

Avoiding tipping points in the management of ecological systems: a non-parametric Bayesian approach

Carl Boettiger^{a,*}, Marc Mangel^a, Stephan B. Munch^b

^a*Center for Stock Assessment Research, Department of Applied Math and Statistics, University of California, Mail Stop SOE-2, Santa Cruz, CA 95064, USA*

^b*Southwest Fisheries Science Center, National Oceanic and Atmospheric Administration, 110 Shaffer Road, Santa Cruz, CA 95060, USA*

Abstract

Model uncertainty and limited data coverage are fundamental challenges to robust ecosystem management. These challenges are acutely highlighted by concerns that many ecological systems may contain tipping points. Before a collapse, we do not know where the tipping points lie, if they exist at all. Hence, we know neither a complete model of the system dynamics nor do we have access to data in some large region of state-space where such a tipping point might exist. These two sources of uncertainty frustrate state-of-the-art parametric approaches to decision theory and optimal control. I will illustrate how a non-parametric approach using a Gaussian Process prior provides a more flexible representation of this inherent uncertainty. Consequently, we can adapt the Gaussian Process prior to a stochastic dynamic programming framework in order to make robust management predictions under both model and uncertainty and limited data.

Introduction

Decision making under uncertainty is a ubiquitous challenge of natural resource management and conservation. Ecological dynamics are frequently complex and difficult to measure, making uncertainty in our understanding a prediction a persistent challenge to effective management. Decision-theoretic approaches provide a framework to determine the best sequence of actions in face of uncertainty, but only when that uncertainty can be meaningfully quantified (Fischer et al. 2009). The sudden collapse of fisheries and other ecosystems has increasingly emphasized the difficulties of formulating even qualitatively correct models of the underlying processes.

Uncertainty enters the decision-making process at many levels: intrinsic stochasticity in biological processes, measurements, and implementation of policy (*e.g.* Reed 1979; C. W. Clark and Kirkwood 1986; Roughgarden and Smith 1996; Sethi et al. 2005), parameteric uncertainty (*e.g.* Ludwig and Walters 1982; Hilborn and Mangel

*Corresponding author
Email address: cboettig@ucsc.edu (Carl Boettiger)

1997; McAllister 1998; Schapaugh and Tyre 2013), and model or structural uncertainty (*e.g.* B. K. Williams 2001; Cressie et al. 2009; Athanassoglou and Xepapadeas 2012). Of these, structural uncertainty incorporates the least a priori knowledge or assumptions and is generally the hardest to quantify. Typical approaches assume a weak notion of model uncertainty in which the correct model (or reasonable approximation) of the dynamics must be identified from among a handful of alternative models. Here we consider an approach that addresses uncertainty at each of these levels without assuming the dynamics follow a particular (*i.e.* parametric) structure. Model uncertainty is particularly insidious when model predictions must be made outside of the range of data on which the model was estimated.

An additional source of uncertainty that has received less attention^[1] arises when applying a dynamical model outside the range of data on which it has been estimated. This extrapolation uncertainty is felt most keenly in decision-theoretic applications, as (a) exploring the potential action space typically involves considering actions that may move the system outside the range of observed behavior, and (b) decision-theoretic algorithms rely not only on reasonable estimates of the expected outcomes, but depend on the weights given to all possible outcomes (*e.g.* Weitzman 2013).

Concerns over the potential for tipping points in ecological dynamics (Scheffer et al. 2001) highlight the dangers of uncertainty in ecological management and pose a substantial challenge to existing decision-theoretic approaches (Brozović and Schlenker 2011). Because intervention is often too late after a tipping point has been crossed (but see Hughes et al. (2013)), management is most often concerned with avoiding potentially catastrophic tipping points before any data is available at or following a transition that would more clearly reveal these regime shift dynamics (*e.g.* Bestelmeyer et al. 2012).

Here we illustrate how a stochastic dynamic programming (SDP) algorithm (Mangel and Clark 1988; Marescot et al. 2013) can be driven by the predictions from a Bayesian non-parametric (BNP) approach (Munch, Kottas, and Mangel 2005). This provides two distinct advantages compared with contemporary approaches. First, using a BNP sidesteps the need for an accurate model-based description of the system dynamics. Second, the BNP can better reflect uncertainty that arises when extrapolating a model outside of the data on which it was fit. We illustrate that when the correct model is not known, this latter feature is crucial to providing a robust decision-theoretic approach in face of substantial structural uncertainty.

This paper represents the first time the SDP decision-making framework has been used without an a priori model of the underlying dynamics through the use of the BNP approach. In contrast to parametric models which can only reflect uncertainty in parameter estimates, the BNP approach provides a more state-space dependent representation of uncertainty. This permits a much greater uncertainty far from the observed data than near the observed data. These features allow the GP-SDP approach to find robust management solutions in face of limited data and without knowledge of the correct model structure.

The idea that any approach can perform well without either having to know the model or have particularly

good data should immediately draw suspicion. The reader must bear in mind that the strength of our approach comes not from black-box predictive power from such limited information, but rather, by providing a more honest expression of uncertainty outside the observed data without sacrificing the predictive capacity near the observed data. By coupling this more accurate description of what is known and unknown to the decision-making under uncertainty framework provided by stochastic dynamic programming, we are able to obtain more robust management policies than with common parametric modeling approaches.

- Note on: Why fisheries

The economic value and ecological concern have made marine fisheries the crucible for much of the founding work (H. S. Gordon 1954; Reed 1979; May et al. 1979; Ludwig and Walters 1982) in managing ecosystems under uncertainty. Global trends (Worm et al. 2006) and controversy (Hilborn 2007; Worm et al. 2009) have made understanding these challenges all the more pressing.

- *Note on comparing models* (via value function rather than by “fit”).

The nature of decision-making problems provides a convenient way to compare models. Rather than compare models in terms of best fit to data or fret over the appropriate penalty for model complexity, model performance is defined in the concrete terms of the decision-maker’s objective function, which we will take as given. (Much argument can be made over the ‘correct’ objective function, e.g. how to account for the social value of fish left in the sea vs. the commercial value of fish harvested; see Halpern et al. (2013) for further discussion of this issue. Alternatively, we can always compare model performance across multiple potential objective functions.) The decision-maker does not necessarily need a model that provides the best mechanistic understanding or the best long-term outcome, but rather the one that best estimates the probabilities of being in different states as a result of the possible actions.

Background on the Gaussian Process

Addressing the difficulty posed by extrapolation without knowing the true model requires a nonparametric approach to model fitting: one that does not assume a fixed structure but rather depends on the size of the data (e.g. non-parametric regression or a Dirichlet process). This established terminology is nevertheless unfortunate, as (a) this approach still involves the estimation of parameters, and (b), Statisticians use non-parametric to mean both this property (structure is not fixed by the parameters) and an entirely different (and probably more familiar) case in which the model does not assume any distribution (e.g. non-parametric bootstrap, order statistics). Some literature thus uses the term semi-parametric, which merely adds ambiguity to the confusion.

This non-parametric property – having a structure explicitly dependent on the data – is precisely the property that makes this approach attractive in face of the limited data sampling challenges discussed above. Having fit

a parametric model to some data, the model is completely described by the values (or posterior distributions) of its parameters. The non-parametric model is not captured by its parameter values or distributions alone. Either the model scales with the complexity of the data on which it is estimated (e.g. nonparametric hierarchical approaches such as the Dirichlet process) or the data points become themselves part of the model specification, as in the nonparametric regression used here. we shall see here.

The use of Gaussian process (GP) regression (or “kriging” in the geospatial literature) to formulate a predictive model is relatively new in the context of modeling dynamical systems (Kocijan et al. 2005), and was first introduced in the context ecological modeling and fisheries management in Munch et al. (2005). An accessible and thorough introduction to the formulation and use of GPs can be found in Rasmussen and Williams (2006).

The essence of the GP approach can be captured in the following thought experiment: An exhaustive parametric approach to the challenge of structural uncertainty might proceed by writing down all possible functional forms for the underlying dynamical system with all possible parameter values for each form, and then consider searching over this huge space to select the most likely model and parameters; or using a Bayesian approach, assign priors to each of these possible models and infer the posterior distribution of possible models. The GP approach can be thought of as a computationally efficient approximation to this approach. GPs represent a large class of models that can be thought of as capturing or reasonably approximating the set of models in this collection. By modeling at the level of the process, rather than the level of parametric equation, we can more concisely capture the possible behavior of these curves. In place of a parametric model of the dynamical system, the GP approach postulates a prior distribution of (n-dimensional) curves that can be thought of as approximations to a range of possible (parametric) models that might describe the data. The GP allows us to consider probabilities on a large set of possible curves simultaneously.

