

# Avoiding tipping points in fisheries management through Gaussian Process Dynamic Programming

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## Abstract

Model uncertainty and limited data are fundamental challenges to robust management of human intervention in a natural system. These challenges are acutely highlighted by concerns that many ecological systems may contain tipping points, such as Allee population sizes. Before a collapse, we do not know where the tipping points lie, if they exist at all. Hence, we know neither a complete model of the system dynamics nor do we have access to data in some large region of state-space where such a tipping point might exist. We illustrate how a Bayesian Non-Parametric (BNP) approach using a Gaussian Process (GP) prior provides a flexible representation of this inherent uncertainty. We embed GPs in a Stochastic Dynamic Programming (SDP) framework in order to make robust management predictions with both model uncertainty and limited data. We use simulations to evaluate this approach as compared with the standard approach of using model selection to choose from a set of candidate models. We find that model selection erroneously favors models without tipping points – leading to harvest policies that guarantee extinction. The GPDP performs nearly as well as the true model and significantly outperforms standard approaches. We illustrate this using examples of simulated single-species dynamics, where the standard model selection approach should be most effective, and find that it still fails to account for uncertainty appropriately and leads to population crashes, while management based on the GPDP does not, since it does not underestimate the uncertainty outside of the observed data.

**Key words:** Bayesian, Structural Uncertainty, Nonparametric, Optimal Control, Decision Theory, Gaussian Processes, Fisheries Management,

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## 14 Introduction

15 Decision making under uncertainty is a ubiquitous challenge in the management of human  
16 intervention in natural resources and conservation. Decision-theoretic approaches provide a  
17 framework to determine the best sequence of actions in face of uncertainty, but only when  
18 that uncertainty can be meaningfully quantified (Fischer et al. 2009). Over the last four  
19 decades (beginning with Clark (1976), Clark (2009) and Walters and Hilborn (1978)) dynamic  
20 optimization methods, particularly Stochastic Dynamic Programming (SDP), have become  
21 increasingly important as a means of understanding how to manage human intervention into  
22 natural systems. Simultaneously, there has been increasing recognition of the importance of  
23 multiple steady states or ‘tipping points’ (Scheffer et al. 2001, 2009, Polasky et al. 2011) in  
24 ecological systems.

25 We develop a novel approach to address these concerns in the context of fisheries; although  
26 the challenges and methods are germane to other problems of conservation or natural resource  
27 exploitation. Economic value and ecological concern have made marine fisheries the crucible for  
28 much of the founding work on management under uncertainty (Gordon 1954, Clark 1976, 2009,  
29 May et al. 1979, Reed 1979, Ludwig and Walters 1982).

30 Even if we know the proper deterministic description of the biological system, there is intrinsic  
31 stochasticity in biological dynamics, measurements, and implementation of policy (*e.g.* Reed  
32 1979, Clark and Kirkwood 1986, Roughgarden and Smith 1996, Sethi et al. 2005). We may also  
33 lack knowledge about the parameters of the biological dynamics (parametric uncertainty, *e.g.*  
34 Ludwig and Walters 1982, Hilborn and Mangel 1997, McAllister 1998, Schapaugh and Tyre 2013),  
35 or even not know which model is proper description of the system (structural uncertainty, *e.g.*  
36 Williams 2001, Cressie et al. 2009, Athanassoglou and Xepapadeas 2012). Of these, the latter  
37 is generally the hardest to quantify. Typical approaches confront the data with a collection of  
38 models, assuming that the true dynamics (or reasonable approximation) is among the collection  
39 and then use model choice or model averaging to arrive at a conclusion (Williams 2001, Cressie  
40 et al. 2009, Athanassoglou and Xepapadeas 2012). Even setting aside other concerns (see Cressie  
41 et al. (2009)), these approaches are unable to describe uncertainty outside the observed data  
42 range.

43 Structural uncertainty is particularly insidious when we try to predict outside of the range of  
 44 observed data (Mangel et al. 2001) because we are extrapolating into unknown regions. In  
 45 management applications, this extrapolation uncertainty is particularly important since (a)  
 46 management involves considering actions that may move the system outside the range of observed  
 47 behavior, and (b) the decision tools (optimal control theory, SDP) rely on both reasonable  
 48 estimates of the expected outcomes and on the weights given to those outcomes (*e.g.* Weitzman  
 49 2013). Thus characterizing uncertainty is as important as characterizing the expected outcome.  
 50 Tipping points in ecological dynamics (Scheffer et al. 2001, Polasky et al. 2011) highlight this  
 51 problem because precise models are not available and data are limited such as around high stock  
 52 levels or an otherwise desirable state. With perfect information, one would know just how far a  
 53 system could be pushed before crossing the tipping point, and management would be simple.  
 54 But we face imperfect models and limited data and, with tipping points,  
 55 even small errors can have very large consequences, as we shall illustrate later. Because  
 56 intervention may be too late once a tipping point has been crossed (but see Hughes et al. (2013)),  
 57 management is often concerned with avoiding tipping points before any data about them are  
 58 available.

59 The dual concerns of model uncertainty and incomplete data create a substantial challenge  
 60 to existing decision-theoretic approaches (Brozović and Schlenker 2011). We illustrate how  
 61 Stochastic Dynamic Programming (SDP) (Mangel and Clark 1988, Marescot et al. 2013) can be  
 62 implemented using a Bayesian Non-Parametric (BNP) model of population dynamics (Munch et  
 63 al. 2005a). The BNP method has two distinct advantages. First, using a BNP model sidesteps  
 64 the need for an accurate model-based description of the system dynamics. Second, a BNP model  
 65 better reflects uncertainty when extrapolating beyond the observed data. This is crucial to  
 66 providing robust decision-making when the correct model is not known (as is almost always  
 67 true). [We use *robust* to characterize approaches that provide nearly optimal solutions without  
 68 being sensitive to the choice of the (unknown) underlying model.]

69 This paper is the first ecological application of the SDP without an *a priori* model of the underlying  
 70 dynamics. Unlike parametric approaches that can only reflect uncertainty in parameter estimates,  
 71 the BNP method provides a broader representation of uncertainty, including uncertainty beyond

the observed data. We will show that Gaussian Process Dynamic Programming (GPDP) allows us to find robust management solutions in face of limited data without knowing the correct model structure.

For comparisons, we consider the performance of management based on GPDP against management policies derived under several alternative parametric models (Reed 1979, Ludwig and Walters 1982, Mangel and Clark 1988). Rather than compare models in terms of best fit to data, we compare model performance in the concrete terms of the decision-maker’s objectives.

## Approach and Methods

We first describe the requirements of dynamic optimization for the management of human intervention in natural resource systems. After that we describe three parametric models for population dynamics and the Gaussian Process (GP)<sup>1</sup> description of population dynamics. All computer code used here has been embedded in the manuscript sources (see Xie (2013)), and an implementation of the GPDP approach is provided as an accompanying R package. Source code, R package and the CSV data files corresponding each figure are archived in the supplement (Munch; 2014).

