Homework #3

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1 Number 1: Exercise 5.1.2

Show leftmost and rightmost derivations given the grammar:

$$\begin{aligned} S &\rightarrow A1B \\ A &\rightarrow 0A|\varepsilon \\ B &\rightarrow 0B|1B|\varepsilon \end{aligned} \tag{1}$$

a. 00101 found in Table 1

b. 1001 found in Table 2

c. 00011 found in Table 3

Leftmost	Rightmost
S	S
$A1B$	$A1B$
$0A1B$	$A10B$
$00A1B$	$A101B$
$00\varepsilon 1B$	$A101\varepsilon$
$0010B$	$0A101$
$00101B$	$00A101$
00101ε	$00\varepsilon 101$
00101	00101

Table 1: Right and leftmost expansions for 00101

Leftmost	Rightmost
S	S
$A1B$	$A1B$
$\varepsilon 1B$	$A10B$
$10B$	$A100B$
$100B$	$A1001B$
$1001B$	$A1001\varepsilon$
1001ε	$\varepsilon 1001$
1001	1001

Table 2: Right and leftmost expansions for 1001

Leftmost	Rightmost
S	S
$A1B$	$A1B$
$0A1B$	$A11B$
$00A1B$	$A11\varepsilon$
$000A1B$	$0A11$
$000\varepsilon 1B$	$00A11$
$00011B$	$000A11$
00011ε	$000\varepsilon 11$
00011	00011

Table 3: Right and leftmost expansions for 00011

2 Number 2: Exercise 5.1.7

a. Prove by induction that:

$$w = x\text{ba}y \notin L(G) \ni G = S \to \text{a}S|S\text{b}|\text{a}|\text{b} \tag{2}$$

The shortest possible candidate string is when $x = y = \varepsilon$; $w = \text{ba}$. This $w \notin L(G)$. If $|x| = 1$, then x will consist of a single a or b... In short, the expansion of a's will always occur at the front of the string and will not follow any b's because the a's are growing to the right and the b's to the left and there isn't a way for the two to cross.

b. The regular expression for this language is:

$$L(G) = (a^*b^+|a^+b^*) \tag{3}$$

3 Number 3: Exercise 5.3.1

Prove that a set of parenthesis is balanced iff it is generated by the grammar (uses iso ebnf syntax to avoid ambiguity):

$$B = (B,B)|'(',B,')'|; \tag{4}$$

A string of parenthesis is balanced iff it is scan balanced, that is if the number of open parenthesis counted scanning from left to right is always greater than or equal to the number of close parenthesis and the total numbers are equal.

There is only one expansion that actually inserts parentheses in this grammar and it inserts an open parenthesis before a close parenthesis. This means that the count of open parenthses will always have to be greater than or equal, and also the totals will have to be equal. Therefore the strings produced will be scan balanced and thus balanced.

4 Number 4: Exercise 5.3.5

Give a grammar for (uses iso ebnf syntax to avoid ambiguity):
DOCTYPE CourseSpecs [
ELEMENT COURSES (COURSE+)
ELEMENT COURSE (CNAME, PROF, STUDENT*, TA?)
ELEMENT CNAME (#PCDATA)
ELEMENT STUDENT (#PCDATA)
ELEMENT TA (#PCDATA)]

Derivation One	Derivation Two
S	S
aS	aSbS
aa$S$bS	aa$S$bS
aaεbS	aaεbS
aabε	aabε
aab	aab

Table 4: Different leftmost derivations for aab

$$\begin{aligned}
\text{courses} &= \text{course list;} \quad (5)\\
\text{course list} &= \text{course, course list;} \quad (6)\\
\text{course} &= \text{name, professor, optional student list, optional assistant;} \quad (7)\\
\text{name} &= {}'\text{character data}'; \quad (8)\\
\text{professor} &= {}'\text{character data}'; \quad (9)\\
\text{optional student list} &= \text{student list}|; \quad (10)\\
\text{student list} &= \text{student, student list;} \quad (11)\\
\text{optional assistant} &= \text{assistant}|; \quad (12)\\
\text{assistant} &= {}'\text{character data}'; \quad (13)
\end{aligned}$$

