

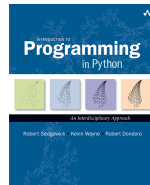
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## 1.3 Conditionals and Loops

In the programs that we have examined to this point, each of the statements is executed once, in the order given. Most programs are more complicated because the sequence of statements and the number of times each is executed can vary. We use the term *control flow* to refer to statement sequencing in a program.

### If statements.

Most computations require different actions for different inputs.

- The following code fragment uses an `if` statement to put the smaller of two `int` values in `x` and the larger of the two values in `y`, by exchanging the values in the two variables if necessary.

```

boolean
expression
  ↓
if ( x > y )
{
sequence of statements → int t = x;
                        x = y;
                        y = t;
}

```

- [Flip.java](#) uses `Math.random()` and an `if-else` statement to print the results of a coin flip.
- The table below summarizes some typical situations where you might need to use an `if` or `if-else` statement.

<i>absolute value</i>	<code>if (x &lt; 0) x = -x;</code>
<i>put the smaller value in x and the larger value in y</i>	<pre> if (x &gt; y) {     int t = x;     x = y;     y = t; } </pre>
<i>maximum of x and y</i>	<pre> if (x &gt; y) max = x; else      max = y; </pre>
<i>error check for division operation</i>	<pre> if (den == 0) System.out.println("Division by zero"); else          System.out.println("Quotient = " + num/den); </pre>
<i>error check for quadratic formula</i>	<pre> double discriminant = b*b - 4.0*c; if (discriminant &lt; 0.0) {     System.out.println("No real roots"); } else {     System.out.println((-b + Math.sqrt(discriminant))/2.0);     System.out.println((-b - Math.sqrt(discriminant))/2.0); } </pre>

## While loops.

Many computations are inherently repetitive. The `while` loop enables us to execute a group of statements many times. This enables us to express lengthy computations without writing lots of code.

- The following code fragment computes the largest power of 2 that is less than or equal to a given positive integer `n`.

```

initialization is a
separate statement
  ↓
int power = 1;
while ( loop-continuation
      condition
      power <= n/2 )
{
braces are optional
when body is a single
statement → power = 2*power;
      body
}

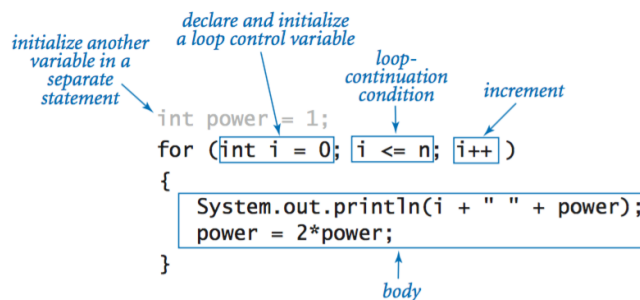
```

- [TenHellos.java](#) prints "Hello World" 10 times.
- [PowersOfTwo.java](#) takes an integer command-line argument  $n$  and prints all of the powers of 2 less than or equal to  $n$ .

## For loops.

The *for loop* is an alternate Java construct that allows us even more flexibility when writing loops.

- *For notation.* Many loops follow the same basic scheme: initialize an index variable to some value and then use a `while` loop to test an exit condition involving the index variable, using the last statement in the `while` loop to modify the index variable. Java's `for` loop is a direct way to express such loops.



- *Compound assignment idioms.* The idiom `i++` is a shorthand notation for `i = i + 1`.
- *Scope.* The *scope* of a variable is the part of the program that can refer to that variable by name. Generally the scope of a variable comprises the statements that follow the declaration in the same block as the declaration. For this purpose, the code in the `for` loop header is considered to be in the same block as the `for` loop body.

## Nesting.

The `if`, `while`, and `for` statements have the same status as assignment statements or any other statements in Java; that is, we can use them wherever a statement is called for. In particular, we can use one or more of them in the body of another statement to make *compound statements*. To emphasize the nesting, we use indentation in the program code.

- [DivisorPattern.java](#) has a `for` loop whose body contains a `for` loop (whose body is an `if-else` statement) and a print statement. It prints a pattern of asterisks where the  $i$ th row has an asterisk in each position corresponding to divisors of  $i$  (the same holds true for the columns).
- [MarginalTaxRate.java](#) computes the marginal tax rate for a given income. It uses several nested `if-else` statements to test from among a number of mutually exclusive possibilities.

## Loop examples.

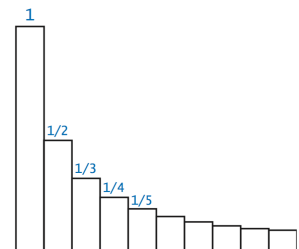
compute the largest power of 2 less than or equal to $n$	<pre>int power = 1; while (power &lt;= n/2)     power = 2*power; System.out.println(power);</pre>
compute a finite sum ( $1 + 2 + \dots + n$ )	<pre>int sum = 0; for (int i = 1; i &lt;= n; i++)     sum += i; System.out.println(sum);</pre>
compute a finite product ( $n! = 1 \times 2 \times \dots \times n$ )	<pre>int product = 1; for (int i = 1; i &lt;= n; i++)     product *= i; System.out.println(product);</pre>
print a table of function values	<pre>for (int i = 0; i &lt;= n; i++)     System.out.println(i + " " + 2*Math.PI*i/n);</pre>
compute the ruler function (see PROGRAM 1.2.1)	<pre>String ruler = "1"; for (int i = 2; i &lt;= n; i++)     ruler = ruler + " " + i + " " + ruler; System.out.println(ruler);</pre>

## Applications.

The ability to program with loops and conditionals immediately opens up the world of computation to us.

