

# Avoiding tipping points in fisheries management through Gaussian Process Dynamic Programming

Carl Boettiger<sup>\*,a</sup>, Marc Mangel<sup>a</sup>, Stephan Munch<sup>c</sup>

<sup>a</sup>*Center for Stock Assessment Research, Department of Applied Math and Statistics, University of California, Mail Stop SOE-2, Santa Cruz, CA 95064, USA*

<sup>b</sup>*Center for Stock Assessment Research, Department of Applied Math and Statistics, University of California, Mail Stop SOE-2, Santa Cruz, CA 95064, USA and Department of Biology, University of Bergen, Bergen, Norway 9020*

<sup>c</sup>*Southwest Fisheries Science Center, National Oceanic and Atmospheric Administration, 110 Shaffer Road, Santa Cruz, CA 95060, USA*

## Abstract

Model uncertainty and limited data are fundamental challenges to robust management of human intervention in a natural system. These challenges are acutely highlighted by concerns that many ecological systems may contain tipping points, such as Allee population sizes. Before a collapse, we do not know where the tipping points lie, if they exist at all. Hence, we know neither a complete model of the system dynamics nor do we have access to data in some large region of state-space where such a tipping point might exist. We illustrate how a Bayesian Non-Parametric (BNP) approach using a Gaussian Process (GP) prior provides a flexible representation of this inherent uncertainty. We embed GPs in a Stochastic Dynamic Programming (SDP) framework in order to make robust management predictions with both model uncertainty and limited data. We use simulations to evaluate this approach as compared with the standard approach of using model selection to choose from a set of candidate models. We find that model selection erroneously favors models without tipping points – leading to harvest policies that guarantee extinction. The GPDP performs nearly as well as the true model and significantly outperforms standard approaches. We illustrate this using examples of simulated single-species dynamics, where the standard model selection approach should be most effective, and find that it still fails to account for uncertainty appropriately and leads to population crashes, while management based on the GPDP does not, since it does not underestimate the uncertainty outside of the observed data.

**Key words:** Bayesian, Structural Uncertainty, Nonparametric, Optimal Control, Decision Theory, Gaussian Processes, Fisheries Management,

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<sup>\*</sup>Corresponding author

Email address: cboettig(at)gmail.com (Carl Boettiger)

## 14 Introduction

15 Decision making under uncertainty is a ubiquitous challenge in the management of human  
16 intervention in natural resources and conservation. Decision-theoretic approaches provide a  
17 framework to determine the best sequence of actions in face of uncertainty, but only when  
18 that uncertainty can be meaningfully quantified (Fischer et al. 2009). Over the last four  
19 decades (beginning with Clark (1976), Clark (2009) and Walters and Hilborn (1978)) dynamic  
20 optimization methods, particularly Stochastic Dynamic Programming (SDP), have become  
21 increasingly important as a means of understanding how to manage human intervention into  
22 natural systems. Simultaneously, there has been increasing recognition of the importance of  
23 multiple steady states or ‘tipping points’ (Scheffer et al. 2001, 2009, Polasky et al. 2011) in  
24 ecological systems.

25 We develop a novel approach to address these concerns in the context of fisheries; although  
26 the challenges and methods are germane to other problems of conservation or natural resource  
27 exploitation. Economic value and ecological concern have made marine fisheries the crucible for  
28 much of the founding work on management under uncertainty (Gordon 1954, Clark 1976, 2009,  
29 May et al. 1979, Reed 1979, Ludwig and Walters 1982)

30 Even if we know the proper deterministic description of the biological system, there is intrinsic  
31 stochasticity in biological dynamics, measurements, and implementation of policy (*e.g.* Reed  
32 1979, Clark and Kirkwood 1986, Roughgarden and Smith 1996, Sethi et al. 2005). We may also  
33 lack knowledge about the parameters of the biological dynamics (parametric uncertainty, *e.g.*  
34 Ludwig and Walters 1982, Hilborn and Mangel 1997, McAllister 1998, Schapaugh and Tyre 2013),  
35 or even not know which model is proper description of the system (structural uncertainty, *e.g.*  
36 Williams 2001, Cressie et al. 2009, Athanassoglou and Xepapadeas 2012). Of these, the latter  
37 is generally the hardest to quantify. Typical approaches confront the data with a collection of  
38 models, assuming that the true dynamics (or reasonable approximation) is among the collection  
39 and then use model choice or model averaging to arrive at a conclusion (Williams 2001, Cressie  
40 et al. 2009, Athanassoglou and Xepapadeas 2012). Even setting aside other concerns (see Cressie  
41 et al. (2009)), these approaches are unable to describe uncertainty outside the observed data  
42 range.

43 Structural uncertainty is particularly insidious when we try to predict outside of the range of  
 44 observed data (Mangel et al. 2001) because we are extrapolating into unknown regions. In  
 45 management applications, this extrapolation uncertainty is particularly important since (a)  
 46 management involves considering actions that may move the system outside the range of observed  
 47 behavior, and (b) the decision tools (optimal control theory, SDP) rely on both reasonable  
 48 estimates of the expected outcomes and on the weights given to those outcomes (*e.g.* Weitzman  
 49 2013). Thus characterizing uncertainty is as important as characterizing the expected outcome.  
 50 Tipping points in ecological dynamics (Scheffer et al. 2001, Polasky et al. 2011) highlight this  
 51 problem because precise models are not available and data are limited such as around high stock  
 52 levels or an otherwise desirable state. With perfect information, one would know just how far a  
 53 system could be pushed before crossing the tipping point, and management would be simple.  
 54 But we face imperfect models and limited data and, with tipping points,  
 55 even small errors can have very large consequences, as we shall illustrate later. Because  
 56 intervention is often (but not always, see Hughes et al. (2013)) too late after a tipping point has  
 57 been crossed, management is often concerned with avoiding tipping points before any data about  
 58 them is available or the transition that would more clearly reveal a tipping point occurs (*e.g.*  
 59 Bestelmeyer et al. 2012).  
 60 The dual concerns of model uncertainty and incomplete data create a substantial challenge  
 61 to existing decision-theoretic approaches (Brozović and Schlenker 2011). We illustrate how  
 62 Stochastic Dynamic Programming (SDP) (Mangel and Clark 1988, Marescot et al. 2013) can be  
 63 implemented using a Bayesian Non-Parametric (BNP) model of population dynamics (Munch et  
 64 al. 2005a). The BNP method has two distinct advantages. First, using a BNP model sidesteps  
 65 the need for an accurate model-based description of the system dynamics. Second, a BNP model  
 66 better reflects uncertainty when extrapolating beyond the observed data. This is crucial to  
 67 providing robust decision-making when the correct model is not known (as is almost always  
 68 true). [We use *robust* to characterize approaches that provide nearly optimal solutions without  
 69 being sensitive to the choice of the (unknown) underlying model.]  
 70 This paper is the first ecological application of the SDP without an *a priori* model of the underlying  
 71 dynamics. Unlike parametric approaches that can only reflect uncertainty in parameter estimates,

the BNP method provides a broader representation of uncertainty, including uncertainty beyond the observed data. We will show that Gaussian Process Dynamic Programming (GPDP) allows us to find robust management solutions in face of limited data without knowing the correct model structure.

