Homework #1

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1 Number 1

Prove that $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$ iff $\overline{(A \cap B)} \subseteq \bar{A} \cup \bar{B}$ and $\overline{(A \cap B)} \supseteq \bar{A} \cup \bar{B}$

Let $x \in \overline{(A \cap B)}$, show that $x \in \bar{A} \cup \bar{B}$.
Since $x \in \overline{(A \cap B)}$, $x \notin (A \cap B)$ by definition of not.
By definition of intersection, $x \notin A$ or $x \notin B$.
By definition of union, $x \in \bar{A} \cup \bar{B}$.
Since x is arbitrary, by definition of subset, $\overline{(A \cap B)} \subseteq \bar{A} \cup \bar{B}$.

Let $x \in \bar{A} \cup \bar{B}$, show that $x \in \overline{(A \cap B)}$.
Since $x \in \bar{A} \cup \bar{B}$, $x \in \bar{A}$ or $x \in \bar{B}$ by definition of union.
By definition of not, $x \notin A$ or $x \notin B$
By definition of intersection, $x \notin A \cap B$
By definition of not, $x \in \overline{(A \cap B)}$
Since x is arbitrary, by definition of subset, $\bar{A} \cup \bar{B} \subseteq \overline{(A \cap B)}$.

Since $\overline{(A \cap B)} \subseteq \bar{A} \cup \bar{B}$ and $\bar{A} \cup \bar{B} \subseteq \overline{(A \cap B)}$, $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$ by definition of set equality.

2 Number 2 : Exercise 2.2.1

1. The automata can be defined as follows:
   $M = (Q, \Sigma, \delta, q_0, F) \ni$
   $Q = \{(x_1, x_2, x_3, a) | x_i \in \{l, r\}; a \in \{A, B\}\} \cup \{q_0\}$
   $\Sigma = \{A, B\}$
   δ is described by Table 1
   $F = \hat{\delta}((r, r, x_3, a), A) \cup \hat{\delta}((x_1, x_2, r, a), B) \cup \hat{\delta}((x_1, r, l, a), B)$
   $\ni x_i \in \{l, r\}; a \in \{A, B\}$

State	A	B
$> q_0$	(r,l,l,A)	(l,r,r,B)
$(l, l, l, x)\|x \in \{A, B\}\}$	(r,l,l,A)	(l,r,r,B)
$(l, l, r, x)\|x \in \{A, B\}\}$	(r,l,r,A)	(l,r,l,B)
$(l, r, l, x)\|x \in \{A, B\}\}$	(r,l,l,A)	(l,l,r,B)
$(l, r, r, x)\|x \in \{A, B\}\}$	(r,r,r,A)	(l,r,l,B)
$(r, l, l, x)\|x \in \{A, B\}\}$	(l,l,l,A)	(r,r,r,B)
$(r, l, r, x)\|x \in \{A, B\}\}$	(l,l,r,A)	(r,r,l,B)
$(r, r, l, x)\|x \in \{A, B\}\}$	(l,r,l,A)	(r,l,r,B)
$(r, r, r, x)\|x \in \{A, B\}\}$	(l,r,r,A)	(r,r,l,B)

Table 1: Transitions for δ

3 Number 3 : Exercise 2.2.4

1. The automata can be represented as Figure 1
2. The automata can be represented as Figure 2
3. The automama can be represented as Figure 3

4 Number 4 : Exercise 2.2.5

1. The automata can be defined as follows: [This is for each block of 5 containing at least two zeroes. ($|w|\%5 = 0$)]
   $M = (Q, \Sigma, \delta, q_0, F) \ni$
   $Q = \{(z, c)|z, c \in \mathbb{N}; z, c < 5\}$
   $\Sigma = \{0, 1\}$
   $\delta((z, c), 0) = (z + 1, c + 1) \ni c \leq 3$
   $\delta((z, c), 1) = (z, c + 1) \ni c \leq 3$
   $\delta((z, 4), 0) = (0, 0) \ni z \geq 1$
   $\delta((z, 4), 1) = (0, 0) \ni z \geq 2$
   $q_0 = (0, 0)$
   $F = \{(0, 0)\}$
2. The automata can be represented as Figure 4
3. The automata can be represented as Figure 5
4. The automata can be defined as follows:
   $M = (Q, \Sigma, \delta, q_0, F) \ni$
   $Q = \{(z, n)|z, n \in \mathbb{N}; z < 5; n < 3\}$