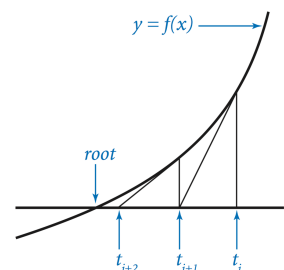
- *Ruler subdivisions.* [RulerN.java](#) takes an integer command-line argument  $n$  and prints the string of ruler subdivision lengths. This program illustrates one of the essential characteristics of loops—the program could hardly be simpler, but it can produce a huge amount of output.
- *Finite sums.* The computational paradigm used in [PowersOfTwo.java](#) is one that you will use frequently. It uses two variables—one as an index that controls a loop, and the other to accumulate a computational result. Program [HarmonicNumber.java](#) uses the same paradigm to evaluate the sum

$$H_n = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$$



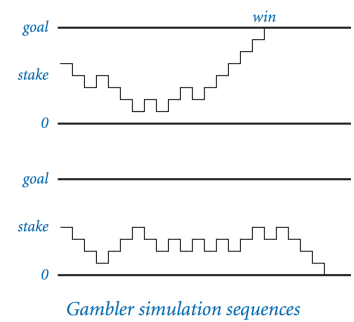
These numbers, which are known as the *harmonic numbers*, arise frequently in the analysis of algorithms.

- *Newton's method.* [Sqrt.java](#) uses a classic iterative technique known as *Newton's method* to compute the square root of a positive number  $x$ : Start with an estimate  $t$ . If  $t$  is equal to  $x/t$  (up to machine precision), then  $t$  is equal to a square root of  $x$ , so the computation is complete. If not, refine the estimate by replacing  $t$  with the average of  $t$  and  $x/t$ . Each time we perform this update, we get closer to the desired answer.
- *Number conversion.* [Binary.java](#) prints the binary (base 2) representation of the decimal number typed as the command-line argument. It is based on decomposing the number into a sum of powers of 2. For example, the binary representation of 106 is  $1101010_2$ , which is the same as saying that  $106 = 64 + 32 + 8 + 2$ . To compute the binary representation of  $n$ , we consider the powers of 2 less than or equal to  $n$  in decreasing order to determine which belong in the binary decomposition (and therefore correspond to a 1 bit in the binary representation).
- *Gambler's ruin.* Suppose a gambler makes a series of fair \$1 bets, starting with \$50, and continue to play until she either goes broke or has \$250. What are the chances that she will go home with \$250, and how many bets might she expect to make before winning or losing? [Gambler.java](#) is a simulation that can help answer these questions. It takes three command-line arguments, the initial stake (\$50), the



goal amount (\$250), and the number of times we want to simulate the game.

- *Prime factorization.* [Factors.java](#) takes an integer command-line argument  $n$  and prints its prime factorization. In contrast to many of the other programs that we have seen (which we could do in a few minutes with a calculator or pencil and paper), this computation would not be feasible without a computer.



## Other conditional and loop constructs.

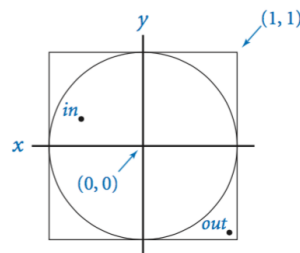
To be complete, we consider four more Java constructs related to conditionals and loops. They are used much less frequently than the `if`, `while`, and `for` statements that we've been working with, but it is worthwhile to be aware of them.

- *Break statements.* In some situations, we want to immediately exit a loop without letting it run to completion. Java provides the `break` statement for this purpose. [Prime.java](#) takes an integer command-line argument  $n$  and prints `true` if  $n$  is prime, and `false` otherwise. There are two different ways to leave this loop: either the `break` statement is executed (because  $n$  is not prime) or the loop-continuation condition is not satisfied (because  $n$  is prime).

Note that the `break` statement does not apply to `if` or `if-else` statements. In a [famous programming bug](#), the U.S. telephone network crashed because a programmer intended to use a `break` statement to exit a complicated `if` statement.

- *Continue statements.* Java also provides a way to skip to the next iteration of a loop: the `continue` statement. When a `continue` is executed within the body of a `for` loop, the flow of control transfers directly to the increment statement for the next iteration of the loop.
- *Switch statements.* The `if` and `if-else` statements allow one or two alternatives. Sometimes, a computation naturally suggests more than two mutually exclusive alternatives. Java provides the `switch` statement for this purpose. [NameOfDay.java](#) takes an integer between 0 and 6 as a command-line argument and uses a `switch` statement to print the corresponding name of the day (Sunday to Saturday).
- *Do-while loops.* A `do-while` loop is almost the same as a `while` loop except that the loop-continuation condition is omitted the first time through the loop. [RandomPointInCircle.java](#) sets  $x$  and  $y$  so that  $(x, y)$  is randomly distributed inside the circle centered at  $(0, 0)$  with radius 1.

```
do
{ // Scale x and y to be random in (-1, 1).
  x = 2.0*Math.random() - 1.0;
  y = 2.0*Math.random() - 1.0;
} while (Math.sqrt(x*x + y*y) > 1.0);
```



With `Math.random()` we get points that are randomly distributed in the 2-by-2 square center at  $(0, 0)$ . We just generate points in this region until we find one that lies inside the unit disk. We always want to generate at least one point so a `do-while` loop is most appropriate. We must declare  $x$  and  $y$  outside the loop since we will want to access their values after the loop terminates.

We don't use the following two flow control statements in this textbook, but include them here for completeness.

- *Conditional operator.* The conditional operator `?:` is a ternary operator (three operands) that enables you to embed a conditional within an expression. The three operands are separated by the `?` and `:` symbols. If the first operand (a boolean expression) is `true`, the result has the value of the second expression; otherwise it has the value of the third expression.

```
int min = (x < y) ? x : y;
```

- *Labeled break and continue statements.* The `break` and `continue` statements apply to the innermost `for` or `while` loop. Sometimes we want to jump out of several levels of nested loops. Java provides the labeled `break` and labeled `continue` statements to accomplish this. Here is an [example](#).

