

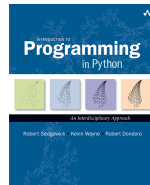
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## 1.2 Built-in Types of Data

A *data type* is a set of values and a set of operations defined on them. For example, we are familiar with numbers and with operations defined on them such as addition and multiplication. There are eight different built-in types of data in Java, mostly different kinds of numbers. We use the system type for strings of characters so frequently that we also consider it here.

<i>type</i>	<i>set of values</i>	<i>common operators</i>	<i>sample literal values</i>
int	integers	+ - * / %	99 12 2147483647
double	floating-point numbers	+ - * /	3.14 2.5 6.022e23
boolean	boolean values	&&    !	true false
char	characters		'A' '1' '%' '\n'
String	sequences of characters	+	"AB" "Hello" "2.5"

**Terminology.** We use the following code fragment to introduce some terminology:

```
int a, b, c;
a = 1234;
b = 99;
c = a + b;
```

The first line is a *declaration statement* that declares the names of three *variables* using the *identifiers* `a`, `b`, and `c` and their type to be `int`. The next three lines are *assignment statements* that change the values of the variables, using the *literals* `1234` and `99`, and the *expression* `a + b`, with the end result that `c` has the value `1333`.

**Characters and strings.** A `char` is an alphanumeric character or symbol, like the ones that you type. We usually do not perform any operations on characters other than assigning values to variables. A `String` is a sequence of characters. The most common operation that we perform on strings is known as *concatenation*: given two strings, chain them together to make a new string. For example, consider the following Java program fragment:

```
String a, b, c;
a = "Hello,";
b = " Bob";
c = a + b;
```

The first statement declares three variables to be of type `String`. The next three statements assign values to them, with the end result that `c` has the value `"Hello, Bob"`. Using string concatenation, [Ruler.java](#) prints the relative lengths of the subdivisions on a ruler.

**Integers.** An `int` is an integer (whole number) between  $-2^{31}$  and  $2^{31} - 1$  ( $-2,147,483,648$  to  $2,147,483,647$ ). We use `ints` frequently not just because they occur frequently in the real world, but also they naturally arise when expressing algorithms. Standard arithmetic operators for addition, multiplication, and division, for integers are built into Java, as illustrated in [IntOps.java](#) and the following table:

<i>expression</i>	<i>value</i>	<i>comment</i>
99	99	<i>integer literal</i>
+99	99	<i>positive sign</i>
-99	-99	<i>negative sign</i>
5 + 3	8	<i>addition</i>
5 - 3	2	<i>subtraction</i>
5 * 3	15	<i>multiplication</i>
5 / 3	1	<i>no fractional part</i>
5 % 3	2	<i>remainder</i>
1 / 0		<i>run-time error</i>
3 * 5 - 2	13	<i>* has precedence</i>
3 + 5 / 2	5	<i>/ has precedence</i>
3 - 5 - 2	-4	<i>left associative</i>
( 3 - 5 ) - 2	-4	<i>better style</i>
3 - ( 5 - 2 )	0	<i>unambiguous</i>

**Floating-point numbers.** The `double` type is for representing *floating-point* numbers, e.g., for use in scientific applications. The internal representation is like scientific notation, so that we can compute with real numbers in a huge range. We can specify a floating point number using either a string of digits with a decimal point, e.g., 3.14159 for a six-digit approximation to the mathematical constant pi, or with a notation like scientific notation, e.g., 6.022E23 for Avogadro's constant  $6.022 \times 10^{23}$ . Standard arithmetic operators for addition, multiplication, and division, for doubles are built in to Java, as illustrated in [DoubleOps.java](#) and the following table:

<i>expression</i>	<i>value</i>
3.141 + 2.0	5.141
3.141 - 2.0	1.111
3.141 / 2.0	1.5705
5.0 / 3.0	1.6666666666666667
10.0 % 3.141	0.577
1.0 / 0.0	Infinity
Math.sqrt(2.0)	1.4142135623730951
Math.sqrt(-1.0)	NaN

[Quadratic.java](#) shows the use of doubles in computing the two roots of a quadratic equation using the quadratic formula.

**Booleans.** The `boolean` type has just two values: `true` or `false`. The apparent simplicity is deceiving—booleans lie at the foundation of computer science. The most important operators defined for the `boolean` are for *and*, *or*, and *not*.