The posterior distribution for the hyper-parameters of the Gaussian process model are estimated by Metropolis-Hastings algorithm, again with details and code provided in the Appendix. Rasmussen and Williams (2006) provides an excellent general introduction to Gaussian Processes and Munch et al. (2005) first discusses their application in the context of population dynamics models such as fisheries stock-recruitment relationships.

Approach and Methods

Statement of the optimal control problem

- Underlying model
- Available data

For simplicity we assume profit is simply linear in the realized harvest (only enforcing the restriction that harvest can not exceed available stock)

Parametric models

We consider three candidate parametric models of the stock-recruitment dynamics: The Ricker model, the Allen model [Allen 2005](#), the Myers model. The familiar Ricker model involves two parameters, corresponding to a growth rate and a carrying capacity, and cannot support alternative stable state dynamics (though as growth rate increases it exhibits a periodic attractor that proceeds through period-doubling into chaos. We will generally focus on dynamics below the chaotic threshold for the purposes of this analysis.) The Allen model resembles the Ricker dynamics with an added Allee effect parameter [Courchamp](#), below which the population cannot persist. The Myers model also has three parameters and contains an Allee threshold, but has compensatory rather than over-compensatory density dependence (resembling a Beverton-Holt curve rather than a Ricker curve at high densities.)

We assume multiplicative log-normal noise perturbs the growth predicted by the each of the deterministic model skeletons described above. This introduces one additional parameter σ that must be estimated by each model.

As we simulate training data from the Allen model (ref section), we will refer to this as the structurally correct model. The Ricker model is thus a reasonable approximation of these dynamics far from the Allee threshold (but lacks threshold dynamics), while the Myers model shares the essential feature of a threshold but differs in the structure. Thus we have three potential parametric models of the stock dynamics.

We infer posterior distributions for the parameters of each model in a Bayesian context using Gibbs sampling (implemented in R (Team 2013) using jags, (Su and Yajima 2013)). We choose uninformative uniform priors for all parameters (See Appendix, Figures S1-S3, and Table S1, and the R code provided). One-step-ahead predictions of these model fits are shown in Figure 1.

An optimal policy function is then inferred through stochastic dynamic programming for each model given the posterior distributions of the parameter estimates. This policy maximizes the expectation of the value function integrated over the parameter uncertainty. (code implementing this algorithm provided in the Appendix).

The Gaussian Process model

We also estimate a simple Gaussian Process defined by a radial basis function kernel of two parameters: ℓ , which gives the characteristic length-scale over which correlation between two points in state-space decays, and σ , which gives the scale of the process noise by which observations Y_{t+1} may differ from their predicted values X_{t+1} given an observation of the previous state, X_t . Munch, Kottas, and Mangel (2005) gives an accessible introduction to the use of Gaussian Processes in providing a Bayesian nonparametric description of the stock-recruitment relationship.

In contrast to the parametric models, this posterior distribution is still conditional on the training data. As such, the uncertainty near the observed data.

We use a Metropolis-Hastings Markov Chain Monte Carlo to infer posterior distributions of the two parameters of the GP (Figure S4, code in appendix), under weakly informative Gaussian priors (see parameters in table S5). As the posterior distributions differ substantially from the priors (Figure S4), we can be assured that most of the information in the posterior comes from the data rather than the prior belief.

Though we are unaware of prior application of this type, it is reasonably straight-forward to adapt the Gaussian Process for Stochastic Dynamic Programming. Recall that unlike the parametric models the Gaussian process with fixed parameters already predicts a distribution of curves rather than a single curve. We must first integrate over this distribution of curves given a sampling of parameter values drawn from the posterior distribution of the two GP parameters, before integrating over the posterior of those parameters themselves.

Results

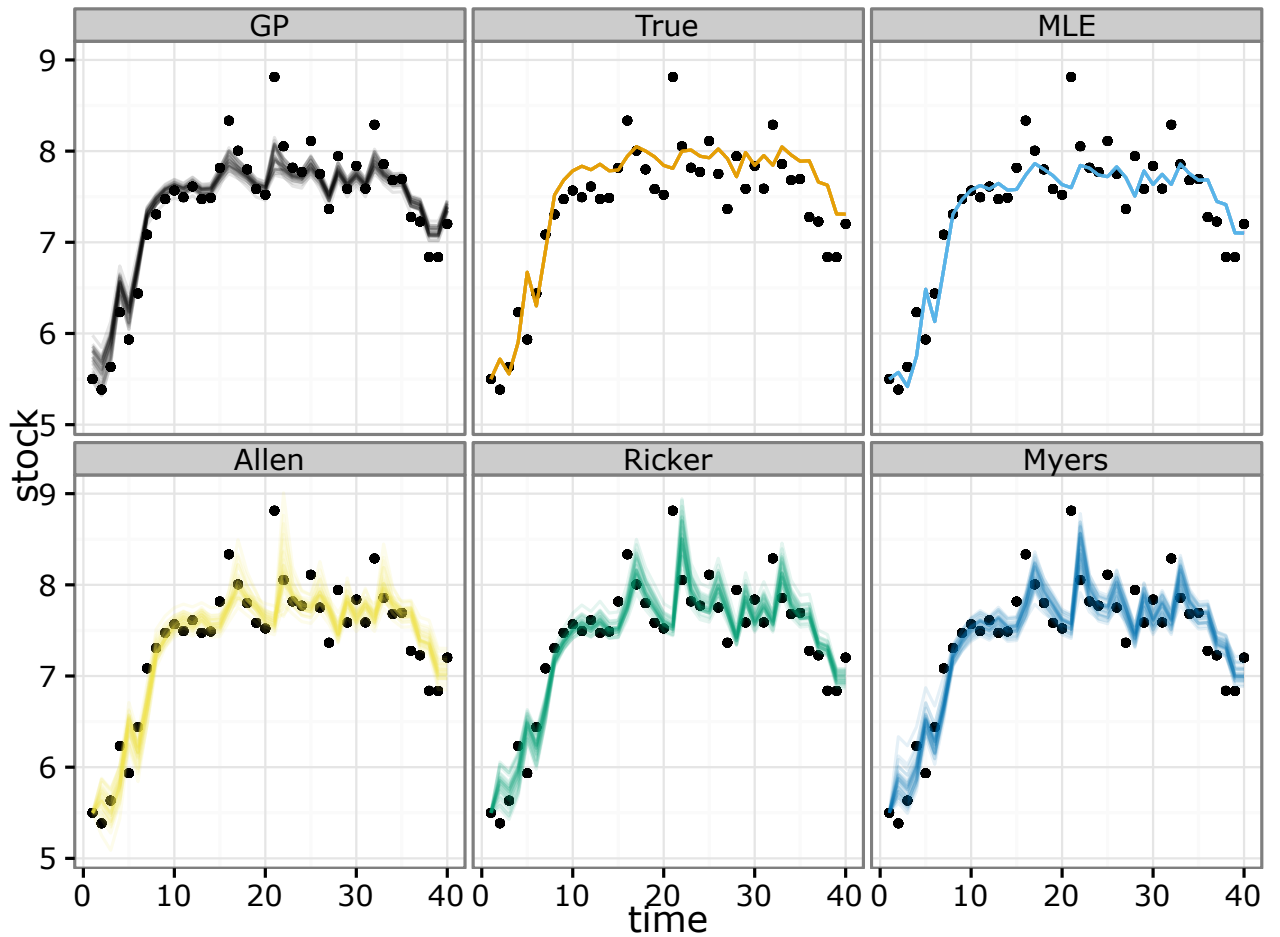


Figure 1: Points show the training data of stock-size over time. Curves show the posterior step-ahead predictions based on each of the estimated models.

All models fit the observed data rather closely and with relatively small uncertainty, as illustrated in the posterior

predictive curves in Figure 1. Figure 1 shows the training data of stock sizes observed over time as points, overlaid with the step-ahead predictions of each estimated model using the parameters sampled from their posterior distributions. Each model manages to fit the observed data rather closely. Compared to the expected value of the true model most estimates appear to overfit, predicting fluctuations that are actually due purely to stochasticity in growth rate. Model-choice criteria shown in Table 1 penalize more complex models and show a slight preference for the simpler Ricker model over the more complicated alternate stable state models (Allen and Myers). Details on MCMC estimates for each model, traces, and posterior distributions can be found in the appendix.

% latex table generated in R 3.0.2 by xtable 1.7-1 package % Thu Nov 14 14:10:28 2013

	Allen	Ricker	Myers
DIC	50.14	49.45	50.61
AIC	-24.60	-30.07	-27.19
BIC	-17.85	-25.00	-20.44

The mean inferred state space dynamics of each model relative to the true model used to generate the data is shown in Figure 2, predicting the relationship between observed stock size (x-axis) to the stock size after recruitment the following year. Note that in contrast to the other models shown, the expected Gaussian process corresponds to a distribution of curves - as indicated by the gray band - which itself has a mean shown in black. Parameter uncertainty (not shown) spreads out the estimates further. The observed data from which each model is estimated is also shown. The observations come from only a limited region of state space corresponding to unharvested or weakly harvested system. No observations occur at the theoretical optimum harvest rate or near the tipping point.