## Exercises

1. Write a program [AllEqual.java](#) that takes three integer command-line arguments and prints `equal` if all three are equal, and `not equal` otherwise.
6. Write a program [RollLoadedDie.java](#) that prints the result of rolling a loaded die such that the probability of getting a 1, 2, 3, 4, or 5 is  $1/8$  and the probability of getting a 6 is  $3/8$ .
8. Rewrite [TenHellos.java](#) to make a program [Hellos.java](#) that takes the number of lines to print as a command-line argument. You may assume that the argument is less than 1000. Hint: consider using `i % 10` and `i % 100` to determine whether to use "st", "nd", "rd", or "th" for printing the *i*th Hello.
9. Write a program [FivePerLine.java](#) that, using one `for` loop and one `if` statement, prints the integers from 1000 to 2000 with five integers per line. *Hint:* use the `%` operator.
12. Write a program [FunctionGrowth.java](#) that prints a table of the values of  $\ln n$ ,  $n$ ,  $n \ln n$ ,  $n^2$ ,  $n^3$ , and  $2^n$  for  $n = 16, 32, 64, \dots, 2048$ . Use tabs ('`\t`' characters) to line up columns.
13. What is the value of `m` and `n` after executing the [following code](#)?

```
int n = 123456789;
int m = 0;
while (n != 0) {
    m = (10 * m) + (n % 10);
    n = n / 10;
}
```

14. What does the [following code](#) print out?

```
int f = 0, g = 1;
for (int i = 0; i <= 15; i++) {
    System.out.println(f);
    f = f + g;
    g = f - g;
}
```

18. Unlike the harmonic numbers, the sum  $1/1 + 1/4 + 1/9 + 1/16 + \dots + 1/n^2$  *does* converge to a constant as  $n$  grows to infinity. (Indeed, the constant is  $\pi^2 / 6$ , so this formula can be used to estimate the value of  $\pi$ .) Which of the following [for loops](#) computes this sum? Assume that `n` is an `int` initialized to 1000000 and `sum` is a `double` initialized to 0.

- (a) 

```
for (int i = 1; i <= n; i++)
    sum = sum + 1 / (i * i);
```
- (b) 

```
for (int i = 1; i <= n; i++)
    sum = sum + 1.0 / i * i;
```
- (c) 

```
for (int i = 1; i <= n; i++)
    sum = sum + 1.0 / (i * i);
```
- (d) 

```
for (int i = 1; i <= n; i++)
    sum = sum + 1 / (1.0 * i * i);
```



21. Modify [Binary.java](#) to get a program `ModifyKary.java` that takes a second command-line argument `k` and converts the first argument to base `k`. Assume the base is between 2 and 16. For bases greater than 10, use the letters A through F to represent the 11th through 16th digits, respectively.
22. Write a program [code fragment](#) that puts the binary representation of a positive integer `n` into a `String` variable `s`.

## Creative Exercises

34. **Ramanujan's taxi.** S. Ramanujan was an Indian mathematician who became famous for his intuition for numbers. When the English mathematician G. H. Hardy came to visit him in the hospital one day, Hardy remarked that the number of his taxi was 1729, a rather dull number. To which Ramanujan replied, "No, Hardy! No, Hardy! It is a very interesting number. It is the smallest number expressible as the sum of two cubes in two different ways." Verify this claim by writing a program [Ramanujan.java](#) that takes an integer command-line argument `n` and prints all integers less than or equal to `n` that can be expressed as the sum of two cubes in two different ways - find distinct positive integers `a`, `b`, `c`, and `d` such that  $a^3 + b^3 = c^3 + d^3$ . Use four nested for loops.

Now, the license plate 87539319 seems like a rather dull number. Determine why it's not.

35. **Checksums.** The International Standard Book Number ([ISBN](#)) is a 10 digit code that uniquely specifies a book. The rightmost digit is a *checksum* digit which can be uniquely determined from the other 9 digits from the condition that  $d_1 + 2d_2 + 3d_3 + \dots + 10d_{10}$  must be a multiple of 11 (here  $d_i$  denotes the *i*th digit from the right). The checksum digit  $d_1$  can be any value from 0 to 10: the ISBN convention is to use the value X to denote 10. *Example:* the checksum digit corresponding to 020131452 is 5 since is the only value of  $d_1$  between 0 and 10 for which  $d_1 + 2*2 + 3*5 + 4*4 + 5*1 + 6*3 + 7*1 + 8*0 + 9*2 + 10*0$  is a multiple of 11. Write a program [ISBN.java](#) that takes a 9-digit integer as a command-line argument, computes the checksum, and prints the 10-digit ISBN number. It's ok if you don't print any leading 0s.
38. **Exponential function.** Assume that `x` is a positive variable of type `double`. Write a program [Exp.java](#) that computes  $e^x$  using the Taylor series expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3} + \frac{x^4}{4!} + \dots$$

39. **Trigonometric functions.** Write two programs [Sin.java](#) and `Cos.java` that compute  $\sin x$  and  $\cos x$  using the Taylor series expansions

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

42. **Game simulation.** In the game show *Let's Make a Deal*, a contestant is presented with three doors. Behind one door is a valuable prize, behind the other two are gag gifts. After the contestant chooses a door, the host opens up one of the other two doors (never revealing the prize, of course). The contestant is then given the opportunity to switch to the other unopened door. Should the contestant do so? Intuitively, it might seem that the contestant's initial choice door and the other unopened door are equally likely to contain the prize, so there would be no incentive to switch. Write a program [MonteHall.java](#) to test this intuition by simulation. Your program should take an integer command-line argument `n`, play the game `n` times using each of the two strategies (switch or don't switch) and print the chance of success for each strategy. Or you can [play the game here](#).
46. **Euler's sum-of-powers conjecture.** In 1769 Leonhard Euler formulated a generalized version of Fermat's Last Theorem, conjecturing that at least  $n$   $n$ th powers are needed to obtain a sum that is itself an  $n$ th power, for  $n > 2$ . Write a program [Euler.java](#) to disprove Euler's conjecture (which stood until 1967), using a quintuply nested loop to find four positive integers whose 5th power sums to the 5th



power of another positive integer. That is, find  $a, b, c, d$ , and  $e$  such that  $a^5 + b^5 + c^5 + d^5 = e^5$ . Use the long data type.