- *and*: `a && b` is true if both `a` and `b` are true, and false otherwise.
- *or*: `a || b` is true if either `a` or `b` is true (or both are true), and false otherwise
- *not*: `!a` is true if `a` is false, and false otherwise.

Although these definitions are intuitive and easy to understand, it is worthwhile to fully specify each possibility for each operation in a *truth table*.

<i>a</i>	<i>!a</i>	<i>a</i>	<i>b</i>	<i>a &amp;&amp; b</i>	<i>a    b</i>
true	false	false	false	false	false
false	true	false	true	false	true
		true	false	false	true
		true	true	true	true

**Comparisons.** The *comparison* operators are *mixed-type* operations that take operands of one type (e.g., `int` or `double`) and produce a result of type `boolean`. These operations play a critical role in the process of developing more sophisticated programs.

<i>op</i>	<i>meaning</i>	<i>true</i>	<i>false</i>
<code>==</code>	<i>equal</i>	<code>2 == 2</code>	<code>2 == 3</code>
<code>!=</code>	<i>not equal</i>	<code>3 != 2</code>	<code>2 != 2</code>
<code>&lt;</code>	<i>less than</i>	<code>2 &lt; 13</code>	<code>2 &lt; 2</code>
<code>&lt;=</code>	<i>less than or equal</i>	<code>2 &lt;= 2</code>	<code>3 &lt;= 2</code>
<code>&gt;</code>	<i>greater than</i>	<code>13 &gt; 2</code>	<code>2 &gt; 13</code>
<code>&gt;=</code>	<i>greater than or equal</i>	<code>3 &gt;= 2</code>	<code>2 &gt;= 3</code>

[LeapYear.java](#) tests whether an integer corresponds to a leap year in the Gregorian calendar.

## Library methods and APIs.

Many programming tasks involve using Java library methods in addition to the built-in operators. An *application programming interface* is a table summarizing the methods in a library.

- *Printing strings to the terminal window.*

<code>void System.out.print(String s)</code>	<i>print s</i>
<code>void System.out.println(String s)</code>	<i>print s, followed by a newline</i>
<code>void System.out.println()</code>	<i>print a newline</i>

- *Converting strings to primitive types.*

<code>int Integer.parseInt(String s)</code>	<i>convert s to an int value</i>
<code>double Double.parseDouble(String s)</code>	<i>convert s to a double value</i>
<code>long Long.parseLong(String s)</code>	<i>convert s to a long value</i>

- *Mathematical functions.*

<code>public class Math</code>	
<code>double abs(double a)</code>	<i>absolute value of a</i>
<code>double max(double a, double b)</code>	<i>maximum of a and b</i>
<code>double min(double a, double b)</code>	<i>minimum of a and b</i>
<code>double sin(double theta)</code>	<i>sine of theta</i>
<code>double cos(double theta)</code>	<i>cosine of theta</i>
<code>double tan(double theta)</code>	<i>tangent of theta</i>
<code>double toRadians(double degrees)</code>	<i>convert angle from degrees to radians</i>
<code>double toDegrees(double radians)</code>	<i>convert angle from radians to degrees</i>
<code>double exp(double a)</code>	<i>exponential (<math>e^a</math>)</i>
<code>double log(double a)</code>	<i>natural log (<math>\log_e a</math>, or <math>\ln a</math>)</i>
<code>double pow(double a, double b)</code>	<i>raise a to the bth power (<math>a^b</math>)</i>
<code>long round(double a)</code>	<i>round a to the nearest integer</i>
<code>double random()</code>	<i>random number in <math>[0, 1)</math></i>
<code>double sqrt(double a)</code>	<i>square root of a</i>
<code>double E</code>	<i>value of e (constant)</i>
<code>double PI</code>	<i>value of <math>\pi</math> (constant)</i>

You can call a method by typing its name followed by *arguments*, enclosed in parentheses and separated by commas. Here are some examples:

<i>method call</i>	<i>library</i>	<i>return type</i>	<i>value</i>
<code>Integer.parseInt("123")</code>	Integer	int	123
<code>Double.parseDouble("1.5")</code>	Double	double	1.5
<code>Math.sqrt(5.0*5.0 - 4.0*4.0)</code>	Math	double	3.0
<code>Math.log(Math.E)</code>	Math	double	1.0
<code>Math.random()</code>	Math	double	<i>random in <math>[0, 1)</math></i>
<code>Math.round(3.14159)</code>	Math	long	3
<code>Math.max(1.0, 9.0)</code>	Math	double	9.0

We often find ourselves converting data from one type to another using one of the following approaches.

