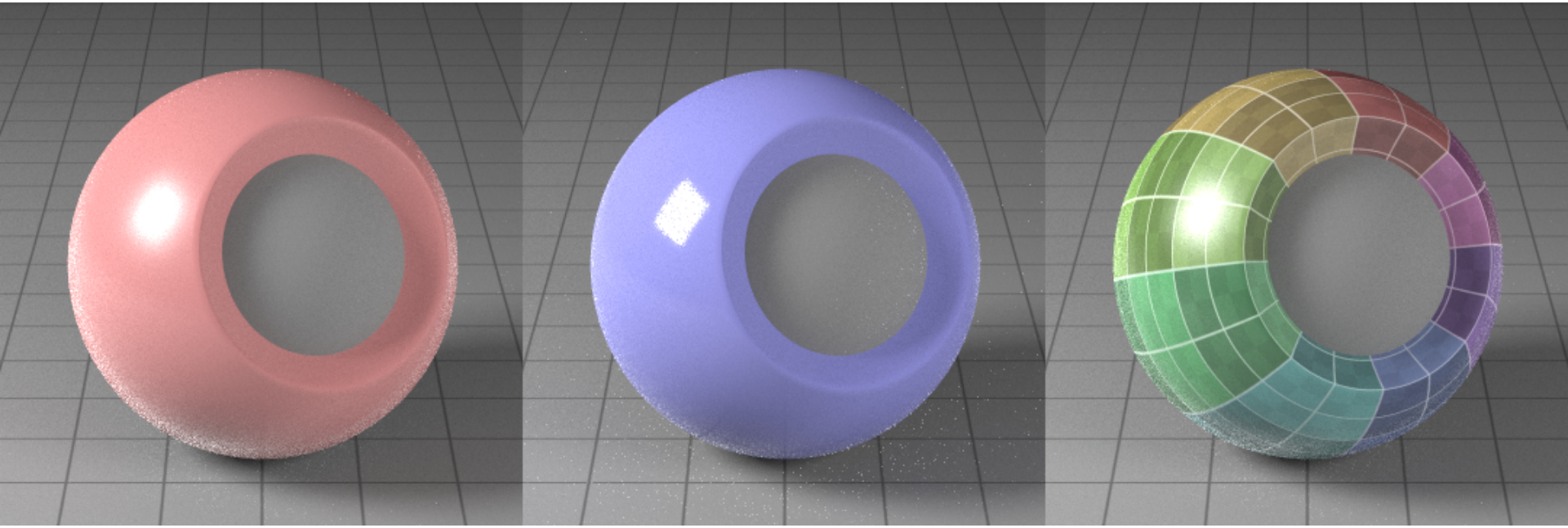
Metal surfaces

- Change color by changing the specular coefficient
- Change reflection blur by changing roughness



Plastic surfaces

- Change color by changing diffuse coefficient
- Change highlights by changing roughness
  - set highlight color to a small value



Representing Materials

- Legacy materials stored coefficients directly
- Store textures as pointers, so they can be shared between materials

```
struct texture {
  image<vec3f> hdr; // either hdr, e.g. for emission
  image<vec3b> ldr; // or ldr, e.g for colors
};

struct legacy_material {
  vec3f emission;          // emission
  vec3f diffuse;           // diffuse
  vec3f specular;          // specular
  float roughness;         // roughness
  texture* emission_tex;   // emission texture
  texture* diffuse_tex;    // diffuse texture
  texture* specular_tex;   // specular texture
  texture* roughness_tex; // roughness texture
};
```

Representing Materials

- Current trends use one main color and weights to define behaviors

```
struct texture {
  image<vec3f> colorf;  // for color values
  image<vec3b> colorb;
  image<float> scalarf; // for scalar values
  image<byte>  scalarb;
};

struct material {
  vec3f emission;          // emission
  vec3f color;             // color
  float metallic;          // metallic
  float roughness;         // roughness
  texture* diffuse_tex;    // diffuse texture
  texture* metallic_tex;   // metallic texture
  texture* roughness_tex;  // roughness texture
};
```

Representing Lights

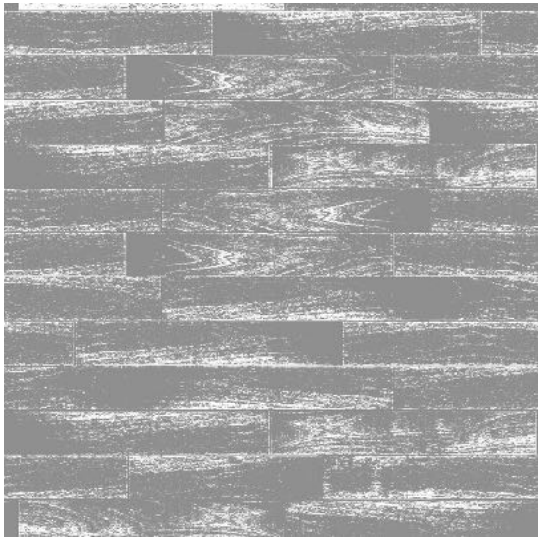
- No need for special objects: lights are just surfaces that emit light
- Stored as emission in the material