## Web Exercises

1. Write a program [RollDie.java](#) that generates the result of rolling a fair six-sided die (an integer between 1 and 6).
2. Write a program that takes three integer command-line arguments  $a, b$ , and  $c$  and print the number of distinct values (1, 2, or 3) among  $a, b$ , and  $c$ .
3. Write a program that takes five integer command-line arguments and prints the *median* (the third largest one).
4. (hard) Now, try to compute the median of 5 elements such that when executed, it never makes more than 6 total comparisons.
5. How can I create an infinite loop with a for loop?

*Solution:* `for(;;)` is the same as `while(true)`.

6. What's wrong with the following loop?

```
boolean done = false;
while (done = false) {
    ...
}
```

The while loop condition uses `=` instead of `==` so it is an assignment statement (which makes `done` always `false` and the body of the loop will never be executed). It's better to style to avoid using `==`.

```
boolean done = false;
while (!done) {
    ...
}
```

7. What's wrong with the following loop that is intended to compute the sum of the integers 1 through 100?

```
for (int i = 1; i <= N; i++) {
    int sum = 0;
    sum = sum + i;
}
System.out.println(sum);
```

The variable `sum` should be defined outside the loop. By defining it inside the loop, a new variable `sum` is initialized to 0 each time through the loop; also it is not even accessible outside the loop.

8. Write a program [Hurricane.java](#) that takes the wind speed (in miles per hour) as an integer command-line argument and prints whether it qualifies as a hurricane, and if so, whether it is a Category 1, 2, 3, 4, or 5 hurricane. Below is a table of the wind speeds according to the [Saffir-Simpson scale](#).

Category	Wind Speed (mph)
1	74 - 95
2	96 - 110
3	111 - 130

- 4            131 - 155
- 5            155 and above

9. What is wrong with the following code fragment?

```
double x = -32.2;
boolean isPositive = (x > 0);
if (isPositive = true) System.out.println(x + " is positive");
else System.out.println(x + " is not positive");
```

*Solution:* It uses the assignment operator = instead of the equality operator ==. A better solution is to write `if (isPositive)`.

10. Change/add one character so that the following program prints 20 xs. There are two different solutions.

```
int i = 0, n = 20;
for (i = 0; i < n; i--)
    System.out.print("x");
```

*Solution:* Replace the `i < n` condition with `-i < n`. Replace the `i--` with `n--`. ( In C, there is a third: replace the `<` with a `+`.)

11. What does the following code fragment do?

```
if (x > 0);
    System.out.println("positive");
```

*Solution:* always prints `positive` regardless of the value of `x` because of the extra semicolon after the `if` statement.

12. **RGB to HSB converter.** Write a program `RGBtoHSB.java` that takes an RGB color (three integers between 0 and 255) and transforms it to an [HSB color](#) (three different integers between 0 and 255).

Write a program `HSBtoRGB.java` that applies the inverse transformation.

13. **Boys and girls.** A couple beginning a family decides to keep having children until they have at least one of either sex. Estimate the average number of children they will have via simulation. Also estimate the most common outcome (record the frequency counts for 2, 3, and 4 children, and also for 5 and above). Assume that the probability  $p$  of having a boy or girl is  $1/2$ .

14. What does the following program do?

```
public static void main(String[] args) {
    int N = Integer.parseInt(args[0]);
    int x = 1;
    while (N >= 1) {
        System.out.println(x);
        x = 2 * x;
        N = N / 2;
    }
}
```

*Solution:* Prints all of the powers of 2 less than or equal to  $n$ .

15. **Boys and girls.** Repeat the previous question, but assume the couple keeps having children until they have another child which is of the same sex as the first child. How does your answer change if  $p$  is different from  $1/2$ ?

*Surprisingly, the average number of children is 2 if  $p = 0$  or 1, and 3 for all other values of  $p$ . But the most likely value is 2 for all values of  $p$ .*

16. Given two positive integers  $a$  and  $b$ , what result does the following code fragment leave in  $c$

```
c = 0;
while (b > 0) {
    if (b % 2 == 1) c = c + a;
    b = b / 2;
    a = a + a;
}
```

*Solution:*  $a * b$ .

17. Write a program using a loop and four conditionals to print

```
12 midnight
1am
2am
...
12 noon
1pm
...
11pm
```

18. What does the following program print?

```
public class Test {
    public static void main(String[] args) {
        if (10 > 5);
        else; {
            System.out.println("Here");
        };
    }
}
```

19. Alice tosses a fair coin until she sees two consecutive heads. Bob tosses another fair coin until he sees a head followed by a tail. Write a program to estimate the probability that Alice will make fewer tosses than Bob? *Solution:* [39/121](#).
20. Rewrite [DayOfWeek.java](#) from Exercise 1.2.29 so that it prints the day of the week as Sunday, Monday, and so forth instead of an integer between 0 and 6. Use a `switch` statement.
21. **Number-to-English.** Write a program to read in a command line integer between -999,999,999 and 999,999,999 and print the English equivalent. Here is an exhaustive list of words that your program should use: negative, zero, one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, hundred, thousand, million. Don't use hundred, when you can use thousand, e.g., use one thousand five hundred instead of fifteen hundred. [Reference](#).
22. **Gymnastics judging.** A gymnast's score is determined by a panel of 6 judges who each decide a score between 0.0 and 10.0. The final score is determined by discarding the high and low scores, and averaging the remaining 4. Write a program `GymnasticsScorer.java` that takes 6 real command line inputs representing the 6 scores and prints their average, after throwing out the high and low scores.
23. **Quarterback rating.** To compare NFL quarterbacks, the NFL devised a the [quarterback rating](#) formula based on the quarterbacks number of completed passes ( $A$ ), pass attempts ( $B$ ), passing yards ( $C$ ), touchdown passes ( $D$ ), and interception ( $E$ ) as follows:
1. Completion ratio:  $W = 250/3 * ((A / B) - 0.3)$ .
  2. Yards per pass:  $X = 25/6 * ((C / B) - 3)$ .
  3. Touchdown ratio:  $Y = 1000/3 * (D / B)$