### *Requirements of dynamic optimization*

Dynamic optimization equires characterizing the dynamics of a state variable (or variables), a control action, and a value function. For simplicity, we consider only a single state variable. This is a best-case scenario for the parametric models because we simulate underlying dynamics from one of the three parametric models , whereas in the natural world we never know the “true” model. In addition, by choosing one-dimensional models with just a few parameters, we limit the chance that poor performance will be due to inability to estimate parameters accurately, something that becomes a more severe problem for higher-dimensional parametric models. Finally, the

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<sup>1</sup>We abbreviate Gaussian Process as GP, which refers to the statistical model we use to approximate the population dynamics, and we use the term Gaussian Process Dynamic Programming (GPDP), to refer to the *use* of a GP as the underlying process model when solving a Dynamic Programming equation. Hence we will refer to the models as: GP, Ricker, Allen, etc, and the novel method we put forward here as GPDP.

parametric models we consider are commonly used in modeling stock-recruitment dynamics or to model sudden transitions between alternative stable states.

We let  $X(t)$  denote the size (numbers or biomass) of the focal population at time  $t$  and assume that in the absence of take its dynamics are:

$$X(t+1) = Z(t)f(X(t), \mathbf{p}) \quad (1)$$

Where  $Z(t)$  is log-normally distributed process stochasticity (Reed 1979) and  $\mathbf{p}$  is a vector of parameters to be estimated from the data. We describe the three choices for  $f(X(t), \mathbf{p})$  in the next section.

The control action is a harvest or take,  $h(t)$ , measured in the same units as  $X$ , at time  $t$ . Thus, in the presence of take, the population size on the right hand side of Eqn 1 is replaced by  $S(t) = X(t) - h(t)$ .

To construct the value function, we consider a return when  $X(t) = x(t)$  and harvest  $h(t) = h$  denoted as the reward,  $R(x(t), h)$ . For example, if the return is the harvest at time  $t$ , then  $R(x(t), h(t)) = \min(x(t), h(t))$ . We assume that future harvests are discounted relative to current ones at a constant rate of discount  $\delta$  and ask for the harvest policy that maximizes total discounted harvest between the current time  $t$  and a final time  $T$ . That is, we seek to maximize over choices of harvest  $E[\sum_{t=0}^T R(X(t), h(t), t)\delta^t]$ , where the state dynamics are given by Eqn 1 and  $E$  denotes the expectation over future population states.

In order to find that policy, we introduce the value function  $V(x(t), t)$  representing the total discounted catch from time  $t$  onwards given that  $X(t) = x(t)$ . This value function satisfies an equation of SDP (Mangel and Clark 1988, Clark and Mangel 2000, Clark 2009, Mangel 2014),

$$V(x(t), t) = \max_{h(t)} \{R(h(t), x(t)) + \delta \cdot E_{X(t+1)} [V(X(t+1), t+1)|x(t), h(t)]\} \quad (2)$$

where expectation is taken over all possible values of the next state,  $X(t+1)$ , and maximized over all possible choices of harvest,  $h(t)$ . That is, at time  $t$ , when population size is  $x(t)$  and harvest  $h(t)$  is applied, the immediate return is  $R(h(t), x(t))$ . When the sole source of uncertainty

118 is the process stochasticity term,  $Z$ , and thus the expectation in Eqn 2 is equivalent to taking  
 119 expectations over  $Z(t)$ . That is

$$E_{X(t+1)} [V(X(t+1), t+1)|x(t), h(t)] = E_{Z(t)} [V(Z(t)f(x(t) - h(t))|\mathbf{p}), t+1|x(t), h(t)] \quad (3)$$

120 where the population size after the take is  $x(t) - h(t)$ , which is then translated into  $X(t+1)$  by  
 121 Eqn 1 (that is, we replace  $X(t+1)$  by  $Z(t)f(x(t) - h(t)|\mathbf{p})$ ).

122 When the parameters governing the dynamics are also uncertain, we take the expectation over  
 123 the posterior distribution for the parameters:

$$E_{X(t+1)} [V(X(t+1), t+1)|x(t), h(t)] = E_{\mathbf{p}|\text{data}} \{E_{Z(t)|\mathbf{p}, \text{data}} [V(Z(t)f(x(t) - h(t))|\mathbf{p}), t]\} \quad (4)$$

124 When the underlying population dynamics are unknown (the case of structural uncertainty), the  
 125 function  $f$  itself is uncertain and the expectation for the next state includes uncertainty in  $f$  as  
 126 well. That is

$$E_{X(t+1)} [V(X(t+1), t+1)|x(t), h(t)] = E_{\mathbf{p}|\text{data}} \{E_{f, Z(t)|\mathbf{p}, \text{data}} [V(Z(t)f(x(t) - h(t))|\mathbf{p}), t]\} \quad (5)$$

127 We consider the finite time problem with  $T = 1000$ , which we solve using the standard value  
 128 iteration algorithm (see Mangel and Clark 1988, Clark and Mangel 2000).

### 129 *Parametric Models*

130 We consider three candidate parametric models for the population dynamics: The Ricker model,  
 131 the Allen model (Allen and Tanner 2005), and the Myers model (Myers et al. 1995), Eqns (6)-(8).  
 132 In all three, we let  $K$  denote the carrying capacity and  $r$  the maximum per capita growth rate.  
 133 The Ricker model has two parameters and the right hand side of Eqn 1 is

$$f(S(t)|r, K) = S(t)e^{r(1-\frac{S(t)}{K})} \quad (6)$$

134 The Allen model has three parameters

$$f(S(t)|r, K, X_c) = S(t)e^{r(1-\frac{S(t)}{K})(S(t)-X_c)} \quad (7)$$

135 where  $X_c$  denotes the location of the unstable steady state (i.e., the tipping point).

136 The Myers model also has three parameters

$$f(S(t)|r, K, \theta) = \frac{rS(t)^\theta}{1 + \frac{S(t)^\theta}{K}} \quad (8)$$

137 where  $\theta = 1$  corresponds to Beverton-Holt dynamics and  $\theta > 2$  leads to Allee effects and multiple  
138 stable states.

139 The Ricker model does not lead to multiple steady states. The Allen model resembles the Ricker  
140 dynamics with an added Allee effect parameter (Courchamp et al. 2008), below which the  
141 population cannot persist. The Myers model also has three parameters and contains an Allee  
142 threshold, but unlike the Ricker model saturates at high population size. The multiplicative  
143 log-normal stochasticity in Eqn 1 introduces one additional parameter  $\sigma$  that must be estimated.  
144 Because of our interest in management performance in the presence of tipping points, all of our  
145 simulations are based on the Allen model. The Allen model is thus the state of nature and is  
146 expected to provide the best-case scenario. The Ricker model is a reasonable approximation of  
147 these dynamics far from the Allee threshold (but lacks threshold dynamics), while the Myers  
148 model shares the essential feature of a threshold but differs in structure from the Allen model.  
149 Throughout, we refer to the “True” model when the underlying parameters *are known without*  
150 *error*, and refer to the “Allen” model when these parameters have been estimated from the  
151 sample data.

152 We consider a period of 40 in which data are obtained to estimate the parameters or the GP.  
153 This is long enough that the estimates do not depend on the particular realization, and longer  
154 times are not likely to provide substantial improvement. Each of the models is fit to the same  
155 data (Figure 1).