For comparisons, we consider the performance of management based on GPDP against management policies derived under several alternative parametric models (Reed 1979, Ludwig and Walters 1982, Mangel and Clark 1988). Rather than compare models in terms of best fit to data, we compare model performance in the concrete terms of the decision-maker’s objectives.

## Approach and Methods

We first describe the requirements of dynamic optimization for the management of human intervention in natural resource systems. After that we describe three parametric models for population dynamics and the Gaussian Process (GP)<sup>1</sup> description of population dynamics.

### *Requirements of dynamic optimization*

Dynamic optimization requires characterizing the dynamics of a state variable (or variables), a control action, and a value function. For simplicity, we consider only a single state variable, which allows us to focus on one-dimensional models of population dynamics for the comparison of parametric models and the GP. This is a best-case scenario for the parametric models because we simulate underlying dynamics from one of the three parametric models, whereas in the natural world we never know the “true” mode. In addition, by choosing one-dimensional models with just a few parameters, we limit the chance that poor performance will be due to inability to estimate parameters accurately, something that becomes a more severe problem for higher-dimensional parametric models. Finally, the parametric models we consider are those most commonly used in

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<sup>1</sup>We abbreviate Gaussian Process as GP, which refers to the statistical model we use to approximate the population dynamics, and we use the term Gaussian Process Dynamic Programming [GPDP], to refer to the use of a GP as the underlying process model when solving a Dynamic Programming equation. Hence we will refer to the models as: GP, Ricker, Allen, etc, and the novel method we put forward here as GPDP.

94 modeling stock-recruitment dynamics or to model sudden transitions between alternative stable  
 95 states.

## 96 *Parametric models*

97 We let  $X(t)$  denote the size (numbers or biomass) of the focal population at time  $t$  and assume  
 98 that in the absence of take its dynamics are. .

$$X(t+1) = Z(t)f(X(t), p) \quad (1)$$

99 Where  $Z(t)$  is log-normally distributed process stochasticity (Reed 1979) and  $p$  is a vector of  
 100 parameters to be estimated from the data. We describe the three choices for  $f(X(t), p)$  in the  
 101 next section.

102 The control action is a harvest or take,  $h(t)$ , measured in the same units as  $X$ , at time  $t$ . Thus,  
 103 in the presence of take, the population size on the right hand side of Eqn 1 is replaced by  
 104  $S(t) = X(t) - h(t)$ .

105 To construct the value function, we consider a return when  $X(t) = x(t)$  and harvest  $h(t) = h$   
 106 denoted as the reward,  $R(x(t), h)$ . For example, if the return is the harvest at time  $t$ , then  
 107  $R(x(t), h(t)) = \min(x(t), h(t))$ . We assume that future harvests are discounted relative to  
 108 current ones at a constant rate of discount  $\delta$  and ask for the harvest policy that maximizes total  
 109 discounted harvest between the current time  $t$  and a final time  $T$ . That is, we seek to maximize  
 110 over choices of harvest  $E_{X(t+1)}[\sum_{t=0}^T R(X(t), h(t), t)\delta^t]$ , where the state dynamics are given by  
 111 Eqn 1 and  $E$  denotes the expectation over the process stochasticity in the future population  
 112 state.

113 In order to find that policy, we introduce the value function  $V(x(t), t)$  representing the total  
 114 discounted catch from time  $t$  onwards given that  $X(t) = x(t)$ . This value function satisfies an  
 115 equation of SDP (Mangel and Clark 1988, Clark and Mangel 2000, Clark 2009 Mangel 2014),

$$V(x(t), t) = \max_{h(t)} \{R(h(t), x(t)) + \delta \cdot E_{X(t+1)} [V(X(t+1), t+1)|x(t), h(t)]\} \quad (2)$$

116 where expectation is taken over all possible values of the next state,  $X(t + 1)$ , and maximized  
 117 over all possible choices of harvest,  $h(t)$ . That is, at time  $t$ , when population size is  $x(t)$  and  
 118 harvest  $h(t)$  is applied, the immediate return is  $R(h(t), x(t))$ . When the sole source of uncertainty  
 119 is the process stochasticity term,  $Z$ , and thus the expectation in Eqn 2 is equivalent to taking  
 120 expectations over  $Z(t)$ . That is

$$E_{X(t+1)} [V(X(t + 1), t + 1)|x(t), h(t)] = E_{Z(t)} [V(Z(t)f(x(t) - h(t))|p), t + 1|x(t), h(t)] \quad (3)$$

121 where the population size after the take is  $x(t) - h(t)$ , which is then translated into  $X(t + 1)$  by  
 122 Eqn 1 (that is, we replace  $X(t + 1)$  by  $Z(t)f(x(t) - h(t)|p)$ ).

123 When the parameters governing the dynamics are also uncertain (the case of parametric un-  
 124 certainty), the expectation in Eqn 2 involves averaging over the posterior distribution for the  
 125 parameters. That is,

$$E_{X(t+1)} [V(X(t + 1), t + 1)|x(t), h(t)] = E_{p|\text{data}} \{E_{Z(t)|p, \text{data}} [V(Z(t)f(x(t) - h(t)|p), t)]\} \quad (4)$$

126 When the underlying population dynamics are unknown (the case of structural uncertainty), the  
 127 function  $f$  itself is uncertain and the expectation for the next state includes uncertainty in  $f$  as  
 128 well. That is

$$E_{X(t+1)} [V(X(t + 1), t + 1)|x(t), h(t)] = E_{p|\text{data}} \{E_{f, Z(t)|p, \text{data}} [V(Z(t)f(x(t) - h(t)|p), t)]\} \quad (5)$$

129 We consider the finite time problem with  $T = 1000$ , which we solve using the standard value  
 130 iteration algorithm (see Mangel and Clark 1988, Clark and Mangel 2000).

### 131 *Parametric models*

132 We consider three candidate parametric models for the population dynamics: The Ricker model,  
 133 the Allen model (Allen and Tanner 2005), and the Myers model (Myers et al. 1995), Eqns (6)-(8).

