Avoiding tipping points in the management of ecological systems: a non-parametric Bayesian approach

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Abstract

Model uncertainty and limited data coverage are fundamental challenges to robust ecosystem management. These challenges are acutely highlighted by concerns that many ecological systems may contain tipping points. Before a collapse, we do not know where the tipping points lie, if the exist at all. Hence, we know neither a complete model of the system dynamics nor do we have access to data in some large region of state-space where such a tipping point might exist. These two sources of uncertainty frustrate state-of-the-art parametric approaches to decision theory and optimal control. I will illustrate how a non-parametric approach using a Gaussian Process prior provides a more flexible representation of this inherent uncertainty. Consequently, we can adapt the Gaussian Process prior to a stochastic dynamic programming framework in order to make robust management predictions under both model and uncertainty and limited data.

Introduction

Decision making under uncertainty is a ubiquitous challenge of natural resource management and conservation. Ecological dynamics are frequently complex and difficult to measure, making uncertainty in our understanding a prediction a persistent challenge to effective management. Decision-theoretic approaches provide a framework to determine the best sequence of actions in face of uncertainty, but only when that uncertainty can be meaningfully quantified (**???**). The sudden collapse of fisheries and other ecosystems has increasingly emphasized the difficulties of formulating even qualitatively correct models of the underlying processes.

We develop these concerns in the context of fisheries, though the underlying challenges and methods are germane to many other conservation and resource management problems. The economic value and ecological concern have made marine fisheries the crucible for much of the founding work (**???**; **???**; **???**; **???**) in managing ecosystems under uncertainty. Global trends (**???**) and controversy (**???**; **???**) have made understanding these challenges all the more pressing.

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Uncertainty enters the decision-making process at many levels: intrinsic stochasticity in biological processes, measurements, and implementation of policy (**???**; **???**; **???**; **???**), parameteric uncertainty (**???**; **???**; **???**; **???**), and model or structural uncertainty (**???**; **???**; **???**). Of these, structural uncertainty incorporates the least a priori knowledge or assumptions and is generally the hardest to quantify. Typical approaches assume a weak notion of model uncertainty in which the correct model (or reasonable approximation) of the dynamics must be identified from among a handful of alternative models. Here we consider an approach that addresses uncertainty at each of these levels without assuming the dynamics follow a particular (i.e. parametric) structure.

Cut the next three paragraphs, since they are covered more consisely in the above paragraph?

Process, measurement, and implementation error

Resource management and conservation planning seek to determine the optimal set of feasible actions to maximize the value of some objectives (e.g (**???**)). Process error, measurement error, implementation error (**???**). These sources of stochasticity in turn mean that model parameters can only be estimated approximately, requiring parametric uncertainty also be considered (**???**).

Parametric uncertainty

As the parameter values for these models must be estimated from limited data, there will always be some uncertainty associated with these values. This uncertainty further compounds the intrinsic variability introduced by demographic or environmental noise. The degree of uncertainty in the parameter values can be inferred from the data and reflected in the estimates of the transition probabilities (**???**; **???**; **???**; **???**).

Structural (model) uncertainty

Estimates of parameter uncertainty are only as good as the parametric models themselves. Often we do not understand the system dynamics well enough to know if a model provides a good approximation over the relevant range of states and timescales (criteria that we loosely refer to as defining the “right” or “true” model.) So called structural or model uncertainty is a more difficult problem than parametric uncertainty. Typical solutions involve either model choice, model averaging, or introducing yet greater model complexity of which others may be special cases (model averaging being one such way to construct such a model) (**???**; **???**; **???**). Even setting aside other computational and statistical concerns (e.g. (**???**)), these approaches do not address our second concern - representing uncertainty outside the observed data range.

Model uncertainty is particularly insidious when model predictions must be made outside of the range of data on which the model was estimated. This extrapolation uncertainty is felt most keenly in decision-theoretic (or optimal control) applications, since (a) exploring the potential action space typically involves considering actions that may move the system outside the range of observed behavior, and (b) decision-theoretic alogrithms rely not only on reasonable estimates of the expected outcomes, but depend on the weights given to all possible outcomes

(**???**). If we are observing the fluctuations of a given fish stock over many years under a fixed harvesting pressure, we might develop and test a model that could reasonably predict the frequency of a deviation of a given size, even when such a deviation has not been previously observed. Yet such predictions are far less reliable when extrapolated to a harvest pressure that has not yet been observed. Thus, model uncertainty can be particularly challenging in the management and decision-making context.

This difficult position of having neither the true model nor data that covers the full range of possible states is unfortunately the rule more than the exception. The potential concern of tipping points in ecological dyanmics (**???**; **???**) reflects these concerns – as either knowledge of the true model or more complete sampling of the state space would make it easy to identify if a tipping point existed. If we do not know but cannot rule out such a possibility, then we face decision-making under this dual challege of model uncertainty and incomplete data coverage.

These dual concerns pose a substantial challenge to existing decision-theoretic approaches (**???**). Because intervention is often too late after a tipping point has been crossed (but see (**???**)), management is most often concerned with avoiding potentially catastrophic tipping points before any data is available at or following a transition that would more clearly reveal these regime shift dynamics (**???**).

Here we illustrate how a stochastic dynamic programming (SDP) algorithm (**???**; **???**) can be driven by the predictions from a Bayesian non-parametric (BNP) approach (**???**). This provides two distinct advantages compared with contemporary approaches. First, using a BNP sidesteps the need for an accurate model-based description of the system dynamics. Second, the BNP can better reflect uncertainty that arises when extrapolating a model outside of the data on which it was fit. We illustrate that when the correct model is not known, this latter feature is crucial to providing a robust decision-theoretic approach in face of substantial structural uncertainty.

This paper represents the first time the SDP decision-making framework has been used without an a priori model of the underlying dynamics through the use of the BNP approach. In contrast to parametric models which can only reflect uncertainty in parameter estimates, the BNP approach provides a more state-space dependent representation of uncertainty. This permits a much greater uncertainty far from the observed data than near the observed data. These features allow the GP-SDP approach to find robust management solutions in face of limited data and without knowledge of the correct model structure.

The idea that any approach can perform well without either having to know the model or have particularly good data should immediately draw suspicion. The reader must bear in mind that the strength of our approach comes not from black-box predictive power from such limited information, but rather, by providing a more honest expression of uncertainty outside the observed data without sacrificing the predictive capacity near the observed data. By coupling this more accurate description of what is known and unknown to the decision-making under uncertainty framework provided by stochastic dynamic programming, we are able to obtain more robust management policies than with common parametric modeling approaches.

The nature of decision-making problems provides a convenient way to compare models. Rather than compare models in terms of best fit to data or fret over the appropriate penalty for model complexity, model performance is defined in the concrete terms of the decision-maker's objective function, which we will take as given. (Much argument can be made over the 'correct' objective function, e.g. how to account for the social value of fish left in the sea vs. the commercial value of fish harvested; see (**???**) for further discussion of this issue. Alternatively, we can always compare model performance across multiple potential objective functions.) The decision-maker does not necessarily need a model that provides the best mechanistic understanding or the best long-term outcome, but rather the one that best estimates the probabilities of being in different states as a result of the possible actions.

Background on the Gaussian Process

Addressing the difficulty posed by extrapolation without knowing the true model requires a nonparametric approach to model fitting: one that does not assume a fixed structure but rather depends on the size of the data (e.g. non-parametric regression or a Dirichlet process). This established terminology is nevertheless unfortunate, as (a) this approach still involves the estimation of parameters, and (b), Statisticians use non-parametric to mean both this property (structure is not fixed by the parameters) and an entirely different (and probably more familiar) case in which the model does not assume any distribution (e.g. non-parametric bootstrap, order statistics). Some literature thus uses the term semi-parametric, which merely adds ambiguity to the confusion.

This non-parametric property – having a structure explicitly dependent on the data – is precisely the property that makes this approach attractive in face of the limited data sampling challenges discussed above. Having fit a parametric model to some data, the model is completely described by the values (or posterior distributions) of it's parameters. The non-parametric model is not captured by its parameter values or distributions alone. Either the model scales with the complexity of the data on which it is estimated (e.g. nonparametric heirarchical approaches such as the Dirchlet process) or the data points become themselves part of the model specification, as in the nonparametric regression used here.

The use of Gaussian process (GP) regression (or "kriging" in the geospatial literature) to formulate a predictive model is relatively new in the context of modeling dynamical systems (**???**), and was first introduced in the context ecological modeling and fisheries management in (**???**). An accessible and thorough introduction to the formulation and use of GPs can be found in (**???**).

The posterior distribution for the hyper-parameters of the Gaussian process model are estimated by Metropolis-Hastings algorithm, again with details and code provided in the Appendix. (**???**) provides an excellent general introduction to Gaussian Processes and (**???**) first discusses their application in the context of population dynamics models such as fisheries stock-recruitment relationships.