156 We inferred posterior distributions for the parameters of each model by Gibbs sampling (Gelman  
 157 et al. (2003) implemented in R (R Core Team 2013) using `jags`, (Su and Masanao Yajima 2013)).  
 158 We choose uniform priors for all parameters of the parametric models (See appendix Tables  
 159 S1-S3; R code provided). We show one-step-ahead predictions of these model fits in Figure 1.  
 160 We tested each chain for Gelman-Rubin convergence and results were robust to longer runs. For  
 161 each simulation we also applied several commonly used model selection criteria (AIC, BIC, DIC,  
 162 see Burnham and Anderson (2002)) to identify the best fitting model.

163 Additionally, we compute the maximum likelihood estimate (MLE, as we will refer to this model  
 164 in the figures) of the parameters for the (structurally correct) Allen model. Comparing this to  
 165 using the posterior distribution of parameters inferred from MCMC for the same model gives  
 166 some indication of the importance of this uncertainty in the dynamic programming.

#### 167 *The Gaussian Process model*

168 The core difference for our purpose between the estimated GP and the estimated parametric  
 169 models is that the estimated GP model is defined explicitly in reference to the observed data.  
 170 As a result, uncertainty arises in the GP model not only from uncertainty in the parameters,  
 171 but is also increases in regions farther from the observed states, such as low population sizes  
 172 in the example illustrated here. The estimated parametric models, by contrast, are completely  
 173 specified by the parameters.

174 The use of GPs to characterize dynamical systems is relatively new (Kocijan et al. 2005), and  
 175 was first introduced in the context ecological modeling and fisheries management in Munch et  
 176 al. (2005b). GP models have subsequently been used to test for the presence of Allee effects  
 177 (Sugeno and Munch 2013a), estimate the maximum reproductive rate (Sugeno and Munch  
 178 2013b), determine temporal variation in food availability (Sigourney et al. 2012), and provide a  
 179 basis for identifying model-misspecification (Thorson et al. 2014). An accessible and thorough  
 180 introduction to the formulation and use of GPs can be found in Rasmussen and Williams (2006).

181 A GP is a stochastic process for which any realization consisting of  $n$  points follows a multivariate  
 182 normal distribution of dimension  $n$ . To characterize the GP we need a mean function and a  
 183 covariance function. We proceed as follows.



184 As before, we assume that the data  $X(t)$  are observed with process stochasticity around a mean  
 185 function  $g(X(t))$

$$X(t+1) = g(X(t)) + \varepsilon, \quad (9)$$

186 where  $\varepsilon$  are IID normal random variables with zero-mean and variance  $\sigma^2$ . Note that we have  
 187 chosen to assume additive stochasticity. While we could consider log-normal stochasticity as in  
 188 the parametric models, we make this choice to emphasize that the Gaussian process approach  
 189 need not have structurally correct stochasticity to be effective.

190 In order to make predictions, we update the GP based on the observed set of transitions. To  
 191 do so, we collect the time series of observed states into a vector of “current” states,  $\mathbf{X}_{\text{obs}} =$   
 192  $\{X(1), \dots, X(T-1)\}$  and a vector of “next” states  $\mathbf{Y}_{\text{obs}} = \{X(2), \dots, X(T)\}$  where  $T$  is the  
 193 time of the final observation. Conditional on these observations, the predicted next state,  $g(X_p)$ ,  
 194 for any given “current” state,  $X_p$  follows a normal distribution with mean  $E$  and variance  $C$   
 195 determined using the standard rules for conditioning in multivariate normals, i.e.

$$E = K(X_p, \mathbf{X}_{\text{obs}}) (K(\mathbf{X}_{\text{obs}}, \mathbf{X}_{\text{obs}}) - \sigma \mathbb{I}_n)^{-1} \mathbf{Y}_{\text{obs}} \quad (10)$$

196 and

$$C = K(X_p, X_p) - K(X_p, \mathbf{X}_{\text{obs}}) (K(\mathbf{X}_{\text{obs}}, \mathbf{X}_{\text{obs}}) - \sigma \mathbb{I})^{-1} K(\mathbf{X}_{\text{obs}}, X_p) \quad (11)$$

197 NA

$$K_{i,j}(\mathbf{x}, \mathbf{y}) = \exp \left( \frac{-(x_i - y_j)^2}{2\ell^2} \right) \quad (12)$$

198 so that  $\ell$  gives the characteristic length-scale over which correlation between two observations  
 199 decays. See Rasmussen and Williams (2006) for other choices of covariance kernels and their  
 200 properties. Note that this simple formulation assumes a prior mean of zero. For the parameters  
 201 we use inverse Gamma priors on both the length-scale  $\ell$  and  $\sigma$ , thus for example

$$f(\ell; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \ell^{-\alpha-1} \exp\left(-\frac{\beta}{\ell}\right) \quad (13)$$

For the prior on  $\ell$ ,  $\alpha = 10$  and  $\beta = 10$ . The prior on  $\sigma$ ,  $\alpha = 5$  and  $\beta = 5$ .

We use a Metropolis-Hastings Markov Chain Monte Carlo (Gelman et al. (2003)) to infer posterior distributions of the parameters of the GP (Figure S4, code in appendix). Since the posterior distributions differ substantially from the priors (Figure S4), most of the information in the posterior comes from the data rather than the prior belief.

### *The method of Gaussian Process Dynamic Programming (GPDP)*

We derive the harvest policy from the estimated GP by inserting it into a SDP algorithm. Given the GP posteriors, we construct the transition matrix representing the probability of going to each state  $X(t+1)$  given any current state  $x(t)$  and any harvest  $h(t)$  (See the function `gp_transition_matrix()` in the provided R package). Given this transition matrix, we use the same value iteration algorithm as in the parametric case to determine the optimal policy. In doing so, the uncertainty in the future state under the GP,  $X(t+1)$ , includes both process uncertainty (based on the estimation of  $\sigma$ ) and structural uncertainty of the posterior collection of curves.

## **Results**

### *Parametric and GP models for population dynamics*

To ensure our results are robust to the choice of parameters, we will consider 96 different scenarios. To help better understand the process, we first describe in detail the results of a single scenario. All of the models fit the observed data rather closely and with relatively small uncertainty. In Figure 1, we show the posterior predictive curves. The training data of stock sizes observed over time are points, overlaid with the step-ahead predictions of each estimated model using the parameters sampled from their posterior distributions. Compared to the true model most estimates appear to over-fit, predicting patterns that are actually due purely to stochasticity. Model selection criteria (Table 1) penalize more complex models and show a preference for