Despite the similarities in model fits to the observed data, the policies inferred under each model differ widely, as shown in Figure 3. Policies are shown in terms of target escapement, S_t . Under models such as this a constant escapement policy is expected to be optimal (Reed 1979), whereby population levels below a certain size S are unharvested, while above that size the harvest strategy aims to return the population to S , resulting in the hockey-stick shaped policies shown. Only the structurally correct model (Allen model) and the GP produce policies close to the true optimum policy (where both the underlying model structure and parameter values are known without error).

The consequences of managing 100 replicate realizations of the simulated fishery under each of the policies estimated is shown in Figure 4. As expected from the policy curves, the structurally correct model under-harvests, leaving the stock to vary around it's un-fished optimum. The structurally incorrect Ricker model over-harvests the population passed the tipping point consistently, resulting in the immediate crash of the stock and thus derives minimal profits.

These results are robust across a range of stochastic realizations, models, and parameter values. The results

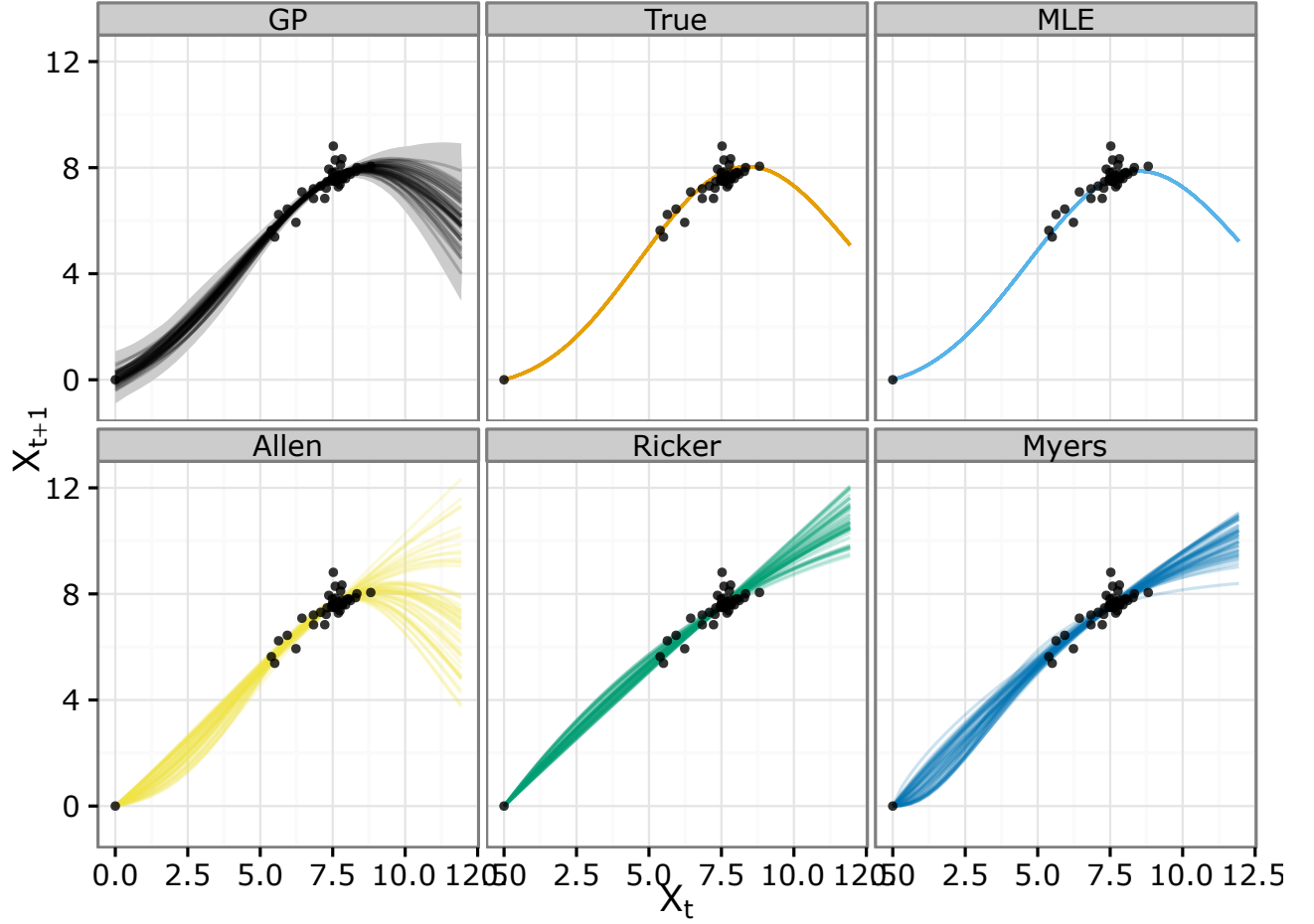


Figure 2: Graph of the inferred Gaussian process compared to the true process and maximum-likelihood estimated process. Graph shows the expected value for the function f under each model. Two standard deviations from the estimated Gaussian process covariance with (light grey) and without (darker grey) measurement error are also shown. The training data is also shown as black points. The GP is conditioned on $(0,0)$, shown as a pseudo-data point.