Approach and Methods

Statement of the optimal control problem

To illustrate the application of the BNP-SDP approach and compare to the predictions of the alternative parametric models we focus on the classical problem of selecting the appropriate harvest level given an observation of the stock size in the previous year (**???**; **???**; **???**). Given this observation and the model (together with the parameter uncertainty) of the stock recruitment process, the manager seeks to maximize the value of the fishery over a fixed time interval of 50 years at a discount rate of 0.01. The value function (profits) at time t depends on the true stock size x_t and the chosen harvest level h_t. For simplicity we assume profit is simply proportional in the realized harvest (only enforcing the restriction that harvest can not exceed available stock).

Parametric models

We consider three candidate parametric models of the stock-recruitment dynamics: The Ricker model, the Allen model (**???**), the Myers model (**???**). The familiar Ricker model involves two parameters, corresponding to a growth rate and a carrying capacity, and cannot support alternative stable state dynamics (though as growth rate increases it exhibits a periodic attractor that proceeds through period-doubling into chaos. We will generally focus on dynamics below the chaotic threshold for the purposes of this analysis.) The Allen model resembles the Ricker dynamics with an added Allee effect parameter (**???**), below which the population cannot persist. The Myers model also has three parameters and contains an Allee threshold, but has compensatory rather than over-compensatory density dependence (resembling a Beverton-Holt curve rather than a Ricker curve at high densities.)

We assume multiplicative log-normal noise perturbs the growth predicted by the each of the deterministic model skeletons described above. This introduces one additional parameter σ that must be estimated by each model.

As we simulate training data from the Allen model, we will refer to this as the structurally correct model. The Ricker model is thus a reasonable approximation of these dynamics far from the Allee threshold (but lacks threshold dynamics), while the Myers model shares the essential feature of a threshold but differs in the structure. Thus we have three potential parametric models of the stock dynamics.

We introduce parameteric uncertainty by first estimating each of the candidate models from data on unexploited stock dynamics following some pertubation (non-equilibrium initial condition) over several time steps. This training data could be generated in several different ways (such as known variable exploitation rates, etc), as long as it reflects the dynamics in some limited region of state space without impacting the problem. We consider a period of 40 years of training data: long enough that the estimates are not dependent on the particular realization, while longer times are not likely to provide substantial improvement (i.e. the results are not sensitive to this interval). Each of the models (described below) is fit to the same training data, as shown in Figure 1.

We infer posterior distributions for the parameters of each model in a Bayesian context using Gibbs sampling (implemented in R (**???**) using jags, (**???**)). We choose uninformative uniform priors for all parameters (See Appendix, Figures S1-S3, and Table S1, and the R code provided). One-step-ahead predictions of these model fits are shown in Figure 1.

An optimal policy function is then inferred through stochastic dynamic programming for each model given the posterior distributions of the parameter estimates. This policy maximizes the expectation of the value function integrated over the parameter uncertainty. (code implementing this algorithm provided in the Appendix).

The Gaussian Process model

We also estimate a simple Gaussian Process defined by a radial basis function kernel of two parameters: ℓ, which gives the characteristic length-scale over which correlation between two points in state-space decays, and σ, which gives the scale of the process noise by which observations Y_{t+1} may differ from their predicted values X_{t+1} given an observation of the previous state, X_t. (**???**) gives an accessible introduction to the use of Gaussian Processes in providing a Bayesian nonparametric description of the stock-recruitment relationship.

We use a Metropolis-Hastings Markov Chain Monte Carlo to infer posterior distributions of the two parameters of the GP (Figure S13, code in appendix), under weakly informative Gaussian priors (see parameters in table S5). As the posterior distributions differ substantially from the priors (Figure S13), we can be assured that most of the information in the posterior comes from the data rather than the prior belief.

Though we are unaware of prior application of this type, it is reasonably straight-forward to adapt the Gaussian Process for Stochastic Dynamic Programming. Recall that unlike the parametric models the Gaussian process with fixed parameters already predicts a distribution of curves rather than a single curve. We must first integrate over this distribution of curves given a sampling of parameter values drawn from the posterior distribution of the two GP parameters, before integrating over the posterior of those parameters themselves.

Results

All models fit the observed data rather closely and with relatively small uncertainty, as illustrated in the posterior predictive curves in Figure 1. Figure 1 shows the training data of stock sizes observed over time as points, overlaid with the step-ahead predictions of each estimated model using the parameters sampled from their posterior distributions. Each model manages to fit the observed data rather closely. Compared to the expected value of the true model most estimates appear to overfit, predicting fluctuations that are actually due purely to stochasticity in growth rate. Model-choice criteria shown in Table 1 penalize more complex models and show a slight preference for the simpler Ricker model over the more complicated alternate stable state models (Allen and Myers). Details on MCMC estimates for each model, traces, and posterior distributions can be found in the appendix.

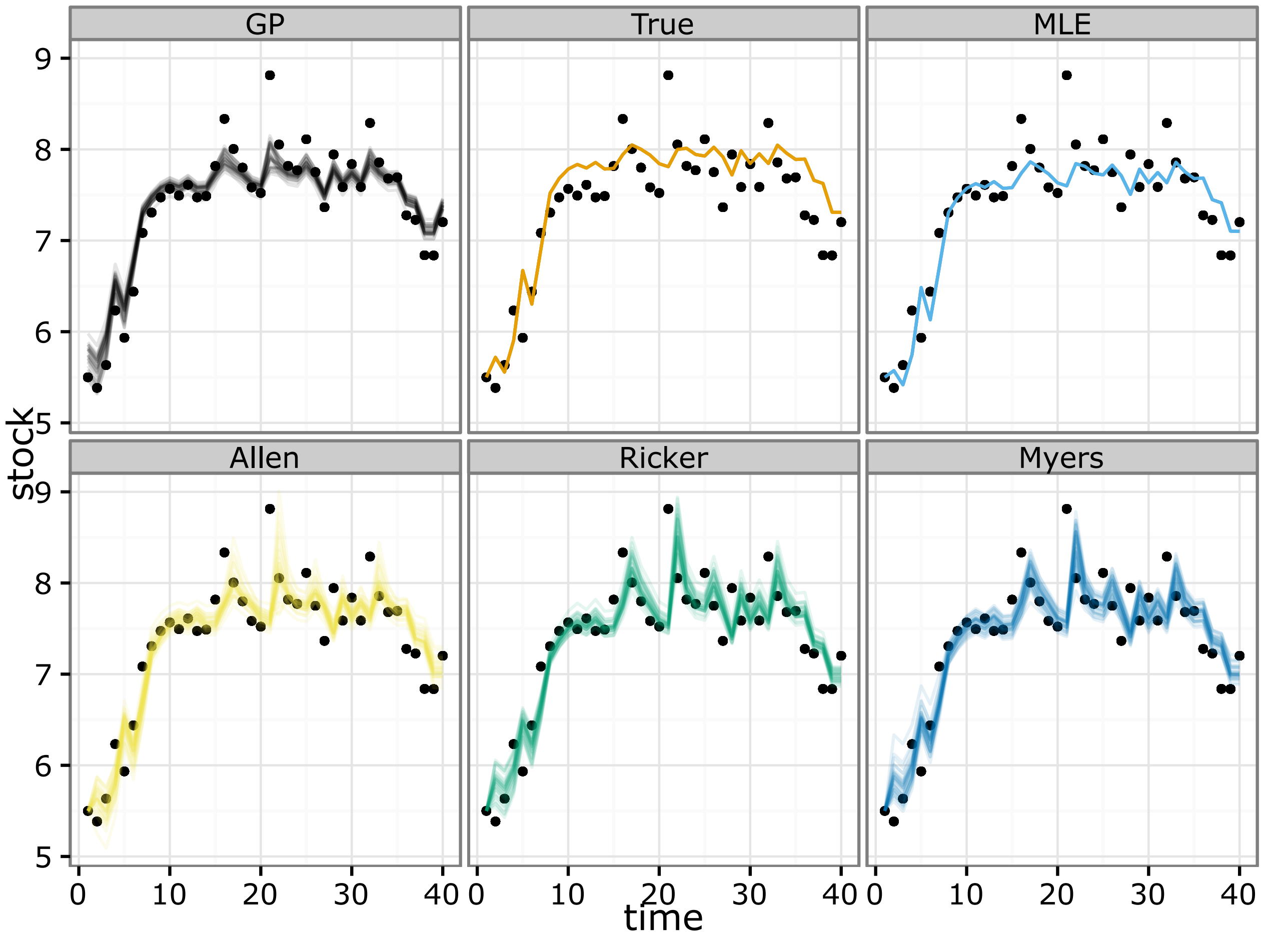

Figure 1: Points show the training data of stock-size over time. Curves show the posterior step-ahead predictions based on each of the estimated models.

