A Thermodynamic Paradox

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Abstract

The line of reasoning with which people usually justify the second law of thermodynamics does not succeed, or so I shall argue. I will supplement my theoretical criticism by a computer simulation, which verifies that, under the assumptions of said reasoning, a low-entropy state would most likely descent from a high entropy state, contrary to both the second law and to observation.

1 Introduction

I shall present a criticism of the usual reasoning for the second law. I am a bit ambivalent about it, since the criticism feels a bit too straightforward to have been overlooked by generations of researchers. Nevertheless, I cannot see a flaw in it, and I have therefore decided to give it a chance by presenting it here.

2 The nature of the problem

The standard argument for the second law of thermodynamics seems to work as follows:

> (A) Suppose that a physical system is initially in a low entropy state, but like any other system it has a non-zero temperature and therefore a little bit of randomized microscopic motion. If we wait long enough, then there is no reason to regard the individual microstates corresponding to a high entropy macrostate as much less likely, than those which correspond to a low entropy macrostate (so long as they are not ruled out by some constraint, such as energy conservation or an obstacle, which blocks the motion). However, the vast majority of microstates corresponds to a high entropy macrostate, and thus it is essentially certain that the system will end up in a high entropy macrostate.

I've never been fully satisfied by this argument, for it is supposed to show how irreversibility can emerge from an underlying set of reversible fundamental laws. However, if the fundamental laws are reversible, then the following argument should be just as sound as (A).

> (B) Suppose that a physical system ends in a low entropy state, but like any other system it also ends with a non-zero temperature and therefore a little bit of randomized microscopic motion. If we turn our clock back long enough, then there is no reason to regard the individual microstates corresponding to a high entropy macrostate as much less likely, than those which correspond to a low entropy macrostate (so long as they are not ruled out by some constraint, such as energy conservation or an obstacle, which blocks the motion). However, the vast majority of microstates corresponds to a high entropy macrostate, and thus it is essentially certain that the system will descent from a high entropy macrostate.

(B) is obtained from (A) by switching past and future. Consequently, if the fundamental laws cannot distinguish past from future, then they cannot distinguish (B) from (A). Some may object that this line of reasoning is moot, since (A) fits the data, while (B) does not, which ought to close the question. Admittedly, The claim about the data is correct, but I don't think it closes the question, since it seems worthwhile to ask whether the data is actually compatible with the assumption of reversibility in the basic laws.

To test my criticism of the practise of using (A) as an argument for the second law, I have simulated a gas of billiard balls in a two-dimensional box with rigid walls. At $t = 0$ all the balls where located in a corner (see the middle part of figure 1) and each component of each particle's velocity is normally distributed with mean 0 and a constant variance. Momentum and energy conservation together with spherical symmetry of the billiard balls suffices to conclude that:

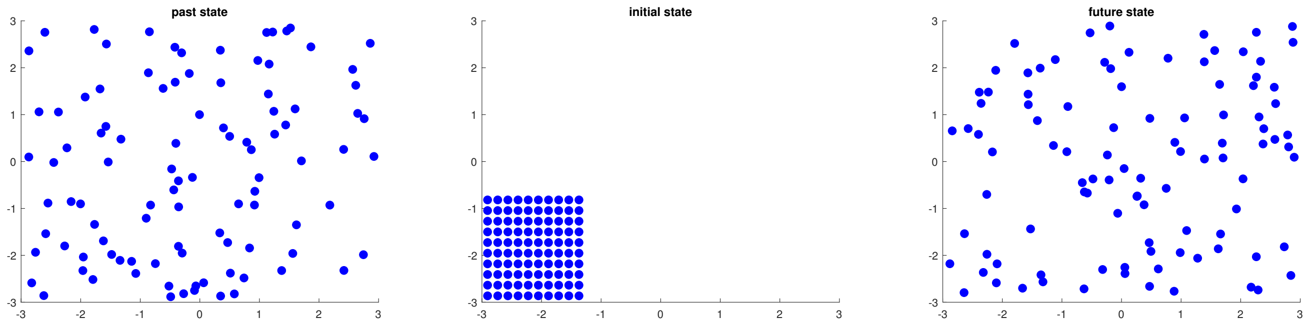

Figure 1: This figure shows the past, present and future (from left to right) of a gas, which is at present in a low entropy state, albeit with a little thermal motion. This figure has been produced by a simulation in the attached Matlab code.