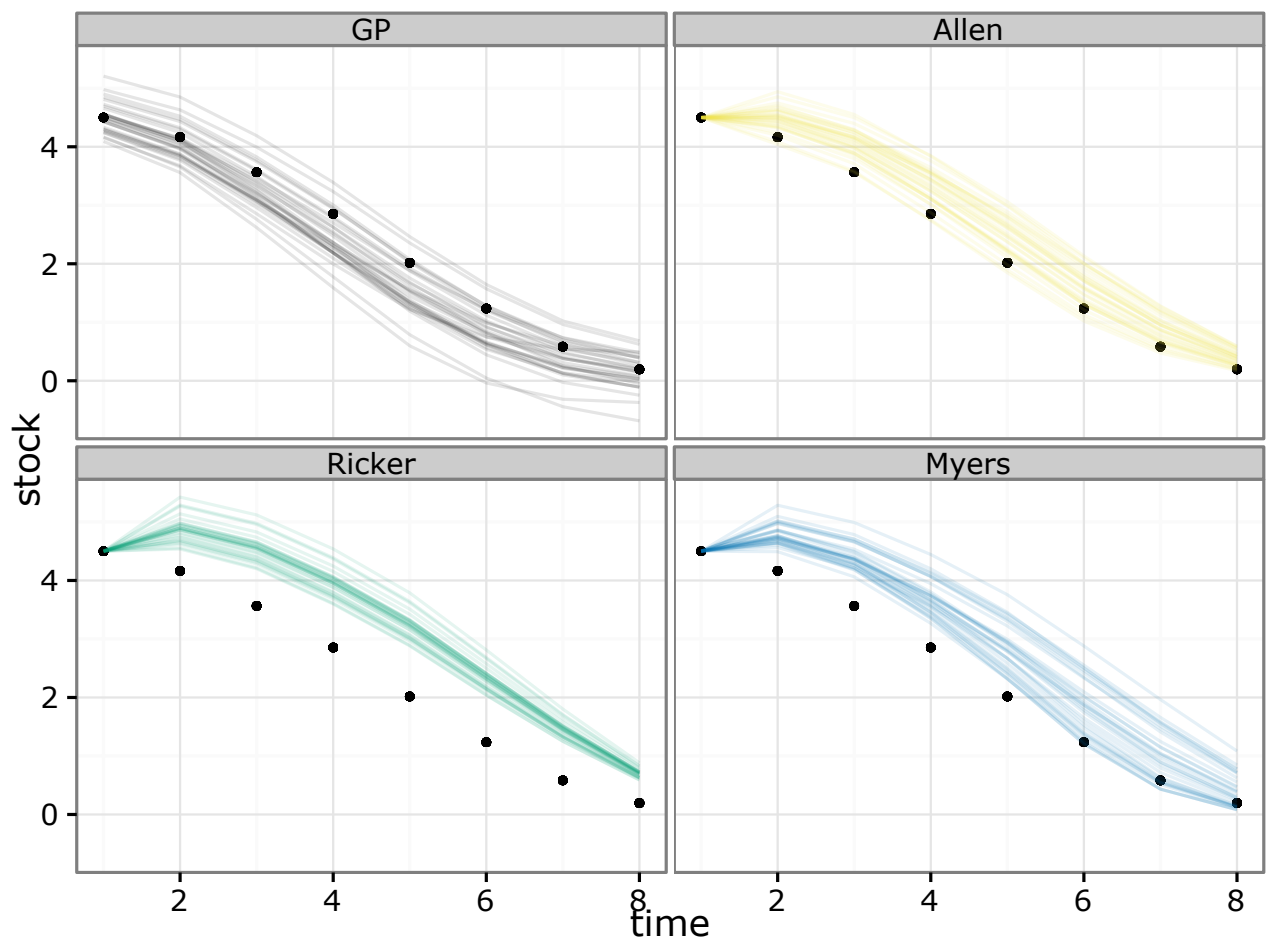


Figure 3: plot of chunk out_of_sample_predictions

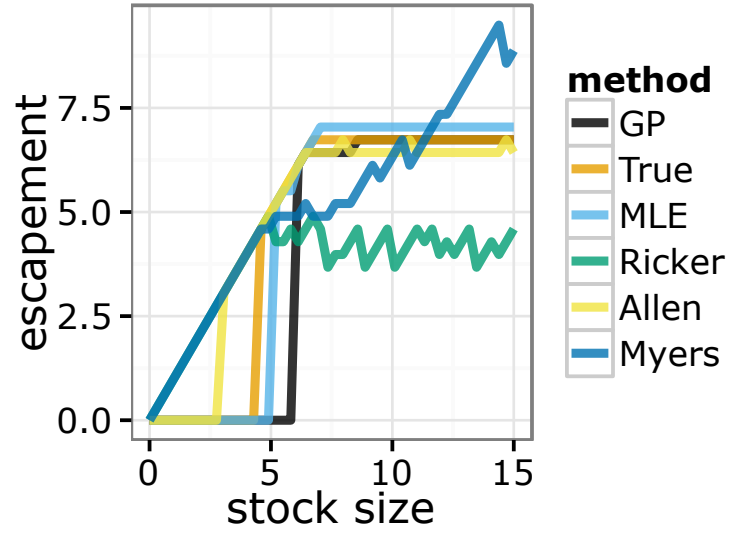


Figure 4: The steady-state optimal policy (infinite boundary) calculated under each model. Policies are shown in terms of target escapement, S_t , as under models such as this a constant escapement policy is expected to be optimal (Reed 1979).

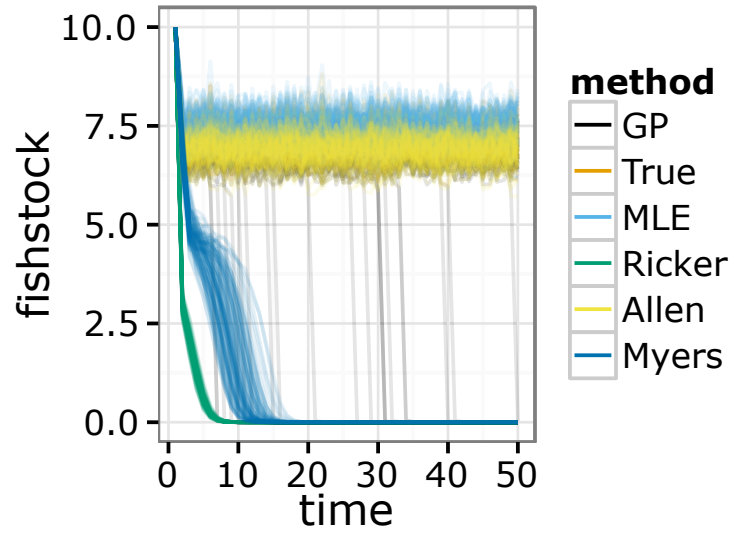


Figure 5: Gaussian process inference outperforms parametric estimates. Shown are 100 replicate simulations of the stock dynamics (eq 1) under the policies derived from each of the estimated models, as well as the policy based on the exact underlying model.

across this range can most easily be compared by using the relative differences in net present value realized by each of the model, as shown in Figure 5. The BNP-SDP approach most consistently realizes a value close to the optimal solution, and importantly avoids ever driving the system across the tipping point, which results in the near-zero value cases in the parametric models.

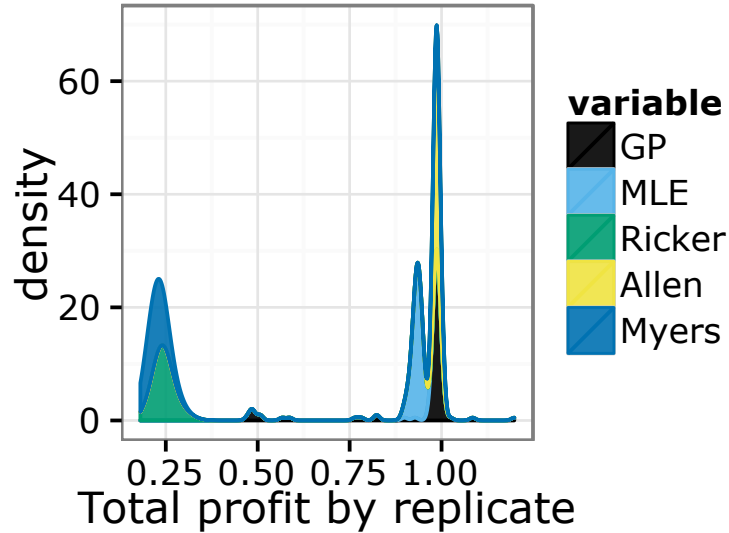


Figure 6: Histograms of the realized net present value of the fishery over a range of simulated data and resulting parameter estimates. For each data set, the three models are estimated as described above. Values plotted are the averages of a given policy over 100 replicate simulations. Details and code provided in the supplement.

Discussion

In any modeling effort, models must be chosen for the task at hand.

Though simple mechanistically motivated models offer the greatest potential to increase our basic understanding of ecological processes (Cuddington, Fortin, and Gerber 2013; Geritz and Kisdi 2012), such models can be not only inaccurate but misleading when relied upon in a quantitative decision making framework.

In this paper we have tackled two aspects of uncertainty that are both common to many ecological decision-making problems and fundamentally challenging to existing approaches which largely rely on parametric models:

1. We do not know what the correct models are for ecological systems.
2. We have limited data from which to estimate the model – in particular, such models may be misleading in predicting the probability of outcomes outside the training data.

We have illustrated how the use of non-parametric approaches can provide more reliable solutions in the sequential decision-making problem.

Traditional model-choice approaches can be positively misleading.

These results illustrate that model-choice approaches would be positively misleading – supporting simpler models that cannot express tipping point dynamics merely on account of them being similar. As the data shown comes only from the basin of attraction near the unfished equilibrium, near which all of the models are approximately linear and approximately identical.

Model choice approaches trade off model complexity and fit to the data. When the data come from a limited region of state-space – as is necessarily the case whenever there is a potential concern about tipping point dynamics – simpler models can fit just as well and will tend to outperform more complex ones. This approach would be appropriate when the dynamics can be expected to remain in the region of the training data; for instance, if we only considered the forecasting accuracy of the unfished population dynamics under each model.

In contrast, the decision-maker’s problem of setting appropriate harvest levels cannot exclude regions of state-space outside the observed range when integrating over all possible decisions to find the optimal choice. Such problems are not constrained to fisheries management but ubiquitous across ecological decision-making and conservation where the greatest concerns involve entering previously unobserved regions of state-space – whether that is the collapse of a fishery, the spread of an invasive, or the loss of habitat.

BNP-SDP expresses larger uncertainty in regions where the data are poor

The parametric models perform worst when they propose a management strategy outside the range of the observed data. The non-parametric Bayesian approach, in contrast, allows a predictive model that expresses a great deal of uncertainty about the probable dynamics outside the observed range, while retaining very good predictive accuracy in the range observed. The management policy dictated by the GP balance this uncertainty against the immediate value of the harvest, and act to stabilize the population dynamics in a region of state space in which the predictions can be reliably reflected by the data.

BNP-SDP has good predictive accuracy where data are good

While expressing larger uncertainty outside the observed data, the GP can also provide a better fit with smaller uncertainty inside the range of the observed data. This arises from the greater flexibility of the Gaussian process, which describes a large family of possible curves. Despite this flexibility, the GP can be described in relatively few parameters and is thus far less likely to overfit.

Risk-prone and risk-adverse value functions

The degree to which the decision-making part of the algorithm (the SDP) chooses to explore or avoid the resulting region of uncertainty can also be influenced by the curvature of the value (profit) function Π . Both to simplify the intuition and avoid biasing this result, we have chosen a profits that are linear in the catch and

thus neither risk-prone nor risk adverse. Making this function concave, representing the typical assumption of diminishing returns, would make the SDP more risk-adverse (as larger-than-expected stock sizes offer diminished returns relative to the cost of smaller-than-expected stock sizes), and strengthen the result shown here in which the BNP solution tends to avoid the region of uncertainty. Sufficiently convex or risk-prone functions could lead the SDP to attempt higher exploitation rates despite the uncertainty. Understanding the relative roles of such functions would be a promising direction for future investigation.