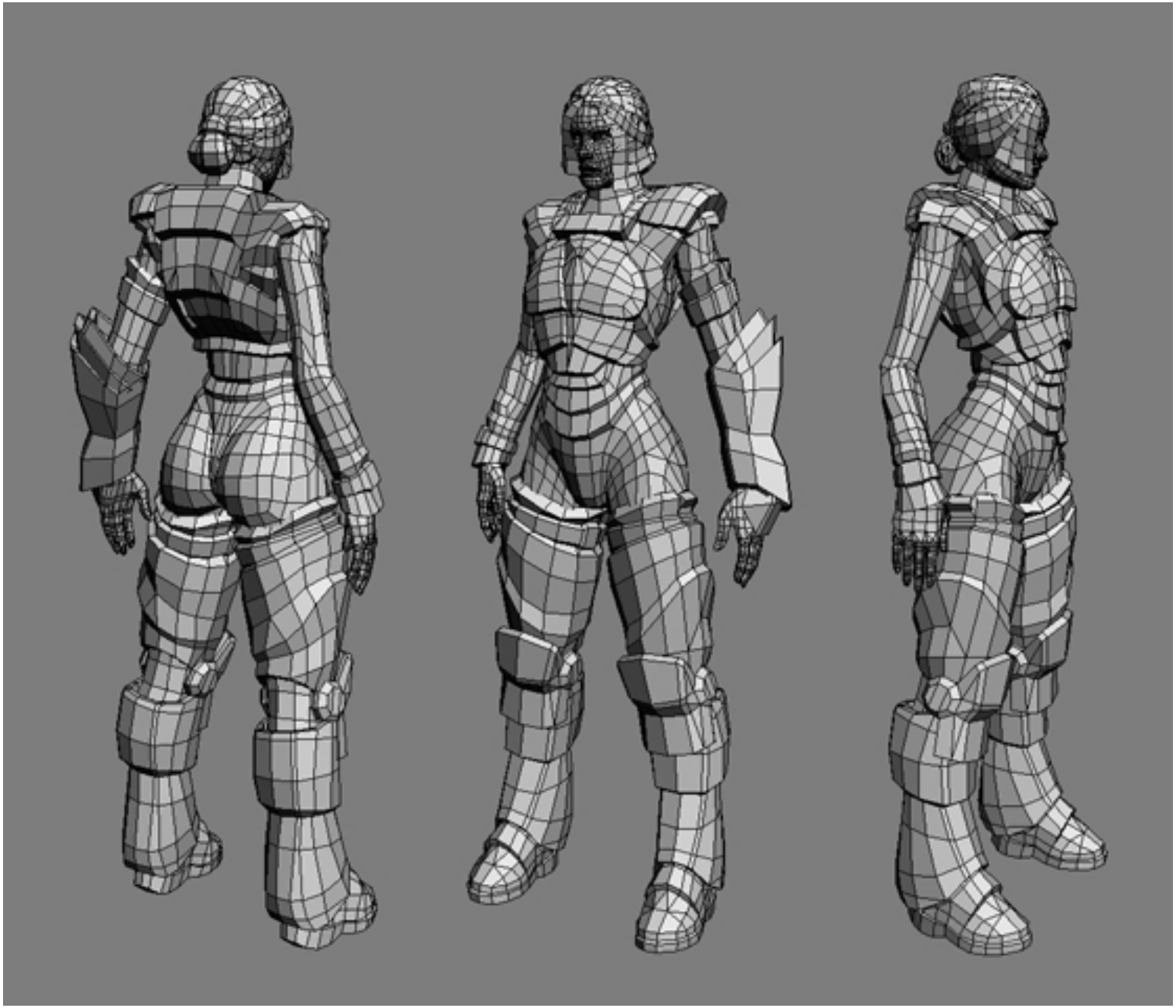
```
struct texture {
  image<vec3f> colorf;  // for color values
  image<vec3b> colorb;
  image<float> scalarf; // for scalar values
  image<vec3b> scalarb;
};
struct material {
  vec3f emission;          // emission
  vec3f color;             // color
  float metallic;          // metallic
  float roughness;         // roughness
  texture* emission_tex;   // emission texture
  texture* diffuse_tex;    // diffuse texture
  texture* metallic_tex;   // metallic texture
  texture* roughness_tex;  // roughness texture
};
```