134 In all three, we let  $K$  denote the carrying capacity and  $r$  the maximum per capita growth rate.  
 135 The Ricker model has two parameters and the right hand side of Eqn 1 is

$$f(S(t)|r, K) = S(t)e^{r(1-\frac{S(t)}{K})} \quad (6)$$

136 The Allen model has three parameters

$$f(S(t)|r, K, X_c) = S(t)e^{r(1-\frac{S(t)}{K})(S(t)-X_c)} \quad (7)$$

137 where  $X_c$  denotes the location of the unstable steady state (i.e., the tipping point).

138 The Myers model also has three parameters

$$f(S(t)|r, K, \theta) = \frac{rS(t)^\theta}{1 + \frac{S(t)^\theta}{K}} \quad (8)$$

139 where  $\theta = 1$  corresponds to Beverton-Holt dynamics and  $\theta > 2$  leads to Allee effects and multiple  
 140 stable states.

141 The Ricker model does not lead to multiple steady states. The Allen model resembles the Ricker  
 142 dynamics with an added Allee effect parameter (Courchamp et al. 2008), below which the  
 143 population cannot persist. The Myers model also has three parameters and contains an Allee  
 144 threshold, but unlike the Ricker model saturates at high population size. The multiplicative  
 145 log-normal stochasticity in Eqn 1 introduces one additional parameter  $\sigma$  that must be estimated.  
 146 Because of our interest in management performance in the presence of tipping points, all of our  
 147 simulations are based on the Allen model. The Allen model is thus the state of nature and is  
 148 expected to provide the best-case scenario. The Ricker model is a reasonable approximation of  
 149 these dynamics far from the Allee threshold (but lacks threshold dynamics), while the Myers  
 150 model shares the essential feature of a threshold but differs in structure from the Allen model.  
 151 Throughout, we refer to the “True” model when the underlying parameters *are known without*  
 152 *error*, and refer to the “Allen” model when these parameters have been estimated from the  
 153 sample data.

154 We consider a period of 40 in which data are obtained to estimate the parameters of the GP.  
155 This is long enough that the estimates do not depend on the particular realization, and longer  
156 times are not likely to provide substantial improvement.

157 We fit each of the models is fit to the same data (Figure 1).

158 We inferred posterior distributions for the parameters of each model by Gibbs sampling (Gelman  
159 et al. (2003) implemented in R (R Core Team 2013) using `jags`, (Su and Masanao Yajima 2013)).  
160 We choose uniform priors for all parameters (See appendix Tables S1-S3; R code provided).  
161 We show one-step-ahead predictions of these model fits in Figure 1. We tested each chain for  
162 Gelman-Rubin convergence and results were robust to longer runs. For each simulation we  
163 also applied several commonly used model selection criteria (AIC, BIC, DIC, see Burnham and  
164 Anderson (2002)) to identify the best fitting model.

#### 165 *The Gaussian Process model*

166 The core difference for our purpose between the GP and the parametric models is that the GP  
167 model is defined explicitly by the observed data. That is, it cannot be specified by the value of  
168 parameters alone.

169 The use of GPs to characterize dynamical systems is relatively new (Kocijan et al. 2005), and  
170 was first introduced in the context ecological modeling and fisheries management in Munch et  
171 al. (2005b). GP models have subsequently been used to test for the presence of Allee effects  
172 (Sugeno and Munch 2013a), estimate the maximum reproductive rate (Sugeno and Munch  
173 2013b), determine temporal variation in food availability (Sigourney et al. 2012), and provide a  
174 basis for identifying model-misspecification (Thorson et al. 2014). An accessible and thorough  
175 introduction to the formulation and use of GPs can be found in Rasmussen and Williams (2006).

176 A GP is a stochastic process for which any realization of for a finite sample of points  $n$  is a  
177 multivariate normal distribution of dimension  $n$ . Thus, to characterize the GP we need a mean  
178 function and a covariance function. We proceed as follows.

179 As before, we assume that the data  $X(t)$  are observed with process stochasticity around a mean  
180 function  $g(X(t))$



$$X(t+1) = g(X(t)) + \varepsilon, \quad (9)$$

where  $\varepsilon$  are IID normal random variables with zero-mean and variance  $\sigma_n^2$ . Note that we have chosen to assume additive stochasticity. While we could consider log-normal stochasticity as in the parametric models, we make this choice to emphasize that the Gaussian process approach need not have structurally correct stochasticity to be effective.

Now consider the situation where we have a set of previous observations of the *timeseries*,  $X_{\text{obs}}(t)$ , and wish to *predict* future values  $X_p(t)$ . We consider the observations as pairs of points  $X_{\text{obs}}(t), X_{\text{obs}}(t+1)$  for  $t = 1, 2, \dots$  up through final observation time  $T$ . As described above, in the GP, a finite sample of these predictions at  $n$  points into the future,  $X_p(\tau)$  for  $\tau \in [T+1, T+2, \dots, T+n]$ , is a multivariate normal ( $\mathcal{N}$ ):

$$g(X_p|X_{\text{obs}}(t), X_{\text{obs}}(t+1)) \sim \mathcal{N}(E, C) \quad (10)$$

with mean and covariance matrix given by (see Eqn 2.22 of Rasmussen and Williams (2006) for a more detailed derivation)

$$E = K(X_p(\tau), X_{\text{obs}}(t)) (K(X_{\text{obs}}(t), X_{\text{obs}}(t)) - \sigma \mathbb{I})^{-1} X_{\text{obs}}(t+1) \quad (11)$$

$$C = K(X_p(\tau), X_p(\tau)) - K(X_p(\tau), X_{\text{obs}}(t)) (K(X_{\text{obs}}(t), X_{\text{obs}}(t)) - \sigma \mathbb{I})^{-1} K(X_{\text{obs}}(t), X_p(\tau)) \quad (12)$$

Where  $K$ , called the covariance kernel, measures how much influence one observation has on another. In this paper, for  $K$  use a matrix whose  $(i, j)$ th element is given by:

$$K_{i,j}(x_i, y_j) = \exp\left(\frac{-(x_i - y_j)^2}{2\ell^2}\right) \quad (13)$$

so that  $\ell$  gives the characteristic length-scale over which correlation between two observations decays.

196 We use inverse Gamma priors on both the length-scale  $\ell$  and  $\sigma$ , thus for example

$$f(\ell; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \ell^{-\alpha-1} \exp\left(-\frac{\beta}{\ell}\right) \quad (14)$$

197 For the prior on  $\ell$ ,  $\alpha = 10$  and  $\beta = 10$ . The prior on  $\sigma$ ,  $\alpha = 5$  and  $\beta = 5$ .

