

Homework #1

Will Holcomb CSC445 - Homework #1

August 25, 2002

1 Number 1

Prove that $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$ iff $\overline{(A \cap B)} \subseteq \bar{A} \cup \bar{B}$ and $\overline{(A \cap B)} \supseteq \bar{A} \cup \bar{B}$

Let $x \in \overline{(A \cap B)}$, show that $x \in \bar{A} \cup \bar{B}$.

Since $x \in \overline{(A \cap B)}$, $x \notin (A \cap B)$ by definition of not.

By definition of intersection, $x \notin A$ or $x \notin B$.

By definition of union, $x \in \bar{A} \cup \bar{B}$.

Since x is arbitrary, by definition of subset, $\overline{(A \cap B)} \subseteq \bar{A} \cup \bar{B}$.

Let $x \in \bar{A} \cup \bar{B}$, show that $x \in \overline{(A \cap B)}$.

Since $x \in \bar{A} \cup \bar{B}$, $x \in \bar{A}$ or $x \in \bar{B}$ by definition of union.

By definition of not, $x \notin A$ or $x \notin B$

By definition of intersection, $x \notin A \cap B$

By definition of not, $x \in \overline{(A \cap B)}$

Since x is arbitrary, by definition of subset, $\bar{A} \cup \bar{B} \subseteq \overline{(A \cap B)}$.

Since $\overline{(A \cap B)} \subseteq \bar{A} \cup \bar{B}$ and $\bar{A} \cup \bar{B} \subseteq \overline{(A \cap B)}$, $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$ by definition of set equality.

2 Number 2 : Exercise 2.2.1

1. The automata can be defined as follows:

$$M = (Q, \Sigma, \delta, q_0, F) \ni$$

$$Q = \{(x_1, x_2, x_3, a) | x_i \in \{l, r\}; a \in \{A, B\}\} \cup \{q_0\}$$

$$\Sigma = \{A, B\}$$

δ is described by Table 1

$$F = \hat{\delta}((r, r, x_3, a), A) \cup \hat{\delta}((x_1, x_2, r, a), B) \cup \hat{\delta}((x_1, r, l, a), B)$$

$$\ni x_i \in \{l, r\}; a \in \{A, B\}$$

State	A	B
$> q_0$	(r,l,l,A)	(l,r,r,B)
$(l, l, l, x) x \in \{A, B\}$	(r,l,l,A)	(l,r,r,B)
$(l, l, r, x) x \in \{A, B\}$	(r,l,r,A)	(l,r,l,B)
$(l, r, l, x) x \in \{A, B\}$	(r,l,l,A)	(l,l,r,B)
$(l, r, r, x) x \in \{A, B\}$	(r,r,r,A)	(l,r,l,B)
$(r, l, l, x) x \in \{A, B\}$	(l,l,l,A)	(r,r,r,B)
$(r, l, r, x) x \in \{A, B\}$	(l,l,r,A)	(r,r,l,B)
$(r, r, l, x) x \in \{A, B\}$	(l,r,l,A)	(r,l,r,B)
$(r, r, r, x) x \in \{A, B\}$	(l,r,r,A)	(r,r,l,B)

Table 1: Transitions for δ

3 Number 3 : Exercise 2.2.4

1. The automata can be represented as Figure 1
2. The automata can be represented as Figure 2
3. The automama can be represented as Figure 3

4 Number 4 : Exercise 2.2.5

1. The automata can be defined as follows: [This is for each block of 5 containing at least two zeroes. ($|w|\%5 = 0$)]
 $M = (Q, \Sigma, \delta, q_0, F) \ni$
 $Q = \{(z, c) | z, c \in \mathbb{N}; z, c < 5\}$
 $\Sigma = \{0, 1\}$
 $\delta((z, c), 0) = (z + 1, c + 1) \ni c \leq 3$
 $\delta((z, c), 1) = (z, c + 1) \ni c \leq 3$
 $\delta((z, 4), 0) = (0, 0) \ni z \geq 1$
 $\delta((z, 4), 1) = (0, 0) \ni z \geq 2$
 $q_0 = (0, 0)$
 $F = \{(0, 0)\}$
2. The automata can be represented as Figure 4
3. The automata can be represented as Figure 5
4. The automata can be defined as follows:
 $M = (Q, \Sigma, \delta, q_0, F) \ni$
 $Q = \{(z, n) | z, n \in \mathbb{N}; z < 5; n < 3\}$