Textures

Texture mapping

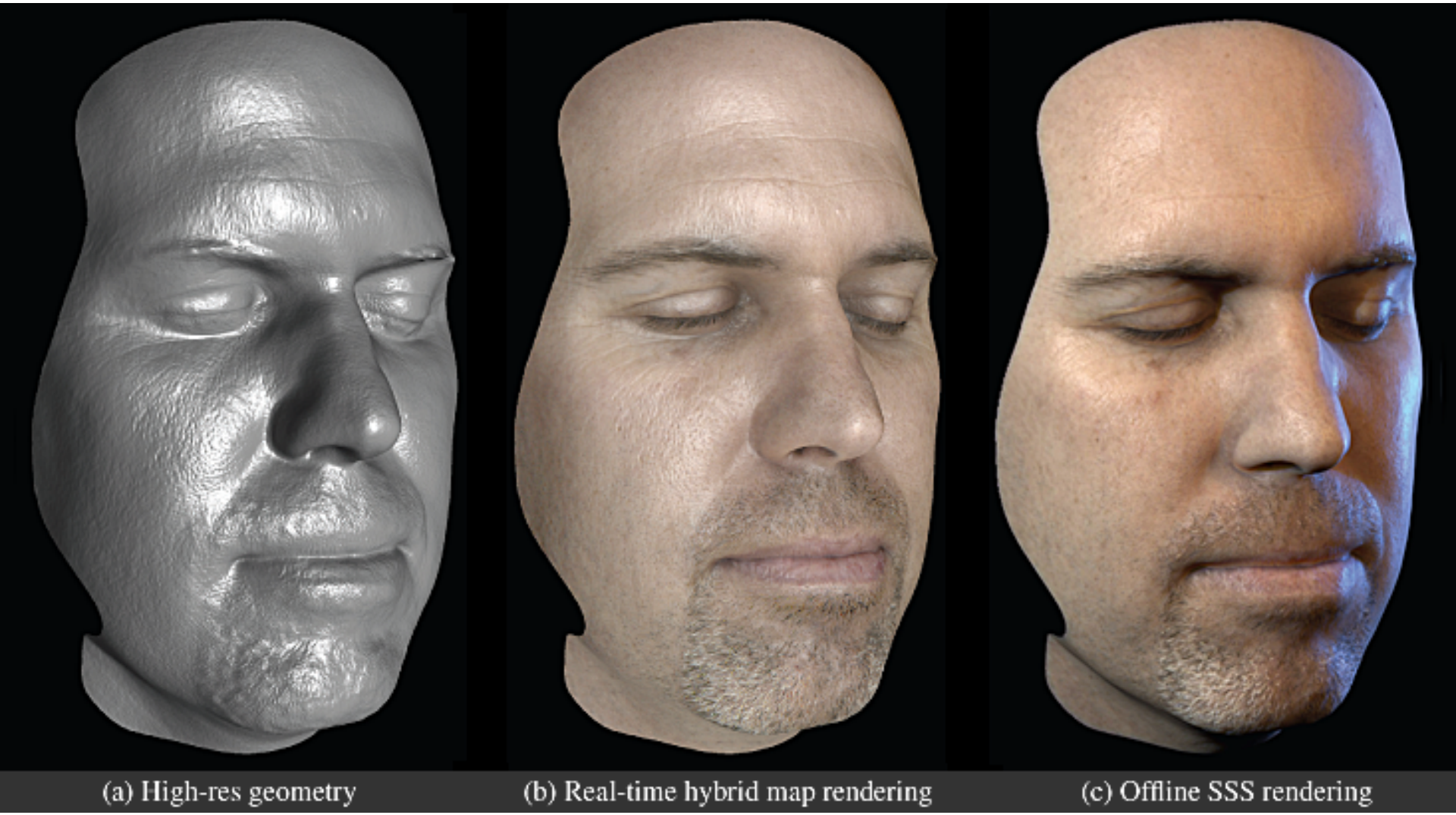
- Objects have properties that vary across the surface
- Apply images to vary material parameters over the surface





[source unknown]

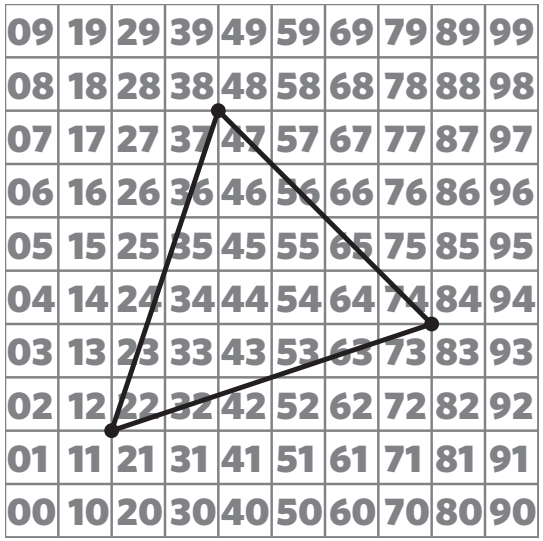
[source unknown]

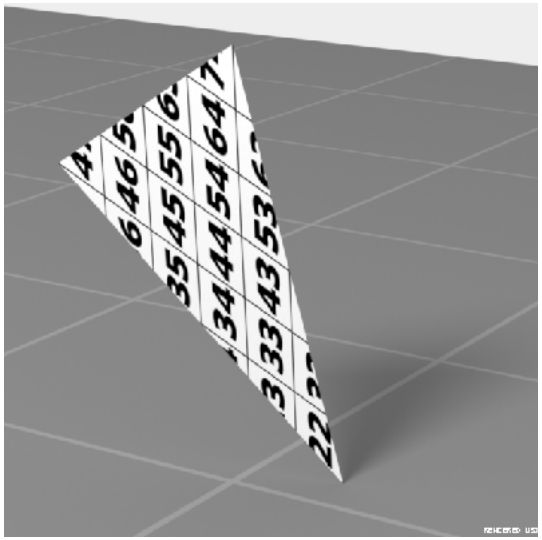


[Paul Debevec]

