Avoiding tipping points in the management of ecological systems: a non-parametric Bayesian approach

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Abstract

Model uncertainty and limited data coverage are fundamental challenges to robust ecosystem management. These challenges are acutely highlighted by concerns that many ecological systems may contain tipping points. Before a collapse, we do not know where the tipping points lie, if the exist at all. Hence, we know neither a complete model of the system dynamics nor do we have access to data in some large region of state-space where such a tipping point might exist. These two sources of uncertainty frustrate state-of-the-art parametric approaches to decision theory and optimal control. I will illustrate how a non-parametric approach using a Gaussian Process prior provides a more flexible representation of this inherent uncertainty. Consequently, we can adapt the Gaussian Process prior to a stochastic dynamic programming framework in order to make robust management predictions under both model and uncertainty and limited data.

Keywords: Bayesian, Structural Uncertainty, Nonparametric, Optimal Control, Decision Theory, Gaussian Processes, Fisheries Management,

Introduction

Decision making under uncertainty is a ubiquitous challenge of natural resource management and conservation. Ecological dynamics are frequently complex and difficult to measure, making uncertainty in our understanding a prediction a persistent challenge to effective management. Decision-theoretic approaches provide a framework to determine the best sequence of actions in face of uncertainty, but only when that uncertainty can be meaningfully quantified (Fischer et al. 2009). The sudden collapse of fisheries and other ecosystems has increasingly emphasized the difficulties of formulating even qualitatively correct models of the underlying processes.

We develop a novel approach to address these concerns in the context of fisheries; though the underlying challenges and methods are germane to many other conservation and resource management problems. The economic value

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and ecological concern have made marine fisheries the crucible for much of the founding work (H. S. Gordon 1954; Reed 1979; **???**; Ludwig and Walters 1982) in managing ecosystems under uncertainty. Global trends (Worm et al. 2006) and controversy (Hilborn 2007; **???**) have made understanding these challenges all the more pressing.

Uncertainty enters the decision-making process at many levels: intrinsic stochasticity in biological processes, measurements, and implementation of policy (*e.g.* Reed 1979; C. W. Clark and Kirkwood 1986; Roughgarden and Smith 1996; Sethi et al. 2005), parametric uncertainty (*e.g.* Ludwig and Walters 1982; **???**; McAllister 1998; **???**), and model or structural uncertainty (*e.g.* B. K. Williams 2001; Cressie et al. 2009; Athanassoglou and Xepapadeas 2012). Of these, structural uncertainty incorporates the least a priori knowledge or assumptions and is generally the hardest to quantify. Typical approaches assume a weak notion of model uncertainty in which the correct model (or reasonable approximation) of the dynamics must be identified from among a handful of alternative models. Here we consider an approach that addresses uncertainty at each of these levels without assuming the dynamics follow a particular (i.e. parametric) structure.

Cut the next three paragraphs, since they are covered more concisely in the above paragraph?

Process, measurement, and implementation error

Resource management and conservation planning seek to determine the optimal set of feasible actions to maximize the value of some objectives (e.g Halpern et al. (2013)). Process error, measurement error, implementation error (Reed 1979). These sources of stochasticity in turn mean that model parameters can only be estimated approximately, requiring parametric uncertainty also be considered (Ludwig and Walters 1982).

Parametric uncertainty

As the parameter values for these models must be estimated from limited data, there will always be some uncertainty associated with these values. This uncertainty further compounds the intrinsic variability introduced by demographic or environmental noise. The degree of uncertainty in the parameter values can be inferred from the data and reflected in the estimates of the transition probabilities (Ludwig and Walters 1982; Mangel and Clark 1988; **???**; **???**).

Structural (model) uncertainty

Estimates of parameter uncertainty are only as good as the parametric models themselves. Often we do not understand the system dynamics well enough to know if a model provides a good approximation over the relevant range of states and timescales (criteria that we loosely refer to as defining the "right" or "true" model.) So called structural or model uncertainty is a more difficult problem than parametric uncertainty. Typical solutions involve either model choice, model averaging, or introducing yet greater model complexity of which others may be special cases (model averaging being one such way to construct such a model) (B. K. Williams 2001; Athanassoglou and Xepapadeas 2012; Cressie et al. 2009). Even setting aside other computational and statistical concerns (e.g. (Cressie et al. 2009)), these approaches do not address our second concern - representing uncertainty outside the observed data range.

Model uncertainty is particularly insidious when model predictions must be made outside of the range of data on which the model was estimated. This extrapolation uncertainty is felt most keenly in decision-theoretic (or optimal control) applications, since (a) exploring the potential action space typically involves considering actions that may move the system outside the range of observed behavior, and (b) decision-theoretic alogrithms rely not only on reasonable estimates of the expected outcomes, but depend on the weights given to all possible outcomes (*e.g.* Weitzman 2013). If we are observing the fluctuations of a given fish stock over many years under a fixed harvesting pressure, we might develop and test a model that could reasonably predict the frequency of a deviation of a given size, even when such a deviation has not been previously observed. Yet such predictions are far less reliable when extrapolated to a harvest pressure that has not yet been observed. Thus, model uncertainty can be particularly challenging in the management and decision-making context.