4. Interception ratio:  $Z = 1250/3 * (0.095 - (E / B))$

The *quarterback rating* is computed by summing up the above four quantities, but rounding up or down each value so that it is at least 0 and at most 475/12. Write a program `QuarterbackRating.java` that takes five command line inputs A, B, C, D, and E, and prints the quarterback rating. Use your program to compute Steve Young's 1994 record-setting season (112.8) in which he completed 324 of 461 passes for 3,969 yards, and threw 35 touchdowns and 10 interceptions. As of 2014, the all-time single-season record is 122.5 by Aaron Rodgers in 2011.

24. **Decimal expansion of rational numbers.** Given two integers  $p$  and  $q$ , the decimal expansion of  $p/q$  has an infinitely repeating cycle. For example,  $1/33 = 0.03030303\dots$ . We use the notation  $0.(03)$  to denote that 03 repeats indefinitely. As another example,  $8639/70000 = 0.1234(142857)$ . Write a program `DecimalExpansion.java` that reads in two command line integers  $p$  and  $q$  and prints the decimal expansion of  $p/q$  using the above notation. *Hint*: use Floyd's rule.
25. **Friday the 13th.** What is the maximum number of consecutive days in which no Friday the 13th occurs? *Hint*: The Gregorian calendar repeats itself every 400 years (146097 days) so you only need to worry about a 400 year interval.

*Solution*: 426 (e.g., from 8/13/1999 to 10/13/2000).

26. **January 1.** Is January 1 more likely to fall on a Saturday or Sunday? Write a program to determine the number of times each occurs in a 400 year interval.

*Solution*: Sunday (58 times) is more likely than Saturday (56 times).

27. What do the following two code fragments do?

```
for (int i = 0; i < N; i++)
    for (int j = 0; j < N; j++)
        if (i != j) System.out.println(i + ", " + j);

for (int i = 0; i < N; i++)
    for (int j = 0; (i != j) && (j < N); j++)
        System.out.println(i + ", " + j);
```

28. Determine what value gets printed out without using a computer. Choose the correct answer from 0, 100, 101, 517, or 1000.

```
int cnt = 0;
for (int i = 0; i < 10; i++)
    for (int j = 0; j < 10; j++)
        for (int k = 0; k < 10; k++)
            if (2*i + j >= 3*k)
                cnt++;
System.out.println(cnt);
```

29. Rewrite [CarLoan.java](#) from Creative Exercise XYZ so that it properly handles an interest rate of 0% and avoids dividing by 0.
30. Write the shortest Java program you can that takes an integer command-line argument  $n$  and prints true if  $(1 + 2 + \dots + n)^2$  is equal to  $(1^3 + 2^3 + \dots + n^3)$ .

*Solution*: Always print true.

31. Modify [Sqrt.java](#) so that it reports an error if the user enters a negative number and works properly if the user enters zero.
32. What happens if we initialize  $t$  to  $-x$  instead of  $x$  in program [Sqrt.java](#)?
33. **Sample standard deviation of uniform distribution.** Modify Exercise 8 so that it prints the sample standard deviation in addition to the average.

34. **Sample standard deviation of normal distribution.** that takes an integer  $N$  as a command-line argument and uses Web Exercise 1 from [Section 1.2](#) to print  $N$  standard normal random variables, and their average value, and sample standard deviation.
35. **Loaded dice.** [Stephen Rudich] Suppose you have three, three sided dice. A: {2, 6, 7}, B: {1, 5, 9}, and C: {3, 4, 8}. Two players roll a die and the one with the highest value wins. Which die would you choose? *Solution:* A beats B with probability  $5/9$ , B beats C with probability  $5/9$  and C beats A with probability  $5/9$ . Be sure to choose second!
36. **Thue–Morse sequence.** Write a program [ThueMorse.java](#) that reads in a command line integer  $n$  and prints the [Thue–Morse sequence](#) of order  $n$ . The first few strings are 0, 01, 0110, 01101001. Each successive string is obtained by flipping all of the bits of the previous string and concatenating the result to the end of the previous string. The sequence has many amazing properties. For example, it is a binary sequence that is *cube-free*: it does not contain 000, 111, 010101, or  $sss$  where  $s$  is any string. It is *self-similar*: if you delete every other bit, you get another Thue–Morse sequence. It arises in diverse areas of mathematics as well as chess, graphic design, [weaving patterns](#), and music composition.
37. Program [Binary.java](#) prints the binary representation of a decimal number  $n$  by casting out powers of 2. Write an alternate version Program [Binary2.java](#) that is based on the following method: Write 1 if  $n$  is odd, 0 if  $n$  is even. Divide  $n$  by 2, throwing away the remainder. Repeat until  $n = 0$  and read the answer backwards. Use % to determine whether  $n$  is even, and use string concatenation to form the answer in reverse order.
38. What does the following code fragment do?

```
int digits = 0;
do {
    digits++;
    n = n / 10;
} while (n > 0);
```

*Solution:* The number of bits in the binary representation of a natural number  $n$ . We use a do-while loop so that code output 1 if  $n = 0$ .