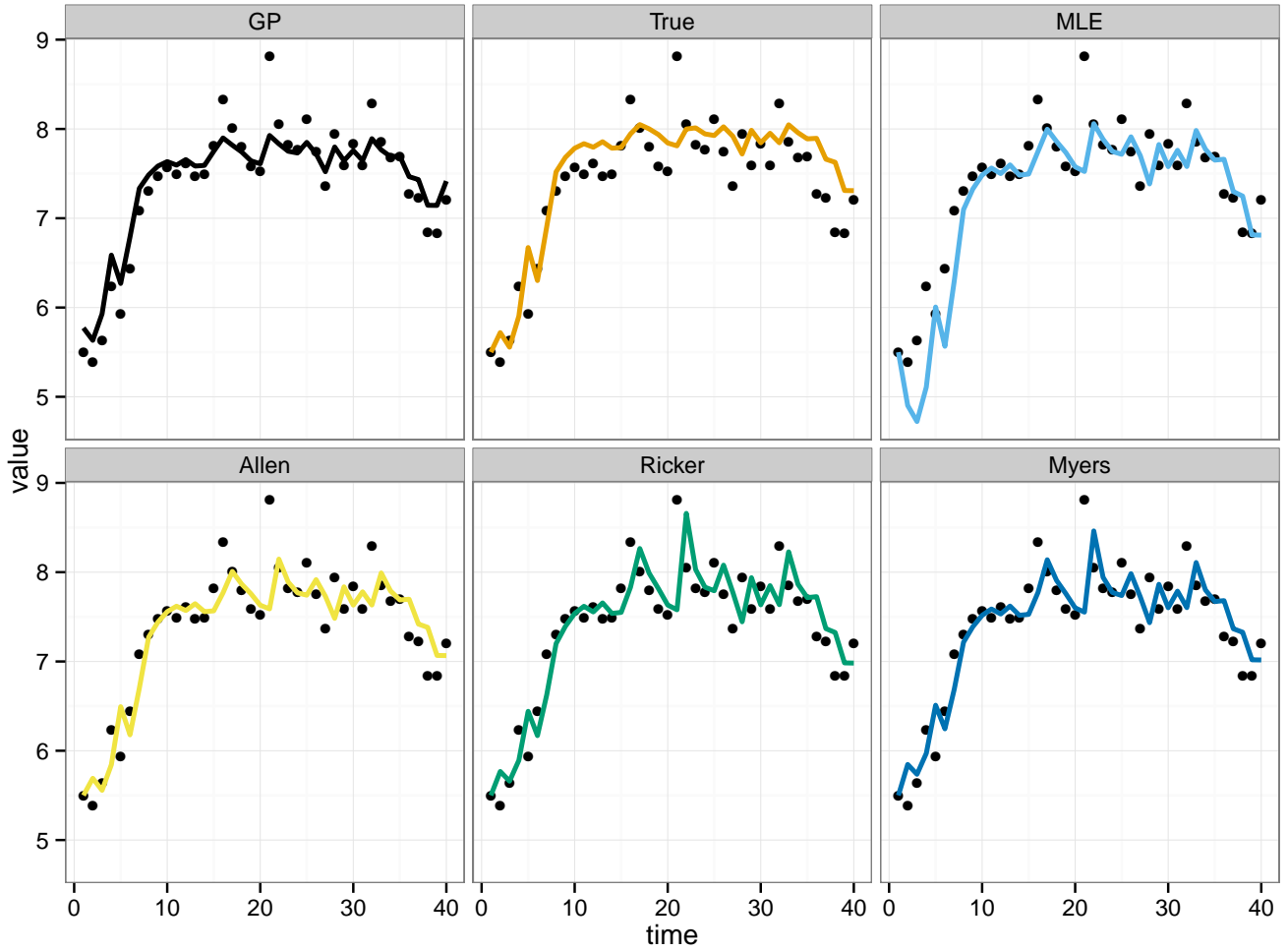


Figure 1: Points show the training data of stock-size over time. Curves show the expectations over the posterior step-ahead predictions based on each of the estimated models. (Observe that all models are fitting the data reasonably well.)

the simpler Ricker model over the models with alternative stable states (Allen and Myers). Supplement provides details on the model estimates.

Table 1: Model selection scores for several common criteria (DIC: Deviance Information Criterion, AIC: Akaike Information Criterion, BIC: Bayesian Information Criterion) all select the wrong model. As the true (Allen) model is not distinguishable from the simpler (Ricker) model in the region of the observed data, this error cannot be avoided regardless of the model choice criterion. This highlights the danger of model choice when the selected model will be used outside of the observed range of the data.

	Allen	Ricker	Myers
<b>DIC</b>	50.75	50.45	50.41
<b>AIC</b>	-24.51	-30.13	-27.01
<b>BIC</b>	-17.75	-25.06	-20.25

We show the mean inferred population dynamics of each model relative to the true model used to generate the data in Figure 2, predicting the relationship between observed population size (x-axis) to the population size after recruitment the following year.

In addition to the raw data, the GP is conditioned on going through the point 0,0 without error. All parametric models also make this assumption. Conditioning on (0,0) is equivalent to making the assumption that the population is closed, so that once it hits 0 it stays at 0, despite the lack of any data in the observed sequence to justify this. This assumption illustrates how the GP can capture common-sense biology without having to assume more explicit details about the dynamics at low population numbers that have never been observed. If the population were not closed, one could repeat the entire analysis without this assumption. Unlike parametric models, the GP