Texture coordinates

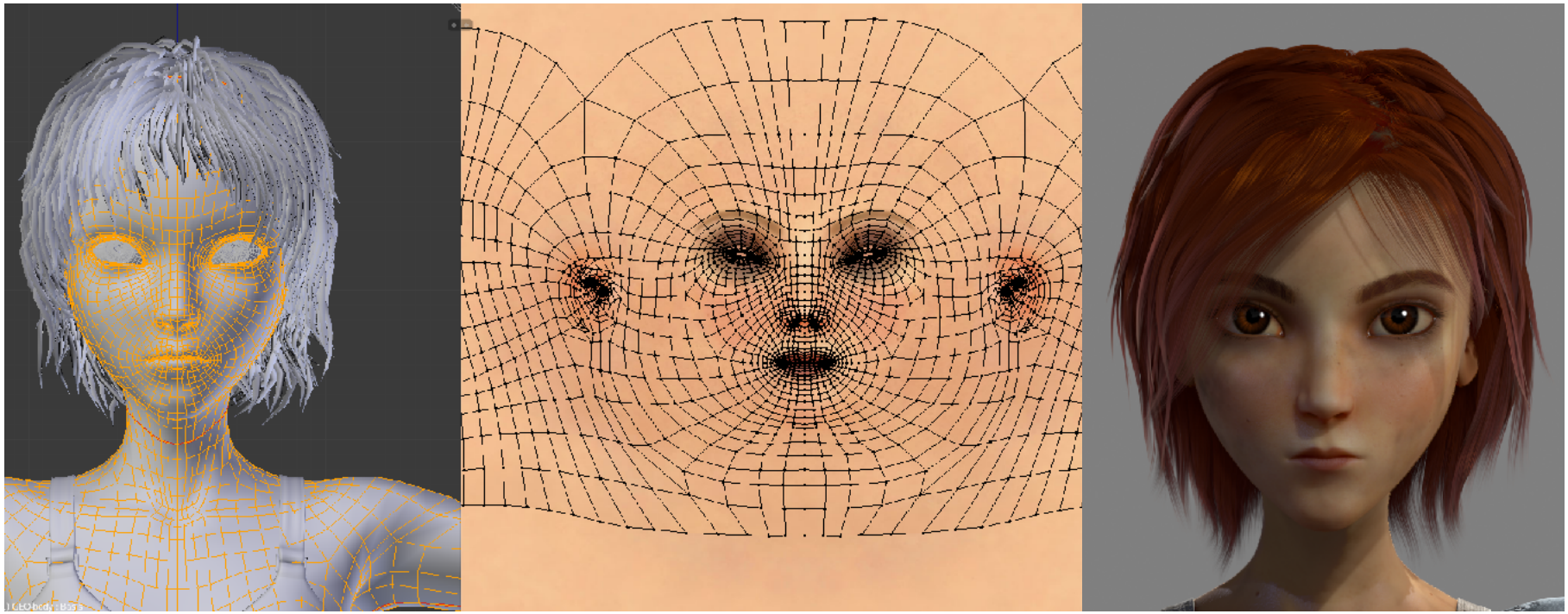
- Texture coordinates are per-vertex data like vertex positions
  - can think of them as a second position: each vertex has a position in 3D space and in 2D texture space
- How to come up with (*u*,*v*)s for points inside triangles?





