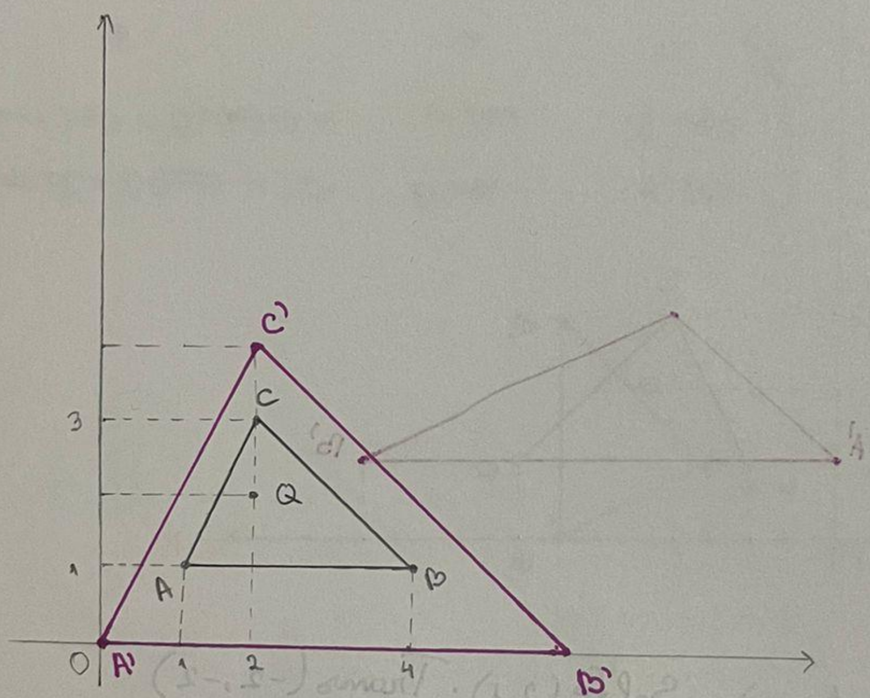
Aplicații ale geometriei în informatică

Laborator 1

Probleme

ΔABC, $A(1,1)$, $B(4,1)$, $C(2,3)$

2. Determinați imaginea triunghiului ABC printr-o scalare uniformă de factor de scală 2 relativ la punctul $Q(2,2)$.



$T_1 = \text{Trans}(2,2) \cdot \text{Scale}(2,2) \cdot \text{Trans}(-2,-2)$

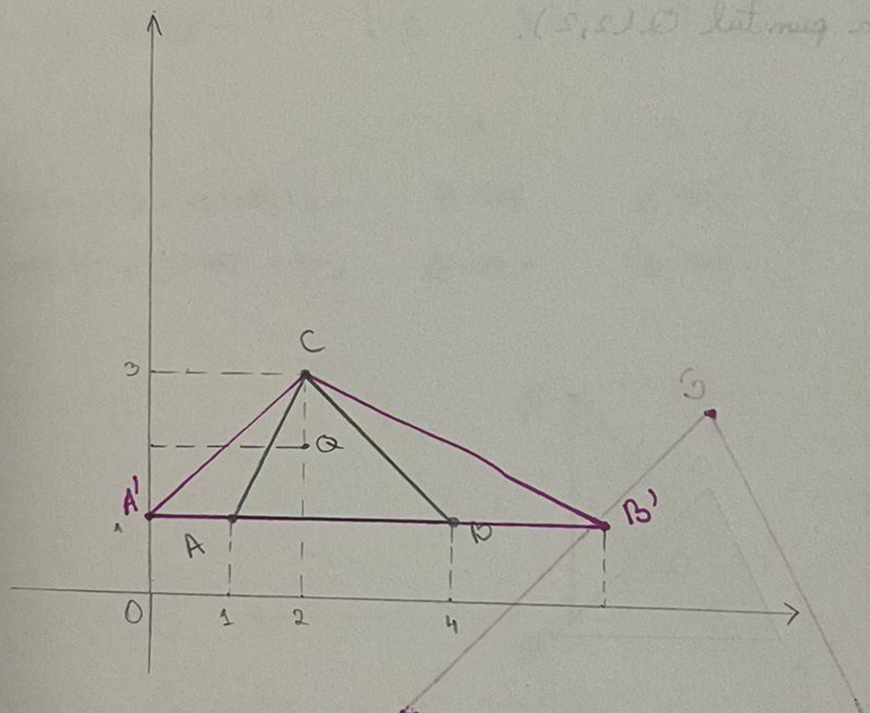
$$T_1 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow T_1 = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$[A' \; B' \; C'] = T_1 \cdot [A \; B \; C] = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$[A' \; B' \; C'] = \begin{pmatrix} 0 & 6 & 2 \\ 0 & 0 & 4 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow A'(0,0) \quad B'(6,0) \quad C'(2,4)$$