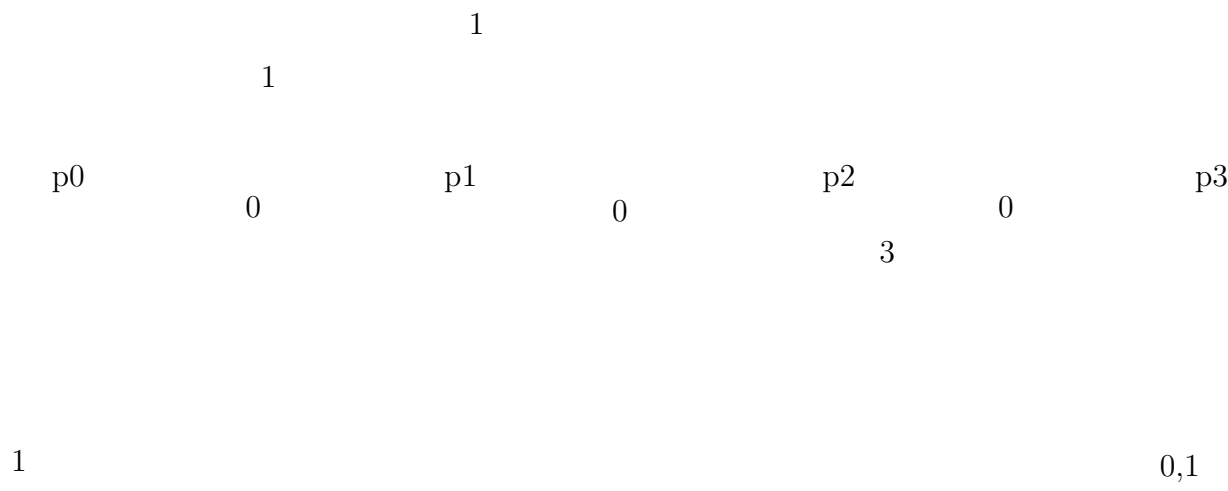


Figure 1: An automata for all strings ending in 00

Figure 2: An automata for all strings with 000 as a substring

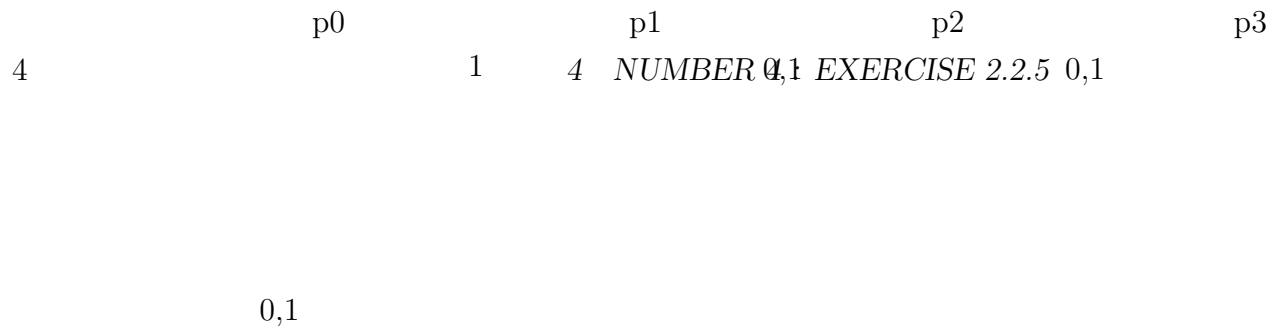


Figure 3: An automata for all strings with 011 as a substring

Figure 4: An automata for strings with 1 as the 10th character to the right

Figure 5: An automata for strings that begin and/or end with 01

$$\begin{aligned}
 \Sigma &= \{0, 1\} \\
 \delta((z, n), 0) &= ((z + 1) \% 5, n) \\
 \delta((z, n), 1) &= (z, (n + 1) \% 3) \\
 q_0 &= (0, 0) \\
 F &= \{(0, 0)\}
 \end{aligned}$$

5 Number 5 : Exercise 2.3.5

Prove that assuming if $\delta_D(q, a) = p$ then $\delta_N(q, a) = \{p\}$, it follows that if $\hat{\delta}_D(q_0, w) = p$ then $\hat{\delta}_N(q_0, w) = \{p\}$.