Texture coordinates

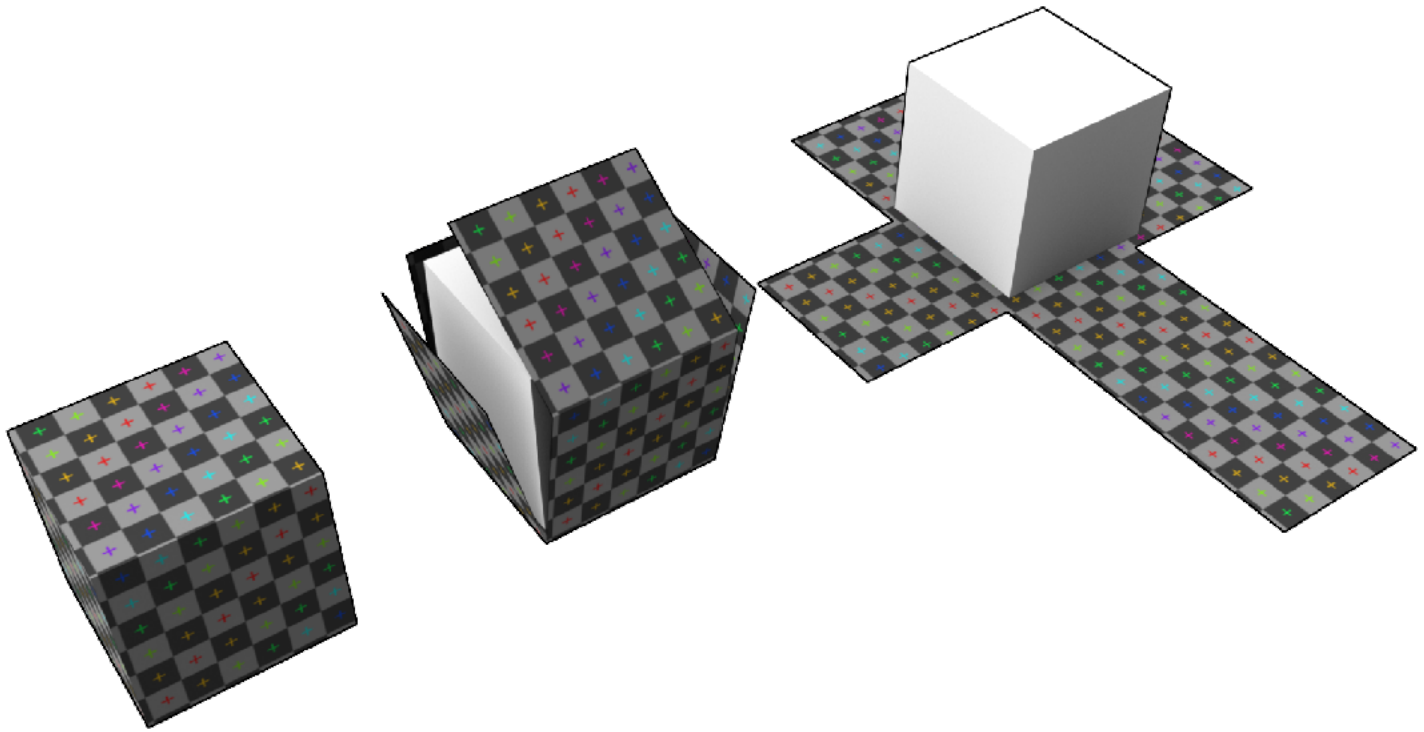
- For each mesh vertex, assign a position in the image plane $[0,1]^2$
- Induces a mapping functions from 3D vertices to 2D pixels



[Blender]

Texture coordinates

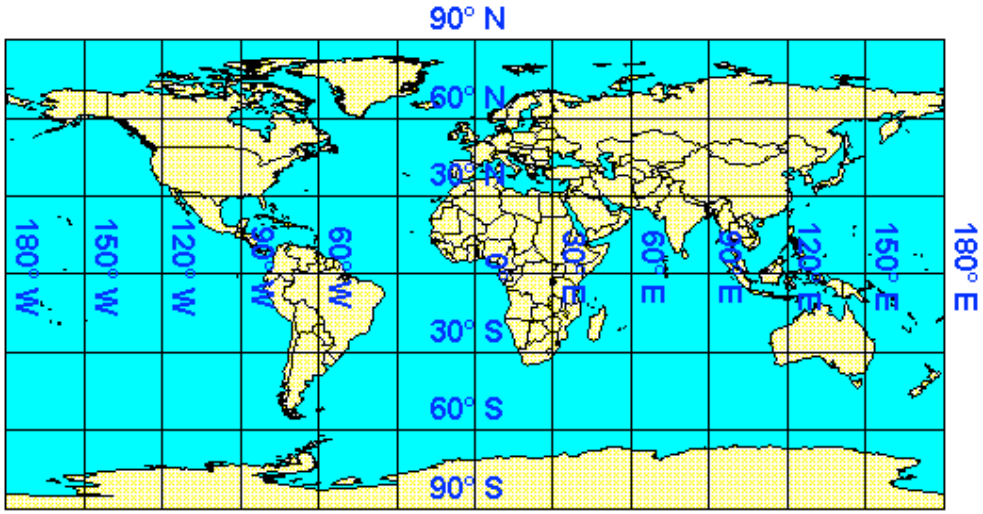
- Assign this mapping is like “unrolling" the surface onto a plane
- Often done by hand, but some tools exist to help
- Will have discontinuities in most cases



[Zepheris@WikimediaCommons]

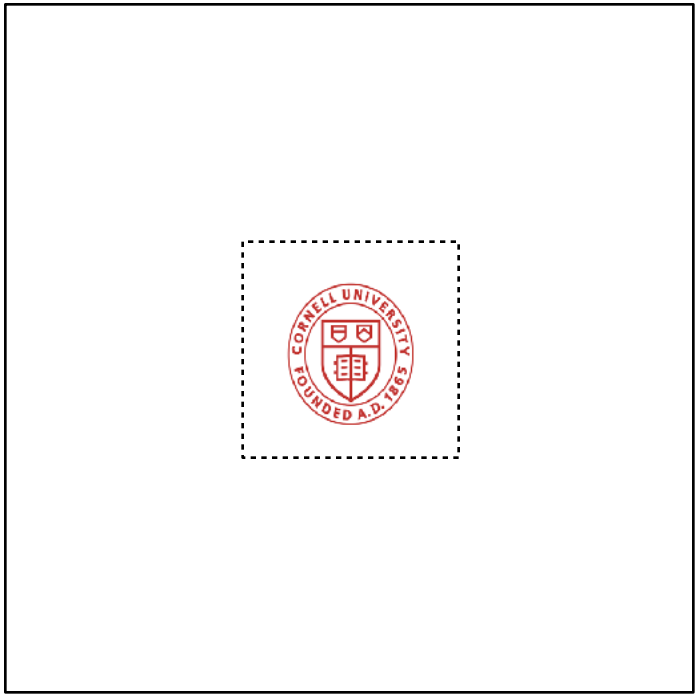
Texture coordinates

- Will also have distortions in most cases
  - Greenland looks really big in the map