This difficult position of having neither the true model nor data that covers the full range of possible states is unfortunately the rule more than the exception. The potential concern of tipping points in ecological dynamics (Scheffer et al. 2001; Polasky et al. 2011) reflects these concerns – as either knowledge of the true model or more complete sampling of the state space would make it easy to identify if a tipping point existed. If we do not know but cannot rule out such a possibility, then we face decision-making under this dual challege of model uncertainty and incomplete data coverage.

These dual concerns pose a substantial challenge to existing decision-theoretic approaches (Brozović and Schlenker 2011). Because intervention is often too late after a tipping point has been crossed (but see Hughes et al. (2013)), management is most often concerned with avoiding potentially catastrophic tipping points before any data is available at or following a transition that would more clearly reveal these regime shift dynamics (e.g. Bestelmeyer et al. 2012).

Here we illustrate how a stochastic dynamic programming (SDP) algorithm (Mangel and Clark 1988; Marescot et al. 2013) can be driven by the predictions from a Bayesian non-parametric (BNP) approach (Munch, Kottas, and Mangel 2005). This provides two distinct advantages compared with contemporary approaches. First, using a BNP sidesteps the need for an accurate model-based description of the system dynamics. Second, the BNP can better reflect uncertainty that arises when extrapolating a model outside of the data on which it was fit. We illustrate that when the correct model is not known, this latter feature is crucial to providing a robust decision-theoretic approach in face of substantial structural uncertainty.

This paper represents the first time the SDP decision-making framework has been used without an a priori model of the underlying dynamics through the use of the BNP approach. In contrast to parametric models which can only reflect uncertainty in parameter estimates, the BNP approach provides a more state-space dependent representation of uncertainty. This permits a much greater uncertainty far from the observed data than near the observed data. These features allow the BNP-SDP approach to find robust management solutions in face of limited data and without knowledge of the correct model structure.

The idea that any approach can perform well without either having to know the model or have particularly good data should immediately draw suspicion. The reader must bear in mind that the strength of our approach comes not from black-box predictive power from such limited information, but rather, by providing a more honest expression of uncertainty outside the observed data without sacrificing the predictive capacity near the observed data. By coupling this more accurate description of what is known and unknown to the decision-making under uncertainty framework provided by stochastic dynamic programming, we are able to obtain more robust management policies than with common parametric modeling approaches.

The nature of decision-making problems provides a convenient way to compare models. Rather than compare models in terms of best fit to data or fret over the appropriate penalty for model complexity, model performance is defined in the concrete terms of the decision-maker's objective function, which we will take as given. (Much argument can be made over the 'correct' objective function, e.g. how to account for the social value of fish left in the sea vs. the commercial value of fish harvested; see Halpern et al. (2013) for further discussion of this issue. Alternatively, we can always compare model performance across multiple potential objective functions.) The decision-maker does not necessarily need a model that provides the best mechanistic understanding or the best long-term outcome, but rather the one that best estimates the probabilities of being in different states as a result of the possible actions.

Background on the Gaussian Process

Addressing the difficulty posed by extrapolation without knowing the true model requires a nonparametric approach to model fitting: one that does not assume a fixed structure but rather depends on the size of the data (e.g. non-parametric regression or a Dirichlet process). This established terminology is nevertheless unfortunate, as (a) this approach still involves the estimation of parameters, and (b), Statisticians use non-parametric to mean both this property (structure is not fixed by the parameters) and an entirely different (and probably more familiar) case in which the model does not assume any distribution (e.g. non-parametric bootstrap, order statistics). Some literature thus uses the term semi-parametric, which merely adds ambiguity to the confusion.

This non-parametric property – having a structure explicitly dependent on the data – is precisely the property that makes this approach attractive in face of the limited data sampling challenges discussed above. Having fit a parametric model to some data, the model is completely described by the values (or posterior distributions) of it's parameters. The non-parametric model is not captured by its parameter values or distributions alone. Either the model scales with the complexity of the data on which it is estimated (e.g. nonparametric hierarchical approaches such as the Dirchlet process) or the data points become themselves part of the model specification, as in the nonparametric regression used here.

The use of Gaussian process (GP) regression (or "kriging" in the geospatial literature) to formulate a predictive model is relatively new in the context of modeling dynamical systems (**???**), and was first introduced in the context ecological modeling and fisheries management in Munch et al. (2005). An accessible and thorough introduction to the formulation and use of GPs can be found in Rasmussen and Williams (2006).