1

1

p0 0 p1 0 p2 0 p3

3

1 0,1

Figure 1: An automata for all strings ending in 00

Figure 2: An automata for all strings with 000 as a substring

0,1

p0 p1 p2 p3

4 1 0,1 0,1

0,1

Figure 3: An automata for all strings with 011 as a substring

Figure 4: An automata for strings with 1 as the 10th character to the right

Figure 5: An automata for strings that begin and/or end with 01

$\Sigma = \{0, 1\}$
$\delta((z, n), 0) = ((z + 1)\%5, n)$
$\delta((z, n), 1) = (z, (n + 1)\%3)$
$q_0 = (0, 0)$
$F = \{(0, 0)\}$

5 Number 5 : Exercise 2.3.5

Prove that assuming if $\delta_D(q, a) = p$ then $\delta_N(q, a) = \{p\}$, it follows that if $\hat{\delta}_D(q_0, w) = p$ then $\hat{\delta}_N(q_0, w) = \{p\}$.
Start with $w = \epsilon$. By definition, $\hat{\delta}_D(q_0, \epsilon) = q_0$ and $\hat{\delta}_N(q_0, \epsilon) = \{q_0\}$.
Consider then $|w| \geq 1$; let x be a string and a a letter such that $w = xa$.
By the inductive hypothesis and the definition of $\hat{\delta}_D$, $\hat{\delta}_D(q_0, w) = \hat{\delta}_D(q_0, xa) = \delta_D(\hat{\delta}(q_0, x), a)$.
By the inductive hypothesis and the definition of $\hat{\delta}_N$, $\hat{\delta}_N(\{q_0\}, w) = \hat{\delta}_N(\{q_0\}, xa) = \delta_N(\hat{\delta}_N(q_0, x), a)$.
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Figure 6: An automata for the strings abc, abd, and aacd

6 Number 6 : Exercise 2.4.1

1. The automata can be represented as Figure 6

2. The automata can be represented as Figure 7

3. The automata can be represented as Figure 8

7 Number 7 : Exercise 2.5.1

1. The ϵ-closures are:
   $\epsilon_c(p) = \{p\}$
   $\epsilon_c(q) = \{p, q\}$
   $\epsilon_c(r) = \{p, q, r\}$

2. All strings of length 3 or less $= \{w||w| \leq 3; (count_b(w) \geq 2$ and/or $count_c(w) \geq 1)\}$

3. As a DFA:

   $F = \{\{p, q, r\}\}$

Figure 7: An automata for the strings 0101, 011, and 101

State	a	b	c
$\rightarrow \{p\}$	$\{p\}$	$\{p, q\}$	$\{p, q, r\}$
$\{p, q\}$	$\{p, q\}$	$\{p, q, r\}$	$\{p, q, r\}$
$\{p, q, r\}$	$\{p, q, r\}$	$\{p, q, r\}$	$\{p, q, r\}$

Table 2: Transitions for δ_D

a

b

q2

Figure 8: An automata for the strings ab, bc, ca

State	a	b	c
$\rightarrow \{p,q,r\}$	$\{p\}$	$\{q,r\}$	$\{p,q,r\}$
$\{p\}$	$\emptyset$	$\{q\}$	$\{r\}$
$\{q,r\}$	$\{p,q,r\}$	$\{r\}$	$\{p,q,r\}$
$\{q\}$	$\{p,q,r\}$	$\{r\}$	$\{p,q,r\}$
$\{r\}$	$\emptyset$	$\emptyset$	$\emptyset$

Table 3: Transitions for δ_D

8 Number 8 : Exercise 2.5.2

1. The ϵ-closures are:
   $\epsilon_c(p) = \{p,q,r\}$
   $\epsilon_c(q) = \{q\}$
   $\epsilon_c(p) = \{r\}$

2. All strings of length 3 or less $= \{x_1x_2x_3 | x_1 \in \{a,b,c\}; x_2 \in \{a,c\}; x_3 \in \{a,b,c,\epsilon\}\} \cup \{abb, abc, bb, a, b, c, \epsilon\}$

3. As a DFA:

   $F = \{\{p,q,r\}, \{q,r\}, \{r\}\}$