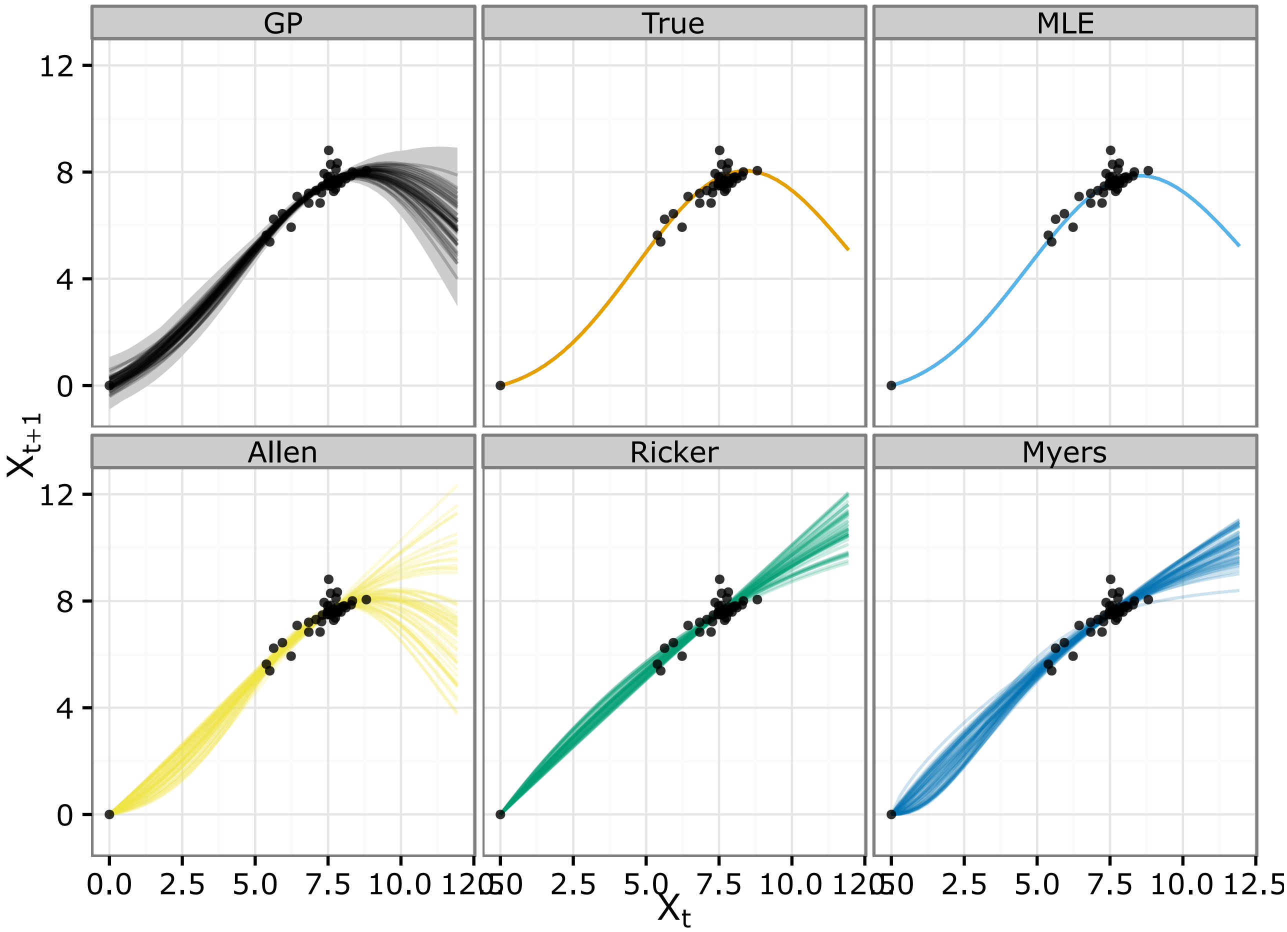

Figure 2: Graph of the inferred Gaussian process compared to the true process and maximum-likelihood estimated process. Graph shows the expected value for the function f under each model. Two standard deviations from the estimated Gaussian process covariance with (light grey) and without (darker grey) measurement error are also shown. The training data is also shown as black points. The GP is conditioned on (0,0), shown as a pseudo-data point.

	Allen	Ricker	Myers
DIC	50.14	49.45	50.61
AIC	-24.60	-30.07	-27.19
BIC	-17.85	-25.00	-20.44

Table 1: Model choice scores for several common criteria all (wrongly) select the simplest model. As the true (Allen) model is not distinguishable from the simpler (Ricker) model in the region of the observed data, this error cannot be avoided regardless of the model choice criterion. This highlights the danger of model choice when the selected model will be used outside of the observed range of the data.

The mean inferred state space dynamics of each model relative to the true model used to generate the data is shown in Figure 2, predicting the relationship between observed stock size (x-axis) to the stock size after recruitment the following year. Note that in contrast to the other models shown, the expected Gaussian process corresponds to a distribution of curves - as indicated by the gray band - which itself has a mean shown in black. Parameter uncertainty (not shown) spreads out the estimates further. The observed data from which each model is estimated is also shown. The observations come from only a limited region of state space corresponding to unharvested or weakly harvested system. No observations occur at the theoretical optimum harvest rate or near the tipping point.

Despite the similarities in model fits to the observed data, the policies inferred under each model differ widely, as shown in Figure 3. Policies are shown in terms of target escapement, S_t. Under models such as this a constant escapement policy is expected to be optimal (**???**), whereby population levels below a certain size S are unharvested, while above that size the harvest strategy aims to return the population to S, resulting in the hockey-stick shaped policies shown. Only the structurally correct model (Allen model) and the GP produce policies close to the true optimum policy (where both the underlying model structure and parameter values are known without error).

The consequences of managing 100 replicate realizations of the simulated fishery under each of the policies estimated is shown in Figure 4. As expected from the policy curves, the structurally correct model under-harvests, leaving the stock to vary around it's un-fished optimum. The structurally incorrect Ricker model over-harvests the population passed the tipping point consistently, resulting in the immediate crash of the stock and thus derives minimal profits.

These results are robust across a range of stochastic realizations, models, and parameter values. The results across this range can most easily be compared by using the relative differences in net present value realized by each of the model, as shown in Figure 5. The BNP-SDP approach most consistently realizes a value close to the optimal solution, and importantly avoids ever driving the system across the tipping point, which results in the near-zero value cases in the parametric models.

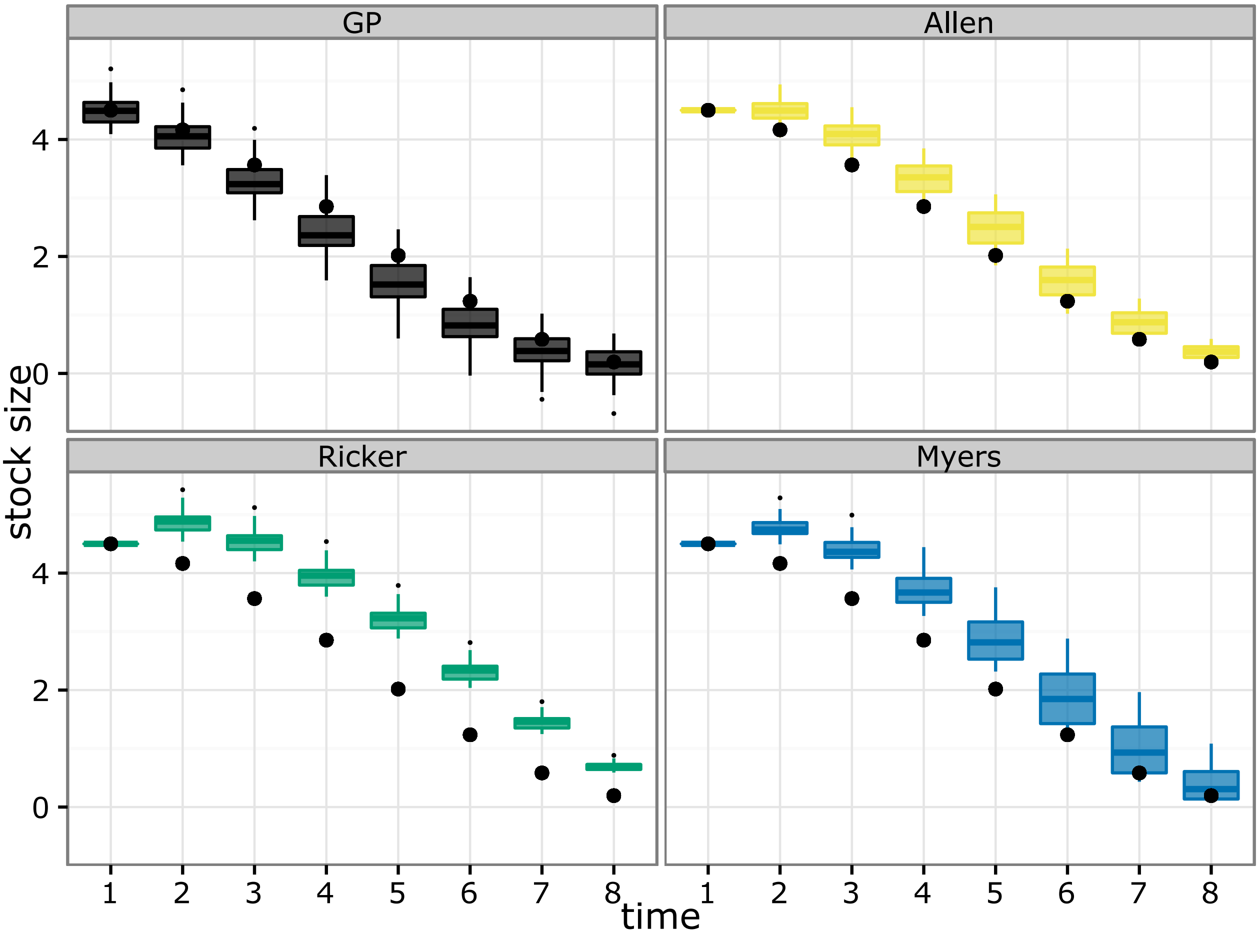

Figure 3: Out of sample predictions of the dynamics under each model. Points show the stock size simulated by the true model. Overlay shows the range of states predicted by each model, based on the state observed in the previous time step. The Ricker model always predicts population growth, while the actual population shrinks in each step as the initial condition falls below the Allee threshold of the underlying model (Allen). Note that the GP is both more pessimistic and more uncertain about the future state than the parameteric models, while the realized state often falls outside of the expected range forecasted by the structurally incorrect Myers and Ricker models.