## Type conversion.

We often find ourselves converting data from one type to another using one of the following approaches.

- *Explicit type conversion.* Call methods such as `Math.round()`, `Integer.parseInt()`, and `Double.parseDouble()`.

- *Automatic type conversion.* For primitive numeric types, the system automatically performs type conversion when we use a value whose type has a larger range of values than expected.
- *Explicit casts.* Java also has some built-in type conversion methods for primitive types that you can use when you are aware that you might lose information, but you have to make your intention using something called a *cast*. [RandomInt.java](#) reads an integer command-line argument  $n$  and prints a "random" integer between 0 and  $n-1$ .
- *Automatic conversions for strings.* The built-in type `String` obeys special rules. One of these special rules is that you can easily convert any type of data to a `String` by using the `+` operator.

<i>expression</i>	<i>expression type</i>	<i>expression value</i>
<code>(1 + 2 + 3 + 4) / 4.0</code>	<code>double</code>	2.5
<code>Math.sqrt(4)</code>	<code>double</code>	2.0
<code>"1234" + 99</code>	<code>String</code>	"123499"
<code>11 * 0.25</code>	<code>double</code>	2.75
<code>(int) 11 * 0.25</code>	<code>double</code>	2.75
<code>11 * (int) 0.25</code>	<code>int</code>	0
<code>(int) (11 * 0.25)</code>	<code>int</code>	2
<code>(int) 2.71828</code>	<code>int</code>	2
<code>Math.round(2.71828)</code>	<code>long</code>	3
<code>(int) Math.round(2.71828)</code>	<code>int</code>	3
<code>Integer.parseInt("1234")</code>	<code>int</code>	1234

## Exercises

1. Suppose that `a` and `b` are `int` values. What does the following sequence of statements do?

```
int t = a;
b = t;
a = b;
```

*Solution:* sets `a`, `b`, and `t` equal to the original value of `a`.

4. Suppose that `a` and `b` are `int` values. Simplify the following expression: `!(a < b) && !(a > b)`

*Solution:* `(a == b)`

5. The *exclusive or* operator `^` for `boolean` operands is defined to be `true` if they are different, `false` if they are the same. Give a truth table for this function.
6. Why does `10/3` give 3 and not 3.33333333?

*Solution:* Since both 10 and 3 are integer literals, Java sees no need for type conversion and uses integer division. You should write `10.0/3.0` if you mean the numbers to be `double` literals. If you write `10/3.0` or `10.0/3`, Java does implicit conversion to get the same result.

7. What do each of the following print?
  - a. `System.out.println(2 + "bc");` prints: 2bc
  - b. `System.out.println(2 + 3 + "bc");` prints: 5bc
  - c. `System.out.println((2+3) + "bc");` prints: 5bc
  - d. `System.out.println("bc" + (2+3));` prints: bc5
  - e. `System.out.println("bc" + 2 + 3);` prints: bc23

Explain each outcome.

8. Explain how to use [Quadratic.java](#) to find the square root of a number.

*Solution:* to find the square root of  $c$ , find the roots of  $x^2 + 0x - c$ .

16. A physics student gets unexpected results when using the code

```
double force = G * mass1 * mass2 / r * r;
```

to compute values according to the formula  $F = Gm_1m_2 / r^2$ . Explain the problem and correct the code.

*Solution:* It divides by  $r$ , then multiplies by  $r$  (instead of dividing by  $r * r$ ). Use parentheses:

```
double force = G * mass1 * mass2 / (r * r);
```

18. Write a program [Distance.java](#) that takes two integer command-line arguments  $x$  and  $y$  and prints the Euclidean distance from the point  $(x, y)$  to the origin  $(0, 0)$ .
20. Write a program [SumOfTwoDice.java](#) that prints the sum of two random integers between 1 and 6 (such as you might get when rolling dice).
21. Write a program [SumOfSines.java](#) that takes a double command-line argument  $t$  (in degrees) and prints the value of  $\sin(2t) + \sin(3t)$ .
23. Write a program [SpringSeason.java](#) that takes two `int` values  $m$  and  $d$  from the command line and prints `true` if day  $d$  of month  $m$  is between March 20 ( $m = 3, d = 20$ ) and June 20 ( $m = 6, d = 20$ ), `false` otherwise.