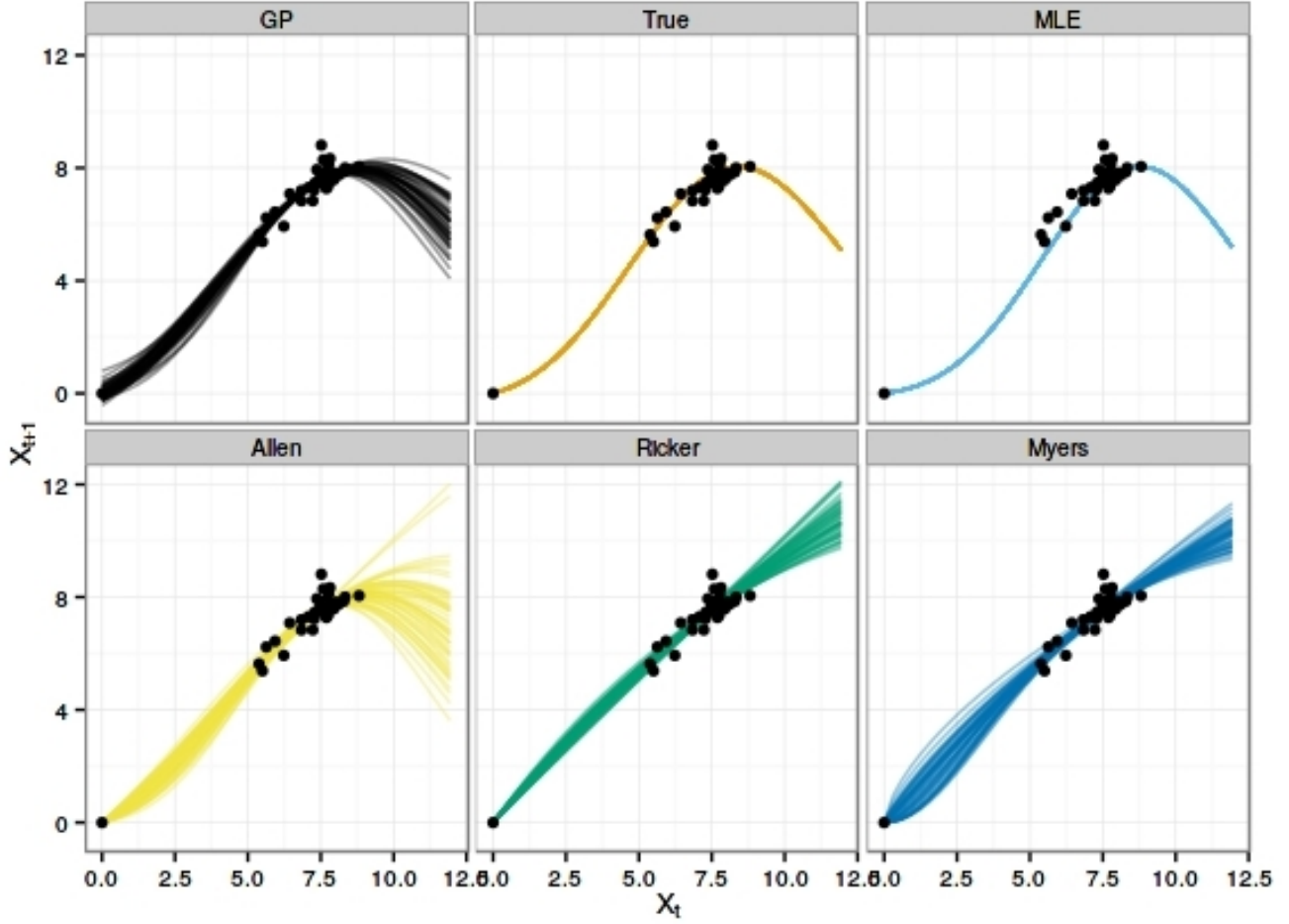


Figure 2: The inferred Gaussian process compared to the true process and maximum-likelihood estimated (MLE) process. We show the expected value for the function  $f$  under each model. Plots show replicates drawn from the posterior distributions in order to convey uncertainty of the estimates. Note the MLE is a point estimate of parameters and so reflects no uncertainty in the distribution. The training data are also shown as black points. The GP is conditioned on  $(0,0)$ , shown as a pseudo-data point.

corresponds to a distribution of curves, of which this plot only shows the means. Uncertainty in the parameters of the GP (not shown) further widens the band of possible population sizes. In Figure S1 (see supplement), we show the performance of the models outside the observed training data.

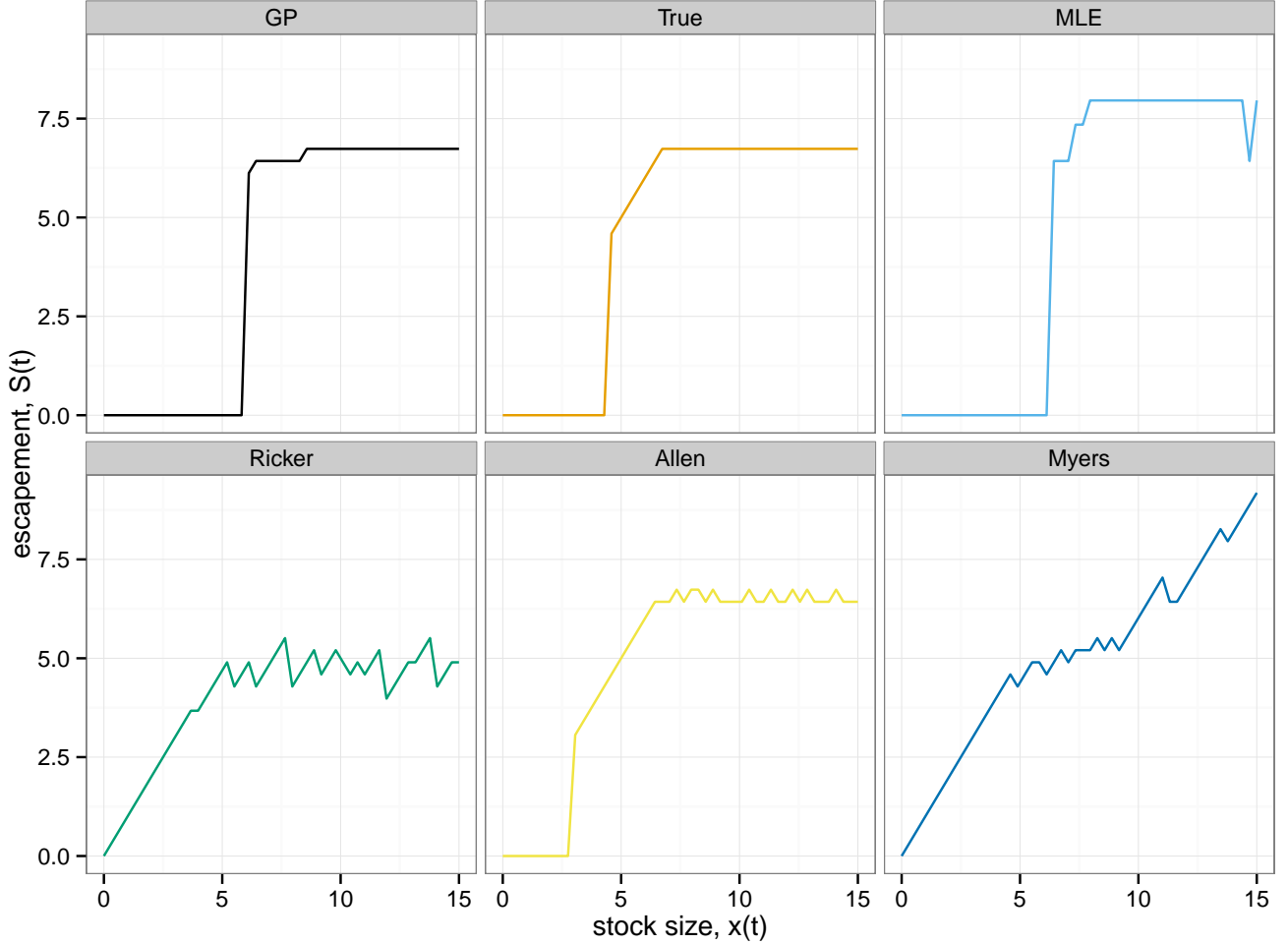


Figure 3: The steady-state optimal policy (infinite boundary) calculated under each model. Policies are shown in terms of target escapement,  $S(t)$ , as under models such as this a constant escapement policy is expected to be optimal (Reed 1979). Several policies show a numerical jitter due to the discretization of states in the dynamic programming algorithm; doubling the number of grid points did not qualitatively change the results.

Despite the similarities in model fits to the observed data, the policies inferred under each model differ widely (Figure 3). Policies are shown in terms of target escapement,  $S(t) = x_t - h$ . Under models such as this a constant escapement policy is expected to be optimal (Reed 1979), whereby population levels below a certain size  $S$  are unharvested, while above that size the

246 harvest strategy aims to return the population to  $S$ . Whenever a model predicts that the  
 247 population will not persist below a certain threshold, the optimal solution is to harvest the entire  
 248 population immediately, resulting in an escapement  $S = 0$ , as seen in the true (correct form,  
 249 exact parameters) model, the Allen model (correct form, estimated parameters) and the GP.  
 250 Only the structurally correct model (Allen model) and the GP produce policies close to the true  
 251 optimum policy.