39. Write a program `NPerLine.java` that takes an integer command-line argument  $n$  and prints the integers from 10 to 99 with  $n$  integers per line.
40. Modify `NPerLine.java` so that it prints the integers from 1 to 1000 with  $n$  integers per line. Make the integers line up by printing the right number of spaces before an integer (e.g., three for 1-9, two for 10-99, and one for 100-999).
41. Suppose  $a$ ,  $b$ , and  $c$  are random number uniformly distributed between 0 and 1. What is the probability that  $a$ ,  $b$ , and  $c$  form the side length of some triangle? *Hint:* they will form a triangle if and only if the sum of every two values is larger than the third.
42. Repeat the previous question, but calculate the probability that the resulting triangle is obtuse, given that the three numbers form a triangle. *Hint:* the three lengths will form an obtuse triangle if and only if (i) the sum of every two values is larger than the third and (ii) the sum of the squares of every two side lengths is greater than or equal to the square of the third.

[Answer.](#)

43. What is the value of  $s$  after executing the [following code](#)?

```
int M = 987654321;
String s = "";
while (M != 0) {
    int digit = M % 10;
    s = s + digit;
    M = M / 10;
}
```

44. What is the value of  $i$  after the following [confusing code](#) is executed?

```
int i = 10;
i = i++;
i = ++i;
i = i++ + ++i;
```

Moral: don't write code like this.

45. **Formatted ISBN number.** Write a program [ISBN2.java](#) that reads in a 9 digit integer from a command-line argument, computes the check digit, and prints the fully formatted ISBN number, e.g, 0-201-31452-5.
46. **UPC codes.** The Universal Product Code ([UPC](#)) is a 12 digit code that uniquely specifies a product. The least significant digit  $d_1$  (rightmost one) is a check digit which is the uniquely determined by making the following expression a multiple of 10:

$$(d_1 + d_3 + d_5 + d_7 + d_9 + d_{11}) + 3(d_2 + d_4 + d_6 + d_8 + d_{10} + d_{12})$$

As an example, the check digit corresponding to 0-48500-00102 (Tropicana Pure Premium Orange Juice) is 8 since

$$(8 + 0 + 0 + 0 + 5 + 4) + 3(2 + 1 + 0 + 0 + 8 + 0) = 50$$

and 50 is a multiple of 10. Write a program that reads in a 11 digit integer from a command line parameter, computes the check digit, and prints the the full UPC. *Hint:* use a variable of type `long` to store the 11 digit number.

47. Write a program that reads in the wind speed (in knots) as a command line argument and prints its force according to the [Beaufort scale](#). Use a `switch` statement.
48. **Making change.** Write a program that reads in a command line integer  $N$  (number of pennies) and prints the best way (fewest number of coins) to make change using US coins (quarters, dimes, nickels, and pennies only). For example, if  $N = 73$  then print

```
2 quarters
2 dimes
3 pennies
```

*Hint:* use the greedy algorithm. That is, dispense as many quarters as possible, then dimes, then nickels, and finally pennies.

49. Write a program [Triangle.java](#) that takes a command-line argument  $N$  and prints an  $N$ -by- $N$  triangular pattern like the one below.

```
* * * * *
. * * * *
. . * * *
. . . * *
. . . . *
. . . . .
```

50. Write a program [Ex.java](#) that takes a command-line argument  $N$  and prints a  $(2N + 1)$ -by- $(2N + 1)$  ex like the one below. Use two `for` loops and one `if-else` statement.

```
* . . . . *
. * . . * .
. . * . * .
. . . * . .
. . * . * .
```

```

. * . . . * .
* . . . . *

```

51. Write a program [BowTie.java](#) that takes a command-line argument  $N$  and prints a  $(2N + 1)$ -by- $(2N + 1)$  bowtie like the one below. Use two `for` loops and one `if-else` statement.

```

* . . . . *
* * . . * *
* * * . * *
* * * * *
* * * . * *
* * . . * *
* . . . . *

```

52. Write a program [Diamond.java](#) that takes a command-line argument  $N$  and prints a  $(2N + 1)$ -by- $(2N + 1)$  diamond like the one below.

```

% java Diamond 4
. . . . * . . . .
. . . * * * . . .
. . * * * * * . .
. * * * * * * * .
* * * * * * * *
. * * * * * * * .
. . * * * * * . .
. . . * * * . . .
. . . . * . . . .

```

53. Write a program [Heart.java](#) that takes a command-line argument  $N$  and prints a heart.  
 54. What does the program [Circle.java](#) print out when  $N = 5$ ?

```

for (int i = -N; i <= N; i++) {
    for (int j = -N; j <= N; j++) {
        if (i*i + j*j <= N*N) System.out.print("* ");
        else System.out.print(". ");
    }
    System.out.println();
}

```

55. **Seasons.** Write a program `Season.java` that takes two command line integers  $M$  and  $D$  and prints the season corresponding to month  $M$  (1 = January, 12 = December) and day  $D$  in the northern hemisphere. Use the following table

SEASON	FROM	TO
Spring	March 21	June 20
Summer	June 21	September 22
Fall	September 23	December 21
Winter	December 21	March 20

56. **Zodiac signs.** Write a program `zodiac.java` that takes two command line integers  $M$  and  $D$  and prints the Zodiac sign corresponding to month  $M$  (1 = January, 12 = December) and day  $D$ . Use the following table



<b>SIGN</b>	<b>FROM</b>	<b>TO</b>
Capricorn	December 22	January 19
Aquarius	January 20	February 17
Pisces	February 18	March 19
Aries	March 20	April 19
Taurus	April 20	May 20
Gemini	May 21	June 20
Cancer	June 21	July 22
Leo	July 23	August 22
Virgo	August 23	September 22
Libra	September 23	October 22
Scorpio	October 23	November 21
Sagittarius	November 22	December 21

57. **Muay Thai kickboxing.** Write a program that reads in the weight of a Muay Thai kickboxer (in pounds) as a command-line argument and prints their weight class. Use a `switch` statement.