## Creative Exercises

25. **Wind chill.** Given the temperature  $t$  (in Fahrenheit) and the wind speed  $v$  (in miles per hour), the National Weather Service defines the [wind chill](#) to be:

$$w = 35.74 + 0.6215 t + (0.4275 t - 35.75) v^{0.16}$$

Write a program [WindChill.java](#) that takes two `double` command-line arguments  $t$  and  $v$  and prints the wind chill. Use `Math.pow(a, b)` to compute  $a^b$ . Note: the formula is not valid if  $t$  is larger than 50 in absolute value or if  $v$  is larger than 120 or less than 3 (you may assume that the values you get are in that range).

26. **Polar coordinates.** Write a program [CartesianToPolar.java](#) that converts from Cartesian to [polar coordinates](#). Your program should take two real numbers  $x$  and  $y$  on the command line and print the polar coordinates  $r$  and  $\theta$ . Use the Java method `Math.atan2(y, x)`, which computes the arctangent value of  $y/x$  that is in the range from  $-\pi$  to  $\pi$ .
29. **Day of the week.** Write a program [DayOfWeek.java](#) that takes a date as input and prints the day of the week that date falls on. Your program should take three command-line arguments:  $m$  (month),  $d$  (day), and  $y$  (year). For  $m$  use 1 for January, 2 for February, and so forth. For output print 0 for Sunday, 1 for Monday, 2 for Tuesday, and so forth. Use the following formulas, for the Gregorian calendar (where  $/$  denotes integer division):

$$\begin{aligned} y_0 &= y - (14 - m) / 12 \\ x &= y_0 + y_0/4 - y_0/100 + y_0/400 \\ m_0 &= m + 12 \times ((14 - m) / 12) - 2 \\ d_0 &= (d + x + 31m_0 / 12) \bmod 7 \end{aligned}$$

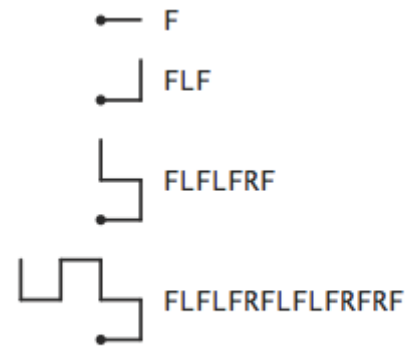
For example, on which day of the week was August 2, 1953?

$$\begin{aligned} y_0 &= 1953 - 0 = 1953 \\ x &= 1953 + 1953/4 - 1953/100 + 1953/400 = 2426 \\ m_0 &= 8 + 12 \times 0 - 2 = 6 \\ d_0 &= (2 + 2426 + (31 \times 6) / 12) \bmod 7 = 2443 \bmod 7 = 0 \quad (\text{Sunday}) \end{aligned}$$

30. **Uniform random numbers.** Write a program [Stats5.java](#) that prints five uniform random values between 0 and 1, their average value, and their minimum and maximum value. Use `Math.random()`, `Math.min()`, and `Math.max()`.



34. **Three-sort.** Write a program [ThreeSort.java](#) that takes three int values from the command line and prints them in ascending order. Use `Math.min()` and `Math.max()`.
35. **Dragon curves.** Write a program [Dragon.java](#) to print the instructions for drawing the [dragon curves](#) of order 0 through 5. The instructions are strings of the characters F, L, and R, where F means "draw line while moving 1 unit forward", L means "turn left", and R means turn right. A dragon curve of order  $n$  is formed when you fold a strip of paper in half  $n$  times, then unfold to right angles. The key to solving this problem is to note that a curve of order  $n$  is a curve of order  $n-1$  followed by an L followed by a curve of order  $n-1$  traversed in reverse order, and then to figure out a similar description of the reverse curve.



*Dragon curves of order 0, 1, 2, and 3*

## Web Exercises

- Write a program [Swap.java](#) that takes two integer command-line arguments  $a$  and  $b$  and swaps their values using the swapping idiom described on p. 17. After each assignment statement, use `System.out.println()` to print a trace of the variables.
- What does the following statement do where `grade` is a variable of type `int`?

```
boolean isA = (90 <= grade <= 100);
```

*Solution:* Syntax error since `<=` is a binary operator. You can rewrite the expression as `(90 <= grade && grade <= 100)`.