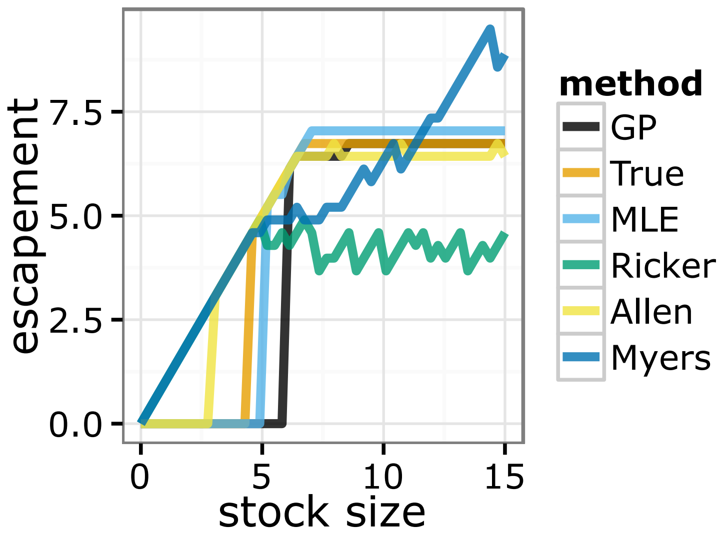

Figure 4: The steady-state optimal policy (infinite boundary) calculated under each model. Policies are shown in terms of target escapement, S_t, as under models such as this a constant escapement policy is expected to be optimal (**???**).

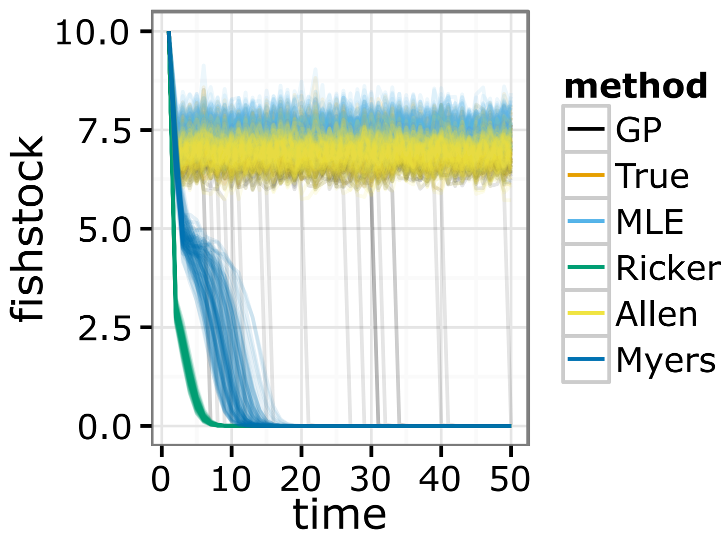

Figure 5: Gaussian process inference outperforms parametric estimates. Shown are 100 replicate simulations of the stock dynamics (eq 1) under the policies derived from each of the estimated models, as well as the policy based on the exact underlying model.

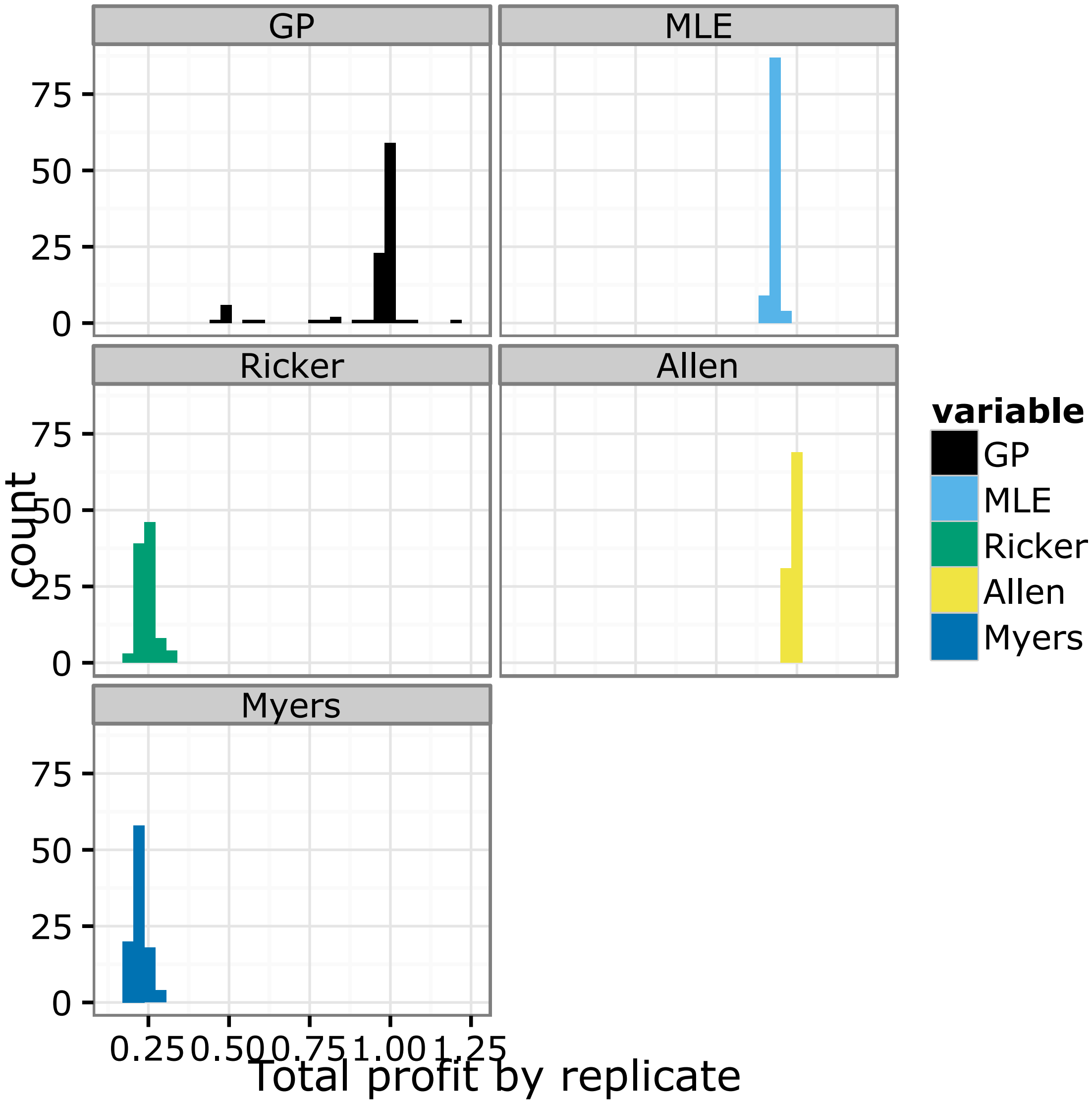

Figure 6: Histograms of the realized net present value of the fishery over a range of simulated data and resulting parameter estimates. For each data set, the three models are estimated as described above. Values plotted are the averages of a given policy over 100 replicate simulations. Details and code provided in the supplement.

Discussion

In any modeling effort, models must be chosen for the task at hand. Though simple mechanistically motivated models offer the greatest potential to increase our basic understanding of ecological processes (**???**; **???**), such models can be not only inaccurate but misleading when relied upon in a quantitative decision making framework. In this paper we have tackled two aspects of uncertainty that are both common to many ecological desicion-making problems and fundamentally challenging to existing approaches which largely rely on parametric models:

1. We do not know what the correct models are for ecological systems.
2. We have limited data from which to estimate the model – in particular, such models may be misleading in predicting the probability of outcomes outside the training data.

We have illustrated how the use of non-parametric approaches can provide more reliable solutions in the sequential decision-making problem.

Traditional model-choice approaches can be positively misleading.

These results illustrate that model-choice approaches would be positively misleading – supporting simpler models that cannot express tipping point dynamics merely on account of them being similar. As the data shown comes only from the basin of attraction near the unfished equilibrium, near which all of the models are approximately linear and approximately identical.