198 We use a Metropolis-Hastings Markov Chain Monte Carlo (Gelman et al. (2003)) to infer  
199 posterior distributions of the parameters of the GP (Figure S4, code in appendix). Since the  
200 posterior distributions differ substantially from the priors (Figure S4), most of the information  
201 in the posterior comes from the data rather than the prior belief.

## 202 *The method of Gaussian Process Dynamic Programming (GPDP)*

203 We derive the harvest policy from the estimated GP by inserting it into a SDP algorithm. Given  
204 the GP posteriors, we construct the transition matrix representing the probability of going  
205 to each state  $X(t+1)$  given any current state  $x(t)$  and any harvest  $h(t)$  (See the function  
206 `gp_transition_matrix()` in the provided R package). Given this transition matrix, we use the  
207 same value iteration algorithm as in the parametric case to determine the optimal policy.

208 In doing so, the uncertainty in the future state under the GP,  $X(t+1)$ , includes both process  
209 uncertainty (based on the estimation of  $\sigma$ ) and structural uncertainty of the posterior collection  
210 of curves.

211 Unlike the case of a parametric model, which is a single curve given the parameters, the GP  
212 remains a distribution of curves.

## 213 **Results**

### 214 *Parametric and GP models for population dynamics*

215 To ensure our results are robust to the choice of parameters, we consider 96 different scenarios,  
216 described in detail below. To help better understand the process, we first describe in detail the  
217 results of a single scenario.

218 All of the models fit the observed data rather closely and with relatively small uncertainty. In  
219 Figure 1, we show the posterior predictive curves. The training data of stock sizes observed

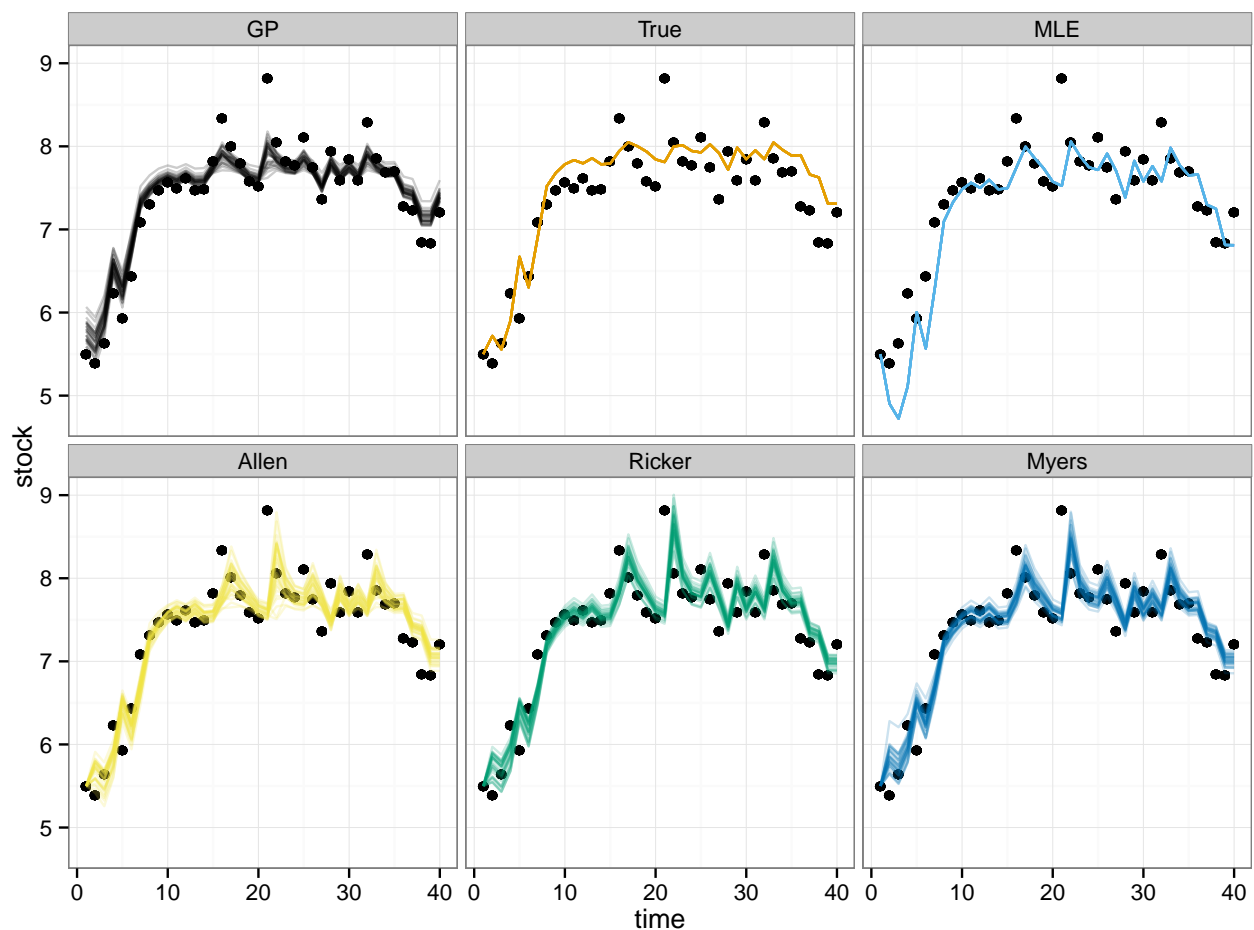


Figure 1: Points show the training data of stock-size over time. Curves show the posterior step-ahead predictions based on each of the estimated models. Observe that all models are fitting the data reasonably well. In the GP panel, black lines are means of the GP for random samples from the posterior distributions, while grey bands around them reflect the standard deviations of the GP distribution. (Recall that while given a particular set of parameters, a parametric model gives just one curve, the GP is always a distribution, whose mean we plot in black.)

over time are points, overlaid with the step-ahead predictions of each estimated model using the parameters sampled from their posterior distributions. Compared to the true model most estimates appear to over-fit, predicting patterns that are actually due purely to stochasticity. Model selection criteria (Table 1) penalize more complex models and show a preference for the simpler Ricker model over the models with alternative stable states (Allen and Myers). Details on MCMC estimates for each model, traces, and posterior distributions can be found in the appendix.

	Allen	Ricker	Myers
<b>DIC</b>	50.75	50.45	50.41
<b>AIC</b>	-24.51	-30.13	-27.01
<b>BIC</b>	-17.75	-25.06	-20.25

Table 1: Model selection scores for several common criteria (DIC: Deviance Information Criterion, AIC: Akaike Information Criterion, BIC: Bayesian Information Criterion) all select the wrong model. As the true (Allen) model is not distinguishable from the simpler (Ricker) model in the region of the observed data, this error cannot be avoided regardless of the model choice criterion. This highlights the danger of model choice when the selected model will be used outside of the observed range of the data.