- RGB to YIQ color converter.** Write a program `RGBtoYIQ.java` that takes an RGB color (three integers between 0 and 255) and transforms it to a [YIQ color](#) (three different real numbers  $Y$ ,  $I$ , and  $Q$ , with  $0 \leq Y \leq 1$ ,  $-0.5957 \leq I \leq 0.5957$ , and  $-0.5226 \leq Q \leq 0.5226$ ). Write a program `YIQtoRGB.java` that applies the inverse transformation.
- CMYK to RGB color matching.** Write a program `CMYKtoRGB` that reads in four command line inputs  $C$ ,  $M$ ,  $Y$ , and  $K$  between 0 and 1, and prints the corresponding RGB parameters. Devise the appropriate formula by "inverting" the RGB to CMYK conversion formula.
- What does the following code fragment print?

```
double x = (double) (3/5);
System.out.println(x);
```

*Solution:* It prints `0.0` since the integer division is done before the cast.

- Why doesn't the following program print  $4294967296 = 2^{32}$ ?

```
int x = 65536;
long y = x * x;
System.out.println(y);
```

*Solution:* The product of the two `int` values is computed as an `int`, and then automatically converted to a `long`. But  $65536 * 65536 = 2^{32}$  overflows a 32 bit `int` before it gets converted.

- What is the value of `(Math.sqrt(2) * Math.sqrt(2) == 2)`?
- Write a program [DivideByZero.java](#) to see what happens when you divide an `int` or `double` by zero.

*Solution:*



- $(17 / 0)$  and  $(17 \% 0)$  result in a division by zero exception;
  - $(17.0 / 0.0)$  results in a value `Infinity`;
  - $(17.0 \% 0.0)$  results in a value `NaN` that stands for "not a number."
9. **Guess the biggest number.** Consider the following game. Alice writes down two integers between 0 and 100 on two cards. Bob gets to select one of the two cards and see its value. After looking at the value, Bob commits to one of the two cards. If he chooses a card with the largest value, he wins; otherwise he loses. Devise a strategy (and corresponding computer program) for Bob so that he guarantees to win strictly more than half the time.
10. **Fibonacci word.** Write a program `FibonacciWord.java` that prints the Fibonacci word of order 0 through 10.  $f(0) = "a"$ ,  $f(1) = "b"$ ,  $f(2) = "ba"$ ,  $f(3) = "bab"$ ,  $f(4) = "babba"$ , and in general  $f(n) = f(n-1)$  followed by  $f(n-2)$ . Use string concatenation.
11. **Standard deviation.** Modify Exercise 1.2.30 so that it prints the sample standard deviation in addition to the average.
12. Write a program that reads in three parameters and prints `true` if all three are equal, and `false` otherwise.
13. What happens if you compile `LeapYear.java` and execute it with
1. `java LeapYear`
  2. `java LeapYear 1975.5`
  3. `java LeapYear -1975`
  4. `java LeapYear 1975 1976 1977`
14. What does the compiler do if you try to write the following expression:

```
int a = 27 * "three";
```

15. What does the compiler do if you try to write the following expression:

```
double x;
System.out.println(x);
```

*Solution:* The compiler complains that the variable `x` might not have been initialized. Variables within `main` are not automatically initialized.

16. What does the following code fragment print.

```
int threeInt = 3;
int fourInt = 4;
double threeDouble = 3.0;
double fourDouble = 4.0;
System.out.println(threeInt / fourInt);
System.out.println(threeInt / fourDouble);
System.out.println(threeDouble / fourInt);
System.out.println(threeDouble / fourDouble);
```

17. Write a program that takes four real-valued command line parameters `x1`, `y1`, `x2`, and `y2` and prints the Euclidean distance between the points  $(x1, y1)$  and  $(x2, y2)$ . Use `Math.sqrt()`.
18. Write a program `Ordered.java` that reads in three integer command line arguments, `x`, `y`, and `z`. Create a boolean variable `b` that is `true` if the three values are either in ascending or in descending order, and `false` otherwise. Print the variable `b`.
19. Write a program [Divisibility.java](#) that reads in two command line inputs and prints `true` if both are divisible by 7, and `false` otherwise.
20. **Area of a triangle.** Write a program `TriangleArea.java` that takes three command line inputs `a`, `b`, and `c`, representing the side lengths of a triangle, and prints the area of the triangle using Heron's formula:  $\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$ , where  $s = (a + b + c) / 2$ .