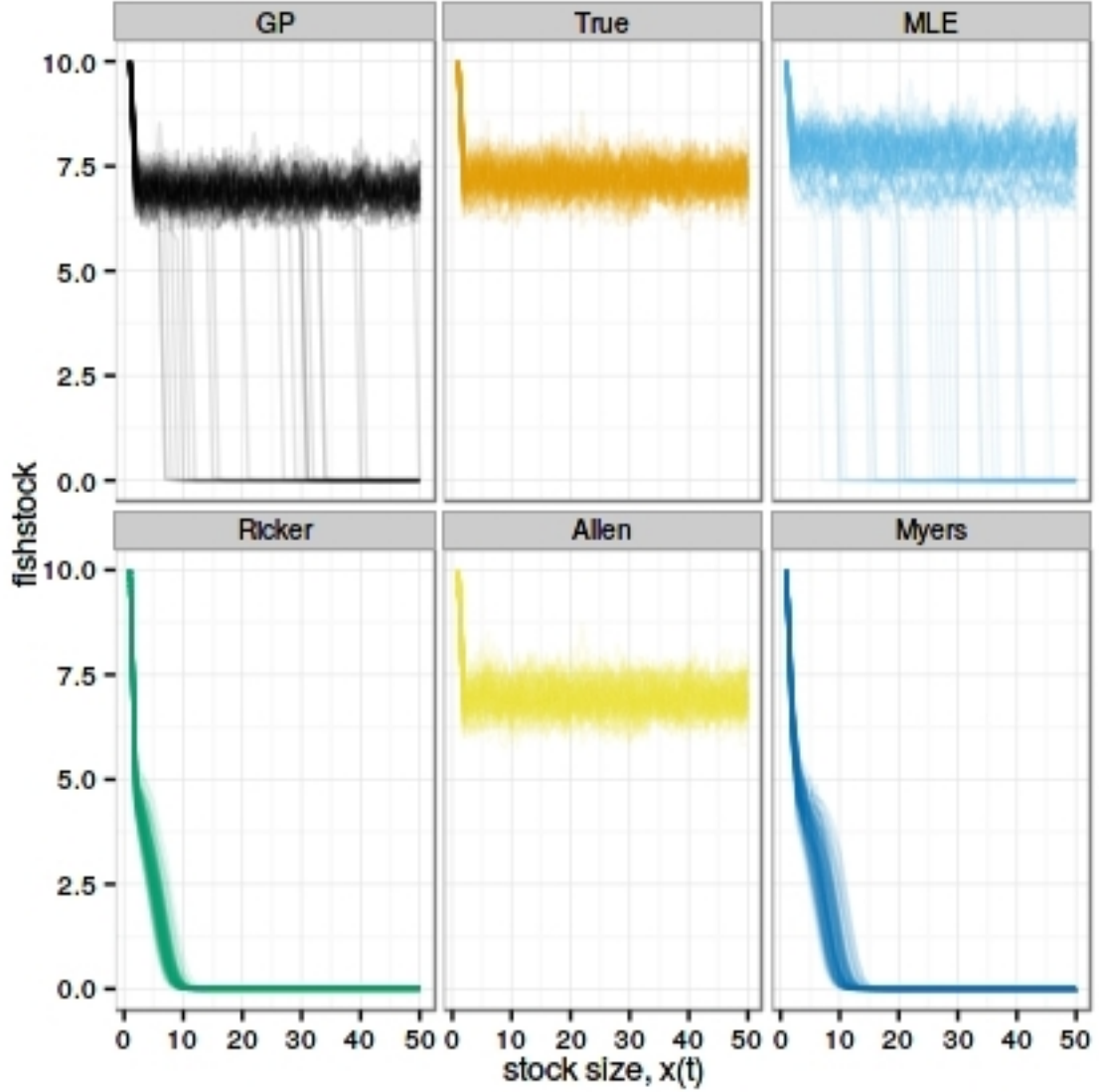


Figure 4: In the management context, GPDP outperforms approaches based on parametric models. We show 100 replicate simulations of the stock dynamics (Eqn 1) under the policies derived from each of the estimated models, as well as the policy based on the exact underlying model.

252 In Figure 4, we show the consequences of managing 100 replicate realizations of the simulated  
253 fishery under policies derived from each model. The structurally correct model under-harvests,  
254 leaving the stock to vary around its unfished optimum. The structurally incorrect Ricker model  
255 over-harvests the population past the tipping point consistently, resulting in the immediate crash  
256 of the stock and thus leads to minimal long-term catch.

257 The results across replicate stochastic simulations are most easily compared by using the relative  
258 differences in net present value realized by each of the model (Figure 5). Although not perfect,  
259 the GPDP consistently realizes a value close to the optimal solution, and avoids ever driving  
260 the system across the tipping point, which results in the near-zero value cases in the parametric  
261 models.

### 262 *Sensitivity Analysis*

263 These results are not sensitive to the modeling details of the simulation. The GPDP estimate  
264 remains very close to the optimal solution (obtained by knowing the true model) across changes  
265 to the training simulation, scale of stochasticity, parameters or structure of the underlying  
266 model. In the Supplement, we consider both a Latin hypercube approach and a more focused  
267 investigation of the effects of the relative distance to the Allee threshold and the variance of  
268 process stochasticity.

269 The GPDP is only weakly influenced by increasing stochasticity or increasing Allee effects over  
270 much of the range (Figure S2). Larger  $\sigma$  or higher Allee levels make even the optimal solution  
271 without any model or parameter uncertainty unable to harvest the population effectively (e.g. the  
272 stochasticity is large enough to violate the self-sustaining criterion of Reed (1979)).

## 273 **Discussion**

274 Simple, mechanistically motivated models offer the potential to increase our basic understanding  
275 of ecological processes (Geritz and Kisdi 2012, Cuddington et al. 2013). But such models can be  
276 both inaccurate and misleading when used in a decision making framework. In this paper we  
277 tackled two aspects of uncertainty that are common to many ecological decision-making problems  
278 and fundamentally challenging to existing approaches that largely rely on parametric models:



