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Contents

Abstract

In this Project we focus on management in a large project using Latex.In large projects, such as reports, keeping parts of your document in several .tex files makes the task of correcting errors and making further changes easier. It's simpler to locate a specific word or element in a short file. For this purpose this report was created as an example of managing big projects.

In large LATEX documents one usually has several .tex files, one for each chapter or section, and then they are joined together to generate a single output. This helps to keep everything organized and makes easier to debug the document, but as the document gets larger the compilation takes longer. This can be frustrating since one is usually only interested in working in a particular file each time.

You can find labs folder with laboratory works that were done during course.Those are samples of what has been studied and used hereinafter.Such as math,tables,multi-columns, references packages used correspondingly,implemented in chapters within report.

Me as a student get to know LATEX basics and its possibilities as edit tool in organization of scientific and research work.That was good pre-diploma practice and I consider using LATEX over Word.My most favourite part in this tool is mathematics formulas edit.

Chapter 1

Lab1

1. Если с − постоянное число и функция f(x) интегрируема на $[a; b]$, то

$$\int_a^b c \cdot f(x)dx = c \cdot \int_a^b f(x)dx \tag{2.4}$$

то есть постоянный множитель с можно выносить за знак определенного интеграла.
2. Если функции $f_1(x)$ и $f_2(x)$ интегрируемы на $[a; b]$,тогда интегрируема на [a;b] их алгебраическая сумма и

$$\int_a^b (f_1(x) \pm f_2(x))dx = \int_a^b (f_1(x) \pm \int_a^b f_2(x))dx \tag{2.5}$$

то есть интеграл от алгебраической суммы равен алгебраической сумме интегралов. Это свойство распространяется на сумму любого конечного числа слагаемых.
3. При перестановке пределов интегрирования знак интеграла изменяется на противоположный:

$$\int_a^b f(x)dx = -\int_a^b f(x)dx. \tag{2.6}$$

Chapter 2

Lab2

Section 2.2 Continuity 67

Applied to Example 2.2.16, Corollary 2.2.13 implies that the function $g(x) = cos1\backslash x$ is uniformly continuous on $[\rho, 1]$ if $0 < \rho < 1$.

More About Monotonic Functions

Theorem 1. *implies that if f is monotonic on an interval I , then f is either continuous or has a jump discontinuity at each x0 in I . This and 2 provide the key to the proof of the following theorem.*

Theorem 2. *If f is monotonic and non constant on* $[a, b]$ *, then f is continuous on* $[a, b]$ *if and only if its range* $\tilde{R}_f = \{f(x) \parallel x \in (a, b)\}$ *is the closed interval with endpoints* $f(a)$ *and* $f(b)$*.*

Proof. We assume that f is non decreasing, and leave the case where f is non increasing to you (Exercise 34). 2 implies that the set$\tilde{R}_f$
is a subset of the open interval $(f(a+), f(b-))$. Therefore,

$$R_f = \{f(a)\} \cup \tilde{R}_f \cup \{f(b)\} \subset \{f(a)\} \cup \{f(a+)\}, \{f(b-)\} \cup \{f(b)\} \tag{15}$$

□

Now suppose that f is continuous on $[a, b]$. Then $f(a) = f(a+), f(b-) = f(b)$, so (2) implies that $R_f \subset [f(a), f(b)]. If f(a) < \mu < f(b)$, then 7 implies that
$\mu = f(x) for some x in (a, b). Hence, R_f = [f(a), f(b)]$.

For the converse, suppose that $R_f = [f(a), f(b)]$. Since $f(a) \leq f(a+) and f(b-) \leq f(b)$, (2) implies that $f(a) = f(a+) and f(b-)$. We know from 2 that if f is non-decreasing and
$a < x_0 < b$,then

$$f(x_0-) \leq f(x_0) \leq f(x_0-).$$

If either of these inequalities is strict, Rf cannot be an interval. Since this contradicts our assumption. $f(x_0-) = f(x_0) = f(x_0+_{)}$.Therefore,f is continuous at x_0(Exercise(2). We can now conclude that f is continuous on $[a, b]$.

2 implies the following theorem

1 Suppose that f is increasing and continuous on $[a, b]$ and let $f(a) = c$ and $f(b) = d$ Then there is a unique function g defined on $[c, d]$ such that

$$g(f(x)) = x, a \leq x \leq b, \tag{16}$$

and

$$f(g(y) = y,\ \ c \leq y \leq d. \tag{17}$$

Moreover, g is continuous and increasing on $[c, d]$.

