# Homework #5

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November 5, 2002

## 1 Number 1: Exercise 7.7.2

How does the context-free pumping lemma fail for each of the following languages?

a. $L = \{00, 11\}$

   It is not possible to pick an arbitrary n because $\{w||w| \neq 2; w \in L\} = \emptyset$

b. $L = \{0^n 1^n | n \geq 1\}$

   With:

$$\begin{aligned}
w &= uvxyz && (1)\\
|w| &\geq 2 && (2)\\
u &= 0^{n_u} \ni n_u = \frac{|w|}{2} - 1 && (3)\\
v &= 0 && (4)\\
x &= \epsilon && (5)\\
y &= 1 && (6)\\
z &= 1^{n_z} \ni n_z = \frac{|w|}{2} - 1 && (7)\\
w_i &= uv^i xy^i z \in L && (8)
\end{aligned}$$

   The pumping lemma for context free languages is satisfied $\therefore$ L is context-free.

c. The set of all palindromes where $\Sigma = \{0, 1\}$. That is:

$$L = \{waw^R | w \in \Sigma^*; a \in \Sigma \cup \{\epsilon\}\} \quad (9)$$

With:

$$\begin{aligned}
w &= uvxyz & (10)\\
|w| &\geq 3 & (11)\\
u &= \epsilon & (12)\\
v &= a \ni a \in \Sigma & (13)\\
x &\in L & (14)\\
y &= a & (15)\\
z &= \epsilon & (16)\\
w_i &= uv^i xy^i z \in L & (17)
\end{aligned}$$

The pumping lemma for context free languages is satisfied $\therefore$ L is context-free.

# 2 Number 2: Exercise 7.3.1

Show the context free languages are closed under $init \ni$

$$init(L) = \{w | \exists x \ni wx \in L\} \quad (18)$$

# 3 Number 3: Exercise 7.3.2

Consider the languages:

$$\begin{aligned}
L_1 &= \{a^n b^{2n} c^m | n, m \in \mathbb{N}\} & (19)\\
L_2 &= \{a^n b^m c^{2m} | n, m \in \mathbb{N}\} & (20)
\end{aligned}$$

a. Show that $L_1$ and $L_2$ are context-free using grammars:

$$\begin{aligned}
S_1 &\rightarrow AC\\
A &\rightarrow aAbb|\epsilon\\
C &\rightarrow cC|\epsilon
\end{aligned}$$

$$\begin{aligned}
S_2 &\rightarrow AB\\
A &\rightarrow aA|\epsilon\\
C &\rightarrow bBcc|\epsilon
\end{aligned}$$

| State | 0 | 1 | X | Y | B |
|---|---|---|---|---|---|
| $> q_0$ | ($q_1$,X,R) | - | - | ($q_3$,Y,R) | - |
| $q_1$ | ($q_1$,0,R) | ($q_2$,Y,L) | - | ($q_1$,Y,R) | - |
| $q_2$ | ($q_2$,0,L) | - | ($q_0$,X,R) | ($q_2$,Y,L) | - |
| $q_3$ | - | - | - | ($q_3$,Y,R) | ($q_4$,B,R) |
| $q_4$* | - | - | - | - | - |

Table 1: Turing machine to accept $\{0^n1^n | n \geq 1\}$

b. Is $L_1 \cap L_2$ context-free? Why?

$$L_1 \cap L_2 = \{a^ib^jc^k | 2i = j; 2j = k; i, j, k \in \mathbb{N}\} = \{a^nb^{2n}c^{4n} | n \in \mathbb{N}\}$$

# 4 Number 4: Exercise 7.4.2

Give linear time algorithms answeing the following questions:

a. Which symbols in a context-free grammar appear in some sentential form?

b. Which symbols in a context-free grammar are nullable?