The role of the prior

Lastly, it should be noted that outside the data, the NBP reverts to the prior, and consequently the choice of the prior can also play a significant role in determining the optimal policy inferred by the SDP. In the examples shown here we have selected a prior that is both relatively uninformative (due to the broad priors placed on its parameters ℓ and σ and simple (mean zero, radial basis function kernel). In practice, both the choice of mean and the choice of the covariance function may be chosen to confer particular biological properties, as well as more biologically informed priors for ℓ and σ . In principle, this may allow a manager to improve the performance of the BNP-SDP approach by adding only enough additional detail as is justified. For instance, it would be possible to use a linear or a Ricker-shaped mean in the prior without making the much stronger assumption that the Ricker is the structurally correct model. However, this influence raises challenges as well. For instance, in choosing a trivial mean-zero prior, we bias the dynamics to transitions to 0 in step X_{t+1} from any stock size X_t in the prior year, in the absence of any other data. Future research must make sure that both the prior and the value function are chosen appropriately for the problem at hand.

Future directions

Higher dimensions

In this simulated example, the underlying dynamics are truly governed by a simple parametric model, allowing the parametric approaches to be more accurate. Similarly, because the dynamics are one-dimensional dynamics and lead to stable nodes (rather than other attractors such as limit-cycles resulting in oscillations), the training data provides relatively limited information about the dynamics. For these reasons, we anticipate that in higher-dimensional examples characteristic of ecosystem management problems that the machine learning approach will prove even more valuable.

Real-time learning

In our treatment here we have ignored the possibility of learning during the management phase, in which the additional observations of the stock size could potentially improve parameter estimates. Of particular interest in the context of the extreme uncertainty considered here is the notion of “active adaptive management” or “adaptive probing”, which may actively seek to reduce uncertainty. The concept of adaptive probing is one

area that has explicitly addressed the extrapolation uncertainty addressed here, though typically without the additional issue of model uncertainty. As a result, adaptive probing strategies suggest rather opposite conclusions than what we observe here. Such adaptive probing or Dual Control (e.g. Ludwig and Walters (1982)) approaches trade off short term utility by choosing actions that can reduce uncertainty. Adaptive probing strategies arise when it is valuable to intentionally force a system far from the observed values even when the expected value such actions is low, as it provides much faster learning and consequent reduction of model uncertainty that can allow greater value to be derived later on. For instance, Ludwig and Walters (1982) show that it may be advantageous to fish an unexploited population very heavily at first to obtain a better estimate of the recruitment rate. This intuitive strategy when a population is governed by a Ricker or Beverton-Holt-like dynamic would clearly be disastrous if instead the dynamics contained an unforeseen tipping point. The best way to learn where the edge lies may be to walk up to it, but it is also the most dangerous. Future work should attempt to understand when such active adaptive learning is valuable, and when it will increase the risk of an irreversible transition.

Acknowledgments

This work was partially supported by the Center for Stock Assessment Research, a partnership between the University of California Santa Cruz and the Fisheries Ecology Division, Southwest Fisheries Science Center, Santa Cruz, CA and by NSF grant EF-0924195 to MM and NSF grant DBI-1306697 to CB.

Appendix

Model definitions and estimation

Equation S1: Ricker model.

$$X_{t+1} = Z_t X_t e^{r(1-\frac{S_t}{K})}$$

Figure S1: Ricker model: prior and posterior distributions for parameter estimates.

Table S1: Parameterization of the priors

% latex table generated in R 3.0.2 by xtable 1.7-1 package % Thu Nov 14 00:55:33 2013

	parameter	lower_bound	upper_bound
1	r0	0.00	10.00
2	K	0.00	40.00
3	sigma	0.00	100.00

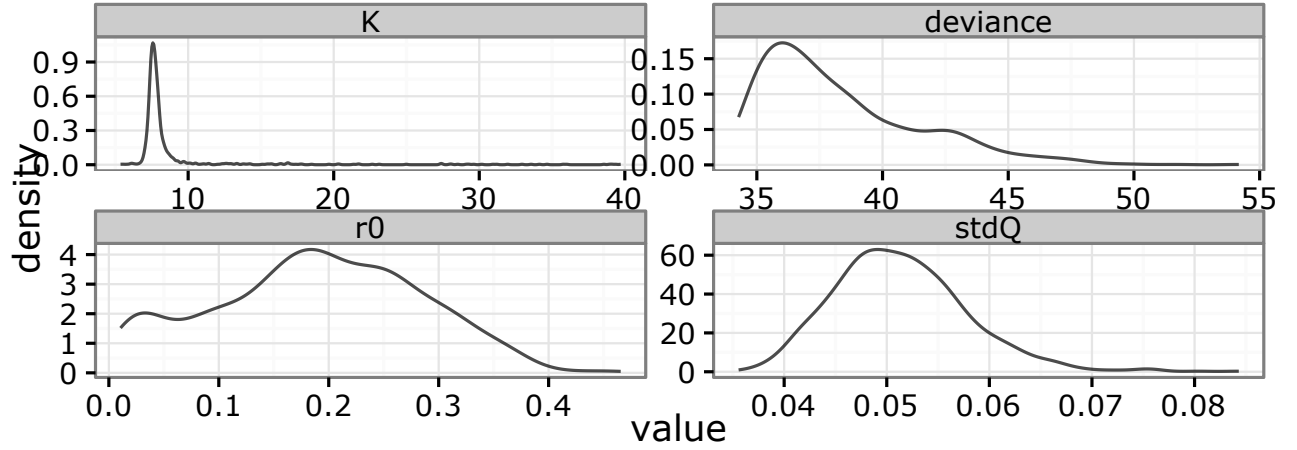


Figure 7: Posteriors from the MCMC estimate of the Ricker model

$$X_{t+1} = Z_t \frac{rS_t^\theta}{1 - \frac{S_t^\theta}{K}}$$

Eq S2: Myers model Figure S2: Myers model: Traces, prior and posterior distributions for parameter estimates.

Table S2: Parameterization of the priors % latex table generated in R 3.0.2 by xtable 1.7-1 package % Thu Nov 14 00:55:36 2013

	parameter	lower_bound	upper_bound
1	r0	0.00	10.00
2	K	0.00	40.00
3	theta	0.00	10.00
4	sigma	0.00	100.00

Eq S3: Allen model

$$f(S_t) = S_t e^{r(1 - \frac{S_t}{K})(S_t - C)}$$

Figure S3: Allen model: prior and posterior distributions for parameter estimates.

Table S3: Parameterization of the priors

% latex table generated in R 3.0.2 by xtable 1.7-1 package % Thu Nov 14 00:55:40 2013

Eq S4: GP model Figure S4: GP model: prior and posterior distributions for parameter estimates.