Proof. We first show that there is a function g satisfying (2) and (2). Since f is continuous, Theorem 2.2.14 implies that for each y_0 in $[c, d]$ there is an x_0 in $[a, b]$ such that

□

$$f(x_0) = y(0), \tag{18}$$

Chapter 3

Lab3

Section 2.2 Continuity 67

Applied to Example 2.2.16, Corollary 2.2.13 implies that the function $g(x) = cos1\backslash x$ is uniformly continuous on $[\rho, 1]$ if $0 < \rho < 1$.

More About Monotonic Functions

Theorem 3. *implies that if f is monotonic on an interval I , then f is either continuous or has a jump discontinuity at each x0 in I . This and 2 provide the key to the proof of the following theorem.*

Theorem 4. *If f is monotonic and non constant on* $[ab]$ *, then f is continuous on* $[a, b]$ *if and only if its range* $\tilde{R}_f = \{f(x) \parallel x \in (a, b)\}$ *is the closed interval with endpoints* $f(a)$ *and* $f(b)$*.*

Proof. We assume that f is non decreasing, and leave the case where f is non increasing to you (Exercise 34). 3 implies that the set$\tilde{R}_f$ is a subset of the open interval $(f(a+), f(b-))$. Therefore, □

$$R_f = \{f(a)\} \cup \tilde{R}_f \cup \{f(b)\} \subset \{f(a)\} \cup \{f(a+) \}, \{f(b-)\} \cup \{f(b)\} \tag{3.1}$$

Now suppose that f is continuous on $[a, b]$. Then $f(a) = f(a+), f(b-) = f(b)$, so (3.1) implies that$R_f \subset [f(a), f(b)].If f(a) < \mu < f(b)$, then 4 implies that $\mu = f(x) for some x in (a, b). Hence, R_f = [f(a), f(b)]$.

For the converse, suppose that $R_f = [f(a), f(b)]$. Since $f(a) \leq f(a+) and f(b-) \leq f(b)$, 3.1 implies that $f(a) = f(a+) and f(b-)$. We know from 7 that if f is non-decreasing and $a < x_0 < b$,then

$$f(x_0-) \leq f(x_0) \leq f(x_0-).$$

If either of these inequalities is strict, Rf cannot be an interval. Since this contradicts our assumption.$f(x_0-) = f(x_0) = f(x_0+_{)}$.Therefore,f is continuous at x_0(Exercise(2). We can now conclude that f is continuous on $[a, b]$.

4 implies the following theorem

Theorem 5. *Suppose that f is increasing and continuous on* $[a, b]$ *and let* $f(a) = c$ *and* $f(b) = d$ *Then there is a unique function g defined on* $[c, d]$ *such that*

$$g(f(x)) = x, a \leq x \leq b, \tag{3.2}$$

and

$$f(g(y) = y, \ c \leq y \leq d. \tag{3.3}$$

Moreover, g is continuous and increasing on $[c, d]$.

Proof. We first show that there is a function g satisfying (3.2) and (3.3). Since f is continuous, 5 implies that for each y_0 in $[c, d]$ there is an x_0 in $[a, b]$ such that □

$$f(x_0) = y(0), \tag{3.4}$$

Chapter 4

Lab4

Next, let us turn to 2×2 matrices, of the form

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

We shall use elementary row operations to find out when the matrix A is invertible. So we consider the array

$$(A|I2) = \begin{pmatrix} a & b & 1 & 0 \\ c & d & 0 & 1 \end{pmatrix} \tag{1}$$

and try to use elementary row operations to reduce the left hand half of the array to $I2$. Suppose first of all that $a = c = 0$. Then the array becomes

$$\begin{pmatrix} 0 & b & 1 & 0 \\ c & d & 0 & 1 \end{pmatrix}$$

and so it is impossible to reduce the left hand half of the array by elementary row operations to the matrix I_2. Consider next the case $a \neq 0$. Multiplying row 2 of the array (4) by a, we obtain

$$\begin{pmatrix} a & b & 1 & 0 \\ ac & ad & 0 & a \end{pmatrix}$$

Adding $-c$ times row 1 to row 2, we obtain

$$\begin{pmatrix} a & b & 1 & 0 \\ 0 & ad - bc & -c & a \end{pmatrix} \tag{2}$$

If $D = ad - bc = 0$, then this becomes

$$\begin{pmatrix} a & b & 1 & 0 \\ 0 & 0 & -c & a \end{pmatrix}$$

and so it is impossible to reduce the left hand half of the array by elementary row operations to the matrix I_2. On the other hand, if $D = ad - bc \neq 0$, then the array (4) can be reduced by elementary row operations to

