2-1

Вычислить предел числовой последовательности:

$$\lim_{n\to\infty}\frac{(3-n)^2+(3+n)^2}{(3-n)^2-(3+n)^2}$$

$$\lim_{n\to\infty}\frac{(3-n)^2+(3+n)^2}{(3-n)^2-(3+n)^2}=\lim_{n\to\infty}\frac{9-6n+n^2+9+6n+n^2}{9-6n+n^2-(9+6n+n^2)}=$$

$$=\lim_{n\to\infty}\frac{2(9+n^2)}{9-6n+n^2-9-6n-n^2)}=\lim_{n\to\infty}\frac{2(9+n^2)}{-12n}=$$

$$=-\frac{2}{12}\lim_{n\to\infty}\left(\frac{9}{n}+n\right)=-\infty$$

2-2

Вычислить предел числовой последовательности:

$$\lim_{n\to\infty}\frac{(3-n)^4-(2-n)^4}{(1-n)^4-(1+n)^4}$$

$$\lim_{n\to\infty}\frac{(3-n)^4-(2-n)^4}{(1-n)^4-(1+n)^4}=$$

$$=\lim_{n\to\infty}\frac{((3-n)^2-(2-n)^2)\cdot((3-n)^2+(2-n)^2)}{((1-n)^2-(1+n)^2)\cdot((1-n)^2+(1+n^2))}=$$

$$=\lim_{n\to\infty}\frac{(9-6n+n^2-4+4n-n^2)\cdot(9-6n+n^2+4-4n+n^2)}{(1-2n+n^2-1-2n-n^2)\cdot(1-2n+n^2+1+2n+n^2)}=$$

$$=\lim_{n\to\infty}\frac{(5-2n)\cdot(13-10n+2n^2)}{-4n\cdot(2+2n^2)}=$$

$$=\lim_{n\to\infty}\frac{n^3\left(\left(\frac{5}{n}-2\right)\cdot\left(\frac{13}{n^2}-\frac{10}{n}+2\right)\right)}{n^3\left(-4\left(\frac{2}{n^2}+2\right)\right)}=$$

$$=\lim_{n\to\infty}\frac{\left(\frac{5}{n}-2\right)\cdot\left(\frac{13}{n^2}-\frac{10}{n}+2\right)}{-4\left(\frac{2}{n^2}+2\right)}=\frac{(0-2)\cdot(0-0+2)}{-4(0+2)}=\frac{-4}{-8}=\frac{1}{2}$$

2-3

Вычислить предел числовой последовательности:

$$\lim_{n\to\infty}\frac{(3-n)^4-(2-n)^4}{(1-n)^3-(1+n)^3}$$

$$\lim_{n\to\infty}\frac{(3-n)^4-(2-n)^4}{(1-n)^3-(1+n)^3}=$$

$$=\lim_{n\to\infty}\frac{((3-n)^2-(2-n)^2)\cdot((3-n)^2+(2-n)^2)}{1-3n+3n^2-n^3-1-3n-3n^2-n^3}=$$

$$=\lim_{n\to\infty}\frac{(9-6n+n^2-4+4n-n^2)\cdot(9-6n+n^2+4-4n+n^2)}{-6n-2n^3}=$$

$$=\lim_{n\to\infty}\frac{(5-2n)\cdot(13-10n+2n^2)}{-2n\cdot(3+n^2)}=$$

$$=\lim_{n\to\infty}\frac{n^3\left(\left(\frac{5}{n}-2\right)\cdot\left(\frac{13}{n^2}-\frac{10}{n}+2\right)\right)}{n^3\left(-2\left(\frac{3}{n^2}+1\right)\right)}=$$

$$=\lim_{n\to\infty}\frac{\left(\frac{5}{n}-2\right)\cdot\left(\frac{13}{n^2}-\frac{10}{n}+2\right)}{-2\left(\frac{3}{n^2}+1\right)}=\frac{(0-2)\cdot(0-0+2)}{-2(0+1)}=\frac{-4}{-2}=2$$

2-4

Вычислить предел числовой последовательности:

$$\lim_{n\to\infty}\frac{(1-n)^4-(1+n)^4}{(1+n)^3-(1-n)^3}$$

$$\lim_{n\to\infty}\frac{(1-n)^4-(1+n)^4}{(1+n)^3-(1-n)^3}=$$

$$=\lim_{n\to\infty}\frac{((1-n)^2-(1+n)^2)\cdot((1-n)^2+(1+n)^2)}{1+3n+3n^2+n^3-1+3n-3n^2+n^3}=$$

$$=\lim_{n\to\infty}\frac{(1-2n+n^2-1-2n-n^2)\cdot(1-2n+n^2+1+2n+n^2)}{6n+2n^3}=$$

$$=\lim_{n\to\infty}\frac{-4n(2+2n^2)}{2n(3+n^2)}=$$

$$=\lim_{n\to\infty}\frac{-4(1+n^2)}{3+n^2}=\lim_{n\to\infty}\frac{n^2\left(-4\left(\frac{1}{n^2}+1\right)\right)}{n^2\left(\frac{3}{n^2}+1\right)}=$$

$$= \lim_{n\to\infty} \frac{-4\left(\frac{1}{n^2}+1\right)}{\frac{3}{n^2}+1} = \frac{-4(0+1)}{0+1} = -4$$

