

(a) Prove the following theorem (by contradiction):

Theorem 1: For functions  $f: \mathbb{R} \rightarrow (0, \infty)$

$g: (0, \infty) \rightarrow \mathbb{R}$ ,

if  $g$  is monotonically increasing, then:

$$\operatorname{argmin}_x f(x) = \operatorname{argmin}_x g(f(x))$$

(Note: a function  $g$  is monotonically increasing iff  $y_1 > y_2 \Leftrightarrow g(y_1) > g(y_2)$  for all  $y \in \operatorname{dom}(g)$ )

(b) Prove the following corollary of Thm 1:

Corollary: For function  $f: \mathbb{R} \rightarrow (0, \infty)$ :

$$\operatorname{argmin}_x f(x) = \operatorname{argmin}_x \log f(x)$$