$$\begin{pmatrix} 1 & 0 & d/D & -b/D \\ 0 & 1 & -c/D & a/D \end{pmatrix}$$

so that

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Consider finally the case $c \neq 0$. Interchanging rows 1 and 2 of the array (4), we obtain

$$\begin{pmatrix} c & d & 0 & 1 \\ ac & bc & c & 0 \end{pmatrix}$$

Adding $-a$ times row 1 to row 2, we obtain

$$\begin{pmatrix} c & d & 0 & 1 \\ 0 & bc - ad & c & -a \end{pmatrix}$$

Chapter 5

Lab5

146 Chapter 3 Integral Calculus of Functions of One Variable
is an antiderivative of $f(\phi(t)\phi'(t)$ on $[c, d]$ and 7 implies that

$$\int_c^d f(\phi(t))\phi' t(t)dt = G(d) = F(\phi(d)) - F(\phi(c)) \tag{5.1}$$

$$= F(\beta) - F(\alpha).$$

Comparing this with (22) yields (21).
Example 3.3.2 To evaluate the integral

$$I = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} (1 - 2x^2)(1 - 2x^2)^{-1/2} dx$$

we let

$$f(x) = (1 - 2x^2)(1 - 2x^2)^{-1/2}, -1/\sqrt{2} \le x \le 1/\sqrt{2},$$

and

$$x = \phi(t) = sin(t), -\pi/4 \le t \le \pi/4.$$

Then $phi(t) = cos\backslash t$ and

$$\begin{cases} I = \int_{-1\sqrt{2}}^{1\sqrt{2}} f(x)dx = \int_{-\pi/4}^{\pi/4} f(sint)cos\ t\ dt \\ = \int_{-\pi/4}^{\pi/4} (1 - 2sin^2 t)(1 - 2sin^2 t)^{-1/2} cos\ t\ dt. \\ (1 - 2sin^2 t)^{1/2} = \\ cost, -\pi/4 \le t \le \pi/4 \end{cases}$$

and

$$1 - 2sin^2 t = cos2t,$$

(23) yields

$$I = \int_{-\pi/4}^{\pi/4} xcos2\ tdt = \frac{sin2t}{2}\bigg|_{-\pi/4}^{\pi/4} = 1.$$

Example 3.3.3 To evaluate the integral

$$I = \int_0^{5\pi} \frac{sin\ t}{2 + cos\ t}$$

we take $\phi(t) = cost$. Then $\phi'(t) = -sin\ t$

$$I = -\int_0^{5\pi} \frac{\phi'(t)}{2 + \phi(t)} dt = -\int_0^{5\pi} f(\phi(t))\phi'(t)dt,$$

where

$$f(x) = \frac{1}{2 + x}.$$

Proof. The existence and uniqueness of $\bar{s}$ and $\underline{s}$ follow from 6 and 2 If $\bar{s}$ and $\underline{s}$ are both finite, then 3.2 and 3.4 imply that □

$$\underline{s} - \epsilon < \bar{s} + \epsilon$$

for every $\epsilon > 0$, which implies (21).If $\underline{s} = -\infty$ or $\bar{s} = \infty$, then(21) is obvious.If $\underline{s} = \infty$ or $\bar{s} = \infty$, then (21) follows immediately from Definition 4.1.10.

Example 4.1.13

$$\overline{\lim_{n\to\infty}} r^n = \begin{cases} \infty, & |r| > 1, \\ 1, & |r| = 1, \\ 0, & |r| < 1; \end{cases}$$

and

$$\underline{\lim_{n\to\infty}} r^n = \begin{cases} \infty, & r > 1, \\ 1, & r = 1, \\ 0, & |r| < 1, \\ -1, & r = -1, \\ -\infty, & r < -1. \end{cases}$$

Also,

$$\overline{\lim_{n\to\infty}} n^2 = \underline{\lim_{n\to\infty}} n^2 = \infty,$$

$$\overline{\lim_{n\to\infty}} (-1)^n (1 - \frac{1}{n}) = 1, \underline{\lim_{n\to\infty}} (-1)^n (n - \frac{1}{n}) = -1,$$

and

$$\overline{\lim_{n\to\infty}} [1 + (-1)^n] n^2 = \infty, \underline{\lim_{n\to\infty}} [1 + (-1)^n] n^2 = 0.$$