$$\boldsymbol{v}_{a2} = \boldsymbol{v}_{b1} \quad , \text{ when } a \text{ and } b \text{ collide,} \tag{1}$$
$$v_{a2x} = -v_{a1x} \quad , \text{ when } a \text{ hits a wall in the } x \text{ direction. (and similarly for } y\text{).} \tag{2}$$

Where the numbers 1 and 2 refer to the time before and after the collision, while the letters a and b label the particles.
This allows us to propagate the system towards both the future and the past. The results are shown in figure 1 and they can also be seen by running the attached Matlab code. The gas begins in a high entropy state, whereupon it collects itself in a corner and then flies apart again. This result verifies both (A) and (B), since they predict that a low entropy state will evolve into and have evolved from a high entropy state, whenever such states are possible.

This shouldn't be too surprising. After all, equation 1 and 2 implies that the function f, which for every particle a maps the positions and velocities $(\mathbf{x}_{at}, \mathbf{v}_{at})$ at time t into $(\mathbf{x}_{a,t+\mathrm{d}t}, \mathbf{v}_{a,t+\mathrm{d}t})$ is equivalent to the function which maps $(\mathbf{x}_{at}, -\mathbf{v}_{at})$ into $(\mathbf{x}_{a,t-\mathrm{d}t}, -\mathbf{v}_{a,t-\mathrm{d}t})$. Thus we have that:

$$(\mathbf{x}_{at}, \mathbf{v}_{at}) = f^N(\mathbf{x}_{a0}, \mathbf{v}_{a0}) \tag{3}$$
$$(\mathbf{x}_{a-t}, -\mathbf{v}_{a,-t}) = f^N(\mathbf{x}_{a0}, -\mathbf{v}_{a0}) \tag{4}$$

Where $N = \frac{t}{\mathrm{d}t}$ is the number of time-steps. This shows that we can interchange the past and future state of the gas, simply by flipping the direction of the initial velocity. However, the initial state $(\mathbf{x}_{a0}, -\mathbf{v}_{a0})$ is no less realistic than $(\mathbf{x}_{a0}, \mathbf{v}_{a0})$, and consequently the operation of switching past and future, should not render the time evolution any less realistic.
Remember that macroscopic quantities, such as temperature and pressure are uniquely determined by the exact microstate, so one cannot respond: "sure, the microstate $(\mathbf{x}_{a0}, -\mathbf{v}_{a0})$ evolves in a time-reversible fashion, but the macrostate does not". Furthermore, f describes the time-evolution of the entire set of particles, so one cannot respond, "sure, the individual particles evolve in a time-reversible fashion, but the entire collection does not".

If (A) and (B) are both sound and if the basic laws are time-reversible, then any system which is initially in a low entropy state must both evolve into and have evolved from a high entropy state (whenever this state does not violate some constraint, such as a conservation law or the presence of an obstacle). If we apply the logic in (A) and (B) to a system in a high (instead of low) entropy initial state, then we must likewise conclude that the system both evolves into and descents from a high entropy state. Consequently, any system's future and past has high entropy, regardless of the initial state. This can only be consistent for every choice of $t = 0$, if there have never been, nor ever will be, any low entropy states. Consequently, the heat death of the universe began at the Big Bang.

This conclusion is obviously contrary to observation, but if the fundamental laws are reversible, then it is hard to see how it could be otherwise. This is a thermodynamic paradox.

3 Conclusion

I have argued that the standard argument (A) for the second law fails, since it can be no more sound than (B). However, if both are sound, then the conclusion displays no temporal asymmetry.