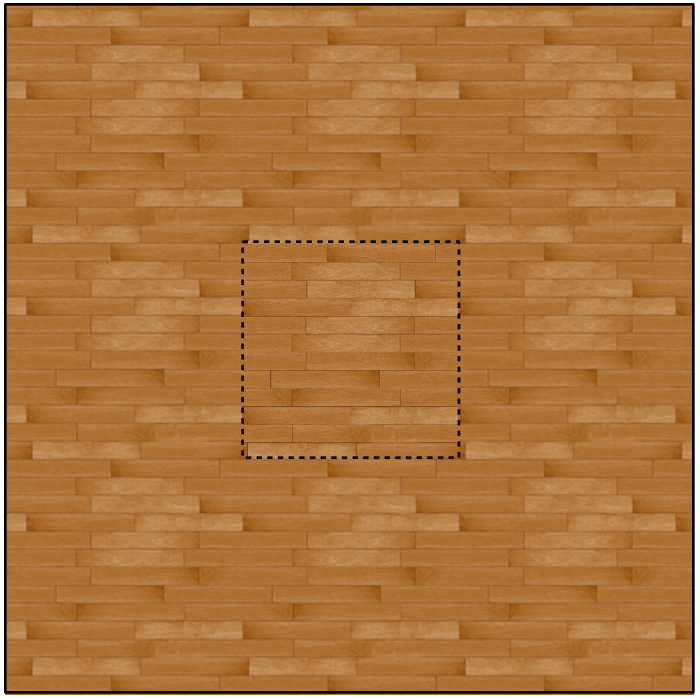


[map: Peter H. Dana]

Wrapping modes



clamp



tiled

Environment Maps

Environment Maps

- Store far-away scene in an 360 panoramic image
- Main reasons: optimization and produces realistic rendering when used as a light source

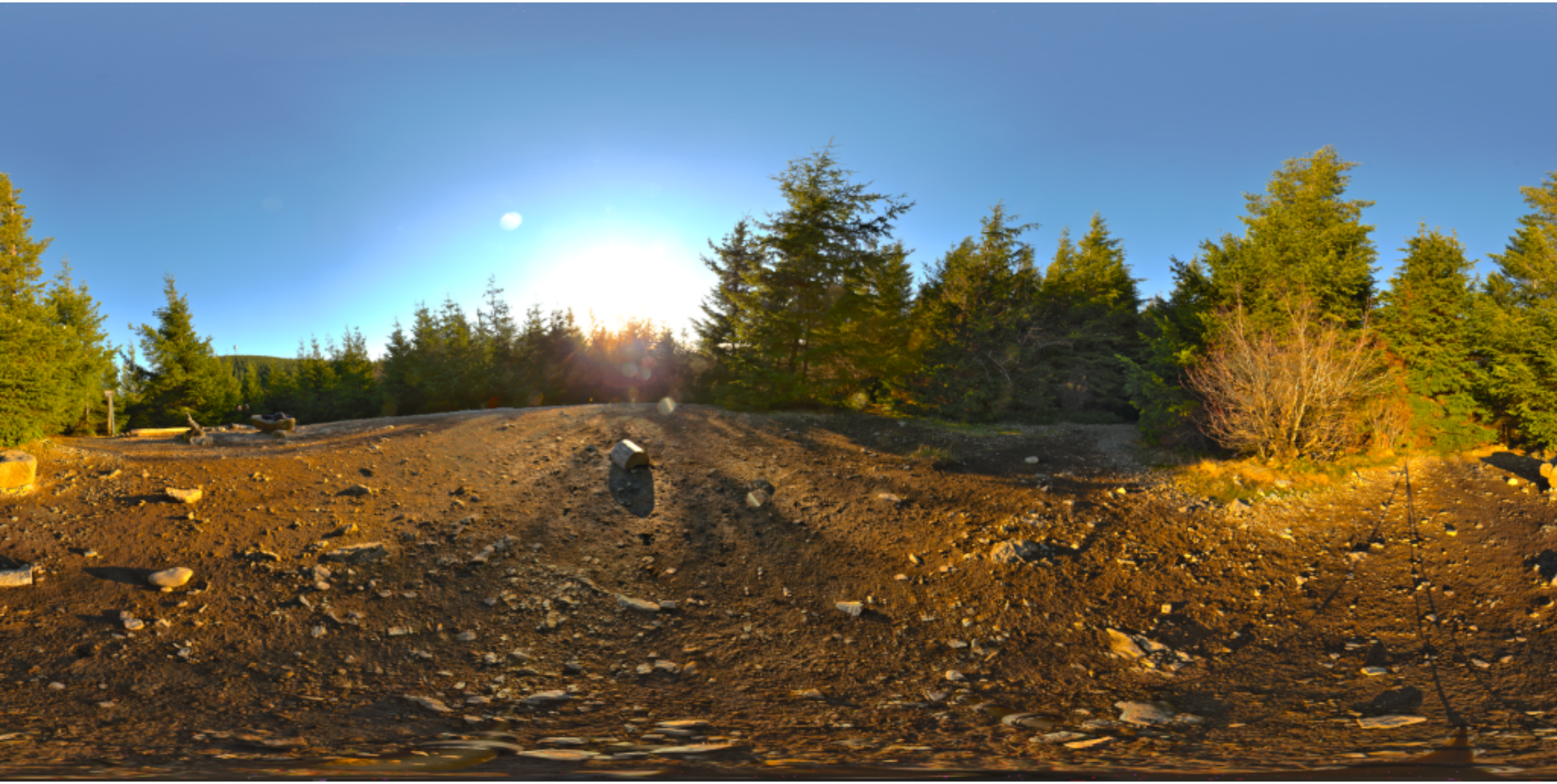
[Paul Debevec]