<b>CLASS</b>	<b>FROM</b>	<b>TO</b>
Flyweight	0	112
Super flyweight	112	115
Bantamweight	115	118
Super bantamweight	118	122
Featherweight	122	126
Super featherweight	126	130
Lightweight	130	135
Super lightweight	135	140
Welterweight	140	147
Super welterweight	147	154
Middleweight	154	160
Super middleweight	160	167
Light heavyweight	167	175
Super light heavyweight	175	183
Cruiserweight	183	190
Heavyweight	190	220
Super heavyweight	220	-

58. **Euler's sum of powers conjecture.** In 1769 Euler generalized Fermat's Last Theorem and conjectured that it is impossible to find three 4th powers whose sum is a 4th power, or four 5th powers whose sum is a 5th power, etc. The conjecture was disproved in 1966 by exhaustive computer search. Disprove the

conjecture by finding positive integers  $a, b, c, d$ , and  $e$  such that  $a^5 + b^5 + c^5 + d^5 = e^5$ . Write a program [Euler.java](#) that reads in a command line parameter  $N$  and exhaustively searches for all such solutions with  $a, b, c, d$ , and  $e$  less than or equal to  $N$ . No counterexamples are known for powers greater than 5, but you can join [EulerNet](#), a distributed computing effort to find a counterexample for sixth powers.

59. **Blackjack.** Write a program `Blackjack.java` that takes three command line integers  $x, y$ , and  $z$  representing your two blackjack cards  $x$  and  $y$ , and the dealers face-up card  $z$ , and prints the "standard strategy" for a 6 card deck in Atlantic city. Assume that  $x, y$ , and  $z$  are integers between 1 and 10, representing an ace through a face card. Report whether the player should hit, stand, or split according to these [strategy tables](#). (When you learn about arrays, you will encounter an alternate strategy that does not involve as many if-else statements).
60. **Blackjack with doubling.** Modify the previous exercise to allow *doubling*.
61. **Projectile motion.** The following equation gives the trajectory of a ballistic missile as a function of the initial angle  $\theta$  and windspeed:  $xxxx$ . Write a java program to print the  $(x, y)$  position of the missile at each time step  $t$ . Use trial and error to determine at what angle you should aim the missile if you hope to incinerate a target located 100 miles due east of your current location and at the same elevation. Assume the windspeed is 20 mph due east.
62. **World series.** The baseball world series is a best of 7 competition, where the first team to win four games wins the World Series. Suppose the stronger team has probability  $p > 1/2$  of winning each game. Write a program to estimate the chance that the weaker teams wins the World Series and to estimate how many games on average it will take.
63. Consider the equation  $(9/4)^x = x^{(9/4)}$ . One solution is  $9/4$ . Can you find another one using Newton's method?
64. **Sorting networks.** Write a program [Sort3.java](#) with three `if` statements (and no loops) that reads in three integers  $a, b$ , and  $c$  from the command line and prints them out in ascending order.

```
if (a > b) swap a and b
if (a > c) swap a and c
if (b > c) swap b and c
```

65. **Oblivious sorting network.** Convince yourself that the following code fragment rearranges the integers stored in the variables  $A, B, C$ , and  $D$  so that  $A \leq B \leq C \leq D$ .

```
if (A > B) { t = A; A = B; B = t; }
if (B > C) { t = B; B = C; C = t; }
if (A > B) { t = A; A = B; B = t; }
if (C > D) { t = C; C = D; D = t; }
if (B > C) { t = B; B = C; C = t; }
if (A > B) { t = A; A = B; B = t; }
if (D > E) { t = D; D = E; E = t; }
if (C > D) { t = C; C = D; D = t; }
if (B > C) { t = B; B = C; C = t; }
if (A > B) { t = A; A = B; B = t; }
```

Devise a sequence of statements that would sort 5 integers. How many `if` statements does your program use?

66. **Optimal oblivious sorting networks.** Create a program that sorts four integers using only 5 `if` statements, and one that sorts five integers using only 9 `if` statements of the type above? Oblivious sorting networks are useful for implementing sorting algorithms in hardware. How can you check that your program works for all inputs?

*Solution:* [Sort4.java](#) sorts 4 elements using 5 compare-exchanges. [Sort5.java](#) sorts 5 elements using 9 compare-exchanges.

The *0-1 principle* asserts that you can verify the correctness of a (deterministic) sorting algorithm by checking whether it correctly sorts an input that is a sequence of 0s and 1s. Thus, to check that `Sort5.java` works, you only need to test it on the  $2^5 = 32$  possible inputs of 0s and 1s.

67. **Optimal oblivious sorting (challenging).** Find an optimal sorting network for 6, 7, and 8 inputs, using 12, 16, and 19 `if` statements of the form in the previous problem, respectively.

*Solution:* [Sort6.java](#) is the solution for sorting 6 elements.

68. **Optimal non-oblivious sorting.** Write a program that sorts 5 inputs using only 7 comparisons. *Hint:* First compare the first two numbers, the second two numbers, and the larger of the two groups, and label them so that  $a < b < d$  and  $c < d$ . Second, insert the remaining element  $e$  into its proper place in the chain  $a < b < d$  by first comparing against  $b$ , then either  $a$  or  $d$  depending on the outcome. Third, insert  $c$  into the proper place in the chain involving  $a, b, d$ , and  $e$  in the same manner that you inserted  $e$  (with the knowledge that  $c < d$ ). This uses 3 (first step) + 2 (second step) + 2 (third step) = 7 comparisons. This method was first discovered by H. B. Demuth in 1956.
69. **Weather balloon.** (Etter and Ingber, p. 123) Suppose that  $h(t) = 0.12t^4 + 12t^3 - 380t^2 + 4100t + 220$  represents the height of a weather balloon at time  $t$  (measured in hours) for the first 48 hours after its launch. Create a table of the height at time  $t$  for  $t = 0$  to 48. What is its maximum height? *Solution:*  $t = 5$ .
70. Will the following code fragment compile? If so, what will it do?

```
int a = 10, b = 18;
if (a = b) System.out.println("equal");
else      System.out.println("not equal");
```

*Solution:* It uses the assignment operator `=` instead of the equality operator `==` in the conditional. In Java, the result of this statement is an integer, but the compiler expects a boolean. As a result, the program will not compile. In some languages (notably C and C++), this code fragment will set the variable `a` to 18 and print `equal` without an error.