21. **Equatorial to horizontal coordinates.** The *equatorial coordinate system* is widely used by astronomers to indicate the position of a star on the celestial sphere. The position is specified by its declination  $\delta$ , its hour angle  $H$ , and its latitude  $\phi$ . The *horizontal coordinate system* (a.k.a. Alt/Az coordinate system) is useful for determining the setting/rising time of celestial objects. The position is specified by its altitude (vertical angle from the horizon) and its azimuth (horizontal angle). Given a star's position using equatorial coordinates, find its position in horizontal coordinates using the formulas below.

$$\begin{aligned}\text{Altitude} &= \arcsin(\sin \phi \sin \delta + \cos \phi \cos \delta \cos H) \\ \text{Azimuth} &= \arccos((\cos \phi \sin \delta - \sin \phi \cos \delta \cos H) / \cos(\text{Altitude}))\end{aligned}$$

22. **Body mass index.** The [body mass index](#) (BMI) is the ratio of the weight of a person (in kilograms) to the square of the height (in meters). Write a program `BMI.java` that takes two command-line arguments, `weight` and `height`, and prints the BMI.
23. **Temperature conversion.** What is wrong with the following code fragment to convert from Fahrenheit (F) to Celsius (C)?

```
double C = (F - 32) * (5 / 9);
```

24. **Exponentiation.** What is wrong with the following code fragment to compute  $a^2$ , where `a` is of type `double`?

```
double b = a^2;
```

*Solution:* In Java, `^` does not mean exponentiation (it's the exclusive or function from logic). Use `a*a` instead. To compute  $a^x$ , use `Math.pow(a, x)`. Note that `Math.pow()` returns a `double` so that you would need an explicit cast if `a` and `b` in the above example were integers.

25. What of the following statements is legal?

```
boolean b = 1;
boolean b = true;
boolean b = "true";
boolean b = True;
```

*Solution:* Only the second one.

26. Barring overflow, give a code fragment to compute the maximum of two integers `a` and `b` without using `Math.max()` or `if`.

```
int max = (a + b + Math.abs(a - b)) / 2;
```

27. **Discriminant of cubic polynomial.** Given the coefficients `b`, `c`, and `d` of the cubic polynomial  $x^3 + bx^2 + cx + d$ , write an expression to compute the *discriminant*  $b^2c^2 - 4c^3 - 4b^3d - 27d^2 + 18bcd$ .
28. **Barycenter.** In a two-body system, the [barycenter](#) is the center of gravity about which the two celestial bodies orbit each other. Given the masses  $m_1$  and  $m_2$  of two bodies, and the shortest distance  $a$  between the two bodies, write a program to compute the distance from the center of the first (more massive) body to the barycenter:  $r_1 = a m_2 / (m_1 + m_2)$ .

Here are a few examples. Masses are in earth-mass units, distances are in kilometers.

Earth-moon:  $m_1 = 1, m_2 = .0123, a = 384,000, r_1 = 4,670, R_1 = 6,380$ .

Pluto-Charon:  $m_1 = .0021, m_2 = .000254, a = 19,600, r_1 = 2,110, R_1 = 1,150$ .

Sun-Earth:  $m_1 = 333,000, m_2 = 1, a = 150,000,000, r_1 = 449, R_1 = 696,000$ .

Note that if  $r_1$  is less than the radius of the first body  $R_1$ , then the barycenter lies within the first body.

29. **Poison parentheses.** Find a legal Java expression that leads to a compile-time error when you add parentheses around a subexpression to document the order of evaluation that would take place in their absence.

*Solution:* The literal value 2147483648 ( $2^{31}$ ) is permitted only as an operand of the unary minus operator, i.e., -2147483648. Enclosing it in parentheses, i.e., -(2147483648), leads to a compile-time error. Similar ideas with the literal value 9223372036854775808L ( $2^{63}$ ).

30. **Car loan payments.** Write a program [CarLoan.java](#) that reads in three command-line arguments P, Y, and R and calculates the monthly payments you would have to make over Y years to pay off a P dollar loan at R per cent interest compounded monthly. The formula is

$$\text{payment} = \frac{P r}{1 - (1 + r)^{-n}}, \quad \text{where } n = 12 * Y, \quad r = (R / 100) / 12$$

*Caveat:* in Chapter 9, we will consider more accurate ways to compute this quantity, so before you go off to run an online bank, be sure to learn about roundoff error.

31. Write a program [Trig.java](#) to illustrate various trigonometric functions in the `Math` library, such as `Math.sin()`, `Math.cos()`, and `Math.toRadians()`.

*Last modified on April 08, 2019.*

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