5 Number 5: Exercise 5.4.1

Show, using the string aab, that the following is ambiguous:

$$S \to \mathrm{a}S|\mathrm{a}S\mathrm{b}S|\varepsilon \tag{14}$$

a. Using parse trees in Figure 1 and Figure 2

b. Using leftmost derivations in Table 4

c. Using rightmost derivations in Table 5

6 Number 6: Exercise 5.4.7.a

Usng the string +*-xyxy and the grammar:

$$E \to +EE|*EE|-EE|\mathrm{x}|\mathrm{y} \tag{15}$$

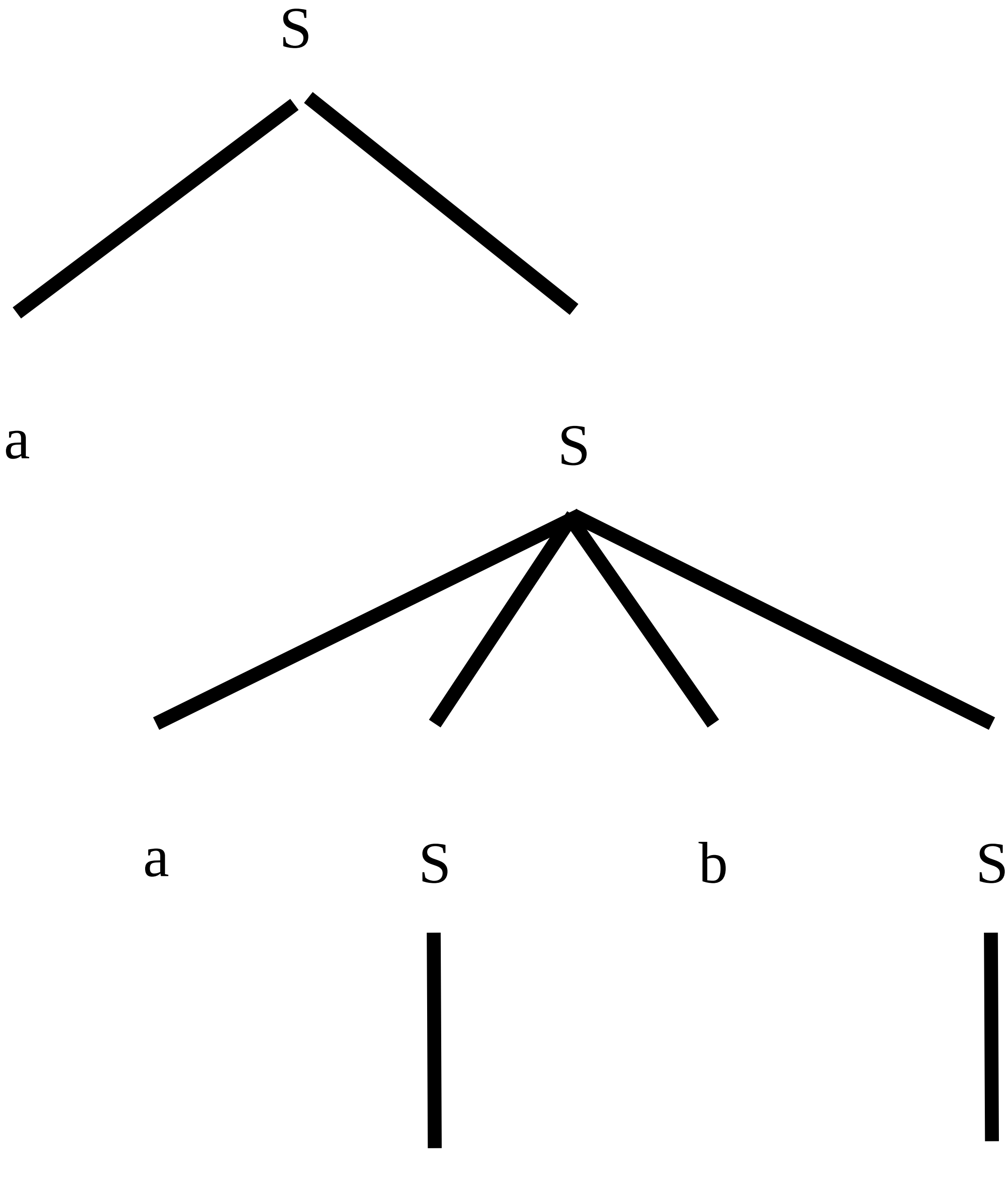

Figure 1: A possible parse tree for aab

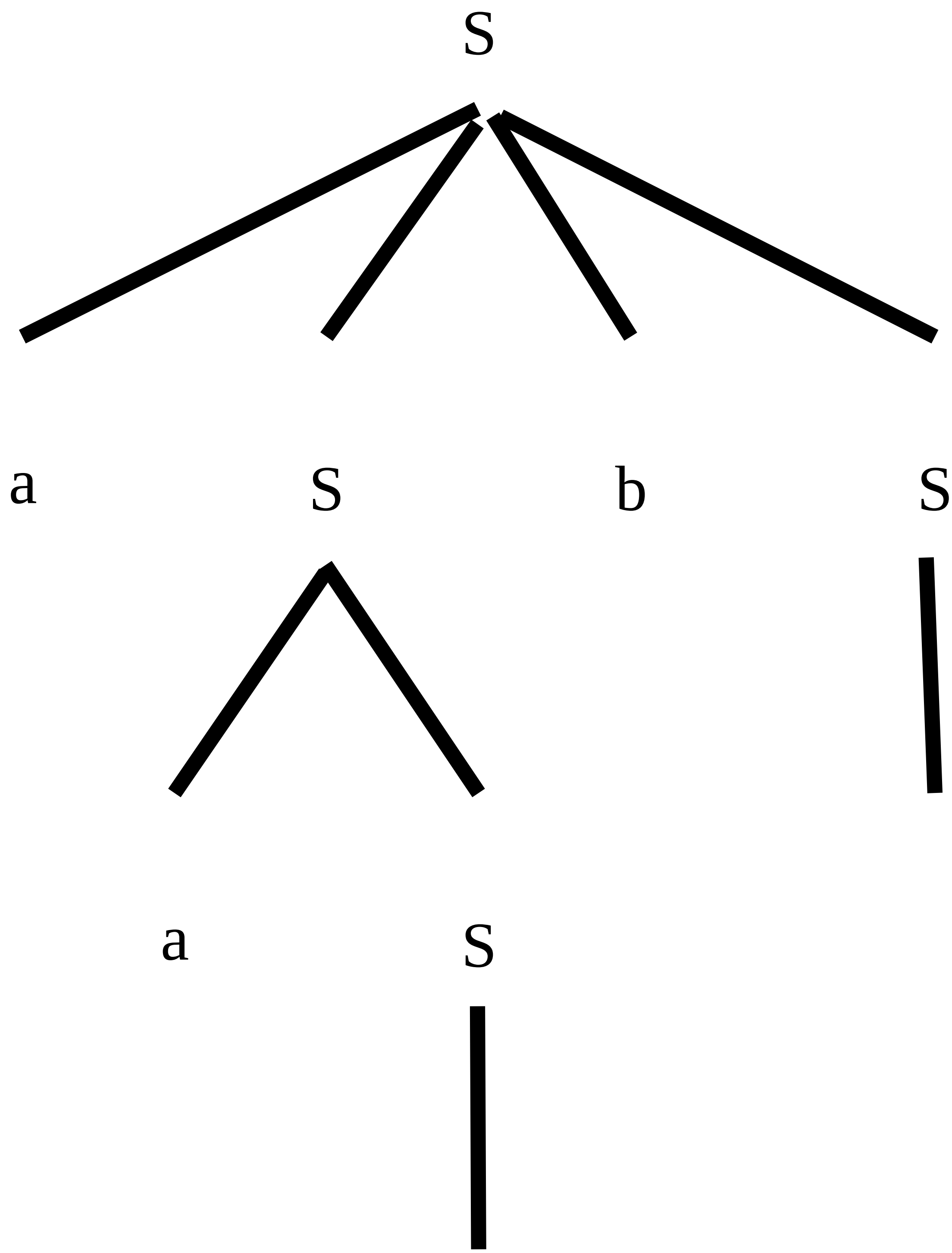

Figure 2: A possible parse tree for aab

Derivation One	Derivation Two
S	S
aS	aSbS
aa$S$bS	aa$S$bS
aa$S$bε	aa$S$bε
aaεb	aaεb
aab	aab

Table 5: Different rightmost derivations for aab

E
$+EE$
$+*EEE$
$+*-EEEE$
$+*-\mathrm{x}EEE$
$+*-\mathrm{xy}EE$
$+*-\mathrm{xyx}E$
$+*-\mathrm{xyxy}$

