CSCI 378 HW7

Suppose we define a probability distribution as any function h whose range is negative and for which

(a) $\int_{-\infty}^{\infty} h(x)\,dx = 1$ if $\text{dom}(X) \neq \mathbb{R}$

(b) $\sum_{x \in \text{dom}(X)} h(x) = 1$ if $\text{dom}(X)$ is discrete

One strategy we discussed to "create" a distribution is to "normalize" a function g whose range is the nonnegative reals. To normalize, we divide $g(x)$ by some constant K such that $h(x) = \frac{g(x)}{K}$ and

one of the above conditions (a) or (b) hold.

Which of the following functions can be normalized into probability distributions using this technique? What is the equation for the resulting distribution h?

(i) $g(x) = \frac{1}{2^x}$ for $x \in \{0, 1, 2, \ldots\}$ ← i.e. the set of non-negative integers

(ii)

(ii) $g(x) = \frac{1}{x}$ for $x \in \{1, 2, \ldots\}$

(iii) $g(x) = \frac{\pi^2}{x^2}$ for $x \in \{1, 2, \ldots\}$ [hint: look up the "Basel problem"]