Theorem 6. *If $\{s_n\}$ is a sequence of real numbers ,then*

$$\lim_{n\to\infty} s_n = s \tag{22}$$

if and only if

$$\overline{\lim_{n\to\infty}} s_n = \underline{\lim_{n\to\infty}} s_n = s. \tag{23}$$

Proof. If $s = \pm\infty$, the equivalence of (5) and (5) follows immediately from their definitions. If $\lim_{n\to\infty} S_n = s$ (finite), then Definition 4.1.1 implies that (16)–(19) hold with $\overline{s}$ and *underlines* replaced by s. Hence, (5) follows from the uniqueness of $\overline{s}$ and *underlines*. For the converse, suppose that s D s and let s denote their common value. Then 3.2 and 3.4 imply that □

$$s - \epsilon < s_n < s + \epsilon$$

for large n, and (22) follows from Definition 4.1.1 and the uniqueness of $lim_{n\to\infty} s_n$ 7.

Chapter 6

Lab6

Chapter 2 Differential Calculus of Functions of One Variable
The definitions of

$$f(x_0-) = \lim_{x \to x_0-} f(x), f(x_0+) = \lim_{x \to x_0+} f(x), and \lim_{x \to x_0} f(x)$$

do not involve $f(x_0)$ or even require that it be defined. However, the case where $f(x_0)$ is defined and equal to one or more of these quantities is important

Definition 1. *a. We say that f is continuous at x_0 if f is defined on an open interval(a, b) containing x_0 and $\lim_{x \to x_0} f(x) = f(x_0)$.*

b. We say that f is continuous from the left at x_0 if f is defined on an open interval $(a, x0)$ and $f(x_0-) = f(x_0)$.

c. We say that f is continuous from the right at x_0 if f is defined on an open interval $(x0, b)$ and $f(x_0+) = f(x_0)$.

The following theorem provides a method for determining whether these definitions are satisfied. The proof, which we leave to you (Exercise 1), rests on 2 1

Theorem 7. *a. A function f is continuous at x_0 if and only if f is defined on an open interval (a, b) containing x_0 and for each $\epsilon > 0$ there is a $\delta > 0$ such that*

$$|f(x) - f(x_0)| < \epsilon \tag{6.1}$$

whenever $\mid x - x_0 \mid < \delta$

b. A function f is continuous from the right at x_0 if and only if f is defined on an interval $(x0, b)$ and for each $\epsilon > 0$ there is a$\delta > 0$ such that 6.1 holds whenever $x0 \leq x < x0 + \delta$

c. A function f is continuous from the left at x_0 if and only if f is defined on an interval $[x_0, b)$ and for each $\epsilon > 0$ there is a $\delta > 0$ such that 6.1 holds whenever $x_0 - \delta < x \leq x_0$.

From 2 and 7, f is continuous at x_0 if and only if

$$f(x_0-) = f(x_0+) = f(x_0)$$

or, equivalently, if and only if it is continuous from the right and left at x_0 (Exercise 2).

Example 2.2.1 Let f be defined on $[0, 2]$

$$f(x) = \begin{cases} x^2 & 0 \leq x < 1, \\ x + 1 & 1 \leq x \leq 2 \end{cases}$$

then

$$\begin{aligned} f(0+) &= 0 = f(0), \\ f(1-) &= 1 \neq f(1) = 2, \\ f(1+) &= 2 = f(1), \\ f(2-) &= 3 = f(2). \end{aligned}$$

Therefore, f is continuous from the right at 0 and 1 and continuous from the left at 2, but not at 1. If$0 < x, x0 < 1$, then

$$\begin{aligned} |f(x) = f(x_0)| = |x^2 - x_0^2| &= |x - x_0||x + x_0| \\ &\leq 2|x - x_0| < \epsilon if |x - x_0| < \epsilon/2. \end{aligned}$$

Hence, f is continuous at each x_0 in $(0, 1)$. If $1 < x, x_0 < 2$ then

$$\begin{aligned} |f(x) - f(x_0)| = |(x+1) - (x_0+1) &= |x + x_0| \\ &< \epsilon \quad if \quad |x - x_0| < \epsilon. \end{aligned}$$