3. Determinați imaginea triunghiului ABC printr-o scalare simplă neuniformă de factori de scală (2,1), relativ la punctul Q(2,2)



$$T_1 = \text{Trans}(2,2) \cdot \text{Scale}(2,1) \cdot \text{Trans}(-2,-2)$$

$$T_1 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow T_1 = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

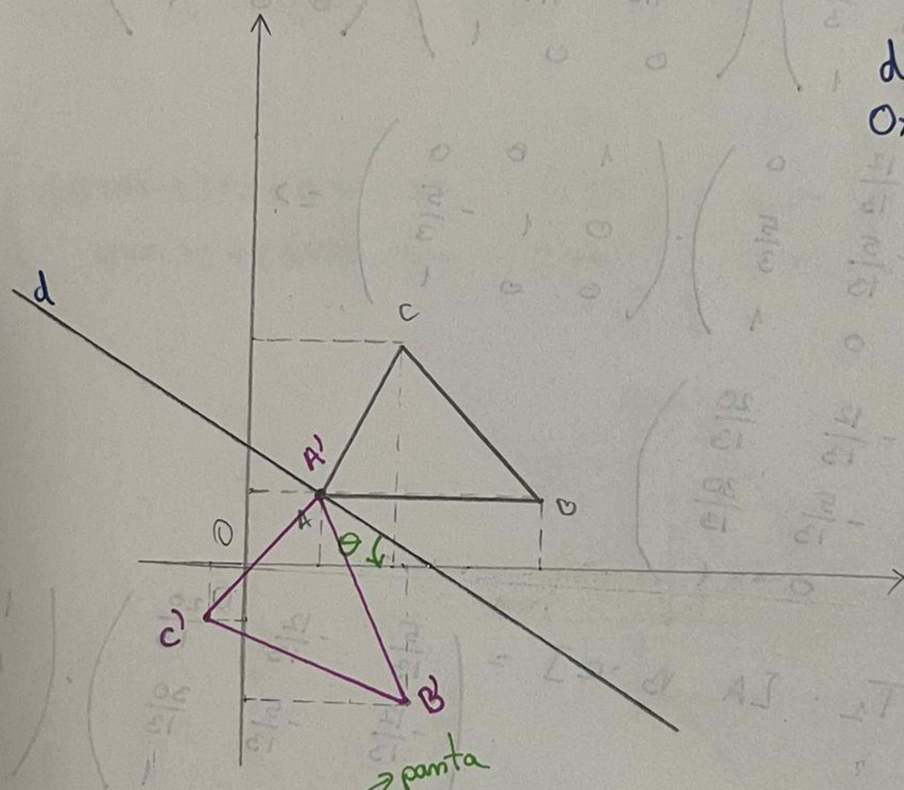
$$[A' \ B' \ C'] = T_1 \cdot [A \ B \ C] = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$[A' \ B' \ C'] = \begin{pmatrix} 0 & 6 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow A'(0,1) \quad B'(6,1) \quad C'(2,3)$$

7. Determinați imaginea triunghiului ABC prin reflexia relativă la dreapta $2x+3y-5=0$

$d: 2x+3y-5=0$

$Ox: y=0$



$$T_1 = Trans\left(0, \frac{5}{3}\right) \cdot Rot(\theta) \cdot R_x \cdot Rot(-\theta) \cdot Trans\left(0, -\frac{5}{3}\right)$$

$$\cos\theta = \frac{a_1 \cdot a_2 + b_1 \cdot b_2}{\sqrt{a_1^2+b_1^2} \cdot \sqrt{a_2^2+b_2^2}} = -\frac{3}{\sqrt{13}} \qquad \sin\theta = \frac{2}{\sqrt{13}}$$

$$T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -\frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} & 0 \\ +\frac{2}{\sqrt{13}} & -\frac{3}{\sqrt{13}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} -\frac{3}{\sqrt{13}} & \frac{2}{\sqrt{13}} & 0 \\ -\frac{2}{\sqrt{13}} & -\frac{3}{\sqrt{13}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{5}{3} \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_1 = \begin{pmatrix} -\frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} & 0 \\ \frac{2}{\sqrt{13}} & -\frac{3}{\sqrt{13}} & \frac{5}{3} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{3}{\sqrt{13}} & \frac{2}{\sqrt{13}} & 0 \\ -\frac{2}{\sqrt{13}} & -\frac{3}{\sqrt{13}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{5}{3} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{3}{\sqrt{13}} & \frac{2}{\sqrt{13}} & 0 \\ \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} & \frac{5}{3} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -\frac{3}{\sqrt{13}} & \frac{2}{\sqrt{13}} & 0 \\ -\frac{2}{\sqrt{13}} & -\frac{3}{\sqrt{13}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{5}{3} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} & 0 \\ -\frac{12}{13} & -\frac{5}{13} & \frac{5}{3} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{5}{3} \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow$$