\$traces_plot

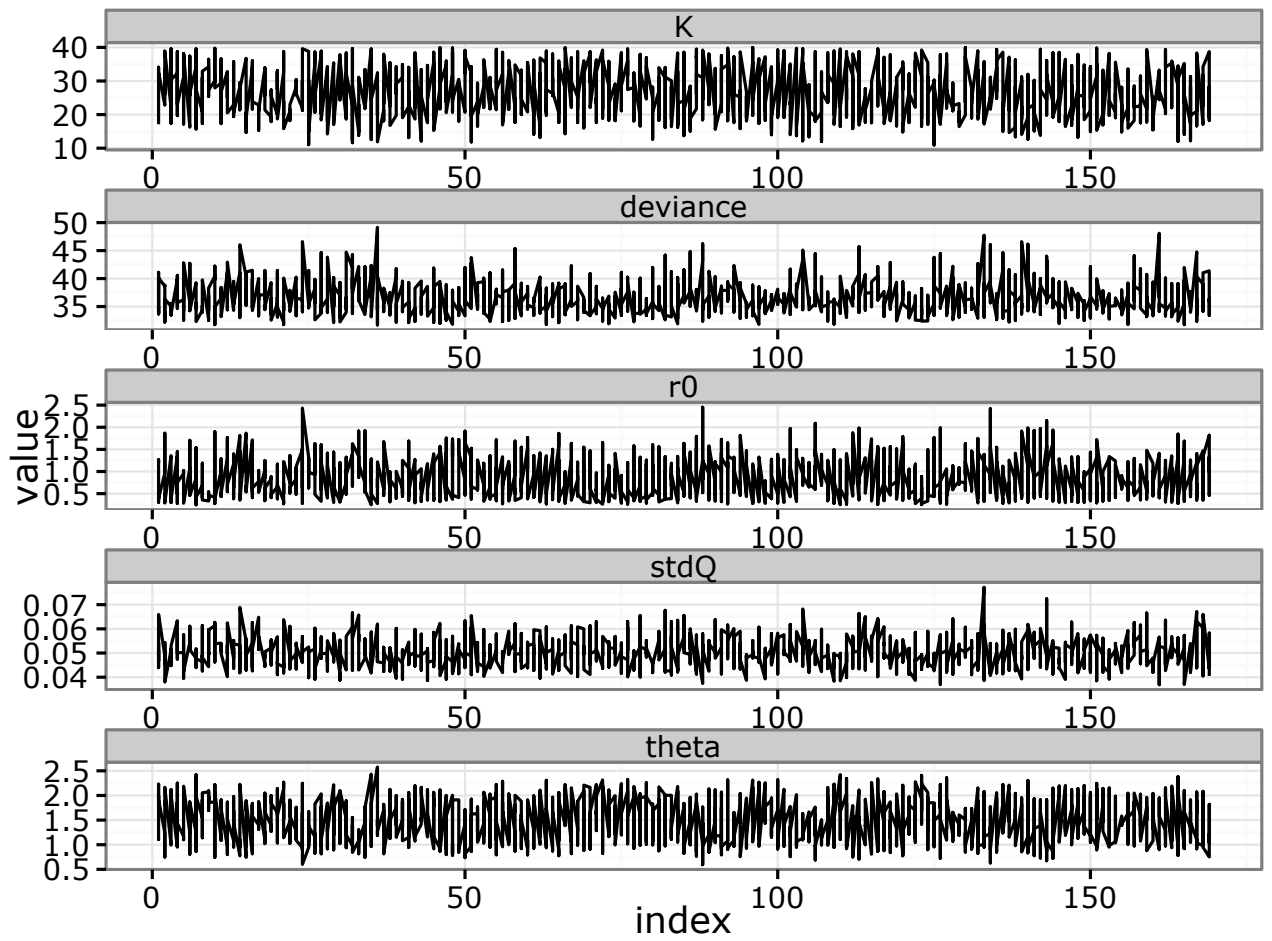


Figure 8: Traces from the MCMC estimate of the Myers model

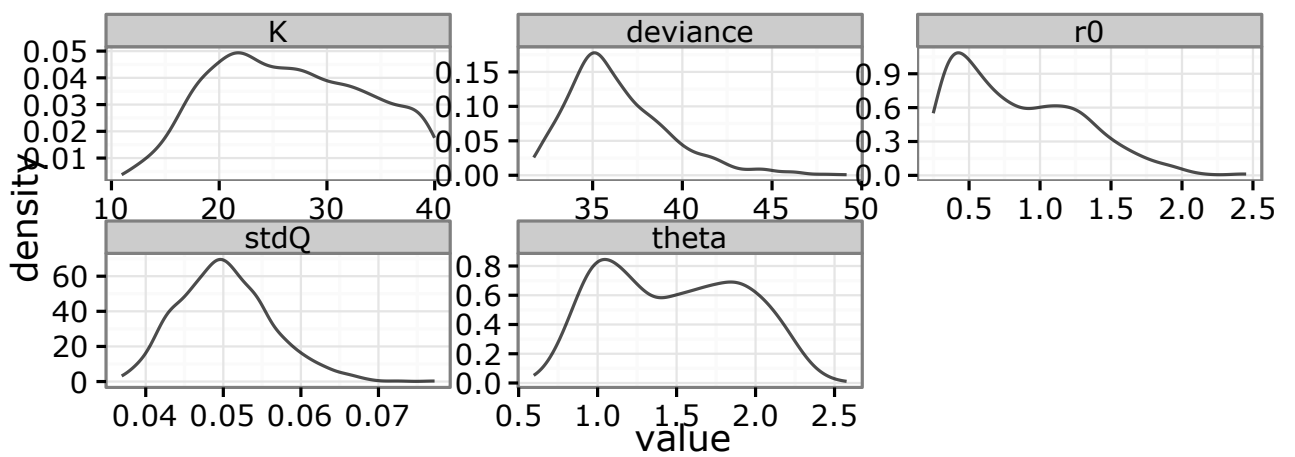


Figure 9: Posterior distributions from the MCMC estimates of the Myers model

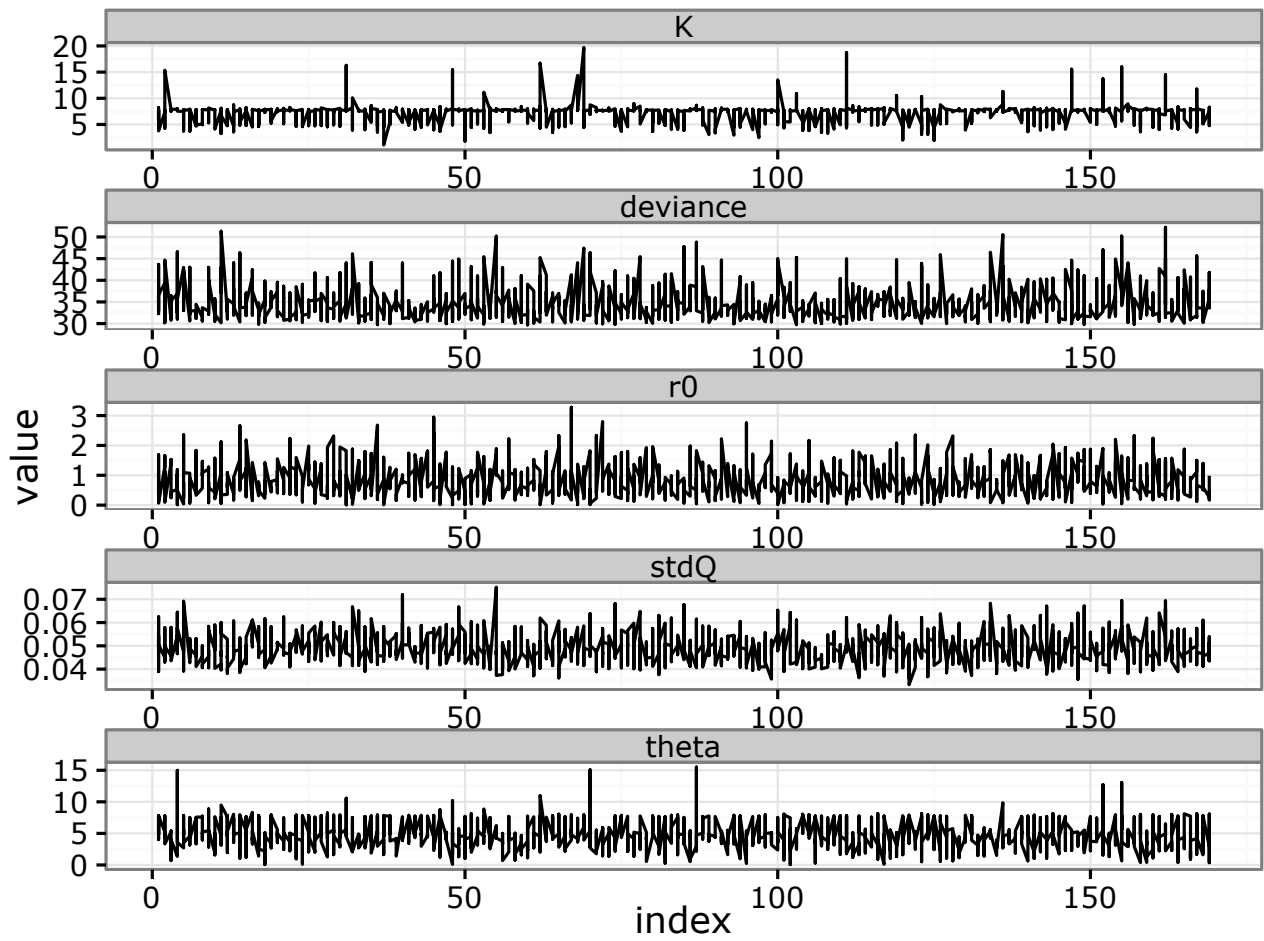


Figure 10: Traces from the MCMC estimate of the Allen model

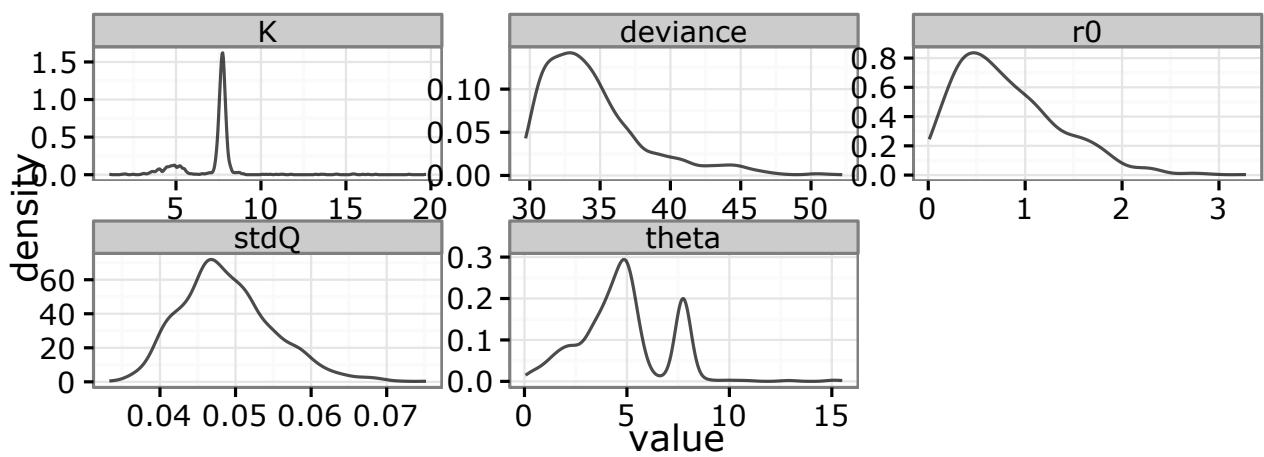


Figure 11: Posteriors from the MCMC estimate of the Allen model

	parameter	lower_bound	upper_bound
1	r0	0.00	10.00
2	K	0.00	40.00
3	theta	0.00	10.00
4	sigma	0.00	100.00

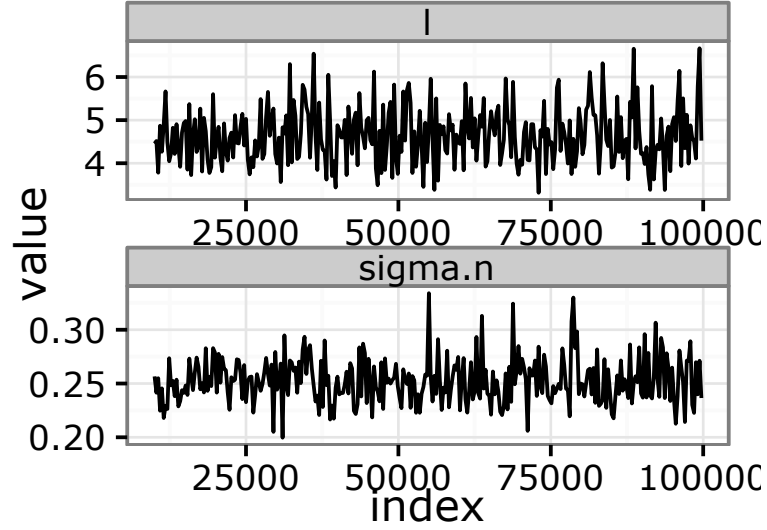


Figure 12: plot of chunk unnamed-chunk-10

\$posteriors_plot

Table S4: Parameterization of the priors

Optimal Control Problem

We seek the harvest policy $h(x)$ that maximizes:

$$\max_{h_t} \sum_{t=0}^{\infty} \Pi_t(X_t, h_t) \delta^t$$

subject to the profit function $\Pi(X_t, h)$, discount rate δ , and the state equation

$$X_{t+1} = Z_t f(S_t)$$

$$S_t = X_t - h_t$$

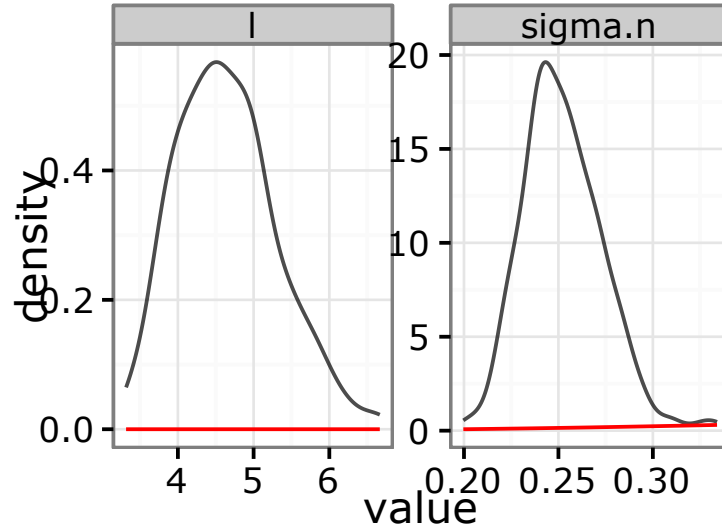


Figure 13: plot of chunk unnamed-chunk-10

Where Z_t is multiplicative noise function with mean 1, representing stochastic growth. We will consider log-normal noise with shape parameter σ_g .

Form this we can write down the Bellman recursion as:

$$V_t(x_t) = \max_h \mathbf{E}(\Pi(h_t, x_t) + \delta V_{t+1}(Z_{t+1}f(x_t - h_t)))$$

For simplicity we assume profit is simply linear in the realized harvest (only enforcing the restriction that harvest can not exceed available stock), $\Pi(h, x) = \min(h, x)$.

Pseudocode for the Bellman iteration

```
V1 <- sapply(1:length(h_grid), function(h){
  delta * F[[h]] %*% V + profit(x_grid, h_grid[h])
})
# find havest, h that gives the maximum value
out <- sapply(1:gridsize, function(j){
  value <- max(V1[j,], na.rm = T) # each col is a diff h, max over these
  index <- which.max(V1[j,]) # store index so we can recover h's
  c(value, index) # returns both profit value & index of optimal h.
})
# Sets V[t+1] = max_h V[t] at each possible state value, x
```

```
V <- out[1,] # The new value-to-go
D[,OptTime-time+1] <- out[2,] # The index positions
```

Training data