Hence, f is continuous at each x_0 in $(1, 2)$

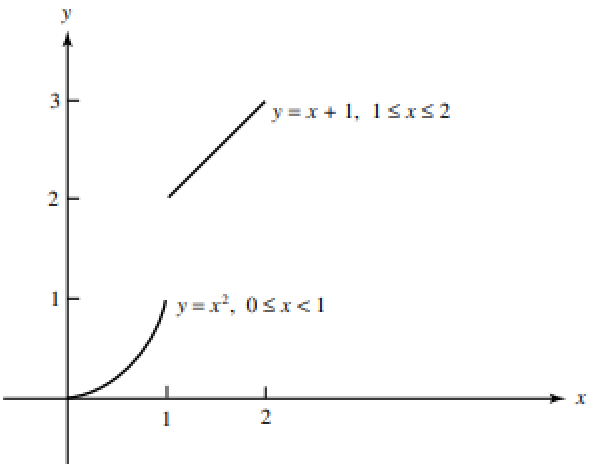

Figure 2.2.1

Definition 2. *A function f is continuous on an open interval (a, b) if it is continuous at every point in (a, b) If, in addition,*

$$f(b-) = f(b) \tag{6.2}$$

or

$$f(a+) = f(a) \tag{6.3}$$

Chapter 7

Lab7

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Traumatic brain injury (TBI) is a critical public health and socioeconomic problem throughout the world. Cognitive rehabilitation (CR) has become the treatment of choice for cognitive impairments after TBI. It consists of hierarchically organized tasks that require repetitive use of impaired cognitive functions. One important focus for CR professionals is the number of repetitions and the type of task performed throughout treatment leading to functional recovery. However, very little research is available that quantifies the amount and type of practice. The Neurorehabilitation Range (NRR) and the Sectorized and Annotated Plane (SAP) have been introduced as a means of identifying formal operational models in order to provide therapists with decision support information for assigning the most appropriate CR plan. In this paper we present a novel methodology based on combining SAP and NRR to solve what we call the Neurorehabilitation Range Maximal Regions (NRRMR) problem and to generate analytical and visual tools enabling the automatic identification of NRR. A new SAP representation is introduced and applied to overcome the drawbacks identified with existing methods. The results obtained show patterns of response to treatment that might lead to reconsideration of some of the current clinical hypotheses.

7.1 Introduction

Traumatic brain injury (TBI) is a critical public health and socioeconomic problem throughout the world. Although high-quality prevalence data are scarce, it is estimated that in the USA around 5.3 million people are living with a TBIrelated disability, and in the European Union approximately 7.7 million people who have experienced a TBI have disabilities [1]. TBI is considered a silent epidemic, because society is largely unaware of the magnitude of the problem [2]. The World Health Organization predicts that, by the year 2020, TBI and road traffic accidents will be the third greatest cause of disease and injury worldwide [3].

The consequences of TBI vary from case to case but can include motor, cognitive, and behavioral deficits in the patient, disrupting their daily life activities at personal, social, and professional levels. The most important cognitive deficits after suffering a TBI are those related to attention, decrease in memory and learning capacity, worsening of the capacity to schedule and to solve problems, a reduction in abstract thinking, communication problems, and a lack of awareness of one's own limitations. These cognitive impairments hamper the path to functional independence and a productive lifestyle for the person with TBI. New techniques of early intervention and the development of intensive TBI care have improved the survival rate noticeably. However, despite these advances, brain injuries

still have no surgical or pharmacological treatment to reestablish lost functions [4]. In this context, cognitive rehabilitation (CR) is defined as a process whereby people with brain injury work together with health service professionals and others to remedy or alleviate cognitive deficits arising from a neurological injury [5].

The structure of the paper is as follows: 7.2 briefly presents the state of the art and the starting point of the proposal. 7.3 introduces the proposed analysis methodology and 7.4 its application to the CR context; 7.5 presents a discussion of the obtained results and a comparison with the previous and 7.5 the conclusions and future lines of research.

The process by which neuronal circuits are modified by experience, learning, or injury is referred to as neuroplasticity [7]. While task repetition is not the only important feature, it is becoming clear that neuroplastic change and functional improvement occur after a number of specific tasks are performed but do not occur with other numbers [8, 9] Thus, one important focus for rehabilitation professionals is the number of repetitions and the type of task performed during treatment. However, there is very little research to quantify the amount and type of practice that occurs during clinical rehabilitation treatment and its relationship to rehabilitation outcomes [10, 11].