We show the mean inferred population dynamics of each model relative to the true model used to generate the data in Figure 2, predicting the relationship between observed population size (x-axis) to the population size after recruitment the following year. In addition to the raw data, the GP is conditioned on going through the point 0,0 without error. This conditioning on (0,0) is equivalent to making the assumption that the population is closed,

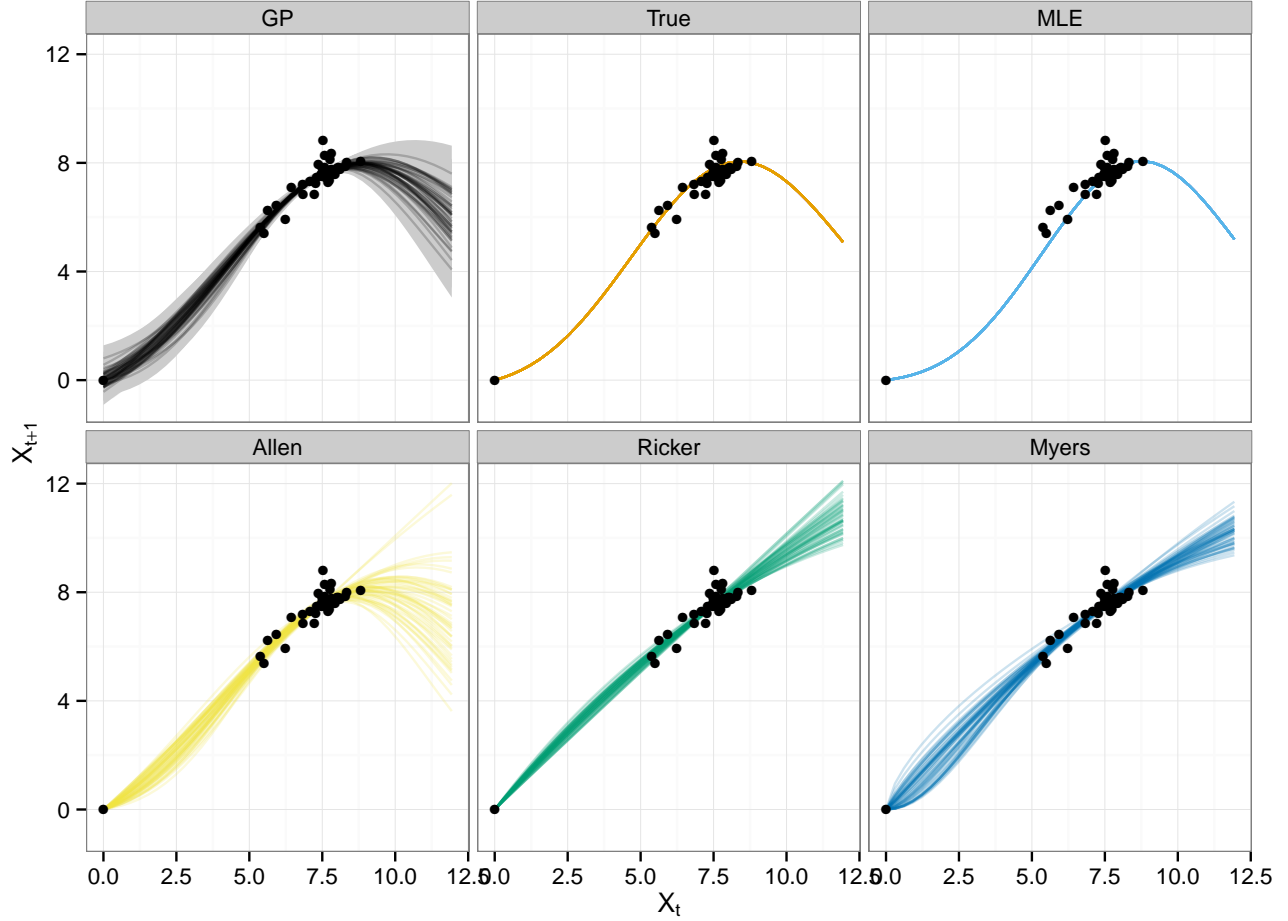


Figure 2: The inferred Gaussian process compared to the true process and maximum-likelihood estimated process. We show the expected value for the function  $f$  under each model. Two standard deviations from the estimated Gaussian process covariance with (light grey) and without (darker grey) measurement error are also shown. The training data are also shown as black points. The GP is conditioned on  $(0,0)$ , shown as a pseudo-data point, as explained in the text

232 so that once it hits 0 it stays at 0, despite the lack of any data in the observed sequence to justify  
 233 this. This assumption illustrates how the GP can capture common-sense biology without having  
 234 to assume more explicit details about the dynamics at low population numbers that have never  
 235 been observed. If the population were not closed, one could repeat the entire analysis without  
 236 this assumption. Unlike parametric models, the GP corresponds to a distribution of curves - the  
 237 gray band with mean shown in black. Uncertainty in the parameters of the GP (not shown)  
 238 further widens the band of possible population sizes. In Figure S1 (see supplement), we show  
 239 the performance of the models outside the observed training data.