Model choice approaches trade off model complexity and fit to the data. When the data come from a limited region of state-space – as is necessarily the case whenever there is a potential concern about tipping point dynamics – simpler models can fit just as well and will tend to outperform more complex ones. This approach would be appropriate when the dynamics can be expected to remain in the region of the training data; for instance, if we only considered the forecasting accuracy of the unfished population dynamics under each model.

In contrast, the decision-maker's problem of setting appropriate harvest levels cannot exclude regions of state-space outside the observed range when integrating over all possible decisions to find the optimal choice. Such problems are not constrained to fisheries management but ubiquitous across ecological decision-making and conservation where the greatest concerns involve entering previously unobserved regions of state-space – whether that is the collapse of a fishery, the spread of an invasive, or the loss of habitat.

BNP-SDP expresses larger uncertainty in regions where the data are poor

The parametric models perform worst when they propose a management strategy outside the range of the observed data. The non-parametric Bayesian approach, in contrast, allows a predictive model that expresses a great deal of uncertainty about the probable dynamics outside the observed range, while retaining very good predictive accuracy in the range observed. The management policy dictated by the GP balance this uncertainty against the immediate value of the harvest, and act to stabilize the population dynamics in a region of state space in which the predictions can be reliably reflected by the data.

BNP-SDP has good predictive accuracy where data are good

While expressing larger uncertainty outside the observed data, the GP can also provide a better fit with smaller uncertainty inside the range of the observed data. This arises from the greater flexibility of the Gaussian process, which describes a large family of possible curves. Despite this flexibility, the GP can be described in relatively few parameters and is thus far less likely to overfit.

Risk-prone and risk-adverse value functions

The degree to which the decision-making part of the algorithm (the SDP) chooses to explore or avoid the resulting region of uncertainty can also be influenced by the curvature of the value (profit) function Π. Both to simplify the intution and avoid biasing this result, we have chosen a profits that are linear in the catch and thus neither risk-prone nor risk adverse. Making this function concave, representing the typical assumption of diminishing returns, would make the SDP more risk-adverse (as larger-than-expected stock sizes offer dimished returns relative to the cost of smaller-than-expected stock sizes), and strengthen the result shown here in which the BNP solution tends to avoid the region of uncertainty. Sufficiently convex or risk-prone functions could lead the SDP to attempt higher exploitation rates despite the uncertainty. Understanding the relative roles of such functions would be a promising direction for future investigation.

The role of the prior

Lastly, it should be noted that outside the data, the NBP reverts to the prior, and consequently the choice of the prior can also play a significant role in determining the optimal policy inferred by the SDP. In the examples shown here we have selected a prior that is both relatively uninformative (due to the broad priors placed on its parameters ℓ and σ and simple (mean zero, radial basis function kernel). In practice, both the choice of mean and the choice of the covariance function may be chosen to confer particular biological properties, as well as more biologically informed priors for ℓ and σ. In principle, this may allow a manager to improve the performance of the BNP-SDP approach by adding only enough additional detail as is justified. For instance, it would be possible to use a linear or a Ricker-shaped mean in the prior without making the much stronger assumption that the Ricker is the structurally correct model. However, this influence raises challenges as well. For instance, in choosing a trivial mean-zero prior, we bias the dynamics to transitions to 0 in step X_{t+1} from any stock size X_t in the prior year, in the absence of any other data. Future research must make sure that both the prior and the value function are chosen appropriately for the problem at hand.

Future directions

In this simulated example, the underlying dynamics are truly governed by a simple parametric model, allowing the parametric approaches to be more accurate. Similarly, because the dynamics are one-dimensional dynamics and lead to stable nodes (rather than other attractors such as limit-cycles resulting in oscillations), the training data provides relatively limited information about the dynamics. For these reasons, we anticipate that in higher-dimensional examples characteristic of ecosystem management problems that the machine learning approach will prove even more valuable.

In our treatment here we have ignored the possibility of learning during the management phase, in which the additional observations of the stock size could potentially improve parameter estimates. Of particular interest in the context of the extreme uncertainty considered here is the notion of "active adaptive management" or "adaptive probing", which may actively seek to reduce uncertainty. The concept of adaptive probing is one area that has explicitly addressed the extrapolation uncertainty addressed here, though typically without the additional issue of model uncertainty. As a result, adaptive probing strategies suggest rather opposite conclusions than what we observe here. Such adaptive probing or Dual Control (e.g. (**???**)) approaches trade off short term utility by choosing actions that can reduce uncertainty. Adaptive probing strategies arise when it is valuable to intentionally force a system far from the observed values even when the expected value such actions is low, as it provides much faster learning and consequent reduction of model uncertainty that can allow greater value to be derived later on. For instance, (**???**) show that it may be advantageous to fish an unexploited population very heavily at first to obtain a better estimate of the recruitment rate. This intuitive strategy when a population is governed by a Ricker or Beverton-Holt-like dynamic would clearly be disastrous if instead the dynamics contained an unforeseen tipping point. The best way to learn where the edge lies may be to walk up to it, but it is also the most dangerous. Future work should attempt to understand when such active adaptive learning is valuable, and when it will increase the risk of an irreversible transition.

Acknowledgments

This work was partially supported by the Center for Stock Assessment Research, a partnership between the University of California Santa Cruz and the Fisheries Ecology Division, Southwest Fisheries Science Center, Santa Cruz, CA and by NSF grant EF-0924195 to MM and NSF grant DBI-1306697 to CB.

Model definitions and estimation

Ricker Model

The Ricker model is given by

(1)

$$X_{t+1} = Z_t X_t e^{r\left(1-\frac{S_t}{K}\right)}$$

where Z_t is log-normal noise of mean unity and log standard deviation σ, representing the stochastic growth, X_t the stock size at time t, S_t the escapement (unharvested population that will recruit in the following year, $S_t = X_t - h_t$). We place uniform priors on the growth rate r, carrying capacity K, and log-normal standard deviation parameter σ, over ranges given in Table 1. Posteriors are inferred by Gibbs sampling using Jags (**???**) (see provided code).

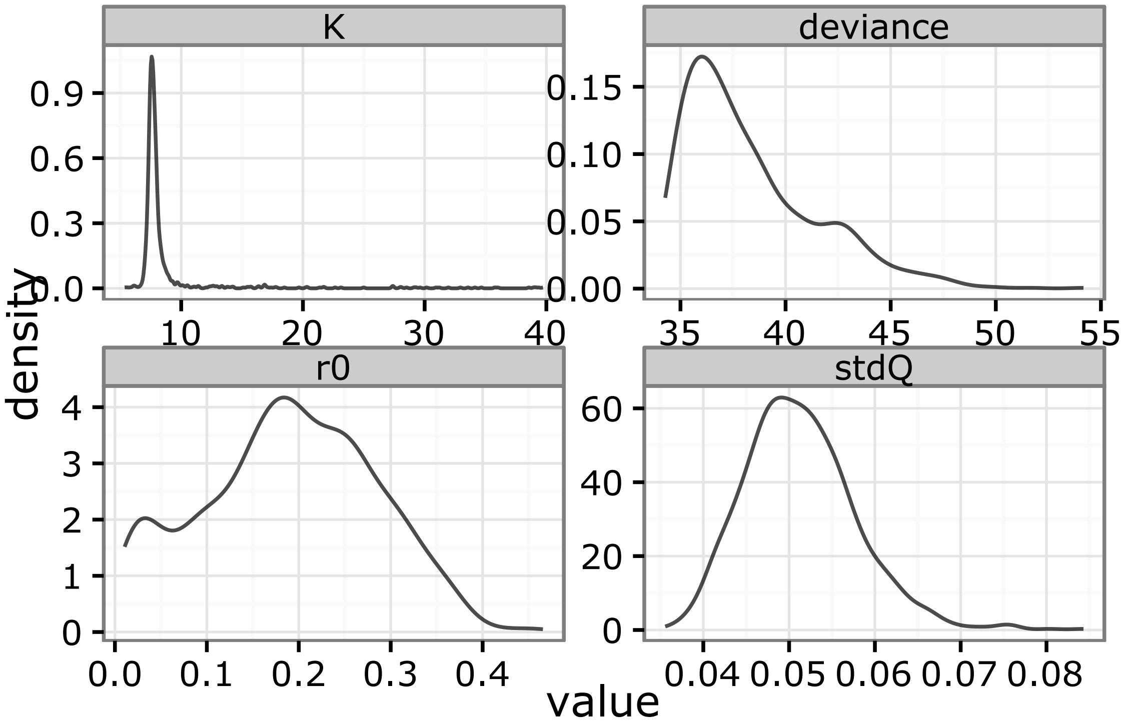

Figure .7: Posteriors from the MCMC estimate of the Ricker model

	parameter	lower.bound	upper.bound
1	r0	0.01	20.00
2	K	0.01	40.00
3	sigma	0.00	100.00

Table .2: parameterization range for the uniform priors in the Ricker model

Myers Model

The Myers model (**???**) is given by

(2)

$$X_{t+1} = Z_t \frac{r S_t^\theta}{1 - \frac{S_t^\theta}{K}}$$

where Z_t is log-normal noise of mean unity and log standard deviation σ, representing the stochastic growth, X_t the stock size at time t, S_t the escapement (unharvested population that will recruit in the following year, $S_t = X_t - h_t$). We place uniform priors on the growth rate r, carrying capacity K, θ controls the strength of the nonlinearity, exhibiting an allee effect for $\theta \geq 2$, and log-normal standard deviation parameter σ, over ranges given in Table 1. Posteriors are inferred by Gibbs sampling using Jags (**???**) (see code provided).