Environment Maps

- Store far-away scene in an 360 panoramic image
- Main reasons: optimization and produces realistic rendering when used as a light source

```
struct environment {
  frame3f frame;
  vec3f emission;           // emission
  texture* emission_tex;    // emission map
};
```

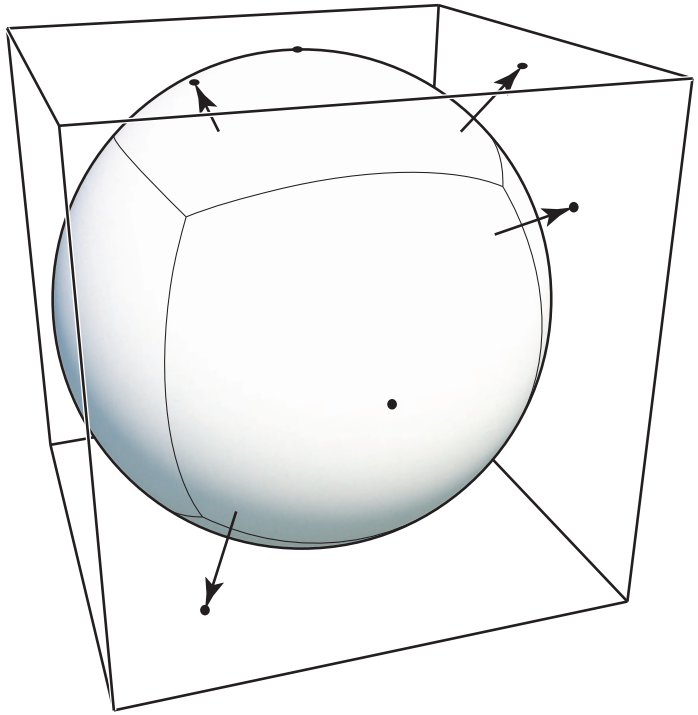

Lat-long Parametrization

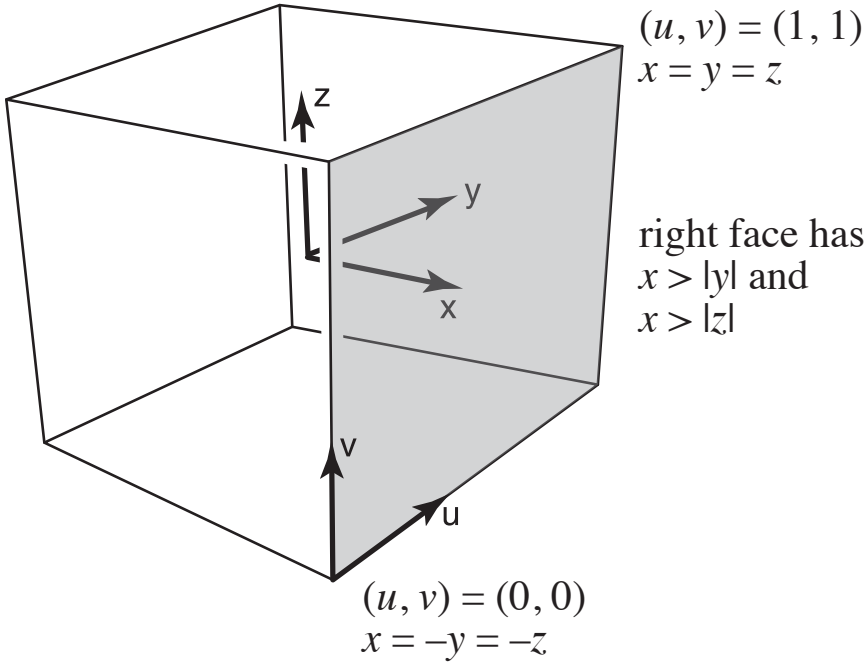


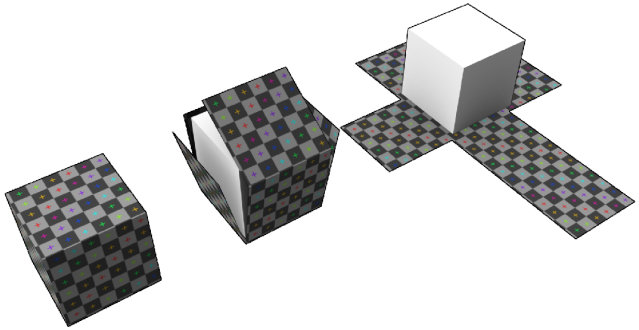
[GiantCowFilms]