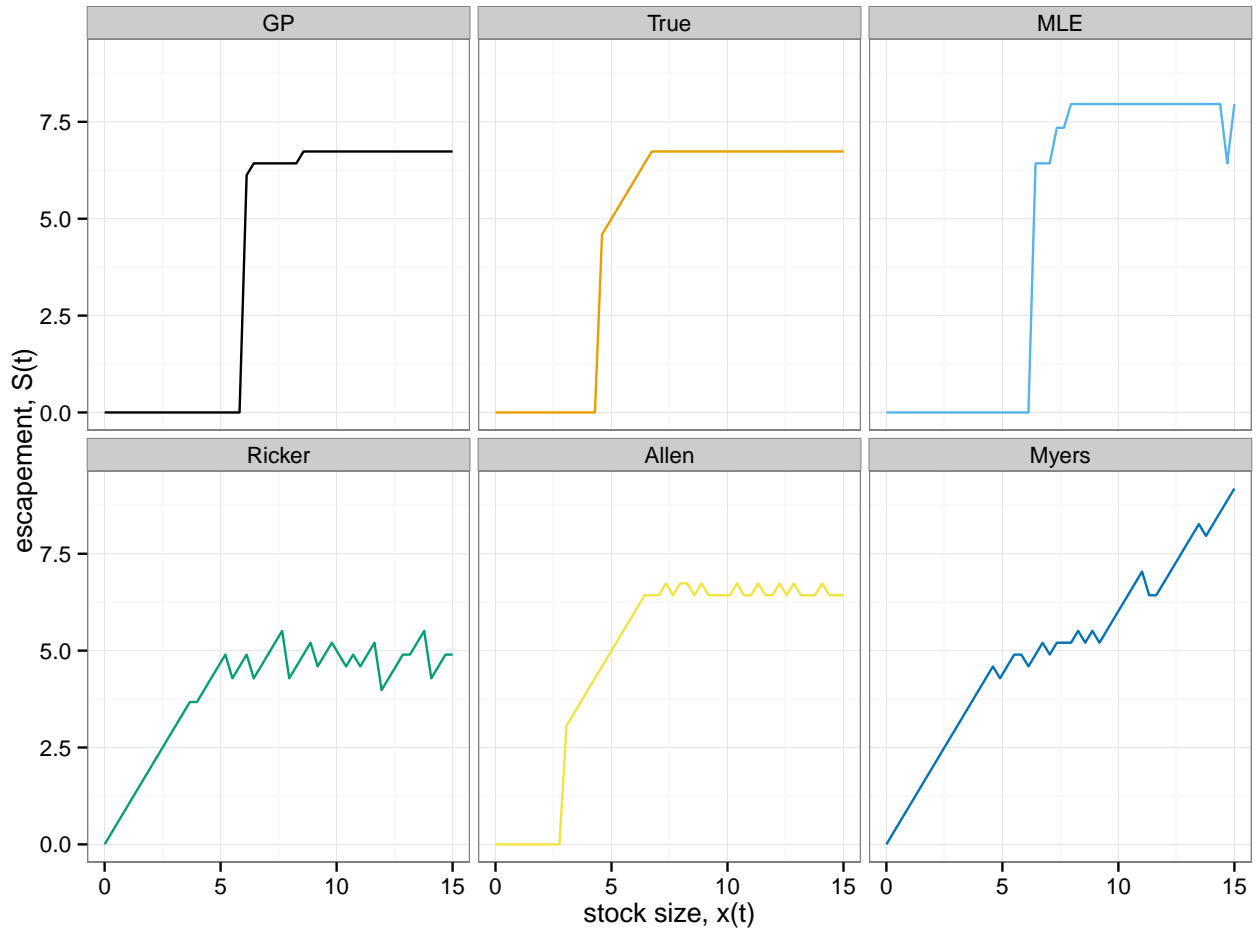


Figure 3: The steady-state optimal policy (infinite boundary) calculated under each model. Policies are shown in terms of target escapement,  $S(t)$ , as under models such as this a constant escapement policy is expected to be optimal (Reed 1979). Several policies show a numerical jitter due to the discretization of states in the dynamic programming algorithm; doubling the number of grid points did not qualitatively change the results.

240 Despite the similarities in model fits to the observed data, the policies inferred under each  
 241 model differ widely (Figure 3). Policies are shown in terms of target escapement,  $S(t) = x_t - h$ .  
 242 Under models such as this a constant escapement policy is expected to be optimal (Reed 1979),  
 243 whereby population levels below a certain size  $S$  are unharvested, while above that size the  
 244 harvest strategy aims to return the population to  $S$ . Whenever a model predicts that the  
 245 population will not persist below a certain threshold, the optimal solution is to harvest the entire  
 246 population immediately, resulting in an escapement  $S = 0$ , as seen in the true (correct form,  
 247 exact parameters) model, the Allen model (correct form, estimated parameters) and the GP.  
 248 Only the structurally correct model (Allen model) and the GP produce policies close to the true  
 249 optimum policy.

250 In Figure 4, we show the consequences of managing 100 replicate realizations of the simulated  
 251 fishery under policies derived from each model. The structurally correct model under-harvests,  
 252 leaving the stock to vary around its unfished optimum. The structurally incorrect Ricker model  
 253 over-harvests the population past the tipping point consistently, resulting in the immediate crash  
 254 of the stock and thus leads to minimal long-term catch.

255 The results across replicate stochastic simulations are most easily compared by using the relative  
 256 differences in net present value realized by each of the model (Figure 5). Although not perfect,  
 257 the GPDP consistently realizes a value close to the optimal solution, and avoids ever driving  
 258 the system across the tipping point, which results in the near-zero value cases in the parametric  
 259 models.

## 260 *Sensitivity Analysis*

261 These results are not sensitive to the modeling details of the simulation. The GPDP estimate  
 262 remains very close to the optimal solution (obtained by knowing the true model) across changes  
 263 to the training simulation, scale of stochasticity, parameters or structure of the underlying  
 264 model. In the Supplement, we consider both a Latin hypercube approach and a more focused  
 265 investigation of the effects of the relative distance to the Allee threshold and the variance of  
 266 process stochasticity.