Start with $w = \epsilon$. By definition, $\hat{\delta}_D(q_0, \epsilon) = q_0$ and $\hat{\delta}_N(q_0, \epsilon) = \{q_0\}$.

Consider then $|w| \geq 1$; let x be a string and a a letter such that $w = xa$.

By the inductive hypothesis and the definition of $\hat{\delta}_D$, $\hat{\delta}_D(q_0, w) = \hat{\delta}_D(q_0, xa) = \delta_D(\hat{\delta}(q_0, x), a)$.

By the inductive hypothesis and the definition of $\hat{\delta}_N$, $\hat{\delta}_N(\{q_0\}, w) = \hat{\delta}_N(\{q_0\}, xa) = \delta_N(\hat{\delta}_N(q_0, x), a)$.

...

Figure 6: An automata for the strings abc, abd, and aacd

6 Number 6 : Exercise 2.4.1

1. The automata can be represented as Figure 6
2. The automata can be represented as Figure 7
3. The automata can be represented as Figure 8

7 Number 7 : Exercise 2.5.1

1. The ϵ -closures are:

$$\begin{aligned}\epsilon_c(p) &= \{p\} \\ \epsilon_c(q) &= \{p, q\} \\ \epsilon_c(r) &= \{p, q, r\}\end{aligned}$$
2. All strings of length 3 or less = $\{w \mid |w| \leq 3; (\text{count}_b(w) \geq 2 \text{ and/or } \text{count}_c(w) \geq 1)\}$
3. As a DFA:

$$F = \{\{p, q, r\}\}$$

Figure 7: An automata for the strings 0101, 011, and 101

State	a	b	c
$\rightarrow \{p\}$	$\{p\}$	$\{p, q\}$	$\{p, q, r\}$
$\{p, q\}$	$\{p, q\}$	$\{p, q, r\}$	$\{p, q, r\}$
$\{p, q, r\}$	$\{p, q, r\}$	$\{p, q, r\}$	$\{p, q, r\}$

Table 2: Transitions for δ_D

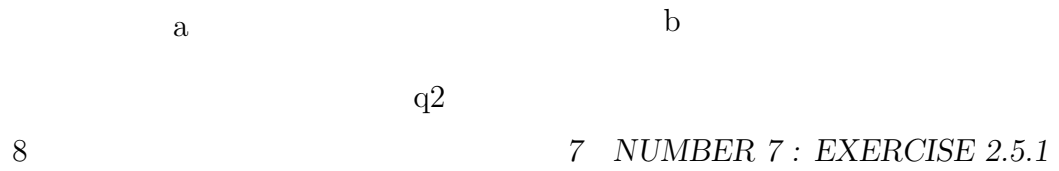


Figure 8: An automata for the strings ab, bc, ca

State	a	b	c
$\rightarrow \{p, q, r\}$	$\{p\}$	$\{q, r\}$	$\{p, q, r\}$
$\{p\}$	\emptyset	$\{q\}$	$\{r\}$
$\{q, r\}$	$\{p, q, r\}$	$\{r\}$	$\{p, q, r\}$
$\{q\}$	$\{p, q, r\}$	$\{r\}$	$\{p, q, r\}$
$\{r\}$	\emptyset	\emptyset	\emptyset

Table 3: Transitions for δ_D

8 Number 8 : Exercise 2.5.2

1. The ϵ -closures are:

$$\epsilon_c(p) = \{p, q, r\}$$

$$\epsilon_c(q) = \{q\}$$

$$\epsilon_c(r) = \{r\}$$

2. All strings of length 3 or less = $\{x_1x_2x_3 | x_1 \in \{a, b, c\}; x_2 \in \{a, c\}; x_3 \in \{a, b, c, \epsilon\}\} \cup \{abb, abc, bb, a, b, c, \epsilon\}$

3. As a DFA:

$$F = \{\{p, q, r\}, \{q, r\}, \{r\}\}$$