Table 6: Leftmost derivation for +*-xyxy

a. Find the leftmost derivation in Table 6

b. Find the rightmost derivation in Table 7

c. Derivation tree in Figure 3

E
$+EE$
$+E\mathrm{y}$
$+*EE\mathrm{y}$
$+*E\mathrm{xy}$
$+*-EE\mathrm{xy}$
$+*-E\mathrm{yxy}$
$+*-\mathrm{xyxy}$

Table 7: Rightmost derivation for +*-xyxy

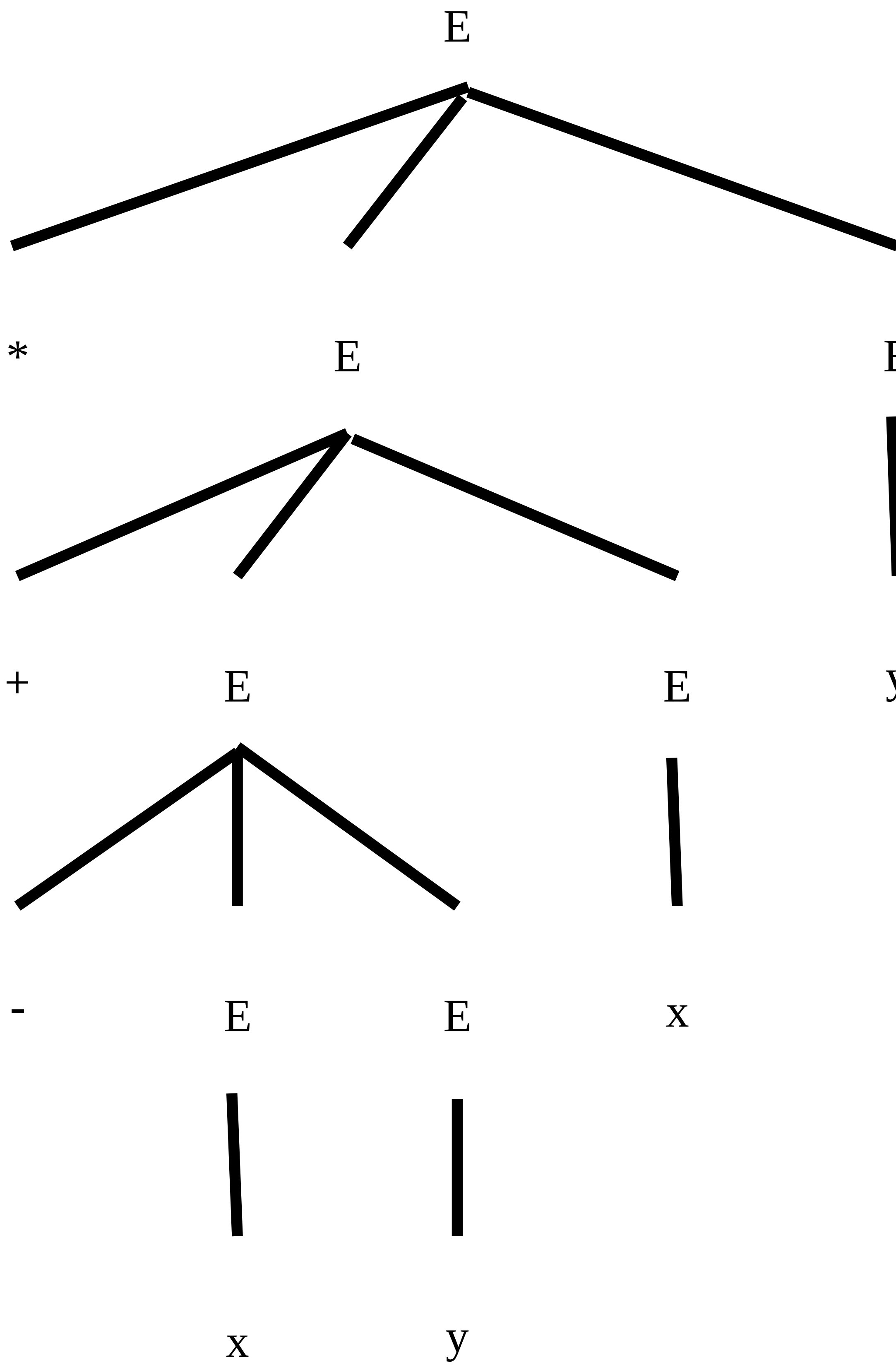

Figure 3: Derivation tree for +*-xyxy