The posterior distribution for the hyper-parameters of the Gaussian process model are estimated by Metropolis-Hastings algorithm, again with details and code provided in the Appendix. Rasmussen and Williams (2006) provides an excellent general introduction to Gaussian Processes and Munch, Kottas, and Mangel (2005) first discusses their application in the context of population dynamics models such as fisheries stock-recruitment relationships.

Approach and Methods

Statement of the optimal control problem

To illustrate the application of the BNP-SDP approach and compare to the predictions of the alternative parametric models we focus on the classical problem of selecting the appropriate harvest level given an observation of the stock size in the previous year (Reed 1979; Ludwig and Walters 1982; Mangel and Clark 1988). Given this observation and the model (together with the parameter uncertainty) of the stock recruitment process, the manager seeks to maximize the value of the fishery over a fixed time interval of 50 years at a discount rate of 0.01. The value function (profits) at time t depends on the true stock size x_t and the chosen harvest level h_t. For simplicity we assume profit is simply proportional in the realized harvest (only enforcing the restriction that harvest can not exceed available stock).

Parametric models

We consider three candidate parametric models of the stock-recruitment dynamics: The Ricker model, the Allen model (Allen and Tanner 2005), the Myers model (Myers et al. 1995). The familiar Ricker model involves two parameters, corresponding to a growth rate and a carrying capacity, and cannot support alternative stable state dynamics (though as growth rate increases it exhibits a periodic attractor that proceeds through period-doubling into chaos. We will generally focus on dynamics below the chaotic threshold for the purposes of this analysis.) The Allen model resembles the Ricker dynamics with an added Allee effect parameter (**???**), below which the population cannot persist. The Myers model also has three parameters and contains an Allee threshold, but has compensatory rather than over-compensatory density dependence (resembling a Beverton-Holt curve rather than a Ricker curve at high densities.)
We assume multiplicative log-normal noise perturbs the growth predicted by the each of the deterministic model skeletons described above. This introduces one additional parameter σ that must be estimated by each model.

As we simulate training data from the Allen model, we will refer to this as the structurally correct model. The Ricker model is thus a reasonable approximation of these dynamics far from the Allee threshold (but lacks threshold dynamics), while the Myers model shares the essential feature of a threshold but differs in the structure. Thus we have three potential parametric models of the stock dynamics.

We introduce parametric uncertainty by first estimating each of the candidate models from data on unexploited stock dynamics following some perturbation (non-equilibrium initial condition) over several time steps. This training data could be generated in several different ways (such as known variable exploitation rates, etc.), as long as it reflects the dynamics in some limited region of state space without impacting the problem. We consider a period of 40 years of training data: long enough that the estimates are not dependent on the particular realization, while longer times are not likely to provide substantial improvement (i.e. the results are not sensitive to this interval). Each of the models (described below) is fit to the same training data, as shown in Figure 1.

We infer posterior distributions for the parameters of each model in a Bayesian context using Gibbs sampling (implemented in R (Team 2013) using jags, (Su and Yajima 2013)). We choose uninformative uniform priors for all parameters (See Appendix, Figures and tables, and the R code provided). One-step-ahead predictions of these model fits are shown in Figure 1. While alternative approaches to the estimation of the posteriors (such as integrating out the rate parameter r analytically and then performing a grid search over the remaining parameter space), the approach of using a standard Gibbs sampler routine is both more general and representitive of common practice in estimating posteriors for such models. Each sampling is tested for Gelman-Rubin convergence and results are robust to longer runs.

An optimal policy function is then inferred through stochastic dynamic programming for each model given the posterior distributions of the parameter estimates. This policy maximizes the expectation of the value function integrated over the parameter uncertainty. (code implementing this algorithm provided in the Appendix).

The Gaussian Process model

We also estimate a simple Gaussian Process defined by a radial basis function kernel of two parameters: ℓ, which gives the characteristic length-scale over which correlation between two points in state-space decays, and σ, which gives the scale of the process noise by which observations Y_{t+1} may differ from their predicted values X_{t+1} given an observation of the previous state, X_t. Munch, Kottas, and Mangel (2005) gives an accessible introduction to the use of Gaussian Processes in providing a Bayesian nonparametric description of the stock-recruitment relationship.

We use a Metropolis-Hastings Markov Chain Monte Carlo to infer posterior distributions of the two parameters of the GP (Figure S13, code in appendix), under weakly informative Gaussian priors (see parameters in table S5). As the posterior distributions differ substantially from the priors (Figure S13), we can be assured that most of the information in the posterior comes from the data rather than the prior belief.

Though we are unaware of prior application of this type, it is reasonably straight-forward to adapt the Gaussian Process for Stochastic Dynamic Programming. Recall that unlike the parametric models the Gaussian process with fixed parameters already predicts a distribution of curves rather than a single curve. We must first integrate over this distribution of curves given a sampling of parameter values drawn from the posterior distribution of the two GP parameters, before integrating over the posterior of those parameters themselves.

Results

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