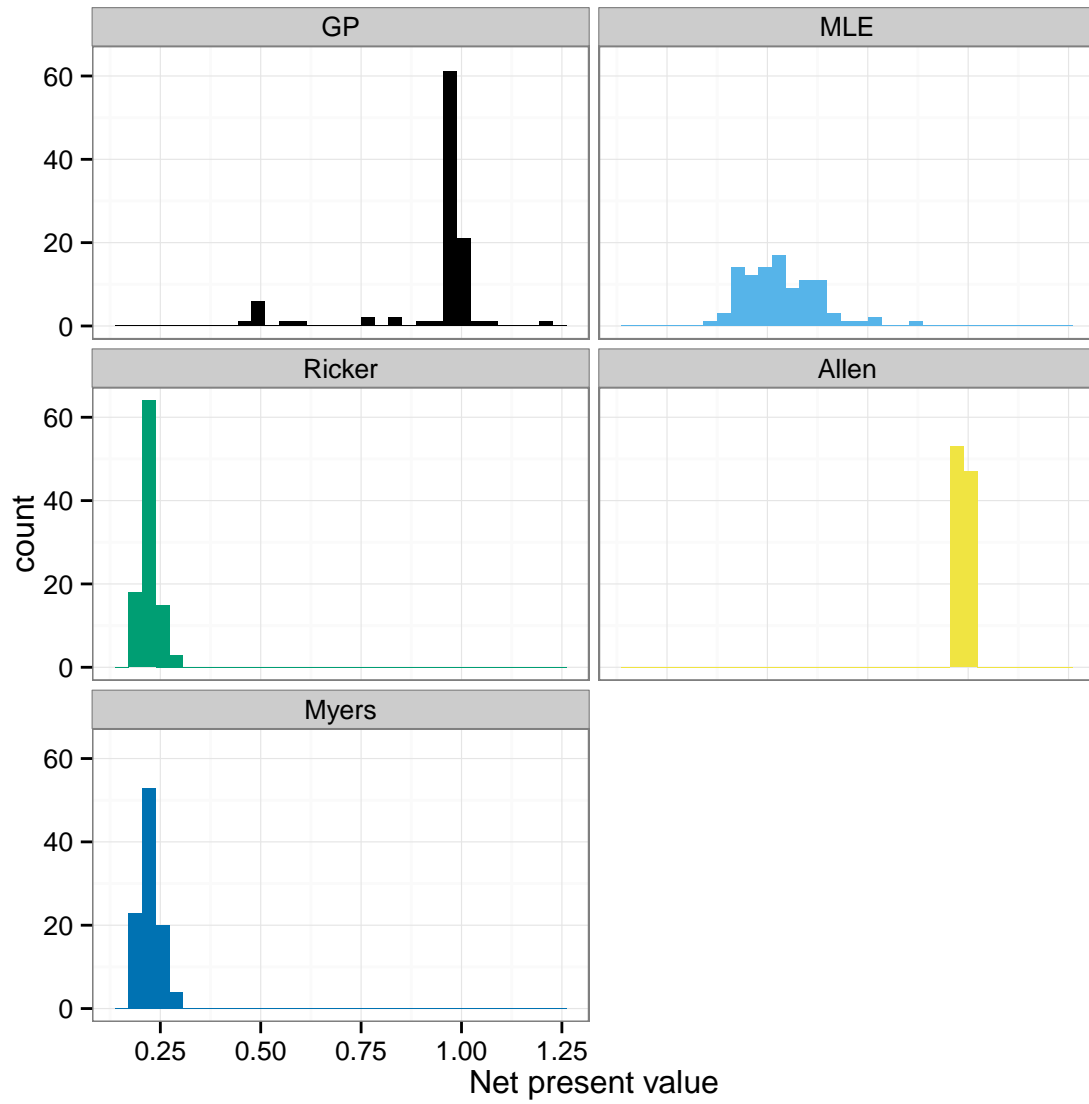


Figure 5: Histograms of the realized net present value of the fishery over a range of simulated data and resulting parameter estimates. For each data set, the three models are estimated as described above.

1. We do not know the correct models for ecological systems.

2. We have limited data from which to estimate the model.

We have illustrated how the use of non-parametric methods provides more reliable solutions in the sequential decision-making problem.

*Traditional model-choice approaches can be positively misleading.*

Our results illustrate that model-choice approaches can be absolutely NA dynamics because they have fewer parameters and the data are far from the tipping point. That is, when the data come from around the stable steady state, all the parametric models are approximately linear and approximately identical. Thus, it is intuitive that all model selection methods choose the simplest model. In a complex world, the result is suboptimal. But in a world that might contain tipping points, the result could be disastrous.

Many managers both in fisheries and beyond face a similar situation: they have not observed the population dynamics at all possible densities. A lack of comprehensive data at all population sizes, combined with the inability to formulate accurate population models for low population sizes in the absence of data, makes this situation the rule more than the exception. Relying on parametric models and model choice processes that favor simplicity ignores this basic reality. For a long time, Carl Walters (e.g. Walters and Hilborn 1978) has argued that if we began by fishing any newly exploited population down to very low levels and then let it recover, we would be much better at estimating population dynamics and thus predicting the optimal harvest levels. While certainly true, this presents a rather risky policy in the face of potential tipping points. The GPDP offers a risk-adverse alternative.

*GPDP population dynamics capture larger uncertainty in regions where the data are poor*

Parametric models perform most poorly when we seek a management strategy outside the range of the observed data. The GPDP, in contrast, leads to a predictive model that expresses a great deal of uncertainty about the probable dynamics *outside* the range of the observed data, while retaining very good predictive accuracy *inside* the range. The management policy based on by the GPDP balances uncertainty outside the range of the observed data against the immediate

306 value of the harvest, and acts to stabilize the population dynamics in a region of state space in  
307 which the predictions are reliably reflected by the data.

308 Such problems are ubiquitous across ecological decision-making and conservation where the  
309 greatest concerns involve decisions that lead to population sizes that have never been observed  
310 and for which we do not know the response – whether this is the collapse of a fishery, the spread  
311 of an invasive, or the loss of habitat.

### 312 *The role of the prior*

313 Outside of the observed range of the data, the GP reverts to the prior, and consequently the  
314 choice of the prior can also play a significant role in determining the optimal policy. In the  
315 examples shown here we have selected a prior that is both relatively uninformative (due to the  
316 broad priors placed on its parameters  $\ell$  and  $\sigma$  and simple (the choice of our covariance function,  
317 Eqns 12 and 13 ). In practice, these should be chosen to confer particular biological properties.  
318 In principle, this may allow a manager to improve the performance of the GPDP by adding detail  
319 as is justified. For instance, it would be possible to use a linear or a Ricker-shaped mean in the  
320 prior without making the much stronger assumption that the Ricker is the structurally correct  
321 model (Sugeno and Munch 2013a). One fruitful avenue of future research is identifying criteria  
322 to ensure the prior and the reward function are chosen appropriately for the problem at hand.

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328 Allen, D., and K. Tanner. 2005. Infusing active learning into the large-enrollment biology class:  
329 seven strategies, from the simple to complex. *Cell biology education* 4:262–8.

330 Athanassoglou, S., and A. Xepapadeas. 2012. Pollution control with uncertain stock dynamics:  
331 When, and how, to be precautionary. *Journal of Environmental Economics and Management*  
332 63:304–320.

333 Brozović, N., and W. Schlenker. 2011. Optimal management of an ecosystem with an unknown  
334 threshold. *Ecological Economics*:1–14.

335 Burnham, K. P., and D. R. Anderson. 2002. *Model Selection and Multi-Model Inference*. Page  
336 496. Springer.

337 Clark, C. W. 1976. *Mathematical Bioeconomics*. WileyNew York.

338 Clark, C. W. 2009. *Mathematical Bioeconomics*. WileyNew York.

339 Clark, C. W., and G. P. Kirkwood. 1986. On uncertain renewable resource stocks: Optimal  
340 harvest policies and the value of stock surveys. *Journal of Environmental Economics and*  
341 *Management* 13:235–244.

342 Clark, C. W., and M. Mangel. 2000. *Dynamic state variable models in ecology*. Oxford University  
343 PressOxford.

344 Courchamp, F., L. Berec, and J. Gascoigne. 2008. *Allee Effects in Ecology and Conservation*.  
345 Page 256. Oxford University Press, USA.

346 Cressie, N., C. a Calder, J. S. Clark, J. M. Ver Hoef, and C. K. Wikle. 2009. Accounting  
347 for uncertainty in ecological analysis: the strengths and limitations of hierarchical statistical  
348 modeling. *Ecological Applications* 19:553–70.

349 Cuddington, K. M., M. Fortin, and L. Gerber. 2013. Process-based models are required to  
350 manage ecological systems in a changing world. *Ecosphere* 4:1–12.