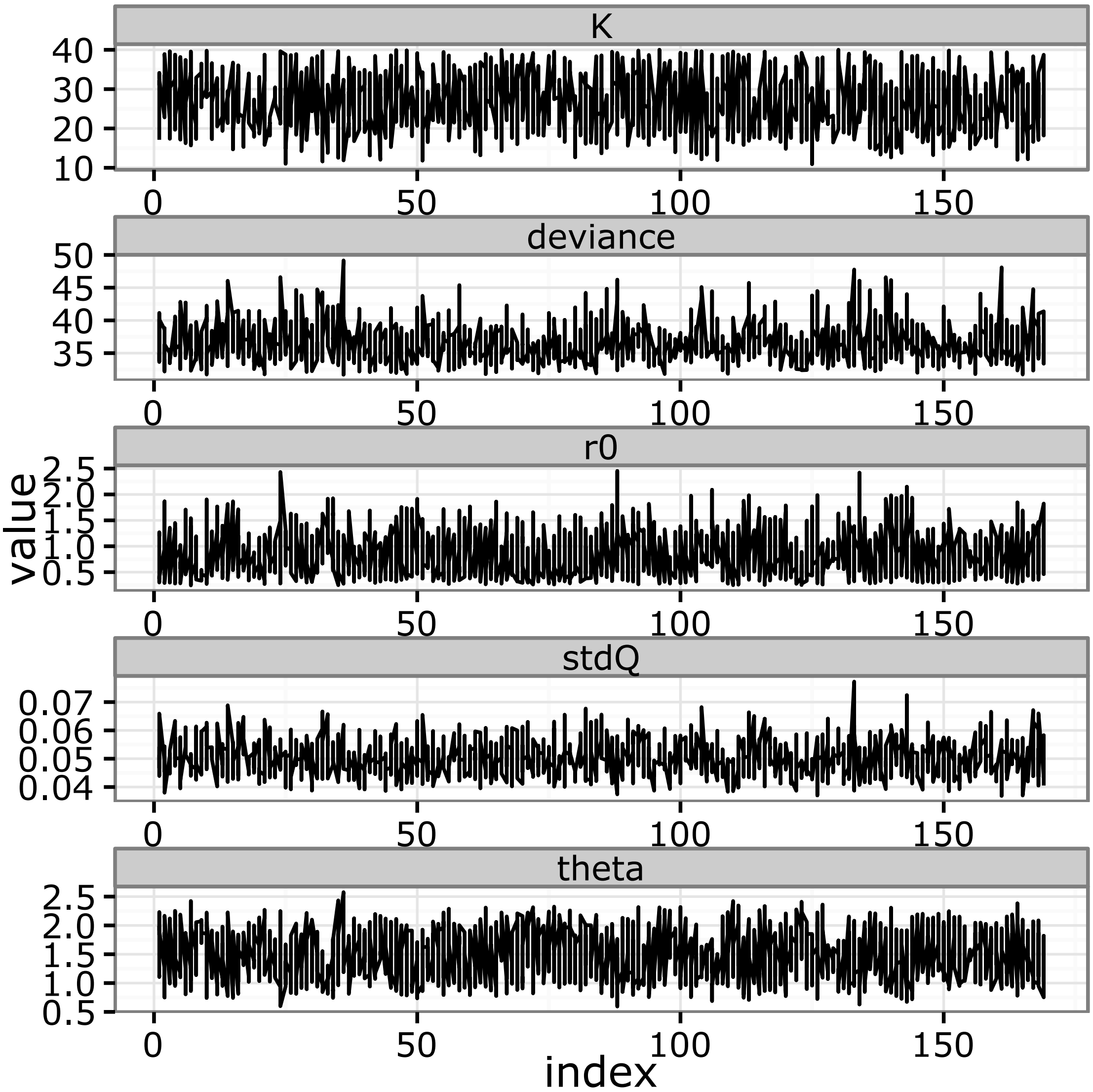

Figure .8: Traces from the MCMC estimate of the Myers model

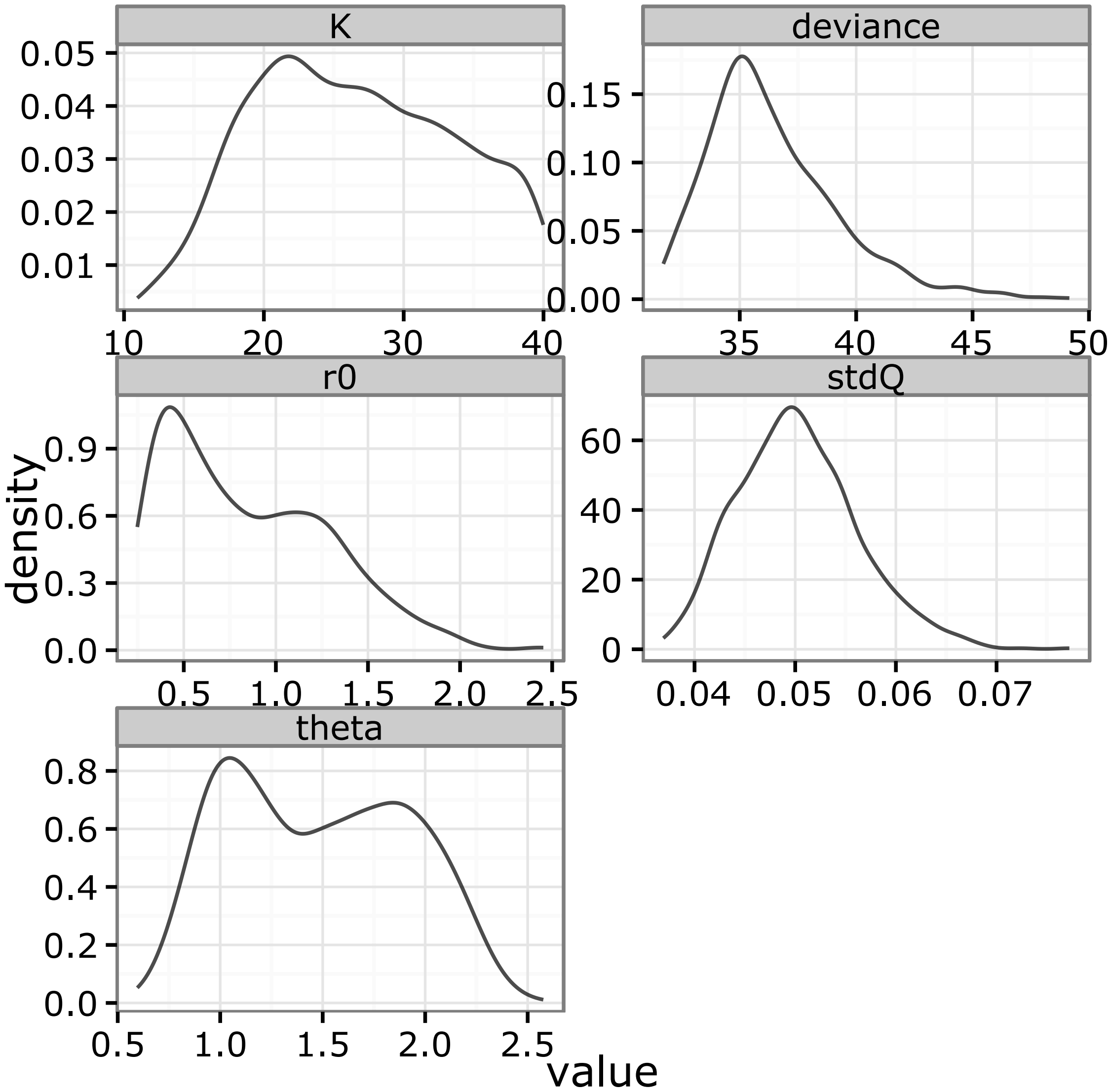

Figure .9: Posterior distributions from the MCMC estimates of the Myers model

	parameter	lower.bound	upper.bound
1	r0	0.00	10.00
2	K	0.00	40.00
3	theta	0.00	10.00
4	sigma	0.00	100.00

Table .3: parameterization range for the uniform priors in the Myers model

Allen model

The Allen model (**???**) is given by

(3)

$$f(S_t) = S_t e^{r\left(1 - \frac{S_t}{K}\right)(S_t - C)}$$

where Z_t is log-normal noise of mean unity and log standard deviation σ, representing the stochastic growth, X_t the stock size at time t, S_t the escapement (unharvested population that will recruit in the following year, $S_t = X_t - h_t$). We place uniform priors on the growth rate r, carrying capacity K, allee threshold C, and log-normal standard deviation parameter σ, over ranges given in Table 1. Posteriors are inferred by Gibbs sampling using Jags (**???**) (see code provided).