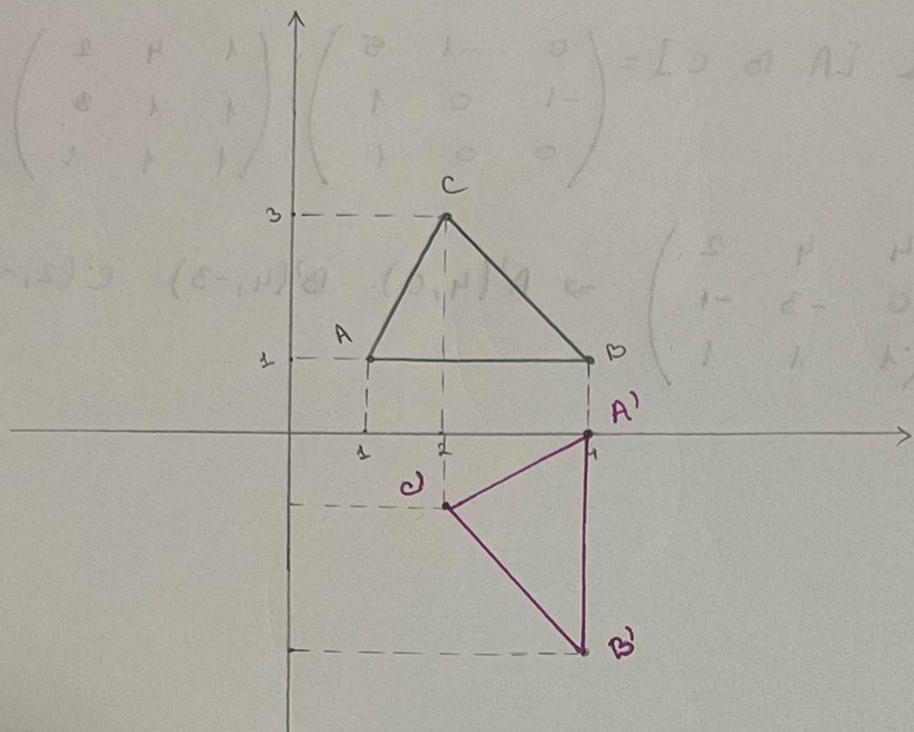
$$\Rightarrow T_1 = \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} & \frac{20}{13} \\ -\frac{12}{13} & -\frac{5}{13} & \frac{30}{13} \\ 0 & 0 & 1 \end{pmatrix}$$

$$[A' \quad B' \quad C'] = T_1 \cdot [A \quad B \quad C] = \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} & \frac{20}{13} \\ -\frac{12}{13} & -\frac{5}{13} & \frac{30}{13} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\Rightarrow [A' \quad B' \quad C'] = \begin{pmatrix} 1 & \frac{28}{13} & -\frac{6}{13} \\ 1 & -\frac{23}{13} & -\frac{9}{13} \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow A'(1,1) \quad B'\left(\frac{28}{13}, -\frac{23}{13}\right)$$

$$C'\left(-\frac{6}{13}, -\frac{9}{13}\right)$$

10. Determinați imaginea triunghiului ABC prin rotația cu 90° în jurul punctului C, urmată de reflexia relativ la dreapta AB.



$$Rot(c,\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & c_1(1-\cos\theta)+c_2\sin\theta \\ \sin\theta & \cos\theta & -c_1\sin\theta + c_2(1-\cos\theta) \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow Rot(c, 90°) = \begin{pmatrix} 0 & -1 & 5 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$ → matricea de rotație

AB: $y = 1 \Rightarrow y - 1 = 0$ → dreaptă paralelă cu Ox → nu mai trebuie să o rotim

$$T = Trans\left(0, -\frac{c}{b}\right) \cdot R_x \cdot Trans\left(0, \frac{c}{b}\right)$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$ → matricea de reflexie

$$T_1 = T \cdot Rot(C, 90^\circ) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 & 5 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 5 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$[A' \quad B' \quad C'] = T_1 \cdot [A \quad B \quad C] = \begin{pmatrix} 0 & -1 & 5 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\Rightarrow [A' \quad B' \quad C'] = \begin{pmatrix} 4 & 4 & 2 \\ 0 & -3 & -1 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow A'(4,0) \quad B'(4,-3) \quad C'(2,-1)$$