Cube Map

- Cube texture with six texture maps



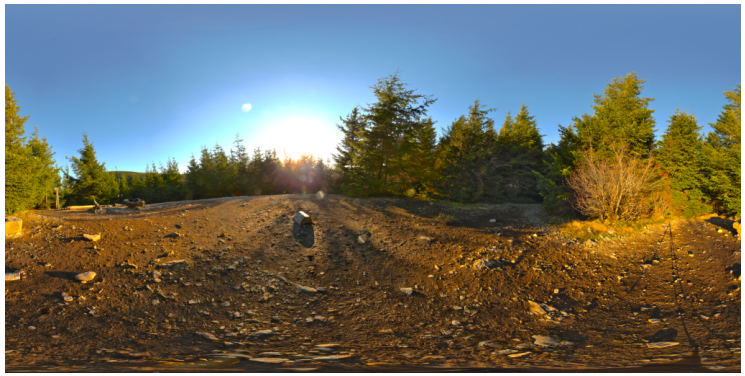




[Zepheris@WikimediaCommons]

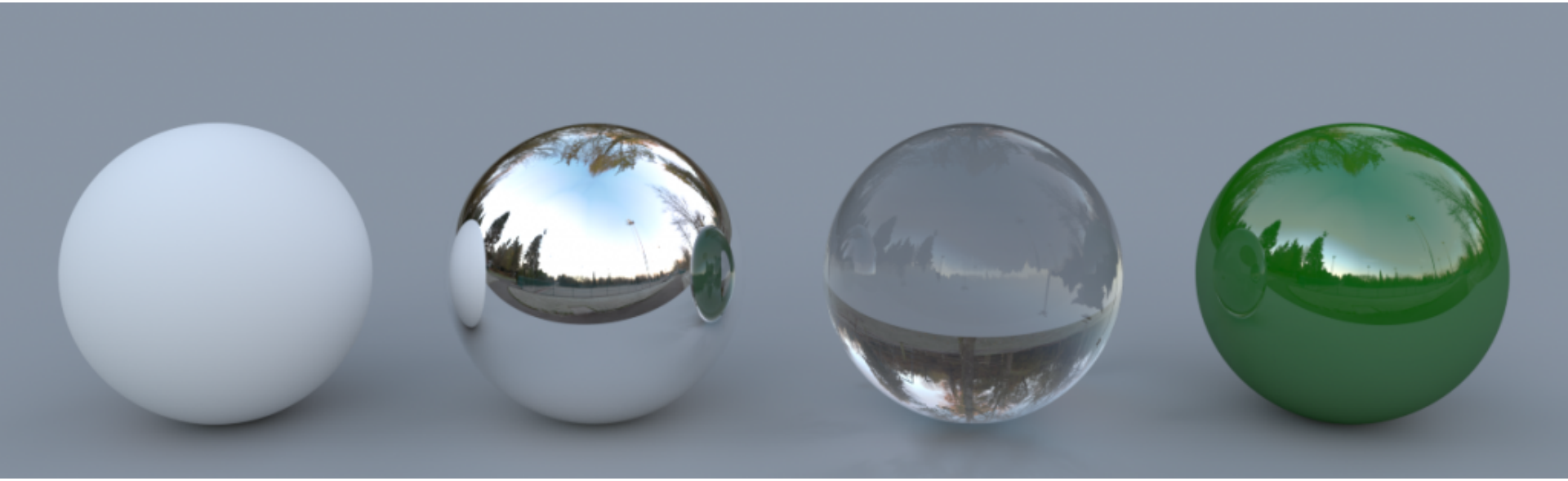
[Emil Persson]

HDRI illumination



[GiantCowFilms]

HDRI illumination



[GiantCowFilms]

Representing Scenes

Representing Objects

- Need a way to associate geometry and materials
- Option 1: Add material inside a mesh
  - mixing geometry and shading
  - cannot re-used shape data
- Option 2: Bundle a mesh, a material and a frame into an object
  - with the coordinate frame, we can reposition the mesh
  - *instancing:* can represent large scenes by placing same objects

```
struct object {
  frame3f frame;      // coordinate frame
  shape*  shape;      // shape data
  material* material; // material
};
```


Instancing

Representing Scenes

- For now, we consider scenes as lists of objects, textures and materials and include a camera and environment map
- Later, we will introduce more complex scene modeling

```
struct scene {
  vector<camera*>      cameras;
  vector<object*>      objects;
  vector<shape*>       shapes;
  vector<material*>    materials;
  vector<texture*>     textures;
  vector<environment*> environments;
};
```