2-5

Вычислить предел числовой последовательности:

$$\lim_{n\to\infty} \frac{(6-n)^2-(6+n)^2}{(6+n)^2-(1-n)^2}$$

$$\lim_{n\to\infty} \frac{(6-n)^2-(6+n)^2}{(6+n)^2-(1-n)^2} = \lim_{n\to\infty} \frac{36-12n+n^2-36-12n-n^2}{36+12n+n^2-1+2n-n^2)} =$$

$$= \lim_{n\to\infty} \frac{-24n}{35+14n} = \lim_{n\to\infty} \frac{-24}{\frac{35}{n}+14} = \frac{-24}{0+14} = -\frac{12}{7} = -1\frac{5}{7}$$

2-6

Вычислить предел числовой последовательности:

$$\lim_{n\to\infty} \frac{(n+1)^3-(n+1)^2}{(n-1)^3-(n+1)^3}$$

$$\lim_{n\to\infty} \frac{(n+1)^3-(n+1)^2}{(n-1)^3-(n+1)^3} = \lim_{n\to\infty} \frac{(n+1)^2\cdot((n+1)-1)}{n^3-3n^2+3n-1-n^3-3n^2-3n-1} =$$

$$= \lim_{n\to\infty} \frac{n(n+1)^2}{-6n^2-2} = \lim_{n\to\infty} \frac{n^3\left(1+\frac{1}{n}\right)^2}{-2n^3\left(\frac{3}{n}+\frac{1}{n^2}\right)} = -\frac{1}{2}\lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)^2}{\left(\frac{3}{n}+\frac{1}{n^2}\right)} = -\infty$$

2-7

Вычислить предел числовой последовательности:

$$\lim_{n\to\infty} \frac{(1+2n)^3-8n^3}{(1+2n)^2+4n^2}$$

$$\lim_{n\to\infty} \frac{(1+2n)^3-8n^3}{(1+2n)^2+4n^2} = \lim_{n\to\infty} \frac{1+6n+12n^2+8n^3-8n^3}{1+4n+4n^2+4n^2} =$$

$$= \lim_{n\to\infty} \frac{1+6n+12n^2}{1+4n+8n^2} = \lim_{n\to\infty} \frac{n^2\left(\frac{1}{n^2}+\frac{6}{n}+12\right)}{n^2\left(\frac{1}{n^2}+\frac{4}{n}+8\right)} =$$
$$= \lim_{n\to\infty} \frac{\frac{1}{n^2}+\frac{6}{n}+12}{\frac{1}{n^2}+\frac{4}{n}+8} = \frac{0+0+12}{0+8} = \frac{12}{8} = 1{,}5$$

2-8

Вычислить предел числовой последовательности:

$$\lim_{n\to\infty} \frac{(3-4n)^2}{(n-3)^3-(n+3)^3}$$
$$\lim_{n\to\infty} \frac{(3-4n)^2}{(n-3)^3-(n+3)^3} =$$
$$\lim_{n\to\infty} \frac{9-24n+16n^2}{n^3-3\cdot n^2\cdot 3+3\cdot n\cdot 3^2-3^3-n^3-3\cdot n^2\cdot 3-3\cdot n\cdot 3^2-3^3} =$$
$$= \lim_{n\to\infty} \frac{9-24n+16n^2}{-3\cdot n^2\cdot 3-3^3-3\cdot n^2\cdot 3-3^3} = \lim_{n\to\infty} \frac{9-24n+16n^2}{-2\cdot 3^2\left(n^2+3\right)} =$$
$$= \lim_{n\to\infty} \frac{\frac{1}{n^2}\left(9-24n+16n^2\right)}{\frac{1}{n^2}(-2)\cdot 3^2\left(n^2+3\right)} = \lim_{n\to\infty} \frac{\frac{9}{n^2}-\frac{24}{n}+16}{-2\cdot 3^2\left(1+\frac{3}{n^2}\right)} =$$
$$= \frac{0-0+16}{-2\cdot 3^2(1+0)} = \frac{16}{-2\cdot 3^2} = -\frac{8}{9}$$

2-9

Вычислить предел числовой последовательности:

$$\lim_{n\to\infty} \frac{(3-n)^3}{(n+1)^2-(n+1)^3}$$
$$\lim_{n\to\infty} \frac{(3-n)^3}{(n+1)^2-(n+1)^3} = \lim_{n\to\infty} \frac{\frac{1}{n^3}(3-n)^3}{\frac{1}{n^3}\left((n+1)^2-(n+1)^3\right)} =$$
$$= \lim_{n\to\infty} \frac{(\frac{3}{n}-1)^3}{\frac{1}{n}(1+\frac{1}{n})^2-(1+\frac{1}{n})^3} = \lim_{n\to\infty} \frac{-1^3}{0\cdot 1^2-1^3} = 1$$