# 5 Number 5: Exercise 8.2.1

For the turing machine in Table 5:

Give the instantaneous descriptions of the following strings:

a. 00

$$\begin{aligned} q_0 \frac{\dagger}{0} 0 &\vdash q_1 X \frac{\dagger}{0} \\ &\vdash q_2 X 0 \frac{\dagger}{\ } \end{aligned}$$

b. 00011

$$
\begin{aligned}
q_0\overset{\dagger}{\underline{0}}0011 &\vdash q_1X\overset{\dagger}{\underline{0}}011 \\
&\vdash q_1X0\overset{\dagger}{\underline{0}}11 \\
&\vdash q_1X00\overset{\dagger}{\underline{1}}1 \\
&\vdash q_2X0\overset{\dagger}{\underline{0}}Y1 \\
&\vdash q_2X\overset{\dagger}{\underline{0}}0Y1 \\
&\vdash q_2\overset{\dagger}{\underline{X}}00Y1 \\
&\vdash q_0X\overset{\dagger}{\underline{0}}0Y1 \\
&\vdash q_1XX\overset{\dagger}{\underline{0}}Y1 \\
&\vdash q_1XX0\overset{\dagger}{\underline{Y}}1 \\
&\vdash q_1XX0Y\overset{\dagger}{\underline{1}} \\
&\vdash q_2XX0\overset{\dagger}{\underline{Y}}Y \\
&\vdash q_2XX\overset{\dagger}{\underline{0}}YY \\
&\vdash q_2X\overset{\dagger}{\underline{X}}0YY \\
&\vdash q_0XX\overset{\dagger}{\underline{0}}YY \\
&\vdash q_1XXX\overset{\dagger}{\underline{Y}}Y \\
&\vdash q_1XXXY\overset{\dagger}{\underline{Y}} \\
&\vdash q_1XXXYY\overset{\dagger}{\underline{\ }}
\end{aligned}
$$

c. 00111

$$
\begin{aligned}
q_0\frac{\dagger}{0}0111 &\vdash q_1X\frac{\dagger}{0}111\\
&\vdash q_1X0\frac{\dagger}{1}11\\
&\vdash q_2X\frac{\dagger}{0}Y11\\
&\vdash q_2\frac{\dagger}{X}0Y11\\
&\vdash q_0X\frac{\dagger}{0}Y11\\
&\vdash q_1XX\frac{\dagger}{Y}11\\
&\vdash q_1XXY\frac{\dagger}{1}1\\
&\vdash q_2XX\frac{\dagger}{Y}Y1\\
&\vdash q_2X\frac{\dagger}{X}YY1\\
&\vdash q_0XX\frac{\dagger}{Y}Y1\\
&\vdash q_3XXY\frac{\dagger}{Y}1\\
&\vdash q_3XXYY\frac{\dagger}{1}
\end{aligned}
$$

# 6 Number 6: Exercise 8.2.2

Design Turing machines for the following languages:

a. $\{w|\text{count}_0(w) = \text{count}_1(w)\}$

The JFlap file for this Turing machine is available at:
http://odin.himinbi.org/classes/csc445/8.2.2.a.TM

b. $\{a^nb^nc^n|n \in \mathbb{N}\}$

The JFlap file for this Turing machine is available at:
http://odin.himinbi.org/classes/csc445/8.2.2.a.TM

c. $\{ww^R|w \in \Sigma^*; \Sigma = \{0, 1\}\}$

The JFlap file for this Turing machine is available at:
http://odin.himinbi.org/classes/csc445/8.2.2.a.TM

# 7 Number 7: Exercise 8.2.3

Design a Turing machine which adds one to a given number in binary.

a. What are the transitions in this machine and the purpose of each?

   The JFlap file for this Turing machine is available at:
   http://odin.himinbi.org/classes/csc445/8.2.3.TM

   $q_0 \rightarrow$ Start and move to the left side of the string
   $q_1 \rightarrow$ Move back to the left over 1's changing them to 0's
   $q_2^* \rightarrow$ On a 0 on B, that digit is the end of the ripple and will become a 1, leaving the rest of the string unaltered

b. Show the sequence of instantaneous descriptions for the string 111:

$$
\begin{aligned}
q_0\frac{\dagger}{1}11 &\vdash q_0 1\frac{\dagger}{1}1 \\
&\vdash q_0 11\frac{\dagger}{1} \\
&\vdash q_0 111\frac{\dagger}{} \\
&\vdash q_1 11\frac{\dagger}{1} \\
&\vdash q_1 1\frac{\dagger}{1}0 \\
&\vdash q_1 \frac{\dagger}{1}00 \\
&\vdash q_1 \frac{\dagger}{}000 \\
&\vdash q_2 1\frac{\dagger}{0}00
\end{aligned}
$$