7.2 State of the Art

There is a common belief that CR is effective for TBI patients, based on a large number of studies and extensive clinical experience. Different statistical methodologies and predictive data mining methods have been applied to predict clinical outcomes of the rehabilitation of patients with TBI [14, 16] Most of these studies focus on determining survival, predicting disability or the recovery of patients, and looking for the factors that better predict the patient's condition after TBI

7.3 Materials and Methods

The proposed methods present two strategies for the analytical and graphical identification and visualization of NRR and non-NRR based on the notion of SAP as introduced in [12] and on the classical MER problem, respectively.

3.1.Sectorized and Annotated Plane (SAP). Given three variables y, x_1, and x_2, where y is a qualitative response variable, with values $\{y_1, y_2, ...\}$, and x_1, y_2 numerical explanatory variables, the SAP is a

2-dimensional plot with x_1 in the axis, $x2$ in the -axis and rectangular regions with constant y displayed and labeled with y values as outlined in Figure 1. An SAP is therefore a graphical support tool aimed at visualization, where the response variable is constant in certain regions of the x_1\times$y_2$ space. Eventually, allowing a relaxation of strict constant in the marked regions, the SAP might include an indicator of region purity, adding the probability of occurrence of the labeling value.

7.4 Application and Results

4.1 Clinical Context. This work is based on the same context as in [12], the Neuropsychological Department of Institut Guttmann Neurorehabilitation Hospital (IG). The Information Technology framework for CR treatments in this clinical setting is therefore the PREVIRNEC ©platform [31]. It is specifically designed to operate CR plans assigned to subjects, as well as to manage precise follow-up information about the process.

7.5 Discussion

This work aims to identify the conditions in which performing a certain cognitive rehabilitation task (or a group of tasks) guarantees better potential for the activation of brain plasticity and therefore helps bring about improvements in the assessed cognitive functions after CR treatment. As this research takes our previous research as a starting point, the results comparison is provided below and the pros and cons are discussed

Conclusions and Future Work

This work builds on our previous contribution towards the design, implementation, and execution of personalized, predictable, and data-driven CR programs. We wish to identify NRR for cognitive rehabilitation tasks that lead to patient improvement.

Conflict of Interests

No competing financial interests exist

7.6 Acknowledgments

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Chapter 8

Lab8

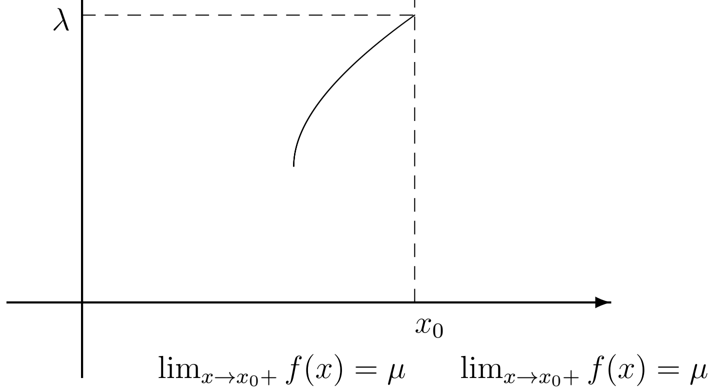

Figure 8.1: Figure 2.1.2

satisfies the inequality

$$|f(x)| < \epsilon$$

if $0 < x < \delta = \epsilon/2$. However, this does not mean that $\lim_{x\to 0} f(x) = 0$ since f is not defined for negative x, as it must be to satisfy the conditions of Definition 2.1.2 with $x_0 = 0$ and $L = 0$. The function

$$g(x) = x + \frac{|x|}{x}, \neq 0,$$

can be rewritten as

$$g(x)x = \begin{cases} x+1 & x > 0 \\ x-1 & x < 0 \end{cases}$$

hence, every open interval containing $x_0 = 0$ also contains points x1 and x2 such that $|g(x_1) - g(x_2)|$ is as close to 2 as we please. Therefore, $\lim_{x\to x_0} g(x)$ does not exist (Exercise 26)

Although $f(x)$ and $g(x)$ do not approach limits as x approaches zero, they each exhibit a definite sort of limiting behavior for small positive values of x, as does $g(x)$ for small negative values of x. The kind of behavior we have in mind is defined precisely as follows.