	parameter	lower.bound	upper.bound
1	r0	0.01	6.00
2	K	0.01	20.00
3	theta	0.01	20.00
4	sigma	0.00	100.00

Table .4: parameterization range for the uniform priors in the Allen model

Assuming the data are known with some process noise,

$$y = f(x) + \varepsilon$$

ε IID normal, variance σ_n^2. Then under the GP,

$$x|y \sim \mathcal{N}(E, C)$$

$$E = K(X_p, X_o)\left(K(X_o, X_o) - \sigma\mathbb{I}\right)^{-1} y$$

$$C = K(X_p, X_p) - K(X_p, X_o)K(X_o, X_o)^{-1}K(X_o, X_p)$$

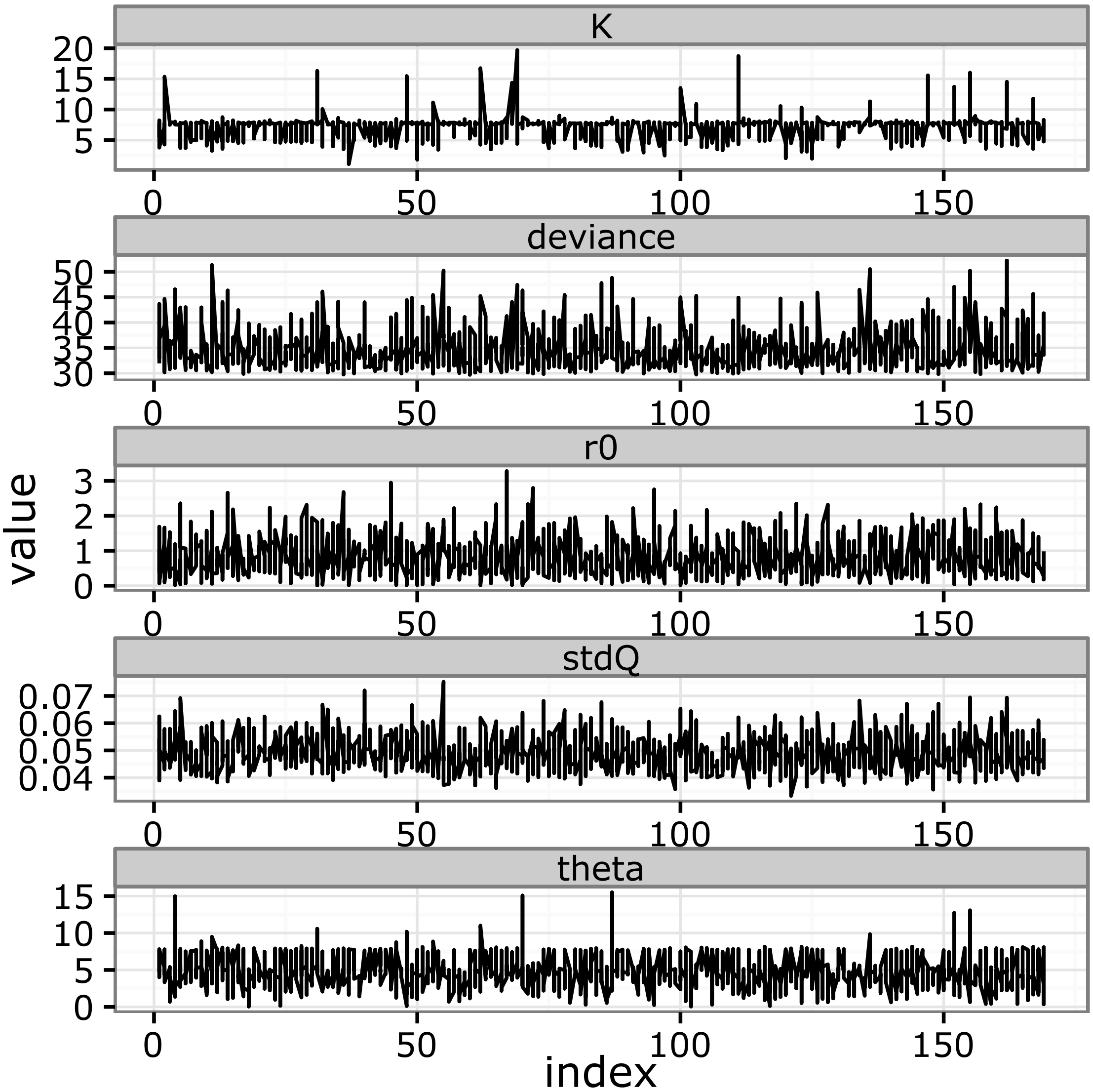

Figure .10: Traces from the MCMC estimate of the Allen model

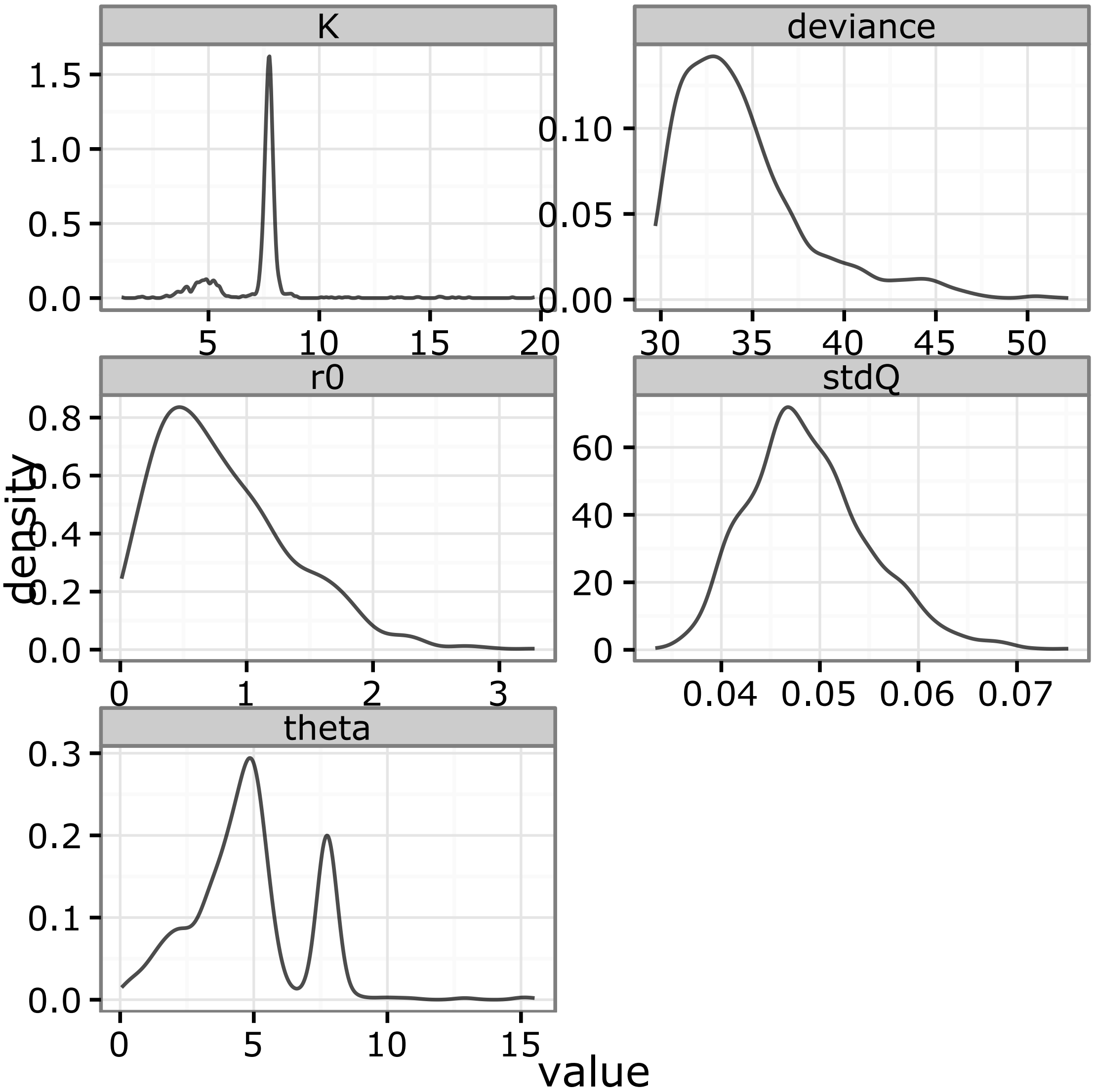

Figure .11: Posteriors from the MCMC estimate of the Allen model

}

The marginal likelihood is then given by:

$$\log(p(y|X)) = -\frac{1}{2}\mathbf{y}^T(K + \sigma_n^2\mathbf{I})^{-1}y - \frac{1}{2}\log\left|K + \sigma_n^2\mathbf{I}\right| - \frac{n}{2}\log 2\pi$$

See (**???**) or (**???**) for a more detailed introduction.