Each of our models $f(S_t)$ must be estimated from training data, which we simulate from the Allen model with parameters $r = \mathbf{r_p}[1]$, $K = \mathbf{r_p}[2]$, $C = \mathbf{r_p}[3]$, and $\sigma_g = \mathbf{r_sigma_g}$ for $T = 40$ timesteps, starting at initial condition $X_0 = 5.5$. The training data can be seen in Figure 1.

Sensitivity Analysis

Athanassoglou, Stergios, and Anastasios Xepapadeas. 2012. “Pollution Control with Uncertain Stock Dynamics: When, and How, to Be Precautious.” *Journal of Environmental Economics and Management* 63 (3) (may): 304–320. doi:10.1016/j.jeem.2011.11.001. <http://linkinghub.elsevier.com/retrieve/pii/S0095069611001409>.

Bestelmeyer, Brandon T., Michael C. Duniway, Darren K. James, Laura M. Burkett, and Kris M. Havstad. 2012. “A Test of Critical Thresholds and Their Indicators in a Desertification-prone Ecosystem: More Resilience Than We Thought.” Edited by Katharine Suding. *Ecology Letters* (dec). doi:10.1111/ele.12045. <http://doi.wiley.com/10.1111/ele.12045>.

Brozović, Nicholas, and Wolfram Schlenker. 2011. “Optimal Management of an Ecosystem with an Unknown Threshold.” *Ecological Economics* (jan): 1–14. doi:10.1016/j.ecolecon.2010.10.001. <http://linkinghub.elsevier.com/retrieve/pii/S0921800910004167>.

Clark, Colin W., and Geoffrey P. Kirkwood. 1986. “On Uncertain Renewable Resource Stocks: Optimal Harvest Policies and the Value of Stock Surveys.” *Journal of Environmental Economics and Management* 13 (3) (sep): 235–244. doi:10.1016/0095-0696(86)90024-0. <http://linkinghub.elsevier.com/retrieve/pii/0095069686900240>.

Cressie, Noel, Catherine a Calder, James S. Clark, Jay M. Ver Hoef, and Christopher K. Wikle. 2009. “Accounting for Uncertainty in Ecological Analysis: the Strengths and Limitations of Hierarchical Statistical Modeling.” *Ecological Applications* 19 (3) (apr): 553–70. doi:10.1890/07-0744.1. <http://www.ncbi.nlm.nih.gov/pubmed/19425416>.

Cuddington, Kim M., M. J. Fortin, and L. R. Gerber. 2013. “Process-based Models Are Required to Manage Ecological Systems in a Changing World.” *Ecosphere* 4 (February): 1–12. doi:10.1890/ES12-00178.1. <http://www.esajournals.org/doi/abs/10.1890/ES12-00178.1>.