267 Changing the intensity of the stochasticity or the distance between stable and unstable steady

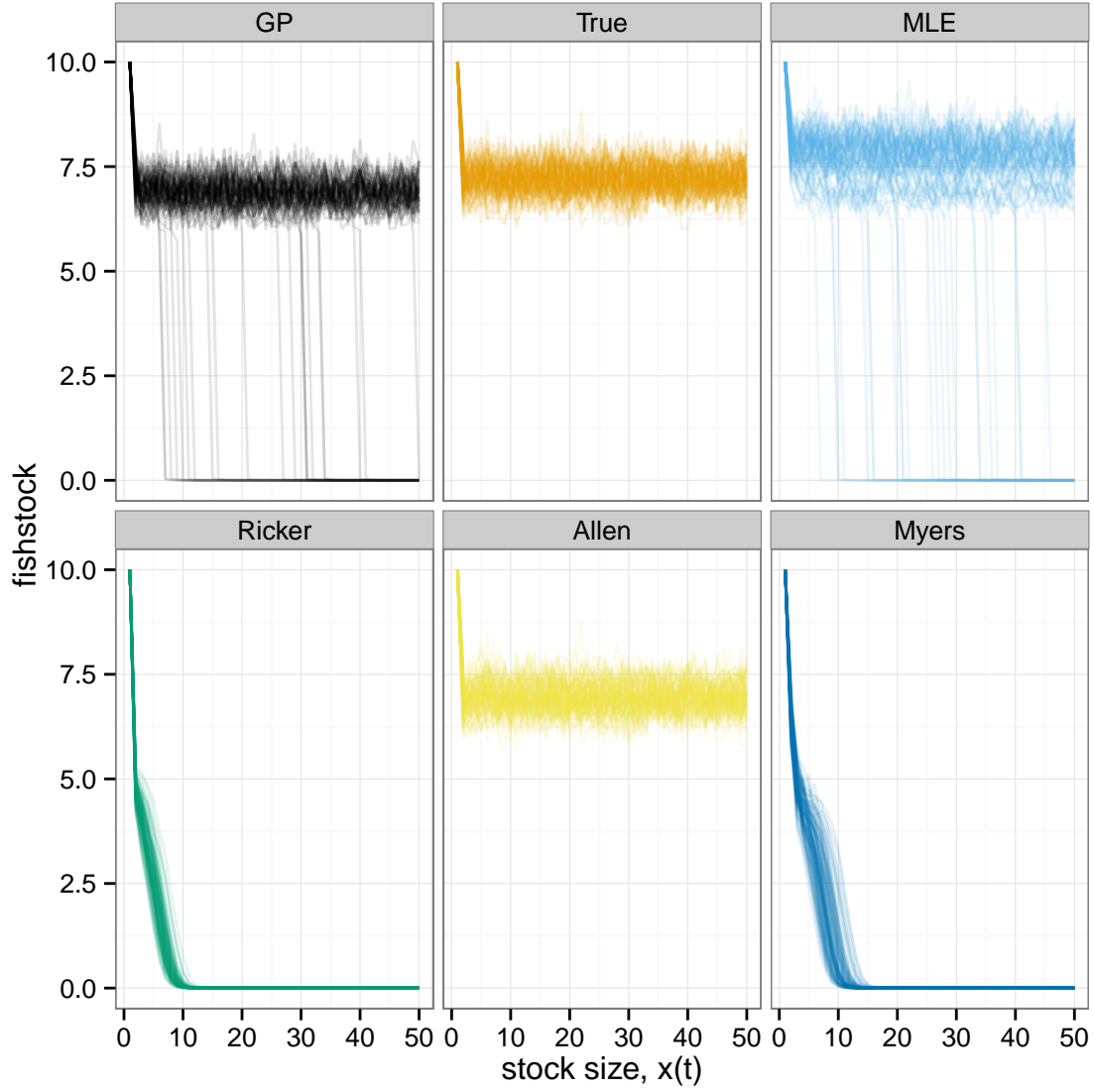


Figure 4: In the management context, GPDP outperforms approaches based on parametric models. We show 100 replicate simulations of the stock dynamics (Eqn 1) under the policies derived from each of the estimated models, as well as the policy based on the exact underlying model.



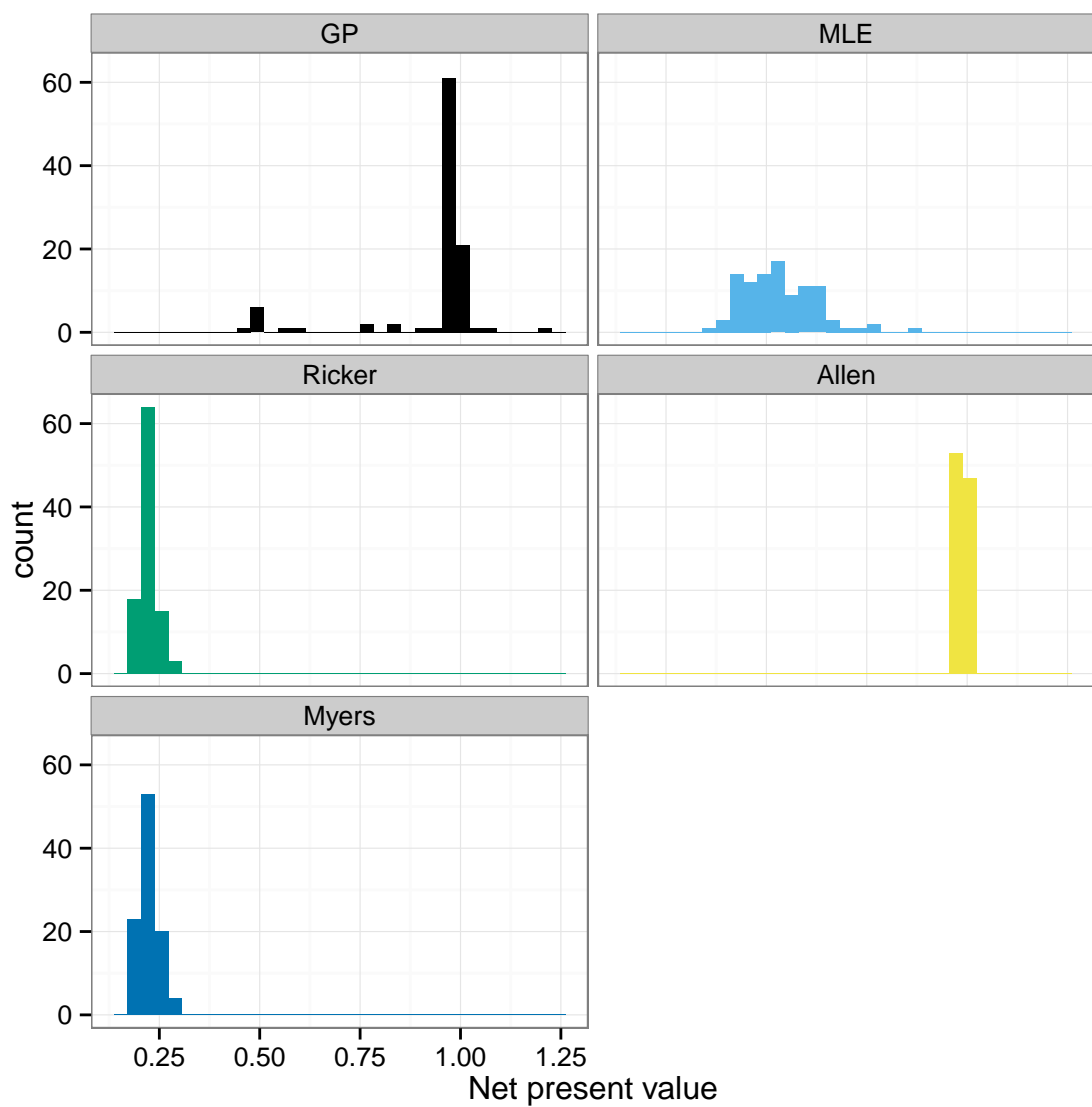


Figure 5: Histograms of the realized net present value of the fishery over a range of simulated data and resulting parameter estimates. For each data set, the three models are estimated as described above.

states does not impact the performance of the GP relative to the optimal solution obtained from  
 the true model and true parameters. The parametric models are more sensitive to this difference.  
 Large values of  $\sigma$  relative to the distance between the stable and unstable point increases the  
 chance of a stochastic transition. More precisely, if we let  $L$  is the distance between the stable  
 and unstable steady states, then the probability that fluctuations drive the population across  
 the unstable steady state scales as  $\exp\left(\frac{L^2}{\sigma^2}\right)$   
 (see Gardiner (2009) or Mangel (2006) for the derivation). Thus, the impact of using a model  
 that underestimates the risk of harvesting beyond the critical point is considerable, since  
 this such a situation occurs more often. Conversely, with large enough distance between the  
 optimal escapement and unstable steady state relative to  $\sigma$ , the chance of a transition becomes  
 vanishingly small and all models can be estimated near-optimally. Models that underestimate  
 the cost incurred by population sizes fluctuating significantly below the optimal escapement level  
 will not perform poorly as long as those fluctuations are sufficiently small.  
 The GPDP is only weakly influenced by increasing stochasticity or increasing Allee effects over  
 much of the range (Figure 6). Larger  $\sigma$  or higher Allee levels make even the optimal solution  
 without any model or parameter uncertainty unable to harvest the population effectively (e.g. the  
 stochasticity is large enough to violate the self-sustaining criterion of Reed (1979)).