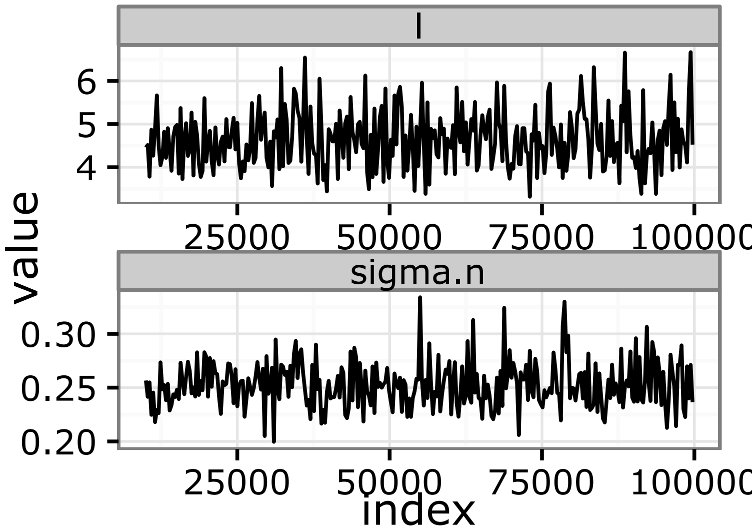

Figure .12: Traces from the MCMC estimates of the GP model

The Gaussian process priors on both the lengthscale ℓ and process noise σ are are inverse Gamma distributed,

$$f(x;\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}x^{-\alpha-1}\exp\left(-\frac{\beta}{x}\right)$$

For the σ prior, $\alpha = 5$ and $\beta = 5$. For ℓ prior, $\alpha = 10$ and $\beta = 10$.

Optimal Control Problem Specification

We seek the harvest policy $h(x)$ that maximizes:

$$\max_{h_t}\sum_{t\in 0}^{\infty}\Pi_t(X_t,h_t)\delta^t$$

subject to the profit function $\Pi(X_t,h)$, discount rate δ, and the state equation

$$X_{t+1} = Z_t f(S_t)$$

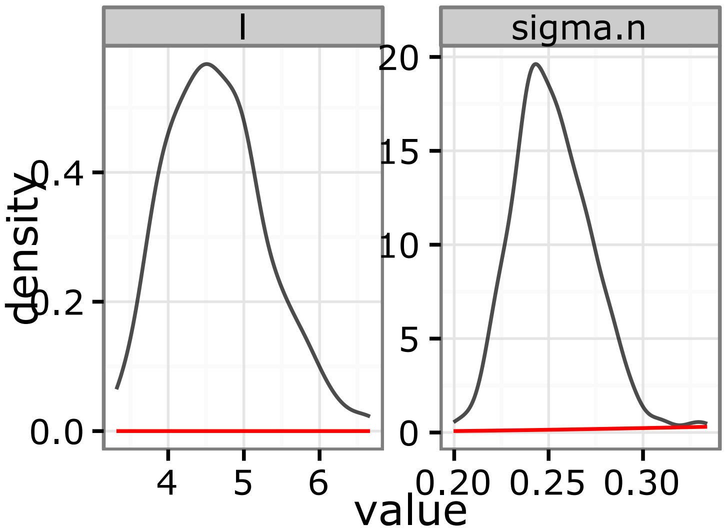

Figure .13: Posterior distributions from the MCMC estimate of the GP model. Prior curves shown in red.

$$S_t = X_t - h_t$$

Where Z_t is multiplicative noise function with mean 1, representing stochastic growth. We will consider log-normal noise with shape parameter σ_g.

Form this we can write down the Bellman recursion as:

$$V_t(x_t) = \max_h \mathbf{E} \left(\Pi(h_t, x_t) + \delta V_{t+1}(Z_{t+1} f(x_t - h_t)) \right)$$

For simplicity we assume profit is simply linear in the realized harvest (only enforcing the restriction that harvest can not exceed available stock), $\Pi(h, x) = min(h, x)$.

Pseudocode for the Bellman iteration

```
# compute the value for each possible harvest
for(h in 1:length(h_grid)){
  V1[h] = delta * F[[h]] %*% V +  profit(x_grid, h_grid[h])
}
# find havest h that gives the maximum value
for(j in 1:gridsize){
  value = max(V1[j,], na.rm = T)  # each col is a different h, max over these
  index = which.max(V1[j,])       # store index so we can recover h's
  output[,j] = c(value, index)    # returns both profit value & index of optimal h.
```

```
}
# Sets V[t+1] = max_h(V[t]) at each possible state value, x
V = out[1,]                            # The new value-to-go
D[,OptTime-time+1] = out[2,]           # The index positions
```

Training data

Eacho of our models $f(S_t)$ must be estimated from training data, which we simulate from the Allen model with parameters $r = 2$, $K = 8$, $C = 5$, and $\sigma_g = 0.05$ for $T = 40$ timesteps, starting at initial condition $X_0 = 5.5$. The training data can be seen in Figure 1.

Code

A copy of the script to reconstruct the simulations and analysis shown here is provided in the supplemental materials, and through this version-stable link to the project's Github code repostitory, nonparameteric-bayes.R. This code is dynamically embedded into the manuscript using Knitr, (**???**). The script relies on custom routines for executing the estimation of the Gaussian process and the for solving the stochastic dynamic programming problem. These routines are provided as an R package, nonparameteric-bayes, also available on Github.

Sensitivity Analysis

These results are not sensitive to the modeling details of the simulation. The Gaussian process estimate remains very close to the optimal solution obtained by knowning the true model across changes to the training simulation, noise scale, parameters or structure of the underlying model, as seen in the figure. Results are pooled across different random seeds (111, 222, 333), noise values of 0.01, 0.05, 0.1, and 0.2, 3 different randomly generated parameters sets for each model, and using either the Myers or Allen as the underlying structure.

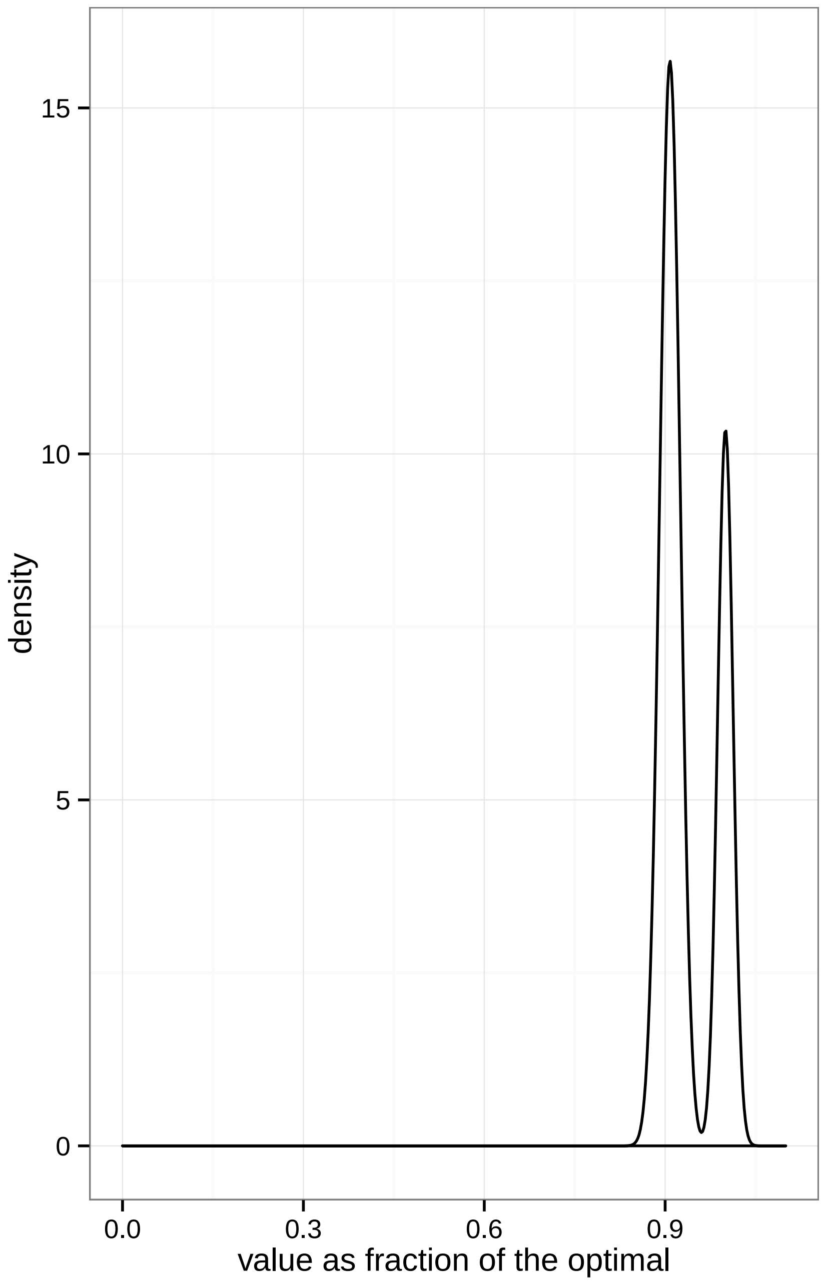

Figure .14: Sensitivity Analysis