Definition 2.1.2

We say that $f(x)$ approaches the left-hand limit L as x approaches x_0 from the left, and write

$$\lim_{x\to x_0} f(x) = L.$$

if f is defined on some open interval (a, x_0) and, for each $\epsilon > 0$, there is a $\delta > 0$ such that

$$|f(x) - L| < \epsilon \text{ if } x_0 - \delta < x < x_0.$$

Chapter 9

Lab9

Listening

Questions 17-20

Complete the form below.

Write **ONE WORD ONLY** for each answer.

ITINERARY	
Day 1	arrive in Kishba
Day 2	rest day
Day 3	spend all day in a **17** ....
Day 4	visit a school
Day 5	rest day
Day 6	see a **18** .... with old carvings
Day 7	rest day
Day 8	swim in a **19** ....
Day 9	visit a **20** ....
Day 10	depart from Kishba

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Bibliography

[1] B. Roozenbeek, A. I. R. Maas, and D. K. Menon, "Changing patterns in the epidemiology of traumatic brain injury," Nature Reviews Neurology, vol. 9, no. 4, pp. 231–236, 2013.

[2] J. A. Langlois and R. W. Sattin, "Traumatic brain injury in the United States: research and programs of the Centers for Disease Control and Prevention (CDC)," Journal of Head Trauma Rehabilitation, vol. 20, no. 3, pp. 187–188, 2005.

[3] S. A. Tabish and N. Syed, "Traumatic brain injury: the neglected epidemic of modern society," International Journal of Science and Research, vol. 3, no. 12, 2014

[4] D. T. Stuss, G. Winocur, and I. H. Robertson, Cognitive NeuroRehabilitation: Evidence and Application, Cambridge University Press, Cambridge, UK, 2nd edition, 2008.

[5] B. A. Wilson, "La readaption cognitive chez les c ´ er´ ebro-l ´ es´ es," in ´ Neuropsychologie Clinique et Neurologie du Comportement, M. I. Botez, Ed., pp. 637–652, Les Presses de l'Universite de Montreal, ´ Montreal, Canada, 2nd edition, 1996.

[6] M. M. Sohlberg, Cognitive Rehabilitation. An interactive Neuropsychological Approach, 2001, edited by: C. A. Mateer.

[7] R. J. Nudo, "Adaptive plasticity in motor cortex: implications for rehabilitation after brain injury," Journal of Rehabilitation Medicine, no. 41, supplement, pp. 7–10, 2003.

[8] J. R. Carey, W. K. Durfee, E. Bhatt et al., "Comparison of finger tracking versus simple movement training via telerehabilitation to alter hand function and cortical reorganization after stroke," Neurorehabilitation and Neural Repair, vol. 21, no. 3, pp. 216– 232, 2007.

[9] S. L. Wolf, C. J. Winstein, J. P. Miller et al., "Effect of constraint induced movement therapy on upper extremity function 3 to 9 months after stroke: the EXCITE randomized clinical trial," Journal of the American Medical Association, vol. 296, no. 17, pp. 2095–2104, 2006.

[10] C. K. English, S. L. Hillier, K. R. Stiller, and A. WardenFlood, "Circuit class therapy versus individual physiotherapy sessions during inpatient stroke rehabilitation: a controlled trial," Archives of Physical Medicine and Rehabilitation, vol. 88, no. 8, pp. 955–963, 2007.

[11] S. Kuys, S. Brauer, and L. Ada, "Routine physiotherapy does not induce a cardio respiratory training effect post-stroke, regardless of walking ability," Physiotherapy Research International, vol. 11, no. 4, pp. 219–227, 2006.

[12] A. Garc´ıa-Rudolph and K. Gibert, "A data mining approach to identify cognitive NeuroRehabilitation Range in Traumatic Brain Injury patients," Expert Systems with Applications, vol. 41, no. 11, pp. 5238–5251, 2014.

[13] A. Naamad, D. T. Lee, and W.-L. Hsu, "On the maximum empty rectangle problem," Discrete Applied Mathematics, vol. 8, no. 3, pp. 267–277, 1984.

[14] A. I. Rughani, T. S. M. Dumont, Z. Lu et al., "Use of an artificial neural network to predict head injury outcome," Journal of Neurosurgery, vol. 113, no. 3, pp. 585–590, 2010

[15] S.-Y. Ji, R. Smith, T. Huynh, and K. Najarian, "A comparative analysis of multi-level computer-assisted decision making systems for traumatic injuries," BMC Medical Informatics and Decision Making, vol. 9, no. 1, article 2, 2009.

