

# A QUICK REVIEW OF BASIC PROBABILITY FORMULAS - M. Hopkins

- ① Suppose we have a  $5 \times 5$  cabinet of drawers, each of which contains a single object:

$\triangle$	$\circ$	$\bullet$	$\triangle$	$\square$
$\blacksquare$	$\square$	$\triangle$	$\square$	$\triangle$
$\bullet$	$\square$	$\bullet$	$\triangle$	$\square$
$\blacktriangle$	$\triangle$	$\square$	$\triangle$	$\blacktriangle$
$\triangle$	$\triangle$	$\blacksquare$	$\triangle$	$\circ$

The drawers are closed and we don't know what's inside them.

- ② Suppose we now open a drawer. Let  $C$  represent the color of the object we find. Let  $S$  be the shape of the object we find. It is easy to express the joint probability  $P(C, S)$  as a table:

$C$	$S$	$P(C, S)$
$b$	$\triangle$	$\frac{2}{25} = .08$
$b$	$\circ$	$\frac{3}{25} = .12$
$b$	$\square$	$\frac{2}{25} = .08$
$w$	$\triangle$	$\frac{10}{25} = .40$
$w$	$\circ$	$\frac{2}{25} = .08$
$w$	$\square$	$\frac{6}{25} = .24$

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- ③ The law of total probability allows us to express  $P(C)$  in terms of the joint probability  $P(C, S)$ :

$$P(C) = P(C, S=\Delta) + P(C, S=O) + P(C, S=\square)$$

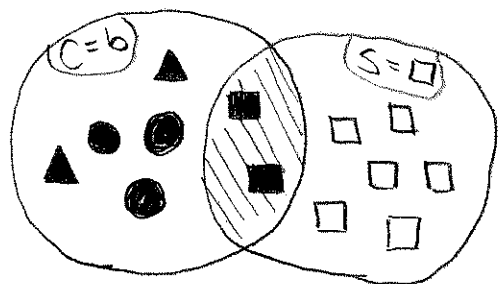
In other words, we can compute the probability that the object's color is white as a case analysis.

<u>C</u>	<u>S</u>	<u><math>P(C, S)</math></u>	<u><math>P(C)</math></u>
b	$\Delta$	.08	} = .28
b	O	.12	
b	$\square$	.08	
w	$\Delta$	.40	} = .72
w	O	.08	
w	$\square$	.24	

- ④ We define the conditional probability  $P(C|S)$  as:

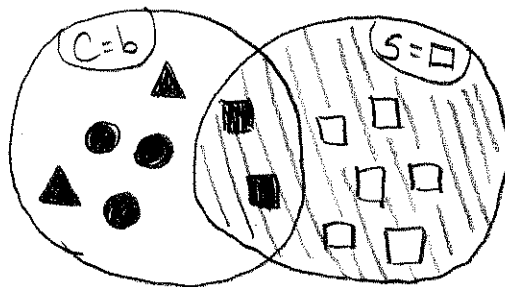
$$P(C|S) \triangleq \frac{P(C, S)}{P(S)}$$

e.g.  $P(C=b|S=\square) =$



$$P(C=b, S=\square)$$

$\div$



$$P(S=\square)$$

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⑤ From the definition of conditional probability, we see:

$$P(C|S) = \frac{P(C,S)}{P(S)} \Rightarrow P(C,S) = P(C|S)P(S)$$

$$P(S|C) = \frac{P(S,C)}{P(C)} \Rightarrow P(S,C) = P(S|C)P(C)$$

Because  $P(C,S) = P(S,C)$ , this means that

$$P(C|S)P(S) = P(S|C)P(C)$$

A simple rearrangement gives us Bayes rule:

$$P(C|S) = \frac{P(S|C)P(C)}{P(S)}$$