Fischer, Joern, Garry D. Peterson, Toby a Gardner, Line J. Gordon, Ioan Fazey, Thomas Elmqvist, Adam Felton, Carl Folke, and Stephen Dovers. 2009. “Integrating Resilience Thinking and Optimisation for Conservation.” *Trends in Ecology & Evolution* 24 (10) (oct): 549–54. doi:10.1016/j.tree.2009.03.020. <http://www.ncbi.nlm.nih.gov/pubmed/19665820>.

- Geritz, Stefan A. H., and Eva Kisdi. 2012. "Mathematical Ecology: Why Mechanistic Models?" *Journal of Mathematical Biology* 65 (6-7) (dec): 1411–5. doi:10.1007/s00285-011-0496-3. <http://www.ncbi.nlm.nih.gov/pubmed/22159789>.
- Gordon, H. S. 1954. "The Economic Theory of a Common-property Resource: the Fishery." *The Journal of Political Economy* 62 (2): 124–142. <http://www.jstor.org/stable/10.2307/1825571>.
- Halpern, Benjamin S., Carissa J. Klein, Christopher J. Brown, Maria Beger, Hedley S. Grantham, Sangeeta Mangubhai, Mary Ruckelshaus, et al. 2013. "Achieving the Triple Bottom Line in the Face of Inherent Trade-offs Among Social Equity, Economic Return, and Conservation." *Proceedings of the National Academy of Sciences of the United States of America* 110 (15) (apr): 6229–34. doi:10.1073/pnas.1217689110. <http://www.ncbi.nlm.nih.gov/pubmed/23530207> <http://www.pubmedcentral.nih.gov/articlerender.fcgi?artid=3625307> \&t
- Hilborn, Ray. 2007. "Reinterpreting the State of Fisheries and Their Management." *Ecosystems* 10 (8) (oct): 1362–1369. doi:10.1007/s10021-007-9100-5. <http://www.springerlink.com/index/10.1007/s10021-007-9100-5>.
- Hilborn, Ray, and Marc Mangel. 1997. *The Ecological Detective: Confronting Models with Data*. Princeton University Press. <http://www.amazon.com/The-Ecological-Detective-Confronting-Models/dp/0691034974>.
- Hughes, Terry P., Cristina Linares, Vasilis Dakos, Ingrid a van de Leemput, and Egbert H. van Nes. 2013. "Living Dangerously on Borrowed Time During Slow, Unrecognized Regime Shifts." *Trends in Ecology & Evolution* 28 (3) (mar): 149–55. doi:10.1016/j.tree.2012.08.022. <http://www.ncbi.nlm.nih.gov/pubmed/22995893>.
- Kocijan, Juš, Agathe Girard, Bla\ vz Banko, and Roderick Murray-Smith. 2005. "Dynamic Systems Identification with Gaussian Processes." *Mathematical and Computer Modelling of Dynamical Systems* 11 (4): 411–424. doi:10.1080/13873950500068567. <http://www.tandfonline.com/doi/abs/10.1080/13873950500068567>.
- Ludwig, Donald, and Carl J. Walters. 1982. "Optimal Harvesting with Imprecise Parameter Estimates." *Ecological Modelling* 14 (3-4) (jan): 273–292. doi:10.1016/0304-3800(82)90023-0. <http://linkinghub.elsevier.com/retrieve/pii/0304380082900230>.
- Mangel, Marc, and Colin W. Clark. 1988. *Dynamic Modeling in Behavioral Ecology*. Edited by John Krebs and Tim Clutton-Brock. Princeton: Princeton University Press.
- Marescot, Lucile, Guillaume Chapron, Iadine Chadès, Paul L. Fackler, Christophe Duchamp, Eric Marboutin, and Olivier Gimenez. 2013. "Complex Decisions Made Simple: a Primer on Stochastic Dynamic Programming." *Methods in Ecology and Evolution* (jun). doi:10.1111/2041-210X.12082. <http://doi.wiley.com/10.1111/2041-210X.12082>.
- May, Robert M., John R. Beddington, Colin W. Clark, Sidney J. Holt, and R. M. Laws. 1979. "Management of Multispecies Fisheries." *Science (New York, N.Y.)* 205 (4403) (jul): 267–77. doi:10.1126/science.205.4403.267. <http://www.ncbi.nlm.nih.gov/pubmed/17747032>.

- McAllister, M. K. 1998. “Bayesian Stock Assessment: a Review and Example Application Using the Logistic Model.” *ICES Journal of Marine Science* 55 (6) (dec): 1031–1060. doi:10.1006/jmsc.1998.0425. <http://icesjms.oxfordjournals.org/content/55/6/1031.short> <http://icesjms.oxfordjournals.org/cgi/doi/10.1006/jmsc.1998.0425>
- Munch, Stephan B., Athanasios Kottas, and Marc Mangel. 2005. “Bayesian Nonparametric Analysis of Stock recruitment Relationships.” *Canadian Journal of Fisheries and Aquatic Sciences* 62 (8) (aug): 1808–1821. doi:10.1139/f05-073. <http://www.nrcresearchpress.com/doi/abs/10.1139/f05-073>.
- Munch, Stephan B., Melissa L. Snover, George M. Watters, and Marc Mangel. 2005. “A Unified Treatment of Top-down and Bottom-up Control of Reproduction in Populations.” *Ecology Letters* 8 (7) (may): 691–695. doi:10.1111/j.1461-0248.2005.00766.x. <http://doi.wiley.com/10.1111/j.1461-0248.2005.00766.x>.
- Rasmussen, Carl Edward, and C. K. I. Williams. 2006. *Gaussian Processes for Machine Learning*. Edited by Thomas Dietterich. Boston: MIT Press,. www.GaussianProcess.org/gpml.
- Reed, William J. 1979. “Optimal Escapement Levels in Stochastic and Deterministic Harvesting Models.” *Journal of Environmental Economics and Management* 6 (4) (dec): 350–363. doi:10.1016/0095-0696(79)90014-7. <http://www.sciencedirect.com/science/article/pii/0095069679900147> <http://linkinghub.elsevier.com/retrieve/pii/0095069679900147>
- Roughgarden, Joan E., and F. Smith. 1996. “Why Fisheries Collapse and What to Do About It.” *Proceedings of the National Academy of Sciences of the United States of America* 93 (10): 5078. <http://www.pnas.org/content/93/10/5078.short>.
- Schapaugh, Adam W., and Andrew J. Tyre. 2013. “Accounting for Parametric Uncertainty in Markov Decision Processes.” *Ecological Modelling* 254 (apr): 15–21. doi:10.1016/j.ecolmodel.2013.01.003. <http://linkinghub.elsevier.com/retrieve/pii/S0304380013000306>.
- Scheffer, Marten, Stephen R. Carpenter, J. A. Foley, C. Folke, and B. Walker. 2001. “Catastrophic Shifts in Ecosystems.” *Nature* 413 (6856) (oct): 591–6. doi:10.1038/35098000. <http://www.ncbi.nlm.nih.gov/pubmed/11595939>.
- Sethi, Gautam, Christopher Costello, Anthony Fisher, Michael Hanemann, and Larry Karp. 2005. “Fishery Management Under Multiple Uncertainty.” *Journal of Environmental Economics and Management* 50 (2) (sep): 300–318. doi:10.1016/j.jeem.2004.11.005. <http://linkinghub.elsevier.com/retrieve/pii/S0095069605000057>.
- Su, Yu-Sung, and Masanao Yajima. 2013. *R2jags: A Package for Running Jags from R*. <http://cran.r-project.org/package=R2jags>.
- Team, R. Core. 2013. *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. <http://www.r-project.org/>.
- Weitzman, Martin L. 2013. “A Precautionary Tale of Uncertain Tail Fattening.” *Environmental and Resource Economics* 55 (2) (mar): 159–173. doi:10.1007/s10640-013-9646-y. <http://link.springer.com/10.1007/s10640-013-9646-y>.

Williams, Byron K. 2001. "Uncertainty , Learning , and the Optimal Management of Wildlife." *Environmental and Ecological Statistics* 8: 269–288. doi:10.1023/A:1011395725123.

Worm, Boris, Edward B. Barbier, Nicola Beaumont, J. Emmett Duffy, Carl Folke, Benjamin S. Halpern, Jeremy B. C. Jackson, et al. 2006. "Impacts of Biodiversity Loss on Ocean Ecosystem Services." *Science (New York, N.Y.)* 314 (5800) (nov): 787–90. doi:10.1126/science.1132294. <http://www.ncbi.nlm.nih.gov/pubmed/17082450>.

Worm, Boris, Ray Hilborn, Julia K. Baum, Trevor A. Branch, Jeremy S. Collie, Christopher Costello, Michael J. Fogarty, et al. 2009. "Rebuilding Global Fisheries." *Science (New York, N.Y.)* 325 (5940) (jul): 578–85. doi:10.1126/science.1173146. <http://www.ncbi.nlm.nih.gov/pubmed/19644114>.