[16] B. C. Pang, V. Kuralmani, R. Joshi et al., "Hybrid outcome prediction model for severe traumatic brain injury," Journal of Neurotrauma, vol. 24, no. 1, pp. 136–146, 2007

[17] M. L. Rohling, M. E. Faust, B. Beverly, and G. Demakis, "Effectiveness of cognitive rehabilitation following acquired brain injury: a meta-analytic re-examination of cicerone et al.'s (2000, 2005) systematic reviews," Neuropsychology, vol. 23, no. 1, pp. 20–39, 2009

[18] J. Whyte and T. Hart, "It's more than a black box; it's a Russian doll: defining rehabilitation treatments," American Journal of Physical Medicine Rehabilitation, vol. 82, no. 8, pp. 639–652, 2003.

[19] K. D. Cicerone, D. M. Langenbahn, C. Braden et al., "Evidencebased cognitive rehabilitation: updated review of the literature from 2003 through 2008," Archives of Physical Medicine and Rehabilitation, vol. 92, no. 4, pp. 519–530, 2011.

[20] K. Gibert and A. Garc´ıa-Rudolph, "Desarrollo de herramientas para evaluar el resultado de las tecnolog´ıas aplicadas al proceso rehabilitador Estudio a partir de dos modelos concretos: Lesion Medular y Da ´ no Cerebral Adquirido," in ˜ Posibilidades de Aplicacion de Miner ´ ´ıa de Datos para el Descubrimiento de Conocimiento a Partir de la Practica Cl ´ ´ınica, Informes de Evaluacion de Tecnologias Sanitarias, AATRM n ´ um. 2006/11, Cap 6, Plan Nacional para el Sistema Nacional de Salud del Ministerio de Sanidad y Consumo, Madrid, Spain; Agencia ' d'Avaluacio de Tecnologia I Recerca M ´ ediques, Barcelona, ' Spain, 2007

[21] J. Serra, J. L. Arcos, A. Garcia-Rudolph, A. Garc´ıa-Molina, T. Roig, and J. M. Tormos, "Cognitive prognosis of acquired brain injury patients using machine learning techniques," in Proceedings of the International Conference on Advanced Cognitive Technologies and Applications (COGNITIVE '13), pp. 108–113, IARIA, Valencia, Spain, 2013

[22] A. Marcano-Cedeno, P. Chausa, A. Garc ˜ ´ıa, C. Caceres, J. M. ´ Tormos, and E. J. Gomez, "Data mining applied to the cognitive ´ rehabilitation of patients with acquired brain injury," Expert Systems with Applications, vol. 40, no. 4, pp. 1054–1060, 2013.

[23] V. Jagaroo, Neuroinformatics for Neuropsychologists, Springer, 1st edition, 2009.

[24] A. Dumitrescu and M. Jiang, "On the largest empty axis-parallel box amidst points," Algorithmica, vol. 66, no. 2, pp. 225–248, 2013

[25] B. Chazelle, R. L. Drysdale, and D. T. Lee, “Computing the largest empty rectangle,” SIAM Journal on Computing, vol. 15, no. 1, pp. 300–315, 1986.

[26] A. Aggarwal and S. Suri, “Fast algorithms for computing the largest empty rectangle,” in Proceedings of the 3rd Annual Symposium on Computational Geometry, pp. 278–290, Waterloo, Canada, June 1987

[27] M. McKenna, J. O’Rourke, and S. Suri, “Finding the largest rectangle in an orthogonal polygon,” in Proceedings of the 23rd Annual Allerton Conference on Communication, Control and Computing, pp. 486–495, Urbana Champaign, Ill, USA, October 1985.

[28] H. S. Baird, S. E. Jones, and S. J. Fortune, “Image segmentation by shape-directed covers,” in Proceedings of the 10th International Conference on Pattern Recognition, vol. 1, pp. 820–825, June 1990.

[29] J. Edmonds, J. Gryz, D. Liang, and R. J. Miller, “Mining for empty spaces in large data sets,” Theoretical Computer Science, vol. 296, no. 3, pp. 435–452, 2003.

[30] J. Augustine, S. Das, A. Maheshwari, S. C. Nandy, S. Roy, and S. Sarvattomananda, “Recognizing the largest empty circle and axis-parallel rectangle in a desired location,” CoRR, abs/1004.0558v2, 2010.

[31] J. M. Tormos, A. Garcia-Molina, A. Garcia Rudolph, and T. Roig, “Information and communications technology in learning development and rehabilitation,” International Journal of Integrated Care, vol. 9, 2009.