351 Fischer, J., G. D. Peterson, T. a Gardner, L. J. Gordon, I. Fazey, T. Elmqvist, A. Felton, C.  
352 Folke, and S. Dovers. 2009. Integrating resilience thinking and optimisation for conservation.  
353 *Trends in ecology & evolution* 24:549–54.

354 Gelman, A., J. B. Carlin, H. S. Stern, and D. B. Rubin. 2003. *Bayesian Data Analysis*. 2nd  
355 editions. Chapman; Hall/CRC.

356 Geritz, S. A. H., and E. Kisdi. 2012. Mathematical ecology: why mechanistic models? *Journal*  
357 *of mathematical biology* 65:1411–5.

358 Gordon, H. 1954. The economic theory of a common-property resource: the fishery. *The Journal*  
359 *of Political Economy* 62:124–142.

360 Hilborn, R., and M. Mangel. 1997. The Ecological Detective: Confronting Models with data.  
 361 Page 330. Princeton University Press.

362 Hughes, T. P., C. Linares, V. Dakos, I. a van de Leemput, and E. H. van Nes. 2013. Living  
 363 dangerously on borrowed time during slow, unrecognized regime shifts. Trends in ecology &  
 364 evolution 28:149–55.

365 Kocijan, J., A. Girard, B. Banko, and R. Murray-Smith. 2005. Dynamic systems identification  
 366 with Gaussian processes. Mathematical and Computer Modelling of Dynamical Systems 11:411–  
 367 424.

368 Ludwig, D., and C. J. Walters. 1982. Optimal harvesting with imprecise parameter estimates.  
 369 Ecological Modelling 14:273–292.

370 Mangel, M. 2014. Stochastic Dynamic Programming Illuminates the Link Between Environment.  
 371 Bulletin of Mathematical Biology in press.

372 Mangel, M., and C. W. Clark. 1988. Dynamic Modeling in Behavioral Ecology. (J. Krebs and T.  
 373 Clutton-Brock, Eds.). Princeton University PressPrinceton.

374 Mangel, M., O. Fiksen, and J. Giske. 2001. Theoretical and statistical models in natural  
 375 resource management and research. Pages 57–71 *in* T. M. Shenk and A. B. Franklin, editors.  
 376 Modeling in natural resource management, development, interpretation and application. Island  
 377 PressWashington DC.

378 Marescot, L., G. Chapron, I. Chadès, P. L. Fackler, C. Duchamp, E. Marboutin, and O. Gimenez.  
 379 2013. Complex decisions made simple: a primer on stochastic dynamic programming. Methods  
 380 in Ecology and Evolution:n/a–n/a.

381 May, R. M., J. R. Beddington, C. W. Clark, S. J. Holt, and R. M. Laws. 1979. Management of  
 382 multispecies fisheries. Science (New York, N.Y.) 205:267–77.

383 McAllister, M. 1998. Bayesian stock assessment: a review and example application using the  
 384 logistic model. ICES Journal of Marine Science 55:1031–1060.

385 Munch, S. B., A. Kottas, and M. Mangel. 2005a. Bayesian nonparametric analysis of stock-  
 386 recruitment relationships. Canadian Journal of Fisheries and Aquatic Sciences 62:1808–1821.

387 Munch, S. B., M. L. Snover, G. M. Watters, and M. Mangel. 2005b. A unified treatment of  
388 top-down and bottom-up control of reproduction in populations. *Ecology Letters* 8:691–695.

389 Munch; C. B. M. M. S. 2014. nonparametric-bayes v0.1.0. ZENODO. [http://dx.doi.org/10.5281/](http://dx.doi.org/10.5281/zenodo.12669)  
390 [zenodo.12669](http://dx.doi.org/10.5281/zenodo.12669).

391 Myers, R. A., N. J. Barrowman, J. A. Hutchings, and A. a Rosenberg. 1995. Population dynamics  
392 of exploited fish stocks at low population levels. *Science (New York, N.Y.)* 269:1106–8.

393 Polasky, S., S. R. Carpenter, C. Folke, and B. Keeler. 2011. Decision-making under great  
394 uncertainty: environmental management in an era of global change. *Trends in ecology &*  
395 *evolution*:1–7.

396 R Core Team. 2013. R: A Language and Environment for Statistical Computing. R Foundation  
397 for Statistical ComputingVienna, Austria.

398 Rasmussen, C. E., and C. K. I. Williams. 2006. *Gaussian Processes for Machine Learning*.  
399 (Thomas Dietterich, Ed.). MIT Press,Boston.

400 Reed, W. J. 1979. Optimal escapement levels in stochastic and deterministic harvesting models.  
401 *Journal of Environmental Economics and Management* 6:350–363.

402 Roughgarden, J. E., and F. Smith. 1996. Why fisheries collapse and what to do about it.  
403 *Proceedings of the National Academy of Sciences of the United States of America* 93:5078.

404 Schapaugh, A. W., and A. J. Tyre. 2013. Accounting for parametric uncertainty in Markov  
405 decision processes. *Ecological Modelling* 254:15–21.

406 Scheffer, M., J. Bascompte, W. A. Brock, V. Brovkin, S. R. Carpenter, V. Dakos, H. Held, E.  
407 H. van Nes, M. Rietkerk, and G. Sugihara. 2009. Early-warning signals for critical transitions.  
408 *Nature* 461:53–9.

409 Scheffer, M., S. R. Carpenter, J. A. Foley, C. Folke, and B. Walker. 2001. Catastrophic shifts in  
410 ecosystems. *Nature* 413:591–6.

411 Sethi, G., C. Costello, A. Fisher, M. Hanemann, and L. Karp. 2005. Fishery management under  
412 multiple uncertainty. *Journal of Environmental Economics and Management* 50:300–318.

413 Sigourney, D. B., S. B. Munch, and B. H. Letcher. 2012. Combining a Bayesian nonparametric

414 method with a hierarchical framework to estimate individual and temporal variation in growth.  
 415 *Ecological Modelling* 247:125–134.

416 Su, Y.-S., and Masanao Yajima. 2013. R2jags: A Package for Running jags from R.

417 Sugeno, M., and S. B. Munch. 2013a. A semiparametric Bayesian method for detecting Allee  
 418 effects. *Ecology* 94:1196–1204.

419 Sugeno, M., and S. B. Munch. 2013b. A semiparametric Bayesian approach to estimating  
 420 maximum reproductive rates at low population sizes. *Ecological applications* : a publication of  
 421 the Ecological Society of America 23:699–709.

422 Thorson, J. T., K. Ono, and S. B. Munch. 2014. A Bayesian approach to identifying and  
 423 compensating for model misspecification in population models. *Ecology* 95:329–41.

424 Walters, C. J., and R. Hilborn. 1978. *Ecological Optimization and Adaptive Management*.  
 425 *Annual Review of Ecology and Systematics* 9:157–188.

426 Weitzman, M. L. 2013. A Precautionary Tale of Uncertain Tail Fattening. *Environmental and*  
 427 *Resource Economics* 55:159–173.

428 Williams, B. K. 2001. Uncertainty , learning , and the optimal management of wildlife. *Environ-*  
 429 *mental and Ecological Statistics* 8:269–288.

430 Xie, Y. 2013. *Dynamic documents with R and knitr*. Chapman; Hall/CRC Boca Raton, Florida.