## Discussion

Simple, mechanistically motivated models offer the potential to increase our basic understanding  
 of ecological processes (Geritz and Kisdi 2012, Cuddington et al. 2013). But such models can be  
 both inaccurate and misleading when used in a decision making framework. In this paper we  
 tackled two aspects of uncertainty that are common to many ecological decision-making problems  
 and fundamentally challenging to existing approaches that largely rely on parametric models:

1. We do not know the correct models for ecological systems.
2. We have limited data from which to estimate the model.

We have illustrated how the use of non-parametric methods provides more reliable solutions in  
 the sequential decision-making problem.

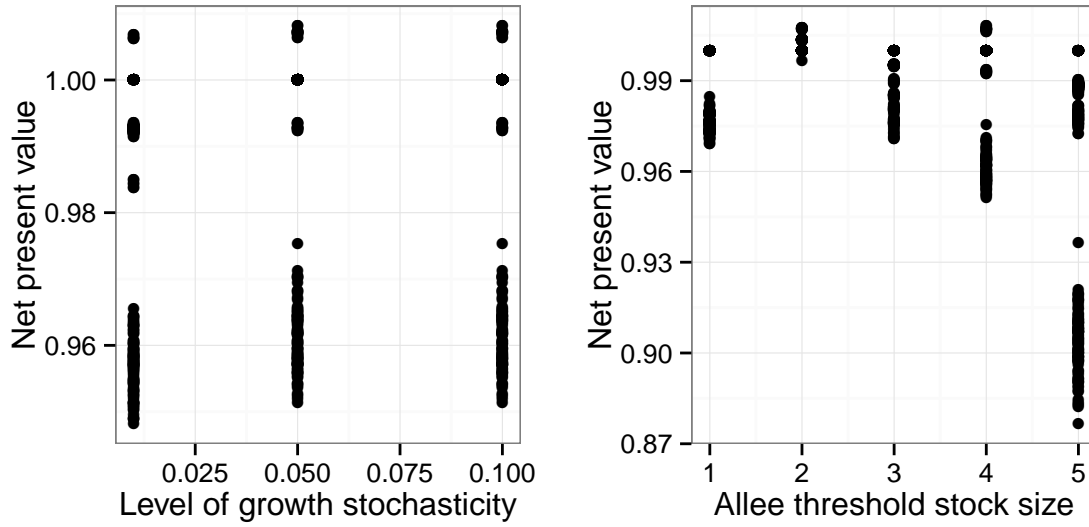


Figure 6: The effect of increasing noise or decreasing Allee threshold levels on the net present value of the fishery when managed under the GPDP, relative to managing under the true model (with known parameters). Other than the focal parameter (stochasticity, Allee threshold), other parameters are held fixed as above to illustrate this effect.

*Traditional model-choice approaches can be positively misleading.*

Our results illustrate that model-choice approaches can be absolutely misleading – by providing support to models that cannot capture tipping point dynamics because they have fewer parameters and the data are far from the tipping point. That is, when the data come from around the stable steady state, all the parametric models are approximately linear and approximately identical. Thus, it is intuitive that all model selection methods choose the simplest model. In a complex world, the result is suboptimal. But in a world that might contain tipping points, the result is disastrous.

Many managers both in fisheries and beyond face a similar situation: they have not observed the population dynamics at all possible densities. A lack of comprehensive data at all population sizes, combined with the inability to formulate accurate population models for low population sizes in the absence of data, makes this situation the rule more than the exception. Relying on parametric models and model choice processes that favor simplicity ignores this basic reality. For a long time, Carl Walters (e.g. Walters and Hilborn 1978) has argued that if we began by fishing any newly exploited population down to very low levels and then let it recover, we would be

much better at estimating population dynamics and thus predicting the optimal harvest levels. While certainly true, this presents a rather risky policy in the face of potential tipping points. The GPDP offers an alternative way to acknowledge this uncertainty.

### *GPDP population dynamics capture larger uncertainty in regions where the data are poor*

Parametric models perform most poorly when we seek a management strategy outside the range of the observed data. The GPDP, in contrast, leads to a predictive model that expresses a great deal of uncertainty about the probable dynamics *outside* the range of the observed data, while retaining very good predictive accuracy *inside* the range. The management policy based on by the GPDP balances uncertainty outside the range of the observed data against the immediate value of the harvest, and acts to stabilize the population dynamics in a region of state space in which the predictions are reliably reflected by the data.

Such problems are ubiquitous across ecological decision-making and conservation where the greatest concerns involve decisions that lead to population sizes that have never been observed and for which we do not know the response – whether this is the collapse of a fishery, the spread of an invasive, or the loss of habitat.

### *The role of the prior*

Outside of the observed range of the data, the GP reverts to the prior, and consequently the choice of the prior can also play a significant role in determining the optimal policy inferred by the SDP. In the examples shown here we have selected a prior that is both relatively uninformative (due to the broad priors placed on its parameters  $\ell$  and  $\sigma$  and simple (the choice of our covariance function, Eqns 12 and 13 ). In practice, these should be chosen to confer particular biological properties. In principle, this may allow a manager to improve the performance of the GPDP by adding detail as is justified. For instance, it would be possible to use a linear or a Ricker-shaped mean in the prior without making the much stronger assumption that the Ricker is the structurally correct model (Sugeno and Munch 2013a). One fruitful avenue of future research is identifying criteria to ensure the prior and the value function are chosen appropriately for the problem at hand.

## 337 Acknowledgments

338 This work was partially supported by NOAA-IAM grant to SM and Alec McCall and administered  
339 through the Center for Stock Assessment Research, a partnership between the University of  
340 California Santa Cruz and the Fisheries Ecology Division, Southwest Fisheries Science Center,  
341 Santa Cruz, CA and by NSF grant EF-0924195 to MM and NSF grant DBI-1306697 to CB.

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