71. **Gotcha 1.** What does the following code fragment do?

```
boolean a = false;
if (a = true) System.out.println("yes");
else          System.out.println("no");
```

*Solution:* it prints `yes`. Note that the conditional uses `=` instead of `==`. This means that `a` is assigned the value `true`. As a result, the conditional expression evaluates to `true`. Java is not immune to the `=` vs. `==` error described in the previous exercise. For this reason, it is much better style to use `if (a)` or `if (!a)` when testing booleans.

72. **Gotcha 2.** What does the following code fragment do?

```
int a = 17, x = 5, y = 12;
if (x > y);
{
    a = 13;
    x = 23;
}
System.out.println(a);
```

*Solution:* Always prints 13 since there is a spurious semicolon after the `if` statement. Thus, the assignment statement `a = 13;` will be executed even though `(x <= y)`. It is legal (but uncommon) to have a block that does not belong to a conditional statement, loop, or method.

73. **Gotcha 3.** What does the following code fragment do?

```
for (int x = 0; x < 100; x += 0.5) {
    System.out.println(x);
}
```

*Solution:* It goes into an infinite loop printing 0. The compound assignment statement `x += 0.5` is equivalent to `x = (int) (x + 0.5)`.

74. What does the following code fragment do?

```
int income = Integer.parseInt(args[0]);
if (income >= 311950) rate = .35;
if (income >= 174700) rate = .33;
if (income >= 114650) rate = .28;
if (income >= 47450) rate = .25;
if (income >= 0) rate = .22;
System.out.println(rate);
```

It does not compile because the compiler cannot guarantee that `rate` is initialized. Use `if-else` instead.

75. **Application of Newton's method.** Write a program `BohrRadius.java` that finds the radii where the probability of finding the electron in the 4s excited state of hydrogen is zero. The probability is given by:  $(1 - 3r/4 + r^2/8 - r^3/192)^2 e^{-r/2}$ , where  $r$  is the radius in units of the Bohr radius (0.529173E-8 cm). Use Newton's method. By starting Newton's method at different values of  $r$ , you can discover all three roots. *Hint:* use initial values of  $r = 0, 5$ , and  $13$ . *Challenge:* explain what happens if you use an initial value of  $r = 4$  or  $12$ .

76. **Pepys problem.** In 1693, Samuel Pepys asked Isaac Newton which was more likely: getting at least one 1 when rolling a fair die 6 times or getting at least two 1's when rolling a fair die 12 times. Write a program `Pepys.java` that uses simulation to determine the correct answer.

77. What is the value of the variable `s` after running the following loop when  $N = 1, 2, 3, 4$ , and  $5$ .

```
String s = "";
for (int i = 1; i <= N; i++) {
    if (i % 2 == 0) s = s + i + s;
    else           s = i + s + i;
}
```

*Solution:* [Palindrome.java](#).

78. **Body mass index.** The [body mass index](#) (BMI) is the ratio of the weight of a person (in kilograms) to the square of the height (in meters). Write a program `BMI.java` that takes two command-line arguments, `weight` and `height`, computes the BMI, and prints the corresponding BMI category:

- Starvation: less than 15
- Anorexic: less than 17.5
- Underweight: less than 18.5
- Ideal: greater than or equal to 18.5 but less than 25
- Overweight: greater than or equal to 25 but less than 30
- Obese: greater than or equal to 30 but less than 40
- Morbidly Obese: greater than or equal to 40

79. **Reynolds number.** The *Reynolds number* is the ratio of inertial forces to viscous forces and is an important quantity in fluid dynamics. Write a program that takes in 4 command-line arguments, the diameter `d`, the velocity `v`, the density `rho`, and the viscosity `mu`, and prints the Reynold's number  $d * v * rho / mu$  (assuming all arguments are in SI units). If the Reynold's number is less than 2000, print `laminar flow`, if it's between 2000 and 4000, print `transient flow`, and if it's more than 4000, print `turbulent flow`.

80. **Wind chill revisited.** The wind chill formula from Exercise 1.2.14 is only valid if the wind speed is above 3MPH and below 110MPH and the temperature is below 50 degrees Fahrenheit and above -50 degrees. Modify your solution to print an error message if the user types in a value outside the allowable range.

81. **Point on a sphere.** Write a program to print the  $(x, y, z)$  coordinates of a random point on the surface of a sphere. Use [Marsaglia's method](#): pick a random point  $(a, b)$  in the unit circle as in the `do-while` example. Then, set  $x = 2a \sqrt{1 - a^2 - b^2}$ ,  $y = 2b \sqrt{1 - a^2 - b^2}$ ,  $z = 1 - 2(a^2 + b^2)$ .

82. **Powers of k.** Write a program `PowersOfK.java` that takes an integer `k` as command-line argument and prints all the positive powers of `k` in the Java `long` data type. *Note:* the constant `Long.MAX_VALUE` is the value of the largest integer in `long`.

83. **Square root, revisited.** Why not use the loop-continuation condition `(Math.abs(t*t - c) > EPSILON)` in [Sqrt.java](#) instead of `Math.abs(t - c/t) > t*EPSILON`?

*Solution:* Surprisingly, it can lead to inaccurate results or worse. For example, if you supply [SqrtBug.java](#) with the command-line argument `1e-50`, you get `1e-50` as the answer (instead of `1e-25`); if you supply `16664444`, you get an infinite loop!

84. What happens when you try to compile the following code fragment?

```
double x;  
if (a >= 0) x = 3.14;  
if (a < 0) x = 2.71;  
System.out.println(x);
```

*Solution:* It complains that the variable `x` might not have been initialized (even though we can clearly see that `x` will be initialized by one of the two `if` statements). You can avoid this problem here by using `if-else`.

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